

Exploring Grade 12 teachers' pedagogical content knowledge of Similarity in Euclidean geometry

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DECLARATION

I, **Goodness Njabulo Shoba**, declare that this study, **Exploring Grade 12 Teachers' Pedagogical Content Knowledge of similarity in Euclidean Geometry**, is my work apart from where sources are acknowledged.

This study is for the degree Master of Education, submitted to the Faculty of Education, North-West University. It has not been previously submitted by me or any other person for a degree at any alternative institute.

Signature:

DEDICATION

This work is dedicated to:

- Both my late parents, mother (Priscilla Shoba) and father (Willy Shoba), for their prayers, endless love, and support. May their precious souls continue to rest in peace.
- My husband, Sibusiso Hendry and my daughter, Brilliant Silindokuhle, for their continued support, believing in me and being my source of strength.

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ABSTRACT

Studies show that Euclidean geometry is a section of school mathematics that takes more work to learn and teach in South Africa. Learners need to do better in Grade 12 matric examinations. The role of teachers in the learning and teaching process is critical for learners' success. This study explored Grade 12 teachers' Pedagogical Content Knowledge (PCK) about similarity in Euclidean geometry, as their depth in a subject-specific PCK influences their classroom practices. Issues raised in the Grade 12 NSC diagnostic reports of the five consecutive years, 2017 to 2021, formed a basis for what to pursue. A qualitative research method was followed to establish the teachers' PCK via their ability to identify and correct the misconceptions and errors of the Grade 12 learners using written work of similarity concepts in Euclidean geometry. Eight secondary school teachers who offer mathematics in the Gauteng North district in the Gauteng province were purposively sampled and participated in the study.

The study's findings revealed that with some of the teachers, their PCK depth of similarity in Euclidean geometry is lacking in identifying and addressing learner misconceptions and errors. Their challenges are similar to those of learners, as reported in the NSC diagnostic reports. These findings have several implications. This highlights the greater challenge of breaking the cycle of learners' poor performance in Euclidean geometry in South African schools. Secondly, the study's outcomes identify where one can intervene in improving the teachers' PCK. Lastly, it poses the challenge of revisiting the school curriculum by the Department of Basic Education and for South African tertiary institutions that train secondary school teachers to also re-visit their training programmes.

KEY TERMS AND CLARIFICATIONS

Euclidean Geometry: It is a branch of mathematics that focuses on the study of the properties of geometrical figures and requires that learners deliver proofs based on axioms and theorems which were developed by a Greek mathematician, Euclid who lived 300 years BC. However, there have been changes in the structure over the years. (Machisi, 2021:2).

Pedagogical Content Knowledge (PCK) refers to the depth of Knowledge of ways of representing and formulating the subject that makes it comprehensible to others and an understanding of what makes the learning of topics easy or difficult: the concepts and preoccupations that different ages and backgrounds bring with them (Shulman, 1986:6).

Similarity: According to Amaral-Schio and Hollebrand (2021:3), polygons are similar if their corresponding angles are equal when measured and the side lengths are proportionate.

Proportionality: Van Dooren *et al.* (2018:14) believe that proportionality is the capstone of numbers, concepts in measurements and elementary arithmetic. It is one of the most elementary understanding that is needed for advanced mathematics. It describes the equality of ratios; however, a ratio describes the relationship between two quantities with the same units (Siyavula, 2015).

Error: Errors are mistakes, faults, or slips that learners or teachers usually commit during the learning process (Brodie, 2011:29). Sometimes, errors could be caused by the teachers' inadequate explanation of concepts. However, this is not always the case, as errors can result from learners' misconceptions, inadequate listening skills or limited understanding of certain concepts (Brodie, 2011:31).

Misconception: It is defined as an unavoidable and not easy-to-resolve problem that learners encounter, which may obstruct the understanding of concepts (Dikgomo, 1994:27). Furthermore, misconceptions are misunderstandings and misinterpretations based on incorrect meaning (Ojose, 2015:8).

Content Knowledge (CK): The knowledge of the subject matter and specific subject teachers must teach (Sherrie & Mawela, 2022:50).

Topic- Specific Pedagogical Content Knowledge (TSPCK) refers to teachers' pedagogical content knowledge focusing on specific Content (Rollnick & Mavhunga, 2015:137).

Examination guideline Is a document that helps teachers prepare learners for their final external examinations (For example, the Grade 12 National Senior Certificate (NSC) Examination) as a guide or scope. It clarifies how deep the teachers should go about assessing the content in a particular Grade. However, it does not substitute the national Curriculum and Assessment Policy Statement (CAPS) document used by school teachers (DBE, 2021:3).

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LIST OF ACRONYMS

NCAPS	National Curriculum and Assessment Policy Statement
DBE	Department of Basic Education
FET	Further Education and Training
TIMSS	Trends in International Mathematics and Science Study
SSIP	Secondary School Improvement Programme
NSC	National Senior Certificate
PCK	Pedagogical Content Knowledge
CK	Content Knowledge
KCT	Knowledge of Curriculum and Teaching
TSPCK	Topic Specific Pedagogical Content Knowledge
KCC	Knowledge of Content and Curriculum
ANA	Annual National Assessment
KCS	Knowledge of Curriculum and Student
ATP	Annual Teaching Plan
SBA	School Based Assessment

1.1. INTRODUCTION

Globally, mathematics education is challenging in terms of performance. For example, Authors reported on studies done in countries such as to cite few: United States of America (USA), Nigeria, New Zealand and Australia that high school learners perform very poorly when it comes to geometry (Adolphus, 2011:145; Atebe, 2008:45; Combrinck, 2003: 54; Roux, 2003:367; Siyepu, 2005:69). South Africa as a country is not immune to the challenges of mathematics education. This is more so in school leaving examinations, especially in Euclidean geometry (DBE, 2017; DBE, 2018; DBE, 2019; DBE, 2020; DBE, 2021).

The average achievement of mathematics learners in each Grade has been ranging between 30% and 40% or even beyond that across the South African schooling system nationwide (Okitowamba *et al.*, 2018:43). Over the years, South Africa has been participating in various mathematics assessment studies such as Trends in International Mathematics and Science Study (TIMSS) internationally, and nationally with the Annual National Assessment (ANA) (Van der Berg, 2015) with the aim of monitoring and evaluating the quality of its education system, the results based on these studies are that South African learners perform inadequately in mathematics across some of the Grades (Chikiwa, 2017:39).

Amongst the 11 key subject areas, which include Accounting, Agricultural sciences, Business Studies, Economics, English First Additional Language, Geography, History, Life Sciences, Mathematical literacy and Physical Sciences, mathematics has been ranked the lowest of them all in terms of performance in South Africa (van der Berg, 2015). Furthermore, the national pass percentage in mathematics has been ranging between 40% and 60% year in and year out ever since the year 2010 (DBE, 2012:5; DBE, 2017:6; DBE, 2018:9; DBE, 2021:5). Though the mathematics performance may show a slight improvement which is from 54% in the year 2020 to 58% in 2021, South Africa is still facing a challenge in terms of producing quality results in mathematics (DBE, 2021:184). This is indicated in Table 1 below, whereby learners who obtained 40% and above are still below 40% from 2017 to 2021.

**Table 1: Overall performance in NSC mathematics from the year 2017- 2021:
Extracted from 2021 diagnostic report (DBE, 2021:184)**

Year	No. wrote	No. achieved at 30% and above	% achieved at 30% and above	No. achieved at 40% and above	% achieved at 40% and above
2017	245 103	127 197	51,9	86 096	35,1
2018	233 858	135 638	58,0	86 874	37,1
2019	222 034	121 179	54,6	77 751	35,0
2020	233 315	125 526	53,8	82 964	35,6
2021	259 143	149 177	57,6	97 561	37,6

From Table 1 above, some learners still achieve around 30% in mathematics, which could be much better by any standards. Some learners intending to pursue careers requiring an achievement score of 30% and above after completing Grade 12 find it difficult to follow those careers.

South Africa is one of the many countries that have participated in TIMSS 1995 and subsequent years. South African learners have consistently performed poorly in mathematics, particularly geometry (Reddy, 2005:125). The 2014 to 2016 Grade 12 National Senior Certificate (NSC) diagnostic reports also indicated that Euclidean geometry had been identified as one of the most challenging topics in mathematics, particularly in the proportionality and similarity sections, as learners performed at an average of 33% in it (Chihambakwe, 2017:44).

The condition of similarity between two polygons entails equal corresponding angles and corresponding side lengths being proportional. It appears that learners need help with the proportionality component. As much as similarity entails proportionality, proportionality does not only apply to similarity in geometry. That is why this study sometimes talks of proportionality and similarity as separate. Even from 2017 to 2021, National Senior Certificate diagnostic reports indicate that Grade 12 learners performed below par (below 50%) in Euclidean geometry's proportionality and similarity sections. Euclidean geometry as a subsection of mathematics constitutes 50 out of the 150 marks in paper 2 of the mathematics examination, and therefore, about 17% of the entire subject matter in the

mathematics examination, which is made up of paper 1 and paper 2 (DBE, 2011:20). Therefore, Euclidean geometry forms a larger part of mathematics paper 2 in the matric examination (DBE, 2017:164; DBE, 2018:143; DBE, 2019:192; DBE, 2020:195; DBE, 2021:198). It is given more attention in terms of teaching time and assessment weight compared to the other topics. When learners need to obtain marks in this section or can only attain minimal marks, this reduces their chances to succeed significantly. Table 2 below indicates how the mathematics topics and their mark allocation are distributed in the mathematics examination. The examination is made up of two papers, that is paper 1 and paper 2.

Table 2: Allocation of marks per topic in Grade 12 mathematics paper 1 and paper 2: Extracted from the CAPS document Grade 10-12 (DBE, 2011:20).

Paper 1: 3 hours		Paper 2: 3 hours	
Algebraic expressions and equations (and inequalities)	25	Euclidean Geometry and measurement	50
Number patterns	25	Analytical Geometry	40
Functions and graphs	35	Trigonometry	40
Finance, growth and decay	15	Statistics	20
Differential Calculus	35		
Counting and probability	15		
Total marks	150	Total marks	150

Euclidean geometry not only forms a significant part of paper 2 but learners need to perform better in it. This is more so for the proportionality and similarity sections. The Diagnostic Reports of examination papers also reveal that learners tend to attempt something other than the sections of proportionality and similarity in Euclidean geometry. There could be several reasons why learners perform badly in the sections above. However, with its limited scope for this study, I have focused on the Grade 12 mathematics teachers' pedagogical content knowledge of similarity in Euclidean geometry.

1.2. Rationale of the study

Based on the researcher's experience and observations as a teacher, moderator, and marker of the NSC examination, learners would either not attempt to answer the geometry sections on proportionality and similarity by leaving blank spaces on their answer sheets or write incorrect solutions and score very low marks. Though some would try their best to ensure that they give answers to the questions, they would be showing some misconceptions and errors.

To state a few examples: some learners would prove a theorem without justifying the construction procedure or drawing any construction, leading to an incomplete argument (justification); hence, they forfeit all the marks allocated for that theorem. A theorem in Euclidean geometry is allocated about five to six marks. Learners would provide incorrect reasons for their statements, and others would give answers to the questions based on their assumptions about the given diagrams without further reading through the given instructions of that diagram. For example, when given a circle with a chord on it, they would conclude that the chord is the circle's diameter, even though the centre of that circle is not given.

During the first term of each year, Grade 12 teachers across all the districts in Gauteng province are invited to attend a roadshow, whereby the examiners present all the misconceptions and errors that the learners have committed in their previous year's NSC examination. Teachers would be provided with examination guidelines documents after the roadshow so that they can use them in conjunction with the CAPS document, Annual Teaching Plan (ATP), and their CAPS-compliant textbooks during the preparations and presentations of the lessons in their respective schools. The national diagnostic report is also provided for every year's NSC examination; it indicates the common misconceptions and errors that learners commit during the writing of their NSC examination papers for every topic in every subject and the suggestions for improvement (DBE, 2017:164; DBE, 2018:143; DBE, 2019:192; DBE, 2020:195; DBE, 2021:198). To my surprise, some misconceptions and errors stated on the mathematics diagnostic reports or shared during the roadshows persist yearly.

Orippa (2020:74) conducted a study on exploring the pedagogical content knowledge of Further Education and Training (FET) mathematics teachers at the Grade 10 level; her findings were that teachers were able to adequately display all the categories of pedagogical content knowledge, which are knowledge of Content and Curriculum (KCC), Knowledge of Content and Students (KCS) as well as the Knowledge of Content and Teaching (KCT).

However, they only displayed minimal knowledge regarding KCT and KCS since they followed the lecturing method when teaching their learners. They could identify but needed help to correct and address misconceptions that learners commit. According to Rollnick and Mavhunga (2017:507), pedagogical content knowledge refers to the depth of teachers' knowledge of how content should be taught.

Ball *et al.* (2008:398) propose that teachers need knowledge of the subject and how that subject can be taught meaningfully. Hence, Orippa (2020:5) affirms that pedagogical content knowledge is significant for teachers because it combines knowledge of the subject and how it can be taught.

Teachers' inadequate Content knowledge and inadequate pedagogical content knowledge are of the underlying factors that result in learners' poor performance in mathematics nationwide (Adler & Venkat, 2012:5; Banjo, 2019:12; Bansilal *et al.*, 2014:41). According to other researchers, some of the other factors resulting in poor learner performance in mathematics are teachers' limited mathematical Content Knowledge and lack of teaching methods (Pournara *et al.*, 2015:197), lack of interest and motivation by learners (Cook, 2017:9), inadequate and old-fashioned teaching methods (Khumalo *et al.*, 2016:28). Most of the researchers have identified some of the categories of pedagogical content knowledge as factors leading to poor performance in mathematics. This has motivated the researcher to explore the depth of the Grade 12 teachers' pedagogical content knowledge in identifying and correcting the learners' misconceptions and errors in Euclidean geometry, similarity in particular.

1.2.1. Background of the study

Throughout the eight years of teaching mathematics and as head of the department in a school, the researcher has observed challenges in teaching and learning Euclidean

geometry in South Africa. This observation has been upheld as not unique to the one school while working as a marker for the National Senior Certificate (NSC) mathematics paper 2. As a moderator for the School-Based Assessments (SBA) in Gauteng province for six years, it was further observed that the problem of poor performance in Euclidean geometry is widespread and occurs throughout the Gauteng North District.

After this realisation, mathematics teachers have extended their teaching commitments by providing extra lessons to learners. Their motives were to provide extra help to learners to alleviate the problems that led to poor performance in school mathematics. Parents have also tried to intervene by paying private tutors to assist their children. The Gauteng Department of Education has also implemented an intervention plan called the Secondary School Improvement Programme (SSIP) to improve the Grade 12 results. Despite this, learners still perform unfavourably in school mathematics (Nkomo, 2014:2). SSIP is an intervention programme which involves weekend classes at selected priority schools in the province. Schools classified as priority have a matric pass rate below the provincial set mark of 70%.

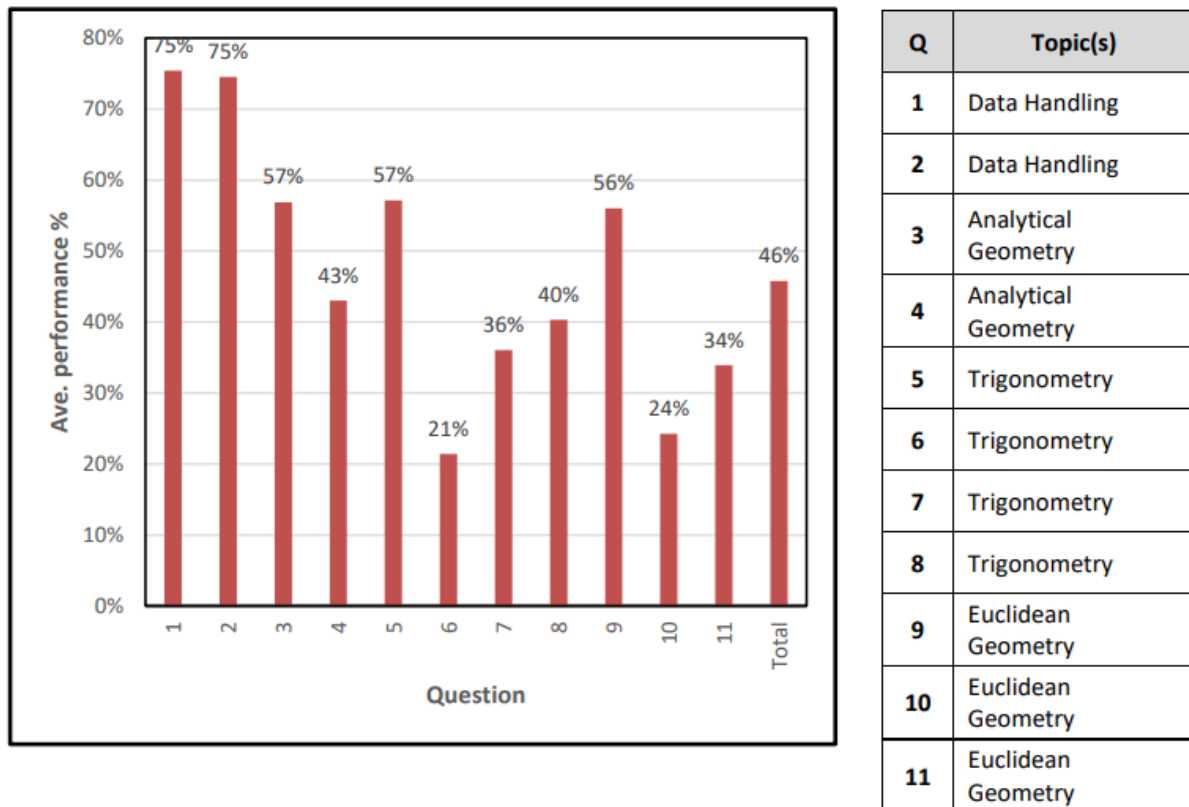
1.2.2. Statement of the problem

The researcher observed over eight years, as discussed earlier in section 1.2.1. above, most Grade 12 learners perform poorly in the Euclidean geometry section of the National Senior Certificate examination, and the Gauteng North District is no exception. Learners need to perform better in Euclidean geometry, particularly in the section on proportionality and similarity. The Proportionality and Similarity sections are always assessed in the last questions of the paper 2 NSC mathematics examination, which could be questions 9, 10, 11 and 12, depending on the number of questions the paper has. Question 9 and 10 from Table 3 below indicates the learner performance in the proportionality and Similarity sections for 2021. The performance is at 24% and 34%, respectively, which is not unique to the previous years (refer to Table 3 below).

The performance has never been above 50% in the proportionality and similarity sections since the previous five presiding years. In 2020, learners performed 43% and 51% in the proportionality and similarity sections in questions 9 and 10, respectively. In 2019, learners performed at 44% and 44% in questions 9 and 10, which form part of the proportionality and similarity sections. The same applies to 2018 proportionality and

similarity, as learners performed at 49% and 31% in questions 9 and 10, respectively. Lastly, the 2017 performance in proportionality and similarity was at 34% for question 9, 34% for question 10 and 41% for question 11 (DBE, 2017:164; DBE, 2018:143; DBE, 2019:192; DBE, 2020:195; DBE, 2021:198).

Table 3: 2021 NSC analysis per question for mathematics paper 2. Extract from (DBE, 2021:198)



The researcher acknowledges that there could be many other factors contributing to the poor performance of the learners. Based on the findings made by Tachie (2020:297), most mathematics teachers were unable to effectively teach geometry due to a lack of subject content knowledge and pedagogical content knowledge. According to Siyepu (2005:5), lack of basic skills and knowledge from primary schools, traditional teaching methods or poor teaching skills, and language problems are seen by other researchers as the underlying causes of poor performance in High school geometry. Siyepu (2005:19) believes that in mathematics, learners should be taught for conceptual understanding instead of promoting memorisation. Of significant is the fact that Euclidean geometry has

been in and out of the South African school curriculum, Section 2.5. further elaborates on this occurrence.

Partly, that is why, in this study, the researcher explored the Grade 12 teachers' PCK depth by analysing the teachers' responses concerning learners' work in Euclidean geometry. The analysis focused on their ability to identify learners' misconceptions and errors and intervene in improving the understanding of Euclidean geometry- specifically, in similarity.

According to Tachie (2020:299), Euclidean geometry was introduced as a main topic in the national school curriculum to help those who are studying it to be geometrically inclined so that they can be able to have long-term memory restoration, have improved level of reasoning as well as thinking capacity and be able to relate well to the other branches of mathematics in higher studies easily. Machisi (2021:2) says that the Department of Basic Education has implemented a CAPS training programme to develop all mathematics teachers in South Africa on the new subject content, such as Statistical regression, Euclidean geometry and Probability that came with the CAPS to try to address the concerns by some of the teachers.

Ndlovu (2013:277) states that only some trained teachers were fully satisfied with the training. There has been a survey that explored the views of South African mathematics teachers on the CAPS training, which was held in 2012, and 60% of the 150 teachers who participated in it said they were still not comfortable nor had confidence in teaching Euclidean geometry (Olivier, 2014:2). Dube (2016:13) also added that the CAPS training facilitators seemed to have lack of Content Knowledge and the required skills to develop the teachers adequately. She further states that the facilitators would even train the teachers for only three days on the content that should be covered for the year. In South Africa, some teachers in schools did not do Euclidean geometry in high school, college or university and yet they are expected to teach the topic in the CAPS (Govender, 2014:2). It is partly for that reason that the teaching of Euclidean geometry in high school remains a challenge in South Africa.

1.2.3. Research aims and objectives

This study addressed the main aim via the objectives. The following subsections are an elaboration on the aim and objectives.

1.2.3.1. Main objective

The study aims to explore a sample of Grade 12 teachers' pedagogical content knowledge concerning similarity in Euclidean geometry. In achieving the study's main aim, the following objectives were pursued.

1.2.3.2. Secondary objectives

- To explore Grade 12 mathematics teachers' competence in identifying and correcting misconceptions and errors from Grade 12 learners' written work focusing on similarity.
- To explore how Grade 12 mathematics teachers apply their pedagogical content knowledge of Euclidean geometry when identifying and correcting misconceptions and errors in problems related to similarity.
- To explore Grade 12 teachers' experiences when identifying and correcting errors and misconceptions in Euclidean geometry problems related to similarity.

1.2.4. Research questions

The following is the research question of the study.

1.2.4.1. Main question

The main research question which has been addressed in the current study is:

What is the depth of the Grade 12 teachers' pedagogical content knowledge concerning similarity in Euclidean geometry?

To answer the research question, the following sub-research questions were formulated.

1.2.4.2. Sub- research questions

- What is the competence of Grade 12 mathematics teachers in identifying and correcting misconceptions and errors from Grade 12 learners' written work focusing on similarity?
- How do Grade 12 mathematics teachers apply their pedagogical content knowledge of Euclidean geometry when identifying and correcting misconceptions and errors in problems related to similarity?
- What are the experiences of Grade 12 teachers' when identifying and correcting misconceptions and errors in learners' responses related to similarity?

1.3. Significance of the study

This study will greatly impact the teaching and learning of mathematics in the FET phase in Grade 12 Euclidean geometry. The researcher believes that this study will finally make a huge contribution in many areas. Firstly, it will improve both teacher knowledge and learner performance in mathematics. Secondly, it will serve as an eye opener for those intending/ planning to conduct content training/ development or in-service training workshops for teachers. Thirdly, it will enlighten teachers on what PCK entails and help them understand the importance of PCK when teaching and addressing problems in mathematics. Fourthly, it will assist teachers to improve their level of PCK in Euclidean geometry to improve learner performance in Euclidean geometry and mathematics. Lastly, but not least, it will increase the number of studies conducted on teachers' pedagogical content knowledge in identifying and correcting learners' misconceptions and errors in Euclidean geometry.

The researcher will share the research findings and recommendations with the Gauteng North district at large to ensure an improvement in Euclidean geometry and mathematics results for the Grade 12 learners in all schools under the Gauteng North district. The following stakeholders will benefit from this study: Mathematics teachers, learners, mathematics student teachers, mathematics Senior Education specialists, mathematics Deputy chief education specialists as well as researchers/scholars. The next chapter will

deliberate on Literature review, followed by the methodology chapter, then data analysis and the last chapter will be based on the discussion of findings and recommendations.

1.4. SUMMARY

In this chapter, the researcher has highlighted the introduction, rationale background of the study, statement of the problem, research aims and objectives and significance of the study. The next chapter provides the literature reviewed on the depth of Grade 12 teachers' pedagogical content knowledge by analysing learners' written work by identifying and correcting learners' misconceptions and errors in Euclidean geometry. It further alludes to the following: Studies conducted in South Africa and international studies about Euclidean geometry.

CHAPTER 2: LITERATURE REVIEW

2.1. Introduction

This chapter will deliberate on what Content Knowledge (CK) is, pedagogical content knowledge (PCK), some studies conducted on teachers' pedagogical content knowledge, topic-specific pedagogical content knowledge (TSPCK), the history of Euclidean geometry, the National Curriculum and Assessment policy statement on Euclidean geometry, the teaching and learning of similarity and congruency, the teaching and learning of proportionality, misconceptions and errors in general, misconceptions and errors in Euclidean geometry, studies conducted in South Africa about Euclidean geometry, studies conducted internationally about Euclidean geometry and lastly, the theoretical framework of the proposed study as well as the summary.

2.2. Content Knowledge (CK) and Pedagogical content knowledge (PCK)

Section 2.2 will have an introduction stating that although this study is on pedagogical content knowledge, one cannot talk of PCK without mentioning or elaborating on CK as they are intrinsically linked. Section 2.2.1 will be on Content Knowledge, whilst section 2.2.2 will be on pedagogical content knowledge.

2.2.1. Content Knowledge (CK)

Content Knowledge is the teachers' knowledge of the subject, including the rules, concepts and procedures used for problem-solving in a subject (Brijlall, 2011). Shulman (1986:9) describes Content Knowledge as "going beyond the knowledge of the facts or concepts of a domain. It requires an understanding of the structures of the subject matter". Content Knowledge is differentiated into three categories that are: pedagogical knowledge, subject matter knowledge and curriculum knowledge (Mavhunga, 2012:16). On the other hand, Seherrie and Mawela (2022:50) define Content Knowledge as subject matter knowledge (SMK) that is acquired and developed by a teacher within his/ her field of specialisation.

In a study conducted by Bansilal *et al.* (2014:41), some KwaZulu Natal mathematics teachers were given a Grade 12 NSC past examination question paper to answer and

explore their mathematics Content Knowledge. They could have performed better. It has been revealed that most of the 253 teachers who took part in the study obtained an average of approximately 59%. Nearly 25% scored below 39% on the test (Bansilal *et al.*, 2014:52). The findings show that some South African teachers still need to improve their Content Knowledge in mathematics, to address poor learning performance. Most mathematics teachers lack Content Knowledge and strong teaching skills due to content training, which they did not properly receive (Bernstein, 2014:38). She further alludes that learner performance is mainly determined by the teacher's pedagogical content knowledge depth in a subject.

In their findings, Krauss *et al.* (2008:888) state that a teacher with very strong subject matter knowledge can have good pedagogical content knowledge. However, this is only sometimes the case, as Krauss *et al.* further allude that a teacher can have good pedagogical content knowledge and at the same time display poor Content Knowledge in a certain area of the topic or subject, so to say, because usually, teachers are trained to specialise in or teach only two subjects. From these findings, it is believed that PCK combines Content and pedagogical knowledge (Krauss *et al.*, 2008:888). It has been indicated that knowledge of learners' thinking and teachers' pedagogic skills in a subject are key aspects of PCK (Krauss *et al.*, 2008:888).

