

An analysis of students' knowledge of graphs in mathematics and kinematics

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- Dedication

I dedicate this study to the following people who are no longer in this world: my mother Kenaope Koikoi, my grandmother Keorapetse Phage, my brother Golekwamang, my uncle Kenalemogwe Phage, my son Letlotlo, ntate Solomon Dibe, bra Oupa Petlele and all those I have not mentioned here, who played a great role in my life. May their Soul Rest in Peace!

SUMMARY

Physics education research found that graphs in kinematics have been a problem to students, even at university level. The study hence investigates what deficiencies first-year Physics students at the Central University of Technology, Bloemfontein, South Africa have in terms of transferring mathematics knowledge and understanding when solving kinematics problems. According to the National Department of Education (DoE, 2003), mathematics enables learners to have creative and logical reasoning about problems in the physical and social worlds. Graphs in kinematics are one of the domains that need that skill in mathematics. DoE (2011) further emphasises that learners should be able to collect, analyze, organize and critically evaluate information at the end of their FET sector and that include graphing in kinematics.

The study started by exploring graph sense and comprehension from literature. The study further explored from a literature review students' problems and possible solutions in transferring their mathematics understanding and knowledge to solve physics problems.

The literature study served as conceptual framework for the empirical study, i.e. the design and interpretation of questionnaires, and interview questions. The mathematics and kinematics questions of the questionnaire were divided into four constructs, namely area, gradient, reading coordinates and form/expression of graphs. The participants undertook the questionnaire and interviews voluntarily according to the research ethics. Hundred and fifty two (152) out of 234 students registered for first-year physics from the faculties of humanities (natural science), health and environmental science and engineering and information technology undertook the questionnaire. The researcher interviewed 14 students of these participants as a follow up to the responses of the questionnaire.

The responses of the participants were analysed statistically to conclude this study. The average percentages of the questionnaire showed that the majority (62.7% participants) have the mathematics knowledge compared to the low percentage of 34.7 % on physics

knowledge. With regard to the constructs the participants generally performed similarly on gradient, reading coordinates and form/expression, i.e. they could either answer both the corresponding mathematics and physics questions and neither of them. In the area construct, most participants with the mathematics knowledge did not transfer it to the physics context. The study further revealed that the majority of interviewees do not have an understanding of the basic physics concepts such as average velocity and acceleration. The researcher therefore recommends that physical science teachers in the FET schools should also undergo constant training in data handling and graphs by subject specialists and academic professionals from Higher Education Institutions. Other remedial actions are also proposed in the dissertation.

Keywords: graphs in kinematics and mathematics, transferring mathematics understanding and knowledge, area, gradient, reading coordinates, form/expression, basic kinematics concepts.

OPSOMMING

Navorsing in fisika onderrig het bevind dat kinematika grafieke probleme skep vir studente, selfs op universiteitsvlak. Daarom ondersoek die studie tekortkomings by eerstejaar fisika studente van die Central University of Technology, Bloemfontein, Suid-Afrika met betrekking tot oordrag van wiskunde kennis en begrip in die oplos van kinematika probleme. Volgens die Nasionale Departement van Onderrig (DoE, 2003) bemagtig wiskunde leerders met kreatiewe en logiese beredenering van probleme in die fisiese en sosiale wêreld. Kinematika grafieke is een van die gebiede wat wiskundige vaardighede benodig. DoE (2011) beklemtoon ook dat leerders teen die einde van hul VOO fase die vermoë moet besit om inligting te versamel, analiseer, organiseer en krities te evalueer. Dit sluit kinematika grafieke in.

Die studie begin met literatuur oor bewustheid en verstaan van grafieke. Die literatuuroorsig vors ook studente se probleme en moontlike oplossings m.b.t. oordrag van hul wiskunde kennis en begrip in die oplos van fisika probleme na.

Die literatuurstudie dien as begripsraamwerk vir die empiriese studie, naamlik die samestelling en interpretasie van die vraelys en onderhoudsvrae. Die wiskunde en kinematika vrae in die vraelys is verdeel in vier konstrekte, naamlik oppervlak, gradient, aflees van koördinate en vorm/uitdrukking van grafieke. Die deelnemers het vrywillig aan die vraelys en onderhoude deelgeneem in ooreenstemming met navorsingsetiek. Honderd twee-en-vyftig (152) uit 234 geregistreerde eerste jaar fisika studente van die fakulteite menslike wetenskappe (spesialisasie natuurwetenskappe), gesondheids- en omgewingswetenskappe en ingenieurs- en inligtingstegnologie het die vraelys beantwoord. Die navorser het 14 van hierdie deelnemers ondervra om hul antwoorde van die vraelys op te volg.

Die deelnemers se antwoorde is statisties ontleed om gevolgtrekkings van die studie te maak. Die gemiddelde persentasies van die vraelys toon dat die meerderheid deelnemers (62.7%) die wiskunde kennis het teenoor die lae persentasie van 34.7 % met fisika kennis. Met betrekking tot die konstrekte het die deelnemers oor die

algemeen soortgelyk presteer in gradient, aflees van koördinate en vorm/uitdrukking, d.w.s hulle het of beide die ooreenstemmende wiskunde en fisika vrae korrek of foutief beantwoord. In die oppervlak konstrueer het die meeste deelnemers met die wiskunde kennis dit nie na die fisika konteks oorgedra nie. Die studie toon verder dat die meerderheid van die studente met wie onderhoude gevoer is, nie die basiese fisika begrippe soos gemiddelde snelheid en versnelling verstaan nie. Die navorser beveel dus aan dat VOO fisiese wetenskappe onderwysers voortdurend opleiding in datahantering en grafieke moet kry by vakspesialiste en akademiese professionele persone by hoër onderwysinstellings. Bykomende remediëringswyses word ook in die verhandelng voorgestel.

Sleutelwoorde: grafieke in kinematika en wiskunde, oordrag van wiskunde begrip en kennis, oppervlak, gradient, aflees van koördinate, vorm/uitdrukking, basiese kinematika begrippe.

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LIST OF ACRONYMS AND ABBREVIATION

CA	Cronbach Alpha Coefficient
CAPS	Curriculum and Assessment Policy Statements
CEO	Corporate Europe Observation
COP	Community of Practice
CUT, FS	Central University of Technology, Free State
DoE	Department of Education
FET	Further Education and Training
MBL	Micro-computer Based Laboratory
NCS	National Curriculum Statement
NQF	National Qualification Framework
NWU	North-West University
SD	Standard Deviation
TUG-K	Test of Understanding Graphs in Kinematics
ZPD	Zone of Proximal Development

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CHAPTER 1 - INTRODUCTION TO THE STUDY

1.1 Introduction

Many first-year university students in the science-related fields have a problem in identifying and applying the fundamental mathematical concepts that they have previously learnt in solving science problems (Freitas, Jiménez & Mellado, 2004). According to this research, the majority of students, mostly from public schools, do not seem to relate the two fields of study. Komati and Phage (2011) showed that the majority of students that participated in their study did not integrate knowledge from different scientific disciplines when solving some specific problems and doing experimental analyses.

Mathematics is an essential tool in studying physics, i.e., it will be difficult to study Physics without the sound basics of Mathematics (Pietrocola, 2008). Mathematics is even called the “language of physics” (Redish, 2005). Redish (2005) stated that Physicists blend conceptual physics with mathematical skills and use them to solve and interpret equations and graphs. For instance, in kinematics, different aspects from mathematics such as knowledge of functions and the solving of equations are combined with physics concepts.

This research study investigated first-year undergraduate Physics students’ existing knowledge on mathematics and kinematics graphs and how effectively they integrated their knowledge on graphs. In this chapter, the researcher defines and discusses the following:

- Motivation and statement of the problem
- Background of the Problem
- Aims and objectives of the study.
- Research Questions
- Hypotheses
- Research Design
- Data analysis

- Significance of the study.

The chapter will close with a short description of the chapters of the dissertation.

1.2 Background of the problem

From personal observation of the researcher, first-year university Physics students at the Central University of Technology (CUT) in the Free State, South Africa are unable to relate their mathematical understanding of graphs with graphs in Physics especially with regard to kinematics. This may result in most students being unable to understand and interpret kinematics graphs or present it clearly in their laboratory experiment reports.

1.3 Motivation and statement of the problem

Physicists use mathematical concepts, representations and techniques to describe physical concepts and situations (Redish, 2005). For instance, in kinematics the mathematical concept and representation of a function are used to describe the change in displacement, velocity or acceleration of a moving object with time. Physics requires a transfer and re-interpretation of mathematical concepts from the mathematical context to the context of the subject (Meredith & Marrongelle, 2008, Redish, 2005).

Physics education researchers found that the learning of physics is often hampered by a lack of understanding of the underlying mathematical concepts, and that this may block students' understanding of the physics and their ability to solve problems (De Mul, Battle & Rinzema, 2004). Hence students may arrive at university with a disjunct knowledge structure and have serious difficulties to use the interpretation of mathematical representations such as graphs in a physical situation. Students may emerge at university with serious gaps in their understanding of important topics like graphs (McDermott, Rosenquist & Van Zee, 1987).

Cumming, Laws, Redish and Cooney (2004) argue that students see physics as a set of disconnected mathematical equations that each apply only to a small number of specific situations. Students seem to fail to comprehend what knowledge they have to use to interpret graphs and what information they can depict from a graph (Shah & Hoeffner,

2002). Some students think that graphs are just mathematics without any direct links with physics (Kohl, 2001).

The National Curriculum Statement (NCS) for Grade 10 to 12 (Department of Education, 2003) that was followed by the participants of this study is adopting integration of knowledge and skills and applied competencies across subjects and terrains of practice as defined by the National Qualification Framework (NQF). The applied competence aims at integrating practical, foundational (learning of theory) and reflective competencies (NCS Grade 10–12 of DoE). These competencies are the key factors on which this research is based. Hence the plotting, use, analysis and interpretation of graphs should incite and initiate these skills and competencies among first-year undergraduate physics students.

Many first-year university physics students perform poorly on the use of mathematical skills and knowledge in their interpretations of graphs in physics. Two possible reasons may be that (Tuminaro & Redish, 2003):

- (1) Students lack the necessary conceptual and representational mathematical knowledge needed to solve the physics problems.
- (2) Students do not transfer their mathematical knowledge to the context of physics.

The problem investigated in this study is how these two factors impact on a group of first-year university Physics students' understanding of kinematics graphs. Implications for improved integration and application of mathematics knowledge and skills in the learning and representation of kinematics graphs followed from the results.

1.4 Aims and objectives of the study

The aim of the research is to investigate the conceptual knowledge and understanding of first-year physics students at the CUT with regard to graphical representations and their interpretation, i.e. what deductions can they make from a given graph in mathematics and kinematics.

The objectives of the research are to:

- Analyse the students' knowledge of graphs in a mathematical and a kinematics context with the aid of questionnaires.
- Compare their competencies in the two different contexts using descriptive statistics.
- Probe the understanding of selected students during interviews
- Come up with recommendations to improve the teaching and learning of kinematics graphs.

1.5 Research questions

The research questions investigated in the empirical study were:

- What deficiencies do the group of first-year physics students at CUT have with regard to knowledge of graphs in mathematics and kinematics?
- How effectively do they transfer knowledge from mathematics to kinematics
- What possible reasons can be given for the problems that the participants experience with kinematics graphs?

1.6 Hypotheses

The majority of the first-year Physics students who participated in the study has the necessary mathematical knowledge on graphs, but cannot effectively apply it in kinematics.

1.7 Research design

In the empirical study the researcher used a mixed method approach consisting of a quantitative part that utilizes questionnaires and a qualitative part in which interviews were conducted. The study was sequential and the qualitative part followed the quantitative in order to enhance the researcher's understanding of the students' responses to the questionnaire.

1.7.1 Sample

All first-year physics students enrolled in 2011 at the CUT were requested to participate in the study. These physics students were enrolled in the faculties of Education, Engineering and Health and Environmental Sciences. The questionnaire was completed by 152 students where after 14 students were selected for interviews.

1.7.2 Data collection

Students were given two questionnaires that assessed corresponding aspects of linear functions and graphs (Mathematics) and kinematics graphs and equations (Physics). The Physics questionnaire contained relevant questions from Beichner's standardized questionnaire on kinematic graphs (Beichner, 1994) and was used as a basis. From this questionnaire, an equivalent questionnaire was devised in the context of Mathematical graphs.

After students completed the questionnaires, interviews were conducted with selected students to probe their misunderstanding shown in their questionnaire answers.

1.7.2.1 Aspects investigated

The following four aspects of graphs were assessed in the mathematics and kinematics parts of the questionnaire.

- Integration / Area under graph
- Differentiation / Gradient
- Reading data from graphs
- Form of graphs/ Expressions

1.7.2.2 Funding

Funding of statistical processing of the quantitative data was paid by the NRF funds of the supervisor.

1.8 Data analysis

The data of the two questionnaires were analysed statistically to determine coherence in the students' answers and the transfer that occurred between mathematics and physics. The interviews were analysed for patterns and trends in the data.

1.9 Significance of the study

The study investigated possible deficiencies that first-year physics students at CUT have in order to use knowledge of linear functions and graphs in mathematics to solve, analyse and interpret kinematic functions and graphs in Physics. The results of this research are expected to have far-reaching implications relating to first-year physics students' learning of kinematics graphs. These findings can serve as a basis for a need to improve the teaching of physical sciences and mathematics in schools, especially in the teaching and learning of science through graphic representations.

1.10 Description of chapters

This section outlines and gives a description of the titles of the various chapters of the dissertation. It gives a clear and proper reflection on the appearance of the dissertation.

Chapter 1. Overview and problem statement:

This chapter addressed aspects such as the motivation and research questions of the study and shortly described the research design.

Chapters 2 and 3 Literature study and theoretical framework on the use of graphs in physics:

A literature study relating to the topic is fully outlined and discussed with regard to the use of graphs as visual representation of relations between variables.

Problems that students may encounter in the interpretation of line graphs as well as aspects relating to knowledge of both mathematics and physics regarding graphs are discussed.

Chapter 4: Research design:

In this chapter a discussion of the mixed-method approach which involves both quantitative and qualitative data collection and analysis are done.

Chapter 5: Results and discussion of results:

A representation of the students' performance in the questionnaires and focus group interviews are given and presented compositely and graphically. The statistical analysis of the quantitative results as well as the patterns and trends shown in the focus group discussions will be discussed.

Chapter 6: Conclusions and recommendations:

From the obtained results and the analysis, conclusions are drawn on which to base the recommendations to overcome the gap between these two subjects and to effectively integrate mathematics and physics knowledge in kinematics graphs.

CHAPTER 2 - GRAPH COMPREHENSION AND GRAPH SENSE

2.1 Introduction and overview

This chapter presents a review on graph comprehension and graph sense. It provides a theoretical background for the empirical study reported in this dissertation. At the outset the researcher engaged in gathering information related to the research problem. The body of information gathered provided the researcher with additional insights as related to the analysis of first-year undergraduate students' interpretation of graphs in mathematics and kinematics. In this chapter various aspects regarding graph comprehension (section 2.2) and graph sense (section 2.3) are discussed. In section 2.4, the research looked and identified different strategies on how graph comprehension and graph sense are taught.

2.2 Graph comprehension

Analysis of quantitative data depends mainly on the graphical representation as a visual display of that quantitative data (Shaughnessy, Garfield & Greer, 1996). Fry (1984) defines a graph as information displayed or transmitted by the position of a point, a line or area on a two dimensional surface or three-dimensional volume.

The framework of the graph consisting of axes, grids, scales and reference markings gives information about the kinds of measurements used and data measured. Maps, plans and geometrical drawings use spatial characteristics (shape or distance) to represent spatial relations whereas graphs use spatial characteristics (height and length) to represent quantity (Gillian & Lewis, 1994).

Graphs are characterised by visual dimensions called specifiers that represent data values and a background in the form of colouring, grids and pictures (Friel, Curcio & Bright, 2001). Specifiers can therefore be the lines on a line graph, the bars on a bar graph, or other marks. They give particular relations among the data presented. The structure and visual display of the tables link with the structure of the graph.

2.2.1 What graph comprehension is

Graph comprehension is the ability to derive meaning from a created graph. It is when graph users like students can read and interpret a graph created by themselves or others and it includes structural construction, invention and choice. Graph comprehension is made up of three different levels that involve three kinds of behaviours in the context of literacy, namely the elementary level, intermediate level and the advanced level (Friel et al. 2001).

2.2.1.1 Elementary level

The elementary level involves translation of the graph, e.g., describing the contents of a table in words or interpreting the graph at a descriptive level and commenting on specific structures of the graph. In other words, the elementary level entails reading the graph.

2.2.1.2 Intermediate level

The intermediate level involves the interpretation of the graph by arranging materials and sorting the important factors from the less important factors. Students have to look for a relationship among the specifiers in a graph or between a specifier and a labelled axis. Therefore, the intermediate entails reading between the data and finding the relation between the variables.

2.2.1.3 Advanced level

The advanced level involves the extrapolation and interpolation of information from a created graph by stating the essence of communication in order to identify some of the consequences, i.e., read beyond the graph or data by analysing the relationship between the variables

There are also aspects of processing information in the graph and this is done by locating, integrating and generating that information. Locating information in the form of translation is done by finding information based on specific conditions or features.

Integrating information in the form of interpretation will be by pulling together two or more pieces of information while generating new information by means of extrapolation. Interpolation is a way of processing information in a document and makes document-based inferences or draw personal background knowledge, i.e., make own conclusions or interpretations based on previous knowledge (Friel et al. 2001).

2.2.2 Requirements of graph comprehension

For students to comprehend graphs, they have to ask the right or relevant questions. Questioning is a fundamental component of cognition and central part of text comprehension. There are two types of questioning. Low level questioning addresses the content and interpretation of explicit materials while the deep level questioning involves inferences, application, synthesis and evaluation of information (Friel et al. 2001).

Comprehending text involves asking questions that identify gaps, contradictions, incongruities, anomalies and ambiguities in knowledge and text itself. A framework needs to be developed for consideration of graph comprehension within which students have to think about questions to be asked, to them or by them (Friel et al. 2001).

Graph comprehension is or can be hampered by students' difficulty with 'read the data' (elementary level) and they make errors with 'read between data' (intermediate level) due to a lack of mathematics knowledge, reading or language errors, scale errors, or reading the axes errors (Friel et al. 2001).

The inferences graph users need to make in graph comprehension on the advanced level are to compare and contrast the data set, to make a prediction about the unknown, to generalise a population and to identify a trend (Friel et al. 2001).

2.2.3 Factors influencing graph comprehension

According to Friel et al. (2001) four critical factors were found and reported to assist in and influence the comprehension and instructional implication of a graph, namely, the purpose for using graphs, its task characteristics, discipline characteristics and reader

characteristics. These factors were identified through and after a thorough synthesis of information about the nature and structure of graphs were provided.

2.2.3.1 Purpose of graphs

Friel et al. (2001) distinguished between two purposes for graphs, namely comprehension as analysis and communication. A graph is meant to measure whether there is any change in experimental data.

- **Analysis**

Analysis as purpose refers to what you can do with the graph. A graph is a discovery tool that makes sense of data as well as detects important and unusual features. An alternative plot can be used to contribute to graph comprehension. Graph instruction is used as an analysis tool of data that promotes graph comprehension by flexible, fluid and generalizable understanding of graphs. It detects unusual or important features in the data, i.e. notice the unexpected (Friel et al. 2001).

A graph is a meaningful picture that gives powerful visual pattern recognition to see trends and subtle differences in shape (Beichner, 1994). The shape of a graph represents a specific meaning to a relationship between variables and also has a specific bearing to its meaning. The analysis and interpretation of a graph is largely dependent on its shape. Graphs are used by scientists in their visual pattern recognition facilities to see trends and spot subtle differences in shape as stated by Mokros and Tinker (1987). With data obtained in a lab experiment, a lot of information can therefore be deduced from its shape, its recognized patterns and trends as well as the subtle differences in those complex variables.

“Graphs are used as a powerful statistical tool to facilitate pattern recognition in complex data” (Eshach, 2010, Van Tonder, 2010 quoting Chambers, Cleveland, Kleiner, & Tukey, 1983). Graphs are hence used to summarize large amounts of information at the same time resolving them into details. Graphing is a skill to be used by both experts and

novices (laboratory students) to solve problems. It is a tool for data analysis and interpretation.

- **Communication**

Graphs are also used as visual displays communicating collected data, i.e., information about numbers and their relationships in order to answer questions of interest. It is used as a summary of statistics rather than the original data in form and content. Information is structured and communicated by an external source. A graph is a picture conveying information about numbers and relations among numbers. It gives a summary of data, is simple in form and content, and display patterns (Friel et al. 2001).

It is imperative to remember the old adage, "a picture is worth a thousand words" when considering to use a graph or chart as a summary in a report of an experimental investigation. This means that a graph is a handy tool for a researcher to convey critical key points easily and quickly. The point here is that your graph should serve as the picture that should save a thousand words. That is, the graph or chart should be seen to supplement the text and it should not be explained ad nauseum in the text. Therefore, the text related to the graph should be relevant and supported rather than being detracted from it (Lavinsky, 2010). Researchers and scientists have often used tables and graphs to report findings and observations of their research. These tools are also often used to support an argument or a fact in newspapers, magazine articles and on television (Kuswanto, 2012).

Graphs have been used not only in science, engineering and mathematics fields but in all other fields of study like in business, social studies, etc. In business field, sales persons and stockbrokers use graphs to gain and win trust of their clients as well as to predict and market their products and or companies (Joyce, Neill, Watson & Fisher, 2008). In this manner these people use graphs to complement the text so that the audience can be able to quickly and easily digest the information, and as always it should interest the audience in taking the next step (e.g., scheduling an in-person meeting) in the investment process.

2.2.3.2 Task characteristics of graphs

Graph perception is where visual perception of the graph is used to analyse the graphs (Legge et al. 1989, p. 365). Data can be obtained from a given graph or it can be used to plot a required graph. It is therefore imperative that we predict what information to obtain from a graph. From the data, obtained information such as the shape of the graph can be predicted. From the shape of the given graph can be deduced how variables are expected to change or what conclusions can be made from the graph. The visual perception of a given or expected graph will enhance curiosity on the research. Tasks done on the graph to enhance visual perception include visual decoding, judgement and context.

- **Visual decoding**

Visual decoding is getting information from a graph by just looking at the graph. It works on first impression, and early and mental representation in the head. It is the syntax of (rules for) graph perception, decoding what have been encoded. Here the choice of graph plays an important role. It also helps to identify physical dimensional and visual processing of materials (Friel et al. 2001).

Graphs as visual representations, should be used to organise information and to show patterns and relationships. This information is represented by the shape of the graph (Joyce et al. 2008).

Students will have to know about tables and graphs, as they are useful tools for helping people to make informed decisions. If not enough information is provided, researchers have to infer from the data obtained and displayed to have a full and complete understanding. In the same way, students should be able to clearly identify what information graphs and/or tables give. It is imperative that they are also able to identify what information is missing to have a complete understanding. With all these information available, the reader will be able to identify and decide what information they need and what information they can discard if evidence is not enough to support the argument. From this perspective, students can be helped to know how to analyse and critique the given data or how to present it (Friel et al. 2001).

A graph is a clear application and presentation of advanced theoretical and experimental results in all spheres of life. As mentioned, the primary purpose of the graph is to give a clear picture from data obtained of many things, which might not be clear from a given table of data (The University of Reading, 2000) and could answer questions like (Joyce et, 2008):

- **How does change in one variable lead to a change in the other variable?**

Is there any fluctuation trend in the variables and is this in accordance with some physical law? Is there any correlation between plotted quantities? Was a wrong variable used or the wrong experiment conducted?

- **Do we have sufficient data?**

Is the information or data conducted enough to give a conclusion? Is the range of the graph wide and legible enough to give a clear picture?

- **Is there a region of interest that suggests further analysis?**

Is the shape of the graph as expected or not visible enough? Do we need to take more data or perform an experiment to clear the uncertainties? Does this give enough reason to understand and answer the aim of the experiment?

These questions reflect that a graph as a meaningful picture aids in visual pattern recognition to see trends and differences in shape (Few, 2006). As a qualitative description, it helps to establish if there is a mathematical relationship between variables. A functional relationship can be identified with a mathematical relation being established by theory (Connery, 2007).

• **Judgement**

Judgement acknowledges the importance of operations in the use of syntactic properties of graphs, i.e. doing things with graphs like calculation comparisons. Calculations results in deduction inferences about different aspects of a graph, i.e., the non-obvious properties of the graph (Friel et al. 2001).

Judgement operates on representation and compares different aspects of a graph like reading data points, performing computations and identifying trends. In this way, it helps getting information by doing calculations on the graph (Friel et al. 2001). A graph measures if there is any change in experimental data or results as observed in a laboratory or any continuous occurring situation like a business practice (Joyce et al. 2008).

In general, graphs are used to find the relationship between two variables. Graph users look at how these variables relate and what other variable(s) can be obtained from them, i.e. graph users analyse and interpret a graph to formulate or prove a given or known mathematical or physics equation(s) or function(s). In this way scientists and mathematicians are able to identify and substantiate the relation between the two variables to give meaning to the third or other variables. The meaning of such variables will depend on the shape of a graph obtained. Many factors can be observed and obtained from the shape of the graph (Joyce et al. 2008).

From an analysis of the obtained shape of graph, an interpretation can be made whether the graphical relationship between the variables is linear, parabolic, cyclic, exponential, logarithmic, etc. The shapes of graphs therefore have different meanings to the interpretation of the relationship between the variables (Joyce et al. 2008). A graph of a physical event gives a glimpse of trends, which cannot easily be recognized in a table of the same data (Beichner, 1994). From characteristics of the type of graph interpretations can be made in terms of gradient, slanting or curving upward or downward and so on.

