Examining interest rate volatility and bond yields in South Africa

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Dissertation submitted in fulfilment of the requirements for the degree *Master of Commerce in Risk Management* at the North-West University

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DECLARATION

I, Lizzy Zoleka Ndolela, student number 23246618, declare that this dissertation titled "Examining interest rate volatility and bond yields in South Africa" which I hereby submit in fulfilment for the degree, Masters of Commerce in Risk Management at North West University, is my own work and that it will not be presented at any other university for a similar or any other degree.

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To whom it may concern

This is to confirm that I, the undersigned, have language edited the d**issertation** of

LZ Ndolela

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The responsibility of implementing the recommended language changes rests with the author of the dissertation.

Yours truly,

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Proverbs 3:5

ABSTRACT

This study examined interest rate volatility and bond yields in South Africa. Two independent variables were employed, the 91 days T-bill and short-term interest rate volatility. The short-term interest rate volatility was estimated by the use of the generalised autoregressive conditional heteroscedasticity GARCH (1,1) model. The average bond yields were used as dependent variables, with the following ranges: zero to three year bond yields (short-term) and 10 year and above bond yields (long-term). The sample period of the study spanned from January 2004 to December 2017 with a total of 168 observations. The choice for the study period was to incorporate periods before, during and after the 2007-2009 financial crisis. The relationship between interest rates, interest rate volatility and bond yields were examined by the use of the Cox-Ingersoll-Ross (CIR) one-factor (1985) model and the Longstaff and Schwartz (1992) two-factor model. Both these models have been used extensively in the term structure of interest rate studies in the literature. In order to evaluate these models the autoregressive distributive lag (ARDL) model was employed as the unit root results from the study indicated that the variables were integrated at different orders, namely I(0) and I(1).

The results from the study demonstrated that there is a statistically significant positive relationship between short-term interest rates and bond yields in South Africa. These results are consistent with the expectation hypothesis theory that long-term interest rates are an average of the current- and future expected short-term interest rates. These results were obtained using the CIR (1985) one-factor model. The use of the Longstaff and Schwartz (1992) two-factor model revealed that there is a statistically insignificant positive relationship between interest rate volatility and bond yields in South Africa and the effect is greater for the (three to five year) medium-term bond yields.

Investors make their decisions based on the prediction of the level of future interest rates. The results of this study verify that the expectation hypothesis theory holds in South Africa. This means it is possible for bond investors to predict the future changes in interest rates by a consideration made to the yield curve. Thus, these results may aid South African bond investors to hedge themselves against unfavourable interest rate volatility in the future, with reference made to the yield curve.

Key words: interest rates, interest rate volatility, bond yields, autoregressive distributed lag model

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LIST OF ABBREVIATIONS

ACF:	Autocorrelation Function
ADF:	Augmented Dickey Fuller
ANC:	African National Congress
ARDL:	Autoregressive Distributive Lag
ARMA:	Autoregressive moving average
BESA :	Bond Exchange of South Africa
CAPM:	Capital Asset Pricing Model
CIR:	Cox-Ingersoll-Ross
DF:	Dickey-Fuller
ECM:	Error Correction Model
ECT:	Error Correction Term
FED:	Federal Reserve Bank
GARCH:	Generalized Autoregressive Conditional Heteroscedasticity
GMM:	Generalized Method of Movement
IMF:	International Monetary Fund
JB:	Jarque-Bera
Jibar:	Johannesburg Interbank Agreed Rate
JSE:	Johannesburg Stock Exchange
KPSS:	Kwiatkowski-Phillips-Schmidt-Shin
OLS:	Ordinary Least Squares
PP:	Phillips-Perron

SARB: South African Reserve Bank

T-bills: Treasury Bills

YTM: Yield to maturity

US: United States

CHAPTER 1: INTRODUCTION

1.1 BACKGROUND OF THE STUDY

During the 2007-2009 financial crisis, the level and volatility of interest rates fluctuated significantly (Gerlach-Kristen & Rudolf, 2010). As a result, short-term interest rate volatility has been a dilemma that fixed income portfolio managers devote much of their time to solve, which is done in order to enhance their portfolio returns. In consequence, this process gives investors more knowledge of the manner in which to mitigate interest rate risk associated with bond portfolios.

Brousseau and Durré (2013:4) state that interest rate volatility has become an important element in financial market analysis. The measurement of interest rate volatility is important as it affects investment decisions (Ariff &Sarkar, 2002:667). For central banks, the analysis of the short-term interest rate volatility is fundamental, since the monetary policy is implemented by the continuous influence of the short-term interest rates, which influence long-term interest rates (Brousseau & Durre, 2013:5). Cox *et al*. (1985) claim that high interest rate volatility stems from higher interest rates as well as when the yield curve reveals higher curvature. In effect, higher debt that results from increased interest rates leads to a high rate of classified loans (Aver, 2008; Louzis *et al*., 2012; Nkusu, 2011).

The volatility of short-term interest rate is extensively observed by Cox *et al*. (1985); Litterman *et al*. (1991); Longstaff and Schwartz (1992); Brenner *et al*. (1996); Anderson and Lund (1997); Ball and Torous (1999); Olan and Sandy (2005); Turan and Liuren (2005); Ariff and Sarkar (2002); and Olweny (2011), among others, as it affects the term structure of interest rates. The term structure of interest rates is important in a sense that economists and investors believe that the shape of the yield curve predicts the future market expectations for monetary policy decisions on interest rates (Omondi, 2016:4). Thus, any variation in short-term interest rates has the power to change the shape of the yield curve. Particularly, during the 2007-2009 financial crisis, the South African Reserve Bank (SARB) increased short-term interest rates continually, which reached a peak in June 2008 of 11.42 percent. In consequence, the increase in the short-term interest rates caused an increase in the bond yields. The short-term interest rates were higher than the bond yields, which resulted in an inverted yield curve. Andersen (2018) claims that the inverted yield curve indicates the presence of recessionary pressure in the economy. After the recession period of 2007-2009, short-term interest rates were lower than long-term interest rates, which resulted in normal yield curves.

High interest rate volatility has an effect on bond yields, especially during periods of higher interest rate volatility (Sundaresan, 2009:138). This makes short-term interest rate volatility a key element when valuing bond yields (Strickland, 1993:1). As a result, many authors such as Vasicek (1977); Cox *et al*. (1985); and Longstaff and Schwartz (1992), have developed theories that explain the relationship between short-term interest rate volatility and bond yields.

Sundaresan (2009:105) explains that the process in which the term structure of interest rate is modelled involves the following interest rate properties: (1) short-term interest rates are mean reverted, (2) short-term interest rates are more volatile than long-term interest rates and (3) short-term interest rate models used have no arbitrage opportunities. The first implemented term structure models were the single-factor models, which follow a mean-reverting process of short-term interest rates. The models developed by Vasicek (1977) and Cox-Ingersoll-Ross (CIR) (1985) are single-factor models that only consider short-term interest rates to have an effect on the entire term structure of interest rates. However, both these models have shortfalls that make them less desirable when the term structure of interest rates is estimated. The Vasicek (1977) model allows short-term interest rates to become negative (Maranga *et al*., 2018:46) while the CIR (1985) model only recognises a one state variable (short-term interest rates) to have an effect on the whole term structure of interest rates.

From the CIR (1985) model, the Longstaff and Schwartz (1992) model was developed. This model uses two state variables, the short-term interest rate and the volatility of the short-term interest rate. The Longstaff and Schwartz (1993) model explains that the increase in short-term interest rate level and the volatility of short-term interest rates are indeterminate. This means that the no-arbitrage approach does not result in accurate predictions with regards to the interest rate level-volatility relationship (Ariff and Sarkar, 2002: 668).

After the introduction of these models, a number of authors tested the effect of interest rate volatility and bond yields (Longstaff & Schwartz, 1992; Ariff & Sarkar, 2002; Bhat & Fahad, 2016). Longstaff and Schwartz (1993) corroborated by Sarkar and Ariff (2002) found a negative and significant relationship between short-term interest rate volatility and bond yields, while Bhat and Fahad (2016) found a positive relationship between short-term interest volatility and sovereign bond yields. In addition, Ariff and Sarkar (2002) claim that the significance of this relationship depends on a number of factors, which include (1) the maturity of the yields, (2) the size of the government bond relative to corporate bonds in the country and (3) liquidity of the bond market. The results from previous studies reveal that there is still an inconclusive relationship between interest rate volatility and bond yields.

The aim of this study is to examine interest rate volatility and bond yields in South Africa. It is important to understand the manner in which interest rate volatility may affect bond prices and to test whether the effect is significant on bond yields. This will aid investors to manage the exposure of interest rate risk. Little attention has been devoted to test whether short-term interest rate volatility has a significant effect on bond yields in South Africa. In the South African context, interest rates are influenced by the decision of the monetary policy committee. The committee makes a decision on the movement of the short-term interest rates, which as a result influence the movement of long-term interest rates (Dube & Zhou, 2013). Although some South African authors looked at the relationship between short-term interest rates and long-term interest rates in South Africa, (Nel, 1996; Arize *et al.*, 2002; Aziakpono & Khomo, 2005; Bonga-Bonga, 2010), none of the authors studied the relationship between short-term interest rate volatility and bond yields in South Africa.

Omondi (2016:2) explains that the volatility of short-term interest rates is strongly related to the shape of the yields curve. In South Africa, attention was focused on interest rates because of the 1997-1998 Asian crisis, when short-term real interest rates were extremely high (Kahn & Farrell, 2002,1). Arize *et al*. (2002) investigated the long-term co-movements of short-term and long-term interest rate series for South Africa and 19 other countries. The study found that short and long-term interest rates move together in the long run. However, on a regular basis, short-term interest rates are lower than long-term interest rates. As a result, this is reflective of high inflation-risk premiums that investors demand for long-term bonds (Bonga-Bonga, 2010:45). The expectations hypothesis theory confirms that the monetary policy influences the direction of long-term interest rates by the decisions made on whether to increase or reduce short-term interest rates and by changing market expectations of future short-term interest rates (Walsh, 2003).

Since short-term interest rate volatility signifies a critical role in the monetary policy decision as well as fixed-income portfolio management, the results of this study will aid economists and investors to hedge against undesirable uncertainties associated with interest rate changes.

1.2 PROBLEM STATEMENT

There are considerable studies that explain the relationship between short-term interest rate volatility and bond yields (Longstaff & Schwartz, 1992; Ariff & Sarkar, 2002; Bhat & Fhad, 2016), however, there is still no accurate conclusion between short-term interest rate volatility and bond yields as previous studies have shown mixed results. The result of the 2007-2009 financial crisis has caused uncertainty in the financial markets and, as a result, banks and investors have become more risk-averse, especially to interest rate risk (Guiso *et al*., 2013:1).

Alam and Uddin (2009:43) argue that price movements of fixed-income assets are largely influenced by interest rate volatility. Moreover, several studies have investigated the empirical estimations of interest rate volatility. Interest rate models such as those of Merton (1973) and Vasicek (1977) assume that short-term interest rate volatility is independent on the different maturities of interest rates. In contrast, later models developed by Cox *et al*. (1985) and Black and Karasinski (1991) explain that volatility of interest rates depend on the different maturities of interest rates. Moreover, Brenner *et al*. (1996) found evidence that interest rate volatility depends on the level of interest rates as well as on the generalised autoregressive conditional heteroscedasticity (GARCH) processes used.

When a government borrows money from the public on a long-term basis, the process takes place by the exchange or issuance of bonds (Bodie *et al*., 2010). The bond's cash flow remains the same, as the maturity date and coupon rate are specified upon issuance. When interest rates increase, the value of the said bond decreases, therefore, there is an inverse relationship between interest rates and bond prices (Brealey *et al*., 2014:174). While there are several theories, studies and models that explain the dependence of short-term interest rate volatility on the level of the short-term interest rates, the results remain inconclusive.

It is important to understand the level of interest rate risk linked with different types of bonds in the market as well as the manner in which the level of interest rate affects the expected return. There is a trade-off between risk and return. Bonds with a higher level of risk will generally have a rate of return much higher than bonds with a lower level of risk. Moreover, most bonds pay lower returns than shares and other riskier investments (International Monetary Fund, 2002:14).

The results of this study will contribute to the body of knowledge, which fixed income managers can used to hedge against the potential uncertainty of interest rates, essentially during periods of high interest rate volatility. This study answered the question of best bond yield maturity during periods of high interest rate volatility in South Africa. By analysing the South African short-term interest rates and bond yields, the following two research questions are posed: Is there a positive or negative relationship between short-term interest rate volatility and bond yields in South Africa? And: Is the effect stronger on short-, medium- or long-term bond yields?

1.3 OBJECTIVES OF THE STUDY

The following objectives have been identified and outlined for the study.

1.3.1 Primary objective

The primary objective of this study is to examine interest rate volatility and bond yields in South Africa.

1.3.2 Theoretical objectives

The following objectives have been formulated for the study:

- Provide the background and history of interest rates and bonds yields in South Africa;
- Describe the relationship between short-term and long-term rates;
- Review the relationship of interest rates and bonds by use of the term structure of interest rates and related theories;
- Provide a theoretical framework of the relationship between interest rates and bond yields; and
- Review and report on the findings from previous empirical studies on the relationship between interest rates and bond yields.

1.3.3 Empirical objectives

To achieve the primary objective of the study, the subsequent empirical objectives have been identified:

- Evaluate the movement between interest rate volatility and bond yields in South Africa;
- Evaluate and interpret graphically the effect of short-term interest rate volatility on bond yields before, during and after the 2007-2009 financial crisis;
- Evaluate whether there is a positive or negative relationship between interest rate volatility and bond yields;
- Determine whether the effect of interest rate volatility is stronger on short-, medium- or long-term bond yields in South Africa; and

- Determine whether short-term interest volatility has a short- or long-run effect on bond yields.

1.4 RESEARCH DESIGN AND METHODOLOGY

The research method used in this study included both the literature review and empirical study.

1.4.1 Literature review

The literature review section used data extracted from books and articles published in peer-reviewed journals in order to achieve the theoretical objectives of the study. The literature chapter described the conceptualisation and theory behind interest rates, interest rate volatility, bond yields and the term structure of interest rates. These theories and concepts better explain the relationship between short-term interest rate volatility and bond yields. The literature also included reviews from previous empirical studies that investigated the similar relationship between interest rates and bond yields.

1.5 EMPIRICAL STUDY

1.5.1 Data collection and sampling

This study examined interest rate volatility and bond yields in South Africa. The study followed a quantitative design. The sampling data spans from January 2004 to December 2017. The reason behind the chosen sample period was to examine the relationship between short-term interest rate volatility and bond yields before, during and after the 2007-2009 financial crisis. The sample period gives an explanation of the manner in which interest rate volatility behaved during shock and non-shock periods in the South African economy. The study used secondary data in order to achieve the study objective. The monthly short-term interest rate and monthly average bond yields data were collected from the International Monetary Fund (IMF) and the South African Reserve Bank (SARB), respectively. The bond yields consist of monthly average bond yields that range from zero to three and 10 year and above bond maturities. The main variable used is the short-term interest rate series, which, in South Africa, is the Central Bank 91 days T-bill rate. Short-term interest rate volatility was estimated using a GARCH (1,1) model.

1.5.2 Data analysis

The data analysis section examined and discussed the results in order to achieve the study objectives. Several statistical tests were conducted before the regression model was employed.

These tests comprised graphs, descriptive statistics, correlation analysis and unit root tests. The Longstaff and Schwartz (1992) two-factor theoretical model is widely used to explain the relationship between short-term interest rate volatility and bond yields. This model specifies two factors that affect the entire term-structure of interest rates, which are short-term interest rates and the volatility of short-term interest rates. In the said model, volatility of the short-term interest rates is the function of the lagged value of the short-term interest rate and square of the last expected change in the short-term interest rate. Ariff and Sarkar (2002) confirm that the issuance of the bond is based on the level of interest rates and its volatility. Thus, in order to use the data, the issuers will base their decision on the previous month’s data. To test the Longstaff and Schwartz (1992) theoretical model, the autoregressive distributed lag (ARDL) was used in order to achieve the objective of examining short-term interest rate volatility and bond yields in South Africa.

1.6 ETHICAL CONSIDERATIONS

The study is conducted in accordance to the ethical guidelines and principles as prescribed by the North-West University. Ethical clearance was obtained from the Social and Technological Sciences Research Ethics Committee. The ethics clearance number for this study is: ECONIT-2017-048.

1.7 CHAPTER CLASSIFICATION

The study comprises the following chapters:

Chapter 1: Introduction and background to the study − The first chapter focuses on the background of interest rate volatility and the manner in which it affects bond yields. Thereafter, the problem statement, primary objective and the research design and methodology are briefly discussed.

Chapter 2: Literature review − Chapter 2 focuses on the dynamic relationship between interest rates and bonds and the manner in which the volatility of interest rates affects bond yields. Furthermore, the chapter provides a review on the concept of investments in bonds by investors.

Chapter 3: Methodology – The methodology used in the study is described. Regression models are used to evaluate the effect of short-term interest rate volatility and short-term interest rates on different maturity government bonds from January 2004 to December 2017. The ARDL model is used to examine the relationship between short-term interest rate volatility and bond yields in

South Africa. The data used in this study was extracted from the International Monetary Fund (IMF) and SARB databases.

Chapter 4: Results and findings - This chapter details the results of the study to examine the relationship between short-term interest rate volatility and bond yields in South Africa. The results of this study are also compared with results from previous studies.

Chapter 5: Conclusion and recommendation – The final chapter concludes the study by providing a summary of the main findings and relating them to the objectives of the study. The overall conclusion of the study is presented. Recommendations for future research and limitations of the present study are also presented.

CHAPTER 2: OVERVIEW OF INTEREST RATES AND BOND YIELDS

2.1 INTRODUCTION

The determination of the relationship between interest rates and different terms of maturities, in other words, the term structure of interest rates, is of concern to researchers and practitioners in economics and finance (Maranga *et al*., 2018). The concern stems from their belief that the shape of the yield curve predicts the future market expectations for interest rates and monetary policy decisions (Omondi, 2016:4). As a result, the term structure of interest rates gives an overview of the relationship between zero coupon bonds that differ in terms of maturity (Olweny, 2011:291). Longstaff and Schwartz (1992) influenced the research of short-term interest rate volatility and the term structure of interest rates. They developed a two-factor model that explains the relationship between short-term interest rate volatility and the term structure of interest rates.

In South Africa, the term structure of interest is influenced by the South African Reserve Bank's (SARB) notion that changes in the repo rate prompt similar changes in money market instruments and long-term bond yields (Dube and Zhou, 2013:188). There is little attention focused on the term structure of interest in South Africa, more specifically between the relationship of interest rate volatility and bond yields. Similarly, researchers (Longstaff & Schwartz, 1992; Ariff and Sarkar, 2002; Bhat & Fahad, 2016) focused on the relationship between interest rate volatility and bond yields. Thus, this chapter will provide an overview of the relationship between short-term interest rate volatility and bond yields.

The aim is to first explore the background of interest rates where the emphasis is on the conceptualisation of interest rates, followed by an explanation of short-term interest rates, short-term interest rate volatility and long-term interest rates. This chapter also investigates the features of bonds and examines the different bond measurements (i.e. duration, Macaulay and modified duration) that explain the sensitivity of bond prices to changes in interest rates. In addition, South African government bonds are discussed and explored from 2004 to 2017.

The relationship between short-term interest rates and bond yields is explained by means of the term structure of interest rates. The term structure of interest rates is clarified in detail by the use of (1) the yield curve, (2) the four theories of the term structure of interest and (3) the theoretical framework of term structure models that give an understanding of how short-term interest rate volatility affects long-term bonds. The chapter concludes with the factors that affect the relationship between interest rates and bonds, the risk associated with the bond portfolio and the reports from previous studies.

2.2 DEFINITION AND TYPE OF INTEREST RATES

Interest rates can be defined as the cost of borrowed funds, for example, interest on credit extended to the borrower (Bean, 2017:3). Thus, the larger the interest on the borrowed funds the more expensive the cost of borrowed funds (Kucera, 2014:7). The interest rate process explains the movement of savings from investors to debtors. This is primarily influenced by the exchange of interest rates; for this reason, interest rates have a significant role in both the capital and money market (Van der Merwe *et al*., 2014:102). Accordingly, interest rates explain that savings and investments are equivalent, assuming the existence of the capital market (Olweny, 2011:289).

In addition, Bean (2016:3) views interest rates as either the repayment received for income not spent; for instance, money saved rather then spent or either cost of consumption when resources are not available, which implies a credit card usage for purchases. This makes interest rates the foundation of many financial economic models, such as the term structure models, which value prices of bonds and derivative pricing models (Olweny, 2011:289). The term structure model explains the relationship between short-term and long-term interest rates (Sundaresans, 2009:132).

The difference between short-term and long-term interest rates is in the prediction of future economic activities (Bauer & Mertens, 2018:11). Furthermore, short-term interest rates influence the changes in long-term interest rates (Akram, 2016:2). As a result, the relationship between short and long-term rates determines the shape of the yield curve (Aling & Hassan, 2012:2).

2.2.1 Short-term interest rates

One of the mandatory rules of the SARB is to influence the movements of repo rates, also known as short-term interest rates, over time (Dube & Zhou, 2013:189). The repo rate is the rate that the SARB charges commercial banks when they borrow money (Van Wyk *et al*., 2015). Arguably, a short-term interest rate is the fundamental tool used by the SARB to influence the supply of money in the economy. This is accomplished by expansionary or contractionary monetary policy where short-term interest rates are increased or decreased to influence the economy (Bataa *et al*., 2015:2). Short-term interest rates form part of the money market since maturity is less than 12 months (Van Wyk *et al*., 2015:314). These money market instruments include, but are not limited to, commercial papers, treasury bills (T-bills) and repurchase agreements (Van Zyl *et al*., 2006:252). A T-bill is issued by the government, in South Arica that is the National Treasury (Van Wyk *et al*., 2015:270). In South Africa, the T-bill is used as a proxy for short-term interest rates (Aling & Hassan, 2012:12).

Short-term interest rates change in response to economic events. These might include international and domestic crises, changes in the Federal Reserve Bank (FED) policy rate and changes in the expectations of future long-term inflation rates and economic growth (Wambua, 2013:6). In consequence, changes in short-term interest rates affect other macroeconomic variables. These might include economic growth, unemployment, price levels and the balance of payments (Khurshid & Suyuan, 2015:81). Moreover, change in short-term interest rates influence the direction of the long-term interest rates and may have a positive or negative influence on long-term interest rates (Bodie e*t al*., 2011).

When short-term interest rates are higher than long-term interest rates, the yield curve forms an inverted yield curve (Grasso & Natoli, 2018:4). Conversely, when long-term interest rates are higher than short-term interest rates, the shape of the yield curve is normal (Motloung, 2013:7). During an upswing period in the economy, the increase in short-term interest rates influences an increase in the long-term interest rates, however, by an amount less than the current change in short-term rates (Dube & Zhou, 2013:192).

Interest rates fluctuate substantially during periods of economic downswing, for example, during the 2007-2009 financial crisis, interest rates were volatile, which caused uncertainty in the market. This led to many researchers (Bali, 2007; Chiarella *et al*., 2015; Olweny, 2011; Ondieki, 2014; Maranga *et al*., 2018) focusing on the dynamics and models of changes in short-term interest rates. These studies reflect that short-term interest rates influence long-term interest rates/bond yields.

A fixed income portfolio is affected by interest rates in such a way that when short-term interest rates are expected to decrease, investors shift their investments decision from short-term bonds to long-term bonds. Investors then sell the short-term bonds in order to finance the purchase of long-term bonds (Clay & Keeton, 2011:4).

In South Africa, short-term interest rates are affected by the monetary policy decisions. It is important for bondholders to know the short-term interest rate movements, as they have a major impact on bond portfolios.

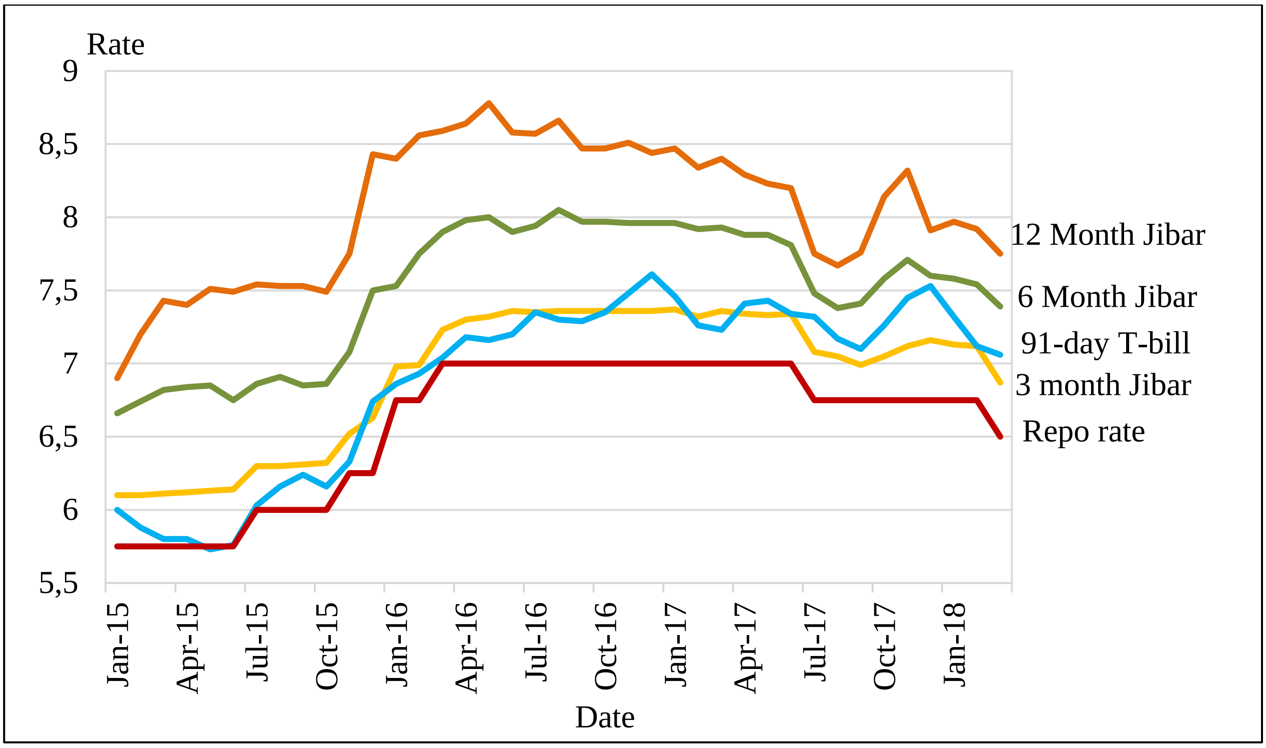

Figure 2-1: South African money market rates

Source: Compiled by author from SARB (2018)

Figure 2.1 depicts short-term money market interest rates in South Africa from January 2015 to January 2018. From the graph, it might be deduced that the repo rate influences the direction of the other short-term money market interest rates over time. The repo rate changes move in conjunction with the monetary policy decisions to either stimulate or depress the economy (Dube & Zhou, 2013:190).