2.2.2. Pedagogical content knowledge (PCK)

Shulman (1986:6) defines pedagogical content knowledge (PCK) as the ways of representing and formulating the subject that make it comprehensible to others and an understanding of what makes the learning of topics easy or difficult: the concepts and preoccupations that different ages and backgrounds bring with them. He also advocated that teachers must understand the subject matter for themselves and be able to elucidate that knowledge in new ways, metaphors, exercises, examples and demonstrations so that it can be grasped by the learners (Shulman 1987:12). He then further suggested several key elements of PCK that is:

1. Knowledge and understanding of subject matter (content knowledge);
2. Understanding of learners' conceptions of the subject and the learning and teaching implications that were associated with the specific subject matter; and
3. General pedagogical knowledge (or teaching strategies/ methodologies).

To complete what he called the knowledge base for teaching, he included other elements:

1. Curriculum knowledge
2. Knowledge of educational contexts
3. Knowledge of the purposes of education (Shulman, 1987).

According to Shulman (1986:8), pedagogical content knowledge is the category that normally differentiates the level of understanding between content specialists and teachers. Other education scholars have expanded and promoted the development of PCK (Shulman, 1987:13). That has led to valuable insights on teaching and learning, among which is mathematical knowledge for teaching. Ball *et al.* (2008:395) see mathematical knowledge for teaching as the knowledge needed to carry out effective mathematics teaching. "Teaching" refers to everything teachers should do to support their learners' learning. It involves guiding learners on how to answer questions, monitoring learners' work and showing them how to solve problems; it also requires knowing school curriculum matters (Ball *et al.*, 2008:395).

Here, a careful look at what PCK is, as originally defined by Shulman and discussed earlier, one can find the elements of mathematical knowledge for teaching attended to in the definition. In short, PCK, as originally conceptualised, includes mathematical knowledge for teaching. This study is not about promoting a particular expansion on PCK since 1986 but to use its tenants to establish what profound knowledge or understanding teachers have of similarity in Euclidean geometry that can enable them to present it to learners in an understandable or comprehensible way. Through this study, one has attempted to establish the possible link that led to the learners' poor performance in Euclidean geometry, focusing on similarity.

For the South African situation, as alluded to above, Tachie (2020:309) states in the findings of his study that most of the teachers lacked mathematical Content Knowledge as well as pedagogical content knowledge to teach geometry effectively. Teachers need to have a rich conceptual understanding of the subject content that they teach to recognise and value the development of their pedagogical content knowledge (Loughran *et al.*, 2012:7). According to Shulman (1985:447), to become a teacher, one needs to

have a wide and highly structured body of knowledge. This includes categories of knowledge such as general pedagogical knowledge, knowledge of learners and their characteristics, curriculum knowledge, knowledge of educational ends, pedagogical content knowledge, Content Knowledge, and knowledge of educational contexts.

As alluded to in the previous Chapter, South Africa is facing a serious challenge in terms of learner performance in mathematics (Bansilal *et al.*, 2014:37; Fleisch, 2008:11; Kazima *et al.*, 2008:294; McAuliffe & Lubben, 2013:157; Mullis *et al.*, 2020:9; Reddy, 2006:17; Spaull, 2013:4). Based on the findings from the other studies it has been stated that many factors cause the underperformance in mathematics, however some of the researchers have identified lack of subject Content Knowledge and pedagogical content knowledge by the teachers as one of the root causes of poor learner performance in mathematics in South Africa (Fleisch, 2008:12; Graven, 2016:11; Hugo *et al.*, 2010:38; Mabena *et al.*, 2021:11; Scholar, 2008:16; Spaull, 2013:5; Taylor & Taylor, 2013:16; Venkat & Spaull, 2015:127). This problem of the identified root causes of underperformance in mathematics must be addressed urgently to improve learner performance.

The current study sought to establish pointers to an intervention strategy for poor mathematics performance. Hence, it explored the Grade 12 teachers' pedagogical content knowledge through their analysis of learners' written work. Chikiwa (2017:4) affirms that mathematics has been the most poorly performed subject among the key subject areas in the NSC examination results since 2013. As a result, this causes a decline in the number of learners opting for mathematics as their compulsory subject, as most learners choose to write mathematical literacy (Adler & Pillay, 2016:24).

Although some of these learners would continue to do mathematics, most of them are still underperforming, which reduces their chances of furthering their studies at the different institutions of higher learning, in particular the Sciences and Technology universities (Adler & Pillay, 2016:30). Taylor (2008:24) declared that as a teacher you cannot teach something that you do not know, hence teachers' professional development should be prioritised. Teachers should be knowledgeable about their subject to avoid imparting wrong information to the learners because that results in misconceptions and errors, hence, underperformance by learners.

Dube (2016:9) identifies the misallocation of teachers as one factor that results in poor learner performance in mathematics. In most cases, you will find that a teacher still needs to specialise in FET mathematics. The teacher is teaching the subject at the FET phase. They often have Knowledge of the Student, Knowledge of the Curriculum, and not Knowledge of the Subject. According to the researcher, pedagogical content knowledge is a special knowledge that a teacher often has about a certain subject so that he/she can impart it rightfully to the learners and make them understand. This implies that as a teacher, you need not only to possess academic knowledge but also to know the subject matter and have all the necessary skills and techniques to make it more comprehensible to learners (Dube, 2016:21).

2.3. Some studies conducted on teachers' pedagogical content knowledge

In a study conducted by Hill *et al.* (2005:371), it has been reported that teachers' mathematical knowledge for teaching is related to learner achievement. Policymakers can use this finding to work on putting more effort into improving teachers' knowledge of the subject to improve learner achievement. Not only that, but also to assist the new upcoming scholars in answering questions on how teachers' pedagogical content knowledge might affect learner results in a subject (Hill *et al.*, 2005:399).

Based on the observations made by Shulman (1986:399), teachers need not only to know the subject matter to teach a particular subject but also require special knowledge on how content can be taught and learnt. These findings are believed to be supporting and, at the same time, challenging the policy inventors (Hill *et al.*, 2005:400).

Teachers' Content Knowledge influences teaching and learning (Alex, 2019:1). He further affirms that various researchers believe that teachers' Content Knowledge impacts learners' performance and achievement. It has since been a cause for concern that most South African learners are underperforming in mathematics (Alex: 2019:2; Pournara *et al.*, 2015). Other researchers claim that teachers' poor Content Knowledge is the resulting cause of learners' underperformance in mathematics, and by so saying, they believe that if the teachers can be well trained in their mathematical content, then the learner's results can improve drastically (Alex, 2019:2; Pournara *et al.*, 2015).

Mji and Makgatho (2006:254) state that South Africa needs more qualified teachers, scientists, medical doctors, engineers, and other science-orientated professionals. As a result, South Africa is currently unable to expand and improve economically because whenever the country needs more scientifically and technologically oriented professionals, they must import foreign nationals (Frantz *et al.*, 1996:254; Gardner & Hill, 1999; Mji & Makgatho, 2006:254; Pratzner, 1994; Ramsuran, 2005). Both poor teaching skills and lack of Content Knowledge are the resulting factors of poor teaching standards (Mji & Makgatho, 2006:254).

In a study by Waller (2012:5), early intervention in mathematics is encouraged and believed to be very important because learner success during the early Grades of schooling strongly correlates to learner achievement outcomes in the future. According to the researcher, an intervention is support given to learners identified at a school level and closely monitored to improve or improve their results in a subject.

From the findings of the study by Waller (2012:118), there is a strong correlation between PCK and mathematics Intervention teachers' training; this validates the fact that educational training programmes help improve teachers' PCK depth. The same thing applies to the relationship between PCK and teacher experience, as it validated the benefits of having experience as a mathematics teacher (Waller, 2012:119). It has also been stated that professional learning communities (Wood, 2007:291) and coaching amongst colleagues play a very crucial role, and it also helps in terms of developing the new teachers' depth of PCK (Gilbertson *et al.*, 2007: 315; Jenkins & Veal, 2002:63; Wood, 2007:297). Ball and Cohen (1999:26) suggest that giving support during implementation is another crucial aspect of improving teacher training to improve learning.

A careful look at the definition of PCK, as given by Shulman (1986), shows that several elements contribute to what PCK is. Amongst them are ways of representing and formulating the subject that make it comprehensible to others and an understanding of what makes learning topics easy or difficult. These five knowledge components assist expert teachers in thinking about the content to be taught and its transformation (Mavhunga and Rollnick, 2016:834). These and other elements constituting PCK have

given rise to several sub-branches of PCK. Amongst them is topic-specific pedagogical content knowledge. The next subsection is an elaboration of such.

2.4. Topic-specific pedagogical content knowledge (TSPCK)

Topic-specific pedagogical content knowledge (TSPCK) is a sub-branch of PCK defined at a topic level and is known to enable a content topic to be transformed into a form reachable to learners (Mavhunga & Rollnick, 2016:831). TSPCK consists of teachers understanding learners' prior knowledge, curricular saliency, what makes a topic simple or complex to teach, demonstrations including powerful analogies and examples, and conceptual teaching strategies (Mavhunga, 2012:34).

TSPCK is the knowledge that allows teachers to convert their understanding of Content Knowledge of a particular topic (Mavhunga & Rollnick, 2016:834). In addition to the previous citation, Mavhunga and Rollnick (2013:119) further define TSPCK as the capability to change knowledge of a particular topic in a subject to a suitable teaching form. Its focus is based mainly on transforming subject matter or specific concepts at a topic level (Malcolm *et al.*, 2015:8). For this study, TSPCK is taken as the ability of teachers to transform the teaching of Euclidean geometry section to a form that is much more understandable to the learners.

TSPCK is unique from other PCKs such as Domain Specific PCK or General PCK, since its focus is on transforming subject matter or specific concepts at a topic level (Malcolm *et al.*, 2015:11). Teachers' lack of PCK has a negative impact towards the learner performance (Ndlovu, 2017:8), as Mamali (2015:1) believes that the challenge of poor learner performance in mathematics cannot be separated from how teachers and their learners interact during the learning process.

Rollnick and Mavhunga (2015:137) define topic-specific pedagogical content knowledge (TSPCK) as pedagogic knowledge more focused on a particular specific content. It is a subset of PCK. It embraces 5 pillars of teacher professional knowledge, which includes:

1. Assessment Knowledge
2. Pedagogical Knowledge
3. Content Knowledge

4. Curricular Knowledge; and
5. Knowledge of learners (Rollnick & Mavhunga, 2015:137)

The teacher's topic-specific pedagogical content knowledge (TSPCK), a subset of PCK, is displayed by the teacher in these forms or components:

1. Knowledge of conceptual teaching strategy
2. Knowledge of curriculum salience
3. Knowing what is difficult to teach
4. Learners prior knowledge
5. Representations and analogies (Rollnick & Mavhunga, 2015:139)

This knowledge informs classroom practice where the teachers' personal PCK interacts with classroom context and is amplified through learners' beliefs, prior knowledge and behaviour and seen in the learners' outcomes (Rollnick & Mavhunga, 2015:137). This study has focused on the above, in which the Content Knowledge can be displayed by depicting errors in the tasks given or by presenting solutions without errors. The pedagogical knowledge is exhibited by explaining what should be done to correct those errors and showing an understanding of learner thinking from written tasks.

2.5. The history of Euclidean geometry

Bassarear (2015:425) states that geometry studies shapes and their relationships and properties. Geometry is a branch of mathematics concerned with the properties of formations of circles, lines, angles, and points, as well as the basics of these shapes (Brannan *et al.*, 2002:298). In geometry, the focus or emphasis is on shape identification, measuring those shapes in different aspects and classifying them according to their properties (Tachie, 2020:298).

Geometry was used by most of the ancient cultures where it originated, such as Egyptian, Chinese, Indian and Babylonian cultures, to measure heights, distances, directions, lengths and angles (Ding & Jones, 2006:43). On the other hand, according to Mamali

(2015:6) Euclidean geometry deals with proofs and axioms of theorems through logical thinking. It includes the study of triangles and other polygons, circles, quadrilaterals, lines and angles.

Changes have affected all levels of education in South Africa before and after the advent of a democratic dispensation in 1994, including mathematics. These changes impacted teachers' pedagogical content knowledge and Content Knowledge as some of the newly introduced topics in mathematics, such as Euclidean geometry, were not studied at all by many teachers in their teacher training phase (Tachie, 2020:297). The South African schooling system has previously included geometry across all the General Education and Training (GET) and Further Education and Training (FET) Grades in the mathematics curriculum. However, it was made optional and assessed under mathematics paper 3 from 2008 until 2011 in all the FET Grades (Ngirishi, 2015:5). The education authorities decided to make geometry compulsory from 2011 once more for all the FET Grades (DBE, 2011:14).

The new curriculum, the Curriculum and Assessment Policy Statement (CAPS), was then implemented in 2012. Therefore, Euclidean geometry was compulsory in the mathematics curriculum from Grades 10 to 12. In 2012, Euclidean geometry was only introduced in Grade 10; the following year, in 2013, it was gradually introduced up to Grade 11 and finally introduced to Grade 12 in 2014 (DBE, 2011). This means that all learners from the FET phase have been studying Euclidean geometry as a compulsory section in mathematics since 2012. Most of the Grade 12 learners decided not to study Euclidean geometry in 2008 since it was made optional and included in Paper 3 due to the poor performance which was displayed by the Grade 12 learners in mathematics (Bansilal & Ubub, 2019:1; Makhubele *et al.*, 2015:26).

According to Bowie (2009:23), teachers were suspected of lacking pedagogical content knowledge in Euclidean geometry, hence the poor performance in the mathematics Grade 12 Examination. Tachie (2020:298) affirms that geometry is difficult for teachers who have to teach it and problematic for learners who have to learn it. One of the reasons for reinstating Euclidean geometry into the mathematics Curriculum was that numerous studies established that students in institutions of higher learning who did not study geometry in high school had poor mathematical skills compared to those who had a background of the study from high school level (Padayachee *et al.*, 2011:5).

2.6. Curriculum and Assessment Policy Statement (CAPS) focus on Euclidean geometry (DBE, 2011)

The Curriculum and Assessment Policy Statement (CAPS) (DBE, 2011), popularly known as the CAPS document, is a detailed South African education policy document which gives teachers clear guidance on what to teach, how to clarify concepts, what to emphasise and how, when and what to assess across all the Grades and per subject. It emphasises notional time per learning area and highlights the content areas teachers should focus on. The next paragraph briefly details how the CAPS presents Euclidean geometry from the General Education and Training (GET) to the Further Education and Training (FET) phase. According to the CAPS document (DBE, 2011:14), the Grade 12 Paper 2 carries a maximum mark of 150. The weighting of Euclidean geometry is 50 ± 3 marks in Grade 12 and should only be taught for two weeks. The topics which are the focus of this study are dealt with from Grade 10 to 12. See Table 4 below for Content specification per Grade:

Table 4: The South African geometry school curriculum (DBE, 2011)

PHASE	GRADE	TOPIC	CONCEPTS	VAN HIELE LEVEL
Foundation phase	R- 3	Space and shapes	1. Position, orientation, and views 2. 3-D objects 3. 2-D shapes 4. Symmetry	0
Intermediate phase	4- 6	Space and shapes	1. Properties of 2-D shapes 2. Properties of 3-D shapes 3. Symmetry 4. Transformations 5. Viewing of objects Position and movement	1
Senior phase	7- 9	Space and shapes	1. Geometry of 2-D shapes 2. Geometry of 3-D objects 3. Geometry of straight lines 4. Transformation Geometry Constructions of geometric figures	2
Further Education and Training phase (FET)	10	Euclidean geometry	1. Revise basic results established in earlier Grades regarding lines, angles, and triangles, especially the similarity and congruence of	3

			shapes. 2. The midpoint theorem 3. Properties of quadrilateral: kite, rectangle, rhombus, parallelogram, square and trapezium 4. Solve problems and prove riders using the properties of parallel lines, triangles, and quadrilaterals	
	11	Euclidean geometry	1. Accept results established in earlier Grades as axioms. 2. Investigate and prove the theorems of the geometry of circles. 3. Use the theorems and their converses, where they exist, to solve riders.	3 & 4
	12	Euclidean geometry	1. Conditions for polygons to be similar. 2. Prove the proportionality and similarity of shapes and the Pythagorean theorem by a similar triangle	3& 4

Most of the schoolteachers skipped the section on geometry and decided not to teach it to their learners. At the same time, it was previously made optional, so some learners would try to study and cover the topic on their own with slight success (Ngirishi, 2015:3). Teachers should teach geometry in such a manner that they can consider different aspects such as how their learners think about geometry as well as how it can be perceived and understood by their learners (Ngirishi, 2015:8). Teachers' mathematical knowledge for teaching is very important during the teaching and learning of geometry.

2.7. Visual “Stereotypes” in geometry

Hamilton and Troler (1986:133) define stereotype as “a cognitive structure that contains perceiver’s knowledge, beliefs and expectations about a human group”. In this study, “stereotypes” refer to the erroneous ideas or beliefs many people have about images or objects in Euclidean geometry based on their exterior appearances, which may be partly true or false.

Visualisation or recognition plays a very important part in teaching and learning geometry. Based on the Van Hiele learning theory, there are five levels of geometric reasoning, namely: visualisation, analysis, order, deduction, and rigour (See table 5 below) (Clements & Battista, 1992:20; Han, 1986:2; Howse & Howse, 2015:306; Van Hiele,

1984). Geometry cannot be properly mastered at Van-Hiele levels two to five if level one has not been attained (Han, 1986:7). According to Clements (2003:152), at level one (Visualisation) learners can recognise that a given figure is a rectangle if it resembles a door, without having to reason based on the properties of that shape.

These levels cannot be bypassed but should be sequentially followed by learners to enhance and reach an understanding of geometry (Alex & Mammen, 2016:182; Clements, 2003:152). A learner can only be taught to logically prove that a quadrilateral is a parallelogram, rectangle, or rhombus. In contrast, the learner can only visually recognise with the knowledge of the properties of shapes as to whether a given shape is a rectangle or rhombus. Some of the researchers believe that the van-Hiele levels are more convenient when it comes to describing learners' conceptual development in geometry (Burger & Shaughnessy, 1986:46; Clements & Battista, 1992:22; Clements, 2003:153; Fuys *et al.*, 1988:133).

Table 5: Van-Hiele levels of geometric taught (Howse & Howse, 2015:306)

Level	Description	Ability of student
1	Visual	Describes shapes on the basis of their appearance
2	Analysis	Describes shapes on the basis of their properties
3	Abstraction	Recognizes the importance of properties and the relationships among them, which assist students in logically ordering the properties of the shapes
4	Deduction	Attains logical reasoning ability and proves theorems deductively
5	Rigor	Establishes and analyzes theorems in different postulation systems

Teachers need to take into consideration the Van Hiele levels of geometric thinking model during teaching and learning since geometry deals with visualisation, positions, and shapes (Ngirishi, 2015:8). According to Genz (cited by Dongwi, 2018:41), "The Van Hiele theory is a learning model that describes the levels of geometric thinking learners go through as they move from a holistic perception of geometric shapes to a refined

understanding of geometric proof". The main aim of Van Hiele theory is to assist teachers in improving the teaching of geometry and help learners improve their understanding of geometry (Alex & Mammen, 2018:3). The Van Hiele's categorised their learners' thinking by levels (Alex & Mammen, 2016:175).

In the view of some, teachers should use the Van Hiele theory-based approach in teaching and learning Euclidean geometry. They should allow learners to share their inputs to help teachers change their teaching methods in the mathematics classroom. These will enable teachers to understand and identify the Van Hiele- levels at which their learners are operating as well as their learning needs (McAndrew *et al.*, 2017:48). I concur with that view as long it allows teachers to be able to create a good learning environment.

In most cases, learners prefer using concept images instead of defining the concepts based on their reasoning (Clements, 2003:155). The researcher affirms that, at times, that becomes a challenge because textbook diagrams could often be better represented, which might lead to misconceptions and errors. For example, most of the Grade 12 prescribed South African textbooks have stated that for similarity theorem "If two triangles are equiangular, then their corresponding sides are in proportion and the triangles are similar" (Aird *et al.*, 2013:283; Bowie *et al.*, 2015:241; DBE, 2021:10; DBE, 2011:14; DBE, 2021: 10; Malan *et al.*, 2020:91; Philips *et al.*, 2013:260; Smith, 2014:255), but the diagrams represented are equilateral triangles.

The researcher has observed that most South African textbooks entail the sketch of equilateral and isosceles triangles for the proofs of the proportionality and similarity theorems instead of representing the diagrams of general scalene triangles. Other researchers have observed that mathematics teachers often rely on textbook chalkboard teaching strategies when teaching geometric concepts (Armah & Kissi, 2019:2; De Villiers, 2012:13), which does not offer major spatial experience (Alex & Mammen, 2016:185; Erdoğan & Durmus, 2009:158).

For the same reasons, (Clements, 2003:155) advises that teachers avoid over-reliance on textbooks. Hershkowitz (1987) and Hershkowitz and Vinner (1980) add that some textbooks show prototyped examples, which may result in poor understanding of concepts. Furthermore, Kajander and Lovric (2009) affirm that some textbooks need

proper definitions of concepts and examples to show excellent cases of concepts. For that reason, teachers should possess a good level of Content Knowledge and pedagogical content knowledge for them to be able to address the misconceptions, errors and content gaps that are found in the textbooks or any other teaching materials used daily as they arise. Teachers should allow their learners to discover things independently instead of promoting memorisation. This is supported by a comment made by Alex and Mammen (2014) and Mtonjeni and Siyepu (2014:4) that the methods of teaching that are used in South Africa often promote rote learning and memorisation instead of promoting problem-solving skills and development of conceptual understanding.

2.8. The teaching and learning of similarity and congruency

The study focuses on the Grade 12 similarity section of Euclidean geometry. However, both similarity and congruency concepts form part of the study since they are inseparable. This is supported by the fact that Conditions for congruency can be applied to both similarity and congruency. Still, conditions for similarity cannot be applied in congruency because congruency is a special case of similarity. The notation for both similarity and congruency is ($///$) and ($\equiv$), respectively.

Figures are similar if one figure can be exactly-mapped on the other one when dilated (enlarged or reduced) (Philips *et al.*, 2013:259) in combination with reflections, rotations and translations (Seago *et al.*, 2014:2). However, if one figure can be exactly mapped onto the other only using a combination of reflections, rotations or translations they are congruent to each other (Kanold *et al.*, 2015:827). Polygons like rectangles, squares, hexagons, etc., are considered similar only if both the conditions of similarity are met.

Dündar and Gündüz (2017:39) established that it is difficult for teachers to relate similarity and congruency to real-life contexts. Siyepu (2005:61) believes you need to know lines, angles and shapes to learn Similarities and Congruencies in geometry. He further affirms that it is much preferable to start by introducing the angle measurement part in geometry before similarities and congruencies can be introduced (Siyepu, 2005:62). There are two conditions for similar polygons that both must be met (Amaral-Schio & Hollebrand, 2021:11), namely:

- 1) The corresponding angles are equal and

2) The corresponding sides are in the same proportion.

For example, Consider the two polygons below.

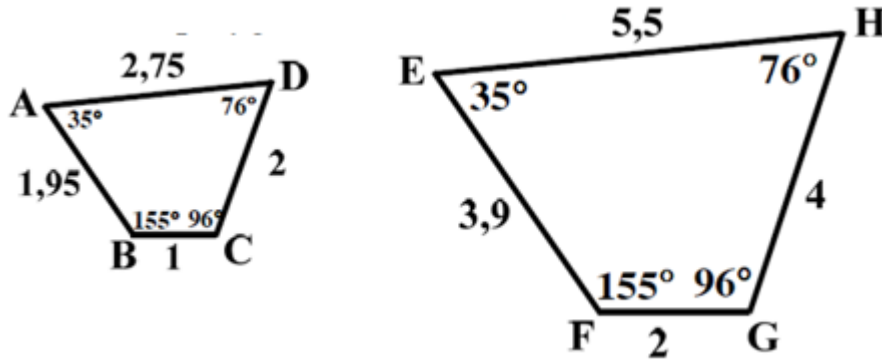


Figure 2.8.1. Diagrams for similar Polygons. (Philips *et al.*, 2013:259)

From figure 2.8.1 above: Since $\hat{A} = \hat{E} = 35^\circ$, $\hat{B} = \hat{F} = 155^\circ$, $\hat{C} = \hat{G} = 96^\circ$ and $\hat{D} = \hat{H} = 76^\circ$, corresponding angles are equal, the first condition is met. Since $\frac{AB}{EF} = \frac{1,95}{3,9} = \frac{1}{2}$, $\frac{BC}{FG} = \frac{1}{2}$, $\frac{CD}{GH} = \frac{2}{4} = \frac{1}{2}$ and $\frac{AD}{EH} = \frac{2,75}{5,5} = \frac{1}{2}$, the sides are in the same proportion, and the second condition is also met. Since both Condition 1 and Condition 2 have met, we can now conclude that $ABCD \sim EFGH$ (i.e. the two polygons are similar to each other). According to the similarity theorem, if two triangles have equal corresponding angles, their corresponding sides are in proportion, and the triangles are similar (Singh, 2018).

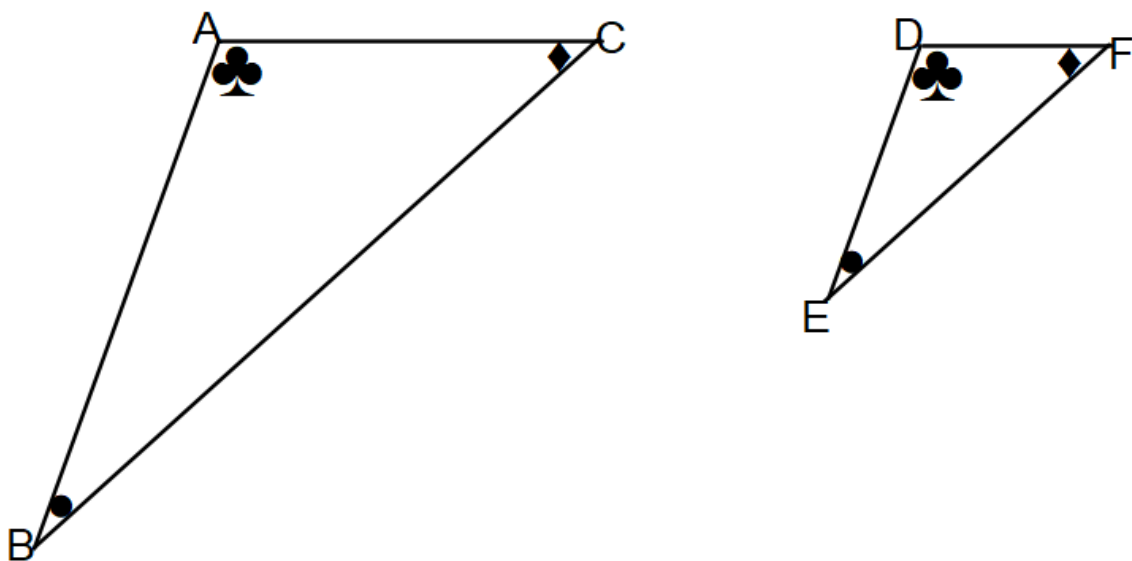


Figure 2.8.2.: Diagrams for similarity theorem

From the diagrams above which are $\triangle ABC$ and $\triangle DEF$, if it is given that $\hat{B} = \hat{E}$, $\hat{C} = \hat{F}$, and $\hat{A} = \hat{D}$ then it can be proven that $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$. As converse to the above theorem, Philips *et al.* (2013:275) state that if the corresponding sides of two triangles are in the same proportion, the corresponding angles are equal, and therefore the triangles are Similar. It simply means that if it is given from the above figure 2.8.2. that $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$ then it can be presumed that $\triangle ABC \sim \triangle DEF$.

2.9. The teaching and learning of proportionality

Proportionality describes the equality of two or more ratios. For example, $\frac{a}{b} = \frac{p}{q} = \frac{x}{y} = t$ (where t denotes a constant). However, a ratio describes the relationship between two quantities with the same units, $\frac{a}{b} = t$ (where t denotes a constant). Ratios can be used to compare heights, weights, currencies, lengths, etc. (Siyavula, 2015). There is a deep connection between proportionality and similarity naturally (Lee & Yim, 2014:542). Heath (1956:124) confirms that the similarity definition is based on proportion. On the other hand, Mbatha (2022:12) believes that to understand the concept of similarity, you need to master the concept of ratio and proportion since they form the foundation of similarity. For instance, a learner may be unable to prove a similarity theorem if he/ she lacks knowledge of proportionality.

