

CFD airfoil profile drag calculation using far-field wake analysis

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ABSTRACT

TITLE: CFD airfoil profile drag calculation using far-field wake analysis.

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With the ever increasing cost of experimental data and the ultimate decrease of project lifespan from design to implementation it has become the aerodynamicist's main priority to design and model new ideas with precise accuracy. There is thus no room for discrepancies that arise between various computational fluid dynamic simulation packages. However, as we live in a realistic world these discrepancies do turn up from time to time, and it is the sole purpose of engineers to minimize and ultimately eliminate these discrepancies.

In the field of CFD simulations the main discrepancies that give rise to never-ending headaches are those found when comparing the experimental drag to the drag predicted by simulation software implementing panel codes and the near-field method – in other words the differences in drag predicted by the Squire-Young model, the near-field model and the far-field, also referred to as the “wake rake model”.

One such example of differences between drag predicted by the Squire-Young model and the near-field model can be seen in **Figure 1**, Chapter 1 where two aerofoils were analysed at a Reynolds number of 1 million and a Mach number of 0. In **Figure 1** it can clearly be seen that for the XFOIL simulation with 120 panels there is a clear distinction between the performance of the ST1 and OPT110 aerofoils. As for the Star-CCM+ simulation set-up with a fine mesh and the SST $k - \omega$, turbulence and the $\gamma - Re_\theta$ transition model, the results show that both aerofoils have similar performance characteristics. One of the main reasons for these discrepancies can be ascribed to the over-sensitivity of the pressure drag component to the level of grid refinement used in the simulation. The reason why drag accuracy is sensitive to grid definition is because an inadequate grid at positions of high curvature means that the boundary layer in these portions of the model is not accurately

solved. These inaccuracies are then superimposed when the near-field method surface integrates these erroneously solved flow variables to find the profile drag.

Historically the far-field methods implemented by experimentalists have hinted at a solution to solve this overdependence of drag on the level of grid refinement. This is because rather than using the locally solved flow variables that can be contaminated by spurious drag, the far-field method uses a momentum deficit at the far-downstream wake to calculate the total profile drag. Although this method potentially solves the discrepancies between the near-field and far-field method, a concise procedure to implement the far-field method to a converged viscous unstructured grid study has not yet been developed. Thus the main theme of this thesis is to address the problem of discrepancies arising in the drag predicted by the near-field and far-field method.

In this thesis reliable methods of far-field drag extraction from a 2-D viscous unstructured CFD study will be developed and validated. To validate the performance of the proposed far-field methods against the performance of the panel code XFOIL and the near-field method of Star-CCM+, it was, in the first place, necessary to acquire reliable experimental data. The aerofoil data was acquired from the UIUC low-speed subsonic wind tunnel. Experiments were conducted in 1996, 1997 and 2002 respectively (Selig & McGranahan, 2003). The simulations for our own validation purposes were conducted between Reynolds numbers of 400 000 and 500 000 and can be regarded as low Reynolds number simulations. The aerofoils used in this thesis were the E231, S834 and FX 63-137 profiles designed for small wind turbine applications. As a first objective the performance of XFOIL was validated with regard to the UIUC low-speed subsonic wind tunnel experimental data for the three above mentioned aerofoils. This was done in two phases:

- I. The first was to analyse the reliability of the standard XFOIL simulation against our benchmark wind tunnel data.
- II. The second was to confirm the performance of a new proposed geometry importation method, aimed to rectify the problem of XFOIL to over predict lift and under predict drag in high separation areas.

The results of the XFOIL validation phase carried out in preparation for this thesis are discussed in **Chapter 3**, and the newly proposed method of geometry importation to non-linearly cluster aerofoil panel nodes more densely in areas of high interest, appear in **Appendix B**.

The main objective of this thesis is to report on the development, implementation and validation of various far-field drag extraction methods incorporated into a converged unstructured grid CFD simulation. **Chapter 4** deals with the development and **Chapter 5** with the implementation of these far-field drag extraction methods. **Tables 22, 27 and 30 Chapter 5** show some promising results in favour of the proposed far-field method above the near-field method for drag calculation. In these tables the maximum drag count error (for angle of attack range simulated) of the near-field and various far-field methods are displayed with regard to experimental data.

The reason why the various proposed far-field methods of drag prediction exhibit such a significant improvement over the accuracy of the drag prediction of the near-field method is due to a reduced sensitivity to spurious drag. In the cases investigated the far-field method has a reduced sensitivity to the mesh refinement in areas of high curvature. For this reason the validity of the pressure drag calculation is conserved to a higher degree. As artificial spurious drag is implicitly added to the pressure drag term of the near-field method, it can clearly be seen from the figures in **Chapter 5** and **Chapter 6** that the far-field drag extraction yields profile drag values lower than the near-field method, and are ultimately closer to the experimental values.

Keywords: computational fluid dynamics, panel codes, near-field drag, far-field drag, Squire-Young model, wake-rake, boundary layer, Trefftz plane analysis, pressure drag, drag extraction

STATEMENT OF ORIGINALITY

This thesis is submitted for the degree of Masters of Engineering in the Department of Mechanical and Nuclear Engineering at the North West University. The research reported herein was carried out, unless otherwise stated, by the author, between January 2018 and January 2019 under the supervision of Dr Johannes J. Bosman. No part of this thesis has been submitted for a degree to any other university or educational establishment.

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NOMENCLATURE

Symbols

C_l airfoil lift coefficient

C_d airfoil drag coefficient

C_f skin friction coefficient

C_p pressure coefficient

Re Reynolds number based on airfoil chord

ρ density

μ dynamic viscosity

q local dynamic pressure

u, v, w mean velocity components in x, y, z directions, respectively

y^+ dimensionless wall distance

δ_1 Displacement thickness

δ_2 Momentum thickness

Y_w Wake width

Abbreviations

LE Leading Edge

TE Trailing Edge

CFD Computational Fluid Dynamics

RANS Reynolds Averaged Navier Stokes

AIAA American Institute of Aeronautics

Chapter 1: Introduction

1.1. INTRODUCTION

A constant eagerness exists for aerodynamicists to investigate, understand and predict the behaviour and nature of flow in the near vicinity of the body of an aerofoil. This ensures that the conceptual design phase will provide accurate performance information regarding the newly designed body. Various mathematical models have been derived and, to a certain extent, validated for the prediction of laminar and turbulent flow patterns with empirical and semi-empirical corrections for a variety of complex flow properties. These complex flow properties include, but are not limited to, laminar/turbulent transition zones, existence of laminar separation bubbles and viscous/inviscid flow effects (Cummings, Morton, Mason, & McDaniel, 2015). With the ever increasing cost of experimental data, computational methods have begun to displace wind tunnel testing in various areas of the design phase.

In the preliminary design phase computational codes such as vortex panel methods, vortex lattice methods and full-potential CFD codes are used to determine the aerodynamic performance of an aerodynamic body in fluid motion. The two workhorses of the aerodynamic design phase consist of the vortex panel code XFOIL and the full potential CFD codes of ANSYS FLUENT, Star-CCM+ and OpenFoam (TickTutor, 2018). Although both CFD codes and panel methods have shown success in the field of aerodynamics the methods of drag prediction in these codes differ fundamentally.

These differences in computational method have led to discrepancies in the prediction of total profile drag of an aerodynamic body. One such example of differences between drag predicted by XFOIL and Star-CCM+ can be seen in **Figure 1** where two aerofoils were analysed at a Reynolds number of 1 million and a Mach number of 0. In **Figure 1** it can clearly be seen that for the XFOIL simulation with 120 panels there is a clear distinction between the performance of the ST1 and OPT110 aerofoils. As for the Star-CCM+ simulation set-up with a fine mesh and the $\gamma - Re_\theta$ transition model, the results show that both aerofoils have similar performance characteristics.

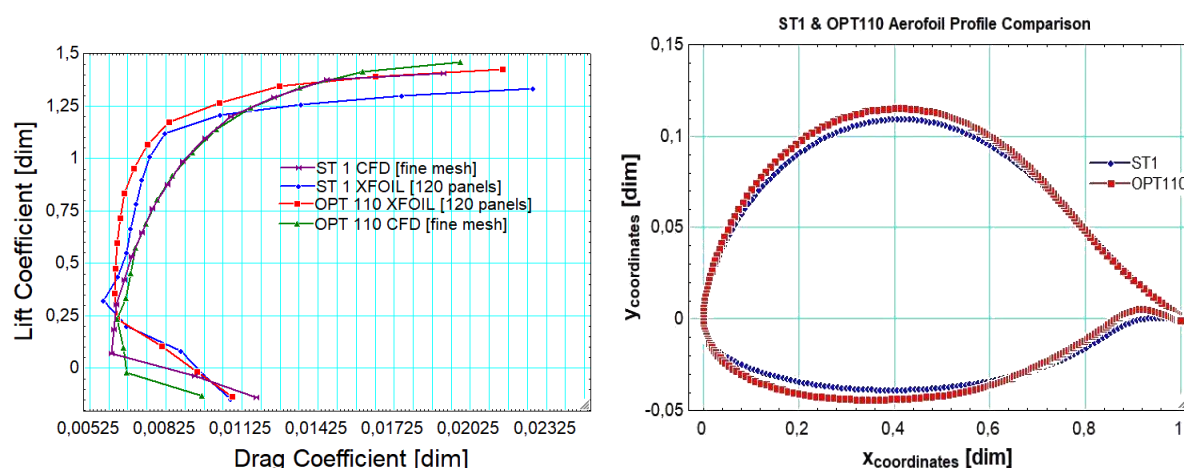


Figure 1: Discrepancies for CFD and XFOIL prediction (left); ST1 & OPT110 profile comparison (right)

As an introduction to address these discrepancies, the differences between the methods of drag computation of XFOIL and classical CFD codes will briefly be discussed.

In XFOIL the drag coefficient is obtained by applying the Squire-Young model to the last point in the wake. In effect the Squire-Young formula uses the momentum deficit at the trailing edge to extrapolate for the momentum deficit at downstream infinity where the wake is stabilized. This means the extrapolated momentum deficit represents the drag that would be calculated with a control-volume momentum balance around the aerofoil (Coder & Maughmer, 2015).

As opposed to the Squire-Young method of drag calculation, traditional CFD codes use the near-field method to determine the drag of an aerodynamic body. The near-field method typically consists of surface integration of the pressure and friction stresses exerted on the aerodynamic body. Through the years of design and simulation via the near-field method it was noted that this method yielded very high accuracy lift coefficient predictions. However, the drag coefficients predicted by this method lagged in terms of accuracy. This is true due to the fact that lift coefficients are typically in the order of 15 to 25 times that of the drag coefficient. This means discretization and truncation errors arising from inadequate grid definition have smaller effects on the lift prediction than on the drag prediction. Counter intuitively, this means the accuracy of the near-field method for drag determination tends to decay as a result of inadequate grid and geometry specification (Snyder, 2012). One of the main reasons why drag accuracy is sensitive to grid definition is because an inadequate grid at positions of high curvature means that the boundary layer in these portions of the model is not accurately solved. These inaccuracies are then superimposed when the near-field method surface integrates these erroneously solved flow variables to find the profile drag.

With clear differences between the drag predicted by XFOIL and Star-CCM+ it will be necessary to study yet another model of drag prediction in order to establish which model, XFOIL or Star-CCM+, yields the highest accuracy predictions (Snyder, 2012).

The third model that will be studied is that of the far-field method as typically employed by early wind tunnel experimentalists. This method has, over the years, proven to be highly reliable, but to date has not been implemented into commercial simulation software.

The far-field method (also known as the wake-integral method) of drag computation consists of calculating the drag of an aerodynamic body by integrating over the Trefftz plane. The Trefftz plane is an arbitrary cross flow plane placed in the wake of the aerodynamic body (Karamcheti., 1980). Because the wake-integral method uses integration of flow variables over the Trefftz plane, the numerical noise as a result of the misdirection of projected pressures or internal cancellation of pressures very close in magnitude is avoided. This leads to reduced artificial drag build up as simulation iteration progresses (Snyder, 2012). Not only does the wake-integral method reduce the presence of spurious drag or artificial drag, but it also gives a physical rather than a mechanical breakdown of the various drag sources, i.e. wave drag and viscous drag (Cummings, 1996). Thus far only limited research has been done on the application of the Squire-Young model and wake-rake analysis methods to determine drag using the flow variable outputs from a converged viscous CFD solution.

1.2. PROBLEM STATEMENT

Although panel methods and CFD codes have been used successfully in the design and optimization of aerofoils, variance still exists in the prediction of aerofoil performance as calculated by these two methods. These variances in prediction accuracy between the two codes become more adverse where separation in the flow regime dominates for instance at stall conditions. The disagreement or conflict between the drag predictions made by CFD codes and panel methods are a result of the different mathematical models deployed in the respective methods.

All the methods of drag prediction known to date can either be classified as near-field or far-field methods. The near-field method has gained traction in the world of commercial CFD application and is deployed in the majority commercial CFD codes to predict the lift and drag coefficient of an aerodynamic body.

Although accurate, the near-field method tends to be overly sensitive to grid definition and yields artificial drag known as spurious drag. This artificial drag term stems from the numerical dissipation of CFD simulations caused by the discretizing schemes deployed to yield stable numerical performance as the iterative solution of the flow field commences. Various studies have shown that the implementation of the far-field method for drag calculation reduces or eliminates the presence of spurious drag. These studies are found in the appendices and form the basis of how the far-field models portrayed in this thesis are implemented. Far-field methods such as wind tunnel wake survey and the Squire-Young model have reduced sensitivity to the far-field boundary treatment of the domain and hence lower errors are introduced at the LE stagnation point. Since errors in the LE stagnation point mainly manifest in the pressure drag component, the total profile drag computed by near-field methods tends to be overestimated. This phenomenon will thus be greatly reduced by the far-field method of drag calculation and, overall, a more accurate prediction of the profile drag component will be the result.

Additional problems that arise with CFD codes are the vast number of iterations and the computing time required to solve for the potential flow field. It is known that an explicitly defined grid must be adequately defined in the regions of interest during the simulation whilst adhering to wall treatment conditions. The simulation accuracy of the near-field method is directly proportional to the refinement of the mesh. Also, the computational time to solve the potential flow Navier-Stokes equation is proportional to the refinement of the mesh; therefore an extremely fine mesh will render the effects of spurious drag negligible, but will also increase the computational time.

Thus, to ensure the conceptual design phase is cost-effective, a method to identify areas of spurious drag production must be developed. The far-field methods such as the wake-survey methods for wind tunnels or the Squire-Young model of wake analysis for drag extraction provides the advantage of using a coarser mesh to arrive at more accurate force predictions. Additionally, these far-field methods also carry the property of identifying the regions of spurious drag production, and this may assist design engineers to evaluate the feasibility of the grid definition.

An algorithm to apply far-field methods to a CFD study for drag extraction therefore needs to be developed. To date much research has been done on developing research-codes which can determine drag from an inviscid Euler simulation, as this method captures the freely deforming wake. Inviscid Euler simulations are, however, computationally expensive and

inviscid of nature and therefore viscous corrections must be made to arrive at a feasible solution.

Thus the advantage of applying a far-field drag computation directly to a converged CFD solution is that no additional computational effort is added to the simulation as the wake is identified explicitly via mathematical models rather than implicitly as in the case of the Euler simulation. Also, seeing that stand-alone CFD software like Ansys Fluent and Star-CCM+ incorporate transitional viscous models, the far-field method can extract drag from simulations representing real-world scenarios rather than using correction factors to supplement the draw back of an inviscid Euler simulation (Snyder, 2012).

1.3. RESEARCH OBJECTIVES

The following are the primary objectives of this research thesis:

- Investigate and evaluate the precision of the Squire-Young model as implemented in XFOIL. The XFOIL simulation results will be compared to the benchmark wind tunnel results identified.
- Develop a method for applying far-field drag extraction to a viscous two-dimensional unstructured CFD simulation.
- Develop a method for applying the Squire-Young model for drag extraction to a viscous two-dimensional unstructured CFD simulation.
- Investigate the pressure drag and artificial spurious drag calculation errors in CFD.
- Quantify the production of spurious drag in a viscous two-dimensional unstructured CFD simulation.
- Improve lift coefficient predictions of XFOIL in regions with adverse separation.
- Investigate the solution sensitivity to grid refinement of near-field, far-field and Squire-Young models.

1.4. RESEARCH APPROACH AND METHODOLOGY

I. To develop a far-field algorithm to be implemented in a CFD simulation, the basis of the formulation which includes the assumptions made during the derivations of the model must

first be understood. Therefore it is of utmost importance to, in the first instance, study the models and derivations of the Squire-Young and wake survey methods of drag prediction. It is also important for the successful implementation of the far-field methods in CFD that the concepts of numerical dissipation, discretization errors and grid sensitivity of the near-field method are clearly understood. For this reason separate attention will be given in the literature review to each of these phenomena.

II. XFOIL has the ever-present property to over predict the lift coefficient and to under predict the drag coefficient as the AOA increases. Thus, as a sub-category of the study, a method was developed to counter this effect in order to yield the highest possible accurate solution from XFOIL. This method can be viewed in the Appendices.

III. With an in-depth understanding and possible improvement of XFOIL predictions in low Re range, its use for three aerofoils will be validated. These aerofoils will serve as the basis for the research presented in this thesis, as all calculations will be validated with the wind tunnel data available for these specific aerofoils.

IV. A method will then be developed to apply the momentum thickness integral, Squire-Young model and wake survey methods to an unstructured 2-d viscous simulation of the three abovementioned aerofoils. The findings will be validated against the experimental data of the three aerofoils.

V. Finally, the best far-field, near-field and XFOIL methods as chosen from the above sections' findings will be deployed in a case study to determine the discrepancies in force predictions between these methods and to establish which of the methods yields the highest accuracy predictions.

1.5. THESIS DISPOSITION

This thesis will comprise of 7 chapters, each building on the information and findings of the previous chapters. **Chapter 1** will serve as an introduction and broad overview of near-field and far-field methods. This chapter will progressively lead onto the limitations and application of these methods. The research proposal and the motivation behind the thesis will also form part of this chapter. To ensure the fluidity of this thesis is maintained, the bulk of the far-field investigation simulations are placed in the appendices. Here studies can be found such as mesh size effect and Trefftz plane location effect on the accuracy of the far-

field solution. It is important these findings be studied thoroughly to ensure chapter 4 to 6 are correctly interpreted.

Chapter 2 will serve as a literature survey on the mathematical derivations and assumptions made in the near-field, far-field and Squire-Young models. The far-field literature survey will also include research on wind tunnel wake analyses methods and the correlation between wake momentum thickness and profile drag of an aerodynamic body. Moreover, this chapter will include literature on the formation and cause of formation of spurious drag in CFD simulations. The relation between spurious drag, pressure drag errors, numerical dissipation and numerical discretization in the numerical schemes deployed in CFD will be examined. It is a well-known fact that it is difficult to declare which region of the wake is regarded as viscous in the CFD simulation. Therefore an investigation into methods to identify these regions in CFD solution will be conducted. This is important, because in the far-field methods of wake analysis only the viscous regions need to be integrated to arrive at accurate profile drag predictions.

Chapter 3 will serve as a validation of both XFOIL accuracy and Squire-Young model accuracy for low Reynolds number flow. In this section a validation will be undertaken of the accuracy of XFOIL for three different aerofoil profiles with wind tunnel results from UIUC low-speed subsonic wind tunnel conducted in 1996, 1997 and 2002 respectively (Selig & McGranahan, 2003).

Chapter 4 contains the derivation and descriptions of the application of various far-field methods to unstructured 2d viscous flow simulations. After the description of the far-field methods that are applied to CFD simulation, the attention will shift to the way in which the model predicts drag compared to traditional near-field methods. A special focus will be cast upon the grid sensitivity of the model and on the identification of spurious drag. The next part of the chapter deals with the evaluation and discussion of the differences in drag component predictions via the near-field and far-field methods.

Chapter 5 is the section in which the developed far-field methods and classical near-field methods will be applied to the three aerofoils mentioned in **Chapter 3**. This will be done to establish which method yields the highest accurate solutions and reasons why the results acclaimed have the observed tendencies will be discussed.

In **Chapter 6** the most accurate CFD and panel code method will be used in a case study to determine the discrepancies between the force predictions made by XFOIL and Star-CCM+

for two different aerofoils. This will ensure that the most accurate CFD and XFOIL methods are verified for the design and development phases of aerodynamic bodies.

Chapter 7 summarises the thesis outcomes and elaborates upon the direction in which further research should be undertaken.

Chapter 2: Literature review

The main focus of this thesis is on two dimensional unstructured grid CFD simulations with laminar to turbulent transition. Moreover, the SST $k - \omega$ turbulence model of Menter with the $\gamma - Re_\theta$ transition model will be used (Menter et. al., 2006). Therefore the literature survey will firstly focus on the broad overview of the turbulence and transition models as derived by Menter and the applicability and accuracy thereof. Attention will also be given to previous research on quantifying differences between XFOIL, near-field and far-field simulations, as well as the models of drag prediction via the near-field, far-field and Squire-Young models. As Trefftz plane placement and numerical spurious drag is very sensitive to the numerical dissipation and discretization schemes used in CFD, a thorough literature survey will also be conducted on these subjects. In the far-field method of drag extraction the flow-variables are numerically integrated and thus the most applicable numerical integration methods need to be studied. Also, with the far-field drag extraction method a viscous region will be identified in the wake that needs to be integrated and therefore a literature survey will be undertaken on mathematical methods to identify the viscous wake region.

2.1. PREVIOUS RESEARCH DONE IN NEAR-FIELD/FAR-FIELD DRAG ANALYSIS

Research in the field of drag prediction has escalated due to the clear discrepancy between the predicted aerodynamic coefficients of panel methods and commercial CFD codes (Monsch, 2007). These studies mainly deal with the variations in drag coefficient predictions of the near-field method and the far-field method. Additionally, many researchers and commercial developers aim to quantify the existence of spurious drag, and ultimately reduce its existence. In the early 2000's a working group of members of the Applied Aerodynamics Technical Committee (AIAA) initiated the first drag prediction workshop or DPW (Vassberg, 2008). The basis of the DPW was, and still is, to improve the accuracy of drag prediction from commercial and research CFD codes. Vos and Sanchi (2010) stated that the results to date stemming from the DPW clearly show that the uncertainty in the drag prediction of CFD codes is a result of the errors in the numerical and discretisation schemes deployed. They

followed up on this statement indicating that “*the drag prediction workshops clearly showed that this was the major source of uncertainty in the results before other sources as turbulence and transition modelling*”. Consequently, the need arises to study more accurate methods of drag predictions that are less reluctant to error due to grid definitions.

One such study was led by Esquieu (2007) of ONERA. Esquieu used the ONERA code for drag extraction in a study of inviscid flow fields around the NACA0012 aerofoil to identify the spurious drag formation in a CFD solution. In this study Esquieu used a coarse, medium and fine structured mesh to identify the linkages between spurious drag production and its dependency on mesh quality, surface discretization and the influence of these parameters on the pressure drag. The drag calculation was done by the ONERA-elsa tool and the flow conditions simulated were transonic ($M_\infty = 0.77$ and $\alpha = 0$). He found a large variation in pressure drag between the coarse and fine grid simulations (4.3 drag counts difference between the two meshing schemes). The work of Esquieu thus clearly indicated a high dependence of pressure drag formulation on the order of mesh used in the simulation.

One limitation of the implementation of the far-field wake survey is the choice of Trefftz plane placement downstream of the aerodynamic body. If the downstream Trefftz plane is placed too far from the TE of the body in fluid motion the wake-integral will yield inaccurate drag predictions due to numerical diffusion (Snyder, 2012). Numerical diffusion of the wake is caused by the discretization of the flow field to introduce stability in the numerical solution. If the Trefftz plane is placed in a region where the wake strength is reduced due to the discretization scheme employed in the solution, then the wake integration will result in a smeared solution.

Makota et al. (2011) describe the process of the far-field boundary entropy production as being contaminated by entropy oscillation due to the effects of numerical diffusion. In this study they deployed a grid convergence study where the cut-off or Trefftz plane was placed at 11 chords behind the TE of the aerodynamic body. It was found that at this position almost all spurious drag was removed from the solution for a coarse, medium and fine grid. In this instance the findings were that for all the grid cases simulated the error was in 2 drag counts of converged near-field solution. It was also noted that the solution of the far-field analysis converged more rapidly than the near-field solution with a lower dependency on grid refinement schemes.

Monsch (2007) used the far-field wake analysis method to predict the induced drag of a finite rectangular wing. In this study a second order Euler simulation was used to solve for

the flow field around a NACA0012 untwisted wing of $AR = 6.7$ with no flap deflections. The induced drag was extracted from the Euler simulation by employing a wake integral at the Trefftz plane. A comparison was then undertaken of the induced drag predicted by an in-house developed lifting line code, a CFD near-field simulation, and the Trefftz plane analysis. In the event of the Euler code being a second order non-viscous solution, near wake and compressible flow corrections were applied to improve the accuracy of the induced drag prediction by wake survey. Monsch once again showed that for transonic simulations the location of the Trefftz plane is of cardinal importance due to the numerical dissipation of the wake in the downstream of the body in fluid motion.

Yamakazi et al. (2005) used the far-field and mid-field method to decompose drag into profile, wave and induced drag components. The mid-field method originates from applying the Gaussian divergence theorem to the far-field definition. This method is often referred to as the volume integration process. By applying the mid-field method to a CFD study they were able to sub-divide the entropy drag into the drag components and visualize the generated position as well as the entropy drag strength in the flow-field. In their study the mid-field method was used on a viscous 2d structured mesh simulation. The far-field and near-field methods were applied on a 3d inviscid unstructured mesh simulation. It was found for both the inviscid 3d and viscous 2d cases that as the mesh became refined, a rapid decline in spurious drag was observed for the predicted drag values of the near-field, far-field and mid-field methods.

Thus far the attention was directed at the discrepancies between the drag prediction of the near-field, far-field and experimental methods. However, the same inconsistency referred to above is observed between the drag computed by the Squire-Young and the near-field methods. The Squire-Young equation is successfully implemented in the panel code of XFOIL. XFOIL makes use of a potential flow panel method in combination with an integral boundary layer formulation to evaluate the flow-field around an aerofoil. XFOIL being a panel code, panels are used to discretize the aerofoil surface for flow field calculation. Morgado et al. (2016) deployed a study in XFOIL, evaluating solution sensitivity to number of panel nodes in order to describe the surface of the aerofoils. They found that more than 150 nodes did not yield a significant difference in the final calculated aerodynamic coefficients. They also studied the performance of XFOIL compared to the SST $k - \omega$ turbulence model with the refurbished $k - kl - \omega$ transition model of Ansys Fluent[®]. They concluded that for the low Reynolds number spectrum evaluated, the performance of XFOIL outperformed Ansys Fluent[®] w.r.t the set benchmark.

A similar study by Yavuz et al. (2016) was deployed to evaluate the accuracy of XFOIL and the SST $k - \omega$ transition model of Ansys Fluent® in a Reynolds number range of 3×10^5 - 4×10^5 . The aerofoil analysed was the SG6040 aerofoil specifically designed for horizontal axis wind turbines with small blades. The aerofoil has a maximum thickness of 16% and a maximum camber of 2.5%. The study found that as separation increased, XFOIL tended to overpredict the lift coefficient and under predict the drag coefficient, but high accuracy was found in the linear regions of AOA, i.e. 0-10 degrees. The concluding remarks, as summarised from the paper, were that for low Reynolds numbers XFOIL and CFD results are comparable with each other until stall angle.

Coder & Maughmer (2010) conducted a study to compare theoretical methods for predicting aerofoil aerodynamic characteristics. In the study they compared the prediction accuracy of the panel code XFOIL and the Euler solver/integral boundary-layer method, MSES 3.05 with the PSU experimental results of the E 387 aerofoil at a Reynolds number of 300 000 (Maughmer & Coder, 2010). As seen in the figure 2, both XFOIL and MSES have similar drag predictions and, as noted by the authors, typically within 10 drag counts of the experimental values. It can also be seen that in the linear range both methods accurately predict the pitching moment coefficients. This research paper once again illustrated that XFOIL is a powerful and very precise tool in the calculation of aerodynamic coefficients.

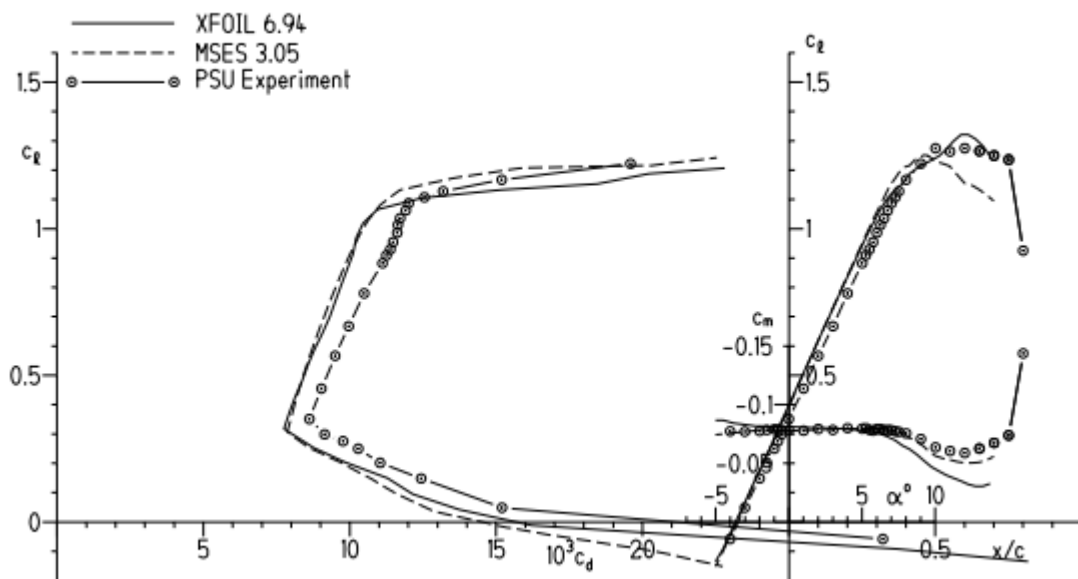


Figure 2: Comparison of predicted and measured aerodynamic characteristics for the E 387 aerofoil, $R = 300,000$.

Coder & Maughmer (2015) studied the application of the Squire-Young equation for drag prediction from the flow-field variables calculated in a CFD solution. The study was not concerned with validating the accuracy of the Squire-Young method from CFD solutions to the experimental values, but rather endeavoured to evaluate the accuracy of the method with regard to the near-field solution as calculated by the commercial CFD code OVERFLOW 2.2f solver (Nichols & Buning, 2014). The findings of this study were that the profile drag computed in the low drag region via the CFD Squire-Young model were in a 2-3% agreement with the near-field method of profile drag prediction from OVERFLOW. It was also found that solutions from the CFD Squire-Young method were insensitive to spurious drag that occurs due to far-field boundary conditions and numerical dissipation in the discretizing scheme.

2.2. SST (MENTER) $k - \omega$ TURBULENCE MODEL OVERVIEW

The traditional $k - \omega$ turbulence model was problematic, due to over sensitivity to free-stream and inlet conditions. In the mid 1990's Menter realized that the epsilon transport equation could be transformed to an omega transport equation by the implementation of a variable substitution (Menter, 1994). This transformed equation bore similarity to the standard $k - \omega$ model, but an additional non-conservative cross-diffusion term was added. This term contained the dot product $\nabla k \cdot \nabla \omega$ (Steve Portal, 2016). The addition of this term in the omega transport equation results in the $k - \omega$ model yielding identical results to the $k - \varepsilon$ model.

The suggestion of adding a blending function which includes wall distance functions was proposed by Menter (2006). This blending function would then include the cross-diffusion term far from walls, but would exclude the cross-diffusion term near the wall. The result of using this blending function was that the $k - \varepsilon$ model in the far-field was now blended with a $k - \omega$ model near the wall. The result of the blending function as proposed by Menter ensures that the $k - \omega$ model can be applied to practical flow simulations.

Menter (2006) went further in improving the $k - \omega$ model by adding a modification to the linear constitutive equation. This modification is widely known as the shear-stress transport (SST) $k - \omega$ model. Whilst the SST $k - \omega$ model is an improvement on the $k - \varepsilon$ model, the problem of the linear relationship between the Reynolds stresses and the mean strain rate to strongly under predict the anisotropy of turbulence still exists (Steve Portal, 2016).

The anisotropic behaviour of turbulence is compensated for by Star-CCM+ with non-linear constitutive relations such as the quadratic relation suggested by Spalart or the cubic relation formulated by Wallin and Johansson (Wallin, 2000).

The SST $k - \omega$ model has seen success in a wide application of uses, including the aerospace industry where there is a need to solve for viscous flow regimes with turbulent boundary layers.

2.3. $\gamma - Re_\theta$ TRANSITION MODEL OVERVIEW AND APPLICATION

The detection of transition onset differs fundamentally in the application of its use. In aerodynamic flows the detection of transition is typically the result of flow instability. These instabilities are commonly related to the Tollmien-Schlichting waves or cross-flow instability where the growth of the instability leads to the nonlinear breakdown in turbulence. Transition detection in turbomachinery is commonly the result of bypass transition (Morkovin, 1969). Bypass transition is a result of high level turbulence imposed on the boundary layer as a result of the turbulence in the inlet-stream coming from the upstream blade rows. Yet another form of transition is the mechanism of separation induced transition (Mayle, 1996). Separation induced transition is a result of laminar boundary layer separation under strong pressure gradients, and transition develops within the shear-layer which may or may not reattach.

Consequently it is difficult to detect transition with a generic code for such a wide range of applications. Menter et al. (2006) defined the main requirements for a fully CFD-compatible transition model as follows:

- I. Allow the *calibrated* prediction of the onset and the length of transition.
- II. Allow the inclusion of the different transition mechanisms.
- III. Avoid multiple solutions (same solution for initially laminar or turbulent boundary layer).
- IV. Do not affect the underlying turbulence model in fully turbulent regimes.
- V. Allow a robust integration with similar convergence as underlying turbulence model.

The $\gamma - Re_\theta$ model was developed with these 5 points mentioned above in mind. In this model the transport equation for intermittency is used to trigger transition locally. The intermittency function is used to turn on the production term of turbulent kinetic energy downstream of the transition point in the boundary layer. In addition to the transport equation of intermittency, the second transport equation of the transition onset momentum-thickness

Reynolds number is solved. This captures the non-local influence of the turbulent intensity which changes as the turbulent kinetic energy in the free-stream decays (Menter et. al., 2006). According to the authors of the model the $\gamma - Re_\theta$ model of transition detection is most suitable for transition detection for open flow simulations of low to medium Mach numbers and delivers accurate results for 2d and 3d unstructured grids.

The $\gamma - Re_\theta$ model has since its development been used successfully to detect the onset of transition in aerodynamic flow simulations. Benini et al. (2011) investigated the capability of the $\gamma - Re_\theta$ model for predicting laminar/turbulent transition in the boundary layer of a supercritical airfoil. The study covered a fully transonic regime, i.e. Mach 0.3 – 0.825. It was found that for low to medium Mach numbers the SST turbulence model with the $\gamma - Re_\theta$ transition model accurately predicted the lift and drag coefficients and the onset of laminar to turbulent transition. It was also found that as the Mach number increases, the discrepancy in both the lift and drag coefficient predictions becomes larger. Also, the prediction of transition on both suction and pressure sides of the aerofoil were concluded to be in agreement with expected values. This was the case even if the correlation on the pressure distribution was less satisfactory. Figure 3 shows the findings of Benini et al. (2011); this figure displays the precision of the evaluated models to predict the pressure coefficient in the transonic regimes.

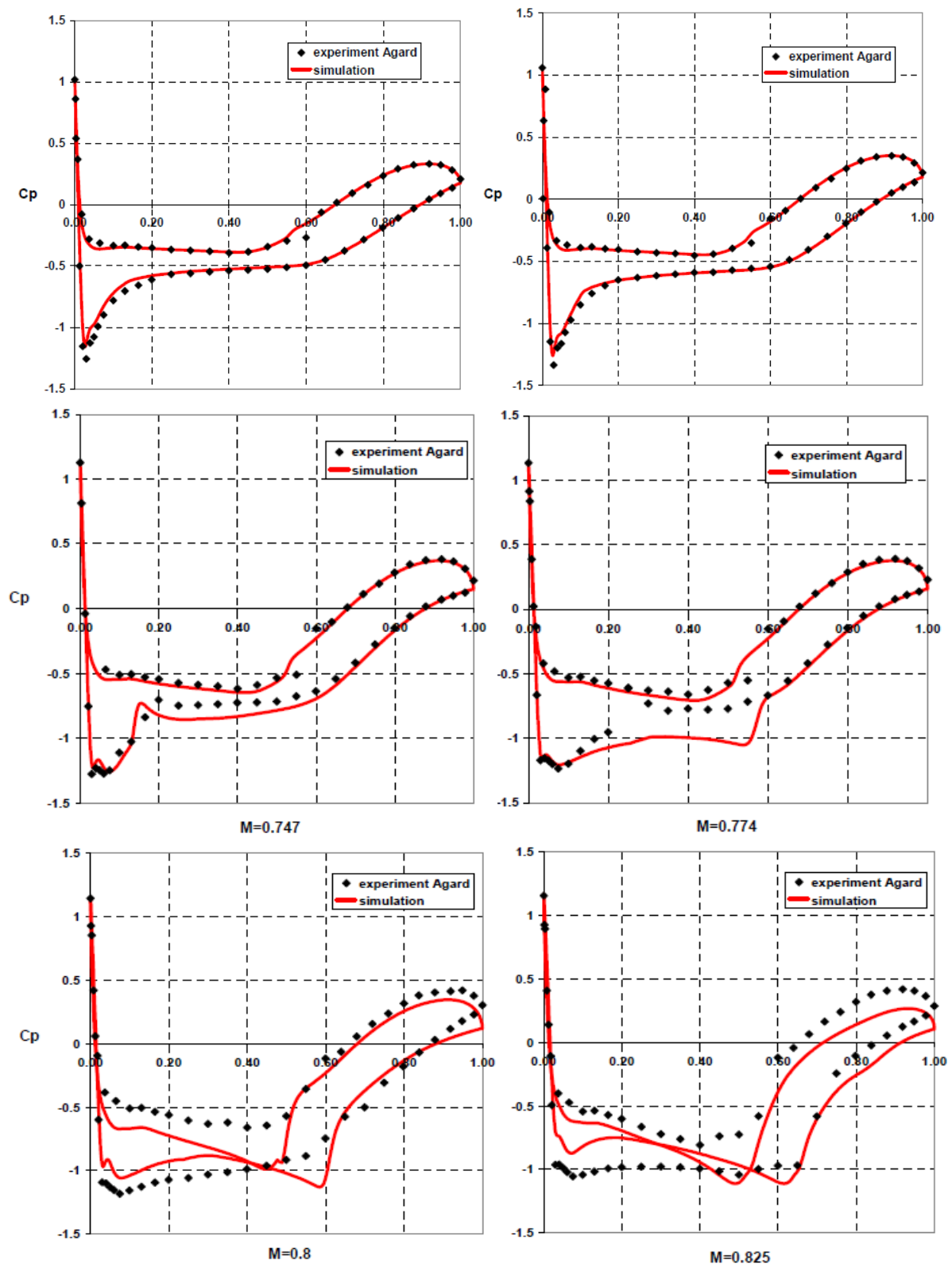


Figure 3: Simulation C_p VS Experimental C_p (Benini et al., 2011)

Gamboa (2010) applied CFD to calculate the flow around a wing sail aerofoil. He compared the simulation results of a NACA0015 aerofoil at a Reynolds number of two million with experimental results at four AOA (3, 5, 7 and 10 degrees). The turbulence models that were investigated were the Spallart-Allmaras (Standard), $k - \varepsilon$ (Standard and Low Re variant), $k - \omega$ (SST and $\gamma - Re_\theta$ variant) and the Reynolds Stress Turbulence models. In this study the numerical result of C_L and C_D was compared with the experimental results, and it was concluded that although all models predicted the lift coefficient to an expected degree of precision, the drag coefficient, with the exception of the $\gamma - Re_\theta$ model, could not be predicted with this expected degree of precision. Gamboa found that the $\gamma - Re_\theta$ transition model coupled with the standard SST turbulence model was the only model to accurately predict the lift as well as the drag coefficient.

Mazharul et al. (2015) studied the $\gamma - Re_\theta$ transition model accuracy by using different correlations to calculate the two parameters required for the $\gamma - Re_\theta$ model solution. They analysed the NACA 4415 aerofoil for a low free-stream turbulence intensity of 0.03% and Reynolds number of 700 000 that would cause natural transition. The following correlations were used in the SST $\gamma - Re_\theta$ study: Sørensen (2009), Malan et al. (2009), Suluksna et al. (2009), Langtry and Menter (2009) and Tomac et al. (2013).

The results were compared with experimental data as well as with XFOIL results. The turbulence model used was the $k - \omega$ model. The results for angle of attack cases of -8 degrees, 0 degrees and 8 degrees can be seen in the following tables:

	C_L	C_D
$\gamma - Re_\theta$ (Langtry and Menter, 2009)	-0.34	0.0216
$\gamma - Re_\theta$ (Malan et al., 2009)	-0.34	0.0211
$\gamma - Re_\theta$ (Sørensen, 2009)	-0.38	0.0139
$\gamma - Re_\theta$ (Suluksna et al., 2009)	-0.36	0.0171
$\gamma - Re_\theta$ (Tomac et al., 2013)	-0.39	0.0132
XFOIL 6.99	-0.39	0.0135

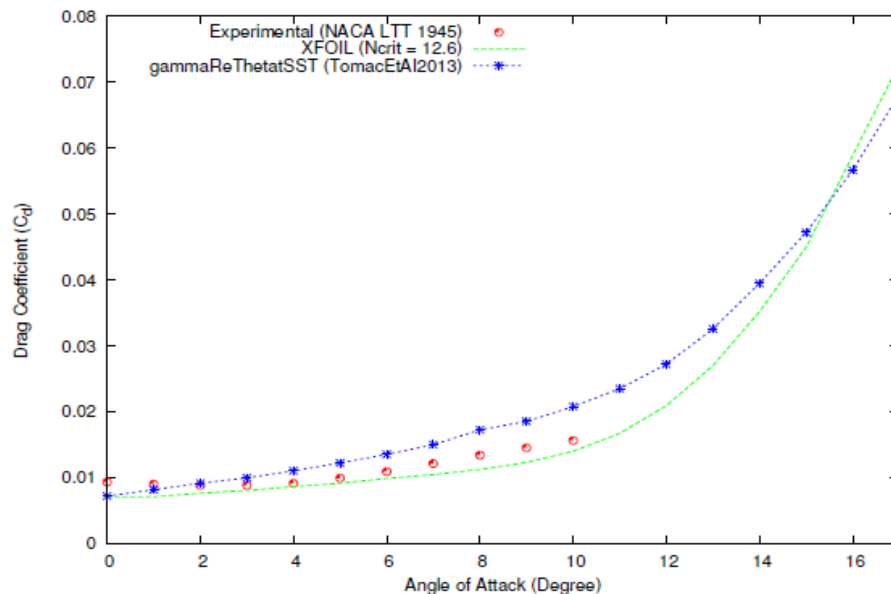
Table 1: -8 degrees AOA results (Mazharul et al., 2015)

	C_l	C_d
$\gamma - Re_\theta$ (Langtry and Menter, 2009)	0.43	0.0074
$\gamma - Re_\theta$ (Malan et al., 2009)	0.43	0.0077
$\gamma - Re_\theta$ (Sørensen, 2009)	0.44	0.0074
$\gamma - Re_\theta$ (Suluksna et al., 2009)	0.42	0.0067
$\gamma - Re_\theta$ (Tomac et al., 2013)	0.41	0.0073
Experimental (Miley, 1982)	0.45	0.0094
XFOIL 6.99	0.47	0.0075

Table 2: 0 degrees AOA results (Mazharul et al 2015)

	C_l	C_d
$\gamma - Re_\theta$ (Langtry and Menter, 2009)	1.19	0.0176
$\gamma - Re_\theta$ (Malan et al., 2009)	1.19	0.0176
$\gamma - Re_\theta$ (Sørensen, 2009)	1.21	0.0171
$\gamma - Re_\theta$ (Suluksna et al., 2009)	1.20	0.0173
$\gamma - Re_\theta$ (Tomac et al., 2013)	1.22	0.0169
Experimental (Miley, 1982)	1.16	0.0135
XFOIL 6.99	1.31	0.0113

Table 3: 8 degrees AOA results (Mazharul et al., 2015)

Figure 4: Cd VS AOA with Tomac et. al. (2013) correlations for $\gamma - Re_\theta$ model

From **Tables 1, 2 and 3** and figure **4** it is clear that the correlations of Tomac et al. and XFOIL show the best overall performance with the expected tendency of XFOIL to overestimate the lift coefficient and underestimate the drag coefficient in high separation zones. **Figure 4** clearly demonstrates that for low Re numbers experimental drag results lie between the XFOIL and CFD predictions.

2.4. PREVIOUS WORK ON CFD VS PANEL CODE ACCURACY

As this thesis is mainly concerned with Reynolds numbers between 400 000 – 1 000 000 (the region where XFOIL is most accurate), a literature survey will be conducted on the comparison between the performance of XFOIL and CFD for this low Reynolds number regime. This is important, because wind tunnel data is not always readily available and because the performances of newly developed far-field drag extraction tools has to be validated by a reliable source. Hence this part of the literature survey will focus on the validation of the use of XFOIL for low Reynolds numbers only.

2.4.1. VALIDATION OF XFOIL USE FOR LOW RE NUMBERS

Various studies have been undertaken to quantify the accuracy of panel codes and CFD codes. One of these studies was deployed by Parezanovic et al. (2008) of the University of Belgrade. They thoroughly investigated the accuracy of the lift, drag and moment coefficient predictions as done by XFOIL and FLUENT– the results were validated by wind tunnel test results (Parezanovic, 2008). The XFOIL simulations presented in the paper were undertaken with 120 panels, and were obtained from Riso National Laboratory, Denmark (Bertagnolio, 2001). They investigated the aerodynamic coefficients of the NACA 63(2)215, FFA-W3-211 and the Aerospatiale A-aerofoil.

In the case of the NACA 63(2)215 experimental data was obtained from the NASA low-turbulence wind tunnel, and the XFOIL, FLUENT and wind tunnel turbulence intensity was 0.07% (Parezanovic, Rasuo, & Adzic, 2005). In the case of FLUENT the fully turbulent $k - \omega$ SST model was used with 11970 quadrilateral cells, of which 146 are on the surface of the aerofoil. For this specific NACA aerofoil the simulations and wind tunnel tests were undertaken in the linear region, i.e. where separation effects do not yet have a significant influence on the simulation performance. It was noted that both XFOIL and FLUENT predict the lift and moment coefficients accurately compared to the experimental results. FLUENT overpredicted the drag compared to experimental data and XFOIL simulation results. This

was ascribed to the fact that the entire boundary layer was simulated as turbulent with no laminar/turbulent transition effects accounted for.

Subsequently, the FFA-W3-211 aerofoil was investigated in the linear region, this time with a turbulence intensity of 0.15%. The same $k - \omega$ SST fully turbulent model was deployed in the FLUENT simulation, but due to the performance characteristics of the FFA-W3-221 aerofoil in laminar/turbulent conditions both XFOIL and FLUENT simulations accurately predicted the drag and lift coefficients with regard to wind tunnel experiments.

Finally the Aerospatial A-aerofoil was investigated at a turbulence intensity of 0.07%. Wind tunnel results were carried out at the ONERA/FAUGA (Haase, 1997). From the wind tunnel results the upper and lower surface transition position was measured, and this was set in the FLUENT simulation. With the transition positions now properly set, the XFOIL, FLUENT and wind tunnel test data was in close agreement in terms of the lift and drag coefficients. The research indicated that both XFOIL and FLUENT predictions were comparable in fluid flow regimes where no significant separation occurs (given the transition points are properly set in FLUENT).

