



Asteroseismic and Astrometric Characterization of Members in the NGC 6866 Open Cluster

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Abstract

Kepler, a NASA space telescope, has revolutionized our understanding of star clusters. Among the four open star clusters (OSCs) in its field of view (FOV), New General Catalogue (NGC) 6866 is the youngest and least studied. Young open clusters like NGC 6866 are expected to harbor a rich population of δ Scuti and binary stars, primarily main-sequence stars. However, studying OSCs poses a challenge in identifying true cluster members. How can asteroseismic and astrometric characterization of member stars in the NGC 6866 OSC, particularly focusing on binary systems, be used to estimate the age and distance of the cluster?

In this work, we aimed to address this challenge by distinguishing between members and non-members of NGC 6866. Identifying member stars is crucial for searching for binary stars, which can then be used to determine the age and distance of the cluster. *Gaia*, the European Space Agency’s (ESA) flagship astrometric mission, has enabled unprecedented insights into the Milky Way galaxy, including its structure, dynamics, and evolution. Data from the *Gaia* and *Kepler* satellites were employed to distinguish between stars that belong to the NGC 6866 open cluster and those that do not. We obtained the *Gaia* satellite data from the VizieR database for stars located within a 10 arcmin radius from the cluster’s center. Additionally, we analyzed *Kepler* photometric data extracted by Balona et al. (2013a). To identify potential members, we calculated the distance moduli (using parallax) for the selected stars and plotted them on a Hertzsprung-Russell (H-R) diagram, which helped us determine their positions within the cluster. We also used proper motion as an additional criterion for discriminating between cluster members and non-members. Our analysis revealed 14 newly categorized δ Scuti stars situated along the main sequence of the NGC 6866 OSC. Furthermore, we identified a single binary star system, *Kepler* Input Catalogue (KIC) 8196903, among the δ Scuti stars using the binarogram technique. Ultimately, our estimation placed the age of the NGC 6866 OSC at approximately 0.86 Gyrs and determined its distance to be roughly 1497.90 parsecs (pc). This study highlights the need for larger datasets, longer observation periods, novel techniques, and utilizing the latest data releases to achieve a more comprehensive understanding of Open Star Clusters.

Keywords: *Gaia*; parallax; distance; *Kepler*; main sequence; Delta Scuti; members.

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Table of abbreviations

A table containing a list of abbreviations that will be used throughout this work.

AU	Astronomical Unit
BCS	Blue component stars
c/d	Cycles per day
CCD	Charge Coupled Device
CMD	Color-magnitude diagram
CoRoT	Convection, rotation and planetary transit
DEB	Detached eclipsing binaries
Dec.	Declination
DFT	Discrete fourier transform
Dias	Dublin Institute for Advanced Studies
DSC	Discrete Source Classifier
eGDR3	early <i>Gaia</i> data release 3
FOV	Field of view
<i>Gaia</i>	Global Astrometric Interferometer for Astrophysics
GDR1	<i>Gaia</i> data release 1
GDR2	<i>Gaia</i> data release 2
GDR3	<i>Gaia</i> data release 3
Gyr	Giga year
H-R	Hertzsprung Russell
JWST	James web space telescope
KASC	<i>Kepler</i> Asteroseismic Science Consortium
KIC	<i>Kepler</i> Input Catalog
LC	Long cadence
Lum	Luminosity
MAST	Mikulski Archive for Space Telescopes
MESA	Modules for Experiments in Stellar Astrophysics
MOST	Microvariability and Oscillations of Stars Telescope
MSTO	Main sequence turn-off
NGC	New General Catalog
OC	Open cluster
OSC	Open star cluster
pc	Parsecs
pmDec.	Proper motion in declination
pmRA	Proper motion in right ascension
RA	Right ascension
RCS	Red component stars
RDP	Radial density profile
RV	Radial velocity
SC	Short cadence
sdB	Type B subdwarf
SolLum	Solar Luminosity
SSO	Solar system object
T_{eff}	Effective temperature
UBVRI	Ultraviolet, Blue, Visible, Red, Infrared
UCAC5	US Naval observatory,s fifth CCD astrograph catalog
WD	White dwarf
WISE	Wide-field Infrared Survey Explorer

Chapter 1

Introduction

1.1 Background

Asteroseismology is the study of stars based on their oscillations. Stars can be used as clocks and galactic calipers to accurately determine the ages, distances, sizes, masses and internal structures of the stars (Tang et al., 2018). The study of asteroseismology is closely associated with a branch of astronomy known as astrometry. It involves precise measurements of the positions of the stars and other celestial bodies and also involves precise measurements of their movements (Govind et al., 2018), (Crosta, 2019). This study uses both methods to estimate the distance and age of the young open star clusters (OSCs), New General Catalogue (NGC) 6866.

The observable universe is made up of approximately 200 billion galaxies. Each galaxy is made up of roughly hundreds to thousands of star clusters. Star clusters are the building blocks of galaxies, and each cluster contains hundreds to thousands of gravitationally (Camargo et al., 2015) and loosely bound stars. By studying a small sample of member stars, the global parameters¹ of the entire cluster can be estimated, such as its age and distance. Member stars of a cluster can be identified by comparing their astrophysical parameters (such as reddening, distance, age, and metallicity) to the cluster’s astrophysical properties. This can be done by analyzing variations in light curves and radial velocity. All member stars of a cluster have similar properties to the cluster. Stars with very different properties are likely non-members, field stars, or members of another overlapping cluster.

The NGC is a catalogue that contains a list of 7840 astronomical objects, ordered with reference to their properties. The astronomical objects include galaxies, nebulae, supernova remnants, and star clusters (Steinicke, 2010). Various types of clusters are known, but the most commonly known cluster types are globular clusters and OSCs. Globular clusters have a symmetrical, roughly spherical appearance, luminously dense core, and consist of old stars that are closely packed. They are the largest star clusters (Gratton et al., 2019). In contrast, OSCs have no distinct shape. OSCs tend to have lower density and a smaller mass range compared to Globular clusters, which are much denser and contain a wider range of masses, often with significantly higher total masses.

¹“Properties” and “parameters” are used interchangeably throughout this work.

OSCs are less massive and consist of young stars with similar properties, such as age, chemical composition/ metallicity $[\text{Fe}/\text{H}]$, radial velocity, and proper motion. Member stars are formed from the same nebulae (molecular clouds), around the same period, with the same physical and chemical properties. This makes OSCs more attractive observational targets for gaining insights into formation and evolution of galaxies (Stott, 2018; Rangwal et al., 2019).

Studying OSCs in a galaxy is vital for understanding the formation history of stars and the parent star clusters' nature. The age of the cluster can be estimated by comparing a star cluster's color-magnitude diagram (CMD) with theoretical evolutionary models and also the evolutionary information of stars of about the same age and chemical composition (Joshi et al., 2012).

In this dissertation, a study of the NGC 6866 OSC is presented. This cluster is found in the constellation Cygnus (Koroleva, 1973). It was discovered by Caroline Herschel on 23 July 1783. The NGC 6866 OSC is positioned at Right Ascension (RA) = 20:03:55, Declination (Dec.) = +44:09:30; as well as the galactic longitude (l) = $79^\circ.560$, and latitude (b) = $+6^\circ.839$ (Janes et al., 2014; Abedigamba, 2016). The radius of the NGC 6866 OSC is approximately equal to seven arcmin (Joshi et al., 2012; Balona et al., 2013a). In the *Kepler* field of view (FOV), NGC 6866 OSC is the youngest (Stello, et al., 2010) among the four OSCs and one of the least studied (Güneş et al., 2012). Tadross (2011) highlights that many clusters in the observable universe are least investigated (that is, they have unknown parameters such as age and distance) due to either one or a combination of the following reasons: having a small population of stars (Dias et al., 2018), having densely populated surroundings, being too distant, being obscured, or having a greater number of low mass stars. In instances where optical data is used to estimate parameters of obscured distant star clusters, differential reddening occurs, resulting in a biased output. Such biases can be countered by detection in the infrared band. However, this problem is further exacerbated when cluster members have low stellar density due to a huge background field (Tadross, 2011). NGC 6866 does not suffer from any of the abovementioned reasons for low interest, however, publications are yet to reach consensus on its membership determination, age, and distance estimations.

This work focuses on the main-sequence of the cluster, where δ Scuti stars are typically abundant in young OSCs. These young, pulsating stars within OSCs are instrumental in determining key cluster parameters such as radius, age, and distance. The main-sequence phase is the longest phase in the life of the stars. In fact, over 90% of all stars in the universe, including our Sun, belong to the main-sequence category. Within the cores of main-sequence stars, helium atoms are generated through the fusion of hydrogen atoms. This shows that most of the stars spend their lives burning hydrogen into helium. Fusion reactions are highly sensitive to temperature changes, and even a slight increase in temperature can significantly accelerate the fusion process. Consequently, fusion primarily transpires in the hot and compact central core of a main-sequence star (Karttunen et al., 2007). Stars along the main-sequence exhibit variations in temperature and luminosity, as depicted in the Hertzsprung-Russell (H-R) diagram (Figure 1.1). The H-R diagram is a graphical representation plotting a star's luminosity against its effective temperature. The main-sequence forms a continuous and distinctive diagonal band on the H-R diagram, extending from small, low-mass, cool, and dim stars (positioned in the lower right corner) to large, high-mass, hot, and brilliant stars (located in the upper left corner). A star's brightness is inherently

linked to its radius, with larger stars generally being brighter and hotter.

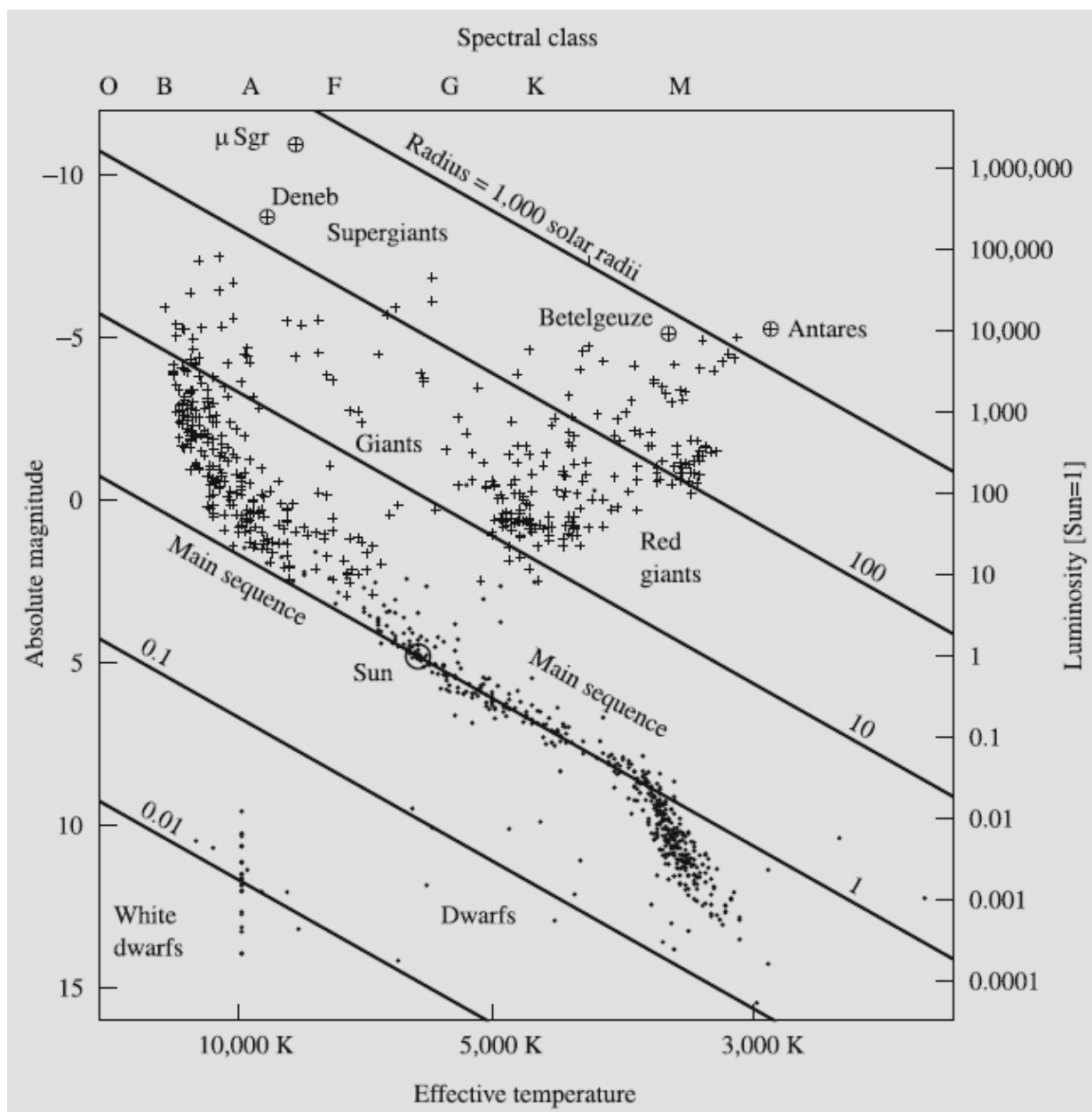


Figure 1.1: An example of the HR-diagram showing the distribution of stars in a star cluster (Karttunen et al., 2007).

1.1.1 The *Kepler* Space Missions

The primary aim of the *Kepler* space mission, as outlined in the works by Koch et al. (2010) and Borucki et al. (2010), involved the extensive examination of regions near our solar system within the Milky Way. The mission's overarching goal was to identify Earth-sized or larger planets, with a particular focus on those residing within or near the habitable zone of stars similar to our Sun (specifically FGKM spectral type dwarfs ranging between 3500 and 7500 Kelvins). The mission also sought to uncover the prevalence of such planets among the billions of stars within our galaxy. The mission, launched in 2009, was further extended by the K2 mission, which operated from

2014 to 2018, ending due to the failure of two of its reaction wheels. During the K2 mission, the spacecraft observed a series of different target fields, including star clusters like NGC 6866, NGC 6819, NGC 6791, and NGC 6811, in addition to other types of targets such as exoplanet candidates, asteroids, and supernovae. The discoveries made by the mission integrated our solar system into the broader context of planetary systems within the Milky Way.

The *Kepler* Mission’s primary objective was to investigate the composition and diversity of planetary systems beyond our solar system. To achieve this, a substantial and diverse array of stars was observed. These observations facilitated estimates of the prevalence of planets in multi-star systems and the ratio of Earth-like and larger planets located within or near the habitable zone of a wide range of stars exhibiting distinct spectroscopic properties. The star samples also enabled the identification of additional members in each planetary system detected through photometric methods. Furthermore, they allowed for the determination of the densities, masses, and sizes of close-in giant planets and provided insights into the characteristics of stars hosting planetary systems (Borucki, 2016). Asteroseismology was a secondary objective. However, it makes a significant contribution to characterization of planets by providing data on the host stars. According to Stello et al. (2009), asteroseismology also provides information on the radius of solar-like pulsars, allowing for deeper investigation of the star’s interior.

The precision of *Kepler*’s space-based photometry, with an accuracy on the order of a few micro-magnitudes (also, the limiting magnitude for detecting exoplanets is around magnitude 15 to 16), contributes to the advancement of stellar research. Space-based observations offer a degree of accuracy that ground-based observations cannot achieve. Both the Convection, Rotation and planetary Transit (CoRoT) spacecraft (Baglin et al., 2006; Auvergne et al., 2009) and the Microvariability and Oscillations of Stars (MOST) satellite (Walker et al., 2003) delivered accurate photometry from orbit prior to *Kepler*’s launch. However, due to their orbits around the Earth, the data required considerable adjustment before they could be used because the noise levels were frequently quite high. *Kepler*’s orbit around the Sun and its larger telescope aperture enabled exceptionally low noise levels and even more accurate photometric measurements. The numerous targets from the satellites in various Milky Way areas enable population studies like those pioneered by (Miglio et al., 2009).

1.1.1.1 The *Kepler* spacecraft’s field of view

The NGC 6866 OSC is found in the field that was observed by *Kepler*. *Kepler*’s FOV allowed four OSCs to be observed during its K2 mission: NGC 6791, NGC 6811, NGC 6819 and NGC 6866 (Janes et al., 2014), as illustrated in Figure 1.2. In studying stellar pulsation, the *Kepler* mission made significant contributions. This mission recorded fluctuations in the brightness of numerous stars in the Lyra and Cygnus regions within a 105-square-degree field, as detailed by Joshi et al. (2012) and Borucki (2016). *Kepler* operated in two modes: Short Cadence (SC) and Long Cadence (LC). The majority of star observations were conducted through the *Kepler* space mission employed an LC mode with an exposure duration of approximately 30 minutes. Some stars have been observed using an SC mode, an exposure time of about 1 minute (Balona et al., 2013a). Searching for stellar objects that may potentially harbour life, the spacecraft has now

catalogued over 4000 exoplanets situated within the habitable zone of their host star.

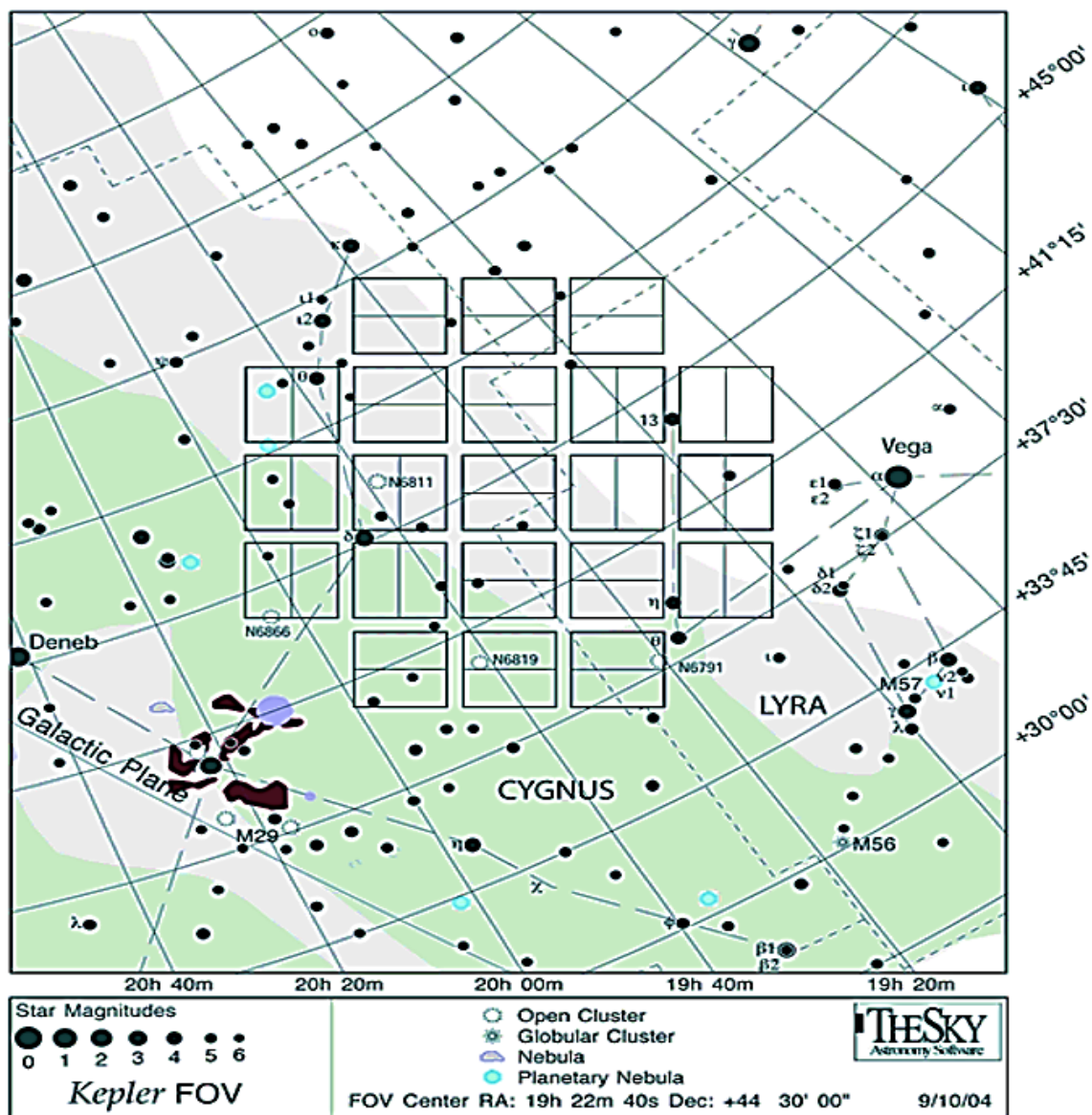


Figure 1.2: *Kepler* spacecraft FOV (<https://keplergo.github.io/KeplerScienceWebsite/the-kepler-space-telescope.html>).

1.1.1.2 The *Kepler* space telescope's data-gathering

The quest for Earth-sized exoplanets orbiting distant stars presents significant challenges when relying on ground-based telescopes, primarily due to the Earth's atmosphere, which is constantly in motion, causing distortions in telescope imagery. To overcome these limitations, the transit method was employed. *Kepler's* transit method is an observational technique that relies on the periodic reduction in a star's luminosity caused by an exoplanet passing in front of it. To ensure optimal positioning of its solar panels and sun shields in relation to the Sun, the *Kepler*

spacecraft undergoes a 90-degree rotation around one of its axes approximately every 90 days. To transfer the stored data to Earth, the spacecraft was directed toward our planet every three months. Consequently, the data are organized into quarters, each spanning 90 days. Furthermore, the SC data are subdivided into three one-month segments, identified as Q_{nm} , while the larger quarterly divisions are labeled as Q_n . These divisions account for the presence of data gaps caused by the monthly downlink periods and the quarterly reorientations. The first 10 days of scientific data collected during the initial commissioning phase are designated as Q_0 , followed by the commencement of the operational data set, Q_1 . Subsequent quarters are sequentially labeled as Q_2 , Q_3 , and so forth.

1.1.1.3 The *Kepler's* instrument

The *Kepler's* Instrument is a photometer with a 1.4-meter primary mirror and a 0.95-meter aperture Schmidt-type telescope. The spacecraft could continuously cover its $(115.6^\circ)^2$ FOV because of its heliocentric Earth-trailing orbit with a 372.5-day period. A total of 42 Charge Coupled Devices (CCDs), each covering four arcsec, are stacked in an array over 21 modules in the photometer camera. To ensure that the telescope continues to cover the same region of the sky after each quarterly roll and that the same stars were visible during the mission, the 42 CCDs were arranged in a four-fold symmetrical pattern on the sky. The *Kepler* spacecraft had an exceptional FOV, spanning a 15° diameter. *Kepler's* observations were carried out at two separate sampling rates, each following a regular schedule determined by the spacecraft's internal clock. As mentioned, these rates included an SC mode with a 58.8-second integration time and LC mode with a 29.43-minute integration period. Nearly continuous LC observations make up the majority of *Kepler's* photometry. This dissertation only contains data in LC mode.

The *Kepler* spacecraft is shown in Figure 1.3. Some of its key features are as follows:

- **Telescope:** *Kepler* was equipped with a 95-megapixel camera, which was the largest ever sent into space at the time of its launch. The camera was designed to continuously monitor a single FOV, about 12 degrees by 12 degrees, in the constellation Cygnus. This is the main instrument/detector.
- **Photometer:** The photometer was the heart of the *Kepler* spacecraft's scientific instrument, responsible for measuring the brightness of stars in its FOV. It was designed to detect tiny changes in brightness caused by planets passing in front of their host stars, known as transits.
- **Reaction wheels:** Four reaction wheels installed on the spacecraft were used to point the spacecraft accurately at its target field. The reaction wheels spun at high speed to counteract the torque from the Sun and other sources.
- **Data management:** *Kepler* generated enormous amounts of data, up to one terabyte per day. To manage this data, the spacecraft was equipped with a sophisticated onboard computer and data storage system.



Figure 1.3: *Kepler* spacecraft (<https://hotcore.info/babki/nasa-kepler-satellite.html>).

- **Communications:** *Kepler* used a high-gain antenna to communicate with Earth. The antenna was designed to transmit data at rates of up to 4.33 megabits per second.

Overall, the *Kepler* spacecraft was a remarkable achievement in the field of exoplanet science, and its findings have significantly enhanced our perception of our place in the universe, showing us that we are just one planet among many in a vast and diverse galaxy.

1.1.1.4 Data Gaps

Kepler's data is almost continuous, with a precision of micro-magnitudes, due to its heliocentric orbit. However, there are some gaps in the data, some of which are planned and some of which are not. Every quarter (four times per orbit), the spacecraft rotated 90° to ensure that its solar panels are facing the Sun. This maximizes power generation and temperature control, but it also causes brief gaps in the light curve data. Additionally, the stars are spread across various CCD modules, each of which has a different sensitivity level and produces a different measured flux. In addition to the gap caused by the quarterly roll, there are also two more gaps each quarter, at about 30-day intervals. These gaps are caused by downlinks, which divide the data into three months each quarter. Both LC and SC recorded data onboard and downlinked every 30 days. This caused the photometer to stop collecting data for intervals of up to ~ 24 hours. Other gaps in the data include occasional safe-mode events. For example, at the start of Q8, there was a data shortage of about 18 days from the end of Q7 (García et al., 2014).

1.1.2 δ Scuti stars

The NGC 6866 OSC is an intermediate-age star cluster. Intermediate-age star clusters can be utilized to investigate short-period variability in their γ Doradus and δ Scuti stars. At certain points in their evolution, some cluster members exhibit distinct types of diversity. For instance, OSCs include a variety of pulsating variable (PV) stars, including β Cepheid stars and δ Scuti stars. Observations of PV stars can be used to deduce a number of significant stellar properties, including mass, radius, and brightness of the stars (Joshi et al., 2012). This project focuses on δ Scuti stars to determine such stellar properties.

Pulsating variables are referred to as δ Scuti stars if they exhibit spectral types A to F, pulsation durations of 18 minutes to 8 hours, and brightness variations of thousandths to tenths of a magnitude. These stars are in the 1.5 to 2.5 $M_{\odot}$ range. Based on their location on the H-R diagram, they are denser than large-amplitude pulsating variables, such as Cepheids. Pulsating stars are found in the Cepheid instability strip located above and below the main-sequence stars (on the vertical, narrow band) instability strip of the H-R Diagram (Yang et al., 2011). δ Scuti stars exhibit multiple frequency pulsations with a cycle rate of 5 to 50 per day. Some pulsating δ Scuti stars have periods in the range of 0.01-0.3 days (Balona et al., 2013a). Stars that are still forming, known as pre-main-sequence stars, can also pulsate like δ Scuti stars, along with main-sequence stars and sub-giants that are further along in their evolution. This gives us important insights into how stars evolve in their early stages (Zwintz et al., 2014).

It is now recognized that thousands of stars are variable. One of the mechanisms causing variability is pulsation. A star's brightness can change, or pulsate, when the size and temperature of its surface layers change. The simplest type of pulsation is radial pulsation: a simple, spherically symmetric expansion and contraction of a star, like it is breathing. Non-radial pulsation is a different type of pulsation that occurs when a star changes shape instead of volume. There are several million pulsating stars in the Milky Way. Hundreds of billions of stars in our galaxy suggests that stellar pulsation is a temporary rather than permanent characteristic of stars (Ostlie and Carroll, 1996).

1.1.3 Binary stars

A pair of gravitationally bound stars that revolve around a common center of mass are called a binary star system. They have been investigated for centuries. In 1650, Jean Baptiste Riccioli discovered the first binary star system, Ursae Majoris (Mizar) (Aitken, 1924). Algol was among the first binary star systems ever found where one star passes in front of the other, causing the luminosity of the system to dim (eclipsing binaries) (Goodricke, 1783). Herschel (1802) coined the term “binary” star and used it to refer to a mutually attracted union of two stars that form one system. Two stars that appear close together in a telescope are often called binary stars. There are two reasons why binary stars may appear close together. First, they may actually be close together in space, orbiting each other due to their mutual gravity. Second, the two stars may be far apart, but they may appear to be traveling in the same direction when observed from Earth. This implies that the stars could be close together or hundreds of astronomical units or

more apart. A binary star's two components typically differ in brightness. The companion star is the fainter of the two stars; the primary star is brighter.

Binary stars can have a variety of orbits, from circular to elliptical. The inclination of the orbit can range from 0° (face-on) to 90° (edge-on). This orbit is also elliptical and follows *Kepler's* third law. Detecting a single star in a binary system, enables us to learn about the system's orbital parameters, such as the shape of the orbit and the masses of the stars. This is because the motion of the visible star around the barycenter is the same as the motion of a single star around a more massive companion star. If both stars in a binary system can be detected, the masses of the stars can be directly measured using *Kepler's* third law. A lot can be learnt about the stars' properties by studying their orbital parameters. There are a variety of methods used to interpret the orbital data of binary star systems, depending on the distance to the system, the shape of the orbit, and the relative masses and brightnesses of the components (Ostlie and Carroll, 1996).

Some binary star systems may have orbital parameters with light curves that do not illustrate an eclipse or only a single eclipse (Soydugan et al., 2020). Some binaries do not form partial or full eclipses when observed, which means that, to reduce uncertainties when identifying binary stars, other techniques should be utilized.

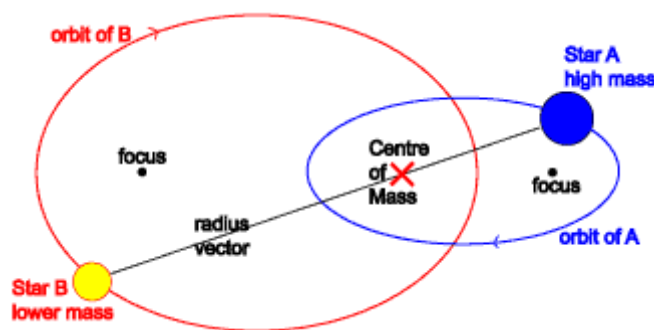


Figure 1.4: Binary Star system diagram showing two Stars, A and B orbiting about their barycenter (https://www.atnf.csiro.au/outreach/education/senior/astrophysics/binary_intro.html).

