

Development of a variable-fill hydraulic dynamometer test bench

GM BENSON



orcid.org/0000-0002-2029-331X

Dissertation submitted in fulfilment of the requirements for the
degree *Master of Engineering in Mechanical Engineering* at the
North-West University

Supervisor: Dr JH Kruger

Graduation: June 2021

Student number: 23514922

Abstract

The WJ Engineering company required a torque absorption device to be used in their newly developed Hydro Testing Facility. The torque absorption device is intended to load hydraulic turbine models that will be tested in the facility, with the purpose of validating and verifying the turbine's performance and control system. To do this, the torque absorption device must be able to measure the torque and power delivered by the models and be able to change the load during continuous operation. The variable-fill hydraulic dynamometer was subsequently identified as possible candidate for application.

The hydraulic dynamometer is a torque measuring device that converts mechanical energy into heat by means of friction, turbulence and incidence losses of a fluid mass circulating inside working compartments within a rotor (connected to the machine under test) and two stators (fixed to the casing). The conversion of energy produces torque that acts on the dynamometer casing which is resisted by a torque arm with load cell, thus enabling torque measurements.

The fluid pressure generated inside the working compartments forces a throughflow of fluid, which dissipates heat to allow for cooling of the dynamometer. The torque absorption and throughflow of the dynamometer can be controlled by an outlet control valve. The entire setup, with the dynamometer, the control valve and an external cooling cycle is known as a variable-fill hydraulic dynamometer test bench.

These test benches are not readily available off-the-shelf and therefore the aim of this study was to develop one suitable for the testing facility. There exists little open literature on the simulation and design of variable-fill hydraulic dynamometers, and therefore almost no established theory to explain and predict their behaviour. However, a one-dimensional theoretical model presented by Hodgson (1991) and J.K. Raine & P.G. Hodgson (1991,1992) claimed to accurately simulate the behaviour of variable-fill hydraulic dynamometers. Their method was implemented during this study to analyse and develop the test bench, and the method was evaluated for its suitability as design tool and applicability in system simulation.

WJ Engineering specified device requirements of a torque loading up to 300 [Nm] at an operating speed of 1200 [rpm] and power absorption capability of 38 [kW]. Due to time constraints WJ Engineering already designed and manufactured the rotor and stators prior to the current study. These rotor and stators were subsequently analysed and used as a basis from which the additional test bench components were developed. Simulation of the WJ Dynamometer by means of the theoretical model, predicted that it would achieve a maximum torque absorption of 1154 [Nm] at 1200 [rpm], hence a 145 [kW] power capability. The 300 [Nm] and 38 [kW] torque and power requirement from WJ Engineering was met at dynamometer percentage fill conditions of as low as 30% according to the simulation.

Additional requirements included a throughflow of 1 [l/s] at low torque and 4 [l/s] at high torque for proper cooling. It was determined that a control valve with a pressure drop of 39 – 45 [kPa] at 1 [l/s] flow rate, and 5 – 15 [kPa] at 4 [l/s], should allow the required throughflow. A control valve and flow meter were manufactured and tested to ensure specifications were satisfied.

After commissioning the test bench, it was found that torque absorption far exceeded the original specification, with an ability to absorb torque between 313.9 to 372.8 [Nm] at a rotational speed

as low as 800 [rpm]. Even though the torque was above specification, the test bench did not achieve the full power requirement, only reaching a power absorption of 32 [kW] within load cell limits. The throughflow achieved during testing ranged from 0.8 to 1.8 [l/s] and consequently the dynamometer overheated after a limited period of operation. The outlet control valve was also found to exhibit unsatisfactory control behaviour with relation to the torque absorption of the test bench.

Based upon these results it was concluded that the test bench, without further modifications, was not yet suited for use in the WJ Hydro Testing Facility. The theoretical model was deemed to possibly be a sufficient analysis and design tool, since the results fell within the range of predicted values of the simulations, but this cannot be confirmed due to the improper flow control characteristics of the outlet control valve which influences the torque characteristic of the dynamometer. It is suspected that the lack of control on the inlet flow of the dynamometer could be the reason that adequate throughflow for cooling and torque control was not reached. The lack of inlet flow control and other shortcomings thus warrant future research.

Nevertheless, a functioning variable-fill hydraulic dynamometer test bench was successfully developed with the capability to absorb load from devices such as a hydraulic turbine, albeit for a slightly smaller power envelope than originally intended.

Keywords: hydraulic testing cycle, hydraulic turbines, variable-fill dynamometers, torque measurement

Dedication

All that I am is due to the love of our Lord and God. I believe that nothing exists without Him and that Jesus is the only truth. Therefore, I dedicate this work to the glory of His being.

To my wife, Marijke, for all your love and support.

To my father and mother. Johan and Karien, for being there from the beginning.

To my father and mother in law, Eric and Magda for all your thoughts and prayers.

Declaration

I, **G.M. Benson** hereby declare that

1. I understand what plagiarism is and am aware of the University's policy in this regard.
2. I know that "plagiarism" means using another person's work and ideas without acknowledgement, and pretending that it is one's own. I know that plagiarism not only includes verbatim copying, but also the extensive (albeit paraphrased) use of another person's ideas without acknowledgement. I know that plagiarism covers this sort of use of material found in textbooks, journal articles, theses AND on the internet.
3. "MY PROJECT TITLE" submitted in the fulfilment in the requirement for the Master's degree in Mechanical Engineering (M-Eng) is my own original work.
4. Where other people's work has been used (either from a printed source, Internet or any other source), this has been properly acknowledged and referenced in accordance with departmental requirements.
5. I have not used work previously produced by another student or any other person to hand in as my own.
6. I have not allowed, and will not allow, anyone to copy my work with the intention of passing it off as his or her own work.

G.M. Benson

STUDENT

2020/12/18

DATE

Acknowledgements

I want to thank Dr. Jan-Hendrik Kruger for his guidance, valuable inputs and all the opportunities that he has provided me throughout the study.

I also wish to thank Mr. Werkman for the knowledge and expertise he provided and the opportunity to do the project at his company.

Appreciation also goes to Ederik, Juandre and Theo for their assistance during my time at WJ Engineering.

Contents

List of Figures	ix
List of Tables	xii
Nomenclature	xiii
1. Introduction	1
1.1. Background	1
1.1.1. Hydraulic turbine testing facility	1
1.1.2. Validation and verification of model hydraulic turbines	3
1.2. WJ Engineering hydro testing facility	4
1.3. Problem statement	7
1.4. Research aim and objectives	7
1.4.1. Primary objectives:	7
1.4.2. Secondary objectives:	8
1.5. Delineations and limitations	8
1.5.1. Delineations:	8
1.5.2. Limitations:	9
1.6. Significance (rationale)	9
1.6.1. Industrial significance:	9
1.6.2. Academic significance:	9
1.7. Chapter overviews	9
1.8. Summary	10
2. Literature review	11
2.1. Overview	11
2.2. Torque absorbers	11
2.3. Hydraulic dynamometers	13
2.4. Variable-fill hydraulic dynamometers	16
2.5. Variable-fill hydraulic dynamometer simulation	20
2.6. Summary	20
3. Theory	22
3.1. Introduction	22
3.1.1. Basic features of one-dimensional dynamometer model	22
3.1.2. Velocity diagrams for rotor and stator	25
3.1.3. Overview of energy dissipation mechanisms	28
3.1.4. Assumptions required by one-dimensional model	28
3.2. Conservation laws	29
3.2.1. Conservation of mass	29
3.2.2. Conservation of energy	31
3.2.3. Conservation of angular momentum	32
3.2.4. Unsteady flow component and dynamic operation	36
3.3. Energy Dissipation	36
3.3.1. Incidence losses	37

3.3.2.	Frictional losses	40
3.3.3.	Secondary circulation losses	45
3.3.4.	Empirical loss coefficients	47
3.4.	Supplementary equations	49
3.4.1.	Partial and varying fill	49
3.4.2.	Fluid pressure	53
3.4.3.	Fluid outflow from working compartment	64
3.5.	Summary	74
4.	Research methodology	75
4.1.	Overview	75
4.2.	Verification of integrated one-dimensional model	76
4.2.1.	Verification of vortex characteristics	78
4.2.2.	Verification of torque and power	80
4.3.	Integrated one dimensional model analysis (Steady state)	81
4.3.1.	Vortex characteristics	81
4.3.2.	Torque and power	82
4.3.3.	Vortex pressure	84
4.3.4.	Fluid inflow	86
4.3.5.	Fluid outflow considerations	86
4.4.	Torque control and flow rate measurement	90
4.4.1.	Outlet control valve	91
4.4.2.	Venturi flow meter	95
4.4.3.	Validation of control valve and Venturi flow meter	97
4.5.	Dynamometer test bench for simulation validation	106
4.5.1.	Dynamometer test bench development	106
4.5.2.	Dynamometer characterisation method	110
4.6.	Summary	110
5.	Results and discussion	111
5.1.	Overview	111
5.2.	Venturi and control valve characterisation	111
5.2.1.	Venturi flow meter	112
5.2.2.	Outlet control valve	113
5.3.	Dynamometer characterisation	114
5.3.1.	Torque and power	118
5.3.2.	Flow rate and pressure	119
5.4.	Summary	122
6.	Conclusion and recommendations	123
6.1.	Overview	123
6.2.	Summary of findings	123
6.2.1.	Dynamometer performance simulation	123
6.2.2.	Dynamometer torque control	124
6.2.3.	Dynamometer torque and power	125
6.2.4.	Dynamometer cooling	125
6.3.	Conclusion	125
6.3.1.	Variable-fill hydraulic dynamometer test bench	126
6.3.2.	One dimensional theoretical model	126
6.3.3.	Suitability of test bench for use in the hydro testing facility	128
6.4.	Recommendations for further study	129
6.5.	Summary of contributions	130

7. REFERENCES	131
A. Identities for cup boundary geometric functions	135
B. Definite integrals	138
C. Air-water interface calculation in MATLAB	139
D. Steady state torque calculation in MATLAB	141
E. Vortex Pressure Calculation in MATLAB	149
F. Fluid outflow and pressure loss calculation Matlab	152
G. FloEFD setup for angle seat valve simulation	156
G.1. Geometrical properties	156
G.2. Initial settings	157
G.3. Meshing	157
G.4. Boundary conditions and goals	157
G.5. Simulation results	159
H. FloEFD setup for Venturi simulation	162
H.1. Geometrical properties	162
H.2. Simulation properties	162
H.3. Boundary conditions and goals	162
H.4. Simulation results	163

List of Figures

1.1. Scheme of typical hydro test facility and test loop (Thapar, 2002:135)	2
1.2. WJ Engineering hydro test facility layout (WJ Engineering, 2019)	2
1.3. Francis turbine model efficiency hill chart for water head of (a) 100 m, (b) 200 m, (c) 300 m, and (d) 500 m. (Guo et al.,2017:4)	3
1.4. Model Francis Turbine	4
1.5. WJ Engineering Hydro Test Facility layout	5
1.6. WJ Hydro Test Facility: axial pump, VFD and low pressure water tank	6
1.7. WJ Hydro Test Facility: outflow pipe from high pressure water tank, turbine housing and data logging system	6
2.1. Schenck, dry gap, disc type eddy-current dynamometer 1. Rotor; 2. Rotor shaft; 3. Coupling flange; 4. Water outlet with thermostat; 5. Excitation coil; 6. Dynamometer housing; 7. Cooling chamber; 8. Air gap; 9. Speed pick-up; 10. Flexure support; 11. Base; 12. Water inlet; 13. Joint; 14. Water outlet pipe (Martyr & Plint, 2007:147)	12
2.2. Hydrokinetic dynamometer, principle of operation: (a) section through dynamometer; (b) end view of rotor; (c) development of section a-a of rotor and casing; (d) representation of toroidal vortex (Martyr & Plint, 2007:172)	14
2.3. Dynamometer vanes and water flow path. (Hodgson, 1991:4)	15
2.4. Schematic section of dynamometer with end view of rotor. (Hodgson, 1991:4)	15
2.5. Open loop torque characteristic of dynamometer. (Raine & Hodgson, 1991:155)	17
2.6. Dynamometer torque curve combined with turbine torque curve	17
2.7. Torque-speed characteristic at constant control valve settings for F24 dynamometer with back pressure valve control (Raine & Hodgson, 1991:155).	18
2.8. Cross-section diagram of F0351 Dynamometer. (Hodgson, 1991:4)	19
2.9. Torque arm and load cell. (Horiba, 2006:9).	20
3.1. Uniform Velocity Distribution and Effective Radii Geometry (Raine & Hodgson, 1991:16)	23
3.2. Linear Distributed Flow and Working Compartment Geometry (Raine & Hodgson, 1991:18)	24
3.3. Full rotor dynamometer velocity diagrams (Hodgson, 1991:19)	25
3.4. Velocity diagram for rotor incidence and angled vanes	26
3.5. Velocity diagram for stator incidence and angled vanes	26
3.6. Velocity diagram for rotor incidence and axial vanes	27
3.7. Velocity diagram for stator incidence and axial vanes	27
3.8. Vane effect on flow area (Hodgson, 1991:19)	29
3.9. Rotor Control Volume and Angular Momentum Components (Raine & Hodgson, 1991:26)	34
3.10. Friction surfaces and streamtubes (Raine & Hodgson, 1991:41)	40
3.11. Rotor and stator quadrant division (Hodgson, 1991:42)	41
3.12. Circumferential view used in the determination of the friction factor (Raine & Hodgson, 1991:50)	48
3.13. Mass Integral in Rotor Control Volume (Raine & Hodgson, 1991:29)	50

3.14. Fluid Position, Velocity and Acceleration Vectors (Raine & Hodgson, 1991:58)	54
3.15. Pressure Forces on a Fluid Element (Raine & Hodgson, 1991:59)	55
3.16. Fluid Acceleration and Vector Cross Products (Raine & Hodgson, 1991:60)	56
3.17. Directions of Vector Cross Product Radial Components (Raine & Hodgson, 1991:62)	58
3.18. Determination of Distance to Air-Water Interface (Raine & Hodgson, 1991:66)	62
3.19. Cup and Vortex Angle Conversion (Raine & Hodgson, 1991:66)	63
3.20. Inflow and Outflow of Dynamometer	65
3.21. Fluid Outflow Paths from Dynamometer Working Compartment	66
3.22. Flow-path Features Inside Dynamometer	67
3.23. Flow-path Features Outside Dynamometer	67
3.24. Flow Velocity at Rotor Cup Periphery (Raine & Hodgson, 1991:76)	68
3.25. Combining flow K_{1-2} (Sharp <i>et. al.</i> 2010:52 52)	72
3.26. Combining flow K_{1-4} (Sharp <i>et. al.</i> 2010:52)	72
4.1. Calculation flow diagram	77
4.2. Reynolds number vs. RPM for F020	79
4.3. Friction factor vs. Reynolds number for F020	79
4.4. Torque vs. speed comparison between model and Hodgson	80
4.5. Variation of air-water interface radii with changes in percentage fill	81
4.6. Variation of vortex velocity related to rotor speed for different fill percentages	82
4.7. Simulated distribution of Torque Characteristic with Percentage Fill of WJ Dynamometer	83
4.8. Simulated distribution of power characteristic for different fill percentages of WJ Dynamometer	84
4.9. Detailed view of simulated power of WJ Engineering Dynamometer. Dashed lines indicate power level of 38 [kW] at 1200 [rpm] and 30% PF.	84
4.10. Variation of fluid pressure distribution with different fill percentages at 1200 [rpm]	85
4.11. Fluid pressure locations in stator for varying speed and different fill percentages	86
4.12. Fluid energy at cross inlet legs for 1[l/s] total flow	88
4.13. Fluid energy at cross inlet legs for 4[l/s] total flow	89
4.14. Desired valve pressure loss at total rate of fluid flow, for different fill percentages	90
4.15. Off the shelf needle valve	91
4.16. Angle seat valve model	92
4.17. Simulated flow trajectories of angle seat valve at 2 [l/s] flow and 10 turns valve closure	93
4.18. Simulated pressure loss characteristic of angle seat valve	93
4.19. Simulated vs desired pressure loss characteristics for angle seat valve	94
4.20. WJ Engineering Venturi flow meter CAD model	95
4.21. Pressure distribution through Venturi flow meter	96
4.22. Venturi flow meter pressure drop characteristic	97
4.23. Control valve and Venturi flow meter characterisation cycle CAD model	98
4.24. Control valve and Venturi flow meter characterisation cycle schematic	99
4.25. Rapid Allweiler CLT40-250 centrifugal pump before refurbishment	101
4.26. Pump curves at different impeller speeds vs. system curves for almost fully open and closed valve (With closed needle valve)	101
4.27. CLT40-250 pump after refurbishment	102
4.28. Druck PTX1400 transducer and IFM Electronic PX9118 electronic pressure sensor	103
4.29. Endress+Hauser Memograph M RSG45	103
4.30. IFM PLC and data logger installed at WJ Engineering	104
4.31. Pressure sensors installed for calibration	104
4.32. WJ Engineering Hydro Test Facility large water tank	105

4.33. Dynamometer test bench characterisation, torque and speed measurement	107
4.34. Section of dynamometer casing	108
4.35. Detail of dynamometer casing showing shaft and bearing arrangement	108
4.36. Dynamometer test bench auxiliary cooling cycle	109
5.1. Control valve and Venturi flow meter characterisation cycle	111
5.2. Venturi flow meter with installed pressure transducers	112
5.3. Venturi pressure drop characteristic, experimental vs. theoretical results	112
5.4. Valve pressure drop characteristic, experimental vs. theoretical results	113
5.5. Outlet control valve manufactured from custom design	114
5.6. Variable-fill hydraulic dynamometer test bench	115
5.7. Dynamometer load cell and proximity speed sensor	116
5.8. Cooling cycle pipework running adjacent to water tank	116
5.9. Submerged radiator inside water tank	117
5.10. Inline filter to clean inlet water	117
5.11. Dynamometer torque characteristic experimental results	118
5.12. Dynamometer torque characteristic experimental vs. theoretical results	119
5.13. Venturi pressure measurements from dynamometer characterisation cycle	120
5.14. Dynamometer outflow rate measured from dynamometer characterisation cycle .	121
A.1. Mass Integral in Rotor Control Volume (Raine & Hodgson, 1991:29)	135
G.1. Various lids used to seal openings in the control valve CAD model	156
G.2. Valve simulation mesh for open valve	158
G.3. Valve simulation mesh refined during valve closure	158
G.4. Valve simulation boundary conditions and surface goals	159
G.5. Valve pressure drop simulation for 10 valve turns and 1 [l/s]	160
G.6. Valve pressure drop simulation for 10 valve turns and 4 [l/s]	160
G.7. Valve pressure drop simulation for fully open	161
G.8. Valve pressure drop simulation for almost fully closed	161
H.1. Venturi CAD model for simulation	162
H.2. Venturi simulation boundary conditions and goals	163
H.3. Venturi simulation pressure contours	163
H.4. Venturi simulation flow trajectories	163
H.5. Venturi simulation parametric study results	164

List of Tables

4.1. Comparison of ω/ω_p	78
4.2. Air-water interface boundaries comparison	78
4.3. Surface roughness compared to Torque, Hodgson vs. Model	80
4.4. Air-Water interface radii for WJ Engineering Dynamometer	81
4.5. Calculated pressures at various fill percentages in dynamometer working compartment	87

Nomenclature

Acronyms and Abbreviations

CAD	Computer Aided Design
CFD	Computational Fluid Dynamics
CLT	Chemical Process Pumps
CS	Control Setting
EFD	Engineering Fluid Dynamics
HTGS	Hydraulic Turbine Governing System
PID	Proportional Integral Derivative
PLC	Programmable Logic Controller
RMS	Root-Mean-Square
rpm	Rotation Per Minute
VFD	Variable Frequency Drive

List of Symbols

$\dot{M}, \dot{m}$	mass flow rate	kg/s
$\dot{P}_F$	power loss due to friction	W
$\dot{P}_I$	input power	W
$\dot{P}_{SC}$	power loss due to secondary circulation	W
A	area	m^2
a	acceleration	rad/s^2
C	loss coefficient; integration constant; cross Product cofactor	—
CS	control surface	—
CV	control volume	—
d_h	hydraulic diameter	m
D_L	arbitrary thickness of streamtube, lower half	—
D_U	arbitrary thickness of streamtube, upper half	—
E	equivalent fluid energy	J
f	friction factor	—
F_P	pressure force	N
F_S	surface force	N

h	mass conservation for stator as function of R_{CI} and R_{CO}	—
J	Jacobian matrix	—
j	definition of percentage fill as a function of R_{CI} and R_{CO}	—
K	loss factor	—
L	flow path length	m
P	power; fluid pressure; pressure loss	$W; Pa; Pa$
PF	percentage fluid fill	%
Q	volume flow rate	m^3/s
R	radius from machine axis	m
r	radius from machine axis, position vector	m
R_C	radius to centre of working compartment	m
R_I	working compartment inner radius	m
R_M	mean flow path radius	m
R_O	working compartment outer radius	m
r_o	radius to where pressure equals atmospheric pressure	m
r_v	radius from vortex centre	m
R_Y	radius to centre of air pocket	m
R_{CI}	air-water interface inner radius	m
R_{CO}	air-water interface outer radius	m
R_{EI}	inner effective radii of vortex	m
R_{EO}	outer effective radii of vortex	m
Re	Reynolds number	—
T	torque	Nm
t	dynamometer vane thickness	m
V	absolute fluid velocities	m/s
$V_{\theta I}$	inner tangential water velocity	m/s
$V_{\theta O}$	outer tangential water velocity	m/s
V_{xyz}	relative fluid velocities	m/s
W_A	axial flow velocity	m/s
W_R	relative fluid velocity from rotor	m/s
W_S	relative fluid velocity from stator	m/s
Z	dynamometer vane number	—

List of Greek symbols

α	dynamometer vane angle	degrees
----------	------------------------	---------

ϵ	pipe relative roughness	m
μ	dynamic viscosity of fluid	$\frac{kg}{ms}$
ω	vortex angular velocity	rad/s
ω_p	dynamometer rotor angular velocity	rad/s
Φ	dimensionless flow coefficient	—
Ψ	dimensionless head coefficient	—
ρ	density of fluid	kg/m^3
θ	angle around working compartment	$degrees$
θ_D	angle for distance to air-water interface as pressure boundary condition	$degrees$

List of Subscripts

1	at inlet to cross from flow path C
2;3	at inlet to cross from flow path B
4	at outlet from cross to flow path D
B	flow Path B, from rotor periphery
C	flow path C, from stator outlet holes
cp	flexible corrugated pipe
D	flow path D, from outlet of cross to outlet control valve
R	rotor
S	stator
vent	venturi
WV; WR; WWR	cross product cofactors

Definitions

CAD-cleanup The process by which a CAD model is simplified for simulation purposes. Usually by removing parts, cutouts or protrusions which should have little to no effect on the result.

Fluid coupling In this study fluid coupling or hydraulic coupling refers to a hydrodynamic or 'hydrokinetic' device that is used to transmit rotating mechanical power.

Front Loading CFD This refers to the practice of moving CFD simulation early into the design process where it is useful to engineers to analyse tendencies and eliminate design deficiencies early in the development process (FloEFD, 2018)

Hydraulic turbine model According to Munson's Fundamentals of Fluid Mechanics (Munson *et al.*, 2013:368) a model is a representation of a physical system that may be used to predict the behaviour of the system in some desired respect. Therefore, a hydraulic turbine model as referred to in this study is a small-scale model of a prototype hydraulic turbine design to be used in the hydro testing facility to validate and verify the predicted performance of the prototype turbine's design.

Prototype hydraulic turbine A prototype is the physical system for which performance predictions are made (Munson *et al.*, 2013:368). The prototype hydraulic turbines mentioned in this study are the new turbine designs to be developed and utilised by WJ Engineering.

1. Introduction

1.1. Background

“Africa, in particular, has barely begun to exploit its renewable energy potential. Energy analysts say that solar, wind, hydro, geothermal and biomass sources could more than supply the energy needs of every continent” Jeremy Rifkin

In our energy hungry world, the need for abundant and renewable energy has become even more dire. One such energy source that is readily available and can be found in abundance in Sub-Saharan Africa is hydro power, yet it remains mostly unused (Murenzi & Ustun, 2015:1-6).

For this reason WJ Engineering, situated in Potchefstroom, seeks to develop efficient and cost-effective hydro power turbines and facilities to be implemented in Sub-Saharan African countries with an abundance of hydro power opportunities that have not yet been exploited.

1.1.1. Hydraulic turbine testing facility

For the past couple of years WJ Engineering was developing a hydro testing facility to be used in the design, development and testing of hydraulic turbines. At the WJ hydro testing facility, homologous hydraulic turbine models will be designed, manufactured and tested. These hydraulic turbine models are to be used as the starting point for consecutive designs, from which they seek to develop hydro power generating facilities for Sub-Saharan Africa.

In hydro testing facilities such as the WJ testing facility, model testing serves several purposes, but are primarily used to validate and verify the predicted performance based on the turbine's design and the control system of the hydraulic turbine (Arter & Meier, 1990:66). Correctly interpreted the model test data can be used as a reliable indication of the performance of the units in the field (Steele, 2007:3), which allows the suitability of the turbine design to be confirmed for its specific application.

The main performance parameters of a hydraulic turbine model to be tested for are power, flow rate, efficiency and specific energy (Kavurmaci et al., 2017:70). Therefore, in a performance test, the torque and rotational speed of the model are measured alongside the operational head and flow rate to allow for the calculation of the efficiency from which a resulting hill chart of the turbine's performance can be generated (Kavurmaci et al., 2013:876).

In addition to the performance tests, it is also required to test and verify the flow control mechanism of the turbine models. Hydraulic turbines are usually coupled directly to an electric generator and since the generator must run at a constant speed, the speed of the turbine should also remain constant when the load changes (Sayers, 1990:88). To achieve the constant operating speed, reaction turbines varies the flow rate across the turbine runner through the use of adjustable guide vanes, such as in the case of Francis type turbines. An alternative technique giving more control is to use adjustable guide vanes in conjunction with adjustable turbine blades, such as in the case of Kaplan turbines (Munson et al., 2013:665).

A governing system is used to adjust the gate and blade angles in response to turbine speed changes resulting from load or head changes. The governing system must be of sufficient accuracy and sensitivity to account for changes in speed due to load changes, even in cases when there is a sudden rejection of the full load, such as when the generator breaker trips (Steele, 2007:3).

Thus, in the testing of both the performance of the turbine and the control mechanism there is the need to have a device that is capable of exerting a load on the turbine which can be measured as well as adjusted. Fig. 1.1 shows a typical layout of a hydro testing facility with Fig. 1.2 that shows a CAD rendering of the Hydro Testing Facility that was developed at WJ Engineering, derived from the system layout shown in Fig. 1.1.

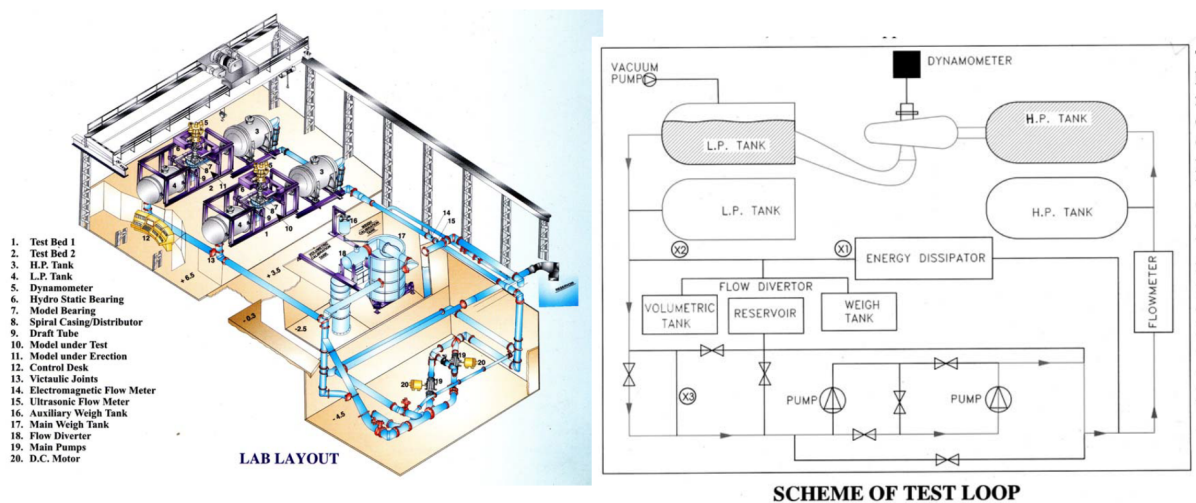


Figure 1.1. Scheme of typical hydro test facility and test loop (Thapar, 2002:135)



Figure 1.2. WJ Engineering hydro test facility layout (WJ Engineering, 2019)

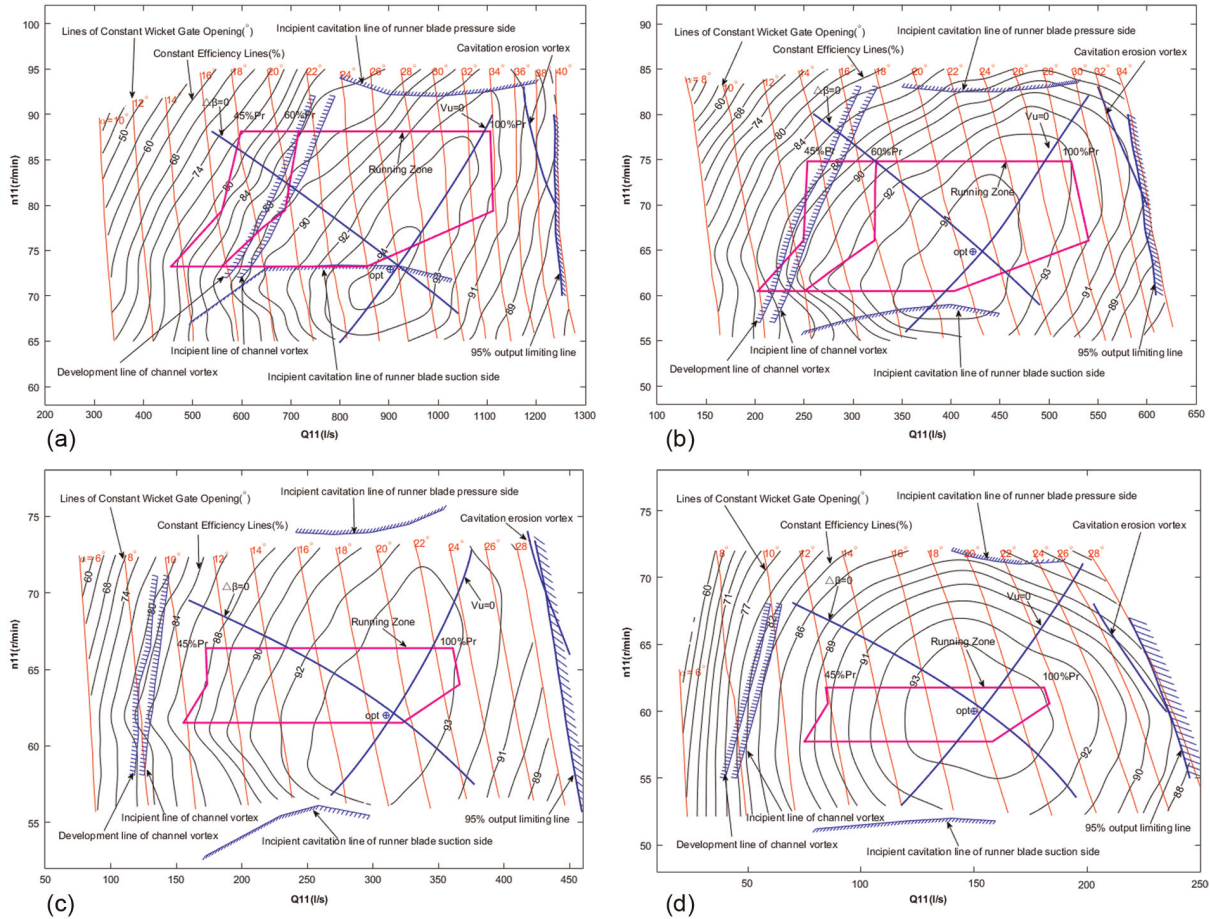


Figure 1.3. Francis turbine model efficiency hill chart for water head of (a) 100 m, (b) 200 m, (c) 300 m, and (d) 500 m. (Guo et al.,2017:4)

1.1.2. Validation and verification of model hydraulic turbines

For validation purposes, it is important to note that all model tests should be done in accordance with the IEC 60193:1999 standard (Kavurmaci et al., 2017:70). At a facility such as shown in Fig. 1.1 the head and flow characteristics for which the turbine is designed should thus be simulated in order to test the behaviour of the turbine under those conditions. A torque absorption device is used to load the turbine at varying rotational speeds and control settings in order to obtain the power produced by the turbine at these different head and flow characteristics. The torque absorption device therefore allows the calculation of the turbine efficiency at different rotational speeds, heads, flow rates and guide vane angles which can be used to generate the model turbine's specific hill charts. In Fig. 1.3 a typical turbine hill chart is shown, with characteristic data plotted under four different water head conditions.

The model's efficiency hill chart will give insight into the hydraulic performance of the turbine prototype as the external characteristics of head and flow change; this provides engineers with a guide to determining the suitability of the turbine for optimum performance and the limitations of using the turbine under certain operating conditions (Guo et al., 2017:1).

The hydraulic turbine flow control mechanism — also called the hydraulic turbine governing system (HTGS) — is one of the most important parts of a hydropower plant. The HTGS plays a key role in maintaining safe, stable and economical operation of the hydropower plant, therefore, the study of the governing system becomes a necessary and important issue (Zhang et al., 2015:128).



Figure 1.4. Model Francis Turbine

A turbine will experience load changes caused by the generator as the electrical consumption on the grid varies, which can cause the turbine to run away as the load decreases or to lose speed as the load increases. In such cases the turbine speed should be corrected by the governing system to allow the generator to deliver electricity at the required frequency of 50 Hz. The turbine testing facility should be capable of simulating these load changes during continuous operation to allow for the verification of the governing system, a requirement to be achieved by the use of the torque absorption device.

Therefore, the torque absorption device implemented at the hydro testing facility must have the characteristics of absorbing a changing load from the turbine at varying rotational speeds to determine the turbine performance at different external flow and head characteristics, and should also have the ability to change its load during continuous operation to test the turbine governing system's responsiveness. From these two requirements we thus find the purpose of the study, which is to develop a torque absorption device that is suitable for use in the hydro testing facility, with the appropriate properties of absorbing and measuring a changeable load during continuous operation.

There are various torque absorption devices that have the characteristics required in the hydro testing facility and are considered in more detail in the literature study see Section 2.2. However the research shows that hydraulic dynamometers, torque absorption devices that work based on energy transfer to a hydraulic fluid in order to absorb power, would probably be the most suitable as will be explained in Section 2.3.

In the next section the details are discussed of the WJ Engineering Hydro Testing Facility, for which the torque absorption device is required.

1.2. WJ Engineering hydro testing facility

As stated in the previous section, the hydro testing facility at WJ Engineering was developed with the purpose of generating different flow rates and pressure heads for a turbine model in order to perform the desired model tests. The hydro testing facility however lacks a device with the ability to load the model turbine for the purpose of performance tests, hence the purpose of the study to develop such a device.

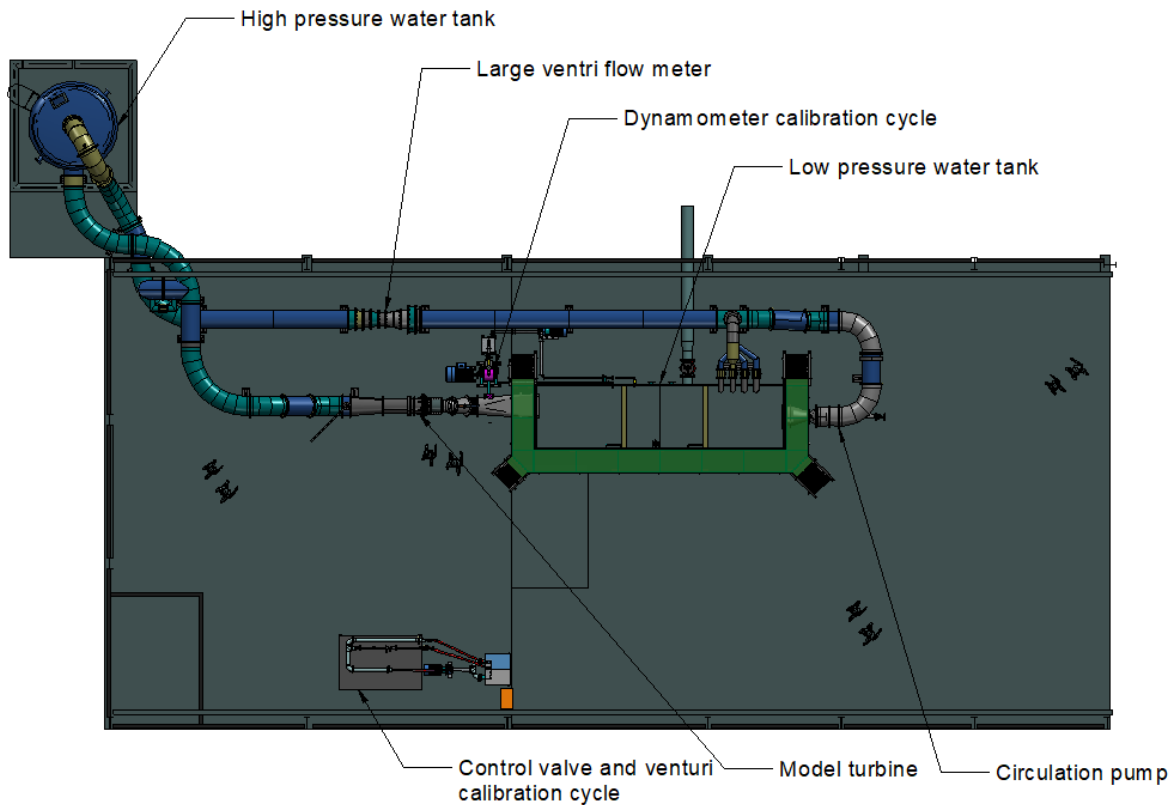


Figure 1.5. WJ Engineering Hydro Test Facility layout

The Hydro Testing Facility employs an axial pump — of which the speed is controlled by a Variable Frequency Drive (VFD) — to pump water from a low-pressure water tank into a high-pressure water tank in order to simulate the pressure head feeding the model turbine to be tested. Flow control valves and the speed-controlled pump are responsible for providing and varying the water flow rate required for the model tests. A Venturi flow meter is employed to accurately measure the flow rate produced. (Refer to Fig. 1.2 and Fig. 1.5 for the facility layout.)

Measurements, such as pressure, flow rate and rotational velocity can be taken instantaneously at all relevant points in the cycle, when used during model tests and evaluation. Fig. 1.6 and Fig. 1.7 show the physical hydro testing facility currently found at WJ Engineering.

The WJ Hydro Test Facility was developed according to the following specifications:

- To provide the equivalent of 10 [m] pressure head onto the turbine at 300 [l/s] flow rate.
- To provide the capability to test model turbines of up to 38 [kW] power generation capacity.

Based on the specifications of the hydro test facility as prescribed by WJ Engineering, a torque absorption and measuring test bench with the following capabilities are thus needed:

- To resist and measure the model turbine power output of 38 [kW] at 1200 [rpm] with a corresponding torque of 300 [Nm].
- The device should be able to vary the torque resistance during continuous operation and at varying rotational speeds.



Figure 1.6. WJ Hydro Test Facility: axial pump, VFD and low pressure water tank



Figure 1.7. WJ Hydro Test Facility: outflow pipe from high pressure water tank, turbine housing and data logging system

Prior to the current study, WJ Engineering did their own independent design calculations to find the rotor and stator geometries of a variable-fill hydraulic dynamometer, that would be suitable for the purpose of loading a turbine according to the required specifications. Thus the manufacturing process of a rotor and stator of a variable-fill hydraulic torque measuring device was already under commencement when the author became a part of the research project.

Existing literature was therefore investigated as part of the current study to confirm if the torque measuring device chosen by WJ Engineering was suitable, and the subsequent analysis and development of a suitable test bench was done based upon the rotor and stator as manufactured by WJ Engineering.

The next section provides the problem statement of the study, derived directly from the requirements and specifications of the WJ Engineering Hydro Testing Facility.

1.3. Problem statement

Firstly, the hydro testing facility constructed at WJ Engineering requires a torque measuring test bench with specific characteristics to be able to accurately validate and verify the performance of model hydraulic turbines during the envisaged testing operations.

This torque measuring test bench still needs to be developed, manufactured and commissioned in order to be able to perform the required control and testing of turbines under the specified loading conditions.

Secondly, preliminary investigations by WJ Engineering has identified that a device known as a variable-fill hydraulic dynamometer has the operational qualities needed to fulfil the requirements of the torque measuring test bench.

Because these types of dynamometers are not readily available as off-the-shelf equipment and due to time constraints, the dynamometer rotor and stators were already designed and manufactured by WJ Engineering prior to the study based on their own design calculations, done without the cooperation of the author. However, in order to develop a fully functional variable-fill hydraulic dynamometer test bench a number of additional components are also required. A mathematical model is thus needed that represents the existing dynamometer, as well as the hydraulic system, and from which the design parameters of the auxiliary components could be obtained. The mathematical model also needs to be validated and evaluated for applicability under operational conditions.

1.4. Research aim and objectives

The aim of this study is to develop and manufacture a torque absorption and measuring test bench with suitable characteristics for testing the performance and governor of a model hydraulic turbine in the hydro testing facility at WJ Engineering according to the required specifications.

From the aim of the study the following primary objectives were identified:

1.4.1. Primary objectives:

- To develop and manufacture a variable-fill hydraulic dynamometer test bench, comprising of the hydraulic cycle, electric motor drive and the control and instrumentation equipment.

- To verify and validate the theoretical model as a design and analysis method for a variable-fill hydraulic dynamometer.
- To verify and validate the variable-fill hydraulic dynamometer test bench for its suitability to be used in the hydro test facility.

From these primary objectives the following set of secondary objectives have been identified, (the significance of which is explained further in the study):

1.4.2. Secondary objectives:

- To design, manufacture and characterise a suitable control valve and cooling cycle to be used in the test bench.
- In order to verify and validate the theoretical method, measuring methods of the following parameters required development: Torque absorption, rotational speed, dynamometer outlet pressure and hydraulic flow rate.
- In order to verify and validate the suitability of the dynamometer test bench it is necessary to determine the hydraulic dynamometer characteristic curves.

1.5. Delineations and limitations

Due to the complexity of the hydro testing facility certain factors will not form part of the scope of the study, the delineations explain what the author of the study will and will not be responsible for. There are also limitations to the study derived directly from the delineations and based on financial and methodical constraints, which will be explained in the limitations.

1.5.1. Delineations:

- WJ Engineering developed the hydro testing facility, therefore it is not the responsibility of the author to design and develop the contributing hydro testing cycle, only to incorporate the hydraulic dynamometer test bench into the hydro testing facility.
- The author will not do a detailed design of the heat exchanger used, only develop a cooling cycle that is sufficient enough to cool the heated water produced by the hydraulic dynamometer.
- At the time of the completion of the study WJ Engineering will not have a model turbine available, therefore the study will not include actual testing of a hydraulic model turbine by use of the hydraulic dynamometer test bench.
- The rotor and stator of the hydraulic dynamometer, including the rotor and stator cup geometry was designed and manufactured by WJ Engineering prior to the study, therefore it will not be the responsibility of the author to design the rotor or stator with the rotor and stator cup geometry. The author will be responsible for designing, developing, manufacturing and assembling all the additional components of the test bench as well as analysing the rotor and stator theoretically and experimentally in order to validate and verify the rotor and stator of the dynamometer and all additional components of the test bench for its suitability in the hydro testing facility. The author will also be responsible for analysing the theoretical method for its use as a design tool to design the rotor and stator with the rotor and stator cup geometry.

- Commercial dynamometers have PID controlled servo valves, the author will however only be responsible for developing a manually operated control valve.
- Data logging and measuring devices are provided by WJ Engineering, the author is only responsible for choosing the most suitable measuring and data logging methods from the devices available.

1.5.2. Limitations:

- Due to financial and technical constraints it will not be possible to measure the vortex pressure found inside the working compartment of the hydraulic dynamometer.
- The use of a manually operated control valve will limit the dynamometer test bench to steady state operation. Therefore, the characterisation and theoretical analysis will only include steady state operation of the hydraulic dynamometer test bench.
- The data logging and measuring devices provided by WJ Engineering, do not allow for high resolution continuous measuring and data logging, which limits the data analysis capabilities.
- The load cell available to measure the torque of the dynamometer has a maximum load of 50 [kg], this prevents any dynamometer testing at torques higher than 372.8 [Nm].

1.6. Significance (rationale)

1.6.1. Industrial significance:

Since WJ Engineering intends to produce hydraulic turbines, the development of a variable-fill hydraulic dynamometer test bench will allow WJ Engineering to properly test their model hydraulic turbines in order to verify and validate the designs.

WJ Engineering will gain valuable knowledge and experience that will allow the production of hydraulic dynamometers, intended to be sold commercially.

The analysis of the variable-fill hydraulic dynamometer itself, will give WJ Engineering valuable insight into the design and manufacturing of the device, which can be used to review the suitability thereof. In addition, the analysis of the theoretical model used as basis for the design, can possibly refine the development methodology for subsequent dynamometers and their application in testing facilities.

1.6.2. Academic significance:

The academic significance of the study will be to verify and validate the theoretical methodology used for the analysis and design of a variable-fill hydraulic dynamometer.

1.7. Chapter overviews

- **Chapter 2: Literature Review** gives a literature background on various torque absorbers and their respective applications, and investigates the suitability of these devices for use specifically in the hydro testing facility.

- **Chapter 3: Theory**, provides and discusses the theoretical model with which the variable-fill hydraulic dynamometer is analyzed and which provides the basis for design and simulation of the device.
- **Chapter 4: Research Methodology**, discusses the research methods employed to verify and validate the suitability of variable-fill hydraulic dynamometer test bench hydraulic cycle and equipment, as well as the suitability of the dynamometer's theoretical model for design purposes.
- **Chapter 5: Results and Discussions**, provides the results obtained by the research methodology and follows with the discussion and interpretation thereof.
- **Chapter 6: Conclusions and Recommendations**. In this chapter the final conclusion considers the success of the outcomes of the investigation when revisiting the original problem statement and research objectives of this study.

From these findings, recommendations for further study and improvements to the device and research methodology are also suggested.

1.8. Summary

In the introductory chapter it was explained that WJ Engineering developed a hydro testing facility for the purpose of testing model hydraulic turbines to be developed by them. However, to properly verify and validate these hydraulic turbine designs a device is needed that would be able to load the turbine during continuous operation and at varying loads; one such a device is known as a variable-fill hydraulic dynamometer.

The specifications of the Hydro Testing Facility of WJ Engineering are provided, and the requirements of the torque absorption and measuring device explained. The problem statement was subsequently derived from the need to develop, manufacture and validate a variable-fill hydraulic dynamometer test bench.

From the problem statement the study objectives, delineations, limitations and contribution of the study was obtained and described. The next chapter continues with a detailed literature review on current research and previous work on torque absorbers and related concepts.

2. Literature review

2.1. Overview

In this chapter, previous studies that have investigated the topics applicable to this study are presented. The content of this chapter will focus on the following areas:

- Different types of torque absorbers
- Hydraulic dynamometers in general (Operating principles and types)
- Variable-fill hydraulic dynamometers (Control mechanisms and characteristics)
- Theoretical modelling of variable-fill hydraulic dynamometers

2.2. Torque absorbers

There are various devices that can be used as torque absorbers, which would have the required characteristics of absorbing and measuring torque from the turbine at varying rotational speeds, and which would be able to change the load during continuous operation. These include most of the different types of dynamometers.

Mechanical power measuring devices (dynamometers) can be divided into two types: Firstly, those that absorb power and dissipate it as heat and secondly those that transmit the measured power. In both cases the measurement of torque and shaft speed are involved, of which the speed can be measured directly by means of a tachometer and the torque can be measured by balancing weights against a lever arm or attaching it to a load cell (Boyes, 2007:35). In the case of the WJ Engineering Hydro Test Facility the absorption type dynamometer would be the most suitable type: the torque and speed can then be measured directly and used to calculate the turbine's delivered power.

Since the hydraulic turbines are expected to deliver relatively high torque at relatively high rotational velocities and the torque absorption device is usually connected directly onto the turbine it should have similar properties. It is suggested that the torque produced by turbines under these specific conditions can be measured by a torque meter and a brake, hydraulic or electrical dynamometer (Arter & Meier, 1990:66).

A brake dynamometer such as a Prony brake or disc brake makes use of friction surfaces, such as wood blocks, flexible bands, asbestos etc. to apply a friction load onto the shaft of the prime mover (Boyes, 2007:40). Because of practical limitations related to the heat removal from the dynamometer at high speeds, this type of dynamometer is better suited for low-speed applications (Martyr & Plint, 2007:154). It is therefore not ideal for use in the hydro testing facility where the turbine tested will rotate at relatively high speeds.

Electric dynamometers include eddy-current brakes, calibrated generators and motors or cradle-mounted generators and motors. In the calibrated machines the mechanical power measurement can be made by measuring the electrical power input (or output) to the machine.

of developing substantial braking torque, making them an appropriate device for use in the hydro testing facility. Such dynamometers are however costly to install and fairly expensive to maintain, additionally they also require elaborate instrumentation and an initially high capital investment (Draughan, 1962). Designing and manufacturing an eddy-current dynamometer is an expensive process and WJ Engineering does not have the in-house manufacturing capability to produce the parts needed to build one themselves; thus such a device will not be considered in this study.

The final class of device to consider is known as a hydraulic dynamometer. Hydraulic dynamometers are commonly used in the automotive industry to test for motor vehicle engine power output and fuel economy (Kadhim, 2013:2955). The operating principle of hydraulic dynamometers is to dissipate the power from the machine being tested into the fluid circulating throughout the dynamometer (Horiba, 2006:9). Hydraulic dynamometers have not commonly been used in hydraulic turbine performance testing, but companies such as AVL ZöLNERR MARINE in Germany has produced commercial dynamometers for use in gas and hydro turbine testing applications, and explain that these devices are versatile enough to be used in almost any performance measuring setup (AVL ZöLNERR MARINE, 2018).

The construction and use of hydraulic dynamometers afford more economical operation, considerably lower capital investment costs, and greatly reduced maintenance costs, especially when compared with electrical dynamometers. Hydraulic dynamometers also require less complex instrumentation to allow torque measurements than most other types of dynamometers (Kadhim, 2013:2955). Therefore, an hydraulic dynamometer would be well suited as a torque absorption device for the WJ Engineering Hydro Testing Facility.

2.3. Hydraulic dynamometers

Water brake dynamometers were originally developed by William Froude in 1877 to measure power delivered to the propeller of large ships (1877:237). Modern water brakes are still based upon the basic working principle developed by William Froude and offer a cost effective alternative to other types of brakes (Vetr *et. al.*, 2010).

Hydrokinetic or hydraulic dynamometers (water brakes), with the exception of the disc dynamometer all work on similar principles. The typical hydraulic dynamometer consists of two essential parts: a rotor (composed of a turbine wheel and shaft) driven by the machine under test; as well as a stator unit composed of the housing and working fluid supply.

The hydraulic dynamometer operates based on the Föttinger principle (Passenbrunner, 2011:4598), essentially making the dynamometer work as an inefficient centrifugal pump to convert mechanical energy into heat (Boyes, 2007:43). The working compartment of the hydraulic dynamometer is shaped as an elliptical torus containing two sets of oppositely angled vanes, one set found in the rotor and the other in the stator as shown in Fig. 2.2 schematic a, b and c.

When considering the section through the toroidal working compartment Fig. 2.2 :(a), it can be seen that it is approximately circular. During operation the rotor pumps water from its periphery to the stator cups, which return the flow to the inner diameter of the rotor cups. This produces a high velocity toroidal vortex flow pattern, that is periodically interrupted each time a vane passes leading to turbulence (Raine & Hodgson, 1992:1181). This combined with wall friction, fluid friction, incidence losses between rotor and stator, and eddies caused by the curved flow path, results in power being transferred to the internal energy of the working fluid as can be seen in Fig. 2.3 (Hodgson, 1991:4).

The energy dissipation process happens entirely in the working chamber and since the hydraulic

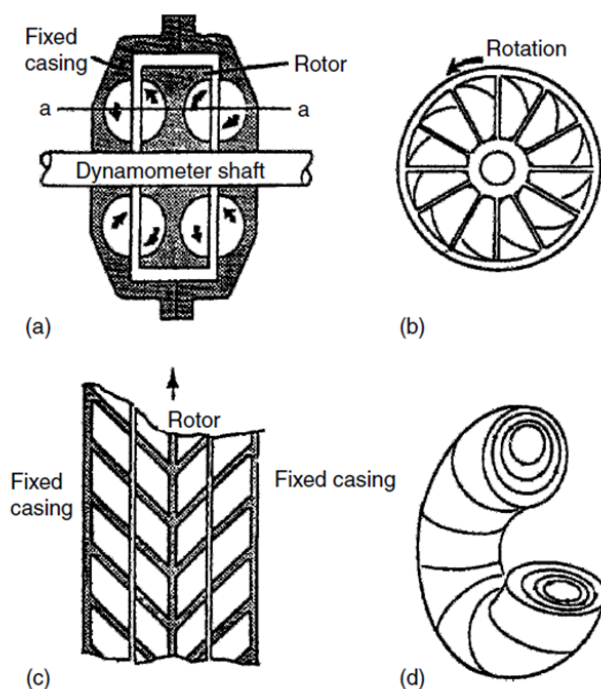


Figure 2.2. Hydrokinetic dynamometer, principle of operation: (a) section through dynamometer; (b) end view of rotor; (c) development of section a-a of rotor and casing; (d) representation of toroidal vortex (Martyr & Plint, 2007:172)

dynamometer has only one rotating part, the slip calculated by the relative difference in rotor and stator speed is always 100%. This results in a large thermal impact on the fluid, leading to an increase in working fluid temperature (Gatiss, 1971:60).

Incidence losses that occur when the fluid passes the gap between the rotor and stator accounts for most of the temperature rise of the fluid, but additional heating also happens due to the friction and secondary circulation losses (Passenbrunner, 2011:4598). This temperature rise necessitates the need to maintain a sufficient fluid throughflow to limit the resultant temperature rise (Hodgson, 1991:4).

Holes found in the stator vanes, approximately at the centre of the toroidal vortex, provide for the cooling water inlet while others vent the centre of the vortex to the atmosphere. The water is discharged (pumped) from the working compartment, by the pressure from the vortex action, through the annular gap between the outer rims of the rotor and stator, or from holes in the back of the rotor or stator cups as shown in Fig. 2.4 (Raine & Hodgson, 1992:1181).

Hydraulic dynamometers are of two kinds, depending on the means by which the resisting torque is varied. The first type is known as constant fill machines. In this machine, variation in torque absorption is achieved by inserting or withdrawing pairs of thin sluice plates between rotor and stator. The number of sluice plates present will affect the extent to which toroidal vortices will develop and reduce the area for fluid mass transfer, thus controlling the amount of torque absorption (Hodgson, 1991:4). The second type is known as variable-fill machines. In these machines, the torque absorbed is varied by adjusting the amount (mass) of water that circulates within the casing of the hydraulic dynamometer. A valve, usually found at the water outlet, is used to control the quantity (mass) of water present. The specific advantage of variable-fill hydraulic dynamometers is that the torque absorbed can be varied much faster and during continuous operation as compared to the case of sluice plate control (Martyr & Plint, 2007:162).

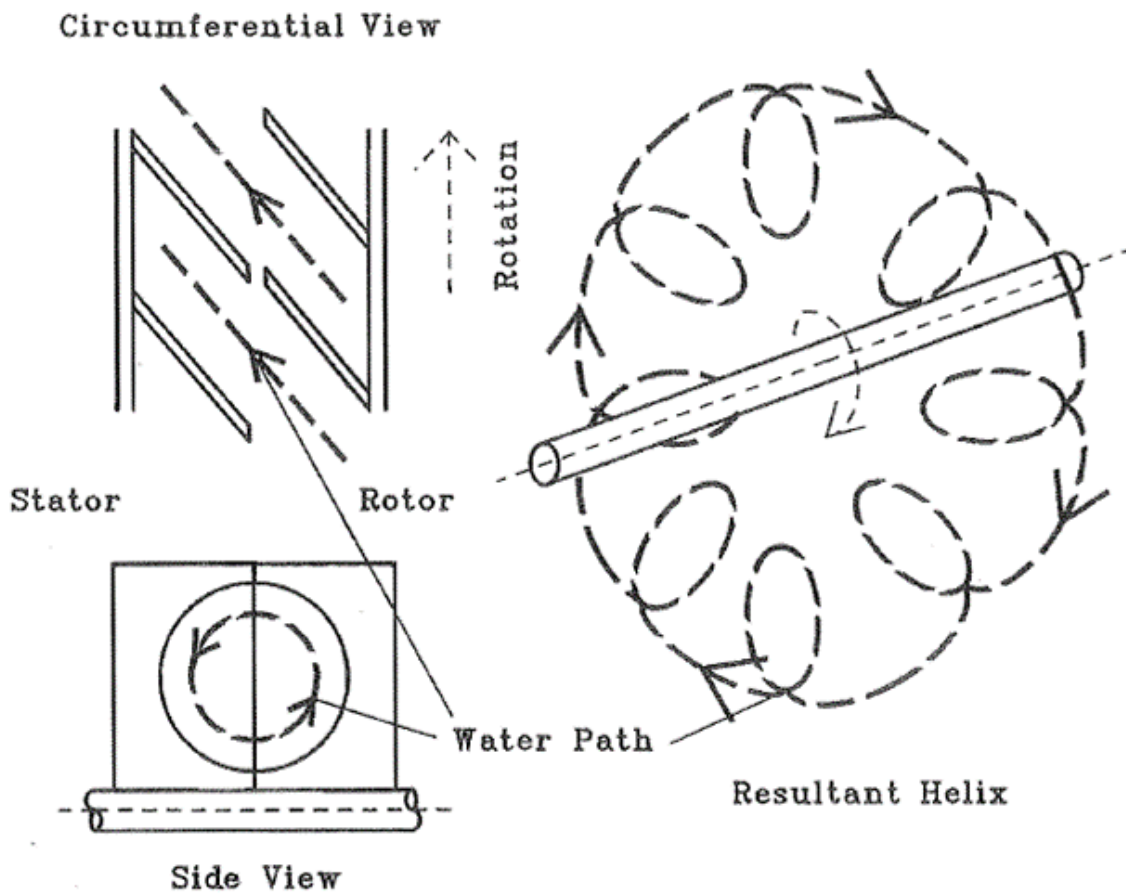


Figure 2.3. Dynamometer vanes and water flow path. (Hodgson, 1991:4)

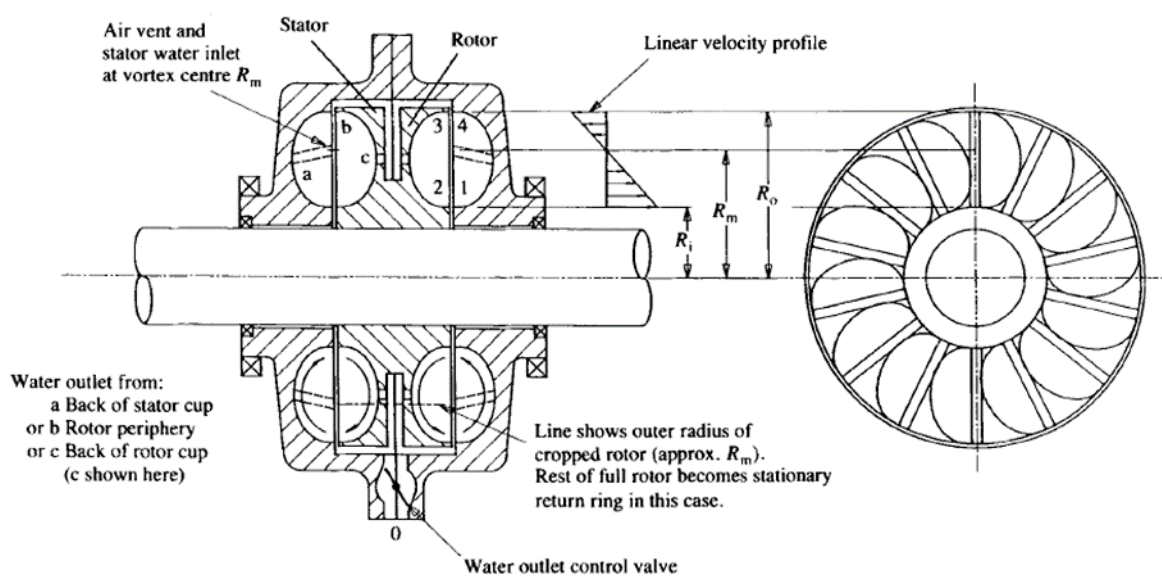


Figure 2.4. Schematic section of dynamometer with end view of rotor. (Hodgson, 1991:4)

Hydraulic dynamometers have high torque absorption capacity per unit rotational inertia (Raine & Hodgson, 1991:155) and therefore offer an excellent alternative to torque measurement (Passenbrunner, 2011:4598). The superior transient response and the reduced bulk of variable-fill machines have superseded the sluice gate controlled machines and the higher torque capacity to rotational inertia of these machines also make their performance superior to most electrical dynamometers in transient testing (Hodgson, 1991:4).

Hydraulic dynamometers can also readily be manufactured at a relatively low cost in house at WJ Engineering, consequently making this type of dynamometer the most appropriate and chosen torque absorption device to be used in the WJ Engineering Hydro test facility. It is also stated that the hydro test facility should verify & validate both the performance of the turbine and its governor. Thus, the ability to change the dynamometer torque absorption characteristic rapidly and during continuous operation is highly desirable. This makes the use of a variable-fill hydraulic dynamometer ideally suitable for consideration in the current study.

2.4. Variable-fill hydraulic dynamometers

The braking power of a variable fill hydraulic dynamometer is controlled via the fill-factor (Horiba, 2006:9). The fill-factor pertains to the amount of fluid present in the working compartment of the dynamometer. As the fill-factor increases i.e. the amount of fluid in the working compartment becomes more, the amount of torque absorption of the dynamometer increases (Voith, 2015).

The control of the fill-factor is achieved with a control valve, mostly found at the outlet of the dynamometer, associated with control systems of widely varying complexity (Martyr & Plint, 2007:162). Due to the volume of the fluid in the working compartment being only a small fraction of the throughflow at any time, a minor change in the flow rate can produce a considerable effect on the torque absorbed (Hodgson, 1991:4). Therefore, the design of the outlet control valve should be carefully considered to ensure proper adjustment of the hydraulic dynamometer torque.