In a similar paper by Günel et al. (2016) the performance and accuracy of XFOIL and FLUENT aerodynamic coefficient prediction of the SG6040 aerofoil was compared to wind tunnel results (Günel, 2016). XFOIL and ANSYS FLUENT simulations were both set up at low Reynolds numbers, i.e. 3×10^5 and 4×10^5 . In XFOIL 250 points were used to define the geometry of the aerofoil, and the number of calculation iterations was set to 100. The critical amplification factor was set to 9, the standard in XFOIL. In FLUENT the SST $k - \omega$ transition turbulence model was used to solve for the boundary layer elements and the numerical convergence was controlled by monitoring the numerical error in the CFD solution. The boundary was set up as an O-ring domain (the external domain was set up with a 25 m diameter and the boundary was defined as a velocity inlet condition). A total of 42 layers were used to describe the boundary layer thickness with the first layer 0.005 m from the wall. The XFOIL and FLUENT simulations closely agreed with the experimental results. For the Reynolds number of 3×10^5 XFOIL predicted the lift coefficient more accurately than the FLUENT simulation with the wind tunnel results as reference. However, for a Reynolds number of 3×10^5 XFOIL under predicted the drag and in this case, FLUENT predicted the drag coefficient more accurately. For the Reynolds number of 4×10^5 XFOIL over predicted the lift coefficient and here the accuracy of the FLUENT results triumphed over XFOIL (again with wind tunnel results as reference). For this Reynolds number XFOIL once again under

predicted the drag, and again, in the case of drag calculation, FLUENT predicted the drag coefficient more accurately. The reduced accuracy of XFOIL for the higher Reynolds number simulation was ascribed to the methods used in XFOIL's separation calculation and ultimately the limitations in the post-stall calculation methodology. Although XFOIL's accuracy decreased when the Reynolds number increased, the results were still feasibly comparable with the wind tunnel results and the authors concluded that XFOIL is a fast, accurate and powerful tool for aerofoil analysis.

In mid-2009 M. Serdar Genç and Ünver Kaynak investigated the control of the laminar separation bubble over a NACA 2415 aerofoil at low transitional flow using blowing/suction (Serdar Genç, 2009). Wind tunnel test results for the NACA 2415 were acquired from the University of Bath and the University of TOBB ETU. The Bath stall angle was at 12 degrees with a maximum lift coefficient of 1.33, whereas the TOBB ETU results equated to a stall angle of 14 degrees and a maximum lift coefficient of 1.35 (Genç, 2008). The simulations to validate the wind tunnel results were done with XFOIL and FLUENT at a Reynolds number of 2×10^5 .

In the research of Genç and Kaynak the examination of various ANSYS FLUENT – low Reynolds number turbulent, fully turbulent and transition models were deployed to quantify the accuracy of the e^N -XFOIL method of transition detection. The study used the $k - \varepsilon$ RNG model and the low Reynolds number $k - \omega$ model to predict the performance of the aerofoil under the fully turbulent boundary layer assumption. From this point onwards the $k - \omega$ SST transition and $k-k_L-\omega$ transition models were used in the simulation under the transport model assumption. All models yielded predictions within expected precision, with discrepancies in the model prediction accuracy as stall effects became more dominant. It was clear that the low Re number $k - \omega$ and $k - \omega$ SST transition models overpredicted and the fully turbulent $k - \varepsilon$ RNG model underpredicted the stall characteristics (Huang, 2004).

2.5. CFD NEAR-FIELD DRAG CALCULATION

The near-field force calculation may be defined as the process where the force components acting on a body are numerically computed via surface integration of the stresses in the normal and tangential directions (Paparone & Tognaccini, 2002).

To derive a simple expression for the near-field forces acting on a body, we consider an unpowered aircraft with steady state fluid flow at a free stream velocity of V_∞ . These assumptions imply that the external forces acting on the body are only due to the fluid flow. Thus the integral formulation of the momentum balance acting on a control volume Ω surrounding the body equates to:

$$\int_S [(\rho \vec{V} \vec{V}) \cdot \vec{n} + p \vec{n} - (\vec{\tau} \cdot \vec{n})] dS = 0 \quad \dots(2.5.1)$$

Where ρ , $\vec{V}$, p and $\vec{\tau}$ specify the density, velocity vector, static pressure and the viscous tensor respectively. In this formulation S is the control surface bounded by the control volume Ω and $\vec{n}$ is the unit vector in the direction facing outwards from the control volume Ω . If we then decompose the surface into the components $S = S_{body} \cup S_{far}$ with S_{body} representing the body surface and S_{far} representing the external surface bounding the control volume Ω it is possible to formulate the integral of the forces acting on the body, according to Paparone & Tognaccini (2002), as:

$$\vec{F}_{body} = \int_{S_{body}} [\rho \vec{n} - (\vec{\tau} \cdot \vec{n})] dS = - \int_{S_{far}} [(\rho \vec{V} \vec{V}) \cdot \vec{n} + p \vec{n} - (\vec{\tau} \cdot \vec{n})] dS \quad \dots(2.5.2)$$

Upon inspecting **Equation 2.5.2** it is clear that the external forces acting on a body can be calculated by evaluating the integral of the stresses acting on the body surface for the near-field method, or by evaluating the net momentum flux across the surface for the far-field method. The latter will be discussed in the following section.

On the other hand, it is, also known that the near-field method of force calculation tends to yield inaccurate results even if the solution is locally accurate in terms of pressure and velocity profile predictions (Wang, Wang, Liu, & Jiang, 2018). This phenomenon can be ascribed to the presence of two factors, the first being the existence of numerical dissipation which sprouts from numerical noise in the approximated integral and gives rise to additional “artificial” drag. To eliminate the existence of spurious drag the grid needs to be set infinitely

dense, but this is impractical as it would lead to infinitely prolonged calculation times (Ueno et. al., 2011).

The second problem with using a near-field computation to approximate the external forces acting on a body, is that the near-field computation only accounts for the drag terms resulting from pressure and friction components. This means the production of spurious drag cannot be separated as it is implicitly added to these components.

2.6. CFD FAR-FIELD DRAG CALCULATION

The far-field analysis aims to eliminate or quantify the amount of artificial drag that accumulates in traditional CFD simulations. Spurious drag is dependent not only on the grid size, but also on the quality of the grid. Therefore a mere increase of grid size may decrease or increase the presence of spurious drag if a near-field analysis is utilized with an improper grid quality definition (Ueno et. al., 2011). The far-field approach typically yields more accurate drag coefficient predictions due to the following:

- I. A very precise approximation of the spurious drag count can be calculated (Destarac & van der Vooren, 2004).
- II. The drag production can accurately be visualized (Tognaccini, 2003).
- III. The quality of the grid can be visualized by visualizing the drag production (van der Vooren & Slooff, 1990).

In this section of the literature review the historical development of far-field methods for drag prediction will be discussed, I shall concentrate on the far-field formulations proposed by Betz, Jones, Oswatitsch, Maskell and Van der Vooren, focusing on the assumptions used and the domain of application of each method.

Originally the far-field approach was inspired by Von Kármán. He suggested applying the conservation laws on a control volume surrounding an aerodynamic body. The work done on the aerodynamic body could thus be equivalently evaluated by integration in the fluid domain. This allowed for a phenomenological breakdown of the drag components.

2.6.1. BETZ FAR-FIELD FORMULATION

In 1925 Betz was the first person to implement the far-field drag breakdown (Betz, 1925). His aim was to directly determine the profile drag of an aircraft from experimental measurements. He thus aimed to calculate drag from downstream wake measurements, though his formulation is only valid for incompressible flow measurements. In the formulation of Betz it is assumed that the velocity projected on the wake plane S_d is aligned with the free-stream velocity and that the variations in stagnation temperature are negligible:

- $v = w = 0$
- $T_i = T_{i\infty}$

The resulting expression is as follows:

$$D_{vw}^{Betz} = \int_{S_d} (p_{t\infty} - p_t) dS - \frac{\rho}{2} \int_{S_d} (u' - u)(2u_\infty - u' - u) dS \quad \dots 2.6.1.$$

Where p_t denotes the total pressure, $p_{t\infty}$ denotes the total pressure upstream of the body, and u' the velocity of a potential flow which would be identical to the real flow outside the vortical region. Thus the second integral limits the vortical region. Betz argues that the second term is negligible, being of order magnitude 1/20 of the first term.

2.6.2. JONES FAR-FIELD FORMULATION

In 1936 Jones proposed another far-field method only valid for incompressible flows (Jones, 1936). He makes use of more restrictive assumptions to eliminate the second integral of Betz's formulation. He assumes homogeneous static pressure on the wake plane S_d and that this pressure is equal to the free-stream pressure. Jones also assumes that the wake projected velocity is aligned with the free-stream velocity:

- $p = p_\infty$
- $v = w = 0$

The formulation of Jones only depends on the dynamic and static pressure for the calculation of the profile drag and can be expressed as:

$$D_{vw}^{Jones} = p d_\infty \int_{S_d} 2\sqrt{p_d^* - p^*} (1 - \sqrt{p_d^*}) dS \quad \dots 2.6.2$$

Where p_d^* and p^* are the dimensionless dynamic and static pressures respectively:

$$p_d^* = \frac{p_d}{p_{d\infty}} \text{ and } p^* = \frac{p}{p_{d\infty}} \text{ with the dynamic pressure being defined as } p_d = \frac{1}{2}\rho u^2 = p_t - p.$$

Thus, the surface S_d can now be limited to the wake boundary of the stream tubes enclosing the body, the boundary layer and the wake. Jones advised that this formulation be applied to a wake defined close to the body trailing edge. He also observed that this formulation does not depend on the location of the wake plane except in an area he referred to as the dead zone. The “dead zone” refers to regions where p_d^* becomes negative.

2.6.3. OSWATITSCH FAR-FIELD FORMULATION

By the end of the 1950's Oswatitsch developed a formulation to calculate profile drag via wake analysis which differs fundamentally from the formulations of Jones and Betz (Oswatitsch, 1956). Instead of assuming no transverse velocity on the wake plane, Oswatitsch used thermodynamic properties. In other words, he expressed the velocity and pressure as a function of thermodynamic variables H and s . He assumed isenthalpic flow and neglected second order terms to arrive at the following expression for total profile drag:

$$d_{vw}^{Oswatitsch} = \frac{T_\infty}{u_\infty} \int_{S_d} \rho u \Delta s \, dS \quad \dots 2.6.3$$

This definition allows the avoidance of the restrictive assumptions of the wake plane. Meheut proved that the formulations of Betz, Jones and Oswatitsch are equivalent at first order (Méheut, 2006).

2.6.4. MASKELL FAR-FIELD FORMULATION

Maskell furthered the field of far-field drag prediction by proposing a formulation to account for the induced drag (Maskell, 1973). The method of Maskell calculates the expression of total drag accumulated using the definition of total pressure and conserves the cross component of the velocity vector. This expression bears strong resemblance to Betz's formulation; however, in Maskell's formulation a blocking velocity u_b is introduced. This allows taking the blocking of the wind tunnel into account for the experimental case. The formulation of Maskell can be expressed as:

$$D_{vw}^{Maskell} = \int_{S_d} (p_{t\infty} - p_t) dS + \frac{\rho}{2} \int_{S_d} (u' - u)(u' + u - 2(u_\infty + u_b)) dS \quad \dots 2.6.4.$$

The expression for the induced drag is derived from the difference between the expression for total drag and the Maskell formulation of profile drag. An alternative expression as a function of the stream function ψ , the velocity potential ϕ , the axial vorticity $\zeta = \frac{dw}{dy} - \frac{dv}{dz}$, and the source term $\sigma = \frac{dv}{dy} + \frac{dw}{dz}$ can be written as:

$$D_i^{Maskell} = \rho \int_{S_d} (v^2 + w^2) dS = \rho \int_{S_d} (\psi \zeta - \phi \sigma) dS \quad \dots 2.6.5$$

Maskell's expression for induced drag allows us to reduce the integration surface S_d to the downstream boundary of the stream tube enclosing the body, the boundary layer and the wake.

All the formulations described up to this point were aimed at the calculation of drag from experimental results using wake surveys. The objective in focussing on the derivations of these methods was to ultimately obtain the most accurate results using the smallest wake measuring plane. With CFD solving the entire flow field in all domain positions, better suited methods needed to be developed to obtain accurate drag predictions via far-field approaches (Toubin, 2016).

2.6.5. VAN DER VOOREN AND DESTARAC FAR-FIELD FORMULATION

For the formulation of the far-field method of Van der Vooren and Destarac, please refer to **Figure 5**. The far-field equation is obtained by applying the conservation of the momentum and mass in a control volume V with boundaries $S_A \cup S_F \cup S_D$. The near-field and far-field definition can then be stated as:

$$\int_{S_A} ((p - p_\infty)(i \cdot n) - (\tau_x \cdot n)) dS = \int_{S_F \cup S_D} (p(u - u_\infty)(q \cdot n) + (p - p_\infty)(i \cdot n) - (\tau_x \cdot n)) dS \quad 2.6.6$$

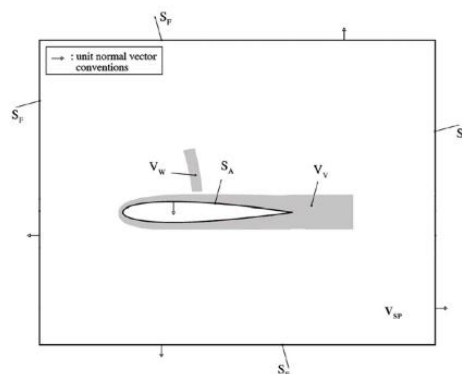


Figure 5: Control volume and boundaries

The assumptions that aim to break down the drag definition in its phenomenological components are:

- $\tau_x = 0$
- $V = w = 0$
- $p = p_\infty$

Free flow of vortices is assumed, i.e. the case where only profile drag is exerted on the body. Therefore the flows on the wake plane SD far from the sources are such as described by the above assumptions. Under these assumptions the axial velocity defect $u - u_\infty$ can be expressed as:

$$\Delta \bar{u} = u_\infty \sqrt{1 + \frac{2\Delta H}{u_\infty^2} - \frac{2}{(\gamma-1)M_\infty^2} (e^{\left(\frac{\gamma-1}{\gamma} \frac{\Delta s}{r}\right)} - 1)} - u_\infty \quad \dots 2.6.7.$$

The notation $\Delta \bar{u}$ is linked to the notation u_{irr} by the simple relation $\Delta \bar{u} = u_{irr} - u_\infty$. The far-field drag which is equal to the profile drag in the case of flow free vortices can now be derived with this assumption as:

$$D_{vw} = - \int_{SD} \rho \Delta \bar{u} (q \cdot n) dS \quad \dots 2.6.8.$$

From here, the wake surface integral is transformed into a volume integral over the respective volumes V_w and V_v as can be observed in **Figure 5**. The assumptions are for inviscid flow outside the volumes V_w and V_v , and, applying the divergence theorem, the following integrals for wave drag and viscous drag respectively can be postulated:

$$D_w = - \int_{V_w} \nabla \cdot (\rho \Delta \bar{u} q) dV \quad \dots 2.6.9.$$

$$D_v = - \int_{V_v} \nabla \cdot (\rho \Delta \bar{u} q) dV \quad \dots 2.6.10$$

Similarly, the induced drag can be expressed as a function of the near-field components:

$$D_i = - \int_{V_w \cup V_v} \nabla \cdot (\rho (u - u_\infty - \Delta \bar{u}) q + (p - p_\infty) i - \tau_x) dV - D_p - D_f \quad \dots 2.6.11$$

The above expression was formulated by using the vector property of:

$$f = -(\rho(u - u_\infty)q - (p - p_\infty)i + \tau_x) \quad \dots 2.6.12$$

$$\nabla \cdot f = 0 \quad \dots 2.6.13$$

This means that the balance between the near-field drag and far-field drag is theoretically ensured. The balance between the near-field drag and the far-field drag for a three dimensional case can be expressed as:

$$D_{nf} = D_{ff} = D_p + D_f = D_w + D_v + D_i + D_{sp} \quad \dots 2.6.14$$

2.7. DRAG COMPUTATION VIA EXPERIMENTAL WAKE ANALYSIS

The calculation of drag through wake analysis is widely implemented in wind tunnel experiments. The core of classic wind tunnel analysis through integrating wake rake methods comprises of a series of Pitot tubes stationed some distance from the TE of the aerodynamic body under scrutiny, which feeds into a single manifold to measure the averaged total pressure deficit in the wake of an aerofoil (Pifer & Bramesfeld, 2012).

Recently Son and Centiner studied the principles of drag prediction in a near-wake of a circular cylinder with the use of mean velocity profiles. They also investigated the closest location where a wake survey would produce the highest order accuracy result (Son & Centiner, 2016). A breakthrough study by Dimotakis showed that it is indeed plausible to determine the drag coefficient from the use of mean velocity profiles and fluctuation terms (Dimotakis, 1977). The method described by Dimotakis is given as:

$$c_d = \frac{\frac{2h}{d}}{\left(1 - \left(\frac{\delta^*}{h}\right)\right)^2} \left[\left(\int_0^1 \frac{U}{U_\infty} \left(1 - \frac{U}{U_\infty}\right) d\eta - \int_0^1 \frac{u'^2 - v'^2}{U_\infty^2} d\eta + \frac{1}{2} \left(\frac{\delta^*}{h}\right)^2 \right) \right] \quad \dots 2.7.1$$

Where u' and v' are the streamwise and cross-stream root mean square (RMS) velocities, h is the visualization height and δ^* is the displacement thickness. Another variation for drag analysis of the wake of a cylinder was developed by Townsend and verified for the wake of a circular cylinder in an independent study by Antonia and Rajagopalan (Townsend, 1980) (Antonia & Rajagopalan, 1990). This variation is displayed in **Equation 2.7.2**:

$$c_d = 2 \int_{-\infty}^{\infty} \frac{U}{U_\infty} \left(1 - \frac{U}{U_\infty}\right) d\eta + 2 \int_{-\infty}^{\infty} \frac{\overline{v'^2} - \overline{u'^2}}{U_\infty^2} d\eta \quad \dots 2.7.2$$

In **Equation 2.7.2** we may define the first term as the momentum deficit, i.e. momentum thickness of the time averaged flow field and the second term is the contribution of the streamwise u' and cross-stream v' turbulent fluctuation. The momentum deficit term has proven its applicability to drag calculation in a region far from the TE where the static pressure is nearly recovered to its free-stream value. In the near-wake region the negative pressure gradient term needs to be accounted for and this is done with the implementation of the second term of **Equation 2.7.2**. However, the further away from the TE, the more the influence of the second term degrades and becomes negligible at approximately 30 diameters from the cylinder (Antonia & Rajagopalan, 1990).

Nevertheless, this thesis is only concerned with the calculation of profile drag of an aerofoil and thus the attention will henceforth be turned to the experimental methods used to determine profile drag. One such method is that of Pope et al. (1984). This method differs fundamentally from those described in **Equation 2.7.1** and **2.7.2**, since it uses wake width, free stream dynamic pressure and local dynamic pressure to estimate the drag of an aerofoil (Rae & Pope, 1984). The method can easily be implemented in CFD simulations because the integral is defined over the wake width. **Figure 6** illustrates the derivation of this method :

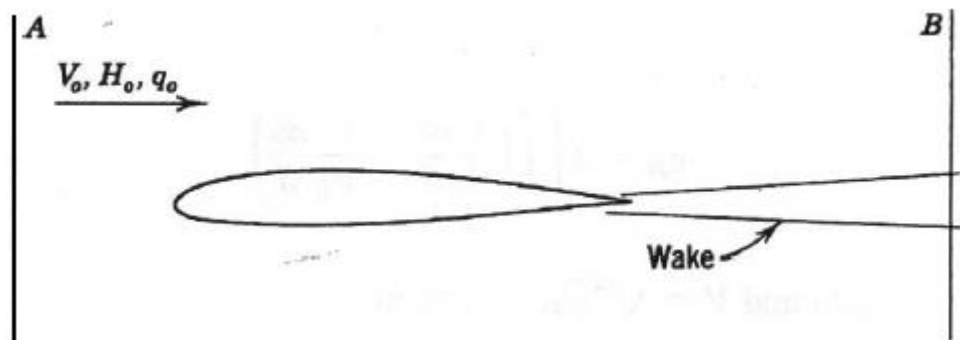


Figure 6: Control volume for derivation of Pope et al., profile drag formulation

The model is derived by comparing the momentum of the air ahead of the aerofoil with momentum in the air behind the aerofoil. The assumption is that the walls are parallel and that the shear stress on the wind tunnel walls can be neglected. This is the base assumption for almost all derivations of two dimensional profile drag equations. It is clear that as the air travels over the aerofoil body it suffers momentum loss; this loss is equal to the profile drag and can be calculated as:

$$D_{2D_{Profile}} = \iint \rho V da (V_o - V) \quad \dots 2.7.3$$

Where V_0 is the initial airspeed at position A, V is the final air speed in the wake at position B and da is the wake area perpendicular to the free airstream. Hence:

$$D_{2D_{Profile}} = \iint (\rho V V_0 da - \rho V^2 da) \quad \dots 2.7.4.$$

and

$$c_{d0} = 2 \iint \left(\frac{V}{V_0} \frac{da}{s} - \frac{V^2}{V_0^2} \frac{da}{s} \right) \quad \dots 2.7.5$$

It can also be stated that:

$$V_0 \sqrt{\frac{2q_0}{p}} \text{ and } V = \sqrt{\frac{2q}{p}} \quad \dots 2.7.7.$$

and therefore the drag for unit chord length and area $da = dy \times 1$ can be expressed as:

$$c_{d0} = 2 \int \left(\sqrt{\frac{q}{q_0}} - \frac{q}{q_0} \right) \frac{dy}{s} \quad \dots 2.7.8.$$

which can be re-written in terms of wake length Y_w , free stream dynamic pressure q_∞ , and local dynamic pressure q as:

$$C_d = \frac{Y_w}{c} - \frac{1}{q_\infty c} \int_{-0.5Y_w}^{0.5Y_w} q \, dy \quad \dots 2.7.9.$$

2.8. SQUIRE-YOUNG MODEL FOR DRAG PREDICTION

Finally, another wake method that has proven to be very successful in far-field drag analysis will be discussed, namely the Squire-Young model. A recent paper published by Coder et al. (2015) investigates and validates the Squire-Young model numerically for profile drag prediction. Even though the authors only investigated the accuracy of the Squire-Young model for profile drag prediction with reference to CFD simulations rather than wind tunnel experiment results, the paper still lays the ground for further extension into this area. Coder et al. (2015), showed that the Squire-Young model predicts drag with 2-3% deviation from the near-field method results. Since no wind tunnel experiments are used for validation, the paper does not give insight into the case that the model of Squire-Young should outperform the classical CFD near-field method. The authors do, however, confirm that the model of Squire and Young is less sensitive to spurious drag that can arise due to the far-field

boundary condition, and therefore the possibility exists for this model to outperform CFD when calculating 2d inviscid or viscid profile drag.

As previously discussed, the Squire-Young model is in effect an extrapolation of the momentum deficit at the TE of an aerofoil to the deficit that would exist in a wake far downstream. This is argued with the assumption that the momentum thickness behaves asymptotically from the TE to downstream infinity. The Squire-Young formula for incompressible flow can be expressed as:

$$c_d = 2 \left(\frac{\delta_{2TE}}{C} \right) \left(\frac{U_{TE}}{U_\infty} \right)^{\frac{5+H_{TE}}{2}} \quad \dots 2.8.1$$

with δ_2 and δ_1 the momentum thickness and displacement thickness respectively, U_{TE} the local velocity at the trailing edge, U_∞ the free stream velocity and H_{TE} the shape factor at the trailing edge. If the Squire-Young model is scrutinized, we notice the resemblance of the first term to the momentum thickness expression, followed by the second term which is the extrapolation function of the momentum deficit from the TE to downstream infinity. To ensure the Squire-Young model is implementable in a CFD simulation, Coder and Maughmer (2015) formulated a closure relationship with the assumption of an isentropic process within the equivalent inviscid flowfield from a known state to the local conditions. In the absence of shock waves the far upstream conditions may be used to estimate the equivalent inviscid density:

$$\rho_I = \left[\frac{\rho_\infty^Y}{\rho_\infty} \right]^{\frac{1}{Y}} \quad \dots 2.8.2$$

With the closure relationship defined, the displacement thickness, momentum thickness and shape factor may now be defined as:

$$\delta_1 = \int_0^\infty \left(1 - \frac{\rho U}{\rho_I U_I} \right) dy \quad \dots 2.8.3$$

$$\delta_2 = \int_0^\infty \frac{\rho U}{\rho_I U_I} \left(1 - \frac{U}{U_I} \right) dy \quad \dots 2.8.4$$

$$H = \frac{\int_0^\infty \left(1 - \frac{\rho U}{\rho_I U_I} \right) dy}{\int_0^\infty \frac{\rho U}{\rho_I U_I} \left(1 - \frac{U}{U_I} \right) dy} \quad \dots 2.8.5$$

2.9. PRESSURE AND SPURIOUS DRAG ERRORS

The pressure drag component is a result of the imbalance between the pressure forces acting in the drag direction ($C_{p \cdot n_x} > 0$), and the suction forces acting in the thrust direction ($C_{p \cdot n_x} < 0$). Generally the suction is underestimated, leading to an overestimation of the pressure drag (Esquieu, 2007). Very small numerical deviations on the pressure distribution will have a large unwanted effect on the pressure drag distribution, which in the near-field method adds to the artificial drag component. Spurious drag is an artificial drag source which does not arise in experimental calculation and leads to an overestimation of the total drag. Spurious drag can be ascribed to the following reasons:

- Poor mesh discretization or quality of the mesh
- Numerical scheme errors and truncation errors inherent to CFD codes
- Far-field boundary condition setting

The main problem with the near-field method is that it cannot distinguish spurious drag contributions to the physical drag contributions, because the numerical errors are implicitly contained in the aerodynamic variables used for integral calculation. The far-field method provides a method to decompose the physical drag contributions and identify and eliminate the production of spurious drag. By identifying the spurious drag via far-field integration, the aerodynamicist can use this parameter as a quantification method to justify the feasibility of the mesh used in the near-field solution.

Esquieu (2007) investigated the production of spurious drag which leads to the overestimation of pressure drag. Part of this study was to investigate the drag prediction of the near-field and far-field method for an AS28 wing/body configuration. The simulation was done under transonic conditions with the structured ONERA-elsA code. Two meshes were used in this study, namely a structured fine mesh and a patched grid mesh.

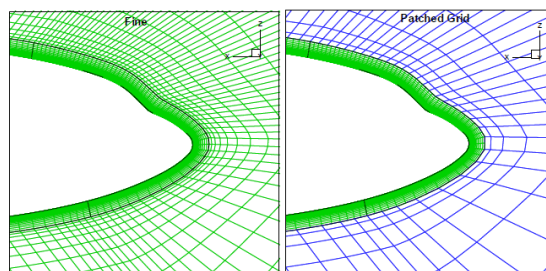


Figure 7: Fine and Patched Grids (Esquieu, 2007)

The near-field and far-field drag values as obtained by Esquieu can be seen in **Table 4:**

	CL	CD_p	CD_f	$CD_p + CD_f$
Fine	0.549	194.3	96.6	290.9
PG	0.547	203.5	96.4	299.9

	CD_w	CD_v	CD_i	CD_{ffd}	CD_{sp}
Fine	26.0	149.7	110.1	285.8	5.1
PG	25.0	150.0	107.9	282.9	17.1

Table 4: Near-field and far-field drag values (Esquieu, 2007)

Esquieu found that the near-field drag value is 9 drag counts higher in the patched grid case compared to that of the fine grid case. This was due to the over prediction of the pressure drag. The far-field drag breakdown shows different levels of spurious drag for the fine and patched grid case with 5.1 drag counts for the fine grid and 17.1 for the patched grid case. Esquieu concluded that the near-field drag values are much more dependent on grid refinement than the far-field drag values. He also stated that in general spurious drag is produced around stagnation points in the flow field, typically at the leading edge, fuselage nose and nacelle tips where the pressure gradients are very high and the grids are not refined enough to capture the actual curvature. Because of the inadequate meshing scheme, the surface pressure exerted on the body is modified and the equilibrium between the front and rear parts of the aerodynamic configuration is slightly changed. This leads to different pressure drag values obtained from solutions with different grid refinements.

2.10. NUMERICAL DIFFUSION

Historically surface integration or near-field method was met with difficulties, especially for the complex configuration, due to the need to approximate the curved surfaces of the body with flat faces. This can cause significant errors introduced by the numerical viscosity and discretization error of the numerical solution (Wang, Wang, Liu, & Jiang, 2018). To overcome this problem researches have begun to look at the experimental wake integral method, derived from the momentum equations of fluid mechanics (Dam, 1999). The main advantage of this method is that there is no need for detailed information on the surface geometry of the configuration (Zhu, Wang, Liu, & Liu, 2007).

The limiting factor of the far-field application to CFD codes is the existence of numerical diffusion. Numerical diffusion results in gradual smearing of the wake further away from the

trailing edge of the simulation geometry and is either explicitly or implicitly incorporated in CFD codes and discretization schemes to yield higher solution stability. This smearing of the wake reduces the wake interaction with the Trefftz plane and can arise to spurious drag in the far-field solution. Therefore it is necessary to implement counteractive measures to ensure that the wake strength is justifiable to, and representative of real flow when it reaches the Trefftz plane.

One of the methods to reduce the presence of numerical dissipation in the wake is to reduce the magnitude of the truncation error associated with the discretization. This is achieved by using a higher order discretization scheme in the downstream wake of the aerodynamic body. Although this is an effective solution, it will also lead to a significant increase in computational cost and simulation duration.

The second method to counteract the effects of numerical diffusion is to place the Trefftz plane closer to the trailing edge of the simulation geometry. This will ensure that the flow variables used for far-field integration are extracted before the wake becomes artificially smeared. It is, however, important to note that if the Trefftz plane is placed too close to the trailing edge, the trailing edge vortices may still be developing and gradients along the free-stream direction may be significant. A general rule is to place the Trefftz plane far enough downstream so that the flow is no longer varying in the free-stream direction.

2.11. NUMERICAL INTEGRATION MEHTODS

As the entire research thesis balances on the shoulders of the precision of the integration method applied, it is of high importance to study the most applicable numerical integration methods. For the reason that an unstructured meshing scheme is used in this thesis and that the wake-plane is explicitly defined in the downstream of the aerofoil, the data set accumulated for the pressure and velocity distribution on the defined wake-plane is spaced non-uniformly. The numerical integration process that is used must thus be able to accommodate such unequally spaced data-sets.

Generally speaking, the process of numerical integration is an approximation of the area below a curve. The most common methods for numerical integration are interpolatory, which means they are derived from the strategy of interpolating a function by a polynomial from the nodes (datasets) of the function (Runborg, 2013). Due to the nature of the way Star CCM+ structures its mesh grid, unequally spaced data will ultimately be obtained. This means that

numerical integration methods derived for non-uniform grid spacing, i.e. the trapezoidal and Simpson rule, will be discussed.

2.11.1. TRAPEZOIDAL RULE FOR INTEGRATION

The trapezoidal rule estimates the area under a curve by approximating the region under the function as a trapezoid. Another way of defining the trapezoidal rule is to obtain the integrant by using the averaged left and right Riemann sums. To increase the accuracy of the general trapezoid rule one can partition the integration interval. This means applying the trapezoidal rule to each subinterval on the function or data set under scrutiny and then summing the result. This method of application of the trapezoidal rule is referred to as the composite trapezoid rule (Atkinson, 1989). To formulate the trapezoidal rule for a non-uniform grid, a partition $\{x_k\}$ on interval $[a, b]$ is defined such that $a = x_0 < x_1 < \dots < x_{N-1} < x_N = b$. Δx_k is defined as the length of the $k - th$ subinterval that is $\Delta x_k = x_k - x_{k-1}$. The composite trapezoid rule for unequal spacing thus becomes:

$$\int_a^b f(x)dx \approx \sum_{k=1}^N \frac{f(x_{k-1}) + f(x_k)}{2} \Delta x_k \quad \dots 2.11.1$$

Equation 2.11.1 converges to the analytical solution as the resolution of the partition increases, that is as the length of the subinterval decreases (Weideman, 2002).

2.11.2. SIMPSON RULE FOR INTEGRATION

Simpson's rule is a numerical method that approximates the value of a definite integral by using quadratic polynomials. Simpson's rule will provide an adequate approximation if the function being integrated is smooth and non-oscillatory over the entire interval $[a, b]$. If the function that is being integrated is not smooth or highly oscillatory over the interval $[a, b]$ or the function lacks derivatives at certain points, then the composite Simpson's rule approach needs to be taken.

With the composite Simpson's rule the interval $[a, b]$ is divided into small subintervals. The Simpson's rule is then applied to each subinterval, and the resultant of each subinterval calculation is summed to produce an approximation of the integrant over the entire interval $[a, b]$. To define the composite Simpson's rule for unequal spacing we assume three points are given: $\{(-h, f(-h)); (0, f(0)); (k, f(k))\}$. Using divided differences Cartwright (2015) showed that.:

$$I = \int_{-h}^k f(x)dx \approx \frac{1}{6}(h+k) \left[\left\{2 - \frac{k}{h}\right\}f(-h) + \frac{(h+k)^2}{hk}f(0) + \left\{2 - \frac{h}{k}\right\}f(k) \right] \quad \dots 2.11.2$$

2.12. CONCLUSION

As can be observed throughout the literature study, the utilization of far-field drag analysis methods such as the Squire-Young model implemented in XFOIL show great promise. Regrettably few studies have been undertaken on ways and means to implement these methods to an unstructured two dimensional CFD simulation, and it is clear that a concise method still needs to be developed. It has been illustrated, furthermore, that the Squire-Young model has been verified to a great extent by means of wind tunnel results. This method will be used as one of the basic far-field models to be implemented in Star CCM+ in order to calculate profile drag via a Trefftz plane analysis. The other methods which also showed promise, and which will be implemented in this thesis, are the momentum deficit, wake rake and dynamic pressure wake-rake methods. From general aerodynamic knowledge it is a well-known fact that the wake downstream in a viscous simulation tends to have a parabolic profile as seen from the y-direction, and therefore the composite Simpson's rule will be used to evaluate the integrals numerically. The procedure and methodology for evaluating the proposed far-field drag analysis in Star-CCM+ will be discussed in **Chapter 4**. It is also important to note that the simulations in Appendix C be studied before studying **Chapter 4** to **Chapter 6**. This is because in this section, the validity of the far-field methods developed are discussed and explored. The results deduced in this section are then used in **Chapter 4** to **Chapter 6** to ensure optimal practices are used when applying the far-field methods in a CFD simulation.

Chapter 3: XFOIL Accuracy Validation

This section is concerned with the validation of the accuracy of XFOIL for three different aerofoil profiles with wind tunnel results from UIUC low-speed subsonic wind tunnel conducted in 1996, 1997 and 2002 respectively (Selig & McGranahan, 2003). The tests for validation purposes were conducted between Reynolds numbers of 400 000 and 500 000, and can be regarded as low Reynolds number tests. The aerofoils under scrutiny are the E231, S834 and FX 63-137 profiles designed for small wind turbine applications.

In the validation of the accuracy of XFOIL for the three named aerofoils the best XFOIL practices are used to achieve the highest accuracy results. The literature on the subject, as expounded in Chapter 2, is unanimous on the known tendency of XFOIL to over predict lift and underpredict drag in high separation areas. Thus a simple method of bunching the panels in areas of high importance was developed and discussed (see **Appendix B**). This method differs from the traditional method of bunching panels in XFOIL, since XFOIL only uses a first order bunching algorithm. Also, in XFOIL the first order bunching algorithm can only be applied to two positions – on the top and bottom of the aerofoil. The bunching method described in **Appendix B** extends to a third order method where every continuous set of x and y coordinates of aerofoil geometry can be clustered at multiple positions. The proposed method also allows for the user to cluster panels at more than two positions, for instance in regions of high curvature, the LE and TE, at top and bottom transition locations, etc.

Both best practice methods for XFOIL simulations and the newly proposed method of panel bunching are used to achieve highest accuracy XFOIL lift and drag prediction results with respect to the experimental results of the three aerofoils.

3.1. FX 63-137 AEROFOIL WIND TUNNEL TESTS VS XFOIL SIMULATIONS

The FX 63-137 has a maximum thickness of 13.66% and a maximum camber of 5.79%. This aerofoil is mainly used for wind turbine applications. In the **Figure 8** the black curve represents the standard FX 63-137 profile and the red curve represents the curve with x and y coordinates interpolated in 1st, 2nd and 3rd order (as per **Appendix B**). An XFOIL simulation was set up with all the parameters (except the Reynolds number) set as in **Table 5**. For this case the Reynolds number was set to 499818, as this is where the UIUC conducted their wind tunnel tests. The aerodynamic coefficients were noted for angles of attack ranging from -5.08 to 5.14 degrees in increasing increments of approximately 1 degree (see **Tables 6-8**).

Parameter	Value	Unit
Critical Amplification Factor (Percentage of Turbulence Level)	0.07	%
Viscous Acceleration Parameter	0.01	dimensionless
Mach Number	0	dimensionless
Panel Bunching Parameter	1	dimensionless

Table 5: XFOIL simulation constants

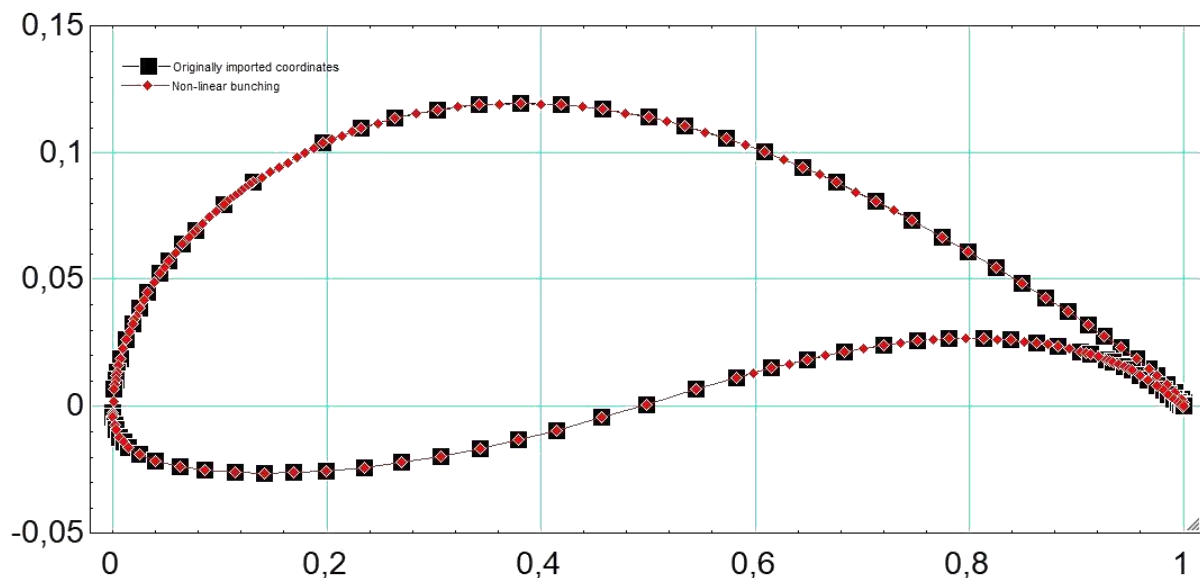


Figure 8: FX 63-137 Aerofoil standard and interpolated geometry plot

Angle of Attack [°]	Lift coefficient [dim]	Drag coefficient [dim]
-5.0800	0.2590	0.0123
-4.0900	0.3810	0.0108
-2.9800	0.5070	0.0094
-1.9600	0.6300	0.0087
-1.0500	0.7250	0.0090
0.0700	0.8410	0.0096
1.1100	0.9480	0.0100
2.1000	1.0520	0.0106
3.1700	1.1620	0.0109
4.2500	1.2740	0.0116
5.1400	1.3600	0.0124

Table 6: FX 63-137 UIUC Wind tunnel test results

Angle of Attack [°]	Lift coefficient (Wind Tunnel) [dim]	Lift coefficient (Standard XFOIL) [162 panels] [dim]	Lift coefficient (Panels bunched) [162 panels] [dim]
-5.0800	0.2590	0.2816	0.2570
-4.0900	0.3810	0.4068	0.3772
-2.9800	0.5070	0.5528	0.5097
-1.9600	0.6300	0.6798	0.6301
-1.0500	0.7250	0.7849	0.7251
0.0700	0.8410	0.9119	0.8736
1.1100	0.9480	1.0283	0.9745
2.1000	1.0520	1.1370	1.0470
3.1700	1.1620	1.2517	1.1478
4.2500	1.2740	1.3628	1.2985
5.1400	1.3600	1.4500	1.3806

Table 7: FX 63-137 Lift coefficient comparison between wind tunnel and XFOIL simulations

Angle of Attack	Drag coefficient (Wind Tunnel)	Drag coefficient (Standard XFOIL) [120 panels]	Drag coefficient (Panels Bunched) [120 panels]
[°]	[dim]	[dim]	[dim]
-5.0800	0.0123	0.0117	0.0108
-4.0900	0.0108	0.0105	0.0101
-2.9800	0.0094	0.0091	0.0099
-1.9600	0.0087	0.0085	0.0100
-1.0500	0.0090	0.0087	0.0103
0.0700	0.0096	0.0090	0.0089
1.1100	0.0100	0.0093	0.0100
2.1000	0.0106	0.0097	0.0132
3.1700	0.0109	0.0102	0.0138
4.2500	0.0116	0.0109	0.0124
5.1400	0.0124	0.0117	0.0136

Table 8: FX 63-137 Drag coefficient comparison between wind tunnel and XFOIL simulation

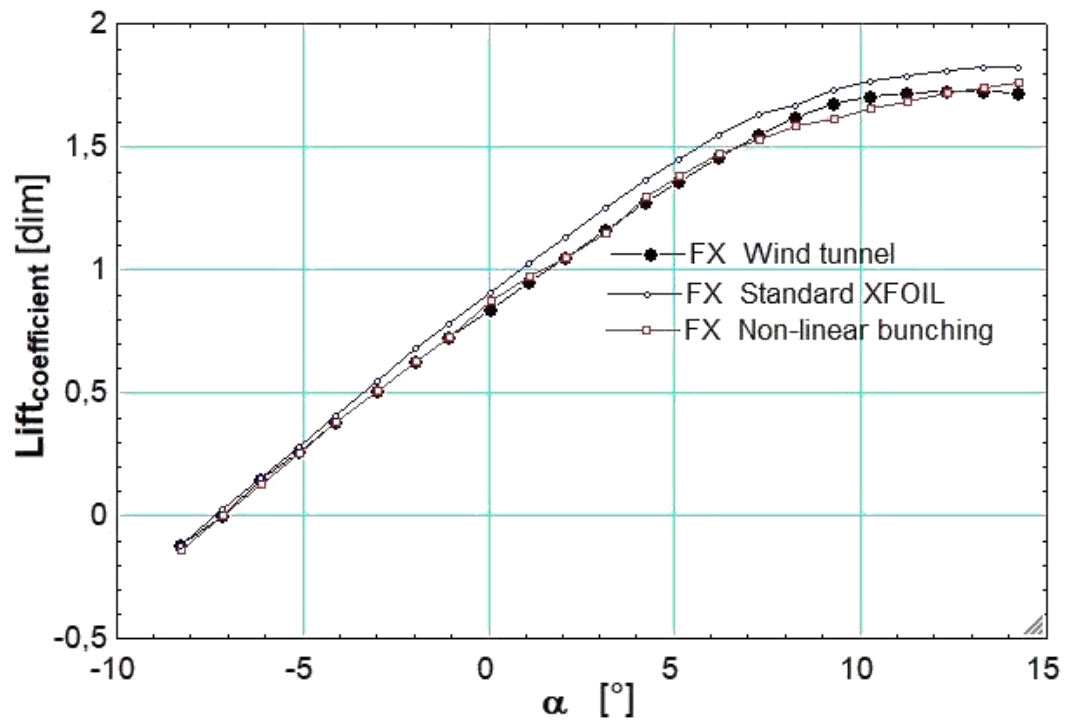


Figure 9: Lift coefficient of FX 63-137 wind tunnel, XFOIL standard and interpolation simulation

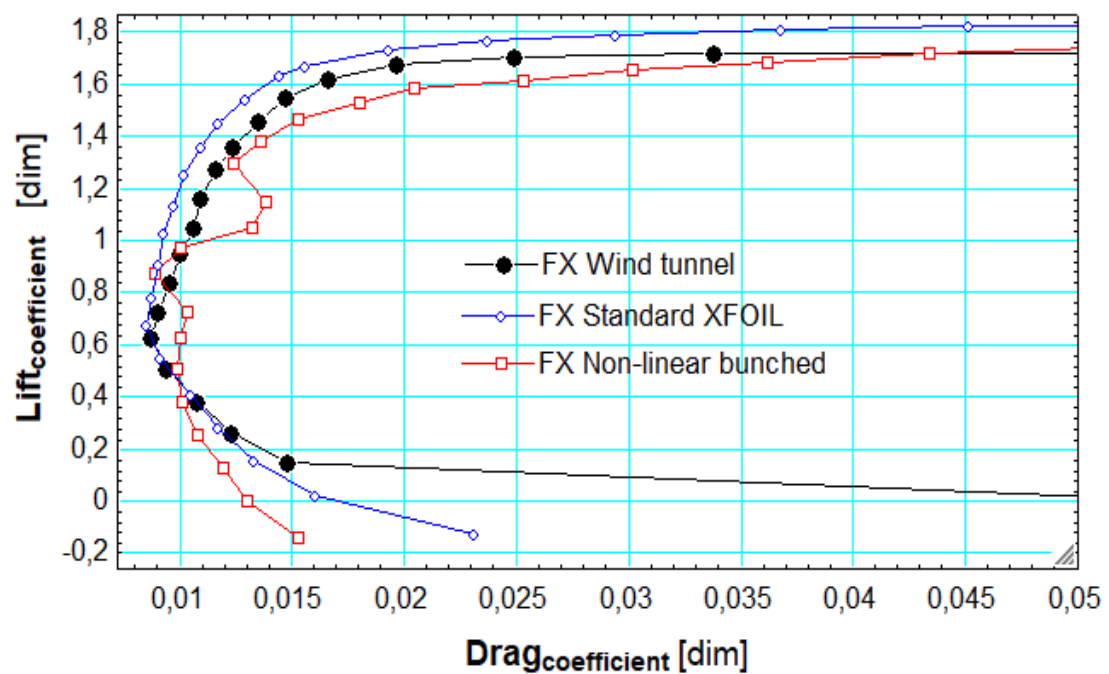


Figure 10: Cd VS Cl of FX 63-137 Wind tunnel, XFOIL standard and interpolation simulation

3.1.1. FX 63-137 XFOIL ACCURACY DISCUSSION

Table 7 and **Figure 9** respectively show that XFOIL is accurate in predicting the lift curve slope and stall angles for the simulation case of the FX 63 -137 aerofoil. It is clear that for the standard XFOIL simulation of the aerofoil under scrutiny with a Reynolds number of 499818 and 162 panel nodes the lift curve slope and maximum lift coefficient is overestimated. It will also be noted that the accuracy of the XFOIL simulation tends to degrade as separation effects increase.

The simulation of the aerofoil described by the non-linear bunched panels more densely clustered at the surface regions with high curvature tends to better predict the lift coefficient. The average absolute relative error of lift coefficient prediction of the standard XFOIL simulation w.r.t the wind tunnel results is 7.91% whereas this value for the non-linear XFOIL simulation case is 1.29%. As can be seen in **Figure 9**, the lift coefficient prediction (for this non-linear bunched panel case) under predicts the lift from 7 to 10 degrees and over predicts from 0 to 2 degrees. For the rest of the AOA simulation spectrum the non-linear bunching case almost exactly predicts the lift coefficient with greatly improved accuracy in the post stall regions.

For the drag coefficient VS lift coefficient plot in **Figure 10** and **Table 8** it can be seen that the standard XFOIL simulation agrees more with the benchmark results than in the case of the non-linear bunching routine. For the standard simulation case the average absolute relative error of the drag coefficient is within 5.12% of the benchmark results and for the non-linear case this value is within 11.67% of the benchmark results.

It is also evident that the standard XFOIL simulation tends to greatly overestimate the lift and underestimate the drag as separation increases. The non-linear bunched simulation case tends to underestimate the lift and overestimate the drag as separation increases, but with greater precision in post stall regions where separation effects become more adverse.