Figure 1.4 shows how two stars of different masses (star A being more massive than star B) orbit each other. The stars revolve around the system's center of mass, or barycenter. The line connecting the two stars always passes through the barycenter. There are many different pairs of stars, from open binary systems to close optical pairs. According to recent studies of young stars (Ghez, 1993; Leinert et al., 1993; Duchêne et al., 2007) neighbouring star-forming regions contain large binary star populations. Duquennoy and Mayor (1991) report that young binary stars are more populous than field stars in the solar vicinity.

Studying binary stars is essential to understanding how stars are structured and how they evolve. Binary stars are vital in determining the basic astrophysical properties of stars. Eclipsing binaries are the main source of fundamental stellar parameters, such as mass and radius. The existence of planetary bodies (exoplanets) in binary star systems is another important feature (Lowrance et al., 2002). In the 1970s, binary star observation underwent a revolution with the development of speckle interferometry. The *Hipparcos* spacecraft launched in 1989, ushered in a new age of

binary star discoveries (Mason et al., 1999).

Binary star systems have been observed using various techniques. Techniques which have helped simplify binary star classifications. Binary stars are categorized based on the manner of observation.

The seven types of binary star systems are as follows (Karttunen et al., 2007; Lea, 2023):

- Binary stars that can be directly observed through a telescope are called optical binaries.
- Binary stars that have a wide enough separation that they can be individually resolved in a telescope are called visual binaries.
- Binary stars that are in a close enough orbit that they periodically eclipse each other, causing their brightness to change, are called eclipsing binaries.
- Binary stars detected by the effect they have on the motion of their center of mass are called astrometric binaries.
- Binary stars detected by the Doppler shifting of their spectral lines caused by their orbital motion are called spectroscopic binaries.
- Binary stars are detected by the presence of two sets of spectral lines in their spectrum, indicating that they are two separate stars, are called spectrum binaries.
- Binary stars that are in such a close orbit that their surfaces are in contact are referred to as contact binaries.

The abovementioned classifications are only based on the manner of observation, but additional distinct methodologies have been proposed. For example, eclipsing binaries are spectroscopic binaries at the very least. Spectroscopic observation can also reveal unresolved visual binaries. There are three ways to observe some systems. A particular binary can typically be classified in more than one of the categories mentioned above.

1.1.4 The *Gaia* mission

The Global Astrometric Interferometer for Astrophysics, or *Gaia*, mission is a science-driven program initiated by the European Space Agency (ESA). The program aims to produce a three-dimensional map of our galaxy, considering all objects, their parameters, locations, distances, and movements (positions, motions, and parallax of stars, asteroids, meteoroids, debris, and so forth) in space. The data acquired from mapping our galaxy is then shared with the research community in the *Gaia* Archive (*Gaia* collaboration et al., 2016).

Gaia's first data were released on 13 September 2016. *Gaia*'s mission of identifying, locating and tracking movement of a billion stars was unprecedented at the time of its first data release. The spacecraft was assigned to orbit around the Sun at the second Sun-Earth Lagrangian point – L2 (Borucki, 2016). The satellite rotates perpendicular to the Sun and Earth while scanning, ensuring

that each star is scanned twice per rotation thanks to *Gaia* having two separate telescopic beams pointed in two different directions, 106.5 degrees apart, for greater scanning accuracy. Within a period of five years, each single star (and other objects) is scanned an average of 70 times. The spacecraft features a focal plane that is a meter in length and 60 centimetres wide, with a stable gigapixel detector that merges light from the telescope mirrors. The data are collected from the stars and transmitted to Earth using ESA antennas on the S-Trac network in Argentina, Australia, Spain and other countries. The data are then sent to Darmstadt in Germany, where the spacecraft is controlled, and thereafter taken to Madrid's data-processing centre. After initial data-processing, the data are further distributed to specialized data-processing centres across Europe. The resulting data are sent to ESA, which then approves and finally releases the detailed data to the science community.

1.1.4.1 *Gaia* data releases (GDR2 and GDR3)

When the first batch (*Gaia* data release 1, or GDR1) of data was released in 2016, GDR2 was already underway in its first processing phase because the releases run concurrently. From the first *Gaia* data release, every subsequent release leaps forward (is an improvement in terms of quantity and quality) in many respects. A new release contains new or updated information, rendering the previous release lacking (Brown et al., 2018). Caution is advised when using previous data releases.

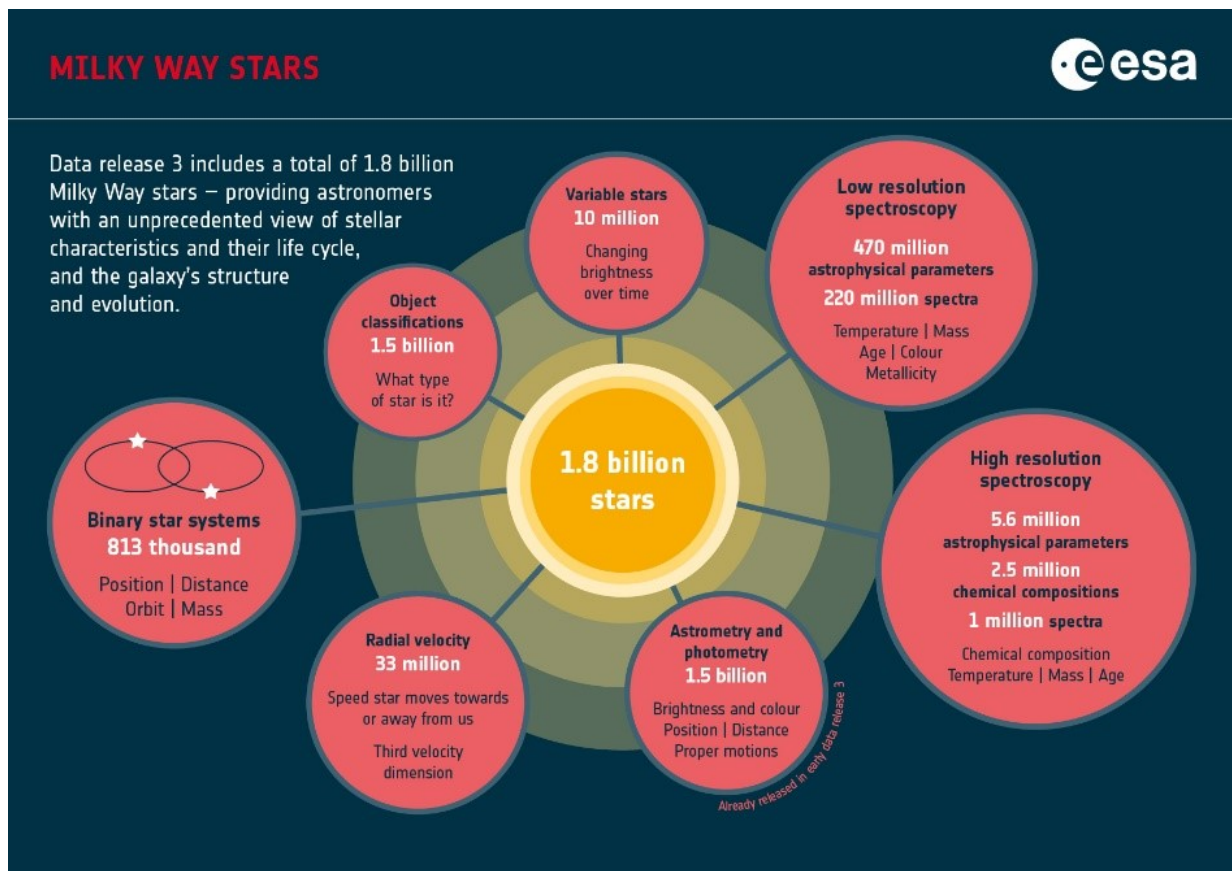


Figure 1.5: Summary of data sources found in GDR3 (Young, 2022).

GDR3 had scanned over 1.8 billion stars (see Figure 1.6) in the Milky Way and beyond. The multi-dimensional mapping will indicate every object in the Milky Way along with their astrophysical properties such as location, distance, velocity, luminosity and temperature (star classification properties), proper motion, type and chemical composition. The satellite also captures macro-molecular dust and gas particles between stars. Early GDR3 was released on 3 December 2020, and the full, latest GDR3 data were released on 13 June 2022. GDR3 contains updated information and a great number of additional stellar objects, spectra, binary stars, and so forth when compared to GDR2. This is due to improvements in the calibration of the spacecraft’s instruments and in the data-processing pipeline. GDR3 also contains data acquired beyond the Milky Way, such as data on quasars and other galaxies (for example the Andromeda galaxy).

Table 1.1: Comparison of data sources in GDR2 and GDR3
(Brown et al., 2018; Vallenari et al., 2023)

Data product or source type	Number of sources	
	GDR2	GDR3
5-parameter astrometry	1 331 909 727	585 416 709
6-parameter astrometry	-	882 328 109
2-parameter astrometry	361 009 408	343 964 953
ICRF3 sources	2 820 (prototypes)	2007 (frame orientation)
<i>Gaia</i> -CRF2 sources	556 869	-
<i>Gaia</i> -CRF3 sources for frame spin	-	428 034
G-band	1 692 919 135	1 806 254 432
G _{BP} -band	1 381 964 755	1 542 033 472
G _{RP} -band	1 383 551 713	1 554 997 939
Photometric time series	-	11 754 237
<i>Gaia</i> Andromeda Photometric Survey	-	1 257 319
Radial velocity	7 224 631	33 812 183
G _{RVS} -band	-	32 232 187
V _{broad}	-	3 524 677
Radial velocity time series	-	1898
BP/RP mean spectra	-	219 197 643
RVS mean spectra	-	999 645
Classified as variable sources	550 737	10 509 536
Variable type estimated	363 969	-
Detailed characterization of light curve	390 529	-
Effective temperature T _{eff}	161 497 595	470
Extinction AG	87 733 672	-
color excess E(G _{BP} – G _{RP})	87 733 672	-
Radius	76 956 778	470
Luminosity	76 956 778	-
Object DSC classification	-	1 590 760 469
Self-organized map of poorly classified sources	-	1

APs from mean BP/RP spectra	-	470 759 263
APs from mean RVS spectra	-	5 591 594
Chemical abundances from mean RVS spectra	-	2 513 593
DIBs from mean RVS spectra	-	472 584
Non-single stars	-	813 687
QSO candidates	-	6 649 162
QSO redshifts	-	6 375 063
QSO host galaxy detected	-	64 498
QSO host galaxy profile	-	15 867
Galaxy candidates	-	4 842 342
Galaxy redshifts	-	1 367 153
Galaxy profiles	-	914 837
SSO reflectance spectra	-	60 518
SSO epoch astrometry and photometry	14 099	158 152
Total galactic extinction maps	-	5
Total	1 692 919 135	1 811 709 771

GDR3 contains measurements for approximately 1.8 billion objects compared to 1.7 billion in GDR2, as indicated in Table 1.1. GDR3 includes new data products, such as radial velocities and astrophysical parameters, which were not available in GDR2. GDR3 provides a more detailed and accurate view of our galaxy and its stellar populations, allowing for more precise investigations of the galaxy’s structure, formation, and evolution.

• Selecting GDR2 over GDR3

GDR2 does have some benefits that we have considered for its selection due to its longevity within the research community, such as having datasets that are established and validated; well known parameters; availability of comprehensive analysis tools, resources, documentation and support; and it is compatible with existing research. The following five reasons were considered in choosing to work mainly with GDR2 data.

- GDR2 provides an established and validated data set. GDR2 has been extensively used and validated by the scientific community since its release. Numerous studies have relied on GDR2 data, demonstrating its reliability and accuracy. By utilizing a well-established dataset, we can build upon existing knowledge and compare our findings with established research more effectively.
- There is an improved understanding of GDR2 parameters. Given the widespread use of GDR2 data, the scientific community has gained a deeper understanding of its parameters, including luminosity. This familiarity allows us to leverage existing expertise and implement proven analysis techniques, thereby minimizing potential errors and discrepancies.

- Comprehensive analysis tools and resources are available for GDR2. Over time, a wide range of analysis tools and resources have been developed specifically for GDR2. These tools provide efficient and reliable ways to extract meaningful insights from the data, allowing us to optimize our research process and enhance the overall productivity of our project.
- Comprehensive documentation and support are available for GDR2. It has comprehensive documentation, including technical papers and user guides, which can guide us through the data-analysis process. Additionally, due to its extensive use, it is more likely to find expert support and assistance when encountering challenges or seeking clarification.
- GDR2 is compatible with existing research. By utilizing GDR2 parameters, we align ourselves with a vast body of existing research that has used this dataset. This compatibility facilitates better comparison and collaboration with other researchers in the field, enhancing the potential impact and visibility of our findings.

While GDR3 offers several advancements over GDR2, such as improved astrometric measurements and additional data products, it is essential to weigh the benefits against the potential risks and uncertainties associated with adopting a relatively new data set. Given the critical nature of our research project, it is advisable to proceed with the more established and extensively validated GDR2 parameters, particularly luminosity, to ensure the reliability and integrity of our results.

1.1.4.2 Observing techniques employed by *Gaia* satellite

Generally, *Gaia* can observe stars down to magnitude 20 or fainter in its main survey, but it can detect even fainter objects with longer exposure times or in specific observations. *Gaia* uses astrometry as previously defined, photometry, and spectroscopy to observe stellar objects.

- Photometry is a technique that measures the brightness emitted by stars, planets, satellites, nebulae and galaxies. Some of the main applications of photometry include determining the brightness and variability of celestial objects, measuring the distance and size of celestial objects, studying the properties of exoplanets, identifying and studying transient events, and mapping the universe’s structure by measuring the distribution of galaxies and other celestial objects across the sky (Bessell, 2005).
- Spectroscopy is a fundamental tool in astrophysics that allows astronomers to study the properties of celestial objects by analyzing their spectra, produced by the interaction of electromagnetic radiation with matter. Some of the main applications of spectroscopy include determining the chemical composition of celestial objects, studying the physical conditions of celestial objects (measuring their temperature, density, pressure, and other physical properties), identifying the motion of celestial objects (that is, velocities of stars, galaxies, and so forth by measuring the Doppler shift of their spectral lines), detecting and characterizing exoplanets by measuring the periodic variations in the spectra of their host stars caused by the gravitational tug of the planet, and studying the early universe (Zwitter and Munari, 2004).

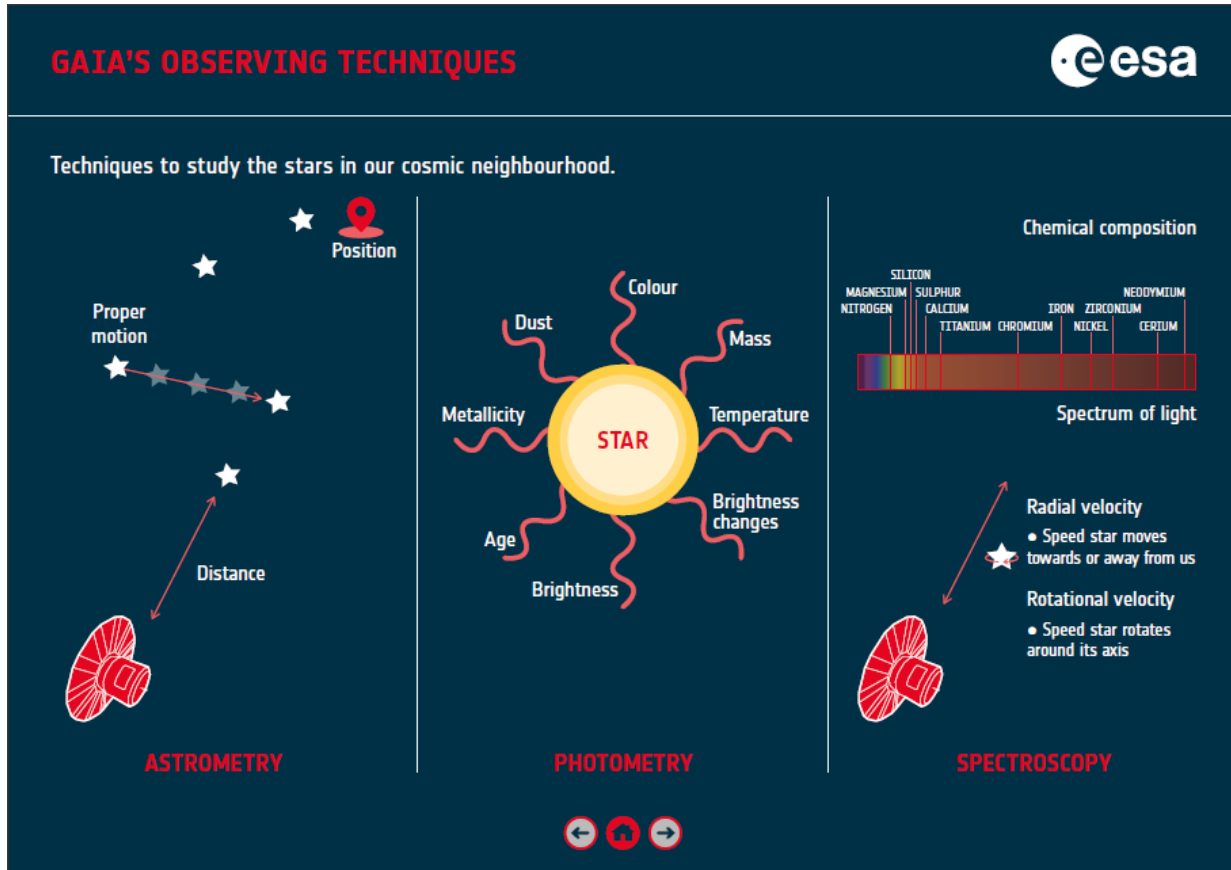


Figure 1.6: *Gaia*'s observing techniques (https://www.esa.int/ESA_Multimedia/Images/2022/06/Gaia_s_observing_techniques).

1.1.5 Periodogram

Any dynamic data sequence can be modeled as a composite of sine or cosine waves with different amplitudes and frequencies. A periodogram is a graph of amplitude against frequency. It serves as a tool for the identification of the most prominent periods (or frequencies) in a time series by isolating the strongest signals. This technique is particularly useful for uncovering the primary cyclical patterns in a time series, especially when the cycles deviate from the typical seasonal patterns (VanderPlas, 2018). Periodograms are similar to Fourier transforms, but they are designed for irregularly sampled data and irregularly shaped periodic signals. Irregularly sampled data are particularly common in astronomy, where a target may rise and set over multiple nights, change brightness during the day, or be affected by weather or natural data gaps (such as the time it takes to send data from the *Kepler* spacecraft to Earth).

A Schuster periodogram is the name given to the traditional periodogram, which was first established in 1898. This periodogram is a straightforward technique and fundamental tool for analyzing the power spectra if the data are consistently spaced in time (Terebizh, 1992). It usually works properly for frequency analysis as well. The Discrete Fourier Transform (DFT) and Fast Fourier Transform (FFT) are effective methods for computing the Fourier transform and its inverse, especially when the input is sampled at equal time intervals. Although FFT is often employed in imaging and signal processing, there are just a few situations in which it may be

directly applied to time series in astronomy. When data are almost evenly spread, interpolation can be used to create evenly spaced data points and then use FFT techniques to analyze them.

A Lomb-Scargle periodogram is an alternative data analysis technique. Lomb introduced this periodogram in 1976, and Ferraz-Mello and Scargle enhanced it in the following years (Lomb, 1976; Ferraz-Mello, 1981; Scargle, 1982). Its definition differs from traditional periodograms. It computes slowly and typically provides little to no benefit over the Schuster periodogram. Lomb-Scargle is an algorithm typically used for identifying and distinguishing periodic signals in data that are irregularly sampled. It is a method that enables researchers to efficiently compute Fourier power spectrum from irregularly sampled data, yielding an effective approach to determine oscillation period (VanderPlas, 2018). The Periodogram techniques covered thus far work well for pulse types that are roughly sinusoidal. These algorithms execute inadequately when the pulse shape deviates much from sinusoidal because power is shared by several Fourier harmonics. Phase dispersion minimization can be used in this instance. One may divide the light curve into, say, 10 phase bins based on the frequency values and compute the scatter in each bin. The frequency with the greatest match is one which reduces the total variations in the phase bins. Useful information can still be obtained regarding a pulse that is not sinusoidal by using this method, if the number of bins is chosen correctly. The patterns in the pulsation frequencies allow us to infer astrophysical properties of stars (Aerts et al., 2010). For asteroseismic analysis using FFT or related transforms (Lomb-Scargle periodogram), García et al. (2014) recommends proper mitigation of the regular gaps present in Kepler data.

1.1.6 Binarogram

A binarogram is an efficient time-delay method developed to identify time-delay effects using pulsation frequencies as a timer. The method is used for detecting celestial objects ranging from as low as $\sim 2 M_J$ (Jupiter mass object) to $\sim 1.5 M_\odot$ (solar mass object). It can be used, for instance, to search for low-mass objects (missing planets) belonging to white dwarfs and subdwarfs in binary-star planetary systems Krzesinski and Blokesz (2019) using the data acquired from the *Kepler* space telescope. The method can also be used to detect δ Scuti stars with orbital periods ranging from 0 to 50+ Earth days, and binary star systems through binarogram computations of light curves or Kepler photometric data (Mohammed, 2019). The output is typically a diagram plotting the fraction of binary stars as a function of the primary star's mass or magnitude in an OSC. It can be used to estimate the distance and age of an OSC by comparing its binarogram to the binarograms of clusters with known parameters.

When resulting binarograms appear contaminated by artefact orbital frequencies due to a star's variable pulsation amplitudes and frequencies, binarograms for different frequencies should be computed individually to eliminate artefacts. Alternatively, the artefacts can be eliminated by including first or any orbital frequency harmonics in the binarogram computations.

1.2 Problem statement

Member stars of the NGC 6866 open cluster should possess common parameters, such as age, distance, chemical composition/ metallicity, radial velocity and proper motion. All stars in the main-sequence (members and non-members) within the NGC 6866 OSC have the same proper motion. Therefore, this property alone cannot be used alone to distinguish members from non-members (Abedigamba, 2016). With ground-based observations, it is not easy to identify the members of a cluster in crowded galactic fields because the stars are closely packed together, making it difficult to distinguish between cluster members and field stars. Identifying which stars are members of a cluster is one of the hardest parts of studying it (Stott, 2018). In this study, we faced the challenge of distinguishing between members and non-members in the NGC 6866 open cluster. To address this, data from the *Kepler* and *Gaia* satellites were utilized. According to Balona et al. (2013a), *Kepler* satellite data alone has not been sufficient to definitively distinguish between member stars and non-member stars (also called field stars) within the NGC 6866. Knowing the cluster's members allowed for a targeted search for binary stars within the cluster. The properties of stars and OSC's age and distance have been determined from binary stars (Sandquist et al., 2016; Kiran et al., 2016). According to the available literature, no similar study had been done for the NGC 6866 using binary stars.

1.3 Aim

This research aims to achieve an asteroseismic and astrometric characterization of members in the NGC 6866 OSC, with the goal of estimating the cluster's age and distance using the presence of binary stars within the cluster.

1.4 Objectives

1. To identify member stars and estimate their properties;
2. To characterize binary Delta Scuti stars; and
3. To estimate the age of the NGC 6866 OSC.

1.5 Outline of the dissertation

This work is divided into five chapters. Chapter 2 reviews existing research on cluster membership determination, focusing on methodologies and their contributions. This review informs the selection of a suitable method for this study. Chapter 3 outlines the methodologies used to address the research objectives. Here, results of the analysis are presented, along with detailed interpretations of the data. Chapter 4 discusses the results, highlights limitations, and offers recommendations. This work is summarized and concluded in Chapter 5.

Chapter 2

Literature Review

2.1 Introduction

The asteroseismic and astrometric characterization of members in the NGC 6866 OSC involves studying the properties of stars within the cluster using two specific techniques: asteroseismology and astrometry. Stars pulsate and vibrate in characteristic patterns, and these oscillations provide valuable information about their internal structure, composition, and evolutionary stage. By analyzing the frequencies and amplitudes of these oscillations, precise measurements of stellar properties can be extracted such as mass, radius, chemical composition (Christensen-Dalsgaard, 2004), and age (Basu and Chaplin, 2017). In the context of NGC 6866, asteroseismology can be used to characterize the members of the OSC by their stellar properties. By studying the oscillations of stars within NGC 6866, we can determine their masses, radii, and ages more accurately than through traditional methods. This information helps in understanding the evolutionary state of stars in the cluster, identifying different stellar populations within NGC 6866, and investigating the effects of stellar evolution and dynamics on the cluster’s overall properties.

High-precision astrometry techniques, such as those employed by space-based observatories like *Gaia*, can provide accurate measurements of stellar positions and proper motions. By tracking the motions of stars within NGC 6866 over time, we can determine the cluster’s kinematics and study its dynamical evolution. Astrometry is also crucial for determining the distances to individual stars in NGC 6866. Accurate distances are essential for calculating the cluster’s overall distance and placing it in the context of the Milky Way. The precise knowledge of distances to cluster members helps refine the cosmic distance ladder, improves our understanding of stellar evolution, and provide valuable data for galactic structure and cosmological studies.

Combining asteroseismic and astrometric techniques allows us to obtain a comprehensive characterization of the members in the NGC 6866 OSC. By studying the oscillations and internal structures of stars through asteroseismology and analyzing their positions, motions, and distances through astrometry, we can gain insights into the stellar populations, evolutionary processes, dynamics, and overall parameters of NGC 6866.

We aim to estimate two parameters of the NGC 6866, namely its age and distance. Estimating

the age and distance of an OSC is crucial for studying stellar populations, galaxy evolution, and cosmology. Determining these parameters enables us to analyze stellar populations and provide insight into the formation and evolution of galaxies (Tadross, 2011; Karataş et al., 2023).

2.2 Open Star Clusters (OSCs)

Open star clusters, also known as open clusters or galactic clusters, are groups of stars that are loosely bound and share a common origin (Gieles and Zwart, 2011). They are typically young clusters of stars, with ages ranging from a few million to a few hundred million years, and are often found in the arms of spiral galaxies like the Milky Way. They typically contain between a few hundred to a few thousand stars, but some can have up to tens of thousands of members. They are generally less massive and more loosely bound than globular clusters. Stars in OSCs are distributed in a dense core and a more scattered outer region. They are also typically associated with interstellar gas and dust, which can be observed as emission and absorption lines in their spectra (Waldek, 2021).

They form from giant molecular clouds of gas and dust in the interstellar medium (Stott, 2018). As these clouds collapse under their own gravity, they fragment into smaller clumps that eventually become individual stars. The stars in OSCs are believed to have a common origin, with similar ages and compositions, as they formed from the same cloud of gas and dust. Over time, the cluster may disperse due to gravitational interactions with other stars or the galactic tidal field, and the stars will eventually become part of the general population of stars in the galaxy (Karttunen et al., 2007; Waldek, 2021).

The stars in OSCs have a similar chemical composition, which reflects the composition of the original molecular cloud from which they formed. Studies of OSCs have provided important insights into the chemical evolution of galaxies, as they can be used to trace the distribution and evolution of heavy elements (metals) in the galaxy. OSCs also contain a range of multiple star systems and star types, including main-sequence stars, red giants, and white dwarfs, providing a diverse sample for studying stellar evolution (Waldek, 2021).

2.3 Methods to determine age and distance of an open star cluster

There are a number of methods that can be used to determine the age and distance of an OSC. Some of the most common methods used for determining the distance of an OSC include parallax measurements, luminosity function, photometric distance estimators (such as main-sequence fitting), and spectroscopic distance indicators. The most common of these methods are main-sequence fitting and parallax. Some methods, such as the color-magnitude diagram and main-sequence fitting, are capable of determining both the age and distance of star clusters. This study focuses on estimating these parameters using parallax and main-sequence fitting. Recent advances in determining the parameters, such as the use of new data sources and improved the-

oretical models, are also discussed.

2.3.1 Parallax

Parallax is the apparent shift or displacement of a celestial object observed from different positions in space. It is used to measure the distance to nearby stars and estimate the age of star clusters. By measuring the parallax angle, which represents the shift in a star's position due to the Earth's orbit around the Sun, we can calculate the star's distance using trigonometry. For star clusters, knowing their distance helps place them in a spatial context and infer the age of the stars within the cluster, as they typically formed from the same molecular cloud and share similar ages. Parallax measurements are most effective for nearby stars, while alternative techniques are better for more distant objects to estimate distances and determine cluster ages (Luri et al., 2018).

2.3.2 Main-sequence fitting

The main-sequence fitting method can be used to estimate both the age and the distance of an OSC. By comparing the apparent magnitude of the stars in a cluster to their absolute magnitudes, the distance to the cluster can be estimated. This is based on the fact that the absolute magnitude of a star on the main sequence depends on its mass. Stars of a particular mass will have a particular location on the main sequence. The age of the cluster can be estimated by fitting theoretical isochrones to the observed main sequence of a cluster.

2.3.3 Recent advances in determining the age and distance of open star clusters

2.3.3.1 Observations

New observational techniques have advanced determining the age and distance of OSCs. Some of the developments include the *Gaia* mission, high-resolution spectroscopy, infrared and X-ray observations. The *Gaia* mission has provided unprecedented astrometric data for billions of stars, including many OSCs. This data can be utilized to measure the parallaxes of cluster members, which can then be used to determine the distances to clusters with high precision. Advances in high-resolution spectroscopy have provided a way to measure the chemical compositions and radial velocities of cluster members with great accuracy. This information can be employed to constrain the ages and distances of clusters, as well as their evolutionary histories. Infrared and X-ray observations are typically used to detect young pre-main-sequence stars and evolved white dwarfs in OSCs. The ages of these stars can be used to constrain the ages of the clusters themselves (Foschini et al., 2008).

