

CONTINUOUS-TIME STOCHASTIC MODELLING OF CAPITAL ADEQUACY RATIOS FOR BANKS

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Uittreksel

Kontinue-Tyd Stogasties Modelling van Kapitaal Bevoegdheidsverhoudings vir Banke

Regulering van die kapitaal benodighede vir banke is besonder belangrik in vandag se banksektor. In hierdie opsig is een maatstaf om bank solvensie te meet die Kapitaal Bevoegdheidsverhouding(KBV).

Ons beskou twee tipes KBV's, een wat risiko gebaseer is en een wat nie-risiko gebaseer is nie. Ons kan die risiko gebaseerde KBV verder opdeel in Basel II en Reeks 1 verhoudings en ook die nie-risiko gebaseerde KBV in hefboom en billikheids verhoudings.

In die algemeen is hierdie verhoudings 'n breuk met die teller 'n maatstaf van 'n bank se kapitaal en die noemer is 'n maatstaf van die risiko waaraan die bank blootgestel is.

Ons hoofdoel is om kontinue-tyd stogastiese modelle te formuleer vir die bogenoemde verhoudings en ons hoofresultaat is die modellering van die Basel II KBV.

Preface

One of the contributions made by North-West University (Potchefstroom Campus) to the activities of the control theory community in South Africa has been the establishment of an active research group that has an interest in financial mathematics. Under the guidance of my supervisor, Prof. Mark A. Petersen, this group has recently made valuable contributions to the existing knowledge about the stochastic control of financial systems in pensions, insurance and banking.

The work in this dissertation originated from our interest in the connections between concepts that arise in systems and (stochastic) control theory and financial models. In this regard, the interests of the group lie with the stochastic controllability of interest rate models, stochastic control of continuous- and discrete-time pension funds, the solvency of dividend equalization funds and the solvency, profitability and operational control of commercial banks.

The most important outcomes of this project were collected in 1 peer-reviewed accredited journal article (Applied Stochastic Models in Business and Industry) and 2 peer-reviewed conference proceedings papers (IFAC World Congress on Automatic Control 2005) that appeared in international financial mathematics publications. Furthermore, a second journal article is currently under review in an accredited mathematical economics journal.

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Chapter 1

Problem Statement and Aim of Study

1.1 OVERVIEW

1.2 PROBLEM STATEMENT

1.3 AIM

1.4 HYPOTHESIS

1.5 STRUCTURE OF DISSERTATION

1.1 OVERVIEW

Regulation related to capital requirements is an important issue in the banking sector. In this regard, one of the indices used to measure how susceptible a bank is to failure, is the capital adequacy ratio (CAR). We consider two types of such ratios, viz., non-risk-based (NRBCARs) and risk-based (RBCARs) CARs. According to the US Federal Deposit Insurance Corporation (FDIC), we can further categorize NRBCARs into leverage and equity capital ratios and RBCARs into Basel II and Tier 1 ratios. In general, these indices are calculated by dividing a measure of bank capital by an indicator of the level of bank risk. Our primary objective is to construct continuous-time stochastic models for the dynamics of each of the aforementioned ratios with the main achievement being the modelling of the Basel II capital adequacy ratio (Basel II CAR). This ratio is obtained by dividing the bank's eligible regulatory capital (ERC) by its risk-weighted assets (RWAs) from credit, market and operational risk. In the main, our discussions conform to the qualitative and quantitative standards prescribed by the Basel II Capital Accord. Also, we find that our models are consistent with data from FDIC-insured institutions. Finally, we demonstrate how our main results may be applied in the banking sector.

1.2 PROBLEM STATEMENT

The main objectives which are solved in this dissertation are as follows.

We strive to achieve the following.

Problem 1.2.1 (Leverage and Equity CARs): *How, if possible, can the dynamics of the leverage and equity CARs of banks be stochastically modelled in continuous-time ? (see Theorem 2.3.1 and Corollary 2.3.2 for a solution).*

Problem 1.2.2 (Basel II and Tier 1 CARs): *How, if possible, can the dynamics of the Basel II and Tier 1 CARs of banks be stochastically modelled in continuous-time ? (see Theorem 2.4.2 and Corollary 2.4.3 for solutions).*

1.3 AIM

To find continuous-time stochastic models for the capital adequacy ratios of banks.

1.4 HYPOTHESIS

The following hypothesis was set for this study:

We can model the Basel II capital adequacy ratios for a bank.

1.5 STRUCTURE OF DISSERTATION

The current chapter is introductory in nature. In addition, a complete list of references will be given in the bibliography contained at the end of each chapter.

In the second chapter we define all the variables used to construct the stochastic model. We first have a small introduction to state the problems at hand. Then in Section 2.2 of Chapter 2 we explain the main issues to consider in the stochastic modeling of a bank. The next two sections are devoted to capital adequacy, both risk-based and non-risk-based. In Section 2.3 of Chapter 2 we describe non-risk-based CARs and the following section is aimed at risk-based CARs. The final section in Chapter 2 is the interpretation of the main issues.

Chapter 3 provides us with the conclusions and also shows the research that may be the subject of a future investigation.

In Chapter 4 all references are provided in the bibliography.

Chapter 2

Continuous-Time Stochastic Modelling of Capital Adequacy Ratios for Banks

2.1 INTRODUCTION

2.2 STOCHASTIC BANKING MODEL

2.3 NON-RISK BASED CAPITAL ADEQUACY RATIOS

2.4 RISK BASED CAPITAL ADEQUACY RATIOS

2.5 INTERPRETATION OF MAIN ISSUES

2.1 INTRODUCTION

One of the first serious attempts to develop regulation of the banking industry on a global scale was the 1988 Basel Accord (see [3] and its amendments [4], [5] and [6]) that was drafted by the Basel Committee on Banking Supervision (BCBS). In essence, the Basel I Capital Accord suggested that banks should hold a capital-to-risk-weighted assets ratio of at least 8 % (see, for instance, [15] and [22]). The recommended ratio was intended to protect depositors and deposit insurance schemes from the ravages of inadequate or reckless portfolio management and to prevent systemic instabilities arising from bank failures. However, Basel I's prescribed capital requirements attracted widespread criticism that focussed on the view that the accord was oversimplified and out of step with the ever-improving standards for bank management and regulation. The accord was further criticized for treating all credit risk-types in the same manner which potentially could lead to regulatory arbitrage and that it did not take modern credit risk management techniques into account. Moreover, Basel I was deemed to be inept as regards the dynamic distortions of capital regulation and complementary regulatory instruments such as supervisory monitoring or prompt corrective regulatory action (see, for instance, [1] and [33]). The BCBS bore the above opinions in mind

when making several adjustments to the original capital accord. This culminated in a first consultative paper in June 1999 (see [7]). The aforementioned document led to experiments being conducted in the banking sector which resulted in second and third consultative papers in January 2001 (see [8]) and April 2003 (see [13]), respectively. These endeavours were undertaken in an effort to finalize the new accord and implement it globally by the end of the year 2006. The Basel II capital accord (see, for instance, [14]) secures international convergence on revisions to supervisory regulations governing the capital adequacy of banks. In this process, the ratio of bank capital to assets, also called the **capital adequacy ratio (CAR)** plays a major role as an index of capital adequacy of banks. This ratio is the primary issue discussed in our dissertation and is described in great detail in the ensuing analysis.

In this contribution, we investigate the role of capital adequacy ratios in bank regulation where

$$\text{Capital Adequacy Ratio (CAR)} = \frac{\text{Indicator of Absolute Amount of Bank Capital}}{\text{Indicator of Absolute Level of Bank Risk}}.$$

This equation suggests that CARs allow us to determine whether the absolute amount of bank capital is adequate when compared to a measure of absolute risk. In our dissertation, we consider equity and leverage CARs (collectively known as NRBCARs; defining formulae given by (2.24) in Subsection 2.3.2) and Basel II and Tier 1 CARs (together classified as RBCARs; defining formulae given by (2.35) in Subsection 2.4.4) as identified by the FDIC (see, for instance, [27]). The definitions of these CARs are provided in ensuing discussions, while Figures 2 and 3 in the appendix highlight and explain differences between them. At this point, the jury is still out on whether RBCARs are superior to NRBCARs in determining the bank's overall risk. Despite this, the Basel II capital accord (see, [9] and [14]) considers the Basel II CAR to be the cornerstone of bank supervision and risk management. In this case, we have

$$\begin{aligned} \text{Basel II Capital Adequacy Ratio (Basel II CAR)} = \\ \frac{\text{Eligible Regulatory Capital (ERC)}}{\text{Total Risk-Weighted Assets (TRWAs)}}. \end{aligned} \quad (2.1)$$

In (2.1), TRWAs are constituted by the capital charges for credit, market and operational risk while the ERC is determined by considering Tier 1, 2 and 3 capital as stipulated in [14]. In most countries, Basel II-compliant banks usually compute their CARs with the aim of reporting its value to a national supervisory organization. In situations where the Basel II CAR drops below 8 %, the supervisory organization can order the bank to take certain actions that, in some cases, may eventually lead to its closure. The fact that the Basel II accord will become legally binding among all major banks in most countries from the year 2006 onwards, acts as the major motivation for this study. The main problems that we pose are related to the capital adequacy issue and is stated below.

Problem 2.1.1 (Leverage and Equity CARs): *How, if possible, can the dynamics of the leverage and equity CARs of banks be stochastically modelled in continuous-time ? (see Theorem 2.3.1 and Corollary 2.3.2 for solutions).*

Problem 2.1.2 (Basel II and Tier 1 CARs): *How, if possible, can the dynamics of the Basel II and Tier 1 CARs of banks be stochastically modelled in continuous-time ? (see Theorem 2.4.2 and Corollary 2.4.3 for solutions).*

As far as RBCARs are concerned, we note that the primary risks that banks bear are **credit** (risk that loans will not be repaid; see, for instance, [14]), **operational** (risk of loss resulting from inadequate or failed internal processes, people and systems or from external events; see, for instance, [9]) and **market** (risk of losses in on- and off-balance sheet positions arising from movements in market prices, including equities; see, for instance, [4]) risks.

The main novelty of this dissertation is the construction of stochastic models for the dynamics of RBCARs and NRBCARs in continuous-time (see Sections 2.2, 2.3 and 2.4 for more details). For each of the aforementioned types of CARs we produce a stochastic differential equation (SDE) that highlights some of the dynamic features of the bank's on- and off-balance sheet activities. Our motivation for representing the evolution of these CARs in this way, emanates from the seminal work done by Karatzas, Lehoczky, Sethi and Shreve and Merton in [34] and [39] (and the references contained therein), respectively. Furthermore, the stochastic models derived in our research serve as an essential precursor to some recent work on optimal (credit, operational, market and interest rate) risk management of banks as in [29] and [42] (see, also, [15], [25] and [26]). Of course, in practice, these types of models have their limitations, but they do have a notable history of use in recent banking studies related to the dynamics of portfolio and capital structure management and the interrelation between capital requirements and regulation (see, for instance, [19] and [21]). Our contribution is distinct from the results contained in the latter mentioned papers in that each of the banking items (asset, liabilities, bank capital, off-balance sheet items) is modelled separately with the dynamics of the CAR ultimately being expressed in terms of these components. This feature distinguishes the SDEs derived in our study from models that have appeared in related literature up to this point. We note that this dissertation consists of both a theoretical as well as a case component. With regard to the former, we make use of stochastic calculus to formalize the most important properties of the aforementioned banking items. Discussions about the case component is mainly restricted to Section 2.4, where credit (see Subsection 2.4.1), market and operational risk (see Subsection 2.4.2) are considered from the idiosyncratic viewpoint of the internal ratings based, internal model and standardized approaches, respectively. In Section 2.5, we provide an economic interpretation of the main issues raised in the preceding discussions. The final section presents some concluding remarks and outlines a few possibilities for further study.