If an equation fits the data, it can be used to predict the behaviour of an experiment by extrapolation or interpolation. Interpolation is relatively straightforward with extrapolation being risky as the regression result is valid only within the range of the data considered. If a mathematical relation is known, then the predicted y-value may be completely wrong. Regression analysis only offers mathematical analysis and it is up to the graph user to offer physical meaning (Deacon, 1999).

- **Context**

Context deals with the semantic (understanding) of graphs set in a real world situation, for example, interpreting graphs in kinematics, and getting the meaning of representation, e.g., velocity. This means the context is integrated with the representation. Context free graphs with unlabelled specifiers means that the units of measures cannot be determined. Information on the graph cannot be interpreted as data (Friel et al. 2001).

As stated explicitly by Bing and Redish (2007, p.26) “An important sign of physics students’ progress is their combining of the symbols and structures of mathematics with their physical knowledge and intuition, enhancing both. The numbers, variables, and equations of the mathematics come to represent physical ideas and relations.” Graphs have been found to foster strong links between experimental data and subject-related explanations or theories (Dori & Sasson, 2008:242). Data obtained from an experiment goes through several rounds of transformation using complex and indirect reasoning processes to make valid inferences before they can be interpreted (Charney, et al. 2007).

A qualitative description, in terms of physical meaning of the variables and shape of graphs, gives meaning and helps to establish if there is a mathematical relationship between investigated variables (Dori & Sasson, 2008). A functional relationship can be identified with a mathematical relation being established by theory. This underlying theory or results of a curve fitting procedure can be used to extract some meaningful physics from the data. For example, the change in position of a free falling object under gravity is described by a polynomial function whose coefficients represent gravitational acceleration, initial velocity and initial position. The qualitative understanding of velocity ($\Delta s/\Delta t$) and acceleration ($\Delta v/\Delta t$) as ratios is the thrust of the study to which individuals apply to the interpretation of motion of real objects (Trowbridge & McDermott, 1980 & 1981) and hence are able to justify it by means of a graph. The skills of interpretation of graphs are important as students should be able to apply the meaning of intercepts and slope in the nature of linear relationships of quantities used as variables in a graph (You, 2009).

The Physics graph and the Mathematics equation are phenomena that can help to predict the behaviour of physical systems as conditions change (Deacon, 1999). A well-drawn graph will bridge these two phenomena and skills are required by laboratory students to plot and interpret a meaningful graph. It is possible with somewhat erratic data to find a function that fits the original values very well. A function must have physical interpretation and meaning irrespective of the number produced by the computer. Mathematical analysis must be simple and should not make the problem complicated (Deacon, 1999).

The theory behind most school and undergraduate physics laboratory experiments is usually simple enough to be explained and described by a graph (especially straight line graphs). Unfortunately, students tend to focus on what is plotted rather than on the gradient of the graph which should give the required results (Giri, 2005).

Laboratory researchers and scientists therefore use graphs as a way to validate data and make quantitative measurements in order to find a relationship between variables. The slope and intercepts of a graph give meaning to its physical interpretation (Columbia University, 2011).

2.2.3.3 Discipline characteristics of graphs

Discipline characteristics deals with the characteristics of the discipline in which the graph is used. It deals with the influences in the comprehension of the graphs, e.g., mathematic functions or kinematic graphs. Statistics involves the systematic study of data, by collecting it, describe and present it and draw up conclusions from it (Moore, 1991).

A graph is a collection of vertices and a collection of edges that connect a pair of vertices. There are four types of graphs in Physics used for experimental results. They are the comparison line graph, the compound graph, Cartesian plane graph and the scatter graph. In introductory Physics, we mainly use Cartesian plane graphs to give and interpret results. These graphs are represented using the coordinate system constructed by means of two perpendicular lines, the horizontal line known as x-axis and vertical known as the y-axis (Shalatov, 2008). Where the two lines intercept, it is

known as the origin with coordinates (0; 0). Increasing positive coordinate values are represented to the right and upward from the origin, while to the left and downward the negative coordinate values are represented. An investigation done by Mudaly (2011) showed that many teachers had difficulty with drawing a graph on a Cartesian plane and as a result they confuse gradient with height. Common mistakes may be due to incorrect labelling and disregard of units of axes.

- **Spread and variation of data**

The spread and variation of the data determines the structure of information (Friel et al. 2001). Representations of graphs as visual quantitative displays also depend on the data analysis (Shaughnessy et al. 1996). It is therefore imperative and important that teachers understand how to read, analyse and interpret data, including the following aspects:

- **Reduction**

Reduction is the transposition of raw data from tabular and/or graphical representations to be presented as grouped data or other aggregate summary representations such as graphs.

- **Scaling**

Scaling is a means or type of data reduction, i.e., it is a reduction of data to meaningful summaries (Ehrenberg, 1975). Scale can be in the form of frequency or percentage. It is used to reduce the data presented. Often student graph users can read a scale, but do not know how to choose the best scale for the data (Rangecroft, 1994).

Scaling includes the use of appropriate scale and scale units (Fry, 1984) and the reading or drawing of a scale as well as the choice of the scale for a given data set, (Rangecroft, 1994). These are important attributes to get or interpret information. The inability to use scaling in line graphs can lead to the inability to interpret asymmetric scales and to make good use of space for graphing (Dunham & Osborne, 1991).

Leinhardt et al. (1990) pointed out that the shape of a graph changes when the scale changes. This affects the mental image that a graph user makes of a graph. If a graph user does not attend to the effect a change of scale has on the shape of the graph, it limits and inhibits the user's graph comprehension.

- **Type and size of data**

The type and size of data influences the choice or type of graph (Landwehr & Watkins, 1986), e.g., kinematics line graphs.

- **Graph complexity**

Graph complexity deals with the aspects of different types of graphs that learners find difficult to understand. Graph complexity concerns that in a graph that makes it difficult to read and interpret, and the way in which the graph provides structure to data. The type of graph is determined by the structure of the data, e.g., a line graph gives a relationship between variables. According to Bell et al. (1987), line graphs are more difficult for learners to comprehend than other types of graphs because it is a big step for students to realise that a line on a Cartesian graph represents a relationship between two variables. This makes it more difficult for learners to understand (Bell et al. 1987).

2.2.3.5 Reader abilities

This is a very important aspect according to most researchers (Carpenter & Shah, 1998, Meyer et al. 1997, Peterson & Shramm, 1954) that influences graph comprehension.

The reader abilities or characteristics of concern are:

- **Cognitive ability – the different levels of understanding**

Cognitive abilities include logical thinking, proportional reasoning, graph construction and abstract reasoning abilities (Berg & Phillips, 1994). Therefore a logical progression from simple to complex graphing needs to be established (Wavering, 1989).

- **Graph experience**

Graph experience is a result of application or build-up from prior knowledge of graphs by learners. Experience or knowledge of graphs plays a large role in the individual differences of comprehending and variation processes of determining graph properties (Carpenter & Shah, 1998). The skill required with graph experience is the ability to do abstract reasoning of line graphs. (Dillashaw & Okey, 1980; Padilla et al. 1986).

- **Practical applications of graphs**

When graph users create a graph for a practical application, it is found that they gain more graph comprehension than when just creating a graph for the sake of graphing. Therefore studying graphing in practice from a cognitive perspective by learners is important (Roth & McGinn, 1997) since less opportunities in graph practice has shown less competence in their interpretation.

The review by Joyce et al. (2008) has also reported a study on whether the undergraduate students are able to link their experimental results in the form of data or a table. In doing so, they should be able to present or represent the data graphically and hence they will also be able to record and relate the sequence of events they followed when conducting the experiment in any Science or Physics laboratory. The research was used in collaboration with physics learning theories, using undergraduate students' understanding to strengthen such theories. Students' perceptions have been found to be limited on how they can use graphs to analyse the experimental results and to present and interpret it. The laboratory results were used as guiding/reference tool to conclude the investigation (Bramble, 2007).

- **Context familiarity**

Familiarity with the context of the graph improves the comprehension thereof. Therefore graph interpretation practices goes beyond well-developed domains of knowledge/experience to more complex notions than originally imagined (Roth, 1998). According to Mokros and Tinker (1987), students using appropriate microcomputer-based laboratory (MBL) investigations will learn to communicate graphs. They will spend less time gathering information but instead they will spend more time to interpret and evaluate data and be able to improve the experiment. They will therefore have acquired the critical thinking, problem solving and self-monitoring skills in this manner.

- **General intelligence**

Vernon (1946) (as cited by Friel et al. 2001) general intelligence might influence graph comprehension. Up to now, there is no evidence that general intelligence plays a role in graph comprehension (Friel et al. 2001). In order for researchers to measure the general level of intelligence of a learner, they must find a way of understanding the manner of interpretation of information by learners.

- **Mathematics knowledge and number knowledge**

When reading graphs for quantitative purposes, various arithmetic operations (Gillian & Lewis, 1994) such as counting, measuring, classifying, number concept, relationship and fundamental operation are used.

The knowledge and background of mathematics on functions and graphs can be used as a guiding factor in acquiring data, drawing and labelling of axes up to scale, plotting of data, drawing of a graph from plotted data (Shah & Carpenter, 1995 and Carpenter & Shah, 1998) and finally the interpretation of the graph whether it is a straight line, parabolic, hyperbolic, circular, etc.

The fundamental laws of physics are described using mathematics as the language of instruction. In the same way as it is not easy to teach physics to students using English as the language of instruction while they can hardly comprehend English (Gollub et al.

2002), it is difficult for physics students to be able to “speak” mathematics. On the other hand, physics students who can use the language of mathematics will be able to manipulate algebraic equations and understand their meaning in the physics context. For example, students should know that linear relationships of variables in an equation will result in straight-line graphs and that if the graph of variables is curved it means that the relationship between these variables cannot be of linear nature. The understanding and development of mathematical thinking (Schoenfeld, 1992) will invoke in the students their conscience as to how well they can present and/or interpret their experimental or laboratory report as well as using graphs to come up or prove a mathematical and/or scientific equation/formula or any other known theories and postulates.

2.3 Graph sense

2.3.1 What is graph sense?

Graph sense is a way of thinking about or working with graphs and being able to characterise the nature of comprehending a graph (Friel et al. 2001). Such comprehension is a required development within a schooling system or situation. For any given graph, students or learners have to be able to read and make sense out of it. Students will develop graph sense if they are able to plot graphs or use already known graphs to solve problems that require getting information out of data.

2.3.2 Difference between graph comprehension and graph sense

Graph comprehension involves reading and interpreting graphs created by oneself or by others. It also involves the considerations and making sense for constructing a graph, specifically as a tool for structuring data and determining the optimal choice of a given graph (Meyer et al. 1997). It includes the task characteristics namely visual decoding, judgement and context. Graph sense is developed while busy with graphs, either by creating a new one or working on an already designed graph.

2.4 How graph comprehension and graph sense are taught

According to Friel et al. (2001) the following can be done to teach graph comprehension and graph sense.

- Teach learners to use a table as a display type and as an organising tool in presenting data.
- Know the learners' mathematical knowledge level, and the development thereof.
- Use data that is fit for the learners. Start with simple data going to more complex data, i.e., apply instruction by progression thereby considering their existing mathematics knowledge and the complexity of the data explored.
- Introduce technology for drawing graphs.
- Attend to the three task characteristics, namely visual decoding, judgement and context.
- Give guidelines for designing graphs.
- Teach learners to be inquisitive and to ask the right questions.
- Let students collect their own data to ensure familiarity with context.

2.5 Summary of chapter

A graph is a summarized and conclusive result of a scientific investigation in any experiment or research whether scientific or not (Deacon, 1999). Shrake et al. (2006) when quoting AHD (2000) concluded that graphs can be used in observations, descriptions, identifications, experimental investigations and theoretical explanations of natural phenomena and to communicate results. A graph in essence therefore can summarize all these aspects in one.

Therefore, it is agreed that the understanding of graphs from mathematics and a physics perspective is an all important and necessary skill and tool undergraduate students must have. Hence, presentation and interpretation of graphs by undergraduates in their studies should play a significant role in achieving a higher level of academic and scholarly excellence and prepare a public awareness and

understanding of science and science processes, thus in a way prepare and produce future scientists (physicists) and or scientific researchers.

CHAPTER 3 - THEORETICAL FRAMEWORK

3.1 Introduction

Several scientists have done research on how students learn graphs, both from a mathematical and scientific point of view. Earlier research (e.g. Basson, 2002; Beichner, 1994) reports on the problems that undergraduate students tend to have with the comprehension, reading, analysis and interpretation of graphs in physics. One of the greatest observations by experienced physics instructors is that students have serious gaps in their understanding of various topics, including graphs (McDermott & Redish, 1999). This chapter inspects such problems and the possible solutions as reported in the available literature. The focus of the discussion is on the teaching and learning of graphs. It starts-off with a discussion of social constructivism as a learning theory in section 3.2. Section 3.3 deals with students' difficulties with kinematics graphs, while possible teaching strategies to address such learning difficulties are discussed in section 3.4.

3.2 Social constructivism as learning theory

The theory of constructivism is based on the idea that all constructed knowledge is built on previous knowledge and experiences. Meaningful learning involves the active creation and modification of knowledge structures, instead of learners' passive absorption of information (Carey, 1985). Constructivism can therefore be regarded as a process where learners use their existing knowledge, beliefs, interests and goals to interpret new information, which in turn may result in their ideas becoming modified or revised. In this way learning proceeds as each individual's conceptual schemes are progressively reconstructed as he or she becomes exposed to new experiences and ideas (Driver *et al.* 1994).

Roth (1993) defines two tenets of constructivism:

- ✓ Knowledge is constructed by students instead of being transmitted from the educator to the student.
- ✓ Learning is an adaptive process.

Social constructivism can furthermore be defined as the recognition of the importance that a learner plays an active role in his or her learning instead of a receptive role, as in a situation where the lesson is teacher-centred. This means that the learning should be the full responsibility of the learner (Von Glasersfeld, 1989). As an instructor, it is better to use facilitation strategies in the classroom than to do the actual teaching (Bauersfeld, 1995). Wertsch (1997) states that social constructivism acknowledges the uniqueness and complexity of the learner by considering his or her background and culture. These attributes become an integral part of the learning process, the learner shapes the knowledge and truth he or she has acquired during the learning process. Learners are exposed to and taught about different worlds and will therefore be able to reproduce learning content and give structure to their approach of doing or seeing things in their daily lives, i.e., a problem solving discovery by learners (Jonassen, 1991). Vygotsky (1935) defines social constructivism as the effects one's environment (family, friends, culture and background) has on learning. Knowledge is shaped by cultural influences and evolves through participation in communities of practice (COP) (Lave & Wenger, 1991; Vygotsky, 1935).

Von Glasersfeld (1989) acknowledges that with motivation the confidence of a student's learning potential is strengthened. Prawat & Floden (1994) agree that with motivation and external acknowledgement, students feel competent and they believe that they have the potential to solve new problems. This confidence emanates from first-hand experience with the mastery of problems in the past. This links up with Vygotsky's "zone of proximal development" (ZPD) (Vygotsky, 1935), a process that differentiates between what a student can do without help and what he or she can do with help.

Today's learning requires that students apply or relate what they have learned to what is happening in their daily lives, environment, communities, etc. It is therefore important for them to understand what is happening in their society in terms of culture and context and they must have a constructed knowledge based on this understanding, as noted by

Kim (2012), Derry (1999) and McMahon (1997). Kim (2012) further points out that students and educators should be involved in social activities for meaningful learning.

Unfortunately students often find themselves in an educator-centred environment where the educator just relates all the information. The students just listen to the educator and take notes from the educator without any active participation or involvement. Instead, teaching should be student-centred so that students can ask questions and are engaged by the educator in their learning (Roth, 1993).

Traditional ways of teaching and learning has been found to be based on the educator's view of the subject and his or her perception of the students (McDermott, 1993). McDermott (1993) argues that the disadvantage is that students are not actively involved in the abstraction and generalisation process, which entails inductive thinking and reasoning. She laments that students lack qualitative reasoning, which is the skill that ultimately enables learners to apply concepts.

3.3 Students' difficulties with kinematics graphs

3.3.1 Introduction

Friel *et al.* (2001), as discussed in chapter 2, identify four critical factors that seem to influence the comprehension of graphs and its instructional implications. These four factors are the purpose of using graphs, task characteristics, discipline characteristics and reader characteristics. From using these four critical factors, as well as making sense of quantitative information in graphs, Friel *et al.* (2001) conclude that issues and ways of instruction can be altered in order to promote graph sense. This section focuses on student difficulties with kinematics graphs.

Data analysis and interpretation in the form of kinematics graphs rely greatly on graphical representations (Shaughnessy *et al.* 1996). The use of visual displays of quantitative data and results of experimental investigations are pervasive in our highly technological society (Friel *et al.* 2001). Students are expected to make predictions from data and to know what type of graph they will need to present the information they have. Unfortunately many students are not able to make such predictions, although this is a

skill that undergraduate physics students should have from their previous mathematics knowledge and background on linear functions and graphs.

Several researchers have investigated and reported on the problems that undergraduate students may encounter with the interpretation and analysis of graphs in physics (e.g., McDermott & Redish, 1999; Basson, 2002; Beichner, 1994). One of the discoveries by the above-mentioned research is that students' difficulties with position, velocity and acceleration versus time graphs can be due to their (Beichner, 1994):

- misinterpretation of graphs as pictures;
- confusion about slope and/or height;
- inability to find the slope if a graph is not passing through the origin; and
- inability to interpret the area under different types of graphs.

Beichner (1994) elaborates on this by saying that some students are unable to do calculations of the slope or use inappropriate axis values when calculating the area under the graph.

The variables in kinematics, like displacement and time, or velocity and time, can also be very difficult to interpret (Palmquist, 2001) and hence difficult to present. Data obtained from a laboratory experiment goes through several rounds of transformation that entail complex and indirect reasoning processes to make valid inferences before they can be interpreted (Charney *et al.* 2007).

McDermott *et al.* (1987) describe the above-mentioned and continue to identify more difficulties that students may experience with kinematics graphs. The following aspects were investigated by McDermott *et al.* (1987):

- Discriminating between the slope and the height of a graph;
- Interpreting changes in height and slope;
- Relating one type of graph to another;
- Matching narrative information with relevant features of a graph;
- Interpreting the area under a graph;
- Connecting graphs to real world situations;

- Connecting mathematics and science;
- Difficulties with different representations.

These problems are subsequently discussed in short below.

3.3.2 Discriminating between the slope and the height of a graph

Students have to identify which features of a graph are characteristic of a particular physical concept for correct interpretation of a physics graph (McDermott *et al.* 1987). Such features include the coordinates of a point, differences between the coordinates of two points and the gradient of a given straight line. For example, students often do not know whether they must use the height or the slope of a graph in order to answer questions about the graph. Beichner (1994) called this problem slope-height confusion.

In comparing two straight line graphs of displacement versus time, students are unable to pick up that the gradient of the lines represents velocity and the steeper the line, the greater the velocity (McDermott *et al.* 1987). Students make the mistake of interpreting the height as the velocity and the difference between the height at different points as the slope or gradient, which they then use to determine the greater speed between these lines. Students further fail to realise that since the gradients are not the same, the velocities are not the same either, even at the point of intersection, which in turn is an indication that the objects are at the same position.

Trowbridge and McDermott (1980; 1981) investigated the qualitative understanding of velocity ($\Delta s/\Delta t$) and acceleration ($\Delta v/\Delta t$) as ratios. In their study individuals had to apply these concepts to the interpretation of motion of real objects and had to be able to justify it by means of a graph. They report that undergraduate students confuse position with velocity and velocity with acceleration, even after instruction. Students also had difficulty with acceleration in experiments with two masses, on internal and external forces, and in simple harmonic motions (McDermott *et al.* 1994).

Beichner (1994) confirms what other studies found concerning students' difficulty with position, velocity and acceleration versus time graphs. Among other things, students confuse the slope and height of a graph and they have difficulty in determining the

gradient of a line passing through the origin. He suggests that this is due to students reading off values from the vertical axis and assigning them directly to the slope. The confusion of students and their inability to properly interpret change in height and change in slope emanate from the confusion they have about instantaneous velocity and average velocity, as well as their failure to take corresponding times into cognisance. This is an indication of insufficient qualitative understanding of the concepts of velocity and acceleration.

Based on their mathematics knowledge undergraduate students should be able to distinguish and discriminate between the slope and the height of a graph and what these mean from a physics perspective. This will only be attainable if teachers develop their own skill in using and interpreting formal representations, such as graphs, diagrams and equations (McDermott *et al.* 2000). With this, teachers will be able to make the formalism of physics meaningful to students by utilising their prior mathematics knowledge.

3.3.3 Interpreting changes in height and slope

First-year students encounter difficulties with the interpretation of changes in height and changes in slope (McDermott *et al.* 1987). Changes in height and slope are helpful when determining velocity and acceleration from displacement versus time and velocity versus time graphs respectively. Trowbridge and McDermott (1981) mention that prior knowledge on the dependence of velocity or acceleration on the slope of the graph gives a clue to making the correct comparisons. However, students were unable to make the distinction between the concepts of velocity and change in velocity. Trowbridge and McDermott (1981) further state that the problem is that students do not associate velocity and change in velocity with the relationship between the numerator and denominator of a fraction.

Students have difficulty in interpreting curved graphs when changes in slope and changes in height are involved, as compared to straight line graphs (McDermott *et al.* 1987). Some common features of the curved graphs that students are having difficulty with include the points on the curve where the velocity is lowest or highest, and whether

the object is decelerating or turning around. They are unable to notice that on a displacement versus time graph, velocity is the lowest at the smallest slope. The greater the slope, the higher the velocity is and *vice versa*. Students tend to think that if in a curved graph the height increases, the slope increases as well, and they assume that when the height is zero the slope is also zero. The other assumption students make is that a negative slope on a displacement versus time graph means that an object is slowing down. Instead of looking at the magnitude and sign of the slope, they use changes in height.

3.3.4 Relating one type of graph to another

There are different kinds of algebraic graphs that can be obtained from kinematics and students are unable to relate them. They cannot draw from a position versus time graph, a velocity versus time graph or vice versa (McDermott *et al.* 1987). They do not realise that the slope of a displacement versus time graph is the height of a velocity versus time graph, nor do they know that the increase in the slope means an increase in the respective height. Students often ignore the shape of the graph when plotting one graph from the other. In the research of McDermott *et al.* (1987), students were unable to manipulate information from a given graph for a new graph.

3.3.5 Matching narrative information with relevant features of a graph

Matching narrative information with relevant features of a graph is another difficulty students encounter (McDermott *et al.* 1987). Students are unable to recognise that the acceleration of a given motion of an object is denoted by the gradient of a given velocity versus time graph. If they have to calculate the slope of a graph, they are unable to match that slope with the correct type of motion (acceleration, retardation or constant velocity), especially if different motion graphs are given.

McDermott *et al.* (1987) further state that students experience difficulty to notice that if the slope of a velocity-time graph is zero, it means the acceleration is zero. Students are also unable to recognise that to calculate the acceleration of a given velocity versus time graph using the slope, they must calculate the ratio of change in velocity (dv)

versus change in time (dt). Instead they find the ratio of the coordinates (v and t). They fail to notice the ratio v/t is different from the ratio dv/dt .

A meaningful interpretation of numerical results requires a sound qualitative understanding of the underlying physics. In studies involving students, the value of quantitative results also depends on a student's understanding of qualitative issues, which are usually less well understood in the case of physical systems. To be able to determine the depth of students' knowledge and the nature of their difficulties, it is necessary to probe the reasoning that lies behind their answers (McDermott & Redish, 1999). McDermott and Redish (1999) conclude that incorrect interpretation might also be due to the analysis of numerical data only.

Beichner (1994) investigated and described the problems that students encounter with the interpretation of kinematics graphs using the development and analysis of the model, 'Test of Understanding Graphs in Kinematics (TUG-K)'. He reports that such problems emanate from the tendency of physics undergraduate lecturers to use graphs as some sort of second language, expecting undergraduate students to extract most of the rich information from it (Beichner, 1994).

Some problems that require simple recall also pose difficulties for students. However, memory alone is not enough to solve problems that need detailed interpretation of a given graph (McDermott *et al.* 1987). For instance, graphical skills and conceptual knowledge are necessary to understand features like what a particular slope represents.

3.3.6 Interpreting the area under a graph

Another common difficulty students have is to determine or interpret the area under a given graph (McDermott *et al.* 1987). Beichner (1994) also found that students are unable to interpret the area under various types of graphs. McDermott *et al.* (1987) further state that students are unable to associate the area under a velocity versus time graph with the position of an object at a particular time. Students cannot figure out an oscillation motion when a velocity-time graph is changing from positive to negative displacements, i.e. when it is above and below the $v = 0$ axis. They cannot see that

equal areas below and above the $v=0$ axis (negative and positive areas) mean that the object has returned to its original position.

In addition to the fact that students cannot visualise a velocity versus time graph, they make mistakes in extracting information about displacement from that graph. This may be due to the problem that students cannot associate a physics quantity with the square units of an area in mathematics (McDermott *et al.* 1987). If the area under the velocity versus time graph is represented by several blocks of squares, they may be able to calculate the area of one block, but they do not know what to do with other blocks. If they do, they do not know which blocks not to count. They also forget that positive area is displacement in a positive direction and negative area is displacement in the opposite direction (McDermott *et al.* 1987).