For the period under review, South African short-term interest rates were at the highest in May 2016 at 8.78 percent in the 12-month Johannesburg Interbank Agreed Rate (Jibar) and were at the lowest by 5.75 percent as reflected by the repo rate. The interest rates also reflect that low duration interest rates have low interest rates and higher duration interest rates have higher interest rates.

In December 2017, short-term money market interest rates fluctuated, which was caused by the effect of the political event held by the South African ruling party, the African National Congress (ANC). The decrease in the interest rates reflected an improvement in the investor's confidence, which was consequently followed by a repo rate reduction. The three-month Jibar rate was 7.16 percent in December 2017. In addition, the movements of the 12-month Jibar rate were more volatile than the three- and six-month Jibar rates, which is the result of the value of the rand (SARB, 2018).

2.2.2 Short-term interest rate volatility

Volatility measures the variability of interest rates in relation to the expected interest rate mean (Sundaresan, 2009:136). Therefore, volatility in interest rates explains the unpredictability in the money market that results in higher risk. It also calculates the risk in terms of the spread of asset returns from expected values (Hajilee *et al.*, 2015:1740). Volatility is comparable to the compounded forward rate, in order for forward rates to be positive. Ultimately, this means that volatility of interest rates is high during periods of high interest rates; consequently, volatility of interest rates is low during low interest rate periods (Donald *et al*., 2013).

Dabale and Jagero (2013:28) state that the continuous increase in interest rates eventually decreases the returns on investments in the real economy and perpetuates trading in financial instruments. The empirical evidence by Ondieki (2014) expresses that the volatility of interest rates affect government, investment, pension funds and personal decisions. Thus, the movement of short-term interest rates is determined by a multitude of factors, for instance, inflation rates, demand and supply, global, fiscal, monetary and political factors.

A number of interest rate volatility characteristics have been observed in the literature: (1) interest rate volatility is stochastic in nature, (2) interest rate volatility contains unspanned components and (3) interest rate changes are correlated to interest rate volatility. Maranga *et al*. (2018) investigated the dynamics of interest rates, the study uncovered that relative interest rate volatility is negatively correlated to interest rates, while, absolute interest rate volatility is positively correlated to interest rates.

The study on the correlation between short-term interest rates and volatility of short-term interest rates was corroborated by Brenner *et al*. (1996), Olan and Sandy (2005), Turan and Liure (2005) and Olweny (2011) who found a positive relationship between interest rates and volatility of interest rates. Drousseau and Durre (2013:5) state that the comparison of interest rate volatility at specific maturities with the average volatility across the whole maturity spectrum allows central banks to detect typical movements in some segments in financial markets. Sundaresan (2009:138) examined the volatility of interest rates of various maturities and found that short-term interest rates are more volatile than long-term rates. The observation made is necessary for a number of reasons: (1) when estimating the term structure of interest rates, it is important to include the volatility in order to have a satisfactory model and (2) volatility significantly influences the hedging and pricing of interest rate derivative products.

Estimating the volatility of interest rates predicts the uncertainty that encompasses the market's expectation, particularly regarding the future path of monetary policy interest rates. The analysis of the volatility of interest rates is crucial for central banks, since monetary policy implements the decisions to either increase or decrease short-term interest rates. These decisions aid in determining the market expectations of future values of those short-term interest rates (Vincent & Allain, 2013).

Olweny (2011) affirms that short-term interest rate volatility has consequences on the yield curve, which stems from two factors. The first factor pertains to higher short-term interest rate volatility and may instigate higher expected rates for longer-term maturities. The second factor pertains to the increased convexity of discount factor function as a result of higher short-term interest rates, which decreases the yields for longer-term maturities.

2.2.3 Long-term interest rates

Van der Merwe and Mollentze (2010:94) define long-term interest rates as rates that are longer than one year and are traded dominantly in the capital market. Long-term rates are influenced by changes in the short-term interest rates; however, the short-term interest rates are more sensitive to these changes. This is because long-term interest rates are influenced by the average of the current and future short-term interest rates (Van der Merwe *et al*., 2015).

The expectation hypothesis theory describes long-term interest rates as an average of the present- and future expected short-term interest rates (Jablonsky, 2012:271). Long-term interest rate estimation measures the uncertainty in market expectations (Brousseau & Durre, 2013:3). In South Africa, long-term interest rates are volatile and have frequently been in real terms, however, in 2004, the long-term rates declined because of the FEDs decision to decrease interest rates, which resulted into a spill over to other economies (Ahmed & Ricci, 2006:211).

2.3 DEFINITION AND TYPE OF BONDS

Mpofu *et al*. (2013:205) define a bond as a long-term debt with fixed interest payments that allow investors to loan money to the government or a company in exchange for a predetermined interest rate. Further, the issuer of the bond commits to make continuous interest payments to the investor at a specified rate until a specified date. Once a bond is issued, money flows from surplus units to the borrower (Reilly & Brown, 2012:570). Further to this, the borrower receives the price of the bond, which has the possibility of being lower, higher, or equal to the principal amount (Van Wyk *et al*., 2015:314).

South African bonds are quoted in domestic currency and traded under a flexible exchange rate regime (Mpofu *et al.*, 2013:206). The South African borrower's ability to raise foreign currency liability is discouraged by fluctuations in interest rates. Moreover, these fluctuations reveal a negative relationship between government bonds and domestic interest rates (Dube & Zhou, 2013). Mu *et al*. (2013:6) state that interest rate volatility is positively related to the corporate bond market and inversely related to the government bond market.

Mpofu *et al*. (2013:206) state that in South Africa the bond market is regulated and monitored by the Bond Exchange of South Africa (BESA), which is one of the subsidiaries of the Johannesburg Stock Exchange (JSE). It is the responsibility of BESA to ensure the effective operation and regulation of the secondary market. The secondary market consists of institutions such as pension funds, banks, insurance companies and fund management companies (JSE, 2013).

2.3.1 Bonds valuation and features

To evaluate the impact of the current market price of bond portfolios the calculation of fixed income assets becomes important. Van Zyl *et al*. (2006:322) explain that the volatility of the price of a bond specifies the sensitivity of the bond to a change in the yield rate. Therefore, a bond price that pays annual coupons can be expressed as:

$$p = \sum_{t-1}^{n} \frac{C}{(1+r)^t} + \frac{M}{(1+r)^n} \tag{2.1}$$

Where:

P = bond price;

C = coupon payment;

M = maturity payment value;

n = maturity period of debt instrument; and

r = yield to maturity

From Equation 2.1, it may be deduced that there is an inverse relationship between interest rates and bonds. As a result, the yield of the bond increase results in the decline of the bond price, likewise, when the yield of the bond decreases, it leads to an increase in bond price (Dornbusch *et al*., 2014:461). Economists and investors hold the view that the shape of the yield curve estimates future expectations for interest rates and monetary policy conditions. In addition, Motloung (2013:6) states that the shape of the yield could be either upward, downward, flat or humped.

2.3.2 Duration, coupon and yield of bonds

Bonds are debt instruments that are issued on the basis that interest will be paid on a regular basis by the issuer to the holder and the capital payment will be paid in full at some point in time (Mohr *et al*., 2008:326). The bond's features may include coupon rate, principal and maturity date (Van Wyk *et al*., 2015:316).

2.3.2.1 Duration

A bond's duration is a weighted average of dates of each cash flow. This implies that the weights of the bonds are divided by the sum of the weights (Reilly & Brown, 2012). Moreover, Reilly and Brown (2012) together with Mpofu *et al*. (2008:226) define duration by the sensitivity of bonds to interest rate changes. This enables portfolio managers to manage the price sensitivity of bonds. The duration measurement of a bond is calculated as (Mpofu *et al*., 2008:226):

$$D=\frac{V_{-}-V_{+}}{2V_0(\frac{\Delta y}{100})} \qquad (2.2)$$

V_{-}=value of a bond should the yield decrease byΔy

V_{+}= value of a bond should the yield increase byΔy

V_0= price of a bond and

Δy= calculates percentage change in yield for $V_{-} and\ V_{+}$.

Moreover, the duration of the bond explains that a one-percentage change in the market price will change the bond price by one percent. This calculation explains the inverse relationship between bond prices and interest rates. The longer the duration of the bond, the more the bond is sensitive to interest rate movements (Mpofu *et al*., 2008:226).

2.3.2.2 Macaulay duration

In 1983, Federick Macaulay developed the Macaulay duration equation for bonds to measure the duration of cash flows from the bond (Reilly & Brown, 2012:635). Crouhy *et al*. (2014:216) explain that the Macaulay duration is calculated by using the yield-to-maturity (YTM) as the suitable interest rate. As a result, the yield curves are assumed flat, which indicates that there is an equivalent amount of change in interest rates across all maturities.

Van Wyk *et al*. (2015:324) explain that Macaulay durations and modified durations are the foundation to the calculation of bond sensitivity to changes in interest rates. The subsequent assumption can be made under the Macaulay duration:

$$D=\sum_{t=1}^{n} \frac{(PV)(CF_t)\times t}{Market\ price\ of\ bond} \tag{2.3}$$

Where $(PV)(CF_t)$ denotes the present value of the bond at period $_t$, while t represents the time of each cash flow and n represents the period of maturity. This equation explains the number of years it will take to recover the real price of a bond given the present value of the coupon price and future expected principal payment.

Reilly and Brown (2012:638) state that the following are key assumption of Macaulay duration:

- The maturity and duration of a zero coupon bond is equivalent;
- The duration of a zero coupon bond will always be lower than the maturity of a bond;
- An inverse relationship exists between the duration and coupon of a bond;
- A positive relationship exists between the duration and maturity of a bond; and
- There is a negative relationship between duration and YTM.

2.3.2.3 Modified duration

Crouhy *et al*. (2014:216) argue that the modified duration is frequently used to calculate the value of the bond. Van der Merwe *et al*. (2014) opine that movements in bond prices will equivalently differ with modified duration and cause small changes in yields.

Modified duration is calculated as:

$$D^* = \frac{D}{1+y} \tag{2.4}$$

Where:

- D= Macaulay duration; and
- y= current YTM.

Crouhy *et al*. (2014:217) also state that a linear relationship exists between price- and yield changes of a bond. Price volatility is highly correlated to higher duration, which implies that bonds with a longer maturity are more sensitive to changes in volatility.

2.3.2.4 Convexity and duration of bonds

Convexity of a bond explains the manner in which bond duration is sensitive to changes in interest rates (Nazir, 2009:13). In line with this, the modified duration explains the slope of a curve at a given yield while convexity details changes in duration (Crouhy *et al*., 2014:643).

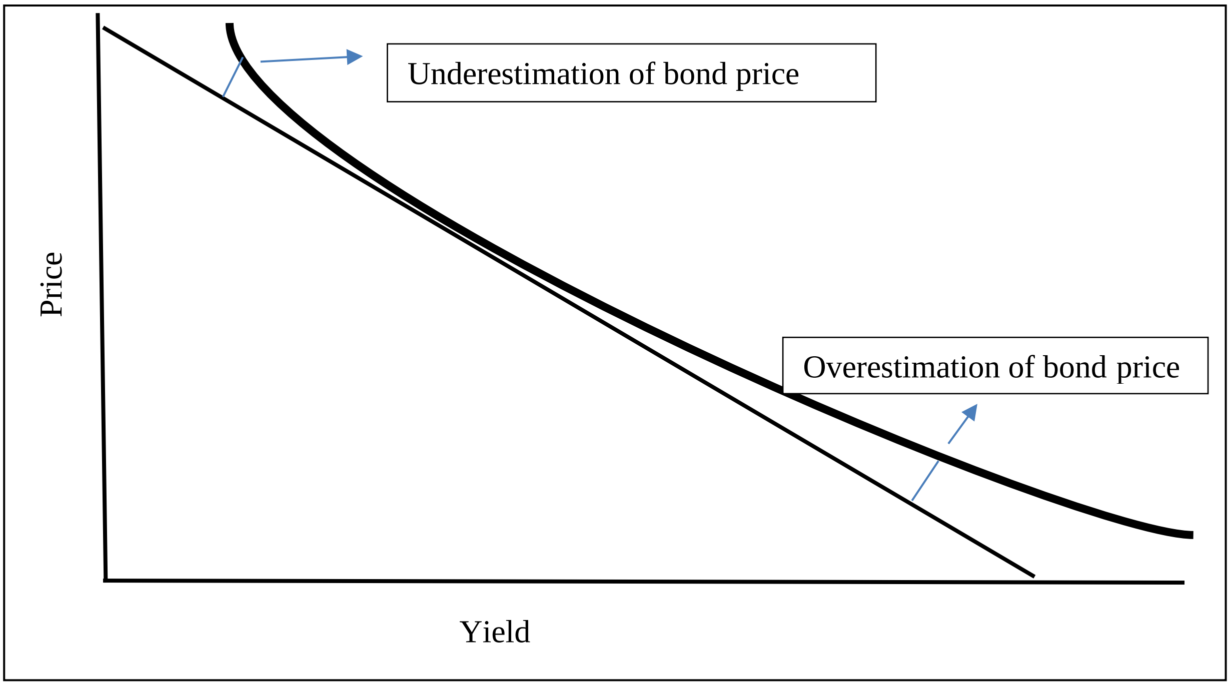

Figure 2-2: Bond convexity

Source: Adapted from Mpofu *et al.* (2013)

Figure 2.2 depicts the relationship between the price and yield of a bond. In cases where one bond is more convex, then another more convex bond is the result of an increased price for a specific decline in yield. In contrast, the less convex bond is a result of a decreased price for a specific increase in yield (Van Wyk *et al*., 2015:326). There is a positive relationship between convexity and duration, which indicates that a less convex value is correlated to less duration values and high convex value is correlated to higher duration values (Mpofu *et al*., 2013:227).

Hou and Suardi (2011) opine that short-term interest rates have higher volatility than long-term interest rates. Theoretically, this makes sense, because investor’s expectation about low future interest rates will cause the yield curve to exhibit an upward trend and high expectations about future interest rates will cause the yield curve to descend.

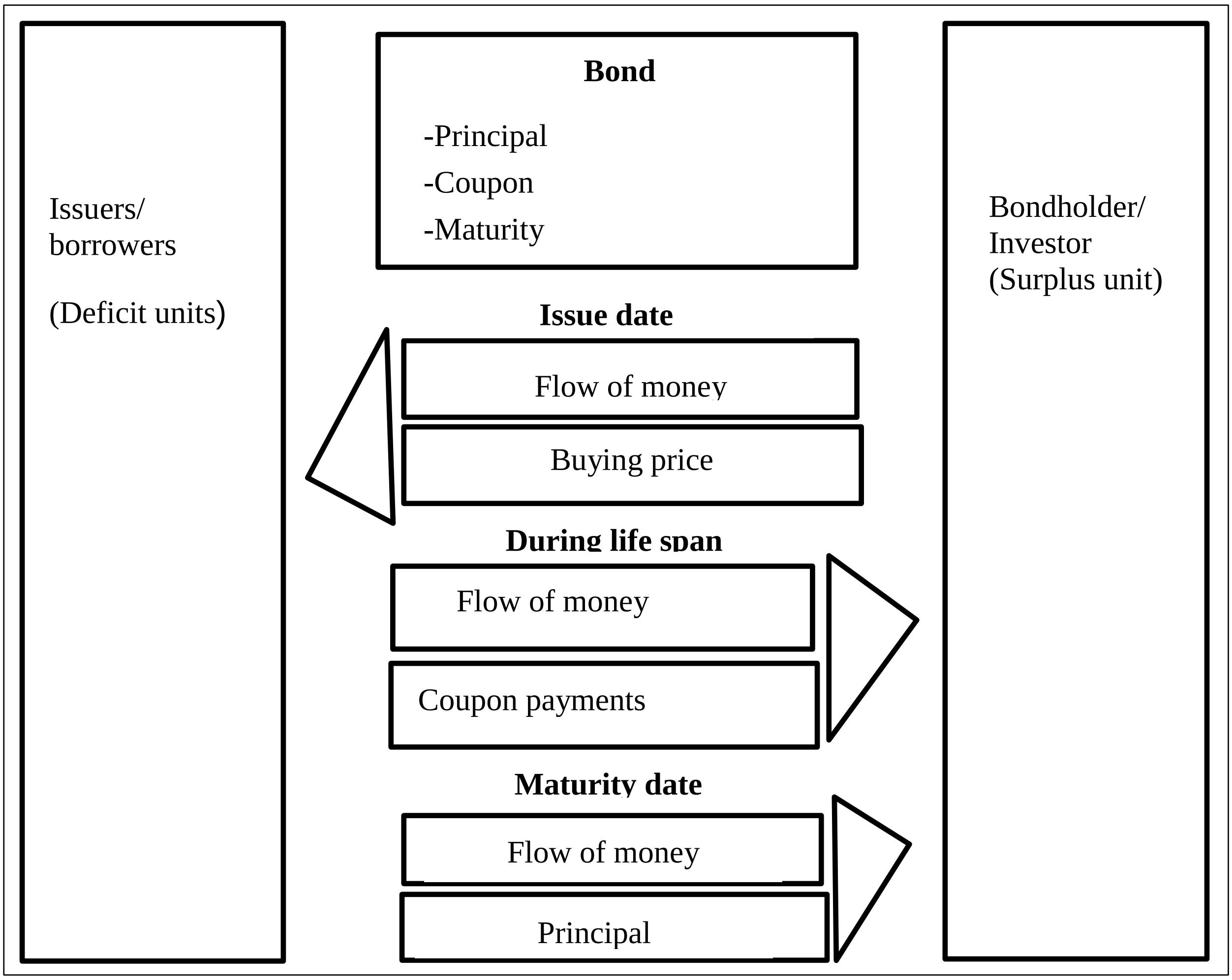

Figure 2-3: Flow of funds concerning bonds over its life span

Source: Adapted from Van Wyk *et al*. (2015)

Figure 2.3 depicts the flow of funds throughout the lifespan of a bond. Money flows from surplus units to the borrower, when a bond is issued. The borrower receives the buying price of the bond, which has the possibility of being lower, higher, or equal to the principal. When the buying price of the bond is lower than the principal is, the bond is issued at a discount. When the buying price of the bond is higher than the principal, the bond is issued at a premium. When the buying price of the bond is equal to the principal, it is issued at par (Van Wyk *et al*., 2015:314). A bond's lifespan has an effect on its default risk and yield − the longer the maturity of a bond, the lower the possibility of a downgrade. Long-term bonds with higher coupons have lower yields (Gajjala, 2006:99). During the lifespan of the bond, the issuer makes coupon payments to the bondholder. These coupons are a specified percentage of the principal. The coupon payments may be a variable- or fixed percentage (Hyman *et al*., 2015:19).

2.3.3 Government bonds

Government bonds are fixed income instruments that are issued by the government at a rate of return measured by the government bond yield (Dube & Zhou, 2013:189). Government bonds are classified as safe financial investments and fall into the following four main categories: (1) treasury inflation protected securities, (2) treasury notes, (3) T-bills and (4) treasury bonds. An example of a government bond in South Africa is the R157 bond, which matured in 2014 (Mpofu *et al*., 2013:206).

The differences between these categories are the maturities of the instruments and the payment structure (Oji, 2015:7). In terms of the maturity structure, government bonds cover a varied range of maturities, from short- to long-term bonds, which provide a more reliable bond yield curve for pricing deriving forward rates and corporate bonds (Liu, 2013).

The South African bond market is sophisticated and has shown a remarkable growth since 1980 when the government issued government bonds on demand. At that time government benchmark bonds did not exist nor any refined yield curve (Guma, 2007:1). While the significance of government bond markets has resulted in numerous studies of markets in the United States (US) and other developed countries (Fleming, 2000; Campbell & Taksler, 2004 and Andritzky, 2012) there are few studies of emerging government bond markets (Bai *et al*., 2013:1). However, Mpofu *et al*. (2013:205) state that the South African bond market is perceived as the largest bond market compared to other African countries such as Zimbabwe and Mozambique.

The SARB is an agent of the government in terms of introducing new bonds on auction through a primary dealer’s system. The JSE (2017) confirms that government bonds are issued over a longer term; thus, the yields on these bonds represent a measure of long-term interest rates. In 1996, government bonds in South Africa accounted for over 80 percent of the total bonds listed on BESA. That ratio substantially declined to 66 percent by mid-2006 (Mboweni, 2006:8). Before the 2007-2009 financial crisis, the supply of government bonds was reduced and bought by non-resident investors. This weakened the bids from domestic accounts. However, since the start of the 2007-2009 financial crisis, supply of government bonds was increased by domestic residents (Andritzky, 2012:3).

After the 2007-2009 financial crisis, central banks, for example SARB, Bank of England, Bank of Amsterdam and the Reserve Bank of Australia, became key role players in the government bond market as a result of quantitative easing. Private banks started to acquire government bonds, specifically for collateral, notwithstanding a decline in the bank's balance sheet. Considering the decline in financial globalisation as well as global imbalances, the fundamental drivers of enlarged non-resident holdings have weakened (Andritzky, 2012:3).

Since 2009, factors such as the introduction and inclusion of the South African government bonds in sovereign credit rating and the Citi World Government Bond Index have increased the attractiveness of the South African bond market to foreign investors. In consequence, the foreign investment activities in the bond market have increased significantly over the years. The non-resident holdings of South African government bonds accounted for more than 30 percent of bond issuance in 2012 (Hassan, 2013:6). Colombo (2014:2) claims that the yields on the three-month T-bills and the 10-year government bonds are a scale to measure the short-term and long-term interest rates. Short-term interest rates are lower than long-term interest rates and short-term rates are more volatile than long-term rates.

During the 2007-2009 financial crisis, the long-term interest rates were lower than short-term interest rates, which is an indication of a recession where the yield curve is inverted. The inverted yield curve supports the expectation hypothesis theory (Dube & Zhou, 2013:192). Post the financial crisis period (2007-2009), long-term interest rates were higher than short-term interest rates, which indicate a normal yield curve. Van Wyk *et al*. (2015) state that the yield curve may have a downward slope if the SARB increases the short-term interest rate and there is a perception that this will bring down the inflation rate over the long-term. This implies that interest rates are expected to decrease; therefore, the long-term rates may be lower than the short-term rates.

2.4 THE RELATIONSHIP BETWEEN SHORT- AND LONG-TERM INTEREST RATES

The term structure of interest rates explains the manner in which different maturity rates are correlated. In addition, it also describes the possible shapes of the yield curve as a result of the different relations of the rates (Van der Merwe *et al*., 2015:115). The term structure of interests is key in the formation of the monetary policy short-term rates. In consequence, long-term interest rates indicate the level at which central banks can achieve price stability.

The South African short-term interest rates and bond yields move in the same direction, which indicates that an increase in short-term interest rate increases will lead to an increase the long-term bond yields. Short-term interest rates as well as the long-term bonds were high during the

2007-2009 financial crisis, which reflected a rate of 9.51 percent for government bonds, 11.42 percent for T-bills and 12 percent for repo rates. During the 2007-2009 financial crisis, the South African yield curve was inverted, as short-term interest rates were higher than bond yields. However, the South African economy recovered from the 2007-2009 financial crisis as reflected by a downward trend of the short- and long-term interest rates after 2009. In addition, the South African yield curve was normal after the 2007-2009 financial crisis as bond yields were higher than short-term interest rates.

2.4.1 Defining the term structure of interest rates

Mpofu *et al*. (2013:223) define the term structure of interest rates as a plot of bonds with different maturities but with similar characteristics. In addition, Olweny (2011:291) confirms that the term structure of interest rates explains the relation of zero coupon bond- and spot yields and diverse maturities of the bonds. Similarly, the term structure of interest rates is a tool that establishes a robust relation between interest rates and the yield curve. Medeiros and Rodriquez (2011:5) state that the term structure of interest rates should have the presence of a set of short-, medium- and long-term rates.

In addition, Olweny (2011) and Maranga *et al*. (2018) claim that the term structure of interest rates has the subsequent observations (1) bonds with different maturities move together over time, (2) in occurrences where short-term interest rates are lower than long-term interest rates, the yield curve slopes upward, (3) in occurrences where short-term interest rates are higher than long-term interest rates, the yield curve slopes downward and, (4) commonly, the yield curve shape is upward sloped.

2.4.2 Defining the yield curve

The yield curve is designed by plotting the interest rates of bonds against their different terms to maturities, which gives the entire view of the current market yields (Van Wyk *et al*., 2015:115). The shape of the yield curve demonstrates the expectations of the future movements and uncertainties of short-term rates (Andersen, 2018:11). Therefore, in an anticipated recession, expected future short- and long-term interest rates may be low as a result of expectations of market participants (Motloung, 2013:10).

2.4.2.1 Different shapes of the yield curve

Andersen (2018:10) states that the yield curve shape frequently changes in response to economic conditions. These are to include interest rates changes and liquidity fluctuations in the economy. Various shapes of the yield curve have been observed as upward, downward, flat or humped in the subsequent section (Reilly & Brown, 2012:622).

