

Investment and operational optimisation of an energy recovery engineering plant

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Abstract

Burnable off-gases generated from operational processes in engineering plants are regularly utilised as energy sources. A common practice is to use this for steam production in boiler houses, where excess steam is allocated to power generation turbines. Fluctuations in off-gas productions may, however, result in turbines shutting down, due to insufficient steam. Some investment models exist, which are typically based on cost minimisations or for the purpose of meeting energy demands. These models do not, however, take into account plant-specific steam flow patterns and typically use average-based profiles for decision making.

The operational control of turbines are typically performed by means of a *fixed-sequence philosophy*. Turbines are loaded in a predetermined order until a fixed set point. This operational philosophy incurs significant power generation losses from turbine shut-downs, as a result of the inability to distribute steam dynamically. Such an operating philosophy is easy to implement and, therefore, commonly used in industry. Another philosophy is that of *dynamic control* where steam distribution between the turbines are computed at each time period by means of an in-time operating control algorithm.

In this thesis, a number of novel model formulations are proposed, which addresses optimal power generation and turbine investments under a fixed-sequence philosophy, as well as dynamic control. Seven conceptual formulations are applied to demonstrate basic power generation optimisation. These conceptual formulations are to incorporate either a fixed-sequence philosophy, dynamic control or both.

The main contributions of this study entail a further seven formulations where six are optimisation models. Each of these six formulations utilises plant signature steam flow profiles in order to determine, either optimal power generation, or optimal turbine investments in terms of the *net present value* (NPV), or both. All investment formulations include turbine shut-downs in the decision making process by penalising the NPV with each occurrence.

The first three of these model formulations are for turbines operating under a fixed-sequence philosophy, two for optimal power generation and one for optimal turbine investments. For optimal power generation the formulations determine in which fixed order the turbines should operate. Optimal turbine investments, as determined by the third formulation, is the combination that yields the highest NPV. For this combination the optimal fixed turbine order and power generation are determined. Optimal investment results indicate that when future trip costs are taken into account as an NPV penalisation, a single turbine should rather be procured.

The final three of these proposed formulations are for turbines operating under dynamic control. The first formulation optimises power generation between any number of turbines. The second formulation optimises turbine investments in terms of NPV. Comparing optimal power generation results, increases between 3.5% and 13.2% are observed for turbines under dynamic control compared to a fixed-sequence philosophy. Optimal NPV's under dynamic control are between 7.3% and 19.3% higher than those of a fixed-sequence philosophy. All optimal outcomes yield that two turbines should be procured.

A formulation is proposed that optimises turbine investments, which incorporates the procurement of a supplementary energy resource to assist during low off-gas, and therefore, low steam flow periods. Such a resource is typically very expensive and does not make sense to procure under normal operating conditions. However, in a fluctuating steam flow environment it proves to increase the NPV, while safeguarding turbines from shut-down occurrences. Depending on the procurement and projected shut-down costs, results indicate that an investment into a supplementary resource under

optimal investments can yield an NPV improvement 10.6% to 118.0% versus a fixed-sequence philosophy, and 3.0% to 82.7% compared to dynamic control. Results further indicate that involuntary turbine shut-downs, owing to low steam flow periods, are reduced up to a 100%.

Keywords: dynamic control, energy recovery, fixed-sequence philosophy, historic steam profiles, mixed integer linear programming, net present value, off-gases, optimal investment, power generation, supplementary energy resource, turbines

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Contents

1	Background and problem statement	2
1.1	Introduction	2
1.2	Focus of this study	4
1.3	Method of investigation	5
1.4	Contributions of this study	5
1.4.1	Main contributions of this study	5
1.5	Layout of the thesis	6
2	Engineering considerations	8
2.1	Relevant engineering definitions and theory	8
2.1.1	Control volume	8
2.1.2	A fluid	8
2.1.3	Steady state and transient flow	9
2.1.4	Pressure and total pressure	9
2.1.5	Total temperature	9
2.1.6	Enthalpy	9
2.1.7	Entropy	10
2.1.8	Conservation laws for a control volume	10
2.2	Relevant engineering component theory	11
2.2.1	Turbines and generators	11
2.2.2	Pumps	12
2.2.3	Heat exchangers	13
2.2.4	Boiler houses	13
2.3	The basic Rankine cycle	13
2.4	The energy recovery plant	14
2.5	Average versus signature steam profiles	15
2.6	Steam shortages for turbines	17
2.7	The fixed-sequence philosophy	18
2.8	Dynamic control	19
2.9	Gas storage vessels	19
2.10	Hypothetical steam profile	21
2.11	Real-world data	21
2.12	Relevant literature	22
2.13	Summary	23
3	Conceptual mathematical model development	24
3.1	The Power Concept Model (PCM)	24
3.2	The Power Time Concept Model (PTCM)	25
3.3	The Power Steam Availability Model (PSAM)	28
3.4	The Double Sequence Model (DSM)	28
3.4.1	Results for DSM	31
3.5	Summary	32

4	Optimal fixed-sequence loading hierarchy	33
4.1	Power generation under a fixed-sequence philosophy	33
4.2	Plant Operational Queueing Model (POQM)	33
4.2.1	Results for POQM	36
4.3	The Optimal Sequence Model (OSM)	40
4.3.1	Results for OSM	42
4.4	Summary	47
5	Optimal investments under fixed-sequence philosophy	49
5.1	The Plant Operational Investment Queueing Model (POIQM)	49
5.2	Results for POIQM	51
5.2.1	POIQM results with a zero trip penalty	52
5.2.2	POIQM results for a fixed trip penalty	53
5.2.3	POIQM results with proportional trip penalty	54
5.2.4	POQIM results under increased costs	56
5.2.5	POIQM results for more efficient turbines	57
5.3	Summary and conclusion	58
6	Conceptual dynamic model explanation	60
6.1	A case to justify MILP optimisations	60
6.2	Dynamically controlled turbines	63
6.2.1	The Dynamic Concept Model (DCM)	64
6.2.2	The Dynamic Time Model (DTM)	66
6.2.3	The Double Dynamic Model (DDM)	67
6.3	Double Dynamic Logic Control (DDLCC)	69
6.4	Summary	72
7	Optimal power generation	74
7.1	The Optimal Power Model (OPM)	74
7.2	Optimisation results for OPM	76
7.2.1	Comparison of OPM and DDLCC results	81
7.3	Summary	82
8	Optimal turbine investments	84
8.1	The Optimal Power Investment Model (OPIM)	84
8.2	Computational Results for OPIM	85
8.2.1	Solving OPIM for a zero trip penalty	86
8.2.2	OPIM results under a fixed trip penalty	87
8.2.3	OPIM results under a proportional trip penalty	88
8.2.4	OPIM results under increased costs	89
8.2.5	Investments under improved isentropic efficiencies	90
8.3	Summary	91
9	Optimal investments under an additional resource	93
9.1	The Optimal Power and Gas Investment Model (OPGIM)	93
9.2	OPGIM scenario results	95
9.2.1	Solving OPGIM under a zero trip penalty	95
9.2.2	Solving OPGIM under fixed trip costs	99
9.2.3	Solving OPGIM under proportional trip costs	102
9.2.4	Sensitivity towards costs	103
9.2.5	Sensitivity towards resource cost increases	104
9.2.6	Investments under improved isentropic efficiencies	105
9.3	Summary	106

10 Summary and conclusion	108
10.1 Chapter summaries	108
10.2 Future research	110

List of Figures

1.1	A simplistic engineering plant layout	3
2.1	The basic Rankine cycle	14
2.2	Two hypothetical steam profiles, A and B.	16
2.3	The average steam profile between profiles A and B.	17
2.4	Operating limits of the medium capacity turbine with the profiles A and B.	18
2.5	Operating limits of the larger capacity turbine with the profiles A and B.	19
2.6	A hypothetical steam scenario with an energy recovery turbine.	20
2.7	Hypothetical steam profile from Table 2.2 plotted over time.	21
2.8	A steam profile experienced by the Works.	22
3.1	Steam distribution determined by PCM.	26
3.2	Steam distribution to Turbine I for PTCM.	27
3.3	Steam distribution for PSAM.	29
3.4	DSM steam distributions for Case 1.	31
3.5	Flow chart of MILP's in Chapter 3.	32
4.1	A hypothetical steam flow scenario.	34
4.2	Case 1 ^q : Power generation for Turbine I when $\alpha_{1q} = 0.8$	37
4.3	Case 1 ^q : Power generation for Turbine II when $\alpha_{1q} = 0.8$	37
4.4	Case 3 ^q : Power generation for Turbine I when $\alpha_{1q} = 1.0$	38
4.5	Case 3 ^q : Power generation for Turbine II when $\alpha_{1q} = 1.0$	39
4.6	Case 3 ^q : Steam not utilised by Turbine I or II.	39
4.7	Case 2 ^f : Power generation for Turbine III.	43
4.8	Case 6 ^f : Power generation for Turbine III as first receiver.	44
4.9	Case 6 ^f : Power generation for Turbine IV as second receiver after III.	45
4.10	Case 6 ^f : Power generation for Turbine VIII as third receiver after III and IV.	45
4.11	Case 6 ^f : Steam not utilised by Turbine III, IV or VIII.	46
4.12	Flow chart of MILP's in Chapters 3 and 4.	47
5.1	Case 11 ^f : Steam utilised if only Turbine VI is in operation.	55
5.2	Case 11 ^f : Excess available steam over that cannot be utilised by Turbine VI.	56
5.3	Flow chart of MILP's in Chapters 3 and 4 that led to POIQM.	59
6.1	Steam distributions for DCM.	66
6.2	Steam distributions for DTM.	67
6.3	Steam distributions for DDM.	68
6.4	Flow chart of MILP's from Chapter 4 used in Chapter 6.	72
7.1	Case 1 ^d : Power generation over time for Turbine I, the 5MW, under dynamic control.	77
7.2	Case 1 ^d : Power generation over time for Turbine II, the 30MW, under dynamic control.	78
7.3	Case 1 ^d : Unutilised steam flow for Turbine II and VII over time, under dynamic control.	78
7.4	Case 6 ^d : Power generation for Turbine III, the 10MW.	79

7.5	Case 6 ^d : Power generation for Turbine IV, the 20MW.	80
7.6	Case 6 ^d : Power generation for Turbine VIII, the 5MW.	80
7.7	Case 6 ^d : Unutilised steam amongst Turbine III, IV and VIII over time.	81
7.8	Formulation from Chapters 3 and 6 that led to OPM	83
8.1	Formulations from Chapters 3, 6 and 7 that built towards OPIM.	92
9.1	Case 2 ^g : Power generation for Turbine II under natural gas assistance.	97
9.2	Case 2 ^g : Power generation for Turbine VII under natural gas assistance.	97
9.3	Unutilised steam flow for Case 2 ^g under natural gas assistance.	98
9.4	Power generation for Turbine VIII, without natural gas assistance.	99
9.5	Unutilised steam flows by Turbine VIII if natural gas is not used.	100
9.6	Case 3 ^g : Steam produced by natural gas for Turbine VIII.	100
9.7	Case 3 ^g : Steam additionally utilised by Turbine VIII due to natural gas.	101
9.8	Case 3 ^g : Power generation by Turbine VIII when natural gas is utilised.	101
9.9	Formulations from Chapters 3 and 6 to 8 that built towards OPGIM.	107

List of Tables

2.1	Turbine operating limits.	15
2.2	A fluctuating hypothetical steam profile.	21
3.1	Power generation and trips determined by PCM, PTCM and PSAM.	28
3.2	Operating parameters used for Case 1 and 2.	30
3.3	Results for DSM.	31
4.1	Turbine limits	36
4.2	Scenario results for Case 1^q to 3^q	38
4.3	Operating limits for Turbine I to VIII.	42
4.4	OSM results for Case 1^f to 7^f	43
5.1	Operating limits for Turbine I to X.	51
5.2	Procurement costs for Turbine I to X.	52
5.3	Scenario results for Case 1^f to 5^f for a zero trip penalty.	53
5.4	Scenario results for Case 6^f to 10^f for a fixed penalty.	54
5.5	Scenario results for Case 11^f to 15^f for a proportional penalty	55
5.6	Scenario results for Case 16^f to 19^f under increased costs.	57
5.7	Operational parameters for Turbine VI and VI*.	58
5.8	Allowable procurement price increases for Case 1^{fi} to 4^{fi}	58
6.1	Steam profiles for a hypothetical Works.	61
6.2	Steam profile one with binary operational status variables.	61
6.3	Steam profile two with binary operational status variables.	62
6.4	Steam profile three with binary operational status variables.	63
6.5	Operating parameters for Turbines I and II.	65
6.6	Power generation and number of trips for DCM, DTM, DDM and DSM.	65
7.1	Operational parameters for Turbine I to VIII.	76
7.2	Summary of OSM results.	76
7.3	OPM results for Case 1^d to 7^d	77
8.1	Operating parameters for Turbines I to X.	86
8.2	Scenario results for Case 1^d to 5^d where $C_i^h = 0$	86
8.3	Scenario results for Case 6^d to 10^d where $C_i^h = 50000$	87
8.4	Scenario results for Case 11^d to 15^d , where $C_i^h = C_i^T/500$	88
8.5	Scenario results for Case 16^d to 19^d under increased costs.	89
8.6	Operational parameters for more efficient turbines.	90
8.7	Allowable procurement price increases for Case 1^{di} to 4^{di}	91
9.1	Scenario results for Case 1^g to 5^g	96
9.2	Scenario results for Case 6^g to 10^g when $C_i^h = 50000$	102
9.3	Scenario results for Case 11^g to 15^g when $C_i^h = C_i^T/500$	103
9.4	Scenario results for Case 16^g to 19^g under higher costs.	104
9.5	Scenario results for Case 1^{g*} to 5^{g*} when $C_i^h = C_i^T/250$	105

9.6	Allowable procurement price increases for Case 1^{g^i} to 5^{g^i}	106
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Chapter 1

Background and problem statement

1.1 Introduction

In the engineering manufacturing industry there are several different types of production plant layouts. A plant refers to a process or a subset of processes delivering any number of products, where the set of all plants are known as *the Works*. Plants can operate as independent entities or form part of an integrated process with other plants. These processes typically have integrated production chains, from which a number of by-products may be generated. If such a by-product is present in a gaseous form and can combust when ignited in an oxygen enriched environment, it is known as a burnable off-gas. A burnable off-gas possesses the potential to release heat during ignition and may therefore be utilised as an energy source for an engineering plant [9, 11, 22].

Figure 1.1 shows a simplistic layout of a typical engineering plant. A number of raw materials are fed to the process plants where end-products are eventually produced. In some cases off-gases form that are used in other processes. The off-gases not utilised in any plant processes, *i.e.* residual off-gases, are either used to generate steam in boiler houses or flared into the environment [9, 11, 14, 20, 22, 25]. Steam is vital for various plant processes, mainly heating, and any excess steam may be utilised for power generation.

Excess steam is directed to turbines, which are coupled to generators, to generate electricity. Since the additional electricity generated has typically no influence on the engineering plant's production outputs and is small in comparison to plant electricity usages, it will be referred to in this thesis as *energy recovery*. The energy recovered, as a by-product, is typically significantly less than the combined plant usages. Therefore, no energy demand and supply side ever need to be met. Power is sold back at exactly the procurement price. Power fed into the grid is therefore just subtracted from the electricity bill.

Variation in the quantities of raw material that feed to a process plant, or deviation of the chemical qualities thereof, may result in fluctuating off-gas productions over time. In principle, fluctuating energy recovery is not necessarily problematic, unless power generation potential goes to waste, due to inefficient resource utilisation. It should be noted that due to the continuous operating nature of an engineering plant, unutilised steam cannot be stored for later use and its power generation potential will consequently go to waste [9, 11, 22, 25].

The Utilisation of energy sources or resources more efficiently has become prominent all over the globe, which includes the engineering manufacturing sector. Improvements in either equipment investment models or operational philosophies will typically not only yield higher profit margins or investment *net present values* (NPV's), but coincide with environmental conscious decisions. If an energy recovery plant operates under fluctuating off-gas and steam productions that result in inefficient energy source usages, potential power generation will go to waste. The need, therefore, exists for investment models and operational philosophies that will improve energy utilisation. As a result, increased power generation may follow, which directly implies less electricity expenses and higher profit margins.

A number of different turbine investment planning approaches can be followed for an energy recov-

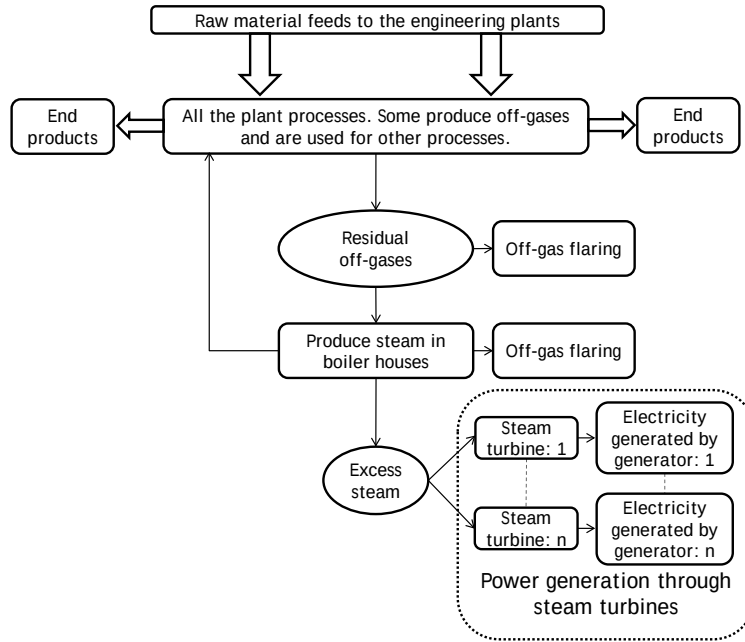


Figure 1.1: A simplistic engineering plant layout demonstrating the concept of fluctuating energy recovery from excess steam.

ery plant operating in a fluctuating steam flow environment. Most methods used for investment planning are typically based on the averaging of steam profiles. An average can be estimated based on a plant personnel's experience, in terms of what a representative steam value will be. In this instance a general fixed plant steam usage is subtracted from the boiler houses' production capacity and used to specify the turbines that should be invested in.

Rather than this crude technique, a steam profile can be obtained and used to calculate the average. For both approaches the numerical value can be used to assist the investment decision-making process. Turbine investment selections, best suited for the expected steam flow, can then be made. A more elegant approach is where an average steam profile is computed by acquiring multiple excess steam profiles from the Works over equivalent time horizons. The profile is calculated by taking the average for every time period between the respective profiles and can then be used in the turbine investment decision making process. Such a profile will exert variations within the time periods and yield a more true reflection of the fluctuating nature of steam availability for the energy recovery plant, compared to only a single steam value. Unless extreme fluctuations occur at the same time periods, it may be flattened out due to averaging, and therefore, not taken into account in the investment decision making process.

The method of investment planning will influence the quantity, operating efficiency and capacity of the turbines to be procured. An improved method of investment planning may result in better suited turbine procurements for an energy recovery plant. More suitable procurements can typically be deemed as investments that will result in increased power generation or higher NPV profits, when compared to other possible outcomes. Coinciding with the investments are the plant's operational philosophy, which is how the steam flow distributions to the turbines at each time period are determined.

A typical operational philosophy for an energy recovery plant is where turbines are loaded with steam in a preordained fixed hierarchy. For this hierarchy each turbine is assigned a specific number, from first to last, which corresponds to the order in which steam is distributed. The first turbine in line, referred to as the *first receiver*, is loaded with steam at each time period until a fixed set point, provided that sufficient steam is available. The *second receiver* can only receive steam if the first receiver is loaded until its fixed set point. The second receiver is then loaded upon availability until its fixed set point, whereafter the *third receiver* is loaded, and so forth. A turbine lower down in

the loading hierarchy can, therefore, only receive steam if all of the prior receivers are operational at their fixed set points. The loading of turbines in this preordained hierarchy or order will be referred to as the plant's *fixed-sequence philosophy* throughout this thesis.

Obtaining an improved operational philosophy, as opposed to turbines being loaded in a preordained fixed hierarchy, may reflect positively in terms of power generation and, consequently profit margins. An approach is required to realistically compare different investments and operational philosophies with each other. If such a technique indicates a better outcome, either with increased power generation or a higher NPV value, the question arises as to what can be deemed as acceptable improvements? Considering the improvements, is there a 'best obtainable' solution and is it implementable?

Furthermore, apart from turbine procurements, the Works may explore the possibility of other investments that aid the power generation process to obtain the best solution. These investments may include additional energy resources that can assist times of low off-gas availability.

It is, therefore, clear that there is a need for decision support systems to aid power generation investment and control decision making support. If power generation under an energy recovery plant's chosen operating policy can be accurately modelled, the expected outcomes of different investment decisions can be determined to support the procurement process. If the optimal control of a chosen operating policy is modelled and implemented, power generation investments can be optimised. Furthermore, if control under the optimal operating policy is modelled, investment optimisation under such circumstances can be determined.

1.2 Focus of this study

For both investment and operational philosophies, the aim should be to investigate various possible outcomes and identify the best suited solution for the energy recovery plant. Appropriate model formulations are critical to guide the investigation process. The outcomes of such an investigation should include equipment procurement options for the chosen operational policy that will yield the best expected results, with an objective to either maximise power generation or NPV, or both.

This study will aim to achieve the above mentioned through mathematical modelling, by incorporating steam flow profiles. As mentioned earlier, the two approaches commonly used are based on working with an average value or an average flow profile. In an energy recovery environment where off-gas and steam flow fluctuations are evident, an average off-gas or steam value will not allow uncertainty to be incorporated into the decision-making process. Using an averaged profile will be more realistic; however, it may still average out steam flow values during time periods of either higher or lower availability. Average off-gas or steam profiles may therefore not truly reflect the signature flow profiles of an engineering plant. These shortcomings will be illustrated and evaluated in more detail.

It is important to notice that various techniques can be implemented to determine the outcome of a mathematical model. One approach is to use a heuristic method, which will yield a feasible solution. Such a solution, however, is not guaranteed to be optimal. All mathematical models proposed in this thesis will be solved by a branch-and-bound algorithm, which is an exact approach that determines mathematically proven optimal solutions.

This thesis will focus on the optimisation of investment and operating policy control challenges for an energy recovery plant in the manufacturing industry. Power generation and turbine investments of such a plant, which operates under a typical fixed-sequence philosophy, will be optimised by means of mathematical modelling. Further focus will be on optimising power generation and investments for turbines under dynamic control with pre-knowledge of steam availability. Investments under dynamic control will include turbines and a supplementary resource, which can assist steam productions in times of low off-gas availability. Optimisation results are based on data obtained from an engineering manufacturing plant where excess steam, generated from residual fluctuating burnable off-gases, is available.

To address the optimisation problems that are specific to an energy recovery plant, this study will propose a number of novel mathematical models. All models will be subjected to physical

conservation laws, equipment design parameters and a chosen operating philosophy.

1.3 Method of investigation

This study will propose a number of novel mathematical optimisation formulations, using *mixed integer linear programming* (MILP) as basis. Relevant information on engineering considerations and principles will precede all model formulations.

For the proposed model formulations the objective function will either be to optimise power generation between turbines or to maximise NPV. All model formulations will incorporate binary decision variables at each time period that simulate the operational status of each turbine. The IBM product, CPLEX v12.6 [13] is used as MILP solver.

In order to determine optimal investments, steam control will be optimised between the turbines under the chosen operating philosophy. Model formulations in this thesis will comprise two operating philosophies, namely the fixed-sequence philosophy and dynamic control. For each operating philosophy the model formulations will firstly be for optimal control of steam flow to the turbines, followed by optimal investments where NPV will be used as a benchmark.

1.4 Contributions of this study

This thesis proposes various novel contributions towards power generation optimisation. The contributions are MILP formulations for turbines operating either under a fixed-sequence philosophy or dynamic control. For each philosophy steam flows to the turbines are optimised for each time period. This is to obtain optimality, either for power generation or investments that use NPV as performance criterion.

The thesis contains seven conceptual MILP formulations. The first three formulations are explanatory models, which introduce the concept of power generation optimisation for a single turbine in a fluctuating steam flow environment. The models demonstrate how a turbine reacts to steam shortages that forces it to trip and how start-up constraints are formulated under sufficient steam availability. The three formulations provide a basis on which all further model formulations of the thesis builds upon. The fourth explanatory model formulation introduces the concept of two turbines operating under a fixed-sequence philosophy. The final three explanatory MILP formulations follow for two turbines operating under dynamic control, explaining how start-up under sufficient steam availability is modelled.

1.4.1 Main contributions of this study

Six of the novel contributions of this study are MILP formulations. All the main MILP formulations of this thesis solve for any number of turbines. The turbines to be included in the MILP models may comprise any operating limits or efficiencies. These models allow for any fixed time period of sufficient steam that needs to be present before turbine start-up is allowed. The contributions are as follows:

- i A model formulation is presented that yields the optimal loading hierarchy for turbines operating under a fixed-sequence philosophy. The objective function is to optimise power generation through optimal steam flow distribution among turbines for each time period. All the possible loading hierarchies and the fixed set point per turbine are input parameters to the model.
- ii A second model formulation is proposed that determines for the optimal loading hierarchy and the optimal fixed set points for turbines operating under a fixed-sequence philosophy. Similar to the first main contribution, the objective function is to optimise power generation through optimal steam flow distribution among turbines for each time period. For this model, however, the fixed set point per turbine is treated as a variable. The model formulation,

furthermore, does not require any loading hierarchies as input parameters. Both the first and second model can be used by any energy recovery plant, operating under a fixed-sequence philosophy, to determine the optimal loading hierarchy and expected power generation for the turbines present in the plant. Furthermore, the additional power generation effect can be simulated if the Works considers future turbine investments, and what that optimal loading hierarchy will be.

- iii A third model formulation is proposed that determines optimal turbine investments under a fixed-sequence philosophy, where NPV is used as a benchmark. The model furthermore yields the optimal loading hierarchy for the investments. This is determined by optimising power generation through steam flow distributions to the turbines for each time period. The model is applicable under a fixed-sequence philosophy, for either an existing energy recovery plant seeking to expand power generation capabilities, or when the Works investigates the possibility of investing in energy recovery. All possible turbine loading hierarchies are input parameters towards this model.
- iv The fourth contribution is a model formulation that determines optimal power generation for turbines under dynamic control. This model optimises steam flow distributions between the turbines for each time period. The model formulation, furthermore, determines which turbine to trip when sufficient steam is not available or when to restart. The model can be used by any energy recovery plant, operating under dynamic control, to determine the optimal expected power generation for the turbines present in the plant. Furthermore, the additional power generation effect can be simulated if the Works considers future turbine investments.
- v The fifth contribution is an investment optimisation model, which is proposed for turbines operating under dynamic control, using NPV as a benchmark. Steam flow distributions between the turbines for each time period are optimised. The model, furthermore, simulates the trip and restart of turbines. This model may be applied for either an existing energy recovery plant, investigating the option to expand power generation capabilities or for the Works exploring the possibility of investing in energy recovery.
- vi The final contribution is an extended optimal investment formulation based on the above model, under dynamic control. Intertwined with optimal turbine investments, the model incorporates the possibility of investing and procuring a supplementary energy resource. This resource can be utilised to produce additional steam in times of low steam availability. Such a resource is typically very expensive and does not make sense to invest in under normal operating conditions. The model can be used similar to the previous one, with the inclusion of exploring the possibility to investigate steam production assistance.

The seventh novel contribution of this thesis is the formulation of a logic control algorithm. The algorithm maximises power generation between two dynamically controlled turbines. Accurate steam flow predictions of two time periods into the future are assumed. The algorithm determines how steam should be distributed at each time period between the turbines in order to achieve maximum power generation. Furthermore, the algorithm incorporates decision variables that determine which turbines to trip when sufficient steam is not available, or when to restart.

1.5 Layout of the thesis

Basic engineering and equipment principles are given in Chapter 2. The provided background is necessary to fully understand the mathematical model formulations of Chapter 3 and onwards. The chapter ends with a literature survey relevant to this thesis.

Chapter 3 provides four concept MILP formulations to emphasise some working principles of mathematical models that follow in later chapters. The first three models demonstrate power generation optimisation for a single turbine in a fluctuating steam flow environment. Operational and time constraints are introduced. The fourth MILP formulation demonstrates optimal power generation for two turbines that operate under a fixed-sequence philosophy.

Two MILP's are formulated in Chapter 4. Both formulations determine the optimal turbine loading hierarchy through power generation optimisation, for any number of turbines. The turbines may comprise any operating capacities or efficiencies. The second formulation, furthermore, determines the optimal fixed set point at which the turbines should be loaded. Scenario analysis are performed with both models, demonstrating how optimal loading hierarchies are determined for the turbines. An optimal turbine investment MILP is formulated in Chapter 5, which uses NPV as a benchmark. Investments for any number of turbines, operating under a fixed-sequence philosophy, can be determined. The turbines can be of any operating capacity or efficiency. The optimal loading hierarchy between the turbines are, furthermore, determined through power generation optimisation. Turbine trips can influence the decision-making process, by penalising the NPV for each occurrence.

The final three conceptual MILP formulations are presented in Chapter 6. These models demonstrate the working principles of power generation optimisation under dynamic control for two turbines. This chapter further provides a logic control algorithm that maximises power generation between two turbines.

In Chapter 7 a MILP formulation is proposed, which optimises power generation amongst any number of dynamically controlled turbines. The turbines can be of any capacity or operating efficiency. Optimal power generation and the number of trip occurrences are determined for various scenarios and compared to outcomes from Chapter 4. Some results are also compared to the outcomes from the control algorithm proposed in Chapter 6.

An optimal turbine investment MILP formulation is proposed in Chapter 8, which uses NPV as a benchmark. This formulation optimises investments for any number of turbines operating under dynamic control. The turbines may, furthermore, comprise any operating limits or efficiencies. Trips can influence the decision-making process, by penalising the NPV for each occurrence. Various scenario outcomes are determined and compared to results reported in Chapter 5.

The final contribution of this thesis is the MILP formulation proposed in Chapter 9. This formulation optimises turbine investments for any number of dynamically controlled turbines and uses NPV as a benchmark. The turbines may consist of any operating limits or efficiencies. The formulation, furthermore, allows for an intertwined supplementary energy resource investment and the timely procurement thereof. This resource can assist with additional steam production, in times of insufficient off-gas availability, to prevent turbine trips. Such a resource is typically very expensive and not a viable option under normal operating conditions. Trips can influence the decision-making process, by penalising the NPV for each occurrence. Various scenario outcomes are determined and compared to results reported in Chapters 5 and 8.

The thesis concludes in Chapter 10 with a summary on the work presented in this study and recommendations for future research.

Chapter 2

Engineering considerations

Chapter 1 identified the need for power generation optimisation models, which can be incorporated to assist the turbine investment decision-making process in a fluctuating steam flow availability environment. In order to determine optimal turbine investments for an engineering Works, typical steam profiles that are experienced need to be incorporated in the decision-making process. For this an understanding of the engineering operations are required. This includes all relevant processes and laws of physics, which governs the fundamental working principles of equipment that form part of the power generation process. Understanding these processes and underlying scientific working principles, is paramount in the formulation reasoning process and calculation of the input parameters, required by these models.

This chapter provides the relevant engineering background needed to follow the rationale behind all formulations to follow. Steam profile data and the use of signature profiles rather than a typical averaging are furthermore stressed in this chapter. Information provided from Sections 2.1 to 2.9 are found in literature reported by [2, 3, 6, 9, 11, 14, 16, 17, 19, 20, 21, 22, 25, 28].

2.1 Relevant engineering definitions and theory

In this section some basic theory on the working principles of relevant engineering equipment is given, which is paramount to the power generation process. To understand its working principles, and therefore, the basis on which the model formulations follow in later chapters, the following fundamental thermal-fluid definitions are required.

2.1.1 Control volume

For thermal-fluid modelling a control volume approach is used, which represents an imaginary surface or volume with regards to the system under consideration. This volume can be either fixed in space or move at a constant velocity.

2.1.2 A fluid

A fluid is a substance where the molecules can move past each other so that it continually distorts when shear stress is applied. Fluids are therefore not bound to a fixed shape. For this study a fluid will either refer to one of three states, which are firstly a liquid, secondly a gas and thirdly a mixture of liquid and gas. A mixture of a liquid and gas phase of the same substance is also known as a two-phase mixture. It is important to note that a same-substance mixture of a gas and a solid or a liquid and a solid are also known as two-phase mixtures. If mention is made in this study regarding a two-phase mixture, it will only imply a liquid-gas phase.

Unless stated otherwise, the fluid referred to in this thesis will be steam, which is water in a gas or vapour phase. All gases are deemed compressible, whereas liquids are incompressible.

2.1.3 Steady state and transient flow

Steady state is referred to as the condition where no mass or energy is accumulated in the control volume over a time period. Therefore, at any point within a system, all thermodynamic properties in the control volume are independent over time. When a fluid experiences variations in volume or mass flows over time it is known as transient- or non steady-state flow conditions. Under such circumstances it is possible for mass and energy to accumulate for some time period at any point or component in the system.

If an engineering plant operates under fluctuating steam availability, transient flow will be evident. In order to model such a phenomenon the entire engineering plant setup with all its component geometries are needed, which include pipe lengths, diameters and elbows. This is not part of the focus of the study and therefore a systems, rather than a component approach, will be applied. Furthermore, changes in plant processes that cause fluctuations are deemed slow in comparison towards in-time changes. For this reason steady state conditions are always assumed. All flow values will therefore be discrete values under the assumption of smooth system flow changes between time periods.

2.1.4 Pressure and total pressure

Pressure is the amount of force (mass-length per time unit squared) applied perpendicular to a surface area (length squared). The total pressure p_0 comprise two components, *i.e.* the static p_s and the dynamic p_d pressure. Static pressure is the force applied to a perpendicular area, where the force is at rest relative towards the area. Dynamic pressure is defined as the kinetic energy per volume unit of the fluid. The total pressure is given by:

$$p_0 = p_s + \frac{1}{2}\rho V^2 \quad (2.1)$$

where $1/2\rho V^2$ represents the dynamic pressure, p_d . The density of a fluid is given by ρ in mass per volume and V is the velocity, which is the displacement over time.

2.1.5 Total temperature

The total temperature T_0 for a compressible substance is defined by:

$$T_0 = T + \frac{V^2}{2c_p} \quad (2.2)$$

where T represents the temperature and c_p (energy per mass and temperature) is a measure of the capacity of a fluid to absorb energy while undergoing a constant pressure process, also known as the specific heat at constant pressure.

2.1.6 Enthalpy

An important thermodynamic property in any thermal-fluid system analysis is the specific static enthalpy h (energy per mass). The static enthalpy provides a measure of the internal energy u (energy per mass) contained in the fluid, in combination with the potential flow energy of a fluid and is defined by:

$$h = u + \frac{p}{\rho} \quad (2.3)$$

The total enthalpy (energy per mass) is defined by:

$$h_0 = h + \frac{1}{2}V^2 \quad (2.4)$$

2.1.7 Entropy

For an internally reversible system the change in specific entropy S (energy per temperature) is defined by:

$$dS = \frac{\partial Q}{T} \quad (2.5)$$

where Q represents the heat transfer (energy) to a fluid and T the temperature measured in Kelvin.

2.1.8 Conservation laws for a control volume

The field of thermal-fluid modelling contains three fundamental conservation laws. These laws are the conservation of mass, momentum and energy.

Conservation of mass

The following generic equation is valid for the conservation of mass:

$$V \frac{\partial \rho}{\partial t} + \dot{m}_e - \dot{m}_i = 0 \quad (2.6)$$

where $\dot{m}$ represents the mass flow rate over time t (mass per time). The subscripts "e" and "i" refer to the outlet and inlet of a fluid towards the control volume. Under the assumption of steady state flow it follows that $\partial \rho / \partial t = 0$, so that conservation of mass reduces to

$$\dot{m}_e - \dot{m}_i = 0. \quad (2.7)$$

From Equation (2.7) it follows that the outlet and inlet mass flow rates are equal under steady state conditions. The mass flow rate throughout the control volume is therefore constant so that

$$\dot{m} = \dot{m}_e = \dot{m}_i. \quad (2.8)$$

The mass flow rate $\dot{m}$ can be calculated by the following equation:

$$\dot{m} = \rho V A_{ff} \quad (2.9)$$

where the free flow area perpendicular to the fluid's flow is represented by A_{ff} . Engineering plants typically use round pipes for steam transport, so that the free flow area, A_{ff} , is calculated by:

$$A_{ff} = \frac{1}{4} \pi D^2 \quad (2.10)$$

where the parameter D represents the inner pipe or flow diameter.

Conservation of momentum

The following generic equation is valid for compressible flow:

$$\rho L \frac{\partial V}{\partial t} + \frac{p}{p_0} (p_{0e} - p_{0i}) + \frac{\rho V^2}{2T_0} (T_{0e} - T_{0i}) + \rho g (z_e - z_i) + \Delta p_{0L} = 0 \quad (2.11)$$

where L (displacement) is the length of the control volume, g (displacement per time squared) the constant gravitational acceleration, z (displacement) the elevation height and Δp_{0L} (force per area) the total pressure loss over the length of the control volume. If a steady state prevails, then, $\partial V / \partial t = 0$ and the following equation will be valid for the conservation of momentum:

$$\frac{p}{p_0} (p_{0e} - p_{0i}) + \frac{\rho V^2}{2T_0} (T_{0e} - T_{0i}) + \rho g (z_e - z_i) + \Delta p_{0L} = 0 \quad (2.12)$$

Conservation of energy

The following generic equation is valid for the conservation of energy:

$$\dot{Q} + \dot{W} = V \frac{\partial(\rho h_0 - p)}{\partial t} + \dot{m}_e h_{0e} - \dot{m}_i h_{0i} + \dot{m}_e g z_e - \dot{m}_i g z_i \quad (2.13)$$

where $\dot{Q}$ (energy per time) is the total rate of heat transfer to the fluid and $\dot{W}$ (energy per time) the total rate of work done on the fluid. For steady state conditions $\partial(\rho h_0 - p)/\partial t = 0$, so that the following equation holds for the conservation of energy:

$$\dot{Q} + \dot{W} = \dot{m}_e h_{0e} - \dot{m}_i h_{0i} + \dot{m}_e g z_e - \dot{m}_i g z_i. \quad (2.14)$$

For a steady state control volume where (2.8) is applicable and the change in elevation height from the inlet to the outlet is negligible, the conservation of energy reduces to

$$\dot{Q} + \dot{W} = \dot{m}(h_{0e} - h_{0i}). \quad (2.15)$$

Equation (2.15) states that the change in enthalpy or flow energy within a fluid can be determined through the combination of the rate that work is performed on the fluid, as well as the rate that heat is added towards it.

Following these fundamental thermal-fluid principles, elementary definitions are provided in the next section regarding some basic engineering equipment involved in a power generation process.

2.2 Relevant engineering component theory

Some theoretical background is supplied regarding engineering equipment relevant to this study.

2.2.1 Turbines and generators

A turbine is a rotary machine that extracts energy from a fluid stream. Fluid with high potential energy flows over the turbine blades, which are connected to a rotor. The fluid expands as it moves over the blades, due to a drop in temperature and pressure as a result of energy transfer. This energy transfer allows the rotor to rotate. When a turbine is coupled to a generator this rotation energy can be converted to electricity.

The work extracted from the fluid stream can be calculated by equation (2.14). Note that heat is not added to a turbine while a fluid flows over its blades, therefore, $\dot{Q} = 0$. If the change in elevation between the turbine's inlet and outlet is negligible and equation (2.8) holds, the energy extracted from the fluid can be calculated by:

$$\dot{W}_T = \dot{m}(h_{0e} - h_{0i}) \quad (2.16)$$

The energy extracted from the fluid by the turbine is therefore given by $\dot{W}_T$. In an ideal system when flow is adiabatic and reversible throughout a turbine, which is not practically possible, the energy extracted is equal to the isentropic energy. The isentropic extracted energy is calculated by:

$$\dot{W}_{isen} = \dot{m}(h_{0isen} - h_{0i}) \quad (2.17)$$

The isentropic energy extracted, $\dot{W}_{isen}$, can be calculated by determining what the total outlet enthalpy, h_{0isen} , would have been if the process was reversible and adiabatic. It should be noted, for the calculation of the isentropic outlet enthalpy, no entropy change may occur between the inlet and outlet of the turbine. For a turbine, the isentropic efficiency is defined as:

$$\eta_{isen} = \frac{\dot{W}_T}{\dot{W}_{isen}} \quad (2.18)$$

A turbine's isentropic efficiency, η_{isen} , is therefore a ratio of the actual energy extracted by a turbine over the maximum theoretical possible energy that could have been extracted.

To produce electricity, a generator is coupled to a turbine. The generator converts the energy extracted by the turbine to electricity at some efficiency, $0 < \eta_G \leq 1$, so that:

$$\dot{W} = \eta_G \dot{W}_T \quad (2.19)$$

The electricity produced, $\dot{W}$ is referred to as *power generation* throughout this thesis. Given that the isentropic efficiency for each turbine and the efficiency of a generator stay unchanged, a constant conversion rate of flow units per power unit generated can be calculated for each machine so that:

$$\frac{\dot{m}}{\eta_{conversion}} = \dot{W} \quad (2.20)$$

In equation (2.20), the unit of $\eta_{conversion}$ is then flow units per power unit generated. This constant value, which can be determined for each turbine, will be referred to as a turbine's *conversion rate* or *efficiency* for the remainder of this thesis.

Note from equation (2.18) that a turbine's isentropic efficiency value lies between 0 and 1. A higher isentropic efficiency therefore implies a turbine that extracts energy more efficiently. It should be noted, however, that $\eta_{conversion}$ can take on any positive number. Therefore, a smaller $\eta_{conversion}$ value in equation (2.20) implies an improved conversion rate, which is a more efficient turbine, *i.e.* a higher isentropic efficiency.

A turbine is designed according to certain operating limits. One of these limits is the isentropic efficiency and when coupled to a generator, with a fixed efficiency, the conversion rate. Two other important design parameters are the maximum and minimum allowable flow limits. A turbine may under no circumstances receive flow below its minimum allowable limit and protection measures are typically in place to enforce this. Flow below the minimum limit places a turbine at risk of experiencing unwanted vibrations, which could be damaging to the machine.

If the flow should drop below the minimum limit during any time period the protection measures will instantaneously shut-off all flow towards the turbine by closing the inlet valves. This instance is referred to as a trip throughout this thesis and it is also customary to refer to a turbine that goes off-line or is halted. A turbine start or restart will always refer to the occurrence when a turbine is brought back into operation or back online, following a trip.

Further protection measures are in place to ensure that a turbine cannot receive more than its maximum allowable flow limit. As a direct result, energy extraction and, therefore, power generation cannot exceed the maximum capacity.

For the remainder of this thesis mention of a turbine or machine in this contexts will explicitly imply the combination of a generator coupled to a turbine. Therefore the term "energy extraction" will not further be used, but only power generation from turbines, which implies the process explained through equations (2.16) to (2.20) in this section.

2.2.2 Pumps

A pump is a mechanical device that increases the flow pressure of a liquid, through work performed on the fluid. Each pump consists of a pressure ratio rating above unity. This ratio represents the fluid's pressure exiting the pump, *i.e.* the outlet, over the pressure entering the pump, *i.e.* the inlet. The equation for the pressure ratio, PR, is given by:

$$PR = \frac{p_{0e}}{p_{0i}} \quad (2.21)$$

The work or rate of energy required by a pump, $\dot{W}_P$ (energy per time) can be calculated by:

$$\dot{W}_P = \frac{\dot{m}}{\rho} \frac{1}{\eta_P} (p_{0e} - p_{0i}) \quad (2.22)$$

where η_P is the isentropic efficiency of the pump. Note that the term on the righthand side of equation (2.22) is divided by the pump's isentropic efficiency, since work, or an energy rate, is performed on the fluid, whereas for a turbine it is extracted.

2.2.3 Heat exchangers

A heat exchanger is used to transfer thermal energy or heat between any number of fluids. Heat can only be transferred from higher to lower temperatures. The fluids can come into direct contact, but are usually separated through a solid medium.

To calculate the heat transfer rate to a fluid between the inlet and outlet of its control volume, equation (2.14) is used. Take note that work is not performed on a fluid throughout a heat exchanger so that $\dot{W} = 0$. Furthermore, if the change in elevation between the two points are negligible and (2.8) is taken into account, the heat transfer calculation reduces to:

$$\dot{Q} = \dot{m}(h_{0e} - h_{0i}). \quad (2.23)$$

2.2.4 Boiler houses

A boiler or boiler house is a closed vessel used to heat up and in most cases boil a fluid. Boiler houses consist of any number of energy sources that are ignited with some oxygen mixture, typically air, within the confinements of the vessel. This ignited mixture is referred to as flue gas. Through a number of heat exchangers a quantity of the thermal energy from the flue gas is transferred to the fluid.

A boiler house is designed to produce a maximum and, for safety precautions, a minimum steam flow per ring main. A ring main refers to steam with exact thermodynamic properties. If a boiler house trips, some time will elapse before it can be operational again. To ensure that unburned fuels are not trapped within the confinements of a boiler house, it needs to be purged for a prolonged time and only thereafter brought back online. The purging is followed by boiler heat-up, where steady-state operational conditions need to be reached before normal steam production can commence. Each boiler house consists of auxiliary equipment to keep it operational and if the equipment is driven by steam, a quantity of steam flow from a specific ring main is required. Furthermore, each fuel has a minimum and maximum allowable flow rating. The possibility therefore exists that some fuels can generate more steam than others in a boiler house.

2.3 The basic Rankine cycle

Two well known power generation processes are the Brayton- and Rankine cycles where both can be used to produce electricity from the same underlying thermal-fluid concept. The basic principle on which these cycles operate is to increase the working fluid's pressure, followed by a heating process, whereafter energy is extracted in a process where the fluid is expanded through a turbine and then cooled. The Brayton cycle operates only in a gaseous state where the Rankine cycle's working fluid has three fluid conditions, *i.e.* vapour, two-phase and a liquid phase. The typical fluid utilised by a Rankine cycle is water, and therefore, steam is generated at the boiler houses that are connected to the turbines. The Rankine cycle is used for all power generation discussions from this study.

Figure 2.1 depicts the basic layout of a closed Rankine cycle. Between points 1 and 2 is a pump that pressurises the fluid, typically water, and circulates it through the system. The high pressure fluid is heated, boiled and superheated in the boiler house between points 3 and 6. The superheated vapour is then expanded through a turbine between points 7 and 8, where it is coupled to a generator that generates electricity from the extracted energy. The low pressure exhaust fluid, typically a two-phase mixture, is then condensed through a heat exchanger, known as the condenser, between points 9 and 10 so that a liquid can enter the pump at point 1.

Note that for the boiler some external energy source is required, in order to be ignited with an oxygen enriched mixture. Further note that in the condenser heat is rejected to an external fluid, *i.e.* the cooling medium, which is typically water.

