POWER COMPARISONS FOR GOODNESS-OF-FIT TESTS UNDER LOCAL ALTERNATIVES

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ABSTRACT

The bootstrap method is applied to discrete multivariate data and the power divergence family of test statistics (PDFS). For a symmetric null hypothesis against a local alternative, exact power values determined by Read and Cressie (1988:76-78) are used as a basis for a comparative power study between the AE approximation of power derived by Taneichi *et al.* (2002), and a bootstrap method which involves the use of newly calculated bootstrap critical values for power calculations. Also, traditional chi-square critical values are used to determine power for these hypotheses and are compared with the methods mentioned above. The study focuses on small sample sizes.

UITTREKSEL

Die skoenlusmetode word toegepas op diskrete meerveranderlike data en die magsdiskrepansie familie van toetsstatistieke (PDFS). Vir 'n simmetriese nul hipotese teenoor 'n lokale alternatief, word eksakte onderskeidingsvermoë-waardes, wat deur Read en Cressie (1988:76-78) bepaal is, gebruik as basis vir 'n vergelykende onderskeidingsvermoë-studie tussen die sogenaamde AE benadering van onderskeidingsvermoë, voorgestel deur Taneichi *et al.* (2002), en 'n skoenlusmetode wat die gebruik van nuwe skoenlus-kritiekewaardes behels vir die berekening van onderskeidingsvermoë. Ook word tradisionele chi-kwadraat kritieke waardes gebruik om onderskeidingsvermoë te bepaal vir dié hipoteses, wat vergelyk word met die metodes hierbo beskryf. Die studie fokus op klein steekproewe.

OPSOMMING

In hierdie studie word die skoenlusmetode aangewend in die veld van diskrete meerveranderlike data analise, en wel by die gebruik van die magsdiskrepansie-familie van statistieke (PDFS). Eksakte onderskeidingsvermoë waardes, bereken deur Read en Cressie (1988:76-78), vir 'n simmetriese nul hipotese teenoor 'n lokale alternatief, word gebruik as basis vir 'n vergelykende studie, waarin daar gefokus word op klein steekproefgroottes. Nuut-berekende skoenlus-kritiekewaardes, asook tradisionele chi-kwadraat kritiekewaardes word gebruik om die onderskeidingsvermoë te bepaal vir toetse by die betrokke hipoteses, en die resultate word vergelyk met die gedrag van onderskeidingsvermoë-benaderings wat afgelei is deur Taneichi *et al.* (2002).

Hoofstukke 1 tot 4 bevat algemene inligting en literatuurstudie. Nuwe benaderings word gedefinieer, naamlik die skoenlus onderskeidingsvermoë-benadering en die AE benaderingsmetode. In Hoofstuk 5, word die skoenlus benaderingsmetode gedefinieer en die resultate van die studie word ontleed en bespreek. 'n Kort opsomming oor die inhoud van elke hoofstuk volg nou.

Die nie-parametriese skoenlusmetode word bespreek in Hoofstuk 1. Daar word aan die volgende konsepte aandag gegee: die skoenlus-steekproef, die skoenlus-prosedure, die skoenlus-beraming van standaardfout, en 'n aantal handige skoenlus-vertrouensintervalle word gedefinieer vir die statistiese gebruiker.

Hoofstuk 2 bevat 'n opsomming van tradisioneel-populêre passingstoetse vir diskrete meerveranderlike data. Belangrike diskrete verdelings word aangehaal, naamlik die binomiaal, Poisson, hipergeometriese en multinomiaal verdelings. 'n Voorbeeld van moontlike toepassingsveld van die resultate wat uit die studie voortspruit, is die log-lineêre model, wat kortliks bespreek word in Hoofstuk 2.

Die magsdiskrepansie-familie van statistieke en verwante sake word bespreek in Hoofstuk 3. Ter saaklike stellings en bewyse uit Read *et al.* (1984, 1988) word aangehaal en bestudeer, naamlik die afleiding van die limietverdeling van die Pearson

chi-kwadraat statistiek, stellings rakende Birch se reëlmatigheidsvoorwaardes (1964), en die afleiding van die limietverdeling van die magsdiskrepansie-familie van statistieke onder die nul hipotese sowel as onder die alternatiewe hipotese. Read (1984) se studies oor die klein-steekproef gedrag van die magsdiskrepansie-familie van statistieke en Read en Cressie (1988) se pogings om te verbeter op die betroubaarheid van dié toetse vir klein steekproewe, word uitgelig.

Verskeie ander benaderings tot die verdeling van die magsdiskrepansie-familie van statistieke word bespreek in Hoofstuk 4, naamlik die Edgeworth benadering, die AE benadering van Taneichi *et al.* (2002), die benadering van Drost *et al.* (1989) en die sogenaamde NT benadering van Sekiya *et al.* (1999).

In Hoofstuk 5, word die skoenlus-benadering om onderskeidingsvermoë te bepaal, verduidelik, asook die metode wat gebruik word om skoenlus-kritiekewaardes te bepaal. Resultate van die vergelykende studie tussen die skoenlus en die AE benaderings om onderskeidingsvermoë te bepaal, sowel as die vergelyking van die effektiwiteit van die tradisionele chi-kwadraat kritiekewaardes en die skoenlus kritiekewaardes, ten opsigte van eksakte kritiekewaardes bereken deur Read en Cressie (1988), word bespreek. Resultate van verdere vergelykende studies word verskaf, gevolg deur opmerkings en gevolgtrekkings, wat soos volg saamgevat kan word: Die skoenlusmetode om onderskeidingsvermoë te bepaal, wat die berekening en gebruik van skoenlus-kritiekewaardes behels, is 'n maklik uitvoerbare, betroubare en stabiele alternatief vir die gebruik van tradisionele metodes, wat gebruik maak van chi-kwadraat kritiekewaardes. Laasgenoemde metode lewer dikwels, veral vir klein steekproewe, toetse van betekenispeil wat beduidend verskil van 'n voorgeskrewe peil α. Dit word ook aangetoon dat 'n gekompliseerde benadering om onderskeidingsvermoë te bepaal, naamlik die AE benadering, onstabiele onderskeidingsvermoë-berekenings voortbring wat dikwels konserwatief is, en gevolglik nie aanbeveel kan word vir algemene gebruik in die geval van klein steekproewe nie.

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CHAPTER 1

THE BOOTSTRAP METHOD

1.1 Introduction

The bootstrap method introduced by Efron (1979), has found application into many areas of statistics. It is used by statisticians as well as by quantitative researchers in the life-sciences, medical sciences, social sciences, business, econometrics and other areas where statistical analysis is needed. The bootstrap has several admirable properties. For example, fewer assumptions are made regarding the underlying distribution of the data, and the availability of high-speed personal computers and programming tools makes the bootstrap a very efficient and practical tool. The most admirable property of the bootstrap is the ease and flexibility in which it can be applied to more complicated statistics, and the derivation of measures of accuracy. The bootstrap can be applied in a parametric or non-parametric way. The non-parametric bootstrap is usually applied in fields where no particular mathematical model is available, with adjustable constants and parameters, which completely defines the distribution function. Furthermore, the non-parametric bootstrap offers a solution to cases where known distributions are used and where the statistics of interest are too complex to calculate theoretically. In ideal parametric situations traditional ways or parametric methods such as the parametric bootstrap, may be more applicable due to the fact that more information is known about the underlying distributions and more accurate statistical inference procedures will be the result.

In §1.2 of this chapter the non-parametric bootstrap procedure is discussed together with the bootstrap mean and the bootstrap variance. The way the bootstrap procedure is applied to calculate the standard error, is explained in §1.3 and a discussion of the bootstrap confidence intervals follows in §1.4.

1.2 The non-parametric bootstrap

Consider a finite, random sample of size n, consisting of independent and identically distributed observations denoted by $\mathbf{X}_n = (X_1, X_2, ..., X_n)$, which are obtained from an unknown distribution function F. We are often interested in some statistic $\theta = T_n(\mathbf{X}_n, F)$, which depends on this unknown distribution.

One estimator of the unknown distribution, F, is the empirical distribution (EDF) F_n. The empirical distribution is a discrete distribution, which allocates to each observation in the sample a mass of $1/n$. The EDF is defined as

$$F_n(x) = n^{-1}\sum_{i=1}^{n} I(X_i \leq x),$$

where $I(\cdot)$ denotes the indicator function. Efron and Tibshirani (1993:32) showed that all the information about F contained in the data is also contained in F_n. Furthermore, the Glivenko-Chantelli Theorem states that this estimator possesses good large sample properties i.e.,

$$\sup_{-\infty<x<\infty} |F_n(x) - F(x)| \to 0 \quad \text{a.s.} \quad n \to \infty .$$

Kernel estimation methods also provides trustworthy estimators for F, and is defined by

$$\hat{F}(x) = F_{n,c}(x) = n^{-1}\sum_{i=1}^{n} K\left(\frac{x - X_i}{c}\right),$$

where $c = c_n$ is a sequence of smoothing parameters such that $c_n \to 0$ as $n \to \infty$ and K is a known continuous cumulative distribution function symmetric about zero. Asymptotic improvements in estimating F by $F_{n,c}$ instead of F_n provided certain regularity conditions on F are met and that the sequence $\{c_n\}$ converges at a specific rate to zero, was shown by Azzalini (1981:326). The best choice of the smoothing parameter remains an important research problem.

The basic concepts of the bootstrap procedure will now be discussed. Throughout this discussion we assume that a sample $\mathbf{X}_n = (X_1, X_2, ..., X_n)$ of size n is available.

1.2.1 The bootstrap sample

A bootstrap sample is defined to be a sample, usually of the same size n as the original sample, drawn randomly and with replacement from the original sample.

Swanepoel (1986, b) introduced the modified bootstrap procedure, using sample size m where $m \neq n$ and recommends this method for cases where the classical bootstrap fails. The unknown distribution function of the data can be approximated by the empirical distribution function F_n, which is defined in §1.2. Random number generator methods are used to obtain random indexes from 1 to n, which corresponds to the respective data elements in the original sample of size n. Each of the original observations can appear once, more than once, or not at all, in the bootstrap sample. The bootstrap sample will be denoted by $\mathbf{X}_n^* = (X_1^*, X_2^*, \ldots, X_n^*)$, and

$$\mathrm{P}^*(X_j^* = X_i) = \frac{1}{n} \quad \text{for } i, j = 1, \ldots, n \ ,$$

where P^* denotes probability under F_n.

1.2.2 The bootstrap procedure

Let $T_n(\mathbf{X}_n, F)$ be some variable of interest, which may depend on the unknown F. The sampling distribution of $T_n(\mathbf{X}_n, F)$ under F can then be approximated by the bootstrap distribution of $T_n(\mathbf{X}_n^*, F_n)$ under F_n, i.e. $\mathrm{P}_F\left(T_n(\mathbf{X}_n, F) \in B\right) \approx \mathrm{P}_{F_n}\left(T_n(\mathbf{X}_n^*, F_n) \in B\right)$ for any set B. To calculate the latter bootstrap probability the following Monte Carlo algorithm is used:

Step 1: Draw n observations with replacement from F_n, to produce the first bootstrap sample, $\mathbf{X}_n^*(1) = (X_{11}^*, X_{12}^*, \ldots, X_{1n}^*)$.

Step 2: From this first bootstrap sample, calculate $\hat{\theta}^*(1) = T_n(\mathbf{X}_n^*(1), F_n)$.

Step 3: Repeat the above two steps B times to obtain bootstrap samples $\mathbf{X}_n^*(1) = (X_{11}^*, X_{12}^*, \ldots, X_{1n}^*)$, $\mathbf{X}_n^*(2) = (X_{21}^*, X_{22}^*, \ldots, X_{2n}^*)$, ... , $\mathbf{X}_n^*(B) = (X_{B1}^*, X_{B2}^*, \ldots, X_{Bn}^*)$ and the respective bootstrap replications, $\hat{\theta}^*(1) = T_n(\mathbf{X}_n^*(1), F_n)$, $\hat{\theta}^*(2) = T_n(\mathbf{X}_n^*(2), F_n)$, ... , $\hat{\theta}^*(B) = T_n(\mathbf{X}_n^*(B), F_n)$.

The distribution of these bootstrap replications $\hat{\theta}^*(i) = T_n(\mathbf{X}_n^*(i), F_n)$, $i = 1, 2, \ldots, B$ is then an approximation to the true sampling distribution of the statistic $\theta = T_n(\mathbf{X}_n, F)$. To assess the accuracy of a bootstrap estimator of some parameter of interest, its standard error and bias is calculated. Other measures of interest such as estimates of location, spread as well as confidence intervals can also be determined by using the bootstrap method.

1.3 The bootstrap estimate of standard error

Suppose θ is some unknown parameter and $\hat{\theta}$ an estimate of θ. The standard error of $\hat{\theta}$ is defined as

$$\sigma(F) = [Var_F(\hat{\theta})]^{1/2}, \tag{1.1}$$

and the bootstrap estimate of $\sigma(F)$ is then defined as

$$\sigma(F_n) = [Var_{F_n}(\hat{\theta}^*)]^{1/2}. \tag{1.2}$$

The following procedure is used to approximate $\sigma(F_n)$, using the nonparametric bootstrap method:

Step 1: Draw n observations independently and with replacement from the original data sample, i.e. $\mathbf{X}_n^*(1) = (X_{11}^*, X_{12}^*, \ldots, X_{1n}^*)$.

Step 2: From this bootstrap sample, calculate $\hat{\theta}^*(1) = \hat{\theta}(\mathbf{X}_n^*(1))$.

Step 3: We repeat the above two steps a large number, B, times, to obtain bootstrap samples $\mathbf{X}_n^*(1)$, $\mathbf{X}_n^*(2)$, ..., $\mathbf{X}_n^*(B)$ and their respective statistic's $\hat{\theta}^*(1)$, $\hat{\theta}^*(2)$, ..., $\hat{\theta}^*(B)$.

Step 4: Calculate $$\hat{\sigma}_B = \left[\frac{1}{B-1}\sum_{b=1}^{B}\left(\hat{\theta}^*(b) - \hat{\theta}^*(\cdot)\right)^2\right]^{\frac{1}{2}}, \tag{1.3}$$

where $\hat{\theta}^*(\cdot) = \frac{1}{B}\sum_{b=1}^{B}\hat{\theta}^*(b)$.

According to Efron (1981:589), $\hat{\sigma}_B \to \sigma(F_n)$ as $B \to \infty$, and values of B between 50 and 200 are usually adequate in estimating standard errors. Several other methods of approximating $\hat{\sigma}_B$ exists in the literature namely the Tukey's jackknife method of standard errors or a method based by Frangos and Schucany (1990:1-11), based on estimates of the influence functions.

1.4 Bootstrap confidence intervals

A $100(1-\alpha)\%$ confidence interval for the parameter of interest is another popular measure of reliability of the estimator $\hat{\theta}$, and the bootstrap can be used successfully to obtain reliable nonparametric confidence intervals. The estimated standard error plays a vital role in defining confidence intervals for the parameter θ. Much work has been done on bootstrap confidence intervals. Singh (1981), Abramovitch and Singh (1985), Bickel & Freedman (1981), Efron (1982, 1981), Beran (1985, 1987a, 1987b), Hall (1988a, 1988b), DiCiccio and Romano (1988) are but a few. The bootstrap t-interval, the percentile interval, the bias-corrected percentile and the accelerated bias-corrected percentile confidence intervals will be discussed briefly in this section. In the percentile, bias-corrected and accelerated bias-corrected intervals the cumulative distribution function of the bootstrap estimator, $\hat{\theta}^* = \hat{\theta}(X_1^*,\ldots,X_n^*)$, based on the bootstrap sample, is used and this distribution is defined as

$$\hat{G}(t) = P^*(\hat{\theta}^* \le t), \tag{1.4}$$

where P^* indicates probability computed according to the bootstrap distribution of $\hat{\theta}^*$.

1.4.1 The bootstrap t-interval

For pivotal statistics of the form

$$T = \frac{\hat{\theta}-\theta}{\sigma\left(\hat{\theta}\right)}, \tag{1.5}$$

Abramovitch & Singh (1985) found that bootstrapping (1.5) improves the normal approximation of the distribution of T. Let $H(s)$ be the distribution of T, and $\hat{H}$ the bootstrap approximation of H, i.e., $\hat{H}$ is the bootstrap distribution of

$$T^* = \frac{\hat{\theta}^* - \hat{\theta}}{\hat{\sigma}_B}, \tag{1.6}$$

i.e., $\hat{H}(s)=B^{-1}\sum_{i=1}^{B}I\left(T_i^*\le s\right)$. To calculate $\hat{H}(s)$, the following procedure is sugessted:

Repeat steps 1 to 4 of the procedure discussed in §1.3.

Step 5: By using (1.6) calculate B values, (B large), of T_i^* for each bootstrap replication $i = 1, 2, \ldots, B$.

Let $T_{(i)}^*$ denote the order statistics of T_i^*. Then $\hat{H}^{-1}(1-\alpha/2)$ and $\hat{H}^{-1}(\alpha/2)$ can be approximated by the $\left[B(1-\alpha/2)\right]$-th and $\left[B(\alpha/2)\right]$-th order statistics of the T_i^* values, with $[z]$ denoting the largest integer less than or equal to z.

The $100(1-\alpha)\%$ bootstrap t-interval for θ is then given by

$$\left[\hat{\theta}-\hat{H}^{-1}(1-\alpha/2)\hat{\sigma}_B;\ \hat{\theta}-\hat{H}^{-1}(\alpha/2)\hat{\sigma}_B\right]. \tag{1.7}$$

Any of the estimators for σ_B mentioned in §1.3 can be used.

1.4.2 The Percentile confidence interval

The percentile $100(1-\alpha)\%$ confidence interval for θ is given by

$$\left[\hat{G}^{-1}(\alpha/2);\ \hat{G}^{-1}(1-\alpha/2)\right], \tag{1.8}$$

where $\hat{G}$ is defined in (1.4).

This interval can be approximated by the following Monte Carlo algorithm:

Step 1: Obtain B independent bootstrap samples, of size n from F_n. For each sample calculate $\hat{\theta}^*(1),\ \hat{\theta}^*(2),\ \ldots,\ \hat{\theta}^*(B)$, as before.

Step 2: Find the order statistics $\hat{\theta}_{(1)}^*,\ \hat{\theta}_{(2)}^*,\ \ldots,\ \hat{\theta}_{(B)}^*$ of $\hat{\theta}^*(1),\ \hat{\theta}^*(2),\ \ldots,\ \hat{\theta}^*(B)$.

Step 3: (1.8) is then approximated by $\left[\hat{\theta}_{(r)}^*;\ \hat{\theta}_{(s)}^*\right]$, where $r=\left[B(\alpha/2)\right]$ and $s=\left[B(1-\alpha/2)\right]$, with $[z]$ denoting the largest integer less than or equal to z.

Efron and Tibshirani (1986:170) pointed out that, if the original estimator $\hat{\theta}$ is distributed according to $N\left(\theta,\sigma^2\right)$, then the percentile and standard confidence intervals will coincide. It was also shown that instead of $\hat{\theta}$ having a $N\left(\theta,\sigma^2\right)$ distribution, it holds for all $\hat{\theta}$, that $\hat{\phi}$ is distributed according to $N\left(\phi,c^2\right)$, for some monotone transformation $\hat{\phi}=m\left(\hat{\theta}\right)$, $\phi=m\left(\theta\right)$, where c^2 is constant, then the standard intervals will be grossly inaccurate but the percentile intervals will be correct. This idea carries through for both the coverage probability and for inverse mapping. The advantage of this method is that the correct transformation does not have to be known only that it exists.

1.4.3 The bias-corrected percentile confidence interval

The bias-corrected $100(1-\alpha)\%$ confidence interval for θ is given by

$$\left[\hat{G}^{-1}\left\{\Phi\left(2z_0-z\left(\alpha/2\right)\right)\right\};\ \hat{G}^{-1}\left\{\Phi\left(2z_0+z\left(\alpha/2\right)\right)\right\}\ \right],\tag{1.9}$$

where Φ is the standard normal distribution function, $\Phi\left(z\left(\alpha/2\right)\right)=1-\left(\alpha/2\right)$ and $z_0=\Phi^{-1}\left(\hat{G}(\hat{\theta})\right)$. This interval is an adjustment of the percentile interval in that it takes into account the bias of the bootstrap distribution of $\hat{\theta}^*$. If $\hat{G}(\hat{\theta})$ = 0.5, the median unbiased case, then z_0 = 0, and this interval is reduced to the percentile interval in (1.8). The bootstrap approximation for (1.9) is obtained in the following way:

Repeat steps 1 to 3 as is described in §1.4.2, but replace r and s with the following values:

$$r=\left[B\Phi\left(2z_0-z\left(\alpha/2\right)\right)\right],$$

and

$$s=\left[B\Phi\left(2z_0+z\left(\alpha/2\right)\right)\right],$$

and

$$\hat{G}(\hat{\theta})=\frac{1}{B}\sum_{b=1}^{B}I(\hat{\theta}_b^*\leq\hat{\theta}).$$

1.4.4 The Accelerated bias-corrected percentile confidence interval

The accelerated bias-corrected $100(1-\alpha)\%$ confidence interval for θ is given by

$$\left[\hat{G}^{-1}\left\{\Phi\left(2z_0-b\left(\alpha/2\right)\right)\right\};\ \hat{G}^{-1}\left\{\Phi\left(2z_0+c\left(\alpha/2\right)\right)\right\}\right] \tag{1.10}$$

where

$$b(\alpha/2)=\frac{\left\{z(\alpha/2)-z_0\right\}}{1-a\left(z_0-z(\alpha/2)\right)}+z_0;$$

$$c(\alpha/2)=\frac{\left\{z(\alpha/2)+z_0\right\}}{1-a\left(z_0+z(\alpha/2)\right)}-z_0$$

and a is some constant depending on F. If a = 0, a measure of skewness, then this interval is reduced to the bias-corrected percentile interval, (1.9). Efron (1982:41) discusses this method in detail. Efron (1987:171) suggested an estimate $\hat{a}=\frac{1}{6}\frac{\sum_{i-1}^{n}I_i^3}{\left(\sum_{i-1}^{n}I_i^2\right)^{3/2}}$, where I_i is the empirical influence function of $\hat{\theta}$ evaluated at $X_i=x_i$, i = 1, 2, …, n. The estimation of a does leave this method open for criticism. DiCiccio and Romano (1988:343) have considered procedures which approximates this interval without the calculation of z_0 and a.

Efron & Tibshirane (1993:162) asserts that B in the order of 1000 is required when calculating the bias-corrected and accelerated bias-corrected confidence intervals, but B = 250 provides useful results for the percentile interval.

Of these three intervals the accelerated bias-corrected percentile generally performs very well. Much work has been done in this regard as is clear from Hall (1988a, 1988b), Singh (1981) and Hartigan (1986) and many more.

CHAPTER 2

GOODNESS-OF-FIT TESTS FOR DISCRETE MULTIVARIATE DATA

2.1 Introduction

Two main approaches are employed in testing goodness-of-fit. One method is the exploratory or graphical technique and the other is the numerical technique. Graphical techniques are usually used as a starting point in analysis to indicate the characteristics of the data, such as the form of the population's distribution. D'Agostino and Stephens (1986) discussed these techniques and further suggested that these techniques should not be used on their own, but in conjunction with formal numerical tests. In this chapter, numerical test methods of testing hypothesis which are of interest for the present study will be discussed.

In this chapter, §2.2, discrete distributions are explained, in §2.3 an application of the discrete distributions is discussed, i.e. the log-linear model. In §2.4 popular test statistics are introduced.

2.2 Discrete distributions

A random variable is said to be a discrete random variable if it takes on only a finite or at most a countably infinite number of values. Some well known discrete distributions will now be discussed briefly.

2.2.1 The Binomial distribution

Suppose that n independent trials are performed, where n is fixed, and that each trial results in either a "success" or "failure", with probability, p, and $1-p$ respectively. Let X denote the total number of successes in the n independent trials. Then X follows a binomial random variable with the parameters n and p. The probability that $X = r$ can be found in the following way:

$$\Pr(X=r)=\binom{n}{r}p^r(1-p)^{n-r}; \qquad \text{for } r=0,\ 1,\ldots,\ n,$$

where $\binom{n}{r}$ is the total number of such sequences. The maximum likelihood estimate for *p* is given by $\hat{p}=X/n$.

2.2.2 The Poisson Distribution

A random variable has a Poisson distribution with parameter $\lambda>0$, if its distribution can be described as

$$\Pr(X=r)=\frac{\lambda^r}{r!}e^{-\lambda}; \qquad \text{for } r=0,\ 1,\ 2,\ \ldots\ .$$

The Poisson distribution can be derived as the limit of the binomial distribution, if the number of trials n approaches infinity and the probability of success on each trial, p, approaches 0 in such a way that $\lambda=np$. The Poisson distribution describes rare events. The maximum likelihood estimator of λ is X/n.

2.2.3 The Hypergeometric Distribution

The Hypergeometric distribution can be explained as follows. Suppose we have a population of *N* objects of which *r* are of a certain type, say type 1, and the rest, *N* - *r*, of the objects are of another type. A sample of size *n* is drawn without replacement from this population. Let *X* denote the number of type 1 objects in the sample. Then X has a hypergeometric distribution with parameters *r*, *N*, *n* and

$$\Pr(X=x)=\frac{\binom{r}{x}\binom{N-r}{n-x}}{\binom{N}{n}}.$$

This distribution can also be derived from a conditional distribution of the sum of two binomial distributions with the same probability but different sample sizes.

2.2.4 The Multinomial Distribution

When the binomial distribution is generalized, the multinomial distribution is obtained in the following way. Suppose there are *n* independent trials which can result in *r* types of outcomes. On each of the trials the probability of obtaining the *r* outcomes are $p_1, p_2, ..., p_r$. Define X_i to be the total number of outcomes of types *i* in the *n* trials, *i* = 1, 2,…, *r*. Note that any particular sequence of trials giving rise to $X_1 = x_1$, $X_2 = x_2$, ..., $X_r = x_r$ occurs with probability $p_1^{x_1} p_2^{x_2} ... p_r^{x_r}$. Note also that there are $\frac{n!}{x_1!x_2!...x_r!}$ such sequences. The joint frequency distribution is then

$$\Pr(X_1 = x_1, X_2 = x_2, ..., X_r = x_r) = \binom{n}{x_1 x_2 ... x_r} p_1^{x_1} p_2^{x_2} ... p_r^{x_r}.$$

To obtain the maximum likelihood estimator $\hat{\mathbf{p}}$ of $\mathbf{p}$, we maximize the logarithm $\sum_{i=i}^{r} x_i \log p_i$ of the likelihood function with respect to p_i, with $p_i \geq 0$ for *i* = 1, 2,…, *r* and $\sum_{i=i}^{r} p_i = 1$. The estimators are then $\hat{p}_i = \frac{x_i}{n}$ for *i* = 1, 2, …, *r*.

2.3 An Application: The Log-linear model

Applications of discrete data are found in the analysis of log-linear models, Logit, Probit and Logistic models. A brief illustration of the multinomial distribution as it is used in log-linear models are now presented.

Suppose we have data from a population where the individuals are classified as falling into one of *r* categories which are mutually exclusive. Let $p_1, p_2, ..., p_r$ be the probabilities of an individual falling into that particular category, i.e., p_i is the probability of an individual falling into the *i*-th category. Then $\sum_{i=i}^{r} p_i = 1$. If x_i denotes the number of individuals in the *i*-th category, then $\sum_{i=i}^{r} x_i = n$. Furthermore, expected counts for the *i*-th category is defined by $m_1, m_2, ..., m_r$, where $E(x_i) = m_i$ for $i = 1, 2,..., r$ and $m_i = np_i$.

For a 2 x 2 situation (i = 1, 2 and j = 1, 2) data from the sample can be represented as follows:

	1	2	total
1	x_{11}	x_{12}	x_{+1}
2	x_{21}	x_{22}	x_{2+}
total	x_{+1}	x_{+2}	n

.

The respective probabilities are represented by

	1	2	total
1	p_{11}	p_{12}	p_{1+}
2	p_{21}	p_{22}	p_{2+}
total	p_{+1}	p_{+2}	1

and the expected cell counts by

	1	2
1	$\hat{m}_{11} = np_{11}$	$\hat{m}_{12} = np_{12}$
2	$\hat{m}_{21} = np_{21}$	$\hat{m}_{22} = np_{22}$

.

The cross-product ratio of this table is then $\alpha = \dfrac{p_{11}p_{22}}{p_{12}p_{21}}$. Taking the logarithm we obtain $\log\alpha = \log p_{11} + \log p_{22} - \log p_{12} - \log p_{21}$ with $\sum_{i,j=i}^{2} p_{ij} = 1$. The log-linear model is then defined as

$$\log p_{ij} = u + u_{1(i)} + u_{2(j)} + u_{12(ij)} \quad \text{for } i = 1, 2 \text{ and } j = 1, 2,$$

where u is the grand mean of the logarithms:

$$u = (1/4)\left(\log p_{11} + \log p_{22} + \log p_{12} + \log p_{21}\right),$$

the mean of the logarithms of the probabilities at level i is then:

$$u + u_{1(i)} = (1/2)\left(\log p_{i1} + \log p_{i2}\right) \quad \text{for } i = 1, 2,$$

similarly for level j it holds that

$$u+u_{2(j)}=(1/2)\left(\log p_{1j}+\log p_{2j}\right) \text{ for } j=1,2.$$

The constraints to this model are:

$$u_{1(1)}+u_{1(2)}=u_{2(1)}+u_{2(2)}=0,$$

since they represent deviations from the mean.

For a complete table, i.e., each cell has a non-zero probability of an individual occurring in that cell, the null hypothesis that the two variables are independent, is written as $H_0: p_{ij}=p_{i+}p_{+j}$, where p_{i+} and p_{+j} are marginal probabilities, and are defined as $p_{i+}=\sum_{j=1}^{J} p_{ij}$ and $p_{+j}=\sum_{i=1}^{I} p_{ij}$. Under H_0 the maximum likelihood estimator for m_{ij} is given by

$$\hat{m}_{ij}=\frac{x_{i+}x_{+j}}{x_{++}},$$

where $x_{i+}=\sum_{j=1}^{J} x_{ij}$ is the row total (summed over *j*), $x_{+j}=\sum_{i=1}^{I} x_{ij}$ is the column total (summed over *i*) and $x_{++}=\sum_{i=1}^{I}\sum_{j=1}^{J} x_{ij}$ is the grand total (summed over *i* and *j*). To test the hypothesis that variable 1 has no effect we then have the model

$$\log m_{ij}=u+u_{2(j)},$$

and $\hat{m}_{ij}=\frac{x_{+j}}{I}$. One way to obtain such direct estimates is to first obtain sufficient statistics. This method is discussed completely by Bishop *et al.* (1975:64)

2.4 Goodness-of-fit statistics

To compare m_{ij} with $\hat{m}_{ij}$ in §2.3, goodness-of-fit statistics play an important role. Two traditional statistics are the Pearson's X^2 statistic and the log-likelihood statistic G^2. We will discuss these briefly, using the notation of §2.3.

2.4.1 Well-known tests

The Pearson's X^2 Statistic is defined by

$$X^2 = \sum_{i=1}^{r} \frac{(x_i - \hat{m}_i)^2}{\hat{m}_i}.$$

The Likelihood Ratio G^2 Statistic is defined by

$$G^2 = -2\sum_{i=1}^{r} x_i \log\frac{\hat{m}_i}{x_i}$$

$$= 2\sum_{i=1}^{r} x_i \log\frac{x_i}{\hat{m}_i}.$$

Both X^2 and the G^2 are asymptotically χ^2 distributed under the null hypothesis with $r-s-1$ degrees of freedom, where r denotes the number of possible outcomes and s the number of parameters to be estimated.

Other popular goodness-of-fit statistics include the following:
The Freeman Tukey Statistic is defined by

$$F^2 = 4\sum_{i=1}^{r}\left(\sqrt{x_i} - \sqrt{\hat{m}_i}\right)^2.$$

The Modified Likelihood Ratio Statistic is defined by

$$GM^2 = 2\sum_{i=1}^{r} \hat{m}_i \log\frac{\hat{m}_i}{x_i}.$$

The Neyman-modified X^2 Statistic is defined by

$$NM^2 = \sum_{i=1}^{r} \frac{(x_i - \hat{m}_i)^2}{x_i}.$$

F^2, GM^2 and NM^2 are also asymptotically χ^2 distributed under the null hypothesis, similar to X^2 and G^2, under certain conditions (Read & Cressie 1988:45). The null hypothesis is rejected if the test statistic exceeds $\chi^2_{(r-s-1)}(\alpha)$. The test statistic with the highest power or the smallest variance is usually preferred.

2.4.2 The Power Divergence Statistic

Cressie and Read (1984:929) defined a class of multinomial goodness-of-fit statistics, $2nI^{\lambda}$, which is based on the power divergence measures. This family combines all the statistics defined in §2.4.1 and will now be discussed. Since 1984, approximations to $2nI^{\lambda}$ were suggested in literature.

Many papers have been published on the fit, accuracy and application of various goodness-of-fit statistics for discrete multivariate data. However, the power divergence family of tests provides an innovative way to unify and extend literature by linking the traditional test statistics through a single real-valued family parameter.

Let $\mathbf{X}_k = (X_1, X_2, ..., X_k)$ be distributed according to a multinomial distribution with parameters $(n, \pi_1, \pi_2, ..., \pi_k)$, where $\sum_{j=1}^{k} X_j = n$, $\sum_{j=1}^{k} \pi_j = 1$, $0 \le \pi_j \le 1$, (j = 1, …, k) and $(\pi_1, \pi_2, ..., \pi_k)$ is an unknown probability vector. Furthermore, suppose that the null-hypothesis $H_0 : \boldsymbol{\pi} \in \boldsymbol{\Pi}_0$, where $\boldsymbol{\Pi}_0$ represents a specified set of probability vectors that are hypothesised for $\boldsymbol{\pi}$. The estimated probability vector is denoted by $\hat{\boldsymbol{\pi}}$. The power divergence family, is then defined as:

$$2nI^{\lambda}(\mathbf{X}/n : \boldsymbol{\pi}) = \frac{2}{\lambda(\lambda+1)} \sum_{i=1}^{k} X_i \left[\left(\frac{X_i}{n\hat{\pi}_i} \right)^{\lambda} - 1 \right]; \quad -\infty < \lambda < \infty, \tag{2.1}$$

where λ is the family parameter. This statistic measures the divergence of $\mathbf{X}/n$ from $\hat{\boldsymbol{\pi}}$ through a weighted sum of powers of the terms $X_i/n\pi_i$ for i = 1, 2, …, k. One family of measures specifies the divergence of the probability distributions between $\mathbf{p} = (p_1, p_2, ..., p_k)$ and $\mathbf{q} = (q_1, q_2, ..., q_k)$ as

$$2nI^{\lambda}(\mathbf{p} : \mathbf{q}) = \frac{2}{\lambda(\lambda+1)} \sum_{i=1}^{k} p_i \left[\left(\frac{p_i}{q_i} \right)^{\lambda} - 1 \right]; \quad -\infty < \lambda < \infty. \tag{2.2}$$

In comparing the cell frequency vector $\mathbf{X}$ against the expected frequency vector $\hat{\mathbf{m}} = n\hat{\boldsymbol{\pi}}$, the power divergence statistic can be written as

$$2nI^{\lambda}(\mathbf{X} : \mathbf{m}) = \frac{2}{\lambda(\lambda+1)} \sum_{i=1}^{k} X_i \left[\left(\frac{X_i}{\hat{m}_i} \right)^{\lambda} - 1 \right]; \quad -\infty < \lambda < \infty \tag{2.3}$$

where $\hat{m}_i$ is the expected cell counts and $\sum_{i=1}^{k} \hat{m}_i = \sum_{i=1}^{k} X_i$.

Remark 2.1

When $\lambda = 1$, Pearson's X^2 statistic with k - 2 degrees of freedom is derived from (2.3). When $\lambda \to 0$, the log-likelihood ratio statistic with k - s - 1 degrees of freedom is obtained, and where s denotes the number of parameters to be estimated, i.e. $\lim_{\lambda \to 0} 2nI^{\lambda} = G^2$. When $\lambda \to -1$, the modified log likelihood ratio statistic is obtained, $\lim_{\lambda \to -1} 2nI^{\lambda} = GM^2$. When $\lambda = -1/2$, the Freeman-Tukey statistic is derived. For the optimal test statistic, Reed and Cressie (1988:63) suggested that $\lambda \in (-1, 2]$ is suitable in most cases where there is some knowledge of possible alternative models. According to Reed & Cressie $\lambda = 2/3$ is always a good choice.

The null hypothesis is rejected if the test statistic is larger than the $\chi^2_{k-s-1}(1-\alpha)$ where α is the significance level of the test and s is the number of parameters estimated in the model.

The Power divergence test statistic will be further discussed in chapter 3, in particular its asymptotic distributions, and some large and small sample results.

CHAPTER 3

GOODNESS-OF-FIT AND THE POWER DIVERGENCE STATISTICS (PDS)

3.1 Introduction

Throughout this chapter, Read & Cressie (1988) is used to view important aspects of the power divergence family of test statistics. The aim of this chapter is to point out how limiting distributions can be derived for the PDS's, both if H_0 is assumed to be true, as well as when H_A is true, where H_A denotes the alternative hypothesis.

In §3.2.1, the limiting distribution of Pearson's X^2 statistic will be derived as a preliminary result. In §3.2.2, Birch's (1964) regularity conditions will be stated and discussed briefly. In §3.2.3, the limiting distribution of the PDS's will be derived under H_0 and in §3.2.4 limiting non-central chi-square distributions are discussed. In §3.3.1 small sample comparisons for the PDS under H_A is discussed briefly. In §3.3.2 a method of improving the accuracy of the test when the sample size is small, is provided.

3.2 Limiting Distributions

Large sample theory is important in goodness-of-fit analysis as will become evident in the paragraph below.

3.2.1 Limiting chi-square distribution of the Pearson's X^2 test statistic

Throughout this chapter we will use the following notation: Suppose $\mathbf{X} = (X_1, X_2, \ldots, X_k)$ is a multinomial random vector from a $\text{Mult}_k(n, k)$ distribution, where n is the total number of counts over the k cells. Let $\boldsymbol{\pi} = (\pi_1, \pi_2, \ldots, \pi_k)$ be the unknown probability vector for the k cells, and let $\mathbf{x} = (x_1, x_2, \ldots, x_k)$ be the vector of observed values.

The following null hypothesis is of interest:

$$H_0 : \boldsymbol{\pi} = \boldsymbol{\pi}_0 \tag{3.1}$$

where $\boldsymbol{\pi}_0 = (\pi_{01}, \pi_{02}, ..., \pi_{0k})$, is a completely specified probability vector with each $\pi_{0i} > 0$ for all $i = 1,2,...,k$.

Theorem 1:

Under H_0, the Pearson's X^2 statistic, i.e.

$$X^2 = \sum_{i=1}^{k} \frac{(X_i - n\pi_{0i})^2}{n\pi_{0i}},$$

which can be written as a quadratic form in $\sqrt{n}\left(\mathbf{X}/n - \boldsymbol{\pi}_0\right)$, converges in distribution to a central chi-square random variable with $k - 1$ degrees of freedom as $n \to \infty$. This proof is divided into three sections, called Lemma 1, Lemma 2 and Lemma 3.

Lemma 1: Assume $\boldsymbol{X}$ is a random row vector with a multinomial distribution $\text{Mult}_k(n, \boldsymbol{\pi})$ and (3.1) holds. Then $\mathbf{W}_n = \sqrt{n}\left(\mathbf{X}/n - \boldsymbol{\pi}_0\right)$ converges in distribution to a multivariate normal random vector $\mathbf{W}$ as $n \to \infty$. The mean vector and covariance matrix of $\mathbf{W}_n$ and $\mathbf{W}$ are

$$E(\mathbf{W}_n) = \mathbf{0}$$

$$\text{cov}(\mathbf{W}_n) = D_{\boldsymbol{\pi}_0} - \boldsymbol{\pi}_0' \boldsymbol{\pi}_0, \tag{3.2}$$

where $D_{\boldsymbol{\pi}_0}$ is the k x k diagonal matrix based on $\boldsymbol{\pi}_0$.

Proof of Lemma 1: The mean and covariance of $\mathbf{W}_n$ in (3.2) are derived from

$$E(X_i) = n\pi_{0i},$$

$$var(X_i) = n\pi_{0i}(1 - \pi_{0i}),$$

$$\text{cov}(X_i, X_j) = -n\pi_{0i}\pi_{0j}$$

and therefore $E(\mathbf{X}) = n\boldsymbol{\pi}_0$ and $\text{cov}(\mathbf{X}) = n\left(D_{\boldsymbol{\pi}_0} - \boldsymbol{\pi}_0'\boldsymbol{\pi}_0\right)$. The asymptotic normality of $\mathbf{W}_n$ follows by showing that the moment generating function (MGF) of $\mathbf{W}_n$ converges to the MGF of $\mathbf{W}$ with mean and variance as in (3.2).

The MGF of $\mathbf{W}_n$ is

$$\begin{aligned} M_{\mathbf{W}_n}(\mathbf{v}) &= E\left[\exp\left(\mathbf{v}\mathbf{W}_n'\right)\right] \\ &= \exp\left(-n^{1/2}\mathbf{v}\boldsymbol{\pi}_0'\right)E\left[\exp\left(n^{-1/2}\mathbf{v}\mathbf{X}'\right)\right] \\ &= \exp\left(-n^{1/2}\mathbf{v}\boldsymbol{\pi}_0'\right)M_{\mathbf{X}}\left(n^{-1/2}\mathbf{v}\right), \end{aligned}$$

where $M_{\mathbf{X}}(\mathbf{v}) = \left[\sum_{i=1}^{k}\pi_{0i}\exp(v_i)\right]^n$ is the MGF of the multinomial random vector $\mathbf{X}$.

Therefore

$$M_{\mathbf{W}_n}(\mathbf{v}) = \left[\sum_{i=1}^{k}\pi_{0i}\exp\left(n^{-1/2}\left(v_i - \mathbf{v}\boldsymbol{\pi}_0'\right)\right)\right]^n$$

and expanding this in a Taylor series gives

$$\begin{aligned} M_{\mathbf{W}_n}(\mathbf{v}) &= \left[1 + \frac{1}{\sqrt{n}}\sum_{i=1}^{k}\pi_{0i}\left(v_i - \mathbf{v}\boldsymbol{\pi}_0'\right) + \frac{1}{2n}\sum_{i=1}^{k}\pi_{0i}\left(v_i - \mathbf{v}\boldsymbol{\pi}_0'\right)^2 + o(n^{-1})\right]^n \\ &= \left[1 + \mathbf{v}\left(D_{\boldsymbol{\pi}_0} - \boldsymbol{\pi}_0'\boldsymbol{\pi}_0\right)\mathbf{v}'/2n + o(n^{-1})\right]^n \\ &\to \exp\left[\mathbf{v}\left(D_{\boldsymbol{\pi}_0} - \boldsymbol{\pi}_0'\boldsymbol{\pi}_0\right)\mathbf{v}'/2\right] \qquad \text{as } n \to \infty \end{aligned}$$

which is the MGF of the multivariate normal random vector $\mathbf{W}$, with mean vector $\mathbf{0}$ and covariance matrix $\left(D_{\boldsymbol{\pi}_0} - \boldsymbol{\pi}_0'\boldsymbol{\pi}_0\right)$.

Lemma 2:
$$X^2 = \sum_{i=1}^{k}\frac{\left(X_i - n\pi_{0i}\right)^2}{n\pi_{0i}}$$

can be written in quadratic form in $\mathbf{W}_n = \sqrt{n}\left(\mathbf{X}/n - \pi_0\right)$ and X^2 converges in distribution (as $n \to \infty$) to the quadratic form of the multivariate normal random vector $\mathbf{W}$ in Lemma 1.

Proof of Lemma 2: It can be shown that

$$X^2 = \sum_{i=1}^{k}\sqrt{n}\left(\frac{X_i}{n} - \pi_{0i}\right)\frac{1}{\pi_{0i}}\sqrt{n}\left(\frac{X_i}{n} - \pi_{0i}\right)$$

$$= \mathbf{W}_n \left(D'_{\boldsymbol{\pi}_0}\right)^{-1} \mathbf{W}'_n$$

From Lemma 1, $\mathbf{W}_n$ converges in distribution to $\mathbf{W}$, which is a multivariate normal random vector with mean and variance as (3.2). This result can be generalised to any continuous function $g(\cdot)$, i.e. where $g(\mathbf{W}_n)$ converges in distribution to $g(\mathbf{W})$, (Rao, 1973:124) and consequently $\mathbf{W}_n \left(D_{\boldsymbol{\pi}_0}\right)^{-1} \mathbf{W}'_n$ converges in distribution to $\mathbf{W}\left(D'_{\boldsymbol{\pi}_0}\right)^{-1} \mathbf{W}'$.

Lemma 3: $X^2 = \mathbf{W}_n \left(D_{\boldsymbol{\pi}_0}\right)^{-1} \mathbf{W}'_n$ converges in distribution to the central chi-square random variable with k - 1 degrees of freedom.

Proof of Lemma 3: The proof of this theorem uses the result from Bishop *et al.* (1975:473). Assuming $\mathbf{U} = (U_1, U_2, ..., U_k)$ has a discrete multivariate normal distribution with mean vector **0,** covariance matrix Σ and $Y = \mathbf{U}B\mathbf{U}'$ for some symmetric matrix B. Then Y has the same distribution as $\sum_{i=1}^{k} \varsigma_i Z_i^2$, where the Z_i^2 s are independent chi-square random variables, each with one degree of freedom, and the ς_i's are the eigenvalues of $B^{1/2}\Sigma\left(B^{1/2}\right)'$.

In the present case, we have

$$\mathbf{U} = \mathbf{W},$$

$$B = \left(D_{\boldsymbol{\pi}_0}\right)^{-1},$$

$$\Sigma = D_{\pi_0} - \boldsymbol{\pi}'_0\boldsymbol{\pi}_0,$$

and

$$B^{1/2}\Sigma\left(B^{1/2}\right)' = \left(D_{\pi_0}\right)^{-1/2}\left(D_{\pi_0} - \boldsymbol{\pi}'_0\boldsymbol{\pi}_0\right)\left(D_{\pi_0}\right)^{-1/2}$$

$$= I - \sqrt{\boldsymbol{\pi}'_0}\sqrt{\boldsymbol{\pi}_0},$$

where I is the k x k identity matrix, and $\sqrt{\boldsymbol{\pi}_0} = \left(\sqrt{\pi_{01}}, \sqrt{\pi_{02}}, ..., \sqrt{\pi_{0k}}\right)$. The k - 1 eigenvalues of $I - \sqrt{\boldsymbol{\pi}'_0}\sqrt{\boldsymbol{\pi}_0}$ are equal to 1 and the rest equals 0. Thus the distribution of $\mathbf{W}_n\left(D_{\pi_0}\right)^{-1}\mathbf{W}_n'$ is the same as that of $\sum_{i=1}^{k} Z_i^2$, which is chi-square with k - 1 degrees of freedom. From Lemma 2, this is also is the asymptotic distribution of $X^2 = \mathbf{W}_n\left(D_{\pi_0}\right)^{-1}\mathbf{W}_n'$.

Theorem 1 enables the practitioner to use the chi-square distribution to obtain critical values for rejecting or accepting the null hypothesis as will be illustrated later in this chapter.

3.2.2 BAN estimates and Birch's (1964) regularity conditions

In order to derive the asymptotic distribution of $2nI^{\lambda}\left(\mathbf{X}/n:\hat{\boldsymbol{\pi}}\right)$, the concept of BAN (best asymptotically normal) estimators and reparameterization, as well as Birch's regularity conditions must be introduced briefly.

