A model-based control system design for a coffee roasting process

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Abstract

Coffee roasting is both a science and an art. Various models have been proposed in order to model the coffee roasting process. A model developed by Schwartzberg (2002) and validated by Vosloo (2016) is used in this study to develop a control strategy for a batch rotating-drum coffee roaster. The control strategy is based on experimentally determined parameters, and validated by simulated data. In addition to the control strategy the controllability of the process is investigated.

It was experimentally determined that there is an initial time frame (90 seconds) during the roasting process that is deemed uncontrollable, most likely due to evaporative cooling taking place during the initial drying phase of the roasting process.

The coffee roasting process was determined to be a lag-dominant first-order plus time delay process which may be approximated, and therefore modelled as a pure integrating system with an average dead-time of 20 seconds. This is in accordance with Hugo (2017) and Ruscio (2010). Initially a relative gain array (RGA) analysis was conducted based on simulated data to determine the best pairing of manipulated and controlled variables, in order to meet the control objective which is the recreating of a roast profile (i.e. the time vs. temperature plot of the roasted bean batch). The final control strategy is based on the RGA results recommending a single-input single-output (SISO) control system utilising the derivative of the roast profile as controlled variable, and the liquid petroleum gas (LPG) flow to the system as manipulated variable. A possible threshold control strategy to be used in combination with the developed control strategy is discussed qualitatively.

The control design methods used were the internal model control (IMC) (based on the pure integrating approximation of the process), the Cohen and Coon and integral of the time-weighted absolute error (ITAE) methods (based on the actual lag-dominant first-order plus time delay behaviour of the process).

The best performing controller was determined to be an IMC-based PI controller that utilises the average determined process parameters. This controller was fine-tuned in order to enhance its performance. The stability margins of the final controller were analysed using Bode plots.

Keywords: Model-based control, pure integrator, coffee roasting, roast profile, roast profile derivative, parameter scheduling.
**Opsomming**

Om koffie te rooster is net soveel 'n kuns as wat dit 'n wetenskap is. Verskeie modelle word voorgestel om hierdie proses te modelleer. 'n Model ontwikkel deur Schwartzberg (2002) en bevestig deur Vosloo (2016) word in hierdie studie gebruik om 'n beheerstrategie vir 'n roterende drom koffie-rooster te ontwikkel. Die beheerstrategie is gebaseer op parameters wat eksperimenteel bepaal is en deur middel van simulasies bevestig is. Addisioneel tot die beheerstrategie is die beheerbaarheid van die proses ondersoek.

Dit was eksperimenteel bepaal dat daar 'n aanvanklike tydsduur (90 sekondes) tydens die proses is wat onbeheerbaar verklaar is, dit is as gevolg van die verdampingsverkoeling wat tydens hierdie aanvanklike drogingsfase van die proses plaasvind.

Dit was vasgestel dat die koffie rooster proses 'n sloer-dominante eerste-orde proses met dooietyd is wat benader kan word, en dus gemodelleer word, as 'n suiwer-integreerende proses met 'n gemiddelde dooietyd van 20 sekondes. Dit is in ooreenstemming met Hugo (2017) en Ruscio (2010). 'n relatiewe winsmatriks (RWM) analyse is aanvanklik opgestel wat op gesimuleerde data gebaseer is om die beste moontlike afparing van prosesveranderlikes te bepaal, en ten einde die beheerdoel te bereik. Die beheerdoel is om die roosterprofiel (ook genoem die tyd teenoor temperatuur kurwe van die geroosterde bone) te herskep. Die finale beheerstrategie is gebaseer op die bevindinge van die RWM analyse wat aanbeveel dat 'n enkel-inset-enkel-uitset strategie gebruik moet word, wat die afgeleide van die roosterprofiel as beheerde veranderlike gebruik, en die vloeibare petroleumgas (LPG) vloei na die sisteem as die manipulateerde veranderlike. 'n Moontlike drumpel beheerstrategie wat in samewerking met die ontwikkelde strategie gebruik kan word, word kwalitatief bespreek.

Die beheer-ontwerp-metodes wat bebruik is, is die IMC (gebaseer op die suiwer integrerende benadering van die proses), die Cohen en Coon en ITAE metodes (gebaseer op die sloer-dominante eerste-orde met dooietyd gedrag van die stelsel).

Die beheerder met die beste uitsette is 'n IMC-gebaseerde PI-beheerder wat die gemiddelde eksperimenteel bepaalde parameters gebruik. Hierdie beheerder was verfyn om die uitset te verbeter. Die stabiliteitsgrense van die finale beheerder is deur middel van Bode-diagramme ondersoek.

*Sleutelwoorde:* Model-gebaseerde beheer, suiwer-integreerder, koffie rooster, roosterprofiel, roosterprofiel-afgeleide, parameter skedulering.
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<tr>
<td>var.</td>
<td>Variance</td>
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<tr>
<th>Acronyms</th>
<th>Description</th>
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<tbody>
<tr>
<td>AR</td>
<td>Amplitude ratio</td>
</tr>
<tr>
<td>ASME</td>
<td>American society of mechanical engineers</td>
</tr>
<tr>
<td>CREMA</td>
<td>Coffee roaster environmental management and automation</td>
</tr>
<tr>
<td>CV</td>
<td>Controlled variable</td>
</tr>
<tr>
<td>IMC</td>
<td>Internal model control</td>
</tr>
<tr>
<td>IPTD</td>
<td>Integral plus time delay</td>
</tr>
<tr>
<td>ITAE</td>
<td>Integral of the time-weighted absolute error</td>
</tr>
<tr>
<td>LPG</td>
<td>Liquid petroleum gas</td>
</tr>
<tr>
<td>L-REA</td>
<td>Lumped-reaction engineering approach</td>
</tr>
<tr>
<td>MIMO</td>
<td>Multi-input multi-output</td>
</tr>
<tr>
<td>MV</td>
<td>Manipulated variable</td>
</tr>
<tr>
<td>PCA</td>
<td>Principal component analysis</td>
</tr>
<tr>
<td>PID</td>
<td>Proportional-integral-derivative</td>
</tr>
<tr>
<td>PLC</td>
<td>Programmable logic controller</td>
</tr>
<tr>
<td>PTR-ToF-MS</td>
<td>Proton transfer reaction time of flight mass spectrometry</td>
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<tr>
<td>RGA</td>
<td>Relative gain array</td>
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<tr>
<td>SISO</td>
<td>Single-input single-output</td>
</tr>
<tr>
<td>TC</td>
<td>Temperature controller</td>
</tr>
<tr>
<td>VOC</td>
<td>Volatile organic compound</td>
</tr>
<tr>
<td>Symbols</td>
<td>Description</td>
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<td>---------</td>
<td>------------------------------------------</td>
</tr>
<tr>
<td>$A$</td>
<td>Surface area</td>
</tr>
<tr>
<td>$Ar$</td>
<td>Arrhenius equation pre-factor</td>
</tr>
<tr>
<td>$Bi$</td>
<td>Biot number</td>
</tr>
<tr>
<td>$b$</td>
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</tr>
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</tr>
<tr>
<td>$D$</td>
<td>Moisture diffusivity</td>
</tr>
<tr>
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<td>Error</td>
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</tr>
<tr>
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</tr>
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<td>Controller transfer function</td>
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<tr>
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<td>Process transfer function</td>
</tr>
<tr>
<td>$G_m$</td>
<td>Measurement element transfer function</td>
</tr>
<tr>
<td>$He$</td>
<td>Amount of heat produced as roasting</td>
</tr>
<tr>
<td>$Het$</td>
<td>Total amount of heat produced during</td>
</tr>
<tr>
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<td>Heat transfer coefficient</td>
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<tr>
<td>$T_{roast}$</td>
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</tr>
<tr>
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<td>Path length of gas flow across a substance</td>
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<td>Frequency</td>
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<td>Phase angle</td>
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<td>Rate of heat flow</td>
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<td>( \sigma )</td>
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<td>( \mu )</td>
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<td>( \xi )</td>
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<tr>
<td>( b )</td>
<td>Roasted medium (coffee bean)</td>
</tr>
<tr>
<td>( b(d, b) )</td>
<td>Dry basis bean estimation</td>
</tr>
<tr>
<td>( eq )</td>
<td>Indicate at equilibrium</td>
</tr>
<tr>
<td>( ev )</td>
<td>Indicate evaporation</td>
</tr>
<tr>
<td>( g )</td>
<td>Indicate roasting gas (air)</td>
</tr>
<tr>
<td>( g, i )</td>
<td>Inlet or initial gas conditions</td>
</tr>
<tr>
<td>( g, o )</td>
<td>Outlet or final gas conditions</td>
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<tr>
<td>( m )</td>
<td>Indicate the metal of the roaster</td>
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<tr>
<td>( r )</td>
<td>Exothermic reactions</td>
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<td>( rp )</td>
<td>Indicate the roast profile</td>
</tr>
<tr>
<td>( R )</td>
<td>Set point value</td>
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<td>( sat )</td>
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CHAPTER 1 - Introduction

Chapter 1 introduces this study and gives a brief historical and economical background on coffee and its origin. It also states the necessity of a well-controlled roasting process. The relevance of this research is explained in light of the relationship between the North-West University and Genio Intelligent Roasters, the supplier of the coffee roasting equipment.
1.1 Background

1.1.1 Introduction and brief history on the origin of coffee

The origin of coffee is largely speculated and based on legends surrounding this beverage. A popular folk legend is about an Ethiopian goat herder, Kaldi, who allegedly took his drove of goats to a new region for grazing. On a particular day he noticed his goats eating strange red berries on a bush. The herder saw that the goats had elevated energy levels; they appeared to be dancing about. When evening came the goats did not settle down, but kept the herder awake throughout the night. He suspected that the peculiar red berries were responsible for the droves’ undeniable energy hype. He decided to try and eat the berries himself and discovered that it had the ability to keep an individual awake for extended hours. After notifying the abbot of the local monastery, they started making a tea-like beverage from the berries and the beans, enabling those who drank it to stay awake longer. Soon it was incorporated into worship rituals to help the tribes pray for prolonged periods in order to bring honour to their gods. (circa A.D.800) (NATGEO, 1999; NCA, 2016; Roland, 2011).

People of the Yemeni district (Arabian Peninsula) were the first in the world to cultivate and trade coffee. By the 15th century, coffee had grown in popularity (mostly due to its cultivation by the Arabian population) and in the 16th century the beverage was already known in Egypt, Persia, Syria and Turkey (NCA, 2016; Roland, 2011). Up until circa. 1600 legend tells that no coffee seed sprouted beyond the borders of Africa or Arabia. The Arabian people went as far as to boil/parch the exported beans to stop any competition. (NATGEO, 1999). The first coffee exports were shipped through the ancient port at Mocha (port city on the Red Sea coast of Yemen, 13°19’N 43°15’E). Mocha was the chief port of Sana’a, Yemen’s capital (Anon., 2009; Ghul et al., 2012).

Circa. 1600 an Indian pilgrim, Baba Budan, smuggled fertile coffee beans out of Arabia. According to legend he tied the beans to his stomach to pass the customs post. His fruitful beans grew successfully and started a worldwide coffee expansion (NATGEO, 1999). Eventually coffee reached Europe in 1615 when a Venetian merchant brought “the drink of black colour” into the country. During the 17th century coffee became so popular in Europe that coffee houses became social communication hubs in England, Austria, France, Germany and Holland. These coffee houses were commonly known as “penny universities”, because a penny purchased a cup of coffee and stimulating conversation (NATGEO, 1999; NCA, 2016).

As the demand for coffee grew, the competition to acquire coffee plantations throughout the world grew. By the end of the 17th century the Dutch had finally planted a successful crop on the Island of Java, Indonesia (NCA, 2016). From there the coffee-monopoly grew into the industry as it is known today. Green coffee is the world’s most widely traded tropical
agricultural commodity with coffee exports reaching 9.69 million 60 kg bags at the end of June 2015 (ICO, 2015a; ICO, 2010). A staggering 149.2 million 60 kg bags of coffee were consumed during the calendar year 2014 (ICO, 2015a).

The coffee plant cultivars most commonly used commercially is the *Coffea Arabica* and the *Coffea canephora* var. *robusta* (Ku Madihah et al., 2012; Tfouni et al., 2013; Zambonin et al., 2005). Zambonin et al. (2005) reports that the *C. Arabica* is the most valuable between these two cultivars and that both are cultivated in various tropical countries worldwide.

1.1.2 Introduction to the coffee roasting process

Our society is filled with various production processes, most of which can be classified as multi-input multi-output (MIMO) systems or processes. MIMO systems have multiple measurable process variables that are required to meet specific standards, which therefore need to be controlled (Nagel & Tustin, 1955). In addition, MIMO systems have a number of variables that will influence these controlled variables. These input variables include manipulated variables and disturbances. The coffee roasting process is typically an example of such a MIMO system. These MIMO processes pose a challenge for automatic control due to their complex nature, with the coffee roasting process being no exception.

Coffee, once roasted, can have a varying shade of dark yellow to dark brown colour with a well-known rich, distinct aroma. Coffee is not consumed in its green, raw form. Consequently the value of coffee beans increases dramatically when roasting the green beans. Yeretzian et al. (2002) estimates that the market value increase is in the range of 100-300 %.

When using the analogy of comparing green coffee beans to a person, the roasting process will classify as the defining moment in that individual’s life, or otherwise stated, the moment the ugly duckling transforms into a wondrous swan. The distinct aroma coffee possesses is acquired during the roasting process. During the roasting process the green coffee beans undergo a series of reactions which transform these tasteless green beans into the characteristic drink millions worldwide love and appreciate (Dutra et al., 2001). With that taken into account, it is no surprise that a profound amount of care and study has been invested into refining and - in a sense - perfecting this coffee roasting process. This includes the design and application of control strategies for the roasting process.

1.1.3 Current applications of coffee roasting control technology

The controller mostly used within batch coffee roaster-machines - by practicing artisan roasters - is a proportional integral derivative (PID) controller. It was however found that, unfortunately, these controllers are rarely set to their optimal controller settings (Davis & Ribich, 2005).
Due to the complexity of the coffee roasting process, a suggested control strategy is real-time control, utilising a proton transfer reaction time-of-flight mass spectrometry (PTR-ToF-MS) analysis as sensory input. This approach allows the on-line monitoring of the flavour formation of the coffee beans as a measured variable to control the roast profile. The roast profile is defined as the temperature vs. time curve of the roasted coffee bean batch. Principal component analysis (PCA) can be used to create a predictive model for control (Feldman et al., 1969). This model in combination with the on-line measuring tool proved to be a successful control strategy able to reproduce roast profiles (Gloess et al., 2014; Rivera et al., 2011; Purdon & McCamey, 1987). However, these methods are very costly and impractical to use within artisan batch roasters. Similar studies using laser mass spectrometry as an on-line sensor for process control merely indicate that the measurement methodology is very successful in determining the on-line degree of roast, but little is known with regards to the effectiveness of any of these control strategies (Dorfner et al., 2004). In most real-world applications thermocouples are used as temperature sensors to monitor the roast profile and so doing monitor the degree of roast (Genio, 2016; Phan, 2012).

Lastly there are patented fluidised bed coffee roasters which use feedback control strategies by measuring the air flow rate and temperature through the fluidised bed (Sewell, 1995). The success rate of these roasters are not high, as coffee beans form very poor fluidised beds due to their shape and size (Bottazzi, 2012:20).

1.2 Purpose of research

There is a need in the roasted coffee manufacturing market for the development of a fully automated coffee-roaster, able to reproduce a light, medium or dark roast at the push of a button. A rotating drum roasting machine, sufficiently automated to replace a skilled coffee-roaster operator, does not yet exist although patents are available that offer fully automated control in fluidised bed roasters (Sewell, 1995). The ultimate purpose of this study is to develop a control strategy which will enable coffee batch-roaster equipment to automatically reproduce specified roast profiles. The Genio 6 Artisan prototype coffee roaster’s control system does not have the ability to recreate a specified roast profile. It can only maintain a fixed temperature set point prior to the commencement of a roast, with subsequent manual control for the duration of the roasting process. The improved system will thus have to automatically reproduce roast profiles by using the temperature vs. time curve (roast profile) of a desired roast as the set point (Yeretzian et al., 2002).
1.3 **Aim and objectives**

This study aims at developing a control strategy for a rotating drum coffee roaster. The control strategy will be designed based on both modelled and experimental data. Ultimately it will be capable of reproducing specified roast profiles directly linked to the desired degree of roast (Feldman *et al*., 1969). Prior to controller development the controllable time frame of the batch coffee roasting process will be determined experimentally.

1.3.1 **Objectives**

The primary (a & b) and secondary (c) objectives of this study are:

a) To determine the time intervals during a roast for which the coffee roasting process is indeed controllable.

b) To develop a control strategy capable of automatically reproducing coffee roast profiles on the Genio 6 Artisan roaster.

c) To validate the developed control strategy.

1.4 **Overview and scope of dissertation**

The application of this control strategy does not necessarily extend to other mechanised coffee roasters. The control strategy aims to enable the Genio 6 Artisan roaster, a batch-rotating drum type of roaster, to reproduce a predefined roast profile. The determination of such a predefined roast profile is based on prior experimental observation linked with modelled results for different degrees of roast of the coffee beans. The development of the control strategy will be preceded by a study on the controllability of the roasting process. All theory regarding the modelling of the roasting process will be acquired from literature and results obtained from the study conducted by Vosloo (2016).
The overview of this study is given in terms of the chosen chapters of this project report, which are listed as:

**Chapter 1 – Introduction:** A brief introduction and background on the history of coffee and coffee roasting is given to provide necessary justification for this study. The aims, objectives and scope is also discussed shortly.

**Chapter 2 – Literature study:** An in depth literature review is given on control strategy development and implementation, and its context within this study as it relates to coffee roaster control strategies. The theory of control is stated and a discussion is given on the relevance of the validated coffee roasting model presented in Vosloo (2016). Motivation is also given as to why the selected control strategy was chosen.

**Chapter 3 – Experimental:** The experimental procedures are discussed for the approach used to meet each objective. All experimentation was conducted on the Genio 6 Artisan coffee roaster. Additional hardware and software used during experimentation are also explained. Initially the results of the RGA are discussed in this chapter.

**Chapter 4 – PID-Controller development:** An account is given of the methodology and thought pattern behind the controller development. Firstly the development of the RGA is detailed. The use of the Simulink® model created by Vosloo (2016) based on Schwartzberg (2002) is discussed shortly in the light of the modifications implemented by this study. The chosen transfer function is stated and verified. Finally the various PI and PID parameters determined by the IMC-based, Cohen and Coon and the ITAE methods are reported.

**Chapter 5 – Results and discussion:** The experimental results are discussed with the initial focus on the controllable time frame of the roasting process. The modified Simulink® modelled results (without the added control strategy) are compared to the experimental results. The performance of the developed controllers are compared and quantitatively discussed with emphasis placed on the controllers' handling of dead-time in the process.

**Chapter 6 – Conclusion and recommendations:** Conclusions are drawn with regards to the validated control strategy and its effectivity on the Genio 6 Artisan roaster. Various recommendations are given regarding the improvement of coffee roaster control.
A schematic representation of the above stated overview is given in Figure 1.1 clarifying the links between the various chapters.

**Figure 1.1: Schematic representation of overview**
1.5 Chapter references


CHAPTER 2 – Literature Study

The literature study itself focuses on feedback control strategies available for single-input single-output (SISO) and multi-input multi-output (MIMO) systems and concentrates on the theory involved with the modelling and development of control strategies for first-order pure integrator systems. The coffee roasting process is discussed in broad terms and current control strategies used within coffee roasters are mentioned. Emphasis is placed on PI/PID controller development and alternative controllers and sensory methods are mentioned in the light of their advantages and disadvantages with regards to coffee roasting.
2.1 Coffee: from the bean to the cup

2.1.1 Preparing the coffee bean for roasting

Mangal (2007) calls coffee one of the “most important cash-crop beverages” in the world. It was already mentioned in Chapter 1 that green coffee beans are a widely traded agricultural commodity (ICO, 2015; ICO, 2010). The coffee economy is second only to crude oil and therefore coffee exports have a significant influence on economies (Esquivel & Jiménez, 2012; ICO, 2015; ICO, 2010; Mangal, 2007; Moldvaer, 2014; Phan, 2012). Hence, the green bean will be the first topic in this overview.

2.1.1.1 The green coffee bean

Green coffee beans (or rather coffee seeds) are extracted from the coffee plant’s fruit, also referred to as the cherries. After these cherries are harvested from the trees (mostly by manual labour) they undergo various time consuming processes in order to remove the outer layers surrounding the seed of the cherry, which is the green coffee bean (Phan, 2012). There are different methods to remove these outer layers; the location of the plantations and resources available mostly determine the method used.

Figure 2.1 portrays a longitudinal section of the coffee fruit (also known as cherry or berry). It consists of a hard outer skin marked (a), known as the pericarp, which is mostly deep red in colour. Just beneath the pericarp the soft, sweet pulp is located, labelled (b). The third layer (c) is slightly yellow and is referred to as the outer mesocarp, found to be highly hydrated covering a layer of mucilage called the pectin layer. The parchment, also a yellowish endocarp, is a thin layer covering the bean itself and is also represented by (c). The silver skin (d) covers each hemisphere of the coffee bean, or endosperm denoted by (e). Lastly, inside the bean the embryo (f) is found.

Figure 2.1: Coffee fruit and bean structure (Adapted from Phan, 2012)
The processes that follow the coffee bean harvesting, i.e. defruiting, drying, sorting and ageing, involve removing each of these layers denoted by (a) to (c) in order to extract the beans and prepare them for export (Esquivel & Jiménez, 2012; Phan, 2012). After these processes the beans have to be packaged and shipped to various consumer countries that use the green beans in coffee production. Coffee, as most will know, is not consumed in its unroasted (raw) form (Bobková et al., 2017; Mangal, 2007; Yeretzian et al., 2002).

### 2.1.1.2 Roasting process

The coffee roasting process is an irreversible thermal treatment of green (raw) coffee beans. The typical duration of this process is between 10-20 min. The roasting is classified by many as a critical and very important step in the production of the coffee beverage (Mangal, 2007; Phan, 2012). The flavour, aroma, composition and colour of coffee beans are dramatically altered during roasting. This is caused by pyrolysis, caramelisation, and Maillard and Strecker reactions occurring. These reactions occur between the first crack (temperatures in the range of 175-185 °C) and second crack (temperatures above 200 °C) stages of the roasting process (Gloess et al., 2014; Rivera et al., 2011). The Maillard reaction is responsible for the flavour formation of the green coffee beans. It is classified as a type of reaction that takes place between the amino acids and reducing sugars present in the green coffee beans at the above mentioned temperature ranges.

The first and second crack stages are characterised by a distinct popping sound. This popping sound is a result of inorganic gasses such as CO₂, CO, N₂ and H₂O abruptly escaping as the bean structure is fractured due to high internal bean pressures. As the bean temperature rises, the pressure inside the bean cavity increases until the bean can no longer withstand the internal pressure build-up. During pyrolysis of the green bean material voids form inside the bean, creating capsules which house the gasses and enable gas accumulation (Dorfner et al., 2004). Crucial flavour components in roasted coffee beans are formed by reactions between polysaccharides, proteins, chlorogenic acids and trigonelline as stated by Dorfner et al. (2004).

By determining the chemical composition of the exhaust gas during the roasting process (i.e. determining the ratios in which polyphenols, amino acids, caffeine and aroma components are present) one can determine the degree of roast of the coffee after roasting (Dorfner et al., 2004; Dutra et al., 2001). In addition, the volume of the beans increase and the moisture content lowers in relation to the degree of roast during roasting. The desired degree of roast strongly depends on personal preference and cultural influence. A classification of degrees of roast was conducted by Phan (2012) and Mangal (2007); the results are combined and listed in Table 2.1.
### Table 2.1: Degree of roast classification (Adapted from Mangal, 2007 & Phan, 2012)

<table>
<thead>
<tr>
<th>Roast style name</th>
<th>Roast style classification</th>
<th>Green bean weight loss (%)</th>
<th>Final colour</th>
<th>Final roasting temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light city, half city</td>
<td>Light</td>
<td>14</td>
<td>Cinnamon</td>
<td>200</td>
</tr>
<tr>
<td>City, American, Breakfast</td>
<td>Medium</td>
<td>15</td>
<td>Brown</td>
<td>210</td>
</tr>
<tr>
<td>Full city</td>
<td>Medium-dark</td>
<td>15.5</td>
<td>Deep brown</td>
<td>210-220</td>
</tr>
<tr>
<td>Brazilian, High, Continental</td>
<td>Dark</td>
<td>16</td>
<td>Dark brown</td>
<td>210-220</td>
</tr>
<tr>
<td>Viennese, Expresso</td>
<td>Dark</td>
<td>17</td>
<td>Very dark brown</td>
<td>225-230</td>
</tr>
<tr>
<td>French, Italian</td>
<td>Dark</td>
<td>18-20</td>
<td>Very dark brown with oil on surface</td>
<td>230-240</td>
</tr>
</tbody>
</table>

It is accepted that the biological active class components in coffee are caffeine and chlorogenic acid. According to Bobková et al. (2017) the beans contain between 0.8-2.8 % caffeine and that this content is not severely affected by the roasting process (Phan, 2012).