In addition to the fill-factor the torque absorption characteristic of the dynamometer also depends on the cup and vane geometry, location of the water inlet and outlet holes and internal surface roughness. When the hydraulic dynamometer runs at steady state, the rate of cooling water inflow and outflow are equal, resulting in a constant fill-factor and therefore a constant torque (Raine & Hodgson, 1992:1181). If accelerated from rest without adjusting the control valve the dynamometer will display a torque characteristic as shown in Fig. 2.5.

Initially the torque will increase with speed, but at a certain point the torque absorbed by the dynamometer will start to lower (Hodgson, 1991:4). The speed where the torque starts to decrease is known as the “cup emptying speed” and at this point the pressure of the working compartment increased to such an extent that the machine pumps water out of the working compartment faster than the rate of inflow, leading to the emptying of the working compartment (Raine & Hodgson, 1991:155).

When considering this characteristic combined with the hydraulic turbine torque curve, certain problems become apparent as can be seen in Fig. 2.6. The model turbine to be tested at the WJ Engineering hydro test facility will have the characteristic of increasing torque with decreasing speed. When the dynamometer has a positive torque gradient, such as at point A, a decrease in system speed would cause a positive turbine torque perturbation and lower the dynamometer torque absorption. This would increase the resulting torque experienced by the system which in turn would cause system acceleration, increasing the dynamometer torque and

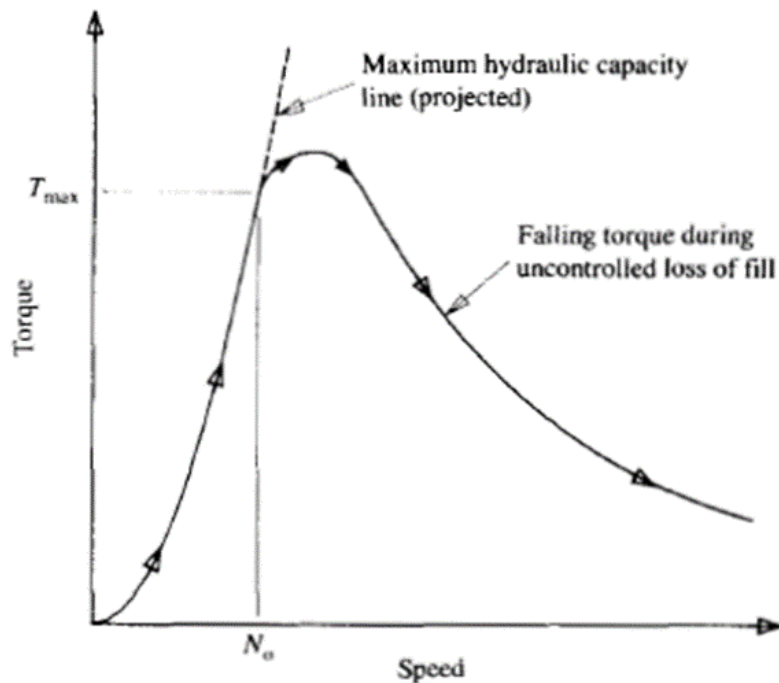


Figure 2.5. Open loop torque characteristic of dynamometer. (Raine & Hodgson, 1991:155)

decreasing the turbine torque, until the resultant torque becomes zero, causing the system to run at the set point. That is, the combination would be self-correcting. Where the dynamometer torque curve is more negative (at point B) than the turbine torque gradient, the increase in speed will lead to a larger decrease in dynamometer torque than the turbine torque, thus further accelerating the system. This would cause the system speed to continue deviating from the desired set point (the turbine would run away). Hence an increasing torque characteristic of the hydraulic dynamometer is desirable for all the required test speeds, to prevent this self-emptying phenomenon, that leads to instability in the system (Hodgson, 1991:4).

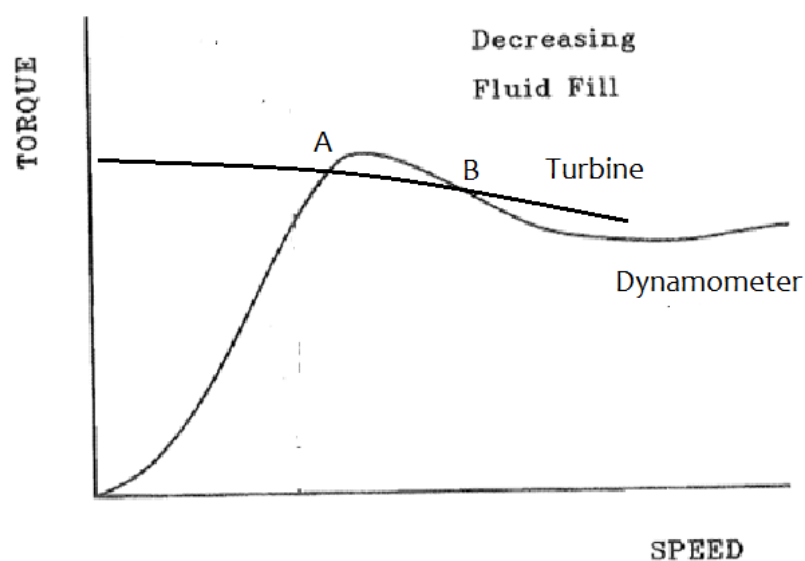


Figure 2.6. Dynamometer torque curve combined with turbine torque curve

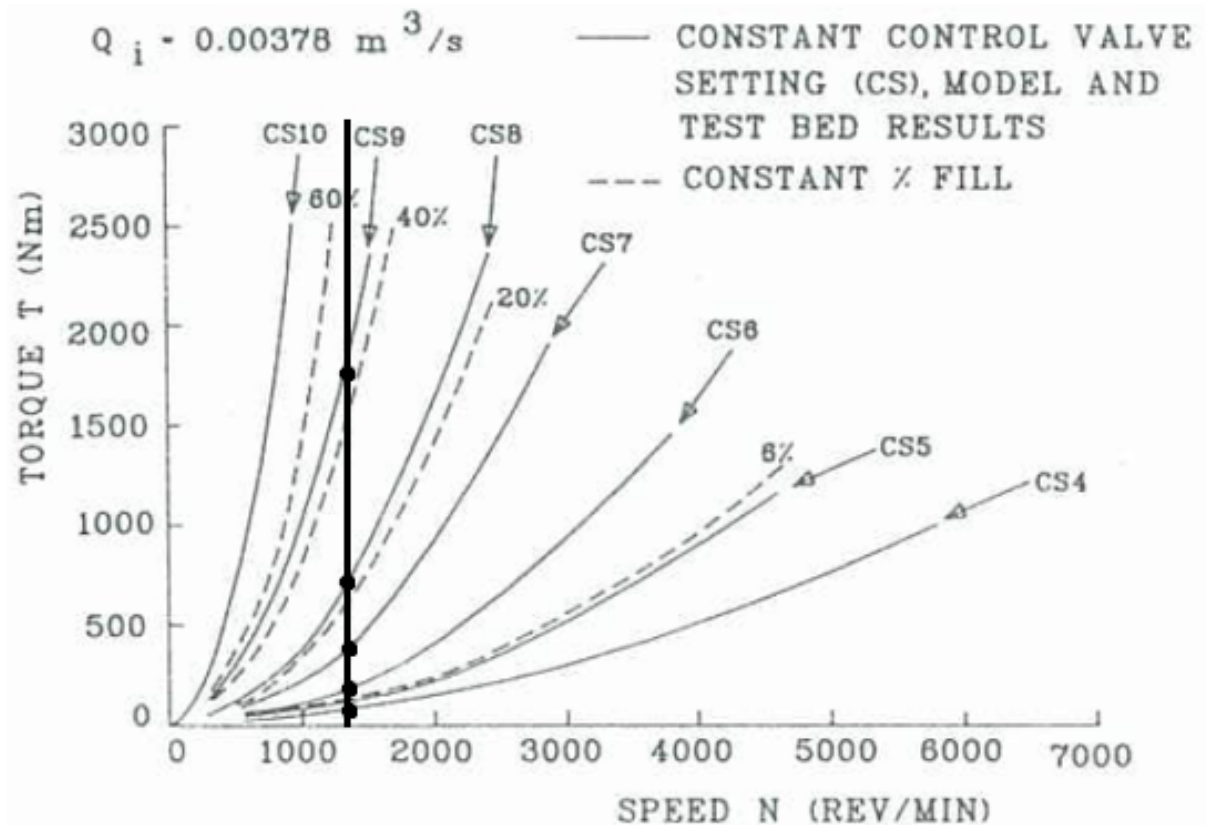


Figure 2.7. Torque-speed characteristic at constant control valve settings for F24 dynamometer with back pressure valve control (Raine & Hodgson, 1991:155).

When the torque curve of the dynamometer is plotted for an entire range of control valve settings (fluid fills), the curves shown in Fig. 2.7 can be obtained. These curves show the positive gradient torque curves for different constant control valve settings (fluid fills). This graph is useful in giving insight into the range of control valve settings where turbine testing via the dynamometer would be suitable.

The WJ Engineering turbine is designed to run at a constant rotational speed of 1200 [rpm]. Consider the torque variation at 1200 [rpm] for different control valve settings as illustrated on Fig. 2.7. The CS10 control valve setting should be avoided as it would lead to a runaway turbine speed (caused by the self-emptying phenomenon); for the remaining control valve settings it can be also be discerned what the torque absorption would be for each setting. This type of torque-speed characteristic curve for different dynamometer control settings is invaluable in determining the suitability of the hydraulic dynamometer for use in the hydro testing facility and needs to be determined empirically.

As shown in Fig. 2.8, where a dynamometer design similar to the model dynamometer developed at WJ Engineering can be seen. In this design the double-sided rotor eliminates axial thrusting and allows for the outlet holes to be in the stator cups for greater hydrodynamic stability (Hodgson, 1991:4). A manually operated outlet control valve will be used to vary the torque absorption and must be carefully designed and calibrated.

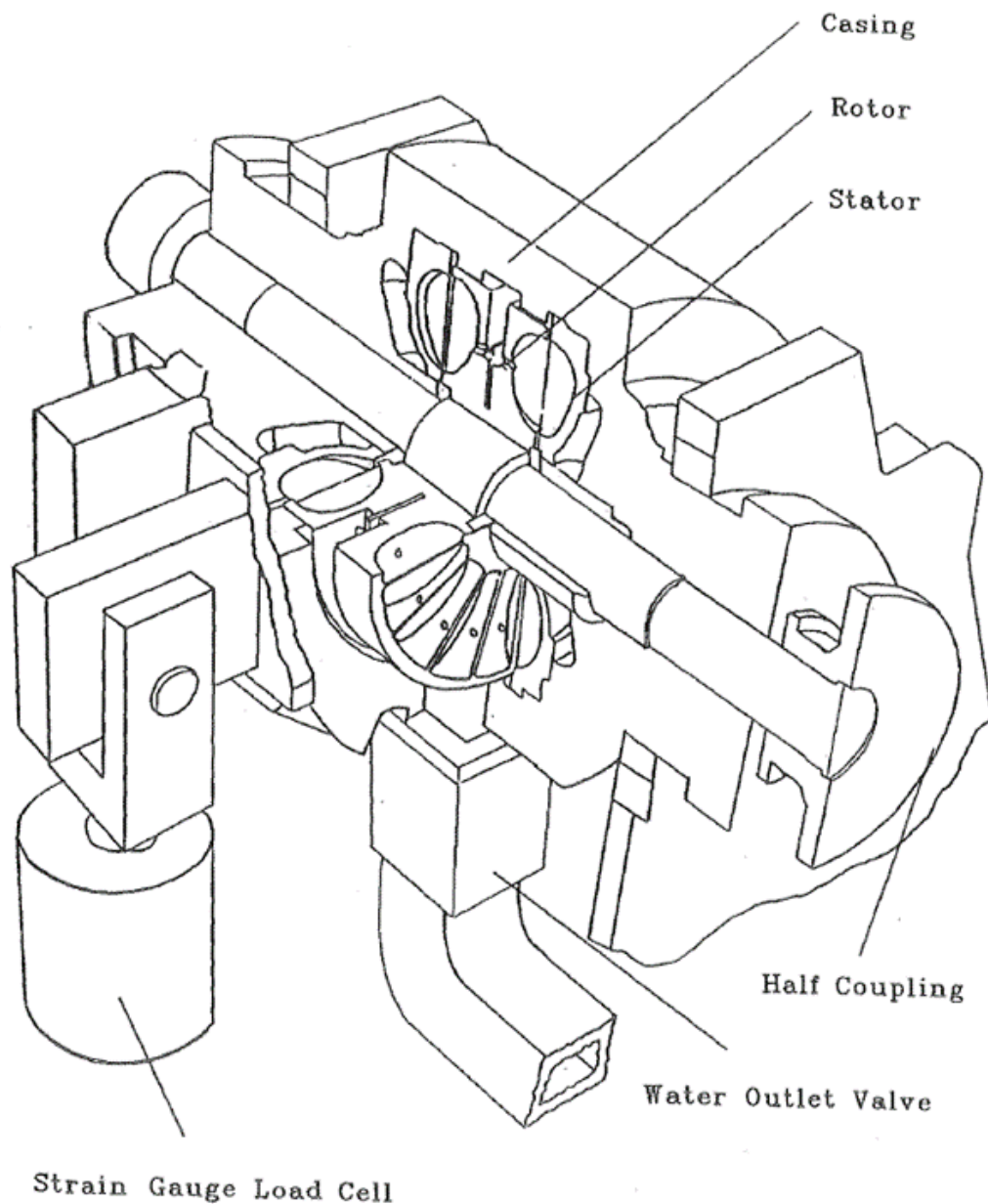


Figure 2.8. Cross-section diagram of F0351 Dynamometer. (Hodgson, 1991:4)

On the left hand side of the device in Fig. 2.8 the lever and load cell for torque measuring can be seen. This is also the method proposed by WJ Engineering for obtaining torque measurements and a further illustration of this method can be seen in Fig. 2.9. The basic principle is that the force generated within the dynamometer is counteracted by the torque arm connected to the load cell — from the load cell a force measurement can then be made and the torque absorbed calculated.

Following the discussion of dynamometers presented in the previous pages, the next section now gives an overview of the theoretical simulation and analysis of a variable-fill hydraulic dynamometer.

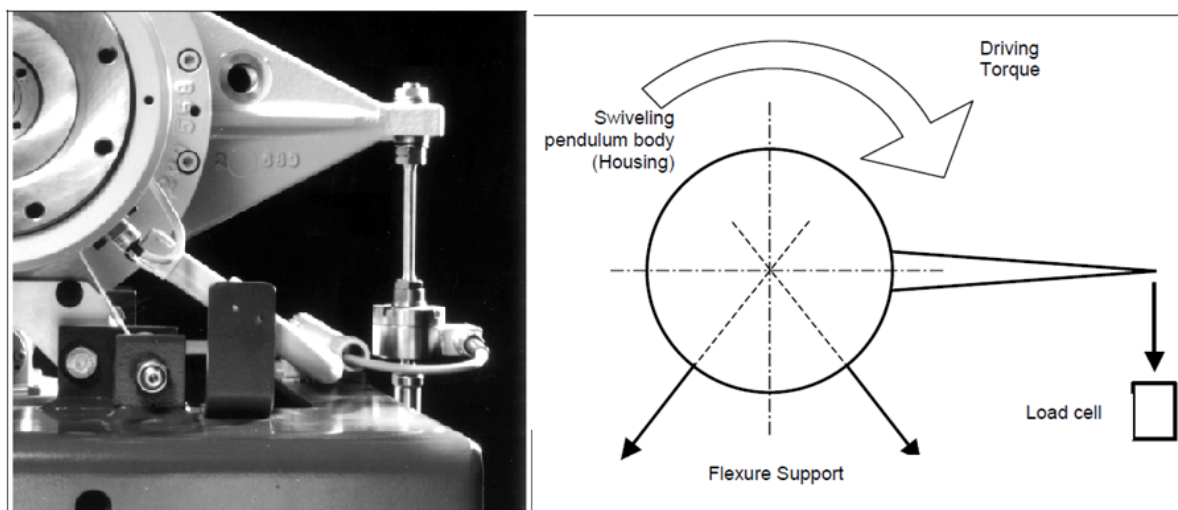


Figure 2.9. Torque arm and load cell. (Horiba, 2006:9).

2.5. Variable-fill hydraulic dynamometer simulation

The control of entire test benches equipped with hydraulic dynamometers has rarely been treated in literature (Passenbrunner, 2013:576), and although studies into fluid coupling performance are well represented in literature, there is very little published research to be found on variable fill dynamometers in open literature.

Studies such as that of Narayan Rao (1968) has focused on running the dynamometer at full performance, but did not consider any transient behaviour, or the control system employed. A model developed by Raine & Hodgson (1991, 1992 a), does however claim to accurately simulate both running full and partial fill behaviour, both for steady state and dynamic operating conditions. Their model also includes a hydromechanical back pressure valve as control mechanism (Hodgson, 1991:15). The model presented by Raine & Hodgson was subsequently used, to great success, to develop an inverse torque control method by Passenbrunner, Sassano, Trogmann, Del Re, Paulweber, Schmidt and Kokal (2011:4598).

The Raine & Hodgson dynamometer model characterises dynamometers through the application of a one-dimensional fluid flow model integrated with a vortex flow model. The model is based on a linear water velocity distribution due to a vortex action of constant angular velocity and a balance between absorbed power and theoretically derived incidence, friction and turbulence losses. In the model the derived torque equation is dynamic, allowing for both change in fill-factor and speed with time (Hodgson, 1991:15). It is therefore proposed that this model can be used to simulate the variable-fill hydraulic dynamometer developed at WJ Engineering. The Raine & Hodgson model can also be proposed as a future design tool, if it can be verified and validated to a suitable level of accuracy. This model is discussed in more detail in the next chapters.

2.6. Summary

In the literature review different torque absorption and measuring devices that would be suitable to use in the hydro testing facility are presented, and for each, with the exception of the variable-fill hydraulic dynamometer, an explanation is given as to why they are not considered for the current study.

From existing literature, hydraulic dynamometers are identified as the most viable solution to the problem statement. The basic operating principles and characteristics of these dynamometers are explained, and reasons given to clearly show why this type of dynamometer is better suited than the others.

Different torque control methods of hydraulic dynamometers are discussed, and existing literature clearly indicated that hydraulic dynamometers with variable-fill control are the most effective for dynamically changing load absorption. Variable-fill hydraulic dynamometers control the torque absorption characteristic by varying the *fill-factor* of the fluid in the working compartment, and this control is achieved by use of a flow rate control valve. This valve also allows for load changes during continuous operation. Therefore variable-fill hydraulic dynamometers are the best suited to be used in the verification and validation of model turbines and the turbine governor.

The simulation and theoretical analysis of variable-fill hydraulic dynamometers have rarely been treated in literature, but a model developed by Raine & Hodgson (1991, 1992 a) claims to accurately simulate variable-fill hydraulic dynamometers. This numerical method is employed in the current study to firstly analyse, and then secondly, simulate the hydraulic dynamometer that was designed and manufactured for use in the WJ Hydro Testing Facility.

The next chapter discusses the theoretical simulation methodology developed by Raine & Hodgson to be employed in the analysis and simulation of the WJ Engineering hydraulic dynamometer device.

3. Theory

The following chapter describes the theory applicable to the study and is based on the theory as presented by Raine & Hodgson (1991, 1992a). Raine and Hodgson developed a model that characterises hydraulic dynamometers by the application of an integrated one-dimensional fluid mechanics theory and a more detailed vortex flow model.

The theoretical chapter consists of three main parts. The first part includes the introduction and conservation laws, which explains the method by which a mean flow path is obtained for the vortex flow and how the conservation laws are implemented to obtain a one-dimensional torque equation for the hydraulic dynamometer. The second part which includes the energy dissipation section describes all the loss coefficients that pertain to the energy being transferred from the dynamometer to the fluid being heated. The third and final part of the theoretical section consists of all the supplementary equations developed, which are used to describe the boundaries of the partial and varying fill, the pressure generated by the vortex flow and the pressure loss due to the fluid outflow from the working compartment.

All three parts of the theoretical section form the basis of the one-dimensional model used to analyse and characterise the variable-fill hydraulic dynamometer, in order to predict its performance, and by which the parameters are obtained from which the additional components of the test bench are designed.

This one-dimensional theory will be validated and verified as part of the outcomes of this study, c.f. Section 1.4.1.

3.1. Introduction

Turbomachines serve in an enormous array of applications in our daily lives and thus play an important role in modern society, for this reason the theoretical characterisation of such machines are of great significance to engineers (Munson et al., 2013:666). Many techniques have been developed for this purpose, especially with the advent of inexpensive computing.

The most important factor when attempting to develop the theory of any turbomachine is to choose the most appropriate technique. Andersson (1982) stated that if performance prediction, instead of flow patterns, is of interest a one-dimensional theory is more appropriate than a two-dimensional theory when studying torque converters. Therefore, an integrated one-dimensional approach developed by Wallace, Whitfield and Sivalingam (1978) is used by Raine & Hodgson as a basis of the model in combination with the work of Raine (1977) and Tan (1980).

3.1.1. Basic features of one-dimensional dynamometer model

The standard torque equation $T = kN^2D^5$ is very convenient for the characterisation of geometrically similar machines once the parameter k is known. Hodgson (1991) makes use of the mean streamline method to predict torque by Eqn. (3.1). The method uses an inner and outer effective radius and a uniform velocity distribution (see Fig. 3.1) as basis for torque predictions.

$$T = \dot{m}(V_{\theta O}R_{EO} - V_{\theta I}R_{EI}) \quad (3.1)$$

where:

$\dot{m}$ = mass flow rate

$V_{\theta O}, V_{\theta I}$ = outer and inner tangential water velocities

R_{EO}, R_{EI} = outer and inner effective radii

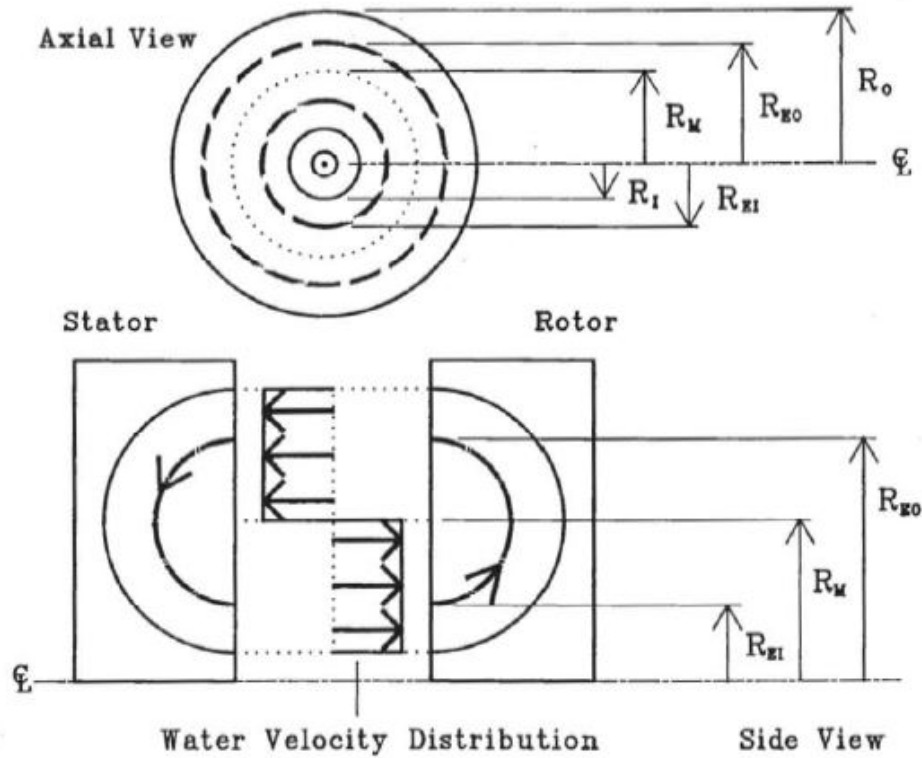


Figure 3.1. Uniform Velocity Distribution and Effective Radii Geometry (Raine & Hodgson, 1991:16)

The centre of the flow path, R_M , is found by equating the areas available for mass flow (Excluding vane thickness for simplicity) to satisfy continuity requirements, to find

$$R_M = \sqrt{\frac{R_O^2 + R_I^2}{2}} \quad (3.2)$$

The position of the inner and outer effective radii is found by a similar method to each of their respective flow areas, to find:

$$R_{EO} = \sqrt{\frac{R_O^2 + R_M^2}{2}} \quad \text{and} \quad R_{EI} = \sqrt{\frac{R_I^2 + R_M^2}{2}}$$

The uniform velocity shows an unrealistic discontinuity at the centre, however Ishihara, Furuya & Mori (1968:496-508) and Numazawa, Ushijima, Fukumura & Ishihara (1968) observed smooth

flows around the working compartment at stall conditions. The smooth flow observed can be represented by a vortex of constant angular velocity, which results in a linear distributed flow, as seen in Fig. 3.2. This linear flow distribution is used by Whitfield, Wallace & Sivlingnam and Raine. The flow velocity is proportional to the linear distance from the vortex centre; therefore, the radius of the vortex centre can be found by equating the inflow and outflow rates of the stator.

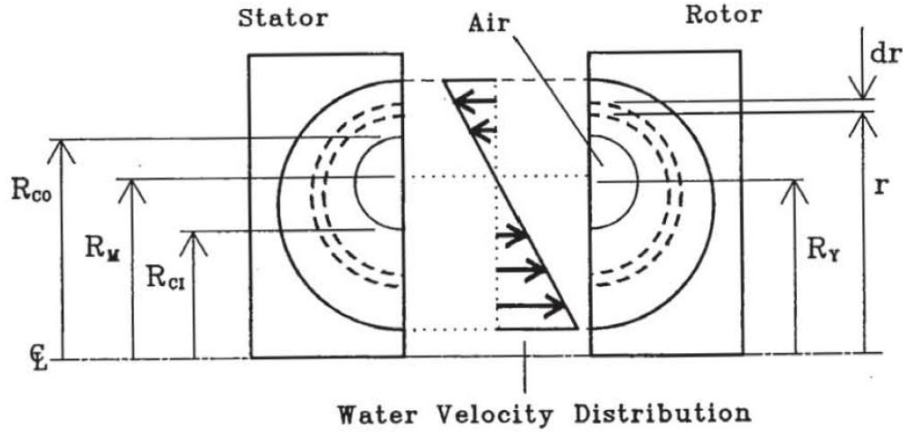


Figure 3.2. Linear Distributed Flow and Working Compartment Geometry (Raine & Hodgson, 1991:18)

As an introduction to the application of the integrated one-dimensional approach, the vortex centre of a fully filled axial vaned machine with zero blade thickness is derived and compared to that for the mean flow path theory above.

A general expression for the quantity of interest is derived for a stream tube, as shown in Fig. 3.2. The general expression is then integrated from the machine cup surface to either the vortex centre (for full fill condition) or the air-water interface (for partial fill condition). In this case the mass flow rate is considered, so for the stream tube

$$\begin{aligned} d\dot{m} &= \text{density} \times \text{flow velocity} \times \text{flow area} \\ &= \rho \times \omega(r - R_M) \times 2\pi r dr \end{aligned} \quad (3.3)$$

Integrate across the inlet,

$$\begin{aligned} \dot{m}_I &= \int_{R_M}^{R_O} 2\pi\rho\omega(r - R_M)r dr \\ &= 2\pi\rho\omega \left[\frac{1}{3}R_O^3 - \frac{1}{2}R_MR_O^2 + \frac{1}{6}R_M^3 \right] \end{aligned} \quad (3.4)$$

The same for stator outflow,

$$\dot{m}_O = 2\pi\rho\omega \left[\frac{1}{3}R_I^3 - \frac{1}{2}R_MR_I^2 + \frac{1}{6}R_M^3 \right] \quad (3.5)$$

Equating inflow and outflow yields

$$R_M = \frac{2}{3} \left[\frac{R_O^3 - R_I^3}{R_O^2 - R_I^2} \right] \quad (3.6)$$

Raine & Hodgson compared this to Eqn. (3.2) for a Froude F020 machine ($R_O = 102.5[mm]$, ($R_I = 43.5[mm]$), and calculated a difference of the mean flow path radius of just over 2%. Although the difference is small it is propagated throughout all the other calculations, for example the torque is dependent on R_M to the fifth power, so the accurate determination of R_M is necessary for the approach chosen.

3.1.2. Velocity diagrams for rotor and stator

The velocity diagrams for the full rotor machine is shown in Fig. 3.3 and were constructed based on the application of the linear velocity distribution of Fig. 3.2. In this section it will be explained how the velocity triangles are defined and how they are utilised to calculate the incidence angles of the flow into and out of the rotor. These calculations are vitally important as they will greatly influence the incidence loss components that causes some of the torque transmission.

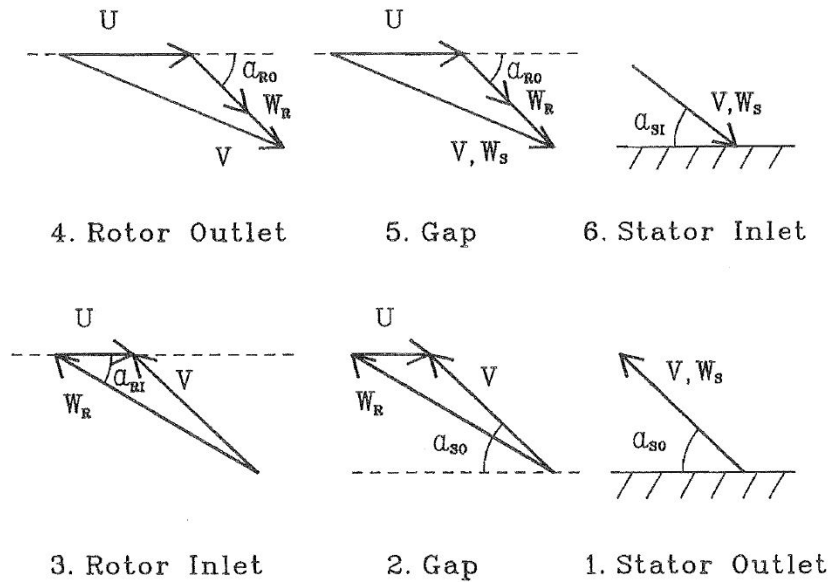


Figure 3.3. Full rotor dynamometer velocity diagrams (Hodgson, 1991:19)

The magnitudes of the relative velocities and the directions of each are indicated on the diagram, for clarification each point is explained:

- At point 1: The moment the water leaves the stator, both the relative and absolute velocities are along the blade surface.
- At point 2: The movement of water between the stator and rotor, the absolute and relative velocity continues at the stator outlet angle, while the velocity relative to the rotor is influenced by the rotational motion.
- At point 3: Water entry into the rotor, the relative velocity is now along the rotor surface and the absolute velocity change in direction is determined by the relative and rotational velocities.

- At point 4: At the rotor outlet, the relative velocity is still along the surface with the rotation contributing to the direction of the absolute velocity.
- At point 5: The movement of the water between the rotor and the stator, the relative velocity continues at the rotor outlet angle, while the stator experiences the absolute velocity, which continues at the same angle as in point 4, as the velocity relative to it.
- At point 6: Entry into the stator, the absolute and relative velocities remain coincident, but the direction of both have changed to that of the stator inlet angle.

It is important to note that even if axial vanes are present, the incidence angles of the flow velocity flowing into the rotor and stator will not be at right angles. The inclusion of the relative velocity component due to the rotor velocity will cause the incidence angles to shift as the rotor speed varies. The incidence angles must therefore be calculated for different rotor speeds, to be used in the equations to follow.

Consider Fig. 3.4, this velocity diagram describes the flow components just before the rotor inlet from the stator outlet.

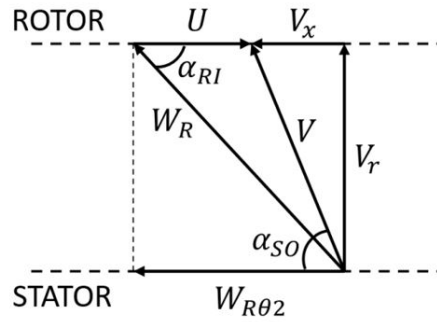


Figure 3.4. Velocity diagram for rotor incidence and angled vanes

Hodgson (1991:39) stated that the tangential relative velocity of the fluid just before the entrance to the rotor to be:

$$W_{R\theta 2} = -(\omega_p r + \omega(R_M - R_I)\cos \alpha_{SO})$$

The absolute velocity follows the direction of the vane of the stator, thus α_{SO} = stator vane angle. The tangential velocity of the rotor is found with $U = \omega_p \times R_I$ from this the incidence angle of the flow just before the rotor is found to be $\alpha_{RI} = \tan^{-1} \frac{V_r}{W_{R\theta 2}}$.

If we consider Fig. 3.5, this velocity diagram describes the flow components just before the inlet to the stator.

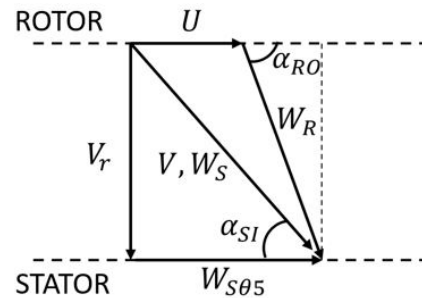


Figure 3.5. Velocity diagram for stator incidence and angled vanes

Hodgson (1991:40) found the tangential relative velocity of the fluid just before the entrance of the stator to be,

$$W_{S\theta 5} = (\omega_p r + \omega(R_O - R_M)\cos \alpha_{RO})$$

The stator relative velocity component follows the angle of the rotor vane, thus α_{RO} = rotor vane angle. At the rotor outlet the tangential velocity of rotation is found as, $U = \omega_p \times R_o$ from this the incidence angle of the flow just before the stator can be found to be $\alpha_{SI} = \tan^{-1} \frac{V_r}{W_{S\theta 5}}$.

For axial vaned dynamometers the velocity diagrams can be represented differently. Fig.3.6 below shows the velocity diagram of an axially vaned dynamometer just before the rotor.

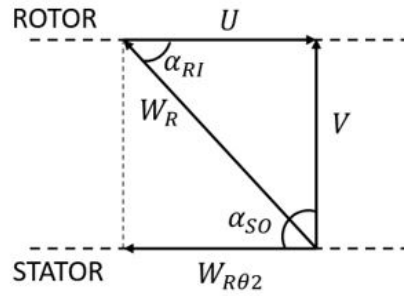


Figure 3.6. Velocity diagram for rotor incidence and axial vanes

If $\alpha_{SO} = 90$ [deg] we can find the incidence angle at the inlet of the rotor. Since $\cos \alpha_{SO} = \cos 90 = 0$. we can find that $U = \omega_p \times R_I = -W_{r\theta 2}$ the relative velocity component then becomes $W_R = ((W_{R\theta 2})^2 + V^2)^{-1/2}$, and the outlet velocity is dependent on the vortex velocity so that

$$V = \omega \times R_I$$

It is then possible to find the rotor entrance incidence angle for axial vanes with,

$$\alpha_{RI} = \cos^{-1} \frac{U}{W_R}$$

Fig. 3.7 shows the velocity diagram of an axially vaned dynamometer just before the stator.

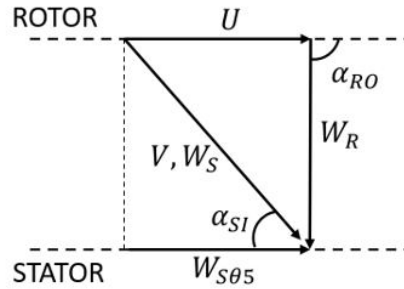


Figure 3.7. Velocity diagram for stator incidence and axial vanes

Since $\cos \alpha_{RO} = \cos 90 = 0$ we can find that $U = \omega_p \times R_o = W_{S\theta 5}$, velocity component relative to the stator then becomes, $W_S = (W_R^2 + W_{S\theta 5}^2)^{-1/2}$, the outlet velocity from the rotor is again dependant on the vortex velocity so that,

$$W_R = \omega \times R_O$$

It is then possible to find the stator entrance incidence angle for axial vanes with,

$$\alpha_{SI} = \cos^{-1} \frac{W_{S\theta 5}}{W_S}$$

Dynamometer vanes are significantly thicker than the blades found in other fluid coupling devices or torque converters, for machines of similar diameters. The dynamometer will further mostly operate in a partial fill condition and the blade blockage to the flow area therefore becomes more pronounced, thus provision must be made for these two factors in the theory from the onset of analysis. Sivalingam (1978:496-501) found that a blade thickness of 1.5 mm caused a 3-4% difference in torque prediction, when compared to calculations using the assumption of zero thickness blades.

To find the vortex centre the principle of conservation of mass is applied to the stator in Section 3.2.1 while considering the addition of vane thickness and non-axial vanes in contrast to Eqn. (3.6). The prediction of torque absorption is found from the conservation of angular momentum of the water flowing across the machine rotor, see Section 3.2.3.

3.1.3. Overview of energy dissipation mechanisms

The power delivered to the dynamometer is absorbed by the water, this energy absorption process is incorporated into the model via three mechanisms:

- Incidence loss: The mechanical energy lost by the fluid stream due to velocity changes between the rotor and stator.
- Frictional loss: The mechanical energy lost by the fluid flowing along the vane and toroidal surface of the working compartment.
- Secondary circulation loss: Mechanical energy lost due to turbulence in the water flow from the curved flow path and vanes breaking the ideal helical flow.

Section 3.2.2 describes the integration of the listed energy dissipation phenomena and the torque equation within the fluid stream's energy balance to determine the angular velocity of the water vortex. These energy diffusion mechanisms are discussed in more detail in Section 3.3.

3.1.4. Assumptions required by one-dimensional model

In order to derive the one-dimensional modelling equations for the dynamometer, certain assumptions must be made, these are listed below:

1. The flow of the water is modelled by a forced vortex of constant angular velocity in the plane of the machine vane, resulting in a linear distribution velocity profile.
2. The centre of the vortex is found for a full-fill condition and is assumed to stay the same as fill changes.

3. The working compartment is assumed to give a circular water flow path. This is mostly true for all practical cases, although sometimes the flow path can become elliptical.
4. It is assumed that there are distinct boundaries between the air and water circulating through the working compartment. This has the implication that the model will only deal with single phase incompressible flow, though the reality is that the water will be highly aerated.
5. Shearing of the vortex passing vanes is not included, however the turbulence caused by this will be included in the secondary circulation loss.
6. Mechanical losses and disk friction are not included.
7. The effect of sudden changes in area of the gap between the rotor and stator is excluded from the model.
8. When analysing the water outflow from the dynamometer outlet, it is assumed that the drain annulus and passages will always be full of water.

The next section will discuss the conservation laws used to find the vortex centre, vortex angular flow velocity and torque equation.

3.2. Conservation laws

Through the conservation laws the fluid dynamics of the dynamometer can be analysed. The first section, the conservation of mass is used to determine the vortex centre inside the working compartment, with the inclusion of vanes. The second section, the conservation of energy is used to find the vortex angular velocity, which is used to determine the torque developed. In the final section, conservation of momentum, the torque equation of the dynamometer is derived.

3.2.1. Conservation of mass

Through application of the conservation of mass principle to the fluid flow inside the stator the vortex centre can be determined, with the location considered to remain constant even as fluid fill varies. At fully filled conditions the mass inflow and outflow from the stator is equal, and the fluid inlet and outlet holes will have no effect. As explained in Section 3.1 a general streamtube expression can be derived for mass flow, which is then integrated. The general expression for mass flow in streamtube between rotor and stator is given by:

$$dm = \text{density} \times \text{axial flow velocity} \times \text{axial flow area}$$

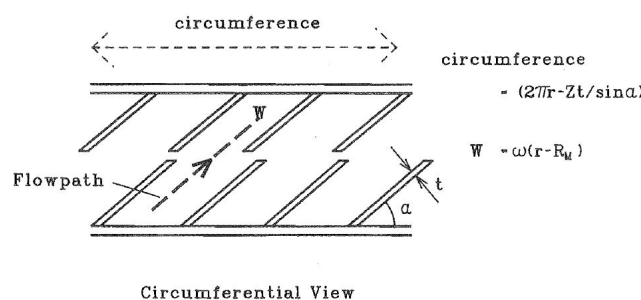


Figure 3.8. Vane effect on flow area (Hodgson, 1991:19)

When considering vane blockage, the axial flow area is found by multiplying the available circumference with the streamtube thickness, as shown in Fig. 3.8.

$$dA = \left(2\pi r - \frac{Zt}{\sin \alpha} \right) dr$$

where:

$$\begin{aligned} Z &= \text{vane number} \\ t &= \text{vane thickness} \\ \alpha &= \text{vane angle} \end{aligned}$$

and the axial flow velocity is

$$W_A = \omega(r - R_M) \sin(\alpha)$$

Hence the elemental mass flow rate between components is

$$d\dot{m} = \rho W_A \left(2\pi r - \frac{Zt}{\sin \alpha} \right) dr \quad (3.7)$$

Therefore the elemental inflow to the stator is

$$d\dot{m}_{SI} = \rho \omega (r - R_M) \sin \alpha_{SI} \left(2\pi r - \frac{Z_S t}{\sin \alpha_{SI}} \right) dr$$

where

$$\begin{aligned} Z_S &= \text{stator vane number} \\ \alpha_{SI} &= \text{stator inlet angle} \end{aligned}$$

This is integrated from vortex centre to cup surface,

$$\dot{M}_{SI} = \int_{R_M}^{R_O} \rho \omega (r - R_M) \sin \alpha_{SI} \left(2\pi r - \frac{Z_S t}{\sin \alpha_{SI}} \right) dr$$

which yields

$$\dot{M}_{SI} = \rho \omega \sin \alpha_{SI} \left[2\pi \left(\frac{1}{3} r^3 - \frac{1}{2} r^2 R_M \right) - \frac{Z_S t}{\sin \alpha_{SI}} \left(\frac{1}{2} r^2 - r R_M \right) \right]_{R_M}^{R_O} \quad (3.8)$$

Similarly for outflow from the stator

$$\dot{M}_{SO} = \int_{R_I}^{R_M} \rho \omega (R_M - r) \sin \alpha_{SO} \left(2\pi r - \frac{Z_S t}{\sin \alpha_{SO}} \right) dr$$

yields

$$\dot{M}_{SO} = \rho \omega \sin \alpha_{SO} \left[2\pi \left(\frac{1}{2} r^2 R_M - \frac{1}{3} r^3 \right) - \frac{Z_s t}{\sin \alpha_{SO}} \left(r R_M - \frac{1}{2} r^2 \right) \right]_{R_I}^{R_M} \quad (3.9)$$

Equating inflow (Eqn. (3.8)) and outflow (Eqn. (3.9)), and rearranging gives

$$\begin{aligned} & \frac{2}{3} \pi (R_O^3 \sin \alpha_{SI} - R_I^3 \sin \alpha_{SO}) - \frac{1}{2} Z_s t (R_O^2 - R_I^2) \\ &= \pi \sin \alpha_{SI} \left(R_M R_O^2 - \frac{1}{3} R_M^3 \right) - \pi \sin \alpha_{SO} \left(R_M R_I^2 - \frac{1}{3} R_M^3 \right) - Z_s t R_M (R_O - R_I) \end{aligned} \quad (3.10)$$

By using the Newton-Raphson method for root finding, R_M can be calculated. If it is the case that the stator vane angles are equal for inlet and outlet, Eqn. (3.10) simplifies to

$$R_M = \frac{\frac{2}{3} \pi \sin \alpha_{SO} (R_O^3 - R_I^3) - \frac{1}{2} Z_s t (R_O^2 - R_I^2)}{\pi \sin \alpha_{SO} (R_O^2 - R_I^2) - Z_s t (R_O - R_I)} \quad (3.11)$$

The above equation can also be used to determine an initial guess value for iteration when Newton-Raphson is used for twisted vanes (unequal inlet and outlet angles). Fortunately the dynamometer developed at WJ Engineering does have equal outlet vs. inlet angles, so it would not be necessary to apply the Newton-Raphson method at this part. Hodgson compared this equation to that of the application of Eqn. (3.6) for the F020 machine commercially available, and it was found that the effect of vane blockage moves the vortex centre to a slightly larger radius.

In the next section the conservation of energy principle is applied to determine the fluid vortex angular velocity, which is needed in the torque equation.

3.2.2. Conservation of energy

Since the dynamometer does not deliver any shaft power all the power input to the rotor must be transferred to the working fluid to maintain the conservation of energy. This energy transfer occurs by means of the energy dissipation outlined in Section 3.3. Eddies in the fluid and an increase in fluid temperature result from the energy transfer.

Input power can be considered as

$$\dot{P}_I = \tau \omega_P \quad (3.12)$$

where

$$\begin{aligned} \tau &= \text{dynamometer torque} \\ \omega_P &= \text{rotor angular velocity} \end{aligned}$$

The power from torque and energy dissipation are functions of fluid fill percentage, fluid and rotor angular velocities, this is illustrated through Eqn. (3.20) found in Section 3.2.3, where a representation of the Euler equation is given.

$$\tau = K_1 \omega \cdot \omega_P + K_2 \omega^2 \quad (3.13)$$

where

$$K_1, K_2 = \text{coefficients dependent on geometry and fill}$$

$$\omega = \text{water angular velocity}$$

In Section 3.3.3, the power loss and the various factors are considered in more detail in Eqn. (3.42). Here it is introduced as part of the overview:

$$\dot{P}_L = (K_{I1} + K_{I4}) \omega \omega_P^2 + (K_{I2} + K_{I5}) \omega^2 \omega_P + (K_{I3} + K_{I6} + K_F + K_{SC}) \omega^3 \quad (3.14)$$

where

$$K_{\#s} = \text{loss coefficients dependent on geometry and fill}$$

Setting the input power and power dissipation equal yields

$$\omega^2 (K_{I3} + K_{I6} + K_F + K_{SC}) + \omega \omega_P (K_{I2} + K_{I5} - K_2) + \omega_P^2 (K_{I1} + K_{I4} - K_I) = 0 \quad (3.15)$$

The above equation is then solved for the fluid vortex angular velocity, ω , using the quadratic formula. The general expression has the form

$$\omega = \omega_P \, fn(K)$$

where

$$fn(K) = \text{combination of geometry and fill dependent coefficients}$$

This means that for certain geometrical properties of the machine and fluid fill the water angular velocity is directly proportional to the rotor angular velocity.

In the next section the conservation of angular momentum will be discussed from which the torque equation for dynamometers are found.

3.2.3. Conservation of angular momentum

The torque equation for the dynamometer is obtained through application of the conservation of angular momentum principle. A control volume is applied around the rotor as shown in Fig. 3.9: for the conservation of angular momentum the torque input along the shaft then equals the rate of change of angular momentum of the fluid passing along the rotor.

Raine and Hodgson (1991,1992) reported that previous work done by Whitfield, Wallace and Sivalingam (1978), and Raine (1977) only considered the steady state flow situation. Raine and Hodgson however also considered the work done by Fox and McDonald (1973) to derive a general control volume formulation for unsteady flow situations. The general control volume formulation is then applied to the fundamental conservation laws of mechanics and thermodynamics to obtain control volume representations of these laws. For the conservation of angular momentum the formulation is then given by:

$$r \times F_S + \iiint_{CV} r \times g \rho dV + T_{shaft} = \frac{\partial}{\partial t} \iiint_{CV} r \times V \rho dV + \iint_{CS} r \times V \rho V_{xyz} \cdot dA$$

where

F_S , g = surface and body forces on the fluid

T_{shaft} = shaft torque

V , V_{xyz} = absolute and relative fluid velocities

dV , dA = elemental volume and -area

CV , CS = control volume and -surface

When neglecting viscosity there will be no surface forces, and due to symmetry the body forces will cancel out. Considering this, the following simplified form is obtained:

$$T_{shaft} = \frac{\partial}{\partial t} \iiint_{CV} r \times V \rho dV + \iint_{CS} r \times V \rho V_{xyz} \cdot dA \quad (3.16)$$

This equation is valid for the rotor as shown in Fig. 3.9. The first term after the equal sign is the time rate of change of angular momentum within the control volume and the second term is the net quantity passing through the control surface. Therefore, the second term represents the change in the angular momentum of the water between rotor inlet and outlet for steady flow.

For analysis and characterisation of the dynamometer considered in this study, it was found sufficient to consider the steady state flow condition. The dynamometer will be characterised in terms of the amount of torque absorbed at different rotor speeds and control valve settings. In order to find the torque quantity of the dynamometer at the different operating conditions, torque measurements will be taken at steady state for all the different rotor speed and control valve settings and plotted to find the dynamometer characterisation curves.

The time dependent term of the conservation of angular momentum equation (Eqn. (3.16)) will be invaluable when considering the dynamometer torque control behaviour for dynamic operation. But for the purpose of the study the dynamic operation torque control of the dynamometer will not be considered. The dynamometer test bench utilised at the WJ Hydro Test Facility will however be used under dynamic operating conditions, but for characterisation of the test bench the dynamic operation is irrelevant, and since the dynamic control of the dynamometer is not an issue for the testing of a hydraulic turbine the time dependent variable of the torque equation will be neglected. Raine & Hodgson (1991:26) stated that the influence of the unsteady flow component on the torque characteristic was found to be very small, thus it is acceptable to ignore its contribution.

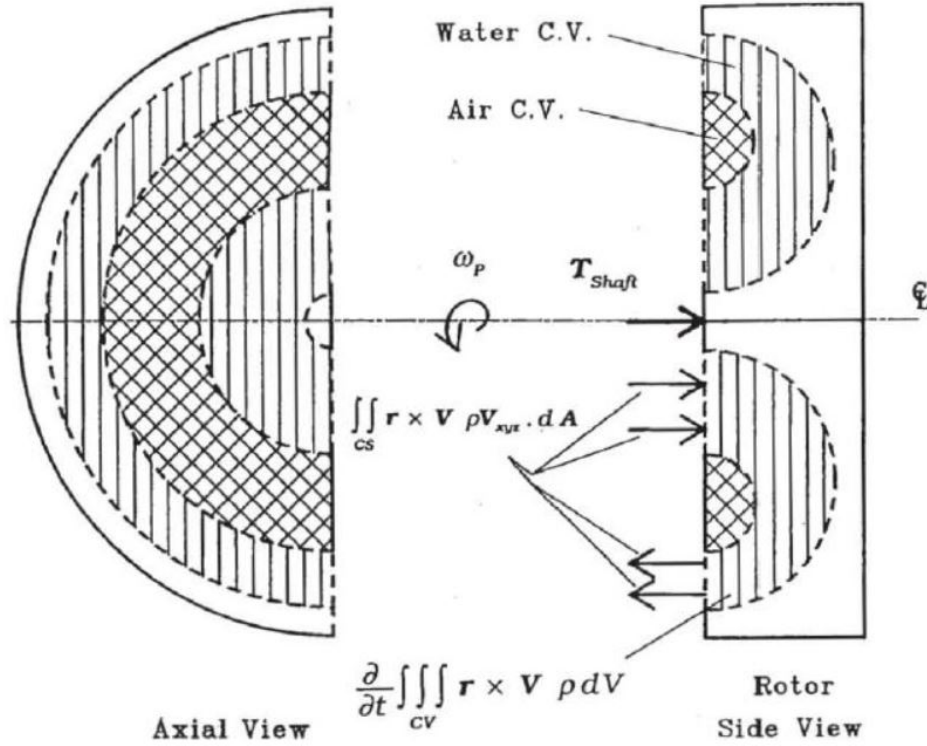


Figure 3.9. Rotor Control Volume and Angular Momentum Components (Raine & Hodgson, 1991:26)

The derivation of the dynamometer torque equation for steady state flow conditions, is discussed in the following section.

Steady flow component

In order to reduce the surface integral of Eqn. (3.16) to a single integral, the elemental mass flow $\rho V_{xyz} \cdot dA$ can be replaced with the streamtube equivalents: for relative velocity, $V_{xyz} = W_A = \omega(r - R_M) \sin \alpha$; elemental area, $dA = (2\pi r - \frac{Zt}{\sin \alpha})dr$ and tangential momentum, $r \times V = rV_\theta$.

Then by integrating from cup surface to air-water interface, the steady flow component becomes

$$\iint_{CS} r \times V \rho V_{xyz} \cdot dA = \int_{R_{CO}}^{R_O} rV_{\theta O} d\dot{m}_O - \int_{R_I}^{R_{CI}} rV_{\theta I} d\dot{m}_I \quad (3.17)$$

where

$$\begin{aligned} d\dot{m}_O &= \rho \omega (r - R_M) \sin \alpha_{RO} \left(2\pi r - \frac{Z_R t}{\sin \alpha_{RO}} \right) dr \\ d\dot{m}_I &= \rho \omega (r - R_M) \sin \alpha_{RI} \left(2\pi r - \frac{Z_R t}{\sin \alpha_{RI}} \right) dr \end{aligned}$$

and with

$V_{\theta O}$, $V_{\theta I}$ = outer and inner tangential absolute velocities
 R_{CO} , R_{CI} = outer and inner air-water interface radii

Considering Fig. 3.3 the tangential components of the absolute water velocity are:

Before the inlet of the rotor (point 2)

$$V_{\theta I} = W_S \cos \alpha_{SO} = -\omega(R_M - r) \cos \alpha_{SO}$$

At the inlet of the rotor (point 4)

$$V_{\theta O} = W_R \cos \alpha_{RO} + U = \omega(r - R_M) \cos \alpha_{RO} + \omega_P r$$

From this the angular momentum of the water flowing into the rotor control volume is

$$\begin{aligned}
 \int_{R_I}^{R_{CI}} r V_{\theta I} d\dot{m}_I &= \int_{R_I}^{R_{CI}} r (-\omega(R_M - r) \cos \alpha_{SO}) \rho \omega(R_M - r) \sin \alpha_{RI} \left(2\pi r - \frac{Z_R t}{\sin \alpha_{RI}} \right) dr \\
 &= -\rho \omega^2 \sin \alpha_{RI} \cos \alpha_{SO} \int_{R_I}^{R_{CI}} \left(2\pi r - \frac{Z_R t}{\sin \alpha_{RI}} \right) (r R_M^2 - 2 r^2 R_M + r^3) dr \\
 &= -\rho \omega^2 \sin \alpha_{RI} \cos \alpha_{SO} \left[2\pi \left(\frac{1}{5} r^5 - \frac{1}{2} r^4 R_M + \frac{1}{3} r^3 R_M^2 \right) \right. \\
 &\quad \left. - \frac{Z_R t}{\sin \alpha_{RI}} \left(\frac{1}{4} r^4 - \frac{2}{3} r^3 R_M + \frac{1}{2} r^2 R_M^2 \right) \right]_{R_I}^{R_{CI}}
 \end{aligned} \tag{3.18}$$

The angular momentum of the water flowing out is

$$\begin{aligned}
 \int_{R_{CO}}^{R_O} r V_{\theta O} d\dot{m}_0 &= \int_{R_{CO}}^{R_O} r (-\omega(r - R_M) \cos \alpha_{RO} + \omega_P r) \\
 &\quad \times \rho \omega(r - R_M) \sin \alpha_{RO} \left(2\pi r - \frac{Z_R t}{\sin \alpha_{RO}} \right) dr \\
 &= \rho \omega \sin \alpha_{RO} \int_{R_{CO}}^{R_O} \left(2\pi - \frac{Z_R t}{\sin \alpha_{RO}} \right) (r^2 - r R_M) \\
 &\quad \times (\omega(r - R_M) \cos \alpha_{RO} + \omega_P r) dr \\
 &= \rho \omega \sin \alpha_{RO} \left\{ \omega_P \left[2\pi \left(\frac{1}{5} r^5 - \frac{1}{4} r^4 R_M \right) - \frac{Z_R t}{\sin \alpha_{RO}} \left(\frac{1}{4} r^4 - \frac{1}{3} r^3 R_M \right) \right]_{R_{CO}}^{R_O} \right. \\
 &\quad \left. + \omega \cos \alpha_{RO} \left[2\pi \left(\frac{1}{5} r^5 - \frac{1}{2} r^4 R_M + \frac{1}{3} r^3 R_M^2 \right) \right. \right. \\
 &\quad \left. \left. - \frac{Z_R t}{\sin \alpha_{RO}} \left(\frac{1}{4} r^4 - \frac{2}{3} r^3 R_M + \frac{1}{2} r^2 R_M^2 \right) \right]_{R_{CO}}^{R_O} \right\}
 \end{aligned} \tag{3.19}$$

By substituting Eqn. (3.18) and Eqn. (3.19) into Eqn. (3.17) in order to get the flow of angular momentum through the control surface and rearranging yields

$$\tau_{ss} = K_1 \omega \cdot \omega_P + K_2 \omega^2 \tag{3.20}$$

where

$$\begin{aligned}
K_1 &= \rho \sin \alpha_{RO} \left[2\pi \left(\frac{1}{5}r^5 - \frac{1}{4}r^4 R_M \right) - \frac{Z_R t}{\sin \alpha_{RO}} \left(\frac{1}{4}r^4 - \frac{1}{3}r^3 R_M \right) \right]_{R_{CO}}^{R_O} \\
K_2 &= \rho \sin \alpha_{RO} \cos \alpha_{RO} \left[2\pi \left(\frac{1}{5}r^5 - \frac{1}{2}r^4 R_M + \frac{1}{3}r^3 R_M^2 \right) \right. \\
&\quad \left. - \frac{Z_R t}{\sin \alpha_{RO}} \left(\frac{1}{4}r^4 - \frac{2}{3}r^3 R_M + \frac{1}{2}r^2 R_M^2 \right) \right]_{R_{CO}}^{R_O} \\
&\quad + \rho \sin \alpha_{RI} \cos \alpha_{SO} \left[2\pi \left(\frac{1}{5}r^5 - \frac{1}{2}r^4 R_M + \frac{1}{3}r^3 R_M^2 \right) \right. \\
&\quad \left. - \frac{Z_R t}{\sin \alpha_{RI}} \left(\frac{1}{4}r^4 - \frac{2}{3}r^3 R_M + \frac{1}{2}r^2 R_M^2 \right) \right]_{R_I}^{R_{CI}}
\end{aligned}$$

It is generally only necessary to apply the torque equation to the water inside the working compartment, the density of water is about one thousand times greater than that of air, thus the influence of air can be ignored as its contribution to the torque transfer is found to be very small (Raine & Hodgson, 1991:28).

3.2.4. Unsteady flow component and dynamic operation

In the model presented by Hodgson (1991) the influence of the unsteady flow component was also included in the torque calculation. The unsteady component is generally valuable when considering the dynamic simulation of any hydraulic dynamometer, as it contributes to the complete operational envelope of the machine.

The unsteady flow component was however neglected for the purpose of the current study. Using the model as a design tool and for characterisation and analysis of the WJ Engineering dynamometer *specifically*, only the steady state operation was validated as the torque measurements could only be taken at steady state conditions at the various speeds.

The turbine governor system will be tested for dynamic operation, but it is not necessary for the dynamometer to be simulated or characterised for dynamic operation as the precise dynamic response of the dynamometer is of no consequence to determine the suitability of the governing system. It is only needed to vary the torque during operation to discern whether the governing system responds appropriately or not.

In the following section the calculation of the energy dissipation in the working compartment of the hydraulic dynamometer will be discussed. The energy dissipation is responsible for transferring the energy from the turbine into the working fluid found inside the dynamometer's working compartments, thus the energy dissipation is the mechanism by which the torque is transmitted.

3.3. Energy Dissipation

Application of the principle of energy conservation in Section 3.2.2 is used to find the angular vortex velocity of the working fluid. This is achieved by equating vortex input power to energy dissipation power. Three different components contribute to the dissipation of energy:

1. Incidence losses

2. Frictional losses
3. Secondary circulation losses

The derivation of these components are described in the following sections.

3.3.1. Incidence losses

When the fluid passes between the rotor and stator a lot of mechanical energy is "lost" or dissipated when the velocity vector realigns with the blading component that the fluid has just entered. Whitfield and Wallace (1978) & Sivalingam (1978) modelled the incidence (shock) loss in several ways, Raine & Hodgson (1991) however applied the kinetic energy model proposed by them. In this model the energy loss is based on the square of the change of the tangential fluid velocity relative to the component being entered.

Incidence losses occur at the entrances of both the rotor and the stator.