Let X be a multinomial $\text{Mult}_k(n,\boldsymbol{\pi})$ random row vector. The hypothesis

$$H_0:\boldsymbol{\pi}\in\boldsymbol{\Pi}_0$$

versus

$$H_1:\boldsymbol{\pi}\notin\boldsymbol{\Pi}_0 \tag{3.3}$$

can be reparameterized by assuming that under H_0 the unknown vector of true probabilities $\boldsymbol{\pi}^* = \left(\pi_1^*,\pi_2^*,\ldots,\pi_k^*\right) \in \boldsymbol{\Pi}_0$ is a function of parameters $\boldsymbol{\theta}^* = \left(\theta_1^*,\theta_2^*,\ldots,\theta_s^*\right)\in\boldsymbol{\Theta}_0$, where $s<k-1$. A function $\mathbf{f}(\boldsymbol{\theta})$ is defined, which maps each element of the subset $\boldsymbol{\Theta}_0\subset R^s$ into the subsets $\Pi_0\subset\Delta_k =$ $\left\{\mathbf{p}=\left(p_1,p_2,\ldots,p_k\right): p_i\geq 0; i=1,2,\ldots,k \text{ and } \sum_{i=1}^{k} p_i = 1\right\}$. Thus the hypothesis in (3.3) above can be reparameterized in terms of the pair $\left(\mathbf{f},\boldsymbol{\Theta}_0\right)$ as

$$H_0: \text{ There exists a } \boldsymbol{\theta}^*\in\boldsymbol{\Theta}_0 \text{ such that } \boldsymbol{\pi}=\mathbf{f}\left(\boldsymbol{\theta}^*\right) \quad \left(\equiv\boldsymbol{\pi}^*\right)$$

versus (3.4)

$$H_1: \text{ No } \boldsymbol{\theta}^* \text{ exists such that } \boldsymbol{\Theta}_0 \text{ for which } \boldsymbol{\pi}=\mathbf{f}\left(\boldsymbol{\theta}^*\right).$$

Instead of describing the estimation of $\boldsymbol{\pi}^*$ in terms of choosing a value $\hat{\boldsymbol{\pi}} \in \boldsymbol{\Pi}_0$ that minimizes a specific objective function, one can consider choosing $\hat{\boldsymbol{\theta}} \in \bar{\boldsymbol{\Theta}}_0$ (where $\bar{\boldsymbol{\Theta}}_0$ represents the closure of $\boldsymbol{\Theta}_0$) for which $\mathbf{f}(\hat{\boldsymbol{\theta}})$ minimizes a specific objective function (e.g. minimum power-divergence estimation) and then set $\hat{\boldsymbol{\pi}} = \mathbf{f}(\hat{\boldsymbol{\theta}})$. This reparameterization helps to describe the properties of the minimum power divergence estimator $\hat{\boldsymbol{\pi}}^{(\lambda)} = \mathbf{f}(\hat{\boldsymbol{\theta}}^{(\lambda)})$ of $\boldsymbol{\pi}^*$, or $\hat{\boldsymbol{\theta}}^{(\lambda)}$ of $\boldsymbol{\theta}^*$ defined by

$$I^{\lambda}\left(\mathbf{X}/n : \mathbf{f}\left(\hat{\boldsymbol{\theta}}^{(\lambda)}\right)\right) = \inf_{\boldsymbol{\theta} \in \boldsymbol{\Theta}_0} I^{\lambda}\left(\mathbf{X}/n : \mathbf{f}(\boldsymbol{\theta})\right) \tag{3.5}$$

It was necessary to define regularity conditions on $\mathbf{f}$ and $\boldsymbol{\Theta}_0$ under H_0, in order to ensure that the minimum power divergence estimator exists and converges in probability to $\boldsymbol{\theta}^*$ as $n \to \infty$. These conditions ensure that the null model really has s parameters and that $\mathbf{f}$ satisfies various smoothness requirements. Assuming H_0, (i.e., there exists a $\boldsymbol{\theta}^* \in \boldsymbol{\Theta}_0$ such that $\boldsymbol{\pi} = \mathbf{f}(\boldsymbol{\theta}^*)$ and that $s < k-1$, the regularity conditions are (Birch, 1964):

1) $\boldsymbol{\theta}^*$ is an interior point of $\boldsymbol{\Theta}_0$ and there is an s-dimensional neighbourhood of $\boldsymbol{\theta}^*$ completely contained in $\boldsymbol{\Theta}_0$;
2) $\pi_i^* \equiv f_i(\boldsymbol{\theta}^*) > 0$ for i = 1, 2,... k. Thus $\boldsymbol{\pi}^*$ is the interior point of the (k-1)-dimensional simplex Δ_k;
3) The mapping $\mathbf{f} : \boldsymbol{\Theta}_0 \to \Delta_k$ is totally differentiable at $\boldsymbol{\theta}^*$, so that the partial derivatives of f_i with respect to each θ_j exists at $\boldsymbol{\theta}^*$ and $\mathbf{f}(\boldsymbol{\theta})$ has a linear approximation at $\boldsymbol{\theta}^*$ given by

$$\mathbf{f}(\boldsymbol{\theta}) = \mathbf{f}(\boldsymbol{\theta}^*) + (\boldsymbol{\theta} - \boldsymbol{\theta}^*)\left(\partial \mathbf{f}(\boldsymbol{\theta}^*)\big/\partial \boldsymbol{\theta}\right)' + o\left(\left|\boldsymbol{\theta} - \boldsymbol{\theta}^*\right|\right) \text{ as } \boldsymbol{\theta} \to \boldsymbol{\theta}^*,$$

   where $\partial \mathbf{f}(\boldsymbol{\theta}^*)/\partial \boldsymbol{\theta}$ is a k x s matrix with (i ,j)-th element $\partial f_i(\boldsymbol{\theta}^*)/\partial \theta_j$;
4) The Jacobian matrix $\partial \mathbf{f}(\boldsymbol{\theta}^*)/\partial \boldsymbol{\theta}$ is of full rank s;
5) The inverse mapping $\mathbf{f}^{-1} : \boldsymbol{\Pi}_0 \to \boldsymbol{\Theta}_0$ is continuous at $\mathbf{f}(\boldsymbol{\theta}^*) = \boldsymbol{\pi}^*$; and
6) The mapping $\mathbf{f} : \boldsymbol{\Theta}_0 \to \Delta_k$ is continuous at every point $\boldsymbol{\theta} \in \boldsymbol{\Theta}_0$

The above conditions are necessary to establish the asymptotic expansion of the power-divergence estimator $\hat{\boldsymbol{\theta}}^{(\lambda)}$ of $\boldsymbol{\theta}^*$ under H_0,

$$\hat{\boldsymbol{\theta}}^{(\lambda)} = \hat{\boldsymbol{\theta}}^* + \left(\mathbf{X}/n - \boldsymbol{\pi}^*\right)\left(D_{\boldsymbol{\pi}^*}\right)^{-1/2} A\left(A'A\right)^{-1} + o_p\left(n^{-1/2}\right) \tag{3.6}$$

where $D_{\boldsymbol{\pi}^*}$ is the k x k diagonal matrix based on $\boldsymbol{\pi}^*$, and A is the k x s matrix with the (i,j)-th element $\left(\pi_i^*\right)^{-1/2} \partial f_i\left(\boldsymbol{\theta}^*\right)/\partial\theta_j$.

An estimator that satisfies (3.6) is called best asymptotically normal (BAN). This expansion plays a central role in deriving the asymptotic distribution of the power-divergence statistic under H_0.

3.2.3 Limiting distribution of the power divergence family of statistics

Reparameterized versions of Lemma 1 - 3 of §3.2.1, i.e. Lemma $1^* - 3^*$ can now be formulated, which are proved in the same way as Lemma 1 – 3.

Lemma 1^*: Assume $\mathbf{X}$ is a random row vector with a multinomial distribution $\text{Mult}_k\left(n, \boldsymbol{\pi}\right)$ and $\boldsymbol{\pi} = \mathbf{f}\left(\boldsymbol{\theta}^*\right) \in \boldsymbol{\Pi}_0$, from (3.4), for some $\boldsymbol{\theta}^* = \left(\theta_1^*, \theta_2^*, \ldots, \theta_s^*\right) \in \boldsymbol{\Theta}_0 \subset R^s$. Provided $\mathbf{f}$ satisfies Birch's regularity conditions (1) – (6) and $\hat{\boldsymbol{\pi}} \in \boldsymbol{\Pi}_0$ is a BAN estimator of $\boldsymbol{\pi}^* \equiv \mathbf{f}\left(\boldsymbol{\theta}^*\right)$, then $\mathbf{W}_n^* = \sqrt{n}\left(\mathbf{X}/n - \hat{\boldsymbol{\pi}}\right)$ converges in distribution to a multivariate normal random vector $\mathbf{W}^*$ as $n \to \infty$. The mean vector and covariance matrix of $\mathbf{W}^*$ are

$$E\left(\mathbf{W}^*\right) = \mathbf{0}$$

$$\text{cov}\left(\mathbf{W}^*\right) = D_{\boldsymbol{\pi}^*} - \boldsymbol{\pi}^{*\prime}\boldsymbol{\pi}^* - \left(D_{\boldsymbol{\pi}^*}\right)^{1/2} A\left(A'A\right)^{-1} A'\left(D_{\boldsymbol{\pi}^*}\right)^{1/2}$$

where $D_{\boldsymbol{\pi}^*}$ is the diagonal matrix based on $\boldsymbol{\pi}^*$ and A is the k x s matrix with (i,j)-th element $\left(\pi_i^*\right)^{-1/2} \partial f_i\left(\boldsymbol{\theta}^*\right)/\partial\theta_j$.

Lemma 2^*:

$$X^2 = \sum_{i=1}^{k} \frac{\left(X_i - n\hat{\pi}_i\right)^2}{n\hat{\pi}_i}$$

can be written as a quadratic form in $\mathbf{W}_n^* = \sqrt{n}\left(\mathbf{X}/n - \hat{\boldsymbol{\pi}}\right)$, and X^2 converges in distribution (as $n \to \infty$) to the quadratic form of the multivariate normal random vector $\mathbf{W}^*$ in Lemma 1^*.

Lemma 3^*: Certain quadratic forms of multivariate normal random vectors are distributed as chi-square random variables. In the case of Lemma 1^* and Lemma 2^*, where $X^2 = \mathbf{W}_n^* \left(D_{\boldsymbol{\pi}^*}\right)^{-1} \mathbf{W}_n^{*'} + o_p(1)$, X^2 converges in distribution to a central chi-square random variable with k - s - 1 degrees of freedom.

The Lemma's above can now be combined to prove the following result:

Theorem 2: Assuming $\mathbf{X}$ is a multinomial $\text{Mult}_k\left(n, \boldsymbol{\pi}\right)$ random vector, then

$$2nI^{\lambda}\left(\mathbf{X}/n : \hat{\boldsymbol{\pi}}\right) = 2nI^{1}\left(\mathbf{X}/n : \hat{\boldsymbol{\pi}}\right) + o_p(1); \qquad -\infty < \lambda < \infty$$

provided $\hat{\boldsymbol{\pi}} = \boldsymbol{\pi}^* + o_p(n^{-1/2})$.

Proof of Theorem 2:

$$2nI^{\lambda}\left(\mathbf{X} : \hat{\boldsymbol{\pi}}\right) = \frac{2n}{\lambda(\lambda+1)} \sum_{i=1}^{k} \frac{X_i}{n} \left[\left(\frac{X_i}{n\hat{\pi}_i}\right)^{\lambda} - 1\right]$$

$$= \frac{2n}{\lambda(\lambda+1)} \sum_{i=1}^{k} \hat{\pi}_i \left[\left(1 + V_i\right)^{\lambda+1} - 1\right] \tag{3.7}$$

where $V_i = \left(X_i/n - \hat{\pi}_i\right)/\hat{\pi}_i$. By assumption, $\hat{\boldsymbol{\pi}} = \boldsymbol{\pi}^* + O_p\left(n^{-1/2}\right)$ and from Lemma 1 of §3.2.1 we know $\mathbf{X}/n = \boldsymbol{\pi}^* + O_p\left(n^{-1/2}\right)$; therefore

$$V_i = O_p\left(n^{-1/2}\right) / \left(\pi_i^* + O_p\left(n^{-1/2}\right)\right) = O_p\left(n^{-1/2}\right),$$

provided $\pi_i^* > 0$. Expanding (3.7) in a Taylor series gives

$$2nI^{\lambda}\left(\mathbf{X}/n : \hat{\boldsymbol{\pi}}\right) = \frac{2n}{\lambda(\lambda+1)} \sum_{i=1}^{k} \hat{\pi}_i \left[\left(\lambda+1\right)V_i + \frac{\lambda(\lambda+1)}{2} V_i^2 + O_p\left(n^{-3/2}\right)\right]$$

$$= n \sum_{i=1}^{k} \hat{\pi}_i V_i^2 + O_p\left(n^{-1/2}\right)$$

$$= 2nI^{1}\left(\mathbf{X}/n : \hat{\boldsymbol{\pi}}\right) + O_p\left(n^{-1/2}\right)$$

as required. This ends the proof.

When $\hat{\boldsymbol{\pi}}$ is BAN, the condition of Theorem 2 is immediately satisfied, and from Lemma 3* it follows that $2nI^1(\mathbf{X}/n:\hat{\boldsymbol{\pi}})$ has an asymptotic chi-square distribution with k - s - 1 degrees of freedom.

3.2.4 Limiting non-central chi-square distribution by Read & Cressie (1988:171)

The distribution of PDS under local alternatives are of great importance in this study. Consider the alternative hypothesis $H_{1,n}:\boldsymbol{\pi}=\boldsymbol{\pi}^*+\frac{\mathbf{c}}{\sqrt{n}}$ where $\boldsymbol{\pi}^*\equiv\mathbf{f}(\boldsymbol{\theta}^*)\in\boldsymbol{\Pi}_0$ is the true but generally unknown value of $\boldsymbol{\pi}$ under $H_0:\boldsymbol{\pi}\in\boldsymbol{\Pi}_0$ and $\sum_{j=1}^{k}c_j=0$. Using a similar argument and set of assumptions to those employed under H_0 in the previous section, it can be shown that (provided $\hat{\boldsymbol{\pi}}=\mathbf{f}(\hat{\boldsymbol{\theta}})$ is BAN under $H_0:\boldsymbol{\pi}\in\boldsymbol{\Pi}_0$) the PDS $2nI^\lambda(\mathbf{X}/n:\hat{\boldsymbol{\pi}})$ has an asymptotic noncentral chi-square distribution under the hypothesis $H_{1,n}$. The degrees of freedom are the same as under H_0 (i.e. k - s - 1), and the noncentrality parameter is $\sum_{j=1}^{k}c^2/\pi_j^*$. The proof follows the following line:

Assuming Birch's regularity conditions and assuming that $\hat{\boldsymbol{\theta}}$ is a best asymptotic normal (BAN) estimator of $\boldsymbol{\theta}^*\in\boldsymbol{\Theta}_0$, then

$$2nI^\lambda\left(\frac{\mathbf{X}}{n}:\hat{\boldsymbol{\pi}}\right)=2nI^1\left(\frac{\mathbf{X}}{n}:\hat{\boldsymbol{\pi}}\right)+o_p(1)\qquad \text{as } n\to\infty$$

under both H_0 and $H_{1,n}$. This can be proved by noting that under $H_{1,n}$,

$\sqrt{n}\left(\frac{\mathbf{X}}{n}-\boldsymbol{\pi}^*-\frac{\mathbf{c}}{\sqrt{n}}\right)$ has an asymptotic multivariate normal distribution and therefore

$\frac{\mathbf{X}}{n}-\boldsymbol{\pi}^*-\frac{\mathbf{c}}{\sqrt{n}}=O_p\left(n^{-1/2}\right)$ which gives

$$\frac{\mathbf{X}}{n}-\boldsymbol{\pi}^*=O_p\left(n^{-1/2}\right). \tag{3.8}$$

The result of this theorem under H_0 is proved by Read & Cressie (1988:169). Now the BAN estimator $\hat{\boldsymbol{\theta}}$ of $\boldsymbol{\theta}^*$ satisfies

$$\hat{\boldsymbol{\theta}} = \boldsymbol{\theta}^* + \left(\frac{\mathbf{X}}{n} - \boldsymbol{\pi}^*\right)\left(D_{\boldsymbol{\pi}^*}\right)^{-1/2} A\left(A'A\right)^{-1} + o_p\left(n^{-1/2}\right)$$

Thus from (3.8) it follows that $\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}^* = O_p\left(n^{-1/2}\right)$ and from Birch's regularity condition 3, we have

$$\begin{aligned}\mathbf{f}\left(\hat{\boldsymbol{\theta}}\right) - \mathbf{f}\left(\boldsymbol{\theta}^*\right) &= \left(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}^*\right)\left(\partial\mathbf{f}\left(\boldsymbol{\theta}^*\right)/\partial\boldsymbol{\theta}\right)' + o\left(\left|\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}^*\right|\right) \\ &= O_p\left(n^{-1/2}\right).\end{aligned}$$

Consequently $\hat{\boldsymbol{\pi}} - \boldsymbol{\pi}^* = O_p\left(n^{-1/2}\right)$, which completes the proof.

3.3 Small-sample comparisons for the power divergence goodness-of-fit statistics

Read (1984:929) studied small sample properties of the power divergence family of goodness-of-fit statistics. He found that the power of well known tests based on G^2 and X^2 can be improved by choosing other members of the family of statistics. In this section these approximations and results will be discussed.

3.3.1 The alternative approximations

Let $\mathbf{X} = (X_1, X_2, \ldots, X_k)$ represent numbers of sequences of n observations that fall into k classes $C_1, C_2, \ldots, C_k$ with $\sum_{j=1}^{k} X_j = n$. If $\boldsymbol{\pi} = \left(\pi_1, \pi_2, \ldots, \pi_k\right)$ with $0 \le \pi_j \le 1$ for $j = 1, \ldots, k$ and $\sum_{j=1}^{k} \pi_j = 1$ are the respective probabilities of any observation's falling into each of the k classes, then the random vector $\mathbf{X}$ follows a multinomial distribution with parameters n, k, and $\boldsymbol{\pi}$. In this section, we will consider testing the simple hypothesis $H_0 : \boldsymbol{\pi} = \boldsymbol{\pi}_0$ versus $H_1 : \boldsymbol{\pi} = \boldsymbol{\pi}_1$.

The power divergence family of statistics can be expressed as

$$2nI^{\lambda}\left(\mathbf{X}/_n : \pi_0\right) = \frac{2}{\left(\lambda(\lambda+1)\right)}\sum_{i=1}^{k} X_i\left[\left(X_i/n\pi_0\right)^{\lambda} - 1\right] \tag{3.9}$$

for testing H_0, where the index parameter $\lambda \in \Re$.

If $F_E(t)$ is the exact distribution of (3.9) for a fixed λ, and $F_{\chi(\upsilon)}(\cdot)$ is the chi-square distribution function on υ degrees of freedom, it then holds that

$$F_E(t) = F_{\chi(k-1)}(\cdot) + o(1) \qquad n \to \infty \tag{3.10}$$

for all t under H_0, and k fixed. This is valid for every value of the family parameter λ.

Two closer approximations to F_E, were discussed by Read & Cressie (1983:59-69). The first is a corrected chi-square distribution, for which the mean and variance agree to second order with the mean and variance of (3.9), is

$$F_C(t) = F_{\chi(k-1)}\left(\frac{t-\gamma_\lambda}{\sqrt{\delta_\lambda}}\right) \tag{3.11}$$

where

$$\delta_\lambda = 1 + \left(6n(k-1)\right)^{-1}\left\{3\left(S-k^2-2k+2\right) + 6(\lambda-1)\left(3S-k^2-6k+4\right)\right.$$
$$\left. + (\lambda-1)^2\left(5S-3k^2-6k+4\right) + 6(\lambda-1)(\lambda-2)(S-2k+1)\right\}$$

and

$$\gamma_\lambda = (k-1)\left(1-\sqrt{\delta_\lambda}\right) + (12n)^{-1}\left\{4(\lambda-1)(S-3k+2) + 3(\lambda-1)(\lambda-2)(S-2k+1)\right\}$$

states the asymptotic variance and mean of (3.9) respectively to $o(1/n)$ with

$S = \sum_{i=1}^{k} \pi_{0i}^{-1}$.

Another closer approximation to F_E is

$$F_S(t) = F_{\chi(k-1)}(t) + \frac{1}{24n}\left[2(1-S)F_{\chi(k-1)}(t) + \left\{3\left(3S-k^2-2k\right) + (\lambda-1)6\left(S-k^2\right)\right.\right.$$
$$\left. + (\lambda-1)^2\left(5S-3k^2-6k+4\right) - (\lambda-1)(\lambda-2)3(S-2k+1)\right\}F_{\chi(k+1)}(t)$$
$$+ \left\{-6\left(2S-k^2-2k+1\right) - (\lambda-1)4\left(4S-3k^2-3k+2\right)\right.$$
$$\left. - (\lambda-1)^2 2\left(5S-3k^2-6k+4\right) + (\lambda-1)(\lambda-2)3(S-2k+1)\right\}F_{\chi(k+3)}(t)$$

$$+\lambda^2\left(5S-3k^2-6k+4\right)F_{\chi(k+5)}(t)\Big]+\left(N_\lambda(t)-n^{(k-1)/2}V_\lambda(t)\right)\frac{e^{-t/2}}{\left[(2\pi n)^{k-1}Q\right]^{1/2}},$$

where $N_\lambda(t)$ is the number of $\mathbf{X}$'s such that $2nI^\lambda\left(\mathbf{X}/n:\pi_0\right)<t$ and

$$V_\lambda(t)=\frac{(\pi t)^{(k-1)/2}}{\Gamma((k-1)/2)}Q^{1/2}\left[1+(t/24(k+1)n)\left\{(\lambda-1)^2\left(5S-3k^2-6k+4\right)\right.\right.$$
$$\left.\left.-3(\lambda-1)(\lambda-2)(S-2k+1)\right\}\right]$$

with $Q=\prod_{i=1}^{k}\pi_{0i}$.

If $k\to\infty$ as $n\to\infty$ so that $n/k\to a$ for $0<a<\infty$ fixed, then (3.10) no longer holds. Read & Cressie (1983:59-69) derived, under the equiprobable hypothesis, $H_0:\pi=\mathbf{1}/k$, where $\mathbf{1}$ is the vector of 1's, and $\lambda>-1$ that

$$F_E(t)=F_N(t)+o(1)\qquad n\to\infty \tag{3.12}$$

where

$$F_N(t)=\Pr\{N(0,1)<(t-\mu_n)/\sigma_n\}$$

and

$$\mu_n=\begin{cases}\dfrac{2n}{(\lambda(\lambda+1))}E\left\{(Y/a)^{\lambda+1}-1\right\} & \lambda>-1,\ \lambda=0\\ 2nE\{(Y/a)\log(Y/a)\} & \lambda=0\end{cases}$$

and

$$\sigma_n^2=\begin{cases}\left(\dfrac{2a}{(\lambda(\lambda+1))}\right)^2k\left[\mathrm{var}\left\{(Y/a)^{\lambda+1}\right\}-a\left(\mathrm{cov}\left\{Y/a,(Y/a)^{\lambda+1}\right\}\right)^2\right] & \lambda>-1,\ \lambda\neq0\\ (2a)^2k\left[\mathrm{var}\{(Y/a)\log(Y/a)\}-a\left(\mathrm{cov}\{Y/a,(Y/a),\log(Y/a)\}\right)^2\right] & \lambda=0\end{cases}$$

where $N(0,1)$ represents the standard normal random variable and Y the Poisson random variable with mean a.

Read (1984:930) then assessed the error incurred by applying each of these four large-sample results to approximate the true distribution function of the family (3.9) in **small samples**. Recommendations were made about which approximation should be used when calculating the test level for tests based on different members of the family (3.9). Exact power functions of tests based on (3.9) were compared for various $\lambda \in \Re$ under specific alternatives, from which different statistics of the family (3.9) were recommended under the alternative H_1.

To compute F_E for any given n, k and λ, under the equiprobable hypothesis, it is necessary to enumerate all possible $\binom{n+k-1}{n}$ partitions $\mathbf{x}$ of n into k parts. With each $\mathbf{x}$ there is an associated statistic value $2nI^{\lambda}\left(\mathbf{x}/n:\mathbf{1}/k\right)$ from (3.9) (where $\mathbf{1}$ is the vector of 1's) and a multinomial probability $\Pr\{\mathbf{X}=\mathbf{x}\mid H_0\}=\dfrac{n!}{k^n\prod_{i=1}^{k}x_i!}$. The statistical values are ranked from smallest to largest and starting from the smallest rank, the associated probabilities are added until just before the inequality $2nI^{\lambda}\left(\mathbf{x}/n:\mathbf{1}/k\right)>t$ is satisfied. The value of the cumulative probability sum equals $F_E(t)$.

Two criteria were used to approximate the errors incurred by the four asymptotic results for small n. The first criterion compares the true versus the approximate significance levels. The second criterion uses the maximum approximation error (Read, 1984:931).

Read (1984:932) concluded that the moment-corrected chi-square approximation based on F_C for statistics based on λ values outside the range [1/3, 3/2], appears to give nominal levels close to the true levels. However, λ values within this range is preferred for values of $n \leq 20$ and $2 < k \leq 6$. For $k = 2$, any λ in the range [-2, 5/2] can be used.

Comparisons of exact power were also carried out against the alternative

$$H_1:\boldsymbol{\pi}=\boldsymbol{\pi}_1 \text{ with } \pi_{1i}=\begin{cases}\dfrac{1-\delta/(k-1)}{k} & i=1,\ldots,k-1\\[2ex] \dfrac{(1+\delta)}{k} & i=k\end{cases}$$

where $-1 \leq \delta \leq k-1$. Three values for δ were chosen, $\delta = 1.5$; $\delta = 0.5$ and $\delta = -0.9$. Exact power values were obtained for $\lambda \in [-5; 5]$, $\alpha = 0.05$, the sample size $n = 20$ and cell number $k = 5$. It was found that λ in the range [1/3, 2/3] gives reasonable power for the extreme values of δ. Read (1984:933) advised that if n is small relative to k, $\lambda > -1$ should be used.

3.3.2 Improving the accuracy of tests with small sample size

In this section we discuss the improvement in accuracy, made by (Read & Cressie 1988:64-80), through more accurate moments of $2nI^{\lambda}(\mathbf{X}/n:\boldsymbol{\pi}_0)$ for $\lambda > -1$, (for $\lambda \leq -1$ moments do not exist, Bishop *et al.* (1975:488)) under the null model $H_0:\boldsymbol{\pi} = \boldsymbol{\pi}_0$. The mean and variance for $\lambda > -1$ are defined as

$$E\left[2nI^{\lambda}(\mathbf{X}/n:\boldsymbol{\pi}_0)\right] = [k-1] + \frac{1}{n}\left[(\lambda-1)\frac{(2-3k+t)}{3} + (\lambda-1)(\lambda-2)\frac{(1-2k+t)}{4}\right]$$
$$+o\left(n^{-1}\right) \tag{3.13}$$

and

$$\operatorname{var}\left[2nI^{\lambda}(\mathbf{X}/n:\boldsymbol{\pi}_0)\right] = [2k-2] + \frac{1}{n}\left[\left(2-2k-k^2+t\right) + (\lambda-1)\left(8-12k-2k^2+6t\right)\right.$$
$$\left. + (\lambda-1)^2\left(4-6k-3k^2+5t\right)/3 + (\lambda-1)(\lambda-2)(2-4k+2t)\right]$$
$$+o\left(n^{-1}\right), \tag{3.14}$$

where $t = \sum_{i=1}^{k} \pi_{0i}^{-1}$.

The first terms of (3.13) and (3.14) are the mean and variance of a chi-square random variable with k -1 degrees of freedom. The second terms are the correction terms of order n^{-1} including the family parameter λ, the number of cells k and the sum of the reciprocal null probabilities t. Denoting the correction term for the mean by $f_m(\lambda,k,t)$ and the correction term for the variance by $f_v(\lambda,k,t)$. Then

$$E\left[2nI^{\lambda}(\mathbf{X}/n:\boldsymbol{\pi}_0)\right] = k-1+n^{-1}f_m(\lambda,k,t)+o\left(n^{-1}\right) \tag{3.15}$$

and

$$\operatorname{var}\left[2nI^{\lambda}(\mathbf{X}/n:\boldsymbol{\pi}_0)\right] = 2k-2+n^{-1}f_v(\lambda,k,t)+o\left(n^{-1}\right). \tag{3.16}$$

It can be seen that $f_m(\lambda,k,t)$ and $f_v(\lambda,k,t)$ control the speed with which the mean and variance of the power divergence statistic converge to the mean and variance of a chi-square random variable with k - 1 degrees of freedom. In carrying out a study to find the values of $\lambda > -1$ for which f_m and f_v are close to 0, Read and Cressie (1988:66), found that under the equiprobable hypothesis, $H_0 : \boldsymbol{\pi} = 1/k$, where $t = \sum_{i=1}^{k} \pi_{0i}^{-1} = k^2$, the Pearson's X^2, ($\lambda = 1$), minimizes the correction terms for $k \geq 20$. However, in cases where t dominates k^2, $\lambda \in [0.61, 0.67]$ results in the smallest mean and variance correction terms.

For a given critical value c, the distribution tail function of the chi-square distribution, with k - 1 degrees of freedom is defined as $T_\chi(c) = \Pr\left(\chi^2_{k-1} \geq c\right)$. Then $T_\chi(c)$ is the asymptotic significance level associated with the critical value c. To improve the small-sample accuracy of this approximation, a moment-corrected distribution tail function based on the moment corrected statistic is defined:

$$\left\{2nI^\lambda\left(\mathbf{X}/n : \boldsymbol{\pi}_0\right) - \mu_\lambda\right\}/\sigma_\lambda \qquad -\infty < \lambda < \infty$$

with

$$\mu_\lambda = (k-1)(1-\sigma_\lambda) + n^{-1} f_m(\lambda,k,t),$$

$$\sigma_\lambda = 1 + \left(2n(k-1)\right)^{-1} f_v(\lambda,k,t),$$

$$f_m(\lambda,k,t) = (\lambda-1)(2-3k+t)/3 + (\lambda-1)(\lambda-2)(1-2k+t)/4,$$

and

$$f_v(\lambda,k,t) = 2 - 2k - k^2 + t + (\lambda-1)\left(8 - 12k + -2k^2 + 6t\right)$$
$$+ (\lambda-1)^2\left(4 - 6k - 3k^2 + 5t\right)/3 + (\lambda-1)(\lambda-2)(2-4k+2t)$$

where $t = \sum_{i=1}^{k} \pi_{0i}^{-1}$. The moment-corrected distribution tail function is then defined as $T_C(c) = T_\chi(c - \mu_\lambda)/\sigma_\lambda$. Read (1988:69) shows that $T_C(c)$ results in a substantial improvement in accuracy for values of λ outside the interval [1/3,3/2], for $\lambda > -1$ and $\lambda \leq -1$.

CHAPTER 4

APPROXIMATIONS TO THE DISTRIBUTIONS OF THE TEST STATISTICS

4.1 Introduction

Various approximations for the distribution of the Power Divergence Test statistic are available in the literature, of which two will be discussed in §4.2 of this chapter, i.e., an Edgeworth approximation and the AE Approximation by Taneichi *et al.* (2002). Power Approximations by Drost *et al.* (1989) and Sekiya *et al.* (1999) will be discussed in §4.3 and §4.4 respectively under a local alternative hypothesis.

4.2 Notation and important results

Consider the multinomial goodness-of-fit test as described in chaper 2. Let $\mathbf{X}=(X_1, X_2, \ldots, X_k)$ be distributed according to a multinomial distribution $\text{Mult}_k(n, \pi_1, \pi_2, \ldots, \pi_k)$, where $\sum_{j=1}^{k} X_j = n$, $\sum_{j=1}^{k} \pi_j = 1$, $0 < \pi_j < 1$ for all j = 1, 2,…, k and $(\pi_1, \pi_2, \ldots, \pi_k)$ is an unknown probability vector. It is of interest to test the simple null hypothesis $H_0 : \pi_j = p_j$ for all j = 1, 2, …, k. where $(p_1, \ldots, p_k)$ is a completely specified probability vector. For this chapter, we redefine the power divergence family of statistics as

$$\mathbf{R}^a = \begin{cases} \dfrac{2n}{a(a+1)} \displaystyle\sum_{j=1}^{k} \frac{X_j}{n} \left\{ \left(\frac{X_j}{np_j} \right)^a - 1 \right\} & (a \neq 0, -1) \\ 2n \displaystyle\sum_{j=1}^{k} \frac{X_j}{n} \log \frac{X_j}{np_j} & (a = 0) \\ 2n \displaystyle\sum_{j=1}^{k} p_j \log \frac{np_j}{X_j} & (a = -1) \end{cases} \tag{4.1}$$

Under H_0, all $\mathbf{R}^a$ are asymptotically distributed according to the central chi-square distribution with k - 1 degrees of freedom.

Furthermore, it is known from Read & Cressie (1988) that under local alternatives

$$H_{1,n} : \pi_j = p_j + \frac{c_j}{\sqrt{n}} \quad \text{for all } j = 1, \ldots, k \quad \text{and} \quad \sum_{j=1}^{k} c_j = 0, \tag{4.2}$$

all $\mathbf{R}^a$ are asymptotically distributed according to the noncentral chi-square distribution with k - 1 degrees of freedom and noncentrality parameter

$$\delta = \sum_{j=1}^{k} \frac{c_j^2}{p_j}. \tag{4.3}$$

To consider asymptotic approximations of the statistics $\mathbf{R}^a$, some important historical results should be noted. Esséen (1945:1-125) derived the asymptotic expansion for the distribution function of the sum of independent and identically distributed lattice-valued random variables by using a Euler-Mclaurin summation formula. Ranga Rao (1961:359-361) proposed a multidimensional extension of the Euler-Mclaurin summation formula. He then derived the asymptotic expansion for the probability that the normalized sum of independent and identically distributed lattice-valued random vectors is contained in a Borel set B.

From Esséen's results and Ranga Rao's expansion, Yarnold (1972:1566-1580) obtained a simple expansion when B is a convex set which includes a term for the number of lattice points inside the region of interest. He also applied it to the null distribution of the Pearson's chi-square test statistic, X^2, and derived an asymptotic expansion for the null distribution of X^2. Siotani and Fujikoshi (1984:115-124) similarly obtained an asymptotic expansion for the distributions of the log-likelihood statistic and the Freeman-Tukey statistic.

Taneichi *et al.* (2002) investigated the asymptotic approximations for the distribution of $\mathbf{R}^a$ under local alternatives of the form $H_{1,n}$. An expression was obtained which consists of continuous and discontinuous terms. The discontinuous term is of a very complicated and impractical form. As a result, a power approximation is attempted by using a multivariate approximation in which the discontinuous term is omitted. Thus the power of $\mathbf{R}^a$ is approximated using the multivariate Edgeworth approximation for a continuous distribution.

4.3 A local Edgeworth approximation according to Taneichi *et al.* (2002)

In this section an explanation is provided of how Taneichi *et al.* (2002) derived a local Edgeworth approximation for the probability of $(X_1, X_2, ..., X_n)$ under $H_{1,n}$.

Let $(X_1, X_2, ..., X_n)$ be distributed according to a $\text{Mult}_k\left(n, p_1 + c_1/\sqrt{n}, p_2 + c_2/\sqrt{n},\right.$ $\left. ..., p_k + c_k/\sqrt{n}\right)$ where $\sum_{j=1}^{k} c_j = 0$. With $r = k$ - 1, and

$$Y_j = \frac{X_j - np_j}{\sqrt{n}},\ j = 1,...,k, \tag{4.4}$$

the lattice random vector $\mathbf{Y} = (Y_1, Y_2, ..., Y_r)'$, takes on values in the set

$L = \left\{ \mathbf{y} = (y_1, ..., y_r)' : \mathbf{y} = \dfrac{\mathbf{x} - np}{\sqrt{n}}, \mathbf{x} \in M \right\}$, where $\mathbf{p} = (p_1, ..., p_r)'$ and

$M = \left\{ \mathbf{x} = (x_1, ..., x_r)' : x_1, ..., x_r \text{ are non-negative integers and } \sum_{j=1}^{r} x_j \le n \right\}$.

Taneichi *et al.* (2002) then proved that for each $\mathbf{x} \in M$, and $\mathbf{y} = \dfrac{\mathbf{x} - n\mathbf{p}}{\sqrt{n}}$ it holds that

$$\Pr\{\mathbf{Y} = \mathbf{y}\} = \left(\frac{1}{n}\right)^{r/2} f(\mathbf{y}) \left\{ 1 + \frac{1}{\sqrt{n}} g_1(\mathbf{y}) + \frac{1}{n} g_2(\mathbf{y}) + o(n^{-1}) \right\}, \tag{4.5}$$

where

$f(\mathbf{y}) = \left(\dfrac{1}{2\pi}\right)^{r/2} \left(\dfrac{1}{\Omega}\right)^{1/2} \exp\left\{ -\dfrac{1}{2}(\mathbf{y} - \mathbf{c})' \dfrac{1}{\Omega}(\mathbf{y} - \mathbf{c}) \right\}$,

$\Omega = diag(p_1, ..., p_r) - \mathbf{pp}'$ and $\mathbf{c} = (c_1, ..., c_r)'$,

$g_1(\mathbf{y}) = -\dfrac{1}{2}\sum_{j=1}^{k} \dfrac{y_j}{p_j} + \dfrac{1}{6}\sum_{j=1}^{k} y_j \left(\dfrac{y_j}{p_j}\right)^2 - \dfrac{1}{2}\sum_{j=1}^{k} y_j \left(\dfrac{c_j}{p_j}\right)^2 + \dfrac{1}{3}\sum_{j=1}^{k} c_j \left(\dfrac{c_j}{p_j}\right)^2$,

$g_2(\mathbf{y}) = \dfrac{1}{2}\{g_1(\mathbf{y})\}^2 + \dfrac{1}{12}\left(1 - \sum_{j=1}^{k} \dfrac{1}{p_j}\right) + \dfrac{1}{4}\sum_{j=1}^{k}\left(\dfrac{y_j}{p_j}\right)^2 - \dfrac{1}{12}\sum_{j=1}^{k} y_j \left(\dfrac{y_j}{p_j}\right)^3 + \dfrac{1}{3}\sum_{j=1}^{k} y_j \left(\dfrac{c_j}{p_j}\right)^3 - \dfrac{1}{4}\sum_{j=1}^{k} c_j \left(\dfrac{c_j}{p_j}\right)^3$

and

$y_k = -\sum_{j=1}^{r} y_j$;

$$c_k = -\sum_{j=1}^{r} c_j .$$

The proof appears in Taneichi *et al.* (2002), Theorem 1.

4.4. Asymptotic approximations for the distributions under local alternatives

Taneichi *et al.* (2002) proceeded to derive approximations for $\mathbf{R}^a$ under $H_{1,n}$. They presented the power divergence statistic $\mathbf{R}^a$, by using the transformation in (4.4)

$$\mathbf{R}^a(\mathbf{Y}) = \begin{cases} \dfrac{2}{a(a+1)}\left(\displaystyle\sum_{j=1}^{k}\frac{\left(np_j+\sqrt{n}Y_j\right)^{a+1}}{\left(np_j\right)^a}-n\right) & (a \neq 0,-1) \\ 2\displaystyle\sum_{j=1}^{k}\left(np_j+\sqrt{n}Y_j\right)\log\left(1+\frac{Y_j}{\sqrt{n}p_j}\right) & (a=0) \\ 2\displaystyle\sum_{j=1}^{k} np_j \log\left(\frac{np_j}{np_j+\sqrt{n}Y_j}\right) & (a=-1) \end{cases} .$$

By defining $B^{(a)} = \left\{\mathbf{y}=(y_1,\ldots,y_r)' :\ \mathrm{R}^a(\mathbf{y}) < b\right\}$, and by using results of Read(1984:59-69), the distribution of the power divergence statistics $\mathbf{R}^a$ under $H_{1,n}$ was then expressed as $\Pr\left\{\mathbf{R}^a < b \mid H_{1,n}\right\} = \Pr\left\{\mathbf{Y} \in B^{(a)} \mid H_{1,n}\right\}$, where for every $l \in \{1, \ldots, r\}$, the set $B^{(a)}$ can be represented as

$$B^{(a)} = \Big\{\mathbf{y} = \left(y_1, y_2, \ldots, y_r\right)' : \omega_l\left(y_l^*\right) < y_l < \theta_l\left(y_l^*\right), \\ y_l^* = \left(y_1, \ldots, y_{l-1}, y_{l+1}, \ldots, y_r\right)' \in B_l\Big\} \tag{4.6}$$

$B_l \subset R^{r-1}$ and θ_l, ω_l are continuous functions on R^{r-1}. $H_{1,n}$ coincides with the null hypothesis H_0, if $\mathbf{c} = 0$.

It is necessary to sum the local Edgeworth expansion (4.5) with $\mathbf{c} = 0$ over all lattice points in $B^{(a)}$, to derive an asymptotic expansion for $\Pr\left\{\mathbf{Y} \in B^{(a)} \mid H_0\right\}$. A summary of historical results of studies of the distributions of $\mathbf{R}^a$ under H_0 now follows.

Ranga Rao (1961:359-361) applied the generalised Euler-Mclaurin summation formula to the local Edgeworth expansion and gave an asymptotic expansion for $\Pr\{\mathbf{Y} \in B\}$ where B is a Borel set. Yarnold (1972:1566-1580) applied Ranga Rao's expansion in case where B is convex and derived an asymptotic expansion for the null distribution of R^1.

Read (1984:59-69) developed an expression for the null distribution of the general $\mathbf{R}^a$. This expansion is expressed in the form

$\Pr\{\mathrm{R}^a < b \mid H_0\} = J_1 + J_2 + J_3 + O(n^{-3/2})$.

The J_1 term is regarded as the multivariate Edgeworth expansion for a continuous distribution. J_2 is a discontinuous term to account for the discontinuity in $\mathbf{Y}$ and is $O(n^{-1/2})$. J_3 is $O(n^{-1})$, discontinuous and very complicated. Yarnold (1972:1566-1580), Siotanu and Fujikoshi (1984:115-124) and Read (1984:59-69) then proposed the approximation

$$\Pr\{\mathbf{R}^a < b \mid H_0\} \approx J_1 + \hat{J}_2 . \tag{4.7}$$

The numerical accuracy of this approximation is shown in Yarnold (1972:1566-1580) for $\mathbf{R}^1$ and in Read (1984:929-935) for general $\mathbf{R}^a$. Refer to Theorem 2 of Taneichi *et al.* (2002:342-343) for evaluation of these terms.

However, under $H_{1,n}$, the asymptotic expansion for the distribution of $\mathbf{R}^a$ can be written as

$$\Pr\{\mathbf{R}^a < b \mid H_0\} \approx J_1^* + J_2^* \tag{4.8}$$

where

$$J_1^* = \int_{B^{(a)}} f(\mathbf{y})\left\{1 + \frac{1}{\sqrt{n}} g_1(\mathbf{y}) + \frac{1}{n} g_2(\mathbf{y})\right\} d\mathbf{y},$$

$$J_2^* = -\frac{1}{\sqrt{n}} \sum_{l=1}^{r} \left(\frac{1}{n}\right)^{(r-l)/2} \sum_{y_{l+1} \in L_{l+1}} \cdots \sum_{y_r \in L_r} \int \cdots \int_{B_l} \to \int_{B_l} \left[S_1\left(\sqrt{n} y_l + np_l\right) f(\mathbf{y}) \right]_{\omega_l(y_l^*)}^{\theta_l(y_l^*)} dy_1, \ldots, dy_{l-1}$$

where

$$S_1(x) = x - [x] - \frac{1}{2},$$

$$[h(\mathbf{y})]_{\omega_l(y_l^*)}^{\theta_l(y_l^*)} = h\left(y_1,...,y_{l-1},\theta_l(y_l^*),y_{l+1},...,y_r\right) - h\left(y_1,...,y_{l-1},\omega_l(y_l^*),y_{l+1},...,y_r\right)$$

and $L_l = \left\{ y_l : y_l = \dfrac{n_l - np_l}{\sqrt{n}}, \quad n_l \text{ is an integer} \right\}$,

where ω_l are continuous functions on $\mathbf{R}^{r-l}$ (Read, 1984:59-69). The J_2^* term turned out to be too complicated if $\mathbf{c} \neq 0$. Taneichi *et al.* (2002:347) only considers J_1^* as an approximation for the distribution of $\mathbf{R}^a$ under $H_{1,n}$ and J_1^* is evaluated as

$$J_1^* = \Pr\left\{\chi_r^2(\delta) < b\right\} + \frac{1}{6\sqrt{n}} \sum_{j=0}^{3} q_j^{(a)} \Pr\left\{\chi_{r+2j}^2(\delta) < b\right\}$$

$$+ \frac{1}{72} \sum_{j=0}^{3} r_j^{(a)} \Pr\left\{\chi_{r+2j}^2(\delta) < b\right\} + o\left(n^{-1}\right), \tag{4.9}$$

where the $q_j^{(a)}$ and $r_j^{(a)}$ are functions of the c_j's and p_j's which are defined as follows:

$$q_0^{(a)} = 2S_2^3$$

$$q_1^{(a)} = -3\left(S_2^3 + aS_1^1\right),$$

$$q_2^{(a)} = (1-a)S_2^3 + 3aS_1^1,$$

$$q_3^{(a)} = aS_2^3,$$

$$r_0^{(a)} = 4\left(S_2^3\right)^2 - 18S_3^4 - 6\left(S_1^0 - 1\right),$$

$$r_1^{(a)} = -12\left(S_2^3\right)^2 - 9\left(S_1^2\right)^2 - 12aS_1^1S_2^3 + 33S_3^4 + 18aS_2^2 - 18akS_1^2$$
$$+3\left\{(a+2)(2a+1)S_1^0 - 3ak(ak+2) + (a-1)(a+2)\right\},$$

$$r_2^{(a)} = (13-4a)\left(S_2^3\right)^2 - 18(a-1)\left(S_1^2\right)^2 + 9a^2\left(S_1^1\right)^2 + 30aS_1^1S_2^3 + 18(a-1)S_3^4$$
$$+18\mathrm{a}(\mathrm{a}\text{-}1)S_2^2 - 18a\left\{(a-2)k + a - 1\right\}S_1^2$$
$$-3a\left\{(7a+5)S_1^0 - 6k(ak+a+1) + 5a + 1\right\}$$

$$r_3^{(a)} = 2(5a-3)\left(S_2^3\right)^2 - 9\left(a^2 - 4a + 1\right)\left(S_1^2\right)^2 - 18a^2\left(S_1^1\right)^2$$
$$+6\mathrm{a}(\mathrm{a}-4)S_1^1S_2^3 + 3\left(2a^2 - 9a + 1\right)S_3^4 - 54a^2S_2^2$$
$$+18a\left\{(2a-1)k + 3a - 1\right\}S_1^2 + 3a^2\left(5S_1^0 - 3k^2 - 6k + 4\right),$$

$$r_4^{(a)} = \left(a^2 - 8a + 1\right)\left(S_2^3\right)^2 + 18a(a-1)\left(S_1^2\right)^2 + 9a^2\left(S_1^1\right)^2,$$

$$-6a(2a-1)S_1^1S_2^3-3a(5a-3)S_3^4+36a^2S_2^2-18a^2(k+2)S_1^2,$$

$$r_5^{(a)}=-2a(a-1)\left(S_2^3\right)^2-9a\left(S_1^2\right)^2+6a^2S_1^1S_2^3+9a^2S_3^4,$$

$$r_6^{(a)}=a^2\left(S_2^3\right)^2,$$

$$S_m^l=\sum_{j=1}^{k}\frac{\left(c_j\right)^l}{\left(p_j\right)^m},$$

δ is defined by (4.3) and χ_r^2 denotes the non-centrality parameter with r degrees of freedom and noncentrality parameter δ.