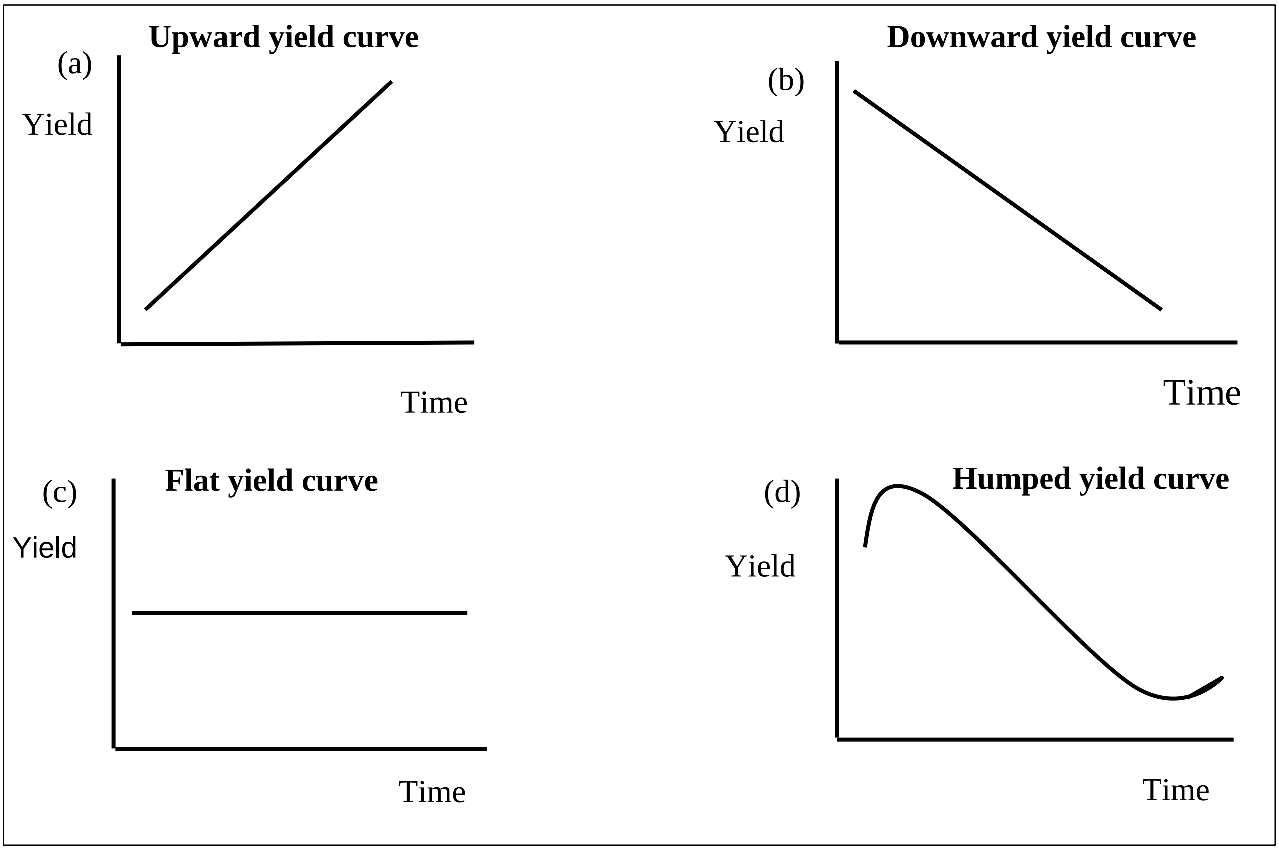

Figure 2-4: Different shapes of the yield curve

Source Compiled by author (2018)

Figure 2.4 depicts the various shapes of the yield curve. Figure 2.4 (a) indicates the upward slope of the yield curve, which implies that short-term interest rates are lower than long-term interest rates (Mpofu *et al*., 2013:224). Specifically, long-term interest rates are frequently higher than short-term interest rates; hence, the normal yield curve. The upward yield curve demonstrates the premium demanded by investors for long-term bonds due to higher inflation risks (De Rezende, 2017:108). Crouhy *et al*. (2013:206) corroborated by Mishkin (2001) argue that yield curves are predominantly upward sloped. Moreover, Crouhy *et al*. (2013:206) state that short-term rates are lower than long-term rates.

Figure 2.4 (b) exhibits the downward yield curve. The shape of the yield curve is sloped downwards when short-term interest rates are higher than long-term interests are. This occurs when the investors expect interest rates to deteriorate (Motloung, 2013). As such, long-term investors will invest in lower yields if the expectations are that the economic conditions will decrease in future (Mpofu *et al*., 2012:224).

An inverted curve is a sign of a weakening economic state. Dube and Zhou (2013:192) state that when the SARB increases the repo rate, it causes slow inflationary pressure (Mpofu *et al*., 2012:224). An inverted yield curve is caused by higher short-term interest rates compared to long-term interest rates (Itrurk, 2006).

Figure 2.4 (c) depicts the flat yield curve. Rose and Hudgins (2013:224) explain that the flat shape of the yield curve is the result of equilibrium between the short- and long-term interest rates. This results in an insignificant distinction between short- and long-term interest rates that transpires when interest rates increase as a result of tighter monetary policy and higher inflation expectations (Motloung, 2013). A flat yield curve is undesirable, as investors do not have the incentive to hold long-term bonds over short-term bonds especially in the absence of a yield premium (Choudhry, 2004:99).

Figure 2.4 (d) describes the humped yield curve. A humped yield curve occurs when the intermediate interest rates are higher than the short- and long-term interest rates (Mpofu *et al*., 2013:224). Van der Merwe *et al*. (2015:116) further explain that the humped yield curve rarely occurs and is a sign of a slow economic growth.

2.4.2.2 Trends in the South African yield curves

Although the South African yield curve flattened steadily since 2014, it steepened throughout 2017. This behaviour is a result of a decrease in the interest rate cycle, as yields in the short-run decline in anticipation of lower future short-term rates. Yield curve steeping has continued, even as markets have priced out further rate cuts. This is a result of rising risk premiums, which reflect concerns about the creditworthiness of the government. These include higher inflation risks and an increased supply of longer-term treasury bonds. In these circumstances, the steepness of the yield curve should not be taken as evidence that markets believe monetary policy is too loose.

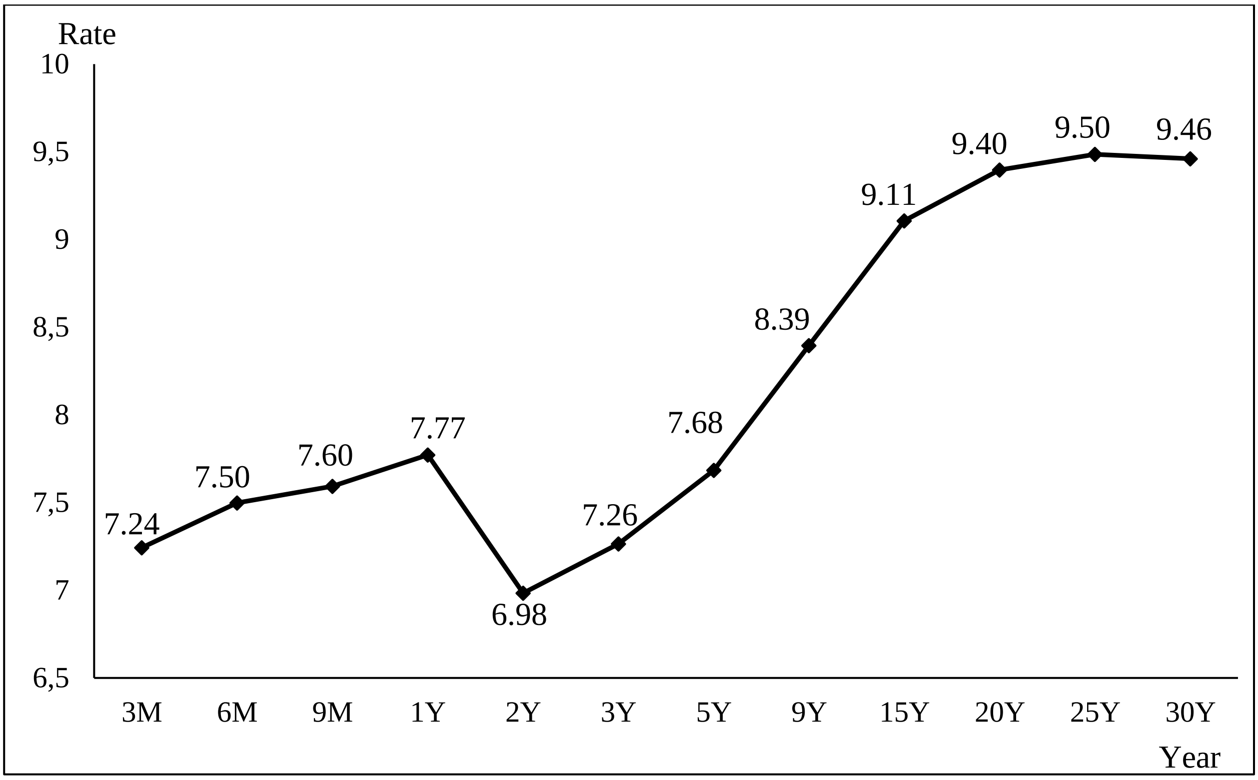

Figure 2-5: South African yield curve

Source: Compiled by author from Bloomberg (2018)

Figure 2.5 illustrates the South African normal yield curve. This means that long-term interest rates rate higher than short-term rates (Bonga-Bonga, 2009:4). Long-term bond yields respond to a range of factors such as market expectations of future short-term interest rates and market price for variables such as inflation risk and sovereign risk. The current South African yield curve indicates that the economy is not in a recession and that investors would benefit from long-term bonds investments.

SARB (2017:3) used the 10-year rand-denominated bond yield on a range of traditional determinants of yields, comprising the repo rate, credit risk, 10-year bond yields and economic growth. The results from the study suggest that during the period from 2016 to 2017 the domestic factors have become more prominent in shaping the long run of the yield curve.

Around two thirds of the variation in the South African 10-year bond yields has been due to South African-specific factors. Credit risk has become one of the most important drivers of local yields, which reflect sovereign rating downgrades, therefore, higher default risk on local bonds (e.g. R186 and R157 bonds). Yield curves incorporate market decisions on the future direction and suitability of the monetary policy. For example, if markets interpret the stance as too loose, the yield curve will be extremely steep; similarly, if policy is perceived to be too tight, the curve will be flat or even inverted (Andersen, 2018).

There are many ways in which the yield curve can be modelled; however, the Bank for International Settlements (1999) states that the Nelson and Siegel (1987) model is predominantly used, essentially by central banks across the globe. Campbell and Shiller (1991) reveal that yield curve between three- and six-month T-bill rates assist in forecasting changes in the three-month T-bill rate. The question that follows is why are there different shapes of the yield curve? However, this concept can currently be explained by the use of the models of the term structure of interest rates.

Mishikin (2007:135) states that the term structure theories should explain the three features of the yield curve: (1) the manner in which interest rates with different maturities move together over time, (2) the manner in which different slopes of the yield curves form and (3) factors that influences the different shapes of the yield curves. There are four theories of the yield curve that attempt to explain this last feature (Olweny, 2011:291).

2.5 TERM STRUCTURE THEORIES

The term structure theories describe the reasons behind the different shapes of the yield (Van Wyk *et al.*, 2015:115). The conceptualisation of the term structure of interest rates will be explored by the four different theories of the term structure of interest rates.

2.5.1 Expectation hypothesis theory

The expectation hypothesis theory is a widely accepted theory that relates to the manner in which short-term interest rates relate to long-term rates (Mpofu *et al.*, 2012:224). The expectation hypothesis theory indicates that long-term interest rates will be equivalent to the average of short-term interest rates that will transpire during the life span of the long-term bond (Mishkin & Eakins, 2012:138). This suggests that changes in the long-term rate are the result of the expected changes in future short-term rates. As such, there is a positive correlation between long-term interest rates and expected future short-term interest rates (Andersen, 2018:10).

Reilly and Brown (2012:628) state that the yield curve under the expectation hypothesis theory is the result of investors' expectation about future interest rates. This states that investors are well informed about the market conditions and ready to make predictions about future short-term rates. Thus, when short-term interest rates are expected to raise the yield curve will have positive slope and when short-term interest rates are expected to decrease, the yield curve will have a negative slope (Mpofu *et al*., 2013:224).

A number of studies (Campa & Chang, 1995; Bekaert *et al*. 2001; Beechey *et al*., 2009; Ghazali & Low, 2010; Guney, 2013) have been conducted over the past years to test whether the assumption of the expectation hypothesis exits in different economies. Though the studies use different state variables for both short- and long-term rates, the results indicate that the expectation hypothesis theory is legitimate and can be proven in different countries.

Gerlach and Smets (1998:215) argue that in economies with a fixed exchange rate, the expectation hypothesis cannot be rejected, because spikes in short-term rates are influenced by regular episodes of exchange rate market pressures. In addition, the increase in short-term rates was temporary; participants in the market expected them to be incomplete, which led to predictability of interest rates that dominated any variability of the term premium.

2.5.2 Liquidity premium theory

The liquidity premium theory expands the expectations hypothesis theory by assuming that investors are risk-averse and, consequently, prone to demand a premium for long-term bonds as a result of interest rate risk (Loukakis, 2012:12). The theory states that risk-aversion can cause forward rates to be higher than expected spot rates. This amount is expressed as an amount that increases with maturity (Omondi, 2016:8).

Hicks (1946) developed the liquidity premium theory, which allows long-term interest rates to deviate from future expected short-term rates. This indicates that additional assumptions are made on the preferences of investors. This theory is based on the assumption that bonds with different maturities are substitutes, however, not perfect substitutes (Mollenze *et al*., 2014:117). This theory is of the view that long-term securities should yield higher returns as compared to short-term instruments, as more aggressive investors are willing to buy securities with longer maturities as a result of higher yields (Reilly & Brown, 2012:630).

Moreover, bonds can be risky assets since it is difficult to predict the expected inflation rates in the market. Bondholders are more focused on the purchasing power of the real return received from bonds, not only on the amount they will receive from the coupon payments (Mishkin & Eakins, 2012:143). Likewise, investors need liquidity premiums to persuade them to inject funds for securities with long-term maturities (Maranga *et al*., 2018:46). Specifically, in order to attract investors, extra returns should be rewarded in the form of interest premium (Van Zyl *et al*., 2006:43).

2.5.3 Market segmentation theory

The market segmentation theory suggests that markets for bonds with different maturities are segmented. The interest rate for every bond with a different maturity is determined by the supply of and demands for that particular bond, since they are not substitutes (Bonga-Bonga, 2009). This implies that the yield curve shape is determined by supply and demand (Mpofu *et al*., 2013:225).

The market segmentation theory explains the manner in which the markets for short- and long-term bonds differ. Short-term bonds' market demand and supply are determined by short-term yields; the same is true for markets for long-term bonds market. This implies that the demand and supply is determined by long-term yields (Olweny, 2011:291). In the theory's strongest form, the theory assumes that investors and borrowers have strong preferences for maturity range and would not take advantage of yield differentials (Reilly & Brown, 2012:631).

Market participants have particular investment preferences. This implies that they have certain periods in which they prefer to borrow or to lend financial instruments. The fundamental assumption of this theory is that bonds that differ in maturity terms cannot be perfect substitutes and thus investors are more interested in bonds with higher returns (Andersen, 2018:11).

The market segmentation theory confirms that although investors are risk-averse they prefer short-term bonds over long-term bonds. However, long-term bonds are subject to higher returns due to less demand since short-term bonds are preferred over long-term bonds (Omondi, 2016:9).

2.5.4 Preferred habitat theory

The preferred habitat theory is an extension of the market segmentation theory. However, the preferred habitat theory focuses on different investor behaviour, with an interest in specific yield curve maturities and not on a fixed maturity range (Motloung, 2013:13). In addition, investors would prefer short-term bonds over long-term bonds (Kariuki & Kitati, 2016:1). The preferred habitat theory is based on the assumption that the yield curve is influenced by a risk premium and interest rate expectations to persuade participants in the market. Thus, the theory accepts that the shape of the yield curve should be up, down, or flat (Bonga-Bonga, 2009).

The expectation hypothesis theory is the most preferred theory amongst the other theories. In South Africa, several studies (Arize *et al*., 2002; Mposelwa, 2016) tested whether the expectation hypothesis theory holds in South Africa and the results of the studies revealed that there is support of the expectation hypothesis theory in South Africa. In consequence, Ghazali and Low (2010) opine that when there is no support of the expectation hypothesis in a country that would imply that there is difficulty in the evaluation of the county's interest rate dynamics.

2.6 TERM STRUCTURE MODELS

The success of term structure models relies on the model's capability to estimate future volatility accurately. The traditional term structure models can be classified into two categories, namely the implied volatility category and the historical volatility category (Mohamed & Samsudin, 2016:9). Ahlgrim *et al*. (2004) state that the differences among the term structure models are the free-arbitrage models developed by Ho and Lee (1986) and Hull and White (1990), the equilibrium model implemented by Vasciek (1977) and the Cox-Ingersoll-Ross (CIR) model (1985) among others and the ability of the models to depict real or nominal interest rates as well as other factors the model has. As an extension of the economic framework, equilibrium models are able to develop a mean reversion process for short-term interest rates. The term structure models do not necessarily exhibit current interest rates, however, based on one or two factors (short-term interest rates and the volatility of interest rates) the models have a more comprehensive way to estimate the term structure of interest rate.

2.6.1 General framework of Cox, Ingersoll and Ross model

Based on the term structure theories, economists and researchers (Cox *et al*. 1985; Brown & Schaefer, 1994; Maranga *et al*. 2018) view the term structure shortfall as a fundamental equilibrium theory problem (Irturk, 2006:28). The rational expectations equilibrium model developed by Lucas (1978) and the intertemporal capital asset pricing model established by Merton (1973) have influenced researchers to consider the equilibrium models of the term structure (Gibsson & Ramaswamy, 1993:620). The term structure of interest rates is modelled by different methods, however, in this study, two models are used, namely the CIR (1985) one-factor model and Longstaff and Schwartz (1992) two-factor model.

The CIR (1985) model is among the most popular models used in the financial markets (Donald *et al*., 2013). This model assumes that short-term rates are the single stochastic factors that influence interest rate direction and that the variance of the short rate of interest is proportional to the level of interest rates. The CIR (1985) model expresses that a bond yield increases in relation to interest rate level and is indeterminate with regards to interest rate volatility. Moreover, the model is explained as a diffusion process suitable for modelling the term structure of interest (Overbeck & Ryden, 1997:430).

The CIR (1985) model was developed to correct the shortfall of the Vasicek (1977) model. The Vasicek model assumes that short rates will be adverse. To correct for this problem, Cox *et al.* (1985) developed a model, which is based on the notion that real interest rates cannot be negative. The difference between the two models is based on the behaviour of the state variable (Omondi, 2016:3). Benninga and Wiener (1998:6) argue that the term structure of the CIR (1985) model is considered to be upward- or downward sloping and can produce humped term structures. The model is in line with no arbitrage opportunities, positive interest rates and the general equilibrium for asset pricing (Chen & Scott, 1992:614).

Equilibrium models are known to be extremely complicated, which takes into account the assumption of the market structure and preferences of investors in relation to future expectations. The term structure has an advantage on these models as it enables the characterisation of returns on zero coupon bonds, which include all maturities. In order to make it possible to estimate the equilibrium models, the interest rate risk premium must first be estimated, as the estimation will make it possible to distinguish among distinctive models (Jarrow, 2009:70).

The CIR (1985) model has a number of characteristics. First, the model is parsimonious in a number of parameters and state variables. Secondly, short-term rates result to non-negative rates and a stationary process, which indicate that the model uses a closed-form approach in obtaining prices for European options and discounted bonds. Lastly, the closed-form approach is also used to obtain the conditional distribution and the movements of the yield to maturity, bond returns and forward rates. This results in straightforward modelling and testing of the model (Constantinides, 1992:531).

The CIR (1985) model has a number of assumptions, which prompted the theoretical framework of the model. The model explains that in an economy there are endlessly lived and indistinguishable people who fully optimise the reduced predictable utility of the use of a single good. This single good is stochastically produced by several technologies, which demonstrate continuous stochastic return to scale. The firms invest consumer’s wealth and consumers have a choice between the investment allocation rule and the consumption rule in the maximisation of expected returns. The firms’ estimation in the economy develop gradually as a vector process, whose covariance matrix and drift rate is determined by the evolution of a vector of the state variable (Gibbsons & Ramaswamy, 1993:622).

A number of academic studies tested the effectiveness of the CIR (1985) model (Pearson & Sun, 1994; Brown & Dybvig, 1986 and Gibson & Ramaswamy, 1993). Pearson and Sun (1994) corroborated with Brown and Dybvig (1986) found that the CIR (1985) model fits the estimation of T-bills as compared to other treasury issues, even though it overprices short-term interest rates. In contrast, the Pearson and Sun (1994) study found that the CIR (1985) model provides an unsuitable description of the treasury market.

2.6.1.1 Properties of the CIR model:

The CIR model has the following properties:

- Interest rates are always positive;
- Mean reverting nature;
- Rates in the CIR (1985) model converge at some point; and
- The incremental variance is relative to the present value of X.

Moreover, models developed by Longstaff and Schwartz (1992), Chen and Scott (1992) and Bibby and Sorensen (1995) have other approaches in estimating the CIR (1985) model. The extensions of the CIR (1985) model explain the term structure more proficiently.

2.6.1.2 Advantages and disadvantages of the CIR (1985) model

As indicated by Nazir (2009:13) the CIR (1985) model has the following advantages:

- The model objects to negative interest rates;
- The restricted volatility is determined by short rate level;
- Explicit formulae for the term structure; and
- The model is based on a rational statistical model for interest rate behaviour.

According to Nazir (2009:13), the CIR (1985) model has the following disadvantages:

- The model is too simple;

- The model is not flexible enough to fit the observed term structure;
- Bond prices in this model are correlated, which is empirically incorrect; and
- This model is not analytically traceable.

Moreover, the main disadvantage of one-factor models is the fact that only short-term rate (one state variable) is considered, however, two-factor models considers two state variables such as the Brennan and Schwartz (1979) and Longstaff and Schwartz (1992) two-factor models.

2.6.2 General framework of the Longstaff-Schwartz two-factor model

The Longstaff and Schwartz (1992) model was developed by means of the CIR (1985) framework, which uses only one state variable. In the CIR (1985) model, bond prices are 100 percent correlated, which is empirically untrue, consequently, the model fails to explain the entire term structure. To solve this problem Longstaff and Schwartz (1992) introduced another state variable (volatility of interest rate) to explain the term structure more effectively.

The theoretical framework from the Longstaff and Schwartz (1992) model is similar to the one used by the CIR (1985) model. This framework considers fundamental economic issues, which relate to investment opportunities, financial market nature, changes in technology, uncertainties in the investment world and preferences of investment. Since the required expected return on interest sensitive bonds is determined by the market price of interest rates, this model is designed to solve this issue. As a result, the equilibrium price for bonds is determined once the expected rate of return is known. This is done by discounted cash flows of bonds at a suitable discount rate.

Since Longstaff and Schwartz (1992) is a two-factor model, the model gives an explanation on the manner in which bond prices depend on two state variables in addition to the maturity of the bonds. The two variables included are short-term interest rates and the volatility of the short-term interest rates (Longstaff & Schwartz, 1993:72). The two-factor model provides additional information of the term structure that leads to improved hedging and pricing as compared to one-factor models.

In contrast to one-factor models, the Longstaff-Schwartz (1992) model is able to describe more complex movements of the yield curve as well as the shifts of the yield curve. One-factor models are known for their incapability of pricing consistency that depends on unrealistic correlation between rates (Rebonato & Cooper, 1995). Chan *et al*. (1992) argue that models that allow for a level effect relationship between interest rates and the volatility of interest rates perform better than the models that do not allow for this relation.

2.6.2.1 Advantages of the Longstaff and Schwartz model

The following are the advantages for Longstaff and Schwartz model (1992:260):

- The two-factor model suggests that bond returns are correlated across all maturities of the term structure;
- Interest rate volatility is explicitly endogenously determined;
- Since the model has a nature of capturing observed properties of the term structure it allows yields to depend on interest rate volatility; and
- The model has a dynamic feature that allows flexibility in fitting observed term structure and the parameters of the model can capture the unpredictability of the term structure (Chen & Scott, 1992:613).

2.7 FACTORS THAT INFLUENCE THE RELATIONSHIP BETWEEN INTEREST RATES AND BONDS

Bonds are traded on an interest rate or yield basis in South Africa. Therefore, the yield of bonds determines the value at which the bonds are priced (Van Wyk *et al*., 2015:327). In addition, bond investors are attracted to higher bond returns. Thus, factors that influence the relationship between interest rates and bonds are important since they affect bond returns.

2.7.1 Interest rate levels

The interest rate of a bond should be in line with the interest rates on other financial market assets. Interest rates in the financial markets are determined by economic variables to include exchange rates, inflation rates and tax policies (Van Wyk *et al*., 2015:327).

2.7.2 Expectations

Although, monetary expansions appear to reduce uncertainty and risk perceptions (Bekaert *et al*., 2013), continuous low interest rates could result in negative effects on the expectations and confidence of investors. These expectations do not only provide forecasts of inflation rates, but they provide a measure of financial markets' perceptions of the current monetary stance

(Breedon, 1995:160). Zekaite and Kontonikas (2016:2) opine that there are relatively small effects of the monetary policy shocks on expected future inflation rates, essentially when explaining the negative effect of rising interest rate on current bond returns.

Subsequently, Loukakis (2012:13) expresses that the expectations theory of the yield curve suggests that the yield on a long-term bond has expectations of future short-term interest rates. Thus, this theory is based on an investor's decision-making process based on future expectations on whether to invest in short- or long-term bonds (Andersen, 2018:11). In addition, the SARB (2007:2) explains that when inflation rates are expected to be high, the buying power of market participants decreases. The lender or suppliers will pursue protection against the erosive power of inflation by demanding a higher interest rate. Thus, high-expected inflation rates result in high interest rates.

2.7.3 Maturity of the bond

The yield of a bond is predominately affected by the bond's maturity (Van Wyk *et al*., 2015:327). Based on the segmented markets theory, there is a separate market for specific maturities along the term structure and the interest rates for these maturities are set by market forces of demand and supply (Choudhry, 2004:125).

2.7.4 Market forces

The business cycle provides a different phase that offers different pillars of supply and demand of funds in the market. During economic peak periods of the business cycle, the demand for funds becomes greater than the supply of funds and this increases interest rates. On the contrary, when there is a construction on the business cycle, the level of interest rates decline because of a slump in business activities (Uche, 2012:16).

2.7.5 Credit risk

Credit risk is determined by interest rates because it influences the debt burden of borrowers (Ajibola *et al*., 2013:27). Louzis and Metaxas (2012) explain that this implies a positive trade-off between credit risk and interest rates. In fact, an increase in debt influenced by increased interest rates can lead to a higher loan rate. Above all, this implies that an increase in interest rates can trigger the investment cost and, consequently, lead to a higher possibility of default, which may result in non-performance.