In Euclidean geometry, the proportionality theorem statement states, "A line drawn parallel to one side of a triangle divides the other two sides of the triangle in the same proportion" (Phillips *et al.*, 2013:247). According to Phillips *et al.* (2016:169), a theorem is a true statement that can be validated using acceptable mathematical operations and arguments. Verifying that a theorem is true (or explaining logically deductively why it is true) is called proof.

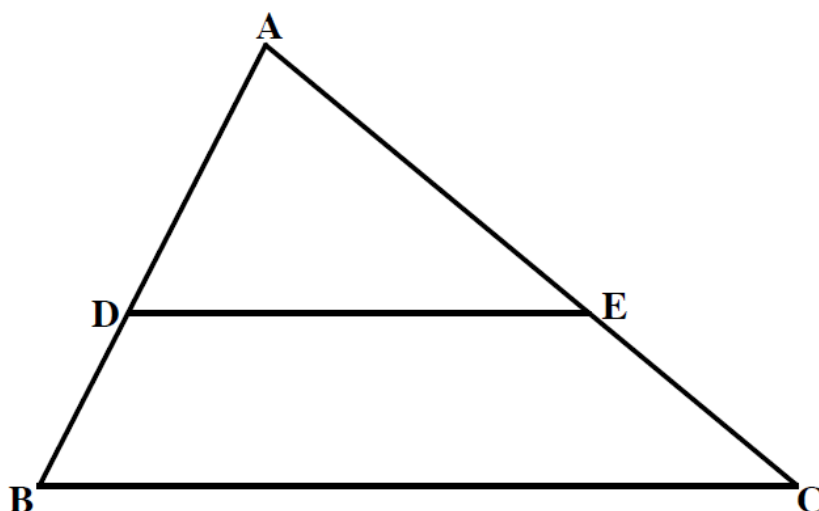


Figure 2.9.1.: A diagram for proportionality theorem: (Phillips *et al.*, 2013:247).

From Figure 2.9.1. above: If it is given that $\triangle ABC$ has $DE \parallel BC$, then it can be proven that $\frac{AD}{DB} = \frac{AE}{EC}$. Conversely, from the above diagram in Figure 2.9.1. it can also be proven that “If a line divides two sides of a triangle proportionally, then the line is parallel to the third side” (Phillips *et al.*, 2013:254). This means that from $\triangle ABC$ above in Figure 2.9.1. , it can deduced that if $\frac{BD}{AD} = \frac{CE}{AE}$ then it can be concluded that $DE \parallel BC$ (Phillips *et al.*, 2013:254).

The understanding of proportionality is not only crucial when working with mathematical courses such as probability or similarity in geometry, but it also plays a very significant role in our everyday life (Van Dooren *et al.*, 2018:14). The idea of proportionality from both secondary and higher education underlies other topics such as the usage of linear models in both statistics and calculus, abstraction in a vector space sense and linear algebra (Van Dooren *et al.*, 2018:16).

2.10. Misconceptions and Errors in general

This section deals with misconceptions and errors in general. In particular, it deliberates on them in teaching and learning Euclidean geometry, their definitions, and what causes such misconceptions and errors.

2.10.1. Misconceptions

Nesher (1987:35) sees a misconception as a “line of thinking that causes a series of errors all resulting from an incorrect underlying premise rather than sporadic unconnected and non-systematic errors”. Thabane (1994:6) argues that misconceptions can be a stumbling block in understanding mathematics concepts.

According to Ngirishi (2015:8), misunderstandings usually occur between the teacher and the learner during the learning process. Incorrect usage of a formula or lack of understanding during learning in a particular situation can cause misconceptions (Drews, 2005:8). Nesher (1987:7) is of the view that one misconception can cause a series of errors because, in mathematics, errors and ideas are closely attached.

Misconceptions should be rectified as soon as they are discovered because they can result in more complex and numerous problems if they are not corrected early (Ngirishi, 2015:7). He further emphasises that it is very important to study misconceptions and errors in mathematics, particularly in the section of geometry. Chazan (1993:9) believes that teachers often encounter numerous misconceptions that their learners commit during their daily lessons in geometry. People often view mathematics as challenging and dull because of the resulting misconceptions (Confrey, 1987:8), and they have a negative attitude towards the subject (Shaughnessy & Burger, 1985:8).

According to Nieuwoudt and Van der Sandt (2003:9), teachers should play a vital role in assisting their learners to overcome misconceptions to enhance conceptual understanding. An incorrect perception of a concept is often believed to result in misconceptions (Confrey, 1987:9). Confrey’s view about misconceptions is supported by Ngirishi (2015:9), who believes that misconceptions partly cause errors. In this study, a misconception is viewed as a misunderstanding or an incorrect opinion of a concept due to faulty understanding.

2.10.2. Errors

An error is defined as the state of believing what is false or perhaps deviating from accuracy (Makhubele, 2014:44). He further indicates that an error can still be referred to as a mistake, oversight, fault, slip, trespass, transgression, misdeed or blunder. Errors

are seen as mistakes that learners commit because of poor memory, lack of concentration or conceptual understanding (Ngirishi, 2015:9). Brodie (2011:29) defines errors as mistakes or slips that learners or teachers commit during the learning process. She further affirms that no one should be blamed for committing an error because that is part of development and learning for everyone involved in the learning process.

Some resulting errors could be due to the teachers' poor explanations to the learners. However, this is not always the case, as errors can also be the result of learners' misconceptions or poor understanding of certain concepts (Brodie, 2011:31). The word error means to wander, lost or mislaid (Harper, 2010:8). Errors are the signs of the learning difficulties that the learners experience during the learning process (Luneta, 2008:9). However, Swan (2001:9) defines an error as the misunderstanding of the given information.

Errors are divided into two types, namely systematic and non-systematic (Riccomini, 2005:239). He describes non-systematic errors as mistakes that learners commit unintentionally, and they can easily rectify them all by themselves. Such errors are often called lapses, unintended mistakes or slips (Riccomini, 2005:239).

Olivier (1989:9) supports Riccomini's view by defining non-systematic errors as unplanned mistakes that can be easily rectified when identified. Sometimes, being impulsive, overconfident, or hurried can lead learners to make many mistakes when they are engaged in a particular task (Makhubele, 2014:45).

Systematic errors may often be repeated, constructed or reconstructed systematically over a certain period as a result of preconceptions or misconceptions when dealing with a particular topic in a subject (Idris, 2011:43). Makhubele (2014:45) defines systematic errors as persistent and systematic. They are not responsive to re-explanation of concepts. He affirms that most studies have shown that most learners often make systematic errors while learning geometry. Errors that are commonly done by learners, which could be because of misconceptions, must be deconstructed.

2.10.3. Causes of Misconceptions and Errors

Luneta and Makonye (2010:36) state that misconceptions and errors are connected. However, they are not the same. For the smooth running of mathematics teaching and

learning, teachers should be aware of the misconceptions, try to deal with them as they occur and address them because they result in errors. It is much easier for learners to commit errors if they need to understand a certain mathematics concept.

Makhubele *et al.* (2015:27) affirm that misconceptions and errors may be triggered by the following sources: faulty schema, prior knowledge, negative attitude, faulty reasoning, knowledge acquisition, procedural and conceptual knowledge, mathematical anxiety, language, teacher, curricular factor, learning concepts and mathematical tasks. On the other hand, Luneta (2008:10) believes that misconceptions are caused by the following factors: different learning styles, lack of prior knowledge of the concept, lack of conceptual knowledge, different levels of cognition, language barriers and lack of focus during instruction.

2.10.4. Misconceptions and Errors in Euclidean geometry

As alluded to earlier, examination reports state that learners have been performing extremely poor in Euclidean geometry (DBE, 2017:172; DBE, 2018:150; DBE, 2019:199; DBE, 2020:195; DBE, 2021:198). Further, the national examination diagnostic reports cite some of the following misconceptions and errors as committed by Grade 12 learners, particularly in proportionality and similarity:

- Learners cannot differentiate between a theorem and its converse
- They provide incorrect reasons for their statements
- Learners confuse similarity with congruency
- Incorrect labelling of angles is still a challenge (For example, they would write $\hat{A}$ instead of $\hat{A}_1$ or, $\hat{B} = \hat{C}$ instead of $\hat{B}_1 = \hat{C}_1$, or $\hat{A}_{1+2}$ instead of $\hat{A}_1 + \hat{A}_2$)
- Learners would prove a theorem without marking/ writing any construction, which is a total breakdown and leads to the awarding of zero marks.
- They are unable to identify the correct ratios from the given diagrams.
- Many learners could not select the correct pairs of equal angles that will result in two triangles being similar. Instead, they state several angles that are each equal to

x and then conclude that the two triangles are similar (DBE, 2017:172; DBE, 2018:150; DBE, 2019:199; DBE, 2020:195; DBE, 2021:198).

The study used the above misconceptions and errors in the Grade 12 examination diagnostic reports to probe the teachers' pedagogical content knowledge. In addition, semi-structured interviews supporting the written task-based interviews have also been used to explore the teachers' PCK in Euclidean geometry, focusing on the similarity section. It was hoped that the outcomes would identify where intervention must focus on improving learners' performance.

South African high school learners were discovered to need to improve when understanding the basics of geometry, such as the properties and characteristics of shapes (De Villiers, 1997; Roux, 2003:365). Hence, they are unable to identify and name simple shapes like rhombus, parallelogram, triangle, kite and trapezium (Siyepu, 2005:65). Based on a study conducted by Marchiş (2008:137), it has been discovered that learners cannot define basic shapes in geometry because they cannot recognise and state their correct properties. Those learners failed to perceive that a square is a rhombus and a rectangle is a parallelogram (Marchiş, 2008:139). Learners are also displaying misconceptions about the concepts in geometry (Siyepu, 2005:65); as such, they still find it very difficult to understand the geometric concepts (Cunningham & Roberts, 2010; Kabaca *et al.*, 2011:47).

Poor understanding of concepts by learners in mathematics (Renne, 2004:260) and incorrect spelling of geometric concepts have been identified as the other main causes of misconceptions and errors in Euclidean geometry (Atebe, 2008:46). As stated by Kabaca *et al.* (2011:49), some of the learners are still finding it difficult to differentiate between $AB=CD$ (equal) and $AB\parallel CD$ (parallel). Hence, they still confuse the parallel with an equal symbol. Makhubele (2014:182) found that learners still need help to prove a geometric theorem, and they cannot deal with problems that require them to reason deductively and present a coherent argument. Other researchers also argue that most learners still need help to mark or state a construction when proving a geometric theorem (Healy & Hoyles, 2000:428; Lin, 2005:11; Weber, 2004).

The usage of addition and subtraction is one of the most often reported errors when it comes to items involving proportional reasoning (Van Dooren *et al.*, 2018:24). Hence,

learners still find it difficult to use proportional reasoning where appropriate (Van Dooren *et al.*, 2018:26).

Van Dooren *et al.* (2018:26) argue that there is still a shortage of research on “teachers’ understanding of proportionality”. Other studies have shown that not all teachers display the good content knowledge and pedagogical knowledge necessary to deal with proportionality problems, such as learners’ additive errors; hence, the primary and secondary Pre-service teachers commit errors when dealing with proportionality problems, especially those entailing similar shapes (Van Dooren *et al.*, 2018:28). Research shows that most of the teachers still find it difficult to differentiate between non-proportional and proportional situations (Van Dooren *et al.*, 2018:29). Learners could easily grasp the concept of proportionality at the high school level if it can be informally introduced at a very young age (Van Dooren *et al.*, 2018:29). This is supported by a claim made by Piaget that usually at the ages of 10 and a half to 11 years the child already has an idea of proportionality in mind (Spyrou & Kospentaris, 2005:1).

Cox (2013:2) conducted a study on learners' understanding of similarity topics, he found that learners' use of visual reasoning showed a more developed notion of similarity. The understanding of proportionality and similarity plays a very crucial role as it forms the basis of other areas in mathematics like trigonometry (Amaral-Schio & Hollebrands, 2021:15). On the other side, Chazan (1987:5) identified the following learning difficulties about similarity namely: 1. defining similarity, ratio and proportions, 2. Learners not understanding what proportional relationship means, 3. Geometric problems that entail enlargements and reductions.

Cox (2013:3) suggests that similarity tasks can be approached using the following strategies: visual, unitizing, functional scaling, avoidance, blending, additive and pattern building. She also proposes that when giving tasks, teachers should give tasks that require learners to scale more difficult geometric figures. Based on the study conducted by Lamon (1993:47), the problem of enlargement and reductions that require learners to use multiplicative reasoning was discovered to be the most difficult.

2.11. Studies conducted in South Africa about Euclidean geometry

Several studies on Euclidean geometry have been conducted in South Africa. They include a few: Dlamini (2012:2) investigated the “Grade 12 Teachers’ understanding of Euclidean geometry”, and he then suggested that Euclidean geometry should be made compulsory to all teacher education courses in the institutions of higher learning; Tachie (2020:298) has focused on the “Challenges faced by South African teachers in Teaching Euclidean geometry” (See “section 2.2.2. on page 3” for findings). Machisi (2021:2) explored the “Students’ reflection on their learning of Euclidean geometry”; Mbatha (2022) explored the “Pre-service teachers’ understanding of similarity and proofs in Euclidean geometry”, and Makhubele (2014:26) conducted a study based on misconceptions and errors displayed by Grade 11 learners in the learning of geometry.

Various researchers believe that teachers play a crucial role in the educational system more than the other stakeholders. This indicates that teachers lack of pedagogical content knowledge, Subject Content Knowledge and Curriculum Knowledge results in a negative impact towards learner performance in teaching and learning (Atebe, 2008:54; Nieuwoudt & van der Sandt, 2003:199; Stoker, 2003:11).

Perhaps because many mathematics learners dislike geometry as they consider it irrelevant, challenging, boring as well and too formal (Atebe, 2008:41), most of the teachers also find it difficult to teach it (Pegg, 1995:87). Even the potentially gifted learners still find Euclidean geometry difficult and too complicated, hence most of them study it through memorisation, which leaves them with little or no understanding of it all (Atebe, 2008:43).

In the South African schooling system, the geometry results at the high school level could improve if geometry can informally be introduced, thoroughly and properly be taught to the lower Grades at the Primary school level (De Villiers, 1996:13). In fact, teachers need to lay a very strong foundation from the Primary schools in geometry for learners to succeed well with their high school geometry.

2.12. Studies conducted internationally about Euclidean geometry

This section deliberates on studies done in other countries on Euclidean geometry. Most mathematics teachers from other countries such as New Zealand, Australia and America claim they still need proper NCS training in teaching geometry. Hence, they need help to teach it effectively (Combrinck, 2003:54). According to Tachie (2020:301), teachers in Zimbabwe with a General Certificate of Education (GCE) qualification find it very challenging to understand and teach Euclidean geometry in Zimbabwean Secondary schools as it was not properly taught in their former schooling and tertiary education.

In Chinese education, one of the focus areas in the teaching of mathematics was that all learners needed to study Euclidean geometry because, based on the observations by the traditional Chinese culture, there has been poor thinking ability and the need to do mathematics-related training about Euclidean geometry (Tachie, 2020:301). According to Adolphus (2011:145), the problem of poor performance by learners in Euclidean geometry in the Nigerian schooling system has been a major concern mainly to the following stakeholders' parents, mathematics teachers and the government. Adolphus (2011:151) studied "Problems of Teaching and Learning of Geometry in Secondary Schools in Rivers State, Nigeria". The following findings were discovered to be the challenges in the teaching and learning of Euclidean geometry:

1. Poor subject Content Knowledge displayed by most mathematics teachers in geometry.
2. Limited basic knowledge or poor foundation in mathematics by learners.
3. The environment could be more conducive to teaching and learning, so there needs to be more infrastructure and basic teaching and learning facilities.
4. Learners display a dismissive attitude towards learning. They need to be more willing and ready to learn.
5. Lack of motivation received by teachers, and as a result, they need to be committed to their work.
6. There needs to be proper monitoring and support that is taking place between the learners and teachers. If there could be any, then all these problems would have been eradicated (Adolphus, 2011:151).

Atebe (2008:45), Roux (2003:367), and Siyepu (2005:69) state that high school learners from the USA perform very poor when it comes to the section of geometry, particularly in items involving the basics of geometry, understanding the properties and characteristics of different shapes. Based on the research conducted in Nigeria, there is no effective teaching and learning of mathematics in general and geometry, specifically in Nigerian high schools (Atebe, 2008:56). Several factors were discovered to be the underlying factors contributing to poor performance in high school mathematics in the Nigerian schooling system. This includes the school environment, the learner's family background, teachers' qualifications, and the curriculum (Atebe, 2008:56).

Learner performance in geometry is much better in other countries like Japan than in other countries, including South Africa (De Villiers, 1996:13). In Japan, the basics of geometry are thoroughly instilled as early as from Grade 1. For example, the concept of similarity and congruency is formally introduced as early as Grade 5, whereas with the South African curriculum, such concepts are only introduced from Grade 8 -9 (De Villiers, 1996:13).

Different approaches to teaching similarity have been introduced, which include infusing transformation geometry in similarity in both the United States and Brazil since 2010 (Amaral-Schio & Hollebrands, 2021:7). Similarity is studied by learners in Grade 9 or 10 in the United States. The same applies to the Brazilian schooling system. However, it is considered the final Grade of middle school in Brazil (Amaral-Schio & Hollebrands, 2021:7).

2.13. Theoretical framework: (Social-Constructivism)

Bezuidenhout *et al.* (2021:64) define a theoretical framework as a detailed gathering of thoughts and theories about the phenomenon at hand or under investigation. According to Weegar and Pacis (2012:11), constructivism is "the philosophy that learners create their knowledge based on interactions with their environment including their interactions with other people". On the other hand, Janssen (1994:36) argues that constructivism is a learning theory that advocates that learners should be taught for conceptual understanding rather than taught only to reproduce theorems. The following types of

constructivism are key in a classroom setting namely, Cognitive constructivism, based on the work of Jean Piaget; and social constructivism, based on the work of Lev Vygotsky (Powell & Kalina, 2009: 241; Shonhiwa, 2020: 57).

For this study, I have chosen to align myself with social constructivism based on the work of Lev Vygotsky. Vygotsky's socio-cultural theory views human development as a socially mediated process in which children acquire their cultural values, beliefs, and problem-solving strategies through collaborative dialogues with more knowledgeable members of society (Vygotsky, 1978). He further states that learning is believed to be most effective when working within the Zone of Proximal Development (ZPD). Vygotsky defines the Zone of Proximal Development as the difference between a child's "actual developmental level as determined by independent problem solving" and the child's "potential development as determined through problem-solving under adult guidance or in collaboration with more capable peers" (Vygotsky 1978:38).

Other philosophers are of the view that learning happens fluidly and naturally. Still, Vygotsky believed that social interaction is an important aspect of learning (Vygotsky, 1978:26). Vygotsky further affirms that another way learners can acquire knowledge is by interacting with their peers or teachers, the outside influence within the social world, and they can also learn from the attitudes and beliefs that are shaped by their culture. Vygotsky (1978:26) affirmed that teachers should facilitate learning in the educational context to see more growth and motivate their learners.

This study supports the theory of Vygotsky. However, teachers should have and display a high level of PCK so that learners can learn from the best. Their level of PCK should be good, such that they know the subject matter, have various teaching methods and techniques, and can identify learners' misconceptions and errors and correct them. The motivation for choosing social constructivism as a theoretical framework is an acknowledgement that knowledge is socially constructed through interaction with others. In this study, the knowledge demonstrated by learners in the NSC examinations and later identified by the diagnostic reports was as a result of their interaction with their teachers. Chambliss (1996:137) reports that Piaget's work is considered cognitive constructivism. The researcher believes that learners gain knowledge and understanding not by the

teacher “pumping” it into them but by actively constructing their meaning of the topic. For example, when teaching similarity, understanding is created when learners struggle to construct their meaning of the concept of similarity, thereby deriving meaning for themselves before they are taught how to solve riders and geometric proofs.

Siyepu (2005:5) attests that lack of basic skills and knowledge from primary schools, traditional teaching methods or poor teaching skills, and language problems are seen by other researchers as the underlying causes of poor performance in High school geometry. Siyepu (2005:19) believes that in mathematics, learners should be taught for conceptual understanding instead of promoting memorisation. Teachers should allow learners to investigate, explore and discover things independently (DOE, 2002b:1).

In a constructivist approach, language is regarded as a “tool for taught and as central to learning” (Siyepu, 2005:31; Vygotsky, 1978:28). Siyepu (2005:61) is of the view that to learn Similarities and Congruencies in geometry, you need to know lines, angles and shapes. He further affirms that it is much preferable to start by introducing the angle measurement part in geometry before similarities and Congruencies can be introduced (Siyepu, 2005:62).

According to the researcher’s experience, teachers tend to provide learners with too much information on Euclidean geometry instead of allowing them to undergo a process of their investigations about the topic so that they can be corrected on what they have already discovered and learned from their mistakes. This idea, which is looking at the best way in which geometry can be taught in secondary schools, will be investigated. The current study focused on Exploring teachers’ PCK depth concerning similarity in Euclidean geometry.

Most teachers and textbook authors often provide learners with ready-made theorems and proofs without deriving and showing them the process of how to go about proving the theorems (De Villiers, 1996:17). De Villiers (1996:18) believes that the Constructivist learning theory needs to be taken seriously. For instance, he believes knowledge cannot be transferred directly from the teacher to the learner. Instead, learners should be allowed to re-construct the knowledge on their own so that it can be more meaningful. De Villiers

(1996:18) affirms that with the Constructivist approach, learners are not presented with ready-made content but are given an opportunity for self-discovery.

2.14. Summary

This study has shown that teachers' pedagogical content knowledge is one factor that plays a crucial role in their student learning. Learners cannot develop learning interests or perform outstandingly well if teachers possess poor teaching skills and lack Content Knowledge and knowledge about curriculum matters in mathematics and geometry. If teachers can master elements of PCK, then it can be easier for them to identify what is missing in their learners' prior knowledge, be able to identify and correct misconceptions and errors, make learning interesting and comprehensible to every learner and avoid the creation of undesirable content gaps in a subject amongst the learners. The geometry study is very important because it helps improve learners' conceptual understanding, thinking capacity level and reasoning skills. The next chapter will elaborate on this study's research methodology.

CHAPTER 3: RESEARCH METHODOLOGY

3.1. Introduction

This chapter is organised under the following sub-headings: the research design, research paradigm, research methods, research site, population and sampling methods, research instrument and data collection strategies, data analysis process, ethical considerations, and trustworthiness. The study sought to **explore the Grade 12 teachers' pedagogical content knowledge of similarity in Euclidean geometry**. The following main research question guided the study: *What is the Grade 12 teachers' pedagogical content knowledge depth concerning similarity in Euclidean geometry?* Furthermore, in addressing and answering the sub-research questions of this study, the Qualitative research method was used. This enabled the researcher to get answers to the phenomenon at hand.

3.2. Research design

According to Poti (2020:39), a research design is a blueprint or plan which states how the research will be conducted. Furthermore, Nieuwenhuis (2007) defines a research design as a plan or strategy which moves from the underlying philosophical assumptions to specifying the selection of respondents, the data-gathering techniques to be used, and the data analysis to be done. Asenahabi (2019:76) affirms that a suitable research design must be chosen for the researcher to conduct a research study successfully. Its essence is to convert the research problem into data for analysis to offer appropriate responses to the research questions at the lowest cost (Asenahabi, 2019:78).

Marczyk *et al.* (2005:23) present various ways of classifying research designs; they usually fall into one of three general categories: experimental, quasi-experimental, and non-experimental. This study followed a non-experimental design, a phenomenology strategy, because it will allow the researcher to identify the teacher's pedagogical content knowledge depth by analysing learners' written work. Polit and Beck (2010:563) define phenomenology as "a qualitative research tradition with roots in philosophy that focuses on the lived experience of humans." In this study, the lived experience is that of learners through their interaction with their teachers. How learners respond in the NSC

examinations, where they exhibit errors and misconceptions in Euclidean geometry, is as a result of their lived experience with their teachers when they were learning the subject in class. By its nature, one cannot rule out extraneous variables as the cause of what is being investigated. This is also because the researcher does not have control over the variables and the environments under study.

3.3. Research paradigm

According to Bertram and Christiansen (2014 :24), a research paradigm is “a particular worldview that defines for the researchers who hold this view, what is acceptable to research and how this should be done”. Creswell (2014:8) affirms that a research paradigm is a philosophical worldview the researcher holds about the participants, the data to be generated, the world in which the research is conducted, and the researcher’s values.

Philosophical worldviews are known to be the first ideas that are needed before a study can be developed. They inform our choice of theories that guide our study or research (Creswell, 2014:16). The study will employ social constructivism because the researcher seeks to understand the interpretation of learners’ work (Creswell, 2016:24). Hence, the purpose of the study was to explore the Grade 12 teachers’ PCK depth of similarity when identifying and correcting the learners’ misconceptions and errors in Euclidean geometry. The study's goal was to rely more on participants' views. Since knowledge is subjective, data was gathered through semi-structured interviews, written task-based interviews, and document analysis. Knowledge has been constructed by both the researcher and the participants.

3.4. Research method

A method refers to the range of approaches in research which are used to gather data that is to be used as a basis for prediction and explanation, interpretation, and inference (Kaplan, 1973:42). On the other hand, Creswell (2014:16) describes methodology as a research approach which describes how a study is to be conducted, how data is to be generated and from whom it shall be generated. The methodology aims to assist

everyone in understanding the product of scientific enquiry and the research process (Merton & Kendall, 1953:543).

Qualitative research intends to build rather than test concepts, theories, and hypotheses (Merriam,1998:45). For Creswell (2016:47), qualitative research can explore any problems or issues about any research phenomenon. It is the method deemed most suitable for data collection and analysis (Merriam,1998:44). For this study, a qualitative research approach has been selected because the data collected and analysed was in a non-numeric form (text) to have a subjective understanding of concepts, views of participants or construct meaning from their experiences. The qualitative research approach enables one to collect data that is not measurable (non-numeric) by using open-ended questions (Asenahabi, 2019:81). The Qualitative research method has been chosen for this study because of the nature of the data and its source. In this study, there have been interviews with open-ended questions for the semi-structured interviews as well as the task-based interviews for the selected participants to have an in-depth exploration of the phenomenon at hand. The open-ended questions enabled the interviewer (researcher) to adapt to the participants' responses easily.

3.4.1. The research site.

This study has been conducted in six selected secondary schools from the Bronkhorstspuit area in the Gauteng North district. The Gauteng North district is one of 13 districts from Gauteng province and has 20 secondary schools, of which only 17 offer mathematics. Bronkhorstspuit is a small developing town located 50 km east of Pretoria, Gauteng, South Africa, along the N4 highway towards Emalahleni. It lies on the border between the Gauteng and Mpumalanga provinces. It comprises one village area, Sokhulum, and three townships: Zithobeni, Rethabiseng and Ekangala.