Flament (2002) proposes that the optimal roast occurs slightly after the beans have reached their lowest pH value during roasting. There is a pH decline during the roasting process from 5.8 to about 4.8. From various other sources and contemporary coffee roasting magazines it is clear that the definition of an optimal roast is nevertheless subjective. In some cases and places throughout the world Flament’s theory as stated above could be true, even though it is surely not a universally accepted fact.

Modern day roasting relies heavily on the roast profile of a batch of coffee, primarily to apply manual control and in order to determine, to some extent, the degree of roast. The roast profile is defined in laymen’s terms as the temperature measured online within the batch of beans vs. time plot of a batch of coffee being roasted. More formally it is stated as “the evolution of the bean-probe temperature as the roasting process progresses” (Rao, 2014; Schwartzberg, 2002; Vosloo, 2016).

There are five distinct stages throughout the roasting process as indicated and summarised in Figure 2.2.
These stages during the roasting process can be identified using a typical roast profile as illustrated by Rao (2014) and shown in Figure 2.3. During the first time interval (0-150 seconds) in the figure below the drying stage occurs. After that the yellowing stage takes place between 150 and 350 seconds. At about 350 seconds the first crack usually takes place with the second crack happening at 530 seconds. The development stage occurs between the first and second crack.

*Figure 2.2: Schematic representation of five stages of roasting (Adapted from Vosloo, 2016)*

*Figure 2.3: Typical roast profile (—) (Adapted from Rao, 2014)*
A study was conducted in 2003 to determine what volatile organic compounds (VOC’s) are released during the roasting process. These VOC’s are responsible for the coffees’ distinct taste and aroma. (Dorfer et al., 2003). Unfortunately the majority of research projects conducted on the roasting process make no attempt to replicate a specific roast profile in order to optimise the flavour of the coffee. Temperature within the roaster is merely left to evolve unregulated for set periods. Studies therefore primarily focus on the uncontrolled, unregulated emissions and phenomenon taking place when coffee beans are heated. Little scientific information is available detailing the process as it would take place in real world scenarios with manual control being applied. During real world roasting processes the temperature within the roaster is not kept constant, but manipulated by the master roaster in order to produce coffee with specific flavours in mind, meaning that the effect of step changes in manipulated variables within the coffee roasted process is not well discussed in literature. All literature found and aimed at coffee roasting control, address the process modelling phase, or sensory methods developed for control and none focus on the actual development of the control strategy itself.

2.1.2 Coffee roasting equipment

The control strategies applied to the coffee roasting process depend greatly on the type of roaster used. Roaster machines are classified as one of two main types; either continuous or batch equipment. In our modern day coffee roasting has become increasingly popular and it is seen as a speciality profession to be an artisan roaster. These speciality artisan roasters tend to use batch roasting machines rather than continuous machines. It is further noted that specific coffee roaster types include rotating drum roasters, with or without perforated walls, rotating-bowl roasters, fluidised bed roasters, swirling-bed roasters and spouted-bed roasters (Bottazzi, 2012; Fabbri et al., 2011; Schwartzberg, 2002; Vosloo, 2016).

Patents for roaster control have been developed during earlier years for fluidised bed roasters (Sewell, 1995). These have been rather unsuccessful because coffee beans form very poor fluidised beds due to their size, shape and density (Bottazzi, 2012:20). Continuous roasters are almost only used by large coffee roasting companies to produce standardised coffee. The control for these applications are not considered in this study; the focus is rather placed on batch coffee roasting machines, specifically the perforated wall, rotating drum type.

Schwartzberg (2002) states that horizontal, rotating drums are most frequently used. Spiral flights within the rotating drum ensure axial mixing of the beans. The drum rotates at speeds low enough to warrant that the centrifugal forces will not cause the beans to adhere to the drum wall.
In modern coffee roasters the main heat source is a gas burner (mostly liquid petroleum gas (LPG)) heating air that passes through layers of coffee beans. A typical rotating-drum roaster with perforated walls is shown in Figure 2.4. Roasting times within these types of roasters range between 8.5 and 15 min according to Schwartzberg (2002).

A variation of these roasters also enable the hot air flow (often referred to as the hot gas) to enter the drum only through perforations along a certain area of the drum and only allows the air to exit after passing through the bean batch. These roasters report typical roasting times of between 10 and 15 min (Schwartzberg, 2002).
2.2 Batch coffee roasting: process control

The preceding section provided a general introduction on coffee processing and the roasting process. The following section focuses on the development of a control strategy for a batch coffee roasting process. Emphasis is placed on the coffee roasting process model development and the development and implementation of a control strategy for said process.

2.2.1 Control system development

Various methods and steps are proposed in order to design a control strategy for a particular control problem. Seborg et al. (1989:11) proposes an overview of the steps followed in order to design a controller, whereas Goodwin et al. (2000) and Åström & Hägglund (1995) focus on the detail of the design process. This section starts off with an overview inspired by the steps set out in Seborg et al. (1989:11) and then delves deeper into the detail of the matter.

The methodology suggested by Seborg et al. (1989:11) is illustrated in Figure 2.5. The blocks with broken lines (- - -) fall outside the scope of this study as explained in Chapter 1. The development of the process model was conducted by Vosloo (2016) and used in the development of the control strategy. The study conducted by Vosloo (2016) used computer simulations, coffee roasting data and the physical and chemical properties of the process in order to develop the process model. Informal information regarding experience with Genio coffee roasters was collected throughout the period of the study, therefore it does not form part of the scope of the study. It was the roasting experimentation results together with results obtained from the model that were the main contributor to the control strategy development.

The steps shown in Figure 2.5 were followed as far as possible with regards to physical development and testing of the control strategy. For most part this outline was also used in the structuring of the report to make the reading of the development process simple and logic. The following section in this chapter aims at formulating the control objectives of the coffee roasting process by utilising information about the coffee roasting process and the beneficiary’s requests.
2.2.2 Control objectives during coffee roasting

Formulating the control objectives is the initial step to almost any control problem. In order to define or identify the control objectives the process itself needs to be understood. Therefore a brief overview of the coffee roasting industry and equipment is provided in Section 2.1 of this chapter, in order to better understand the key process, i.e. roasting.
It is clear that the roasting process is responsible for the flavour formation in the coffee beans (Flament, 2002; Jokanović et al., 2012). In order to increase the quality of the coffee produced by coffee roasting equipment, the flavour produced has to be consistent throughout various roasted batches. There are numerous manipulated variables during a coffee roasting process that influence the quality and taste of the brewed coffee (Eggers & Pietsch, 2001).

Chapter 1 briefly mentioned studies conducted by Dorfner et al. (2004), Fabbri et al. (2011), Gloess et al. (2014) and Wieland et al. (2011) aimed at validating measuring tools to use within a control strategy. Some studies proposed measuring VOC’s via proton transfer reaction time-of-flight mass spectrometry (PTR-ToF-MS). This approach allows the on-line monitoring of the flavour formation of the coffee beans as a measured variable to control the roast profile. This approach is not feasible for small scale batch roasting equipment as the measuring equipment for on-line monitoring of VOC’s are too costly and impractical.

Roasting is induced by heat added to the raw coffee beans. In effect the heat added to the system, as measured by the temperature inside the bean batch, is a primary factor in the roasting process. In practice this fact has been confirmed worldwide as the majority, if not all, master roasters and artisans use the temperature progression of the beans during a roast to monitor and recreate their own desired roasted batch of coffee (Heyd et al., 2007; Jokanovic et al., 2012).

By merely placing green coffee beans into a roaster and keeping the initial roasting temperature constant for a set amount of time will not necessarily ensure repeatability of a roast. Neither is it proved that a roasted batch will be replicated by recreating a previous batch’s environmental and manipulated conditions. In order to replicate a batch of roasted coffee it is required to replicate the roast profile of the reference batch. Implying that the change of temperature as time progresses, of the batch-bean temperature has to be identical to that of the reference roast throughout the duration of the roast. What is meant by the batch-bean temperature is the temperature measured by a thermocouple inserted into the batch of coffee beans being roasted (Dorfner et al., 2003; Jokanovic et al., 2012).

To summarise, the main control objective during the roasting of coffee would be to replicate the flavour and taste of a desired (well) roasted batch, using the roast profile of the desired reference batch to do so.

A list of parameters influencing the roast profile and could act as manipulated or controlled variables when utilising feedback control, are given in Table 2.2.
Table 2.2: High-level control system summary indicating subsystem connections and possible variables to be used within control loops (Adapted from Genio, 2017)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Type of variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bean temperature</td>
<td>Controlled (CV)</td>
</tr>
<tr>
<td>Bean temperature derivative (Rate of rise)</td>
<td>Controlled (CV)</td>
</tr>
<tr>
<td>Environmental temperature</td>
<td>Disturbance</td>
</tr>
<tr>
<td>Environmental temperature derivative (Rate of rise)</td>
<td>Disturbance</td>
</tr>
<tr>
<td>LPG flow</td>
<td>Manipulated (MV)</td>
</tr>
<tr>
<td>Air pressure</td>
<td>Manipulated (MV)</td>
</tr>
<tr>
<td>Air velocity</td>
<td>Manipulated (MV)</td>
</tr>
<tr>
<td>Drum speed</td>
<td>Manipulated (MV)</td>
</tr>
<tr>
<td>Roast time</td>
<td>Manipulated (MV)</td>
</tr>
</tbody>
</table>

2.2.3 Coffee roasting process model

The modelling of systems and processes have been approached in two primary manners throughout the years. The first is to develop a model for a system based on first principles and the second is based upon formulating a model based on (experimental) data from the same system. The latter is known as the system identification approach, it is less time consuming in most cases than the first option, and is preferred by control designers and system engineers (Human, 2009). A combination of both these approaches was followed by Vosloo (2016) in the development of a process model.

2.2.3.1 Model identification and validation

Coffee roasting truly is a scientific art. There are many factors to take into account and Vosloo (2016) rightly stated that years of training and experience of roasting coffee is required before it is mastered. The modelling of such a process is therefore complex and intricate. Vosloo (2016) specifically focussed on three proposed models in order to validate their efficiency in modelling the heat and mass transfer of the coffee roasting process for control purposes. These identified models were validated after various other models were also investigated. The first and most prominent was a semi-physical model developed by Schwartzberg (2002) able to predict a coffee bean batch’s temperature and moisture content. Other investigated models include a model by Heyd et al. (2007) which presents a dynamic heat and mass transfer model.
of a spouted-bed roaster. This model was investigated but found to be less accurate than the first mentioned model. Another considered model, advocated by Fabbri et al. (2011), which used a three-dimensional geometry to predict the heat and mass transfer during roasting was also found inadequate for roast profile prediction. Hernández-Díaz et al. (2008) developed a viable moisture content prediction and lastly Putranto & Chen (2012) proposes an approach where the moisture content of the coffee bean was not assumed to be uniform, but rather as a function of time. This related to a very realistic moisture content prediction. Both the latter models were considered for roast profile prediction by Vosloo (2016).

The coffee roasting process can be divided into two main phases, namely drying (temperatures below 160°C) and roasting (temperatures between 160 and 260°C) (Hernández et al., 2007). During the drying phase there is moisture loss, and during the roasting phase various chemical reactions occur that alter the composition and taste of the beans. The process models discussed in Vosloo (2016) therefore have two main components being modelled, i.e. the bean moisture content and temperature during a roast. Vosloo (2016) established the validity of three separate models capable of accurately predicting the roast profile. These are listed in Table 2.3.

<table>
<thead>
<tr>
<th>Bean temperature model</th>
<th>Moisture content model</th>
<th>Model name as presented by Vosloo (2016)</th>
</tr>
</thead>
</table>

The first model validated by Vosloo (2016), Sch_TX, uses Schwartzberg’s model for both the bean temperature and moisture content section of the model. It was found that the model tended to overestimate the roast profile compared to experimental results. The same was true for the second model utilising Schwartzberg’s bean temperature model combined with the moisture content model from Hernández-Díaz et al. (2008). The third and last model had the best overall fit to experimental data, but overestimated the moisture content to a large degree. Additionally the third model, Put_TX, tended to underestimate the roast profile towards the end of the roast when the other models seemed to correct themselves after overestimation and move closer to experimental results (Vosloo, 2016). The model equations used in each model are stated below.
2.2.3.2 Validated bean temperature model equations

The bean temperature model presented by Schwartzberg (2002) is given by Equation 2.1 in the form of a dynamic energy balance. It denotes the change in bean temperature ($T_b$) over time ($t$). In the equation $F_g$ represents the flow of air through the system and $C_{P_g}$ the heat capacity of the air where $C_{P_b}$ represents the heat capacity of the roasted coffee beans. The inlet ($T_{g,i}$) and outlet ($T_{g,o}$) air temperature influence the energy balance greatly, as do the heat generated by the exothermic reactions ($Q_r$). The mass of the coffee beans ($m_{b(d,b)}$) was estimated on dry basis and the latent heat of vaporisation is represented by $\Delta H_v$, and the moisture content by $X_b$.

\[
\frac{dT_b}{dt} = \frac{F_g C_{P_g} [T_{g,i} - T_{g,o}] + m_{b(d,b)} (Q_r + \Delta H_v \frac{dX_b}{dt})}{m_{b(d,b)} (1 + X_b) C_{P_b}}
\]

Equation 2.1

Putranto & Chen (2012) proposed a bean temperature model given by Equation 2.2. It was assumed that heat transfer only occurred between the hot roasting air and the beans, calculated by determining the difference between the inlet air temperature and the bean temperature. This implies that any heat transfer that may occur between the roasting air and the metal drum, and between the metal and the beans were neglected. These factors were taken into account by the model of Schwartzberg (2002). The model in Equation 2.2 uses a constant heat transfer coefficient ($h$) and takes the surface area of the beans into account ($A_b$).

\[
\frac{dT_b}{dt} = \frac{h A_b (T_{g,i} - T_b) + m_{b(d,b)} \frac{dX_b}{dt} \Delta H_v}{m_{b(d,b)} (1 + X_b) C_{P_b}}
\]

Equation 2.2

2.2.3.3 Validated moisture content model equations

A semi-empirical equation was used by Schwartzberg (2002) to model the moisture loss of the beans during roasting. An Arrhenius type equation was assumed to govern the temperature dependency of the diffusion coefficient (Bottazzi et al., 2012; Schwartzberg, 2002; Vosloo, 2016). This model, given by Equation 2.3, calculates the moisture loss over time as a function of bean temperature and the equivalent sphere diameter of the coffee beans ($d_b$).
\[
- \frac{dX_b}{dt} = \frac{4.32 \times 10^9 \cdot X_b^2}{(d_b \times 10^3)^2} \exp\left[\frac{-9889}{T_b}\right]
\]

Equation 2.3

Hernández-Díaz et al. (2008) implemented a model for moisture loss with an optimised slope and represents a faster drying rate than the model proposed by Equation 2.3. This optimised moisture content model is shown by Equation 2.4, it uses the characteristic length of moisture diffusion \((L)\), bean moisture diffusivity \((D_b)\) and the moisture content at equilibrium \((X_{eq})\) to calculate the moisture content of the bean as time progresses.

\[
\frac{dX_b}{dt} = -0.032D_b \frac{L^2}{X_b - X_{eq}}
\]

Equation 2.4

The last moisture model, developed by Putranto & Chen (2012), uses established data from Keey (1992) in order to first estimate the saturated vapour concentration \((\beta_{sat})\) at various bean temperatures and then used those findings combined with the lumped-reaction engineering approach (L-REA) to define Equation 2.5. The mass transfer coefficient is represented by \(k\) while the activation energy is represented by \(\Delta E\), and the gas law constant is denoted by \(R\). The saturation vapour concentration is used together with the vapour concentration of the air \((\beta_g)\) in order to determine the moisture content of the coffee beans.

\[
\frac{dX_b}{dt} = -k A_b \left[\exp\left(-\frac{\Delta E}{RT_b}\right) \beta_{sat} - \beta_g\right] \frac{1}{m_{b(d,b)}}
\]

Equation 2.5

2.2.3.4 Justification for choice of model

The coffee roasting process model used in this study was developed by Schwartzberg (2002) and validated by Vosloo (2016). In Table 2.3 it is referred to as Sch_TX. When compared with experimental results the model is able to accurately simulate the roast profile of the roasting process. The model was constructed in Simulink®. A comparison between the model’s prediction of the roast profile and experimental results as determined by Schwartzberg (2002) are shown in Figure 2.6. In this figure \(T_b\) represents the internal bean temperature where \(T_{np}\) indicates the bean batch temperature, or roast profile.
It was concluded by Schwartzberg (2002) that the proposed model correlated extremely well with the roast profile obtained from experimental results. The largest difference between the modelled and actual temperatures was a mere 2 °C (Schwartzberg, 2002; Vosloo, 2016). Therefore, despite the less accurate moisture content prediction, this model shows very accurate prediction of the bean-batch temperature throughout the roast, and was thus deemed most suitable for the development of the control strategy based on the significant role the roast profile plays in the control of the process. This specific model is discussed in further detail in the following section.

2.2.3.5 Modelling equations used in the chosen model

The model was constructed in Simulink® using various subsystems that represent different aspects of the coffee roasting process. These subsystems are listed in Table 2.4 on the next page. The model was adapted from Vosloo (2016).
### Table 2.4: Simulink® model: Subsystem definitions and list of equations (Vosloo, 2016)

<table>
<thead>
<tr>
<th>Simulink® subsystem name</th>
<th>Significance of subsystem</th>
<th>Equation used</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heat capacity of air</td>
<td>These four aspects were combined to determine the thermophysical properties of the drying air. The drying air refers to the air that is drawn through the system, heated by the LPG flame and comes into contact with the beans during the roasting process to ensure convection heating.</td>
<td>Equation 2.6</td>
</tr>
<tr>
<td>Thermal conductivity of air</td>
<td></td>
<td>Equation 2.7</td>
</tr>
<tr>
<td>Viscosity of air</td>
<td></td>
<td>Equation 2.8</td>
</tr>
<tr>
<td>Density of air</td>
<td></td>
<td>Equation 2.9</td>
</tr>
<tr>
<td>Moisture loss</td>
<td>During the first phase of the roasting process the beans merely lose moisture. This subsystem calculated this moisture loss and therefore represents the initial phase of roasting.</td>
<td>Equation 2.3</td>
</tr>
<tr>
<td>Exothermic roasting reaction</td>
<td>The second phase of the process consists of various chemical reactions taking place, like the exothermic reactions responsible for heat generation. The heat generated by these reactions are modelled in this subsystem.</td>
<td>Equation 2.10 and Equation 2.11</td>
</tr>
<tr>
<td>Air energy balance</td>
<td>Represents the change of the air temperature as it passes through the drum during roasting.</td>
<td>Equation 2.12</td>
</tr>
<tr>
<td>Bean temperature</td>
<td>Calculates temperature inside the bean.</td>
<td>Equation 2.14</td>
</tr>
<tr>
<td>Roast profile</td>
<td>Uses input from various subsystems to determine final roast profile.</td>
<td>Equation 2.20</td>
</tr>
</tbody>
</table>
2.2.3.6  Convective heat transfer – Air properties

Heat transfer by means of convection is dominant during the roasting process within a rotating drum roaster. The drum rotates, therefore the heat transfer from the drum to the beans via conduction is minimal. The drying air is drawn over the bean batch in the rotating drum and therefore the drying air’s thermophysical properties are of utmost importance to determine an accurate model of the roasting process. The drying air’s equations used in the model are given below. These are all functions of the air temperature \( T_g \).

\[
C_p_g = 5.3091 \times 10^{-17}[T_g^6] - 4.1550 \times 10^{-13}[T_g^5] + 1.3621 \\
\times 10^{-9}[T_g^4] - 2.3267 \times 10^{-6}[T_g^3] + 2.1034 \\
\times 10^{-3}[T_g^2] - 7.2075 \times 10^{-1}[T_g] + 1.0839 \times 10^3
\]

Equation 2.6

\[
\xi_g = 1.3819 \times 10^{-20}[T_g^6] - 9.1506 \times 10^{-17}[T_g^5] + 2.2342 \\
\times 10^{-13}[T_g^4] - 2.2872 \times 10^{-10}[T_g^3] + 6.8867 \\
\times 10^{-8}[T_g^2] + 8.0128 \times 10^{-5}[T_g] + 7.6694 \times 10^{-4}
\]

Equation 2.7

\[
\mu_g = 1.2184 \times 10^{-24}[T_g^6] - 8.1123 \times 10^{-21}[T_g^5] + 1.6089 \\
\times 10^{-17}[T_g^4] + 1.1460 \times 10^{-15}[T_g^3] - 3.9733 \\
\times 10^{-11}[T_g^2] + 7.1226 \times 10^{-8}[T_g] + 4.8855 \times 10^{-7}
\]

Equation 2.8

\[
\rho_g = 353.34 \times T_g^{-1.002}
\]

Equation 2.9

2.2.3.7  Moisture loss during roasting

During the first phase of roasting the coffee beans merely lose moisture, therefore this phase is labelled the drying phase. Schwartzberg (2002) proposed an original semi-empirical equation that assumes the moisture loss is diffusively governed and that the temperature dependence of the diffusion coefficient depends on an Arrhenius type equation. (Bottazzi et al., 2012; Schwartzberg, 2002; Vosloo, 2016). The equation used in this study is a modified version of an original proposed equation. Schwartzberg (2002) corrected the equation based on experimentally determined moisture loss data, with the adapted moisture loss relation stated in Equation 2.3.
2.2.3.8 **Exothermic roasting reactions and heat generated**

The exothermic reactions occurring within the beans are responsible for heat being released as soon as the drying phase has been completed. The heat released during this second phase of the reaction is given by Equation 2.10.

\[ Q_r = Ar \cdot \exp \left( -\frac{\Delta E}{R(T_b)} \right) \left( \frac{Het - He}{Het} \right) \]  

Equation 2.10

The equation calculating the heat generated by the exothermic reactions is used in conjunction with the constant values stated in Table 2.5. These constant values were obtained from experimental data from Raemy (1981) and Raemy & Lambelet (1982) (Vosloo, 2016).

**Table 2.5: Constant values used within model: Heat generation (Taken from Schwartzberg, 2002)**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value of constant</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Ar )</td>
<td>116 200 x 10^3</td>
<td>W/kg</td>
</tr>
<tr>
<td>( \frac{\Delta E}{R} )</td>
<td>5 500</td>
<td>K</td>
</tr>
<tr>
<td>( Het )</td>
<td>232 x 10^3</td>
<td>J/kg</td>
</tr>
<tr>
<td>( \Delta Hv )</td>
<td>2790 x 10^3</td>
<td>J/kg</td>
</tr>
</tbody>
</table>

Furthermore, the following equation is used to determine accumulated heat generated at any given time.

\[ He = \sum Q_r \cdot \Delta t \]  

Equation 2.11

2.2.3.9 **Air energy balance**

The roasting air temperature rises by an average of 85 °C during roasting as it passes through the bean batch and then drops back to room temperature. Therefore, an air energy balance was proposed by Schwartzberg (2002) and stated as:

\[ -G_g C_{pg} \frac{dT_g}{dZ} = he \left( \frac{dA_b}{dZ} \right) \left( T_g - T_b \right) \]  

Equation 2.12
Integrating Equation 2.12 between the entering and exit temperatures of the air as it enters and leaves the rotating drum and simplifying the result yields the following (Vosloo, 2016):

\[
(T_{g,i} - T_{g,o}) = (T_{g,i} - T_b) \left( 1 - \exp \left[ -\frac{heA_b}{G_gC_p g} \right] \right)
\]

Equation 2.13

Schwartzberg (2002) uses the following means to calculate the effective heat transfer coefficient \((he)\):

\[
he = \frac{h}{1 + 0.3Bi}
\]

Equation 2.14

### 2.2.3.10 Bean temperature & roast profile

The temperature within the bean is dependent upon various factors which are listed as:

- Heat transferred by the roasting air \((\psi_{gb})\).
- The rate of heat generated by the exothermic reactions \((\psi_r)\).
- Heat loss due to evaporative cooling \((\psi_{ev})\).
- Weight loss of the bean \((m_bC_p b)\).

Within a rotating drum roaster the heat transferred to the metal before being transferred to the bean is neglected as the metal drum is always preheated prior to any roast. The heat transfer to the bean via conduction is also deemed negligible (Bottazzi et al., 2012; Hernández et al., 2007; Vosloo, 2016). These assumptions yield the following equation for the heat transferred by the roasting air:

\[
\psi_{gb} = G_gC_p g [T_{g,i} - T_{g,o}]
\]

Equation 2.15

In addition the rate of heat flow due to the heat generated by the exothermic reactions is given by:

\[
\psi_r = Q_r m_{b(d,b)}
\]

Equation 2.16

As the moisture loss takes place water evaporates from the surface of the beans, this means there is heat loss during this phase due to evaporative cooling. This heat loss is conveyed in Equation 2.17.
\begin{equation}
\psi_{ev} = \Delta H v \left( - \frac{dX_b}{dt} \right) m_{b(d.b)} \quad \text{Equation 2.17}
\end{equation}

Not only does cooling take place during moisture loss but the mass of the bean will vary according to Equation 2.18.

\begin{equation}
m_b C_p_b = m_{b(d.b)} (1 + X_b) C_p_b \quad \text{Equation 2.18}
\end{equation}

Lastly, taking all the assumptions found in Fabbri et al. (2011), Raemy (1981), Raemy & Lambelet (1982), Schwartzberg (2002) and Incropera et al. (2013) into account— that are briefly stated above and discussed in detail in Vosloo (2016), the following equation is obtained when performing a heat balance over the roasted beans:

\begin{equation}
m_b C_p_b \frac{dT_b}{dt} = \psi_{gb} - \psi_{gm} + \psi_{mb} + \psi_r - \psi_{ev} \quad \text{Equation 2.19}
\end{equation}

When substituting Equation 2.15 to Equation 2.18 into Equation 2.19 the final equation for the bean temperature prediction is given by Equation 2.1 (Schwartzberg, 2002; Vosloo, 2016). Schwartzberg (2002) used the simulated bean temperature, given by Equation 2.1, to model the roast profile. Note that the roast profile constant \((C)\) was determined to be 0.016 by Schwartzberg (2002). This is the desired output from the model needed to develop the control strategy of this study. The relation used in the model for the roast profile is given by:

\begin{equation}
\frac{dT_{rp}}{dt} = C (T_b - T_{rp}) \quad \text{Equation 2.20}
\end{equation}

2.2.4 Control strategy development and implementation

According to the knowledge of the author no other literature on the development of control strategies for a rotating drum coffee roaster could be found, a rather extensive passage is therefore given on a general control strategy development process. The basic control theory discussed here is vital for the development phase used in this study.