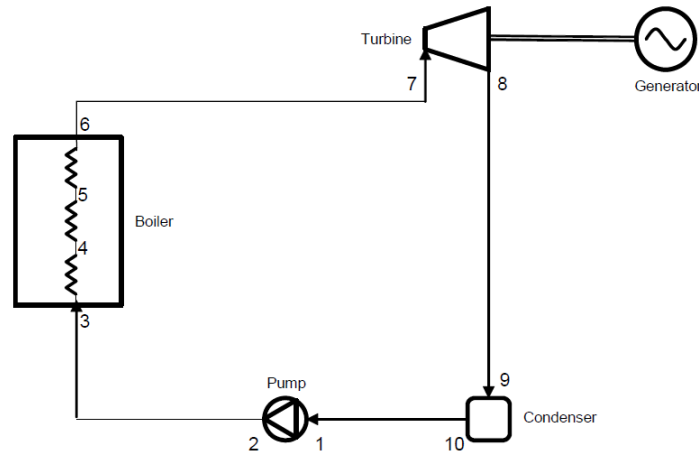


Figure 2.1: Schematic representation of the basic Rankine cycle [17].

2.4 The energy recovery plant

A basic layout of an engineering plant, containing energy recovery, was given in Figure 1.1, Chapter 1. Figure 1.1 shows a simplistic layout of a typical engineering plant considered in this study. For the engineering manufacturing industry, a plant refers to a process or a subset of processes delivering any number of products, where the set of all plants is known as the Works. Depending on the nature of a plant, it can operate independently or intertwined with other plant processes. These processes typically have integrated production chains, from which a number of by-products may be generated.

Some of these by-products have no use and are discarded or dispersed of, while others might be utilised for production purposes elsewhere or sold. It might even be that such a product is used up-stream in the production chain. When a by-product is present in a gaseous form it is also referred to as an off-gas. If an off-gas possesses the potential to ignite within an oxygen enriched environment it is commonly referred to as a burnable off-gas. For the remainder of this thesis, any reference to an off-gas will imply a burnable off-gas.

An off-gas is formed during continuous production processes and should consequently be utilised as such, since its connecting pipelines are open to the environment. If it is not open to the environment, gas built-up will occur over time, with potential hazardous consequences. Various plants over the Works may require off-gas utilisations for production purposes. Given availability, these processes will typically have priority over off-gas usages. The residual off-gases that have not been utilised by other plant processes are flared into the environment, or used to produce steam before the remainder thereof is flared.

Since all off-gas pipelines must be open to atmosphere, either a fixed percentage of the mass or volume flow, or a fixed quantity thereof, must be flared or burned-off into the environment. The rationale behind this is, firstly, attributed to potential health risks, as the burning thereof will result in a typical hydro-carbon chain or ring molecule to break off into water and carbon dioxide molecules. The second reason, is that if an off-gas is not burned and fully utilised by the Works, air will be sucked in through the open pipelines. This will result in pipelines filled with a burnable oxygen enriched mixture, which possesses potential catastrophic and fatal consequences. The process of burning off-gases into the environment is known as flaring.

Precise control need to be executed by a plant to ensure that the minimum quantity off-gases are flared into the environment. If flaring is minimised, the use of off-gases can potentially be maximised throughout the Works. The work presented in this thesis will, however, not investigate the minimisation of flaring. Unless otherwise noted, any further reference to available off-gases will entail residual off-gases available for steam production where flared quantities have been accounted for.

The fluid used in the boiler houses are water, so that superheated steam is produced. Steam is

Table 2.1: Three turbines' operating parameters in terms of maximum and minimum allowable steam limits.

Turbine	Maximum steam limit (flow unit per time)	Minimum steam limit (flow unit per time)
Small capacity	50	16
Medium capacity	60	18
Large capacity	75	22

utilised all over the Works, assisting with various plant processes, mainly for heating purposes. If excess steam is available after all of the Works' usage requirements have been met, it can be used for power generation. Since the additional power generation has typically no influence on the Works' production outputs and is small in comparison to overall electricity usages, it is referred to as *energy recovery* in this thesis.

Note that Figure 2.1 is only used to explain power generation, based on the Rankine cycle. The energy recovery plant receives steam after all other plant processes have been attended to. Therefore, steam exiting the boiler houses is distributed to the rest of the Works and the remainder thereafter is allocated for energy recovery.

Variation in the quantities of raw material that feed to a process plant, or deviation of the chemical qualities thereof, may result in fluctuating off-gas productions over time. As a result, fluctuating power generation will be observed. In principle, fluctuating energy recovery is not necessarily problematic, unless power generation potential goes to waste, due to inefficient resource utilisation. It should be noted that due to the continuous operating nature of an engineering plant, unutilised steam cannot be stored for later use and its power generation potential will consequently go to waste.

2.5 Average versus signature steam profiles

In the engineering sector it is not uncommon to use averages for planning purposes. This section demonstrates the effect that averaging can have on potential turbine investments for an energy recovery plant operating under fluctuating steam availability.

One averaging option is to estimate the available steam for power generation based on experience. A typical intuitive value for plant usages can be subtracted from the combined production capacity of the boiler houses. This projected value can then be used for turbine investment purposes and, furthermore, to approximate the expected power generation. Although experience-based answers might, at times, prove to be 'sufficient', it should not be used if relevant data can be acquired.

Following an average approach, based on calculations, can serve as an investment planning model. If a steam profile or profiles are acquired, the average production and plant usage values can be calculated. The average steam availability is then determined by subtracting these calculated values, *i.e.* plant usage from production. This average steam availability value, rather than an experience-based one, can then be used for investment planning purposes. For energy recovery under fluctuating steam availability this approach may, however, prove to be insufficient since the variations are not accounted for.

To justify the statement, assume a fictitious Works. The signature steam profile of this Works is represented by the two hypothetical available steam profiles, plotted in Figure 2.2 and labeled as *Steam profile A* and *B*, respectively. Both steam profiles display a fluctuating nature, as can be observed from the graph. Assume this fictitious Works pursues the possibility to invest in energy recovery and that one of three turbines can be procured.

Operating parameters, in terms of maximum and minimum allowable steam limits are given in Table 2.1 for a small, medium and large capacity turbine. All three turbines consist of equal conversion rates. It is assumed, only for explanatory purposes, that if steam is available above a turbine's minimum limit, it will be operational. Therefore, during each time period that the steam availability drops below the minimum allowable limit, the turbine experiences a trip.

The average value between the two profiles, also plotted in Figure 2.2, is just below 47 steam

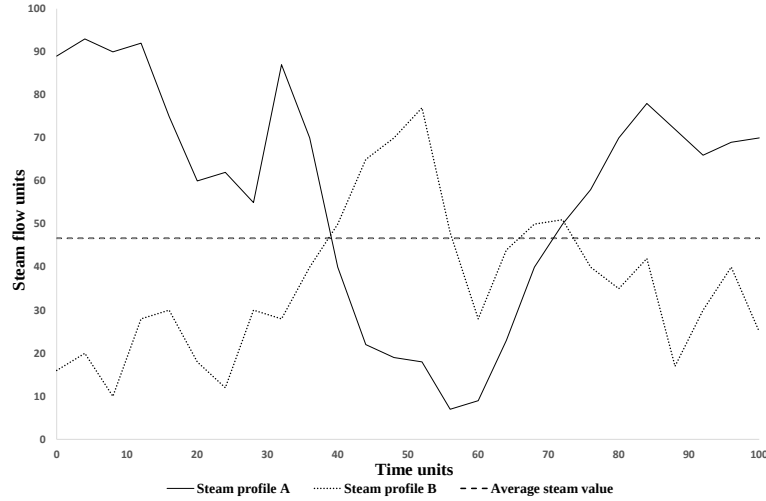


Figure 2.2: Two hypothetical steam profiles over time, denoted as A and B .

flow units per time. Using this value as investment benchmark, the small capacity turbine should be procured, since it can utilise steam up to 50 units per time. If the future steam flow does not fluctuate then all potential power generation will be realised by the small capacity turbine. However, if steam profiles A and B were to realise, then it can be observed from Figure 2.2 that steam above 50 units per time is sent to the turbine. This will result in power generation that goes to waste, due to the turbine's maximum limit. Furthermore, both profiles consist of time periods where steam availability is lower than 16 units per time, indicating turbine trips. From this it can be concluded that for an energy recovery plant, operating under fluctuating steam availability, turbine investments should not be based on an average expected value.

A more advanced investment modelling approach is to consider the average steam profile. The average steam profile can be determined between the two profiles from Figure 2.2 and used for investment planning purposes. This profile is plotted in Figure 2.3 together with the operating limits of the best suited turbine, which is the turbine of medium sized capacity. Note how this fluctuating profile stays within the operating limits of the turbine. As a result, fluctuating power generation is anticipated, however, complete steam utilisation is expected with no trips.

If the average steam profile is used by this fictitious Works for turbine investment planning, the medium capacity turbine will be procured. This turbine, however, will operate under the signature steam profiles A and B . The operating limits of the medium capacity turbine are plotted in Figure 2.4 together with these profiles. Take note, both profiles comprise time periods where either the medium capacity turbine cannot utilise all the steam or trips. This is due to averaging that can, to some extent, smooth out periods of high and low steam availability.

Figure 2.5 displays steam profiles A and B of the fictitious Works together with the operating limits of the large capacity turbine. Note that even for this turbine, time periods exist when steam availability is above its maximum limit. Comparing Figures 2.4 and 2.5 it can be observed that the lower minimum limit of the medium capacity turbine allows for one less trip and additional power generation between the minimum flow limits of 16 (flow units per time) and 18 (flow units per time). It should be noted, however, more power generation is gained between the maximum flow limits of 60 (flow units per time) and 75 (flow units per time). Therefore, the large capacity turbine will incur more power generation. As a result, turbine investment models should incorporate signature steam profiles, as will typically be experienced by the energy recovery plant.

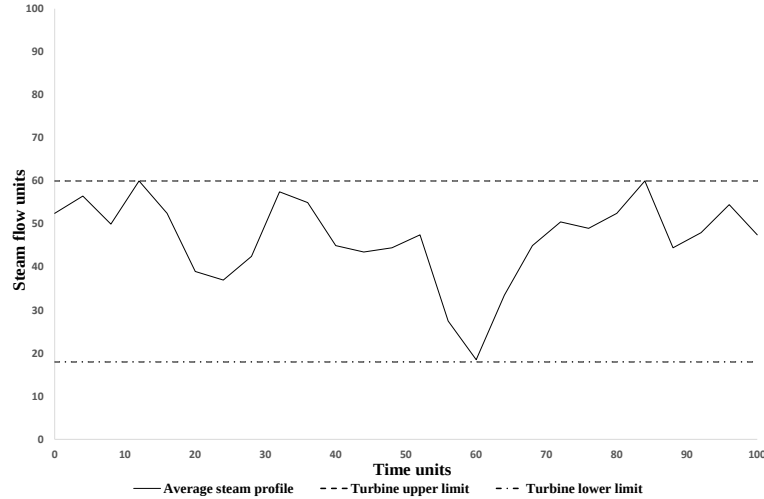


Figure 2.3: The average steam profile between profiles A and B over time, together with the operating limits of the medium capacity turbine.

2.6 Steam shortages for turbines

As mentioned earlier, turbines are high speed rotating machines designed to operate continuously and only to be tripped for maintenance intervals, typically once every two to three years under perfect working conditions. Protection measures are in place to ensure that steam flow to a turbine cannot exceed the maximum allowable limit. Steam flow availability above a turbine's maximum capacity is, therefore, not problematic, but if it is not utilised elsewhere, potential power generation is lost.

Each turbine has a finite number of allowable trips during its lifetime since each trip reduces its life expectancy. In a fluctuating steam flow environment, turbine trips may occur when available steam flow drops below its minimum allowable limit. Start-up protection measures are also in place to prevent a turbine from being brought back online while steam availability is still insufficient. Before a turbine can be restarted, sufficient steam flow must be available for a continuous pre-determined time interval leading up to that event.

As an example, consider an energy recovery plant that operates under fluctuating excess steam availability. Figure 2.6 depicts a hypothetical steam profile over time for such a plant with one turbine operating between a minimum and maximum flow limit of 22 and 75 units respectively. Given that the turbine is operational at time zero, it will generate power up to time period 20 where the available steam flow drops below the minimum threshold, resulting in a trip.

Before the trip there are instances where the steam flow is above 75 units, resulting in potential power generation that cannot be utilised. After the trip at time 20 all power generation potential have gone to waste up to time period 80, whereafter the turbine is brought back online. During this time period there are instances where the turbine could have been operational, but none of these intervals provided an adequate time span to satisfy the plant-specific operating policy for a turbine start-up. Significant power generation capacity is lost under these circumstances and, furthermore, a turbine's life expectancy will be drastically reduced as a result of trips, due to low steam availability.

The challenge is to determine what capacity turbine(s) should be invested in under these conditions. Furthermore, investment decisions must be based on the most cost effective combination of turbines, which will maximise NPV while ensuring that turbine trips are minimised. The decision tradeoff is that an investment can be made in either a less expensive single large generating capacity turbine, or in a number of smaller capacity turbines which may be more costly. For the latter case, however, only partial generation capacity may be lost, compared to a significant loss in power generation

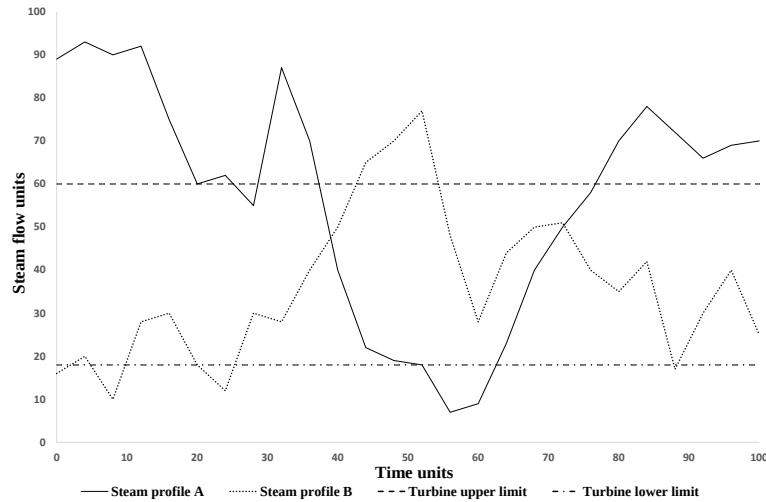


Figure 2.4: The operating limits of the medium capacity turbine under the average steam profile are plotted over time, together with steam profiles A and B.

capacity when a single large turbine trips.

Another decision faced is whether supplementing available off-gases with a supplementary energy resource will result in an overall improvement of NPV. Additional burners may be installed in some boiler houses that will allow for a supplementary energy resource to be burned in times when off-gases cannot produce sufficient steam to keep all or some turbines operational. Note that apart from investment funding considerations, such a resource is typically an expensive commodity to use for power generation. The revenue generated is less than the procurement cost of the gas itself. Therefore, under normal circumstances it does not make sense to investigate any expenditure into such a supplementary resource for this purpose. However, when fluctuating steam availability results in the loss of potential power generation at times, such an investment and periodical supply thereof may prove to be cost efficient.

2.7 The fixed-sequence philosophy

In Chapter 1 the fixed-sequence philosophy, typically found in industry, was briefly discussed. An energy recovery plant requires an operational procedure and typically makes use of a fixed-sequence philosophy, due to its simplicity to incorporate. This operating philosophy does not require any complex control algorithm and the decision making is limited to whether a turbine is loaded until a fixed set point and when a restart may commence. The simplicity of this approach makes it attractive to adopt by an energy recovery plant, as no dynamic control system is required.

In this philosophy each turbine is assigned a specific number, from first to last, which corresponds to the order in which steam is distributed. The first turbine in line, referred to as the *first receiver*, is loaded with steam at each time period until a fixed set point, given sufficient steam availability. The second turbine in line, *i.e.* the *second receiver*, can only receive steam if the first receiver is loaded to its fixed set point. If sufficient steam exists to keep only the second receiver operational, but not the first, none of the turbines will be loaded.

If both the first and second receiver are loaded until their fixed set points, the *third receiver* can acquire steam. The same rationale is applied for the third receiver, so that it cannot be operational if the second receiver is in trip. This holds for all turbines, up to the last receiver. Take note that the fixed load setting per turbine does not necessarily need to be the maximum steam limit and is, therefore, defined as a percentage of the upper limit.

A turbine may only restart if all other receivers before this machine are operational. If a turbine is in

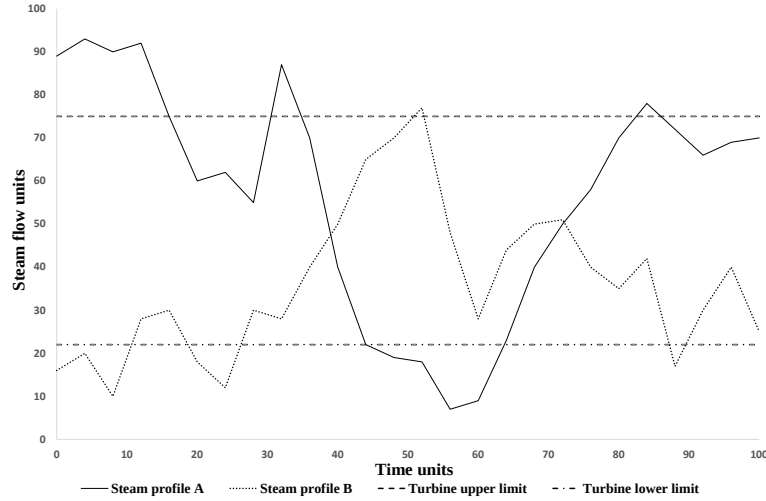


Figure 2.5: The operating limits of the larger capacity turbine with steam profiles A and B plotted over time.

trip, it always implies that all steam receivers following in the hierarchy are also off-line. Therefore, if sufficient steam is not available to restart the next-in-line turbine, but is adequate to allow a receiver lower on the hierarchy to go back online, it is not allowed. This is a direct result of the simplicity, and therefore, the robustness of this operating policy. Take note that multiple turbines, however, are allowed to restart at the same period, given above mentioned rules are satisfied.

2.8 Dynamic control

Another control philosophy that can be followed is dynamic control. Under dynamic control turbines are allowed to be loaded without any pre-set order. For this philosophy a complex control algorithm is required that needs to be programmed into the Works' operating control system. This control algorithm must determine how steam should be distributed, in-time, amongst the turbines and, furthermore, which turbine(s) to trip during time periods of steam shortages. Turbines are not bound to a restart order and may go back online provided sufficient steam availability exists. To restart a turbine sufficient steam needs to be present, similar to the fixed-sequence philosophy, during a predefined fixed time interval. This sufficient steam, however, is defined as steam that could have potentially been utilised by the turbine to be restarted. Therefore, for each time period the potential available steam for start-up is the summation of the unused steam and all the steam loaded to operational turbines above its minimum limits. Take note, steam flow above a turbine's minimum flow limit only influences the quantity of power generation and not the machine's operational status. A turbine may, therefore, receive less steam to assist a restart. It is important to note that a turbine may never be intentionally tripped so that another can come online.

2.9 Gas storage vessels

Gas storage vessels, commonly referred to as gas holders, are used to help regulate off-gas flows and are useful in predicting production and therefore available steam for power generation. A gas holder is typically a large vertical cylinder with inlet and outlet gas pipes connected to it. To regulate the volumes of gas exiting the vessel, a control valve can be fitted to the outlet pipe. This study will not address the control of off-gas flows through such a storage vessel, but will focus on the result thereof to accurately predict near-future flows.

To demonstrate, let an off-gas pipeline consists of an inner pipe diameter of d , which flows into a

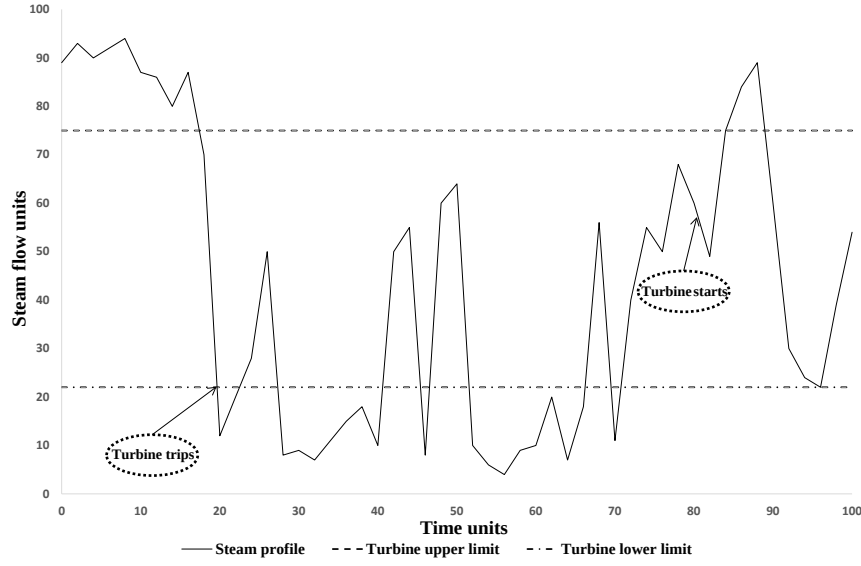


Figure 2.6: A hypothetical excess steam flow scenario over time for a single energy recovery turbine. It demonstrates how power generation capacity goes to waste in a fluctuating availability environment for a turbine operating within the flow limits [22;75].

cylindrical gas holder with an inner diameter D , where $D = 15d$. Gas flows are typically measured in volume per time unit and given by equation (2.24). The volume flow $\dot{F}$ is a function of the free flow cross sectional area A_{ff} , which is perpendicular to the flow and the velocity, *i.e.* the distance traveled over the length of the pipe section per time unit. Let a volume flow over length l of the pipe be needed to transport $\dot{F} = 1$ (volume flow per time unit) of gas and assume the height of the gas holder to be $H = 32l$.

$$\dot{F} = A_{ff}V. \quad (2.24)$$

The volume, F , contained by the pipe segment while flow of $\dot{F} = 1$ is experienced can be calculated by:

$$F = A_{ff}l. \quad (2.25)$$

The working principle of a gas holder is the volume it occupies. If equation (2.25) is used to compare the volume needed to transport $\dot{F} = 1$, to that of the gas holder, the following holds:

$$F_{pipe} : F_{cylinder}, \quad (2.26)$$

since mass conservation must be satisfied, given steady state conditions.

$$\frac{1}{4}\pi d^2l : \frac{1}{4}\pi D^2H, \quad (2.27)$$

$$\frac{1}{4}\pi d^2l : \frac{1}{4}\pi (15d)^2 32l, \quad (2.28)$$

$$1 : 7200. \quad (2.29)$$

Under these assumptions the gas holder contains 7200 flow units, under steady-state conditions. If it is a single flow unit per second, then the gas holder will contain two hours' flow capacity within its boundaries, *i.e.* the control volume. The gas holder may therefore be viewed as a long pipe segment where an off-gas enters at time t and exits at time $t + i$, respectively, where $i = 2$ in

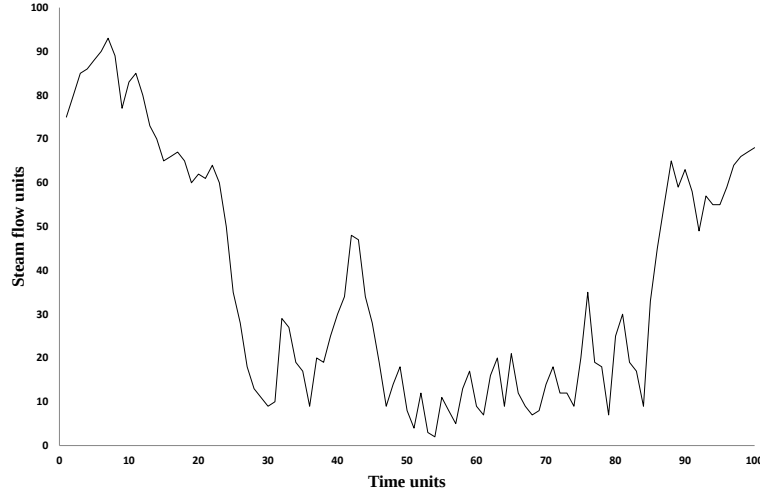


Figure 2.7: Hypothetical steam profile from Table 2.2 plotted over time.

the above mentioned example. This time difference allows for steam production predictions and in conjunction with plant steam usages, an estimated excess steam available at time $t + i$ can be determined.

2.10 Hypothetical steam profile

Chapters 3 and 6 will propose MILP formulations used for explanatory purposes. These formulations will require a steam profile that can be used to emphasise specific model behaviour. A hypothetical steam profile that comprise 100 arbitrary values are chosen for this explanatory purposes. The values are given in Table 2.2 and plotted in Figure 2.7.

Table 2.2: A fluctuating hypothetical steam profile consisting of 100 arbitrarily chosen steam flow values, to be used for conceptual model behaviour explanations in Chapters 3 and 6.

Time intervals	Steam (mass flow unit per time)									
[1..10]	75	80	85	86	88	90	93	89	77	83
[11..20]	85	80	73	70	65	66	67	65	60	62
[21..30]	61	64	60	50	35	28	18	13	11	9
[31..40]	10	29	27	19	17	9	20	19	25	30
[41..50]	34	48	47	34	28	19	9	14	18	8
[51..60]	4	12	3	2	11	8	5	13	17	9
[61..70]	7	16	20	9	21	12	9	7	8	14
[71..80]	18	12	12	9	20	35	19	18	7	25
[81..90]	30	19	17	9	33	45	55	65	59	63
[91..100]	58	49	57	55	55	59	64	66	67	68

Each row in Table 2.2 provides 10 steam values that chronologically follow on each other from left to right. The most left hand column indicates the time interval of the values. In both Table 2.2 and Figure 2.7 the fluctuating nature of the hypothetical steam profile can be observed.

2.11 Real-world data

As mentioned in Chapter 1, this thesis focuses on novel MILP optimisation model formulations. The objective function for each formulation is either to optimise power generation over time or

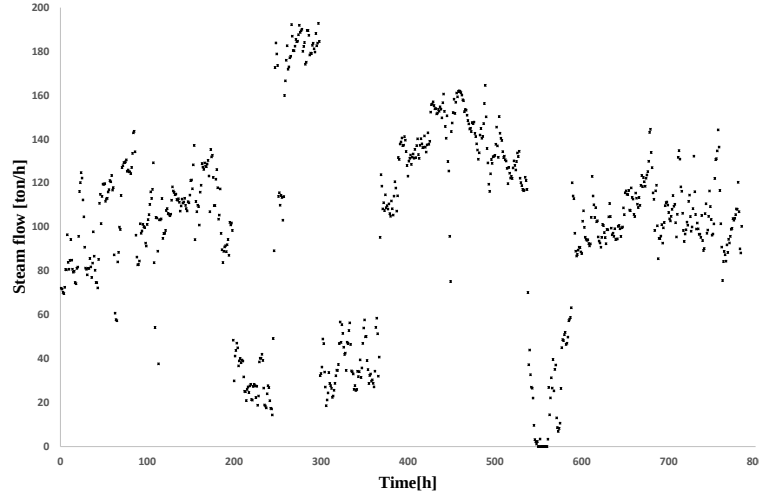


Figure 2.8: An available steam profile experienced by the Works plotted over time.

investment NPV, or both. In order to apply these optimisation models, a specific plant's signature steam profile needs to be simulated. This thesis, however, does not include any investigation into techniques to simulate any future steam profiles.

For the purpose of capturing the uncertainty in future steam flow profiles for an energy recovery plant, historically obtained data are used and considered sufficient. The data are, therefore, assumed to represent future signature steam profiles for an energy recovery plant. As a result, the planning time horizon can be extended to a realistic interval since power generation is a rate and turbine operating parameters do not change over time.

Hourly data are available for this study that comprise three independent excess steam profiles as experienced by an engineering Works. The three profiles are assumed to fully represent the signature steam flow of the Works. Take note that this Works' production processes are subjected to a slow rate of change over time. Hourly data are therefore deemed a true representation of the steam availability for the energy recovery plant during this time period.

The three steam profiles combined comprise 2353 hours of data. The average steam value is 117.7 (ton steam flow per hour) with a standard deviation of 44.8 (ton steam flow per hour). One of the three profiles is plotted in Figure 2.8. Note the fluctuations in the steam profile. The signature steam profile of the Works, *i.e.* the three steam profiles combined, will be referred to as the *real-world data* for the remainder of the study.

Take note, the real-world data represent excess steam that is available for power generation. Unless otherwise stated for the remainder of this thesis all reference towards steam will imply steam that is available for power generation, so that plant usages are already taken into account. Further note, a *time period* of steam is referred to as a single flow instance or value, were as a *time interval* refers to multiple continuous time periods.

2.12 Relevant literature

Various optimisation investment models exist for energy co-generation, which are per definition different than energy recovery plants. These models are typically concerned with multiple power generation mediums and the minimisation of power production's fixed and operational costs in an attempt to determine optimal investment choices [1, 4, 5, 7, 8, 10, 12, 15, 18, 23, 29]. Furthermore, the optimisation models of [1, 7, 4, 8, 10] also focus on linking energy demand with supply. As accustomed to typical engineering investment practises, Henning [10] made use of average-based estimates over time for optimal investment purposes.

A long term optimisation *mixed integer linear programming* (MILP) model for planning purposes was proposed by Thorin *et al.* [24]. Their approach, however, only allowed for the modelling of a number of turbines and boiler houses, but did not focus on investments. The proposed model optimises profits through buying or selling electricity in a spot market. A fixed power generation setup was therefore used with variable electricity costs.

The MILP model proposed by Venter *et al.* [26] optimises power generation from off-gas production through the optimal control of steam flows to any number of turbines in a fluctuating steam production environment. The results show that the involuntary shutdown of turbines is reduced and the utilisation of resources is improved through increased energy recovery. The model does not, however, address any questions regarding what capacity or number of turbines should be invested in and assumes all turbines are already procured and available for power generation. It was demonstrated by Venter *et al.* [27] how the model proposed by Venter *et al.* [26] can be used to determine the power generation effect of various installed turbine options.

No literature could be found on investment or in-time operational optimisation models that utilise an engineering plant's signature gas- or steam flow profiles. There is a definite need for such model formulations, which optimises power generation and investments of an energy recovery plant, based on plant-specific signature steam flow profiles. It should be noted that in the context of energy recovery, all the power is sold back to the engineering plant's power utility. The energy recovered, as a by-product, is typically significantly less than the combined plant usages. Therefore, no energy demand and supply side ever need to be met. Power is sold back at exactly the procurement price. Power fed into the grid is therefore just subtracted from the electricity bill.

2.13 Summary

This chapter provided some basic fundamental engineering principles and equipment definitions needed to comprehend model formulation constraints in following chapters. The working principles of an energy recovery plant that operates under fluctuating steam availability were furthermore discussed, which included power generation losses and turbine trips that occur over time. Literature stressed the absence of relevant optimisation models with regards to optimal energy recovery investments and control.

In the following chapter explanatory optimisation models will be formulated. The modelling of power generation optimisation, which include turbine trips and start-ups, will be demonstrated.

Chapter 3

Conceptual mathematical model development

The previous chapter discussed the basic engineering working principles that are necessary to understand the mathematical model formulations of this thesis. Literature background relevant to this study was also provided. The lack of literature regarding optimisation models on in-time operations and investment choices for an engineering Works, based on plant specific data, was emphasised. This chapter presents explanatory mathematical formulations that form the basis for the modelling philosophy of the following chapters.

3.1 Simplified energy recovery

For an energy recovery plant comprising of power generation turbines, the aim should be to distribute steam in such a way that optimal power is generated. Optimising the power generation directly implies less electricity usages from the national power utility, and therefore, higher cost savings. Once the optimal steam distributions to the turbines over a time horizon are simulated through mathematical modelling, it can be used to determine optimal turbine operations. Furthermore, mathematical models should be formulated to yield the optimal turbine investment choices for an energy recovery plant. The mathematical formulations must support existing energy recovery plants seeking to expand power generation capabilities and plants without any turbines, looking for future expansion.

The simplified concept of energy recovery, and therefore, power generation is demonstrated in this section, through optimal steam distribution to a single turbine. The turbine consists of design constraints that need to be accounted for in the modelling process. These constraints include the minimum and the maximum allowable flow limits, accompanied by the isentropic efficiency used to calculate the steam to power conversion rate. If a turbine receives steam below the minimum limit a trip occurs and no power is generated. Furthermore, it cannot receive steam above the maximum limit and can therefore only generate power until full capacity.

Input parameters to the model are available steam flows over a time horizon, in mass flow per time unit, and a turbine's operational limits. Input parameters for a turbine are the minimum and maximum flow rates, given in mass flow per time unit and the conversion efficiency or rate in mass flow per power unit generated. Power generation optimisation from the model follows by allowing a turbine to be operational for each time period, when steam flow is equal or above the minimum allowable limit. Steam flow to a turbine can never exceed the maximum limit and can, therefore, only be utilised within operational limits. The steam received is converted through the efficiency rate to yield power generation for that time period.

The formulation of the Power Concept Model (PCM) follows. PCM is a MILP that optimises power generation for a single turbine over the time horizon. The primary decision variable is concerned with the operational status of the turbine over time. The following parameters are defined for PCM, with the objective to optimise power generation for a single turbine over time. Parameter and variable definitions are laid out first, followed by the model's mathematical formulation.

For the model formulation the time index is given by $\mathcal{T} = \{1, 2, \dots, |\mathcal{T}|\}$. The parameter for available steam at time t is denoted by m_t^S (mass unit per time period). PCM determines the steam, m_{1t}^S (mass unit per time period), that the turbine receives at time t . The subscript "1" is used in the parameter and variable notation to emphasise the use of a single turbine. This steam is always within the allowable minimum and maximum boundaries, denoted by L_1 (mass unit per time period) and U_1 (mass unit per time period) respectively, or no steam at all. Steam is converted with a corresponding efficiency of η_1^T (flow units per power unit generated) by the turbine. The binary decision variable $y_{1t}^T = 1$ indicates that the turbine is online at time t , whereas a zero implies a non-operational status. The objective of PCM is to:

$$\text{maximise } \sum_{t \in \mathcal{T}} \frac{m_{1t}^S}{|\mathcal{T}| \eta_1^T}, \quad (3.1)$$

subject to

$$m_{1t}^S \leq m_t, \quad \forall t \in \mathcal{T}, \quad (3.2)$$

$$m_{1t}^S \leq U_1 y_{1t}^T, \quad \forall t \in \mathcal{T}, \quad (3.3)$$

$$m_{1t}^S \geq L_1 y_{1t}^T, \quad \forall t \in \mathcal{T}. \quad (3.4)$$

The objective function, *i.e.* optimal power generation over time, is given by equation (3.1). The constraint set (3.2) limits the turbine so that only the available steam at time t may be utilised for power generation. To verify that operations are within allowable limits, constraint sets (3.3) and (3.4) are used. A solution to the PCM (3.1) to (3.4) simulates the optimal operation of a turbine, taking into account a prediction of future steam flows. The solution results may be used to predict optimal future electricity cost savings to be realised by a turbine at an energy recovery plant.

In order to demonstrate the behaviour of PCM, a fictitious energy recovery plant with one turbine, *Turbine I*, is considered. Turbine I comprise an upper flow limit of $U_1 = 50$ (mass unit per time period) and a lower flow limit of $L_1 = 16$ (mass unit per time period). Steam is converted to power at a rate of $\eta_1^T = 5$ (flow units per power unit generated) so that the power generation range is 3.2 to 10.0 power units. It is assumed that the hypothetical steam profile from Table 2.2, Chapter 2 is representative for this fictitious plant. The average steam flow for the profile is 37.5 (flow units per time). Therefore, if all the steam are utilised with $\eta_1^T = 5$, an average of 7.5 power units will be generated over time.

Solving the PCM (3.1) to (3.4) yields an optimal average rate of 5.5 power units with 10 trips over the time horizon. Optimal steam distribution results from PCM and the steam profile provided in Table 2.2 are plotted in Figure 3.1. It can be observed that steam above $U_1 = 50$ and below $L_1 = 16$ cannot be utilised by Turbine I. Note the frequent trip events occur between times 27 and 84. As discussed in Section 2.6, trips are harmful to turbines and should be avoided if possible. An optimisation model is therefore proposed next, which incorporates a protection measure to limit involuntary trips.

3.2 Modelling of a turbine restart after sufficient time

PCM allows a turbine to become operational if sufficient steam is available in that time period. The formulation of PCM, however, does not take into account any steam flow data prior to start-up in the decision making process. To address this shortcoming, the Power Time Concept Model (PTCM) is formulated. PTCM optimises power generation for a single turbine over time with the inclusion of a start-up time constraint. The constraint ensures that when the turbine trips, PTCM will not allow it to become operational for at least two time periods thereafter. This constraint models the protection of a turbine in a fluctuating steam flow environment from instances where availability falls below the minimum operational limit, then momentarily rises above the limit, but

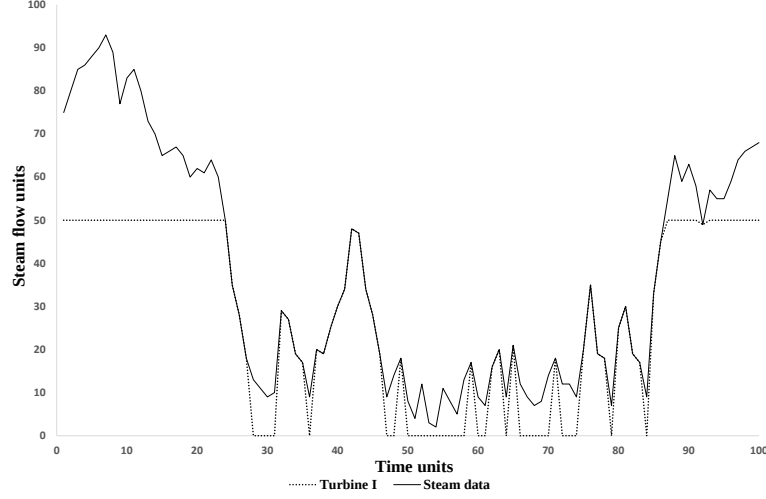


Figure 3.1: Steam distribution determined by PCM over time plotted with the steam profile provided in Table 2.2.

then drops below it again. Due to the conceptual nature of PTCM, this time constraint is a fixed parameter and cannot be changed.

As a result the tripped turbine will stay off-line for at least three time periods, *i.e.* the trip period and the following two. A restart may only commence if sufficient steam is available at that time period. Due to the restart constraint, PTCM will always determine an optimal outcome for which the objective function value and the number of trip occurrences are equal or less than the outcomes of PCM.

The primary decision variables for PTCM are concerned with the operational status of the turbine and if sufficient time has elapsed since a trip occurred. All the parameters and some decision variables are already defined above for PCM and are only restated for completeness:

For PTCM the time index is given by $\mathcal{T} = \{1, 2, \dots, |\mathcal{T}|\}$ and the excess steam available for power generation at time t is denoted by m_t^S (mass unit per time period). The turbine receives m_{1t}^S (mass unit per time period) steam, either within the allowable minimum and maximum boundaries, denoted by L_1 (flow units per time) and U_1 (mass unit per time period), respectively, or none at all. Steam is converted at a rate of η_1^T (mass unit per time period). The decision variable $y_{1t}^T = 1$ indicates that the turbine is online at time t , whereas a zero implies a non-operational status. For a tripped turbine the binary variable $y_{1t}^\delta = 1$ needs to hold before it is allowed back online. The objective of PTCM is to:

$$\text{maximise } \sum_{t \in \mathcal{T}} \frac{m_{1t}^S}{|\mathcal{T}| \eta_1^T}, \quad (3.5)$$

subject to

$$m_{1t}^S \leq m_t, \quad \forall t \in \mathcal{T}, \quad (3.6)$$

$$m_{1t}^S \leq U_1 y_{1t}^T, \quad \forall t \in \mathcal{T}, \quad (3.7)$$

$$m_{1t}^S \geq L_1 y_{1t}^T, \quad \forall t \in \mathcal{T}, \quad (3.8)$$

$$y_{1t}^\delta = 1, \quad \forall t \in \mathcal{T} : t < 4, \quad (3.9)$$

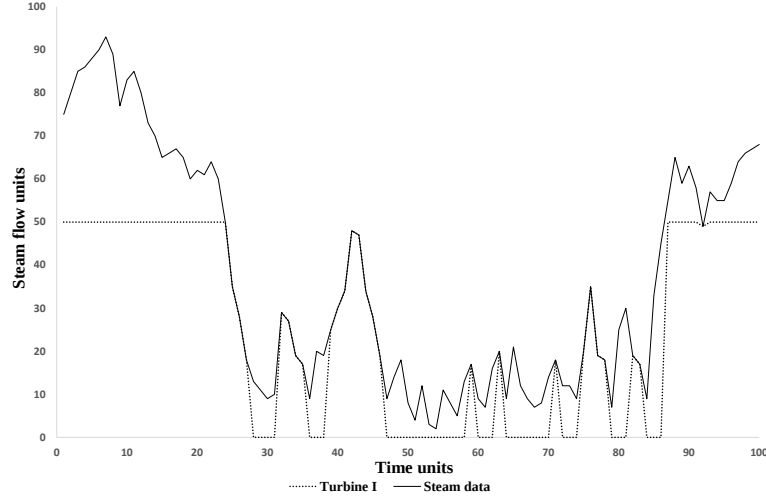


Figure 3.2: Steam distribution to Turbine I for PTCM over time with the steam profile provided in Table 2.2.

$$\sum_{k=t-3}^{t-1} (1 - y_{1k}^T) \geq 3y_{1t}^\delta, \quad \forall t \in \mathcal{T} : t > 3, \quad (3.10)$$

$$y_{1t}^T \leq y_{1t}^\delta + y_{1t-1}^T, \quad \forall t \in \mathcal{T} : t > 1. \quad (3.11)$$

The objective function of PTCM optimises power generation over time for a single turbine and is given by equation (3.5). The turbine may not receive more steam at any time period t than what is available for that period, as per constraint set (3.6). Constraint sets (3.7) and (3.8) ensure that the turbine can only receive steam within the allowable operational limits at any time period t .

If the trip occurs within the first three time periods of the time horizon, a restart will be allowed during the next time period as per constraint set (3.9). From constraint set (3.10) it follows that y_{1t}^δ can only take on the value of one after two time periods following a trip. Therefore, if the turbine trips at time $t > 3$, it follows that $y_{1t+1}^\delta = 0$ and $y_{1t+2}^\delta = 0$. If the turbine is in trip at $t - 1$ and $y_{1t}^\delta = 0$, it will stay off-line at time t , as per constraint set (3.11). It should be noted from constraint set (3.11) that y_{1t}^T is not influenced by y_{1t}^δ if $y_{1t-1}^T = 1$. Note, furthermore, that when $y_{1t}^\delta = 1$ the turbine is eligible to be brought back online, but it is not guaranteed since the minimum steam constraint set of (3.8) must also be satisfied.

Solving PTCM for Turbine I and for the steam profile from Table 2.2 yields an optimal power generation of 5.0 power units with 8 trips. PTCM solves for a lower power generation compared to PCM, but with a reduction in the number of trips. These results are plotted in Figure 3.2 with the steam profile provided in Table 2.2. It can be observed from the plot that at times 39 and 87 by restricting Turbine I to be operational during the time period after a trip occurrence, power generation is lost that could have been utilised. However, as a result of the startup time constraint, two less trips are observed.

As mentioned in Chapter 2, trip occurrences are harmful to a turbine and should be avoided when possible, even if potential power generation is lost. The penalisation of turbine trips follow in later chapters. From Figure 3.2 it is evident between times 47 and 83 that the turbine restarts, only to trip shortly thereafter. This emphasises the need for a time constraint that incorporates sufficient steam availability. The next model formulation allows a turbine to restart only if the preceding time periods consist of sufficient steam. Sufficient steam is any flow quantity equal or above the minimum limit of a turbine.

3.3 Restarting of a turbine provided sufficient steam

A waiting-time period was introduced in the previous section, ensuring that a tripped turbine stays off-line for at least two periods after the trip. Less trips and power generation are experienced under PTCM when compared to PCM. There are, however, still instances where Turbine I is restarted, but trips shortly thereafter as seen between times 47 and 83 of Figure 3.2. Even though PTCM consists of a waiting period before a turbine restart, it does not take steam availability during this time period into account.

In this section a single time constraint set for PTCM is reformulated to include steam availability within the two time periods prior to restart. The Power Steam Availability Model (PSAM), with the objective to optimise power generation for a single turbine over time, only allows a tripped turbine to become operational if the current and preceding two time periods consist of sufficient steam to keep it operational. This constraint set will result in lower optimal power generation in comparison with PCM and PTCM, however, less turbine trips are anticipated.

All parameters and variables definitions for the PSAM are similar to that of the PTCM formulation. Therefore, the model parameters and variables for PSAM are not given again. The PSAM formulation is that of PTCM, except constraint set (3.10) is substituted with:

$$y_{1t}^\delta L_1 \leq m_k, \quad \forall t, k \in \mathcal{T} : t > 3, t - 2 \leq k \leq t - 1, \quad (3.12)$$

To verify that the previous two time periods consist of sufficient steam for a turbine in trip, constraint set (3.12) is used. Take note that this constraint formulation does not influence an operational turbine. Furthermore, if the turbine has not yet been off-line for at least two time periods after the trip, $y_{1t}^\delta = 0$ must hold for the binary decision variable.

Table 3.1: Power generation and number of turbine trips as determined by PCM, PTCM and PSAM for Turbine I operating under the steam profile found in Table 2.2.

Model	Power generation units	Trip occurrences
PCM	5.5	10
PTCM	5.0	8
PSAM	4.7	5

PSAM is solved for Turbine I and the steam profile given in Table 2.2. Results indicate an optimal power generation rate of 4.7 power units with 5 trips. Steam flow under PSAM is plotted with the steam profile provided in Table 2.2 in Figure 3.3. By comparing Figure 3.3 with Figure 3.2, the two-time period steam availability constraint is clearly demonstrated between times 46 to 72. During this time interval there are no preceding two periods with adequate steam to keep Turbine I operational. As a result, three less turbine trips are observed for PSAM when compared to PTCM. A summary on the power generation and turbine trips for the three model outcomes is provided in Table 3.1. From Table 3.1 it can be seen that the formulation of start-up constraints reduce the number of trips, including the optimal rate of power generation.

3.4 Power generation for two turbines under a fixed-sequence philosophy - the Double Sequence Model (DSM)

Recall from the discussion in Chapter 2; a fixed-sequence philosophy implies that turbines receive steam until a fixed setting in a preordained order. The simplicity of operations under this philosophy makes it attractive to adopt for an energy recovery plant. This section provides a formulation to optimise power generation for two turbines operating under a fixed-sequence philosophy.

The objective of the Double Sequence Model (DSM) is to optimise power generation for two turbines under a fixed-sequence philosophy over the time horizon. The two turbines will be referred to as first and second receiver, which respectively refer to the steam loading hierarchy. The first receiver is loaded with steam until a fixed set point, whereafter, the second receiver can acquire steam. It

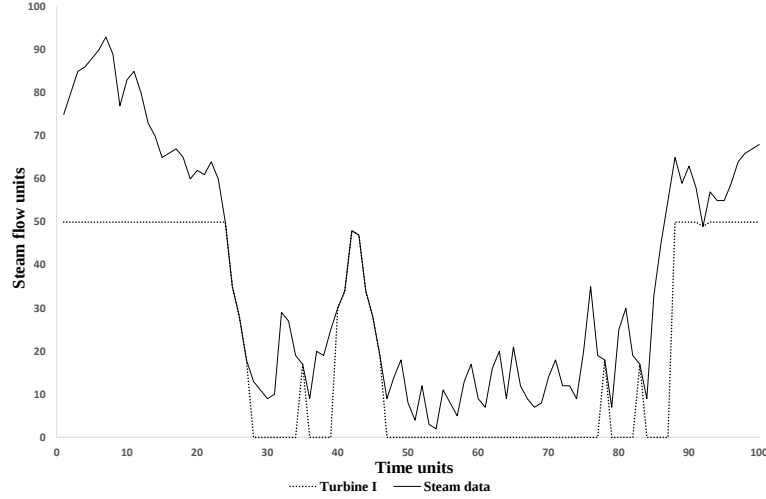


Figure 3.3: Steam distribution for PSAM over time together with the steam profile provided in Table 2.2.

therefore follows directly that the second receiver can only be operational if the first receiver is loaded until its fixed set point.