3.3.7 Connecting graphs to real world situations

McDermott *et al.* (1987) identified and reported that students have difficulty relating kinematic concepts and their graphical representations to the motions of real objects. An example of where students struggle to connect real motion with its formal representation in a graph is the association of Newton's first equation of motion: $v = at + u$ with the algebraic equation for a straight line graph: $y = mx + c$. Students have difficulty associating the variables in an algebraic equation with those in a kinematic equation: In mathematics, "m" represents slope, while in kinematics "a" represents the slope, since it is a physical variable known as acceleration. "x" and "y" are variables where the value of "y" will depend on the value of "x", where "m" and "c" are constant values. In kinematics the value of "v", the final velocity, will depend on the value "t", the time taken while "a", acceleration, and "u", initial velocity, are constants. Positive "a" indicates forward acceleration, while negative "a" indicates opposite or backward acceleration.

Clement (1997) defines the so-called graph-as-picture error where students would associate the point where two velocity-time graphs intercept as the position where two cars would be passing each other. They tend to map a local visual feature of the problem and fail to interpret a graph as the relationship between variables. Beichner

(1994) states that the difficulty is that, students consider a graph to be more of a photograph of a situation, rather than an abstract representation of mathematics as it should be. They regard it as an exact duplicate of a motion event.

Connery (2007) supports Beichner's argument by identifying that students cannot develop clear connections between independent and dependent variables. He further states that they are unable to collect and analyse a great amount of quantitative data, let alone to make sense of it.

3.3.8 Connecting mathematics and science

Students struggle with the scientific and mathematical concepts underlying laboratory investigations (Connery, 2007). Connery ascribes these problems to both ineffective pre-lab preparation of students and failure to explicitly link science and mathematics by the physics teacher. He suggested that to remedy this, the physics teacher can ask students, before collecting and plotting a graph of their actual data, to predict what they think their data would look like and draw a rough sketch of a graph of their prediction.

First-year university students in physics show an inability to use linear relations and graphs from mathematics to solve kinematic equations and graphs. Reasons may include complexities/ difficulties in comprehending graphs, students' inadequacies with skills and knowledge related to mathematics (Basson, 2002) in general, and the application of these mathematics skills and knowledge in solving physics problems. Molefe (2006) states that this is due to a lack of algebraic knowledge and skills in physics among teachers. As a result they tend to treat mathematics and physics as separate entities that are not related.

Khan *et al.* (2011) report that physics students have difficulty recognising that mathematics knowledge can be used to solve problems by calculating physical quantities from other non-constant quantities, whether symbolic or graphical. Undergraduate students are unable to recognise that, like differentiation, integration in kinematics equations can yield the required results in interpreting, analysing and solving physics graphs. They are also unable to notice that integration is the reverse of differentiation and *vice versa*.

In their review of the construction and interpretation of graphs, Leinhardt *et al.* (1990) conclude that these aspects are important as they are included in the curriculum of early mathematics, like geometry. They even suggest that it is important to consider a subject matter orientated approach in the review of teaching and learning, including introducing graphing and functions in the upper elementary school system. They also state that there is a discontinuity with the use of graphs, graphing and functions in physical science as compared to its use in mathematics. The review by Leinhardt *et al.* (1990) was prompted by “the symbolic connections that represent potentials for increased understanding between graphical and algebraic worlds”.

The actual causes of the serious and apparent deficiencies in the mathematics capabilities of students are not clear (Scott, 2012). Scott (2012) relates that when a test was done using calculators in mathematics and chemistry, the significant errors were from poor understanding of basic operations. On increasing the level of difficulty, students were able to perform better in mathematics questions than in chemistry questions. The reason for this was argued to be poor understanding of underlying mathematical concepts, resulting in an inability among students to solve problems in other subjects. Kinematics graphs are one of the aspects in physics that students need to be able to apply the underlying mathematical concepts. However, students are often not able to relate kinematics equations and their corresponding graphs to the mathematical linear equations and graphs (Basson, 2002).

3.3.9 Difficulties with different representations

Studies have been conducted with respect to the interpretation, analysis and construction of functions and graphs in terms of their representations. Kohl (2001) argues that representations are highly valued in physics, but that research studies have shown that undergraduate university physics students can struggle with the representations typically required in solving physics problems. These interpretation, analysis and construction of functions are either represented in algebraic, tabular or graphical form. These are mathematical concepts that can also be used in physics.

Incorrect representation of information from graphs and tables can be due to (Joyce et al. 2008):

- Important information being left out.
- Construction of the graph in such a way that it misrepresents relationships.

These misrepresentations may be intentional to create an argument or may be due to poor skills in plotting or representing a graph or a table.

- Data that is misrepresented in the construction of the graph or table.
- Poor reading skills of graphs and tables.

As a result of the above factors, students often regard tables and graphs as an end in themselves (Joyce et al. 2008). Few students see them as a source of evidence, or as a way of exploring patterns and relationships in data or information.

3.4. Constructivist teaching strategies to address learners' problems with graphs

3.4.1 Learner-centred instruction

Roth (1993) argues that physics teaching and learning should move away from educator-centred learning, which may contribute to problems in the integration of mathematics and science. Students should construct knowledge themselves, instead of knowledge being transmitted from educator to student. He further states that learning should be regarded as an adaptive process. Kuo *et al.* (2004) says that this means that the beliefs and values of instructors about the teaching and learning of problem solving in physics from a curriculum point of view should be investigated. It means that if this approach to teaching and learning is part of the curriculum, it will enforce students to construct their own knowledge.

In physics teaching and learning, the teacher should focus on what students already know, either from experiences in society or from their previous classroom learning. Students can then build on existing knowledge and incorporate the new information. They should therefore be encouraged to develop cognitive skills of learning through experience, that is, to make sense of meaningful reception learning through experience and integration of concepts (Kim, 2012).

The construction of knowledge should mean that physics educators should value meta-cognition in students' problem solving approaches. The educators should not concentrate on how they themselves actually solve problems in physics or how they teach the concept, but should infer from how students talk when describing the problem-solving process (Fernandez *et al.* 1994).

Angell (2004) suggests that learning ideas in the form of misconceptions that occur in physics problems solving have to be explored (developed and defined). He states that quantitative and qualitative approaches might be used together to probe student thinking and reasoning. Angell (2004) further studied some TIMSS items (both multiple choice and free response items) from the physics Specialists Test. In his study, he was able to show that students have the potential to understand physics and that it helped to explore and clarify students' thinking and the nature of the misconceptions they have.

Friel *et al.* (2001) investigated brought together perspectives concerning the processing and use of statistical graphs to assist teachers and students to identify critical factors that appear to influence graph comprehension and to suggest instructional implications, i.e., these will help teachers interpret students' thinking and to explicitly define the nature of difficulties students have in reading graphs. As discussed in chapter 2, they identified these perspectives as the purpose of using graphs, task characteristics, discipline characteristics and reader characteristics.

Without motivation, students do not see a reason to learn and to be inquisitive so that they can know more and perform well. Kinematics graphs as a concept in physics seems like an abstract subject to students. Great motivation will therefore be necessary to change students' perspective or approach to learning kinematics graphs. Maimane (2006) argue that motivation plays a role as students must be actively involved in their learning and it will help students to clarify and develop learning concepts and skills.

In physics education, recent research centred on the effects of various types of learner-centred teaching interventions that could help students' conceptual understanding (Jimoyiannis & Komis, 2001). Several applications in physics teaching have been found to support a powerful modelling environment for students' conceptual understanding and analysis of physics. Computer simulations as one of the alternative instructional

tools have been found to help students deal with cognitive constraints and functional development in conceptual understanding of physics (Jimoyiannis & Komis, 2001). Another strategy is inquiry learning, which is discussed in the following paragraph.

3.4.2 Inquiry learning

McDermott (1996) reports that a laboratory-based module that emphasises the development of conceptual and scientific reasoning skills through inquiry, enhances physics learning. It helps students when they do collaborative work as they conduct investigations and to make their own observations, which ultimately help them to construct scientific models. Prospective and practicing physics and physical science teachers at pre-tertiary level can be prepared well using these instructional and scientific materials and models. McDermott *et al.* (1998) also state that worksheets and tutorials should guide students, who work collaboratively in small groups, to acquire the required reasoning for the development and application of important physics concepts and principles.

Learning in groups, e.g. peer learning, has been found to facilitate constructed knowledge and overcome deficiencies in the interpretation of kinematics graphs (Basson, 2002). In addition, Larkin *et al.* (1980) investigated the role of physical intuition in problem solving. They concluded that expert science teachers often “use highly structured patterns of information to index and apply their knowledge”.

3.4.3 Technology-aided learning

Graphs can be used to prompt the reconsideration of the roles of technology in mathematics teaching and learning (Edwards *et al.* 2008). Some software products are specifically designed to make graphs easy to create and understand (Lymer, 2003). Grayson and McDermott (1996) describe the use of the computer as an instructional aid and as a research tool to examine student reasoning. Most experimental observations and data can be read into a computer to give a visual picture of the final results.

Computational modelling activities have also been used to investigate undergraduate students' performance and interpretation of kinematics graphs and to improve their

physics learning (Araujo *et al.* 2008). During the research, only conventional teaching methods were used on the control group, while the experimental group was subjected to complementary computational modelling activities. The results proved that the performance of students in the experimental group showed statistically significant improvement as compared to students in the control group. Students' perceptions, which resulted from their experimental activities, were found to have played a fundamental role in the conceptual understanding of mathematical relations to physics, as well as their motivation to learn from them (Araujo *et al.* 2008).

One of the solutions to the problems related to the interpretation of graphs is the use of sonic range finders and graphing calculators (Palmquist, 2001). It can help students to acquire a richer and better understanding of motion graphs when they act out motion. Sonic range finders and graphing calculators are said to be easy to use to analyse motion, since students are able to visualise the motion and can consequently discuss the graphs that represent the motion.

3.4.4 Subject-related aspects

Graphing, and in particular the interpretation of graphs in physics, can be classified as part of learning fundamental science concepts. Hubber *et al.* (2010) state that the role of language and the importance of personal and contextual aspects of understanding science as emphasised by cognitive science, is a challenge to the purely conceptual view of learning. They argue that the recognition and development of students' representational resources and skills are the tools required for learning. They further report on the pedagogical and epistemological challenges encountered by educators while utilising a representational focus and how such a focus supports students' learning effectively. This means that educators' beliefs about the complex and ever-changing nature of knowledge could be associated with more learner-centred pedagogical practices, as well as the use of technology in the classrooms. Educators' beliefs about learning are also responsible for shaping the pedagogical practices in the classroom, not only their epistemological beliefs about knowledge and knowing.

As stated before, kinematics graphs are one of the aspects in physics to which students need to be able to apply underlying mathematical concepts. Scott (2012) suggests that there is a need for better communication between teachers in mathematics and science departments, with a less algorithmic approach to the teaching of mathematics as key to the solution to the problem.

Beichner (1994) further argues that since students cannot use graphs fluently as expected from their mathematics knowledge, physics teachers and undergraduate lecturers should acknowledge first that students have to understand graphs before using them as a language of instruction. He recommended that educators should teach students to use graphs so that students can understand and know the meaning of kinematic variables.

3.5 Summary

The chapter investigated and reported on problems and solutions to students' difficulties with the interpretation and analysis of kinematics graphs (e.g. Basson, 2002, Beichner, 1994). In addition, the chapter discussed problems concerning gaps and deficiencies in students' knowledge of kinematic graphs and in graphs in general, as well as constructivist teaching strategies to address these problems.

All the ideas discussed in this chapter are applicable to the teaching and learning of laboratory experiments to undergraduate physics students. The discussion on the empirical research in the next chapter examines how teaching and learning, as well as the application of mathematics knowledge can be used best by physics students of CUT to present and interpret kinematics graphs from their laboratory experiments. The empirical study focused specifically on the perceptions and understanding of graphs by first-year undergraduate physics students at CUT. The investigation probed how these students integrate their mathematical knowledge and skills.

CHAPTER 4 - RESEARCH DESIGN

4.1 Introduction

The research aim of the empirical study reported in this dissertation was to probe first-year CUT students' conceptual knowledge of graphs in kinematics and mathematics. This chapter offers a discussion of the mixed-method approach, which involves both quantitative and qualitative aspects of data collection and analysis.

The quantitative part of the research was based on a questionnaire (Appendices C and D) composed of two sections related to graphs. Section A (Appendix C) was on linear functions and graphs (mathematics) and section B (Appendix D) was on kinematics graphs and equations (physics). All willing 2012 first-year physics students at CUT, Bloemfontein campus were given this questionnaire to respond to it. The 152 participants' responses to the questionnaire were supported by interviews with a focus group of participants to investigate what prompted their responses, i.e. investigate their reasoning in terms of their choices for corresponding questions in the questionnaire.

The literature study of the previous two chapters was used as a basis for conducting this research design and analysis with the emphasis on the qualitative and quantitative approach of the investigation. With regard to kinematics graphing, Beichner (1994) compiled a questionnaire to determine the extent to which undergraduate students are able to interpret and analyse kinematics graphs and suggested possible teaching methods of this topic. Beichner uncovered common misconceptions of students. His questionnaire served as the evaluation instrument in this empirical study. The misconceptions of the CUT students who took part in the study were compared with the findings of Beichner and other researchers, as summarised in Chapter 3.

The research focussed on two of the four critical factors in the comprehension of graphs as identified by Friel *et al.* (2001), namely task characteristics and discipline characteristics. These characteristics were discussed in Chapter 2. Part of the aim was to test whether the CUT undergraduate physics students are competent with regard to

these critical factors. The critical factors should be attended to when students perform kinematics experiments and plot resulting experimental graphs in their reports.

The following sections of this chapter present the empirical research conducted in this study. The discussion involves the population that was targeted for this study, the instruments used to collect data and how it was analysed.

The aim of the empirical study was to identify the mathematical knowledge and skills that first-year undergraduate physics students at CUT must have to be able to interpret and analyse kinematics graphs.

4.2 Empirical study

4.2.1 Target population

According to Best and Khan (2003), a proportion of the selected population known as the sample can be used for observations and analysis. Such a sampling should be a fair and accurate representation of the target population (Charles & Mertler, 2008; Takona, 2002).

In this study, the researcher used as target population, first-year undergraduate physics students enrolled at CUT, Bloemfontein campus in 2012. Physics at CUT is compulsory in the first-year for all students in three schools, i.e. the School of Health and Environmental Sciences, the School of Engineering and Information Technology, and the School of Teacher Education (Natural Science).

The sample was selected based on convenience. According to Anderson (1998) this means that the population is easy, quick and convenient to access. The researcher found it convenient to select first-year physics students at CUT as a convenient population, rather than to visit all the universities in South Africa. The decision was influenced by time, financial and logistical constraints. Gall *et al.* (1996) maintain that the largest possible sample should be used for quantitative research, while at the same time bearing in mind financial and time constraints. A smaller sample group should not be seen as of less value, according to Lawson (1997).

Completion of the questionnaire was confidential and not compulsory and the researcher explained this to them. As a result, not all students in the target population completed the questionnaire. Out of 234 students enrolled for first-year physics in 2012, 152 students completed the questionnaire. The study population or available sample therefore consisted of 152 CUT first-year students. Although participants' biographical background was included as part of the research, the questionnaire was not discriminatory in any respect. The study population who participated in the research completed a consent form and the results were used for research purposes only.

4.2.2 Methodology

Beichner's standardised questionnaire on kinematic graphs (Beichner, 1994) was used as a basis for the physics questions in this study. An equivalent questionnaire was devised to address linear equations (functions) and graphs in mathematics. The data of the physics and mathematics parts of the questionnaire were analysed statistically to determine the deficiencies as well as the coherence and integration of the participants' knowledge.

After the participants completed the questionnaire, a focus group of 14 participants from the sample were interviewed. The purpose of the interviews was to probe their misconceptions and reasons for the deficiencies that became apparent from their answers and to determine to which extent they applied their mathematics knowledge in kinematics when answering the questionnaire. The interviews were transcribed and analysed for patterns and trends in the data.

Data was therefore collected using a mixed method approach that involves quantitative and qualitative instruments to answer the research questions (Tashakkori & Teddlie, 2003). Tashakkori and Teddlie (2003) state that a mixed method offers wider and broader views and inferences regarding the research questions. It is imperative that there must be interaction between processes of data collection, interpretation and the analysis of the same data (Gay & Airasian 2000).

Quantitative and qualitative methods complement each other to give results that are balanced and more reliable. Quantitative methods are objective since it gives results in

terms of numbers, while qualitative methods entail more subjective interpretation of the research data in words (Dahlberg & McCaig: 2010; Taylor, *et al.* 2010). Qualitative methods are inquiry-based and explore human and or social problems (Creswell, 1997) in order to understand them.

4.2.3 Graph comprehension

Task and discipline characteristics (refer to 2.2.3.1 and 2.2.3.3) were investigated in the questionnaire to determine the participants' competence in:

- applying previously learned mathematics and physics knowledge in the questionnaires;
- relating mathematics equations and graphs with those in physics;
- extrapolating mathematical and physical variables from given graphs;
- naming and relating variables and concepts (e.g. gradient) in mathematics with corresponding variables and concepts in physics;
- showing a proper understanding of physics knowledge instead of using rote knowledge.

4.3 Quantitative methods

4.3.1 A questionnaire as quantitative research tool

Maree (2007) and Salkind (2009) see the questionnaire as a method that is relatively economical (i.e. questions are the same for all participants) and that can be treated anonymously. Participants can respond freely to questions in the questionnaire (Maree, 2007). The basic objective of a questionnaire is to gather facts and opinions about a concept from people who are informed about a particular issue (Delpont, 2005). Even if a questionnaire uses statements or questions, respondents in most cases respond to something written for specific purposes (McMillan & Schumacher, 2006). Quantitative methods such as questionnaires give precise and measurable data that can be reported in the form of tables and graphs.

McMillan and Schumacher (2006) state that questions or statements can take on different formats, such as a closed-ended or an open-ended format. According to Fraenkel & Wallen (2003), closed-ended questions can measure opinions, attitudes or knowledge. Cohen *et al.* (2000) and Maree (2007) add that the close-ended questions in a questionnaire have a high user-value as they are easy to complete, with little effort and time. The respondents are kept focused on the topic that appears relatively objective and the responses are easy to table and analyse (Cohen *et al.* 2000; Maree, 2007). Closed-ended questions that consist of multiple-choice questions allow participants to choose answers from a number of options (Fraenkel & Wallen, 2003). For quantitative purposes the researcher used closed-ended questions where participants had to choose between predetermined responses as part of this study.

The questions used in this research study contained quantitative and qualitative graphs. Qualitative graphs are graphs where numerical values are not necessarily used to represent situations. Essential elements of a situation are represented graphically. For instance, the shape of a qualitative graph can represent the linear motion of an object. A horizontal line on a velocity-time graph depicts constant velocity and zero acceleration. A change into a downward inclined line depicts decreasing velocity and constant acceleration (Glencoe University, 2010).

Delpont (2005) identifies the different types of questionnaires as mailed questionnaires, telephonic questionnaires, self-administered questionnaires, questionnaires delivered by hand and group-administered questionnaires. The researcher delivered the questionnaires to the participants by hand and personally administered the questionnaires to the participants under his invigilation and supervision. The participants had to complete the questionnaire within two hours and return them to the researcher.

4.3.2 Pilot Study

Christensen and Johnson (2012) state that one of the important rules of research is that, a questionnaire must be piloted before use in research to determine that it is adequate. Pilot testing helps to generate approaches to questions for the sake of the credibility of the research study (Conrad & Serlin, 2011). Since it is difficult for a

researcher to criticise his or her own work (Anderson, 2002), it is imperative to get comments from a small group of intended respondents. This will assist in spotting ambiguities, wording omissions in the instructions, questions and multiple-choice options. When there has been a pilot study, data is not biased (Strydom, 2005b).

The researcher therefore piloted this study with a class of 30 first-year physics participants at CUT in 2011. Their confidentiality was maintained and they were assured that their responses would only be used for research purposes. This was explained to them before they undertook the pilot study questionnaire. The researcher himself, who also marked the responses, administered the pilot questionnaire. The results of the pilot study were discussed with the study leaders and modifications were made according to their inputs for the final questionnaire, which is discussed in the following section (4.3.3).

4.3.3 The questionnaire used in the empirical study

The questionnaire with which data was gathered for the empirical study is attached as Appendices B, C and D. The questionnaire was split into two sections: Section A was on graphs in mathematics and section B was on kinematics graphs (physics). The aspects tested in the questionnaire, as well as the physics and mathematics questions used to investigate the participants' knowledge of these aspects are summarised in Table 4.1. This table shows the aspects tested in terms of categories of physical quantities (variables) in symbols and their mathematical definition or relation, the facets assessed as well as the numbers for both physics and mathematics questions of the questionnaire.

Table 4.1 Aspects/ constructs tested in the physics and mathematics parts of the questionnaire

Aspect/ constructs		Mathematics questions	Physics questions	
Area under graph	Smallest/Largest (Qualitative)	1.1; 1.2; 7.1; 7.2	$s = \int v dt,$	1
			$v = \int a dt,$	3
	Calculation (Quantitative)	12.1; 13.5	$s = \int v dt,$	8.2; 13, 18, 19
			$v = \int a dt,$	17.1; 17.2
Gradient of graph	Smallest/ Largest	8	$v = ds/dt,$	7, 12
	Increasing/ Decreasing/ Constant	2; 10.4	$v = ds/dt,$	2; 9;14; 15
			$a = dv/dt,$	8.1; 10.2; 14; 15; 16
	+, -, 0	9.2; 13.2; 13.4	$a = dv/dt,$	4.1; 4.2, 12
Calculation (Quantitative)	5; 6; 11.1; 11.2; 12.2	$v = ds/dt,$	6.2; 6.4	
		$a = dv/dt,$	8.3; 10.1	
Reading coordinates from graph		10.1; 10.2; 10.3		6.1; 6.3
Reading changes in function values from graphs		9.1; 13.1; 13.3		5, 14, 15, 16
Form of graphs / Expression		3; 4.1; 4.2; 4.3		11; 20.1; 20.2; 20.3

Table 4.2 identifies the different kinematics graphs in the physics section of the questionnaire, including the graph form and meaning and which questions specifically targeted the participants' knowledge of the graph type or form. The participants were given displacement-time (s-t) and velocity-time (v-t) graphs to determine velocity (v), acceleration (a), total displacement (s), etc. as shown in Table 4.2. Furthermore, they also had to differentiate between instantaneous velocity, uniform or average velocity or acceleration.

Table 4.2 Knowledge of forms of graphs tested in the questionnaire

Graph form:	Meaning	Physics Questions
s-t: horizontal line	constant displacement (not moving) zero velocity zero acceleration	3, 7, 8, 10
s-t: slant line	uniformly changing displacement constant velocity zero acceleration	3, 7, 8, 10, 11, 13
s-t: parabolic	changing displacement uniformly changing velocity constant acceleration	3, 8, 13
v-t: horizontal line	uniformly changing displacement constant velocity zero acceleration	6.2, 10, 11, 13, 15
v-t: slant line	changing displacement uniformly changing velocity constant acceleration	6.2, 11, 13, 15
v-t: parabolic	changing displacement changing velocity changing acceleration	6.2, 15
a-t: horizontal line	changing displacement uniformly changing velocity constant acceleration	11, 13
a-t: slant line	changing displacement changing velocity changing acceleration	11, 13

Other concepts tested under kinematics graphs include:

- Relationships of displacement and velocity, i.e., when is velocity negative, positive, constant or zero.
- Relationships of velocity and acceleration, i.e., when is acceleration negative, positive, constant or zero.

- What a given graph represents in terms of y-variable and x-variable (The x-variable is the time in all kinematics graphs).

4.3.4 Corresponding questions in the questionnaire

Mathematics generally deals with variables that do not necessarily have a specific physical meaning as they do in physics. The two questions below provide an example of equivalent graphical situations used in the questionnaire to compare students' mathematics and physics knowledge. The graph in the mathematics question (section A, question 2 of the questionnaire) that is shown in Figure 1, is a linear graph of variables y versus x . The variables x and y do not refer to anything in the physical world. However, the graph in the physics question (section B, question 9) that is shown in Figure 2, is a kinematic graph of the variable's position versus time (abbreviated as s vs t).

In the mathematics graph (Figure 1), students were asked about the gradient, but in kinematics (Figure 2) they were asked about the interpretation of the gradient in the kinematics situation. Students were tested on whether they can see a similarity or relation between such corresponding graphs.

Section A: Mathematics

2. From the graph below, the gradient is:

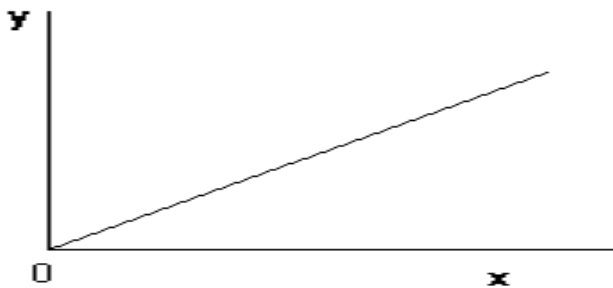


Figure 1

(A) Zero

- (B) Increasing
- (C) Decreasing
- (D) Constant
- (E) Varying

Section B: Physics

9. Below is a graph of an object's motion. Which sentence is the best interpretation?

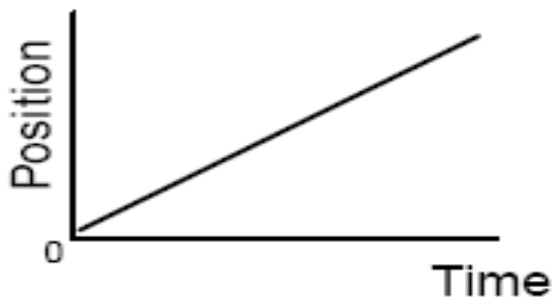


Figure 2

- (A) The object is moving with a constant, non-zero acceleration.
- (B) The object does not move.
- (C) The object is moving with a uniformly increasing velocity.
- (D) The object is moving with a constant velocity.
- (E) The object is moving with a uniformly increasing acceleration.