3.2. E231 AEROFOIL WIND TUNNEL VS XFOIL

The E231 aerofoil has a maximum thickness of 12.34% and a maximum camber of 2.46%. The experimental lift and drag coefficients for this aerofoil were evaluated by Professor Michael Selig and his team with the UIUC wind tunnel facilities and published in “Summary of Low-Speed Airfoil Data – Volume 3” (Lyon, Broeren, Giguere, Gopalarathnam, & Selig, 1998). This is the third instalment in a series of books documenting the on-going work of the University of Illinois at Urbana-Champaign Low-Speed Aerofoil Tests (UIUC LSATs). The tests were conducted at a Reynolds number of 400400 and the results are displayed in **Table 9**. Once again these wind tunnel test results will be used as reference against which the XFOIL simulation accuracy will be evaluated. The XFOIL simulation was run with the constants as in **Table 5** and Reynolds number of 400400. Two simulations were run with angles of attack ranging from -4.35 to 4.78 in increments of approximately 1 degree. The first simulation was a standard XFOIL simulation with 228 linear panels. In the second simulation 228 panels were also used, but the bunching algorithm described in **Appendix B** was used to bunch the panels at areas of interest. The results of both methods are presented in **Figure 11**:

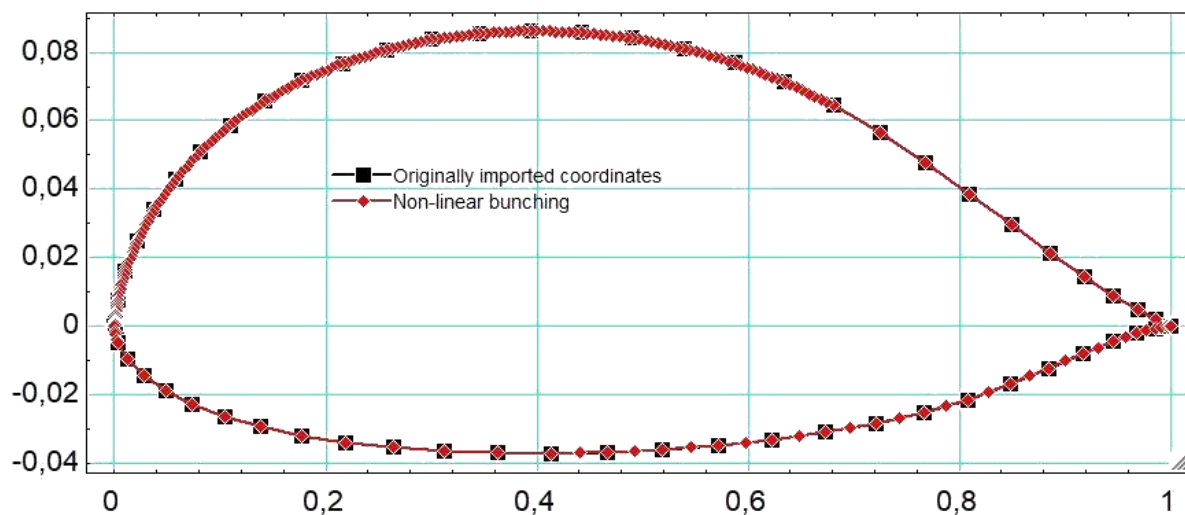


Figure 11: E231 Aerofoil standard and interpolated geometry plot

Angle of Attack [°]	Lift coefficient [dim]	Drag coefficient [dim]
-4.5300	-0.3020	0.0146
-3.4800	-0.2030	0.0133
-2.4700	-0.1100	0.0113
-1.4400	-0.0340	0.0089
-0.3400	0.0640	0.0094
0.6400	0.1820	0.0097
1.6900	0.3140	0.0088
2.6600	0.4390	0.0084
3.7500	0.5890	0.0093
4.7800	0.7240	0.0085

Table 9: E231 Wind tunnel test results

Angle of Attack	Lift coefficient	Lift coefficient	Lift coefficient
	(Wind Tunnel)	(Standard XFOIL) [228 panels]	(Bunched Panels) [228 panels]
[°]	[dim]	[dim]	[dim]
-4.53	-0.302	-0.1944	-0.18
-3.48	-0.203	-0.0985	-0.1021
-2.47	-0.11	-0.04	-0.0469
-1.44	-0.034	0.0615	0.0529
-0.34	0.064	0.2131	0.1986
0.64	0.182	0.3704	0.3547
1.69	0.314	0.517	0.4912
2.66	0.439	0.6395	0.6223
3.75	0.589	0.7716	0.7568
4.78	0.724	0.8917	0.8643

Table 10: E231 Lift coefficient comparison between wind tunnel and XFOIL simulations

Angle of Attack	Drag coefficient	Drag coefficient	Drag coefficient
	(Wind Tunnel)	(Standard XFOIL)	(Bunched Panels)
		[228 panels]	[228 panels]
[°]	[dim]	[dim]	[dim]
-4.53	0.0146	0.0147	0.01573
-3.48	0.0133	0.01215	0.01288
-2.47	0.0113	0.00852	0.00881
-1.44	0.0089	0.00931	0.00978
-0.34	0.0094	0.01007	0.01034
0.64	0.0097	0.01008	0.01018
1.69	0.0088	0.00979	0.00985
2.66	0.0084	0.0094	0.00921
3.75	0.0093	0.00907	0.009
4.78	0.0085	0.00924	0.01025

Table 11: E231 Drag coefficient comparison between wind tunnel and XFOIL simulations

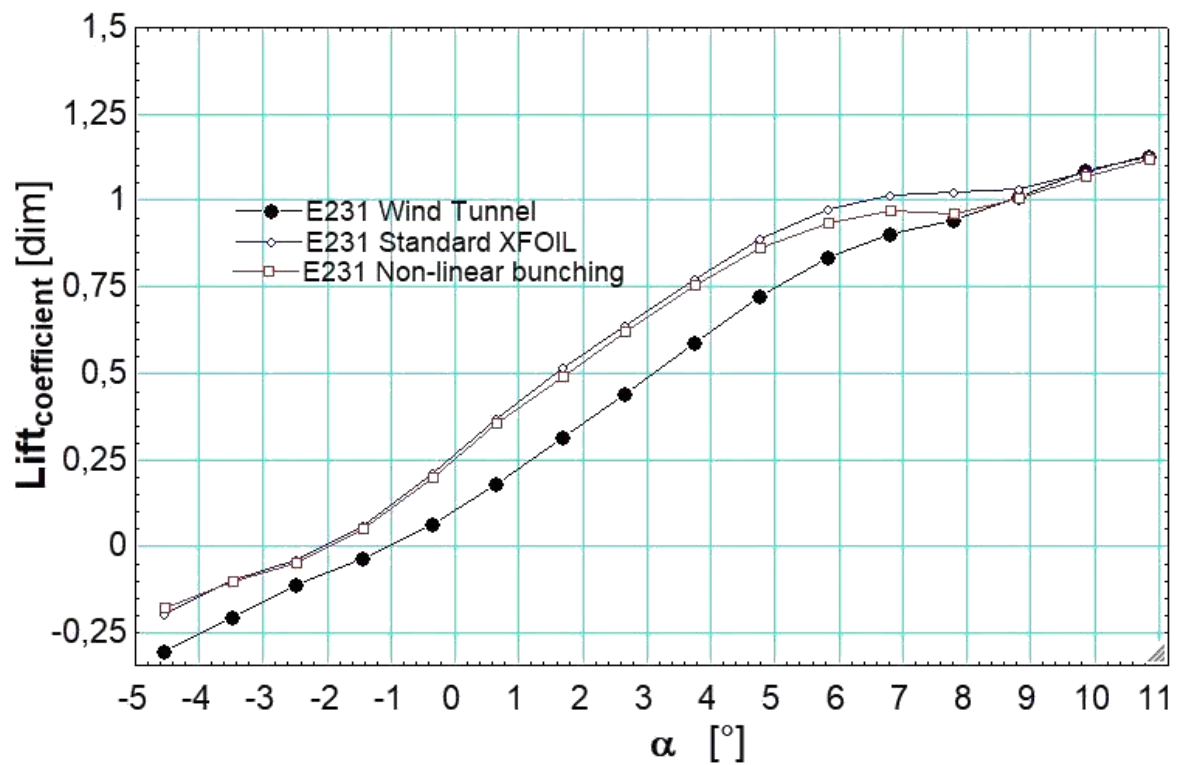


Figure 12: Lift coefficient of E231 wind tunnel, XFOIL standard and interpolation simulation

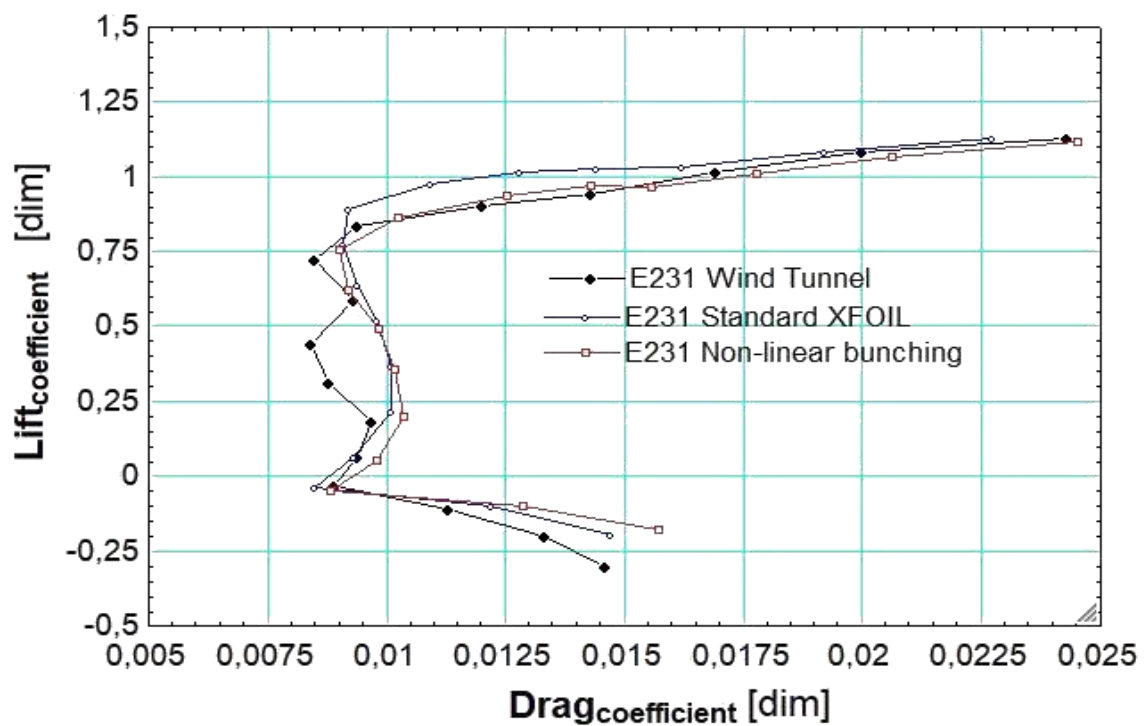


Figure 13: Cd VS Cl of E231 wind tunnel, XFOIL standard and interpolation simulation

3.2.1. E231 XFOIL ACCURACY DISCUSSION

If the lift curve slope presented in **Figure 12** is thoroughly studied, it can be seen that the standard XFOIL simulation does not agree with the benchmark lift coefficient results. This can also be said for the case of the non-linearly bunched simulation. For the standard simulation case the average absolute relative error of the drag coefficient is within 8.39 % of the benchmark results and for the non-linear case this value is within 10.32 % of the benchmark results.

The best lift results were found when setting the critical envelope factor to 6, however the lift results shown in figure 12 is for a critical envelope factor of 9. In this thesis we are mainly concerned with drag results. It was therefore decided to keep the envelope factor at 9 because this is where the best drag results were found and.

Studying **Figure 13**, it is evident that as separation effects strengthen as higher angles of attack are simulated, so the accuracy of the non-linear bunching case increases. However, in regions of angles of attack reaching the minimum of -4.53 degrees, so the accuracy of the non-linear bunching case decreases with the standard simulation triumphing over the non-linear bunching case. The slightly reduced accuracy in regions of low angles of attack, and improved accuracy in regions of high angle of attack for the non-linear bunched case, is due to the fact that one bunching method is used for both high and low angles of attack.

This means that if better predictions are required for negative angles of attack the bunching should change accordingly for this case, and then again be adjusted for the higher angles of attack. This will ensure that the separation bubble is captured more precisely and therefore it will lead to more accurate results..

3.3. S834 AEROFOIL WIND TUNEL VS XFOIL

The S834 aerofoil has a maximum thickness of 15% and a maximum camber of 1.63%. This aerofoil is designed to be quiet with natural-laminar-flow for application in one to three meter diameter variable pitch horizontal-axis wind turbines. The two primary objectives of this aerofoil are high maximum lift and low sensitivity to surface roughness accompanied with reduced profile drag (Somers, 2005). In 2002-2003 Selig and his team analysed the aerodynamic characteristics of this aerofoil at various Reynolds numbers in the UIUC wind tunnel (Selig & McGranahan, 2003). The wind tunnel test results at a Reynolds number of 499497 are displayed in **Table 12**, and have been used as the basis on which the XFOIL accuracy with which this study is concerned will be measured. **Figure 14** displays the standard geometry and non-linear bunched geometry of the S834 aerofoil used in our XFOIL simulations.

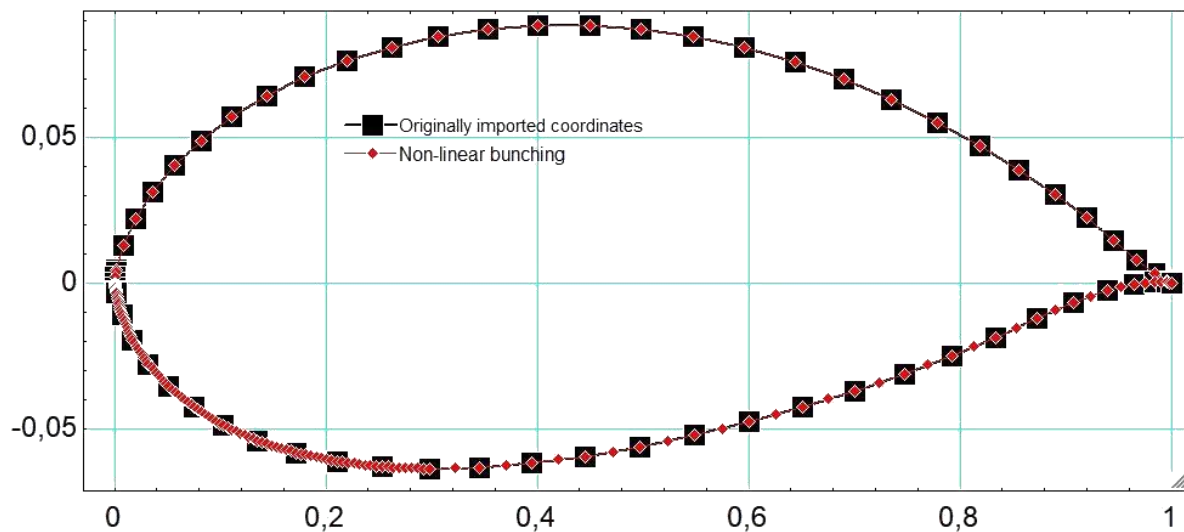


Figure 14: S834 Aerofoil standard and interpolated geometry plot

AOA [°]	CL [dim]	CD [dim]
-5.060	-0.276	0.011
-4.000	-0.173	0.010
-3.010	-0.073	0.009
-1.960	0.039	0.009
-0.980	0.140	0.008
0.070	0.246	0.008
1.040	0.345	0.008
2.140	0.461	0.008
3.150	0.566	0.008
4.170	0.662	0.009
5.200	0.747	0.009

Table 12: S834 Aerofoil wind tunnel test results

Angle of Attack	Lift coefficient	Lift coefficient	Lift coefficient
	(Wind Tunnel)	(Standard XFOIL) [182 panels]	(Bunched Panels) [182 panels]
[°]	[dim]	[dim]	[dim]
-5.06	-0.276	-0.3038	-0.2906
-4	-0.173	-0.1879	-0.1803
-3.01	-0.073	-0.0787	-0.0862
-1.96	0.039	0.0406	0.0263
-0.98	0.14	0.1456	0.1411
0.07	0.246	0.2635	0.2637
1.04	0.345	0.3661	0.3652
2.14	0.461	0.4795	0.4788
3.15	0.566	0.5742	0.5835
4.17	0.662	0.6582	0.6532
5.2	0.747	0.7167	0.7578

Table 13: S834 Lift coefficient comparison between wind tunnel and XFOIL simulations

Angle of Attack	Drag coefficient	Drag coefficient	Drag coefficient
	(Wind Tunnel)	(Standard XFOIL) [174 panels]	(Bunched Panels) [174 panels]
[°]	[dim]	[dim]	[dim]
-5.06	0.0108	0.01047	0.01132
-4	0.0099	0.00935	0.00991
-3.01	0.0091	0.00851	0.00904
-1.96	0.0085	0.00784	0.00824
-0.98	0.0081	0.00753	0.00776
0.07	0.0078	0.00738	0.00754
1.04	0.0077	0.00744	0.0075
2.14	0.0078	0.00771	0.00771
3.15	0.0081	0.00801	0.00794
4.17	0.0085	0.00835	0.0082
5.2	0.0094	0.00979	0.00892

Table 14: S834 Drag coefficient comparison between wind tunnel and XFOIL simulations

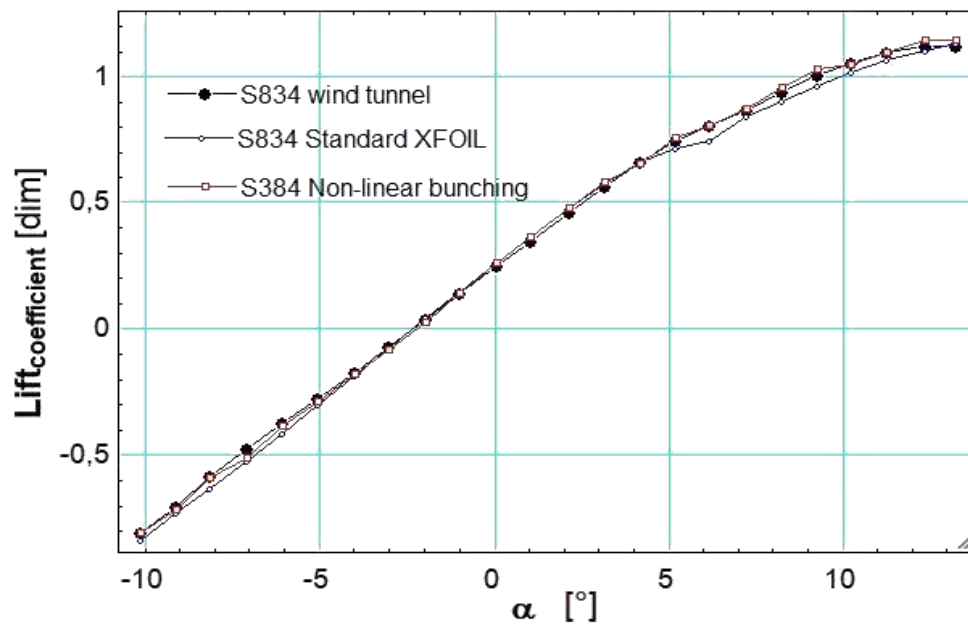


Figure 15: Lift coefficient of S834 wind tunnel, XFOIL standard and interpolation simulation

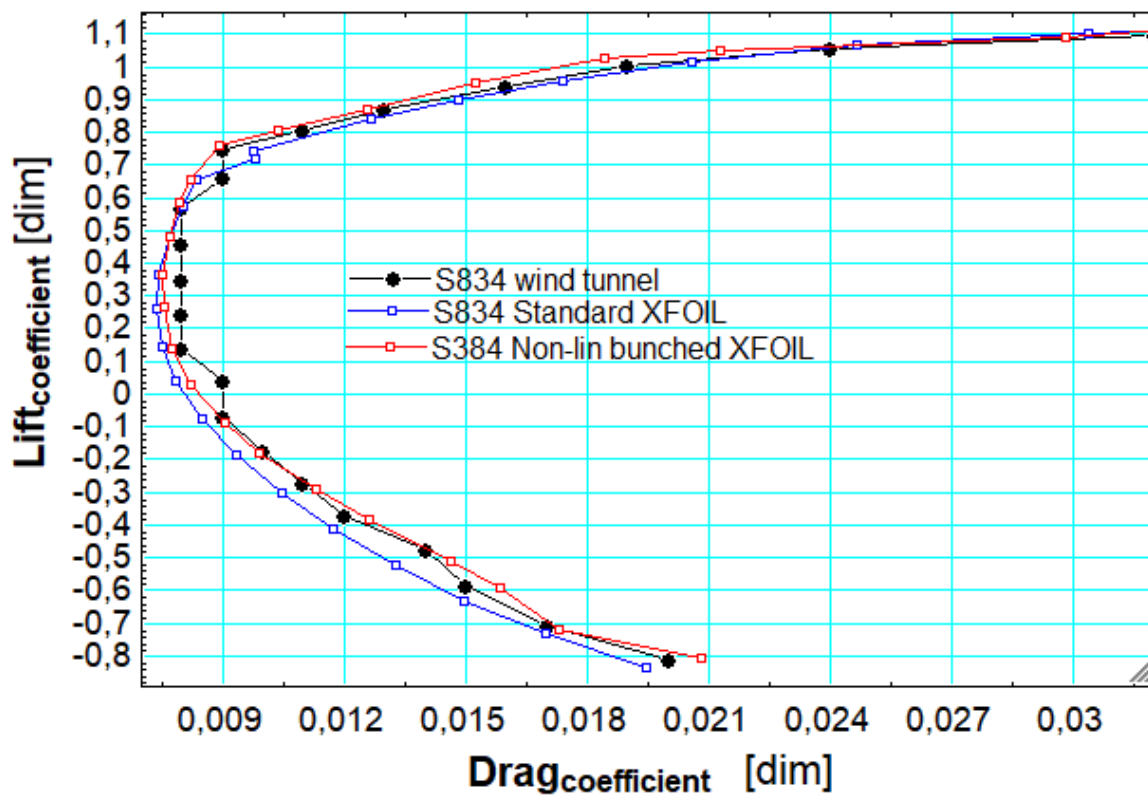


Figure 16: Cd VS Cl of S834 wind tunnel, XFOIL standard and interpolation simulation

3.3.1. S834 XFOIL ACCURACY DISCUSSION

Studying the wind tunnel test results, standard XFOIL simulation, and interpolated XFOIL simulation results in the figures of **Section 3.3** for the lift and drag coefficient of the S834 aerofoil with 182 panels and a Reynolds number of 499497 the following was noted:

- The average absolute relative error of lift coefficient prediction of the standard XFOIL simulation w.r.t the wind tunnel results is 5.27% whereas this value for the non-linear XFOIL simulation case is 7.61%. For the standard simulation case the average absolute relative error of the drag coefficient is within 4.26% of the benchmark results and for the non-linear case this value is within 2.78% of the benchmark results.
- **Figures 15–16** show improved lift and drag predictions for the case of the non-linear bunched simulation– especially in regions where more adverse separation exist.
- For the S834 aerofoil with a Reynolds number of 499497, both standards XFOIL and the non-linear bunched XFOIL simulations show highly accurate lift and drag coefficient predictions in the linear range of AOA w.r.t benchmark results.

3.4. CONCLUSION

With the wind tunnel test results of the S834, E231 and FX 63-137 aerofoils now reproduced with the XFOIL standard and interpolated geometry simulations, it was observed that for a Reynolds number between 4×10^5 and 5×10^5 , both methods yield desirable results in predicting the lift and drag coefficients. In the case of the three aerofoil simulations it was noted that the interpolated geometry simulation triumphs the standard simulation in the lift coefficient calculation, but slightly loses efficiency with regard to the standard simulation in drag coefficient predictions. All in all it is clear from the C_l VS C_d plots in **Chapter 3** that the interpolated simulation offers the best overall predictions. However, the time spent to accurately interpolate the data before importing it to XFOIL with the proposed method is a rather tedious process compared to the C_l and C_d prediction improvements it yields. It has therefore been decided that for the comparison of the XFOIL accuracy with the CFD accuracy in the aerodynamic coefficient calculations the standard XFOIL simulations will be used as wind tunnel data. From the simulations in **Chapter 3** it can be concurred that the highest XFOIL accuracy is in the AOA range between -5 and 5 degrees. This is usually where the least flow separation occurs for a Reynolds number between 4×10^5 and 5×10^5 with aerofoil max thickness between 12% and 15% and max camber between 1% and 6%.

Chapter 4: Far-Field Drag Analysis

Algorithm and Procedure

4.1. ALGORITHM & PROCEDURE FOR FAR-FIELD DRAG ANALYSIS IN STAR CCM+

It is of utmost importance **Figure 17** be studied in detail to understand the far-field procedure used in this thesis. Equally as important is to study Appendix C, Appendix E and Appendix F in great detail. Here all numerical investigations and simulations show how the performance of the far-field method is influenced by parameters such as wake width, location and length of Trefftz plane, mesh size and numerical integration techniques. These findings are then used in Chapter 5 and Chapter 6 to ensure the best performing far-field results are obtained.

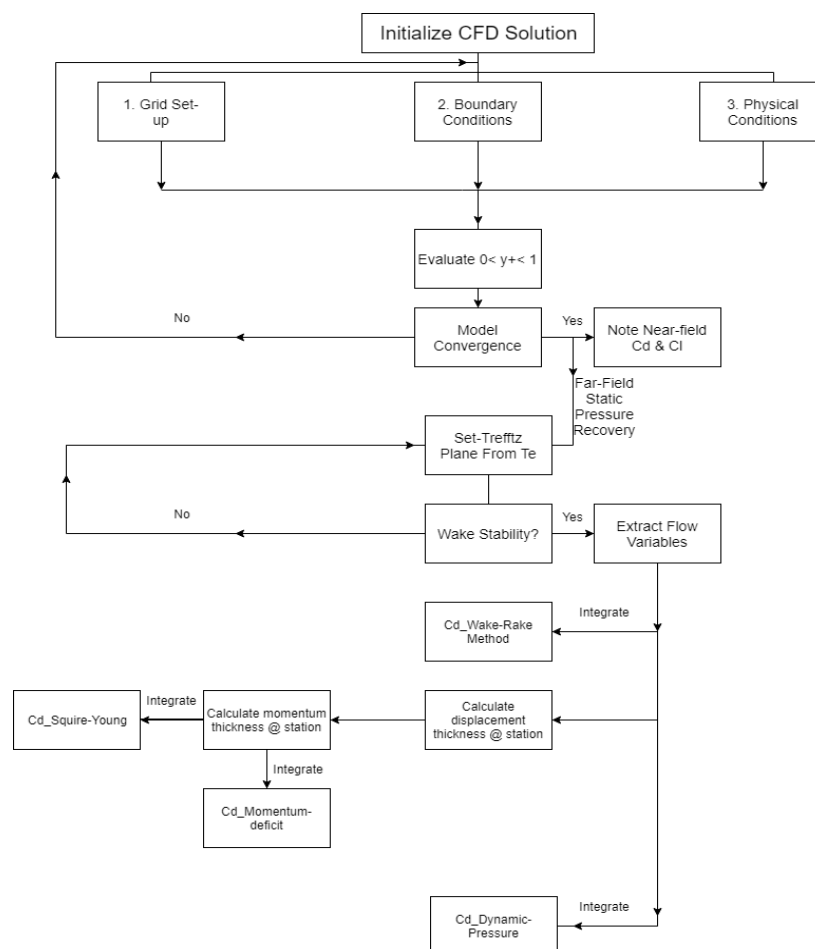


Figure 17: Algorithm & Procedure for far-field drag analysis in Star CCM+

Referring to the above figure which depicts the methodology of how the near-field simulation is set-up to extract flow variables for far-field drag calculation, the following is highlighted:

- The near-field simulation is set up with a feasible grid (which means that a grid convergence study is implemented).
- The grid should provide a y^+ between 0 and 1 (as per requirements of transition model $\gamma - Re_\theta$).
- The wake mesh is set up very densely with fine meshers to avoid numerical dispersion.
- The Trefftz plane is set up in the downstream of the simulation geometry.
- The Trefftz plane should be set where total static pressure recovery is visible.
- Flow variables are extracted according to the calculated viscous region of the wake:
 - The viscous region can be estimated by evaluating the following condition : $T_1 < \frac{\mu_{turbulent} + \mu_{laminar}}{\mu_{laminar}} < T_2$
 - Where T_1 and T_2 represent the upper and lower threshold for the turbulent/laminar sensor.
- The flow variables are used in **Equations 4.1.1 – 4.1.5**, displayed below, and are integrated with the composite Simpsons 1/3 rule with an Excel sheet.
- The Trefftz plane is moved and the threshold values are changed in the first few iterations to calibrate the method for the general application for all AOA cases used.

$$D_{Wake_{Wake}} = \int_{-\frac{L_w}{2}}^{\frac{L_w}{2}} ((\rho * V_0) * (V_0 - V_\infty)) dy \quad \dots 4.1.1$$

$$D_{Momentum_{Deficit}} = 2 \int_{-\frac{L_w}{2}}^{\frac{L_w}{2}} \left(\left(\frac{V_0}{V_I} \right) * \left(1 - \frac{V_0}{V_I} \right) \right) dy \quad \dots 4.1.2$$

$$H_{Shape_{Factor}} = \frac{\int_{-\frac{L_w}{2}}^{\frac{L_w}{2}} \left(1 - \frac{V_0}{V_I} \right) dy}{\int_{-\frac{L_w}{2}}^{\frac{L_w}{2}} \left(\left(\frac{V_0}{V_I} \right) * \left(1 - \frac{V_0}{V_I} \right) \right) dy} \quad \dots 4.1.3$$

$$C_{dynamic_{pressure}} = \frac{L_w}{C} - \frac{1}{q_\infty C} \int_{-0.5L_w}^{0.5L_w} q dy \quad \dots 4.1.4.$$

$$c_{dSquireYoung} = 2 \left(\frac{\int \frac{L_w}{2} \left(\left(\frac{V_0}{V_1} \right) * \left(1 - \frac{V_0}{V_1} \right) \right) dy}{c} \right) \left(\frac{V_0}{V_\infty} \right)^{\frac{5 + \frac{\int \frac{L_w}{2} \left(\left(\frac{V_0}{V_1} \right) * \left(1 - \frac{V_0}{V_1} \right) \right) dy}{\int \frac{L_w}{2} \left(\left(\frac{V_0}{V_1} \right) * \left(1 - \frac{V_0}{V_1} \right) \right) dy}}{2}} \right) \dots 4.1.5$$

4.2.1. SIMULATION PROCEDURE: MESH SET-UP AND Y^+ VALUE

As this thesis only focuses on the validation of XFOIL, near-field and far-field drag methods for two-dimensional simulations, our simulations will be set up to represent only this case. Seeing that each aerofoil is different in terms of physical parameters, a convergence study needs to be done for each case to such a point where the converged results become independent of boundary size. Such a study has indeed been deployed and it was found that the general mesh parameters for aerofoils with 1 meter chord length and a maximum thickness between 11-16% with Reynolds number in the order of 400 000 to 500 000 can be set as follows (**Tables 15 -17**):

Property	Value
Mesher type	Triangular mesher
Distribution mode	Wall thickness
Near core layer aspect ratio	0.6
Surface growth rate	1.05
Number of prism layers	15
Prism layer total thickness [mm]	14

Table 15: Boundary mesh constants under assumed constraints

Property	Value	Unit
Target surface size	4	mm
Minimum surface size	2	mm

Table 16: Aerofoil surface mesh controls under assumed constraints

Property	Value	Unit
Spread angle	0.17	Radians
Isotropic size	30	mm
Wake refinement growth rate	1.05	dim

Table 17: Wake refinement mesh controls under assumed constraints

In this research thesis the SST $K - \omega$ turbulence model with $\gamma - Re_\theta$ transition model for unstructured meshes will be used. Making use of this model, it is of utmost importance to keep the y^+ value on the aerofoil surface between 0 and 1, as this has an enormous influence on the accuracy of the end results. The y^+ value is dependent on the near wall prism layer thickness (Δs), the wall shear stress (τ_{wall}), the dynamic viscosity (μ), the free stream density (ρ) and the Reynolds number (Re).

To speed up the simulation process, flat plate boundary layer theory can be used as an estimate of the initial distance of the near wall prism layer (Δs) with a chosen y^+ value of 1. The simulation can then be executed until converged and the appropriate changes can be made to the near wall distance until a sufficient y^+ value is reached. With all values except the near wall prism layer thickness known, the following equations can be used in the iterative process of calculation (Frank, 2002):

$$Re = \frac{\rho U_\infty L}{\mu} \quad \dots 4.2.1$$

$$C_f = \frac{0.026}{Re^{\frac{1}{7}}} \quad \dots 4.2.2$$

$$\tau_{wall} = \frac{C_f \rho U_{\infty}^2}{2} \quad \dots 4.2.3$$

$$U_{fric} = \sqrt{\frac{\tau_{wall}}{\rho}} \quad \dots 4.2.4$$

$$\Delta s = \frac{y^+ \mu}{U_{fric} \rho} \quad \dots 4.2.5$$

In the CFD simulation a histogram plot of the y^+ value distribution over the surface of the aerofoil will typically look like the one displayed in **Figure 18**. It is clear from this figure that the y^+ lies well below 1 and in the range between 0 and 1, which can be expected in a feasible simulation. This method of y^+ representation is chosen for its ease of assessment ability to evaluate graphically where the majority of the y^+ values are spread over the aerofoil surface. The method is also computationally inexpensive and will therefore not add additional strain on precious CPU and RAM resources.

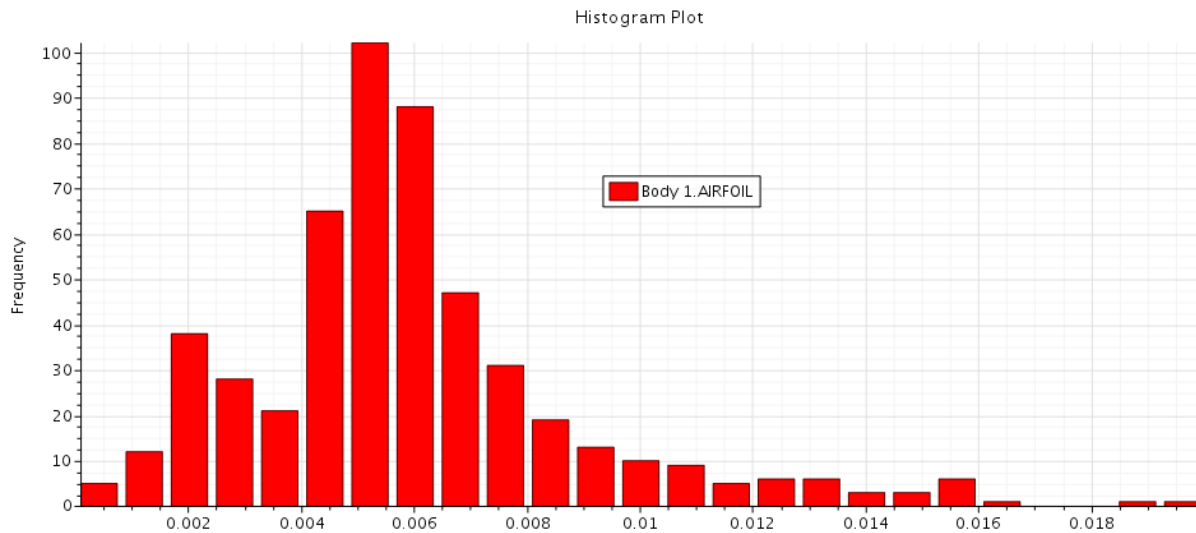


Figure 18: Acceptable y^+ values between 0 and 1 for a converged simulation

4.2.2. FAR-FIELD DRAG ANALYSIS PROCEDURE

This section contains a discussion of the methodology that will later be implemented to determine the drag of an aerofoil by virtue of a far-field analysis from data accumulated from a CFD simulation. It is important to note that for the far-field variables extracted and integrated, the procedures in **Appendix C**, **Appendix E** and **Appendix F** should be followed to ensure that the viscous region is estimated accurately.

Due to the nature of the far-field drag equations which require integration with respect to y , it is important to set up a method to extract wake data in the y -direction from the CFD simulation. To do this, a plane normal to the aerofoil in the y -direction will be created by means of the derived part selection with an offset to the leading edge in the x -direction. The input parts will be the aerofoil and the simulation domain. The choice of the origin will be a crucial tool in the wake analysis, as this distance needs to be applied as far as possible from the trailing edge to render the viscous drag effects in the far-field drag analysis negligible. This distance will be set to 7 [m] from the LE as a first iteration. As it will be demonstrated later, for a linear region of AOA with minimal separation, this position is generally where the wake stabilises in the CFD simulation for low Reynolds numbers.

Nodes	Values
 Plane @ 6C	
Origin	[7.0, 0.0, 0.0] m, m, m
Coordinate System	Laboratory
Normal	[1.0, 0.0, 0.0] m, m, m
Parts	[Airfoil_Domain, Airfoil_Domain.Airfoil_S834,
Section Mode	Single Section
Displayed Index	-1
Tags	[]
 Single section	

Figure 19: Creating a plane from a derived part for wake analysis

With the plane created, a threshold can now be set with the plane as the input part. To ensure that the data extracted from the threshold part is normal to the y -direction, the scalar field is set to the field variable Y -Centroid which is pre-defined in Star-CCM+ version 13.

This threshold will now act as a line from where all data extraction will commence. It will later be shown that the range of the threshold choice is yet again a critical tool in the far-field drag

analysis, since this is the limit which binds the integration region. Please refer to **Appendix C** and **Appendix F** in order to understand the process of selecting the bounding regions for the integration process. To establish the length of the Trefftz plane is in essence to capture the viscous wake. Numerical investigations in the appendices show how trigger functions are used to establish the viscous wake region. The python model in the appendices show how to extract the viscous wake coordinates if the entire boundary region is extracted rather than explicitly defining the Trefftz plane length as in **Figure 20**.

Nodes	Values
 Line @ 6C	
Parts	[Plane @ 6C]  
Scalar Field	Centroid[Y] 
Range	[-0.26, 0.18] m 
Mode	Between 

Figure 20: Creating a line from a derived plane for wake analysis

By carefully studying the far-field drag equations of **Section 4.1**, it is clear that defining boundaries be set for the velocity magnitude and the dynamic pressure as a function of y in the wake for integration purposes. Star-CCM+ has the advantage of using field functions to represent these quantities graphically in the direction required if the user specifies the functions and plots with care.

Figures 21-22 are examples of how a user can define the dynamic pressure via a field function and plot the data sets in the direction required respectively.

Nodes	Values
 Dynamic_Pressure	
Function Name	UserFieldFunction_2 
Type	Scalar 
Dimensions	Dimensionless 
Definition	$0.5 * (\$Velocity.mag() * \$Velocity.mag() * \$Density)$ 
Tags	[] 
Inverse Distance Weight	☐
Assembly Code	$(* 0.5 (* (* (mag \$\{Velocity\}) (mag \$\{Velocity\})) \$\{Density\}))$ 

Figure 21: Setting a field function for dynamic pressure in Star-CCM+

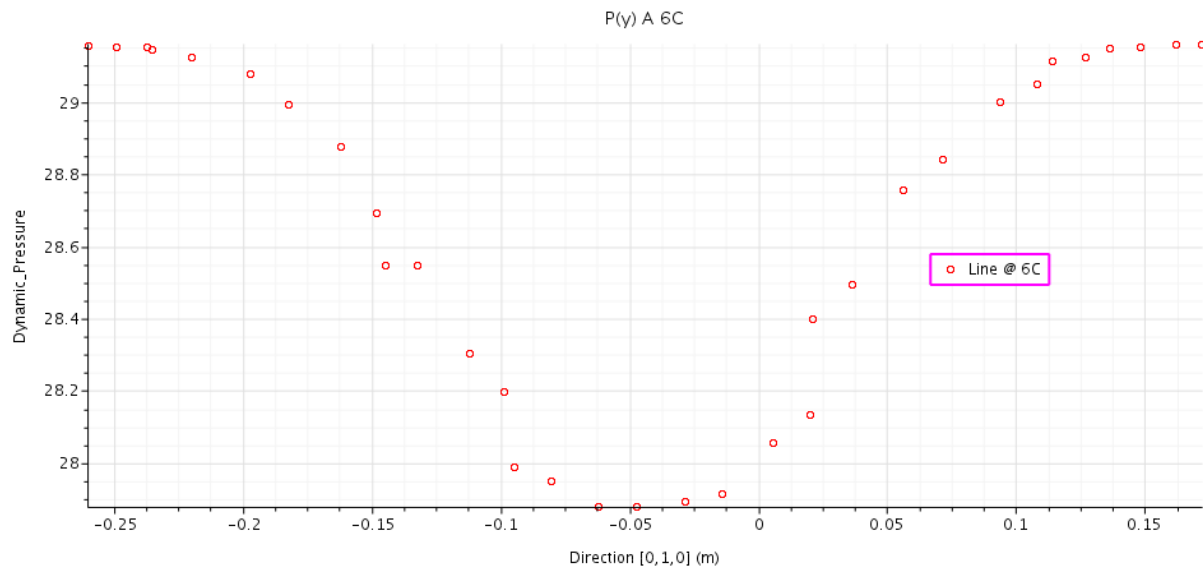


Figure 22: Dynamic pressure plot as a function of y

It can be derived from **Figure 22** that the threshold distance from the TE and the range (or wake length) are chosen adequately, because the wake is stabilized in terms of free stream to free stream position. With the velocity magnitude and dynamic pressure now displayed as a function of y , the data can be extracted and third party software such as Python or Excel can be used to sort the data in the correct ascending order and to numerically integrate the respective far-field drag equations by means of the composite Simpson's rule for unequal spacing over the wake region chosen. Please refer to **Appendix C** and **Appendix F** for clarity on this procedure.

Tabular Data		
	Line @ 6C: Direction [0,1,0] (m)	Line @ 6C: Dynamic_Pressur
0	0.135848	2.914654E+1
1	0.161493	2.915873E+1
2	0.12656	2.912348E+1
3	0.147952	2.91523E+1
4	0.171796	2.916014E+1
5	0.056039	2.875731E+1
6	0.093399	2.900117E+1
7	0.113459	2.911413E+1
8	0.035867	2.849329E+1
9	0.071131	2.884172E+1
10	0.107552	2.90479E+1

Figure 23: First 10 nodes of unsorted dynamic pressure data as a function of y

Equation 4.1.2 shows that there is a need for the inviscid local velocity. To obtain the local inviscid velocity as a function of y , the same procedure as described previously in this section will be used; this time with the physics set to inviscid. It is important to note that in ideal circumstances the viscous and inviscid simulations have to be identical in order to render the integration process feasible over the same wake lengths and data sets. This will only be true if the mesh and domain settings of both these simulations are identical, because this will imply that the same velocity magnitude calculations will be done at the same cells with different physics models, which internally will ensure that the extracted arrays are of the same length.

4.1.3. CONCLUSION

With the methodology and procedures now properly developed, it is possible to utilize the above mentioned methods to evaluate and quantify the accuracy of XFOIL drag prediction, as well as the newly implemented drag extraction tools with respect to the wind tunnel experiment data. Since this thesis is only concerned with regions with minor separation, the wind tunnel experiments, standard XFOIL drag predictions, standard CFD drag predictions and the drag prediction of the far-field analysis will be evaluated only in the AOA spectrum between -5 degrees and 5 degrees. From the wind tunnel data in **Chapter 3** it is clear that this is where the drag bucket stays somewhat linear.

The linear region of angle of attack will be used, because here the wind tunnel prediction from wake rake methods is most accurate, meaning the proposed algorithm will be justified in a region where the data accumulated has a high order of accuracy. Another reason for only evaluating the drag prediction algorithm in the linear region of angle of attack is the fact that separation inevitably increases computing time.

Chapter 5: Far-field drag analysis implementation

5.1. FX 63-137 AEROFOIL

To start the far-field drag analysis for the FX 63-137 aerofoil, the first step is to set up an accurate and converged CFD simulation using Star-CCM+ according to the methodology described in **Section 4.1**. The CAD model of the aerofoil is created in Siemens PLM NX 11 software from the aerofoil coordinate files in **Appendix D**. Unlike XFOIL, the positioning of the coordinates used to describe the aerofoil shape is not of high importance in Star-CCM+. This is because in XFOIL the coordinate density and position are used as a meshing parameter in the panel node generation and in Star-CCM+ the mesh is generated around the aerofoil surface. The level of refinement in Star-CCM+ is rather defined with tessellation density parameter. From the 3d CAD model in **Figure 24** the appropriate CFD simulation can now be set up.



Figure 24: FX 63-137 Aerofoil CAD model for Star-CCM+ simulation

The simulation domain size chosen was a 2d square with a width of 32 chord lengths and a height of 32 chord lengths with the aerofoil LE positioned in the centre of the domain. The mesh parameters were all retained as in **Section 4, Tables 15** through **17**. The prism layer near wall thickness was chosen as $1.0\text{E-}5$ meters for the first iteration. In **Figure 25** it can be seen that the chosen mesh parameter equates to a mesh more intensely clustered around the aerofoil surface and the leading and trailing edges. This will ensure that the simulation behaves in a more stable way and will yield refined wake calculations. Upon closer inspection of the Figures, it is noted that the derived plane and line is at 9 meters from the LE with a position of $[-0.59, -0.096]$ meters with respect to the y-centroid. This is the position from where the wake data will be extracted for the far-field drag analysis. In **Figure 25** it can also be seen that the downstream wake is discretised more finely than the free-stream. Here it is evident that the wake mesh has a spread angle. This is to counter the effects of numerical dispersion at the Trefftz plane.

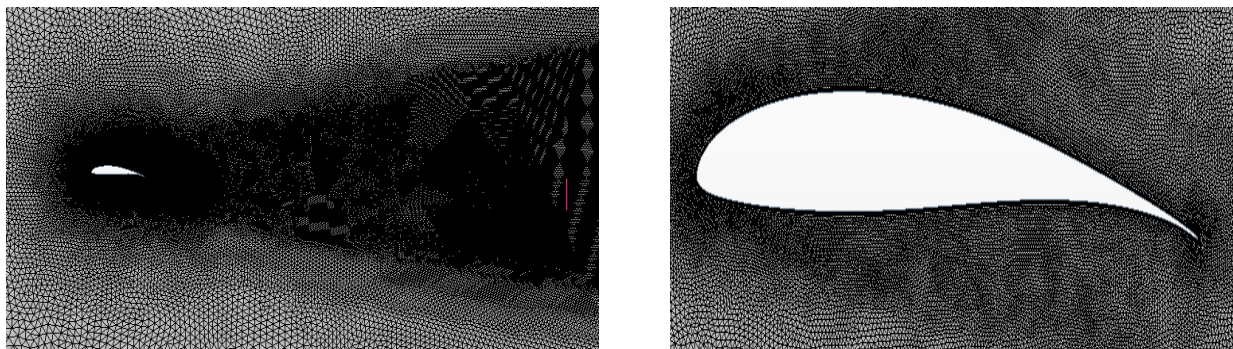


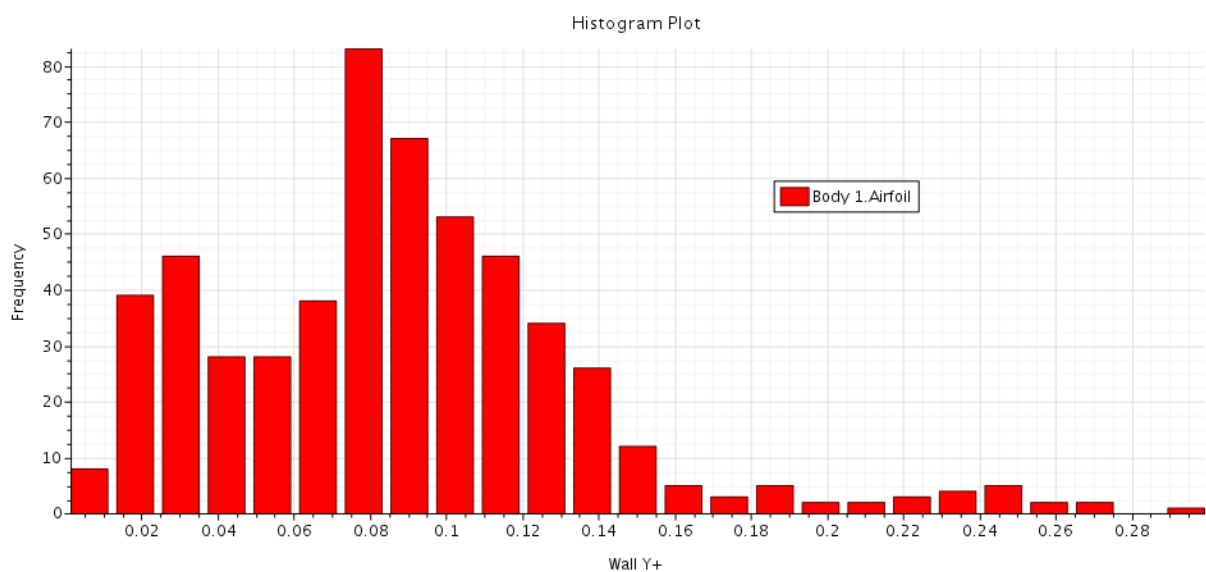
Figure 25: Dense mesh around aerofoil (left); (right) clustered mesh at LE and TE (right)

The simulation physics is set up according to **Table 18**. Therefore the Reynolds number is the same as for the wind tunnel experiment. Consequently we can quantify our converged simulation accuracy with the Selig wind tunnel results for this aerofoil (Selig & McGranahan, 2003).