The following approximation for the distribution of $\mathbf{R}^a$ under $H_{1,n}$ is derived from (4.9):

$$\Pr\left\{\mathbf{R}^a<b\mid H_{1,n}\right\}\approx\Pr\left\{\chi_r^2(\delta)<b\right\}-\frac{1}{6\sqrt{n}}\sum_{j=0}^{3}q_j^{(a)}\Pr\left\{\chi_{r+2j}^2(\delta)<b\right\}$$

$$+\frac{1}{72n}\sum_{j=0}^{6}r_j^{(a)}\Pr\left\{\chi_{r+2j}^2(\delta)<b\right\} \tag{4.10}$$

4.5. Asymptotic approximations of the power of $\mathbf{R}^a$ under local alternatives

The power approximation of $\mathbf{R}^a$ against $H_{1,n}$ is derived from (4.10) as

$$1-\Pr\left\{\chi_r^2(\delta)<c_o^{(a)}\right\}-\frac{1}{6\sqrt{n}}\sum_{j=0}^{3}q_j^{(a)}\Pr\left\{\chi_{r+2j}^2(\delta)<c_o^{(a)}\right\}$$

$$-\frac{1}{72n}\sum_{j=0}^{6}r_j^{(a)}\Pr\left\{\chi_{r+2j}^2(\delta)<c_o^{(a)}\right\} \tag{4.11}$$

where $c_o^{(a)}$ is the critical value for $\mathbf{R}^a$. This approximation is referred to as the AE approximation. Taneichi *et al.* (2002:347-353) then carried out a numerical comparisons of this AE method with the true power calculation discussed in Read & Cressie (1988:76-77).

4.6 Two Power Approximations by Drost *et al.* (1989)

Drost *et al.* (1989:130-1410) proposed two power approximations, the A^a and B^a approximations. The A^a approximation of $\mathbf{R}^a$ consists of only linear and quadratic terms and is formed from a linear combination of mutually independent noncentral chi-square random variables. The B^a approximation is more easily computed and is obtained by taking the quadratic terms to have equal coefficients chosen so that the B^a has the same first moment as A^a, i.e. it is formed by a single noncentral chi-square random variable which has the same moment as A^a.

A description of these approximations to the power of $\mathbf{R}^a$ against the alternative $H_1 : \pi = \tau \quad (\tau \neq \mathbf{p})$, where $\boldsymbol{\pi} = (\pi_1,\ldots,\pi_k)'$, $\boldsymbol{\tau} = (\tau_1,\ldots,\tau_k)'$, and $\tilde{\mathbf{p}} = (p_1,\ldots,p_r,p_k)'$.

Rewriting the $\mathbf{R}^a$ statistic as

$$I^a(\boldsymbol{\tau},\tilde{\mathbf{p}}) = \begin{cases} \dfrac{1}{a(a+1)} \displaystyle\sum_{j=1}^{k} \tau_j \left[\left(\dfrac{\tau_j}{p_j} \right)^a - 1 \right] & (a \neq 0,\ 1) \\ \displaystyle\sum_{j=1}^{k} \tau_j \log \dfrac{\tau_j}{p_j} & (a = 0) \\ \displaystyle\sum_{j=1}^{k} p_j \log \dfrac{p_j}{\tau_j} & (a = -1), \end{cases}$$

where $\tau^{1/2} = \left(\tau_1^{1/2},\ldots,\tau_k^{1/2}\right)'$, $\rho_j = \dfrac{\tau_j}{p_j}$ $(j = 1,\ldots,k)$, $Q^{(a)} = diag\left(\rho_1^{a/2},\ldots,\rho_k^{a/2}\right)$ and

$$\zeta^{(a)} = a^{-1} n^{1/2} \left(\tau_1^{1/2}\left(1-\rho_1^{-a}\right),\ldots,\tau_k^{1/2}\left(1-\rho_k^{-a}\right) \right)'.$$

Let $\beta_1^{(a)},\ldots,\beta_r^{(a)}$ be the non-zero eigenvalues and $S^{(a)}$ the k x k orthogonal matrix of eigenvalues of $Q^{(a)}\left(E - \tau^{1/2}\left(\tau^{1/2}\right)'\right)Q^{(a)} : \left(S^{(a)}\right)' Q^{(a)}\left(E - \tau^{1/2}\left(\tau^{1/2}\right)'\right)Q^{(a)}S^{(a)}$

$= diag\left(\beta_1^{(a)},\ldots,\beta_r^{(a)},0\right)$, where E denotes the k-dimensional unit matrix. Let

$$\boldsymbol{\eta}^{(a)} = \left(\eta_1^{(a)},\ldots,\eta_k^{(a)}\right)' = \left(S^{(a)}\right)' Q^{(a)} \zeta^{(a)}.$$

The A^a approximation of power is then

$$1 - \Pr\left\{ \sum_{j=1}^{r} \beta_j^{(a)} \chi_1^2 \left(\frac{\left(\eta_j^{(a)}\right)^2}{\beta_j^{(a)}} \right) < c_0^{(a)} - 2nI^a(\boldsymbol{\tau},\tilde{\mathbf{p}}) + \sum_{j=1}^{r} \left(\eta_j^{(a)}\right)^2 \right\}. \qquad (4.12)$$

Let

$$\Lambda^{(a)} = \frac{1}{r} \sum_{j=1}^{k} \left(1 - \tau_j\right) \rho_j^a$$

and

$$\omega^{(a)} = \frac{n}{\left(a\Lambda^{(a)}\right)^2}\left[\sum_{j=1}^{k}\tau_j\left(p_j^a-1\right)^2-\left\{\sum_{j=1}^{k}\tau_j\left(p_j^a-1\right)\right\}^2\right]$$

then the B^a approximation of power is

$$1-\Pr\left\{\chi_r^2\left(\omega^{(a)}\right)<\frac{c_0^{(a)}-2nI^a\left(\boldsymbol{\tau},\tilde{\mathbf{p}}\right)+\Lambda^{(a)}\omega^{(a)}}{\Lambda^{(a)}}\right\}. \qquad (4.13)$$

4.7. The NT Approximation by Sekiya *et al.* (1999)

Sekiya *et al.* (1999) proposed the NT approximation, which is a normal approximation based on the normalizing transformation of $\mathbf{R}^a$.

Let $H_1 : \pi = \tau \quad (\tau \neq \mathbf{p})$,

$$A_j^{(a)} = \begin{cases} \dfrac{p_j^a}{a} & (a \neq 0) \\ \log p_j^a & (a = 0) \end{cases},$$

$$B_j^{(a)} = \frac{p_j^a}{\tau_j},$$

$M_{jl}^{(2)} = \delta_{jl}\tau_j - \tau_j\tau_l$, with the δ values defined in Sekiya *et at.* (1999) (in terms of 1's and 0's),

$M_{jlm}^{(3)} = \delta_{jl}\delta_{jm}\tau_j - \left(\delta_{jl}\tau_j\tau_m + \delta_{jm}\tau_j\tau_l + \delta_{lm}\tau_j\tau_l\right) + 2\tau_j\tau_l\tau_m$,

$\upsilon^{(a)} = I^{(a)}\left(\boldsymbol{\tau},\tilde{\mathbf{p}}\right)$,

$$\upsilon_1^{(a)} = \frac{1}{2}\sum_{j=1}^{k} B_j^{(a)} M_{jj}^{(2)},$$

$$\upsilon_2^{(a)} = \sum_{j=1}^{k}\sum_{l=1}^{k} A_j^{(a)} A_l^{(a)} M_{jl}^{(2)},$$

$$\upsilon_3^{(a)} = \sum_{j=1}^{k}\sum_{l=1}^{k}\sum_{m=1}^{k}\left(A_j^{(a)} A_l^{(a)} A_m^{(a)} M_{jlm}^{(3)} + 3A_j^{(a)} A_l^{(a)} B_m^{(a)} M_{jm}^{(2)} M_{lm}^{(2)}\right),$$

$$\Gamma^{(a)} = -\frac{\upsilon^{(a)}\upsilon_3^{(a)}}{3\left(\upsilon_2^{(a)}\right)^2} \text{ and}$$

$$\varphi_a(x)=\begin{cases}\dfrac{\upsilon^{(a)}}{\Gamma^{(a)}+1}\left(\left(\dfrac{x}{\upsilon^{(a)}}\right)^{\Gamma^{(a)}+1}-1\right) & \left(\Gamma^{(a)}\neq-1\right)\\ \upsilon^{(a)}\log\dfrac{x}{\upsilon^{(a)}} & \left(\Gamma^{(a)}=-1\right).\end{cases}$$

Then the NT Approximation is represented as

$$1-\phi\left(\sqrt{\frac{n}{\upsilon_2^{(a)}}}\left\{\varphi_a\left(\frac{c_0^{(a)}}{2n}\right)-\frac{1}{n}\left(\upsilon_1^{(a)}-\frac{\upsilon_3^{(a)}}{6\upsilon_2^{(a)}}\right)\right\}\right) \tag{4.14}$$

where $\phi(\cdot)$ denotes the standard normal distribution function.

If we take $\tau_j = p_j+\frac{c_j}{\sqrt{n}}$, $(j=1,\dots,k)$, then (4.14) represents the NT approximation to the power of $\mathbf{R}^a$ against $H_{1,n}$.

CHAPTER 5

SIMULATION STUDIES

5.1 Introduction

Read & Cressie (1988:76) restricted themselves to the equiprobable null model:

$$H_0 : \boldsymbol{\pi} = 1/k \tag{5.1}$$

with k the number of cells, and the alternative hypothesis where 1 of k probabilities is perturbed and the rest are adjusted so that they still sum to 1:

$$H_1 : \pi_i = \begin{cases} \dfrac{1-\delta/(k-1)}{k} & i = 1,\ldots,k-1 \\ \dfrac{(1+\delta)}{k} & i = k \end{cases} \tag{5.2}$$

where $-1 \le \delta \le k-1$. They then determined exact power functions using (5.1) and (5.2) for α = 0.05; the sample size n = 20 and k = 5. H_1 is a special case of $H_1 : \pi_j = p_j + c_j/\sqrt{n}$, j = 1, 2, …, k, which was referred to in (4.2).

The main objective of this chapter is to conduct simulation studies to compare the bootstrap approximation of power to the AE approximation of Taneichi *et al.* (2002) (which was discussed in Chapter 4) as well as with the exact power values of Read & Cressie (1988:78). Taneichi *et al.* (2002:352) found that the AE approximation of power performed better than the A^a, B^a and NT approximation of power mentioned in §4.4 and §4.6. Therefore, only the AE approximation was considered in the present comparative study, which is discussed in §5.3. The bootstrap power approximation is fully discussed in §5.2. Additional comparisons of interest are performed in the rest of this chapter. For example, since very limited exact results are available for evaluating the first objective of this chapter, a wider Monte Carlo study is conducted for general comparative purposes. Furthermore, the performance and appropriateness of the bootstrap critical values for the power divergence statistic family are to be compared to that of the traditional chi-square critical values used in literature, especially for small samples where asymptotic theory is not applicable. These estimates are discussed in §5.3. A summary of the conclusions of this study follows in §5.5.

The bootstrap approximation of power, the AE approximation and the exact power values obtained by Read & Cressie (1988:78) are compared for the following cases regarding the hypothesis defined in (5.1):

(a) For the alternative hypothesis (5.2), for $\delta = 1.5$ and 0.5, for $n = 20$, 30, 50 and 100, and $k = 3$, 4 and 5. Only exact values for the cases $\delta = 1.5$ and 0.5, $n = 20$ and $k = 5$ are available for comparison.

(b) For the alternative hypothesis (5.2), for $\delta = 3k/4$, k, $5k/4$ for $n = 20$, 30, 50 and 100 and $k = 3$, 4 and 5, as was done by Taneichi *et al.* (2002:348).

(c) Also for various values of c_j, $j = 1, 2, \ldots, k$, under the alternative hypothesis $H_{1,n} : \pi_j = p_j + c_j/\sqrt{n}$, simulation studies were performed in order to derive the general behaviour patterns of the power approximations concerned.

The c_j values were chosen so that

$$\delta = \sum_{j=1}^{k} \frac{c_j^2}{p_j} \quad \text{and} \quad \sum_{j=1}^{k} c_j = 0 \quad \text{for all } j = 1, 2, \ldots, k. \tag{5.3}$$

Attention was also given to the performance of traditional chi-square values for small samples, and deductions which can be made from the results (see §5.4 for discussions).

1000 Monte Carlo repetitions and 2000 bootstrap replications were chosen for this study, i.e. $MC = 2000$ and $B = 1000$ in §5.2 below. These studies were performed using the Microsoft Fortran Power Station 4.0 and the MSIMSL library on Pentium iv personal computers.

5.2 The bootstrap power approximation

The application of the bootstrap procedure will now be discussed under the following points of interest:

5.2.1 Bootstrap critical values

In general, the bootstrap estimate of power is defined as

$$P_{H_1}\left(T_\theta^\lambda(\mathbf{X}_N) \geq C_B^\lambda(\mathbf{X}_N, \alpha)\right),$$

where $T_\theta^\lambda(\mathbf{X}_N)$ denotes the test statistic of interest and $C_B^\lambda(\mathbf{X}_N,\alpha)$ is the bootstrap approximation of the exact critical value $C_E^\lambda(\mathbf{X}_N,\alpha)$, which is usually unknown.

A parametric bootstrap approximation for C_E^λ, say $C_B^\lambda(\mathbf{X}_N,\alpha)$, is defined by

$$P^*\left(T_\theta^\lambda(\mathbf{X}_N^*) \geq C_B^\lambda(\mathbf{X}_N,\alpha)\right) = \alpha$$

where $\mathbf{X}_N^* = (X_1^*, X_2^*, ..., X_N^*)$ is a bootstrap sample generated parametrically from a $\text{Mult}_k(N, \hat{\pi}_{0i}, i = 1,...,k)$ distribution. $C_B^\lambda(\mathbf{X}_N,\alpha)$ is then determined by using the following algorithm:

Step 1: A bootstrap sample $\mathbf{X}_N^* = (X_1^*, X_2^*, ..., X_N^*)$ is generated from a $\text{Mult}_k(N, \hat{\pi}_{0i}, i = 1,...,k)$ distribution and the statistic $T_\theta^\lambda(\mathbf{X}_N^*) \equiv T_\theta^{\lambda*}$ is calculated from here.

Step 2: Repeat step 1, a large number, B, times to obtain $T_\theta^{\lambda*}(1)$, $T_\theta^{\lambda*}(2)$, . . . , $T_\theta^{\lambda*}(B)$.

Step 3: These statistics are then ordered to obtain $T_\theta^{\lambda*}{}_{(1)} \leq T_\theta^{\lambda*}{}_{(2)} \leq ... \leq T_\theta^{\lambda*}{}_{(B)}$ where $T_\theta^{\lambda*}{}_{(1)}$, $T_\theta^{\lambda*}{}_{(2)}$, . . . , $T_\theta^{\lambda*}{}_{(B)}$ are the ordered statistics of $T_\theta^{\lambda*}(1)$, $T_\theta^{\lambda*}(2)$, . . . , $T_\theta^{\lambda*}(B)$.

$C_B^\lambda(\mathbf{X}_N,\alpha)$ is then approximated by $T_\theta^{\lambda*}{}_{[B(1-\alpha)]}$ where $[z]$ denotes the largest integer less than or equal to z.

5.2.2 Bootstrap approximation of power

Monte Carlo comparisons are used to approximate the power of the test. The test is conducted as follows, when testing the null hypothesis $H_0 : \pi_j = p_j$ = $1/k$, versus the alternative hypothesis $H_{1,n} : \pi_j = p_j + \frac{c_j}{\sqrt{n}}$, $j = 1,...,k$, $\sum_{j=1}^{k} c_j = 0$ and the number of cells $k = 5$: H_0 is rejected if the test statistic is larger than the critical value and H_0 is not rejected if the test statistic is smaller than the critical value.

This procedure involves the following steps:

Step 1: Generate data under **null hypothesis**, say $\mathbf{X}_N = (X_1, X_2, ..., X_N)$, and calculate bootstrap critical values $C_B^{\lambda}(\mathbf{X}_N, \alpha)$, for all λ values as is described in §5.2.1.

Step 2: Data is then generated under the **alternative hypothesis** from a $\text{Mult}_k(n, \pi_1, \pi_2, ..., \pi_k)$ distribution and this sample is denoted by $\mathbf{X}_N^{Alt}(1) = (X_1, X_2, ..., X_N)$.

Step 3: From this sample, $\mathbf{X}_N^{Alt}(1)$, the power divergence test statistic is calculated for various λ values. (For this study λ values are taken from -2.0 to 2.0 in steps of 0.2). Since the power divergence statistic is not evaluated at $\lambda = 0$ and $\lambda = -1$, as mentioned in remark 2.1, we used:

for $\lambda = 0$: $$G^2 = -2\sum_{i=1}^{5} X_i \log \frac{\hat{m}_i}{X_i},$$

for $\lambda = -1$: $$GM^2 = 2\sum_{i=1}^{5} \hat{m}_i \log \frac{\hat{m}_i}{X_i}$$

and for all other λ: $$2nI^{\lambda} = \frac{2}{\lambda(\lambda+1)} \sum_{i=1}^{5} X_i \left[\left(\frac{X_i}{\hat{m}_i} \right)^{\lambda} - 1 \right],$$

where $\hat{m}_i = n\pi_i$. We denote in general the test statistic by $T_{Alt}^{\lambda}(\mathbf{X}_N^{Alt}(1)) \equiv T_{Alt}^{\lambda}(1)$.

Step 4: The above two steps are repeated a large number of times, say *MC* times, to obtain $T_{Alt}^{\lambda}(1)$, $T_{Alt}^{\lambda}(2)$, ..., $T_{Alt}^{\lambda}(MC)$, for all values of λ.

Step 5: The bootstrap power is then determined by

$$\frac{1}{MC}\sum_{m=1}^{MC} I\left(T_{Alt}^{\lambda}(m) > C_B^{\lambda}(\mathbf{X}_N, \alpha)\right), \text{ for all } \lambda \text{ values.}$$

5.3 Results

5.3.1 Trustworthiness of the chi-square critical values

Extractions from the full tables (Table 11 and Table 12 on pages 90-93) in Appendix B are shown below. For Table 5.1, parameters were chosen as follows: δ = 1.5, n =

20, α = 0.05, k = 5 and for Table 5.2 the values were chosen as δ = 0.5, n = 20, α = 0.05, k = 5. These tables show the comparison of the bootstrap approximation and the AE approximation of power with the exact power performance obtained by Read & Cressie (1988:78). The tables also show the power performance when traditional chi-square critical values were used in comparison when bootstrap critical values were used instead.

Table 5.1 Comparison of exact power with the Bootstrap and AE approximation of power, for n = 20, α = 0.05, k = 5 and δ = 1.5, under the hypothesis (5.1) and (5.2)

λ	-2	-1	0	1	2
Bootstrap approximation i.e. with the bootstrap critical values	0.259	0.259	0.622	0.713	0.707
AE approximation	-0.0001	-0.0001	0.0464	0.0817	0.0361
Exact values	0.2253	0.2253	0.61	0.6997	0.7306
Chi-square critical values	0.792	0.693	0.675	0.713	0.765

Table 5.2 Comparison of exact power with the Bootstrap and AE approximation of power, for n = 20, α = 0.05, k = 5 and δ = 0.5, under the hypothesis (5.1) and (5.2)

λ	-2	-1	0	1	2
Bootstrap approximation i.e. with the bootstrap critical values	0.077	0.072	0.111	0.121	0.103
AE approximation	0	0	0.0488	0.0857	0.0425
Exact values	0.0742	0.0742	0.1073	0.1228	0.1278
Chi-square critical values	0.396	0.173	0.138	0.114	0.132

Remark 5.3.1

- For δ = 1.5 and δ = 0.5 the bootstrap approximation gives remarkably close values to the exact power values for the range of λ values under comparison.
- The AE approximation fails to reject for lower λ values and provides much lower power values than the exact values and are therefore too conservative.
- When using chi-square critical values instead of the bootstrap values, the power estimates are much larger than the exact power values, and are not to be trusted. In this regard, see §5.4.
- From tables 5.1 and 5.2 above, it is clear that the chi-square critical values are not trustworthy for small samples, since the power approximations derived from it are too large in comparison with the exact power of the test.

- The bootstrap critical values, on the other hand, encourage confidence, since power approximations obtained from it result in values near the exact power calculations of Read and Cressie (1988:78).

5.3.2 Power comparisons between the AE approximation and the bootstrap approximation

Further power approximations between the bootstrap and the AE approximation methods, follow below. Also the behaviour of traditional chi-square critical values is studied throughout. The following parameter combinations are used, for various λ values: $k = 3$, 4 and 5, δ performed for 1.5, 0.5 as well as for values $3k/4$, $5k/4$ and k, as was done by Taneichi *et al.* (2002). Three sizes of tests were conducted throughout, i.e. $\alpha = 0.01, 0.05, 0.10$. The reader is strongly referred to the Appendix B (tables 1 – 16) for a better understanding of the remarks which are to follow below. The discussions are aimed at capturing general features of the procedures discussed in chapter 4.

Remark 5.3.2

a) For $k = 3$ (refer to Appendix B, tables 1 – 5, pages 70 - 79, rows printed in red):

- The bootstrap method critical power values gives very good power estimates for n = 20, 30, 50 and 100 for all values of α, δ and c_j (j = 1, 2, ..., k) values considered.
- At first glance it appears as if the chi-square critical values provide higher power approximations than the bootstrap method, which uses bootstrap critical values. This may imply that it is better to use chi-square critical values than the bootstrap critical values. This phenomenon is investigated further in §5.4.
- The AE approximation fails to reject H_0 if H_1 is true, especially for small sample sizes for most values of λ. Although the situation improves slightly for larger λ, it is safe to say that this method is too conservative overall. For larger sample sizes the situation improves for all λ.
- Also, for small λ values, negative power values are obtained, which is not allowed.

b) For $k = 4$ (refer to Appendix B, tables 6 – 10, pages 80 - 89 , rows printed in red):

- The AE approximation behaves unstable: it either fails to reject, or it rejects too easily, i.e. it results in un-expected high power values, even for small sample sizes, $n = 20, 30$, $\alpha = 0.05$, $\alpha = 0.10$, $\delta = 1.5, 0.5$ and the c_j ($j = 1, 2, \ldots, k$) values obtained from (5.2). This phenomenon does not repeat itself with the rest of the AE approximation's behaviour throughout this study. In §5.4 a closer look at the critical values does provide some explanation.

c) For $k = 5$ (refer to Appendix B, tables 11 – 15, pages 90 - 99 , rows printed in red):

- The AE power estimates seem to vary widely over the range of λ values under review. The estimates are very low for low λ values but increase substantially as λ increases. This behaviour is unstable, as was observed in the previous two cases.
- The bootstrap approximation produces stable power estimates for $k = 5$.
- The chi-square critical values appear to provide much higher power approximations than the bootstrap method, which will be discussed later.

d) It is expected that it is more difficult to reject the null hypothesis for c_j ($j = 1, \ldots, k$) values close to zero. This was noticed to be true throughout the studies, and well reflected by the bootstrap method.

e) From the above, it is evident that the behaviour of the AE approximation is inconsistent throughout the studies. The bootstrap approximation provides stable behaviour. The chi-square critical values appear to provide high power values in general.

f) The number of cells (i.e. the value of k) has no influence on the general results. Instead the values of c_j ($j = 1, 2, \ldots, k$) influence power.

5.4 Results when using the chi-square critical values (Table 16, Appendix B, pages 100-101)

To study the apparent successful behaviour of the chi-square critical values, simulation studies were conducted with $c_j = 0$ for all $j = 1, 2, \ldots, k$. This implies that $H_0 = H_1$ (see 4.2), and

$$\begin{aligned}\alpha &= P(\text{Type 1 error}) \\ &= P_{H_0}(\text{Reject } H_0), \\ &= P_{H_1}(\text{Reject } H_0) \\ &= \text{Power of the test.}\end{aligned}$$

The power of the test in this case should be close to α, i.e. 0.01 or 0.05 or 0.10. Table 5.3, an extraction from Table 16 in Appendix B (pages 100 - 101), shows interesting results.

Table 5.3 Comparison of exact power with the Bootstrap and AE approximation of power, for $\alpha = 0.05$, $k = 5$, $\delta = 0.2$ and $c_j = 0$ for all $j = 1, 2, \ldots, k$

λ	-2	-1	0	1	2
			n = 20		
Bootstrap approximation i.e. with the bootstrap critical values	0.057	0.049	0.034	0.041	0.033
AE approximation	0	0	0.0278	0.0464	0.0231
Chi-square critical values	0.299	0.099	0.07	0.042	0.059
			n = 30		
Bootstrap approximation i.e. with the bootstrap critical values	0.058	0.058	0.068	0.073	0.066
AE approximation	0.0046	0.0194	0.0606	0.0698	0.0536
Chi-square critical values	0.177	0.102	0.066	0.06	0.079
			n = 50		
Bootstrap approximation i.e. with the bootstrap critical values	0.043	0.047	0.05	0.036	0.036
AE approximation	0.0599	0.0888	0.111	0.1138	0.1004
Chi-square critical values	0.113	0.07	0.051	0.035	0.045
			n = 100		
Bootstrap approximation	0.061	0.056	0.055	0.049	0.048
AE approximation	0.0843	0.1006	0.1115	0.1143	0.1093
Chi-square critical values	0.077	0.063	0.046	0.039	0.047

- For small values of n, it is clear that using chi-square critical values, will provide tests which result in very high power (i.e. 29.9%, 17.7%) for $n = 20$ and 30, instead of 5%. When n increases, the situation improves because the PDF's have asymptotic chi-square distributions. By using chi-square critical values for small n, the size of the tests **cannot** be compared with tests of size $\alpha = 0.05$.

- Therefore, throughout the simulation studies (Tables 1 – 16), the use of chi-square critical values provided tests, **which were never to be compared with the bootstrap or AE approximations of power. Using chi-square critical values provide tests at a different size, i.e. $\alpha > 0.01, 0.05$ and 0.10.**
- The power of the tests obtained by using chi-square critical values was therefore **not** to be compared with the bootstrap and AE approximation methods.
- The apparent high power values obtained by using chi-square critical values were not applicable in the comparative studies.
- The AE approximation behaves in an unstable way throughout.
- For $n = 100$, the chi-square critical values gave power values near 0.05.
- From Table 5.3 it can clearly be seen that the bootstrap approximation gives values close to the required α value of 0.05 for $k = 5$, $\delta = 0.2$. The bootstrap method can be trusted and applied with confidence. Although the chi-square power values are large, it is clear that the power values obtained are not at the required level and cannot be trusted.

5.5 Conclusions

The previous chapters and paragraphs can now be summarised and concluded as follows:

1. The AE approximation and the other approximations by Drost *et al.* (1989) and Sekiya (1999) are complicated procedures. The bootstrap approximation is relatively easy to implement if complex computer codes are available. In this study we developed our own Fortran code, (see Appendix A). Standard bootstrap codes exist in most computer packages, for example S-Plus and R (which is a free cloned version of S-Plus), of which the basic package can easily be downloaded and installed from the website http://cran.t-project.org. The bootstrap library of Davidson and Hinkley, obtainable from http://dmawww.epfl.ch/dvison.mosaic is also very useful. SAS, and Statistica too have developed standard bootstrap functions as many other user-friendly packages.

2. The bootstrap critical values are to be trusted in studies of this nature, while traditional chi-square methods are not providing the desired level of tests in small samples. Even for $n = 100$, the bootstrap critical values are more trustworthy than the chi-square critical values, for most λ values.
3. The bootstrap approximation to power was remarkably close to the exact power as it is given by Read and Cressie (1988:78).
4. Due to points 1 – 3 above, it is recommended that for similar hypotheses, in goodness-of-fit studies regarding discrete data-analysis, bootstrap critical values should be used instead of traditional chi-square critical values.
5. Furthermore, statistical practitioners can be assured that decisions based on point 4 will be based on tests with high power.

APPENDIX A:

FORTRAN CODE

A1: Introduction

The Fortran routines used in this study are included here. The main program is called **PowerTest**, (pages 55 to 64), which is used to test the coverage probability of the bootstrap approximation and the AE approximation as well as the chi-square test. It calls a subroutine named **CriticalValues**, (page 65 to 67), which calculate bootstrap critical values for the test. The results are written to a file called "output.txt", the critical values are written to a file called "criticalval.txt" and a file used for the subroutine parameters is called "input.txt". The test statistics from the subroutine are written to a file called "stats.txt".

A2: Powertest

```
Program Powertest
use msimsl
USE PORTLIB
implicit none
!&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&
!Power comparison test for various lamda's
!&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&

INTEGER   mcK, mcLDIR
INTEGER   mcI,k, md(9), mcJ, mcN_vek(5),ml,mcNR,mcN,mcm,mcB, mbp
INTEGER,ALLOCATABLE::mcIR(:),mcMAT(:)
REAL,ALLOCATABLE::mcP(:),mcKolsom(:),mcpnul(:)
INTEGER mcplek,mcMC_aantal,mcMc,mcl
REAL mclambda(21),mclambda_i
REAL mcXIJ,mcmeankritiek(3,21),mcstddevkritiek(3,21)
REAL,ALLOCATABLE::mcteststats(:,:),mcP_druk(:)
REAL,ALLOCATABLE::mckritiek1(:),mckritiek5(:),mckritiek10(:),mckritiekk(:)
REAL,ALLOCATABLE::mccover1(:),mccover5(:),mccover10(:), somm(:)
REAL,ALLOCATABLE::mccover_e1(:),mccover_e5(:),mccover_e10(:)
REAL,ALLOCATABLE::power1(:),power5(:),power10(:)
REAL,ALLOCATABLE::chsq1(:),chsq5(:),chsq10(:)
REAL mcalpha(3),chi_s(3),mcc
REAL s32, s11,s43,s22, s01,s21, som32,som11, som43,som01, som21,som22
REAL, ALLOCATABLE :: q(:,:),mccc(:),r(:,:)
REAL, ALLOCATABLE :: gr1(:), gr5(:), gr10(:)
REAL df,mcq,alam,scom
REAL, ALLOCATABLE::somb1(:), somc1(:), somb5(:), somc5(:),somb10(:), somc10(:)
INTEGER mckorrek
REAL mcratio,mcr,som
REAL mcratio1,mcratio11
REAL(8) elapsed_time

!The constant value
mcc=0.000001

open (3,file="c:\Temp\output.doc",status="unknown",access="sequential")

!assigning lamda's, sample sizes and alpha values
mcplek=0
do mclambda_i = -2.0,2.2,0.2
        mcplek=mcplek+1
        mclambda(mcplek)=mclambda_i
enddo

mcN_vek(1)=20
mcN_vek(2)=30
mcN_vek(3)=40
mcN_vek(4)=50
mcN_vek(5)=100

!Screen counter
md(1)=100      md(2)=200
md(3)=300      md(4)=400
md(5)=500      md(6)=600
md(7)=700      md(8)=800
md(9)=900
```

```
mcalpha(1)=0.01
mcalpha(2)=0.05
mcalpha(3)=0.1

!allocating matrices sizes
mcMC_aantal = 1000
mcB=2000

mcK=5                                   !no. of cells
df=mck-1.0d0                          !degrees of freedom
allocate(mckritiek1(21))
allocate(mckritiek5(21))
allocate(mckritiek10(21))
allocate(mckritiekk(63))

allocate(mccover1(21))
allocate(mccover5(21))
allocate(mccover10(21))
allocate(mccover_e1(21))
allocate(mccover_e5(21))
allocate(mccover_e10(21))
allocate(chsq1(21))
allocate(chsq5(21))
allocate(chsq10(21))
allocate(power1(21))
allocate(power5(21))
allocate(power10(21))
allocate(gr1(21))
allocate(gr5(21))
allocate(gr10(21))
allocate(mcteststats(mcMC_aantal,21))

!allocating proportions
allocate(mcP(mcK))
allocate(mcpnul(mcK))
allocate(mcP_druk(mcK))
allocate(mcIR(mcK))

!Mat is a matrix with
allocate(mcMat(mcK))
allocate(mcKolsom(mcK))
allocate(somm(21))
allocate(q(4,21))
allocate(r(7,21))
allocate (mccc(mcK))
allocate(somb1(21))
allocate (somc1(21))
allocate(somb5(21))
allocate (somc5(21))
allocate(somb10(21))
allocate (somc10(21))

!alternative hypothesis
mcP(1) = 0.2
mcP(2) = 0.2
mcP(3) = 0.2
mcP(4) = 0.2
mcP(5) = 0.2
```

```
!cj values under the alternative hypothesis
mccc(1)= 0
mccc(2)= 0
mccc(3)= 0
mccc(4)= 0
mccc(5)= 0

write(3,*)" "
write(3,*)"Monte Carlo repetitons = ",mcmc_aantal
write(3,*)"Bootstrap repetitions  = ",mcB
write(3,*)""
write(3,*)"The proportions under the alternative hypothesis: "
mcp_druk=mcp
write(3,'(F8.4,F8.4,F8.4,F8.4,F8.4,F8.4)')mcP_druk
write(3,*)" "
write(3,*)"The lambda values: "
write(3,"(F8.2,F8.2,F8.2,F8.2,F8.2,F8.2,F8.2,F8.2,F8.2,F8.2,F8.2,F8.2,F8.2,F8.2,F8.2,F8.2,F8.2,F8.2,F
8.2,F8.2,F8.2,A7)")mclambda,"min"

write(3,*)" "
write(3,*)"the cj values are: "
write(3,"(F8.4,F8.4,F8.4,F8.4,F8.4,A7)")mccc
write(3,*)""
write(3,*)"delta = 0.2"


do mcl=1,1
!Over the sample sizes

 mcN=mcN_vek(mcl)
 mcteststats=0.0
 call rnset(0)

 open (1,file="c:\Temp\input.txt",status="unknown",access="sequential")
 write(1,*)mcB
 write(1,*)mcN
 write(1,*)mcK
 close(1)

 !taneichi et.al.  pre-calculations
  scom=0.0
  som32=0.0
  som11=0.0
  som43=0.0
  som01=0.0
  som21=0.0
  som22=0.0

        do mci=1,mck
                mcpnul(mci)= 1.0d0/mck
        enddo


  do mcj=1,mck
                som32=som32+(((mccc(mcj))**3)/(mcpnul(mcj)**2))
                som11=som11+(((mccc(mcj))**1)/(mcpnul(mcj)**1))
                som43=som43+(((mccc(mcj))**4)/(mcpnul(mcj)**3))
                som01=som01+(((mccc(mcj))**0)/(mcpnul(mcj)**1))
                som21=som21+(((mccc(mcj))**2)/(mcpnul(mcj)**1))
```

```
            som22=som22+(((mccc(mcj))**2)/(mcpnul(mcj)**2))
            scom=scom+((mccc(mcj)**2)/mcpnul(mcj))
enddo

s32=som32
s11=som11
s43=som43
s01=som01
s21=som21
s22=som22
alam=scom

    do ml=1,21                                  !over all lambda's
            do mcq = 1,4
                    if (mcq .eq. 1) then
                      q(1,ml)=2*s32
                    endif
                    if (mcq.eq.2) then
                      q(2,ml)=-3*(s32 + ml*s11)
                    endif
                    if (mcq.eq.3) then
                            q(3,ml)=(1-ml)*s32 + 3*ml*s11
                    endif
                    if (mcq.eq.4) then
                            q(4,ml)=ml*s32
                    endif
            enddo
    enddo

do ml=1,21
    do mcr=1,7
            if (mcr .eq. 1) then
                    r(1,ml)=4*(s32**2)-18*s43-6*(s01-1.0)
            endif
            if (mcr .eq. 2) then
                    r(2,ml)=-12*(s32**2) -9*(s21**2)-12*ml*s11*s32+ 33*s43+18*ml*s22-
                    18*ml*mck*s21+3*((ml+2)*(2*ml+1)*s01-3*ml*mck*(ml*mck+2)+(ml-
                    1)*(ml+2))
            endif
            if (mcr .eq. 3) then
                    r(3,ml)=(13-4*ml)*(s32**2)-18*(ml-
                    1)*(s21**2)+9*(ml**2)*(s11**2)+30*ml*s11*s32+18*(ml-
                    1)*s43+18*ml*(ml-1)*s22-18*ml*((ml-2)*mck+ml-1)*s21-
                    3*ml*((7*ml+5)*s01-6*mck*(ml*mck+ml+1)+5*ml+1)
            endif
            if (mcr .eq. 4) then
                    r(4,ml)=2*(5+ml-3)*(s32**2)-9*((ml**2-4*ml+1)*(s21**2)) -
                    18*(ml**2)*(s11**2) +6*ml*(ml-4)*s11*s32 +3*(2*(ml**2)-
                    9*ml+1)*s43-54*(ml**2)*s22+18*ml*((2*ml-1)*mck+3*ml-
                    1)*s21+3*(ml**2)*(5*s01-3*(mck**2)-6*mck+4)
            endif
            if (mcr .eq. 5) then
                    r(5,ml)=((ml**2)-8*ml+1)*(s32**2)+18*ml*(ml-
                    1)*(s21**2)+9*(ml**2)*(s11**2)-6*ml*(2*ml-1)*s11*s32-3*ml*(5*ml-
                    3)*s43+36*(ml**2)*s22-18*(ml**2)*(mck+2)*s21
            endif
            if (mcr .eq. 6) then
                    r(6,ml)=-2*ml*(ml-1)*(s32**2)-
                    9*(ml**2)*(s21**2)+6*(ml**2)*s11*s32+9*(ml**2)*s43
            endif
```

```
			if (mcr .eq. 7) then
				r(7,ml)=(ml**2)*(s32**2)
			endif
		enddo
	enddo


!Call a subroutine, bootstrap loop with the data
 call CriticalValues

!store the critical values in the matrix for later comparison with teststat
open (2,file="c:\Temp\criticalval.txt",status="old",access="sequential")
do mci=1,63
	read(2,*)mckritiekk(mci)
enddo
close(2)

do mci=1,21
	!sort this matrix for the three alpha values
	mckritiek1(mci)=mckritiekk(mci)
	mckritiek5(mci)=mckritiekk(mci+21)
	mckritiek10(mci)=mckritiekk(mci+42)

	chsq1(mci)=mckritiek1(mci)
	chsq5(mci)=mckritiek5(mci)
	chsq10(mci)=mckritiek10(mci)
enddo

somb1=0.0
somc1=0.0
power1=0.0
somb5=0.0
somc5=0.0
power5=0.0
somb10=0.0
somc10=0.0
power10=0.0
gr1=0.0
gr5=0.0
gr10=0.0

!Calculating AE approximation eqn (4.1) pg. 347 taneich et.al.

do mbp=1,3				!alpha = 1%
 if (mbp.eq.1)then

	do ml=1,21
		gr1(ml)=CSNDF(CHSQ1(ml), DF, ALAM)

		do mci=1,4
			somb1(ml)=somb1(ml)+q(mci,mcl)*CSNDF(CHSQ1(ml), DF+2*mci,
			ALAM)
		enddo
		somb1(ml)=somb1(ml)/(6*sqrt(real(mcn)))

		do mci=1,7
			somc1(ml)=somc1(ml)+r(mci,mcl)*CSNDF(CHSQ1(ml), DF+2*mci,
			ALAM)
		enddo
```

```
            somc1(ml)=somc1(ml)/(72*mcn)
            power1(ml)=1.0d0-(gr1(ml)+somb1(ml)+somc1(ml))
        enddo

  endif

  if (mbp.eq.2)then                    !alpha = 5%

        do ml=1,21
            gr5(ml)=CSNDF(CHSQ5(ml), DF, ALAM)

            do mci=1,4
            somb5(ml)=somb5(ml)+q(mci,mcl)*CSNDF(CHSQ5(ml), DF+2*mci, ALAM)
            enddo
            somb5(ml)=somb5(ml)/(6*sqrt(real(mcn)))

            do mci=1,7
                somc5(ml)=somc5(ml)+r(mci,mcl)*CSNDF(CHSQ5(ml), DF+2*mci,
                ALAM)
            enddo
            somc5(ml)=somc5(ml)/(72*mcn)
            power5(ml)=1.0d0-(gr5(ml)+somb5(ml)+somc5(ml))
        enddo

  endif

  if (mbp.eq.3)then                    !alpha = 10%
        do ml=1,21
            gr10(ml)=CSNDF(CHSQ10(ml), DF, ALAM)
            do mci=1,3
                somb10(ml)=somb10(ml)+q(mci,mcl)*CSNDF(CHSQ10(ml), DF+2*mci,
                ALAM)
            enddo
            somb10(ml)=somb10(ml)/(6*sqrt(real(mcn)))

            do mci=1,6
                somc10(ml)=somc10(ml)+r(mci,mcl)*CSNDF(CHSQ10(ml), DF+2*mci,
                ALAM)
            enddo
            somc10(ml)=somc10(ml)/(72*mcn)
            power10(ml)=1.0d0-(gr10(ml)+somb10(ml)+somc10(ml))
        enddo
  endif

enddo


!The MC study:
do mcmc=1,mcMC_aantal

        !print MC loop number to screen
        do k=1,9
        if (mcmc .eq. md(k)) then
                print*, mcmc
        endif
        enddo

        mcNR = 1
        mcLDIR = 1
        CALL RNMTN (mcNR, mcN, mcK, mcP, mcIR, mcLDIR)
```

```
!check that no negative values are generated
mckorrek = 2
do while (mckorrek .ne.1)
        do mci=1,mcK
                if (mcIR(mci) .lt. 0.0) then
                 mckorrek=2
                 exit
                else
                 mckorrek=1
                endif
        enddo
        if (mckorrek .eq. 2) then
           CALL RNMTN (mcNR, mcN, mcK, mcP, mcIR, mcLDIR)
        endif
enddo

!assigning the output to the matrix
mcmat = mcIR

!calculate the statistics for the sample
do mcm=1,21
        if (mclambda(mcm) .ge. -0.0001 .and. mclambda(mcm) .le. 0.0001) then

        !teststat for mclambda=0  = G square
                som = 0.0
                do mcj=1,mcK
                   mcXIJ=real(mcmat(mcj)+mcc)
                        mcratio1=(mcXIJ)/(mcN*real(mcPnul(mcj)+mcc))
                        mcratio1=log(mcratio1)
                        mcratio11=real(mcratio1)
                        som = som + (mcratio1*mcXIJ)
                enddo !j
                mcteststats(mcmc,mcm)=som*2.0

        elseif (mclambda(mcm) .ge. -1.0001 .and. mclambda(mcm) .le. -0.9999) then

        !teststat for mclambda=-1  = GM square
                som=0.0
                do mcj=1,mcK
                        mcXIJ=real(mcmat(mcj)+mcc)
                        mcratio=(mcN*real(mcPnul(mcj)+mcc))
                        mcratio1=mcratio/mcXIJ
                        mcratio1=log(mcratio1)
                        mcratio11=real(mcratio1)
                        som = som + mcratio11*mcratio
                enddo !j
                mcteststats(mcmc,mcm)=som*2.0

        else
        !all other lambda's
                somm(mcm) = 0.0
                do mcj=1,mcK
                mcXIJ=real(mcmat(mcj)+mcc )
                        !Hier bereken ek nou die teststat
                        mcratio= (mcXIJ)/real(mcN*(mcPnul(mcj)))
                        mcratio11=real(mcratio)
                        somm(mcm)=somm(mcm) +
                        (mcXIJ)*((mcratio11**mclambda(mcm))-1.0)
                enddo !j
```

```
                mcteststats(mcmc,mcm)=somm(mcm)*2.0/(mclambda(mcm)*
                (mclambda(mcm)+1.0))
                endif
        enddo   !the m's, different teststats


enddo !The MC loop

!calculating coverage probabilities
mcmeankritiek=0.0
mcstddevkritiek=0.0
mccover1=0.0
mccover5=0.0
mccover10=0.0

do mcmc=1,mcMC_aantal
        do mcm = 1,21
                if (mcteststats(mcmc,mcm) .ge. mckritiek1(mcm)) then
                        mccover1(mcm)=mccover1(mcm)+1
                endif

                if (mcteststats(mcmc,mcm) .ge. mckritiek5(mcm)) then
                        mccover5(mcm)=mccover5(mcm)+1
                endif
                if (mcteststats(mcmc,mcm) .ge. mckritiek10(mcm)) then
                        mccover10(mcm)=mccover10(mcm)+1
                endif
        enddo
enddo

!calculating tradisional coverage
mccover_e1=0.0
mccover_e5=0.0
mccover_e10=0.0
chi_s(1)=CHIIN(0.99, real(mcK-1.0))
chi_s(2)=CHIIN(0.95, real(mcK-1.0))
chi_s(3)=CHIIN(0.9, real(mcK-1.0) )

do mcmc=1,mcMC_aantal
        do mcm = 1,21
                if (mcteststats(mcmc,mcm) .ge. chi_s(1)) then
                        mccover_e1(mcm)=mccover_e1(mcm)+1
                endif
                if (mcteststats(mcmc,mcm) .ge. chi_s(2)) then
                        mccover_e5(mcm)=mccover_e5(mcm)+1
                endif
                if (mcteststats(mcmc,mcm) .ge. chi_s(3)) then
                        mccover_e10(mcm)=mccover_e10(mcm)+1
                endif
        enddo
enddo


mccover1=mccover1/real(mcMC_aantal)
mccover5=mccover5/real(mcMC_aantal)
mccover10=mccover10/real(mcMC_aantal)
mccover_e1=mccover_e1/real(mcMC_aantal)
mccover_e5=mccover_e5/real(mcMC_aantal)
mccover_e10=mccover_e10/real(mcMC_aantal)
```

```
!printing the bootstrap results to file
write(3,*)""
write(3,"(A184)")"########################################################################
##########"
write(3,*)"For Sample Size = ",mcN

write(3,*)" "
write(3,"(A184)")"--------------------------------------------------------------------------------"
write(3,*)"The bootstrap estimated power at alpha = 0.01:"
write(3,"(F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F
8.4,F8.4,F8.4,F8.4)")mccover1(1:21)
write(3,*)""
write(3,*)"The bootstrap estimated power at alpha = 0.05:"
write(3,"(F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F
8.4,F8.4,F8.4,F8.4)")mccover5(1:21)
write(3,*)""
write(3,*)"The bootstrap estimated power at alpha = 0.1:"
write(3,"(F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F
8.4,F8.4,F8.4,F8.4)")mccover10(1:21)
write(3,*)""
write(3,*)""

!The AE results
write(3,"(A184)")"--------------------------------------------------------------------------------"
write(3,*)"The AE approximation of power at alpha = 0.01: "
write(3,"(F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F
8.4,F8.4,F8.4,F8.4)")power1(1:21)
write(3,*)""
write(3,*)"The AE approximation of power at alpha = 0.05:"
write(3,"(F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F
8.4,F8.4,F8.4,F8.4)")power5(1:21)
write(3,*)""
write(3,*)"The AE approximation of power at alpha = 0.1:"
write(3,"(F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F
8.4,F8.4,F8.4,F8.4)")power10(1:21)
write(3,*)" "
write(3,*)" "

!The traditional coverage
write(3,"(A184)")"--------------------------------------------------------------------------------"
write(3,*)"The traditional estimated power at alpha = 0.01 (Chi-square=",chi_s(1),"): "
write(3,"(F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F
8.4,F8.4,F8.4,F8.4)")mccover_e1(1:21)
write(3,*)" "
write(3,*)"The traditional estimated power at alpha = 0.05 (Chi-square=",chi_s(2),"):"
write(3,"(F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F
8.4,F8.4,F8.4,F8.4)")mccover_e5(1:21)
write(3,*)" "
write(3,*)"The traditional estimated power at alpha = 0.1 (Chi-square=",chi_s(3),"):"
write(3,"(F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F
8.4,F8.4,F8.4,F8.4)")mccover_e10(1:21)
write(3,*)" "
write(3,*)" "

!The standard error of the bootstrap coverage
write(3,"(A184)")"--------------------------------------------------------------------------------"
write(3,*)"The bootstrap estimated standard errors for the power at alpha = 0.01: "
```

```
write(3,"(F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F
8.4,F8.4,F8.4,F8.4)")sqrt((mccover1(1:21)*(1.0-mccover1(1:21)))/real(mcmc_aantal))
write(3,*)""
write(3,*)"The bootstrap estimated standard errors for the power at alpha = 0.05:"
write(3,"(F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F
8.4,F8.4,F8.4,F8.4)")sqrt((mccover5(1:21)*(1.0-mccover5(1:21)))/real(mcmc_aantal))
write(3,*)""
write(3,*)"The bootstrap estimated standard errors for the power at alpha = 0.1:"
write(3,"(F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F
8.4,F8.4,F8.4,F8.4)")sqrt((mccover10(1:21)*(1.0-mccover10(1:21)))/real(mcmc_aantal))
write(3,*)" "
write(3,*)" "

!The standard errors of the classical method
write(3,"(A184)")"------------------------------------------------------------------------------"
write(3,*)"The traditional estimated standard errors for the power at alpha = 0.01:"
write(3,"(F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F
8.4,F8.4,F8.4,F8.4)")sqrt((mccover_e1(1:21)*(1.0-mccover_e1(1:21)))/real(mcmc_aantal))
write(3,*)""
write(3,*)"The traditional estimated standard errors for the power at alpha = 0.05:"
write(3,"(F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F
8.4,F8.4,F8.4,F8.4)")sqrt((mccover_e5(1:21)*(1.0-mccover_e5(1:21)))/real(mcmc_aantal))
write(3,*)""
write(3,*)"The traditional estimated standard errors for the power at alpha = 0.1:"
write(3,"(F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F
8.4,F8.4,F8.4,F8.4)")sqrt((mccover_e10(1:21)*(1.0-mccover_e10(1:21)))/real(mcmc_aantal))
write(3,*)" "
write(3,*)" "

write(3,"(A184)")"------------------------------------------------------------------------------"
enddo !Die N

elapsed_time = TIMEF()
write(3,*)" "
write(3,"(A14,F15.2,A7)")"Running time ",elapsed_time/60.0d0," minute"

close(3)
deallocate(mcP)
deallocate(mcP_druk)
deallocate(mcIR)
deallocate(mcteststats)
deallocate(mckritiek1)
deallocate(mckritiek5)
deallocate(mckritiek10)
deallocate(mccover1)
deallocate(mccover5)
deallocate(mccover10)
deallocate(mccover_e1)
deallocate(mccover_e5)
deallocate(mccover_e10)
deallocate(mcpnul)
deallocate(somm)
deallocate(mcMat)
deallocate(mcKolsom)


END
```

