

## Chapter 7

# Discussion and conclusion

In the previous Chapter the constraints on the geometric parameters for the six pulsars, in addition to the associated best-fit profiles, were reported. The LC fits obtained in this study generally reproduce the required characteristics of the observed  $\gamma$ -ray and radio LCs, their shapes and the radio-to- $\gamma$  phase lag, for the six pulsars studied, but do so to varying degrees. As examples of this consider PSR J0659+1414 and PSR J1509–5850 (see Figure 6.1). For the former the predicted LCs of both the OG and TPC models fail to reproduce the radio-to- $\gamma$  phase lag, while for the latter, specifically for the TPC model, both the  $\gamma$ -ray and radio profile shapes, and radio-to- $\gamma$  phase lag, is reproduced strikingly well.

For PSR J0659+1414 a case can be made for the predicted OG LC as the better candidate by considering how well it would reproduce both the peak shape and the background emission level if the radio-to- $\gamma$  phase lag constraint were sufficiently relaxed. For PSR J0631+1036 and PSR J1718–3825 neither the OG nor TPC model is able to reproduce the roughly triangular shape of their  $\gamma$ -ray profiles. Looking at the TPC and OG atlases (see Section 4.1.3) it can be seen that profiles that would better reproduce this shape would either imply a markedly different radio-to- $\gamma$  phase lag (e.g., the profiles to the lower left on the TPC atlas) or imply no visible radio emission (e.g., the profile at  $[90^\circ, 20^\circ]$  on the OG atlas). This means that a refinement of these models may be necessary for a good fit to be possible. Possible refinements to the models include the introduction of different magnetic field geometries and the consideration of non-uniform emissivity profiles (see, e.g., Kalapotharakos et al., 2012). For PSR J1420–6048 and PSR J1718–3825 it is also evident that the radio model employed is not sufficiently robust to enable good fits as it can only produce roughly symmetric profiles, which don't fit profiles such as the ones observed for these pulsars. This shortcoming of the radio model, besides negatively affecting the quality of the radio-to- $\gamma$  phase lag reproduction, necessitates a stopgap approach to fitting the radio profile shapes where only the peak multiplicity is considered and the overall shape of the profile is ignored.

The values derived for  $|\beta|$  are generally quite small ( $\leq 20^\circ$ ). This is to be expected considering that radio emission is observed for all six pulsars. Furthermore,  $\beta < 0$  in most cases, so that the observer samples radiation coming from above the magnetic axis (i.e.,  $\zeta < \alpha$ ). This is easily understood in the context of the  $\gamma$ -ray phaseplots in Section 4.1.2 where it can be seen that the portion of the caustic which is generally responsible for producing the  $\gamma$ -ray profiles in the best fits lies at lower  $\zeta$  than does the PC. The difference in gap position between the OG and TPC models, with the gap lying farther inward toward the magnetic field axis in the OG case, results in

the OG-predicted contours generally lying farther away from the origin than the TPC contours in  $(\alpha, \zeta)$  space.

The qualitative knowledge of the geometric models gained in Chapter 4 serves very well when trying to evaluate observed as well as predicted LCs, as is done in the previous paragraph. The strength of this knowledge becomes especially apparent when looking at the process by which the contour boundaries were located in Chapter 5, the most relevant insight being that LC features seem to reside on roughly perpendicular lines on the  $\gamma$ -ray and radio atlases. The best example of this is the location of the OG and radio visibility boundaries. The OG boundary lies roughly on an “anti-diagonal”, while the radio boundaries lie on diagonals. This tendency of the radio and  $\gamma$ -ray models to give “mutually perpendicular” constraints make their simultaneous use very effective.

Another useful by-product of the by-eye approach employed in this study is the capability to associate each boundary generating the resultant solution contours with a specific LC feature. This not only gives insight into why the solution contours look the way they do, but also provides a good starting point when considering possible refinements to the models.

Lastly, the values obtained for  $f_\Omega$  for each pulsar, with their associated errors, correspond well to values implied by other studies (e.g., Watters et al., 2009), and are especially noteworthy considering that values for  $f_\Omega$  generally do not have errors accompanying them.