The two turbines are both subjected to a two-time period sufficient steam constraint, similar to PSAM in the previous section. Therefore, if the first receiver is off-line, adequate steam must be present during the previous two and the current time period for it to restart. The same philosophy is applicable for the second receiver. The available steam for the second receiver, however, is the steam present after the first receiver has been loaded until the fixed setting.

For DSM the time index is given by $\mathcal{T} = \{1, 2, \dots, |\mathcal{T}|\}$ and the available steam at time t is denoted by m_t^S (mass unit per time period). The first receiver receives m_{1t}^S (mass unit per time period) and the second obtains m_{2t}^S (mass unit per time period), each within the operational limits of the machine. For the first receiver the minimum and maximum limits are given by L_1 (mass unit per time period) and U_1 (mass unit per time period), where the second receiver's limits are denoted by L_2 (mass unit per time period) and U_2 (mass unit per time period), respectively. Steam is converted by the turbines with corresponding efficiencies of η_1^T (flow units per power unit generated) and η_2^T (flow units per power unit generated) for the first and second receiver, respectively. The first receiver is loaded until full capacity.

The decision variable $y_{1t}^T = 1$ indicates that the first receiver is online at time t and $y_{2t}^T = 1$ states that the second receiver is operational, whereas a zero for both implies a non-operational status. To determine if the first receiver may be allowed back online at time t after sufficient steam availability, $y_{1t}^b = 1$ must hold and $y_{2t}^b = 1$ is required for the second receiver to be restarted. Note, therefore, the superscript “ b ” refers to “back online”. The objective of DSM is to:

$$\text{maximise } \sum_{t \in \mathcal{T}} \left(\frac{m_{1t}^S}{|\mathcal{T}| \eta_1^T} + \frac{m_{2t}^S}{|\mathcal{T}| \eta_2^T} \right), \quad (3.13)$$

subject to

$$m_{1t}^S \leq m_t^S, \quad \forall t \in \mathcal{T}, \quad (3.14)$$

$$m_{2t}^S \leq m_t^S - m_{1t}^S, \quad \forall t \in \mathcal{T}, \quad (3.15)$$

$$y_{1t}^T \geq y_{2t}^T, \quad \forall t \in \mathcal{T}, \quad (3.16)$$

$$m_{1t}^S \geq U_1 y_{2t}^T, \quad \forall t \in \mathcal{T}, \quad (3.17)$$

$$m_{1t}^S \leq U_1 y_{1t}^T, \quad \forall t \in \mathcal{T}, \quad (3.18)$$

$$m_{1t}^S \geq L_1 y_{1t}^T, \quad \forall t \in \mathcal{T}, \quad (3.19)$$

$$m_{2t}^S \leq U_2 y_{2t}^T, \quad \forall t \in \mathcal{T}, \quad (3.20)$$

$$m_{2t}^S \geq L_2 y_{2t}^T, \quad \forall t \in \mathcal{T}, \quad (3.21)$$

$$y_{1t}^b = 1, \quad \forall t \in \mathcal{T} : t < 4, \quad (3.22)$$

$$y_{1t}^b L_1 \leq m_k, \quad \forall t \in \mathcal{T} : t > 3, t - 2 \leq k \leq t - 1, \quad (3.23)$$

$$y_{1t}^b \geq y_{1t}^T - y_{1t-1}^T, \quad \forall t \in \mathcal{T} : t > 3, \quad (3.24)$$

$$y_{2t}^b = 1, \quad \forall t \in \mathcal{T} : t < 4, \quad (3.25)$$

$$y_{2t}^b L_2 \leq m_{t-k} - m_{1t}^S, \quad \forall t \in \mathcal{T} : t > 3, t - 2 \leq k \leq t - 1, \quad (3.26)$$

$$y_{2t}^b \geq y_{2t}^T - y_{2t-1}^T, \quad \forall t \in \mathcal{T} : t > 3. \quad (3.27)$$

The objective function of DSM is to optimise power generation for two turbines operating under a fixed-sequence loading hierarchy and is given by equation (3.13). The two constraint sets for available steam are defined by (3.14) and (3.15). Steam flow to the first receiver cannot be more than what is available at time t , as per constraint set (3.14). Steam available for the second receiver is the residual quantity after the first receiver has received steam as ensured by constraint set (3.15). Constraint (3.16) ensures that the second receiver can only be operational if the first receiver is online. Furthermore, the second receiver can only be operational if the first receiver is loaded up to its maximum capacity, as per constraint set (3.17). The sets of constraints from (3.18) to (3.21) ensure both turbines operate within permissible machine limits.

To determine if sufficient steam is available to restart the first receiver, constraint sets (3.22) to (3.24) are used. The sets of constraints (3.25) to (3.27) ensure that the second receiver can only be brought back online if satisfactory steam is present for the previous two, and current time period. Constraint (3.26) ensures that for the second receiver to become operational, following a trip, sufficient steam must be available after the first receiver has been loaded.

Table 3.2: Operating parameters for Case 1 and 2, *i.e.* the upper and lower allowable flow limits, rate of steam flow per power unit generation, and the maximum and minimum power generation limits.

Case		U (mass unit per time period)	L (mass unit per time period)	η^T (mass flow per power unit generated)	Maximum (power unit)	Minimum (power unit)
1	First receiver	50	16	5.0	10	3.2
	Second receiver	50	20	4.8	$10\frac{5}{12}$	$4\frac{1}{6}$
2	First receiver	50	20	4.8	$10\frac{5}{12}$	$4\frac{1}{6}$
	Second receiver	50	16	5.0	10	3.2

3.4.1 Results for DSM

DSM is solved for two scenarios, **Case 1** and **2**, for the steam profile provided in Table 2.2. The operational parameters for the first and second receiver turbines in Case 1 and 2 are given in Table 3.2. Turbine I is the first receiver for Case 1 and the second receiver for Case 2. Another turbine, *Turbine II*, is the second receiver for Case 1 and the first receiver for Case 2. Turbine II operates within allowable steam flow limits of 16 (flow units per time) to 50 (flow units per time) and converts steam to power at a rate of 4.8 (flow units per power unit generated). The two turbines combined can utilise up to 100 steam flow units per time period, whereas the maximum flow value in the steam profile from Table 2.2 is 93 flow units. Optimal results from DSM for both scenarios are given in Table 3.3.

Table 3.3: The turbine hierarchy obtained by DSM for Turbine I and II under the hypothetical steam profile from Section 2.10, followed by the trips per turbine, power generation units per turbine and the total rate of power generation.

Case	Trips for the first and second receiver		Power generation units for the first and second receiver		Combined power generation units
1	5	1	4.7	1.0	5.7
2	2	1	4.5	0.9	5.4

Solving DSM for **Case 1**, yields six trips where Turbine I, the first receiver, contributes five of these occurrences. The total power generation is 5.7 power units of which Turbine II, the second receiver, generates 1.0 of these units. Even though the combination of both turbines are able to accommodate the maximum available steam from the profile provided in Table 2.2, the additional power generation is less than 10% of Turbine II's full capacity.

The steam flow distributions, as determined by DSM to the first and second receiver for Case 1, are plotted in Figure 3.4, together with the steam profile from Table 2.2. From the graph it can be seen that whenever Turbine II is operational, Turbine I is at full capacity. Note that even though sufficient steam flow exists at time 15 to keep both machines operational, *i.e.* 65 (mass unit per time period) where $L_1 + L_2 = 16 + 20 = 36$ (mass unit per time period), Turbine II trips due to the constraint that ensures Turbine I is fully loaded. It should be noted from Table 3.2 that Turbine II converts steam at a better efficiency than Turbine I, it might therefore be deemed as a trivial decision to rather allocate Turbine II as the first receiver.

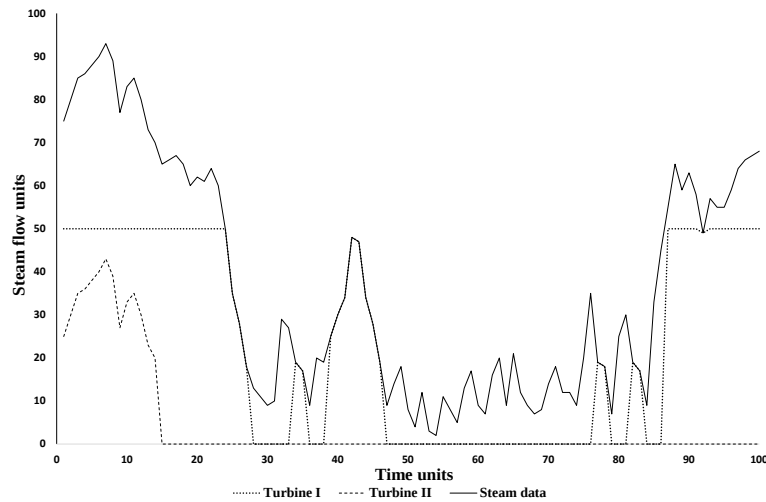


Figure 3.4: DSM steam distributions for Case 1, together with the steam profile provided in Table 2.2, over time.

Solving DSM is for **Case 2**, where Turbine II is the first receiver and Turbine I the second receiver, a lower power generation is observed. This scenario yields a rate of 5.4 power units, but with only

3 trips. Therefore, under a fixed-sequence philosophy it is not necessarily the more efficient turbine that should be allocated as the first steam receiver.

For an energy recovery plant it is necessary to determine the optimal loading hierarchy of a fixed-sequence philosophy, in order to optimise power generation. The need, therefore, exists for the formulation of an optimal loading model that optimises power generation for any number of turbines with different conversion efficiencies over time. Such a formulation should, furthermore, allow for any preferred predetermined time interval of sufficient steam availability. The number of trips should, however, also be noted. The impact of trips is addressed in Chapter 5.

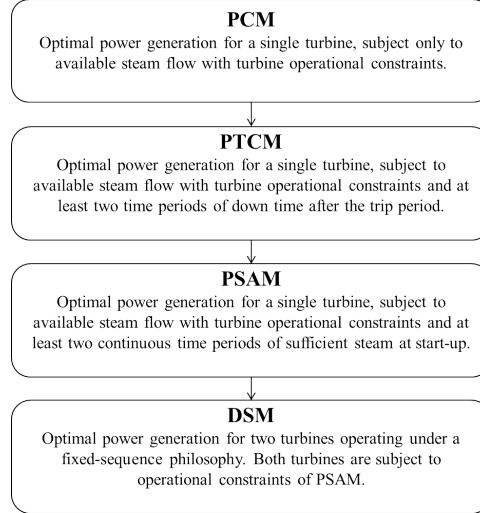


Figure 3.5: Flow chart for the four MILP concept formulations found in this chapter.

Figure 3.5 displays a flow chart of the four MILP optimisation formulations found in this chapter. PCM demonstrates the concept of energy recovery for a single turbine. The PTCM formulations enforce a two-time waiting period constraint for a single turbine by not allowing it to become operational during the two time periods following a trip occurrence. To ensure that the two waiting time periods before start-up comprise sufficient steam availability, the PSAM was formulated. The DSM formulation optimises power generation between two turbines operating under a fixed-sequence philosophy. Both turbines from DSM operate under the constraints formulated for the PSAM.

3.5 Summary

In this chapter four MILP optimisation formulations were presented. The concept of power generation for a single turbine operating in a fluctuating steam flow environment over time was demonstrated by PCM. PCM is followed by the formulation of PTCM that forces a turbine to be off-line for at least two time periods after a trip has occurred. Power generation decreased from 5.5 power units to 5.0 power units for the hypothetical steam profile depicted in Table 2.2. Two fewer trip occurrences were experienced between PTCM and PCM. PTCM was followed by the formulation of PSAM, which allows a turbine to restart only if the previous two and current time period provided sufficient steam to keep it operational. Optimal power generation for PSAM determined at 4.7 power units with three less trips, compared to PTCM.

The fourth MILP model formulation, DSM, optimises power generation for two turbines operating under a fixed-sequence philosophy where two time periods of sufficient steam are required for a turbine restart. Results indicate that optimal power generation does not necessarily occur when the more efficient turbine is the first steam receiver. Furthermore, the results show that optimal power generation between two turbines under a fixed-sequence operating philosophy does not imply minimum trip occurrences. The next chapter provides two model formulations that determine the optimal loading hierarchy for an energy recovery plant that operates under a fixed-sequence philosophy.

Chapter 4

Optimal hierarchy for the fixed-sequence loading philosophy

Power generation concept models for turbines operating under a fluctuating steam flow environment were introduced in Chapter 3. DSM, the final MILP model presented, optimises power generation for two turbines operating under a fixed-sequence philosophy. In this chapter two MILP formulations are proposed. The objective of both models is to determine the optimal loading hierarchy between any number of turbines, operating under the fixed-sequence philosophy, as discussed in Section 2.7.

4.1 Power generation under a fixed-sequence philosophy

The operational procedure of the fixed-sequence philosophy is discussed in Section 2.7. It is mentioned that a turbine is loaded up to a fixed set point, which is not necessarily the maximum flow limit, whereafter the next machine may acquire steam. As a result, operations under a fixed-sequence philosophy can dictate that a turbine is never loaded up to full capacity. The formulations that follow in this chapter, therefore, should include the option that a turbine might not necessarily be loaded to maximum capacity.

To explain the rationale for a decision not to load a turbine to full capacity, consider the hypothetical steam scenario depicted in Figure 4.1. Let this steam scenario capture the signature flow profile of a plant where two turbines are operational under a fixed-sequence philosophy. Furthermore, let the first receiver operate within allowable steam limits of 12 (mass unit per time period) to 25 (mass unit per time period) and the second receiver between 40 (mass unit per time period) and 150 (mass unit per time period), respectively.

If the first receiver is loaded to full capacity, followed by the second receiver, $25+40=65$ steam flow units are required per time period to keep both machines operational. The dashed line in Figure 4.1 indicates 65 steam flow units per time period. Note that the hypothetical steam flow drops seven times below the dashed line, indicating power generation losses due to trips. If the energy recovery plant, however, changes the operating policy and load the first receiver to only 80% of maximum capacity, *i.e.* $0.8 \cdot 25 = 20$, then $20+40=60$ steam flow units are needed per time period to allow both turbines to stay operational. The dotted line indicates 60 steam flow units per time period. Note that the steam availability drops below 60 flow units only once, indicating that an 80% set point yields less trips and therefore more power generation.

The real-world data discussed in Section 2.11 is used as available steam parameter inputs for all simulation scenarios reported in this chapter. A sufficient steam time limit before start-up of 15 hours, as typically found in industry, is used for all scenarios.

4.2 Plant operational procedure model

The formulation of DSM in Section 3.4 optimises power generation amongst two turbines under a fixed-sequence philosophy. The first and second receiver are assigned as input parameters for

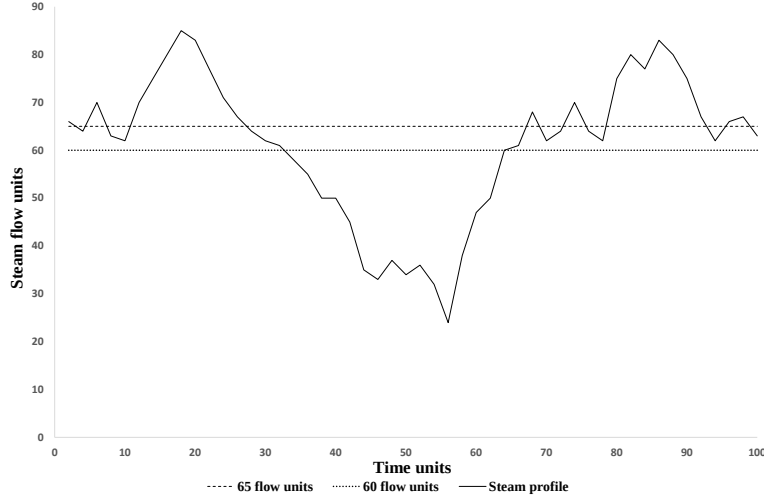


Figure 4.1: A hypothetical steam flow over time scenario plotted with two constant lines, *i.e.* a 60 (flow units) and a 65 (flow units).

DSM. The MILP formulation that follows in this section determines the optimal loading hierarchy for any number of turbines under a fixed-sequence philosophy. A turbine can comprise any operating capacity or efficiency. The operational philosophy dictates that the turbines are loaded according to a specific hierarchy. Each turbine is loaded until a fixed percentage of its maximum allowable limit and when this level is reached, the next turbine in line may acquire steam. All fixed load settings are percentage input parameters towards the model.

Tripped turbines are restarted according to the loading hierarchy. Therefore, if more than one turbine is off-line and insufficient steam exists to bring the next-in-line machine back into operation, no restart may occur. Even if adequate steam is present to restart a smaller capacity turbine, which is not the next-in-line, a restart will not commence. This model is referred to as the *Plant Operational Queueing Model* (POQM), where the objective of POQM is to determine optimal power generation over the time horizon from the optimal turbine loading hierarchy.

Turbine capacities, efficiencies, different loading hierarchies and fixed load points are input parameters. The following parameter definitions are required to formulate POQM: The time index is represented by $\mathcal{T} = \{1, 2, \dots, |\mathcal{T}|\}$. The index set of all turbines is given by $\mathcal{I}^T = \{1, 2, \dots, |\mathcal{I}^T|\}$, and the index set of all different turbine queues is defined by $\mathcal{I}^Q = \{1, 2, \dots, |\mathcal{I}^Q|\}$. The loading hierarchies with the corresponding turbine numbers from $\mathcal{I}^T$ are defined by $\mathcal{I}^T(q)$, where $q \in \mathcal{I}^Q$. Note, therefore, every vector in $\mathcal{I}^T(q)$ refers to a unique loading hierarchy subset from the set of available turbines.

The input parameter m_t^S denotes the available steam for power generation at time t that can be distributed amongst the optimal loading hierarchy from $\mathcal{I}^T(q)$. Turbine $i \in \mathcal{I}^T$ can operate within the allowable minimum and maximum limits of L_i (mass unit per time period) and U_i (mass unit per time period) respectively, where steam is converted at an efficiency rate of η_i^T (mass unit flow per power unit generated). If a turbine trips, a minimum time of δ^T , which coincides with sufficient steam availability, needs to elapse before the tripped turbine may be brought back online.

For a specific queue $q \in \mathcal{I}^Q$ each turbine $i \in \mathcal{I}^T$ in the j^{th} position of the loading hierarchy $\mathcal{I}^T(q)$ receives steam up to a fixed fraction of its upper limit, α_{jq} , before the next-in-line may be loaded where $j \in \mathcal{I}^T(q)$ and $q \in \mathcal{I}^Q$. Note that every queue $q \in \mathcal{I}^Q$ has a unique set of α_{jq} parameters, where each $j \in \mathcal{I}^T(q)$ refers to a turbine $i \in \mathcal{I}^T$. Furthermore, for every $j \in \mathcal{I}^T(q)$ and $q \in \mathcal{I}^Q$ it follows that $0 \leq \alpha_{jq} \leq 1$.

There are two main groups of decision variables defined for POQM. The first group relates to the distribution of steam to the turbines. Let $m_{iqt}^S \geq 0$ be the steam flow to turbine $i \in \mathcal{I}^T(q)$ for queue $q \in \mathcal{I}^Q$ at time $t \in \mathcal{T}$. The binary decision variable $y_{iqt}^T = 1$ specifies that turbine $i \in \mathcal{I}^T(q)$ for queue $q \in \mathcal{I}^Q$ is operational at time t , and a zero value relates to a non-operational status. The

second binary variable decision group is λ_q correlates to a queue $q \in \mathcal{I}^Q$, indicating which fixed-sequence hierarchy delivers optimal power generation. Note, therefore, only a single λ_q variable is non-zero that correlates with the optimal loading hierarchy, where $q \in \mathcal{I}^Q$. The formulation of POQM is to:

$$\text{maximise } \sum_{i \in \mathcal{I}^T(q)} \sum_{q \in \mathcal{I}^Q} \sum_{t \in \mathcal{T}} \frac{m_{iqt}^S}{\eta_i^T |\mathcal{T}|}, \quad (4.1)$$

subject to

$$\sum_{q \in \mathcal{I}^Q} \lambda_q = 1, \quad (4.2)$$

$$\sum_{i \in \mathcal{I}^T(q)} m_{iqt}^S \leq \lambda_q m_t^S, \quad \forall q \in \mathcal{I}^Q, t \in \mathcal{T}, \quad (4.3)$$

$$m_{iqt}^S \leq \alpha_{iq} U_i y_{iqt}^T, \quad \forall i \in \mathcal{I}^T(q), q \in \mathcal{I}^Q, t \in \mathcal{T}, \quad (4.4)$$

$$m_{iqt}^S \geq L_i y_{iqt}^T, \quad \forall i \in \mathcal{I}^T(q), q \in \mathcal{I}^Q, t \in \mathcal{T}. \quad (4.5)$$

$$y_{iqt}^T \leq y_{jq,t}^T, \quad \forall i \in \mathcal{I}^T(q), j \in \mathcal{I}^T(q) : i > j, q \in \mathcal{I}^Q, t \in \mathcal{T}, \quad (4.6)$$

$$m_{iqt}^S \geq \alpha_{iq} U_i y_{i+1qt}^T, \quad \forall i \in \mathcal{I}^T(q) : i < |\mathcal{I}^T(q)|, q \in \mathcal{I}^Q, t \in \mathcal{T}, \quad (4.7)$$

$$\lambda_q m_k^S \geq L_i y_{iqt}^T - (1 - y_{iqt}^T - y_{iqt-1}^T)M, \quad \forall i \in \mathcal{I}^T(q) : i = 1, q \in \mathcal{I}^Q, \begin{matrix} t \in \mathcal{T}, \\ t > \delta^T, \\ k \geq t - \delta^T, \\ k \leq t - 1, \end{matrix} \quad (4.8)$$

$$\lambda_q m_k^S - \sum_{j \in \mathcal{I}^T(q) : j < i} \alpha_{jq} U_j \geq L_i y_{iqt}^T - (1 - y_{iqt}^T - y_{iqt-1}^T)M, \quad \forall i \in \mathcal{I}^T(q) : i > 1, \begin{matrix} q \in \mathcal{I}^Q, \\ t \in \mathcal{T}, \\ k \geq t - \delta^T, \\ k \leq t - 1, \end{matrix} \quad (4.9)$$

The objective of POQM is to optimise power generation over the time horizon $\mathcal{T}$, *i.e.* the average steam flow to each turbine over the efficiency of that turbine, given by equation (4.1). Constraint (4.2) ensures that only one fixed-sequence hierarchy is chosen. Constraint set (4.3) ensures that the steam distribution between turbines for queue $q \in \mathcal{I}^Q$ at time $t \in \mathcal{T}$, may not exceed the available steam for that time period. From the sets of constraints (4.2) and (4.3) it follow that POQM allows only one turbine loading hierarchy to receive potential steam for power generation.

The sets of constraints (4.4) and (4.5) guarantee that a turbine can only operate within its allowable limits. The α_{iq} parameters in constraint set (4.4) ensure that every turbine $i \in \mathcal{I}^T(q)$ for queue $q \in \mathcal{I}^Q$ can only be loaded up to the pre-determined fixed fraction of its maximum allowable limit. For queue $q \in \mathcal{I}^Q$ constraint set (4.6) allows a turbine only to be operational if the previous machine in the hierarchy is online. Constraint set (4.7) ensures that a turbine must be loaded until the fixed setting before the next-in-line machine may receive steam.

Before a turbine may be brought back online, the sets of constraints (4.8) and (4.9) must be satisfied. M corresponds to a sufficient large value. When all turbines are in trip, constraint set (4.8) ensures that steam was present during each of the previous δ^T time periods that could have kept the first receiver operational. From the second receiver and onwards, the formulation of constraint set (4.9) allows a turbine to become operational if sufficient steam existed for each of previous δ^T time periods, after all prior receivers have been loaded to its fixed set points.

Table 4.1: Maximum and minimum power generation capabilities, followed by the efficiency, together with the upper and lower limits for Turbine I and II.

Turbine	Max [MW]	Min [MW]	η_i [ton/(h·MW)]	U_i [ton/h]	L_i [ton/h]
I	5.0	2.0	5.0	25.0	10.0
II	30.0	8.0	5.0	150.0	40.0

4.2.1 Results for POQM

As mentioned above, all scenarios in this chapter use the real-world data from Section 2.11 as steam input parameters. Each scenario refers to either a different set of turbines, different fixed set points, or both. Some simulation results are reported for each scenario and individually labeled as a *Case*. Throughout this study, results for various Case numbers will be reported. For POQM each Case number will be given a superscript “ q ”. The ability of POQM to determine optimal power generation results is now demonstrated.

Assume an energy recovery plant consists of two turbines, operating under a fixed-sequence philosophy. Let the turbines be a 5MW and a 30MW machine, where both operate at an efficiency of 5 ton steam per hour per Mega Watt generated. The plant enforces a waiting period of 15 hours of sufficient steam before a turbine startup is allowed. Operational parameters for these two turbines are given in Table 4.1. The maximum and minimum power generation limits are given in the table, followed by the turbine’s efficiency and the corresponding upper and lower flow limits. For the remainder of this chapter these two turbines will be referred to as *Turbine I* and *Turbine II*.

Furthermore, assume that this plant operates its first receiver at an 80% fixed load setting of the maximum capacity. Therefore, if Turbine I is the first receiver it will be loaded up to 4.00MW, whereas if Turbine II is the first receiver it will be loaded to 24.00MW. In order to demonstrate the behaviour of POQM, three Cases of parameter settings are considered. All of the results from this section are reported in Table 4.2. The first column indicates the Case number, followed by the optimal turbine loading hierarchy, the corresponding α parameters, trips experienced per turbine, power generation per turbine (Mega Watts) and the total power generation (Mega Watts).

Take note, throughout this thesis values regarding Mega Watts (MW) are reported. If the value does not contain a decimal number, *i.e.* 5MW or 30MW it will always refer to a turbine name, therefore the 5MW turbine and 30MW turbine in this instance. A Mega Watt value containing a single decimal number, *i.e.* 5.0MW or 30.0MW refers to either the minimum or maximum capacity of that turbine, as seen in Table 4.1. If the reported Mega Watt value contains two decimal numbers, *i.e.* 4.00MW or 24.00MW it either refers to quantity of power that can potentially be generated or the outcome as determined by the model formulation.

Case 1^q: POQM is solved for this plant’s two operational turbines, that is Turbine I and II, where the first receiver is only loaded until 80%, *i.e.* $\alpha_{1q} = 0.8$, where $\alpha_{2q} = 1.0$ for $q \in \mathcal{I}^Q$. Note that it is not sensible for the α parameter of the last turbine in the hierarchy to be less than 1.0, since none of the excess steam can be utilised for power generation after this turbine has been loaded. Optimal results indicate that Turbine I must be the first receiver, followed by Turbine II.

Power generation for Turbine I and II are plotted in Figures 4.2 and 4.3, respectively. The effect of $\alpha_{1q} = 0.8$ for $i \in \mathcal{I}^T$ on maximum power generation for Turbine I is clearly demonstrated in Figure 4.2 where a maximum of only 4.00MW is allowed. Note that for each period where Turbine I is not loaded up to 4.00MW, Turbine II is off-line. The fluctuating nature of the real-world data is evident from the power generation depicted in Figure 4.3. The frequent trips, 14 in total, of Turbine II can furthermore be observed in Figure 4.3.

The results in Table 4.2 indicate that during the time of operation, Turbine I experiences 4 trips and generates power totalling 3.81MW. Turbine II trips 14 times and contributes 16.99MW to the total of 20.80MW over the time horizon. The fluctuations in steam availability are evident in the total of 18 trips between the two turbines. The optimal results as determined by POQM are bound to the prerequisite of $\alpha_{1q} = 0.8$ for $i \in \mathcal{I}^T$. By changing the value of α_{1q} , a different optimal loading hierarchy might be obtained. Furthermore, POQM may solve for another optimal power

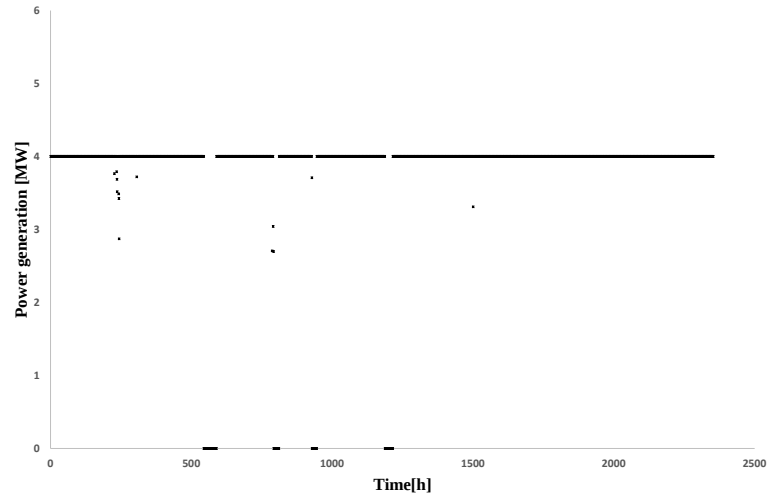


Figure 4.2: Case 1^q: Power generation over time for Turbine I as the first receiver where $\alpha_{1q} = 0.8$ for $i \in \mathcal{I}^T$.

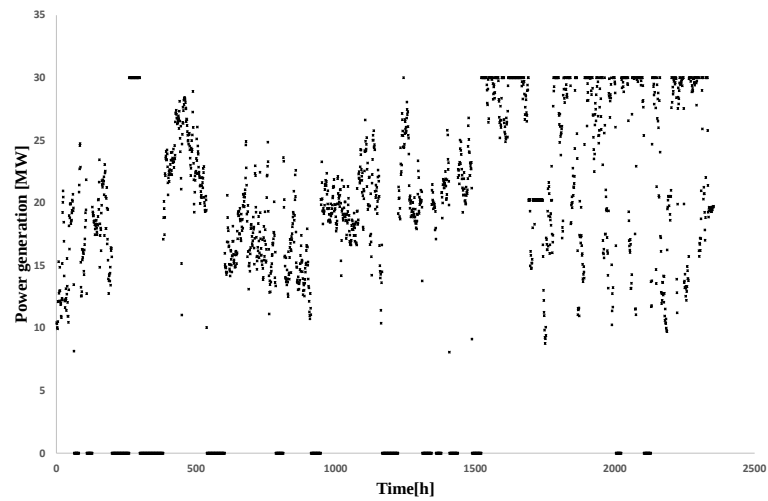


Figure 4.3: Case 1^q: Power generation over time for Turbine II as the second steam receiver where $\alpha_{1q} = 0.8$ for $i \in \mathcal{I}^T$.

Table 4.2: Scenario results for Case 1^q to 3^q . The results include the optimal turbine hierarchy, loading factors, trips, individual and total power generation for Turbine I and II.

Case number	Turbine hierarchy		α		Trips per turbine		Power generation per turbine [MW]		Combined power generation [MW]
1^q	I	II	0.8	1.0	4	14	3.81	16.99	20.80
2^q	I	II	0.9	1.0	4	14	4.28	16.65	20.93
3^q	I	II	1.0	1.0	4	15	4.76	16.20	20.96

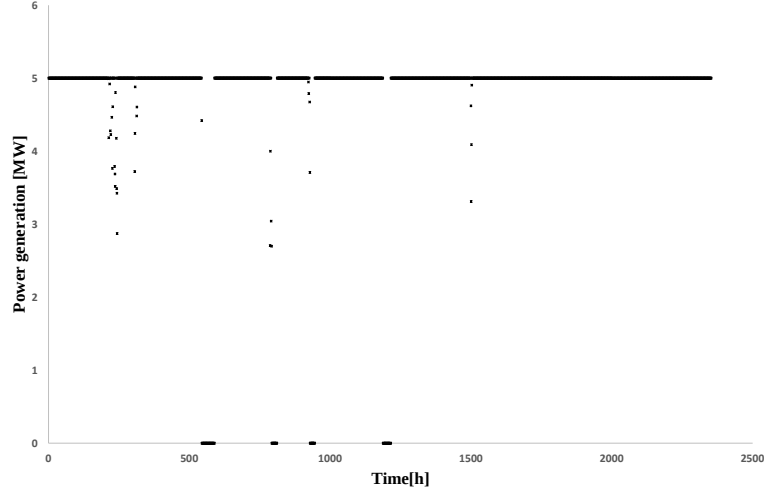


Figure 4.4: Case 3^q : Power generation over time for Turbine I where $\alpha_{1q} = 1.0$ for $i \in \mathcal{I}^T$.

generation and a different number of trips.

Case 2^q : The fixed set point is changed to $\alpha_{1q} = 0.9$ for $i \in \mathcal{I}^T$. Solving POQM under these parameter settings, optimal results are obtained and reported in Table 4.2. Results indicate that the optimal turbine loading hierarchy and the number of trips are unchanged. Total power generation, however, increases from 20.80MW to 20.93MW, where Turbine I generates power at 4.28MW and Turbine II at 16.65MW. By increasing the α parameters more power generation is realised.

Case 3^q : POQM is now solved for the scenario where $\alpha_{1q} = 1.0$ for $i \in \mathcal{I}^T$ and outcomes reported in Table 4.2. As can be seen from the results, Turbine I stays the first receiver with the same number of trips compared to Case 1^q and 2^q , whereas Turbine II experiences one more trip, *i.e.* 15. Furthermore, the optimal power generation increases, so that 20.96MW is produced between the two turbines. It should be noted that even though the percentage power generation increase is small, *i.e.* $(20.96 - 20.80)/20.80 = 0.8\%$, it is only due to an operational setting change.

POQM's results for Case 3^q are plotted in Figures 4.4 and 4.5. The similarities for Turbine I in Figures 4.2 and 4.4 are evident, with the exception that power is generated up to the full 5.00MW capacity for Case 3^q . More steam is utilised for Case 3^q when compared to Case 1^q , however, an extra trip occurs for Turbine II.

Figure 4.6 displays the unused steam for Case 3^q . The steam represented in the graph possesses a power generation potential of 2.59MW, which is 12.93 (ton steam per hour) that cannot be utilised by the two turbines, and therefore, goes to waste. Note from the plot that for some periods the unused steam flow is above 150 (ton steam per hour), which is the maximum capacity of Turbine II. Furthermore, for 30.90% of the time there are steam flows that cannot be utilised.

The results from Case 1^q to 3^q show how POQM can be utilised to acquire the optimal turbine loading hierarchy and, therefore, optimal power generation. In Section 4.1 a motivation is provided on why an energy recovery plant may choose not to load a turbine to full capacity. The results given in this section indicate that as the α parameter is increased, so does the optimal power generation. The possibility may exist that for the optimal power generation between Turbine I and II, under

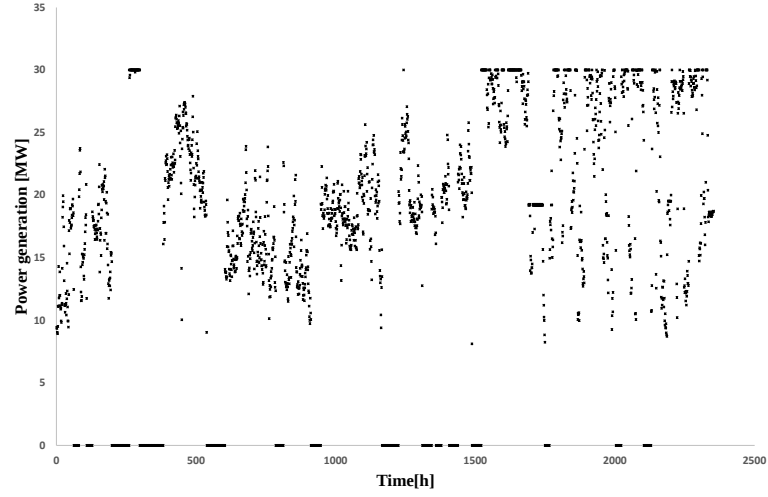


Figure 4.5: Case 3^q: Power generation over time for Turbine II where $\alpha_{1q} = 1.0$ for $i \in \mathcal{I}^T$.

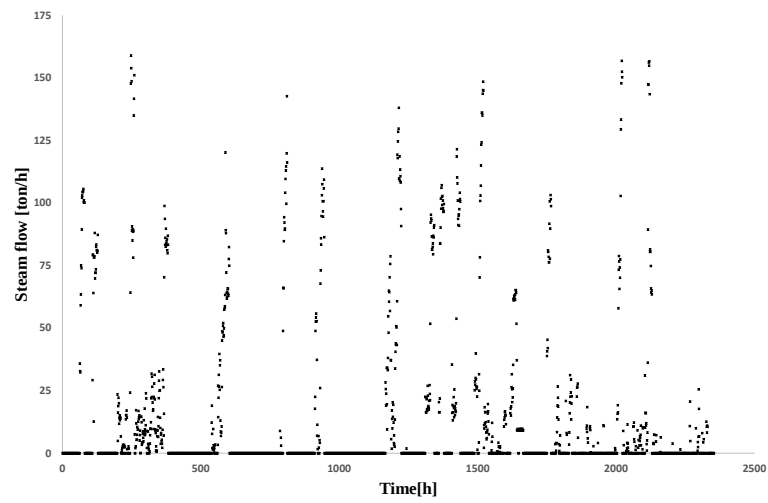


Figure 4.6: Case 3^q: Excess available steam over time that cannot be utilised by Turbine I or II.

a fixed-sequence philosophy, $\alpha_{1q} < 1$ for $i \in \mathcal{I}^T$. In the next section a formulation follows where α is treated as a decision variable.

4.3 Optimal loading sequence for queueing turbines

This section provides a MILP formulation that optimises power generation for any number of turbines operating under a fixed-sequence philosophy. Similar to POQM, turbines of any operating capacities or efficiencies are allowed. In the previous section optimal results are shown for two turbines when $\alpha_{1q} = 0.8$, $\alpha_{1q} = 0.9$ or $\alpha_{1q} = 1.0$ with $q \in \mathcal{I}^q$. The optimal power generation results improve as α increases, however, the possibility may exist that the optimal loading factor is less than 1.0. To investigate the uncertainty regarding the optimal α loading factors, the Optimal Sequence Model (OSM) from this section treats it as decision variables. The formulation of OSM, furthermore, allows to solve for an optimal turbine loading hierarchy, without the requirement to load different hierarchies as input parameters.

In order to model the operation, some parameters and decision variables need to be defined. For completeness, all definitions for OSM are given. The time index is represented by $\mathcal{T} = \{1, 2, \dots, |\mathcal{T}|\}$. The set of all turbines is $\mathcal{I}^T = \{1, 2, \dots, |\mathcal{I}^T|\}$. The mass flow of excess steam at time t that may be utilised for power generation is represented by m_t^S (mass unit per time period), where each turbine i receives m_{it}^S (mass unit per time period) at time t .

Turbine $i \in \mathcal{I}^T$ can only be operational if steam flow is equal or above the minimum threshold L_i (mass unit per time period) and it cannot receive more than a fraction α_i of the maximum limit of U_i (mass unit per time period). Steam is converted with an efficiency of η_i^T (mass flow per power generated) for each turbine. Before a turbine may be restarted, sufficient steam must be available for δ^T time.

The optimal loading hierarchy's upper limit vector is given by U_i^P (flow units per time). Turbines according to the optimal hierarchy of operation are chronologically assigned to this vector. The corresponding L_i^P (flow units per time) represents the minimum limits and η_i^P (flow units per power generated) the conversion efficiencies. The superscript "P" signifies the *position* in the optimal loading hierarchy. Each turbine in the hierarchy is loaded with a fraction of α_i of the upper limit. The positional element W_{ij} of matrix $\mathbf{W}$ is the power generated from the i^{th} ranked turbine in the optimal hierarchy, where $i \in \mathcal{I}^T$, by the j^{th} machine defined in $\mathcal{I}^T$.

Two binary decision variables are defined. To indicate that the i^{th} turbine in the hierarchy is defined in the j^{th} position and operational at time t , $y_{ijt}^T = 1$. If $y_{ijt}^T = 0$ it is either an indication that the i^{th} ranked position is not occupied by the j^{th} defined turbine or that the machine is off-line. If the i^{th} ranked position in the optimal hierarchy is occupied by the j^{th} defined turbine in $\mathcal{I}^T$, $y_{ij}^P = 1$, else $y_{ij}^P = 0$. The objective of OSM is to:

$$\text{maximise } \sum_{i \in \mathcal{I}^T} \sum_{j \in \mathcal{I}^T} W_{ij}, \quad (4.10)$$

subject to

$$\alpha_i \leq 1, \quad \forall i \in \mathcal{I}^T, \quad (4.11)$$

$$U_i^P = \sum_{j \in \mathcal{I}^T} y_{ij}^P \alpha_i U_j, \quad \forall i \in \mathcal{I}^T, \quad (4.12)$$

$$L_i^P = \sum_{j \in \mathcal{I}^T} y_{ij}^P L_j, \quad \forall i \in \mathcal{I}^T, \quad (4.13)$$

$$\eta_i^P = \sum_{j \in \mathcal{I}^T} y_{ij}^P \eta_j^P, \quad \forall i \in \mathcal{I}^T, \quad (4.14)$$

$$\sum_{j \in \mathcal{I}^T} y_{ij}^P = 1, \quad \forall i \in \mathcal{I}^T, \quad (4.15)$$

$$\sum_{i \in \mathcal{I}^T} y_{ij}^P = 1, \quad \forall j \in \mathcal{I}^T, \quad (4.16)$$

$$y_{ij}^P \geq y_{ij}^T, \quad \forall i, j \in \mathcal{I}^T, t \in \mathcal{T}, \quad (4.17)$$

$$\sum_{k \in \mathcal{I}^T} y_{ikt}^T \leq \sum_{k \in \mathcal{I}^T} y_{jkt}^T, \quad \forall i, j \in \mathcal{I}^T : 1 < i < j, t \in \mathcal{T}, \quad (4.18)$$

$$m_{it}^S - U_i^P \geq \sum_{k \in \mathcal{I}^T} (y_{i+1kt}^T - 1)M, \quad \forall i \in \mathcal{I}^T : i < |\mathcal{I}^T|, t \in \mathcal{T}, \quad (4.19)$$

$$m_h^S - \sum_{j \in \mathcal{I}^T : j < i} U_j^P \geq \sum_{k \in \mathcal{I}^T} L_k y_{ikt}^T - (1 - \sum_{k \in \mathcal{I}^T} y_{ikt}^T)M, \quad \forall i \in \mathcal{I}^T : i > 1, h \geq t - \delta^T, \quad (4.20)$$

$$t \in \mathcal{T} : \delta^T < t < 2\delta^T, \\ h \leq t - 1,$$

$$m_h^S - \sum_{j \in \mathcal{I}^T : j < i} U_j^P \geq \sum_{k \in \mathcal{I}^T} L_k y_{ikt}^T - (1 - \sum_{k \in \mathcal{I}^T} y_{ikt}^T - y_{ikt-1}^T)M, \quad \forall i \in \mathcal{I}^T : i > 1, h \geq t - \delta^T, \quad (4.21)$$

$$t \in \mathcal{T} : t \geq \delta^T, \\ h \leq t - 1,$$

$$m_h^S \geq \sum_{k \in \mathcal{I}^T} L_k y_{ikt}^T - (1 - \sum_{k \in \mathcal{I}^T} y_{ikt}^T)M, \quad \forall i \in \mathcal{I}^T : i = 1, h \geq t - \delta^T, \quad (4.22)$$

$$t \in \mathcal{T} : \delta^T < t < 2\delta^T, \\ h \leq t - 1,$$

$$m_h^S \geq \sum_{k \in \mathcal{I}^T} L_k y_{ikt}^T - (1 - \sum_{k \in \mathcal{I}^T} y_{ikt}^T - y_{ikt-1}^T)M, \quad \forall i \in \mathcal{I}^T : i = 1, h \geq t - \delta^T, \quad (4.23)$$

$$t \in \mathcal{T} : t \geq \delta^T, \\ h \leq t - 1,$$

$$\sum_{i \in \mathcal{I}^T} m_{it}^S \leq m_t^S, \quad \forall t \in \mathcal{T}, \quad (4.24)$$

$$m_{it}^S \leq \sum_{j \in \mathcal{I}^T} U_j y_{ijt}^T, \quad \forall i \in \mathcal{I}^T, t \in \mathcal{T}, \quad (4.25)$$

$$m_{it}^S \geq \sum_{j \in \mathcal{I}^T} L_j y_{ijt}^T, \quad \forall i \in \mathcal{I}^T, t \in \mathcal{T}, \quad (4.26)$$

$$W_{ij} \leq U_j y_{ij}^P, \quad \forall i, j \in \mathcal{I}^T, \quad (4.27)$$

$$W_{ij} \eta_j |\mathcal{T}| \leq \sum_{t \in \mathcal{T}} m_{it}^S, \quad \forall i, j \in \mathcal{I}^T. \quad (4.28)$$

Equation (4.10) defines the objective function of OSM, which is to optimise power generation. Constraint set (4.11) ensures that the fixed loading setting per turbine does not exceed 100% of its full capacity. The sets of constraints (4.12) to (4.14) guarantee that turbines from $\mathcal{I}^T$ are assigned to the optimal loading hierarchy. Constraint sets (4.12) to (4.14) assign the upper limit together with the corresponding lower limit and steam conversion rates of turbines in $\mathcal{I}^T$ to the optimal loading hierarchy. Every turbine $i \in \mathcal{I}^T$ must be assigned to the optimal hierarchy, as ensured by constraint sets (4.15) and (4.16).

Constraint set (4.17) ensures that a turbine can only be operational in a specific ranking position in the optimal hierarchy if it is assigned there. To ensure that turbine operations adhere to the fixed-sequence philosophy, constraint sets (4.18) and (4.19) are used. M represents a sufficient large number. Constraint set (4.18) allows a machine to be operational, only if the prior turbine in

Table 4.3: Maximum and minimum power generation capabilities, followed by the efficiency, together with the upper and lower limits for Turbine I to VIII.

Turbine	Max [MW]	Min [MW]	η_i^T [ton/(h·MW)]	U_i [ton/h]	L_i [ton/h]
I	5.0	2.0	5.0	25.0	10.0
II	30.0	8.0	5.0	150.0	40.0
III	10.0	$3\frac{5}{9}$	4.5	45.0	16.0
IV	20.0	$6\frac{2}{9}$	4.5	90.0	28.0
V	30.0	$8\frac{8}{9}$	4.5	135.0	40.0
VI	10.0	3.2	5.0	50.0	16.0
VII	20.0	5.6	5.0	100.0	28.0
VIII	5.0	$2\frac{2}{9}$	4.5	22.5	10.0

the loading hierarchy is online. For the next-in-line turbine to be operational, the current ranked machine must be loaded until its fixed set point as per constraint set (4.19).