A3: CriticalValues

```
subroutine CriticalValues
use msimsl
implicit none
!&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&
!bootstrap critical values
!&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&

INTEGER   K, LDIR
INTEGER   I, J, NR,N,m !IPERM(21)
INTEGER,ALLOCATABLE::IR(:),MAT(:),data_mat(:)
REAL,ALLOCATABLE::mcpnul(:),somm(:),stats(:,:)
INTEGER plek,B_aantal,B,korrek
REAL lambda(21),lambda_i
REAL XIJ,kritieke_wrds(3,21)
REAL,ALLOCATABLE::teststats(:,:),teststat_min(:),teststats_sort(:)
REAL alpha(3),c,ratio,som
REAL ratio1,ratio11

!The constant value
c=0.000001

!assigning the lambdas, sample sizes and alpha values
plek=0
do lambda_i = -2.0,2.2,0.2
        plek=plek+1
        lambda(plek)=lambda_i
enddo

alpha(1)=0.01
alpha(2)=0.05
alpha(3)=0.1

!Reading the data required for the matrix
open (1,file="c:\Temp\input.txt",status="old",access="sequential")
Read(1,*)B_aantal
Read(1,*)N
Read(1,*)K


!Printing the critical results obtained into this file
open (2,file="c:\Temp\criticalval.txt",status="unknown",access="sequential")

!allocating the matrix values
allocate(stats(B_aantal,21))
allocate(teststats(B_aantal,21))
allocate(teststats_sort(B_aantal))
allocate(teststat_min(B_aantal))

!allocating the proportions
allocate(mcpnul(K))
allocate(IR(K))

allocate(Mat(K))
allocate(data_Mat(K))
allocate(somm(21))

!equiprobable null hypothesis
```

```
do i=1,K
        mcpnul(i)=real(1.0d0/K)
enddo


!Bootstrap loop begins
teststats=0.0
open (4,file="c:\Temp\stats.txt",status="unknown")

do B=1,B_aantal

    NR = 1
    LDIR = 1
    CALL RNMTN (NR, N, K, mcpnul, IR, LDIR)
    korrek = 2
    do while (korrek .ne.1)
        do i=1,K
            if (IR(i) .lt. 0.0) then
                korrek=2
                exit
            else
                korrek=1
            endif
        enddo
        if (korrek .eq. 2) then
                CALL RNMTN (NR, N, K, mcpnul, IR, LDIR)
        endif
    enddo

    mat = IR

    !calculating the teststats
    do m=1,21
        if (lambda(m) .ge. -0.0001 .and. lambda(m) .le. 0.0001) then
           !teststat for lambda 0
           som = 0.0
           do j=1,K
                XIJ=real(mat(j)+c)
                ratio1=(XIJ)/(N*real(mcpnul(j)+c))
                ratio1=log(ratio1)
                ratio11=real(ratio1)
                som = som + (ratio11*XIJ)
           enddo !j
           stats(B,m)=som*2.0
           teststats(B,m)=stats(B,m)

        elseif (lambda(m) .ge. -1.0001 .and. lambda(m) .le. -0.9999) then
           !teststat for lambda -1
           som=0.0
           do j=1,K
                XIJ=real(mat(j)+c)
                ratio=(N*real(mcpnul(j)+c))
                ratio1=ratio/XIJ
                ratio1=log(ratio1)
                ratio11=real(ratio1)
                som = som + ratio11*ratio
           enddo !j
           stats(B,m)=som*2.0
           teststats(B,m)=stats(B,m)
        else
```

```
            !teststat for all other lambda
             somm(m) = 0.0
            do j=1,K
                  XIJ=real(mat(j)+c )
                  ratio= (XIJ)/real(N*(mcpnul(j)))
                  ratio11=real(ratio)
                  somm(m)=somm(m) + (XIJ)*((ratio11**lambda(m))-1.0)
               enddo !j
               stats(B,m)=somm(m)*2.0/(lambda(m)*(lambda(m)+1.0))
               teststats(B,m)=stats(B,m)
            endif

            write(4,*)B,m,teststats(B,m),"   B,m,teststats(B,m)"

         enddo     !the m's

enddo ! B loop
close(4)

!calculating the critical values for the fixed lambdas
do m =1,21

  do B=1,B_aantal
      teststats_sort(B)=teststats(B,m)
  enddo

           !sort and obtain the quantile
           CALL SVRGN (B_aantal, teststats_sort, teststats_sort)

           kritieke_wrds(1,m)=teststats_sort(ceiling(B_aantal*(1.0-alpha(1))))
           kritieke_wrds(2,m)=teststats_sort(ceiling(B_aantal*(1.0-alpha(2))))
           kritieke_wrds(3,m)=teststats_sort(ceiling(B_aantal*(1.0-alpha(3))))
enddo

!printing the results to file
do i=1,3
   do j=1,21
            write(2,*)kritieke_wrds(i,j)
   enddo
enddo

close(2)
deallocate(mcpnul)
deallocate(IR)
deallocate(Mat)
deallocate(stats)
deallocate(somm)
deallocate(teststats)
deallocate(teststats_sort)
deallocate(teststat_min)
deallocate(data_Mat)


END
```

APPENDIX B:

SIMULATION RESULTS

B1: Introduction

This appendix consists of the AE and Bootstrap comparisons as well as the bootstrap and chi-square power estimates. The results are represented here in tabular form for easy comparisons. The tables are as follows:

For each table, the bootstrap approximation and the AE approximation of power estimates are shown for sample sizes 20, 30, 50 and 100 over a range of lambda values from -2 to 2. Also the specific c_j (for all j = 1, 2, …, k) and proportions under both the null hypothesis and alternative hypothesis are specified at the beginning of the tables. The approximated power values are shown for three sizes i.e. α = 0.05, 0.01 and 0.10. Further, the power values are shown when the traditional chi-square values are used instead of the bootstrap critical values. For easier reference, the power values are printed in red and their standard errors are printed in black.

B2: Note

The purpose of Table 16, is to ascertain the true α value, or the $P(\text{Type 1 error})$, for the bootstrap and AE approximates and by using the chi-square test. The power of the test in this case should be close to $\alpha = 0.05$. Refer to §5.4 for a detailed explanation.

Table 1

Proportions under the null hypothesis:					0.2	0.2	0.2														
The proportions under the alternative hypothesis:					0.25	0.25	0.5														
n =	20			c(j) =	0.158	0.158	-0.316						delta =	0.5			k =	3			
lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
alpha	The bootstrap estimated power																				
0.01:	0.043	0.043	0.041	0.043	0.041	0.047	0.047	0.045	0.069	0.08	0.082	0.107	0.135	0.128	0.128	0.128	0.128	0.122	0.121	0.127	0.126
std errors:	0.0064	0.0064	0.0063	0.0064	0.0063	0.0067	0.0067	0.0066	0.008	0.0086	0.0087	0.0098	0.0108	0.0106	0.0106	0.0106	0.0106	0.0103	0.0103	0.0105	0.0105
0.05:	0.271	0.209	0.21	0.209	0.209	0.271	0.243	0.271	0.209	0.266	0.266	0.277	0.277	0.277	0.256	0.277	0.277	0.277	0.276	0.277	0.276
std errors:	0.0141	0.0129	0.0129	0.0129	0.0129	0.0141	0.0136	0.0141	0.0129	0.014	0.014	0.0142	0.0142	0.0142	0.0138	0.0142	0.0142	0.0142	0.0141	0.0142	0.0141
0.1:	0.352	0.395	0.394	0.394	0.371	0.372	0.395	0.391	0.373	0.374	0.432	0.435	0.435	0.435	0.435	0.435	0.352	0.435	0.43	0.347	0.43
std errors:	0.0151	0.0155	0.0155	0.0155	0.0153	0.0153	0.0155	0.0154	0.0153	0.0153	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0151	0.0157	0.0157	0.0151	0.0157
alpha	The AE approximation of power																				
0.01	-0.0001	-0.0001	-0.0001	-0.0003	-0.0005	-0.0005	0	0.0014	0.004	0.0078	0.0127	0.0186	0.023	0.0216	0.0199	0.0181	0.0195	0.0205	0.021	0.0209	0.0204
0.05	0.0432	0.0477	0.0519	0.0558	0.0592	0.0622	0.0646	0.0664	0.0677	0.0684	0.076	0.0837	0.0807	0.0774	0.0765	0.08	0.0827	0.0846	0.0857	0.086	0.0857
0.1	0.0987	0.1104	0.122	0.1334	0.1444	0.155	0.1651	0.1746	0.1835	0.1917	0.1949	0.1919	0.1885	0.1847	0.1803	0.1755	0.1703	0.1647	0.1587	0.1523	0.1457
alpha	The traditional estimated power, chi-square																				
0.01:	0.209	0.179	0.179	0.179	0.155	0.134	0.107	0.107	0.107	0.107	0.107	0.105	0.098	0.098	0.098	0.092	0.122	0.121	0.12	0.12	0.12
std errors:	0.0129	0.0121	0.0121	0.0121	0.0114	0.0108	0.0098	0.0098	0.0098	0.0098	0.0098	0.0097	0.0094	0.0094	0.0094	0.0091	0.0103	0.0103	0.0103	0.0103	0.0103
0.05:	0.322	0.322	0.271	0.271	0.271	0.271	0.271	0.271	0.271	0.271	0.266	0.277	0.277	0.277	0.277	0.277	0.277	0.256	0.256	0.256	0.256
std errors:	0.0148	0.0148	0.0141	0.0141	0.0141	0.0141	0.0141	0.0141	0.0141	0.0141	0.014	0.0142	0.0142	0.0142	0.0142	0.0142	0.0142	0.0138	0.0138	0.0138	0.0138
0.1:	0.434	0.434	0.434	0.395	0.395	0.352	0.352	0.352	0.352	0.352	0.352	0.352	0.352	0.352	0.352	0.352	0.352	0.352	0.347	0.43	0.43
std errors:	0.0157	0.0157	0.0157	0.0155	0.0155	0.0151	0.0151	0.0151	0.0151	0.0151	0.0151	0.0151	0.0151	0.0151	0.0151	0.0151	0.0151	0.0151	0.0151	0.0157	0.0157
The proportions under the alternative hypothesis:					0.2	0.2	0.2														
n =	30			c(j) =	0.158	0.158	-0.316						delta =	0.5			k =	3			
lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
alpha	The bootstrap estimated power																				
0.01:	0.076	0.072	0.087	0.08	0.092	0.107	0.107	0.127	0.143	0.137	0.143	0.136	0.155	0.155	0.155	0.155	0.161	0.174	0.173	0.173	0.173
std errors:	0.0084	0.0082	0.0089	0.0086	0.0091	0.0098	0.0098	0.0105	0.0111	0.0109	0.0111	0.0108	0.0114	0.0114	0.0114	0.0114	0.0116	0.012	0.012	0.012	0.012
0.05:	0.288	0.288	0.288	0.288	0.288	0.27	0.285	0.285	0.285	0.341	0.341	0.339	0.371	0.371	0.371	0.371	0.371	0.357	0.371	0.383	0.357
std errors:	0.0143	0.0143	0.0143	0.0143	0.0143	0.014	0.0143	0.0143	0.0143	0.015	0.015	0.015	0.0153	0.0153	0.0153	0.0153	0.0153	0.0152	0.0153	0.0154	0.0152
0.1:	0.446	0.439	0.458	0.439	0.439	0.483	0.484	0.483	0.505	0.505	0.505	0.505	0.505	0.505	0.505	0.505	0.47	0.505	0.505	0.502	0.505
std errors:	0.0157	0.0157	0.0158	0.0157	0.0157	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158
alpha	The AE approximation of power																				
0.01	0.0008	0.0012	0.0018	0.0026	0.0038	0.0054	0.0074	0.0099	0.0128	0.0161	0.0196	0.0197	0.019	0.018	0.017	0.0158	0.0163	0.0166	0.0166	0.0163	0.0158
0.05	0.0298	0.0322	0.0345	0.0366	0.0385	0.0401	0.0424	0.0489	0.0557	0.0625	0.0694	0.0761	0.079	0.0791	0.0787	0.0779	0.081	0.0837	0.0858	0.0863	0.0821
0.1	0.0819	0.0916	0.1015	0.1113	0.1138	0.117	0.1252	0.133	0.1315	0.129	0.1263	0.1233	0.1233	0.1258	0.1278	0.1293	0.1302	0.1307	0.1307	0.1324	0.1373
alpha	The traditional estimated power, chi-square																				
0.01:	0.234	0.234	0.231	0.206	0.206	0.192	0.169	0.169	0.148	0.148	0.166	0.159	0.159	0.177	0.177	0.177	0.177	0.174	0.174	0.174	0.174
std errors:	0.0134	0.0134	0.0133	0.0128	0.0128	0.0125	0.0119	0.0119	0.0112	0.0112	0.0118	0.0116	0.0116	0.0121	0.0121	0.0121	0.0121	0.012	0.012	0.012	0.012
0.05:	0.458	0.439	0.42	0.42	0.385	0.385	0.385	0.385	0.385	0.385	0.385	0.385	0.382	0.371	0.371	0.371	0.371	0.371	0.357	0.357	0.411
std errors:	0.0158	0.0157	0.0156	0.0156	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0153	0.0153	0.0153	0.0153	0.0153	0.0152	0.0152	0.0156
0.1:	0.543	0.543	0.543	0.543	0.543	0.543	0.543	0.543	0.524	0.524	0.505	0.505	0.505	0.505	0.505	0.505	0.505	0.505	0.505	0.505	0.505
std errors:	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158

Proportions under the null hypothesis: 0.2 0.2 0.2

The proportions under the alternative hypothesis:

n = **50** c(j) = 0.158 0.158 -0.316 delta = 0.5 k = 3

lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
alpha	The bootstrap estimated power																				
0.01:	0.26	0.26	0.266	0.309	0.308	0.317	0.317	0.317	0.317	0.307	0.325	0.325	0.354	0.354	0.36	0.359	0.356	0.359	0.359	0.378	0.378
std errors:	0.0139	0.0139	0.014	0.0146	0.0146	0.0147	0.0147	0.0147	0.0147	0.0146	0.0148	0.0148	0.0151	0.0151	0.0152	0.0152	0.0151	0.0152	0.0152	0.0153	0.0153
0.05:	0.523	0.534	0.534	0.534	0.534	0.514	0.534	0.54	0.54	0.54	0.536	0.564	0.58	0.58	0.584	0.58	0.584	0.61	0.58	0.61	0.61
std errors:	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0157	0.0156	0.0156	0.0156	0.0156	0.0156	0.0154	0.0156	0.0154	0.0154
0.1:	0.649	0.628	0.647	0.647	0.647	0.649	0.647	0.649	0.652	0.67	0.67	0.664	0.688	0.682	0.688	0.688	0.688	0.688	0.707	0.707	0.707
std errors:	0.0151	0.0153	0.0151	0.0151	0.0151	0.0151	0.0151	0.0151	0.0151	0.0149	0.0149	0.0149	0.0147	0.0147	0.0147	0.0147	0.0147	0.0147	0.0144	0.0144	0.0144
alpha	The AE approximation of power																				
0.01	0.0206	0.023	0.0255	0.0276	0.0284	0.0291	0.031	0.0329	0.0348	0.0365	0.0381	0.039	0.0385	0.0414	0.0416	0.0411	0.0405	0.0418	0.0431	0.0428	0.0412
0.05	0.0791	0.0822	0.0851	0.0878	0.0904	0.0927	0.0949	0.0948	0.0941	0.0985	0.1011	0.102	0.1054	0.1094	0.1132	0.1169	0.1204	0.1235	0.1214	0.119	0.1165
0.1	0.1238	0.1256	0.1279	0.1346	0.1412	0.1478	0.1543	0.1606	0.1593	0.1576	0.1556	0.1577	0.1615	0.1651	0.1685	0.1715	0.1744	0.1769	0.1768	0.1754	0.1738
alpha	The traditional estimated power, chi-square																				
0.01:	0.365	0.351	0.351	0.351	0.351	0.35	0.341	0.335	0.335	0.335	0.325	0.325	0.353	0.352	0.352	0.351	0.351	0.351	0.351	0.358	0.358
std errors:	0.0152	0.0151	0.0151	0.0151	0.0151	0.0151	0.015	0.0149	0.0149	0.0149	0.0148	0.0148	0.0151	0.0151	0.0151	0.0151	0.0151	0.0151	0.0151	0.0152	0.0152
0.05:	0.576	0.576	0.56	0.56	0.56	0.56	0.56	0.56	0.56	0.56	0.536	0.532	0.56	0.56	0.56	0.56	0.553	0.553	0.553	0.573	0.573
std errors:	0.0156	0.0156	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0158	0.0158	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0156	0.0156
0.1:	0.674	0.674	0.649	0.649	0.649	0.679	0.679	0.679	0.677	0.67	0.67	0.688	0.688	0.682	0.682	0.682	0.682	0.682	0.682	0.682	0.682
std errors:	0.0148	0.0148	0.0151	0.0151	0.0151	0.0148	0.0148	0.0148	0.0148	0.0149	0.0149	0.0147	0.0147	0.0147	0.0147	0.0147	0.0147	0.0147	0.0147	0.0147	0.0147

The proportions under the alternative hypothesis: 0.2 0.2 0.2

n = **100** c(j) = 0.158 0.158 -0.316 delta = 0.5 k = 3

lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
alpha	The bootstrap estimated power																				
0.01:	0.573	0.585	0.585	0.611	0.632	0.649	0.651	0.652	0.648	0.659	0.663	0.69	0.722	0.722	0.73	0.73	0.73	0.721	0.729	0.729	0.744
std errors:	0.0156	0.0156	0.0156	0.0154	0.0153	0.0151	0.0151	0.0151	0.0151	0.015	0.0149	0.0146	0.0142	0.0142	0.014	0.014	0.014	0.0142	0.0141	0.0141	0.0138
0.05:	0.84	0.84	0.841	0.84	0.847	0.847	0.853	0.853	0.847	0.853	0.852	0.854	0.86	0.86	0.874	0.873	0.872	0.871	0.871	0.872	0.872
std errors:	0.0116	0.0116	0.0116	0.0116	0.0114	0.0114	0.0112	0.0112	0.0114	0.0112	0.0112	0.0112	0.011	0.011	0.0105	0.0105	0.0106	0.0106	0.0106	0.0106	0.0106
0.1:	0.903	0.903	0.903	0.913	0.913	0.913	0.913	0.915	0.915	0.915	0.915	0.915	0.915	0.927	0.915	0.927	0.927	0.915	0.927	0.927	0.927
std errors:	0.0094	0.0094	0.0094	0.0089	0.0089	0.0089	0.0089	0.0088	0.0088	0.0088	0.0088	0.0088	0.0088	0.0082	0.0088	0.0082	0.0082	0.0088	0.0082	0.0082	0.0082
alpha	The AE approximation of power																				
0.01	0.022	0.0237	0.0255	0.0272	0.029	0.0302	0.031	0.0318	0.032	0.0327	0.0341	0.0353	0.0365	0.0377	0.0386	0.0379	0.0372	0.0365	0.036	0.0364	0.0366
0.05	0.0909	0.0922	0.0933	0.0939	0.0946	0.0959	0.0966	0.0971	0.0976	0.0978	0.098	0.0979	0.098	0.099	0.0983	0.0991	0.0997	0.1003	0.1007	0.101	0.1011
0.1	0.1505	0.1547	0.154	0.1528	0.1514	0.15	0.1516	0.1543	0.157	0.1595	0.1619	0.1642	0.1664	0.1674	0.166	0.1645	0.1629	0.1612	0.1594	0.1622	0.1649
alpha	The traditional estimated power, chi-square																				
0.01:	0.683	0.683	0.678	0.678	0.676	0.695	0.695	0.695	0.692	0.692	0.706	0.723	0.723	0.722	0.722	0.722	0.722	0.729	0.729	0.729	0.744
std errors:	0.0147	0.0147	0.0148	0.0148	0.0148	0.0146	0.0146	0.0146	0.0146	0.0146	0.0144	0.0142	0.0142	0.0142	0.0142	0.0142	0.0142	0.0141	0.0141	0.0141	0.0138
0.05:	0.854	0.854	0.854	0.854	0.854	0.854	0.854	0.853	0.853	0.853	0.86	0.86	0.86	0.874	0.874	0.874	0.872	0.871	0.871	0.871	0.871
std errors:	0.0112	0.0112	0.0112	0.0112	0.0112	0.0112	0.0112	0.0112	0.0112	0.0112	0.011	0.011	0.011	0.0105	0.0105	0.0105	0.0106	0.0106	0.0106	0.0106	0.0106
0.1:	0.917	0.917	0.917	0.916	0.916	0.916	0.915	0.915	0.915	0.915	0.915	0.915	0.915	0.915	0.915	0.915	0.927	0.927	0.927	0.927	0.927
std errors:	0.0087	0.0087	0.0087	0.0088	0.0088	0.0088	0.0088	0.0088	0.0088	0.0088	0.0088	0.0088	0.0088	0.0088	0.0088	0.0088	0.0082	0.0082	0.0082	0.0082	0.0082

Proportions under the null hypothesis: 0.2 0.2 0.2

The proportions under the alternative hypothesis: 0.0833 0.0833 0.8334

n = 20 c(j) = 0.913 0.913 -1.826 delta = 1.5 k = 3

lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
alpha	The bootstrap estimated power																				
0.01:	0.787	0.787	0.787	0.862	0.862	0.862	0.938	0.938	0.959	0.959	0.977	0.977	0.977	0.977	0.977	0.983	0.983	0.977	0.983	0.983	0.977
std errors:	0.0129	0.0129	0.0129	0.0109	0.0109	0.0109	0.0076	0.0076	0.0063	0.0063	0.0047	0.0047	0.0047	0.0047	0.0047	0.0041	0.0041	0.0047	0.0041	0.0041	0.0047
0.05:	0.984	0.984	0.984	0.984	0.984	0.993	0.993	0.993	0.993	0.993	0.993	0.996	0.996	0.996	0.999	0.999	0.999	0.996	0.999	0.999	0.999
std errors:	0.004	0.004	0.004	0.004	0.004	0.0026	0.0026	0.0026	0.0026	0.0026	0.0026	0.002	0.002	0.002	0.001	0.001	0.001	0.002	0.001	0.001	0.001
0.1:	0.996	0.997	0.997	1	1	1	1	1	1	0.999	1	1	1	1	1	1	1	1	1	1	1
std errors:	0.002	0.0017	0.0017	0	0	0	0	0	0	0.001	0	0	0	0	0	0	0	0	0	0	0
alpha	The AE approximation of power																				
0.01	-2.0692	-1.6847	-1.1595	-0.6087	-0.1297	0.2368	0.4951	0.6679	0.7804	0.8369	0.8474	0.8841	0.9089	0.9258	0.925	0.9209	0.9145	0.9058	0.8942	0.8792	0.86
0.05	0.7483	0.8345	0.8921	0.9299	0.9546	0.9706	0.9809	0.9876	0.9919	0.9946	0.9951	0.995	0.9949	0.9958	0.996	0.9953	0.9944	0.9933	0.994	0.9951	0.9959
0.1	0.9926	0.9943	0.9955	0.9963	0.9969	0.9974	0.9977	0.9979	0.9981	0.9982	0.9983	0.9984	0.9984	0.9984	0.9984	0.9984	0.9984	0.9983	0.9985	0.9986	0.9986
alpha	The traditional estimated power, chi-square																				
0.01:	0.993	0.984	0.984	0.984	0.984	0.984	0.983	0.983	0.983	0.983	0.983	0.983	0.983	0.983	0.983	0.983	0.992	0.992	0.992	0.992	0.992
std errors:	0.0026	0.004	0.004	0.004	0.004	0.004	0.0041	0.0041	0.0041	0.0041	0.0041	0.0041	0.0041	0.0041	0.0041	0.0041	0.0028	0.0028	0.0028	0.0028	0.0028
0.05:	0.997	0.997	0.996	0.996	0.996	0.996	0.996	0.996	0.996	0.996	0.996	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999
std errors:	0.0017	0.0017	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
0.1:	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
std errors:	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

The proportions under the alternative hypothesis: 0.0833 0.0833 0.8334

n = 30 c(j) = 0.913 0.913 -1.826 delta = 1.5 k = 3

lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
alpha	The bootstrap estimated power																				
0.01:	0.988	0.988	0.992	0.997	0.997	0.999	0.999	1	1	1	1	1	1	1	1	1	1	1	1	1	1
std errors:	0.0034	0.0034	0.0028	0.0017	0.0017	0.001	0.001	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.05:	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
std errors:	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.1:	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
std errors:	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
alpha	The AE approximation of power																				
0.01	1.4054	1.2472	1.1453	1.0818	1.0432	1.0204	1.007	1.0018	0.998	0.9971	0.9946	0.9931	0.9922	0.9917	0.9914	0.9912	0.9913	0.9913	0.9914	0.9915	0.9917
0.05	0.9921	0.9921	0.9922	0.9922	0.9923	0.9924	0.9925	0.9927	0.9927	0.9928	0.9929	0.9929	0.9931	0.9933	0.9935	0.9937	0.9936	0.9935	0.9933	0.9932	0.993
0.1	0.9885	0.9897	0.9907	0.9915	0.9917	0.9919	0.9924	0.9929	0.9933	0.9936	0.9939	0.9941	0.9943	0.9945	0.9947	0.9948	0.9947	0.9947	0.9946	0.9945	0.9943
alpha	The traditional estimated power, chi-square																				
0.01:	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
std errors:	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.05:	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
std errors:	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.1:	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
std errors:	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 2

Proportions under the null hypothesis: 0.2 0.2 0.2

The proportions under the alternative hypothesis: 0.0833 0.0833 0.8334

n = 50 c(j) = 0.913 0.913 -1.826 delta = 1.5 k = 3

lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
alpha	The bootstrap estimated power																				
0.01:	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
std errors:	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.05:	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
std errors:	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.1:	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
std errors:	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
alpha	The AE approximation of power																				
0.01	1.2461	1.1963	1.1573	1.1266	1.1024	1.0833	1.0777	1.0744	1.0723	1.0713	1.0562	1.0423	1.0315	1.023	1.0225	1.0237	1.0254	1.0276	1.0304	1.0339	1.0382
0.05	0.9905	0.99	0.9897	0.9894	0.9892	0.9891	0.989	0.989	0.989	0.9891	0.9889	0.9888	0.9889	0.9889	0.989	0.9892	0.9893	0.9896	0.9894	0.9891	0.989
0.1	0.9839	0.9847	0.9854	0.9861	0.9867	0.9873	0.9879	0.9884	0.9887	0.9886	0.9884	0.9886	0.9887	0.9885	0.9882	0.9879	0.9876	0.9873	0.9869	0.9872	0.9875
alpha	The traditional estimated power, chi-square																				
0.01:	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
std errors:	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.05:	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
std errors:	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.1:	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
std errors:	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

The proportions under the alternative hypothesis: 0.0833 0.0833 0.8334

n = 100 c(j) = 0.913 0.913 -1.826 delta = 1.5 k = 3

lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
alpha	The bootstrap estimated power																				
0.01:	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
std errors:	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.05:	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
std errors:	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.1:	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
std errors:	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
alpha	The AE approximation of power																				
0.01	0.9988	0.9883	0.9801	0.9739	0.9693	0.9654	0.9628	0.9624	0.9621	0.962	0.9619	0.962	0.9621	0.9623	0.9626	0.9617	0.9611	0.9605	0.9604	0.9607	0.961
0.05	0.9703	0.9712	0.9722	0.9731	0.973	0.9727	0.9725	0.9722	0.9725	0.9729	0.9725	0.9721	0.9716	0.9719	0.9726	0.9733	0.974	0.974	0.9746	0.9751	0.975
0.1	0.9801	0.9805	0.9812	0.9818	0.9825	0.983	0.9834	0.9833	0.9833	0.9837	0.984	0.9841	0.9842	0.9842	0.9843	0.9843	0.984	0.9837	0.9838	0.9839	0.9835
alpha	The traditional estimated power, chi-square																				
0.01:	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
std errors:	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.05:	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
std errors:	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.1:	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
std errors:	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Appendix B

Table 3

Proportions under the null hypothesis: 0.2 0.2 0.2

The proportions under the alternative hypothesis: 0.391 0.391 0.218

n = 20 c(j) = 0.2598 0.2598 -0.5196 delta = 3k/4 k = 3

lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
alpha	The bootstrap estimated power																				
0.01:	0.049	0.041	0.041	0.041	0.055	0.047	0.052	0.047	0.055	0.05	0.051	0.055	0.052	0.052	0.052	0.042	0.046	0.042	0.048	0.042	0.048
std errors:	0.0068	0.0063	0.0063	0.0063	0.0072	0.0067	0.007	0.0067	0.0072	0.0069	0.007	0.0072	0.007	0.007	0.007	0.0063	0.0066	0.0063	0.0068	0.0063	0.0068
0.05:	0.156	0.127	0.156	0.156	0.156	0.165	0.164	0.137	0.137	0.158	0.142	0.155	0.168	0.154	0.171	0.137	0.137	0.171	0.179	0.179	0.139
std errors:	0.0115	0.0105	0.0115	0.0115	0.0115	0.0117	0.0117	0.0109	0.0109	0.0115	0.011	0.0114	0.0118	0.0114	0.0119	0.0109	0.0109	0.0119	0.0121	0.0121	0.0109
0.1:	0.235	0.242	0.242	0.242	0.242	0.242	0.242	0.242	0.242	0.242	0.242	0.242	0.242	0.242	0.24	0.242	0.242	0.242	0.256	0.213	0.256
std errors:	0.0134	0.0135	0.0135	0.0135	0.0135	0.0135	0.0135	0.0135	0.0135	0.0135	0.0135	0.0135	0.0135	0.0135	0.0135	0.0135	0.0135	0.0135	0.0138	0.0129	0.0138
alpha	The AE approximation of power																				
0.01	-0.0023	-0.0006	0.002	0.0053	0.009	0.0131	0.0171	0.0209	0.0244	0.0272	0.0294	0.0407	0.0496	0.0521	0.0615	0.07	0.0776	0.0777	0.0752	0.0719	0.0678
0.05	0.005	0.0125	0.0225	0.035	0.0495	0.0655	0.0826	0.1003	0.1181	0.1286	0.1407	0.1554	0.1693	0.1821	0.1939	0.2045	0.214	0.2224	0.2189	0.2144	0.2091
0.1	0.1523	0.1555	0.1579	0.169	0.1794	0.1888	0.1971	0.2044	0.2107	0.2158	0.2199	0.223	0.225	0.2259	0.2259	0.2249	0.237	0.248	0.2559	0.2476	0.2387
alpha	The traditional estimated power, chi-square																				
0.01:	0.165	0.156	0.156	0.156	0.093	0.061	0.055	0.055	0.055	0.055	0.055	0.043	0.033	0.033	0.033	0.029	0.038	0.034	0.034	0.034	0.034
std errors:	0.0117	0.0115	0.0115	0.0115	0.0092	0.0076	0.0072	0.0072	0.0072	0.0072	0.0072	0.0064	0.0056	0.0056	0.0056	0.0053	0.006	0.0057	0.0057	0.0057	0.0057
0.05:	0.229	0.229	0.187	0.187	0.187	0.187	0.187	0.187	0.187	0.187	0.158	0.137	0.137	0.137	0.137	0.137	0.137	0.105	0.105	0.105	0.105
std errors:	0.0133	0.0133	0.0123	0.0123	0.0123	0.0123	0.0123	0.0123	0.0123	0.0123	0.0115	0.0109	0.0109	0.0109	0.0109	0.0109	0.0109	0.0097	0.0097	0.0097	0.0097
0.1:	0.363	0.363	0.363	0.298	0.298	0.242	0.242	0.242	0.242	0.242	0.242	0.242	0.242	0.242	0.242	0.242	0.242	0.242	0.213	0.256	0.256
std errors:	0.0152	0.0152	0.0152	0.0145	0.0145	0.0135	0.0135	0.0135	0.0135	0.0135	0.0135	0.0135	0.0135	0.0135	0.0135	0.0135	0.0135	0.0135	0.0129	0.0138	0.0138

The proportions under the alternative hypothesis: 0.381 0.381 0.238

n = 30 c(j) = 0.2598 0.2598 -0.5196 delta = 3k/4 k = 3

lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
alpha	The bootstrap estimated power																				
0.01:	0.06	0.051	0.051	0.053	0.062	0.054	0.063	0.06	0.07	0.07	0.063	0.047	0.038	0.05	0.041	0.053	0.053	0.053	0.048	0.042	0.042
std errors:	0.0075	0.007	0.007	0.0071	0.0076	0.0071	0.0077	0.0075	0.0081	0.0081	0.0077	0.0067	0.006	0.0069	0.0063	0.0071	0.0071	0.0071	0.0068	0.0063	0.0063
0.05:	0.186	0.178	0.177	0.169	0.196	0.196	0.18	0.196	0.196	0.176	0.188	0.17	0.166	0.166	0.166	0.166	0.166	0.166	0.166	0.147	0.143
std errors:	0.0123	0.0121	0.0121	0.0119	0.0126	0.0126	0.0121	0.0126	0.0126	0.012	0.0124	0.0119	0.0118	0.0118	0.0118	0.0118	0.0118	0.0118	0.0118	0.0112	0.0111
0.1:	0.258	0.258	0.28	0.282	0.274	0.274	0.274	0.274	0.284	0.282	0.251	0.251	0.284	0.284	0.284	0.251	0.283	0.255	0.266	0.283	0.251
std errors:	0.0138	0.0138	0.0142	0.0142	0.0141	0.0141	0.0141	0.0141	0.0143	0.0142	0.0137	0.0137	0.0143	0.0143	0.0143	0.0137	0.0142	0.0138	0.014	0.0142	0.0137
alpha	The AE approximation of power																				
0.01	0.0026	0.0028	0.0036	0.0051	0.0076	0.0112	0.016	0.022	0.0289	0.0365	0.0408	0.0396	0.0405	0.0453	0.0497	0.0536	0.0528	0.0514	0.0496	0.0474	0.0448
0.05	0.0893	0.0966	0.1036	0.1102	0.1162	0.1216	0.1265	0.1307	0.1343	0.1372	0.1394	0.1408	0.1416	0.1417	0.1412	0.14	0.1448	0.1488	0.1521	0.1528	0.1464
0.1	0.1417	0.1569	0.1718	0.1864	0.1901	0.1947	0.2063	0.2173	0.2276	0.2374	0.2465	0.2549	0.2627	0.2698	0.2763	0.2822	0.2794	0.2761	0.2723	0.268	0.2632
alpha	The traditional estimated power, chi-square																				
0.01:	0.152	0.152	0.141	0.096	0.096	0.084	0.082	0.082	0.07	0.07	0.063	0.047	0.047	0.041	0.041	0.041	0.041	0.03	0.03	0.03	0.03
std errors:	0.0114	0.0114	0.011	0.0093	0.0093	0.0088	0.0087	0.0087	0.0081	0.0081	0.0077	0.0067	0.0067	0.0063	0.0063	0.0063	0.0063	0.0054	0.0054	0.0054	0.0054
0.05:	0.304	0.257	0.224	0.224	0.199	0.199	0.199	0.199	0.199	0.199	0.199	0.199	0.188	0.166	0.166	0.166	0.166	0.166	0.143	0.143	0.16
std errors:	0.0145	0.0138	0.0132	0.0132	0.0126	0.0126	0.0126	0.0126	0.0126	0.0126	0.0126	0.0126	0.0124	0.0118	0.0118	0.0118	0.0118	0.0118	0.0111	0.0111	0.0116
0.1:	0.331	0.331	0.331	0.331	0.331	0.331	0.331	0.331	0.284	0.284	0.251	0.251	0.251	0.251	0.251	0.251	0.251	0.251	0.251	0.251	0.251
std errors:	0.0149	0.0149	0.0149	0.0149	0.0149	0.0149	0.0149	0.0149	0.0143	0.0143	0.0137	0.0137	0.0137	0.0137	0.0137	0.0137	0.0137	0.0137	0.0137	0.0137	0.0137

Proportions under the null hypothesis: 0.2 0.2 0.2

The proportions under the alternative hypothesis: 0.37 0.37 0.26

n = 50 c(j) = 0.2598 0.2598 -0.5196 delta = 3k/4 k = 3

lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
alpha	The bootstrap estimated power																				
0.01:	0.053	0.064	0.064	0.055	0.057	0.057	0.057	0.057	0.056	0.057	0.056	0.058	0.053	0.054	0.053	0.054	0.054	0.054	0.06	0.053	0.048
std errors:	0.0071	0.0077	0.0077	0.0072	0.0073	0.0073	0.0073	0.0073	0.0073	0.0073	0.0073	0.0074	0.0071	0.0071	0.0071	0.0071	0.0071	0.0071	0.0075	0.0071	0.0068
0.05:	0.182	0.182	0.178	0.182	0.179	0.182	0.175	0.181	0.181	0.181	0.167	0.175	0.16	0.168	0.168	0.161	0.168	0.152	0.16	0.153	0.16
std errors:	0.0122	0.0122	0.0121	0.0122	0.0121	0.0122	0.012	0.0122	0.0122	0.0122	0.0118	0.012	0.0116	0.0118	0.0118	0.0116	0.0118	0.0114	0.0116	0.0114	0.0116
0.1:	0.263	0.263	0.265	0.265	0.249	0.249	0.265	0.265	0.252	0.25	0.26	0.244	0.253	0.253	0.253	0.244	0.253	0.253	0.253	0.253	0.239
std errors:	0.0139	0.0139	0.014	0.014	0.0137	0.0137	0.014	0.014	0.0137	0.0137	0.0139	0.0136	0.0137	0.0137	0.0137	0.0136	0.0137	0.0137	0.0137	0.0137	0.0135
alpha	The AE approximation of power																				
0.01	0.0365	0.0402	0.0441	0.0474	0.0488	0.0499	0.053	0.0562	0.0593	0.0622	0.0622	0.061	0.0645	0.0665	0.0707	0.0747	0.0757	0.0764	0.0755	0.0729	0.0701
0.05	0.1327	0.1375	0.142	0.1463	0.1502	0.1538	0.1571	0.1569	0.1559	0.1626	0.1664	0.1666	0.1688	0.1695	0.1697	0.1696	0.1691	0.1682	0.167	0.1654	0.1635
0.1	0.1948	0.1974	0.1997	0.205	0.2141	0.2229	0.2314	0.2397	0.2453	0.2428	0.2402	0.2431	0.2452	0.241	0.2366	0.2319	0.227	0.2219	0.2171	0.2205	0.2256
alpha	The traditional estimated power, chi-square																				
0.01:	0.102	0.097	0.097	0.097	0.097	0.081	0.07	0.059	0.059	0.059	0.058	0.058	0.054	0.05	0.05	0.043	0.043	0.043	0.043	0.046	0.044
std errors:	0.0096	0.0094	0.0094	0.0094	0.0094	0.0086	0.0081	0.0075	0.0075	0.0075	0.0074	0.0074	0.0071	0.0069	0.0069	0.0064	0.0064	0.0064	0.0064	0.0066	0.0065
0.05:	0.21	0.21	0.189	0.189	0.189	0.189	0.189	0.189	0.189	0.189	0.167	0.152	0.16	0.16	0.16	0.16	0.152	0.152	0.152	0.16	0.16
std errors:	0.0129	0.0129	0.0124	0.0124	0.0124	0.0124	0.0124	0.0124	0.0124	0.0124	0.0118	0.0114	0.0116	0.0116	0.0116	0.0116	0.0114	0.0114	0.0114	0.0116	0.0116
0.1:	0.311	0.311	0.278	0.278	0.278	0.289	0.289	0.289	0.276	0.26	0.26	0.269	0.269	0.253	0.253	0.253	0.253	0.253	0.253	0.253	0.253
std errors:	0.0146	0.0146	0.0142	0.0142	0.0142	0.0143	0.0143	0.0143	0.0141	0.0139	0.0139	0.014	0.014	0.0137	0.0137	0.0137	0.0137	0.0137	0.0137	0.0137	0.0137

The proportions under the alternative hypothesis: 0.359 0.359 0.282

n = 100 c(j) = 0.2598 0.2598 -0.5196 delta = 3k/4 k = 3

lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
alpha	The bootstrap estimated power																				
0.01:	0.046	0.044	0.043	0.045	0.045	0.046	0.046	0.048	0.049	0.048	0.051	0.051	0.051	0.051	0.049	0.048	0.048	0.047	0.044	0.044	0.042
std errors:	0.0066	0.0065	0.0064	0.0066	0.0066	0.0066	0.0066	0.0068	0.0068	0.0068	0.007	0.007	0.007	0.007	0.0068	0.0068	0.0068	0.0067	0.0065	0.0065	0.0063
0.05:	0.161	0.156	0.156	0.159	0.154	0.153	0.154	0.154	0.154	0.15	0.154	0.154	0.158	0.159	0.153	0.145	0.145	0.145	0.145	0.145	0.145
std errors:	0.0116	0.0115	0.0115	0.0116	0.0114	0.0114	0.0114	0.0114	0.0114	0.0113	0.0114	0.0114	0.0115	0.0116	0.0114	0.0111	0.0111	0.0111	0.0111	0.0111	0.0111
0.1:	0.272	0.271	0.27	0.272	0.272	0.254	0.265	0.271	0.271	0.261	0.269	0.259	0.268	0.266	0.259	0.256	0.256	0.266	0.256	0.253	0.25
std errors:	0.0141	0.0141	0.014	0.0141	0.0141	0.0138	0.014	0.0141	0.0141	0.0139	0.014	0.0139	0.014	0.014	0.0139	0.0138	0.0138	0.014	0.0138	0.0137	0.0137
alpha	The AE approximation of power																				
0.01	0.0445	0.0453	0.0464	0.0496	0.0523	0.0555	0.0588	0.0622	0.0655	0.0663	0.0694	0.0706	0.0719	0.0731	0.0739	0.0725	0.0708	0.069	0.0711	0.0689	0.0667
0.05	0.1515	0.1533	0.155	0.1559	0.1556	0.155	0.1543	0.1535	0.1524	0.1538	0.156	0.159	0.1607	0.1618	0.1613	0.1607	0.1598	0.1588	0.1577	0.1585	0.1605
0.1	0.2407	0.2393	0.2445	0.2501	0.2555	0.2608	0.2655	0.267	0.2683	0.2695	0.2711	0.275	0.2788	0.2783	0.2756	0.2728	0.2727	0.2725	0.2721	0.2694	0.2656
alpha	The traditional estimated power, chi-square																				
0.01:	0.075	0.065	0.062	0.055	0.053	0.054	0.054	0.054	0.047	0.043	0.043	0.045	0.045	0.044	0.044	0.044	0.041	0.042	0.042	0.04	0.042
std errors:	0.0083	0.0078	0.0076	0.0072	0.0071	0.0071	0.0071	0.0071	0.0067	0.0064	0.0064	0.0066	0.0066	0.0065	0.0065	0.0065	0.0063	0.0063	0.0063	0.0062	0.0063
0.05:	0.183	0.172	0.17	0.162	0.162	0.162	0.162	0.157	0.157	0.157	0.163	0.163	0.163	0.162	0.16	0.16	0.149	0.145	0.145	0.145	0.145
std errors:	0.0122	0.0119	0.0119	0.0117	0.0117	0.0117	0.0117	0.0115	0.0115	0.0115	0.0117	0.0117	0.0117	0.0117	0.0116	0.0116	0.0113	0.0111	0.0111	0.0111	0.0111
0.1:	0.276	0.276	0.276	0.254	0.254	0.254	0.242	0.242	0.242	0.242	0.242	0.234	0.234	0.234	0.234	0.234	0.241	0.241	0.241	0.241	0.23
std errors:	0.0141	0.0141	0.0141	0.0138	0.0138	0.0138	0.0135	0.0135	0.0135	0.0135	0.0135	0.0134	0.0134	0.0134	0.0134	0.0134	0.0135	0.0135	0.0135	0.0135	0.0133