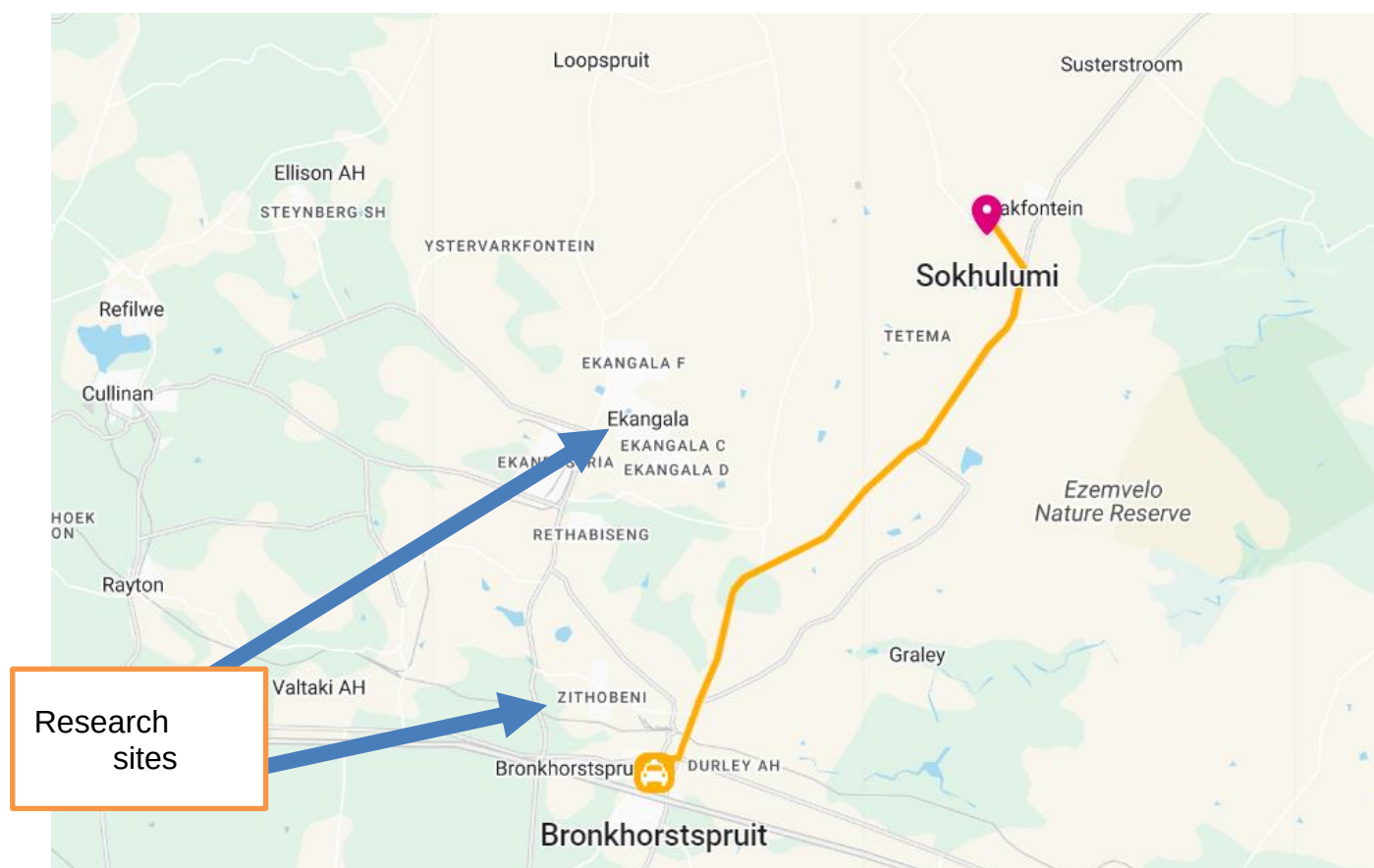


Figure 3.4.1: A map indicating where the research sites (Ekangala and Zithobeni) are situated.

This study took place in the schools of the selected participants, and the interviews were conducted after school hours on the school premises. The researcher and the participants were using rooms allocated by the school principal. The researcher ensured that all the necessary COVID-19 protocols were properly followed and encouraged the participants to do the same to ensure the well-being of everyone. The data was collected during the third term, and during that time, the participants were busy with the Grade 12 revisions in preparation for the preliminary examination. The study's teachers indicated they were done with the Grade 12 syllabus.

3.4.2. Population and Sampling Methods

The term population in research studies refers to the entire group of interest. According to Robson (2002:135), population “refers to all the cases”. This study's population is all Grade 12 mathematics teachers in the Gauteng North district. According to Robson (2002:136), sampling refers to the selected people from the population list. Before sampling can be performed, a general problem should be identified. The task of selecting the unit analysis should follow (Merriam,1998:60). Sampling methods that fit the study are purposive and convenient. Convenience sampling is when a sample is selected based on currency, location, time, availability of respondents or cities, and so forth (Merriam,1998:63).

Robson (2002:141) states that convenience sampling involves selecting the most convenient and the nearest person or place to act as a participant. According to Poti (2020:5), purposive sampling involves identifying and selecting individuals or a group of knowledgeable people about a singularity of interest. Purposive sampling, or theoretical sampling, has a built-up sample that enables the researcher to satisfy his/her needs in a particular project (Robson, 2002:141). To respond to the research questions, a few participants were selected with a focus on context, background and teaching experience due to time and other constraints.

In this case, eight Grade 12 mathematics teachers from six conveniently sampled schools in the Gauteng North District were purposively selected to participate in this study. The teachers were selected according to the following criteria: He/she must be offering mathematics at the Grade 12 level and must have a mathematics teaching qualification in the FET phase. However, Participants from the schools not part of Gauteng North district and not offering Grade 12 mathematics in Gauteng North district were not sampled to participate in this study. The schools selected are close to each other, making it ideal for the researcher in terms of travelling and time. The schools conveniently sampled by the researcher under the Gauteng North District were as follows: School A, School B, School C, School D, School E and School F. The six sampled schools fall under quintiles 1, 2 and 4.

3.4.3. Instruments and data collection strategies

Data collection is all about utilising the selected methods of investigation (Robson, 2002:304). For this study, the following collection instruments were used to obtain data: the misconceptions and errors stated in section 2.10.4 (see Chapter 2), as revealed in the Grade 12 mathematics examination diagnostic reports, which informed the probing of the teachers' pedagogical content knowledge depth. Individual face-to-face semi-structured interviews supporting the written task-based interviews in Euclidean geometry and similarity, in particular, were used to explore the teachers' depth of PCK. It was hoped that the outcomes would identify where intervention needs to focus in improving Grade 12 learners' performance in Euclidean geometry and the similarity section.

An individual semi-structured interview schedule (see Appendix G- Section B) contains the interview questions in support of Appendix G- Section C (written task-based interview). Appendix H was used as a document analysis guide. As part of data collection, teachers were given written task-based interviews entailing Grade 12 Euclidean geometry questions extracted from the various past examination question papers, together with the corresponding learner written responses created by the researcher (see Appendix G- Section C). The researcher has compiled a test (Written task) based on similarity questions extracted from the previous question papers and created typical exemplars of learners' written responses to those questions. The researcher ensured that those typical exemplars of learner responses contained some of the misconceptions and errors stated in the 2017- 2021 National Diagnostic Report. The participants identified and corrected misconceptions and errors in the learner solutions on similarity in Euclidean geometry. They then had to explain in writing and verbally the possible causes for the resulting misconceptions and errors and how they are corrected in class.

Furthermore, semi-structured interviews supported Appendix G- Section C, which contained general questions based more on the similarity concept in Euclidean geometry (see Appendix G- Section B). Here, the participants were required to respond to the questions based on their general knowledge. The participants were expected to respond to the interviews, which took approximately 2 hours. The first hour was for responding to the written task, and the other was for responding to the semi-structured interview questions. There needed to be a break between completing the task and interviews. The task-based and semi-structured interviews had two long and four short follow-up

questions, respectively. Other than this break, between tasks, the participants were made aware that it shall be given at any time they wish to request a break.

A section summarised misconceptions and errors that have commonly occurred from the 2017- 2021 National Diagnostic Report when solving Euclidean geometry questions, particularly similarity (see Appendix H). However, the researcher only used Appendix H to determine the teachers' PCK based on their responses to the given task (learners' written work). The responses were evaluated based on the teachers' ability to identify and correct misconceptions and errors as indicated on the diagnostic reports published by the DBE. The data obtained was in the qualitative (non-numerical) form. The teachers' task sheets and verbal and written responses formed part of the data generation for this study. The interviews were recorded using standard tape and digital voice recorders and then transcribed.

3.4.4. Data analysis process

The study employed qualitative data analysis. Cohen *et al.* (2018:643) assert that this includes "organising, describing, understanding, accounting for, and explaining data, making sense of data in terms of participants' definitions of the situation, noting patterns, themes, categories and regularities..." The interview transcripts, teachers' task sheets and written responses correcting the identified misconceptions and errors, and their explanations of how they can address these misconceptions and errors encapsulated the data analysis process. The misconceptions and errors, as cited in the Grade 12 examination diagnostic reports and also mentioned in section 2.10.4. have informed the formation of the themes and categories. The misconceptions and errors were coded, and the information was put into themes to reduce it to certain categories used to answer the research question. Task sheets were used to back up claims from teachers' explanations in the face-to-face interviews.

3.4.5. Ethical considerations

Cavan (1977:810) defines ethics as "The matter of sensitivity to the right of others". Furthermore, it says, "while the truth is good, respect for human dignity is better". Ethics show both the correct and incorrect actions or the moral research procedures that are engaged in the study (Ngirishi, 2015:31). According to Maziri and Madinga (2016:5),

ethical issues are conditions that are elucidated by following principles and moral guidelines of the study conducted. The researcher applied to the Faculty of Education Ethics Committee and wrote a letter to the Department of Basic Education requesting permission to conduct research in the Gauteng North district (See Appendix B). She was granted ethical clearance by both institutions (See Appendix A and Appendix C, respectively). The researcher applied for consent letters to be sent to school principals, SGB chairpersons and the Grade 12 teacher participants (see Appendix D and E). Confidentiality, privacy, and the personal rights of participating teachers were protected by ensuring that the researcher kept every single document to herself and the supervisor and respected the participants' autonomy. The anonymity of the participant's data was protected by concealing their identity, and pseudonyms were used in the study.

The researcher ensured that she remained honest and respected the rights of the participants. The research participants (teachers) signed the informed consent letters as they were fully aware of the purpose of the study and were obtained from them during the beginning of the study (see Appendix F). There was an independent person to recruit and explain the study to the participants—an independent person assisted with recruiting participants (who are only teachers).

The independent person (a recruiter of participants) explained fully to potential participants (teachers) the implications and risks associated with participation in the study. Further, the participation was voluntary, and they may withdraw from the study when they feel uncomfortable. Lastly, the recruiter also explained to potential participants the study's implications. The independent person has obtained participants' written (signed) consent after agreeing to be part of the study.

The researcher also re-emphasised similar points, as raised above, before conducting the interviews with participants. The study occurred in Gauteng province under the Gauteng North district in South Africa. The participants were teachers who have done their tertiary, college or university studies in English as a medium of instruction. The research was not carried out in the mother tongue. Code-switching was only allowed during interviews if the participant felt that he/she could express a view better in the mother tongue. The neutral person was used because participants were teachers in the

district where the researcher worked as a teacher, and that was not featured in the consenting. This occurred from the 24th of August, 2023, to the 5th of September, 2023.

3.5. Trustworthiness

Research is generally concerned with producing trustworthy findings. Schwandt *et al.* (cited by Poti, 2020:52) see trustworthiness as the measure of a researcher's trust in qualitative data, whether credible, transferable, or dependable. Sutton and Austin (2015:229) state that “trustworthiness aims to evaluate the following from the research:

1. Credibility (believing in the truth from the findings);
 2. Transferability (the findings of the research may apply in any other context);
 3. Dependability (research findings show consistency, and the findings may be repeated);
- and
4. Confirmability (no researcher influences the research findings, and the findings are only shaped by respondents).” Trustworthiness is the necessary procedure that is followed to check the credibility and reliability of the study.

Criteria for judging research are generally limited to questions about the specific aspects of methodology, such as reliability (replicability) and validity (correspondence between descriptions and what is intended to be described). Qualitative research, followed in this study, is one approach based on different assumptions about reality and a different worldview, and one should consider validity and reliability from a perspective congruent with the philosophical assumptions underlying the paradigm. Therefore, the canons (standards) by which quantitative studies are judged could be more appropriate for judging the merit of qualitative studies. On the other hand, Korstjens and Moser (2018:120) posit that “The quality criteria for all qualitative research are credibility, transferability, dependability, and confirmability.”

Howitt (2007) define confirmability as a way of controlling research bias in an objective and neutral manner. Furthermore, Kumar (2011:172) defines Confirmability as the state in which others might corroborate the study's results. In this study, the data was transcribed and analysed, and the description of the findings was related to the literature

to achieve confirmability. The supervisor and the co-supervisor confirmed the findings of the study to ensure that there is credibility. The researcher also did member-checking by double-checking the audios of the participants to ensure that what was recorded was correctly transcribed. Plagiarism was avoided, and sources were properly acknowledged according to the rules and regulations of the Institution.

For reliability in this study, one uses Lincoln and Guba's (1985:288) suggestion that we may need to consider naming the concept differently. They then suggest using "dependability" or "consistency" of results obtained from data instead of reliability. To this, Merriam (1998:206) had this to say, "Rather than demanding that outsiders (to the study) get the same results, a researcher wishes outsiders to concur that, given the data collected, the results make sense - they are consistent and dependable." The question then is not whether findings will be found again but whether the results are consistent with the data collected. In this study, one sought to describe and explain the world as those in the world experience it by providing rich and detailed or thick descriptions of the research phenomena from the views of the teacher participants. Furthermore, this study confirmed the dependability using the teachers' original written responses to the research questions.

Lastly, in a dilemma of generalizability and transferability in studies such as mine, I used the suggestion by Merriam (1998:211-212), where she says it could be enhanced by providing a rich, thick description by making available to the reader enough detailed descriptions of the study's context to enable a reader to compare and fit with their situation. Similarly, Du Plooy-Cilliers *et al.* (2014:258) define transferability as the capability to apply results to similar settings and yield the same results. This study's findings can be transferred to similar settings since the essential information was given, such as the research method used in this study, inserting the participants' work, including the direct quotes from the interviews, the research site and the detailed description of the research findings.

3.6. Summary

This chapter has presented the methods, techniques, tools, population and sample criteria, the ethical compliance used in the study to collect data, and their justification or motivation for selection. The issues around trustworthiness have also been discussed. In

summary, the steps and the procedures that were followed to collect and analyse data have been thoroughly explained in Chapter 3. The next chapter (Chapter 4) will discuss this study's data analysis and interpretations.

CHAPTER 4: DATA ANALYSIS AND INTERPRETATION

4.1. Introduction

This chapter presents the findings of the study. It entails the introduction, biographical details of the participants, teachers' responses to learner misconceptions and errors, visualisation of geometric diagrams, and summary. The study aims to explore a sample of Grade 12 teachers' pedagogical content knowledge depth concerning similarity in Euclidean geometry. Both semi-structured interviews and task-based interviews were used to collect data. A qualitative method was used to analyse data through thematic analysis.

The following research question and its sub-questions guided the study. They were also used to analyse and present the findings of the study.

Main question

The main research question being addressed in the current study is:

What is the depth of the Grade 12 teachers' pedagogical content knowledge concerning similarity in Euclidean geometry?

To answer this main research question, the following sub-research questions are addressed.

- What is the competence of Grade 12 mathematics teachers in identifying and correcting misconceptions and errors from Grade 12 learners' written work focusing on similarity?
- How do Grade 12 mathematics teachers apply their pedagogical content knowledge of Euclidean geometry when identifying and correcting misconceptions and errors in problems related to similarity?
- What are the experiences of Grade 12 teachers' when identifying and correcting misconceptions and errors in learners' responses related to similarity?

In finding the answers to the sub-questions above, one must have answers to the main question. The depth of the teacher's pedagogical content knowledge was established by giving the teachers written tasks with answers provided by learners and asking them to comment. The teachers' comments are a window into their pedagogical content knowledge depth. The tasks given to teachers are laden with the learner's misconceptions and errors as identified in various diagnostic reports. As mentioned, and discussed in section 2.10.4. They are as follows:

- a. Inability to distinguish between a theorem and its converse
- b. Inability to differentiate between similarity and congruency
- c. Provision of incorrect reasoning for the statements in Euclidean geometry
- d. Inability to identify the correct ratios from a given diagram
- e. , Inability to select the correct pairs of equal angles that will result in two triangles being similar.
- f. Incorrect labelling of angles
- g. Proving a theorem without making/ writing any construction

4.2. Biographic details of participants

The table below shows the participants' information about their teaching experience and their qualifications in teaching mathematics. The table shows that all the participants are qualified to teach the subject at the Grade 12 level, and most are well-experienced.

Table 4.2: Biographical details of participants

Participant	Gender	Race	Total number of years in the field of teaching	Total number of years of teaching grade 12 mathematics	Highest qualification(s)
Teacher 1 (T1)	Male	African	7 years	1 year, 6 months	BSc in Chemistry, and (PGCE in progress)
Teacher 2 (T2)	Male	African	7 years	7 years	Bed FET (Hons) in Mathematics
Teacher 3 (T3)	Male	African	1 year, 6 months	1 year, 6 months	Diploma in chemistry, and PGCE
Teacher 4 (T4)	Male	African	20 years	15 years	Post Graduate Diploma (PGD) in Mathematics Education
Teacher 5 (T5)	Male	African	9 years	5 years	Bed (FET) specialisation
Teacher 6 (T6)	Female	African	23 years	18 years	Bachelor of Arts in Education (BA Ed), and ACE in Mathematics and Science
Teacher 7 (T7)	Female	African	23 years	15 years	ACE in Mathematics
Teacher 8 (T8)	Male	African	7 years	7 years	BSc in Applied mathematics (Honours), and PGCE

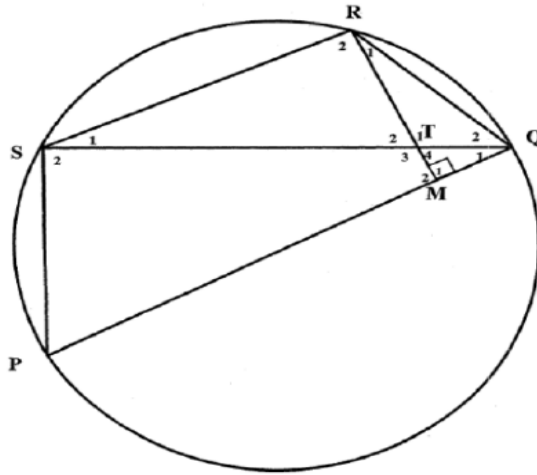
4.3. Responses of teachers to learners' misconceptions and errors

As indicated in Chapter 2, learners are challenged in dealing with some of the following misconceptions and errors in Euclidean geometry. As already stated, responses to questions by learners were given to the teachers to establish their PCK in terms of identifying and correcting the misconceptions and errors that learners usually commit (see Appendix G, section C). Subsequently, teachers were given general questions based more on the similarity section to establish their knowledge of the concept (see Appendix G, section B). Below are the teacher responses per section from the interviews and their written responses.

4.3.1. The difference between a theorem and its converse

As the diagnostic reports show, learners cannot differentiate between a theorem and a converse. Question 1.2.2. of the task (see Appendix G, section C) was given to the teachers with the hope that their responses to the questions would be able to tell whether they could identify and correct the misconceptions and errors which are indicated in the given task.

1.2. In the diagram below, PQ is a diameter of the circle. R and S are points on the circle and M is a point on PQ such that $RM \perp PQ$. RM and SQ intersect at T. (extracted from Grade 12 prelim 2017 question paper, from North West province).



Van Hiele level -3

Prove that:

$$1.2.2. \hat{RTS} = \hat{SPQ}$$

(3)

$$1.2.2. \hat{S}_2 = \hat{M}_2 \text{ (both } = 90^\circ)$$

PMTS is a cyclic quad (ext $\angle$ of a cyclic quad)

$$\therefore \hat{T}_2 = \hat{P}$$

Figure 4.1: Question 1.2.2

Among the 8 teachers, 5 could identify and correct the learner misconceptions and errors in question 1.2.2, whereas 3 could only identify some possible errors from the learner response. Teachers 1, 3 and 8 could not differentiate between a theorem and its converse when providing reasons for the statements. These 3 teachers were unable to identify all the misconceptions and errors. As a result, their corrections contained some misconceptions and errors in return.

On the very same question 1.2.2. Teacher 7 stated during the interviews that when learners must prove that a diagram is a cyclic quadrilateral, they should first add “converse” to the reason at the end. The same applies to Teachers 2, 4, 5 and 6, who have also indicated during the interviews and in writing the term “converse” when stating

the reason for their statements in question 1.2.2. Whereas Teachers 1, 3 and 8 did not mention anything about the term “converse” during the interviews and in writing. They just gave reasons for the converse of a cyclic quadrilateral theorem as though they were dealing with the original theorem. Below are some of the direct quotes from the teachers’ responses:

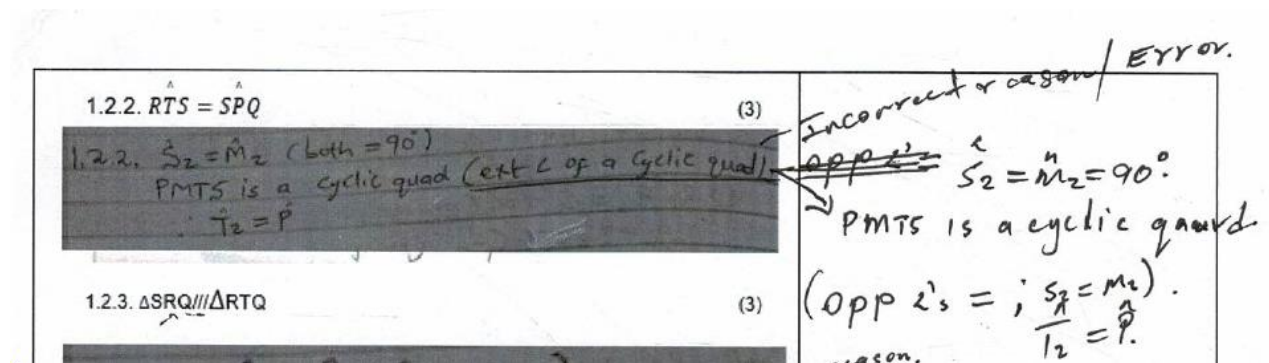


Figure 4.1.1: Teacher.1 response to Question 1.2.2

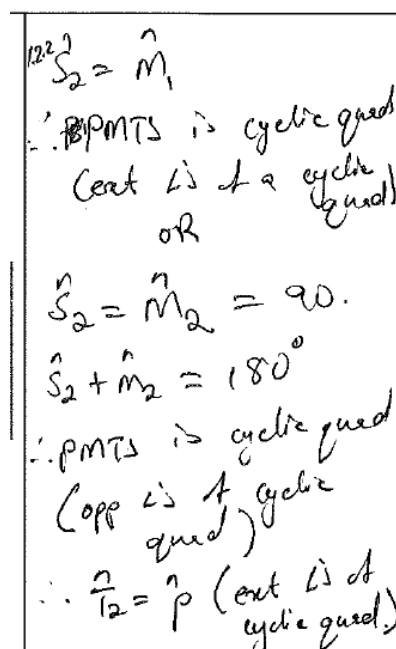


Figure 4.1.2: Teacher 3 response to Question 1.2.2

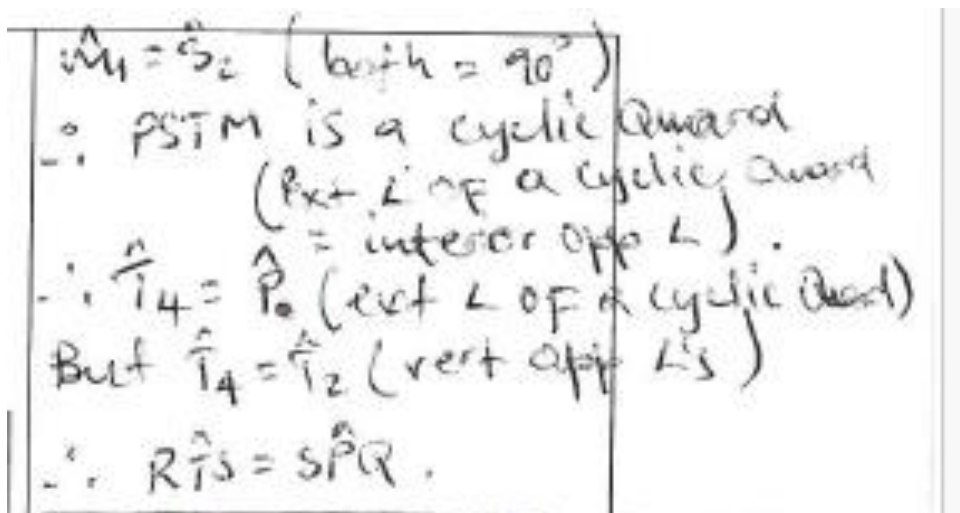


Figure 4.1.3: Teacher 8 response to Question 1.2.2.

Figure 4.1.4: Teacher 5 response to Question 1.2.2.

Teacher 5: The one that says PMTS is a cyclic quad. The reason must be converse, opposite angles of a cyclic quad. And then the other thing, angle T2 is equals to angle P. It should be the exterior angle of the cyclic quad.

Interviewer: Even if there's no converse, right?

Teacher 5: Yes.

Interviewer: Okay.

Teacher 5: When you are proving that it's cyclic.

Interviewer: Oh, it's a cyclic quad.

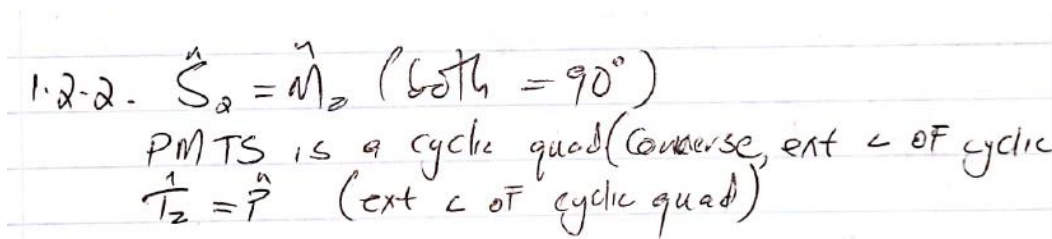
Teacher 5: Once you prove it's cyclic, don't use Converse. You use the normal reason.

Interviewer: Because normally they don't indicate Converse.

Teacher 5: Yes, they don't use Converse.

Interviewer: But what's the reason? Just for them to be familiar with Converse. Because normally says exterior, just like this one.

Teacher 5: No, you must emphasise that if they are still proving, the word converse must be there. If they are proving, and then if they are convinced or have now proved that it's a cyclic quad, you take the original reason as it is.



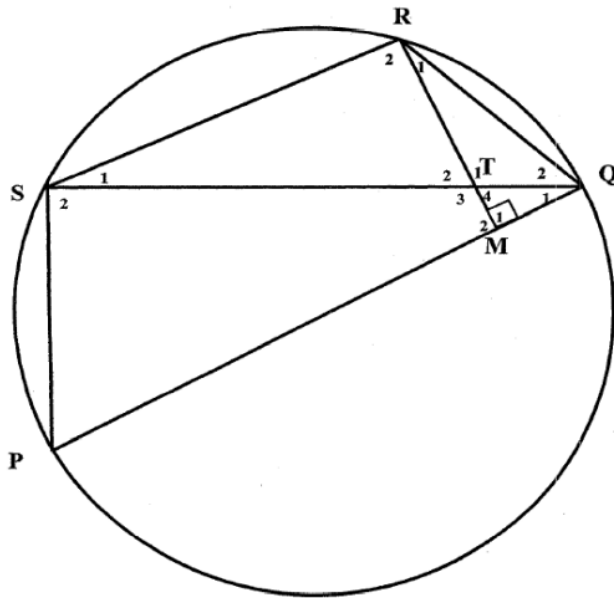
1.2-2. $S_2 = M_2$ (both = 90°)
PMTS is a cyclic quad (converse, ext $\angle$ of cyclic quad)
 $\frac{1}{2} = P$ (ext $\angle$ of cyclic quad)

Figure 4.1.5: Teacher 2 response to Question 1.2.2.

4.3.2. Inability to differentiate between similarity and congruency

As stated in section 2.10.4. In the study, some learners needed help differentiating between similarity and congruency, Questions 1.2.3, 10. a and 10.b. below (see Appendix G, section C) were used to test the teacher's PCK as to whether they were able to differentiate between similarity and congruency.