The sets of constraints (4.20) to (4.23) are used to verify if sufficient steam was available during the previous δ^T time that would have kept a tripped turbine operational. Constraint set (4.24) ensures that steam distribution to the turbines at time t cannot exceed what is available during that time step. The sets of constraints (4.25) and (4.26) guarantee that a turbine can only operate within allowable limits. The power generated per turbine is determined by constraint sets (4.27) and (4.28).

4.3.1 Results for OSM

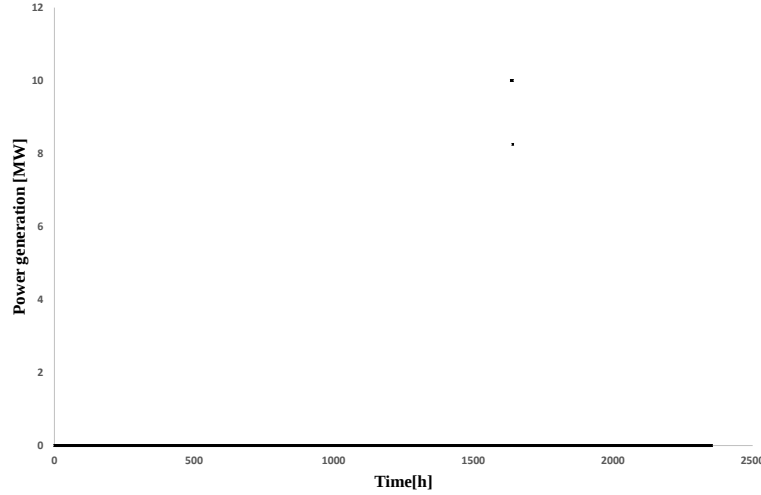
As mentioned above, the real-world data is used as steam input parameters for OSM. All turbines that are used for this section's scenario Cases are defined in Table 4.3. The turbines are defined in the chronological order in which they are introduced in this section. Two of each capacity 5MW, 10MW, 20MW and 30MW turbines are given in Table 4.3, where any two same size capacity machines differ in efficiency. Information is supplied for turbines that convert steam at an efficiency of 5 (ton per hour per Mega Watt generated) and for turbines converting steam at an improved or lower rate of 4.5 (ton per hour per Mega Watt generated).

For a fixed capacity turbine the upper limit for the corresponding turbine reduces as the conversion rate improves, however, the lower limit stays unchanged. Therefore, for a more efficient turbine the ratio of L/U is typically higher when compared to a lower efficient machine. Furthermore, Turbine I and II in Table 4.3 are identical to those defined in Table 4.1. All results from this section are given in Table 4.4.

Case 1^f: OSM is solved for the two turbines from the hypothetical plant from Case 1^q to 3^q, *i.e.* Turbine I and II from Table 4.3. Results show that for optimal power generation, Turbine I must be fully loaded as first receiver, followed by Turbine II. The results are therefore equivalent to that of Case 3^q reported in Table 4.2, that is 19 (4+15) accumulative trips and a combined power generation of 20.96MW (4.76MW+16.20MW). OSM is now solved for a number of scenarios where additional turbines are introduced to further demonstrate the novelty of this formulation to yield optimal power generation results for turbines operating under a fixed-sequence philosophy. All of the scenario results are tabulated in Table 4.4. Take note that all of these results can be determined by POQM, given the optimal fixed set point for each turbine is known.

Case 2^f: The operating parameters for Turbine I to III are fixed as inputs to OSM. Turbine III can generate up to 10.00MW and converts steam to power at a rate of 4.5 (ton per hour per Mega Watt generated). It might, therefore, be intuitive to predict that the optimal loading hierarchy will include Turbine III as the first receiver, since the conversion efficiency is a 10% improvement compared to that of Turbine I and II. OSM is now solved for the three turbines. Results indicate, however, that optimal power generation is obtained if Turbine I is the first receiver, followed by Turbine II and lastly Turbine III.

For optimal power generation all three turbines should be fully loaded, *i.e.* $\alpha_1 = 1.0$, $\alpha_2 = 1.0$ and $\alpha_3 = 1.0$. The results in Table 4.4 show that Turbine III generates only 0.02MW from a potential

Figure 4.7: Case 2^f: Power generation over time for Turbine III.Table 4.4: Optimal OSM results for Case 1^f to 7^f. The solved turbine loading hierarchy is given, together with the loading factors, trips and power generation per turbine, followed by the total power generation.

Case number	Turbine hierarchy			α			Trips per turbine			Power generation per turbine [MW]			Combined power generation [MW]
1 ^f	I	II		1.0	1.0	1.0	4	15		4.76	16.20		20.96
2 ^f	I	II	III	1.0	1.0	1.0	4	15	1	4.76	16.20	0.02	20.98
3 ^f	I	IV	-	1.0	1.0	-	4	15	-	4.76	14.70	-	19.46
4 ^f	V	I	-	1.0	1.0	-	11	14	-	21.72	0.57	-	22.29
5 ^f	I	VII	VI	1.0	1.0	1.0	4	17	18	4.76	14.13	1.20	20.09
6 ^f	III	IV	VIII	1.0	1.0	1.0	5	17	23	9.25	12.31	0.57	22.13
7 ^f	I	IV	VI	1.0	1.0	1.0	4	17	18	4.76	14.70	1.49	20.95

10.00MW. From these results it is evident that the signature steam profile from Section 2.11, *i.e.* the real-world data, does not have the ability to operate this combined turbine capacity of 45.00MW (5.00MW+30.00MW+10.00MW) under a fixed-sequence philosophy. Furthermore, a seemingly trivial decision to assign the more efficient Turbine III as the first receiver will not result in optimal power generation.

Figure 4.7 displays power generation, or the lack thereof, for Turbine III. Power is generated for only six hours or 0.26% of the time horizon. This clearly displays the inability to accommodate Turbine III together with Turbine I and II under a fixed-sequence philosophy.

Case 3^f: Case 2^f demonstrates that the inclusion of a more efficient turbine does not necessarily result in it being assigned as the first receiver. What is also evident from Case 2^f is that the hypothetical energy recovery plant will not benefit in terms of power generation with the inclusion of a third, 10MW, turbine under the fixed-sequence operating philosophy. For Case 3^f OSM is solved for Turbine I and IV. Turbine IV is a 20MW machine with an efficiency of 4.5 (ton steam per hour per Mega Watt generated). The intuitive decision, yet again, might seem to allocate the more efficient Turbine IV as the first receiver, followed by Turbine I.

Optimal results, however, show that Turbine I should be the first receiver, with $\alpha_1 = 1.0$, before Turbine IV is loaded. A total of 19 trips are reported and optimal power generation of 19.46MW. Lower power generation is to be expected in comparison with Case 1^f and 2^f, since Turbine IV's maximum capacity is 10.00MW less than that of Turbine II that is used in the previous Cases. Take note that power generation is only 1.52MW less for Case 3^f compared to Case 2^f, where the total potential capacity is 20.00MW higher (5.00MW + 30.00MW + 10.00MW - 5.00MW - 20.00MW). This brings forth the question as to what combination of turbine procurement will be optimal? This question will be addressed in the next chapter.

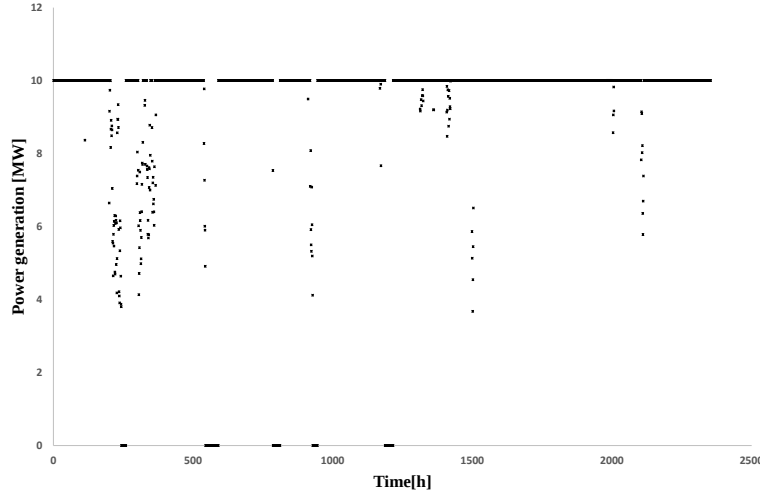


Figure 4.8: Case 6^f: Power generation over time for Turbine III as the first receiver.

Case 4^f: Taking Case 1^f to 3^f into consideration it might, perhaps, be concluded that a lower capacity turbine should be favored as a first receiver for the fixed-sequence philosophy. Case 4^f is now solved for Turbine I and V as input parameters towards OSM. Turbine V is a 30MW machine, converting steam to power at a rate of 4.5 (ton steam per hour per Mega Watt generated). Solving OSM for the two turbines result in optimal power generation of 22.29MW with combined trips of 25. Results indicate that Turbine V should be the first receiver with $\alpha_1 = 1.0$. For this Case optimal power generation realises when the larger and more efficient turbine is the first receiver, but the accumulated trips are higher compared to the previous Cases. The results, furthermore, emphasise that even if only two turbines are operating under a fixed-sequence philosophy, the optimal loading hierarchy is not necessarily trivial.

Case 5^f: Three turbines are operational for each of the following three presented Cases. Each Case comprises total power generation capacity of 35.00MW and consists of a 5MW, 10MW and 20MW turbine combination from Table 4.3. OSM is firstly solved for Turbine I, VI and VII, which are the 5MW, 10MW and 20MW machines with efficiencies of 5 (ton per hour per Mega Watt generated).

Optimal power generation results show that the small capacity turbine, *i.e.* the 5MW, should be the first receiver with $\alpha_1 = 1.0$. Turbine VII, the 20MW should be the second receiver, with $\alpha_2 = 1.0$, followed by Turbine VI. A total of 39 trips are determined with a combined power generation of 20.09MW. Take note, for Case 1^f where both the 5MW and 30MW turbine operates at 5 (ton steam per hour per Mega Watt generated), 19 trips are reported under optimal conditions with total power generation of 20.96MW. The results indicate, therefore, if the combined power generating capacity stays constant, but more turbines are used, higher optimal power generation will not necessarily follow under a fixed-sequence philosophy.

Case 6^f: OSM is now solved for a 5MW, 10MW and 20MW turbine, where all three operate at an efficiency of 4.5 (ton per hour per Mega Watt generated). These turbines correspond to Turbine VIII, III and IV in Table 4.3. Knowing the optimal outcomes of Case 5^f it might be assumed that OSM will determine an equivalent capacity optimal loading hierarchy for Case 6^f. Results indicate, however, that an equivalent capacity optimal loading hierarchy, compared to that of Case 5^f, is not obtained.

Optimal results indicate that Turbine III, the 10MW, should be the first receiver and loaded fully before Turbine IV, the 20MW is fully loaded. Note, to obtain optimality under a lower turbine efficiency, the 10MW is loaded last for Case 5^f. The 5MW turbine is the third receiver under the improved efficiency, whereas it is the first receiver for Case 5^f. Under these improved efficiencies optimal power is generated at a rate of 22.13MW, which is 10.2% more than optimal results for Case 5^f with less efficient turbines. Note that 45 turbine trips are observed.

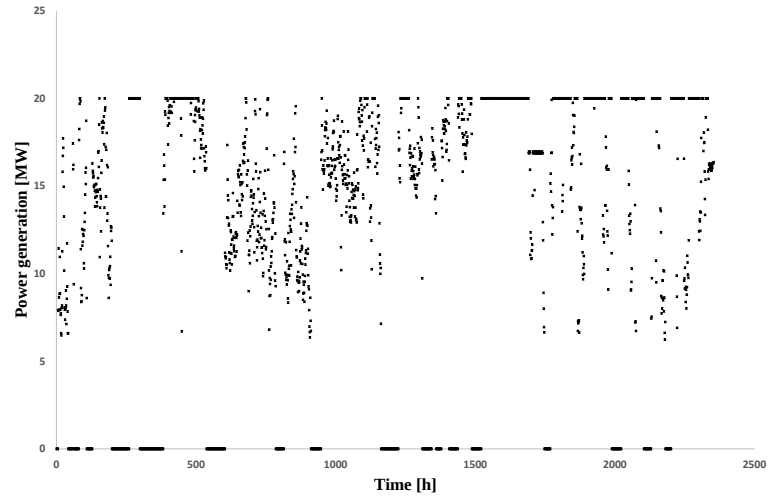


Figure 4.9: Case 6^f: Power generation over time for Turbine IV as the second receiver after Turbine III.

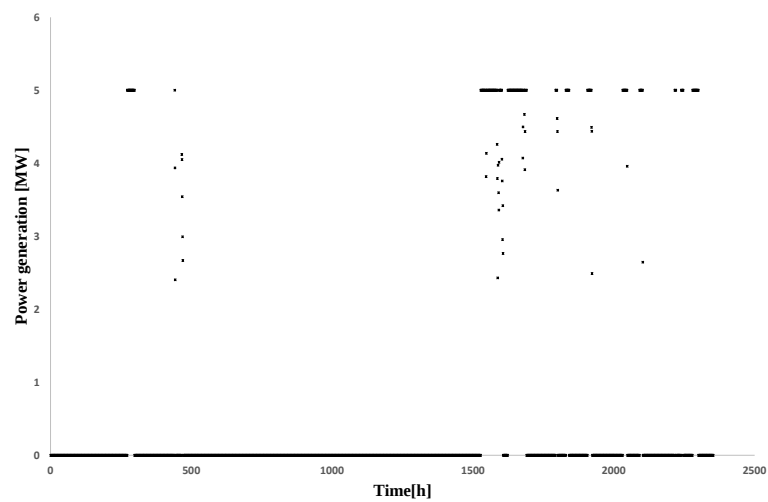


Figure 4.10: Case 6^f: Power generation over time for Turbine VIII as the third receiver after Turbine III and IV.

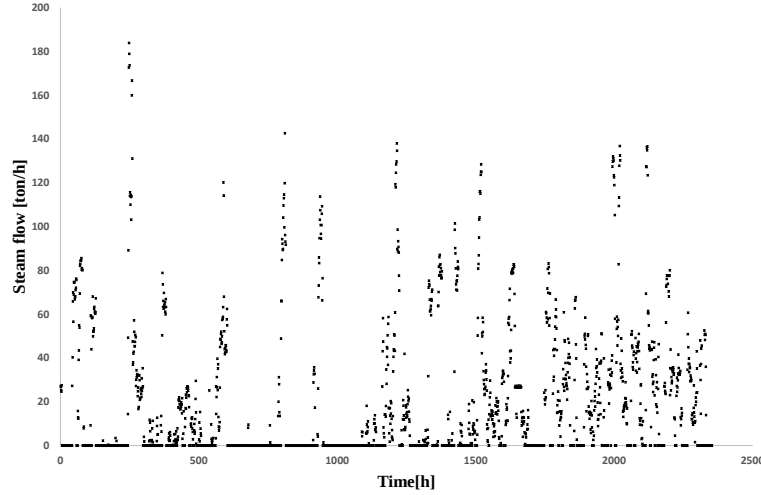


Figure 4.11: Case 6^f : Excess available steam over time that cannot be utilised by either Turbine III, IV or VIII.

This optimal power generation is the second largest when compared to Case 1^f to 5^f , however, with the most trip occurrences. Power generation results for Case 6^f are plotted for Turbine III, IV and VIII in Figures 4.8, 4.9 and 4.10, respectively. Note that the same power generation pattern is observed for Figure 4.8 when compared to Figures 4.2 and 4.4, *i.e.* the first receivers. Figure 4.9 corresponds to that of Figures 4.3 and 4.5, whereas Figures 4.7 and 4.10 possess the same trend. Figure 4.10 indicates limited power generation for Turbine VIII with 23 trips.

Figure 4.11 represents the steam flows that could not be utilised by either Turbine III, IV or VIII. The total power generation capacity between these turbines, as mentioned, is 35.00MW, which is equivalent to that of Turbine I and II. The reported power generation for Case 6^f is 22.13MW, compared 20.96MW for Case 1^f . This 5.58% power generation increase for Case 6^f is, however, only a result of more efficient turbines. When the data points from Figure 4.6 and 4.11 are analysed and compared, it follows that more power generation potential is lost for Case 6^f .

Steam of 18.06 (ton per hour) or 4.00MW cannot be used by these three more efficient turbines from Case 6^f , compared to the 12.93 (ton per hour) for Case 1^f . Either partial or full power generation goes to waste for 47.47% of the time for Case 6^f , compared to the 30.90% for Case 1^f . Higher power generation from more efficient turbines, therefore, does not necessarily imply improved steam utilisation. Furthermore, the question may arise whether more efficient turbines will necessarily be the optimal investment choice when the Works is seeking to invest in energy recovery or the expansion thereof? This question will also be addressed later on in Chapter 5.

Case 7^f : Turbine I, IV and VI are fixed as input parameter towards OSM. The 20MW, *i.e.* Turbine VI operates at an efficiency of 4.5 (ton per hour per Mega Watt generated) and the other two at 5 (ton per hour per Mega Watt generated). For both Case 5^f and 6^f the 20MW turbine is the second receiver when all three turbines operate at equivalent efficiencies. Case 7^f investigates whether a more efficient 20MW turbine, compared to the 5MW and 10MW machine, will solve as the first receiver. OSM is now solved and optimal power generation results show that the 20MW turbine should still be operated as the second receiver and fully loaded. The 5MW is the first receiver and should also be loaded to full capacity. The results from the last three Cases further emphasise that an optimal loading hierarchy is not a trivial decision.

From the results reported for Case 5^f to 7^f in Table 4.4 and Figure 4.10 it can be observed that the third receiver does not generate significant power when compared to the first two turbines. Power for the third receivers of 1.20MW, 0.57MW and 1.49MW are generated by a 10MW, 5MW and 10MW turbine, respectively. For each Case the sensibility of including a third turbine must be questioned. Results indicate, furthermore, increased trip occurrences when three turbines are operational, compared to two. The need, therefore, arises to determine not only the optimal loading

hierarchy and power generation, but the optimal turbine investment choices. Furthermore, trips induce additional maintenance expenses and decrease a turbine's life expectancy, as discussed in Chapter 2. Trip occurrences are therefore critical and should be included in the decision making process, when these investment choices are made.

Note from all the reported results in Table 4.4 that for no Case does OSM determine an α loading factor less than 1.0. It can therefore be concluded that it does not make sense to load any of the turbines under a fixed-sequence philosophy below its maximum capacity. Referring back to the example in Section 4.1, it can be concluded that the real-world data do not experience a signature fluctuation as portrayed in Figure 4.1. Taking this factor into account and the excessive solving times observed for OSM when compared to POQM, POQM will be used as basis in the next chapter to formulate an optimal investment model. Note that solving times are not discussed in this thesis. The results in Table 4.4 indicate a significant increase for turbine trips when three, rather than two machines are used. Case 2^f, however, is an exception, due to oversized turbine capacities. In Section 2.11 it is mentioned that the real-world data comprise an average of 117 (ton per hour), with a standard deviation of 44.8 (ton per hour). The combined steam flow capacity for Turbine I, II and III, however, is 220 (ton per hour). As a result, the more efficient Turbine III as third receiver is only operational once. This furthermore emphasises the need for an optimal investment model formulation.

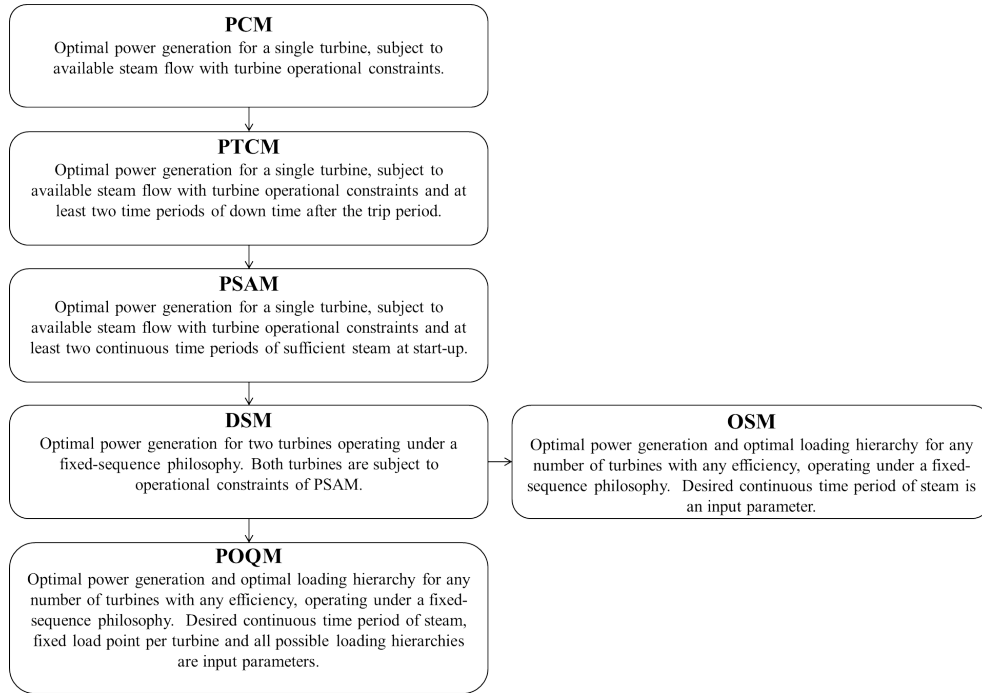


Figure 4.12: Flow chart for the four MILP concept formulations from Chapter 3 and two fixed-sequence optimal power generation models presented in this chapter.

4.4 Summary

This chapter presented two novel MILP formulations. Both models determine the optimal loading hierarchy and optimise power generation for any number of turbines operating under a fixed-sequence philosophy. The POQM formulation requires, as additional input parameters, each turbine's fixed loading point as well as every possible loading hierarchy. Turbine fixed set points are treated as variables by OSM and the formulation does not require any loading hierarchy as input parameters. Optimal results show that it is not sensible to load any turbine below its maximum capacity. Results further indicate that an intuitive turbine placement in the loading hierarchy is not necessarily optimal and, therefore, not a trivial process.

From the results it follows that if a higher capacity turbine is the first receiver, more trips result

and these occurrences increase further if three rather than two turbines are present. To summarise the MILP formulations provided up to now, Figure 4.12 displays a flow chart that starts at the basic PCM formulation until that of POQM and OSM. DSM is used as a modelling platform for both POQM and OSM, however, these two models are formulated independently. As a result, two different formulation paths are indicated after DSM.

POQM and OSM can be utilised to determine how turbines in a plant should be loaded, however, it does not indicate which turbine investment choice will be optimal. It should be noted that all optimal results generated by OSM yielded α values equal to unity. Furthermore, significant larger solving times are required when OSM is compared to POQM. POQM will therefore be used as basis for the optimal investment model formulation that follows in the next chapter. This formulation will optimise investment choices for any number of turbines operating under a fixed-sequence philosophy.

Chapter 5

Optimal turbine investments for the fixed-sequence loading philosophy

The POQM and OSM formulations proposed in Chapter 4, optimise turbine loading hierarchies and power generation under a fixed-sequence philosophy. The results in Chapter 4 indicate how turbines, apart from the first receiver, are on average not loaded close to maximum capacity. Results further show that a more efficient turbine, which is typically higher priced, is not necessarily the optimal first receiver. For such a scenario the question must be asked if the more efficient, but expensive turbine falls, within the optimal investment choices? It is therefore not only important to be able to determine the optimal loading hierarchy and power generation for turbines operating under a fixed-sequence philosophy, but also the optimal investment choices.

This chapter proposes a MILP formulation that optimises turbine investment choices for operations under a fixed-sequence philosophy. The optimal loading hierarchy and power generation are also determined.

5.1 Plant operational procedure investment model

In this section a MILP formulation is proposed that solves the optimal turbine investment problem for an energy recovery plant operating under a fixed-sequence philosophy. Optimal investment results are obtained, which includes the optimal turbine loading hierarchy and power generation. Owing to excessive solving times for OSM in the previous chapter, POQM's formulation is used as basis for the Plant Operational Investment Queueing Model (POIQM). Take note that all optimal results from OSM in Chapter 4 indicate that $\alpha = 1$ for each turbine. For this reason, and since a smaller capacity turbine with a lower minimum steam flow limit can rather be procured at a less expensive price, POIQM utilises each turbine to full capacity. Turbine operational parameters and procurement costs are input parameters to POIQM, including all possible loading hierarchies. Results reported throughout Chapter 4 indicate that trip occurrences are frequently observed. In order to address this problem in the optimal decision making process of POIQM, trip costs are formulated in terms of monetary penalties. The penalties represent an NPV value of projected future income losses owing to maintenance down-times and costs.

The formulation of POQM in Section 4.2 defines indices, variables and parameters to be repeated for POIQM, for the sake of completeness. The index set of all turbines are given by $\mathcal{I}^T = \{1, 2, \dots, |\mathcal{I}^T|\}$, and the index set of all different turbine queues are defined by $\mathcal{I}^Q = \{1, 2, \dots, |\mathcal{I}^Q|\}$. The loading hierarchies with the corresponding turbine numbers from $\mathcal{I}^T$ are defined by $\mathcal{I}^T(q)$, where $q \in \mathcal{I}^Q$. The operation of the turbines are modelled over a time set $\mathcal{T} = \{1, 2, \dots, |\mathcal{T}|\}$.

The input parameter m_t^S denotes the available steam for power generation at time t . Turbine $i \in \mathcal{I}^T$ can operate within the allowable minimum and maximum limits of L_i (mass unit per time period) and U_i (mass unit per time period), respectively, where steam is converted at an efficiency rate of η_i^T (mass flow per power unit generated). If a turbine trips, a minimum time of δ^T , which coincides with sufficient steam availability, needs to elapse before the tripped turbine may be brought back online. M corresponds to a sufficient large value.

The income per power generation unit at time t is given by C_t^E . The capital cost of turbine $i \in \mathcal{I}^T$ at time zero is given by C_i^T . The present value trip cost for turbine $i \in \mathcal{I}^T$ is given by C_i^h at time zero for each future trip. It is therefore assumed that trip costs increase according to the real interest rate. The real interest rate applied for NPV discounting is given by r .

There are three main groups of decision variables defined for POIQM. The first group relates to the distribution of steam to turbines. Let $m_{igt}^S \geq 0$ be the steam flow to turbine $i \in \mathcal{I}^T$ for queue $q \in \mathcal{I}^Q$ at time $t \in \mathcal{T}$. The binary decision variable $y_{igt}^T = 1$ specifies that turbine $i \in \mathcal{I}^T$ for queue $q \in \mathcal{I}^Q$ is operational at time t , and a zero value relates to a non-operational status. If the binary trip (halt) variable $y_{igt}^h = 1$, then turbine $i \in \mathcal{I}^T$ for queue $q \in \mathcal{I}^Q$ trips from time $t-1$ to t . The second binary variable decision group is λ_q , indicating which fixed-sequence hierarchy delivers optimal power generation. Element X_{iq} in the binary matrix X indicates if an investment of turbine $i \in \mathcal{I}^T$ is made for queue $q \in \mathcal{I}^Q$. The third decision group model optimal investment choices. If $x_i = 1$ an investment is made to procure turbine $i \in \mathcal{I}^T$, and $x_i = 0$ if not.

The objective of POIQM is to:

$$\text{maximise } \sum_{i \in \mathcal{I}^T(q)} \sum_{q \in \mathcal{I}^Q} \sum_{t \in \mathcal{T}} \left(\frac{m_{igt}^S}{\eta_i^T |\mathcal{T}| (1+r)^t} C_t^E - C_i^h y_{igt}^h \right) - \sum_{i \in \mathcal{I}^T} \sum_{q \in \mathcal{I}^Q} X_{iq} C_i^T, \quad (5.1)$$

subject to

$$\sum_{t \in \mathcal{T}} y_{igt}^T \leq |\mathcal{T}| X_{iq}, \quad \forall i \in \mathcal{I}^T(q), q \in \mathcal{I}^Q, \quad (5.2)$$

$$m_{igt}^S \geq U_{Q_i} y_{i+1,gt}^T, \quad \forall i \in \mathcal{I}^T(q) : i < |\mathcal{I}^T(q)|, q \in \mathcal{I}^Q, t \in \mathcal{T}, \quad (5.3)$$

$$\lambda_q m_k^S - \sum_{j \in \mathcal{I}^T(q) : j < i} U_i \geq L_i y_{igt}^T - (1 - y_{igt}^T - y_{i+1,gt}^T) M, \quad \forall i \in \mathcal{I}^T(q) : i > 1, t > \delta^T, \quad (5.4)$$

$q \in \mathcal{I}^Q,$
 $t \in \mathcal{T},$
 $k \geq t - \delta^T,$
 $k \leq t - 1,$

$$m_{igt}^S \leq U_i y_{igt}^T, \quad \forall i \in \mathcal{I}^T(q), q \in \mathcal{I}^Q, t \in \mathcal{T}, \quad (5.5)$$

$$y_{igt}^h \geq y_{i+1,gt}^T - y_{igt}^T, \quad \forall i \in \mathcal{I}^T(q), q \in \mathcal{I}^Q, t \in \mathcal{T} : t > 1. \quad (5.6)$$

The remainder of the constraint sets are equivalent to that defined by (4.2), (4.6), (4.9), (4.3), (4.5).

The objective of POIQM is to optimise NPV profit over the time horizon $\mathcal{T}$, given by equation (5.1). To ensure that a turbine can only be operational if it is invested in, constraint set (5.2) is used. Constraint (5.3) ensures that a turbine must be loaded until full capacity before the next-in-line machine may receive steam.

The formulation of constraint set (4.2) allows for only one fixed-sequence hierarchy to be chosen. For the chosen queue $q \in \mathcal{I}^Q$, constraint set (4.6) allows a turbine only to be operational if the previous machine in the hierarchy is online. Constraint set (4.7) ensures that a turbine is fully loaded before the next-in-line machine can acquire steam.

Before a turbine may be brought back online, the sets of constraints (5.4) and (4.9) must be satisfied. When all turbines are in trip, constraint set (5.4) ensures that steam was present during each of the previous δ^T time periods that would have been sufficient to have kept the first receiver operational. From the second receiver and onwards, the formulation of constraint set (4.9) allows a turbine to become operational if sufficient steam existed for each of previous δ^T time periods, after all prior receivers have been loaded at its fixed set points. Constraint set (4.3) ensures that the steam distribution between turbines for queue $q \in \mathcal{I}^Q$ at time $t \in \mathcal{T}$ may not exceed the available steam for that time period. The sets of constraints (5.5) and (4.5) guarantee that a turbine can only operate within its allowable limits. The formulation of constraint set (5.6) accounts for each turbine trip.

Table 5.1: Maximum and minimum power generation capacities, followed by the steam conversion rate, together with the upper and lower limits for Turbine I to X.

Turbine	Max [MW]	Min [MW]	η_i^T [ton/(h·MW)]	U_i [ton/h]	L_i [ton/h]
I	3.0	1.2	5	15.0	6.0
II	5.0	2.0	5	25.0	10.0
III	10.0	3.2	5	50.0	16.0
VI	15.0	4.4	5	75.0	22.0
V	20.0	5.6	5	100.0	28.0
VI	25.0	6.8	5	125.0	34.0
VII	30.0	8.0	5	150.0	40.0
VIII	35.0	9.2	5	175.0	46.0
IX	40.0	10.4	5	200.0	52.0
X	50.0	12.8	5	250.0	64.0

5.2 Results for POIQM

This section demonstrates how POIQM can solve the optimal turbine investment problem, which includes the optimal loading hierarchy and power generation for an energy recovery plant operating under a fixed-sequence philosophy. Prior to making an investment in power generation capabilities, a market research will typically be conducted to acquire information on a number of turbines from different suppliers. The turbine information should include steam flow limits, efficiencies and procurement prices. To exhibit POIQM's functionality, operational limits for 10 turbines are defined in Table 5.1. Unless otherwise stated, all scenario results from this chapter will be performed on these 10 turbines. These turbines are chosen to demonstrate POIQM's ability, which is generating optimal investment, loading hierarchy and power generation results. Optimal investments are given in terms of NPV.

Operational properties for the 10 turbines, *i.e.* maximum and minimum power generation, steam conversion rate followed by the upper and lower bounds for *Turbine I* to *X* are provided in Table 5.1. All 10 turbines have equivalent conversion rates, even though POIQM can accommodate any isentropic efficiencies. A later discussion in this chapter will make provision for when a more efficient turbine is used. Procurement costs for the turbines, only for scenario demonstration purposes, are found in Table 5.2. For all scenarios in this chapter it is assumed that no existing turbine is in operation, even though it can be accommodated by POIQM.

It should be noted that for a turbine, the ratio L_i/U_i typically reduces as L_i or U_i increase for the same steam conversion rate. Furthermore, it is important to consider that a large number of turbine manufacturers exist over the world, each one of them producing a variety of power generating capacities at different efficiencies. Workmanship qualities, maintenance plans and the number of allowable trips per turbine are just some of the properties that may vary between turbine manufacturers and influence the procurement costs.

All optimisation results from this chapter will be performed by utilising the real-world data, as discussed in Section 2.11, as the steam input parameters. Since this profile is assumed to accurately represent all future steam flow predictions, it can be extended to a time line suitable for long term investments. The investment time horizon is chosen over a 10 year period, which is $365.25 \cdot 24 \cdot 10 = 87660$ hours.

All scenarios are performed under the assumption that the current price of electricity that the Works pays to its power supplier is conservative at R500.00 per Mega Watt hour utilised. Furthermore, it is assumed that electricity cost inflation is 5%¹ per year, annually compounded, whereas the real investment rate is assumed to be 5% per year, continuously (hourly) compounded. For all scenario simulations in this chapter a typical industry standard for adequate steam availability of 15 hours will be applied. Similar to Chapter 4, scenarios are labeled as *Case numbers* with a superscript "f" throughout this chapter. Take note that none of these Cases are compared to that of Chapter 4 and, therefore, should not be confused with the same numbering system.

¹This is a conservative value for the current electricity price hikes experienced in South Africa. Note that a higher price inflation will yield higher NPV's. The aim of this chapter is, however, to demonstrate the functionality of POIQM and not to make exact predictions on accurate obtained parameter values.

Table 5.2: Procurement costs used to generate scenario results for Turbine I to X in millions of Rand.

Turbine	Procurement cost in R · 10 ⁶
I	18.0
II	25.0
III	47.5
IV	70.0
V	90.0
VI	110.0
VII	130.0
VIII	150.0
IX	167.5
X	185.0

Unless otherwise stated, each result section will contain outcomes to five different Cases. The first Case reported is for optimal investment where any turbines from Table 5.1 can be procured. In order to investigate how close the second to optimal solution is, POIQM will be resolved where the optimal investment vector is not allowed. Optimality solved for by POIQM will then yield the second best investment combination from the turbine list. To acquire the third best investment vector, both optimal solutions from the previous two Cases are excluded. The process is repeated until the five best investment vectors are obtained.

The first set of reported results for POIQM in Section 5.2.1 is for when a zero trip penalty is solved for. The rationale is to set a platform used to compare other Case instances where the penalty influences the optimal investment choices in terms of a cost. It should be noted, these penalties represent projected future costs to be incurred owing to turbine trips. These include maintenance expenditures and additional down-times where power cannot be generated, irrespective of steam availability.

Section 5.2.2 follows where each trip penalises the NPV with a fixed cost. For Sections 5.2.3 and 5.2.4 cost penalisation is a proportion of the procurement price. The various penalisation methods are used to demonstrate the sensitivity towards expected NPV's and, furthermore, optimal investment choices.

5.2.1 POIQM results with a zero trip penalty

All the results from this section are reported in Table 5.3. The results indicate the turbine number from Table 5.1 that should be invested in, which includes the optimal loading hierarchy. This is followed, respectively, by the number of trips per turbine over the 10 year interval and the optimal power generation. The combined power generation is given, followed by the NPV in terms of millions of Rand (R·10⁶). For all simulation results reported in this section zero trip penalties are taken into account.

It should be noted that all the NPV values provided are in terms of electricity savings for the Works. The off-gases do not contain a direct procurement cost for energy recovery, because it is a by-product paid for by other plant processes. The electricity savings from energy recovery can therefore be more accurately interpreted as the subsidising of production costs for the plant processes that generate the off-gases.

Case 1^f: POIQM is now solved for Turbine I to X from Table 5.1 under the procurement costs given in Table 5.2. As mentioned, trip penalties are not taken into consideration, that is $C_i^h = 0 \forall i \in \mathcal{I}^T$. Out of a possibility of 10 turbines the optimal investment matrix, X_{iq} , yields two non-zero values, indicating two turbine procurements. Optimal results from POIQM yields investments for Turbine II and VII, *i.e.* the 5MW and 30MW machines.

Results indicate that Turbine II should be loaded before Turbine VII, similar to what was obtained by POQM and OSM in Chapter 4. It should be noted, however, these optimal investment results are subjected to the turbines chosen for Table 5.1, under the costs given in Table 5.2 and do not account for trip penalties.

Under the assumption that the real-world data, as discussed in Section 2.11, is representative of the

Table 5.3: Scenario results for Case 1^f to 5^f where trip costs are not considered. The results include the optimal turbine investment and loading hierarchy, trips and power generation per turbine, total power generation and the expected NPV profit for 10 years.

Case number	Turbine hierarchy	Trips per turbine		Power generation per turbine [MW]		Total power generation [MW]	NPV R·10 ⁶
1 ^f	II VII	149	560	4.75	16.17	20.92	734.7
2 ^f	VII	411		20.33		20.33	734.5
3 ^f	I VII	149	485	2.88	17.65	20.63	729.1
4 ^f	VIII	485		20.62		20.62	726.6
5 ^f	II VI	149	374	2.88	17.65	20.54	723.0

signature steam profile for the Works over 10 years, Turbine II and VII are projected to trip 149 and 560 times, respectively. The rates of power generation are 4.75MW and 16.17MW, respectively, with a combined 20.92MW out of a potential 35.00MW. The NPV of the power generation profit is determined to be R734.7 million. Note that a significant number of trips occur between the turbines, *i.e.* 709, however, before a non-zero trip penalty is used, four additional Cases are simulated with POIQM.

Case 2^f to 5^f: Optimal investment results that maximise the NPV are reported in Case 1^f. The results, however, do not indicate by what margin it is optimal towards other investment choices. POIQM is, therefore, solved in these Cases by removing the option to invest only in Turbine II and VII from the list of potential investment options.

Case 2^f: By solving POIQM optimal investment results are obtained, which indicate the procurement of only Turbine VII. For Turbine VII, 411 trips are expected over 10 years with a power generation rate of 20.33MW. The NPV profit is determined to be R734.5 million. Therefore, taking the results of Case 1^f into consideration, if the procurement price of Turbine II is increased by more than R200,000.00, the optimal investment choice will be to procure only Turbine VII. If Turbine VII alone is procured, significant less turbine trips are incurred, which is 411 compared to 709 for Case 1^f.

Case 3^f: Solving POIQM and not allowing the investment combinations of Case 1^f or 2^f, optimal results yield that Turbine I and VII should be procured. The lower capacity Turbine I is predicted to trip 149 times, equivalent to Turbine II in Case 1^f as the first steam receiver. Less trips, however, are solved for Turbine VII when compared to Case 1^f, which is 485 against 560. The total rate of power generation is determined to be 20.63MW and the resulting NPV is R729.1 million. Take note, based on Case 2^f, which provided higher optimal NPV and which consists only of Turbine VII, it does not make sense to also procure Turbine I, as a lower NPV will be obtained.

Case 4^f and 5^f: Repeating the process for the next two POIQM simulations, optimal results show that for Case 4^f Turbine VIII should be invested in with an NPV of R726.6 million. Accumulated trips over the 10 years are expected to be 485 with a power generation rate of 20.62MW. Optimal investments for Case 5^f indicate procurements of Turbine II and VI with an NPV of R723.0 million. When the number of trips for Case 5^f is compared to that of Case 1^f and 3^f, a significant reduction is observed for the second steam receiver.

The results from Case 1^f to 5^f, summarised in Table 5.3, demonstrate the ability of POIQM to yield optimal turbine investment results that include the optimal loading hierarchy. The results, however, are generated with a zero trip penalty. Given the number of trips experienced for all of the reported investment choices, over a 10 year period, a non-zero penalty should be applied in order to generate more realistic NPV values. A non-zero penalty will, furthermore, demonstrate the sensitivity towards optimal investment choices.

5.2.2 POIQM results for a fixed trip penalty

In this section a fixed cost per trip is assigned to the C_i^h parameters. For Case 6^f to 10^f each trip penalises the NPV with R50,000.00 at time zero, so that $C_i^h = 50000 \forall i \in \mathcal{I}^T$.

The same approach as in Section 5.2.1 is followed to generate the top five optimal investment results with POIQM. All results from this section are summarised in Table 5.4. Similar to Table 5.3, the

Table 5.4: Scenario results for Case 6^f to 10^f where trip are penalised with a R50,000.00 cost at time zero for each future occurrence. The results include the optimal turbine investment and loading hierarchy, trips and power generation per turbine, total power generation and the expected NPV profit over 10 years.

Case number	Turbine hierarchy	Trips per turbine		Power generation per turbine [MW]		Total power generation [MW]	NPV R·10 ⁶
6^f	VII	411		20.33		20.33	714.0
7^f	VIII	485		20.62		20.62	702.3
8^f	II VII	149	560	4.75	16.17	20.92	699.3
9^f	I VII	149	485	2.88	17.75	20.63	697.4
10^f	VI	337		19.34		19.34	695.7

optimal investment and loading hierarchy are given followed by the number of trips per turbine over a 10 year period and the average expected individual rates of power generation. The combined power generation is given, followed by NPV in terms of millions of Rand. Note that if a specific turbine investment hierarchy is present in both Tables 5.3 and 5.4, all values except for the NPV will be equivalent.

Case 6^f to 10^f : Solving POIQM when a trip penalty of R50,000.00 is included per occurrence, yields that an optimal investment is for Turbine VII alone in Case 6^f . The projected NPV of R714.0 million is 2.8% lower than the NPV reported in Table 5.3 for Case 1^f without a trip penalty. The result generation process is repeated from Section 5.2.1 for **Case 7^f to 10^f** , *i.e.* optimal investment results from each prior Case is not allowed for the following scenarios, in order to generate the next four optimal turbine investment configurations. Take note that these investment combination omissions are only relevant towards results from Case 6^f to 10^f . These scenario results are summarised in Table 5.4.

For optimal results without trip penalties, POIQM yields that Turbine II and VII should be procured for Case 1^f . When the NPV is subjected to a fixed trip penalisation, this investment combination is the third best option. Therefore, according to the results in Table 5.4 it does not make sense to procure Turbine VII in combination with either Turbine I or II. Procuring Turbine VII alone results in a higher expected NPV and less investment capital is needed.

When the results from Tables 5.3 and 5.4 are compared, four out of the five investment combinations are repeated in Table 5.4. As mentioned above, POIQM indicates that an investment into Turbine VII alone should be made for optimal investment purposes. The second best option is a single investment into Turbine VIII, followed by the initial optimal investment where no trips costs are considered, *i.e.* Turbine II and VII at almost 5% lower NPV than that of Case 1^f . The results indicate that when trips are penalised with the anticipation of future costs to be incurred under a fixed-sequence philosophy, optimal investments comprise a single, rather than multiple turbine investments. It should be noted, if a single turbine is procured, all power generation potential will be lost if it trips and no steam above its maximum limits will be utilised.

In conclusion, the extra income generated from potential steam, which is lost owing to trips or the inability to capture high flow instances, is not necessarily sufficient to cover the expenses of an additional turbine investment when fixed trip costs are incurred. It should be noted, however, that trip costs will typically not be fixed for different turbines. A more realistic approach will be to penalise each trip occurrence with a proportion of the initial investment cost. The next section investigates the effect on optimal turbine investment outcomes if each trip incurs a penalty equal to $1/500^{th}$ of the procurement price.

5.2.3 POIQM results with proportional trip penalty

It would be expected that additional maintenance costs, which are incurred over time owing to trips, are related towards turbine investment costs and are therefore not fixed. More frequent trips will result in more increased maintenance stops and shorter periods between minor and major overhaul outings, where a turbine is taken off-line and serviced for a prolonged period of time. Being off-line directly implies no power can be generated, irrespective of steam availability. Depending

Table 5.5: Scenario results for Case 11^f to 15^f where each future trip penalises the NPV at time zero with $1/500^{th}$ of the procurement cost. The results include the optimal turbine investment and loading hierarchy, trips and power generation per turbine, total power generation and the expected NPV profit for 10 years.

Case number	Turbine hierarchy	Trips per turbine		Power generation per turbine [MW]		Total power generation [MW]	NPV R·10 ⁶
11 ^f	VI	337		19.34		19.34	638.4
12 ^f	VII	411		20.33		20.33	627.7
13 ^f	VI I	337	484	19.34	0.49	19.84	624.0
14 ^f	VI II	337	521	19.34	0.73	20.07	618.3
15 ^f	VII I	411	445	20.33	0.24	20.57	603.7

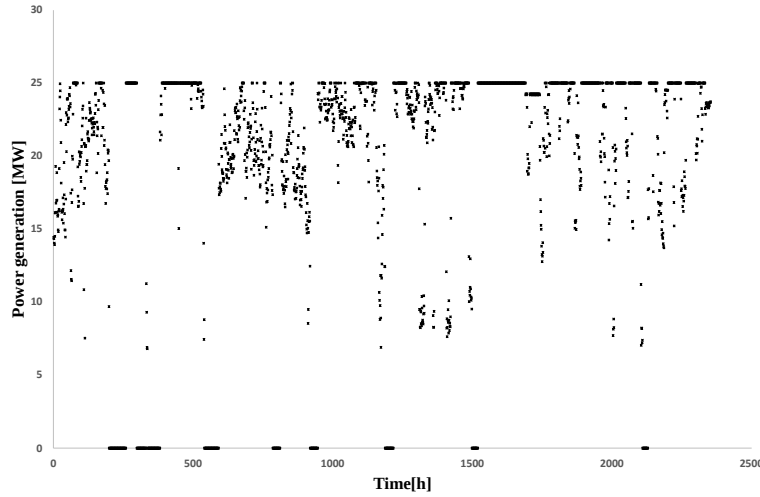


Figure 5.1: Case 11^f: Steam utilised if only Turbine VI is in operation.

on the operational strain placed on the machine, the overhauls may vary from seal and bearing inspections to turbine-blade refurbishments. The costs and when it occurs are not explicitly modelled. The costs over time, however, are approximated through POIQM by penalising each trip with a proportion of the procurement price at time zero. For all scenario results generated in this section, the NPV is penalised with a trip cost of $1/500^{th}$ of the procurement cost at time zero for each occurrence.²

An optimal result generation process, similar to that from Sections 5.2.1 and 5.2.2 are followed. Optimal investment combinations obtained from POIQM in this section are not allowed in the follow-up simulations, in order to generate the top five investment results based on the NPV. All results from this section are summarised in Table 5.5.