## 7.1 Comparison to Weltevrede et al. (2010)

As detailed in Section 1.3, Weltevrede et al. (2010) generated best-fit contours for five of the six pulsars being considered using the radio profiles and polarisation data available. Figure 7.1 shows the contours obtained by Weltevrede et al. (2010), with the solution contours acquired in this study superimposed as coloured rectangles. Note that Weltevrede et al. (2010) obtained solution contours in  $(\alpha, \beta)$ -space, while the solution contours found in this study were originally in  $(\alpha, \zeta)$ -space. It is important to realise that the rectangles shown in Figure 7.1 correspond to non-rectangular regions in  $(\alpha, \zeta)$  space which are smaller than the rectangular regions implied by the errors on the best-fit  $(\alpha, \zeta)$  values as listed in Table 6.1. This is due to the fact that the solution contour boundaries can be more naturally described by giving  $\beta$ , with errors, along with either  $\alpha$  or  $\zeta$ , than they can be described using only  $\alpha$  and  $\zeta$  values.

Furthermore, in some cases (e.g., PSR J1420–6048) the symmetry contained in the radio and  $\gamma$ -ray models needs to be taken into account correctly to compare the solutions found in this study with those of Weltevrede et al. (2010). Specifically, the fact that emission is assumed to be the same from both the southern and northern hemispheres of the pulsar, but shifted in phase by  $\pi$ , leads to the profile produced at  $(\alpha, \zeta)$  being identical to that produced at  $(\pi - \alpha, \pi - \zeta)$  under the transformation  $\phi \rightarrow \pi + \phi$ . The important feature of this symmetry is that the value of  $\beta$  changes sign under the relevant transformations ( $\beta = \zeta - \alpha \rightarrow (\pi - \zeta) - (\pi - \alpha) = -\beta$ ). Thus, to compare these solutions with those obtained by Weltevrede et al. (2010), the transformation  $(\alpha, \zeta) \rightarrow (\pi - \alpha, \pi - \zeta)$  must be applied for the cases where the sign of the predicted  $\beta$  in this study differs from that of Weltevrede et al. (2010). For instance, for PSR J1420–6048 the derived value for  $\beta$  (for the OG case) is negative ( $-22^\circ \pm 9^\circ$ , see Table 6.1), whereas the contours derived by Weltevrede et al. (2010) correspond to positive  $\beta$  values. Thus, to compare this contour to that of

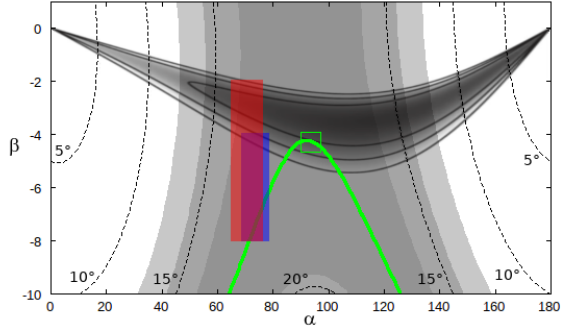
**Table 7.1:** Values for  $\alpha$ ,  $\zeta$ ,  $\beta$ , and  $f_\Omega$  derived for the six pulsars for both the OG and TPC models.