Proportions under the null hypothesis:					0.2	0.2	0.2														
The proportions under the alternative hypothesis:					0.4083	0.4083	0.1834														
n =	20		c(j) =		0.346	0.346	-0.693					delta =	5k/4			k =	3				
lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
alpha	The bootstrap estimated power																				
0.01:	0.117	0.117	0.117	0.117	0.112	0.117	0.112	0.112	0.117	0.082	0.121	0.094	0.094	0.081	0.08	0.076	0.058	0.052	0.052	0.045	0.039
std errors:	0.0102	0.0102	0.0102	0.0102	0.01	0.0102	0.01	0.01	0.0102	0.0087	0.0103	0.0092	0.0092	0.0086	0.0086	0.0084	0.0074	0.007	0.007	0.0066	0.0061
0.05:	0.312	0.31	0.305	0.307	0.305	0.312	0.312	0.28	0.28	0.303	0.271	0.246	0.243	0.246	0.204	0.246	0.246	0.246	0.204	0.204	0.224
std errors:	0.0147	0.0146	0.0146	0.0146	0.0146	0.0147	0.0147	0.0142	0.0142	0.0145	0.0141	0.0136	0.0136	0.0136	0.0127	0.0136	0.0136	0.0136	0.0127	0.0127	0.0132
0.1:	0.375	0.381	0.381	0.381	0.381	0.381	0.381	0.381	0.381	0.381	0.381	0.381	0.381	0.381	0.376	0.381	0.381	0.381	0.394	0.349	0.394
std errors:	0.0153	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0153	0.0154	0.0154	0.0154	0.0155	0.0151	0.0155
alpha	The AE approximation of power																				
0.01	-0.0113	-0.0099	-0.0076	-0.0046	-0.0012	0.0025	0.006	0.0091	0.0201	0.0385	0.0593	0.0659	0.0838	0.0916	0.0858	0.0793	0.072	0.0642	0.056	0.0476	0.0392
0.05	0.0709	0.0785	0.0854	0.0913	0.0963	0.1227	0.1497	0.1765	0.2025	0.2174	0.2341	0.2501	0.244	0.237	0.2351	0.2424	0.248	0.2519	0.2542	0.2549	0.2541
0.1	0.2478	0.252	0.2554	0.2702	0.2837	0.2958	0.3065	0.3157	0.3235	0.3299	0.3349	0.3386	0.3411	0.3422	0.3422	0.341	0.3556	0.3687	0.378	0.3682	0.3576
alpha	The traditional estimated power, chi-square																				
0.01:	0.312	0.305	0.305	0.305	0.207	0.165	0.133	0.133	0.133	0.133	0.133	0.094	0.074	0.074	0.074	0.051	0.058	0.056	0.052	0.052	0.052
std errors:	0.0147	0.0146	0.0146	0.0146	0.0128	0.0117	0.0107	0.0107	0.0107	0.0107	0.0107	0.0092	0.0083	0.0083	0.0083	0.007	0.0074	0.0073	0.007	0.007	0.007
0.05:	0.372	0.372	0.335	0.335	0.335	0.335	0.335	0.335	0.335	0.335	0.303	0.246	0.246	0.246	0.246	0.246	0.246	0.204	0.204	0.204	0.204
std errors:	0.0153	0.0153	0.0149	0.0149	0.0149	0.0149	0.0149	0.0149	0.0149	0.0149	0.0145	0.0136	0.0136	0.0136	0.0136	0.0136	0.0136	0.0127	0.0127	0.0127	0.0127
0.1:	0.537	0.537	0.537	0.45	0.45	0.381	0.381	0.381	0.381	0.381	0.381	0.381	0.381	0.381	0.381	0.381	0.381	0.381	0.349	0.394	0.394
std errors:	0.0158	0.0158	0.0158	0.0157	0.0157	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0151	0.0155	0.0155
The proportions under the alternative hypothesis:					0.3946	0.3946	0.2108														
n =	30		c(j) =		0.346	0.346	-0.693					delta =	5k/4			k =	3				
lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
alpha	The bootstrap estimated power																				
0.01:	0.128	0.12	0.12	0.121	0.12	0.131	0.121	0.131	0.124	0.124	0.11	0.111	0.105	0.108	0.098	0.096	0.096	0.096	0.089	0.082	0.082
std errors:	0.0106	0.0103	0.0103	0.0103	0.0103	0.0107	0.0103	0.0107	0.0104	0.0104	0.0099	0.0099	0.0097	0.0098	0.0094	0.0093	0.0093	0.0093	0.009	0.0087	0.0087
0.05:	0.267	0.267	0.267	0.265	0.272	0.272	0.25	0.272	0.272	0.247	0.259	0.231	0.255	0.255	0.243	0.229	0.216	0.234	0.216	0.234	0.222
std errors:	0.014	0.014	0.014	0.014	0.0141	0.0141	0.0137	0.0141	0.0141	0.0136	0.0139	0.0133	0.0138	0.0138	0.0136	0.0133	0.013	0.0134	0.013	0.0134	0.0131
0.1:	0.366	0.364	0.388	0.386	0.382	0.402	0.389	0.394	0.394	0.392	0.35	0.35	0.394	0.394	0.394	0.429	0.385	0.351	0.369	0.385	0.35
std errors:	0.0152	0.0152	0.0154	0.0154	0.0154	0.0155	0.0154	0.0155	0.0155	0.0154	0.0151	0.0151	0.0155	0.0155	0.0155	0.0157	0.0154	0.0151	0.0153	0.0154	0.0151
alpha	The AE approximation of power																				
0.01	0.0265	0.033	0.0401	0.0476	0.0553	0.0628	0.0701	0.0768	0.0809	0.0802	0.0789	0.0889	0.0986	0.1001	0.1007	0.1003	0.099	0.0968	0.0938	0.0901	0.0857
0.05	0.1562	0.1672	0.1775	0.1869	0.1956	0.2033	0.2102	0.216	0.221	0.2249	0.2279	0.2299	0.2389	0.2518	0.2639	0.2749	0.2686	0.2617	0.2542	0.2486	0.248
0.1	0.2276	0.2483	0.2682	0.2873	0.3055	0.3228	0.3304	0.3273	0.339	0.3508	0.3617	0.3717	0.3808	0.3891	0.3966	0.4033	0.4001	0.3963	0.392	0.387	0.3815
alpha	The traditional estimated power, chi-square																				
0.01:	0.23	0.23	0.211	0.148	0.148	0.135	0.131	0.131	0.12	0.12	0.11	0.096	0.096	0.085	0.085	0.085	0.085	0.071	0.071	0.071	0.071
std errors:	0.0133	0.0133	0.0129	0.0112	0.0112	0.0108	0.0107	0.0107	0.0103	0.0103	0.0099	0.0093	0.0093	0.0088	0.0088	0.0088	0.0088	0.0081	0.0081	0.0081	0.0081
0.05:	0.408	0.364	0.32	0.32	0.274	0.274	0.274	0.274	0.274	0.274	0.274	0.274	0.255	0.216	0.216	0.216	0.216	0.216	0.192	0.192	0.21
std errors:	0.0155	0.0152	0.0148	0.0148	0.0141	0.0141	0.0141	0.0141	0.0141	0.0141	0.0141	0.0141	0.0138	0.013	0.013	0.013	0.013	0.013	0.0125	0.0125	0.0129
0.1:	0.438	0.438	0.438	0.438	0.438	0.438	0.438	0.438	0.394	0.394	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35
std errors:	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0155	0.0155	0.0151	0.0151	0.0151	0.0151	0.0151	0.0151	0.0151	0.0151	0.0151	0.0151	0.0151

Proportions under the null hypothesis:					0.2	0.2	0.2														
The proportions under the alternative hypothesis:					0.3808	0.3808	0.2384														
n =	50			c(j) =	0.346	0.346	-0.693					delta =	5k/4			k =	3				
lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
alpha	The bootstrap estimated power																				
0.01:	0.097	0.106	0.106	0.103	0.101	0.103	0.099	0.103	0.103	0.103	0.105	0.097	0.084	0.078	0.078	0.078	0.078	0.076	0.076	0.076	0.076
std errors:	0.0094	0.0097	0.0097	0.0096	0.0095	0.0096	0.0094	0.0096	0.0096	0.0096	0.0097	0.0094	0.0088	0.0085	0.0085	0.0085	0.0085	0.0084	0.0084	0.0084	0.0084
0.05:	0.25	0.263	0.257	0.25	0.25	0.25	0.25	0.255	0.261	0.238	0.238	0.219	0.22	0.22	0.22	0.22	0.22	0.205	0.218	0.205	0.218
std errors:	0.0137	0.0139	0.0138	0.0137	0.0137	0.0137	0.0137	0.0138	0.0139	0.0135	0.0135	0.0131	0.0131	0.0131	0.0131	0.0131	0.0131	0.0128	0.0131	0.0128	0.0131
0.1:	0.364	0.364	0.364	0.348	0.348	0.347	0.348	0.375	0.362	0.356	0.362	0.328	0.365	0.331	0.365	0.365	0.365	0.365	0.349	0.349	0.349
std errors:	0.0152	0.0152	0.0152	0.0151	0.0151	0.0151	0.0151	0.0153	0.0152	0.0151	0.0152	0.0148	0.0152	0.0149	0.0152	0.0152	0.0152	0.0152	0.0151	0.0151	0.0151
alpha	The AE approximation of power																				
0.01	0.0665	0.0717	0.0773	0.0823	0.0843	0.086	0.0907	0.0956	0.1004	0.1049	0.1091	0.1112	0.1083	0.1053	0.1056	0.1102	0.1145	0.1133	0.111	0.1083	0.1105
0.05	0.2091	0.2158	0.2221	0.228	0.2334	0.2383	0.2427	0.2425	0.2412	0.2502	0.2553	0.2548	0.2504	0.2457	0.2463	0.2512	0.2555	0.2577	0.256	0.2539	0.2514
0.1	0.2637	0.2768	0.2894	0.3016	0.3132	0.3244	0.3351	0.3453	0.3521	0.3492	0.3459	0.3494	0.3558	0.3617	0.3671	0.372	0.3765	0.3805	0.3803	0.3781	0.3755
alpha	The traditional estimated power, chi-square																				
0.01:	0.159	0.155	0.155	0.155	0.155	0.13	0.114	0.105	0.105	0.105	0.096	0.096	0.086	0.078	0.078	0.072	0.072	0.072	0.072	0.076	0.076
std errors:	0.0116	0.0114	0.0114	0.0114	0.0114	0.0106	0.0101	0.0097	0.0097	0.0097	0.0093	0.0093	0.0089	0.0085	0.0085	0.0082	0.0082	0.0082	0.0082	0.0084	0.0084
0.05:	0.297	0.297	0.274	0.274	0.274	0.274	0.274	0.274	0.274	0.274	0.238	0.215	0.22	0.22	0.22	0.22	0.205	0.205	0.205	0.218	0.218
std errors:	0.0144	0.0144	0.0141	0.0141	0.0141	0.0141	0.0141	0.0141	0.0141	0.0141	0.0135	0.013	0.0131	0.0131	0.0131	0.0131	0.0128	0.0128	0.0128	0.0131	0.0131
0.1:	0.408	0.408	0.39	0.39	0.39	0.405	0.405	0.405	0.39	0.362	0.362	0.365	0.365	0.331	0.331	0.331	0.331	0.331	0.331	0.331	0.331
std errors:	0.0155	0.0155	0.0154	0.0154	0.0154	0.0155	0.0155	0.0155	0.0154	0.0152	0.0152	0.0152	0.0152	0.0149	0.0149	0.0149	0.0149	0.0149	0.0149	0.0149	0.0149

The proportions under the alternative hypothesis:					0.3669	0.3669	0.2662														
n =	100			c(j) =	0.3354	0.3354	-0.6708					delta =	5k/4			k =	3				
lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
alpha	The bootstrap estimated power																				
0.01:	0.099	0.099	0.099	0.096	0.098	0.101	0.085	0.084	0.083	0.083	0.083	0.084	0.084	0.089	0.089	0.084	0.083	0.082	0.077	0.077	0.077
std errors:	0.0094	0.0094	0.0094	0.0093	0.0094	0.0095	0.0088	0.0088	0.0087	0.0087	0.0087	0.0088	0.0088	0.009	0.009	0.0088	0.0087	0.0087	0.0084	0.0084	0.0084
0.05:	0.251	0.239	0.243	0.243	0.243	0.243	0.243	0.235	0.235	0.236	0.237	0.237	0.23	0.225	0.219	0.22	0.23	0.224	0.224	0.224	0.228
std errors:	0.0137	0.0135	0.0136	0.0136	0.0136	0.0136	0.0136	0.0134	0.0134	0.0134	0.0134	0.0134	0.0133	0.0132	0.0131	0.0131	0.0133	0.0132	0.0132	0.0132	0.0133
0.1:	0.344	0.344	0.337	0.337	0.337	0.34	0.34	0.34	0.34	0.338	0.338	0.338	0.338	0.338	0.338	0.326	0.326	0.338	0.332	0.332	0.332
std errors:	0.015	0.015	0.0149	0.0149	0.0149	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.0148	0.0148	0.015	0.0149	0.0149	0.0149
alpha	The AE approximation of power																				
0.01	0.0711	0.0738	0.0764	0.0791	0.0839	0.0887	0.0898	0.0928	0.0933	0.0936	0.0937	0.0939	0.095	0.0958	0.0965	0.0969	0.0971	0.0971	0.0971	0.0964	0.0958
0.05	0.2217	0.2281	0.228	0.2277	0.2292	0.234	0.2387	0.2432	0.2434	0.2409	0.2383	0.2355	0.234	0.2353	0.2347	0.2378	0.2421	0.2462	0.2501	0.2524	0.2494
0.1	0.3212	0.3282	0.327	0.3289	0.3348	0.3405	0.3461	0.3515	0.3566	0.3554	0.354	0.3523	0.3505	0.3501	0.3532	0.356	0.3585	0.3609	0.3575	0.3536	0.3496
alpha	The traditional estimated power, chi-square																				
0.01:	0.13	0.119	0.118	0.111	0.109	0.112	0.112	0.112	0.099	0.087	0.09	0.091	0.091	0.089	0.089	0.089	0.084	0.086	0.086	0.085	0.088
std errors:	0.0106	0.0102	0.0102	0.0099	0.0099	0.01	0.01	0.01	0.0094	0.0089	0.009	0.0091	0.0091	0.009	0.009	0.009	0.0088	0.0089	0.0089	0.0088	0.009
0.05:	0.276	0.269	0.258	0.244	0.244	0.244	0.244	0.235	0.235	0.235	0.238	0.238	0.238	0.236	0.23	0.23	0.224	0.214	0.214	0.214	0.214
std errors:	0.0141	0.014	0.0138	0.0136	0.0136	0.0136	0.0136	0.0134	0.0134	0.0134	0.0135	0.0135	0.0135	0.0134	0.0133	0.0133	0.0132	0.013	0.013	0.013	0.013
0.1:	0.36	0.36	0.36	0.34	0.34	0.34	0.333	0.333	0.333	0.333	0.333	0.321	0.321	0.321	0.321	0.321	0.326	0.326	0.326	0.326	0.319
std errors:	0.0152	0.0152	0.0152	0.015	0.015	0.015	0.0149	0.0149	0.0149	0.0149	0.0149	0.0148	0.0148	0.0148	0.0148	0.0148	0.0148	0.0148	0.0148	0.0148	0.0147

Table 5

Proportions under the null hypothesis: 0.2 0.2 0.2

The proportions under the alternative hypothesis: 0.4108 0.4108 0.1784

n = **20** c(j) = 0.346 0.346 -0.693 delta = k k = 3

lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
alpha	The bootstrap estimated power																				
0.01:	0.072	0.047	0.072	0.06	0.072	0.072	0.072	0.047	0.053	0.081	0.081	0.072	0.072	0.072	0.072	0.075	0.061	0.052	0.052	0.048	0.044
std errors:	0.0082	0.0067	0.0082	0.0075	0.0082	0.0082	0.0082	0.0067	0.0071	0.0086	0.0086	0.0082	0.0082	0.0082	0.0082	0.0083	0.0076	0.007	0.007	0.0068	0.0065
0.05:	0.314	0.272	0.314	0.314	0.314	0.276	0.223	0.276	0.276	0.251	0.275	0.259	0.257	0.259	0.206	0.259	0.259	0.259	0.207	0.208	0.234
std errors:	0.0147	0.0141	0.0147	0.0147	0.0147	0.0141	0.0132	0.0141	0.0141	0.0137	0.0141	0.0139	0.0138	0.0139	0.0128	0.0139	0.0139	0.0139	0.0128	0.0128	0.0134
0.1:	0.483	0.483	0.407	0.407	0.403	0.483	0.483	0.404	0.48	0.431	0.425	0.445	0.445	0.445	0.432	0.445	0.403	0.445	0.403	0.361	0.403
std errors:	0.0158	0.0158	0.0155	0.0155	0.0155	0.0158	0.0158	0.0155	0.0158	0.0157	0.0156	0.0157	0.0157	0.0157	0.0157	0.0157	0.0155	0.0157	0.0155	0.0152	0.0155
alpha	The AE approximation of power																				
0.01	-0.0071	-0.0075	-0.0087	-0.0104	-0.0118	-0.011	-0.0066	0.0021	0.0146	0.0303	0.0477	0.0633	0.0645	0.0642	0.0721	0.0793	0.072	0.0642	0.056	0.0476	0.0392
0.05	0.0116	0.0278	0.0478	0.0709	0.0962	0.115	0.1403	0.1653	0.1895	0.2125	0.2341	0.2501	0.244	0.237	0.2351	0.2424	0.248	0.2519	0.2542	0.2549	0.2541
0.1	0.2749	0.2972	0.318	0.3375	0.3556	0.3723	0.3876	0.4015	0.4142	0.4256	0.4299	0.426	0.4213	0.4159	0.4098	0.4029	0.3953	0.387	0.378	0.3682	0.3576
alpha	The traditional estimated power, chi-square																				
0.01:	0.318	0.314	0.314	0.314	0.219	0.166	0.136	0.136	0.136	0.136	0.136	0.097	0.072	0.072	0.072	0.057	0.061	0.056	0.052	0.052	0.052
std errors:	0.0147	0.0147	0.0147	0.0147	0.0131	0.0118	0.0108	0.0108	0.0108	0.0108	0.0108	0.0094	0.0082	0.0082	0.0082	0.0073	0.0076	0.0073	0.007	0.007	0.007
0.05:	0.394	0.394	0.345	0.345	0.345	0.345	0.345	0.345	0.345	0.345	0.303	0.259	0.259	0.259	0.259	0.259	0.259	0.206	0.206	0.206	0.206
std errors:	0.0155	0.0155	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.0145	0.0139	0.0139	0.0139	0.0139	0.0139	0.0139	0.0128	0.0128	0.0128	0.0128
0.1:	0.561	0.561	0.561	0.483	0.483	0.403	0.403	0.403	0.403	0.403	0.403	0.403	0.403	0.403	0.403	0.403	0.403	0.403	0.361	0.403	0.403
std errors:	0.0157	0.0157	0.0157	0.0158	0.0158	0.0155	0.0155	0.0155	0.0155	0.0155	0.0155	0.0155	0.0155	0.0155	0.0155	0.0155	0.0155	0.0155	0.0152	0.0155	0.0155

The proportions under the alternative hypothesis: 0.3966 0.3966 0.2068

n = 30 c(j) = 0.346 0.346 -0.693 delta = k k = 3

lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
alpha	The bootstrap estimated power																				
0.01:	0.104	0.081	0.083	0.083	0.083	0.084	0.107	0.097	0.115	0.115	0.097	0.102	0.096	0.096	0.096	0.096	0.096	0.096	0.081	0.098	0.083
std errors:	0.0097	0.0086	0.0087	0.0087	0.0087	0.0088	0.0098	0.0094	0.0101	0.0101	0.0094	0.0096	0.0093	0.0093	0.0093	0.0093	0.0093	0.0093	0.0086	0.0094	0.0087
0.05:	0.257	0.257	0.257	0.257	0.274	0.274	0.259	0.274	0.274	0.259	0.268	0.244	0.259	0.259	0.242	0.234	0.226	0.245	0.226	0.201	0.2
std errors:	0.0138	0.0138	0.0138	0.0138	0.0141	0.0141	0.0139	0.0141	0.0141	0.0139	0.014	0.0136	0.0139	0.0139	0.0135	0.0134	0.0132	0.0136	0.0132	0.0127	0.0126
0.1:	0.356	0.357	0.379	0.387	0.375	0.375	0.375	0.375	0.387	0.386	0.346	0.346	0.387	0.387	0.387	0.346	0.375	0.349	0.366	0.375	0.346
std errors:	0.0151	0.0152	0.0153	0.0154	0.0153	0.0153	0.0153	0.0153	0.0154	0.0154	0.015	0.015	0.0154	0.0154	0.0154	0.015	0.0153	0.0151	0.0152	0.0153	0.015
alpha	The AE approximation of power																				
0.01	0.0106	0.0101	0.0109	0.0136	0.0184	0.0254	0.0346	0.0456	0.0581	0.0716	0.0789	0.0889	0.0986	0.1001	0.1007	0.1003	0.1057	0.1102	0.1135	0.1111	0.1041
0.05	0.1562	0.1672	0.1775	0.1869	0.1956	0.2033	0.2102	0.216	0.221	0.2249	0.2279	0.2299	0.2389	0.2518	0.2639	0.2749	0.2686	0.2617	0.2542	0.2461	0.2375
0.1	0.2276	0.2483	0.2682	0.2873	0.292	0.2979	0.3126	0.3262	0.339	0.3508	0.3617	0.3717	0.3808	0.3891	0.3966	0.4033	0.4001	0.3963	0.392	0.387	0.3815
alpha	The traditional estimated power, chi-square																				
0.01:	0.22	0.22	0.205	0.146	0.146	0.128	0.123	0.123	0.115	0.115	0.097	0.083	0.083	0.073	0.073	0.073	0.073	0.065	0.065	0.065	0.065
std errors:	0.0131	0.0131	0.0128	0.0112	0.0112	0.0106	0.0104	0.0104	0.0101	0.0101	0.0094	0.0087	0.0087	0.0082	0.0082	0.0082	0.0082	0.0078	0.0078	0.0078	0.0078
0.05:	0.41	0.356	0.315	0.315	0.274	0.274	0.274	0.274	0.274	0.274	0.274	0.274	0.259	0.226	0.226	0.226	0.226	0.226	0.2	0.2	0.219
std errors:	0.0156	0.0151	0.0147	0.0147	0.0141	0.0141	0.0141	0.0141	0.0141	0.0141	0.0141	0.0141	0.0139	0.0132	0.0132	0.0132	0.0132	0.0132	0.0126	0.0126	0.0131
0.1:	0.441	0.441	0.441	0.441	0.441	0.441	0.441	0.441	0.387	0.387	0.346	0.346	0.346	0.346	0.346	0.346	0.346	0.346	0.346	0.346	0.346
std errors:	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0154	0.0154	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015

Proportions under the null hypothesis: 0.2 0.2 0.2

The proportions under the alternative hypothesis: 0.3823 0.3823 0.2354

n = 50 c(j) = 0.346 0.346 -0.693 delta = k k = 3

lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
alpha	The bootstrap estimated power																				
0.01:	0.121	0.107	0.108	0.111	0.108	0.111	0.103	0.111	0.11	0.111	0.111	0.107	0.094	0.086	0.086	0.086	0.086	0.08	0.089	0.084	0.081
std errors:	0.0103	0.0098	0.0098	0.0099	0.0098	0.0099	0.0096	0.0099	0.0099	0.0099	0.0099	0.0098	0.0092	0.0089	0.0089	0.0089	0.0089	0.0086	0.009	0.0088	0.0086
0.05:	0.256	0.256	0.256	0.256	0.257	0.257	0.23	0.257	0.249	0.235	0.238	0.248	0.24	0.24	0.24	0.24	0.24	0.225	0.24	0.225	0.24
std errors:	0.0138	0.0138	0.0138	0.0138	0.0138	0.0138	0.0133	0.0138	0.0137	0.0134	0.0135	0.0137	0.0135	0.0135	0.0135	0.0135	0.0135	0.0132	0.0135	0.0132	0.0135
0.1:	0.369	0.382	0.371	0.371	0.382	0.382	0.371	0.382	0.351	0.348	0.351	0.331	0.359	0.339	0.359	0.358	0.359	0.359	0.357	0.356	0.357
std errors:	0.0153	0.0154	0.0153	0.0153	0.0154	0.0154	0.0153	0.0154	0.0151	0.0151	0.0151	0.0149	0.0152	0.015	0.0152	0.0152	0.0152	0.0152	0.0152	0.0151	0.0152
alpha	The AE approximation of power																				
0.01	0.0698	0.0758	0.0801	0.0834	0.0897	0.0963	0.1029	0.1096	0.1156	0.1162	0.1139	0.1112	0.1083	0.1053	0.1056	0.1102	0.1145	0.1185	0.122	0.1212	0.1169
0.05	0.178	0.1787	0.1821	0.1859	0.195	0.2065	0.2178	0.2289	0.2397	0.2394	0.2469	0.2556	0.2585	0.2594	0.2597	0.2595	0.2588	0.2593	0.2626	0.2653	0.2676
0.1	0.2882	0.2917	0.2962	0.3088	0.3209	0.3325	0.3436	0.3543	0.3521	0.3492	0.3459	0.3494	0.3558	0.3617	0.3671	0.372	0.3765	0.3805	0.3803	0.3781	0.3755
alpha	The traditional estimated power, chi-square																				
0.01:	0.168	0.159	0.159	0.159	0.159	0.133	0.116	0.108	0.108	0.108	0.104	0.104	0.094	0.086	0.086	0.077	0.077	0.077	0.077	0.08	0.077
std errors:	0.0118	0.0116	0.0116	0.0116	0.0116	0.0107	0.0101	0.0098	0.0098	0.0098	0.0097	0.0097	0.0092	0.0089	0.0089	0.0084	0.0084	0.0084	0.0084	0.0086	0.0084
0.05:	0.298	0.298	0.276	0.276	0.276	0.276	0.276	0.276	0.276	0.276	0.238	0.219	0.229	0.229	0.229	0.229	0.214	0.214	0.214	0.225	0.225
std errors:	0.0145	0.0145	0.0141	0.0141	0.0141	0.0141	0.0141	0.0141	0.0141	0.0141	0.0135	0.0131	0.0133	0.0133	0.0133	0.0133	0.013	0.013	0.013	0.0132	0.0132
0.1:	0.4	0.4	0.382	0.382	0.382	0.392	0.392	0.392	0.381	0.351	0.351	0.359	0.359	0.339	0.339	0.339	0.339	0.339	0.339	0.339	0.339
std errors:	0.0155	0.0155	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0151	0.0151	0.0152	0.0152	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015

The proportions under the alternative hypothesis: 0.368 0.368 0.264

n = 100 c(j) = 0.3641 0.3641 -0.6929 delta = k k = 3

lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
alpha	The bootstrap estimated power																				
0.01:	0.091	0.089	0.091	0.094	0.097	0.099	0.096	0.098	0.095	0.095	0.094	0.091	0.091	0.091	0.081	0.081	0.081	0.088	0.085	0.082	0.082
std errors:	0.0091	0.009	0.0091	0.0092	0.0094	0.0094	0.0093	0.0094	0.0093	0.0093	0.0092	0.0091	0.0091	0.0091	0.0086	0.0086	0.0086	0.009	0.0088	0.0087	0.0087
0.05:	0.234	0.228	0.234	0.235	0.237	0.239	0.239	0.239	0.235	0.235	0.235	0.237	0.221	0.219	0.223	0.225	0.225	0.225	0.225	0.225	0.225
std errors:	0.0134	0.0133	0.0134	0.0134	0.0134	0.0135	0.0135	0.0135	0.0134	0.0134	0.0134	0.0134	0.0131	0.0131	0.0132	0.0132	0.0132	0.0132	0.0132	0.0132	0.0132
0.1:	0.366	0.362	0.366	0.369	0.359	0.37	0.355	0.355	0.366	0.361	0.363	0.363	0.355	0.363	0.35	0.351	0.351	0.363	0.356	0.356	0.356
std errors:	0.0152	0.0152	0.0152	0.0153	0.0152	0.0153	0.0151	0.0151	0.0152	0.0152	0.0152	0.0152	0.0151	0.0152	0.0151	0.0151	0.0151	0.0152	0.0151	0.0151	0.0151
alpha	The AE approximation of power																				
0.01	0.0783	0.0831	0.0881	0.0932	0.0984	0.1037	0.106	0.1092	0.1098	0.1102	0.1103	0.1095	0.1075	0.1083	0.1087	0.1077	0.1111	0.1144	0.114	0.1116	0.1113
0.05	0.2137	0.2186	0.2233	0.2278	0.2321	0.2361	0.2399	0.2435	0.2468	0.2499	0.2527	0.2553	0.2576	0.2597	0.2599	0.2591	0.258	0.2566	0.2551	0.2562	0.2588
0.1	0.3372	0.3439	0.3503	0.3565	0.3625	0.3682	0.3738	0.3792	0.3844	0.3832	0.3818	0.3801	0.3783	0.3779	0.3809	0.3837	0.3863	0.3887	0.3853	0.3814	0.3773
alpha	The traditional estimated power, chi-square																				
0.01:	0.133	0.118	0.116	0.114	0.107	0.11	0.11	0.11	0.104	0.101	0.098	0.098	0.098	0.096	0.096	0.096	0.093	0.094	0.094	0.09	0.09
std errors:	0.0107	0.0102	0.0101	0.0101	0.0098	0.0099	0.0099	0.0099	0.0097	0.0095	0.0094	0.0094	0.0094	0.0093	0.0093	0.0093	0.0092	0.0092	0.0092	0.009	0.009
0.05:	0.289	0.274	0.261	0.251	0.251	0.251	0.251	0.248	0.248	0.248	0.252	0.252	0.252	0.252	0.236	0.236	0.231	0.225	0.225	0.225	0.225
std errors:	0.0143	0.0141	0.0139	0.0137	0.0137	0.0137	0.0137	0.0137	0.0137	0.0137	0.0137	0.0137	0.0137	0.0137	0.0134	0.0134	0.0133	0.0132	0.0132	0.0132	0.0132
0.1:	0.404	0.404	0.404	0.37	0.37	0.37	0.355	0.355	0.355	0.355	0.355	0.342	0.342	0.342	0.342	0.342	0.35	0.35	0.35	0.35	0.335
std errors:	0.0155	0.0155	0.0155	0.0153	0.0153	0.0153	0.0151	0.0151	0.0151	0.0151	0.0151	0.015	0.015	0.015	0.015	0.015	0.0151	0.0151	0.0151	0.0151	0.0149

Proportions under the null hypothesis:					0.2	0.2	0.2	0.2													
The proportions under the alternative hypothesis:					0.208	0.208	0.208	0.376													
n =	20			c(j) =	0.707	-0.707	0.707	-0.707				delta =	0.5			k =	4				
lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
alpha	The bootstrap estimated power																				
0.01:	0.029	0.019	0.028	0.028	0.028	0.024	0.028	0.024	0.027	0.036	0.053	0.054	0.06	0.071	0.071	0.093	0.086	0.091	0.091	0.092	0.091
std errors:	0.0053	0.0043	0.0052	0.0052	0.0052	0.0048	0.0052	0.0048	0.0051	0.0059	0.0071	0.0071	0.0075	0.0081	0.0081	0.0092	0.0089	0.0091	0.0091	0.0091	0.0091
0.05:	0.137	0.138	0.142	0.141	0.144	0.15	0.15	0.14	0.178	0.167	0.169	0.169	0.16	0.187	0.184	0.18	0.183	0.18	0.17	0.173	0.185
std errors:	0.0109	0.0109	0.011	0.011	0.0111	0.0113	0.0113	0.011	0.0121	0.0118	0.0119	0.0119	0.0116	0.0123	0.0123	0.0121	0.0122	0.0121	0.0119	0.012	0.0123
0.1:	0.244	0.226	0.219	0.237	0.245	0.225	0.25	0.224	0.251	0.255	0.245	0.245	0.272	0.271	0.244	0.259	0.222	0.25	0.25	0.249	0.252
std errors:	0.0136	0.0132	0.0131	0.0134	0.0136	0.0132	0.0137	0.0132	0.0137	0.0138	0.0136	0.0136	0.0141	0.0141	0.0136	0.0139	0.0131	0.0137	0.0137	0.0137	0.0137
alpha	The AE approximation of power																				
0.01	0	0	0	0	0	0	-0.0004	-0.0095	0.0797	0.2421	0.3908	0.438	0.4609	0.5034	0.5165	0.5259	0.5091	0.4993	0.5066	0.5075	0.4965
0.05	0.0991	0.18	0.2693	0.3576	0.4389	0.5067	0.569	0.6212	0.6644	0.6819	0.6946	0.712	0.7277	0.7213	0.7129	0.7304	0.7169	0.7011	0.683	0.6782	0.6563
0.1	0.6494	0.6694	0.6863	0.7003	0.7117	0.7207	0.7275	0.7323	0.7544	0.7716	0.7782	0.8007	0.8109	0.8114	0.8107	0.809	0.8063	0.8058	0.8225	0.8309	0.8283
alpha	The traditional estimated power, chi-square																				
0.01:	0.191	0.18	0.176	0.176	0.121	0.112	0.088	0.079	0.073	0.066	0.061	0.054	0.046	0.043	0.043	0.043	0.05	0.06	0.06	0.088	0.091
std errors:	0.0124	0.0121	0.012	0.012	0.0103	0.01	0.009	0.0085	0.0082	0.0079	0.0076	0.0071	0.0066	0.0064	0.0064	0.0064	0.0069	0.0075	0.0075	0.009	0.0091
0.05:	0.245	0.245	0.245	0.218	0.218	0.218	0.218	0.204	0.186	0.169	0.158	0.158	0.156	0.156	0.148	0.143	0.143	0.17	0.178	0.195	0.197
std errors:	0.0136	0.0136	0.0136	0.0131	0.0131	0.0131	0.0131	0.0127	0.0123	0.0119	0.0115	0.0115	0.0115	0.0115	0.0112	0.0111	0.0111	0.0119	0.0121	0.0125	0.0126
0.1:	0.354	0.354	0.317	0.317	0.317	0.287	0.287	0.287	0.259	0.259	0.245	0.227	0.227	0.244	0.222	0.222	0.222	0.221	0.221	0.245	0.245
std errors:	0.0151	0.0151	0.0147	0.0147	0.0147	0.0143	0.0143	0.0143	0.0139	0.0139	0.0136	0.0132	0.0132	0.0136	0.0131	0.0131	0.0131	0.0131	0.0131	0.0136	0.0136
The proportions under the alternative hypothesis:					0.208	0.208	0.208	0.376													
n =	30			c(j) =	0.707	0.707	-0.707	-0.707				delta =	0.5			k =	4				
lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
alpha	The bootstrap estimated power																				
0.01:	0.057	0.061	0.061	0.065	0.064	0.063	0.069	0.071	0.073	0.082	0.09	0.096	0.092	0.093	0.094	0.099	0.095	0.106	0.099	0.099	0.092
std errors:	0.0073	0.0076	0.0076	0.0078	0.0077	0.0077	0.008	0.0081	0.0082	0.0087	0.009	0.0093	0.0091	0.0092	0.0092	0.0094	0.0093	0.0097	0.0094	0.0094	0.0091
0.05:	0.165	0.165	0.159	0.159	0.162	0.163	0.182	0.186	0.201	0.222	0.228	0.219	0.224	0.223	0.224	0.237	0.242	0.234	0.237	0.231	0.229
std errors:	0.0117	0.0117	0.0116	0.0116	0.0117	0.0117	0.0122	0.0123	0.0127	0.0131	0.0133	0.0131	0.0132	0.0132	0.0132	0.0134	0.0135	0.0134	0.0134	0.0133	0.0133
0.1:	0.267	0.274	0.286	0.292	0.291	0.299	0.296	0.304	0.306	0.305	0.307	0.311	0.31	0.311	0.328	0.324	0.328	0.324	0.324	0.338	0.353
std errors:	0.014	0.0141	0.0143	0.0144	0.0144	0.0145	0.0144	0.0145	0.0146	0.0146	0.0146	0.0146	0.0146	0.0146	0.0148	0.0148	0.0148	0.0148	0.0148	0.015	0.0151
alpha	The AE approximation of power																				
0.01	0.0988	0.1364	0.1772	0.2191	0.2513	0.2721	0.2931	0.3188	0.3403	0.3947	0.4075	0.4366	0.4358	0.4303	0.4591	0.457	0.4447	0.4341	0.4104	0.3846	0.3568
0.05	0.4118	0.435	0.455	0.4719	0.4857	0.5124	0.5653	0.6001	0.6378	0.6426	0.6647	0.6702	0.6888	0.6814	0.6769	0.6761	0.6738	0.6641	0.664	0.6582	0.6501
0.1	0.6178	0.6382	0.6665	0.6916	0.7137	0.7229	0.7306	0.7446	0.7412	0.7431	0.749	0.7477	0.7665	0.7759	0.7714	0.766	0.7772	0.7848	0.7758	0.7673	0.766
alpha	The traditional estimated power, chi-square																				
0.01:	0.165	0.165	0.159	0.149	0.141	0.125	0.114	0.108	0.102	0.098	0.1	0.096	0.094	0.094	0.093	0.089	0.095	0.106	0.121	0.124	0.127
std errors:	0.0117	0.0117	0.0116	0.0113	0.011	0.0105	0.0101	0.0098	0.0096	0.0094	0.0095	0.0093	0.0092	0.0092	0.0092	0.009	0.0093	0.0097	0.0103	0.0104	0.0105
0.05:	0.329	0.306	0.295	0.282	0.267	0.27	0.254	0.254	0.241	0.236	0.233	0.228	0.223	0.223	0.235	0.246	0.25	0.25	0.257	0.255	0.253
std errors:	0.0149	0.0146	0.0144	0.0142	0.014	0.014	0.0138	0.0138	0.0135	0.0134	0.0134	0.0133	0.0132	0.0132	0.0134	0.0136	0.0137	0.0137	0.0138	0.0138	0.0137
0.1:	0.394	0.394	0.394	0.394	0.379	0.367	0.356	0.343	0.332	0.325	0.325	0.346	0.336	0.331	0.328	0.328	0.328	0.324	0.339	0.354	0.376
std errors:	0.0155	0.0155	0.0155	0.0155	0.0153	0.0152	0.0151	0.015	0.0149	0.0148	0.0148	0.015	0.0149	0.0149	0.0148	0.0148	0.0148	0.0148	0.015	0.0151	0.0153

Proportions under the null hypothesis: 0.2 0.2 0.2 0.2

The proportions under the alternative hypothesis: 0.208 0.208 0.208 0.376

n = 50 c(j) = 0.707 0.707 -0.707 -0.707 delta = 0.5 k = 4

lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
alpha	The bootstrap estimated power																				
0.01:	0.1	0.107	0.107	0.117	0.12	0.127	0.134	0.144	0.152	0.154	0.154	0.167	0.185	0.184	0.19	0.189	0.197	0.197	0.197	0.201	0.198
std errors:	0.0095	0.0098	0.0098	0.0102	0.0103	0.0105	0.0108	0.0111	0.0114	0.0114	0.0114	0.0118	0.0123	0.0123	0.0124	0.0124	0.0126	0.0126	0.0126	0.0127	0.0126
0.05:	0.266	0.278	0.283	0.287	0.288	0.297	0.301	0.307	0.314	0.341	0.342	0.342	0.353	0.367	0.372	0.381	0.375	0.381	0.381	0.387	0.386
std errors:	0.014	0.0142	0.0142	0.0143	0.0143	0.0144	0.0145	0.0146	0.0147	0.015	0.015	0.015	0.0151	0.0152	0.0153	0.0154	0.0153	0.0154	0.0154	0.0154	0.0154
0.1:	0.411	0.414	0.414	0.417	0.428	0.432	0.436	0.439	0.443	0.443	0.448	0.455	0.459	0.459	0.464	0.46	0.465	0.474	0.481	0.48	0.48
std errors:	0.0156	0.0156	0.0156	0.0156	0.0156	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158
alpha	The AE approximation of power																				
0.01	0.3755	0.398	0.419	0.4383	0.4559	0.4687	0.4813	0.499	0.5165	0.5195	0.5297	0.5337	0.5419	0.5418	0.5437	0.537	0.5358	0.5263	0.5176	0.5122	0.5025
0.05	0.6225	0.6303	0.6324	0.6368	0.6508	0.6594	0.666	0.6701	0.6783	0.696	0.7064	0.7021	0.7127	0.7178	0.7167	0.7254	0.7211	0.7161	0.7186	0.7171	0.709
0.1	0.7299	0.7317	0.7378	0.7456	0.7552	0.7627	0.7698	0.7705	0.7792	0.7891	0.7937	0.7931	0.7966	0.7965	0.8014	0.7969	0.7943	0.7969	0.7998	0.7968	0.7933
alpha	The traditional estimated power, chi-square																				
0.01:	0.197	0.192	0.181	0.175	0.171	0.167	0.163	0.161	0.157	0.156	0.153	0.159	0.165	0.167	0.182	0.182	0.188	0.199	0.203	0.216	0.233
std errors:	0.0126	0.0125	0.0122	0.012	0.0119	0.0118	0.0117	0.0116	0.0115	0.0115	0.0114	0.0116	0.0117	0.0118	0.0122	0.0122	0.0124	0.0126	0.0127	0.013	0.0134
0.05:	0.388	0.384	0.37	0.362	0.351	0.358	0.35	0.35	0.348	0.345	0.342	0.349	0.347	0.346	0.363	0.368	0.371	0.371	0.369	0.374	0.374
std errors:	0.0154	0.0154	0.0153	0.0152	0.0151	0.0152	0.0151	0.0151	0.0151	0.015	0.015	0.0151	0.0151	0.015	0.0152	0.0153	0.0153	0.0153	0.0153	0.0153	0.0153
0.1:	0.484	0.471	0.471	0.468	0.472	0.461	0.459	0.451	0.447	0.443	0.441	0.453	0.453	0.456	0.455	0.454	0.465	0.474	0.473	0.473	0.473
std errors:	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0158	0.0157	0.0157	0.0158	0.0158	0.0158	0.0158	0.0158

The proportions under the alternative hypothesis: 0.208 0.208 0.208 0.376

n = 100 c(j) = 0.707 0.707 -0.707 -0.707 delta = 0.5 k = 4

lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
alpha	The bootstrap estimated power																				
0.01:	0.248	0.26	0.278	0.292	0.305	0.332	0.337	0.343	0.355	0.371	0.37	0.37	0.377	0.381	0.396	0.402	0.413	0.423	0.426	0.421	0.433
std errors:	0.0137	0.0139	0.0142	0.0144	0.0146	0.0149	0.0149	0.015	0.0151	0.0153	0.0153	0.0153	0.0153	0.0154	0.0155	0.0155	0.0156	0.0156	0.0156	0.0156	0.0157
0.05:	0.514	0.526	0.535	0.54	0.547	0.549	0.554	0.559	0.568	0.578	0.588	0.598	0.607	0.618	0.624	0.625	0.626	0.64	0.645	0.647	0.65
std errors:	0.0158	0.0158	0.0158	0.0158	0.0157	0.0157	0.0157	0.0157	0.0157	0.0156	0.0156	0.0155	0.0154	0.0154	0.0153	0.0153	0.0153	0.0152	0.0151	0.0151	0.0151
0.1:	0.657	0.662	0.665	0.666	0.666	0.675	0.681	0.687	0.701	0.704	0.706	0.71	0.719	0.718	0.726	0.73	0.736	0.737	0.74	0.74	0.746
std errors:	0.015	0.015	0.0149	0.0149	0.0149	0.0148	0.0147	0.0147	0.0145	0.0144	0.0144	0.0143	0.0142	0.0142	0.0141	0.014	0.0139	0.0139	0.0139	0.0139	0.0138
alpha	The AE approximation of power																				
0.01	0.4001	0.4167	0.4285	0.441	0.4545	0.4672	0.475	0.4782	0.4875	0.4912	0.4882	0.4892	0.49	0.4935	0.4921	0.4944	0.496	0.4968	0.4943	0.4841	0.4832
0.05	0.6196	0.6278	0.6345	0.6378	0.6408	0.6455	0.6479	0.6551	0.6622	0.667	0.6682	0.6679	0.6725	0.6773	0.6788	0.6808	0.6818	0.6861	0.6838	0.6825	0.6835
0.1	0.7289	0.7294	0.7311	0.7343	0.7407	0.7472	0.751	0.7561	0.7649	0.7689	0.7683	0.7709	0.7738	0.7743	0.7709	0.7719	0.7684	0.771	0.7704	0.7706	0.7704
alpha	The traditional estimated power, chi-square																				
0.01:	0.39	0.391	0.392	0.391	0.394	0.396	0.393	0.391	0.395	0.398	0.396	0.4	0.407	0.42	0.424	0.434	0.438	0.443	0.454	0.463	0.466
std errors:	0.0154	0.0154	0.0154	0.0154	0.0155	0.0155	0.0154	0.0154	0.0155	0.0155	0.0155	0.0155	0.0155	0.0156	0.0156	0.0157	0.0157	0.0157	0.0157	0.0158	0.0158
0.05:	0.616	0.62	0.615	0.614	0.613	0.619	0.622	0.626	0.625	0.631	0.629	0.629	0.633	0.629	0.632	0.639	0.65	0.651	0.653	0.657	0.657
std errors:	0.0154	0.0153	0.0154	0.0154	0.0154	0.0154	0.0153	0.0153	0.0153	0.0153	0.0153	0.0153	0.0152	0.0153	0.0153	0.0152	0.0151	0.0151	0.0151	0.015	0.015
0.1:	0.732	0.732	0.726	0.725	0.723	0.723	0.725	0.724	0.722	0.728	0.726	0.735	0.738	0.742	0.744	0.744	0.744	0.75	0.751	0.753	0.753
std errors:	0.014	0.014	0.0141	0.0141	0.0142	0.0142	0.0141	0.0141	0.0142	0.0141	0.0141	0.014	0.0139	0.0138	0.0138	0.0138	0.0138	0.0137	0.0137	0.0136	0.0136

Table 7

Proportions under the null hypothesis:	0.2	0.2	0.2	0.2	
The proportions under the alternative hypothesis:	0.125	0.125	0.125	0.625	
n = 50	c(j) = 1.225	-1.225	-1.225	1.225	delta = 1.5 k = 4

lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
0.01:	0.192	0.192	0.191	0.192	0.192	0.192	0.192	0.227	0.314	0.484	0.559	0.651	0.698	0.731	0.745	0.745	0.744	0.743	0.743	0.742	0.742
std errors:	0.0125	0.0125	0.0124	0.0125	0.0125	0.0125	0.0125	0.0132	0.0147	0.0158	0.0157	0.0151	0.0145	0.014	0.0138	0.0138	0.0138	0.0138	0.0138	0.0138	0.0138
0.05:	0.642	0.642	0.683	0.683	0.713	0.774	0.774	0.781	0.83	0.826	0.841	0.841	0.841	0.841	0.841	0.871	0.871	0.868	0.868	0.879	0.882
std errors:	0.0152	0.0152	0.0147	0.0147	0.0143	0.0132	0.0132	0.0131	0.0119	0.012	0.0116	0.0116	0.0116	0.0116	0.0116	0.0106	0.0106	0.0107	0.0107	0.0103	0.0102
0.1:	0.645	0.686	0.686	0.716	0.777	0.782	0.834	0.834	0.849	0.878	0.893	0.893	0.931	0.931	0.931	0.937	0.931	0.935	0.937	0.935	0.937
std errors:	0.0151	0.0147	0.0147	0.0143	0.0132	0.0131	0.0118	0.0118	0.0113	0.0103	0.0098	0.0098	0.008	0.008	0.008	0.0077	0.008	0.0078	0.0077	0.0078	0.0077
alpha	The AE approximation of power																				
0.01	0	0	0	0	0	0	-0.0657	0.4819	0.8943	0.9856	1.003	1.0062	1.0067	1.0066	1.0066	1.0067	1.0067	1.0067	1.0066	1.0061	1.0049
0.05	0.9327	0.9775	0.9972	1.0046	1.0066	1.0065	1.0057	1.0048	1.0046	1.0039	1.0037	1.0035	1.0034	1.0033	1.0032	1.0032	1.0034	1.0036	1.0036	1.0039	1.0044
0.1	0.9572	0.9908	1.0033	1.0065	1.0064	1.0052	1.0044	1.0034	1.0026	1.0023	1.0018	1.0015	1.0016	1.0017	1.0015	1.0014	1.0014	1.0015	1.0015	1.0016	1.0018
alpha	The traditional estimated power, chi-square																				
0.01:	0.736	0.731	0.716	0.716	0.713	0.708	0.698	0.689	0.681	0.656	0.653	0.698	0.697	0.696	0.695	0.695	0.72	0.743	0.743	0.81	0.811
std errors:	0.0139	0.014	0.0143	0.0143	0.0143	0.0144	0.0145	0.0146	0.0147	0.015	0.0151	0.0145	0.0145	0.0145	0.0146	0.0146	0.0142	0.0138	0.0138	0.0124	0.0124
0.05:	0.864	0.864	0.864	0.834	0.834	0.834	0.834	0.834	0.833	0.846	0.841	0.841	0.841	0.841	0.841	0.841	0.841	0.871	0.884	0.923	0.923
std errors:	0.0108	0.0108	0.0108	0.0118	0.0118	0.0118	0.0118	0.0118	0.0118	0.0114	0.0116	0.0116	0.0116	0.0116	0.0116	0.0116	0.0116	0.0106	0.0101	0.0084	0.0084
0.1:	0.913	0.913	0.901	0.901	0.901	0.899	0.899	0.899	0.893	0.893	0.893	0.892	0.892	0.931	0.931	0.931	0.931	0.931	0.931	0.937	0.937
std errors:	0.0089	0.0089	0.0094	0.0094	0.0094	0.0095	0.0095	0.0095	0.0098	0.0098	0.0098	0.0098	0.0098	0.008	0.008	0.008	0.008	0.008	0.008	0.0077	0.0077