2.8 RISKS ASSOCIATED WITH A BOND PORTFOLIO

To measure risk, it is essential to identify key factors that might cause volatile returns from the bond portfolio. When an equity portfolio is evaluated, the risk factor will be the volatility of the stock price and may be evaluated in various ways (Crouhy, 2014:6).

2.8.1 Interest rate risk

Interest rate risk may be defined as the risk that stems from changes in interest rates. This risk has the potential of reduce the market value of a bond (Roberts, 2017:8).

2.8.2 Credit risk

Credit risk analyses the credit worthiness and capability of the borrower should they default on an agreement and, as such, measures the borrower's solvency (Liechtenstein Bankers Association, 2008:5). Brown and Moles (2016:16) argue that three characteristics define credit risk, (1) the possible exposure to the borrower that may default on the agreement, (2) the probability of default that the borrower will default on its obligations and (3) the recovery rate, which assess the amount that can be retrieved if the borrower defaults.

2.8.3 Downgrade risk

Downgrade risk, like credit risk assesses the borrower's creditworthiness. In general, poor creditworthiness interprets into a downgrade by the rating agencies, such as Standard and Poor's (S&P), Moody's, or Fitch in South Africa and an increase in the risk premium (Crouhy *et al*., 2014:30).

2.8.4 Tax risk

Before making investment decisions, investors must consider tax deductions on bonds. Profits on government bond sales are subject to tax deductions. Interest rates charged on corporate bonds are vulnerable to government tax (Roberts, 2017:10).

2.8.5 Reinvestment risk

Reinvestment risk explains the process where market rates differ from the yield to maturity (Dahl, 2007). However, reinvestment risk is contradictory to price risk and applies regardless of the holding term (Mpofu *et al*., 2013:209).

2.8.6 Default risk

Default risk is related to the borrower's inability or unwillingness to meet their debt obligation (Feldhutter & Nielsen, 2012). This is when the predetermined period of the loan has lapsed, which is commonly 60 days in the banking industry (Crouhy *et al.*, 2014:30).

2.8.7 Trade-off between risk and return

Practitioners agree that the most important thought in investment planning is the relationship between expected risk and return. A large number of investment alternatives are available to the investor, which start from nearly riskless government instruments like T-bills to extremely risky derivative instruments like credit default swaps, options and future contracts (Mpofu *et al.*, 2013:15).

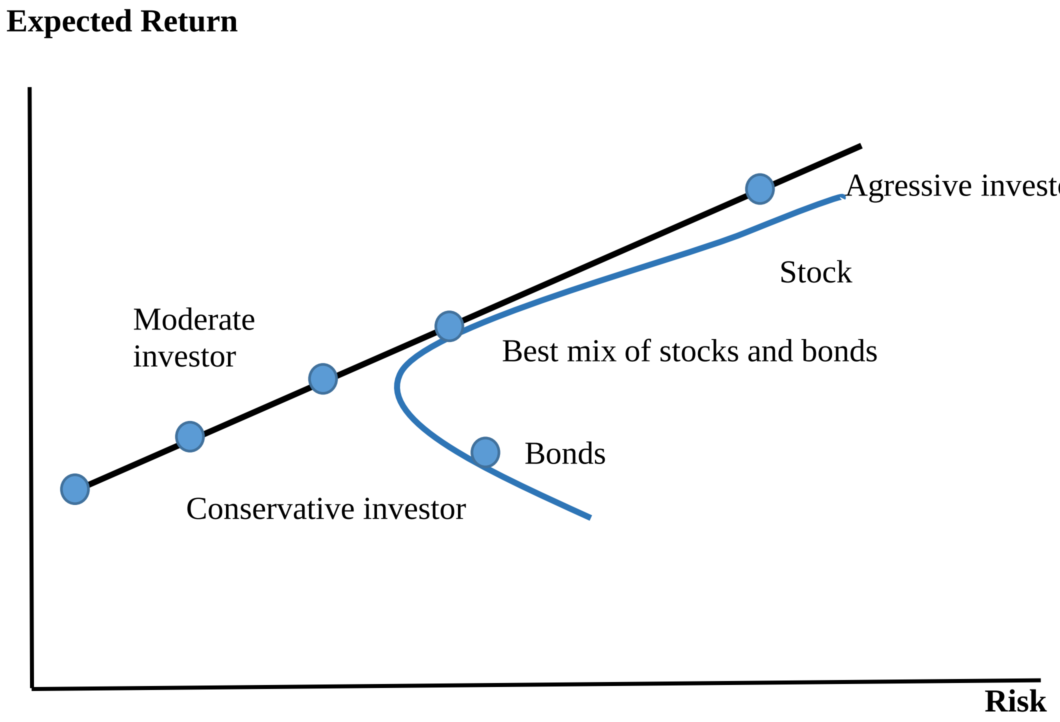

Figure 2-6 : Trade-off between risk and return

Source: Adapted from Mpofu *et al.* (2013)

Figure 2.6 illustrates the relationship between the risk and return of investments. The vertical axis describes the expected return and the horizontal axis describes the risk. The relationship between risk and return of assets has an important role in investment management (Nwude, 2012:138). Investors in the capital market prefer a high return for a high-risk investment, as a result, risk and return are strongly correlated (Lenz, 2010:4).

Asset pricing theory (i.e. the capital asset pricing model (CAPM) of Sharpe 1964 and Lintner, 1965) suggests a positive correlation between risk and expected returns. The CAPM model is used to describe the correlation between risk and return in an efficient market (Nhleko, 2015:6). Moreover, Wang *et al.* (2013:1) discovered that low-risk entities have a tendency to earn higher average returns when risk is measured by stock return volatility or CAPM beta.

2.9 EVIDENCE FROM PREVIOUS STUDIES

Several studies confirm the link of short-term interest rate volatility and the term structure of interest rates. The theories of the term structure have often interested both investment analysts and theorists; however, the results from previous studies are inconclusive, creating an opportunity for more research.

The structural model by Roley (1981) assumes that the relationship between short- and long-term rates, as well as treasury yields, is positive. In contrast to Roley (1981), Cox *et al*. (1985) modelled short-term interest rates and the interest rate effect on the term structure. The results reflect a negative relationship between short-term interest rates and the term structure interest rates. Moreover, the volatility rate is lower when interest rates are low and when the yield curve reflects lower curvature.

Longstaff and Schwartz (1992) developed a two-factor model to test the relationship between short-term interest rate volatility and the term structure. The results were tested using the USA one-month T-bill yields and the treasury notes spanning from six months to 30 year maturities. They found that short-term interest rate volatility has a negative-significant effect on longer-dated bonds, as compared to shorter-dated bonds. They also show that short-term interest rates are positive across all maturities.

Longstaff and Schwartz (1993) found that the yield curve shifts are significantly different from parallel shifts, convexity analyses and the fundamental assumptions of the standard duration. Moreover, the results exhibit that there is a strong negative relationship between interest rate volatility and the yield curve for bonds with longer maturities. This confirms that interest rate volatility, as well as the level of interest rate, influence bond prices. This means that an increase in uncertainty about interest rate changes results in a decrease in bond prices.

Gibbsons and Ramaswamy (1993) used the CIR (1985) model to test the term structure theory with the use of the index bond prices. Hansen’s generalised method of movement (GMM) method was used in this process. This method makes it possible to estimate a continuous-time model based on discretely sampled data. The study concurs with other studies (Brown and Dybvig, 1986; Brown & Schaefer, 1994) that the T-bill returns perform well when using the CIR (1985) model. Furthermore, the results indicate that term structure premiums are positive and that yield curves can take several shapes. In contrast, there is a serial correlation issue when estimating the T-bill return, which indicates that the model is not a good fit.

Nowma's (1997) study is consistent with other studies in explaining that the short-term interest rate volatility is sensitive to the level of rates in the US. However, short-term interest rate volatility is not sensitive to the level of interest rates in UK. The two countries provide opposite results. The results were obtained using the Gaussian estimation of continuous time dynamic models.

Apte (2001) tested the one-factor models using data from India. The study concluded that the different one-factor models give different results, these models also give rise to yield curves, which depart substantially from those extracted from market data using standard curve fitting procedures. In contrast to other studies, the results also show that T-bill yield does not perform well under the given models, namely Vasicek (1977) and CIR (1985). The study further suggests that more work must be done in order to find a model that better fits the Indian data, as the used term structure models provide undesirable results.

Ariff and Sarkar (2002) tested the effect of interest rate volatility on treasury yields. The study concurs with Longstaff and Schwartz (1992 and 1993) that there is a negative relationship between interest rate volatility and bond yields. Moreover, the uncertainty in interest rate volatility is stronger for longer-maturity bonds as compared to shorter-maturity bonds. To analyse the results Ariff and Sarkar (2002) used the two-factor model developed by Longstaff and Schwartz (1992).

Bonga-Bonga (2009) focused on responses of short- and long-term rates on demand and supply as well as the monetary shocks for South Africa. The study found that the monetary shocks and demand result in simultaneous movements of long- and short-term rates. However, when considering the supply shocks for both short- and long-term rates, they move in the opposite direction.

Collin-Duferense *et al*. (2009) found that there is a negative relationship between the volatility from yields with different maturities using the free-arbitrage model. In contrast, Jacobs and Karoui (2009) and Almeida *et al*. (2011) found that volatility from affine models (CIR, 1985;

Longstaff & Schwartz, 1992) have a positive correlation. Espada and Francia (2008) tested the manner in which various macroeconomic shocks affect the term structure of interest rates. With the use of the no-arbitrage model, they found that shocks perceived to have a persistent effect on inflation might affect the level of the yield curve. Thus, an increase in short-term rates affects the medium- and long-term yields. Furthermore, the demand shocks in the economy reveal that positive shocks result in upward shifts in the yield curve. These shifts in the yield curve are explained better by the responses of the monetary policy as well as the time-varying term premium.

Olweny (2011), on modelling interest rate volatility in Kenya, established that there exists a link between the level of short-term interest rates and their volatility in Kenya. With the use of South African data, Aling and Hassan (2012) conclude that short-term interest rates direct the movement of the term structure. This behaviour is explained in a free-arbitrage market. However, the study finds support only for diffusions where the interest rate volatility is abstemiously sensitive to interest rate levels. Other well-known models with limitations that can either restrict or prevent the effect to be extremely high, are not a suitable fit for the data. Furthermore, diffusion models that allow the conditional interest rate volatility to be abstemiously determined by the level of interest rates provide the best empirical results for South African data.

Joslin and Le (2012) investigated the relationship between interest rate volatility and no-arbitrage term structure models. The study used US T-bill data spanning from March 1984 to December 2006. The study shows that under the arbitrage affine term structure models, there is a strong correlation between yields with different maturities and the set of valid volatility instruments.

Khramov (2013) estimated the parameters of real short-term interest rate models. The three-month T-bill spanning from January 1934 to December 2012 was used. The results show that the relationship between changes in interest rate volatility and the level of interest rate plays a significant role in explaining the dynamics of interest rates. The results agree with Gibbsons and Ramaswamy (1993) and other authors (Brown & Dybvig, 1986; Brown & Schaefer, 1994; Maranga *et al*. 2018) that the CIR (1985) model gives a good estimation for short-term interest rates and thus is enough to capture the dependence of volatility of the level of interest rates.

2.10 CHAPTER SUMMARY

This chapter introduced the interest rates and bonds market in South Africa. Interest rates remain the main driver of the monetary policy and, in consequence, influence the capital market. More so, the SARB uses the T-bill as a proxy to measure short-term interest rates in South Africa. Correspondingly, the short-term interest rate volatility was defined as a conditional variance of short-term interest rates, which effect bond yields.

The South African bond market is the largest debt market in emerging markets and accounts for more than 90 percent of JSE listed bonds. The trends of South African government bonds and short-term interest rates were examined. Further, during the 2007-2009 financial crisis, short-term interest rates were higher than long-term government bonds; this further explained that the yield curve was inverted. However, after the 2007-2009 financial crisis, bond yields recovered and were higher than short-term interest rates; this revealed that the yield curve was normal.

The term structure of interest rates was used to explain the relationship between short- and long-term interest rates. Mpofu *et al*. (2013:223) define the term structure of interest rates as a plot of bonds with different maturities, but with similar characteristics such as zero coupon bonds. Thereafter, the yield curve was described as a graphical plot of different bond yield maturities and was further described by the four different shapes.

The four shapes of the yield curve were analysed; (1) the normal yield curve where short-term interest rates and lower than long-term interest rates, (2) the inverted shape of the yield curve where short-term interest rates are higher than long-term interest rates, (3) the flat yield curve where short- and long-term interest rates are equal and (4) the humped yield curve where intermediate interest rates yield higher interest rates than short-term and long-term interest rates. In addition, there was an analysis of the South African yield curve, which reflected that during the 2007-2009 financial crisis interest rates were extremely volatile and had an impact on the bond price depending on the interest rate level.

Further to this, to explain the reasons behind the different shapes of the yield curve, the four term structure theories were investigated, (1) expectation hypothesis theory, which explains that short-term interest rates and long-term interest rates move together over time and that investors can predict the expected future interest rates, (2) the liquidity premium theory expands the expectations hypothesis by assuming that investors are risk-averse and, consequently, investors are prone to demand a premium for long-term bonds as a result of interest rate risk, (3) the market segmentation theory suggests that markets for bonds with totally different maturity are segmented and (4) the preferred habitat theory assumes individual investors have a preferred range of bond maturity lengths and will only perform beyond the normal range if there is a higher yield. In addition, this theory expresses that investors prefer short-term bonds to long-term bonds.

In addition, the chapter explained the term structure models that are used to analyse the relationship between short-term and long-term interest rates, namely the CIR model (1985) and the Longstaff and Schwartz model (1992). These two models explain the relationship between interest rate volatility and the term structure of interest rates, although one is the one-factor model and the other is the two-factor model. The two-factor model included an additional variable being the volatility of the short-term interest rate.

Further to this, factors that influence the relationship between interest rates and bonds were analysed, namely expectations, maturity of the bonds, market forces and credit risk. This was followed by the risks associated with a bond portfolio such as interest rate risk, credit risk, downgrade risk, reinvestments risk and default risk.

Finally, previous studies were reviewed and reported, which investigated the relationship between interest rate volatility and the term structure of interest rates. The theory of the term structure has often intrigued both investment analysts and theorists, however, the results from previous studies are inconclusive, creating an opportunity for more studies to be researched.

In the South African context, the relationship between short-term interest rate volatility and the term structure of interest rates has been barely analysed. After reviewing what other studies have found, there is still a gap in the South African literature to find out whether short-term interest rate volatility affects bond yields. This study aims to empirically test whether the CIR (1985) model, and Longstaff and Schwartz (1992) model will fit the South African data well.

CHAPTER 3: METHODOLOGY

3.1 INTRODUCTION

The aim of this chapter is to provide the method that will fulfil the objectives of this study. In Chapter 2, two models from the literature were reviewed, which explain the relationship between interest rate volatility and bond yields. These models, CIR (1985) one-factor model and the Longstaff and Schwartz (1992) two-factor model, are used in this study to test empirically the relationship between the observed variables. Both the one-factor and two-factor models follow an autoregressive distributed lag (ARDL) model structure and an ARDL model estimation, therefore, will be followed. ARDLs are standard least squares regressions that include lags of both the dependent and independent variables as regressors (Greene, 2008).

The approach in this chapter is outlined in four sub-sections. The first subsection gives a description of the design, sample size and the sample period observed in this study. The second subsection provides the model specification used in this study. The third subsection explains the diagnostic tests and the last subsection provides the empirical objectives set in this study as well as the methods used in order to achieve these empirical objectives.

3.2 DATA

The study employed monthly time series data for the period January 2004 to December 2017. The study period was chosen in order to observe the movement of interest rate volatility before, during and after the 2007-2009 financial crisis. The choice of the sample period will also give an indication of the extent interest rate volatility affects bond yields in South Africa during periods of shocks in the market. The data used were extracted from the International Monetary Fund and South African Reserve Bank databases.

Table 3-1: Variables description

Variables	Explanation	Source
Short-term interest rates	T-bills are used as short-term interest rates and are an independent variable	International Monetary Fund
Short-term average bond yields	The zero to three year average bond yields are short-term bond yields and used as dependent variables.	South African Reserve Bank

Variables	Explanation	Source
Medium average bond yields	Three to five and five to ten year average bond yields are used as medium-term bond yields and are used as a dependent variable.	South African Reserve Bank
Long-term average bond yields	10 year and above average bond yields are used as long-term bonds yields and are used as a dependent variable	South African Reserve Bank

Source: Compiled by author from IMF and SARB (2018)

The sample period then is split into three sub-periods: before, during and after the financial crisis. The split of the sample period assists in determining possible structural breaks in the data and to observe whether there are different effects during these sub-periods. The yield curve theory states that during periods of recession, short-term interest rates tend to be higher than long-term rates, which results in an inverted yield curve (Andersen, 2018). During non-recession periods, short-term rates are lower than long-term rates, which indicate a normal yield curve and well-performing economy (Motloung, 2010:10). Thakor (2015) states that the 2007-2009 financial crisis started in September 2007 and ended in May 2009. With this consideration, the study is split into the periods explained Table 3.2

Table 3-2: Breakdown of the sample period

Period	Start date	End date
Data observed	January 2004	December 2017
Before financial crisis	January 2004	August 2007
During financial crisis	September 2007	May 2009
After financial crisis	June 2009	December 2017

Source: Compiled by author from Thakor (2015)

3.3 MODEL AND ESTIMATION METHOD

In order to examine interest rate volatility and bond yields, the study employed two models that cater for short-term interest rates, short-term interest rate volatility and bond yields.

3.3.1 CIR one-factor model

In examining the relationship between interest rate volatility and bond yields in South Africa, the study followed both the CIR (1985) and Longstaff and Schwartz (1992) models. The CIR (1985) model considers one independent variable (short-term interest rate).

The CIR (1985) model is explained by the following model:

$$\Delta Y_t = \alpha + \beta_1 \Delta r_t \tag{3.1}$$

Where ΔY_t is the first differenced dependent variable. For this study, the short-term, medium-term and long-term yields are used as the dependent variables in four separate regressions. Δr_t is the first differenced independent variable, which for this study is the short-term interest rate.

The above equation can be written to show the ARDL model components more clearly:

$$Y_t = \alpha + \beta_1 (r_t - r_{t-1}) + \beta_2 Y_{t-1} \tag{3.2}$$

Y_{t-1} is the first order autoregressive component and r_{t-1} is the distributed lag component. These properties are similar to the Longstaff and Schwartz (1992) model properties.

Modelling the term structure of interest rates is emphasised in the CIR (1985) model as this model provides an explanation of the behaviour of the term structure as well as the ability of the term structure to accommodate the four popular theories known as expectation hypothesis, liquidity preference, and habitat and market segmentation theory. This model was later extended to a two-factor model by Longstaff and Schwartz (1992). This model does not only include one state variable but includes two state variables, which are short-term interest rates and short-term interest rate volatility.

3.3.2 Longstaff and Schwartz two-factor model

The Longstaff and Schwartz (1992) model is used to examine the interest rate volatility and bond yields in South Africa. This model uses two state variables (short-term interest rates and the volatility of short-term interest rates).

The Longstaff and Schwartz (1992) model is explained as follows:

$$\Delta Y_t = \alpha + \beta_1 \Delta r_t + \Delta V_t \tag{3.3}$$

Where ΔY_t is the first differenced dependent variable. For this study, the short-term, medium-term and long-term yields are used as the dependent variable in three separate regressions. Δr_t is the first differenced independent variable r, which for this study is the short-term interest rate and ΔV_t is the first differenced independent variable V, which for this study is the volatility of the short-term interest rate.

Interest rate volatility employed in this study is the conditional variance series obtained by estimating a GARCH (1,1) model. Bollerslev (1986) developed the GARCH model. The GARCH (1,1) variance equation permits the variance of the short-term interest rate to be conditionally dependent on its own lagged values. Consequently, volatility follows an autoregressive process since its current value depends on its lagged value (Longstaff & Schwartz, 1992:1275).

3.3.3 Stationary and unit root tests

When examining a time series data set, the first step is to conduct a unit root test. This unit root test will determine the appropriate model to use for the dataset. According to Shrestha and Bhatta (2018:71), a unit root test determines whether the variables are stationary or non-stationary; in addition to this, it also determines the order of integration of each variable.

Figure 3.1 depicts the steps to be followed when modelling time series data. The first step would be to test for stationarity using unit root tests. When the variables are all stationary at I(0), then the ordinary least squares would be appropriate to model the data. Alternatively, when the variables are all non-stationary at I(0) then the variables have to be differenced once in order to obtain stationarity and then OLS regression may be used. If OLS regression is done with non-stationary data without the presence of cointegration, then spurious results will be obtained. However, when the variables are a mixture of I(0) and I(1), then the ARDL model is the best model to use, with the consideration that none of the variables are I(2). This means that the unit root test plays a fundamental role that determines whether the stochastic process of time series can be expected to be invariant over time.

The data is stationary when the variance and mean are constant over time (Alexander, 2008:216). Unit root tests on conditional variance Y_t are measured in order to estimate the stationarity of series and are important to run the empirical tests (Rahmani, 2016:22). According to Martin *et al*. (2013:34) there are two commonly used definitions for stationarity known as weak and strong stationarity. In addition, Aas and Dimakos (2004) opine that X_t variables are stationary when there is no existence of trend and when the covariance does not change over time. This can be explained as:

$$E[X_t] = \mu \text{ for t} \quad (3.4)$$

and

$$Cov(X_t X_{t-k}) = E[(X_t - \mu)(X_{t-k} - \mu)] = y_k \text{ for t and k} \quad (3.5)$$

Sibanda (2012) affirms that the independent and dependent factors of a regression model are stationary and errors have a zero mean and limited variance. The unit root test can be estimated by different unit root techniques used in EViews such as the Augmented Dickey-Fuller test (ADF), Phillips Perron test (PP), Kwiatkowski-Phillips-Schmidt-Shin test (KPSS) and the unit-root with breaks test.

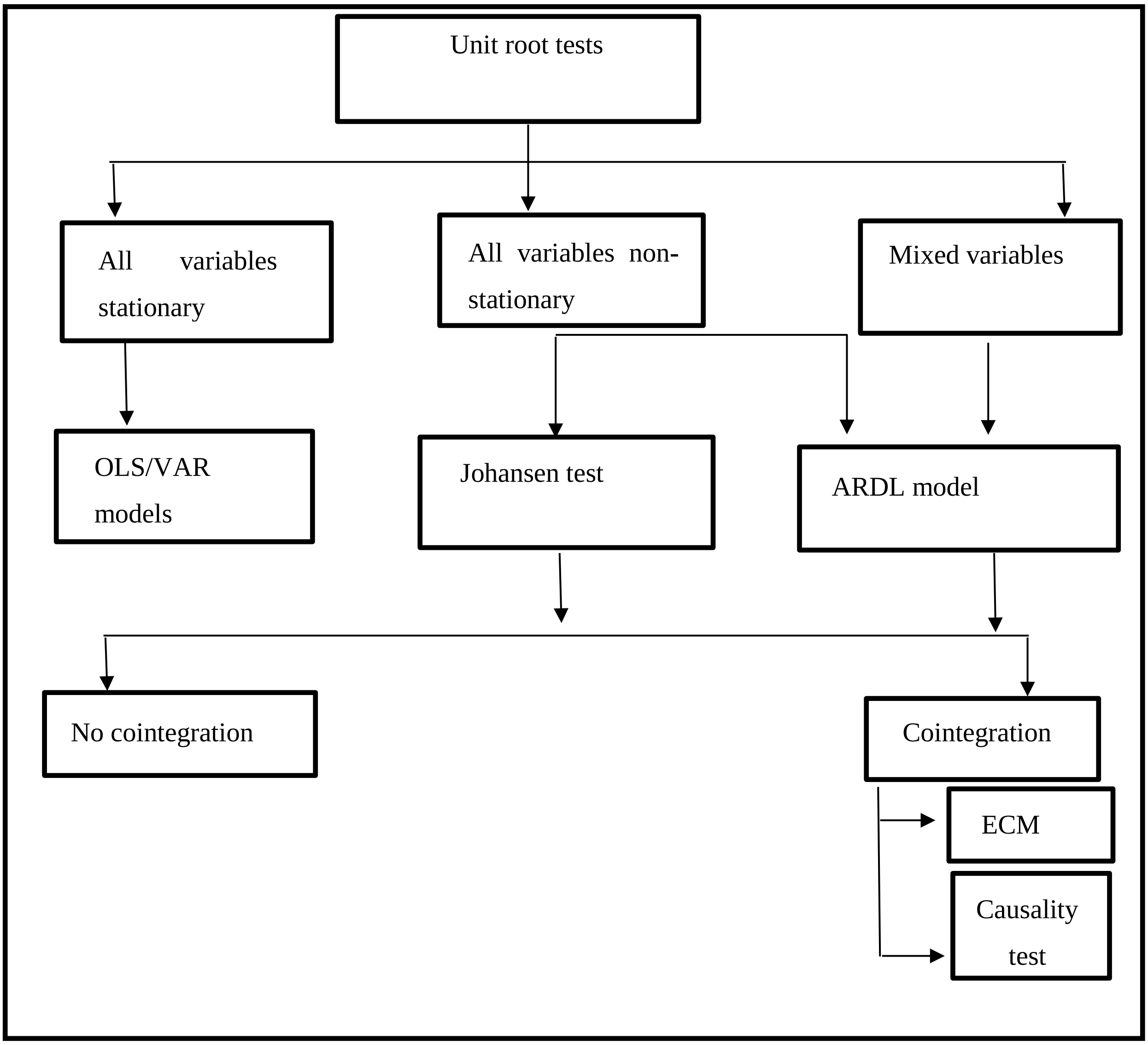

Figure 3-1: Unit root test process

Source: Compiled by author from Shrestha and Bhatta (2018)

3.3.3.1 Augmented Dickey-Fuller unit root test (ADF)

The ADF test is utilised to evaluate whether the data series has the presence of a unit root or not, as well as the order of integration of variables (Gervais and Khraief, 2007:2). Dickey and Fuller (1979, 1981) created a technique to test non-stationarity in view of the presence of a unit root. This test is known as the Dickey-Fuller test (DF) for unit roots (Asteriou *et al*., 2007:295). According to Enders (2010:206), the DF test gives three distinct equations that evaluate the existence of a unit root in the data set, explained as follows:

$$\Delta y_1 = \gamma y_{t-1} + \sum_{i=1}^{p} \beta_i \Delta y_{t-i} + \mu_t \tag{3.6}$$

$$\Delta y_1 = a_0 + \gamma y_{t-1} + \sum_{i=1}^{p} \beta_i \, \Delta y_{t-i} + \mu_t \tag{3.7}$$

$$\Delta y_1 = a_0 + \alpha_1 t + \gamma y_{t-1} + \sum_{i=1}^{p} \beta_i \, \Delta y_{t-i} + \mu_t \tag{3.8}$$

The difference between equations (3.6) to (3.8) is the presence of linear time trend and intercepts, while Equation (3.6) consists of an intercept, Equation (3.7) consists of a deterministic time trend and an intercept. Asteriou *et al*. (2007:296) explain that the principal interest for the three equations is γ and, consequently, if γ = zero, the time series is explained to have a unit root. A substitute to the DF test is the augmented Dickey-Fuller (ADF) test.