At rotor inlet, the elemental incidence power loss is:

$$d\dot{P}_{IR} = \frac{1}{2} d\dot{m}_{IR} (W_{R\theta 2} - W_{R\theta 3})^2 \quad (3.21)$$

where

$W_{R\theta 2}$ = tangential relative velocity just before rotor

$W_{R\theta 3}$ = tangential relative velocity just inside rotor

$d\dot{m}_{IR}$ = elemental fluid flow in rotor

From Fig. 3.3 the relative velocity components are:

$$\begin{aligned} W_{R\theta 2} &= -(\omega_p r + \omega(R_M - r) \cos \alpha_{SO}) \\ W_{R\theta 3} &= -(\omega(R_M - r) \cos \alpha_{RI}) \end{aligned}$$

and the elemental fluid flow is calculated by applying Eqn. (3.7) to the rotor inlet,

$$d\dot{m}_{IR} = \rho \omega (R_M - r) \sin \alpha_{RI} \left(2 \pi r - \frac{Z_R t}{\sin \alpha_{RI}} \right) dr$$

Integrate the elemental loss at the rotor inlet from R_I to R_{CI} ,

$$\begin{aligned} \dot{P}_{IR} &= \int_{R_I}^{R_{CI}} \left[\frac{1}{2} \rho \omega (R_M - r) \sin \alpha_{RI} \left(2 \pi r - \frac{Z_R t}{\sin \alpha_{RI}} \right) \right. \\ &\quad \left. \times \{ \omega_p r + \omega(R_M - r) \cos \alpha_{SO} - \omega(R_M - r) \cos \alpha_{RI} \}^2 \right] dr \end{aligned}$$

Rearranging,

$$\begin{aligned} \dot{P}_{IR} = \frac{1}{2} \rho \sin \alpha_{RI} \int_{R_I}^{R_{CI}} \left(2 \pi r - \frac{Z_R t}{\sin \alpha_{RI}} \right) & \left[\omega \omega_p^2 (R_M r^2 - r^3) \right. \\ & + 2 \omega^2 \omega_p (\cos \alpha_{SO} - \cos \alpha_{RI}) (R_M^2 r - 2 R_M r^2 - r^3) \\ & \left. + \omega^3 (\cos \alpha_{SO} - \cos \alpha_{RI})^2 (R_M^3 - 3 R_M^2 r + 3 R_M r^2 - r^3) \right] dr \end{aligned}$$

which can be rewritten in terms of coefficients

$$\dot{P}_{IR} = K_{I1} \omega \omega_p^2 + K_{I2} \omega^2 \omega_p + K_{I3} \omega^3 \quad (3.22)$$

where

$$\begin{aligned} K_{I1} &= \frac{1}{2} \rho \sin \alpha_{RI} \left[2 \pi \left(\frac{1}{4} R_M r^4 - \frac{1}{5} r^5 \right) - \frac{Z_R t}{\sin \alpha_{RI}} \left(\frac{1}{3} R_M r^3 - \frac{1}{4} r^4 \right) \right]_{R_I}^{R_{CI}} \\ K_{I2} &= \rho \sin \alpha_{RI} (\cos \alpha_{SO} - \cos \alpha_{RI}) \left[2 \pi \left(\frac{1}{3} R_M^2 r^3 - \frac{1}{2} R_M r^4 + \frac{1}{5} r^5 \right) \right. \\ & \quad \left. - \frac{Z_R t}{\sin \alpha_{RI}} \left(\frac{1}{2} R_M^2 r^2 - \frac{2}{3} R_M r^3 + \frac{1}{4} r^4 \right) \right]_{R_I}^{R_{CI}} \\ K_{I3} &= \frac{1}{2} \rho \sin \alpha_{RI} (\cos \alpha_{SO} - \cos \alpha_{RI})^2 \\ & \quad \times \left[2 \pi \left(\frac{1}{2} R_M^3 r^2 - R_M^2 r^3 + \frac{3}{4} R_M r^4 - \frac{1}{5} r^5 \right) \right. \\ & \quad \left. - \frac{Z_R t}{\sin \alpha_{RI}} \left(R_M^3 r - \frac{3}{2} R_M^2 r^2 + R_M r^3 - \frac{1}{4} r^4 \right) \right]_{R_I}^{R_{CI}} \end{aligned}$$

At the stator inlet Eqn. (3.21) becomes

$$d\dot{P}_{IS} = \frac{1}{2} d\dot{m}_{IS} (W_{S \theta 5} - W_{S \theta 6})^2 \quad (3.23)$$

where from Fig. 3.3

$$\begin{aligned} W_{S \theta 5} &= (\omega_p r + \omega(r - R_M) \cos \alpha_{RO}) \\ W_{S \theta 6} &= (\omega(r - R_M) \cos \alpha_{SI}) \end{aligned}$$

and from Eqn. (3.7)

$$d\dot{m}_{IS} = \rho \omega(r - R_M) \sin \alpha_{SI} \left(2 \pi r - \frac{Z_S t}{\sin \alpha_{SI}} \right) dr$$

The incidence loss at the stator inlet is therefore

$$\begin{aligned}
\dot{P}_{IS} &= \int_{R_{CO}}^{R_O} \left[\frac{1}{2} \rho \omega(r - R_M) \sin \alpha_{SI} \left(2 \pi r - \frac{Z_S t}{\sin \alpha_{SI}} \right) \right. \\
&\quad \times \left. \{ \omega_p r + \omega(r - R_M) \cos \alpha_{RO} - \omega(r - R_M) \cos \alpha_{SI} \}^2 \right] dr \\
&= \frac{1}{2} \rho \sin \alpha_{SI} \int_{R_{CO}}^{R_O} \left(2 \pi r - \frac{Z_S t}{\sin \alpha_{SI}} \right) \left[\omega \omega_p^2 (r^3 - R_M r^2) \right. \\
&\quad + 2 \omega^2 \omega_p (\cos \alpha_{RO} - \cos \alpha_{SI}) (r^3 - 2 R_M r^2 + R_M^2 r) \\
&\quad \left. + \omega^3 (\cos \alpha_{RO} - \cos \alpha_{SI})^2 (r^3 - 3 R_M r^2 + 3 R_M^2 r - R_M^3) \right] dr
\end{aligned}$$

which can be rewritten as

$$\dot{P}_{IS} = K_{I4} \omega \cdot \omega_p^2 + K_{I5} \omega^2 \cdot \omega_p + K_{I6} \omega^3 \quad (3.24)$$

where

$$K_{I4} = \frac{1}{2} \rho \sin \alpha_{SI} \left[2 \pi \left(\frac{1}{5} r^5 - \frac{1}{4} R_M r^4 \right) - \frac{Z_S t}{\sin \alpha_{SI}} \left(\frac{1}{4} r^4 - \frac{1}{3} R_M r^3 \right) \right]_{R_{CO}}^{R_O}$$

$$\begin{aligned}
K_{I5} &= \rho \sin \alpha_{SI} (\cos \alpha_{RO} - \cos \alpha_{SI}) \left[2 \pi \left(\frac{1}{5} r^5 - \frac{1}{2} r^4 + \frac{1}{3} R_M^2 r^3 \right) \right. \\
&\quad \left. - \frac{Z_S t}{\sin \alpha_{SI}} \left(\frac{1}{4} r^4 - \frac{2}{3} R_M r^3 + \frac{1}{2} R_M^2 r^2 \right) \right]_{R_{CO}}^{R_O}
\end{aligned}$$

$$\begin{aligned}
K_{I6} &= \frac{1}{2} \rho \sin \alpha_{SI} (\cos \alpha_{RO} - \cos \alpha_{SI})^2 \\
&\quad \times \left[2 \pi \left(\frac{1}{5} r^5 - \frac{3}{4} R_M r^4 + R_M^2 r^3 - \frac{1}{2} R_M^3 r^2 \right) \right. \\
&\quad \left. - \frac{Z_S t}{\sin \alpha_{SI}} \left(\frac{1}{4} r^4 - R_M r^3 + \frac{3}{2} R_M^2 r^2 - R_M^3 r \right) \right]_{R_{CO}}^{R_O}
\end{aligned}$$

Combining the incidence losses for the rotor and stator from Eqn. (3.22) and Eqn. (3.23), this results in the total incidence loss for the dynamometer:

$$\dot{P}_I = (K_{I1} + K_{I4}) \omega \cdot \omega_p^2 + (K_{I2} + K_{I5}) \omega^2 \cdot \omega_p + (K_{I3} + K_{I6}) \omega^3 \quad (3.25)$$

3.3.2. Frictional losses

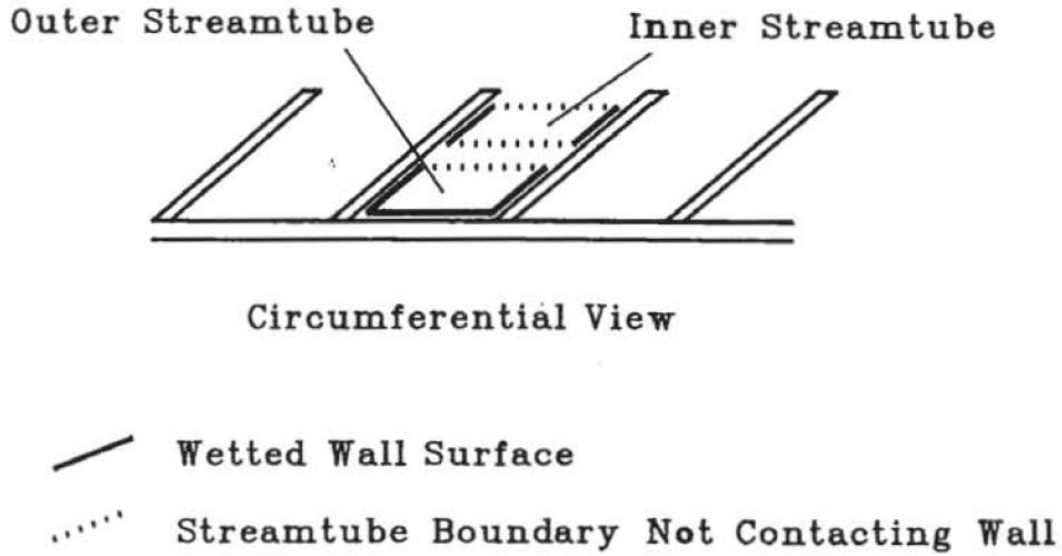


Figure 3.10. Friction surfaces and streamtubes (Raine & Hodgson, 1991:41)

With fluid flow along the surfaces of the machine energy losses occur due to friction. The elemental frictional losses ($d\dot{P}_F$) are found by through the application of the commonly used pipe friction formula,

$$d\dot{P}_F = d\dot{m} \frac{4 f L W^2}{2 d} \quad (3.26)$$

where

L = flow path length

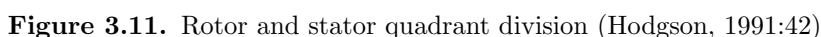
W = relative water velocity

d = hydraulic diameter = $4 \frac{\text{cross sectional area}}{\text{wetted perimeter}}$

f = Fanning friction factor

$\lambda_F = 4 f$

Fig. 3.10 shows that streamtubes next to the machine wall surface (outer streamtubes) have wetted perimeters that differ from those further up the blade surface (inner streamtubes), and thus different hydraulic diameters. Hence the two types are analysed individually. For simplification the rotor and stator are divided into upper (further from axis) and lower halves, as illustrated in Fig. 3.11.



To represent the contact of the fluid with the wall surface and blade surfaces a streamtube of arbitrary thickness is used in Eqn. (3.26), instead of an integrated thickness. For the lower half the arbitrary thickness is $D_L = (R_M - R_I)/50$, and D_U (upper half) is defined by continuity.

$$\begin{aligned} \rho \, \omega (R_M - R_I) \, \sin \alpha_{RI} \left(2 \pi R_I - \frac{Z_R t}{\sin \alpha_{RI}} \right) \\ = \rho \, \omega (R_O - R_M) \, \sin \alpha_{RO} \left(2 \pi R_O - \frac{Z_R t}{\sin \alpha_{RO}} \right) D_{UR} \end{aligned}$$
$$D_{UR} = \frac{(R_M - R_I) \sin \alpha_{RI} \left(2 \pi R_I - \frac{Z_R t}{\sin \alpha_{RI}} \right)}{(R_O - R_M) \sin \alpha_{RO} \left(2 \pi R_O - \frac{Z_R t}{\sin \alpha_{RO}} \right)} D_L$$
$$D_{US} = \frac{(R_M - R_I) \sin \alpha_{SI} \left(2 \pi R_I - \frac{Z_S t}{\sin \alpha_{SI}} \right)}{(R_O - R_M) \sin \alpha_{SO} \left(2 \pi R_O - \frac{Z_S t}{\sin \alpha_{SO}} \right)} D_L$$

Development of a variable-fill hydraulic dynamometer test bench

$$\begin{aligned}
\text{path length} \quad L_L &= \frac{1}{2}\pi(R_M - R_I) \\
\text{hydraulic diameter} \quad d &= 4 \frac{\left(2\pi R_I - \frac{Z_R t}{\sin \alpha_{RI}}\right) D_L}{\left(2\pi R_I - \frac{Z_R t}{\sin \alpha_{RI}}\right) + 2 Z_R D_L} \\
\text{relative velocity} \quad W &= \omega(R_M - R_I) \\
\text{mass flowrate} \quad \dot{m} &= \rho \omega(R_M - R_I) \sin \alpha_{RI} \left(2\pi R_I - \frac{Z_R t}{\sin \alpha_{RI}}\right) D_L
\end{aligned}$$

Hence Eqn. (3.26) for the frictional power loss in the outer streamtubes of the rotor becomes, starting with the lower rotor:

$$\begin{aligned}
\dot{P}_{FORL} &= \frac{1}{2} \lambda_F L_L W^2 \frac{\rho \omega(R_M - R_I) \sin \alpha_{RI} \left(2\pi R_I - \frac{Z_R t}{\sin \alpha_{RI}}\right) D_L}{\left(4 \frac{\left(2\pi R_I - \frac{Z_R t}{\sin \alpha_{RI}}\right) D_L}{\left(2\pi R_I - \frac{Z_R t}{\sin \alpha_{RI}}\right) + 2 Z_R D_L}\right)} \\
&= \frac{1}{16} \lambda_F \rho \omega^3 \pi \sin \alpha_{RI} (R_M - R_I)^4 \left(2\pi R_I - \frac{Z_R t}{\sin \alpha_{RI}} + 2 Z_R D_L\right)
\end{aligned}$$

Which rearranged is

$$\dot{P}_{FORL} = K_{FORL} \omega^3 \quad (3.27)$$

where

$$K_{FORL} = \frac{1}{16} \lambda_F \rho \pi \sin \alpha_{RI} (R_M - R_I)^4 \left(2\pi R_I - \frac{Z_R t}{\sin \alpha_{RI}} + 2 Z_R D_L\right)$$

For the upper rotor:

$$\begin{aligned}
\text{path length} \quad L_U &= \frac{1}{2}\pi(R_O - R_M) \\
\text{hydraulic diameter} \quad d &= 4 \frac{\left(2\pi R_O - \frac{Z_R t}{\sin \alpha_{RO}}\right) D_{UR}}{\left(2\pi R_O - \frac{Z_R t}{\sin \alpha_{RO}}\right) + 2 Z_R D_{UR}} \\
\text{relative velocity} \quad W &= \omega(R_O - R_M) \\
\text{mass flowrate} \quad \dot{m} &= \rho \omega(R_O - R_M) \sin \alpha_{RO} \left(2\pi R_O - \frac{Z_R t}{\sin \alpha_{RO}}\right) D_{UR}
\end{aligned}$$

Again, with the rearrangement in terms of a loss factor coefficient, it becomes:

$$\dot{P}_{FORU} = K_{FORU} \omega^3 \quad (3.28)$$

where

$$K_{FORU} = \frac{1}{16} \lambda_F \rho \pi \sin \alpha_{RO} (R_O - R_M)^4 \left(2 \pi R_O - \frac{Z_R t}{\sin \alpha_{RO}} + 2 Z_R D_{UR} \right)$$

For the lower stator it can be similarly written:

$$\dot{P}_{FOSL} = K_{FOSL} \omega^3 \quad (3.29)$$

where

$$K_{FOSL} = \frac{1}{16} \lambda_F \rho \pi \sin \alpha_{SO} (R_M - R_I)^4 \left(2 \pi R_I - \frac{Z_S t}{\sin \alpha_{SO}} + 2 Z_S D_L \right)$$

and the upper stator,

$$\dot{P}_{FOSU} = K_{FOSU} \omega^3 \quad (3.30)$$

where

$$K_{FOSU} = \frac{1}{16} \lambda_F \rho \pi \sin \alpha_{SI} (R_O - R_M)^4 \left(2 \pi R_O - \frac{Z_S t}{\sin \alpha_{SI}} + 2 Z_S D_{US} \right)$$

Inner streamtubes

Eqn. (3.26) in elemental form is used to determine the frictional losses of the inner streamtubes and is integrated between the air-water interface and the limit of the outer streamtubes. The fluid is only in contact with the blade surface, therefore the hydraulic diameter is considerably different from that of the outer streamtubes.

Again beginning with the lower rotor,

$$\begin{aligned} \text{path length} \quad L &= \frac{1}{2} \pi (R_M - r) \\ \text{hydraulic diameter} \quad d &= 4 \frac{\left(2 \pi r - \frac{Z_R t}{\sin \alpha_{RI}} \right) Dr}{2 Z_R dr} \\ \text{relative velocity} \quad W &= \omega (R_M - r) \\ \text{mass flowrate} \quad \dot{m} &= \rho \omega (R_M - r) \sin \alpha_{RI} \left(2 \pi r - \frac{Z_R t}{\sin \alpha_{RI}} \right) Dr \end{aligned}$$

and integrating Eqn. (3.26) gives the frictional power loss in the inner streamtubes of the rotor becomes, starting with the lower rotor:

$$\begin{aligned}
\dot{P}_{FIRL} &= \int_{R_I+D_L}^{R_{CI}} \frac{1}{2} \lambda_F L W^2 \frac{\rho \omega (R_M - r) \sin \alpha_{RI} \left(2 \pi r - \frac{Z_R t}{\sin \alpha_{RI}} \right)}{\frac{2}{Z_R} \left(2 \pi r - \frac{Z_R t}{\sin \alpha_{RI}} \right)} dr \\
&= \frac{1}{8} \lambda_F Z_R \rho \pi \omega^3 \sin \alpha_{RI} \int_{R_I+D_L}^{R_{CI}} (R_M - r)^4 dr \\
&= \frac{1}{8} \lambda_F Z_R \rho \pi \omega^3 \sin \alpha_{RI} \left[R_M^4 r - 2 R_M^3 r^2 + 2 R_M^2 r^3 - R_M r^4 + \frac{1}{5} r^5 \right]_{R_I+D_L}^{R_{CI}}
\end{aligned}$$

which can be rewritten using a loss coefficient as:

$$\dot{P}_{FIRL} = K_{FIRL} \omega^3 \quad (3.31)$$

where

$$K_{FIRL} = \frac{1}{8} \lambda_F Z_R \rho \pi \sin \alpha_{RI} \left[R_M^4 r - 2 R_M^3 r^2 + 2 R_M^2 r^3 - R_M r^4 + \frac{1}{5} r^5 \right]_{R_I+D_L}^{R_{CI}}$$

For the upper rotor, the parameters are defined as:

$$\begin{aligned}
\text{path length} \quad L &= \frac{1}{2} \pi (r - R_M) \\
\text{hydraulic diameter} \quad d &= \frac{2}{Z_R} \left(2 \pi r - \frac{Z_R t}{\sin \alpha_{RO}} \right) \\
\text{relative velocity} \quad W &= \omega (r - R_M) \\
\text{mass flowrate} \quad \dot{m} &= \rho \omega (r - R_M) \sin \alpha_{RO} \left(2 \pi r - \frac{Z_R t}{\sin \alpha_{RO}} \right) Dr
\end{aligned}$$

and by rearranging it can be written as

$$\dot{P}_{FIRU} = K_{FIRU} \omega^3 \quad (3.32)$$

where

$$K_{FIRU} = \frac{1}{8} \lambda_F Z_R \rho \pi \sin \alpha_{RO} \left[\frac{1}{5} r^5 - R_M r^4 + 2 R_M^2 r^3 - 2 R_M^3 r^2 + R_M^4 r \right]_{R_{CO}}^{R_{O-DUR}}$$

Similarly for the lower stator

$$\dot{P}_{FISL} = K_{FISL} \omega^3 \quad (3.33)$$

where

$$K_{FISL} = \frac{1}{8} \lambda_F Z_S \rho \pi \sin \alpha_{SO} \left[R_M^4 r - 2 R_M^3 r^2 + 2 R_M^2 r^3 - R_M r^4 + \frac{1}{5} r^5 \right]_{R_I+D_L}^{R_{CI}}$$

and for the upper stator,

$$\dot{P}_{FISU} = K_{FISU} \omega^3 \quad (3.34)$$

where

$$K_{FISU} = \frac{1}{8} \lambda_F Z_R \rho \pi \sin \alpha_{SI} \left[\frac{1}{5} r^5 - R_M r^4 + 2 R_M^2 r^3 - 2 R_M^3 r^2 + R_M^4 r \right]_{R_{CO}}^{R_{O-DUS}}$$

Consequently the total power loss ($\dot{P}_F$) dissipated due to frictional losses is calculated as the sum of the preceding equations for the constitutional components, given by Eqn. (3.27) – Eqn. (3.34):

$$\dot{P}_F = K_F \omega^3 \quad (3.35)$$

where

$$K_F = K_{FORL} + K_{FORU} + K_{FOSL} + K_{FOSU} + K_{FIRL} + K_{FIRU} + K_{FISL} + K_{FISU}$$

3.3.3. Secondary circulation losses

The curved nature of the fluid flow path and the interruption of the ideal flow by passing vanes will certainly lead to fluid circulation that is different from the ideal helical vortex flow pattern shown in Fig. 2.3. The rotor operates as a radial flow pump that forms a pressure gradient in the vane to vane direction, and the flow around the bends leads to a radial pressure gradient, both of which cause the appearance of eddy flows that are superimposed onto the ideal helical flow. This flow mixing effect is further enhanced by the turbulence resulting from the disruption of flow caused by the passing vanes.

The common formula for pipe bends (Munson, 2013:437) is used to model the losses caused by the secondary circulation as explained above.

$$\dot{P}_{SC} = \frac{1}{2} K_B W^2 d \dot{m} \quad (3.36)$$

where

W = relative water velocity

K_B = empirically derived loss coefficient

The formulation for the power loss can again be developed by considering the rotor and stator as upper and lower halves and by integrating the general streamtube formulations and then adding the individual components together.

The lower rotor gives

$$\begin{aligned}
\dot{P}_{SCRL} &= \int_{R_I}^{R_{CI}} \frac{1}{2} K_B (\omega(R_M - r))^2 \rho \omega(R_M - r) \sin \alpha_{RI} \left(2\pi r - \frac{Z_R t}{\sin \alpha_{RI}} \right) dr \\
&= \frac{1}{2} K_B \rho \omega^3 \sin \alpha_{RI} \int_{R_I}^{R_{CI}} (R_M - r)^3 \left(2\pi r - \frac{Z_R t}{\sin \alpha_{RI}} \right) dr \\
&= \frac{1}{2} K_B \rho \omega^3 \sin \alpha_{RI} \left[2\pi \left(\frac{1}{2} R_M^3 r^2 - R_M^2 r^3 + \frac{3}{4} R_M r^4 - \frac{1}{5} r^5 \right) \right. \\
&\quad \left. - \frac{Z_R t}{\sin \alpha_{RI}} \left(R_M^3 r - \frac{3}{2} R_M^2 r^2 + R_M r^3 - \frac{1}{4} r^4 \right) \right]_{R_I}^{R_{CI}}
\end{aligned}$$

which, when rearranged gives

$$\dot{P}_{SCRL} = K_{SCRL} \omega^3 \quad (3.37)$$

where

$$\begin{aligned}
K_{SCRL} &= \frac{1}{2} K_B \rho \sin \alpha_{RI} \left[2\pi \left(\frac{1}{2} R_M^3 r^2 - R_M^2 r^3 + \frac{3}{4} R_M r^4 - \frac{1}{5} r^5 \right) \right. \\
&\quad \left. - \frac{Z_R t}{\sin \alpha_{RI}} \left(R_M^3 r - \frac{3}{2} R_M^2 r^2 + R_M r^3 - \frac{1}{4} r^4 \right) \right]_{R_I}^{R_{CI}}
\end{aligned}$$

For the upper rotor,

$$\dot{P}_{SCRL} = \int_{R_{CO}}^{R_O} \frac{1}{2} K_B (\omega(r - R_M))^2 \rho \omega(r - R_M) \sin \alpha_{RO} \left(2\pi r - \frac{Z_R t}{\sin \alpha_{RO}} \right) dr$$

which yields

$$\dot{P}_{SCRU} = K_{SCRU} \omega^3 \quad (3.38)$$

where

$$\begin{aligned}
K_{SCRU} &= \frac{1}{2} K_B \rho \sin \alpha_{RO} \left[2\pi \left(\frac{1}{5} r^5 - \frac{3}{4} R_M r^4 + R_M^2 r^3 - \frac{1}{2} R_M^3 r^2 \right) \right. \\
&\quad \left. - \frac{Z_R t}{\sin \alpha_{RO}} \left(\frac{1}{4} r^4 - R_M^2 r^3 + \frac{3}{2} R_M^2 r^2 - R_M^3 r \right) \right]_{R_{CO}}^{R_O}
\end{aligned}$$

Similarly for the lower stator,

$$\dot{P}_{SCSL} = K_{SCSL} \omega^3 \quad (3.39)$$

where

$$K_{SCSL} = \frac{1}{2} K_B \rho \sin \alpha_{SO} \left[2 \pi \left(\frac{1}{2} R_M^3 r^2 - R_M^2 r^3 + \frac{3}{4} R_M r^4 - \frac{1}{5} r^5 \right) - \frac{Z_S t}{\sin \alpha_{SO}} \left(R_M^3 r - \frac{3}{2} R_M^2 r^2 + R_M r^3 - \frac{1}{4} r^4 \right) \right]_{R_I}^{R_{CI}}$$

and the upper stator

$$\dot{P}_{SCSU} = K_{SCSU} \omega^3 \quad (3.40)$$

$$K_{SCSU} = \frac{1}{2} K_B \rho \sin \alpha_{SI} \left[2 \pi \left(\frac{1}{5} r^5 - \frac{3}{4} R_M r^4 + R_M^2 r^3 - \frac{1}{2} R_M^3 r^2 \right) - \frac{Z_S t}{\sin \alpha_{SI}} \left(\frac{1}{4} r^4 - R_M^2 r^3 + \frac{3}{2} R_M^2 r^2 - R_M^3 r \right) \right]_{R_{CO}}^{R_O}$$

Therefore the total secondary circulation loss is found as the sum of Eqns. (3.37), (3.38), (3.39) and (3.40):

$$\dot{P}_{SC} = K_{SC} \omega^3 \quad (3.41)$$

where

$$K_{SC} = K_{SCRL} + K_{SCRU} + K_{SCSL} + K_{SCSU}$$

The total energy dissipation is therefore the sum of Eqns. (3.25), (3.35) and (3.41)

$$\dot{P}_L = (K_{I1} + K_{I4}) \omega \omega_P^2 + (K_{I2} + K_{I5}) \omega^2 \omega_P + (K_{I3} + K_{I6} + K_F + K_{SC}) \omega^3 \quad (3.42)$$

3.3.4. Empirical loss coefficients

In the preceding two sections describing the model of Raine & Hodgson (1991), the calculations for friction and secondary losses require the use of empirical loss coefficients. The determination of these values are discussed here.

Friction factor

In order to find the friction factor the approach commonly used for fluid friction in pipes is adapted. The fluid flow area is considered as a rectangular duct Fig. 3.12 with a circumferential width at mean radius per fluid flow space given by

$$X = \left(2\pi R_C - \frac{Zt}{\sin \alpha} \right) \quad (3.43)$$

and depth by,

$$Y = \frac{1}{2} (R_O - R_I) \quad (3.44)$$

From these equations an equivalent hydraulic diameter is found

$$d_h = 4 \times \frac{Y X \sin \alpha}{2Y + X} \quad (3.45)$$

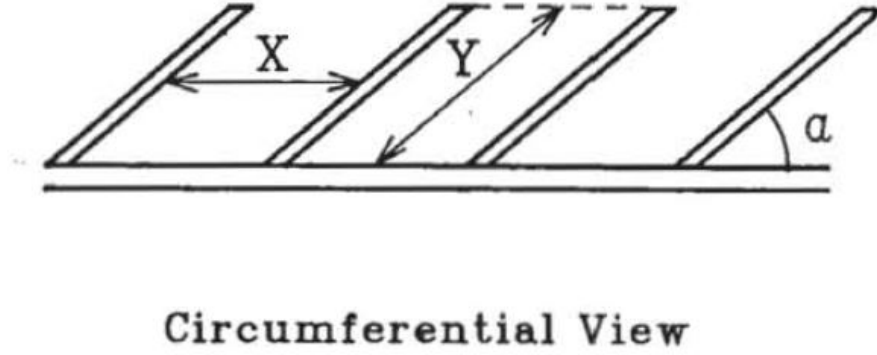


Figure 3.12. Circumferential view used in the determination of the friction factor (Raine & Hodgson, 1991:50)

For the dynamometer at WJ Engineering ($R_O = 144$ [mm], $R_I = 61$ [mm], $t = 5$ [mm], $Z = 12$, $\alpha = 90$ [deg]) these equations give an equivalent hydraulic diameter of 140.7 [mm]. The dynamometer cups have a machine finish that typically have a surface roughness of 0.003 [mm]. By using the water angular velocity of 86.57 [rad/s] at 1200 [rpm] for the WJ Engineering dynamometer, a Reynolds number of 1.1×10^{10} was obtained.

The proper friction factor is then calculated from the Moody formula (Hodgson, 1991:50),

$$f = 0.01375 \left[1 + \left(20000 \frac{k}{d_h} + \frac{10^6}{Re} \right)^{\frac{1}{3}} \right]$$

where

k = surface roughness

d_h = hydraulic diameter

Re = Reynolds number

$\lambda_F = 4f$ = friction constant used in Section 3.3.2

For the dynamometer used at WJ Engineering the appropriate value of λ_F is 0.0465.

Secondary circulation loss coefficient

The values of the secondary circulation loss coefficients depend on the type of machine. Sivalingam *et. al.* (1978) found that a value 0.45 is suitable for retarders which is different from a value of 0.72 commonly used for fluid couplings. Hodgson and Raine (Raine & Hodgson, 1991:51) found that applying a value of 0.45 leads to accurate torque predictions for cropped rotors and 90 deg vane machines.

Therefore a suitable value for the secondary loss coefficient for the dynamometer used at WJ Engineering was chosen as 0.45.

The following section will expand on the supplementary equations needed to be able to utilise the theory presented thus far in this chapter.

3.4. Supplementary equations

In the application of the integrated one dimensional model developed by Raine & Hodgson (1991) the theory must be expanded by supplementary equations that can simulate the self emptying behaviour of hydraulic dynamometers.

The supplementary equations include new methods developed by Raine & Hodgson (1991) to determine the partial fill boundaries derived in Section 3.4.1. Raine & Hodgson includes the effect of acceleration, Coriolis force and non-axial vanes in the derivation of water pressure in the working compartment (Section 3.4.2) in order to expand on previous work. The losses incurred for fluid outflow from the working compartment are analysed in Section 3.4.3.

3.4.1. Partial and varying fill

In the previous sections many of the theoretical derivations use the air-water interface radii as integration limits, therefore Raine & Hodgson developed a method to determine their position, they also expanded upon the method to include the rate of change of the fluid-fill for the purpose of dynamic modelling, this will however not be considered in this study.

The level of the fluid in the working compartment is defined as the percentage fill of the machine:

$$\text{Percentage Fill} = 1 - \frac{\text{Fluid mass displaced by air}}{\text{Maximum Fluid Mass possible}} \quad (3.46)$$

In order to find the fluid mass amounts required, the boundaries of the fluid volume must be clearly defined. A volume integration technique is applied in order to find the mass of liquid in the control volume, namely

$$\iiint_{CV} \rho \, dV$$

The control volume geometry and elemental area are defined in cylindrical coordinates as can be seen in Fig. 3.13 for the rotor with integration required both radially and axially. Due to the machine being symmetrical the elemental values can be taken as equal at all angles around the axis, thus the triple integral is reduced to a double integral, making allowance for vane blockage.

$$\iiint_{CV} \rho \, dV = \int_0^b \int_{g(x)}^{f(x)} \rho \left(2\pi r - \frac{Zt}{\sin \alpha} \right) dr \, dx - \int_0^{b_Y} \int_{g_Y(x)}^{f_Y(x)} \rho \left(2\pi r - \frac{Zt}{\sin \alpha} \right) dr \, dx \quad (3.47)$$

where

$f(x), g(x)$ =geometric functions defining the cup boundary of the control volume
 $fy(x), gy(x)$ =geometric functions defining the air-water interface boundary of the control volume

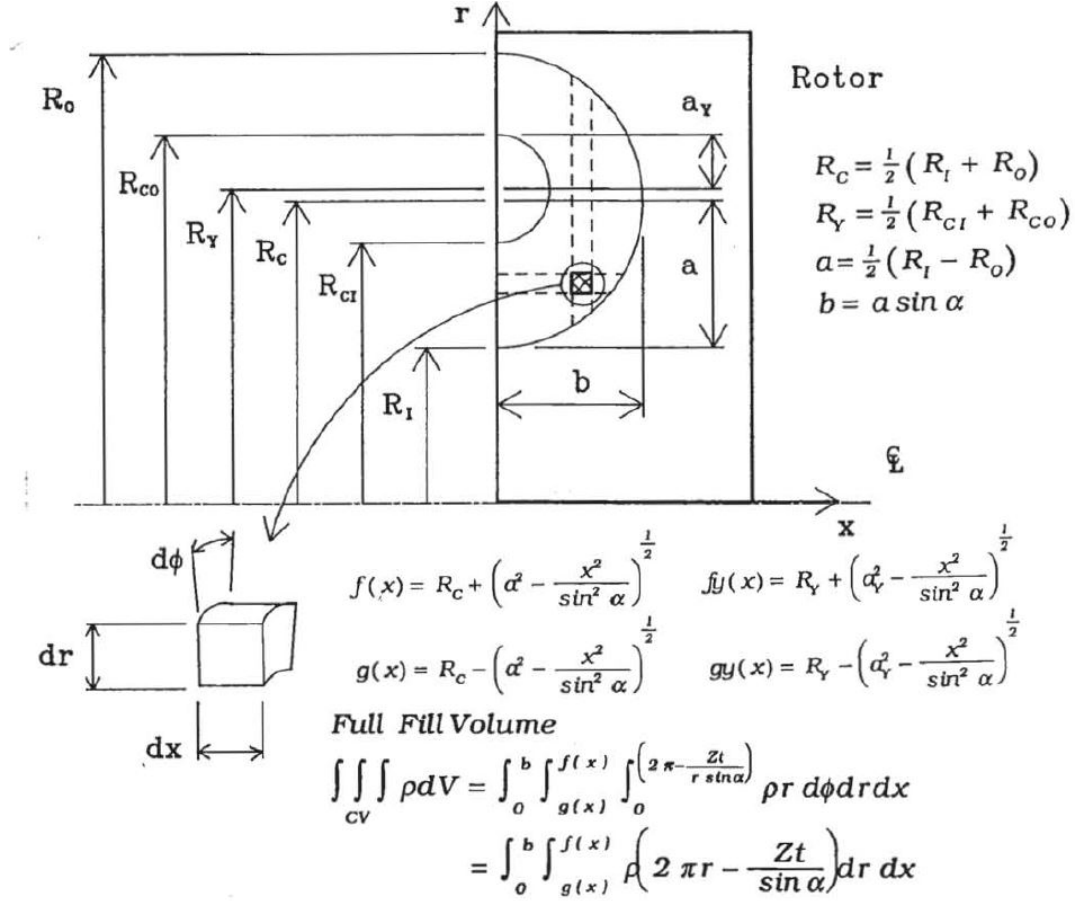


Figure 3.13. Mass Integral in Rotor Control Volume (Raine & Hodgson, 1991:29)

To clarify the function of $f(x)$, $g(x)$, $fy(x)$ and $gy(x)$ are written as f , g , fy and gy respectively from here on. The first term of Eqn. (3.47) becomes

$$\int_0^b \int_g^f \rho \left(2\pi r - \frac{Zt}{\sin \alpha} \right) dr dx = \rho \int_0^b \left[\pi r^2 - \frac{Zt}{\sin \alpha} r \right]_g^f dx \quad (3.48)$$

$$= \rho \int_0^b \left[\pi (f^2 - g^2) - \frac{Zt}{\sin \alpha} (f - g) \right] dx \quad (3.49)$$

From the derivation in Appendix A

$$(f^2 - g^2) = 4 R_c \left(a^2 - \frac{x^2}{\sin^2 \alpha} \right)^{\frac{1}{2}}$$

$$(f - g) = 2 \left(a^2 - \frac{x^2}{\sin^2 \alpha} \right)^{\frac{1}{2}}$$

leading to

$$\int_0^b \int_g^f \rho \left(2 \pi r - \frac{Z t}{\sin \alpha} \right) dr dx = \rho \left(4 \pi R_C - 2 \frac{Z t}{\sin \alpha} \right) \int_0^b \left(a^2 - \frac{x^2}{\sin^2 \alpha} \right)^{\frac{1}{2}} dx$$

By using the identity from Appendix B,

$$\int_0^b \left(a^2 - \frac{x^2}{\sin^2 \alpha} \right)^{\frac{1}{2}} dx = \frac{1}{4} \pi a b \quad (3.50)$$

gives

$$\int_0^b \int_g^f \rho \left(2 \pi r - \frac{Z t}{\sin \alpha} \right) dr dx = \rho \left(2 \pi R_C - \frac{Z t}{\sin \alpha} \right) \frac{1}{2} \pi a b \quad (3.51)$$

for the rotor. If the same analysis is applied to the stator the mass in a fully filled machine is found to be

$$M_F = \rho \left(2 \pi R_C - \frac{Z t}{\sin \alpha} \right) \pi a b \quad (3.52)$$

the same as for an elliptical toroid of mean radius, R_C , with allowance for vane blockage. The derivation is applied in the same way to the second term of Eqn. (3.47). but with f, g, a, b, and R_C replaced with fy, gy, a_Y , b_Y and R_Y respectively, to give the mass of the fluid displaced by air,

$$M_A = \rho \left(2 \pi R_Y - \frac{Z t}{\sin \alpha} \right) \pi a_Y b_Y \quad (3.53)$$

where

$$R_Y = \frac{1}{2} (R_{CO} + R_{CI}), \alpha_Y = \frac{1}{2} (R_{CO} - R_{CI}), b_Y = \frac{1}{2} (R_{CO} - R_{CI}) \sin \alpha$$

Substituting into Eqn. (3.46) yields

$$PF = 1 - \frac{(R_{CO} - R_{CI})^2 \left(\pi (R_{CO} + R_{CO}) - \frac{Z t}{\sin \alpha} \right)}{(R_O - R_I)^2 \left(\pi (R_O + R_I) - \frac{Z t}{\sin \alpha} \right)} \quad (3.54)$$

The radii R_{CO} and R_{CI} define the air water interface for a given percentage fill. These radii are determined by the simultaneous solution of two non-linear equations. The first of which is the definition of fluid percentage fill (Eqn. (3.54)), and the second is an application of the conservation of mass for the stator, as derived in Section 3.2.1, by using the interface radii as integration limits rather than the vortex centre. Hence the mass inflow of Eqn. (3.8) becomes

$$\dot{M}_{SI} = \rho \omega \sin \alpha_{SI} \left[2 \pi \left(\frac{1}{3} r^3 - \frac{1}{2} R_M r^2 \right) - \frac{Z_S t}{\sin \alpha_{SI}} \left(\frac{1}{2} r^2 - R_M r \right) \right]_{R_{CO}}^{R_O}$$

and outflow (Eqn. (3.9)) is

$$\dot{M}_{SO} = \rho \omega \sin \alpha_{SO} \left[2 \pi \left(\frac{1}{2} R_M r^2 - \frac{1}{3} r^3 \right) - \frac{Z_S t}{\sin \alpha_{SO}} \left(R_M r - \frac{1}{2} r^2 \right) \right]_{R_I}^{R_{CI}}$$

Equating inflow and outflow and rearranging gives

$$\begin{aligned} h(R_{CO}, R_{CI}) = & \frac{2}{3} \pi [(R_O^3 - R_{CO}^3) \sin \alpha_{SI} + (R_{CI}^3 - R_I^3) \sin \alpha_{SO}] \\ & - \pi R_M [(R_O^2 - R_{CO}^2) \sin \alpha_{SI} + (R_{CI}^2 - R_I^2) \sin \alpha_{SO}] \\ & - \frac{1}{2} Z_S t (R_O^2 - R_{CO}^2 + R_{CI}^2 - R_I^2) \\ & + Z_S t R_M (R_O - R_{CO} + R_{CI} - R_I) = 0 \quad (3.55) \end{aligned}$$

The percentage fill definition (Eqn. (3.54)) is rearranged,

$$\begin{aligned} j(R_{CO}, R_{CI}) = & (R_{CO} - R_{CI})^2 \left(\pi (R_{CO} + R_{CI}) - \frac{Z t}{\sin \alpha} \right) \\ & - (1 - PF) (R_O - R_I)^2 \left(\pi (R_O + R_I) - \frac{Z t}{\sin \alpha} \right) = 0 \quad (3.56) \end{aligned}$$

These can be solved by applying Newton-Raphson method in two variables, by use of the iteration

$$\begin{bmatrix} R_{CO_{i+1}} \\ R_{CI_{i+1}} \end{bmatrix} = \begin{bmatrix} R_{CO_i} \\ R_{CI_i} \end{bmatrix} - \mathbf{J}_i^{-1} \begin{bmatrix} h(R_{CO_i}, R_{CI_i}) \\ j(R_{CO_i}, R_{CI_i}) \end{bmatrix} \quad (3.57)$$

where $\mathbf{J}$ is the Jacobi matrix,

$$\mathbf{J}_i = \begin{bmatrix} h_{R_{CO_i}} & h_{R_{CI_i}} \\ j_{R_{CO_i}} & j_{R_{CI_i}} \end{bmatrix}$$

the elements of which are the derivatives of $h(R_{CO}, R_{CI})$ and $j(R_{CO}, R_{CI})$ with respect to R_{CO} and R_{CI} ,

$$\begin{aligned} h_{R_{CO}} &= R_{CO} (2 \pi R_M \sin \alpha_{SI} + Z_S t) - 2 \pi \sin \alpha_{SI} R_{CO}^2 - Z_S t R_M \\ h_{R_{CI}} &= 2 \pi \sin \alpha_{SO} R_{CI}^2 - R_{CI} (2 \pi R_M \sin \alpha_{SO} + Z_S t) + Z_S t R_M \\ j_{R_{CO}} &= 2 (R_{CO} - R_{CI}) \left(\pi (R_{CO} + R_{CI}) - \frac{Z t}{\sin \alpha} \right) + \pi (R_{CO} - R_{CI})^2 \\ j_{R_{CI}} &= -2 (R_{CO} - R_{CI}) \left(\pi (R_{CO} + R_{CI}) - \frac{Z t}{\sin \alpha} \right) + \pi (R_{CO} - R_{CI})^2 \end{aligned}$$

After the matrix multiplication of the right hand side the iteration becomes

$$\begin{bmatrix} R_{CO_{i+1}} \\ R_{CI_{i+1}} \end{bmatrix} = \begin{bmatrix} R_{CO_i} \\ R_{CI_i} \end{bmatrix} - \frac{1}{(h_{R_{CO}} j_{R_{CI}} - j_{R_{CO}} h_{R_{CI}})_i} \begin{bmatrix} j_{R_{CI}} h_i & -h_{R_{CI}} j_i \\ -j_{R_{CO}} h_i & +h_{R_{CO}} j_i \end{bmatrix} \quad (3.58)$$

Thus at any percentage fill (at steady state conditions) the position of the air-water interface is evaluated.

In the next section, the fluid pressure in the working compartment will be considered.

3.4.2. Fluid pressure

When considering the fluid in the working compartment of the dynamometer it is clear that the pressure will vary according to the rotational speed of the dynamometer. In Fig. 3.14 it can also be seen that the fluid flow follows a helical path, thus a pressure variation occurs in the fluid as you move about the flow path. This variation in pressure based on the rotational velocity and helical flow path will have a significant effect on the fluid outflow from the working compartment (self-emptying behaviour) and the fluid-fill of the dynamometer. The fluid outflow and the back-pressure has a direct influence on the torque produced by the dynamometer therefore the pressure variation must be studied.

Since the fluid pressure varies along the helical flow path and is found to be lower in the stator than in the rotor, the rate of fluid emptying from the working compartment will change drastically depending on the position of the fluid outlet holes from the working compartment. The fluid flow path from the outlet of the working compartment to the outlet control valve is considered in the next Section 3.4.3, which is used to find the change in fluid through-flow tempo and the desired design parameters used for the design of a suitable control valve.

Raine & Hodgson (1991, 1992a) found that previous work in the area of pressure analysis of the working compartment of torque converters and fluid couplings did not meet the requirements of their one-dimensional model. They expanded upon the previous work by implementing a vector approach to allow for pressure calculations around the working compartment at different vane angles.

Raine & Hodgson reported that the most complete analysis of the fluid pressure was done by Sivalingam (1978) who used the pressure expression to define the free surface profile in partially filled fluid couplings, thus allowing for calculation at different points in the working compartment and including forces due to both machine and vortex rotation (which has not been the case in previous work, in which they would only consider one or the other). However the work of Sivalingam omits the effect of Coriolis force while analysing a fluid element, which is necessary as the fluid element is analysed within the context of a rotating reference frame. In the case of an axial vaned machine such as the dynamometer developed at WJ Engineering the omission is of no concern since the Coriolis component is perpendicular to the plane in which the forces are analysed, but in order to account for the effect when considering the analysis of other vane angles Raine & Hodgson included the Coriolis effect into Sivalingam's approach. Raine & Hodgson also expanded the approach by including the effects of dynamometer acceleration to allow for dynamic modelling, but this is not of interest in this study as only steady state modelling will be done.

For the purpose of calculating the fluid pressure a fluid element, lying in the machine vane plane, is considered. The vane plane of reference rotates at the same speed as the machine part in which the fluid is being examined. The fluid itself follows a circular vortex path within this vane plane of reference. The velocity vectors, acceleration vectors and fluid flow relative to the rotating vane plane are shown in Fig. 3.14.

Due to the assumed curved flow path of the fluid around the vortex the fluid element forces must balance in the direction of the vortex centre. As stated earlier in Section 3.1, the actual flow path does not coincide with a circle as the model vortex centre is different from that of the working compartment centre; due to this there occurs a small difference between the relative

velocity and acceleration vectors used and the actual vectors. Raine & Hodgson compensated for this difference due to the small component of the tangential vector being excluded by not reducing the radial component, thus the simplification only introduces a small error into the pressure determination.

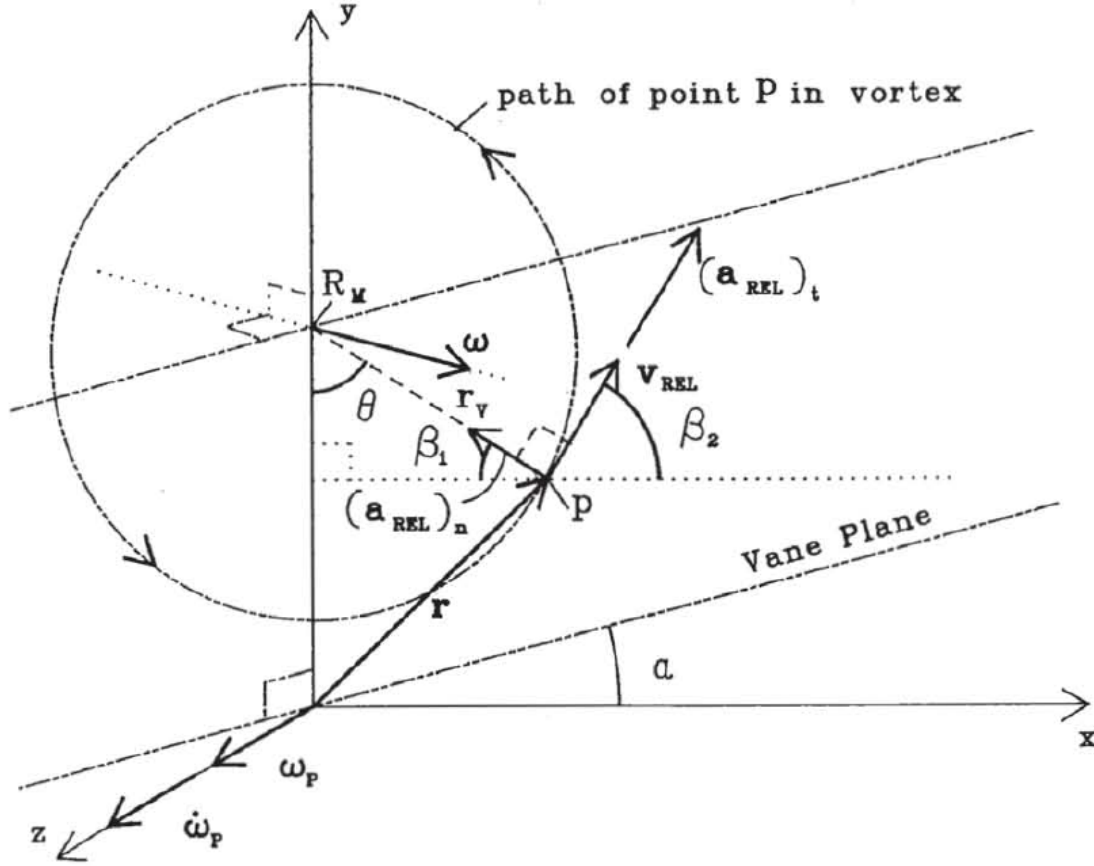


Figure 3.14. Fluid Position, Velocity and Acceleration Vectors (Raine & Hodgson, 1991:58)

A fluid element is considered when analysing the pressure in the working compartment as shown in Fig. 3.15, this figure shows the total pressure force experienced by an fluid element of unit thickness is the sum of all the forces towards the centre of the vortex,

$$F_{PI} = (P + \delta P) \delta \theta (r_v + \delta r_v)$$

$$F_{PI} = P r_v \delta \theta + P \delta r_v \delta \theta + \delta P r_v \delta \theta + \delta P \delta r_v \delta \theta$$

and the force away from R_M ,

$$F_{PO} = -P r_v \delta \theta - 2(P + k \delta P) \delta r_v \sin\left(\frac{1}{2}\delta \theta\right)$$

which yields

$$F_P = P r_v \delta \theta + P \delta r_v \delta \theta + \delta P r_v \delta \theta + \delta P \delta r_v \delta \theta - P r_v \delta \theta - 2(P + k \delta P) \delta r_v \sin\left(\frac{1}{2}\delta \theta\right)$$

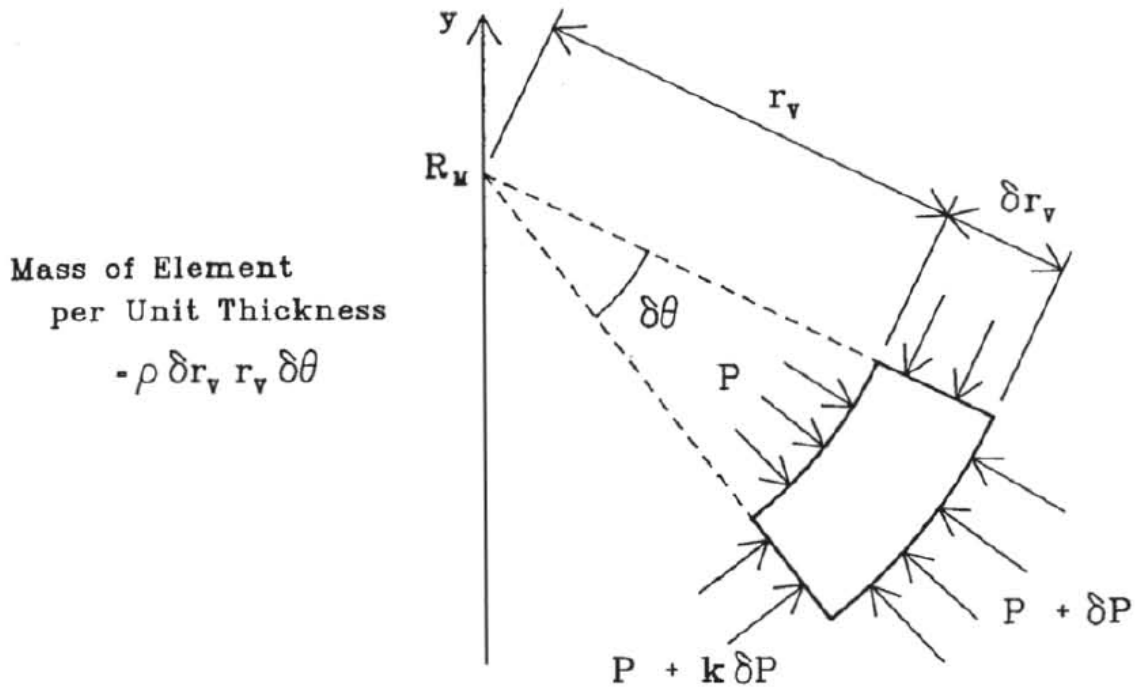


Figure 3.15. Pressure Forces on a Fluid Element (Raine & Hodgson, 1991:59)

Use the small angle approximation for small θ in $\sin \theta$ and neglecting the second order of smallness terms, the simplifications becomes

$$F_P = \delta P r_v \delta \theta \quad (3.59)$$

At a point P in a reference frame rotating with angular velocity ω_P , the acceleration components are

$$\mathbf{a} = \dot{\omega}_P \times \mathbf{r} + \omega_P \times (\omega_P \times \mathbf{r}) + 2 \omega_P \times \mathbf{v}_{REL} + \mathbf{a}_{REL} \quad (3.60)$$

Regarding Fig. 3.16. Raine & Hodgson cited Meriam (1975) to state that the direction of the vector cross products must be considered meticulously as vane angle α and cup angle θ change, since the products normally lie in the rotor plane (X - Y plane in the figure) rather than the vane plane, but occasionally in the line of intersection. For this reason Raine & Hodgson determined directional co-factors to give the radial components of acceleration toward the vortex centre, depending on the geometry, see Fig. 3.16.

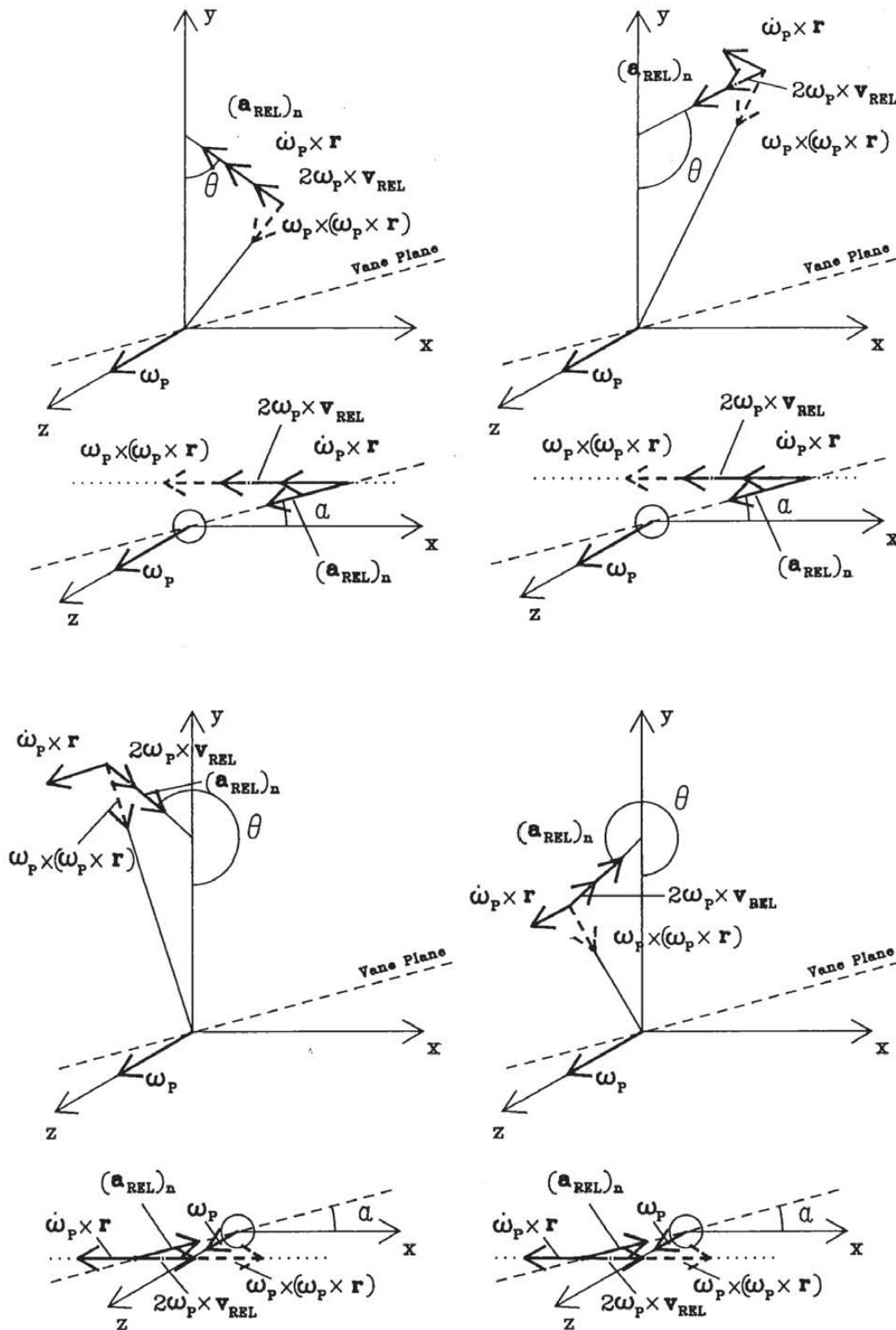


Figure 3.16. Fluid Acceleration and Vector Cross Products (Raine & Hodgson, 1991:60)

In Eqn. (3.60), the first cross product term is found for the acceleration of the reference frame (due to the rotor acceleration) and has a directional cofactor from Fig. 3.16 and Fig. 3.17 of,

$$C_{WR} = K \cos \mu \cos \alpha \quad (3.61)$$

where

$$\mu = \text{absolute}(\zeta - 90^\circ)$$

$$K = \begin{cases} 1, & 0^\circ < \theta < 180^\circ \\ -1, & 180^\circ < \theta < 360^\circ \\ 0, & \theta = 0^\circ \text{ or } 180^\circ \end{cases}$$

Since the stator is stationary the first of the three cross product terms all becomes zero (for $180^\circ < \theta < 360^\circ$ the stator portion of the working compartment) even though their cofactors are non-zero, because the frame of reference is stationary at all times in the stator portion.

The final cross product term for the Coriolis acceleration ($2 \omega_p \times \mathbf{v}_{REL}$) will have a cofactor,

$$C_{WV} = \begin{cases} \cos \alpha, & \theta \neq 0^\circ \text{ or } 180^\circ \\ 1, & \theta = 0^\circ \text{ or } 180^\circ \end{cases} \quad (3.62)$$

since it will always be in the rotor plane with the exception of $\theta = 0^\circ$ and $\theta = 180^\circ$ when it intersects with the rotor and vane planes.

Special care must be taken to find the direction cofactor of the machine rotation term, given by the second vector cross product $\omega_p \times (\omega_p \times \mathbf{r})$, because the resultant direction of this product varies from that of the negative $\mathbf{r}$ vector to the negative y-axis direction as the vane angle α changes from 0° to 90° as seen in Fig. 3.17. The angle ζ is determined by use of the cosine rule,

$$\cos \zeta = \frac{r_v^2 + |\mathbf{r}|^2 - R_M^2}{2 r_v |\mathbf{r}|} \quad (3.63)$$

with $|\mathbf{r}|$ the magnitude of vector $\mathbf{r}$; angle β is found by the following equation and varies linearly with vane angle,

$$\beta = (180^\circ - \zeta) - \frac{\alpha}{90^\circ} (180^\circ - \zeta - \theta) \quad (3.64)$$

As with the Coriolis component the rotation product lies in the rotor plane and the line of intersection at $\theta = 0^\circ$ and $\theta = 180^\circ$. Therefore the machine rotation direction cofactor is

$$C_{WWR} = \begin{cases} -\cos \beta \cos \alpha, & \theta \neq 0^\circ \text{ or } 180^\circ \\ -\cos \beta, & \theta = 0^\circ \text{ or } 180^\circ \end{cases} \quad (3.65)$$

where β is defined by Eqn. (3.63) and Eqn. (3.64).

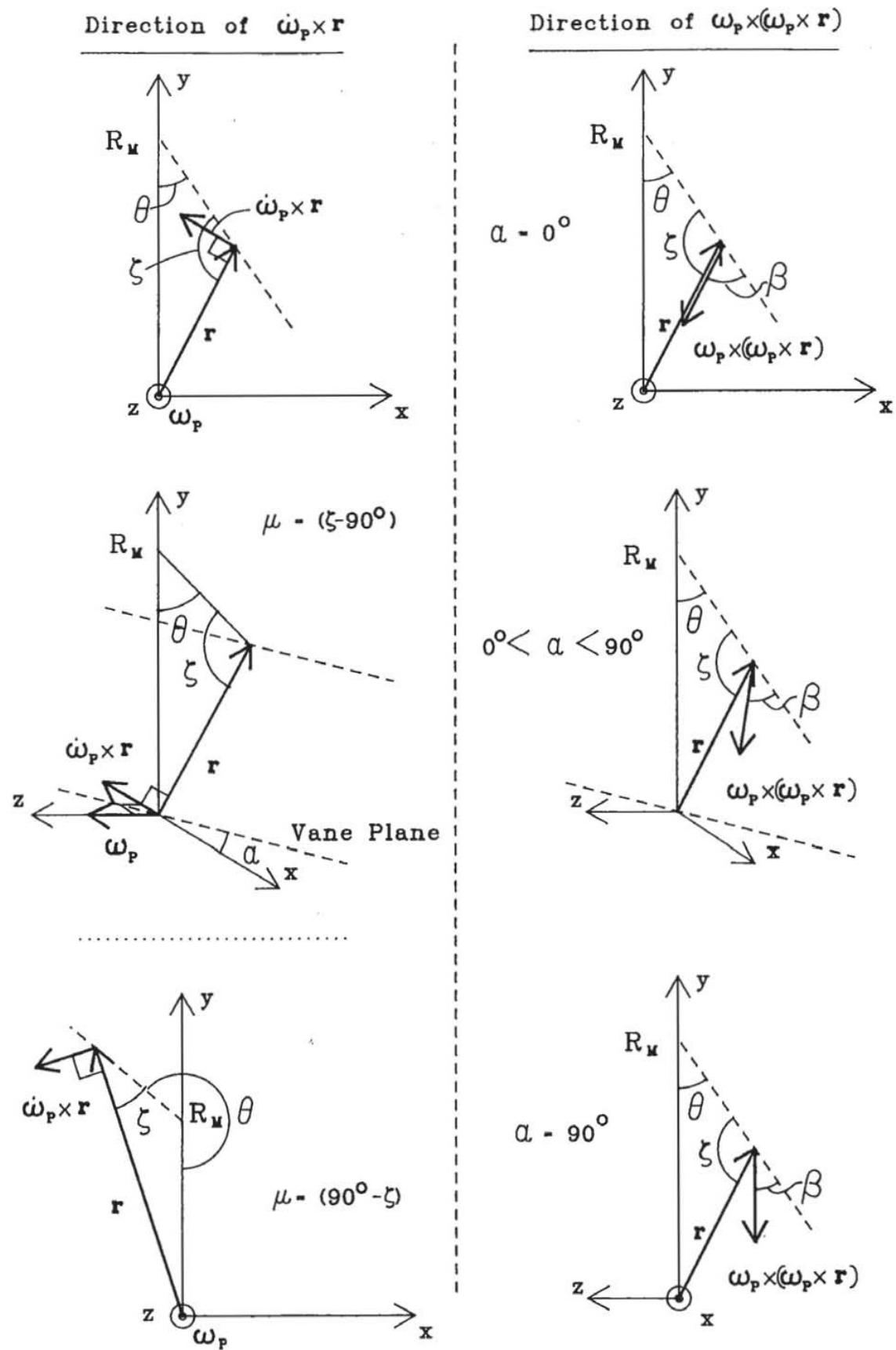


Figure 3.17. Directions of Vector Cross Product Radial Components (Raine & Hodgson, 1991:62)

As can be seen the directional cofactors are all equal to zero for an axially vaned machine (like the one considered in this study), thus for this study it is not actually necessary to determine these cofactors, but for the sake of completeness it is included.

The final term $\mathbf{a}_{REL}$ from Eqn. Eqn. (3.60) is the acceleration relative to the reference frame, due to the fluid vortex. The total relative acceleration consists of as radial and tangential components:

$$\mathbf{a}_{REL} = (\mathbf{a}_{REL})_n + (\mathbf{a}_{REL})_t$$

The fluid velocity is taken as perpendicular to the radial direction, thus the acceleration component due to the fluid vortex is simply the centripetal acceleration, so the direction cofactor is unity for all angles.