2.2.4.1 Process variable pairing: the Bristol RGA

When the control objectives have been formulated and a model obtained, the need for the pairing of process input and output variables in order to apply control using a decentralised architecture is identified. If a particular system or process has a \(n \times n\) input-output model, there are \(n!\) possible pairings (Goodwin et al., 2000:635; Seborg et al., 1989:445).
Therefore it is advisable to apply a proposed methodology in order to determine the optimal pairing of process input and output variables. Bristol (1966) developed one of the first organised methods in order to analyse multivariable process control problems. Seborg et al. (1989:454) notes that this method requires only steady-state data from the process. In addition to the advantage of using only steady-state information, the following conclusions can be drawn from the results obtained by Bristol’s approach (Bristol, 1966; Seborg et al., 1989:454):

1. A measure to what degree the process interacts.
2. A recommendation stating the most favourable/effective pairing of the manipulated and controlled variables in a system.

The final result from Bristol’s method is in the form of a matrix and based on the concept of relative gain (Seborg et al., 1989:454-459). The relative gain, \( \lambda_{ij} \), as defined by Bristol (1966) and stated in Seborg et al. (1989:454) is given by the following relation:

\[
\lambda_{ij} = \left( \frac{\partial C_i / \partial M_j}{\partial C_i / \partial M_j} \right)_{\text{open loop gain}} = \left( \frac{\text{closed loop gain}}{\text{open loop gain}} \right)
\]

Equation 2.21

For \( i = 1, 2, \ldots, n \) and \( j = 1, 2, \ldots, n \).

It is defined to be the dimensionless ratio of two steady-state gains between the controlled variable, \( C_i \), and the manipulated variable, \( M_j \).

These calculated relative gains are then arranged in a relative gain array (RGA). The relative gain array is denoted by \( \Lambda \) (Seborg et al., 1989:455):

\[
\Lambda = \begin{bmatrix}
M_1 & M_2 & \cdots & M_n \\
C_1 & \lambda_{11} & \lambda_{12} & \cdots & \lambda_{1n} \\
C_2 & \lambda_{21} & \lambda_{22} & \cdots & \lambda_{2n} \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
C_n & \lambda_{n1} & \lambda_{n2} & \cdots & \lambda_{nn}
\end{bmatrix}
\]

Equation 2.22

For a typical system consisting of two controlled variables and two manipulated variables, the following RGA will be produced:

\[
\Lambda = \begin{bmatrix}
M_1 & M_2 \\
C_1 & \lambda_{11} & \lambda_{12} \\
C_2 & \lambda_{21} & \lambda_{22}
\end{bmatrix}
\]

Equation 2.23
Based on this 2 x 2 matrix, the calculation of $\lambda_{11}$ simplifies to (Seborg et al., 1989:456):

$$\lambda_{11} = \frac{1}{1 - \frac{K_{12}K_{21}}{K_{11}K_{22}}}$$  \hspace{1cm} \text{Equation 2.24}

Where the values of $K_{11}, K_{12}, K_{21}, K_{22}$ represent the ratio of the change in controlled variable to the change in manipulated variable, for instance:

$$K_{ij} = \frac{\Delta C_i}{\Delta M_j}$$  \hspace{1cm} \text{Equation 2.25}

This relates to a steady-state change in $C_i$ produced by a steady-state change of $\Delta M_j$ in $M_j$. This steady-state gain is a measure of the process sensitivity and relates to the gradient of a pure integrating process, which is also a measure of the process sensitivity (Hu et al., 2011a; Ruscio, 2010).

Seborg et al. (1989: 457) discusses five scenarios with regards to the results of a 2 x 2 RGA. Based on these five cases the ultimate conclusions for the interpretation of an RGA’s results are summarised as follows:

- A controlled variable should only be paired with a manipulated variable if the relative gain between them is greater than or equal to 0.5.
- Finally, the controlled and manipulated variables should be paired in such a way that the corresponding relative gains are positive and as close to one as possible.
- If the relative gain between a controlled variable and a manipulated variable is negative, they should under no circumstances be paired. The closed loop and open loop gains have different signs in this case.
- If the relative gain between a controlled variable and manipulated variable is zero they have no direct effect on each other and should not be paired.

Seborg et al. (1989:457) stated in one of the five possible cases to consider when interpreting a 2 x 2 RGA: Knowing that $M_1$ has a larger effect on $C_1$ than on $C_2$, coupling $M_1$ with $C_1$ is advantageous. Yet a change in $M_2$ will affect the degree to which $C_1$ reacts to a change in $M_1$, see the first summarised point above. Consequently if $C_1$ is to be coupled with $M_1$ without the troubling interference of the second control loop an additional strategy has to be followed.

This relates to the four strategies stated by Seborg et al. (1989:461) which reduces control loop interaction:

- “Detuning” feedback controller(s)
- Changing the selected manipulated and controlled variables
• Using a decoupling controller
• Using a multivariable control scheme

For the purpose of this study emphasis is placed only on the first two strategies. Detuning a control loop (in most cases) means to mitigate the action of the controller by reducing the value of $K_c$ and increasing the value of $\tau_I$. This usually reduces the performance of the detuned controller while reducing the controller interaction (Chien et al., 1999; Seborg et al., 1989:457).

2.2.4.2 Synthesis of PI/PID control

Taking the nature of the coffee roasting process and the results from the developed RGA into account it was determined that the control mode of an on-off controller is too rudimentary to control the roasting process. This is mainly due to the extensive dead-time in the process. The theoretical interpretation of the initial experimental results and a study conducted by Hugo (2017) showed that the controlled process variable yields first-order pure integrator behaviour for the time frame in which roasting takes place, making PI/PID control a more adequate option.

According to Svrcek et al. (2000:114) there is a logical thought process, similar to the one shown in Figure 2.16, one can follow in order to select the type of controller. This proposed decision chart is depicted in Figure 2.7.

Figure 2.7: Controller selection flowchart (Adapted from Svrcek et al., 2000:114)
If the reasoning from Svrcek et al. (2000:114) is applied to the coffee roasting process, it would be concluded that the use of a PI controller is adequate. No offset can be tolerated during the process, as any large deviation from the desired set point will result in a different flavour compound formation (Bottazzi, 2012). The roast profile is key during control of the process. In many real-world instances the derivative of the roast profile is used as controlled variable, which results in the presence of process noise (Rao, 2014). If a filter is used to lessen the noise, there is excessive dead-time observed in the process variable which still leads to the PI controller being recommended by Svrcek et al. (2000:114) according to Figure 2.7. The process capacitance (defined as the ability or capacity of a process unit to retain energy or mass) is also deemed fairly small in comparison with large industrial systems, which also tends towards the selection of a PI controller.

In light of the decision chart results, the advantages and disadvantages of a PI and PID controller are mentioned below.

![Figure 2.8: Typical process response when controller action is taken for P, I and PI controllers (Adapted from Svrcek et al., 2000:104)](image)

Figure 2.8 theoretically depicts that the integral controller eliminates the offset whereas the proportional controller cannot, even though the response time of the proportional controller reacts much faster. In the above figure $\tau_n$ represents the time lapse between the first and second oscillation. The combination of these two control modes ensures a faster response time whilst eliminating the residual offset. Combining the strengths of these two control modes could result in efficient control.
However, comparing the usual responses these controllers have on systems in general, it is noted that the PID controller yields the best control with regards to reaction time combined with eliminating a potential offset in most cases. A comparison between typical process responses is shown in Figure 2.9.

Based on prior observations as depicted by Figure 2.9, a PID controller is found to react quicker to disturbances than PI control. This is due to the derivative action’s advantage added to the control strategy, which examines the rate of the error in order to apply a type of predictive control.

It is very common in modern roasters to find PID controllers, however, unfortunately, the controller parameters are rarely set to an optimal value to ensure sufficient process control (Davis & Ribich, 2005; Ribich, 2006). This control philosophy is being reported for use with spouted-bed, rotating-bowl and swirling-bed roasters. In most applications the final controlled variable is the bean temperature or the roast profile (which directly relates to the bean temperature).

Taking into account that the recommended controlled variable, i.e. the derivative of the roast profile as will be discussed in Chapter 4 and also suggested by Rao (2014), shows first-order pure integrator behaviour during certain time intervals during roasting (Hugo, 2017), the following section focusses on the development of control strategies for such processes with the emphasis on PI/PID controllers.
2.2.4.3 Behaviour and control of first-order pure integrator systems

It was concluded by Hugo (2017) that the rate of temperature change (i.e. the gradient of the roast profile curve) during the coffee roasting process shows first-order pure integrator behaviour for the latter part of the roasting process, i.e. the duration of the roast after the drying phase. Hugo (2017) conducted experiments and analysed the effect of an induced step-change in heat added to the system on the rate of temperature rise, which will be referred to as $\dot{T}_{\text{roast}}$. Some of the results are shown in Figure 2.10.

![Graph showing temperature change over time](image)

*Figure 2.10: Response of rate of temperature change to step input (Taken from Hugo, 2017)*

Figure 2.10 shows the first-order pure integrator behaviour of $\dot{T}_{\text{roast}}$ for step changes later than 90 seconds after commencing the process. The first-order plus time delay behaviour of the variable for step-changes induced at 90 seconds are also visible. The illustrated results are of roasting experiments conducted at a starting temperature of 170 °C. The various step changes were induced at 90, 240 and 300 seconds from the time the roast was started. Ruscio (2010) states that lag-dominant first-order processes with time delays may be approximated using pure integrator with time delay behaviour. Considering a system with the following first-order time constant plus time delay model (Ruscio, 2010):
\[ G_p(s) = \frac{K e^{-\theta s}}{1 + \tau_p s} \]  

Equation 2.26

Where \( K \) is the process gain, \( \tau_p \) is the dominant time constant (or time lag), and \( \theta \) is the time delay. The system is defined as lag-dominant when \( \tau_p \gg \theta \). If this is the case, Equation 2.26 may be approximated with a pure integrator plus time delay model, and the controller tuning could be based on this approximation (Ruscio, 2010), as shown in the following model:

\[ G_p(s) = \frac{K e^{-\theta s}}{s} \]  

Equation 2.27

According to the findings of Hugo (2016) the behaviour of the batch coffee roasting process is that of a lag-dominant first-order process, and Ruscio (2010) states that this process may be approximated with a pure integrator plus time delay model.

Ruscio (2010) further states that the control design of a lag-dominant first-order plus time delay system, approximated using a pure integrator with time delay model, may be based on either the first-order model or the pure integrator model. According to Ruscio (2010) utilising either one of the models and their corresponding design methods yields similar results.

It has been reported by Chakraborty et al. (2017) that integrating plus time delay processes (IPTD) include temperature control, level control, liquid storage tank and bio-reactor processes. This list will now also contain the coffee roasting process on the premise of this study’s findings and the proposed control strategy.

These processes pose various challenges with regards to process control due to their marginal open-loop stability (Chakraborty et al., 2017). Furthermore Veronesi & Visioli (2010) states that there is a particular interest in the tuning of PID controllers for these types of processes due to their lack of asymptotic stability. One last challenge presented by these processes is control input saturation.

This implies that the simulated or calculated controller output is larger than the physical final control element’s capabilities (Chakraborty et al., 2017; Hamamci & Koksal, 2010).

The open-loop response of an IPTD process on a temperature control plant is illustrated by Figure 2.11. This is the typical response of an IPTD process to a step change in the manipulated variable. In this case the valve opening used as the final control element was set to 25%. The valve determined the hot water flow to the secondary water stream of a heat exchanger in order to control the cold water’s (secondary flow) temperature (Chakraborty et al., 2017).
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PID controllers are widely used within IPTD processes (Chakraborty et al., 2017; Shamsuzzoha & Lee, 2008).

Several studies propose the use of the IMC-based method for parameter determination in order to optimise the performance of the controller with regards to integrating processes with time delay (Chakraborty et al., 2017; Shamsuzzoha & Lee, 2008; Veronesi & Visioli, 2010).

Two studies propose the use of a PD controller in order to control an IPTD process (Chakraborty et al., 2017; Hu et al., 2011b). The proposed controller development in Chakraborty et al. (2017) yields a PD controller with good robustness and less overshoot in controller responses than, among others, the controller yielded by the IMC parameter tuning methods.

The controller performance is portrayed in Figure 2.12. The red dotted line (---) represents the controller response of the IMC-based controller and the solid black line (—) represents the all-PD controller developed with the method proposed by Chakraborty et al. (2017). The section of the temperature control plant to be controlled in Chakraborty et al. (2017) was modelled by the transfer function given in Equation 2.28.

\[ G_p(s) = \frac{0.002}{s} e^{-3s} \]  

Equation 2.28

Figure 2.11: IPTD process open loop response (Adapted from Chakraborty et al., 2017)
Another study proposed the use of a retuned PID controller in order to control an IPTD process. A pre-tuned controller is assessed based on its set point and load disturbance step responses. If the assessment results are not satisfactory, the PID parameters are retuned. This ensures better controller performance without the need of special experimentation and utilises routine set point changes or load disturbances in order to retune the controller (Veronesi & Visioli, 2010). Both studies discussed, aimed at developing new tuning methodology which was compared to existing methods in order to validate the proposed design and tuning methods. Selected results from the study conducted by Veronesi & Visioli (2010) are presented in Figure 2.13.

![Figure 2.12: Controller response to step change in temperature control plant (Adapted from Chakraborty et al., 2017); IMC-based controller (---), Proposed PD controller (—)](image)

![Figure 2.13: Load disturbance step response of IPTD process with applied control (Taken from Veronesi & Visioli, 2010); Pre-existing controller (---), retuned controller (—)](image)
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It is clear from Figure 2.13 that the retuned PID controller exhibited a better response time than the pre-existing controller. This method for improving controller performance is quite simple to apply and very effective according to the results obtained from Veronesi & Visioli (2010). The transfer function used in order to obtain the data used in Figure 2.13 is given below (Veronesi & Visioli, 2010):

$$G_p(s) = \frac{1}{s(s+1)^4} e^{-0.5s}$$  \hspace{1cm} \text{Equation 2.29}

This specific experiment conducted by Veronesi & Visioli (2010) only addressed the load disturbance rejection performance of the controller. The performance of a PD controller was also compared to that of a PID controller. The results showed that the PD controller had a faster response time than the PID controller, similar to results obtained in Chakraborty et al. (2017).

2.2.4.4 Tuning methods for PI/PID controllers

It was already stated that the controlled process was identified as a lag-dominant first-order plus time delay process that could be estimated as a pure integrator system. There are various methods available in order to find the most suitable PI/PID controller settings (Skogestad & Grimholt, 2012; Vilanova & Visioli, 2010). This study only focused on the IMC-based, Cohen and Coon and lastly the ITAE relations presented by Seborg et al. (1989:282-284) in order to determine the controller parameters. The transfer function of a first-order pure integrator system with time delay is given by Equation 2.27.

a) IMC-Based PID controller settings

This control parameter design method is based on an assumed process model, like the one depicted in Equation 2.27. The internal model control (IMC) method specifically takes model uncertainty into account and allows the control designer to trade-off on control system performance and control system robustness in order to take model uncertainty into account (Seborg et al., 1989:278).

An IMC controller can be represented as a feedback controller, $G_c$, by:

$$G_c = \frac{G_c^*}{1 - G_c^* G}$$  \hspace{1cm} \text{Equation 2.30}
In the previous relation the actual controller’s transfer function is represented by $G_c^*$ and the transfer function of the process to be controlled is denoted by $\tilde{G}$.

According to Seborg et al. (2011:278) the tuning parameters, utilising this methodology, for a PI controller and first-order pure integrator system are given as:

$$K_c = \frac{2\tau_c + \theta}{K_p(\tau_c + \theta)^2}$$  \hspace{1cm} \text{Equation 2.31}

$$\tau_l = 2\tau_c + \theta$$  \hspace{1cm} \text{Equation 2.32}

Using the IMC-based method Seborg et al. (2011:278) states that the PID parameter equations for a first-order pure integrator system are given as:

$$K_c = \frac{2\tau_c + \theta}{K(\tau_c + \theta/2)}$$  \hspace{1cm} \text{Equation 2.33}

$$\tau_l = 2\tau_c + \theta$$  \hspace{1cm} \text{Equation 2.34}

$$\tau_d = \frac{\tau_c\theta + \theta^2}{2\tau_c + \theta}$$  \hspace{1cm} \text{Equation 2.35}

b) **Cohen and Coon PID controller settings**

In 1953 Cohen and Coon empirically developed a design relation which provides closed-loop responses with a decay ratio of $\frac{1}{4}$ (Noris, 2006; Seborg et al., 1989:282).

These relations for a first-order process with time delay is given as (Seborg et al., 1989:282):

**PI**

$$K_c = \frac{1}{K_p} \cdot \frac{\tau}{\theta} \cdot \left[0.9 + \frac{\theta}{12\tau}\right]$$  \hspace{1cm} \text{Equation 2.36}

$$\tau_l = \frac{\theta[30 + 3(\theta/\tau)]}{9 + 20(\theta/\tau)}$$  \hspace{1cm} \text{Equation 2.37}

**PID**

$$K_c = \frac{1}{K_p} \cdot \frac{\tau}{\theta} \cdot \left[16\tau + \frac{3\theta}{12\tau}\right]$$  \hspace{1cm} \text{Equation 2.38}

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PID

\[
\tau_I = \frac{\theta [32 + 6(\theta / \tau)]}{13 + 8(\theta / \tau)} \quad \text{Equation 2.39}
\]

\[
\tau_D = \frac{4 \theta}{11 + 2(\theta / \tau)} \quad \text{Equation 2.40}
\]

c) **ITAE PID control settings**

This method is based on a performance index that considers the entire closed-loop response of the controlled process. The integral of the time-weighted absolute error (ITAE) is given by:

\[
ITAE = \int_{0}^{\infty} t |e(t)| dt \quad \text{Equation 2.41}
\]

Where \( e(t) \) is the error signal that denotes the difference between the set point and the measured value. Design relations are chosen in order to minimise the ITAE. They are listed in Table 2.6. The equations to determine the parameters of a PI and PID controller of a first-order with time delay system is given by Seborg *et al.* (1989:284) as:

\[
K_c = \frac{A}{K_p} (\theta / \tau)^B \quad \text{Equation 2.42}
\]

\[
\tau_I = \frac{\tau}{A + B (\theta / \tau)} \quad \text{Equation 2.43}
\]

\[
\tau_D = \tau A (\theta / \tau)^B \quad \text{Equation 2.44}
\]

The various values for A and B to be used with the ITAE performance index for a first-order process with time delay is given in Table 2.6.

*Table 2.6: ITAE Performance index values – first-order with time delay process (Adapted from Seborg *et al.*, 1989:284)*

<table>
<thead>
<tr>
<th>Type of controller</th>
<th>Controller parameter</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>PI</td>
<td>( K_c )</td>
<td>0.586</td>
<td>-0.916</td>
</tr>
<tr>
<td></td>
<td>( \tau_I )</td>
<td>1.030</td>
<td>-0.165</td>
</tr>
<tr>
<td>PID</td>
<td>( K_c )</td>
<td>0.965</td>
<td>-0.850</td>
</tr>
<tr>
<td></td>
<td>( \tau_I )</td>
<td>0.796</td>
<td>-0.147</td>
</tr>
<tr>
<td></td>
<td>( \tau_D )</td>
<td>0.308</td>
<td>0.929</td>
</tr>
</tbody>
</table>
2.2.4.5 Analysis of SISO and MIMO controllers

Seborg et al. (1989:445) states that there is no conceptual difference between the modelling of SISO and MIMO systems.

It is without doubt that the coffee roasting process has to be modelled using a MIMO process model. The control strategy, however, may be simplified by using a SISO approach.

![Figure 2.14: SISO process with multiple disturbances](Adapted from Seborg et al., 1989:445)

![Figure 2.15: MIMO process with multiple disturbances](Adapted from Seborg et al., 1989:445)

According to Goodwin et al. (2000) controller analysis deals with the feedback interaction between the controller and the system and what effect this interaction has. Synthesis on the other hand relates to the construction of controllers with certain properties. Analysing a controller would require asking and answering the following questions (Goodwin et al., 2000:119):

- Is the control loop stable?
- How sensitive is the system to various disturbances?

2.2.4.6 Control loop stability

There are numerous methods to apply in order to determine the stability of a controller. Seborg et al. (1989:267) proposes a flowchart that leads the user to a decision as to what method to consider for stability analysis. The flowchart is shown in Figure 2.16.
The process models identified and validated by Vosloo (2016) for the coffee roasting process as well as the experimental data could both yield a linearized approximation of a process transfer function based on the process variable. By incorporating observed time delays in the chosen model the decision making flowchart in Figure 2.16 leads one to decide on either the Routh or frequency response stability criteria.

There are multiple useful features to the frequency response method. It can be applied to dynamic models of any order. The desirable closed-loop response characteristics can be specified by the designer and the information needed on stability margins and sensitivity characteristics is provided (Seborg et al., 1989:363). A downside to this approach is its iterative nature that could be time consuming.

The frequency response method requires analysing the input-output characteristics of feedback controllers with different mode combinations.
To conclude, the use of frequency response methods in the study and design of control systems was originated by Bode in 1920 (Seborg et al., 1989:363). As a result of this work the Bode stability criterion was formulated as:

“A closed-loop system is unstable if the frequency response of the open-loop transfer function $G_{OL} = G_cG_uG_pG_m$ has an amplitude ratio greater than one at the critical frequency. Otherwise the closed-loop system is stable. The critical frequency $\omega_c$ is defined to be the frequency at which the open-loop phase angle is $-180^\circ$. ” (Seborg et al., 1989:363).

2.2.4.7 Control loop sensitivity

In order for a controller to function efficiently it should not be unnecessarily sensitive to small changes within the system. If a model is used in the design of the controller it should also not react unduly sensitive to inaccurate calculations in the process model. In effect the controller needs to be robust enough (not greatly affected by these factors), according to Seborg et al. (1989:375).

2.2.4.8 Controllability of coffee roasting process

Controllability of a process is defined by Faanes (2003) and Ziegler & Nichols (1943) as the ability of a process to achieve acceptable performance and maintain the desired equilibrium value. Furthermore it is noted that controllability is independent of the controller, and solely a property of the process (Skogestad & Postlethwaite, 2001).
2.3 Chapter references


CHAPTER 3 – Experimental

In order to justify the methodology followed during experimentation the relative gain array (RGA) analysis is considered in this chapter. The RGA indicates how the controlled variables were paired and chosen prior to experimentation. This was done in order to minimise the amount of experiments conducted, while utilising the best possible pairing of variables. The experimental equipment, the executing procedures as well as an experimental plan for this study are detailed afterwards.


3.1 RGA analysis

In order to simplify the experimental procedure an RGA analysis was conducted to determine the best pairing of manipulated and controlled variables. After interpretation of the RGA results the effect of the chosen manipulated variable on the corresponding controlled variable was tested in order to develop a control strategy for the coffee roasting process. The detailed RGA procedure is discussed in Chapter 4 and Appendix D.

3.1.1 Concluding recommendations resulting from RGA

Using Bristol’s original reasoning it is concluded from the RGA results that $T_{roast}$ (roast profile derivative) is to be controlled by $F_{gas}$ (gas flow to burner). Denoting the primary control loop. The secondary control loop is recommended as $T_{roast}$ (roast profile) controlled by varying $F_{air}$ (air flow through system) (Bristol, 1966; Seborg et al., 1989:456). Three factors made this original recommendation less desirable:

1. One of the controlled variables is a function of the other, so they are inherently related, i.e. $T_{roast}$ is the derivative of $T_{roast}$.

2. It was experimentally determined (and the results thereof are given in Appendix C) that the effect of $F_{air}$ on $T_{roast}$ was much smaller than the effect of $F_{gas}$ on $T_{roast}$, even though $F_{air}$ had a greater influence on $T_{roast}$ than on $T_{roast}$, as validated by the results of the RGA analysis.

3. The manner in which the Genio 6 Artisan roaster is manufactured does not allow for solitary manipulation of the air flow fan. The induced draft fan speed is interlinked to the rotation speed of the drum, which introduces an additional manipulated variable.

Taking the above stated limitations into account, especially the fact that one controlled variable is a direct function of the other, it was decided to consider a threshold controller, which is qualitatively discussed in Chapter 5. This gives rise to the development of a SISO control strategy, utilising $T_{roast}$ as the controlled variable and $F_{gas}$ as the manipulated variable, with the second control loop being implemented in a threshold controller orientation only. Controlling $T_{roast}$ without interference means the gradient of the desired roast profile will optimally be controlled and therefore, theoretically should yield the required roast profile ($T_{roast}$).