1.2. In the diagram below, PQ is a diameter of the circle. R and S are points on the circle and M is a point on PQ such that $RM \perp PQ$. RM and SQ intersect at T. (extracted from Grade 12 prelim 2017 question paper, from North West province).



Van Hiele level -3

Prove that:

1.2.3. $\triangle SRQ \sim \triangle RTQ$ (3)

1.2.3. $\hat{S} = \hat{R}$ (proven)
 $\hat{Q} = \hat{Q}$ (Common sides)
 $\therefore \triangle SRQ \sim \triangle RTQ$

Figure 4.2: Question 1.2.3

The general trend I have picked up under this theme is that some teachers needed help identifying the learner misconception and the error of stating the congruency symbol instead of the similarity symbol. However, Teachers 1,3, 6 and 8 were able to identify all the errors in this question and were able to correct them accordingly.

Other than that, Teachers 2, 4, 5 and 7 should have mentioned something about the incorrect usage of the congruency ($\equiv$) symbol instead of the similarity Symbol ($\sim$) during the interviews and when identifying the misconceptions and errors. However, the teachers

had identified the following as the only errors committed by the learner in Question 1.2.3. that is, the learner gave only 2 statements to prove similarity instead of mentioning three of them, including the statement of the third angle, incorrect reason for the statement and incorrect labelling of angles. Even though these teachers could not identify all the possible misconceptions and errors, they could correct them perfectly in writing. Below are some of the responses to the question by the various teachers:

Figure 4.2.1: Teacher 8 response to Question 1.2.3.

Teacher 8: The learner was asked to prove similarities, and the learner is concluding by congruence.

Interviewer: Okay. The learner was now proving congruence, neh?

Teacher 8: Yes.

Interviewer: Instead of similarity?

Teacher 8: Yeah.

Interviewer: And then how will you assist them to avoid such misconceptions or errors?

Teacher 8: They should know. They should know that we focus on angles whenever we are proving similarities.

Interviewer: Yes.

Teacher 8: Angles being equal.

Figure 4.2.2: Teacher 2 response to Question 1.2.3.

Interviewer: Now 1.2.3.

Teacher 2: Angle S is equal to angle R.

Interviewer: Proven. Angle S is equal to angle R.

Teacher 2: Where is it proven?

Interviewer: I can't see.

Teacher 2: The first one is wrong. That's not because.

Interviewer: angle S is equal to angle R is wrong.

Teacher 2: Oh. Oh yeah. S is equal to R. Proven. Yeah. It's fine. It's there S1 equals to R1.

Interviewer: Yes.

Teacher 2: Yeah. Proven on 1.2.1. And then so, the statement is correct. Angle Q is equal to angle Q. Yeah. Angle Q is equal to angle Q. These are common angles. The learner is rare. No. Angle Q equals angle Q. The first error is on the second Q. The learner did not put an arrow on...

Interviewer: To indicate that it is an angle.

Teacher 2: It is an angle. And then the second one is that the learner talked about the sides. And you are not talking about sides but angles.

Interviewer: Okay.

Teacher 2: Yes.

Interviewer: All right.

Teacher 2: So, on this one, it's an error. Incorrect. Not mentioning an angle. And then the second one. It's talking about the sides, not the angle.

Interviewer: Not angles.

Teacher 2: Right. $\angle SRQ$ is equal to $\angle RTQ$. Yes. This one is just what they said you must do, but there is an error because there is no reason why they are similar.

Interviewer: Okay.

Teacher 2: Yeah. So it's not an error—no reason for similar.

Interviewer: Okay.

Teacher 2: The correct statement that the learner was supposed to write in 1.2.3. On the first, the learner was supposed to see angle S. Yes. It's the same as angle R. This one is proven. And then the second one says angle Q is equal to angle Q, and it's a common angle. And then lastly, the learner still needs to prove the third angle.

$$\begin{array}{lcl}
 \text{1.2.3)} & \hat{S}_1 = \hat{R}_1 & | \text{ proven.} \\
 & \hat{Q}_2 = \hat{Q}_2 & | \text{ Common.} \\
 & \angle SRQ = \angle RTQ & | \text{ 3rd } \angle \text{ of } \Delta \\
 \therefore & \Delta SRQ \equiv \Delta RTQ & | \text{ } \angle \angle \angle / A A A.
 \end{array}$$

Figure 4.2.3: Teacher 4 response to Question 1.2.3.

Question 10. a and 10. b. of the task-based interview

Question 10 below was also given to the teachers to use their responses to the question to tell whether they can differentiate between similarity and congruency concepts

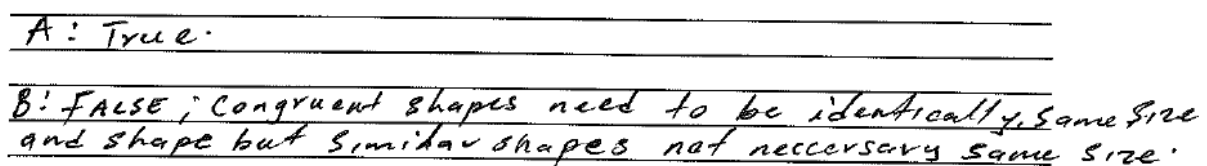
10. State whether the following statements are true or false. If false, state reasons for your answer. (Zhang & Wong, 2021:11).

Van Hiele level -2

- a. All Congruent shapes are similar.
- b. All similar shapes are Congruent.

Figure 4.2.b: Question 10. a and 10. b.

Teachers had different views when responding to this question. For instance, Teachers 1, 2, 3, 4, 5, 7, and 8 believe that all congruent shapes are similar. However, not all similar shapes are congruent due to various reasons. Afterwards, Teacher 8 even understood the concept as he could relate similarity and congruency to a real-life context when responding to the above question. According to Teacher 6, all congruent shapes are not similar, and the reason for that was to say congruent shapes have the same size and shape, but similar shapes have the same shape but different sizes. Below are some of the different responses which the teachers gave about the Question:



A: True.

B: FALSE; Congruent shapes need to be identically, same size and shape but similar shapes not necessary same size.

Figure 4.2.b.1: Teacher 1 response to Question 10.a and 10. b.

Figure 4.2. b.2: Teacher 8 response to Question 10.a and 10. b.

Interviewer: All congruent shapes are similar.

Teacher 8: True.

Interviewer: True, neh?

Teacher 8: Yes. Number B is false.

Interviewer: Please justify your answer...

Teacher 8: When discussing congruency, they must be equal. If I buy a pair of toughees that is size eight and the other person buys size eight before we wear them, they will be equal in every possible way. But when we talk about similarity, we can buy these toughees size eight and size eight. They are still equal in congruence. They are said to be similar in congruence. If you can buy size eight, I buy size four. They are not equal. They are not congruent, but they are similar.

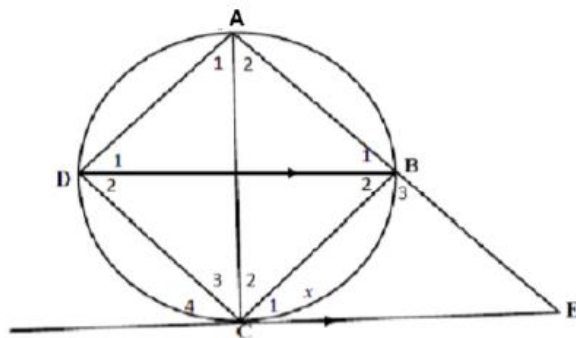
Interviewer Okay.

Teacher 8: But it's the same thing. That means they can be congruent, and congruence can lead to similar but similar cannot lead to congruence. You know that congruence needs to be equal in every possible way.

4.3.3. Provision of incorrect reasoning for the statements in Euclidean geometry

Questions 1.1 and 2.2 discuss the gap in finding answers to this misconception and the error of providing incorrect reasons for the statements in Euclidean geometry. These questions were given to the teachers with the hope that their responses would be able to tell whether they were having the same challenge as their learners. The aim was to check whether such misconceptions and errors emanate from the teachers, learners or both. Below are the attached Questions from the given task.

1.1. In the figure below $BD \parallel EC$, and CE is a tangent to the circle $ABCD$.



1.1.1. If $\angle BCE = x$, name with reasons other four angles equal to x (4)

Van Hiele level -3

1.1.1. $\hat{C}_1 = \hat{A}_2 = x$ (tan-chord)
 $\hat{B}_2 = \hat{A}_2$ (∠'s in the same segment)
 $\hat{C}_1 = \hat{B}_2$ (alt ∠'s)
 $\hat{B}_2 = \hat{A}_1$ (∠'s in the same segment)
 $\hat{B}_2 = \hat{C}_4$ (tan-chord)

Figure 4.3: Question 1.1.

In this question, all eight teachers identified an error of writing an incomplete reason for the third statement from the attached learner response. In correcting it, all 8 teachers emphasised that parallel lines should always be mentioned when dealing with “FUN” angles (such as corresponding, co-interior, and alternating angles). This could imply that this identified error could emanate from the learners due to ignorance or forgetfulness, or perhaps teachers are not emphasising or not often reminding their learners to state the // (parallel) lines. Below are some of the responses given by the teachers in answering to the question:

Figure 4.3.1: Teacher 6 response to Question 1.1.

Teacher 6: Yeah. Yes. So, the first solution the learner said angle C1 is equal to angle A2, which equals X. Tan-cot. However, they require us to say the tan-cot theorem.

Interviewer: Okay. So that can be highlighted. You can highlight. You have your pencil. Highlight and indicate yeah.

Teacher 6: Tan cot theorem. Must I write it here?

Interviewer: Add yeah. You could say tan cot, you know.

Teacher 6: Tan-Cot theorem. And angle D2 is equal to angle A2 in the same segment. That's correct. This one is correct. angle C1 equals angle B2, and the learner said alternate angles, but he or she did not indicate the parallel lines. We should have said DE, DB, it's parallel to CE.

Interviewer: Correct? Can you say that is an error or misconception?

Teacher 6: I think it's an error.

Interviewer: It's an error.

Teacher 6: It's not a misconception.

Interviewer: Yeah.

Teacher 6: Yeah, it's an error. The learner just forgot to indicate the parallel lines.

Figure 4.3.2: Teacher 8 response to Question 1.1.

Interviewer: Can you say the learner has answered correctly?

Teacher 8: The statement... The first statement is correct. Okay. But the problem is with the reason.

Interviewer: Okay.

Teacher 8: The learner wrote tan-chord...

Interviewer: Feel free to use a red pen, maybe to highlight.

Teacher 8: Yet is supposed to say tan cot theorem, D or M.

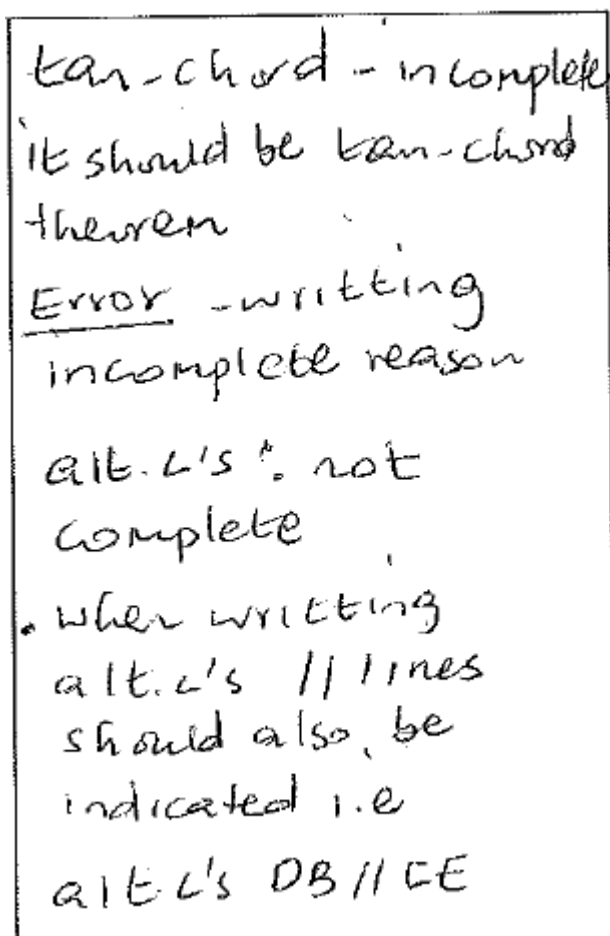
Interviewer: Okay. Okay. I see.

Teacher 8: Then, yes. We come to the second one. Where angle D2 is equal to angle A2. angle D2 is equal to angle A2. angle A2 here is our A2—Angles in the same segment.

Interviewer: Yes.

Teacher 8: That one is correct, yes. angle C1 is equal to angle B2. angle C1 is equal to angle B2—alternate angles. You need to remember the parallel lines.

Interviewer: Forgetting the parallel lines, neh.



tan-chord - incomplete
it should be tan-chord
theorem
Error - writing
incomplete reason
alt. $\angle$'s : not
complete
• when writing
alt. $\angle$'s // lines
should also be
indicated i.e
alt. $\angle$'s DB // CE

Figure 4.3.3: Teacher 7 response to Question 1.1.

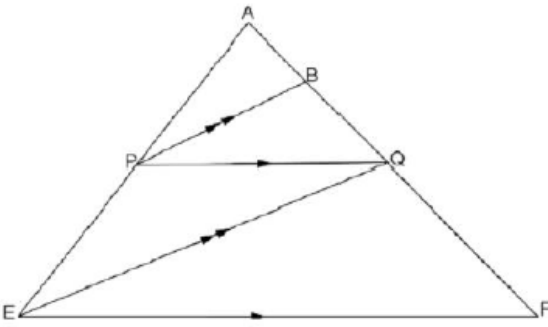
2.2. In the diagram below, $BP \parallel EQ$ and $PQ \parallel EF$. $AP:PE = 2:3$. $AE = 120\text{m}$ and $QF = 60\text{m}$.  2.2.1. Calculate the length of AP (2) Van Hiele level -2 2.2.1. $AP = \frac{2}{5} \times 120\text{m} = 48\text{m}$	
2.2.2. Calculate the length of AQ (4) 2.2.2. $\frac{AQ}{BF} = \frac{AP}{PE}$ (line div sides of Δ prop) $\frac{AQ}{60\text{m} + BQ} = \frac{48\text{m}}{72\text{m}}$ $\therefore AQ = \frac{2}{3} (60\text{m} + BQ)$ $= 40\text{m} + \frac{2}{3} BQ$ 2.2.3. Calculate: $\frac{AB}{BQ}$ (1) 2.2.3. $\frac{AB}{BQ} = \frac{2}{3}$ (line div sides of Δ prop) Van Hiele level -3 Van Hiele level -2	

Figure 4.3. b: Question 2.2.

Regarding question 2.2 above, most teachers needed help properly identifying misconceptions or errors in the learner responses. For Example, some were not even sure whether the given reason of “line div sides of triangles proportionally” is part of the reasons used in Euclidean geometry, whether it is a reason for the converse theorem or does not exist. Teachers 2, 3 and 1 repeated the same mistake of writing the incorrect reason for the statement when correcting the learner's misconception and error in this question. It shows that they are not familiar with the converse of the proportionality theorem. The same applies to Teacher 7 as she argued that such a reason “line divides

sides of a triangle proportionally” does not exist in Euclidean geometry. Furthermore, Teacher 6 stated that she was not sure as to whether it was part of the reasons in geometry or not. This is not true because this reason is for the converse of the proportionality theorem. At least Teachers 4, 5 and 8 were able to respond correctly to question 2.2 and noticed that this is the reason used for the converse of the proportionality theorem. According to Teachers 2, 4, 7 and 8, learners should be given the reasons from the exam guidelines to state the correct and complete reasons for the different theorems. This can assist the learners to familiarise themselves with the reasons used for the statements in geometry. Below are some of the responses by the various teachers:

Figure 4.3. b.1: Teacher 2 response to Question 2.2.

Interviewer: And then going back to AP. Calculate the length of AP.

Teacher 2: The length of AP.

Interviewer: Yes. The learner wrote two over 5 X 120 meters. Which is...

Teacher 2: 2 over 5.

Interviewer: 48 meters.

Teacher 2: This is correct.

Interviewer: We don't need the reason, neh?

Teacher 2: We don't need the reason, yeah. Calculate the length of A, Q. Start by saying AQ over PF.

Interviewer: Yes.

Teacher 2: Is equal to AP over PE.

Interviewer: Mm-hmm.

Teacher 2: Yeah, line...

Interviewer: ...Line divides sides of a triangle proportionally.

Teacher 2: Perfect.

Interviewer: Mm-hmm.

Teacher 2: And then AQ.

Interviewer: Yes.

Teacher 2: Over 60 Plus B, Q Is equal to 48.

2.2.2. $\frac{AQ}{QF} = \frac{AP}{PE}$ (line divides sides of Δ prop)

$$\frac{AQ}{60} = \frac{48}{72}$$
$$AQ = 40m$$

Figure 4.3. b.1: Teacher 2 corrections to Question 2.2. (continue...)

Figure 4.3. b.2: Teacher 5 response to Question 2.2.

Interviewer: Okay. You solved this problem. And then now look at the reason. It's a line, divide, and sides of the triangle.

Teacher 5: It's the converse.

Interviewer: Oh, the converse.

Teacher 5: Yeah. Which is used to prove that lines are parallel. You're supposed to say a line parallel to one side of a triangle—wrong reason.

Interviewer: All right.

Teacher 5: Wrong statement, wrong reason.

Interviewer: Yeah. Of course, everything will be wrong. The same applies to 2.2.3.

2.2.2

in $\triangle AEF$

$$\frac{AQ}{QF} = \frac{AP}{EP} \quad (\text{line } \parallel \text{ to one side } \triangle)$$

$$\frac{AQ}{60} = \frac{2}{3}$$

$$3AQ = 120$$

$$AQ = 40 \text{ m}$$

2.2.3

$$\frac{AB}{BO} = \frac{AP}{PE} \quad (\text{line } \parallel \text{ to one side } \triangle)$$

$$= \frac{2}{3}$$

Figure 4.3. b.2: Teacher 5 corrections to Question 2.2. (continue...)

Figure 4.3. b.3: Teacher 8 response to Question 2.2.

Interviewer: Is it 2 over 3?

Teacher 8: Yes.

Interviewer: Oh, the answer is correct.

Teacher 8: The answer is correct.

Interviewer: But the reasoning?

Teacher 8: It's incorrect.

Interviewer: It's incorrect?

Teacher 8: Yeah.

Interviewer: Okay. What should be the correct reason?

Teacher 8: A line parallel to one side of a triangle.

Interviewer: Mm-hmm. Okay. Underline it. Okay.

$$\begin{aligned}
 &2.2.2 \\
 &\frac{AQ}{QF} = \frac{AP}{PE} = \frac{2}{3} \quad (\text{Prop. Theorem } PQ \parallel EF) \\
 &\frac{3t}{3} = \frac{40}{3} \\
 &t = 20 \text{ m} \\
 &\therefore AQ = 2t = 2(20) = 40 \text{ m} \\
 &2.2.3, \quad Sp = 40 \text{ m} \\
 &\quad p = 8 \text{ m} \\
 &\therefore \frac{AB}{BR} = \frac{2p}{3p} = \frac{2(8)}{3(8)} \quad (\text{Line } \parallel \text{ to one side of } \Delta) \\
 &\quad = \frac{2}{3}
 \end{aligned}$$

Figure 4.3. b.3: Teacher 8 corrections to Question 2.2. (Continue)

4.3.4. Inability to identify the correct ratios from the given diagrams

Teacher responses to Question 2.2.2 were used to establish whether they can identify and correct ratios from the given diagram since it has been stated in section 2.10.4. some of the summarised common misconceptions and errors that learners have a challenge in terms of stating the ratios from the given diagrams.

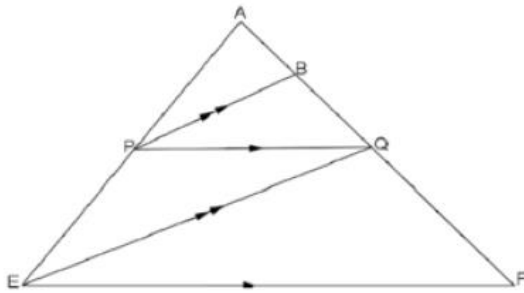
2.2. In the diagram below, $BP \parallel EQ$ and $PQ \parallel EF$. $AP:PE = 2:3$. $AE = 120\text{m}$ and $QF = 60\text{m}$. 	
2.2.2. Calculate the length of AQ (4) 2.2.2. $\frac{AQ}{BF} = \frac{AP}{PE}$ (line div sides of Δ prop) $\frac{AQ}{60\text{m} + BQ} = \frac{48\text{m}}{72\text{m}}$ $\therefore AQ = \frac{2}{3} (60\text{m} + BQ)$ $\approx 40\text{m} + \frac{2}{3} BQ$	Van Hiele level -3

Figure 4.4: Question 2.2.2.

Teachers 4, 5, 6, 7 and 8 did not display much of a challenge when it came to identifying the correct ratios since they were able to address this misconception and error from the given task, apart from Teachers 1, 2 and 3, who did not identify the incorrect reason from the learner response and still did not correct it accordingly. Instead, Teacher 1 identified the reason as incomplete and then added the parallel lines to the same reason when correcting it as “Line div sides of triangle prop, $PQ \parallel EF$ ”, and Teacher 3 used “Prop theorem” for the statement in question 2.2.2. Subsequently, the response from Teacher 2 contained some systematic errors apart from the incorrect reasoning part, in return when correcting the learner's responses on the given task. For example, below are samples of the responses from Teacher 2 with errors and Teachers 4 and 5, who have done it perfectly.

Figure 4.4.1: Teacher 2 response to Question 2.2.2.

Teacher 2: So, Yeah. Our AQ is what we are looking for.

Interviewer: Mm-hmm.

Teacher 2: Our QF It's 60.

Interviewer: Mm-hmm.

Teacher 2: Our AP, AP is what we got to be 48.

Interviewer: Yes.

Teacher 2: And then PE, if the whole thing is 180, uh, it's 120, meaning it will be 120 minus 48. Yeah. And it's minus 48. Then, it means that PE is 72. And then when you cross multiply.

Interviewer: Mm-hmm.

Teacher 2: 60 times 48.

Interviewer: Yes.

Teacher 2: Divided by 72. So it means that our AQ it's going to be 440.

Handwritten mathematical work on lined paper:

$$\begin{aligned} 2.2.2. \quad \frac{AQ}{QF} &= \frac{AP}{PE} \quad (\text{line divides of a prop}) \\ \frac{AQ}{60} &= \frac{48}{72} \\ AQ &= 440m \end{aligned}$$

Figure 4.4.1: Teacher 2 response to Question 2.2.2. (continue...)

$$\begin{aligned}
 2.2.2) \quad \frac{AQ}{QF} &= \frac{AP}{PE} & \left| \text{prop; } PQ \parallel EF. \right. \\
 \frac{AQ}{60} &= \frac{48}{72} \\
 AQ &= \frac{48 \times 60}{72} \\
 AQ &= 40m.
 \end{aligned}$$

Figure 4.4.2: Teacher 4 response to Question 2.2.2.

$$\begin{aligned}
 2.2.2 \\
 \text{in } \triangle AEF \\
 \frac{AQ}{QF} &= \frac{AP}{EP} \quad (\text{line } PQ \parallel \text{to one side } \triangle) \\
 \frac{AQ}{60} &= \frac{2}{3} \\
 3AQ &= 120 \\
 AQ &= 40m
 \end{aligned}$$

Figure 4.4.3: Teacher 5 response to Question 2.2.2.

4.3.5. Inability to select the correct pairs of equal angles that will result in two triangles being similar.

Question 1.1.2. was used to check whether teachers can address the above misconception and errors stated in various diagnostic reports.

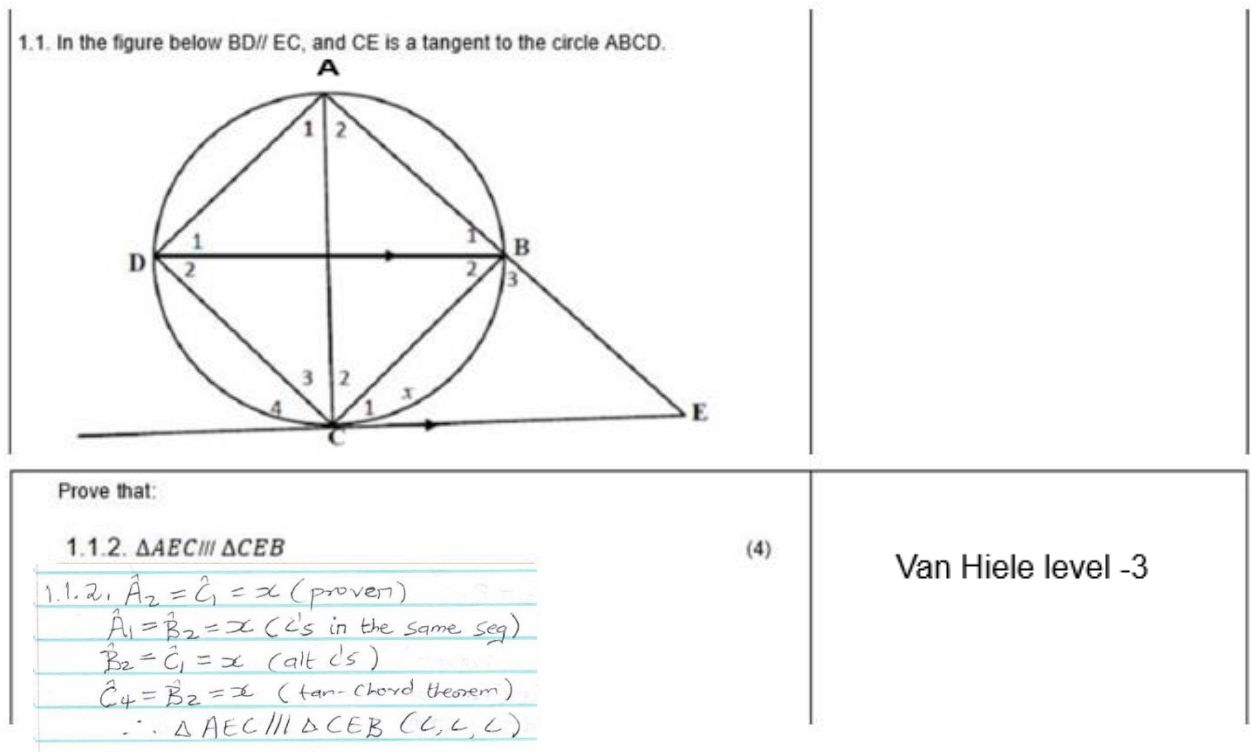


Figure 4.5. Question 1.1.2.

From the results which the researcher found, none of the teachers had a challenge when it came to identifying and correcting the type of misconception or error. They could see that the angles the learner stated have no relationship with the two triangles they are required to prove similarity between. Instead, teachers were able to come up with different ways to avoid the persistent misconceptions and errors that learners commit.

According to Teachers 2, 4, 5, 6, 7 and 8, when dealing with the application of similarity, different highlighters or coloured Pens should be used to be able to highlight the exact triangles to avoid stating angles which are not part of the given triangles. Teachers 1 and 3 did not emphasise the usage of different highlighters; instead, they said learners should

separate the triangles they work with from the original diagram and make a sketch based on that. Below are the comments made and responses given by some of the teachers:

Figure 4.5.1: Teacher 5 response to Question 1.1.2.

Interviewer: ...are similar. According to you, what did the learner miss here?

Teacher 5: He does not know how to identify angles or the expectations regarding similarity.

Interviewer Okay.

Teacher 5: Because if you're proving similarity, they've given you the order. You must prove them according to the way they've arranged the alphabet. So it would be best if you said okay, A must be equal to C. Then you go back to this Okay, in triangle AEC. Which am I talking about in AEC? Yeah, so he should have highlighted a triangle AEC and said, Okay, what is angle A? And this is AEC, which means the A that I'm talking about is A2. Then in triangle CEB, he would have highlighted CEB; when I talk of C, I'm talking about which C It's C1.