2.3.3.2 Theory

New theoretical models of stellar evolution have been developed, which can be applied to more accurately fit isochrones to observed color–magnitude diagrams (CMDs). Some of the advancements include stellar evolutionary models and cluster dynamics models. Stellar evolution models have improved significantly in recent years, providing more accurate predictions for the luminosities, temperatures, and lifetimes of stars. This information helps determine the ages of OSCs from their observed CMDs. Cluster dynamics models are utilizable for simulating the evolution of OSCs, taking into account the effects of stellar interactions, gas expulsion, and tidal disruption. These models can be employed to constrain the ages and distances of clusters, as well as their initial masses and structures. Recent advances in observations and theory have led to a number of significant breakthroughs in the determination of the age and distance of OSCs. For example, *Gaia* data have been used to refine the distances to hundreds of OSCs, with some uncertainties reduced by as much as a factor of 10. High-resolution spectroscopy has been used to determine the ages of OSCs to unprecedented accuracy, revealing that some clusters are significantly older than previously thought (Martines et al., 2020). Infrared and X-ray observations have been used to discover new OSCs and to constrain the ages of clusters that were previously difficult to study (Foschini et al., 2008).

2.4 Related work

2.4.1 NGC 6866 (OSC)

Astrometric and photometric methods are the main approaches used to determine cluster membership. Astrometric methods are used for determining the position and motion of stars, and photometric methods are used for separating cluster stars from field stars using the color and luminosity of stars (Stott, 2018).

2.4.1.1 Stellar variability

NGC 6866 (Figure 2.1) is home to variable stars, including pulsating, δ Scuti, rotating stars, eclipsing binaries (Yang et al., 2011), γ Doradus, WUMa, β Cephei and ellipsoidal variables (Joshi et al., 2012). The variations among these variable stars can span various time scales, from minutes to days, or even months to years, exhibiting intrinsic and extrinsic variations in parameters like brightness, size, and temperature (Molenda-Żakowicz et al., 2009). Joshi (2022) investigated stellar variability within NGC 6866 using phase-folded diagrams. This research harnessed data from the *Hipparcos* and *Tycho* catalogues to determine the positions and brightnesses of the stars within the cluster, further employing All-Sky Automated Survey (ASAS) data to create phase-folded diagrams for a comprehensive investigation of variability (period, amplitude, and shape of the light curve). The majority of variable stars in NGC 6866 were identified as δ Scuti stars, characterized by pulsations with periods ranging from 0.01 to 0.3 days. A few other types of variable stars, including eclipsing binaries and rotating variables, were also detected. The phase-

folded diagrams allowed for the estimation of distances, masses, and ages of variable stars within NGC 6866. δ Scuti stars were typically situated between two and three kpc from Earth, with ages around 100 million years. Eclipsing binaries were found closer to Earth, at around one kpc, and had ages of approximately 50 million years. This study emphasized the utility of phase-folded diagrams for understanding the nature of stellar variability and for estimating crucial stellar parameters in OSCs.

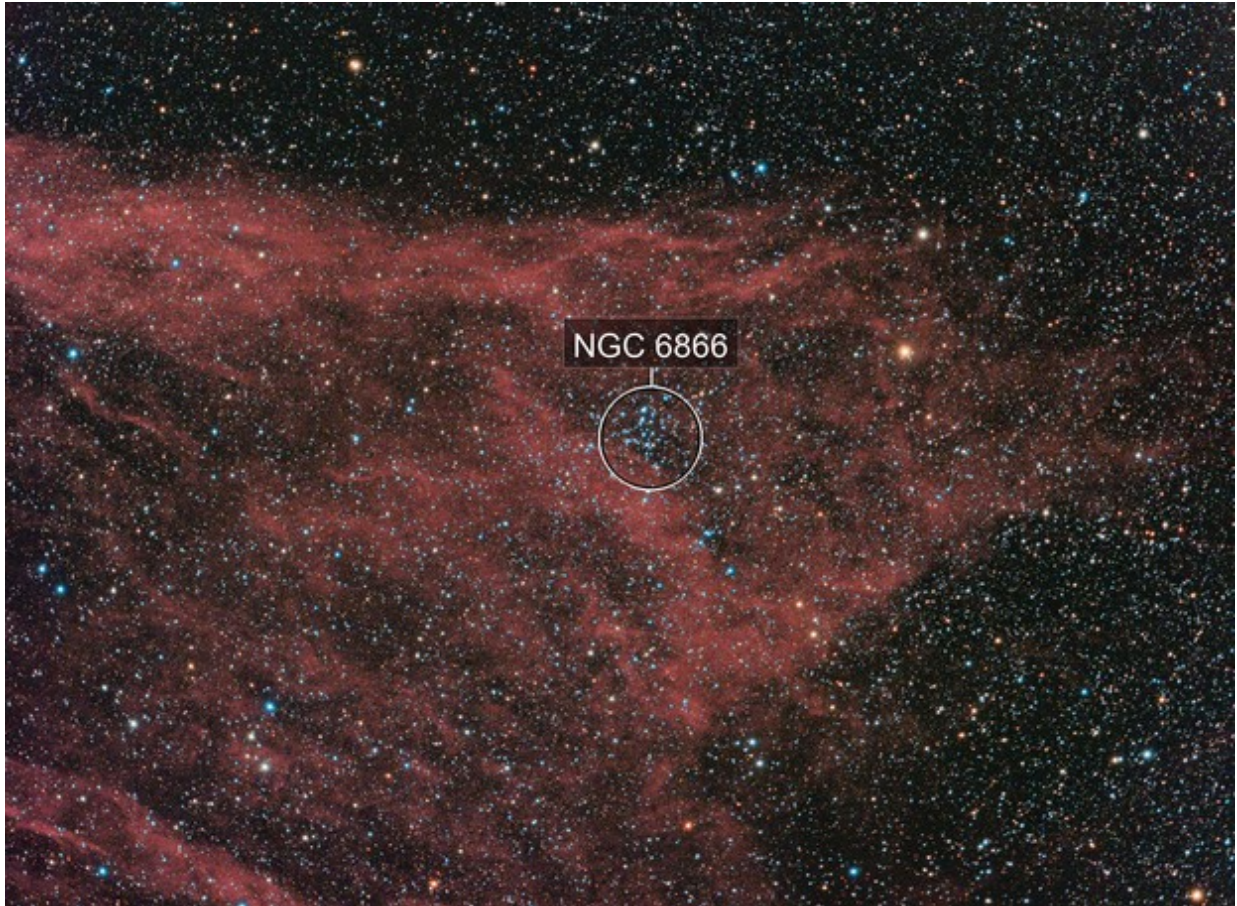


Figure 2.1: An image of NGC 6866 OSC taken from <https://www.astrobin.com/q3im75/>.

Yang et al. (2011) employed a stellar-population synthesis method to study solar-like oscillations in star clusters. They utilized stellar evolution models and Monte Carlo simulations to explore the relationships between oscillation frequencies, age, and metallicity in star clusters. They used data from the *Kepler* satellite to identify and characterize the variable stars in NGC 6866. Phase-folded diagrams, membership analysis, and temporal analysis were used to study the shape and amplitude of the pulsations of these stars. According to Yang et al. (2011), the variable stars in NGC 6866 show evidence of secondary pulsations, which are additional pulsations that occur at a different frequency than the primary pulsation. They argue that these secondary pulsations may be caused by non-radial modes of pulsation, or by the presence of a companion star.

Balona et al. (2013a) used data from the Kepler Input Catalog (KIC) and time-series photometry from the *Kepler* database to study pulsating and rotating stars in the field of the NGC 6866. To analyze the pulsating stars in the sample, they used the following techniques: frequency analysis to identify the frequencies of the pulsations; light curve modeling to determine the properties of the

pulsations, such as the mode, amplitude, and period; and rotational modulation to identify stars that were rotating with spots on their surfaces. They did this by looking for periodic variations in the light curve that were caused by the rotation of the star. Balona et al. (2013a) found that the stars in their sample were a mix of pulsating and rotating stars. They also found eight of γ Doradus stars, which are pulsating stars that have periods in the range of 0.001-0.01 days.

2.4.1.2 Multiple-star systems

According to Karttunen et al. (2003), solitary stars account for less than 50% of all stars; most are part of systems with two or more members. Spectroscopic and imaging investigations conducted by Tokovinin (1997) and Tokovinin et al. (2006) revealed that 40% of the examined multiple-star systems were binaries with orbital periods shorter than 10 days, and 96% contained a more massive tertiary companion. Wilson (1953) and Jaschek and Jaschek (1957) found from the General Catalogue of Radial Velocities (GCRV) that the proportion of spectroscopic binaries among the main-sequence stars varies between 20% and 30%. Petrie (1960) calculated it to be above 50% using a different method for these spectroscopic binaries. Even when the scope is narrowed down to particular subtypes of stars, the ratio seems to hold. For instance, spectroscopic binaries make up 60% to 70% of Am stars (Abt and Levy, 1985; Carquillat and Prieur, 2007; Smalley et al., 2014). According to Loore and Doom (1992), around 30% of main-sequence stars are binaries, whereas 60% of early type stars are binaries with a mass ratio greater than 0.2. At least half of all “stars” in the sky are actually multiple-star systems (Ostlie and Carroll, 1996). Similar to this, around 50% of the visible stars are part of binary or multiple-star systems (Lada, 2006). Loore and Doom (1992) conclude that binary (or multiple-star) systems comprise 50% of the stars seen in the solar neighborhood. Additional research on triple, quadruple, and quintuple system proportions have also been documented in various literature studies (Worley, 1967; Heintz, 1969).

2.4.1.3 Membership determination

In 1973, Koroleva released one of the earliest publications concerning NGC 6866. NGC 6866 is one of the least-researched OSCs (Güneş et al., 2012). Various techniques can be applied to identify member stars of OSCs, such as kinematics and statistical distance methods. Typically, initial membership determination procedures are preceded by potential member isolations. Member stars are isolated from non-members through discrimination regarding stellar radial velocities, proper motions, spatial location, and derived metallicities (Donor et al., 2020). Additionally, photometric approaches (still predominant in membership determination) are used to further narrow down potential member samples.

Koroleva (1973) used Sverdllovsk photometric data to discern the members of NGC 6866. The research identified a total of 93 member stars, of which three were giant stars and 57 were main-sequence stars. Bostanci et al. (2015) isolated potential cluster members through probability calculations and astrometric data analysis and conducted photometric and spectroscopic observations to derive astrophysical properties. Long et al. (2023) used 5D parameter space on GDR3 and *Kepler*/K2 photometric data to determine member stars of 16 clusters, of which 487 had a

probability greater than 60% and 347 had a probability greater than 90% of membership. Karataş et al. (2023) used *Gaia* eDR3 photometric data from VizieR (selecting an area of 40 to 70 arcmin) on the fitCMD algorithm to derive astrophysical parameters, including the membership, age, distance, metallicity, mass, and mass function of OSCs within the *Kepler* FOV (NGC 6791, NGC 6811, NGC 6819, and NGC 6866). With the application of the Gaussian Mixture Model (GMM) and the SCIKIT-learn package, they determined the membership probabilities of the OSCs. For NGC 6866, using probability percentage greater than 90, 67 potential cluster members were obtained. Brogaard et al. (2023) detected six giant stars (KIC 8461659, KIC 8329894, KIC 8395903, KIC 8264549, KIC 7991875, and KIC 8264592) within the NGC 6866 by utilizing photometric data, proper motions, and parallaxes sourced from *Gaia*, along with data from spectroscopic literature. Additionally, they incorporated asteroseismic data obtained from the *Kepler* mission for five of these stars.

Open star clusters are primarily situated in the densely populated region of the Milky Way’s disk. Within the same line of sight, there exist numerous disk stars that are not associated with these clusters. Typically, the determination of cluster membership relies on assessing the spatial arrangement and the similarity in motion determined through proper motions and/or radial velocities. If proper motions for individual stars are not sufficient to distinguish members from non-members of the cluster, the distance modulus of the cluster and the stars within its vicinity should be calculated (Balona et al., 2013a). In their analysis, Balona et al. (2013a) only included stars that had a distance modulus that was consistent with the cluster’s distance modulus. The distance modulus is a valuable tool for isolating member stars of a cluster from non-members because it allows us to identify stars that are likely to be at the same distance as the cluster. However, non-member stars at the same distance can still contaminate the cluster’s CMD and make it difficult to fit isochrones for the cluster.

Joshi (2021) utilized photometric colors to isolate potential member stars of NGC 6866. CMDs were employed, specifically (B-V) vs. V, to isolate blue component stars (BCS) from red component stars (RCS), based on their locations in the CMD plane. Potential cluster members can also be identified using a field star decontamination algorithm applied to 2MASS JHKs photometric data (Güneş et al., 2012). Joshi (2021) categorized BCSs as the most probable cluster members of the cluster, while the RCSs were regarded as field stars, with some being giant stars. Sometimes, stars are designated as field stars due to age differences; for example, the red-giant KIC 8263801, with an age of 1.48 ± 0.21 Giga year (Gyr) (Tang et al., 2018), and the SX Phe star ID 1077, with $\log(\text{age}) = 9.73 \pm 0.12$ yr (Joshi, 2022) or 5.37 Gyrs were found to be older than NGC 6866, of which the age ranges from 0.43 to 1.0 Gyr.

2.4.1.4 Binary star systems identification

This work focuses on isolating potential members, identifying δ Scuti stars within which the aim is to search for binary star systems using the binarogram method. Balona (2014) used the binarogram method to determine orbital parameters of 34 pulsating stars from multiple or binary star systems by applying the time-delay method, due to the changing distance of the pulsating stars. In contrast to other techniques used for determining stellar orbital parameters, the

binarogram method directly derives a periodogram of the orbital frequency, utilizing all available data to extract the time delay from stellar pulsations (Balona, 2014). According to Balona (2014), the method is easy to use when you know the pulsation frequencies. He applied the time-delay method to 34 δ Scuti variables (*Kepler* objects of interest) and detected five binary stars. However, detecting celestial bodies with masses below that of Jupiter is still a challenge. Pulsating stars with significantly high frequencies are required for detection of such low masses. Determining binary star systems that have significant orbital frequencies will enable us to further fit isochrones to determine the age and use stellar parallax to determine the distance of the cluster.

2.4.1.5 Age and distance determination

The age and distance of NGC 6866 have been estimated using a variety of methods. A summary of this data collection is provided in Table 2.1, after which a discussion follows.

Table 2.1: Methodologies, techniques and sources of data used by various authors to determine age, distance, and other parameters for OSC NGC 6866.

Authors	Methodology and/or techniques	Data source(s)	Target(s)	Findings		
				Age (Gyr)	Distance (kpc)	Other
This work	Astroseismic and astrometric analysis, binogram, and isochrone fittings	<i>Kepler</i> and GDR2 extracted from the Vizier website	NGC 6866 (OSC)	0.86	1.497	Membership, mean proper motion, distance moduli, and orbital period
Güneş et al. (2012)	Stellar radial density profile (RDP), 2MASS (J, J – H) CMD	2MASS JHKs photometric data	NGC 2360, NGC 6866 and NGC 7062	0.80	1.65	Membership and reddening
Joshi et al. (2012)	Isochrones fittings, observations, calibrated (B-V), (V-R), (V-I) vs V diagrams and CMD	Observations	NGC 6866 (OSC)	0.63	1.47	Periodic variables
Balona et al. (2013a)	proper motion, parallax and radial velocities	<i>Kepler</i> satellite data	NGC 6866 (OSC)	0.65	2.5	Membership, pulsation periods and rotational periods
Balona (2014)	Time-delay method – Binogram	<i>Kepler</i> satellite data	δ Scuti stars and detecting binary stars	0.65 ± 0.10	-	Orbital parameters, binary star systems and distance modulus

Janes et al. (2014)	Maximum-likelihood approach, Markov chain Monte Carlo algorithm, fitting theoretical isochrones and CMD	UBVRI photometric data	NGC 6866 (OSC)	0.705 ± 0.17	1.25	Apparent distance modulus ($m - M$)V, reddening ($E(B - V)$) and metallicity (Z)
Bostanci et al. (2015)	Astrometric data analysis, theoretical evolutionary isochrones fittings and CMDs	Stellar density data	NGC 6866 (OC)	0.813 ± 0.05	1.189 ± 0.075	Membership, metallicity and reddening
Oralhan et al. (2015)	Observation and reduction techniques	Ground-based telescope and photometry UBV(RI) _C	20 open clusters including NGC 6866 (OSC)	0.75 ± 0.04	1.32 ± 0.01	Reddening and metallicity
Joshi (2017)	Phase-folded diagrams, Membership analysis, temporal analysis, isochrones fitting, parallax and light-curves	Vizier: JOS12 and Time-series photometric data	NGC 6866 (OSC)	$\log(age) = 8.60 \pm 0.07 \sim 0.39$	-	Membership, period, amplitude, frequency, shape of pulsation and largest amplitude of regular fluctuations
Joshi (2021)	(B-V) vs V CMD, King Models best fit of RDP and n ($V - K$ vs V) CMD	Photometric data	NGC 6866 (OC)	$\log(age) = 8.85 \pm 0.05 \sim 0.71$	1.19 ± 0.04	Membership, radius and distance modulus
Donor et al. (2020)	Discrimination stars with regards to stellar radial velocities, proper motions, spatial location and derived metallicities	<i>Gaia</i> DR2, SDSS/APOGEE DR16 and stellar parallaxes from the MWSC catalogue	NGC 6866 (OSC)	0.44	7.87	Membership

Long et al. (2023)	5D parameter space, Isochrones fitting, membership probability calculations, astrometric and photometric data	<i>Gaia</i> DR3 and <i>Kepler</i> /K2 photometric data	16 OSCs including NGC 6866 (OSC)	0.794	-	Membership, binary systems, rotational characteristics and astrometric parameters
Karataş et al. (2023)	fitCMD algorithm, <i>Gaia</i> -eDR3 parallaxes, Gaussian Mixture Model (GMM) and the SCIKIT-learn package	<i>Gaia</i> eDR3 photometric data from VizierR	<i>Kepler</i> (NGC 6791, NGC 6811, NGC 6819, and NGC 6866)	1.0 ± 0.20	1.19 ± 0.21	Membership
Brogaard et al. (2023)	Compared asteroseismic data to stellar-model isochrones	Photometric data, proper motions, and parallaxes sourced from <i>Gaia</i> DR3, TOP CAT, <i>Kepler</i> asteroseismic data and spectroscopic literature	NGC 6866 (OSC): six giant stars	0.43 ± 0.05	1.42	Extent of convective-core overshoot

Most OSC age determination studies depend on fitting isochrones, including the study conducted by Joshi et al. (2012). Joshi et al. (2012) provided parameters that include the age (0.63 Gyr) and distance (1.47 kpc) of NGC 6866. In order for them to determine the cluster’s distance and age, they constructed calibrated (B-V), (VR), and (V-I) vs V diagrams of NGC 6866 using their data and overplotted theoretical isochrones for solar metallicity. A study by Balona et al. (2013a) determined a similar age of 0.65 Gyr through KIC data and time series photometry, but a distance of about 2.5 kpc from Earth. However, Joshi (2021) highlights a discrepancy in stellar magnitude estimates made by Joshi et al. (2012) when compared to the general estimates made by other studies. For example, Long et al. (2023) determined the age of the cluster as 0.794 Gyr through isochrone-fitting, Güneş et al. (2012) estimated the age to be 0.80 Gyr, the reddening values of 0.19, and a distance of 1.65 kpc. They employed stellar radial density profiles and the 2MASS (J, J – H) CMDs.

Bostanci et al. (2015) determined astrophysical parameters of NGC 6866, including age, distance, metallicity, and reddening, using stellar density data. The age of NGC 6866 was found to be 0.813 ± 0.05 Gyr, with a distance of 1.189 ± 0.075 kpc and a distance modulus of 10.60 ± 0.10 mag. Theoretical evolutionary isochrones were fitted to observations represented on the CMDs to simultaneously determine the abovementioned parameters. The study employed multiple methods to mitigate reddening-degeneracy and assumed solar metallicity.

Donor et al. (2020) found the cluster’s age and distance to be 0.44 Gyr and 7.87 kpc. They used data from GDR2 and SDSS/APOGEE DR16. Distance was determined using stellar parallaxes from the Milky Way Star Cluster catalogue (MWSC) and GDR2. Brogaard et al. (2023) compared asteroseismic data to stellar-model isochrones to determine the extent of convective-core overshoot, leading to a more constrained estimation of the cluster’s age of 0.43 ± 0.05 Gyr. The mean distance from the six giant stars studied is 1.42 kpc. Karataş et al. (2023) used *Gaia*-eDR3 parallaxes for the distance 1.19 ± 0.21 kpc, and fitCMD for the age of 1.0 ± 0.20 Gyr.

Janes et al. (2014) developed a maximum-likelihood approach that employs a Markov chain Monte Carlo algorithm for fitting theoretical isochrones to the observed CMDs of NGC 6866. In the context of NGC 6866, an OSC of a specific age range and limited red giant members, Janes et al. (2014) acknowledge that relying solely on photometric data to ascertain cluster properties may yield less precise results in the case of NGC 6866, an OSC of a specific age range and limited red-giant members. Nonetheless, they successfully employed UBVRI photometric data to derive essential cluster parameters: the apparent distance modulus $(m - M)V$ is approximately 10.98 ± 0.24 , the reddening $(E(B - V))$ is roughly 0.16 ± 0.04 (corresponding to a distance of 1.25 kpc), the age is estimated to be around 0.705 ± 0.17 Gyr, and the metallicity (Z) is approximately 0.014 ± 0.005 .

Using a ground-based telescope and UBVRI photometry, Oralhan et al. (2015) determined the fundamental astrophysical parameters (interstellar reddening, metallicity, distance, and age) of 20 OSCs. In the case of NGC 6866, they estimated age and distance to be 0.75 ± 0.04 Gyr and 1.32 ± 0.01 kpc. Oralhan et al. (2015) compared their results to previous studies and found significant differences. These differences are likely due to a variety of factors, including the use of different empirical calibrations, assumptions about cluster metallicity, isochrones, interstellar

reddening, and data reduction and cluster membership techniques. The morphological ages of 12 OSCs, which were determined using morphological indices in the CMDs, were consistent with the isochrone ages. This is, in part, due to the fact that the clusters had good red clump and red-giant star candidates (Oralhan et al., 2015).

Janes et al. (2014) highlight that not knowing which stars are members of a cluster, as well as not knowing their metallicity or reddening, will result in measurements of the cluster's age and distance using photometry producing less accurate results. Missing information can lead to skewed results. For instance, lacking membership information can introduce foreground or background stars into the analysis, making it difficult to fit the cluster's CMD with isochrones. Similarly, an unknown metallicity could lead to selecting inappropriate isochrones, resulting in inaccurate measurements. Finally, uncertainty about the reddening towards the cluster can cause an overestimation or underestimation of its distance.

2.4.2 Other open star clusters

2.4.2.1 Membership determination

Open star clusters are typically studied for cluster membership determination (Gao, 2014). Although, determination of cluster membership is a challenging aspect of studying a cluster (Stott, 2018), various studies have managed to identify members of various clusters. Maia et al. (2010) investigated NGC 1981 OSC using near-IR and UBVRI photometric data. They developed a technique that takes into account field contamination, which allowed photometric membership derivation for cluster stars. They further determined a new cluster center by fitting a Gaussian function to the two-dimensional stellar distribution on the sky and used that new cluster center to derive the radial stellar density profile and the structural parameters. The study showed that a group of six relatively bright stars is related to the NGC 1981 OSC, instead of the nearby Orion nebula cluster.

Gao (2014) determined member stars in the field of the NGC 188 OSC by implementing the density-based spatial clustering of applications with noise (DBSCAN) clustering algorithm. The study identified 472 member stars by applying this algorithm using 3D kinematics. This method requires high-precision kinematic data (proper motions or radial velocities) to produce sound results, but only a handful of OSCs possess a large, homogeneous set of radial velocities. Another way to identify potential members of OSCs is by using proper motions, parallaxes, and CMDs. This information was used by Rangwal et al. (2019) to identify the members of three southern OSCs; NGC 6067, NGC 2506 and IC 4651.

Apart from determining cluster membership, some astrophysicists aim to determine planetary members of multiple star systems. Krzesinski and Blokesz (2019) observed and searched for existence of low-mass objects (exoplanets) orbiting the pulsating white dwarf KIC 8626021 and two eclipsing binary stars KIC 9472174 (made up of a pulsating type B subdwarf and a red dwarf) and KIC 7975824 (made up of a type B subdwarf and a white dwarf). No low-mass objects were detected for KIC 8626021. No exoplanets were identified orbiting KIC 7975824 and

KIC 9472174, within a distance of 0.7 and 1.4 AU. Krzesinski et al. (2020a) were not satisfied with their previous results and repeated the investigation. However, white dwarfs and subdwarfs have the least to none detected exoplanets in their orbits, hence claims brought forth for their existence were met with criticism from other scholars. Krzesinski et al. (2020a) aimed to gauge prowess of analytical techniques on detecting exoplanets orbiting three star systems nearing the end of their evolution (white dwarfs and extended horizontal branch stars). They demonstrated that data analysis of the *Kepler* photometric time series data of light curves using the binarogram method in conjunction with Fourier transforms, and eclipse timing variations can be used to identify Jupiter-mass exoplanets orbiting around binary star systems with short orbital periods, subdwarf and white dwarf stars. However, the authors used light curves from K2 mission (approx. 80 day photometric time-series data) which limited the search to short orbital periods ranging between 10 to 40 days long and object masses ranging between 7 to 20+ M_J objects (Krzesinski et al., 2020a).

2.4.2.2 Age and distance determination

Rangwal et al. (2019) estimated the distances for the OSCs NGC 6067, NGC 2506 and IC 4651 using parallaxes taken from GDR2 as 3.01 ± 0.87 , 3.88 ± 0.42 and 1.00 ± 0.08 kpc. The cluster radii values were determined to be 10' for NGC 6067, 12' for NGC 2506 and 11' for IC 4651. Further, the ages of the clusters were estimated by isochrone-fitting as 66 ± 8 Myr, 2.09 ± 0.14 Gyr and 1.59 ± 0.14 Gyr for NGC 6067, NGC 2506 and IC 4651.

Hendy and Tadross (2020) re-estimated the astrophysical properties of the Kronberger 60 OSC using GDR2. All stars within the radius of 40 arcmin of the cluster centre were selected to ensure that the entire cluster is encompassed. Hendy and Tadross (2020) applied theoretical stellar isochrones to the observed CMDs for the OSC Kronberger 60, and estimated the age, distance modulus, and color excess of the OSC to be 0.80 Gyr, 12.05 mag and 0.30 mag.

Balona et al. (2013b) used isochrone-fitting to estimate the age of NGC 6819 OSC to be about 2.5 Gyr. They selected all stars in the KIC within the radius of 12 arcmin of the cluster center. The area selected contains 640 stars. Sandquist et al. (2016) analyzed data of the OSC NGC 6811 within the same FOV, mainly focusing on the most luminous eclipsing binary star system (KIC 9777062). The primary star was determined to be a metallic-line Am star, while the secondary star was found to be a Γ Dor pulsating star. Measuring stellar masses and radii indicated that the age difference between the binary stars is 15% to 20%. The secondary star was concluded to have lower mass and being the older star. A slow rotation and longer orbital periods ($P = 19.23$ days) have been observed from the two stars as they signal tidal interaction. The age of the cluster was determined to be 0.9 Gyr after the analysis of asteroseismic data in the *Kepler* FOV. However, the authors were not convinced of their age and distance estimations, as various models they used were not consistent with their recorded observations. The methods yielded ages of 1.0 Gyr for the primary star and 1.2 Gyr for the secondary star, which is above the host cluster's age.

Brewer et al. (2016) investigated a triple star system (WOCS 24009), which included a detached

binary system, and eclipsing binaries to define their parameters and further constrain the age of NGC 6819. Photometric and spectroscopic observations of three detached eclipsing binary stars, WOCS 24009, WOCS 40007 and WOCS 23009, were presented. The analyses for the latter two detached binary (DEB) star systems were improved over the authors' previous study while the researchers were busy determining the NGC 6819's distance modulus, found to be $(m - M)_V = 12.38 \pm 0.04$. The former dual star system's orbital period is equivalent to 3.6 Earth days. The triple-lined star system WOCS 24009 contains a non-eclipsing star with a mass of $M_A = 1.251 \pm 0.057 M_\odot$. The eclipsing binary stars have masses of $M_B = 1.090 \pm 0.010 M_\odot$ and $M_C = 1.075 \pm 0.013 M_\odot$ with respective radii of $R_B = 1.099 \pm 0.006 \pm 0.005 R_\odot$ and $R_C = 1.069 \pm 0.006 \pm 0.013 R_\odot$. Individual star masses were determined through radial-velocity measurements. The age of the *Kepler* OSC NGC 6819 was constrained to be $2.38 \pm 0.05 \pm 0.22$ Gyr using the three star systems' aggregate analysis of mass and radii (M-R plane) and $2.21 \pm 0.10 \pm 0.20$ Gyr using CMD. The following strategy was used to identify the three stars: entity A is the brightest of the three with a wide orbit around DEB. The DEB consists of entities B and C; B is larger and brighter than C (Brewer et al., 2016). Radial velocity was used to determine the masses of the three individual stars in the triple-star system. The greater the number of binary stars with predetermined ages, masses, and radii in a cluster, the greater the precision of cluster age determination.