The magnitude of a vector cross product is given by

$$\mathbf{a} \times \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \sin \gamma$$

where

$$\gamma = \text{angle between } \mathbf{a} \text{ and } \mathbf{b} = \cos^{-1} \left(\frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}| |\mathbf{b}|} \right)$$

$$|\mathbf{a}| \text{ \& } |\mathbf{b}| = \text{magnitudes of } \mathbf{a} \text{ \& } \mathbf{b}$$

From Fig. 3.14 the vectors of Eqn. (3.60) are, in Cartesian coordinates,

position

$$\mathbf{r} = (r_v \sin \theta \cos \alpha, R_M - r_v \cos \theta, -r_v \sin \theta \sin \alpha)$$

relative velocity

$$\mathbf{v}_{REL} = (\omega r_v \cos \beta_2 \cos \alpha, \omega r_v \sin \beta_2, -\omega r_v \cos \beta_2 \sin \alpha)$$

radial relative acceleration

$$\begin{aligned} (\mathbf{a}_{REL})_n &= (-\omega^2 r_v \cos \beta_1 \cos \alpha, \omega^2 r_v \sin \beta_1, \omega^2 r_v \cos \beta_1 \sin \alpha) \\ &= (-\omega^2 r_v \sin \theta \cos \alpha, \omega^2 r_v \cos \theta, \omega^2 r_v \sin \theta \sin \alpha) \end{aligned}$$

machine rotational velocity

$$\omega_p = \begin{cases} (0, 0, \omega_p) , & \text{rotor} \\ (0, 0, 0) , & \text{stator} \end{cases}$$

machine rotational acceleration

$$\dot{\omega}_p = \begin{cases} (0, 0, \dot{\omega}_p) , & \text{rotor} \\ (0, 0, 0) , & \text{stator} \end{cases}$$

The magnitudes of which are

$$\begin{aligned}
 |\mathbf{r}| &= (r_v^2 + R_M^2 - 2 R_M r_v \cos \theta)^{\frac{1}{2}} \\
 |\mathbf{v}_{REL}| &= \omega r_v \\
 |(\mathbf{a}_{REL})_n| &= \omega^2 r_v \\
 |\omega_p| &= \begin{cases} \omega_p, & \text{rotor} \\ 0, & \text{stator} \end{cases} \\
 |\dot{\omega}_p| &= \begin{cases} \dot{\omega}_p, & \text{rotor} \\ 0, & \text{stator} \end{cases}
 \end{aligned}$$

Hence the vector cross products are

$$\dot{\omega}_p \times \mathbf{r} = \dot{\omega}_p (r_v^2 + R_M^2 - 2 R_M r_v \cos \theta)^{\frac{1}{2}} \sin \gamma_A$$

where

$$\gamma_A = \cos^{-1} \left(\frac{-r_v \sin \theta \sin \alpha}{(r_v^2 + R_M^2 - 2 R_M r_v \cos \theta)^{\frac{1}{2}}} \right)$$

$$\omega_p \times \mathbf{r} = \dot{\omega}_p (r_v^2 + R_M^2 - 2 R_M r_v \cos \theta)^{\frac{1}{2}} \sin \gamma_R$$

where

$$\gamma_R = \cos^{-1} \left(\frac{-r_v \sin \theta \sin \alpha}{(r_v^2 + R_M^2 - 2 R_M r_v \cos \theta)^{\frac{1}{2}}} \right)$$

$$\omega_p \times (\omega_p \times \mathbf{r}) = \omega_p^2 (r_v^2 + R_M^2 - 2 R_M r_v \cos \theta)^{\frac{1}{2}} \sin \gamma_R \sin \gamma_W$$

where

$$\sin \gamma_W = 1 \text{ as } (\omega_p \times \mathbf{r}) \perp \omega_p$$

$$2 \omega_p \times \mathbf{v}_{REL} = 2 \omega_p \omega r_v \sin \gamma_v$$

where

$$\gamma_V = \cos^{-1} (-\cos \theta \sin \alpha)$$

Applying Newton's second law to the fluid element of Fig. 3.15 in the radial direction,

$$\text{net pressure force} = \text{mass} \times \text{radial acceleration component}$$

gives

$$\begin{aligned} \delta P r_v \delta \theta = \rho \delta r_v r_v \delta \theta \left[\dot{\omega}_p (r_v^2 + R_M^2 - 2 R_M r_v \cos \theta)^{\frac{1}{2}} \sin \gamma_A C_{WR} \right. \\ \left. + \omega_p^2 (r_v^2 + R_M^2 - 2 R_M r_v \cos \theta)^{\frac{1}{2}} \sin \gamma_R C_{WWR} \right. \\ \left. + 2 \omega_p \omega r_v \sin \gamma_V C_{WV} + \omega^2 r_v \right] \end{aligned}$$

Which rearranged is

$$\begin{aligned} \delta P = \rho \left[\omega^2 r_v + 2 \omega_p \omega \sin \gamma_V C_{WV} r_v \right. \\ \left. + (\dot{\omega}_p \sin \gamma_A C_{WR} + \omega_p^2 \sin \gamma_R C_{WWR}) (r_v^2 + R_M^2 - 2 R_M r_v \cos \theta)^{\frac{1}{2}} \right] \delta r_v \quad (3.66) \end{aligned}$$

To simplify the integration of the last term of Eqn. (3.66) the following substitutions are made,

$$u = (r_v - R_M \cos \theta) \text{ and } a^2 = R_M^2 (\cos^2 \theta - 1)$$

and by using the following integration identities

$$\int (u^2 - a^2)^{\frac{1}{2}} du = \frac{1}{2} u (u^2 - a^2)^{\frac{1}{2}} - \frac{1}{2} a^2 \ln \left| u - (u^2 - a^2)^{\frac{1}{2}} \right| + C$$

Also note

$$\ln \left| u + (u^2 - a^2)^{\frac{1}{2}} \right| = \cosh^{-1} \left(\frac{u}{a} \right)$$

Thus Eqn. (3.66) is integrated,

$$\begin{aligned} P = \rho \left[(\omega^2 + 2 \omega_p \omega \sin \gamma_V C_{WV}) \frac{1}{2} r_v^2 + (\dot{\omega}_p \sin \gamma_A C_{WR} + \omega_p^2 \sin \gamma_R C_{WWR}) \right. \\ \times \left\{ \frac{1}{2} (r_v - R_M \cos \theta) (r_v^2 + R_M^2 - 2 R_M r_v \cos \theta)^{\frac{1}{2}} - \frac{1}{2} (\cos^2 \theta - 1) R_M^2 \right. \\ \left. \times \ln \left| (r_v - R_M \cos \theta) + (r_v^2 + R_M^2 - 2 R_M r_v \cos \theta)^{\frac{1}{2}} \right| \right\} \left. \right] + C \quad (3.67) \end{aligned}$$

Since this is the indefinite integral and the constant of integration C is unknown, it is necessary determine the integral at the boundary condition of the air-water interface as well, which is at atmospheric pressure due to the air vents in the dynamometer.

The pocket of air is assumed to be circular, with

$$\text{centre } R_Y = \frac{1}{2} (R_{CO} + R_{CI}) \text{ and radius } a = \frac{1}{2} (R_{CO} - R_{CI})$$

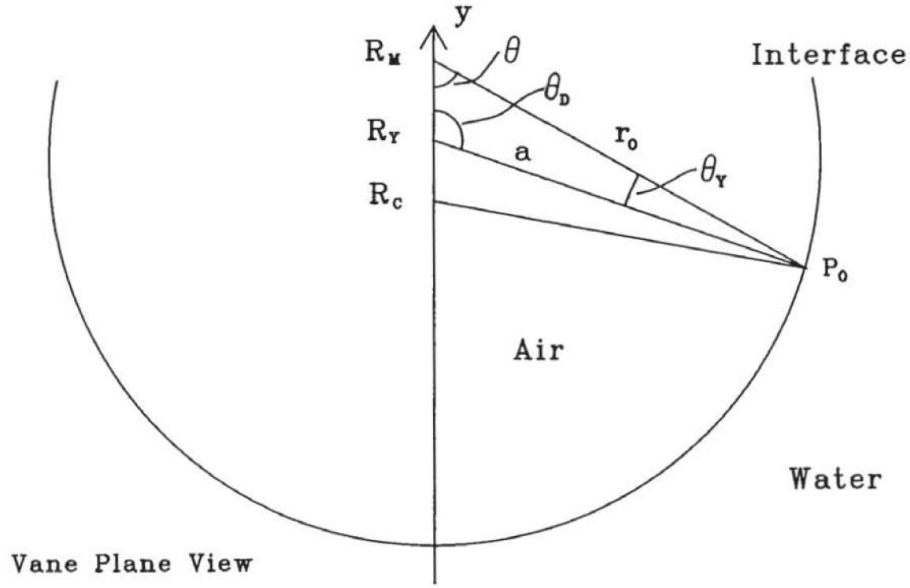


Figure 3.18. Determination of Distance to Air-Water Interface (Raine & Hodgson, 1991:66)

From Fig. 3.18 the point on the interface where the pressure will equal atmospheric pressure is determined by using the sine rule,

$$r_o = a \frac{\sin \theta_D}{\sin \theta} \quad (3.68)$$

where

$$\theta_D = 180^\circ - (\theta_Y + \theta)$$

and

$$\sin \theta_Y = \frac{1}{a} (R_M - R_Y) \sin \theta, \text{ when } \theta > 180^\circ \text{ put } \theta = (360^\circ - \theta)$$

By taking atmospheric pressure P_o at the radial distance r_o as determined above, and applying it to Eqn. (3.67) gives

$$\begin{aligned} P_o = \rho \left[(\omega^2 + 2 \omega_p \omega \sin \gamma_V C_{WV}) \frac{1}{2} r_o^2 + (\dot{\omega}_p \sin \gamma_A C_{WR} + \omega_p^2 \sin \gamma_R C_{WWR}) \right. \\ \times \left\{ \frac{1}{2} (r_o - R_M \cos \theta) (r_o^2 + R_M^2 - 2 R_M r_o \cos \theta)^{\frac{1}{2}} - \frac{1}{2} (\cos^2 \theta - 1) R_M^2 \right. \\ \left. \left. \times \ln \left| (r_o - R_M \cos \theta) + (r_o^2 + R_M^2 - 2 R_M r_o \cos \theta)^{\frac{1}{2}} \right| \right\} \right] + C \quad (3.69) \end{aligned}$$

The gauge pressure at any point along the radial direction of angle θ can be found by subtracting Eqn. (3.69) from Eqn. (3.67). For simulation we would mostly be concerned with the pressure at the surface of the compartment wall, thus a method to calculate the distance r_v to the surface is required, as well as a method to convert angles around the working compartment from its centre (which is defined easier) to angles from the vortex centre.

The angle θ is used to find the working compartment fluid pressure, this angle however is defined from the vortex centre, and in Section 3.1 it is shown that the compartment centre and the vortex centre do not coincide.

It is simpler to present the pressure values in terms of the angle θ_c defined from the working compartment centre as seen in Fig. 3.19, thus θ_c must be converted to its equivalent θ value.

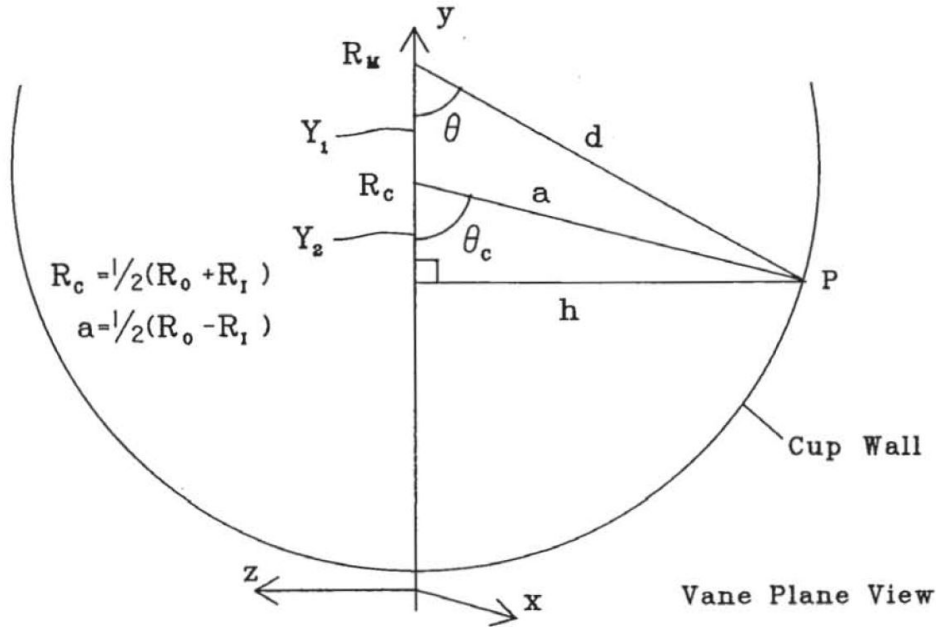


Figure 3.19. Cup and Vortex Angle Conversion (Raine & Hodgson, 1991:66)

From the figure above the following identities are obtained:

$$Y_1 = R_M - R_C$$

$$Y_2 = a \cos \theta_c$$

$$h = a \sin \theta_c$$

*(note that a in the above equations differ from a used in Eqn. (3.68))

The vortex angle can now be calculated by

$$\theta = \tan^{-1} (h / (Y_1 + Y_2))$$

As the position of angle θ_c changes around the working compartment the sign of both the numerator and denominator of the expression will change. Therefore several conditional statements are needed to ensure that the correct value between 0 °and 360 °is calculated. These conditional statements are given below.

If $Y_1 + Y_2 = 0$ then

$$\theta = 90^\circ \text{ if } \theta_c < 180^\circ$$

$$\text{or } \theta = 270^\circ \text{ if } \theta_c > 180^\circ$$

If $\cos \theta_c < 0^\circ$ and $Y_1 + Y_2 < 0$ then

$$\theta = 180^\circ + \tan^{-1} (h / (Y_1 + Y_2))$$

If $\cos \theta_c > 0$ and $\theta_c > 180^\circ$ then

$$\theta = 360^\circ + \tan^{-1} (h / (Y_1 + Y_2))$$

The casing radius r_v is calculated by Pythagoras' Theorem,

$$r_v = \left[(Y_1 + Y_2)^2 + h^2 \right]^{\frac{1}{2}}$$

Thus the vortex pressure at any point along the working compartment can be obtained. This pressure can especially be used to find the pressure at the working compartment outlet holes and the periphery of the rotor and stator cups that will pump water from the working compartment out of the dynamometer. Then by knowing the pressure loss from the flow out of the dynamometer the fluid velocity corresponding to the pressure can be obtained and then used to design a suitable control valve.

In the next section, the fluid outflow from the working compartment will be explained and defined, which is then used in conjunction with the vortex pressure to determine the design parameters of a suitable control valve.

3.4.3. Fluid outflow from working compartment

Having calculated the fluid pressure in the working compartment, this can be used to find approximate values for the design parameters of the desired outlet control valve. The pressure loss coefficient of a suitable outlet control valve is found by analysing the fluid outflow path, at different flow velocities, using Bernoulli's energy equation with allowance for pipe energy losses (Massey, 1979). In order to use this method, it should be assumed that there is no aeration of the water, in order to have constant fluid density, and that the drain passages are full at all times, leading to steady flow (no mass accumulation). Using this approach leads to gross simplification of the complex turbulent two-phase flows that are realistic for such a device, but it will at least lead to adequately accurate answers to provide a starting point for the design of the control valve, which would be verified empirically later in the study.

The outflow control valve is an important component due to its effect on the steady state torque characteristic. As the amount of valve closure is changed the torque absorption is varied during continuous operation. Increased closure of the control valve causes the map of inflow equal to outflow points to shift towards higher torque values, this also leads to a higher rate of decreased fluid-fill when in the self-emptying region (Raine & Hodgson, 1991:125).

The dynamometer should also achieve sufficient through-flow to allow for the cooling of the working compartments; WJ Engineering suggested that these flows should be between 1 [l/s] for low fill and 4 [l/s] for high fill scenarios.

Dynamometers commonly have three different outflow paths: from the back of the rotor cup; from the periphery of the working compartment; and from the back of the stator cup. The WJ Engineering dynamometer outflow path is a combination of fluid flow starting from the back of the stator cups and the rotor periphery see Fig. 3.21, and will be analysed in the

following section in order to find a suitable outlet control valve characteristic at certain fluid outflow rates.

Dynamometer flow paths:

In Fig. 3.20 the inflow and outflow of the working fluid to the dynamometer are illustrated. Fluid flows into the dynamometer, due to gravity and the dynamometer suction, from an inlet pipe with one end submerged in the water bucket. The inlet water pipe delivers water to the circular fluid passage (inlet annulus) found inside the dynamometer that allows water and air flow into the working compartment through holes found in the stator vanes.

Outflow from the dynamometer occurs through the three outlet pipe sections found at the sides and top of the dynamometer, the outflows combine at the cross from where it flows out of the outlet control valve into the water bucket.

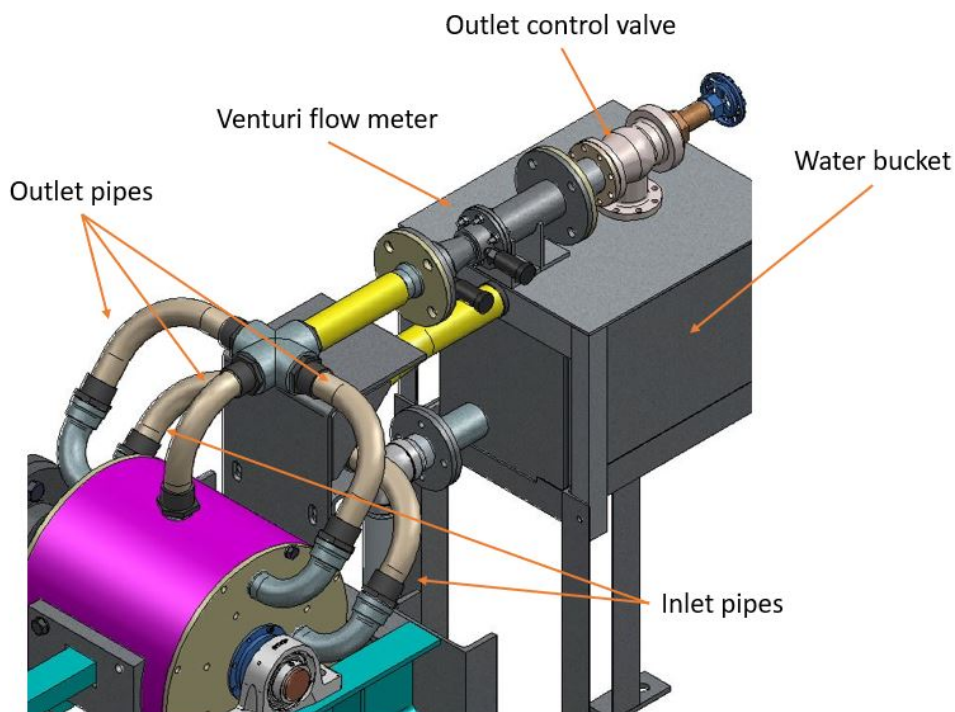


Figure 3.20. Inflow and Outflow of Dynamometer

The outflow paths in combination with the outlet control valve produce certain energy losses, which directly influence the dynamometer through-flow rate and back-pressure experienced at the outflow. The through-flow rate and back-pressure determines the fill-factor of the working compartment and consequently the torque as explained in Section 2.4. Therefore, the outflow valve can be used to vary the dynamometer torque. By analysing the outflow path energy losses and utilising Bernoulli's equation, the desired loss coefficient of the control valve for different through-flow rates can be calculated and used as a design parameter to develop a suitable control valve.

Fig. 3.21 shows the two outflow paths from the dynamometer working compartment to the outlet of the control valve. Path B, is the flow path from the periphery of the rotor cup to the cross-section. Path C, is the flow path from the outlet holes at the back of the rotor cups to the

cross-section. Finally, path D is the flow path from the cross-section, where flow path B and C combine, to the outlet of the control valve. All three flow paths will be analysed and used simultaneously to find the control valve loss coefficient for specific flow rates.

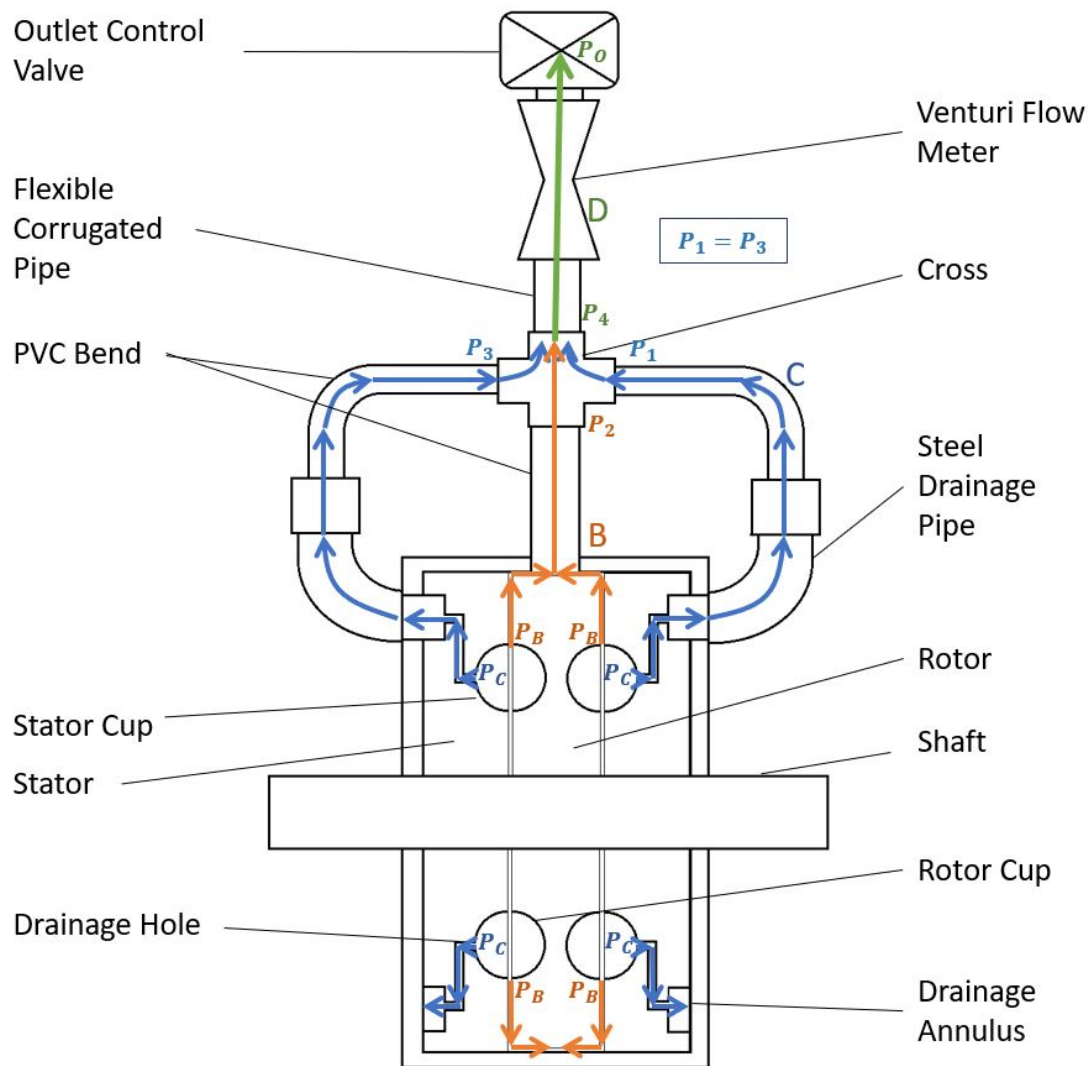


Figure 3.21. Fluid Outflow Paths from Dynamometer Working Compartment

Each feature responsible for empirical pressure losses in each of the flow-paths are marked in Fig. 3.22, inside the dynamometer, and Fig. 3.23, outside the dynamometer.

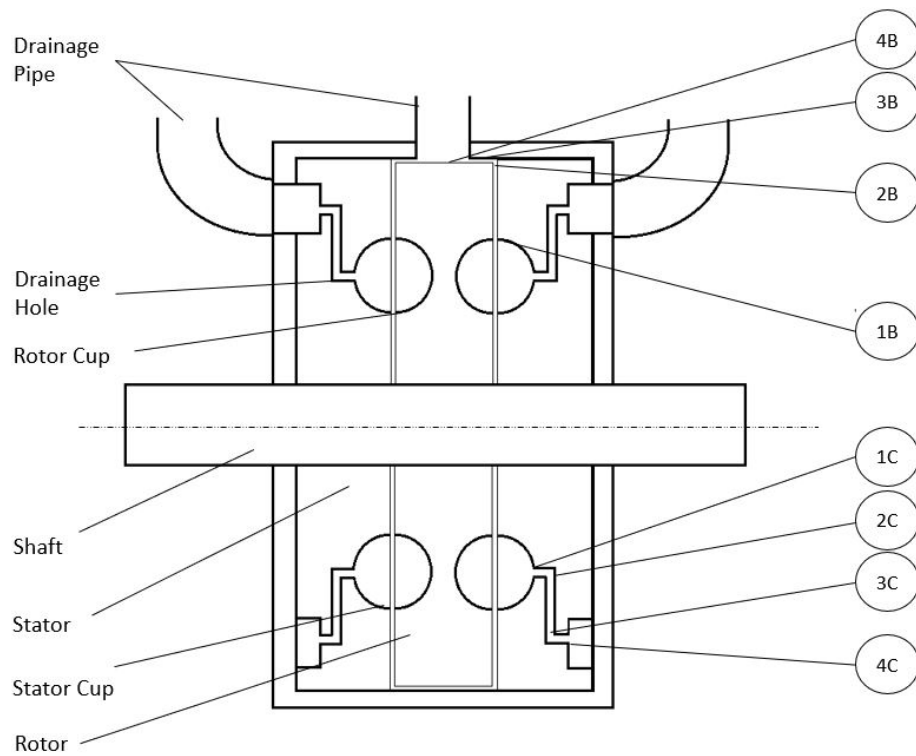


Figure 3.22. Flow-path Features Inside Dynamometer

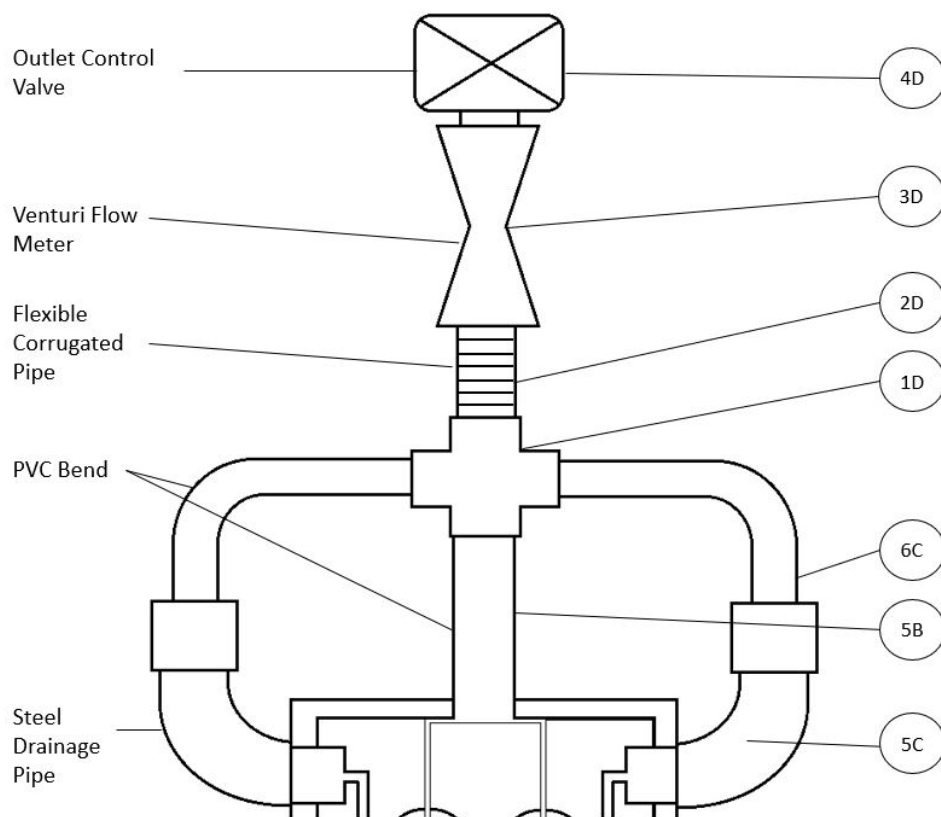


Figure 3.23. Flow-path Features Outside Dynamometer

Cup periphery fluid outlet, flow-path B

The flow from point B, the cup periphery (outer edge of the rotor cup), the fluid leaves the working compartment between the rotor and the stator with a tangential flow velocity component as shown in Fig. 3.24 for flow between a moving plate and a stationary plate. The flow velocity at point B is thus,

$$V_B^2 = \left(\frac{Q_B}{A_B} \right)^2 + \left(\frac{1}{2} \omega_p R_o \right)^2 \quad (3.70)$$

where

A_B = the total gap area available for fluid outflow between the rotor and the stator

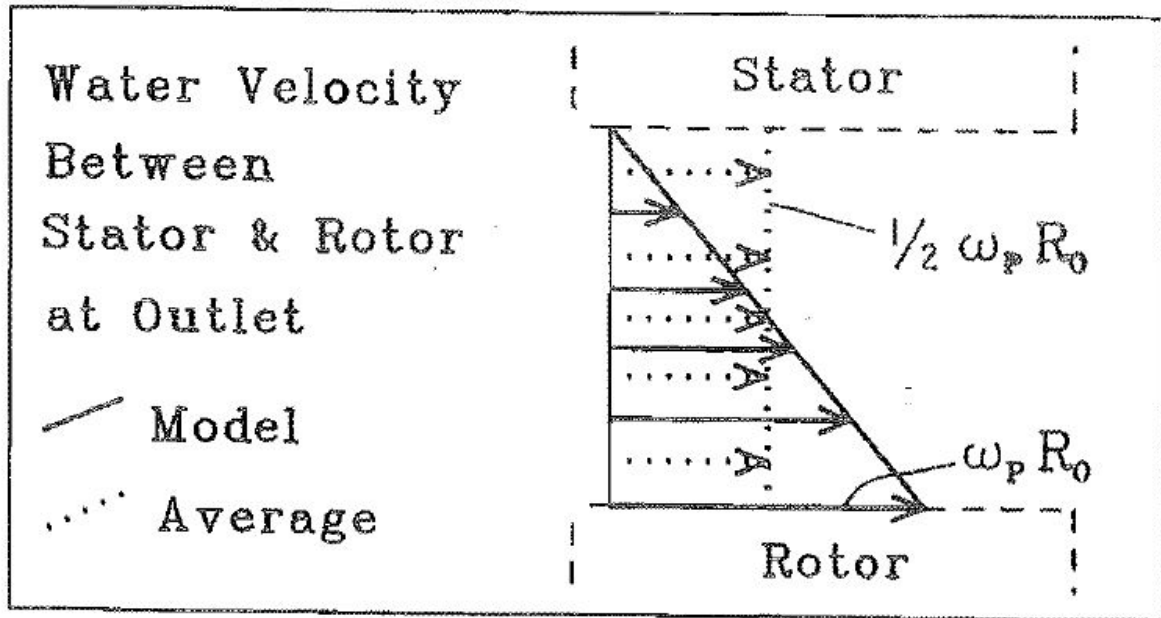


Figure 3.24. Flow Velocity at Rotor Cup Periphery (Raine & Hodgson, 1991:76)

The flow-path B is analysed using Bernoulli's equation (neglecting gravitational effects), from the point B at the cup periphery to point P_2 just at the inlet of the cross, to find the equivalent fluid energy at that point.

$$E_2 = \frac{P_B}{\rho} + \frac{1}{2} \left[\left(\frac{Q_B}{A_B} \right)^2 + \left(\frac{1}{2} \omega_p R_o \right)^2 \right] - P_{lossB \rightarrow 2} \quad (3.71)$$

where,

P_C = Fluid pressure at point B, from Section 3.4.2

V_C = Fluid velocity along the flow path being considered

$P_{lossB \rightarrow 2}$ = Empirical pressure losses along the flow path

The empirical pressure losses along flow path B are evaluated for each feature marked in Fig. 3.22 and Fig. 3.23.

1B: Entry into the drain gap, the standard loss is (Munson, 2013:432)

$$P_{1B} = \frac{1}{2} C_{1B} \left(\frac{Q_B}{A_B} \right)^2 \quad \text{with } C_{1B} = 0.5$$

2B: Exit from drain gap, the standard loss is (Munson, 2013:434)

$$P_{2B} = \frac{1}{2} C_{2B} \left(\frac{Q_B}{A_B} \right)^2 \quad \text{with } C_{2B} = 1.0$$

3B: Flow around drain annulus, this flow is similar to flow around a bend. The loss coefficient is calculated for the WJ Dynamometer ($R_o = 144$ [mm], annulus width = 98 [mm], annulus depth = 5 [mm]).

To calculate the hydraulic diameter of the drain annulus,

$$d_e = 4 \frac{98 \times 5}{2 \times (98 + 5)} = 9.5 \text{ [mm]}$$

hence the bend radius of curvature is

$$R_c = 144 + 9.5 = 153.5 \text{ [mm]}$$

Ward-Smith (1980) in his Fig. E24 gives bend loss coefficients for different geometries, and since the downstream distance for flow to re-establish is very short the values for $L_d/d_e + 1 = 1$, since $L_d \approx 0$. In this case the aspect ratio is 1.72 and $R_c/h = 30.7$ which leads to a loss-coefficient of 0.3. Thus the loss in the drain annulus is given by:

$$P_{3B} = \frac{1}{2} C_{3B} \left(\frac{Q_B}{A_Z} \right)^2 \quad \text{with } C_{3B} = 0.3$$

where

A_Z = Cross-sectional area of the drain annulus

4B: Bend of annulus into one outlet, using standard bend loss (Munson, 2013:436)

$$P_{4B} = \frac{1}{2} C_{4B} \left(\frac{Q_B}{A_A} \right)^2 \quad \text{with } C_{4B} = 0.8$$

where

A_A = Cross-sectional area of the PVC pipe

5B: Bend loss in outlet PVC pipe, using standard bend loss (Munson, 2013:437)

$$P_{5B} = \frac{1}{2} C_{5B} \left(\frac{Q_B}{A_A} \right)^2 \quad \text{with } C_{5B} = 0.5$$

This leads to the total loss equation,

$$P_{lossB \rightarrow 2} = P_{1B} + P_{2B} + P_{3B} + P_{4B} + P_{5B} \quad (3.72)$$

Therefore, the empirical pressure losses are summed and substituted in Eqn. (3.71) to give

$$E_2 = \frac{P_B}{\rho} + \frac{1}{2} \frac{Q_B^2}{A_B^2} \left[\frac{1 - C_{1B} - C_{2B}}{A_B^2} - \frac{C_{3B}}{A_Z^2} - \frac{4(C_{4B} + C_{5B})}{A_A^2} \right] + \frac{1}{8} (\omega_p R_o)^2 \quad (3.73)$$

This equation can then be used to analyse the fluid energy at point P_2 just before the cross.

Stator cup fluid outlet, flow-path C

In most modern dynamometers the flow will leave the dynamometer from the back of the stator cups alone, which leads to more stable behaviour. The dynamometer designed at WJ Engineering will however allow fluid to flow from the cup periphery and the stator cups. This will be done to allow the through-flow rate to be higher in order to ensure that the cooling of the dynamometer working compartment is sufficient.

The water outlet flow at point C can be analysed with,

$$V_C^2 = \left(\frac{Q_C}{A_C} \right)^2 \quad (3.74)$$

where

A_C = Combined area of one of the stator cup outlet holes

The initial flow velocity will be much less than for the flow from the rotor cup periphery in flow-path B from point B, the pressure will also be far less, indicating that the fluid outflow rate from this point will also be a lot less than the fluid outflow from the cup periphery.

Utilising Bernoulli's equation (neglecting gravitational forces) again, the flow-path C is analysed from the stator cup outlet to the point P_1 just before the cross, to find the fluid energy at this point. The energy at P_1 will be the same at P_3 as the same flow path applies for both.

$$E_1 = \left[\frac{P_C}{\rho} + \frac{1}{2} \left(\frac{Q_C}{A_C} \right)^2 - P_{lossC \rightarrow 1} \right] \quad (3.75)$$

Evaluate again the empirical losses for each feature marked in Fig. 3.22 and Fig. 3.23 along flow-path C.

1C: Entry into drain holes, the standard loss (Munson, 2013:432)

$$P_{1C} = \frac{1}{2} C_{1C} \left(\frac{Q_C}{A_C} \right)^2 \quad \text{with } C_{1C} = 0.5$$

2C and 3C: Bends in drain holes, bend loss with some turbulence, the standard loss (Munson, 2013:436)

$$P_{2C} = P_{3C} = \frac{1}{2} C_{2C} \left(\frac{Q_C}{A_C} \right)^2 \quad \text{with } C_{2C} = 0.8$$

4C: Exit from drain holes, same as section (3.4.3)

$$P_{4C} = \frac{1}{2} C_{4C} \left(\frac{Q_C}{A_C} \right)^2 \quad \text{with } C_{4C} = 0.8$$

5C: Bend in steel drainage pipe, bend loss same as section (3.4.3)

$$P_{5C} = \frac{1}{2} C_{5C} \left(\frac{Q_C}{A_{DP}} \right)^2 \quad \text{with } C_{5C} = 0.5$$

where

A_{DP} = Cross-sectional area of steel drainage pipe

6C: Bend in PVC pipe, bend loss same as section (3.4.3)

$$P_{6C} = \frac{1}{2} C_{6C} \left(\frac{Q_C}{A_A} \right)^2 \quad \text{with } C_{6C} = 0.5$$

The empirical losses are summed again and substituted into Eqn. (3.75) to give

$$E_1 = \frac{P_C}{\rho} + \frac{Q_C^2}{2} \left[\frac{1 - C_{1C} - 2C_{2C} + C_{4C}}{A_C^2} - \frac{C_{5C}}{A_{DP}^2} - \frac{C_{6C}}{A_A^2} \right] \quad (3.76)$$

This equation is then used to determine the fluid energy at points P_1 and P_3 just before the inlet at the cross arms.

Cross junction energy loss, 1D

The flows from flow-path B and C combine in the cross junction to form flow path D that flows through the outlet control valve into the water bucket. Before the flow path D can be analysed the energy losses in the cross junction is analysed to find the energy at point P_4 at the outlet of the cross.

Sharp, Johnson, Barfuss and Rahmeyer (2010:50) gives a method for finding the energy loss in cross junctions dependent on the flow rates at the cross legs and the energy at at least one of the legs. By utilising the contour plots in Fig. 3.25 and Fig. 3.26 the energy losses for the cross junction can be determined with the following equation (again ignoring gravitational effects),

$$E_1 - E_i = K_{1-i} \frac{V_i^2}{2} \quad (3.77)$$

where

K_{1-i} =energy loss coefficient from leg 1 to leg i

V_i =average flow velocity in leg i

E_4 is found with an iterative process. Since the ratios of fluid flow in paths B and C are unknown, an initial value for the fluid flow in either of the flow paths must be chosen and adjusted for each iteration until the values for the energy in each of the legs is found without needing to vary the values for k_{1-2} and k_{1-4} from the figures.

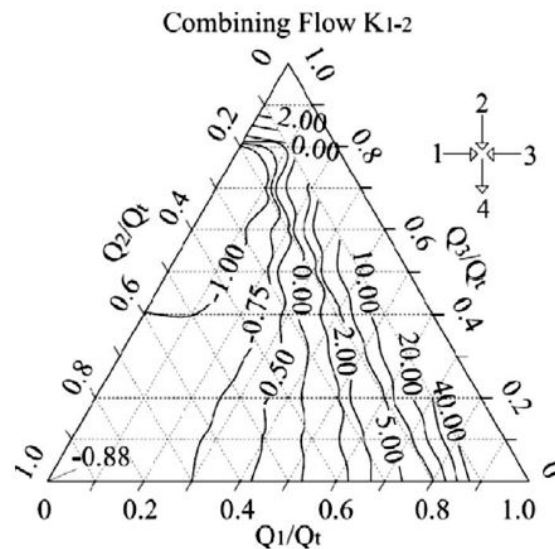


Figure 3.25. Combining flow K_{1-2} (Sharp *et. al.* 2010:52 52)

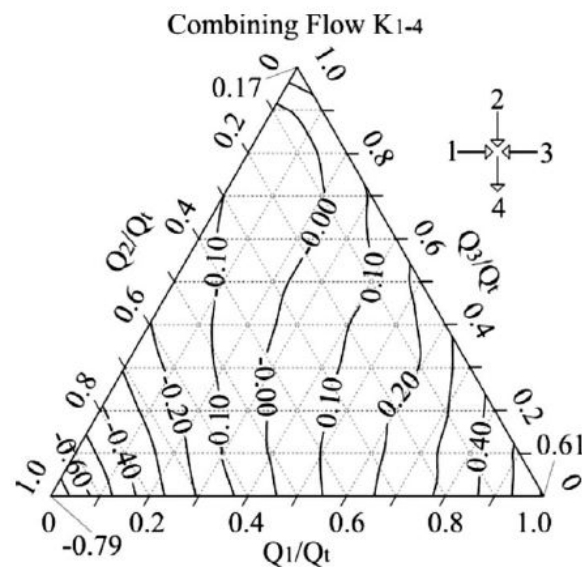


Figure 3.26. Combining flow K_{1-4} (Sharp *et. al.* 2010:52)

Control valve fluid outlet, flow-path D

Fluid flow from flow-path B and flow-path C combines in the cross junction to form flow path D, from conservation of mass we have

$$Q_D = Q_B + Q_C \quad (3.78)$$

The fluid flow from the dynamometer is adjusted by use of the outlet control valve, by adjusting the loss through valve closure. By analysing flow path D we can find the valve characteristic required to achieve certain flow rates.

Bernoulli's equation (ignoring gravitational effects) is applied for flow path D, from the outlet of the cross to the outlet of the control valve, into the water bucket.

$$E_4 = \frac{P_o}{\rho} + \frac{1}{2} \left(\frac{Q_D}{A_o} \right)^2 + P_{lossD \rightarrow O} \quad (3.79)$$

where

P_o = Atmospheric pressure

A_o = Outlet area of control valve

$P_{lossD \rightarrow O}$ = Major and minor losses in flow path D

where:

$$P_{lossD \rightarrow O} = P_{2D} + P_{3D} + P_{4D}$$

Evaluate the empirical losses for each feature in Fig. 3.23.

2D: Major loss in flexible corrugated pipe, by use of the Darcy-Weisbach equation, neglecting gravitational effects (Munson, 2013:427)

$$P_{2D} = f_{cp} \frac{L_{cp}}{D_{cp}} \frac{(Q_D/A_{cp})^2}{2}$$

where

D_{cp} = Diameter of flexible corrugated pipe

L_{cp} = Length of the flexible corrugated pipe

f_{cp} = Friction factor of flexible corrugated pipe

The corrugated pipe friction factor is found with the Colebrook equation (Munson, 2013:429)

$$\frac{1}{\sqrt{f_{cp}}} = -1.8 \log \left[\left(\frac{\epsilon/D_{cp}}{3.7} \right)^{1.11} + \frac{6.9}{Re} \right]$$

where

$$Re = \frac{\rho Q_D D_{cp}}{A_{cp} \mu} = \text{Reynolds number}$$

μ = Dynamic viscosity of fluid

ϵ = relative roughness of pipe

3D: Loss in Venturi flow meter

$$P_{3D} = (C_{vent} Q_D)^2 / \rho \quad \text{with } C_{vent} = 14834 \text{ determined in section (4.4.2)} \quad (3.80)$$

4D: Loss for Control Valve

$P_{4D} \times \rho$ will give the pressure drop required from the valve at its respective settings to produce the flow rate chosen. P_{4D} will be a function of fluid flow and a loss coefficient dependent on the valve characteristics for different valve closures.

By summing the empirical loss coefficients and substituting it into Eqn. (3.79) with the energy value from Eqn. (3.76) for E_4 we can evaluate the valve characteristic for the outlet control valve for certain chosen flow rates.

3.5. Summary

A full description of the mathematical model employed to simulate the variable-fill hydraulic dynamometer is given in this chapter. This theoretical model describes the dynamometer working parameters based on an integrated one-dimensional theory used to predict the dynamometer performance. Supplementary equations are also described, which is employed to predict additional dynamometer characteristics, such as the air-water interface boundaries for varying fill required for torque prediction, working compartment pressure variation, and the evaluation of the fluid outflow. These equations will be used to find the design parameters desired to develop a suitable dynamometer test bench for the WJ Hydro Testing Facility.

In the next chapter the research methodology is discussed, which relates to the application of the theory and experimental methods used to develop, verify and validate the variable fill hydraulic dynamometer test bench.

4. Research methodology

4.1. Overview

Based on the problem statement from chapter 1, the primary purpose of this study is to develop a variable-fill hydraulic dynamometer test bench, suitable for the testing of a model hydraulic turbine, which will be utilized in the WJ Hydro Testing Facility. This chapter will explain and describe the research methodology employed in order to address this problem and achieve the study aims and objectives.

The chapter is divided into four sections:

- Verification of integrated one-dimensional model, Section [4.2](#)
- Integrated one dimensional model analysis, Section [4.3](#)
- Torque control and flow rate measurement, Section [4.4](#)
- Dynamometer characterisation, Section [4.5](#)

In the first section, the implementation of the theoretical model and the theoretical model itself was first verified based on the results provided by Hodgson and actual dynamometer characteristics to ensure that the model was implemented correctly and corresponds to that of Hodgson.

In the second section, the simulation of the WJ Dynamometer by the use of the integrated one-dimensional analysis is discussed. The theoretical model developed by Raine & Hodgson (1991,1992) was applied to predict the dynamometer performance characteristics. This provided an initial dynamometer characteristic to be examined, and was compared to the experimental results in order to verify the numerical model. The numerical predictions also provided design parameters for certain test bench features, from which the Venturi flow meter and outlet control valve were developed.

Control of the dynamometer torque characteristic is vital to the suitability of the test bench for the testing of model hydraulic turbines, thus the second section explains how the design parameters from the simulation in the first section is utilised to design a suitable control valve and Venturi flow meter, and the experimental method used to characterise both (the valve and Venturi characterisation cycle). The control valve will be responsible for varying the torque characteristic of the dynamometer during continuous operation and the function of the Venturi flow meter is to measure the dynamometer throughflow rate and outlet pressure to analyse the validity of the model prediction and the control valve design.

In the final section, the experimental setup and method to characterise the entire variable-fill hydraulic dynamometer test bench is described and explained. By means of this test setup, the variable-fill hydraulic test bench is then verified and validated, to see if it is suitable to be used in the hydro test facility for its intended purpose. The theoretical model employed will also be validated with this experimental setup. With this experimental setup (the dynamometer characterisation cycle) the dynamometer performance will be determined and compared to the predicted and desired performance.

4.2. Verification of integrated one-dimensional model

For the simulation of the hydraulic dynamometer developed by WJ Engineering to be valid and useful the implementation of the Hodgson one-dimensional theoretical model must first be verified. This is done by comparing the simulation of this study to the results obtained by Hodgson (1991) and actual dynamometer characteristics, also provided by Hodgson.

In order to predict dynamometer performance the theory developed in the previous chapters was formulated as a computer program. The theoretical model was implemented into Matlab version R2017b (MATLAB, 2017) and used to simulate a hydraulic dynamometer performance. The computer program is divided into ten subroutines that calculates the different dynamometer characteristics separately, with each using the outputs from the previous subroutine as inputs. The Matlab code used in the simulations are provided in Appendix C, D, E and F.

The step by step sequence of calculation of the theoretical model, to accompany the calculation flow diagram of Fig. 4.1, and with reference to the relevant theoretical sections and equations follows:

1. Define the geometrical inputs, (R_o , R_I , t , Z , α)
2. First step is to find the mean flow radius (R_m), this is done with the concept of conservation of mass (Section 3.2.1) and Eqn. (3.11) (Hodgson, 1991:24) for axial vanes and Eqn. (3.10) (Hodgson, 1991:23) which is solved using the Newton method, for angled vanes. [newton_calc.m program is used here].
3. Then, to find the air-water interface boundaries (Section 3.4.1), the desired fluid-fill is specified as an input and the newton method is used with Eqn. (3.58) (Hodgson, 1991:54) to find the air-water interface boundaries for each chosen percentage fill value. [System.m and NewtonrhapsonSystem.m programmes are used here]
4. Then, to find the vortex velocity across a range of rotational velocities. This part will be divided into sub sections. [vortex_calc_actual program is used here]
 - a) Inputs: geometrical properties (see 1.), R_m (see 2.), air-water interface boundaries of the percentage fill interested in (see 3.) and range of rotor velocities. Important note, use guess value for incidence angles to start iteration.
 - b) Steady state momentum coefficients: Calculate geometric parameters K_1 and K_2 for the calculation of the torque by means of Eqn. (3.20) (Hodgson, 1991:28), this is done by means of the conservation of momentum [Section 3.2.3].
 - c) Incidence loss coefficients: Calculate the parameters K_{I1} , K_{I2} , K_{I3} , K_{I4} , K_{I5} and K_{I6} to give the incidence losses [Section 3.3].
 - d) Friction loss coefficients: Calculate the parameters K_{FORL} , K_{FORU} , K_{FOSL} , K_{FOSU} , K_{FIRL} , K_{FIRU} , K_{FISL} and K_{FISU} to give the friction losses [Section 3.3.2]. Important note, use guess value for friction coefficient to start iteration.
 - e) Secondary loss coefficients: Calculate parameters K_{SCRL} , K_{SCRU} , K_{SCSL} and K_{SCSU} , to give the secondary circulation losses [Section 3.3.3].
 - f) Vortex velocity: Calculate the vortex velocity by means of Eqn. (3.15) (Hodgson, 1991:21) this is done with the quadratic formula, the principle of the conservation of energy is used here [Section 3.2.2].
 - g) Torque and Power: The torque is then found with Eqn. (3.20) (Hodgson, 1991:28) and consequently the power.

5. With a first iteration of the vortex velocity calculated, the incidence angles can then be calculated. [angle_calc_actual program is used here].
6. The incidence angles are then introduced as inputs to the torque calculation and the friction factor is then calculated and updated (based on Re values).
7. The incidence angles are then again calculated, reintroduced into the torque calc and this is repeated until convergence.
8. Output of torque and power across a range of velocities for different Percentage Fill values.
9. Find pressure along surface with Eqn. (3.66) and Eqn. (3.69) (Hodgson, 1991:65), [Section 3.4.2]
10. Use pressure at surface and pressure loss along flow path to find desired valve characteristic for specified flow rates (i.e. the flow rates defined by WJ as specification).

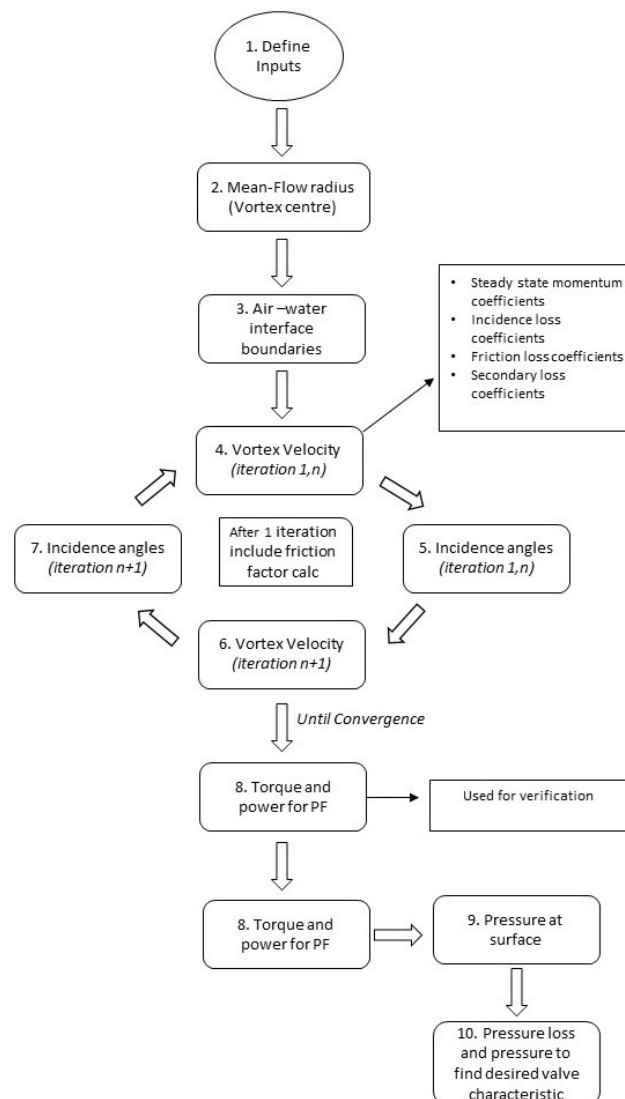


Figure 4.1. Calculation flow diagram

4.2.1. Verification of vortex characteristics

It is vitally important to verify the vortex characteristics since they form the basis from where the torque calculation is done. The vortex characteristics include the vortex velocity ratio, the Reynolds number and friction factor of the flow in the working compartments and the air-water interface boundaries for the different fluid fills. These characteristics are simulated with the program developed for his thesis and compared to the known values provided by Hodgson to verify that the theoretical model was implemented correctly.

Hodgson (1991:24) provided the geometrical properties for the F020 dynamometer as ($R_o = 102.5 \text{ mm}$, $R_I = 43.5 \text{ mm}$, $t = 5.0 \text{ mm}$, $Z_S = 12$, $\alpha_{SO} = 90 \text{ degrees}$). The F020 dynamometer was simulated by Hodgson and a number of the results of vortex characteristics obtained from the simulation were provided.

Vortex velocity

Hodgson simulated the rotor speed velocity ratio for the F020 dynamometer and illustrated the results on Fig. 10 (Hodgson, 1991:113), these results are shown and compared to the Matlab simulation shown in Table 4.1.

Table 4.1. Comparison of ω/ω_p

% FILL	100	60	40	20	10	6	2
ω/ω_p Hodgson	10	8.68	7.83	6.69	5.57	4.7	4.7
ω/ω_p Model	10.21	8.64	7.82	6.77	5.64	4.3	2.45

As can be seen from the table the model simulation values correspond very well with that of Hodgson, with only some variation at very low percentage fills (lower than 2 % Percentage Fill).

Air water interface boundaries

Hodgson (1991:55) provided the geometrical properties for a F24 machine as ($R_o = 121.0 \text{ mm}$, $R_I = 51.2 \text{ mm}$, $\alpha = 45 \text{ degrees}$). The F24 machine was simulated by Hodgson to find the air water interface boundaries of the vortex flow, these values are provided and compared to the Matlab simulation results in Table 4.2.

Table 4.2. Air-water interface boundaries comparison

% FILL	0	25	50	75	100
R_{CO} Hodgson [mm]	121	117.5	113.2	107.3	94.49
R_{CO} Model [mm]	121	117.5	113.2	107.1	93.4
R_{CI} Hodgson [mm]	51.2	57.68	64.78	73.31	94.49
R_{CI} Model [mm]	51.19	57.68	64.78	73.1	91.2

It can be seen that the model values correspond almost perfectly with that of Hodgson's results with only slight deviation at 100 % fill.

Reynolds number and friction factor

The Reynolds number will vary with changes in the rotor speed and the friction factor used will be dependant on the Reynolds number. The Reynolds number must therefore be recalculated for each change in the rotor speed in the model and the resulting friction factor then verified to that of the simulated by Hodgson.

Fig. 4.2 illustrates how the Reynolds number changes in the model for the F020 as the rotor rotational velocity varies.

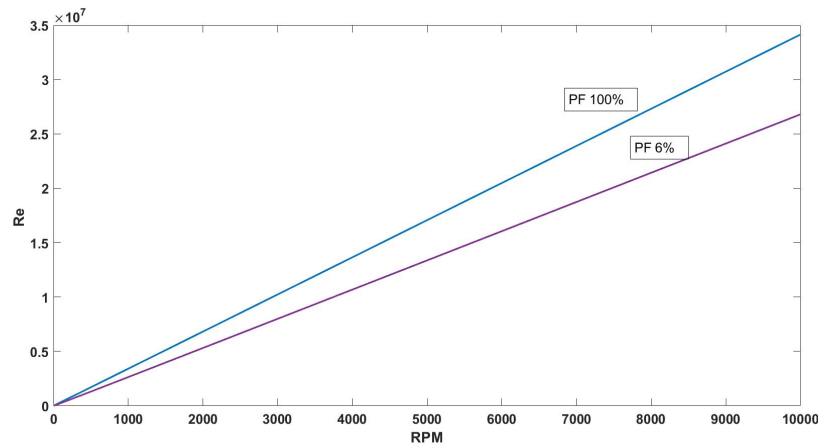


Figure 4.2. Reynolds number vs. RPM for F020

The friction factor will therefore also change according to the changes in the Reynolds number. Hodgson suggested that the friction factor be calculated with the Moody formula (1991:50), it is however suggested that the Swamee and Jain formula be used, since the Moody formula is suggested to have a error of about 15 % (Haaland, 1983). Fig. 4.3 illustrates the variation in the friction factor as the Reynolds number changes.

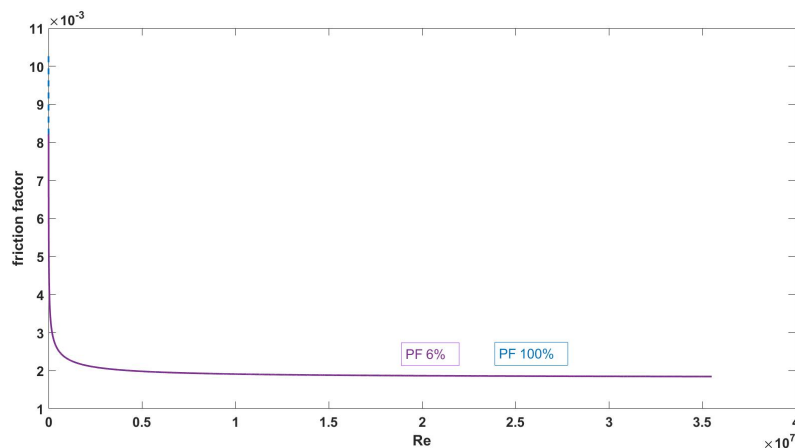


Figure 4.3. Friction factor vs. Reynolds number for F020

Hodgson provided the results for the influence of a change in the surface roughness on the friction factor and the torque absorption of the F020 dynamometer, in Table 4.3 the results of the model are compared to that provided by Hodgson (1991:11) in order to verify that the friction factor was implemented correctly into the model.

Table 4.3. Surface roughness compared to Torque, Hodgson vs. Model

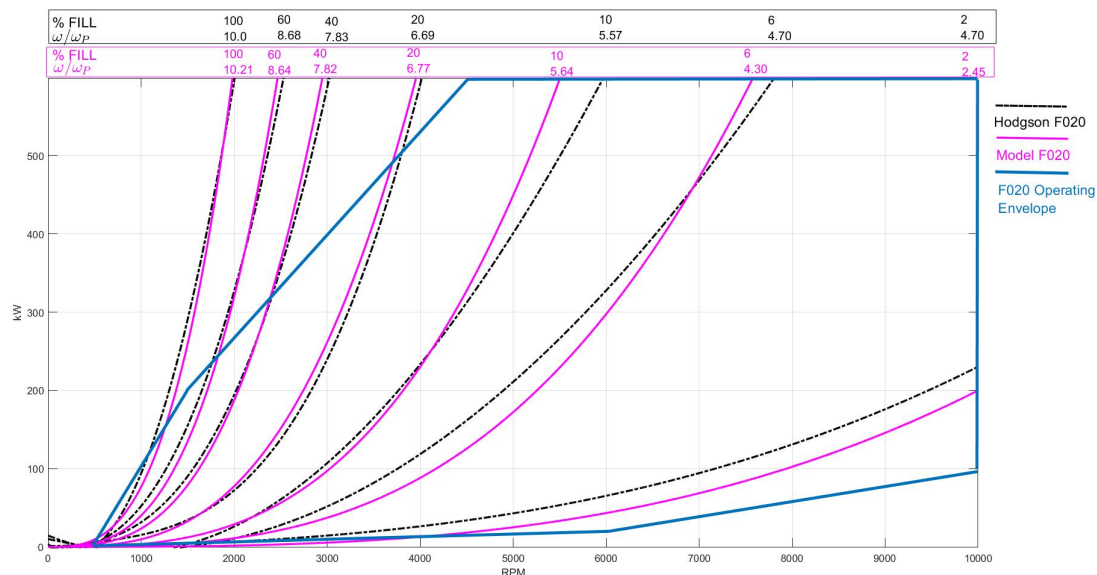
Roughness [mm]	ff Hodgson	ff Model	Torque Hodgson (Nm)	Torque Model (Nm)
0.1270	0.0305	0.0216	1253	1388
0.1016	0.0287	0.0204	1280	1409
0.0762	0.0266	0.0192	1313	1434
0.0508	0.0239	0.0172	1358	1467
0.0254	0.0201	0.0148	1426	1516
0.0015	0.0112	0.0096	1612	1638

The friction factor values of the model and the corresponding torque values differ somewhat from that of Hodgson, but the values are however in the same order size and still considered as adequately accurate. The calculation of the Reynolds number and friction factors are therefore considered as verified to that of the Hodgson model.

4.2.2. Verification of torque and power

The main outcome of the theoretical model simulation is to find the torque and power characteristic curves. It is therefore critical that the torque and power prediction of the model be validated against the results provided by Hodgson. In Table 4.3 it could already be seen that the model torque corresponds well to that of Hodgson, but the form of the characteristic curves for changes in rotor velocity and fluid fill must also be verified.

Hodgson provided the characteristic curves for the F020 hydraulic dynamometer in Fig. 5.1 of his thesis (1991:104). The Matlab simulation is used to predict the characteristic curves of the F020 dynamometer and compared to that of Hodgson in Fig. 4.4.


Figure 4.4. Torque vs. speed comparison between model and Hodgson

It can be seen that the results of the model simulation compare very favourably to the results provided by Hodgson, with only slightly larger deviations at low fills. The model is thus expected to accurately predict the hydraulic dynamometer torque based on Hodgsons results and therefore the model is considered as verified.

4.3. Integrated one dimensional model analysis (Steady state)

In this section the one-dimensional theoretical model discussed in Chapter 3 is applied to simulate the WJ dynamometer performance. The simulation results were then used to theoretically predict the dynamometer performance and to obtain the design parameters of certain test bench components such as the control valve and flow meter. The model predictions were experimentally validated against the actual dynamometer test results as final step in the study, with results described in Chapter 5.

4.3.1. Vortex characteristics

The position of the vortex centre, the vortex angular velocity and the air-water boundaries greatly influence the performance of the dynamometer. These parameters must be analysed and understood before any torque predictions can be made.

