NEW MODELING OF GALACTIC PROTON MODULATION DURING THE MINIMUM OF SOLAR CYCLE 23/24

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ABSTRACT

During the recent prolonged solar minimum of cycle 23/24, the PAMELA detector measured 27-day averaged Galactic proton energy spectra over the energy range that is important for solar modulation. By comparing these spectra to computed spectra from a three-dimensional model that contains all of the important heliospheric modulation processes, the recent minimum can be studied in detail from a modulation perspective. This was done by setting up a realistic heliosphere in the model, and reproducing a representative selection of seven intermittent PAMELA spectra, separated by approximately six months, from 2006 July to 2009 December. Additionally, a new very local interstellar proton spectrum was constructed using measurements below 600 MeV from Voyager 1, taken beyond the heliopause, combined with PAMELA and AMS-02 measurements above 30 GeV at the Earth. As a result of the extreme minimum modulation conditions that governed the recent solar minimum, the highest ever Galactic cosmic ray spectrum at Earth was observed by PAMELA at the end of 2009. It was found that, apart from the self-consistent changes in the heliospheric current sheet and the heliospheric magnetic field over time, additional increases in the mean free paths during this period were required below ~4 GV in order to reproduce the intensities observed by PAMELA.

Key words: cosmic rays – stars: activity – Sun: heliosphere

1. INTRODUCTION

With the availability of accurate proton energy spectra measured by PAMELA during the recent unusual minimum of 2009 (Adriani et al. 2013), this solar minimum period can be studied in detail with regard to how the proton energy spectrum developed. Potgieter et al. (2014) conducted a similar study, aiming to uncover the extent to which various modulation processes contributed to the high intensities observed by PAMELA, as well as to prove the interplay that exists among these processes. These authors also studied the modulation of Galactic electrons between 2006 and 2009, and concluded that even though the solar minimum of cycle 23/24 seemed to have been diffusion-dominated, all modulation processes still played important roles, including gradient, curvature, and current-sheet drifts.

A major development that influences modeling results are the recent Galactic cosmic-ray (GCR) observations made by Voyager 1 in the very local interstellar medium (LISM) after it crossed the heliopause (HP) in 2012 August (Stone et al. 2013). Since these observations are largely unaffected by solar modulation, they enable us to adjust the proton very local interstellar spectrum (LIS) between 3 and 600 MeV accordingly. It has also been shown that the amount of modulation experienced by GCRs above 30–50 GeV becomes negligible, so that PAMELA and AMS-02 observations at the Earth in this energy range can be considered as accurate intensity levels for the very LISM.

A newly constructed very LIS for protons, based on the above mentioned observations, is presented here, and used as an input spectrum for modeling solar modulation. The modulation model used here also utilizes a modification to the heliospheric magnetic field (HMF) as proposed by Smith & Bieber (1991).

This study aims to broaden the work of Potgieter et al. (2014) by reproducing PAMELA spectra during the recent solar minimum of cycle 23/24 over smaller intervals, while also taking into account Voyager measurements at different radial distances and spatial gradients in the inner heliosphere. These improvements resulted in quantitative changes to the diffusion coefficients (DCs) compared with Potgieter et al. (2014).

The results of a detailed study of the Voyager radial profiles and the radial and latitudinal gradients between PAMELA and the position of Ulysses will be presented in an upcoming publication.

2. A NEW LOCAL INTERSTELLAR PROTON SPECTRUM

Many attempts have been made to obtain reliable LIS estimates for protons in the energy range important for solar modulation. Only with the recent availability of in situ measurements by Voyager 1 from beyond the HP this has become possible.

Figure 1 gives the proton very LIS used in this study. Voyager 1 observations (diamonds) below ~600 MeV were used to set the absolute value of the spectrum beyond the HP (Stone et al. 2013; Webber & McDonald 2013), while PAMELA and AMS-02 measurements (Adriani et al. 2013; Aguilar et al. 2015) were used to normalize the very LIS above 30 GeV, where solar modulation is considered negligible (shaded band). GALPROP solutions (see, e.g., Moskalenko et al. 2002) were used as a guide between 600 MeV and 30 GeV. The top and bottom panels of Figure 1 give the differential intensity and corresponding spectral index, respectively.