Photometric analysis of six star clusters observed with the Hubble Space Telescope Advanced Camera for Surveys (ACS) in the F555W and F841W filters was presented by Glatt et al. (2008). The observed star clusters were Lindsay 1, Kron 3, NGC 339, NGC 416, Lindsay 38, and NGC 419, in the Small Magellanic Cloud. Three isochrone models, namely Padova, Teramo, and Dartmouth, all included in the ACS photometric system were used to determine cluster ages. The researchers produced a homogenous and distinctive collection of low-metallicity, single-age fiducial isochrones by fitting observed ridgelines for each cluster. Except for NGC 419, where the Padova isochrones offered the closest fit, all cluster CMDs are best approximated by the Dartmouth isochrones. They determined ages of 7.5 ± 0.5 Gyr (Lindsay 1), 6.5 ± 0.5 Gyr (Kron 3), 6 ± 0.5 Gyr (NGC 339), 6 ± 0.5 Gyr (NGC 416), and 6.5 ± 0.5 Gyr (Lindsay 38) using Dartmouth isochrones Glatt et al. (2008).

The *Kepler* mission and *Gaia* missions are the largest, but not the only missions that identify and provide photometric and astrometric data regarding celestial objects within the Milky Way and beyond. Other notable examples include VISTA Variables in the Via Lactea survey (VVV), Galactic Legacy Infrared Mid-Plane Survey Extraordinaire (GLIMPSE), Wide-field Infrared Survey Explorer (WISE), Variability Phenomena in the High Astronomical Survey (VPHAS), Transiting Exoplanet Survey Satellite (TESS). A study by Bisht et al. (2021) used broad-band data from the American Association of Variable Star Observers (AAVSO), Photometric All-Sky Survey (APASS), GLIMPSE, VPHAS, VVV, WISE, while photometric and astrometric data were acquired from eGDR3. eGDR3 and GDR3 feature high-precision astrometric and photometric data compared to previous releases. The data were utilized to determine the age, distance, galactic orbits, luminosity function (LF), and mass function (MF) among others, for two pairs of binary star clusters: NGC 5617 and Trumpler 22 and NGC 3293 and NGC 3324. The age and distance were estimated using solar metallicity. To confirm and validate the results, the distance was further calculated using both parallax and distance modulus. Moreover, the scientists identified 18

stars in NGC 5617 and 44 located in Trumpler 22.

Dias et al. (2018) studied seven OSCs using photometric data from observations (UBVRI CCD) and GDR2. The UBVRI is a photometric system used in astronomy to measure the brightness of stars in different filters, and stands for the following wavelengths of light: Ultraviolet (320-380 nm), Blue (400-500 nm), Visual (500-550 nm), Red (600-700 nm), and Infrared (700-900 nm). The authors used proper motions acquired from the Fifth USNO (US Naval Observatory) CCD Astrograph Catalog (UCAC5) to estimate the probability of photometric membership, and they determined kinematic membership. They also used proper motions and parallaxes from GDR2 to find stellar membership. For individual data sets (UCAC5 and GDR2) and membership determination, they fitted objective theoretical isochrones in their global optimization tool for the OSCs Collinder 307, Dias 6, IC 4651, NGC 5138, NGC 6087, and NGC 6178 to estimate distance in pc (1819 ± 497 , 2627 ± 182 , 924 ± 52 , 1857 ± 95 , 1009 ± 97 , and 817 ± 108), age, binary fraction, metallicity, and reddening of the clusters. The GDR2 results showed more precise estimations for parameters and membership selection. They further discovered two new star clusters, Dias 4a and Dias 4b, in a location previously known as Dias 4.

Soydugan et al. (2020) investigated double-lined binary stars in NGC 6819 using spectroscopic and photometric data. They performed a thorough examination of light and radial-velocity curves to determine the fundamental parameters of the binary systems' components. They also evaluated the evolutionary status of these components using Modules for Experiments in Stellar Astrophysics (MESA) evolutionary tracks and isochrones, determining that KIC 5111815 and KIC 5113146 have ages of approximately 2.50 and 1.95 Gyrs. Photometric distances are calculated to be around 2.85 kpc for KIC 5113146 and 3.12 kpc for KIC 5111815. Overall, the results indicate that both KIC 5113146 and KIC 5111815 are likely members of the NGC 6819 OSC after they had used radial velocities and astrometric data from GDR2. Binary star systems provide a great advantage to narrowly constrain astrophysical parameters of star clusters.

Tadross (2011) determined the following parameters for 120 previously unstudied NGC OSCs: angular diameter, distance modulus, Galactic geometric distances, reddening and age. The JHK Near-IR 2MASS photometric observations data were extracted from VizieR (WEBDA and DIAS catalogue) for this investigation. Astrophysical parameters (distance modulus, reddening and age) for each OSC were determined by applying theoretical Padova isochrones of solar metallicity on each OSC's CMD. The same procedures employed in the investigation were initially utilized for calibration purposes on previously studied OSCs. Table 2.2 provides a concise overview of this sub-section.

Table 2.2: Methodologies, techniques and sources of data used by authors to determine age, distance, and other parameters of other OSCs.

Authors	Methodology and/or techniques	Data set(s)/ source(s)	Target(s)	Findings
Gao (2014)	DBSCAN clustering algorithm, 3D kinematics, proper motions or radial velocities	High-precision data from old photographic plates of various large aperture reflectors	NGC 188 (OSC)	Membership
Maia et al. (2010)	Photometric membership derivation and Gaussian fittings	Near-IR and BV (RI) photometric data	NGC 1981 (OSC)	Membership, cluster centre, radial stellar density profile and structural parameters
Rangwal et al. (2019)	color-magnitude diagrams, isochrones fittings, proper motions, parallaxes	<i>Gaia</i> DR2	NGC 6067, NGC 2506 and IC 4651 (OSCs)	Membership, distance, radii, and age
Hendy and Tadross (2020)	Theoretical stellar isochrones and color-magnitude diagram	<i>Gaia</i> DR2	Kronberger 60 (OSC)	Age, distance modulus, and color excess
Balona et al. (2013b)	Isochrones fittings	<i>Kepler</i> Input Catalogue	NGC 6819 (OSC)	Age
Sandquist et al. (2016)	Analysis of asteroseismic data	<i>Kepler</i> data (asteroseismic data in the <i>Kepler</i> FOV)	NGC 6811 (OSC), eclipsing binary star system (KIC 9777062)	Stellar masses, radii and age
Brewer et al. (2016)	Radial-velocity measurements, three star systems' aggregate analysis of Mass and Radii (M – R plane)	Photometric and spectroscopic observations	Triple star system (WOSCS 24009, WOSCS 40007 and WOSCS 23009) and NGC 6819 (OSC)	Mass, radii and age

Bisht et al. (2021)	Solar metallicity isochrones fittings, parallax and distance modulus	Broad-band data from (APASS), GLIMPSE, VPHAS, VVV, WISE and photometric, astrometric data from eGDR3	Two binary star clusters: (NGC 5617 and Trumpler 22) and (NGC 3293 and NGC 3324), 18 stars in NGC 5617 and 44 stars located on Trumpler 22	Age, distance, Galactic orbits, luminosity function (LF) and mass function (MF)
Dias et al. (2018)	Photometric data, proper motion, parallax and objective theoretical isochrones fittings	Observations from (UBVR1 CCD) and <i>Gaia</i> DR2, UCAC5	Two new star clusters in location previously known as Dias 4	Photometric, stellar, kinematic membership, age, binary fraction, distance, metallicity, and reddening
Glatt et al. (2008)	Photometric analysis, three isochrone models: Padova, Teramo and Dartmouth, and ACS photometric system	Observations by the Hubble Space Telescope Advanced Camera for Surveys in the F555W and F814W filters	Star clusters: Lindsay 1, Kron 3, NGC 339, NGC 416, Lindsay 38, and NGC 419, in the Small Magellanic Cloud	Age and metallicity
Tadross (2011)	CMD and solar metallicity isochrones fitting	Sample of 145 JHK-2MASS observations, JHK Near-IR photometry, VizieR (WEBDA and DIAS catalogue) – 2MASS data extraction	120 NGC OSCs	Parameters for 120 OSCs: angular diameter, distance modulus, Galactic geometric distances, reddening and age
Krzesinski and Blokesz (2019)	Binogram method	<i>Kepler</i> space telescope	Pulsating WD (faint KIC 8626021) and eclipsing binaries (KIC 7975824 and bright KIC 9472174)	Identifying low-mass objects
Krzesinski et al. (2020b)	Binogram method, light curves from K2 mission, and Fourier transforms	<i>Kepler</i> photometric time series data of light curves	KIC 8626021, KIC 7975824 and KIC 9472174	Identifying Jupiter-mass exoplanets orbiting around binary star systems

Soydugan et al. (2020)	Analysis of light and radial velocity curves	Spectroscopic and photometric data	Double-lined binary stars located in the NGC 6819	Age, distance, mass and radius
Koroleva (1973)	Photometry, CMD and Artyukhina's stellar proper motion	Sverdlovsk photometric data and Washington data	NGC 6866 (OSC)	Membership, proper motion and radius
Joshi (2022)	Parallax, phase-folded diagrams, membership analysis, visual inspection, temporal analysis, CMD and isochrones fittings	Vizier, phase diagram analysis, higher frequency spectrum and eGDR3	Variable stars found in the NGC 6866 (OSC): ID 00016, ID 1077, ID 1088, ID 1292, ID 1421, ID 1583 and ID 1274	Membership probability, period, amplitude, frequency, shape of pulsation, age and distance
Tang et al. (2018)	Asteroseismic analysis and evolutionary models	<i>Kepler</i> light curve	NGC 6866 (OC): KIC 8263801 (red-giant star)	Mass, age, and radius
Yang et al. (2011)	Stellar population synthesis, stellar evolution model, Eggleton's stellar evolution code, ADIPLS pulsation code, and Monte Carlo method	Simulation	Simulated star clusters	Shape of light curves, distance, age, mass, radius, metallicity and $v_{\max}$, and Δv

2.4.3 Challenges and Uncertainties

Age estimations of star clusters are all we can achieve with current techniques and methodologies. Ascertaining the age of a cluster to the exact value (all the way down to the units) is not yet possible - as age estimations are typically rounded off to Myrs and Gyr. The difficulty becomes establishing at what point a set of stars began forming a star cluster. How many stars can be regarded as a cluster? Is it when they become bound to each other (i.e., loosely or gravitationally)? Does a cluster's inception commence before or after the star formation process? Are clusters born from rogue stars that end up at the same spot? Is a cluster's inception point when nebula-cloud dust starts forming stars? The dust itself is gravitationally bound, so do we use the age when the first stars were fully formed as the age of the cluster? If so, does that mean the time-period prior the birth of the "first-born" stars (maybe red dwarfs and red giants) is not considered when estimating the age of the cluster? Does this make using red giants or red dwarfs the best bet for determining a more constrained age estimation of a cluster? That point is hard to determine. Star cluster age and distance estimations can, for now, only be a range, not a specific value. Instruments, techniques, methods, and technologies that enable us to investigate parameters of star clusters advance with time, thus we can only report best findings with our current but soon-to-be outdated or improved tools.

2.4.4 Conclusion

NGC 6866 is an OSC located approximately 4,600 to 5,000 lightyears (or 1.41 to 1.53 kpc) from Earth, with a core radius of 5.22 ± 0.29 arcmin (Joshi, 2021). It is home to a number of variable stars, including δ Scuti stars, eclipsing binaries, and rotating variables. The age and distance of NGC 6866 have been estimated using a variety of methods, but the results are still somewhat uncertain: that is, there is 100-million-years age difference between estimations made by various studies. The age and distance estimation of star clusters involves a combination of factors, including when stars become gravitationally bound within a common space, the role of molecular clouds, and the use of certain star types like the earliest fully formed stars to narrow down the age range. However, pinpointing a specific age and distance is not yet possible, resulting in estimations typically presented as ranges and not specific values. The more research outputs regarding these parameters, the narrower the estimation range can be constrained. Such parameters are much sought after for their significant applicability when investigating cluster formation rate, galactic geometry, star formation, stellar and galactic disk evolution (Joshi, 2021), and cluster-dissolution time scale.

Chapter 3

Analysis and Results

This chapter discusses how astrometric and asteroseismic data can be used to identify binary star systems using a method called a binarogram. The chapter also outlines the methodologies used to address the research objectives. Here, results of the analysis are presented, along with detailed interpretations of the data.

3.1 Data

3.1.1 Astrometric Data

Astrometry is key for identifying members in loosely bound open clusters. It measures stellar positions over time, revealing stars that move together – a strong indication of cluster membership. This data includes positions (RA, Dec., distance) and motions (proper motion, transverse velocity, parallax) (*Gaia* collaboration et al., 2016).

3.1.2 Asteroseismic Data

Stars vibrate in unique ways, revealing their internal structure through the frequencies of these vibrations. Asteroseismic data, obtained by measuring these frequencies, allows us to probe a star’s interior, including mass, radius, density, chemical composition (Christensen-Dalsgaard, 2004), age (Basu and Chaplin, 2017), distance, and metallicity (Stello et al., 2010). This powerful tool has been applied to stars like the Sun and aids in understanding stellar evolution and even exoplanet detection (Christensen-Dalsgaard, 2004).

In this project, both *Kepler* and *Gaia* satellite data have been used. The *Gaia* mission observed stars with faint magnitudes and the *Kepler* mission observed stars with bright magnitudes. The selected columns in GDR2 were the distance (r) of the probable members from the cluster centre, distance modulus (σ), the RA, Dec., parallax, proper motion in right ascension (pmRA), proper motion in declination (pmDec.), effective temperature (T_{eff}) and luminosity (Lum). The stars with negative values of parallax were not considered in the sample because negative parallax

values give meaningless negative distance. The sample was filtered by eliminating all stars with no parallax, effective temperature and luminosity (Table C.1).

The KIC numbers of the probable members of the Cluster were determined. The RA and Dec. of stars in both *Kepler* and *Gaia* satellite data were used to obtain the KIC numbers of the stars from *Kepler* data source. The stars with no KIC numbers were removed from the sample. After filtering out all the stars that were not probable members of the NGC 6866 OSC, a sample of 119 probable member stars was left. Table 3.1 consists of 24 old δ Scuti stars. These stars are considered old because they were first categorized by Balona et al. (2013a). The selected columns are the same as the ones from Table C.1.

Table 3.1: 24 old δ Scuti stars found in the NGC 6866 open cluster. The columns are the KIC number; the distance (r) of the probable members from the cluster centre (arcminutes); distance modulus (σ); the right ascension (RA) and declination (Dec.) (J2000); parallax (milliarcseconds); proper motion in RA (pmRA) and proper motion in Dec. (pmDec.) (milliarcseconds per year); effective temperature (T_{eff}) (Kelvin) and luminosity (Lum) ($L_{\odot}$).

KIC	r (arcmin)	σ	RA (h:m:s)	Dec. (d:m:s)	Parallax (mas)	pmRA (mas/yr)	pmDec. (mas/yr)	T_{eff} (K)	Lum ($L_{\odot}$)
8197355	5.1949	10.5	20:03:43	+44:04:44	0.7866	-1.631	-5.862	7133.00	4.695
8197788	4.9011	10.6	20:04:11	+44:05:33	0.7479	-1.537	-5.764	7809.67	9.306
8264623	1.7885	10.6	20:03:60	+44:07:55	0.7520	-1.299	-5.690	7428.67	12.685
8264698	1.8132	10.8	20:04:04	+44:10:20	0.6884	-1.323	-5.780	7629.00	18.878
8264546	0.3376	10.8	20:03:55	+44:09:50	0.6824	-1.560	-5.528	7327.50	7.408
8264588	0.9951	10.8	20:03:58	+44:08:37	0.6815	-1.442	-5.776	7171.10	80.868
8264492	1.0600	10.9	20:03:52	+44:08:38	0.6708	-1.313	-5.638	7509.50	15.407
8264494	1.0099	10.7	20:03:52	+44:10:19	0.7201	-1.407	-5.844	7589.33	32.151
8264543	1.3808	10.8	20:03:55	+44:10:53	0.6875	-1.278	-5.661	7850.00	45.019
8196903	8.1270	10.9	20:03:17	+44:05:00	0.6676	-1.535	-5.046	7672.00	12.569
8196911	8.4345	10.8	20:03:18	+44:04:15	0.7053	-1.287	-5.719	7970.50	17.131
8197220	5.6461	10.8	20:03:35	+44:05:08	0.6791	-1.414	-5.810	7120.50	6.652
8264419	1.2710	10.6	20:03:48	+44:09:32	0.7520	5.042	-5.486	5330.00	1.012
8264404	1.4138	10.8	20:03:47	+44:09:26	0.6864	-1.158	-5.598	7726.50	22.088
8265377	8.8729	10.7	20:04:44	+44:08:58	0.7139	-1.443	-5.688	7518.75	8.139
8264949	4.2077	10.8	20:04:18	+44:09:52	0.7035	-1.393	-5.980	7605.67	22.286
8265022	5.1048	10.8	20:04:23	+44:10:31	0.6960	-1.456	-5.781	7927.33	43.764
8264824	3.1090	10.8	20:04:11	+44:10:35	0.6967	-1.414	-5.793	7522.75	6.775
8330778	4.5876	10.9	20:04:16	+44:12:05	0.6726	-1.254	-5.767	7270.50	7.215
8330712	4.1156	10.8	20:04:13	+44:12:05	0.7061	-1.343	-5.643	7195.00	6.969
8330790	5.0582	10.7	20:04:17	+44:12:44	0.7292	-0.063	-3.498	7821.67	8.337
8330910	7.7265	10.9	20:04:23	+44:15:22	0.6736	-1.166	-5.713	7181.67	7.126
8330740	8.8045	10.9	20:04:14	+44:17:37	0.6720	-1.441	-5.741	7338.75	9.851
8330102	8.6039	10.8	20:03:36	+44:17:22	0.7042	-1.435	-5.508	7145.75	6.453

3.2 Methods

This research project has adopted the multiple-methods approach, where both qualitative and quantitative methods are applied concurrently. Qualitative research methods are used to understand the underlying reasons for a phenomenon. They are often used to gather data that is difficult to quantify, such as literature reviews, and surveys, interviews. In this regard, OSCs and historic data regarding estimations of the age and distance have been studied. To estimate the age and distance of the NGC 6866, a quantitative research method has been employed. Quantitative research methods are used for collecting, measuring, and analyzing data. For this, *Gaia* and *Kepler* satellite data were collected, parameters were used to select and eliminate stars, identify δ Scuti stars, use the binarogram method identify binary star systems from δ Scuti stars and estimate the age and distance of the NGC 6866. Findings will be triangulated, that is to say, quantitative data acquired from data analysis are compared with findings in literature that guide and validate the most probable estimation of age and distance of NGC 6866 (Creswell and Creswell, 2017).

3.2.1 Identifying member stars of NGC 6866

The first objective is to use data from Kepler and Gaia to distinguish between member and non-member stars of the NGC 6866 OSC. This objective further incorporates constructing the H-R diagram and identifying Delta Scuti stars within the cluster using parallax and proper motion data.

3.2.1.1 *Gaia* Satellite Data

The VizieR website was used to download all *Gaia* satellite data for the stars in the area around the NGC 6866 OSC. All stars within a radius of 10 arcmin of the cluster centre were selected (9581 stars). Among the selected stars, narrowing down the set to seclude members from non-members of the NGC 6866 required using particular parameters for non-member elimination. Stars with no effective temperature and luminosity were not considered in the sample of stars, because the H-R diagrams cannot be constructed with stars with no effective temperature and luminosity. Stars with no or negative values of parallax were also eliminated, because negative values of parallax are negligible. After manually eliminating 8,859 stars (due to limited coding skills), 722 stars were left. An H-R diagram was then constructed to determine their locations on the cluster, as shown on Figure 3.1, top panel. Stars are color-coded in H-R diagrams based on their surface temperature. Hotter stars emit more blue light (left-most stars), while cooler stars emit more red light (right-most stars). This means that the hottest stars on the H-R diagram are blue, while the coolest stars are red.

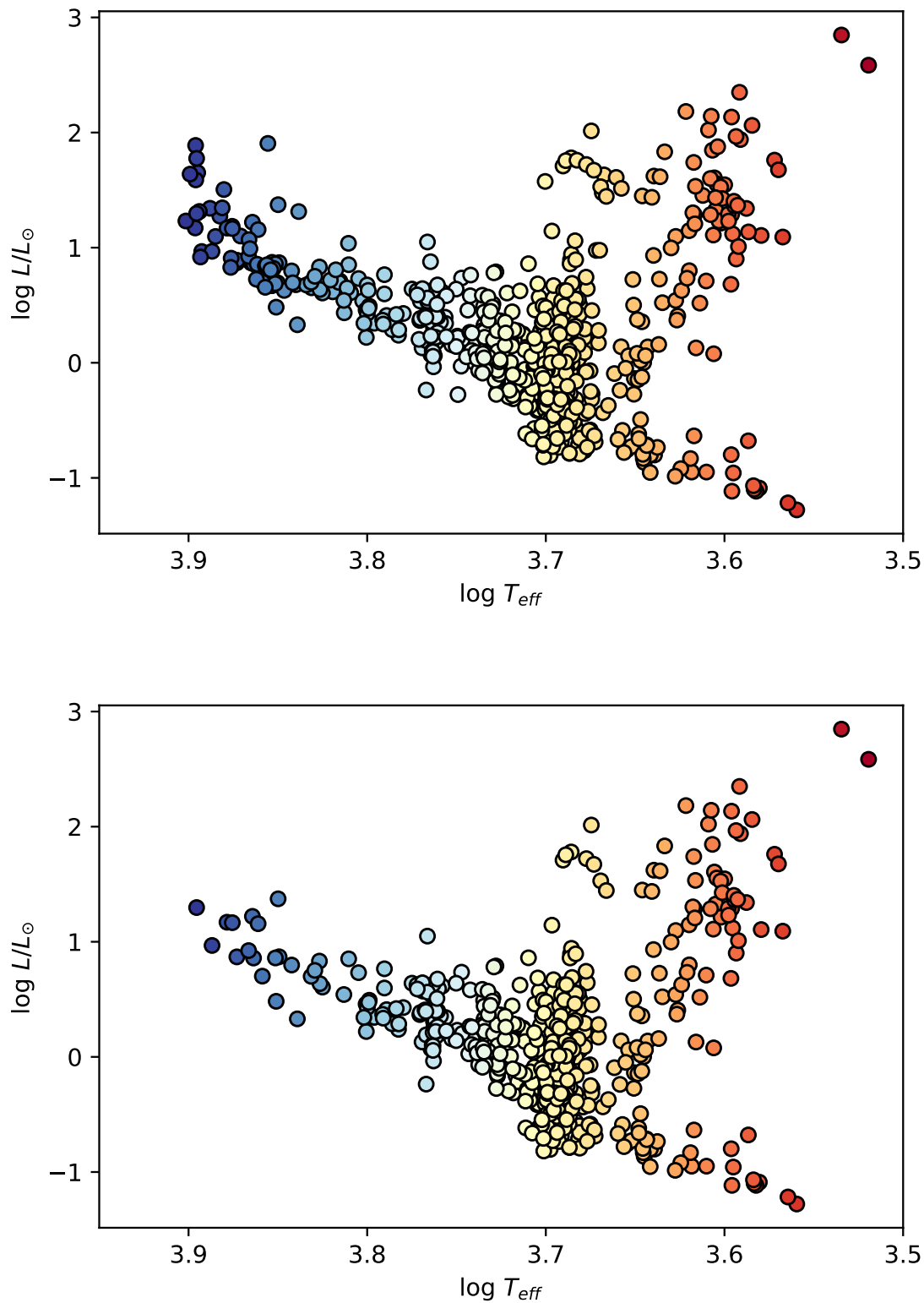


Figure 3.1: Top: The H-R diagram constructed with 722 stars. Bottom: The H-R diagram constructed with 584 stars. The continuous and distinctive diagonal band, extends from small, low-mass, cool, and dim stars (positioned in the lower right corner) to large, high-mass, hot, and brilliant stars (located in the upper left corner).

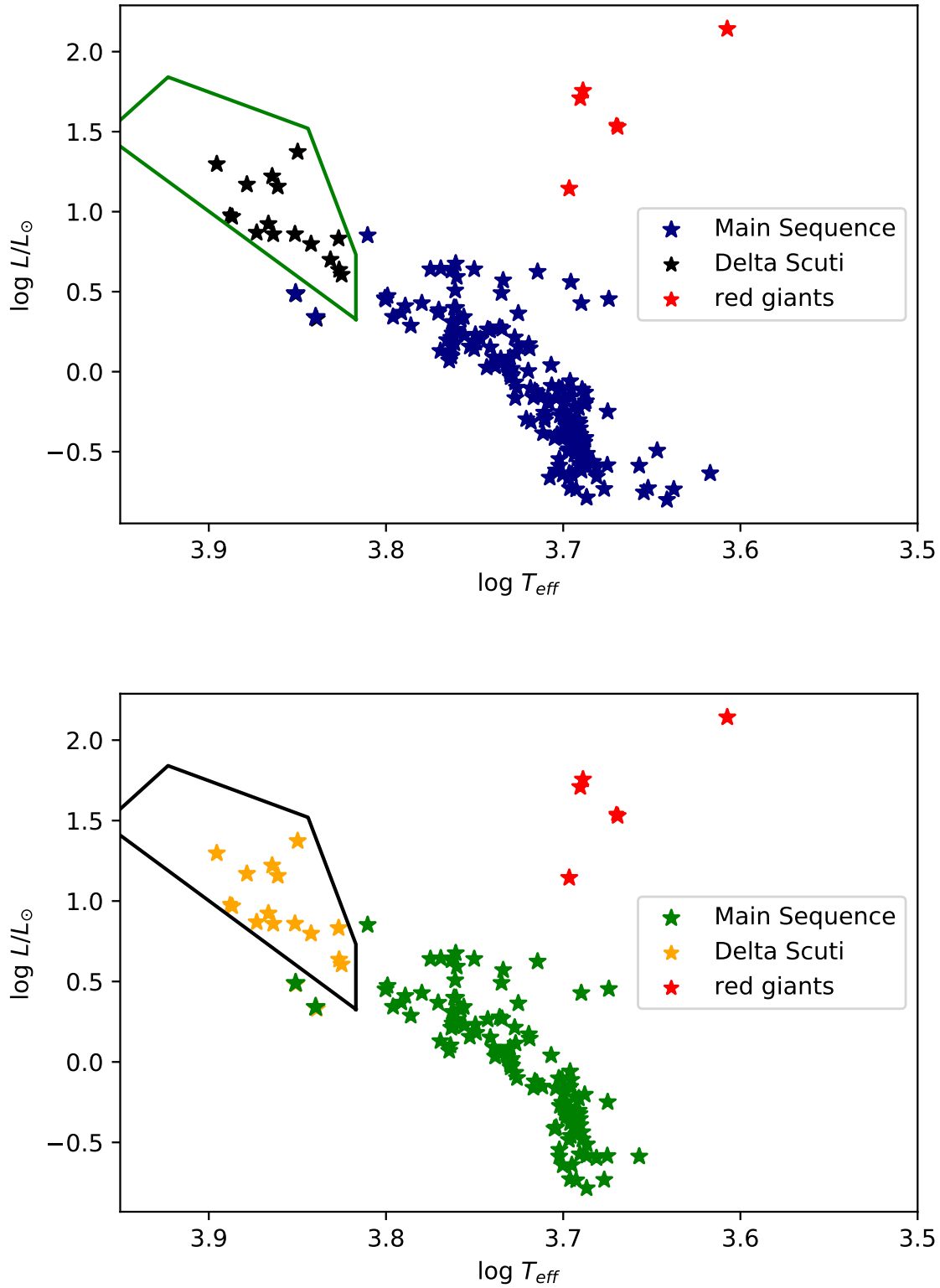


Figure 3.2: Top: The H-R diagram constructed with 192 stars. Bottom: The H-R diagram constructed with 132 stars. The trapezoidal regions (denoted by green and black solid lines) on the H-R diagrams show the general location of *Kepler* δ Scuti stars. The black and orange stars represent new δ Scuti stars, the blue and green stars represent the main sequence stars and the red stars represent the red giants.