| Pulsar         | Weltevrede et al. (2010)       |                     | This study |                           |                           |                            |                                    |
|----------------|--------------------------------|---------------------|------------|---------------------------|---------------------------|----------------------------|------------------------------------|
|                | $(\alpha, \beta)$ ( $^\circ$ ) | $\rho$ ( $^\circ$ ) | Model      | $\alpha$ ( $^\circ$ )     | $\zeta$ ( $^\circ$ )      | $\beta$ ( $^\circ$ )       | $f_\Omega$                         |
| PSR J0631+1036 | (95, -4)                       | 18                  | OG         | $74 \pm 5$                | $67 \pm 4$                | $-6 \pm 2$                 | $0.93 \pm 0.06$                    |
|                |                                |                     | TPC        | $71 \pm 6$                | $66 \pm 7$                | $-5 \pm 3$                 | $1.04 \pm 0.04$                    |
| PSR J0659+1414 | (50, -8)<br>(143, -11)         | 22                  | OG         | $59 \pm 3$                | $48 \pm 3$                | $-12 \pm 5$                | $1.16 \pm 0.53$                    |
|                |                                |                     | TPC        | $50 \pm 4$                | $39 \pm 4$                | $-13 \pm 6$                | $1.64 \pm 0.04$                    |
|                |                                |                     |            | $142 \pm 1$               | $130 \pm 4$               | $-11 \pm 4$                | $1.63 \pm 0.05$                    |
| PSR J0742-2822 | (124, -6)                      | 13                  | OG         | $86 \pm 3$<br>$109 \pm 6$ | $71 \pm 5$<br>$94 \pm 4$  | $-16 \pm 6$<br>$-16 \pm 6$ | $0.99 \pm 0.10$<br>$0.81 \pm 0.09$ |
|                |                                |                     | TPC        | $116 \pm 8$               | $100 \pm 4$               | $-15 \pm 6$                | $0.88 \pm 0.41$                    |
| PSR J1420-6048 | (14, 3)                        | 11                  | OG         | $113 \pm 5$               | $135 \pm 7$               | $22 \pm 9$                 | $0.77 \pm 0.13$                    |
|                |                                |                     | TPC        | $116 \pm 6$               | $137 \pm 8$               | $21 \pm 9$                 | $0.90 \pm 0.10$                    |
|                |                                |                     |            | $42 \pm 5$                | $63 \pm 5$                | $21 \pm 9$                 | $0.77 \pm 0.06$                    |
| PSR J1509-5850 | -                              | -                   | OG         | $66 \pm 4$                | $50 \pm 7$                | $-18 \pm 8$                | $0.77 \pm 0.11$                    |
|                |                                |                     | TPC        | $61 \pm 5$                | $44 \pm 7$                | $-18 \pm 8$                | $0.89 \pm 0.10$                    |
| PSR J1718-3825 | (19, 4)<br>(157, 5)            | 13                  | OG         | $113 \pm 6$               | $132 \pm 6$               | $19 \pm 8$                 | $0.76 \pm 0.12$                    |
|                |                                |                     | TPC        | $119 \pm 5$<br>$42 \pm 6$ | $137 \pm 6$<br>$62 \pm 5$ | $19 \pm 8$<br>$19 \pm 7$   | $0.86 \pm 0.07$<br>$0.83 \pm 0.10$ |

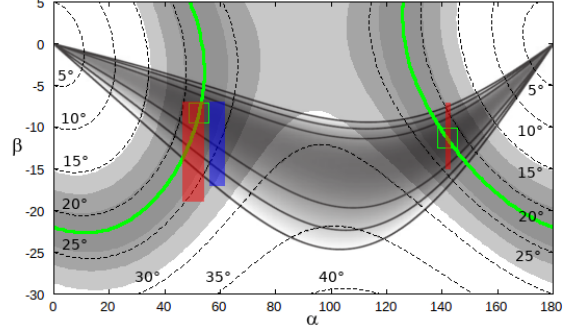
Weltevrede et al. (2010), the transformed  $(\alpha, \beta)$ -contour must be considered, so that the result for the OG fit for PSR J1420-6048 becomes  $\alpha = 113^\circ \pm 5^\circ$ ,  $\zeta = 135^\circ \pm 7^\circ$ , and  $\beta = 22^\circ \pm 9^\circ$ .

Lastly, note that the emission heights from which the  $\rho$  values for PSR J0742-2822 and PSR J1420-6048 were derived, were incorrectly reported by Weltevrede & Johnston (2008), from whom Weltevrede et al. (2010) cited the relevant emission heights (Weltevrede 2012, private communication). The values for  $\rho$  reported by Weltevrede & Johnston (2008) (in their Table 3), were too large by a factor of 2. This means that the values of  $\rho$  derived using these heights are a factor  $\sqrt{2}$  too large. The reported values for  $\rho$  indicated in Figure 7.1 (by the green  $x$ 's) have been adjusted accordingly.