3.3.3.2 Phillips Perron test (PP)

The main distinction between ADF and (PP) unit root is the way heteroscedasticity and serial correlation are handled in the errors. Particularly, the ADF test employs parametric autoregression to estimate auto regressive moving average (ARMA) structure of the errors in the regression.

The tests handle the serial correlation and heteroscedasticity in the errors. All the more particularly, the ADF test utilises parametric autoregression to estimate the ARMA structure of the errors in the test regression. The PP tests overlook the serial correlation in the regression test. The regression test for the PP tests is:

$$\Delta y_t = \pi y_{t-1} + B_i D_{t-i} + u_t \tag{3.9}$$

Where y_t denotes the time series and u_t represent I(0) with mean and which may possibly be heteroscedastic. While $B_i D_{t-i}$ represent the deterministic trend component. The PP tests amends for heteroscedasticity in the errors u_t and serial correlation of the test regression by specifically altering the test statistics.

3.3.3.3 Kwiatkowski-Phillips-Schmidt-Shin test (KPSS)

Kwiatkowski *et al*. (1992) developed the KPSS model, which was motivated by the Dickey and Fuller (1979), Dickey and Fuller (1981) and Said and Dickey (1984) models, which is commonly used in the economic time series. The KPSS test differs from the ADF test with a null hypothesis of the unit root. Unlike the ADF test, the KPSS assumes that there is a stationary process under the null hypothesis (Ding, 2006:20). The KPSS test is established on residuals from the OLS regression and takes the form of:

$qt = X_t'\delta + \varepsilon_t$ (3.10)

The KPSS test is defined as:

$\varphi_\varepsilon = \sum_t S(t)^2/(T^2 f_0)$ (3.11)

Where f_0 is an estimator of the residual spectrum at frequency zero and where $S(t)^2$ is a cumulative residual function:

Furthermore, the KPSS model suggested that trend stationary should be considered as the null hypothesis and the unit root as the alternative hypothesis. In the functional setting, the null hypothesis of trend stationarity is stated as follows:

H0: $Xn(t) = \mu(t) + n\xi(t) + nm(t)$ (3.12)

Where, N refers to day, month, quarter, or year, while t refers to time for each function. For instance, intraday price curves, where t is the time within a trading day, which is measured in hours and minutes. The time t, does not represent the actual time, but the time until expiration, for example the expiration of the bond maturity. Moreover, μ and ξ correspond, respectively, to the intercept and slope. While ηm are the functions, which model random departures of the observed functions of Xn which form a deterministic model.

3.3.3.4 Unit root test in the presence of structural break

Shershtah and Bhatta (2018:75) define structural breaks as a rise or decrease in an economic time series; this occurs as a result of shocks such as the financial crisis as well as changes in policy direction. The structural breaks can make it difficult to conduct a unit root test; the discussion on the hypothesis of unit root has experienced new interest following the findings of Nelson and Plosser (1982). The known view of unit root hypothesis was that current shocks were temporary and that the shocks did not affect the time series data in the long-run.

Nelson and Plosser's (1982) findings reflect that there are permanent and long-run effects on economic data caused by random shocks; this means that the fluctuations are not temporary. However, Perrson (1989) challenged the findings by Nelson and Plosser (1982) and argues that in the occurrence of structural breaks, the ADF tests are biased towards the non-rejection of the null hypothesis.

In addition, the importance of structural breaks was first emphasised by Perron (1989), and Rappoport and Reichlin (1989). Perron (1989) states that the structural breaks in time series data could influence the unit root results.

Harvey (2001:21) explains that suppose there is a structural break in the trend at a known time r + 1 and let λ= r/T represent the segment of the sample before the structural break occurs. Consider the model:

Model 1: $y_t = \mu_t + \delta W_t + \varepsilon_t$ (3.13)

Model 2: $y_t = \mu_t + B_t + \delta W_t + \delta_B(W_t t) + \varepsilon_t$ (3.14)

Model 2(a): $y_t = \mu_t + B_t + \delta W_t + \varepsilon_t$ (3.15)

Model 2(b): $y_t = \mu_t + B_t + \delta_{BZt} + \varepsilon_t$ (3.16)

Where μ_t denotes the random walk and ε_t represents the white noise. While $\delta\ and\ \delta B$ are parameters and t are the observed series for which stationarity is tested. Model 1 has no slope and thus the structural break is level, while the other three models have a time trend. In model two, the structural break occurs in the slope and the level. Model 2(a) has a structural break in level only and model 2(b) links to the piecewise linear trend (Harvey, 2001:21).

All the different unit root tests explained above are key to the ARDL model. Once all the unit root tests are conducted, then the appropriate model will be selected. For the purpose of this study, the focus will be on the ARDL model as it caters for variables that are integrated at I(0) and I(1) to be modelled together.

3.3.4 Autoregressive distributed lag model (ARDL)

ARDL model was presented by Pesaran *et al*. (2001). The purpose of this model is to cater for variables that are a mixture of I(0) and I(1) in the same estimation, which is impossible in other econometrics models. Therefore, the ARDL model has the advantage of not requiring a specific specification of the order of the underlying data (Atif *et al*., 2010:32). In the ARDL model, the variable of interest is assumed to be a function of the past values of itself and the current and past values of other variables (Shrestha & Bhatta, 2018:79).

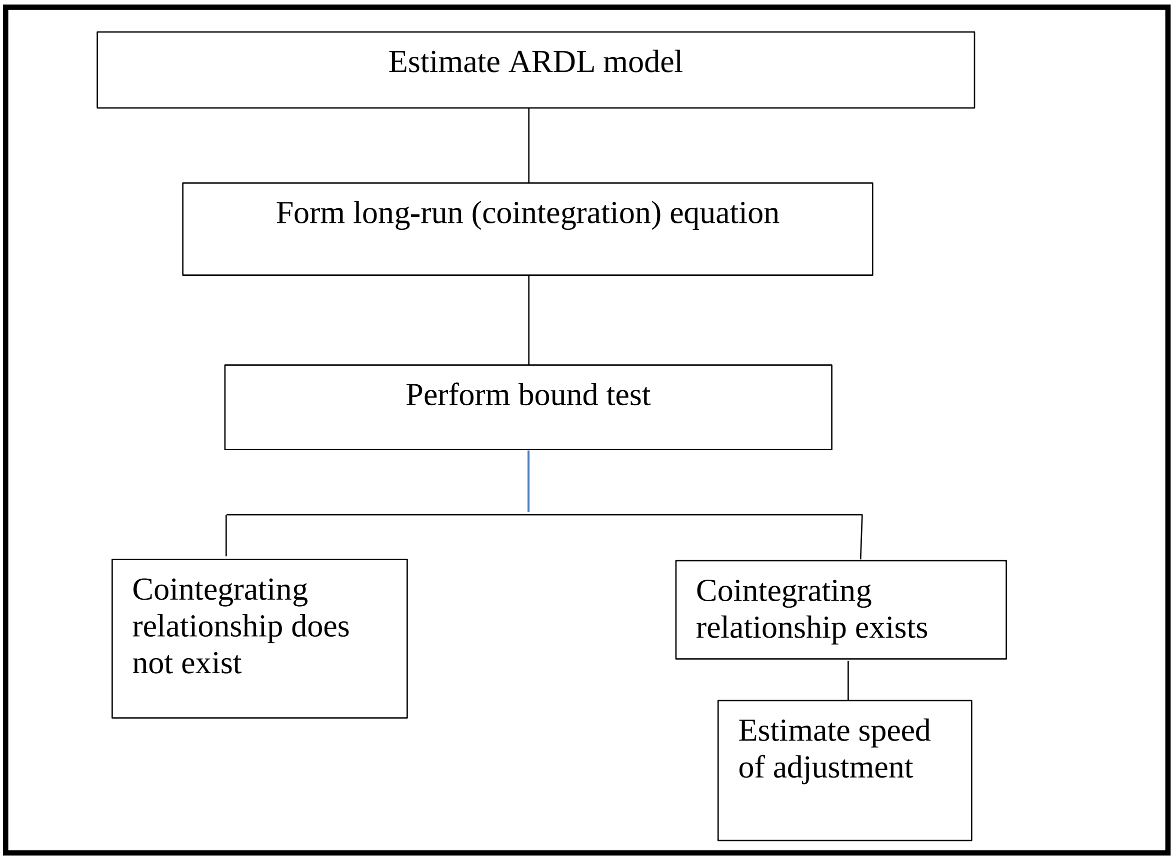

Figure 3-2: Graphical illustration of the ARDL model

Source: Compiled by author from Pesaran *et al*. (2001)

Figure 3.2 depicts the process followed when estimating the ARDL model. Pesaran *et al*. (2001) encouraged the use of the ARDL model for the estimation of variables that are integrated at different orders I(0) and I(1), but not I(2), because the model advocates that once the order of the ARDL has been recognised, the relationship can be estimated by OLS. Once the variables are integrated at different orders then an ARDL test may be conducted. To test for the long-run relationship between the variables the bound test follows. Bound tests allows a mixture of variables as regressors. Once the test shows the presence of the long-run relationship among the variables through the F-statistic, the rule is that the F-statistic has to be higher than the lower bound I (0) and the upper bound I (1). According to Engle and Granger (1987), when two or more variables are cointegrated they rule out the possibility of the results being spurious.

The general specification of the ARDL model is explained as follows:

$$y_t = \sum_{i=1}^{p} \alpha_i \, \Delta y_{t-i} + \sum_{i=0}^{n} c_i' \, \Delta x_{t-i} + u_t \qquad (3.17)$$

Where y_t denotes the dependent variable and X represents the independent variable while Δ represents the first difference of the variables, p and n are the lag lengths and e_t is a scalar mean error term. In light of the above discussion, the study aims to examine the relationship between short-term interest rate volatility and bond yields in South Africa and will follow the following ARDL model equation:

$$\Delta Y1_t = \alpha_0 + \sum_{j=1}^{k} B_j \, \Delta Y1_{t-j} + \sum_{j=1}^{k} \gamma_j \Delta r_{t-j} + \sum_{j=1}^{k} \delta_j \Delta V_{t-1} + \varphi_1 \Delta Y_{t-1} + \varphi_2 r_{t-1} + \varphi_3 \, V_{t-1} + \mu_t \qquad (3.18)$$

$$\Delta Y2_t = \alpha_0 + \sum_{j=1}^{k} B_j \, \Delta Y2_{t-j} + \sum_{j=1}^{k} \gamma_j \Delta r_{t-j} + \sum_{j=1}^{k} \delta_j \Delta V_{t-1} + \varphi_1 \Delta Y2_{t-1} + \varphi_2 r_{t-1} + \varphi_3 \, V_{t-1} + \mu_t \qquad (3.19)$$

$$\Delta Y3_t = \alpha_0 + \sum_{j=1}^{k} B_j \, \Delta Y3_{t-j} + \sum_{j=1}^{k} \gamma_j \Delta r_{t-j} + \sum_{j=1}^{k} \delta_j \Delta V_{t-1} + \varphi_1 \Delta Y3_{t-1} + \varphi_2 r_{t-1} + \varphi_3 V_{t-1} + \mu_t \qquad (3.20)$$

$$\Delta \, Y4_t = \alpha_0 + \sum_{j=1}^{k} B_j \, \Delta Y4_{t-j} + \sum_{j=1}^{k} \gamma_j \Delta r_{t-j} + \sum_{j=1}^{k} \delta_j \Delta V_{t-1} + \varphi_1 \Delta Y4_{t-1} + \varphi_2 r_{t-1} + \varphi_3 \, V_{t-1} + \mu_t \qquad (3.21)$$

Y1 denotes the zero to three year bond yields, Y2 denotes the three to five year bond yields, Y3 denotes the five to ten year bond yields and Y4 denotes 10 year and above bond yields. In addition, r denotes interest rates, while V denotes interest rate volatility.

From equations 3.18 to 3.21 the following explain the variables used: $\Delta Y1_t, \Delta Y2_t, \Delta Y3_t$ and $\Delta \, Y4_t$ represents changes in bond yields, Δr_{t-j}= changes in short-term interest rates, ΔV_{t-1} = changes in interest rate volatility, while the intercept is denoted by α_0 and number of lags by k. Furthermore, $B_j \; and \; \gamma_j$ represent short-run dynamics and $\varphi_1, \varphi_2 \; and \varphi_3$ represent the long-run relationship.

From equations 3.18 and 3.21 the following hypotheses are formulated:

H_0: no existence of cointegration: $\varphi_1 = \varphi_2 = \varphi_3 = 0$

H_1: existence of cointegration: $\varphi_1 \neq \varphi_2 \neq \varphi_3 \neq 0$

The hypotheses are tested by means of the bound test in equation 3.18 to 3.21, respectively. The critical values of the F-statistics for a different number of variables (K) and whether the ARDL model contains an intercept and/or trend are available in the Pesaran (1996a) and Pesaran *et al.* (2001) test. The cointegration equations are conducted by the use of the bound test. The rule of the test is that the F-statistics should be greater than the upper bound critical value (Persaran *et al.*, 2001). Thus, if the F-statistics is greater than the upper bound critical value the null hypothesis is rejected. The cointegration among the variables proposes that there is a long-run relationship between the variables. In consequence, when the F-statistic is lower than the upper bound critical value the null hypothesis is accepted, which favours the alternative hypothesis.

After testing for the cointegration test, the next step is to conduct the short-run effect through the error correction model (ECM); in addition, the ECM is derived from the ARDL model. Shrestha and Bhatta (2018:79) explain that the ECM incorporates the short-run dynamics with the long-run equilibrium without losing long-run information.

3.3.4.1 Advantages and disadvantages of the ARDL model

The ARDL model has characteristics that make it a suitable model for the observed data. In addition, what makes a model more attractive is when the advantages are more than its disadvantages and can solve parameters that other cointegration models cannot solve (Pesaran & Pesaran, 1997).

3.3.4.1.1 Advantages of the ARDL model

According to Nkoro and Uko (2016:79), the following are the advantages of the ARDL model. First, endogeneity is not a problem in the ARDL model since it is free from residual correlation, because the underlying variables represent a single equation. In this case, variables are expected to be endogenous.

Secondly, in cases where there is a distinct long-run relationship between the variables, the ARDL model process is able to differentiate between the independent and dependent variables. This means that the ARDL model assumes that there exists a single reduced form equation relationship between the independent and dependent variables (Pesaran *et al*., 2001).

Thirdly, the ARDL model has an advantage of identifying the cointegrating vectors in cases where there are multiple cointegrating vectors.

Lastly, the ARDL model derives the error correction model (ECM) through a linear transformation; this process incorporates the short-run adjustments with the long-run equilibrium and ensures that the long-run information is not lost.

3.3.4.1.2 Disadvantages of the ARDL model

The ARDL model only considers variables that are stationary at I(1) or I(0) or variables that are integrated at both I(1) and I(0) in a single linear equation, however, variables that are stationary at I(2) are not considered. Variables that are stationary at I(2) contain two unit roots and thus require more than one differencing to make them stationary (Brooks, 2014:360). According to Muchapondwa & Pimhidzai (2011), the other disadvantage of the ARDL model is that it does not consider the possibilities of other cointegration equations, since the model incorporates only a single equation.

3.3.4.2 Error correction term

This model is estimated if the bound test confirms the presence of cointegration among the variables. In this study four ECM equations are used, which explains the short-run relationship between short-term interest rate volatility and bond yields in South Africa. If variables in equations 3.18 to 3.21 co-integrate, the error correction model is conducted. The following is the equation for the error correction.

$$\Delta Y1_t = \alpha_0 + \sum_{j=1}^{k} B_j \, \Delta Y1_{t-j} + \sum_{j=1}^{k} \gamma_j \Delta r_{t-j} + \sum_{j=1}^{k} \delta_j \Delta V_{t-1} + \delta ECT_{t-1} + \mu_t \quad (3.22)$$

$$\Delta Y2_t = \alpha_0 + \sum_{j=1}^{k} B_j \, \Delta Y2_{t-j} + \sum_{j=1}^{k} \gamma_j \Delta r_{t-j} + \sum_{j=1}^{k} \delta_j \Delta V_{t-1} + \delta ECT_{t-1} + \mu_t \quad (3.23)$$

$$\Delta Y3_t = \alpha_0 + \sum_{j=1}^{k} B_j \, \Delta Y3_{t-j} + \sum_{j=1}^{k} \gamma_j \Delta r_{t-j} + \sum_{j=1}^{k} \delta_j \Delta V_{t-1} + \delta ECT_{t-1} + \mu_t \quad (3.24)$$

$$\Delta Y4_t = \alpha_0 + \sum_{j=1}^{k} B_j \, \Delta Y4_{t-j} + \sum_{j=1}^{k} \gamma_j \Delta r_{t-j} + \sum_{j=1}^{k} \delta_j \Delta V_{t-1} + \delta ECT_{t-1} + \mu_t \quad (3.25)$$

Where δ denotes the coefficient of ECT and ECT is the error correction term. The ECT specifies the speed of adjustment it would take the model to adjust to the equilibrium. Thus, the ECM is only used if a long-run relationship is identified between the short-term interest rate volatility and bond yields in South Africa.

3.4 DIAGNOSTIC TESTS

After the model estimation, diagnostic tests should be performed. Performing diagnostic tests enables the researchers to identify misspecifications of the stability of the regression coefficients (Tandrayen *et al.*, 2011). The diagnostic tests are used to determine whether the underlying ARDL model assumptions hold and whether the estimated results are non-spurious. Since this study employed the ARDL model, the diagnostic test will ensure that the ARDL model assumptions are not violated. The different types of diagnostic tests also depend on the modelling technique used. Moreover, the diagnostic test can help in reducing the amount of errors in the model.

3.4.1 Goodness of fit

Goodness of fit of the model describes the robustness of the model used and can be tested by how well the regression line explains the data. In addition, the R^2 is the most used goodness of fit of the model; it shows how much of the variance in the dependent variable is explained by the independent variables.

3.4.2 Serial correlation

Serial correlation test is a vital limitation in time series examination (Akinlawon *et al*., 2008:52). The autocorrelation function (ACF) tests whether the data series is independent. When the ACF yields results that are substantially diverse from zero, it means that there is dependence among the observations. In consequence, the ACF is a primary instrument for testing the independence of variables (Janacek & Swift, 1993). The residual autocorrection is requested for regulating whether the residuals are white noise or not. The Durbin-Watson (DW) is one of the early signs that the data used might be correlated serially. The Ljung-Box Q statistic is used as a substitute to check for serial correlation and is given as (Ljung & Box, 1978):

$$Q_k = n(n+2)\sum_{i-1}^{k}\frac{r_i}{(n-i)} \tag{3.26}$$

Where n represent the sample size, k represents number of lags and r_i represents the i^{th} autocorrelation. When Q_k is large then the likelihood that the process has uncorrelated data declines. Q_k is asymptotically approximate.

3.4.3 Heteroscedasticity

One of the assumptions of the ARDL model is that the data should be homoscedastic, which means that the residuals should have constant variance. Heteroscedasticity happens when the error term do not have constant variance (Williams, 2015: 1). When this happens, the OLS estimates are unbiased and become inefficient, which lead to wrong estimations (White, 1980). The presence of heteroscedasticity causes data from highly variable areas to have a larger effect on reducing the weighted criterion in OLS (Gelfand, 2013:4).

Heteroscedasticity can be examined by the following diagnostic tests: Breusch-Pagan LM test, White's test, Glesjer LM test, Harvey-Godfrey LM test, Park LM test and Goldfeld-Quandt test. These tests, however, produce different results in the context of economic time series. The Breusch-Pagan LM test is commonly used when examining the data for the possibility of heteroscedasticity. The Breusch-Pagan LM test was implemented by Breusch and Pagan in 1979. The Breusch-Pagan LM heteroscedasticity test is calculated in the following form:

$$\text{Var}\ (\varepsilon_t^2) = \sigma_i^2, i = 1 \dots, n \qquad (3.27)$$

$$\text{Cov}\ (\varepsilon_i \varepsilon_j)\text{=0,i} \neq j = 1,2, \dots n \qquad (3.28)$$

3.4.4 Normality test

Jarque-Bera (JB) examines whether the data series is normally distributed. The test estimates the distinction between the skewness and kurtosis of the data series in comparison to the normal distribution. The JB test relies on the assumption that skewness and kurtosis are equivalent to zero (Mlambo, 2013:71). The JB test is figured as:

$$jb = \frac{T-K}{6}\ (s^2 + \frac{(K-3)^2}{4}) \qquad (3.29)$$

Where T represents observations, K denotes number of estimated parameters, while S and K represent skewness and kurtosis, respectively. The JB test explains that the bigger the JB the lower the likelihood that the data series is drawn from normal distribution (Hamann, 2001).

3.4.5 Ramsey's RESET test

The Ramsey RESET test (1969) was designed to detect if there any neglected nonlinearities in the regression model. This test specifically, checks whether non-linear combinations to the fitted values help explain the response variable. Given a multiple regression model the Ramsey RESET test is given as:

$$y = B_0 + B_1 x_1 + B_2 x_2 + \cdots + B_k x_k + u \qquad (3.30)$$

Then add the fitted values y_1^2 to the multi regression model and re-estimate as:

$$y = B_0 + B_1 x_1 + B_2 x_2 + \cdots + B_k x_k + y_1^2 + u \qquad (3.31)$$

Where y is the independent variable and u is assumed to have the multivariate normal distribution. Moreover, B_1, B_2 and B_k are the least square estimators.

3.5 STUDY EMPIRICAL OBJECTIVES

The empirical objectives in Table 3.3 are tested in Chapter 4 (results and findings). These empirical objectives are used to test the relationship between short-term interest rate volatility and bond yields in South Africa. The process will be done through the methods explained in this chapter.

Table 3-3: Empirical objectives

Empirical objective	Method
Evaluate the movement between interest rate volatility and bond yields in South Africa	Graphs
Evaluate and interpret graphically the effect of short-term interest rate volatility on bond yields before, during and after the 2007-2009 financial crisis	Graphs
Evaluate whether there is a positive or negative relationship between interest rate volatility and bond yields	ARDL model: the calculations are specified in Section 3.7 Note: Interest rate volatility is measured by the conditional GARCH (1,1) model.
Determine whether the effect of interest rate volatility is stronger on shorter-, medium- or longer-term bond yields in South Africa	ARDL model
Determine whether short-term interest volatility has a short- or long-run effect of bond yields	Long-run bound test and error correction model.

Source: Compiled by author from IMF and SARB (2018)

3.6 CHAPTER SUMMARY

The aim of this study is to examine interest rate volatility and bond yields in South Africa. This chapter explained the methodological procedures followed to achieve the objectives set in this study. In addition, the style and type of data set used as well as the observed period to meet the objectives was explained. The time span for this study is from January 2004 to December 2017, which amounts to a total of 168 observations. The study period was split for periods before, during and after the 2007-2009 financial crisis analysis purposes.

Moreover, the CIR (1985) and Longstaff and Schwartz (1992) models were explained as they give a view of the relationship between short-term interest rate volatility and bond yields. The only difference between these two models is that one uses one variable (short-term interest rates), while the other one uses two variables (short-term interest rates and short-term interest rate volatility). In addition to this, the GARCH (1,1) model was elaborated upon, as this model is used to estimate the volatility of short-term interest rates.

Subsequently, the chapter explained the model selection process, which consists of various steps before finding the best model that fits the data used in this study. This process began with the unit root test explanations. In order to avoid spurious results, the variables must be stationary, thus the unit root tests help to identify any stationarity in the data set as well as the order of integration of variables. The results of the unit root tests, therefore, will suggest the appropriate model to use. This study employed the ARDL model, as it caters for variables with mixed order of integration.

The ARDL model, unlike other models (OLS/VAR and Johansen), allows the mixture of variables to be used I(0) and I(1), but not I(2). Once a cointegration relationship is discovered through the use of the F-statistics, among the variables the step that follows is to test for the long-run and short-run effect. The short-run test also measures the speed of adjustment it takes variables to adjust back to their equilibrium.

Finally, in order to check the authenticity of the ARDL model, the diagnostic tests were explained, which also validate whether the ARDL assumptions were not violated. Chapter 4 will proceed to test the procedures explained in this chapter in order to achieve the study's empirical objectives.

CHAPTER 4: EMPIRICAL RESULTS AND FINDINGS

4.1 INTRODUCTION

To examine interest rate volatility and the bond yields in South Africa, the study employed data series that span from January 2004 to December 2017. The choice of the data observation was chosen to look at the periods before, during and after the 2007-2009 financial crisis. Interest rate volatility was expected to be high during the 2007-2009 financial crisis period.

Since the 2007-2009 financial crisis, the capture of interest rate movement has become more significant for banks and financial institutions across the globe. The volatility of these interest rate movements usually reflect signs of risk in the provision of the level of volatility in the market.

This study used the conditional variance from a GARCH (1,1) estimation of T-bill rates to observe the short-term interest rate volatility. The bond yields used in the study consist of the average monthly bond yields with different maturities. The literature pertaining to the relationship between short-term interest rate volatility and the term structure of interest rates is inconclusive and thus this study aims to observe the relationship empirically with the use of South African data.

CIR (1985) developed a one-factor model that evaluates the effect of short-term interest rates on the term structure of interest rates. The results reflected a negative relationship between short-term interest rates and the term structure interest rates. Moreover, the results indicated that volatility of short-term interest rate is lower when interest rates are low and when the yield curve reflects lower curvature, likewise, for high interest rate volatility.