4.3.5 Quantitative data analysis

The data collected in this study consisted of 152 participants available sample out of a target population of 234, i.e., out of 234 first-year Physics students at CUT, 152 of them voluntarily participated in the study. This data was analysed by the North-West University's Statistical Consultation Services at the Potchefstroom Campus.

The researcher marked the questions of the questionnaire completed by participants and used the information to analyse the data. Descriptive statistics, like frequencies, mean and standard deviation of the variables were calculated. Cronbach Alpha coefficients were calculated to assure reliability of the constructs in table 4.3. CA values of 0.6 and above (i.e. $CA \geq 0.6$) in table 5.5 are regarded as an acceptable measure of the reliability and consistencies of the constructs (Field, 2005).

Since no random sampling (only available sampling) was done, interpretation of results will be done using effect sizes. Effect sizes yield important results in any empirical study and can be used to give the practical significance of such results (Lakens, 2013). However, p-values will be reported for completeness sake.

In this study's case comparisons between differences in proportions between mathematics and physics successes will be interpreted according to Cohen's effect sizes

$$w = \sqrt{\frac{\chi^2}{n}}$$

where n is the total number of participants and the χ^2 -value with one degree of freedom retained from the McNemar test (Stokes, Davis & Koch, 2000).

This effect size determines whether there is a practical significant difference between the proportion of students who succeeded in answering the mathematics correctly versus the proportion of students who succeeded in answering the physics correctly.

The w-values are interpreted as follows:

$w = 0.1$ is small effect,

$w = 0.3$ medium effect and

$w = 0.5$ is a large effect.

$w \geq 0.5$ is considered as practical significant.

For this study a small effect size indicates that the mathematics and physics questions were answered similarly, either both correct or both incorrect. A large effect size means

that the mathematics and physics questions were answered practical significantly different, either the mathematics correctly and the physics incorrectly or vice versa.

4.4 Qualitative methods

4.4.1 Qualitative research tool

Data obtained by means of quantitative methods can be better understood and put into perspective when followed by a qualitative method (Tashakkori & Teddlie, 2010; Dahlberg & McCaig, 2010). With qualitative methods, a traditional process of inquiry and understanding is used to determine the social or human problem (Creswell, 1997). The researcher used a phenomenological approach (Leedy & Ormrod, 2001) to determine the participants' ability to use their mathematics knowledge and skills to interpret and analyse kinematics graphs. The qualitative research tool was focus group interviews.

Open-ended questions make room for more individual responses, which are often more difficult to interpret and hard to score, as aptly pointed out by Fraenkel and Wallen (2003). The responses obtained from open-ended questions vary, because participants write down any response they want. Both formats of open- and closed-ended questions can be combined in a single questionnaire, which is useful for researchers (Fraenkel & Wallen, 2003).

4.4.2 Pilot interview

Thirty (30) participants completed the questionnaire for the pilot interview in 2011. The purpose of this was to check if any questions included in the questionnaire were ambiguous or too difficult to answer. These were corrected in the final questionnaire that was conducted with 152 participants in 2012. The pilot questionnaire was structured as follows:

Physics conceptual knowledge:

- Explain what displacement is.
- Explain what velocity is.
- Explain what acceleration is.

Area and gradient of a graph:

- Give a position-time graph - What is the meaning of the gradient? Explain why.
- Give a velocity-time graph:
 - What is the meaning of the gradient? Explain why.
 - What is the meaning of the area under graph? Explain why?
- Give an acceleration-time graph: What is the meaning of the area under graph?

4.4.3 Interviews

After the participants had completed the questionnaire and the results were available, a group of fourteen (14) participants from different schools were interviewed to support their answers, to probe their misconceptions and misunderstanding and to determine to which extent they applied their mathematics knowledge in kinematics. The aim was to get insight into the conceptual knowledge and transfer of knowledge of participants with different abilities. Participants that obtained high marks, average marks and low marks in the questionnaire were included. The different abilities and courses of the participants gave a wider range of coverage of conceptual understanding and reasoning, interpretation and analysis of kinematics graphs using their mathematical knowledge and skills. The researcher used open-ended questions to conduct the interviews (Appendix E). The interview questions were structured as in table 4.3 below.

Table 4.3: Interviews questions

Constructs	Relation	Interview
1. Integration / Area under graph	M7.1 x P3 M7.2 x P1 M12.1 x P17.2 M13.5 x P13 M13.5 x P17.1 P18 x P19	P1: Explain answer P3: Explain answer P17.2: Explain answer
2. Differentiation / Gradient		
a. Qualitative		
i. Kind: positive/negative/zero	P4.1 x P4.2 M9.1 x M9.2	M9.1 & M9.2 (Confusion gradient and function value) M13.2: Explain answer
ii. Largest/smallest	M8 x P7	M8: Explain answer P7: Explain answer
iii. Increasing/decreasing/constant		P16: Answer 2 (Confusion gradient & height?)
b. Quantitative (calculation)		
i. $\frac{y}{x} = \frac{\Delta y}{\Delta x}$	M11.1 x P6.2	M11.1: Explain answer
ii. $\frac{y}{x} \neq \frac{\Delta y}{\Delta x}$	M11.2 x P6.4	M11.2: Explain answer M12.2: Explain answer P6.4: Explain answer P10N1: Answer 3 (Confusion gradient & height?)
3. Reading from graph		
a. coordinates	M10.2 x P6.1 M10.2 x P6.3	
b. Value increasing/decreasing/constant	P5 x P21	P21: Explain answer 4
4. Form of graphs / Expressions	M4.1 x P20.1 M4.2 x P20.3 M4.3 x P20.2	
		P21: Explain answer 4

4.4.4 Aspects evaluated

Results of the questionnaire and pilot interviews indicated that the participants had problems with basic knowledge of kinematics concepts, interpretation of kinematics graphs and applications of their mathematics knowledge in kinematics graphs. The

interviews therefore focussed on the following aspects in order to probe deeper into the participants' conceptual knowledge:

➤ **Basic theoretical knowledge of kinematics**

- Definitions and units
- Equations of motion
- Average and instantaneous velocity

➤ **Basic interpretation of kinematics graphs**

- Gradient s-t graph
- Gradient v-t graph
- Area v-t graph
- Average and instantaneous velocity

➤ **Mathematics knowledge applied in kinematics graphs**

- Equation of v-t graph
- Area of v-t graph
- Area of a-t graph
- Gradient of s-t graph
- Gradient of v-t graph

4.4.5 Data analysis

Interviews were transcribed, coded, analysed and interpreted after data was gathered and participants' responses were summarised in a table. The qualitative results were combined with the quantitative results to give substantiated answers to the research questions of this study.

4.5 Ethical aspects

The completion of the questionnaire by participants was voluntary. Participants gave their informed consent and questionnaires were completed anonymously. Though participants did write their names on the questionnaire, they were informed that it was for research purpose only. The researcher explained that the questionnaire would not influence their studies and/or be to their disadvantage in any way. They only signed a consent form to complete the questionnaire. The consent form is attached as Appendix A.

4.6 Summary

This chapter outlined the research design and methodology employed in this study. It kicked off with a background to and definition of the mixed method used to do the research. The background includes a literature review associated with the research methods. Both quantitative and qualitative methods of collecting and analysing data, including ethical aspects, were discussed. Instruments and aspects used to collect and analyse data were detailed and accompanied by the relevant literature. The next chapter focuses on presenting and discussing the results of the empirical survey conducted in this research study.

CHAPTER 5 - RESULTS OF THE EMPIRICAL SURVEY AND DISCUSSION OF RESULTS

5.1 Introduction

This chapter provides a representation of the participants' performance in the questionnaires and interviews. The researcher analysed and discusses the results of the empirical study and survey as stated in chapter 4 with a view to addressing the aims and objectives in paragraph 1.4 and answering the research questions in paragraph 1.5 in chapter 1. The pilot study was conducted in 2011 and the questionnaire was administered in 2012.

The results of the quantitative study were statistically analysed using Cronbach's alpha coefficient and effect sizes. Trochim (2005) states that such descriptive statistics provides information about a sample by describing what the data shows. In this study, descriptive statistics were used to analyse, describe and discuss the data obtained.

The qualitative results from interviews were coded, analysed and interpreted to compare the understanding of participants in their response to the questionnaire. The results of the questionnaire and interviews are presented and summarised in tables. Section 5.2 presents the biographical data, section 5.3 the quantitative results of the questionnaire and the results of the interviews are presented in section 5.4.

5.2 Results: Biographical information

Table 5.1 below reports on the results of the bibliographical information of the 152 participants based on gender, age, etc. Results shown here and elsewhere in the chapter reflect the study conducted in 2012. The biographical data helps to get the general background of who the participants are and not to use for comparison.

The results indicate that, of the 152 participants, more or less equal numbers were males and females. The number of males is slightly more than that of females, although the percentages did not differ much. The majority of participants, about 66%, are within

the age of starting university (18 - 20) with about 22 % of them turning 18 years old in that year, while about 10% are over 20 years of age.

Table 5.1 A Results: Biographic information – Gender, Age, Nationality, School type, Matric completion year

Variable	Options	Number of participants (out of 152)	Percentage students
Gender	Male	77	53.47
	Female	67	46.53
	Missing	8	
Age	16	2	1.39
	17	32	22.22
	18	56	38.89
	19	24	16.67
	20	15	10.42
	21	15	10.42
	22 and older	0	0.00
	Missing	8	
Nationality	South African	144	96.00
	Non-South African	6	4.00
	Missing	2	
School type	Rural (Village/farm)	15	10.34
	Semi-Urban (Township)	71	48.97
	Urban (Town or Model C)	59	40.69
	Missing	7	
Matric Completion Year	2011	77	51.33
	2010	37	24.67
	2009	16	10.67
	2008	7	4.67
	2007	5	3.33
	Other	8	5.33
	Missing	2	

Table 5.1 B Results: Biographic information – Mathematics and Physics Pass

Levels and Faculty

Variable	Options	Number of participants (out of 152)	Percentage students
Mathematics Pass level	0-29	1	0.67
	30-39	5	3.36
	40-49	38	25.50
	50-59	54	36.24
	60-69	32	21.48
	70-79	17	11.41
	80-100	2	1.34
	Missing	3	
Physics Pass level	0-29	1	0.68
	30-39	6	4.05
	40-49	31	20.95
	50-59	56	37.84
	60-69	37	25.00
	70-79	13	8.78
	80-100	4	2.70
	Missing	4	
Faculty	Humanities	34	23.13
	Engineering and Information Technology	35	23.81
	Health and Environmental Sciences	78	53.06
	Missing	5	

Table 5.1 C Results: Biographic information – English Proficiency and Home Language

Variable	Options	Number of participants (out of 152)	Percentage students
English proficiency	1st Language	71	48.63
	2nd Language	74	50.68
	Other	1	0.68
	Missing	6	
Home language	English	4	2.72
	Afrikaans	20	13.61
	Sesotho	68	46.26
	Tswana	31	21.09
	Sepedi	1	0.68
	Xhosa	11	7.48
	Zulu	6	4.08
	Ndebele	1	0.68
	Swazi	0	0.00
	Venda	1	0.68
	Tsonga	1	0.68
	Other (SA)	0	0.00
	Other (Non-SA)	3	2.04
	Missing	5	

96% of participants were South African citizens, while 4% are foreigners. Only a small percentage of participants come from rural areas (10%), while 49% are from semi-urban areas and 41% are urban. Though only 3% of participants' home language is English, 49% of participants indicated that their English proficiency is at a first language level, while 51% indicated that it is at a second language level. This is important because the

study was conducted in English. Also, of importance is that about 90% of participants are from township and city schools, giving them an added English language advantage as semi-urban and urban schools.

Even though the minority of the participants use English as their home language, English is the language of instruction, tuition and assessment in the institution, i.e., CUT, South Africa. This may affect their understanding of what they have been taught in schools. The majority of them used English as a medium of instruction, and a few used Afrikaans as a parallel medium of instruction in terms of the National Department of Education (DoE) policy.

Of the study population who filled out the questionnaire, 51% completed matric (Grade 12) the previous year (2011), 25% completed in 2010, while about 24% completed their matric before 2010. This implies that about half of the participants probably had a gap in their physics training, which may affect their progression and knowledge retention.

74% of the participants had passed their Grade 12 physical science with 50% and above and 21% of them passed with between 40 and 49% while 5% got below 40%. 70% of the participants had passed their mathematics by 50% and above, 26% passed with between 40 and 49% while about 4% got below 40%. Therefore, because the pass percentage in physical science is higher than mathematics in grade 12, it seems as if participants are a little bit more competent in physical science than in mathematics based on their grade 12 background. This also indicates that the participants were competent enough to complete the questionnaire of the study.

Sesotho, followed by Setswana and Afrikaans respectively, was the predominant language among the participants of this study. This is because the geographical area in which the study was conducted, is dominated by these languages. There were 68 Sesotho speaking (46.26%), 31 Setswana speaking (21%) and 20 Afrikaans speaking (14%) participants. There were 11 Xhosa and 6 Zulu speaking participants, i.e. 7% and 4% respectively. Only four participants were English speaking (3%) followed by three non-South African languages (2%) while Ndebele, Venda and Tsonga had one student

each (0.7%). Five participants did not give their home language, so it is recorded as missing data.

5.3. Results of the questionnaire

5.3.1 Comparison of mathematics and physics sections

Below is a comparative results table of the participants' responses to the mathematics and physics sections of the questionnaire (Table 5.2). This is as explained in paragraph 4.3.3 in chapter 4. The questionnaire is attached as appendices C and D. The corresponding results of the Cronbach's alpha coefficients (abbreviated CA), average percentage performances and standard deviations are compared for the mathematics and physics sections.

Table 5.2 Mathematics and physics questionnaires: Comparison of results

Questionnaire	Cronbach's Alpha coefficient (CA)	Average percentage	Standard deviation
Mathematics	0.75	62.7	16.0
Physics	0.69	34.7	12.8

The average percentage and standard deviation are higher for mathematics, 62.7 % and 16.0 respectively, as compared to 34.7 % and 12.8 respectively for physics.

The internal consistency or average correlations of variables used in the study are determined to measure reliability and consistency by means of CA (Santos, 1999), i.e., an indication of the manner of answering the questions. A CA value of 0.7 and more is acceptable for test reliability and an indication of consistent responses. The CA value for mathematics (0.75) is more than the cut-off and acceptable reliability value, while the CA value for physics (0.69) is almost the cut-off. Both sections can thus be considered as reliable and be statistically compared.

The average percentage performance in mathematics at 62.7% is high compared to that of physics, which is low at 34.7%, an indication that participants did better in the

mathematics than physics questions on graphs. This is confirmed by other researchers in section 3.1 of chapter 3 (e.g. Basson, 2002 and Beichner, 1994).

Standard deviation is a measure of the amount of variation or dispersion in the data from the average. A low standard deviation (SD) shows that the data values seem to be very close to the average/mean (small variance), while a high standard deviation shows that the data values are scattered over a large range of values and might be unrelated. A high SD value means that the result of the sample is not representative and is therefore not reliable. A high SD value also means that there is a wider data distribution between the minimum and maximum values. In this case the standard deviations are lower or smaller than the average percentage performance in both mathematics and physics, giving a relatively small variance, which implies a high precision, i.e., that the results are reliable and representative.

5.3.2 Students' performances in mathematics questions

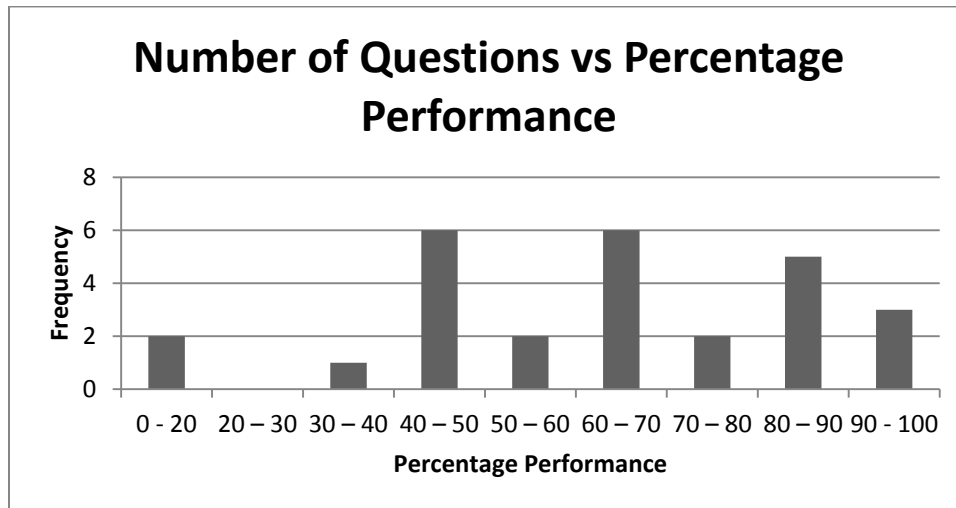
Table 5.3 below illustrates the results of the average percentage correct and standard deviation per question of the performance in the mathematics section of the questionnaire.

A histogram (Graph 5.1) was constructed to illustrate the distribution of participants' performance in the mathematics questions.

Table 5.3 Mathematics questions: Average percentages and standard deviation per question

Question number	Correct option	Average % correct	Standard deviation
M1.1	B	81.6	38.9
M1.2	D	82.9	37.8
M2	B	48.7	50.1
M3	C	66.4	47.4
M4.1	A	67.1	47.1
M4.2	E	72.4	44.9
M4.3	C	69.7	46.1
M5	D	61.2	48.9
M6	C	43.4	49.7
M7.1	E	72.4	44.9
M7.2	D	69.1	46.4
M8	B	46.7	50.1
M9.1	E	42.1	49.5
M9.2	A	32.9	47.1
M10.1	B	92.8	26.0
M10.2	B	93.4	24.9
M10.3	C	48.7	50.1
M10.4	A	17.1	37.8
M11.1	C	82.9	37.8
M11.2	A	16.4	37.2
M12.1	C	57.9	49.5
M12.2	A	42.8	49.6
M13.1	D	91.4	28.1
M13.2	B	57.9	49.5
M13.3	B	89.5	30.8
M13.4	C	82.9	37.8
M13.5	A	63.8	48.2

Graph 5.1: Histogram of number of questions against percentage performance for the mathematics questions



Performance of over 90% was obtained in three questions (M10.1, M10.2, and M13.1) and between 80% and 90% in five questions (M1.1, M1.2, M11.1, M13.3, and M13.4), while between 70% and 80% in two questions (M4.2 and M7.1). Performance of between 60% and 70% was obtained in six questions (M3, M 4.1, M4.3, M5, M7.2 and M13.5) and between 50% and 60% in two questions (M12.1 and M13.2), while scores between 40% and 50% was obtained in six questions (M2, M6, M8, M9.1, M10.3 and M12.2). Of all the questions, only two questions (M10.4 and M11.2) were performed at below 20% while 30% to 40% was obtained in one question (M9.2). The highest performance percentage in a question was 93.4% (M10.2), while the lowest was 16.4% (M11.2). Only one-third of the questions was performed below or equal to 50%. The implication is that participants do not seem to have many problems with mathematics and they do have prior mathematics knowledge. This refers to the problems and efficiency participants encountered in their previous learning or studies, as explained in paragraph 3 of chapter 3. The average of 62.7% indicates that mathematics problems did occur, although much less than with physics.

Of the 27 mathematics questions in the questionnaire, the average performance of three questions (M9.2, M10.4 and M11.2) were less than 40% and the average of six questions obtained above 40% but below 50%. Those questions are question M2, on

determining what will happen to a graph if the x-coordinate increases (48.7), question M6 on determining the gradient of a graph slanting to the left where axes intercepts were given (43.4%), while question M8 on determining intervals where the gradient is the largest (46.7%). Question M9.1 was on determining intervals where a function is increasing (42.1%), question M10.3 on determining corresponding x-values if y-value is known (48.7%) and question M12.2 on determining the gradient for a set of x-coordinates, where the gradient is zero (42.8%).

From these results, it is evident that participants have a problem with discriminating between the slope and height of a graph, i.e. the slope-height confusion problem stated by Beichner (1994). McDermott *et al.* (1987) further confirm the problem with identifying features of graphs like coordinates of a point, differences between coordinates of two points and the gradient of a given straight line. Another reason might be that participants are unable to develop clear connections between dependent and independent variables (Connery, 2007). The results of the questionnaire show this deficiency in participants' responses.

The standard deviation in three questions is between 50 and 55, between 40 and 50 in eleven questions, 30 and 40 in seven questions while it is between 20 and 30 in three questions. The maximum standard deviation is 50.1 and the minimum is 24.9.

The standard deviation always depends on the average/mean and therefore standard deviations cannot be compared with each other. Depending on how close standard deviations are and how much less they are than the means/averages (like in most cases in this study), they provide an indication that the results of the study are representative, reliable and have a high precision.

The results of groups of items that investigated the different aspects listed in Table 4.1 from chapter 4 are discussed in the following paragraphs. The participants' performance for the mathematics questions are given and compared with those of the physics questions in Table 5.5.

Area under graph

81.6% and 82.9% of participants were able to determine the geometric shapes with the greatest and smallest areas respectively in questions M1.1 and M1.2. This means that they can compare the areas of shapes by inspection. In questions M7.1 and M7.2, 72.4% and 69.1% of participants were able to determine the smallest and greatest areas respectively under graphs of different shapes. When the domain is given, 63.8% were able to calculate the area under a graph for the same domain in question M13.5, while 57.9% were able to determine the area for a given x-interval in question M12.1.

Participants seem not to have many mathematical problems with comparing the areas of geometric shapes. They experienced more difficulty with comparing or calculating areas beneath a line graph in a given domain or interval.

Gradient of graph

82.9% of the participants were able to find the gradient at a point in question M11.1 if the graph is a straight line, but when the graph is curved, only 16.4% (the lowest performance in the mathematics questions) managed to find the gradient at a point in question M11.2. Here the gradient is zero. The majority calculated it as $\frac{y}{x}$. This means participants experience more difficulty with finding the gradient at a point of a curved graph than a straight line graph. This is in contradiction with Beichner's (1994) findings that participants are unable to calculate the slope of the graph unless he was not referring to straight line graphs. The result is a confirmation that participants have more difficulty with interpreting curved graphs where changes in slope and changes in height are involved, as compared to straight line graphs (McDermott *et al.* 1987).

For a given x-interval, 42.8% of the participants were able to determine the gradient in question M12.2. 61.2% of participants were able to determine the value of the gradient if the change in y and the change in x are given like in question M5, but only 43.4% were able to determine the value of the gradient if the x-intercept and the y-intercept are given as in question M6.

In questions M13.2 and M13.4, 57.9% and 82.9% of participants respectively were able to determine whether the gradient of a graph is increasing, decreasing, constant or zero in a given x-coordinate domain. In M13.2 the gradient was zero and in M13.4 it was positive, but still constant. It shows that many participants can identify positive and negative gradients, but get confused when the gradient is zero. Question M10.4 was the second lowest performance with only 17.1% of participants able to determine how the gradient of curve changes within a given set of x-coordinates.

The third lowest performance (below 40%) was in question M9.2 where only 32.9% correctly identified the part of a continuous curving graph where the gradient is positive. Only 46.7% participants were able to determine the domain in which the gradient of a graph is the largest such as in question M8, while 48.7% were able to determine that the gradient is constant such as in question M2.

A mixed inference is obtained from these results in that for straight lines through the origin participants were able to find the gradient, but they struggled with zero gradients, gradients of curved graphs and straight lines not passing the origin. This result could be due to inappropriate use of axis values when calculating the gradient (Beichner 1994).

Reading coordinates from a graph

The highest percentages of 92.8% and 93.4% were obtained in questions M10.1 and M10.2 respectively, meaning the participants did not have difficulty in finding the coordinates of a point and finding the other coordinate if one coordinate is given. Only 48.7% of participants were able to determine the x-values for the given y-value in question M10.3. This low percentage was probably because there were two x-values corresponding to the same y-value. The results show that more or less 20% chose option 3, which is an indication of not estimating the correct x-values and more or less 25% chose option 5, which is choosing only one value for x instead of two.

Deficiencies in reading coordinates from graphs might be another reason why participants use inappropriate axis values when calculating the area under the graph or the gradient of a graph, as is suggested by Beichner (1994).

Reading changes in function values

In question M13.1 a percentage of 91.4% of participants was able to determine the change in the y-value when the x-coordinate changes in the horizontal line graph. For a slanting graph in questions M13.3, 89.5% were able to determine changes in the values of y in a given set of x-values (domain). Only 42.1% were able to determine within which intervals of a graph/function the gradient is increasing in question M9.1, probably because there were multiple intervals to indicate in this question and some of the participants confuse positive function values with positive gradient. This problem corresponds with the height-gradient confusion found by Beichner (1994) and McDermott *et al.* (1987).

Form of graph / Expression

66.4% of participants were able to give the equation of a horizontal straight line in question M3 and 67.1%, 72.1% and 69.7% were able to match graphs to given equations in questions M4.1, M4.2 and M4.3 respectively. This means that between 66% and 73% participants did not have a problem to find equations of given graphs or labelling a given graph in mathematics.