Case 11^f: POIQM is solved under a trip penalty of $1/500^{th}$ of the procurement cost, incurred per trip at time zero. Optimal results show that an investment should be made into Turbine VI alone, the 25MW machine. Take note that under a zero-trip penalty Turbine VI is not within the top five optimal investment solutions for the given parameters and is the 5th best option under a fixed cost penalty of R50,000.00. The NPV of R638.4 million is significant lower than NPV's from Tables 5.3 and 5.4, *i.e.* 10.6% and 13.1% below that of Case 6^f Case 1^f, respectively.

Figure 5.1 shows a plot of power generation over time for the real-world data, as discussed in Section 2.11, if Turbine VI alone is operational. Take note that this is not over 10 years, but over the time horizon of the real-world data. From the results shown in Figure 5.1 it follows that

²It is acknowledged that higher manufacturing quality turbines, which are more expensive, may incur lower future maintenance costs, compared to less expensive ones. For such an instance the trip cost can be adjusted for the corresponding turbine, since it is a parameter. This chapter's aim is only to demonstrate the functionality of POIQM and not to investigate all possible turbine investment outcomes. This however, given sufficient parameter information and computer solving capacity, is possible for the POIQM formulation.

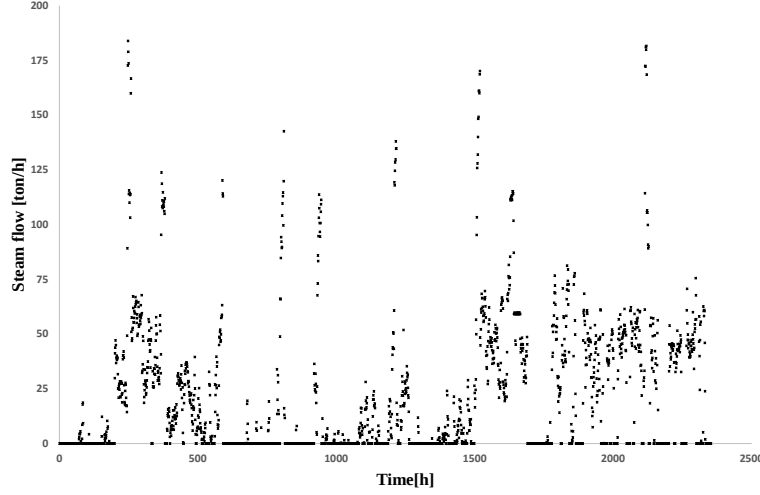


Figure 5.2: Case 11^f: Excess available steam over that cannot be utilised by Turbine VI.

Turbine VI is operational for 86.7% of the time at an average of 77.4% of full capacity.

The steam that cannot be utilised by Turbine VI is plotted in Figure 5.2. From this graph it can be observed that significant potential power generation goes to waste. An average of 20.84 ton steam per hour cannot be used for power generation, which is 4.17MW at a turbine efficiency of 5 (ton per Mega Watt generated). These results indicate the effect of incorporating a proportional trip penalty into the investment decision making process.

As a result, optimal turbine investment choices will not necessarily coincide with optimal power generation. From the POIQM results it can be concluded that for power generation under a fixed-sequence philosophy in a fluctuating steam flow environment, it is better to invest in a single turbine. Such a single investment, however, will evidently result in the inability to utilise periods of high steam availability, since another turbine will not be able to assist. Furthermore, all of the steam availability goes to waste, for power generation purposes, when the turbine is off-line.

Case 12^f: Solving POIQM by not allowing for a procurement into Turbine VI alone, the optimal outcome is to invest in Turbine VII, which is the optimal choice under a fixed trip cost penalty, *i.e.* Case 6^f. Take note, on average Turbine VII generates 0.99MW more than Turbine VI. The NPV income over 10 years for a constant 0.99MW is R43.4 million, whereas the procurement price between Turbine VI and VII differs by R20 million. Even though, the NPV from Case 11^f is almost R11 million higher due to projected trip expenditures.

Case 13^f to 15^f: From POIQM's results, as reported in Table 5.5, it can be observed that for these three Cases the investment combinations do not make sense to pursue. Either Turbine VI or VII in combination with Turbine I or II form part of the optimal solution for Case 13^f to 15^f. The procurement of an additional smaller capacity turbine for these Cases, therefore, reflects negative towards the single investments from Case 11^f or 12^f. It should be noted that the optimal investment results obtained for Case 1^f, Turbine II and VII, are not within the top five investment options when a proportional trip penalty is incurred.

5.2.4 POIQM results under increased investment and penalty costs

In the previous sections the sensitivity of optimal investment results are displayed toward trip penalty choices. In this section POIQM is solved for when investment and trip penalties are increased. The investment costs from Table 5.2 are doubled and each trip is, furthermore, penalised with 1/250th of the procurement price. In this section the top three and not five investment choices are reported. The fourth investment combination, *i.e.* Case 19^f, is not the fourth best investment option, but for comparative purposes only. Case 19^f is used to demonstrate how the optimal investment results from Case 1^f, with a zero trip penalty, perform under the increased procurement

Table 5.6: Scenario results for Case 16^f to 19^f where investment costs are doubled and each future trip is penalised at time zero with 1/250th of the procurement price. The results include the optimal turbine investment and loading hierarchy, trips and power generation per turbine, total power generation and the expected NPV profit for 10 years.

Case number	Turbine hierarchy	Trips per turbine		Power generation per turbine [MW]		Total power generation [MW]	NPV R ·10 ⁶
16 ^f	VI	337		19.34		19.34	306.0
17 ^f	IV	262		13.25		13.25	276.5
18 ^f	V	375		16.75		16.75	262.1
19 ^f	II VII	149	560	4.75	16.17	20.92	-32.5

prices and trip penalties.

Case 16^f: POIQM is solved for the increased procurement and trip costs. Optimal results show that Turbine VI alone should be invested in with an expected NPV of R306.0 million. Take note, even though the NPV is reduced by more than 52%, compared to that of Case 11^f, an equivalent optimal investment is solved for. This indicates that the optimal investment solution for the real-world data under the given turbines are not sensitive to a 100% procurement price inflation and on top of that a 100% proportional trip costs increase.

Case 17^f and 18^f: By repeating the processes of the previous sections, the second and third best investment choices are determined from POIQM and tabulated in Table 5.6. Case 17^f indicates that Turbine IV, the 15MW machine is the second best investment choice with an expected NPV of almost 10% below that obtained for Case 16^f. Note that this investment yields an estimated power generation of only 13.25MW over time, however, the total trip occurrences of 262 is considerably less in comparison to all other single-turbine Cases reported in this chapter. The third best investment option, reported by Case 18^f, is for Turbine V, the 20MW machine.

Case 19^f is where POIQM is solved for the initial investments of Case 1^f, *i.e.* Turbine II and VII. The optimal investment matrix for Case 1^f is therefore fixed as input parameter to POIQM. Results indicate a negative NPV of R32.5 million. Take note, as mentioned earlier, the trip costs are deducted from the NPV after POIQM has yielded optimal results. This is to allow more realistic outcomes without future knowledge of trip events.

As a result, under higher costs, simulation outcomes indicate that if Turbine II and VII are to be invested in, a net loss is expected to realise. It should be noted, without a trip penalty the expected NPV is R581.1 million when the procurement prices are doubled. The potential future costs of turbine trips can, therefore, significantly influence the NPV, to such an extend that a realistic net loss is experienced where an optimal NPV was expected.

These results demonstrate the need for turbines to be dynamically controlled and not according to a hierarchy with fixed set points. If steam flow to the turbines can be dynamically controlled so that a machine is not necessarily loaded to full capacity, in order for another to stay operational, fewer trips and more power generation may result. Less trips will incur fewer potential power generation wastage, including lower maintenance, and therefore trip costs over time.

5.2.5 POIQM results for more efficient turbines

Results reported in Sections 5.2.1 to 5.2.4 demonstrate the unique working ability of POIQM to yield optimal investment choices, for an energy recovery plant operating under a fixed-sequence philosophy. These results, furthermore, demonstrate that if expected future turbine trip costs are taken into consideration, a single turbine investment should rather be selected. POIQM yields that Turbine VI should be procured to optimise the NPV profit over 10 years when trip costs are linked to the procurement costs. This section shows how the interpretation of turbine trip cost penalties influence the optimal investment decision when a more efficient turbine can be procured.

Scenarios are investigated when Turbine VI is chosen as the investment option. An investment into Turbine VI is compared to *Turbine VI**. Turbine VI* is a 25MW turbine, which converts steam to power at a rate of 4.5 (ton per Mega Watt generated) with the same lower limit as Turbine VI. The operational parameters of Turbine VI are repeated in Table 5.7 together with that of Turbine VI*.

Table 5.7: Maximum and minimum power generation capabilities, followed by the steam conversion rate, together with the upper and lower limits for Turbine VI and VI*.

Turbine	Max [MW]	Min [MW]	η_i^T [ton/MW]	U_i [ton/h]	L_i [ton/h]
VI	25.0	6.8	5	125.0	34.0
VI*	25.0	$7\frac{5}{9}$	4.5	112.5	34.0

Table 5.8: Allowable procurement price increases based on results for Case 1^{fi} to 4^{fi}. The percentage increase indicates the maximum price increase from Turbine VI to VI* that will allow Turbine VI* to be a better investment based on NPV profit over 10 years.

	Case 1 ^{fi}	Case 2 ^{fi}	Case 3 ^{fi}	Case 4 ^{fi}
Percentage price increase	31.2	31.2	18.6	6.6

POIQM is solved when an investment into Turbine VI* alone is allowed. Optimal results yield that Turbine VI* will generate power at a rate of 20.15MW compared to Turbine VI's 19.34MW. This section consists of four Cases, where the scenario numbering includes a superscript "fi" that refers to a change in the *isentropic efficiency* of a turbine under a fixed-sequence philosophy.

Case 1^{fi} to 4^{fi}: POIQM is solved for Turbine VI and VI* under three trip cost parameters. For each Case the maximum percentage price increase from Turbine VI to VI* that an energy recovery plant should be willing to pay is determined. A price increase above the percentage indicates that the procurement of Turbine VI will yield a higher NPV. For Case 1^{fi} to 3^{fi} the procurement price of Turbine VI is equal to that given in Table 5.2 and for Case 4^{fi} it is doubled. POIQM is solved for a zero trip penalty in Case 1^{fi} and a penalisation of R50,000.00 per occurrence in Case 2^{fi}. Case 3^{fi} and 4^{fi} incorporate trip costs of 1/500th and 1/250th of the procurement prices respectively. From the results reported in Table 5.8 it can be observed that under the initial procurement price for Turbine VI with zero or a fixed penalty cost, an energy recovery plant should be willing to pay 31.2% more for Turbine VI*. Since the difference in income is equivalent for Case 1^{fi} and 2^{fi}, the allowable percentage relative towards Turbine VI's procurement price is unchanged at 31.2%. If a more realistic cost is incurred, in terms of a proportion of the procurement price, POIQM yields that for a 1/500th trip penalty, the price increase for Turbine VI* should be below 18.6%. If the investment cost of Turbine VI is doubled and trip costs are determined to be 1/250th of the procurement cost, a price increase of only 6.6% can be accommodated.

The results reported in this section demonstrate POIQM's ability to yield optimal investment choices when turbines operate at different efficiencies. Furthermore, the results show how the interpretation of trip costs can influence the optimal investment decision, in terms of a price increase that the energy recovery plant should be willing to pay for a more efficient turbine.

5.3 Summary and conclusion

This chapter demonstrated how optimal turbine investments can be determined by POIQM for an energy recovery plant operating under a fixed-sequence philosophy. Results indicate that higher power generation does not necessarily correlate with an increased NPV. Furthermore, results show that a smaller capacity turbine should be invested in to avoid expensive trip costs of larger capacity machines, even if lower power generation will result. Significant quantities of steam and therefore potential power generation are wasted owing to the inability to capture the fluctuating steam profiles with a single operational turbine.

To conclude, it is more profitable to let potential power generation go to waste, than operating a larger capacity or multiple turbines under a fixed-sequence philosophy. Figure 5.3 displays a flow chart of the MILP models used in the process to formulate POIQM. The four concept formulations proposed in Chapter 3 are used as basis for POQM in Chapter 4, which is used for POIQM.

If an energy recovery plant could, therefore, dynamically control steam flow to the turbines and is not forced to load a machine until full capacity in a fixed-sequence, it might be possible to capture the fluctuations in steam profiles without experiencing excessive turbine trips. The following chapter will demonstrate, through conceptual MILP models, the concept of dynamic steam control between two turbines and how it can be incorporated to increase power generation.

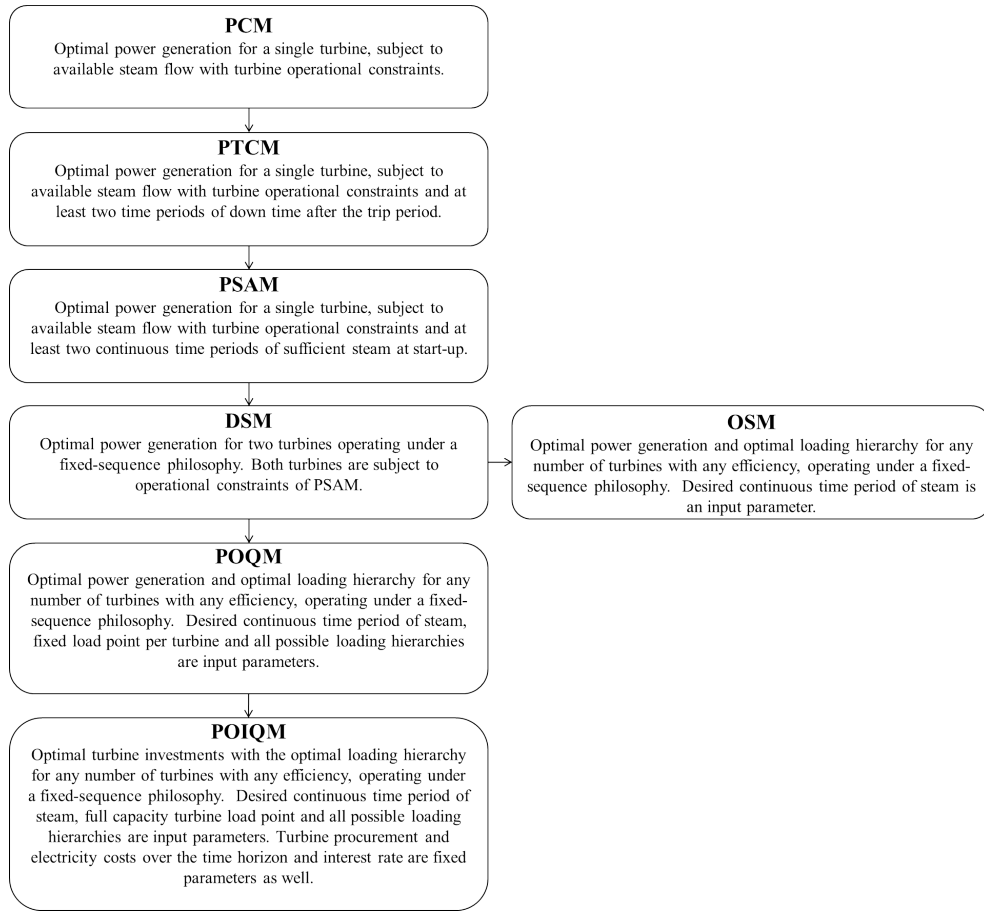


Figure 5.3: Flow chart of the MILP's building up to POIQM, from the four concept formulations presented in Chapter 3 and POQM in Chapter 4.

Chapter 6

Conceptual dynamic model explanation

Chapters 4 and 5 demonstrated optimal power generation and optimal investments for turbines operating under a fixed-sequence loading hierarchy. Through real-world data it was demonstrated how frequent trips can occur under fluctuating steam availability that led to power generation losses. Chapter 5 showed that the cost of frequent trips influence optimal investments so that only a single turbine should be procured, rather than attempting to capture increased potential power generation with multiple machines. The need therefore arises for steam flow to be dynamically controlled between turbines in an attempt to improve power generation and reduce turbine trips. This chapter provides explanatory MILP model formulations for turbines that are dynamically controlled, *i.e.* loaded according to steam availability. In the section to follow an example will be used to demonstrate that the remaining MILP models applied in this study allow for a level of foresight, which may be considered unrealistic. In order to justify the use of MILP models for the purpose of deciding on optimal turbine investments, a dynamic control logic algorithm is proposed that allows the implementation of turbine operating policies to be more in line with the MILP results. The dynamic control logic algorithm will maximise power generation where accurate steam flow predictions for future time periods are allowed. Results from the dynamic control logic will be compared with optimal results.

6.1 A case to justify the use of dynamically controlled MILP optimisations

Results obtained in the previous chapters demonstrate how frequent trip events are experienced under a fixed-sequence philosophy and the inability at times to sufficiently utilise the available steam. When potential future trip costs were taken into account in Chapter 5, optimal outcomes show that it is financially sensible to only invest in a single turbine. As a result, significant potential power generation is wasted.

In this chapter optimal power generation models are formulated for two turbines operating under dynamic control. The two turbines may be loaded in any order at each time period. Note that all scenario results in this chapter are performed with the hypothetical steam profile discussed in Section 2.10.

In order to formulate optimal investment models in following chapters, optimal power generation needs to be determined. These optimal results can then be used in the decision making process to determine which turbines should be invested in. It is, therefore, imperative for the real-life power generation to be in close proximity to the optimal power generation outcome, in order for the investment choices to be sensible. To demonstrate the level of foresight, which the MILP comprise and how that compares with realistic obtainable power generation under dynamic control, an example follows.

Consider a hypothetical Works where two turbines are operational in a fluctuating steam flow environment. The minimum and maximum steam flow limits of the first turbine are 10 (mass units

Table 6.1: Three steam profiles for a hypothetical Works that operates with two turbines.

Time	Steam flow units per time									
	1	2	3	4	5	6	7	8	9	10
Profile one	60	45	40	38	38	50	60	58	53	52
Profile two	60	45	38	40	40	50	60	58	53	52
Profile three	60	45	38	40	38	50	60	58	53	52

Table 6.2: Steam profile one with binary operational status variables for the three dynamic control philosophies.

Time		Steam flow units per time										Accumulated steam flow units
		1	2	3	4	5	6	7	8	9	10	
Profile one		60	45	40	38	38	50	60	58	53	52	494
Philosophy one	y_{1t}^T	1	1	1	1	1	1	1	1	1	1	373
	y_{2t}^T	1	0	0	0	0	0	0	1	1	1	
Philosophy two	y_{1t}^T	1	0	0	0	0	1	1	1	1	1	358
	y_{2t}^T	1	1	1	0	0	0	0	1	1	1	
Philosophy three	y_{1t}^T	1	0	0	0	0	1	1	1	1	1	358
	y_{2t}^T	1	1	1	0	0	0	0	1	1	1	

per time period) and 25 (mass units per time period), respectively. The second turbine operates within the allowable steam flow limits of 40 (mass units per time period) and 150 (mass units per time period). The operational statuses of the first and second turbine at time t are denoted by the binary variables y_{1t}^T and y_{2t}^T , respectively. Note, if the first turbine is operational at time t , $y_{1t}^T = 1$, else $y_{1t}^T = 0$. The second turbine is operational at time t if $y_{2t}^T = 1$, else $y_{2t}^T = 0$.

Table 6.1 depicts three hypothetical steam profiles for this Works that can be utilised for power generation. Each profile comprises steam flow values over 10 time periods and will be referred to as profile one, two and three. Take note, for all three profiles, only the steam flows from times three until five differ. At time period one, both turbines are operational for each profile so that $y_{11}^T = 1$ and $y_{21}^T = 1$. The Works requires that sufficient steam must have been present in the previous two time periods to allow a restart at time t .

Steam flow distributions to the turbines, and therefore power generation at each time period, are determined for three different dynamic control philosophies. Each control philosophy determines at time t which turbine to trip if sufficient steam is not available, and when to restart. A turbine can only trip if sufficient steam is not available to keep it operational. Furthermore, a turbine may not be tripped for the other one to restart. In the examples to follow, each philosophy allows both turbines to be operational if sufficient steam is present.

Philosophy one optimises power generation over the 10 time periods, by utilising accurate knowledge at time t of all future steam flows. Power generation is maximised in philosophy two, through accurate knowledge of steam flow, one time period in advance. Therefore, the steam distribution decision at time t can be influenced by the future flow value at time $t + 1$. If one of the two turbines must be tripped at time t , but not both, this philosophy will determine which operational status will result in the most power generation for the times t and $t + 1$. The turbine that will result in the most power generation for time t and $t + 1$ combined, is kept operational. Take note, in Section 2.9 it was demonstrated by means of a gas holder, that some level of accurate off-gas and therefore steam predictions can be obtained for the near future flows.

For philosophy three, no knowledge of future steam flows are present. If sufficient steam is not present to keep both turbines operational, this philosophy will trip the one resulting in the least power generation. Each steam profile is now solved for all three control philosophies and the results reported in Tables 6.2 to 6.4. Take note, only the operational statuses of both turbines are indicated at each time period and the accumulated steam units utilised are provided.

Results from the three philosophies for turbine operations under profile one are given in Table 6.2.

Table 6.3: Steam profile two with binary operational status variables for the three dynamic control philosophies.

Time Profile two		Steam flow units per time										Accumulated steam flow units
		1	2	3	4	5	6	7	8	9	10	
		60	45	38	40	40	50	60	58	53	52	496
Philosophy one	y_{1t}^T	1	0	0	0	0	0	0	1	1	1	378
	y_{2t}^T	1	1	0	0	0	1	1	1	1	1	
Philosophy two	y_{1t}^T	1	1	1	1	1	1	1	1	1	1	373
	y_{2t}^T	1	0	0	0	0	0	0	1	1	1	
Philosophy three	y_{1t}^T	1	0	0	0	1	1	1	1	1	1	343
	y_{2t}^T	1	1	0	0	0	0	0	1	1	1	

It can be seen that the optimal accumulated steam utilised by philosophy one is 373 steam flow units over time. Both philosophies two and three uses 358 steam flow units over time for power generation. Take note, at time period two, one of the turbines must trip owing to insufficient steam. The optimal choice will be to trip the larger capacity turbine, even though all the available steam at that time period cannot be used by the smaller capacity turbine. This decision can be made, because of knowledge regarding the future steam flows. Note that time periods six and seven comprise sufficient steam to keep both turbines operational, therefore, the second machine is restarted at time period eight for all three philosophies.

At time period two, the second philosophy needs to determine how to maximise the steam distribution for the current and next time period. If the first turbine is kept operational, as is the optimal choice, the second machine will trip and a total of 50 steam units will be utilised during the two time periods. By tripping the first turbine, however, 85 steam units will be utilised between the two time periods by the second machine. The maximum power generation choice is, therefore, to trip the first turbine, which is not optimal.

The 38 steam flow units at time period four is not sufficient to keep the second turbine operational and as a result it trips. Time period four's steam flow, however, would have been sufficient to keep the first turbine operational. As a result no power is generated during times four and five, where both periods are sufficient to load the first turbine at full capacity. At time period six the first turbine is allowed to restart, owing to adequate steam that is present at time periods four and five. The second turbine is allowed back online at time period eight.

Philosophy three does not comprise any future knowledge and maximises power generation for each time period. At time period two, the decision to maximise power generation will therefore result in the first turbine being tripped, since the second one will utilise all 45 steam flow units. The same decision will, therefore, result at time period two for the second and third philosophy, irrespective of future knowledge into the next time period. For the following time periods the same logic applies as for the second philosophy, so that both yield an equivalent power generation over the time horizon.

Results are now obtained for all three philosophies when the turbines operate under profile two and reported in Table 6.3. Profile two is relative similar to the first one, with the exception that the steam values between periods three and four are switched and at time five the flow is increased by two units. As a result, the first philosophy determines to trip turbine one at time period two, even though the second turbine will trip during the next time period. The trip of the second turbine results in three time periods without power generation, whereafter it is restarted at time period six. Note that the first turbine could have started at time period five, however, this would have prevented the second turbine from restarting at time period six. The first turbine restarts at time period eight and the optimal accumulated steam flow units utilised are 378 for this steam profile.

For the second philosophy, the decision at time period two is to either keep turbine one operational, resulting in 50 units of steam to be utilised between time periods two and three, or turbine two that will use 45 units. To maximise power generation, turbine two is therefore tripped. As a result the first turbine is operational throughout the time horizon and at time period eight, the second turbine is restarted. The maximised accumulated steam flow units utilised are 373.

Table 6.4: Steam profile three with binary operational status variables for the three dynamic control philosophies.

Time Profile one		Steam flow units per time										Accumulated steam flow units
		1	2	3	4	5	6	7	8	9	10	
		60	45	38	40	38	50	60	58	53	52	494
Philosophy one	y_{1t}^T	1	1	1	1	1	1	1	1	1	1	373
	y_{2t}^T	1	0	0	0	0	0	0	1	1	1	
Philosophy two	y_{1t}^T	1	1	1	1	1	1	1	1	1	1	373
	y_{2t}^T	1	0	0	0	0	0	0	1	1	1	
Philosophy three	y_{1t}^T	1	0	0	0	1	1	1	1	1	1	343
	y_{2t}^T	1	1	0	0	0	0	0	1	1	1	

The third philosophy trips the first turbine at time period two, as for the optimal case, since more power is generated by the first turbine for that period. Note, however, at time period five the first turbine is restarted, since it maximises power generation for that period. This, however, is not the optimal decision, since it prevents the second turbine from starting at time period six. The third philosophy allows for 343 steam units to be used.

For the last profile, both the first and second philosophy solve to trip turbine two at the second time period. As a result, both philosophies yield equivalent power generation of an accumulated 373 steam flow units. The decision making process of the third philosophy is equivalent to that under the second steam profile and, therefore, 343 steam flow units are utilised over the time horizon.

The three profiles depict different outcomes for the three dynamic control philosophies as discussed above. Let the three profiles be combined into a single profile of 30 time periods. For the first and optimal philosophy, 1124 steam units will be utilised. For the second philosophy, where one future steam flow value is known in advance, 1104 units are used and 1044 units for the third philosophy. As a result, the second philosophy will generate 98.2% of the optimal power and the third philosophy 92.9%.

It should be noted, however, that the examples depicted in this section are used to demonstrate trip and restart occurrences. During prolonged time periods where sufficient steam is available to keep both turbines operational, all three philosophies will allow for equivalent power generation. As a result, the percentages in the above mentioned paragraph will increase. Furthermore, this is not the case for the fixed-sequence philosophy, since during this hypothetical prolonged time period a turbine may trip owing to the first receiver's fixed set point.

This example, however, demonstrates not only the rationale behind dynamic control, but that realistic maximised power generation can be achieved. As a result, MILP optimisation models for dynamic control should be formulated to yield optimal investments from optimal power generation. It is, however, imperative for a dynamic control logic algorithm to be formulated, which can be programmed in a real-life operating control system to maximise power generation.

6.2 Model formulation for two dynamically controlled turbines

This section provides three MILP formulations that demonstrate the concept of optimising power generation amongst two dynamically controlled turbines. Turbine loading is not dependent on a hierarchy or a fixed limit setting. Both turbines can therefore be loaded to any allowable setting at time t , subject to steam availability and operational status. If steam flow is insufficient to keep both turbines operational, any one of the two can be tripped. Furthermore, when both turbines are in trip there is no prerequisite start-up order.

For a tripped turbine to become operational the model formulation determines if sufficient steam is present that could have kept it online during the preceding two time periods. Steam utilised by the other turbine, if operational, during this time is the minimum quantity at each period between its maximum limit and the available steam flow. Take note, however, that any steam received by an operational turbine above its minimum limit could potentially have been utilised by the turbine

in trip. Therefore, if a turbine is in trip, the steam used by the other machine should only be considered up to its minimum allowable limit.

6.2.1 The Dynamic Concept Model (DCM)

The first model formulation provided in this chapter is that of the Dynamic Concept Model (DCM). DCM optimises power generation between two dynamically controlled turbines over time. A turbine is only allowed to be operational if sufficient steam is available at that time period. The sufficient steam startup constraints used to formulate DSM in Section 3.4 are used for DCM.

Parameter and variable definitions for the DCM are laid out, followed by the mathematical formulation. The time index set is given by $\mathcal{T} = \{1, 2, \dots, |\mathcal{T}|\}$ with m_t^S (mass units per time period) the excess steam available for power generation at time t . Steam is distributed between two turbines, denoted by a subscript “1” and “2” respectively. Turbine one receives m_{1t}^S (mass units per time period) and turbine two m_{2t}^S (mass units per time period) at time t . Turbine one utilises steam within the minimum and maximum operational limits of L_1 (mass units per time period) and U_1 (mass units per time period), respectively and generates power at a rate of η_1^T (flow units per power unit generated). The corresponding limits and rate of power generation for turbine two are given by L_2 (mass units per time period), U_2 (mass units per time period) and η_2^T (flow units per power unit generated).

When turbine one is operational at time t , the decision variable $y_{1t}^T = 1$, otherwise $y_{1t}^T = 0$. The same holds for turbine two and y_{2t}^T . Turbine one that is in trip at time $t - 1$ is eligible for a restart at time t only if $y_{1t}^b = 1$ and a tripped turbine two at time $t - 1$ may only go back online if $y_{2t}^b = 1$. The formulation of DCM follows:

$$\text{maximise } \sum_{t \in \mathcal{T}} \left(\frac{m_{1t}^S}{|\mathcal{T}| \eta_1^T} + \frac{m_{2t}^S}{|\mathcal{T}| \eta_2^T} \right), \quad (6.1)$$

subject to

$$m_{1t}^S + m_{2t}^S \leq m_t^S, \quad \forall t \in \mathcal{T}, \quad (6.2)$$

$$m_{1t}^S \leq U_1 y_{1t}^T, \quad \forall t \in \mathcal{T}, \quad (6.3)$$

$$m_{1t}^S \geq L_1 y_{1t}^T, \quad \forall t \in \mathcal{T}, \quad (6.4)$$

$$m_{2t}^S \leq U_2 y_{2t}^T, \quad \forall t \in \mathcal{T}, \quad (6.5)$$

$$m_{2t}^S \geq L_2 y_{2t}^T, \quad \forall t \in \mathcal{T}, \quad (6.6)$$

$$y_{1t}^b = 1, \quad \forall t \in \mathcal{T} : t < 4, \quad (6.7)$$

$$y_{1t}^b L_1 + y_{2k}^T L_2 \leq m_k, \quad \forall t \in \mathcal{T} : t > 3, t - 2 \leq k \leq t - 1, \quad (6.8)$$

$$y_{1t}^b \geq y_{1t}^T - y_{1t-1}^T, \quad \forall t \in \mathcal{T} : t > 3, \quad (6.9)$$

$$y_{2t}^b = 1, \quad \forall t \in \mathcal{T} : t < 4, \quad (6.10)$$

$$y_{2t}^b L_2 + y_{1k}^T L_1 \leq m_k, \quad \forall t \in \mathcal{T} : t > 3, t - 2 \leq k \leq t - 1, \quad (6.11)$$

$$y_{2t}^b \geq y_{2t}^T - y_{2t-1}^T, \quad \forall t \in \mathcal{T} : t > 3. \quad (6.12)$$

The objective function of DCM is given by equation (6.1) and optimises power generation between the two turbines over the time horizon, *i.e.* the summation of the average steam flow over the efficiency, for each turbine. The combined steam distribution at time t between the turbines must be less or equal to what is available, as can be seen from constraint set (6.2). The sets of constraints (6.3) to (6.6) ensure that each turbine operates within allowable steam limits at all times.

To verify that turbine one in trip may be brought back online, constraint sets (6.7) to (6.9) are used. For turbine one in trip to become operational at time t , $y_{1t}^b = 1$ needs to hold. Constraint set (6.7) allows turbine one to restart within the first three time periods, irrespective of preceding steam availability. Sufficient steam for turbine one is verified by constraint set (6.8) and note that only potential steam utilised by turbine two until its minimum limit is taken into consideration. Variable y_{1t}^b can only affect a non-operational status for turbine one, as can be seen from constraint set (6.9). Equivalent constraint sets for turbine two to become operational at time t , where the operational status of turbine one is taken into consideration, are given by (6.10) to (6.12).

Table 6.5: Operating parameters for Turbines I and II, *i.e.* the upper and lower flow limits, rate of steam flow to power unit generation, the maximum and minimum power generation limits.

	U (mass unit per time period)	L (mass unit per time period)	η (flow units per power unit generated)	Maximum (power units)	Minimum (power units)
Turbine I	50	16	5.0	10	3.2
Turbine II	50	20	4.8	$10\frac{5}{12}$	$4\frac{1}{6}$

Turbine I and *Turbine II*, as defined in Chapter 3, are used to demonstrate the working ability of all model formulations presented in this chapter. The operational parameters for the turbines were initially provided in Table 3.2 and are restated in Table 6.5. Furthermore, all the MILP formulation results from this chapter are summarised in Table 6.6.

Solving DCM for Turbine I and II yields a total optimal power generation of 6.288 (power generation units). Turbine I generates 2.5 power units and Turbine II 3.8 power units when rounded, with seven and one trip occurrences, respectively. From the results it is clear that Turbine II, the more efficient machine, receives more steam and the DCM formulation aims to keep it operational above Turbine I. Steam distributions for DCM are plotted in Figure 6.1 against the steam profile provided in Table 2.2.

Take note that Turbine I trips at time period 24 and restarts at time period 25, and the same recurrence at time periods 92 and 93. The sufficient steam availability constraints dictate that adequate steam needs to be present in the previous two time periods. For DSM in Section 3.4 the constraints sets (3.23) and (3.26) ensure that an adequate time interval before turbine start-up is present. The sets of constraints (6.8) and (6.11) for DCM are equivalent, however, Turbine I trips at time t and goes back online at time $t + 1$ at two instances. To address this issue the next MILP formulation is proposed where a tripped turbine is forced to stay off-line for at least two time periods following the occurrence.

Table 6.6: Power generation and number of turbine trips as determined by the DCM, DTM and DDM for Turbine I and II, including results for DSM from Section 3.4.

Model	Trips		Power generation units		Total power generation units
	Turbine I	Turbine II	Turbine I	Turbine II	
DCM	7	1	2.5	3.8	6.288
DTM	5	1	2.6	3.7	6.286
DDM	5	1	2.6	3.7	6.277
DSM	5	1	4.7	1.0	5.654

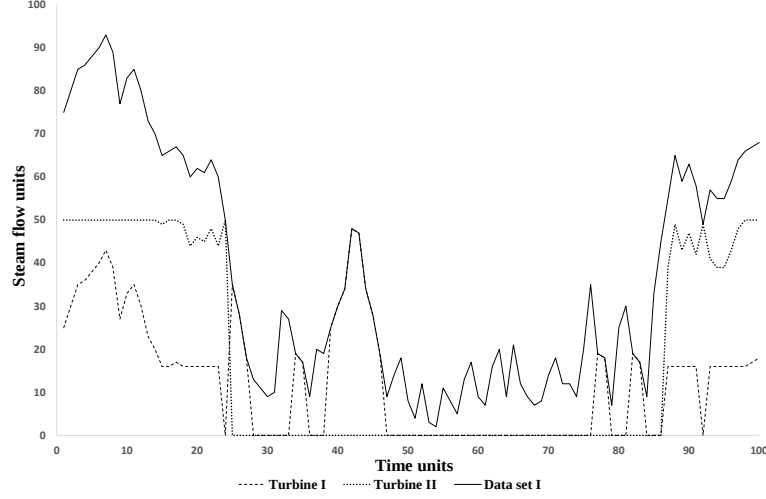


Figure 6.1: Steam distributions for DCM over time plotted against the steam profile provided in Table 2.2.

6.2.2 The Dynamic Time Model (DTM)

To ensure that a tripped turbine stays off-line for the minimum prerequisite interval, the Dynamic Time Model (DTM) is formulated. DTM optimises power generation between two dynamically controlled turbines and addresses the restart problem mentioned from DCM. All the indices, parameters and variables that are defined in the previous section for DCM, are used for DTM. An additional decision variable, however, is defined for DTM.

The binary decision variable y_{1t}^δ can only be equal to one if turbine one has been off-line for at least three time periods prior to time t , *i.e.* the trip period and two time periods thereafter. Likewise, for turbine two y_{2t}^δ can only be equal to one after it has been in trip for at least three time periods. The model formulation follows for DTM, where the objective is to:

$$\text{maximise } \sum_{t \in \mathcal{T}} \left(\frac{m_{1t}^S}{|\mathcal{T}| \eta_1^T} + \frac{m_{2t}^S}{|\mathcal{T}| \eta_2^T} \right), \quad (6.13)$$

subject to constraint sets (6.2), (6.3), (6.4), (6.5), (6.6), (6.7), (6.8), (6.9), (6.10), (6.11) and (6.12) and

$$y_{1t}^\delta = 1, \quad \forall t \in \mathcal{T} : t < 4, \quad (6.14)$$

$$(1 - y_{1t-3}^T) + (1 - y_{1t-2}^T) + (1 - y_{1t-1}^T) \geq 3y_{1t}^\delta, \quad \forall t \in \mathcal{T} : t > 3, \quad (6.15)$$

$$y_{1t}^\delta \geq y_{1t}^T - y_{1t-1}^T, \quad \forall t \in \mathcal{T} : t > 3. \quad (6.16)$$

$$y_{2t}^\delta = 1, \quad \forall t \in \mathcal{T} : t < 4, \quad (6.17)$$

$$(1 - y_{2t-3}^T) + (1 - y_{2t-2}^T) + (1 - y_{2t-1}^T) \geq 3y_{2t}^\delta, \quad \forall t \in \mathcal{T} : t > 3, \quad (6.18)$$

$$y_{2t}^\delta \geq y_{2t}^T - y_{2t-1}^T, \quad \forall t \in \mathcal{T} : t > 3. \quad (6.19)$$

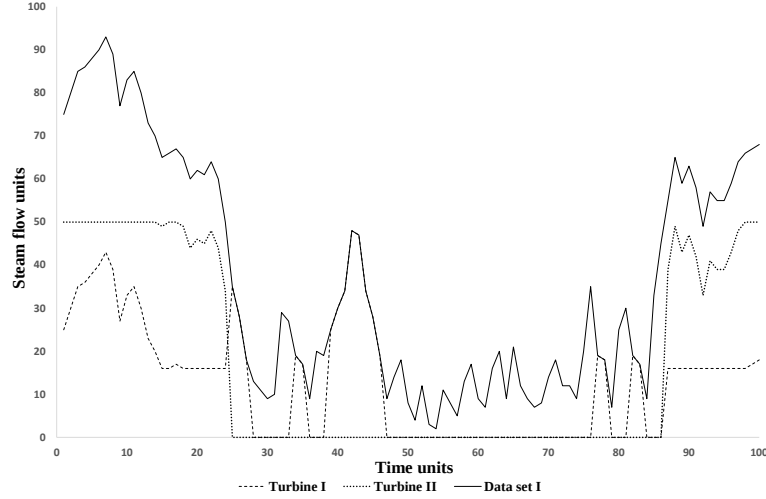


Figure 6.2: Steam distributions for DTM over time plotted against the steam profile provided in Table 2.2.

The objective function of DTM, given by (6.13) optimises power generation between two dynamically controlled turbines over time. The sets of constraints (6.2) to (6.12) are discussed in Section 6.2.1 for DCM and are not repeated. Constraint set (6.14) allows $y_{1t}^\delta = 1$ for turbine one within the first three time periods, whereas (6.15) ensures that turbine one must be in trip for at least three time periods before $y_{1t}^\delta = 1$. From constraint set (6.16) it follows that when turbine one is in trip it can only be brought back online at time t if $y_{1t}^\delta = 1$. As for turbine one, the sets of constraints (6.17) to (6.19) ensure that turbine two must be off-line for a minimum of three time periods before it is allowed back into operation, unless it is within the first three time periods.

If DTM is solved for Turbine I and II, optimal power generation yields 6.286 power units with a total of six trips. The power generation is 99.97% of DCM with two less trips. The steam distributions of the turbines against the steam profile provided in Table 2.2 are plotted in Figure 6.2. From the figure it can be observed that whenever a turbine trips, it stays off-line for at least two time periods thereafter.

DTM's restart constraints are adequate to ensure that a turbine stays off-line for the minimum required interval. Note, however, at time period 87 both turbines restart at the same time. Steam availability at time periods 85 and 86 are 33 (mass units per time period) and 45 (mass units per time period), respectively. The minimum allowable steam flow for Turbine I is 16 (mass units per time period) and for Turbine II 20 (mass units per time period). For both time periods satisfactory steam is available to keep only one turbine operational, where at time period 87 adequate steam is available for both machines. The DTM formulation, therefore, can determine if steam was available to potentially have kept a tripped turbine online during the previous two time periods. However, when both turbines are eligible to restart at time t , constraint sets (6.15) and (6.18) fail to account for steam that could potentially have been utilised by both turbines. The following concept model addresses sufficient steam availability for when both turbines are in trip.

6.2.3 The Double Dynamic Model (DDM)

In order to address the restart constraint shortcoming of DTM, the Double Dynamic Model (DDM) is formulated. DDM optimises power generation between two dynamic turbines over time and allows a turbine to become operational if the previous two time periods possess sufficient steam. Note that for sufficient steam both turbines need to be taken into account, not only for an operational status but also for a possible restart that can occur, i.e. the y_{1t}^b and y_{2t}^b variables.

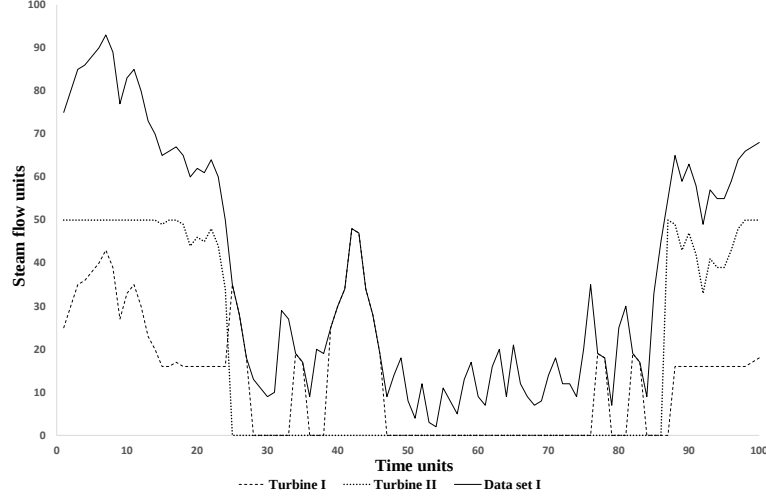


Figure 6.3: Steam distributions for DDM over time against the steam profile provided in Table 2.2.

DDM uses every index, parameter and variable defined for DCM and DTM in Section 6.2.1 and 6.2.2. As for DCM and DTM, the objective is to:

$$\text{maximise } \sum_{t \in \mathcal{T}} \left(\frac{m_{1t}^s}{|\mathcal{T}| \eta_1^T} + \frac{m_{2t}^s}{|\mathcal{T}| \eta_2^T} \right), \quad (6.20)$$

subject to the following constraint sets given by (6.2), (6.3), (6.4), (6.5), (6.6), (6.7), (6.9), (6.10) and (6.12),

$$(y_{1k}^T + y_{1t}^b)L_1 + (y_{2k}^T + y_{2t}^b)L_2 \leq m_k, \quad \forall t \in \mathcal{T} : t > 3, t - 2 \leq k \leq t - 1, \quad (6.21)$$

and final sets of constraints (6.14), (6.15), (6.16), (6.17), (6.18) and (6.19).

All sets of constraints are defined in either Section 6.2.1 or 6.2.2, with the exception of (6.21). Constraint set (6.21) ensures that both turbines' operational status and possibility of restarting at time t are taken into consideration. Therefore, if both turbines are brought back into operation at time t , steam flow equal or above $L_1 + L_2$ needs to be present at times $t - 2$ to t .

Solving DDM for Turbine I and II yields a total optimal power generation of 6.277 power units and six combined turbine trips. In Table 6.6 optimal results are given for DCM, DTM and DDM, including DSM results from Section 3.4. The steam distributions to the turbines under DDM and the steam profile provided in Table 2.2 are shown in Figure 6.3. It is evident from the figure that both turbines adhere to the plant's operational rules. Furthermore, DDM and DSM both yield a total of six turbine trips between Turbine I and II, however, DDM solves for more than an 11% power generation increase owing to dynamic control.

The results reported for DDM demonstrate the positive effect that dynamic turbine control possesses on power generation, compared to a fixed-sequence philosophy. As mentioned earlier, the dynamic MILP formulations requires foresight into future steam flows. This however, is not practically viable. To illustrate the ability of the MILP formulations to yield optimal results that can realistically be achieved within close proximity, a maximisation logic control model is formulated in the next section. Realistic maximisation results are then compared to the optimal result in order to interpret the relevancy of the MILP outcomes.

6.3 Dynamic power generation through logic control

In Section 6.1 the formulation of power generation optimisation MILP's for turbines under dynamic control was justified by means of comparative maximisation examples. As a result, the conceptual MILP formulations of this chapter followed. From these models, optimal power generation and investment formulations will follow in the next three chapters. In order for the upcoming models to yield outcomes that can realistically be achieved, a dynamic maximisation logic control algorithm is required. By programming such an algorithm on the Works' operating control system, power generation in close proximity of optimality should be attainable.

A discussion followed in Section 2.9, which focused on near-future off-gas flow predictions for when an engineering plant consists of a gas holder. Under the assumption that accurate gas predictions will allow a level of certainty for estimating some future steam flows, a logical control algorithm is formulated in this section. The Double Dynamic Logic Control (DDLC) maximises power generation between two dynamically controlled turbines with any operating limits or efficiencies. DDLC adheres to an energy recovery plant's sufficient steam availability needed before start-up and can incorporate future flow predictions. Take note, future steam predictions are not a requirement for the algorithm. The pseudo code of the algorithm is provided in Algorithm 1.

For DDLC, the steam distribution at time t is influenced by foreknowledge of the availability for m time periods into the future, *i.e.* $t + m$. Power generation is maximised by taking into account that a turbine is not allowed to trip if sufficient steam is available to keep it operational. Furthermore, for a restart to commence, the plant's prerequisite minimum sufficient steam time limit must be adhered to. Trip occurrences are unwanted and should be avoided if possible, therefore, DDLC does not allow a restart if it will result in a trip within the next m time periods.

If the potential restart of a second turbine will not yield more power generation at that time period, the machine will be kept off-line. Such an instance may arise if either the turbines operate at different efficiencies or if all the steam can be utilised by the first operational machine.

The formulation of Algorithm 1 is as follows: DDLC receives steam input parameters for times t to $t + m$, where $m \in \mathbb{N}_0$. The steam distribution between the two turbines is then determined for time t , through logic control, with the goal of maximum power generation. The steam available for power generation at time t is denoted by m_t^S (mass units per time period), where turbines one and two receive m_{1t}^S (mass units per time period) and m_{2t}^S (mass units per time period). The binary y_{1t}^T and y_{2t}^T variables indicate whether turbines one and two are operational at time t by taking on the value of one or a zero if in trip. Turbine one in trip can only go back online if the binary variable $y_{1t}^\delta = 1$ and the same holds for turbine two, *i.e.* $y_{2t}^\delta = 1$. For either y_{1t}^δ or y_{2t}^δ to be equal to one, sufficient steam must have been available during the previous n time periods to have kept it operational, where $n \in \mathbb{N}_0$.