Property	Value	Unit
Pressure (Gauge)	0	Pa
Free-stream velocity	6.74734821	m/s
Chord length	1	m
Density	1.283	Kg/m ³
Dynamic viscosity	1.7325E-5	Pa-s
Reynolds number	499818	dimensionless

Table 18: FX 63-137 Simulation parameters

The simulation was run at the same angles of attack at which the Selig wind tunnel experiments were performed, each time ensuring the y^+ values were in an acceptable range. Also, in each iteration the wake shape was evaluated for its feasibility for far-field drag analysis. Feasibility in this context means the capturing of the viscous wake. The parameters that were changed in each iteration to achieve an acceptable wake were the length of the plane from the LE, the wake density and the threshold line length, i.e. Trefftz plane length. These were changed to render viscous effects negligible and to achieve a well-rounded and easily integratable wake shape. In each simulation of angle of attack the wake and y^+ values were in the range of those displayed in **Figures 26-27**, with differences in the wake height, width, position and in the frequency and magnitude of the wall y^+ values.

Figure 26: Wall y^+ values for FX 63-137 simulation at AOA of 4.25

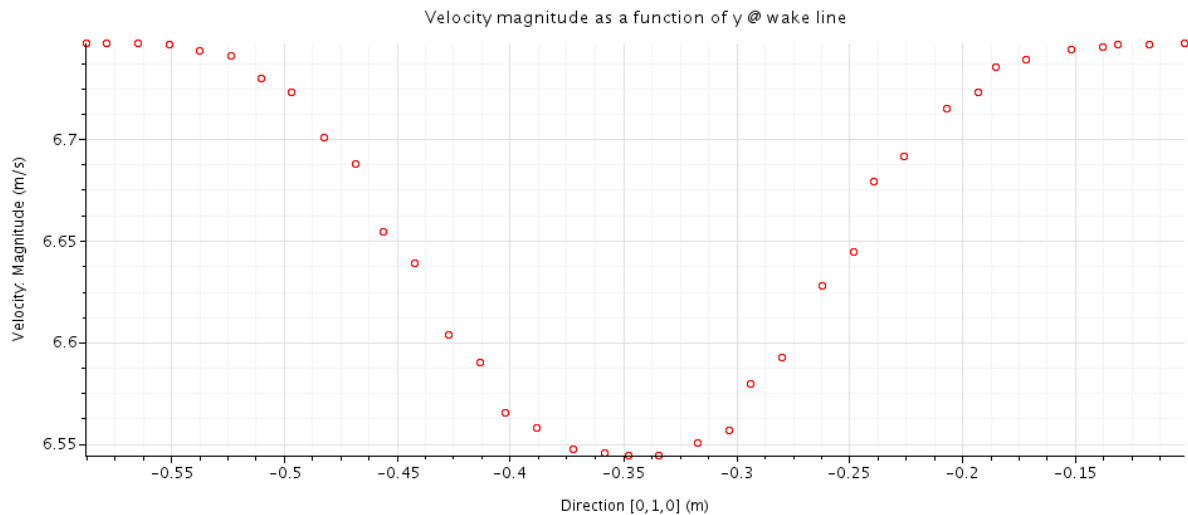


Figure 27: $V(y)$ values for FX 63-137 simulation at AOA of 4.25

To evaluate the feasibility of the simulation please refer to **Table 19** and **Figure 28**, which represent the converged results for the lift coefficient from the CFD simulation. It is evident that as separation increases so does the accuracy of the CFD simulation decay with respect to the wind tunnel results (higher AOA, larger relative error). However, it is also clear that the largest error between the CFD lift coefficient and wind tunnel lift coefficient is in the order of 0.037 [dim] at AOA 5.14. This leads to the conclusion that in the range of angles of attack the simulation yields lift coefficient predictions in the desired range when compared to the benchmark results.

Angle of Attack	Lift coefficient (Wind Tunnel)	Lift Star-CCM+ standard (Near-Field)
[°]	[dim]	[dim]
-5.08	0.259	0.253
-4.09	0.381	0.388
-2.98	0.507	0.521
-1.96	0.63	0.634
-1.05	0.725	0.740
0.07	0.841	0.865
1.11	0.948	0.978
2.1	1.052	1.093
3.17	1.162	1.201
4.25	1.274	1.311
5.14	1.36	1.397

Table 19: FX 63-137 Wind tunnel CI compared to standard Star-CCM+ CI

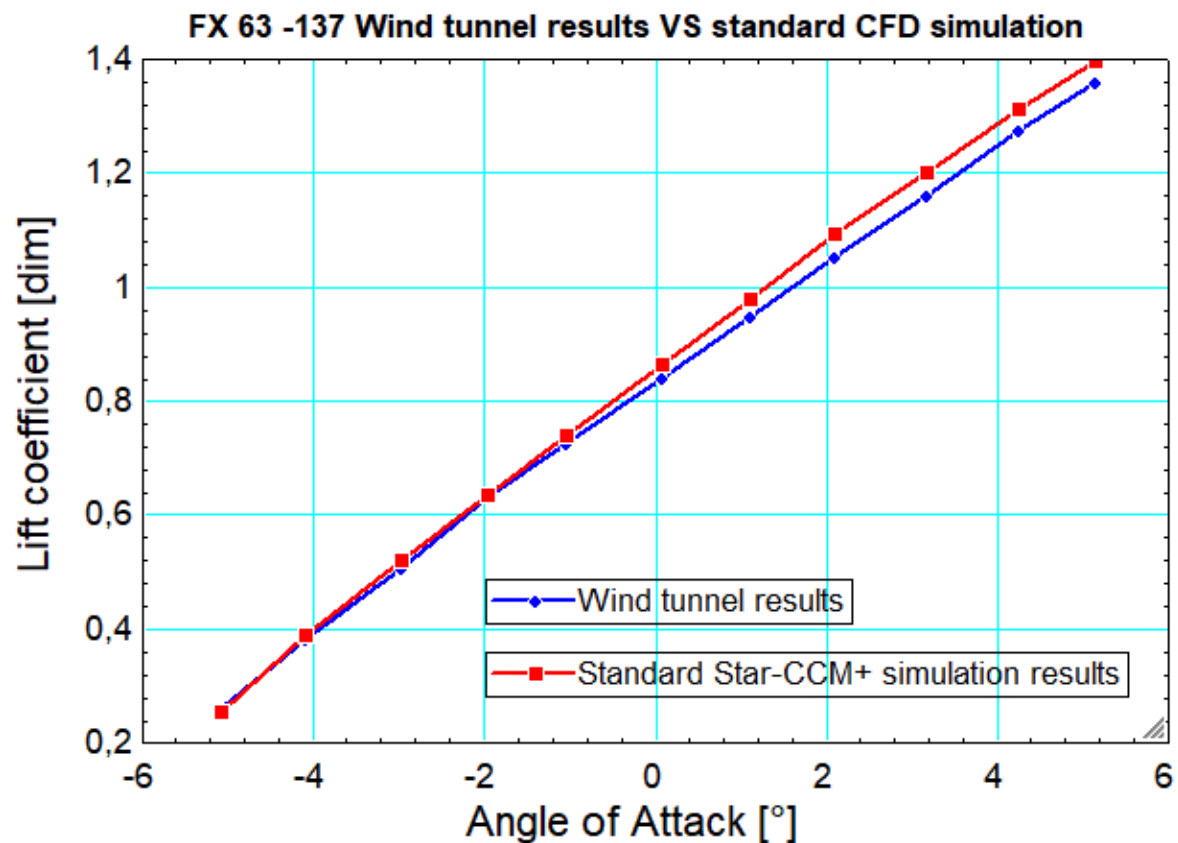


Figure 28: FX 63-137 Wind tunnel C_L compared to standard Star-CCM+ C_L

Upon inspection of **Table 20** and **Figure 29**, it can be observed that the prediction of the drag coefficient, as predicted by the Star-CCM+ simulation, tends to be inaccurate to a higher order as that found in the lift coefficient comparison. The maximum error of the drag coefficient as estimated by our CFD simulation is in the order of 28 drag counts at the position of highest separation. The reason why the drag coefficient predictions are further away from the benchmark drag results even though the lift coefficient predictions seem close to experimental values is due to the presence of spurious drag arising from inadequacies in the mesh at the leading edge. This effect causes erroneous effects in the prediction of the pressure drag whilst the skin friction drag is unaffected. Because the total drag is quantified as the sum of the skin friction and pressure drag, the total drag yields inaccurate results.

AOA [Degrees]	Cd wind [dim]	Cd Star-CCM+ Standard [dim]
-5.08	0.0123	0.0144
-4.09	0.0108	0.0113
-2.98	0.0094	0.0098
-1.96	0.0087	0.01
-1.05	0.009	0.0103
0.07	0.0096	0.0108
1.11	0.01	0.0115
2.1	0.0106	0.0121
3.17	0.0109	0.0131
4.25	0.0116	0.0142
5.14	0.0124	0.0152

Table 20: FX 63-137 Wind tunnel Cd compared to standard Star-CCM+ Cd

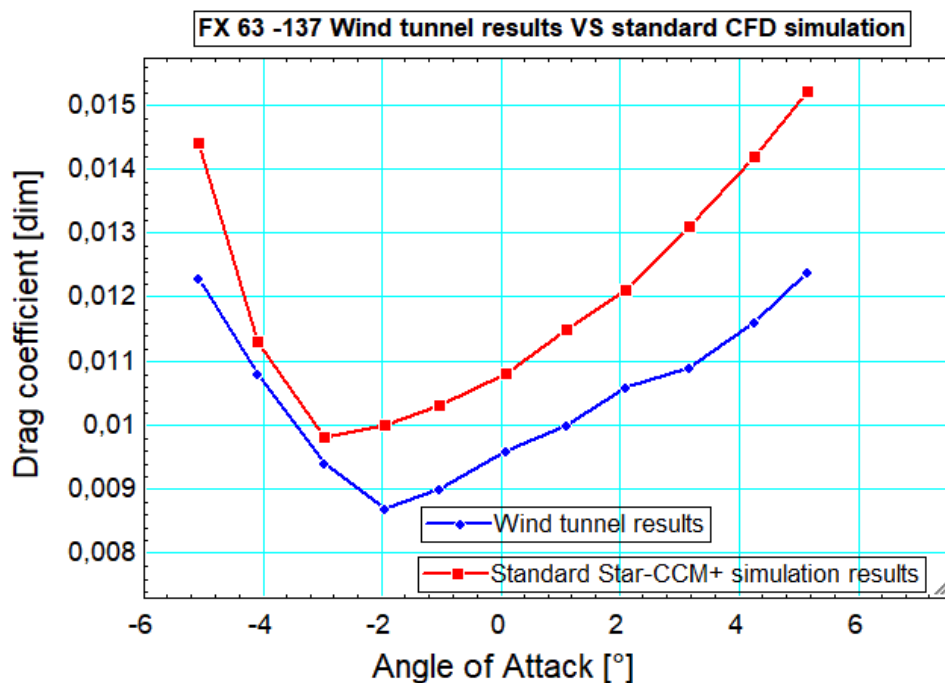


Figure 29: FX 63-137 Wind tunnel Cd compared to standard Star-CCM+ Cd

At this point the attention will shift to the implementation of the four derived far-field drag equations in the simulations under scrutiny. The far-field analysis for the FX 63-137 aerofoil yielded the results depicted in **Tables 21-22** in the angle of attack range.

AOA	Cd wind	Cd Star-CCM+	Cd Squire Approx.	Cd Dynamic Pressure	Cd Wake Rake	Cd Momentum Thickness
[Degrees]	[dim]	[dim]	[dim]	[dim]	[dim]	[dim]
-5.08	0.0123	0.01441	0.01338	0.01417	0.01398	0.01342
-4.09	0.0108	0.01133	0.00959	0.01027	0.01016	0.00963
-2.98	0.0094	0.00981	0.00869	0.00942	0.00934	0.00869
-1.96	0.0087	0.00999	0.00863	0.00947	0.00938	0.00864
-1.05	0.009	0.01032	0.00861	0.00951	0.00942	0.00863
0.07	0.0096	0.01082	0.01024	0.01007	0.00996	0.01025
1.11	0.01	0.01147	0.00955	0.01016	0.01007	0.00952
2.1	0.0106	0.01214	0.01028	0.01136	0.01077	0.01028
3.17	0.0109	0.01307	0.01031	0.01113	0.01101	0.01031
4.25	0.0116	0.01421	0.01066	0.01193	0.01587	0.01066
5.14	0.0124	0.01524	0.01151	0.01277	0.01262	0.01151
AOA	Error Cd Wind Tunnel	Error Cd Star-CCM+	Error Cd Squire Approx.	Error Cd Dynamic Pressure	Error Cd Wake Rake	Error Cd Momentum Thickness
[Degrees]	[dim]	[dim]	[dim]	[dim]	[dim]	[dim]
-5.0800	0	0.0021	0.0011	0.0019	0.0017	0.0011
-4.0900	0	0.0005	0.0012	0.0005	0.0006	0.0012
-2.9800	0	0.0004	0.0007	0.0000	0.0001	0.0007
-1.9600	0	0.0013	0.0001	0.0008	0.0007	0.0001
-1.0500	0	0.0013	0.0004	0.0005	0.0004	0.0004
0.0700	0	0.0012	0.0006	0.0005	0.0004	0.0006
1.1100	0	0.0015	0.0004	0.0002	0.0001	0.0005
2.1000	0	0.0015	0.0003	0.0008	0.0002	0.0003
3.1700	0	0.0022	0.0006	0.0002	0.0001	0.0006
4.2500	0	0.0026	0.0009	0.0003	0.0002	0.0009
5.1400	0	0.0028	0.0009	0.0004	0.0002	0.0009

Table 21: FX 63-137 Far-field drag analysis results from CFD simulation and proposed algorithm

	Error Cd Star-CCM+ Standard [dim]	Error Cd Squire Approx. [dim]	Error Cd Dynamic Pressure [dim]	Error Cd Wake Rake [dim]	Error Cd Momentum Thickness [dim]
Max Cd error	0.0028	0.0012	0.0019	0.0017	0.0012
Min Cd error	0.0004	0.0001	0.0000	0.0001	0.0001
Max drag count diff.	28.0000	12.1276	18.6629	16.7833	11.6550
Min drag count diff.	4.0000	0.7207	0.1741	0.6367	0.5604

Table 22: FX 63-137 Far-field drag analysis results from CFD simulation and proposed algorithm

Tables 21 and **22** indicate that the maximum error occurs as expected at instances where the angles of attack are high in either direction. The error is defined as the absolute difference of simulation result with the wind tunnel experiment results. The standard converged CFD simulation has a maximum error of 28 drag counts.

The Squire-Young, wake rake and momentum thickness methods, have a maximum drag count difference of 12.13, 16.78 and 11.66 drag counts respectively. The dynamic pressure analysis method has a max drag count difference of 18.66 drag counts. The standard Star-CCM+ simulation have a minimum error of 4 drag counts; the dynamic pressure analysis method has a minimum error of 0.1741 drag counts. The wake rake, Squire-Young and momentum thickness far-field analysis methods have a minimum difference of between 0.5 and 0.75 drag counts with respect to experimental results. By thoroughly studying **Figure 30**, it can clearly be seen that the best method of analysis for this particular aerofoil, at the angles of attack spectrum chosen, is the wake-rake analysis method. The conclusion can thus be reached that for the aerofoil and Reynolds number under discussion the proposed wake-rake method yields a 40.06% improvement on the maximum drag prediction error compared to the near-field method.

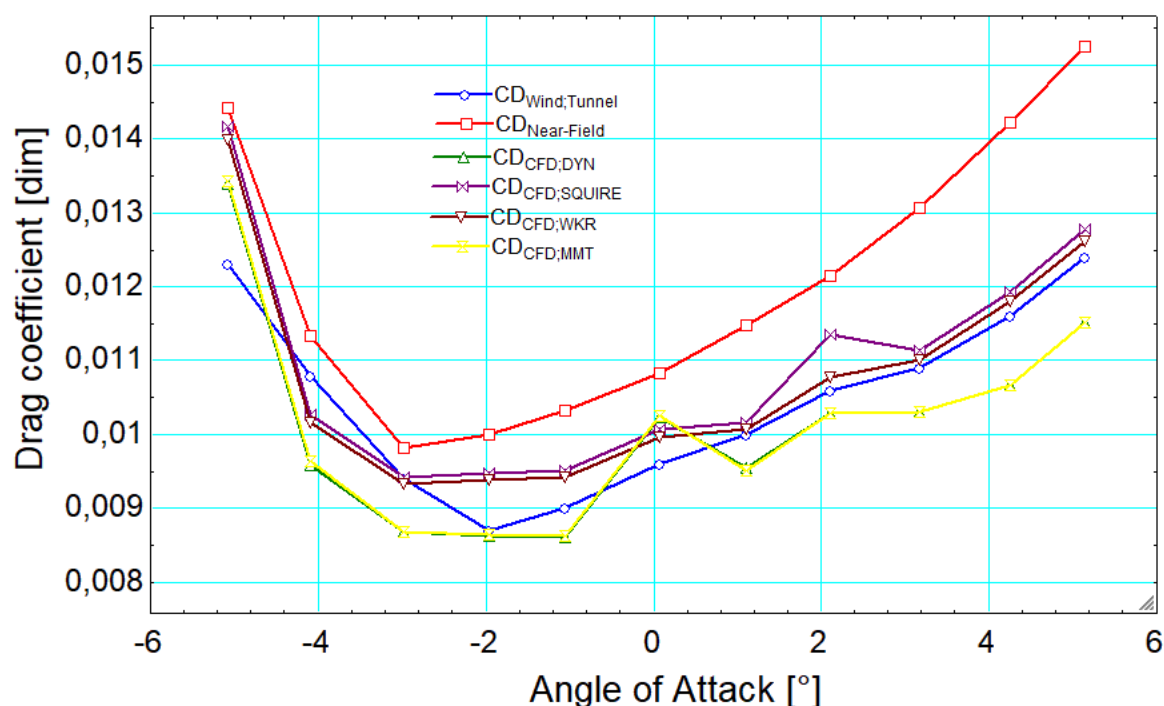


Figure 30: FX 63-137 Far-field drag analysis results from CFD simulation and proposed algorithm

It is important to note that all parameters in the simulation were kept the same in the simulations portrayed in Figure 30. The only change was the method of integration for the far-field procedure. The improvement of the drag prediction can be ascribed to the nature of the far-field analysis methods applied, which ensures minimal numerical noise in the calculation procedure and a reduced dependence on mesh refinement. To further understand the independence of the far-field method to mesh refinement, please refer to **Appendix C**. In the case of the dynamic pressure and wake rake methods the drag estimation from these methods yield results more in agreement with our benchmark results, because the drag is calculated by integrating the respective parameters once over the wake length. These far-field integrals are evaluated with the flow variables of velocity and pressure distribution extracted from the CFD solution. The variables of velocity and pressure are accurately solved by the CFD code and are less dependent on the stagnation point errors due to inadequate discretization and boundary condition setting (Snyder, 2012) (Yamakazi, Matsushima, & Nakahashi, 2005). This means that errors in the pressure drag calculation are avoided in the far-field method and the final drag solution has a reduced presence of spurious drag. The reduction of artificial drag means that lower drag values are obtained by the far-field method when compared to the near-field method. In the case of the momentum thickness and Squire-Young method the accuracy is of a higher order – for the same reason.

The reason why the momentum thickness method and Squire-Young method are less accurate with respect to the wake rake and dynamic pressure methods is because of the presence of an inviscid solution term in the calculation. Both the momentum thickness and Squire-Young method use the inviscid velocity profile as well as the viscous velocity profile extracted from CFD simulations. The errors calculated in both the inviscid and viscous simulation superimpose in the integrant evaluation and lead to reduced accuracy with regard to the wake rake and dynamic pressure methods. The tendency of the far-field method to yield lower profile drag due to reduced spurious drag can be seen in **Figure 30**. This tendency holds for all the far-field methods over the entirety of the AOA cases simulated.

5.2. E231 AEROFOIL

The next step is to investigate the accuracy of the proposed far-field drag extraction tool on the E231 aerofoil with regard to the wind tunnel data of Selig as explained in **Chapter 3**. For a thorough investigation, the accuracy of a standard Star-CCM+ near-field CFD simulation with respect to the Selig wind tunnel data will be evaluated. The data for the far-field drag analysis will be extracted from the same near-field CFD simulation. The aerofoil geometry is created in the CAD software package PLM NX 11 from the coordinate files in **Appendix D** and can be seen in the figure below. The geometry part is then imported into Star-CCM+ for the appropriate mesh and physics set up for the CFD simulation.

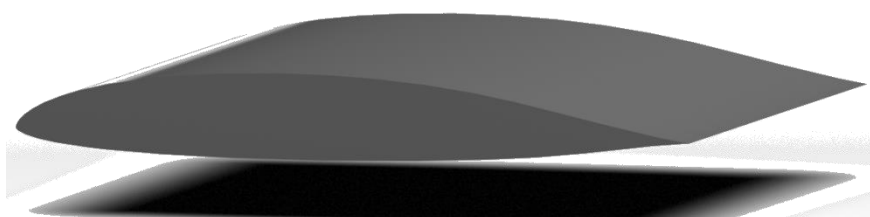


Figure 31: E231 Aerofoil CAD model for Star-CCM+ simulation

The mesh is set up in Star-CCM+ from the imported CAD geometry with a square simulation domain with a height and width of 32 meters. The basic mesh parameters were set as discussed in **Section 4, Tables 15 through 17**, with a near wall prism layer thickness of $9.04\text{E-}7$ meters for the first iteration. From **Figure 32** it is quite obvious that the mesh parameters chosen lead to an unstructured mesh densely clustered at the LE and TE edges. A refined wake area with a spread angle of 0.17 radians can also be seen. This is to capture

the wake interaction on a very fine scale at the defined position. The derived plane set at 6 chord lengths behind the aerofoil with coordinates of $[-0.416, 0.031]$ meters defined from the y-centroid of the domain is also visible.

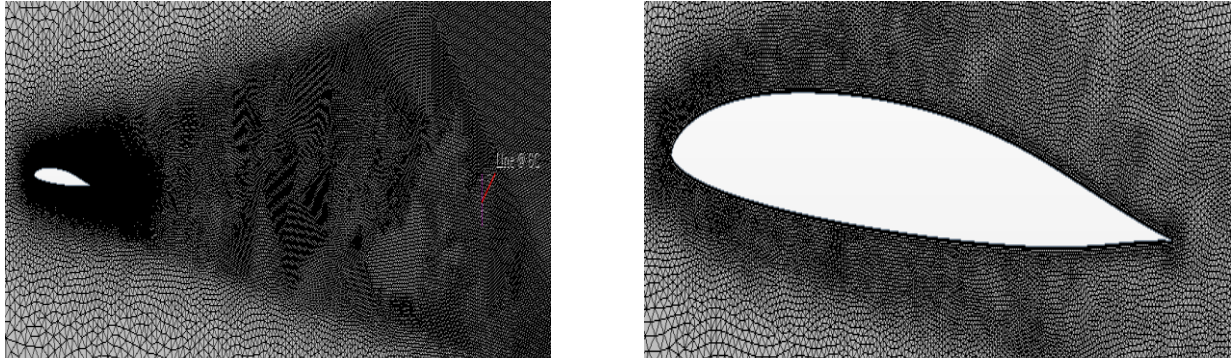


Figure 32: Dense mesh around aerofoil (left); clustered mesh at LE and TE (right)

The simulation physics was set up according to **Table 23**. Thus the Reynolds number is the same as for the wind tunnel experiment and therefore it is possible to quantify the converged simulation accuracy with the Selig wind tunnel results (Selig & McGranahan, 2003).

Property	Value	Unit
Pressure	0	Pa
Free-stream velocity	5.405243959	m/s
Chord length	1	m
Density	1.283	Kg/m ³
Dynamic viscosity	1.7325E-5	Pa-s
Reynolds number	400400	dimensionless

Table 23: E231 Simulation parameters

A similar simulation was set up, as shown above, for each angle of attack at which the Selig wind tunnel experiments were conducted. In each iteration, the y^+ values as well as the wake shape and length were evaluated. The wake length and distance from the TE were varied in each simulation to ensure a stable and uniform wake was used in the far-field drag analysis algorithm (Important to see appendices). It is interesting to note that the highest accuracy results for the case of the E231 simulations at a Reynolds number of 400400 were

achieved with a wake plane set considerably closer to that of the simulation in **Section 5.1**. This is ascribed to the reason that a lower Reynolds number yields a resulting wake that tends to dissipate more abruptly as the distance from the TE increases. In **Section 5.1** the average point of wake data extraction was at 6 chord lengths from the TE, whereas the average for this section was at approximately 4 chord lengths. In each simulation of angle of attack the wake shape and y^+ values were in the range of those displayed in **Figures 33-34**, with differences in the wake height, width and position, and in the frequency and magnitude of the wall y^+ values

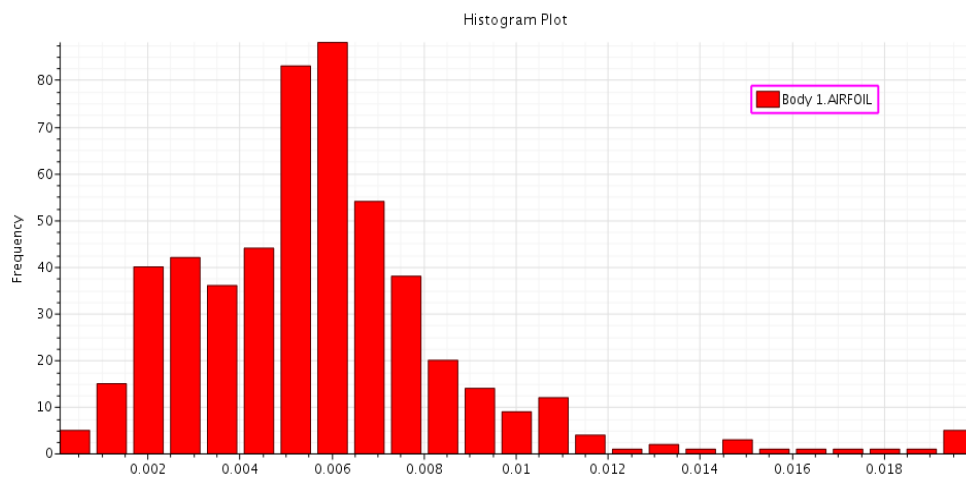


Figure 33: Wall y^+ values for E231 simulation at AOA of 4.78

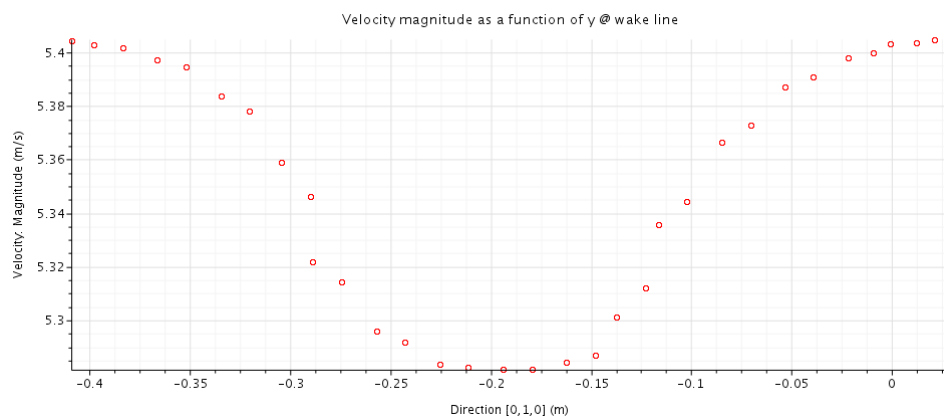


Figure 34: $V(y)$ values for E231 simulation at AOA of 4.78

The simulations were run until convergence was met in an AOA range of -4.53 to 4.78 degrees in increasing instances of approximately 1 degree. **Table 24** and **Figure 35**

demonstrate that the accuracy of the CFD simulation with respect to the wind tunnel experiments are high at high angles of attack and tend to degrade as the angle of attack is lowered. The maximum difference between our simulation lift coefficient and wind tunnel lift coefficient is for the case of -0.34 degrees AOA and is in the order of 0.112 [dim]. This error is still within high order accuracy with regard to the wind tunnel experiments.

Angle of Attack [°]	Lift coefficient (Wind Tunnel) [dim]	Lift coefficient (Std. Star-CCM+) [dim]
-4.53	-0.302	-0.216
-3.48	-0.203	-0.126
-2.47	-0.11	-0.057
-1.44	-0.034	0.057
-0.34	0.064	0.175
0.64	0.182	0.277
1.69	0.314	0.394
2.66	0.439	0.489
3.75	0.589	0.605
4.78	0.724	0.712

Table 24: E231 Wind tunnel CI compared to standard Star-CCM+ CI

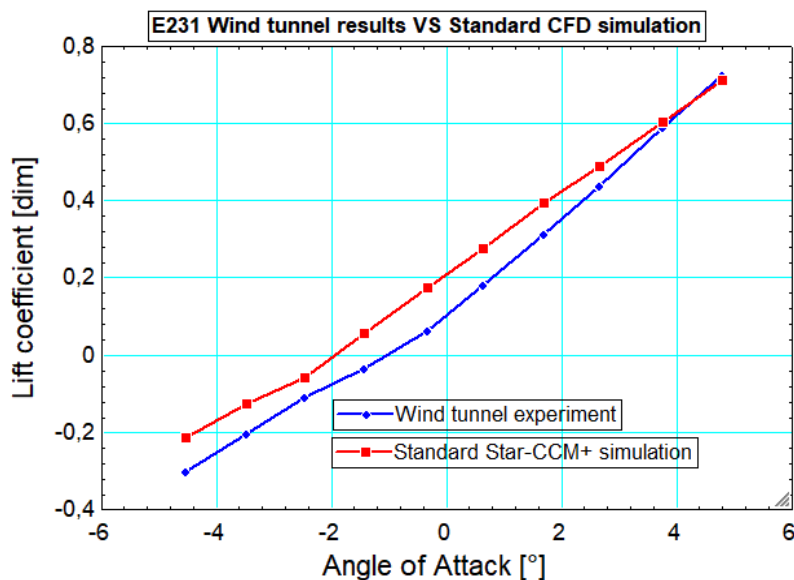


Figure 35: E231 Wind tunnel CI compared to standard Star-CCM+ CI

In the following section the accuracy of the drag prediction as computed by our Star-CCM+ simulation will be investigated. As can be observed in Figure 35, the mesh parameters chosen for the E231 case at various angles of attack yielded accurate lift coefficient predictions w.r.t our benchmark; this is, however, not the case for the drag coefficient prediction. The following figure and table confirm that although the CFD simulation is accurate in the lift coefficient, there are some erroneous predictions in the drag coefficient in our angle of attack range.

The CFD simulation tends to under predict the drag in the AOA range from -4 degrees to 2 degrees and to over predict the drag from 3 to 5 degrees. There are also discrepancies in the trend line of the simulation results with respect to the wind tunnel results. This error can be ascribed to the phenomenon of spurious drag which occurs at simulations where the drag is computed by integrating over the surface boundary of the aerofoil. The maximum drag error between the CFD simulation and the wind tunnel drag results are in the order of 29 drag counts, which is not ideal for aerofoil design or for optimizing where precision in prediction is of high importance.

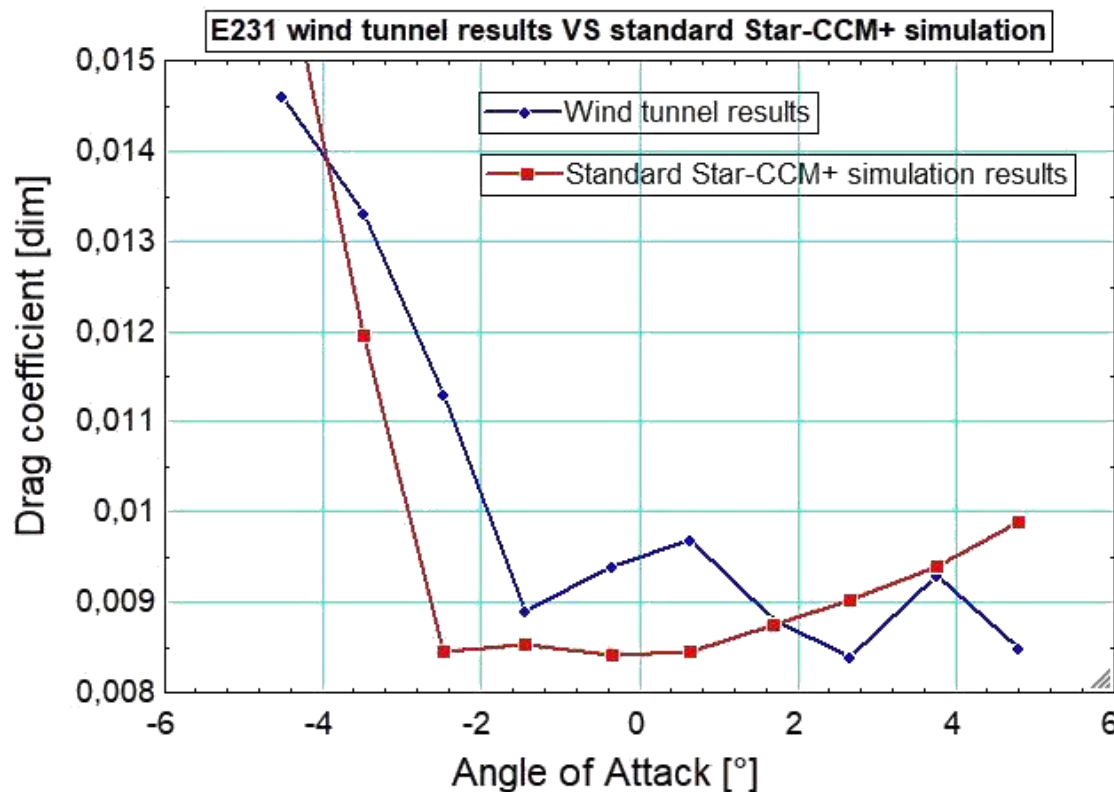


Figure 36: E231 Wind tunnel Cd compared to standard Star-CCM+ Cd

AOA [Degrees]	Cd wind [dim]	Cd Star-CCM+ Standard [dim]
-4.53	0.0146	0.01627
-3.48	0.0133	0.01195
-2.47	0.0113	0.00845
-1.44	0.0089	0.00853
-0.34	0.0094	0.00842
0.64	0.0097	0.00845
1.69	0.0088	0.00874
2.66	0.0084	0.00903
3.75	0.0093	0.00939
4.78	0.0085	0.00989

Table 25: E231 Wind tunnel Cd compared to standard Star-CCM+ Cd

In order to evaluate our proposed methods for far-field drag analysis, the procedures of **Chapter 4** were applied to each AOA of **Table 24**. The results (and accompanied errors) with respect to the wind tunnel experiment were calculated as displayed below:

AOA [Degrees]	Cd wind [dim]	Cd Star-CCM+ [dim]	Cd Squire Approx. [dim]	Cd Dynamic Pressure [dim]	Cd Wake Rake [dim]	Cd Momentum Thickness [dim]
-4.53	0.0146	0.01627	0.01444	0.01746	0.01692	0.01458
-3.48	0.0133	0.01195	0.01063	0.01327	0.01308	0.01062
-2.47	0.0113	0.00845	0.00774	0.00946	0.00931	0.00782
-1.44	0.0089	0.00853	0.00773	0.00866	0.00859	0.00775
-0.34	0.0094	0.00842	0.00834	0.00928	0.00917	0.0084
0.64	0.0097	0.00845	0.00804	0.00855	0.00846	0.00805
1.69	0.0088	0.00874	0.00816	0.00863	0.00855	0.00817
2.66	0.0084	0.00903	0.00824	0.00872	0.00863	0.00825
3.75	0.0093	0.00939	0.00845	0.00892	0.00884	0.00846
4.78	0.0085	0.00989	0.00871	0.00935	0.00926	0.00872
AOA [Degrees]	Error Cd Wind Tunnel [dim]	Error Cd Star- CCM+ [dim]	Error Cd Squire Approx. [dim]	Error Cd Dynamic Pressure [dim]	Error Cd Wake Rake [dim]	Error Cd Momentum Thickness [dim]
-4.53	0	0.00167	0.00016	0.00286	0.00232	0.00002
-3.48	0	0.00135	0.00267	0.00003	0.00022	0.00268
-2.47	0	0.00285	0.00356	0.00184	0.00199	0.00348

-1.44	0	0.00037	0.00117	0.00024	0.00031	0.00115
-0.34	0	0.00098	0.00106	0.00012	0.00023	0.001
0.64	0	0.00125	0.00166	0.00115	0.00124	0.00165
1.69	0	0.00006	0.00064	0.00017	0.00025	0.00063
2.66	0	0.00063	0.00016	0.00032	0.00023	0.00015
3.75	0	0.00009	0.00085	0.00038	0.00046	0.00084
4.78	0	0.00139	0.00021	0.00085	0.00076	0.00022

Table 26: E231 Far-field drag analysis results from CFD simulation and proposed algorithm

In **Table 27** the maximum and minimum errors as accumulated from our far-field drag analysis are calculated. It is clear that for this case the Squire-Young and momentum thickness methods yielded drag values that were inaccurate to a greater extent than those of the standard CFD simulation as opposed to the wake rake and dynamic pressure methods that resulted in drag values that were superior to those of the standard CFD simulation.

Property	Error Cd Star-CCM+ Standard [dim]	Error Cd Squire Approx. [dim]	Error Cd Dynamic Pressure [dim]	Error Cd Wake Rake [dim]	Error Cd Momentum Thickness [dim]
Max Cd error	0.00285	0.00356	0.00286	0.00232	0.00348
Min Cd error	0.00006	0.00016	0.00003	0.00022	0.00002
Max drag count diff.	29	36	29	23	35
Min drag count diff.	0.60	1.58	0.34	2.25	0.15

Table 27: E231 Drag count errors of near-field and proposed far-field methods

Upon inspection of **Figure 46** it can clearly be seen that the dynamic pressure and wake rake analysis had the upper hand in predicting the drag coefficient over our range of angles of attack. The dynamic pressure far-field method predicted a maximum error w.r.t benchmark results in the simulation case where the AOA was -4.53 degrees. The dynamic pressure far-field method yielded an average relative error (w.r.t benchmark results) of 7.94 drag counts. For the wake rake method and standard CFD simulation this value was 8.02 and 10.66 drag counts respectively. This indicates that for the same physics set-up, boundary conditions and mesh settings the far-field dynamic pressure method and far-field wake rake method gave on average, drag coefficient predictions with an error of between 2-3 drag counts lower than the standard simulation.

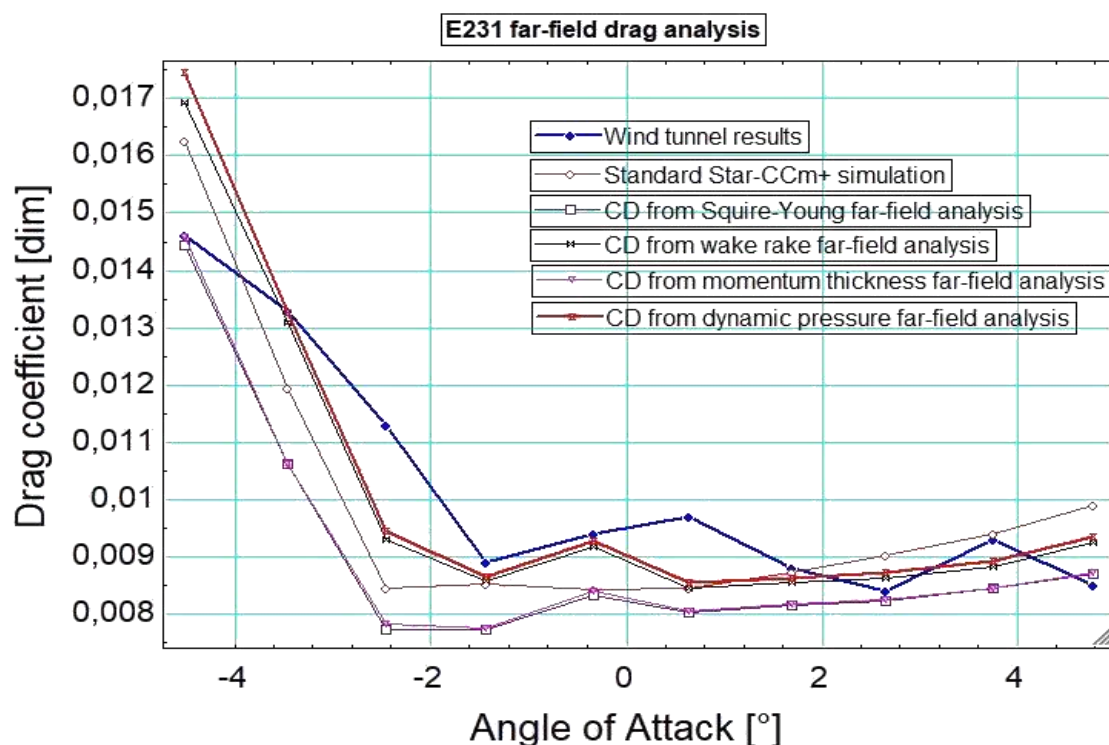


Figure 37: E231 Far-field drag analysis results from CFD simulation and proposed algorithm

Figure 46 clearly shows that the wake rake and pressure drag far-field methods approximate the wind tunnel results to the highest degree of accuracy in terms of trend line and accumulated error. The reason for the higher order of drag coefficient underprediction by the Squire-Young method and the momentum thickness method was caused by the position of application of these methods. For the case of the E231 aerofoil the Reynolds number was significantly lower than that of the FX 63-137 case which was at 499818. The lower Reynolds number resulted in a wake which degraded far more significantly as the distance from the TE was increased. Because the Squire-Young formula is an extrapolation of the momentum thickness far downstream under the assumption that the profile drag behaves asymptotically in the downstream, the position of application is of high importance.

The Squire-Young formula can, however, by definition, not be applied in regions too close to the TE (typically within 1 chord length of the TE) as this violates the assumptions under which the equation is derived (Coder & Maughmer, 2015). Thus a discrepancy is created in the application of the Squire-Young formula in CFD codes where the Reynolds number is of such a nature that the wake rapidly degrades. Further investigation needs to be conducted in order to arrive at a method for the application of the Squire-Young and momentum thickness far-field analysis methods in CFD to overcome this special case.

However, this special case does not apply to panel codes such as XFOIL, the reason being that although XFOIL applies the Squire-Young equation at roughly one chord length from the trailing edge, it does not use the solution of the Navier-Stokes equation to calculate the momentum thickness, shape factor and pressure distribution. It rather uses a two-equation integral formulation based on dissipation closure for both the laminar and turbulent flow regimes over the aerofoil (Balleur, 1981).

5.3. S834 AEROFOIL

In the last instance the attention is turned to the final aerofoil case for the investigation of the accuracy of the far-field drag extraction methods discussed previously. As usual, the first step is to model the aerofoil geometry in NX PLM 11 from the coordinate file displayed in **Appendix D**. This 3 dimensional model is then imported into Star-CCM+ for the appropriate mesh and physics set-up for a 2 dimensional simulation.

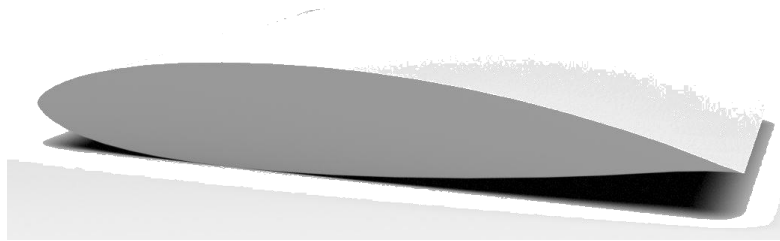


Figure 38: S834 3-d CAD geometry for Star-CCM+ simulation

For this case the chosen simulation domain was a square with height and width of 32 chord lengths. The reason why the 32 by 32 chord length square domain is acceptable for the S834, E231 and FX 63-137 aerofoil simulations, is that all three aerofoils have a chord length of 1 meter and a thickness in the range of 12-15% with camber between 1-6%. The basic mesh parameters were set as defined in **Chapter 4** and **Tables 15 - 17**, with a near wall prism layer thickness of $1.483\text{E-}5$ meters for the first iteration. Referring to **Figure 39**, it is clear that the chosen mesh parameters yield an acceptable mesh with refined mesh zones at the LE, TE, aerofoil surface and downstream wake. For the case displayed, i.e. 4.17° degrees AOA, the wake length projected on the y-axis lay between -0.403 and 0.030 meters with the y-extraction plane 8 meters from the LE. The distance of 8 meters was chosen due to the fact that this is where the wake stabilises in terms of the top and bottom free stream static pressure which is a requirement for accurate far-field drag analysis. The wake length as projected on the y-axis is chosen in such a manner that only the first top and bottom free stream velocity is captured in the numerical integration process as this yields the most accurate prediction. A thorough study and explanation of various methods to choose the top and bottom cut-off positions for far-field integration can be seen in **Appendix C**.

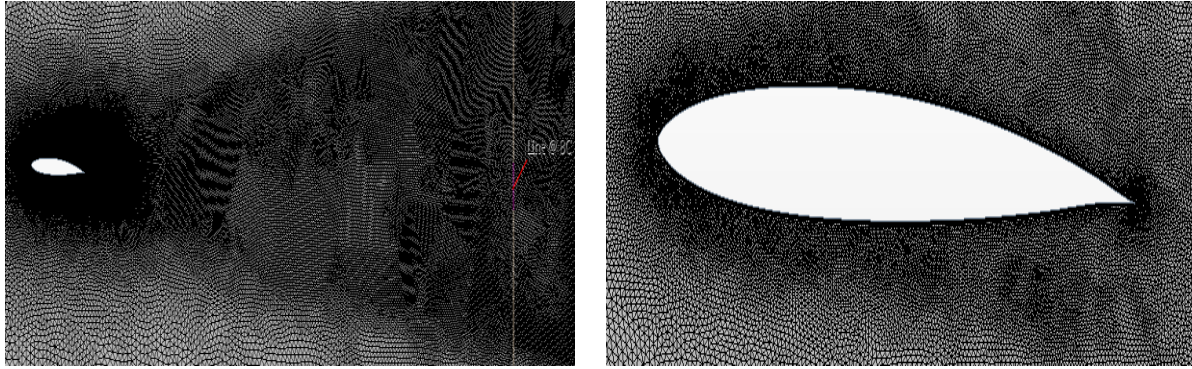


Figure 39: Dense mesh around aerofoil(left); clustered mesh at LE and TE (right)

Furthermore, the mesh parameters and near wall prism layer thickness chosen for this case resulted in a wall y^+ value between 0 and 1 which is the requirement for our transition model. In the first of the two following figures the range and frequency of the y^+ values over the aerofoil surface are displayed for the case of 4.17 AOA. The second of the following two figures shows the wake form and the top and bottom ranges where the wake is captured for the integration process.

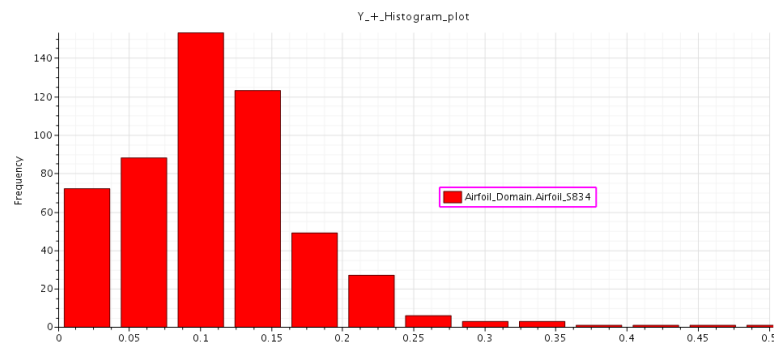


Figure 40: Y+ distribution over S83 aerofoil for 4.17 degree AOA case

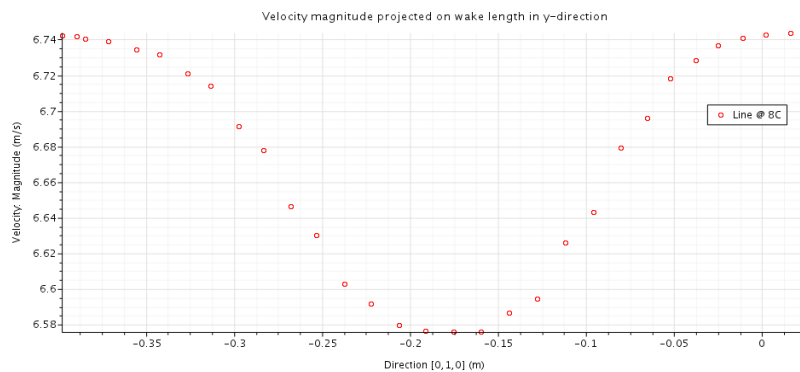


Figure 41: Velocity magnitude projected on wake length in y-direction for 4.17 [deg.] AOA case

With the wind tunnel data for the S834 aerofoil laying between -5.06 and 5.2 degrees the same procedure as for the 4.17 degree case was followed for each of the angles of attack in this range. For each simulation the drag and lift coefficients predicted by our CFD code was evaluated against the wind tunnel results of Selig (Selig & McGranahan, 2003). It is clear from **Figure 42** that the experimental lift coefficients are approximated extremely accurately by our CFD simulation. The maximum absolute error between the wind tunnel experiment CL and our CFD simulation CL is at -3.01 degrees and has a magnitude of 0.0318 [dim]. This error is so insignificant that it can be concluded that our meshing routine and y^+ values are in an acceptable range for each simulation set-up, seeing that our theoretical results and experimental results correlate to such a high degree of accuracy.