2.2 STOCHASTIC BANKING MODEL

At the outset, we assume that we are working with a probability space $(\Omega, \mathcal{F}, \mathbf{P})$ on a time period $T = [t_0, t_1]$, where $\mathbf{P}$ is a probability measure on Ω . Banks operate via the process of **asset transformation** which entails the selling of liabilities (sources of funds) with certain properties (combination of liquidity, risk, size and return) and the use of the proceeds to buy assets (uses of funds) with other characteristics (see [40] for more details). In this regard, we consider a continuous-time dynamic model in which the bank holds assets and has liabilities that behave in a stochastic manner (see [45] for an analogue in insurance theory). This behaviour is consistent with the uncertainty associated with items appearing on the balance sheet, namely, the reserves, loans and wealth (assets) and deposits and borrowings (liabilities). The aforementioned items are balanced by the core capital according to the well-known relation

$$\text{Total Assets} = \text{Total Liabilities} + \text{Tier 1 Capital}.$$

In this regard, a stylized balance sheet of a typical commercial bank at time t can be represented as

$$R(t) + L(t) + a_{nr}(t) = D(t) + B(t) + c_{T1}(t), \quad (2.2)$$

where we have that

$$\begin{aligned} R : \Omega \times T &\rightarrow \mathbb{R}_+ & :- \text{Reserves}, & \quad L : \Omega \times T &\rightarrow \mathbb{R}_+ & :- \text{Loan Demand}, \\ a_{nr} : \Omega \times T &\rightarrow \mathbb{R}_+ & :- \text{NRWAs}, & \quad D : \Omega \times T &\rightarrow \mathbb{R}_+ & :- \text{Deposits}, \\ B : \Omega \times T &\rightarrow \mathbb{R}_+ & :- \text{Borrowings}, & \quad c_{T1} : \Omega \times T &\rightarrow \mathbb{R}_+ & :- \text{Tier 1 Capital}. \end{aligned}$$

2.2.1 BANK ASSETS

Our main objective in this subsection is to model the items constituting bank assets in a realistic fashion. In this regard, we use stochastic calculus to describe bank loans (see, for instance, [20], [31], [35] and [50] and more generally [16], [30], [39] and [40]) and wealth (see, for instance, [18], [19], [21], [31] and [41]). Our stochastic model for bank assets is distinct from all of those developed in the aforementioned literature. In our analysis of the balance sheet it is superfluous to describe bank reserves and as a result such a discussion is omitted.

Loans

We consider a **bank loan** to be a financial contract between a bank and a debtor that has the features outlined below. At a given time, the bank lends the debtor an amount, s_0 , called the **principal**, that has been mutually agreed upon. Subsequently, the debtor pays **amortizations** back to the bank at a **nominal interest rate** of r , which is constantly being adapted to uncertain market conditions. This interest rate is endogeneously decided by the bank by taking the directives from the federal reserve or central bank into account. In the

sequel, we will denote the total amount of amortization repayments at time $t \geq 0$ at rate r by, $A(t, r(t))$. Some properties of the **amortization function**, A , is that it is right-continuous, has a finite value, is non-decreasing and $A(0, r(0)) = 0$. The amortization is designed in such a way that its value at $t = 0$ is equal to the principal value, $s_0 = 1$, which by convention may be set to 1 monetary unit. When accounting and taxation considerations are factored in, the loan contract needs to give a precise description of how the amortization function, A , is decomposed into repayments and interest. In this regard, A may be represented by

$$A(t, r(t)) = F(t) + I(t, r(t)),$$

where F and I are the **loan repayment** and **interest functions** of time, respectively. F and I are both non-negative, right-continuous and non-decreasing with $F(0) = I(0, r(0)) = 0$.

In our model, the loans applied for are exogenously determined with dynamics given by

$$dL_a(t) = r_{L_a} dt + \sigma_{L_a}(t) dW_{L_a}(t), \quad L_a(t_0) = L_{a0},$$

where $L_a : \Omega \times T \rightarrow \mathbb{R}$ is a stochastic process, r_{L_a} is the rate at which loans are applied for, $\sigma_{L_a}(t)$ is the volatility in the loans applied for and $W_{L_a} : \Omega \times T \rightarrow \mathbb{R}$ is a standard Brownian motion whose value at time t is denoted by $W_{L_a}(t)$. The stochastic process $l : \Omega \times T \rightarrow \mathbb{R}$ is the **loans issuing rate** whose value at time t is denoted by $l(t)$, $\sigma_l(t)$ is the volatility and $W_l : \Omega \times T \rightarrow \mathbb{R}$ is a standard Brownian motion whose value at time t is denoted by $W_l(t)$. In the sequel, we have that $\mathcal{F} = \{\mathcal{F}_t\}_{t \geq 0}$ is a complete, right continuous filtration generated by the one-dimensional Brownian motion $\{W_l(t)\}_{t \geq 0}$. The dynamics of the loans issuing rate can be described as follows,

$$dl(t) = l(t) \left[\kappa \left(\epsilon - \ln l(t) \right) dt + \sigma_l dW_l(t) \right], \quad l(t_0) = l_0, \quad (2.3)$$

where κ is the rate of mean reversion to the equilibrium level of the loan issuing rate given by ϵ . The mean reversion rate can be estimated by regressing changes in loan issuing on the previous values of the loans issued. This choice of model enables us to illustrate how general results about loan issuing may be specialized to real-world situations where, for instance, mean-reverting models are appropriate. Supporting evidence for the mean-reversion of loan issuing rates is provided by recent FDIC data (see, for instance, [27]). This trend was shown to be particularly prevalent among credit types such as project, object and commodities finance. Next, we model the **loan demand** which is denoted by $L : \Omega \times T \rightarrow \mathbb{R}$ and expressed as

$$dL(t) = l(t)dt - dA(t) + \eta dM(t) - N_t; \quad dA(t) = \alpha l(t)dt \quad \text{where} \quad 0 \leq \alpha \leq 1. \quad (2.4)$$

Here A is the amortization paid back, η is a constant related to macroeconomic activity, M is a time-dependent variable representing the level of macroeconomic activity, α is the

proportion of loans issued that are paid back and N_t is said to be a Poisson process with intensity λ . The latter process is used to model loans that default. Moreover, (2.4) leads to the expression

$$dL(t) = (1 - \alpha)l(t)dt + \eta dM(t) - N_t. \quad (2.5)$$

Lending is likely to be more responsive to macroeconomic conditions under Basel II than Basel I.

Wealth

Banks are allowed to invest in both **riskless** and **risky** assets (see, for instance, the treatment of bank allocation in [18], [19], [21], [31] and [41]). We assume that our bank invests in a market with $n + 1$ financial assets. In our contribution, the risk-free rate of interest is modelled as a one-factor diffusion process (see, for instance, [44]) that may be represented by the time-homogeneous SDE

$$dr(t) = \mu_r(r(t))dt + \sum_{j=1}^n \sigma_{rj}(r(t))dZ_j(t),$$

where

$$Z(t) = (Z_1(t), \dots, Z_n(t))^T \quad (2.6)$$

and the Z_j 's are independent standard Brownian motion. Furthermore, we define

$$\sigma_r(r) = (\sigma_{r_1}(r), \dots, \sigma_{r_n}(r))^T,$$

where σ_r is the r -th row of the $n \times n$ volatility matrix $(\sigma_{rj})_{r,j=1}^n$. In this case, the value of the monetary units in the money market fund at t is given by

$$x_0(t) = x_0(0) \exp \left\{ \int_0^t r(s)ds \right\}.$$

We describe the n risky asset categories next. Let $x_i(t)$ be the total return on an investment in the risky asset category i (amount of a single investment in asset i with reinvestment of dividend income) where

$$dx_i(t) = x_i(t) \left[\left(r(t) + \sum_{j=1}^n \sigma_{ij} \theta_j \right) dt + \sum_{j=1}^n \sigma_{ij} dZ_j(t) \right], \quad 1 \leq i \leq n.$$

Here the volatility matrix and the market prices of risk, given by

$$C = (\sigma_{ij})_{i,j=1}^n \text{ and } \theta = (\theta_1, \dots, \theta_n)^T, \quad (2.7)$$

and

$$\theta = (\theta_1, \dots, \theta_n)^T, \quad (2.8)$$

respectively, are assumed to be constant.

2.2.2 BANK LIABILITIES

Liabilities constitute the sources of funds for banks. In the main, these funds are used to purchase income-earning assets, issue loans and accumulate reserves. The dynamics of the bank's liabilities is stochastic because its value has a reliance on, for instance, deposits and borrowings that both have randomness associated with them. Evidence supporting the forms of liability item models derived subsequently may be found in general banking literature such as [30] and [40].

Borrowings

In the sequel, $B : \Omega \times T \rightarrow \mathbb{R}$ denotes **borrowing from other banks and the federal reserve** whose value at time t is denoted by $B(t)$. Changes in B will mainly be driven by liquidity needs. However, it is an indisputable fact that the returns on the bank's investments in risky assets and the evolution of borrowings are closely related. How this relationship may be quantified requires further investigation. In this regard, a realistic assumption is that a change in borrowings relative to NRWAs may be expressed as

$$\frac{dB(t)}{a_{nr}(t)} = \left(r(t) + p(t)^T C \theta \right) dt + p(t)^T C dZ(t), \quad (2.9)$$

where

$$p(t) = (p_1(t), \dots, p_n(t))^T \quad (2.10)$$

reflects the proportions of bank investments in the n different risky asset categories. In this regard, it is possible in an environment of falling interest rates that investors might be switching out of bank deposits into equities/long term bonds. This may necessitate that banks substitute interbank borrowing for deposits, while the bank is making very good returns on its liquid portfolio w .

2.2.3 BANK CAPITAL

Basel I and II suggest strong connections between bank capital, loan issuing and macroeconomic factors. The uncertainty inherent in the latter two activities, is a motivation for modelling bank capital stochastically (see, for instance, [20], [21], [24], [31], [32], [37] and [47] for a few examples). In addition, bank capital is likely to be less variable under Basel II than under Basel I.