5.3.3 Participants' performance in physics questions

Table 5.4 below illustrates the results of the average percentage correct and standard deviation per question to show the performance by participants in the physics section of the questionnaire.

The percentage performance (table below) in the physics questions varies between 92.1% as the highest and 1.3% as the lowest. Participants scored above 90% in one question (P6.1) and between 80% and 90% in another one (P6.3). Participants did not score between 70% and 80% in any question, three questions (P6.2, P11 and P20.1) were between 60% and 70%, five questions (P4.1, P12, P13, P18 and P50.7) were between 50% and 60%, between 40% and 50% was obtained in two questions (P3 and P4.2), between 30% and 40% in four questions (P7, P20.2, P21 and P22.1), between

Table 5.4: Physics: Average percentages and standard deviation per question

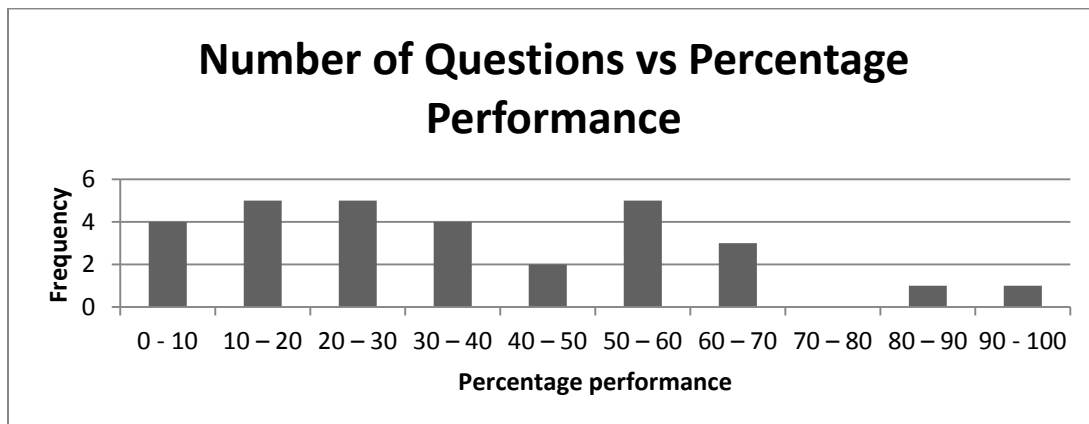
Question number		Average % correct	Standard deviation
P1	B	27.0	44.5
P2	C	9.9	29.9
P3	D	41.4	49.4
P4.1	B	59.2	49.3
P4.2	D	48.7	50.1
P5	C	9.9	29.9
P6.1	A	92.1	27.1
P6.2	E	64.5	48.0
P6.3		84.2	36.6
P6.4		7.9	27.1
P7	D	34.2	47.6
P10.1	B	14.4	35.3
P10.2		11.2	31.6
P11	B	63.2	48.4
P12	D	59.2	23.7
P13	C	50.0	50.2
P14	C	20.4	40.4
P15	B	12.5	33.2
P16	B	1.3	11.4
P17.1	A	12.5	33.2
P17.2	C	29.6	45.8
P18		59.2	49.3
P19		50.7	50.2
P20.1		65.1	47.8
P20.2		38.2	48.7
P20.3		28.9	45.5
P21		34.9	47.8
P22.1		30.9	46.4
P22.2		10.5	30.8
P22.3		23.7	42.7

20% and 30% in five questions (P1, P14, P17.2, P20.3 and P22.3), between 10% and 20% in five questions (P10.1, P10.2, P15, P17.1 and P22.2) and between 0% and 10% were obtained in four questions (P2, P5, P6.4 and P16). The percentage performance of the participants in the physics questions is summarised in the histogram below (Graph 5.2).

Participants performed poorly in question P16 (1.3%), where they were asked to interpret a velocity-time graph showing constant negative acceleration. Questions that were performed below 10% includes question P2 on determining the position-time graph showing a constant positive acceleration (9.9%), question P5 on describing the motion depicted by a position-time graph. The changes with time from zero velocity at a positive position to constant negative velocity until it remains stationary at the origin, and question P6.4 on determining velocity from a position-time graph at an estimated time (7.9%). Three of the four questions for which the students' percentages were below 10 % entailed interpretations of the gradient of kinematics graphs. They performed the best with percentages above 80 % in the two physics questions (6.1 and 6.5) where they had to read coordinates from a given position-time graph.

The highest standard deviation is 50.2 and the lowest is 23.7 in the physics questions. Standard deviations of between 50 and 55 were obtained in three questions, between 40 and 50 were obtained in fifteen questions, between 30 and 40 in six questions, 20 and 30 in five question, while only one question has the lowest standard deviation of 11.4.

Graph 5.2: Histogram of number of questions against percentage performance for the physics questions



The results show that more than half of the participants had two-thirds of the physics questions incorrect and only one-third correct. This result differs from the mathematics results (Graph 5.2), where the vast majority of participants performed above 40 %.

5.3.4.1 Results per construct: Comparison of physics and mathematics questions

Constructs that relate corresponding concepts in mathematics and physics sections of the questionnaire were identified and classified in Table 5.5 below. The percentage performance (%) and the Cronbach's alpha (CA) of each construct are compared for both subjects. There were more questions in the physics section than in the mathematics section, but still a comparison was made as shown in Table 5.5. Qualitative and quantitative analyses were conducted on the first two constructs, namely area under graph (integration) and gradient/slope of a graph (differentiation).

Some of the questions in the questionnaire had to be omitted in the comparison. For instance, analysis of mathematics questions M5 and M6 were omitted because participants did not answer their corresponding physics questions. The corresponding physics questions (P8.1, P8.2, P8.3 and P9) were accidentally not printed out and this was only discovered during the marking and evaluation of the questionnaire. Physics questions P14 and P15 were also omitted since too many aspects had to be considered

Table 5.5: Results per construct: Percentage performance and Cronbach's Alpha values

Construct	Mathematics			Physics		
	Q #	%	CA	Q #	%	CA
1. Integration / Area under graph						
a. Qualitative (largest/smallest)	M1.1 M1.2 M7.1 M7.2	82 83 72 69	0.7	P1 P3	27 41	< 0.6
b. Quantitative (calculation)	M12.1 M13.5	58 64	< 0.6	P13 P18 P19 P17.1 P17.2	50 59 50 13 30	< 0.6 < 0.6
2. Differentiation / Gradient						
a. Qualitative						
i. Kind: positive/negative/zero	M9.2, M13.2 M13.4	33 58 83	< 0.6	P4.1 P4.2 P12	59 49 6	< 0.6
ii. Largest/smallest	M8	47		P7 P12	34 6	< 0.6
iii. Increasing/decreasing/ constant	M2, M10.4	49 17	< 0.6	P2 P10.2 P16 (P14 P15)	10 11 1 20 13	< 0.6
b. Quantitative (calculation)	(M5 M6)	61 43				
i. Line intercepts the origin	M11.1	83		P6.2	64	
ii. Line does not intercept the origin	M11.2 M12.2	16 43	< 0.6	P6.4 P10.1	8 14	< 0.6
3. Reading from graph			< 0.6			
a. Coordinates	M10.1 M10.2 M10.3	93 93 49	< 0.6	P6.1 P6.3	92 84	0.7
b. Value increasing/decreasing/ constant	M9.1 M13.1 M13.3	42 91 89	< 0.6	P5 P16 (P14 P15)	10 1 20 13	< 0.6
4. Form of graphs/ Expressions	M3 M4.1 M4.2 M4.3	66 67 72 70	0.60	P11 P20.1 P20.2 P20.3	63 65 38 29	< 0.6

to answer them. Questions P21 and P22 in the physics section test knowledge of physics and do not have corresponding mathematics questions.

Area under graph

A qualitative analysis of the results concerning area under a graph in the mathematics questions M1.1, M1.2, M7.1, and M7.2 showed a high percentage performance of 82%, 83%, 72% and 69% respectively and a high CA of 0.7, an indication of consistent responses. In the physics questions P1 and P3 there is a low percentage performance of 27% and 41% respectively and a low CA of less than 0.6, an indication of inconsistent responses. The consistency in participants' responses on the mathematics questions in this construct was reliably acceptable, while the participants answered the two physics questions differently. The poor performance in the physics questions is in accord with the assertion of McDermott *et al.* (1987) and Beichner (1994) that students have problems with interpreting the area of kinematics graphs. Beichner (1994) further argues that this may be because students use inappropriate axis values when calculating the area under the graph.

The quantitative analysis of the construct where areas had to be calculated showed in the mathematics questions M12.1 and M13.5 a percentage performance of 58% and 64%, while in the physics questions P13, P18 and P19, the percentages are 50%, 59%, and 50% respectively. In these questions, the participants obtained similar percentages in physics and mathematics. In all these three physics questions, the students had to describe how displacement is determined from a velocity-time graph. No calculations had to be done. The responses on calculations of the area of acceleration-time graphs, i.e. questions, P17.1 and P17.2, yielded small percentages of 13% and 30% respectively. The results of these two questions indicate that students either lack the physics knowledge that average velocity is calculated from the area of an acceleration-time graph or they do not apply the mathematics knowledge to calculate the areas, or both.

Gradient of graph

Under the gradient construct, a qualitative analysis in mathematics was classified into three sub-constructs namely positive/negative/zero, largest/smallest and increasing/decreasing/constant. None of the constructs had CA values larger than 0.6. The percentages obtained for the questions varied significantly. Questions M9.2, M13.2 and M13.4 (positive/negative/zero) have percentages of 33%, 58% and 83% respectively, question M8 (largest/smallest) has a percentage of 47% and questions M2 and M10.4 (increasing/decreasing/constant) have percentages of 49% and 17% respectively. In physics, questions P4.1, P4.2 and P12 (positive/negative/zero) have percentages of 59%, 49% and 6% respectively, question P7 and P12 (largest/smallest) have percentages of 34% and 6% respectively and questions P2, P10.2, and P16 (increasing/decreasing/constant) have percentages of 10%, 11% and 1% respectively.

The results of items P2, P5, P6.2, P6.4 and P7 indicate that many participants revealed the height-gradient confusion when they did not distinguish between the y-coordinate value of an s-t graph (displacement) and the gradient (velocity). In items P1, P10.1, P10.2, P13, P17.1, P17.2, P22.1 and P22.3, the participants should have shown the physics knowledge that positive, constant acceleration means increasing velocity, which means increasing gradient of a s-t graph (concave up). Participants probably struggled because they did not realise that $a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$ and they interpreted the acceleration as the gradient of the s-t graph and velocity as the y-value of the s-t graph. Furthermore, participants seem not to know that constant velocity means zero acceleration and think a straight horizontal line graph has a positive gradient. This is another indication that they use coordinate values where they have to work with gradient.

The quantitative analysis of gradient in mathematics questions M5 and M6 show percentages of 61% and 43% respectively, with question M11.1 for a line graph intercepting the origin ($\frac{y}{x} = \frac{\Delta y}{\Delta x}$) showing a percentage of 83%, while questions M11.2 and M12.2 ($\frac{y}{x} \neq \frac{\Delta y}{\Delta x}$) on graphs not passing the origin show the much lower percentages

of 16% and 43% respectively. The same tendency appears in the physics questions. In physics question P6.2 for a line through the origin shows a higher percentage of 64%, compared to questions P6.4 and P10.1 which show percentages of 8% and 14% respectively for lines not intercepting the origin. In both mathematics and physics the participants performed much better in the questions on functions intercepting the origin than on those that do not. This result confirms the height-gradient confusion found in the previous questions. When the origin is a point on a straight-line graph, $\frac{y}{x} = \frac{\Delta y}{\Delta x}$ and participants with height-gradient confusion can give the correct answer. However, for line segments that do not intercept the origin, $\frac{y}{x} \neq \frac{\Delta y}{\Delta x}$, and the participants' answers are incorrect.

Reading coordinates from a graph

The construct of reading coordinates from the graph showed percentages of 93%, 93% and 49% respectively. On increasing/decreasing/constant function values, percentages of 42%, 91% and 89% were obtained respectively. Apart from questions M10.3 and M9.1, high percentages were obtained. Students seem to have problems with the definition of intervals, hence poor or low performance in M9.1 and M10.3. Physics questions on reading coordinates from kinematics graphs show high percentages of 92% and 84% respectively, but on increasing/ decreasing/ constant values show low percentages of 10%, 1%, 20%, and 13%. The participants did well on the physics questions that entail the reading of coordinate values. However, in the increasing/decreasing/constant questions they either did not transfer their mathematics knowledge, or could not interpret the kinematics graphs.

Form of graph / Expression

The forms of graph/expression construct showed in mathematics questions M3, M4.1, M4.2, and M4.3 percentages of 66%, 67%, 72%, and 70% respectively, which indicate consistencies in participants' response. In physics questions P11, P12, P20.1, P20.2 and P20.3 percentages of 63%, 59%, 65%, 38% and 29% respectively were obtained. This percentage performance might be due to inconsistent responses, which is evident

in the variation in percentage performance. More students answered questions P11 and P20.1 on the equation for a straight-line graph correctly than questions 20.2 and 20.3 on quadratic and hyperbolic forms of graphs. It thus seems that students readily transfer their knowledge of the straight-line equation to the kinematics graphs, but not for other graph forms.

5.3.4.2 Discussion of comparison of results per construct

From table 5.5 we can observe that almost all CA values were smaller than 0.6. This is probably due to too small a number of questions per construct and varying difficulty between the questions in a construct. Exceptions are the three constructs of questions M1.1, M1.2, M7.1 and M7.2 (CA=0.7), P6.1 and P6.3 with CA of 0.7 and M3, M4.1, M4.2 and M4.3 with CA of 0.6. Percentage performances in most mathematics and physics constructs vary, with some high and other very low percentages. This may indicate different levels of difficulty of the questions in the construct. The results show in which questions the participants struggled and in which they did well. Due to the too small CA values, the researcher could not compare the constructs statistically and instead focussed on trends revealed by the individual questions

The results for construct 1 (Table 5.5) indicate that participants have the mathematical knowledge and skills to determine whether the area under the graph is the largest or smallest, and this is an acceptable and reliable result. The participants seem not to use this knowledge from mathematics in answering questions on area in kinematics graphs. In P17.1 and P17.2 they struggled to do calculations. This is probably because most (58%) of them did not recognise the linear relation (straight line) and the constant acceleration (horizontal line) or did not know the meaning of the area in acceleration-time graphs.

According to the results of construct 2 in question M13.2, they had to indicate that the gradient is zero and in M13.4 it is positive. In both these cases only one interval was involved and that is most probably why they performed well. In M9.2, they had to indicate in which intervals the gradient was positive. Most probably, they did not

perform so well in this question because there were multiple intervals to indicate and some of them confused positive function values with positive gradient (approximately 20% of the participants chose this answer). The participants could determine negative acceleration (P4.1) and negative gradient (P4.2) in kinematics (physics) maybe because there were no multiple intervals to choose from.

Determining the positive gradient in algebra (M13.4) went somewhat better than in kinematics (P4.2), but they struggled more to interpret a zero gradient in kinematics (P12), while they managed to find it in algebra (M13.2). They further struggled to find the largest/smallest (M8, P7 and P12) and increasing/decreasing/constant (M2, M10.4, P2, P10.2 and- P16) gradient in both algebra and kinematics.

The relation, $\frac{y}{x} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$, means that the average gradient over an interval is equal to the ratio of the y and x coordinates, in which case $y_2 = y$, $x_2 = x$, $y_1 = 0$, $x_1 = 0$. This is only true for functions intercepting the origin. The ratio $\frac{y}{x} \neq \frac{\Delta y}{\Delta x}$ is where $y_2 = y$, $x_2 = x$ and $y_1 \neq 0$ or $x_1 \neq 0$. This holds true for functions not going through the origin or when the interval over which the gradient should be calculated, is not starting at the origin. Participants get the latter wrong because they take the gradient as the ratio of the coordinates of y and x instead of the ratio of the change in coordinates. This result indicates that height-gradient confusion may be due to incomprehension of the gradient of a graph.

In construct 3 participants did not experience difficulties to read the coordinates from the given graphs and to determine the y-coordinate when the x-value is given (construct 3a) in both algebra (M10.1 and M10.2) and kinematics (P6.1 and P6.3), except for question M10.3 of the mathematics section. This was probably because for one y-value there are two x-values. This might have confused them. The kinematics results were acceptable for reliability, but not the results in algebra. According to construct 3b, about 90% of the participants were able to read increasing (M13.3) and constant values (M13.1) from a graph, but some (58%) were unable to read increasing

values from the graph in M9.1, which contained multiple intervals. They struggle to read the increasing/decreasing/constant (P5 and P16) value from the graphs in kinematics. These questions required interpretation of the gradient or area of the graphs, which are aspects that they experienced difficulties with in constructs 1 and 2.

According to construct 4, participants (between 66 and 72% correct) were able to find the forms or expression of graphs in algebra (M3, M4.1, M4.2 and M4.3), but in kinematics they were able to find it only in P11 and P20.1. They could not find the correct answers in P20.2 and P20.3. A possible reason is that in the case where the relation is linear, they were able to identify it as a straight line and apply the mathematical expression for a straight line graph, but they could not in the case where the relation is quadratic or inverse proportional.

Results of table 5.5 give an indication of students' difficulties with kinematics graphs. They do not have all the necessary mathematics (algebraic) knowledge, background and skills required to solve kinematics graphs, or they do not use those algebraic skills and knowledge effectively and efficiently in the contexts of kinematics. Previous research studies reported in chapter 3 were confirmed and new problems were also indicated. The research problems of this study were investigated in more depth by determining effect sizes of paired questions (in section 5.3.5) and with the aid of interviews (section 5.4).

5.3.5.1 Results of the paired questions

Corresponding mathematics and physics questions that test the same skill or knowledge were chosen from the constructs (Table 5.5) and the performances compared with the aid of the effect sizes, w -values. The method of calculating effect sizes are explained and given in paragraph 4.3.5 of chapter 4. In the results (Table 5.6) the effect sizes that indicate a medium effect is marked with an asterisk, while practical significant effects are indicated with a double asterisk.

Table 5.6 below gives the results of the pairing of questions in mathematics (M) and physics (P). Some of the gaps between these pairs are very large, while in other pairs performances are closer to one another.

Table 5.6 Results of the effect sizes: Paired Questions, Mathematics and Physics

Construct	Paired questions	Total M correct	Total P correct	p – value	w-value
1a. Area qualitative	M7.1 x P3	72.37	41.4	<0.0001	0.42*
	M7.2 x P1	69.08	26.98	< 0.0001	0.63**
1b. Area quantitative	M12.1 x P17.2	56.90	29.61	<0.0001	0.43*
	M13.5 x P13	63.81	50.00	0.0082	0.21
	M13.5 x P17.1	63.82	12.5	<0.0001	0.67**
2a. Gradient qualitative	M8 x P7	46.71	34.21	0.0282	0.18
2b. Gradient quantitative	M11.1 x P6.2	82.90	64.48	0.0002	0.30*
	M11.2 x P6.4	16.45	7.90	0.0067	0.22
3. Reading coordinates	M10.2 x P6.1	93.42	92.11	0.6171	0.04
	M10.2 x P6.3	93.42	84.21	0.0060	0.22
4. Form of graph/ Expressions	M4.1 x P20.1	61.07	65.13	0.7180	0.03
	M4.2 x P20.3	22.37	28.95	<0.0001	0.58**
	M4.3 x P20.2	69.76	38.16	<0.0001	0.44*

*The McNemar test value (effect size) is calculated as described in paragraph 4.3.5

*Statistically significant at 0.05 level according to t-test results for independent groups (Stokes *et al.* 2000).

The effect size (w) is calculated (Ellis & Steyn, 2003) similarly as described in paragraph 4.3.5 as follows:

$$w = \sqrt{\frac{Q_n}{n}},$$

Where

$$Q_n = X^2,$$

$$X^2 = \frac{(n_{12} - n_{21})^2}{n_{12} + n_{21}}$$

The variance in effect has the following meanings in terms of difference in responses to mathematics and physics questions:

$w = 0.1$ is a small effect, meaning participants answered similarly, i.e. either both correctly or both incorrectly,

$w = 0.3$ is a medium effect, meaning participants answered with a medium degree of difference and,

$w = 0.5$ is a large effect, meaning participants answered significantly differently, i.e., either they got mathematics correct and physics incorrect or vice versa.

$w \geq 0.5$ is considered as a practically significant effect.

5.3.5.2 Discussion of results summarised in Table 5.6

Three pairs of questions were answered practically significantly differently, i.e., $w \geq 0.5$., M7.2 and P1 tested on identifying the largest area in a given interval and on the greatest change in position (displacement) respectively under the given graphs. M13.5 was on calculating the greatest area in a given interval, while P17.1 was on calculating change in velocity within a given interval. M4.2 and P20.3 were on choosing the correct graph that represents the given functions or equations. In all these pairs, participants performed practically significantly better in the mathematics questions than in the

physics questions. This result indicates that the participants had the mathematics knowledge, but did not transfer it to the physics contexts.

In the pairs M10.2 x P6.1 and M4.1 x P20.1, the effect size was very small, meaning the students performed similarly in each pair of mathematics and physics questions. M10.2 was on finding the y-coordinate when the x-coordinate is given and P6.1 was on calculating the instantaneous velocity at a given time. The percentage performances in both M10.2 and P6.1 were both very good (the percentage of participants who had both correct was high), meaning participants were able to apply their mathematics knowledge in the physics questions. Questions M4.1 and P20.1 were on choosing the graphs that represent the given linear equations. . In this pair the percentages correct were similar (61 % and 65 % respectively).

In the pair M11.2 x P6.4, a high percentage of students answered both wrong, indicating that the participants lacked the necessary mathematics knowledge and therefore they could not apply it in the physics questions.

A comparison of the results obtained per construct show medium to large effect sizes in 4 out of the 5 pairs on area and 2 out of the 3 pairs on form/expression. For these constructs it thus seems that students did not transfer their mathematics knowledge to the corresponding physics setting. With regard to the constructs gradient and reading of coordinates, the pairs of questions yielded small effect sizes (i.e. similar results), indicating that the students who possessed the mathematics knowledge also applied it in the physics contexts.

Table 5.7A shows a pairing of a few selected physics questions (P). The pairing is done based on the physics aspect or conceptual understanding the paired questions carry. Percentages of total correct in each question of the pairs and the corresponding coefficients of the effect sizes are tabulated in Table 5.7A.

Table 5.7A Results of the effect sizes, w-values: Some paired Questions, Physics only

Aspect	Paired questions (P1 x P2)	Total P1 correct	Total P2 correct	p-value	w-value
Area under graph/displacement	P18 x P19	59.21	50.66	0.0741	0.14
Form of graph/Qualitative gradient	P4.1 x P4.2	59.21	48.69	0.0325	0.17
Reading from graph	P5 x P21	9.87	34.87	<0.0001	0.44

In the pair P5 x P21, the effect size is $0.3 < w < 0.5$, meaning participants answered differently with medium effect size. The questions tested the interpretation of a given position-time graph and the meaning of the constant velocity of given magnitude. Total P1 correct is lower than total P2 correct, meaning the participants could to some extent interpret the given magnitude of constant velocity, but struggled to interpret the position-time graph. This result shows a lack of conceptual application and understanding of both these physics concepts.

For the pairs P18 x P19 and P4.1 x P4.2, the effect sizes are $0.1 < w < 0.3$, meaning the participants answered with small difference.. P18 x P19 tested what the area of a v-t graph represents and how to obtain displacement in a v-t graph respectively. Questions P4.1 x P4.2 tested on negative acceleration and negative gradient of a given v - t graph. The percentage correct in P18 and P4.1 is a little bit higher than in P19 and P4.2. This means participants are confused between these concepts' meanings to a lesser extent.

At P5 and P21 where the participants answered differently to a larger extent, the percentage for both wrong is high. The percentages correct are 9.9% and 34.9% respectively. This indicates that the participants could not answer these questions and did not understand how to think about the questions. It is therefore an indication that participants have difficulty in interpreting the motion of the given graph (P5) and have difficulty in determining the type of motion if the velocity is constant (P21). It seems

participants are more confused if they have to explain motion from a given graph than when it is stated in words.

Table 5.7 B Results of the effect sizes, w-values: Some paired Questions, Mathematics only

Aspect	Paired questions (M9.1 x M9.2)	Total M9.1 correct	Total M9.2 correct	P-value	w-value
Reading the value vs Qualitative gradient	M9.1 x M9.2	32.11	32.90	0.0348	0.17

In Table 5.7B above, mathematics questions M9.1 and M9.2 are compared. The highest percentage was obtained for participants answering both wrong. The small effect size, $0.1 < w < 0.3$ indicates that participants answered the questions similarly. In both questions the percentage correct, 32.1 and 32.9 respectively, is low. This means participants have difficulty in interpreting curved graphs. It is clear that the misconception or misunderstanding that causes the participants to answer wrong (low percentage of total correct) was the same for both cases. Maybe it was the multiple intervals over which the function changed that confused them. If it was that, they didn't associate increasing function with positive gradient, and the percentages of answering differently would have been the highest and the effect would have been larger.

The results prove that participants have difficulty with analysis and interpretation of algebraic and kinematics graphs, as well as relating them, or they lack mathematics (algebraic) knowledge and skills to solve physics (kinematics) graphs as reported in chapter 3.

Participants performed better in mathematics questions than in physics questions in all constructs. This aspect is discussed in chapter 2 under graph comprehension and is addressed under the problems and solutions (interventions) participants apply in kinematics graphs in paragraph 3 of chapter 3.