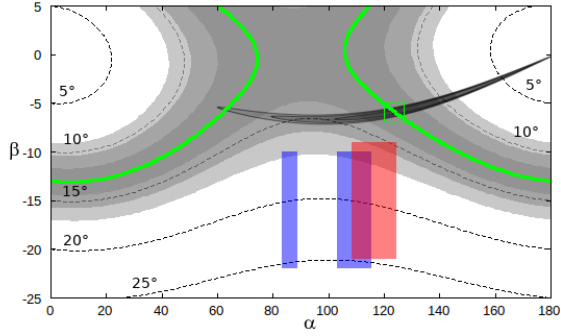
Generally, the best-fit  $(\alpha, \beta)$  from this study compare favourably with those inferred by Weltevrede et al. (2010). Comparison is hampered, however, by uncertainties in estimating the half-opening angle  $\rho$ , which influences the optimal solutions obtained by Weltevrede et al. (2010). In Figure 7.1 it can be seen that for PSR J0631+1036, PSR J0659+1414, and PSR J0742-2822 even a conservative error of 10% – 20% on  $\rho$  leads to relatively large errors on the allowed  $\alpha$ , large enough so that the solution contours from this study would then be included in their inferred parameter ranges. In the case of PSR J1420-6048 and PSR J1718-3825 there is less agreement between the results of this study and those of Weltevrede et al. (2010). It is interesting to note that for PSR J1420-6048 and PSR J1718-3825 different approaches for estimating the radio pulse width  $W$  were used in the two studies. In this study only the most prominent radio peak is modelled, while Weltevrede et al. (2010) measured  $W$  as the width of the total two-peak radio profile. In both cases this discrepancy amounts to a factor  $\sim 2$  difference in  $W$ . Figure 7.2 shows the effect on the contours for  $\rho$  of halving the  $W$  reported by Weltevrede et al. (2010) for PSR J1420-6048 and PSR J1718-3825. In both cases the introduction of the factor 2 change in  $W$  improves the agreement between the results from this study and those from Weltevrede et al. (2010), and in the case of PSR J1718-3825 the contours from this study almost overlap with those of Weltevrede et al.