Angle of Attack [°]	CL coefficient (Wind Tunnel) [dim]	CL Star-CCM+ [dim]
-5.06	-0.2760	-0.2859
-4	-0.1730	-0.1896
-3.01	-0.0730	-0.1048
-1.96	0.0390	0.0106
-0.98	0.1400	0.1140
0.07	0.2460	0.2229
1.04	0.3450	0.3318
2.14	0.4610	0.4471
3.15	0.5660	0.5500
4.17	0.6620	0.6572
5.2	0.7470	0.7603

Table 28: S834 Cl wind tunnel results VS standard CFD simulation Cl results

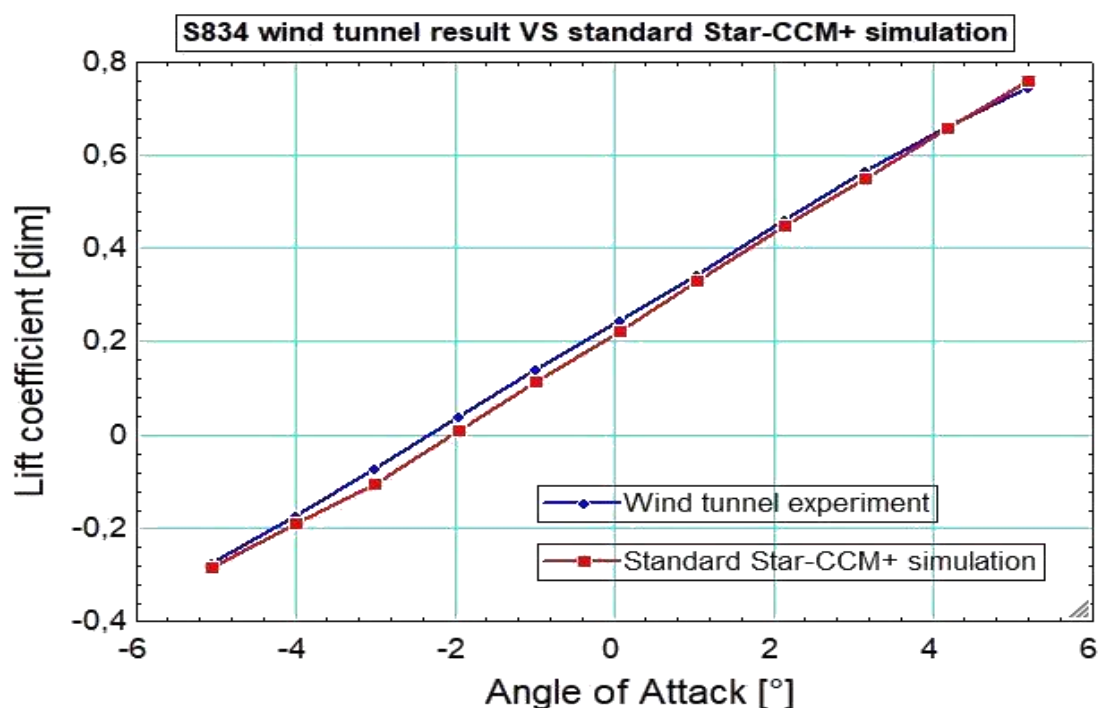


Figure 42: S834 CI wind tunnel results VS standard CFD simulation CI results

On the other hand, referring to **Figure 43**, it is evident that although our CFD lift coefficient predictions are well in agreement with the experimental results, this is not the case for the drag coefficients. Here the maximum error between the CFD simulation drag coefficient and the wind tunnel experiment's drag coefficient is at 4.17 degrees and has a magnitude of approximately 14.6 drag counts. This inaccuracy of the drag coefficient prediction can once again be ascribed to the numerical noise in the pressure drag portion of the profile drag calculation.

As in the cases of the S834, E231 and FX 63-137 aerofoils, with similar thickness and Reynolds numbers, the profile drag as computed by our CFD simulation strongly diverges from the wind tunnel results as the effects of separation become more dominant. The more dominant the separation becomes, the higher the presence of the spurious drag term, and thus the higher the inaccuracy of the drag final solution. Therefore classical CFD methods have weaker accuracy on drag prediction in regions with very high AOA in both directions – the positive and negative.

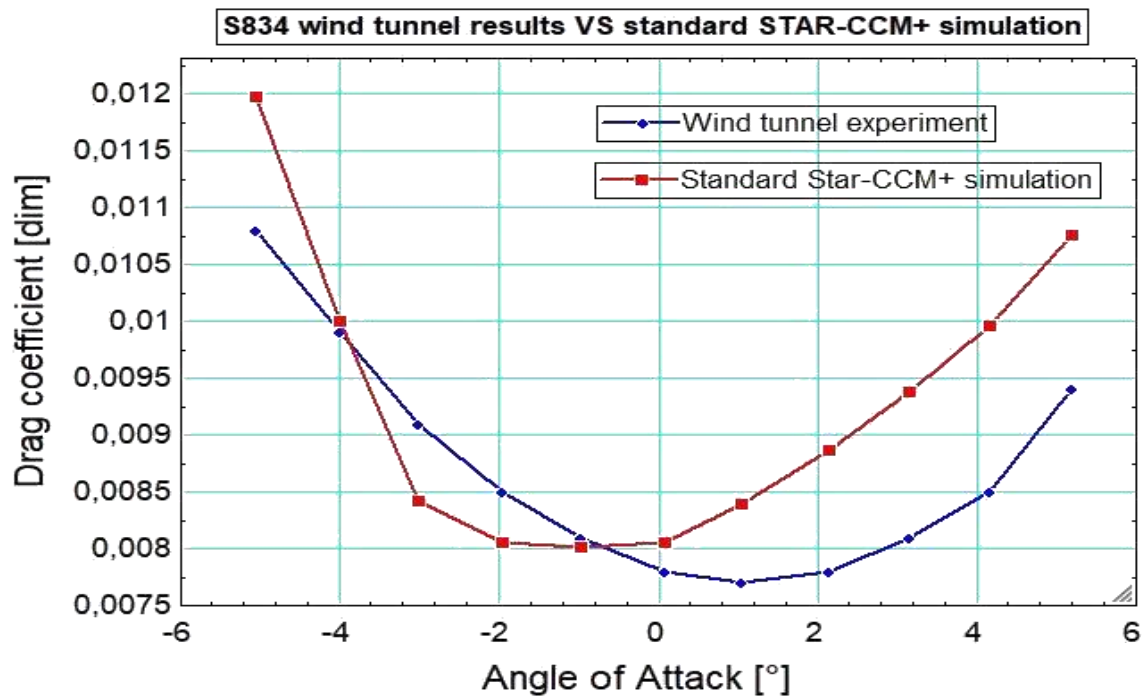


Figure 43: S834 Cd wind tunnel results VS standard CFD simulation Cd results

The next part of the discussion is to focus on the implementation of various far-field drag analysis methods in CFD. The algorithm and procedures described in **Chapter 4** were followed. In each angle of attack simulation case described in the above section, the velocity magnitude of the viscous and inviscid simulations as well as the dynamic pressure distribution as projected on the defined wake plane was extracted. For each simulation case the length of the wake extraction plane from the TE and the width of the total wake were varied to ensure a stable wake was captured in the integration process. The result of the four equations defining the far-field drag analysis can be seen in **Figure 44** and **Table 29** below:

AOA	Cd wind	Cd Star-CCM+	Cd Squire Approx.	Cd Dynamic Pressure	Cd Wake Rake	Cd Momentum Thickness
[Degrees]	[dim]	[dim]	[dim]	[dim]	[dim]	[dim]
-5.06	0.0108	0.01197	0.01006	0.01154	0.01145	0.01008
-4	0.0099	0.01	0.00933	0.01017	0.0101	0.00933
-3.01	0.0091	0.00842	0.0079	0.00835	0.00828	0.00791
-1.96	0.0085	0.00806	0.00793	0.0086	0.00854	0.00795
-0.98	0.0081	0.00801	0.00765	0.00806	0.00799	0.00766
0.07	0.0078	0.00805	0.00731	0.00776	0.0077	0.00732
1.04	0.0077	0.00839	0.00778	0.00824	0.00818	0.00779
2.14	0.0078	0.00887	0.00802	0.00839	0.00833	0.00803

3.15	0.0081	0.00938	0.00821	0.00899	0.00891	0.00824
4.17	0.0085	0.00996	0.00854	0.00931	0.00922	0.00854
5.2	0.0094	0.01076	0.00853	0.00969	0.00961	0.00852
AOA	Error Cd Wind Tunnel	Error Cd Star-CCM+	Error Cd Squire Approx.	Error Cd Dynamic Pressure	Error Cd Wake Rake	Error Cd Momentum Thickness
[Degrees]	[dim]	[dim]	[dim]	[dim]	[dim]	[dim]
-4.53	0	0.001169934	0.000739826	0.00074096	0.000646627	0.00072351
-3.48	0	0.000095963	0.000572748	0.000272941	0.000197055	0.000572976
-2.47	0	0.000679881	0.001199237	0.000754141	0.000818324	0.001187945
-1.44	0	0.000439493	0.000566403	0.000103019	0.000042458	0.000551046
-0.34	0	0.000086594	0.000449977	0.000042768	0.000108242	0.000442576
0.64	0	0.000245793	0.000491963	0.000038708	0.000103377	0.000482643
1.69	0	0.000693451	0.000076532	0.000539403	0.000478101	0.000092204
2.66	0	0.001066066	0.000221839	0.00059481	0.000527425	0.000226362
3.75	0	0.001282859	0.000112955	0.000886741	0.00081445	0.000140385
4.78	0	0.001464328	0.000035775	0.000810159	0.000719999	0.00003949
4.78	0	0.001361982	0.000870806	0.000287362	0.00020723	0.000877041

Table 29: S834 Far-field drag analysis results from CFD simulation and proposed algorithm

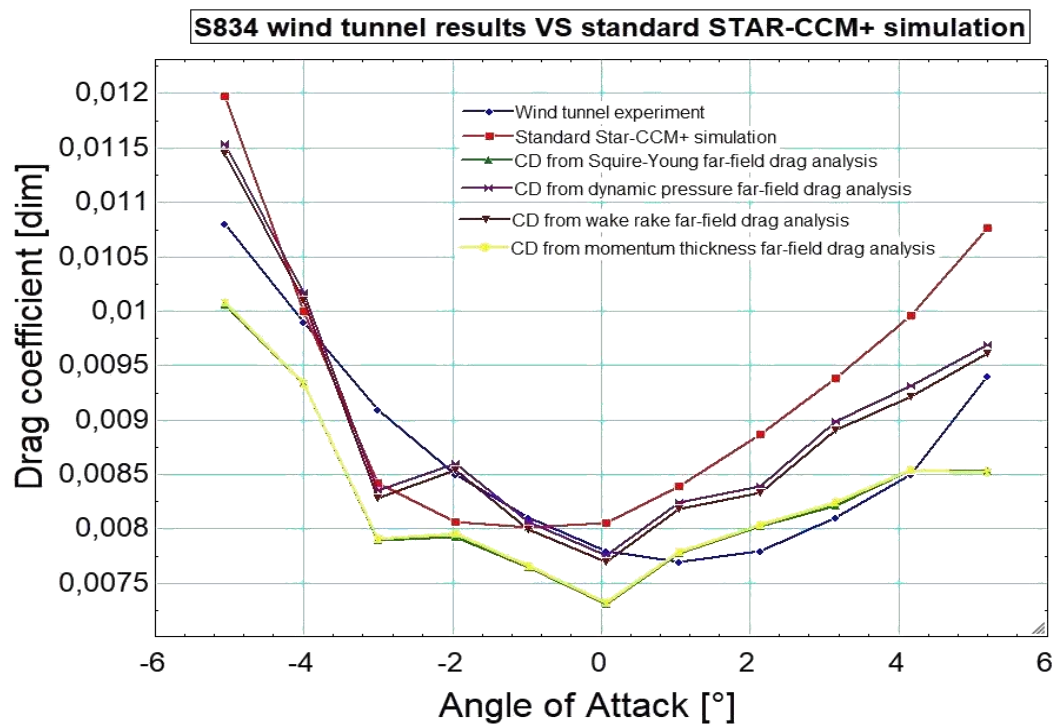


Figure 44: S834 Wind tunnel drag results VS far-field drag analysis results

The dynamic pressure far-field method predicted a maximum error w.r.t benchmark results in the simulation case where the AOA was 4.17 degrees. The dynamic pressure far-field method yielded an average relative error (w.r.t benchmark results) of 4.61 drag counts. For the wake rake method and standard CFD simulation this value was 4.23 and 7.81 drag counts respectively. For the Squire-Young method and momentum thickness method the average relative error (w.r.t benchmark results) was 4.852 and 4.851 drag counts respectively. This once again indicates that for the same physics set-up, boundary conditions and mesh settings the far-field methods gave on average, drag coefficient predictions with an error of between 3-4 drag counts lower than the standard simulation.

Again referring to **Figure 44**, it is evident that the most accurate results of drag prediction comes by the far-field methods of the velocity profile integration over the wake, i.e. the wake rake method, and the dynamic pressure method. Both methods stem from obtaining the velocity and dynamic pressure distribution from a converged CFD simulation over the wake length projected on the y-axis respectively. These velocity magnitude- and dynamic pressure distributions as a function of y are then extracted and a third party software Excel or Python (see **Appendix F**) is used to evaluate the integral numerically. According to **Table 30** the most accurate analysis for drag prediction in this case is the wake rake method with a maximum CD difference of 8.18 drag counts with respect to wind tunnel experiment results.

This means that the most accurate far-field drag analysis method equates to a relative accuracy increase of 44.12% with regard to standard CFD simulations. This demonstrates, once again, that using a converged simulations far-field wake to determine the drag instead of integrating each iteration over the aerofoil surface, yields accuracy improvement in the prediction of the drag coefficient.

Property	Error Cd Star-CCM+ Standard [dim]	Error Cd Squire Approx. [dim]	Error Cd Dynamic Pressure [dim]	Error Cd Wake Rake [dim]	Error Cd Momentum Thickness [dim]
Max Cd error	0.0015	0.0012	0.0009	0.0008	0.0012
Min Cd error	0.0001	0.0000	0.0000	0.0000	0.0000
Max drag count diff.	14.64	11.99	8.87	8.18	11.88
Min drag count diff.	0.866	0.358	0.387	0.425	0.395

Table 30: S834 Aerofoil far-field drag analysis error summary

5.4. CONCLUSION

This chapter was devoted to an investigation of the accuracy of standard CFD simulations and far-field drag analysis methods on three different aerofoil cases at AOA ranges between -5 to 5 degrees and Reynolds numbers in the range of 400 000 to 500 000. Studying the polar plots in **Figures 45-47** below, it becomes clear that for each converged CFD simulation the far-field drag analysis applied to that same converged simulation yields drag values more in agreement with our benchmark results than the corresponding standard CFD solution. The polar plots for each aerofoil case displayed was set up by plotting the wind tunnel lift against the wind tunnel drag and then on the same axis overlay plotting the standard CFD simulation drag and far-field drag analysis results against the CFD lift coefficient. By using the lift coefficients as predicted by the CFD simulation for the polar plot of the far-field analysis and standard CFD simulation, it is ensured that the curve tendency can feasibly be evaluated for a prediction improvement. It is clear from the polar plots that the far-field results have a saw-tooth tendency. This occurrence is due to the numerical errors of the composite Simpsons rule for numerical integration. **Appendix F** gives an in-depth investigation on methods to arrive at more natural curves i.e. numerical noise causing saw-tooth curves are removed. Some of the points discussed in **Appendix F** include different methods to determine the viscous region, various methods to interpolate the viscous region and ultimately using the newly interpolated viscous region functions to be integrated by a wide variety of numerical integration methods.

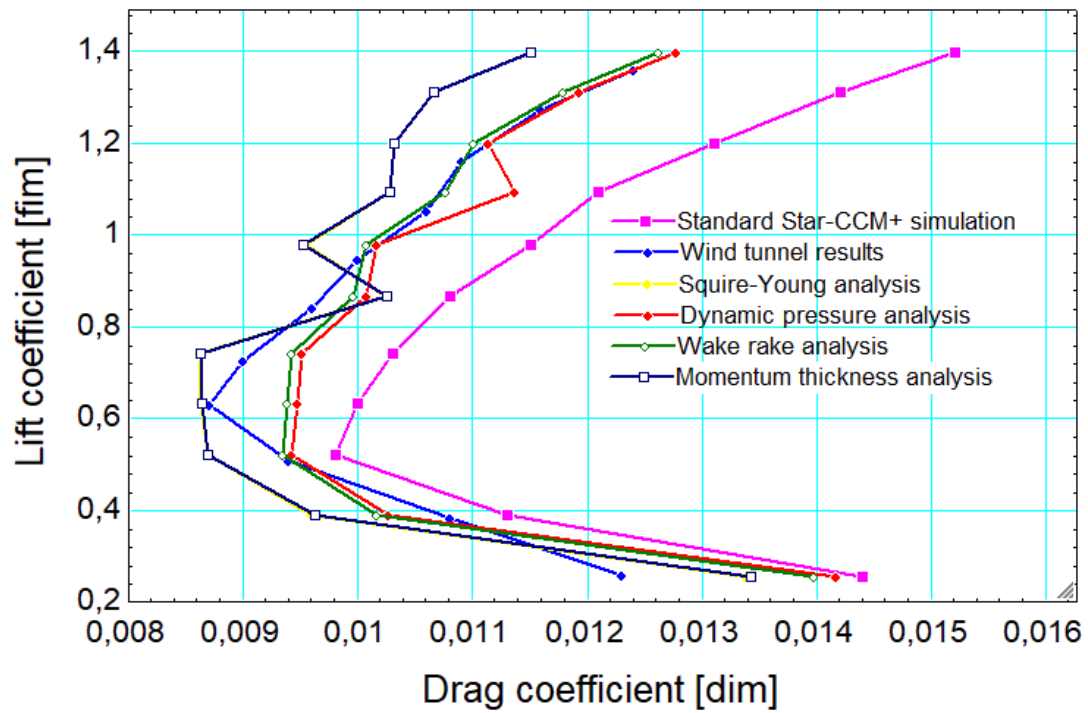


Figure 45: FX 64-137 Polar plots for wind tunnel results, std. CFD simulation & far-field analysis

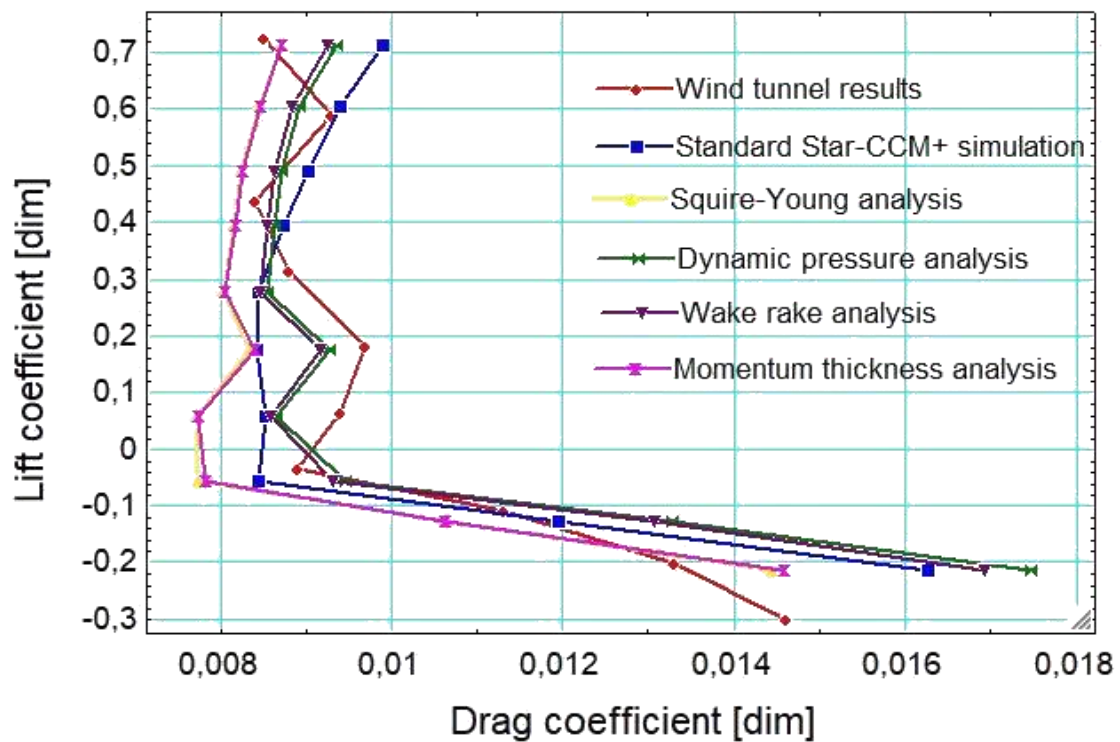


Figure 46: E231 Polar plots for wind tunnel results, std. CFD simulation & far-field analysis

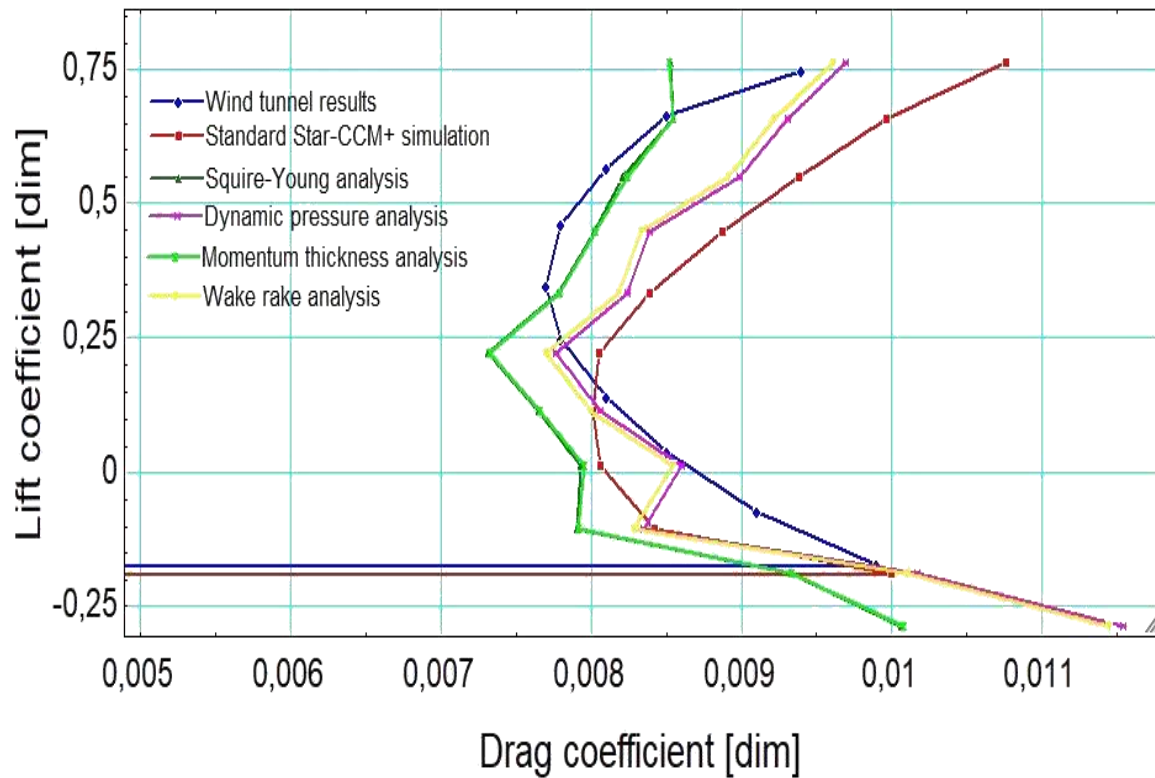


Figure 47: S834 Polar plots for wind tunnel results, std. CFD simulation & far-field analysis

From **Figures 45-47** the following should be clear:

- I. All four far-field drag analysis methods perform better in drag prediction w.r.t benchmark results. This is especially true for cases with high separation – as AOA increases and separation increases so does the accuracy of the proposed far-field methods.
- II. The best performing far-field methods investigated of the four methods are the wake rake method and the dynamic pressure method with the dynamic pressure method slightly more accurate than the wake rake method.
- III. The improvements are due to the following reasons:
 - 1) Reduction or total elimination of spurious drag in the solution
 - 2) Using highly accurate numerical integration methods for 2d wake profiles

- 3) Minimization of the numerical error in surface integration due to boundary layer separation
- 4) Reduced dependency on mesh refinement

Figure 47 demonstrates a discrepancy in the far-field drag analysis applied at the lowest AOA simulation. This discrepancy or inaccuracy was the result of applying all four far-field drag methods to a position that was situated too far from the TE. This means that, at this position, the wake was in the process of dissipating and the drag values calculated from this wake were higher than that of the wind tunnel- and CFD results. This phenomenon expresses the need for the numerical studies deployed in Appendix C to Appendix F. Here the method of application of the far-field drag analysis at the correct position to capture the downstream wake accurately is investigated.

The conclusion that can be made at the end of this section is that the overall best performing method for drag calculation in our linear region of AOA and low Reynolds numbers is the dynamic pressure method. This method is deemed most accurate in terms of its tendency to predict the polar curve with high precision compared to benchmark results, and with minimal error in the drag coefficient calculated with regard to our benchmark results.

Chapter 6: Analysis of the ST1 and OPT110 aerofoils

Up to this point the accuracy of XFOIL, the near-field model and various far-field models to predict drag for low Reynolds numbers have been validated. Subsequently the best XFOIL and CFD practices will be utilised to analyse the performance characteristics of the ST1 and OPT110 aerofoils. It has been demonstrated throughout this thesis that there is a discrepancy in the accuracy of the pressure drag as predicted by CFD methods due to the existence of spurious drag. This discrepancy will also be investigated in this chapter. For the purpose of the case study it will therefore be assumed that the skin friction drag as predicted by CFD is accurate and will then compute the profile drag by means of the far-field algorithm. The new pressure drag will then be computed by subtracting the CFD skin friction drag from the far-field drag. As reference for our simulations it will be assumed that XFOIL simulation results can be regarded as benchmark results. Thus our standard Star-CCM+ simulation and far-field drag simulation will be validated against the XFOIL simulation results.

6.1. ST1 & OPT110 AEROFOILS

In **Figure 48** the aerofoil profiles of the ST1 and the OPT110 are depicted. It is clear that the OPT110 exhibits modification of position of max thickness, max camber and overall thickness distribution when compared to the ST1 aerofoil.

Although the geometry modifications do not seem to result in major differences between the two aerofoils, these geometry manipulations result in major changes in the aerodynamic performance of the aerofoils. These differences in aerodynamic performance between the two aerofoils will be investigated with CFD and panel methods to establish to which extent the geometry modifications have altered the performance characteristics. XFOIL and a CFD near-field and various CFD far-field methods will also be employed to determine which of the simulation methods closely agree, and which can be assumed to be most accurate.

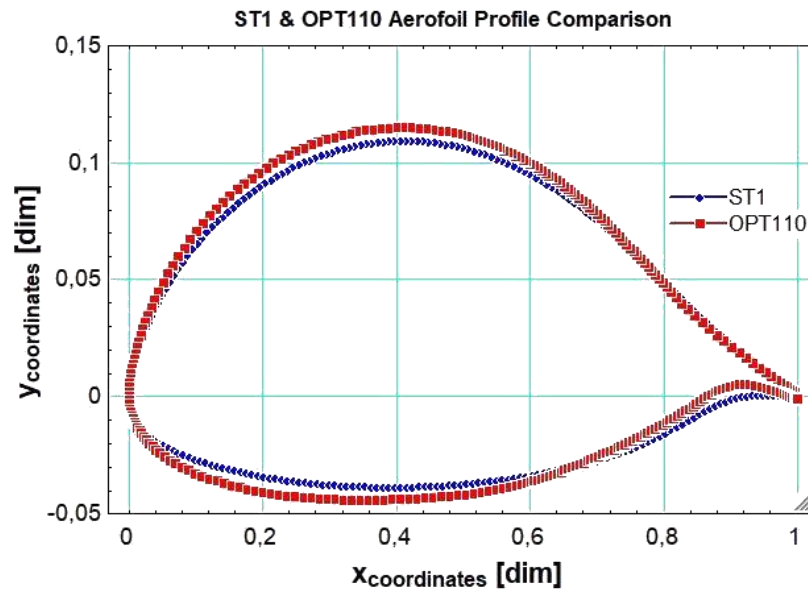


Figure 48: ST1 & OPT110 Aerofoil Comparison

6.2. ST1 & OPT110 XFOIL SIMULATION

Both the ST1 and OPT110 aerofoils will be simulated in XFOIL under the same constraints and constants. As previously stated, the XFOIL simulation results will be used as reference data against which our other simulations will be validated. This assumption can be made because XFOIL has proven itself as the workhorse of aerofoil design in the industry. References to this assumption appear in **Chapter 2**. The **Table 31** displays the constants under which the XFOIL simulation was conducted:

Parameter	Value
Critical Amplification Factor [%]	0.07
Viscous Acceleration Parameter	0.01
Reynolds Number	1 000 000
Mach Number	0
Panel Bunching Parameter	1
Angle of Attack Range [degrees]	-5 to 10
Number of panels	120

Table 31: ST1 & OPT110 XFOIL Simulation Parameters

With the simulation set-up and run with the parameter inputs shown in **Table 31**, the performance of the two aerofoils can now be investigated in terms of the lift and drag coefficients.

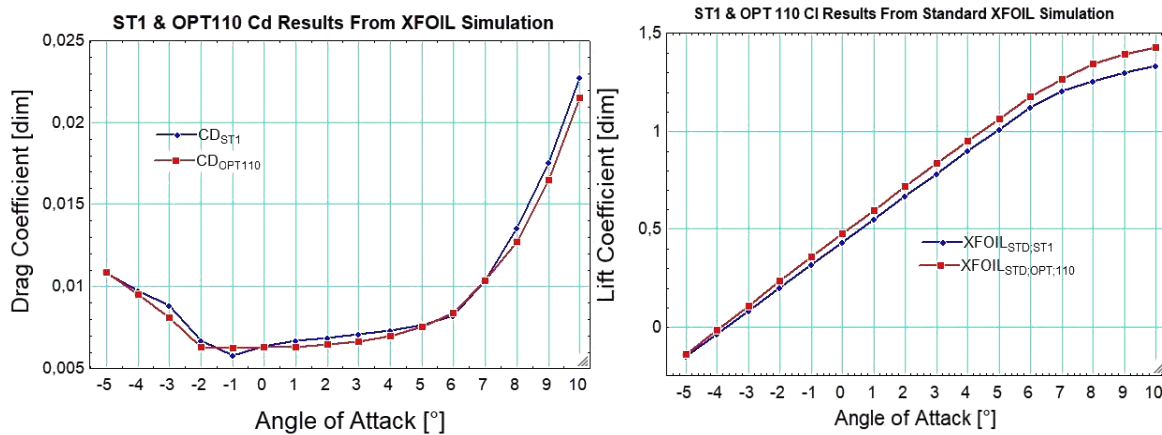


Figure 49: Cd results (left); Cl results from XFOIL simulation (right)

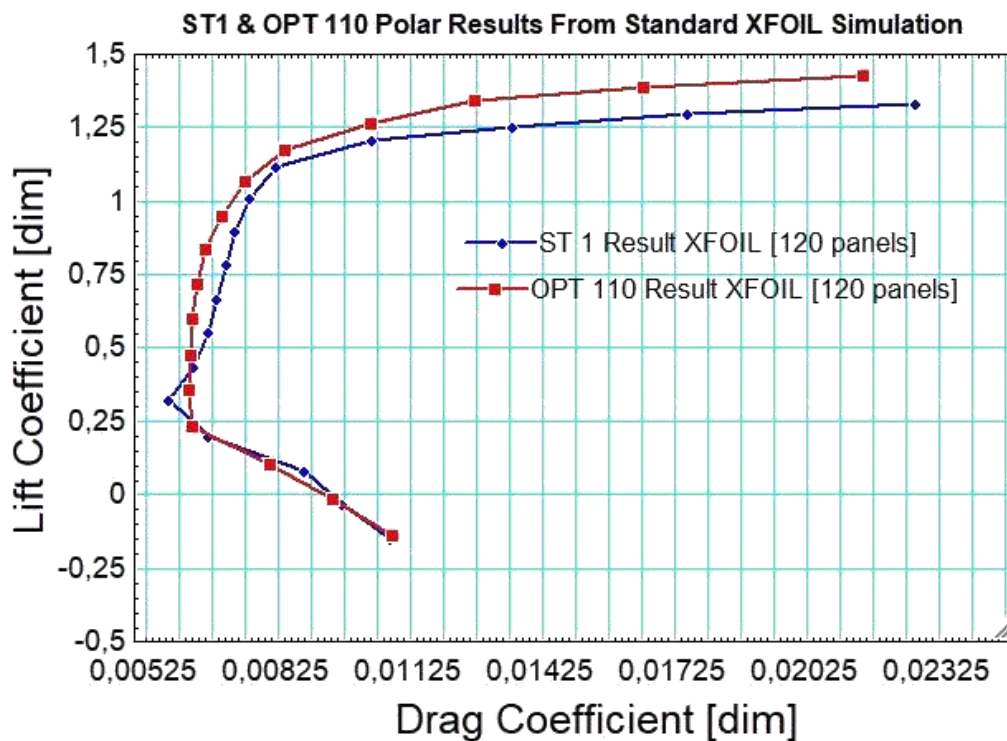


Figure 50: ST1 & OPT110 Polar Results from XFOIL Simulation

6.2.1. ST1 & OPT110 XFOIL SIMULATION RESULTS DISCUSSION

In **Figure 50** it can be seen that the OPT110 outperforms the ST1 aerofoil in terms of lift coefficient where the OPT110 has a higher lift coefficient at every instance of angle of attack. In this figure it can also be observed that the OPT110 aerofoil has a lower drag coefficient for the AOA range of -5 to -2 degrees and again for the AOA range of 0 to 10 degrees. In **Figure 50** it is clear from the polar plots of the ST1 and the OPT110 that the CL VS CD curve of the OPT110 aerofoil is shifted to the left and slightly lifted from the ST1 polar curve.

6.3. ST1 & OPT110 STANDARD STAR-CCM+ SIMULATION

The next step of the case study is to perform a Star-CCM+ CFD simulation of the OPT110 and ST1 aerofoils at the same AOA range and Re number as that of the XFOIL simulation. The simulation results will then be evaluated against the results of the XFOIL simulation.

6.3.1. STAR-CCM+ SIMULATION SET-UP

To evaluate the accuracy of the predictions made for the lift and drag coefficients as in our CFD simulation, a mesh convergence study will be performed. The mesh convergence study will be carried out for the ST1 aerofoil under the constraints as set in the **Table 32**. Using our XFOIL data as reference, the feasibility of each domain size will be evaluated against the XFOIL predicted aerodynamic coefficient in the same angle of attack.

The turbulence model used for our 2d simulation is the SST (Menter) $k - \omega$ model coupled with the $\gamma - Re_{\theta}$ transition model. For this model, the low- y^+ wall model must be chosen and the wall distance in the boundary layer must be set up in such a manner that the y^+ value stays between 0 and 1 for the entire duration of the simulation. The free stream is defined as all triangular meshers larger than 4 mm. Therefore the target surface size and surface size of our aerofoil can be defined as 3 mm and 1 mm respectively.

Property	Value
Mesher type	Triangular mesher
Distribution mode	Wall thickness
Near core layer aspect ratio	0.6
Surface growth rate	1.05
Number of prism layers	15
Prism layer total thickness	14
Reynolds number	1e6
Target surface size	3
Minimum surface size	1
Wake spread angle [radians]	0.17
Wake isentropic size [mm]	30
Wake growth rate	1.05

Table 32: Star-CCM+ Simulation parameters

We simulated the ST1 aerofoil with the constraints of **Table 32** until convergence of the residuals was met. **Table 33** indicates that if the square domain is increased from a 18 m by 18 m domain to a 32 m by 32 m domain the change in lift coefficient is 1.16%; the change in drag coefficient is 2.52%.

The larger domain has a smaller absolute relative error with respect to the XFOIL simulation case and therefore the 32 m by 32 m square domain will be used for our Star-CCM+ simulation study in both the ST1 and OPT110 simulations.

Case Study	AOA [degrees]	Cl [dim]	Cd [dim]	Cl Relative abs error [%]	Cd Relative abs error [%]
XFOIL	0	0.4341	0.00636	0	0
CFD (18mx18m domain)	0	0.4142	0.00675	4.58	6.13
CFD (32mx32m domain)	0	0.4190	0.00658	3.48	3.46

Table 33: Convergence study for ST1

6.4. ST1 & OPT110 STAR-CCM+ SIMULATION

The simulations for the various angles of attack for the ST1 and OPT110 aerofoils were carried out using the 32 m by 32 m square domain accompanied by the models and constants as defined in **Section 6.3**. From **Figure 51** the same prediction tendency of lift coefficient where the OPT110 outperforms the ST1 aerofoil for every instance of AOA simulated, is evident. This tendency almost exactly correlates with that predicted in our XFOIL simulation for the same range of AOA and Re number. However, for the drag coefficient the Star-CCM+ simulation predicts that the OPT110 has a higher drag coefficient from the AOA range of -2 to 10 degrees which is in direct contradiction to that predicted by the XFOIL simulation.

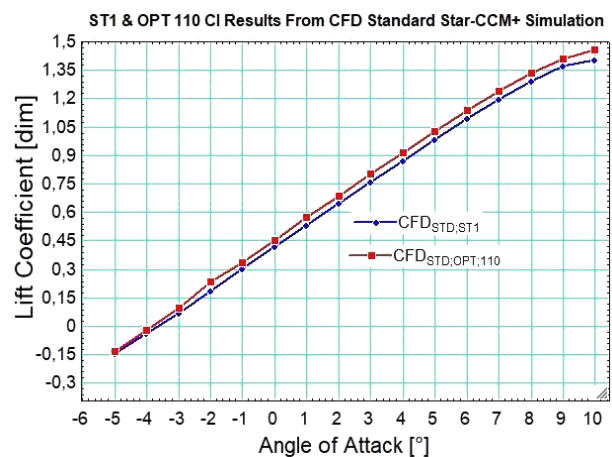
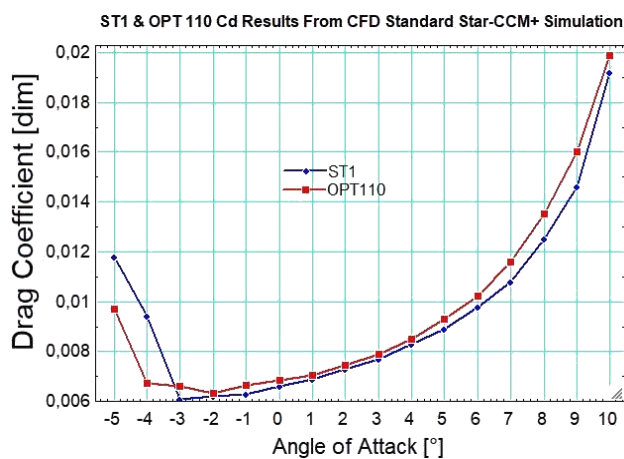


Figure 51: Cd Results (left); Cl Results from Star-CCM+ simulation (right)

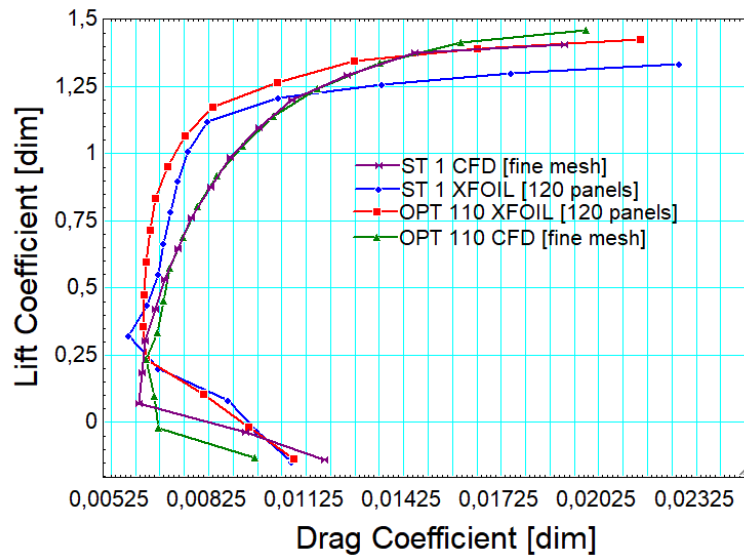


Figure 52: ST1 & OPT110 Polar results from Star-CCM+ & XFOIL simulation

Referring to **Figure 52**, it should be clear that the Star-CCM+ predicts that the aerodynamic performance of the ST1 and the OPT110 are almost exactly the same over the majority of angles of attack simulated. As can be seen, for the case of the XFOIL simulation, there was a clear distinction between the OPT110 and ST1 performance. This discrepancy between XFOIL and CFD is the base motivation for the development of the far-field drag extraction tool.

6.5. EVALUATION OF STAR-CCM+ AND XFOIL DRAG RESULTS

This main point will be dedicated to the discrepancies in the differences of drag coefficients as predicted by XFOIL and Star-CCM+. In **Section 6.4** it is demonstrated that the lift coefficients as predicted by XFOIL and Star-CCM+ correlate with almost exact precision. The maximum absolute difference for the CL prediction between XFOIL and Star-CCM+ for the ST1 aerofoil case equates to 0.076, and for the OPT110 aerofoil case this difference equates to 0.036. The maximum absolute difference for both cases is deemed sufficiently small and we can assume that the XFOIL and Star-CCM+ models chosen are in the same order of accuracy for lift coefficient prediction.

Angle of attack	Cl ST1 (XFOIL)	Cl ST1 (Star-CCM+)	Absolute difference	Cl OPT110 (XFOIL)	Cl OPT110(Star-CCM+)	Absolute difference
[Degrees]	[dim]	[dim]	[dim]	[dim]	[dim]	[dim]
-5.00000	-0.14940	-0.14071	0.00869	-0.13740	-0.13360	0.00380
-4.00000	-0.03460	-0.03635	0.00175	-0.01740	-0.02135	0.00395
-3.00000	0.08260	0.06818	0.01442	0.10520	0.09762	0.00758
-2.00000	0.20100	0.18522	0.01578	0.23460	0.23460	0.00000
-1.00000	0.32060	0.30341	0.01719	0.35620	0.33322	0.02298
0.00000	0.43410	0.41898	0.01512	0.47600	0.45185	0.02415
1.00000	0.55090	0.53298	0.01792	0.59580	0.57281	0.02299
2.00000	0.66600	0.64765	0.01835	0.71530	0.68631	0.02899
3.00000	0.78210	0.76122	0.02088	0.83340	0.80211	0.03129
4.00000	0.89780	0.87480	0.02300	0.95070	0.91677	0.03393
5.00000	1.00980	0.98640	0.02340	1.06370	1.02794	0.03576
6.00000	1.11940	1.09419	0.02521	1.17350	1.13716	0.03634
7.00000	1.20560	1.19772	0.00788	1.26520	1.23993	0.02527
8.00000	1.25490	1.29135	0.03645	1.34440	1.33426	0.01014
9.00000	1.29890	1.37437	0.07547	1.38950	1.41038	0.02088
10.00000	1.33180	1.40320	0.07140	1.42510	1.45869	0.03359
Maximum error [dim]			0.07547	-		0.03634

Table 34: Lift coefficient prediction difference between XFOIL and Star-CCM+

Investigating the skin friction drag coefficient (Cdf) as calculated by XFOIL and Star-CCM+ for the ST1 and OPT110 aerofoils in the AOA range between -5 to 10 degrees at a Re of $1e6$ [dim], it is evident that the results correlate for both simulations with minor differences. The average drag count difference for the Cdf predictions between XFOIL and Star-CCM+ for the ST1 aerofoil case equates to 8.92, and for the OPT110 aerofoil case this difference equates to 6.96.

For the case of the pressure drag the average absolute drag count difference between XFOIL and Star-CCM+ for the ST1 aerofoil is 9.46 and for the OPT110 aerofoil this value is 9.69 drag counts. There is thus a larger average difference between the pressure drag coefficients than those of the skin friction drag coefficient as predicted by the two methods. It is therefore feasible to assume that the skin friction drag of Star-CCM+ is more accurate, and therefore, in the following section, the larger pressure drag difference will be rectified with the far-field drag procedure.

Angle of attack	Cdf ST1 (XFOIL)	Cdf ST1 (Star-CCM+)	Absolute difference	Cdf OPT110 (XFOIL)	Cdf OPT110(Star-CCM+)	Absolute difference
[Degrees]	[dim]	[dim]	[dim]	[dim]	[dim]	[dim]
-5.00000	0.00609	0.00516	0.00093	0.00621	0.00486	0.00135
-4.00000	0.00622	0.00483	0.00139	0.00603	0.00327	0.00276
-3.00000	0.00621	0.00322	0.00299	0.00531	0.00365	0.00166
-2.00000	0.00479	0.00351	0.00128	0.00393	0.00393	0.00000
-1.00000	0.00386	0.00373	0.00013	0.00402	0.00391	0.00011
0.00000	0.00387	0.00385	0.00002	0.00399	0.00397	0.00002
1.00000	0.00399	0.00394	0.00005	0.00404	0.00398	0.00006
2.00000	0.00402	0.00397	0.00005	0.00409	0.00398	0.00011
3.00000	0.00404	0.00399	0.00005	0.00405	0.00397	0.00008
4.00000	0.00405	0.00397	0.00008	0.00411	0.00394	0.00017
5.00000	0.00403	0.00390	0.00013	0.00411	0.00391	0.00020
6.00000	0.00408	0.00380	0.00028	0.00422	0.00388	0.00034
7.00000	0.00476	0.00370	0.00106	0.00469	0.00391	0.00078
8.00000	0.00557	0.00385	0.00172	0.00507	0.00416	0.00091
9.00000	0.00605	0.00392	0.00213	0.00545	0.00420	0.00125
10.00000	0.00647	0.00449	0.00198	0.00584	0.00453	0.00131
Average difference [dim]			0.00089	-		0.00070
Average drag count difference [dim]			8.92	-		6.96

Table 35: Skin drag coefficient prediction difference between XFOIL and Star-CCM+

Angle of attack	Cdp ST1 (XFOIL)	Cdp ST1 (Star-CCM+)	Absolute difference	Cdp OPT110 (XFOIL)	Cdp OPT110(Star-CCM+)	Absolute difference
[Degrees]	[dim]	[dim]	[dim]	[dim]	[dim]	[dim]
-5.00000	0.00473	0.00665	0.00192	0.00465	0.00482	0.00017
-4.00000	0.00350	0.00459	0.00109	0.00345	0.00344	0.00001
-3.00000	0.00264	0.00290	0.00026	0.00277	0.00294	0.00017
-2.00000	0.00190	0.00264	0.00074	0.00239	0.00239	0.00000
-1.00000	0.00194	0.00259	0.00065	0.00221	0.00276	0.00055
0.00000	0.00249	0.00273	0.00024	0.00228	0.00288	0.00060
1.00000	0.00270	0.00295	0.00025	0.00227	0.00309	0.00082
2.00000	0.00284	0.00329	0.00045	0.00235	0.00347	0.00112
3.00000	0.00305	0.00374	0.00069	0.00256	0.00394	0.00138
4.00000	0.00324	0.00432	0.00108	0.00287	0.00457	0.00170
5.00000	0.00359	0.00505	0.00146	0.00340	0.00538	0.00198
6.00000	0.00415	0.00595	0.00180	0.00417	0.00636	0.00219
7.00000	0.00562	0.00711	0.00149	0.00567	0.00767	0.00200
8.00000	0.00800	0.00869	0.00069	0.00765	0.00936	0.00171
9.00000	0.01149	0.01069	0.00080	0.01107	0.01182	0.00075
10.00000	0.01626	0.01474	0.00152	0.01569	0.01534	0.00035
Average difference [dim]			0.00095	-		0.00097
Average drag count difference [dim]			9.46	-		9.69

Table 36: Pressure drag coefficient prediction difference between XFOIL and Star-CCM+

6.6. ST1 & OPT110 FAR-FIELD SIMULATION

In this section it will be demonstrated how the far-field procedure as described in **Chapter 4** has been utilised to determine the drag of the ST1 and OPT110 aerofoils from analysis of the far-field wake. Because, as previously explained, the lift coefficient predictions done by the standard Star-CCM+ simulation are in close agreement with those predicted by XFOIL, the Star-CCM+ CL values and the far-field Cd values will be used to plot the polar results for the St1 and OPT110 aerofoils. This will ensure clear distinctions between any differences in the predicted polars resulting from the drag predicted from the far-field method will be visualised.