The spectral index from the top panel of Figure 1 remains mostly constant at ~2.78 between 30 and 50 GeV, and corresponds to that reported for PAMELA and AMS-02. Below 10 MeV the shape levels off to a fairly steady index of 0.12. The difference between the LIS and the PAMELA and AMS-02 observations below 30 GeV is indicative of solar modulation.
This very LIS is given by

\[ j_{\text{LIS}} = 2.70 \frac{E^{1.12}}{\beta^2} \left( \frac{E + 0.67}{1.67} \right)^{-3.93}, \]

where \( E \) is the kinetic energy in GeV, \( \beta = v/c \) the ratio of particle speed relative to the speed of light, and \( j_{\text{LIS}} = P^2 f \) is the differential intensity given in units of particles \( \text{m}^{-2} \text{s}^{-1} \text{sr}^{-1} \text{MeV}^{-1} \), with \( P \) as the rigidity in GV and \( f \) as the GCR distribution function.

In the model, the very LIS is specified at the HP, taken at 122 AU. Modulation beyond the HP is not considered here; see, e.g., Strauss et al. (2013) and Luo et al. (2015).

3. THE PAMELA PROTON SPECTRA

The top panel of Figure 2 gives an overview of the proton energy spectra measured by PAMELA, averaged over Carrington rotations, from 2006 July to the end of 2009 (Adriani et al. 2013). The bottom panel gives the intensity ratios relative to 2006 July. The change in color represents the development with time, as indicated by the colorbar, with 2006 July given in blue and the beginning of 2010 January given in red.

As expected, the lowest energy GCRs, being the most responsive to changes in modulation conditions, have undergone the largest increase throughout this minimum period—by a factor of \( \sim 3 \) around 100 MeV. At higher energies, this increase becomes less pronounced, with intensity variations below 20% for GCRs above 10 GeV, from the bottom panel of Figure 2. The assumption that GCR modulation can be neglected above 30 GeV is supported by the fact that ratios in this energy range show very little to no observable changes; see also Strauss & Potgieter (2014a).

As a result of solar activity reaching extremely low levels during the recent prolonged minimum of solar cycle 23/24 (e.g., Mewaldt et al. 2010; Kane 2011) PAMELA measured the highest ever GCR spectrum at Earth at the end of 2009. This spectrum is shown in Figure 3 by the solid blue circles, and clearly exceeds proton spectra from other experiments taken at different times in the solar cycle (see the legend). PAMELA observations also show a clear consensus with other experiments with regard to normalization above 30 GeV.

The blue, orange, and red bands in Figure 3 indicate what are considered to be minimum, moderate, and maximum modulation regimes, respectively, classified according to the heliospheric current-sheet (HCS) tilt angles, which is a very good proxy for solar activity and the subsequent GCR modulation. Minimum modulation usually occurs for tilt angles below \( \sim 15^{\circ} \), while maximum modulation occurs for tilt angles larger than \( \sim 50^{\circ} \), with moderate modulation in between. See, e.g., Strauss & Potgieter (2014b) for a comparison of \( A < 0 \) and \( A > 0 \) spectra observed during solar minimum activity.

The primary objective of this work is to reproduce intermittent PAMELA proton spectra measured between 2006 July and 2009 December, using a three-dimensional (3D) modulation model, and utilizing what is known about...
modulation conditions in the heliosphere. A representative selection of 27-day averaged PAMELA proton spectra, taken at the end of each semester, is shown in Figure 4. The time-periods of these spectra are given in the legend, and are from hereon referred to as the 2006e, 2007m, 2007e, 2008m, 2008e, 2009m, 2009e spectra, where the "m" and "e" suffixes denote the middle and end of each year, respectively.