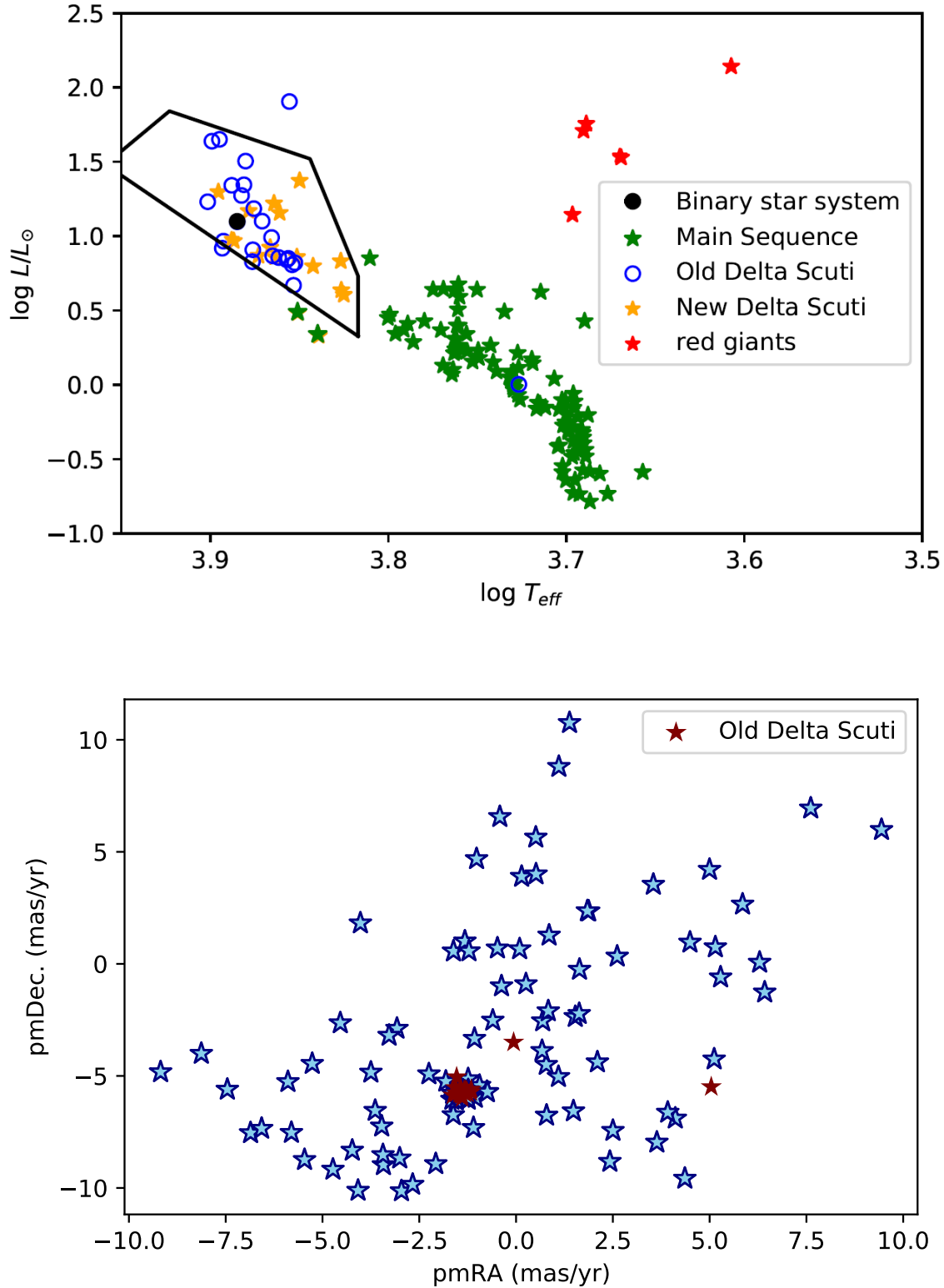


Figure 3.3: Top: The H-R diagram for 119 stars and 24 old δ Scuti stars. Blue open circles represent old δ Scuti stars and a black dot represents a binary star system KIC 8196903. Bottom: Proper motions in RA and Dec. for the new probable members and old δ Scuti stars. Blue stars represent the new probable members and maroon stars represent old δ Scuti stars.

3.2.1.2 *Kepler* Satellite Data

Kepler data extracted by Balona et al. (2013a) were used to eliminate the known probable member stars. If the RA and Dec. of the sample of stars in *Kepler* and *Gaia* are the same, it means that the stars are known. After eliminating the known probable member stars with reference to *Kepler*, a set of 584 sample stars was left. The second H–R diagram was constructed with 584 stars (Figure 3.1, bottom panel). Stars were also eliminated using the following three steps:

Step 1: Distance Modulus

To further narrow down the list of most probable members of the cluster, the distance modulus was calculated. Most probable members of the NGC 6866 OSC were determined by calculating the distance moduli (from parallax) of the selected stars. According to Balona et al. (2013a), a star is assumed to be a member of the NGC 6866 OSC if its distance modulus is in the range $10.0 < \sigma < 11.0$ mag. The following two formulas were used to calculate the distance modulus of each star in the sample of 584 stars:

$$\sigma = m - M = 5 \log(d) - 5 \quad (3.1)$$

$$\sigma = m - M = 5 \log\left(\frac{r}{10pc}\right) \quad (3.2)$$

In this formula, m is the apparent magnitude of the object, M is the absolute magnitude of the object, and $d = r$ is the distance to the object in parsecs.

All stars with distance moduli less than or equal to 10.0 mag. and greater than or equal to 11.0 mag. were eliminated from the sample of stars. From 584 stars, 192 stars were left in the sample, meaning that 392 stars were eliminated. The third H–R diagram was constructed to determine the locations of 192 stars on the cluster (Figure 3.2, top panel). KIC numbers for all 192 probable members of the cluster were determined.

Step 2: *Kepler* Archive

The MAST *Kepler* archive website was used to determine the KIC numbers of 192 stars in *Gaia*. To determine the KIC numbers of the stars, the RA and Dec. of the stars must be known. The KIC numbers were determined by entering the RA and Dec. of the stars in *Gaia*. Sixty stars out of 192 stars have no KIC numbers and were not considered in the sample of stars. The fourth H–R diagram and the proper motion in RA vs proper motion in Dec. graph were then constructed with 132 probable member stars of the NGC 6866 OSC (Figure 3.2, bottom panel).

Step 3: Proper Motion as an Additional Parameter

Proper motion was used as an additional parameter to identify the new members of the NGC 6866 open cluster. The mean proper motion in RA and declination were calculated with their standard deviations. The average proper motion of the cluster was then calculated with its standard deviation. The stars which deviated by more than three standard deviations were eliminated from the sample of the stars (Joshi et al., 2012). From 132 stars in the sample, only 13 stars were not considered. The H–R diagram was then constructed with 119 new members of the NGC 6866

OSC, including 24 δ Scuti stars determined by Balona et al. (2013a), considered old δ Scuti stars (Figure 3.3, top panel). The graph of proper motion was plotted in RA vs proper motion in Dec. including old δ Scuti stars (Figure 3.3, bottom panel). With this graph, the radius of the cluster can be estimated.

3.2.2 Characterizing binary δ Scuti stars in NGC 6866

The second objective focuses on using a binarogram technique to identify binary systems among the confirmed δ Scuti member stars.

3.2.2.1 Finding δ Scuti stars

δ Scuti stars were then selected among the sample of stars on the main-sequence of the NGC 6866 OSC. The trapezoidal regions on the H-R diagrams show the general location of *Kepler* δ Scuti stars (Balona and Dziembowski, 2011). All stars outside these regions are not considered as δ Scuti stars. Fourteen newly categorized δ Scuti stars were found on the trapezoidal region (Figure 3.2). These stars are young and are expected to be positioned on the main-sequence of the cluster.

3.2.2.2 Retrieving *Kepler* photometric data

Kepler data is organized into LC and SC segments, each with its associated pixel type (target, background, or collateral). To adhere to astronomical data standards, *Kepler* data is converted into the Flexible Image Transport System (FITS) format. A variety of processed data products are archived for public access and download, including: light curves derived from photometric analysis and corrected using Pre-search Data Conditioning (PDC), full-frame images generated from all 84 detector array channels, and target pixel images, which are individual pixel masks collected for each observation during a quarter and calibrated using Calibration CAL.

In *Kepler* data, light curve files are created for each target using a simple aperture photometry technique. These light curve files contain time-series data. Simultaneously, *Kepler* can observe over 160,000 LC targets and up to 512 SC targets. Data collected by the *Kepler* photometer are recorded on orbit, transmitted to Earth, archived, processed, and calibrated through the *Kepler* Science Pipeline. The pipeline primarily focuses on scientifically relevant data sets within the optical, ultraviolet, and near-infrared portions of the electromagnetic spectrum. The archived data includes raw and calibrated pixel values, background pixels, calibrated and corrected light curves, and associated ancillary engineering data.

To retrieve *Kepler* photometric data (light curves), the KIC numbers of the stars must be known. All the stars with no KIC numbers were excluded in the sample of the stars. *Kepler* photometric data for 38 δ Scuti stars were retrieved using *Kepler* public archive which is available on the MAST website. The retrieved data belongs to the LC sampling rate and has been tabulated within FITS files. To access them, a special reader was needed. A simpler way to get the data

was to use a script. A Fortran program was used to extract the required data from the FITS files, in to an easy and usable five columns: the barycentric Julian Date relative to JD 2455495.0, the raw count, the error in the raw count, the corrected count, and the error in the corrected count. Figure 3.4, top panel, shows an example of *Kepler* light curve for the identified binary star system (KIC 8196903).

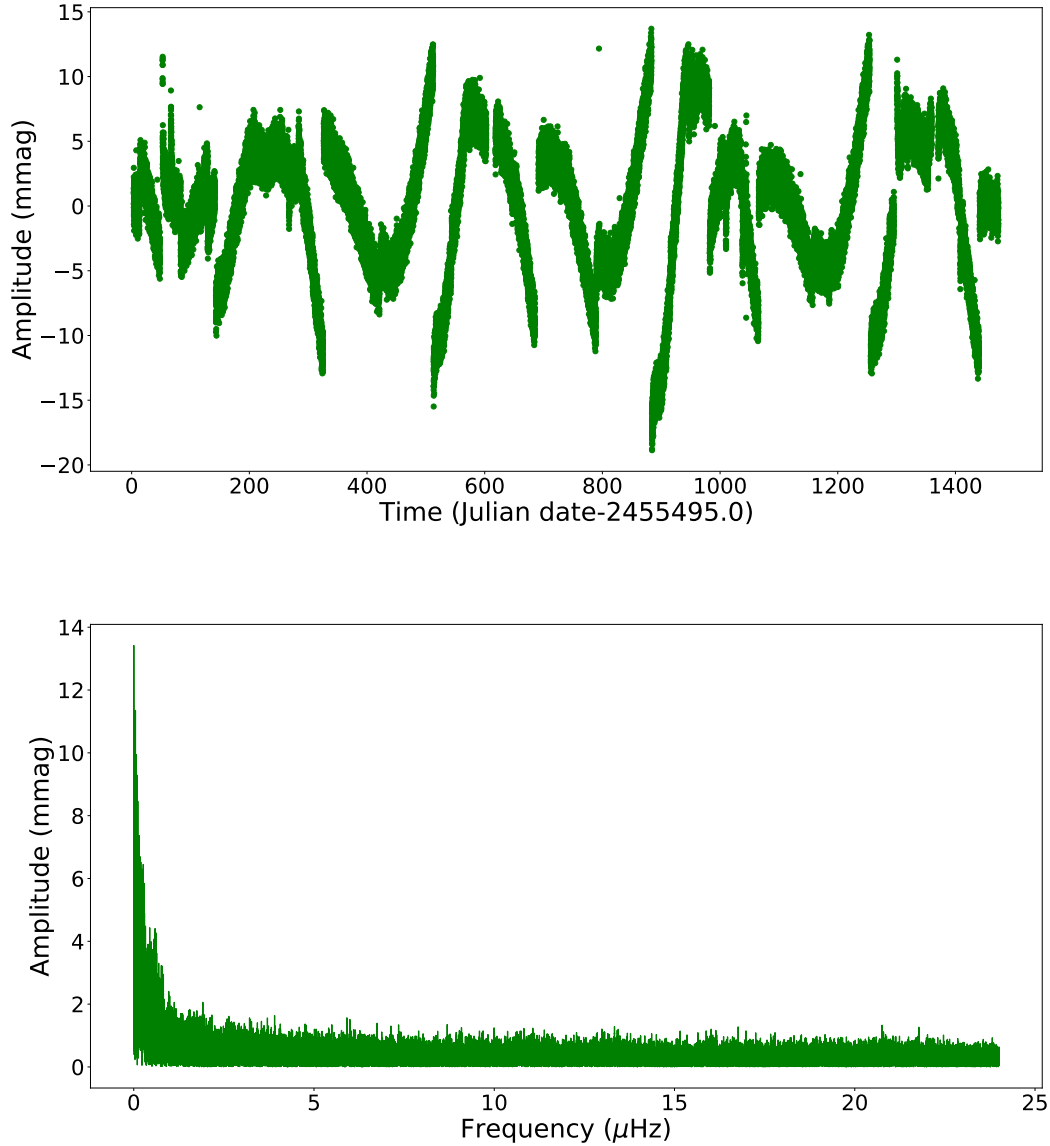


Figure 3.4: KIC 8196903. Top panel: light curves; bottom panel: periodogram

To find the pulsation frequencies of a pulsating star, a Fortran program was used to compute the periodogram of the star. The periodogram is a graph that shows the power of the star's light at different frequencies. PDC data was used to create the periodogram. PDC rectifies systematic errors introduced by the telescope and spacecraft, including pointing drift, focus variations, and thermal fluctuations. The periodogram was calculated from 0 to 24 cycles per day (c/d). The calculation of the periodogram terminated upon reaching either 1,000 frequencies or a specific

probability threshold. Figure 3.4, bottom panel, depicts a periodogram for KIC 8196903, and Figure 3.5 illustrates a binarogram for the same star. The pulsation frequencies of the star system are $f_1 = 5.995$, $f_2 = 5.912$, and $f_3 = 20.776$ c/d.

3.2.3 Estimating the Age and Distance of NGC 6866

The last objective focuses on using the Binarogram method to determine binary star systems, calculate orbital parameters and estimate the age of the NGC 6866 OSC. Additionally, it incorporates using stellar parallax to determine the distance of the cluster.

3.2.3.1 Binarogram Method

The binarogram method was used to identify the binary star systems in the cluster. The mathematical operation of a binarogram is provided below (see Equation 3.3 – 3.9). A star's distance (z) varies depending on the frame of reference as taken into account in the equation below:

$$z = a \sin i \frac{(1 - e^2) \sin(v + \omega)}{1 + e \cos v} \quad (3.3)$$

where $a \sin i$ represents projected semi-major axis, e represents orbit eccentricity, v represents true anomaly and ω represents periastron angle. Time-delay ($\tau = z/c$) for the circular orbit is,

$$\tau = \frac{a \sin i}{c} \sin(\Omega t + \omega) = \alpha_1 \sin \Omega t + \alpha_2 \cos \Omega t \quad (3.4)$$

where $\Omega = 2\pi/P_{orb}$ is the orbital angular frequency, P_{orb} is the orbital period and α_1 and α_2 are coefficients given as,

$$\alpha_1 = \frac{a \sin i}{c} \cos \omega, \alpha_2 = \frac{a \sin i}{c} \sin \omega \quad (3.5)$$

The brightness variation y of a multiperiodic pulsating star at a given time t can be represented by a truncated Fourier series as follows:

$$y = a_1 + \sum_{j=1}^q [a_{2j} \sin \omega_j(t + \tau) + a_{2j+1} \cos \omega_j(t + \tau)] \quad (3.6)$$

where $\omega_j = 2\pi f_j$. With the assumption that the pulsation frequencies, f_j and orbital period, P_{orb} are predetermined, this means that y becomes a function of the two coefficients, $y = y(\alpha_1, \alpha_2)$. Further assuming that estimates of α_1 and α_2 are already known, the adjustments to these estimates are,

$$\Delta y = \frac{\partial y}{\partial \alpha_1} \Delta \alpha_1 + \frac{\partial y}{\partial \alpha_2} \Delta \alpha_2 \quad (3.7)$$

where Δy represents the discrepancy between the computed and observed brightness variations. This expression employs the linear least-squares approach. Since this is a linear least-squares type of expression, the corrections $\Delta\alpha_1$ and $\Delta\alpha_2$ can be solved. The partial derivatives are expressed as

$$\frac{\partial y}{\partial \alpha_1} = u \sin k\Omega t, \frac{\partial y}{\partial \alpha_2} = u \cos k\Omega t' \quad (3.8)$$

and

$$\frac{\partial y}{\partial \tau} = u = \sum_{j=1}^q \omega_j [a_{2j} \cos \omega_j(t + \tau) - a_{2j+1} \sin \omega_j(t + \tau)]. \quad (3.9)$$

To solve the problem above, a periodic time delay with frequency f_{orb} is identified, and a value for f_{orb} is selected to use the least squares. The computation gives α_1 and α_2 and the values of $a \sin i$ are obtained. The process can be iteratively repeated by selecting different values of f_{orb} until the corrections become sufficiently small. If there is variation in time delay for that frequency in the data, the function f_{orb} as a function of $a \sin i$ will exhibit peaks. This is the essence of the binarogram method.

A Fortran program that computes the binarogram of pulsating stars was utilized for this research project. Inputs and assumptions were asserted as follows: first, critical pulsation frequencies have to be identified, assuming that the binary star system at least contains one pulsating star. Frequencies were computed with the periodogram program. To maximize the signal-to-noise ratio, it is vital to utilize highest amplitude pulsation frequencies that are isolated. Isolated frequencies were selected by ensuring that all neighboring peaks are farther than 0.1 c/d, having amplitudes above 0.1 of the peak amplitude. Selected frequencies ranged from 5 to 100 c/d. Orbital frequency step was set to 0.001 c/d, ranging from 0 to 0.10 c/d.

To run a binarogram Fortran program, a time-delay configuration file (tdauto.conf) had to be created. This file contains the content shown in Table 3.2.

Table 3.2: Time-delay configuration for multiple stars.

TDAUTO CONFIGURATION	
5.0 100.0	<FRANGE> Frequency range
0.10	<FRAMP> Fraction of maximum amplitude to use
0.05	<DELFREQ> Minimum frequency difference
5	<NUMFREQ> Number of frequencies to use
3	<DVARNO> Column number for data
0.0010000	<EPSILON> Threshold correction
0.00 0.20 0.0001	<F1F2DF> Starting, stopping frequencies and increment

The Fortran binarogram program (tdauto.f) was configured to calculate the binarograms of the pulsating stars automatically. The program works without the intervention of the user, hence the

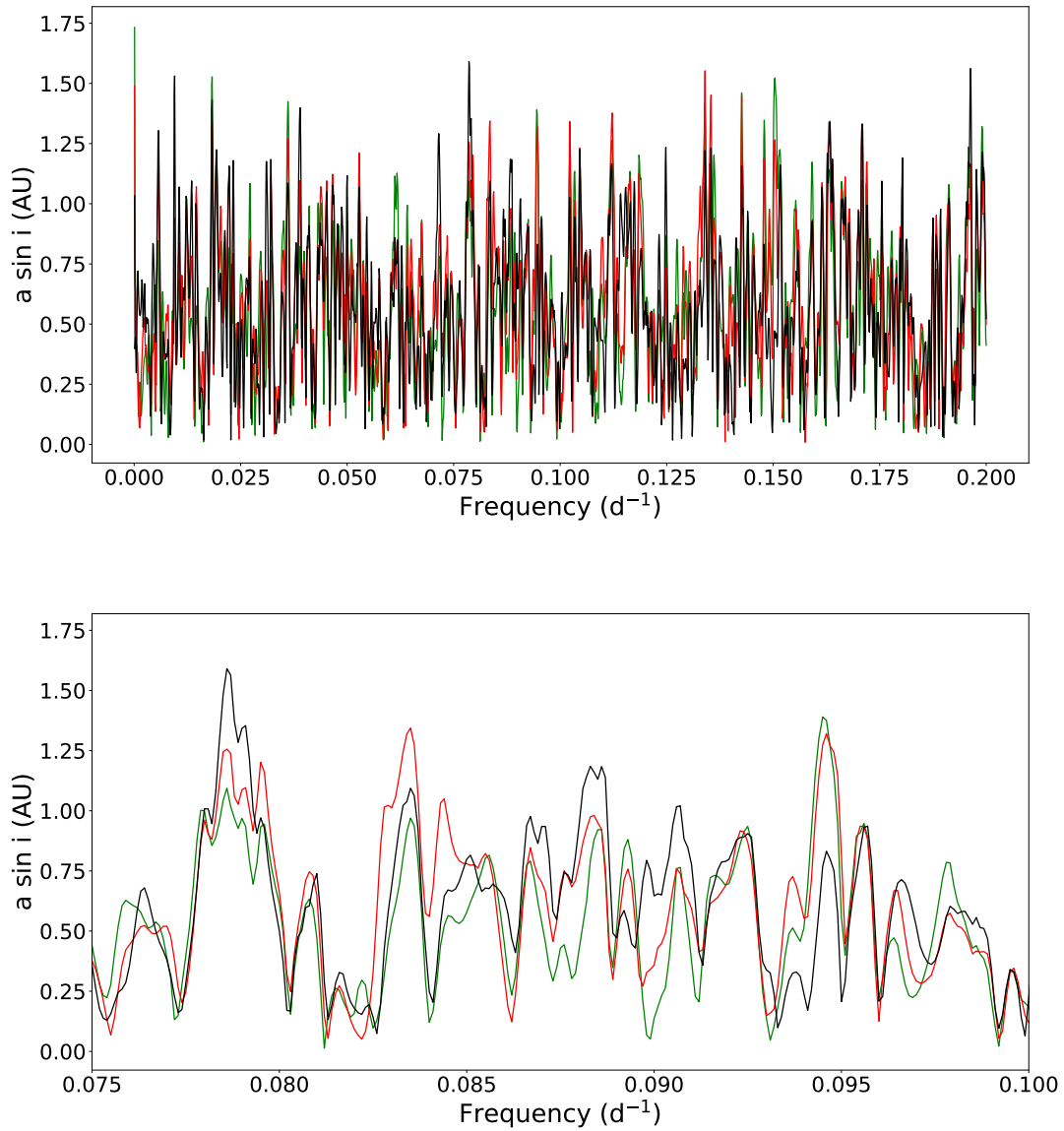


Figure 3.5: Top: Binarogram of KIC 8196903 showing peak value at orbital frequency of 0.9926000 ± 0.0000700 c/d for three pulsation frequencies. Bottom panel: a zoomed-in binarogram figure of KIC 8196903.

name (tdauto.f). Initially, the program was affected by pulsation frequencies that were close to one another. This meant that a peak does not indicate a binary by default; it may be due to a neighboring pulsation interference.

The possibility of the program to read peaks that occur as a result of interference was countered by setting the program to calculate up to three binarograms for each star. Each of the three binarograms use a set of different pulsation frequencies. The peak will not be seen in all three binarograms, if it is due to the interference. But if the peak is real due to the binary motion, it will be visible in all three binarograms.

The first row, “<FRANGE>”, in the Table 3.2 indicates the starting and ending pulsation fre-

quencies in cycles per day (c/d), that were used in the analysis of the binarograms. The range 5 to 100 c/d was found reasonable. Anything less or above the range was not considered.

It is important to use pulsation frequencies with the highest amplitudes. The second row, “<FRAMP>”, gives the fraction of the highest amplitude ($A_{\max}$) that was used throughout. All the frequencies with amplitudes less than $\text{FRAMP} \times A_{\max}$ were not considered in the time-day analysis.

As mentioned above, it is not advisable to use pulsation frequencies that are very close to one another, because the time-delay signal will be interrupted. The third row, “<DELFREQ>”, denotes a minimum frequency difference used. The peaks within a $\text{<DELFREQ>} = 0.05$ were ignored.

The program picks only five frequencies in the desired range. If there is only one pulsation frequency, it is your output. The maximum number of pulsation frequencies to be used is given in the fourth row, “<NUMFREQ>”.

From the produced *Kepler* photometric data, there are .dat files that contain four columns. The fifth row, “<DVARNO>”, denotes the column number from the .dat files. To analyze the time-delay, column 3 must always be used.

The threshold value for convergence to obtain α_1 and α_2 in equation (3.2), is given in the sixth row “<EPSILON>” as 0.001.

The starting, stopping orbital frequencies and the frequency step are given in the seventh row, “<F1F2DF>”. A frequency step of 0.0001 was used.

Running the program

The name “tdauto.conf” must always be present. A list of the full path names to the data was created before running the program.

Creating a list file

If, for instance, the files are kept in separate folders with names like “008196903” and are all placed in the directory “/home/Desktop/datafiles”, then the following command will generate a list file:

```
ls/home/Desktop/datafiles> list
```

The entries in the “list” file will then look like this:

```
/home/Desktop/datafiles/008196903
/home/Desktop/datafiles/008196840
/home/Desktop/datafiles/008196911
and so on.
```

Each folder contains files with extensions: .001, .dat and .frq. For example, the folder “008196903” contains three files:

```
008196903.001
008196903.dat
008196903.frq
```

The program searches for .dat and .frq files in this directory “/home/Desktop/datafiles/008196903”. After creating the list file, the program could be run using the command line ./tdauto list.

The program then worked automatically. The likelihood that a star system is a binary star system increases with the number of binarograms produced. From the stars in the sample, three binarograms were generated from only one star. A conclusion was drawn that this star is a binary star system because its pulsation frequencies gave rise to the same peak, as shown in the bottom panel of Figure 3.5. All pulsation frequencies for a true binary should result in the same peak in the binarogram.

The results for each star were written to a separate folder in the location where the program was started. Every folder was created when stars were processed. Some folders were empty, while some had one or more binarograms. The folder “008196903” contained files like 008196903.td01, 008196903.fr01, 008196903.td02, 008196903.fr02, 008196903.td03, and 008196903.fr03, where files with the extensions.td, are binarograms and files with the extensions.fr contain the pulsation frequencies and amplitudes used in constructing the corresponding binarograms.

3.2.3.2 Calculation of the orbital parameters

To determine the orbital parameters for a binary star system, a time-delay configuration file (td.conf) must be created. This file contains the content shown in Table 3.3.

Table 3.3: Time-delay configuration for a single star.

TD CONFIGURATION	
008196903.dat	<DATAFILE> Time series data file
3	<DVARNO> Column number for data
frq	<FREQFILE> Pulsation frequency file
0.00010000	<EPSILON> Threshold correction
0.00 1.00 0.0001	<F1F2DF> Starting, stopping frequencies and increment

The first row, “<DATAFILE>”, is the produced *Kepler* data file as a .dat file. This file contains four columns: first, the time (days), second, photometry, the third column was taken from the.in file that was produced (corrected with *Kepler*’s own correction), and the fourth column was taken from the.rw file which is the uncorrected simple aperture photometry.

From the four columns, the third column is always used. The numeral 3 in the second row in the table above indicates the column number from.dat file.

The third row, “<FREQFILE>”, gives the name of the file (frq) that lists all the pulsation

frequencies to use. This file consists of only one column.

The fourth row, “<EPSILON>” and the fifth row, “<F1F2DF>”, have the same explanation as those in Table 3.2.

Running the program

The file that contained the pulsation frequencies was created before running the program, as it was needed in the row “<FREQFILE>”. The frequencies in the file were produced when the binarograms were determined.

The first thing that was asked when running the program, was to refine these frequencies. The first column was the frequencies, the second was the frequency steps used to refine the frequencies, and the third was asini. The important thing was to reduce the values in the third column to their lowest possible values. The frequencies were saved in “td.frq” file during the frequency refinement process. The stage where asini was no longer changing was reached and then “y” was typed to discontinue the running process of the program.

Thereafter, the program operated independently, outputting the orbital frequency and the asini value. It took between 20 and 30 minutes to complete the binarogram.

The peak frequency and the period were produced at the end. The program prompted for “Peak frequency (f1,f2)?”, for which the two peak frequencies were entered. Thereafter, the orbital parameters were produced.

3.2.3.3 Age estimation

Stellar isochrones are essential tools in astrophysics for estimating the age of star clusters and studying stellar evolution. An isochrone connects stars of the same age but different masses on the HR diagram, showing their expected luminosity and temperature at a given time. Isochrones are generated from theoretical models of stellar evolution, which incorporate various physical processes and input parameters (Swastik et al., 2024). Key models include Padova, Geneva, BaSTI, Dartmouth Stellar Evolution Program (DSEP), and PAdova and tRieste Stellar Evolutionary Code (PARSEC), each offering unique features. The Padova models incorporate mass loss, overshooting, and varying initial chemical compositions, covering a wide range of stellar masses and ages. Geneva models focus on the effects of rotation, particularly useful for studying young, massive stars. BaSTI offers comprehensive grids for different metallicities, including diffusion and rotation effects. The Dartmouth Stellar Evolution Program (DSEP) provides isochrones for low and intermediate-mass stars, considering alpha-element enhancement and helium abundance. PARSEC, an update from the Padova group, includes detailed models for low-mass and very metal-poor stars, covering a wide range of stellar ages (Bressan et al., 2012).

Isochrones vary significantly with age and stellar mass. Young isochrones are steep, extending from the main sequence to pre-main-sequence phases, while older isochrones show a noticeable turn-off point and a pronounced red giant branch. High mass stars evolve rapidly, resulting in shorter isochrones, while intermediate and low mass stars evolve more slowly, with well-defined

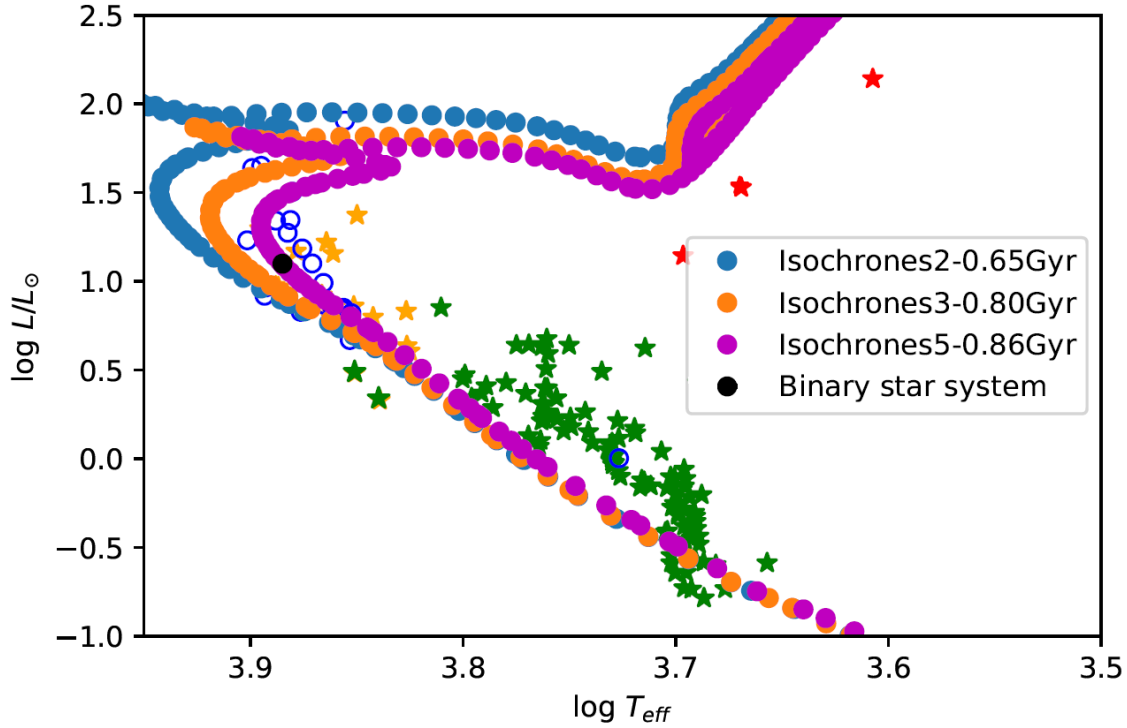


Figure 3.6: The H-R diagram with three over-plotted isochrones of different ages. Blue, orange and purple dotted lines represent isochrones with ages 0.65 Gyr, 0.80 Gyr and 0.86 Gyr, respectively. A black dot represents a binary star system KIC 8196903. Blue open circles represent old δ Scuti stars, orange stars represent new δ Scuti stars and green stars are the main sequence stars.

main sequences and turn-off points. Variations in metallicity also affect isochrones, necessitating consideration of chemical composition when estimating ages. These variations highlight the complexity and precision involved in using isochrones for understanding stellar populations and their evolutionary histories (Swastik et al., 2024).