Turbine one's maximum and minimum flow limits are given by U_1 (flow units per time period) and L_1 (flow units per time period), respectively, and its steam conversion rate by η_1^T (flow units per power unit generated). The corresponding parameters for turbine two are depicted by U_2 (flow units per time period), L_2 (flow units per time period) and η_2^T (flow units per power unit generated). Steam distributions to the turbines at time t are determined by:

Algorithm 1 Double Dynamic Logic Control (DDLCL)

```

 $L = L_1 + L_2$ 
for  $t \in \mathcal{T} : t > n$  do
   $y_{1t}^T = 0.$ 
   $y_{2t}^T = 0.$ 
   $y_1^\delta = 0.$ 
   $y_2^\delta = 0.$ 
   $y^\delta = 0.$ 
  for  $i=0..m$  do
     $y_{1i}^p = 0$ 
     $y_{2i}^p = 0$ 
  end for
  if  $m_t^s \geq L_1$  then
     $y_{10}^p = 1$ 
  end if
  if  $m_t^s \geq L_2$  then
     $y_{20}^p = 1$ 
  end if
  for  $i=1..m$  do
    if  $m_{t+i}^s \geq L_1$  then
       $y_{1i}^p = y_{1i-1}^p$ 
    end if
    if  $m_{t+i}^s \geq L_2$  then
       $y_{2i}^p = y_{2i-1}^p$ 
    end if
  end for
  if  $\sum_{i=1}^{i=n} y_{1t-i}^T = 0$  and  $m_{t-i}^S - y_{2t-i}^T L_2 \geq L_1$  and  $m_{t+j}^S \geq L_1$  for  $i = (1..n), j = (1..m)$  then
     $y_1^\delta = 1$ 
  end if
  if  $\sum_{i=1}^{i=n} y_{2t-i}^T = 0$  and  $m_{t-i}^S - y_{1t-i}^T L_1 \geq L_2$  and  $m_{t+j}^S \geq L_2$  for  $i = (1..n), j = (1..m)$  then
     $y_2^\delta = 1$ 
  end if
  if  $m_{t-i}^S \geq L$  for  $i = (-m..n)$  then
     $y^\delta = 1$ 
     $y_{1t}^T = 1.$ 
     $y_{2t}^T = 1.$ 
  end if
  if  $y_1^\delta = 1$  and  $\{[y_{2t-1}^T = 1 \text{ and } (m_{t+i}^S \geq L \text{ or } \min(m_{t+i}^S) < L_2)] \text{ or } [y_{2t-1}^T = 0 \text{ and } y_2^\delta = 0]\}$  for  $i = (0..m)$  then
     $y_{1t}^T = 1.$ 
  end if
  if  $\frac{\sum_{i=0}^{i=m} \min(U_1, m_{t+i}^S) y_{1i}^p}{\eta_1^T} \geq \frac{\sum_{i=0}^{i=m} \min(U_2, m_{t+i}^S) y_{2i}^p}{\eta_2^T}$  and  $[y_1^\delta + y_2^\delta + y^\delta = 2 \text{ or } (y_{1t-1}^T = 1 \text{ and } y_{2t-1}^T = 1 \text{ and } m_t^S < L)]$  then
     $y_{1t}^T = 1.$ 
     $y_2^\delta = 0.$ 
  end if
  if  $y_{1t-1}^T = 1$  and  $m_t^S \geq L_1$  and  $[(y_{2t-1}^T = 1 \text{ and } \frac{\sum_{i=0}^{i=m} \min(U_1, m_{t+i}^S) y_{1i}^p}{\eta_1^T} \geq \frac{\sum_{i=0}^{i=m} \min(U_2, m_{t+i}^S) y_{2i}^p}{\eta_2^T}) \text{ or } m_t^S \geq L \text{ or } y_{2t-1}^T = 0]$  then
     $y_{1t}^T = 1.$ 
  end if

```

Algorithm 1 Double Dynamic Logic Control (DDLC) is continued

```

if  $y_1^\delta = 1$  and  $y_{2t-1}^T = 1$  and  $m_t^S \geq L_2$  and  $[(m_t^S \leq U_2 \text{ and } \eta_2^T \leq \eta_1^T) \text{ or } \frac{\min(U_2, m_t^S)}{\eta_2^T} \geq \frac{\min(U_2, m_t^S - \min(U_1, m_t^S - L_2))}{\eta_2^T} + \frac{\min(U_1, m_t^S - L_2)}{\eta_1^T}]$  then
   $y_{1t}^T = 0.$ 
end if
if  $y^\delta = 1$  and  $\frac{\min(U_1, m_t^S)}{\eta_1^T} \geq \frac{\min(U_1, m_t^S - L_2)}{\eta_1^T} + \frac{\min(U_2, m_t^S - \min(U_1, m_t^S - L_2))}{\eta_2^T}$  then
   $y_2^\delta = 0.$ 
end if
if  $y^\delta = 1$  and  $\frac{\min(U_2, m_t^S)}{\eta_2^T} \geq \frac{\min(U_1, m_t^S - \min(U_2, m_t^S - L_1))}{\eta_1^T} + \frac{\min(U_2, m_t^S - L_1)}{\eta_2^T}$  then
   $y_{1t}^T = 0.$ 
end if
if  $y_{1t}^T = 1$  and  $[(y_{2t-1}^T = 0 \text{ and } y_2^\delta = 0) \text{ or } m_t^S < L]$  then
   $m_{1t}^S = \min(U_1, m_t^S).$ 
end if
if  $y_{1t}^T = 1$  and  $y_{2t-1}^T = 1$  and  $m_t^S \geq L$  then
  if  $\eta_1^T \geq \eta_2^T$  then
     $m_{1t}^S = \min(U_1, m_t^S - L_2).$ 
  else
     $m_{1t}^S = \max(L_1, m_t^S - \min(U_2, m_t^S - L_1)).$ 
  end if
end if
if  $y_{1t}^T = 1$  and  $y^{\delta_2=1}$  then
  if  $\frac{\min(U_1, m_t^S)}{\eta_1^T} \geq \frac{\min(U_1, m_t^S - L_2)}{\eta_1^T} + \frac{\min(U_2, m_t^S - \min(U_1, m_t^S - L_2))}{\eta_2^T}$  then
     $m_{1t}^S = \min(U_1, m_t^S).$ 
  else
    if  $\eta_1^T \geq \eta_2^T$  then
       $m_{1t}^S = \min(U_1, m_t^S - L_2).$ 
    else
       $m_{1t}^S = \max(L_1, m_t^S - \min(U_2, m_t^S - L_1)).$ 
    end if
  end if
end if
if  $[(y_{2t-1}^T = 1 \text{ or } y^\delta = 1) \text{ and } m_t^S - m_{1t}^S \geq L_2] \text{ or } [y_2^\delta = 1 \text{ and } y_{1t}^T = 1 \text{ and } (\min(m_t^S + i) \geq L \text{ or } (L_2 < L_1 \text{ and } L_2 \leq \min(m_t^S + i) \leq L_1))]$  for  $i = (0..m)$  then
   $m_{2t}^S = \max(U_2, m_t^S - m_{1t}^S).$ 
end if
if  $(y_2^\delta = 1 \text{ and } y_{1t}^T = 0)$  then
   $m_{2t}^S = \max(U_2, m_t^S).$ 
end if
 $t=t+1$ 
end for

```

The algorithm of DDLC, *i.e.* Algorithm 1 is now applied to determine maximum power generation between Turbine I and II, operating under the hypothetical steam profile given in Table 2.2 of Section 2.10. It is assumed that two future steam flows are known at time t and that two time periods of sufficient steam is required, *i.e.* $m = 2$ and $n = 2$. The maximum total power generation yields 6.246 power units, which is within 0.6% of the optimal 6.277 power units, yielded by DDM. Similar to DDM, Turbine I solves for five trips and Turbine II for one. The results from DDLC, therefore, validate the applicability of DDM and therefore the ability of the MILP optimisation model to solve for realistic obtainable power generation outcomes. Take note, the DDLC can never exceed the results from DDM. These results, however, demonstrate that DDLC can solve within

close proximity of DDM.

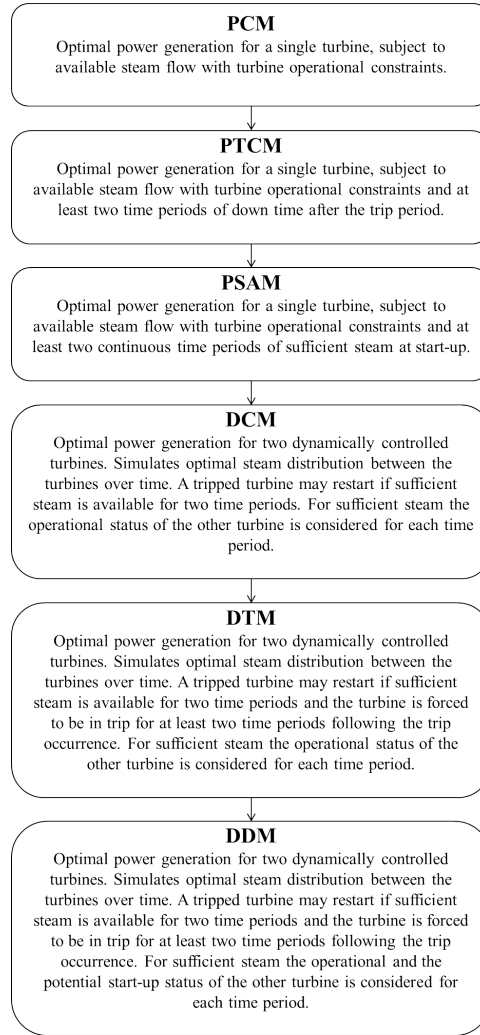


Figure 6.4: Flow chart for three of the MILP concept formulations from Chapter 3 that were used as foundation for the three conceptual dynamic MILP's presented in this chapter.

6.4 Summary

Three concept MILP formulations were developed in this chapter that demonstrate dynamic steam distribution between two turbines with the objective of optimising power generation. DCM showed how constraint sets used for the fixed-sequence philosophy do not ensure that a turbine stays off-line for at least two time periods following a trip occurrence. Results from DTM demonstrated that a turbine did not restart within the two time periods that followed a trip. DTM, however, permitted both turbines to become operational where sufficient steam was present for each individual machine, but not the combination thereof during the previous two time periods. The third MILP formulation, DDM, optimises power generation and adheres to all the energy recovery plant's operational rules. DDM yielded an optimal power generation increase above 11% when compared to DSM under a fixed-sequence philosophy, and equivalent trips for Turbine I and II.

The flow chart in Figure 6.4 indicates the three power generation optimisation MILP concept formulations from Chapter 3 that were used as foundation for the MILP's presented in this chapter. A logic control algorithm, *i.e.* DDLC, which maximises power generation between two dynamically controlled turbines was, furthermore, provided by Algorithm 1. DDLC can incorporate future steam flow predictions and maximises power generation. Solving DDLC for Turbine I and II, when operational under the hypothetical steam profile from Section 2.10, yielded maximum power gen-

eration within 99.4% of optimality when compared to DDM, and the same number of trips. Two accurate future steam predictions were allowed. The following chapter will provide a MILP formulation that optimises power generation over time between any number of dynamically controlled turbines consisting of any operating efficiencies. Steam flows to the turbines will be determined at each time period, where every machine adheres to a plant's specific operational policy of sufficient steam availability.

Chapter 7

Optimal power generation for an energy recovery plant

Three conceptual MILP formulations and a dynamic logic control algorithm were formulated in the previous chapter. The final MILP formulation, DDM, demonstrated power generation optimisation between two dynamically controlled turbines, where both adhere to an energy recovery plant's operational constraints. This chapter presents a MILP formulation that optimises power generation over time between any number of dynamically controlled turbines. The turbines can be of any operating capacities or efficiencies.

7.1 Optimal power generation amongst any number of turbines

Results from Chapter 6 indicate a power generation increase when turbines are controlled dynamically rather than sequentially. The DDM MILP formulation defined in Chapter 6, yields an improved optimal power generation of more than 11% when compared to DSM from Chapter 3. DDM, however, can only accommodate two turbines. In order to determine optimal power generation between any number of turbines, a new model formulation is required.

Take note that all further model formulations that follow in this thesis are for dynamically controlled turbines. The term "Dynamic" will, therefore, not reoccur in a model name, nor will it necessarily be stated that steam flow is dynamically controlled. Unless stated otherwise, it should be assumed that all future steam distributions mentioned are under dynamic control. All results from this chapter are computed by using the real-world data as defined in Section 2.11.

The Optimal Power Model (OPM) presented below, optimises power generation amongst any number of turbines and the optimal solution prescribes the distribution of steam flow at each time period. Turbines may be of any operating capacity or efficiency. Take note, each turbine adheres to an energy recovery plant's specific operational rules. These rules include that a turbine may not trip if sufficient steam exists to keep it operational. Before a turbine is restarted, adequate steam must be present for a predetermined time interval, as discussed in Section 2.8. If the steam were to be used during this time interval by the turbine to be restarted, it should have kept this turbine online. Note further, if this turbine is kept online during the said time interval, the operational status of all the other turbines have to stay unchanged.

The formulation of OPM does not permit a turbine to trip when sufficient steam is available to keep it operational, even if a higher optimal power generation can be obtained. A turbine may also not be taken off-line for another to restart. Under insufficient steam flow the OPM formulation determines which turbine(s) to keep operational and when to allow a restart. Unlike for the fixed-sequence philosophy, a turbine is not forced to trip or start in a preordained order.

The index set of all turbines is given by $\mathcal{I} = \{1, 2, \dots, |\mathcal{I}^T|\}$. The operation of these turbines are modelled over a time set $\mathcal{T} = \{1, 2, \dots, |\mathcal{T}|\}$.

The excess steam for power generation at time t is given by m_t^S (mass units per time period), where each turbine $i \in \mathcal{I}^T$ receives m_{it}^S (mass units per time period). The i^{th} machine operates between minimum and maximum allowable limits of L_i (mass units per time period) and U_i (mass

units per time period), respectively, under a conversion efficiency of η_i^T (flow units per power unit generated). Once a turbine trips, a minimum time of δ^T , which coincides with sufficient steam availability, needs to elapse before the tripped turbine may be brought back online.

There are three main groups of decision variables defined for OPM. The first group of binary decision variables is required to model the operational status of each turbine. Let $y_{it}^T = 1$ if turbine $i \in \mathcal{I}^T$ is operational at time $t \in \mathcal{T}$. The following auxiliary variables are necessary to facilitate the operational policy related to the shut-down and start-up of turbines. Let $y_{it}^h = 1$ if turbine $i \in \mathcal{I}^T$ is tripped (halted) from time $t - 1 \in \mathcal{T}$ to $t \in \mathcal{T}$ and let $y_{it}^\delta = 1$ if turbine $i \in \mathcal{I}$ is in trip for at least δ^T until time $t \in \mathcal{T}$. The variable $y_{it}^b = 1$ if turbine $i \in \mathcal{I}^T$, which has been in trip at time $t - 1 \in \mathcal{T}$, may be brought back online at time $t \in \mathcal{T}$. The objective of OPM is to:

$$\text{maximise } \sum_{i \in \mathcal{I}^T} \sum_{t \in \mathcal{T}} \frac{m_{it}^S}{\eta_i^T |\mathcal{T}|}, \quad (7.1)$$

subject to

$$\sum_{i \in \mathcal{I}^T} m_{it}^S \leq m_t^s, \quad \forall t \in \mathcal{T}, \quad (7.2)$$

$$m_{it}^S \geq L_i y_{it}^T, \quad \forall i \in \mathcal{I}^T, t \in \mathcal{T}, \quad (7.3)$$

$$m_{it}^S \leq U_i y_{it}^T, \quad \forall i \in \mathcal{I}^T, t \in \mathcal{T}, \quad (7.4)$$

$$y_{it}^h \geq y_{it-1}^T - y_{it}^T, \quad \forall i \in \mathcal{I}^T, t \in \mathcal{T} : t > 1, \quad (7.5)$$

$$y_{it}^h \leq y_{it-1}^T, \quad \forall i \in \mathcal{I}^T, t \in \mathcal{T} : t > 1, \quad (7.6)$$

$$y_{it}^h \leq 1 - y_{it}^T, \quad \forall i \in \mathcal{I}^T, t \in \mathcal{T} : t > 1, \quad (7.7)$$

$$m_t^s y_{it}^h - \sum_{j \in \mathcal{I}^T} y_{jt}^T L_j + \epsilon \leq y_{it}^h L_i, \quad \forall i \in \mathcal{I}^T, t \in \mathcal{T} : t > 1, \quad (7.8)$$

$$\sum_{k=t-\delta^T}^{t-1} (1 - y_{ik}^T) \geq \delta^T y_{it}^\delta, \quad \forall i \in \mathcal{I}^T, t \in \mathcal{T} : t > \delta^T, \quad (7.9)$$

$$\sum_{k=t-\delta^T}^{t-1} (1 - y_{ik}^T) \leq \delta^T - c_0 + y_{it}^\delta, \quad \forall i \in \mathcal{I}^T, t \in \mathcal{T} : t > \delta^T, \quad (7.10)$$

$$y_{it}^\delta = 1, \quad \forall i \in \mathcal{I}^T, t \in \mathcal{T} : t < \delta^T + 1, \quad (7.11)$$

$$y_{it}^T \leq y_{it}^\delta + y_{it-1}^T, \quad \forall i \in \mathcal{I}^T, t \in \mathcal{T} : t > 1, \quad (7.12)$$

$$\sum_{i \in \mathcal{I}^T} (y_{ik}^T + y_{it}^b) L_i \leq m_k, \quad \forall t \in \mathcal{T} : t > \delta^T, \begin{matrix} k \geq t - \delta^T, \\ k \leq t - 1, \end{matrix} \quad (7.13)$$

$$y_{it}^b \geq y_{it}^T - y_{it-1}^T, \quad \forall i \in \mathcal{I}^T, t \in \mathcal{T} : t > \delta^T. \quad (7.14)$$

The objective function of OPM, given by equation (7.1), is to optimise power generation over the time horizon $\mathcal{T}$, *i.e.* the total summation of steam flow to each turbine over the efficiency of that turbine divided by the length of the time horizon. Constraint set (7.2) ensures that the accumulated steam flow to the turbines is less than or equal to the available steam flow at time t . Constraint

Table 7.1: Maximum and minimum power generation capabilities, followed by the efficiency, together with the upper and lower limits for Turbine I to VIII. This is equivalent to Table 4.3.

Turbine	Max [MW]	Min [MW]	η_i [ton/(h·MW)]	U_i [ton/h]	L_i [ton/h]
1	5.0	2.0	5.0	25.0	10.0
2	30.0	8.0	5.0	150.0	40.0
3	10.0	$3\frac{5}{9}$	4.5	45.0	16.0
4	20.0	$6\frac{2}{9}$	4.5	90.0	28.0
5	30.0	$8\frac{8}{9}$	4.5	135.0	40.0
6	10.0	3.2	5.0	50.0	16.0
7	20.0	5.6	5.0	100.0	28.0
8	5.0	$2\frac{2}{9}$	4.5	22.5	10.0

Table 7.2: Summary of Table 4.4 with turbines used either for POQM or OSM, the combined trips and power generation.

Case number	Turbine selection			Combined turbine trips	Combined power generation [MW]
1^f	I	II		19	20.96
2^f	I	II	III	20	20.98
3^f	I	IV		19	19.46
4^f	I	V		25	22.29
5^f	I	VI	VII	39	20.09
6^f	III	IV	VIII	45	22.13
7^f	I	IV	VI	39	20.95

sets (7.3) and (7.4) ensure that steam flow to any turbine $i \in \mathcal{I}^T$ is always within the allowable minimum and maximum operational limits or zero.

Constraint sets (7.5) to (7.7) determine if turbine $i \in \mathcal{I}^T$ tripped from time $t - 1$ to t , and the formulation of constraint set (7.8) guarantees that a turbine is not tripped if sufficient steam flow is available. Note for (7.8) that ϵ is a very small positive number, just to ensure that an inequality is obtained for the constraint set. The set of constraints (7.9) to (7.11) determine if at least a δ^T time has elapsed for a tripped turbine $i \in \mathcal{I}^T$. The constant $0 < c_0 < 1$ in constraint set (7.10) is to ensure an inequality. If the minimum δ^T has not yet passed for a turbine in trip, constraint set (7.12) ensures that the turbine cannot go back online. To determine if sufficient steam flow was available for a turbine in trip during the preceding δ^T period and whether it may be brought back online, constraint sets (7.13) and (7.14) are applied.

7.2 Optimisation results for OPM

In order to sensibly compare turbine operations under dynamic control, the results from OPM are matched against that of POQM in Chapter 4 for the fixed-sequence philosophy. For this reason, all turbine scenarios given in Chapter 4 are repeated for OPM. Every turbine used in this chapter is, therefore, found in Table 4.3, and reappear in Table 7.1.

Seven *Cases* are solved with OPM and compared with those outcomes from OSM¹ under a fixed-sequence philosophy. The results from Case 1^f to 7^f are summarised in Table 7.2. Take note that the second column of Table 7.2 comprises only the chronologically defined turbine number and not necessarily the optimal loading hierarchy as determined by either POQM or OSM. OPM is solved for the seven corresponding turbine configurations and the results are reported in Table 7.3. OPM's scenarios are labeled as *Case 1^d* to *7^d* . The Case d implies “dynamic control”.

Case 1^d OPM is solved for Turbine I and II, which are the 5MW and 30MW machines. Optimal power generation yields 21.70MW with 15 trips in total. These results are four trips less and 0.74MW increased power generation when compared to that from Case 1^f for equivalent turbines.

¹Note that POQM yields equivalent outcomes.

Table 7.3: OPM results for Case 1^d to 7^d, with the turbines for each scenario, followed by individual and combined trips and power generation.

Case number	Turbine number			Trips per turbine		Combined turbine trips	Power generation per turbine [MW]			Combined power generation [MW]
1 ^d	I	II		4	11	15	2.41	19.29		21.70
2 ^d	I	II	III	9	11	7	1.67	12.61	8.73	23.31
3 ^d	I	IV		5	8	13	3.96	16.53		20.49
4 ^d	I	V		5	11	16	2.34	20.99		23.33
5 ^d	I	VI	VII	7	10	8	2.81	6.66	12.96	22.43
6 ^d	III	IV	VIII	10	8	10	5.39	15.01	3.80	24.20
7 ^d	I	IV	VI	8	8	10	2.07	16.14	5.50	23.71

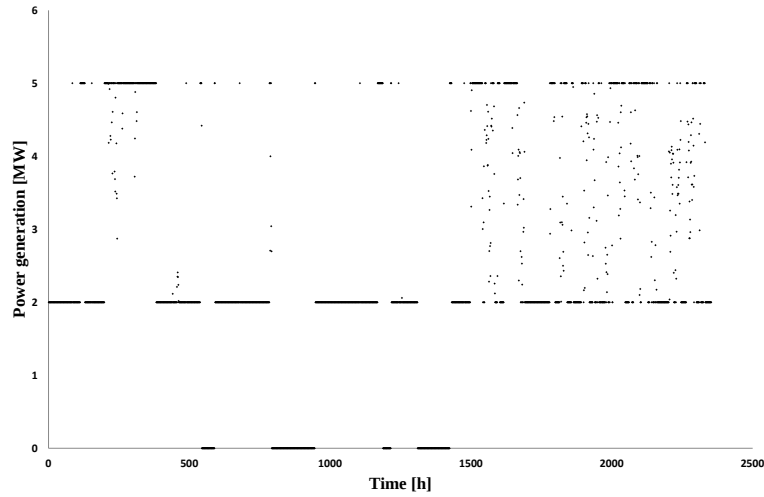


Figure 7.1: Case 1^d: Power generation over time for Turbine I, the 5MW, under dynamic control.

Therefore, by changing the turbine steam loading from a simplistic fixed-sequence philosophy to a complex dynamic control, power generation increases by 3.53% for these two turbines. It is acknowledged that the implementation costs of a dynamic control system will incur some investment costs, however, a significant increase in power generation may be realised.

Figures 7.1 and 7.2 display the power generation for Turbine I and II over the time horizon. Comparing Figures 7.1 and 7.2 to Figures 4.4 and 4.5 for Turbine I and II, the effect of dynamic steam control is clearly observed. In Figure 4.4 Turbine I is loaded to full capacity if sufficient steam is available. Dynamic behaviour can be observed in Figures 7.1 and 7.2 for Turbine I and II. It should be noted that the loading pattern observed for Turbine II in Figure 4.5 is owed to the fluctuating nature of the plant's signature steam profile. Figure 4.5 should not be mistaken for dynamic control and must, therefore be, simultaneously interpreted with Figure 4.4.

The excess steam that cannot be utilised by either Turbine I or II for Case 1^d is depicted in Figure 7.3. Less unused steam is evident in this graph, compared to Figure 4.6, which contains the respective unutilised steam under a fixed-sequence philosophy for Turbine I and II. On average 9.21 (ton per hour) cannot contribute towards power generation, compared to 12.93 (ton per hour) for Case 1^f.

Case 2^d: By solving OPM for Turbine I, II and III, optimal power generation of 23.31MW is obtained with 27 trips. This is an 11.1% increase compared to Case 2^f, however, with seven more trips. Take note, the 5MW turbine receives, percentage wise, the lowest proportion towards maximum capacity, which is 33.4%. For Case 2^f, Turbine III generates only 0.2% of its capacity, compared to the 87.3% for Case 2^d.

Case 3^d to 7^d: OPM is solved for the final five turbine configurations that correspond to Case 3^f to 7^f in Chapter 4. For each scenario higher optimal power generation is observed, which varies between 4.67% and 13.17%. It should be noted that for Case 1^d, 3^d and 4^d, where only two turbines

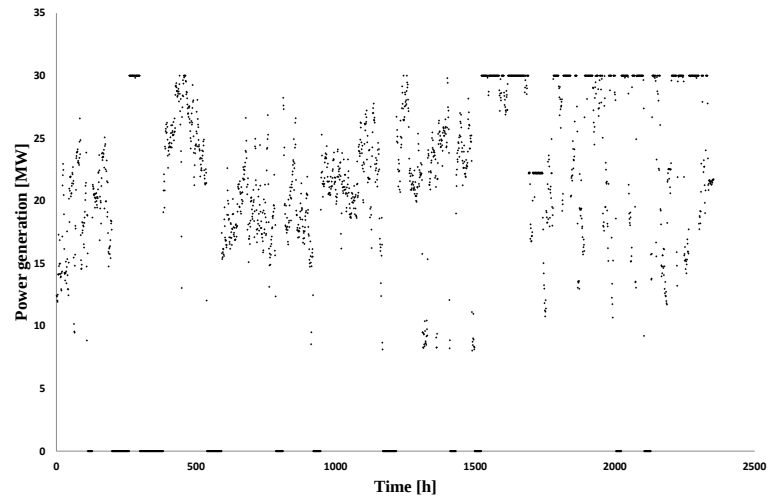


Figure 7.2: Case 1^d: Power generation over time for Turbine II, the 30MW, under dynamic control.

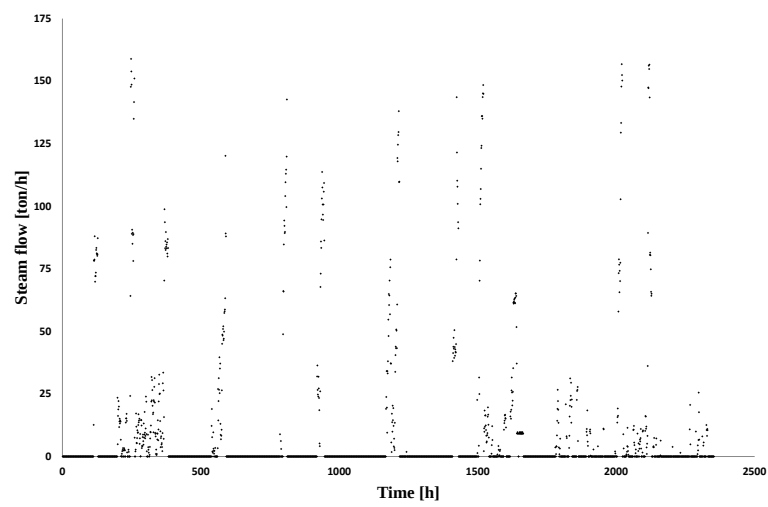


Figure 7.3: Case 1^d: Unutilised steam flow for Turbine II and VII over time, under dynamic control.

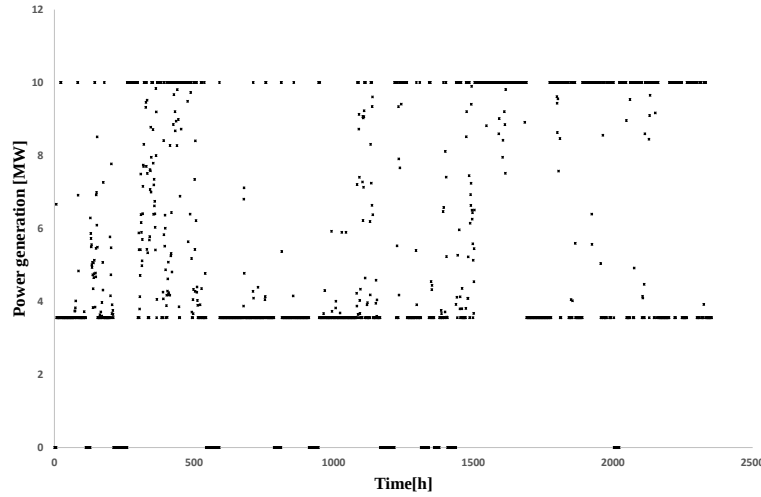


Figure 7.4: Case 6^d: Power generation over time for Turbine III, the 10MW, under dynamic control.

are used, power generation increases are between 3.53% to 5.29%. For the instances where three turbines are used, *i.e.* Case 2^d and 5^d until 7^d, a power generation increase of 9.35% to 13.17% is observed. It can therefore be concluded that as the number of turbines increases under the fixed-sequence philosophy, the percentage gap increases with regards to optimal dynamically controlled outcomes.

The reported results clearly demonstrate the positive effect that power generation under dynamic control has over that of turbines under a fixed-sequence philosophy. It is important to note, this positive effect is due to a change in the operating philosophy for the same turbines.

Take note that only for Case 2^d does a solution to OPM have more trips when compared to the respective Cases from Chapter 4. In Chapter 4 the oversize capacity of the three turbines is discussed, which results in the third receiver to be operational only once. It is therefore not sensible to compare trip results of Case 2^f with that of Case 2^d, since such a turbine combination should not be operational under the real-world data from Section 2.11. It is therefore concluded that selective scenarios exist where fewer trips will occur under a fixed-sequence philosophy. Dynamic control, however, will predominantly result in fewer trips and always higher power generation.

From the results discussion in Chapter 3, it is mentioned that for Case 5^f lower power generation is observed compared to Case 1^f. The total combined turbine capacities for both Cases are the same and all turbines have equivalent efficiencies. Optimal results show that under a fixed-sequence philosophy 4.15% less power is generated by the combination of three turbines, compared to two turbines. For dynamic control optimal results for the same turbines, *i.e.* Cases 1^d and 5^d, indicate higher optimal power generation by 3.36% if three, rather than two turbines are used. Case 5^d, however, results in 25 trips compared to the 15 of Case 1^d. This further emphasises the ability of dynamic control to yield improved power generation.

For a further demonstration of power generation under dynamic control, consider Figures 7.4 to 7.7. Power generation for Turbines III, IV and VIII, as solved by OPM in Case 6^d, are plotted in Figures 7.4 up to 7.6. Comparing these graphs with those of Figures 4.8 to 4.10 further demonstrates the consequence of dynamic control.

The decrease in loading periods are evident in Figures 4.8 to 4.10. POQM results show that for Case 6^f, Turbine III, IV and VIII are operational for 93.4%, 73.8% and 11.9% of the time, respectively. For Case 6^d under dynamic control Turbine III, IV and VIII are operational for 86.2%, 85.8% and 89.4% of the time, respectively. These results demonstrate the ability of OPM to distribute steam dynamically amongst the turbines, and therefore yield higher optimal power generation and less turbine trips.

Figure 7.7 depicts the unused steam for Case 6^d. When this figure is compared to that of Figure 4.11, plotted in Section 4.3.1, it is clearly evident that more steam is utilised by Case 6^d, compared to 6^f.

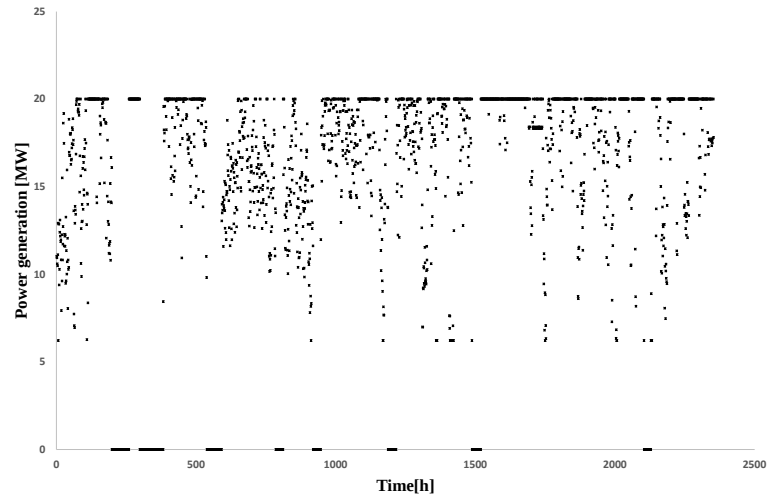


Figure 7.5: Case 6^d: Power generation over time for Turbine IV, the 20MW, under dynamic control.

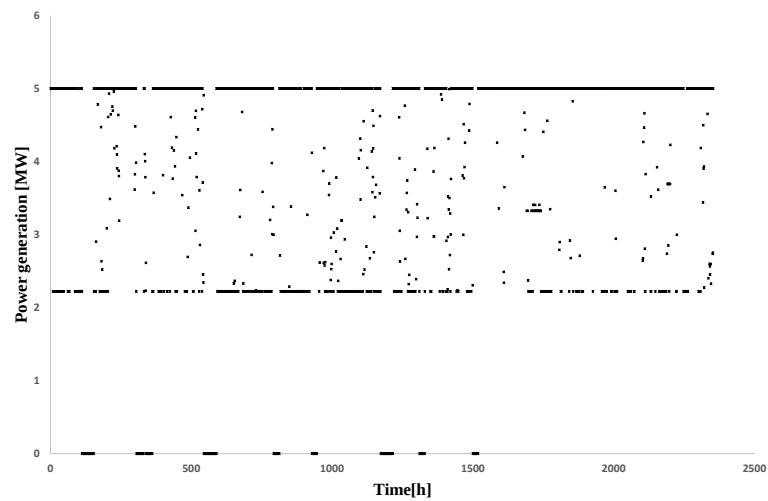


Figure 7.6: Case 6^d: Power generation over time for Turbine VIII, the 5MW, under dynamic control.

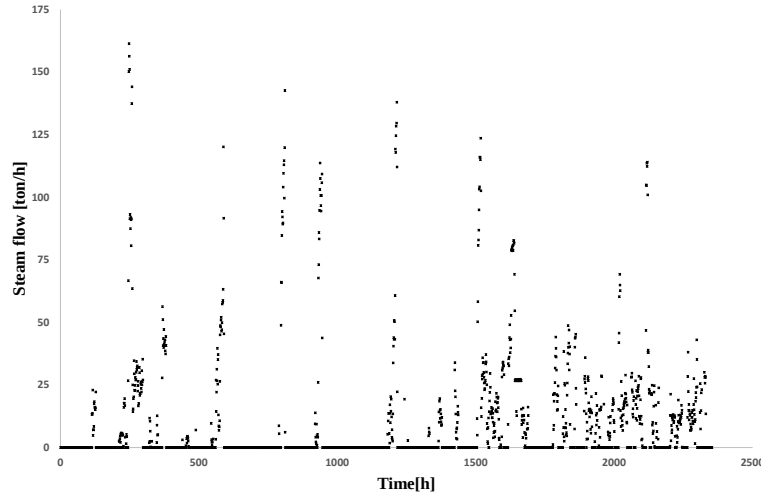


Figure 7.7: Case 6^d: Unutilised steam amongst Turbine III, IV and VIII over time.

For Case 6^d an average steam flow of 8.79 (ton per hour) cannot be utilised for power generation, as opposed to Case 6^f's 18.06 (ton per hour).

In Chapter 4, emphasise is placed on unused steam between Case 1^f and 6^f. It was noted that the two lesser efficient turbines utilised more of the steam flow, even though lower power generation was observed. The results from Case 6^d, compared to that of 1^d, however, indicate that more steam is utilised by the latter Case for the higher efficient turbines. This is, furthermore, a result of dynamic control.

OPM solves for power generation of 24.20MW, compared to 21.70MW for Case 6^d and 1^d, respectively. This is an 11.5% power generation increase for the 5MW, 10MW and 20MW turbines that operates at 4.5 (ton per hour per Mega Watt generated), when compared to the 5MW and 30MW machines that operate under 5 (ton per hour per Mega Watt generated). Case 6^d's power generation results, furthermore, show an increase of 3.7% when compared to Case 4^d. Note that Case 4^d operates with a 5MW at an efficiency of 5 (ton per hour per Mega Watt generated) and a 30MW with 4.5 (ton per hour per Mega Watt generated).

For Case 1^d and 4^d a 5MW and a 30MW turbine are operational, where Case 4^d yields optimal results of 1.63MW higher under a 30MW turbine with a 10% improvement in the steam conversion rate. The highest optimal power generation from the results is 24.20MW for a 5MW, 10MW and 20MW turbine combination, where all three machines convert steam flow to Mega Watt generated at a rate of 4.5. Note that 28 trips are experienced between the turbines over the time horizon of the real-world data. None of the results from OPM, however, give an indication as to which turbine selection should be chosen for an energy recovery plant. The need therefore exists to formulate a turbine investment optimisation model to assist the procurement decision making process.

The number of trips between Case 1^d, 4^d and 6^d are 15, 16 and 28, respectively. Even though Case 6^d results in higher power generation, and probably more expensive turbine costs, it results in higher trip occurrences. OPM can solve for optimal power generation, however, these results do not give an indication as to which turbines should be procured. A model formulation is, therefore, required that optimises turbine investment choices. This model should, furthermore, take into account the trips, since these unwanted occurrences will result in additional future maintenance costs. In the next chapter an optimal turbine investment model formulation will be proposed to address these issues.

7.2.1 Comparison of OPM and DDLC results

All results from OPM are determined with foresight into the predicted future steam flow profile. In Section 6.3 it is demonstrated through DDLC that the maximisation of power generation results

are within 0.6% of optimality. These results follow from Chapter 6's two turbines, which use the hypothetical steam profile from Section 2.10 as input parameters.

The formulation of DDLC is used where 15 time periods of sufficient steam is required before a turbine may go back online. Take note, the two-time period future knowledge of steam flow stays unchanged. This allows DDLC to solve for realistic results that can be compared with that of OPM. Case 1^d, mentioned previously, reported optimal power generation results for when Turbine I and II are considered, using the real-world data from Section 2.11 for the steam input parameters. Results reported in Table 7.3 indicate optimal power generation of 21.70MW with 15 combined trips, where Turbine I contributes to four of those occurrences.

The DDLC, from Algorithm 1, was solved for Turbine I and II, using the real-world data from Section 2.11 as steam signature profile. As a result, a combined power generation of 21.51MW is obtained. This indicates that power generation within 0.9% of optimality of Case 1^d can be obtained if two future time periods can be accurately predicted. DDLC solves for the same number of trips, *i.e.* four for Turbine I and 15 for Turbine II.

DDLC was solved by not allowing any future steam predictions. Maximum power generation of 21.45MW is obtained with a total of 19 trips. Therefore, if the Works does not comprise a gas holder or any other means to predict near future steam flows, power generation at 98.9% of optimality can be achieved.

It is mentioned previously that by changing from a fixed-sequence philosophy to that of dynamic control, a potential increase in power generation of 3.53% may be achieved. DDLC outcomes suggest that a realistic increase of 2.42% can be achieved by changing the operating philosophy and 2.62% if steam flow can be accurately predicted for two future time periods.

7.3 Summary

This chapter presented OPM, which optimises power generation amongst any number of dynamically controlled turbines. Figure 7.8 displays a flow chart of model formulations that led up to OPM, from PCM onwards.

The results reported in this chapter demonstrate the advantage that dynamic turbine control possesses over that of a fixed-sequence philosophy. For all Cases improved power generation results are achieved when OPM is compared to POQM and OSM. For only one of the Cases did OPM result in more turbine trips. As discussed, for this Case three turbines are present with an oversized combined power generation capacity.

DDLC was used with a 15 hour sufficient steam prerequisite and demonstrated how power generation close to optimality can be achieved when steam is predicted accurately for two hours in advance. Results showed that maximum power generation could be obtained within 99.1% of optimality. Without knowledge of future steam availability, DDLC yielded power generation within 98.9% of optimality.

In the following chapter an optimal power generation investment model is formulated. The model determines from any number of turbines the procurement choices that optimise the NPV of the investments over the time horizon. The effect of trips is taken into consideration when optimal investment choices are determined by penalising the NPV for each occurrence.

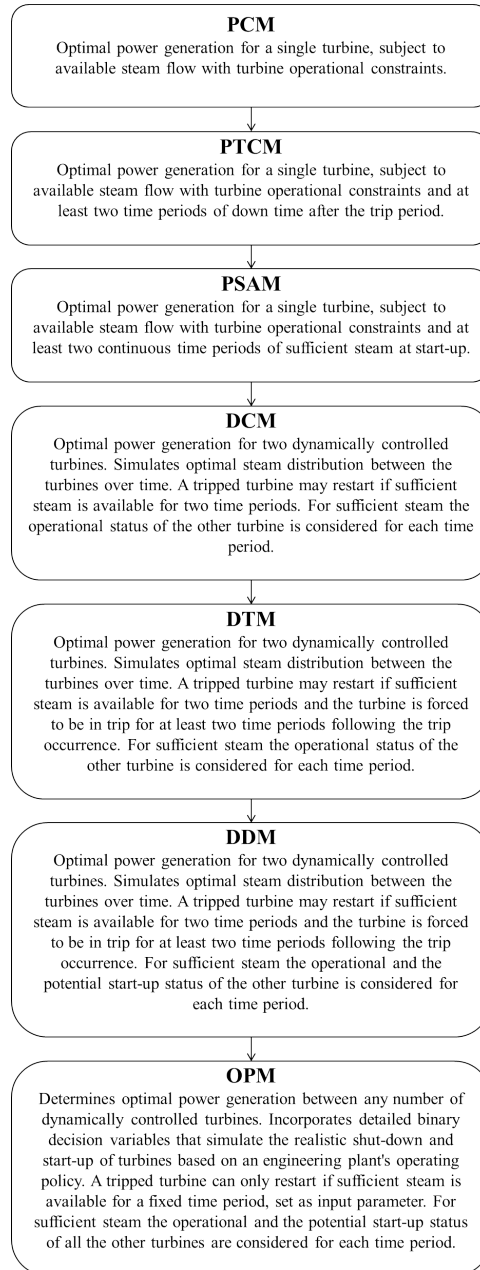


Figure 7.8: Flow chart for the MILP formulations from Chapters 3 and 6 that led to OPM.

Chapter 8

Optimal turbine investments for power generation

The previous chapter provided a MILP formulation that optimises power generation between any number of dynamically controlled turbines. Optimal results indicated a power generation increase between 4.67% to 13.17% for optimal operations under a dynamic, as opposed to a fixed-sequence loading hierarchy. This chapter provides a MILP formulation that determines the optimal turbine investment configuration, which yields the highest NPV over time.

8.1 Modelling approach

Results from the previous chapter demonstrated that dynamic controlled turbines yield higher power generation, compared to the fixed-sequence philosophy, however, no investment configurations were addressed. To determine the NPV of optimal investments for dynamically controlled turbines and whether such a procurement configuration differs from results yielded by POIQM in Chapter 5, an investment-based optimisation model needs to be formulated. Results from such a formulation need to be compared with that obtained in Chapter 5, which includes investment configurations under various trip penalties.

A MILP model is presented that uses historic plant-specific signature steam flow data to determine what the optimal configuration of turbine investments should be when utilising fluctuating excess steam for energy recovery. The primary decision variables are concerned with the optimal investment choices through the optimal distribution of steam flow to the selected turbines. The optimal investment choices are the combination of turbines that yield the highest NPV over time, *i.e.* profit from power generation after investment and trip costs. The MILP model incorporates detailed binary decision variables that simulate the realistic shut-down and start-up of turbines based on an engineering plant's operating policy. This model is referred to as the *Optimal Power Investment Model* (OPIM).

Turbine capacities, efficiencies and costs are input parameters. The following parameter definitions are required to formulate OPIM: The index set of all turbines is given by $\mathcal{I}^T = \{1, 2, \dots, |\mathcal{I}^T|\}$. The operation of these turbines is modelled over a time set $\mathcal{T} = \{1, 2, \dots, |\mathcal{T}|\}$. The total excess steam at time $t \in \mathcal{T}$ is given by m_t^S .

Turbine $i \in \mathcal{I}^T$ can operate within the allowable minimum and maximum limits of L_i (mass units per time period) and U_i (mass units per time period), respectively, where steam is converted at an efficiency rate of η_i^T (flow units per power unit generated). Once a turbine trips, a minimum time of δ^T , which coincides with sufficient steam availability, needs to elapse before the tripped turbine may be brought back online.

The income per unit, generated from power production, is equal to the unit cost C_t^E (cost per power unit generated) that a power utility would have been paid at time t . The capital cost of turbine $i \in \mathcal{I}^T$ at time zero is given by C_i^T . The present value trip cost per turbine is given by C_i^h at time zero for each future trip. The real interest rate applied for NPV discounting is given by r . There are three main groups of decision variables defined for OPIM. The first group relates to

the optimal distribution of steam to turbines. Let $m_{it}^S \geq 0$ be the steam flow to turbine $i \in \mathcal{I}^T$ at time $t \in \mathcal{T}$. The second group of variables is necessary for modelling the different investment options. These variables are all binary variables. Let $x_i = 1$ if an investment is made to procure turbine $i \in \mathcal{I}^T$, and $x_i = 0$ if not. The third group of binary decision variables is required to model the operational status of each turbine. Let $y_{it}^T = 1$ if turbine $i \in \mathcal{I}^T$ is operational at time $t \in \mathcal{T}$. The following auxiliary variables are necessary to facilitate the operational policy related to the shut-down and start-up of turbines. Let $y_{it}^h = 1$ if turbine $i \in \mathcal{I}^T$ is tripped (halted) from time $t - 1 \in \mathcal{T}$ to $t \in \mathcal{T}$ and let $y_{it}^\delta = 1$ if turbine $i \in \mathcal{I}^T$ is in trip for at least δ^T until time $t \in \mathcal{T}$. The variable $y_{it}^b = 1$ if turbine $i \in \mathcal{I}^T$, which has been in trip at time $t - 1 \in \mathcal{T}$, may be brought back online at time $t \in \mathcal{T}$.