6.6.1. FAR-FIELD DRAG ANALYSIS SIMULATION SETUP

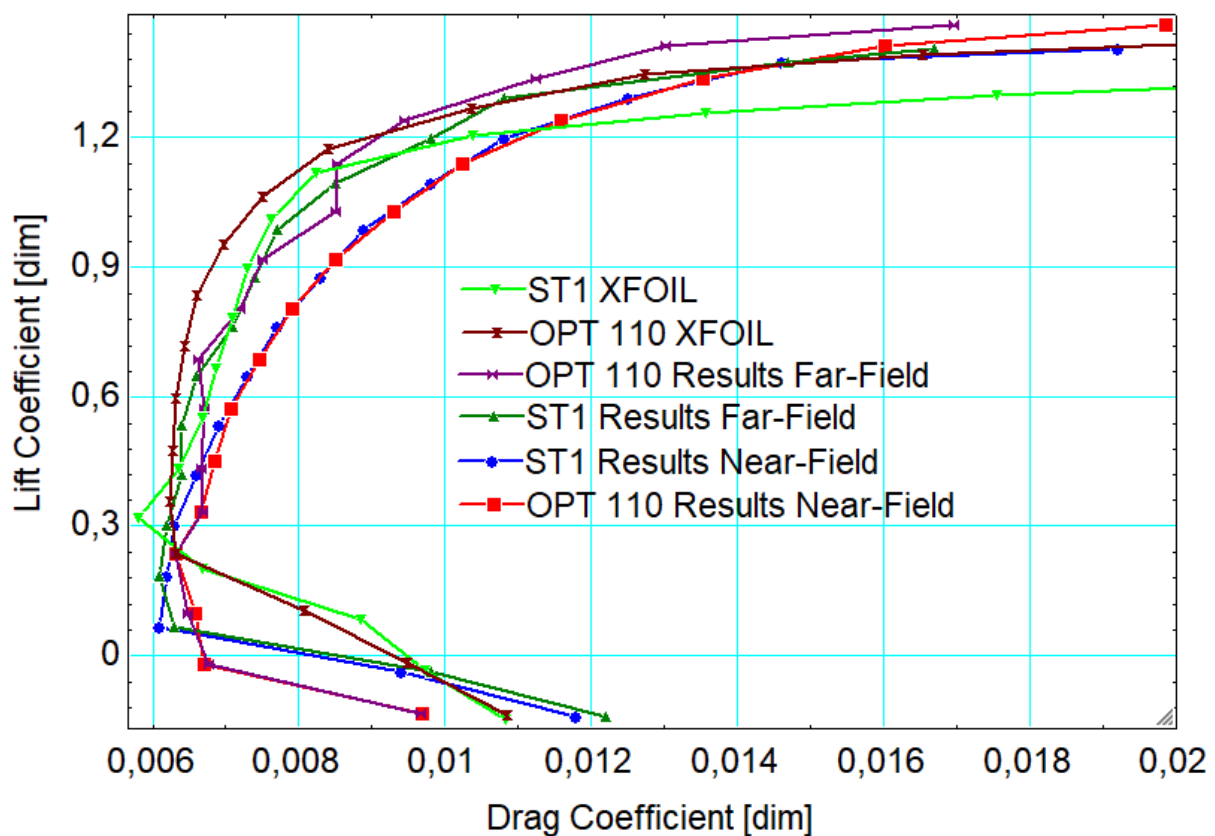
For the far-field results to be relatable with the standard Star-CCM+ simulation for every instance of AOA, the extraction method has been applied to each converged simulation of **Section 6.4**. This means that the Star-CCM+ lift coefficient and the far-field drag coefficient can be utilised to plot the polar curves for each aerofoil. By applying the far-field drag algorithm to each converged Star-CCM+ simulation also means that all the meshing, domain and physics parameters may be kept constant. This implies that the difference in drag accumulated was a result of the different methods used rather than varying parameters in the simulation setup.

6.6.2. ST1 & OPT110 FAR-FIELD DRAG ANALYSIS RESULTS

Studying **Figure 53**, resulting from the far-field drag analysis, it is evident that the polar curve of the far-field method is more in agreement with the XFOIL polar curve than with the standard Star-CCM+ polar curve. The far-field polar curves of the ST1 and OPT110 aerofoils lie between the XFOIL and standard Star-CCM+ curves. **Figure 53** also illustrates that the far-field method predicts a clear difference between the aerodynamic characteristics of the ST1 and OPT110 aerofoils. However, it has previously been pointed out that the standard Star-CCM+ simulation does not predict a difference between the performance of these two aerofoils. Some of the reasons why the results of the far-field and near-field methods differ here, can be ascribed to the following:

- The OPT110 has a higher degree of curvature at the leading edge, trailing edge and at the position of maximum thickness.

- This means that an even finer mesh is needed to describe the OPT110 geometry in order to accurately capture the pressure distribution at the regions of higher curvature.
- Seeing that the meshing parameters used for the ST1 and OPT110 simulations were exactly the same and already very fine, an even more refined mesh would result in extremely long simulation times.
- The far-field method yields higher independence on mesh refinement to accurately capture the total drag as it uses the pressure deficit at the far-downstream wake. This means that the errors in pressure drag associated with inadequate mesh refinement in the near-field method is not included in the far-field method. Thus it is clear that the far-field method yielded more accurate results compared to the near-field simulation



results.

Figure 53: ST1 & OPT110 Polar results from far-field analysis, XFOIL and Star-CCM+

6.7. XFOIL, CFD & FAR-FIELD DRAG COMPARISON

As previously stated and illustrated, the skin friction drag as computed by XFOIL and Star-CCM+ is in close agreement. For this reason the total profile drag coefficient as calculated from the wake analysis will be used, and the Star-CCM+ skin friction drag coefficient will be subtracted from this value to arrive at the far-field pressure drag coefficient.

This method has been implemented for each angle of attack and the average absolute pressure drag coefficient error of the far-field pressure drag coefficient (C_{dp}) was calculated. The error was calculated with regard to the XFOIL pressure drag coefficient, similar to that in **Section 6.5**. Studying **Table 37** below, it is clear that the average drag count difference for the C_{dp} predictions between XFOIL and the far-field analysis for the ST1 aerofoil case equates to 8.31 which is 1.15 drag counts lower than the standard Star-CCM+ simulation. For the OPT110 aerofoil case this difference equates to 7.03 which is 2.66 drag counts lower than the standard Star-CCM+ simulation. Inspection of **Figure 54** clearly indicates that the pressure drag coefficient prediction of the far-field method is more in agreement with the benchmark data than the corresponding CFD value. The curve tendency of the C_{dp} from the far-field analysis method closely correlates with the XFOIL predicted curve. Due to this pressure drag improvement there is an improvement in the overall accuracy and performance of the profile drag coefficient of the far-field analysis method compared to the near-field method.

Angle of attack	C_{dp} ST1 (XFOIL)	C_{dp} ST1 (Far-Field)	Absolute difference	C_{dp} OPT110 (XFOIL)	C_{dp} OPT110 (Far-Field)	Absolute difference
[Degrees]	[dim]	[dim]	[dim]	[dim]	[dim]	[dim]
-5.00000	0.00473	0.00707	0.00234	0.00465	0.00481	0.00016
-4.00000	0.00350	0.00492	0.00142	0.00345	0.00349	0.00004
-3.00000	0.00264	0.00309	0.00045	0.00277	0.00282	0.00005
-2.00000	0.00190	0.00257	0.00067	0.00239	0.00239	0.00000
-1.00000	0.00194	0.00250	0.00056	0.00221	0.00278	0.00057
0.00000	0.00249	0.00258	0.00009	0.00228	0.00271	0.00043
1.00000	0.00270	0.00244	0.00026	0.00227	0.00272	0.00045
2.00000	0.00284	0.00264	0.00020	0.00235	0.00265	0.00030
3.00000	0.00305	0.00309	0.00004	0.00256	0.00322	0.00066
4.00000	0.00324	0.00341	0.00017	0.00287	0.00356	0.00069
5.00000	0.00359	0.00384	0.00025	0.00340	0.00460	0.00120
6.00000	0.00415	0.00469	0.00054	0.00417	0.00463	0.00046
7.00000	0.00562	0.00607	0.00045	0.00567	0.00553	0.00014

8.00000	0.00800	0.00691	0.00109	0.00765	0.00708	0.00057
9.00000	0.01149	0.01079	0.00070	0.01107	0.00882	0.00225
10.00000	0.01626	0.01220	0.00406	0.01569	0.01242	0.00327
Average difference [dim]			0.00083	-		0.00070
Average drag count difference [dim]			8.31	-		7.03

Table 37: Pressure drag coefficient prediction difference between XFOIL and far-field analysis

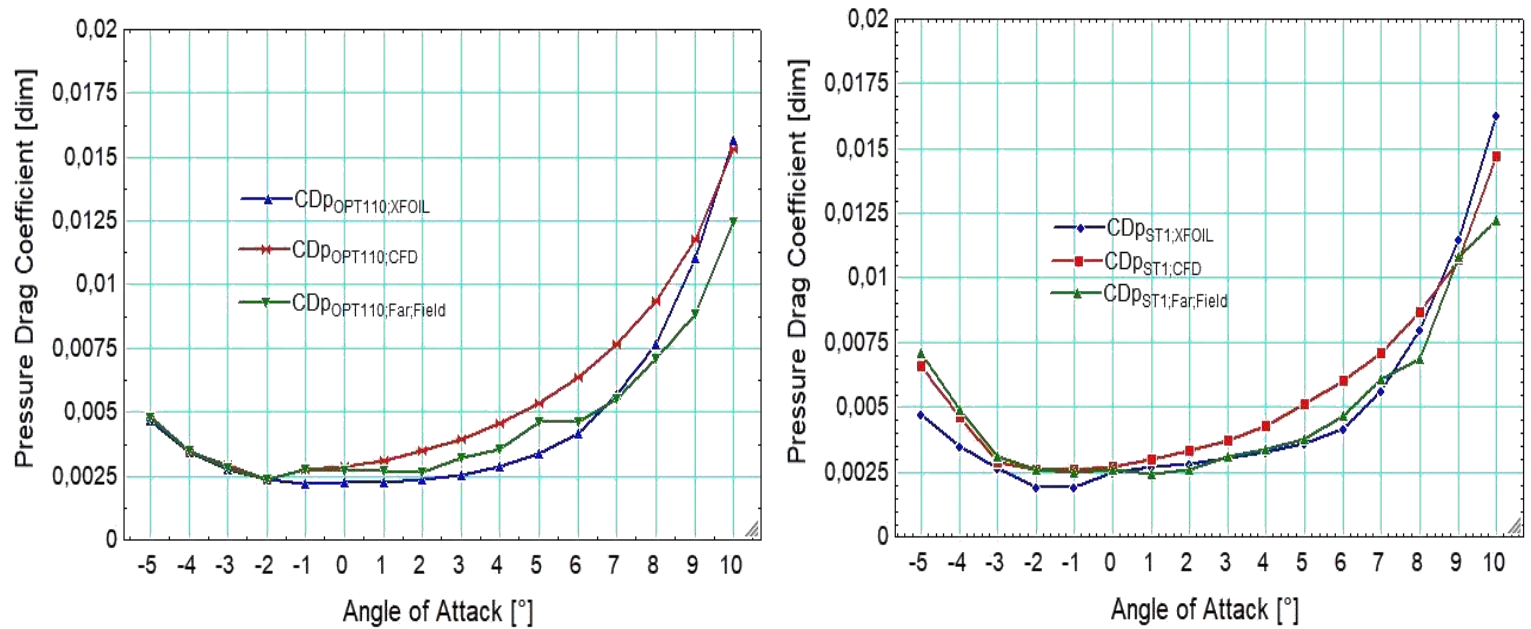


Figure 54: (ST1: Left), (OPT110: Right) Pressure drag coefficient prediction from XFOIL, Star-CCM+ & Far-Field Analysis

6.8. CONCLUSION

In this chapter it was clearly demonstrated that both XFOIL and Star-CCM+ predict lift coefficients to the same order magnitude for both the ST1 and OPT110 aerofoils at an AOA range of -5 to 10 degrees and a Re number of one million. A major discrepancy between XFOIL and Star-CCM+ was observed in the drag coefficient predictions for these two aerofoils.

Investigating the skin friction drag and pressure drag components of the total profile drag as predicted by both XFOIL and Star-CCM+, it was noted that there is a larger difference between the pressure drag coefficients predicted by both programs. After investigation it was concluded that the Star-CCM+ skin friction drag coefficient is more in agreement with the benchmark data than the Star-CCM+ pressure drag coefficient. Therefore, the far-field drag algorithm was deployed to rectify the error in the pressure drag coefficient.

As XFOIL has proven its accuracy on numerous occasions, the feasibility of the newly calculated pressure drag coefficients with regard to the XFOIL values for the same instance of angle of attack were evaluated. A major improvement of the pressure drag coefficient was noted with the far-field analysis approach.

It is important to note that the method of profile drag coefficient calculation as implemented in XFOIL and the far-field drag analysis method used by us differ greatly. XFOIL uses the Squire-Young model to extrapolate the momentum deficit to downstream infinity in order to evaluate the profile drag coefficient, whereas our far-field drag analysis method utilizes the CFD solution of the dynamic pressure profile in a stabilized region to determine the profile drag coefficient. Star-CCM+ calculates the profile drag via surface integration of the stresses in the normal and tangential directions.

The reason why the far-field drag analysis method yields profile drag values in close agreement with that of XFOIL can be described as follows:

- The dynamic pressure magnitudes use in the far-field integration process is locally solved directly and accurately by the Star-CCM+ models.
- Lower numerical and truncation errors and reduced sensitivity to boundary conditions means that more accurate drag terms is calculated (Zhu, Wang, Liu, & Liu, 2007) (Wang, Wang, Liu, & Jiang, 2018).

Differences between the profile drag of the far-field analysis method and XFOIL can be described as follows:

- The Squire-Young method uses a function to extrapolate the local fluid conditions to downstream infinity to estimate the profile drag.
- XFOIL uses a global iterative Newton method to solve for the lift and drag coefficients whereas the far-field method uses an unequally spaced Simpsons rule. There is thus a difference in the error of approximation of the two methods.
- The Squire-Young model of XFOIL is implemented one chord length from the TE, whereas the far-field analysis is applied at an average of six to eight chord lengths from the TE for Reynolds numbers between 5×10^5 to 1×10^6 .

Chapter 7: Conclusion and Recommendations

7.1. PANEL CODE CONCLUSION

In **Chapter 3** it was shown that for low Reynolds numbers the panel code XFOIL predicted the lift and drag coefficients of aerofoils with great precision when compared to benchmark results. Here however, there were discrepancies found in the lift coefficient predictions for the E231 aerofoil. This was due to the envelope factor setting as it was chosen that this factor need be set to accurately model drag rather than lift. The simulations in **Chapter 3** clearly displayed the known tendency of XFOIL to under predict drag and over predict lift as the angle of attack is increased. A method to counter this effect was developed (see **Appendix B**). The method consisted of interpolating each data set describing the aerofoil surface with a first, second or third order routine. This means that the user could define regions of high curvature and areas of transition with a more descriptive panel distribution, thus allowing XFOIL to more accurately capture characteristics like laminar separation bubble formation, turbulent re-attachment and laminar-turbulent transition. The method was deployed to cluster panel nodes at regions of high curvature for the aerofoils used in **Chapter 3**. With the performance of XFOIL validated for low Reynolds number simulations, and the benchmark results approximately reproduced, these simulation results were used as reference data where experimental data was not available. One such case where the XFOIL simulation results were used as reference data was discussed in **Chapter 6**, where the near-field and proposed far-field methods for drag prediction were used to determine the aerodynamic performance characteristics of the ST1 and OPT110 aerofoils. For these two aerofoils, i.e. the ST1 and the OPT110, classical CFD codes such as Star-CCM+ yielded discrepancies in the performance predictions of said aerofoils. XFOIL simulation results were thus used as benchmark data to evaluate the errors in near-field predictions of the pressure and friction drag components. These errors were then rectified with the implementation of the proposed far-field drag prediction methods. The coordinate files for the ST1 and OPT110 aerofoils are redacted from this thesis as they are considered proprietary information.

7.2. CFD AND FAR-FIELD CODE CONCLUSION

In **Chapter 4** various methodologies and procedures were discussed to model drag via far-field wake analysis methods. These methods include the use of the dynamic pressure and velocity distribution in the far-field viscous wake. The far-field method makes use of a classical Trefftz plane analysis from where the flow variables projected onto the defined Trefftz plane are extracted into third party software where the far-field integration processes described in **Chapter 4** are used to determine the profile drag. As the definition of the Trefftz plane length and Trefftz plane location from the trailing edge highly influences the accuracy of the far-field methods, a supplementary study (see **Appendix C**) was conducted. In this supplementary study it is demonstrated how the far-field method has a reduced dependency on the level of mesh refinement used to describe the simulation geometry. This reduced sensitivity of the far-field method on mesh refinement lies in the nature of the methods used. In the far-field method flow variables in the far downstream wake are extracted and integrated. These far-field procedures rely on a momentum deficit approach. Thus errors in flow calculations near stagnation points due to inaccurate mesh refinement, at these stagnation locations, have a reduced effect when a far-field method, i.e. momentum deficit approach, is used. In the supplementary study it is also demonstrated how the position of the Trefftz plane from the trailing edge influences the accuracy of the drag predicted by the far-field methods proposed. It is shown that the results are most accurate when the Trefftz plane is placed at such a distance from the trailing edge that total free-stream static pressure recovery is realized. Furthermore, the supplementary study illustrates how three different methods can be utilized to determine the upper and lower integral bounds of the far-field drag integrals. The most accurate far-field integrals originate from defining the upper and lower integral bounds via noting the first upper and lower node of static pressure recovery in the viscous wake; or by using the laminar and turbulent dynamic viscosity ratios to determine the most oscillatory coordinates in the wake. The best practises for the far-field methods abstracted from the supplementary study was used in **Chapter 5** where it was shown that the proposed far-field drag methods outperform the near-field drag method for the low Reynolds number cases simulated with regard to experimental results. It was also noted that not only does the far-field accuracy outweigh the near-field accuracy, but the far-field accuracy increases as separation becomes more significant. This is because the more separation occurs, the more important the flow variables solved in the boundary layer become, hence the more dependent the solution is on how fine the mesh is around positions of high curvature. Thus, as separation occurs and the geometry is not properly discretised in

these regions, the near-field error due to spurious drag increases. This erroneous process is avoided when using a far-field analysis. Finally, it can be concluded that the far-field methods proposed in this thesis are powerful tools to determine the profile drag of a 2d aerofoil and that these methods can be used to evaluate the feasibility of the grid refinement used in a CFD simulation.

7.3. RECOMMENDATIONS

This thesis has paved the way for implementing various far-field methods for drag prediction in a viscous unstructured two dimensional CFD simulation. The following research ensuing from this research project is recommended:

1. Taking into account that the methods pertained in this thesis only concern low Reynolds number simulations, i.e. between 400 000 and 500 000, it is recommended that research be undertaken to validate the accuracy of the proposed methods for Reynolds numbers between 1-3 million with the appropriate experimental results.
2. It is recommended that the methods used in this thesis be extended and validated for the three dimensional case.
3. The methods of determining the wake length, i.e. the upper and lower bounds for the far-field integral, are developed and discussed in **Appendix C.3**. These are, however, only validated for low Reynolds numbers. Thus these methods should further be researched for simulation cases with higher Reynolds numbers.
4. The method discussed for determining upper and lower far-field integral bounds should also be validated for use in the three dimensional case and additional corrections should be made to the method to ensure that the validity and accuracy of the method is preserved for 3d cases.
5. The bunching method described in **Appendix B** shows promise to improve the accuracy of XFOIL predictions for low Reynolds numbers. This method should be studied in more detail to establish correlations between the density and bunching position regarding accuracy and convergence improvement.
6. The far-field methods developed in this thesis were validated in regions of minor separation. Further research needs to be done on the accuracy of these methods for cases where strong separation occurs.

7. Also, the far-field methods pertained in this thesis was not studied for compressible flow, and therefore these methods need to be studied for flow conditions with Mach numbers between 0.3 and 1.

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APPENDIX A: CFD AND PANEL CODE DEVELOPMENT

A.1. BACKGROUND

Originally panel codes were developed as low-order methods to determine incompressible subsonic flow. In 1968 the first panel program, A203, was described in a paper written by Paul Rubbert and Gary Saaris of Boeing Aircraft (Rubbert, 1972). Later developments led to three dimensional higher order panel code developments, amongst these code developments PMRAC from NASA and PANAIR from Boeing (Carmichael, 1981) (Ashby, Dudley, Iguchi, & Browne, 1991) .

From Boeing's A203 to the latest version of XFLR5 (a C++ version of XFOIL) there was significant progress in the panel code development with the inviscid-viscid coupling methods greatly improving the accuracy of the simulation results. In the 1980's two pioneers individually contributed to significant development in the field of panel code implementation. One of these pioneers was Professor Richard Eppler of the University of Stuttgart with his program PROFIL. The code was developed with the help of NASA funding and released in the early 1980's (Eppler & Somers, 1980). Although the Eppler code was successful in accurately solving the potential flow problem with panel methods, it was limited in the sense that the program could not solve the boundary layer flow. Then, later in the 1980's, Professor Mark Drela of MIT developed the iconic panel method program, XFOIL (Drela, 1989).

XFOIL has been one of the leaders in the field of aerofoil design and optimization. XFOIL is written in FORTRAN and its accuracy and calculation methodology have been the subject of study in various research projects. The code makes use of a linear-vorticity stream function formulation which provides adequate environment for the successful implementation of a model for the viscous displacement effects. By superimposing source distributions on the aerofoil surface and wake, Drela was able to model the viscous layer influence on the potential flow. With a wall transpiration model, the XFOIL code approximates the displacement effect on the outer most inviscid flow. Through the implementation of a two equation lagged dissipation integral method, the viscous and boundary layers are represented and modelled. Finally, with the help of a modified e^n envelope method and the Squire-Young model, the laminar-turbulent transition location and the profile drag can be predicted respectively. The boundary layer equations are solved simultaneously and iteratively with the inviscid flow field by a global Newton method (Drela M. , 2001).

Although XFOIL has had huge success in the research and preliminary design fields since its birth, it has also had some difficulties. XFOIL was very popular amongst users in terms of design, development and optimization of aerofoils of all shapes and sizes. Increased use of XFOIL for aerofoil design and development meant increased availability of research to validate XFOIL's results. It became clear that XFOIL tended to overpredict the lift-slope and underpredict the drag in post-stall regions. This phenomenon has been ascribed to various reasons, amongst the most popular being:

- The wake geometry is determined from inviscid calculations (Giridhar Ramanujam, 2017).
- Presence of laminar separation bubbles (Thomas, 2004).
- Errors in the Falker-Skan boundary layer profile calculations due to the linear envelope approximations and solution methods implemented to solve the stability equation (Wazzen, 1968).

Although XFOIL is the workhorse of the preliminary 2d aerofoil design phase, this method has its limitations as stated above. Therefore, a design tool for better flow visualization and more refined design and optimization processes is needed. The capability of 3d extension for better understanding of the more complex flow field interaction and behaviour will ensure a competent aerodynamicist will be able to design a specialized aerofoil, wing or aircraft with meticulous precision. This is where CFD programs came into perspective.

CFD and its commercial use were familiarized by Brian Spalding and Brian Launder of Imperial College. Their success seeded from the research they did on the $k - \epsilon$ model to popularize its industrial use. Spalding's work on the $k - \epsilon$ model for industrial implementation developed to a point where he founded a company named Concentration Heat and Momentum (CHAM) in Wimbeldon, England, in 1974 which enabled him to capitalize on his research (CHAM, 2017). In 1981 CHAM saw to it that all its CFD codes were consolidated into a single code called Parabolic Hyperbolic or Elliptic Numerical Integration Code Series (PHOENICS). With PHOENICS's launch it set the benchmark as the first commercial-purpose CFD package.

At the time of PHOENICS's release, a group of researchers at the Sheffield University in England was developing CFD codes for combustion analysis under the hand of Jim Swithenbank and Ferit Boysan (Creare, 2016). Their work came to the attention of Bart Patel

of Creare. Creare was, and still is, a technology consultancy situated in Hanover, New Hampshire. Upon recommendations from Bryan Patel, Swithenbank's team began to develop a CFD package called FLUENT with heat transfer, combustion and laminar/turbulent analysis being its main attributes. As time progressed more and more complex commercial CFD codes became available. One of these was the all-purpose CFD simulation package of ANSYS FLUENT.

The ANSYS FLUENT CFD package is based on the principal of solving a set of governing equations numerically. These mathematical equations include conservation of mass, energy, momentum and effects of body forces. CFD solvers as implemented in FLUENT, CFX and Star-CCM+ are based on the finite volume method. This method entails the discretization of the fluid domain into a finite set of control volumes. From there conservation or transport equations for mass, momentum, energy, etc. are solved on this set of control volumes. These transport equations are typically in the form of partial differential equations and they are discretised into a system of algebraic equations. Finally these matrices of algebraic equations are solved for numerically to render the solution field. In CFD programs such as FLUENT and Star-CCM+ the simulation is only as accurate as the mesh and its underlying elements. Element size, shape and spacing specified in the mesh generation phase of simulations should be done in such a manner that a point exists where there is result independence on additional mesh refinement. In CFD codes, simulation convergence relies on appropriateness and accuracy used in the simulation to describe the physics of the model, mesh resolution/independence and numerical errors.

As with XFOIL, CFD codes have had remarkable success in the field of aerodynamics and have also proven themselves a worthy tool for designing and optimizing aerofoils. One of the capabilities of CFD codes (apart from 3d visualization) that trumps XFOIL, is the ability to solve for flow of an object travelling at supersonic and hypersonic speeds. Also, CFD codes provide the user with multiple options of solvers to use in the calculations of laminar/turbulent calculations, namely transition predictions and calculation of the presence of laminar separation bubbles.

APPENDIX B: NON-LINEAR PANEL BUNCHING METHOD

In this section a method of increasing panel density at upper and lower aerofoil surface boundaries before importing the geometry into XFOIL for further analysis will be developed.

The following section will thoroughly explain the methodology implemented to cluster the panels in a non-linear fashion near locations of interest.

B.1. NON-LINEAR PANEL BUNCHING METHODOLOGY

The method implemented for non-linear panel bunching requires some user intuition. The user starts by importing a geometry file specifying the aerofoil's coordinates. A basic viscous-XFOIL simulation is set up at user defined Reynolds number and standard XFOIL parameter settings. From there the user notes the highest top and bottom x-coordinate where transition occurs (in the desired range of angles of attack), as this will be the starting boundary condition for the non-linear bunching method. With the transition location now specified a lower limit is chosen, as this is the end boundary condition for the non-linear bunching procedure. The method is then implemented as follows:

Aerofoil coordinate file in form of:

Node position	$X_{coordinate}$	$Y_{coordinate}$
1	x_1	y_1
.	.	.
.	.	.
n_i	x_i	y_i
n_n	x_n	y_n
.	.	.
n_{XT_2}	x_{XT_2}	y_{XT_2}

Table 38: XFOIL Standard format of aerofoil coordinates

Specify top & bottom start and stop boundary condition: $[XT_1 \ XT_2]$; $[XB_1 \ XB_2]$

*Specify order of bunching: **1, 2 or 3***

For 1st order bunching the following is applicable:

Node position	$X_{coordinate}$	$Y_{coordinate}$
1	x_1	y_1
.	.	.
n_i	x_i	y_i
n_k	x_k	y_k
n_n	x_n	y_n
.	.	.
n_{XT_2}	x_{XT_2}	y_{XT_2}

Nearest position to XT_1 (top surface x-coordinate of interest)

The yellow cells represent the 1st order approximated bunched coordinate.

Table 39: XFOIL Coordinate format for 1st order bunching between 2 data-sets

With:
$$x_k = \frac{x_n + x_i}{2} \text{ and } y_k = \frac{y_n + y_i}{2}$$

This is done until the bottom-coordinate criteria, XT_2 , is reached.

For 2nd order bunching the following is applicable:

Node position	$X_{coordinate}$	$Y_{coordinate}$
1	x_1	y_1
.	.	.
.	.	.
n_i	x_i	y_i
n_{k-1}	x_{k-1}	y_{k-1}
n_k	x_k	y_k
n_{k+1}	x_{k+1}	y_{k+1}
n_n	x_n	y_n
.	.	.
n_{XT_2}	x_{XT_2}	y_{XT_2}

Nearest position to XT_1 (top surface x-coordinate of interest)

The blue cells represent the 2nd order approximated bunched coordinate.

Table 40: XFOIL Coordinate format for 2nd order bunching between 2 data-sets

$$x_k = \frac{x_{k+1} + x_{k-1}}{2} \text{ and } y_k = \frac{y_{k+1} + y_{k-1}}{2}$$

$$x_{k-1} = \frac{x_k + x_i}{2} \text{ and } y_{k-1} = \frac{y_k + y_i}{2}$$

$$x_{k+1} = \frac{x_n + x_k}{2} \text{ and } y_{k+1} = \frac{y_n + y_k}{2}$$

B.2. NON-LINEAR PANEL BUNCHING RESULTS

For a first order bunching setting the algorithm works by calculating a coordinate average between each set of coordinates, and proceeds to do so for the next set of coordinates. This loop is continued from the user specified starting coordinate until the stopping criteria on the top and bottom surface is reached. This means that each original coordinate set is now described by one additional coordinate between them.

For a second order approximation the coordinate array from the first order bunching program is used, and an 'average coordinate' is calculated between the first order approximated coordinate and the original coordinate. This means that each original coordinate set is now described by three additional coordinates between them.

For a third order approximation the coordinate array from the second order bunching program is used, and an 'average coordinate' is calculated between the second order approximated coordinate and the original coordinate. This means that each original coordinate set is now described by seven additional coordinates between them.

The higher the order of approximation the smoother the region of interest is represented in the XFOIL calculation procedures. The ST1 baseline aerofoil coordinate file was subjected to the new bunching algorithm and the results can be seen in the following figures:

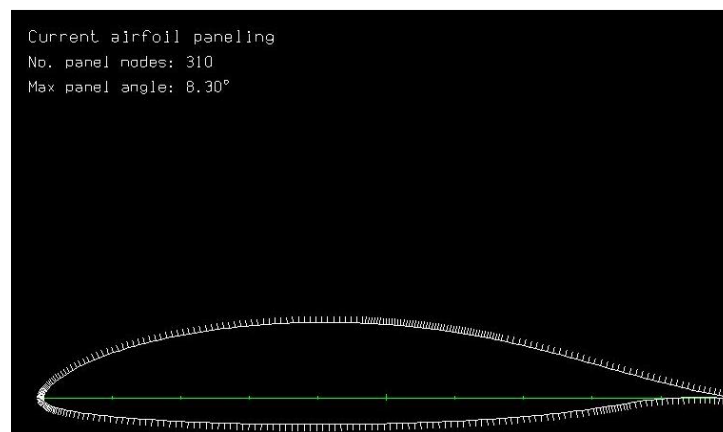


Figure 55: ST1 1st order panel bunching at top and bottom transition locations

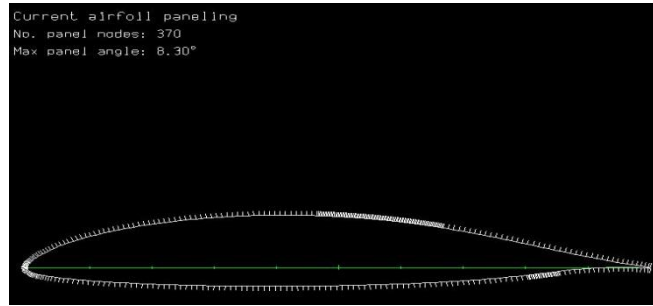


Figure 56: ST1 2nd order panel bunching at top and bottom transition locations

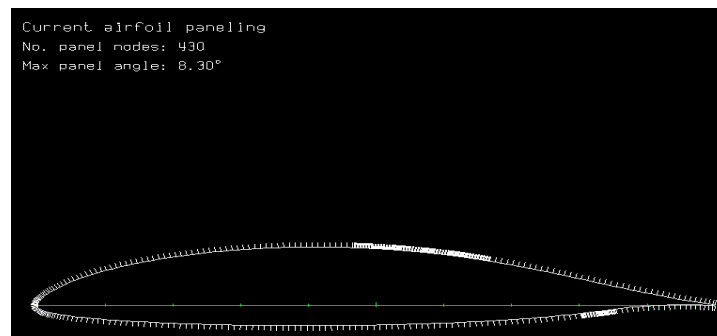


Figure 57: ST1 3rd order panel bunching at top and bottom transition locations (showing every second 3rd order coordinate)

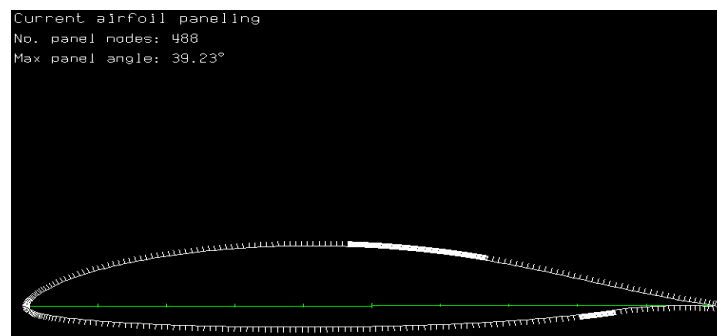


Figure 58: ST1 3rd order panel bunching, at top and bottom transition locations (showing every 3rd order coordinate)

The method as described in this section is used in **Chapter 3** where the panels are bunched before the geometry is imported into XFOIL. Here the XFOIL simulation results from the normal coordinate file and the bunched coordinate file are compared to one another and also to the experimental data.

APPENDIX C: SUPPLEMENT STUDY OF FAR-FIELD DRAG EXTRACTION PROCEDURE

In this section several topics of interest regarding the methodology and accuracy of the far-field drag extraction methods proposed will be investigated. The wind tunnel data of the FX 63-137 aerofoil will be used for the angles of attack of -5.08, 0.07 and 5.14 degrees. The topics of interest will include a study on:

- I. The reduced sensitivity to mesh refinement of the proposed far-field method as opposed to the traditional near-field method
- II. The effects of Trefftz plane placement on the accuracy of the proposed far-field drag extraction method
- III. The establishment of a “safe” region where far-field drag integration can be initialized and ceased to ensure the viscous wake region is fully captured in the drag calculation progress

C.1. MESH REFINEMENT INFLUENCE ON FAR-FIELD DRAG EXTRACTION ACCURACY

As discussed throughout this thesis, the far-field method shows improved accuracy of drag prediction when compared to the near-field method, because it has a lower dependence on mesh resolution to accurately capture the pressure drag component. To demonstrate this, a coarse, medium and fine grid simulation was set up in Star-CCM+ for the angle of attack cases of -5.08, 0.07 and 5.14 degrees for the FX 63-137 aerofoil. In each case the experimental results are compared to the accuracy of the near-field and various far-field methods. The far-field methods used were those described in **Chapter 4** and include the wake-rake, dynamic pressure, momentum thickness and Squire-Young models. As the momentum thickness method and the Squire-Young model require an inviscid velocity, an inviscid simulation was first set-up for these cases and the average inviscid velocity at the respective points of interest was calculated to be used later in the integration process. The coarse, medium and fine grid properties for the mentioned angle of attack cases can be seen in **Tables 41-43**. For all simulation cases in this section, the Reynolds number was set to 499818.

	Triangular cells	Quadrilateral cells	Interior faces	Vertices	Total Cells
Coarse	125630	5246	198555	68443	130876
Medium	203887	6369	318132	108749	210256
Fine	660693	10033	1010490	340995	670726

Table 41: -5.08 degrees AOA coarse, medium and fine mesh properties

	Triangular cells	Quadrilateral cells	Interior faces	Vertices	Total Cells
Coarse	116208	5214	184362	63700	121423
Medium	195759	6390	305982	104706	202149
Fine	667709	10045	1021038	344515	677754

Table 42: 0.07 degrees AOA coarse, medium and fine mesh properties

	Triangular cells	Quadrilateral cells	Interior faces	Vertices	Total Cells
Coarse	128840	5229	203338	70029	134069
Medium	197519	6339	308523	105532	203858
Fine	692955	9939	1058700	357027	702894

Table 43: 5.14 degrees AOA coarse, medium and fine mesh properties

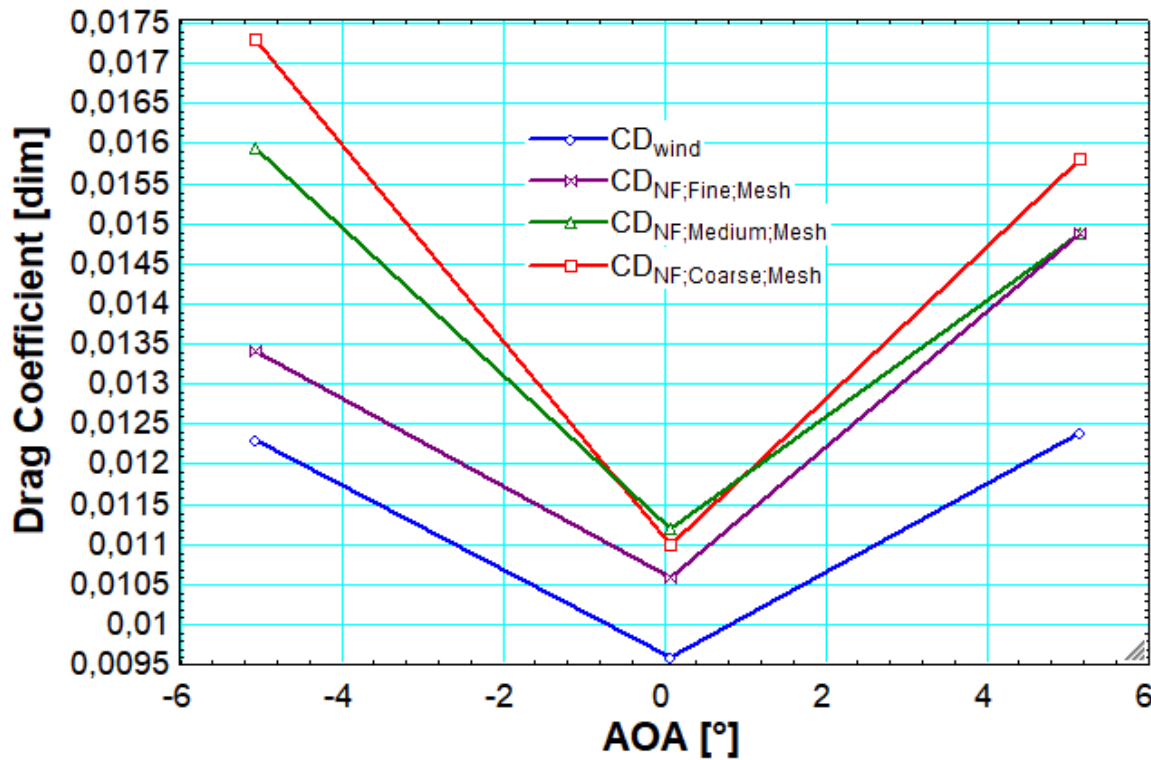


Figure 59: Near-field drag results for coarse, medium and fine grid simulations

The results of the simulations for our angles of attack for the coarse, medium and fine grid can be seen in **Figure 59**. As expected, as the mesh becomes more refined, so does the accuracy of the near-field profile drag predictions increase. If the grid is refined even more than in the case of our fine grid, the accuracy of the near-field method for profile drag prediction will approximate the experimental results. However, this is impractical from a preliminary design perspective as this extensively adds to the computational expense.

To evaluate if our proposed far-field methods yield lower dependence on grid refinement whilst maintaining high order accuracy predictions, the velocity profiles and dynamic pressure profiles along with the averaged inviscid flow velocity were extracted on a Trefftz plane no more than 8 chord lengths from the trailing edge of the aerofoil. This distance was chosen arbitrarily as this is where approximate total static pressure recovery was firstly noted. Integration of the flow variables were initialized and ceased where the wake region first stabilized in the positive and negative free-stream directions. The results of the far-field drag extraction methods can be seen in **Tables 44-46** and in **Figures 60-63**.

Angle of Attack	CD (Wind Tunnel)	CD (Near-Field)	CD (Squire-Young)	CD (Dynamic Pressure)	CD (Wake Rake)	CD (Momentum Thickness)	CL (Wind Tunnel)	CL (Near Field)	Trefftz Plane Dist.
[°]	[dim]	[dim]	[dim]	[dim]	[dim]	[dim]	[dim]	[dim]	[m from TE]
-5.08	0.0123	0.017309462	0.016691014	0.017697631	0.017435681	0.016674709	0.2590	0.234158332	8
0.07	0.0096	0.010996769	0.010321545	0.009897122	0.009826367	0.010322211	0.8410	0.871378159	8
5.14	0.0124	0.015808058	0.011583358	0.013096558	0.012964749	0.011560098	1.3600	1.396629083	8

Table 44: Coarse grid simulation results for near-field and far-field methods

Angle of Attack	CD (Wind Tunnel)	CD (Near-Field)	CD (Squire-Young)	CD (Dynamic Pressure)	CD (Wake Rake)	CD (Momentum Thickness)	CL (Wind Tunnel)	CL (Near Field)	Trefftz Plane Dist.
[°]	[dim]	[dim]	[dim]	[dim]	[dim]	[dim]	[dim]	[dim]	[m from TE]
-5.08	0.0123	0.015948291	0.015192993	0.016091786	0.01587461	0.01523948	0.2590	0.234175146	8
0.07	0.0096	0.011204597	0.010286793	0.009853389	0.009765124	0.010281968	0.8410	0.880219371	8
5.14	0.0124	0.014881799	0.011409031	0.012515021	0.012301836	0.011414234	1.3600	1.403318525	8

Table 45: Medium grid simulation results for near-field and far-field methods

Angle of Attack	CD (Wind Tunnel)	CD (Near-Field)	CD (Squire-Young)	CD (Dynamic Pressure)	CD (Wake Rake)	CD (Momentum Thickness)	CL (Wind Tunnel)	CL (Near Field)	Trefftz Plane Dist.
[°]	[dim]	[dim]	[dim]	[dim]	[dim]	[dim]	[dim]	[dim]	[m from TE]
-5.08	0.0123	0.013399753	0.012843511	0.013492309	0.013229994	0.012758018	0.2590	0.238692457	8
0.07	0.0096	0.010585682	0.010739377	0.010224098	0.010115867	0.010731261	0.8410	0.863277366	8
5.14	0.0124	0.014881659	0.011240754	0.012512627	0.012299441	0.01124588	1.3600	1.40332228	8

Table 46: Fine grid simulation results for near-field and far-field methods

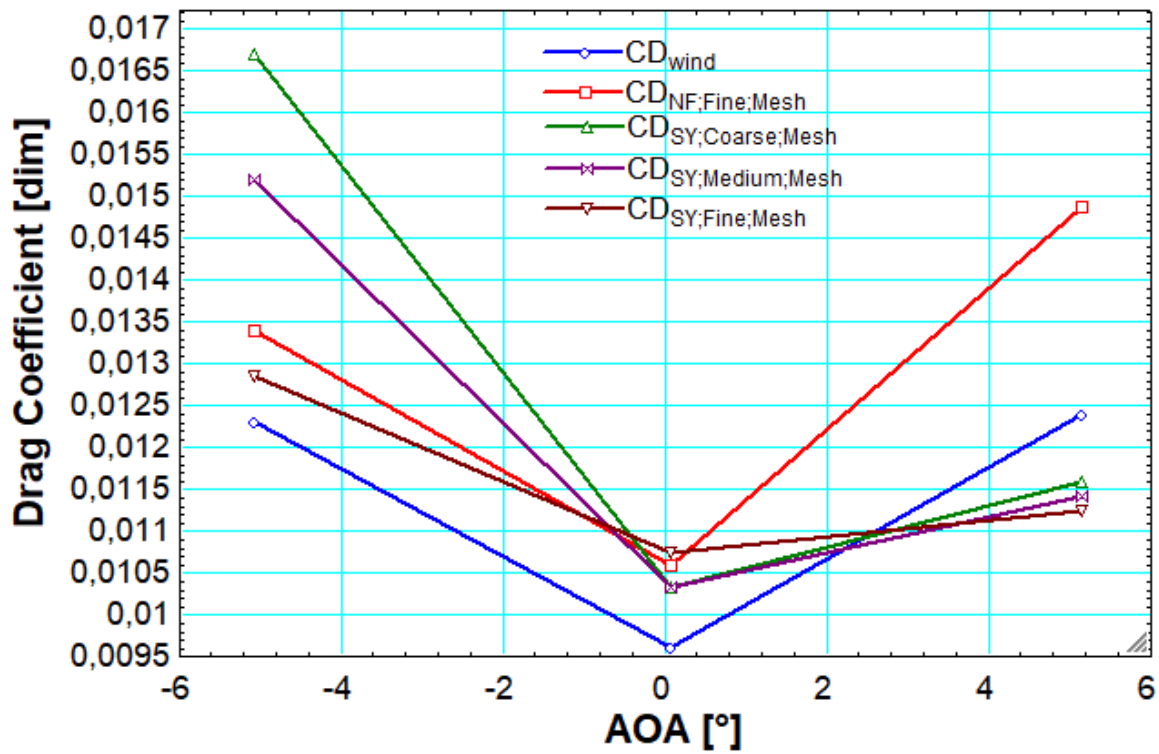


Figure 60: Coarse, medium & fine grid simulation results for Squire-Young far-field model

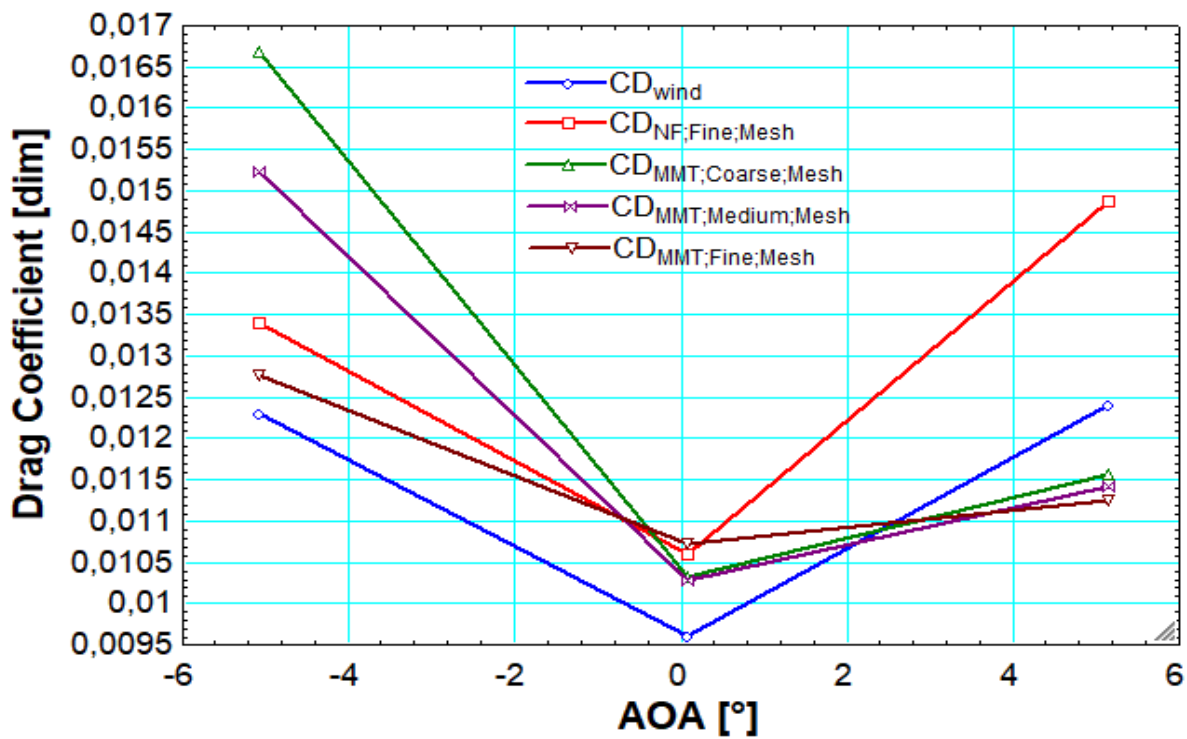


Figure 61: Coarse, medium & fine grid simulation results for momentum thickness far-field model