Equity Capital

Equity capital consists of issued and paid ordinary shares and noncumulative perpetual preferred stock. Such capital also includes instruments that are not redeemable at the option of the bank. Under the Basel II framework, this type of capital is considered to be the most important. The first reason for this is that it is the only form of capital that is common

to all G-10 countries. Also, in keeping with the third pillar of the Basel II Capital Accord about disclosure, it should be reported in any bank's published statements. Furthermore, information about equity capital is indispensable when calculating profit margins and how competitive a bank is. In the sequel, the stochastic process $c_{eq} : \Omega \times T \rightarrow \mathbb{R}_+$ is taken to be the **equity capital**, whose value at time t is denoted by $c_{eq}(t)$. A realistic assumption about this type of capital is that its evolution will be affected by disruptive and unexpected events that are related to the investment philosophy of shareholders, general state of the economy or profitability of the bank. In this case, c_{eq} can be considered to be random and we may choose to represent it by means of geometric Brownian motion thus making the analysis more tractable. In this regard, c_{eq} will reflect reality by, for instance, always having positive values. Also, its increments will follow lognormal distributions. Moreover, it seems plausible to model c_{eq} as a path-continuous scalar Itô process acting on the probability space $(\Omega, \mathcal{F}, \mathbf{P})$ considered on the time interval $[t_0, t_1]$. We describe the dynamics of the equity capital by means of the SDE

$$dc_{eq}(t) = c_{eq}(t) \left[\left(r(t) + \mu_{eq}(t) \right) dt + \sigma_{eq_0} dZ_0(t) + \sigma_{eq}^T dZ(t) \right], \quad c_{eq,0} = c_{eq}(t_0), \quad (2.11)$$

where μ_{eq} is a deterministic function of time and

$$\sigma_{eq} = (\sigma_{eq_1}, \dots, \sigma_{eq_n})^T, \text{ where } \sigma_{eq_j} \text{ constant for all } j.$$

Also, Z_0 is a standard Brownian motion independent of $Z(t)$. The volatility σ_{eq} allows for a possible correlation between the equity capital and returns on investments. In the situation described by (2.11), $c_{eq}(t)$ may be decomposed as

$$c_{eq}(t) = c_{eq_h}(t) c_{eq_n}(t), \quad (2.12)$$

where c_{eq_h} is the hedgeable factor of c_{eq} that may be expressed as

$$c_{eq_h}(t) = c_{eq}(0) \exp \left[\left(r(t) + \mu_{eq}(t) - \frac{1}{2} \sigma_{eq}^2 \right) t + \sigma_{eq}^T dZ(t) \right] \quad (2.13)$$

and c_{eq_n} the non-hedgeable factor of c_{eq} that is given by

$$c_{eq_n}(t) = \exp \left[-\frac{1}{2} \sigma_{eq_0}^2 t + \sigma_{eq_0} dZ_0(t) \right]. \quad (2.14)$$

Tier 1 or Core Capital

According to [27], **Tier 1 (T1)** consists of common equity capital plus noncumulative perpetual preferred stock plus minority interest in consolidated subsidiaries minus certain deductions. As is well-known these deductions consist of goodwill and investments in subsidiaries which are engaged in banking or other financial activities but which are not consolidated when determining the bank's capital adequacy requirements. T1 capital is a term

used to describe the capital adequacy of a bank, is always available and acts as a buffer against losses without a bank being required to cease trading. Also, the amount of T1 capital affects returns for shareholders in the bank while a minimum amount of such capital is required by regulatory authorities. The description of T1 capital below, is analogous to that for equity capital (compare, equations (2.11), (2.12), (2.13) and (2.14)). In the ensuing analysis, $c_{T1} : \Omega \times T \rightarrow \mathbb{R}_+$ is the **Tier 1 capital**, whose value at time t is denoted by $c_{T1}(t)$. We represent the dynamics of T1 capital by

$$dc_{T1}(t) = c_{T1}(t) \left[\left(r(t) + \mu_{T1}(t) \right) dt + \sum_{j=1}^n \sigma_{T1,j} dZ_j(t) \right], \quad c_{T1,0} = c_{T1}(t_0). \quad (2.15)$$

Here μ_{T1} is a deterministic function of time that is dependent on $\mu_{eq}(t)$ and the $\sigma_{T1,j}$'s are constants such that

$$\sigma_{T1} = (\sigma_{T1,1}, \dots, \sigma_{T1,n})^T.$$

In fact, we can rewrite (2.15) in the form

$$dc_{T1}(t) = c_{T1}(t) \left[\left(r(t) + \mu_{T1}(t) \right) dt + \sigma_{T1}^T dZ(t) \right]. \quad (2.16)$$

Supplementary (Tier 2 and 3) Capital

Tier 2 (T2) capital includes unaudited retained earnings; revaluation reserves; general provisions for bad debts; perpetual cumulative preference shares (i.e., preference shares with no maturity date whose dividends accrue for future payment even if the bank's financial condition does not support immediate payment) and perpetual subordinated debt (i.e., debt with no maturity date which ranks in priority behind all creditors except shareholders). Tier 2 capital or supplementary capital can absorb losses in the event of a wind-up and so provides a lesser degree of protection to depositors. **Tier 3 (T3) capital** consists of subordinated debt with a term of at least 5 years and redeemable preference shares which may not be redeemed for at least 5 years. T3 capital can be used to provide a hedge against losses caused by market risks if T1 and T2 capital are insufficient for this. In a manner analogous to that described in the previous section for T1 capital, the dynamics of the T2 and T3 capital may be represented by

$$dc_{Tk}(t) = c_{Tk}(t) \left[\mu_{Tk}(t) dt + \sigma_{Tk}^T dZ(t) \right], \quad c_{Tk,0} = c_{Tk}(t_0),$$

where μ_{Tk} is a deterministic function of time and

$$\sigma_{Tk} = (\sigma_{Tk,1}, \dots, \sigma_{Tk,n})^T, \text{ where } \sigma_{Tk,j} \text{ constant for all } j.$$

Eligible Regulatory Capital

The definition of the forms of capital that are eligible to be held as regulatory capital can be found in [9] (see, also, [14], [15] and [22]). In this regard, the ERC, $c_{er} : \Omega \times T \rightarrow \mathbb{R}_+$, can be described as total capital minus certain regulatory deductions as in

$$c_{er}(t) = c_{T1}(t) + c_{T2}(t) + c_{T3}(t) - c_d(t),$$

where we assume that changes in the deductions, $c_d \in \mathbb{R}$, remain constant, i.e.,

$$dc_d(t) = 0, \quad c_d(t_0) = c_{d,0}.$$

As was alluded to before, c_d consist of goodwill (deducted from total T1 capital) and investments in subsidiaries which are engaged in banking or other financial activities but which are not consolidated when determining the bank's capital adequacy requirements (deducted from the total of T1 and T2 capital). The amount of eligible intangibles (including servicing rights) included in ERC is limited in accordance with supervisory capital regulations. The stochastic (control) system for the ERC can be deduced from the above models for T1, T2 and T3 capital and regulatory deductions. As a result, the dynamics of the ERC can be represented by

$$\begin{aligned} dc_{er}(t) = & c_{er}(t) \left(\frac{c_{T1}(t)}{c_{er}(t)} \left[\left(r(t) + \mu_{T1}(t) \right) dt + \sigma_{T1}^T dZ(t) \right] \right. \\ & \left. + \sum_{k=1}^2 \frac{c_{Tk}(t)}{c_{er}(t)} \left[\mu_{Tk}(t) dt + \sigma_{Tk}^T dZ(t) \right] \right), \end{aligned} \quad (2.17)$$

where $c_{er}(t_0) = c_{er,0}$. For ease of computation, we choose to express the dynamics of the ERC, c_{er} , given in (2.17) in the simplified form

$$dc_{er}(t) = c_{er}(t) [r_{c_{er}}(t) dt + \sigma_{c_{er}}(t) dW_{c_{er}}(t)], \quad (2.18)$$

where

$$r_{c_{er}}(t) = r(t) + \sum_{k=1}^3 \frac{c_{Tk}(t)}{c_{er}(t)} \mu_{Tk}(t) \quad \text{and} \quad \sigma_{c_{er}}(t) dW_{c_{er}}(t) = \sum_{k=1}^3 \frac{c_{Tk}(t)}{c_{er}(t)} \sigma_{Tk}^T dZ(t). \quad (2.19)$$

2.3 NON-RISK-BASED CAPITAL ADEQUACY RATIOS

Our main objective in this section is to provide a description of the non-risk-based capital adequacy ratios, viz., leverage and equity CARs (see Theorem 2.3.1 and Corollary 2.3.2).

2.3.1 DYNAMICS OF NON-RISK-WEIGHTED ASSETS

Equation (2.2) implies that the dynamics of the stylized balance sheet may be represented as

$$dR(t) + dL(t) + da_{nr}(t) = dD(t) + dB(t) + dc_{T1}(t). \quad (2.20)$$

If we apply the banking principle (see, for instance, [40] for more details) that when a bank receives additional deposits it gains an equal amount of reserves (also, when it loses deposits, it loses an equal amount of reserves) we have that $dR(t) = dD(t)$, $t_0 < t < t_1$, in (2.20) and

$$da_{nr}(t) = dB(t) + dc_{T1}(t) - dL(t). \quad (2.21)$$

From (2.9) and (2.21) we can deduce that the dynamics of the value of the total non-risk weighted assets (TNRWAs), a_{nr} , may be represented by

$$da_{nr}(t) = a_{nr}(t) \left[\left(r(t) + p(t)^T C \theta \right) dt + p(t)^T C dZ(t) \right] - dL(t) + dc_{T1}(t). \quad (2.22)$$

A further modification can be made to (2.22) if we take (2.5) and (2.16) into account and set $\eta = N_t = 0$. In this case, we can re-write (2.22) as

$$\begin{aligned} da_{nr}(t) = & a_{nr}(t) \left[\left(r(t) + p(t)^T C \theta \right) dt + p(t)^T C dZ(t) \right] + (\alpha - 1)l(t)dt \\ & + c_{T1}(t) \left[\left(r(t) + \mu_{T1}(t) \right) dt + \sigma_{T1}^T dZ(t) \right]. \end{aligned} \quad (2.23)$$

2.3.2 STOCHASTIC MODELLING OF NRBCARs

In this subsection, we deduce a stochastic model for the NRBCAR dynamics of a bank. We denote the leverage and equity CARs by $z_l : \Omega \times T \rightarrow \mathbb{R}$ and $z_{eq} : \Omega \times T \rightarrow \mathbb{R}$ respectively. Also, their defining formulas are given by

$$z_l(t) = \frac{c_{T1}(t)}{a_{nr}(t)} \text{ and } z_{eq}(t) = \frac{c_{eq}(t)}{a_{nr}(t)}, \quad (2.24)$$

respectively. In the following results we compute z_l and z_{eq} , respectively. Strictly speaking, the denominator of the formulas for the leverage and equity CARs presented in (2.24) should be the total assets $R + L + a_{nr}$. Our alternative choice of denominator is justified by the fact that the NRWAs, a_{nr} , by itself effectively encapsulates market risk and, by definition, includes a component reflecting credit risk.