The vortex centre calculation is described in Section 3.1 and the air-water interface boundaries calculation in Section 3.4.1. Table 4.4 lists the values calculated for the the air water interface boundaries for different magnitudes of fill percentage.

Table 4.4. Air-Water interface radii for WJ Engineering Dynamometer

% FILL	0	10	30	40	50	60	70	80	90	100
R_{CO} [m]	0.142	0.1407	0.1389	0.1369	0.1347	0.1322	0.1292	0.1257	0.1209	0.1088
R_{CI} [m]	0.064	0.0671	0.0702	0.0735	0.077	0.0808	0.0849	0.0896	0.0955	0.1087

The water in the working compartment will not likely be a streamtube flowing around the outer surface of the cups, but rather a spray of fluid circulating in a vortex. For this reason, the air-water boundaries are accepted to not be indicative of the actual flow at low fill conditions, but is indeed used for modelling purposes although not intended to be applicable to actual machines. From Fig. 4.5 it can be seen how the amount of fluid in the working compartment will increase and the air in the working compartment lowers as the boundaries tend to come closer as the percentage fill increases. The vortex centre remains constant regardless of percentage fill and velocity variation. The amount of fluid fill will directly influence the torque of the dynamometer.

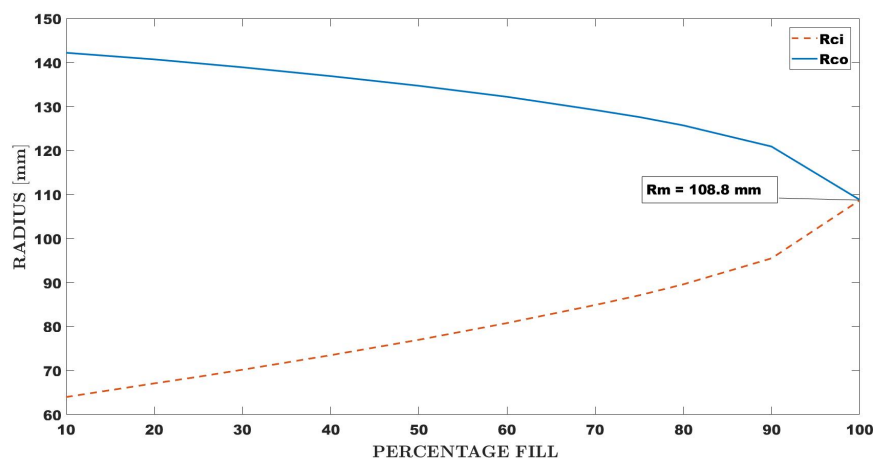


Figure 4.5. Variation of air-water interface radii with changes in percentage fill

The vortex velocity is described in Section 3.2.2 and in Fig. 4.6 it is shown how the vortex velocity varies in a narrow band as the percentage fill changes. The torque will however also vary greatly due to small variations in vortex velocity. Variations in fluid fill will have a linear impact on the vortex velocity for changes in the rotational velocity. It can also be seen that the vortex velocity increases with increases in the fluid fill, which illustrates how the increase of the fluid fill condition can lead to increases in the torque absorption of the dynamometer.

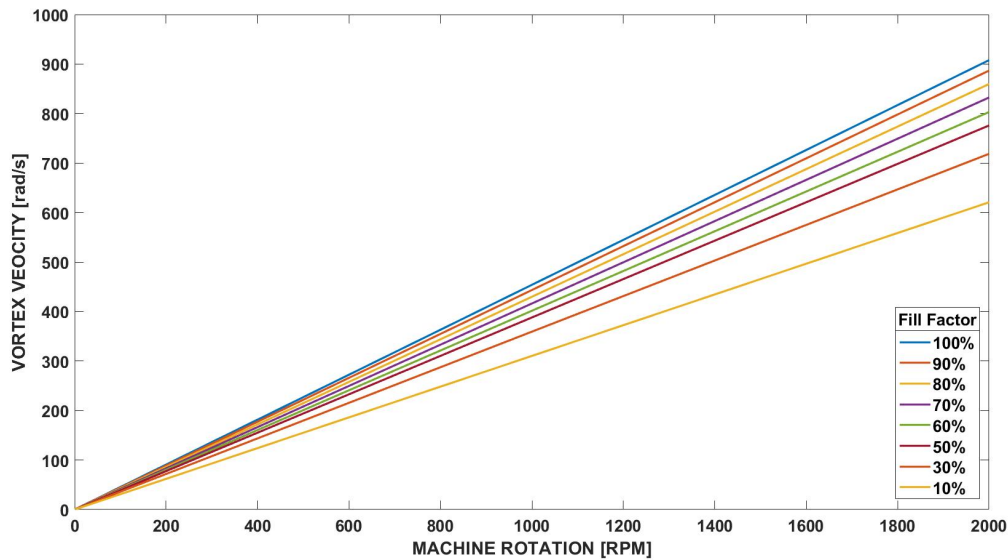


Figure 4.6. Variation of vortex velocity related to rotor speed for different fill percentages

4.3.2. Torque and power

The attainment of the dynamometer torque and power characteristics are considered as one of the main aims of this study. Understanding the torque and power properties of the dynamometer is vital to the verification and validation of the dynamometer for its suitability to be used in the model hydraulic turbine tests. In any test setup the shape of the dynamometer torque-speed characteristics greatly influences the operational stability of the integrated system (Rakopoulos, 2009) and indicates if the dynamometer is able load the turbine at it's design point.

The variable-fill hydraulic dynamometer torque characteristic takes the form of cascading curves, with torque increasing exponentially as rotor speed increases, for different fluid fill conditions. The torque curves vs. speed for the WJ Dynamometer is simulated using the theoretical model, developed by Hodgson (1991) presented in Chapter 3.

Fig. 4.7 illustrates the results of the simulations, that show how the torque varies with speed for different constant fill conditions. The varying dynamometer torque for different fluid-fill percentages is supported by the variation in vortex velocity for different fluid fills as shown in Fig. 4.6 since this will certainly lead to a change in torque.

It is notable that the fill lines show slightly higher variation at low percentage fills as compared to high percentage fills, so small changes in fluid fill will have a slightly larger influence on the torque than at higher fluid fills. The fill lines are however evenly spaced across the entire operating range of the dynamometer, suggesting that relatively accurate torque control can be achieved for all fluid fill conditions (effected by valve closure settings, refer to Section 2.4 and Section 3.4.3), more so than for most other commercially available dynamometers (Hodgson, 1991).

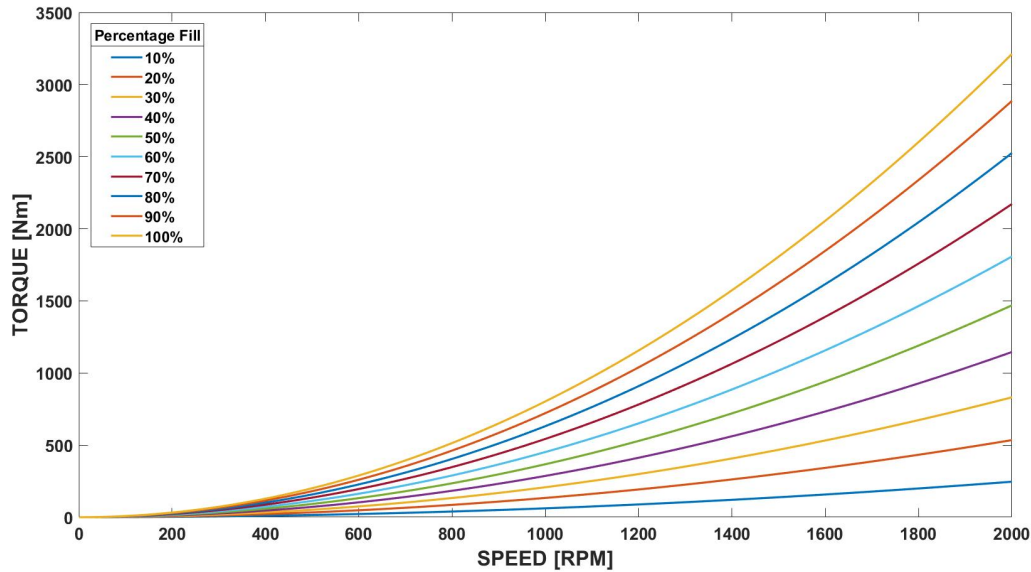


Figure 4.7. Simulated distribution of Torque Characteristic with Percentage Fill of WJ Dynamometer

It is important to consider the gradients of the torque curves, this is because of the requirement to test the responsiveness of the HTGS. If the gradients are very steep small variations in the rotational velocity of the system can lead to large changes in the torque absorption. If the torque changes are too excessive the governing system will have difficulty controlling the system's rotational velocity. It is not known what torque gradient would be too excessive for the governor tests, since no governor system exists at WJ Engineering to be tested. What is however important to take note of is that the gradients of the constant fill lines seem to increase as the fluid fill increases, so the dynamometer will become more sensitive with higher fluid fills.

Fig. 4.8 illustrates the simulated power of the WJ Dynamometer at different fluid-fill conditions. The torque and power curves for different fluid-fills follow the same pattern. WJ Engineering designed the dynamometer rotor and stator expecting it to resist a maximum of 38 [kW] and 300 [Nm] at 1200 [rpm], as per the requirement of the model hydraulic turbine to deliver 20 [kW] at the same operating speed. The simulated curves show that this design requirement is already met at the very low percentage fill of just less than 30% at 1200 [rpm]. Therefore the WJ dynamometer can, according to the simulation, be expected to deliver a torque in excess of that which is required by the specification. With a power absorption of about 145 [kW] at a fluid fill of 100% achievable at 1200 [rpm]. The experimental validation of the simulated results will thus add insight into possible reasons and accuracy of the extremely high torque prediction based on the numerical model.

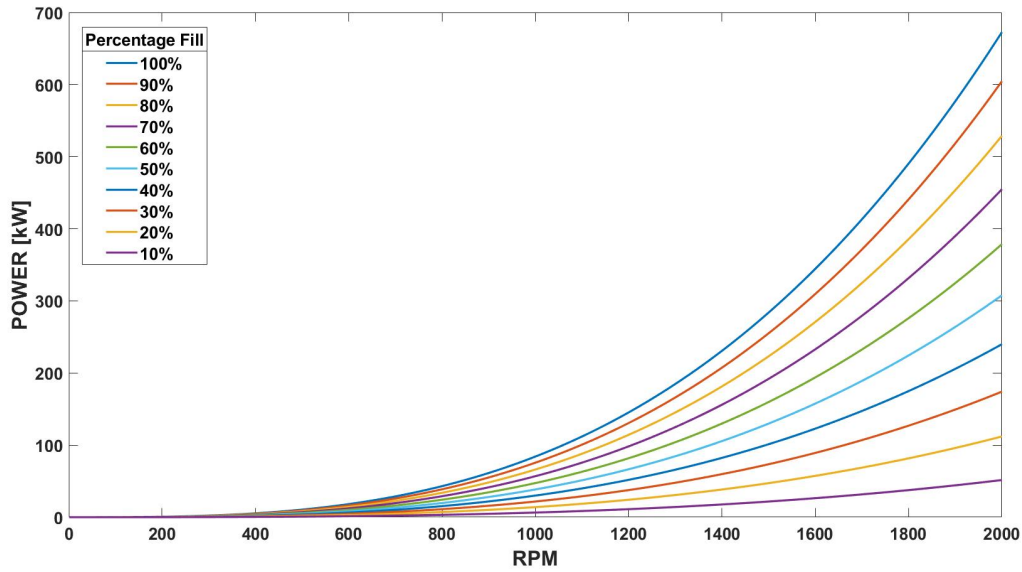


Figure 4.8. Simulated distribution of power characteristic for different fill percentages of WJ Dynamometer

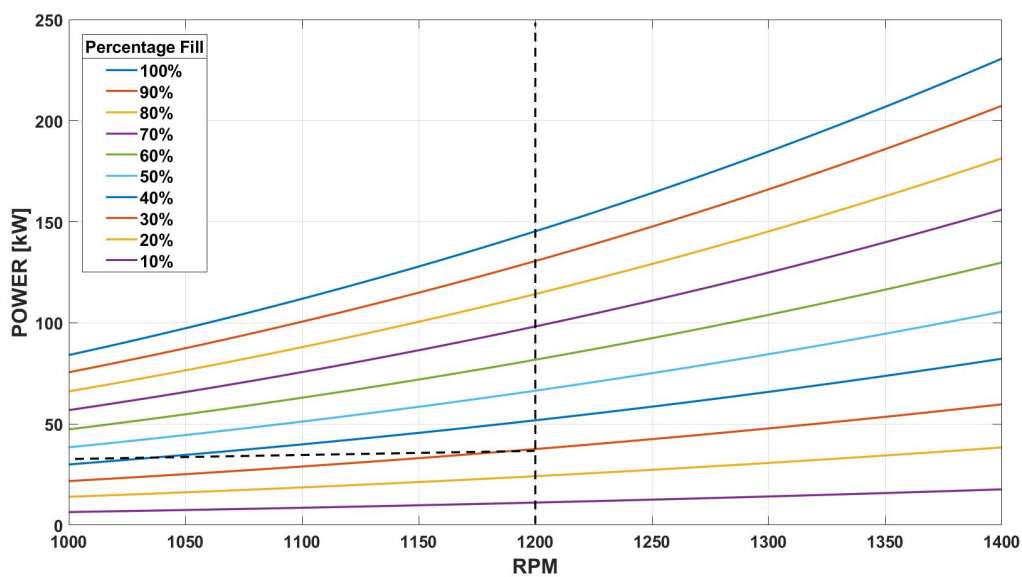


Figure 4.9. Detailed view of simulated power of WJ Engineering Dynamometer.
Dashed lines indicate power level of 38 [kW] at 1200 [rpm] and 30% PF.

4.3.3. Vortex pressure

Analysing the pressure at the cup surface is very important, since it is used to find the fluid outflow and pressure loss characteristic of the dynamometer test bench from which the control valve can be designed. The pressure derivations from Section 3.4.2 is utilised to investigate the effect of changes in fluid-fill and speed on the pressure at the cup surface.

Fig. 4.10 illustrates the simulated distribution of pressure profiles with varying fill. Interestingly the pressure profiles of the rotor and stator are symmetrical and show no visible pressure loss

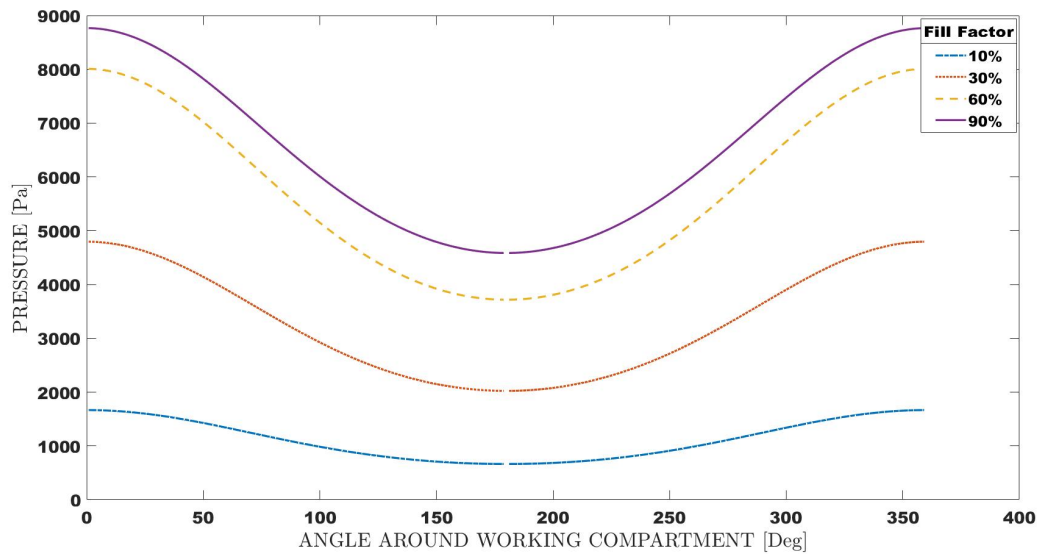


Figure 4.10. Variation of fluid pressure distribution with different fill percentages at 1200 [rpm]

between the rotor and the stator — this is however not the case with most dynamometers simulated by Raine & Hodgson where some pressure loss is observable between the rotor and stator; and the rotor and stator pressure profiles differ slightly due to incidence losses (Hodgson, 1991:120). A reason for this can be the axially angled vanes of the dynamometer which give rise to no significant incidence loss between the rotor and stator cups and leads to a very evenly distributed vortex velocity profile for the entire working compartment.

Instinctively it is expected that the maximum fluid pressure in the rotor will be at the outer radius, where the angular rotation is highest. However, due to the vortex action the pressure peak is found at the inner radius of the rotor and stator cups. This can be explained by the surge in the acceleration of the fluid as the vortex moves about the working compartment to the outer edges of the rotor cup leading to a drop in pressure up to the outer radius and the deceleration of fluid as it moves along the stator surface causing the pressure to increase to the inner radius.

The pressure in the working compartment leads to the self-emptying behaviour of the dynamometer as explained in Section 2.4. If the pressure in the working compartment increases to such an extent that the cup outflow is higher than the inflow it would lead to self-emptying behaviour of the dynamometer, this causes unstable torque control and must be avoided. In Fig. 4.11 it is seen how an increase in operational speed of the dynamometer will lead to exponential increase in the working compartment pressure. It is therefore desired to find the cup emptying speed during the experimental tests to determine the operational control envelope of the dynamometer, for use during turbine tests, to ensure stable operation throughout the dynamometer's use.

From Fig. 4.11 it is clear that the pressure at the rotor outlet is much lower than the pressure at the back of the stator cups, this will have some influence on emptying behaviour of the WJ Dynamometer since there are two outflow paths available as shown in Section 3.4.3, one from the back of the stator cup and one from the cup periphery. The influence of the pressure difference between these two points and the characteristics of these flow paths will be investigated in the coming sections to determine the throughflow characteristic of the dynamometer.

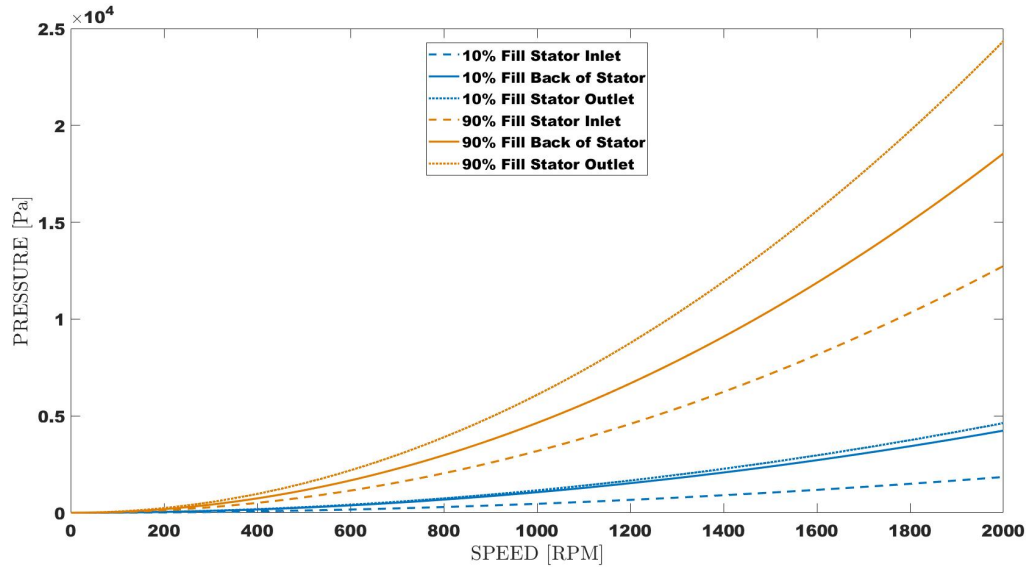


Figure 4.11. Fluid pressure locations in stator for varying speed and different fill percentages

4.3.4. Fluid inflow

Fluid inflow is a very important characteristic to consider in variable-fill hydraulic dynamometers. As stated in Section 2.4 the self-emptying of a variable-fill hydraulic dynamometer occurs when the pressure in the working compartment increases to such an extent that it leads to the fluid outflow exceeding the fluid inflow. This should be avoided as it leads to a rapid drop in the fill-factor of the dynamometer causing torque loss and unstable operation specifically when loading a hydraulic turbine.

The point when the self-emptying of the dynamometer occurs depends on the fluid inflow rate, the higher the fluid inflow rate that can be sustained the higher the pressure in the working compartment must be before the fluid outflow causes the onset of self-emptying (Hodgson, 1991:125). The fluid pressure that leads to the self-emptying behaviour increases as the rotor speed increases, so self-emptying is bound to occur as long as the rotational velocity increases.

Fluid inflow in the WJ Engineering dynamometer is achieved through gravity — since the inlet water pipe entrance is situated above the dynamometer working compartments — as well as the suction force created by the dynamometer. The fluid inflow of the dynamometer is however not studied in the dissertation, but it will be attempted to determine experimentally the points where self-emptying will commence for the WJ Dynamometer, since the self-emptying characteristic of a dynamometer gives a clear indication of the operating range and will have a big influence on the suitability of the dynamometer for use in the hydro testing facility.

4.3.5. Fluid outflow considerations

When operating at a constant speed and load, steady state will be achieved for all operating conditions before the onset of the self-emptying phenomena, this means that the inflow, due to gravity and the dynamometer suction, and the dynamometer outflow will tend to become equal.

Back pressure at outflow

The back pressure experienced at the outflow of the dynamometer will thus have an influence on the inflow and outflow rates to and from the dynamometer working compartments. Thus the influence of the back-pressure experienced at the outflow of the dynamometer, is the governing factor that determines the torque absorption, since it will directly influence the amount of fluid circulating inside the working compartments. It is stated by Raine and Hodgson (1992:328) that the operation of an outlet control valve varies the mass of working fluid in circulation and the higher fluid mass thus leads to higher dynamometer fill percentage. (The influence of fluid fill on the torque absorption is explained in Section 2.4).

Therefore, by controlling the back-pressure via an outlet control valve, the torque absorption of the dynamometer can be adjusted. This is an important operational quality of variable-fill hydraulic dynamometers, since the adjustment of its torque characteristic allows the testing of machines at different operating points.

For this reason, the fluid outflow from the dynamometer is studied in order to determine the suitable outflow valve characteristics that will allow proper dynamometer torque control. This is accomplished through the application of the analysis presented in Section 3.4.2 and Section 3.4.3, from which the outlet valve characteristics can be obtained that will deliver the desired back pressures at specified outlet flow rates.

Vortex pressures in working compartment

The vortex pressures for different fluid-fill percentages are calculated at the relevant points in the dynamometer working compartment: namely the back of the stator cups and the cup periphery, from where the fluid outflow via the two outflow paths shown in Fig. 3.21 will occur. The pressures at each point for the different fluid fill percentages and rotational speed of 1200 [rpm] are presented in Table 4.5.

Table 4.5. Calculated pressures at various fill percentages in dynamometer working compartment

Pressure in working compartment		
% Fill	Back of stator cup outlet (P_C @ $\theta = 270^\circ$) [Pa]	Cup periphery outlet (P_B @ $\theta = 180^\circ$) [Pa]
10	1142	665
30	3361	2023
60	5826	3718
90	6670	4586

The pressure at these points pump the fluid out of the dynamometer working compartment along the proposed flow paths to the outlet. The rate of fluid flow in each flow path is dependent on the pressure losses developed. The pressure losses along the flow paths is essentially variable and can be controlled by adjusting the outlet control valve opening, thereby directly influencing the outflow rate. By using a range of desired flow rates, it is possible to determine the pressure losses that the control valve should develop at each different setting to achieve the flow rates specified.

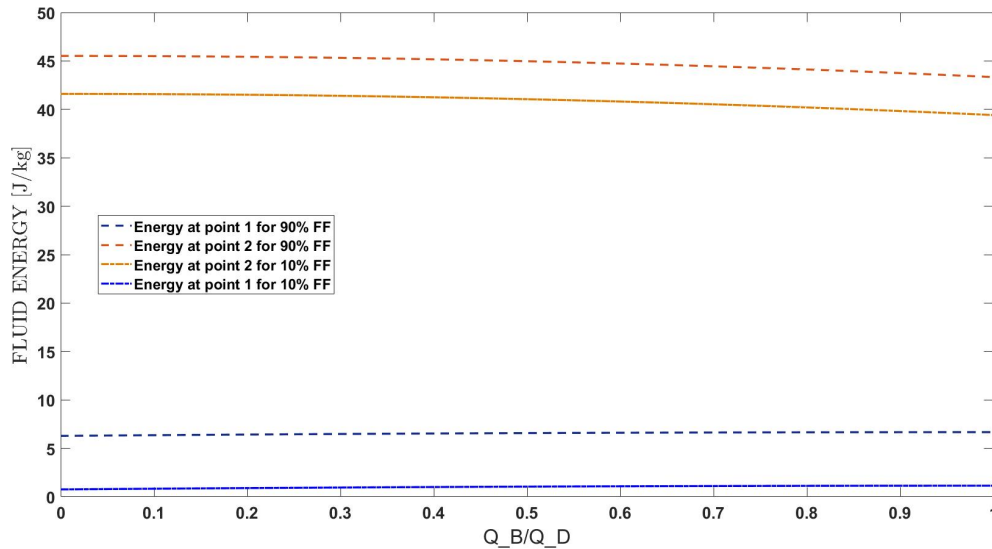


Figure 4.12. Fluid energy at cross inlet legs for 1[l/s] total flow

Flow rate distribution in outlet flow paths

WJ Engineering proposed that a throughflow rate from a minimum of 1 [l/s] to a maximum of 4 [l/s] should be developed by the dynamometer to ensure proper heat dissipation. Through their own independent calculations and unknown to the author — but by means of a different method than the one set forth by Raine & Hodgson (1991) — WJ Engineering determined the required valve pressure loss parameters to achieve the desired flow rates. Based upon the assumptions made by WJ Engineering and their own rotor and stator design a valve pressure drop of 15 [kPa] was predicted for a total flow rate of 4 [l/s] and 100 [kPa] at 1 [l/s].

The method outlined in Section 3.4.2 was implemented to find the valve pressure loss characteristics at the proposed flow rates and different fluid-fill percentages and compared that to the values found by WJ Engineering. The method proposed in this study produced some interesting results. First of all regardless of the flow rate used, the equivalent fluid energy calculated with the Bernoulli equation at point 2 (the entrance to the cross, from the cup periphery flow path) was always much higher than the equivalent fluid energy calculated at point 1 (the entrance to the cross, from the back of the stator cup flow path). Fig. 4.12 and Fig. 4.13 illustrates the fluid energies at point 1 and 2 where the fluid will flow into the cross-section as the ratio of flow in each leg. The flow rates at the cross is varied for a total flow rate throughout the dynamometer of 1 [l/s] and 4 [l/s] respectively.

From both Fig. 4.12 and Fig. 4.13 it can be seen how the energy values at the two entrances to the cross tend to converge as the fluid flow rate in flow path B increases. However, regardless of the flow ratios in each flow path, the fluid energy at point 2, where the fluid from flow path B enters the cross junction, will always remain much higher than the fluid energy at point 1, where the fluid from path C enters the junction. This is due to the much higher kinetic energy component introduced to the fluid in flow path D, from the rotational velocity of the rotor, shown in Eqn. (3.70). From this the assumption is made that almost all of the fluid flow out of the dynamometer working compartments will happen in flow path B, thus from the cup periphery.

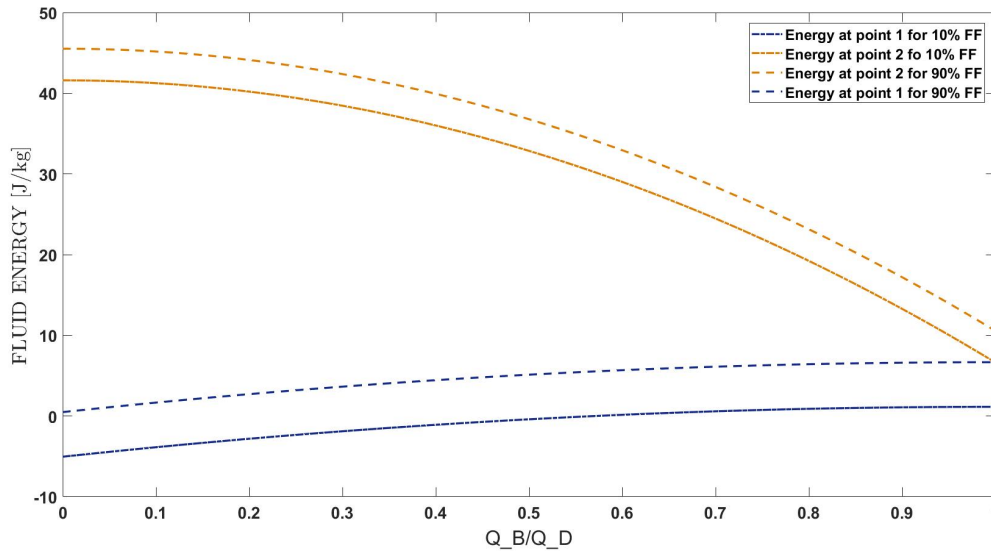


Figure 4.13. Fluid energy at cross inlet legs for 4[l/s] total flow

The excessively high fluid energy in flow path B compared to the fluid energy in flow path C illustrates some design trade-offs in the WJ Dynamometer, listed below for clarity:

1. It is for instance unnecessary to have two available fluid flow paths out of the dynamometer working compartment, in this specific case the fluid flow from the back of the stator cups are redundant.
2. It is also a possibility that due to the high fluid energy at the exit of flow path B compared to that of the exit of flow path C that the fluid will be pumped into flow path C through the cross from flow path B and be recirculated into the stator cup via the stator cup outlet holes. This backflow could be extremely detrimental to the torque produced as such recirculation might interrupt the vortex flow developed.
3. Lastly, when allowing fluid flow out of the dynamometer working compartments via holes in the back of the stator cups alone it leads to much more stable torque generation and control than fluid flow from the cup periphery (Hodgson, 1991:120).

However, the decision to keep both flow paths was made due to the concern that the throughflow rate of the concept dynamometer will not be enough with just one flow path, and that proper cooling of the working compartments would not be attainable.

Pressure loss characteristic required for outlet valve

For further calculations it was assumed that no such recirculation from flow path B into flow path C will occur and that most of the flow from flow path B will flow directly into flow path D from where it will flow out into the water bucket. With the majority of flow coming from flow path B (the cup periphery outlet) the desired amount of pressure drop from the outlet control valve can be found for the different flow rates and is shown in Fig. 4.14. The outlet valve must therefore be able to give rise to similar pressure drops at the different flow rates by controlling the amount of valve closure.

In Fig. 4.14 the valve characteristics needed for each fluid-fill condition to achieve a flow rate between 1 [l/s] and 4 [l/s] are shown. The control valve must be responsible for adjusting the

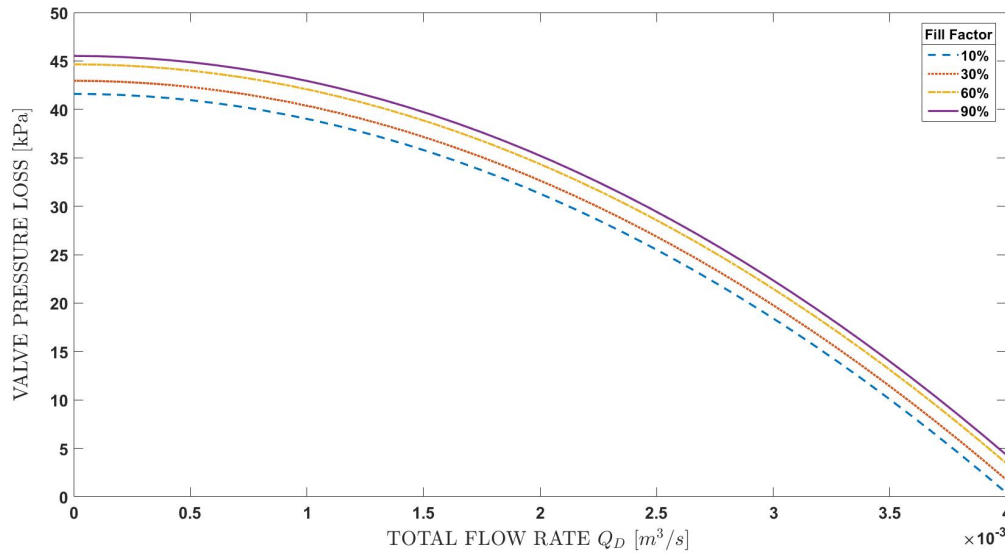


Figure 4.14. Desired valve pressure loss at total rate of fluid flow, for different fill percentages

fluid-fill condition, so therefore the fluid-fill conditions are also varied in order to determine the desired behaviour for pressure drop across the valve.

The amount of valve travel from the fully open position to the fully closed position, also known as the valve closure, is responsible for influencing the amount of back pressure experienced by the fluid exiting the dynamometer working compartment and conversely the amount of fluid-fill. Intuitively it would seem to make sense that the fluid-fill decreases as the valve closure increases — however the literature suggests that the opposite will in fact happen — that is, an increase in the outlet valve closure will ultimately result in higher torque absorption (Hodgson & Raine, 1991:160) and (Hodgson & Raine, 1992:50).

From interpretation of Fig. 4.14 the desired valve pressure drop characteristic can be discerned. It can be seen that in order to achieve a fluid flow of 1 [l/s] the valve pressure drop produced must be between approximately 39 [kPa] and 45 [kPa], for any of the fluid-fill conditions, when the valve is almost fully closed. To achieve a fluid flow of 4 [l/s] the valve pressure drop produced must be between approximately 0 [kPa] and 5 [kPa] (thus when the valve is almost fully open) for any fluid-fill condition.

The valve parameters determined by WJ Engineering are different from the calculated parameters found through the method proposed in the current study; however the values are of the same order of magnitude in size and a decision was made to find a control valve that is able to roughly satisfy both requirements. The development and experimental validation of a suitable control valve based on the identified parameter ranges, is discussed in the next section.

4.4. Torque control and flow rate measurement

By utilising the one-dimensional approach proposed by Hodgson & Raine (1991, 1992) the desired valve characteristic of the WJ Dynamometer was found in Fig. 4.14 as described in Section 4.3.5. The torque characteristic can only be validated if a suitable control valve is employed. In this section the development and validation of such a valve configuration is discussed.

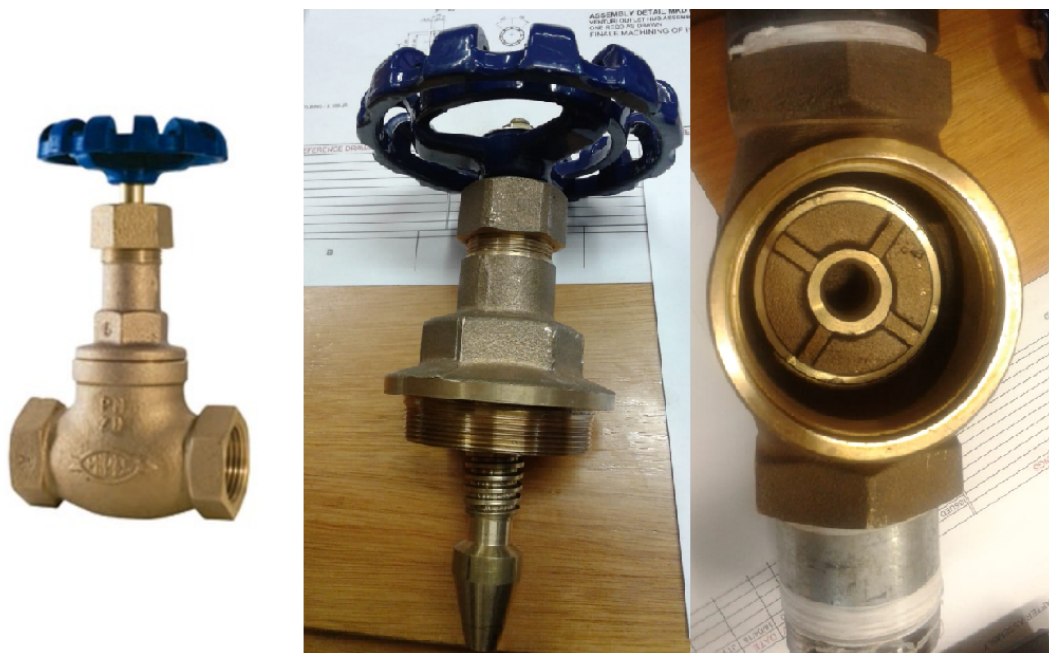


Figure 4.15. Off the shelf needle valve

4.4.1. Outlet control valve

To identify a suitable control valve, the most suitable commercial option was first considered, but found to be lacking in terms of compliance with required specifications. A custom-built angle seat valve was then designed, constructed and validated to obtain improved control characteristics.

Initial concept: Off-the-shelf needle valve

Initially the commercially available needle valve shown in Fig. 4.15 was purchased and analysed using Computational Fluid Dynamics (CFD) to determine the suitability of the valve as an outlet control valve for the WJ Dynamometer. A needle valve was chosen based on its pressure drop characteristic over the range of the needle's movement, that could foreseeably provide the control behaviour required.

FloEFD (Engineering Fluid Dynamics) is a so called "front-loading" CFD package, that can be used to do rapid fluid simulations on CAD models. The FloEFD (2018) Solid Edge package was used throughout this study.

Through simulation of the off the shelf valve with the FloEFD simulation tool, it however showed that the pressure drops achieved for flow rates from 1 [l/s] to 4 [l/s] were excessively high. The pressure drop is the most important parameter where any potential control valve should have specification compliance, and this finding eliminated the needle valve as suitable candidate.

Final concept: Custom-built angle seat valve

Based upon the simulation results of the off-the-shelf valve and recommendations by the author, WJ Engineering devised and manufactured a new angle seat valve, shown in Fig. 4.16. This was accomplished by combining some of the parts from the off-the-shelf needle valve with a custom

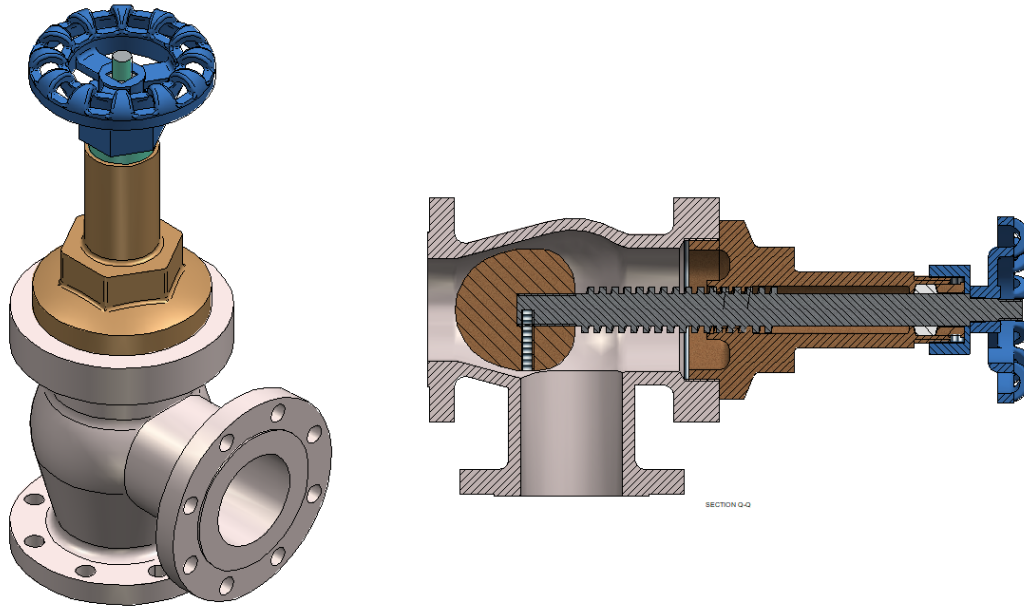


Figure 4.16. Angle seat valve model

manufactured housing into the new valve configuration. The valve plug size and geometry were designed to ensure a more linear pressure drop characteristic and the seat diameter was especially enlarged to ensure that the pressure drop is low enough to fall within specification.

By employing the FloEFD simulation tool again, the angle seat valve was simulated in order to find the valve pressure drop characteristics at different valve closures, refer to Appendix G for a detailed explanation of the model setup for simulation of the angle seat valve. The valve characteristic must agree with the parameters found from the outflow simulation for the valve to be a viable control valve to be used in the test bench. From the outflow simulation the valve should produce a pressure drop between 39 [kPa] and 100 [kPa] at about 1 [l/s] throughflow rate, when almost fully closed; and between 5 [kPa] and 15 [kPa] at about 4 [l/s] throughflow rate, when almost fully open. Compliance would then satisfy requirements from the current study as well as requirements based on the original WJ Engineering predictions.

The valve produced by WJ Engineering needs just less than 13 full turns to go from fully open to fully closed, with a pitch of 6 mm, thus a total valve plug travel of 78 mm is achieved. The valve is simulated for flow rates from 1 [l/s] to 4 [l/s] and all valve turns, from 0-turns to 13-turns, with 0-turns being fully open and 13-turns fully closed. Fig. 4.17 illustrates an example of the simulated valve flow trajectories and the simulated overall pressure drop characteristic of the valve is shown in Fig. 4.18.

For convenience the square root of the pressure drop is plotted for the different flow rates, to make interpretation of the graph easier — in actual fact the pressure drop will vary exponentially with a change in flow rate. The simulation of the angle seat valve showed that the valve closure will have little effect on the pressure drop from 9 turns of the valve and downward, however starting from 9 turns the valve closures effect on the pressure drop will become more significant and increase exponentially as the valve closes. Thus, the interpretation was made that the valve will only influence the torque between the valve closure range from 9 to 13 (at full valve closure) turns when connected to the dynamometer.

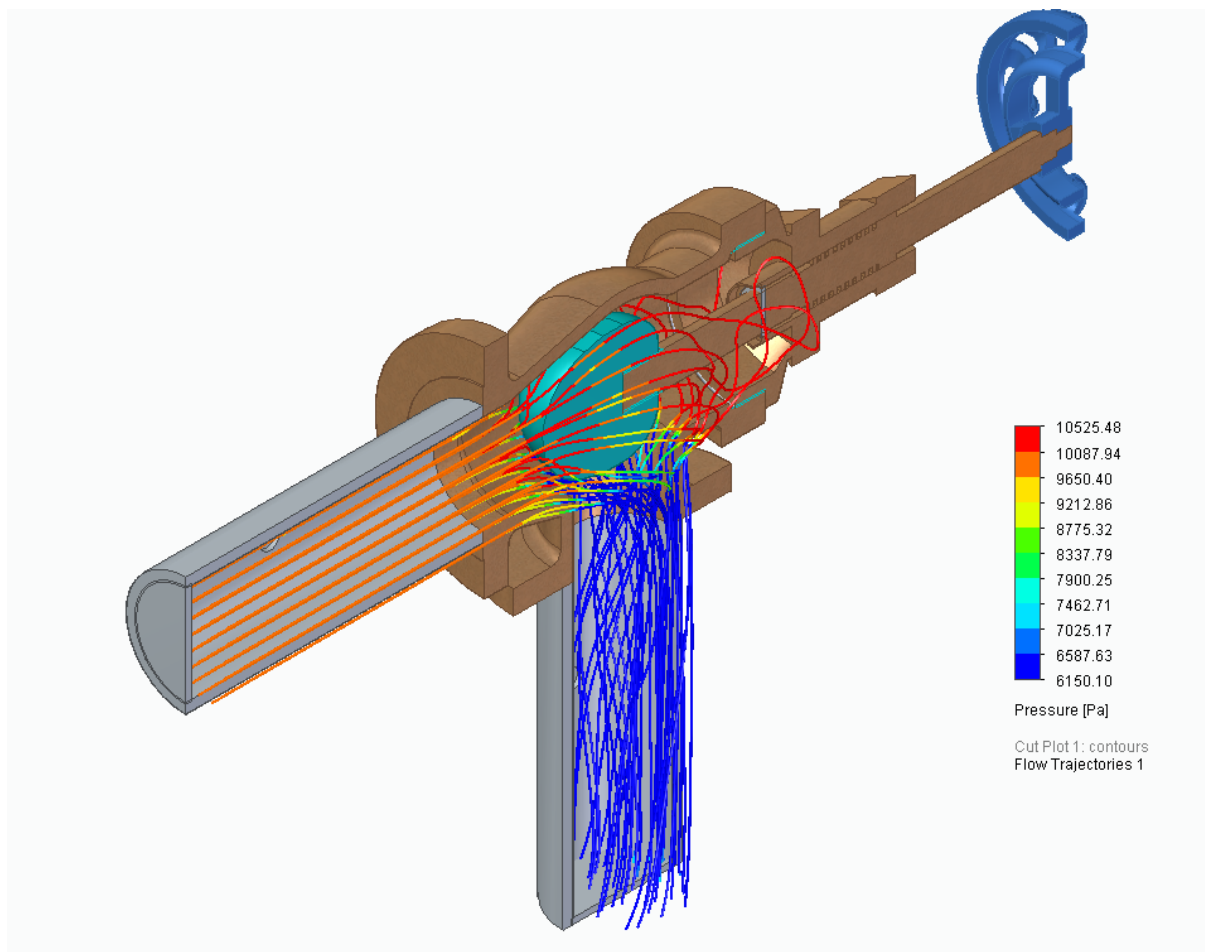


Figure 4.17. Simulated flow trajectories of angle seat valve at 2 [l/s] flow and 10 turns valve closure

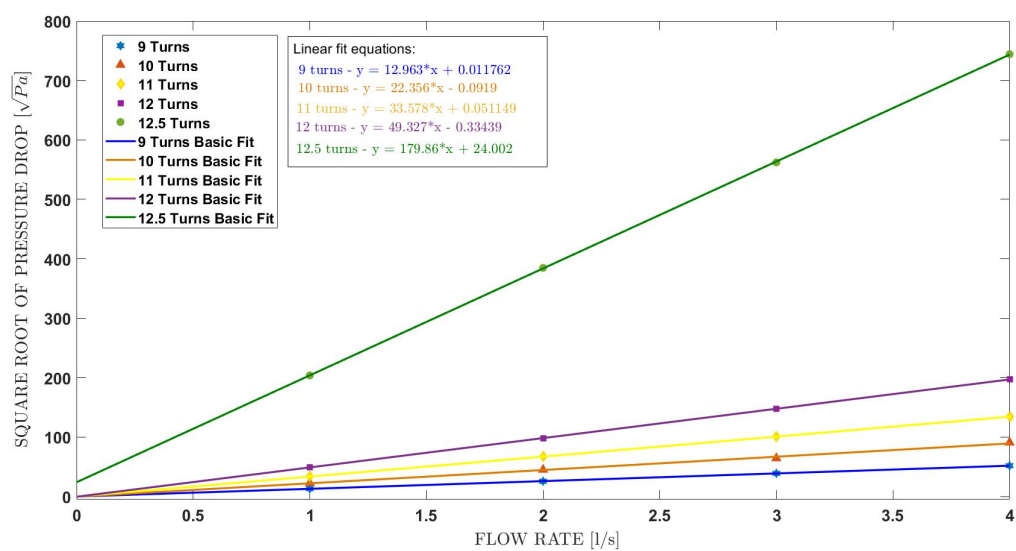


Figure 4.18. Simulated pressure loss characteristic of angle seat valve

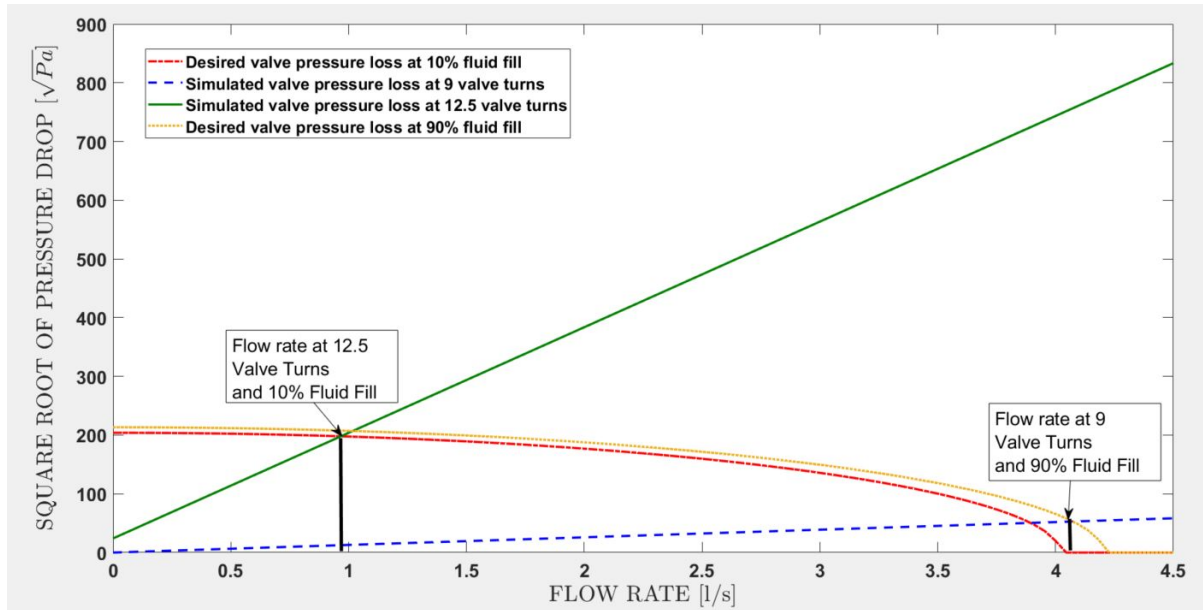


Figure 4.19. Simulated vs desired pressure loss characteristics for angle seat valve

Further assumptions were made that for the specific angle seat valve the dynamometer will be at close to 90% fluid-fill when the valve closure is at 12.5 turns (almost fully closed) and 10% fluid-fill when the valve closure is at 9 turns. We can compare the plot for the desired pressure drop from the valve (Fig. 4.14), with the plot for the simulated pressure drop of the angle seat valve in order to see if the valve will deliver the desired flow rates at the chosen valve closure settings. Fig. 4.19 shows an overlay plot comparing Fig. 4.14 & Fig. 4.18 for 9 and 12.5 valve turns at 10% and 90% fluid-fill.

From the figure we can see that based on the fluid outflow simulation from Section 3.4.3 and the CFD simulation of the angle seat valve, that the current angle seat valve will deliver a throughflow rate of just less than 1 [l/s] at 12.5 valve turns (almost fully closed) depending on the fluid-fill and a throughflow rate of just less or just more than 4 [l/s] at 9 valve turns (almost fully open) depending on fluid-fill.

The results of the custom valve appear favourable and confirmed that the current design will be able to deliver the desired flow rates between 1 [l/s] and 4 [l/s], at the different valve settings, required for proper cooling of the dynamometer. Thus, it is expected that the angle seat valve will be suitable to control the torque of the dynamometer while maintaining an acceptable throughflow rate.

The only major downside of the angle seat valve is the very small range of valve plug movement that will effect the pressure drop of the valve. The simulation of the angle seat valve showed that the valve will cause almost no pressure drop variation for each of the different flow rates from 9 valve turns downward, thus for a large portion of the valve plug travel the valve will be useless in varying the torque of the dynamometer. Fig. 4.18 also illustrates how the valve will only influence the pressure loss slightly between 9 and 12 turns of the valve and then suddenly have a significant effect on the pressure loss from 12 turns upward. This means that highly sensitive and high-resolution control of the torque will be difficult to achieve.

This became even more clear when taking into account the results from Fig. 4.14, the pressure drop requirement of the valve for different throughflow rates. Since the pressure requirement of the valve varies in such a narrow band for changes in fluid-fill, it means that even if the valve setting greatly influences the fluid-fill percentage, the influence on the throughflow rate will

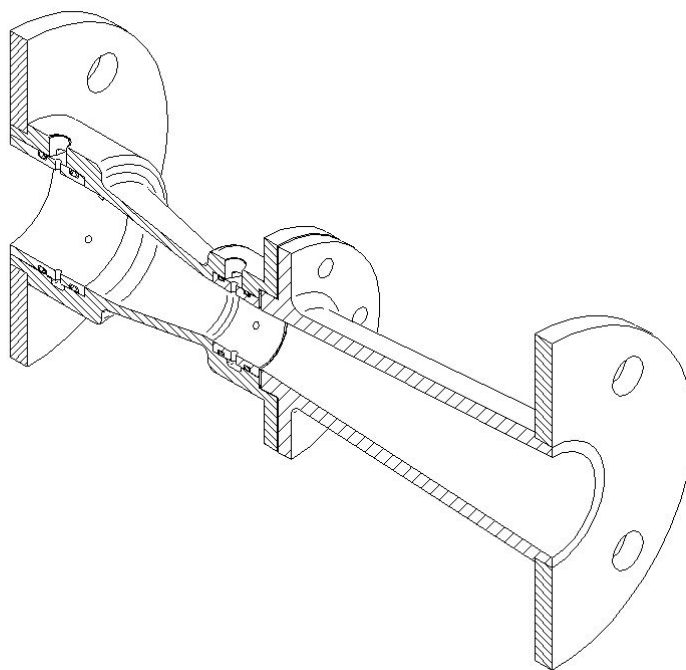


Figure 4.20. WJ Engineering Venturi flow meter CAD model

still be very disproportionate for valve closure above 12 turns as compared to the valve closure below 12 turns. However, regardless of these undesirable properties of the valve, the decision was made to use it in the test bench as it was still expected to exercise an adequate amount of control over the torque absorption.

The angle seat valve had to be validated experimentally to confirm that it is suitable as a control valve; however to achieve this, a flow measuring device was required. In the next section the development of such a device is discussed.

4.4.2. Venturi flow meter

The throughflow rate of the dynamometer influences both the dissipation of the heat generated when loading a machine and the fluid-fill rate which affects the torque absorption. In the previous section it is shown how the throughflow rate is crucial to the development of a proper outlet control valve. For these reasons a method is required to measure the dynamometer throughflow rate, not only to ensure that a proper throughflow rate is achieved for cooling but also to be used in the verification and validation of the outlet control valve design.

A wide variety of fluid flow measuring devices are available of which obstruction type flow meters are considered to be some of the least expensive, easiest to use and most accurate, when one is only interested in measuring the bulk fluid velocity (Tavoularas, 2005). Of the obstruction type flow measuring devices a Venturi flow rate meter is considered one of the most precise (Munson, 2013:459). A Venturi flow meter can be manufactured and used quite easily, Venturi flow meters also leads to the least head loss, which is ideal, thus the decision to use a Venturi nozzle as the chosen device to measure the fluid throughflow rate.

The British Standard for the measurements of fluid in pipes (British Standard, 1964) was used by the author to design the smallest possible Venturi nozzle flow meter, with the design shown in Fig. 4.20.

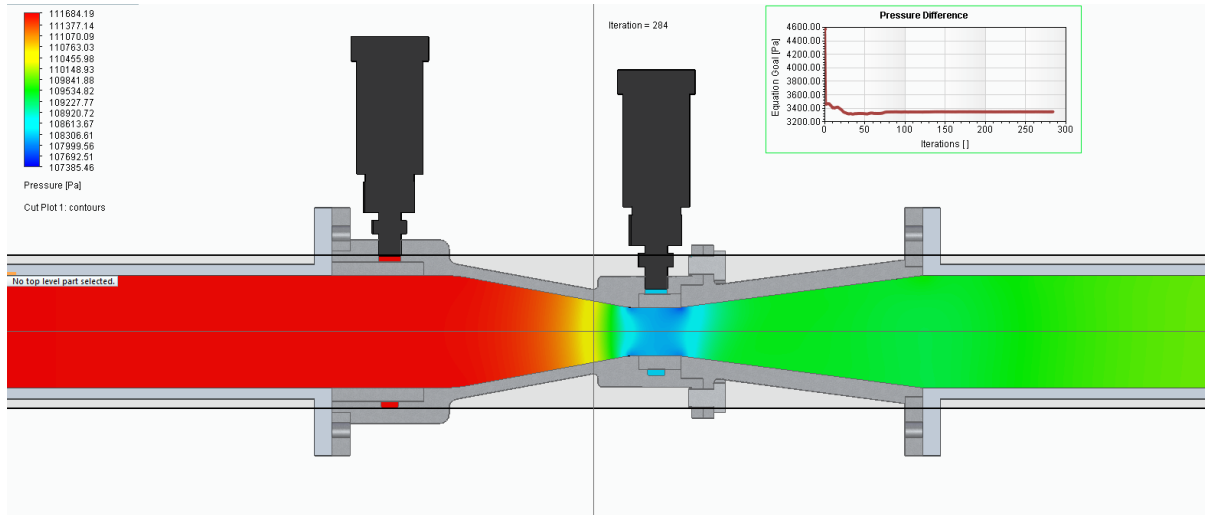


Figure 4.21. Pressure distribution through Venturi flow meter

The FloEFD simulation tool was again used for the analysis of the flow meter design to determine whether the Venturi would be suitable as a flow measuring device for the Dynamometer test bench. An example of such a simulation can be seen in Fig. 4.21, a detailed explanation of the simulation method is provided in Appendix H. The pressure drop in the Venturi throat should be sufficient enough to be properly measured and discerned with pressure measuring devices, while not allowing the total pressure drop across the Venturi to become so excessive that it would inhibit the flow rate from the dynamometer.

Bernoulli's equation can be applied, assuming steady, inviscid, and incompressible flow to analytically determine the pressure drop across the throat of the Venturi:

$$\Delta P = \frac{\rho(1 - \beta^4)}{2} \times (Q/A_2)^2$$

where:

A_2 = Area of Venturi throat

β = Ratio between Venturi inlet diameter and throat diameter

The simulation results were used in conjunction with the application of the Bernoulli equation to find the square root of the pressure drop between the inlet and the throat of the Venturi. The simulation values and the values determined from the Bernoulli equation are shown in Fig. 4.22.

The simulated and theoretically determined results as seen in Fig. 4.22 corresponded very well. From the figure it can be seen that for a total flow rate of 1 [l/s] the pressure drop developed in the throat is about 871 [Pa], and for a flow rate of 4 [l/s] the pressure drop developed in the throat is about 13679 [Pa]. Installation of a pressure transducer that is able to measure pressure values in this range should enable sufficiently accurate flow measurements to be made.

It is also shown in Fig. 4.22 that the total pressure drop across the Venturi will at most be 3230 [Pa] at a flow rate of 4 [l/s], this is a relatively low total pressure drop due to the divergent outlet section of the Venturi, thus the Venturi will not have an adverse effect on the fluid outflow

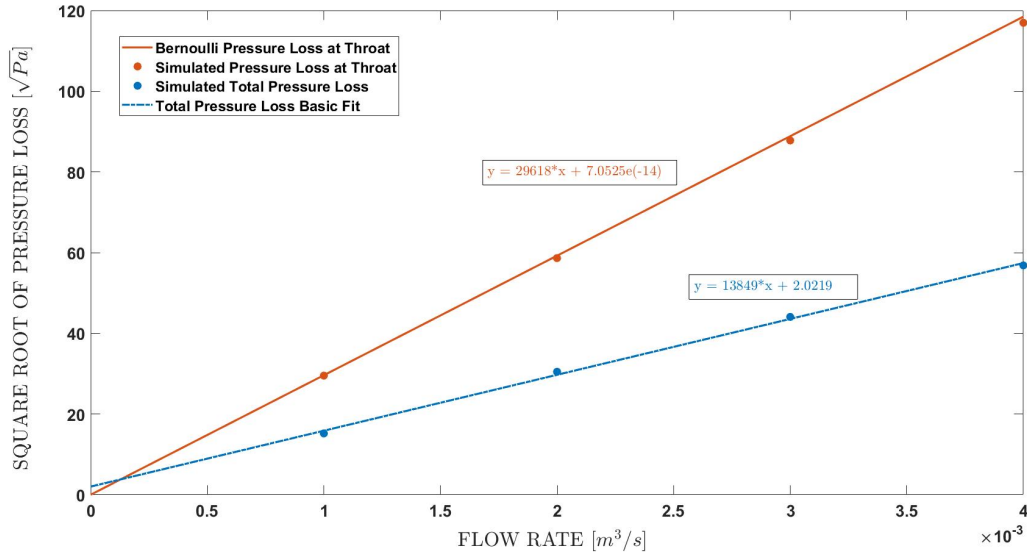


Figure 4.22. Venturi flow meter pressure drop characteristic

from the dynamometer. Utilising the curve for the total pressure drop, the Venturi pressure drop characteristic equation was found:

$$\text{Total Venturi pressure loss} = (C_{vent} Q)^2 / \rho$$

with the loss coefficient given by $C_{vent} = 14834$.

This Venturi loss coefficient is used in Section 3.4.3 as part of the fluid outflow analysis.

The Venturi design should also be experimentally validated and calibrated to ensure the suitability of its use in the test bench. The verification and calibration methods for the Venturi flow meter and outlet control valve are discussed in the next section.

4.4.3. Validation of control valve and Venturi flow meter

Characterisation of the control valve and Venturi flow meter is crucial in verifying and validating their respective designs. For this reason, a separate test bench with procedure was developed to experimentally characterise the valve and Venturi. The measurements were then compared to the theoretical and simulated results presented in the previous sections.

The experimental setup was designed in such a way that the flow meter and control valve were analysed simultaneously to save time and money. The setup allowed the flow rate through the Venturi and valve to be varied and measured while measuring the pressure drops across both devices. Through this method the actual pressure drop characteristics of the control valve and Venturi flow meter for different fluid flow rates were found. The experimental setup is shown in Fig. 4.23 and the schematic layout is illustrated in Fig. 4.24.

The characterisation cycle includes a centrifugal pump connected to a Variable Frequency Drive (VFD), a large water bucket divided into two partitions (denoted by numbers 1 and 2), two gate valves, the Venturi flow meter, the angle seat valve and an additional needle valve.