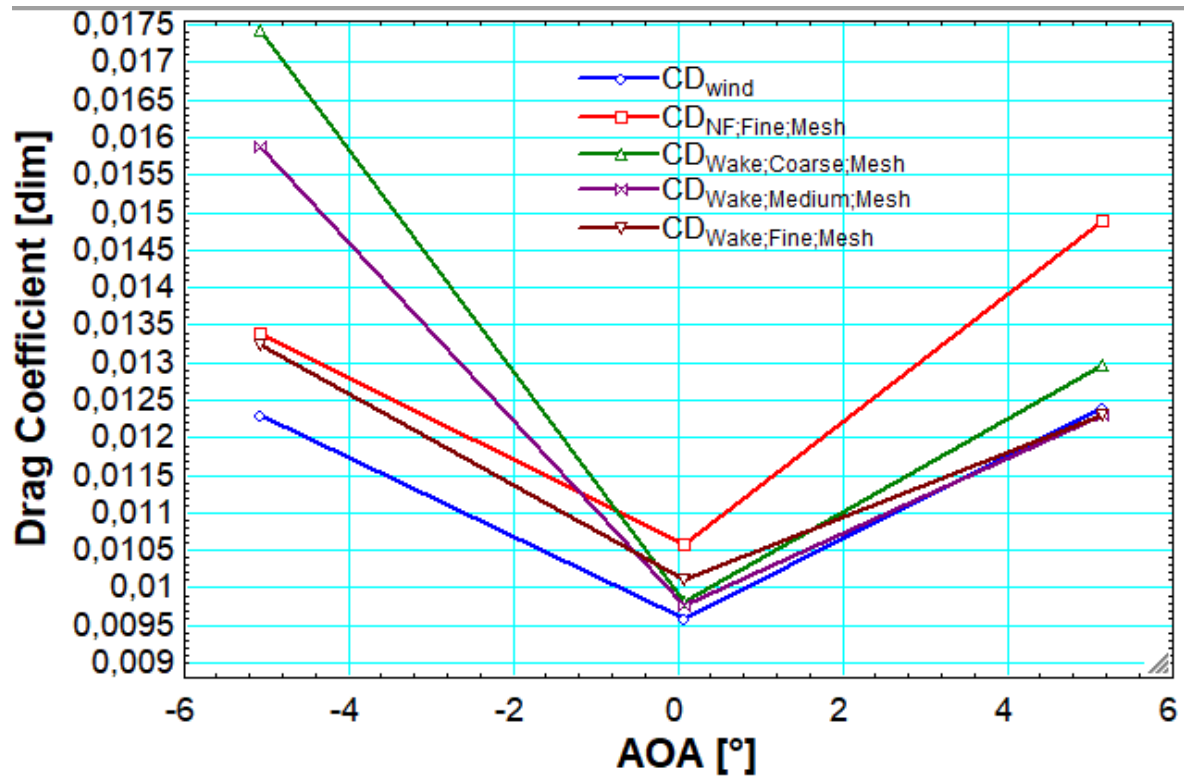


Figure 62: Coarse, medium & fine grid simulation results for Wake-rake far-field model

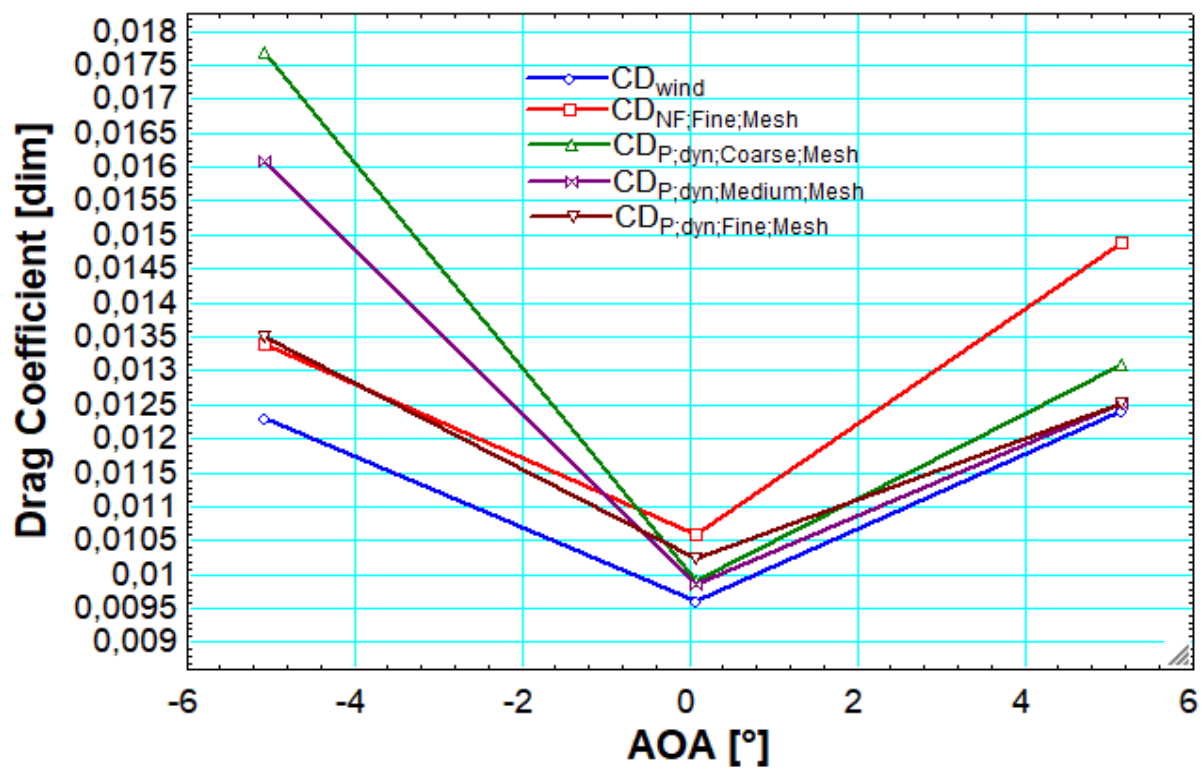


Figure 63: Coarse, medium & fine grid simulation results for Dynamic pressure far-field model

C.1.1. MESH REFINEMENT CONCLUSION

In **Figures 60-63** only the near-field profile drag results are plotted for the fine grid simulation which was the most accurate with regard to experimental results. It is clear upon inspection of **Tables 44-46** and **Figures 60-63** that all the proposed far-field models (for the fine mesh case) predict the profile drag more accurately than the near-field method. The most accurate of the far-field models were the wake-rake and the dynamic pressure models. Upon closer inspection a discrepancy of the Squire-Young and momentum thickness models with regard to the experimental and near-field method is evident. This discrepancy is due to the averaged inviscid velocity solution used in the integration process. In this section a general Trefftz plane position was assumed with an estimation position used for the upper and lower bounds for the integration process. Even with these generalising assumptions, all proposed far-field methods show a reduced sensitivity to mesh refinement and this clearly displays why the proposed far-field methods are superior to the near-field method when it comes to profile drag calculation.

C.2. TREFFTZ PLANE POSITION INFLUENCE ON FAR-FIELD DRAG ACCURACY

To establish the influence of the Trefftz plane position on the accuracy of the far-field drag method, the fine grid simulation set-up as defined in **Section C.1** will be used. Also, taking into account that the most accurate method has been found to be the dynamic pressure method, only this model will be used in the preceding parts of the investigation. The Trefftz plane will be varied in 2 chord length increments and the same assumption that was previously used for upper and lower integration bounds will be used here. The results for these simulations can be seen in **Table 47** and **Figure 64**. Here it is clear that the results from integrating at the Trefftz plane positioned at 1 chord length from the TE have the lowest accuracy when compared to the fine mesh near field results and experimental results. This is because here, due to the artificial dissipation and separation effects, the static pressure has not yet recovered and the wake is not yet stabilized. Similarly, from **Figure 64** it can be deduced that the far-field drag predicted is most accurate when the Trefftz plane is placed between 8 and 14 chord lengths from the trailing edge of the aerofoil. This is because here total static pressure recovery is achieved and a stable wake results. Hence the assumption of wake upper and lower bounds in the integration process is of highest accuracy. Referring

to **Figure 65**, the stable wake and static pressure recovery which validate our integration assumption can be seen.

Angle of Attack [°]	CD (Wind Tunnel) [dim]	CD (Near-Field) [dim]	CD Station 0 (1 m from TE) [dim]	CD Station 1 (2 m from TE) [dim]	CD Station 2 (4 m from TE) [dim]	CD Station 3 (6 m from TE) [dim]	CD Station 4 (8 m from TE) [dim]	CD Station 5 (10 m from TE) [dim]	CD Station 6 (12 m from TE) [dim]	CD Station 7 (14 m from TE) [m from TE]
-5.080	0.012300	0.013400	0.015882	0.014956	0.014070	0.013573	0.013492	0.013280	0.012636	0.012865
0.070	0.009600	0.010586	0.012349	0.010717	0.010305	0.010189	0.010224	0.010023	0.010161	0.009960
5.140	0.012400	0.014882	0.015551	0.014018	0.012903	0.012831	0.012513	0.012418	0.012206	0.012200

Table 47: Trefftz plane position influence on far-field drag calculation accuracy

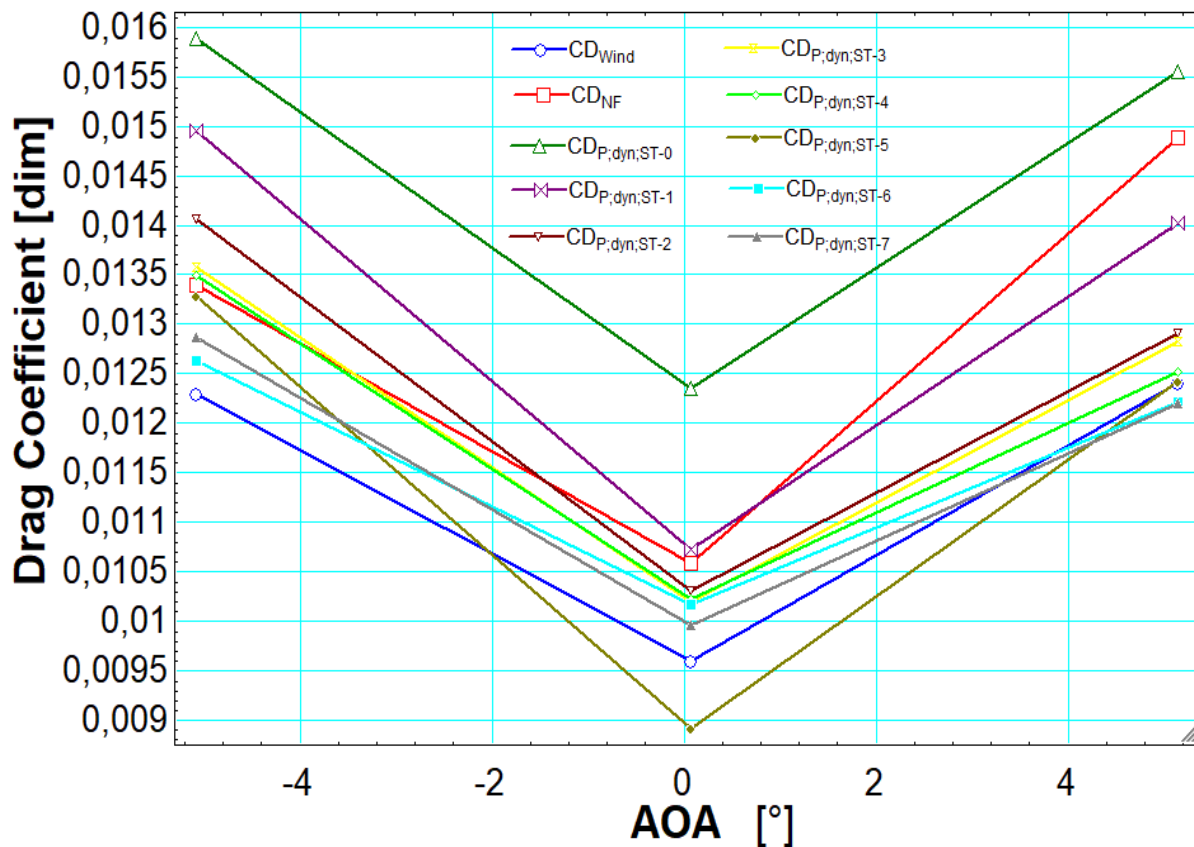


Figure 64: Trefftz plane position influence on far-field drag calculation accuracy

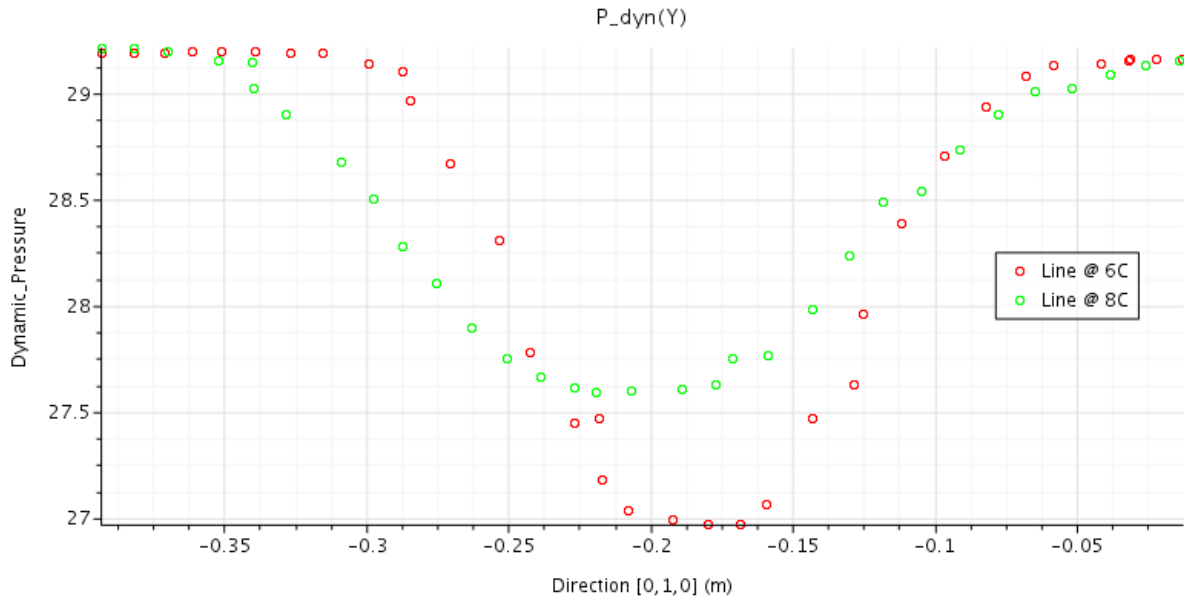


Figure 65: Stable wake and recovered static pressure for Trefftz plane @ 6[m], 8[m]

C.3. ESTABLISHMENT OF UPPER AND LOWER BOUND FOR WAKE INTEGRATION

The establishment of the upper and lower bounds for the far-field integrals greatly impact the accuracy of the solution. If the wake region chosen– the upper and lower integral bounds – are far apart, then the far-field method tends to underestimate the profile drag. This is because, in the integral, regions are evaluated where free-stream static pressure has recovered and is not interactive with the viscous wake.

In this section the fine mesh simulation set-up as used in **Section C.1** will be employed, and the Trefftz plane will be placed six chord lengths from the trailing edge, as this has constantly yielded the most accurate results in **Section C.2**. The first method that will be utilised to determine the top and bottom bounds for the integral is making use of the laminar to viscous dynamic viscosity ratio, defined as:

$$\frac{\mu_{t_{local}} + \mu_{l_{local}}}{\mu_{l_{local}}} > \frac{\mu_{t_{\infty}} + \mu_{l_{\infty}}}{\mu_{l_{\infty}}} * \varepsilon \quad \dots \text{C.3.1}$$

Where: $\mu_{t_{\infty}}$ is the turbulent dynamic viscosity in the free-stream, $\mu_{l_{\infty}}$ is the laminar dynamic viscosity in the free-stream, $\mu_{t_{local}}$ is the local turbulent dynamic viscosity on the Trefftz

plane, μ_{local} is the local laminar dynamic viscosity on the Trefftz plane and ε is the arbitrary constant value.

Equation C.3.1 simply means that the position where the local viscosity ratio diverges from the free stream viscosity ratio times ε is where the integration starts – the upper bound. For the upper bound this equation is evaluated from the top to the bottom of the Trefftz plane and for the lower bound this equation is evaluated from the bottom to the top of the Trefftz plane. Refer to the figure below:

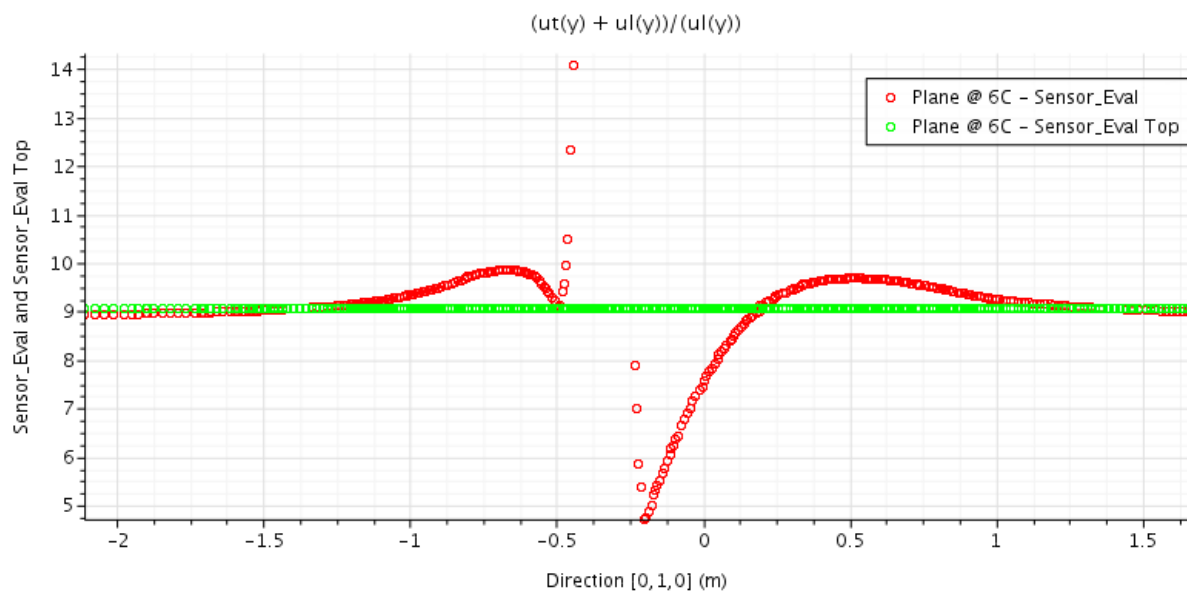


Figure 66: Example of determining far-field integration bounds from equation C.3.1

If, stepping along the above figure from left to right, it in effect means stepping from the top to the bottom of the Trefftz plane and vice versa if stepping from the right to the left. Thus evaluating **Equation C.3.1** by stepping left to right, it is evident that the lower bound equates to roughly -1.49700, and by stepping right to left and evaluating the equation, the upper bound equates to 1.495. These bounds are then used to define the Trefftz plane length which will be used in the far-field drag calculation process.

This procedure was followed for the FX 63-13 aerofoil at AOA -5.08, 0.07 and 5.14 degrees with a Reynolds number of 499818. Each time the value of epsilon is increased, it in effect means decreasing the length of the Trefftz plane on which the flow variables are projected. The results appear in **Table 48** and **Figure 67**. In **Table 48** the last column represents the results for setting the value of epsilon to represent top and bottom integral bounds respectively. This means two functions are evaluated when stepping along the Trefftz plane from top to bottom and from bottom to top. This is graphically expressed in **Figure 68**.

Angle of Attack	CD (Wind Tunnel)	CD (Near-Field)	CD Sensor 0 $\left[\left\{\frac{ut+ul}{ul}\right\}\right]$ * 1.01	CD Sensor 1 $\left[\left\{\frac{ut+ul}{ul}\right\}\right]$ * 1.02	CD Sensor 2 $\left[\left\{\frac{ut+ul}{ul}\right\}\right]$ * 1.03	CD Sensor 3 $\left[\left\{\frac{ut+ul}{ul}\right\}\right]$ * 1.04	CD Sensor 4 $\left[\left\{\frac{ut+ul}{ul}\right\}\right]$ * 1.05	CD Sensor 5 $\left[\left\{\frac{ut+ul}{ul}\right\}\right]$ * 1.06	CD Sensor 6 $\left[\left\{\frac{ut+ul}{ul}\right\}\right]$ * 1.07	CD Sensor 7 $\left[\left\{\frac{ut+ul}{ul}\right\}\right]$ * 1.08	CD Station 8 [upper & lower bounds]
[°]	[dim]	[dim]	[dim]	[dim]	[dim]	[dim]	[dim]	[dim]	[dim]	[dim]	[dim]
-5.080	0.012300	0.013400	0.013362	0.013429	0.013429	0.013471	0.013561	0.013484	0.013294	0.013294	0.013494
0.070	0.009600	0.010586	0.009555	0.009798	0.010138	0.010044	0.010072	0.010222	0.010301	0.010334	0.009710
5.140	0.012400	0.014882	0.011882	0.012434	0.012580	0.012630	0.012638	0.012574	0.012711	0.013341	0.012773

Table 48: Far-field drag accuracy dependence on wake integration bounds

C.3.1. FIRST METHOD OF BOUND ESTABLISHMENT DISCUSSION

Studying the results of **Figure 67** and **Table 48**, it is obvious that, for the angles of attack 0.07 and 5.14, as the value of epsilon is increased and thus decreasing the length of the Trefftz plane, results of higher accuracy are achieved with regard to the experimental results. It is also evident that for an epsilon value of $1.06 < \epsilon < 1.08$ the far-field drag calculation method outperforms the near-field drag calculation method. The reason is that for these values, at our Reynolds number, the viscous wake is captured more accurately in terms of top and bottom free-stream static pressure recovery. An interesting phenomenon can be seen in **Table 48**, namely that for the AOA case of 5.14 we achieved high accuracy far-field drag calculation for a larger wake region than for the AOA case of 0.07. This is because in the case of 5.14 degrees AOA, more separation occurs and thus a more viscous wake, meaning the value of epsilon should be smaller to capture a large wake region. These findings agree with prior knowledge from this thesis where it is evident that higher separation cases require a wider Trefftz plane to capture total static pressure recovery on the top and bottom of the plane. It is also clear that for the angle of attack of -5.08, the far-field drag closely approximates the near-field drag values (rather than experimental), and this is due to inadequate wake discretization in order to achieve high accuracy flow variable extraction for the far-field drag calculation process.

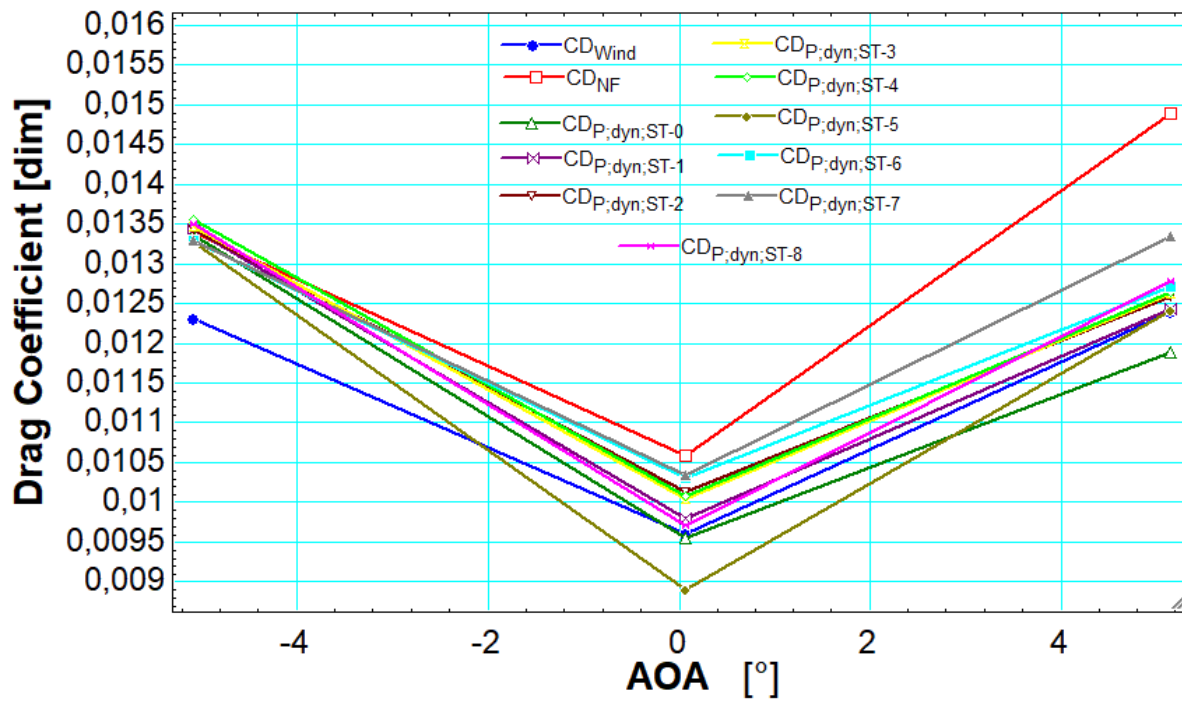


Figure 67: Far-field drag accuracy dependence on wake integration bounds

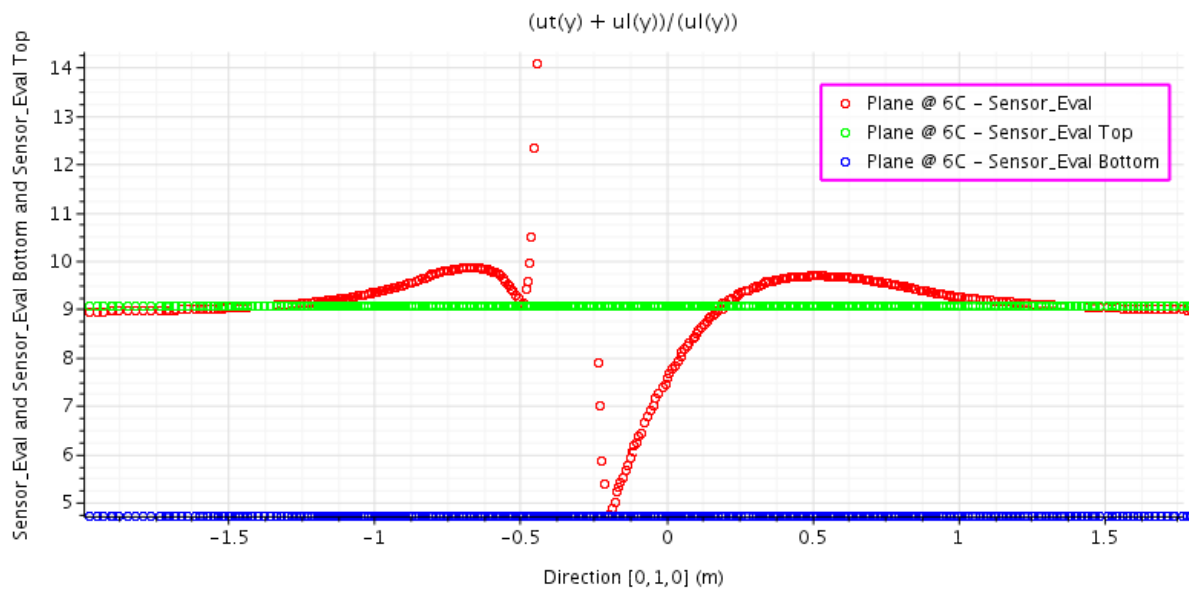


Figure 68: Example of determining far-field integration upper and lower bounds from equation C.3.1

C.4. NUMERICAL METHOD TO DETERMINE UPPER AND LOWER BOUNDS OF WAKE REGION

The method described in **Section C.3** is adequate to determine the upper and lower integration bounds accurately, however this method requires user intuition to determine where the threshold function value of **Equation C.3.1** crosses the free-stream function value. Thus, to remove the requirement of user intuition, as this may lead to human error, we want to develop a simple method to numerically determine the upper and lower integration bounds. Refer to the flow chart below describing the proposed numerical method to determine the upper and lower integration bounds. The far-field variables in this section were integrated via the wake rake method of **Chapter 4** with the fine mesh settings of section **C.1**.

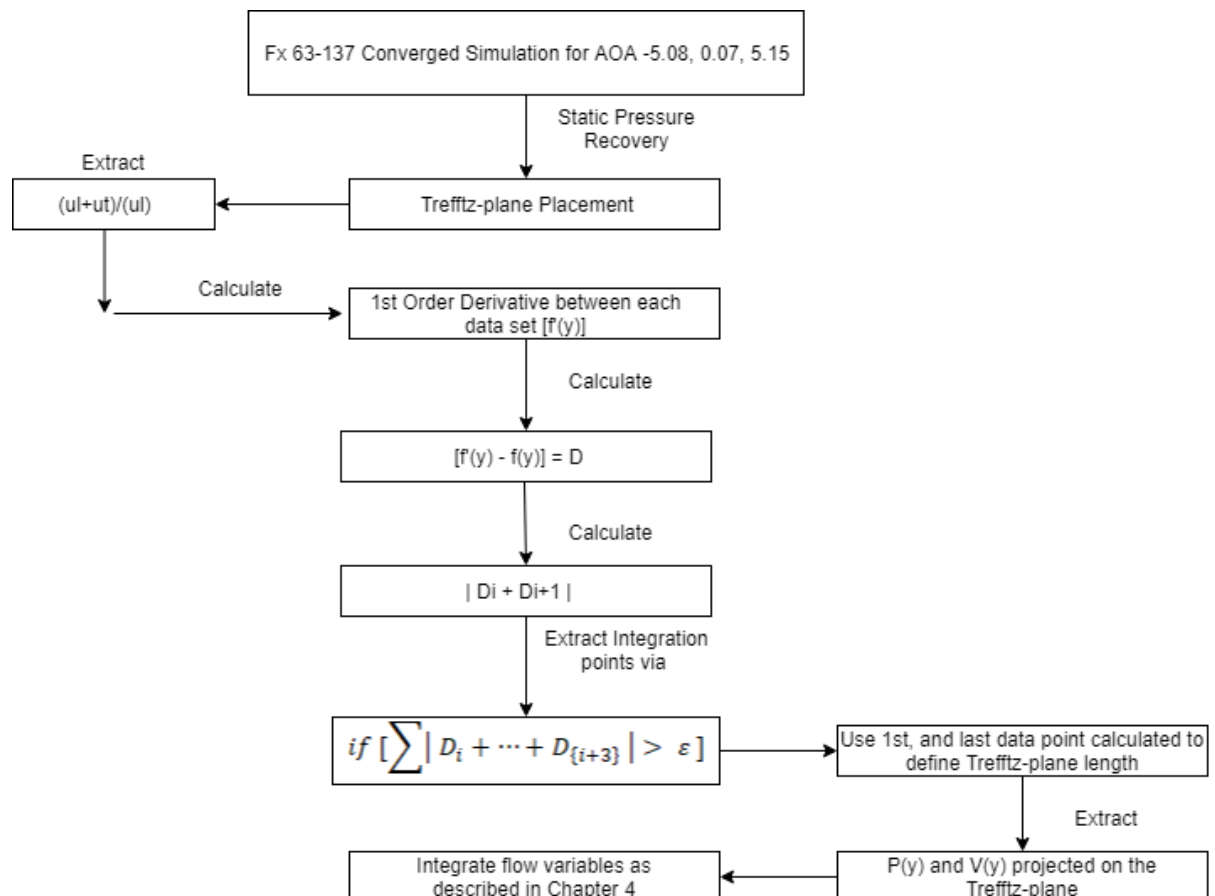


Figure 69: Flow chart of numerical method to determine upper and lower integration bounds

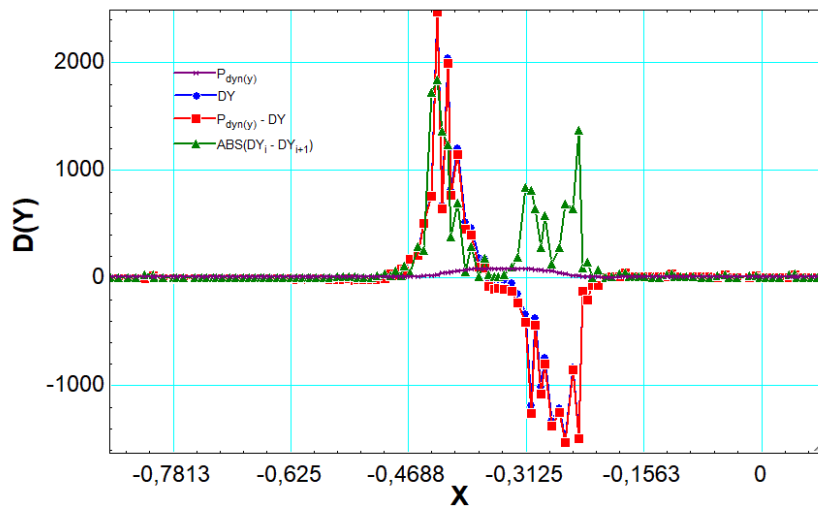


Figure 70: Numerical method to determine upper and lower integration bounds for AOA 5.14 [deg]

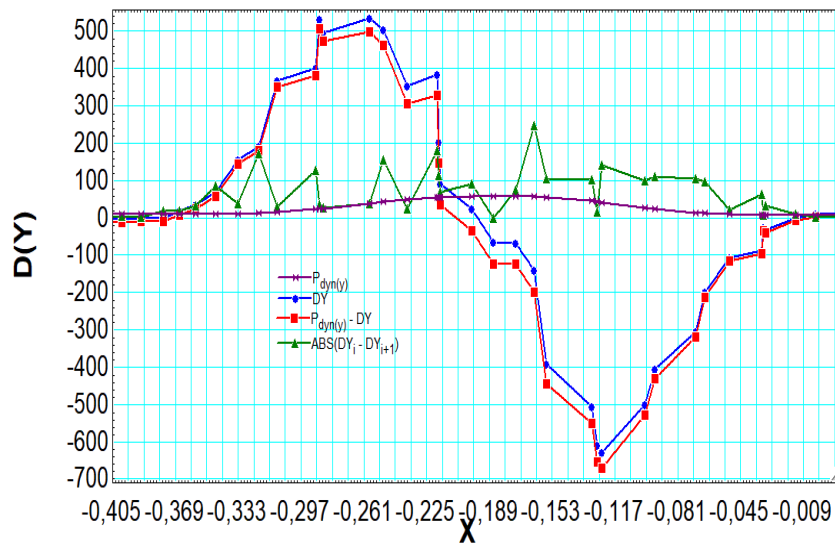


Figure 71: Numerical method to determine upper and lower integration bounds for AOA 0.07 [deg]

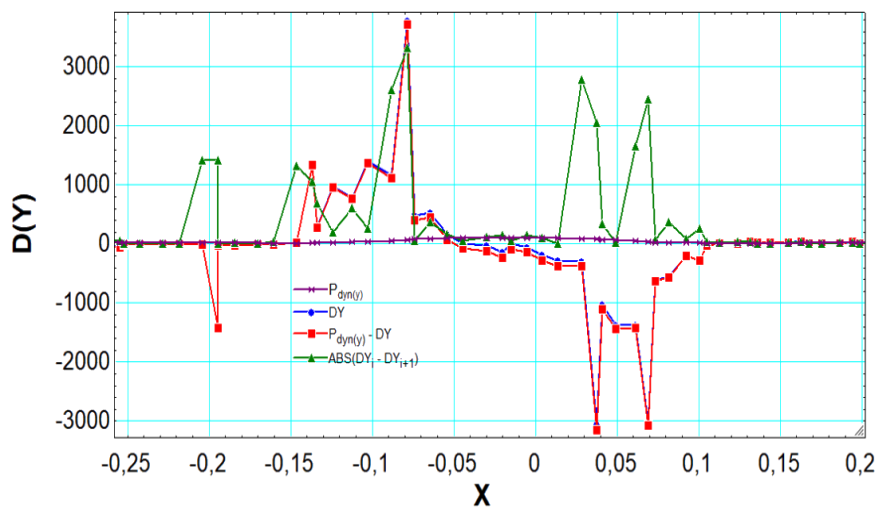


Figure 72: Numerical method to determine upper and lower integration bounds for AOA -5.08 [deg]

C.4.1. NUMERICAL METHOD TO DETERMINE UPPER AND LOWER BOUNDS OF WAKE REGION CONCLUSION

When studying **Figures 70-72** it is clear that the green cells, i.e. function $|Dy_i - Dy_{i+1}|$, that lie above the function line $P_{dyn}(y)$ represent the absolute function values with the most adverse gradient. This region thus represents the most viscous parts of the wake and the first and last x-coordinates of the function $|Dy_i - Dy_{i+1}|$ can be noted and used to define the length of the Trefftz plane. It is important to note that the first x-coordinate should be used as the bottom integration boundary and the last x-coordinate should be used as the top integration boundary.

To accurately extract the most viscous part of the wake, i.e. the regions with the highest value for $|Dy_i - Dy_{i+1}|$, only coordinate values larger than the threshold parameter ε are extracted. As separation increases, so do the function values of $|Dy_i - Dy_{i+1}|$ increase. This means that to accurately extract the first and last wake coordinate where static pressure recovery has resulted, the threshold value ε has to be increased. This can be seen in **Table 49**, where the threshold value is low for the AOA of 0.07 where separation is less significant, and high for AOA -5.08 and 5.14 where separation effects are more adverse. This method of identifying the coordinates to be integrated coupled with the wake rake method of far-field drag calculation yielded extremely high accuracy. For AOA case 0.07 the difference between the far-field drag and experimental drag equated to 0.000368 or 3.68 drag counts. This is opposed to the near-field drag difference with experimental drag of 10 drag counts. For AOA case 5.14 the difference between the far-field drag and experimental drag equated 0.7 drag counts. This is opposed to the near-field drag difference with experimental drag of 25 drag counts. For AOA case -5.08 the difference between the far-field drag and experimental drag equated 8.79 drag counts. This is opposed to the near-field drag difference with experimental drag of 11 drag counts.

Angle of Attack [°]	CD (Wind Tunnel) [dim]	CD (Near-Field) [dim]	CD (Far-field) [dim]	ε [dim]
-5.08	0.0123	0.013400	0.013179	200.0
0.07	0.0096	0.010600	0.009968	5.0
5.14	0.0124	0.014900	0.012470	100.0

Table 49: Results of numerical method to determine upper and lower integration bounds

C.5. CONCLUSION

From this section it is clear that the best practice for far-field drag analysis can be achieved by implementing either the medium or fine mesh parameters as displayed in **Section C.1** to define the grid. Together with these mesh settings the Trefftz plane should be placed in regions where total static pressure recovery to the free-stream values is achieved. For the low Reynolds number case displayed in this section this distance is typically between 6 and 10 chord lengths from the trailing edge. To accurately define the length of the Trefftz plane on which the flow variables are projected and extracted for integration, one of the following methods can be deployed:

- I. Use **Equation C.3.1.** and vary the top threshold parameter ε iteratively until a region of total static pressure recovery in the wake itself is visualized at the top and bottom plane positions.
- II. Vary the Trefftz plane length iteratively in Star-CCM+ until total static pressure recovery is realized.
- III. Use **Equation C.3.1.** and vary the top and bottom threshold parameter ε iteratively until a region of total static pressure recovery in the wake itself is visualized at the top and bottom plane positions.
- IV. Use the method of **Section C.4.1** to numerically calculate the first derivative and extract the coordinates of the first and last highest oscillating function term $|Dy_i - Dy_{\{i+1\}}|$ (with reference to the first derivative) and use these positions to define the Trefftz plane length.

APPENDIX D: SIMULATION GEOMETRY COORDINATES

FX63-137 Non-Linear Panels		E231 Non-Linear Panels		S834 Non-Linear Panels	
1	0.001313	1	0	1	0
0.998244	0.002655	0.9963	0.00039	0.996064	0.000606
0.99187	0.005587	0.98554	0.00187	0.984816	0.003101
0.984488	0.008637	0.96845	0.00477	0.967539	0.007914
0.975411	0.012071	0.94569	0.00903	0.945483	0.014649
0.967994	0.01467	0.91774	0.0146	0.919407	0.02247
0.956278	0.018539	0.88524	0.02148	0.889501	0.030561
0.940983	0.0233	0.84897	0.02952	0.855652	0.038699
0.925965	0.027731	0.80968	0.03837	0.818293	0.046926
0.91053	0.032107	0.7681	0.04759	0.777966	0.05503
0.892025	0.037167	0.72481	0.05653	0.735169	0.062727
0.871596	0.042586	0.68007	0.06458	0.690338	0.069726
0.849029	0.048423	0.674305	0.065436	0.643856	0.075777
0.824949	0.054516	0.66854	0.066293	0.596064	0.080708
0.799019	0.060894	0.662775	0.067149	0.54733	0.084531
0.774258	0.066824	0.65701	0.068005	0.498288	0.087141
0.745917	0.073388	0.651245	0.068861	0.449272	0.088341
0.729284	0.077068	0.64548	0.069718	0.400592	0.088199
0.71265	0.080748	0.639715	0.070574	0.352742	0.086855
0.694036	0.084584	0.63395	0.07143	0.306248	0.084371
0.675421	0.088419	0.628045	0.072128	0.261604	0.080802
0.659808	0.09137	0.62214	0.072825	0.219296	0.076196
0.644195	0.09432	0.616235	0.073523	0.179743	0.070615
0.626723	0.097292	0.61033	0.07422	0.143372	0.064126
0.60925	0.100263	0.604425	0.074918	0.110502	0.056817
0.590933	0.102977	0.59852	0.075615	0.081485	0.048795
0.572616	0.105691	0.592615	0.076313	0.056518	0.040187
0.553603	0.108067	0.58671	0.07701	0.035873	0.031189
0.534589	0.110443	0.580703	0.077541	0.01966	0.022005
0.517364	0.11218	0.574695	0.078073	0.008118	0.012967
0.500139	0.113917	0.568688	0.078604	0.001394	0.004469
0.478977	0.115487	0.56268	0.079135	0.001039	0.003754
0.457814	0.117057	0.556673	0.079666	0.000308	0.001855
0.438454	0.11793	0.550665	0.080198	0.000001	0.000091
0.419094	0.118802	0.544658	0.080729	3.21E-05	-0.00013
0.399802	0.119108	0.53865	0.08126	6.33E-05	-0.00035
0.38051	0.119413	0.532589	0.081626	9.44E-05	-0.00057
0.361183	0.119122	0.526528	0.081993	0.000126	-0.00079
0.341855	0.118831	0.520466	0.082359	0.000157	-0.00101
0.322625	0.117883	0.514405	0.082725	0.000188	-0.00123
0.303394	0.116934	0.508344	0.083091	0.000219	-0.00145
0.28334	0.115194	0.502283	0.083458	0.00025	-0.00167
0.263286	0.113454	0.496221	0.083824	0.000298	-0.00182
0.247733	0.111526	0.49016	0.08419	0.000346	-0.00197
0.23218	0.109598	0.484095	0.084396	0.000393	-0.00212
0.223172	0.108126	0.47803	0.084603	0.000441	-0.00227
0.214165	0.106654	0.471965	0.084809	0.000489	-0.00242
0.205157	0.105181	0.4659	0.085015	0.000537	-0.00256
0.196149	0.103709	0.459835	0.085221	0.000584	-0.00271
0.188028	0.101797	0.45377	0.085428	0.000632	-0.00286
0.179907	0.099885	0.447705	0.085634	0.000667	-0.00295
0.171785	0.097973	0.44164	0.08584	0.000703	-0.00304
0.163664	0.096061	0.435631	0.085895	0.000738	-0.00312
0.155543	0.094148	0.429623	0.08595	0.000774	-0.00321
0.147422	0.092236	0.423614	0.086005	0.000809	-0.0033
0.1393	0.090324	0.417605	0.08606	0.000844	-0.00338
0.131179	0.088412	0.411596	0.086115	0.00088	-0.00347
0.12774	0.08731	0.405588	0.08617	0.000915	-0.00356
0.124302	0.086208	0.399579	0.086225	0.001563	-0.00449
0.120863	0.085105	0.39357	0.08628	0.002211	-0.00542
0.117424	0.084003	0.387681	0.08619	0.002859	-0.00635
0.113985	0.082901	0.381793	0.0861	0.003507	-0.00728
0.110547	0.081799	0.375904	0.08601	0.004155	-0.00821
0.107108	0.080696	0.370015	0.08592	0.004803	-0.00914
0.103669	0.079594	0.364126	0.08583	0.005451	-0.01007
0.097094	0.077032	0.358238	0.08574	0.006099	-0.011
0.090519	0.07447	0.352349	0.08565	0.007383	-0.01205
0.083943	0.071908	0.34646	0.08556	0.008667	-0.01311
0.077368	0.069346	0.340754	0.085328	0.009951	-0.01417
0.074345	0.067968	0.335048	0.085095	0.011235	-0.01522
0.071322	0.06659	0.329341	0.084863	0.012519	-0.01628
0.065275	0.063834	0.323635	0.08463	0.013803	-0.01734
0.058834	0.060515	0.317929	0.084398	0.015087	-0.01839
0.052393	0.057195	0.312223	0.084165	0.016371	-0.01945
0.048316	0.054838	0.306516	0.083933	0.018243	-0.02049
0.044238	0.052481	0.30081	0.0837	0.020115	-0.02154