This chapter will first explore the graphical analysis of short-term interest rates, volatility of short-term interest rates and the bond yields. These graphical analyses will give the first view of the relationship between interest rate volatility and bond yields. However, the graphical analyses are not enough to determine the relationship between the variables. In that case, the unit root tests will be estimated in order to avoid spurious results. These unit root tests will also determine the order of integration of variables, which is important when selecting an appropriate model.

The results of the unit root test will be followed by the ARDL model estimation. The steps to be followed were explained in Chapter 3. The ARDL model will first be employed with the use of the CIR (1985) model and then be employed with the use of the Longstaff and Schwartz (1992) model.

4.2 GRAPHICAL ANALYSIS

The graph discussion depicts the relationship between the variables before proceeding with other discussions. The aim of this study is to examine interest rate volatility and bond yields in South Africa. The graphical analysis gives an overview of the manner in which short-term interest rates, short-term interest rate volatility and bond yields performed during the observed period. The observed period will also investigate the performance of the variables before, during and after the financial crisis. In addition, the graphical analysis will explain the theory of the yield curve empirically as was explained in Section 2.4.2.

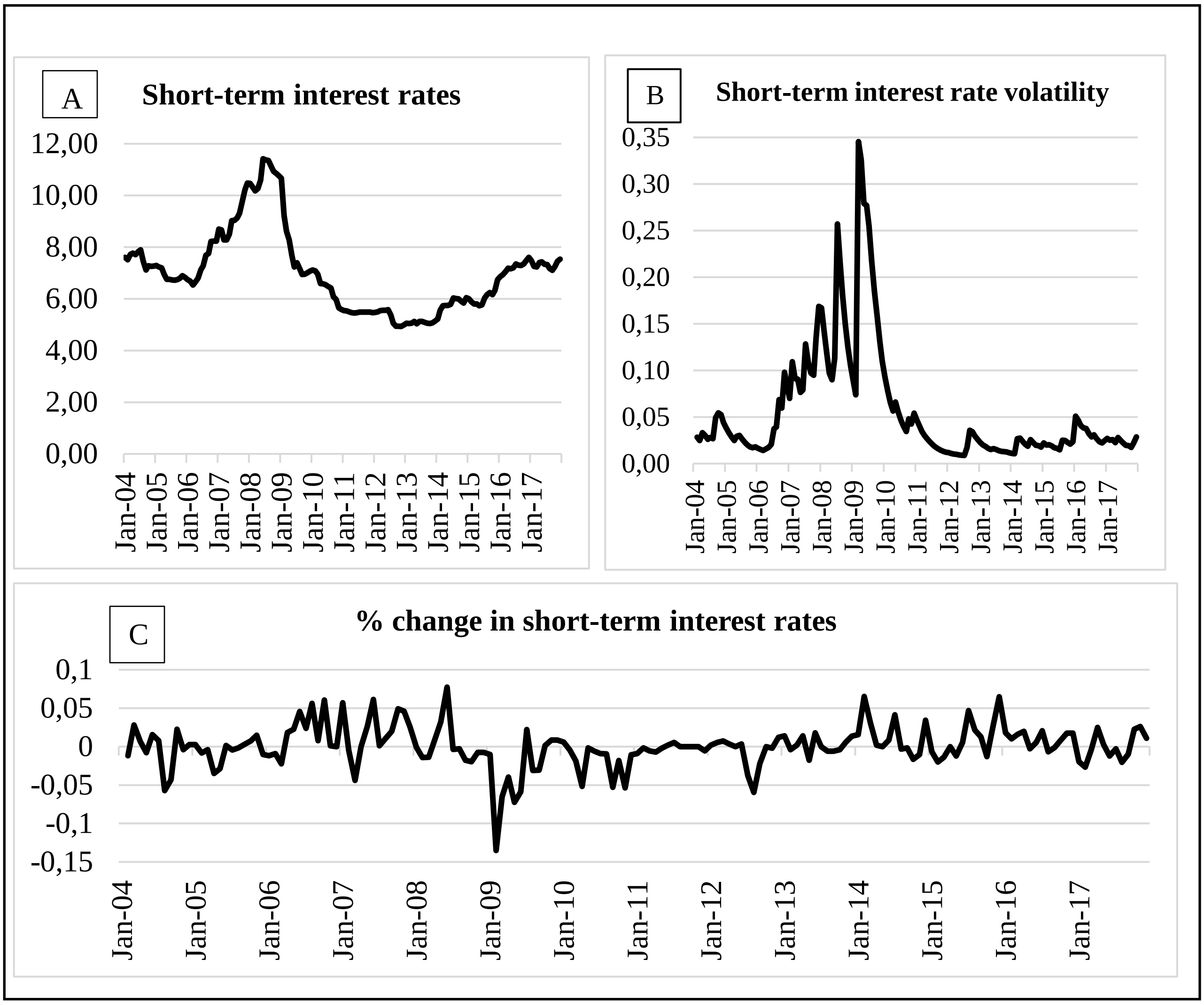

Figure 4-1: Volatility of short-term interest rates

Source: Compiled by author from IMF and SARB (2018)

Figure 4.1 has three different graphs. Graph A depicts short-term interest rates, while graph B depicts the volatility of short-term interest rates and graph C illustrates the changes in short-term interest rates for a period that spans from January 2004 to December 2017. All these graphs reflect the fluctuations of the short-term rates.

Graph A, illustrates the movements of short-term rates in South Africa during the periods spanning from January 2004 to December 2017. The short-term interest rates were very high during the 2007-2009 financial crisis, peaking in June 2008. The decrease in the South African inflation outlook motivated the SARB to increase the repo rate in April and June 2008 by 50 basis points. After a few months of successive increases in the repo rates during 2008, the SARB decreased the repo rate in December 2008. By June 2009 the repo rate had decreased by an aggregate of 450 basis points.

In August 2009, the SARB decreased the repo rate by a further 50 basis points to 7 percent per annum. Other money market interest rates responded in the same direction; money-market liquidity conditions remained stable during that time. Subsequently, short-term interest rates were on a downward trend after the financial crisis, which meant there was less pressure for South African consumers, especially for consumers who had credit facilities with numerous financial institutions.

From Figure 4.1, there is no evidence of a specific trend that the short-term interest rates may be following. Investment decisions are a result of money given up in the present moment for the promise of a greater return in the future. Thus, high interest rates explain that investment that promises fixed payments in the future is worth less than today’s money, this results in a decrease in investment prices. The decision for the SARB to increase interest rate during the financial crisis was caused by the inflation outlook.

Graph B indicates the short-term interest rate volatility, measured by the GARCH (1,1) as the conditional variance. For the period, observed interest rates were more volatile during the 2007-2009 financial crisis and reached the highest point in June 2009. After the 2007-2009 financial crisis, interest rate volatility was on a downward trend, which meant that the economy had recovered after the shock of the 2007-2009 financial crisis. High interest rate volatility is associated with a greater chance of an increase in interest rates, which, in consequence, will result in a reduction in asset prices.

Therefore, the volatility of interest rates creates uncertainties for investors, since this volatility has a direct impact on bond yields. The sensitivity of bond prices to changes in interest rate is called duration. The higher the duration, the more the bond price moves in response to interest rate changes. The volatility of the short-term rates is caused by the changes in interest rates.

Graph C depicts the changes of short-term interest rates on a monthly basis. During the financial crisis, the SARB increased and decreased repo rates continually, which caused T-bill rates to react in the same manner. This is seen by the large increases and decreases in graph C. The changes in interest rates reflect the assumption in the Longstaff and Schwartz (1992) model, where short-term interest rates are classified as mean reverting. Interest rates are known to provide support predictability of the yield curve. The increase or decrease in short-term interest rates affects bond yields even though the positive or negative effect differs in each country.

Figure 4.2 illustrates the different bond yields for varying maturities in South Africa, with maturities that range from zero to three year and 10 year and above bond yields. From these graphs, the short to medium bond yields were more volatile than the long-term bond yields. The changes in the short to medium bond yields ranged from 4.5 to 12.2 while the longer bond yields ranged from 6.0 to 10.2. These analyses reflect that in South Africa short- to medium-term bond yields fluctuate more than the longer-term bond yields.

During January 2008, the yield curve sloped upwards for all bond yields with varying maturities, as the bond yields rise in response to the tightening of the monetary policy and high inflation rate expectations. During July 2008, the yield curve level, particularly at the long end of the maturity, shifted down, while in the short end continued being attached to the repo rate. After July 2008, bond yields decreased as a result of lower oil prices, rand appreciation and improved inflation outlook. Consequently, a yield curve gap contracted in January 2008 and then widened in August 2008 by a negative 351 basis points (SARB, 2008).

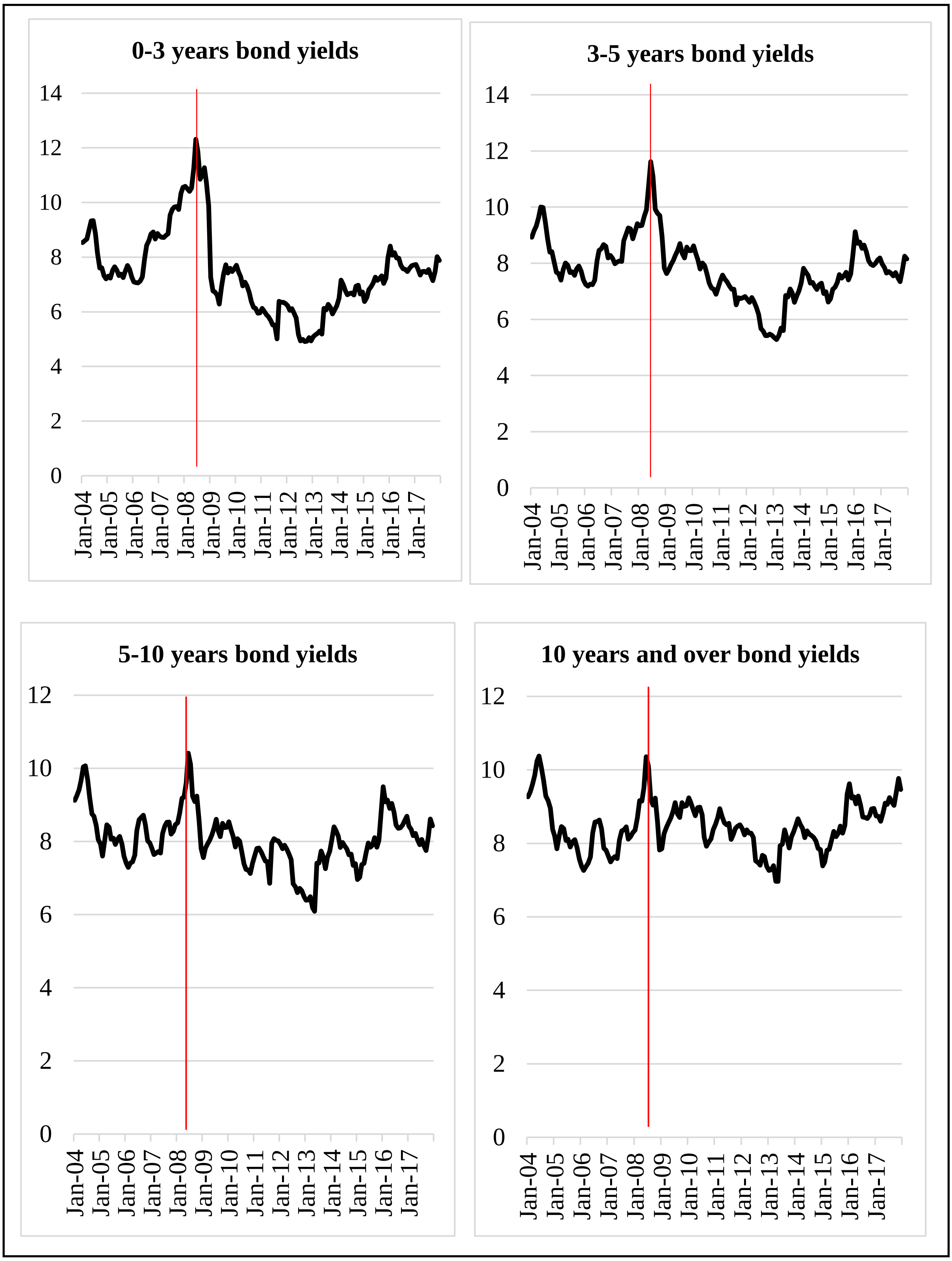

Figure 4-2: Trends of the South African bond yields

Compiled by author from IMF and SARB (2018)

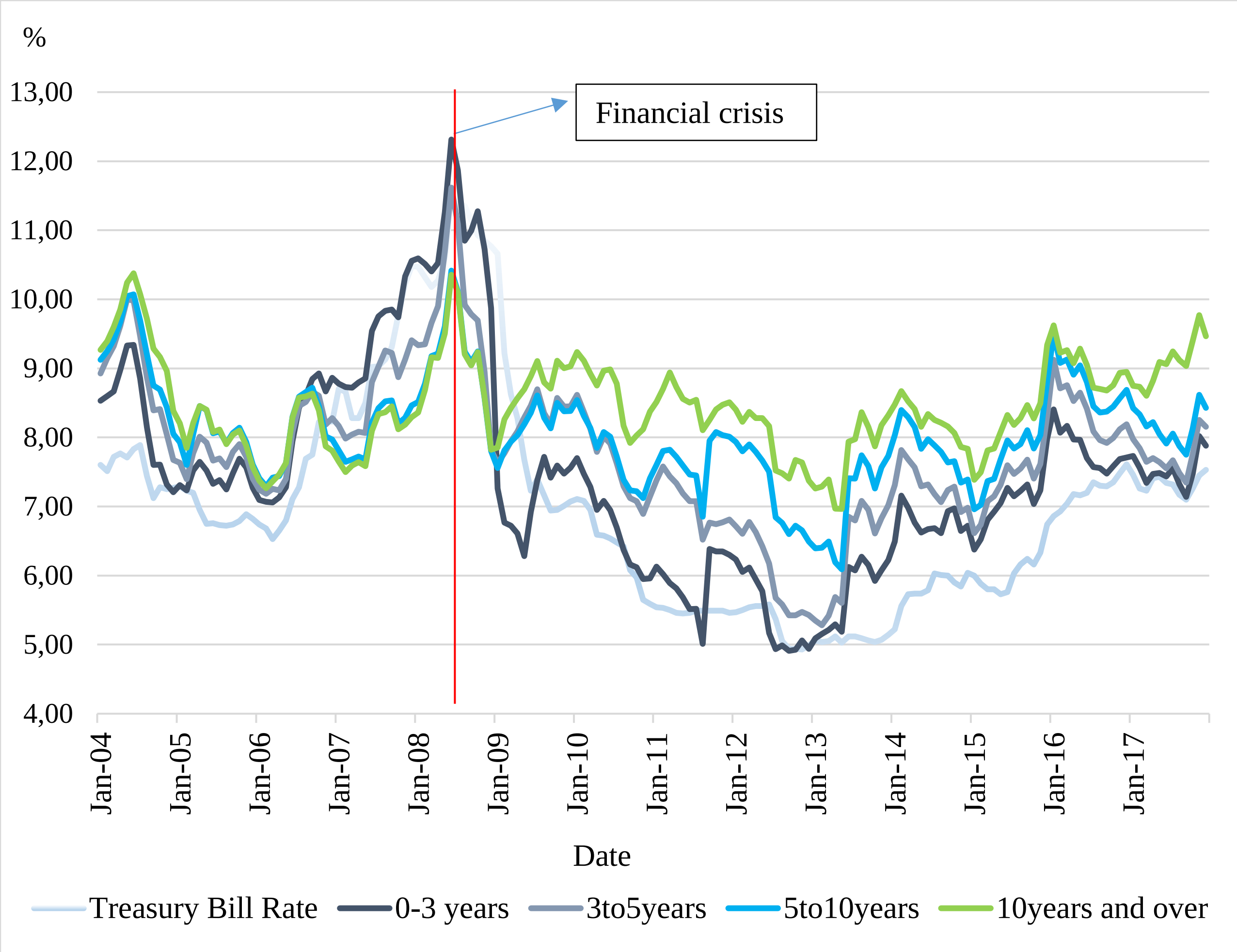

Figure 4-3: **Co-movements of short-term interest rates and bond yields in South Africa**

Source: Compiled by Author from IMF and SARB (2018)

Figure 4.3 illustrates the simultaneous movements of the South African government bond yields with different maturities and the short-term interest rates. The rates move together in the same direction over time. Before and after the financial crisis, the medium- and long-term bond yields in South Africa moved closely together. After the financial crisis, long-term bond yields were higher than medium-term bonds yields, which illustrated normal yield curves in South Africa. Normally, yield curves are positively sloped, because there is more risk involved with long-term investments, therefore, investors would need a higher return to convince them to invest in longer-term bond yields (Van Wyk *et al*.,2015:360). After the 2007-2009 financial crisis, the yield curve was normal as long-term bonds were higher than the short-term interest rates and reflected high-risk premiums, which are demanded by investors on long-term assets (Bonga-Bonga, 2009:4; Nel, 1996:162). Upward yield curves characterise an increase in economic activities and downward yields characterise a decline in economic activities (Aziakpono & Khomo, 2007:198).

When a security's historical volatility rises or becomes higher than normal, it means prices are moving up and down further/more than usual and it is an indication that something is expected to change, or has already changed with regards to the underlying security. From Figure 4.3, it is evident that the highest point of volatility in June 2009 is the indication of what had already happened in June 2008.

10 year and longer bond yields were below all the other bond yields, which peaked at just over 10 percent in June 2008, which meant that medium- and long-term bond yields were less sensitive to the movement of short-term interest rates as compared to the shorter-term bond yields. Theoretically, when long-term rates are lower than short-term rates it results in an inverted yield curve and the economy is expected to go into a recessionary phase. This was the case in South Africa during the 2007-2009 financial crisis as reflected in Figure 4.3. This occurrence indicated changes in inflation-risk premiums, which influenced the expectation of short-term interest rates in the future by the market participants.

In addition, Figure 4.3 indicates that there was a positive relationship between short-term interest and bond yields across the varying maturities. This relationship means that when short-term interest rates increase, the bond yields also follow in the same direction. When short-term interest rates decrease, bond yields will also decrease. This means that the relationship between short-term interest rates and bond yields in South Africa supports the argument of the expectation hypothesis theory. Mishkin (2004) opines that when the central bank employs a restrictive monetary policy by the decision to increase short-term interest rates in recessionary periods, participants in the market view this as a temporary shock and thus they will expect future interest rates to increase by less than the current changes of short-term interest rates. In addition, the expectation hypothesis theory explains that long-term interest rates will increase by less than the current short-term interest rates in recessionary periods. However, high interest rate expectations result in an increase in future short-term interest rates during upswing periods by more than the current short-term interest rates. Consequently, long-term interest rates increase more than the current short-term interest rates.

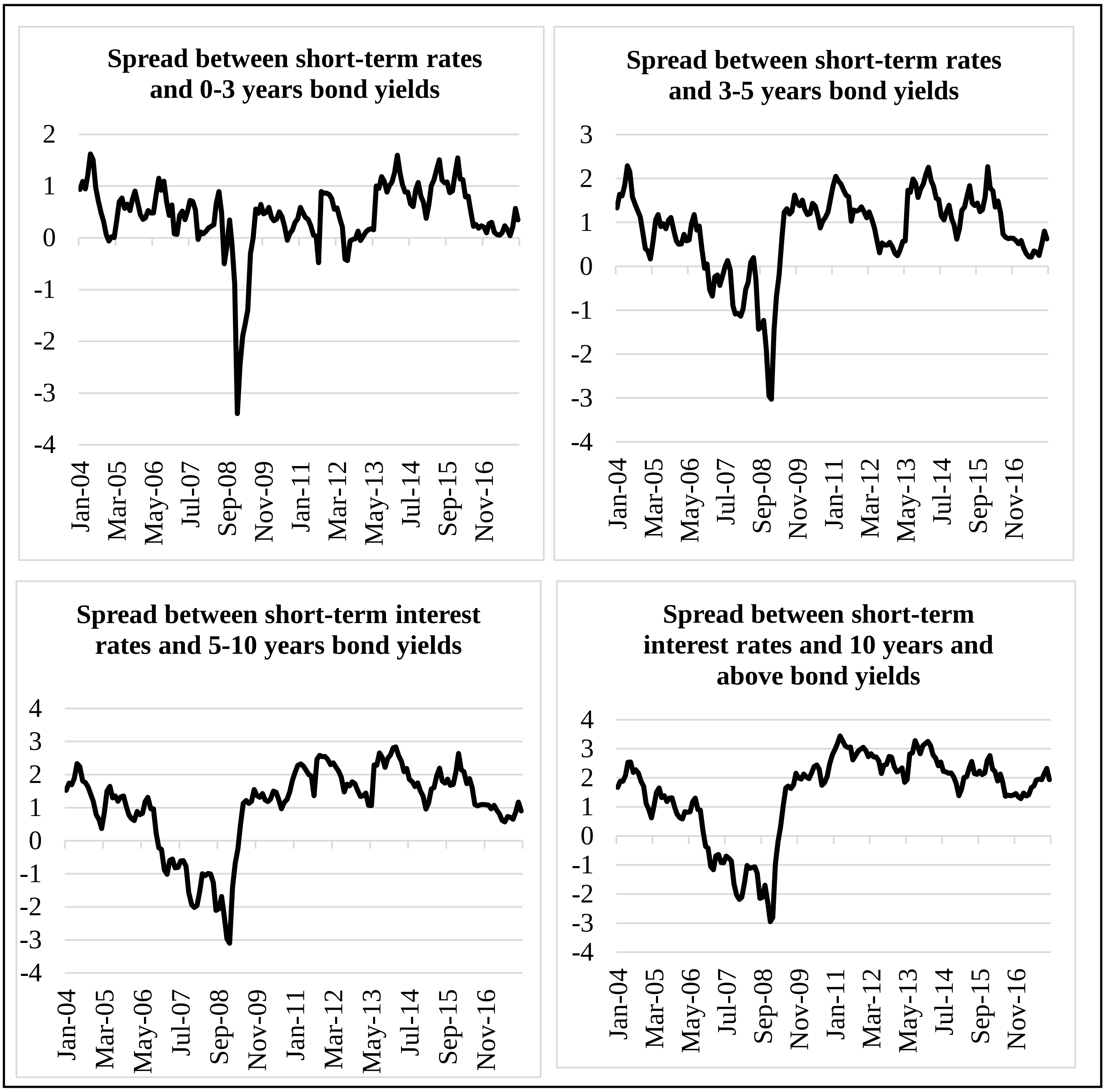

Figure 4-4: Spread between short-term interest rates and bond yields

Source: Compiled by author from IMF and SARB (2018)

Large positive yield spreads mean that investors have a perception that interest rates will rise in the future and that short-term rates will rise at a faster pace than long-term interest rates, while large negative spreads imply that interest rates will decrease in the future.

From Figure 4.4 it can be seen that the South African bond yields had negative spreads during the financial crisis, which implies that investors anticipated a fall in interest rates in the future. A yield spread indicates the likelihood of a recession or recovery one year forward. The yield spreads were at the lowest point during the 2007-2009 financial crisis. These low yield spreads indicated an economic slowdown. Furthermore, this meant that bond investors expected low growth because of the movement of short-term interest rates and other economic activities. After January 2009, the yield spreads increased in South Africa and were positive as at April 2009, which indicated an improvement in the economy. This was good for bond market investors as they are driven by higher returns on investments, however, this caused a negative effect on inflation rates. In consequence, during July to September 2009, the yield curve moved downward for all bond yield maturities. Subsequently, the yield curve moved upwards with the exception of the short-end, which was attached to the repo rate that remained unchanged by the SARB (SARB, 2009). Accordingly, the difference between the long- and short-term interest rates broadened.

In 2009, there was an increase in oil prices and expected inflation rates, which contributed to the yield curve gap in bond yields. The expected inflation rates increased from February to July 2009 by 2.7 percent as real yields decreased and nominal yields increased. In consequence, the proxy for expected inflation rate declined in August 2009 by 5.94 percent, as there was a decline in nominal yields (SARB, 2009).

Clay and Keeton (2011) state that the 2007-2009 financial crisis was predicted successfully by the yield curve and the predictions were better than other prediction parameters since 1980. This means that the yield spread is a powerful tool for forecasting and predicting economic slumps in South Africa.

4.3 DESCRIPTIVE STATISTICS

This section elaborates on the descriptive statistics for short-term interest, short-term interest rate volatility and bond yields in South Africa. The descriptive statistics offer an explanation of the characteristics of the data used for the study; in this case, this section will look at the mean, median, maximum, minimum and standard deviation of the variables. Table 4.1 has the descriptive statistics for all the variables used in this study.

Table 4-1: Descriptive statistics for short-term interest rates and bond yields

	Short-term	Volatility	0-3 years	3-5 years	5-10 years	10 years +
Mean	7.02	0.06	7.45	7.80	8.06	8.47
Median	6.9	0.03	7.30	7.71	8.03	8.40

	Short-term	Volatility	0-3 years	3-5 years	5-10 years	10 years +
Maximum	11.42	0.35	12.31	11.62	10.42	10.38
Minimum	4.93	0.01	4.91	5.28	6.09	7.00
Std. Dev.	1.58	0.06	1.51	1.11	0.76	0.68
Skewness	1.01	2.42	0.80	0.27	0.26	0.32
Kurtosis	3.60	9.136	3.63	3.867657	3.76	3.04
Jarque-Bera	30.93	424.62	20.81	7.244853	5.88	2.91
Probability	0.00	0.00	0.00	0.026718	0.05	0.23
Sum	1178.65	9.19	1251.62	1310.328	1354.62	1423.25
Sum Sq. Dev.	416.88	0.66	378.41	204.3095	96.58	77.23
Observations	168	167	168	168	168	168

Source: Compiled by author from IMF and SARB (2018)

A summary of the descriptive statistics is elaborated on in Table 4.1 for short-term interest rates, short-term interest rate volatility and bond yields with different maturities in South Africa. The total observation for all bond yields and short-term interest rates is 168 and total observations for short-term interest rate volatility is 167 since the volatility of interest rates is calculated as the one month lagged observation of short-term interest rates.

The mean values for all the rates excluding interest rate volatility range from 7.02 to 8.47, these rates reflect the average value for the variables. The rates also indicate that, on average, 10 year and above bond yields have the highest bond returns. This is in line with theory, as long-term rates have higher returns than short-term rates. In contrast, short-term interest rates have the lowest mean value of the other rates, which indicates the nature of short-term rates, since they have a lifespan of less than a year and are less risky than longer-term bond yields. These results are consistent with the study by Modena (2008:7), which explains that the average of spread between short-term interest rates and long-term bond yields rises with maturity.