The age was determined by leveraging the identified binary star system within the cluster, KIC 8196903, using isochrones fitting of solar metallicity 0.021 (as suggested by Vagnozzi (2019)). Parameters of this binary star system were compared to a set of 19 different isochrones in PARSEC v1.2S + COLIBRI PR16 (Bressan et al., 2012; Marigo et al., 2017), mass-loss = 0.0 and with different metallicities. The age of NGC 6866 was estimated to be 0.86 Gyr. Figure 3.6 illustrates how these various isochrones were plotted and compared to the properties of the binary system to find the best fit, ultimately revealing the age of the cluster.

Appendix B shows a PYTHON code that was written to eliminate non-member stars of the NGC 6866 OSC and also that was used to plot the H-R diagrams.

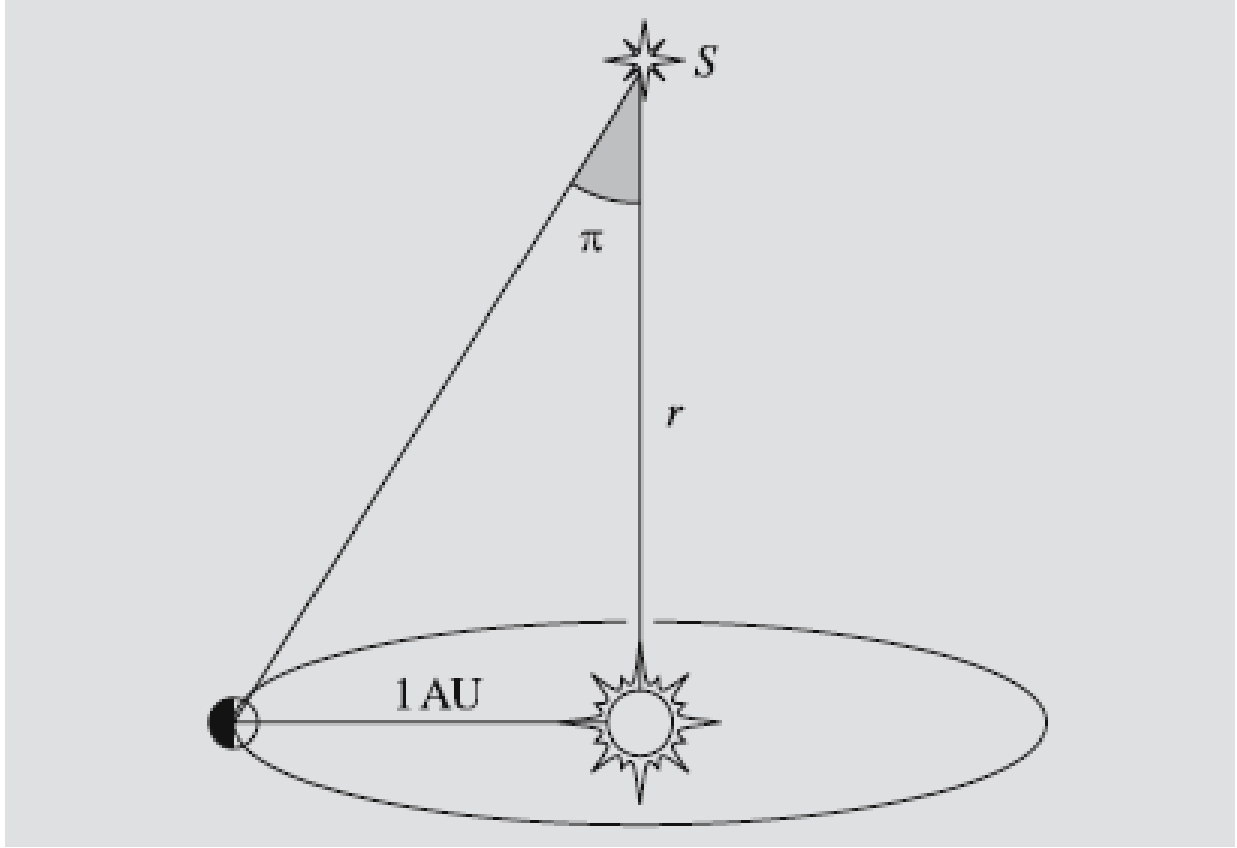


Figure 3.7: Stellar parallax diagram (Karttunen et al., 2007). 1 Astronomical Unit (AU) is the distance from Earth to the Sun; r is the distance from the center of the Earth's orbit to the star S ; π is the parallax measured in arcseconds.

3.2.3.4 Distance calculation

The angle or semi-angle between the two lines of sight from the observer to the object is used to measure parallax. The difference in a star's direction as observed from two distant sites is known as a stellar parallax. These two distinct locations are located on the Earth's orbit and were produced by the planet being in two different orbital positions, as will be explained below. Using the formula:

$$r = \frac{1}{\tan(\pi)} \approx \frac{1}{\pi} \quad (3.10)$$

the parallax of a celestial body can be used to determine an approximation of distance. π is the observed parallax angle measured in arcseconds and r is the actual distance measured in parsecs (Figure 3.7). The distance at which an object exhibits a 1-arcsecond stellar parallax is known as a parsec. In other words, a parsec is the distance at which an angular size of one arcsecond will be present for a disk with a diameter of one astronomical unit.

The last objective was to compute the cluster's distance. The parallax of the binary star system was used to compute the approximate distance of the cluster using the stellar-distance formula.

Using equation 3.10, the distance of NGC 6866 open cluster was determined to be 1497.90 parsecs.

3.3 Results

In this section, the results obtained to achieve the objectives are presented. *Kepler* and *Gaia* satellite data were both used to identify 14 new δ Scuti stars, including one binary star system.

3.3.1 Identifying potential member stars

Figure 3.1 top panel, is the first H-R diagram that was constructed. It includes members and non-members of the NGC 6866 OSC. This diagram was constructed before applying several techniques to eliminate the non-member stars.

To accomplish the identification of the potential members of the cluster, data was harnessed from both the *Kepler* and *Gaia* satellites. *Kepler* focused on brighter magnitude stars, while *Gaia* extended its observations to fainter magnitudes. By cross-referencing the two datasets, known stars catalogued by *Kepler* were eliminated and new potential members that *Kepler* had missed were identified. This effort culminated in the identification of 119 new potential members, underscoring the extensive overlap between the observations made by *Kepler* and *Gaia*, with a relatively smaller fraction of stars being exclusive to *Gaia*'s dataset.

3.3.2 Classifying stars using Hertzsprung–Russell diagrams

The construction of the H-R diagram played a pivotal role in the analysis, shedding light on the evolutionary stages of the observed stars. Notably, Figure 3.2 top panel, revealed that the majority of stars in the NGC 6866 OSC resided within the main-sequence region, with only five situated in the region of red giants. This distribution is a reflection of the cluster's intermediate age, further emphasizing the significance of the H-R diagrams in understanding the characteristics and properties of stars.

Understanding a star's position on the H–R diagram is of paramount importance, as it serves as a gateway to determining a multitude of crucial characteristics, including temperature, color, brightness, mass, spectral class, radius, and the nature of nuclear reactions transpiring within stellar cores. When a new star comes under scrutiny, one of the initial steps is to ascertain its placement on an H–R diagram, as this knowledge greatly facilitates the subsequent determination of its physical attributes.

3.3.3 Delta Scuti stars

Relying solely on proper motion for distinguishing cluster members from non-members yields unsatisfactory results. However, the integration of both proper motion and distance modulus

techniques has shown a significant enhancement in results. Moreover, further improvements in discriminating between members and non-members can be achieved by employing advanced techniques like the Bayesian analysis. The calculation of the cluster's mean proper motion was based on a sample of 132 stars, resulting in an estimate of 7.01 ± 0.01 mas/yr, with a standard deviation (σ) = 3.75 mag. Specifically, the mean pmRA and pmDec. were determined to be -1.21 mas/yr and -4.58 mas/yr, accompanied by standard deviations of 4.01 mag and 4.97 mag. The mean pmRA value is higher than the pmDec. value. The graph of proper motion in RA vs proper motion in Dec. including old δ Scuti stars (Figure 3.3 bottom panel) was plotted. This graph is more relevant for this kind of study because it can be used to estimate the radius of the cluster. Fourteen new δ Scuti stars were found in the trapezoidal region, indicated by a green box in Figure 3.2 top panel. Table 3.4 provides more details on these 14 stars.

Table 3.4: 14 new δ Scuti stars found in the NGC 6866 open cluster

KIC	pmRA (mas/yr)	pmRA error	pmDec. (mas/yr)	pmDec. error	pm (mas/yr)	pm error	$\log(T_{\text{eff}})$ (K)	$\log(\text{Lum})$ ($L_{\odot}$)
8264706	-1.450	0.029	-5.710	0.034	5.891	0.034	3.825036441	0.6060477285
8264708	-1.346	0.025	-5.736	0.025	5.892	0.025	3.851375164	0.8595686834
8264581	-1.450	0.035	-5.730	0.028	5.911	0.028	3.826730934	0.8320927985
8264304	-1.237	0.031	-5.626	0.028	5.760	0.028	3.826442177	0.6364245725
8264326	-0.740	0.038	-5.715	0.038	5.760	0.040	3.878478709	1.1697544950
8330286	-1.324	0.025	-5.834	0.028	5.982	0.028	3.872960575	0.8687829471
8264526	-1.640	0.044	-5.784	0.049	6.010	0.050	3.861056245	1.1569117180
8263940	2.502	0.024	-7.441	0.024	7.850	0.024	3.863620220	0.8589713044
8329790	-1.420	0.029	-5.725	0.025	5.898	0.025	3.842297134	0.7980402633
8329888	-0.481	0.037	0.700	0.037	0.850	0.040	3.895486636	1.2972429840
8265204	1.482	0.042	-6.569	0.049	6.730	0.050	3.810366954	0.8517986977
8265244	-1.366	0.027	-5.707	0.029	5.868	0.029	3.866375962	0.9232540440
8264767	-1.268	0.029	-5.890	0.028	6.025	0.028	3.886847602	0.9680825620
8396259	-2.252	0.041	-4.916	0.042	5.410	0.040	3.849849196	1.3731981060

3.3.4 Light curves and periodograms

The *Kepler* light curves were successfully retrieved for 38 δ Scuti stars and plotted together with their corresponding periodograms (Appendix A). Figure 3.4 shows the light curve and periodogram of a binary star system, KIC 8196903.

3.3.5 Binary stars amongst δ Scuti stars

In part, this study is an attempt to identify binary star systems among a group of 24 old δ Scuti stars, as determined by Balona et al. (2013a) and 14 newly discovered δ Scuti stars (38 in total). In the pursuit of identifying binary stars, a criterion that required the presence of

three pulsation frequencies was employed during testing to classify a star as a binary system. This approach provided a robust means of identifying these binary configurations. Employing a binarogram method, one binary star system, KIC 8196903, was successfully identified. It was characterized by the presence of three *time-delays* and three pulsation frequencies. Figure 3.5 shows the binarogram of KIC 8196903, with the top panel showing peak value at orbital frequency of 0.9926000 ± 0.0000700 c/d for three pulsation frequencies ($f_1 = 5.9952659$ c/d, $f_2 = 5.9121509$ c/d and $f_3 = 20.7761649$ c/d). The bottom is a zoomed-in binarogram figure of KIC 8196903.

3.3.6 KIC 8196903

Within the dataset of 38 δ Scuti stars presented in Figure 3.8, a binary star is identified, constituting a proportion of 2.6 % within the sample. In addition, when examining Table 3.1, it becomes evident that KIC 8196903 exhibits a scorching temperature of 7672.00 K, rendering the system notably hot and relatively luminous at $12.569 L_{\odot}$. The binary star system was identified from the 24 old δ Scuti stars. KIC 8196903 exhibits typical characteristics of a δ Scuti star: it is hotter, brighter, more massive, and denser than other types of pulsating stars, such as Cepheids (Yang et al., 2011).

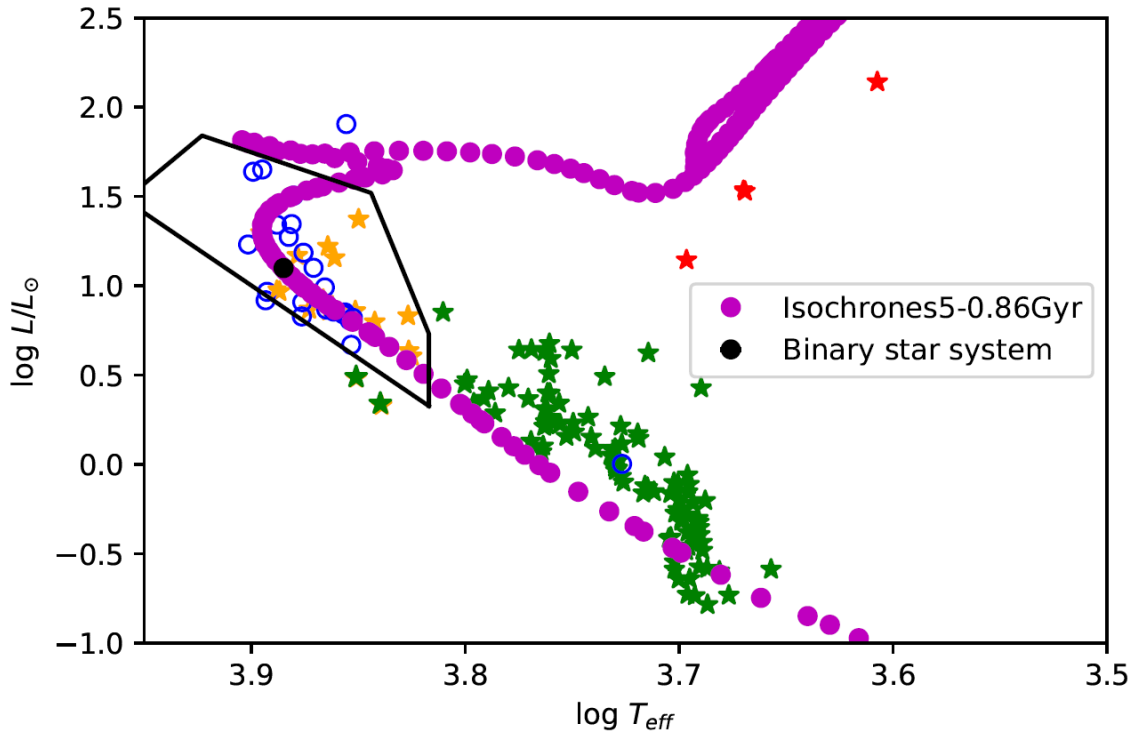


Figure 3.8: The H-R diagram and isochrones of the best perfect fit. Purple dotted line represents isochrones with age 0.86 Gyr.

The time-delay method is limited to determining orbital periods shorter than about 50 days and is often used in conjunction with other techniques to identify cluster members. The short orbital period implies the spread of a large frequency and a high chance of including neighboring

frequencies. The KIC 8196903 binary star has a short orbital period of 1.1935 days, as depicted in Table 3.5. This means that the two stars in the system orbit each other very quickly. In this case, the two stars are so close together that they are tidally locked, meaning that the same side of each star always faces the other star. Tidally locked binary star systems with a short orbital period can be very unstable. The gravitational forces between the two stars can be so strong that they can cause the stars to merge into a single star.

Table 3.5: Orbital parameters of a binary star system KIC 8196903.

a sin i (AU)	1.7149 ± 0.7820
P (d)	1.1935 ± 0.0001
f (c/d)	0.8379000 ± 0.0000907
K (km/s)	15632.3811 ± 7128.6102
f(M) ($M_{\odot}$)	$0.4724\text{E}+06 \pm 0.6462\text{E}+06$

3.3.7 Age and distance of the NGC 6866

Lastly, through analysis (overplotting isochrones fittings), the age of the NGC 6866 OSC was estimated to be approximately 0.86 Gyr (Figure 3.8), with a calculated distance of approximately 1497.90 pc using stellar parallax. These findings contribute valuable insights into the cluster's properties and further our understanding of stellar evolution within it.

3.3.8 Effective temperature and luminosity comparisons between GDR2 and GDR3.

GDR2 data and GDR3 data were compared for only 11 out of 14 probable members (δ Scuti stars) of the cluster (Table C.2). The other three δ Scuti stars are not captured in GDR3 archive, thus further justifies choosing the GDR2 database. GDR3 contains updated values for parallax and effective temperature, but the luminosity of the stars was not captured. Luminosity and effective temperature of the stars play important roles in this work. Considering two parallaxes from GDR2 and GDR3, the distances are approximately the same. Parallax changes were considered and included in this research, but kept the other values unchanged from GDR2.

Chapter 4

Discussion

The discussion provides a comprehensive understanding of the study’s findings and implications. This study successfully estimated the age of the NGC 6866 OSC to be approximately 0.86 Gyr and determined its distance at approximately 1497.90 pc. The age was estimated using a combination of observational data, proper motion analysis, and H–R diagram placement. The distance was estimated using *Gaia* parallax substituted into the stellar distance formula. Based on the OSCs characteristic stellar populations, the age of NGC 6866 falls within the range of a few hundred million to a billion years. Additionally, given its location in the Milky Way, the cluster’s distance is expected to be roughly around 1500 pc. The estimated distance of 1497.90 pc or 1.498 kpc for NGC 6866 is consistent with these premises and aligns with previous studies that used different methodologies. The estimated distances in the literature range from 1.189 to 1.65 kpc, with a few outliers such as 2.5 and 7.87 kpc reported by Balona et al. (2013a) and Donor et al. (2020), respectively.

The estimated age of 0.86 Gyr for NGC 6866 is consistent with findings reported by previous studies (ranging from 0.39 to 1.0 Gyr) that used different methodologies (for example, using CMDs instead H-R diagrams). Isochrone fittings were a popular choice for age estimation studies of the NGC 6866 OSC: Joshi et al. (2012); Janes et al. (2014); Bostanci et al. (2015); Joshi (2017); Long et al. (2023) and Brogaard et al. (2023) all made use of this method. The determined age is closer but slightly greater than ages 0.80 and 0.813 Gyr determined by Güneş et al. (2012) and Bostanci et al. (2015), respectively, but it is less than 1.0 Gyr recently reported by Karataş et al. (2023). This agreement with prior research strengthens the reliability of the findings. Moreover, it aligns with the intermediate age classification typically associated with NGC 6866, given its location in the Milky Way.

The age determination was primarily based on the position of stars in the H-R diagram by overplotting isochrone fitting; that is, identifying the main-sequence turnoff (MSTO) stars. The MSTO stars are the most massive stars in the cluster that are still on the main-sequence. As stars age, they evolve off the main-sequence and become red giants. The MSTO stars are the last stars in the cluster to evolve off the main-sequence, so their age is a good estimate of the age of the cluster (Wu et al., 2017). The majority of stars in the cluster were found in the main-sequence region, indicative of its intermediate age and suggesting they are young, stable

burning stars. Their distribution within a specific region of the diagram might indicate a similar evolutionary stage. Further analysis of their masses, ages, and metallicities compared to other stars in the cluster could reveal trends related to cluster formation and early evolution. The number of identified δ Scuti stars in NGC 6866 (14) is higher than what is typically found in young open clusters. This finding suggests a potentially higher fraction of δ Scuti stars within NGC 6866 compared to similar clusters. Further studies comparing δ Scuti populations across various young OSCs are needed to solidify this observation.

The calculation of the distance relied on stellar parallax measurements. To measure stellar parallax, the position of a star relative to a background star at two different times of year, must be measured. The difference in position is the parallax angle (Karttunen et al., 2007). The parallax angle was then used to calculate the distance to the star.

4.1 Recommendations

Despite the significant progress made in this study, there are some limitations that should be noted and addressed.

4.1.1 Accuracy

The limitation to our approach is that it relies on the accuracy of the *Gaia* parallax data, which can introduce uncertainties, especially for distant stars. Additionally, a binary star system was successfully identified using the binarogram method; it is possible that more binary systems exist within the cluster that were not identified in this analysis. For instance, Balona (2014) detected five binary stars within the NGC 6866. The binarogram technique might not be sensitive enough to detect all binary systems, especially those with longer orbital periods or lower mass ratios. Additionally, noise in the data or limitations in the observing cadence could have masked the signatures of some candidate binaries. Alternatively, the "missed" binaries could belong to a different population within the cluster with distinct properties that require a different detection method. Future observations with higher resolution or different techniques might be necessary to identify these potential binaries. New techniques could be developed to improve the characterization of OSCs. For example, new methods could be developed to identify binary stars more accurately or to estimate the age of OSCs more precisely by, for example, using machine learning. Machine learning can also be used to develop new methods for identifying binary stars, estimating the age of OSCs, and other tasks related to OSC characterization. Future studies could also use data from longer time periods to get a better understanding of the variability of stars in OSCs. This could be done by combining data from multiple *Kepler* missions or by using ground-based telescopes to monitor stars for longer periods of time, as done by the *Kepler* Asteroseismic Science Consortium Working Groups (Stello et al., 2010).

4.1.2 Applicability

Another limitation of our methodology is that it is only applicable to OSCs. OSCs are relatively young clusters of stars that are loosely bound together. Globular clusters are older clusters of stars that are tightly bound together. The methodology used to estimate the age and distance of NGC 6866 cannot be used to estimate the age and distance of globular clusters. Future research could explore modifications to the methodology to extend its applicability to globular clusters. This might involve incorporating additional data sources or considerations specific to the denser environments and older stellar populations found in globular clusters.

4.1.3 Data quality

The *Kepler* and *Gaia* satellite data used in the study is of high quality, but there are some limitations. For example, the *Kepler* data is only available for a limited time, and the *Gaia* DR2 does not have the same precision for all stars as the more recent data releases. The most recent *Gaia* data can be used to identify new members of OSCs and to study their variability. *Gaia* data is available for millions of stars in OSCs, which provides a great opportunity to identify new members and to study their variability.

4.1.4 Sample size

The sample of stars in the study is relatively small, especially for the binary star search. This means that the results may not be representative of the entire cluster. Future studies could use larger sample sizes to get more representative results. This could be done by combining data from multiple satellites or by using ground-based telescopes to observe more stars (Stello et al., 2010).

4.1.5 Manual elimination of stars

Eliminating stars manually was a tedious, time-consuming, human-error prone task for identifying potential member stars. It took several days to eliminate stars with no T_{eff} , luminosity, parallax and negative values of parallax. This process could potentially be automated through a PYTHON code, which was a challenge due to limited coding skills.

The main limitation of the methodology is that it relies on the accuracy of the H-R diagram and the parallax measurements. The methodology can be generalized to other OSCs, but the accuracy of the age and distance estimates will depend on the accuracy of the H-R diagram and the parallax measurements. While limitations exist, these findings contribute to our understanding of stellar evolution within NGC 6866 and serve as a valuable tool for similar studies in the field.

Chapter 5

Conclusion

5.1 Summary

This study has made significant contributions to our understanding of δ Scuti stars and the NGC 6866 OSC. To achieve the study's objectives, a combination of *Kepler* and *Gaia* satellite data was used to discriminate between member and non-member stars of the NGC 6866 OSC. H-R diagrams were constructed for the star samples in the field of the NGC 6866 OSC to classify the stars accordingly. New δ Scuti stars (members) were redefined and identified using *Gaia* parallax and proper motion in the field of the NGC 6866 OSC. *Kepler* photometric data were retrieved for the new and old δ Scuti stars to search for binary stars amongst the δ Scuti stars using a binarogram method. The distance of the cluster was determined using stellar parallax, and isochrone fittings were overplotted to estimate the age of the NGC 6866 OSC.

The construction of the H-R diagram revealed that most stars in the NGC 6866 cluster are in the main-sequence, indicative of its intermediate age. Understanding a star's position on an H-R diagram is vital, as it provides key information about its properties. Using a criterion of three pulsation frequencies to classify a star as a binary system, a single binary star system, KIC 8196903, was successfully identified. KIC 8196903 is a hot (7672.00 Kelvin) and luminous ($12.569 L_{\odot}$) binary star system with a short orbital period of 1.1935 Earth days. KIC 8196903 was found among the 24 old δ Scuti stars that were previously identified by Balona et al. (2013a), but not among the 14 newly classified δ Scuti stars.

Hundred and nineteen new cluster members were identified by cross-referencing the *Kepler* and *Gaia* datasets. This demonstrates the effectiveness of using both proper motion and distance modulus techniques to discriminate between members and non-members of the cluster. The cluster's mean proper motion was estimated to be 7.01 ± 0.01 mas/yr with a standard deviation (σ) = 3.75 mas. The mean pmRA and the mean pmDec were estimated to be -1.21 mas/yr and -4.58 mas/yr with standard deviations of 4.01 mas and 4.97 mas, respectively.

By overplotting isochrone fittings to the H-R diagram, the age of the NGC 6866 was estimated to be approximately 0.86 Gyr, with a distance of approximately 1497.90 pc. These findings enhance our understanding of the cluster's characteristics and stellar evolution within it.

In conclusion, the potential member stars of the NGC 6866 open cluster using *Kepler* and *Gaia* satellite data have been identified. The H-R diagrams for the star samples are constructed and distance modulus and proper motion are used to identify new δ Scuti stars. *Kepler* photometric data for all δ Scuti stars were retrieved and binary stars among δ Scuti stars were searched using the binarogram method. The distance to the cluster was determined using stellar parallax and its age was also estimated by over-plotting isochrones.

The methodology used in this study, combining proper motion analysis and H-R diagram placement, can be applied to other OSCs for age determination. Our findings regarding the age and distance of NGC 6866 can be generalized to similar intermediate-age OSCs in the Milky Way. Clusters exhibiting characteristics akin to NGC 6866 are likely to have comparable age and distance properties. However, the specific age and distance obtained will depend on the cluster's individual characteristics and evolutionary history. It's essential to adapt the approach accordingly. It is also important to note that the accuracy of the age and distance estimates will depend on the accuracy of the H-R diagram and the parallax measurements.

Overall, this study has made significant contributions to our understanding of δ Scuti stars and the NGC 6866 OSC. By addressing the limitations of the current study and pursuing the potential avenues for future research outlined, we can improve our understanding of OSCs and their role in the Milky Way Galaxy.

References

- Abedigamba, O. (2016). Kic 8263801: A clump star in the *Kepler* open cluster ngc 6866 field? *New Astron.*, 46:21–24.
- Abt, H. and Levy, S. (1985). Improved study of metallic-line binaries. *ApJS*, 59:229–247.
- Aerts, C., Christensen-Dalsgaard, J., and Kurtz, D. (2010). *Asteroseismology*. Springer Science & Business Media, Heidelberg London, New York.
- Aitken, R. (1924). The constellation ursa majoris. *PASP*, 36:53.
- Auvergne, M., Bodin, P., Boissard, L., Buey, J., Chaintreuil, S., Epstein, G., Joutet, M., Lam-Trong, T., Levacher, P., Magnan, A., and Perez, R. (2009). The corot satellite in flight: description and performance. *A&A*, 506(1):411–424.
- Baglin, A., Auvergne, M., Boissard, L., Lam-Trong, T., Barge, P., Catala, C., Deleuil, M., Michel, E., and Weiss, W. (2006). CoRoT: a high precision photometer for stellar evolution and exoplanet finding. In *36th COSPAR Scientific Assembly*, volume 36, page 3749.
- Balona, L. (2014). Binary star detection using the time-delay method: application to 34 *Kepler* objects of interest. *MNRAS*, 443(3):1946–1954.
- Balona, L. and Dziembowski, W. (2011). *Kepler* observations of δ scuti stars. *MNRAS*, 417(1):591–601.
- Balona, L., Joshi, S., Joshi, Y., and Sagar, R. (2013a). Pulsation and rotation of *Kepler* stars in the ngc 6866 field. *MNRAS*, 429(2):1466–1478.
- Balona, L., Medupe, T., Abedigamba, O., Ayane, G., Keeley, L., Matsididi, M., Mekonnen, G., Nhlapo, M. D., and Sithole, N. (2013b). *Kepler* observations of the open cluster ngc 6819. *MNRAS*, 430(4):3472–3482.
- Basu, S. and Chaplin, W. (2017). *Asteroseismic data analysis: Foundations and Techniques*. Princeton University Press, Princeton.
- Bessell, M. (2005). Standard photometric systems. *Annu. Rev. Astron. Astrophys.*, 43:293–336.
- Bisht, D., Zhu, Q., , Yadav, R., Ganesh, S., Rangwal, G., Durgapal, A., Saria, D., and Jiang, I. (2021). Multicolour photometry and gaiaedr3 astrometry of two couples of binary clusters (ngc 5617 and trumpler 22) and (ngc 3293 and ngc 3324). *MNRAS*, 503(4):5929–5947.