Interviewer Okay.

Teacher 5: Then he must look for a reason why C1 is equal to A2. Look for a reason. Because he was correct with the first one, isn't A1 equal to C1. he gave a correct reason. Then he moves to the next alphabet and E in AEC; which E are we talking about? There's only one E, okay, that E must be equal to E in CEB, there is C. is the same angle E. So, he could have said E equals to E, common angle, because it's the same angle and in the two triangles.

Interviewer Okay.

4.3.6. Incorrect labelling of angles

According to the various diagnostic reports, learners are challenged to name or label angles in Questions 1.1.4 and 1.2.1 incorrectly. The task was given to the teachers with the hope that their responses to the questions would tell whether they could identify and correct such a misconception and error as it comes.

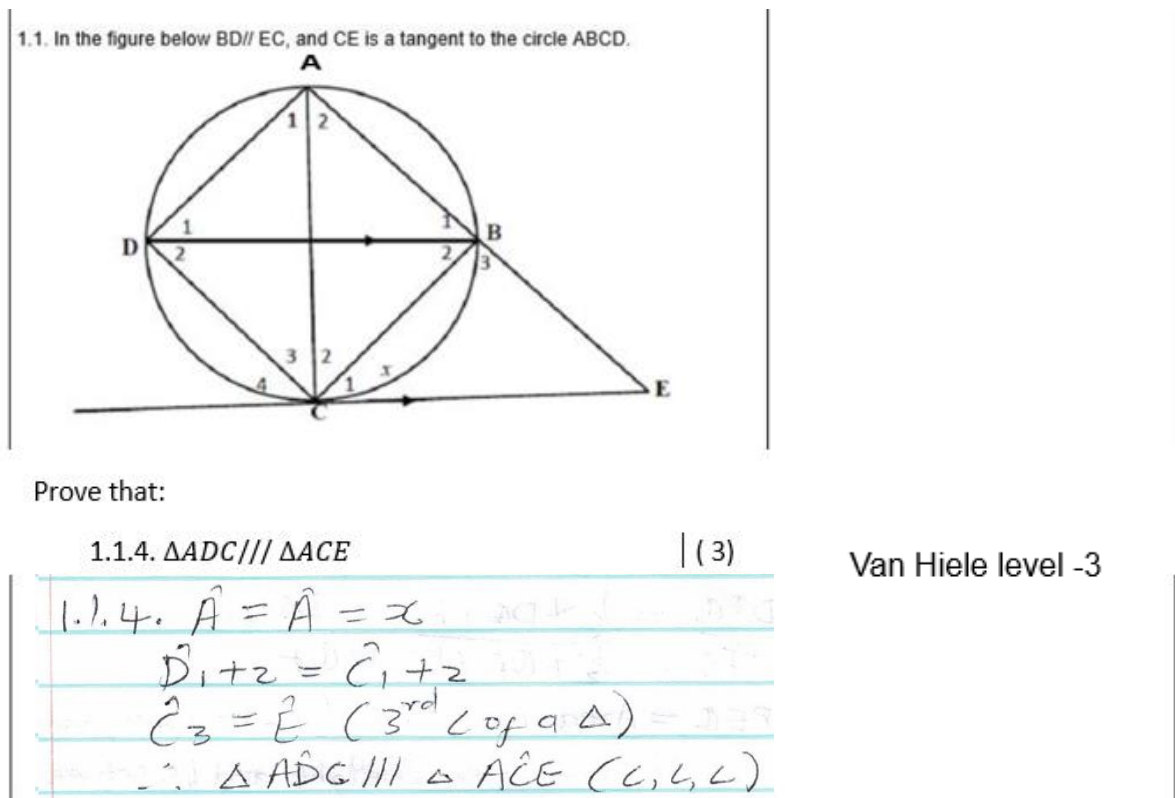


Figure 4.6: Question 1.1.4.

From the results obtained, it has been observed that amongst the eight teachers, only Teacher 1 and 3 had a challenge of labelling angles correctly in the section of Euclidean geometry. The researcher saw this during the interviews with the teachers and from their written responses. Teachers 2, 4, 5, 6, 7 and 8 addressed the possible misconceptions and errors from the learner response. Below are the supporting examples of some teachers' corrections during the interviews.

1.1.4 $\hat{A}_1 = \hat{A}_2 = x$ proven in 1.1.1

$\hat{D}_{21} = \hat{C}_2$ $\angle$'s in same chord.

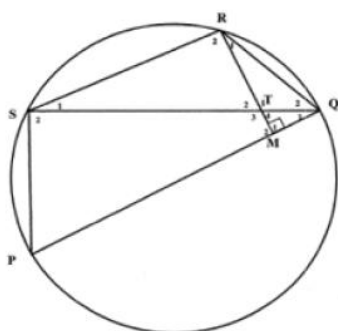
$\hat{D}_{1+2} = \hat{C}_{1+2}$ ($\hat{C}_1 = \hat{B}_2$ & $\hat{C}_2 = \hat{B}_1$)

$\hat{C}_3 = \hat{E}$ (3rd $\angle$ of Δ)

ded $\therefore \Delta ADC \parallel \Delta ACE$ LLL

Figure 4.6.1: Teacher 3 response to Question 1.1.4.

1.2. In the diagram below, PQ is a diameter of the circle. R and S are points on the circle and M is a point on PQ such that $RM \perp PQ$. RM and SQ intersect at T. (extracted from Grade 12 prelim 2017 question paper, from North West province).



Prove that:

1.2.1. $\hat{RST} = \hat{MRQ}$

(5)

Van Hiele level -3

1.2.1. $\hat{S} = 90^\circ$ ($\angle$'s in the same seg)

$\hat{Q}_{1+2} = 90^\circ - \hat{R}_1$ ($\angle$'s in ΔRTQ)

$\hat{S}_{1+2} + \hat{Q}_{1+2} = 180$ (Opp $\angle$ s of a cyclic quad)

$\therefore \hat{S} = \hat{R}$

Figure 4.6. b: Question 1.2.1.

From Question 1.2.1. above, the researcher noticed that almost all eight teachers still, were able to address the learner misconceptions and errors from the given task

accordingly. However, Teachers 1 and 3 still needed to work correctly labelling angles. This shows that the 2 identified teachers did not commit this error unintentionally or systematically in the previous Question 1.1.4, but they are used to labelling the angles in that way. The teachers have identified the following misconceptions and errors from the given task, and the corrections to those misconceptions and errors were done as follows:

1.2.1.
 $\hat{S} = 90$ (Semi-Circle)
 $\hat{Q}_{1+2} = 90 - \hat{R}$ (Sum of $\angle$'s in $\triangle RMA$)
 $\hat{S}_{1+2} + \hat{Q}_{1+2} = 180$ (opp $\angle$'s of a cyclic quad)
 $\hat{S} = \hat{R}$

Figure 4.6. b.1: Teacher 1 response to Question 1.2.1.

Prove that:
 1.2.1. $\hat{RST} = \hat{MRQ}$

1.2.1. $\hat{S} = 90$ ($\angle$'s in the same seg) $\leftarrow$ error (wrong reason)
 $\hat{Q}_{1+2} = 90 - \hat{R}$ ($\angle$'s in $\triangle RMA$) $\leftarrow$ misconception ($\angle$ not mentioned)
 $\hat{S}_{1+2} + \hat{Q}_{1+2} = 180$ (opp $\angle$'s of a cyclic quad) $\leftarrow$ misconception (Incorrect naming of $\angle$'s)
 $\hat{S} = \hat{R}$ $\leftarrow$ Incorrect naming

Figure 4.6. b.2: Teacher 2 identified misconceptions and errors to Question 1.2.1.

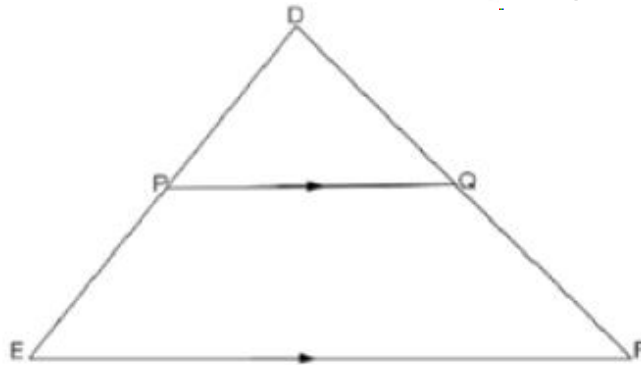
$$\begin{aligned}
 1.2.1. \quad \hat{S}_2 &= 90 \text{ (L' subt by diameter)} \\
 \hat{Q} &= 90 - \hat{R}_1 \text{ (L's in } \Delta RMA) \\
 \text{A} \quad \hat{Q}_1 + \hat{Q}_2 + \hat{S}_1 + S_2 &= 180 \text{ (opp L's of cyclic quad)} \\
 90 - \hat{R}_1 + S_1 + 90 &= 180 \\
 \hat{S}_1 &= \hat{R}_1
 \end{aligned}$$

Figure 4.6. b.2: Teacher 2 corrections to Question 1.2.1. (Continue...)

4.3.7. Proving a theorem without making/ writing any construction

As it appears in the diagnostic reports, learners prove a theorem without making any construction, Question 2.1. was given to the teachers with the hope that their responses would be able to tell whether they could identify and correct such an error.

2.1. In the diagram below, P is a point on DE and Q is a point on DF in $\triangle DEF$. Prove the theorem which states that : If $PQ \parallel EF$, then $\frac{DP}{PE} = \frac{DQ}{QF}$ (5)



Van Hiele level -4

$$2.1. \frac{\text{Area of } \triangle DPQ}{\text{Area of } \triangle PEQ} = \frac{\frac{1}{2} \times DP \times h}{\frac{1}{2} \times PE \times h} = \frac{DP}{PE}$$

$$\frac{\text{Area of } \triangle DPQ}{\text{Area of } \triangle PFQ} = \frac{\frac{1}{2} \times DQ \times h}{\frac{1}{2} \times QF \times h} = \frac{DQ}{QF}$$

but Area of $\triangle PEQ$ = Area of $\triangle PFQ$ (Same base, Same height and lie between // lines)

$$\frac{\text{Area of } \triangle DPQ}{\text{Area of } \triangle PEQ} = \frac{\text{Area of } \triangle DPQ}{\text{Area of } \triangle PFQ}$$

$$\therefore \frac{DP}{PE} = \frac{DQ}{QF}$$

Figure 4.7: Question 2.1.

The researcher noticed that most teachers could identify the error of proving without construction. However, only Teachers 4 and 2 could correct the construction error and further identify the other existing misconceptions and errors from the learner's written solution apart from the construction part on the diagram in 2.1. Even though Teacher 4 made a systematic error of naming the given triangle as $\triangle DET$ instead of $\triangle DEF$ during corrections, the logic behind what has been written by the teacher is mathematically correct and acceptable. On the other hand, Teachers 1, 3, 5, 6, 7 and 8 did not attempt

to correct the identified error in detail. Instead, they just concluded by saying that if there is no construction, there is no need to continue looking at the written statement. All eight teachers confirmed that when learners write proof without construction, that is a total breakdown, which makes the learner forfeit all the marks allocated for that particular theorem. So, they could give reasons for the error and devise ways to avoid it. Below are some of the teacher's responses to Question 2.1. from the given task:

Figure 4.7.1: Teacher 8 response to Question 2.1.

Teacher 8: Proving the diagram.

Interviewer: Yeah.

Teacher 8: One. One, there is no construction.

Interviewer: There is no construction. Very good. I see there is no construction. And then, if that is the case, how do we allocate marks?

Teacher 8: Zero.

Interviewer: There is no construction at all.

Teacher 8: It's a breakdown.

Interviewer: The learner has proven everything. Sometimes, you will find that the learner is correct.

Teacher 8: It's a breakdown.

Interviewer: They are memorising.

Teacher 8: No. Remember, when we mark, we mark according to the construction. The construction can be done by writing, I mean, being constructed or by writing.

Interviewer: By writing it down.

Teacher 8: Yes, writing it down to say I'm constructing a line from here to here.

Interviewer: Okay.

Teacher 8: But if you provide the answer without construction, then it's a breakdown.

Interviewer: Okay. Is that a misconception or an error? So, in that case, the problem lies with maybe the teacher, like they've forgotten to explain to the learner, or maybe the learner just decided, or maybe the learner needs to remember to write to construct.

Teacher 8: The learner needs to remember to construct.

Interviewer: Because they normally forget to construct.

Teacher 8: They always need to remember to construct, yes.

Interviewer: Okay.

Teacher 8: But giving learners more, more priority tests.

Interviewer: Okay. In every test, there should be a theorem to prove.

Interviewer: Okay. And they know that they must construct.

Teacher 8: They know I will get a zero daily if I don't construct. If I don't construct, I'm going to; if you do that, it will be done twice a month. Twice a month. They know that I need to get some marks for the construction. This is a free mark.

Interviewer: Okay. Then so far, it means here the challenge is only construction.

Teacher 8: Construction.

Interviewer: Apart from that, the statements and everything. That's fine.

201)

$$\frac{\text{Area } \triangle DPA}{\text{Area } \triangle PEA} = \frac{\frac{1}{2} DP h_1}{\frac{1}{2} PE h_1} = \frac{DP}{PE} \quad \left| \begin{array}{l} \text{Same Vertex} \\ \text{Same height} \end{array} \right.$$

$$\frac{\text{Area } \triangle DQA}{\text{Area } \triangle QTA} = \frac{\frac{1}{2} DQ h_2}{\frac{1}{2} QT h_2} = \frac{DQ}{QT} \quad \left| \begin{array}{l} \text{Same Vertex} \\ \text{Same height} \end{array} \right.$$

Area $\triangle DPA$ = Area $\triangle DQA$ | same Δ
 Area $\triangle PEA$ = Area $\triangle QTA$ | Same base

$$\Rightarrow \frac{\text{Area } \triangle DPA}{\text{Area } \triangle PEA} = \frac{\text{Area } \triangle DQA}{\text{Area } \triangle QTA}$$

$$= \frac{DP}{PE} = \frac{DQ}{QT}$$

Figure 4.7.2: Teacher 4 response to Question 2.1.

4.3.8. The teachers' Content Knowledge of similarity in Euclidean geometry

Teachers' content knowledge is very important in teaching and learning Euclidean geometry. The following question was posed during the interviews to test the teachers' PCK depth of similarity in general:

9. Are the two rectangles below similar? Justify your answer.

Van Hiele level -2

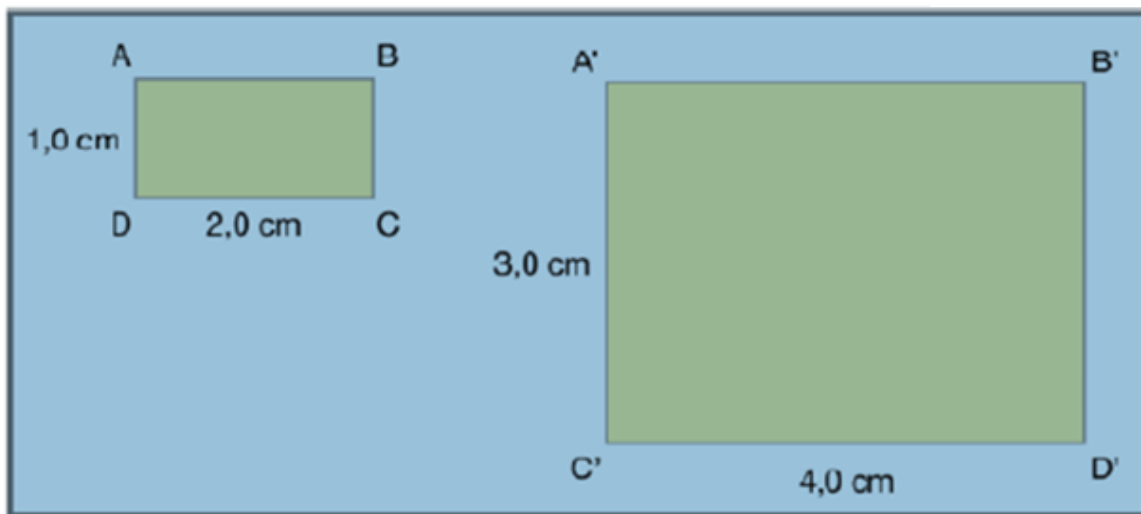


Figure 4.8: Question 9

While conducting the interviews with the teachers, the researcher observed that out of the eight teachers, only 5 showed adequate understanding of Similar polygons, while some needed more understanding. Their responses displayed different views about the concept of similar polygons. For some reason, Teacher 7 did not respond to question 9 altogether.

Teachers 3, 4, 5, 6 and 8 reported that the two rectangles could not be considered similar since their corresponding sides are not in proportion. In contrast, Teacher 1 and 2 were of the view that the two polygons are similar. Furthermore, Teacher 2 has stated that they are similar because a scale factor enlarged the first diagram to obtain the new one. Subsequently, Teacher 1 also stated that the two rectangles are similar because they have equal corresponding angles of 90 degrees. Among the participants who said “No” to the question, Teacher 5 clarified further the conditions of similarity to say that when proving similarity between polygons like quadrilaterals, it is very important to prove by using both conditions of similarity, but when proving similar triangles only one condition can be proven because triangles are special polygons. Below are some of the teacher's responses to the question:

~~AD~~ $\neq$ ~~AC~~ The two rectangles are similar since similarity means two shapes that look the same but not necessarily the same size; since all the $\angle$'s $= 90^\circ$.

Figure 4.8.1: Teacher 1 response to Question 9

Yes They are, They are enlarged by a scale factor of 2 AD $\rightarrow$ AC being $1\text{cm} \rightarrow 2\text{cm}$ and DC $\rightarrow$ CD being enlarged by 2 from $2\text{cm} \rightarrow 4\text{cm}$

Figure 4.8.2: Teacher 2 response to Question 9

Teacher 5 below responded correctly by saying the two diagrams are not similar due to the sides that are not in proportion even though they have equal corresponding angles. The teacher shows an understanding of proving similar polygons that both conditions need to be proven since rectangles are not special polygons.

Not similar, Although the corresponding angles are equal the corresponding sides are not in proportion.

Figure 4.8.3: Teacher 5 response to Question 9

Figure 4.8.3: Teacher 5 response to Question 9 (continue)

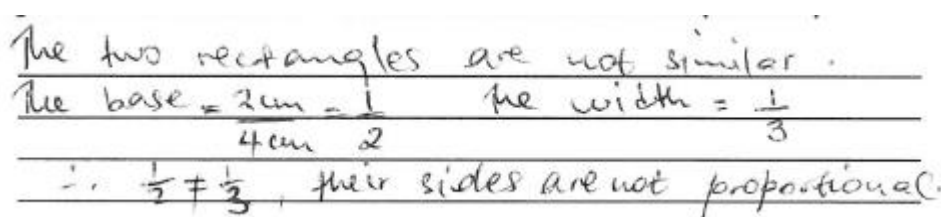
Interviewer

Alright. And the last, this is the last question. Question nine. Oh, the second last question. Are there two rectangles below similar? Justify your answer.

Teacher 5: Okay, for rectangles, you can't just conclude based on sides only or angles only. You have to prove that both the angles and the sides are unlike in triangles, where you can prove one. Choose either between angles and sides. You can prove that angles corresponding angles are equal. Then, you can conclude that the triangles are similar. Or if you can prove that the sides are in proportion, you can conclude that the triangles are similar to the other polygons.

Interviewer: Okay, you must prove both.

Teacher 5: You must prove both sides and both angles. So, triangles are special polygons. Because for them, you only prove that you choose one. It's up to you which one you want to see.



The two rectangles are not similar.
 The base = $\frac{2\text{cm}}{4\text{cm}} = \frac{1}{2}$ the width = $\frac{1}{3}$
 $\therefore \frac{1}{2} \neq \frac{1}{3}$, their sides are not proportional.

Figure 4.8.4: Teacher 8 response to Question 9

Furthermore, when testing the teachers' PCK of similarity Question 10.c. and 10.d. of the given task were used to gather their responses.

10. State whether the following statements are true or false. If false, state reasons for your answer. (Zhang & Wong, 2021:11).

Van Hiele level -2

- c. All rhombi are similar.
- d. All equilateral triangles are similar.

Figure 4.8. b: Question 10.c and 10.d.

Based on the results obtained by the researcher on the response to question 10 above, Teachers 2, 3, 4 and 1 believe that all rhombi are similar, and all equilateral triangles are also similar. Whereas Teachers 5, 7 and 8 argue that all rhombi cannot be similar since they can have different angles, all equilateral triangles are similar. On the other hand, Teacher 6 affirms that for all rhombi to be considered similar, it will depend on whether they satisfy both conditions of similarity or not. However, she could not state whether the answer was true or false. She just said, "It depends". Below are the comments made by some of the teachers about the question above:

The image shows a handwritten response on lined paper. The first line contains 'c - true.' and the second line contains 'd - true.'.

Figure 4.8.b.1: Teacher 4 response to Question 10.c and 10.d.

Figure 4.8.b.2: Teacher 5 response to Question 10. c and 10.d.

Teacher 5: No, can I withdraw my conclusion? All rhombi will be similar. I think so.

Interviewer: Should I change this to be true?

Teacher 5: Want to check? Can't I have a situation where the angles have changed? I have 70, 70. And then I have one 80, one 80. It will still be a rhombus. And the angles are different. So they will not be equal. So I maintain. Yeah, not all rhombi are similar.

Interviewer: Yeah.

Teacher 5: Because of that.

Interviewer: Okay. It's false.

Teacher 5: *It's false, yeah. All equilateral triangles are similar. Yes.*

Interviewer: *True, right?*

Teacher 5: *Yeah, true.*

Figure 4.8.b.3: Teacher 6 response to Question 10. c and 10.d.

Interviewer: *Alright, that is noted...All rhombi are similar.*

Teacher 6: *When we talk of similarities, we refer to triangles.... So, when you are now talking of a rhombus, it means referring to the quadrilaterals....*

Interviewer: *So does it mean that similarity can only be applied to triangles?*

Teacher 6: *It depends.... If they satisfy the condition, then they can be similar.*

Interviewer: *So can you say the answer is true or false?*

Teacher 6: *It depends on the condition.... If you say they are similar, do they satisfy the condition? Remember that... we have the conditions for similarity and the ones for congruency...*

Interviewer: *Okay...*

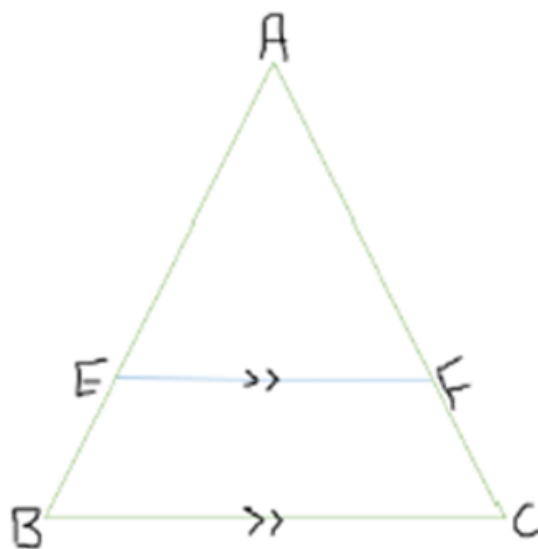
4.3.9. Visualisation of geometric figures

Apart from the misconceptions and errors already mentioned, it has been noted from Section 2.7. that learners often prefer using concept images instead of defining concepts based on their reasoning (Clement, 2003:155); in most cases, that becomes a challenge because some of the diagrams are poorly represented in most of the South Africa textbooks. That can sometimes become a serious challenge, often leading to misconceptions and errors. For example, most Grade 12 prescribed South African textbooks have an equilateral triangle and isosceles triangles for both the proportionality

and similarity proofs or theorems (Aird *et al.*, 2013:283; Bowie *et al.*, 2015:241; DBE, 2021:10; DBE, 2011:14; DBE, 2021: 10; Malan *et al.*, 2020:91; Maskew Miller & Heinemann, 2021:12; Philips *et al.*, 2013:260; Smith, 2014:255), of which the diagrams represented are supposed to be of a general scalene triangle.

So, in this case, Question 8 below was used to test the teacher's way of visualising diagrams in geometry. They were asked to respond to the following question about proving a theorem:

8. If it is given that $\triangle ABC$ below has $EF \parallel BC$, can it be proven that $\frac{AE}{EB} = \frac{AF}{FC}$? Justify your answer.



Van Hiele level -2

Figure 4.9: Question 8

All eight teacher participants, from Teacher 1 to 8, argued that any triangle can prove the proportionality theorem as long as parallel lines are given or indicated on the diagram. Teacher 2 continued to say that even with the similarity theorem, any triangle can be used to prove the theorem as long as the two given triangles, which are said to be similar to each other, are of the same kind. Below are some of the responses from the teachers:

Figure 4.9.1: Teacher 7 response to Question 8

Interviewer: According to you, what type of a triangle was mentioned here in question 8?

Teacher 7: ... I think it's an Isosceles triangle, but because it was never mentioned in the instructions, we can assume it is a scalene triangle with no equal sides.

Interviewer: Okay...

Teacher 7: Yes, if it was mentioned that $AB = AC$, then it is an isosceles triangle.

Interviewer: Ohhh... so if they did not mention it, it means it is a scalene, neh?

Teacher 7: Yes. It's a scalene.

Interviewer: Okay...but when you prove a proportionality theorem, does the type of triangle matter? ...Can any triangle be used to prove a proportionality theorem?

Teacher 7: Yes. Any triangle can be used to prove the proportionality theorem. It doesn't matter whether it is an Isosceles, scalene, or equilateral.... As long as there are // (parallel) lines given.

Interviewer: So, what is your response to question 8? Can it be proven that $\frac{AE}{EB} = \frac{AF}{FC}$?

Teacher 7: Yes, it can be proven.... Given that EF is parallel to BC, the theorem states, "Line // to one side of a triangle divides the sides of a triangle proportionally.".

Interviewer: Alright...

Figure 4.9.2: Teacher 2 response to Question 8

Interviewer: Can it be proven that AE over EB equals AF over FC?

Teacher 2: Yeah, it can be proven.

Interviewer: Okay, justify your answer. Why?

Teacher 2: You can... The reason why it can be proven is because... because we have parallel lines. And then, as we are proving, we will construct. And then, as we are constructing, we can prove. This is the proportionality theorem.

Interviewer: This is the proportionality theorem. Okay. So, it doesn't matter, regardless of the triangle. It doesn't matter which triangle is given.

Teacher 2: Yeah, it doesn't...

- It can be proven because it is proportionality Theorem at hand once parallel lines are mentioned. ✓
 - As a result we can prove the proportion of $\triangle AEF$ and $\triangle ABC$.
 - Similarity can be proven using similar triangles (e.g. equilateral and equilateral $\triangle$), whereas proportionality can be proved using any triangle as long as parallel lines are mentioned.
 - On similar triangles example, scalene & scalene, equilateral and equilateral, Isosceles and isosceles.

Figure 4.9.2: Teacher 2 response to Question 8 (continuation)

4.6. Summary

The findings of this study, as presented in this chapter, show that some of the teachers involved in this study have similar challenges to those of learners about similarity in geometry, as established by the NSC national diagnostic reports. This is cause for concern when one considers teachers' critical role in facilitating learning in mathematics, and learners look up to these teachers for guidance. The next chapter will entail a more detailed discussion of the research findings about the research questions of this study.

Chapter 5: Findings, conclusion & recommendations

5.1. Introduction

In this chapter, the researcher will discuss the findings, limitations of the study, areas of future research, recommendations, and conclusion. This chapter discusses the findings of the main research question as presented in Chapter 4.

The main question is:

What is the depth of the Grade 12 teachers' pedagogical content knowledge concerning similarity in Euclidean geometry?