SDE for the Leverage CAR

The leverage ratio has a long history and assumes implicitly that the capital needs of a bank are directly proportional to its level of assets. A leverage ratio stipulation affects the asset allocation of banks that are constrained by the requirement.

Theorem 2.3.1 (SDE for the Leverage CAR of a Bank) *Suppose that Z , θ , C and p are given by (2.6), (2.7), (2.8) and (2.10), respectively. Furthermore, assume that the Tier 1 (core) capital and TNRWAs are as defined by (2.16) and (2.23), respectively. A system*

with constant volatilities that describes the stochastic dynamics of the leverage CAR of a bank may be represented by the SDE

$$dz_l(t) = z_l(t) \left\{ \left[(1 - z_l(t))\mu_{T1} - z_l(t)r(t) - p^T(t)C\theta + a_{nr}^{-1}(t)(1 - \alpha)l(t) + C^T p(t)p^T(t)C + C^T p(t)\sigma_{T1} \right] dt - p^T(t)CdZ(t) + (1 - z_l(t))\sigma_{T1}dZ(t) \right\}. \quad (2.25)$$

Proof. A comprehensive proof is included in the appendix. $\square$

SDE for the Equity CAR

The equity ratio also has a long history and supposes that the capital contributed by shareholders are directly proportional to the bank's level of assets.

Corollary 2.3.2 (SDE for the Equity CAR of a Bank) *Assume that Z , θ , C and p are defined by (2.6), (2.7), (2.8) and (2.10), respectively. Furthermore, suppose that the equity capital and TNRWAs are as given by (2.11) and (2.23), respectively. A system with constant volatilities that describes the stochastic dynamics of the equity CAR of a bank may be represented by the SDE*

$$dz_{eq}(t) = z_{eq}(t) \left\{ \left[(1 - z_l(t))\mu_{T1} - z_l(t)r(t) - p^T(t)C\theta + a_{nr}^{-1}(t)(1 - \alpha)l(t) + \sigma_{T1}^2 + C^T p(t)p^T(t)C + C^T p(t)\sigma_{T1} + \sigma_{T1}^T p(t)^T C - \sigma_{eq_0}^2 - \sigma_{eq}^2 - \sigma_{eq}^T p(t)^T C \right] dt - p^T(t)CdZ(t) + (1 - z_l(t))\sigma_{T1}dZ(t) \right\}. \quad (2.26)$$

2.4 RISK-BASED CAPITAL ADEQUACY RATIOS

In this section, we consider risk-based capital adequacy ratios which require that we extend the dynamics of the NRWAs to the situation where assets are risk-weighted (see, also, [41]). However, by contrast to the analysis in Section 2.3, we deem it more appropriate to make exclusive use of summation formulas in the current section.

2.4.1 CREDIT RISK-WEIGHTED ASSETS

The discussions in this subsection have been distilled from a formalization of certain aspects of "Section III. Credit Risk - The Internal Ratings Based Approach" of "Part 2: The First Pillar - Minimum Capital Requirements" contained in [14] (see, also, [6] and [10] for more details). According to Basel II, the measurement of credit risk exposures via the IRB approach requires that amendments be made to the value of bank assets displayed on a balance sheet as given by (2.2). In this regard, the different categories of loans a

bank issues are weighted according to their general degree of riskiness. Off-balance sheet contracts, such as guarantees and foreign exchange contracts, also carry credit risks. These exposures are converted to credit equivalent amounts which are also weighted in the same way as on-balance sheet credit exposures.

Credit Risk Exposure Types

Under the IRB approach, banks must categorize banking-book exposures into broad classes of assets with different underlying risk characteristics (see [6], [10] and [14]). In this regard, 15 credit risk exposure types may be identified that may be listed as follows.

- $i = 1$: Project Finance (PF);
- $i = 2$: Object Finance (OF);
- $i = 3$: Commodities Finance (CF);
- $i = 4$: Income Producing Real Estate (IPRE);
- $i = 5$: Specialized Lending High Volatility Commercial Real Estate (SLHVCRE);
- $i = 6$: Specialized Lending Not Including
High Volatility Commercial Real Estate (SLNIHVCRE);
- $i = 7$: Bank Exposure (BE);
- $i = 8$: Sovereign Exposure (SE);
- $i = 9$: Retail Residential Mortgage (RRM);
- $i = 10$: Home Equity Line of Credit (HELOC);
- $i = 11$: Other Retail Exposure (ORE);
- $i = 12$: Qualifying Revolving Retail Exposure (QRRE);
- $i = 13$: Small to Medium Size Enterprises with Corporate Treatment (SMECT);
- $i = 14$: Small to Medium Size Enterprises with Retail Treatment (SMERT);
- $i = 15$: Equity Exposure Not Held in the Trading Book (EENHTB)

with $i = 1-6$ and $i = 9-12$ constituting corporate and retail exposures, respectively. The precise definitions of the foregoing credit risk categories are provided in [14] (see, also, [10]). With certain minimum conditions and disclosure requirements in place, banks that have received supervisory approval to use the Internal Ratings Based (IRB) approach may use their own internal estimates of risk components in determining the capital requirement for a given exposure. The derivation of RWAs for the aforementioned credit risk categories is dependent on estimates of risk components such as the probability of default (PD), loss given default (LGD), exposure at default (EAD) and, in some cases, effective maturity (EM). In the sequel, the actual values of PD, LGD, EAD and EM are denoted by p_d , l_d , e_d and m , respectively. Throughout we have that

$$0 \leq p_d \leq 1, \quad 0 \leq l_d \leq 1$$

and that e_d is measured in a monetary unit. Also, the unit of measurement of the effective maturity, m , is years. In some cases, banks may be required to use a supervisory value as opposed to an internal estimate for one or more of the risk components.

UL Capital Requirements for Credit Risk Exposures

The IRB approach produces a statistical measurement of both the ULs and ELs that banks face in relation to their credit risk exposures. In particular, the framework set out in [13] incorporated both UL and EL components into the IRB capital requirement. After consultation, the Basel Committee on Banking Supervision (BCBS) decided that the separate treatment of ULs and ELs within the IRB framework will result in a setting that is superior and more consistent. Under this modified approach, the measurement of RWAs are based solely on the ULs portion of the IRB calculations. In the sequel, we discuss the UL capital requirements for credit risk exposures that are not in default and the cases where they are. The former situation is treated by considering a risk-weighted function (RWF) that provides the means by which risk components are transformed into RWAs and ultimately capital requirements.

For *credit risk exposures not in default*, seven categories of UL RWFs for calculating RWAs can be distinguished. The first component is the **weighted correlation for the exposure** given by

$$R = c_1 w + c_2(1 - w), \quad (2.27)$$

where the **weight for the exposure**, w , is given by

$$w = \frac{1 - \exp\{Jp_d\}}{1 - \exp\{J\}}.$$

Furthermore, for SMECT and EENHTB, a **firm-size adjustment** can be made by subtracting

$$0,04 \left[1 - \frac{s-5}{45} \right], \quad s_1 = 5 \leq s \leq s_2 = 50,$$

from (2.27). The **maturity adjustment for the exposure** may be represented as

$$b = (p_A + p_B \times \ln(p_d))^2.$$

In this case, the **capital requirement for the exposure** has the form

$$k = l_d \left[N \left[G(p_d) \sqrt{\frac{1}{1-R}} + G(0,999) \sqrt{\frac{R}{1-R}} - p_d \right] \left[\frac{1 + (m-2,5)b}{1-1,5b} \right] \right],$$

where $N(x)$ denotes the cumulative distribution function for a standard normal random variable while $G(z)$ denotes the inverse cumulative function for a standard normal random variable. Finally, we have that the **value of the RWAs for the exposure**, is given by

$$a_c = 12,5ke_d.$$

The capital requirement, k_i^{def} , $i = 1, \dots, 15$, for *defaulted credit risk exposures* is subject to the following condition:

$$k_i^{def} = \max\{0, l_{d_i}^{def} - l_{e_i}^{def\epsilon}\}, \quad i = 1, \dots, 15,$$

where

- $l_{d_i}^{def}$: Value of the LGD of Credit Risk Exposures in Default;
- $l_{e_i}^{def\epsilon}$: Bank's Best Estimate of ELs for Defaulted Credit Risk Exposures.

The value of the RWAs for *defaulted credit risk exposures* is

$$a_{c_i}^{def} = 12,5 k_i^{def} e_{d_i}^{def}, \quad i = 1, \dots, 15.$$

For each defaulted asset, the bank's best estimates of ELs are based on prevailing economic circumstances and institutional status.

2.4.2 MARKET AND OPERATIONAL RWAs

The contribution [41] computes *market TRWAs* via the *internal model approach* that involves Value-at-Risk (VaR) models and describes the capital requirement for *operational risk* from the viewpoint of the *standardized approach* (see, for instance, [11]). The aforementioned paper calculates the value of the market and operational TRWAs which, in the sequel, are denoted by a_m and a_o , respectively.

Market RWAs

Market risk is defined as the risk of losses in on- and off-balance sheet positions arising from movements in market prices. Market risks include risks of losses on foreign exchange and interest rate contracts caused by changes in foreign exchange rates and interest rates. In our dissertation, a version of the well-known Value-at-Risk (VaR) model is used to describe the capital charge for *market risk*. By way of definition, for a given time horizon T and confidence level p the **VaR** is the loss in the market value over T that is exceeded with probability $1 - p$. A VaR model that is used by many banks in G-10 countries is

$$a_{m_p}(t) = \max[VaR(t_-) + d(t)ASR^{VaR}(t_-), \quad (2.28)$$

$$M(t)\frac{1}{60} \sum_{p=1}^{60} VaR((t-p)_-) + d(t)\frac{1}{60} \sum_{p=1}^{60} ASR^{VaR}((t-p)_-)],$$

where

- $VaR(s)$: Value-at-Risk at Time s ;
- $VaR(s_-)$: Value-at-Risk 24-Hrs Before Time s ;
- $d(t)$: 0-1 Indicator Function Related to Estimation of Specific Risk
Measured Through Additional Specific Risk (ASR) Measure from VaR;
- $M(t)$: Multiplier for Stress Factor, $M(t) \geq 3$;
- p : Days, $1 \leq p \leq 60$.

The choice of VaR formula in (2.28) satisfies the qualitative standards for the model approach to market risk outlined in [4] (see, also, [5]). We also note that (2.28) falls within the class of VaR models that depend on random changes in the prices of the underlying instruments, like equity indices, interest rates, foreign exchange rates and commodity indices.