- Borucki, W. (2016). *KEPLER* mission: development and overview. reports on progress in physics. *Rep. Prog. Phys.*, 79(3):036901.
- Borucki, W., Koch, D., Basri, G., Batalha, N., Brown, T., Caldwell, D., Caldwell, J., Christensen-Dalsgaard, J., Cochran, W., DeVore, E., and Dunham, E. (2010). *Kepler* planet-detection mission: introduction and first results. *Science*, 327(5968):977–980.
- Bostanci, Z., Ak, T., Yontan, T., Bilir, S., Güver, T., Ak, S., Ö. Cakirli, Özdarcan, O., Paunzen, E., Cat, P. D., Fu, J., Zhang, Y., Hou, Y., Li, G., Wang, Y., Zhang, W., Shi, J., and Wu, Y. (2015). A comprehensive study of the open cluster ngc 6866. *MNRAS*, 453(1):1095–1107.
- Bressan, A., Marigo, P., Girardi, L., Cero, B. S. C. D., Rubele, S., and Nanni, A. (2012). Parsec: stellar tracks and isochrones with the padova and trieste stellar evolution code. *MNRAS*, 427(1):127–145.
- Brewer, L., Sandquist, E., Mathieu, R., Milliman, K., Geller, A., Jeffries, M., Jr., Orosz, J., Brogaard, K., Platais, I., Bruntt, H., Grundahl, F., Stello, D., and Frandsen, S. (2016). Determining the age of the *Kepler* open cluster ngc 6819 with a new triple system and other eclipsing binary stars. *AJ*, 151(3):66.
- Brogaard, K., Arentoft, T., Miglio, A., Casali, G., Tailo, J. T. M., and Sandquist, E. (2023). An asteroseismic age estimate of the open cluster ngc 6866 using *Kepler* and *gaia*. arxiv preprint arxiv:2308.12731. *A&A*.
- Brown, A., Vallenari, A., Prusti, T., Bruijne, J. D., Babusiaux, C., Bailer-Jones, C., Biermann, M., Evans, D., Eyer, L., Jansen, F., and Jordi, C. (2018). *Gaia* data release 2-summary of the contents and survey properties. *A&A*, 616:A1.
- Camargo, D., Bica, E., and Bonatto, C. (2015). New galactic embedded clusters and candidates from a wise survey. *New Astron.*, 34:84–97.
- Carquillat, J. and Prieur, J. (2007). Contribution to the search for binaries among am stars–viii. new spectroscopic orbits of eight systems and statistical study of a sample of 91 am stars. *MNRAS*, 380(3):1064–1078.
- Christensen-Dalsgaard, J. (2004). An overview of helio-and asteroseismology. *SOHO 14 Helio-and Asteroseismology*, 559:1.
- Creswell, J. and Creswell, J. (2017). *Research design: Qualitative, quantitative, and mixed methods approaches*. Sage publications.
- Crosta, M. (2019). Astrometry in the 21st century. from hipparchus to einstein. *Riv. Nuovo Cimento*, 42(10):443–510.
- Dias, W., Monteiro, H., Lépine, J., Prates, R., Gneiding, C., and Sacchi, M. (2018). Astrometric and photometric study of dias 4, dias 6, and other five open clusters using ground-based and *Gaia* dr2 data. *MNRAS*, 481(3):3887–3901.
- Donor, J., Frinchaboy, P., Cunha, K., O’Connell, J., Prieto, C., Almeida, A., Anders, F., Beaton, R., Bizyaev, D., Broenstein, J., and Carrera, R. (2020). The open cluster chemical abundances and mapping survey. iv. abundances for 128 open clusters using sdss/apogee dr16. *AJ*, 159(5):199.

- Duchêne, G., Bontemps, S., Bouvier, J., André, P., Djupvik, A. A., and Ghez, A. M. (2007). Multiple protostellar systems-ii. a high resolution near-infrared imaging survey in nearby star-forming regions. *A&A*, 476(1):229–242.
- Duquennoy, A. and Mayor, M. (1991). Multiplicity among solar-type stars in the solar neighbourhood. ii-distribution of the orbital elements in an unbiased sample. *A&A*, 248:485–524.
- Ferraz-Mello, S. (1981). Estimation of periods from unequally spaced observations. *AJ*, 86:619.
- Foschini, L., Treves, A., Tavecchio, F., Impiombato, D., Ghisellini, G., Covino, S., Tosti, G., Gliozzi, M., Bianchin, V., Cocco, G. D., and Malaguti, G. (2008). Infrared to x-ray observations of pks 2155-304 in a low state. *arXiv preprint arXiv:0805.0190*.
- Gao, X. (2014). Membership determination of open cluster ngc 188 based on the dbscan clustering algorithm. *RAA*, 14(2):159–164.
- García, R., Mathur, S., Pires, S., Régulo, C., Bellamy, B., Pallé, P., Ballot, J., Forteza, S., Beck, P., Bedding, T., and Ceillier, T. (2014). Impact on asteroseismic analyses of regular gaps in *Kepler* data. *A&A*, 568:A10.
- Güneş, O., Karataş, Y., and Bonatto, C. (2012). Astrophysical and structural parameters of the open clusters ngc 6866, ngc 7062, and ngc 2360. *New Astron.*, 17(8):720–731.
- Ghez, A. (1993). *The Multiplicity of T Tauri Stars in the Star Forming Regions Taurus-Auriga and Ophiucus-Scorpius: A 2.2 μ m Speckle Imaging Survey*. Dissertation (Ph.D.), California Institute of Technology, California.
- Gieles, M. and Zwart, S. P. (2011). The distinction between star clusters and associations. *MNRAS*, 410(1):L6–L7.
- Glatt, K., Grebel, E., Sabbi, E., Gallagher, J., Nota, A., Sirianni, M., Clementini, G., Tosi, M., Harbeck, D., Koch, A., Kayser, A., and Costa, G. D. (2008). Age determination of six intermediate-age small magellanic cloud star clusters with hst/acs. *AJ*, 136(4):1703–1703.
- Goodricke, J. (1783). Xxvi. a series of observations on, and a discovery of, the period of the variation of the light of the bright star in the head of medusa, called algol. in a letter from john goodricke, esq. to the rev. anthony shepherd, d. d. f. r. s. and plumian professor at cambridge. *Philos. Trans. R. Soc of London*, 73:474–482.
- Govind, A., Choudhury, D., Mathew, B., and Paul, K. (2018). Astrometry: The foundation for observational astronomy. *MJS*, 17(1):1–10.
- Gratton, R., Bragaglia, A., Carretta, E., D’Orazi, V., Lucatello, S., and Sollima, A. (2019). What is a globular cluster? an observational perspective. *A&AR*, 27(1):8.
- Heintz, W. (1969). A statistical study of binary stars. *JRASC*, 63:275.
- Hendy, Y. and Tadross, A. (2020). Characteristics of the open star cluster kronberger 60 using gaia dr2. *NRIAG 11421*.

- Herschel, W. (1802). Catalogue of 500 new nebulae, nebulous stars, planetary nebulae, and clusters of stars; with remarks on the construction of the heavens. *Philos. Trans. R. Soc*, 92:477–528.
- Janes, K., Barnes, S., Meibom, S., and Hoq, S. (2014). Open clusters in the *KEPLER* field. ii. ngc 6866. *AJ*, 147(6):139.
- Jaschek, C. and Jaschek, M. (1957). The frequency of spectroscopic binaries. *PASP*, 69(411):546–551.
- Joshi, G. (2017). Test of depending frequencies of the variable stars of oc: Ngc 6866. arxiv preprint arxiv:1712.00701. *Department of Physics, Kumaun University (Nainital)*.
- Joshi, G. (2021). Effect of blue component stars in parameters of ngc 6866. arxiv preprint arxiv:2106.04754. *P.P.S.V.M.I. College (India)*.
- Joshi, G. (2022). Implication of phase-folded diagrams for validation of the nature of stellar variability i: the case of ngc 6866. *BAJ*, 37(01).
- Joshi, Y., Joshi, S., Kumar, B., Mondal, S., and Balona, L. (2012). Photometric study and detection of variable stars in the open clusters – i. ngc 6866. *MNRAS*, 419(3):2379–2390.
- Karataş, Y., Çakmak, H., İ. Akkaya Oralhan, Bonatto, C., Michel, R., and Netopil, M. (2023). Study of the open clusters in *Kepler* prime field. *MNRAS*, 521(2):2408–2426.
- Karttunen, H., Kröger, P., Oja, H., Poutanen, M., and Donner, K. (2003). *Stellar Spectra*. In *Fundamental Astronomy*, Springer, Berlin, Heidelberg.
- Karttunen, H., Kröger, P., Oja, H., Poutanen, M., and Donner, K. (2007). *Fundamental astronomy*. Springer Science & Business Media, Springer Berlin Heidelberg New York.
- Kiran, E., Harmanec, P., Ö.L. Değirmenci, Wolf, M., Nemravová, J., Šlechta, M., and Koubský, P. (2016). The orbital elements and physical properties of the eclipsing binary bd+36°3317, a probable member of δ lyrae cluster. *A&A*, 587:A127.
- Koch, D., Borucki, W., Basri, G., Batalha, N., Brown, T., Caldwell, D., Christensen-Dalsgaard, J., Cochran, W., DeVore, E., Dunham, E., and Gautier, T. (2010). *Kepler* mission design, realized photometric performance, and early science. *ApJL*, 713(2):L79.
- Koroleva, L. (1973). The open cluster ngc 6866. *SvA*, 16:643.
- Krzesinski, J. and Blokesz, A. (2019). Searching for low-mass companions around white dwarfs and subdwarfs from *Kepler* field. *Proc IAU*, 15(s357):41–44.
- Krzesinski, J., Blokesz, A., Siwak, M., and Stachowski, G. (2020a). The quest for planets around subdwarfs and white dwarfs from *Kepler* space telescope fields-i. techniques and tests of the methods. *A&A*, 642:A105.
- Krzesinski, J., Blokesz, A., Siwak, M., and Stachowski, G. (2020b). The quest for planets around subdwarfs and white dwarfs from *Kepler* space telescope fields-i. techniques and tests of the methods. *A&A*, 642:A105.

- Lada, C. (2006). Stellar multiplicity and the initial mass function: most stars are single. *ApJ*, 640(1):L63.
- Lea, R. (2023). What are binary stars? <https://www.space.com/22509-binary-stars.html>. [Online; accessed 19-May-2024].
- Leinert, C., Zinnecker, H., Weitzel, N., Christou, J., Ridgway, S., Jameson, R., Haas, M., and Lenzen, R. (1993). A systematic approach for young binaries in taurus. *A&A*, 278:129–149.
- Lomb, N. (1976). Least-squares frequency analysis of unequally spaced data. *Ap&SS*, 39(2):447–462.
- Long, L., Bi, S., Zhang, J., Zhang, X., Zhang, L., Ge, Z., and Zhou, J. (2023). Investigating 16 open clusters in the *Kepler*/k2–gaia dr3 field. i. membership, binary systems, and rotation. *ApJS*, 268(1):30.
- Loore, C. D. and Doom, C. (1992). *Structure and evolution of single and binary stars (Vol. 179)*. Springer Science & Business Media.
- Lowrance, P., Kirkpatrick, J., and Beichman, C. (2002). A distant stellar companion in the ν andromedae system. *ApJ*, 572(1):L79.
- Luri, X., Brown, A., Sarro, L., Arenou, F., Bailer-Jones, C., Castro-Ginard, A., and Delgado, H. (2018). Gaia data release 2-using gaia parallaxes. *A&A*, 616:A9.
- Maia, F., Corradi, W., and Santos, J. J. (2010). Characterization and photometric membership of the open cluster ngc 1981. *MNRAS*, 407(3):1875–1886.
- Marigo, P., Girardi, L., Bressan, A., Rosenfield, P., Aringer, B., Chen, Y., Dussin, M., Nanni, A., Pastorelli, G., Rodrigues, T., and Trabucchi, M. (2017). A new generation of parsec-colibri stellar isochrones including the tp-agb phase. *AJ*, 835(1):77.
- Martines, C., Holanda, N., Pereira, C., and Drake, N. (2020). High-resolution spectroscopy of red giants and ‘yellow stragglers’ in the southern open cluster ngc 2539. *MNRAS*, 494(1):1470–1489.
- Mason, B., Martin, C., Hartkopf, W., Barry, D., Germain, M., Douglass, G., Worley, C., Wycoff, G., Brummelaar, T. T., and Franz, O. (1999). Speckle interferometry of new and problem hipparcos binaries. *AJ*, 117(4):1890.
- Miglio, A., Montalbán, J., Baudin, F., Eggenberger, P., Noels, A., Hekker, S., Ridder, J. D., Weiss, W., and Baglin, A. (2009). Probing populations of red giants in the galactic disk with corot. *A&A*, 503(3):L21–L24.
- Mohammed, A. (2019). The study of short orbital period of δ scuti pulsating variable stars. *Proc IAU*, 15(s356):407–407.
- Molenda-Żakowicz, J., Kopacki, G., Stęślicki, M., and Narwid, A. (2009). Photometric study of variable stars in the open cluster ngc6866. arxiv preprint arxiv:0907.0813. *Astronomical Institute, University of Wrocław (Poland)*.
- Oralhan, I., Karataş, Y., Schuster, W., Michel, R., and Chavarría, C. (2015). Ccd ubv (ri) c photometry of twenty open clusters. *New Astron.*, 34:195–210.

- Ostlie, D. A. and Carroll, B. W. (1996). *An Introduction to Modern Stellar Astrophysics*.
- Petrie, R. (1960). February. probable errors of stellar radial velocities and the frequency of variable velocities. *In Annales d'Astrophysique*, 23:744.
- Rangwal, G., Yadav, R., Durgapal, A., Bisht, D., and Nardiello, D. (2019). Astrometric and photometric study of ngc 6067, ngc 2506, and ic 4651 open clusters based on wide-field ground and gaia dr2 data. *MNRAS*, 490(1):1383–1396.
- Sandquist, E., Jessen-Hansen, J., Shetrone, M., Brogaard, K., Meibom, S., Leitner, M., Stello, D., Bruntt, H., Antoci, V., Orosz, J., Grundahl, F., and Frandsen, S. (2016). The age and distance of the *KEPLER* open cluster ngc 6811 from an eclipsing binary, turnoff star pulsation, and giant asteroseismology. *AJ*, 831(1):11.
- Scargle, J. (1982). Studies in astronomical time series analysis. ii-statistical aspects of spectral analysis of unevenly spaced data. *ApJ*, 263:835–853.
- Smalley, B., Southworth, J., Pintado, O., Gillon, M., Holdsworth, D., Anderson, D., Barros, S., Cameron, A., Delrez, L., Faedi, F., and Haswell, C. (2014). Eclipsing am binary systems in the superwasp survey. *A&A*, 564:A69.
- Soydugan, E., Aliçavuş, F., Soydugan, F., and Bilir, S. (2020). *Kepler* binary stars in the ngc 6819 open cluster: Kic 5113146 and kic 5111815. *AJ*, 160(6):245.
- Steinicke, W. (2010). Observing and cataloguing nebulae and star clusters: from herschel to dreyer's new general catalogue. *CUP*.
- Stello, D., Basu, S., Bedding, T., Brogaard, K., Bruntt, H., Chaplin, W., Christensen-Dalsgaard, J., Demarque, P., Elsworth, Y., García, R., and Gilliland, R. (2010). Solar-like oscillations in cluster stars. *Astron. Nachr*, 331(9–10):985–988.
- Stello, D., Chaplin, W., Bruntt, H., Creevey, O., García-Hernández, A., Monteiro, M., Moya, A., Quirion, P., Sousa, S., Suárez, J., and Appourchaux, T. (2009). Radius determination of solar-type stars using asteroseismology: What to expect from the *Kepler* mission. *ApJ*, 700(2):1589.
- Stott, J. (2018). Determining open cluster membership - a bayesian framework for quantitative member classification. *A&A*, 609:A36.
- Swastik, C., Banyal, R., Narang, M., Unni, A., and Sivarani, T. (2024). Age analysis of extrasolar planets: Insight from stellar isochrone models. *AJ*, 167(6):270.
- Tadross, A. (2011). A catalog of 120 ngc open star clusters. arxiv preprint arxiv:1108.2134. *JKAS*.
- Tang, Y., Basu, S., Davies, G., Bellinger, E., and Gai, N. (2018). Asteroseismology of kic 8263801: Is it a member of ngc 6866 and a red clump star? *AJ*, 866(1):59.
- Terebizh, V. (1992). *Time series analysis in astrophysics*. Main Editorial Board for Physical and Mathematical Literature, Moscow, Nauka.

- Gaia* collaboration, Prusti, T., Bruijne, J. D., Brown, A., Vallenari, A., Babusiaux, C., Bailer-Jones, C., Bastian, U., Biermann, M., Evans, D., Eyer, L., and Jansen, F. (2016). The *gaia* mission. arxiv preprint arxiv:1609.04153. *A&A*.
- Tokovinin, A. (1997). On the multiplicity of spectroscopic binary stars. *Astronomy Letters*, 23:727–730.
- Tokovinin, A., Thomas, S., Sterzik, M., and Udry, S. (2006). Tertiary companions to close spectroscopic binaries. *A&A*, 450(2):681–693.
- Vagnozzi, S. (2019). New solar metallicity measurements. *Atoms*, 7(2):41.
- Vallenari, A., Brown, A., Prusti, T., Bruijne, J. D., Arenou, F., Babusiaux, C., Biermann, M., Creevey, O., Evans, C. D. D., and Eyer, L. (2023). *Gaia* data release 3-summary of the content and survey properties. *A&A*, 674:A1.
- VanderPlas, J. (2018). Understanding the lomb–scargle periodogram. *ApJS*, 236(1):16.
- Waldek, S. (2021). What are star clusters?
- Walker, G., Matthews, J., Kuschnig, R., Johnson, R., Rucinski, S., Pazder, J., Burley, G., Walker, A., Skaret, K., Zee, R., and Grocott, S. (2003). The most asteroseismology mission: ultraprecise photometry from space. *PASP*, 115(811):1023.
- Wilson, R. (1953). *General catalogue of stellar radial velocities (Vol. 601)*. Carnegie Institution of Washington, Washington, DC: Carnegie Institution of Washington.
- Worley, C. (1967). Multiple star systems as related to double stars: Multiple stars among the visual binaries. in on the evolution of double stars. *IAU*, 17:221.
- Wu, Y., Xiang, M., Zhang, X., Li, T., Bi, S., Liu, X., Fu, J., Huang, Y., Tian, Z., Liu, K., and Ge, Z. (2017). Stellar parameters of main sequence turn-off star candidates observed with *lamost* and *Kepler*. *RAA*, 17(1):5.
- Yang, W., Li, Z., Meng, X., and Bi, S. (2011). Characteristics of solar-like oscillations of clusters simulated by stellar population synthesis. *MNRAS*, 414(2):1769–1776.
- Young, M. (2022). The *gaia* revolution: New data and strange stars - sky & telescope - sky & telescope.
- Zwintz, K., Fossati, L., Ryabchikova, T., Guenther, D., Aerts, C., Barnes, T., Themeßl, N., Lorenz, D., Cameron, C., Kuschnig, R., and Pollack-Drs, S. (2014). Echography of young stars reveals their evolution. *Science*, 345(6196):550–553.
- Zwitter, T. and Munari, U. (2004). Spectroscopy, photometry and micro-arcsec astrometry of binaries with the *gaia* space mission and with the *rave* experiment. *IAUC*, 191:251–258.

Appendix A

Light curves and periodograms

The light curves and periodograms of the new and old Delta Scuti stars in the field of the NGC 6866 open cluster. Top panel: light curves and bottom panel: periodograms of the stars.

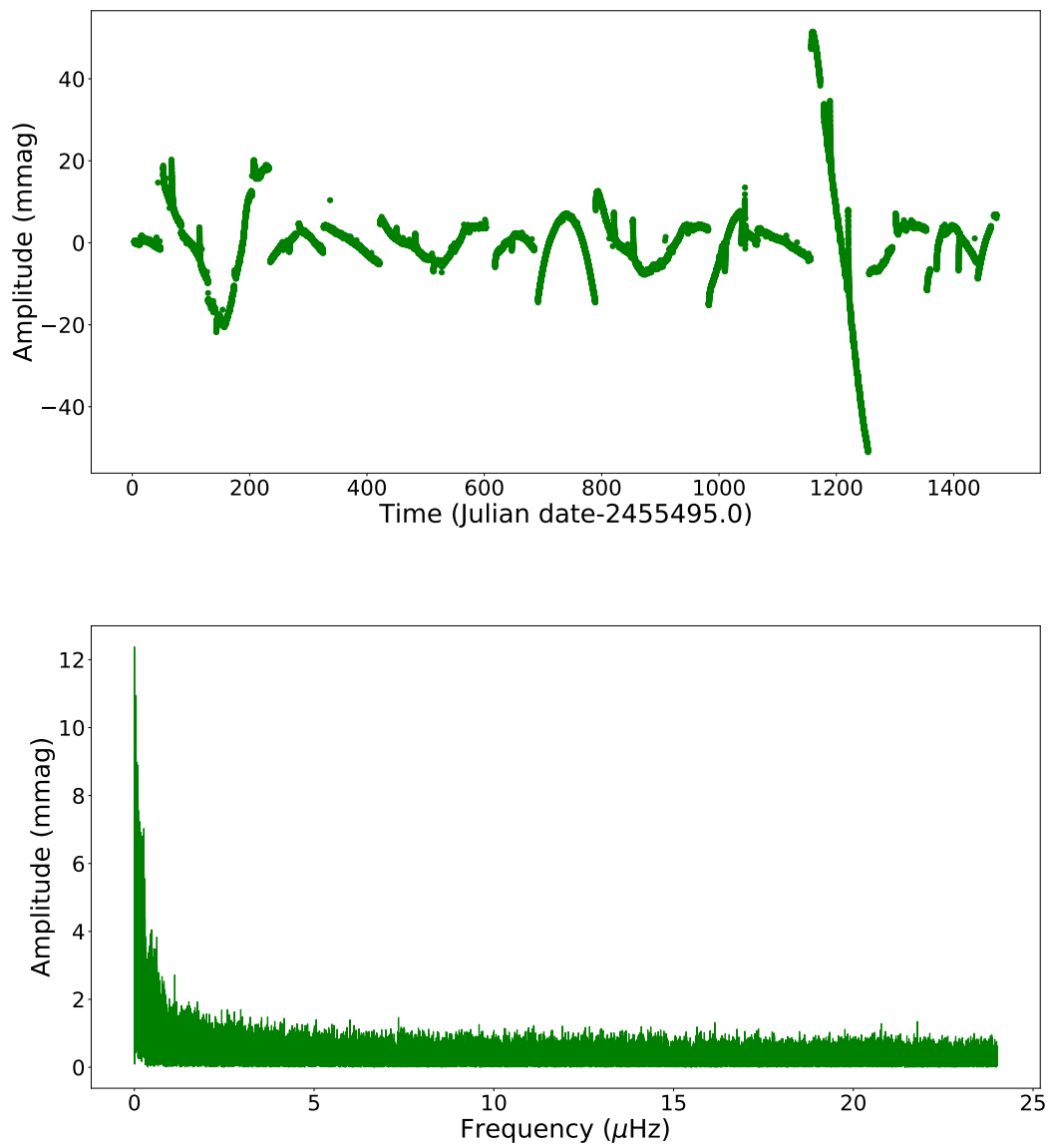


Figure A.1: KIC 8196911.

Additional figures for light curves and periodograms can be accessed using the following link:
<https://drive.google.com/file/d/1pgvASiuJlV40rVYuZ5sm5kx2FLUUwbEO/view?usp=sharing>

Appendix B

Code

A PYTHON program to find the new members of the NGC 6866 open cluster, calculate the distance moduli and the mean proper motions of the stars and plot the H-R, periodogram and light curve diagrams.

Code

June 3, 2024

```

In [128]: # A PYTHON program to find the new members of the NGC 6866 open cluster, calculate
          # the distance moduli and the mean proper motions of the stars and plot the H-R,
          # periodogram and light curve diagrams.

In [129]: import numpy as np
          import matplotlib.pyplot as plt
          from pylab import *
          import math

In [130]: #The H-R diagram plotted after filtering out stars without effective temperature and
          # luminosity manually

          eff_T= np.loadtxt("/home/user/Downloads/2ndcode.csv", delimiter=',', usecols=9)
          #Effective Temperature in Kelven

          Lum= np.loadtxt("/home/user/Downloads/2ndcode.csv", delimiter=',', usecols=10)
          #Luminosity in solLum (solar Luminosity)

          eff_T1=np.log10(eff_T)
          Lum_Sun= 3.846*10**33      # Sun luminosity in ergs/s
          solLum= 3.826*10**33      # ergs/s (1 Solar Luminosity = 3.826*10**33 ergs/s)
          Lum1= Lum*solLum
          Lumi=log10(Lum1/Lum_Sun)
          xmaxScale_factor= 3.95
          xminScale_factor= 3.50

In [131]: # The H-R diagram plotted after eliminating the known stars

          eff_T2= np.loadtxt("/home/user/Downloads/4thcode.csv", delimiter=',', usecols=10)
          # Effective Temperature in Kelven

          Lum2= np.loadtxt("/home/user/Downloads/4thcode.csv", delimiter=',', usecols=11)
          # Luminosity in solLum (solar Luminosity)

          eff_T3=log10(eff_T2)
          Lum3=Lum2*solLum
          Lum4= log10(Lum3/Lum_Sun)
          xmaxScale_factor= 3.95
          xminScale_factor= 3.50

```

```

In [132]: # calculating distance moduli of the stars

Parallax= np.loadtxt("/home/user/Downloads/4thcode.csv", delimiter=',', usecols=5)
#(mas)

d=1/(Parallax/1000)      # converting distance to parsecs
D_modulus= 5*log10(d)-5  #Distance modulus in magnitude (mag)

In [133]: D_modulus.mean()  #to calculate the mean of the distance modulus

Out[133]: 10.869906735134279

In [134]: # The H-R diagram plotted after eliminating the stars with non-acceptable distance
# moduli
eff_Ta= np.loadtxt("/home/user/Downloads/8thcode.csv", delimiter=',', usecols=11)
Luma= np.loadtxt("/home/user/Downloads/8thcode.csv", delimiter=',', usecols=12)
eff_Taa=log10(eff_Ta)
Lumaa=Luma*sollum
Lum4a= log10(Lumaa/Lum_Sun)
array=[3.817,3.817,3.844,3.923,3.959,3.817]
array1=[0.327,0.732,1.520,1.841,1.482,0.327]
array2=[3.851,3.8397]
array3=[0.49,0.341]
xmaxScale_factor= 3.95
xminScale_factor= 3.50
col=np.where(eff_Taa>3.82, 'k', np.where(Lum4a<1.1, 'navy', 'red'))

In [135]: # The H-R diagram plotted after eliminating the stars withno KIC numbers manually

eff_T22a= np.loadtxt("/home/user/Downloads/10thcode.csv", delimiter=',', usecols=12)
Lum22a= np.loadtxt("/home/user/Downloads/10thcode.csv", delimiter=',', usecols=13)
eff_T33a=log10(eff_T22a)
Lum33a=Lum22a*sollum
Lum44a= log10(Lum33a/Lum_Sun)
xmaxScale_factor= 3.95
xminScale_factor= 3.50
col1=np.where(eff_T33a>3.82, 'orange', np.where(Lum44a<1.1, 'g', 'r'))

In [136]: # Using proper motion as an additional parameter to identify memebers of the Cluster

import pandas as pd
import uncertainties
from uncertainties.umath import *
from uncertainties import ufloat

In [137]: pmRA13= pd.read_csv("/home/user/Downloads/7bthcode.csv")#Proper motion in RA (mas/yr.
c=pd.read_csv("/home/user/Downloads/9thcode.csv")
Tef=c['C13']
lum=c['C14']

```

```

In [174]: #pmRA13

In [139]: #for i in range(len(pmRA13['C2'])):
           #print(((ufloat((pmRA13['C1'][i], pmRA13['C2'][i]))**2
           #+ (ufloat((pmRA13['C3'][i], pmRA13['C4'][i]))**2)**0.5) # calculating
           # proper motions of the star.

In [140]: # Adding more columns to pmRA13

           pmRA13['m']=m
           pmRA13['m1']=m1
           pmRA13['Tef']=Tef
           pmRA13['lum']=lum

In [144]: #pmRA13

In [145]: m2 = sum(m)
           print (m2)

925.3490000000002

In [146]: m3 = np.zeros(len(m1)) # creating placeholders

           for i in range(len(m1)):
               m3[i] = (pmRA13['m1'][i])**2

In [147]: m4 = sum(m3)
           m5 = (m4)**0.5

In [148]: ufloat(m2, m5)/132 # mean pm of the cluster

Out[148]: 7.010219696969698+/-0.005940939811193207

In [149]: #((ufloat((-3.86, 0.16))**2 + (ufloat((-4.63, 0.17))**2)**0.5

In [150]: #((ufloat((-3.47, 0.41))**2 + (ufloat((-3.30, 0.35))**2)**0.5

In [151]: n = sum(pmRA13['C1'])
           print (n)

-159.133

In [152]: n1 = np.zeros(len(pmRA13['C2'])) # creating placeholders

           for i in range(len(pmRA13['C2'])):
               n1[i] = (pmRA13['C2'][i])**2
           len (n1)