The objective of OPIM is to:

$$\text{maximise } \sum_{i \in \mathcal{I}} \sum_{t \in \mathcal{T}} \left(\frac{m_{it}^S}{\eta_i^T (1+r)^t} C_t^E - C_i^h y_{it}^h \right) - \sum_{i \in \mathcal{I}} x_i C_i^T, \quad (8.1)$$

subject to

$$\sum_{t \in \mathcal{T}} y_{it}^T \leq |\mathcal{T}| x_i, \quad \forall i \in \mathcal{I}, \quad (8.2)$$

the remainder of the sets of constraints for OPIM are equivalent to that defined by (7.2), (7.3), (7.4), (7.5), (7.6), (7.7), (7.8), (7.9), (7.10), (7.11), (7.12), (7.13) and (7.14).

As mentioned previously, the objective of OPIM is to optimise the NPV over the time horizon, given by equation (8.1). The NPV is determined by the power generation income with deductions of turbine investment related and trip costs. Constraint set (8.2) ensures that a turbine $i \in \mathcal{I}^T$ may only be operational if it is invested in. For each time $t \in \mathcal{T}$, constraint set (7.2) guarantees that the accumulated steam flow to the turbines are less or equal than the available steam flow. Constraint sets (7.3) and (7.4) ensure that steam flow to any turbine $i \in \mathcal{I}^T$ is always within the allowable minimum and maximum operational limits or zero.

Constraint sets (7.5) to (7.7) determine if turbine $i \in \mathcal{I}^T$ tripped from time $t - 1$ to t , and constraint set (7.8) ensures that the turbine is not tripped if sufficient steam flow is available. Note for constraint set (7.8) that ϵ is a very small positive number, just to ensure an inequality is obtained for the constraint set. The set of constraints (7.9) to (7.11) determine if at least a δ^T time has elapsed for a tripped turbine $i \in \mathcal{I}^T$. If the minimum δ^T has not yet passed for a turbine in trip, constraint set (7.12) ensures that the turbine cannot go back online. To determine whether sufficient steam flow was available for a turbine in trip during the preceding δ^T and whether this turbine may be brought back online, constraint sets (7.13) and (7.14) are applied.

The next section demonstrates the ability of OPIM to yield optimal investment results.

8.2 Computational Results for OPIM

In order to demonstrate how OPIM is applied to generate optimal investment results, investment scenarios are investigated with the 10 turbines given in Table 5.1. Information on the maximum capacity (Mega Watt), minimum capacity (Mega Watt), the steam upper limit U_i (ton per hour) and the steam lower limit L_i (ton per hour) of 10 turbines are repeated in Table 8.1. Unless stated otherwise, scenario results are generated under the applicable costs from Table 5.2.

All OPIM scenarios in this chapter are solved by utilising the real-world data, as discussed in Section 2.11, for the steam input parameters. Similar to Chapter 5, all investment NPV's are determined over a 10 year interval. In order to compare the scenario results with those reported in Chapter 5, the cost of electricity at time zero is R500.00 per Mega Watt hour utilised. The cost of electricity is annually inflated by 5.00% and the real interest rate is 5.00%, continuously (hourly) compounded. A 15 hour sufficient steam availability is required before a turbine can restart. Note that all trip costs occur at time zero and reflects as a NPV penalisation.

For each subsection, unless stated otherwise, optimal investment results are determined for the first *Case*. Any number of turbines can be invested in, with the objective to optimise the NPV over

Table 8.1: Maximum and minimum power generation capabilities, together with the upper and lower bounds for the 10 turbines, $\mathcal{I}^T$, to be used in determining optimal turbine investment choices. The parameters are extracted from Table 5.1.

Turbine	Max [MW]	Min [MW]	U_i [ton/h]	L_i [ton/h]
I	3.0	1.2	15.0	6.0
II	5.0	2.0	25.0	10.0
III	10.0	3.2	50.0	16.0
IV	15.0	4.4	75.0	22.0
V	20.0	5.6	100.0	28.0
VI	25.0	6.8	125.0	34.0
VII	30.0	8.0	150.0	40.0
VIII	35.0	9.2	175.0	46.0
IX	40.0	10.4	200.0	52.0
X	50.0	12.8	250.0	64.0

Table 8.2: Scenario results for Case 1^d to 5^d where $C_i^h = 0$. The results include the optimal investment vector, trips and power generation per turbine, total power generation and the expected NPV profit for 10 years.

Case number	Investment vector	Trips per turbine		Power generation per turbine [MW]		Total power generation [MW]	NPV R · 10 ⁶
1^d	$x^{C_1^d} = [0, 0, 1, 0, 0, 1, 0, 0, 0, 0]$	373	299	4.46	17.79	22.25	788.7
2^d	$x^{C_2^d} = [0, 0, 0, 1, 1, 0, 0, 0, 0, 0]$	448	299	6.75	15.35	22.10	779.8
3^d	$x^{C_3^d} = [0, 0, 0, 1, 0, 1, 0, 0, 0, 0]$	448	299	7.71	14.68	22.39	772.2
4^d	$x^{C_4^d} = [0, 0, 1, 0, 1, 0, 0, 0, 0, 0]$	373	299	5.53	15.79	21.32	769.1
5^d	$x^{C_5^d} = [0, 1, 0, 0, 0, 1, 0, 0, 0, 0]$	372	299	2.79	18.44	21.23	767.8

a 10 year period. For the second scenario, the investment vector obtained in the first Case is not allowed to be solved for by OPIM. This ensures that the next optimal solution in line, with its corresponding NPV, is determined. Solving for the third best investment outcome, the first two optimal investment vectors are not allowed, and so forth for the remaining scenarios. Take note, each scenario is solved for every turbine $i \in \mathcal{I}^T$, however, the previous investment vectors of that subsection is prohibited. Case scenarios for OPIM are numbered with a superscript “ d ” to indicate “dynamic control”.

8.2.1 Solving OPIM for a zero trip penalty

For the first five scenarios a zero trip penalty is included into the objective function so that $C_i^h = 0 \forall i \in \mathcal{I}^T$. All of the results reported in this section are summarised in Table 8.2. It should be noted that all outcome comparisons are either with results from this section or that of Case 1^f to 5^f from Section 5.2.1.

Case 1^d : Optimal NPV results are realised for investments in Turbine III and VI, the 10MW and 25MW machines, when solving OPIM. The results from Chapter 5 yield that optimal investments are the procurement of Turbine II and VII, the 5MW and 30MW machines. These results indicate different turbine investments to that of Case 1^f , however, with an equivalent combined power generation capacity of 35.00MW. Let the binary investment vector for Case 1^d be denoted by $x^{C_1} = [0, 0, 1, 0, 0, 1, 0, 0, 0, 0]$. Total power generation yields 22.25MW, which is 6.4% higher than the 20.92MW of Case 1^f from Section 5.2.1 for fixed-sequence investments with a zero trip penalty. The NPV is determined at R788.7 million, 7.3% higher than that of Case 1^f . In Case 1^d , Turbine III trips 373 times over 10 years and generates power at 4.46MW, while Turbine VI trips 299 times with optimal power generation of 17.79MW. Accumulated trips of 672 are experienced in comparison with Case 1^f 's 709, which is a 5.2% reduction.

Table 8.3: Scenario results for Case 6^d to 10^d where a trip is penalised by R50,000.00 at time zero. The results include the optimal investment vector, trips and power generation per turbine, total power generation and the expected NPV for 10 years.

Case number	Investment vector	Trips per turbine		Power generation per turbine [MW]		Total power generation [MW]	NPV R · 10 ⁶
6^d	$x^{C_6^d} = [0, 0, 1, 0, 0, 1, 0, 0, 0, 0]$	373	299	4.46	17.79	22.25	755.1
7^d	$x^{C_7^d} = [0, 0, 0, 1, 1, 0, 0, 0, 0, 0]$	448	299	6.75	15.35	22.10	742.4
8^d	$x^{C_8^d} = [0, 1, 0, 0, 0, 0, 1, 0, 0, 0]$	149	411	2.52	19.15	21.70	738.3
9^d	$x^{C_9^d} = [0, 0, 1, 0, 1, 0, 0, 0, 0, 0]$	373	299	5.53	15.79	21.32	735.5
10^d	$x^{C_{10}^d} = [0, 0, 0, 1, 0, 1, 0, 0, 0, 0]$	448	299	7.71	14.68	22.39	734.8

Case 2^d : The binary investment vector $x^{C_1} = [0, 0, 1, 0, 0, 1, 0, 0, 0, 0]$ is not permitted as a solution, whereafter OPIM is solved. An optimal outcome yields an investment vector of $x^{C_2} = [0, 0, 0, 1, 1, 0, 0, 0, 0, 0]$, *i.e.* the 15MW and 20MW turbines. The second optimal investment vector, therefore, also indicates procurements of two turbines with a combined capacity of 35.00MW. The NPV is solved at R779.8 million with 448 and 299 trip occurrences for Turbine IV and V, respectively. Note that 747 trip occurrences take place between the two turbines, which is 11.2% higher than that of Case 1^d .

Case 3^d to 5^d : The above procedure to generate optimal solutions is repeated to obtain the third to fifth best possible investment results from OPIM. For each scenario OPIM determines that two turbine investments are optimal when the signature steam flow profile from the real-world data, as discussed in Section 2.11, is used as steam input parameters. It is evident in Table 8.2 that Case 3^d yields the highest power generation at 22.39MW with the shared highest accumulated trip count of 747 instances over 10 years. This demonstrates that the optimal investments do not necessarily coincide with optimal power generation. The combined power generation capacity for Case 1^d is 40.00MW, whereas the last two Cases have a maximum limit of 30.00MW.

In Section 5.2.1 two of the top five investment options yield a single turbine procurement. Take note, for every Case in Table 8.2 the NPV is higher than that of Case 1^f given in Table 5.3. The results demonstrate the ability of additional energy recovery that can be utilised under dynamic turbine control. Furthermore, Cases 1^d to 5^d do not solve for a turbine procurement above 25MW, whereas POIQM solves for either Turbine VII or VIII, *i.e.* the 30MW and 35MW machines, in Case 1^f to 4^f .

DDLC is now modified to yield the expected NPV value of an investment. The investment vector $x^{C_1^d}$, *i.e.* Turbine III and VI is solved using the real-world data from Section 2.11, where steam availability is known two hours in advance. A maximised NPV of R782.8 million is computed, which is 0.7% below optimal results reported for Case 1^d and 6.6% higher than that of Case 1^f . Take note, power generation for Turbine III and VI is 0.5% below optimality.

Solving DDLC when no future knowledge into steam availability is allowed, yields an NPV of R778.5 which is 1.3% below optimality. Power generation at 22.04MW is within 99.0% of optimal results. The results obtained from the modified DDLC demonstrate the realistic monetary advantage that can be achieved under dynamic turbine control. All further NPV's reported are, therefore, assumed to be in close proximity of practical obtainable outcomes.

Results from this section demonstrate the ability of OPIM to determine optimal investment outcomes with realistic NPV, however, trip costs are not considered. In order to predict more realistic NPV over the time horizon, the next section incorporates a fixed penalty cost at time zero for each turbine trip.

8.2.2 OPIM results under a fixed trip penalty

In scenario results from Section 5.2.2 a fixed trip cost, or penalty, of R50,000.00 per occurrence is incorporated at time zero, *i.e.* $C_i^h = 50000 \forall i \in \mathcal{I}^T$. OPIM is solved for the next five Cases, yielding the optimal investment outcomes for each scenario where, $C_i^h = 50000 \forall i \in \mathcal{I}^T$. Results

for Case 6^d to 10^d are summarised in Table 8.3.

Case 6^d : Solving OPIM yields $x^{C_6^d} = x^{C_1^d}$ with an optimal NPV of R755.1 million, which is 4.3% lower than the solution obtained for Case 1^d above. The result is 5.8% higher than the solution found in Table 5.4 for Case 6^f . Take note, optimal results for Case 6^f indicate an investment into Turbine VII with 411 trips, whereas the combination of Turbine III and VI for Case 6^d trips 672 times. Therefore, even though 261 additional trip penalties of R50,000.00 each are incurred for Case 6^d , compared to 6^f , the additional 1.48MW of power generation results in an NPV increase of R40.1 million.

Case 7^d : OPIM is solved under the constraints that the investment vectors $x^{C_5^d}$ and $x^{C_6^d}$ are not allowed. An investment vector $x^{C_7^d} = x^{C_2^d}$ is obtained, with an NPV of R742.4 million. Note that this NPV, even though trip penalties are considered, is higher than that of Case 1^f in Section 5.2.1 where $C_i^h = 0 \forall i \in \mathcal{I}^T$. Optimal results for Case 7^d is 1.7% below that of Case 6^d with 75 more trips.

Case 8^d to 10^d : The results from solving OPIM for the next three scenarios are obtained and summarised in Table 8.3. Note that all NPV's are greater than that of Case 1^f . Furthermore, note that the first four optimal investment vectors obtained under a zero trip penalty, *i.e.* $x^{C_1^d}$ to $x^{C_4^d}$ are present within $x^{C_6^d}$ up to $x^{C_{10}^d}$. This is an indication that the optimal investment vectors are relatively insensitive towards a zero trip penalty and that of $C_i^h = 50000 \forall i \in \mathcal{I}^T$ for each occurrence. The total power generation capacity for Case 11^d to 15^d ranges from 30.00MW to 40.00MW. Further note, the investment vector $x^{C_8^d} = [0, 1, 0, 0, 0, 0, 1, 0, 0, 0]$, which is for Turbine II and VII, is solved by POIQM as the optimal investment choices under a fixed-sequence philosophy when $C_i^h = 0 \forall i \in \mathcal{I}^T$.

Results from Section 5.2.3 indicate that under a fixed-sequence operating philosophy it is not sensible to invest in more than one turbine when trip costs are proportionally accounted for. The following section investigates how proportional trip penalties influence optimal investments under dynamic turbine control.

8.2.3 OPIM results under a proportional trip penalty

Table 8.4: Scenario results for Case 11^d to 15^d where trips are penalised by $1/500^{th}$ of the turbine's procurement price. The results include the optimal investment vector, trips and power generation per turbine, total power generation and the expected NPV profit for 10 years.

Case number	Investment vector	Trips per turbine		Power generation per turbine [MW]		Total power generation [MW]	NPV R · 10 ⁶
11^d	$x^{C_{11}^d} = [0, 0, 1, 0, 0, 1, 0, 0, 0, 0]$	373	299	4.46	17.79	22.25	687.5
12^d	$x^{C_{12}^d} = [0, 1, 0, 0, 0, 1, 0, 0, 0, 0]$	372	299	2.79	18.44	21.23	683.4
13^d	$x^{C_{13}^d} = [0, 0, 1, 0, 1, 0, 0, 0, 0, 0]$	373	299	5.53	15.79	21.32	679.9
14^d	$x^{C_{14}^d} = [1, 0, 0, 0, 0, 1, 0, 0, 0, 0]$	261	299	1.80	18.80	20.60	672.2
15^d	$x^{C_{15}^d} = [0, 0, 1, 1, 0, 0, 0, 0, 0, 0]$	262	262	7.25	12.66	19.91	667.3

OPIM is solved for Case 11^d to 15^d where trips are penalised, at time zero, with a proportion of the turbine's procurement price. For each trip the NPV is penalised at time zero with $1/500^{th}$ of the turbine's investment cost. Optimal scenario results for Case 11^d to 15^d are summarised in Table 8.4.

Case 11^d : Solving OPIM when $C_i^h = C_i^T/500 \forall i \in \mathcal{I}^T$ yields an optimal investment vector equivalent to that of Case 1^d and 6^d , so that $x^{C_1} = x^{C_6} = x^{C_{11}} = [0, 0, 1, 0, 0, 1, 0, 0, 0, 0]$. This indicates that optimal investments under dynamic control are less sensitive toward trip costs, compared to the fixed-sequence philosophy. Recall that all three investment vectors obtained for Case 1^f , 6^f and 11^f under the fixed-sequence philosophy are dissimilar. The optimal investment vector, $x^{C_{11}}$ further indicates that under dynamic control it is sensible to invest in more than one turbine, even if more trips occur, *i.e.* 672 compared 337 of Turbine VI for Case 11^f . Therefore, multiple dynamically

Table 8.5: Scenario results for Case 16^d to 19^d where investment costs are doubled and a trip cost is equal to 1/250th of the turbine's procurement price. The results include the optimal investment vector, trips and power generation per turbine, total power generation and the expected NPV for 10 years.

Case number	Investment vector	Trips per turbine		Power generation per turbine [MW]		Total power generation [MW]	NPV R · 10 ⁶
16 ^d	$x^{C_{16}^d} = [0, 0, 1, 1, 0, 0, 0, 0, 0, 0]$	262	262	7.25	12.66	19.91	365.1
17 ^d	$x^{C_{17}^d} = [0, 1, 0, 0, 1, 0, 0, 0, 0, 0]$	187	299	3.63	16.13	19.76	357.4
18 ^d	$x^{C_{18}^d} = [0, 1, 0, 1, 0, 0, 0, 0, 0, 0]$	224	262	4.20	13.00	17.20	349.8
19 ^d	$x^{C_{19}^d} = [0, 1, 0, 0, 0, 0, 1, 0, 0, 0]$	149	411	2.52	19.15	21.70	154.1

controlled turbines may yield sufficient additional power generation income, which compensates for increased trip costs to be incurred.

The optimal NPV for Case 11^d is determined at R687.5 million, which is 7.7% above the NPV of Case 11^f's single turbine. The NPV, however, is almost 13% lower than that of Case 1^d with a zero trip penalty. This, furthermore, demonstrates how potential future trip costs might affect the NPV of the investment choices. Even though an investment vector equivalent to that of Case 1^d is obtained from OPIM, the Works will not realise the expected profits if $C_i^h = 0 \forall i \in \mathcal{I}^T$ is used as benchmark.

Case 12^d to 15^d: OPIM is solved for the following four Cases and relevant results are summarised in Table 8.4. Take note, OPIM results yield that an investment into two turbines should be made for each Case outcome. Furthermore, the NPV's from Table 8.4 are between 4.5% and 7.7% higher than that of Case 11^f reported in Table 5.5. The total power generation capacity for Case 11^d to 15^d ranges from 25.00MW to 35.00MW. Note that investment vectors $x^{C_{11}}$ and $x^{C_{12}}$ are present within x^{C_1} until x^{C_5} and $x^{C_{11}}$ and $x^{C_{13}}$ are found within x^{C_6} to $x^{C_{10}}$.

OPIM is solved in the next section for four Cases, where costs are increased, in order to investigate the effect on the optimal investment vectors and the NPV's. Turbine procurement costs are doubled and, furthermore, a proportional trip penalty of $C_i^h = C_i^T/250 \forall i \in \mathcal{I}^T$ is provided as input parameters towards OPIM.

8.2.4 OPIM results under increased investment costs and a higher fractional trip penalty

Results from Case 1^d, 6^d and 11^d indicate that under the chosen turbine costs in Table 5.2 and respective trip penalties, the optimal investment vectors obtained by solving OPIM, are equivalent. In this section the turbine procurement prices are doubled and, furthermore, each trip is penalised by $C_i^h = C_i^T/250 \forall i \in \mathcal{I}^T$.

In Section 5.2.4 the top three optimal results for POIQM are tabulated in Table 5.6 for when POIQM was solved under double costs. POIQM was, furthermore, solved for a fourth scenario where the optimal turbine investments from Case 1^f, *i.e.* Turbine II and VII, were fixed as input parameters towards the model. In this section OPIM is also solved for the top three optimal investment vectors and in the fourth scenario, where Turbine II and VII are fixed as input parameters, so that $|\mathcal{I}^T| = 2$.

Case 16^d: Solving OPIM under double procurement costs and a 1/250th proportional trip penalty, yields an optimal investment vector of $x^{C_{16}} = [0, 0, 1, 1, 0, 0, 0, 0, 0, 0]$, which is for the 10MW and the 15MW turbine. Take note, optimal investments are into two turbines and the combined maximum power generation capacity is equal to 25.00MW, compared to the 35.00MW for Case 1^d, 6^d and 11^d. The optimal NPV outcome is R365.1 million, which is 53.7% lower than the optimality of Case 1^d. This is, however, 19.3% higher than optimal investment results reported for Case 16^f, in Table 5.6. Turbines operating in a dynamically controlled environment, therefore, not only yield improved NPV's, as opposed to that of the fixed-sequence philosophy, but as the procurement and trip costs escalate, the percentage improvement on the NPV increases.

Table 8.6: Maximum and minimum power generation capabilities, together with the upper and lower bounds for the 10 turbines to be used in determining the maximum percentage price increase that yields higher NPV values.

Turbine	Max [MW]	Min [MW]	η_i^T [ton/MW]	U_i [ton/h]	L_i [ton/h]
I*	3.0	1	4.5	13.5	6.0
II*	5.0	2	4.5	22.5	10.0
III*	10.0	3	4.5	45.0	16.0
IV*	15.0	4	4.5	67.5	22.0
V*	20.0	6	4.5	90.0	28.0
VI*	25.0	7	4.5	112.5	34.0
VII*	30.0	8	4.5	135.0	40.0
VIII*	35.0	10	4.5	157.5	46.0
IX*	40.0	11	4.5	180.0	52.0
X*	50.0	14	4.5	225.0	64.0

Case 17^d and 18^d: Optimal OPIM results indicate that for Case 17^d investments should be made into Turbine II and V, which are the 5MW and 20MW machines. Similar to Case 16^d the total power generation capacity is 25.00MW. Results of Case 18^d suggest the procurement of Turbine II and IV, *i.e.* the 5MW and 15MW machines. Note that when the NPV's are compared to that obtained for Case 16^f, Case 17^d and 18^d yield an increase in NPV outcomes of 16.8% and 14.3%, respectively.

These optimal investment results are for less combined power generation when compared to the previous sections. Power generation and number of trips for Case 1^d to 3^d vary from 22.10MW to 22.39MW and 672 to 747 occurrences, respectively. For Case 16^d to 18^d, power generation between 17.20MW and 19.91MW and trips from 486 to 524 are obtained from OPIM outcomes. The smaller capacity turbines result in less power generation, however, with fewer trips and, therefore, reduced expected future maintenance expenditures.

Case 19^d: In Chapter 5, for Case 1^f where $C_i^h = 0 \forall i \in \mathcal{I}^T$, POIQM yields an optimal NPV when Turbine II and VII are procured. In Section 5.2.4, POIQM results from Case 19^f indicate that when the procurement prices from Table 5.2 are doubled and each trip penalised with $C_i^h = C_i^T/250 \forall i \in \mathcal{I}^T$, this turbine investment combination results in an NPV of negative R32.5 million over 10 years. The investment vector $x^{C_{19}^d} = [0, 1, 0, 0, 0, 0, 1, 0, 0, 0]$ is fixed as input parameter towards OPIM, and solved. The optimal NPV is determined at R154.1 million, which is 57.8 % lower than that of Case 16^d, however, a positive result is obtained whereas the NPV of Case 19^f is negative. The difference in NPV between Case 19^d and 19^f is R186.6 million. Similar to all other scenario results reported in this chapter, these outcomes demonstrate the positive effect that dynamic control has on power generation and the profits thereof.

8.2.5 Investments under improved isentropic efficiencies

All turbines from Table 5.2 operate at the same isentropic efficiency, converting 5.0 ton steam flow per Mega Watt generated. In this section OPIM is used to determine at what percentage price increase an improved isentropic efficient turbine yields higher NPV's. For this section only, all the turbines operate at a 10.0% reduction in the conversion rate so that 4.5 ton steam flow per Mega Watt generated is required. Operating parameters for these turbines are given in Table 8.6. Note that the corresponding numbers between turbines from Tables 8.1 and 8.6 possess equivalent capacities, η_i^T , and minimum flow limits, L_i . Therefore, $\eta_i^T = 4.5 \forall i \in \mathcal{I}^T$ and the parameters for the upper limit vector, U , are changed to the values given in Table 8.6.

Optimal turbine investment vectors for Case 1^d, 6^d, 11^d and 16^d are used for the scenarios depicted in this section. Each investment vector is individually fixed into OPIM as a parameter, under the corresponding costs from Table 5.2. The maximum percentage price increase that the Works should be willing to pay for each cost scenario is determined for the corresponding Cases. The four scenarios of this section are numbered with a superscripts “*di*” to indicate *dynamic* turbine control

Table 8.7: Allowable procurement price increases based on results for Case 1^{di} to 4^{di}. The percentage increase indicates the maximum total price increase for the turbines to yield an improved NPV over 10 years.

	Case 1 ^{di}	Case 2 ^{di}	Case 3 ^{di}	Case 4 ^{di}
Percentage increase	48.3	48.3	29.4	14.5

with improved *isentropic efficiencies*.

OPIM is solved for all four scenarios, from which the maximum allowable price increases are calculated. The corresponding price increase percentage from Case 1^d, 6^d, 11^d and 16^d are reported for Case 1^{di} to 4^{di}, respectively. For Case 1^{di} to 3^{di}, Turbine III* and VI* are the fixed investment parameters, whereas Turbine III* and IV* are fixed for Case 4^{di}. The percentage price increase results are summarised in Table 8.7.

Case 1^{di} and 2^{di}: OPIM is solved for the first two scenarios. If either $C_i^h = 0$ or $C_i^h = 50000 \forall i \in \mathcal{I}^T$ for each trip, the Works should be willing to pay 48.3% more for the two turbines investments. An equal number of trips result for Case 1^d, 6^d, 1^{di} and 2^{di} for equivalent turbine investment vectors. As a result, the allowable percentage increase between the two scenarios, Case 1^{di} and 2^{di} are unchanged. Note that when turbines are controlled under a fixed-sequence philosophy, for the scenarios where each trip is penalised with either $C_i^h = 0$ or $C_i^h = 50000 \forall i \in \mathcal{I}^T$, allowable price increases of 31.2% are reported for Case 1^{fi} and 2^{fi} in Section 5.2.5.

Case 3^{di}: If a trip is penalised with 1/500th of the procurement prices at time zero, a 29.4% increase in investments costs is acceptable. Under double the investment costs and a 1/250th trip penalisation, **Case 4^{di}** yields a 14.5% allowable price increase. The respective price increases under a fixed-sequence policy, reported in Table 5.8, are 18.6% and 6.6%. Results from this section demonstrate that when turbines are dynamically controlled a higher procurement price increase can be paid for a more efficient machine, compared to operations under the fixed-sequence philosophy. Take note, however, the results from Table 8.7 show that as potential trip costs increase, the willingness to pay more for a higher isentropic efficient turbine decreases.

All scenario results from OPIM demonstrate that dynamically controlled turbines yield higher power generation and, therefore, NPV's when compared to a fixed-sequence philosophy. The fluctuating nature of the signature steam profile from the real-world data discussed in Section 2.11, however, result in imminent trips. The occurrences reflect negative on the NPV due to potential future trip expenses and unutilised steam. As the cost of trips increases, the combined optimal turbine investment capacity decreases and therefore, potential power generation losses above maximum capacity intensify. In the following chapter a MILP formulation is proposed that will optimise turbine investments with the intertwined ability to allow for an investment and the procurements of a supplementary energy resource, to assist periods of low steam production.

8.3 Summary

OPIM is formulated in this chapter and its unique working ability is demonstrated through a number of Cases. Optimal turbine investments are determined from OPIM, for turbines operating under dynamic control, where NPV is used as benchmark. Figure 8.1 provides a layout of all the preceding MILP formulations from Chapters 3, 6 and 7, which built on one another, leading up to OPIM.

The next chapter proposes a MILP formulation that optimises NPV for the intertwined combination of turbine and a supplementary energy resource investments. This formulation will investigate the profitability of investments into acquiring a typically very expensive, additional energy resource to be timely procured for assisting instances of low steam productions.

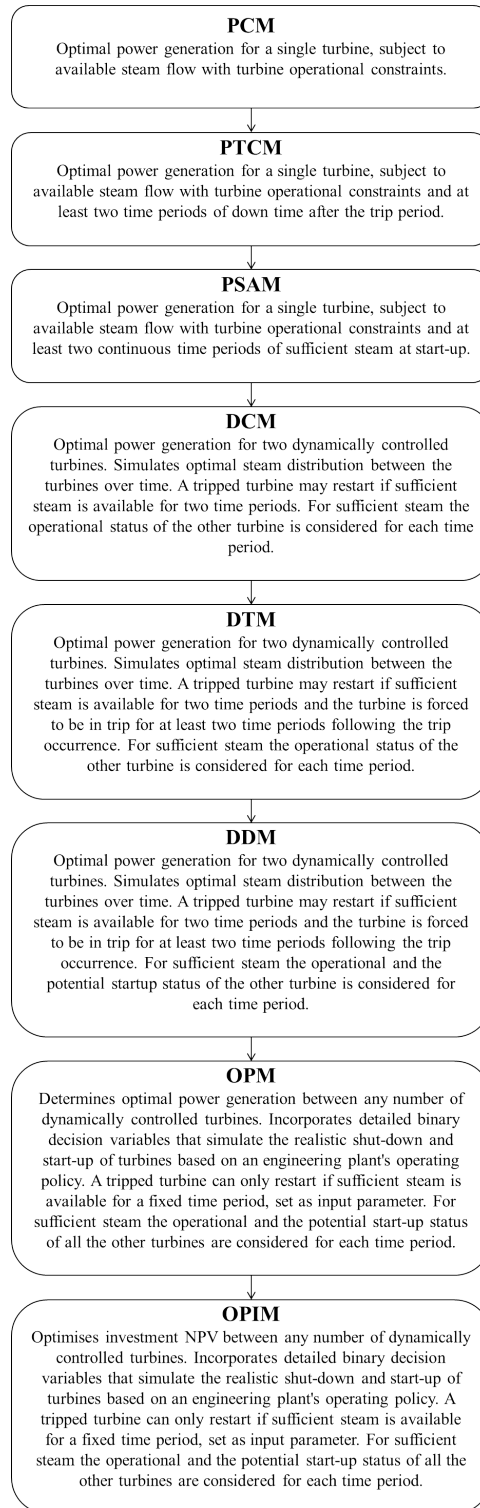


Figure 8.1: A flow chart for the model formulations from Chapters 3, 6 and 7 that built towards the formulation of OPIM.

Chapter 9

Optimal investments under an additional energy resource

An optimal turbine investment MILP was proposed in the previous chapter. Through scenario results the negative influence on NPV, was demonstrated when potential future trip expenditures were considered. This chapter investigates the possibility of allowing for an additional energy resource to be invested in and procured in order to assist steam production at periods of low availability. The opportunity may exist where additional burners can be installed in boiler houses to utilise a supplementary resource. Another possibility is for an energy resource to be used in combination with off-gases for an unrelated plant process. If the plant process can use more of this energy resource, increased volumes of off-gases will be available for steam productions. Such a resource is typically very expensive and under normal circumstances it is not a viable option to produce additional steam to generate more power.

A model formulation follows where turbine investments are optimised and NPV is used as the benchmark. This formulation further allows for an intertwined investment into a supplementary energy resource with the option of timely procurements. This resource can be utilised to increase steam production in order to prevent turbine trips. Take note, if an investment into such a resource is made, it could potentially influence the optimal turbine investment vector.

9.1 Model formulation

A MILP model is presented that uses historic plant-specific signature steam flow data to determine what the optimal configuration of turbine investments should be when utilising fluctuating excess steam for energy recovery. The model further allows for an intertwined investigation into a supplementary energy resource investment, incorporating both fixed and variable expenses. The primary decision variables are concerned with the optimal investment choices through the optimal distribution of steam flow to the selected turbines. The optimal investment choices are the combination of turbines and the supplementary energy resource investments that yield the highest NPV over time, *i.e.* profit from power generation after the investment and procurement costs. The MILP model incorporates detailed binary decision variables that simulate the realistic shut-down and start-up of turbines based on an engineering plant's operating policy. This model is referred to as the *Optimal Power and Gas Investment Model* (OPGIM).

Turbine capacities, efficiencies and costs are input parameters. The following parameter definitions are required to formulate OPGIM: the index set of all turbines are given by $\mathcal{I}^T = \{1, 2, \dots, |\mathcal{I}^T|\}$, and the operation of these turbines are modelled over a time set $\mathcal{T} = \{1, 2, \dots, |\mathcal{T}|\}$.

Turbine $i \in \mathcal{I}^T$ can operate within the allowable minimum and maximum limits of L_i (mass unit per time period) and U_i (mass unit per time period), respectively, where steam is converted at an efficiency rate of η_i^T (mass flow per power unit generated). Once a turbine trips, a minimum time of δ^T , which coincides with sufficient steam availability, needs to elapse before the tripped turbine may be brought back online.

The income per unit, generated from power production, is equal to the unit cost C_t^E (cost per

power unit generated) that the power utility would have been paid at time t . The capital cost of turbine $i \in \mathcal{I}^T$ at time zero is given by C_i^T . The initial investment cost for equipment and installation of a supplementary energy resource is denoted by C^R . The operational costs involved for the supplementary resource is given by C_t^R (cost per generated steam flow unit) at time $t \in \mathcal{T}$. The real interest rate applied for NPV discounting is given by r . The present value cost of a turbine trip is given by C_i^h at time zero for each future occurrence. The real interest rate applied for NPV discounting is given by r .

There are three main groups of decision variables defined for OPGIM. The first group relates to the optimal distribution of steam to turbines. Let $m_{it}^S \geq 0$ be the steam flow produced by off-gases to turbine $i \in \mathcal{I}^T$ at time $t \in \mathcal{T}$ and let $m_{it}^{SR} \geq 0$ be the steam produced from the additional resource for turbine $i \in \mathcal{I}^T$ at time $t \in \mathcal{T}$. The second group of variables is necessary for modelling the different investment options. These variables are all binary variables. Let $z = 1$ if an investment is made into the supplementary resource's infrastructure, $z = 0$ if not. If $x_i = 1$ an investment is made to procure turbine $i \in \mathcal{I}^T$, and $x_i = 0$ if not. The third group of binary decision variables is required to model the operational status of each turbine. Let $y_{it}^T = 1$ if turbine $i \in \mathcal{I}^T$ is operational at time $t \in \mathcal{T}$. The following auxiliary variables are necessary to facilitate the operational policy related to the shut-down and start-up of turbines: let $y_{it}^h = 1$ if turbine $i \in \mathcal{I}$ is tripped (halted) from time $t-1 \in \mathcal{T}$ to $t \in \mathcal{T}$ and let $y_{it}^\delta = 1$ if turbine $i \in \mathcal{I}^T$ is in trip for at least δ^T until time $t \in \mathcal{T}$. The variable $y_{it}^b = 1$ if turbine $i \in \mathcal{I}$ which has been in trip at time $t-1 \in \mathcal{T}$, may be brought back online at time $t \in \mathcal{T}$.

The objective is to maximise NPV over the time horizon $\mathcal{T}$. For this purpose let

$$\gamma_{it} = \frac{(m_{it}^{SR} + m_{it}^S)}{\eta_i^T} C_t^E - m_{it}^{SR} C_t^R$$

be the operating profit for turbine i in time period t with $(m_{it}^{SR} + m_{it}^S)/\eta_i^T$ the rate of power generated by turbine i .

The objective of OPGIM is to:

$$\text{maximise } \sum_{i \in \mathcal{I}} \sum_{t \in \mathcal{T}} \frac{\gamma_{it}}{(1+r)^t} - C_i^h y_{it}^h - \sum_{i \in \mathcal{I}} x_i C_i^T - C^I z, \quad (9.1)$$

subject to:

$$\sum_{t \in \mathcal{T}} m_{it}^{SR} \leq U_i |\mathcal{T}| z, \quad \forall i \in \mathcal{I}, \quad (9.2)$$

$$m_{it}^S + m_{it}^{SR} \geq L_i y_{it}^T, \quad \forall i \in \mathcal{I}, t \in \mathcal{T}, \quad (9.3)$$

$$m_{it}^S + m_{it}^{SR} \leq U_i y_{it}^T, \quad \forall i \in \mathcal{I}, t \in \mathcal{T}, \quad (9.4)$$

the remainder of the sets of constraints for OPGIM are equivalent to that defined by (8.2), (7.2), (7.5), (7.6), (7.7), (7.8), (7.9), (7.10), (7.11), (7.12), (7.13) and (7.14).

As mentioned previously, the objective of OPGIM is to optimise the NPV over the time horizon, given by equation (9.1). The NPV is determined by the power generation income and supplementary resource procurement cost term γ with the deductions of turbine investment related costs and initial supplementary resource investment costs. Constraint set (8.2) ensures that a turbine $i \in \mathcal{I}^T$ may only be operational if it is invested in and constraint (9.2) allows procurement into a supplementary resource only if an investment in that is made. For each time $t \in \mathcal{T}$, constraint set (7.2) guarantees that the accumulated steam flow to the turbines produced from off-gases are less than or equal to the available steam flow from off-gases. Constraint sets (9.3) and (9.4) ensure that steam flow to any turbine $i \in \mathcal{I}^T$ is always within the allowable minimum and maximum operational limits or zero.

Constraint sets (7.5) to (7.7) determine if turbine $i \in \mathcal{I}^T$ tripped from time $t-1$ to t , and constraint set (7.8) ensures that the turbine is not tripped if sufficient steam flow is available. The set of

constraints (7.9) to (7.11) determine whether at least a δ^T time has elapsed for a tripped turbine $i \in \mathcal{I}^T$. If the minimum δ^T has not yet passed for a turbine in trip, constraint set (7.12) ensures that it cannot go back online. To determine if sufficient steam flow was available for a turbine in trip during the preceding δ^T and whether this turbine may be brought back online, constraint sets (7.13) and (7.14) are applied.

9.2 OPGIM scenario results

The question arises as to why an energy recovery plant does not utilise a supplementary energy resource to refrain turbines from trip occurrences and boost power generation to maximum capacity? Such a resource is typically far more expensive to acquire than its power generation income. Therefore, if power is generated from only such a resource a net loss will realise. The rationale behind an investigation into the possibility of investing in an additional resource, with the procurement thereof at certain time periods, is to capture potential power generation losses in times of turbine trips and prevent future maintenance costs.

For the scenario results in this chapter, *natural gas* is chosen as the supplementary energy resource. The cost to income ratio of natural gas is, unless stated otherwise, assumed to be a typical market related 2.20:1.00 under the Works' current boiler houses' steam production efficiencies. Therefore, for every R 2.20 spend on natural gas, a direct income of R 1.00 can be achieved. For the first set of scenario results the assumption of a zero trip penalty is made, so that $C_i^h = 0 \forall i \in \mathcal{I}^T$. Under this assumption investment and the acquirement of natural gas are only viable if the effect of gaining unutilised steam is higher than the costs incurred to do so. Take note, this is not applicable for steam flows above a turbine's maximum capacity.

All OPGIM scenarios in this chapter are solved by utilising the real-world data, as discussed in Section 2.11, for the steam input parameters. Similar to Chapters 5 and 8, all investment NPV's are determined over a 10 year interval. In order to compare the scenario results with those reported in Chapters 5 and 8, the cost of electricity at time zero is assumed to be R500.00 per Mega Watt hour utilised. The cost of electricity is annually inflated by 5.00% and the real interest rate is 5.00%, continuously (hourly) compounded. A 15 hour of sufficient steam availability is required before a turbine can restart. Note that all trip costs occur at time zero and reflects as a NPV penalisation.

9.2.1 Solving OPGIM under a zero trip penalty

OPGIM is solved for turbines from Table 8.1 with respective costs given in Table 5.2. Any number of turbines can be invested in, including the option to invest in and at times the procurement of natural gas. If natural gas is used elsewhere at the Works the possibility exists, as discussed previously, for more off-gases to be utilised in the boiler houses. Such an option will, typically, comprise limited fixed costs. However, for all scenarios to follow it is assumed that natural gas infrastructure investment costs at time zero amounts to R10 million. This amount is assumed to cover all necessary boiler house burner installations, all relevant piping and control valves. Similar to the turbines, the utilisation of natural gas is dynamically controlled.

Optimal investment choices are the combination of turbine and natural gas procurements that yield the highest NPV profit over 10 years. OPGIM determines which turbines must be procured, if it is financially sensible to invest into natural gas and when to procure whatever quantity thereof. Simulation results from OPGIM are labeled under *Cases* with a superscript "*g*" to indicate an additional "*gas*" can be procured. Results are summarised in Table 9.1. OPGIM is now solved for the five optimal instances where the trip penalties are zero, *i.e.* $C_i^h = 0 \forall i \in \mathcal{I}^T$.

Case 1^g: Solving OPGIM with the possible inclusion of natural gas results in a turbine investment vector of $x^{C_1^g} = [0, 0, 1, 0, 0, 1, 0, 0, 0, 0]$. The same investment result is obtained when OPIM is solved with a zero trip penalty, so that $x^{C_1^g} = x^{C_1^d}$. OPGIM further yields $z^{C_1^g} = 1$, so that a R10 million investment into natural gas must be made. Results of Case 1^d reveals a combined trip quantity of 672, whereas Case 1^g has 412 occurrences. The 260 trip reductions are a direct result

Table 9.1: Scenario results for Case 1^g to 5^g where natural gas investments and procurements thereof are allowed. A zero trip penalty is solved for. The results include the optimal turbine and natural gas investment vectors, trips and power generation per turbine, total power generation and the expected NPV for 10 years.

Case number	Investment vector	Trips per turbine		Power generation per turbine [MW]		Total power generation [MW]	NPV R · 10 ⁶
1^g	$x^{C_1^g} = [0, 0, 1, 0, 0, 1, 0, 0, 0, 0]$ $z^{C_1^g} = 1$	261	151	7.73	15.68	23.41	812.5
2^g	$x^{C_2^g} = [0, 1, 0, 0, 0, 0, 1, 0, 0, 0]$ $z^{C_2^g} = 1$	261	38	1.85	21.53	23.38	811.1
3^g	$x^{C_3^g} = [0, 0, 0, 0, 0, 0, 0, 1, 0, 0]$ $z^{C_3^g} = 1$	0		23.62		23.62	810.8
4^g	$x^{C_4^g} = [1, 0, 0, 0, 0, 0, 1, 0, 0, 0]$ $z^{C_4^g} = 1$	262	0	1.20	21.90	23.10	806.0
5^g	$x^{C_5^g} = [0, 0, 0, 0, 0, 0, 0, 0, 1, 0]$ $z^{C_5^g} = 1$	0		24.00		24.00	803.3

of expensive natural gas being utilised, and furthermore, 23.41MW is generated opposed to Case 1^d 's 22.25MW.

An increase in the NPV of 3.0% is observed between Case 1^g and 1^d , where power generation is 5.2% higher. Additionally, a 38.7% reduction in trips are observed. As a result, more than one in three trips are prevented by utilising natural gas as a supplementary energy resource.

Take note, since zero trip penalties are incurred, the 260 trips prevented through the use of natural gas are instances where the income from otherwise unutilised steam outweighs the expensive natural gas costs. The NPV is reported at R812.5 million, so that the net effect of the natural gas investment and procurements are R23.8 million in comparison to the NPV from Case 1^d . As a result, if future trips costs are assumed to be zero, the Works can accommodate natural gas investment costs of R33.8 (23.8+10) million at time zero.

Case 2^g : OPGIM is solved for when the optimal investment vector $x^{C_1^g}$ is not allowed. Results yield an NPV of R811.1 million for an optimal turbine investment vector $x^{C_2^g} = [0, 1, 0, 0, 0, 0, 1, 0, 0, 0]$ and $z^{C_2^g} = 1$. Note that $x^{C_2^g} = x^{C_4^d}$ and by utilising natural gas as an additional energy resource, power generation increases by 8.0% to 23.38MW. Furthermore, in Case 2^g 299 trips are observed, which is a 46.6% reduction compared to the 560 of Case 8^d without natural gas.

In Chapter 5 POIQM is solved for turbines under a fixed-sequence philosophy when $C_i^h = 0 \forall i \in \mathcal{I}^T$. The optimal turbine investments obtained for Case 1^f are the procurements of Turbine II and VII, equivalent to that of $x^{C_1^g}$. Under dynamic control with the inclusion of a natural gas investment and procurements, solving OPGIM results in an 11.8% power generation increase, compared to Case 1^f . Furthermore, the NPV determined by OPGIM is 10.6% higher than POIQM results and trips are reduced by 57.8%.

Optimal power generation for a 5MW and a 30MW turbine are plotted in Figures 4.4 and 4.5 for operations under a fixed-sequence philosophy. Take note, these two turbines possess the identical operating limits to that of Turbine II and VII. In Chapter 7, Figures 7.1 and 7.2 depict power generation for Turbine II and VII, which operate under dynamic control. Figures 9.1 and 9.2 display the power generation for Turbine II and VII when natural gas is utilised as an additional energy resource. Note that these two graphs are plotted over the time line of the real-world data, as discussed in Section 2.11, and not the extended 10 year interval.

By comparing Figure 7.1 to Figure 9.1, a difference can be observed in the number of trips. When natural gas is allowed, more trips occur for the smaller capacity, *i.e.* the 5MW turbine. To explain this increase in trips, Figures 7.2 and 9.2 are used. The main difference between these two figures is that only a single trip is evident in Figure 9.2. Natural gas is, therefore, used at times to keep the 30MW operational, that would otherwise not have been possible. As a result more time periods exist where insufficient steam is available for the 5MW turbine. Take note, OPGIM determines that

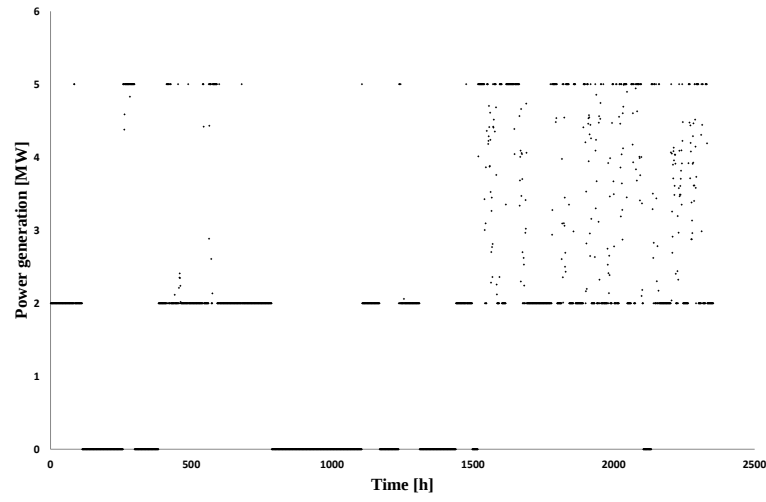


Figure 9.1: Case 2^g: Power generation over time for Turbine II, the 5MW, when natural gas can assist steam production.

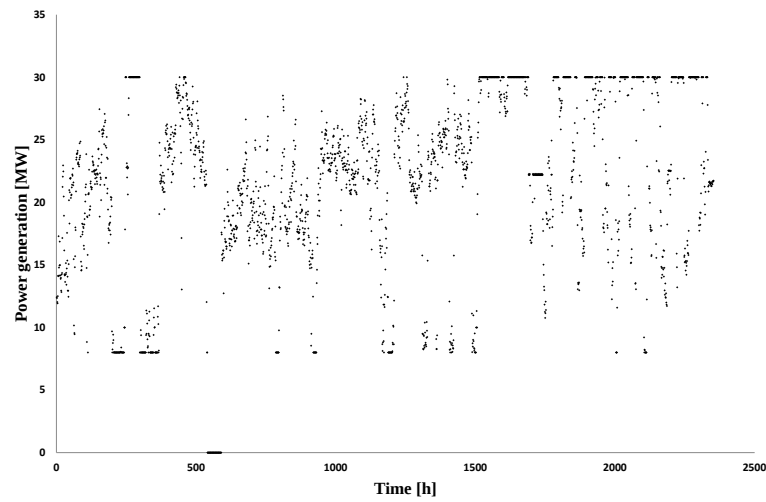


Figure 9.2: Case 2^g: Power generation over time for Turbine VII, the 30MW, when natural gas can assist steam production.