4. MODELING THE PAMELA PROTON SPECTRA

A full 3D modulation model is used to compute differential intensities of GCR protons at the Earth, and is based on the numerical solution of the heliospheric transport equation from Parker (1965):

$$\frac{\partial f}{\partial t} = -(V + \langle v_D \rangle) \cdot \nabla f + \nabla \cdot (K_v \cdot \nabla f)$$

$$+ \frac{1}{3} (\nabla \cdot V) \frac{\partial f}{\partial \ln P},$$

with $f$ as the GCR distribution function, and $t$ as the time, where we study the steady-state case $\partial f/\partial t = 0$, for modulation during solar minimum when modulation parameters change gradually. The terms on the right-hand side, respectively, represent convection, with $V$ as the solar wind (SW) velocity; averaged particle drift velocity $\langle v_D \rangle$ caused by gradients, curvatures, and HCS drifts in the global HMF; diffusion, with $K_v$ as the symmetric diffusion tensor; adiabatic cooling, with $P$ as the rigidity.

The numerical model used in this study is further described in detail by Potgieter et al. (2014, 2015). The modulation parameters as modeled are discussed below. For a comprehensive review of the underlying theory, see Potgieter (2013).

4.1. Calculating the Intrinsic Parameters

When working with a steady-state modulation model, it is often a challenge to determine representative values for time-varying modulation parameters. The HCS tilt angle and the HMF at Earth changed pronouncedly over a four-year span prior to 2009. These time-dependent changes are accounted for by setting up realistic modulation conditions in the model that coincide with the 27-day averaged spectra given in Figure 4. Moving averages were used to approximate these conditions at the times when the selected spectra were observed.

Figure 5 gives the HCS tilt angle and HMF at Earth (top and bottom panels, respectively), along with moving averages (red lines and circles). The calculation of these averages is based on the time it takes for the tilt angles and the frozen-in HMF to travel from the Sun to the HP, as they are carried outward by the SW. These propagation times serve as a window over which preceding averages are calculated. The HCS is mostly confined to the ecliptic region during moderate to minimum solar activity (within an ~30° latitudinal extent) and, therefore, remains in the slow SW region. Knowing the averaged SW speed in the slow SW region (~430 km s$^{-1}$ upwind of the termination shock, TS), the propagation time for the tilt angle is calculated to be ~16 months. For the HMF, both the slow and fast (750 km s$^{-1}$) SW regions are taken into account by means of a weighted moving average, where the weights are determined by the volume occupied by the different SW regions. Following this approach, the HMF’s propagation time calculates to ~10 months.
progressively softer from 2006 to 2009, reaching an intensity of $2.7 \times 10^{-2}$ particles m$^{-2}$ s$^{-1}$ sr$^{-1}$ MeV$^{-1}$ at the end of 2009, with an accompanying shift of the spectrum peak down to 270 MeV. Consequent predictions of intensity levels can be made for energies below 80 MeV, where PAMELA measurements are unavailable. At 10 MeV, 2006e intensities are estimated at $\sim 0.09$ particles m$^{-2}$ s$^{-1}$ sr$^{-1}$ MeV$^{-1}$, and increased to $\sim 0.3$ particles m$^{-2}$ s$^{-1}$ sr$^{-1}$ MeV$^{-1}$ at the end of 2009.

### 4.3. Intensity Ratios

Figure 8 shows the ratios of the consecutive reproduced spectra from Figure 7 relative to the 2006e reference spectrum in Figure 6, and gives a quantitative measure of the total variability. Good agreement between measurements and computed spectra is reflected in the fact that the computed spectra adheres to the constraints of the statistical errors from the observations.

At 10 MeV it is estimated that intensities in 2009e increased by a factor of 3.3 relative to 2006e. The largest semesterly increase of $\sim 65\%$ occurred between 2008e and 2009m, likely due to the recovery of GCRs following the transient decrease during the first semester of 2008. Conversely, the sudden increase in tilt angle during 2008, and the corresponding solar activity, temporarily suppressed intensities, resulting in almost identical spectra for 2007e (green) and 2008m (brown). Since the HMF continued to decrease during 2008 (bottom panel of Figure 5), this transient effect experienced by GCRs is linked to current-sheet drifts and supports the presence of such a modulation mechanism during the recent solar minimum.