5.4 Results of the interviews

Table 5.8 Results from interview questions (interviews)

Aspects evaluated	Questions	Number of participants (out of 14)			
		Correct	Partially correct	Incorrect	Not answered
Basic theoretical knowledge of kinematics					
Definitions & units: displacement velocity	1	10	3	0	1
		12	2	0	0
Equations of motion	2	3	10	0	1
Average and instantaneous velocity	6	0	5	7	2
		3	0	9	2
Average and instantaneous velocity	7	2	5	2	5
		0	0	7	7
Basic interpretation of kinematics graphs					
Gradient s-t graph	3	0	11	1	2
Gradient v-t graph	4	5	4	5	0
Area v-t graph	5	1	5	5	3
Average and instantaneous velocity	8	8	0	0	6
Mathematics knowledge applied in kinematics graphs					
Equation of v-t graph	9 10	2	5	4	3
		5	2	5	2
Area of v-t graph	11	-	-	-	-
Area of a-t graph	12	3	1	6	4
Gradient of s-t graph	14	0	2	1	11
Gradient of v-t graph	13 14	0	9	3	2
		4	1	1	8

A sample of 14 participants taken randomly from the population sample that undertook the questionnaire, participated in the interviews as discussed in section 4.4.4 of chapter 4. Their responses to the questionnaire in Appendix C and D were recorded in writing and analysed. The results are summarised in table 5.8 and discussed below.

Question 1

Almost all the participants (ten out of fourteen) could define displacement in question 1, and thirteen of them were able to give its SI unit. The definitions that they generally

gave for displacement were “distance between point A and point B”, “difference between two points”, “a path from beginning to end” and “product between velocity and time”. The first quote mentioned above indicates that the student could not differentiate between distance and displacement. The third quote reveals knowledge of displacement having direction, but neglects the magnitude of the displacement. In the last quote, the mathematics expression of displacement is used.

Twelve participants defined velocity (change in displacement in a certain time) correctly and had the unit (m/s) correct. The other two participants defined velocity partially correct. Their definitions were “a quantity with both magnitude and direction, speed of an object”, which is the description of all vectors, not only velocity, and “distance over time”. The latter quote neglects the words “change in” distance and “change in” time and did not refer to its direction. All the participants were able to state the SI unit of velocity. This shows that participants have a fair knowledge of basic kinematics concepts.

Question 2

Five of the participants gave all four kinematics equations correct of which two didn't define the symbols used. Three of the participants gave three of the four equations correct and defined the symbols. The equations they gave incorrectly were $v = d/t$, $\bar{v} = (u + v)/t$ and $a = (u + v)/t$. A further four could give two of the equations correct, of which two of them also defined the symbols correct. None of them gave the equation $s = ut + at^2/2$ and three of them didn't give $s = (u + v)t/2$ with the fourth one giving this equation incorrectly as $s = (u + v)t/x$. One of them also gave the incorrect equation $a = (v - u)/x + t$ and another repeated $v = u + at$ as $a = (v - u)/t$. One participant only gave one equation ($v = u + at$) and one did not answer the question. The answers of the participants do indicate that most of them know most of the kinematics equations and the meaning of the symbols.

Question 3

This question is based on construct 2 in tables 4.3 in chapter 4 and table 5.5 in chapter 5, which tests the qualitative understanding of the gradient of a displacement-time (s-t) graph. Only one student defined it correctly as “average velocity”, while nine defined it

simply as “velocity”. Another one defined it in terms of a formula, $\frac{\Delta s}{\Delta t}$, to calculate it from the given graph. This shows a misconception or misunderstanding between average velocity (average gradient over an interval) and instantaneous velocity (gradient of secant). Two participants could not answer the question, while one said it is a slope. Except for confusion of the terms average velocity and instantaneous velocity, most participants seem to know what the gradient of an s-t graph is or how to find it. Participants defined the gradient of the displacement-time graph as the velocity, not knowing that instantaneous velocity is the gradient at a point (gradient of secant), while average velocity is the average gradient over an interval. This further indicates confusion of the ratio of the coordinates instead of the ratio of the change in coordinates, as has been found in the questionnaire results. Many participants could pick up that velocity is the gradient of displacement-time graph.

Question 4

This question is also based on the qualitative and quantitative understanding of gradient of a graph according to construct 2 of tables 4.3 and 5.5. Five participants gave the correct answer as $\Delta v/\Delta t$, “gradient” or “slope”. Four had it partially correct giving expressions v/t and $\Delta v/t$. The latter participants used only the coordinates v and t instead of the change in coordinates Δv and Δt . One participant’s response was “look at the line”, the other as “area under the graph” while another defined it as “points on the graph (points of intersection)” and another defined it as the “x factor”. The implication of the responses in this question is similar to that of question 3 above. The only difference is that it is the reverse, i.e., the answer for question 3 is question 4 and *vice versa*. Only one-third of students calculated acceleration correctly from the velocity-time graph and another one-third were confused as to how to find the acceleration, though some do not know the term and some confuse the formulas, i.e., the relationships between the concepts. The lack of conceptual and procedural knowledge about acceleration is evident from their responses.

Question 5

This question is based on construct 1 of tables 4.3 and 5.5 on the meaning and determination of the area under the graph. Only one student got it correct, five defined the displacement as product of velocity and time or $s = vt$, four defined it as gradient/slope or $s = \Delta v/t$ or $s = \Delta v/2a$, one defined it as points on the graph, while three did not give any answer. It seems most of the participants do not know the meaning of area and how to find the area, or could not associate or relate the mathematics term *area* with the physics term displacement, i.e., they don't know how to calculate displacement from a velocity-time graph.

Question 6

This question is based on constructs 2 and 3 of tables 4.3 and 5.5. None of the participants could correctly define average velocity. Four did indicate it as $(u + v)/2$ but this only holds over time intervals when the acceleration is constant. Another participant also had it partially correct indicating it as " $v = s/t$ ". Three of the participants wrongly said it is the sum of velocities divided by the number of velocities, one just said it is the sum of velocities and the other said it is the sum of the initial and final velocity. Other wrong answers were "average speed" and "velocity that increases over time". Three participants correctly indicated that the instantaneous velocity is the velocity at a certain instant. Two participants wrongly described it as the exact or instantaneous speed. Other wrong answers were "the velocity that remains constant" (three participants), "average speed divided by average time", "just a velocity", "single velocity" and "velocity of just one situation". Two participants didn't answer this question. This shows confusion between the average velocity and instantaneous velocity, though most can work out or define average velocity. The majority also have a problem with instantaneous velocity.

Participants defined the gradient of the displacement-time graph as the velocity, implying instantaneous velocity, instead of specifying by saying average velocity. They also could not tell how to find the average velocity and acceleration from a displacement-time graph and velocity-time graph respectively (Questions 3 and 4). Although most could tell that the gradient of the displacement-time graph represents the

velocity, only some of them could tell that the gradient of the velocity-time graph represents acceleration. Since they did not know how to find displacement from the velocity-time graph (Question 5) they could not tell when it is smaller or greater. Participants showed that they do not know the difference between average velocity and instantaneous velocity. This result gives insight into the problems that the participants experienced with answering questions on constructs 2 and 3 of the questionnaire.

Question 7

This question is related to question 6 above. In this case, the participants had to give the quantitative definition of the instantaneous and average velocity and acceleration. Two of the participants correctly indicated that the average velocity can be calculated by $\Delta s/\Delta t$ and the other five got it partially correct by using the expression $(u + v)/2$ which actually only holds when the acceleration is constant. Wrong answers given were “getting final to initial” and “sum of initial and final velocity”. Five of the participants did not show how to calculate the average velocity. No participant could correctly or partially correct indicate how to calculate the instantaneous velocity. Seven gave wrong answers and seven did not answer. The wrong expressions given were “ s/t ” (two participants), “ dv/t ”, “ v_1 ” (two participants), “ $d/\Delta t$ ” and “ $\lim_{\Delta t \rightarrow 0}(v - u)/\Delta t$ ”. This is a further proof of the inference stated in question 6 above, i.e., confusion and lack of knowledge about determining instantaneous velocity as compared to average velocity. They are confused on calculations of average and instantaneous velocities.

Question 8

Question 8 tested constructs 2b, 3a and 4 according to tables 4.3 and 5.5. Three participants said the average velocity can be calculated from the displacement-time graph and that the instantaneous velocity can be calculated from the velocity-time graph. Otherwise, to this four participants said the average velocity can be calculated from the velocity-time graph and the instantaneous velocity from the displacement-time graph of which one indicated that the instantaneous velocity can also be calculated from the velocity-time graph. One participant said that both the average and instantaneous

velocity can be calculated from the displacement-time graph. Six participants did not give any answers to this question.

After analysing the answers of the participants it was realised that average and instantaneous velocity can be calculated from both. From the velocity-time graph the instantaneous velocity can just be read off on the vertical axis and the average velocity can be calculated by $\left[\int_{t_1}^{t_2} v(t) dt \right] / (t_2 - t_1)$. Most likely, the participants would not have known the latter since it is only taught in first year mathematics and usually in the second semester. In this case however, since the acceleration is constant, the average velocity can be calculated by $[v(t_2) + v(t_1)]/2$. From the displacement-time graph, the average velocity can be calculated by $[s(t_2) - s(t_1)] / (t_2 - t_1)$ (average gradient) and the instantaneous velocity by ds/dt (gradient at the point – gradient of the tangent) at the instant asked. The participants should know both these calculations and should be able to do them. In this specific case shown, the average and instantaneous velocity in the displacement-time graph are equal since the gradient is constant. For the graphs shown, the participants should be able to calculate both average and instantaneous velocity from both the graphs. Therefore, all answers are correct and by no means it is possible to tell whether they really understood what they answered or whether it was just a guess. Therefore, no true deduction or implication can be made from the answers to this question.

Question 9

This question was based on construct 4 in tables 4.3 and 5.5, testing which kinematic equation will give a straight - line graph. Only two participants had this question completely correct giving the equation $v = u + at$ indicating a as the gradient and u as the intercept on the velocity axis. Five participants had this question partially correct giving the correct equation but with incorrect explanation (two) or no explanation (three). The incorrect explanations given were that velocity is directly proportional to time and the acceleration is constant. Four participants gave the incorrect equations $v^2 + u^2 + 2as$ and $s = tu + at^2/2$ with no explanation, $d/t = s/t, \Delta v/t$ explaining that these equations will give straight lines and because velocity increases as time increases and

with $v = \Delta s/t$ explaining that the velocity is directly proportional to time and with the acceleration as the gradient. Three participants did not answer this question. This shows that more than half of the participants could relate kinematic equations and algebraic equations for a straight - line graph, though 3 of them did or could not explain their choice. Another confirmation as to why participants could not calculate or determine acceleration from a velocity-time graph.

Question 10

This question is similar to question 9, but asked the meaning of the variables from an algebraic perspective to kinematics, i.e., state what the algebraic terms or symbols stand for in kinematics terms. Five participants got this question correct and another two got acceleration correct not giving the meaning of the y-intercept (correct). The rest (five) were completely incorrect. They incorrectly gave the gradient of a v-t graph as “at” and “ v_i ”, displacement as “ v_i ”. Others gave explanations that are irrelevant to the question, like “velocity is directly proportional to time taken”, “velocity is greater and directly proportional to displacement therefore displacement is greater”, “y-intercept is the displacement”, “ $\Delta v/t = \text{gradient}$ ” and “ $\Delta x = (v_1 + v_2)/2 \Delta t$ ”. This shows confusion and a lack of conceptual understanding of a relation/similarity between physics terms or symbols or equations with mathematics terms or symbols or equations.

Question 11

This question was based on construct 1 in table 4.3. From the answers given by the participants, it is clear that they did not understand the question correctly and the results could not be used to make any deductions.

Question 12

This question is based on constructs 1 in table 4.3. Three participants gave correct answers. One participant gave a partially correct answer saying, “gradient is change in velocity”. Six participants gave wrong answers and four did not answer. The wrong answers were “velocity constant and acceleration zero”, “continuous acceleration”, velocity increasing as time increasing over the interval 0 to 5 and acceleration

decreasing as time increasing over interval 7 to 10, “change in velocity zero”, “displacement” and “negative”. Clearly most participants did not know what could be determined from the acceleration-time graph, neither the acceleration nor the velocity.

Question 13

This question is based on constructs 2b and 3b. None of the participants could answer this question correct. Nine of the participants correctly indicated that the gradient of a velocity-time graph slanting upwards (downwards) presents positive (negative) acceleration. Six of these participants also indicated the positive (negative) acceleration as increasing (decreasing) and the others did not indicate whether it is increasing/decreasing/constant. There were three wrong answers stating “negative”, “constant speed and no acceleration” and line “DE”. Two participants did not answer the question. Still this shows a lack of conceptual knowledge and understanding from graphs, what a positive or negative acceleration stand for, and they are confusing acceleration and velocity in terms of their gradients. Participants could tell that the gradient of a graph slanting upwards is positive, but could not tell that it represents a constant acceleration and why. This is an indication of confusion between slope and height in a graph.

Question 14

This question is based on construct 2 in table 4.3 and is similar to question 13 above in terms of testing for what a certain type of gradient means. Four participants were able to tell that zero gradient in a velocity-time graph means the velocity is constant, the acceleration is zero, and one only said the velocity is constant. No one answered the part about the displacement–time graph correctly. Two had this part of the question partially correct stating zero velocity and constant displacement respectively. Four gave various responses, viz., “slope is totally horizontal”, “it is constant”, “velocity is zero” and “negative”. This shows that the majority of participants lack conceptual knowledge and understanding of what a constant gradient is or what it means from both the velocity-time graph and displacement-time graph.

Participants seem to lack knowledge and understanding of physics concepts and equations. Hence, they failed to interpret the questions. For that reason, they were unable to answer the question correctly or they gave irrelevant answers. Even with mathematics concepts and perspectives, they were unable to associate the meaning of a gradient when the shape of a straight line graph changes, i.e., horizontal, slanting upward or downwards, as constant or zero, increasing or decreasing. Majority (more than 70 %) struggled with answering questions on zero gradients.

5.5 Integrated discussion of results

This section gives an interpretation and reflection of the results of the statistical analysis of the responses of the questionnaire and from the interviews, i.e. the quantitative and qualitative investigations. The results are supported in paragraphs 3.3.2 and 3.3.3 in chapter 3 by the findings of previous researchers (McDermott *et al.* 1987; Towbridge & McDermott, 1980 and 1981, etc.). The results show that students have difficulty interpreting changes in height/ position and changes in slope/ gradient (McDermott *et al.* 1987). Towbridge and McDermott (1981) also state that another difficulty is the inability of students to associate these changes with the numerator and denominator in gradient calculations. The observations made from the constructs investigated are as discussed in the following sub-sections.

5.5.1 Averages

The results shown in Table 5.2 and a comparison of Graph 5.1 and Graph 5.2 clearly indicate that the participants performed better in the mathematics questions than in physics. This is evident from the fact that average percentage performances in mathematics questions (62.7 %) are higher than that in physics questions (34.7 %) an indication that participants possess or have mathematics skills and knowledge of functions and line graphs, but don't always know how to use those skills and knowledge efficiently to solve kinematics equations and graphs.

The finding of this study that participants performed better in mathematics (Table 5.2) seems contradictory to the biographical data (Table 5.1) according to which they performed better in Grade 12 physical science than in mathematics. This might be because grade 12 physical science is composed of both chemistry and physics,

meaning they might have been helped by chemistry to perform better in physical science than in mathematics.

Tables 5.6, 5.7A and 5.7B show in general effect sizes of less than 0.6. The majority of participants' responses have a small effect size. This means the majority of the participants answered the pairs of questions similarly, i.e. either both the mathematics and physics questions wrong, or both correct. This was not the case with the constructs on reading of coordinates and gradient pairs, in which they answered differently. Medium to large effect sizes mainly occurred in the constructs area and forms/expressions of graphs.

5.5.2 Area under the graph

When coming to given mathematical shapes, the majority (over 70%) of participants did well in determining which shape has the smallest and largest area, meaning they understand or know how to determine the area by inspection. An average of more than 60% of students was able to calculate the smallest and largest areas from a given mathematics graph. In the physics graphs, the majority of participants were unable to determine the largest and smallest areas. This might be because in physics the area is given a meaning and therefore participants are unable to associate displacement with area under the v-t graph and velocity with area under the a-t graph. The same problem was proven in the interviews, as participants could not explain how to calculate area (displacement) in a velocity-time graph. In the questionnaire, 30% and less students could calculate the change in velocity from an acceleration-time graph.

5.5.3 Gradient of a graph

The results show that the majority of participants were unable to determine in both the mathematics and physics sections the x-intervals (domain) of a graph with positive/negative/constant gradient. It seems that participants confused gradient and function value in mathematics and consequently confused gradient and height in physics questions. This is confirmed in the interviews where most students, more than 80% of them, could not differentiate between these concepts in their answer in the interviews. Various researchers (McDermott et al. 1987; Towbridge & McDermott, 1980

and 1981, etc. as stated in paragraph 3.3.2 in chapter 3) support the height-gradient confusion.

The majority of participants could find the gradient of a velocity-time graph in some of the items, but struggled to find the gradient in a displacement-time graph in question 14 of the interviews. This is unlike in item 13 of the interviews where 8 out of 14 were able to find the gradient of v-t graph correctly.

Participants were also able to find the gradient if $\frac{y}{x} = \frac{\Delta y}{\Delta x}$, i.e., when the graph intercepts the origin in both mathematics and kinematics questions, but they could not find the gradient of a section of a line graph that does not pass through the origin, i.e., when $\frac{y}{x} \neq \frac{\Delta y}{\Delta x}$. The interview responses confirm that they incorrectly use ratios of quantities instead of ratios of changes in the quantities. This is confirmed by Trowbridge and McDermott (1980), and Trowbridge and McDermott (1981), as discussed in paragraph 3.3.2 in chapter 3.

5.5.4 Reading from the graph

The results show that the vast majority of students, over 80%, do not have difficulty with reading coordinates from the graph, both in mathematics and physics. This means they have the required mathematics (algebraic) skill and knowledge and they are able to use that skill and knowledge to apply, solve and answer kinematics graphs. Participants only seem to encounter a problem where two x-values have the same y-value, as a little less than 50% got it correctly. The participants might also be confused with finding the domain if the range is given, not realising that the procedure of finding x-value if y-value is known is the same as finding the y-value if the x-value is given. This might emanate from teaching of graphs in the classroom where their curriculum and questioning is mainly about finding the range when the domain is given or known.

5.5.5 Form of graphs / Expression

Participants did not have difficulty in finding or naming the form of graphs in mathematics and they could also find or name a form of linear graphs in kinematics. They could not associate variables in the non-linear forms or expressions in kinematics with variables in non-linear algebraic forms / expressions. They could not explain this in

the interview questions. This is an indication that they do have mathematical knowledge, but somehow they could not apply this knowledge in solving non-linear kinematics problems. This is one inference not encountered in the literature review.

5.6 Conclusion

This chapter shows how the available sample at CUT struggle with difficulties in the analysis and interpretation of algebraic and kinematics functions and graphs. The results confirm the problems stated in chapter 3 by previous research studies about participants' difficulties with kinematics graphs. Basson (2002) and Beichner (1994) found that undergraduate students tend to have difficulty with the comprehension, reading, analysis and interpretation of graphs in Physics. Even on university level students have serious gaps in their understanding of various topics, including kinematics graphs (McDermott & Redish, 1999). Friel *et al.* (2001) say that students are unable to use the prediction skills in solving physics graphs that physics students have from their previous mathematics knowledge and background on linear functions and graphs.

The next chapter, chapter 6, will give a conclusive summary of the findings of the analysis of these results and provide answers to the research questions. The chapter will also try to suggest solutions to alleviate the students' difficulties, as well as come up with implications for teaching and learning.

CHAPTER 6 - FINDINGS, CONCLUSIONS, AND IMPLICATIONS OF THE STUDY

6.1 Introduction

The main focus of the study is to determine how the learning of algebraic graphs in mathematics affects the learning of kinematics graphs in physics. The participants were 152 first-year undergraduate physics students at CUT, South Africa.

In chapter 1 of this dissertation, the researcher gave an overview of the motivation, objectives, and research design that assisted to reach the aim of this study. A literature review (chapters 2 and 3) served as theoretical framework in the design of the investigation (chapter 4) and analysis of the results (chapter 5). Chapter 5 presents, analyses and discusses the qualitative and quantitative data obtained in an effort to explore the research questions.

This final chapter serves to tie all the loose ends and to present answers to the research questions. The researcher does this by summarising the findings, discussing the interpretation of the results, and presenting the conclusions and implications that relate to the literature and the empirical investigation in respect of the research questions. Limitations of the study and suggestions for further research are also given. Lastly, the significance of the study is related to both the institutional (local) and national contexts.

6.2. The literature study as the framework of the empirical study

The literature study served as a framework to compile the research instruments and interpret the obtained results. In chapter 2 various aspects regarding graph comprehension (section 2.2) and graph sense (section 2.3) are discussed and these were covered by the research instruments. The students' conceptions of these aspects were identified and discussed in the form of constructs (table 4.4 and table 5.4). In section 2.4, the literature study covers different strategies and interventions on how graph comprehension and graph sense can be taught. Graph comprehension and sense will only become feasible through effective strategies and methods of teaching of

graphs (see 2.4). The implications (see 6.5 below) give possible solutions to the problems identified in the results of the study.

Friel *et al.* (2001) further identified four critical factors of graph comprehension, viz., purpose of using graphs, task characteristics, discipline characteristics, and reader characteristics. This study focussed mainly on task and discipline characteristics of graphs. The research determined what constitutes a graph in terms of graph comprehension and graph sense in the context of mathematics and kinematics on first-year university level.

Students usually come to understand the meaning of a graph from a mathematics perspective before they are expected to present and interpret it in kinematics or any scientific or non-scientific research or experiment. They should therefore be able to use a graph to summarise and conclude on a scientific or non-scientific investigation of any experiment and/or research (Deacon, 1999).

The structure of the questionnaire and interviews relate to the term graph comprehension. An investigation of graph comprehension (see 2.2), its meaning together with needs and factors associated with the meaning, gives an overview of what should be or is tested and studied in the research. Furthermore, the research aim was accomplished using graph sense, i.e., the thinking behind working with graphs (see 2.3). This is cemented in the structuring of the constructs used and in the formulation of interview questions under the research design (see table 4.3 in chapter 4).

The literature review included an overview of the constructivist teaching strategies (3.4) to address learning gaps and deficiencies of students in the understanding and knowledge of kinematics graphs (3.3). Combining these gaps and deficiencies (problems) with the suggested solutions, an unbiased and objective study was conducted based on the research design (chapter 4) and in the discussion of the results (chapter 5).

In chapter 3 the literature review emphasised the importance of social constructivism as a learning theory (section 3.2) according to which previous knowledge and experience are used to construct new knowledge. In the learning process students infer from

previous or personal background knowledge, i.e., making own conclusions or interpretations based on previous knowledge (Friel *et al.* 2001). This is important in the empirical study, since the kinematics questionnaires and interviews were based on what the participants should have known, learnt, be taught or constructed from the mathematics and physical science classes in the secondary school. Constructivist teaching strategies (section 3.4) intends to address learning gaps and deficiencies of students. The empirical study was used to investigate if the participants are encountering difficulties with kinematics graphs such as finding gradients of line graphs not passing through the origin and interpreting the meaning of the area under the graphs (Beichner, 2007; McDermott *et al.* 1987).

6.3 Design of the empirical research

Problems with regard to using mathematics knowledge and skills of graphs in solving kinematics graphs were investigated in the undergraduate physics students at CUT, South Africa. The research methodology in chapter 4 explains in detail how these problems and solutions were investigated in the study.

To obtain answers to the research questions, the researcher followed and described a mixed method approach consisting of quantitative and qualitative methods to gather and analyse the data. The quantitative data collection strategy helped in obtaining numerical data about participants' responses to the questionnaires that could be analysed statistically, while the qualitative data analysis comprises of interpretation of the results of interviews. To test for validity and reliability of the results, statistical procedures were used.

The participants' competencies in mathematics and kinematics graphs were compared using descriptive statistics. The effect sizes of pairs of questions give evidence of this. An effect size, $w = 0.1$ means a small effect, $w = 0.3$ is a medium effect, $w = 0.5$ is a large effect, while $w \geq 0.5$ is considered as a practically significant effect (see 5.3.5).

6.4 Results of the empirical study

The literature review, as well as the empirical research enabled the researcher to investigate and come up with answers to the research questions of the study

6.4.1 First research question

What deficiencies do the group of first-year physics students at CUT have with regard to conceptual knowledge of graphs in mathematics and in kinematics?

The first-year undergraduate physics participants performed poorly in kinematics graphs with an average percentage of 34.7%. The average percentage performance in mathematics was 62.7%. The constructs (Table 4.1) and the discussion of their results (Tables 5.5 and 5.6) gave an indication of the deficiencies participants have with graphs in mathematics and kinematics. They performed well (approximately 60% and above) on aspects related to reading from graph - coordinates (3a), gradient - quantitative (Line intercepts the origin) (2.b.i) and half of form of graphs/ expressions (4), while in the rest of the aspects the performance was far below 50% to as low as 1%. They lack the necessary conceptual understanding, knowledge (tools) and skills of mathematics to solve kinematics graphs, especially in constructs on integration / area under graph (1a), differentiation / gradient - qualitative (2a) and quantitative (line does not intercept the origin) (2b.ii) and reading from graph - value (increasing/ decreasing/ constant) (3b). The findings prove that learners could not comprehend mathematics graphs and struggled to use their previous knowledge in mathematics from their lower grades (high school) to answer some of the physics questions, for example, question P2 of questionnaire.