For the observed period, short-term interest rates have the highest maximum, which is the result of the 2007-2009 financial crisis. Under normal circumstances, with the absences of shocks in the economy, long-term bond yields have a higher maximum value than short-term interest rates. The zero to three year bond yields have the highest maximum rates of 12.31 and the lowest minimum rate of 4.91, which indicates that the zero to three year bond yields are more sensitive to structural changes in the economy.

The standard deviation calculation measures the manner in which variables deviate from their mean; the higher the value of the standard deviation, the more volatile the variable is compared to other variables. From Table 4.1, short-term interest rates were more volatile than bond yields. This is in contrast with theory, where long-term bond yields are perceived to be more volatile than short-term interest rates as there is more uncertainty with long-term bonds in the future. Moreover, the zero to three and three to five year bond yields have higher standard deviations than the five to 10 year and 10 year and above bond yields. Therefore, this means that the zero to three and three to five year bond yields are more sensitive to the changes in short-term interest rates. Overall, the standard deviation values are lower than the mean values, which means the volatility is not high over the observed period; this means variables are not distributed away from their mean, thus making them less volatile.

4.4 CORRELATION ANALYSIS

The correlation analysis enables the study to investigate the relationship between the variables whether there is a strong or weak or a positive or negative relationship between the variables. Table 4.2 illustrates the correlation analysis between short-term interest rates and short-term interest rate volatility and the bond yields in South Africa. The focus of this analysis is to find out whether the variables move in the same direction or whether they move in isolation.

Table 4-2: Correlation analysis

	Short-term interest rates	volatility	0-3 years	3-5 years	5-10 years	10 years +
Short-term interest rates	1.00					
volatility	0.61	1.00				
0-3 years	0.91	0.45	1.00			
3-5 years	0.82	0.50	0.91	1.00		
5-10 years	0.63	0.32	0.77	0.92	1.00	
10 years +	0.38	0.22	0.48	0.72	0.86	1.00

Source: Compiled by author from IMF and SARB (2018)

Table 4.2 depicts the correlation analysis for all the variables used in this study. There is an overall positive relationship amongst the variables, which indicates that the variables move in the same direction. When comparing the relation of the bond yields to short-term interest rates, the zero to three year bond yields are strongly correlated to short-term interest rates with a correlation coefficient of 0.91, followed by the three to five year bond yields with a correlation coefficient of 0.82. The five to ten year bond yields also have a strong correlation with short-term interest rates, while in contrast, the 10 year and above bond yields have a weak positive relationship with short-term interest rates.

The other observation made is that short-term interest rates are positively correlated to interest rate volatility. This is consistent with studies conducted by Olan and Sandy (2005), Turan and Liuren (2005) and Olweny (2011), where volatility was found to be positively correlated with the level of short-term interest rates in Kenya.

In addition, short-term interest rate volatility is positively correlated to all the bond yields in South Africa. More specifically, short-term interest rate volatility is strongly correlated to the three to five year bond yields with a correlation coefficient of 0.50, followed by the zero to three year bond yields with a correlation coefficient of 0.45. There is a weaker correlation between short-term interest rate volatility and five to ten and 10 year and above bond yields with a correlation coefficient of 0.32 and 0.22, respectively.

The overall positive relationship among the variables means that when short-term interest rates increase, the bond yields also move in the same direction, however, the movements differ with maturities. The results are, however, in contrast with the duration theory where longer bond yields are more sensitive to changes in short-term interest rates; however, in Longstaff and Schwartz's (1992) study they found that interest rate volatility was more correlated to the medium-term bond yields, which is the case with this study. Notably, for the study period, short-term interest rates were highly influenced by the SARB inflation outlook, especially during the 2007-2009 financial crisis.

4.5 UNIT ROOT TESTS

This section investigates the unit root process undergone in the study to establish whether variables are stationary or not, as well as to establish the order of integration of these variables. According to Stock and Watson (1988), interest rates commonly follow the unit root process of I(1). Thus, this section will use four unit root tests to ensure that the correct model is used as the result of the integration of the variables. The ADF, KPSS, PP and unit root with breaks will be used in this section to investigate the order of integration of the variables.

Table 4.3 to 4.5 illustrate the unit root test results for all the variables in the regression, the short-term interest rates and short-term interest rate volatility as well as the bond yields with different maturities. The unit root process conducted is to ensure or verify whether the variables are stationary at I(0) or I(1). If all the variable have a mixture of I(0) and I(1) there is a possibility that the variables may be cointegrated. When variables provide a mixture of I(0) and I(1), then the best model to use is the ARDL model, with the condition that none of the variables is I(2).

The analysis will first begin with the commonly known unit root tests and then proceed with the unit root with structural breaks, since the data have periods where there were shocks in the economy.

Table 4-3: Unit root test augmented Dicker-fuller (ADF)

	Level intercept		**Level with intercept and trend**		**1st difference intercept**		**Order of integration**
Variables	t-stat.	p-value	t-stat.	p-value	t-stat.	p-value	
Short-term interest rates	1.42	0.57	1.38	0.86	7.99	0.00**	I(1)
Volatility	2.88	0.05**	3.04	0.13	12.70	0.00**	I(1)
0-3 years	2.21	0.20	2.22	0.47	9.62	0.00**	I(1)
3-5 years	2.69	0.08*	2.681	0.25	9.53	0.00**	I(1)
5-10 years	3.48	0.01**					I(0)
10 years +	3.63	0.01**					I(0)

Note * and ** statistically significant at 10% and 5% respectively.

Source: Compiled by author from IMF and SARB (2018)

Table 4-4: Phillips-Perron (PP)

	Level intercept		Level with intercept and trend		1st difference intercept		Order of integration
Variables	t-stat.	p-value	t-stat.	p-value	t-stat.	p-value	
Short-term interest rates	1.49	0.54	1.44	0.84	8.12	0.00**	I(1)
Volatility	2.95	0.04**					I(0)
0-3 years	1.98	0.30	1.96	0.62	9.55	0.00**	I(1)
3-5 years	2.30	0.17	2.32	0.42	9.43	0.00**	I(1)
5-10 years	3.09	0.03**					I(0)
10 years +	3.00	0.04**					I(0)

Note * and ** statistically significant at 10% and 5% respectively.

Source: Compiled by author from IMF and SARB (2018)

Table 4-5: Kwiatkowski-Phillips-Schmidt-Shin (KPSS)

	Level intercept			Level with intercept and trend			1st difference intercept		
Variables	L.M Stat.	Crit. value	Outcome	L.M Stat.	Crit. Value	Outcome	L.M Stat.	Critic value	Outcome
Short-term interest rates	0.49	0.46	Unit root	0.16	0.15	Unit root	0.13	0.46**	Stationary
Volatility	0.32	0.46**	Stationary						
0-3 years	0.48	0.46	Unit root	0.17	0.14	Unit root	0.08	0.46**	Stationary
3-5 years	0.49	0.46	Unit root	0.16	0.14	Unit root	0.08	0.46**	Stationary
5-10 years	0.35	0.46**	Stationary						
10 years +	0.12	0.46**	Stationary						

Note * and ** failure to reject the null hypothesis of stationarity at 5% and 10%, respectively.

Source: Compiled by Author from IMF and SARB (2018)

The results from Table 4.3 were obtained through the ADF unit root test. This test is used to investigate whether the variables have a unit root. If the variables have a unit root then the variables are classified to be non-stationary and if variables are classified to have the absence of a unit root, it then implies that the variables are stationary. The null hypothesis for the ADF test is that variables have a unit root. The null hypothesis should be rejected in order to conclude that the variables do not have a unit root. Table 4.3 observes the different integration order of the variables; it first observes whether the variables are stationary at level and intercept. The results show that only the three to five year and the 10 year and above bond yields are stationary at level and intercept, while the short-term interest rates, volatility of the short-term interest rates and the zero to three and three to five year bond yields are all non-stationary at level.

Table 4.3 also indicates that the variables are all stationary at first difference, this means that the variables have no unit root and are thus all stationary at I(1). At first difference all the variables probability values are significant at 5 percent level.

Table 4.4 illustrates the unit root test through the Phillips-Perron (PP), which also reflects results consistent with the ADF test, that the five to ten and 10 year and above bond yields are stationary at level, this means that the variables are I(0). In addition, the short-term interest rates, short-term interest rate volatility and the zero to three and three to five year bond yields are not stationary at level, which means the variables are not I(0). Thus, the variables are all stationary at first difference, which implies that there is no unit root found in the variables.

In addition, Table 4.5 reflects the unit root test through the KPSS. The KPSS test for the unit root is conducted in order to compare the outcome of results with that of the ADF and PP unit root test. With the KPSS unit root test, under the null hypothesis the series is stationary, while under the alternative hypothesis the series has a unit root, therefore, is not stationary. Overall, the KPSS results for the variables are consistent with the ADF unit root test; however, there are a few differences. The five to ten and the 10 year and above bond yields are stationary at level like the ADF test, however, the five to ten year bond yields have unit root at intercept. While the volatility of interest rates is stationary at level, all the variables are stationary at first difference, which means there was no unit root at first difference for all the variables.

Table 4-6: Unit root with structural breaks

	Intercept		Trend and intercept		1st Difference Trend	
Variables	t-stat.	p-value	t-stat	p-value	t-stat	p-value
Short-term interest rates	3.26	0.53	4.43	0.16	9.93	<0.01**
Volatility	7.74	<0.01**				
0-3 years	3.92	0.19	4.65	0.09*	12.41	<0.01**
3-5 years	4.40	0.14	3.87	0.46	10.21	<0.01**
5-10 years	4.33	0.07*	4.25	0.23	11.08	<0.01**
10 years +	4.15	0.11	4.22	0.25	10.17	<0.01**

Note * and ** statistically significant at 10% and 5% respectively.

Source: Compiled by author from IMF and SARB (2018)

Structural changes affect time series data and may happen due to events such as the financial crisis and changes in policy, among others in the country. During the observed period, the financial crisis had a direct impact on short-term interest rates and bond yields. Table 4.6 illustrates the unit root with breaks for short-term interest rates, volatility of short-term interest rates and bond yields in South Africa.

Short-term interest rate volatility was stationary at level and intercept while all other variables had a unit root at level and intercept. At first difference, all the variables are stationary. These results are almost like the ADF test. The breaks for most of the variables happened in June 2008, which was during the 2007-2009 financial crisis. While the 10 year and above bonds had a break around July 2009. This means that bond yields that are shorter than the 10 year and above responded faster to the 2007-2009 financial crisis than the 10 year and above bond yields.

Because the variables are integrated at different orders, the study will proceed to use the ARDL model. The ARDL model caters for variables that are I(0) and I(1) but not for variables that are I(2).

4.6 COINTEGRATION TESTS: CIR (1985) MODEL

Since the variables used in this study are I(0) and I(1), the study will proceed with the cointegration test. The cointegration test is done through the ARDL model. The first step is to determine the model and lag selection for the study, which is followed by the bound test, which determines if the variables have a long-run relationship. If variables have a long-run relationship then the step that follows is to estimate the ECM model, which checks if variables have a short-run relationship and determines the speed of adjustments for the variables to return to their equilibrium.

4.6.1 Cox-Ingersoll-Ross (1985) CIR model

The study begins with the CIR (1985) one-factor model where the short-term interest rate is used as the only independent variable that influences the entire term-structure of interest rates.

4.6.1.1 ARDL model selection

In order to choose the most suitable ARDL model for estimation and optimum lags, the ARDL model chose the best model for the variables. The model selection took into consideration the ARDL assumptions. The assumptions were not violated, such as there is no serial correlation, no heteroscedasticity and normal distribution of the variables. The tests used EViews 10 and its automatic ARDL model selection function.

Table 4-7: Model selection

Bond yields	Model	Model selection criteria	Trend specification	R-Squared
0-3 years	ARDL (4.1)	AIC	Constant	0.95
3-5 years	ARDL (4.1)	AIC	Constant	0.94
5-10 years	ARDL (4.1)	AIC	Constant	0.86
10 years +	ARDL (4.1)	AIC	Constant	0.86

Source: Compiled by author from IMF and SARB (2018)

The results illustrated in Table 4.7 indicate that the model lag selection was generated by the AIC criterion. The best selection for all the variables was ARDL (4, 1). An adjustment had to be made for the independent variable (short-term interest rates). A fixed lag selection was chosen for the dependent variables because the data consists of contemporaneous yields. Hence, one month lagged values were used for the independent variables. The R-squared values for all variables are all above 80 percent, which indicates that the selected model criteria for the all the variables are a good fit for the data.

4.6.1.2 Bound test

This section investigates whether a cointegration relationship exists with variables used in this study. First, this section will investigate whether the variables are cointegrated or not through the bounds tests. The rule is that the F-statistic has to be higher than the upper bound and lower bound of the results. If the F-statistic is higher than the upper bound and lower bound then there exists a long-run cointegration relationship among the variables.

The long-run regression results are illustrated in Table 4.8. These results give an indication whether short-term interest rates affect bond yields in South Africa, for the period under review. Furthermore, the results of the long-run cointegration tests represent the probability values of the variables. These probability values are used to determine whether short-term interest rates have a significant influence on the bond yields in the long run.

Table 4-8: Bound test

Bond yields	0-3 years		3-5 years		5-10 years		10 years +	
Bound test	I(0)	I(1)	I(0)	I(1)	I(0)	I(1)	I(0)	I(1)
5%	3.62	4.16	3.62	4.16	3.62	4.16	3.62	4.16
10%	3.02	3.51	3.02	3.51	3.02	3.51	3.02	3.51
F-stat	5.07		5.80		7.04		4.96	

I(0) and I(1) denotes lower and upper bound, respectively at a statistical significance level of 5% and 10%.

Source: Compiled by Author from IMF and SARB (2018)

Table 4.8 illustrates the bound test results for variables used in the study and from the results the bound test indicates that there is a long-run relationship among the dependent variables and independent variables. The F-statistics were 5.07, 5.80, 7.04 and 4.96. These values exceed the upper bound critical at both 10 percent and 5 percent significance levels. The null hypothesis for the bounds test is that there is no cointegration relationship amongst the variables. The null hypothesis is rejected since the F-statistic exceeds the upper bound value. This implies there is cointegration between short-term rates and bond yields. This means that there is a cointegration long-run relationship among the variables.

Since the results from Table 4.8 indicate that there is a long-run relationship between the variables, the study then proceeds with the long-run relationship test. This test will determine whether there is a positive or negative and significant effect of short-term interest rate and bond yields.

4.6.1.3 Long-run cointegration analysis

The long-run cointegration will give a view into the manner in which the variables react in the long run.

Table 4-9: Long-run relationship

0-3 years				
Variable	Coefficient	Std. error	t-static	Prob.
Short-term interest rate	0.76	0.08	9.72	0.00**

C	2.06	0.56	3.70	0.00**
3-5 years				
Short-term interest rate	0.49	0.09	5.72	0.00**
C	4.31	0.61	7.06	0.00**
5-10 years				
Short-term interest rate	0.25	0.07	3.43	0.00**
C	6.30	0.51	12.31	0.00**
10 years +				
Short-term interest rate	0.12	0.09	1.31	0.19
C	7.59	0.66	11.50	0.00**

Note * and ** statistically significant at 10% and 5%, respectively.

Source: Compiled by author from IMF and SARB (2018)

Table 4.9 depicts the long-run analysis for the short-term interest rates and respective bond yields. The zero to three year bond yields results indicate a long-run coefficient of 2.06 and a positive coefficient short-term interest rates of 0.76. This means that a 1 percent increase in short-term interest rate leads to a 76 percent increase in the zero to three year bond yields in the long-run. This increase is significant as the p-value is below the 0.05 percent value.

The three to five year bond yields results indicate a long-run coefficient of 4.31 and a positive coefficient for short-term interest rate of 0.49. This implies that a 1 percent increase in short-term interest rate will lead to a 49 percent increase of the three to five year bond yields. This increase is significant as the p-value is below the 0.05 percent value.

The five to ten year bond yields results indicate a long-run coefficient of 6.30 and a positive coefficient short-term interest rates of 0.25. This means that a 1 percent increase in short-term interest rate leads to a 25 percent increase in the five to ten year bond yields in the long-run. This increase is significant as the p-value is below the 0.05 percent value.

The 10 year and above bond yields result specify a long-run coefficient of 7.59 and positive coefficient short-term interest rates of 0.12. This means that a 1 percent increase in short-term interest rate volatility leads to a 12 percent increase in the 10 year and above bond yields in the long-run. This increase is not significant as the p-value is above the 0.05 percent value.

4.6.1.4 Error correction model (ECM) analysis

The variables indicate that there is a long-run relationship between short-term interest rates and short-term interest rate volatility and bond yields. However, in this section the ECM is used to correct the disequilibrium that may have occurred in a previous period, which is the disequilibrium in the short-run. Table 4.10 indicates the short-run relationship between short-term and bond yields. Moreover, the ECM also tests how fast the variables return to equilibrium. Furthermore, the results in this section denote the probability values of all the regression equations.

Table 4-10: Error correction

Error correction model				
Variable	Coefficient of the Error Correction term	Std. error	t-statistic	Prob.
0-3 years	-0.24	0.06	-3.92	0.00**
3-5 years	-0.17	0.04	-4.20	0.00**
5-10 years	-0.20	0.04	-4.63	0.00**
10 years +	-0.14	0.04	-3.88	0.00**

Note * and ** statistically significant at 10% and 5%, respectively.

Source: Compiled by author from IMF and SARB (2018)

The ECM results represented in Table 4.10 determine the short-run relationship between the variables. The requirement for this test is that the Coefficient of the Error Correction term must be negative and significant and this is the case for all the regression equations used in this study. Thus, in the short-run, the zero to three year bond yields are responsive to the adjustments towards equilibrium correcting approximately 24 percent of its previous period of disequilibrium, this means that it takes approximately 4.23 months (1/0.236239) for the zero to three year bond yields to fully adjust back to equilibrium, given shocks to the short-term interest rate.

In the short run, the three to five year bond yields are responsive to the adjustments towards equilibrium correcting approximately 17 percent of its previous period of disequilibrium; this means that short-term interest rates take approximately 5.6 months (1/0.168521) to fully adjust back to equilibrium.

In the short run, the five to ten year bond yields are responsive to the adjustments towards equilibrium correcting approximately by 20 percent of its previous period of disequilibrium, this means that short-term interest rates take 5.09 months (1/0.196372) to fully adjust back to equilibrium.

In the short run the 10 year and above bond yields are responsive to the adjustments towards equilibrium correcting approximately by 14 percent of its previous period of disequilibrium, this means that short-term interest rates take 7.23 months (1/0.138267) to fully adjust back to equilibrium.

Above all, there is a positive relationship between short-term interest rate and bond yields in the short-run. However, short-term interest rate is significant for the zero to three, three to five and five to ten year bond yields. The 10 year and above bond yields had an insignificant p-value since it is above 0.05 percent. The speed of adjustment is faster for the zero to three year bond yields.

4.7 DIAGNOSTIC TESTS

The diagnostic tests are used to check that the ARDL models used do not violate the regression assumptions of: (1) serially independent residuals, and (2) homoscedastic residuals. The ARDL assumption will be used to test the diagnostic test such as the test for serial correlation, heteroscedasticity and normality test. If none of the regression assumptions is violated, we may infer that the results are non-spurious.

This section will first explore the test for serial correlation and then proceed with the heteroscedasticity and normality test for the variables used in this study.

Table 4-11: Breusch-Godfrey LM test for serial correlation

	0-3 years	**3-5 years**	**5-10 years**	**10 years +**
Prob. chi-square	0.47	0.70	0.80	0.75

Source: Compiled by author from IMF and SARB (2018)

Autocorrelations do not exist if the error terms are uncorrelated with each other (Brooks, 2008:139). The null hypothesis is that there is no serial correlation. The results show the null hypothesis of no serial correlation is not rejected as none of the p-values is below the 5 percent level of significance.

Table 4-12: Breusch-Pagan: Heteroscedasticity

	0-3 years	3-5 years	5-10 years	10 years +
Prob. chi-square	0.46	0.11	0.10	0.29

Source: Compiled by author from IMF & SARB (2018)

The null hypothesis is that there is no heteroscedasticity, which means that the residuals should be homoscedastic. In this case the null hypothesis cannot be rejected, as all the p-values are above the 5 percent significant level. These results indicate that the ARDL model assumptions were not violated; therefore, the results may be used for inferences.

Table 4-13: Ramsey's RESET test

	0-3 years	3-5 years	5-10 years	10 years+
F-statistic	0.11	2.43	0.06	0.11
P-value	0.75	0.12	0.81	0.74

Source: Compiled by author from IMF & SARB (2018)

The Ramsey's RESET test is designed to identify whether the model suffers from omitted variable bias. If the variables are statically significant then there is likely an omitted variable bias. All variables presented in table 4.13 are more than 5% significant level. Therefore, do not have an omitted variable bias

4.8 COINTEGRATION TESTS: LONGSTAFF AND SCHWARTZ (1992) MODELS

Since the variables used in this study are I(0) and I(1), the study will proceed with the ARDL model. The same procedure will be followed for the Longstaff and Schwartz (1992) model as the CIR (1985) model to test for cointegration among the variables. The only difference with the Longstaff and Schwartz (1992) model is the inclusion of two independent variables (short-term interest rates and short-term interest rate volatility).

Should the results reflect the presence of cointegration through the bound test, the study will proceed with the long-run and short-run tests.

4.8.1 Longstaff and Schwartz (1992) two factor model

This section will first explore the ARDL model selection; this section will use two state variables, (1) the short-term interest rates and (2) the volatility of short-term interest rates.

4.8.1.1 ARDL model selection

Table 4-14: ARDL model selection

Bond yields	Model	Model selection criteria	Trend specification	R-Squared
0-3 years	ARDL (4.1.1)	AIC	Constant	0.95
3-5 years	ARDL (4.1.1)	AIC	Constant	0.94
5-10 years	ARDL (4.1.1)	AIC	Constant	0.86
10 years +	ARDL (4.1.1)	AIC	Constant	0.86

Compiled by author from IMF and SARB (2018)

The results illustrated in Table 4.14 indicate that the model lag selection was generated by the AIC criterion. The best selection for all the variables was ARDL (4, 1, 1). An adjustment had to be made for the independent variables (short-term interest rates and volatility of the short-term interest rates). A fixed lag selection was chosen for the dependent variables because the data consist of contemporaneous yields, therefore, one month lagged values were used for the independent variables. According to Ariff and Sarkar (2002:670), the argument is that if the issuer's decision is based on the interest levels and/or the volatility of interest rates, then these will be observed by the issuer. Therefore, in order for the issuer to make an investment decision, the issuer relies on the previous month's short-term interest rates, hence the lagged independent variables. The R-squared values for all variables are all above 80 percent, which indicates that the selected models are a good fit for the data.

4.8.1.2 Bound test

Table 4-15: Bound test

Bond yields	0-3 years		3-5 years		5-10 years		10 years +	
Bound test	I(0)	I(1)	I(0)	I(1)	I(0)	I(1)	I(0)	I(1)
5%	3.1	3.87	3.1	3.87	3.1	3.87	3.1	3.87
10%	2.63	3.35	2.63	3.35	2.63	3.35	2.63	3.35

F-stat	3.88	4.65	5.23	3.94

I(0) and I(1) denote lower and upper bound, respectively at a statistical significance level of 5% and 10%.

Source: Compiled by author from IMF and SARB (2018)

Table 4.15 demonstrates the bound test results for variables used in the study and from the results the bound test indicates that there is a long-run relationship among the dependent variables and independent variables. The F-stats were 3.88, 4.47, 5.24 and 3.94. The values for the bond yields exceed the upper bound critical at both 10 percent and 5 percent significance levels. The null hypothesis for the bounds test is that there is no cointegration relationship amongst the variables. The null hypothesis is rejected since the F-statistic exceeds the upper bound value for all the bond yields. This implies there is cointegration between short-term interest rates and volatility of short-term rates and bond yields.

Since the results from Table 4.15 indicate that there is a long-run relationship between the variables, the study then proceeds with the long-run relationship test. This test will determine whether there is a positive or negative and significant effect of short-term interest rate volatility and bond yields.

4.8.1.3 Long-run cointegration analysis

Table 4-16: Long-run analysis

0-3 years				
Variable	Coefficient	Std. error	t-static	Prob.
Short-term interest rate	0.71	0.14	5.00	0.00**
Volatility	1.67	3.34	0.50	0.62
C	2.36	0.85	2.76	0.01**
3-5 years				
Short-term interest rate	0.40	0.12	3.22	0.00**
Volatility	3.60	3.18	1.14	0.26
C	4.76	0.75	6.31	0.00**
5-10 years				
Short-term interest rate	0.22	0.10	2.16	0.03**
Volatility	0.95	2.65	0.36	0.72

C	6.43	0.62	10.30	0.00**
10 years +				
Short-term interest rate	0.08	0.13	0.61	0.54
Volatility	1.65	3.44	0.48	0.63
C	7.79	0.79	9.84	0.00**

Note * and ** statistically significant at 10% and 5% respectively.

Source: Compiled by author from IMF and SARB (2018)

Table 4.16 depicts the long-run cointegration analysis using the Longstaff and Schwartz (1992) model where there are two state variables (short-term interest rates and volatility of short-term interest rates). The zero to three year bond yields results indicate a long-run coefficient of 2.36 and a positive coefficient short-term interest rate volatility of 1.67 and positive coefficient of short-term rates of 0.71. This means that a 1 percent increase in short-term interest rate volatility leads to a 167 percent increase in the zero to three year yields in the long-run. However, this increase is insignificant as the p-value is above the 0.05 percent value. The results also show the effect of short-term interest rates on the zero to three year bond yields and indicate that a 1 percent in short-term interest rates will lead to a 71 percent in the zero to three year bond yields in the long run.

The three to five year bond yields results indicate a long-run coefficient of 4.76 and a positive coefficient for short-term interest rate volatility and short-term interest rates of 3.60 and 0.40, respectively. This implies that a 1 percent increase in short-term interest rate volatility will lead to a 360 percent increase of the three to five year bond yields. The short-term interest rates indicate that a 1 percent increase in short-term interest rates will lead to a 40 percent of the three to five year bond yields.