To summarise:

Due to three limiting factors identified during the study it was decided, on the grounds of an RGA determination and interpretation, to develop a SISO control strategy using the primary control loop and recommending a threshold controller utilising the secondary control loop. Hence the main focus is on the development of the primary control loop.
3.2 Equipment and experimental setup

In order to determine the controllable time of roast and the effect of the chosen parameters on the process, various coffee roasting experiments were conducted. The details concerning these experimental roasts are provided in the section to follow.

3.2.1 Green beans used

The green beans roasted during experimentation are from exactly the same origin as the beans used in the study by Vosloo (2016). Cultivated in Brazil, the Arabica coffee beans grew at altitudes between 600 and 1000 m in a tropical climate. The final bean classification is unwashed Arabica. The beans are graded as Brazil Santos, denoting a Brazilian grading term that means the beans are of higher quality than “Brazilian coffee” (Sevenoaks Trading, 2015; Vosloo, 2016).

The average properties of the green beans used are presented in Table 3.1. These properties were experimentally determined by Vosloo (2016).

Table 3.1: Physical properties of green beans used during experimentation (Adapted from Vosloo, 2016)

<table>
<thead>
<tr>
<th>Property</th>
<th>Average value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single bean mass</td>
<td>$1.54 \times 10^{-4}$ kg</td>
</tr>
<tr>
<td>Moisture content</td>
<td>9.09 wt%</td>
</tr>
<tr>
<td>Longitudinal diameter</td>
<td>$9.23 \times 10^{-3}$ m</td>
</tr>
<tr>
<td>Equatorial diameter</td>
<td>$6.97 \times 10^{-3}$ m</td>
</tr>
<tr>
<td>Thickness</td>
<td>$3.56 \times 10^{-3}$ m</td>
</tr>
</tbody>
</table>

3.2.2 Coffee roasting equipment

All roasting experiments were conducted on the Genio 6 Artisan coffee roaster, manufactured by Genio Intelligent Roasters (2016). The roasting equipment has a rotating drum design where the drum (of 6 kg green bean capacity) containing the coffee beans is rotated over an open LPG flame. This LPG flame is the only heat source in the system. Air is heated over the flame and enters the perforated drum from the flat side to provide the convection heat required for the roasting of the beans. A schematic representation of the roaster is depicted in Figure 3.1. The various components of the roaster, indicated with numeric values on Figure 3.1, are listed in Table 3.2.
Table 3.2: Genio 6 Artisan roaster component specification (Genio Intelligent Roasters, 2016)

<table>
<thead>
<tr>
<th>Component number</th>
<th>Component name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Control panel</td>
<td>Connected to roaster, employing a Siemens S7-1200 PLC controller.</td>
</tr>
<tr>
<td>2</td>
<td>Gas burner</td>
<td>Gas burner fed with LPG that functions as heat source to process.</td>
</tr>
<tr>
<td>3</td>
<td>Roasting drum</td>
<td>Known as roasting chamber, contains beans during roast.</td>
</tr>
<tr>
<td>4</td>
<td>Roasting fan</td>
<td>Draws air through system, over LPG flame and beans to be roasted.</td>
</tr>
<tr>
<td>5</td>
<td>Exhaust duct</td>
<td>Duct leading to fan containing air exiting the drum.</td>
</tr>
<tr>
<td>6</td>
<td>Cyclone</td>
<td>Separates solid chaff particles from air leaving through duct.</td>
</tr>
<tr>
<td>7</td>
<td>Chaff bin</td>
<td>Captures solid particles (chaff and light beans) from cyclone.</td>
</tr>
<tr>
<td>8</td>
<td>Bean Hopper</td>
<td>Entrance point of beans into roasting drum.</td>
</tr>
<tr>
<td>9</td>
<td>Sampling port</td>
<td>Hand-held, extractable sampling cup placed inside of roasting drum.</td>
</tr>
<tr>
<td>10</td>
<td>Cooling bin</td>
<td>Round container used to cool beans rapidly after roasting process.</td>
</tr>
<tr>
<td>11</td>
<td>Cooling fan</td>
<td>Draws air through bean batch in cooling bin as final step in roasting process.</td>
</tr>
</tbody>
</table>
Figure 3.2 illustrates the frontal view of the rotating drum and also indicates the position of the two main thermocouples used during experimentation. The first thermocouple (ET Thermocouple) measures the temperature of air in the drum around the beans, also referred to as the environmental temperature of the air drawn through the system. It was inserted inside the rotating drum but did not touch the batch of roasting beans during the roasting process. The second, BB thermocouple, measures the bean batch temperature during roasting. The thermocouple was placed inside the batch of beans being roasted. The shaded area on the figure indicates the region inside the rotating drum where the beans are mostly found throughout the roasting process. The spiral flights ensure proper axial mixing of the roasted coffee beans. The position of the LPG burner inside the equipment is also shown in the figure. The controller display screen displays online curves of the bean and environmental temperature as functions of time. An external data logger, as detailed in Table 3.5, was incorporated during experimentation in order to log the temperature profiles of all the experimental runs. The additional instrumentation is listed in Table 3.5.
Table 3.3: Summary of roaster specifications (Adapted from Genio Intelligent Roasters, 2016)

<table>
<thead>
<tr>
<th>Genio 6 Artisan Roaster specifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bean capacity</td>
</tr>
<tr>
<td>Maximum of 6 kg</td>
</tr>
<tr>
<td>Heat source</td>
</tr>
<tr>
<td>25 kW gas burner</td>
</tr>
<tr>
<td>Gas source used</td>
</tr>
<tr>
<td>Natural gas (LPG)</td>
</tr>
<tr>
<td>Roasting fan</td>
</tr>
<tr>
<td>660 m³/hr at 1100 Pa</td>
</tr>
<tr>
<td>Cooling fan</td>
</tr>
<tr>
<td>1000 m³/hr at 1600 Pa</td>
</tr>
<tr>
<td>Heat transfer method</td>
</tr>
<tr>
<td>Conduction and convection</td>
</tr>
</tbody>
</table>

3.2.2.1 Operation of roasting equipment

Initially the roasting machine is primed at a pre-set temperature. This entails heating the interior of the empty drum to the initial roasting temperature. After the prime temperature has been reached the green beans are inserted via the bean hopper. The beans fall into the already heated, already rotating drum (depicted in Figure 3.2). At this point the roasting process has commenced.

The roasting fan draws air through the system, firstly drafting the inlet air over the LPG flame. The heated air then enters the perforated walls of the rotating drum, is drawn over the beans and ensures heat transfer from the air to the bean via convection. Note that the beans inside the drum are also heated by conduction due to their contact with the drum wall. After the exchange of heat with the beans, the air leaves the drum through the exhaust duct, from where it enters the cyclone where impurities in the form of chaff and small lightweight bean particles are separated from the air that is to enter the atmosphere. Air is drawn through the system throughout the duration of the roasting process. The roaster drum also continues its rotation over the open flame for this period. At the completion of the roasting process, the beans are extracted from the rotating drum and they fall into the cooling bin. The cooling bin starts rotating the raking arms at the start of the cooling phase of the process while air is continuously drawn through the cooling tray, which is key to ensure that the chemical processes are stopped as soon as possible after the beans’ extraction. The cooling bin has a perforated bed ensuring the continuous and even flow of air through the batch of beans.

The Siemens S7-1200 programmable logic controller (PLC) does not apply any control during the roasting process. It is merely active during the priming phase of the roasting process. During this phase it keeps the air temperature within the empty rotating drum at a constant set point. The control panel has three main settings tabulated in Table 3.4. The first is the “Prime” setting where the drum is preheated. The second is the “Roast” setting where the process is under manual control only. This setting is employed as soon as the roasting process
commences. The operator is able to manually vary the LPG flow to the burner and the drum rotation speed in conjunction with the roasting fan speed while the controller is set to this setting. The last setting is the “Cool” setting, which activates the cooling fan and the rotating cooling bin rakes.

<table>
<thead>
<tr>
<th>Controller setting</th>
<th>Process phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prime</td>
<td>Preheating of empty roasting drum, controller keeps environmental temperature at set point.</td>
</tr>
<tr>
<td>Roast</td>
<td>Beans are inserted – comprises entire roasting time. Manual control.</td>
</tr>
<tr>
<td>Cool</td>
<td>Cooling bin rakes rotate while cooling fan pulls air through hot beans, no control required.</td>
</tr>
</tbody>
</table>

3.2.2.2 Measuring instruments

The measuring instruments used during the roasting processes are listed in Table 3.5.

<table>
<thead>
<tr>
<th>Instrument</th>
<th>Application</th>
<th>Resolution (Maximum)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type K thermocouples</td>
<td>Temperature measurements of environmental air and bean batch.</td>
<td>0.025 °C (1370 °C)</td>
</tr>
<tr>
<td>Data logger: Pico technology USB TC-08</td>
<td>Logs online temperature profiles and stores data in compatible computer file.</td>
<td>0.025 °C (1370 °C)</td>
</tr>
<tr>
<td>Kern laboratory balances</td>
<td>Weigh bean batch for roasting purposes.</td>
<td>1 g (6000 g)</td>
</tr>
<tr>
<td>Fluke 922 Airflow meter with pilot tube</td>
<td>Measure gas flow through the system.</td>
<td>1 m³/hr (99 999 m³/hr)</td>
</tr>
</tbody>
</table>

3.3 Experimental plan

The Experiments were firstly aimed at determining the controllable time frame of the roasting process and secondly to test the effect of manipulated variables on the system. The experimental plan was set up according to the results obtained from the RGA. There are, however, various parameters during the roasting process that can be manipulated that will affect the roasting process. These manipulated variables that were fixed include, amongst
others, the position of the LPG burner relative to the drum. This parameter was kept constant for all experiments conducted. The parameter has the greatest influence on the conductive heat transfer to the beans. As it is preferable for the beans to be heated via convection this parameter was not considered and the burner was placed at the furthest point from the drum. If the LPG burner is placed too close to the drum it could result in bean scorching. Maree (2014) distinguishes between scorcing, tipping and cratering, where tipping refers to the tip of the beans being burnt due to the environmental energy exceeding the beans’ heat transfer coefficient. Tipping could be desirable in some cases, and varies according to artisan taste. It can be caused by either convection or conduction heat. On the other hand scorching is when the beans burn on the flat surface of the bean due to excessive localised conduction heat, translating to the drum surface being too hot or the rotation speed too slow (Maree, 2014).

The manner in which the control panel was connected to the roasting machine did not allow the operator to manipulate the drum rotation speed and the roasting fan speed independent of one another. These two parameters could only be varied in conjunction with each other. The LPG flow to the burner – directly influencing the heat input to the system – could be varied along with the initial roasting temperature and the time the operator chose to conduct the process (roasting time).

The process parameters are listed in Table 3.6. The parameters tested and therefore varied are also listed in Table 3.6.

<table>
<thead>
<tr>
<th>Process parameter</th>
<th>Parameter setting during experimentation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position of LPG burner (relative to drum)</td>
<td>Fixed (Furthest from drum)</td>
</tr>
<tr>
<td>Roasting fan speed &amp; drum rotation speed</td>
<td>Fixed (TMS value of 0, or maximum of both)</td>
</tr>
<tr>
<td>LPG flow to burner</td>
<td>Varied (Primary manipulated variable)</td>
</tr>
<tr>
<td>Initial roast temperature</td>
<td>Varied</td>
</tr>
<tr>
<td>Roasting time</td>
<td>No induced variation (Varied due to final roast temperature being fixed)</td>
</tr>
<tr>
<td>Final roast temperature</td>
<td>Fixed</td>
</tr>
<tr>
<td>Time of induced step change</td>
<td>Varied</td>
</tr>
</tbody>
</table>
| Magnitude of step change                       | Fixed (50-100 % on LPG burner feed valve)
In an attempt to keep the degree of roast uniform the final roast temperature was kept constant at approximately 200 °C for all experimental runs. This means that the beans were extracted as soon as the batch reached a temperature of 200 °C. Experiments with lower initial roasting temperatures therefore had a slightly longer roasting time than the experiments with the higher initial temperatures. First and second crack was identified by listening to the crescendo of the batch’s popping sounds. The model presented by Vosloo (2016) and used in this study does not account for the rotation speed of the drum and therefore it was also decided to keep this parameter constant at the same setting as the study conducted by Vosloo (2016). A value of TMS = 0 denotes the maximum rotation speed of the drum and fan of the Genio 6 Artisan coffee roaster. The time of induced step change refers to the time at which a step change in the LPG flow rate valve was induced, from 50 % to 100 %.

The roasting experiments conducted by Vosloo (2016) were used as baseline roasts at each of the corresponding initial roasting temperatures. The experimental data from the study conducted by Vosloo (2016) was analysed and average baseline values for roasts at 170, 180, 200, 210 and 230 °C were obtained. These average baseline roast profiles were used as the reference roast profiles for generating set point values in order to determine the effect of an induced step change on the process and lastly to test the controller. The experimental plan of the experiments conducted by Vosloo (2016) is stated in Table 3.7. The experimental runs used to determine the average roast profile baselines are highlighted in the table. The constructed baseline roast profiles are depicted in Appendix B.

<table>
<thead>
<tr>
<th>Initial roast temperature</th>
<th>Initial Moisture content</th>
<th>Roasting times</th>
<th>Repeats</th>
<th>Number of roasts</th>
</tr>
</thead>
<tbody>
<tr>
<td>170, 180, 190, 200, 210, 220, 230 and 240 °C</td>
<td>9.09 wt%</td>
<td>10 minutes</td>
<td>3</td>
<td>24</td>
</tr>
<tr>
<td>170, 180, 190, 200, 210, 220, 230 and 240 °C</td>
<td>9.09 wt%</td>
<td>After second crack (10 to 15 minutes)</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>170, 190, 210, and 230 °C</td>
<td>12.74, 13.29 and 14.54 wt%</td>
<td>After second crack (11 to 17 minutes)</td>
<td>1</td>
<td>12</td>
</tr>
</tbody>
</table>
The experimental plan executed by this study is shown in Table 3.8.

**Table 3.8: Experimental plan for controller development**

<table>
<thead>
<tr>
<th>Initial roast temperature</th>
<th>Type of step change</th>
<th>Time of step change</th>
<th>Number of roasts</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>170, 180, 200, 210, 230°C</strong></td>
<td>TMS = 0 &amp; Step LPG 50-100%</td>
<td>90 s (1.5 min)</td>
<td>5</td>
</tr>
<tr>
<td>170°C</td>
<td>LPG = 50% &amp; Step TMS = 0-50;</td>
<td>180 s (3 min)</td>
<td>1</td>
</tr>
<tr>
<td>170°C</td>
<td>TMS = 0 &amp; Step LPG 100-50%</td>
<td>240 s (4 min)</td>
<td>1</td>
</tr>
<tr>
<td><strong>170, 180, 200, 210, 230°C</strong></td>
<td>TMS = 0 &amp; Step LPG 50-100%</td>
<td>240 s (4 min)</td>
<td>5</td>
</tr>
<tr>
<td><strong>170, 180, 200, 210, 230°C</strong></td>
<td>TMS = 0 &amp; Step LPG 50-100%</td>
<td>300 s (5 min)</td>
<td>5</td>
</tr>
<tr>
<td>170°C</td>
<td>None (Control experiment)</td>
<td>---</td>
<td>3</td>
</tr>
</tbody>
</table>

Following the RGA recommendations during the experimental plan, experiments only tested the effect of the LPG flow to the burner on the roast profile. The derivative of the roast profile was then easily calculated from the logged roast profile data.

All initial experiments were conducted at an initial roast (prime) temperature of 170 °C. The effect of the TMS setting was determined experimentally with the help of a single experiment in combination with simulation results. Testing the effect of this variable on the process was omitted based on the RGA results. Therefore the bulk of the experiments were conducted while keeping the TMS value at a constant.

The initial roasting temperatures chosen correspond to the initial roasting temperatures chosen by Vosloo (2016) to enable the experimental results of the baseline roasts to be compared to that of the induced step change experiments. The experimental procedure followed, is stated in Appendix C.
3.4 Chapter references


CHAPTER 4 – PID-controller development

The design method used to develop the PI/PID controller for the Genio 6 Artisan roaster is discussed in Chapter 4. Firstly, the development of the RGA analysis is discussed after which the first-order pure integrator behaviour approximation of the system is proven in order to verify the chosen transfer function. The Simulink® model is explained in light of the set point generator and PI/PID parameter validation. The experimental results used in order to obtain the controller parameters are discussed. Lastly the IMC, Cohen-Coon and ITAE controller parameters are presented.
4.1 Development of RGA analysis

An RGA was not set up by using experimental data, but rather by utilising a modified version of the process model developed by the parallel study of Vosloo (2016).

The possible controlled and manipulated variables were chosen from literature. After identifying these variables an RGA was set up in order to determine the interaction between these variables and to identify the best pairing of variables. Recommended manipulated variables were identified as LPG flow rate to the burner and the air flow rate through the system (Ribich, 2006). There were many possibilities for the controlled variables. Using information from literature in combination with knowledge about the mechanics of the Genio 6 Artisan coffee roaster, the controlled variables were determined to be the bean batch temperature vs. time curve (i.e. the roast profile) and the rate of temperature change of the bean batch vs. time curve (Smith, 2016; Vosloo, 2016). Table 4.1 gives a brief summary of the listed controlled and manipulated variables initially considered in this analysis.

<table>
<thead>
<tr>
<th>Variable name</th>
<th>Variable description</th>
<th>Controlled/Manipulated</th>
<th>Variable Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate of Temperature change</td>
<td>The derivative of the roast profile (bean temperature vs. time) curve. Also produced in real time during roast.</td>
<td>Controlled</td>
<td>$\dot{T}_{roast}$</td>
</tr>
<tr>
<td>Roast profile</td>
<td>Temperature vs. time curve, produced in real time. Temperature taken of the coffee bean batch being roasted.</td>
<td>Controlled</td>
<td>$T_{roast}$</td>
</tr>
<tr>
<td>Burner LPG flow</td>
<td>The roaster burner operates with LPG, the valve determining the flow of LPG will be opened or closed in order to increase or decrease LPG flow.</td>
<td>Manipulated</td>
<td>$F_{gas}$</td>
</tr>
<tr>
<td>Air flow</td>
<td>The beans are heated by air that first flows over the LPG burner and then over the beans transferring heat to the beans (i.e. convection heat). An induced draft fan regulating the flow of air through the system will be set to draw less or more air through the roaster.</td>
<td>Manipulated</td>
<td>$F_{air}$</td>
</tr>
</tbody>
</table>
4.1.1 **RGA development**

A total of two controlled and two manipulated variables were initially identified to design the control strategy of the Artisan 6 coffee roaster.

The data required to set up the RGA of the given system was produced using a process model created in Simulink®. The model created in Simulink® was taken from the study conducted by Vosloo (2016) and adapted in order to get suitable output to set up an RGA. The Simulink® model is based on the semi-physical model developed by Schwartzberg (2002). It is denoted by Sch_TX in Vosloo (2016). This specific model was compared to experimental data from studies done by Hernández *et al.* (2007), Bottazzi *et al.* (2012) and Alonso-Torres *et al.* (2013). All of these authors found that the model correlated well with the experimental data, concluding that Schwartzberg (2002) accurately predicted the bean temperature and moisture content of the beans during a batch roast.
4.1.2 Obtaining process gain values from the Simulink® model

A schematic representation of the process as it is modelled in Simulink® is shown in Figure 4.1. Each block shown represents a set of calculations in Simulink® that calculates the parameter value stated on the block. In Figure 4.1 the temperature of the air surrounding the batch of beans in the drum, also referred to as the environmental air temperature, relates directly to the LPG flow of the burner \( F_{\text{gas}} \). Initial experimental results yielded a direct linear correlation between the LPG flow rate and the environmental air temperature. A linear output for the environmental air temperature was observed when a step change was induced in the LPG flow rate to the burner. The results from an initial experimental roast showing this correlation is given in Figure 4.2. Therefore the environmental air temperature was utilised as a manipulated variable inside the model in order to test the effect of the LPG flow of the burner on the two controlled variables, \( \dot{T}_{\text{roast}} \) and \( T_{\text{roast}} \). These two controlled variables are \( C_1 \) and \( C_2 \) respectively. The reason for using the environmental air temperature instead of directly using the LPG flow of the burner is due to the complexity of the model itself. The Simulink® model uses the derivative of various input values during calculation, seeing that a step-change does not have a gradient at the moment of the step, the model cannot compute the roast profile when a step-input is inserted as an initial value.

Figure 4.2 shows a step change of 50-100 % in the LPG flow rate of the burner at 240 seconds during the roast. It is clear that there is a linear increase in the environmental air temperature shortly after the step change is induced. This roast was conducted at an initial temperature, i.e. prime temperature, of 200 °C.

The trend illustrated in Figure 4.2 was present in all the physical roasts conducted, regardless of the initial roasting temperature or the time at which the step change was induced. These results are depicted in Appendix C. An average range of values were obtained for the gradient of the linear graph produced by the environmental air temperature plot. A ramp block was used in Simulink® to model the environmental air temperature’s reaction to a step change in LPG flow rate. These average gradient values are reported in Table 4.2.
Table 4.2: Experimentally determined average gradients for environmental air temperature for a 50% increase in LPG flow

<table>
<thead>
<tr>
<th>Initial roasting temperature</th>
<th>Average gradient of environmental air temperature ($^\circ$C/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>170°C</td>
<td>$9.153 \times 10^2$</td>
</tr>
<tr>
<td>180°C</td>
<td>$9.621 \times 10^2$</td>
</tr>
<tr>
<td>200°C</td>
<td>$7.746 \times 10^2$</td>
</tr>
<tr>
<td>210°C</td>
<td>$8.223 \times 10^2$</td>
</tr>
<tr>
<td>230°C</td>
<td>$7.620 \times 10^2$</td>
</tr>
<tr>
<td>170-230°C</td>
<td>$8.473 \times 10^2$</td>
</tr>
<tr>
<td>(Average)</td>
<td></td>
</tr>
</tbody>
</table>
The average gradient for all initial priming temperatures was used in Simulink® as the slope value for the ramp block (Hu et al., 2011). The ramp block represented the environmental air temperature and the induced gradient was the effect of a step change in the LPG flow of the burner ($M_1$). The roast profile and the derivative of the roast profile ($T_{roast}$ and $\dot{T}_{roast}$) are available as direct outputs of the model, and subsequently used to calculate the values of $K_{11}$ and $K_{21}$ based on the formula presented in Equation 2.25.

The inlet air flow is represented as $F_{air}$ and has a unit of kg.s$^{-1}$ in the model. This inlet air flow is the second chosen manipulated variable ($M_2$). A simple step change was induced from 43 kg.s$^{-1}$ to 50 kg.s$^{-1}$. This approach was used to calculate $K_{12}$ and $K_{22}$.

Both the gradient of the environmental air temperature ($M_1$) and the step changes of the air mass flow ($M_2$) were initiated at 240 seconds during the modelling of the process. The block parameters and further details regarding the Simulink® model is provided in Appendix A, and the detailed calculation of $K_{ij}$ in Appendix D.

### 4.1.3 Final RGA calculations

After considering that an RGA is normalised and therefore the sum of each row and each column will always equal to one, it follows that a $2 \times 2$ RGA can simplify to (Seborg et al., 1989:456):

$$\Lambda = \begin{bmatrix} \lambda_{11} & 1 - \lambda_{11} \\ 1 - \lambda_{11} & \lambda_{11} \end{bmatrix}$$

Equation 4.1

The detailed steps of the mathematical derivation and the simplifications are stated in Appendix E and the simulated data used in order to calculate these parameters are given in Appendix D.

The final calculated values for $K_{11}$; $K_{12}$; $K_{21}$; $K_{22}$ are given in Table 4.3.

<table>
<thead>
<tr>
<th>Process gain based on slopes</th>
<th>Calculated value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{11}$</td>
<td>$1.592 \times 10^{-3}$</td>
</tr>
<tr>
<td>$K_{12}$</td>
<td>$3.301 \times 10^{-5}$</td>
</tr>
<tr>
<td>$K_{21}$</td>
<td>$-1.443 \times 10^{-1}$</td>
</tr>
<tr>
<td>$K_{22}$</td>
<td>$8.336 \times 10^{-3}$</td>
</tr>
</tbody>
</table>
The final RGA for this particular system yields the following:

\[ \Lambda_{\text{roast}} = \begin{bmatrix} 0.736 & 0.264 \\ 0.264 & 0.736 \end{bmatrix} \]

*Table 4.4: Calculated values of relative gains extracted from RGA*

<table>
<thead>
<tr>
<th>Relative gain</th>
<th>Controlled and manipulated variables involved</th>
<th>Calculated relative gain value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda_{11} )</td>
<td>Change in ( F_{\text{gas}} ) effecting change in ( \dot{T}_{\text{roast}} )</td>
<td>0.736</td>
</tr>
<tr>
<td>( \lambda_{12} )</td>
<td>Change in ( F_{\text{air}} ) effecting change in ( \dot{T}_{\text{roast}} )</td>
<td>0.264</td>
</tr>
<tr>
<td>( \lambda_{21} )</td>
<td>Change in ( F_{\text{gas}} ) effecting change in ( T_{\text{roast}} )</td>
<td>0.264</td>
</tr>
<tr>
<td>( \lambda_{22} )</td>
<td>Change in ( F_{\text{air}} ) effecting change in ( T_{\text{roast}} )</td>
<td>0.736</td>
</tr>
</tbody>
</table>

These results were interpreted using the five cases regarding the interpretation of an RGA listed by Seborg *et al.* (1989:457) together with the four strategies mitigating loop interaction; also given by Seborg *et al.* (1989:461). The final suggested pairing is to control \( \dot{T}_{\text{roast}} \) with \( F_{\text{gas}} \) and \( T_{\text{roast}} \) with \( F_{\text{air}} \). Due to the large control loop interaction resulting from the one controlled variable being a direct function of the other, a threshold controller is recommended rather than two separate control loops. These findings were used during the setup of the experimental plan as discussed in Chapter 3.
4.2 Coffee roasting: process transfer function

The coffee roasting process model developed by Schwartzberg (2002) and validated by Vosloo (2016) was used in combination with experimental data to determine the process transfer function of the coffee roasting process. It was decided to determine the transfer function of the process based on the behaviour of the chosen controlled variable, which is the rate of change of the roast temperature ($\dot{T}_{\text{roast}}$). The results of the RGA conducted on the roasting process confirmed the applicability of incorporating a SISO continuous control strategy with $\dot{T}_{\text{roast}}$ as the only controlled variable. This section depicts the development of a SISO control strategy with the aim of reproducing specified roast profiles.