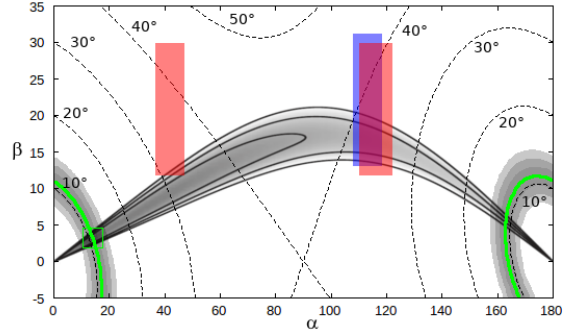
(a) PSR J0631+1036



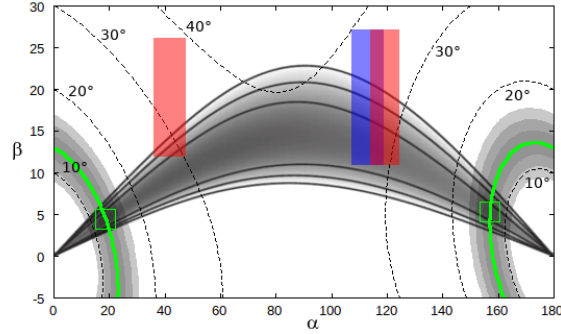
(b) PSR J0659+1414



(c) PSR J0742-2822

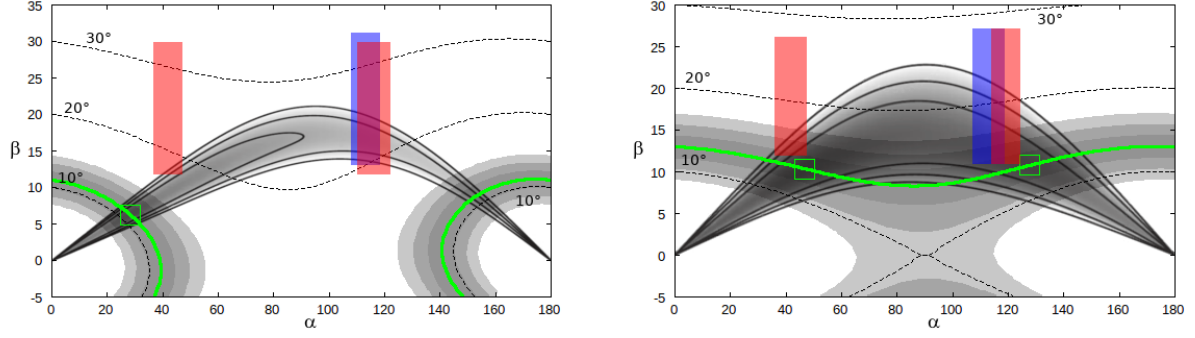


(d) PSR J1420-6048



(e) PSR J1718-3825

**Figure 7.1:** Independent model constraints on  $\alpha$  and  $\beta = \zeta - \alpha$  for each of the pulsars considered in this study. The filled contours with their tips at  $(\alpha, \beta) = (0^\circ, 0^\circ)$  and/or  $(180^\circ, 0^\circ)$  are the RVM  $\chi^2$  contours reproduced from Weltevrede et al. (2010). The dashed contours are the contours of  $\rho$ , and the green line indicates the contour corresponding to the value of  $\rho$  reported by Weltevrede et al. (2010), with the green square indicating where it most favourably intersects the RVM contours. The second set of filled contours, lying along the green contour, indicate 10%, 20%, and 30% errors on  $\rho$  respectively. The blue areas are for the OG model, and red for the TPC model and represent solution contours from this work.



**Figure 7.2:** Comparison with Weltevrede et al. (2010) for PSR J1420–6048 and PSR J1718–3825 after considering the different methods for measuring  $W$ .

(2010). This doesn't imply an error in their estimation of  $W$ , of course, merely that our solution contours are consistent with the value they derived for  $\rho$ . Comparison with Weltevrede et al. (2010) is not possible for PSR J1509–5850 as the lack of polarisation data rendered the inference of a best-fit  $(\alpha, \beta)$  by these authors impossible.

It is also interesting to note that in all five cases shown in Figure 7.1 the contours obtained in this study lie at slightly higher  $|\beta|$  than the RVM  $\chi^2$  contours derived by Weltevrede et al. (2010). This difference is most likely due to the difference in magnetic field assumed in the two studies. In this study a retarded dipole field is assumed while in Weltevrede et al. (2010) a static dipole field is used. The cause of this separation in  $\beta$  between the two sets of constraints can, to a certain extent, be understood by considering Figure 3.2. Assuming that the shape and size of the PC is correlated to the shape and size of the conal ring (from where the radio emission originates in the model used in this study) means that a larger PC at a given  $\alpha$  would lead to a larger ring on the phaseplots. This would then mean that to obtain comparable radio profiles from the two regimes (one where a static dipole is assumed and one where a retarded dipole is assumed) would require that the  $|\beta|$  of the retarded dipole profile be larger than that of the static dipole profile. The precise behaviour of the predicted  $\beta$  ranges when changing from one regime to another is beyond the scope of this study.

Looking at the plots in Figure 7.1 individually allows for a more detailed comparison of the results obtained in this study and those obtained by Weltevrede et al. (2010) for each pulsar. For **PSR J0631+1036** both the OG and TPC contours obtained in this study lie within  $3^\circ$  of the  $\rho$  derived by Weltevrede et al. (2010). The TPC solution contour overlaps the  $< 2\chi_{\min}^2$  region of the RVM contour, while the OG contour only overlaps the  $< 3\chi_{\min}^2$  and  $< 4\chi_{\min}^2$  regions. The value of  $\alpha$  is constrained very well by the methods used in this study when compared to the range of possible  $\alpha$  values implied by the RVM contours and 10%  $\rho$  error contour for Weltevrede et al. (2010): a range of  $\sim 10^\circ$  compared to a range of  $\sim 50^\circ$ . The solution contours from this study contain the  $\beta$  range found by Weltevrede et al. (2010).