The five to ten year bond yields results indicate a long-run coefficient of 6.43 and a positive coefficient short-term interest rate volatility of 0.95 and positive coefficient of short-term interest rates of 0.22. This means that a 1 percent increase in short-term interest rate volatility leads to a 95 percent increase in the five to ten year bond yields in the long-run. The results also show the effect of short-term interest rates on the five to ten year bond yields and indicate that a 1 percent in short-term interest rates will lead to a 22 percent increase in the five to ten year bond yields in the long-run.

The 10 year and above bond yields results indicate a long-run coefficient of 7.79 and a positive coefficient short-term interest rate volatility of 1.65 and positive coefficient of short-term interest rates of 0.08. This means that a 1 percent increase in short-term interest rate volatility leads to a 165 percent increase in the 10 year and above bond yields in the long-run. The results also show the effect of short-term interest rates on the 10 year and above bond yields and indicate that a 1 percent in short-term interest rates will lead to an 80 percent increase in the 10 year and above bond yields in the long-run.

All these results indicate that there is a positive relationship between short-term interest rate volatility and the different bond yield maturities in South Africa. However, this positive relationship is not statistically significant at 5 percent and 10 percent levels. These results contradict results from the Longstaff and Schwartz (1992) study and Ariff and Sarkar (2002), which found that there is a negative and statistically significant relationship between short-term interest rates and bond yields in US. These results, however, are consistent with results from the study by Bhat and Fahad (2016) that found a statistically significant positive relationship between short-term interest rate bond yields in India. However, the results from this study were statistically insignificant.

In addition, the results indicate that short-term interest rate volatility affects three to five year bond yields more than other bond yields. Therefore, this means that in South Africa, bond yields are not sensitive to interest rate volatility.

4.8.1.4 Error correction model (ECM) analysis

The variables indicate that there is a long-run relationship between short-term interest rates and short-term interest rate volatility and bond yields. Table 4.17 specifies the short-run relationship between short-term interest rates, short-term interest volatility and bond yields. Moreover, the ECM also tests how fast the variables return back to equilibrium.

Table 4-17: Error correction

Error correction model				
Variable	Coefficient of the Error Correction Term	Std. error	t-statistic	Prob.
0-3 years	-0.22	0.05	-3.98	0.00**

3-5 years	-0.17	0.04	-4.36	0.00**
5-10 years	-0.19	0.04	-4.62	0.00**
10 years +	-0.14	0.03	-3.89	0.00**

Note * and ** statistically significant at 10% and 5% respectively.

Source: Compiled by author from IMF and SARB (2018)

The ECM results represented in Table 4.17 determine the short-run relationship between the variables. The requirement for this test is that the Coefficient of the Error Correction Term must be negative and significant. Thus, in the short-run the zero to three year bond yields are responsive to the adjustments towards equilibrium correcting approximately by 22 percent of its previous period of disequilibrium. This means that it takes an average of 4.57 months for the zero to three year bond yields to fully adjust back to equilibrium, given shocks to the short-term interest rate and short-term interest rate volatility. The short-run coefficient lag of the zero to three years bond yields is significant at 0.05 significant value, which indicates a strong short-run relationship.

In the short run, the three to five year bond yields is responsive to the adjustments towards equilibrium, correcting approximately by 17 percent of its previous period of disequilibrium, this means that short-term interest rates and short-term interest rate volatility take 6.02 months (1/0.16598) to fully adjust to equilibrium.

In the short-run, the five to ten year bond yields is responsive to the adjustments towards equilibrium correcting approximately by 19 percent of its previous period of disequilibrium, this means that short-term interest rates and short-term interest rate volatility take 5.19 months (1/0.19277) to fully adjust to the equilibrium of the five to ten year bond yields.

The 10 year and above bond yields is responsive to the adjustments towards equilibrium, correcting approximately by 14 percent of its previous period of disequilibrium, this means that short-term interest rates and short-term interest rate volatility takes 7.35 months (1/0.136064) to fully adjust to equilibrium.

Above all, there is a positive relationship between short-term interest rate volatility and bond yields in the short run. However, short-term interest rate volatility is positive and insignificant for all bond yields, while the short-term interest rates have a positive and significant relationship with the bond yields just like in the long-run effect. The speed of adjustment is faster for the zero to three year bond yields. The finding suggest that short-term interest rate volatility does not have a statistically significant effect on bond yields in South Africa in the short run, however, short-term interest rates do statistically affect bond yields and move in the same direction in the short run.

4.9 DIAGNOSTIC TESTS

This section will first explore the test for serial correlation and then proceed with the heteroscedasticity test for the variables used in this study.

Table 4-18: Breusch-Godfrey LM test for serial correlation

	0-3 years	3-5 years	5-10 years	10 years +
Prob. chi-square	0.30	0.73	0.76	0.72

Source: Compiled by author from IMF and SARB (2018)

Autocorrelations do not exist if the error terms are uncorrelated with each other (Brooks, 2008:139). The null hypothesis is that there is no serial correlation. The results show the null hypothesis of no serial correlation is not rejected as none of the p-values are below the 5 percent level of significance.

Table 4-19: Breusch-Pagan: Heteroscedasticity

	0-3 years	3-5 years	5-10 years	10 years +
Prob. chi-square	0.59	0.14	0.61	0.40

Source: Compiled by Author from IMF and SARB (2018)

The null hypothesis is that there is no heteroscedasticity, meaning that the residuals should be homoscedastic. In this case, the null hypothesis cannot be rejected, as all the p-values are above the 5 percent significant level. These results indicate that the ARDL model assumptions were not violated; therefore, the results may be used for inferences.

Table 4-20: Ramsey's RESET test

	0-3 years	3-5 years	5-10 years	10 years+
F-statistic	0.03	1.94	0.16	0.22
P-value	0.87	0.17	0.69	0.64

Source: Compiled by Author from IMF and SARB (2018)

The Ramsey's RESET test is designed to identify whether the model suffers from omitted variable bias. If the variables are statically significant then there is likely an omitted variable bias. All variables presented in table 4.20 are more than 5% significant level. Therefore, do not have an omitted variable bias.

4.10 DISCUSSION OF RESULTS

Based on the graphical investigation, short-term interest rates and bond yields move together over time. This implies that there is a positive relationship between short-term interest rates and bond yields in South Africa. The graphs also explain the manner in which short-term interest rates behave before, during and after the 2007-2009 financial crisis. Before and after the financial crisis the yield curve was normal, as short-term interest rates were lower than the bond yields. However, during the 2007-2009 crisis the yield curve was inverted, which indicated recessionary pressure in the economy. These results do match with the yield curve theory, as the different shapes of the yield curve are inspired by what happens in the economy. The graphs also indicated that short-term interest rates and bond yields move together over time. These results are consistent with studies by MacDonald and Speight (1988), Hall *et al*. (1992), and Mandeno and Giles (1995) that found that the interest rate series move together. These results are also consistent with the expectation hypothesis theory, where an increase in short-term interest rates is often followed by a small increase in the long-term rate (Estrella & Trubin, 2006).

The yield spreads, which is the difference between the short-term interest rates and bond yields were low (negative) during the 2007-2009 financial crisis, which was an indication that there was a recessionary pressure in the South African economy. After the 2007-2009 financial crisis the yield spreads were high (positive), which indicated that the economy recovered from the recessionary pressure.

During the 2007-2009 financial crisis, the SARB increased the repo rate in a number of occasions as a result of the inflation outlook. The increase in the repo rate, consequently, resulted in an increase in the 91 day T-bill that further influenced the direction of the bond yields. These results further explained that the expectation hypothesis theory holds in South Africa, as long-term interest rates are an average of future expected short-term interest rates. The descriptive statistics indicated that the bond yields had the highest maximum value as expected, because bond yields are normally higher than short-term interest rates. In addition, short-term interest rates had the lowest minimum values, which were also expected as short-term interest rates have lower values than bond yields under normal circumstances.

The unit root results indicated the presence of a unit root for some of the variables. Hall *et al*. (1992) opine that short-term interest rates are generally a I(1) process. Which was the case with the T-bill data used in this study. Since some of the variables were integrated at I(0) and others at I(0) the ARDL model was employed as it caters for variables that are integrated at order I(0) and I(1) or a mixture of both.

The ARDL first employed using the CIR (1985) model where there is only one state variable (short-term interest rates). The results from the long-run cointegration analysis suggest that there is a positive relationship between short-term interest rates and bond yields in South Africa. In addition, the results from the zero to three, three to five and five to ten year bond yields were positive and significant. This meant that short-term interest rates had an effect on these bond yields. However, the effect of short-term interest rates on the 10 year and above bond yields was statistically insignificant. This effect was stronger on the zero to three year bond yields as compared to the other bond yield maturities. This means that the zero to three year bond yields are more sensitive to changes in short-term interest rates. While the 10 year and above bond yields were less sensitive to changes in short-term interest rates. According to the duration theory, since long-term bonds have a longer duration than short-term bonds, changes in interest rates should affect the long-term bond more than the short-term bonds. However, the results from this study are contradictory to the duration theory as medium-term bond yields were more sensitive to changes in interest rates in the long run.

The cointegration results of this study are consistent with studies by Iyke (2015) and Mposelwa (2015) who found that there is a cointegration relationship between short- and long-term interest rates in South Africa. The error correction model investigated the speed of adjustment it will take the variables to return to their equilibrium. The results indicate that the short-term interest rates are able to influence the bond yields in the short run, which allows the predictions of the bond yields. Both the short-term interest rates and bond yields adjust to restore to equilibrium, however after different periods. The 10 year and above bond yields adjust back to equilibrium faster than the other bond yields in the short run.

The ARDL model was also employed with the use of the Longstaff and Schwartz (1992) model where two state variables were used (short-term interest rates and short-term interest rate volatility. The results from the cointegration analysis revealed that the short-term interest rate volatility has a statistically insignificant positive relationship with bond yields in South Africa. This positive effect is consistent with Bhat and Fahad (2016) who found a statistically significant positive relationship between interest rate volatility and bond yields in India, although this study found an insignificant relationship. However, Longstaff and Schwartz (1992) and Ariff and Sarkar (2002) found a statistically significant negative relationship between interest rate volatility and bond yields using US data. Mposelwa (2015) opines that expectation hypothesis holds in India and South Africa, but the results from US were inconclusive. The differing results of the studies may be the case of whether the expectation hypothesis theory holds in those particular countries and the monetary policy decision. In addition, India and South Africa are developing countries while US is a developed country with a well liquidised bond market compared to developing countries. The error correction model was used to investigate the short-run effect of the variables. All the bond yields reverted to the equilibrium in the short run.

Overall the results of this study found that short-term interest rate volatility has no effect on the bond yields for the observed period, this relationship bears little relation to the convexity and duration of the bond, just like the Longstaff and Schwartz (1993) study where medium-term (three to five year) bonds were more sensitive to the effects of interest rate volatility.

Since investors are concerned with the effect of changes in interest rate volatility on bond yields, in South Africa the effect is not significant for the observed period, however, short-term interest rates do have a significant effect on bond yields. As short-term interest rates increase, bond yields also follow the same direction. Accordingly, in South Africa the expectation hypothesis holds and thus to make informed investment decision, fixed income mangers may consider the expectation hypothesis theory. This means that by observing the expectation hypothesis it may be possible to predict the future shape of the yield curve. For central banks, the analysis of the short-term interest rates and the volatility of the short-term interest rates will aid them to make more informed decisions as their decisions may have a positive or negative effect on bond yields. From the graphical analysis, it was noted that short-term interest rate movements predicted the shape of the yield curve, which, in addition, explained the economic performance.

Moreover, the 2007-2009 financial crisis did not have a rapid effect on South African interest rates as compared to more developed economies. The short-term interest rate volatility is possibly due to low demand for exports and slow global economic growth. Holmes *et al*. (2011)

suggest that there is a strong influence of global monetary policy on local monetary policy as well as the influence of the term structure of interest rates.

4.11 CHAPTER SUMMARY

This chapter focused on the testing and evaluating of data used to achieve the objectives set in this study. The chapter first explored the different graphs in order to have the first view of the direction of short-term interest rates and bond yields and how these rates reacted during the periods of shocks in the economy.

From the graphical analysis it was clear that short-term interest rates and bond yields move in the same direction and during the financial crisis, both short-term interest rates and bond yields had an upward movement. In normal cases, yield curves are normal, where short-term interest rates are lower than long-term bond yields. This was the case with South Africa as well, however, during the financial crisis South Africa experienced an inverted yield curve, where the short-term interest rates were higher than the long-term bond yields. Interestingly, the zero to three year bond yields were higher than the short-term interest rates and all the other bond yields used in this study. In addition, the section focused on the co-movements of the bond yields and from the analyses it was determined that bond yields all move in the same direction, meaning that when short-term interest rates increase, all the bond yields follow in the same direction.

The chapter also explored the descriptive statistics of the variables used in the study. From the analysis, it was determined that the mean values for all the rates, excluding interest rate volatility ranged from 7.00 to 8.47; these rates reflect the average value for the variables. The rates also indicate that, on average, 10 year and above bond yields have the highest bond rates.

The chapter explored the different unit root tests in order to find out the order of integration of the variables. The variables were integrated at different orders I(0) and I(1). This meant that there was a mixture of order of integration among the variables. The chapter then proceeded with the ARDL model, which is the most appropriate model to estimate variables with different orders of integration.

The ARDL estimation method was applied to the CIR (1985), and Longstaff and Schwartz (1992) models. From the ARDL model results, all the tests done for different bond yields, short-term interest rate and interest rate volatility, there was a long-run relationship between the variables, as the F-statistic was higher than the lower and the upper bound of the test. The results from both the CIR (1985), and Longstaff and Schwartz (1992) models revealed a positive relationship between the variables used in this study.

Furthermore, the chapter proceeded to conduct the short-run test for all the variables. The results from the test are similar to the long-run test results. However, the 10 year and above bond yields demonstrated statistically insignificant values for both the short-term interest rates and short-term interest rate volatility. These results also investigated the speed of adjustment it would take the variables to return to equilibrium. The results revealed that the 10 year and above bond yields took the shortest period of approximately 0.73 months to return to equilibrium. In addition, the diagnostic tests conducted suggest that the ARDL model assumptions were not violated.

CHAPTER 5: SUMMARY, CONCLUSION AND RECOMMENDATIONS

5.1 INTRODUCTION

Investors are aware of the risk that pertains to interest rate uncertainty. This uncertainly differs with the level of interest rates, this means that the higher the interest rates move the higher the interest rate volatility. Further to this, the uncertainty in interest rates affects the attractiveness of bond yields. It is therefore not surprising that interest rate volatility plays a central role in the pricing of bonds. This role has become significant as witnessed in several academic studies (Longstaff & Schwartz, 1992; 1993, Anderson & Lund, 1997; Ball, 1999; Olweny, 2011). Interest rate volatility is not only limited to its significant effect in bond pricing, but also the shape of the yield curve. The shape of the yield curve is used as a tool to predict economic performance (i.e. growth and slowdown). In periods of slowdown in the economy, there would be an inverted yield curve and during economic growth, the yield curve would reflect a normal yield curve.

During the 2007-2009, financial crisis, interest rates were more volatile as compared to non-financial crises periods. In light of this event, the primary objective of this study was to examine the relationship between short-term interest rate volatility and bond yields in South Africa. Interest rates change as a result of economic events which may be global (i.e. financial crisis) or local (i.e. change in monetary policy). In addition, the uncertainty about changes in interest rates affect investment decisions and the implication thereof may differ across countries.

Interest rate volatility affects investment decisions made by economists and investors on whether to save or invest as well as the type of investment they should make (i.e. bonds or stocks). Thus, understanding the implication of interest rate volatility aids economists and investors to make more accurate predictions about the shape of the yield curve in the future.

5.2 SUMMARY

Chapter 1 provided the background of the study, which included the problem statement, research objectives, research design and method, ethical consideration, as well as the outline of the chapters presented in this study.

Chapter 2 presented the definitions and conceptualisation of interest rates and bond yields. This chapter aided to achieve the theoretical objectives set in this study. The chapter first explained the definition and concept of interest rates, since interest rates are the driver of the monetary policy and, in consequence, affect the capital market. This section also viewed the co-movement of different short-term interest rates in South Africa. This illustration included the repo rate, three-month Jibar rate, six-month Jibar rate and the 91-day T-bill. The graphical illustration of these variables reflected that the different short-term interest rates move together over time in South Africa. Thereafter, the definition of short-term interest rate volatility was provided and the manner in which short-term interest rate volatility affects bond yields. The chapter further provided the definition and conceptualisation of bonds. For the nature and scope of this study, the government bonds were explained in depth. Bonds were described by the use of different valuation methods such as duration, Macaulay duration, modified duration and convexity.

The relationship between the two variables was explained by the use of the term structure of interest rates. The term structure of interest rates was defined as the relationship between bond yields with different maturities and similar characteristics such as zero coupon bonds. The relationship is better illustrated by the use of the yield curve. The four shapes of the yield curve were analysed:

- the normal yield curve where short-term interest rates are lower than long-term interest rates
- the inverted shape of the yield curve where short-term interest rates are higher than long-term interest rates
- the flat yield curve where short- and long-term interest rates are equal
- the humped yield curve where intermediate interest rates yield higher interest rates than short-term and long-term interest rates.

The term structure of interest rates was further explained by the four theories of the term structure of interest rates, namely expectation hypothesis, liquidity premium, market segmentation and preferred habitat theory. These theories explain the reaction of investors to the increase or decrease of short-term interest rates, which, consequently, affect their investment decisions.

Several models have been used in an attempt to explain the relationship between short-term interest rate volatility and bond yields. However, this study focused on the CIR (1985) model and the Longstaff and Schwartz (1992) model. These two models, the one-factor model and the two-factor model, explain the relationship between short-term interest rates, interest rate volatility and the term structure of interest rates. The two-factor model includes, in addition to the one-factor model, the volatility of the short-term interest rate.

Finally, previous empirical studies (Longstaff and Schwartz, 1992; Ariff and Sarkar, 2002 and Olweny, 2011), which investigated the relationship between interest rate volatility and the term structure of interest rates, were reviewed and reported on. The theories of the term structure have often intrigued both investment analysts and theorists; however, the results from previous studies are inconclusive, creating an opportunity for further research.

In the South African context, the relationship between short-term interest rate volatility and the term structure of interest rates has barely been analysed. After reviewing what other studies have found, there is still a gap in the South African literature to find out whether short-term interest rate volatility affects bond yields. This study aims to empirically test whether the CIR (1985) model and Longstaff and Schwartz (1992) model will fit the South African data well.

Chapter 3 provided an overview of the research design and methodology employed in this study. The justification of the use of the ARDL model was explained. Further to this, the chapter discussed the process when an ARDL model is employed. The sample period was selected based on the observation of the period before, during and after the 2007-2009 financial crisis.

Chapter 4 presented the research findings and results. Graphs were used to explain the relationship between the variables. The results from the graphs showed that there is a positive relationship between the short-term interest rate, short-term interest rate volatility and bond yields. Further, a number of statistics (i.e. descriptive statistics, correlations) were conducted to explain the relationships between the variables. The graphical illustrations, likewise, showed that during the 2007-2009 financial crisis the yield curve was inverted, as short-term interest rates were higher than bond yields, these results correspond with the yield curve theory. The unit roots test reflected that the variables used are not integrated at the same order, which results in a mixture of integration of variables used. The unit root tests were a strong justification of the use of the ARDL model with the use of South African data. The ARDL model results presented contrasting results from previous studies, as the results showed a positive and insignificant relationship between short-term interest rate volatility and bond yields. Further to this, the effect was stronger for the medium-term (three to five year) bond yields. However, these results were consistent with the study by Bhat and Fahad (2016) who found a positive relationship between short-term interest rate and bond yields in India. Ariff and Sarkar (2002) claim that the significance of this relationship depends on a number of factors; these factors include the maturity of the yields, the size of the government bond relative to corporate bonds in the country and liquidity of the bond market. The results from previous studies reveal that there is still an inconclusive relationship between interest rate volatility and bond yields.

5.3 REALISATION OF THE STUDY OBJECTIVES

The section provides an overview of the objectives set in this study and the manner in which they were achieved. The primary objective was to examine the relationship between short-term interest rate volatility and bond yields in South Africa, which was achieved through the methods used in this study.

5.3.1 Theoretical objectives

In order to achieve the study objectives, there were five theoretical objectives outlined:

5.3.1.1 Provide the background and history of interest rates and bonds yields in South Africa.

This objective was accomplished by using definitions and theoretical concepts. In addition, an in-depth explanation was used to differentiate between short-term and long-term interest rates.

5.3.1.2 Describe the relationship between short-term and long-term rates.

This objective was described through the use of the yield curve and the four shapes of the yield curve (upward, downward, flat and humped).

5.3.1.3 Review the relationship of interest rates and bonds by use of the term structure of interest rates and related theories.

The term structure of interest rates explained the relationship of the bond yields with varying maturities. The term structure of interest rates was further explained by the four theories (expectation hypothesis, liquidity premium, market segmentation and preferred habitat-theory) of the term structure of interest rates. The study of the term structure of interest rates confirmed that the expectation hypothesis holds in South Africa, as short-term interest rates affect the direction of the long-term interest rates; this was confirmed by various studies (Bonga-Bonga, 2009; Dube & Zhou, 2013; Mposelwa, 2016).

5.3.1.4 Provide a theoretical framework of the relationship between interest rates and bond yields.

This objective was achieved through the discussion of the CIR (1985) and Longstaff and Schwartz (1992) which explain the manner in which short-term interest rates, short-term interest rate volatility and bond yields are related. These equilibrium models are able to develop a mean reversion process for short-term interest rates, where there is consideration made about the type of financial market, risk in investment preferences, investment opportunities, changes in technology and uncertainty of investment in the economy. The difference between the two models is that one considers only interest rates while the other considers both short-term interest rates and the volatility of the short-term interest rates. Both these models have been used extensively in the literature (Gibsson & Ramaswamy, 1993; Olweny, 2011; Maranga et al., 2018) in an attempt to understand the relationship between interest rates and bond yields. Investors and economists assume that the shape of the yield curve replicates future expectations of interest rates and monetary policy conditions. Thus, modelling the term structure of interest rates assists in the interpretation of the behaviour of interest rates. The final theoretical objective was to review and report on the empirical studies on the relationship between interest rate volatility and bond yields.

5.3.1.5 Review and report on the findings from previous empirical studies on the relationship between interest rates and bond yields.

This theoretical objective was achieved by a review on previous studies, the study confirmed that the expectation hypothesis holds in South Africa, as short-term interest rates affect the direction of the long-term interest rates; this was confirmed by various studies (Bonga-Bonga, 2009; Dube & Zhou, 2013; Mposelwa, 2016).

5.3.2 Empirical objectives

In order to test the study objectives empirically, five empirical objectives were employed:

5.3.2.1 Evaluate the movement between interest rate volatility and bond yields in South Africa.

This empirical objective was achieved through graphical illustrations, descriptive statistics and correlation analysis, which revealed a positive relationship between interest rate volatility and bond yields.

5.3.2.2 Evaluate and interpret graphically the effect of short-term interest rate volatility on bond yields before, during and after the 2007-2009 financial crisis.

The graphical illustration aided in evaluating the movement of interest rates, interest rate volatility and bond yields in South Africa, before, during and after the financial crisis. The results thereof reflected that during the 2007-2009 financial crisis, interest rates and bond yields fluctuated as a sign of the economic shock and uncertainty. In consequence, interest rate volatility was higher during the financial crisis than periods before and after the 2007-2009 financial crisis.

5.3.2.3 Evaluate whether there is a positive or negative relationship between interest rate volatility and bond yields.

This empirical objective was achieved through graphical illustrations and the ARDL model. The results revealed that there is a positive relationship between interest rate volatility and bond yields. The results further reflected an insignificant relationship between interest rate volatility and bond yields.

5.3.2.4 Determine whether the effect of interest rate volatility is stronger on short-, medium- or long-term bond yields in South Africa.

The ARDL model was employed to determine whether the relationship between interest rate volatility and bond yields was more significant on either the short-, medium- or long-term bond yields in South Africa. The results of the ARDL using the CIR (1985) model revealed that there was a positive and significant long-run and short-run relationship between short-term interest rates and bond yields in South Africa. While the results from the Longstaff and Schwartz (1992) model revealed that, there is a positive and insignificant relationship between short-term interest rate volatility and bond yields in South Africa. In addition, the effect was more on the medium-term bond yields as compared to the short- and long-term bond yields.

5.3.2.5 Determine whether short-term interest volatility has a short- or long-run effect on bond yields.

The empirical objective was achieved through the use of the long-run bound test and the ECM test. The results revealed that there is both the short-and long-run effect of short-term interest rate volatility on bond yields.

5.4 CONCLUSION

The results of the study discovered that short-term interest rates have a significant influence on bond yields, which reveal that the expectation hypothesis theory holds in South Africa. This was also discovered by a study by Mposelwa (2016), which revealed that the expectation hypothesis holds in developing countries such as South Africa and India and the results from developed countries (i.e. US and UK) were inconclusive. Therefore, the inconclusive results from these countries may be because of the monetary policy decision and the type of term structure theory that holds in those particular countries. In addition, a number of studies found that it is easy to reject the expectation hypothesis in US. Kugler (1990) also rejected the expectation hypothesis in US.

These results also reveal that monetary policy has been more effective during the 2007-2009 financial crisis. Thus, when the monetary policy decides to increase or decrease interest rates, they have to consider the implications of changes in interest rates, which further affect the expectation of interest rates in the future. This effect further affects investment decisions by fixed income portfolio investors.

5.5 LIMITATIONS

- This study only focused on one country, namely South Africa. Including more countries, especially trading partners of South Africa could provide further insights.
- The sample span only included the 2007-2009 financial crisis episode. A longer sample period and more shocks/crises could provide further insights.
- The study only examined the one-factor and two-factor models, the three-factor models were not examined, which could provide different results.

5.6 RECOMMENDATIONS

Future studies may focus on the effect of interest rate volatility on bond yields in different countries. The problem of interest rate volatility starts with the monetary policy decision to either decrease or increase interest rates given certain economic circumstances. Countries may face the same shocks, such as the financial crisis, but the effect may be different in each country, which leads to different policy decisions.

Future studies may also want to consider the use of other term structure models to test for the effect of interest rate volatility and bond yields. This study used the one-factor and two-factor models. There are also three-factor models that may be employed when analysing the term structure of interest rates.

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