4.2.1 Verification of first-order behaviour of coffee roasting process

Taking into account that the coffee roasting process within a rotating drum roaster has a 8-15 min duration, it was proven that the response of $\dot{T}_{\text{roast}}$ to an induced step change in the heat flow to the system, showed first-order pure integrator behaviour for certain time intervals during the roasting process. These results agree with the results obtained by Hugo (2017) as illustrated in Figure 2.10. This first-order pure integrator behaviour is, however, only seen after 90 seconds from the time the roasting process was started. Before this time instance, the process shows clear first-order behaviour with a time delay, being lag-dominant. According to Ruscio (2010), first-order plus time delay processes, which are lag-dominant, may be estimated by using a first-order pure integrator transfer function. Figure 4.5 and Figure 4.6 illustrate the first-order plus time delay and pure integrator response of $\dot{T}_{\text{roast}}$ respectively.

Firstly this section details how Figure 4.5 and Figure 4.6 were constructed, and then elaborates on the phenomenon seen in these figures. Baseline roast profiles were used as the reference point. For each prime temperature a baseline roast profile was constructed by determining the average roast profile of three individually conducted roasts. No step change was induced during the baseline roasts. The baseline roast for each prime temperature was then assumed to be the general, unregulated behaviour of the controlled variable. The derivative of the baseline roast profile was calculated and used as the reference point to determine the deviation of $\dot{T}_{\text{roast}}$ from general, unregulated behaviour when a step-change in the heat flow to the system is induced.

The baseline roast profiles and $\dot{T}_{\text{roast}}$ curves for the 170, 180 and 200 °C prime temperatures are shown in Figure 4.3 and Figure 4.4 respectively.
Figure 4.3: Average baseline roast profiles; 170 °C (●), 180 °C (●), 200 °C (●)

Figure 4.3 indicates the roast profile to be reproduced whereas Figure 4.4 indicates the derivative of the roast profile (i.e. the rate of temperature change) to be reproduced as controlled variable, in order to also reproduce the specified roast profile of Figure 4.3. The $\dot{T}_{\text{roast}}$ curves shown below were eventually used as the set point value fed to the controller.

Figure 4.4: Baseline rate of temperature change, depicting $\dot{T}_{\text{roast}}$; 170 °C (●), 180 °C (●), 200 °C (●)
Figure 4.5: First-order plus time delay behaviour of roasting process, step-change induced at 90 s; 170 °C (●), 200 °C (●), 230 °C (●)

Figure 4.5 depicts the experimentally determined response of the controlled variable, $\dot{T}_{roast}$, to an induced step change 90 seconds after commencement of the roasting process. It is evident that when a manipulation is induced this early in the process, the controlled variable shows first-order with time delay behaviour.

Experimental results proving the first-order pure integrator behaviour (after 90 seconds of roasting) of the controlled variable is shown in Figure 4.6. Taking Figure 4.6 into account, which shows the response of the controlled variable when the step change is induced later than 90 seconds after commencing the roast, it can be concluded that in reality $\dot{T}_{roast}$ shows lag-dominant first-order plus time delay behaviour. Therefore the system may be estimated by using a first-order pure integrator transfer function (Ruscio, 2010).

Results from three various experiments are depicted below, for each experiment the step-change was induced 240 seconds after the commencement of the roasting process. The prime temperature (temperature of roasting machine prior to the insertion of green beans) of each experiment was varied. A prime temperature of 170, 180 and 200 °C was chosen. Each experimental roast ended between 500-570 seconds. The trend visible in Figure 4.6 was seen for every induced step-change experiment conducted, with the step-change being made later than 90 seconds after commencing the roast. These results are available in Appendix C.
4.2.2 Process transfer function

A key step during the controller development phase, was to determine a suitable transfer function for the coffee roasting process. The transfer function was used within a Simulink® model to validate the performance of the different control parameters of a PI and PID controller. After completing experimentation, the results of the experimental plan, which is stated in Table 3.8, indicate that a step-change in LPG flow resulted in pure integrator behaviour by the controlled variable, \( \hat{T}_{\text{roast}} \) (i.e. the derivative of the roast profile), for step changes induced later than 90 seconds. As shown in the previous section. This is due to the overall system showing lag-dominant first-order plus time delay behaviour, therefore Ruscio (2010) states that the process could be approximated and modelled by Equation 2.27 stated in Chapter 2:

\[
G_{p(\text{roast})} = \frac{Ke^{-\theta s}}{s}
\]

Equation 2.27

The value of \( K \) was determined by calculating the average gradient of the controlled variable (\( \hat{T}_{\text{roast}} \)) vs. time (t) plot of the experimental data. These experimentally determined gradients were then compared to plots produced by the Schwartzberg (2002) Simulink® process model’s results. A detailed discussion on the determination of the \( K \) and \( \theta \) values are stated in the following section.

Figure 4.6: Pure integrator response of \( \hat{T}_{\text{roast}} \) to step-change at 240 s; 170 °C (●), 180 °C (●), 200 °C (●)
4.2.3 Determination of $K$, $K_p$, and $\theta$ values

The Schwartzberg (2002) process model in Simulink® was used to generate simulated roast profiles of roasts with various prime temperatures. The experimental $T_{roast}$ vs. $t$ plots were created by inducing a step change of 50-100% in the LPG valve opening at various time intervals during experimentation. The simulation results were obtained by inserting the experimental profile of the environmental air temperature – for every corresponding experiment – into the model as input, in order to simulate the exact ramp (observed in the environmental temperature) caused by the step-change induced in the LPG flow. This manipulation is discussed in detail in the previous section. An overall average of the slope obtained between the experimental and simulation results was then used as the value for $K$ to be used in the IMC-based design method. In order to determine the value for $K_p$ to be used in the Cohen and Coon and ITAE methods, the average process gain was determined using

![Figure 4.7: Step-change results (200 °C prime temperature); Experimental data (●), Simulation data (—)](image)

![Figure 4.8: Step-change results (170 °C prime temperature); Experimental data (●), Simulation data (—)](image)
the experiments conducted with step-changes made at, or prior to 90 seconds of commencing the roasting process. The average dead-time was determined experimentally for each prime temperature, and an overall average value was used as the final value for $\theta$.

Figure 4.5 depicts the first-order with time delay behaviour of the process, allowing for the determination of the average process gain value, $K_p$, used in the Cohen and Coon and ITAE design methods.

Figure 4.7 and Figure 4.8 show some of the experimental and simulation data used in order to calculate $K$ and $\theta$ to be used in the IMC method. The step-change in LPG flow for both experiments were induced 240 seconds after commencement of the process.

The time interval between instant A and B, indicated on the figures, depicts the dead-time. The Simulink$^®$ model did not take dead-time into account, therefore the effect of the step-change is immediately visible in the simulation results. After the induced experimental step-change, a certain time period proceeds until there is a clear increase perceived in the gradient of the data; the time difference between this instant and the time of the induced step-change is determined graphically as the value of the dead-time.

The dead-time results are summarised in Table C.1 in Appendix C. The overall average dead-time was calculated as 20 seconds. This is fairly large as the entire process only lasts 8-15 min, and signifies that the dead-time consists of approximately 3 % of the duration of the process.

The $K$ value used in the IMC-based method was determined by graphically calculating the average gradient of both the experimental and simulated data for each experiment conducted, as shown in Figure 4.7 and Figure 4.8, and lastly finding an overall average gradient.

A noticeable trend was present within both the dead-time ($\theta$) averages and the process gradient ($K$) averages. As time of the step-change increases, the dead-time and the gradient also increased.

These trends are depicted in Table 4.5, and the final summarised values for the gradient of the process is given in Table 4.6.

<table>
<thead>
<tr>
<th>Time of step-change</th>
<th>Average dead-time ($\theta$)</th>
<th>Average gradient ($K$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>90 s</td>
<td>17.2 s</td>
<td>$1.97 \times 10^{-4}$ °C/s²</td>
</tr>
<tr>
<td>240 s</td>
<td>20.4 s</td>
<td>$3.48 \times 10^{-4}$ °C/s²</td>
</tr>
<tr>
<td>300 s</td>
<td>23.2 s</td>
<td>$4.42 \times 10^{-4}$ °C/s²</td>
</tr>
</tbody>
</table>
Table 4.6: Average determined gradient of $\dot{t}_{\text{roast}}$

<table>
<thead>
<tr>
<th></th>
<th>Average determined gradient of $\dot{t}_{\text{roast}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Experimental average</strong></td>
<td>$3.36 \times 10^{-4} , ^\circ C/s^2$</td>
</tr>
<tr>
<td><strong>Simulation average</strong></td>
<td>$3.22 \times 10^{-4} , ^\circ C/s^2$</td>
</tr>
<tr>
<td><strong>Overall average</strong></td>
<td>$3.29 \times 10^{-4} , ^\circ C/s^2$</td>
</tr>
</tbody>
</table>

Inserting the final $K$ and $\theta$ values into Equation 2.27 yields the transfer function:

$$G_{p(\text{roast})} = \frac{(3.29 \times 10^{-4})e^{-20s}}{s}$$  \hspace{1cm} \text{Equation 4.2}

The IMC expressions for determining controller tuning parameters were based on Equation 4.2. The process controlled in Simulink® was represented by Equation 4.2 in a transfer function model, and the Schwartzberg (2002) model created a baseline roast profile. This generated roast profile’s derivative acted as a scheduled set point that was used by the controller.

The average $K_p$ values used in the Cohen and Coon and ITAE methods are summarised in Table 4.7.

Table 4.7: Summarised average process gain values

<table>
<thead>
<tr>
<th>Prime temperature</th>
<th>Average change in process output for 50 % change in valve opening</th>
<th>Average process gain ($K_p$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$170 , ^\circ C$</td>
<td>$7.378 \times 10^{-2} , ^\circ C/s$</td>
<td>$1.476 \times 10^{-3} , ^\circ C/kg$</td>
</tr>
<tr>
<td>$180 , ^\circ C$</td>
<td>$7.191 \times 10^{-2} , ^\circ C/s$</td>
<td>$1.438 \times 10^{-3} , ^\circ C/kg$</td>
</tr>
<tr>
<td>$200 , ^\circ C$</td>
<td>$9.856 \times 10^{-2} , ^\circ C/s$</td>
<td>$1.971 \times 10^{-3} , ^\circ C/kg$</td>
</tr>
<tr>
<td>$210 , ^\circ C$</td>
<td>$7.675 \times 10^{-2} , ^\circ C/s$</td>
<td>$1.535 \times 10^{-3} , ^\circ C/kg$</td>
</tr>
<tr>
<td>$230 , ^\circ C$</td>
<td>$3.758 \times 10^{-2} , ^\circ C/s$</td>
<td>$7.520 \times 10^{-4} , ^\circ C/kg$</td>
</tr>
<tr>
<td><strong>Overall average process gain</strong></td>
<td></td>
<td>$1.434 \times 10^{-3} , ^\circ C/kg$</td>
</tr>
</tbody>
</table>

4.2.4 Simulink® model: transfer function application

The roasting process model discussed in Chapter 2 and schematically depicted in Figure 4.1 was used as a subsystem in the final model to simulate the controller. This section of the report explains the design procedure followed in Simulink®.
The green subsystem in Figure 4.9 depicts the Schwartzberg (2002) model. It generates a roast profile of which the derivative is calculated and sent as a set point value to the PID controller. The controller then applies control to the inserted process transfer function (labelled “Roast process”) in Figure 4.9. A PID block, available within Simulink®, is used to model the controller. The parameter settings are set to “external”, to enable the designer to specify values for the $K_c$, $\tau_I$ and $\tau_D$ parameters from sources outside the PID block. The process transfer function as inserted in Simulink® does not take the process dead-time into account, necessitating the use of a Transport Delay block. This Simulink® setup was used in order to validate the performance of the various controller parameters in order to select the best performing controller. The results of these validations are discussed in Chapter 5 of this report.

**Figure 4.9: Final Simulink® setup for parameter determination**

### 4.3 PI/PID parameter determination

The parameters to be determined are for PI and PID controllers utilising the IMC-based, Cohen and Coon and the ITAE methods. The decision for utilising only PI and PID controllers are based upon the decision chart depicted in Svrcek et al. (2000:114), and shown in Figure 2.7.
4.3.1 IMC-based parameters

The average $K$ and $\theta$ values were used in Equation 2.31 and Equation 2.32 to determine the various PI controller parameters, and in Equation 2.33, Equation 2.34 and Equation 2.35 to determine the PID controller parameters. These equations are listed in Table 4.8.

Table 4.8: IMC-based equations for parameter determination (Adapted from Seborg et al., 2011:278)

<table>
<thead>
<tr>
<th>Controller</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>PI</td>
<td>$K_c = \frac{2\tau_c + \theta}{K(\tau_c + \theta)^2}$</td>
</tr>
<tr>
<td>PID</td>
<td>$K_c = \frac{2\tau_c + \theta}{K(\tau_c + \frac{\theta}{2})^2}$</td>
</tr>
<tr>
<td></td>
<td>$\tau_i = 2\tau_c + \theta$</td>
</tr>
<tr>
<td></td>
<td>$\tau_i = 2\tau_c + \theta$</td>
</tr>
<tr>
<td></td>
<td>$\tau_D = \frac{\tau_c\theta + \frac{\theta^2}{4}}{2\tau_c + \theta}$</td>
</tr>
</tbody>
</table>

The value $\tau_c$ represents a desired closed-loop time constant. According to Rivera et al. (1986) the value for $\tau_c$ should be at least 80% of the process dead-time $\theta$, and at least 10% of the open-loop time constant $\tau_p$. Skogestad (2003) stated that $\tau_c$ should equal the process dead-time $\theta$. For the determination of all the parameters Skogestad (2003) was followed and $\tau_c$ was chosen equal to $\theta$. This was done to simplify calculations as utilising any of these methods resulted in fairly similar parameter values.

Three sets of parameters were developed for each controller, one utilising a gain scheduling curve as the parameter values, and two others using constant values. Substituting the values from Table 4.5 into Equation 2.31 to Equation 2.35 yielded gain scheduling curves for the PI and PID controllers. The constant parameter values were calculated using the overall average values of $K$ and $\theta$, and the maximum gain scheduling values of $K$ and $\theta$. These resulting parameters are detailed in Section 4.3.1.1 and Section 4.3.1.2.
4.3.1.1 PI Controller parameters

The gain scheduling curves were obtained using the values for $K$ and $\theta$ found in Table 4.5. These values were substituted into Equation 2.31 and Equation 2.32. For time intervals equal and smaller than 90 seconds, the average $K$ and $\theta$ values determined from the 90 seconds step-change experiments were used to calculate $K_c$ and $\tau_I$, the same methodology was used for the rest of the time intervals in order to construct the gain scheduling curve. These calculated parameters are:

$$K_c = \begin{cases} 221.895 & \text{for } t \leq 90 \text{ s} \\ -0.721x + 284.970 & \text{for } 90 \text{ s} < t \leq 300 \text{ s} \\ 73.189 & \text{for } t > 300 \text{ s} \end{cases}$$  \hspace{1cm} \text{Equation 4.3}

$$\tau_I = \begin{cases} 51.6 & \text{for } t \leq 90 \text{ s} \\ 0.082x + 43.677 & \text{for } 90 \text{ s} < t \leq 300 \text{ s} \\ 69.600 & \text{for } t > 300 \text{ s} \end{cases}$$  \hspace{1cm} \text{Equation 4.4}

The constant values for the PI controller are summarised in Table 4.9. The overall average values of $K$ and $\theta$ were used to determine the average parameter values, and the maximum gain scheduling curve values were used as the maximum parameter values.

<table>
<thead>
<tr>
<th>Controller parameter</th>
<th>Overall average parameter value</th>
<th>Maximum parameter value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_c$</td>
<td>112.526</td>
<td>221.895</td>
</tr>
<tr>
<td>$\tau_I$</td>
<td>60.800 s</td>
<td>69.600 s</td>
</tr>
</tbody>
</table>

4.3.1.2 PID Controller parameters

The same methodology stated in Section 4.3.1.1 was used to determine the PID parameter gain scheduling curves, by substituting the relevant $K$ and $\theta$ values into Equation 2.33 to Equation 2.35. These $K_c$, $\tau_I$ and $\tau_D$ values are:

$$K_c = \begin{cases} 394.479 & \text{for } t \leq 90 \text{ s} \\ -1.282x + 506.620 & \text{for } 90 \text{ s} < t \leq 300 \text{ s} \\ 130.115 & \text{for } t > 300 \text{ s} \end{cases}$$  \hspace{1cm} \text{Equation 4.5}
\[ \tau_I = 51.6 \text{ for } t \leq 90 \text{ s} \]
\[ = 0.082x + 43.677 \text{ for } 90 \text{ s} < t \leq 300 \text{ s} \]
\[ = 69.600 \text{ for } t > 300 \text{ s} \]  
Equation 4.6

\[ \tau_D = 7.167 \text{ for } t \leq 90 \text{ s} \]
\[ = 0.011x + 6.066 \text{ for } 90 \text{ s} < t \leq 300 \text{ s} \]
\[ = 9.667 \text{ for } t > 300 \text{ s} \]  
Equation 4.7

The constant values for the PID controller parameters are summarised in Table 4.10. The same methodology stated in Section 4.3.1.1 was used for the parameter determination.

<table>
<thead>
<tr>
<th>Controller parameter</th>
<th>Overall average parameter value</th>
<th>Maximum parameter value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_c )</td>
<td>200.046</td>
<td>394.479</td>
</tr>
<tr>
<td>( \tau_I )</td>
<td>60.800 s</td>
<td>69.600 s</td>
</tr>
<tr>
<td>( \tau_D )</td>
<td>8.444 s</td>
<td>9.667 s</td>
</tr>
</tbody>
</table>

### 4.3.2 Cohen and Coon parameters

This section discusses the parameter values determined based on the Cohen and Coon method. The response of \( \dot{T}_{roast} \) shows lag-dominant first-order plus time delay behaviour, and is approximated as a first-order pure integrator process. Therefore this method, developed for first-order plus time delay processes was utilised in addition to the IMC-based method, and derived for first-order pure integrator systems. Ruscio (2010) states that most methods for parameter determination can be based on either the first-order plus time delay, or the pure integrator model, and develop similar parameter values, when the system is lag-dominant. For both the gain scheduling, and constant parameter value determination, the average \( K_p \) and \( \theta \) values as stated in Section 4.2.3, were used. The same methodology was followed as detailed in the previous section. The values of \( K_p \) and \( \theta \) were substituted into Equation 2.36 and Equation 2.37 for the PI controller parameter calculations, and into Equation 2.38, Equation 2.39 and Equation 2.40 for the PID controller calculations. These equations are listed in Table 4.11. In all the listed equations the open-loop time constant \( \tau_p \), is assumed to be equal to the closed-loop time constant \( \tau_c \) and therefore equal to the dead-time \( \theta \).
4.3.2.1 **PI Controller parameters**

The gain & integral time scheduling curves for the $K_c$ and $\tau_I$ parameter values based on the Cohen and Coon method are:

$$K_c = \frac{1}{K_p \theta} \left[ 0.9 + \frac{\theta}{12\tau} \right]$$

Equation 2.36

$$\tau_I = \frac{\theta [30 + 3(\theta / \tau)]}{9 + 20(\theta / \tau)}$$

Equation 2.37

$$K_c = \frac{1}{K_p \theta} \left[ \frac{16\tau + 3\theta}{12\tau} \right]$$

Equation 2.38

$$\tau_I = \frac{\theta [32 + 6(\theta / \tau)]}{13 + 8(\theta / \tau)}$$

Equation 2.39

$$\tau_D = \frac{4\theta}{11 + 2(\theta / \tau)}$$

Equation 2.40

The Cohen and Coon constant value parameters are given in Table 4.12.

<table>
<thead>
<tr>
<th>Controller parameter</th>
<th>Overall average parameter value</th>
<th>Maximum parameter value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_c$</td>
<td>685.581</td>
<td>685.581</td>
</tr>
<tr>
<td>$\tau_I$</td>
<td>23.062 s</td>
<td>26.400 s</td>
</tr>
</tbody>
</table>
4.3.2.2 PID Controller parameters

The same methodology stated in Section 4.3.1 was used to determine the PID parameter gain and integral time scheduling curves, by substituting the relevant $K_p$ and $\theta$ values into Equation 2.38 to Equation 2.40. These $K_c$, $\tau_i$ and $\tau_D$ values are:

$$K_c = 1103.902 \text{ for } 90 \text{ s} \leq t \leq 300 \text{ s} \quad \text{Equation 4.10}$$

$$\tau_i = 31.124 \text{ for } t \leq 90 \text{ s}$$
$$= 0.049x + 26.345 \text{ for } 90 \text{ s} < t \leq 300 \text{ s}$$
$$= 41.981 \text{ for } t > 300 \text{ s} \quad \text{Equation 4.11}$$

$$\tau_D = 5.292 \text{ for } t \leq 90 \text{ s}$$
$$= 0.008x + 4.480 \text{ for } 90 \text{ s} < t \leq 300 \text{ s}$$
$$= 7.138 \text{ for } t > 300 \text{ s} \quad \text{Equation 4.12}$$

The constant values for the PID controller parameters are summarised in Table 4.13.

<table>
<thead>
<tr>
<th>Controller parameter</th>
<th>Overall average parameter value</th>
<th>Maximum parameter value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_c$</td>
<td>1103.902</td>
<td>1103.902</td>
</tr>
<tr>
<td>$\tau_i$</td>
<td>36.673 s</td>
<td>41.981 s</td>
</tr>
<tr>
<td>$\tau_D$</td>
<td>6.236 s</td>
<td>7.138 s</td>
</tr>
</tbody>
</table>

4.3.3 ITAE-based parameters

This section details the parameter values determined based on the ITAE method. This method was chosen for similar reasons as stated in Section 4.3.2. For both the gain and integral time scheduling as well as the constant parameter value determination, the same $K_p$ and $\theta$ values stated in Section 4.3.2, were used once again. The same methodology was also followed. The values of $K_p$ and $\theta$ were substituted into Equation 2.42, Equation 2.43 and Equation 2.44 for the PI and PID controller parameter calculations. Substituting the relevant A and B values from Table 2.6 into these equations, yields the relations for the parameter determination. These final relations are listed in Table 4.14.
In all the listed equations the open-loop time constant \( \tau_p \) is assumed to be equal to the closed-loop time constant \( \tau_c \) and therefore equal to the dead-time \( \theta \).

Table 4.14: ITAE-based equations for parameter determination (Adapted from Seborg et al., 1989:284)

<table>
<thead>
<tr>
<th>Controller</th>
<th>Equation 4.13</th>
<th>Equation 4.14</th>
<th>Equation 4.15</th>
<th>Equation 4.16</th>
<th>Equation 4.17</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>PI</strong></td>
<td>[ K_c = \frac{0.586}{K_p} (\theta/\tau)^{-0.916} ]</td>
<td>[ \tau_i = \frac{\tau}{1.030 - 0.165(\theta/\tau)} ]</td>
<td>[ K_c = \frac{0.965}{K_p} (\theta/\tau)^{-0.850} ]</td>
<td>[ \tau_i = \frac{\tau}{0.796 - 0.147(\theta/\tau)} ]</td>
<td>[ \tau_D = 0.308\tau(\theta/\tau)^{0.929} ]</td>
</tr>
</tbody>
</table>

4.3.3.1 **PI Controller parameters**

The scheduling curves for the \( K_c \) and \( \tau_i \) parameter values based on the ITAE method were determined using Equation 4.13 and Equation 4.14. Substituting the corresponding \( K_p \) and \( \theta \) values yields:

\[ K_c = 408.560 \text{ for } 90 \text{ s} \leq t \leq 300 \text{ s} \]  
\[ \tau_i = 16.704 \text{ for } t \leq 90 \text{ s} \]  
\[ = 0.048x + 12.346 \text{ for } 90 \text{ s} < t \leq 300 \text{ s} \]  
\[ = 26.821 \text{ for } t > 300 \text{ s} \]

The ITAE averaged constant parameter values are summarised in Table 4.15.