Operational RWAs

We consider the capital requirement for *operational risk* from the viewpoint of the *standardized approach* (see, for instance, [11]). **Operational risk** is defined as the risk of loss resulting from inadequate or failed internal processes, people and systems or from external events. For the standardized approach, the activities of banks are categorized into eight business lines, viz., corporate finance (corporate finance, municipal/government finance, merchant banking, advisory services), trading and sales (sales, market making, proprietary positions, treasury), retail banking (retail banking, private banking, card services), commercial banking, payment and settlement (external clients), agency services (custody, corporate agency, corporate trust), asset management (discretionary fund management) and retail brokerage. Each of the aforementioned business lines (not the whole institution) uses *gross income* (net interest income + net non-interest income) as a broad indicator of the extent of business operations and thus the probable extent of operational risk exposure. The *capital charge for each business line* is determined by multiplying the business line gross income by a weighting term known as a **beta factor**. This beta factor is an indication of the correlation between the operational risk loss experience and the aggregate level of gross income for that business line taking the whole industry into account. The total capital charge for operational risk under the standardized approach is calculated as the three-year average of the simple summation of the ERC charges across each of the business lines in each year. The capital charge under the standardized approach is expressed as

$$a_{occ} = \max\left[\sum_{k=1}^8 \beta_k g_k, 0\right],$$

where

- a_{occ} : Total Capital Charge for Operational Risk under the Standardized Approach;
- g_{1-8} : Three-Year Average of Gross Income for Each of Eight Business Lines;
- β_{1-8} : Fixed Percentage Relating Level of Required Capital to Level of Gross Income for Each of Eight Business Lines.

The values of the betas for operational risk are provided by Figure 4 in Subsection 2.6.2.

2.4.3 TOTAL RISK-WEIGHTED ASSETS

According to the Basel II Capital Accord (see Part 2 of [14] for more details), the TRWAs of a bank are determined by multiplying the capital charges for market and operational risk by 12,5 and adding the resulting value to the sum of RWAs for credit risk. In the sequel, we denote the value of the TRWAs by a , where, for $1 \leq i \leq 15$, $1 \leq k \leq 8$, we have

$$\begin{aligned}
 a(t) &= \sum_{i=1}^{15} a_{ci}^{tot}(t) + 12,5a_{mcc}(t) + 12,5a_{occ}(t) \\
 &= a_c(t) + a_m(t) + a_o(t).
 \end{aligned} \tag{2.29}$$

Here we specify that

- a : Value of TRWAs Arising From Credit, Market and Operational RWAs;
- a_{c_i} : UL RWAs for i -th Credit Risk Exposure Not In Default,
 $a_{c_i}(t) = 12,5k_i(t)e_{d_i}(t)$, $i = 1, \dots, 15$;
- k_i : UL Capital Requirement for i -th Credit Risk Exposure Not In Default;
- e_{d_i} : UL EAD for i -th Credit Risk Exposure Not In Default;
- $a_{c_i}^{def}$: UL Credit RWAs for i -th Exposure In Default,
 $a_{c_i}^{def}(t) = 12,5k_i^{def}(t)e_{d_i}^{def}(t)$, $i = 1, \dots, 15$;
- k_i^{def} : UL Capital Requirement for i -th Credit Risk Exposure In Default;
- $e_{d_i}^{def}$: UL EAD for i -th Credit Risk Exposure In Default;
- $a_{c_i}^{tot}$: Value of Credit TRWAs for i -th Exposure,
 $a_{c_i}^{tot}(t) = a_{c_i}(t) + a_{c_i}^{def}(t)$;
- a_c : Value of Credit TRWAs,
 $a_c(t) = \sum_{i=1}^{15} a_{c_i}^{tot}(t)$;
- a_m : Value of Market TRWAs;
- a_o : Value of Operational TRWAs.

Total RWA Outflows and Inflows

Outflows from RWAs have uncertainty associated with them and can thus be modelled as a stochastic process that is dependent on several random events like, for instance, need for capital for loan issuing and maintenance of appropriate cash reserve levels. In some respects, the bank is free to choose how the capital outflow rate can be varied. In the sequel, the stochastic process $e : \Omega \times T \rightarrow \mathbb{R}$ is the **rate of capital outflow from RWAs** whose value at time t is denoted by $e(t)$. Inflows to RWAs arise from such sources as chequeable and nontransaction deposits, loan repayments, bank borrowing and bank capital. For the sake of our subsequent analysis, we denote the **rate of capital inflows to RWAs** by $u(t)$. Furthermore, in our study, we assume that u is a measurable adapted process with respect to the filtration, $\mathcal{F} = \{\mathcal{F}_t\}_{t \geq 0}$ and for all $t \in T$

$$\mathbf{E}[|u(t)|] < \infty \text{ and } \int_0^t |u(s)|ds < \infty, \text{ a.s.}$$

Modelling the Uncertainty in Total RWAs

From formula (2.29), we have that the bank's TRWAs consist of assets weighted for credit, operational and market risk. We denote the capital requirements for market risk by $y_m(t)$, while categories for credit and operational risk are denoted by $y_{c1}(t), \dots, y_{c15}(t)$ and

$y_{o_1}(t), \dots, y_{o_8}(t)$), respectively. In this case, we represent the stochastic dynamics of the **RWAs for market risk** by the SDE

$$dy_m(t) = y_m(t)[r_m dt + \sigma_m dW_m(t)], \quad (2.30)$$

where $y_m(0) = 1$. Also, r_m , the rate of change of the market RWAs described in (2.30) may be stochastic and be modelled as a one-factor diffusion process. Furthermore, the evolution of the **credit RWAs** may be described by

$$dy_{c_i}(t) = y_{c_i}(t)[r_{c_i} dt + \sum_{j=1}^{15} \sigma_{c_{ij}} dW_{c_j}(t)], \quad i, j = 1, \dots, 15, \quad (2.31)$$

where $y_{c_i}(0) = y_{c_0}$. Also, r_{c_i} and $\sigma_{c_{ij}}$ are positive constants with $\sigma_{c_{ij}}$ being the i -th row of the 15×15 volatility matrix $(\sigma_{c_{ij}})_{i,j=1}^{15}$. Also, we will represent the dynamics of the **operational RWAs** by

$$dy_{o_k}(t) = y_{o_k}(t)[r_{o_k} dt + \sum_{l=1}^8 \sigma_{o_{kl}} dW_{o_l}(t)], \quad k, l = 1, \dots, 8, \quad (2.32)$$

where $y_{o_k}(0) = y_{o_0}$. Furthermore, r_{o_k} and $\sigma_{o_{kl}}$ are positive constants with $\sigma_{o_{kl}}$ being the k -th row of the 8×8 volatility matrix $(\sigma_{o_{kl}})_{k,l=1}^8$. In this case, the vector

$$(W_m(t), W_{c_1}(t), \dots, W_{c_{15}}(t), W_{o_1}(t), \dots, W_{o_8}(t))^T$$

is an 24-dimensional Brownian motion process with independent components defined on the probability space $(\Omega, \mathcal{G}, \mathbf{P})$, where $\{\mathcal{G}_t\}_{t \geq 0}$ represents the completion of the filtration

$$\sigma\{(W_m(s), W_{c_1}(s), \dots, W_{c_{15}}(s), W_{o_1}(s), \dots, W_{o_8}(s))^T : 0 \leq s \leq t\}.$$

From formula (2.29), the *value of the RWAs for market risks* is given by

$$a(t) = \sum_{i=1}^{15} a_{c_i}^{tot}(t) - \sum_{k=1}^8 a_{o_k}(t).$$

These variables are subject to the regulation contained in the Basel I and II Capital Accords (see [3] and [14]) where assets are risk-weighted in a very particular way.

Proposition 2.4.1 (SDE for the TRWA Dynamics of a Bank) *Suppose that the changes in the value of the bank's TRWAs is solely determined by the changes in capital requirements for credit, operational and market risk and the rate of inflows to and outflows from RWAs. Then the dynamics of the value of the bank's TRWAs may be represented as*

$$\begin{aligned} da(t) = & \left(r_m a(t) + \sum_{i=1}^{15} a_{c_i}^{tot}(t)(r_{c_i} - r_m) + \sum_{k=1}^8 a_{o_k}(t)(r_{o_k} - r_m) + u(t) - e(t) \right) dt \\ & + \sum_{i=1}^{15} \sum_{j=1}^{15} a_{c_i}^{tot}(t) \sigma_{c_{ij}} dW_{c_j}(t) + \sum_{k=1}^8 \sum_{l=1}^8 a_{o_k}(t) \sigma_{o_{kl}} dW_{o_l}(t) \\ & + a_m(t) \sigma_m dW_m(t), \end{aligned} \quad (2.33)$$

with initial condition $a(0) = a_0$.

Proof. Our first observation is that, by making use of the equations (2.30), (2.31) and (2.32), the dynamics of the bank's TRWAs may be given by

$$\begin{aligned}
da(t) &= \sum_{i=1}^{15} a_{c_i}^{tot}(t) \frac{dy_{c_i}(t)}{y_{c_i}(t)} + \sum_{k=1}^8 a_{o_k}(t) \frac{dy_{o_k}(t)}{y_{o_k}(t)} + (a(t) - \sum_{i=1}^{15} a_{c_i}^{tot}(t) - \sum_{k=1}^8 a_{o_k}(t)) \frac{dy_m(t)}{y_m(t)} \\
&\quad + (u(t) - e(t))dt \\
&= \sum_{i=1}^{15} a_{c_i}^{tot}(t) [r_{c_i} dt + \sum_{j=1}^{15} \sigma_{c_{ij}} dW_{c_j}(t)] + \sum_{k=1}^8 a_{o_k}(t) [r_{o_k} dt + \sum_{l=1}^8 \sigma_{o_{kl}} dW_{o_l}(t)] \\
&\quad + (u(t) - e(t))dt \\
&\quad + (a(t) - \sum_{i=1}^{15} a_{c_i}^{tot}(t) - \sum_{k=1}^8 a_{o_k}(t)) [r_m dt + \sigma_m dW_m(t)], \tag{2.34}
\end{aligned}$$

with initial condition $a(0) = a_0$. The proof is completed with equation (2.33) being derived from (2.34). $\square$

2.4.4 STOCHASTIC MODELLING OF RBCARs

In this subsection, we deduce stochastic models for Basel II and Tier 1 CARs defined by

$$x(t) = \frac{c_{er}(t)}{a(t)} \text{ and } x_{T1}(t) = \frac{c_{T1}(t)}{a(t)}, \tag{2.35}$$

respectively.