Figure 9 shows the modulation reduction factor (MRF) of the reproduced spectra, calculated by taking the ratio of the modulated spectra relative to the LIS. At high energies, the MRF approaches unity, indicating how the amount of modulation decreases with increasing energy. It follows that GCR protons experienced less than $\sim 10\%$ solar modulation above 30 GeV, similar to what was found by Strauss & Potgieter (2014a), who used a model based on stochastic differential equations (SDEs) in an independent study. Measurements above 30–50 GeV should therefore reflect astrophysical processes.

Energy ranges where the MRF is larger than 0.9, 0.5, and 0.1 are indicated by the gray shaded bands in Figure 9. Evidently, below 30 GeV, solar modulation effects become increasingly dominant. At the lower end of the spectrum (around 100 MeV), the MRF is already at $\sim 0.03$.

### 4.4. Proton Intensity Over Time

For a qualitative and quantitative understanding of the temporal development of the proton spectrum, Figure 10 shows the observed PAMELA intensities (solid lines) with statistical errors (shaded bands) in the 500 MeV–3 GeV energy range from mid-2006 to the end of 2009, as well as the model intensities at the same energies (dashed lines and filled circles), with the different energy ranges color-coded. Time-dependent modulation diminishes at higher energies, as seen in the comparatively smaller increases and variations toward 3 GeV.

During 2007, intensities increased gradually. In 2008, however, as a result of the sudden increase in the HCS tilt angle, proton intensities decreased notably across all energies and started recovering in the middle of 2008. During 2009,
intensities increased more rapidly than during the previous two years, because of the continued decrease in solar activity.

Figure 11 shows the normalized PAMELA observations relative to 2006e (top panel). Apart from including time-dependent changes in \( \alpha \) and \( B_r \) for reproducing the selected PAMELA spectra (Table 1), additional increases in the DCs were required below \( \sim 4 \) GV in order to reach measured intensity levels (middle panel). When the latter increases were excluded in the model, intensities were found to be \( \sim 75\% \) smaller than PAMELA observations at 100 MeV (bottom panel). See also Potgieter et al. (2014).

5. RIGIDITY AND SPATIAL DEPENDENCE OF THE DCS

As the heliosphere approached solar minimum conditions, it can be inferred that the HMF became more structured in the years leading up to 2009 (e.g., McComas et al. 2008), which effectively reduced the amount of turbulence in the heliosphere and increased particle mean free paths (MFPs). These increases, combined with drifts, are expected to be responsible for the intensity increase observed across the greater part of the proton spectrum between 2006 and 2009. The numerical solutions given in Figures 6 and 7 were obtained by using a phenomenological diffusion approach that approximates QLT (e.g., Potgieter 2000; Shalchi 2009), while still adhering to constraints from recent studies.

Figure 12 shows the rigidity dependence of the parallel \( (\lambda_0) \) and perpendicular \( (\lambda_b) \) MFPs, and the drift scale \( (\lambda_d) \) obtained after reproducing the 2006e to 2009e spectra. With the DCs related to the MFPs by \( \kappa = \lambda (\nu/3) \), with \( \nu \) as the particle speed, the equation for diffusion parallel to the average background HMF is given by

\[
\kappa || = \kappa_{||0} \beta F(r, \theta, \phi) G(P),
\]

with

\[
F(r, \theta, \phi) = \frac{B_0}{B},
\]

and

\[
G(P) = \frac{P}{P_0} \left( \frac{P}{P_0} \right)^c + \frac{P_k}{P_0} \left[ 1 + \frac{P_k}{P_0} \right]^{b-a},
\]

where \( \kappa_{||0} \) is a scaling constant in units of \( \text{cm}^2 \text{s}^{-1} \), \( B \) is the magnetic field magnitude in nT, with \( \beta \) and \( P \) as discussed before. The variables \( a, b, c, \) and \( P_l \) determine the shape of the MFP rigidity dependence, which has the functional form of two combined power laws. The constants \( B_0 = 1 \) nT and \( P_0 = 1 \) GV are to keep \( F \) and \( G \) dimensionless. Most of the parameters in Equation (5) are given in Table 2.