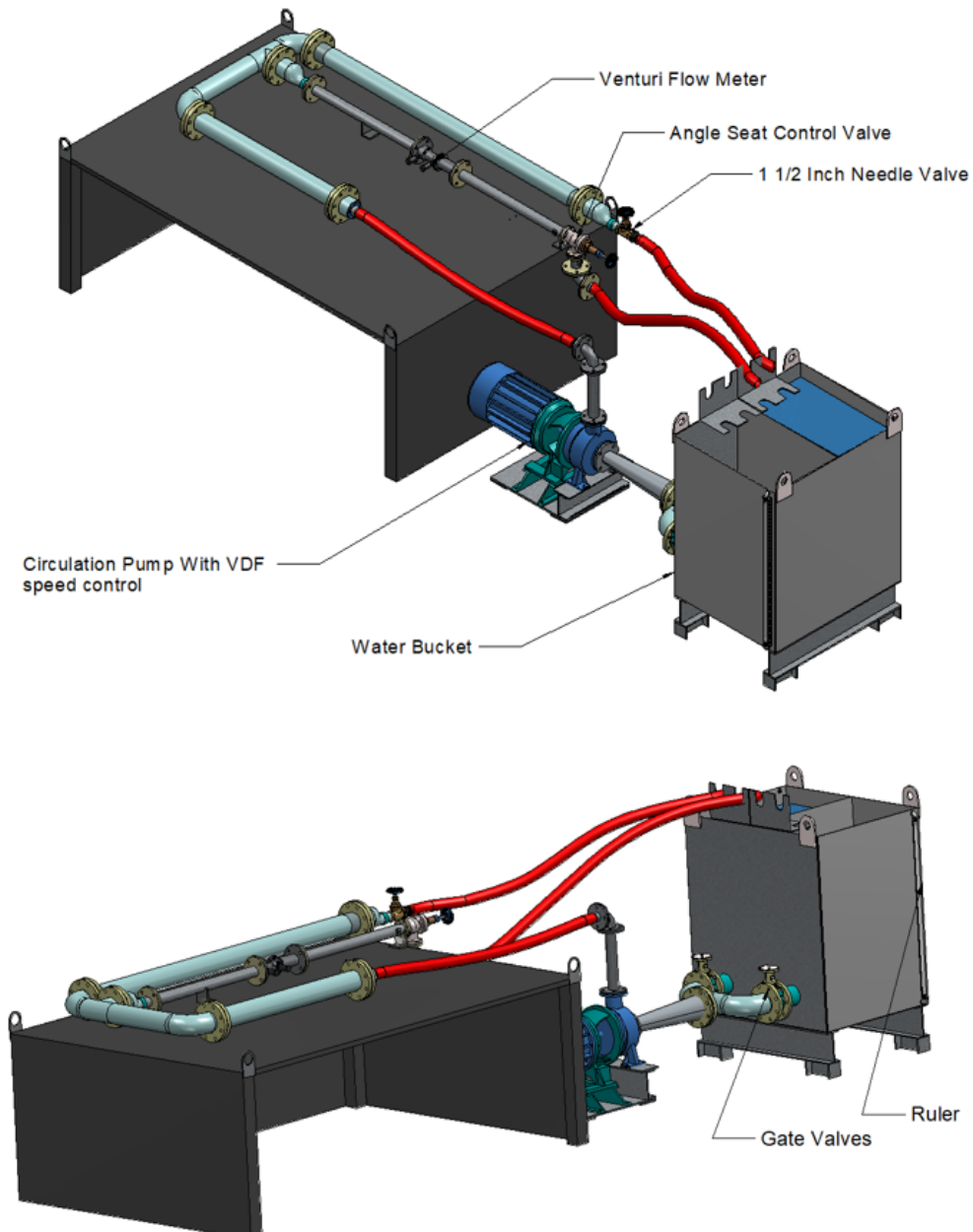


Figure 4.23. Control valve and Venturi flow meter characterisation cycle CAD model

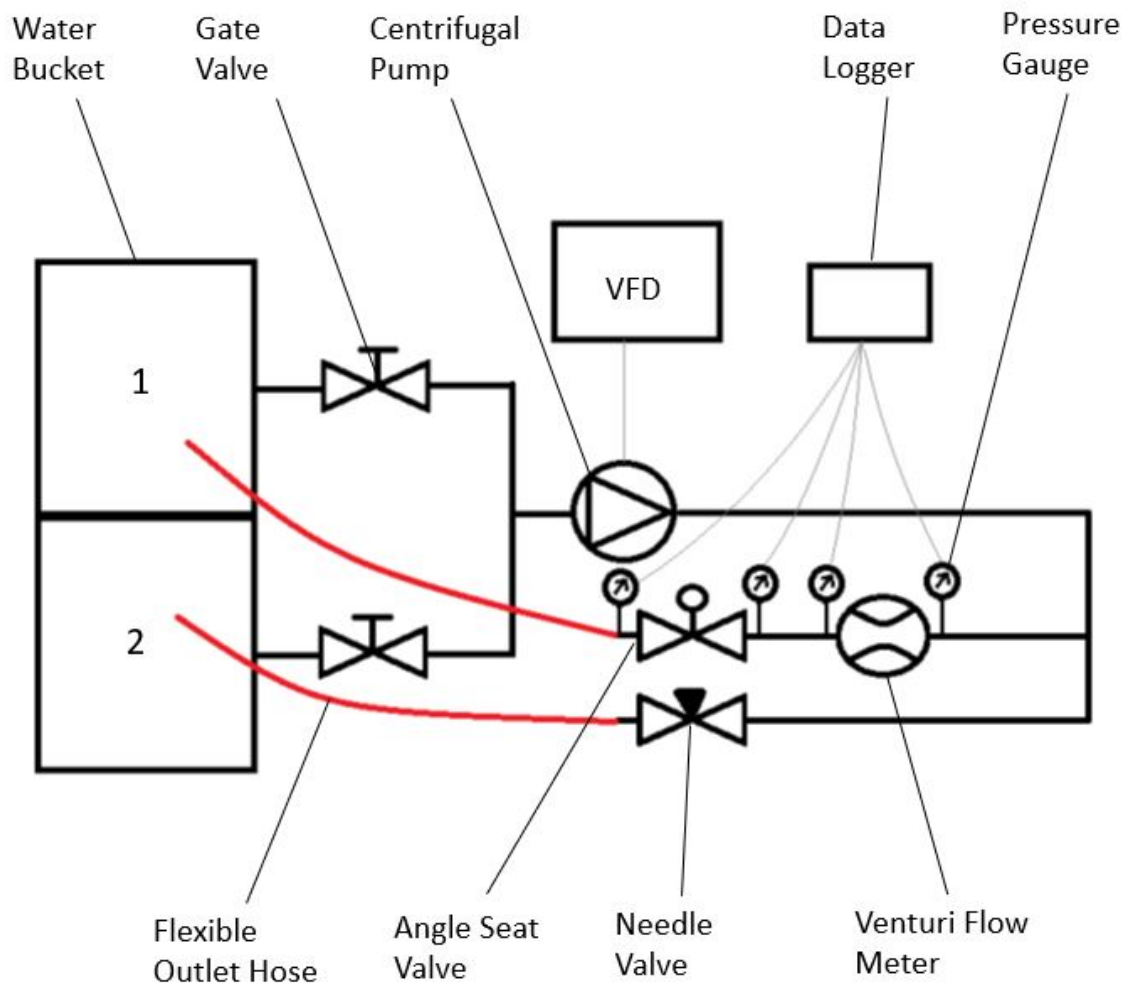


Figure 4.24. Control valve and Venturi flow meter characterisation cycle schematic

The flow rate through the Venturi and angle seat valve was varied by adjusting the centrifugal pump impeller speed through use of the VFD. The flow rate was measured with the bucket-stop-start method, where the amount of time to gain a certain volume of fluid was measured in the water bucket partitions. This was achieved by pumping the water from one water bucket section to the other while ensuring that the gate drain valve of the section into which the water was being pumped was closed, in order to prevent water from flowing out of the section from which the flow rate measurement is being taken. The time it took for the certain amount of water volume to flow into the relevant bucket section was measured with rulers placed on the sides of the water buckets and a stop watch.

To further increase the sensitivity and resolution by which the flow rate could be varied an additional needle valve was employed. By opening or closing the needle valve and allowing the water from the needle valve pipe section to flow into the section from which the water was being pumped, the flow rate through the angle seat valve and Venturi could be adjusted incrementally in addition to the larger-scale changes in flow rate from varying the pump impeller speed. Water could be pumped into one partition or the other by placing the flexible outlet pipes in the section into which the outlet water flow was desired.

The pressure drops across the angle seat valve and Venturi flow meter were then measured with suitable pressure sensors. The pressure drop characteristic for each different setting of the control valve and fluid flow velocity and the Venturi could then be determined. From this the valve characterisation was done. The results of the validation tests are shown and discussed in Section 5.2.1.

Centrifugal pump selection

The centrifugal pump used in the characterisation and validation test bench had to be able to deliver the desired amount of fluid flow for all of the different control valve settings. The valve and Venturi should be characterised for flow rates ranging from 1 [l/s] to 4 [l/s], for the operating conditions as discussed in the previous sections.

A variety of pumps were available at WJ Engineering and based upon the provided pump curve (Rapid Allweiler Pump Curves, 2018) a CLT40 centrifugal pump from Rapid Allweiler was chosen for the characterisation cycle, which is shown in Fig. 4.25. In order to determine whether the pump was suitable, the head loss of the system was calculated and compared to the pump curve. The pump speed was also varied during the characterisation, therefore dimensional analysis was applied to determine the suitability of the proposed pump at each of the different impeller speed settings.

The total head losses for the different flow rates in the cycle remained constant for each valve setting. The expected valve and Venturi characteristic have been determined theoretically in the previous two sections; therefore, the expected pressure drops across the valve for the different flows was known and could be used in conjunction with the head loss across the calibration cycle to find the expected pump flow when adjusting the pump impeller speeds. The pump curve for different impeller speeds was analysed to confirm whether the pump was able to deliver the desired amount of fluid flow for each of the specific control valve settings.

Applying dimensional analysis allowed the prediction of the pump performance at different impeller speeds, which was adjusted by use of the VFD (Sayers, 1990). Comparing the dimensionless flow coefficient $\Phi_D = \frac{Q}{N D^3}$ and head coefficient $\Psi_D = \frac{g H}{N^2 D^2}$ of the pump at different pump impeller speeds to the pump characteristic supplied by the manufacturer, allowed the calculation of the head and flow produced for each of the different impeller speeds.



Figure 4.25. Rapid Allweiler CLT40-250 centrifugal pump before refurbishment

By overlaying the pump and system head loss curves for the valve at almost fully open and almost fully closed conditions, it could be seen whether the pump was able to deliver flow within the desired range of flow rates. The performance curves of the pump at different impeller speeds and the relevant system curves of the characterisation cycle are shown in Fig. 4.26 (The system curves are determined with the needle valve in a fully closed position).

Since the dynamometer is predicted to deliver about 1 [l/s] flow rate for the almost fully closed valve setting and 4 [l/s] flow rate for the almost fully open valve setting, for calibration purposes the pump must deliver the same flow rates for each of the respective valve settings. From Fig. 4.26 it was predicted that the pump would deliver a flow rate of about 1 [l/s] at a pump impeller speed of 2250 [rpm] if the valve was almost fully closed. The pump would also deliver a flow rate of about 1 [l/s] at a pump impeller speed of 250 [rpm] and a flow rate of about 4 [l/s] at a pump impeller speed of 1050 [rpm]], where for both cases the valve was almost fully open.

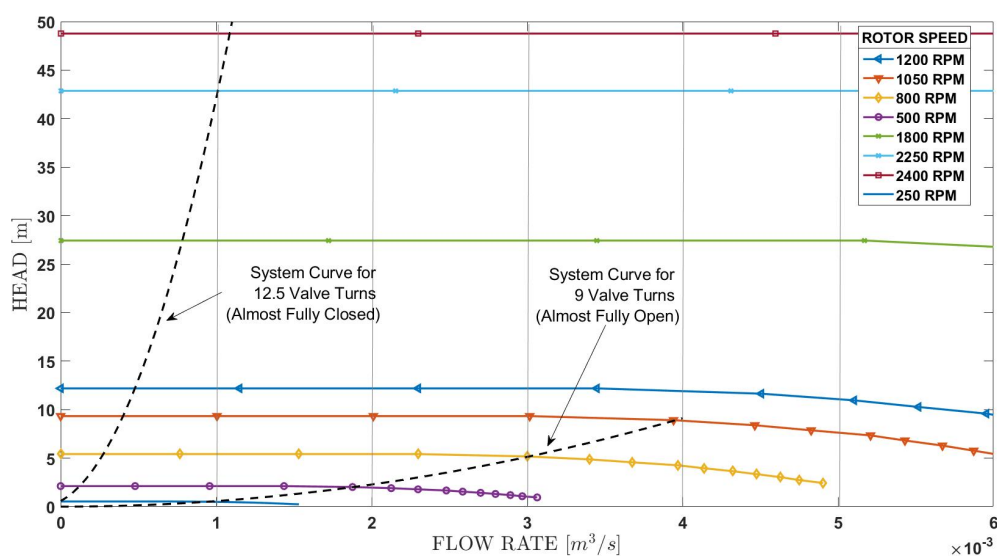


Figure 4.26. Pump curves at different impeller speeds vs. system curves for almost fully open and closed valve (With closed needle valve)

Rotor speeds between 100 and 2600 [rpm] were easily achievable by means of the VFD, without overheating the motor. Therefore, the CLT40-250 pump was found to be ideally suited for the valve and Venturi calibration cycle, since it was shown that the desired flow rates were attainable when utilising this pump. The pump shown in Fig. 4.25 was damaged and worn, so the pump was refurbished and installed as shown in Fig. 4.27.

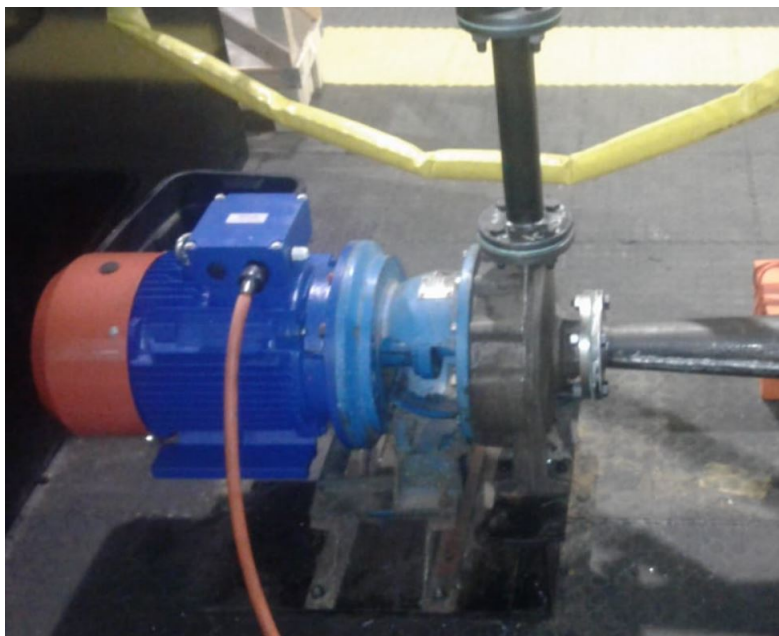


Figure 4.27. CLT40-250 pump after refurbishment

Pressure sensors

For the accurate measurement of the pressure differences across the Venturi flow meter and control valve, at different flow rates and valve settings, suitable pressure transducers were required. When using the correct pressure transducers, it is possible to determine the pressure values at the relevant points and to log the results, allowing processing of the data from which the components can be evaluated. The pressure transducers utilised in the characterisation test bench are shown in Fig. 4.28.

The Druck PTX 1400 and IFM Electronic PX9118 electronic pressure sensors can both measure pressure across the desired ranges and with the needed accuracy and resolution, to satisfy the requirements of the test bench procedures. The measurements taken from the pressure transducers were processed and logged by use of the Memograph M RSG40 data logger from Endress+Hauser (Fig. 4.29) and the data logging and control system of the hydro testing facility, installed by IFM Electronics. The IFM control system utilizes a PLC installed into the hydro testing facility, as seen in Fig. 4.30.

To ensure accurate operation of the pressure transducers, the instruments were calibrated before installation. By utilising the large water tank from the hydro testing cycle, see Fig. 4.32, the pressure sensors were specifically calibrated for the desired range of pressure values. By pumping water into the transparent pipe that runs vertically along the large water tank, and then measuring the precise height of the water level, the hydrostatic pressure could be determined. The pressure sensors were then connected to the transparent pipe and calibrated to ensure that the pressure readings shown by the data logger corresponded to the actual hydrostatic pressure as derived from the height of the water column.



Figure 4.28. Druck PTX1400 transducer and IFM Electronic PX9118 electronic pressure sensor

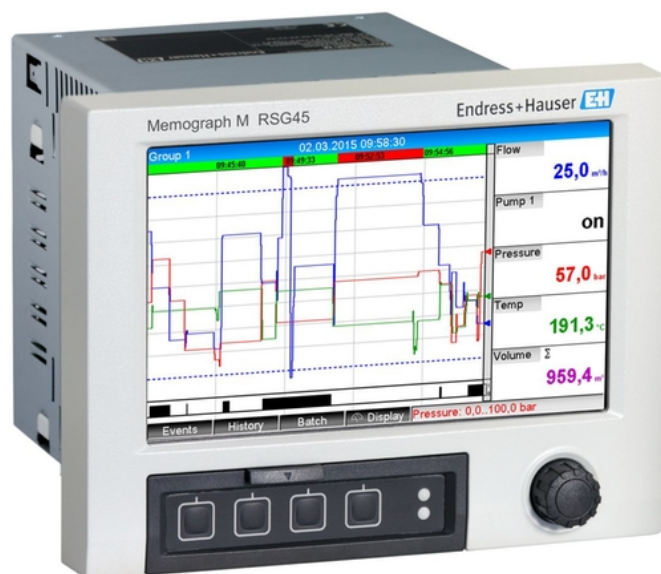


Figure 4.29. Endress+Hauser Memograph M RSG45



Figure 4.30. IFM PLC and data logger installed at WJ Engineering



Figure 4.31. Pressure sensors installed for calibration



Figure 4.32. WJ Engineering Hydro Test Facility large water tank

By means of this method of calibration, it was ensured that the pressure measurements during the characterisation and validation process of the Venturi flow meter, control valve and dynamometer would be accurate. There will however still be some slight inaccuracy inherent to the measuring devices.

4.5. Dynamometer test bench for simulation validation

The hydraulic dynamometer designed and manufactured by WJ Engineering was analysed by means of the integrated one-dimensional model described in Section 4.3.2. The results of the simulation predicted that the prototype dynamometer would not achieve the original design requirements set by WJ Engineering. In order for the simulation results to be useful and give insight into the actual performance of the design, the numerical model needed to be validated. The performance predictions made by the integrated one-dimensional approach were validated through characterisation of the dynamometer as installed in the dynamometer test bench.

The dynamometer test bench was developed to characterise the dynamometer in terms of its most significant performance parameters. The test bench allowed the dynamometer characteristic curves to be experimentally determined and be compared to resulting curves from the theoretical model. Ultimately, from these tests and comparisons, the suitability of the dynamometer for use in the WJ Engineering Hydro Testing Facility could be evaluated. The test bench setup and characterisation procedure will be discussed in the following sections.

4.5.1. Dynamometer test bench development

It was the responsibility of the author to conceptualise, design and facilitate the manufacturing of the entire test bench based on the rotor and stator developed by WJ Engineering. The author also had to assemble and commission the test bench in order to characterise the dynamometer test bench.

The intended goal of the test bench development was to enable the determination of the torque and power absorption of the dynamometer at different fluid-fill levels (corresponding to outlet control valve settings) and rotational speeds. These performance curves gave insight into the power and speed ranges for which the dynamometer would be a suitable loading device.

For characterisation of the dynamometer test bench the torque absorbed by the dynamometer was measured while the rotational speed of the rotor and the control valve settings were adjusted. Additionally during the characterisation, the working fluid absorbed energy that caused it to heat up during the operation and the fluid then had to be cooled sufficiently.

The entire dynamometer test bench configuration is thus made up of the dynamometer, the cooling cycle, the torque arm and load cell. The dynamometer is driven by an electrical motor connected to a Variable Speed Drive (VFD), rather than the hydro turbine to allow the characterisation of the dynamometer torque and power for various speed ranges.

Torque and speed measurement

Fig. 4.33 illustrates the dynamometer test bench characterisation cycle used in the study. For the characterisation setup, the dynamometer was connected directly via a stiff coupling to an electrical motor of sufficient size to drive the dynamometer rotor at the desired speed and power conditions. The motor was connected to a VFD that adjusted the motor speed, allowing the dynamometer to be driven through various speed ranges as required for the study. An electrical

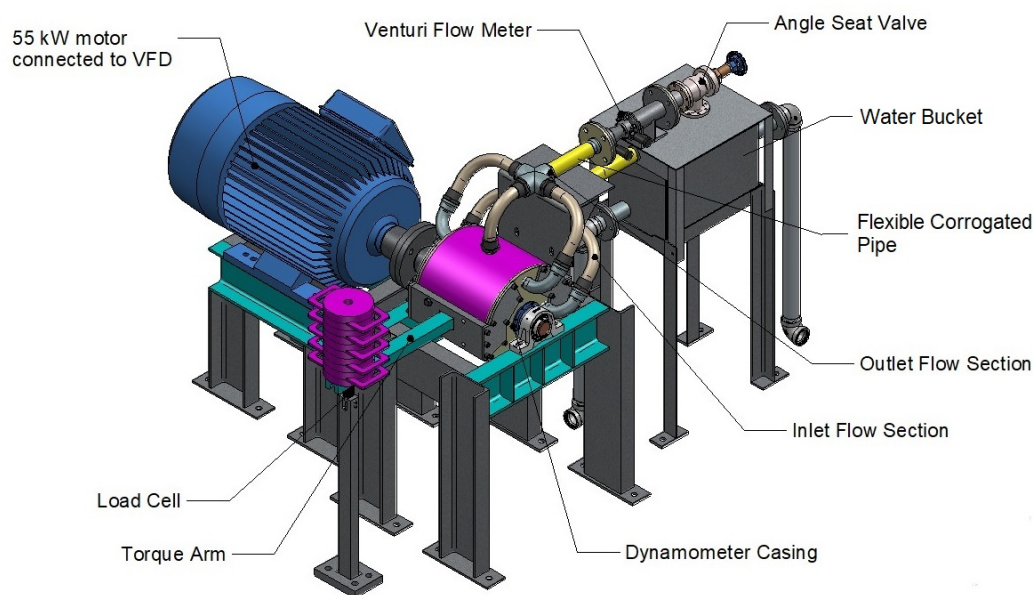


Figure 4.33. Dynamometer test bench characterisation, torque and speed measurement

motor with a 55 [kW] power output was deemed appropriate for this task even though the power expected to be absorbed by the dynamometer would not exceed 20 [kW]. By using a motor that was overpowered it was ensured that the motor can operate safely without overheating at the lower frequency conditions set by the VFD, this also allowed the dynamometer to be characterised at speeds higher than the expected operating speed of the hydraulic turbines. By running the dynamometer at high speeds, the dynamometer self-emptying characteristic discussed in Section 2.4 could be studied as well.

The dynamometer casing was free to rotate about the rotor shaft through the use of bearings mounted onto the shaft; see Fig. 4.34 for an overall view and Fig. 4.35 for detail on the shaft mounting. The dynamometer casing was therefore only held in a fixed position by means of the torque arm. The torque arm was connected, by means of a load cell, to a vertical arm fixed onto the ground. The power from the rotating motor was transmitted from the rotor to the stator of the dynamometer by means of the wall friction, fluid friction, incidence losses and recirculating flow caused by the resulting vortex flow pattern developed in the working compartments of the dynamometer. The torque arm connected to the load cell resisted the rotation, therefore the stator which were fixed to the dynamometer casing, did not rotate. The load cell was subjected to strain caused by the torque on the torque arm and could thus be used to determine the amount of power being absorbed by the dynamometer. Adjustable weights were used to calibrate the load cell, to ensure accurate torque measurements.

Water was supplied to the dynamometer from an external water bucket shown in Fig. 4.33. The inlet water pipe of the dynamometer was submerged in the water bucket, from where water flowed into the dynamometer working compartments by means of both gravity and suction. The inlet pipe was fixed to the water bucket and to ensure that the water bucket did not influence the resisting torque on the load cell, a section of the inlet pipe consisted of a flexible corrugated

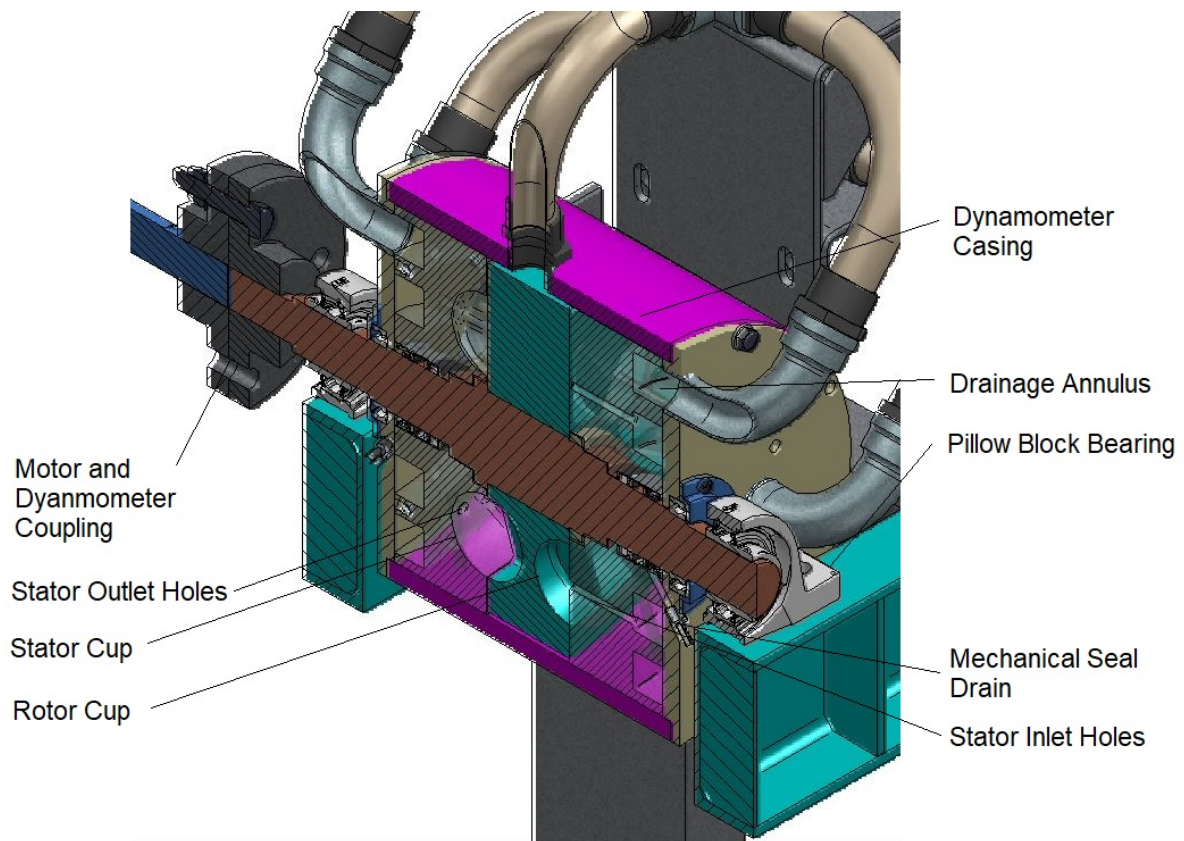


Figure 4.34. Section of dynamometer casing

pipe. The flexible pipe was able to bend and therefore caused the inlet pipe section to have minimum resistance to the torque experienced by the dynamometer.

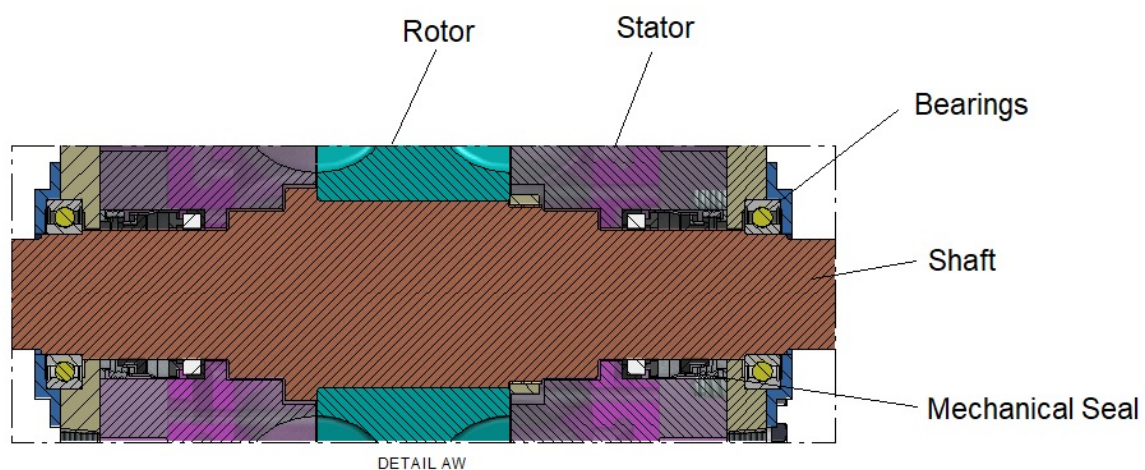


Figure 4.35. Detail of dynamometer casing showing shaft and bearing arrangement

Water flowed out of the dynamometer through a flexible corrugated pipe, similar to the water inflow, to minimise the influence of the outflow pipe section on the resisting torque of the dynamometer. The angle seat valve and Venturi flow meter were fixed to the external water bucket. The outlet flow passed through the Venturi and angle seat valve back into the water

bucket.

Traditionally the flow into a Venturi flow meter must be fully developed to ensure stable and accurate pressure differential readings; this was extremely difficult to achieve in the dynamometer characterisation setup since increasing the outflow pipe length, to allow for fully developed flow to be achieved, would cause the pressure drop along the outflow path to become excessively high. For this reason, it was accepted that the Venturi flow rate measurements would have a certain degree of inaccuracy. For the purposes of the current study, ball park values of the flow rates were however deemed acceptable, since it will still provide insight into the functionality of the dynamometer and control valve. The outlet valve was used to adjust the back pressure on the outlet flow path and controls the flow rate and fluid fill in this manner.

Cooling of working fluid

It is explained in Chapter 1 that the operation of the dynamometer will cause the working fluid to become heated. This is why an auxiliary cooling cycle must also be employed to ensure that the working fluid of the dynamometer is cooled sufficiently to ensure safe operation. Fig. 4.36 depicts the piping system of the cooling cycle.

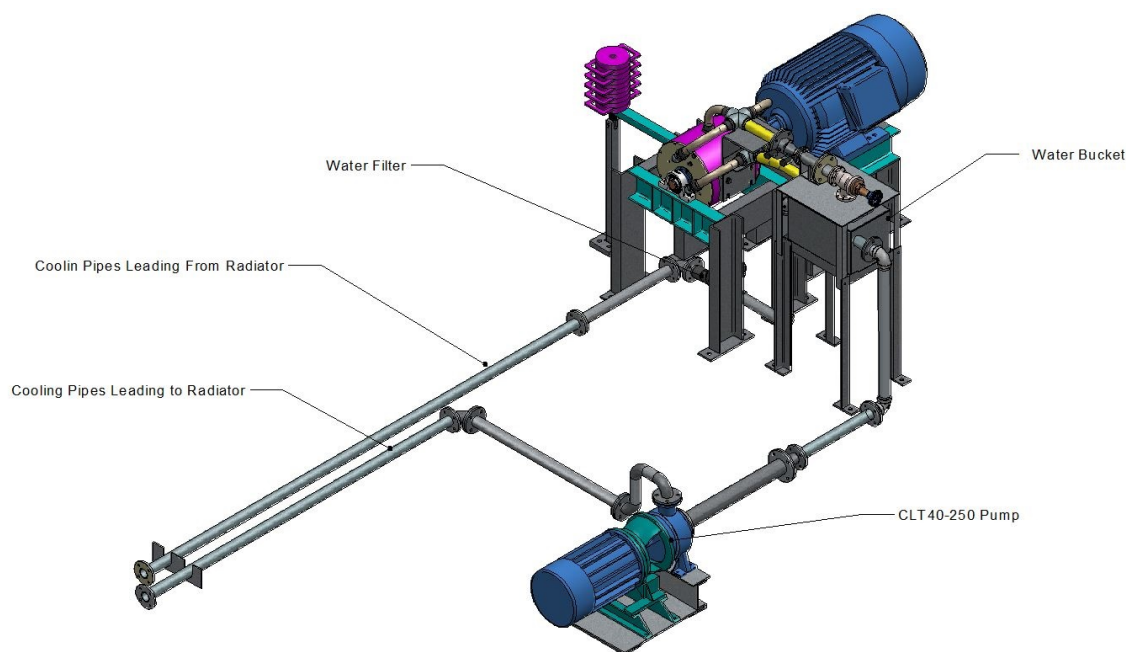


Figure 4.36. Dynamometer test bench auxiliary cooling cycle

The water in the external water bucket was circulated constantly in an auxiliary cycle, separate from the main dynamometer cycle, for cooling purposes. The water was pumped from the water bucket by means of the CLT40-250 centrifugal pump, also used in the valve and Venturi characterisation cycle. This pump was again connected to a VFD to allow the circulating flow rate throughout the cooling cycle to be adjusted. The water was pumped through a commercially available aluminium truck radiator submerged in the large low-pressure water tank of the Hydro Test Facility. The water was cooled within the radiator and circulated

back into the external water bucket from where it was reused by the dynamometer for energy dissipation. The water was also pumped through a filter to eliminate any impurities in the water that would have an adverse effect on the dynamometer function.

A detailed design of the dynamometer cooling cycle fell beyond the scope of this study, but because of the relatively high heat capacity of water, the efficacy of a aluminium truck radiator and the ability to increase the circulating flow rate by means of the VFD, a rather safe assumption was made that the cooling cycle would have the capability to adequately cool the working fluid.

4.5.2. Dynamometer characterisation method

Characterisation of the dynamometer required the measurement of the torque absorption at varying speeds and control valve settings. The dynamometer was driven by the connected motor at a chosen speed by means of the VFD, the speed ranges were from 0 to 2000 [rpm], or depending on where the self-emptying started to occur or the load-cell limit was reached. The torque and throughflow rate of the dynamometer were measured by means of the load-cell and Venturi flow meter at set speeds and valve settings. These steps were repeated at all the desired motor speed and control valve settings and the results were used to determine the dynamometer characterisation curves. These curves were then compared to the predicted operating curves shown in Fig. 4.7 and Fig. 4.8. The comparison would serve to validate the theoretical model and to ascertain whether the dynamometer would be suited for the WJ Engineering Hydro Testing Facility.

If a sufficient operating speed could be reached, within the operational limits of the test bench, the self-emptying of the dynamometer working compartments will start to occur. This phenomenon is incredibly important to predict and will be used to produce the complete hydraulic dynamometer operating envelope.

4.6. Summary

In this chapter the research methodology employed to achieve the aims of the study are discussed. This includes the simulation of the dynamometer to predict the torque absorption and the parameters from which a suitable control valve was designed, the cycle by which the control valve and flow meter designs were validated and lastly the test bench and hydraulic cycle with which the dynamometer could be characterised.

In the following chapter the experimental setups discussed in this chapter are showcased and the results from the various experimental methods are presented and discussed.

5. Results and discussion

5.1. Overview

In this chapter the experimental results are presented and discussed, that was obtained by applying the research methodology described in Chapter 4. A critical evaluation of the results will ultimately confirm whether the outcomes of the study were accomplished and if the research problem was suitably addressed.

The chapter is divided into two main sections, each relating to one of the two test cycle setups detailed in the previous chapter. The first section contains the results and discussion of the Venturi's and outlet control valve's validation experiments. The second section contains the results and discussion of the complete dynamometer test bench's validation experiments.

5.2. Venturi and control valve characterisation

In Section 4.4.3 the design of a characterisation and validation cycle (see Fig. 4.23 and Fig. 4.24) is described by which the control valve and the Venturi flow meter are to be characterised to ensure that both components are suitable for use in the dynamometer test bench. The physical test bench can be seen in Fig. 5.1 with Fig. 5.2 focussing on the Venturi flow meter with installed pressure transducers.

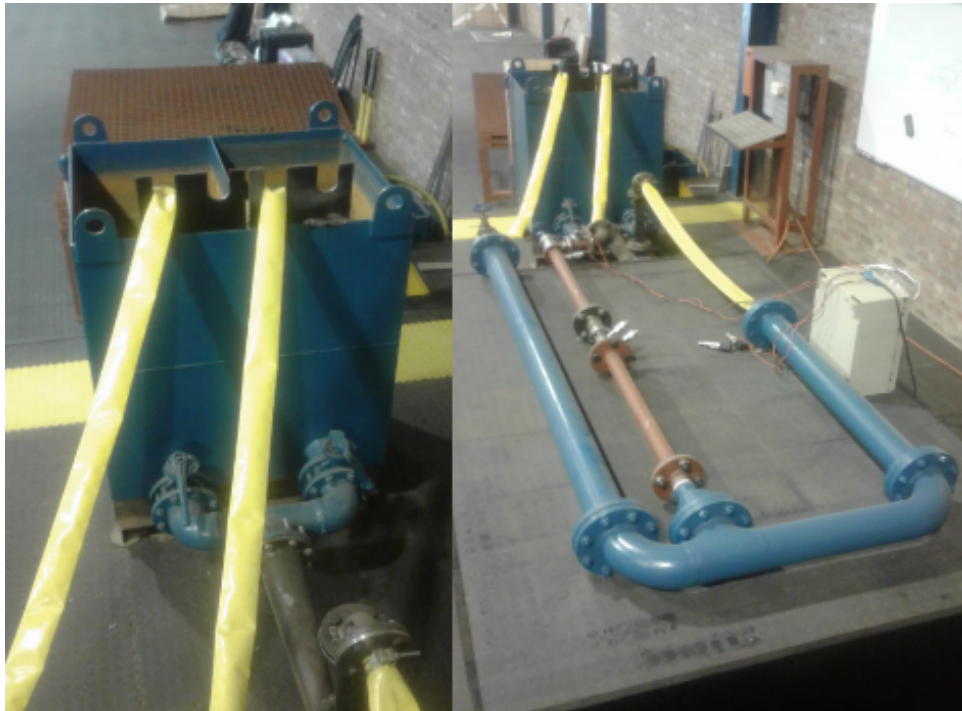


Figure 5.1. Control valve and Venturi flow meter characterisation cycle

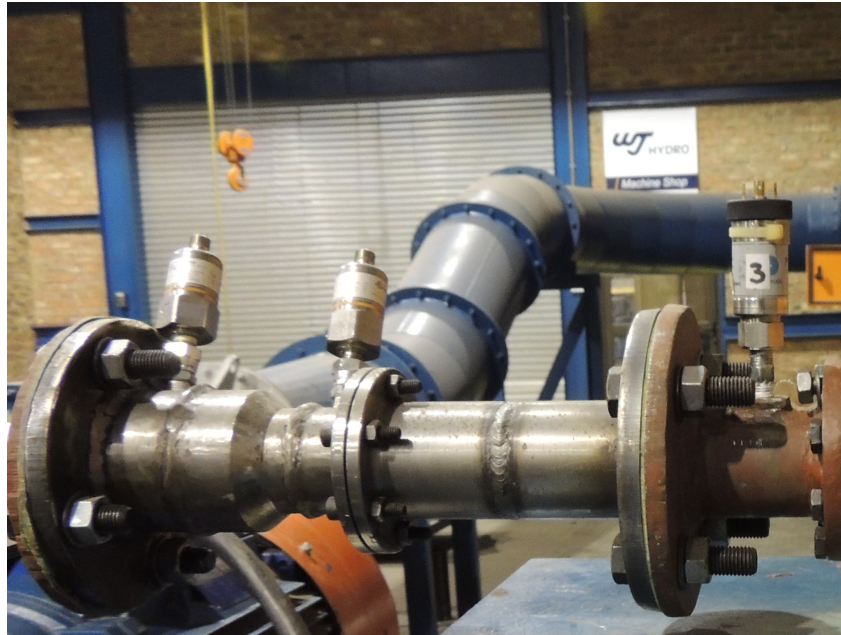


Figure 5.2. Venturi flow meter with installed pressure transducers

5.2.1. Venturi flow meter

The Venturi flow meter design characteristics and specifications are discussed in Section 4.4.2. The pressure drop characteristic predicted theoretically in Chapter 4 (see Fig. 4.22) was to be validated experimentally through measurements taken from the characterisation cycle. Fig. 5.3 shows the experimental results of the characterisation of the Venturi flow meter compared to the theoretically predicted characteristic.

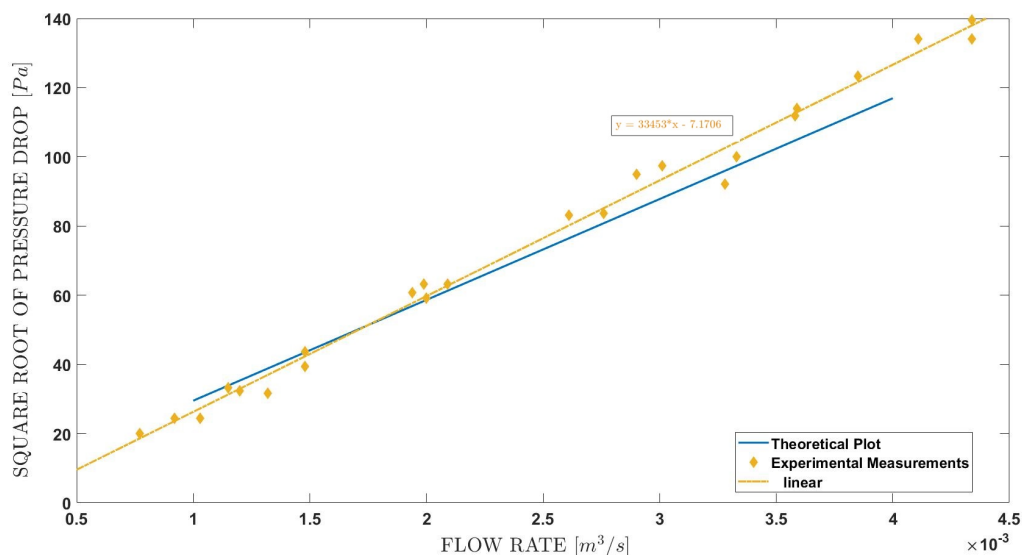


Figure 5.3. Venturi pressure drop characteristic, experimental vs. theoretical results

From Fig. 5.3 it can be seen that the experimentally predicted pressure drops for the various flow rates correspond well with the experimentally obtained pressure drop values. The pressure loss characteristic has the familiar quadratic curve shape expected for constriction-type flow measurement devices.

There were some slight deviations of the experimental values from the theoretical values at the lower and higher ends of the flow rate range. This was deemed to be of no practical significance for the current study as the pressure losses expected from the Venturi were still within acceptable operational limits. Subsequently, in order to ensure accurate flow rate measurements, the experimental characteristic was used in flow rate calculations and the theoretical predictions discarded. It was therefore concluded that the Venturi flow meter as designed and manufactured, was suitable to be used to measure the dynamometer outflow rate.

5.2.2. Outlet control valve

The pressure drop characteristic of the outlet control valve (development described in Section 4.4.1) was also determined by means of the Venturi and outlet valve testing cycle. The most important requirement for the control valve was to vary the back pressure with changes in the amount of valve closure, and to do so within the pressure ranges specified in Section 4.4.1. The pressure ranges for which flow rate control were desired are between 39 [kPa] - 100 [kPa] at a flow rate of 1 [l/s] and 5 [kPa] - 15 [kPa] at a flow rate of 4 [l/s], depending on the fluid-fill condition (also see Section 4.3.5).

The valve design performed well when initially analysed through CFD simulation and this is confirmed by the comparative plots in Fig. 5.4, where the experimental results for the valve pressure loss characteristic corresponds well with values from the CFD predictions for the pressure loss behaviour.

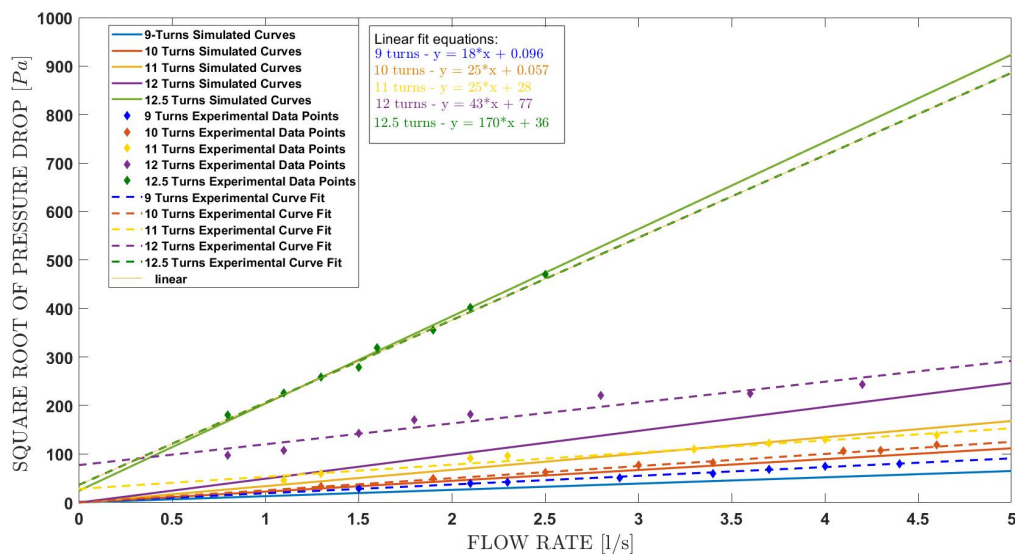


Figure 5.4. Valve pressure drop characteristic, experimental vs. theoretical results

The outlet valve with custom manufactured design, shown in Fig. 5.5, delivered a pressure drop of 60.9 [kPa], at a flow rate of 1 [l/s] and 12.5 valve turns (almost fully closed) and a pressure drop of 7.1 [kPa] at a flow rate of 4 [l/s] and 9 valve turns (almost fully open). The valve design was therefore confirmed to control the back pressure on the working compartments within the desired pressure limits, regardless of the fluid-fill induced.

The experimental investigation found that the valve will only start to influence the pressure drop from about 9 valve turns onward, just as was predicted with the CFD simulations. Therefore the valve will only effectively control the back pressure experienced by the dynamometer — and consequently the fluid-fill — from 9 valve turns up to the fully closed valve setting. Some

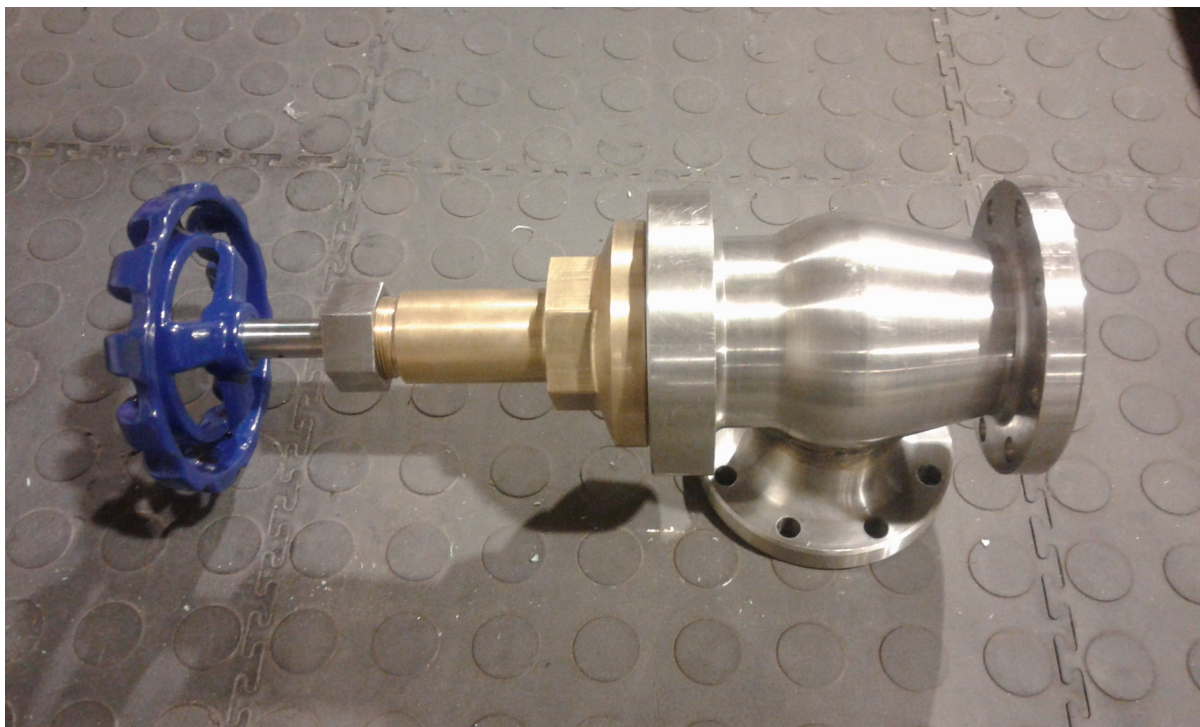


Figure 5.5. Outlet control valve manufactured from custom design

slight deviations was found between experimental measurements and numerical results for the curves at 9 and 12 valve turns, however this was not excessive. It was also interesting to note that the pressure/flow rate gradient kept a relatively gentle slope from 9 valve turns up to 12 valve turns; above which the gradient increased sharply as the valve seat was closed.

The range of values within which the control valve had to influence the back pressure on the working compartments of the dynamometer were broadly defined, since it was unknown what pressure and throughflow rate combinations were responsible for which specific fluid-fill conditions. These ranges were therefore used as design guidelines to determine the suitability of the valve for control purposes. After testing, the physical valve was confirmed to influence the back pressure developed and it was therefore expected that it would influence the torque absorption of the dynamometer from valve settings of 9 turns up until the fully closed valve setting. Thus the valve was deemed to be a suitable outlet control valve for use in the variable-fill dynamometer test bench according to the dynamometer design requirements and specifications.

5.3. Dynamometer characterisation

The torque and power characteristics of the WJ Dynamometer at various fill levels were predicted by means of the one-dimensional theoretical model described in Section 4.3.2, with specific reference to Fig. 4.7 and Fig. 4.8. The purpose of the dynamometer test bench and characterisation cycle was thus to verify the predicted torque and power curves in order to validate the theoretical model and to characterise the WJ Dynamometer to determine whether it was suitable for use in the WJ Hydro Testing Facility.

Fig. 5.6 gives an overall view of the actual test bench where the individual components can be discerned: variable-fill dynamometer, electrical motor, torque arm and load cell, outlet piping with cross junction and Venturi flow meter.

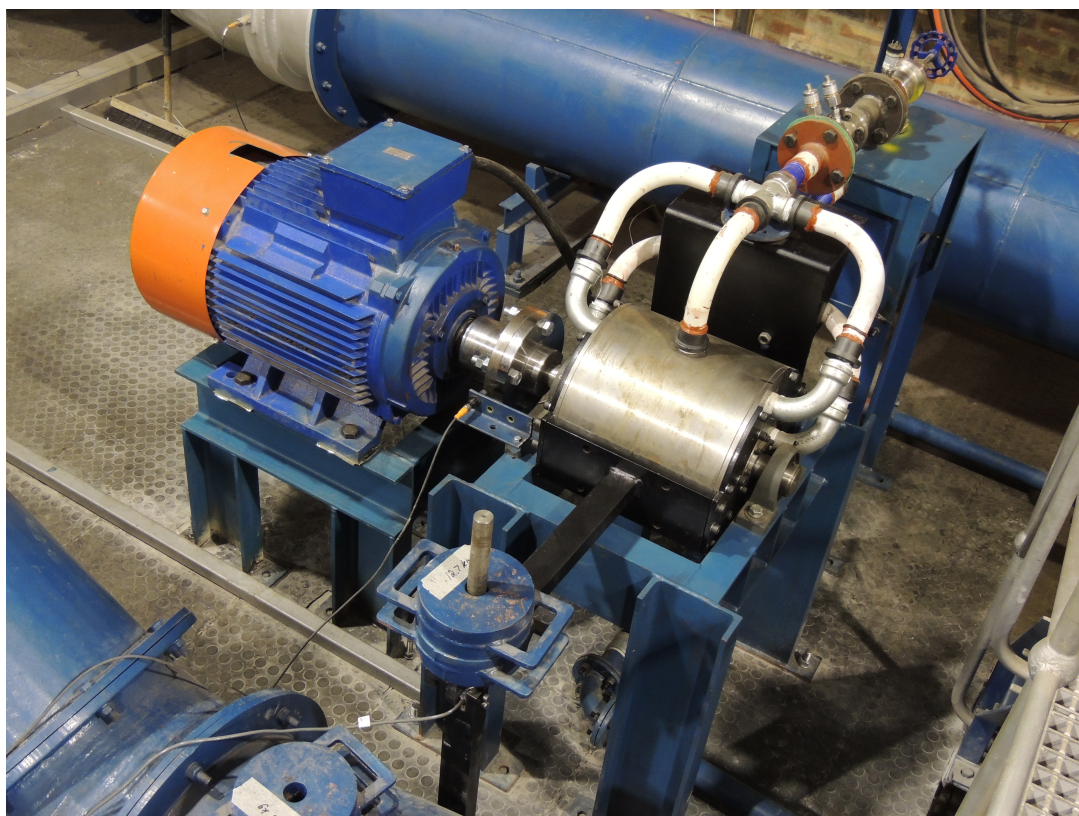


Figure 5.6. Variable-fill hydraulic dynamometer test bench

As stated previously the dynamometer torque was measured by means of a load cell attached to the end of the torque arm. Operation of the dynamometer transmitted torque from the rotor to the stator which caused the dynamometer casing to rotate about the cradle mounted bearings, the rotation was however resisted by the load-cell and torque arm, and generated a force measurement from which the dynamometer torque could be calculated. In order to calculate the power output of the dynamometer the speed of the dynamometer was measured by means of a proximity speed sensor and used in conjunction with the torque measurements. A VFD controller on the electrical motor allowed the dynamometer operating speed to be adjusted and permitted the measurement of the dynamometer torque over a range of speed settings. A closer view of the torque arm, load cell and proximity speed sensor is shown in Fig. 5.7.

The external cooling cycle used to cool the dynamometer working fluid during operation is shown in Fig. 5.8. The circulation pump used to pump water through the submerged radiator (see Fig. 5.9) was connected to a separate VFD in order to adjust the flow rate throughout the cooling cycle, providing a means by which the rate of cooling could be adjusted as desired. An inline filter was also placed in the cooling cycle to clean the water of impurities that could lessen the effectiveness or damage the dynamometer see Fig. 5.10.

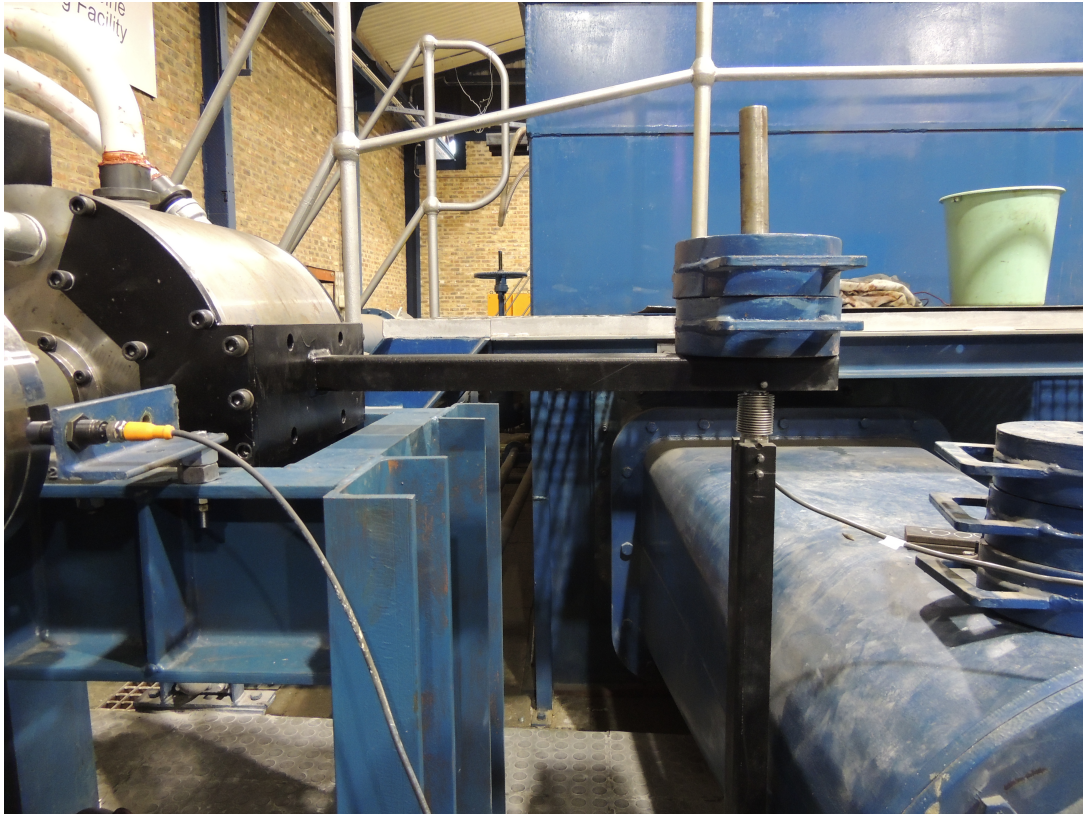


Figure 5.7. Dynamometer load cell and proximity speed sensor



Figure 5.8. Cooling cycle pipework running adjacent to water tank

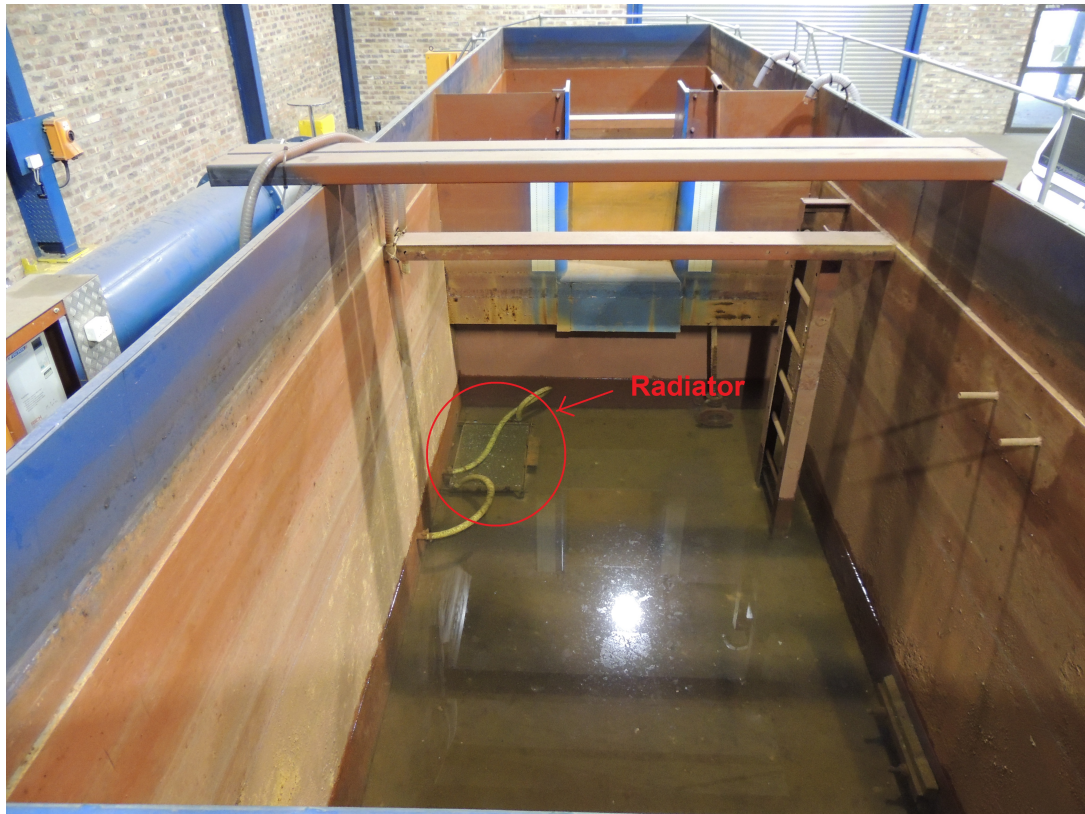


Figure 5.9. Submerged radiator inside water tank



Figure 5.10. Inline filter to clean inlet water

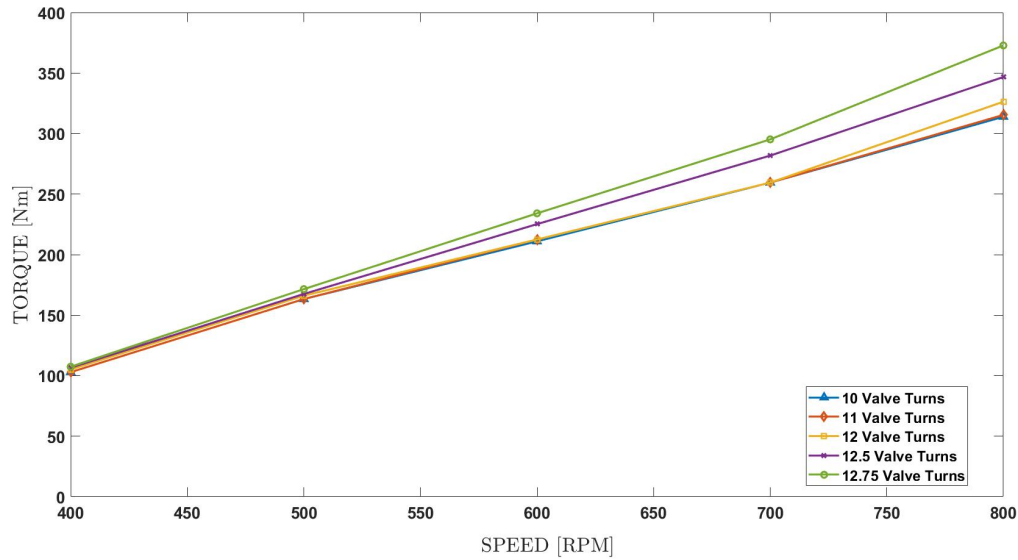


Figure 5.11. Dynamometer torque characteristic experimental results

5.3.1. Torque and power

The variable-fill hydraulic dynamometer developed at WJ Engineering was expected to show an increase in torque absorption as the operating speed increased. The torque characteristic was also expected to be variable with changes in the valve closure of the outlet control valve. Fig. 5.11 shows the torque absorption measured by means of the dynamometer test bench characterisation cycle.

The actual dynamometer torque curve measurements of the WJ Dynamometer showed a somewhat linear increase in torque as the operating speed increased. Only minor changes in torque absorption occurred with changes in the outlet valve closure and these changes only started to happen from 10 valve turns from fully open, to an almost closed valve setting.

Just as was predicted with the theoretical model the initial dynamometer torque absorption was high with an initial torque of about 105 [Nm] at 400 [rpm] for all valve settings. The torque absorption increased up to a minimum of 313.9 [Nm] and a maximum of 372.8 [Nm] at an operating speed of 800 [rpm]. WJ Engineering required a torque absorption of 300 [Nm] at an operating speed of 1200 [rpm], the amount of torque absorption of the dynamometer far exceeds this requirement. The characterisation tests were ceased at a rotational speed of 800 [rpm] as the torque values reached the maximum permitted load of the load cell. Unfortunately for this reason no test could be done at the desired operating speed of 1200 [rpm], but the results contained sufficient data to make a conclusion about the suitability of the dynamometer for the WJ Hydro Test Facility and to verify the theoretical model.