Theorem 2.4.2 (SDE for the Basel II CAR of a Bank) *Suppose that the ERC and the market, credit and operational RWAs are as given in (2.18) and (2.30), (2.31) and (2.32), respectively. A stochastic system that describes the stochastic dynamics of the Basel II CAR of a bank may be represented by the SDE*

$$dx(t) = x(t)[\mu(t)dt + \sigma(t)dW(t)], \tag{2.36}$$

where $x(t_0) = x(0)$,

$$\begin{aligned}
\mu(t) &= -r_m - \sum_{i=1}^{15} \frac{a_{c_i}^{tot}(t)}{a(t)} (r_{c_i} - r_m) + r_{cer} - \sum_{k=1}^8 \frac{a_{o_k}(t)}{a(t)} (r_{o_k} - r_m) - \frac{u(t) - e(t)}{a(t)} \\
&\quad + \sum_{i=1}^{15} \sum_{j=1}^{15} \sum_{k=1}^{15} \frac{a_{c_i}^{tot} a_{c_j}^{tot}}{a(t)^2} (\sigma_{c_{ij}} \sigma_{c_{jk}}) + \sum_{k=1}^8 \sum_{l=1}^8 \sum_{m=1}^8 \frac{a_{o_k} a_{o_l}}{a(t)^2} (\sigma_{o_{kl}} \sigma_{o_{lm}}) \\
&\quad + \frac{a_m(t)^2}{a(t)^2} \sigma_m^2 \tag{2.37}
\end{aligned}$$

and

$$\begin{aligned} \sigma(t)dW(t) = & - \sum_{i=1}^{15} \sum_{j=1}^{15} \frac{a_{c_i}^{tot}(t)}{a(t)} \sigma_{c_{ij}} dW_{c_j}(t) - \sum_{k=1}^8 \sum_{l=1}^8 \frac{a_{o_k}(t)}{a(t)} \sigma_{o_{kl}} dW_{o_k}(t) + \sigma_{c_{er}} dW_{c_{er}}(t) \\ & - \frac{a_m(t)}{a(t)} \sigma_m dW_m(t). \end{aligned} \quad (2.38)$$

Proof. We determine (2.36) by considering the general Ito formula for which (2.44) holds. The method of proof is an analogue of the one for Theorem 2.3.1 and will be omitted here.

□

SDE for the Basel II CAR

The Basel II ratio has a short history and assumes implicitly that the ERC needs of a bank are directly proportional to its level of RWAs.

SDE for the Tier 1 CAR

The Basel II ratio also has a short history and assumes implicitly that the T1 capital needs of a bank are directly proportional to its level of RWAs.

Corollary 2.4.3 (SDE for the Tier 1 CAR of a Bank) *Suppose that the T1 capital and the market, credit and operational RWAs are as given in (2.16) and (2.30), (2.31) and (2.32), respectively. A system that describes the stochastic dynamics of the Tier 1 CAR of a bank may be represented by the SDE*

$$dx_{T1}(t) = x_{T1}(t)[\mu_{T1}(t)dt + \sigma_{T1}(t)dW_{T1}(t)],$$

where $x_{T1}(t_0) = x_{T1}(0)$. Also, explicit formulas for μ_{T1} and $\sigma_{T1}dW_{T1}$ are obtained from Theorem 2.4.2 by replacing r_{cer} in (2.37) and $\sigma_{cer}dW_{cer}$ in (2.38) by r_{cT1} and $\sigma_{cT1}dW_{cT1}$, respectively.

2.5 INTERPRETATION OF MAIN ISSUES

In this section, we comment on the main issues raised in the discussion above. In particular, we consider some features of the stochastic models that we constructed in the preceding sections, we look at Basel II regulation and forge connections between our work and related studies.

2.5.1 STOCHASTIC BANKING MODEL

Our continuous-time stochastic model is based on the stylized balance sheet presented in (2.2). When we apply a simple banking principle that equates the change in deposits and reserves (see, for instance, [40] where $dR(t) = dD(t)$), we obtain a description of the

dynamics of the bank portfolio of the form given in (2.21). A further choice of banking policy which aims to correlate bank borrowings and portfolio features ultimately leads to the SDE (2.23) in the case of NRBCARs.

Bank Assets and Liabilities

There is strong evidence, from recent FDIC US commercial bank data (see [27]), that the choice of a mean-reverting loan issuing rate as in (2.3) is a realistic one for many credit exposure types. For example, this is true for home, object and project finance loans. In addition, the Basel Accord has several capital constraints associated with the issuing of loans. The most important of these is the **total capital constraint** that relates loans, $L(t)$, to the sum of T1, T2 and T3 capital, $c_{T1} + c_{T2} + c_{T3}$, via the inequality

$$c_{T1} + c_{T2} + c_{T3} \geq \rho L(t), \quad (2.39)$$

where ρ denotes the regulatory ratio of total capital to loans. If this policy is violated the bank should introduce some measures to restrict loan issuing.

In reality, bank reserves are the outcome of all the net payment flows affecting the bank balance sheet, i.e., dR will depend on dL and dB as well as on dD and also on virtually all profit and loss variables, including all net interest payments on L , B , D , non-interest income and non-interest expenses, tax payments, etc.

Bank Capital

In order to forge a closer connection between bank profitability, π , and the value of the bank to the investor, V , our model for T1, T2 and T3 capital may be refined in the following way. Suppose that T1 capital, c_{T1} , and the sum of T2 and T3 capital, $c_{T2} + c_{T3}$, are solely constituted by the market value of bank equity, v , and bonds, b (which pay an interest rate of r_b), respectively. Furthermore, let π be the bank's profit which is used to meet obligations such as dividend payments on equity, d_e , and interest and principal payments on bonds, $(1 + r_b)b$. In this case, we may compute the retained earnings, e_r , (constituent of the T1 capital) after these payments as

$$e_r(t) = \pi(t) - d_e(t) - (1 + r_b(t))b(t). \quad (2.40)$$

Suppose that the total cost of the buildings and equipment that the bank invests in, c , depreciates to the extent δc . Improvements to these items may be financed through retained earnings, new equity and bonds according to

$$\delta c(t) = e_r(t) + dv(t) + db(t). \quad (2.41)$$

From (2.40) and (2.41) we can conclude that the net cash flow generated by the bank for an investor, ν , may be given by

$$\nu(t) = \pi(t) - \delta c(t) = d_e(t) + (1 + r_b(t))b(t) - dv(t) - db(t). \quad (2.42)$$

The equation (2.42) suggests that the value of the bank to the investor, V , is equal to the sum of the net cash flow, ν , and the bank's ex-dividend value as given by

$$V(t) = \nu(t) + dv(t) + db(t).$$

2.5.2 NON-RISK-BASED CAPITAL ADEQUACY RATIOS (NRBCARs)

In order to illustrate the economic importance of the results obtained in Theorem 2.3.1 and Corollary 2.3.2 we consider their role in general and in determining an optimal asset allocation strategy for NRBCARs.

Stochastic Modelling of NRBCARs

A more general system than (2.25) that describes the stochastic dynamics of the leverage CAR of a bank with *time-varying volatility* may be represented by the SDE

$$\begin{aligned} dz_l(t) = & z_l(t) \left\{ \left(\mu_{T1}(t) - p(t)^T C \theta - z_l(t)(r(t) + \mu_{T1}(t)) + a_{nr}^{-1}(t)(1 - \alpha)l(t) \right) dt \right. \\ & + \left(dZ(t)^T R(t) + [1 - z_l(t)]\sigma_{T1}^T(t) - p(t)^T C \right) dZ(t) \\ & \left. - dZ(t)^T \sigma_{T1}(t) \left(z_l(t)\sigma_{T1}^T(t) + p(t)^T C - dZ(t)^T R(t) \right) dZ(t) \right\}, \end{aligned}$$

where the symmetric matrix equation

$$\begin{aligned} R(t) = & C^T p(t)p(t)^T C + z_l(t) \left(C^T p(t)\sigma_{T1}^T(t) + \sigma_{T1}(t)p(t)^T C \right) \\ & + z_l(t)^2 \sigma_{T1}(t)\sigma_{T1}^T(t). \end{aligned} \quad (2.43)$$

A more general system than (2.26) that considers the stochastic dynamics of the equity CAR of a bank with *time-varying volatility* may be given by the SDE

$$\begin{aligned} dz_{eq}(t) = & z_{eq}(t) \left\{ \left(\mu_{eq}(t) - p(t)^T C \theta - z_l(t)(r(t) + \mu_{T1}(t)) + a_{nr}^{-1}(t)(1 - \alpha)l(t) \right) dt \right. \\ & + \left(dZ(t)^T R(t) + \sigma_{eq}^T(t) - z_l(t)\sigma_{T1}^T(t) - p(t)^T C \right) dZ(t) \\ & \left. - dZ(t)^T \sigma_{eq}(t) \left(z_l(t)\sigma_{T1}^T(t) + p(t)^T C - dZ(t)^T R(t) \right) dZ(t) \right\}. \end{aligned}$$

The models of banking items related to non-risk based capital adequacy ratios, constructed in the current dissertation, have proven to be most useful in the optimal management of bank risks via stochastic control. Subsequently, we briefly describe the contributions made in [29] and [42] (see, also, [15], [25] and [26]). A first optimization problem may be formulated as follows.

Problem 2.5.1 (Optimization Problem): *For a given time period $[t_0, t_1]$, what optimal level of inflows from Tier 1 capital and returns on assets of a commercial bank is needed to attain a stipulated level of loan issuing at t_1 ?*

The instruments that may be used by the bank to reach the control objectives set out in the above problem are the rate at which bank capital is sourced and the return on investments by means of portfolio choice. A scenario in which the solution to this problem would be useful is described below. For a particular type of credit exposure, a bank wishes to issue a certain amount in loans at t_1 . During the period $[t_0, t_1]$, capital towards the achievement of these stipulated goals is sourced from investment returns and the accessing of core capital. As in [50], the solution to this problem establishes a positive correlation between bank capital and lending. The solution of the stochastic optimal control problem formulated above may be obtained by relying on some of the procedures developed in [28], [36], [39] and [49]. In this spirit, [42] examines a problem related to the optimal management of commercial banks in a stochastic dynamic setting. In particular, we minimize market risk and risks related to the inflow of bank capital that involves the security of the assets held and the stability of sources of funds, respectively. In this regard, we suggest an optimal portfolio choice and rate of bank capital inflow that will keep the levels of loan issuing as close as possible to an actuarially determined reference process. This set-up leads to a nonlinear stochastic optimal control problem whose solution may be determined by means of the dynamic programming algorithm. The accompanying analysis is reliant on the construction of a continuous-time stochastic model that results in a spread method of bank capitalization. Many aspects of this model is consistent with U.S. commercial bank data from the last twenty years as reported by the Federal Deposit Insurance Corporation (FDIC). A second optimization problem is considered in [29], where we illustrate that it is possible to use an analytic approach to optimize asset allocation and location strategies for banks. The connection with the current dissertation is that the aforementioned optimization procedure is dependent on the stochastic modelling of items on the bank's balance sheet, regulatory capital and the capital adequacy ratio. We show that the optimal allocation is constituted by a combination of cash, bonds and equities. On the other hand, the optimal asset location strategy intimates that banks prefer to allocate their entire tax-deferred wealth to the asset with the highest yield.

2.5.3 RISK-BASED CAPITAL ADEQUACY RATIOS (RBCARs)

In this subsection, we briefly discuss aspects of our main results in terms of the stochastic modelling of RBCARs. Furthermore, we interpret the formulas (2.36), (2.37) and (2.38).

Stochastic Modelling of RBCARs

The Basel II Capital Accord recommends a minimum Basel II CAR value of 0.08 in order to ensure that banks can absorb a reasonable level of losses before going insolvent. Applying minimum capital adequacy standards serves to protect depositors and promote the stability

and efficiency of the bank. Banks should be able to access their overall capital adequacy in terms of their risk profile. Furthermore, they should have a procedure in place for maintaining their levels of capital. However, these levels will differ between banks operating normally and those in the process of liquidation. An important question that arises from Theorem 2.4.2 is the following.