In answering the main research question, the following sub-research questions served as a guide:

- What is the competence of Grade 12 mathematics teachers in identifying and correcting misconceptions and errors from Grade 12 learners' written work focusing on similarity?
- How do Grade 12 mathematics teachers apply their pedagogical content knowledge of Euclidean geometry when identifying and correcting misconceptions and errors in problems related to similarity?
- What are the experiences of Grade 12 teachers' when identifying and correcting misconceptions and errors in learners' responses related to similarity?

5.2. Discussion of research findings

As mentioned in Chapter 2, most of the learners in the South African schooling system are not performing well in the subject of mathematics (Alex, 2019:2; Pournara *et al.*, 2015) (See section 2.3 of Chapter 2) and more so in Euclidean geometry. National diagnostic reports are provided every year on the NSC examination. These reports, amongst other things, indicate the common misconceptions and errors that learners commit during the writing of their NSC examination papers for every topic in every subject and provide suggestions for improvement (DBE, 2017:164; DBE, 2018:143; DBE, 2019:192; DBE, 2020:195; DBE, 2021:198). The Department of Basic Education, in turn, every year, using

the contents of the diagnostic reports, runs roadshows for teachers to equip them on what to be careful of in teaching learners. Despite all these attempts, some misconceptions and errors in the mathematics diagnostic reports persist. Amongst other factors contributing to the poor performance of learners in mathematics, some researchers are of the view that a lack of depth of pedagogical content knowledge, Curriculum Knowledge and Subject Content Knowledge also contributes (van der Sandt & Nieuwoudt, 2003:199; Stoker, 2003:11; Atebe, 2008:54) (*See section 2.11 of Chapter 2*).

In considering the critical role that teachers play in providing a learning environment for learners, this study focused on establishing the teachers' depth of pedagogical content knowledge, for it is this depth that informs or guides how teachers create a positive, conducive environment for learners. I also established teachers' pedagogical content knowledge depth of similarity in Euclidean geometry for this study. As discussed in section 1.2, this is one topic where learners perform very poorly.

This section deliberates on the study's research findings about the analysed data in Chapter 4.

5.2.1. The Grade 12 teachers' pedagogical content knowledge depth of similarity in Euclidean geometry

Based on the findings from the analysed data in Chapter 4, it is evident that teachers' PCK depth of similarity needs to improve. It has been found that most of the teachers responded correctly to some of the questions they are familiar with and are part of the FET (Grade 10- 12) content. Even though they had challenges, that was different from the sections of the interviews where teachers were required to show their conceptual understanding of the topic. Most of them were challenged by a certain portion of the Grade 12 content that is examinable and most sections that are also part of the Grade 12 syllabus but not examinable and required their conceptual understanding of similarity.

5.2.2. The competence of Grade 12 mathematics teachers in identifying and correcting the types of misconceptions and errors from Grade 12 learners' written work focusing on similarity

The researcher observed that Teacher 1 and 2 displayed a misconception of labelling angles incorrectly. Both teachers could not identify the incorrect labelling of angles as a

misconception or error from the learner's written work and correct that misconception or error. As presented in section 2.8. this is like Euclidean geometry, where the 2017- 2021 diagnostic reports found that "Other candidates could not name the angles correctly. Instead: they would write $\hat{A}$ instead of $\hat{A}_1$ or, $\hat{B} = \hat{C}$ instead of $\hat{B}_1 = \hat{C}_1$, $\hat{M}_{1+2}$ instead of $\hat{M}_1 + \hat{M}_2$) (DBE, 2017:172; DBE, 2018:150; DBE, 2019:199; DBE, 2020:195; DBE, 2021:198).

On the very same note, teachers were given Question 2.1. of the task-based interview with the hope that their responses will determine whether they can identify and correct such an error from the learner's written work. The researcher discovered that all the teachers could identify and correct the error of proving a theorem without construction. However, most of them could not further see that the heights were labelled the same on the learner's written work. They could not identify other possible errors because there was no construction at all. Some teachers even stated that in most cases, learners fail to construct because of have either memorised that theorem or need to remember to construct. They further stated that learners should be taught to construct when proving a theorem and to state a construction procedure to avoid this error of forgetting to construct when proving geometric theorems.

It has been noted that some teachers need to be made aware of incorrect reasons for the statements in geometry. This shows that teachers must display a deeper understanding of some geometric concepts, which becomes challenging since the learners must learn from the best. On the same note, the researcher has observed that in this study Teachers 1, 3 and 8 were not able to differentiate between a theorem and its converse, particularly when giving reasons for their statements (See chapter 4, Figure 4.1). Similarly, it has been mentioned in various diagnostic reports that learners need to be made aware of being able to differentiate between a theorem and its converse in Euclidean geometry. The researcher affirms that this resulting misconception amongst learners could be due to the teacher's lack of PCK.

As it appears in the various diagnostic reports that learners cannot differentiate between similarity and congruency, the researcher has also observed that some teachers could not identify the error of the incorrect usage of the congruency symbol instead of using the similarity symbol from the learner's written work. Based on the 2017 to 2021 diagnostic

reports, it has been noted that most of the Grade 12 learners attempt to prove theorems without showing the construction (DBE, 2017:172; DBE, 2018:150; DBE, 2019:199; DBE, 2020:195; DBE, 2021:198). Other researchers also found that learners attempt to prove geometric theorems without showing any construction (Healy & Hoyles, 2000:428; Lin, 2005:11; Weber, 2004).

5.2.3. Grade 12 mathematics teachers' experiences and how they apply their pedagogical content knowledge of Euclidean geometry when identifying and correcting misconceptions and errors in problems related to similarity

Based on the data's findings, some teachers displayed insufficient Content Knowledge in some aspects of Euclidean geometry. For instance, teachers were asked to state if two given rectangles are similar or not, and so when responding to the question, they did not display the knowledge of understanding similar polygons. They could not state that when proving the similarity between polygons both conditions need to be proven before it can be concluded that the polygons are similar or not.

However, Teacher 5 was able to mention further that *"You must prove both sides and both angles. So, triangles are special polygons. Because for them, you only prove, you choose one. It's up to you which one you want to see"* (See section 4.3. of Chapter 4). This is also supported by a statement made by Amaral-Schio and Hollebrand (2021:11), who state that polygons can only be similar if both conditions of similarity are met. In this study, some teachers believed that all rhombi are similar, whereas some were against that view. So that, on its own, shows that teachers did not have a common understanding of the concept of similarity. Therefore, this study revealed that some teachers do not display sufficient Content Knowledge of the similarity concept in Euclidean geometry.

However, all the teachers had a common understanding of the similarity between equilateral triangles as they all had the same view that equilateral triangles are similar. In this context, Content knowledge is one of the components of PCK, which refers to the teacher's knowledge and understanding of similarity in Euclidean geometry. The researcher is of the view that teachers' lack of Content Knowledge could have a negative influence towards learner performance. Waller (2012:1) concurs with the above statement

by saying that how certain individual learners learn and how teachers teach during the learning process influences the learner achievement outcomes in a subject.

According to Teacher 5 and 8, learners should be given class tests which contain theorems to get used to the proofs since they are often taught occasionally. Furthermore, Teacher 5 believes that teachers should also show their learners a marking guideline as to how marks are being allocated when marking a theorem, including the construction part so that the learners can begin to take the construction of a theorem very seriously. On the very same note, Teacher 2 is of the view that learners should be taught to construct on the diagram and at the same time write down the construction in words so that in case they forget to construct on the diagram, then at least they will be on the safe side since they will have the construction in words. If the learner writes or states the construction accordingly, he or she will not be penalised for not constructing on the given diagram.

After the teachers identified and corrected the learner's misconceptions and errors from the learner's written work, teachers were also asked to come up with the possible reasons for such a misconception and error and how it can be avoided from persisting in future. Sometimes, they could devise different strategies in their daily teaching and learning. For example, other teachers replied that the reason for the learners to commit this error of not constructing when proving a geometric theorem could be the result of memorisation. However, some teachers confessed that they normally teach their learners to memorise theorems. Other teachers said that the error of not constructing when proving a theorem could be caused by ignorance from the learner side and others said that the learner might have forgotten to do the construction. De Villiers (2004:704) states that learners often experience the challenge of being unable to deal with geometric problems that entail proofs because most textbook authors and teachers provide learners with ready-made products that deny them the opportunity to think deductively. Therefore, he further advises that when teaching proofs, it is very important to understand the cognitive thinking of the learners to avoid memorisation.

Ngirishi and Bansilal (2019:13) affirm that most learners need help to prove theorems, especially when it requires them to show many steps. Based on the findings from a study by Cassim (2006:102), learners use visualisation to prove a theorem. Proofs not only challenge the learners at the high school level but were also found to be challenging to the university students due to insufficient understanding of the proofs (Weber, 2001:117).

5.2.4. The implications for Grade 12 teachers in identifying and correcting learners' misconceptions and errors in Euclidean geometry

Errors in diagrams need to be taken seriously, as that could result in a series of misconceptions. For example, in the previous chapter 4, teachers were asked if any triangle can be used to prove the proportionality or similarity theorem, so all of them affirmed that any triangle can be used to prove the proportionality theorem as long as there are parallel ($//$) lines on the given triangle. That is not the case because no other triangle could be used to prove such theorems apart from a general scalene triangle. A similar misconception has been found in some Grade 12 prescribed South African mathematics textbooks and study materials (Aird *et al.*, 2013:283; Bowie *et al.*, 2015:241; DBE, 2021:10; DBE, 2011:14; DBE, 2021: 10; Malan *et al.*, 2020:91; 2021:12; Philips *et al.*, 2013:260; Smith, 2014:255). It is evident that teachers often need to pay more attention to textbooks instead of infusing their PCK competencies into teaching and learning.

Since the study explores the teachers' PCK depth of similarity, it is basically on teachers' TSPCK. It is very important and beneficial for teachers to be able to identify and correct the learner misconceptions and errors because this will enable them to know what is difficult to teach so that they can improve their knowledge of conceptual teaching strategies and be able to identify the learners' prior knowledge when teaching Euclidean geometry. In that case, the emanating misconceptions and errors can also be eradicated or avoided, and that will assist in improving the learner's performance in mathematics. However, Luneta (2015:2) argues that knowledge of learners' errors is crucial, and teachers should allow their learners to show their mistakes for effective teaching and learning. Committing errors should be considered part of the learning process (Makhubele, 2014:169).

As section 2.7. of the previous chapter (see *Chapter 2*) indicates, visualisation plays a crucial role in teaching and learning geometry. According to Ngirishi and Bansilal (2019:15), during geometry teaching, teachers need to use a variety of representations instead of relying only on definitions or verbal explanations. This will enable the learners to easily connect visual and verbal representations to switch from one representation to another.

Looking at the profile of the teachers as presented in Chapter 4, section 4.2, one can see that most teachers are highly qualified. Yet some of them struggled with the same concepts as their learners, as shown by diagnostic reports. The higher qualifications in mathematics do not translate to being able to present the school content well. What is critical here is having a profound knowledge of school mathematics. The findings also showed that some of the teachers in this study demonstrated misconceptions and errors in Euclidean geometry. Put in the words of Ma (1999:82), "...without a solid knowledge of what to represent, no matter how rich one's knowledge of students' life, no matter how much one is motivated to connect mathematics with students' lives, one still cannot produce a conceptually correct representation".

5.3. Limitations of study

After serious consideration of the research methodology of the study and the context in which the study took place, the following limitations were observed:

- The investigation was restricted to six schools under the Gauteng North district, which is located in Pretoria, in the Gauteng province of South Africa. Though this was considered a limiting factor, conducting the study within the confines of this location allowed the researcher to access the target schools, and these schools fell within a reasonable travelling demand from the researcher's place of residence.
- Only a certain group of Grade 12 mathematics teachers were selected for participation in the study. The selection of this group turned to render the study findings not generalisable to other Grade levels within the Gauteng North district.
- The study solely focused on teachers and thus examined teachers'

PCK depth. The researcher believes that teachers' PCK may influence the nature of learners acquired problem-solving abilities. Hence, this study would not have managed to study the problem holistically, which could have generated more representative and comprehensive findings.

- The fact that the teachers' PCK depth was only explored through analysing the learner's written work and without doing class visits was a limiting factor. By doing the class visits, the researcher would have been able to observe the various classroom teaching practices, teaching strategies, lesson planning and preparations done by the teachers instead of only being able to observe how the teachers handle the misconceptions and errors that learners commit. That would have contributed to the triangulation of the findings.

5.4. Recommendations

The following are the recommendations that are made for this study:

- The subject advisors should consider the aspects mentioned in the diagnostic reports when conducting the workshops for teachers so that whatever misconceptions and errors are addressed immediately with the teacher.
- Subject advisors should give teachers pre-tests and generally present the test results containing various concepts in geometry, considering the aspects mentioned in the diagnostic reports in the form of a workshop, and try to address the findings with the educators. However, the educator names need not to be mentioned to respect their human dignity.
- One-on-one mentorship sessions, perhaps by the subject advisors with the teachers could also help so that the subject advisors can directly address the issues with the direct individuals, particularly for teachers that do not have much experience in the subject.
- The topic of similarity should also be infused into the Grade 11 syllabus so that it can become progressive across all the FET Grades.
- Teachers should always use examination guidelines in line with the prescribed CAPS materials, especially during the teaching of geometry in schools and the

content training workshops. This will enable the teachers to familiarise themselves with the applicable geometric reasons.

- There is a need to revisit the curriculum at schools and tertiary institutions where teachers are trained. One would have expected the teachers to be deeper in their Content Knowledge, bearing in mind that the tertiary institutions would have offered a second opportunity for the trainee teachers to review the topics the second time.
- Given the history of handling Euclidean geometry in South African Schools, as discussed in section 1.2.2 and section 2.6, there is a need to revisit the time allocation to this topic in both schools and tertiary institutions to break the cycle of poor performance.

Lastly, since the current study explored the Grade 12 teachers' PCK of similarity in Euclidean geometry, using a small sample and recruiting teachers from one district, it is recommended that a similar study be conducted about both Grade 12 proportionality and similarity using a larger sample space.

5.5. Conclusion

This study explored the Grade 12 teachers' PCK of similarity in Euclidean geometry in one district under Gauteng province in South Africa. Some of the teachers were fine when they had to identify and correct misconceptions and errors from the learner's written task; maybe that is because of were familiar with the questions and the structure of the entire paper. However, they displayed challenges when responding to general questions, which are also part of the Grade 12 syllabus and other areas based on similarity.

The teacher's PCK depth of similarity in Euclidean geometry needs to be improved and up to the ideal standard. Teachers' PCK play a vital role in teaching and learning. Hence, poor PCK depth by teachers can result in poor performance by learners. Therefore, the researcher believes that if the abovementioned recommendations can be considered and properly followed, future Grade 12 mathematics educators can handle the same challenges. That, on its own, might help to improve the standard and performance of

mathematics subjects in general and geometry in particular in the secondary school curriculum.

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APPENDIX A: ETHICS APPROVAL LETTER



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North-West University
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Senate Committee for Research Ethics

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7 August 2023

Dear Dr Nkhoma

ETHICS APPROVAL LETTER OF STUDY

Based on approval by the North-West University Education, Management and Economic Sciences, Law, Theology, Engineering and Natural Sciences Research Ethics Committee (NWU-EMELTEN-REC) on 7 August 2023, the NWU-EMELTEN-REC hereby approves your study as indicated below. This implies that the NWU-EMELTEN-REC grants its permission that, provided the general conditions specified below are met and pending any other authorisation that may be necessary, the study may be initiated, using the ethics number below.

Study title: Exploring Grade 12 teachers' pedagogical content knowledge of Similarity in Euclidean geometry.																		
Principal Investigator/Study Supervisor/Researcher: Dr Pentecost Nkhoma																		
Co-supervisor: Dr Nothile Kunene																		
Student: Ms Goodness Shoba (40806197)																		
Ethics number: <table border="1"><tr><td>N</td><td>W</td><td>U</td><td>-</td><td>0</td><td>0</td><td>4</td><td>0</td><td>4</td><td>-</td><td>2</td><td>3</td><td>-</td><td>S</td><td>2</td></tr></table>				N	W	U	-	0	0	4	0	4	-	2	3	-	S	2
N	W	U	-	0	0	4	0	4	-	2	3	-	S	2				
Institution Study Number Year Status Status: S = Submission; R = Re-Submission; P = Provisional Authorisation; A = Authorisation																		
Application Type: Single study																		
Risk:		Medium Risk																
Commencement date: 07/08/2023																		
Expiry date: 06/08/2024																		
Approval of the study is provided for a year, after which continuation of the study is dependent on the receipt and review of a six-monthly monitoring report and the concomitant issuing of a letter of continuation.																		

General conditions:

While this ethics approval is subject to all declarations, undertakings and agreements incorporated and signed in the application form, the following general terms and conditions will apply:

- *The principal investigator/study supervisor/researcher must report in the prescribed format to the NWU-EMELTEN-REC:*
 - *twelve-monthly on the monitoring of the study, whereby a letter of continuation will be provided annually, and upon completion of the study; and*
 - *without any delay in case of any adverse event or incident (or any matter that interrupts sound ethical principles) during the course of the study.*
- *The approval applies strictly to the proposal as stipulated in the application form. Should any amendments to the proposal be deemed necessary during the course of the study, the principal investigator/study supervisor/researcher must apply for approval of these amendments at the NWU-EMELTEN-REC, prior to implementation. Should there be any deviations from the study proposal without the necessary approval of such amendments, the ethics approval is immediately and automatically forfeited.*
- *Annually a number of studies may be randomly selected for active monitoring.*
- *The date of approval indicates the first date that the study may be started.*
- *In the interest of ethical responsibility, the NWU-EMELTEN-REC reserves the right to:*
 - *request access to any information or data at any time during the course or after completion of the study;*
 - *to ask further questions, seek additional information, require further modification or monitor the conduct of your research or the informed consent process; – withdraw or postpone approval if:*
 - *any unethical principles or practices of the study are revealed or suspected;*
 - *it becomes apparent that any relevant information was withheld from the NWU-EMELTEN-REC or that information has been false or misrepresented;*
 - *submission of the twelve-monthly monitoring report, the required amendments, or reporting of adverse events or incidents was not done in a timely manner and accurately; and/or*
 - *new institutional rules, national legislation or international conventions deem it necessary.*
- *NWU-EMELTEN-REC can be contacted for further information via Ethics-EMELTEN-apply@nwu.ac.za or 018 285 3037*

The NWU-EMELTEN-REC would like to remain at your service and wishes you well with your study.

Please do not hesitate to contact the NWU-EMELTEN-REC for any further enquiries or requests for assistance.

Yours sincerely,



Prof Erhabor Idemudia
Chairperson NWU-EMELTEN-REC

Current details: (23239522) G:\My Drive\1. Research and Postgraduate Education\9.1.5.4 Templates\9.1.5.4.2_NWU-HREC_EAL.docm 20 August 2019

File Reference: 9.1.5.4.2

APPENDIX B: APPLICATION TO CONDUCT RESEARCH

ENQ: Shoba Goodness
P O Box 22029
Groblersdal
0470
CELL: 079 952 8679
27 January 2022

The District Director
Gauteng North District
Private bag X75
Pretoria
0001

APPLICATION FOR PERMISSION TO DO RESEARCH IN YOUR DISTRICT

Dear Sir/ Madam

I, at this moment, request permission to conduct research in your District. I am a Masters's student at North West University (Potchefstroom Campus). My topic is **Exploring Grade 12 Teachers' Pedagogic Content Knowledge of Similarity in Euclidean Geometry**.

The purpose is to interview Grade 12 Mathematics teachers while I also collect data.

I enclose herein the Ethical Clearance certificate from my University.

Yours faithfully

.....

Shoba G.N

APPENDIX C: APPROVAL TO CONDUCT RESEARCH FROM GDE



GAUTENG PROVINCE

Department: Education
REPUBLIC OF SOUTH AFRICA

8/4/4/1/2

GDE RESEARCH APPROVAL LETTER

Date:	15 August 2023
Validity of Research Approval:	08 February 2023– 30 September 2023 2023/347
Name of Researcher:	Shoba GN
Address of Researcher:	No 46 Sokhulum Bronkospruit
Telephone Number:	079 9528679
Email address:	Goodnessshoba902@gmail .com
Research Topic:	Exploring Grade 12 teacher pedagogical content knowledge of Similarity in Euclidean geometry
Name of University:	Northwest University
Type of qualification	Masters
Number and type of schools:	06 Secondary Schools
District/s/HO	Gauteng North

Re: Approval in Respect of Request to Conduct Research

This letter serves to indicate that approval is hereby granted to the above-mentioned researcher to proceed with research in respect of the study indicated above. The onus rests with the researcher to negotiate appropriate and relevant time schedules with the school/s and/or offices involved to conduct the research. A separate copy of this letter must be presented to both the School (both Principal and SGB) and the District/Head Office Senior Manager confirming that permission has been granted for the research to be conducted.

[Signature] 15/08/2023

The following conditions apply to GDE research. The researcher may proceed with the above study subject to the conditions listed below being met. Approval may be withdrawn should any of the conditions listed below be flouted:

1

Making education a societal priority

Office of the Director: Education Research and Knowledge Management

7th Floor, 17 Simmonds Street, Johannesburg, 2001

Tel: (011) 355 0488

Email: Faith.Tshabalala@gauteng.gov.za

Website: www.education.gpg.gov.za

1. Letter that would indicate that the said researcher/s has/have been granted permission from the Gauteng Department of Education to conduct the research study.
2. The District/Head Office Senior Manager/s must be approached separately, and in writing, for permission to involve District/Head Office Officials in the project.
3. **Because of the relaxation of COVID 19 regulations researchers can collect data online, telephonically, physically access schools or may make arrangements for Zoom with the school Principal. Requests for such arrangements should be submitted to the GDE Education Research and Knowledge Management directorate.**
4. **The Researchers are advised to wear a mask at all times, Social distance at all times, Provide a vaccination certificate or negative COVID-19 test, not older than 72 hours, and Sanitise frequently.**
5. A copy of this letter must be forwarded to the school principal and the chairperson of the School Governing Body (SGB) that would indicate that the researcher/s have been granted permission from the Gauteng Department of Education to conduct the research study.
6. A letter / document that outline the purpose of the research and the anticipated outcomes of such research must be made available to the principals, SGBs and District/Head Office Senior Managers of the schools and districts/offices concerned, respectively.
7. The Researcher will make every effort obtain the goodwill and co-operation of all the GDE officials, principals, and chairpersons of the SGBs, teachers and learners involved. Persons who offer their co-operation will not receive additional remuneration from the Department while those that opt not to participate will not be penalised in any way.
8. Research may only be conducted after school hours so that the normal school programme is not interrupted. The Principal (if at a school) and/or Director (if at a district/head office) must be consulted about an appropriate time when the researcher/s may carry out their research at the sites that they manage.
9. Research may only commence from the second week of February and must be concluded before the beginning of the last quarter of the academic year. If incomplete, an amended Research Approval letter may be requested to conduct research in the following year.
10. Items 6 and 7 will not apply to any research effort being undertaken on behalf of the GDE. Such research will have been commissioned and be paid for by the Gauteng Department of Education.
11. It is the researcher's responsibility to obtain written parental consent of all learners that are expected to participate in the study.
12. The researcher is responsible for supplying and utilising his/her own research resources, such as stationery, photocopies, transport, faxes and telephones and should not depend on the goodwill of the institutions and/or the offices visited for supplying such resources.
13. The names of the GDE officials, schools, principals, parents, teachers and learners that participate in the study may not appear in the research report without the written consent of each of these individuals and/or organisations.
14. On completion of the study the researcher/s must supply the Director: Knowledge Management & Research with one Hard Cover bound and an electronic copy of the research.
15. The researcher may be expected to provide short presentations on the purpose, findings and recommendations of his/her research to both GDE officials and the schools concerned.
16. Should the researcher have been involved with research at a school and/or a district/head office level, the Director concerned must also be supplied with a brief summary of the purpose, findings and recommendations of the research study.

The Gauteng Department of Education wishes you well in this important undertaking and looks forward to examining the findings of your research study.

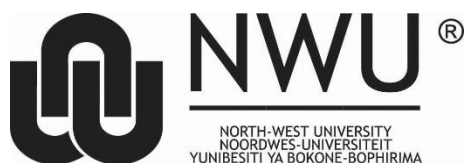
Kind regards



Dr. Gumanj Mukatuni
Acting CES: Education Research and Knowledge Management

DATE: 15/08/2023

APPENDIX D: REQUEST LETTER TO PRINCIPALS



Private Bag X6001, Potchefstroom South Africa 2520

Tel: +2718 299-1111/2222 Web:
<http://www.nwu.ac.za>

(Recipient name).....

(Recipient address).....

(Recipient address).....

(Recipient address).....

**North-West University Education, Management
and Economic Sciences, Law, Theology,
Engineering and Natural Sciences Research Ethics
Office (NWU-EMELTEN-REC)**

Tel: +27 18 285 3037 Email:
Erhabor.Idemudia@nwu.ac.za

Date: 22 June 2023

PERMISSION LETTER: SCHOOL PRINCIPAL/ OTHER RELEVANT PERSON

I request your permission for Grade 12 Mathematics Teachers to participate in this research, which involves completing Mathematics tasks and being interviewed. Participation in this study is voluntary, and confidentiality will be ensured. At any time, teachers can without any penalty, withdraw from the study. The researcher and the independent person will sign a confidentiality agreement at the onset of the study. Before granting permission, please acquaint yourself with the information below.

The details of the research are as follows:

TITLE OF THE RESEARCH PROJECT:

Exploring Grade 12 teachers' pedagogical content knowledge of Similarity in Euclidean geometry

ETHICS APPLICATION NUMBER: NWU-00404-23-S2

PROJECT SUPERVISOR: Dr P. Nkhoma

CO-SUPERVISOR: Dr N. Kunene

ADDRESS: Faculty of Education, Building A9, Room 113

CONTACT NUMBER: 018 389 2297/ 084 843 2205/ 081 434 3747

MEMBER OF PROJECT TEAM MEd-Student: Ms G.N Shoba

ADDRESS: 7019 Tamarix Str Unit 15, Mmabatho, 2735

CONTACT NUMBER: 079 9528 679

FACULTY OF EDUCATION RESEARCH EMELTEN-REC ETHICS COMMITTEE

Contact person: Ms Paballo Molefe, E-mail: Paballo.Molefe@nwu.ac.za , Tel. (018) 285 3037

The Research Ethics Committee of the Faculty of Education of North-West University has approved this study. It will be conducted according to the ethical guidelines of this committee. Permission was also obtained from the provincial Department of Basic Education/other relevant bodies.

What is this research about?

The aims of this research are:

- The researcher seeks to explore the teachers' pedagogical content knowledge in Euclidean geometry, focusing on similarity and its application to Grade 12 learners from the Gauteng North district. More specifically, they can identify and correct learner's misconceptions and errors in the topic. **These objectives will achieve this research aim:**
- To explore Grade 12 mathematics teachers' competence in identifying and correcting misconceptions and errors from Grade 12 learners' written work focusing on similarity.
- To explore how Grade 12 mathematics teachers apply their Pedagogical Content Knowledge of Euclidean geometry when identifying and correcting misconceptions and errors in problems related to similarity.
- To explore Grade 12 teachers' experiences when identifying and correcting errors and misconceptions in Euclidean geometry problems related to similarity.

- To explore the implications of teachers in identifying and correcting learners' misconceptions and errors in Euclidean geometry.
- To explore the teachers' Content Knowledge of similarity in Euclidean geometry.

Participants

- Grade 12 Mathematics educators

What is expected of the participants?

- To identify and correct the types of misconceptions and errors that the Grade 12 learners commit. So, they will be given a task-based interview question with learner solutions to work on.
- The participants will be interviewed.

Benefits to the participants

- They will be developed on the identified misconceptions and errors and will reflect on practice
- Their schools and learners will have teachers who can refocus on these misconceptions and errors to directly correct their learners' challenges to improve mathematics performance.