The torque characteristics for different constant fill percentages were predicted in Section 4.3.2 by means of the one-dimensional integrated theory put forward by Raine & Hodgson (refer to Fig. 4.7 and Fig. 4.8). The predicted torque values for constant fill were superimposed and compared to the experimental results from the dynamometer characterisation cycle in Fig. 5.12.

The actual torque and power absorption of the dynamometer was shown to correspond well to the theoretically predicted torque and power found by means of the one-dimensional theory presented by Raine & Hodgson. In comparison, the theoretical model predicted that the dynamometer torque absorption would be between 300 [Nm] and 365 [Nm] for fluid fills between

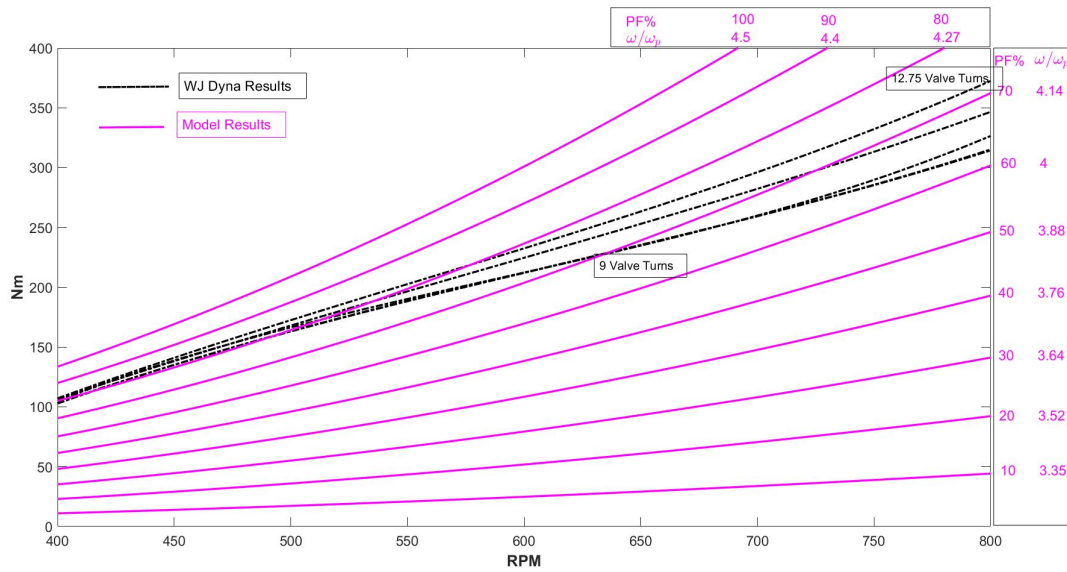


Figure 5.12. Dynamometer torque characteristic experimental vs. theoretical results

60% and 70% at an operating speed of 800 [rpm], while the actual dynamometer tests achieved a load between 300 [Nm] and 375 [Nm], Thus, the actual torque of the dynamometer falls within the ranges of the predicted torque.

The torque and power exceed the values predicted by WJ Engineering. The dynamometer was able to achieve the required load of 300 [Nm] at about 780 [RPM] operating speed for a fully open valve setting, which coincided with a low fill condition, and at about 710 [RPM] operating speed for an almost fully closed valve setting, which coincides with a high fill condition. However, regardless of the fact that the required torque has been achieved at a lower speed than was required, the maximum power attainable by the dynamometer within the limitations, amounts to about 32 [kW] which is much lower than the desired power requirement of 38 [kW] specified by WJ Engineering.

The theoretical model illustrated that changes in the outlet control valve closure had a significant impact on the torque absorption of the dynamometer. The experimental results did however contradict this; although the torque absorption varied with valve closure, it did so within a narrow band. At the maximum rotational speed of 800 [RPM] for which tests were done, the valve was able to vary the torque from a minimum of 313.9 [Nm] to a maximum of 372.8 [Nm], meaning that the control valve was able to vary the torque by about 18.76% of the maximum. This represented a somewhat noticeable influence of torque variation through use of the control valve, but not nearly as significant as was predicted by the theoretical model. When considering that the torque was barely varied at the low speed conditions, the outlet control valve did not have the desired effect.

Unfortunately the onset of the self-emptying phenomena of the dynamometer working compartments could not be determined since the dynamometer speed required to achieve this would produce torque values much higher than that which the load cell could handle. For this reason the complete operating envelope of the dynamometer could not be determined.

5.3.2. Flow rate and pressure

The dynamometer outflow rate was measured by means of the Venturi flow meter. The Venturi flow meter measured the pressure at the inlet of the Venturi and the pressure drop due to the

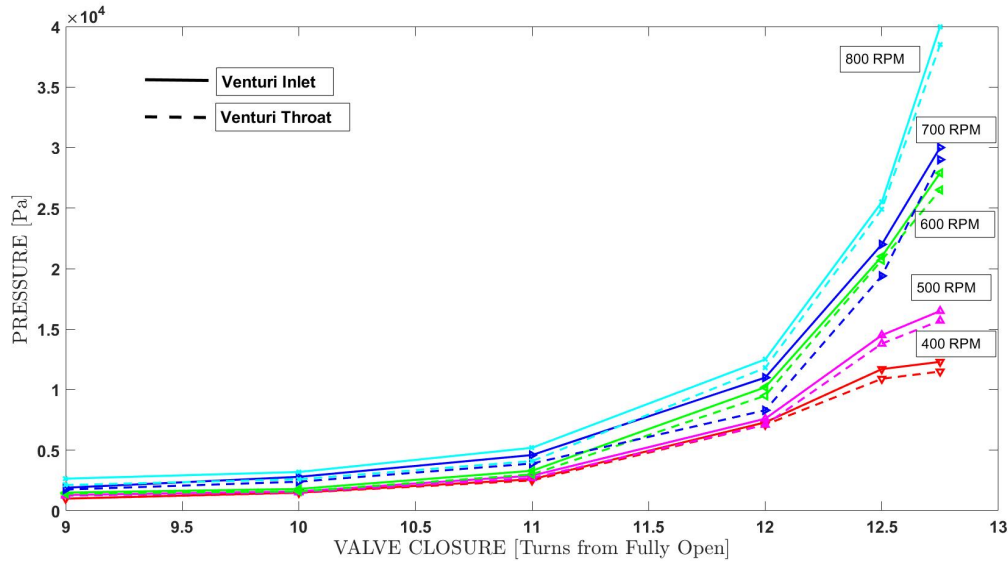


Figure 5.13. Venturi pressure measurements from dynamometer characterisation cycle

constriction. The purpose of the Venturi measurements was to observe the influence of the outlet control valve on the pressure of the working compartments and on the throughflow rate produced.

In Section 3.4.3 and Section 4.3.5 it is explained how the influence of back pressure leads to a change in the fluid-fill of the working compartments and consequently the torque absorption of the dynamometer. It is unknown how the flow rates and pressures relate to the torque absorption, but through the measurement of the throughflow rate and the back pressure, it can be distinguished if there is any correlation between the fluid-fill, the flow rate and back pressure. Fig. 5.13 depicts the change in pressure observed at the inlet and throat of the Venturi during operation at different valve closure and speed conditions.

The results clearly illustrated that the pressure in the dynamometer working compartments increased exponentially with an increase in operating speed. The overall pressure curves also moved to slightly higher-pressure conditions as the valve closure increased.

The increase observed in the working compartment pressure linked to the increase in operating speed, coincided with the theoretical model prediction discussed in Section 4.3.3. In Fig. 4.11, the integrated one-dimensional theory simulation showed a similar exponential increase in working compartment pressure to that seen in the experiment. It was therefore clear that an increase in operating speed, led to higher vortex velocities, which in turn increased the vortex pressure, as was explained in Section 4.3.1 and illustrated with Fig. 4.6.

The rise in the pressure curves with higher valve closure seen from the experimental results, exhibited a similar trend to the simulated increase in pressure curves of the working compartment vortex pressure linked to the increase in fluid-fill as seen in Fig. 4.10. There was therefore a definite correlation between the back pressure — induced by higher outlet valve closure settings — and the fluid-fill condition of the dynamometer working compartments.

A possible reason why the fluid-fill amount increased with higher back pressures, is due to the dynamometer trait that functions to equalize the inflow and outflow rates. When the valve closure increased the back pressure that resisted the fluid outflow from the working compartment, the pressure produced by the working compartment had to increase to allow continuity between

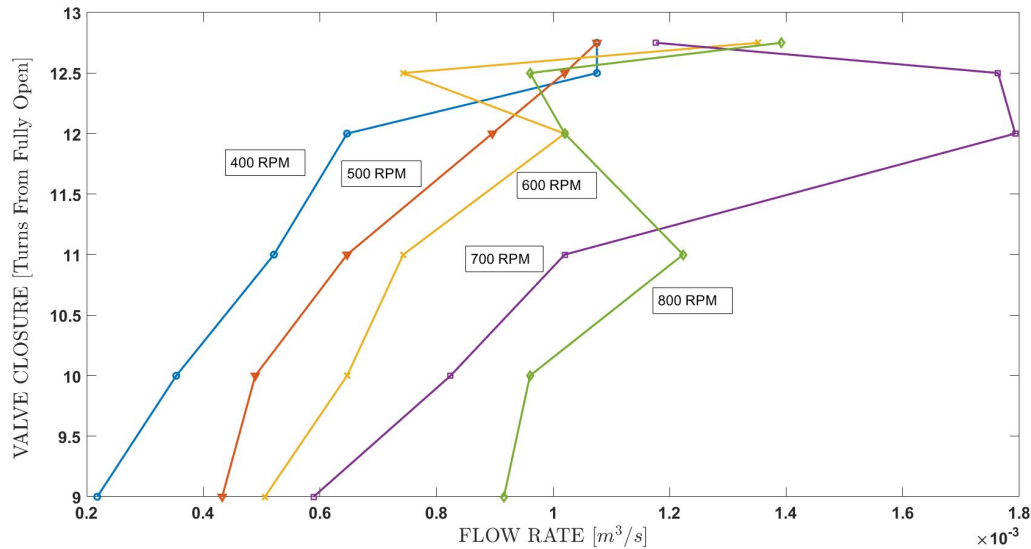


Figure 5.14. Dynamometer outflow rate measured from dynamometer characterisation cycle

fluid outflow and inflow be satisfied. Consequently the amount of fluid mass in the working compartment increased, causing the desired pressure rise.

The accumulation of fluid mass in the working compartment was most likely due to a slight decrease in working compartment outflow, just as the valve closure increased. This led to a build-up of fluid in the working compartment, since the outflow was less than the inflow. The fluid mass accumulated in the working compartment due to this imbalance of flows until the fluid-fill inside the working compartment increased to such an extent that the resulting pressure rise became sufficient to pump water out of the dynamometer working compartment at a rate equal to the inflow rate. This phenomenon could possibly be observed with high resolution data logging of the pressure sensors, unfortunately the test setup was unable to log continuous data from the pressure sensors at a high enough resolution to see minute changes in the outlet pressure.

Fig. 5.14 shows how the outflow rate from the dynamometer increased with operating speed. This was expected, since the vortex pressure increased with higher operating speeds. Increased vortex pressures pumped more fluid from the working compartments.

Fig. 5.14 also shows how the incremental closure of the valve opening leads to slightly higher outflow rates, up until about 12 turns of the valve, from fully open, where the flow rates started to decrease and become erratic. It might seem counter intuitive that the outflow rate would increase with valve closure, but considering the possibility discussed previously that the pressure of the working compartment increased to compensate for the increased back pressure to achieve equal inflow and outflow. The slight increase in outflow rate may be explained by the resulting increase in the working compartment pressure.

The reduction in flow rates when going from 12 valve turns to more, might occur because the almost fully closed condition of the control valve generated such a high back pressure that the amount of pressure rise required from the working compartment to achieve balanced outflow and inflow rates was unachievable due to working compartment geometry constraints. Keeping this in mind, the inflow rate would thus decrease in order to achieve the inflow equal to outflow rate condition. This hypothesis might have been observable if the inflow rate into the dynamometer could also be measured, which was not possible with this specific test bench.

The flow rates generated by the dynamometer was much lower than expected and designed for. The throughflow rates varied from a minimum of 0.2 [l/s] at 400 [rpm] and an almost fully open valve, to a maximum of 1.8 [l/s] at 800 [rpm] and an almost fully closed valve. This was much lower than the desired throughflow rates of 1 [l/s] at a low-fill condition and 4 [l/s] at a high-fill condition in order to achieve adequate heat dissipation of the working compartments.

It became apparent from the experiments that the throughflow rates were not high enough to achieve the required cooling of the dynamometer to allow for long periods of continuous operation. The dynamometer heated up during operation and the water from the working compartments were quite warm after a short time, especially at higher operating speeds. This was due to the throughflow rate not being high enough to allow proper heat dissipation of the dynamometer working compartment. The original design decision that was made to allow for two outflow paths (mentioned in Section 3.4.3) was therefore unsuccessful in achieving a higher throughflow rate.

The external heat exchanger cycle was very effective in reducing the temperature of the water being circulated throughout the cooling cycle, but this had a very small impact on the cooling of the dynamometer itself as not enough water from the dynamometer working compartments could be pumped into the external cooling cycle to allow the dynamometer to be sufficiently cooled.

With the experiment it was confirmed that the throughflow rates related to the constant fill conditions of the dynamometer working compartments. It was seen that the slight increase in throughflow rates, related to the increase in torque of the dynamometer. Therefore, the measuring of flow rates gave an indication of the fluid-fill percentage of the dynamometer.

5.4. Summary

In this chapter various experimental findings from the investigative methodology (in Chapter 4) were discussed. Based on these results it could be seen that the physical dynamometer's torque absorption was 1) within the torque absorption range predicted by the theoretical model, 2) much higher than required by the hydro test facility and lastly 3) that the torque control did not have the desired effect in varying the torque absorption.

The dynamometer throughflow rate was also insufficient to cool the working compartments, regardless of the efficacy of the external cooling cycle. In the next section conclusions based on the experimental results will be made together with some recommendations for improvement of the study or for future studies.

6. Conclusion and recommendations

6.1. Overview

The main purpose of this study was to develop a variable-fill hydraulic dynamometer test bench to be used in the WJ Hydro Testing Facility. In the facility, the test bench is intended to be used to load the model hydraulic turbines and measure the torque output and performance of these turbines in order to verify and validate their designs and the hydraulic turbine governing system (HTGS) employed.

For the test bench to be suitable for use in the hydro testing facility it must have the desired torque absorption capabilities based on the model turbines' expected power outputs. The torque and power requirements were provided by WJ Engineering and based on this the test bench must be able to load the turbine at 300 [Nm] and a corresponding speed of 1200 [rpm], therefore power absorption of at least 38 [kW] should be achievable by the dynamometer. The test bench should also be able to change the torque load during continuous operation to be able to test the envisaged turbine governing system.

The research methodology described in the previous chapters were utilised to develop a variable-fill hydraulic dynamometer test bench and the procedure through which the test bench will be verified and validated in order to achieve the desired outcome of the study. In this chapter the outcomes and results of these methods will be summarised, and then conclusions are made to highlight the benefits and the shortcomings of the research conducted. Lastly recommendations are made, based on the research results and conclusions, for further study and improvements to the overall project.

6.2. Summary of findings

To achieve the outcomes of the study a theoretical model was employed to predict the performance of the dynamometer and from this simulation the predicted power and torque of the dynamometer and matching design parameters of the control valve were obtained. The simulation was used as a first step to verify the suitability of the test bench and then two distinct test cycles were used to characterise the control and the performance of the dynamometer in order to both verify and validate the theoretical model and to determine whether the test bench was suitable to be used in the hydro test facility. A short summary of the findings of the simulation with the theoretical model and the results of the two experimental test cycles are provided in the following sections.

6.2.1. Dynamometer performance simulation

A theoretical model was developed by Raine & Hodgson (1991) that characterises dynamometers through the application of integrated one-dimensional fluid mechanics theory with a vortex flow model. This model was used in this study to simulate the variable-fill hydraulic dynamometer developed and manufactured by WJ Engineering, prior to the current research. The model

was used to predict the torque absorption and power of the dynamometer and could therefore give an indication as to the suitability of the dynamometer for the test bench and provided parameters from which the control valve can be designed. This model was also validated during the study.

Simulation of the dynamometer with the theoretical model produced the expected torque trend curves for these types of dynamometers. The simulation results made clear that the torque absorption of the dynamometer increased exponentially with increased speed and that the amount of torque absorbed will vary in cascading curves based on the fluid-fill percentage of the working compartments.

The torque predicted with the simulation showed that the torque requirement of the dynamometer for use in the hydro testing facility can be achieved with very low fluid fills, and that the power absorption capability of the dynamometer is much higher than required by the hydro testing facility. The simulated expected torque absorption from the dynamometer was shown to produce the desired 300 [Nm] of torque absorption at percentage fills of as low as 30% at the rotational velocity of 1200 [rpm].

6.2.2. Dynamometer torque control

As stated in the previous section the simulation was also employed to find the required outlet valve properties to properly control the dynamometer's torque characteristic. This torque control characteristic would in the future allow turbine performance to be tested for various head and flow conditions that cause the turbine to produce different torques for the same operating speed and also to test the turbine governing system.

The outlet control valve varies the torque absorption of the dynamometer by influencing the fluid-fill in the working compartments. WJ Engineering stated that the control valve should however allow enough fluid throughflow to ensure that the working compartments are properly cooled during operation. The desired throughflow rates were provided to be between 1 [l/s] for low torque conditions and 4 [l/s] for higher torque conditions.

The simulation produced the required pressure loss of the control valve to attain the desired throughflow rates, to be between 39 [kPa] and 45 [kPa] for an almost fully closed valve to achieve a throughflow rate of 1 [l/s] and to be between 5 [kPa] and 15 [kPa] for an almost fully open valve to achieve a throughflow rate of 4 [l/s]. Based upon these specifications the outlet control valve was designed and manufactured.

In order to characterise the outlet control valve to ensure that the design requirements were met, a Venturi and valve characterisation cycle was used (see Fig. 4.23 & Fig. 5.1). This cycle was also used to characterise the Venturi flow meter used to measure the throughflow rate in the dynamometer test bench.

CFD simulation of the valve geometry indicated that the valve will only influence the back pressure on the dynamometer working compartments during a small part of the available plug movement. The valve will therefore only influence the torque control when very near to a fully closed condition.

The results from the Venturi and valve characterisation cycle was very favourable, with the results showing that the control valve delivers a pressure drop of 60,9 [kPa] at a flow rate of 1 [l/s] and a almost fully closed valve, and 7.1 [kPa] at a flow rate of 4 [l/s] and a almost fully open valve. This means that according to the simulation the valve will give the desired pressure drop between 39 [kPa] and 45 [kPa] somewhere between the fully closed and fully open condition to produce the required throughflow rate of 1 [l/s], and that the valve will produce the desired

pressure drop between 5 [kPa] and 15 [kPa] somewhere between the fully closed and fully open condition to produce a required throughflow rate of 4 [l/s]. The expectation that the valve will only control the back pressure during the last part of the valve plug movement was confirmed by the experimental results.

6.2.3. Dynamometer torque and power

The second experimental test bench used in this study is the dynamometer characterisation cycle. This cycle consists of the dynamometer test bench, driven by an electrical motor connected to a Variable Frequency Drive (VFD) instead of a hydro turbine (see Fig. 4.33 and Fig. 5.6). This configuration allowed the dynamometer torque and performance characteristic to be characterised for various operational speeds.

From the dynamometer characterisation cycle, the results showed that the dynamometer test bench produced a minimum torque load of 313.9 [Nm] and a maximum torque load of 372.8 [Nm] at an operating speed of 800 [rpm] (see Fig. 5.11). The tests had to be ceased at operating speeds higher than 800 [rpm], as the torque load started to exceed the limits of the load cell at these speeds and the dynamometer overheated. The maximum power rating of the dynamometer test bench with the current load cell was therefore about 32 [kW].

The dynamometer characterisation results also showed that the control valve has very little effect on the torque absorption especially at the lower operating speeds. The very small range of plug movement that the valve will have an influence on the back pressure of the working compartments also limited the sensitivity of the torque control achievable.

6.2.4. Dynamometer cooling

The fluid in the external heat exchanger cycle of the dynamometer — that is circulated through the radiator in the large low-pressure water tank — exhibited only a slight increase in temperature. The dynamometer itself did however undergo significant warming of metal component parts when operated for an extended time.

The throughflow rates measured by means of the Venturi flow meter, ranged from 0.2 [l/s] and 1.8 [l/s] depending on the valve closure and dynamometer operating speeds. WJ Engineering suggested that the throughflow rates should be between a minimum of 1 [l/s] at low torque conditions and 4 [l/s] at high torque conditions to ensure that there will be proper heat dissipation from the working compartments.

6.3. Conclusion

In this section the study is concluded by discussing the three main outcomes set forth to be achieved in chapter 1 of this study. The three main outcomes of this study were to develop and manufacture a functioning variable-fill hydraulic dynamometer test bench, to verify and validate the theoretical model as a useful analysis and possibly design tool and to verify and validate the test bench in terms of its suitability to be used in the hydro testing facility. In the following sections conclusions are made based on the research results in terms of these three outcomes.

6.3.1. Variable-fill hydraulic dynamometer test bench

A functional variable-fill hydraulic dynamometer test bench was developed during this study. The test bench was able to absorb torque across varying speeds and exhibited the expected torque absorption curves for these types of devices. The test bench was able to exert a load onto the electrical motor and the torque, speed and consequently the power delivered by the motor could be found. This indicates that the test bench can perform the desired functions.

Sufficiently fine torque control by means of an outlet control valve was not achieved. The outlet control valve developed for the test bench had a very small effect on the torque produced, especially at the lower speed ranges. At first, it would seem that the inadequate control of the outlet valve could be contributed to the small range of plug movement for which the valve will have an actual effect on the back pressure of the working compartments. The author suspects that this was not the case, since even extreme closure or opening of the valve did not lead to drastic changes in the torque absorption of the dynamometer. Thus the conclusion was made that *the back pressure on the fluid outflow path alone does not have a substantial effect on the fluid-fill of the working compartments*, but rather that the influence of the fluid inflow, into the working compartments, in combination with the outflow, determines the fluid-fill of the working compartments and therefore the torque characteristic. Since this study only considered the fluid outflow from the working compartments and the influence of changes in the back pressure on the outflow path, it can not be concluded how the combination of the inflow and outflow conditions relates to the torque characteristic of the dynamometer.

The external heat exchanger cycle of the dynamometer was found to be seemingly effective in cooling the water from the dynamometer working compartments, but regardless of the effectiveness of the external cooling cycle the dynamometer overheated during extended periods of operation. This could be contributed to the extremely low throughflow rates achieved. The inability of the fluid throughflow to dissipate the heat generated in the working compartments was thus mainly responsible for the dynamometer overheating.

The inadequate throughflow rates can also possibly be attributed to the influence of the fluid inflow conditions combined with the fluid outflow at the working compartments. For instance if the pressure and flow rate at the inlet of the working compartments increased, the outflow needed to achieve the continuity of inflow vs. outflow at the working compartments must be higher, so by changing the inflow conditions of the dynamometer it is suspected that the throughflow rate of the dynamometer can also be adjusted.

Some major leakages were also present in the existing dynamometer construction. Water leaked from the bearings situated in the rotor and stator housing of the dynamometer. This leakage could presumably be fixed by using gaskets to seal the bearing cups, in future revisions of the dynamometer design.

However regardless of these shortcomings, the dynamometer test bench that was developed had the capability to load a device and measure the resulting torque and power outputs. The use of the dynamometer could therefore be continued by WJ Engineering and employed in tests within the limitations caused by the restricted torque limits, torque control capabilities and dynamometer cooling properties.

6.3.2. One dimensional theoretical model

The one-dimensional theory model utilised to analyse and design the dynamometer test bench had to be verified in this study. The theory was implemented into MATLAB with great success to simulate the dynamometer. Through the simulation it was possible to theoretically

predict the vortex flow properties, such as the vortex boundaries and velocity. By utilising the predicted vortex characteristics, the torque and power characteristics were determined theoretically. The simulation results could also be verified against the results provided by Hodgson to great success.

The actual torque and power characteristic were determined through use of the dynamometer characterisation cycle and compared to the theoretical results obtained from the simulation model. Comparing the results showed that the actual torque produced fell within the range of torques predicted by the theoretical model. The actual results however varied within a very narrow band of the possible torque ranges resulting from the simulation. At 800 [rpm] of the dynamometer rotor the dynamometer produced a torque of between 313.9 [Nm] and 372.8 [Nm] for all the fluid flow ranges achievable, the model predicted that the torque absorption could vary between 48 [Nm] and 3200 [Nm] depending on the percentage fluid fill. This is a much larger range of torques than what the actual dynamometer could produce. This difference in values do not discredit the theoretical model in terms of its capability to accurately predict the dynamometer torque characteristic, but rather illustrates the shortcomings of the torque control method.

Nevertheless, the theoretical model was useful in understanding the behaviour of the dynamometer as the experimental results and the simulated results showed similar torque and power curve trends, albeit with the actual curves being much more linear than the theoretical ones. The higher pressure linked to rotational speed increases, predicted with the simulation, can also be likened to the pressure rise measured from the outflow of the working compartments. The theoretical model was also useful in understanding how the dynamometer properties, such as geometry, materials used, and the working fluid influenced the behaviour of the dynamometer. It is not suggested that the model be used as the sole design tool for hydraulic dynamometers, but instead a supplementary tool to aid in the design.

The actual conditions of the flow in the dynamometer working compartments during operation are still a mystery, for this reason simplifications such as assuming a vortex flow pattern with a constant angular velocity and linearly distributed flow pattern is highly optimistic, since the flow will definitely be exceedingly turbulent, this is true even when including losses due to secondary circulation into the model. The model also assumes that there are distinct boundaries between the fluid and the air inside the working compartments, which is also definitely not the case, since the water will be extremely aerated. The outflow simulation was also done based on the assumption that the flow paths would be completely filled with fluid, which is also likely not what occurs in reality, and this leads to some inaccuracy when predicting the dynamometer performance and properties.

The model as was used in this study, for steady state operation, was also not sufficient in producing the parameters by which a control valve must be designed. This is partly because the model was used in this study without the inclusion of the influence that the inflow has on the working compartments. The overall effect that the inflow in combination with the change in the back pressure on the working compartments have on the vortex or flow conditions was not considered, which was found to likely be an important consideration. Thus it is concluded that it can be beneficial if the dynamometer inflow is also included in the analysis.

In the computer simulation used by Raine and Hodgson (1991) inclusion was made for the influence of dynamic operation and inflow vs. outflow mapping, this was not done for the simulation of this study, since the effect of the dynamic operation on the torque control and performance was assumed to be minimal. The dynamic simulation could however also prove valuable in finding the desired valve characteristics and the steady state torque properties for different valve settings.

6.3.3. Suitability of test bench for use in the hydro testing facility

In the end the main desire of WJ Engineering was to have a functioning variable-fill hydraulic dynamometer test bench that is capable of loading the turbine models tested in the hydro testing facility, according to their specific designed torque and power outputs and to allow validation of the turbines governing system.

The dynamometer had set requirements to be met for it to be useful in determining the performance characteristic of the turbine models and the effectiveness of the turbine governing system. These requirements were that the dynamometer test bench must be able to load the turbines with a torque of at least 300 [Nm] at an operating speed of 1200 [rpm], or to absorb a total of 38 [kW] from the hydraulic turbines. The dynamometer was also specified to have the ability to vary the torque during continuous operation and that the torque curves must not be so steep that small changes in the speed will produce an excessive response from the governing system or lead to runaway turbine speeds or stalling. The operation of the dynamometer in the hydro testing facility should also not lead to the self-emptying phenomena explained in Chapter 2. The test bench could still be used for limited tests, since the model turbines are not anticipated to deliver more than 20 [kW].

Considering that the dynamometer already produced a torque of higher than 300 [Nm] at rotational speeds as low as 800 [rpm], it was clear that the dynamometer torque was in excess of the torque requirements for the hydraulic turbine model tests planned. Seeing that the limit of the load-cell used was already surpassed at operational speeds higher than 800 [rpm], the dynamometer will also never reach the operational speed desired for turbine testing when connected directly to the turbines being tested. The maximum power capacity of the dynamometer with the current load cell limitation was also at about 32 [kW]. For these reasons it can be concluded that the dynamometer test bench was not suitable in its current version for use in the hydro testing facility.

This can however be remedied by using some sort of belt drive, sprocket or gear system that connects the turbine shaft to the dynamometer, that would allow for a speed reduction from the turbine to the dynamometer which can lead to torque loads on the turbine that are more in line with the requirements. But in such a case the load-cell must also be replaced by one with a higher load capacity as the torque requirement from the dynamometer will also increase with such a speed reduction system. This will certainly make the test bench unnecessarily more complex, but v-belt systems and so forth can be implemented rather easily with little extra costs, losses or implications.

The actual dynamometer torque characteristic curves were very steep, this could also lead to unfavourable consequences when testing the performance of the turbines. With such steep torque curves that led to large variations in the torque absorbed for small changes in speed, it can cause unstable testing conditions. A highly sensitive dynamometer test bench such as the one developed in this study can lead to what is referred to as “hunting” of the turbine governor, where the speed constantly fluctuates during turbine loading as the turbine governing system struggles to find an equilibrium. But since there was not yet a governing system in operation it can not be accepted that the current dynamometer is too sensitive for hydraulic turbine tests.

Runaway and stalling of the turbine when loaded by the dynamometer will mostly depend on the self-emptying phenomenon of the dynamometer as explained in Section 2.4. This occurs when the turbine loaded produces a torque and speed characteristic that falls outside the operating envelope of the dynamometer and it leads to runaway speeds or turbine stalling. The self-emptying phenomenon could however not be tested during this study as the dynamometer speed required for self-emptying to occur could never be reached due to the load-cell and cooling

restrictions. But the loads delivered by the dynamometer was so high before self-emptying could be observed that it can safely be concluded that the model turbines envisaged for use in the hydro testing facility would not likely fall outside the operating envelope of the dynamometer.

It was found that the dynamometer overheats after a short period of operation. This will limit the duration of any turbine model tests, but if tests can be done periodically within the limited time frames the dynamometer can be used in the hydro testing facility if time is made available for the dynamometer to cool.

The last question to answer in determining whether the test bench is suitable to be used in the hydro testing facility, is to distinguish if the control valve varied the torque of the dynamometer enough to adjust the absorption to suit different turbine power outputs and for load changing during continuous operation to test the governor control. The simulation illustrated how the changes in fluid-fill can vary the torque characteristic of the dynamometer by large amounts, changes such as those seen in the simulation are desirable for the testing cycle since it would allow for adequate testing of a wide variety of turbine power outputs and instantaneous changes in load specifically for governor tests. The actual experimentation however showed that the control valve was not able to achieve such ample changes in the torque characteristic, and that only very small load variances were possible. It can hence be concluded that the test bench does not adequately control the torque characteristic to allow for proper turbine model tests, but nevertheless it can still be possible to do tests on the functioning of the governor as the control valve is capable of varying the torque by some amount.

To conclude whether this outcome has been met: the dynamometer test bench may well be used in the hydro testing facility to load model turbines for performance tests, *but subjected to certain limitations and adjustments*. By implementing a speed reduction method such as v-belts, gears or sprockets between the turbine under test and the dynamometer and the replacement of the load cell with one of a higher load capacity the desired torque vs. speed and power absorption can be achieved. The governing system must however be sensitive enough to prevent hunting or constant speed fluctuations and the test can not be done across a wide variety of load changes. With the dynamometer test bench as it is, the tests will also be severely limited by the cooling capacity. The inclusion of a method to control the inflow into the dynamometer working compartments, might possibly also lead to more favourable dynamometer characteristics. But nonetheless the test bench can still be utilised to give some indication of the performance of the model turbines being tested, but in order to do proper tests according to the requirements set forth, adjustments must be made to the current test bench or an entirely new test bench should be developed.

6.4. Recommendations for further study

From the experimental results it is suggested that the fluid inflow conditions into the dynamometer working compartments have just as significant an effect on the fluid-fill of the dynamometer as the fluid outflow conditions out of the working compartments. For this reason, it would be advantageous to also study the fluid inflow conditions and the effect that it has on the dynamometer performance.

The influence of the fluid inflow conditions can be studied by means of measuring the fluid inflow rate and pressure, as well as adjusting the fluid inflow by means of a speed controlled feed pump or a inlet valve during operation of the dynamometer. The results can then be used in conjunction with the fluid outflow data to find correlations. The study of the fluid inflow can also provide insight into its influence on the cooling of the dynamometer and the cooling of the dynamometer working compartments themselves can also be studied.

Using the dynamic simulation of the model with the inclusion of inflow vs. outflow mapping in the simulation program can also help to better simulate the dynamometer in order to find the correct valve characteristics required for certain fluid-fill amounts and throughflow rates.

If it is possible, it would be extremely gainful and insightful to study the flow conditions inside the working compartments of the dynamometer to better understand the type of flow present and to determine if a vortex flow pattern is a meaningful assumption to make. Advanced CFD simulation techniques may need to be utilized to achieve this specific goal.

Additional studies can also be done on the outlet control valve, to find a suitable way to design an appropriate control valve, and to include an open loop feedback control system to maintain satisfactory torque control during operation, especially with the inclusion of a servo controlled inlet and outlet valve system.

A more detailed investigation into the heat dissipation mechanisms affecting the fluid inside the working compartments can also be useful in ensuring that the dynamometer working compartments are designed to facilitate the proper amounts of fluid throughflow and heat transfer to achieve adequate cooling of the dynamometer during operation.

6.5. Summary of contributions

The outcome of this study delivered valuable insights into the functioning of a variable-fill hydraulic dynamometer. With these insights, subsequent designs of new variable-fill hydraulic dynamometers can be done more effectively.

The study could not conclusively prove that the one-dimensional theoretical model is a well suited method for the design and analysis of the specific variable-fill hydraulic dynamometer developed at WJ Engineering, this is mainly due to the lack of proper torque control design resulting from the model. The experimental result did however fall within the range of the simulated predicted results, so the model is helpful in the design of a suitable variable-fill hydraulic dynamometer. The theoretical model however also proved valuable in understanding the effect that various parameters have on the functioning of the dynamometer.

The study lead to the development of a functioning variable-fill hydraulic dynamometer test bench that can be used to load devices in order to find the torque and power outputs thereof. Although the dynamometer test-bench lacks the ability to finely control the torque characteristic and it overheats when used for a long period of time, it is still considered to be usable for performance tests when considering the limitations.

7. REFERENCES

Anderson, S. 1982. On Hydrodynamic Torque Converters. Lund Sweden: Technical University. (Dissertation – PhD).

Arter, A. & Meier, U. 1990. Chapter 3: Performance tests on cross flow turbines. In: Arter, A. & Meier, U. eds. *Hydraulics Engineering Manual*. St. Gallen Switzerland: pp. 66-72.

AVL Zöllner Marine. 2018. Hydraulic dynamometers for non-automotive applications. https://www.avl.com/documents/10138/2699442/090619_Folder_ZoellnerMarine_v4_ES.PDF.pdf Date of access: 15 Sep. 2018.

Boyes, W.H. 2007. Instruments. In: Avallone, E.A., Baumeister, T., & Sadegh A., eds. *Marks standard handbook for mechanical engineers*. New York: McGraw-Hill Education. pp. 1-49.

BSI (British Standard Institution). 1964. *Methods for the measurement of fluid flow in pipes: Part 1, orifice plates, nozzles and venturi tubes*. London: BSI. (BSI 1042)

Draughan, J.M. & Van Voorhies, R.L. 1962. *Dynamometer and Process of Using Same*. (Patent: US 3050993A). <https://patents.google.com/patent/US3050993A/en> Date of Access: 26 Jun 2019.

FloEFD Technical Reference, 2018, Mentor Graphics Corporation, Oregon. https://s3.amazonaws.com/s3.mentor.com/public_documents/datasheet/products/mechanical/products/floefd.pdf Date of Access: 25 August 2018.

Fox, R.W. & McDonald, A.T. 2011. *Introduction to Fluid Mechanics*. 8th ed. Hoboken: John Wiley & Sons.

Froude, W. 1877. On a new dynamometer for measuring the power delivered to the screws of large ships. *Proceeding of the Institution of Mechanical Engineers*, 28:237-275.

Gatiss, A.L. 1971. Torque Converters and Hydraulic Couplings. In: Bell, P.C. ed. *Mechanical Power Transmission*. London: Palgrave Macmillan. Available from Springer eBook Collection: https://link.springer.com/chapter/10.1007%2F978-1-349-01197-1_6 Date of access: 3 Dec. 2018.

Guo, P., Wang, Z., Sun, L. & Luo, X. 2017. Characteristic analysis of the efficiency hill chart of Francis turbine for different water heads. *Advances in Mechanical Engineering*, 9(2): 1-8.

Grefstad, M. 2017. *Development of a Francis Turbine Test Rig at Kathmandu University*. Trondheim, Norway: University of Science and Technology. (Thesis – Masters).

Hodgson, P.G. 1991. Theoretical model and dynamic simulation of variable fill hydraulic dynamometers. Christchurch: University of Canterbury. (Thesis-PhD).

Hodgson, P.G., & Raine, J.K. 1991. Computer simulation of a variable fill hydraulic dynamometer Part 1: torque absorption theory and the influence of working compartment geometry on performance. *Journal of Mechanical Engineering Science*, 205(3): 155-163.

Hodgson, P.G., & Raine, J.K. 1992. Computer simulation of a variable fill hydraulic dynamometer Part 2: steady state and dynamic open-loop performance. *Journal of Mechanical Engineering Science*, 206(1): 49-56.

Hodgson, P.G., & Raine, J.K. 1992. Computer simulation of a variable fill hydraulic dynamometer Part 3: closed-loop performance. *Journal of Mechanical Engineering Science*, 206(5): 327-336.

Horiba. 2006. Hydraulic dynamometer type DT: Technical specification. Darmstadt: Horiba automotive test systems.

Ishihara, T., Furuya, S. & Mori, K. 1968. Characteristics of Fluid Couplings. *Bulletin of JSME*, 11(45): 496-508.

Kadhim, N.S. 2013. Experimental investigation of three various sources of diesel fuel on the engine performance. *Journal of Babylon University*, 8(21): 2955–2964.

Kavurmaci, B., Akin, H., Ayli, E., Celebioglu, K. & Aradag, S. 2013. Design of an experimental test stand for Francis type hydraulic turbines. Paper presented at the 4th International Conference on Power Engineering, Energy and Electrical Drives, Istanbul, 13-17 May. https://www.researchgate.net/profile/EceAyli/publication/260225028_Design_of_an_Experimental_Test_Stand_for_Francis_Type_Hydraulic_Turbines/links/0deec5303414c3a27e000000.pdf Date of access: 1 Aug. 2019.

Kavurmaci, B., Celebioglu, K., Aradag, S. & Tascioglu, Y. 2017. Model testing of Francis-type hydraulic turbines. *Measurement and control*, 50(3): 70-73.

Liu, D. & Van Wachem, B. 2019. Comprehensive assessment of the accuracy of CFD-DEM simulations of bubbling fluidized beds. *Powder Technology*, 343(1): 145-158.

Leithold, L. 1981. *The Calculus with Analytic Geometry*. 4th ed. Reston: National Council of Teachers of Mathematics.

Martyr, A.J. & Plint, M.A. 2007. *Engine testing*. 3rd ed. Oxford: Elsevier.

Massey, B.S. 1979. *Mechanics of Fluids*. 4th ed. New York: Elsevier.

MATLAB 2017b, ver.(9.3.0.713579), The MathWorks, Inc., Natick, Massachusetts, United States.

Meriam, J. L. 1975. *Dynamics*. 2nd ed. Hoboken: John Wiley & Sons.

Munson, B.R., Okiishi, T.H., Hiebsch, W.W. & Rothmayer, A.P. 2013. *Fluid mechanics*. 7th ed. Singapore: John Wiley & Sons.

Murenzi, J.P. & Ustun, T.S. The case for microgrids in electrifying Sub-Saharan Africa, Paper delivered at the Sixth International Renewable Energy Congress, Sousse. <https://ieeexplore.ieee.org/abstract/document/7281111> Date of access: 15 Mar. 2018.

Narayan Rao, N. 1968. The basic theory of hydraulic dynamometers and retarders. <https://saemobilus.sae.org/content/680178> Date of access: 18 Nov. 2018.

Numazawa, A., Ushijima, F., Fukumura, K. & Ishihara, T. 1983. An experimental analysis of fluid flow in a torque converter. Paper delivered at the SAE Technical Papers, International Congress and Exposition, Tokyo. <https://doi.org/10.4271/830571> Date of access: 14 Feb 2019.

Passenbrunner, T.E., Sassano, M., Trogmann, H., del Re, L., Paulweber, M., Schmidt, M. & Kokal, H. 2011. Inverse torque control of hydrodynamic dynamometers for combustion engine test benches. Paper delivered at the Proceedings of the 2011 American Control Conference, San Francisco. <https://folk.ntnu.no/skoge/prost/proceedings/acc11/data/papers/0834.pdf> Date of access: 3 Feb. 2019.

Passenbrunner, T.E., Sassano, M. & del Re, L. 2013. Optimal control of internal combustion engine test benches equipped with hydrodynamic dynamometers. Paper delivered at the 7th IFAC symposium on advances on automotive control, Tokyo. <https://www.sciencedirect.com/science/article/pii/S1474667016384348> Date of access: 15 Jan. 2019.

Raine, J.K. 1977. General theory and computer simulation for F type dynamometer performance: technical note 168. London: Froude Eng. Ltd.

Rakopoulos, C.D. & Giakoumis, E.G. 2009. *Diesel Engine Transient Operation: Principles of Operation and Simulation*. 1st ed. Athens Greece: Springer.

Rapid Allweiler. 2005. CLT 40-250 pump curve. http://www.rapidallweiler.co.za/curves/CLT_CLB%20Curves_8.pdf Date of access: 10 Oct. 2018.

Sayers, A.T. 1990. *Hydraulic and compressible flow turbomachines*. London: McGraw-Hill.

Sharp, Z.B., Johnson, M.C., Barfuss, S.L. and Rahmeyer, W.J., 2010. Energy losses in cross junctions. *Journal of Hydraulic Engineering*, 136(1):50-55.

Sivalingam, R. 1978. *Flow modelling and computer aided design of fluid couplings and torque converters*. Scotland: University of Bath. (Thesis – PhD).

Steele, R.D. 2007. Hydraulic Turbines. In: Avallone, E.A., Baumeister, T., & Sadegh A., eds. Marks standard handbook for mechanical engineers. 11th ed. New York: McGraw-Hill Education. pp. 1-30.

Tan, K.C. 1980. *Identification of a Hydraulic Dynamometer*. England: University of Manchester. (Thesis – M.Sc.)

Tavoularis, S. 2005. *Measurement in fluid mechanics*. Cambridge: Cambridge University Press.

Thapar, O.D. 2002. Modern hydroelectric engineering practice in India electro-mechanical works, Chapter 5: Turbine performance characteristics. https://www.iitr.ac.in/departments/HRE/uploads/modern_hydroelectric_engg/vol_1/Chapter-5_Turbine_Performance_Characteristics.pdf Date of access: 31 Aug. 2019.

Vetr, M., Passenbrunner, H., Trogmann, H., Ortner, P., Kokal, H., Schmidt, M. & Paulweber, M. 2010. Control oriented modelling of a water brake dynamometer. Paper delivered at the 2010 IEEE International Conference on Control Applications Part of 2010 IEEE Multi-Conference on Systems and Control, Yokohama. <https://ieeexplore.ieee.org/document/5611137/authors#authors> Date of access, 10 Aug. 2019.

Voith Hydrodynamics Pamphlet. 2015. <https://d2euiryrvxi8z1.cloudfront.net/asset/445934742530/d810385a0e1d0df78e04218fac8441f3> Date of access June 2019.

Ward-Smith, A.J. 1980. *Internal Fluid Flow*. Oxford: Clarendon Press.

Whitfield, A., Sivalingam, R. & Wallace, F.J. 1978. The performance prediction of fluid coupling with the introduction of a base plate. *International Journal of Mechanical Sciences*, 20(10):729-736.

Zhang, H., Chen, D., Xu, B. & Wang, F. 2015. Nonlinear modelling and dynamic analysis of hydro turbine governing system in the process of load rejection transient. *Energy Conversion and Management*, 90(1): 128-137.

A. Identities for cup boundary geometric functions

In Fig. A.1 which is the same as Fig. 3.11 used in Chapter 3, the functions $f(x)$ and $g(x)$ are illustrated. These functions define the cup boundary of the control volume and are used to develop the identities used:

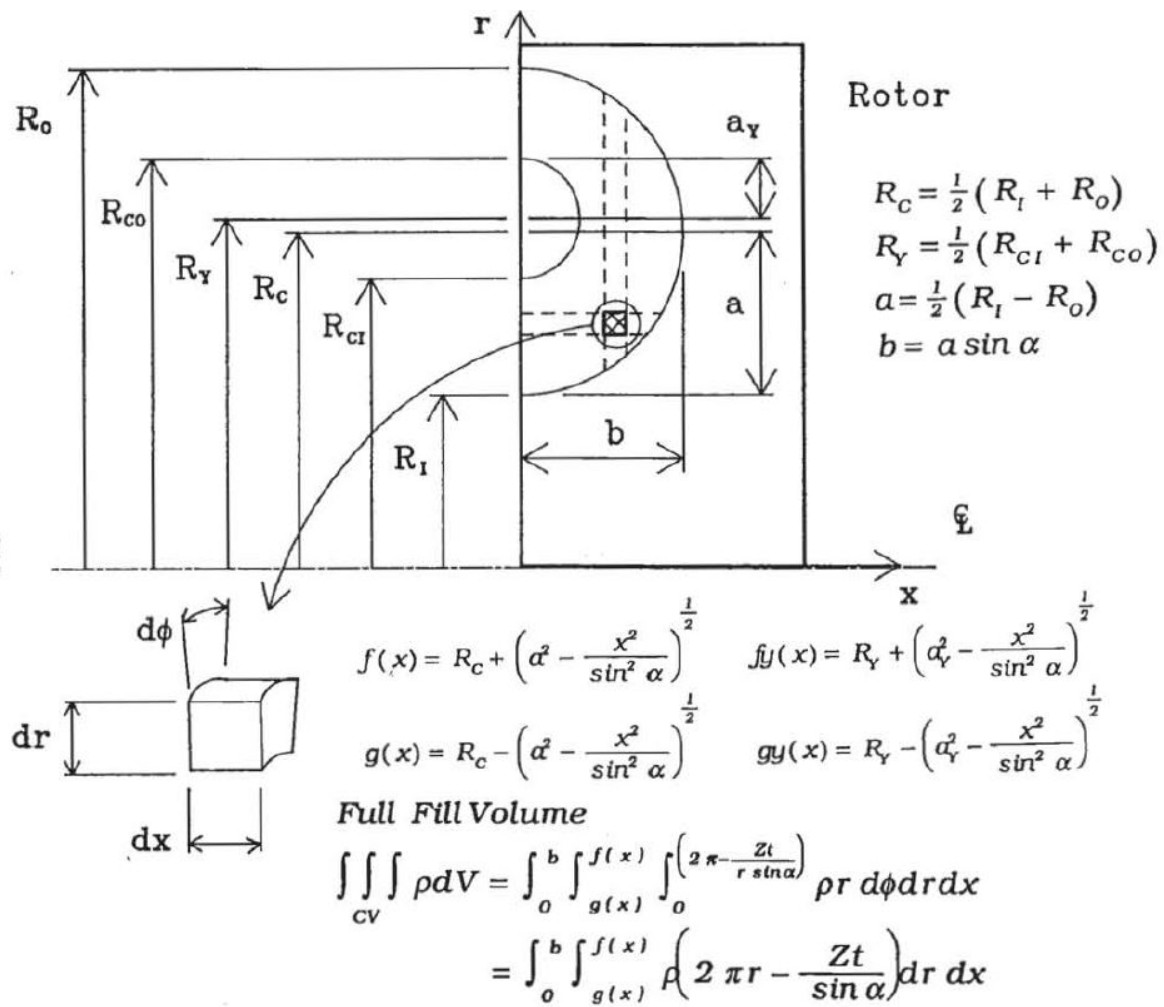


Figure A.1. Mass Integral in Rotor Control Volume (Raine & Hodgson, 1991:29)

Outer Cup Boundary

$$\begin{aligned}
 f &= f(x) = R_C + \left(a^2 - \frac{x^2}{\sin^2 \alpha} \right)^2 \\
 f^2 &= (f(x))^2 = R_C^2 + 2 R_C \left(a^2 - \frac{x^2}{\sin^2 \alpha} \right)^{\frac{1}{2}} + \left(a^2 - \frac{x^2}{\sin^2 \alpha} \right) \\
 f^3 &= (f(x))^3 = R_C^3 + 3 R_C^2 \left(a^2 - \frac{x^2}{\sin^2 \alpha} \right)^{\frac{1}{2}} \\
 &\quad + 3 R_C \left(a^2 - \frac{x^2}{\sin^2 \alpha} \right) + \left(a^2 - \frac{x^2}{\sin^2 \alpha} \right)^{\frac{3}{2}} \\
 f^4 &= (f(x))^4 = R_C^4 + 4 R_C^3 \left(a^2 - \frac{x^2}{\sin^2 \alpha} \right)^{\frac{1}{2}} + 6 R_C^2 \left(a^2 - \frac{x^2}{\sin^2 \alpha} \right) \\
 &\quad + 4 R_C \left(a^2 - \frac{x^2}{\sin^2 \alpha} \right)^{\frac{3}{2}} + \left(a^2 - \frac{x^2}{\sin^2 \alpha} \right)^2
 \end{aligned}$$

Inner Cup Boundary

$$\begin{aligned}
 g &= g(x) = R_C - \left(a^2 - \frac{x^2}{\sin^2 \alpha} \right)^2 \\
 g^2 &= (g(x))^2 = R_C^2 - R_C \left(a^2 - \frac{x^2}{\sin^2 \alpha} \right)^{\frac{1}{2}} + \left(a^2 - \frac{x^2}{\sin^2 \alpha} \right) \\
 g^3 &= (g(x))^3 = R_C^3 - 3 R_C^2 \left(a^2 - \frac{x^2}{\sin^2 \alpha} \right)^{\frac{1}{2}} \\
 &\quad + 3 R_C \left(a^2 - \frac{x^2}{\sin^2 \alpha} \right) - \left(a^2 - \frac{x^2}{\sin^2 \alpha} \right)^{\frac{3}{2}} \\
 g^4 &= (g(x))^4 = R_C^4 - 4 R_C^3 \left(a^2 - \frac{x^2}{\sin^2 \alpha} \right)^{\frac{1}{2}} + 6 R_C^2 \left(a^2 - \frac{x^2}{\sin^2 \alpha} \right) \\
 &\quad - 4 R_C \left(a^2 - \frac{x^2}{\sin^2 \alpha} \right)^{\frac{3}{2}} + \left(a^2 - \frac{x^2}{\sin^2 \alpha} \right)^2
 \end{aligned}$$

Difference between Outer and Inner Cup Boundaries

Now by taking the difference between the outer and inner cup boundaries, we get:

$$\begin{aligned}(f - g) &= 2 \left(a^2 - \frac{x^2}{\sin^2 \alpha} \right)^{\frac{1}{2}} \\(f^2 - g^2) &= 4 R_C \left(a^2 - \frac{x^2}{\sin^2 \alpha} \right)^{\frac{1}{2}} \\(f^3 - g^3) &= 6 R_C^2 \left(a^2 - \frac{x^2}{\sin^2 \alpha} \right)^{\frac{1}{2}} + 2 \left(a^2 - \frac{x^2}{\sin^2 \alpha} \right)^{\frac{3}{2}} \\(f^4 - g^4) &= 8 R_C^3 \left(a^2 - \frac{x^2}{\sin^2 \alpha} \right)^{\frac{1}{2}} + 8 R_C \left(a^2 - \frac{x^2}{\sin^2 \alpha} \right)^{\frac{3}{2}}\end{aligned}$$

B. Definite integrals

The following definite integral identities are obtained from Leithold (1981):

$$\begin{aligned}\int (a^2 - u^2)^{\frac{-1}{2}} du &= \sin^{-1} \left(\frac{u}{a} \right) + C \\ \int (a^2 - u^2)^{\frac{1}{2}} du &= \frac{1}{2} u (a^2 - u^2)^{\frac{1}{2}} + \frac{1}{2} a^2 \sin^{-1} \left(\frac{u}{a} \right) + C \\ \int (a^2 - u^2)^{\frac{3}{2}} du &= \frac{-1}{8} u (2u^2 - 5a^2) (a^2 - u^2)^{\frac{1}{2}} + \frac{3}{8} a^4 \sin^{-1} \left(\frac{u}{a} \right) + C\end{aligned}$$

These identities are used to evaluate the following expressions.

$$\begin{aligned}\int_0^b \left(a^2 - \frac{x^2}{\sin^2 \alpha} \right)^{\frac{-1}{2}} dx &= \frac{1}{2} \pi \sin \alpha \\ \int_0^b \left(a^2 - \frac{x^2}{\sin^2 \alpha} \right)^{\frac{1}{2}} dx &= \frac{1}{4} \pi ba \\ \int_0^b \left(a^2 - \frac{x^2}{\sin^2 \alpha} \right)^{\frac{1}{2}} dx &= \frac{3}{16} \pi ba^3\end{aligned}$$

Other definite integrals evaluated are

$$\begin{aligned}\int_0^b 1 dx &= b \\ \int_0^b \left(a^2 - \frac{x^2}{\sin^2 \alpha} \right) dx &= \frac{2}{3} ba^2 \\ \int_0^b \left(a^2 - \frac{x^2}{\sin^2 \alpha} \right)^2 dx &= \frac{8}{15} ba^4\end{aligned}$$

C. Air-water interface calculation in MATLAB

Based on the theory for the calculation of the air-water interface boundaries presented in Section 3.4.1

```
function [fval,J] = System(X)

%Define variables

Rco = X(1);
Rci = X(2);

Ri = 0.061;
Ro = 0.144;
a = 90;
Z = 12;
t = 0.005;
PF = 0.25;

%Find Rm

Rm = ((2/3)*pi*sin(a)*(Ro.^3-Ri.^3)-(1/2)*Z*t*(Ro.^2-Ri.^2))/(pi*sin(a)
*(Ro.^2-Ri.^2)-Z*t*(Ro-Ri));

%Define f(x)

fval(1,1) = (2/3)*pi*((Ro.^3-Rco.^3)*sin(a)+(Rci.^3-Ri.^3)*sin(a))-pi*Rm
*((Ro.^2-Rco.^2)*sin(a)+(Rci.^2-Ri.^2)*sin(a))-(1/2)*Z*t*(Ro.^2-Rco.^2
+Rci.^2-Ri.^2)+Z*t*Rm*(Ro-Rco+Rci-Ri);

fval(2,1) = (Rco-Rci).^2*(pi*(Rco+Rci)-(Z*t)/sin(a))-(1-PF)*(Ro-Ri).^2
*(pi*(Ro+Ri)-(Z*t)/sin(a));

%Define Jacobian

J = [Rco*(2*pi*Rm*sin(a)+Z*t)-2*pi*sin(a)*Rco.^2-Z*t*Rm ,
      2*pi*sin(a)*Rci.^2-Rci*(2*pi*Rm*sin(a)+Z*t)+Z*t*Rm;
      2*(Rco-Rci)*(pi*(Rco+Rci)-(Z*t)/sin(a))+pi*(Rco-Rci).^2 ,
      -2*(Rco-Rci)*(pi*(Rco+Rci)-(Z*t)/sin(a))+pi*(Rco-Rci).^2];

%Initial conditions

X0 = [0.13;0.065];
maxIter = 200;
tolX = 1e-4;
```

```
%Compute NR

X = X0;
Xold = X0;

for i = 1:maxIter
    [f,J] = System(X);
    X = X - inv(J)*f;
    err(:,i) = abs(X-Xold);
    Xold=X;
    if (err(:,i)<tolX)
        break;
    end
end
```

D. Steady state torque calculation in MATLAB

Based on the theory to calculate the torque and power absorption of the dynamometer for steady state conditions, presented in Section 3.2.3. With the inclusion of the determination of the vortex centre (Section 3.2.1), the fluid angular vortex velocity (Section 3.2.2) and the energy that is dissipated due to the incidence losses (Section 3.3.1), the frictional losses (Section 3.3.2) and the secondary circulation losses (Section 3.3.3). The vortex_calc_actual code was used iteratively in conjunction with the angle_calc_actual code to find the vortex velocity, the incidence angles, the resulting Reynolds numbers and corresponding friction factors and lastly the torque and power for different fluid fills, as shown in Fig. 4.1.

vortex_calc_actual

```
omega_p_rpm = linspace(1,2000,2000); %Rotor speed for which we are interested in

vel = W;

[W,P,Re,f] = vortex_V(omega_p_rpm,vel); %Vortex velocity in [rad/s]

g = W/(omega_p_rpm*(2*pi/60));

%Froude F020 Data
% y_100 = 7.264*10^-9*omega_p_rpm.^3+0.0001837*omega_p_rpm.^2-0.1007*omega_p_rpm+3.498;
% y_60 = 1.754*10^-8*omega_p_rpm.^3+5.868*10^-5*omega_p_rpm.^2-0.02425*omega_p_rpm+0.077;
% y_40 = 1.623*10^-8*omega_p_rpm.^3+1.752*10^-5*omega_p_rpm.^2-0.003669*omega_p_rpm+1.387;
% y_20 = 1.218*10^-8*omega_p_rpm.^3-1.777*10^-5*omega_p_rpm.^2+0.02483*omega_p_rpm-3.965;
% y_10 = -6.38*10^-10*omega_p_rpm.^3+2.832*10^-5*omega_p_rpm.^2-0.04841*omega_p_rpm+14.52;
% y_6 = -1.191*10^-10*omega_p_rpm.^3+1.411*10^-05*omega_p_rpm.^2-0.02744*omega_p_rpm+10.08;
% y_2 = 1.508*10^-10*omega_p_rpm.^3+6.127*10^-7*omega_p_rpm.^2+0.001911*omega_p_rpm-0.8757;

%WJ Dyna Results
% y_10_wj = 1.55*10^-6*omega_p_rpm.^3 - 0.00287*omega_p_rpm.^2 +
2.236*omega_p_rpm - 431.2;
% y_11_wj = 1.675*10.^-6*omega_p_rpm.^3 - 0.003095*omega_p_rpm.^2 +
2.369*omega_p_rpm - 456.8;
% y_12_wj = 2.867*10.^-6*omega_p_rpm.^3 - 0.005073*omega_p_rpm.^2 +
3.43*omega_p_rpm - 638.7;
% y_125_wj = 9.583*10.^-7*omega_p_rpm.^3 - 0.001676*omega_p_rpm.^2 +
1.538*omega_p_rpm - 301.8;
% y_1275_wj = 1.5*10.^-6*omega_p_rpm.^3 - 0.002518*omega_p_rpm.^2 +
2.005*omega_p_rpm -388;

% plot(omega_p_rpm,T);hold on;
% xlabel ('RPM');
% ylabel ('Nm');

% plot(omega_p_rpm,y_10_wj);hold on;
```

```
% xlabel ('RPM');
% ylabel ('Nm');
%
% plot(omega_p_rpm,y_11_wj);hold on;
% xlabel ('RPM');
% ylabel ('Nm');
%
% plot(omega_p_rpm,y_12_wj);hold on;
% xlabel ('RPM');
% ylabel ('Nm');
%
% plot(omega_p_rpm,y_125_wj);hold on;
% xlabel ('RPM');
% ylabel ('Nm');
%
% plot(omega_p_rpm,y_1275_wj);hold on;
% xlabel ('RPM');
% ylabel ('Nm');

% plot(omega_p_rpm,y_100);hold on;
% xlabel ('RPM');
% ylabel ('kW');
%
% plot(omega_p_rpm,y_60);hold on;
% xlabel ('RPM');
% ylabel ('kW');
%
% plot(omega_p_rpm,y_40);hold on;
% xlabel ('RPM');
% ylabel ('kW');
%
% plot(omega_p_rpm,y_20);hold on;
% xlabel ('RPM');
% ylabel ('kW');
%
% plot(omega_p_rpm,y_10);hold on;
% xlabel ('RPM');
% ylabel ('kW');
%
% plot(omega_p_rpm,y_6);hold on;
% xlabel ('RPM');
% ylabel ('kW');
%
% plot(omega_p_rpm,y_2);hold on;
% xlabel ('RPM');
% ylabel ('kW');

% plot(omega_p_rpm,Re);hold on;
% xlabel ('RPM');
% ylabel ('Re [rad/s]');
```

```
% plot(Re,f);hold on;
% xlabel ('Re');
% ylabel ('friction factor');

%Plot vortex velocity
% plot(omega_p_rpm,Vel);hold on;
% xlabel ('MACHINE ROTATION [RPM]');
% ylabel ('VORTEX VELOCITY [rad/s]');

%Plot dyna torque
% plot(omega_p_rpm,T);hold on;
% xlabel ('SPEED [RPM]');
% ylabel ('TORQUE [Nm]');

%Plot dyna power
plot(omega_p_rpm,P);hold on;
xlabel ('RPM');
ylabel ('POWER [kW]');

%Now to start the function for vortex velocity calculation
%Output must be changed from P to T for torque
function [omega,P,Re,f] = vortex_V(omega_p_rpm,vel)

%Inputs

Ri = 0.061; %[m]
Ro = 0.144; %[m]
Z = 12;
t = 0.005; %[m]
Rci = 0.064; %Determined by the NR method and depends on the fill factor
Rco = 0.1424; %Determined by the NR method and depends on the fill factor
% PF = 0.1; %This value is inserted just to keep track of the PF used
rho = 998.2;
a_SO = 90*pi/180 ; %Stator vane angle [rad]
a_RO = 90*pi/180 ; %Rotor vane angle [rad]
a_RI = 60.35204912*pi/180 ; %Rotor inlet angle [rad], this value adjust until
convergence based on the angle values calculated
a_SI = 60.35204912*pi/180 ; %Stator inlet angle [rad], this value adjusts
until convergence based on the angle values calculated

%To find vortex centre
%Rm = ((2/3).*pi.*sin(a_SO).*(Ro.^3 - Ri.^3) - (1/2).*Z.*t.*(Ro.^2 -
Ri.^2))/(pi.*sin(a_SO).*(Ro.^2 - Ri.^2) - Z.*t.*(Ro - Ri));
Rm = 0.1088; %get this value from newton_calc if vanes are angled, not 90 deg

%Steady state momentum coefficients
K1 = rho*sin(a_RO)*(((2*pi*((1/5)*Ro.^5 - (1/4)*Ro.^4*Rm) -
(Z*t/sin(a_RO))*((1/4)*Ro.^4 - (1/3)*Ro.^3*Rm))) - ((2*pi*((1/5)*Rco.^5 -
(1/4)*Rco.^4*Rm) -
```



```

(Z*t/sin(a_R0))*((1/4)*Rco.^4 - (1/3)*Rco.^3*Rm)))) ;
K2 = rho*sin(a_R0)*cos(a_R0)*((2*pi*((1/5)*Ro.^5 - (1/2)*Ro.^4*Rm +
(1/3)*Ro.^3*Rm.^2) - (Z*t/sin(a_R0))*((1/4)*Ro.^4 - (2/3)*Ro.^3*Rm +
(1/2)*Ro.^2*Rm.^2)) -
(2*pi*((1/5)*Rco.^5 - (1/2)*Rco.^4*Rm + (1/3)*Rco.^3*Rm.^2) -
(Z*t/sin(a_R0))*((1/4)*Rco.^4 - (2/3)*Rco.^3*Rm + (1/2)*Rco.^2*Rm.^2))) +
rho*sin(a_RI)*cos(a_S0)*((2*pi*((1/5)*Rci.^5 - (1/2)*Rci.^4*Rm +
(1/3)*Rci.^3*Rm.^2) - (Z*t/sin(a_RI))*((1/4)*Rci.^4 - (2/3)*Rci.^3*Rm +
(1/2)*Rci.^2*Rm.^2)) -
(2*pi*((1/5)*Ri.^5 - (1/2)*Ri.^4*Rm + (1/3)*Ri.^3*Rm.^2) -
(Z*t/sin(a_RI))*((1/4)*Ri.^4 - (2/3)*Ri.^3*Rm + (1/2)*Ri.^2*Rm.^2)));

%Find incidence loss coefficients
%Rotor incidence loss coefficients

Ki1 = (1/2)*rho*sin(a_RI)*((2*pi*((1/4)*Rm*Rci.^4 - (1/5)*Rci.^5) -
(Z*t/sin(a_RI))*((1/3)*Rm*Rci.^3 - (1/4)*Rci.^4)) -
(2*pi*((1/4)*Rm*Ri.^4 -
(1/5)*Ri.^5) - (Z*t/sin(a_RI))*((1/3)*Rm*Ri.^3 - (1/4)*Ri.^4)));
%Ki1 = (1/2)*rho*sin(a)*(((2*pi*((1/4)*Rm*Rci.^4 - (1/5)*Rci.^5) - (Z*t/sin(a
)))*((1/3)*Rm*Rci.^3 - (1/4)*Rci.^4)) -
((2*pi*((1/4)*Rm*Ri.^4 - (1/5)*Ri.^5) -
(Z*t/sin(a))*((1/3)*Rm*Ri.^3 - (1/4)*Ri.^4))));

Ki2 = rho*sin(a_RI)*(cos(a_S0) - cos(a_RI))*((2*pi*((1/3)*Rm.^2*Rci.^3 -
(1/2)*Rm*Rci.^4 + (1/5)*Rci.^5) - (Z*t/sin(a_RI))*((1/2)*Rm.^2*Rci.^2 -
(2/3)*Rm*Rci.^3 + (1/4)*Rci.^4)) - (2*pi*((1/3)*Rm.^2*Ri.^3 -
(1/2)*Rm*Ri.^4 + (1/5)*Ri.^5) - (Z*t/sin(a_RI))*((1/2)*Rm.^2*Ri.^2 -
(2/3)*Rm*Ri.^3 + (1/4)*Ri.^4)));

Ki3 = (1/2)*rho*sin(a_RI)*(cos(a_S0) - cos(a_RI)).^2*(
(2*pi*((1/2)*Rm.^3*Rci.^2 - Rm.^2*Rci.^3 + (3/4)*Rm*Rci.^4 -
(1/5)*Rci.^5) - (Z*t/sin(a_RI))*(Rm.^3*Rci -
(3/2)*Rm.^2*Rci.^2 + Rm*Rci.^3 - (1/4)*Rci.^4)) -
(2*pi*((1/2)*Rm.^3*Ri.^2 - Rm.^2*Ri.^3 + (3/4)*Rm*Ri.^4 - (1/5)*Ri.^5) -
(Z*t/sin(a_RI))*(Rm.^3*Ri - (3/2)*Rm.^2*Ri.^2 + Rm*Ri.^3 -
(1/4)*Ri.^4)));

%Stator incidence coefficients

Ki4 = (1/2)*rho*sin(a_SI)*((2*pi*((1/5)*Ro.^5 - (1/4)*Rm*Ro.^4) - (Z*t/sin(a_SI))
*((1/4)*Ro.^4 - (1/3)*Rm*Ro.^3)) - (2*pi*((1/5)*Rco.^5 -
(1/4)*Rm*Rco.^4) -
(Z*t/sin(a_SI))*((1/4)*Rco.^4 - (1/3)*Rm*Rco.^3)));
%Ki4 = (1/2)*rho*sin(a)*(((2*pi*((1/5)*Ro.^5 - (1/4)*Rm*Ro.^4) - (Z*t/sin(a
))
*((1/4)*Ro.^4 - (1/3)*Rm*Ro.^3)) - ((2*pi*((1/5)*Rco.^5 - (1/4)*Rm*Rco.^4) -
(Z*t/sin(a))*((1/4)*Rco.^4 - (1/3)*Rm*Rco.^3))));

Ki5 = rho*sin(a_SI)*(cos(a_R0) - cos(a_SI))*((2*pi*((1/5)*Ro.^5 -
(1/2)*Rm*Ro.^4 + (1/3)*Rm.^2*Ro.^3) - (Z*t/sin(a_SI))*((1/4)*Ro.^4 -