Problem 2.5.2 (Problem Related to Theorem 2.4.2): *How can the stochastic model of Theorem 2.4.2 assist banks in maintaining a particular Basel II CAR level ?*

A partial answer to this question is provided below. If banks apply control to their credit (lending) operations in such a way that the Basel II CAR remains high they will remain out of the zone in which insolvency may be a possibility. The higher banks set the control objective for the Basel II CAR the lower the probability that it will fall below 8 %. This probability can be computed with the SDE formulated in Theorem 2.4.2. As a guideline the author recommends that the bank sets as control objective to keep its Basel II CAR in the range of [12%, 20%]. Of course, a high value of the ERC may mean a lower income for the bank. The trade-off between observing the Basel II regulations and the income of the bank has to be made by every bank individually. A major benefit of large banks is that the averaging effect of many market, credit and operational operations results in a lower variance for the Basel II CAR than if all credits were taken in the same credit sector. The underlying assumption is that the probabilities of bad events for the different credit operations are as much independent as possible. Because of this lower variance, the probability of the Basel II CAR falling below 8 % is smaller than if all capital is assigned to one credit sector. This minimizing effect of the variance is an important advantage for a banks with a large portfolio of credit operations.

For the formulas (2.36), (2.37) and (2.38) the relative change in the Basel II CAR is specified by the SDE

$$\frac{dx(t)}{x(t)} = \mu(t)dt + \sigma(t)dW(t).$$

If the rate function $\mu(t) > 0$ then x increases so that the Basel II CAR improves. If $\mu(t) < 0$ then x decreases so that the Basel II CAR deteriorates. From the formula (2.37) for the rate function μ one sees that if r_{cer} increases then μ increases hence the Basel II CAR x increases. If either for any $i = 1, \dots, 15$, r_{ci} , or for $k = 1, \dots, 8$, we have that r_{ok} increases, or r_m increases then μ decreases and consequently the Basel II CAR decreases. If e increases then there is increased outflow of the TRWAs and hence μ increases and so does x . If u increases then there is a net inflow of TRWAs so that both μ and the Basel II CAR decreases. The last three terms in (2.38) represent the increase in x due to the volatility. The SDE (2.38) represents the effect of all uncertainty. From an applications viewpoint, it is of interest to have guidelines about what to do if the Basel II CAR is too low or too high. If the Basel II CAR $x(t)$ is too low, then increasing the rate of inflow of ERC, r_{cer} , or of

outflow of TRWAs, e , will result in an increase in the Basel II CAR. If x is too high, then the bank can take on more credit requests with a consequent increase in $a_{c_i}^{tot}$, decrease in μ and decrease in the Basel II CAR.

2.5.4 BASEL I vs BASEL II

As is evident from elements of the preceding discussion, Basel II differs from Basel I in two major respects. Firstly, internal models may be used by banks to make an assessment of their portfolio risk and to determine the capital requirements. We can apply these models to credit and operational risk that may lead to the decentralization of the regulatory capital adequacy requirements for banks. Implementation of these options is contingent on the validation of the internal model by the regulators and the idiosyncratic choice of model made by the bank, respectively. The second difference between the capital accords is the introduction of complementary mechanisms to safeguard against bank failures. In Basel II, the two aspects that come into play in addition to the traditional focus on minimum bank capital (Pillar 1) in Basel I is supervisory review (Pillar 2) and market discipline (Pillar 3). Pillar 2 is intended not only to ensure that banks have adequate capital to support all risks in their business, but also to encourage banks to develop and use improved strategies in monitoring and managing their risks. This process recognizes the responsibility of bank management in developing an internal capital assessment process and setting capital targets that are commensurate with the bank's risk profile and control environment. There are three main areas that are particularly suited to treatment under Pillar 2, viz, risks considered under Pillar 1 that are not fully captured under the first pillar (for example, credit concentration risk); those factors that are not taken into account under the Pillar 1 process (for example, interest rate risk in the banking book, business and strategic risk) and factors external to the bank (for example business cycle effects).

Besides the major differences between Basel I and Basel II highlighted in the previous paragraph, the accords may also be compared on a more specific level. For instance, [19] conjectures that risk-based capital requirements from Basel II offer an improvement upon Basel I-type risk-insensitive prerequisites via the differential impact on supervisory monitoring. On the other hand, [21] shows how focussing on the interaction of capital requirements (Pillar 1 in Basel II) and market discipline (Pillar 3 in Basel II) in regulating moral hazard is superior to the approach from Basel I. The paper [47] demonstrates that in competitive markets with low intermediation margins banks have an incentive to take excessive risks and that both flat-rate capital requirements (Basel I) and interest rate ceilings are effective in eliminating this problem. From a welfare perspective, however, these regulatory instruments worsen the position of depositors due to the associated lowering of equilibrium deposit rates. On the other hand, risk-based capital requirements (Basel II), if they can be implemented by the regulator, are strictly better than the other two instruments because they correct the banks' risk-taking incentives without distorting equilibrium deposit rates. In [38], Lopez shows that Basel II will resolve the trade-off between the flexibility

and regulatory standardization much better than Basel I.

We contend that the type of results reported in this dissertation adds to the growing debate about bank supervisory review. In particular, our work encourages the monitoring of capital levels and loan issuing as a means of assisting the supervision process. Our research has connections with [19], where capital requirements and regulatory activity via a specific audit policy is discussed. As far as we are concerned, our models do not explicitly allow for the risk-shifting and risk-taking incentives that may be recommended subsequent to a supervisory review. This situation may be addressed, for instance, by the introduction of a value function for shareholders which may have some specific convexity features. The BCBS aims to encourage market discipline by developing a set of disclosure requirements which will allow market participants to assess key pieces of information on the scope of application, capital, risk exposures, risk assessment processes and hence the capital adequacy of the institution. Banks have extensive reporting requirements which in principle means that data should be easy to find (see, for instance, the website [27]). Despite this, banking supervisors do not have the capacity to monitor every aspect of the complexity related to banking activities. In fact, the regulators may not even be kept informed of the banks' investment choices, which makes audits of the portfolio risk necessary. Some of the data from [27] that is consistent with our study is provided in the appendix. In our contribution, risk management activities are directly related to loan issuing and asset allocation strategies. In this regard, the FDIC's 2004 banking profile reported a steady increase in asset growth due to increased residential mortgage lending, home equity and credit card lending.

2.5.5 NRBCARs vs RBCARs

In this subsection, we refer to the figures contained in the appendix. Figures 2 and 3 draw comparisons between types of NRBCARs and RBCARs for certain criteria, respectively. In addition, comparative data for the aforementioned CARs from FDIC-insured institutions for the period January 1988 to December 2004 is presented in Figure 5. Figure 6 provides a plot of the leverage and equity CARs for FDIC-insured institutions. In this regard, it is interesting to note that the dramatic increase in the equity CAR in Figure 6 is consistent with the related stochastic models (see, for instance, (2.11) that models equity capital) that we suggest in this dissertation. Moreover, in Figure 7 we make parameter choices for the components of our stochastic model for Basel II CARs in order to compare real and simulated data in Figure 8. The latter figure also provides a plot of Tier 1 capital versus time in years.

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2.6 APPENDICES

2.6.1 APPENDIX OF PROOFS

Proof of Theorem 2.3.1

In this proof we derive (2.25) by mainly using the general Ito formula. In fact, the said formula is useful in calculating both the dynamics of the inverse of the NRWAs, da_{nr}^{-1} , and the leverage CAR, dz_l . In particular, we recall from the aforementioned formula that for Ito processes, B_i and B_j , we have

$$d\langle B_i(t), B_j(t) \rangle = \delta_{ij} dt. \quad (2.44)$$

For a_{nr}^{-1} it follows that

$$da_{nr}^{-1}(t) = -a_{nr}^{-2}(t)da_{nr}(t) + a_{nr}^{-3}(t)d[a_{nr}, a_{nr}]_t, \quad (2.45)$$

where

$$d[a_{nr}, a_{nr}]_t = a_{nr}^2(t) \left\{ C^T p(t) p^T(t) C + C^T p(t) \sigma_{T1} + \sigma_{T1}^T p^T(t) C + \sigma_{T1}^2 \right\} dt.$$

This means that (2.45) becomes

$$\begin{aligned} da_{nr}^{-1}(t) &= a_{nr}^{-1}(t) \left\{ \left(-r(t) - p^T(t)C\theta \right) dt - p^T(t)CdZ(t) + a_{nr}^{-1}(t)(1-\alpha)l(t)dt \right. \\ &\quad \left. + z_l(t) \left[\left(-r(t) - \mu_{T1}(t) \right) dt - \sigma_{T1}dZ(t) \right] \right. \\ &\quad \left. + \left\{ \sigma_{T1}^2 + C^T p(t)p^T(t)C + C^T p(t)\sigma_{T1} + \sigma_{T1}^T p^T(t)C \right\} dt \right\}. \end{aligned}$$

From identity (2.44) we can deduce that

$$d[c_{T1}, a_{nr}^{-1}]_t = -z_l(t) \left(\sigma_{T1}^T \sigma_{T1} + \sigma_{T1}^T p^T(t)C \right) dt.$$

Furthermore, we have that

$$\begin{aligned} c_{T1}(t)a_{nr}^{-2}(t)da_{nr}(t) &= z_l(t)a_{nr}^{-1}(t)da_{nr}(t) \\ &= z_l(t) \left\{ \left(r(t) + p^T(t)C\theta \right) dt + p^T(t)CdZ(t) + a_{nr}^{-1}(t)(\alpha-1)l(t)dt \right. \\ &\quad \left. + z_l(t) \left[\left(r(t) + \mu_{T1}(t) \right) dt + \sigma_{T1}dZ(t) \right] \right\} \end{aligned}$$

and

$$c_{T1}(t)a_{nr}^{-3}(t)d[a_{nr}, a_{nr}]_t = z_l(t) \left\{ C^T p(t)p^T(t)C + C^T p(t)\sigma_{T1} + \sigma_{T1}^T p^T(t)C + \sigma_{T1}^2 \right\} dt$$

It follows that

$$\begin{aligned} c_{T1}(t)da_{nr}^{-1}(t) &= -c_{T1}(t)a_{nr}^{-2}(t)da_{nr}(t) + c_{T1}(t)a_{nr}^{-3}(t)d[a_{nr}, a_{nr}]_t \\ &= z_l(t) \left[\left(-r - p^T(t)C\theta \right) dt - p^T(t)CdZ + a_{nr}^{-1}(t)(1-\alpha)l(t)dt \right. \\ &\quad \left. + z_l(t) \left\{ \left(-r - \mu_{T1} \right) dt - \sigma_{T1}dZ(t) \right\} \right] \\ &\quad + z_l(t) \left\{ C^T p(t)p^T(t)C + C^T p(t)\sigma_{T1} + \sigma_{T1}^T p^T(t)C + \sigma_{T1}^2 \right\} dt. \end{aligned}$$

From stochastic calculus, we have

$$dz_l(t) = c_{T1}(t)da_{nr}^{-1}(t) + a_{nr}^{-1}(t)dc_{T1}(t) + d[c_{T1}, a_{nr}^{-1}]_t,$$

where, in our case, it follows

$$a_{nr}^{-1}(t)dc_{T1}(t) = z_l(t) \left[\left(r(t) + \mu_{T1}(t) \right) dt + \sigma_{T1}dZ(t) \right].$$

From the above, one is easily able to deduce that

$$\begin{aligned}
dz_l(t) = & z_l(t) \left[\left(-r - p^T C \theta \right) dt - p^T(t) C dZ + a_{nr}^{-1}(t) (1 - \alpha) l(t) dt \right. \\
& \left. + z_l(t) \left\{ \left(-r - \mu_{T1} \right) dt - \sigma_{T1} dZ(t) \right\} \right] \\
& + z_l(t) \left\{ C^T p(t) p^T(t) C + C^T p(t) \sigma_{T1} + \sigma_{T1}^T p^T(t) C + \sigma_{T1}^2 \right\} dt \\
& + z_l(t) \left[\left(r + \mu_{T1} \right) dt + \sigma_{T1} dZ(t) \right] \\
& - z_l(t) \left(\sigma_{T1}^T \sigma_{T1} + \sigma_{T1}^T p^T(t) C \right) dt.
\end{aligned}$$

Hence, the statement of Theorem 2.3.1 has been proven.