Risks involved for participants

- The risks to you in this study are that **one of my objectives is to explore Grade 12 Mathematics teachers' competence**, so perhaps you might experience mild discomfort. Therefore, we assure you are free to express any discomfort anytime. Secondly, please note that the responses gained from the interview will not be used against you. Thirdly, your identities will be concealed through the use of pseudonyms.
- There are more gains for you in joining this study than there are risks.

Confidentiality and protection of identity

- The researcher will apply for consent letters to school principals. Confidentiality, privacy and the personal rights of participating educators will be protected by ensuring that I keep every single document to myself and the supervisor. The researcher will ensure that she remains honest and respects the participants' rights.

Dissemination of findings

The results will be sent to the Department of Education, District officials, schools and participants. The overall research project will be made public after being assessed and approved by the university. The participants will be given a summary of the findings.

If you have any further questions or enquiries regarding participating in this research, please contact the researchers for more information.

DECLARATION BY PRINCIPAL/ OTHER RELEVANT PERSON:

By signing below, I agree to permit the research to take place with the identified participants in the study entitled:

[Exploring Grade 12 teachers' pedagogical content knowledge of Similarity in Euclidean geometry]

I declare that:

- I have read this information and consent form and understand what is expected of the participants in the research.
- I have had a chance to ask questions to the researcher and all my questions have been adequately answered.
- I understand that participating in this study is voluntary, and participants will not be pressured to participate.
- Participants may leave the study anytime and will not be penalised or prejudiced.
- Participants may be asked to leave the research process before completion if the researcher feels it is in their best interests or if they do not follow the agreed-upon research procedures.

Signed at (place)_____ on (date) ____/____/20____

Signature of School Principal/ Relevant person

APPENDIX E: REQUEST TO PARTICIPATE IN THE STUDY- TEACHERS



Private Bag X6001, Potchefstroom South Africa 2520

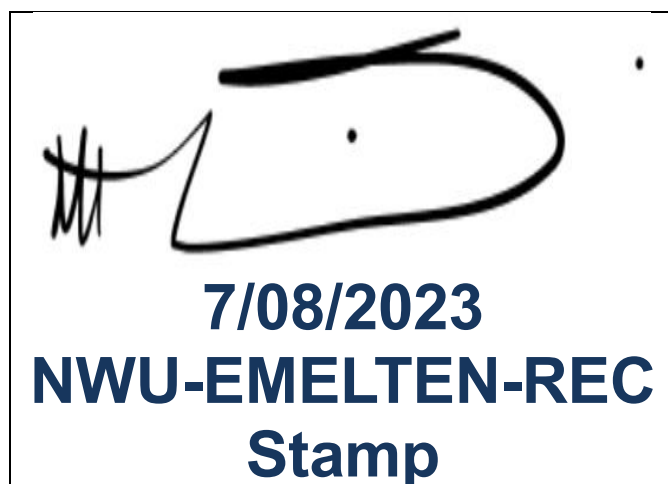
Tel: +2718 299-1111/2222 Web:
<http://www.nwu.ac.za>

**North-West University Education, Management
and Economic Sciences, Law, Theology,
Engineering and Natural Sciences Research Ethics
Office (NWU-EMELTEN-REC)**

Tel: +27 18 285 3037 Email:
Erhabor.Idemudia@nwu.ac.za

Date: 22 June 2023

The Faculty of Health Sciences Ethics Office of the North-West University is acknowledged for the use of their document with minor adjustments made by the North-West University Education, Management and Economic Sciences, Law, Theology, Engineering and Natural Sciences Research Ethics Committee (NWU-EMELTEN-REC).



INFORMED CONSENT DOCUMENTATION

FOR: Grade 12 Mathematics Teachers being interviewed

TITLE OF THE RESEARCH STUDY: Exploring Grade 12 teachers' pedagogical content knowledge of Similarity in Euclidean geometry

ETHICS REFERENCE NUMBERS: NWU-00404-23-S2

PRINCIPAL INVESTIGATOR: Dr Pentecost Nkhoma

POST GRADUATE STUDENT: Ms G.N Shoba

ADDRESS: Corner of university drive and Chief Albert street Luthuli, Mafikeng 2790

CONTACT NUMBER: 018 389 2297 or 0848432205

You are invited to participate in a **research study** that forms part of a Masters study. Please take some time to read the information presented here, which will explain the details of this study. Please ask the researcher or person explaining the research to you any questions about any part of this study you need help understanding. You must be fully satisfied that you clearly understand what this research is about and how you might be involved. Also, your participation is **entirely voluntary**, and you are free to say no to participate. If you say no, this will not affect you negatively in any way whatsoever. You are also free to withdraw from the study at any point, even if you agree to participate now.

This study has been approved by the North-West University Education, Management and Economic Sciences, Law, Theology, Engineering and Natural Sciences Research Ethics Committee (NWU-00404-23-S2). It will be conducted according to the ethical guidelines and principles of Ethics in Health Research: Principles, Processes and Structures (DoH, 2015) and other international ethical guidelines applicable to this study. It might be necessary for the research ethics committee members or other relevant people to inspect the research records.

What is this research study all about?

- *We plan to explore a sample of Grade 12 teachers' pedagogical content knowledge depth concerning similarity in Euclidean geometry.*
- *This study will be conducted at the selected school in the Gauteng North district, in the private Venue of your choice. Depending on the size of each school, at least six participants (educators) will be included in this study.*

Why have you been invited to participate?

- *You have been invited to be part of this research because you are a Grade 12 Mathematics teacher with an FET teaching qualification. Your school is under Gauteng North District.*
- *Unfortunately, You will not be able to participate in this research if your schools are not part of Gauteng North district and are not offering Grade 12 Mathematics in Gauteng North district.*

What will be expected of you?

- *You will be expected to respond to a task-based interview, which may take approximately 2 hours. The first hour is for responding to the tasks, and the other is for responding to the clarity-seeking questions as they will be interviewed. There will be a break between completing the task and the interview. Both the task-based interview and the interviews will have 2 long questions and 4 short follow-up questions, respectively. Other than this break, between tasks, the participant will be made aware that it will be given at any time they wish to request a break.*

Will you gain anything from taking part in this research?

- *The gains for you, if you take part in this study, will be the establishment of an alternative approach to the teaching and learning of school Mathematics in our Gauteng Province schools.*
- *The other gains of the study are for educators to gain new knowledge (different approaches) that will make them better facilitators in mathematics classrooms.*

Are there risks involved in participating in this research, and what will be done to prevent them?

- *The risks to you in this study are that one of my study's objectives is to explore Grade 12 Mathematics teachers' competence, so perhaps you might experience mild discomfort. Therefore, we assure you are free to express any discomfort anytime. Secondly, please note that the responses gained from the interview will not be used against you. Thirdly, your identities will be concealed through the use of pseudonyms.*
- *There are more gains for you in joining this study than risks.*

How will we protect your confidentiality, and who will see your data?

- *The anonymity of your data will be protected by concealing your identity, and pseudonyms will be used in the study. Your privacy will be respected by ensuring that your participation is voluntary and that you can withdraw your involvement in the study at any time you feel uncomfortable. Your results will be kept confidential by not using your real name. Only the researchers and the study supervisor can look at your data. Data will be kept safe by locking hard copies in locked cupboards in the researcher's office, and electronic data will be password protected. (As soon as data has been transcribed, it will be deleted from the recorders.) Data will be stored for 5 years.*

What will happen with the data or samples?

- *The data of this study will be used in future for further studies and to collaborate with other studies in other provinces.*

How will you know about the results of this research?

- *We will give the results of this research project to the Department of Education, District officials, schools and participants directly in person or via email. The overall research project will be made public after being assessed and approved by the university. The participants will be given a summary of the findings.*

Will you be paid to participate in this study, and are there any costs for you?

- *Any outside body does not fund this study. The Master's student is paying for this.*
- *You will only be paid to participate in the study if funds were secured.*
- *You have no travel expenses as the investigator will travel to where you are.*
- *Participants will be given USBs as a token of appreciation for participating in this study and light refreshments.*

Is there anything else that you should know or do?

- You can contact Ms G.N Shoba at 073 4893 084 if you have further questions or problems.
- You can also contact the North-West University Education, Management and Economic Sciences, Law, Theology, Engineering and Natural Sciences Research Ethics Committee via Ms Paballo Molefe at 018 285 3037 or Paballo.Molefe@nwu.ac.za if you have any concerns that were not answered about the research or if you have complaints about the research.
- You will receive a copy of this information and consent form.

APPENDIX F: CONSENT FORM FOR TEACHERS

Declaration by participant

By signing below, I _____ agree to take part in the
a research study titled: **Exploring Grade 12 Teachers' Pedagogical Content Knowledge of Similarity in Euclidean Geometry.**

I declare that:

- I have read this information/it was explained to me by a trusted person in a language I am fluent and comfortable with.
- The research was clearly explained to me.
- I have had a chance to ask questions to both the person getting the consent from me, as well as the researcher and all my questions have been answered.
- I understand that participating in this study is voluntary, and I have not been pressured to participate.
- I may choose to leave the study at any time and will not be handled negatively if I do so.
- I may be asked to leave the study before it has finished if the researcher feels it is in the best interest or if I do not follow the agreed-upon study plan.

Signed at (place) on (date) 20...

.....

Signature of participant

.....

Signature of witness

Declaration by the person obtaining consent.

I (name) declare that:

- I clearly and in detail explained the information in this document to
.....
- I did/did not use an interpreter.

- I encouraged him/her to ask questions and took adequate time to answer them.
- I am satisfied that he/she adequately understands all aspects of the research, as discussed above.
- I gave him/her time to discuss it with others if he/she wished.

Signed at (*place*) on (*date*) 20...

.....

Signature of the person obtaining consent

Declaration by researcher

I (*name*) Goodness Njabulo Shoba declare that:

- I explained the information in this document to, **or** I had it explained by, whom I trained for this purpose.
- I did/did not use an interpreter
- I encouraged him/her to ask questions and took adequate time to answer them or I was available should he/she want to ask further questions.
- An independent person obtained the informed consent.
- I am satisfied that he/she adequately understands all aspects of the research, as described above.
- I am satisfied that he/she had time to discuss it with others if he/she wished to do so.

Signed at (*place*) on (*date*) 20...

.....

Signature of researcher

APPENDIX G: SEMI- STRUCTURED INTERVIEW AND TASK-BASED INTERVIEW SCHEDULE

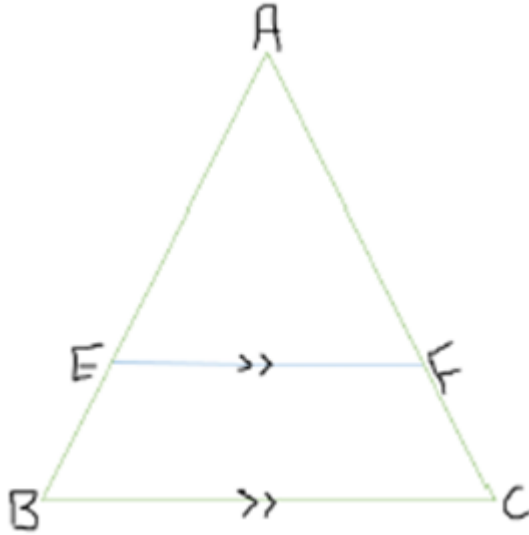
Section A

Introduction, finalizations of the consent form and pseudo-names and explaining the interview process and the use of the recorder (unstructured).

Section B:

1. I have some learner responses to a set of questions on Euclidean geometry. Please go through these sample of learner's response **(See Section C)** below. **(N.B. The responses to no. 2,3,4,5 and 6 below should be practically written / shown in APPENDIX G, section C) on the spaces provided on the right column. You may use an extra answer sheet provided if the space is not enough.**
2. Do you think the learner answered the questions correctly? Are there any errors or misconceptions in the learner's solution?
3. Where is (are) the error(s) or misconception(s)?
4. What could be the reason learners write the solutions like this?
5. Please correct this learner by writing down the correct solutions.
6. How would you explain to the learner to correct this error or misconception?
7. According to you what are errors and misconceptions?

-
-
8. If it is given that $\triangle ABC$ below has $EF \parallel BC$, can it be proven that $\frac{AE}{EB} = \frac{AF}{FC}$? Justify your answer.



9. Are the two rectangles below similar? Justify your answer.

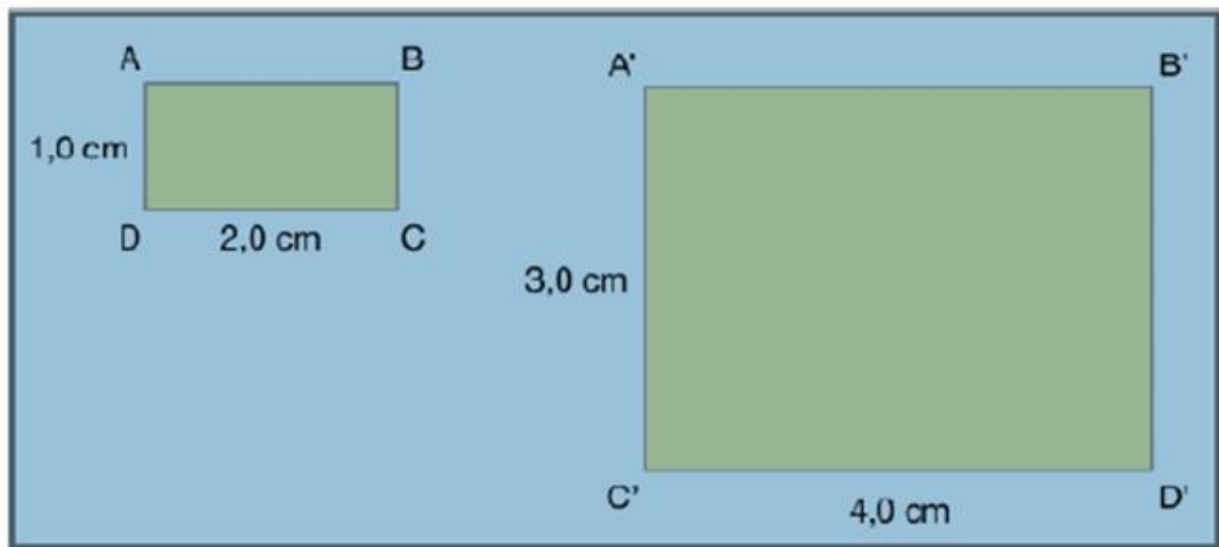
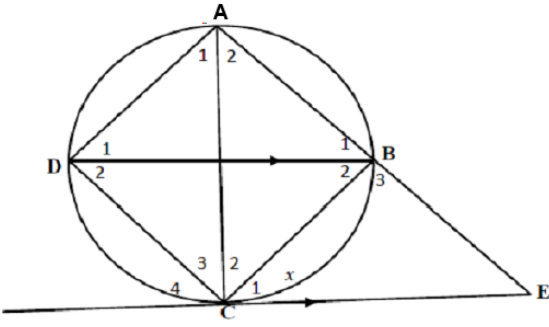


Figure.02: Extracted from Amaral-Schio and Hollebrand (2021: 11)

10. State whether the following statements are true or false. If false, state reasons for your answer. (Zhang & Wong, 2021:11).

- a. All congruent shapes are similar.
- b. All similar shapes are congruent.
- c. All rhombi are similar.
- d. All equilateral triangles are similar.

Section C: Task based interviews

QUESTIONS:	TEACHER RESPONSES
QUESTION 1 [24 Marks] APPENDIX. G (TEST) extracted from the 2017 June question paper from Gauteng and North west province. 1.1. In the figure below $BD \parallel EC$, and CE is a tangent to the circle $ABCD$.  1.1.1. If $\angle BCE = x$, name with reasons other four angles equal to x (4)	
1.1.1. $\hat{C}_1 = \hat{A}_2 = x$ (tan-chord) $\hat{B}_2 = \hat{A}_2$ ($\angle$'s in the same segment) $\hat{C}_1 = \hat{B}_2$ (alt $\angle$'s) $\hat{B} = \hat{A}$ ($\angle$'s in the same segment) $\hat{B}_2 = \hat{C}_4$ (tan-chord) Prove that: 1.1.2. $\triangle AEC \parallel \triangle CEB$ (4) 1.1.2. $\hat{A}_2 = \hat{C}_1 = x$ (proven) $\hat{A}_1 = \hat{B}_2 = x$ ($\angle$'s in the same seg) $\hat{B}_2 = \hat{C}_1 = x$ (alt $\angle$'s) $\hat{C}_4 = \hat{B}_2 = x$ (tan-chord theorem) $\therefore \triangle AEC \parallel \triangle CEB$ ($\angle, \angle, \angle$)	

1.1.3. $EC^2 = EB \cdot EA$

(2)

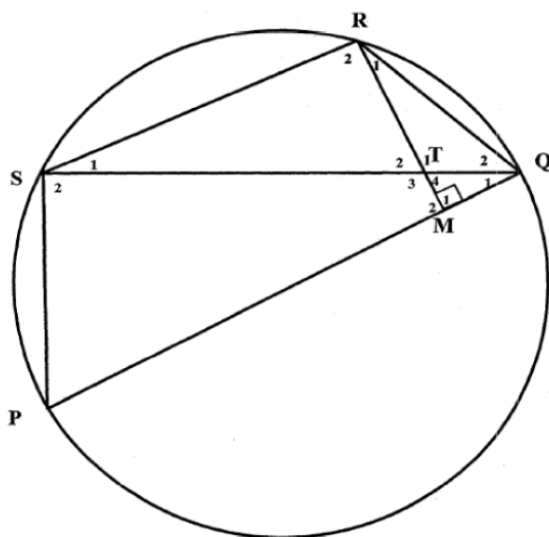
1.1.3. From $\triangle AEC$ and $\triangle CEB$
 $\frac{EC}{EA} = \frac{EB}{EC}$ ($\triangle$'s $\parallel$)
 $\therefore EC^2 = EB \cdot EA$

1.1.4. $\triangle ADC \parallel \triangle ACE$

(3)

1.1.4. $\hat{A} = \hat{A} = x$
 $\hat{D}_1 + 2 = \hat{C}_1 + 2$
 $\hat{C}_3 = \hat{E}$ (3^{rd} $\angle$ of $\triangle$)
 $\therefore \triangle ADC \parallel \triangle ACE$ (C, L, L)

1.2. In the diagram below, PQ is a diameter of the circle. R and S are points on the circle and M is a point on PQ such that $RM \perp PQ$. RM and SQ intersect at T. (extracted from Grade 12 prelim 2017 question paper, from North West province).



Prove that:

$$1.2.1. \hat{RST} = \hat{MRQ} \quad (5)$$

$$\begin{aligned} 1.2.1. \hat{S} &= 90^\circ \text{ (}\angle\text{'s in the same seg)} \\ \hat{Q}_{1+2} &= 90^\circ - \hat{R}_1 \text{ (}\angle\text{'s in } \triangle RMQ) \\ \hat{S}_{1+2} + \hat{Q}_{1+2} &= 180^\circ \text{ (opp } \angle\text{'s of a cyclic quad)} \\ \therefore \hat{S} &= \hat{R} \end{aligned}$$

$$1.2.2. \hat{RTS} = \hat{SPQ} \quad (3)$$

$$\begin{aligned} 1.2.2. \hat{S}_2 &= \hat{M}_2 \text{ (both } = 90^\circ) \\ \text{PMTS is a cyclic quad (ext } \angle \text{ of a cyclic quad)} \\ \therefore \hat{T}_2 &= \hat{P} \end{aligned}$$

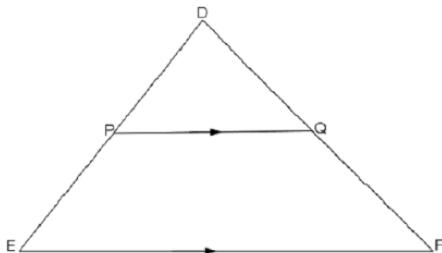
$$1.2.3. \triangle SRQ \parallel \triangle RTQ \quad (3)$$

$$\begin{aligned} 1.2.3. \hat{S} &= \hat{R} \text{ (proven)} \\ \hat{Q} &= \hat{Q} \text{ (Common sides)} \\ \therefore \triangle SRQ &\equiv \triangle RTQ \end{aligned}$$

QUESTION 2 [23 Marks]

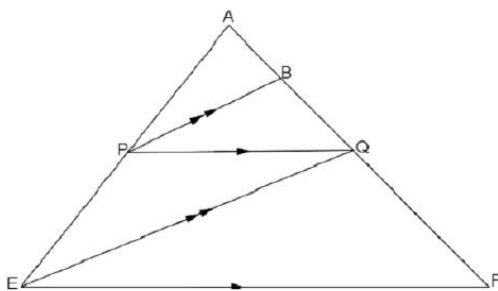
2.1. In the diagram below, P is a point on DE and Q is a point on DF in $\triangle DEF$.

Prove the theorem which states that : If $PQ \parallel EF$, then $\frac{DP}{PE} = \frac{DQ}{QF}$. (5)



2.1. Area of $\triangle DPA = \frac{1}{2} \times DP \times h = \frac{DP}{PE}$
Area of $\triangle PEA = \frac{1}{2} \times PE \times h$
Area of $\triangle DPA = \frac{1}{2} \times DP \times h = \frac{DP}{PE}$
Area of $\triangle PEA = \frac{1}{2} \times PE \times h = \frac{PE}{PE}$
but Area of $\triangle DPA = \text{Area of } \triangle PEA$ (Same base, same height and lie between // lines)
Area of $\triangle DPA = \text{Area of } \triangle PEA$
Area of $\triangle DPA = \text{Area of } \triangle PEA$
 $\therefore \frac{DP}{PE} = \frac{DP}{PE}$

2.2. In the diagram below, $BP \parallel EQ$ and $PQ \parallel EF$. $AP:PE = 2:3$. $AE = 120\text{m}$ and $QF = 60\text{m}$.



2.2.1. Calculate the length of AP

(2)

2.2.1. $AP = \frac{2}{5} \times 120\text{m} = 48\text{m}$

2.2.2. Calculate the length of AQ

(4)

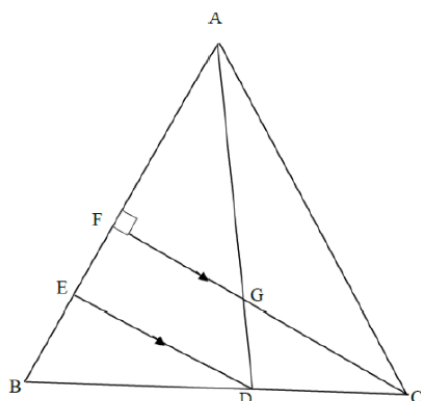
$$\begin{aligned} 2.2.2. \frac{AQ}{BF} &= \frac{AP}{PE} \text{ (line div sides of } \Delta \text{ prop)} \\ \frac{AQ}{60m + BQ} &= \frac{48m}{72m} \\ \therefore AQ &= \frac{2}{3} (60m + BQ) \\ &= 40m + \frac{2}{3} BQ \end{aligned}$$

2.2.3. Calculate : $\frac{AB}{BQ}$

(1)

$$2.2.3. \frac{AB}{BQ} = \frac{2}{3} \text{ (line div sides of } \Delta \text{ prop)}$$

2.3. In the diagram, CF is the perpendicular bisector of AB in $\triangle ABC$. D is a point on BC such that BD:DC=3:2. CF and AD intersect at G. ED//FC. (extracted from Limpopo, prelim question paper, 2017)



Determine, giving reasons:

2.3.1. $\frac{AF}{AE}$

(4)

$$2.3.1. \frac{EF}{BE} = \frac{DC}{BD} = \frac{3}{2} \text{ (line div sides of } \triangle \text{ prop)}$$

$$\frac{BF}{FA} = \frac{5}{5}$$

$$\therefore \frac{AF}{AE} = \frac{5}{7}$$

$$2.3.2. \frac{DE}{FG}$$

(4)

2.3.2. In $\triangle AFG$ and $\triangle AED$

$\hat{A}$ is common

$\angle FGA = \angle EDA$ (alt $\angle$'s)

$\therefore \triangle AFG \equiv \triangle AED$ ($\angle, \angle, \angle$)

$$\frac{DE}{FG} = \frac{AE}{AF}$$

$$\therefore \frac{DE}{FG} = \frac{7}{5}$$

$$2.3.3. \frac{\text{area of } \triangle GFA}{\text{area of } \triangle DEA}$$

(3)

$$2.3.3. \text{Area of } \triangle GFA = \frac{1}{2} \times FG \times FA$$

$$\text{Area of } \triangle DEA = \frac{1}{2} \times ED \times AE$$

$$= \frac{5 \times 5}{7 \times 7}$$

$$= \frac{25}{49}$$

APPENDIX H: DOCUMENT ANALYSIS/ INSTRUMENT

This section entail the summary of the common misconceptions and errors as stated in Section 2.10.4. of Chapter 2, together with the analysis of the types of misconceptions and errors that are displayed on the learner written responses per question in **(Appendix G, section C)**. It will not be given to the teachers, but it will only be used by the researcher.

1. the following are some of the common misconceptions and errors that are committed by Grade 12 learners particularly in proportionality and similarity. (DBE diagnostic report, 2017-2021)

- Learners cannot differentiate between a theorem and its converse. **(Misconception.1)**
- They provide incorrect reasons for their statements. **(Error.1)**
- They are confusing similarity with congruency. **(Misconception.2)**
- Incorrect labelling of angles is still a challenge (For example, they would write $\hat{A}$ instead of $\hat{A}_1$ or, $\hat{B} = \hat{C}$ instead of $\hat{B}_1 = \hat{C}_1$, or $\hat{A}_{1+2}$ instead of $\hat{A}_1 + \hat{A}_2$) **(Error.2)**
- Euclidean Geometry questions containing other topics like trigonometry is a challenge as majority of the learners do not even attempt to answer such questions. **(Misconception.3)**
- Learners would prove a theorem without making/ writing any construction of which that is a total breakdown and leads to the awarding of zero marks. **(Error.3)**
- They are unable to identify the correct ratios from the given diagrams. **(Misconception.4)**
- Many learners could select the correct pairs of equal angles that will result in two triangles being similar to each other. Instead they state a number of angles that are each equal to x and then conclude that $\triangle WRV \parallel \triangle TSV$. **(Misconception.5)**

2. Analysis of misconceptions and errors for Learner. A

Learner solutions with errors and Misconceptions :Question number	Learner solutions (Type of Misconception/ Error)
Question.1	
1.1.1	Error.2
1.1.2.	Misconception.5 and Error.1
1.1.4.	Error.1 and Error.2
1.2.1.	Misconception.1 and Error.1
1.2.2.	Misconception.1
1.2.3.	Error.1 and Error.2

Learner solutions with errors and Misconceptions :Question number	Learner solutions (Type of Error/ Misconception)
Question.2	
2.1.	Error.3
2.2.1.	
2.2.2.	Misconception. 1 and Misconception. 4
2.3.1.	Misconception.1
2.3.2.	Error.1 and Misconception. 2
2.3.3.	Misconception.4

APPENDIX I: LANGUAGE EDITING CERTIFICATE



ALDE UNIT, A13 ADC BUILDING B125 (Department of Academic Literacy)
RS NGUBANE (28383095) ext 892109, Cell 0828178046

28 November 2023

CERTIFICATE OF EDITING A DISSERTATION

To whom it may concern

May this confirm that I have proofread and edited the dissertation by **G.N Shoba, Student number:**

40806197  **orcid.org/0000-0001-9322-7260**, titled, '**Exploring Grade 12 Teachers' Pedagogical Content Knowledge of Similarity in Euclidean Geometry**'

The candidate corrected the language errors identified. The document is now of an acceptable linguistic standard.

I thank you.

A handwritten signature in blue ink that reads 'RS Ngubane'.

.....
RS NGUBANE (Ms)

APPENDIX J: TURNITIN REPORT

SHOBAGN-DISSERTATION-FINAL02--
CHANGES_ACCEPTED_(02)-NOV2023.pdf

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