The proportions under the alternative hypothesis:	0.125	0.125	0.125	0.625	
n = 30	c(j) = 1.225	-1.225	-1.225	1.225	delta = 1.5 k = 4

lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
alpha	The bootstrap estimated power																				
0.01:	0.281	0.31	0.388	0.427	0.522	0.596	0.686	0.756	0.77	0.819	0.841	0.853	0.869	0.853	0.853	0.866	0.896	0.898	0.897	0.899	0.903
std errors:	0.0142	0.0146	0.0154	0.0156	0.0158	0.0155	0.0147	0.0136	0.0133	0.0122	0.0116	0.0112	0.0107	0.0112	0.0112	0.0108	0.0097	0.0096	0.0096	0.0095	0.0094
0.05:	0.9	0.915	0.915	0.915	0.915	0.928	0.936	0.936	0.965	0.968	0.969	0.969	0.968	0.972	0.973	0.976	0.981	0.98	0.98	0.981	0.981
std errors:	0.0095	0.0088	0.0088	0.0088	0.0088	0.0082	0.0077	0.0077	0.0058	0.0056	0.0055	0.0055	0.0056	0.0052	0.0051	0.0048	0.0043	0.0044	0.0044	0.0043	0.0043
0.1:	0.972	0.972	0.972	0.975	0.98	0.98	0.983	0.983	0.987	0.987	0.987	0.987	0.987	0.987	0.987	0.987	0.99	0.99	0.99	0.992	0.991
std errors:	0.0052	0.0052	0.0052	0.0049	0.0044	0.0044	0.0041	0.0041	0.0036	0.0036	0.0036	0.0036	0.0036	0.0036	0.0036	0.0036	0.0031	0.0031	0.0031	0.0028	0.003
alpha	The AE approximation of power																				
0.01	0.023	0.1602	0.3809	0.5935	0.7604	0.8667	0.9214	0.9483	0.9584	0.9735	0.9797	0.9854	0.9867	0.9847	0.9821	0.981	0.9795	0.9799	0.9799	0.9795	0.9787
0.05	0.9937	0.9942	0.9952	0.9958	0.9963	0.9967	0.998	0.9987	0.999	0.9991	0.9994	0.9995	0.9995	0.9994	0.9993	0.9994	0.9994	0.9993	0.9993	0.9992	0.9992
0.1	0.999	0.9993	0.9995	0.9996	0.9997	0.9997	0.9998	0.9998	0.9998	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998
alpha	The traditional estimated power, chi-square																				
0.01:	0.894	0.894	0.885	0.883	0.883	0.879	0.879	0.879	0.876	0.874	0.895	0.904	0.905	0.905	0.905	0.905	0.914	0.924	0.948	0.951	0.951
std errors:	0.0097	0.0097	0.0101	0.0102	0.0102	0.0103	0.0103	0.0103	0.0104	0.0105	0.0097	0.0093	0.0093	0.0093	0.0093	0.0093	0.0089	0.0084	0.007	0.0068	0.0068
0.05:	0.972	0.972	0.971	0.971	0.97	0.973	0.969	0.969	0.969	0.969	0.969	0.968	0.968	0.968	0.974	0.977	0.981	0.981	0.982	0.982	0.982
std errors:	0.0052	0.0052	0.0053	0.0053	0.0054	0.0051	0.0055	0.0055	0.0055	0.0055	0.0055	0.0056	0.0056	0.0056	0.005	0.0047	0.0043	0.0043	0.0042	0.0042	0.0042
0.1:	0.984	0.984	0.984	0.984	0.984	0.984	0.984	0.988	0.987	0.987	0.987	0.987	0.987	0.987	0.987	0.987	0.987	0.987	0.99	0.992	0.993
std errors:	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.0034	0.0036	0.0036	0.0036	0.0036	0.0036	0.0036	0.0036	0.0036	0.0036	0.0036	0.0031	0.0028	0.0026

Table 7

Proportions under the null hypothesis: 0.2 0.2 0.2 0.2

The proportions under the alternative hypothesis: 0.125 0.125 0.125 0.625

n = **50** c(j) = 1.225 -1.225 -1.225 1.225 delta = 1.5 k = 4

lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
alpha	The bootstrap estimated power																				
0.01:	0.958	0.974	0.975	0.975	0.975	0.983	0.985	0.988	0.99	0.991	0.991	0.992	0.994	0.995	0.995	0.995	0.996	0.996	0.996	0.996	0.996
std errors:	0.0063	0.005	0.0049	0.0049	0.0049	0.0041	0.0038	0.0034	0.0031	0.003	0.003	0.0028	0.0024	0.0022	0.0022	0.0022	0.002	0.002	0.002	0.002	0.002
0.05:	0.996	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998
std errors:	0.002	0.0017	0.0017	0.0017	0.0017	0.0017	0.0017	0.0017	0.0017	0.0017	0.0017	0.0017	0.0014	0.0014	0.0014	0.0014	0.0014	0.0014	0.0014	0.0014	0.0014
0.1:	0.997	0.997	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	1	0.999
std errors:	0.0017	0.0017	0.0014	0.0014	0.0014	0.0014	0.0014	0.0014	0.0014	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0	0.001
alpha	The AE approximation of power																				
0.01	0.9762	0.9783	0.9795	0.9807	0.982	0.9845	0.9863	0.9887	0.99	0.9903	0.9908	0.9909	0.991	0.9917	0.9913	0.9908	0.9907	0.9902	0.9901	0.9897	0.9888
0.05	0.9951	0.9954	0.9957	0.9959	0.996	0.9964	0.9966	0.9967	0.9971	0.9972	0.9972	0.9973	0.9972	0.9974	0.9973	0.9972	0.9972	0.9973	0.9974	0.9974	0.9972
0.1	0.9977	0.9978	0.9979	0.998	0.9982	0.9982	0.9984	0.9985	0.9986	0.9986	0.9986	0.9987	0.9988	0.9988	0.9988	0.9989	0.9988	0.9987	0.9987	0.9987	0.9986
alpha	The traditional estimated power, chi-square																				
0.01:	0.989	0.989	0.989	0.991	0.991	0.991	0.991	0.991	0.991	0.991	0.991	0.992	0.994	0.994	0.995	0.995	0.996	0.996	0.996	0.996	0.996
std errors:	0.0033	0.0033	0.0033	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.0028	0.0024	0.0024	0.0022	0.0022	0.002	0.002	0.002	0.002	0.002
0.05:	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998
std errors:	0.0017	0.0017	0.0017	0.0017	0.0017	0.0017	0.0017	0.0017	0.0017	0.0017	0.0017	0.0014	0.0014	0.0014	0.0014	0.0014	0.0014	0.0014	0.0014	0.0014	0.0014
0.1:	1	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	1	1	1	1
std errors:	0	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0	0	0	0

The proportions under the alternative hypothesis: 0.125 0.125 0.125 0.625

n = **100** c(j) = 1.225 -1.225 -1.225 1.225 delta = 1.5 k = 4

lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
alpha	The bootstrap estimated power																				
0.01:	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
std errors:	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.05:	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
std errors:	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.1:	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
std errors:	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
alpha	The AE approximation of power																				
0.01	0.9641	0.9658	0.9677	0.9696	0.9697	0.9732	0.9746	0.9774	0.9795	0.9807	0.9817	0.9826	0.983	0.984	0.9845	0.9842	0.9839	0.9836	0.9837	0.9834	0.9826
0.05	0.9928	0.9932	0.9935	0.9938	0.9941	0.9944	0.9945	0.9946	0.9947	0.9949	0.9948	0.995	0.9954	0.9952	0.9952	0.995	0.9952	0.9951	0.9951	0.9951	0.9949
0.1	0.997	0.9972	0.9973	0.9973	0.9974	0.9975	0.9976	0.9976	0.9976	0.9978	0.9978	0.9978	0.9979	0.9979	0.9979	0.9979	0.9979	0.9979	0.9979	0.9979	0.9978
alpha	The traditional estimated power, chi-square																				
0.01:	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
std errors:	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.05:	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
std errors:	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.1:	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
std errors:	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Proportions under the null hypothesis: 0.2 0.2 0.2 0.2

The proportions under the alternative hypothesis: 0.3468 0.3468 0.1532 0.1532

n = 20 c(j) = 0.433 0.433 -0.433 -0.433 delta = 3k/4 k = 4

lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
alpha	The bootstrap estimated power																				
0.01:	0.085	0.074	0.074	0.078	0.078	0.074	0.078	0.078	0.081	0.09	0.1	0.115	0.12	0.118	0.107	0.094	0.092	0.092	0.092	0.088	0.085
std errors:	0.0088	0.0083	0.0083	0.0085	0.0085	0.0083	0.0085	0.0085	0.0086	0.009	0.0095	0.0101	0.0103	0.0102	0.0098	0.0092	0.0091	0.0091	0.0091	0.009	0.0088
0.05:	0.254	0.283	0.285	0.256	0.262	0.268	0.261	0.268	0.261	0.263	0.258	0.233	0.227	0.25	0.262	0.24	0.24	0.24	0.237	0.237	0.245
std errors:	0.0138	0.0142	0.0143	0.0138	0.0139	0.014	0.0139	0.014	0.0139	0.0139	0.0138	0.0134	0.0132	0.0137	0.0139	0.0135	0.0135	0.0135	0.0134	0.0134	0.0136
0.1:	0.347	0.337	0.349	0.351	0.349	0.359	0.354	0.365	0.366	0.379	0.387	0.387	0.387	0.401	0.401	0.411	0.372	0.409	0.401	0.387	0.403
std errors:	0.0151	0.0149	0.0151	0.0151	0.0151	0.0152	0.0151	0.0152	0.0152	0.0153	0.0154	0.0154	0.0154	0.0155	0.0155	0.0156	0.0153	0.0155	0.0155	0.0154	0.0155
alpha	The AE approximation of power																				
0.01	0	0	0	0	0	0	0	-0.0009	0.0027	0.028	0.0589	0.0783	0.1036	0.1106	0.107	0.1011	0.0933	0.0841	0.0737	0.0628	0.0517
0.05	0.0051	0.0171	0.0372	0.065	0.0987	0.1288	0.156	0.1896	0.2182	0.2136	0.2112	0.2165	0.2268	0.2423	0.2769	0.287	0.2743	0.26	0.2442	0.227	0.2278
0.1	0.0096	0.0276	0.0565	0.0954	0.1305	0.1778	0.2198	0.2583	0.3009	0.3403	0.3639	0.3585	0.3666	0.3874	0.405	0.4198	0.4157	0.4099	0.4026	0.3939	0.3836
alpha	The traditional estimated power, chi-square																				
0.01:	0.38	0.358	0.351	0.351	0.258	0.237	0.178	0.166	0.161	0.157	0.145	0.132	0.109	0.103	0.094	0.094	0.098	0.11	0.11	0.122	0.133
std errors:	0.0153	0.0152	0.0151	0.0151	0.0138	0.0134	0.0121	0.0118	0.0116	0.0115	0.0111	0.0107	0.0099	0.0096	0.0092	0.0092	0.0094	0.0099	0.0099	0.0103	0.0107
0.05:	0.428	0.428	0.428	0.399	0.399	0.399	0.399	0.365	0.345	0.307	0.281	0.281	0.269	0.269	0.244	0.24	0.24	0.269	0.276	0.29	0.302
std errors:	0.0156	0.0156	0.0156	0.0155	0.0155	0.0155	0.0155	0.0152	0.015	0.0146	0.0142	0.0142	0.014	0.014	0.0136	0.0135	0.0135	0.014	0.0141	0.0143	0.0145
0.1:	0.567	0.567	0.532	0.532	0.532	0.479	0.479	0.479	0.441	0.441	0.407	0.387	0.387	0.401	0.372	0.372	0.372	0.371	0.371	0.403	0.403
std errors:	0.0157	0.0157	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0157	0.0157	0.0155	0.0154	0.0154	0.0155	0.0153	0.0153	0.0153	0.0153	0.0153	0.0155	0.0155

The proportions under the alternative hypothesis: 0.3291 0.3291 0.1709 0.1709

n = 30 c(j) = 0.433 0.433 -0.433 -0.433 delta = 3k/4 k = 4

lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
alpha	The bootstrap estimated power																				
0.01:	0.1	0.095	0.098	0.087	0.087	0.086	0.084	0.084	0.091	0.097	0.093	0.093	0.09	0.099	0.092	0.092	0.076	0.075	0.067	0.073	0.073
std errors:	0.0095	0.0093	0.0094	0.0089	0.0089	0.0089	0.0088	0.0088	0.0091	0.0094	0.0092	0.0092	0.009	0.0094	0.0091	0.0091	0.0084	0.0083	0.0079	0.0082	0.0082
0.05:	0.256	0.24	0.24	0.247	0.245	0.241	0.252	0.259	0.276	0.274	0.265	0.259	0.259	0.25	0.254	0.252	0.253	0.251	0.252	0.239	0.239
std errors:	0.0138	0.0135	0.0135	0.0136	0.0136	0.0135	0.0137	0.0139	0.0141	0.0141	0.014	0.0139	0.0139	0.0137	0.0138	0.0137	0.0137	0.0137	0.0137	0.0135	0.0135
0.1:	0.365	0.37	0.37	0.379	0.372	0.37	0.374	0.37	0.365	0.364	0.37	0.374	0.373	0.369	0.369	0.381	0.384	0.369	0.374	0.359	0.35
std errors:	0.0152	0.0153	0.0153	0.0153	0.0153	0.0153	0.0153	0.0153	0.0152	0.0152	0.0153	0.0153	0.0153	0.0153	0.0153	0.0154	0.0154	0.0153	0.0153	0.0152	0.0151
alpha	The AE approximation of power																				
0.01	0.0177	0.0251	0.0327	0.035	0.0392	0.046	0.0496	0.0526	0.0765	0.0948	0.1033	0.1067	0.1084	0.1128	0.1085	0.1033	0.0971	0.0903	0.083	0.0753	0.0693
0.05	0.1268	0.1315	0.1415	0.1501	0.1574	0.1631	0.195	0.2221	0.2448	0.2468	0.2662	0.2712	0.2889	0.2817	0.2774	0.2767	0.2745	0.2657	0.2663	0.2604	0.2533
0.1	0.2377	0.2552	0.2681	0.2916	0.3142	0.3355	0.3539	0.35	0.3632	0.376	0.3873	0.3963	0.3926	0.401	0.4171	0.4209	0.4109	0.3998	0.3877	0.3746	0.3663
alpha	The traditional estimated power, chi-square																				
0.01:	0.239	0.239	0.237	0.218	0.204	0.174	0.157	0.153	0.136	0.121	0.117	0.102	0.106	0.103	0.099	0.092	0.093	0.104	0.109	0.112	0.118
std errors:	0.0135	0.0135	0.0134	0.0131	0.0127	0.012	0.0115	0.0114	0.0108	0.0103	0.0102	0.0096	0.0097	0.0096	0.0094	0.0091	0.0092	0.0097	0.0099	0.01	0.0102
0.05:	0.418	0.392	0.376	0.339	0.318	0.321	0.306	0.306	0.29	0.281	0.276	0.259	0.25	0.25	0.256	0.258	0.259	0.259	0.263	0.256	0.254
std errors:	0.0156	0.0154	0.0153	0.015	0.0147	0.0148	0.0146	0.0146	0.0143	0.0142	0.0141	0.0139	0.0137	0.0137	0.0138	0.0138	0.0139	0.0139	0.0139	0.0138	0.0138
0.1:	0.465	0.465	0.465	0.464	0.449	0.435	0.423	0.415	0.399	0.38	0.38	0.394	0.378	0.369	0.364	0.364	0.364	0.354	0.374	0.382	0.386
std errors:	0.0158	0.0158	0.0158	0.0158	0.0157	0.0157	0.0156	0.0156	0.0155	0.0153	0.0153	0.0155	0.0153	0.0153	0.0152	0.0152	0.0152	0.0151	0.0153	0.0154	0.0154

Table 8

Proportions under the null hypothesis:					0.2	0.2	0.2	0.2													
The proportions under the alternative hypothesis:					0.3112	0.3112	0.1888	0.1888													
n =	50			c(j) =	0.433	0.433	-0.433	-0.433				delta =	3k/4			k =	4				
lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
alpha	The bootstrap estimated power																				
0.01:	0.089	0.089	0.09	0.093	0.095	0.092	0.094	0.097	0.095	0.097	0.095	0.102	0.107	0.106	0.104	0.1	0.102	0.1	0.107	0.107	0.105
std errors:	0.009	0.009	0.009	0.0092	0.0093	0.0091	0.0092	0.0094	0.0093	0.0094	0.0093	0.0096	0.0098	0.0097	0.0097	0.0095	0.0096	0.0095	0.0098	0.0098	0.0097
0.05:	0.265	0.265	0.264	0.266	0.266	0.268	0.268	0.277	0.28	0.281	0.282	0.29	0.289	0.286	0.29	0.29	0.289	0.29	0.29	0.29	0.29
std errors:	0.014	0.014	0.0139	0.014	0.014	0.014	0.014	0.0142	0.0142	0.0142	0.0142	0.0143	0.0143	0.0143	0.0143	0.0143	0.0143	0.0143	0.0143	0.0143	0.0143
0.1:	0.374	0.379	0.384	0.389	0.397	0.399	0.406	0.406	0.417	0.419	0.414	0.413	0.413	0.407	0.414	0.414	0.406	0.406	0.406	0.397	0.406
std errors:	0.0153	0.0153	0.0154	0.0154	0.0155	0.0155	0.0155	0.0155	0.0156	0.0156	0.0156	0.0156	0.0156	0.0155	0.0156	0.0156	0.0155	0.0155	0.0155	0.0155	0.0155
alpha	The AE approximation of power																				
0.01	0.0835	0.0937	0.1033	0.1182	0.1331	0.1428	0.1544	0.161	0.1677	0.1738	0.1776	0.1818	0.195	0.1962	0.1933	0.1897	0.1917	0.1912	0.1911	0.1875	0.1797
0.05	0.233	0.2438	0.2539	0.2609	0.2676	0.2722	0.2831	0.2899	0.2981	0.305	0.311	0.3203	0.3223	0.3235	0.3235	0.3231	0.3213	0.3241	0.3273	0.3282	0.3295
0.1	0.3171	0.3322	0.3456	0.3638	0.3821	0.3862	0.3963	0.4006	0.4114	0.4244	0.4305	0.4256	0.42	0.414	0.4195	0.4114	0.4158	0.4144	0.4126	0.4043	0.4096
alpha	The traditional estimated power, chi-square																				
0.01:	0.222	0.213	0.203	0.19	0.175	0.164	0.151	0.142	0.133	0.129	0.109	0.111	0.107	0.106	0.105	0.103	0.104	0.112	0.115	0.124	0.131
std errors:	0.0131	0.0129	0.0127	0.0124	0.012	0.0117	0.0113	0.011	0.0107	0.0106	0.0099	0.0099	0.0098	0.0097	0.0097	0.0096	0.0097	0.01	0.0101	0.0104	0.0107
0.05:	0.382	0.376	0.355	0.343	0.332	0.329	0.32	0.32	0.319	0.311	0.298	0.302	0.298	0.295	0.298	0.301	0.299	0.298	0.29	0.29	0.288
std errors:	0.0154	0.0153	0.0151	0.015	0.0149	0.0149	0.0148	0.0148	0.0147	0.0146	0.0145	0.0145	0.0145	0.0144	0.0145	0.0145	0.0145	0.0145	0.0143	0.0143	0.0143
0.1:	0.475	0.459	0.459	0.457	0.462	0.446	0.443	0.423	0.422	0.419	0.414	0.415	0.415	0.422	0.416	0.414	0.417	0.42	0.417	0.417	0.417
std errors:	0.0158	0.0158	0.0158	0.0158	0.0158	0.0157	0.0157	0.0156	0.0156	0.0156	0.0156	0.0156	0.0156	0.0156	0.0156	0.0156	0.0156	0.0156	0.0156	0.0156	0.0156
The proportions under the alternative hypothesis:					0.2933	0.2933	0.2067	0.2067													
n =	100			c(j) =	0.433	0.433	-0.433	-0.433				delta =	3k/4			k =	4				
lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
alpha	The bootstrap estimated power																				
0.01:	0.125	0.124	0.13	0.136	0.132	0.131	0.124	0.124	0.125	0.124	0.129	0.12	0.124	0.118	0.12	0.121	0.118	0.114	0.112	0.111	0.111
std errors:	0.0105	0.0104	0.0106	0.0108	0.0107	0.0107	0.0104	0.0104	0.0105	0.0104	0.0106	0.0103	0.0104	0.0102	0.0103	0.0103	0.0102	0.0101	0.01	0.0099	0.0099
0.05:	0.302	0.296	0.296	0.302	0.306	0.302	0.303	0.304	0.302	0.307	0.301	0.3	0.305	0.3	0.3	0.298	0.296	0.293	0.293	0.286	0.281
std errors:	0.0145	0.0144	0.0144	0.0145	0.0146	0.0145	0.0145	0.0145	0.0145	0.0146	0.0145	0.0145	0.0146	0.0145	0.0145	0.0145	0.0144	0.0144	0.0144	0.0143	0.0142
0.1:	0.399	0.4	0.395	0.395	0.395	0.395	0.399	0.399	0.398	0.395	0.393	0.397	0.402	0.403	0.398	0.399	0.396	0.393	0.396	0.396	0.398
std errors:	0.0155	0.0155	0.0155	0.0155	0.0155	0.0155	0.0155	0.0155	0.0155	0.0155	0.0154	0.0155	0.0155	0.0155	0.0155	0.0155	0.0155	0.0154	0.0155	0.0155	0.0155
alpha	The AE approximation of power																				
0.01	0.1535	0.1604	0.1704	0.1807	0.1826	0.1857	0.1857	0.1883	0.1928	0.1948	0.1977	0.1965	0.1983	0.1967	0.1977	0.1954	0.1943	0.191	0.1872	0.1839	0.1827
0.05	0.2928	0.2949	0.3013	0.3109	0.3162	0.3175	0.3191	0.3221	0.3262	0.3343	0.3342	0.3373	0.3394	0.3343	0.3312	0.3278	0.3289	0.3283	0.327	0.3245	0.3203
0.1	0.3865	0.3896	0.3923	0.3945	0.3964	0.3979	0.399	0.4023	0.406	0.4067	0.4072	0.4091	0.411	0.4144	0.4125	0.4084	0.4115	0.4143	0.415	0.4157	0.4141
alpha	The traditional estimated power, chi-square																				
0.01:	0.165	0.16	0.157	0.147	0.143	0.137	0.128	0.124	0.118	0.118	0.116	0.112	0.111	0.113	0.113	0.112	0.112	0.111	0.112	0.12	0.12
std errors:	0.0117	0.0116	0.0115	0.0112	0.0111	0.0109	0.0106	0.0104	0.0102	0.0102	0.0101	0.01	0.0099	0.01	0.01	0.01	0.01	0.0099	0.01	0.0103	0.0103
0.05:	0.337	0.334	0.325	0.322	0.311	0.308	0.305	0.304	0.302	0.295	0.291	0.289	0.289	0.288	0.287	0.286	0.288	0.29	0.287	0.286	0.286
std errors:	0.0149	0.0149	0.0148	0.0148	0.0146	0.0146	0.0146	0.0145	0.0145	0.0144	0.0144	0.0143	0.0143	0.0143	0.0143	0.0143	0.0143	0.0143	0.0143	0.0143	0.0143
0.1:	0.45	0.442	0.431	0.426	0.42	0.419	0.418	0.416	0.419	0.419	0.413	0.411	0.41	0.409	0.408	0.408	0.404	0.405	0.406	0.403	0.403
std errors:	0.0157	0.0157	0.0157	0.0156	0.0156	0.0156	0.0156	0.0156	0.0156	0.0156	0.0156	0.0156	0.0156	0.0155	0.0155	0.0155	0.0155	0.0155	0.0155	0.0155	0.0155

Proportions under the null hypothesis: 0.2 0.2 0.2 0.2

The proportions under the alternative hypothesis: 0.375 0.375 0.125 0.125

n = 20 c(j) = 0.559 0.559 -0.559 -0.559 delta = 5k/4 k = 4

alpha The bootstrap estimated power

lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
0.01:	0.111	0.101	0.109	0.109	0.109	0.107	0.109	0.107	0.117	0.144	0.194	0.245	0.237	0.235	0.248	0.246	0.188	0.181	0.181	0.181	0.154
std errors:	0.0099	0.0095	0.0099	0.0099	0.0099	0.0098	0.0099	0.0098	0.0102	0.0111	0.0125	0.0136	0.0134	0.0134	0.0137	0.0136	0.0124	0.0122	0.0122	0.0122	0.0114
0.05:	0.419	0.432	0.434	0.434	0.421	0.453	0.453	0.46	0.469	0.45	0.486	0.525	0.506	0.512	0.51	0.441	0.484	0.484	0.451	0.418	0.415
std errors:	0.0156	0.0157	0.0157	0.0157	0.0156	0.0157	0.0157	0.0158	0.0158	0.0157	0.0158	0.0158	0.0158	0.0158	0.0158	0.0157	0.0158	0.0158	0.0157	0.0156	0.0156
0.1:	0.565	0.568	0.561	0.566	0.563	0.568	0.568	0.574	0.593	0.593	0.595	0.595	0.594	0.594	0.594	0.618	0.581	0.616	0.614	0.591	0.614
std errors:	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0156	0.0155	0.0155	0.0155	0.0155	0.0155	0.0155	0.0155	0.0154	0.0156	0.0154	0.0154	0.0155	0.0154

alpha The AE approximation of power

alpha	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
0.01	0	0	0	0	0	0	0	-0.004	0.017	0.0808	0.167	0.2326	0.2351	0.2589	0.2554	0.2403	0.2229	0.2033	0.1821	0.1596	0.1364
0.05	0.022	0.0551	0.1009	0.1554	0.214	0.2726	0.3284	0.378	0.3892	0.4247	0.4548	0.4797	0.4987	0.4908	0.4807	0.4683	0.4771	0.4668	0.4459	0.4228	0.3975
0.1	0.3327	0.3493	0.363	0.3739	0.3818	0.3869	0.4188	0.4791	0.5079	0.5504	0.5639	0.5953	0.5867	0.5755	0.5929	0.6073	0.6032	0.5977	0.5906	0.582	0.5717

alpha The traditional estimated power, chi-square

alpha	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
0.01:	0.527	0.484	0.47	0.47	0.421	0.402	0.334	0.307	0.301	0.289	0.238	0.23	0.183	0.18	0.162	0.162	0.174	0.194	0.194	0.216	0.25
std errors:	0.0158	0.0158	0.0158	0.0158	0.0156	0.0155	0.0149	0.0146	0.0145	0.0143	0.0135	0.0133	0.0122	0.0121	0.0117	0.0117	0.012	0.0125	0.0125	0.013	0.0137
0.05:	0.594	0.594	0.594	0.568	0.568	0.568	0.568	0.552	0.539	0.525	0.486	0.486	0.456	0.456	0.434	0.428	0.428	0.454	0.462	0.474	0.504
std errors:	0.0155	0.0155	0.0155	0.0157	0.0157	0.0157	0.0157	0.0157	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0157	0.0156	0.0156	0.0157	0.0158	0.0158	0.0158
0.1:	0.729	0.729	0.708	0.708	0.708	0.648	0.648	0.648	0.611	0.611	0.595	0.582	0.582	0.594	0.581	0.581	0.581	0.579	0.579	0.614	0.614
std errors:	0.0141	0.0141	0.0144	0.0144	0.0144	0.0151	0.0151	0.0151	0.0154	0.0154	0.0155	0.0156	0.0156	0.0155	0.0156	0.0156	0.0156	0.0156	0.0156	0.0154	0.0154

The proportions under the alternative hypothesis: 0.3521 0.3521 0.1479 0.1479

n = 30 c(j) = 0.559 0.559 -0.559 -0.559 delta = 5k/4 k = 4

alpha The bootstrap estimated power

lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
0.01:	0.147	0.15	0.157	0.157	0.157	0.153	0.16	0.16	0.18	0.213	0.211	0.22	0.216	0.224	0.216	0.24	0.226	0.206	0.21	0.201	0.197
std errors:	0.0112	0.0113	0.0115	0.0115	0.0115	0.0114	0.0116	0.0116	0.0121	0.0129	0.0129	0.0131	0.013	0.0132	0.013	0.0135	0.0132	0.0128	0.0129	0.0127	0.0126
0.05:	0.434	0.434	0.411	0.434	0.423	0.44	0.442	0.436	0.439	0.44	0.44	0.457	0.442	0.434	0.426	0.436	0.436	0.44	0.436	0.425	0.42
std errors:	0.0157	0.0157	0.0156	0.0157	0.0156	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0158	0.0157	0.0157	0.0156	0.0157	0.0157	0.0157	0.0157	0.0156	0.0156
0.1:	0.591	0.583	0.583	0.6	0.591	0.581	0.584	0.586	0.586	0.577	0.573	0.577	0.585	0.587	0.569	0.566	0.561	0.559	0.558	0.546	0.538
std errors:	0.0155	0.0156	0.0156	0.0155	0.0155	0.0156	0.0156	0.0156	0.0156	0.0156	0.0156	0.0156	0.0156	0.0156	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0158

alpha The AE approximation of power

alpha	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
0.01	0.0213	0.0307	0.0418	0.0545	0.068	0.082	0.0957	0.1107	0.1482	0.1795	0.2007	0.2215	0.2293	0.2298	0.2386	0.2455	0.2433	0.2263	0.2285	0.2073	0.1859
0.05	0.1777	0.1999	0.2217	0.2429	0.263	0.3013	0.3372	0.3554	0.3786	0.4126	0.4347	0.4428	0.447	0.4495	0.4502	0.4493	0.4468	0.4426	0.437	0.4298	0.4211
0.1	0.4018	0.4094	0.4388	0.4667	0.4924	0.5034	0.5128	0.5301	0.5453	0.5557	0.5444	0.549	0.5754	0.5706	0.5647	0.5575	0.5493	0.5425	0.5446	0.5456	0.5425

alpha The traditional estimated power, chi-square

alpha	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
0.01:	0.443	0.443	0.434	0.394	0.375	0.332	0.312	0.303	0.279	0.256	0.253	0.23	0.236	0.224	0.216	0.2	0.208	0.21	0.21	0.21	0.22
std errors:	0.0157	0.0157	0.0157	0.0155	0.0153	0.0149	0.0147	0.0145	0.0142	0.0138	0.0137	0.0133	0.0134	0.0132	0.013	0.0126	0.0128	0.0129	0.0129	0.0129	0.0131
0.05:	0.635	0.611	0.6	0.564	0.543	0.543	0.525	0.525	0.489	0.479	0.47	0.457	0.442	0.442	0.445	0.447	0.44	0.44	0.448	0.444	0.442
std errors:	0.0152	0.0154	0.0155	0.0157	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157
0.1:	0.669	0.669	0.669	0.668	0.663	0.65	0.639	0.623	0.612	0.593	0.593	0.613	0.597	0.587	0.578	0.578	0.578	0.567	0.578	0.585	0.589
std errors:	0.0149	0.0149	0.0149	0.0149	0.0149	0.0151	0.0152	0.0153	0.0154	0.0155	0.0155	0.0154	0.0155	0.0156	0.0156	0.0156	0.0156	0.0157	0.0156	0.0156	0.0156

Proportions under the null hypothesis: 0.2 0.2 0.2 0.2

The proportions under the alternative hypothesis: 0.3291 0.3291 0.1709 0.1709

n = 50 c(j) = 0.559 0.559 -0.559 -0.559 delta = 5k/4 k = 4

lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
alpha	The bootstrap estimated power																				
0.01:	0.114	0.123	0.134	0.141	0.14	0.154	0.162	0.18	0.184	0.185	0.186	0.186	0.194	0.195	0.185	0.171	0.158	0.166	0.16	0.158	0.158
std errors:	0.0101	0.0104	0.0108	0.011	0.011	0.0114	0.0117	0.0121	0.0123	0.0123	0.0123	0.0123	0.0125	0.0125	0.0123	0.0119	0.0115	0.0118	0.0116	0.0115	0.0115
0.05:	0.387	0.394	0.395	0.394	0.398	0.403	0.403	0.419	0.417	0.429	0.429	0.442	0.448	0.446	0.45	0.449	0.438	0.446	0.446	0.446	0.445
std errors:	0.0154	0.0155	0.0155	0.0155	0.0155	0.0155	0.0155	0.0156	0.0156	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157
0.1:	0.521	0.53	0.537	0.547	0.547	0.551	0.561	0.561	0.56	0.566	0.562	0.572	0.57	0.565	0.562	0.549	0.554	0.559	0.557	0.552	0.557
std errors:	0.0158	0.0158	0.0158	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0156	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157
alpha	The AE approximation of power																				
0.01	0.1213	0.1492	0.1764	0.2	0.2173	0.2391	0.2613	0.281	0.293	0.2986	0.3043	0.3055	0.3288	0.328	0.3172	0.3059	0.3021	0.3085	0.3044	0.297	0.2925
0.05	0.3751	0.391	0.4043	0.4093	0.4208	0.4277	0.4352	0.4542	0.4623	0.471	0.4753	0.488	0.4892	0.4985	0.4968	0.4993	0.4874	0.4941	0.5017	0.4994	0.4993
0.1	0.4769	0.4928	0.5145	0.5247	0.5416	0.5516	0.5638	0.5717	0.5707	0.5729	0.5833	0.5927	0.5944	0.5883	0.5934	0.5857	0.5901	0.5887	0.5788	0.5784	0.5838
alpha	The traditional estimated power, chi-square																				
0.01:	0.345	0.324	0.318	0.295	0.275	0.267	0.247	0.234	0.228	0.224	0.198	0.199	0.194	0.195	0.192	0.188	0.192	0.2	0.21	0.212	0.213
std errors:	0.015	0.0148	0.0147	0.0144	0.0141	0.014	0.0136	0.0134	0.0133	0.0132	0.0126	0.0126	0.0125	0.0125	0.0125	0.0124	0.0125	0.0126	0.0129	0.0129	0.0129
0.05:	0.535	0.53	0.513	0.505	0.49	0.486	0.47	0.47	0.466	0.458	0.449	0.455	0.448	0.442	0.447	0.447	0.45	0.447	0.44	0.439	0.439
std errors:	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157
0.1:	0.619	0.605	0.605	0.605	0.61	0.597	0.593	0.584	0.581	0.579	0.573	0.574	0.574	0.573	0.568	0.565	0.57	0.575	0.575	0.575	0.575
std errors:	0.0154	0.0155	0.0155	0.0155	0.0154	0.0155	0.0155	0.0156	0.0156	0.0156	0.0156	0.0156	0.0156	0.0156	0.0157	0.0157	0.0157	0.0156	0.0156	0.0156	0.0156

The proportions under the alternative hypothesis: 0.3059 0.3059 0.1941 0.1941

n = 100 c(j) = 0.559 0.559 -0.559 -0.559 delta = 5k/4 k = 4

lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
alpha	The bootstrap estimated power																				
0.01:	0.182	0.184	0.194	0.197	0.198	0.202	0.207	0.21	0.211	0.209	0.207	0.213	0.217	0.219	0.22	0.222	0.221	0.226	0.219	0.214	0.211
std errors:	0.0122	0.0123	0.0125	0.0126	0.0126	0.0127	0.0128	0.0129	0.0129	0.0129	0.0128	0.0129	0.013	0.0131	0.0131	0.0131	0.0131	0.0132	0.0131	0.013	0.0129
0.05:	0.403	0.402	0.397	0.398	0.398	0.4	0.397	0.401	0.4	0.399	0.396	0.397	0.402	0.402	0.401	0.401	0.399	0.399	0.4	0.399	0.399
std errors:	0.0155	0.0155	0.0155	0.0155	0.0155	0.0155	0.0155	0.0155	0.0155	0.0155	0.0155	0.0155	0.0155	0.0155	0.0155	0.0155	0.0155	0.0155	0.0155	0.0155	0.0155
0.1:	0.518	0.516	0.516	0.513	0.513	0.512	0.525	0.525	0.527	0.527	0.527	0.524	0.526	0.537	0.534	0.526	0.531	0.528	0.53	0.53	0.532
std errors:	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158
alpha	The AE approximation of power																				
0.01	0.2331	0.2461	0.2587	0.2709	0.2765	0.2865	0.297	0.3003	0.309	0.3095	0.3108	0.3165	0.3187	0.3198	0.32	0.3203	0.3203	0.3216	0.3176	0.3078	0.3011
0.05	0.4219	0.4265	0.4277	0.4311	0.4349	0.4434	0.4436	0.454	0.455	0.4578	0.4605	0.4623	0.4688	0.4692	0.4715	0.4692	0.4664	0.4657	0.4629	0.4655	0.4645
0.1	0.5309	0.5359	0.54	0.5477	0.548	0.55	0.5533	0.5604	0.5626	0.5666	0.5717	0.5755	0.5756	0.5758	0.5737	0.5692	0.5681	0.5714	0.5692	0.5667	0.568
alpha	The traditional estimated power, chi-square																				
0.01:	0.283	0.269	0.261	0.247	0.241	0.235	0.226	0.223	0.22	0.22	0.219	0.216	0.214	0.214	0.215	0.218	0.219	0.218	0.219	0.224	0.224
std errors:	0.0142	0.014	0.0139	0.0136	0.0135	0.0134	0.0132	0.0132	0.0131	0.0131	0.0131	0.013	0.013	0.013	0.013	0.0131	0.0131	0.0131	0.0131	0.0132	0.0132
0.05:	0.466	0.462	0.453	0.448	0.445	0.444	0.443	0.441	0.441	0.436	0.429	0.428	0.429	0.423	0.421	0.424	0.423	0.427	0.428	0.429	0.429
std errors:	0.0158	0.0158	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0156	0.0157	0.0156	0.0156	0.0156	0.0156	0.0156	0.0156	0.0157	0.0157
0.1:	0.6	0.593	0.583	0.579	0.572	0.569	0.569	0.566	0.565	0.567	0.563	0.565	0.564	0.564	0.561	0.561	0.561	0.562	0.565	0.563	0.56
std errors:	0.0155	0.0155	0.0156	0.0156	0.0156	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157

Proportions under the null hypothesis: 0.2 0.2 0.2 0.2

The proportions under the alternative hypothesis: 0.3618 0.3618 0.1382 0.1382

n = 20 c(j) = 0.5 0.5 -0.5 -0.5 delta = k k = 4

lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
alpha	The bootstrap estimated power																				
0.01:	0.09	0.078	0.078	0.087	0.087	0.078	0.087	0.087	0.089	0.101	0.117	0.122	0.124	0.124	0.124	0.116	0.115	0.125	0.125	0.125	0.11
std errors:	0.009	0.0085	0.0085	0.0089	0.0089	0.0085	0.0089	0.0089	0.009	0.0095	0.0102	0.0103	0.0104	0.0104	0.0104	0.0101	0.0101	0.0105	0.0105	0.0105	0.0099
0.05:	0.359	0.359	0.364	0.361	0.366	0.373	0.38	0.379	0.397	0.397	0.375	0.41	0.386	0.406	0.407	0.339	0.384	0.384	0.355	0.36	0.346
std errors:	0.0152	0.0152	0.0152	0.0152	0.0152	0.0153	0.0153	0.0153	0.0155	0.0155	0.0153	0.0156	0.0154	0.0155	0.0155	0.015	0.0154	0.0154	0.0151	0.0152	0.015
0.1:	0.481	0.512	0.482	0.512	0.513	0.515	0.482	0.482	0.513	0.521	0.521	0.524	0.539	0.537	0.503	0.512	0.476	0.511	0.511	0.493	0.509
std errors:	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158
alpha	The AE approximation of power																				
0.01	0	0	0	0	0	0	0	-0.0021	0.0067	0.0413	0.0911	0.1188	0.1292	0.1336	0.1558	0.1548	0.1451	0.1429	0.126	0.1084	0.0906
0.05	0.016	0.0423	0.0672	0.1085	0.1548	0.2028	0.2496	0.2947	0.3242	0.3381	0.3652	0.3892	0.4079	0.4001	0.3902	0.3782	0.3781	0.376	0.3567	0.3516	0.3288
0.1	0.3219	0.3424	0.3603	0.3759	0.3889	0.3995	0.4076	0.4134	0.4413	0.4642	0.4993	0.5105	0.521	0.5216	0.5207	0.5181	0.5138	0.508	0.5007	0.4917	0.4812
alpha	The traditional estimated power, chi-square																				
0.01:	0.45	0.414	0.404	0.404	0.332	0.308	0.238	0.211	0.209	0.202	0.166	0.154	0.128	0.124	0.116	0.116	0.123	0.135	0.135	0.152	0.168
std errors:	0.0157	0.0156	0.0155	0.0155	0.0149	0.0146	0.0135	0.0129	0.0129	0.0127	0.0118	0.0114	0.0106	0.0104	0.0101	0.0101	0.0104	0.0108	0.0108	0.0114	0.0118
0.05:	0.513	0.513	0.513	0.48	0.48	0.48	0.48	0.463	0.442	0.41	0.375	0.375	0.35	0.35	0.335	0.326	0.326	0.359	0.366	0.381	0.406
std errors:	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0157	0.0156	0.0153	0.0153	0.0151	0.0151	0.0149	0.0148	0.0148	0.0152	0.0152	0.0154	0.0155
0.1:	0.64	0.64	0.606	0.606	0.606	0.562	0.562	0.562	0.526	0.526	0.509	0.488	0.488	0.503	0.476	0.476	0.476	0.475	0.475	0.509	0.509
std errors:	0.0152	0.0152	0.0155	0.0155	0.0155	0.0157	0.0157	0.0157	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158

The proportions under the alternative hypothesis: 0.3413 0.3413 0.1587 0.1587

n = 30 c(j) = 0.5 0.5 -0.5 -0.5 delta = k k = 4

lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
alpha	The bootstrap estimated power																				
0.01:	0.071	0.071	0.071	0.075	0.079	0.093	0.107	0.12	0.139	0.138	0.154	0.158	0.175	0.178	0.178	0.184	0.16	0.16	0.153	0.153	0.143
std errors:	0.0081	0.0081	0.0081	0.0083	0.0085	0.0092	0.0098	0.0103	0.0109	0.0109	0.0114	0.0115	0.012	0.0121	0.0121	0.0123	0.0116	0.0116	0.0114	0.0114	0.0111
0.05:	0.306	0.324	0.324	0.324	0.307	0.344	0.348	0.361	0.356	0.378	0.367	0.38	0.385	0.372	0.362	0.36	0.365	0.35	0.35	0.361	0.361
std errors:	0.0146	0.0148	0.0148	0.0148	0.0146	0.015	0.0151	0.0152	0.0151	0.0153	0.0152	0.0153	0.0154	0.0153	0.0152	0.0152	0.0152	0.0151	0.0151	0.0152	0.0152
0.1:	0.489	0.489	0.494	0.513	0.499	0.503	0.501	0.489	0.505	0.497	0.495	0.497	0.51	0.516	0.516	0.526	0.523	0.508	0.512	0.509	0.517
std errors:	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158
alpha	The AE approximation of power																				
0.01	0.0001	0.0006	0.0026	0.0072	0.0159	0.0303	0.0515	0.08	0.0992	0.1221	0.139	0.1504	0.1627	0.1693	0.1808	0.1822	0.1716	0.1664	0.1537	0.1392	0.1276
0.05	0.1232	0.1397	0.1562	0.1722	0.1876	0.22	0.2518	0.2902	0.304	0.3295	0.3516	0.3744	0.3794	0.369	0.3642	0.3634	0.361	0.351	0.3508	0.3486	0.3443
0.1	0.3061	0.3253	0.3534	0.38	0.4048	0.4156	0.4249	0.442	0.4572	0.4677	0.4563	0.4609	0.4878	0.4829	0.4973	0.5167	0.5065	0.4952	0.4827	0.4814	0.4696
alpha	The traditional estimated power, chi-square																				
0.01:	0.346	0.346	0.337	0.307	0.297	0.265	0.23	0.22	0.208	0.191	0.192	0.179	0.184	0.178	0.165	0.151	0.16	0.162	0.167	0.168	0.173
std errors:	0.015	0.015	0.0149	0.0146	0.0144	0.014	0.0133	0.0131	0.0128	0.0124	0.0125	0.0121	0.0123	0.0121	0.0117	0.0113	0.0116	0.0117	0.0118	0.0118	0.012
0.05:	0.543	0.518	0.513	0.471	0.452	0.453	0.434	0.434	0.401	0.397	0.391	0.377	0.372	0.372	0.375	0.381	0.367	0.367	0.373	0.369	0.367
std errors:	0.0158	0.0158	0.0158	0.0158	0.0157	0.0157	0.0157	0.0157	0.0155	0.0155	0.0154	0.0153	0.0153	0.0153	0.0153	0.0154	0.0152	0.0152	0.0153	0.0153	0.0152
0.1:	0.598	0.598	0.598	0.595	0.585	0.571	0.56	0.539	0.534	0.511	0.511	0.534	0.52	0.516	0.51	0.51	0.51	0.499	0.512	0.52	0.527
std errors:	0.0155	0.0155	0.0155	0.0155	0.0156	0.0157	0.0157	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158