Table 4.15: Constant ITAE PI controller parameters

<table>
<thead>
<tr>
<th>Controller parameter</th>
<th>Overall average parameter value</th>
<th>Maximum parameter value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_c )</td>
<td>408.560</td>
<td>408.560</td>
</tr>
<tr>
<td>( \tau_i )</td>
<td>23.430 s</td>
<td>26.821 s</td>
</tr>
</tbody>
</table>
4.3.3.2 PID Controller parameters

The same methodology stated in Section 4.3.2 was used to determine the PID parameter scheduling curves, by substituting the relevant \( K_p \) and \( \theta \) values into Equation 4.15 through Equation 4.17. These \( K_c \), \( \tau_I \) and \( \tau_D \) values are:

\[
K_c = 672.799 \text{ for } 90 \text{ s} \leq t \leq 300 \text{ s} \quad \text{Equation 4.20}
\]

\[
\tau_I = 22.452 \text{ for } t \leq 90 \text{ s} \\
= 0.063x + 16.732 \text{ for } 90 \text{ s} < t \leq 300 \text{ s} \\
= 35.720 \text{ for } t > 300 \text{ s} \quad \text{Equation 4.21}
\]

\[
\tau_D = 5.214 \text{ for } t \leq 90 \text{ s} \\
= 0.009x + 4.367 \text{ for } 90 \text{ s} < t \leq 300 \text{ s} \\
= 7.146 \text{ for } t > 300 \text{ s} \quad \text{Equation 4.22}
\]

The average constant values for the PID controller parameters are summarised in Table 4.16.

<table>
<thead>
<tr>
<th>Controller parameter</th>
<th>Overall average parameter value</th>
<th>Maximum parameter value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_c )</td>
<td>672.799</td>
<td>672.799</td>
</tr>
<tr>
<td>( \tau_I )</td>
<td>31.203 s</td>
<td>35.720 s</td>
</tr>
<tr>
<td>( \tau_D )</td>
<td>6.242 s</td>
<td>7.146 s</td>
</tr>
</tbody>
</table>
4.4 Chapter references


CHAPTER 5 – Results and discussion

Firstly the controllability of the roasting process is discussed. This is followed by a selection of a controller including a qualitative discussion on a secondary threshold control strategy and finally the fine-tuning of the controller is discussed. The results shown and discussed in this chapter were obtained through analysing various roasting simulations, which represent the developed controllers’ performances. These simulation results are compared with each other in order to identify the best performing controller. Bode plots are incorporated to validate the performance of various selected controllers and also to fine-tune the final selected controller.
5.1 Controllability of coffee roasting process

Prior to the development of the control strategy for the batch coffee roasting system, a set of experiments were conducted in order to determine the time frame in which the process is controllable. It was concluded that there is a limited time frame during the coffee roasting process, which is deemed uncontrollable by the definitions provided by Faanes (2003), Ziegler & Nichols (1943) and Skogestad & Postlethwaite (2001).

5.1.1 Uncontrollable time frame of roasting process

The large input time delay together with other factors during the coffee roasting process, causes the process to exhibit little controllability during the first 90 seconds of the process. This is partly due to the rapid changes in the gradient of the roast profile ($\dot{T}_{roast}$) within this initial time frame. The average dead-time for process response was determined to be 20 seconds, as discussed in the previous chapter. The unregulated values of $\dot{T}_{roast}$ during the first 90 seconds increase and decrease multiple times within one 20 second time interval. Figure 5.1 shows the controlled variable’s unregulated response, and depicts the rapid changes within 20 second intervals. The figure below represents the responses of three separate roasting experiments, all three conducted at a prime temperature of 170 °C with no step-change induced during roasting. The graph depicts the deviation of $\dot{T}_{roast}$ from the average roast profile’s gradient. During the initial time frame of 0-150 seconds of roasting, the beans are in the drying phase, and the bean temperature only stabilises near the end of this phase. This could be due to the evaporative cooling occurring during the drying phase, for as soon as the majority of the moisture has evaporated the system temperature stabilises.

Figure 5.1: Initial unregulated behaviour of $\dot{T}_{roast}$ at 170 °C prime temperature; Run 1 (●), Run 2 (●), Run 3 (●)
The rapid changes shown in Figure 5.1 in combination with the large response dead-time in the system, does not create favourable conditions for a controller to stabilise the process. As the controller will only have an effect on the process based on a disturbance 20 seconds in the past, multiple changes have occurred during that single interval.

The uncontrollable time frame corresponds to the time prior to the turning point of the roast profile and is denoted as the point of intercept on the roast profile derivative ($t_{roast}$) graph. This is depicted in Figure 5.2 and Figure 5.3, respectively.

**Figure 5.2**: Unregulated roast profiles at 170 °C prime temperature; Run 1 (●), Run 2 (■), Run 3 (♦)

**Figure 5.3**: Unregulated $t_{roast}$ curve at 170 °C prime temperature; Run 1 (●), Run 2 (■), Run 3 (♦)
The graphs depicted in Figure 5.2 and Figure 5.3 show the experimental results of the same three unregulated roasts. All three experimental roasts were conducted at a prime temperature of 170 °C and no change in the manipulated variable was induced throughout the duration of the roast process. The same trend is visible regarding this uncontrollable time frame, and similar turning points of the roast profile were observed for roasts of all initial temperatures. These results are provided in Appendix B.

5.1.2 Validation of uncontrollable time frame during roasting

Based on the conclusion drawn from the previous section regarding the controllability of the roasting process, various roasting experiments were conducted in order to validate the findings. Experiments were carried out at prime temperatures of 170, 180, 200, 210 and 230 °C, and step-changes were induced at 30, 50 and 90 seconds after commencement of the process.

The step-changes were induced in the LPG flow rate to the system which influences the heat added to the system. The LPG valve opening was changed from 50-100 % at the various step-change times. The response (deviation from unregulated behaviour) of $t_{roast}$ for the experiments conducted at a 170 °C prime temperature is shown in Figure 5.4. It is evident that the three response curves seem to be from a repeated experiment. Each response depicted in Figure 5.4 had a different step-change time, but the step-changes occurred during the uncontrollable time frame of the process, therefore it can be concluded that any change prior to 90 seconds in the roast will not have any effect on the roast profile or its derivative.

![Figure 5.4: Response of induced step-change for various roasts conducted at 170 °C prime temperature; 30 s Step (●), 50 s Step (●), 90 s Step (●)](image-url)
The rapid changes shown in Figure 5.1 still occur during the initial 90 seconds of the process, and, in addition to these rapid changes, the effect of step-changes induced on the system during this time frame is only visible long after the turning point, or inflection point at approximately 90 seconds.

Based on the large process response dead-time (approximately 20 seconds), combined with the rapid changes during the initial 90 seconds of roasting, it is concluded from the theoretical definition of controllability, that the coffee roasting process cannot reach a desired equilibrium value within the initial 90 second time frame. Therefore, the roasting process is deemed theoretically uncontrollable during this initial phase of drying. Not only is the process deemed theoretical uncontrollability, its uncontrollability was experimentally proven, and is illustrated in Figure 5.4. Chapter 6 states a recommendation regarding the uncontrollability.

### 5.2 Optimal controller identification

Three different methods were used in order to develop various controller parameters. The IMC-based, Cohen and Coon and lastly the ITAE method. Parameters were determined for a PI and PID controller utilising all three methods, and for each PI controller and PID controller gain- and time constant scheduling as well as constant parameters were determined and assessed. These various controller parameters are summarised in Table 5.1.

**Table 5.1: Summary of types of determined controller parameters**

<table>
<thead>
<tr>
<th>Design method</th>
<th>Controller type</th>
<th>Parameter type</th>
<th>Controller acronym</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>IMC-based</strong></td>
<td>PI PID</td>
<td>Parameter scheduling curves</td>
<td>IMC_PI_GS IMC_PID_GS</td>
</tr>
<tr>
<td></td>
<td>PI PID</td>
<td>Maximum constant values</td>
<td>IMC_PI_MaxC IMC_PID_MaxC</td>
</tr>
<tr>
<td></td>
<td>PI PID</td>
<td>Average constant values</td>
<td>IMC_PI_AvgC IMC_PID_AvgC</td>
</tr>
<tr>
<td><strong>Cohen and Coon</strong></td>
<td>PI PID</td>
<td>Parameter scheduling curves</td>
<td>CC_PI_GS CC_PID_GS</td>
</tr>
<tr>
<td></td>
<td>PI PID</td>
<td>Maximum constant values</td>
<td>CC_PI_MaxC CC_PID_MaxC</td>
</tr>
<tr>
<td></td>
<td>PI PID</td>
<td>Average constant values</td>
<td>CC_PI_AvgC CC_PID_AvgC</td>
</tr>
<tr>
<td><strong>ITAE</strong></td>
<td>PI PID</td>
<td>Parameter scheduling curves</td>
<td>ITAE_PI_GS ITAE_PID_GS</td>
</tr>
<tr>
<td></td>
<td>PI PID</td>
<td>Maximum constant values</td>
<td>ITAE_PI_MaxC ITAE_PID_MaxC</td>
</tr>
<tr>
<td></td>
<td>PI PID</td>
<td>Average constant values</td>
<td>ITAE_PI_AvgC ITAE_PID_AvgC</td>
</tr>
</tbody>
</table>
CHAPTER 5 – Results and discussion

The controller acronyms notated in Table 5.1 will be used henceforth to refer to the controller being discussed. A comparison was made between these eighteen listed controllers in order to determine the best performing controller, the results from these comparisons being discussed in the following section.

5.2.1 Values used for PID controller block in Simulink®

Figure 4.9 illustrates the Simulink® setup of the model used in order to compare and validate the controller parameters. It is noted that the transfer function used in Simulink® for its built-in PID controller block, slightly deviates from the theoretical transfer function of a PID controller. The theoretical transfer function for a PID controller according to Seborg et al. (2011) is given by:

\[ G_c = K_c (1 + \frac{1}{\tau_I s} + \tau_D s) \]  
Equation 5.1

In Simulink®, the PID transfer function in the Ideal mode of operation within the program is given by:

\[ G_c = P (1 + I \frac{1}{s} + D \frac{N}{1 + N \frac{1}{s}}) \]  
Equation 5.2

Where \( P \) would represent the proportional action, \( I \) the integral action and \( D \) the derivative action of the controller. \( N \) in the above equation is a dimensionless factor which relates to a filter that is connected to the derivative action of the controller. For all simulations no filter was utilised, and therefore \( N = 0 \) was used as the filter coefficient throughout the study.

Equating Equation 5.1 to Equation 5.2 expresses the equations used for \( P, I \) and \( D \), as required by the Simulink® program:

\[ P = K_c \]  
Equation 5.3

\[ I = \frac{1}{\tau_I} \]  
Equation 5.4

\[ D = \tau_D \]  
Equation 5.5
5.2.2 Performance comparison between controller parameters

In order to identify the best performing and most robust controller, the Schwartzberg (2002) Simulink® model validated by Vosloo (2016) was used to generate the set point for a prime temperature of 170 °C in the modified Simulink® model. The model was also utilised during the parameter determination. While the Schwartzberg (2002) Simulink® model generated the desired set point fed to the controller, the process to be controlled was represented by a pure integrating transfer function given by Equation 4.2. In order to identify the optimal controller, the time delay (represented by a Transport delay block within Simulink®) was initially set to 0 seconds and increased incrementally.

5.2.2.1 Parameter scheduling: controller performance

The experimental results showed increasing dead-time (\( \theta \)) and average gradient (\( K \)) values as the progression time of the roast increased. The parameter scheduling curves for the various controller parameters were based on these trends seen in the dead-time and average gradient values provided in Table 4.5. The scheduling curves for the IMC_PI_GS controller parameters are depicted graphically in Figure 5.5, which shows the resulting \( K_c \) and \( \tau_I \) values stated in Equation 4.3 and Equation 4.4. The scheduling curves for each developed parameter scheduling controller is depicted in Appendix D, Section D.1.2.

![Parameter scheduling curves for IMC_PI_GS controller parameters; \( K_c \) (—), \( \tau_I \) (—)](image-url)

*Figure 5.5: Parameter scheduling curves for IMC_PI_GS controller parameters; \( K_c \) (—), \( \tau_I \) (—)*
Figure 5.6 depicts the controller performance of the IMC_PI_GS controller. The blue line in each graph represents the set point value generated by the Schwartzberg (2002) model, whereas the red line indicates the controlled variable’s ($\dot{T}_{\text{roast}}$) response to the controller output. The top left graph depicts the controller performance when there is no dead-time in the process and without any induced step-changes. The top right graph shows the results without dead-time and a single step-change of 0.2 °C.s$^{-1}$ induced 180 seconds after commencement of the process on the set point. The two bottom graphs show the controller performance when dead-time is taken to be 20 seconds, the bottom left without, and the bottom right graph with a step-change of 0.2 °C.s$^{-1}$ carried out at 180 seconds.

The results indicated that the use of parameter scheduling curves, obtained using the IMC method, yielded an aggressive response during the early stages of the controllable time frame. The IMC_PI_GS controller could stabilise the process when dead-time was taken into account, but the time frame in which it did so comprised only of the last 25% of the process duration. As soon as the dead-time was taken to be 20 seconds, the controller was able to reach the set point value after seemingly unstable initial behaviour, with the initial degree of overshoot being fairly large, but still small compared to the overshoot observed for the IMC_PID_GS controller. The degree of overshoot, even though fairly large, was small enough to ensure that
the effect of the step-change of 0.2 °C.s\(^{-1}\) was visible in the controller output. At 250 seconds the effect of the step-change is visible on the bottom right graph. Here, the degree of overshoot on the left bottom graph was found to be larger than on the bottom right graph, proving the difference in controller output due to the step-change at 180 seconds.

Figure 5.5 indicates that the value of \(K_c\) decreases due to the increase in the process gradient as the roasting time progresses, and that the value of \(\tau_I\) increases, which results in a decreasing proportional and integral action executed by the controller. The controller behaviour shown in Figure 5.6 therefore indicated that a smaller integral action (relating to a larger \(\tau_I\) value), is favourable in order to stabilise the process. The large time frame used by the controller in order to stabilise the process and correct the error, indicates that this controller will have difficulty handling disturbances during roasting. Similar controller behaviour is seen for the IMC_PID_GS controller in Figure 5.7. Even though the IMC_PID_GS controller corrects the error between the set point and the process output without the presence of dead-time efficiently, it only stabilises the process 20 seconds prior to the completion of the process, when dead-time is taken to be 20 seconds.

\textbf{Figure 5.7: IMC_PID_GS controller performance; Set point (—), Process output (—)}
Both the IMC_PI_GS and IMC_PID_GS controllers showed efficient performance without the presence of dead-time in the process, but the IMC_PI_GS controller handled the presence of dead-time slightly better, despite the lengthy time taken to correct the error.

Figure 5.7 shows the controller performance of the IMC_PID_GS controller, where the top left graph indicates the controller behaviour when dead-time is neglected and no step-change is induced. The top right graph also neglects dead-time, but a single step input was inserted 180 seconds after commencement of the process. The bottom graphs represent the controller performance when dead-time is taken to be 20 seconds. The left bottom graph shows that the IMC_PID_GS controller is able to correct the error in the process when dead-time is present and no disturbances occur, but also not within a reasonable time frame. The degree of overshoot is 50 times larger than that of the IMC_PI_GS controller. When a step-change of 0.2 °C.s\(^{-1}\) is induced at 180 seconds, the bottom graphs indicate that the controller overshoot becomes too large during the roast for the step-change to show any significant influence on controller output.

Proving that the proportional action is too large during the process, concludes that the parameter scheduling curves applied based on the IMC method is insufficient. Even though it decreases the proportional parameter as time progresses, the initial value of \(K_c\) causes the controller to be too aggressive when dead-time is taken into account.
In the top left corner of Figure 5.8 and Figure 5.9 the Cohen and Coon based controller performances without dead-time and no induced step-changes are illustrated. The controller performance when dead-time is not taken into account, but a step-change of 0.2 °C.s⁻¹ is induced at 180 seconds, is shown in the top right graphs of these figures. Again the two bottom graphs depict the controller performance for various dead-times, in each figure, both bottom figures portray the output without an induced step-change.
The overall performance of the CC_PI_GS controller was observed to be better than the CC_PID_GS controller in each case. Both these controllers stabilised the process and corrected the error when dead-time was neglected, however, both were unable to handle even minimal amounts of dead-time. The CC_PI_GS controller could correct the error and reach the set point value within 50% of the process duration when the dead-time was taken to be less than 6 seconds. Unfortunately, at a value of 6 seconds dead-time the controller could not correct the error during the time frame of the process, even though Figure 5.8 shows that the degree of overshoot decreased as time progressed, deeming the controller stable, but insufficient. Any value larger than 6 seconds allocated as the dead-time, caused the controller to render the process unstable. The results shown in the bottom right graph of Figure 5.8 show exponentially increasing overshoot values. This increasing degree of overshoot worsens as the dead-time is increased. Figure 5.9 illustrates the same controller behaviour for CC_PID_GS, but instead of handling dead-time values of up to 6 seconds, the controller is
only stable for dead-time values equal to, or smaller than 4 seconds. As soon as this value exceeds 4 seconds, the controller overshoot also increases exponentially.

The PI and PID controllers developed using the Cohen and Coon and ITAE methods showed similar results, as seen in Figure 5.8 to Figure 5.11. All the above mentioned controllers corrected the error and stabilised the process within 60 seconds when dead-time was neglected, but had extreme difficulty reaching the set point with even small dead-time values. The ITAE controllers handled the presence of dead-time better than that of the Cohen and Coon controllers. The ITAE controllers are discussed below.

![Graphs showing controller performance](image)

*Figure 5.10: ITAE_PI_GS controller performance; Set point (—), Process output (—)*
The ITAE_PI_GS and ITAE_PID_GS controllers, although performing better than the Cohen and Coon controllers, still could not handle dead-time values larger than 9 seconds and 6 seconds, respectively. This controller design method calculates the parameter for the proportional action in order to eliminate the error as soon as possible, resulting in too aggressive controller behaviour when taking the large dead-time of the process into account. The performance of the ITAE_PID_GS controller is given in Figure 5.11. It can also only stabilise the process if the dead-time is less than 6 s, and is found to be marginally stable at a dead-time of 6 seconds, showing constant amplitude oscillatory behaviour.
Figure 5.12 shows the Bode plots for the IMC_PI_GS controller’s maximum parameter values. This controller performed well in the handling of dead-time and could stabilise the process with the highest amount of dead-time.

Taking the dead-time as 20 seconds, the Bode plot for this controller indicates that the determined $\tau_I$ value is large enough to ensure marginal stability. At the critical frequency ($\omega_c$) of 0.068 rad.s⁻¹ the amplitude ratio (AR) value is slightly above one as indicated in Figure 5.12, proving the marginal stability of the best performing scheduling controller.

Analysing the stability margins of the controller using the constructed Bode plots in Excel®, it was concluded that the lowest $\tau_I$ should be larger than the average dead-time in order for the controller to be stable. Any $\tau_I$ value beneath 20 seconds relates to instability at any frequency.

Based on the simulation results and the analysed Bode plots, the best performing parameter scheduling controller is the IMC_PI_GS controller and will be considered for further investigation.
5.2.2.2  Overall average parameters: controller performance

The controller parameters determined using the overall average values of $K$, $K_p$ and $\theta$, are discussed and compared in the following section. All illustrated results depict a simulated roasting process with a prime temperature of 170 °C. All simulation step-changes were induced on the set point at 180 seconds with a magnitude of 0.2 °C.s\(^{-1}\).

Figure 5.13: IMC_PI_AvgC controller performances; Set point (—), Process output (—)

Figure 5.13 depicts the controller performance of the IMC_PI_AvgC controller. The controller’s performance for various dead-time values were evaluated and are depicted in Appendix D. The top graphs in Figure 5.13 illustrate the controller performance when no dead-time is taken into account, and the controller is able to correct any error within 300 seconds of the commencement of the process, and therefore within 210 seconds of the controllable time frame of the process. The bottom graphs illustrate the results obtained when the dead-time was taken to be 20 seconds. With an average process response dead-time experimentally
determined at 20 seconds, the use of the IMC_PI_AvgC controller is deemed plausible as it is able to stabilise the process with a dead-time even greater than 20 seconds. Even though the controller is able to correct the error and stabilise the process, it does so quite late during the progression of the roast, using almost a third of the process duration to do so.

The degree of overshoot is fairly large, even when dead-time is neglected. The overshoot is, however, small enough to ensure that step-changes induced while taking dead-time into account had an effect on the controller output, as seen in Figure 5.13. The IMC_PI_AvgC performed better than the IMC_PI_GS controller. The parameter scheduling curves decreased the \( K_c \) value and increased the \( \tau_I \) value as time progressed. Despite the trend caused by the scheduling curves, the initial \( K_c \) parameter values as used by the IMC_PI_GS controller was larger than the IMC_PI_AvgC parameter, while the \( \tau_I \) value was smaller, indicating that a larger \( \tau_I \) together with a slightly lower \( K_c \) value is desirable in effective control of this process.

Figure 5.14: IMC_PID_AvgC controller performances; Set point (—), Process output (—)
In Figure 5.14, once again, the top graphs indicate the results when no dead-time is taken into account, and the bottom graphs the results for a dead-time of 18 s (bottom left graph) and 20 seconds (bottom right graph). The IMC_PI_AvgC controller detailed in Figure 5.13 performed significantly better than the IMC_PID_AvgC shown in Figure 5.14, in the sense that the IMC_PI_AvgC was able to both eliminate the error and stabilise the process when dead-time is present. When dead-time is neglected, the degree of overshoot of the IMC_PID_AvgC controller was slightly less than that of the IMC_PI_AvgC controller, due to the larger proportional action taken by the IMC_PID_AvgC controller, which also causes its instability at larger dead-time values.

The CC_PI_AvgC shown in Figure 5.15 and the CC_PID_AvgC shown in Figure 5.16 are subsequently discussed.
When dead-time was not taken into account the performance of the CC_PID_AvgC controller exceeded that of the CC_PI_AvgC controller. The latter controller reached the desired set point later than the first mentioned controller, however, as soon as dead-time is considered the CC_PID_AvgC had a larger overshoot than the CC_PI_AvgC, indicating that the CC_PI_AvgC is better suited for larger dead-time values.

Both the CC_PI_AvgC and CC_PID_AvgC became unstable when dead-time was considered, and both showed undesirable behaviour at rather small dead-time values, 6 seconds and 4 seconds respectively. Neither of these controllers reached the set point value within a reasonable time frame. Therefore none of these controllers will be considered for further investigation, as they cannot stabilise the process and correct the error for dead-time values of 20 seconds, as determined for the coffee roasting process.

The parameters determined using the Cohen and Coon and ITAE methods were in the same ranges. The determined $K_c$ values were much larger than the IMC-based $K_c$ values, and the $\tau_i$ values fairly low in comparison with the integral time constant determined by the IMC-based method.
Figure 5.17 depicts the performance of the PI and PID controllers developed using the ITAE method. It is noted that the PI controller outperformed the PID controller regardless of the design method used. The PI controllers can handle larger dead-time values when applied to a first-order pure integrator process. The same marginal stability as previously discussed for lower dead-time values can be seen in Figure 5.17 and then complete controller instability when the dead-time value is increased. Neither the ITAE_PI_AvgC nor the ITAE_PID_AvgC controller could stabilise the process when taking the dead-time as 20 seconds, as both already exhibited unstable behaviour at dead-time values larger than 10 seconds and 6 seconds respectively.

The results of the CC_PI_AvgC and the ITAE_PID_AvgC are close to identical, as their determined parameter values are very closely related. These controller performances are the same with or without the consideration of dead-time.
This comparison between the various overall average parameter values prove the superior performance of the IMC_PI_AvgC controller. This controller’s performance is due to its larger $\tau_i$ value, which ensures a smaller integral action, added to a fairly large $K_c$ value, still smaller than the other determined $K_c$ values. Relating to a measure of aggressiveness large enough to force the output to the set point value, though small enough as to not cause instability. The integral action of all the controllers discussed in this section was within bounds of stability, but the larger $K_c$ value for all the other controllers, except the IMC_PI_AvgC controller, resulted in instability at a dead-time of 20 seconds.

In order to validate the superior performance of the IMC_PI_AvgC controller, comparative Bode diagrams are shown and discussed below. The Bode plots for the three developed PI controllers are used for the comparison given in Figure 5.18.

![Bode plots](image)

Figure 5.18: Bode plots for overall average PI parameter controllers; IMC_PI_AvgC (—), CC_PI_AvgC (—), ITAE_PI_AvgC (—)

The Bode plots show that the larger $\tau_i$ value of the IMC_PI_AvgC controller results in the phase angle approaching and crossing the -180° value at higher frequency values than the CC_PI_AvgC and ITAE_PI_AvgC controllers. The IMC_PI_AvgC controller is also the only one of the three compared controllers that has a small enough $K_c$ value to ensure that the amplitude ratio is beneath a value of one as the phase angle crosses -180°.
5.2.2.3 Maximum parameter values: controller performance

The results obtained in the following section depict the controller performance of the controllers designed using the maximum values for \( K, K_p \) and \( \theta \). In general, these results show worse overall performance for every design method, as the \( K_c \) values are too large. Each controller in this section had the ability to stabilise the process when dead-time was neglected and had similar results as detailed in the previous two sections. The controllers corrected the error within 20 – 60 seconds after commencement of the process, without the presence of dead-time. Therefore the results neglecting dead-time were omitted from the following discussion.