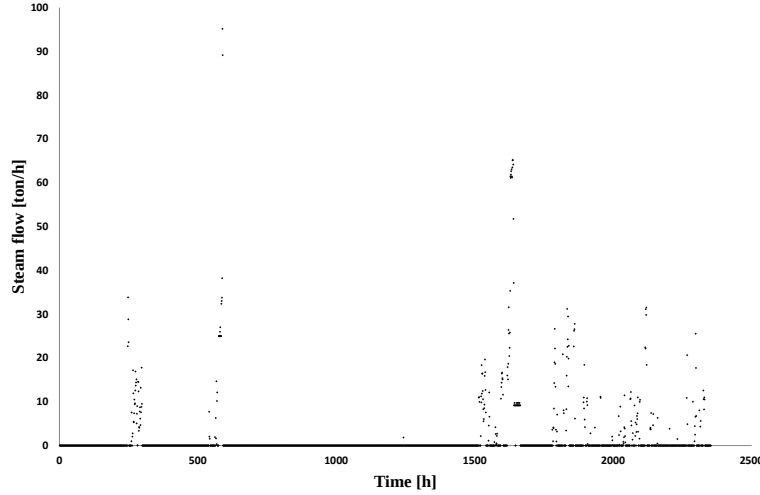


Figure 9.3: Unutilised steam flow for Case 2^g over time when natural gas can assist steam production.

it is more expensive to keep the 5MW operational during these times, compared to the additional power generation income that could have been realised, when $C_i^h = 0 \forall i \in \mathcal{I}^T$.

The difference between power generation from Case 8^d to Case 2^g is clearly visible when Figure 9.3 is compared to Figure 7.3. From Figure 9.3 it follows that more steam quantities are unutilised and, therefore, used for power generation.

Case 3^g to 5^g : OPGIM is solved and yields investment results for the next three optimal outcomes. Each outcome dictates that a R10 million investment into natural gas should realise. Take note that for every outcome in Table 9.1 the projected NPV is above that of Case 1^d in Table 8.2. The NPV improvements between Case 1^g to 5^g over dynamic control without natural gas, from Case 1^d , range from 1.9% to 3.0%.

Results from Case 3^g and 5^g demonstrate that under dynamic natural gas control it is sensible to invest into a single large turbine. For Case 3^g optimal investments are for natural gas and Turbine VIII, *i.e.* the 35MW machine, when $x^{C_1^g}$ and $x^{C_2^g}$ are not allowed. Optimal results from Case 5^g show that if $C_i^h = 0 \forall i \in \mathcal{I}^T$, the procurement of the 40MW machine together with natural gas is the fifth best investment outcome. Take note no other optimal dynamic scenario outcome thus far indicated that a large single capacity turbine should be procured.

It is important to note, however, that for Case 3^g , if natural gas is not invested in and Turbine VIII procured, the NPV is expected to be R726.6 million. If the vectors $x^{C_5^g}$ and $z^{C_5^g} = 0$ are fixed into OPGIM, *i.e.* Turbine IX without natural gas, the optimal NPV is determined at R716.5 million. Case 3^g and 5^g , therefore, result in a respective 11.6% and 12.1% NPV increase when natural gas is invested in. This demonstrates the intertwined dependency of the supplementary energy resource and turbine investment vectors. Therefore, investment vector into $x^{C_j^g}$ must be accompanied by $z^{C_j^g}$.

Note from the results in Table 9.1 that for Case 3^g and 5^g OPGIM yields that the single turbine should always be kept operational to realise an optimal NPV. For Case 1^g , 2^g and 4^g , however, optimal results include trip instances. Under zero trip penalties it is therefore, at times, more profitable to not utilise the expensive natural gas to keep both turbines operational.

The results from Case 3^g is used to graphically display the effect on steam usages and power generation when natural gas is utilised in Figures 9.4 to 9.8. OPM from Chapter 7 is solved for $x^{C_3^g}$, *i.e.* Turbine VIII is fixed as input parameter. Results from OPM are plotted in Figure 9.4, where power is generated at 20.62MW. Note the number of trip intervals, *i.e.* 13 on the x-axis. Over a 10 year period the trips will amount to 485 instances.

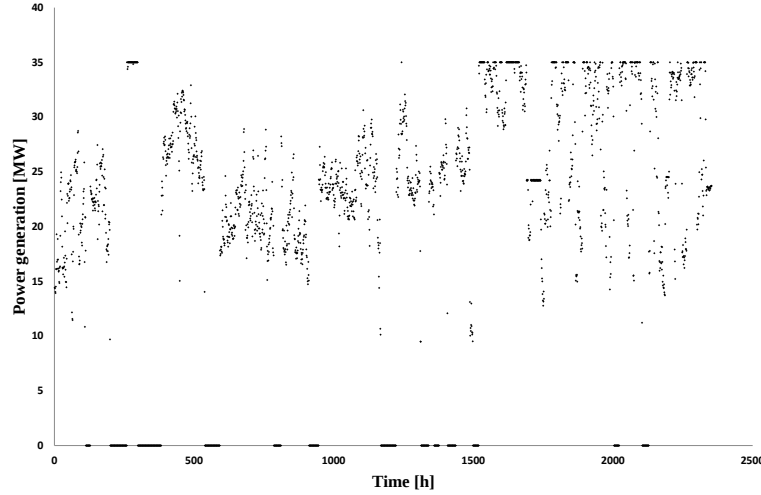


Figure 9.4: Power generation over time for Turbine VIII, as solved by OPM, where natural gas is not used.

The unutilised steam for this OPM scenario is plotted in Figure 9.5. From the graph a number of instances can be observed where steam is not fully utilised by the turbine. Note that instances exist where unused steam is above, $U_8 = 175$ (ton per hour), which is the turbine's maximum flow limit. As a result, even if the turbine is operational at all times, some steam cannot be utilised.

Figure 9.6 displays the steam production taking place from natural gas procurements for Case 3^g. All direct power generation from these steam flows are subjected to a cost to income ratio of 2.20:1.00. It is mentioned previously that OPGIM solves for Turbine VIII to stay operational during the entire time horizon. The steam productions shown in Figure 9.6 are therefore required to prevent an average of 13 trips over the time horizon of the real-world data.

If natural gas procurements are not allowed, some steam quantities are lost due to trips and where $m_t^S > U_8$, as plotted Figure 9.5. When natural gas is utilised to prevent Turbine VIII from trips, some of these steam flow quantities are recovered for power generation. Figure 9.7 displays steam flows that are 'recovered' and therefore additionally used for power generation due to trip prevention for Turbine VIII.

Combining power generation plotted in Figures 9.4, 9.6 and 9.7 yield the OPGIM results plotted in Figure 9.8, which depicts power generation for Turbine VIII under the real-world data when natural gas procurements are allowed. Note from the figure that no trips occur and the inclusion of a number of lower power generation instances when compared to Figure 9.4. The positive effect on power generation, due to natural gas, is clearly evident in Figure 9.8 and reported in Table 9.1. Power generation at 23.62MW for Case 3^g is 3.00MW or 14.5% higher when natural gas is utilised. Take note, only 0.36MW is a direct result by means of natural gas usages and the remaining 2.64MW power generation are from 'recovered' steam flows.

The next section investigates optimal investments when fixed trip penalties are incorporated. OPGIM is used to solve for optimal investments for the five best possible NPV if a trip cost of R50,000.00 is realised at time zero for each instance, *i.e.* $C_i^h = 50000 \forall i \in \mathcal{I}^T$.

9.2.2 Solving OPGIM under fixed trip costs

The previous section demonstrated the positive effect that an expensive supplementary energy resource can exert on power generation and the NPV. In this section optimal investment outcomes are determined by OPGIM, when each turbine trip penalises the NPV with R50,000.00, so that $C_i^h = 50000 \forall i \in \mathcal{I}^T$. The five optimal investment results from OPGIM are summarised in Table 9.2.

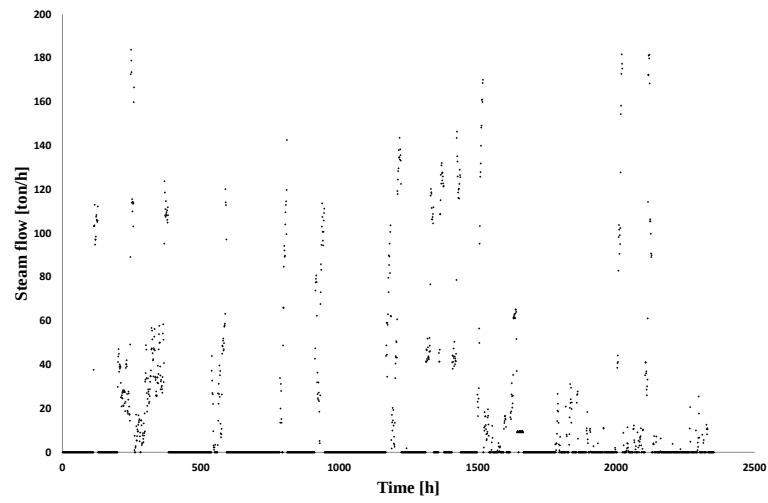


Figure 9.5: Unutilised steam flows by Turbine VIII if natural gas is not used.

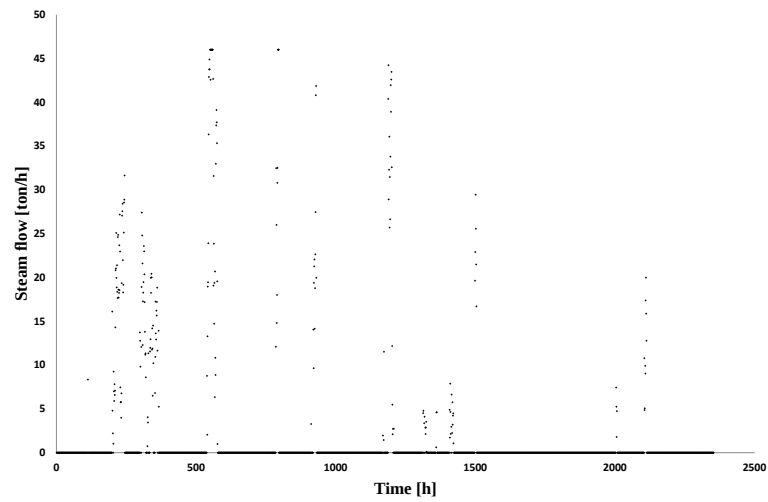


Figure 9.6: Case 3^g: Steam produced by natural gas for Turbine VIII over time.

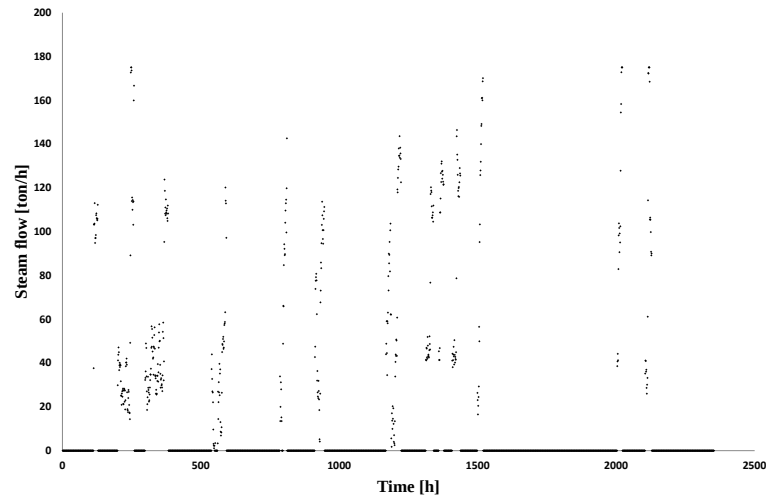


Figure 9.7: Case 3⁹: Steam flows additionally utilised due to steam produced by natural gas for Turbine VIII over time.

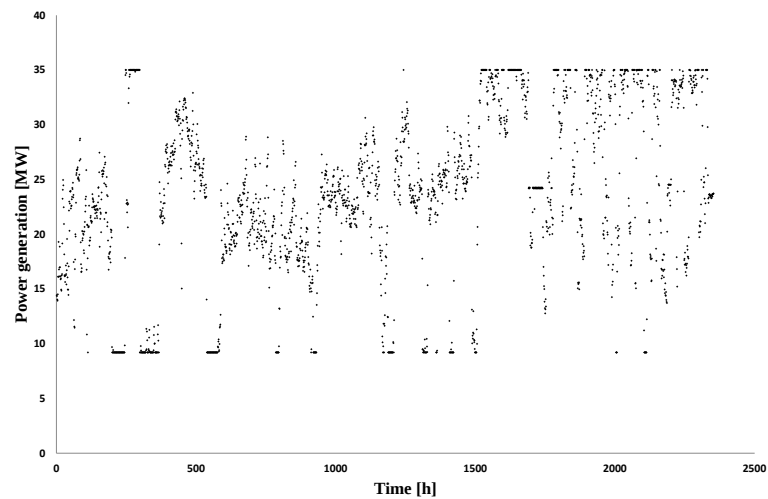


Figure 9.8: Case 3⁹: Power generation by Turbine VIII over time when natural gas investment and procurement thereof is allowed.

Table 9.2: Scenario results for Case 6^g to 10^g where natural gas investments and procurements thereof are allowed. Trip penalty cost of R50,000.00 is incurred per occurrence. The results include the optimal turbine and natural gas investment vectors, trips and power generation per turbine, total power generation and the expected NPV for 10 years.

Case number	Investment vector	Trips per turbine		Power generation per turbine [MW]		Total power generation [MW]	NPV R · 10 ⁶
6 ^g	$x^{C_6^g} = [0, 0, 0, 0, 0, 0, 0, 1, 0, 0]$ $z^{C_6^g} = 1$	0		23.62		23.62	810.8
7 ^g	$x^{C_7^g} = [0, 1, 0, 0, 0, 0, 1, 0, 0, 0]$ $z^{C_7^g} = 1$	38	0	1.21	22.29	23.50	807.1
8 ^g	$x^{C_8^g} = [0, 0, 1, 0, 0, 1, 0, 0, 0, 0]$ $z^{C_8^g} = 1$	38	38	5.46	18.04	23.50	804.4
9 ^g	$x^{C_9^g} = [0, 0, 0, 0, 0, 0, 0, 0, 1, 0]$ $z^{C_9^g} = 1$	0		24.00		24.00	803.3
10 ^g	$x^{C_{10}^g} = [1, 0, 0, 0, 0, 0, 1, 0, 0, 0]$ $z^{C_{10}^g} = 1$	38	0	0.82	22.40	23.62	802.7

Case 6^g: Solving OPGIM under a fixed trip cost of R50,000.00 yields an optimal investment vector of $x^{C_6^g} = [0, 0, 0, 0, 0, 0, 0, 1, 0, 0]$ and $z^{C_6^g} = 1$. The OPGIM results for these scenarios are, therefore, equivalent to that of Case 3^g. The optimal NPV of R810.8 million, compared to Case 1^g, is only 0.2% lower. Therefore, irrespective of the potential trip costs incurred, the OPGIM outcome for investment vector $x^{C_6^g}$ and $z^{C_6^g} = 1$ is unchanged, since natural gas is used to prevent all trip occurrences. Take note, the NPV from Case 6^g is only 0.2% lower than that of Case 1^g and 2.8% higher than that of Case 1^d.

Case 7^g to 10^g: OPGIM is solved for the following four instances and yield forthcoming optimal investment results. Even though not in the same chronological order, all five optimal investment combinations from Case 1^g to 5^g are repeated in Case 6^g to 10^g. These results indicate that with the option to obtain an additional energy resource, optimal investment vectors are less sensitive towards turbine trip costs. OPGIM results for Case 7^g, 8^g and 10^g yield investment vectors containing two turbines. Even though not zero, all three scenarios solve for significant less turbine trips. Take note that the fifth optimal investment outcome, *i.e.* Case 10^g, yields an optimal NPV, which is 1.2% lower than that of Case 1^g. The NPV's from Case 6^g to 10^g are between 6.3% and 7.4% higher than that of Case 6^d.

Outcomes from Case 6^g to 10^g demonstrate how trip prevention by means of an expensive energy resource do not only yield higher NPV's, but less sensitive optimal investment results towards fixed trip expenses. In the next section the sensitivity of OPGIM outcomes is investigated when trip penalties of $C_i^h = C_i^T/500 \forall i \in \mathcal{I}^T$ per instance are assumed.

9.2.3 Solving OPGIM under proportional trip costs

The previous section demonstrates that NPV's as determined by OPGIM are in close proximity for a trip penalty of $C_i^h = 50000 \forall i \in \mathcal{I}^T$, compared to $C_i^h = 0 \forall i \in \mathcal{I}^T$. This is owing to the utilisation of natural gas as a supplementary energy resource. In this section trip expenses are a proportion of the procurement price, so that each occurrence penalises the objective function of OPGIM with $C_i^h = C_i^T/500 \forall i \in \mathcal{I}^T$. Optimal NPV results are expected to be close to those reported in Table 9.2, owing to the possibility to eliminate trips through additional steam productions from natural gas. The top five optimal investment results for this section are summarised in Table 9.3.

Case 11^g to 15^g: The five optimal investment outcomes are obtained by OPGIM under proportional trip penalties. Since zero trips are reported for Case 6^g, it is inevitable for Case 11^g to yield the exact outcome. Take note, similar to the previous section, all turbine investment vectors from Case 1^g to 5^g are present in Case 11^g to 15^g. This, furthermore, indicates that optimal investment selections are more robust when an additional energy resource can be invested in.

All five scenario results from OPGIM indicated zero turbine trips and, therefore, suggest that

Table 9.3: Scenario results for Case 11^g to 15^g where natural gas investments and procurements thereof are allowed. Proportional trip penalties of $C_i^h = C_i^T/500 \forall i \in \mathcal{I}^T$ are incurred as a cost towards OPGIM's objective function for each occurrence. The results include the optimal turbine and natural gas investment vectors, trips and power generation per turbine, total power generation and the expected NPV profit for 10 years.

Case number	Investment vector	Trips per turbine		Power generation per turbine [MW]		Total power generation [MW]	NPV R · 10 ⁶
11 ^g	$x^{C_{11}^g} = [0, 0, 0, 0, 0, 0, 0, 1, 0, 0]$ $z^{C_{11}^g} = 1$	0		23.62		23.62	810.8
12 ^g	$x^{C_{12}^g} = [0, 1, 0, 0, 0, 0, 1, 0, 0, 0]$ $z^{C_{12}^g} = 1$	0	0	1.22	22.29	23.51	805.3
13 ^g	$x^{C_{13}^g} = [0, 0, 0, 0, 0, 0, 0, 0, 1, 0]$ $z^{C_{13}^g} = 1$	0		24.00		24.00	803.3
14 ^g	$x^{C_{14}^g} = [1, 0, 0, 0, 0, 0, 1, 0, 0, 0]$ $z^{C_{14}^g} = 1$	0	0	0.87	22.40	23.67	802.3
15 ^g	$x^{C_{15}^g} = [0, 0, 1, 0, 0, 1, 0, 0, 0, 0]$ $z^{C_{15}^g} = 1$	0	0	4.31	19.34	23.65	800.5

turbines should be protected against all such occurrences. The NPV under proportional trip costs for the five optimal investment outcomes lie within 0.2 % and 1.5 % of optimality of Case 1^g when $C_i^h = 0 \forall i \in \mathcal{I}^T$. Comparing the NPV's from Case 11^g to 15^g with that obtained from OPIM for Case 1^d, 6^d and 11^d, increases are observed from of 1.5% to 17.9%. The results show an NPV increase of 25.4% to 27.0% over that of Case 11^f under a fixed-sequence philosophy. Therefore, the OPGIM outcomes demonstrate the ability of a supplementary resource to protect a turbine from trips and simultaneously increase the NPV. Take note, this is applicable for when the resource, *i.e.* natural gas, consists of a cost to income ratio of 2.20:1.00.

The next section further investigates sensitivity towards expenditure increases. Procurement prices are doubled and the objective function penalised by $C_i^h = C_i^T/250 \forall i \in \mathcal{I}^T$ for each trip occurrence.

9.2.4 Sensitivity towards investment and trip cost changes

In order to investigate the sensitivity of the investment results obtained in this chapter towards procurement increases, OPGIM is solved under increased investment costs. The procurement prices as given in Table 5.2 are doubled for each turbine and OPGIM's objective function is penalised by $C_i^h = C_i^T/250 \forall i \in \mathcal{I}^T$ per trip occurrence. OPGIM is used to determine the three optimal investment options and, fourthly, the NPV when the optimal POIQM investments from Case 1^f are fixed as input parameters, so that $x^{C_{19}^g} = [0, 1, 0, 0, 0, 0, 1, 0, 0, 0]$. The results are summarised in Table 9.4.

Case 16^g: OPGIM is solved to obtain the optimal investment outcome under increased costs. Results indicate that Turbine VII, the 30MW should be invested in, together with natural gas investment and procurements. Even though the optimal investment vector $x^{C_{16}^g}$ is not equal to any one of $x^{C_1^g}$ to $x^{C_{15}^g}$, Turbine VII is present within six of these vectors. The rate of power generation is determined at 22.60MW, which is lower than all other power generation results under natural gas inclusions mentioned in this chapter. Take note that the NPV of R667.0 million is 17.9% lower than that of Case 1^g under lower investment costs.

Case 17^g and 18^g: The following two optimal investment vectors with relevant outcomes are determined by OPGIM and summarised in Table 9.4. It should be noted that both investment vectors are present in Tables 9.1 to 9.3 and $x^{C_{17}^g} = x^{C_3^g} = x^{C_6^g} = x^{C_{11}^g}$. Furthermore, the reported NPV's for Case 16^g to 18^g vary between 79.2% and 82.7% above the respective dynamic scenario of Case 16^d. When the results are compared to Case 16^f under a fixed-sequence philosophy, an increased NPV between 113.8% and 118.0% is observed. Dynamic plant operations with the inclusion of natural gas, under the given cost constraints, therefore yield optimal NPV's more than double when compared to turbines operating under a fixed-sequence philosophy.

Table 9.4: Scenario results for Case 16^g to 19^g where natural gas investments and procurements thereof are allowed. Turbine costs are doubled and OPGIM's objective function is penalised by $C_i^h = C_i^T/250 \forall i \in \mathcal{I}^T$ per trip occurrence. The results include the optimal turbine and natural gas investment vectors, trips and power generation per turbine, total power generation and the expected NPV profit for 10 years.

Case number	Investment vector	Trips per turbine	Power generation per turbine [MW]	Total power generation [MW]	NPV R · 10 ⁶
16 ^g	$x^{C_{16}^g} = [0, 0, 0, 0, 0, 0, 1, 0, 0, 0]$ $z^{C_{16}^g} = 1$	0	22.60	22.60	667.0
17 ^g	$x^{C_{17}^g} = [0, 0, 0, 0, 0, 0, 0, 1, 0, 0]$ $z^{C_{17}^g} = 1$	0	23.62	23.62	660.8
18 ^g	$x^{C_{18}^g} = [1, 0, 0, 0, 0, 0, 1, 0, 0, 0]$ $z^{C_{18}^g} = 1$	0 0	1.56 21.78	23.34	654.3
19 ^g	$x^{C_{19}^g} = [0, 1, 0, 0, 0, 0, 1, 0, 0, 0]$ $z^{C_{19}^g} = 1$	0 0	2.47 21.23	23.70	650.3

In Section 5.2.4 POIQM is solved for Turbine II and VII when investment costs are doubled and each trip penalises the NPV with $C_i^h = C_i^T/250 \forall i \in \mathcal{I}^T$. Results show that the expected NPV is negative R32.5 million for Case 19^f. OPGIM results indicate for **Case 19^g**, under equivalent turbines and costs, an NPV of R650.3 million is expected when the turbines are dynamically controlled and natural gas invested in and procured. Therefore, with the inclusion of an expensive additional energy resource, a net positive effect of R682.8 million is observed when compared to operations under a fixed-sequence philosophy.

Take note that for Case 16^g and 17^g, OPGIM results indicate a single turbine investment, together with the inclusion of natural gas procurements, under double cost conditions. Results from Section 5.2.4 report that when a fixed-sequence philosophy is followed under these costs, single turbine investments should also realise. However, when turbines are dynamically controlled and off-gas shortages cannot be supplemented by an additional energy resource, optimal investments are into two machines, as demonstrated in Chapter 8.

From the results generated by OPGIM the potential positive effect on the NPV profit is clearly demonstrated when a supplementary energy resource is being invested in. It can furthermore be concluded that if trip costs are related to a turbine's procurement price, the positive effect of the resource investment on the NPV increases with regards to non-resource investments. All scenarios are performed with a cost to income ratio of 2.20:1.00. In order to further demonstrate how the fluctuating signature nature of the steam profiles are dependent on a supplementary energy resource, to assist NPV, the cost ratio is increased in the next section.

9.2.5 Sensitivity towards supplementary energy resource cost increases

This section displays how sensitive OPGIM's optimal investment outcomes are towards a significant increase in the supplementary energy resource's procurement costs. For this section alone, a natural gas cost to income ratio of 10.00:1.00 is assumed. In other words, for every R10.00 spend on natural gas procurements, R1.00 worth of electricity can directly be generated.

Scenario results are generated by OPGIM for the top five optimal investment vectors, in terms of NPV. Turbine costs are doubled in relation to those given in Table 5.2 and it is assumed that each machine trip incurs a penalty cost at time zero of $1/250^{th}$ of the procurement price, *i.e.* $C_i^h = C_i^T/250 \forall i \in \mathcal{I}^T$. The sensitivity of the optimal investment outcomes from OPGIM is therefore tested and only on the more extreme turbine and trip expenses. The Cases are labeled with a superscript “*g*” to indicate an additional investment into a “*gas*” can be made at an increased cost to income ratio.

Case 1^{g*} to 5^{g*}: The top five optimal results obtained by solving OPGIM are summarised in Table 9.5. Note that even for the extreme cost to income ratio of 10.00:1.00, all OPGIM results

Table 9.5: Scenario results for Case 1^{g*} to 5^{g*} where natural gas investments and procurements thereof are allowed. Turbine costs are doubled. Proportional trip costs of $1/250^{th}$ of the procurement cost are incurred. The results include the optimal turbine and natural gas investment vectors, trips and power generation per turbine, total power generation and the expected NPV profit for 10 years. The cost to income ratio for natural gas is fixed at 10.00:1.00.

Case number	Investment vector	Trips per turbine	Power generation per turbine [MW]		Total power generation [MW]	NPV R ·10 ⁶	
1 ^{g*}	$x^{C_{1^*}^g} = [0, 0, 0, 0, 0, 1, 0, 0, 0, 0]$ $z^{C_{1^*}^g} = 1$	0	20.95		20.95	587.5	
1 ^{g*}	$x^{C_{2^*}^g} = [0, 1, 0, 0, 0, 1, 0, 0, 0, 0]$ $z^{C_{2^*}^g} = 1$	0	2.64	19.99	22.63	583.0	
3 ^{g*}	$x^{C_{3^*}^g} = [0, 0, 0, 0, 0, 0, 1, 0, 0, 0]$ $z^{C_{3^*}^g} = 1$	0	22.60		22.60	582.7	
4 ^{g*}	$x^{C_{4^*}^g} = [0, 0, 1, 0, 0, 1, 0, 0, 0, 0]$ $z^{C_{4^*}^g} = 1$	0	0	4.31	19.37	23.68	567.3
5 ^{g*}	$x^{C_{5^*}^g} = [0, 1, 0, 0, 0, 0, 1, 0, 0, 0]$ $z^{C_{5^*}^g} = 1$	0	0	1.12	22.29	23.41	562.5

yield a R10 million investment into natural gas. This further demonstrates the dependency on a supplementary energy resource for the signature steam profile to improve NPV over time. Turbine VI is the optimal investment choice with an NPV of R587.5 million with no trips. It should be noted that, even with a 10.00:1.00 natural gas cost to income ratio, it is more profitable for all five investment scenarios to procure sufficient quantities to eliminate all turbine trips. The results further emphasise the importance of an energy recovery plant to incorporate measures, even when expensive, to eliminate turbine trips owing to off-gas, and therefore steam shortages. Note that the NPV's are between 11.9% and 16.7% lower than optimality under a 2.20:1.00 cost ratio, which is Case 16^g .

Comparing these results to the respective optimal NPV for Case 16^d , without a supplementary energy resource, optimal OPGIM outcomes yield an increased NPV ranging from 54.1% to 60.9%. When these results are furthermore compared to the respective optimal NPV under a fixed-sequence philosophy from POIQM in Chapter 5, *i.e.* Case 16^f , the increased NPV's are between 83.8% and 92.0% higher. Therefore, even under extreme resource costs the dynamically controlled NPV's, with the inclusion of natural gas, solve for almost double the highest NPV under a fixed-sequence philosophy. The results further emphasise the effect that a fluctuating steam profile exerts on energy recovery, as a result of turbine trips. Even at high protection costs the NPV's prove to react positively towards extreme cost protection measures. It is therefore imperative to safeguard these expensive machinery from trip occurrences, when possible.

The next section will investigate the effect of an improved turbine efficiency on the NPV, in terms of the percentage price increase that a Works should be willing to pay for such a machine.

9.2.6 Investments under improved isentropic efficiencies

All Case scenarios for this section are performed on turbines from Table 8.6 with $\eta_i^T = 4.5 \forall i \in \mathcal{I}^T$. This section reports on allowable procurement price increases for higher efficient turbines operating under dynamic control where a supplementary energy resource can be invested in. Optimal results are generated from OPGIM for five scenarios and labeled under Case 1^{gi} up to 5^{gi} , where the superscript “*gi*” indicates that *gas* can be procured with turbines under higher *isentropic efficiencies*. Results are summarised in Table 9.6.

OPGIM is solved to determine at what percentage price increase an improved isentropic efficient turbine still yields a higher NPV. Optimal investments vectors as reported for Case 1^g , 6^g , 11^g , 16^g and 1^{g*} in this chapter are forced as inputs towards OPGIM for Case 1^{gi} to 5^{gi} . For each scenario the investment's maximum percentage price increase is determined that still yields an improvement

Table 9.6: Allowable procurement price increases based on results for Case 1^{gi} to 5^{gi} . The percentage increase indicates the maximum price increase that can be allowed if each turbine's conversion rate improves with 10.0% from 5.0 to 4.5.

	Case 1^{gi}	Case 2^{gi}	Case 3^{gi}	Case 4^{gi}	Case 5^{gi}
Percentage increase	52.1	54.6	54.6	51.3	18.2

on the NPV.

Case 1^{gi} to 3^{gi} : OPGIM is now solved for the five Cases. Take note, for the first four Cases the natural gas cost to income ratio is fixed at 2.20:1.00 and for Case 5^{gi} it is 10.00:1.00. The percentage price increase for Case 1^{gi} is 52.1%. Therefore, the combined procurement prices between the improved 10MW and 25MW turbines should be within a 52.1% increase for this turbine vector choice to yield an NPV above that of Case 1^g . For Case 6^g and 11^g OPGIM yields optimal investments into Turbine VIII and natural gas, where no trips occur. As a result, Case 2^{gi} and 3^{gi} yield an equivalent maximum allowable price increase of 54.6% for a more efficient 35MW turbine. If the procurement price is doubled and each trip penalises the NPV with $C_i^h = C_i^T/250 \forall i \in \mathcal{I}^T$, OPGIM outcomes show that the allowable percentage increase is relatively insensitive. The result for **Case 4^{gi}** indicates that a 51.3% increase can be accepted for a 10.0% reduction in the steam to power conversion rate. Comparing the results between Tables 5.8 and 8.7, it is clear that dynamically controlled turbines in a fluctuating steam flow environment, operating under the assistance of an additional energy resource, are less sensitive towards improved isentropic efficiency turbines' price increases when potential future trip costs are taken into account.

For the fixed-sequence operating philosophy, the maximum allowable increase varies between 6.6% and 31.2%. Under dynamic control the allowable percentage price increases are between 14.5% and 48.3%. The addition of an expensive supplementary energy resource, however, results in a maximum price increase between 51.3% and 54.6%.

When the cost to income ratio of natural gas is increased to 10.00:1.00, turbine procurement costs are doubled and trips are penalised at $1/250^{th}$ of the investment price, the maximum allowable increase is 18.2% for **Case 5^{gi}** .

9.3 Summary

OPGIM was formulated in this chapter and its unique working ability was demonstrated through a number of Cases. The intertwined combination of turbine and supplementary energy resource investments, with the timely procurements thereof, is optimised by OPGIM, using NPV as a benchmark. Through various scenarios it was demonstrated how a typically very expensive energy resource, such as natural gas, can increase power generation profits by limiting or eliminating turbine trips due to off-gas and steam shortages.

Figure 9.9 provides a flow of MILP formulations used in the process to formulate OPGIM. The first conceptual model formulations were proposed in Chapter 3, followed by those in Chapter 6. Optimal power generation from OPM in Chapter 7, followed as a result of these models, which in turn was then used for the OPIM formulation in Chapter 8 that resulted in the formulation of OPGIM.

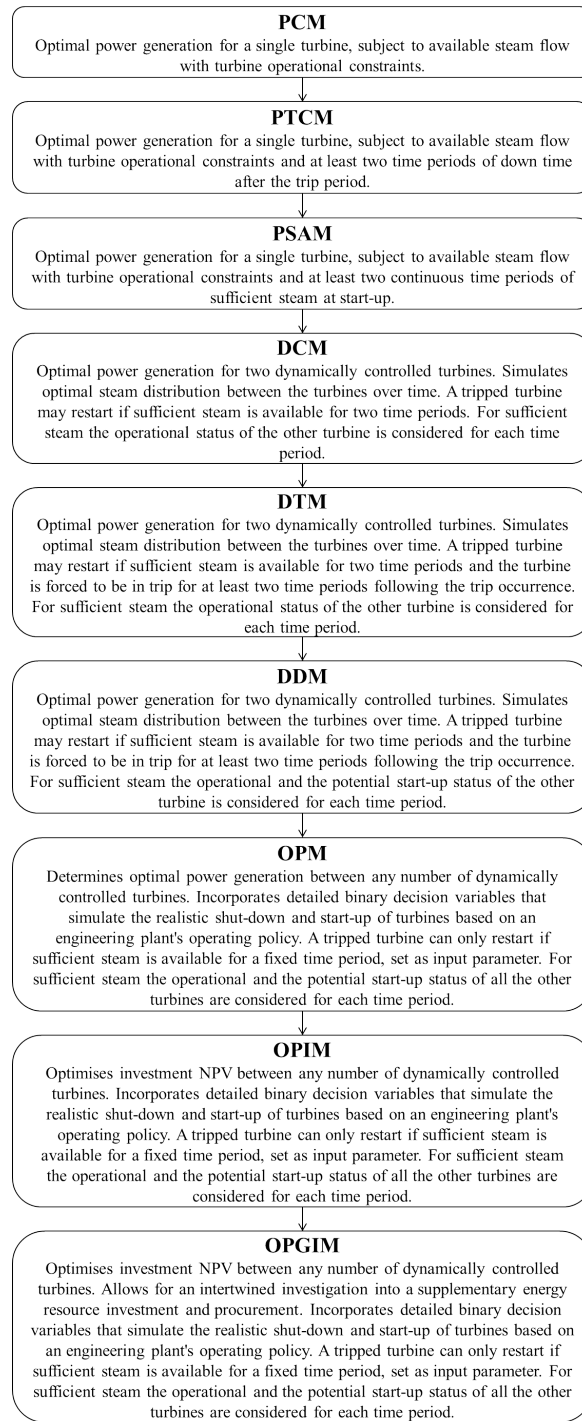


Figure 9.9: Complete flow chart from PCM to PSAM in Chapter 3, followed by DCM to DDM in Chapter 6, OPM in Chapter 7, OPIM in Chapter 8 and OPGIM presented in this chapter.

Chapter 10

Summary and conclusion

The need for optimal power generation control and investment formulation, under a fluctuating steam flow environment, were identified and addressed in this study. Various investment optimisation models exist in the literature, however, averaging of steam profiles is typically incorporated. Nowhere in literature could it be found that plant specific steam profiles were incorporated to determine optimal investments. By achieving it in this study, optimal investments can now be determined for a specific engineering Works under its signature steam flow profiles.

The start-up and shut-down of turbines are simulated and from this, the optimal steam distributions are determined for every time period. From optimal power generation, optimal turbine investments can be modelled. In order to determine optimal turbine investments for an engineering Works, typical steam profiles that are experienced need to be incorporated in the decision-making process. For this an understanding into the engineering operations is required. This includes all relevant processes and laws of physics, which govern the fundamental working principles of equipment that form part of the power generation process. Understanding these processes and underlying scientific working principles, is paramount in the formulation reasoning process and calculation of the fixed input parameters, incorporated by these models.

This thesis proposed novel *mixed integer linear programming* (MILP) formulations to address either optimal power generation, turbine investment decisions, or both for a typical engineering Works where energy recovery takes place in a fluctuating steam flow environment. All of the main contributions can accommodate any number of turbines. Furthermore, it can be of any operating capacities or efficiencies.

The working ability of all main model contributions was demonstrated by solving each for a number of different scenarios, using real-world data, obtained from an engineering Works. All conceptual model formulations of this thesis were solved for a hypothetical data set that comprise arbitrarily chosen steam values.

10.1 Chapter summaries

Chapter 2 provided a discussion on the engineering background relevant to this study, which included a hypothetical steam profile and that of the real-world data for scenario demonstration purposes.

Chapter 3 provided conceptual MILP formulations to illustrate the modelling of power generation optimisation in a fluctuating steam flow environment. The first proposed MILP formulation of this thesis was the *Power Concept Model* (PCM). The formulation of PCM demonstrated how steam flow to a single turbine should be distributed to obtain optimal power generation, while the turbine's operational limits are adhered to. PCM allows a turbine to be operational at every time period where sufficient steam is available.

PCM was followed by the *Power Time Concept Model* (PTCM), which optimises power generation for a single turbine under a two-time period start-up constraint. The third formulation was the *Power Steam Availability Model* (PSAM). PSAM optimises power generation for a single turbine, but once it trips, a restart is only allowed if the previous two time periods consisted of sufficient

steam to have kept it operational during that period. PSAM demonstrated some basic formulation requirements for a MILP to ensure that a turbine can only restart if sufficient steam is and was available during some time interval.

Following PSAM was the *Double Sequence Model* (DSM), which optimises power generation for two turbines operating under a fixed-sequence philosophy. Turbines are assigned as the first and second receiver, where power generation between the two are then optimised. Two time periods of sufficient steam are needed before a start-up.

The first main MILP contribution towards this study was the formulation of the *Power Optimal Queueing Model* (POQM), proposed in Chapter 4. The formulation of POQM solves for the optimal turbine loading hierarchy by optimising power generation. All possible combinations of turbine loading hierarchies are input parameters towards POQM, which include the fixed set points at which each turbine receiver should be loaded.

POQM was solved for two turbines where the fixed load setting varied from 80% to 100%. Higher optimal power generation resulted as the fixed set point increased. To address the uncertainty regarding optimal fixed set points, the second main MILP, the *Optimal Sequence Model* (OSM) was formulated. OSM treats all the turbine fixed set points as variables and solve the optimal values thereof. Furthermore, the formulation of OSM does not require any turbine hierarchies as input parameters. Therefore, OSM solves for the optimal turbine loading hierarchy, by optimising power generation to the turbines under the computed fixed load points.

A number of turbines with different operational limits were solved by using OSM. These results demonstrated that the intuitive turbine loading hierarchy choices are not always optimal, even when only two turbines are used. The results, furthermore, clearly showed the inability of a fixed-loading philosophy to utilise the potential turbine capacities, especially for three operational machines. The optimal loading hierarchy and power generation results did not give any indication as to what turbine choices will be optimal to invest in.

Chapter 5 presented the third main contribution, the *Power Optimal Investment Queueing Model* (POIQM), which yields optimal turbine investments under a fixed-sequence philosophy. To determine these investments, POIQM solves the optimal loading hierarchy and power generation problem. In order to incorporate turbine trips in the optimal decision making process, POIQM penalises the objective function for each occurrence. Optimal investments are the combination of turbine procurements, trip penalties and power generation income that yield the highest net present value (NPV).

The ability of POIQM to yield optimal turbine investments was demonstrated for a number of *Cases*. Results indicated that projected trip costs do influence the optimal investment choices. As trip cost penalties increased, results showed that it is better to procure a single turbine and rather let significant power generation potential go to waste. For no Case did POIQM suggest that more than two turbines should be procured. The results obtained from POIQM, furthermore, motivated the need for turbines to be dynamically controlled.

The three conceptional formulations provided in Chapter 6, demonstrated optimal power generation between two dynamically controlled turbines. The Dynamic Concept Model (DCM) was proposed and used for the formulation of the Dynamic Time Model (DTM). From DTM the Double Dynamic Model (DDM) was formulated, which optimises power generation between two dynamically controlled turbines. If a turbine trips, two time periods of sufficient steam must be present before it can go back online. Both the DSM from Chapter 3 and DDM solved for the same number of trips between the turbines, however, DDM yielded an 11% increase in power generation.

A main contribution followed, the *Double Dynamic Logic Control* (DDLCL) algorithm, which maximises power generation between any two turbines over time. Accurate steam flow predictions are allowed, but not required for the algorithm. The DDLCL algorithm computes how steam should be distributed amongst the turbines at each time period under real-life dynamic control. DDLCL was solved using the hypothetical data and yielded maximum power generation within 99.4% of optimality, as determined with DDM. Both the DDLCL and DDM solved for the same number of trips between the two turbines.

In order to determine optimal power generation between any number of dynamically controlled

turbines, the formulation of the Optimal Power Model (OPM) was proposed in Chapter 7. The working ability of OPM was demonstrated by solving it for the turbine combinations used for OSM. Increased power generation yielded for each scenario with predominantly less turbine trips. Results showed that optimally dynamically controlled turbines yielded an increased power generation of within 3.53% to 5.29% for two turbines, and 9.35% to 13.17% for three turbines, respectively, when compared to a fixed-sequence philosophy.

In order to address investments under dynamic control, the *Optimal Power Investment Model* (OPIM) formulation was proposed in Chapter 8. OPIM yields optimal turbine investments under dynamic control and penalises the objective function with expected future trip costs. The NPV is used as benchmark to determine optimal investments. A number of scenario results were reported and compared to that of POIQM.

Optimal OPIM NPV results for various trip penalisation yielded increased power generation within 7.3% and 19.3% when compared to that of POIQM. All OPIM results indicated that two turbines should be invested in, irrespective of the enforced trip penalisation. The results demonstrated the functionality of OPIM to yield optimal turbine investments.

The DDLC algorithm was solved for the real-world data in Chapters 8 and 9. Maximised power generation within 99.1% optimality when two accurate future steam predictions were allowed. Without allowing for future steam predictions, power generation results within 98.9% were obtained. Maximised NPV within 99.3% of optimality was obtained by DDLC when two future periods of accurate steam flow predictions were allowed. When DDLC was solved without knowledge of future steam flows, an NPV within 98.7% of optimality was determined.

The final contribution from this thesis is the proposed formulation of the Optimal Power Gas Investment Model (OPGIM) in Chapter 9. OPGIM determines optimal turbine investments, together with an investment into a supplementary energy resource, with the possibility of timely procurements. This decision will not be sensible under normal operational conditions, since the cost to generate power from steam produced by a supplementary energy resource will typically outweigh the direct electricity income received.

Solving OPGIM for a number of scenarios showed the positive effect of investing and procuring a supplementary energy resource. A cost to income ratio of 2.20:1.00 was chosen. OPGIM results indicated that such an investment should be made. This energy resource can be used to prevent turbine trips and, therefore, result in additional power generation to take place from steam that would otherwise have gone to waste. OPGIM results further demonstrated that when the trip penalty cost increased, more of the supplementary gas was utilised in order to prevent these occurrences. Even when the cost to income ratio was increased to 10.00:1.00, did OPGIM results yield an investment into natural gas.

Significant NPV increases were obtained by OPGIM, compared to OPIM and POIQM, especially as trip penalties increased. Depending on the trip penalty, OPGIM yielded NPV's between 10.6% and 118.0% higher than that of POIQM, with 41.9% to 100% less trips. Compared to OPIM, NPV increases of 3.0% and 82.7% were observed and trips reduced between 38.7% to 100%. To conclude, it was demonstrated that dynamic control can be used to increase power generation and that OPIM should be used to determine optimal investments. Furthermore, in a fluctuating steam flow environment, if the option exists to invest in a supplementary energy resource, although perceived as expensive, it should be exercised. Under these conditions OPGIM must be used to determine the optimal investment choices.

10.2 Future research

This thesis provided novel MILP formulations for optimal power generation and investments when energy recovery transpires in a fluctuating steam flow environment. A further novel contribution was a dynamic control algorithm, which maximises power generation between two dynamically controlled turbines. Some further research from this thesis is proposed:

- Off-gas flaring should typically be controlled at the Works, so that maximum volume flows can be utilised throughout. Even if all the off-gases are utilised at full capacities, the possibility

might exist that unnecessarily quantities are flared into the environment. Follow up MLIP formulations should accommodate off-gas volume flows in order to solve for the inclusion of optimal boiler house investments. This can result in additional power generation that would otherwise not be possible. An inclusion of boiler houses might result in sufficient quantities of additional steam that will be produced, and as a result, require additional turbine investments.

- Turbine investments for operations under dynamic control showed that two machines should always be invested in. If optimal investment results would indicate to procure additional boiler houses, further turbine investments might be necessary. For an energy recovery plant this might result into the procurement of at least a third turbine. A control algorithm that maximises power generation between three or more turbines is therefore required and should be formulated. This algorithm must determine how steam should be distributed amongst these turbines, when which machine should trip and if a restart may occur.
- A further expansion of the power generation process is required. All formulations for this thesis were for turbines operating on the same ring main (inlet pressure). In industry it is typically found that a number of ring mains is present and therefore steam at various pressures. Steam may be de-superheated from a higher to a lower pressure. This can be achieved by relaxing the steam to a lower pressure. As a result the temperature will typically be higher than the lower ring main and the flow will need to be mixed with additional water at that reduced pressure. This will yield steam with exact thermodynamic properties. The model formulation must solve for time periods where steam can be de-superheated to a lower ring main and either prevent turbines from tripping or provide those turbines with additional steam to increase power generation.
- From the above mentioned proposal, an optimal investment formulation is required where turbines that operate between the different steam rings are allowed, *i.e.* inlets and outlets at multiple ring mains should be possible.
- A further formulation should follow where off-gas profiles are used to determine optimal boiler house and turbine investments throughout the Works, taking into account all the ring mains, steam productions and plant usages. Turbines with an inlet or outlet at multiple ring mains should be incorporated in the model formulation. Boiler houses or plant processes must be allowed to invest in and procure any number of supplementary energy resources to be used when required. By taking all of these factors into account, it will allow a Works to investigate, furthermore, the possibility of a custom designed turbine that might optimise power generation and the NPV.
- For all future investment models it is proposed to take carbon (dioxide) taxes into account. Increased power generation will, therefore, not only reduce electricity costs, but potential carbon taxes as well. If additional resources are procured and utilised, the carbon taxes that will be incurred by the Works should, furthermore, be taken into account.

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