Proportions under the null hypothesis:					0.2	0.2	0.2	0.2													
The proportions under the alternative hypothesis:					0.3207	0.3207	0.1793	0.1793													
n =	50			c(j) =	0.5	0.5	-0.5	-0.5				delta =	k			k =	4				
lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
alpha	The bootstrap estimated power																				
0.01:	0.151	0.151	0.155	0.156	0.151	0.137	0.145	0.151	0.145	0.153	0.159	0.157	0.156	0.149	0.152	0.15	0.163	0.159	0.148	0.146	0.144
std errors:	0.0113	0.0113	0.0114	0.0115	0.0113	0.0109	0.0111	0.0113	0.0111	0.0114	0.0116	0.0115	0.0115	0.0113	0.0114	0.0113	0.0117	0.0116	0.0112	0.0112	0.0111
0.05:	0.345	0.351	0.351	0.356	0.356	0.351	0.351	0.355	0.357	0.36	0.358	0.358	0.35	0.362	0.351	0.347	0.351	0.349	0.348	0.348	0.348
std errors:	0.015	0.0151	0.0151	0.0151	0.0151	0.0151	0.0151	0.0151	0.0152	0.0152	0.0152	0.0152	0.0151	0.0152	0.0151	0.0151	0.0151	0.0151	0.0151	0.0151	0.0151
0.1:	0.49	0.49	0.482	0.496	0.493	0.498	0.5	0.507	0.503	0.51	0.51	0.506	0.504	0.504	0.509	0.508	0.498	0.496	0.489	0.495	0.489
std errors:	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158
alpha	The AE approximation of power																				
0.01	0.16	0.1712	0.1833	0.1945	0.2014	0.2036	0.2186	0.2324	0.2348	0.2467	0.2598	0.2623	0.2622	0.2595	0.2629	0.2616	0.264	0.259	0.2515	0.2471	0.2444
0.05	0.3183	0.3271	0.3367	0.3482	0.3604	0.3684	0.3771	0.3847	0.3913	0.3981	0.4012	0.4063	0.4069	0.4116	0.4085	0.4077	0.4058	0.4034	0.4001	0.3996	0.3986
0.1	0.442	0.4428	0.4458	0.4674	0.4719	0.49	0.493	0.509	0.5241	0.5266	0.5362	0.533	0.5262	0.526	0.5295	0.5265	0.5163	0.5165	0.5138	0.509	0.5034
alpha	The traditional estimated power, chi-square																				
0.01:	0.27	0.256	0.25	0.229	0.214	0.202	0.187	0.179	0.174	0.172	0.157	0.155	0.147	0.149	0.15	0.15	0.155	0.16	0.165	0.168	0.165
std errors:	0.014	0.0138	0.0137	0.0133	0.013	0.0127	0.0123	0.0121	0.012	0.0119	0.0115	0.0114	0.0112	0.0113	0.0113	0.0113	0.0114	0.0116	0.0117	0.0118	0.0117
0.05:	0.463	0.458	0.434	0.425	0.411	0.407	0.392	0.392	0.387	0.382	0.365	0.371	0.364	0.362	0.358	0.36	0.358	0.357	0.353	0.357	0.357
std errors:	0.0158	0.0158	0.0157	0.0156	0.0156	0.0155	0.0154	0.0154	0.0154	0.0154	0.0152	0.0153	0.0152	0.0152	0.0152	0.0152	0.0152	0.0152	0.0151	0.0152	0.0152
0.1:	0.572	0.548	0.548	0.544	0.548	0.529	0.528	0.507	0.503	0.501	0.496	0.501	0.501	0.502	0.495	0.495	0.498	0.508	0.502	0.502	0.502
std errors:	0.0156	0.0157	0.0157	0.0158	0.0157	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158
The proportions under the alternative hypothesis:					0.3	0.3	0.2	0.2													
n =	100			c(j) =	0.5	0.5	-0.5	-0.5				delta =	k			k =	4				
lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
alpha	The bootstrap estimated power																				
0.01:	0.124	0.126	0.131	0.132	0.138	0.144	0.151	0.151	0.15	0.153	0.151	0.15	0.147	0.147	0.149	0.15	0.146	0.142	0.144	0.137	0.147
std errors:	0.0104	0.0105	0.0107	0.0107	0.0109	0.0111	0.0113	0.0113	0.0113	0.0114	0.0113	0.0113	0.0112	0.0112	0.0113	0.0113	0.0112	0.011	0.0111	0.0109	0.0112
0.05:	0.319	0.317	0.319	0.326	0.326	0.328	0.326	0.328	0.331	0.338	0.344	0.343	0.343	0.339	0.342	0.348	0.348	0.337	0.339	0.342	0.336
std errors:	0.0147	0.0147	0.0147	0.0148	0.0148	0.0148	0.0148	0.0148	0.0149	0.015	0.015	0.015	0.015	0.015	0.015	0.0151	0.0151	0.0149	0.015	0.015	0.0149
0.1:	0.472	0.479	0.48	0.483	0.479	0.479	0.473	0.472	0.473	0.47	0.47	0.471	0.474	0.47	0.469	0.469	0.474	0.475	0.475	0.475	0.474
std errors:	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158
alpha	The AE approximation of power																				
0.01	0.1856	0.1917	0.2013	0.2083	0.2158	0.225	0.234	0.236	0.2381	0.2431	0.2477	0.25	0.2523	0.253	0.2548	0.2569	0.2529	0.2461	0.2458	0.2405	0.2441
0.05	0.3681	0.3723	0.3793	0.3887	0.3923	0.3963	0.397	0.4011	0.4059	0.4102	0.4196	0.4221	0.4203	0.4163	0.4153	0.4224	0.4179	0.4131	0.4124	0.4138	0.4097
0.1	0.4937	0.5045	0.5123	0.5169	0.5195	0.5252	0.527	0.5242	0.5302	0.5281	0.5273	0.5294	0.5345	0.5363	0.5359	0.5352	0.5359	0.5362	0.5333	0.5339	0.5312
alpha	The traditional estimated power, chi-square																				
0.01:	0.208	0.201	0.197	0.189	0.175	0.173	0.168	0.163	0.158	0.158	0.155	0.152	0.147	0.147	0.145	0.146	0.146	0.147	0.149	0.151	0.149
std errors:	0.0128	0.0127	0.0126	0.0124	0.012	0.012	0.0118	0.0117	0.0115	0.0115	0.0114	0.0114	0.0112	0.0112	0.0111	0.0112	0.0112	0.0112	0.0113	0.0113	0.0113
0.05:	0.378	0.368	0.352	0.352	0.347	0.341	0.341	0.338	0.34	0.338	0.33	0.328	0.327	0.32	0.321	0.325	0.329	0.332	0.334	0.334	0.334
std errors:	0.0153	0.0153	0.0151	0.0151	0.0151	0.015	0.015	0.015	0.015	0.015	0.0149	0.0148	0.0148	0.0148	0.0148	0.0148	0.0149	0.0149	0.0149	0.0149	0.0149
0.1:	0.512	0.496	0.485	0.48	0.474	0.468	0.466	0.463	0.462	0.46	0.455	0.452	0.447	0.448	0.451	0.451	0.45	0.451	0.449	0.456	0.456
std errors:	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0158	0.0158

Table 11

Proportions under the null hypothesis:					0.2	0.2	0.2	0.2	0.2												
The proportions under the alternative hypothesis:					0.175	0.175	0.175	0.175	0.3												
n =	20			c(j) =	-0.1155	-0.1155	-0.1155	0.1732	0.1732			delta =	0.5			k =	5				
lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
alpha	The bootstrap estimated power																				
0.01:	0.008	0.014	0.013	0.014	0.014	0.02	0.02	0.019	0.02	0.024	0.025	0.024	0.029	0.031	0.032	0.033	0.027	0.027	0.032	0.032	0.031
std errors:	0.0028	0.0037	0.0036	0.0037	0.0037	0.0044	0.0044	0.0043	0.0044	0.0048	0.0049	0.0048	0.0053	0.0055	0.0056	0.0056	0.0051	0.0051	0.0056	0.0056	0.0055
0.05:	0.077	0.077	0.072	0.076	0.076	0.072	0.076	0.073	0.078	0.093	0.111	0.114	0.119	0.113	0.115	0.121	0.114	0.112	0.119	0.114	0.103
std errors:	0.0084	0.0084	0.0082	0.0084	0.0084	0.0082	0.0084	0.0082	0.0085	0.0092	0.0099	0.0101	0.0102	0.01	0.0101	0.0103	0.0101	0.01	0.0102	0.0101	0.0096
0.1:	0.171	0.171	0.186	0.186	0.175	0.176	0.176	0.176	0.176	0.18	0.173	0.187	0.191	0.189	0.188	0.209	0.187	0.201	0.197	0.191	0.191
std errors:	0.0119	0.0119	0.0123	0.0123	0.012	0.012	0.012	0.012	0.012	0.0121	0.012	0.0123	0.0124	0.0124	0.0124	0.0129	0.0123	0.0127	0.0126	0.0124	0.0124
alpha	The AE approximation of power																				
0.01	0	0	0	0	0	0	0	0.0002	0.0014	0.0048	0.0101	0.0112	0.0146	0.0148	0.0153	0.0132	0.0109	0.0086	0.0073	0.006	0.0041
0.05	0	0	0	0	0	0	0	0.0007	0.0082	0.0287	0.0488	0.0642	0.0685	0.0784	0.0818	0.0857	0.0767	0.07	0.0653	0.0534	0.0425
0.1	0.0183	0.0266	0.0372	0.0497	0.0634	0.0776	0.076	0.0821	0.0923	0.1014	0.1092	0.1253	0.1416	0.1369	0.1431	0.1518	0.1487	0.1439	0.1375	0.1298	0.1208
alpha	The traditional estimated power, chi-square																				
0.01:	0.171	0.141	0.118	0.118	0.111	0.102	0.093	0.09	0.089	0.063	0.033	0.027	0.024	0.031	0.031	0.033	0.033	0.048	0.053	0.053	0.054
std errors:	0.0119	0.011	0.0102	0.0102	0.0099	0.0096	0.0092	0.009	0.009	0.0077	0.0056	0.0051	0.0048	0.0055	0.0055	0.0056	0.0056	0.0068	0.0071	0.0071	0.0071
0.05:	0.396	0.337	0.273	0.233	0.193	0.173	0.173	0.144	0.147	0.147	0.138	0.13	0.12	0.105	0.104	0.114	0.103	0.112	0.123	0.131	0.132
std errors:	0.0155	0.0149	0.0141	0.0134	0.0125	0.012	0.012	0.0111	0.0112	0.0112	0.0109	0.0106	0.0103	0.0097	0.0097	0.0101	0.0096	0.01	0.0104	0.0107	0.0107
0.1:	0.409	0.409	0.401	0.353	0.289	0.288	0.254	0.225	0.203	0.203	0.188	0.187	0.182	0.189	0.187	0.186	0.185	0.181	0.182	0.197	0.191
std errors:	0.0155	0.0155	0.0155	0.0151	0.0143	0.0143	0.0138	0.0132	0.0127	0.0127	0.0124	0.0123	0.0122	0.0124	0.0123	0.0123	0.0123	0.0122	0.0122	0.0126	0.0124
alpha	Reed & Cressie Exact Power Functions (1988:78)																				
0.05	0.0742	-	-	-	-	0.0742	-	-	-	-	0.1073	-	-	-	-	0.1228	-	-	-	-	0.1278
The proportions under the alternative hypothesis:					0.175	0.175	0.175	0.175	0.3												
n =	30			c(j) =	-0.1155	-0.1155	-0.1155	0.1732	0.1732			delta =	0.5			k =	5				
lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
alpha	The bootstrap estimated power																				
0.01:	0.012	0.013	0.013	0.011	0.013	0.013	0.013	0.013	0.017	0.02	0.03	0.035	0.035	0.038	0.039	0.051	0.06	0.061	0.058	0.06	0.057
std errors:	0.0034	0.0036	0.0036	0.0033	0.0036	0.0036	0.0036	0.0036	0.0041	0.0044	0.0054	0.0058	0.0058	0.006	0.0061	0.007	0.0075	0.0076	0.0074	0.0075	0.0073
0.05:	0.094	0.094	0.096	0.104	0.115	0.114	0.123	0.143	0.148	0.15	0.146	0.155	0.161	0.164	0.169	0.177	0.18	0.182	0.189	0.188	0.192
std errors:	0.0092	0.0092	0.0093	0.0097	0.0101	0.0101	0.0104	0.0111	0.0112	0.0113	0.0112	0.0114	0.0116	0.0117	0.0119	0.0121	0.0121	0.0122	0.0124	0.0124	0.0125
0.1:	0.234	0.241	0.241	0.239	0.245	0.245	0.242	0.25	0.251	0.253	0.26	0.271	0.261	0.266	0.27	0.281	0.278	0.276	0.271	0.274	0.27
std errors:	0.0134	0.0135	0.0135	0.0135	0.0136	0.0136	0.0135	0.0137	0.0137	0.0137	0.0139	0.0141	0.0139	0.014	0.014	0.0142	0.0142	0.0141	0.0141	0.0141	0.014
alpha	The AE approximation of power																				
0.01	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0003	0.004	0.0107	0.0158	0.0171	0.0201	0.0219	0.0223	0.0243	0.0232	0.0216	0.0199	0.0183	0.0169
0.05	0.0015	0.0039	0.008	0.013	0.0201	0.0285	0.0352	0.0466	0.0539	0.0593	0.0609	0.0701	0.0752	0.0764	0.083	0.0819	0.0846	0.0821	0.0797	0.0737	0.0663
0.1	0.0539	0.0625	0.0667	0.0762	0.086	0.0924	0.0965	0.1062	0.1119	0.1157	0.1306	0.1423	0.147	0.1452	0.1495	0.1541	0.1519	0.1485	0.1439	0.1468	0.139
alpha	The traditional estimated power, chi-square																				
0.01:	0.185	0.171	0.157	0.147	0.144	0.133	0.116	0.082	0.074	0.062	0.067	0.064	0.064	0.066	0.076	0.081	0.078	0.082	0.096	0.105	0.12
std errors:	0.0123	0.0119	0.0115	0.0112	0.0111	0.0107	0.0101	0.0087	0.0083	0.0076	0.0079	0.0077	0.0077	0.0079	0.0084	0.0086	0.0085	0.0087	0.0093	0.0097	0.0103
0.05:	0.333	0.3	0.273	0.26	0.255	0.244	0.223	0.22	0.205	0.191	0.18	0.181	0.176	0.175	0.182	0.177	0.187	0.198	0.203	0.209	0.213
std errors:	0.0149	0.0145	0.0141	0.0139	0.0138	0.0136	0.0132	0.0131	0.0128	0.0124	0.0121	0.0122	0.012	0.012	0.0122	0.0121	0.0123	0.0126	0.0127	0.0129	0.0129
0.1:	0.414	0.408	0.395	0.37	0.333	0.333	0.308	0.302	0.293	0.293	0.292	0.276	0.266	0.272	0.267	0.261	0.273	0.276	0.276	0.278	0.282
std errors:	0.0156	0.0155	0.0155	0.0153	0.0149	0.0149	0.0146	0.0145	0.0144	0.0144	0.0144	0.0141	0.014	0.0141	0.014	0.0139	0.0141	0.0141	0.0141	0.0142	0.0142

Proportions under the null hypothesis:					0.2	0.2	0.2	0.2	0.2												
The proportions under the alternative hypothesis:					0.175	0.175	0.175	0.175	0.3												
n =	50			c(j) =	-0.1155	-0.1155	-0.1155	0.1732	0.1732			delta =	0.5			k =	5				
lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
alpha	The bootstrap estimated power																				
0.01:	0.057	0.062	0.066	0.065	0.068	0.06	0.068	0.072	0.078	0.085	0.087	0.091	0.092	0.092	0.095	0.09	0.098	0.106	0.106	0.113	0.116
std errors:	0.0073	0.0076	0.0079	0.0078	0.008	0.0075	0.008	0.0082	0.0085	0.0088	0.0089	0.0091	0.0091	0.0091	0.0093	0.009	0.0094	0.0097	0.0097	0.01	0.0101
0.05:	0.181	0.183	0.194	0.192	0.201	0.206	0.203	0.216	0.222	0.226	0.228	0.23	0.233	0.245	0.245	0.253	0.255	0.258	0.26	0.253	0.251
std errors:	0.0122	0.0122	0.0125	0.0125	0.0127	0.0128	0.0127	0.013	0.0131	0.0132	0.0133	0.0133	0.0134	0.0136	0.0136	0.0137	0.0138	0.0138	0.0139	0.0137	0.0137
0.1:	0.299	0.302	0.316	0.315	0.324	0.337	0.337	0.342	0.343	0.347	0.348	0.353	0.357	0.363	0.366	0.367	0.372	0.37	0.369	0.372	0.369
std errors:	0.0145	0.0145	0.0147	0.0147	0.0148	0.0149	0.0149	0.015	0.015	0.0151	0.0151	0.0151	0.0152	0.0152	0.0152	0.0152	0.0153	0.0153	0.0153	0.0153	0.0153
alpha	The AE approximation of power																				
0.01	0.0247	0.0342	0.0418	0.0488	0.054	0.0559	0.0645	0.07	0.0779	0.0839	0.0861	0.0875	0.0883	0.0878	0.0856	0.0807	0.0803	0.0799	0.0763	0.0752	0.0727
0.05	0.0936	0.1009	0.1085	0.1132	0.1196	0.1254	0.1293	0.1342	0.139	0.1437	0.148	0.1485	0.1502	0.1517	0.1525	0.1513	0.1537	0.1519	0.1501	0.1439	0.139
0.1	0.1402	0.1481	0.1583	0.1651	0.1692	0.1805	0.1843	0.188	0.198	0.1973	0.2006	0.2032	0.2032	0.2057	0.2063	0.2108	0.2054	0.2054	0.2022	0.1984	0.1956
alpha	The traditional estimated power, chi-square																				
0.01:	0.155	0.149	0.14	0.136	0.121	0.107	0.101	0.091	0.089	0.086	0.082	0.078	0.085	0.088	0.093	0.1	0.108	0.116	0.119	0.126	0.134
std errors:	0.0114	0.0113	0.011	0.0108	0.0103	0.0098	0.0095	0.0091	0.009	0.0089	0.0087	0.0085	0.0088	0.009	0.0092	0.0095	0.0098	0.0101	0.0102	0.0105	0.0108
0.05:	0.325	0.322	0.308	0.296	0.273	0.261	0.26	0.25	0.24	0.234	0.233	0.233	0.228	0.228	0.237	0.236	0.243	0.248	0.257	0.267	0.271
std errors:	0.0148	0.0148	0.0146	0.0144	0.0141	0.0139	0.0139	0.0137	0.0135	0.0134	0.0134	0.0134	0.0133	0.0133	0.0134	0.0134	0.0136	0.0137	0.0138	0.014	0.0141
0.1:	0.408	0.399	0.397	0.388	0.388	0.385	0.379	0.374	0.361	0.357	0.354	0.353	0.357	0.355	0.359	0.367	0.365	0.369	0.369	0.374	0.383
std errors:	0.0155	0.0155	0.0155	0.0154	0.0154	0.0154	0.0153	0.0153	0.0152	0.0152	0.0151	0.0151	0.0152	0.0151	0.0152	0.0152	0.0152	0.0153	0.0153	0.0153	0.0154
The proportions under the alternative hypothesis:					0.175	0.175	0.175	0.175	0.3												
n =	100			c(j) =	-0.1155	-0.1155	-0.1155	0.1732	0.1732			delta =	0.5			k =	5				
lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
alpha	The bootstrap estimated power																				
0.01:	0.146	0.157	0.174	0.194	0.21	0.229	0.241	0.249	0.251	0.252	0.254	0.259	0.261	0.268	0.267	0.264	0.279	0.266	0.271	0.276	0.284
std errors:	0.0112	0.0115	0.012	0.0125	0.0129	0.0133	0.0135	0.0137	0.0137	0.0137	0.0138	0.0139	0.0139	0.014	0.014	0.0139	0.0142	0.014	0.0141	0.0141	0.0143
0.05:	0.406	0.41	0.417	0.424	0.44	0.448	0.459	0.463	0.47	0.479	0.48	0.481	0.489	0.489	0.494	0.497	0.499	0.501	0.51	0.512	0.513
std errors:	0.0155	0.0156	0.0156	0.0156	0.0157	0.0157	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158
0.1:	0.529	0.536	0.545	0.552	0.559	0.57	0.572	0.575	0.583	0.584	0.587	0.595	0.597	0.606	0.611	0.621	0.62	0.621	0.63	0.636	0.639
std errors:	0.0158	0.0158	0.0157	0.0157	0.0157	0.0157	0.0156	0.0156	0.0156	0.0156	0.0156	0.0155	0.0155	0.0155	0.0154	0.0153	0.0153	0.0153	0.0153	0.0152	0.0152
alpha	The AE approximation of power																				
0.01	0.0449	0.0503	0.0573	0.0642	0.0709	0.0756	0.0794	0.0823	0.0829	0.0833	0.0836	0.0839	0.083	0.0829	0.0817	0.0777	0.0783	0.0736	0.0715	0.0692	0.068
0.05	0.1248	0.1289	0.1307	0.1335	0.1387	0.1416	0.1443	0.1476	0.1495	0.1502	0.1498	0.1495	0.1522	0.1516	0.152	0.1497	0.1485	0.1476	0.1473	0.1465	0.1442
0.1	0.1781	0.183	0.1888	0.1926	0.198	0.2014	0.2028	0.204	0.2051	0.2072	0.2088	0.2112	0.2108	0.2136	0.2149	0.2191	0.217	0.2161	0.2163	0.2172	0.2159
alpha	The traditional estimated power, chi-square																				
0.01:	0.243	0.241	0.24	0.235	0.233	0.233	0.233	0.233	0.23	0.228	0.232	0.233	0.239	0.245	0.254	0.261	0.265	0.279	0.293	0.304	0.317
std errors:	0.0136	0.0135	0.0135	0.0134	0.0134	0.0134	0.0134	0.0134	0.0133	0.0133	0.0133	0.0134	0.0135	0.0136	0.0138	0.0139	0.014	0.0142	0.0144	0.0145	0.0147
0.05:	0.457	0.456	0.455	0.454	0.452	0.449	0.449	0.451	0.454	0.454	0.458	0.463	0.468	0.474	0.48	0.487	0.49	0.496	0.497	0.506	0.511
std errors:	0.0158	0.0158	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158
0.1:	0.569	0.57	0.573	0.566	0.565	0.56	0.563	0.567	0.565	0.568	0.568	0.57	0.575	0.58	0.584	0.592	0.594	0.597	0.601	0.609	0.611
std errors:	0.0157	0.0157	0.0156	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0156	0.0156	0.0156	0.0155	0.0155	0.0155	0.0155	0.0154	0.0154

Proportions under the null hypothesis: 0.2 0.2 0.2 0.2 0.2

The proportions under the alternative hypothesis: 0.125 0.125 0.125 0.125 0.5

n = **20** c(j) = -0.2 -0.2 -0.2 0.3 0.3 delta = 1.5 k = 5

lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
alpha:	The bootstrap estimated power																				
0.01:	0.178	0.236	0.24	0.247	0.247	0.247	0.243	0.252	0.311	0.379	0.397	0.476	0.5	0.517	0.562	0.562	0.556	0.575	0.564	0.574	0.573
std errors:	0.0121	0.0134	0.0135	0.0136	0.0136	0.0136	0.0136	0.0137	0.0146	0.0153	0.0155	0.0158	0.0158	0.0158	0.0157	0.0157	0.0157	0.0156	0.0157	0.0156	0.0156
0.05:	0.272	0.272	0.272	0.272	0.272	0.272	0.272	0.293	0.434	0.571	0.643	0.651	0.707	0.72	0.73	0.776	0.767	0.765	0.764	0.771	0.768
std errors:	0.0141	0.0141	0.0141	0.0141	0.0141	0.0141	0.0141	0.0144	0.0157	0.0157	0.0152	0.0151	0.0144	0.0142	0.014	0.0132	0.0134	0.0134	0.0134	0.0133	0.0133
0.1:	0.685	0.685	0.7	0.714	0.712	0.725	0.725	0.763	0.767	0.758	0.758	0.762	0.799	0.797	0.82	0.819	0.82	0.816	0.815	0.823	0.82
std errors:	0.0147	0.0147	0.0145	0.0143	0.0143	0.0141	0.0141	0.0134	0.0134	0.0135	0.0135	0.0135	0.0127	0.0127	0.0121	0.0122	0.0121	0.0123	0.0123	0.0121	0.0121
alpha	The AE approximation of power																				
0.01	-0.0008	-0.0008	-.0008 -	0.0008	-0.0008	-0.0008	-0.0008	0.0001	0.0052	0.0173	0.0258	0.035	0.0454	0.0553	0.0535	0.0456	0.0389	0.035	0.0264	0.0209	0.0143
0.05	-0.0008	-0.0008	-.0008 -	0.0008	-0.0008	-0.0008	-0.0008	0.0016	0.0211	0.063	0.1	0.1202	0.1466	0.1452	0.161	0.1752	0.1571	0.1385	0.1238	0.119	0.1128
0.1	0.0503	0.0646	0.082	0.1014	0.1219	0.1424	0.1621	0.1804	0.1967	0.2107	0.2222	0.231	0.2372	0.2408	0.2418	0.2404	0.2571	0.2614	0.2506	0.238	0.2239
alpha	The traditional estimated power, chi-square																				
0.01:	0.685	0.646	0.568	0.568	0.559	0.501	0.488	0.486	0.477	0.469	0.451	0.463	0.455	0.484	0.484	0.507	0.507	0.565	0.605	0.609	0.616
std errors:	0.0147	0.0151	0.0157	0.0157	0.0157	0.0158	0.0158	0.0158	0.0158	0.0158	0.0157	0.0158	0.0157	0.0158	0.0158	0.0158	0.0158	0.0157	0.0155	0.0154	0.0154
0.05:	0.796	0.783	0.775	0.747	0.731	0.704	0.704	0.678	0.687	0.687	0.683	0.71	0.703	0.697	0.697	0.716	0.711	0.727	0.764	0.773	0.774
std errors:	0.0127	0.013	0.0132	0.0137	0.014	0.0144	0.0144	0.0148	0.0147	0.0147	0.0147	0.0143	0.0144	0.0145	0.0145	0.0143	0.0143	0.0141	0.0134	0.0132	0.0132
0.1:	0.833	0.833	0.833	0.829	0.821	0.821	0.797	0.782	0.773	0.773	0.758	0.758	0.758	0.797	0.795	0.801	0.8	0.797	0.797	0.812	0.812
std errors:	0.0118	0.0118	0.0118	0.0119	0.0121	0.0121	0.0127	0.0131	0.0132	0.0132	0.0135	0.0135	0.0135	0.0127	0.0128	0.0126	0.0126	0.0127	0.0127	0.0124	0.0124
alpha	Reed & Cressie Exact Power Functions (1988:78)																				
0.05	0.2253	-	-	-	-	0.2253	-	-	-	-	0.61	-	-	-	-	0.6997	-	-	-	-	0.7306

The proportions under the alternative hypothesis: 0.125 0.125 0.125 0.125 0.5

n = **30** delta = 1.5 k = 5

lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
alpha	The bootstrap estimated power																				
0.01:	0.311	0.327	0.344	0.362	0.405	0.445	0.457	0.49	0.497	0.576	0.635	0.663	0.673	0.704	0.735	0.738	0.744	0.745	0.742	0.741	0.741
std errors:	0.0146	0.0148	0.015	0.0152	0.0155	0.0157	0.0158	0.0158	0.0158	0.0156	0.0152	0.0149	0.0148	0.0144	0.014	0.0139	0.0138	0.0138	0.0138	0.0139	0.0139
0.05:	0.373	0.428	0.49	0.54	0.589	0.67	0.71	0.753	0.784	0.801	0.815	0.841	0.84	0.855	0.859	0.858	0.859	0.859	0.862	0.861	0.861
std errors:	0.0153	0.0156	0.0158	0.0158	0.0156	0.0149	0.0143	0.0136	0.013	0.0126	0.0123	0.0116	0.0116	0.0111	0.011	0.011	0.011	0.011	0.0109	0.0109	0.0109
0.1:	0.777	0.796	0.809	0.809	0.832	0.842	0.843	0.848	0.859	0.872	0.877	0.882	0.883	0.905	0.909	0.925	0.923	0.923	0.923	0.924	0.924
std errors:	0.0132	0.0127	0.0124	0.0124	0.0118	0.0115	0.0115	0.0114	0.011	0.0106	0.0104	0.0102	0.0102	0.0093	0.0091	0.0083	0.0084	0.0084	0.0084	0.0084	0.0084
alpha	The AE approximation of power																				
0.01	0.0036	0.0085	0.0156	0.0231	0.029	0.0328	0.0351	0.0363	0.037	0.0376	0.0385	0.0393	0.0397	0.0402	0.0406	0.0406	0.0404	0.0396	0.0387	0.0381	0.0376
0.05	0.0112	0.0217	0.0312	0.036	0.0375	0.0393	0.0442	0.0503	0.0607	0.0644	0.0712	0.0817	0.083	0.0876	0.0859	0.0817	0.0843	0.0819	0.0766	0.0685	0.0629
0.1	0.0545	0.0602	0.0679	0.0773	0.0825	0.0908	0.0985	0.1087	0.1166	0.1297	0.1425	0.1474	0.1635	0.1681	0.1688	0.173	0.1701	0.1656	0.1597	0.1525	0.1443
alpha	The traditional estimated power, chi-square																				
0.01:	0.721	0.718	0.692	0.678	0.675	0.67	0.668	0.638	0.659	0.65	0.672	0.668	0.673	0.685	0.709	0.738	0.738	0.745	0.765	0.789	0.818
std errors:	0.0142	0.0142	0.0146	0.0148	0.0148	0.0149	0.0149	0.0152	0.015	0.0151	0.0148	0.0149	0.0148	0.0147	0.0144	0.0139	0.0139	0.0138	0.0134	0.0129	0.0122
0.05:	0.858	0.857	0.85	0.847	0.846	0.842	0.843	0.842	0.835	0.835	0.831	0.842	0.852	0.855	0.859	0.859	0.86	0.868	0.87	0.891	0.907
std errors:	0.011	0.0111	0.0113	0.0114	0.0114	0.0115	0.0115	0.0115	0.0117	0.0117	0.0119	0.0115	0.0112	0.0111	0.011	0.011	0.011	0.0107	0.0106	0.0099	0.0092
0.1:	0.894	0.894	0.894	0.894	0.892	0.892	0.892	0.888	0.885	0.885	0.885	0.883	0.883	0.904	0.903	0.911	0.923	0.924	0.924	0.925	0.928
std errors:	0.0097	0.0097	0.0097	0.0097	0.0098	0.0098	0.0098	0.01	0.0101	0.0101	0.0101	0.0102	0.0102	0.0093	0.0094	0.009	0.0084	0.0084	0.0084	0.0083	0.0082

Proportions under the null hypothesis:					0.2	0.2	0.2	0.2	0.2												
The proportions under the alternative hypothesis:					0.125	0.125	0.125	0.125	0.5												
n =	50			c(j) =	-0.2	-0.2	-0.2	0.3	0.3			delta =	1.5			k =	5				
lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
alpha	The bootstrap estimated power																				
0.01:	0.69	0.717	0.751	0.791	0.82	0.823	0.849	0.863	0.89	0.913	0.913	0.923	0.935	0.935	0.939	0.939	0.947	0.949	0.95	0.953	0.953
std errors:	0.0146	0.0142	0.0137	0.0129	0.0121	0.0121	0.0113	0.0109	0.0099	0.0089	0.0089	0.0084	0.0078	0.0078	0.0076	0.0076	0.0071	0.007	0.0069	0.0067	0.0067
0.05:	0.922	0.934	0.939	0.951	0.959	0.961	0.962	0.966	0.97	0.971	0.974	0.98	0.981	0.982	0.983	0.984	0.986	0.985	0.986	0.987	0.987
std errors:	0.0085	0.0079	0.0076	0.0068	0.0063	0.0061	0.006	0.0057	0.0054	0.0053	0.005	0.0044	0.0043	0.0042	0.0041	0.004	0.0037	0.0038	0.0037	0.0036	0.0036
0.1:	0.965	0.97	0.971	0.977	0.98	0.98	0.984	0.985	0.989	0.989	0.989	0.992	0.992	0.992	0.992	0.992	0.992	0.992	0.992	0.993	0.994
std errors:	0.0058	0.0054	0.0053	0.0047	0.0044	0.0044	0.004	0.0038	0.0033	0.0033	0.0033	0.0028	0.0028	0.0028	0.0028	0.0028	0.0028	0.0028	0.0028	0.0026	0.0024
alpha	The AE approximation of power																				
0.01	0.0463	0.0606	0.0752	0.0895	0.0996	0.1012	0.1109	0.1194	0.13	0.1388	0.1395	0.1422	0.1452	0.1439	0.1421	0.1392	0.1358	0.1317	0.127	0.1216	0.1179
0.05	0.1593	0.1679	0.1753	0.1837	0.1918	0.1991	0.2059	0.2144	0.2172	0.2268	0.2298	0.2343	0.2404	0.2423	0.2429	0.2449	0.2386	0.2375	0.2312	0.2267	0.2232
0.1	0.2353	0.2471	0.256	0.267	0.2726	0.2772	0.2877	0.2961	0.2975	0.3013	0.3053	0.3056	0.3122	0.3191	0.3181	0.3209	0.3172	0.3152	0.3163	0.3158	0.3132
alpha	The traditional estimated power, chi-square																				
0.01:	0.906	0.909	0.908	0.907	0.909	0.908	0.917	0.917	0.915	0.918	0.924	0.935	0.936	0.94	0.945	0.952	0.956	0.958	0.96	0.962	0.965
std errors:	0.0092	0.0091	0.0091	0.0092	0.0091	0.0091	0.0087	0.0087	0.0088	0.0087	0.0084	0.0078	0.0077	0.0075	0.0072	0.0068	0.0065	0.0063	0.0062	0.006	0.0058
0.05:	0.97	0.97	0.97	0.97	0.971	0.97	0.972	0.972	0.971	0.972	0.972	0.976	0.98	0.98	0.98	0.979	0.98	0.981	0.985	0.987	0.987
std errors:	0.0054	0.0054	0.0054	0.0054	0.0053	0.0054	0.0052	0.0052	0.0053	0.0052	0.0052	0.0048	0.0044	0.0044	0.0044	0.0045	0.0044	0.0043	0.0038	0.0036	0.0036
0.1:	0.985	0.985	0.985	0.985	0.986	0.986	0.986	0.989	0.989	0.989	0.989	0.991	0.992	0.992	0.992	0.992	0.992	0.992	0.992	0.992	0.993
std errors:	0.0038	0.0038	0.0038	0.0038	0.0037	0.0037	0.0037	0.0033	0.0033	0.0033	0.0033	0.003	0.0028	0.0028	0.0028	0.0028	0.0028	0.0028	0.0028	0.0028	0.0026
Proportions under the null hypothesis:					0.2	0.2	0.2	0.2	0.2												
n =	100			c(j) =	-0.2	-0.2	-0.2	0.3	0.3			delta =	1.5			k =	5				
lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
alpha	The bootstrap estimated power																				
0.01:	0.995	0.996	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.998	0.998	0.998	0.999	1	1	1	1	1	1	1	1
std errors:	0.0022	0.002	0.0017	0.0017	0.0017	0.0017	0.0017	0.0017	0.0017	0.0014	0.0014	0.0014	0.001	0	0	0	0	0	0	0	0
0.05:	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
std errors:	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.1:	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
std errors:	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
alpha	The AE approximation of power																				
0.01	0.0851	0.0935	0.1021	0.1064	0.1119	0.1158	0.12	0.1272	0.1273	0.1271	0.1302	0.1343	0.1342	0.1337	0.1301	0.1318	0.1294	0.1257	0.125	0.1236	0.1208
0.05	0.1911	0.1933	0.1962	0.2004	0.2051	0.2095	0.2131	0.2176	0.2183	0.2244	0.2275	0.2304	0.2298	0.2316	0.232	0.2309	0.2295	0.2272	0.2294	0.2275	0.2234
0.1	0.2644	0.2723	0.2792	0.2826	0.2843	0.2909	0.2915	0.293	0.2943	0.2936	0.2942	0.2952	0.2961	0.2967	0.2956	0.2922	0.2945	0.2916	0.2892	0.2871	0.2853
alpha	The traditional estimated power, chi-square																				
0.01:	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.999	1	1	1	1	1	1	1	1
std errors:	0.0014	0.0014	0.0014	0.0014	0.0014	0.0014	0.0014	0.0014	0.0014	0.0014	0.0014	0.0014	0.001	0	0	0	0	0	0	0	0
0.05:	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
std errors:	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.1:	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
std errors:	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Proportions under the null hypothesis: 0.2 0.2 0.2 0.2 0.2

The proportions under the alternative hypothesis: 0.1293 0.1293 0.1293 0.3061 0.306

n = 20 c(j) = -0.316 -0.316 -0.316 0.474 0.474 delta = 3k/4 k = 5

lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
The bootstrap estimated power																					
0.01:	0.06	0.093	0.087	0.093	0.093	0.096	0.093	0.09	0.092	0.109	0.111	0.119	0.115	0.122	0.113	0.116	0.113	0.113	0.124	0.104	0.093
std errors:	0.0075	0.0092	0.0089	0.0092	0.0092	0.0093	0.0092	0.009	0.0091	0.0099	0.0099	0.0102	0.0101	0.0103	0.01	0.0101	0.01	0.01	0.0104	0.0097	0.0092
0.05:	0.189	0.189	0.188	0.189	0.189	0.185	0.189	0.189	0.217	0.232	0.26	0.273	0.277	0.285	0.283	0.287	0.28	0.288	0.279	0.279	0.279
std errors:	0.0124	0.0124	0.0124	0.0124	0.0124	0.0123	0.0124	0.0124	0.013	0.0133	0.0139	0.0141	0.0142	0.0143	0.0142	0.0143	0.0142	0.0143	0.0142	0.0142	0.0142
0.1:	0.392	0.392	0.42	0.409	0.408	0.421	0.393	0.403	0.401	0.401	0.396	0.403	0.406	0.411	0.419	0.412	0.412	0.415	0.41	0.424	0.424
std errors:	0.0154	0.0154	0.0156	0.0155	0.0155	0.0156	0.0154	0.0155	0.0155	0.0155	0.0155	0.0155	0.0155	0.0156	0.0156	0.0156	0.0156	0.0156	0.0156	0.0156	0.0156
The AE approximation of power																					
0.01	-0.0122	-0.0122	-.0122 -	0.0122	-0.0122	-0.0122	-0.0121	-0.0087	0.0107	0.0457	0.0743	0.0966	0.1085	0.1222	0.1122	0.0991	0.0973	0.094	0.0746	0.0562	0.0395
0.05	-0.0122	-0.0122	-.0122 -	0.0122	-0.0122	-0.0122	-0.012	-0.0002	0.0701	0.1444	0.1994	0.2366	0.2766	0.275	0.2984	0.2852	0.2862	0.2773	0.2526	0.2375	0.2111
0.1	0.1511	0.1814	0.2147	0.249	0.2824	0.3138	0.3403	0.356	0.345	0.3605	0.3748	0.3752	0.4007	0.4102	0.4178	0.4044	0.4286	0.4286	0.4221	0.4097	0.3949
The traditional estimated power, chi-square																					
0.01:	0.392	0.338	0.301	0.301	0.277	0.257	0.251	0.242	0.233	0.197	0.143	0.121	0.107	0.111	0.109	0.116	0.116	0.134	0.146	0.153	0.174
std errors:	0.0154	0.015	0.0145	0.0145	0.0142	0.0138	0.0137	0.0135	0.0134	0.0126	0.0111	0.0103	0.0098	0.0099	0.0099	0.0101	0.0101	0.0108	0.0112	0.0114	0.012
0.05:	0.614	0.576	0.523	0.477	0.431	0.392	0.392	0.342	0.343	0.343	0.334	0.322	0.309	0.285	0.279	0.299	0.277	0.291	0.306	0.309	0.317
std errors:	0.0154	0.0156	0.0158	0.0158	0.0157	0.0154	0.0154	0.015	0.015	0.015	0.0149	0.0148	0.0146	0.0143	0.0142	0.0145	0.0142	0.0144	0.0146	0.0146	0.0147
0.1:	0.642	0.642	0.639	0.605	0.552	0.548	0.516	0.474	0.448	0.448	0.42	0.42	0.411	0.424	0.419	0.412	0.411	0.406	0.41	0.438	0.426
std errors:	0.0152	0.0152	0.0152	0.0155	0.0157	0.0157	0.0158	0.0158	0.0157	0.0157	0.0156	0.0156	0.0156	0.0156	0.0156	0.0156	0.0156	0.0155	0.0156	0.0157	0.0156

The proportions under the alternative hypothesis: 0.1423 0.1423 0.1423 0.2866 0.2865

n = 30 c(j) = -0.316 -0.316 -0.316 0.474 0.474 delta = 3k/4 k = 5

lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
The bootstrap estimated power																					
0.01:	0.104	0.104	0.107	0.111	0.117	0.117	0.128	0.143	0.13	0.116	0.127	0.137	0.153	0.141	0.137	0.144	0.146	0.148	0.147	0.143	0.147
std errors:	0.0097	0.0097	0.0098	0.0099	0.0102	0.0102	0.0106	0.0111	0.0106	0.0101	0.0105	0.0109	0.0114	0.011	0.0109	0.0111	0.0112	0.0112	0.0112	0.0111	0.0112
0.05:	0.181	0.187	0.204	0.213	0.232	0.244	0.265	0.286	0.302	0.306	0.31	0.32	0.33	0.329	0.337	0.333	0.334	0.341	0.34	0.348	0.342
std errors:	0.0122	0.0123	0.0127	0.0129	0.0133	0.0136	0.014	0.0143	0.0145	0.0146	0.0146	0.0148	0.0149	0.0149	0.0149	0.0149	0.0149	0.015	0.015	0.0151	0.015
0.1:	0.393	0.396	0.4	0.399	0.388	0.397	0.398	0.403	0.407	0.417	0.417	0.441	0.436	0.451	0.464	0.46	0.454	0.453	0.47	0.455	0.456
std errors:	0.0154	0.0155	0.0155	0.0155	0.0154	0.0155	0.0155	0.0155	0.0155	0.0156	0.0156	0.0157	0.0157	0.0157	0.0158	0.0158	0.0157	0.0157	0.0158	0.0157	0.0158
The AE approximation of power																					
0.01	0.0306	0.0382	0.049	0.0621	0.0761	0.092	0.1089	0.1265	0.1319	0.1313	0.1465	0.1625	0.1749	0.1692	0.1653	0.1744	0.1715	0.1674	0.1585	0.1474	0.1357
0.05	0.0412	0.0573	0.0787	0.1038	0.132	0.1655	0.2002	0.2406	0.2649	0.2891	0.3034	0.3095	0.326	0.3269	0.3345	0.3269	0.3315	0.3209	0.3152	0.312	0.2903
0.1	0.2651	0.2827	0.3015	0.3206	0.3301	0.3442	0.3587	0.3716	0.3829	0.4002	0.4162	0.4342	0.4413	0.4462	0.4531	0.4517	0.4522	0.4512	0.4488	0.4452	0.4365
The traditional estimated power, chi-square																					
0.01:	0.321	0.299	0.28	0.266	0.262	0.244	0.224	0.192	0.177	0.163	0.159	0.145	0.133	0.129	0.138	0.144	0.142	0.146	0.162	0.176	0.191
std errors:	0.0148	0.0145	0.0142	0.014	0.0139	0.0136	0.0132	0.0125	0.0121	0.0117	0.0116	0.0111	0.0107	0.0106	0.0109	0.0111	0.011	0.0112	0.0117	0.012	0.0124
0.05:	0.515	0.482	0.465	0.442	0.425	0.397	0.374	0.37	0.342	0.334	0.322	0.32	0.318	0.311	0.317	0.311	0.32	0.337	0.344	0.351	0.363
std errors:	0.0158	0.0158	0.0158	0.0157	0.0156	0.0155	0.0153	0.0153	0.015	0.0149	0.0148	0.0148	0.0147	0.0146	0.0147	0.0146	0.0148	0.0149	0.015	0.0151	0.0152
0.1:	0.596	0.592	0.572	0.557	0.521	0.521	0.502	0.484	0.474	0.474	0.47	0.448	0.443	0.434	0.424	0.424	0.44	0.453	0.453	0.455	0.477
std errors:	0.0155	0.0155	0.0156	0.0157	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0157	0.0157	0.0157	0.0156	0.0156	0.0157	0.0157	0.0157	0.0157	0.0158

Proportions under the null hypothesis:					0.2	0.2	0.2	0.2	0.2												
The proportions under the alternative hypothesis:					0.1553	0.1553	0.1553	0.2671	0.267												
n =	50			c(j) =	-0.316	-0.316	-0.316	0.474	0.474			delta =	3k/4			k =	5				
lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
	The bootstrap estimated power																				
0.01:	0.08	0.083	0.089	0.092	0.097	0.1	0.103	0.102	0.11	0.118	0.126	0.119	0.117	0.126	0.127	0.136	0.133	0.125	0.122	0.119	0.129
std errors:	0.0086	0.0087	0.009	0.0091	0.0094	0.0095	0.0096	0.0096	0.0099	0.0102	0.0105	0.0102	0.0102	0.0105	0.0105	0.0108	0.0107	0.0105	0.0103	0.0102	0.0106
0.05:	0.25	0.249	0.258	0.261	0.264	0.264	0.271	0.273	0.29	0.294	0.293	0.292	0.294	0.292	0.3	0.295	0.297	0.299	0.302	0.297	0.299
std errors:	0.0137	0.0137	0.0138	0.0139	0.0139	0.0139	0.0141	0.0141	0.0143	0.0144	0.0144	0.0144	0.0144	0.0144	0.0145	0.0144	0.0144	0.0145	0.0145	0.0144	0.0145
0.1:	0.349	0.373	0.38	0.378	0.381	0.384	0.384	0.384	0.391	0.388	0.388	0.389	0.391	0.389	0.39	0.395	0.393	0.396	0.396	0.402	0.391
std errors:	0.0151	0.0153	0.0153	0.0153	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0155	0.0154	0.0155	0.0155	0.0155	0.0154
	The AE approximation of power																				
0.01	0.1645	0.1916	0.2168	0.2378	0.2555	0.2709	0.2822	0.2883	0.3014	0.3128	0.3217	0.3197	0.3209	0.3249	0.3238	0.3287	0.3229	0.3165	0.3119	0.3048	0.3049
0.05	0.3255	0.3361	0.3472	0.3546	0.3622	0.3689	0.3771	0.385	0.3986	0.4011	0.4095	0.4104	0.4132	0.4158	0.4194	0.4203	0.4158	0.4152	0.4137	0.4088	0.4039
0.1	0.443	0.4523	0.4593	0.468	0.4746	0.4812	0.4856	0.4905	0.4946	0.4995	0.5002	0.5043	0.5066	0.5071	0.5078	0.5058	0.5096	0.5105	0.5068	0.5042	0.4987
	The traditional estimated power, chi-square																				
0.01:	0.24	0.229	0.208	0.197	0.182	0.168	0.151	0.14	0.134	0.129	0.119	0.117	0.115	0.114	0.115	0.123	0.122	0.133	0.136	0.144	0.153
std errors:	0.0135	0.0133	0.0128	0.0126	0.0122	0.0118	0.0113	0.011	0.0108	0.0106	0.0102	0.0102	0.0101	0.0101	0.0101	0.0104	0.0103	0.0107	0.0108	0.0111	0.0114
0.05:	0.389	0.384	0.38	0.372	0.348	0.341	0.337	0.32	0.316	0.309	0.306	0.3	0.294	0.294	0.298	0.293	0.297	0.302	0.306	0.31	0.315
std errors:	0.0154	0.0154	0.0153	0.0153	0.0151	0.015	0.0149	0.0148	0.0147	0.0146	0.0146	0.0145	0.0144	0.0144	0.0145	0.0144	0.0144	0.0145	0.0146	0.0146	0.0147
0.1:	0.477	0.464	0.453	0.445	0.441	0.433	0.43	0.426	0.411	0.408	0.403	0.4	0.401	0.399	0.403	0.407	0.399	0.401	0.406	0.409	0.419
std errors:	0.0158	0.0158	0.0157	0.0157	0.0157	0.0157	0.0157	0.0156	0.0156	0.0155	0.0155	0.0155	0.0155	0.0155	0.0155	0.0155	0.0155	0.0155	0.0155	0.0155	0.0156
The proportions under the alternative hypothesis:					0.1684	0.1684	0.1684	0.2474	0.2474												
n =	100			c(j) =	-0.316	-0.316	-0.316	0.474	0.474			delta =	3k/4			k =	5				
lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
	The bootstrap estimated power																				
0.01:	0.098	0.104	0.108	0.11	0.112	0.113	0.12	0.128	0.138	0.139	0.147	0.146	0.156	0.163	0.179	0.182	0.177	0.175	0.173	0.174	0.177
std errors:	0.0094	0.0097	0.0098	0.0099	0.01	0.01	0.0103	0.0106	0.0109	0.0109	0.0112	0.0112	0.0115	0.0117	0.0121	0.0122	0.0121	0.012	0.012	0.012	0.0121
0.05:	0.302	0.304	0.305	0.307	0.308	0.308	0.314	0.327	0.328	0.328	0.33	0.332	0.331	0.333	0.335	0.341	0.344	0.342	0.335	0.331	0.33
std errors:	0.0145	0.0145	0.0146	0.0146	0.0146	0.0146	0.0147	0.0148	0.0148	0.0148	0.0149	0.0149	0.0149	0.0149	0.0149	0.015	0.015	0.015	0.0149	0.0149	0.0149
0.1:	0.421	0.427	0.427	0.436	0.433	0.435	0.434	0.437	0.434	0.432	0.44	0.441	0.444	0.446	0.447	0.45	0.447	0.447	0.451	0.451	0.451
std errors:	0.0156	0.0156	0.0156	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157
	The AE approximation of power																				
0.01	0.2283	0.2405	0.2497	0.2581	0.2653	0.2724	0.28	0.2855	0.2902	0.2962	0.3002	0.3004	0.3065	0.312	0.317	0.3197	0.3161	0.3127	0.3087	0.3074	0.3064
0.05	0.3645	0.3685	0.3734	0.3785	0.3829	0.3862	0.394	0.401	0.4042	0.4079	0.4085	0.4091	0.4099	0.4131	0.4143	0.419	0.4167	0.4156	0.409	0.405	0.4037
0.1	0.4704	0.4793	0.4826	0.4878	0.4927	0.4977	0.4998	0.5041	0.5056	0.5071	0.511	0.5123	0.514	0.517	0.517	0.5166	0.5169	0.5185	0.5173	0.5155	0.5146
	The traditional estimated power, chi-square																				
0.01:	0.187	0.182	0.178	0.171	0.16	0.153	0.151	0.149	0.145	0.141	0.141	0.142	0.137	0.136	0.141	0.147	0.148	0.15	0.151	0.159	0.164
std errors:	0.0123	0.0122	0.0121	0.0119	0.0116	0.0114	0.0113	0.0113	0.0111	0.011	0.011	0.011	0.0109	0.0108	0.011	0.0112	0.0112	0.0113	0.0113	0.0116	0.0117
0.05:	0.368	0.364	0.362	0.354	0.348	0.342	0.335	0.332	0.331	0.327	0.325	0.328	0.325	0.326	0.324	0.328	0.325	0.328	0.332	0.335	0.335
std errors:	0.0153	0.0152	0.0152	0.0151	0.0151	0.015	0.0149	0.0149	0.0149	0.0148	0.0148	0.0148	0.0148	0.0148	0.0148	0.0148	0.0148	0.0148	0.0149	0.0149	0.0149
0.1:	0.477	0.477	0.466	0.46	0.46	0.452	0.451	0.446	0.439	0.432	0.432	0.431	0.428	0.431	0.427	0.426	0.429	0.43	0.432	0.44	0.437
std errors:	0.0158	0.0158	0.0158	0.0158	0.0158	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0156	0.0157	0.0156	0.0156	0.0157	0.0157	0.0157	0.0157	0.0157