2.6.2 APPENDIX OF TABLES

Bank Item	Abbreviation	Value
Capital Adequacy Ratio:	CAR	
Commercial Real Estate:	CRE	
Commodities Finance:	CF	
Commodities Finance Not Including SMEs, SL and PRs:	CFNISMESLPR	
Correlation:	R	R
Corporate Exposure:	CE	
Effective Maturity:	EM	m
Eligible Regulatory Capital:	ERC	c_{er}
Equity Exposure Not Held in the Trading Book:	EENHTB	
Expected Losses:	EL	
Exposure at Default:	EAD	e_d
Financial Collateral:	FC	
Home Equity Line of Credit:	HELOC	
High Volatility Commercial Real Estate:	HVCRE	
Income Producing Real Estate:	IPRE	
Income Producing Real Estate Not Including SMEs, SL and PRs:	IPRENISMESLPR	
Internal Ratings-Based:	IRB	
Liquidity Facilities Securitized Assets:	LFSAs	
Loss Given Default:	LGD	l_d
Loss Given Default Floor:	LGD _f	$l_{d_{f1}}$
Maturity Adjustment:	MA	b
Object Finance:	OF	
Object Finance Not Including SMEs, SL and PRs:	OFNISMESLPR	
Other Off-Balance Sheet Exposures:	OOSE	
Other Physical Collateral:	OPC	
Other Retail Exposure:	ORE	
Over the Counter:	OTC	
Probability of Default:	PD	p_d
Probability of Default Floor:	PD _f	$p_{d_{f1}}$
Project Finance:	PF	
Project Finance Not Including SMEs, SL and PRs:	PFNISMESLPR	
Purchased Receivable:	PR	
Qualifying Revolving Retail Exposure:	QRRE	
Residential Real Estate:	RRE	
Retail Residential Mortgage:	RRM	
Risk-Weighted Asset:	RWA	
Risk-Weighted Function:	RWF	
Small to Medium Size Enterprises:	SME	
Small to Medium Size Enterprises with Corporate Treatment:	SMECT	
Small to Medium Size Enterprises with Retail Treatment:	SMERT	
Sovereign Exposure:	SE	
Specialized Lending:	SL	
Specialized Lending High Volatility Commercial Real Estate:	SLHVCRE	
Specialized Lending Not Including High Volatility Commercial Real Estate:	SLNIHVCRE	
Total Risk-Weighted Asset:	TRWA	
Unexpected Losses:	UL	
Value-at-Risk:	VaR	

Figure 2.1: Abbreviations Used During the Calculation of Bank RWAs

Comparisons Between Types of Non-Risk Based Capital Adequacy Ratios	
Leverage CARs	Equity CARs
Leverage CAR = T1 Capital / TNRWAs	Equity CAR = Equity Capital / TNRWAs
Long History of Implementation	Long History of Implementation
Virtually Costless	Inexpensive
Straightforward to Calculate	Simple to Calculate
Very Transparent	Very Transparent
Helps to Distinguish Between Risky and Safe Banks	Distinguishes Between Risky and Safe Banks
Captures Overall Risk of Banks	Captures Overall Risk of Banks
Holding of Risky Assets Not Discouraged	Holding of Risky Assets Not Discouraged
Good Over Short Periods	Good Over Short-Term
Considers On-Balance Sheet Items	Considers On-Balance Sheet Items
Off-Balance Sheet Items Not Considered	Off-Balance Sheet Items Not Considered
Signal of Need for Supervisory Action	Signal of Need for Supervisory Action
Robust Predictor of Bank Failure	Effective Predictor of Bank Failure
May Affect Asset Allocation Strategies	Affects Asset Allocation Strategies
Achieves Highest Pseudo-R ² and Concordance Ratio	Does Not Adjust for Credit Risk Differentials

Figure 2.2: Leverage vs Equity CARs

Comparisons Between Types of Risk Based Capital Adequacy Ratios	
Basel II CARs	Tier 1 CARs
Basel II CAR = ERC / TRWAs	T1 CAR = T1 Capital / TRWAs
Short History of Implementation	Short History of Use
Monitoring and Reporting Costs Expensive	Expensive
Does Not Consistently Outperform NRB Ratios	
Difficult to Calculate	Calculation Problematic
Can Increase Without Raising Capital	Can Increase Without Raising Capital
Distinguishes Between Risky and Safe Banks	Distinguishes Between Risky and Safe Banks
Captures Overall Risk of Banks	Does Not Captures Overall Risk of Banks
Holding of Risky Assets Discouraged	Holding of Risky Assets Discouraged
Good Over Long Periods	Good Over Long-Term
Considers On-Balance Sheet Items	Considers On-Balance Sheet Items
Weakens Relationship Between Ratios and Other Risks	
Considers Off-Balance Sheet Items	Considers Off-Balance Sheet Items
Signal of Need for Supervisory Action	Signal of Need for Supervisory Action
Effective Predictor of Bank Failure	Effective Predictor of Bank Failure
Affects Asset Allocation Strategies	Affects Asset Allocation Strategies
Adjusts for Credit Risk Differentials	Adjusts for Credit Risk Differentials
According to Counterparty and Instrument	According to Counterparty and Instrument

Figure 2.3: Basel II vs Tier 1 CARs

β	Business Lines	β Val
β_1	Corporate Finance	18 %
β_2	Trading and Sales	18 %
β_3	Retail Banking	12 %
β_4	Commercial Banking	15 %
β_5	Payment and Settlement	18 %
β_6	Agency Services	15 %
β_7	Asset Management	12 %
β_8	Retail Brokerage	12 %

Figure 2.4: Business Lines and Operational Risk Factors

Year	Equity	Leverage	Tier 1	Basel II
1988	0,0705	0,0744	0,0896	–
1989	0,0722	0,0761	0,0939	–
1990	0,0707	0,0747	0,0862	–
1991	0,0707	0,0743	0,1002	–
1992	0,0722	0,0760	0,0930	0,1216
1993	0,0784	0,0755	0,0942	0,1237
1994	0,0793	0,0763	0,0956	0,1242
1995	0,0811	0,0769	0,1025	0,1263
1996	0,0827	0,0775	0,0974	0,1227
1997	0,0833	0,0796	0,0982	0,1231
1998	0,0861	0,0785	0,0967	0,1159
1999	0,0836	0,0786	0,0926	0,1127
2000	0,0849	0,0771	0,0942	0,1213
2001	0,0909	0,0779	0,0991	0,1272
2002	0,0921	0,0787	0,1043	0,1300
2003	0,0915	0,0788	0,1047	0,1300
2004	0,1028	0,0812	0,1076	0,1319

Figure 2.5: Yearly Mean CARs for FDIC-Insured Institutions

Figure 2.6: Leverage and Equity CARs for FDIC-Insured Institutions

Item	Value	Item	Value	Item	Value
r_m	0,25	a_m	0,475	σ_m	0,01
r_{o_k}	0	a_o	0,4	$\sigma_{o_{kl}}$	0
r_{c_3}	0,3	a_c	1,5	$\sigma_{c_{3j}}$	0,05
u	0,2	e	0,3	a	2,1

Figure 2.7: Scaled Parameter Choices for Basel II CAR Simulations

Figure 2.8: Basel II and Tier 1 CARs for FDIC-Insured Institutions

Chapter 3

Conclusions and Further Investigations

3.1 CONCLUSIONS

3.2 FURTHER INVESTIGATIONS

3.1 CONCLUSIONS

The centerpiece of this contribution is the derivation of SDEs that describe the evolution of NRBCARs (leverage and equity CARs) and RBCARs (Basel II and Tier 1 CARs). In order to accomplish this, we had to construct stochastic models for the dynamics of bank items (investments, loans, borrowings, core capital) appearing on a stylized balance sheet.

3.2 FURTHER INVESTIGATIONS

Future research will concentrate on developing real-world illustrations of and simulations for formalizations of the recommendations made in the Basel II Capital Accord. This may lead to more advanced results requiring a thorough understanding of such concepts as the transition density of diffusion processes, Levy processes and scaling factors. The recent trend has been a regulatory shift from reserve requirements to capital requirements. In order to better understand this issue we have to further investigate the implications of monetary policy on the behaviour of banks. In this regard, our models may have to be extended to include the interaction between the three pillars of the Basel II capital accord, viz., minimum regulatory capital requirement (Pillar I); supervisory review (Pillar II) and market discipline (Pillar III). Open problems concern also the control of the credit operation, capital requirements and asset allocation of a bank in a stochastic dynamic setting. In this regard, unresolved questions that arise out of our study are posed below.

Question 3.2.1 (Improving Low CARs): *What should a bank do to improve its CAR if it is too low ?*

Question 3.2.2 (Existence of Control Law): *Does a control law for a stochastic banking system exist that will prevent the CAR from dropping below a benchmark threshold value (like 0,08 for Basel II CARs) ?*

Question 3.2.3 (Existence of Control Law): *If the answer to Question 3.2.2 is no, how do we determine a control law such that the drop below the benchmark value is postponed as long as possible ?*

At first glance, the main ways to deal with low CAR values seem to be to lower the issuing of new loans or even to stop taking on such requests for a while and to increase the inflow of ERC by acquiring capital from shareholders. Once the Basel II CAR has been stabilized at a "safe" level, loan issuing can be increased again. It is our belief that a control theoretic formulation with a quantitative control law will be of interest here. Furthermore, we conjecture that the answer to the second question is no. The consequence of this is that every bank that is modelled in the way that we suggest will almost surely default eventually. In this case, the third question poses an interesting challenge.

Chapter 4

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