```

```

Out[152]: 132

In [153]: n2 = sum(n1)
          n3 = (n2)**0.5
          ufloat(n, n3)/132 # mean pmRA

Out[153]: -1.2055530303030304+/-0.005508237424303505

In [154]: s = sum(pmRA13['C3'])
          print (s)

-604.90900000000001

In [155]: s1 = np.zeros(len(pmRA13['C4'])) # creating placeholders

          for i in range(len(pmRA13['C4'])):
              s1[i] = (pmRA13['C4'][i])**2
          len (s1)

Out[155]: 132

In [156]: s2 = sum(s1)
          s3 = (s2)**0.5
          ufloat(s, s3)/132 # mean pmDec

Out[156]: -4.58264393939394+/-0.006074331049944174

In [157]: pRA=pmRA13['C1']

In [158]: pRA.mean()      #the mean of the proper motion in RA

Out[158]: -1.2055530303030304

In [159]: np.std(pRA)     #the standard deviation of the proper motion in RA

Out[159]: 4.0131916005774455

In [160]: pDec=pmRA13['C3']

In [161]: pDec.mean()     #the mean of the proper motion in Dec.

Out[161]: -4.58264393939394

In [162]: np.std(pDec)    #the standard deviation of the proper motion in Dec.

Out[162]: 4.965325629610744

In [163]: pm=pmRA13['m']   #the proper motion of the stars in the NGC 6866 open cluster

In [164]: pm.mean()       #the mean of the proper motion of the stars of the cluster

```

```

Out[164]: 7.010219696969698

In [165]: np.std(pm)           #the standard deviation of the proper motion of the stars

Out[165]: 3.751348315816825

In [166]: thirdstd= 3*(np.std(pm))
          a=np.std(pm) +np.std(pm) + np.std(pm)
          pm=pmRA13[pmRA13['m'] < thirdstd]
          #To filter the data according to the proper motions of the well-known members within
          # the standard deviations values.

In [167]: #pm

In [168]: pm.to_csv('out.csv') # saving pm.csv as out.csv

In [169]: import numpy as np
          import matplotlib.pyplot as plt
          from pylab import *
          import math

In [170]: #The H-R diagram plotted after filtering the data according to the proper motions of
          # the well-known members within the standard deviations values.

          Tef1= np.loadtxt("/home/user/Downloads/aaa.csv", delimiter=',', usecols=6)
          #Effective Temperature in Kelven

          lum1= np.loadtxt("/home/user/Downloads/aaa.csv", delimiter=',', usecols=7)
          #Luminosity in solLum (solar Luminosity)

          Te=log10(Tef1)
          lu=lum1*solLum
          lumin= log10(lu/Lum_Sun)
          old_eT= np.loadtxt("/home/user/Downloads/6thcode.csv", delimiter=',', usecols=12)
          #Old Delta Scuti stars-Effective Temperature in Kelven

          old_Lum= np.loadtxt("/home/user/Downloads/6thcode.csv", delimiter=',', usecols=13)
          #Old Delta Scuti stars-Luminosity in solLum (solar Luminosity)

          eT_old=log10(old_eT)
          lum_old=old_Lum*solLum
          Lum_Old= log10(lum_old/Lum_Sun)
          arrayb=[3.817,3.817,3.844,3.923,3.959,3.817]
          array1b=[0.327,0.732,1.520,1.841,1.482,0.327]
          array2b=[3.851,3.8397]
          array3b=[0.49,0.341]
          col1=np.where(Te>3.82, 'orange', np.where(lumin<1.1, 'g', 'r'))

In [171]: # This graph was plotted after filtering the data according to the proper motions

```

```

from matplotlib.patches import Circle
pmRa= np.loadtxt("/home/user/Downloads/aaa.csv", delimiter=',', usecols=0)
# Proper motion in RA (mas/yr)

pmdeC= np.loadtxt("/home/user/Downloads/aaa.csv", delimiter=',', usecols=2)
# Proper motion in DE (mas/yr)

old_ra= np.loadtxt("/home/user/Downloads/6thcode.csv", delimiter=',', usecols=8)
old_dec= np.loadtxt("/home/user/Downloads/6thcode.csv", delimiter=',', usecols=9)

In [172]: Tef1= np.loadtxt("/home/user/Downloads/aaa.csv", delimiter=',', usecols=6) #T_eff (K.
lum1= np.loadtxt("/home/user/Downloads/aaa.csv", delimiter=',', usecols=7) #Lum in(s
Te=log10(Tef1)
lu=lum1*solLum
lumin= log10(lu/Lum_Sun)
old_eT= np.loadtxt("/home/user/Downloads/6thcode.csv", delimiter=',', usecols=12)
old_Lum= np.loadtxt("/home/user/Downloads/6thcode.csv", delimiter=',', usecols=13)
eT_old=log10(old_eT)
lum_old=old_Lum*solLum
Lum_Old= log10(lum_old/Lum_Sun)
#arrayb=[3.817,3.817,3.844,3.923,3.959,3.817]
#array1b=[0.327,0.732,1.520,1.841,1.482,0.327]
array2b=[3.851,3.8397]
array3b=[0.49,0.341]
col1=np.where(Te>3.82, 'orange', np.where(lumin<1.1, 'g', 'r'))
datfile = np.loadtxt("/home/user/Desktop/Desktop/2.dat", #1.00e9
                    unpack=True, skiprows=0)
datfile1 = np.loadtxt("/home/user/Desktop/Desktop/IS1.dat", #1.00e8
                    unpack=True, skiprows=0)
datfile2 = np.loadtxt("/home/user/Desktop/Desktop/IS_a1.dat", #0.65e9
                    unpack=True, skiprows=0)
datfile3 = np.loadtxt("/home/user/Desktop/Desktop/IS_b1.dat", #0.65e8
                    unpack=True, skiprows=0)
datfile4 = np.loadtxt("/home/user/Desktop/Desktop/IS_c1.dat", #0.60e9
                    unpack=True, skiprows=0)
datfile5 = np.loadtxt("/home/user/Desktop/Desktop/IS_d1.dat", #0.80e9
                    unpack=True, skiprows=0)
datfile6 = np.loadtxt("/home/user/Desktop/Desktop/ISO_1.dat", #1.00e9 - 1.00e10
                    unpack=True, skiprows=0)
datfile6 = np.loadtxt("/home/user/Desktop/Desktop/0.99e.dat", #0.99e9
                    unpack=True, skiprows=0)
datfile7 = np.loadtxt("/home/user/Desktop/Desktop/Isochrones(a).dat", #0.65e9 with
                    unpack=True, skiprows=0) #Z=0.010
datfile8 = np.loadtxt("/home/user/Desktop/Desktop/Isochrones2a.dat", #0.65e9 with
                    unpack=True, skiprows=0) #Z=0.0153
datfile9 = np.loadtxt("/home/user/Desktop/Desktop/Isochrones3a.dat", #0.65e9 with
                    unpack=True, skiprows=0) #Z=0.0154
datfile10 = np.loadtxt("/home/user/Desktop/Desktop/Isochrones4a.dat", #0.75e9 with

```

```

                                unpack=True, skiprows=0)          #Z=0.0154
datfile11 = np.loadtxt("/home/user/Desktop/Desktop/Isochrones5a.dat", #0.85e9 with
                                unpack=True, skiprows=0)          #Z=0.0154
datfile12 = np.loadtxt("/home/user/Desktop/Desktop/Isochrones6a.dat", #0.85e9 with
                                unpack=True, skiprows=0)          #Z=0.0154
datfile13 = np.loadtxt("/home/user/Desktop/Desktop/Isochrones7a.dat", #0.86e9 with
                                unpack=True, skiprows=0)          #Z=0.021
datfile14 = np.loadtxt("/home/user/Desktop/Desktop/Isochrones8a.dat", #0.88e9 with
                                unpack=True, skiprows=0)          #Z=0.021
datfile15 = np.loadtxt("/home/user/Desktop/Desktop/Isochrones9a.dat", #0.85e9 with
                                unpack=True, skiprows=0)          #Z=0.021
datfile16 = np.loadtxt("/home/user/Desktop/Desktop/Isochrones10a.dat", #0.87e9 with
                                unpack=True, skiprows=0)          #Z=0.021
datfile17 = np.loadtxt("/home/user/Desktop/Desktop/Isochrones11a.dat", #0.65e9 with
                                unpack=True, skiprows=0)          #Z=0.021

# the age of the NGC 6866 open cluster was estimated to be 0.86 Gyr
#PARSEC version 1.2S + COLIBRI PR16
#mass loss=0.0
#Av=0

```

In [173]: # the H-R diagram with a perfect fit isochrones of age = 0.86 Gyr was the plotted

Appendix C

Tables

Table C.1: Probable members of the NGC 6866 open cluster with KIC numbers. The columns are the KIC number; the distance (r) of the probable members from the cluster centre (arcminutes); distance modulus (σ); the right ascension (RA) and declination (Dec.) (J2000); parallax (milliarcseconds); proper motion in RA (pmRA) and proper motion in Dec. (pmDec.) (milliarcseconds per year); effective temperature (T_{eff}) (Kelvin) and luminosity (Lum) ($L_{\odot}$).

KIC	r (arcmin)	σ	RA (h:m:s)	Dec. (d:m:s)	Parallax (mas)	pmRA (mas/yr)	pmDec. (mas/yr)	T_{eff} (K)	Lum ($L_{\odot}$)
8198066	9.2149	10.8	20:04:29	+44:02:36	0.6893	-1.020	4.686	4967.48	0.419
8197998	8.1085	10.4	20:04:24	+44:03:21	0.8181	-7.458	-5.604	5658.70	1.443
8198083	7.9818	10.8	20:04:31	+44:04:42	0.6901	-2.960	-10.135	4905.00	0.268
8197481	9.0493	10.8	20:03:50	+44:00:29	0.7021	-1.184	-5.775	5068.54	0.387
8129760	9.5903	10.7	20:03:57	+44:59:55	0.7377	-2.965	-12.657	5418.50	3.740
8197567	8.1161	10.5	20:03:55	+44:01:23	0.8099	-4.231	-8.325	4966.95	0.705
8197313	9.2651	10.5	20:03:41	+44:00:34	0.7799	0.780	-4.496	5896.00	2.345
8197223	9.2848	10.6	20:03:35	+44:00:55	0.7589	-1.346	-5.824	5812.00	1.181
8197351	7.3543	10.3	20:03:43	+44:02:27	0.8757	0.853	1.275	6252.00	2.216
8197343	8.0681	10.4	20:03:43	+44:01:44	0.8511	-7.253	-21.014	5435.00	1.136
8197426	7.1169	10.6	20:03:47	+44:02:32	0.7482	-1.401	-5.876	5058.16	0.393
8197487	7.4784	10.9	20:03:51	+44:02:04	0.6626	-1.423	-5.736	5033.49	0.536
8197502	6.8023	10.3	20:03:52	+44:02:43	0.8515	1.529	-2.358	5509.93	1.427
8197270	6.7915	10.8	20:03:39	+44:03:23	0.6873	-1.320	-5.720	6109.67	1.950
8197326	4.6735	10.1	20:03:42	+44:05:29	0.9349	-3.641	-14.902	5725.00	2.125
8197793	7.8256	10.8	20:04:12	+44:02:16	0.6981	3.546	3.542	5612.10	1.525
8197651	6.7482	10.6	20:04:01	+44:02:50	0.7596	0.092	0.663	5357.95	1.050
8197696	4.1179	10.6	20:04:04	+44:05:44	0.7619	-1.621	-6.741	5209.53	0.695
8264835	3.1142	10.5	20:04:12	+44:08:38	0.7913	-11.860	-10.332	5443.52	1.905
8197461	4.5420	10.4	20:03:50	+44:05:04	0.8173	-1.502	-5.700	6153.48	2.587
8264611	2.9574	10.4	20:03:59	+44:06:38	0.8263	-6.858	-7.543	5357.00	0.957
8197607	3.6185	10.8	20:03:58	+44:05:55	0.6896	-1.346	-5.810	6313.75	2.839
8264592	3.0795	10.9	20:03:58	+44:06:28	0.6555	-1.236	-6.052	4901.99	51.464

8264448	3.5546	10.6	20:03:49	+44:06:06	0.7434	-0.376	-0.990	5060.00	0.690
8197356	4.2007	10.8	20:03:44	+44:05:51	0.7044	-1.657	-6.053	5703.00	2.209
8264476	1.9677	10.5	20:03:51	+44:07:42	0.7859	-5.464	-8.741	5014.91	0.229
8264400	1.9752	10.2	20:03:47	+44:08:11	0.9077	-0.601	-2.522	4956.01	0.230
8264538	2.0155	10.7	20:03:55	+44:07:29	0.7205	-1.238	-5.135	5878.76	1.355
8264589	1.7976	10.9	20:03:58	+44:07:46	0.6639	-1.476	-5.733	5955.50	4.391
8264706	1.7393	10.7	20:04:05	+44:09:30	0.7148	-1.450	-5.710	6684.00	4.058
8264708	1.7589	10.9	20:04:05	+44:09:34	0.6519	-1.346	-5.736	7101.91	7.275
8264655	1.2665	10.9	20:04:02	+44:09:14	0.6645	-0.943	-5.431	5430.19	3.117
8264581	1.2806	10.9	20:03:57	+44:08:17	0.6721	-1.450	-5.730	6710.13	6.829
8264549	1.0989	10.9	20:03:55	+44:08:24	0.6486	-1.131	-5.776	4884.62	57.289
8264541	0.3033	10.5	20:03:55	+44:09:12	0.7979	-1.052	-5.966	4972.33	0.524
8264563	2.1089	10.8	20:03:56	+44:11:36	0.7013	-1.341	-5.883	6022.40	2.695
8264657	1.5764	10.9	20:04:02	+44:10:27	0.6648	-1.313	-5.712	5763.00	1.779
8197011	9.7294	10.9	20:03:23	+44:01:37	0.6744	-9.182	-4.825	4937.86	0.610
8197057	7.9955	10.6	20:03:25	+44:03:35	0.7759	2.423	-8.823	5793.00	2.046
8196976	7.2628	10.5	20:03:21	+44:05:27	0.7832	-3.005	-8.673	4928.33	0.408
8197231	5.1028	10.5	20:03:36	+44:05:42	0.7850	9.440	5.987	5090.86	1.105
8264148	4.8775	10.1	20:03:32	+44:06:46	0.9503	-0.755	-5.703	6904.00	2.151
8263969	6.4908	10.9	20:03:22	+44:06:51	0.6475	0.689	-2.551	5617.50	1.713
8264076	5.5969	10.7	20:03:28	+44:06:35	0.7096	-10.562	-7.012	5426.05	1.867
8263835	7.7221	10.5	20:03:13	+44:07:39	0.7822	6.422	-1.266	5338.00	1.645
8263828	7.7725	10.9	20:03:12	+44:07:59	0.6532	4.113	-6.891	5200.00	0.762
8263787	8.0223	10.6	20:03:10	+44:08:52	0.7584	1.105	8.802	5755.00	3.935
8263793	8.1110	10.1	20:03:11	+44:07:59	0.9375	6.291	0.064	5018.00	0.562
8263821	7.8217	10.8	20:03:12	+44:10:53	0.6931	-2.074	-8.925	7315.61	16.716
8264304	3.7717	10.9	20:03:41	+44:06:40	0.6694	-1.237	-5.626	6705.67	4.352
8264274	2.7510	10.3	20:03:40	+44:09:23	0.8809	-1.513	-5.720	7095.00	3.056
8264326	2.3277	10.8	20:03:42	+44:10:06	0.6820	-0.740	-5.716	7559.25	14.860
8264363	1.9502	10.5	20:03:45	+44:08:47	0.7886	1.631	-2.225	4959.51	0.777
8264376	1.7553	10.7	20:03:46	+44:10:05	0.7289	-1.678	-5.454	5241.00	1.495
8330286	3.1719	10.8	20:03:47	+44:12:20	0.6938	-1.324	-5.834	7463.81	7.431
8330281	2.9592	10.9	20:03:47	+44:12:03	0.6687	-5.893	-5.262	5322.00	0.799
8264437	2.2736	10.7	20:03:49	+44:11:29	0.7104	-4.540	-2.640	5331.67	1.310
8264526	2.0530	10.7	20:03:53	+44:11:32	0.7310	-1.640	-5.784	7262.00	14.427
8263940	6.4054	10.9	20:03:20	+44:10:35	0.6638	2.502	-7.441	7305.00	7.265
8264046	5.4602	10.9	20:03:27	+44:11:33	0.6577	-6.572	-7.348	4888.33	0.332
8264121	4.8373	10.7	20:03:31	+44:11:41	0.7135	-1.311	-5.847	5380.12	1.199
8263901	6.9078	10.7	20:03:17	+44:11:03	0.7242	-5.265	-4.453	4912.50	0.501
8329790	7.8669	10.8	20:03:16	+44:13:06	0.6816	-1.420	-5.725	6955.00	6.314
8329839	7.0922	10.5	20:03:20	+44:12:42	0.7813	4.360	-9.576	5371.47	1.012
8330012	6.2745	10.1	20:03:30	+44:13:58	0.9358	-4.022	-10.538	4910.78	0.597
8330064	5.8429	10.4	20:03:33	+44:13:53	0.8361	-0.419	6.565	5370.00	0.969
8330088	6.1317	10.3	20:03:35	+44:14:26	0.8655	-8.681	-14.512	5313.00	2.326
8329908	8.4779	10.3	20:03:24	+44:15:53	0.8570	-1.231	-5.678	5780.50	2.060
8330073	7.9766	10.8	20:03:34	+44:16:32	0.7032	-8.130	-4.003	4972.67	14.020
8263634	9.7251	10.6	20:03:01	+44:08:04	0.7461	5.117	-4.253	5370.00	1.235
8263674	9.3294	10.9	20:03:03	+44:10:32	0.6521	0.144	3.896	5626.00	4.389

8263732	8.8433	10.7	20:03:07	+44:11:14	0.738	-1.617	0.565	4540.56	0.260
8329784	9.2509	10.9	20:03:16	+44:15:28	0.6603	-1.302	-5.788	5786.50	1.744
8329888	9.4929	10.7	20:03:22	+44:16:58	0.7261	-0.481	0.700	7861.16	19.930
8198117	8.3457	10.8	20:04:33	+44:04:37	0.6790	-1.490	-5.648	5802.49	1.274
8265392	9.4652	10.5	20:04:45	+44:06:27	0.7896	7.612	6.945	4895.37	2.695
8265267	8.4590	10.5	20:04:38	+44:06:10	0.7895	0.672	-3.876	4799.81	0.254
8198108	7.7039	10.6	20:04:32	+44:05:34	0.7558	5.851	2.659	5365.67	0.930
8198116	8.1186	10.9	20:04:32	+44:04:56	0.6581	-1.275	-5.820	6297.00	2.991
8265180	7.1439	10.3	20:04:33	+44:07:40	0.8767	-4.730	-9.173	4929.79	0.384
8265165	6.9947	10.5	20:04:33	+44:07:43	0.7871	-16.922	-1.732	4991.28	0.460
8265421	9.3189	10.4	20:04:47	+44:10:01	0.8148	0.510	5.647	4863.50	0.263
8265272	7.8170	10.2	20:04:39	+44:09:11	0.9059	5.148	0.750	5335.10	0.862
8265278	7.8953	10.3	20:04:39	+44:09:01	0.8629	4.493	0.961	4669.62	34.005
8265204	7.1406	10.5	20:04:35	+44:09:15	0.8128	1.482	-6.569	6462.00	7.146
8265332	8.3715	10.6	20:04:42	+44:09:30	0.7704	-3.426	-8.971	5795.00	4.310
8265274	7.8991	10.9	20:04:39	+44:08:28	0.6720	-3.639	-6.547	5528.05	1.849
8265197	7.1318	10.8	20:04:35	+44:08:49	0.6826	-1.364	-5.648	5485.00	1.237
8265317	8.2449	10.5	20:04:41	+44:09:50	0.7779	-7.560	-10.267	4732.54	0.262
8265428	9.5918	10.5	20:04:47	+44:11:33	0.7950	0.256	-0.897	5763.00	4.771
8331308	9.6720	10.5	20:04:46	+44:12:42	0.8019	-1.074	-3.335	4974.92	0.330
8331309	9.7932	10.4	20:04:46	+44:13:03	0.8413	-3.076	-2.884	4928.34	0.185
8265028	5.5906	10.4	20:04:23	+44:07:11	0.8361	3.918	-6.630	5019.76	0.786
8265061	5.4533	10.8	20:04:25	+44:09:57	0.6821	-3.280	-3.196	5011.50	0.641
8265014	4.9827	10.7	20:04:22	+44:10:17	0.7278	-1.507	-5.655	6185.00	2.354
8265150	6.9309	10.1	20:04:32	+44:11:31	0.9480	1.095	-5.036	5796.00	1.640
8265167	6.9607	10.5	20:04:33	+44:11:06	0.7921	1.875	2.350	4967.27	0.877
8265192	7.3805	10.4	20:04:34	+44:11:49	0.8133	-4.021	1.818	5149.81	0.707
8265244	7.7028	10.8	20:04:37	+44:11:15	0.6890	-1.366	-5.707	7351.50	8.424
8331020	7.7130	10.4	20:04:30	+44:13:59	0.8173	0.832	-2.104	5876.00	4.430
8264816	2.8323	10.1	20:04:11	+44:09:19	0.9398	-5.139	-10.729	4730.50	0.566
8264736	2.7322	10.5	20:04:06	+44:11:18	0.8123	-1.100	-7.312	4902.19	0.407
8264767	3.2247	10.8	20:04:08	+44:11:42	0.6908	-1.268	-5.890	7706.33	9.340
8330723	4.7354	10.5	20:04:13	+44:12:54	0.7951	12.792	-5.299	5475.25	1.087
8264903	4.1198	10.3	20:04:15	+44:11:27	0.8779	3.637	-7.964	4861.60	0.165
8330640	5.0615	10.8	20:04:09	+44:13:56	0.6935	-1.545	-6.081	5760.88	2.516
8330548	4.8075	10.8	20:04:04	+44:14:02	0.6877	-1.511	-5.701	5775.00	2.522
8330775	6.3085	10.6	20:04:16	+44:14:32	0.7462	1.843	2.361	5038.00	0.260
8330880	6.9729	10.9	20:04:21	+44:14:36	0.6679	2.612	0.336	4890.00	0.368
8330749	7.1268	10.6	20:04:15	+44:15:43	0.7478	-5.803	-7.532	5763.00	1.842
8331112	8.2171	10.2	20:04:35	+44:13:26	0.9261	5.283	-0.594	5776.67	1.673
8331274	9.8866	10.5	20:04:43	+44:14:14	0.7824	-1.322	1.010	5397.00	1.130
8331061	9.1709	10.9	20:04:32	+44:15:50	0.6540	0.516	4.006	5235.36	1.405
8331016	9.0591	10.4	20:04:30	+44:16:05	0.8414	-10.108	-14.736	4860.24	0.309
8330892	8.9527	10.9	20:04:22	+44:17:01	0.6538	-4.956	-19.280	4723.13	2.858
8330927	8.9015	10.4	20:04:24	+44:16:41	0.8226	-1.454	-5.854	4962.03	0.349
8330992	8.8590	10.4	20:04:28	+44:16:05	0.8239	1.381	10.759	4967.67	0.188
8396421	9.8412	10.9	20:04:22	+44:18:05	0.6560	-2.671	-9.846	4905.00	0.445
8396311	9.3491	10.6	20:04:15	+44:18:08	0.7421	4.997	4.208	5001.00	0.483

8396288	9.4553	10.9	20:04:13	+44:18:22	0.6572	-1.309	-5.703	4907.20	0.477
8396357	9.8131	10.4	20:04:18	+44:18:25	0.8380	-3.438	-8.523	5186.88	0.747
8330287	7.4988	10.1	20:03:47	+44:16:52	0.9698	2.101	-4.383	5767.00	3.234
8330371	6.2957	10.9	20:03:53	+44:15:47	0.6656	-1.814	-5.243	4049.64	139.424
8330647	8.3673	10.8	20:04:09	+44:17:30	0.6987	-1.260	-5.727	4937.72	0.433
8396143	9.5064	10.9	20:04:02	+44:18:55	0.6508	-3.750	-4.841	5698.33	1.692
8396084	9.0077	10.8	20:03:58	+44:18:30	0.6992	-3.470	-7.247	5001.00	0.496
8330216	7.8489	10.6	20:03:42	+44:17:01	0.7539	-1.217	0.573	5181.56	4.232
8330243	7.2027	10.4	20:03:45	+44:16:27	0.8411	0.788	-6.761	4751.65	0.186
8330218	7.7479	10.5	20:03:43	+44:16:55	0.7787	1.638	-0.251	4873.75	0.630
8395874	9.3755	10.2	20:03:44	+44:18:40	0.9003	-4.082	-10.113	5037.00	0.286
8330051	8.7213	10.8	20:03:33	+44:17:15	0.6938	-1.246	-6.007	5040.38	0.795
8396259	9.7114	10.9	20:04:11	+44:18:48	0.6566	-2.252	-4.916	7077.00	23.739

Table C.2: Data comparisons of 11 probable members of the cluster between GDR2 and GDR3.

KIC 8196903									
	r (arcmin)	σ	RA (h:m:s)	Dec. (d:m:s)	Parallax (mas)	pmRA (mas/yr)	pmDec. (mas/yr)	T _{eff} (K)	Lum (L _☉)
GDR2	8.1270	10.9	20:03:17	+44:05:00	0.6676	-1.535	-5.046	7672.00	12.569
GDR3	8.1270	10.9	20:03:17	+44:05:00	0.6760	-1.365	-5.217	8048.20	-
KIC 8197355									
	r (arcmin)	σ	RA (h:m:s)	Dec. (d:m:s)	Parallax (mas)	pmRA (mas/yr)	pmDec. (mas/yr)	T _{eff} (K)	Lum (L _☉)
GDR2	5.1949	10.5	20:03:43	+44:04:44	0.7866	-1.631	-5.862	7133.00	4.695
GDR3	5.1950	10.6	20:03:43	+44:04:44	0.7668	-1.394	-5.798	7735.50	-
KIC 8198066									
	r (arcmin)	σ	RA (h:m:s)	Dec. (d:m:s)	Parallax (mas)	pmRA (mas/yr)	pmDec. (mas/yr)	T _{eff} (K)	Lum (L _☉)
GDR2	9.2149	10.8	20:04:29	+44:02:36	0.6893	-1.020	4.686	4967.48	0.419
GDR3	9.2149	10.7	20:04:29	+44:02:36	0.7220	-0.958	4.711	5023.20	-
KIC 8197998									
	r (arcmin)	σ	RA (h:m:s)	Dec. (d:m:s)	Parallax (mas)	pmRA (mas/yr)	pmDec. (mas/yr)	T _{eff} (K)	Lum (L _☉)
GDR2	8.1085	10.4	20:04:24	+44:03:21	0.8181	-7.458	-5.604	5658.70	1.443
GDR3	8.1085	10.4	20:04:24	+44:03:21	0.8483	-7.459	-5.655	5741.40	-
KIC 8198083									
	r (arcmin)	σ	RA (h:m:s)	Dec. (d:m:s)	Parallax (mas)	pmRA (mas/yr)	pmDec. (mas/yr)	T _{eff} (K)	Lum (L _☉)
GDR2	7.9818	10.8	20:04:31	+44:04:42	0.6901	-2.960	-10.135	4905.00	0.268
GDR3	7.9819	10.4	20:04:31	+44:04:42	0.8162	-3.002	-10.140	4336.10	-
KIC 8197481									
	r (arcmin)	σ	RA (h:m:s)	Dec. (d:m:s)	Parallax (mas)	pmRA (mas/yr)	pmDec. (mas/yr)	T _{eff} (K)	Lum (L _☉)
GDR2	9.0493	10.8	20:03:50	+44:00:29	0.7021	-1.184	-5.775	5068.54	0.387
GDR3	9.0493	10.7	20:03:50	+44:00:29	0.7234	-1.304	-5.847	4856.60	-
KIC 8129760									
	r (arcmin)	σ	RA (h:m:s)	Dec. (d:m:s)	Parallax (mas)	pmRA (mas/yr)	pmDec. (mas/yr)	T _{eff} (K)	Lum (L _☉)
GDR2	9.5903	10.7	20:03:57	+44:59:55	0.7377	-2.965	-12.657	5418.50	3.740
GDR3	Does not appear in the catalog								
KIC 8197567									
	r (arcmin)	σ	RA (h:m:s)	Dec. (d:m:s)	Parallax (mas)	pmRA (mas/yr)	pmDec. (mas/yr)	T _{eff} (K)	Lum (L _☉)
GDR2	8.1161	10.5	20:03:55	+44:01:23	0.8099	-4.231	-8.325	4966.95	0.705
GDR3	8.1162	10.5	20:03:55	+44:01:23	0.7970	-4.462	-8.489	4985.50	-
KIC 8197313									
	r (arcmin)	σ	RA (h:m:s)	Dec. (d:m:s)	Parallax (mas)	pmRA (mas/yr)	pmDec. (mas/yr)	T _{eff} (K)	Lum (L _☉)
GDR2	9.2651	10.5	20:03:41	+44:00:34	0.7799	0.780	-4.496	5896.00	2.345
GDR3	9.2651	10.5	20:03:41	+44:00:34	0.7886	0.781	-4.567	6212.20	-

KIC 8197223									
	r (arcmin)	σ	RA (h:m:s)	Dec. (d:m:s)	Parallax (mas)	pmRA (mas/yr)	pmDec. (mas/yr)	T _{eff} (K)	Lum (L _⊙)
GDR2	9.2848	10.6	20:03:35	+44:00:55	0.7589	-1.346	-5.824	5812.00	1.181
GDR3	9.2848	10.7	20:03:35	+44:00:55	0.7091	-1.383	-5.815	6057.70	-

KIC 8197351									
	r (arcmin)	σ	RA (h:m:s)	Dec. (d:m:s)	Parallax (mas)	pmRA (mas/yr)	pmDec. (mas/yr)	T _{eff} (K)	Lum (L _⊙)
GDR2	7.3543	10.3	20:03:43	+44:02:27	0.8757	0.853	1.275	6252.00	2.216
GDR3	7.3543	10.2	20:03:43	+44:02:27	0.8944	0.911	1.262	6182.00	-