Proportions under the null hypothesis:					0.2	0.2	0.2	0.2	0.2												
The proportions under the alternative hypothesis:					0.1087	0.1087	0.1088	0.3369	0.3369												
n =	20			c(j) =	-0.4082	-0.4082	-0.4082	0.6124	0.6124			delta =	5k/4				k =	5			
lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
The bootstrap estimated power																					
0.01:	0.137	0.161	0.159	0.159	0.159	0.159	0.159	0.149	0.164	0.164	0.185	0.222	0.237	0.237	0.251	0.242	0.252	0.249	0.248	0.233	0.2
std errors:	0.0109	0.0116	0.0116	0.0116	0.0116	0.0116	0.0116	0.0113	0.0117	0.0117	0.0123	0.0131	0.0134	0.0134	0.0137	0.0135	0.0137	0.0137	0.0137	0.0134	0.0126
0.05:	0.267	0.263	0.265	0.265	0.265	0.265	0.265	0.268	0.299	0.373	0.428	0.476	0.51	0.514	0.519	0.517	0.493	0.495	0.485	0.483	0.47
std errors:	0.014	0.0139	0.014	0.014	0.014	0.014	0.014	0.014	0.0145	0.0153	0.0156	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158
0.1:	0.564	0.579	0.564	0.571	0.577	0.537	0.575	0.58	0.58	0.586	0.586	0.596	0.61	0.624	0.628	0.647	0.629	0.645	0.619	0.624	0.624
std errors:	0.0157	0.0156	0.0157	0.0157	0.0156	0.0158	0.0156	0.0156	0.0156	0.0156	0.0156	0.0155	0.0154	0.0153	0.0153	0.0151	0.0153	0.0151	0.0154	0.0153	0.0153
The AE approximation of power																					
0.01	-0.0566	-0.0566	-0.0566	-0.0566	-0.0566	-0.0566	-0.0564	-0.0442	0.0097	0.0821	0.1468	0.1965	0.2316	0.2471	0.2539	0.2709	0.2401	0.2137	0.1861	0.146	0.1068
0.05	-0.0566	-0.0566	-0.0566	-0.0566	-0.0566	-0.0566	-0.056	-0.0126	0.1409	0.2895	0.3688	0.445	0.471	0.5027	0.526	0.5148	0.4929	0.4959	0.4618	0.448	0.4111
0.1	0.2664	0.3082	0.3508	0.39	0.424	0.4533	0.4816	0.5057	0.5255	0.5415	0.5537	0.575	0.6005	0.6097	0.6171	0.6416	0.6352	0.6306	0.6212	0.6093	0.5948
The traditional estimated power, chi-square																					
0.01:	0.587	0.531	0.486	0.486	0.44	0.402	0.38	0.364	0.349	0.336	0.275	0.241	0.226	0.228	0.223	0.237	0.237	0.255	0.261	0.276	0.307
std errors:	0.0156	0.0158	0.0158	0.0158	0.0157	0.0155	0.0153	0.0152	0.0151	0.0149	0.0141	0.0135	0.0132	0.0133	0.0132	0.0134	0.0134	0.0138	0.0139	0.0141	0.0146
0.05:	0.756	0.734	0.699	0.666	0.618	0.593	0.593	0.537	0.54	0.54	0.526	0.527	0.518	0.488	0.482	0.493	0.46	0.485	0.502	0.505	0.51
std errors:	0.0136	0.014	0.0145	0.0149	0.0154	0.0155	0.0155	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158
0.1:	0.775	0.775	0.771	0.756	0.721	0.718	0.689	0.661	0.631	0.631	0.614	0.614	0.612	0.63	0.628	0.626	0.625	0.615	0.618	0.635	0.625
std errors:	0.0132	0.0132	0.0133	0.0136	0.0142	0.0142	0.0146	0.015	0.0153	0.0153	0.0154	0.0154	0.0154	0.0153	0.0153	0.0153	0.0153	0.0154	0.0154	0.0152	0.0153

The proportions under the alternative hypothesis:					0.1255	0.1255	0.1254	0.3118	0.3118												
n =	30			c(j) =	-0.4082	-0.4082	-0.4082	0.6124	0.6124			delta =	5k/4				k =	5			
lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
The bootstrap estimated power																					
0.01:	0.186	0.188	0.193	0.199	0.208	0.215	0.228	0.244	0.262	0.267	0.26	0.267	0.272	0.27	0.262	0.26	0.265	0.268	0.263	0.27	0.26
std errors:	0.0123	0.0124	0.0125	0.0126	0.0128	0.013	0.0133	0.0136	0.0139	0.014	0.0139	0.014	0.0141	0.014	0.0139	0.0139	0.014	0.014	0.0139	0.014	0.0139
0.05:	0.278	0.291	0.306	0.332	0.363	0.4	0.416	0.444	0.48	0.498	0.5	0.501	0.503	0.514	0.516	0.521	0.515	0.515	0.51	0.516	0.498
std errors:	0.0142	0.0144	0.0146	0.0149	0.0152	0.0155	0.0156	0.0157	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158
0.1:	0.564	0.568	0.576	0.579	0.58	0.587	0.588	0.59	0.594	0.601	0.618	0.616	0.621	0.621	0.611	0.6	0.597	0.612	0.612	0.615	0.615
std errors:	0.0157	0.0157	0.0156	0.0156	0.0156	0.0156	0.0156	0.0156	0.0155	0.0155	0.0154	0.0154	0.0153	0.0153	0.0154	0.0155	0.0155	0.0154	0.0154	0.0154	0.0154
The AE approximation of power																					
0.01	0.1289	0.1415	0.1567	0.1743	0.1949	0.2183	0.2442	0.2713	0.2983	0.3125	0.3187	0.3333	0.3409	0.3394	0.3379	0.336	0.3321	0.3215	0.3128	0.3023	0.2825
0.05	0.1468	0.1695	0.2002	0.237	0.2875	0.3385	0.38	0.4253	0.4534	0.4926	0.5012	0.5153	0.5255	0.5385	0.5317	0.5439	0.5284	0.5163	0.5098	0.5062	0.4818
0.1	0.4692	0.4895	0.509	0.5241	0.5394	0.558	0.5699	0.5794	0.5941	0.612	0.6247	0.6326	0.6418	0.6436	0.6443	0.6486	0.6406	0.6316	0.6342	0.6271	0.6179
The traditional estimated power, chi-square																					
0.01:	0.488	0.46	0.424	0.404	0.398	0.373	0.348	0.318	0.289	0.273	0.269	0.263	0.251	0.247	0.254	0.26	0.26	0.268	0.29	0.309	0.321
std errors:	0.0158	0.0158	0.0156	0.0155	0.0155	0.0153	0.0151	0.0147	0.0143	0.0141	0.014	0.0139	0.0137	0.0136	0.0138	0.0139	0.0139	0.014	0.0143	0.0146	0.0148
0.05:	0.657	0.64	0.623	0.602	0.597	0.573	0.547	0.541	0.527	0.52	0.504	0.497	0.491	0.483	0.487	0.484	0.491	0.511	0.517	0.52	0.524
std errors:	0.015	0.0152	0.0153	0.0155	0.0155	0.0156	0.0157	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158
0.1:	0.717	0.713	0.702	0.694	0.675	0.675	0.653	0.644	0.63	0.63	0.63	0.624	0.615	0.611	0.604	0.598	0.615	0.617	0.617	0.619	0.631
std errors:	0.0142	0.0143	0.0145	0.0146	0.0148	0.0148	0.0151	0.0151	0.0153	0.0153	0.0153	0.0153	0.0154	0.0154	0.0155	0.0155	0.0154	0.0154	0.0154	0.0154	0.0153

Proportions under the null hypothesis:					0.2	0.2	0.2	0.2	0.2												
The proportions under the alternative hypothesis:					0.1423	0.1423	0.1422	0.2866	0.2866												
n =	50			c(j) =	-0.4082	-0.4082	-0.4082	0.6124	0.6124			delta =	5k/4			k =	5				
lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
The bootstrap estimated power																					
0.01:	0.166	0.174	0.183	0.193	0.198	0.194	0.212	0.239	0.258	0.266	0.279	0.298	0.297	0.296	0.291	0.28	0.274	0.27	0.275	0.262	0.265
std errors:	0.0118	0.012	0.0122	0.0125	0.0126	0.0125	0.0129	0.0135	0.0138	0.014	0.0142	0.0145	0.0144	0.0144	0.0144	0.0142	0.0141	0.014	0.0141	0.0139	0.014
0.05:	0.431	0.44	0.439	0.439	0.444	0.45	0.455	0.469	0.473	0.476	0.488	0.49	0.492	0.498	0.502	0.502	0.503	0.504	0.504	0.504	0.507
std errors:	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0157	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158
0.1:	0.557	0.575	0.586	0.584	0.592	0.607	0.615	0.616	0.619	0.615	0.612	0.617	0.622	0.631	0.636	0.637	0.641	0.645	0.644	0.647	0.642
std errors:	0.0157	0.0156	0.0156	0.0156	0.0155	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0153	0.0153	0.0152	0.0152	0.0152	0.0151	0.0151	0.0151	0.0152
The AE approximation of power																					
0.01	0.4343	0.4582	0.4765	0.4894	0.4959	0.5001	0.5084	0.5158	0.5239	0.53	0.5343	0.5396	0.5404	0.5409	0.5404	0.5379	0.5351	0.5321	0.5315	0.5273	0.5261
0.05	0.5376	0.5453	0.5505	0.5561	0.5638	0.5688	0.5754	0.5821	0.5881	0.594	0.5994	0.6041	0.6093	0.6112	0.6109	0.6132	0.6109	0.6099	0.6071	0.6036	0.6013
0.1	0.6554	0.6571	0.6612	0.6648	0.6715	0.6763	0.6837	0.687	0.6892	0.6924	0.6946	0.6983	0.7016	0.7045	0.705	0.7092	0.7055	0.7065	0.7071	0.7033	0.6986
The traditional estimated power, chi-square																					
0.01:	0.41	0.386	0.368	0.356	0.335	0.32	0.303	0.293	0.282	0.277	0.269	0.266	0.263	0.263	0.259	0.262	0.265	0.271	0.275	0.284	0.291
std errors:	0.0156	0.0154	0.0153	0.0151	0.0149	0.0148	0.0145	0.0144	0.0142	0.0142	0.014	0.014	0.0139	0.0139	0.0139	0.0139	0.014	0.0141	0.0141	0.0143	0.0144
0.05:	0.606	0.599	0.589	0.582	0.553	0.536	0.531	0.52	0.517	0.507	0.506	0.498	0.492	0.495	0.502	0.499	0.503	0.504	0.518	0.521	0.524
std errors:	0.0155	0.0155	0.0156	0.0156	0.0157	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158
0.1:	0.712	0.699	0.688	0.682	0.683	0.677	0.675	0.664	0.649	0.644	0.634	0.624	0.628	0.626	0.628	0.636	0.632	0.637	0.638	0.64	0.654
std errors:	0.0143	0.0145	0.0147	0.0147	0.0147	0.0148	0.0148	0.0149	0.0151	0.0151	0.0152	0.0153	0.0153	0.0153	0.0153	0.0152	0.0153	0.0152	0.0152	0.0152	0.015

The proportions under the alternative hypothesis:					0.1592	0.1592	0.1592	0.2612	0.2612												
n =	100			c(j) =	-0.4082	-0.4082	-0.4082	0.6124	0.6124			delta =	5k/4			k =	5				
lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
The bootstrap estimated power																					
0.01:	0.185	0.188	0.194	0.206	0.218	0.218	0.238	0.247	0.253	0.263	0.268	0.274	0.27	0.271	0.27	0.272	0.273	0.272	0.27	0.267	0.268
std errors:	0.0123	0.0124	0.0125	0.0128	0.0131	0.0131	0.0135	0.0136	0.0137	0.0139	0.014	0.0141	0.014	0.0141	0.014	0.0141	0.0141	0.0141	0.014	0.014	0.014
0.05:	0.428	0.433	0.437	0.446	0.451	0.456	0.457	0.457	0.464	0.463	0.464	0.47	0.466	0.466	0.472	0.474	0.472	0.475	0.478	0.484	0.482
std errors:	0.0156	0.0157	0.0157	0.0157	0.0157	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158
0.1:	0.578	0.581	0.583	0.588	0.595	0.597	0.597	0.601	0.6	0.601	0.605	0.603	0.605	0.609	0.608	0.61	0.607	0.603	0.603	0.604	0.604
std errors:	0.0156	0.0156	0.0156	0.0156	0.0155	0.0155	0.0155	0.0155	0.0155	0.0155	0.0155	0.0155	0.0155	0.0154	0.0154	0.0154	0.0154	0.0155	0.0155	0.0155	0.0155
The AE approximation of power																					
0.01	0.4403	0.447	0.4541	0.4614	0.469	0.4728	0.4792	0.4849	0.4882	0.4938	0.4968	0.5001	0.4983	0.4982	0.498	0.4993	0.4992	0.4969	0.4942	0.4917	0.4892
0.05	0.5329	0.5395	0.5453	0.5503	0.5571	0.5617	0.5642	0.5662	0.5685	0.5734	0.5747	0.5799	0.581	0.5802	0.5828	0.5818	0.5821	0.5819	0.5798	0.5809	0.5774
0.1	0.6537	0.6562	0.6589	0.6637	0.6679	0.6707	0.6725	0.6774	0.6784	0.6825	0.6836	0.6834	0.684	0.6851	0.6857	0.6887	0.6854	0.6821	0.6822	0.6808	0.6795
The traditional estimated power, chi-square																					
0.01:	0.344	0.333	0.326	0.311	0.306	0.298	0.292	0.288	0.272	0.268	0.266	0.262	0.264	0.265	0.264	0.266	0.268	0.271	0.277	0.284	0.287
std errors:	0.015	0.0149	0.0148	0.0146	0.0146	0.0145	0.0144	0.0143	0.0141	0.014	0.014	0.0139	0.0139	0.014	0.0139	0.014	0.014	0.0141	0.0142	0.0143	0.0143
0.05:	0.551	0.54	0.531	0.529	0.524	0.521	0.511	0.509	0.505	0.504	0.499	0.494	0.494	0.496	0.494	0.501	0.498	0.501	0.502	0.506	0.51
std errors:	0.0157	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158
0.1:	0.646	0.644	0.642	0.639	0.638	0.633	0.632	0.633	0.631	0.63	0.625	0.622	0.618	0.622	0.622	0.624	0.628	0.627	0.627	0.628	0.625
std errors:	0.0151	0.0151	0.0152	0.0152	0.0152	0.0152	0.0153	0.0152	0.0153	0.0153	0.0153	0.0153	0.0154	0.0153	0.0153	0.0153	0.0153	0.0153	0.0153	0.0153	0.0153

Table 15

Proportions under the null hypothesis:	0.2	0.2	0.2	0.2	0.2
The proportions under the alternative hypothesis:	0.1184	0.1184	0.1184	0.3224	0.3224
n = 20, c(j) =	-0.3651	-0.3651	-0.3651	0.5477	0.5477

delta = k   k = 5

lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
The bootstrap estimated power																					
0.01:	0.094	0.12	0.117	0.117	0.12	0.12	0.12	0.109	0.125	0.126	0.143	0.158	0.156	0.154	0.147	0.157	0.147	0.164	0.166	0.157	0.171
std errors:	0.0092	0.0103	0.0102	0.0102	0.0103	0.0103	0.0103	0.0099	0.0105	0.0105	0.0111	0.0115	0.0115	0.0114	0.0112	0.0115	0.0112	0.0117	0.0118	0.0115	0.0119
0.05:	0.209	0.209	0.209	0.209	0.209	0.202	0.209	0.209	0.24	0.297	0.345	0.363	0.367	0.389	0.38	0.379	0.379	0.37	0.37	0.377	0.361
std errors:	0.0129	0.0129	0.0129	0.0129	0.0129	0.0127	0.0129	0.0129	0.0135	0.0144	0.015	0.0152	0.0152	0.0154	0.0153	0.0153	0.0153	0.0153	0.0153	0.0153	0.0152
0.1:	0.473	0.473	0.509	0.498	0.496	0.514	0.49	0.492	0.497	0.501	0.502	0.488	0.511	0.517	0.511	0.554	0.527	0.55	0.516	0.548	0.548
std errors:	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0157	0.0158	0.0157	0.0158	0.0157	0.0157
The AE approximation of power																					
0.01	-0.0289	-0.0289	-0.0289	-0.0289	-0.0289	-0.0289	-0.0288	-0.0231	0.0067	0.0535	0.0997	0.1357	0.1397	0.1404	0.1389	0.1327	0.1247	0.1145	0.1022	0.0885	0.0739
0.05	-0.0289	-0.0289	-0.0289	-0.0289	-0.0289	-0.0289	-0.0286	-0.004	0.1049	0.2229	0.3072	0.3644	0.3763	0.407	0.4018	0.3813	0.3964	0.3725	0.3658	0.3326	0.2957
0.1	0.2192	0.2588	0.3001	0.3407	0.3789	0.4137	0.4423	0.459	0.4708	0.478	0.4806	0.4904	0.5317	0.526	0.5358	0.5488	0.5443	0.537	0.5271	0.5145	0.4994
The traditional estimated power, chi-square																					
0.01:	0.473	0.408	0.379	0.379	0.35	0.319	0.297	0.283	0.273	0.242	0.211	0.187	0.169	0.174	0.174	0.184	0.184	0.198	0.202	0.213	0.228
std errors:	0.0158	0.0155	0.0153	0.0153	0.0151	0.0147	0.0144	0.0142	0.0141	0.0135	0.0129	0.0123	0.0119	0.012	0.012	0.0123	0.0123	0.0126	0.0127	0.0129	0.0133
0.05:	0.674	0.645	0.6	0.562	0.527	0.478	0.478	0.414	0.417	0.417	0.408	0.394	0.377	0.364	0.363	0.379	0.357	0.379	0.394	0.405	0.409
std errors:	0.0148	0.0151	0.0155	0.0157	0.0158	0.0158	0.0158	0.0156	0.0156	0.0156	0.0155	0.0155	0.0153	0.0152	0.0152	0.0153	0.0152	0.0153	0.0155	0.0155	0.0155
0.1:	0.702	0.702	0.702	0.676	0.631	0.629	0.588	0.563	0.541	0.541	0.505	0.505	0.496	0.517	0.509	0.513	0.512	0.512	0.514	0.55	0.548
std errors:	0.0145	0.0145	0.0145	0.0148	0.0153	0.0153	0.0156	0.0157	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0157	0.0157

The proportions under the alternative hypothesis:	0.1333	0.1333	0.1334	0.3	0.3
n = 30, c(j) =	-0.3651	-0.3651	-0.3651	0.5477	0.5477

delta = k   k = 5

lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
The bootstrap estimated power																					
0.01:	0.132	0.136	0.136	0.14	0.146	0.144	0.143	0.144	0.14	0.137	0.152	0.195	0.189	0.181	0.189	0.175	0.186	0.18	0.179	0.179	0.176
std errors:	0.0107	0.0108	0.0108	0.011	0.0112	0.0111	0.0111	0.0111	0.011	0.0109	0.0114	0.0125	0.0124	0.0122	0.0124	0.012	0.0123	0.0121	0.0121	0.0121	0.012
0.05:	0.221	0.225	0.233	0.259	0.288	0.302	0.321	0.336	0.355	0.368	0.385	0.385	0.39	0.387	0.391	0.404	0.419	0.426	0.44	0.419	0.394
std errors:	0.0131	0.0132	0.0134	0.0139	0.0143	0.0145	0.0148	0.0149	0.0151	0.0153	0.0154	0.0154	0.0154	0.0154	0.0154	0.0155	0.0156	0.0156	0.0157	0.0156	0.0155
0.1:	0.479	0.484	0.491	0.501	0.507	0.509	0.497	0.501	0.492	0.497	0.523	0.529	0.54	0.546	0.547	0.559	0.542	0.555	0.542	0.533	0.529
std errors:	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0157	0.0157	0.0157	0.0158	0.0157	0.0158	0.0158	0.0158
The AE approximation of power																					
0.01	0.0686	0.0793	0.0931	0.1093	0.1275	0.1476	0.1609	0.1723	0.1829	0.1876	0.2049	0.2433	0.2501	0.2451	0.2494	0.2405	0.2393	0.2344	0.223	0.2102	0.1966
0.05	0.0844	0.1056	0.1309	0.1654	0.2088	0.2453	0.2828	0.3328	0.3578	0.3853	0.419	0.423	0.4257	0.4215	0.4349	0.4267	0.436	0.4434	0.4334	0.4105	0.3866
0.1	0.3825	0.4071	0.4325	0.441	0.459	0.4783	0.4825	0.4944	0.4985	0.5059	0.5323	0.5492	0.5539	0.5573	0.5578	0.5565	0.5616	0.5652	0.5564	0.546	0.5343
The traditional estimated power, chi-square																					
0.01:	0.383	0.359	0.333	0.321	0.319	0.295	0.276	0.233	0.223	0.205	0.202	0.193	0.174	0.168	0.176	0.188	0.186	0.188	0.209	0.224	0.238
std errors:	0.0154	0.0152	0.0149	0.0148	0.0147	0.0144	0.0141	0.0134	0.0132	0.0128	0.0127	0.0125	0.012	0.0118	0.012	0.0124	0.0123	0.0124	0.0129	0.0132	0.0135
0.05:	0.571	0.558	0.54	0.512	0.488	0.467	0.446	0.433	0.406	0.39	0.381	0.382	0.377	0.372	0.382	0.378	0.389	0.407	0.413	0.43	0.441
std errors:	0.0157	0.0157	0.0158	0.0158	0.0158	0.0158	0.0157	0.0157	0.0155	0.0154	0.0154	0.0154	0.0153	0.0153	0.0154	0.0153	0.0154	0.0155	0.0156	0.0157	0.0157
0.1:	0.662	0.655	0.641	0.623	0.605	0.605	0.581	0.574	0.553	0.553	0.552	0.527	0.516	0.518	0.504	0.502	0.514	0.529	0.529	0.533	0.548
std errors:	0.015	0.015	0.0152	0.0153	0.0155	0.0155	0.0156	0.0156	0.0157	0.0157	0.0157	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0157

Proportions under the null hypothesis:				0.2	0.2	0.2	0.2	0.2													
The proportions under the alternative hypothesis:				0.1484	0.1484	0.1484	0.2774	0.2774													
n =	50		c(j) =	-0.3651	-0.3651	-0.3651	0.5477	0.5477			delta =	k				k =		5			
lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
The bootstrap estimated power																					
0.01:	0.077	0.081	0.089	0.092	0.096	0.107	0.132	0.144	0.154	0.152	0.162	0.163	0.167	0.172	0.181	0.183	0.182	0.187	0.185	0.178	0.183
std errors:	0.0084	0.0086	0.009	0.0091	0.0093	0.0098	0.0107	0.0111	0.0114	0.0114	0.0117	0.0117	0.0118	0.0119	0.0122	0.0122	0.0122	0.0123	0.0123	0.0121	0.0122
0.05:	0.309	0.314	0.317	0.32	0.324	0.327	0.347	0.356	0.385	0.391	0.389	0.389	0.397	0.398	0.405	0.418	0.412	0.416	0.419	0.411	0.414
std errors:	0.0146	0.0147	0.0147	0.0148	0.0148	0.0148	0.0151	0.0151	0.0154	0.0154	0.0154	0.0154	0.0155	0.0155	0.0155	0.0156	0.0156	0.0156	0.0156	0.0156	0.0156
0.1:	0.443	0.454	0.459	0.472	0.483	0.493	0.504	0.51	0.518	0.519	0.52	0.529	0.532	0.534	0.539	0.537	0.538	0.535	0.534	0.539	0.537
std errors:	0.0157	0.0157	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158
The AE approximation of power																					
0.01	0.2374	0.266	0.2928	0.3168	0.3375	0.3568	0.3803	0.3939	0.4042	0.4099	0.4161	0.42	0.423	0.4251	0.4264	0.4268	0.4241	0.4237	0.4196	0.4129	0.411
0.05	0.4219	0.4316	0.4409	0.4499	0.456	0.4624	0.4747	0.4846	0.4999	0.5013	0.5032	0.5074	0.514	0.5171	0.5185	0.5209	0.5186	0.5183	0.5161	0.5114	0.5102
0.1	0.5549	0.5589	0.5624	0.5671	0.5759	0.5832	0.5895	0.5964	0.5998	0.6046	0.6065	0.6116	0.616	0.6186	0.6215	0.624	0.6195	0.6176	0.6168	0.6109	0.6118
The traditional estimated power, chi-square																					
0.01:	0.32	0.297	0.284	0.271	0.254	0.242	0.232	0.217	0.204	0.195	0.182	0.174	0.175	0.173	0.179	0.183	0.188	0.193	0.199	0.212	0.227
std errors:	0.0148	0.0144	0.0143	0.0141	0.0138	0.0135	0.0133	0.013	0.0127	0.0125	0.0122	0.012	0.012	0.012	0.0121	0.0122	0.0124	0.0125	0.0126	0.0129	0.0132
0.05:	0.498	0.493	0.484	0.472	0.453	0.443	0.438	0.429	0.418	0.411	0.408	0.399	0.398	0.399	0.404	0.401	0.407	0.409	0.419	0.421	0.419
std errors:	0.0158	0.0158	0.0158	0.0158	0.0157	0.0157	0.0157	0.0157	0.0156	0.0156	0.0155	0.0155	0.0155	0.0155	0.0155	0.0155	0.0155	0.0155	0.0156	0.0156	0.0156
0.1:	0.595	0.585	0.577	0.573	0.573	0.559	0.554	0.552	0.54	0.538	0.533	0.533	0.533	0.526	0.527	0.536	0.532	0.535	0.533	0.541	0.547
std errors:	0.0155	0.0156	0.0156	0.0156	0.0156	0.0157	0.0157	0.0157	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0157
The proportions under the alternative hypothesis:				0.1635	0.1635	0.1634	0.2548	0.2548													
n =	100		c(j) =	-0.3651	-0.3651	-0.3651	0.5477	0.5477			delta =	k				k =		5			
lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
The bootstrap estimated power																					
0.01:	0.172	0.176	0.184	0.189	0.191	0.19	0.189	0.182	0.188	0.197	0.201	0.203	0.204	0.202	0.207	0.206	0.206	0.205	0.208	0.208	0.211
std errors:	0.0119	0.012	0.0123	0.0124	0.0124	0.0124	0.0124	0.0122	0.0124	0.0126	0.0127	0.0127	0.0127	0.0127	0.0128	0.0128	0.0128	0.0128	0.0128	0.0128	0.0129
0.05:	0.353	0.354	0.364	0.365	0.368	0.372	0.373	0.379	0.377	0.379	0.384	0.383	0.392	0.396	0.406	0.408	0.406	0.403	0.402	0.405	0.413
std errors:	0.0151	0.0151	0.0152	0.0152	0.0153	0.0153	0.0153	0.0153	0.0153	0.0153	0.0154	0.0154	0.0154	0.0155	0.0155	0.0155	0.0155	0.0155	0.0155	0.0155	0.0156
0.1:	0.481	0.492	0.498	0.498	0.511	0.515	0.515	0.518	0.521	0.524	0.521	0.524	0.52	0.515	0.51	0.514	0.51	0.511	0.512	0.511	0.515
std errors:	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158
The AE approximation of power																					
0.01	0.3432	0.3545	0.3657	0.3768	0.381	0.3842	0.3844	0.3841	0.3892	0.3959	0.3995	0.4049	0.406	0.4061	0.4095	0.4092	0.4083	0.4083	0.407	0.4033	0.4003
0.05	0.4522	0.4589	0.4674	0.4724	0.4795	0.4831	0.4862	0.4902	0.4951	0.4962	0.5016	0.5036	0.5105	0.5134	0.5199	0.5183	0.5163	0.514	0.5161	0.5147	0.5164
0.1	0.569	0.5754	0.5806	0.5842	0.591	0.5962	0.6006	0.6054	0.6068	0.6108	0.6122	0.6135	0.6147	0.6125	0.6103	0.6117	0.6085	0.6085	0.6085	0.6061	0.607
The traditional estimated power, chi-square																					
0.01:	0.252	0.245	0.24	0.229	0.223	0.215	0.209	0.205	0.2	0.199	0.197	0.194	0.192	0.194	0.19	0.191	0.193	0.198	0.2	0.203	0.208
std errors:	0.0137	0.0136	0.0135	0.0133	0.0132	0.013	0.0129	0.0128	0.0126	0.0126	0.0126	0.0125	0.0125	0.0125	0.0124	0.0124	0.0125	0.0126	0.0126	0.0127	0.0128
0.05:	0.451	0.44	0.435	0.431	0.421	0.411	0.402	0.401	0.399	0.392	0.391	0.387	0.386	0.383	0.385	0.389	0.386	0.387	0.391	0.391	0.395
std errors:	0.0157	0.0157	0.0157	0.0157	0.0156	0.0156	0.0155	0.0155	0.0155	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0154	0.0155
0.1:	0.556	0.552	0.548	0.542	0.539	0.533	0.531	0.526	0.521	0.52	0.514	0.51	0.508	0.507	0.508	0.511	0.51	0.511	0.512	0.519	0.516
std errors:	0.0157	0.0157	0.0157	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158

Proportions under the null hypothesis:	0.2	0.2	0.2	0.2	0.2
The proportions under the alternative hypothesis:	0.2	0.2	0.2	0.2	0.2

n = 20 c(j) = 0 0 0 0 0 delta = 0.2 k = 5

lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
alpha	The bootstrap estimated power																				
0.01:	0.006	0.008	0.008	0.008	0.007	0.007	0.007	0.007	0.007	0.005	0.005	0.006	0.006	0.006	0.006	0.007	0.007	0.007	0.007	0.006	0.005
std errors:	0.0024	0.0028	0.0028	0.0028	0.0026	0.0026	0.0026	0.0026	0.0026	0.0022	0.0022	0.0024	0.0024	0.0024	0.0024	0.0026	0.0026	0.0026	0.0026	0.0024	0.0022
0.05:	0.057	0.057	0.051	0.055	0.051	0.049	0.049	0.051	0.049	0.049		0.037	0.036	0.04	0.04	0.041	0.034	0.033	0.033	0.037	0.033
std errors:	0.0073	0.0073	0.007	0.0072	0.007	0.0068	0.0068	0.007	0.0068	0.0068	0.0057	0.006	0.0059	0.0062	0.0062	0.0063	0.0057	0.0056	0.0056	0.006	0.0056
0.1:	0.086	0.09	0.088	0.091	0.088	0.083	0.091	0.088	0.091	0.09	0.09	0.081	0.084	0.085	0.086	0.089	0.083	0.084	0.083	0.083	0.091
std errors:	0.0089	0.009	0.009	0.0091	0.009	0.0087	0.0091	0.009	0.0091	0.009	0.009	0.0086	0.0088	0.0088	0.0089	0.009	0.0087	0.0088	0.0087	0.0087	0.0091
alpha	The AE approximation of power																				
0.01	0	0	0	0	0	0	0	0.0001	0.0003	0.001	0.0029	0.0053	0.0068	0.0077	0.008	0.0087	0.0085	0.0061	0.0042	0.0027	0.0017
0.05	0	0	0	0	0	0	0	0.0002	0.0032	0.0143	0.0278	0.0384	0.0437	0.0432	0.0507	0.0464	0.0401	0.0358	0.0314	0.0296	0.0231
0.1	0.0055	0.0094	0.0148	0.021	0.0277	0.0345	0.042	0.0493	0.056	0.0619	0.0667	0.0703	0.0848	0.0925	0.0965	0.0896	0.095	0.0973	0.0969	0.0923	0.085
alpha	The traditional estimated power, chi-square																				
0.01:	0.099	0.081	0.073	0.073	0.069	0.061	0.059	0.059	0.058	0.026	0.014	0.01	0.006	0.006	0.006	0.007	0.007	0.008	0.01	0.011	0.013
std errors:	0.0094	0.0086	0.0082	0.0082	0.008	0.0076	0.0075	0.0075	0.0074	0.005	0.0037	0.0031	0.0024	0.0024	0.0024	0.0026	0.0026	0.0028	0.0031	0.0033	0.0036
0.05:	0.299	0.226	0.179	0.154	0.115	0.099	0.099	0.083	0.083	0.083	0.07	0.049	0.045	0.04	0.038	0.042	0.037	0.044	0.053	0.058	0.059
std errors:	0.0145	0.0132	0.0121	0.0114	0.0101	0.0094	0.0094	0.0087	0.0087	0.0087	0.0081	0.0068	0.0066	0.0062	0.006	0.0063	0.006	0.0065	0.0071	0.0074	0.0075
0.1:	0.301	0.301	0.286	0.228	0.181	0.18	0.165	0.132	0.116	0.116	0.106	0.104	0.095	0.094	0.086	0.089	0.089	0.084	0.085	0.095	0.092
std errors:	0.0145	0.0145	0.0143	0.0133	0.0122	0.0121	0.0117	0.0107	0.0101	0.0101	0.0097	0.0097	0.0093	0.0092	0.0089	0.009	0.009	0.0088	0.0088	0.0093	0.0091

The proportions under the alternative hypothesis:	0.2	0.2	0.2	0.2	0.2

n = 30 c(j) = 0 0 0 0 0 delta = 0.2 k = 5

lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
alpha	The bootstrap estimated power																				
0.01:	0.018	0.019	0.017	0.017	0.018	0.017	0.017	0.016	0.016	0.014	0.013	0.014	0.013	0.015	0.015	0.014	0.016	0.013	0.013	0.012	0.014
std errors:	0.0042	0.0043	0.0041	0.0041	0.0042	0.0041	0.0041	0.004	0.004	0.0037	0.0036	0.0037	0.0036	0.0038	0.0038	0.0037	0.004	0.0036	0.0036	0.0034	0.0037
0.05:	0.058	0.056	0.058	0.058	0.056	0.058	0.062	0.059	0.065	0.064	0.068	0.068	0.072	0.071	0.07	0.073	0.074	0.071	0.071	0.069	0.066
std errors:	0.0074	0.0073	0.0074	0.0074	0.0073	0.0074	0.0076	0.0075	0.0078	0.0077	0.008	0.008	0.0082	0.0081	0.0081	0.0082	0.0083	0.0081	0.0081	0.008	0.0079
0.1:	0.113	0.107	0.107	0.109	0.112	0.112	0.113	0.112	0.121	0.122	0.118	0.119	0.116	0.112	0.111	0.121	0.12	0.121	0.119	0.118	0.12
std errors:	0.01	0.0098	0.0098	0.0099	0.01	0.01	0.01	0.01	0.0103	0.0103	0.0102	0.0102	0.0101	0.01	0.0099	0.0103	0.0103	0.0103	0.0102	0.0102	0.0103
alpha	The AE approximation of power																				
0.01	0.0002	0.0006	0.0015	0.0028	0.0049	0.007	0.0093	0.0117	0.0139	0.0156	0.016	0.0196	0.0207	0.0238	0.024	0.0244	0.024	0.0219	0.0204	0.0177	0.0154
0.05	0.0046	0.0065	0.0086	0.0109	0.014	0.0194	0.0284	0.036	0.0429	0.0529	0.0606	0.0629	0.0668	0.0685	0.0695	0.0698	0.0678	0.0682	0.0646	0.0592	0.0536
0.1	0.0351	0.0393	0.0468	0.0549	0.0633	0.0716	0.0736	0.0791	0.0859	0.092	0.0983	0.1078	0.1106	0.1126	0.1129	0.1121	0.1151	0.1174	0.112	0.1083	0.1055
alpha	The traditional estimated power, chi-square																				
0.01:	0.081	0.074	0.068	0.066	0.066	0.056	0.034	0.023	0.019	0.018	0.018	0.012	0.009	0.009	0.01	0.011	0.011	0.011	0.012	0.018	0.028
std errors:	0.0086	0.0083	0.008	0.0079	0.0079	0.0073	0.0057	0.0047	0.0043	0.0042	0.0042	0.0034	0.003	0.003	0.0031	0.0033	0.0033	0.0033	0.0034	0.0042	0.0052
0.05:	0.177	0.148	0.128	0.113	0.109	0.102	0.095	0.092	0.082	0.071	0.066	0.065	0.066	0.062	0.063	0.06	0.061	0.065	0.067	0.068	0.079
std errors:	0.0121	0.0112	0.0106	0.01	0.0099	0.0096	0.0093	0.0091	0.0087	0.0081	0.0079	0.0078	0.0079	0.0076	0.0077	0.0075	0.0076	0.0078	0.0079	0.008	0.0085
0.1:	0.264	0.261	0.232	0.204	0.172	0.17	0.149	0.139	0.131	0.131	0.13	0.119	0.114	0.107	0.1	0.1	0.112	0.112	0.112	0.114	0.12
std errors:	0.0139	0.0139	0.0133	0.0127	0.0119	0.0119	0.0113	0.0109	0.0107	0.0107	0.0106	0.0102	0.0101	0.0098	0.0095	0.0095	0.01	0.01	0.01	0.0101	0.0103

Proportions under the null hypothesis:					0.2	0.2	0.2	0.2	0.2												
The proportions under the alternative hypothesis:					0.2	0.2	0.2	0.2	0.2												
n =	50			c(j) =	0	0	0	0	0			delta =	0.2			k =	5				
lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
alpha	The bootstrap estimated power																				
0.01:	0.013	0.013	0.013	0.014	0.014	0.014	0.013	0.013	0.011	0.013	0.014	0.015	0.016	0.016	0.016	0.016	0.014	0.013	0.012	0.013	0.013
std errors:	0.0036	0.0036	0.0036	0.0037	0.0037	0.0037	0.0036	0.0036	0.0033	0.0036	0.0037	0.0038	0.004	0.004	0.004	0.004	0.0037	0.0036	0.0034	0.0036	0.0036
0.05:	0.043	0.043	0.044	0.045	0.047	0.047	0.048	0.049	0.049	0.048	0.05	0.043	0.043	0.039	0.036	0.036	0.036	0.039	0.036	0.036	0.036
std errors:	0.0064	0.0064	0.0065	0.0066	0.0067	0.0067	0.0068	0.0068	0.0068	0.0068	0.0069	0.0064	0.0064	0.0061	0.0059	0.0059	0.0059	0.0061	0.0059	0.0059	0.0059
0.1:	0.09	0.091	0.094	0.096	0.094	0.096	0.098	0.1	0.097	0.097	0.094	0.092	0.089	0.091	0.093	0.093	0.093	0.093	0.085	0.086	0.088
std errors:	0.009	0.0091	0.0092	0.0093	0.0092	0.0093	0.0094	0.0095	0.0094	0.0094	0.0092	0.0091	0.009	0.0091	0.0092	0.0092	0.0092	0.0092	0.0088	0.0089	0.009
alpha	The AE approximation of power																				
0.01	0.0093	0.0138	0.0192	0.0251	0.0312	0.0357	0.0397	0.0442	0.0487	0.054	0.0584	0.0591	0.0609	0.0622	0.0645	0.0628	0.0623	0.0592	0.0557	0.0526	0.0512
0.05	0.0599	0.0665	0.0732	0.0779	0.0832	0.0888	0.0929	0.0969	0.1019	0.1065	0.111	0.111	0.1127	0.1126	0.1114	0.1138	0.1113	0.112	0.1065	0.1048	0.1004
0.1	0.096	0.1048	0.1138	0.1196	0.1252	0.1311	0.138	0.1422	0.1442	0.1467	0.1488	0.1523	0.1509	0.1513	0.1521	0.1559	0.157	0.1548	0.1501	0.1472	0.1451
alpha	The traditional estimated power, chi-square																				
0.01:	0.041	0.041	0.037	0.035	0.03	0.022	0.02	0.019	0.017	0.014	0.013	0.014	0.014	0.015	0.015	0.012	0.012	0.013	0.014	0.015	0.019
std errors:	0.0063	0.0063	0.006	0.0058	0.0054	0.0046	0.0044	0.0043	0.0041	0.0037	0.0036	0.0037	0.0037	0.0038	0.0038	0.0034	0.0034	0.0036	0.0037	0.0038	0.0043
0.05:	0.113	0.106	0.093	0.089	0.077	0.07	0.067	0.063	0.058	0.052	0.051	0.046	0.042	0.038	0.036	0.035	0.036	0.039	0.042	0.043	0.045
std errors:	0.01	0.0097	0.0092	0.009	0.0084	0.0081	0.0079	0.0077	0.0074	0.007	0.007	0.0066	0.0063	0.006	0.0059	0.0058	0.0059	0.0061	0.0063	0.0064	0.0066
0.1:	0.16	0.151	0.141	0.134	0.132	0.125	0.119	0.118	0.111	0.105	0.101	0.096	0.097	0.094	0.094	0.095	0.093	0.096	0.09	0.092	0.1
std errors:	0.0116	0.0113	0.011	0.0108	0.0107	0.0105	0.0102	0.0102	0.0099	0.0097	0.0095	0.0093	0.0094	0.0092	0.0092	0.0093	0.0092	0.0093	0.009	0.0091	0.0095
The proportions under the alternative hypothesis:					0.2	0.2	0.2	0.2	0.2												
n =	100			c(j) =	0	0	0	0	0			delta =	0.2			k =	5				
lamda	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
alpha	The bootstrap estimated power																				
0.01:	0.013	0.013	0.013	0.013	0.013	0.013	0.012	0.012	0.012	0.011	0.011	0.01	0.01	0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.009
std errors:	0.0036	0.0036	0.0036	0.0036	0.0036	0.0036	0.0034	0.0034	0.0034	0.0033	0.0033	0.0031	0.0031	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003
0.05:	0.061	0.062	0.062	0.06	0.059	0.056	0.055	0.057	0.056	0.054	0.055	0.05	0.049	0.046	0.044	0.049	0.048	0.049	0.05	0.05	0.048
std errors:	0.0076	0.0076	0.0076	0.0075	0.0075	0.0073	0.0072	0.0073	0.0073	0.0071	0.0072	0.0069	0.0068	0.0066	0.0065	0.0068	0.0068	0.0068	0.0069	0.0069	0.0068
0.1:	0.1	0.103	0.102	0.108	0.106	0.105	0.103	0.103	0.104	0.101	0.101	0.101	0.102	0.099	0.097	0.094	0.091	0.091	0.086	0.085	0.084
std errors:	0.0095	0.0096	0.0096	0.0098	0.0097	0.0097	0.0096	0.0096	0.0097	0.0095	0.0095	0.0095	0.0096	0.0094	0.0094	0.0092	0.0091	0.0091	0.0089	0.0088	0.0088
alpha	The AE approximation of power																				
0.01	0.0261	0.03	0.031	0.0334	0.0371	0.0412	0.0451	0.0489	0.0515	0.0513	0.0514	0.0532	0.0524	0.0541	0.0539	0.0535	0.053	0.0524	0.05	0.0489	0.048
0.05	0.0843	0.088	0.0912	0.0946	0.0984	0.1006	0.1031	0.1053	0.1067	0.1084	0.1115	0.1125	0.1122	0.1123	0.1118	0.1143	0.1138	0.1132	0.1123	0.1112	0.1093
0.1	0.1272	0.133	0.1353	0.1403	0.1427	0.1447	0.1467	0.1484	0.1498	0.1502	0.1503	0.1529	0.1527	0.1522	0.1522	0.151	0.1504	0.1501	0.1464	0.1466	0.1466
alpha	The traditional estimated power, chi-square																				
0.01:	0.026	0.025	0.022	0.018	0.016	0.015	0.014	0.013	0.012	0.011	0.011	0.01	0.01	0.009	0.009	0.009	0.009	0.009	0.009	0.01	0.012
std errors:	0.005	0.0049	0.0046	0.0042	0.004	0.0038	0.0037	0.0036	0.0034	0.0033	0.0033	0.0031	0.0031	0.003	0.003	0.003	0.003	0.003	0.003	0.0031	0.0034
0.05:	0.077	0.072	0.068	0.067	0.063	0.063	0.058	0.057	0.054	0.052	0.046	0.043	0.042	0.04	0.04	0.039	0.04	0.04	0.043	0.045	0.047
std errors:	0.0084	0.0082	0.008	0.0079	0.0077	0.0077	0.0074	0.0073	0.0071	0.007	0.0066	0.0064	0.0063	0.0062	0.0062	0.0061	0.0062	0.0062	0.0064	0.0066	0.0067
0.1:	0.124	0.122	0.119	0.118	0.114	0.112	0.109	0.108	0.104	0.104	0.101	0.101	0.099	0.099	0.095	0.094	0.092	0.091	0.089	0.089	0.09
std errors:	0.0104	0.0103	0.0102	0.0102	0.0101	0.01	0.0099	0.0098	0.0097	0.0097	0.0095	0.0095	0.0094	0.0094	0.0093	0.0092	0.0091	0.0091	0.009	0.009	0.009

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