For **PSR J0659+1414** both the OG and TPC contours obtained in this study overlap the  $< 2\chi_{\min}^2$  region of the RVM contour, and are within  $5^\circ$  of the  $\rho$  derived by Weltevrede et al. (2010). It is important to note that the promising shape of the OG best-fit profile in this case (when neglecting the radio-to- $\gamma$  phase lag) suggests that the actual radio emission height might be

lower than  $r_{\text{KG}}$  as this would imply a smaller aberration effect and thus a smaller radio-to- $\gamma$  phase lag. A similar but slightly less forceful argument can be made for the two TPC profiles, implying instead a higher emission height. The OG and TPC contours all lie approximately inside the 20%  $\rho$  error regions. Both the TPC contours contain the  $\rho$  value reported by Weltevrede et al. (2010). For the TPC contour at  $\alpha > 90^\circ$  it is important to note that the previously discussed trend of systematically larger  $|\beta|$  relative to the RVM contours found in this study adversely affects the agreement between this contour and the best-fit position identified by Weltevrede et al. (2010). For the OG contour, and the other TPC contour, however, a shift toward smaller  $|\beta|$  would again improve the agreement between the results of the two studies.

For **PSR J0742–2822** neither of the best-fit contours overlap the RVM contours reported by Weltevrede et al. (2010). They again appear at higher  $|\beta|$  values than those at which the RVM contours are encountered. The contours do, however, still lie within  $5^\circ - 10^\circ$  of the  $\rho$  derived from the PA-swing. The range of  $\alpha$  implied by the two best-fit contours at  $\alpha > 90^\circ$  corresponds well with that implied by the RVM contours and the width-derived  $\rho$  reported by Weltevrede et al. (2010).

For **PSR J1420–6048** there is very little agreement between the results from this study and those of Weltevrede et al. (2010), despite the TPC contour at  $\alpha < 90^\circ$  corresponding to one of the lowest predictions for  $\alpha$  in this study. The previously discussed difference in choice of radio pulse width accounts for a large portion of this discrepancy, but the different choice only improves the agreement somewhat. The  $\rho$  value reported by Weltevrede et al. (2010) also lies well beyond the best-fit contours produced in this study, at least  $15^\circ$  before considering the choice of  $W$ , and still at least  $5^\circ$  after it has been taken into account (See Figure 7.2).

For **PSR J1718–3825**, where a similar situation to that seen for PSR J1420–6048 is encountered, the change in choice of  $W$  has a much more beneficial effect, with the  $\alpha$  and  $\beta$  ranges implied by the RVM and width-determined  $\rho$  contours comparing very favourably with that implied by the TPC and OG models (See Figure 7.2).

## 7.2 Future projects

The fits obtained in this study may be improved upon by employing a more rigorous approach than the by-eye approach detailed in this text. Such an approach has thus far been hampered by the large difference in the amount of data available for the radio and  $\gamma$ -ray light curve observations respectively. Future work may take advantage of the ever-increasing amount of  $\gamma$ -ray data generated by *Fermi* LAT (e.g., Johnson et al., 2011).

Another possible extension of this work could be to identify other  $\gamma$ -ray LC characteristics, one possibility being the  $\gamma$ -ray peak separation, that would lead to tightly confined regions in  $(\alpha, \zeta)$  space when combined with the radio constraints. The characteristic by which the six pulsars in this study were chosen, their single-peak  $\gamma$ -ray profile structure, is only one example of an LC feature which appears (roughly) along “anti-diagonals” on the  $\gamma$ -ray atlases. As alluded to earlier, this approach may also yield significant results when applied to an entire population of pulsars. This prospect is rendered even more promising with the upcoming publication of the *Second Fermi LAT Pulsar Catalogue* (Abdo et al., 2013).

The publication of the *Second Fermi LAT Pulsar Catalogue* is also important in the light of the

LCs of PSR J1420–6048 and PSR J1509–5850, which were identified by Weltevrede et al. (2010) as possibly being two-peaked in nature. With the greater number of counts at its disposal the *Second Fermi LAT Pulsar Catalogue* will give more clarity as to the nature of these two LCs, but from the current work it seems to be a reasonable expectation that they will be found to be two-peaked. This classification is shown to be slightly inadequate, however, by the relatively complex composition of the LCs used to fit them in this study. This is most evident for PSR J1509–5850, where the predicted TPC profile consists of three distinct peaks, two of which are associated with the caustic (the two peaks at later phases), and the other with the “bump”.