The expression for the HMF, as modified by Smith & Bieber (1991) is

\[
B = B_a \left( \frac{r_0}{r} \right)^2 \sqrt{1 + \tan \psi^2},
\]

with \( r_0 = 1 \) AU, and \( \tan \psi \) as a function of radial distance \( r \) and polar angle \( \theta \) given by

\[
\tan \psi = \Omega (r - r_b) \sin \theta V(r, \theta) \left( \frac{r V(r, \theta) B_T(r_b)}{r_b V(r, \theta) B_R(r_b)} \right),
\]

Here \( B_a \) is a normalization constant that assures the HMF has the value \( B_a \) at Earth, \( \Omega \) is the angular velocity of the Sun, and \( V \) is the SW speed. With \( r_0 = 0.005 \) AU as the solar radius, the value \( r_b = 10 \) and the ratio \( B_T/B_R = 0.15 \) are constants that determine the HMF modification.

For diffusion perpendicular to the magnetic field lines, a rigidity dependence similar to that of \( \lambda_0 \) is assumed below 4 GV, while a slightly weaker dependence of \( P^{1.58} \) is assumed above 4 GV. A distinction is made between the radial \( \kappa_{r,\theta} \) and polar \( \kappa_{\psi,\theta} \) diffusion directions, where the former and latter are scaled by 2% and 1% of the parallel diffusion respectively. This differs from what Potgieter et al. (2014) used and is in line with what is required from turbulence theory (see, e.g., Burger et al. 2000; Strauss et al. 2013, and Manuel et al. 2014). These
The coefficients are given by
\[ \kappa_{q\theta} = 0.02 \kappa_{\|\theta} F(r, \theta, \phi) G(P), \]  
(8)
and
\[ \kappa_{\perp\theta} = 0.01 \kappa_{\|\theta} F(r, \theta, \phi) G(P) h_{\perp\theta}, \]  
(9)

where
\[ h_{\perp\theta} = A^+ \mp A^- \tanh \left[ 8(\theta_A - 90^\circ \pm \theta_F) \right]. \]  
(10)

with \( A^+ = (3 \pm 1)/2 \), \( \theta_F = 35^\circ \), \( \theta_A = \theta \) for \( \theta \leq 90^\circ \) but \( \theta_A = 180^\circ - \theta \) for \( \theta > 90^\circ \). Equation (10) enhances \( \kappa_{\perp\theta} \) in the polar regions as motivated by, e.g., Potgieter (2000).

Figure 7. Similar to Figure 6, but for the reproduced and observed PAMELA spectra at the end of each semester, between 2007 and 2009. As a reference, the gray circles represent all the previously reproduced PAMELA spectra.

Figure 8. Ratios of consecutive computed (lines) and measured (symbols) proton spectra from Figure 7, relative to the 2006e reference spectrum in Figure 6.

Figure 9. Modulation reduction factor for the reproduced spectra between 2006 November and 2009 December, calculated by taking the ratio of the modulated spectra to the appropriate LIS value.
The rigidity and spatial dependence for the drift coefficient is given by

$$\kappa = \beta \frac{P}{P_0} \left( \frac{P}{P_0} \right)^2,$$

which reduces drifts below $P_0 = 0.55$ GV with respect to the weak scattering case (e.g., Ngobeni & Potgieter 2015). This is required to explain the small latitudinal gradients observed by Ulysses at low rigidities (Heber & Potgieter 2006; De Simone et al. 2011). Table 2 gives the values of the modulation variables obtained after reproducing the year-end spectra in Figures 6 and 7.

As a result of using different rigidity dependencies for $\kappa_\parallel$ and $\kappa_\perp$, as proposed by turbulence theory (e.g., Burger et al. 2000), some differences in the values for these coefficients exist when compared to a similar study from Raath et al. (2015). These differences are also ascribed to dissimilar parameters in the HMF modification.