Figure 5.19: IMC_PI_MaxC Controller performances; Set point (---), Process output (—)

Figure 5.20: IMC_PID_MaxC Controller performances; Set point (---), Process output (—)
The IMC-based controllers performed better once more, and could stabilise the process at higher dead-time values than the Cohen and Coon and the ITAE method. No step-changes were induced on any of the controllers in this section, as none had the ability to stabilise the process and correct the deviation from the set point value when dead-time was simulated at 20 seconds. Each simulation depicts a roasting process with a prime temperature of 170 °C.

Figure 5.19 depicts the controller behaviour of the IMC\_PI\_MaxC controller. The highest dead-time value the controller could handle without rendering the process unstable is 18 seconds. In this case the controller could not correct the error within a reasonable time frame, even though it slightly decreased the degree of overshoot leading to controller stability. The IMC\_PID\_MaxC controller in Figure 5.20 showed similar behaviour, but could only handle dead-time values of up to 10 seconds; any larger dead-time value caused instability and the degree of overshoot increases as the dead-time value was increased. The IMC\_PI\_MaxC controller had less overshoot than the IMC\_PID\_MaxC controller, indicating yet again that the proportional gain of the controller was too large, leading to an overly aggressive action.

![Graphs showing controller behaviour](image)

*Figure 5.19: IMC\_PI\_MaxC and IMC\_PID\_MaxC Controller performances; Set point (—), Process output (—)*

*Figure 5.20: CC\_PI\_MaxC and CC\_PID\_MaxC Controller performances; Set point (—), Process output (—)*
The Cohen and Coon based controllers, illustrated in Figure 5.21, performed slightly worse than the ITAE controllers shown in Figure 5.22. The CC_PI_MaxC controller had a lesser degree of overshoot at dead-time values larger than 6 seconds than the CC_PID_MaxC controller, which also constitutes the largest dead-time value for which the controller could act marginally stable. A dead-time value of 4 seconds resulted in stability of the CC_PID_MaxC controller and any dead-time values larger than 4 seconds increased the overshoot greatly and rendered the controller unstable. Based on the handling of dead-time, these controllers are considered insufficient for use.

![Graph 1](image1.png)

**Figure 5.22: ITAE_PI_MaxC and ITAE_PID_MaxC Controller performances; Set point (—), Process output (—)**

![Graph 2](image2.png)

The ITAE-based controllers performed better than the Cohen and Coon based controllers only in the sense that they could remain stable for larger dead-time values as shown in Figure 5.22. Neither one of the ITAE_PI_MaxC or ITAE_PID_MaxC controller could stabilise the process at a dead-time value of 20 seconds. The ITAE_PI_MaxC became unstable at dead-time values larger than 9 seconds, whereas the ITAE_PID_MaxC became unstable at dead-time values exceeding 6 seconds.
CHAPTER 5 – Results and discussion

5.3 Controller fine-tuning and validation

5.3.1 Fine-tuning utilising Bode plots and measures of stability

The performance comparison of the various controllers indicated that the IMC_PI_AvgC controller had the best overall performance, proving that the controller selection chart shown in Figure 2.7 shows to be accurate (Svrcek et al., 2000:114). It was the only controller able to correct the error and remain stable when the dead-time was taken to be equal to the process average dead-time of 20 seconds. The controller performance, even though superior to the other studied controllers, is not very good, even though the controller corrects the deviation from the set point, the time in which it does so is not favourable for use during the limited time of the coffee roasting process. In order to replicate the taste of a batch of roasted coffee, the roast profile of the desired batch needs to be replicated. This leads to a very low tolerance for offset, for as soon as the process deviates from the set point, the path followed by the process to reach the end temperature varies. This will in turn result in a different flavour profile (Dorfner et al., 2003).

In order to rectify the shortcoming of the IMC_PI_AvgC controller, the parameters were fine-tuned in an attempt to shorten the reaction time of the controller. When analysing the Bode plot of the original and fine-tuned IMC_PI_AvgC controller shown in Figure 5.23, it is evident that a larger $\tau_I$ value results in controller stability at higher frequency values.

According to Seborg et al. (1989:363) if the open-loop transfer function has an amplitude ratio greater than one at the critical frequency ($\omega_c$), the controller is deemed unstable. Therefore Figure 5.23 proves that the original IMC_PI_AvgC together with both fine-tuned controllers are stable, as the amplitude ratio is always less than one at the critical frequency.

This is in agreement with studies conducted by Åström & Hägglund (2004) regarding tuning methods for PID controllers, it was found that, as the time delay (referred to as dead-time in this chapter) becomes larger, the integral action should decrease. This results in an increasing $\tau_I$ value, which justifies the decision to increase the $\tau_I$ value incrementally in order to achieve the desired controller performance.
The original IMC_PI_AvgC parameters were used as the starting point for the fine-tuning of the final controller, and its Bode plot is indicated by the blue line in Figure 5.23. The first fine-tuned controller is illustrated using the red line and the second using the green line in Figure 5.23. As determined by the Bode plots the smallest value of $\tau_I$ before the controller becomes inherently unstable is 20.5 seconds, with a corresponding maximum $K_C$ value of 10.93, which gives the margin of stability.

The controller parameters for these three controllers are summarised in Table 5.2.

*Table 5.2: Summarised fine-tuned controller parameters*

<table>
<thead>
<tr>
<th>Controller</th>
<th>$K_C$</th>
<th>$\tau_I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>IMC_PI_AvgC Original</td>
<td>112.526</td>
<td>60.800</td>
</tr>
<tr>
<td>IMC_PI_AvgC Fine-tuned1</td>
<td>112.526</td>
<td>152.000</td>
</tr>
<tr>
<td>IMC_PI_AvgC Fine-tuned2</td>
<td>112.526</td>
<td>6841.581</td>
</tr>
</tbody>
</table>
Extremely large $\tau_I$ values result in very small integral action. Even though the controller has this extremely small integral action, it is adequate to force the controlled variable to the set point value without causing lag in the controller response. It is noted that all three controllers mentioned in Table 5.2 are able to control the simulated first-order integrating coffee roasting process. The second fine-tuned controller's $\tau_I$ value was calculated using the following equation:

$$\tau_{I(Fine-tuned)} = K_c \tau_I$$  \hspace{1cm} \text{Equation 5.6}

For a small enough integral action able to stabilise the process, and correct the error between the controlled variable and the set point, Equation 5.6 is recommended.

### 5.3.2 Validation of fine-tuned controller parameters

In order to validate the recommended controller (named: IMC_PI_AvgC Fine-tuned2), various simulations were conducted to prove the swift performance and accuracy of the specified controller. Various step-changes of $0.2 \, ^\circ\text{C}. \text{s}^{-1}$ (at 180 seconds) and $0.4 \, ^\circ\text{C}. \text{s}^{-1}$ (at 250 seconds) were induced on a simulated roasting process with a prime temperature of 170 °C, at dead-time values of 15, 20 and 25 s, in order to validate the controller despite the experimental error regarding the dead-time value determination.

*Figure 5.24: Final controller performance – 15 s dead-time with step-changes; Set point (---), Process output (—)*
In Figure 5.24 through Figure 5.26 the performance of the controller is illustrated when dead-time is less, equal to, and more than the average determined dead-time. In each case the controller is able to stabilise the process and correct the error with a maximum, final difference of 0.01 °C/s from the set point value. With a smaller dead-time value, the controller is able to correct the error sooner, as seen in Figure 5.24, when the controller takes approximately 100 seconds during the controllable time frame of the process to correct the error. It also has the ability to handle various step-changes sufficiently.

A larger dead-time leads to a larger degree of overshoot, resulting in a longer time taken to correct errors. The stability of the controller at lower and higher dead-time values, in combination with its response time, proves that the IMC_PI_AvgC_Fine-tuned2 controller is deemed adequate for controlling the coffee roasting process.
Table 5.3: Controller performance summary based on effect of dead-time on various controllers

<table>
<thead>
<tr>
<th>Controller</th>
<th>Maximum dead-time before instability</th>
<th>Error corrected with maximum dead-time present</th>
<th>Contextual factors regarding controller performance when dead-time is taken to be larger than maximum dead-time value.</th>
</tr>
</thead>
<tbody>
<tr>
<td>IMC_PI_GS</td>
<td>20 s</td>
<td>Yes</td>
<td>Error is corrected during last 25% of process duration. Large degree of overshoot.</td>
</tr>
<tr>
<td>IMC_PID_GS</td>
<td>20 s</td>
<td>Yes</td>
<td>Error correction as IMC_PI_GS, with larger overshoot.</td>
</tr>
<tr>
<td>CC_PI_GS</td>
<td>6 s</td>
<td>No</td>
<td>Degree of overshoot increases exponentially.</td>
</tr>
<tr>
<td>CC_PID_GS</td>
<td>4 s</td>
<td>No</td>
<td>Degree of overshoot increases exponentially.</td>
</tr>
<tr>
<td>ITAE_PI_GS</td>
<td>9 s</td>
<td>No</td>
<td>Overshoot increases to lesser degree than CC_PI_GS.</td>
</tr>
<tr>
<td>ITAE_PID_GS</td>
<td>6 s</td>
<td>Yes</td>
<td>Error corrected during last 25% of process duration. Large degree of overshoot.</td>
</tr>
<tr>
<td>IMC_PI_AvgC</td>
<td>20 s</td>
<td>Yes</td>
<td>Shows best overall performance. Smallest degree of overshoot with dead-time taken into account.</td>
</tr>
<tr>
<td>IMC_PID_AvgC</td>
<td>18 s</td>
<td>No</td>
<td>Slight increase in degree of overshoot.</td>
</tr>
<tr>
<td>CC_PI_AvgC</td>
<td>6 s</td>
<td>No</td>
<td>Degree of overshoot increases exponentially.</td>
</tr>
<tr>
<td>CC_PID_AvgC</td>
<td>4 s</td>
<td>No</td>
<td>Degree of overshoot increases exponentially.</td>
</tr>
<tr>
<td>ITAE_PI_AvgC</td>
<td>10 s</td>
<td>No</td>
<td>Degree of overshoot increases exponentially.</td>
</tr>
<tr>
<td>ITAE_PID_AvgC</td>
<td>6 s</td>
<td>Yes</td>
<td>Error corrected during last 10% of process duration.</td>
</tr>
<tr>
<td>IMC_PI_MaxC</td>
<td>18 s</td>
<td>No</td>
<td>Slight increase in degree of overshoot.</td>
</tr>
<tr>
<td>IMC_PID_MaxC</td>
<td>10 s</td>
<td>Yes</td>
<td>Degree of overshoot increases exponentially.</td>
</tr>
<tr>
<td>CC_PI_MaxC</td>
<td>6 s</td>
<td>No</td>
<td>Degree of overshoot increases exponentially.</td>
</tr>
<tr>
<td>CC_PID_MaxC</td>
<td>4 s</td>
<td>No</td>
<td>Degree of overshoot increases exponentially.</td>
</tr>
<tr>
<td>ITAE_PI_MaxC</td>
<td>9 s</td>
<td>No</td>
<td>Degree of overshoot increases exponentially.</td>
</tr>
<tr>
<td>ITAE_PID_MaxC</td>
<td>6 s</td>
<td>Yes</td>
<td>Degree of overshoot increases exponentially.</td>
</tr>
<tr>
<td>Fine-tuned1</td>
<td>20 s</td>
<td>Yes</td>
<td>Corrects error too late during process.</td>
</tr>
<tr>
<td>Fine-tuned2</td>
<td>25 s</td>
<td>Yes</td>
<td>Corrects error within reasonable time frame, little to no overshoot.</td>
</tr>
</tbody>
</table>
5.3.3 Qualitative discussion regarding possible threshold control strategy

The following discussion was not experimentally validated in this study and is based on control theory in order to give relevant recommendations with regards to the developed controller. It does not form part of the listed objectives, but was deemed noteworthy to mention as the RGA analysis indicated that it requires attention.

The chosen controlled variable, $\dot{T}_{roast}$ (derivative of the roast profile), was used as the only controlled variable, on the grounds of the RGA results given in Chapter 3 and 4 of this report. The interaction between the main and a possible, second control loop, utilising the roast profile as the controlled variable, is detrimental to the proper functioning of the overall controller. Utilising such a control strategy seems necessary after initial analysis of the process. This seeming necessity of the second control loop is due to the phenomenon shown in Figure 5.2 and Figure 5.3. These figures illustrate that a function can have different values while having the same gradient, which means that different roast profiles can have the same gradient during the roasting process. Unfortunately this means that even if perfect control is executed by the controller with regards to the derivative of the roast profile (i.e. the gradient of the roast profile), the controller will not necessarily replicate a desired roast profile.

A qualitative motivation as to why a second control loop utilising the roast profile as controlled variable, is not viable, is given below. The motivation is given with the help of Figure 5.27 and Figure 5.28.

![Figure 5.27: Hypothetical roast profiles; Set point (---), Process output (—)](image)
Figure 5.27 illustrates a desired roast profile tendency graph while Figure 5.28 illustrates the corresponding roast profile derivative of the same roast. If two control loops were incorporated in the control strategy, the main control loop acting upon the deviation of the derivative of the roast profile from the set point value, would take no action between 300 – 400 seconds, as the controlled variable is equal to the set point value as seen in Figure 5.28. The second control loop could, however, take action as to force the roast profile value closer to the set point value, resulting in a change in gradient of the roast profile, which will cause an action in the opposite direction executed by the first control loop. The two control loops will therefore work against each other in similar situations.

A proposed solution is to incorporate a threshold control strategy, which uses the second control loop (using the roast profile as the only controlled variable) as the main control loop for the initial 110 seconds of the process, ensuring the controller corrects the error between the roast profile and its set point during this time frame. After the 110 seconds initial phase of the process the controller could switch over to the main control loop, utilising the derivative of the roast profile in the manner proposed by this study.

Another possible strategy could be to affect a trimming action on the roast profile using the flow rate of air through the drum system, while still maintaining the pairing of the derivative of the roast profile with the LPG flow rate as manipulated variable.

Both these strategies could benefit from additional investigations in order to optimise the control of the coffee roasting process.
5.4 Experimental error

The experimental error was determined using the standard error of the mean (\( \sigma_{x} \)), given by (Devore & Farnum, 2005):

\[
\sigma_{x} = \frac{\sigma}{\sqrt{n}}
\]

Equation 5.7

Here, \( \sigma \) represents the standard deviation and \( n \) the number of samples used in the determination. Three experiments were conducted in order to determine the experimental error. A roasting process with a prime temperature of 200 °C was repeated, and a step-change of 50 % in the LPG valve opening was induced at 90 seconds after commencing the roast. The results of the error determination are given in Table 5.4.

<table>
<thead>
<tr>
<th>Dead-time [s]</th>
<th>Roast 1</th>
<th>Roast 2</th>
<th>Roast 3</th>
<th>Average</th>
<th>Standard deviation</th>
<th>Error</th>
<th>Error percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>21.00</td>
<td>17.00</td>
<td>15.00</td>
<td>17.67</td>
<td>2.49</td>
<td>1.44</td>
<td>8.15</td>
</tr>
<tr>
<td>Initial slope [^{°C.s^{-2}}]</td>
<td>3.20 x10(^{-5})</td>
<td>2.87 x 10(^{-4})</td>
<td>3.36 x 10(^{-4})</td>
<td>2.18 x 10(^{-4})</td>
<td>1.33 x 10(^{-4})</td>
<td>7.67 x 10(^{-5})</td>
<td>35.24</td>
</tr>
<tr>
<td>Process gain [^{°C.s^{-1}}]</td>
<td>6.48 x 10(^{-2})</td>
<td>9.88 x 10(^{-2})</td>
<td>4.04 x 10(^{-2})</td>
<td>6.80 x 10(^{-2})</td>
<td>2.40 x 10(^{-2})</td>
<td>1.38 x 10(^{-2})</td>
<td>20.34</td>
</tr>
</tbody>
</table>

The standard error of the mean value for each experimentally determined parameter is shown in Table 5.4, which reports the summarised error values. The standard error of the mean for the determined dead-time is 8.15 %, whereas the standard error of the mean for the initial slope is 35.24 % and the process gain, 20.34 %.

The error percentage of the dead-time implies that a real-world scenario roast could have an 8.15 % difference in value to that of the determined average dead-time measured in seconds. This in turn relates to a real-world roast’s initial slope and process gain that could deviate 25.24 % and 20.34 % respectively, from the average calculated values. This creates a range of values for the PI/PID parameters, within this error margin, that will result in similar control to the experimentally determined PI/PID parameter values.
5.5 Chapter references


The final conclusions regarding the developed control strategy is stated in the following chapter, together with various recommendations regarding further research as well as the implementation of the developed control strategy.


6.1 Conclusions

The model validated by Vosloo (2016) was originally proposed by Schwartzberg (2002) and found to be accurate in its determination of the roast profile prediction, not only with regards to the roast profile values but also with regards to the predicted gradient of the roast profile. The model was found to be a useful tool in the development of a realisable control strategy for the coffee roasting process.

The coffee roasting process may be modelled using a pure integrating transfer function, which stands in agreement with an observation by Hugo (2017) and is based on the findings of Ruscio (2010). According to the findings of this study it is concluded that the coffee roasting process is in fact a lag-dominant first-order system with dead-time, and therefore shows pure integrator behaviour when step-changes are induced later than 90 seconds after the commencement of the roast. Ruscio (2010) states that such processes may be approximated using a pure integrating transfer function.

After analysing the controllability of the coffee roasting process within a rotating drum, batch roaster, It was concluded that there is an initial time frame during the roasting process that is deemed uncontrollable by the definitions stated in Faanes (2003), Skogestad & Postlethwaite (2001) and Ziegler & Nichols (1943). This initial uncontrollable time frame is due to the evaporative cooling occurring during the drying phase of the process, and corresponds to the turning point of the roast profile, and the point of intercept on the x-axis of the derivative of the roast profile plots. These observations indicate that the sooner the process is forced past the turning point of the roast profile, the sooner it will stabilise and be controllable.

The use of the Cohen and Coon method was deemed insufficient and showed overestimated controller parameters, which agrees well with an observation by Seborg et al. (2011:278) that suggests to a control designer not to use this specific design method. The ITAE design method resulted in slightly improved parameter values measured in the performance of the controller, but the IMC-based parameters proved to be far superior compared to the other methods, especially in the handling of systems with large time delays. Both the Cohen and Coon and the ITAE method are however recommended for processes with little to no time delays, as these controllers showed fast response times and very little overshoot under the specified conditions, and were able to correct the errors under these circumstances with ease.

Further analyses of the controller parameters indicated that the integral action resulting from the IMC-based design method was within stability bounds for every IMC-based controller, even though the proportional parameter was overestimated in many instances, which led to the controller’s instability.
Finally the use of PI controllers showed better overall results than the utilisation of PID controllers for this pure integrating system, validating the decision flowchart stated in Svrcek et al. (2000:114), which would lead a designer to the implementation of a PI controller based on the criteria given. The best performing controller was a PI controller designed using the IMC method by substituting the overall average process gradient, and dead-time values into the stipulated IMC functions. After these original parameters were determined, the controller was fine-tuned, which yielded better controller performance.

The final control strategy was proven successful and well able to correct the error and to stabilise the process with a dead-time of 20 seconds. It consists of a SISO controller that controls the derivative of the roast profile by manipulating the LPG flow rate to the system in order to replicate a roast profile of a desired batch of coffee.

### 6.2 Recommendations

The implementation of a threshold controller, as discussed in Section 5.3.3, should be studied in combination with the validated SISO control strategy in order to assess the optimisation of the suggested control strategy.

In order to shorten the uncontrollable time frame of the roasting process, it is also recommended to study the influence of more heat added during the initial time frame (< 90 seconds) of the process, in an attempt to drive off moisture sooner, which will lead to the process stabilising sooner.

Further research in the application of neural-networks in combination with Fuzzy-Logic is recommended in order to fully automate the control of coffee roasting within batch, rotating drum roasters.
6.3 Chapter references


APPENDIX A – Simulink® models
A.1 Schwartzberg (2002) Simulink® model layout

Figure A.1: Schwartzberg (2002) model as presented in Simulink®
Figure A.2: Heat capacity of air as modelled in Simulink® Schwartzberg (2002) model
Figure A.3: Thermal conductivity of air as modelled in Simulink® Schwartzberg (2002) model
Figure A.4: Viscosity of air as modelled in Simulink® Schwartzberg (2002) model
Figure A.5: Density of air as modelled in Simulink® Schwartzberg (2002) model

Figure A.6: Moisture loss during roasting as modelled in Simulink® Schwartzberg (2002) model
Figure A.7: Exothermic roasting reactions as modelled in Simulink® Schwartzberg (2002) model
Figure A.8: Air temperature balance as modelled in Simulink® Schwartzberg (2002) model
Figure A.9: Calculation of $he$ within air temperature balance as modelled in Simulink®

Figure A.10: Calculation of $Bi$ within air temperature balance as modelled in Simulink®
Figure A.11: Calculation of $h$ within air temperature balance as modelled in Simulink®

Figure A.12: Calculation of $Nu$ within $h$ calculation as modelled in Simulink®
Figure A.13: Calculation of $Re$ within $Nu$ calculation as modelled in Simulink®

Figure A.14: Calculation of $Pr$ within $Nu$ calculation as modelled in Simulink®
Figure A.15: Bean temperature calculation as modelled in Simulink®
Figure A.16: Calculation of $C_{pb}$ within bean temperature calculations as modelled in Simulink®

Figure A.17: Roast profile calculation as modelled in Simulink®

### A.2 PI/PID controller: Simulink® layout

The Simulink® model laid out in Section A.1 of this report represented the Simulink® model that acted as the subsystem that generated the set point value for the controller. This controller layout within Simulink® is represented in the following section.
Figure A.18 illustrates the Simulink® model setup used during the development and testing of the constant parameter controllers. The controllers with the gain scheduling curves were developed and tested using the Simulink® setup shown in Figure A.19.
The values fed to the controller block within Simulink® for the $P$, $I$ and $D$ values are defined in Section 5.2.1 of this report. For the constant controller parameter cases the values fed, were mere constant blocks available in Simulink®. For the gain scheduling curves, these values were modelled as Signal Builder blocks, as shown in Figure A.19. The calculated gain scheduling curves were then inserted into the Signal Builder block as illustrated in Figure A.20. This example illustrates the gain scheduling curve for the $I$ component of the PID controller block, utilising the ITAE method.

A Transport Delay block was used to model the dead-time present during the process, and Step blocks were used to model process disturbances in order to validate the controller performance. The roast profile to be controlled was modelled using a transfer function, as discussed in Section 4.1.3 and given in Equation 4.2.
The PID controller form settings were set to ideal, and the source was set to external in order to accommodate the gain scheduling parameters. This must not be confused with the initial condition source, which was set to internal. As explained in detail in Section 5.2.1, and shown in Figure A.21.

Figure A.20: Example of gain scheduling curve inserted in Signal Builder block within Simulink®

![Signal Builder block](image)
Figure A.21: PID controller block settings as modelled in Simulink®
APPENDIX B – Data used from literature
B.1 ITAE performance index used from literature

The parameters, $A$ and $B$, used when the ITAE method was utilised, are taken from Seborg et al. (1989:284) and stated below. This data was used in order to derive Equation 4.13 to Equation 4.17.

<table>
<thead>
<tr>
<th>Type of input</th>
<th>Type of controller</th>
<th>Mode</th>
<th>$A$</th>
<th>$B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load</td>
<td>PI</td>
<td>P</td>
<td>0.859</td>
<td>-0.977</td>
</tr>
<tr>
<td></td>
<td></td>
<td>I</td>
<td>0.674</td>
<td>-0.680</td>
</tr>
<tr>
<td>Load</td>
<td>PID</td>
<td>P</td>
<td>1.357</td>
<td>-0.947</td>
</tr>
<tr>
<td></td>
<td></td>
<td>I</td>
<td>0.842</td>
<td>-0.738</td>
</tr>
<tr>
<td></td>
<td></td>
<td>D</td>
<td>0.381</td>
<td>0.995</td>
</tr>
<tr>
<td>Set point</td>
<td>PI</td>
<td>P</td>
<td>0.586</td>
<td>-0.916</td>
</tr>
<tr>
<td></td>
<td></td>
<td>I</td>
<td>1.03*</td>
<td>-0.165*</td>
</tr>
<tr>
<td>Set point</td>
<td>PID</td>
<td>P</td>
<td>0.965</td>
<td>-0.85</td>
</tr>
<tr>
<td></td>
<td></td>
<td>I</td>
<td>0.796*</td>
<td>-0.1465*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>D</td>
<td>0.308</td>
<td>0.929</td>
</tr>
</tbody>
</table>

The standard ITAE design relation according to Seborg et al. (1989:284) is given as:

$$Y = A(\theta/\tau)^B$$  \hspace{1cm} \text{Equation B.1}

Where the proportional mode would yield a $Y$ value of:

$$Y = K_pK_c$$  \hspace{1cm} \text{Equation B.2}

The integral mode a $Y$ value of:

$$Y = \tau/\tau_I$$  \hspace{1cm} \text{Equation B.3}

The derivative mode yields:

$$Y = \tau_D/\tau$$  \hspace{1cm} \text{Equation B.4}

*Note that in the case of a set point input the standard design relation changes to:

$$Y = A + B(\theta/\tau)$$  \hspace{1cm} \text{Equation B.5}
B.2 Experimental data used from literature

The experimental data used from literature was obtained from the study conducted by Vosloo (2016) in order to determine the baseline roast profile at various prime temperatures.