The results confirm findings by previous researchers (Towbridge & McDermott, 1980:1981, McDermott & Redish, 1999, Basson, 2002; Beichner, 1994, etc.) that students encounter problems with interpretation and analysis of kinematics concepts and graphs. The difficulties found in the empirical study include discriminating between the gradient and the height of a graph (slope-height confusion) and interpreting changes in height and gradient from a graph.

Apart from the difficulties discussed above, the results of the questionnaire and interviews indicate that the participants are confused and are struggling with perceptions and calculation of the kinematics concepts themselves, e.g. average velocity, instantaneous velocity, displacement and acceleration, both in a graph and in equation form. These difficulties and deficiencies were revealed in all the constructs of the questionnaire, as well as the interviews. Participants were consequently unable to link these kinematics concepts with concepts of gradient and the area of kinematics graphs.

Although participants performed better on overall in mathematics questions than in kinematics questions, there is still a need for improvement in terms of conceptual knowledge and understanding of both mathematics and kinematics graphs.

6.4.2 Second research question

How effectively do they transfer knowledge from mathematics to kinematics?

The results of the questionnaire (Table 5.6) indicate that participants effectively transferred their knowledge of mathematics graphs to kinematics graphs in some of the constructs, but not in others. With regard to the three constructs gradient, reading coordinates and form/expression of graphs, students who possessed the mathematics knowledge generally also answered the corresponding physics questions correctly, while students who lacked the mathematics knowledge also performed poorly in the physics question. A different situation occurred with regard to the construct area, where the students did not generally transfer their mathematics knowledge to the physics contexts. The interview results indicate that inadequate knowledge of the kinematics concepts may have contributed to the poor physics performance.

Participants struggled to relate concepts such as area in mathematics with concepts in kinematics graphs. They seem to treat them as unrelated concepts and as separate entities. The ability to transfer knowledge can be associated with the ability to know, analyse and critique the given data/information or graph, or to present that data in graphical form (Friel *et al.* 2001).

Few (2006) and Connery (2007) both state that a graph as a qualitative description helps to establish if there are mathematical and functional relationships between variables, something participants struggled with. This research discovered that many participants could not or did not relate the variables in mathematics with variables in physics. For example, some could not associate velocity and acceleration (kinematics concepts) with gradient (mathematics concept) in s - t and v - t graphs respectively, or deduce displacement (kinematics concept) from the area (mathematics concept) of a v - t graph. Even students who had the mathematics knowledge of concepts such as area could often not transfer it to the physics context. The inability of participants to effectively transfer or use variables might be due to a difficulty to interpret these variables in kinematics.

The researcher deduces that having pre-knowledge and understanding of the concept of graphs does not necessarily lead to effective transfer to another problem or context.

6.4.3 Third research question

What possible reasons can be given for the problems that the participants experience with kinematics graphs?

The research results agree with previous researchers (paragraph 3.3.8) that students' inability to respond effectively to the questions on kinematics graphs is due to several factors. Those factors include that students' inadequacies are due to complexities in comprehending graphs (Basson, 2002; Palmquist, 2001), lack of algebraic knowledge and skills in physics among teachers (Molefe, 2006), and the inability of students to recognise that physical quantities can be solved using mathematics knowledge (Khan et al. 2011). An additional problem found in this study is students' incomprehension of the basic kinematics concepts. For example, students who do not know that acceleration is the rate of change in velocity will find it difficult to understand why the acceleration of an object in linear motion equals the gradient of a velocity-time graph. Apart from the factors given above, deficiencies in conceptual knowledge of the basic kinematics

concepts thus contributed to the problems that the participants experienced with kinematics graphs.

6.5 Implications

From the findings from the literature study, theoretical review, quantitative and qualitative analysis and the interpretation of the results of the questionnaire and interviews, the researcher came to the following conclusions:

(Note that kinematics is part of introductory physics.)

- The two tenets of constructivism, constructed knowledge and adaptive learning, as stated by Roth (1993) can be used to recommend which strategy should be used to learn graphs. Students have to gain constructed knowledge on graphs to make their own interpretations and conclusions. Furthermore, interactive devices in the form of online educational systems have to be used to facilitate students' performances.
- The different teaching strategies and methodologies in mathematics associated with learning and teaching of kinematics (physics) have to be observed. The method must be procedural and conceptual to encompass step-by-step algebraic procedures and manipulation, decision making, intuitive and constructive learning.
- Mathematics teachers should be able to show where the knowledge and skills in mathematics apply to other subjects such as physics, i.e. mathematics teachers in schools should be taught introductory or basic quantitative literacy on graphs as part of their qualification.
- Physics teachers without sufficient background should be taken for advanced graph training. Physics teachers should get to know and understand the links between graph concepts and skills in mathematics and physics to be able to teach graphs more effectively.
- There is a need to incorporate the teaching of graph comprehension and graph sense in high school curricula in mathematics and physics.
- Learners at high school should be engaged in activities that develop problems solving in real life contexts (like kinematics) by collecting data.

- There should be regular in-service training by properly qualified and skilled instructors to empower teachers with challenges facing secondary school and first-year undergraduate students in terms of prior learning.
- Wherever possible or as much as possible automate computations and graphics like simulation lessons should be introduced in school so that learners can understand the importance of graphs in kinematics and mathematics. In so doing they will be able to present, analyse and interpret graphs and know how to obtain, handle and use data (Shaughnessy *et al.* 1996, Friel *et al.* 2001).
- Before kinematics graphs are presented, the teacher should ensure learners' thorough understanding of the relevant basic kinematics concepts and mathematics knowledge.
- During teaching of kinematics graphs instructors should repeatedly focus students' attention on related prior knowledge and correspondences between mathematics and kinematics graphs and equations.

6.6 Limitations of the study

The study was conducted with first-year undergraduate physics students at Central University of Technology, Free State at the Bloemfontein campus only. Therefore, it is not possible to generalise on the results of this research. The study was voluntary and a high percentage, 152 out of 234 of students, participated in the research study.

The researcher therefore acknowledges that only tentatively formulated conclusions can be drawn regarding the teaching and learning of graphs in linear algebra (mathematics) and kinematics (physics) curriculum.

Though data of the study included biographical information, the researcher did not examine its influence on the knowledge and understanding of graphs in linear algebra and kinematics. Such an examination could have provided more information in detail regarding this research.

6.7 Significance of the study

Since the study is on deficiencies with using knowledge of linear function and graphs in mathematics to solve kinematics graphs (see 1.9), it can serve as guiding tool for best practices in the teaching and learning of graphs in mathematics and kinematics at high school and at first-year university level. The results should be used to enhance teachers' professional development level by extending their knowledge in the teaching of graphs in linear algebra and kinematics. It will also benefit first-year university students to be able to apply their prior knowledge to solve kinematics problems and present their laboratory experiment report using graphical representation. The research results indicate how first-year undergraduate physics instructors can be helped to identify their students' problems on graphs. Solutions to effectively analyse and interpret kinematic graphs, especially when writing a laboratory experiment report, are recommended (6.4).

Furthermore, tertiary institutions will also be able to train future scientists (physicists) without having to spend more time on the basics if the foundations are properly laid. As a result, one of the serious gaps in the understanding of important topics is that students emerge with these gaps in their study of Physics. This is demonstrated in the students' responses from the questionnaire.

6.8 Further Research

Since the study was conducted on one campus of one institution (CUT, FS) in South Africa, it can be expanded to include the other campus of CUT, FS, as well as other institutions of higher learning in South Africa. This will help to conclude on the state of knowledge and understanding of linear algebraic (mathematics) and kinematics (physics) graphs in South Africa.

Further research can include factors like:

- Perceptions of grade 10–12 educators/teachers on the teaching and learning of graphs in linear algebra and kinematics

- What conceptual understanding and knowledge are needed to empower mathematics and physics teachers/academics with literacy, reasoning and thinking skills in linear algebraic and kinematics graphs?
- Assessing learners' reasoning and thinking in the understanding and knowledge of graphs in linear algebra and kinematics
- How physics academics can overcome the barriers of undergraduate students with deficiencies in application of linear algebraic graphs into kinematics graphs.

6.9 Final conclusion

From the discussion of results in this chapter, the researcher agrees that the understanding of graphs from a mathematics and a physics perspective is an all important and necessary skill and tool that undergraduate students must have. Hence, presentation and interpretation of graphs by undergraduates in their studies should play a significant role in achieving a higher level of academic and scholarly excellence and in preparing a public awareness and understanding of science and science processes (Ovseiko, Oancea & Buchan, 2012), thus in a way prepare and produce future scientists (physicists) and or scientific researchers.

First-year undergraduate physics students at CUT have difficulty using some of the mathematics skills and knowledge and hampering the effective learning and teaching of kinematics. This is in line with the hypotheses of the research (see 1.6) that undergraduate physics students do have some necessary mathematics knowledge and skills on graphs, but cannot effectively apply it in kinematics. They often fail to integrate mathematics knowledge into kinematics.

It seems as if whatever undergraduate physics students have been taught in their mathematics class either from high school or in their first-year mathematics class, does not link with problem solving in kinematics (physics). The students could hardly associate or think of applying that to solving physics problems. Only a small percentage could figure it out. As a result, it seems students lacked the necessary mathematics skills and knowledge. In some cases they had the required mathematics knowledge

needed for an interpretation of graphs in physics, and in some cases not. Tuminaro and Redish (2003) state two possible reasons (see 1.2), which are both valid according to the results. An additional reason is deficiencies in knowledge of basic kinematics concepts.

The research can therefore, based on the results obtained, concludes that there are deficiencies among first-year undergraduate physics students at CUT, South Africa with regard to their knowledge of functions (mathematics), transfer of mathematics knowledge to graphs in kinematics, as well as knowledge and applications of kinematics concepts in kinematics graphs.

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APPENDIX A

CONSENT FORMS

SECTION A - RESEARCH QUESTIONNAIRE (MATHEMATICS)

This is a survey on mathematics graphs. It forms part of a research study aimed at improving your tuition. Please fill in your name, because the researcher might want to ask clarification of some answers later.

Student number:

Name:.....

INFORMED CONSENT

I understand that participating in this research project is completely voluntary and no pressure may be placed on me to participate. I understand that it is not a test and will not influence my marks. I hereby give permission that the results may be used for research purposes.

.....

.....

Signature

Date

SECTION B - RESEARCH QUESTIONNAIRE (PHYSICS)

This is a survey on graphs in physics. It forms part of a research study aimed at improving your tuition. Please fill in your name, because the researcher might want to ask clarification of some answers later.

Student number:

Name:.....

INFORMED CONSENT

I understand that participating in this research project is completely voluntary and no pressure may be placed on me to participate. I understand that it is not a test and will not influence my marks. I hereby give permission that the results may be used for research purposes only.

.....

Signature

.....

Date

APPENDIX B

BIOGRAPHICAL BACKGROUND

- Make a cross/tick where applicable *in the last column of the following table*.
- You are requested to answer all questions.
- Your participation will be highly appreciated.
- Be assured that confidentiality will be maintained.

Number	Variable	Label	Tick here:
Q1	Your gender	Male	1
		Female	2
Q2	Your age	17	1
		18	2
		19	3
		20	4
		21	5
		22 and older	6

Variable	Text	Label	Tick here:
Q3	Your nationality	South African Non-South African	1 2
Q4	Your school type	Rural(village/farm) Semi-Urban(Township) Urban (Town or Model C)	1 2 3
Q5	Your matric completion	2011 2010 2009 2008 2007 Other	1 2 3 4 5 6
Q6.1	Your pass level in Mathematics	(0-29) (30-39) 40-49 50-59 60-69 70-79 80-100	(1) (2) 3 4 5 6 7
Q6.2	Your pass level in Physical Science	(0-29) (30-39) 40-49 50-59 60-69 70-79 80-100	(1) (2) 3 4 5 6 7

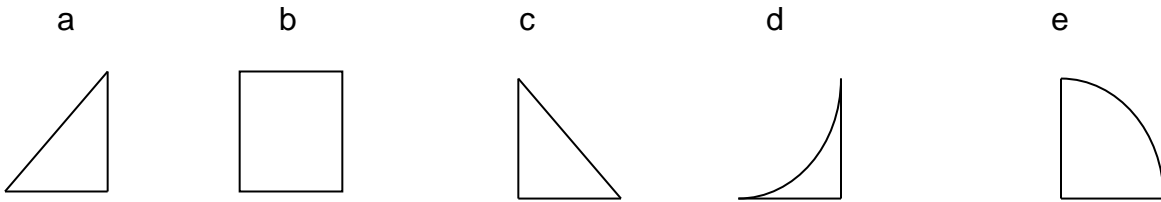
Variable	Text	Label	Tick here:
Q7	Your Faculty	Humanities	1
		Engineering and Information Technology	2
		Health and Environmental Sciences	3
Q8	Your English Proficiency	1st Language	1
		2nd Language	2
		Other	3
Q9	Your Home Language	English	1
		Afrikaans	2
		Sesotho	3
		Tswana	4
		Sepedi	5
		Xhosa	6
		Zulu	7
		Ndebele	8
		Swati	9
		Venda	10
		Tsonga	11
		Other (SA)	12
		Other (Non-SA)	13

APPENDIX C

SECTION A - Questions on mathematics graphs

Five options are given to the multiple choice items below. Please cross the number (1, 2, 3, 4 or 5) of the correct answer

1.1. Of the geometric figures below, which one has the greatest area? Assume that all the figures have the same width and height.

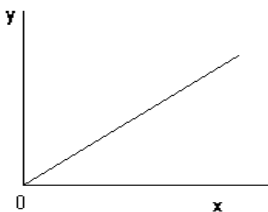


1. a
2. b
3. c
4. d
5. e

1.2. Of the geometric figures in 1.1 above, which one has the smallest area? Assume that all the figures have the same width and height.

1. a
2. b
3. c
4. d
5. e

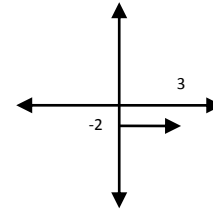
2. Consider the graph below. With increasing x , the gradient is:



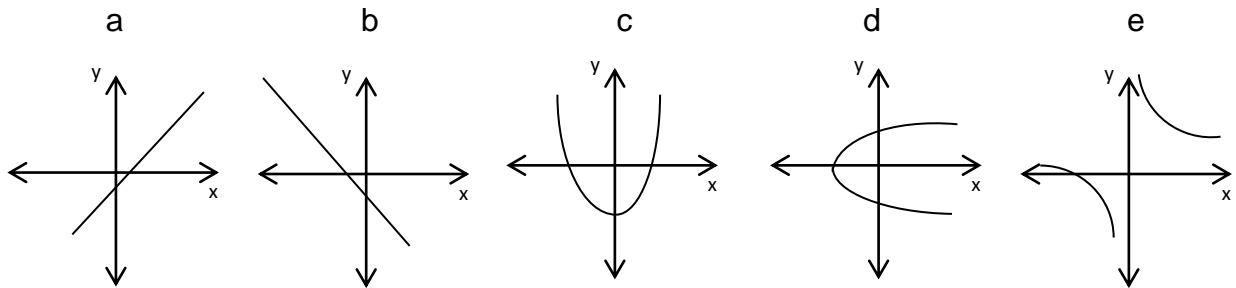
- 1 Zero
- 2 Increasing
- 3 Decreasing
- 4 Constant
- 5 Varying

3. The equation of the given graph is:

- 1 $x = 3$
- 2 $y = -2$
- 3 $y = 3x - 2$
- 4 $y = -\frac{2}{3}x$
- 5 $y = -2x + 3$



4. Choose the answers of the questions below from the following graph forms.



4.1 Which one of the graphs shows the function $f(x) = x - 1$?

- 1 a
- 2 b
- 3 c
- 4 d
- 5 e

4.2 Which one of the graphs shows the function $g(x) = \frac{2}{x}$?

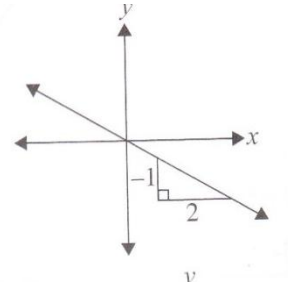
- 1 a
- 2 b
- 3 c
- 4 d
- 5 e

4.3 Which one of the graphs shows the function $h(x) = x^2 - 1$?

- 1 a
- 2 b
- 3 c
- 4 d
- 5 e

5. The gradient of the given graph is:

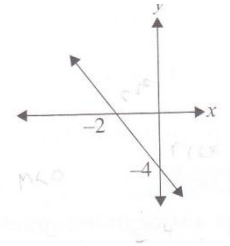
- 1 +2
- 2 -2
- 3 $+\frac{1}{2}$
- 4 $-\frac{1}{2}$
- 5 0



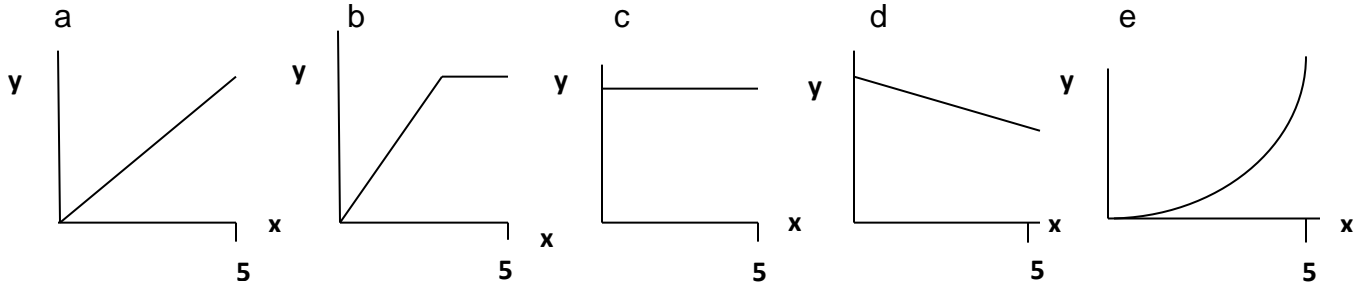
9

6. The gradient of the given graph is:

- 1 +2
- 2 -2
- 3 $+\frac{1}{2}$
- 4 $-\frac{1}{2}$
- 5 -4



7. Choose the answers of questions below from the following graphs. In all these graphs the maximum values for y are the same.



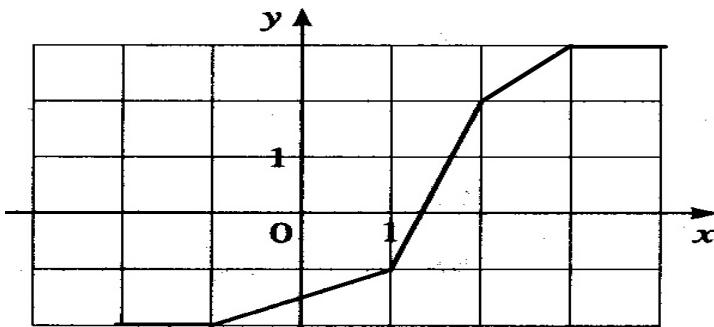
7.1 Which one of the graphs has the smallest area under the graph from $x=0$ to $x=5$?

- 1 a
- 2 b
- 3 c
- 4 D
- 5 e

7.2 Which one of the graphs has the largest area under the graph from $x = 0$ to $x = 5$?

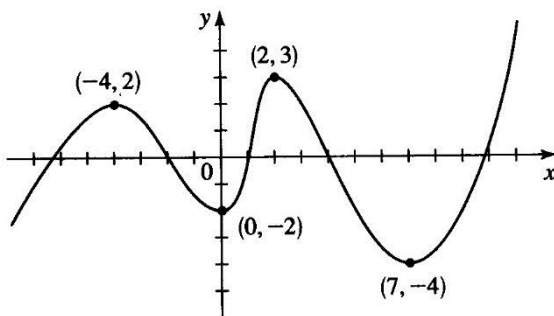
- 1 a
- 2 b
- 3 c
- 4 d
- 5 e

8. In which of the intervals is the gradient of the graph the largest?



- 1 $-2 < x < -1$
- 2 $-1 < x < 1$
- 3 $1 < x < 2$
- 4 $2 < x < 3$
- 5 $3 < x < 4$

9. Consider the graph of a function shown in the figure



In each one of the questions below the intervals are labeled as follows:

- a $-4 < x < -2$
- b $-2 < x < 0$
- c $0 < x < 1$
- d $1 < x < 2$
- e $2 < x < 4$
- f $4 < x < 7$

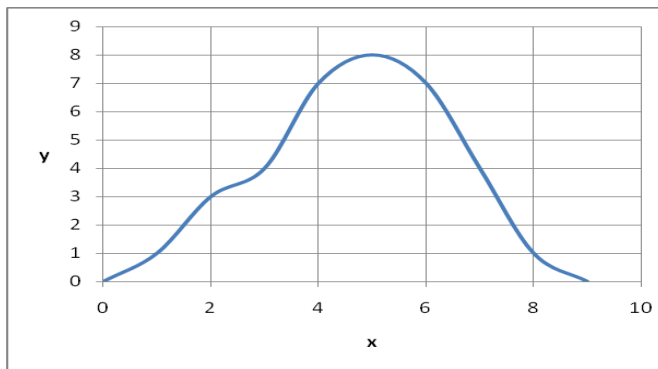
9.1 In which of the intervals is the function increasing?

- 1 c, d, e, f
- 2 only c, d
- 3 a, d, e
- 4 only d

9.2 In which of the intervals is the gradient positive?

- 1 c, d, e, f
- 2 only c, d
- 3 a, d, e
- 4 only d

10. Use the graph below and answer the questions that follow:



10.1 What are the coordinates of the turning point in the graph?

- 1. (0;5)
- 2 (5;8)
- 3 (8;5)
- 4 (0;8)
- 5 (9;8)

10.2 What is the value of y if $x = 4$?

- 1 4
- 2 0
- 3 7
- 4 3
- 5 8

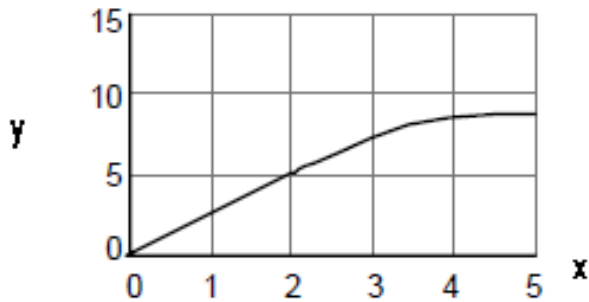
10.3 What are the values of x if $y = 4$?

- 1 3 and 7
- 2 $2\frac{1}{2}$ and $6\frac{1}{2}$
- 3 $3\frac{1}{2}$ and $7\frac{1}{2}$
- 4 Only $2\frac{1}{2}$
- 5 Only 3

10.4 From $x = 4$ to $x = 5$ the gradient

1. increases
2. decreases
3. is constant
4. Not possible to tell

11. What is the gradient of the graph when



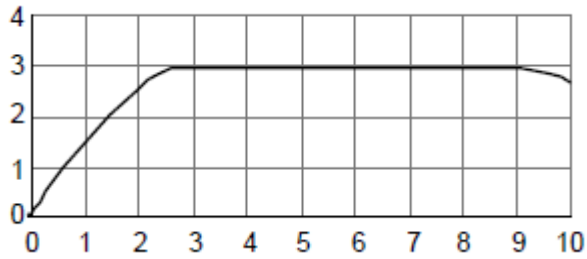
11.1 $x = 2$?

- 1 0.4
- 2 2.0
- 3 2.5
- 4 5.0
- 5 10.0

11.2 $x = 4.5$?

- 1 5
- 2 0
- 3 $8 / 4.5$
- 4 $4.5 / 8$
- 5 8

12. Consider the following graph:



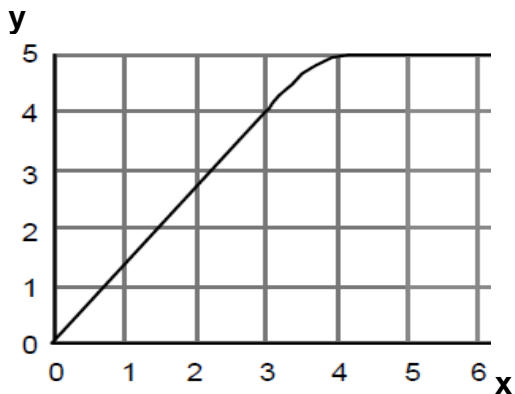
12.1 The area under the graph in the x-interval (4, 8) is:

- 1 0
- 2 1.33
- 3 4.0
- 4 12.0
- 5 24.0

12.2 The gradient of the graph in the x-interval (4, 8) is:

- 1 0
- 2 0.75
- 3 1.33
- 4 4.0
- 5 12.0

13. For the graph below, answer the following questions :



13.1 How does the value of y change between $x = 4$ and $x = 6$?

- 1 Increases
- 2 Decreases
- 3 Remains constant
- 4 Zero
- 5 Not possible to tell

13.2 The gradient for the interval $4 < x < 6$ is:

- 1 Positive
- 2 Negative
- 3 Zero
- 4 Unknown
- 5 Not possible to tell

13.3 How does the value of y change between $x = 0$ and $x = 3$?

- 1 Increases
- 2 Decreases
- 3 Remains constant
- 4 Zero
- 5 Not possible to tell

13.4 The gradient for the interval $0 < x < 3$ is:

- 1 Positive
- 2 Negative
- 3 Zero
- 4 Unknown
- 5 Not possible to tell

13.5 What is the area under the graph for $0 < x < 3$?

- 1 0.75
- 2 1.33
- 3 4.0
- 4 6.0
- 6 12.0

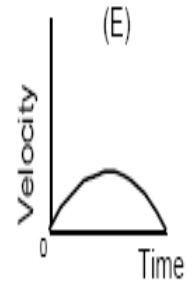
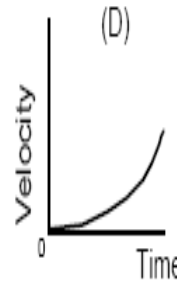
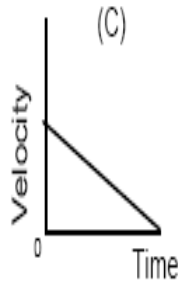
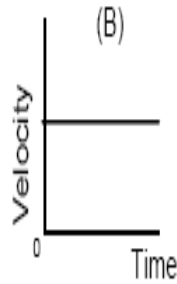
THANK YOU VERY MUCH!