The combined increases that were required for proton MFPs below $\sim 4.0$ GV resulted in a change in slope for the rigidity dependence from $P^{0.9}$ in 2006, to $P^{0.8}$ in 2009. This is a stronger dependence than the $P^{1/3}$ suggested by the random sweeping and damping turbulence models (see also Potgieter 2000). Above $\sim 3.0$ GV the rigidity slope gradually steepens to $P^{2.1}$, similar to the $P^2$ dependence found by Pei et al. (2010) following their QLT-based analysis of $\lambda_\parallel$.

Zhao et al. (2014) similarly studied the PAMELA proton spectra between 2007 and 2009, and reported qualitatively similar results, in particular, larger scaling factors for $\lambda_\parallel$ compared to $\lambda_\perp$. This suggests a more efficient radial diffusion perpendicular to the background HMF in times of weak turbulence found during solar minima. See also Raath et al. (2015) for a comparison of HMF modifications when reproducing PAMELA spectra.

6. SUMMARY AND CONCLUSIONS

With the advent of the 2009 solar minimum, and the availability of 27-day averaged PAMELA GCR proton spectra below 50 GeV, we were able to gain insight into how solar modulation affected GCRs during the recent unusual minimum. This was done using a newly constructed proton very LIS based on in situ measurements from Voyager 1, PAMELA, and AMS-02, along with a 3D heliospheric modulation model. After calculating average representative values for the intrinsic parameters $\alpha$ and $B_\perp$, seven 27-day averaged PAMELA spectra at the end of each semester, between 2006 July and 2009.
December, were successfully reproduced. It was shown what was required in order for the proton spectrum to become significantly softer as modulation reached extreme minimum levels, increasing intensities by a factor of ~3 at 100 MeV.

Although the modulation parameters given in Table 2 are different from what was found by Potgieter et al. (2014), primarily due to the differences in the very LIS, the HMF modification, and a different rigidity dependence for perpendicular diffusion, we come here to the same conclusions made by these authors, namely that the rigidity dependence of the DCs had to be decreased over this solar minimum period in order to simulate the PAMELA observations. In reproducing these spectra, it was also found that additional increases in proton MFPs below ~4 GV were required on top of the self-consistent changes in the tilt angle and the weakening HMF strength. It is consequently clear from both observations and modeling that increasingly more low-energy particles reached the Earth during the approximately four years leading up to 2009. Even though diffusion remained a dominant modulation mechanism during this time, the presence of drifts still had a significant contribution to the record-high spectrum measured by PAMELA at the peak of the 2009 minimum.

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Figure 12. Rigidity dependence of the proton MFPs and drift scale for diffusion parallel and perpendicular to the magnetic field lines at the Earth. Parallel MFPs ($\lambda_p$) are given by the solid lines, while perpendicular MFPs in the radial ($\lambda_{r,\theta}$) and polar ($\lambda_{\phi}$) directions are given by the dashed and dashed–dotted lines, respectively. The drift scale ($\Delta$) is given by the dotted lines. See Equations (3), (8), (9), and (11).

Table 2

Summary of the Parameters Used to Reproduce the Year-end PAMELA Spectra

<table>
<thead>
<tr>
<th>Parameter</th>
<th>2006e</th>
<th>2007e</th>
<th>2008e</th>
<th>2009e</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_p$ (AU)$^a$</td>
<td>0.742</td>
<td>0.888</td>
<td>0.970</td>
<td>1.204</td>
</tr>
<tr>
<td>$a$</td>
<td>0.91</td>
<td>0.88</td>
<td>0.86</td>
<td>0.80</td>
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<tr>
<td>$b$ for $\lambda_r$</td>
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<td>2.10</td>
<td>2.10</td>
<td>2.10</td>
</tr>
<tr>
<td>$b$ for $\lambda_{\theta}$</td>
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<td>1.58</td>
<td>1.58</td>
<td>1.58</td>
</tr>
<tr>
<td>$c$</td>
<td>2.60</td>
<td>2.40</td>
<td>3.0</td>
<td>2.2</td>
</tr>
<tr>
<td>$P_0$ (GV)</td>
<td>4.00</td>
<td>4.05</td>
<td>4.08</td>
<td>4.30</td>
</tr>
</tbody>
</table>

Notes. See Equation (5).

$^a$ MFP values at 1.0 GV at the Earth.