0.038576	0.04872	0.295346	0.08333	0.021987	-0.02258
0.032913	0.044958	0.289883	0.08296	0.023859	-0.02362
0.029006	0.041881	0.284419	0.08259	0.025731	-0.02466
0.025099	0.038803	0.278955	0.08222	0.027603	-0.02571
0.021802	0.035752	0.273491	0.08185	0.029475	-0.02675
0.020153	0.034227	0.268028	0.08148	0.031347	-0.02779
0.018504	0.032701	0.262564	0.08111	0.033775	-0.02877
0.015608	0.029502	0.2571	0.08074	0.036202	-0.02975
0.012711	0.026302	0.251933	0.080236	0.03863	-0.03073
0.00996	0.022543	0.246765	0.079733	0.041057	-0.03171
0.007208	0.018783	0.241598	0.079229	0.043485	-0.03268
0.005735	0.016209	0.23643	0.078725	0.045912	-0.03366
0.004261	0.013634	0.231263	0.078221	0.04834	-0.03464
0.00352	0.012039	0.226095	0.077718	0.050767	-0.03562
0.002778	0.010443	0.220928	0.077214	0.05375	-0.03651
0.0021	0.008625	0.21576	0.07671	0.056733	-0.0374
0.001421	0.006806	0.210941	0.076079	0.059715	-0.03829
0.00083	0.002041	0.206123	0.075448	0.062698	-0.03918
0.000239	-0.00273	0.201304	0.074816	0.065681	-0.04007
0.000397	-0.00345	0.196485	0.074185	0.068664	-0.04096
0.000554	-0.00418	0.191666	0.073554	0.071646	-0.04185
0.001879	-0.00737	0.186848	0.072923	0.074629	-0.04274
0.003306	-0.00932	0.182029	0.072291	0.078133	-0.04351
0.006598	-0.01218	0.17721	0.07166	0.081637	-0.04429
0.010118	-0.01421	0.172786	0.070909	0.08514	-0.04507
0.0148	-0.0162	0.168363	0.070158	0.088644	-0.04584
0.024765	-0.01904	0.163939	0.069406	0.092148	-0.04662
0.0395	-0.02155	0.159515	0.068655	0.095652	-0.0474
0.06272	-0.02387	0.155091	0.067904	0.099155	-0.04817
0.086431	-0.02528	0.150668	0.067153	0.102659	-0.04895
0.113905	-0.02615	0.146244	0.066401	0.106672	-0.0496
0.141969	-0.02639	0.14182	0.06565	0.110686	-0.05025
0.169331	-0.02617	0.137829	0.06479	0.114699	-0.0509
0.199932	-0.02545	0.133838	0.06393	0.118712	-0.05155
0.234658	-0.02412	0.129846	0.06307	0.122725	-0.0522
0.270149	-0.0222	0.125855	0.06221	0.126739	-0.05285
0.306575	-0.01971	0.121864	0.06135	0.130752	-0.0535
0.343245	-0.01669	0.117873	0.06049	0.134765	-0.05415
0.379412	-0.01326	0.113881	0.05963	0.139244	-0.05466
0.415039	-0.00944	0.10989	0.05877	0.143723	-0.05517
0.456772	-0.00453	0.106368	0.057813	0.148202	-0.05568
0.498683	0.000739	0.102845	0.056855	0.152682	-0.05619
0.544736	0.006575	0.099323	0.055898	0.157161	-0.0567
0.582318	0.011214	0.0958	0.05494	0.16164	-0.05722
0.598582	0.013091	0.092278	0.053983	0.166119	-0.05773
0.614845	0.014968	0.088755	0.053025	0.170598	-0.05824
0.631599	0.016695	0.085233	0.052068	0.175518	-0.05861
0.648353	0.018421	0.08171	0.05111	0.180438	-0.05898
0.665681	0.019968	0.078684	0.050074	0.185357	-0.05935
0.683009	0.021515	0.075658	0.049038	0.190277	-0.05972
0.701339	0.02281	0.072631	0.048001	0.195197	-0.06009
0.719669	0.024104	0.069605	0.046965	0.200117	-0.06046
0.735447	0.024892	0.066579	0.045929	0.205036	-0.06082
0.751225	0.025679	0.063553	0.044893	0.209956	-0.06119
0.765906	0.026102	0.060526	0.043856	0.215263	-0.06142
0.780586	0.026524	0.0575	0.04282	0.22057	-0.06164
0.796891	0.026595	0.054989	0.041726	0.225877	-0.06187
0.813195	0.026666	0.052478	0.040633	0.231184	-0.0621
0.825986	0.026387	0.049966	0.039539	0.23649	-0.06232
0.838777	0.026107	0.047455	0.038445	0.241797	-0.06255
0.850802	0.025553	0.044944	0.037351	0.247104	-0.06277
0.862827	0.024999	0.042433	0.036258	0.252411	-0.063
0.872764	0.024287	0.039921	0.035164	0.258067	-0.06308
0.882701	0.023575	0.03741	0.03407	0.263722	-0.06316
0.893295	0.022543	0.035431	0.032946	0.269378	-0.06325
0.903888	0.02151	0.033453	0.031823	0.275034	-0.06333
0.908711	0.020932	0.031474	0.030699	0.280689	-0.06341
0.913534	0.020353	0.029495	0.029575	0.286345	-0.0635
0.92041	0.019403	0.027516	0.028451	0.292	-0.06358
0.927285	0.018453	0.025538	0.027328	0.297656	-0.06367
0.930771	0.017907	0.023559	0.026204	0.321422	-0.06346
0.934256	0.017361	0.02158	0.02508	0.345187	-0.06325
0.939574	0.01643	0.020139	0.023961	0.369896	-0.06253
0.944892	0.015498	0.018698	0.022843	0.394605	-0.06181
0.948267	0.014822	0.017256	0.021724	0.419974	-0.06062
0.951641	0.014145	0.015815	0.020605	0.445343	-0.05943
0.959647	0.0123	0.014374	0.019486	0.471134	-0.05782
0.967	0.010398	0.012933	0.018368	0.496925	-0.05621
0.974946	0.008106	0.011491	0.017249	0.522831	-0.05424
0.981085	0.006159	0.01005	0.01613	0.548737	-0.05227
0.985996	0.004491	0.009149	0.015069	0.574493	-0.04998
0.991222	0.002657	0.008248	0.014008	0.600249	-0.0477
0.995348	0.001256	0.007346	0.012946	0.625539	-0.04514
0.9975	0.000367	0.006445	0.011885	0.650829	-0.04259
0.9995	-0.0002	0.005544	0.010824	0.675402	-0.03981

1	0				
FX63-137 Base-File Panels					
1	0.001313				
0.998244	0.002655				
0.99187	0.005587				
0.984488	0.008637				
0.975411	0.012071				
0.967994	0.01467				
0.956278	0.018539				
0.940983	0.0233				
0.925965	0.027731				
0.91053	0.032107				
0.892025	0.037167				
0.871596	0.042586				
0.849029	0.048423				
0.824949	0.054516				
0.799019	0.060894				
0.774258	0.066824				
0.745917	0.073388				
0.71265	0.080748				
0.675421	0.088419				
0.644195	0.09432				
0.60925	0.100263				
0.572616	0.105691				
0.534589	0.110443				
0.500139	0.113917				
0.457814	0.117057				
0.419094	0.118802				
0.38051	0.119413				
0.341855	0.118831				
0.303394	0.116934				
0.263286	0.113454				
0.23218	0.109598				
0.196149	0.103709				
0.131179	0.088412				
0.103669	0.079594				
0.077368	0.069346				
0.065275	0.063834				
0.052393	0.057195				
0.044238	0.052481				
0.032913	0.044958				
0.025099	0.038803				
0.018504	0.032701				
0.012711	0.026302				
0.007208	0.018783				
0.004261	0.013634				
0.002778	0.010443				
0.001421	0.006806				
0.000239	-0.00273				
0.000554	-0.00418				
0.001879	-0.00737				
0.003306	-0.00932				
0.006598	-0.01218				
0.010118	-0.01421				
0.0148	-0.0162				
0.024765	-0.01904				
0.0395	-0.02155				
0.06272	-0.02387				
0.086431	-0.02528				
0.113905	-0.02615				
0.141969	-0.02639				
0.169331	-0.02617				
0.199932	-0.02545				
0.234658	-0.02412				
0.270149	-0.0222				
0.306575	-0.01971				
0.343245	-0.01669				
0.379412	-0.01326				
0.415039	-0.00944				
0.456772	-0.00453				
0.498683	0.000739				
0.544736	0.006575				
0.582318	0.011214				
0.614845	0.014968				
0.648353	0.018421				
0.683009	0.021515				
0.719669	0.024104				
0.751225	0.025679				
0.780586	0.026524				
0.813195	0.026666				
0.838777	0.026107				
0.862827	0.024999				
		0.004643	0.009763		
		0.003741	0.008701		
		0.00284	0.00764		
		0.002508	0.006875		
		0.002175	0.00611		
		0.001843	0.005345		
		0.00151	0.00458		
		0.001178	0.003815		
		0.000845	0.00305		
		0.000513	0.002285		
		0.00018	0.00152		
		0.000158	0.00133		
		0.000135	0.00114		
		0.000113	0.00095		
		0.00009	0.00076		
		0.000068	0.00057		
		0.000045	0.00038		
		0.000023	0.00019		
		0	0		
		0.0002	-0.00139		
		0.00093	-0.00259		
		0.00217	-0.00382		
		0.00325	-0.00466		
		0.01291	-0.00966		
		0.02804	-0.01448		
		0.04843	-0.0189		
		0.07381	-0.02288		
		0.10394	-0.02638		
		0.13848	-0.02938		
		0.17704	-0.03189		
		0.21921	-0.03389		
		0.2645	-0.03541		
		0.3124	-0.03644		
		0.36238	-0.037		
		0.41385	-0.03711		
		0.440045	-0.03694		
		0.46624	-0.03677		
		0.49258	-0.03638		
		0.51892	-0.03599		
		0.54511	-0.03538		
		0.5713	-0.03477		
		0.597045	-0.03395		
		0.62279	-0.03312		
		0.647785	-0.03208		
		0.67278	-0.03104		
		0.696725	-0.02978		
		0.72067	-0.02851		
		0.74329	-0.02694		
		0.76591	-0.02537		
		0.787085	-0.02342		
		0.80826	-0.02146		
		0.82796	-0.01919		
		0.84766	-0.01692		
		0.86579	-0.01458		
		0.88392	-0.01224		
		0.90025	-0.0101		
		0.91658	-0.00796		
		0.93073	-0.00625		
		0.94488	-0.00453		
		0.956455	-0.00331		
		0.96803	-0.00209		
		0.97671	-0.00136		
		0.98539	-0.00063		
		0.990835	-0.00035		
		0.99628	-0.00007		
		0.99814	-3.5E-05		
		1	0		
E231 Base-File Panels					
1	0				
0.9963	0.00039				
0.98554	0.00187				
0.96845	0.00477				
0.94569	0.00903				
0.91774	0.0146				
0.88524	0.02148				
0.84897	0.02952				
0.80968	0.03837				
0.7681	0.04759				
0.72481	0.05653				
0.68007	0.06458				
0.63395	0.07143				
0.58671	0.07701				
		0.699974	-0.03702		
		0.723551	-0.03406		
		0.747128	-0.03111		
		0.769481	-0.02802		
		0.791833	-0.02494		
		0.812714	-0.02181		
		0.833594	-0.01867		
		0.852924	-0.01542		
		0.872254	-0.01217		
		0.89006	-0.00934		
		0.907866	-0.00651		
		0.923539	-0.00452		
		0.939211	-0.00253		
		0.952116	-0.00143		
		0.96502	-0.00033		
		0.974618	0.000032		
		0.984215	0.000393		
		0.990118	0.000319		
		0.996021	0.000244		
		0.998011	0.000122		
S834 Base-File Panels					
1	0				
0.996064	0.000606				
0.984816	0.003101				
0.967539	0.007914				
0.945483	0.014649				
0.919407	0.02247				
0.889501	0.030561				
0.855652	0.038699				
0.818293	0.046926				
0.777966	0.05503				
0.735169	0.062727				
0.690338	0.069726				
0.643856	0.075777				
0.596064	0.080708				
0.54733	0.084531				
0.498288	0.087141				
0.449272	0.088341				
0.400592	0.088199				
0.352742	0.086855				
0.306248	0.084371				
0.261604	0.080802				
0.219296	0.076196				
0.179743	0.070615				
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0.081485	0.048795				
0.056518	0.040187				
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0.01966	0.022005				
0.008118	0.012967				
0.001394	0.004469				
0.001039	0.003754				
0.000308	0.001855				
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0.102659	-0.04895				
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0.170598	-0.05824				
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0.872254	-0.01217				

0.882701	0.023575	0.53865	0.08126	0.907866	-0.00651
0.903888	0.02151	0.49016	0.08419	0.939211	-0.00253
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N/A	N/A -	0.99628	-0.00007	N/A	N/A -
N/A	N/A -	1	0	N/A	N/A -

APPENDIX E: INVESTIGATION OF WEAVING FAR-FIELD DRAG CURVES

In this thesis numerous far-field methods were developed to determine the viscous drag from the momentum deficit downstream from the trailing edge of an aerofoil. All methods developed showed superior accuracy performance when compared to the traditional near-field method used in the commercial CFD code of Star-CCM+. The most accurate amongst the far-field methods evaluated was the far-field velocity and far-field dynamic pressure integrals. Although accurate the drag results stemming from the far-field integral tends to have an oscillating curve shape. This zigzag C_d VS angle of attack pattern can be ascribed to numerical instability in the CFD code when solving the viscous wake, choice of Trefftz plane length, distance of Trefftz plane downstream and numerical errors in the approximation of the area below the curve when using the non-uniformly distributed Simpsons rule.

To solve the problem it is important to first fully understand the procedure of computing the far-field integral from the output of a converged viscous CFD solution. As a first measure we will be assuming our CFD simulation is set up correctly and the discretisation and physical conditions is representative of the actual problem at hand. With this the far-field method starts by moving the Trefftz plane iteratively downstream to a position where the total static pressure recovery at the downstream is equivalent to the free stream total static pressure. Then the upper and lower far-field integral bounds are noted by either using the first and last node in the viscous wake where the total static pressure recovery is realized or by implementing the methods discussed in **Appendix C**. The problem here is that in each angle of attack case the Trefftz plane length and Trefftz plane distance from the trailing edge varies. This means each time the Trefftz plane distance changes, different threshold trigger values are used to determine the most viscous part of the wake. This means our number of integrable nodes projected on the Trefftz plane varies substantially. This then internally means the approximation errors in our non-uniformly distributed Simpson rule varies and thus we arrive at a jagged C_d VS angle of attack curve. To overcome this problem a standalone Python code was developed and is thoroughly discussed in the following section.

E.1. PYTHON CODE DISCUSSION

The Python code displayed in **Appendix F** uses only 4 input arrays from the converged CFD solution. These input arrays include the y coordinates of the Trefftz plane length for the flow variables of dynamic pressure and dynamic viscosity ratio projected onto the Trefftz plane. The input arrays also include the magnitude of the dynamic pressure and dynamic viscosity ratio in the y direction projected onto the Trefftz plane. For this investigation the Trefftz plane distance from the TE, in the CFD study, was set to six chord lengths downstream. As case study the jagged FX 63-137 aerofoil curve displayed in **Figure 30** will be rectified with the newly developed numerical approach as implemented in the Python code of **Appendix F**.

The new approach starts by using a constant threshold trigger to determine the most positive part of the dynamic viscosity ratio matrix. This is done by calculating the average free stream dynamic viscosity ratio for the first 20-30 nodes and then subtracting the average value plus a threshold value, from the local dynamic viscosity ratio value. The positive values in the range of the input dynamic viscosity ratio matrix thus will represent the most viscous part of the wake. The next step is then to determine the index and value of the index of the first and last positive node in the newly calculated dynamic viscosity ratio matrix. These index positions are then used to determine the x and y values of the corresponding indexes of the original dynamic viscosity ratio as imported from CFD. With this a new dynamic pressure and dynamic viscosity x and y matrix is set up between the upper and lower positive bounds.

The newly calculated dynamic pressure and dynamic viscosity matrices now represent the most viscous part of the wake at six chords downstream of the trailing edge. The next step is to interpolate the newly calculated dynamic pressure nodes as accurate as possible. The interpolation methods used was those available in the Scipy interp1d library and includes cubic, linear, zero, slinear, quadratic and nearest interpolation schemes. The zero, slinear, quadratic and cubic refers to a spline interpolation of zeroth, first, second or third order. In the below standing figures the red dots represent the original full spectrum of the dynamic pressure magnitudes projected onto the full scale Trefftz plane length. The purple 'x' nodes represent the dynamic pressure magnitudes projected onto the Trefftz plane in the viscous region. Also in this figure, the interpolation results from the various mentioned interpolation schemes can be viewed.

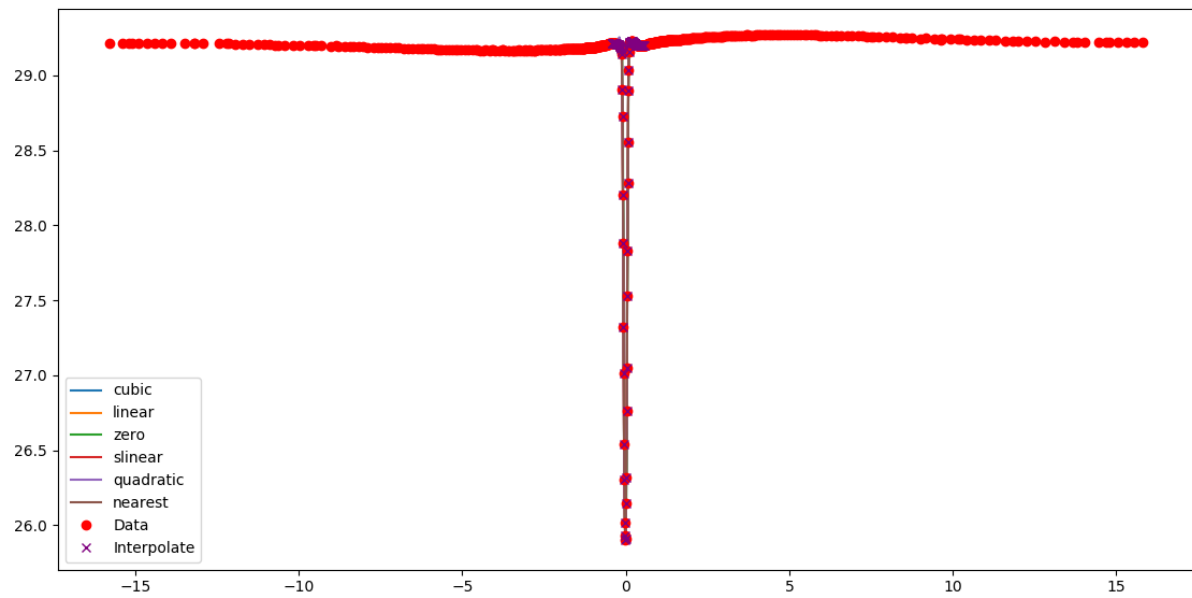


Figure 73: Dynamic pressure projected onto Trefftz plane & interpolation results (1)

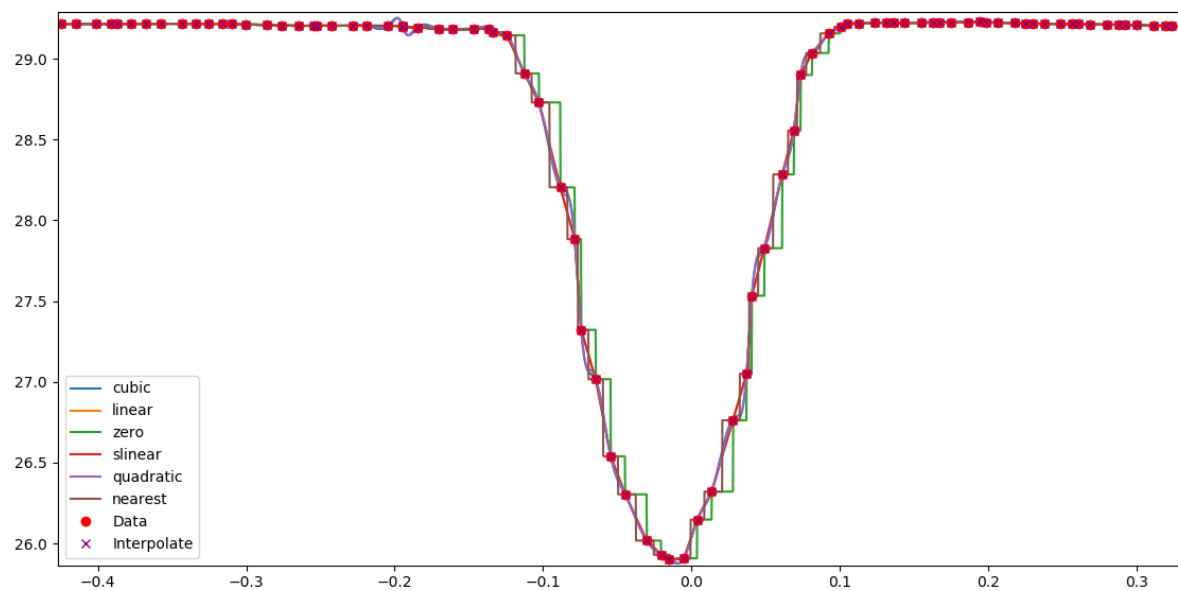


Figure 74: Dynamic pressure projected onto Trefftz plane & interpolation results (2)

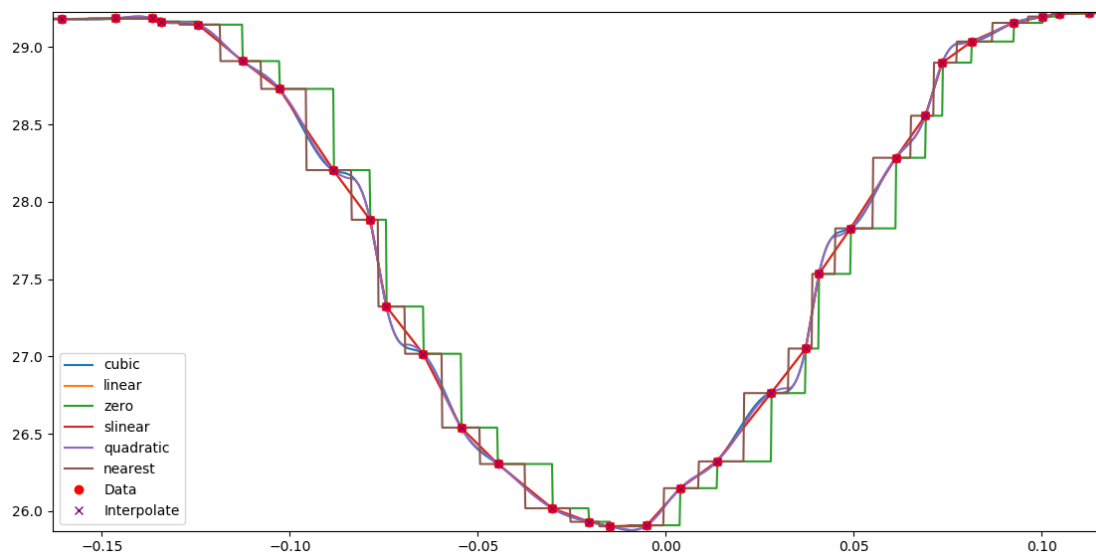


Figure 75: Dynamic pressure projected onto Trefftz plane & interpolation results (3)

From the above figures we see that the most accurate method to interpolate the viscous region is the 'slinear' method and is represented by the red curve. With the interpolation methods now defined the next step is to integrate each interpolation function representing the viscous region with various integration schemes. This is done to determine the best interpolation-integration combination which yields the highest accurate drag results with the least jagged curve tendencies. For the integration part the methods used included the Gauss-quadrature, Rhomberg, Trapezium and Simpson numerical integration algorithms. To demonstrate the dependency of the drag curve oscillation on the numerical interpolation and integration approximations refer to the below standing figures:

Wake-profile approximated with ZERO method

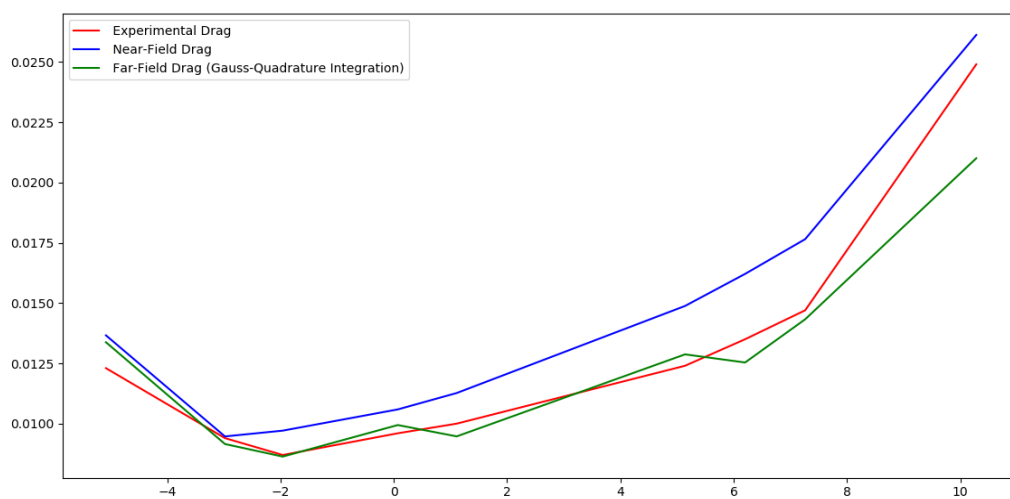


Figure 76: Zero interpolation scheme with Gauss-Quadrature integration

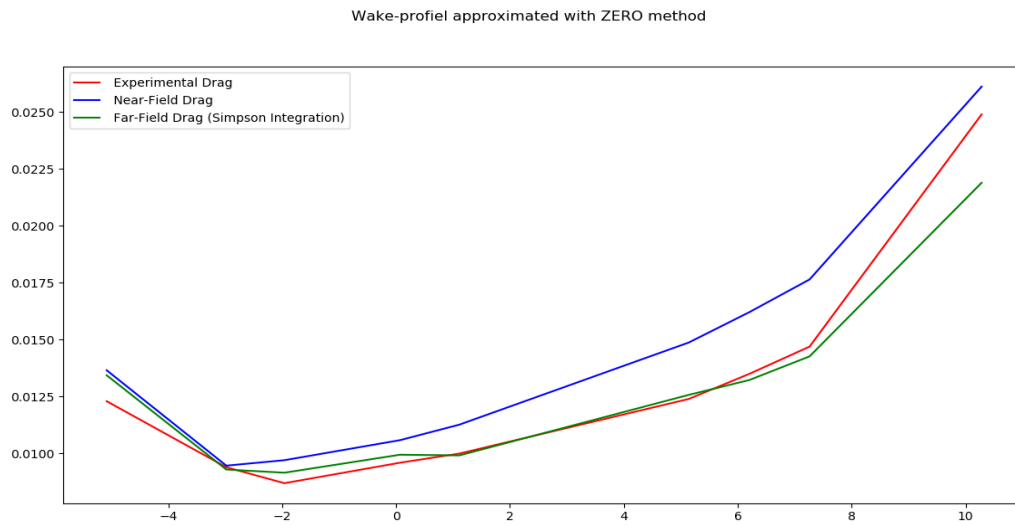


Figure 77: Zero interpolation scheme with Simpson integration scheme

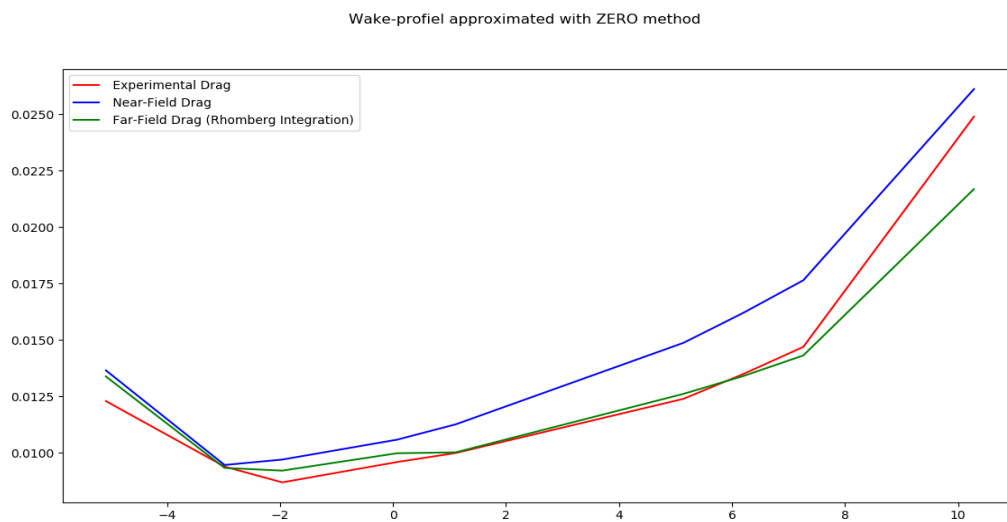


Figure 78: Zero interpolation scheme with Rhombberg integration scheme

E.2. CONCLUSION

Figure 76 shows the results of interpolating the viscous wake with the zero interpolation method and then computing the far-field dynamic pressure integral with the Gauss-quadrature method. It is clear that this method gives rise to numerical instabilities and the momentum deficit computed from one angle of attack to the next tends to oscillate. This is however not the tendency seen in **Figure 77** and **Figure 78**. Here the C_d VS angle of attack curve is much smoother when implementing the same interpolation scheme but using the Rhombberg and Simpson numerical integration algorithms respectively. This indicates that to overcome jagged C_d VS AOA curves the most accurate interpolation and integration algorithms needs to be utilized.

APPENDIX F: PYTHON CODE -5.08 AOA CASE

```

import numpy as np
import matplotlib.pyplot as plt
from scipy.interpolate import interp1d
from scipy.integrate import quad, trapz, simps, romberg, quadrature
from sklearn import preprocessing
from sklearn.preprocessing import MinMaxScaler
from matplotlib import pyplot

density = 1.283
u_dyn = 1.783*10**-5
V_inf = 6.747348215
P_dynam_sq = 0.5*V_inf**2*density
chord_lenght = 1
int_matrix = []
Eddy_inf = 0
eta_thresh_eddy = .5 #Increasing the threshold trigger will enlarge
the viscous wake region to be integrated and visa-versa

x = np.array([-15.79157394, -15.40444491, -15.19092167, -15.10882074, -14.91618859,
-14.63333324, -14.39454972, -14.1887888, -13.91980654, -13.47665002, -13.19780016,
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-8.419513432, -8.325706828, -8.184127227, -8.040004347, -7.899695199, -7.724515905,
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])

for i in range(30):
    Eddy_inf += y_eddy[i]
    Eddy_inf_avg = Eddy_inf/[i+1]

y_int = y - P_dynam_sq
Eddy_int =(y_eddy-eta_thresh_eddy) -(Eddy_inf_avg)
cond = Eddy_int > 0 #Determine the
condition matrix of most viscous wake
Eddy_value = np.extract(cond, Eddy_int) # Out of the Eddy
integration matrix extract values of viscous wake to be integrated
Upper_Eddy_value = (Eddy_value[0]) #First positive
value in Eddy integration matrix
Lower_Eddy_value = (Eddy_value[-1]) #Last positive
value in Eddy integration matrix
Bound_1_index = (np.where(Eddy_int == Upper_Eddy_value)) #Determine index of
upper bound in Eddy integration matrix
Bound_2_index = (np.where(Eddy_int == Lower_Eddy_value)) #Determine index of

```

```

upper bound in Eddy integration matrix
Bound_11_index = Bound_1_index[0] #Get upper bound
index as integer
Bound_22_index = Bound_2_index[0] #Get lower bound
index as integer
a = x_eddy[Bound_11_index] #Lower integral
final bound
b = x_eddy[Bound_22_index] #Upper integral
final bound
Length_Matrix = len(y)
#####
def find_nearest(array, value):
    array = np.asarray(array)
    idx = (np.abs(array - value)).argmin()
    return array[idx]

array = x
value = a
k = find_nearest(array, value)

Bound_3_index = (np.where(x == k))
Bound_33_index = Bound_3_index[0]

#####
#####
#Matrix lenght
Matrix_Del_1 = np.arange(0, Bound_33_index + S_F) #1st Deleting
matrix for interpolation
Matrix_Del_2 = np.arange(Bound_22_index, Length_Matrix) #2nd Deleting
matrix for interpolation
Complete_Del_Matrix = np.concatenate((Matrix_Del_1, Matrix_Del_2), axis = 0)
new_x = np.delete(x, Complete_Del_Matrix) #New x_ function
interpolation nodes for viscous region
new_y = np.delete(y, Complete_Del_Matrix) #New y function
interpolation nodes for viscous region
new_x_eddy = np.delete(x_eddy, Complete_Del_Matrix) #New x_eddy
coordinates to visualise where the viscous eddy function lies
new_y_eddy = np.delete(y_eddy, Complete_Del_Matrix) #New y_eddy
coordinates to visualise where the viscous eddy function lies
x_min = new_x[0] #Bottom value for
interpolation
x_max = new_x[-1] #Top value for
interpolation
Trefftz_lenght = abs(x_max-x_min)

x_plt = np.linspace(x_min, x_max, 5000)
options = ('cubic', 'linear', 'zero', 'slinear', 'quadratic', 'nearest')

for o in options:
    f = interp1d(new_x, new_y, kind=o)
    plt.plot(x_plt, f(x_plt), label=o)

    area_quad = quad(f, x_min, x_max)
    area_romb = romberg(f, x_min, x_max)
    area_Gauss = quadrature(f, x_min, x_max)
    area_trapz = trapz(new_y, new_x)
    area_simps = simps(new_y, new_x)
    print('The area calculated with interpolating method:', o, 'and', 'Quad
integration is = ', area_quad[0],
        'Trapezoidal integration is = ', area_trapz,
        'Simpson integration is = ', area_simps,

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    'Gauss integration is = ', area_Gauss[0],
    'Rhombberg integration is = ', area_romb)

Cd_dynam_qd = (Trefftz_lenght) - (1 / P_dynam_sq) * area_quad[0]
Cd_dynam_romb = (Trefftz_lenght) - (1 / P_dynam_sq) * area_romb
Cd_dynam_Gauss = (Trefftz_lenght) - (1 / P_dynam_sq) * area_Gauss[0]
Cd_dynam_Trapz = (Trefftz_lenght) - (1 / P_dynam_sq) * area_trapz
Cd_dynam_simps = (Trefftz_lenght) - (1 / P_dynam_sq) * area_simps

print('Eddy Free Stream Value is =', Eddy_inf_avg)
print('Eddy integration point matrix is =', Eddy_int)
print('First positive value in Eddy integration matrix is =', Upper_Eddy_value)
print('Last positive value in Eddy integration matrix is =', Lower_Eddy_value)
print('First positive index in Eddy integration matrix is =', Bound_1_index)
print('Last positive index in Eddy integration matrix is =', Bound_2_index)
print('Lower integral final bound is =', a)
print('Upper integral final bound is =', b)
print('Matrix lenght =', Length_Matrix)
#print('First Deleting matrix is =', Matrix_Del_1)
#print('Second Deleting matrix is =', Matrix_Del_2)
print('Deleting matrix to find viscous region is =', Complete_Del_Matrix )
print('Deleting matrix =', new_x )
print('Bottom interpolation node =', x_min)
print('Top interpolation node =', x_max)
print('Quad integral values is =', area_quad[0])
print('Romberg integral values is =', area_romb)
print('Gauss integral value is =', area_Gauss[0])
print('Trapz integral value is =', area_trapz)
print('Simps integral value is =', area_simps)
print('Quad drag value calculated =', Cd_dynam_qd)
print('Gauss drag value calculated =', Cd_dynam_Gauss)
print('Romberg drag value calculated =', Cd_dynam_romb)
print('Trapezoidal drag value calculated =', Cd_dynam_Trapz)
print('Simpson drag value calculated =', Cd_dynam_simps)
print('The nearest value in matrix x to a is =', k)
print(Bound_3_index[0])

#for accurate visualization

Scalar = MinMaxScaler()

x_data = x #x/np.linalg.norm(x, ord=np.inf, axis=0, keepdims=True)
y_data = y#y/np.linalg.norm(y, ord=np.inf, axis=0, keepdims=True)

x_int = new_x#new_x/np.linalg.norm(new_x, ord=np.inf, axis=0, keepdims=True)
y_int = new_y#new_y/np.linalg.norm(new_y, ord=np.inf, axis=0, keepdims=True)

plt.plot(x_data, y_data, 'o', label = 'Data', color = 'red')
plt.plot(x_int, y_int, 'x', label = 'Interpolate', color = 'purple')
plt.legend()
plt.show()

# Integration results

Cd_experiment = 0.0123
CD_DYN_Quad = Cd_dynam_qd
CD_DYN_Gauss = Cd_dynam_Gauss
CD_DYN_Romb = Cd_dynam_romb
CD_DYN_Trap =Cd_dynam_Trapz
CD_DYN_Simps =Cd_dynam_simps

Cd_experiment_m = np.linspace(Cd_experiment, Cd_experiment, 500)
CD_DYN_Quad_m = np.linspace(Cd_dynam_qd, Cd_dynam_qd, 500)

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Cd_DYN_Gauss_m = np.linspace(Cd_dynam_Gauss, Cd_dynam_Gauss, 500)
Cd_DYN_Romb_m = np.linspace(Cd_dynam_romb, Cd_dynam_romb, 500)
Cd_DYN_Trap_m = np.linspace(Cd_dynam_Trapz, Cd_dynam_Trapz, 500)
Cd_DYN_Simps_m = np.linspace(Cd_dynam_simps, Cd_dynam_simps, 500)
xd_plt = np.arange(0, 500)

plt.plot(xd_plt, Cd_experiment_m, 'o', label = 'Cd_experiment')
plt.plot(xd_plt, Cd_DYN_Quad_m, 'o', label = 'Cd_dynam_qd')
plt.plot(xd_plt, Cd_DYN_Gauss_m, 'o', label = 'Cd_dynam_Gauss')
plt.plot(xd_plt, Cd_DYN_Romb_m, 'o', label = 'Cd_dynam_romb')
plt.plot(xd_plt, Cd_DYN_Trap_m, 'o', label = 'Cd_dynam_Trapz')
plt.plot(xd_plt, Cd_DYN_Simps_m, 'o', label = 'Cd_dynam_simps')

plt.legend()
plt.show()

plt.plot(x_eddy, y_eddy, 'x', label = 'Exported Eddy Function', color = 'red')
plt.plot(new_x_eddy, new_y_eddy, 'x', label = 'New Exported Eddy Function', color =
'blue')
plt.legend()
plt.show()

#Monte Carlo integration

degree = 27
weights = np.polyfit(new_x, new_y, degree)
model = np.poly1d(weights)
weight_matrix = np.arange(0, degree + 1, 1)
weight_matrix = list(weight_matrix)
print(weight_matrix)

f_model = np.poly1d([weights[0], weights[1], weights[2], weights[3],
weights[4], weights[5], weights[6], weights[7],
weights[8], weights[9], weights[10], weights[11],
weights[12], weights[13], weights[14], weights[15],
weights[16], weights[17], weights[18], weights[19],
weights[20], weights[21], weights[22], weights[23],
weights[24], weights[25], weights[26], weights[27],

])

x_model = np.linspace(x_min, x_max, 300)
N = 100000
x0 = x_min
x1 = x_max

def f(xx):
    return f_model(xx)

xx = np.linspace(x0, x1, N)

fmax = max(f_model(xx))
fmin = min(f_model(xx))
print(fmax)
x_rand = x0 + (x1 - x0)*np.random.random(N)
y_rand = fmin + (fmax - fmin)*np.random.random(N)

ind_below = np.where(y_rand < f(x_rand))
ind_above = np.where(y_rand >= f(x_rand))

pts_below = plt.scatter(x_rand[ind_below], y_rand[ind_below], color = "green")
pts_above = plt.scatter(x_rand[ind_above], y_rand[ind_above], color = "blue")
vis_1 = plt.plot(xx, f_model(xx), color = "purple", label = 'Interpolated data')
vis_2 = plt.plot(new_x, new_y, color = 'red', label = 'Viscous eddy viscosity
function')
```

```
plt.legend((pts_below, pts_above, vis_1, vis_2),
          ('Nodes below', 'Nodes above'),
          loc='lower left',
          ncol=3,
          fontsize=8)

plt.show()

area_monte = fmax * (x1 - x0)
under_monte = fmax * (x1 - x0) * len(ind_below[0]) / N
print("Number of pts above the curve:", len(ind_above[0]))
print("Number of pts below the curve:", len(ind_below[0]))
print("N. below/N.total:", len(ind_below[0]) / N)
print("Rectangle area:", area_monte)
print("Area under the curve:", under_monte)
print("Monte-Carlo integral value is = ", (Trefftz_lenght) - (1 / P_dynam_sq) *
under_monte)
print(model)
print(len(new_y))

AOA = np.array([-5.0800, -2.9800, -1.9600, 0.0700, 1.1100, 5.1400, 6.2000, 7.2600,
10.2800])
CD_Exp = np.array([0.01230, 0.00940, 0.00870, 0.00960, 0.01000, 0.01240, 0.01350,
0.01470, 0.02490])
CD_NF = np.array([0.01366, 0.00947, 0.00971, 0.01059, 0.01127, 0.01488, 0.01621,
0.01765, 0.02612])
CD_FF_GAUS_SL = np.array([0.013470, 0.009594, 0.008749, 0.009941, 0.009829,
0.012882, 0.012794, 0.014230, 0.021286])
CD_FF_SIMPS_SL = np.array([0.01344, 0.00930, 0.00916, 0.00995, 0.00992, 0.01258,
0.01323, 0.01427, 0.021896296])
CD_FF_TRAPS_SL = np.array([0.01346, 0.00929, 0.00915, 0.00995, 0.00992, 0.01258,
0.01325, 0.01426, 0.021757412])
CD_FF_RHOMB_SL = np.array([0.01346, 0.00929, 0.00915, 0.00996, 0.00992, 0.01258,
0.01325, 0.01426, 0.021753178])

plt.plot(AOA, CD_Exp, label = ' Experimental Drag', color = 'red')
plt.plot(AOA, CD_NF, label = ' Near-Field Drag', color = 'blue')
plt.plot(AOA, CD_FF_GAUS_SL, label = ' Far-Field Drag (Gauss-Quadrature
Integration)', color = 'green')

plt.legend()
plt.suptitle('Wake-profiel approximated with Slinear method')
plt.show()

plt.plot(AOA, CD_Exp, label = ' Experimental Drag', color = 'red')
plt.plot(AOA, CD_NF, label = ' Near-Field Drag', color = 'blue')
plt.plot(AOA, CD_FF_SIMPS_SL, label = ' Far-Field Drag (Simpson Integration)',
color = 'green')

plt.legend()
plt.suptitle('Wake-profiel approximated with Slinear method')
plt.show()

plt.plot(AOA, CD_Exp, label = ' Experimental Drag', color = 'red')
plt.plot(AOA, CD_NF, label = ' Near-Field Drag', color = 'blue')
plt.plot(AOA, CD_FF_TRAPS_SL, label = ' Far-Field Drag (Trapezoidal Integration)',
color = 'green')

plt.legend()
plt.suptitle('Wake-profiel approximated with Slinear method')
plt.show()
```

```

plt.plot(AOA, CD_Exp, label = ' Experimental Drag', color = 'red')
plt.plot(AOA, CD_NF, label = ' Near-Field Drag', color = 'blue')
plt.plot(AOA, CD_FF_RHOMB_SL, label = ' Far-Field Drag (Rhombberg Integration)',
color = 'green')

plt.legend()
plt.suptitle('Wake-profiel approximated with Slinear method')
plt.show()

error_gauss = abs(CD_FF_GAUS_SL - CD_Exp)
error_simps = abs(CD_FF_SIMPS_SL - CD_Exp)
error_traps = abs(CD_FF_TRAPS_SL - CD_Exp)
error_rhomb = abs(CD_FF_RHOMB_SL - CD_Exp)

Gaus_total_err = sum(error_gauss)
Traps_total_err = sum(error_traps)
Simps_total_err = sum(error_simps)
Rhomb_total_err = sum(error_rhomb)

best_method_SL = min(Gaus_total_err, Rhomb_total_err, Simps_total_err,
Traps_total_err)

print('ERRORS', Gaus_total_err, Rhomb_total_err, Simps_total_err, Traps_total_err)
print('The drag count error sum opf the best method is', (best_method_SL)*1000)

CD_FF_GAUS_ZR = np.array([0.013375, 0.009155, 0.008634, 0.009942, 0.009472,
0.012873, 0.012536, 0.014327, 0.021005])
CD_FF_SIMPS_ZR = np.array([0.01344, 0.00930, 0.00916, 0.00995, 0.00992, 0.01258,
0.01323, 0.01427, 0.021896296])
CD_FF_TRAPS_ZR = np.array([0.01346, 0.00929, 0.00915, 0.00995, 0.00992, 0.01258,
0.01325, 0.01426, 0.021757412])
CD_FF_RHOMB_ZR = np.array([0.01339, 0.00934, 0.00922, 0.00999, 0.01003, 0.01262,
0.01341, 0.01432, 0.021689723])

plt.plot(AOA, CD_Exp, label = ' Experimental Drag', color = 'red')
plt.plot(AOA, CD_NF, label = ' Near-Field Drag', color = 'blue')
plt.plot(AOA, CD_FF_GAUS_ZR, label = ' Far-Field Drag (Gauss-Quadrature
Integration)', color = 'green')

plt.legend()
plt.suptitle('Wake-profiel approximated with ZERO method')
plt.show()

plt.plot(AOA, CD_Exp, label = ' Experimental Drag', color = 'red')
plt.plot(AOA, CD_NF, label = ' Near-Field Drag', color = 'blue')
plt.plot(AOA, CD_FF_SIMPS_ZR, label = ' Far-Field Drag (Simpson Integration)',
color = 'green')

plt.legend()
plt.suptitle('Wake-profiel approximated with ZERO method')
plt.show()

plt.plot(AOA, CD_Exp, label = ' Experimental Drag', color = 'red')
plt.plot(AOA, CD_NF, label = ' Near-Field Drag', color = 'blue')
plt.plot(AOA, CD_FF_TRAPS_ZR, label = ' Far-Field Drag (Trapezoidal Integration)',
color = 'green')

plt.legend()
plt.suptitle('Wake-profiel approximated with ZERO method')
plt.show()

plt.plot(AOA, CD_Exp, label = ' Experimental Drag', color = 'red')
plt.plot(AOA, CD_NF, label = ' Near-Field Drag', color = 'blue')

```

```

plt.plot(AOA, CD_FF_RHOMB_ZR, label = ' Far-Field Drag (Rhombberg Integration)',
color = 'green')

plt.legend()
plt.suptitle('Wake-profiel approximated with ZERO method')
plt.show()

error_gauss = abs(CD_FF_GAUS_ZR - CD_Exp)
error_simps = abs(CD_FF_SIMPS_ZR - CD_Exp)
error_traps = abs(CD_FF_TRAPS_ZR - CD_Exp)
error_rhomb = abs(CD_FF_RHOMB_ZR - CD_Exp)

Gaus_total_err = sum(error_gauss)
Traps_total_err = sum(error_traps)
Simps_total_err = sum(error_simps)
Rhomb_total_err = sum(error_rhomb)

best_method_ZR = min(Gaus_total_err, Rhomb_total_err, Simps_total_err,
Traps_total_err)

print('ERRORS', Gaus_total_err, Rhomb_total_err, Simps_total_err, Traps_total_err)
print('The drag count error sum of the best method is', (best_method_ZR)*1000)

CD_FF_GAUS_CB = np.array([0.013592, 0.009743, 0.008166, 0.009346, 0.008886,
0.012932, 0.012024, 0.014191, 0.020708])
CD_FF_SIMPS_CB = np.array([0.01344, 0.00930, 0.00916, 0.00995, 0.00992, 0.01258,
0.01323, 0.01427, 0.021896296])
CD_FF_TRAPS_CB = np.array([0.01346, 0.00929, 0.00915, 0.00995, 0.00992, 0.01258,
0.01325, 0.01426, 0.021757412])
CD_FF_RHOMB_CB = np.array([0.01347, 0.00929, 0.00915, 0.00994, 0.00992, 0.01257,
0.01325, 0.01426, 0.021727729])

plt.plot(AOA, CD_Exp, label = ' Experimental Drag', color = 'red')
plt.plot(AOA, CD_NF, label = ' Near-Field Drag', color = 'blue')
plt.plot(AOA, CD_FF_GAUS_CB, label = ' Far-Field Drag (Gauss-Quadrature
Integration)', color = 'green')

plt.legend()
plt.suptitle('Wake-profiel approximated with CUBIC method')
plt.show()

plt.plot(AOA, CD_Exp, label = ' Experimental Drag', color = 'red')
plt.plot(AOA, CD_NF, label = ' Near-Field Drag', color = 'blue')
plt.plot(AOA, CD_FF_SIMPS_CB, label = ' Far-Field Drag (Simpson Integration)',
color = 'green')

plt.legend()
plt.suptitle('Wake-profiel approximated with CUBIC method')
plt.show()

plt.plot(AOA, CD_Exp, label = ' Experimental Drag', color = 'red')
plt.plot(AOA, CD_NF, label = ' Near-Field Drag', color = 'blue')
plt.plot(AOA, CD_FF_TRAPS_CB, label = ' Far-Field Drag (Trapezoidal Integration)',
color = 'green')

plt.legend()
plt.suptitle('Wake-profiel approximated with CUBIC method')
plt.show()

plt.plot(AOA, CD_Exp, label = ' Experimental Drag', color = 'red')
plt.plot(AOA, CD_NF, label = ' Near-Field Drag', color = 'blue')
plt.plot(AOA, CD_FF_RHOMB_CB, label = ' Far-Field Drag (Rhombberg Integration)',

```

```
color = 'green')

plt.legend()
plt.suptitle('Wake-profile approximated with CUBIC method')
plt.show()

error_gauss = abs(CD_FF_GAUS_CB - CD_Exp)
error_simps = abs(CD_FF_SIMPS_CB - CD_Exp)
error_traps = abs(CD_FF_TRAPS_CB - CD_Exp)
error_rhomb = abs(CD_FF_RHOMB_CB - CD_Exp)

Gaus_total_err = sum(error_gauss)
Traps_total_err = sum(error_traps)
Simps_total_err = sum(error_simps)
Rhomb_total_err = sum(error_rhomb)

best_method_CB = min(Gaus_total_err, Rhomb_total_err, Simps_total_err,
Traps_total_err)

print('ERRORS', Gaus_total_err, Rhomb_total_err, Simps_total_err, Traps_total_err)
print('The drag count error sum of the best method is', (best_method_CB)*1000)
```

APPENDIX G: SOFT COPY

See attached thumb drive for the simulation files and calculation spread sheets used in this thesis.