```

```

(2/3)*Rm*Ro.^3 +
(1/2)*Rm.^2*Ro.^2)) - (2*pi*((1/5)*Rco.^5 - (1/2)*Rm*Rco.^4 +
(1/3)*Rm.^2*Rco.^3 - (Z*t/sin(a_SI))*((1/4)*Rco.^4 - (2/3)*Rm*Rco.^3 +
(1/2)*Rm.^2*Rco.^2)));

Ki6 = (1/2)*rho*sin(a_SI)*(cos(a_RO) - cos(a_SI)).^2*((2*pi*((1/5)*Ro.^5
- (3/4)*Rm*Ro.^4 + Rm.^2*Ro.^3 - (1/2)*Rm.^3*Ro.^2) -
(Z*t/sin(a_SI))*((1/4)*Ro.^4 -
Rm*Ro.^3 + (3/2)*Rm.^2*Ro.^2 - Rm.^3*Ro)) - (2*pi*((1/5)*Rco.^5 -
(3/4)*Rm*Rco.^4 + Rm.^2*Rco.^3 - (1/2)*Rm.^3*Rco.^2) -
(Z*t/sin(a_SI))*((1/4)*Rco.^4 - Rm*Rco.^3 + (3/2)*Rm.^2*Rco.^2 -
Rm.^3*Rco)));

%Find friction loss coefficients
X = 2*pi*(0.5*(Ri+Ro)) - Z*t;
Y = 0.5*(Ro - Ri);
d_h = 4*(X*Y)/(2*Y+X);
v = 1.14*10.^(-6);
U = vel*0.5*(Ri + Ro); %water velocity, this is iterative, since the
omega changes the value
Re = (U*d_h)/v;
%Re = 1*10^7;
k = 0.1270*10^-3;
f = 0.001375.*(1 + (20000.*k./d_h + 10.^6./Re).^(1/3)); %Moody formula
%f = 0.01;
%as is used Hodgson
%f = (0.25)./(log((k./d_h)./3.7 + 5.74./Re.^0.9)).^2; %Swamee Jain formula
sigma = 4*f;
%sigma = 0.0171; %Used for Hodgson F2020

%Hydraulic diameters

Dl = (Rm - Ri)/50;

Dur = ((Rm-Ri)*sin(a_RI)*(2*pi*Ri-(Z*t/sin(a_RI)))*Dl)/((Ro-Rm)*sin(a_RO)
*(2*pi*Ro-(Z*t/sin(a_RO))));

Dus = ((Rm-Ri)*sin(a_SI)*(2*pi*Ri-(Z*t/sin(a_SI)))*Dl)/((Ro-Rm)*sin(a_SO)
*(2*pi*Ro-(Z*t/sin(a_SO))));

%Friction coefficients

Kforl = (1/16)*sigma*rho*pi*sin(a_RI)*(Rm-Ri).^4*(2*pi*Ri-(Z*t/sin(a_RI))
+2*Z*Dl);

Kforu = (1/16)*sigma*rho*pi*sin(a_RO)*(Ro-Rm).^4*(2*pi*Ro-(Z*t/sin(a_RO))
+2*Z*Dur);

Kfosl = (1/16)*sigma*rho*pi*sin(a_SO)*(Rm-Ri).^4*(2*pi*Ri-(Z*t/sin(a_SO))
+2*Z*Dl);

```

```

Kfosu = (1/16)*sigma*rho*pi*sin(a_SI)*(Ro-Rm).^4*(2*pi*Ro-(Z*t/sin(a_SI))
+2*Z*Dus);

Kfirl = (1/8)*sigma*Z*rho*pi*sin(a_RI)*((Rm.^4*Rci-2*Rm.^3*Rci.^2+2*Rm.^2
*Rci.^3-Rm*Rci.^4+(1/5)*Rci.^5) - (Rm.^4*(Ri+Dl)-2*Rm.^3*(Ri+Dl).^2+
2*Rm.^2*(Ri+Dl).^3-Rm*(Ri+Dl).^4+(1/5)*(Ri+Dl).^5));

Kfiru = (1/8)*sigma*Z*rho*pi*sin(a_RO)*(((1/5)*(Ro-Dur).^5-Rm*(Ro-Dur).^4
+2*Rm.^2*(Ro-Dur).^3-2*Rm.^3*(Ro-Dur).^2+Rm.^4*(Ro-Dur))-
((1/5)*Rco.^5-Rm*Rco.^4+2*Rm.^2*Rco.^3-2*Rm.^3*Rco.^2+Rm.^4*Rco));

Kfisl = (1/8)*sigma*Z*rho*pi*sin(a_SO)*((Rm.^4*Rci-2*Rm.^3*Rci.^2+2*Rm.^2
*Rci.^3-Rm*Rci.^4+(1/5)*Rci.^5)-(Rm.^4*(Ri+Dl)-2*Rm.^3*(Ri+Dl).^2+
2*Rm.^2*(Ri+Dl).^3-Rm*(Ri+Dl).^4+(1/5)*(Ri+Dl).^5));

Kfisu = (1/8)*sigma*Z*rho*pi*sin(a_SI)*(((1/5)*(Ro-Dus).^5-Rm*(Ro-Dus).^4
+2*Rm.^2*(Ro-Dus).^3-2*Rm.^3*(Ro-Dus).^2+Rm.^4*(Ro-Dus))-
((1/5)*Rco.^5-Rm*Rco.^4+2*Rm.^2*Rco.^3-2*Rm.^3*Rco.^2+Rm.^4*Rco));

%Friction loss coefficient

Kf = Kforl + Kforu + Kfosl + Kfosu + Kfirl + Kfiru + Kfisl + Kfisu;

%Find secondary circulation losses
%Secondary circulation factor
%Kb = 0.22; %Empirically derived loss coefficient (0.22 found to be
accurate for 45 degree vanes)
Kb = 0.45; %Empirically derived loss coefficient (0.45 found to be
accurate for 90 degree vanes)

%Secondary coefficients:

Kscrl = (1/2)*Kb*rho*sin(a_RI)*((2*pi*((1/2)*Rm.^3*Rci.^2-Rm.^2*Rci.^3+(
3/4)*Rm*Rci.^4-(1/5)*Rci.^5)-(Z*t/sin(a_RI))*(Rm.^3*Rci-(3/2)*Rm.^2*Rci.
^2+
Rm*Rci.^3-(1/4)*Rci.^4)) -
(2*pi*((1/2)*Rm.^3*Ri.^2-Rm.^2*Ri.^3+(3/4)*Rm*Ri.^4-(1/5)*Ri.^5)-(Z*t/si
n(a_RI))*(Rm.^3*Ri-(3/2)*Rm.^2*Ri.^2+Rm*Ri.^3-(1/4)*Ri.^4)));

Kscru = (1/2)*Kb*rho*sin(a_RO)*((2*pi*((1/5)*Ro.^5-(3/4)*Rm*Ro.^4+Rm.^2*
Ro.^3-(1/2)*Rm.^3*Ro.^2)-(Z*t/sin(a_RO))*((1/4)*Ro.^4 -Rm*Ro.^3+
(3/2)*Rm.^2*Ro.^2-Rm.^3*Ro)) -
(2*pi*((1/5)*Rco.^5-(3/4)*Rm*Rco.^4+Rm.^2*Rco.^3-(1/2)*Rm.^3*Rco.^2)-(Z*
t/sin(a_RO))*((1/4)*Rco.^4-Rm*Rco.^3+(3/2)*Rm.^2*Rco.^2-Rm.^3*Rco)));

Kscsl = (1/2)*Kb*rho*sin(a_SO)*((2*pi*((1/2)*Rm.^3*Rci.^2-Rm.^2*Rci.^3+(
3/4)*Rm*Rci.^4-(1/5)*Rci.^5)-(Z*t/sin(a_SO))*(Rm.^3*Rci-(3/2)*Rm.^2*Rci.
^2+
Rm*Rci.^3-(1/4)*Rci.^4)) -
(2*pi*((1/2)*Rm.^3*Ri.^2-Rm.^2*Ri.^3+(3/4)*Rm*Ri.^4-(1/5)*Ri.^5)-(Z*t/si

```

```

n(a_S0))*(Rm.^3*Ri-(3/2)*Rm.^2*Ri.^2+Rm*Ri.^3-(1/4)*Ri.^4)));

Kscsu = (1/2)*Kb*rho*sin(a_SI)*((2*pi*((1/5)*Ro.^5-(3/4)*Rm*Ro.^4+Rm.^2*
Ro.^3-(1/2)*Rm.^3*Ro.^2)-(Z*t/sin(a_SI))*((1/4)*Ro.^4 -Rm*Ro.^3+
(3/2)*Rm.^2*Ro.^2-Rm.^3*Ro)) -
(2*pi*((1/5)*Rco.^5-(3/4)*Rm*Rco.^4+Rm.^2*Rco.^3-(1/2)*Rm.^3*Rco.^2)
-(Z*t/sin(a_SI))*((1/4)*Rco.^4-Rm*Rco.^3+(3/2)*Rm.^2*Rco.^2-Rm.^3*Rco)))
;

%Secondary loss coefficient

Ksc = Kscrl + Kscru + Kscsl + Kscsu;

omega_p = omega_p_rpm*(2*pi/60);

%Find fluid vortex angular velocity

a = Ki3+Ki6+Kf+Ksc;
b = omega_p*(Ki2+Ki5-K2);
c = omega_p.^2*(Ki1+Ki4-K1);

omega_1 = (-b+sqrt(b.^2-4.*a.*c))./(2.*a);
omega_2 = (-b-sqrt(b.^2-4.*a.*c))./(2.*a);

if omega_1 > 0
    omega = omega_1;
else
    omega = omega_2;
end

%Steady state torque equation

tau_ss = K1.*omega.*omega_p + K2.*omega.^2;

T = 2.*tau_ss;
P = T.*omega_p/1000;
end

angle_calc_actual

%Incidence angle calcualtion

o = W;
S = omega_p_rpm;
[a_RI,a_SI]=incidence_angle(o,S);

function [a_RI,a_SI] = incidence_angle(o,S)
%Vane angles
a_S0 = 90*pi/180 ; %Stator vane angle [rad]
a_R0 = 90*pi/180 ; %Rotor vane angle [rad]

```

```

Z = 12;
t = 0.005; %[m]
Ri = 0.061; %[m]
Ro = 0.144; %[m]

omega_p = S*(2*pi/60);
%omega_p_rpm = 2000; %the velocity in which we are interested
%omega_p = omega_p_rpm*(2*pi/60);

Rm = ((2/3)*pi*sin(a_S0)*(Ro.^3 - Ri.^3) -
(1/2)*Z*t*(Ro.^2 - Ri.^2))/(pi*sin(a_S0)*(Ro.^2 - Ri.^2) - Z*t*(Ro - Ri));

if a_S0 > 89*pi/180
    W_R = sqrt((omega_p*Ri).^2 + (o*Ri).^2);
    a_RI = (cos((omega_p*Ri)/(W_R))).^-1.*180/pi;
else
    W_Rtheta2 = (omega_p*Ri + o*(Rm - Ri)*cos(a_S0));
    Ui = omega_p*Ri;
    V_x1 = W_Rtheta2 - Ui;
    V_r1 = V_x1*tan(a_S0);
    a_RI = (tan(V_r1/W_Rtheta2)).^-1.*180/pi;
end

if a_R0 > 89*pi/180
    W_S = sqrt((o*Ro).^2 + (omega_p*Ro).^2);
    W_Stheta5 = omega_p*Ro;
    a_SI = (cos(W_Stheta5/W_S)).^-1.*180/pi;
else
    W_Stheta5 = omega_p*Ro + o*(Ro - Rm)*cos(a_R0);
    Uo = omega_p*Ro;
    V_r2 = W_Stheta5*tan(a_R0);
    a_SI = (tan(V_r2/(Uo + W_Stheta5))).^2.*180/pi;
end
end

```

E. Vortex Pressure Calculation in MATLAB

Based on the theory to calculate the vortex pressure inside the working compartments, as presented in Section 3.4.2.

```
theta_c = linspace(1,360,360);

%Determine the angle theta from the vortex centre in terms of angle theta_c
%from the working compartment centre
alpha = 0.5.*(Ro - Ri);
Rc = 0.5.*(Ro + Ri);
Y_1 = Rm - Rc;
Y_2 = alpha.*cosd(theta_c);
h = alpha.*sind(theta_c);

for i = 1:360
    if (Y_1 + Y_2(i) == 0) && i < 180
        theta(i) = 90;
    elseif (Y_1 + Y_2(i) == 0) && i > 180
        theta(i) = 270;
    elseif (cosd(i) < 0) && (Y_1 + Y_2(i) < 0)
        theta(i) = 180 + atand(h(i)./(Y_1 + Y_2(i)));
    elseif (i > 180)
        theta(i) = 360 + atand(h(i)./(Y_1 + Y_2(i)));
    else
        theta(i) = atand(h(i)./(Y_1 + Y_2(i)));
    end
end

%Casing radius is determined by
r_v = ((Y_1 + Y_2).^2 + h.^2).^0.5;

%Radius for atmospheric pressure found with
Ry = 0.5*(Rco + Rci);
a_1 = 0.5*(Rco - Rci);

theta_Y = asind((1./a_1).*(Rm - Ry).*abs(sind(theta)));

% theta_D = 180 - (theta_Y + theta);

for t = 1:360
    if t < 180
        theta_D(t) = 180 - (theta_Y(t) + theta(t));
    else
        theta_D(t) = 180 + (theta_Y(t) - theta(t));
    end
end
```



```

end

r_o = a_1*(sind(theta_D)./sind(theta));

%Determine co-factors
r_y = Rm - cosd(theta).*r_v;
r_x = sind(theta).*r_v;
R = sqrt(r_y.^2 + r_x.^2);
zeta = acos((r_v.^2 + abs(R).^2 - Rm.^2).*(2.*r_v.*abs(R)));
mu = abs(zeta - 90);

for x = 1:360
    if (0 < x) && (x < 180)
        K(x) = 1;
    elseif (180 < x) && (x < 360)
        K(x) = -1;
    else
        K(x) = 0;
    end
end

C_WR = K.*cosd(mu).*cosd(a);

for y = 1:360
    if y ~= 0 && y ~= 180
        C_WV(y) = cosd(a);
    elseif y == 180
        C_WV(y) = 1;
    end
end

beta = (180 - zeta) - (a./90).*(180 - zeta - theta);

for c = 1:360
    if (c ~= 0) && (c ~= 180)
        C_WWR(c) = 0;
    else
        C_WWR(c) = 1;
    end
end

gamma_A = acosd((-r_v.*sind(theta).*sind(a))./(r_v.^2 + Rm.^2 - 2.*Rm.*r_v
    .*cosd(theta)).^0.5);
gamma_R = acosd((-r_v.*sind(theta).*sind(a))./(r_v.^2 + Rm.^2 - 2.*Rm.*r_v
    .*cosd(theta)).^0.5);
gamma_V = acosd(-cosd(theta).*sind(a));

%Acceleration of rotor, will be 0 for steady state analysis%
omega_p_dot = 0;

% For rotational speed of rotor and no rotational speed of stator

```

```

for s = 1:360
    if (0 <= s) && (s <= 180)
        speed(s) = omega_p;
    else
        speed(s) = 0;
    end
end

%Pressure of vortex throughout the working compartment%
P = rho.*((omega.^2 + 2.*speed.*omega.*sind(gamma_V).*C_WV).*0.5.*r_v.^2 +
(omega_p_dot + sind(gamma_A).*C_WR
+ speed.^2.*sind(gamma_R).*C_WWR).*(0.5.*(r_v - Rm.*cosd(theta))
.*(r_v.^2 + Rm.^2 - 2.*Rm.*r_v.*cosd(theta)).^0.5 - 0.5.*(cosd(theta).^2 - 1)
.*Rm.^2.*log((r_v - Rm.*cosd(theta)) + (r_v.^2 + Rm.^2
- 2.*Rm.*r_v.*cosd(theta)).^0.5))) ;

%Pressure of vortex at atmosphere, thus atmospheric pressure%
P_o = rho.*((omega.^2 + 2.*speed.*omega.*sind(gamma_V).*C_WV).*0.5.*r_o.^2 +
(omega_p_dot + sind(gamma_A).*C_WR
+ speed.^2.*sind(gamma_R).*C_WWR).*(0.5.*(r_o - Rm.*cosd(theta)).*(r_o.^2 + Rm.^2
- 2.*Rm.*r_o.*cosd(theta)).^0.5
- 0.5.*(cosd(theta).^2 - 1).*Rm.^2.*log((r_o - Rm.*cosd(theta)) + (r_o.^2 + Rm.^2
- 2.*Rm.*r_o.*cosd(theta)).^0.5))) ;

P_guage = P - P_o;

% plot(theta,theta_c);hold on;
% xlabel ('Theta');
% ylabel ('Theta_c in deg');hold off;

plot(theta,P_guage);hold on;
xlabel ('Theta in deg');
ylabel ('Pa');

```

F. Fluid outflow and pressure loss calculation

Matlab

Based on the theory to calculate the valve pressure drop requirement presented in Section 3.4.3.

```
% Known variables
rho = 1000;
omega_p = 40*pi; %1200 rpm
R_o = 0.144;

% Pressures in working compartment, as determined by vortex pressure
% calculation, will vary this accoring to percentage fill

%-10 % fill
P_B = 665 ;
P_C = 1142 ;
%30% fill
% P_B = 2023 ;
% P_C = 3361 ;
%60% fill
% P_B = 3718 ;
% P_C = 5826 ;
% -90% fill
% P_B = 4586 ;
% P_C = 6670 ;

% Area calculations
A_B = 2*pi*R_o*0.0005; %Total gap area between the rotor and the stator
D_D = 0.038; %Inside Diameter of fitting PVC fittings at cross etc.
A_D = pi*(D_D/2).^2; %Cross-sectional Area at the cross arms
D_PVC = 0.042; %Inside diameter of PVC pipe
A_A = pi*(D_PVC/2).^2; %Cross-sectional area of PVC pipes
D_C = 0.01; %Diameter of Drainage holes
A_C = 12*(pi*(D_C/2).^2); %Combined area of one of the stators cup outlet holes
D_DP = 0.038; %Inside diameter of drainage pipes
A_DP = pi*(D_DP/2).^2; %Cross-sectional area of drainage pipes
anulw = 0.098; %Width of rainage annulus
anuld = 0.005; %Depth of drainage annulus
d_e = 2*(anulw*anuld)/(anulw+anuld); %Hydraulic diameter of drain annulus
A_Z = anulw*anuld;
%pi*(d_e/2).^2; %Cross-sectional area of drainage annulus

% Loss coefficients
C_1B = 0.5;
C_2B = 1;
```

```

C_3B = 0.3;
C_4B = 0.8;
C_5B = 0.5;

C_1C = 0.5;
C_2C = 0.8;
C_4C = 0.8;
C_5C = 0.5;
C_6C = 0.5;

% Choose value for Q_D (0.001 ; 0.002 ; 0.003 ; 0.004)
Q_D = linspace(0,0.0045) ;
% Q_D = 0.001;
% Q_D = 0.003;
% Q_D = 0.002;
% Q_D = 0.001;
% Choose initial value for Q_B to start iteration
% Q_B = 0.003919; %Converges for Q_D = 0.004
% Q_B = 0.0029999999999999; %Converges for Q_D = 0.003
% Q_B = 0.001999999999999999; %Converges for Q_D = 0.002
% Q_B = 0.000999; %Converges for Q_D = 0.001

Ratio_24 = 0.99;

%Vary the fluid flow ratios
Q_B = Ratio_24*Q_D;

%Then
Q_C = (Q_D - Q_B)/2 ;

%Find fluid energy at 2

P_1B = 0.5*C_1B*(Q_B/A_B).^2 ; %Entry into drain gap
P_2B = 0.5*C_2B*(Q_B/A_B).^2; %Exit from drain gap
P_3B = 0.5*C_3B*(Q_B/A_Z).^2; %Flow around drain annulus
P_4B = 0.5*C_4B*(Q_B/A_A).^2; %Bend of annulus into one outlet
P_5B = 0.5*C_5B*(Q_B/A_A).^2; %Bend loss in outlet PVC pipe

P_loss_Bto2 = P_1B + P_2B + P_3B + P_4B + P_5B ; %Total empirically determined losses

E_2 = P_B/rho + 0.5*((Q_B/A_B).^2 + (0.5*omega_p*R_o).^2) - P_loss_Bto2;

%Find fluid energy at 1

P_1C = 0.5*C_1C*(Q_C/A_C).^2; %Entry into drain holes
P_2C = 0.5*C_2C*(Q_C/A_C).^2; %Bends in drain holes
P_3C = P_2C; %Second bend in drainage hole
P_4C = 0.5*C_4C*(Q_C/A_C).^2; %Exit from drain holes
P_5C = 0.5*C_5C*(Q_C/A_DP).^2; %Bend in steel drainage pipe
P_6C = 0.5*C_6C*(Q_C/A_C).^2; %Bend in PVC pipe

```

```

P_loss_Cto1 = P_1C + P_2C + P_3C + P_4C + P_5C + P_6C;

E_1 = P_C/rho + 0.5*(Q_C/A_C).^2 - P_loss_Cto1;

%Flow ratios in arms
% Ratio_14 = Q_C/(2*Q_D);
% Ratio_24 = Q_B/Q_D;

%from flow ratios choose the appropriate loss coefficient from the graphs

K_12 = -0.88;
K_14 = -0.79;

%Find the energy at the outlet cross leg point 4

E_1a = E_2 + 0.5*K_12*(Q_B/A_D).^2;
E_4 = E_1a - 0.5*K_14*(Q_D/A_D).^2;

%Find the valve loss characteristic required for the chosen flow rate

%Geometric values for flow path D
l_cp = 0.03;
D_cp = 0.05;
D_0 = 0.05;

A_cp = pi*(D_cp/2).^2;
A_0 = pi*(D_0/2).^2;

mu = 0.0007972;

Re = rho*(Q_D/A_cp)*D_cp/mu;

e = 0.08;

f_cp = (1./(-1.8.*log(((e./D_cp)/3.7).^1.11 + 6.9./Re))).^2;

P_2D = f_cp.*(l_cp./D_cp).*(Q_D./A_cp).^2;

C_vent = 14834;

P_3D = ((C_vent*Q_D).^2)/rho;

P_valve = E_4 - (0.5*(Q_D/A_0).^2 + P_2D + P_3D);

P_valve_squared = sqrt(P_valve*1000);

x1 = linspace(0,4);

y_1 = 12.963*x1+0.011762 ;

y_5 = 179.86*x1+24.002;

```

```
plot(x1,P_valve); hold on;  
  
% plot(x1,y_1); hold on;  
%  
% plot(x1,y_5); hold on;
```


G. FloEFD setup for angle seat valve simulation

G.1. Geometrical properties

The FloEFD (2018) simulation package is a robust simulation tool that requires little to almost no initial CAD cleanup before simulation on the model can commence. For this reason the angle seat valve CAD model is used without modifications. The FloEFD for Solid Edge package was used, so the simulation was done directly by using the angle seat valve model in the Solid Edge workspace.

One of the few provisions that must be made for the CAD model is to ensure that there exists no leakages, this implies that all openings including the inlets and outlets must be closed off with a lid surface. Fig. G.1 illustrates the lids placed on the inlet and outlet sections of the valve model.

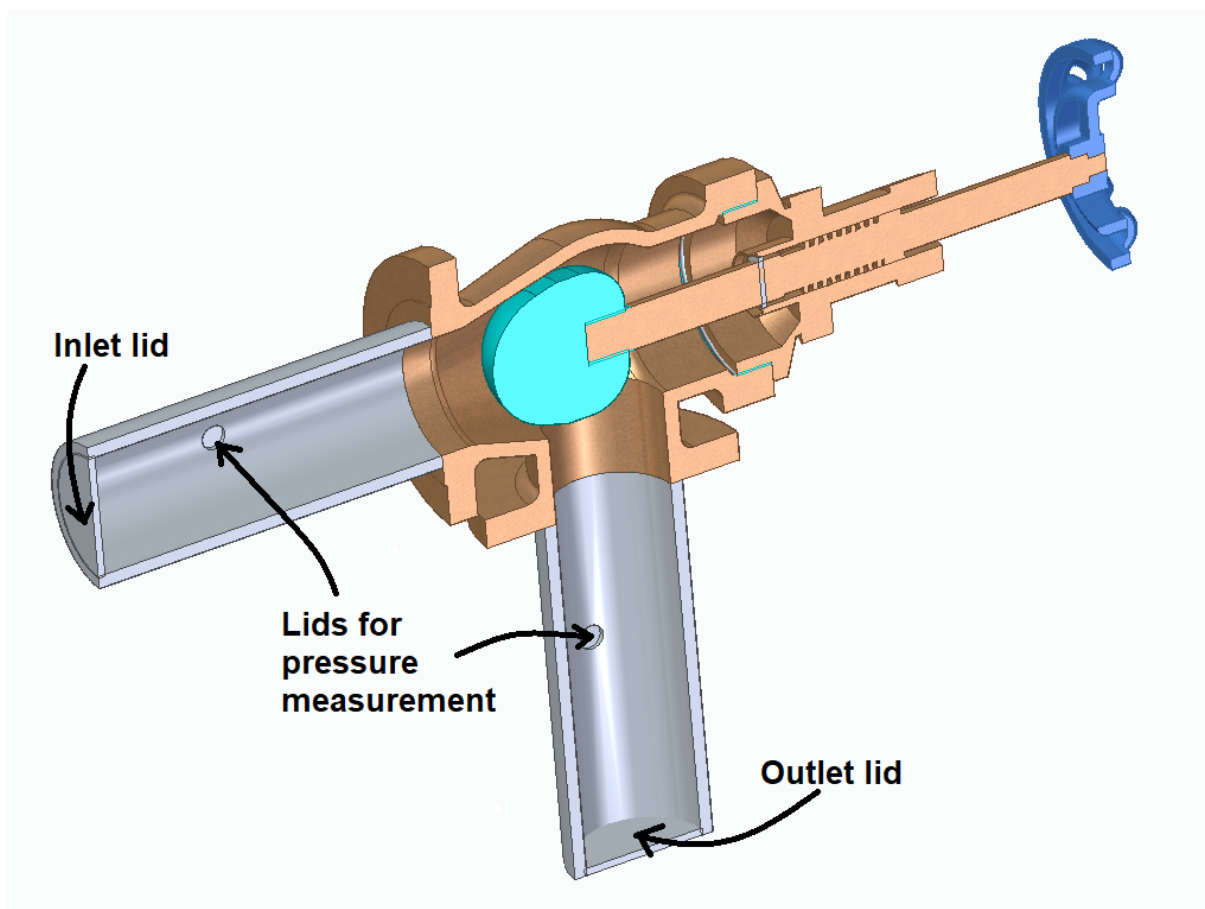


Figure G.1. Various lids used to seal openings in the control valve CAD model

G.2. Initial settings

For the simulation the following initial settings were chosen, in order:

1. The internal analysis type is selected with the option to exclude cavities without flow conditions.
2. Gravity is enabled and set to be $-9.81 \text{ m}^2/\text{s}$ in the Z-direction. The analysis is not time dependant.
3. The working fluid is selected as water and the flow type as laminar and turbulent.
4. The wall thermal condition is selected to be adiabatic
5. The initial conditions are indicated as 21 [deg C] and the pressure as 100900 [Pa] to match the atmospheric conditions encountered at Potchefstroom. Pressure potential is also enabled.

G.3. Meshing

The meshing method employed by FloEFD, generates a number of cubic shaped elements throughout the computational domain. The purpose of the Cartesian mesh is to enable discretisation of the finite volume Navier Stokes equations for each element. The resulting matrix system can then be solved for the unknown variables like pressure and velocity components.

The mesh can be automatically generated based on predefined settings or generated manually according to user preference. The cubic mesh can be made incrementally smaller in areas of the computational domain that are of specific interest. The resolution of the mesh can also be refined automatically based on the high flow gradient areas. The mesh density is therefore only increased in areas where it is required to create a more efficient mesh to save computational time and resources. An example of the mesh grid utilised for the angle seat valve simulation is shown in Fig. G.2 and Fig. G.3 illustrates how the mesh is refined more, as the valve opening is reduced.

G.4. Boundary conditions and goals

The boundary conditions of the setup dictate the flow and pressure properties that the valve will be simulated for. The inlet boundary condition is indicated as the flow rate for which the valve pressure drop is to be determined. This flow rate is varied from 1 [l/s] to 4 [l/s] by use of the parametric study function provided in the FloEFD workspace and then simulated for all the flow rate increments. The outlet boundary condition is chosen as atmospheric pressure (100900 [Pa]) since the outlet of the valve opens to the atmosphere.

The goals of the simulation indicate the properties the analyst is interested in. The goals in the valve simulation are indicated as surface goals at the inlet and the outlet of the valve so that the valve pressure drop can be determined for each simulation. Fig. G.4 shows the areas where the boundary conditions and the surface goals are implemented.

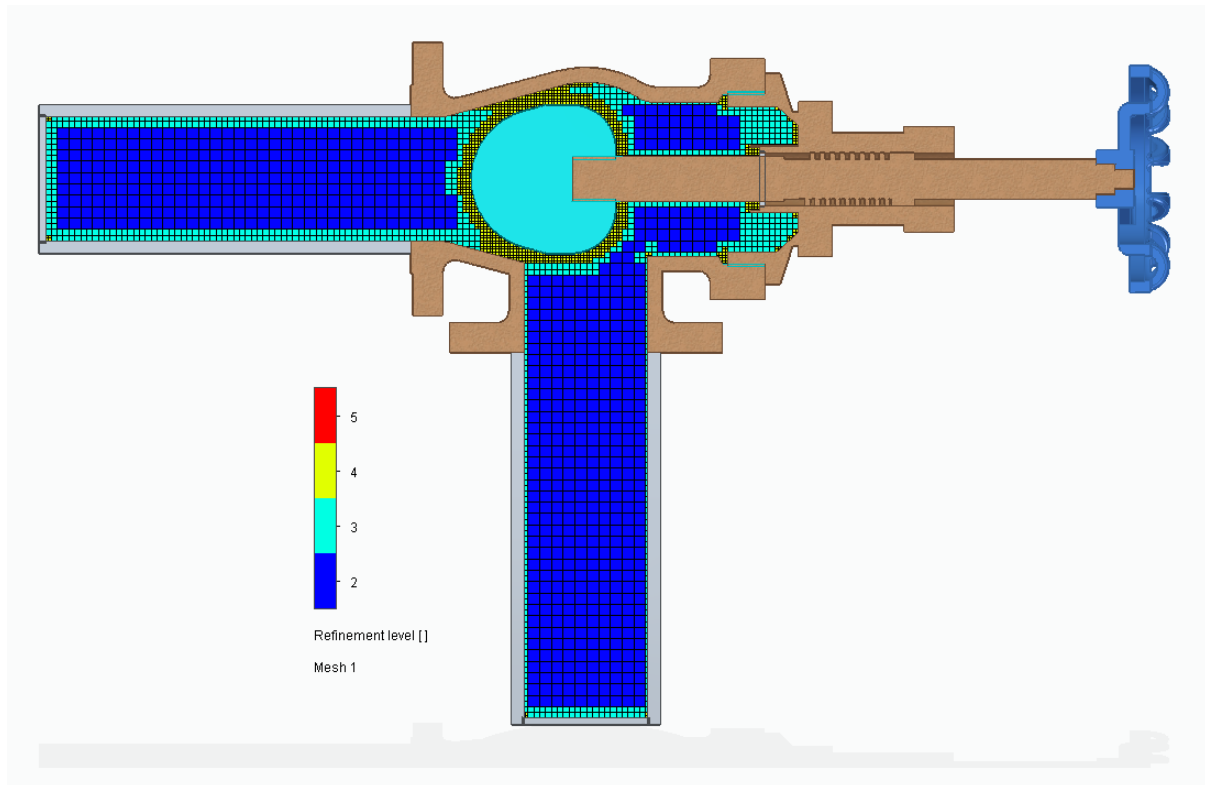


Figure G.2. Valve simulation mesh for open valve

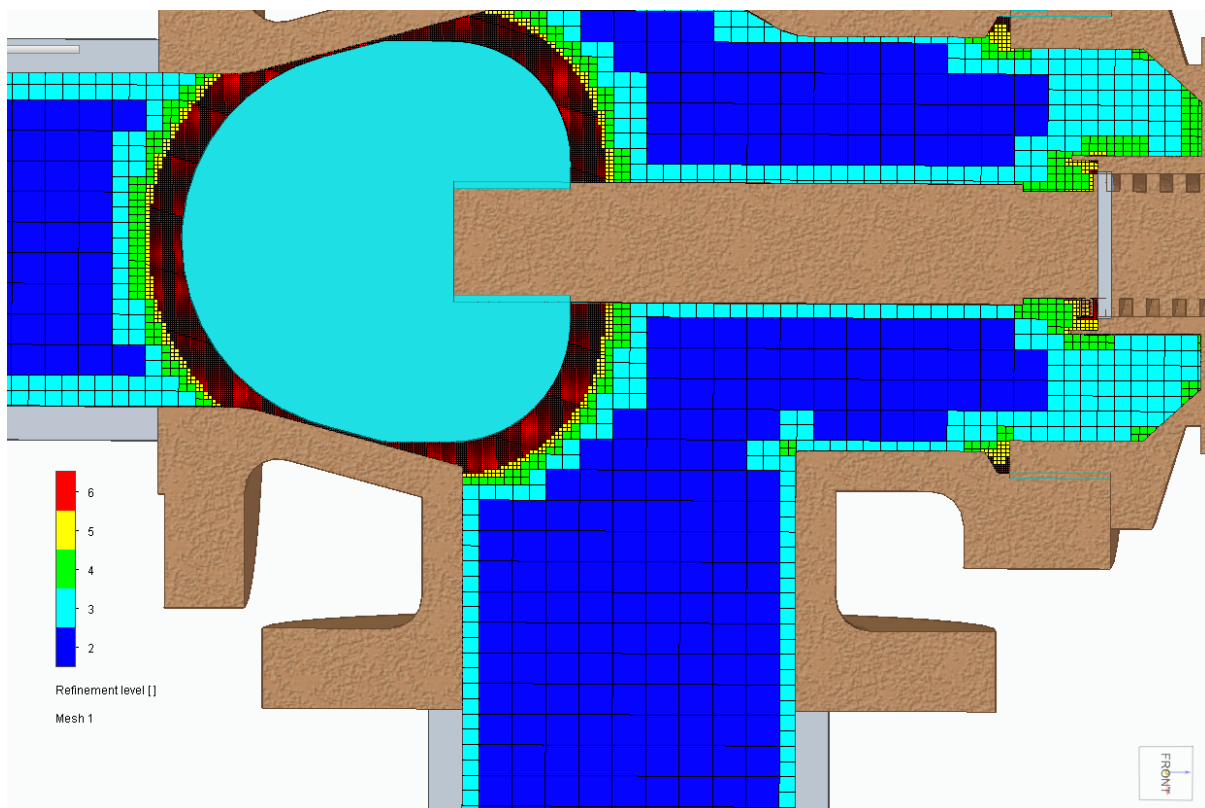


Figure G.3. Valve simulation mesh refined during valve closure

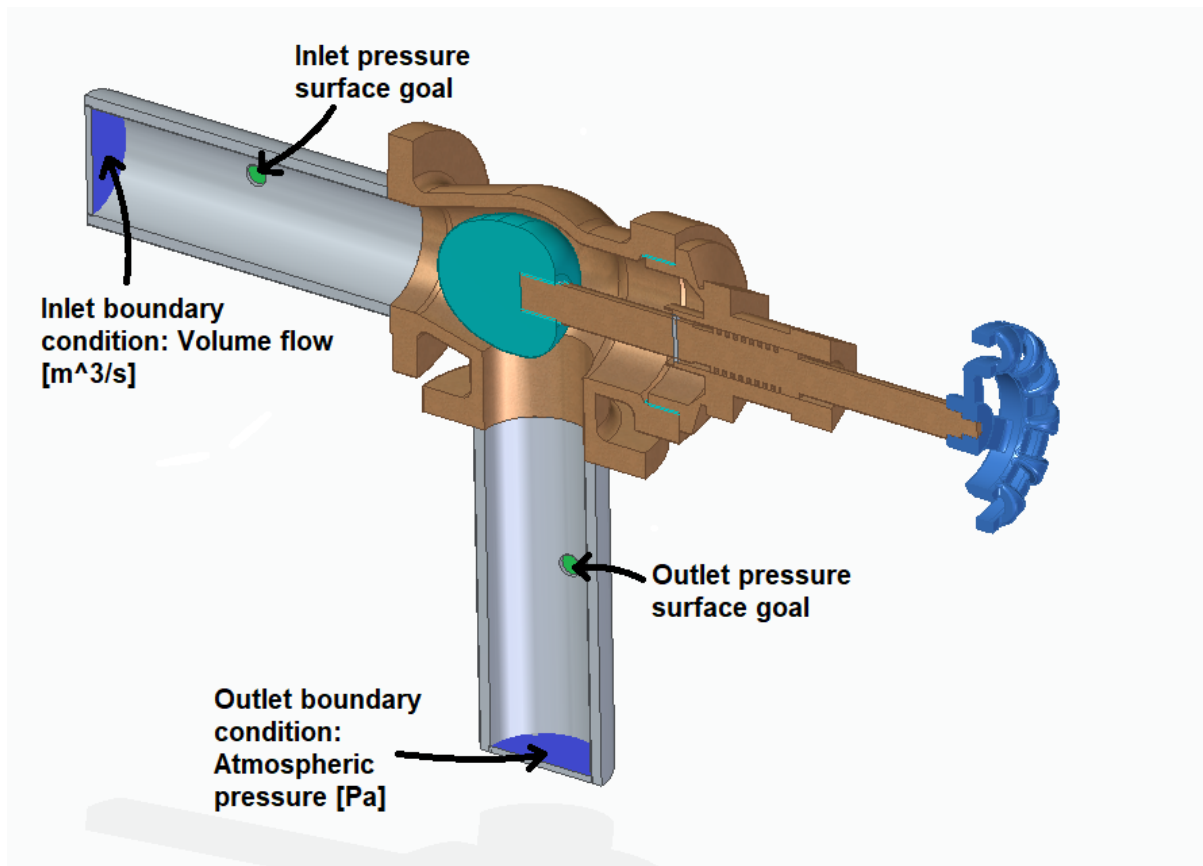


Figure G.4. Valve simulation boundary conditions and surface goals

G.5. Simulation results

The simulation delivered the pressure drop generated by the valve for flow rates between 1 [l/s] and 4 [l/s] and the valve closure from fully open to almost fully closed.

Fig. G.5 and Fig. G.6 shows the pressure contours for different flow rates and Fig. G.7 and Fig. G.8 the pressure contours for different valve closures.

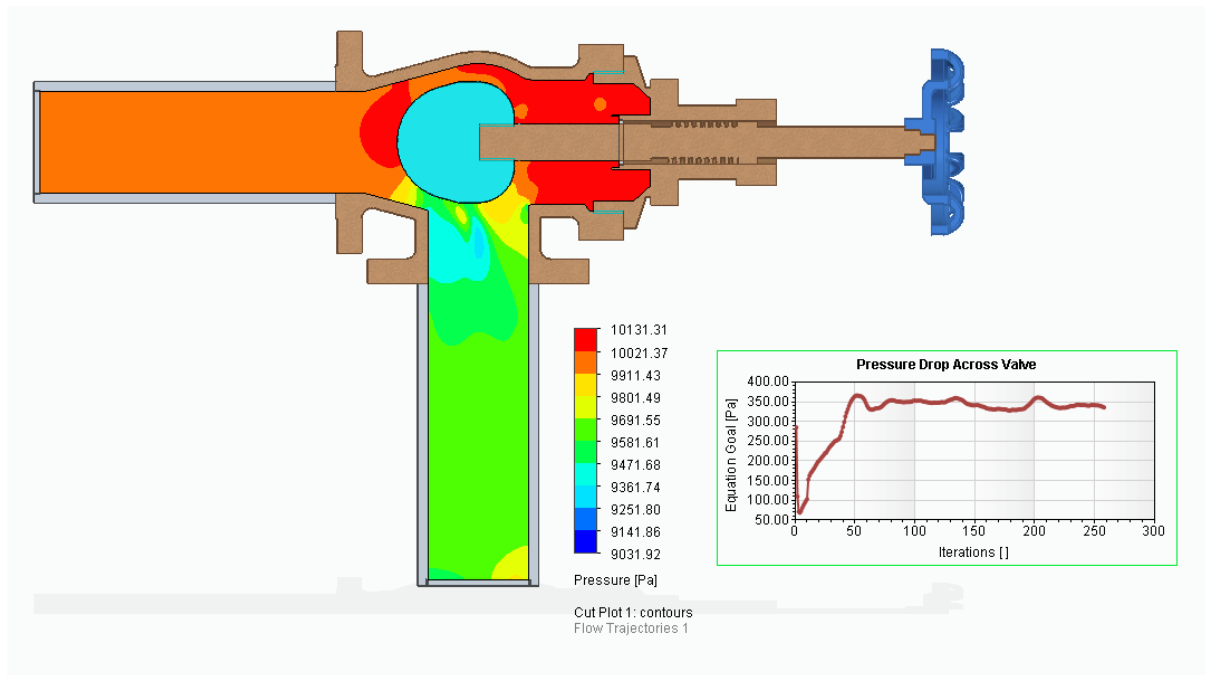


Figure G.5. Valve pressure drop simulation for 10 valve turns and 1 [l/s]

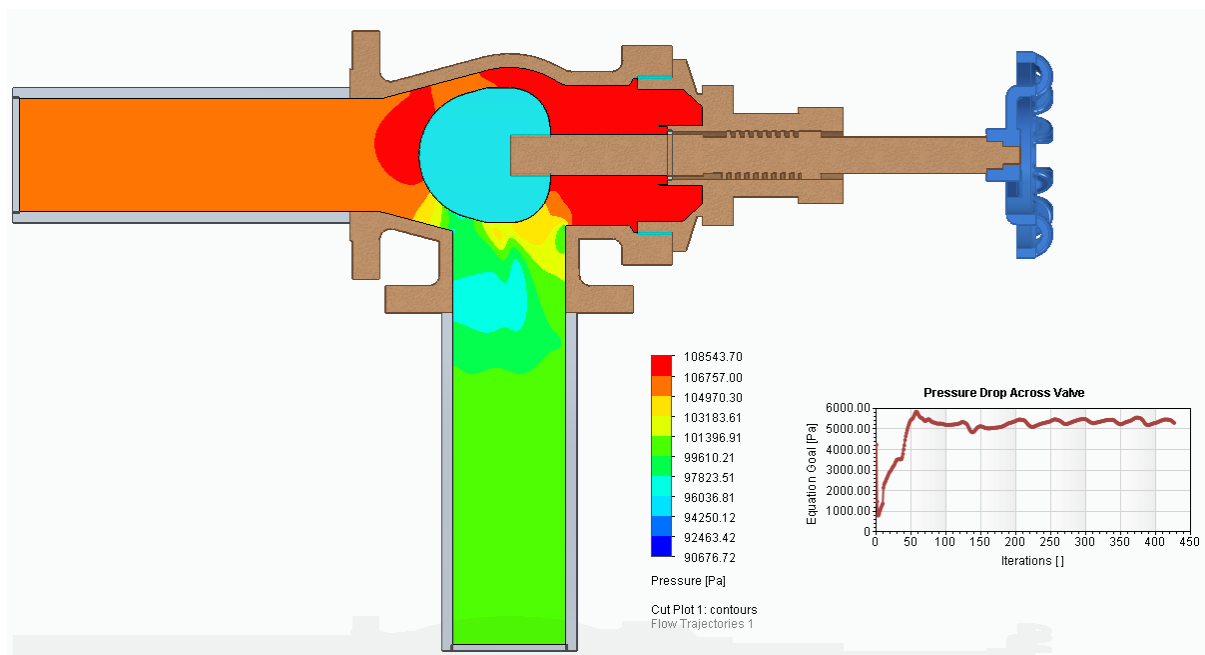


Figure G.6. Valve pressure drop simulation for 10 valve turns and 4 [l/s]

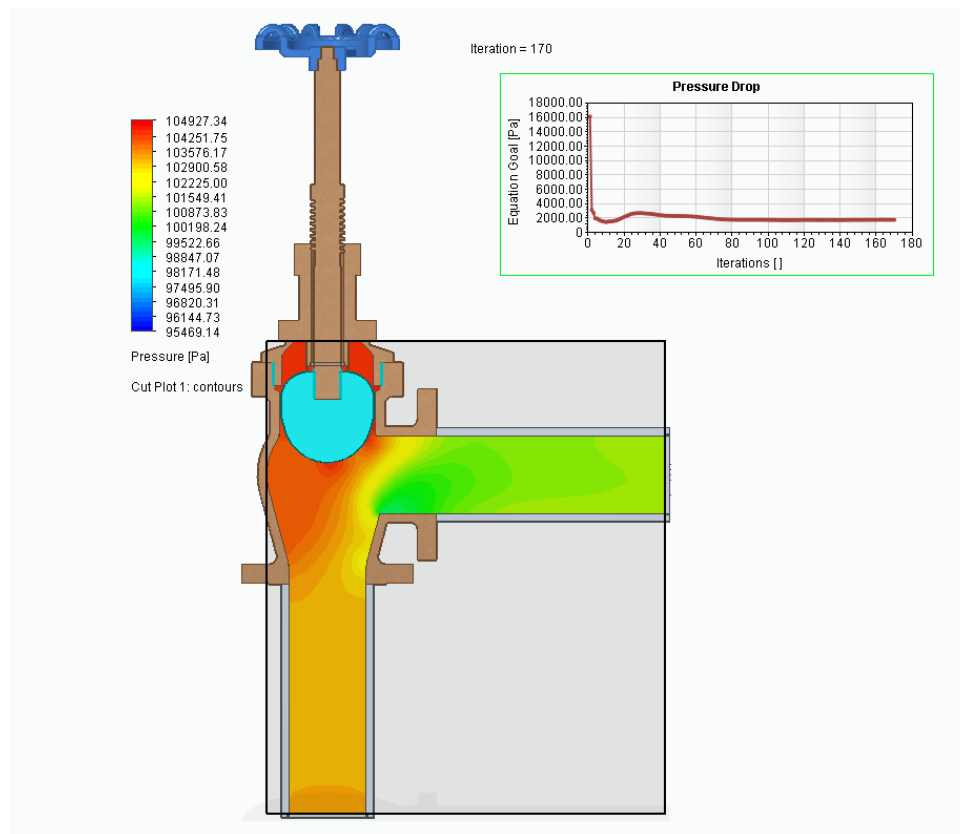


Figure G.7. Valve pressure drop simulation for fully open

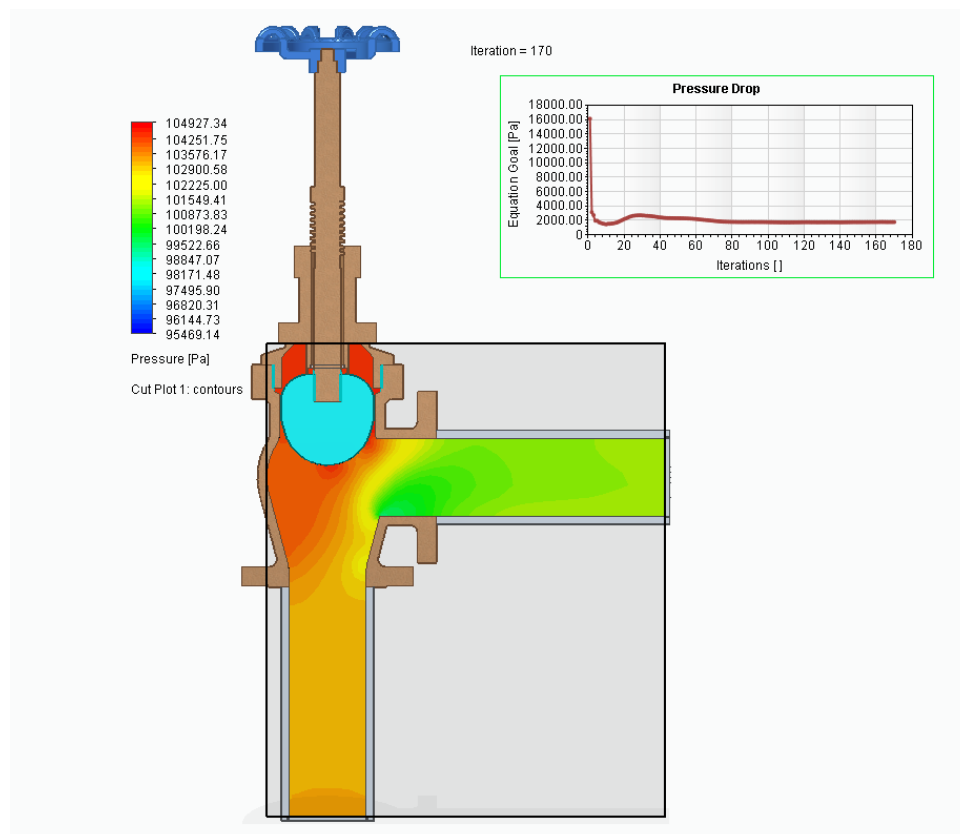


Figure G.8. Valve pressure drop simulation for almost fully closed

H. FloEFD setup for Venturi simulation

H.1. Geometrical properties

Fig. H.1 illustrates the CAD model constructed, that was used in the simulation of the Venturi flow meter. The inlet and outlet lids and pressure sensor surfaces can be seen.

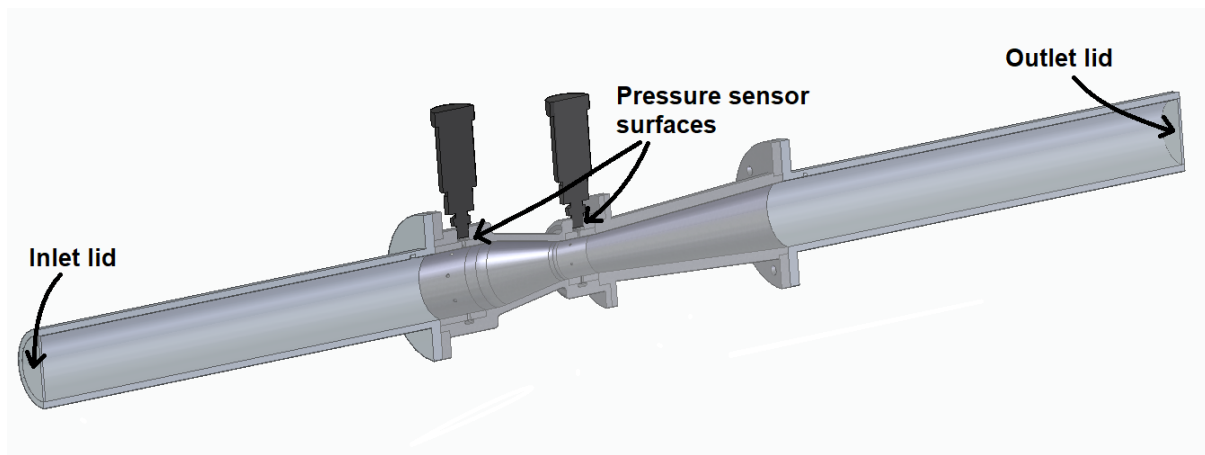


Figure H.1. Venturi CAD model for simulation

H.2. Simulation properties

The properties used in this model to set up the simulation are similar to the properties discussed in Appendix G

H.3. Boundary conditions and goals

The boundary conditions defined for the Venturi simulation are:

- The inlet flow velocity was also varied by means of the parametric study tool, between 1 [l/s] and 4 [l/s].
- The outlet pressure was taken as equal to the atmospheric pressure, the pressure value is irrelevant as the flow rate will determine the pressure drop value between the inlet and the throat; which is the parameter to be determined during this simulation.

The goals are defined with the aid of the surface pressures at the inlet and the throat of the Venturi, and then a calculation is made to obtain the difference between these pressures.

Fig. H.2 illustrates the boundary conditions and goals of this simulation.

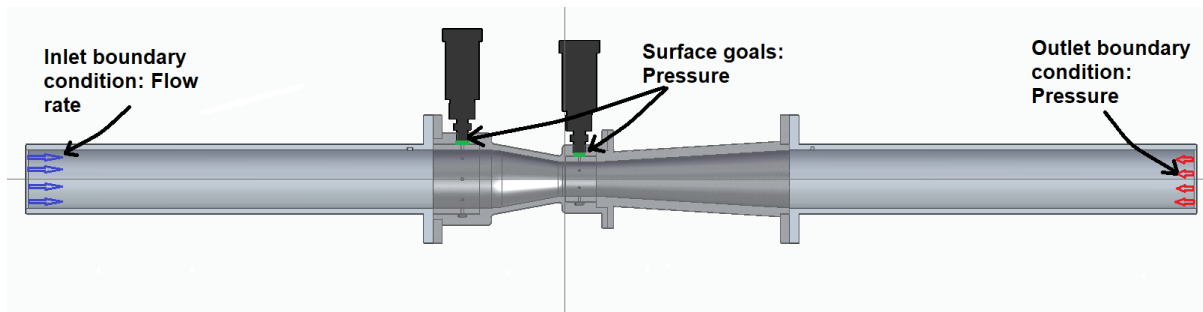


Figure H.2. Venturi simulation boundary conditions and goals

H.4. Simulation results

The simulation produced pressure contours that clearly illustrate the pressure drop experienced by the area contraction in the Venturi and how the pressure is slowly recovered during the decelerated flow in the outlet divergent section. Fig. H.3 shows an example of the pressure contours delivered and Fig. H.4 shows an example of the flow trajectories determined. Fig. H.5 lists the pressure results from the parametric study.

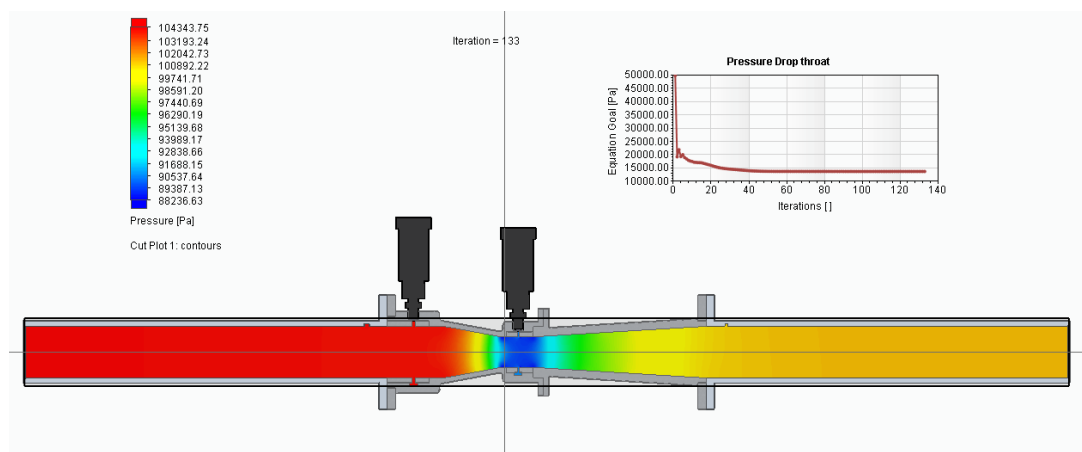


Figure H.3. Venturi simulation pressure contours

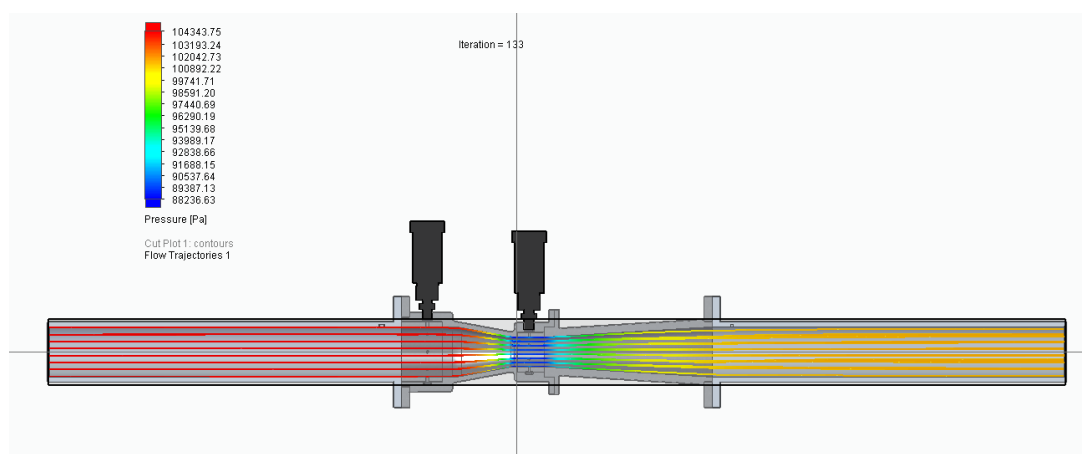


Figure H.4. Venturi simulation flow trajectories

What If Analysis				
What If Analysis 1				
Input Variables	Output Parameters	Scenario	Goals	
Run				
Summary	Design Point 1	Design Point 2	Design Point 3	Design Point 4
Volume flow rate (Inlet Volume Flow 1) [m ³ /s]	0.001	0.002	0.003	0.004
Pressure Drop throat [Pa]	871.047333	3439.16113	7708.76735	13678.5643
Total Pressure Drop [Pa]	231.346404	929.3066	1942.99762	3230.56565
Status	Finished	Finished	Finished	Finished
Run at	This computer	This computer	This computer	This computer
Number of cores	88	88	88	88

Figure H.5. Venturi simulation parametric study results