B.2.1 Baseline roast profile data ($T_{roast}$)

Raw experimental data obtained from the study conducted by Vosloo (2016) was processed in order to construct average baseline roast profiles at prime temperatures of 170, 180, 200, 210 and 230 °C. These baseline roast profiles are illustrated in this section.

B.2.1.1 170 °C Baseline roast profile

Three various roasts, each conducted at the same prime temperature of 170 °C are portrayed in Figure B.1 (a). These roast profiles were averaged in order to obtain the overall average roast profile depicted in Figure B.1 (b). This exact methodology was used in order to construct the baseline roast profiles at all the other prime temperatures.

Figure B.1: Baseline roast profile at 170 °C; a) Roast profiles used to determine average baseline roast profile, b) Average baseline roast profile at 170 °C
**B.2.1.2 180 °C Baseline roast profile**

![Graph](image1)

Figure B.2: Baseline roast profile at 180 °C; a) Roast profiles used to determine average baseline roast profile, b) Average baseline roast profile at 180 °C

**B.2.1.3 200 °C Baseline roast profile**

![Graph](image2)

Figure B.3: Baseline roast profile at 200 °C; a) Roast profiles used to determine average baseline roast profile, b) Average baseline roast profile at 200 °C
B.2.1.4 210 °C Baseline roast profile

Figure B.4: Baseline roast profile at 210 °C; a) Roast profiles used to determine average baseline roast profile, b) Average baseline roast profile at 210 °C

Figure B.5: Baseline roast profile at 230 °C; a) Roast profiles used to determine average baseline roast profile, b) Average baseline roast profile at 230 °C
B.2.2 Baseline derivative of roast profile data ($\dot{T}_{roast}$)

The baseline roast profiles depicted in the previous section were used in order to determine the derivative of the roast profile curve, which yields the rate of temperature change, or rather the $\dot{T}_{roast}$ curve baseline which was used in order to control the coffee roasting process. The $\dot{T}_{roast}$ baseline curves were compared to the simulation and experimental results in order to determine the deviation from unregulated roast conditions, which are represented by the baseline roast curves. These following $\dot{T}_{roast}$ curves were determining the gradient of the roast profile using Excel®.

\[ \begin{array}{c}
\begin{array}{c}
\text{Figure B.6: Baseline of derivative of roast profile at 170 °C prime temperature}
\end{array}
\end{array} \]

\[ \begin{array}{c}
\begin{array}{c}
\text{Figure B.7: Baseline of derivative of roast profile at 180 °C prime temperature}
\end{array}
\end{array} \]
Figure B.8: Baseline of derivative of roast profile at 200 °C prime temperature

Figure B.9: Baseline of derivative of roast profile at 210 °C prime temperature
Figure B.10: Baseline of derivative of roast profile at 230 °C prime temperature
C.1 Experimental procedure

- Prime (preheat) the Genio 6 Artisan roaster, giving specified initial roast temperature (170, 180, 200, 210 or 230°C) as Prime set point temperature. The “Prime” button on the control panel is pressed to set the Prime phase into action.
- Press “Roast” on control panel to switch to manual control of process.
- Insert green coffee beans through hopper, tabulate starting time of roast.
- Leave roast to proceed through various phases. Induce step in LPG valve-opening at specified time intervals (90, 180, 240 or 300s). This is done by turning the “Gas” knob to the desired value indicated on the dial.
- Listen for distinct popping sound and tabulated time of first crack. This may occur before the step change is induced.
- Leave roast to proceed to a final roasting temperature of 200°C.
- Open latch allowing roasted coffee beans to fall into cooling bin, press “Cool” on control panel initiating rotation of cooling bin rakes and starting cooling fan.

C.2 Step-change experimental data

The following section depicts the results obtained for the induced step change experiments. A single step-change was induced in the LPG valve opening, which regulates the heat flow to the process. The LPG valve opening was increased from 50 % to 100 % at various time intervals in each experiment. In order to construct Figure C.1 to Figure C.5, from which the average dead-time and average gradient of the process was determined, the baseline roasts depicted in the previous section was used as the reference values. The reference values was taken to be the zero-point, the deviation from this reference value when a step change was induced is depicted in these figures.

The simulations were constructed by using the environmental temperature from the experiment to be simulated as the initial input to the model, in order to simulate each experimental run accurately.

The experimental data was gathered in Excel® and the simulation results from Simulink® was exported to Excel®, and then constructed therein. The average gradient of the experimental results, as well as the simulation results was determined in order to calculate an overall average gradient to be used within the transfer function of the process model. The dead-time was also calculated from these results, but only utilising the experimental data. The time between the induced step-change and the moment the gradient has a visible increase was taken as the dead-time value for every experimental run.

The final determined values are summarised in Section C.2.2.
APPENDIX C – Experimental data

C.2.1 Induced step-changes: Experimental and simulation results

Figure C.1: Effect of step change on $\dot{\nu}_{\text{roast}}$ at a 170 °C prime temperature; Experimental results ($\bullet$), Simulation results (---), (a) 90 s step, (b) 240 s step, (c) 300 s step
Figure C.2: Effect of step change on $\dot{T}_{\text{roast}}$ at a 180 °C prime temperature; Experimental results (●), Simulation results (—). (a) 90 s step, (b) 240 s step, (c) 300 s step
Figure C.3: Effect of step change on $\dot{t}_{\text{roast}}$ at a 200 °C prime temperature; Experimental results (●).

Simulation results (—), (a) 90 s step, (b) 240 s step, (c) 300 s step
Figure C.4: Effect of step change on $t_{roast}$ at a 210 °C prime temperature; Experimental results (●), Simulation results (—), (a) 90 s step, (b) 240 s step, (c) 300 s step
Figure C.5: Effect of step change on $\dot{t}_{\text{roast}}$ at a 230 °C prime temperature; Experimental results (●), Simulation results (—), (a) 90 s step, (b) 240 s step, (c) 300 s step
C.2.2 Summarised gradient and dead-time results

The experimentally determined dead-time values are given below.

Table C.1: Summarised dead-time values at various step-change times and prime temperatures

<table>
<thead>
<tr>
<th>Dead-time (θ) seconds</th>
<th>170°C</th>
<th>180°C</th>
<th>200°C</th>
<th>210°C</th>
<th>230°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time of step-change</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.5min</td>
<td>19 s</td>
<td>25 s</td>
<td>18 s</td>
<td>14 s</td>
<td>10 s</td>
</tr>
<tr>
<td>4min</td>
<td>28 s</td>
<td>15 s</td>
<td>27 s</td>
<td>18 s</td>
<td>14 s</td>
</tr>
<tr>
<td>5min</td>
<td>20 s</td>
<td>29 s</td>
<td>28 s</td>
<td>21 s</td>
<td>18 s</td>
</tr>
</tbody>
</table>

Table C.2: Summarised experimentally determined gradient values at various step-change times and prime temperatures

<table>
<thead>
<tr>
<th>Gradient (K) °C/s²</th>
<th>170°C</th>
<th>180°C</th>
<th>200°C</th>
<th>210°C</th>
<th>230°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time of step-change</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.5min</td>
<td>1,63E-04</td>
<td>3,04E-04</td>
<td>2,16E-04</td>
<td>3,67E-04</td>
<td>1,03E-04</td>
</tr>
<tr>
<td>4min</td>
<td>3,33E-04</td>
<td>3,57E-04</td>
<td>3,53E-04</td>
<td>3,26E-04</td>
<td>2,54E-04</td>
</tr>
<tr>
<td>5min</td>
<td>4,32E-04</td>
<td>3,76E-04</td>
<td>4,94E-04</td>
<td>3,58E-04</td>
<td>6,05E-04</td>
</tr>
</tbody>
</table>

Table C.3: Summarised simulation results for average gradient values at various step-change times and prime temperatures

<table>
<thead>
<tr>
<th>Gradient (K) °C/s²</th>
<th>170°C</th>
<th>180°C</th>
<th>200°C</th>
<th>210°C</th>
<th>230°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time of step-change</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>1.5min</td>
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<td>1,61E-04</td>
<td>2,07E-04</td>
<td>9,52E-05</td>
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<td>4min</td>
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<td>3,27E-04</td>
<td>3,83E-04</td>
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<tr>
<td>5min</td>
<td>3,30E-04</td>
<td>4,73E-04</td>
<td>4,82E-04</td>
<td>4,58E-04</td>
<td>4,08E-04</td>
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Table C.4: Summarised average gradients

<table>
<thead>
<tr>
<th>Average gradient value °C/s²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overall Average</td>
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<tr>
<td>Experimental Average</td>
</tr>
<tr>
<td>Simulation Average</td>
</tr>
</tbody>
</table>
C.2.3 Experimentally determined process gain prior to 90 seconds of roasting

The coffee roasting process was determined to be a lag-dominant first-order plus time delay process, which therefore shows first-order behaviour during the first 90 seconds of the roasting process. The process gain during these initial 90 seconds was determined and used within the Cohen and Coon and ITAE methods in order to determine the control parameters. The same experimental and simulation results as depicted in Section C.1.1 was used in the determination of the process gain during the initial 90 seconds of roasting. The data for all step-changes induced prior to 90 seconds was used.

The summarised results of the experimentally determined process gain ($K_p$) are given in Chapter 4.

![Figure C.6: Illustration of method used to determine process gain during initial 90 seconds of roasting](image)

The process gain was calculated using the following relation (Svrcek et al., 2000:160):

$$K_p = \frac{\Delta Out}{\Delta In} = \frac{\Delta CV}{\Delta MV}$$  \hspace{1cm} \text{Equation C.1}

Where $\Delta CV$ represents the change in $\dot{T}_{roast}$ that occurs for a certain induced change in LPG valve opening, represented by $\Delta MV$. $\Delta CV$ was calculated as illustrated by Figure C.6, and $\Delta MV$ was taken to 50 % in each case, as there was a change of 50 % in the LPG flow for each induced step-change.
C.2.4 Effect of step-change in LPG flow on environmental temperature

The simulations constructed in Simulink® used the effect observed in the environmental air temperature as input value to the model in order to model a step-change in the LPG flow to the system. The model does not have the ability to calculate the roast profile with a step-input as initial value.

The effect of a step-change of 50 – 100 % in the LPG flow on the environmental temperature is depicted in the following figures which show the experimental results of the experimental plan given in Chapter 3.

![Graph showing the effect of step-change in LPG flow on environmental temperature.](image)

*Figure C.7: Environmental air temperature response to step-change at 90 s in LPG flow: 170 °C (●), 180 °C (●), 200 °C (●), 210 °C (●), 230 °C (●)*
Figure C.9: Environmental air temperature response to step-change at 240 s in LPG flow; 170 °C (●), 180 °C (●), 200 °C (●), 210 °C (●), 230 °C (●).

Figure C.8: Environmental air temperature response to step-change at 300 s in LPG flow; 170 °C (●), 180 °C (●), 200 °C (●), 210 °C (●), 230 °C (●).
C.3 Influence of manipulated variables on roast profile

In Section 3.1 it is stated that the effect of step-change in the LPG flow to the system has a larger effect on the change in the roast profile than a step-change in the ID fan speed.

Three experiments were conducted in order to validate this statement. The first was a roasting experiment conducted at a prime temperature of 170 °C, with an induced step-change in the LPG flow of 50 % to 80 % at 180 seconds, denoted as LPG1. The second experiment was also conducted at a prime temperature of 170 °C, with a step-change induced in the fan speed, of 100 % of the maximum fan speed to 50 % of the maximum fan speed at 180 seconds, denoted as FAN1. The third experiment, also with a prime temperature of 170 °C, had a step-change induced in the LPG flow of 50 % to 20 % at 180 seconds. The last experiment is referred to as LPG2. The results of these three experiments are depicted below. Figure C.10 shows the deviation of the roast profile from unregulated roasting behaviour. It is clear that the step change in the ID fan of the system, though the step-change was larger than that of the LPG flow, had a smaller on the roast profile in both instances.

\[\text{Figure C.10: Response of roast profile to various step-changes induced at 180 s; LPG1 (●), LPG2 (●), FAN1 (★)}\]
APPENDIX D – Parameter determination data
D.1 Determined controller parameter summary

The control parameters determined using the IMC-based, Cohen and Coon and ITAE methods were calculated using the results obtained from the experimental and simulation data, stated in Appendix C.

This section depicts and summarises these parameters for the eighteen developed controllers. A summary of these various controllers are given in Chapter 5 in Table 5.1. The interpretation of the controller acronyms are also available in Table 5.1.

The constant controller parameters are tabulated, and the gain scheduling parameters are given separately.

D.1.1 Constant controller parameter summary

<table>
<thead>
<tr>
<th>Controller acronym</th>
<th>$K_c$</th>
<th>$\tau_I$</th>
<th>$\tau_D$</th>
<th>Equations used in determination</th>
</tr>
</thead>
<tbody>
<tr>
<td>IMC_PI_MaxC</td>
<td>221.895</td>
<td>69.600</td>
<td>---</td>
<td>Equation 2.31 to Equation 2.35</td>
</tr>
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<td>IMC_PID_MaxC</td>
<td>394.479</td>
<td>69.600</td>
<td>9.667</td>
<td></td>
</tr>
<tr>
<td>IMC_PI_AvgC</td>
<td>112.526</td>
<td>60.800</td>
<td>---</td>
<td></td>
</tr>
<tr>
<td>IMC_PID_AvgC</td>
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<td>60.800</td>
<td>8.444</td>
<td></td>
</tr>
<tr>
<td>CC_PI_MaxC</td>
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<td>26.400</td>
<td>---</td>
<td>Equation 2.36 to Equation 2.40</td>
</tr>
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<td>41.981</td>
<td>7.138</td>
<td></td>
</tr>
<tr>
<td>CC_PI_AvgC</td>
<td>685.581</td>
<td>23.062</td>
<td>---</td>
<td></td>
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<tr>
<td>CC_PID_AvgC</td>
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<td>36.673</td>
<td>6.236</td>
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<tr>
<td>ITAE_PI_MaxC</td>
<td>408.560</td>
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<td>---</td>
<td>Equation 4.13 to Equation 4.17</td>
</tr>
<tr>
<td>ITAE_PID_MaxC</td>
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<td>35.720</td>
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</tr>
<tr>
<td>ITAE_PI_AvgC</td>
<td>408.560</td>
<td>23.430</td>
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<tr>
<td>ITAE_PID_AvgC</td>
<td>672.799</td>
<td>31.203</td>
<td>6.242</td>
<td></td>
</tr>
</tbody>
</table>
D.1.2 Scheduling controller parameter summary

The scheduling curves are depicted graphically. They were inserted into Simulink® in this form by utilising a Signal Builder block available within Simulink®.

Figure D.1: IMC_PI_GS controller parameters; (a) $K_c$, (b) $\tau_i$
Figure D.2: IMC_PID_GS controller parameters; (a) $K_c$, (b) $\tau_I$, (c) $\tau_D$
Figure D.3: CC_PI_GS controller parameters; (a) $K_c$, (b) $\tau_i$. 
Figure D.4: CC_PID_GS controller parameters: (a) $K_c$, (b) $\tau_I$, (c) $\tau_D$
Figure D.5: ITAE_PI_GS controller parameters; (a) $K_c$, (b) $\tau_i$. 
Figure D.6: ITAE_PID_GS controller parameters: (a) $K_c$, (b) $\tau_I$, (c) $\tau_D$
D.2 Determination of process gain for RGA analysis

Seborg et al. (2011:64) states that a pure integrating system does not have a steady state gain in the normal sense of the term. The gradient of the pure integrating system is however a measure of the process sensitivity, and can therefore be used within the relative gain calculations of the RGA analysis (Hu et al., 2011a).

When calculating the relative gains of the RGA analysis, instead of determining the steady-state gain of the process when a step-change was induced, the gradient of the pure integrator was determined and used as a measure of the process gain in order to determine which manipulated variable has a larger effect on which controlled variable.

\[ K_{11}, K_{12}, K_{21} \text{ and } K_{22} \] were all calculated by using the slopes (gradients) of the various controlled variables resulting from an induced step-change in one of the manipulated variables.

The process responses when various step-changes were induced were not experimentally determined for the RGA analysis, but rather simulated using the Simulink® model of the roasting process depicted in Appendix A. In order to simulate the step-change in the LPG flow, the effect of this step-change visible in the environmental air was inserted into the model. A step change of 50 to 100 % in LPG valve opening was simulated when \( F_{gas} \) was varied, and a step-change was induced in the air flow to the system modelled in order to simulate the step-change in \( F_{air} \) of 43 to 50 kg.s\(^{-1}\). The results of these simulations are given in the following section.
In order to calculate $K_{11}$ and $K_{21}$ Equation 2.25 was used, but the change in the controlled variable was replaced with the gradient of the controlled variable resulting from the step-change, and the change in the manipulated variable was also replaced with the gradient of the environmental temperature. This in turn results in Equation 2.25 becoming:

$$K_{ij} = \frac{\partial C_i}{\partial t} \frac{\partial M_j}{\partial t}$$

Equation D.1

$K_{12}$ and $K_{22}$ were calculated in a similar fashion, with the exception of the change in the manipulated variable retaining its original form, due to the fact that the step-change in the flow of air through the system could be modelled in Simulink®. In this case Equation 2.25 becomes:

$$K_{ij} = \frac{\partial C_i}{\partial t} \Delta M_j$$

Equation D.2

### D.2.1 Simulation data used in calculation of $K_{ij}$

The raw simulation data used during these calculations are tabulated on the next pages.
Table D.2: Simulation results showing effect of step-change in air mass flow on both controlled variables

<table>
<thead>
<tr>
<th>Time</th>
<th>Roast Temp (T)</th>
<th>Derivative Roast profile (Tdot)</th>
<th>Environmental Temp</th>
<th>Air mass flow (G)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[s]</td>
<td>[°C]</td>
<td>[°C/s]</td>
<td>[°C]</td>
<td>[kg/s]</td>
</tr>
<tr>
<td>0</td>
<td>180</td>
<td>0</td>
<td>180</td>
<td>43</td>
</tr>
<tr>
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<td>43</td>
</tr>
<tr>
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<td>180</td>
<td>43</td>
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<td>43</td>
</tr>
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<td>180</td>
<td>43</td>
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</table>
Table D.3: Simulation results showing effect of step-change in LPG flow on both controlled variables

<table>
<thead>
<tr>
<th>Time [s]</th>
<th>Roast Temp (T) [°C]</th>
<th>Derivative Roast profile [°C/s]</th>
<th>Environmental Temp [°C]</th>
<th>Air mass flow (G) [kg/s]</th>
</tr>
</thead>
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<td>0</td>
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D.3 IMC Controller responses to various dead-time values

This section graphically depicts the IMC_PI_AvgC, IMC_PI_GS and IMC_PID_GS controllers’ performance for various dead-time values. The same simulation procedure was followed as stated in Chapter 5. For each of the following results the step-change occurred at 180 seconds after commencement of the roasting process and had a magnitude of 0.2 °C.s⁻².

![Graphs showing IMC_PI_AvgC controller performances for different dead-times](image)

*Figure D.8: IMC_PI_AvgC controller performances; Set point (—), Process output (—)*
Figure D.9: IMC.PI.GS controller performances; Set point (—), Process output (—)
Figure D.10: IMC_PID_GS controller performances; Set point (—), Process output (—)
E.1 Derivation of relative gain used in RGA analysis

One of the final relations used in the determining of the RGA, was the value of $\lambda_{11}$ given by Equation 2.24. The derivation taken from Seborg et al. (1989:455-456) of this parameter follows:

Consider a 2 × 2 process where a steady-state model is available and can be expressed as:

$$C_1 = K_{11}M_1 + K_{12}M_2$$  \hspace{1cm} \text{Equation E.1}

$$C_2 = K_{21}M_1 + K_{22}M_2$$  \hspace{1cm} \text{Equation E.2}

And $K_{ij}$ denotes the steady-state gain, or in this case the gradient of $C_i$ caused by the step-change in $M_j$.

It follows from Equation E.1 that:

$$\left( \frac{\partial C_1}{\partial M_1} \right)_{M_2} = K_{11}$$  \hspace{1cm} \text{Equation E.3}

First $M_2$ must be eliminated before $K_{11}$ from Equation E.3 will be calculated. This is done by solving Equation E.2 for $M_2$ with $C_2$ held constant at its nominal value, $C_2 = 0$, which yields:

$$M_2 = -\frac{K_{21}}{K_{22}}M_1$$  \hspace{1cm} \text{Equation E.4}

Substituting this value into Equation D.1 yields:

$$C_1 = K_{11} \left(1 - \frac{K_{12}K_{21}}{K_{11}K_{22}}\right) M_1$$  \hspace{1cm} \text{Equation E.5}

It follows that:

$$\left( \frac{\partial C_1}{\partial M_1} \right)_{C_2} = K_{11} \left(1 - \frac{K_{12}K_{21}}{K_{11}K_{22}}\right)$$  \hspace{1cm} \text{Equation E.6}

Now substituting Equation E.6 and Equation E.3 into Equation 2.21 gives a simplified expression for the relative gain in the form of Equation 2.24.
E.2 Derivation of stability boundary functions

The derivation of the PI and PID controller transfer functions are stated below in order to determine the stability boundaries. These derivation were done for a pure integrating process with a transfer function as stated in Equation 2.27.

The Euler identity used in the conjecture to estimate the dead-time element $e^{-\theta s}$ as stated in Seborg et al., (2011:254) is given as:

$$G(j\omega) = \cos \omega \theta - j \sin \omega \theta$$  \hspace{1cm} \text{Equation E.7}

E.2.1 PI Controller transfer function derivation

The open loop transfer function of a PI controller yields:

$$G_{OL}(s) = G_p G_v G_c G_m$$

$$G_{OL}(s) = K_v K_m G_p G_c$$

Let $K' = KK_v K_m$

$$G_{OL}(s) = K' K_c \left( e^{-\theta s} \right) \left( 1 + \frac{1}{\tau_i s} \right)$$

Transforming the transfer function from the $(s)$ domain to $(j\omega)$ domain using the Euler identity given in Equation E.7, after simplifying, the above equation yields:

$$G_{OL}(s) = K' K_c \left[ \frac{\cos \omega \theta - j \sin \omega \theta}{j\omega} + \left( \frac{\cos \omega \theta - j \sin \omega \theta}{-\tau_i \omega^2} \right) \right]$$

$$G_{OL}(s) = K' K_c \left[ \frac{j\tau_i \omega \cos \omega \theta + \tau_i \omega \sin \omega \theta}{-\tau_i \omega^2} + \left( \frac{\cos \omega \theta - j \sin \omega \theta}{-\tau_i \omega^2} \right) \right]$$

$$G_{OL}(s) = K' K_c \left[ \frac{j\tau_i \omega \cos \omega \theta + \tau_i \omega \sin \omega \theta + \cos \omega \theta - j \sin \omega \theta}{-\tau_i \omega^2} \right]$$

$$Re = K' K_c \left( \frac{\tau_i \omega \sin \omega \theta + \cos \omega \theta}{-\tau_i \omega^2} \right)$$

$$Im = K' K_c \left( \frac{\tau_i \omega \cos \omega \theta - \sin \omega \theta}{-\tau_i \omega^2} \right)$$

$$Amplitude Ratio = AR = \sqrt{(Re)^2 + (Im)^2}$$

$$Phase = \phi = \tan^{-1} \left( \frac{Im}{Re} \right)$$
These final functions were inserted in Excel® in order to plot the Bode diagrams of the various controllers.

E.2.2 PID Controller transfer function derivation

The open loop transfer function of a PID controller yields:

\[ G_{OL}(s) = G_p G_v G_c G_m \]
\[ G_{OL}(s) = K_v K_m G_p G_c \]

Let \( K' = K K_v K_m \)

\[ G_{OL}(s) = K'K_c \left( \frac{e^{-\theta s}}{s} \right) \left( 1 + \frac{1}{\tau_i s} - \tau_D s \right) \]

Transforming the transfer function from the \((s)\) domain to \((j\omega)\) domain using the Euler identity given in Equation E.7, after simplifying, the above equation yields:

\[ G_{OL}(j\omega) = K'K_c \left[ \frac{\cos \omega \theta - j \sin \omega \theta}{j \omega} \right] + \left( \frac{\cos \omega \theta - j \sin \omega \theta}{-\tau_i \omega^2} \right) - \left( \frac{\tau_D j \omega (\cos \omega \theta - j \sin \omega \theta)}{j \omega} \right) \]

\[ G_{OL}(j\omega) = K'K_c \left( j \frac{\tau_i \omega \cos \omega \theta + \tau_i \omega \sin \omega \theta + \cos \omega \theta - j \sin \omega \theta + \tau_i \tau_D \omega^2 (\cos \omega \theta - j \sin \omega \theta)}{-\tau_i \omega^2} \right) \]

\[ Re = K'K_c \left( \frac{\tau_i \omega \sin \omega \theta + \cos \omega \theta + \tau_i \tau_D \omega^2 \cos \omega \theta}{-\tau_i \omega^2} \right) \]

\[ Im = K'K_c \left( \frac{\tau_i \omega \cos \omega \theta - \sin \omega \theta - \tau_i \tau_D \omega^2 \sin \omega \theta}{-\tau_i \omega^2} \right) \]

\[ Amplitude Ratio = AR = \sqrt{(Re)^2 + (Im)^2} \]

\[ Phase = \phi = \tan^{-1} \left( \frac{Im}{Re} \right) \]