APPENDIX D

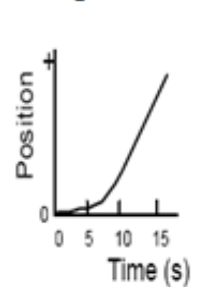
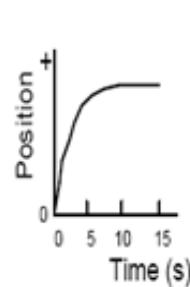
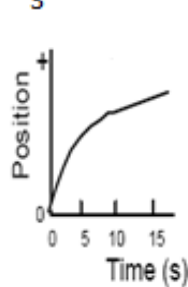
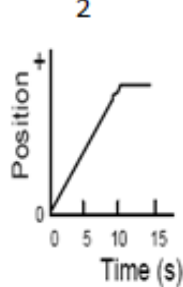
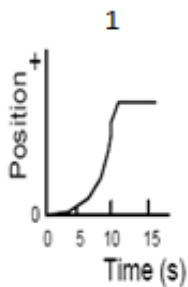
SECTION B - Questions on graphs in physics

Five options are given to the multiple choice items below. Please cross the number (1, 2, 3, 4 or 5) of the correct answer

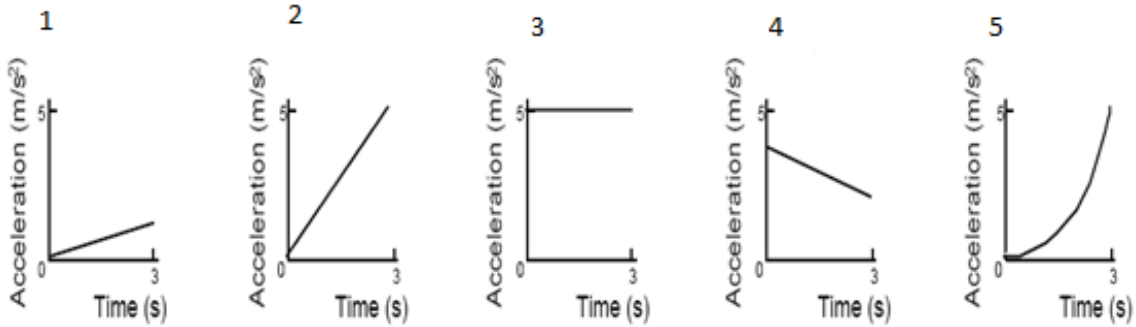
1. Velocity versus time graphs for five objects are shown below. All axes have the same scale. Which object had the greatest change in position (displacement) during the interval?



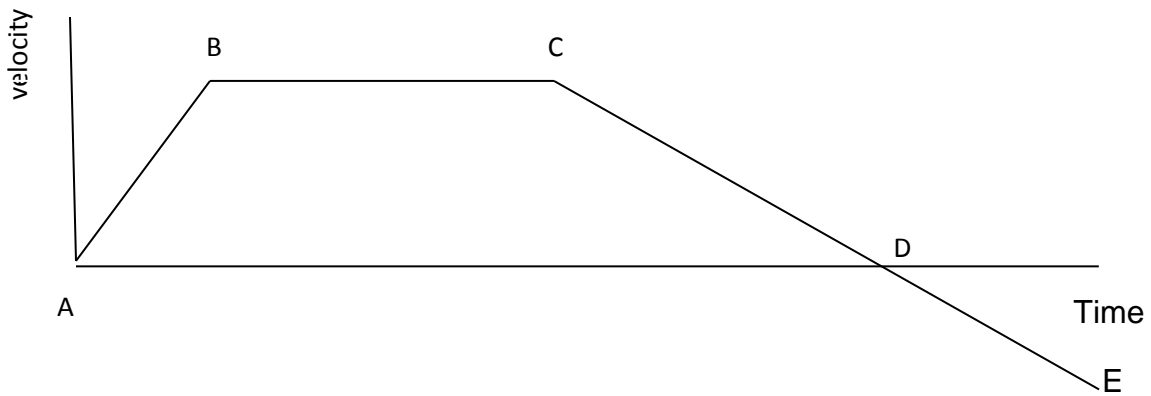
1. (A)
2. (B)
3. (C)
4. (D)
5. (E)
2. An object starts from rest and undergoes a positive, constant acceleration for ten seconds. It then continues with a constant velocity. Which of the following graphs correctly describes this motion?



3. Five objects move according to the following acceleration versus time graphs. Which has the smallest change in velocity during the three second interval?



4. The figure below shows a velocity-time graph of an object's motion.



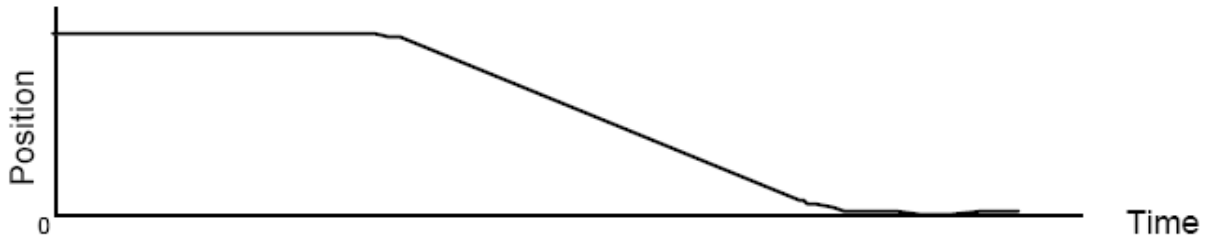
- 4.1 Where is the acceleration negative?

- 1 AB
- 2 BC
- 3 CD and DE
- 4 CD only
- 5 DE only

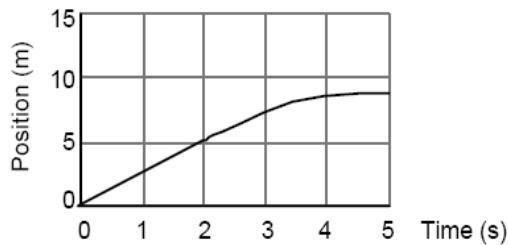
- 4.2 Where is the gradient negative?

- 1 AB
- 2 BC
- 3 CD and DE
- 4 CD only
- 5 DE only

5. The Figure below shows a position-time graph of an object's motion. Which sentence is a correct interpretation of the motion of the object?



- 1 The object rolls along a flat surface. Then it rolls forward down a hill, and then finally stops.
 - 2 The object doesn't move at first. Then it rolls forward down a hill and finally stops.
 - 3 The object is moving at constant velocity. Then it slows down and stops.
 - 4 The object doesn't move at first. Then it moves backwards and then finally stops
 - 5 The object moves along a flat area, moves backwards down a hill, and then it keeps moving.
6. The position-time graph below shows the straight line motion of an object. Answer the following questions:

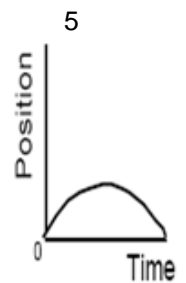
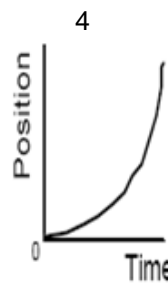
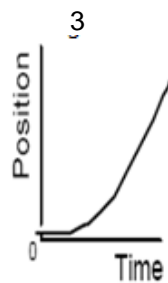
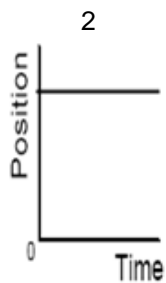


- a. The position at the 2 second point in the position-time graph is most nearly:
 - 1 0.4 m
 - 2 2.0 m
 - 3 2.5 m
 - 4 5.0 m
 - 5 9.0 m
- b. The velocity at the 2 second point in the position-time graph is most nearly:
 - 1 0.4 m/s
 - 2 2.0 m/s
 - 3 2.5 m/s
 - 4 5.0 m/s
 - 5 10.0 m/s

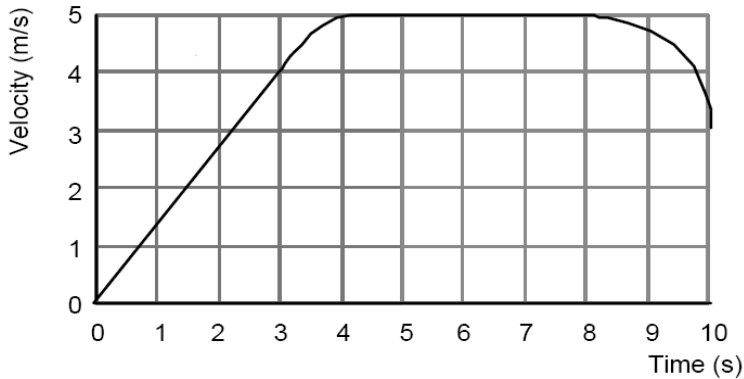
- c. The position at the 5 second point in the position-time graph is most nearly:
- 1 0.0 m
 - 2 0.56 m
 - 3 1.8 m
 - 4 5.0 m
 - 5 9.0 m

- d. The velocity at the 5 second point in the position-time graph is most nearly:
- 1 0.0 m/s
 - 2 0.56 m/s
 - 3 1.8 m/s
 - 4 5.0 m/s
 - 5 9.0 m/s

7. Position versus time graphs for five objects are shown below. All axes have the same scale. Which object has the highest instantaneous velocity in the interval shown?

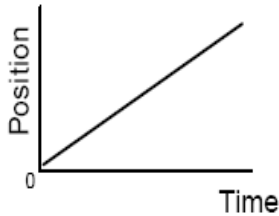


8. An elevator moves from the basement to the tenth floor of a building. The elevator moves as shown in the velocity-time graph below.



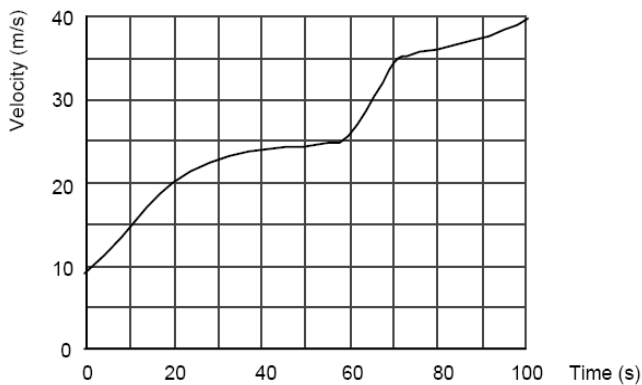
- a. From 4 to 8 seconds the elevator:
- 1 Stands still
 - 2 Moves with constant velocity
 - 3 Moves with increasing velocity
 - 4 Accelerates uniformly
- b. How far does it move during the first three seconds of motion?
- 1 0.75 m
 - 2 1.33 m
 - 3 4.0 m
 - 4 6.0 m
 - 5 12.0 m
- c. What is its acceleration during the first three seconds?
- 1 0.75 m/s^2
 - 2 1.33 m/s^2
 - 3 4.0 m/s^2
 - 4 6.0 m/s^2
 - 5 12.0 m/s^2

9. Below is a graph of an object's motion. Which sentence is the best interpretation of the motion?



- 1 The object is moving with a constant, non-zero acceleration.
- 2 The object does not move.
- 3 The object is moving with a uniformly increasing velocity.
- 4 The object is moving with a constant velocity.
- 5 The object is moving with a uniformly increasing acceleration.

10. The motion of an object traveling in a straight line is represented by the following graph.



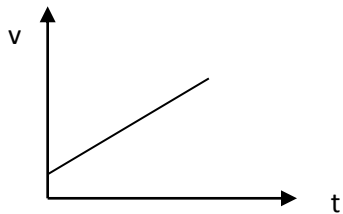
- a. At time = 65 s, the magnitude of the instantaneous acceleration of the object was most nearly:

- 1 +2 m/s²
- 2 +1 m/s²
- 3 +30 m/s²
- 4 +9.8 m/s²
- 5 +34 m/s²

- b. The graph shows that for the three parts of the motion. Which set best describes the object's motion?

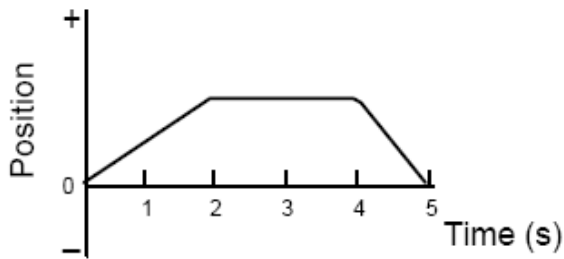
	0 to 60 seconds	60 to 70 seconds	70 to 100 seconds
1	Acceleration decreases	Acceleration remains constant	Acceleration increases
2	Velocity decreases	Velocity remains constant	Velocity increases
3	Acceleration increases	Acceleration increases	Acceleration increases
4	Velocity increases	Velocity remains constant	Velocity decreases
5	Acceleration increases	Acceleration remains constant	Acceleration decreases

11. Which one of the following equations correctly gives the equation for the line graph below?

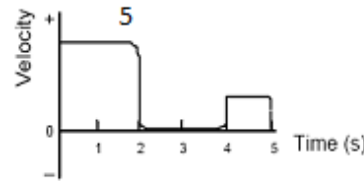
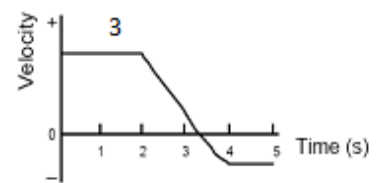
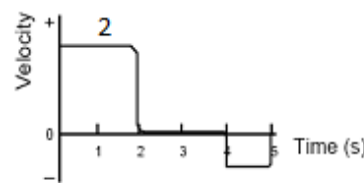
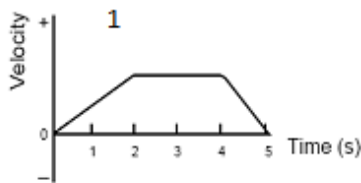


- 1 $v = at$
- 2 $v = \frac{\Delta s}{\Delta t}$
- 3 $v = u + at, u \neq 0$
- 4 $v^2 = u^2 + 2as, u \neq 0$

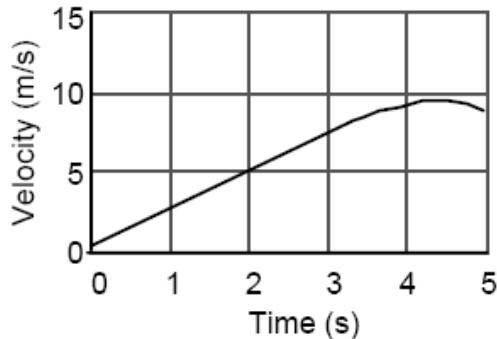
12. The following is a position-time graph for an object during a 5 s time interval.



Which one of the following graphs of velocity versus time would best represent the object's motion during the same time interval?

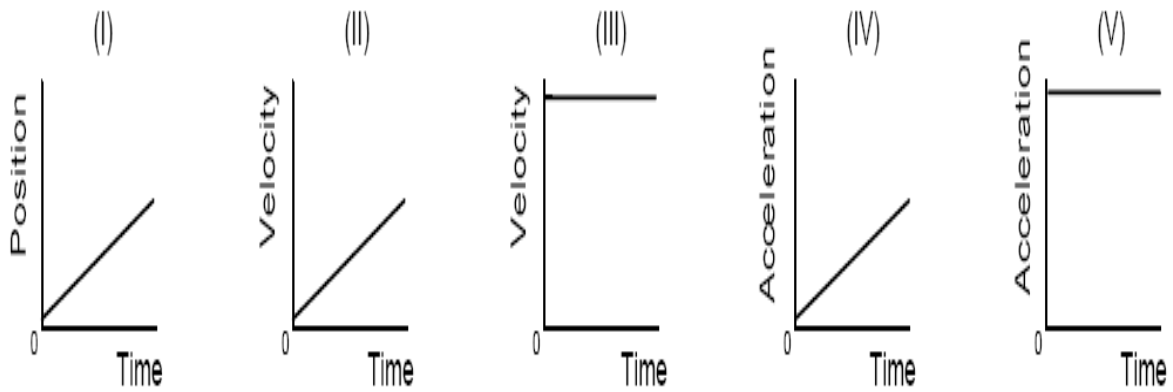


13. If you wanted to know the distance covered during the interval from $t = 0$ s to $t = 2$ s, from the graph below you would:



- 1 Read 5 directly off the vertical axis.
- 2 Find the area between that line segment and the time axis by calculating $(5 \times 2)/2$.
- 3 Find the slope of that line segment by dividing 5 by 2.
- 4 Find the slope of that line segment by dividing 15 by 5.
5. Not enough information to answer.

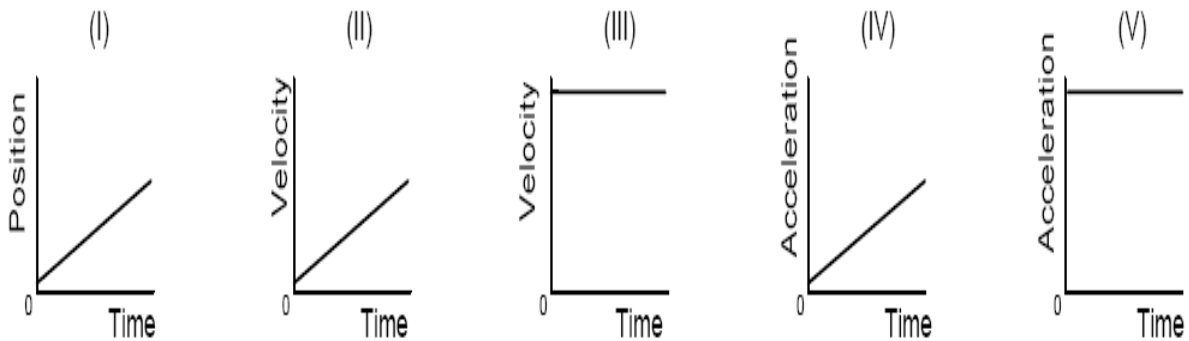
14. Consider the following graphs, noting the different axes:



Which of these represent(s) motion at constant velocity?

- 1 I, II, and IV
- 2 I and III
- 3 III and V
- 4 IV only
- 5 V only

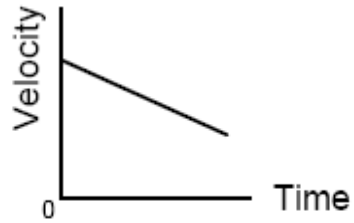
15. Consider the following graphs, noting the different axes:



Which of these represent(s) motion at constant, non-zero acceleration?

- 1 I, II, and IV
- 2 I and III
- 3 II and V
- 4 IV only
- 5 V only

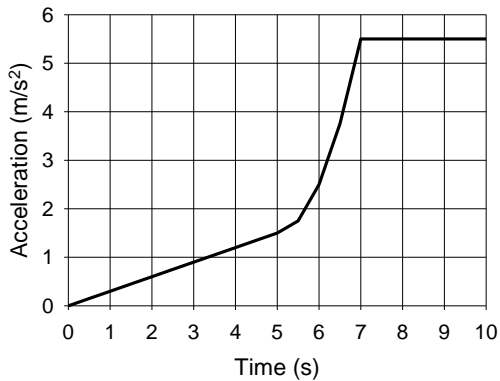
16. Below is a velocity-time graph of an object's motion.



Which sentence is the best interpretation?

- 1 The object is moving with a constant acceleration
- 2 The object is moving with a uniformly decreasing acceleration.
- 3 The object is moving with a uniformly increasing velocity.
- 4 The object is moving at a constant velocity.
- 5 The object does not move

17. The acceleration-time graph below represents the motion of an object travelling in a straight line. Answer the following questions.



a. What is the change in velocity of the object from 0 seconds to 5 seconds?

- 1 1.5 m/s
- 2 5 m/s
- 3 3.33 m/s
- 4 3.75 m/s
- 5 7.5 m/s

b. What is the change in velocity of the object from 7 seconds to 10 seconds?

- 1 5.5 m/s
- 2 16.5 m/s
- 3 1.833 m/s
- 4 0.545 m/s
- 5 55.0 m/s

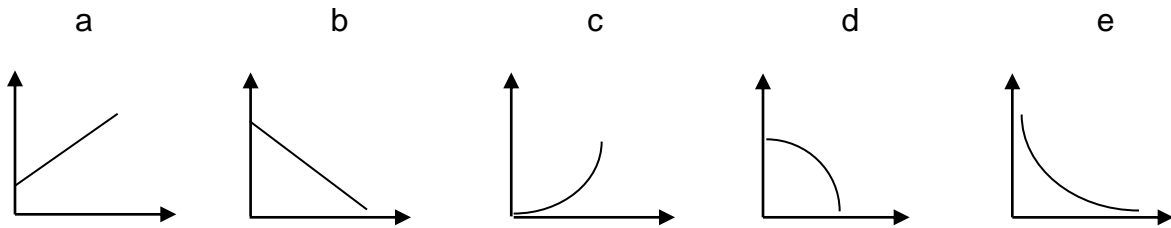
18. The area under a velocity-time graph represents

- 1 acceleration
- 2 change in acceleration
- 3 speed
- 4 change in velocity
- 5 displacement

19. Displacement can be obtained from the:

- 1 gradient of an acceleration-time graph
- 2 gradient of a velocity-time graph
- 3 area under an acceleration-time graph
- 4 area under a velocity-time graph

20. Choose the answers of questions below from the following graph forms.



a. What is the form of the v versus t graph if $v = u + at$ is plotted with u and a positive constants?

- 1 a
- 2 b
- 3 c
- 4 d
- 5 e

b. What is the form of the s versus t graph if $s = ut + \frac{1}{2}at^2$ is plotted with u and a positive constants?

- 1 a
- 2 b
- 3 c
- 4 d
- 5 e

c. What is the form of the v - t graph if $v = \frac{s}{t}$ is plotted with s a positive constant?

- 1 a
- 2 b
- 3 c
- 4 d
- 5 e

21. An object moves with a constant velocity of 6 m/s. Which one of the following statements is true?

- 1. The acceleration of the object is constant and 6 m/s^2 .
- 2. The object undergoes a displacement of 6 m in every second.
- 3. The object does not necessarily travel in a straight line.
- 4. The velocity-time graph of the motion is a straight line through the origin.

22. Starting from rest, an object is accelerated at 4 m/s^2 in a straight line. Answer the following questions.

22.1 Which statement accurately describes the motion of the object?

- 1 The object travels 4 meter during each second
- 2 The object travels 4 meters during the first second only
- 3 The speed of the object increases by 4 m/s during each second
- 4 The acceleration of the object increases by 4 m/s^2 during each second
- 5 The final velocity of the object will be proportional to the distance that the object covers.

22.2 How far has the object travelled after 10 seconds?

- 1 20 m
- 2 40 m
- 3 100 m
- 4 200 m
- 5 400 m

22.3 What is the speed of the particle after it has travelled 8 m?

- 1 4 m/s
- 2 8 m/s
- 3 32 m/s
- 4 64 m/s
- 5 100 m/s

THANK YOU VERY MUCH!!

APPENDIX E

Interview Questions from Research Questionnaire

1. Define displacement and velocity and give their units
2. Name equations of motion and state what each symbol stand for
3. What does the gradient of a displacement – time graph represent?
4. How do you find the acceleration in a given velocity – time graph?
5. How do you find the displacement in a given velocity – time graph?
6. What is the difference between average velocity and instantaneous velocity?;9
7. How do you determine them?
8. Between velocity-time and displacement-time graphs, in which one can you obtain average velocity and in which can you obtain instantaneous velocity?



9. .Which equation of linear motion can give a straight line on a v-t graph? Explain your answer.
10. Given $v_f = v_i + at$ with constant a, which symbol represents the gradient and which one represents the y-intercept on a v-t graph? (If a student cannot answer this, ask him to write down the mathematics equation for a straight line graph and indicate which symbols represent the gradient and y-intercept. If correct, then

you can ask him again about the linear motion equation above and whether he sees any similarities between the maths and physics equations.)

11. From a given velocity-time graph (P1 and P3 in questionnaire), determine whether the displacement is smaller or greater. Explain your answer.
12. In an acceleration-time graph (P17N2), what can be obtained between two time intervals?
13. When a velocity-time graph slants upward, is the acceleration (gradient) positive or negative, decreasing or increasing or constant (P4.1, P4.2, M9N1, M9.2 and M13). Explain your reasoning.
14. How /when do we know that the gradient is zero in a velocity-time graph and displacement time graph? (M13.2). What is the meaning of zero gradient in a velocity-time and displacement-time graph?