OPPORTUNITIES FOR THE DEVELOPMENT OF CRITICAL THINKING SKILLS IN THE MATHEMATICS CLASSROOM

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DECLARATION

I, ANNELIZE DEUCHAR, solemnly declare that this work is original and the result of my own labour. It has never, on any previous occasion, been presented in part or whole to any institution or Board for the award of any Degree.

I further declare that all information used and quoted has been duly acknowledged by complete reference.

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TO WHOM IT MAY CONCERN

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DEDICATION

This thesis is dedicated to my husband, Oswald Deuchar, and my lovely son, Aston Deuchar, who offered me unconditional love and support throughout the course of this study. I also dedicate this work to my parents, Buks and Lettie van Zyl, who have supported me all the way since the beginning of my studies.

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SUMMARY

The nurturing of critical thinking skills is one of the cornerstones of Outcomes-Based Education (OBE). This study investigated to what extent teachers provide opportunities for the development of critical thinking skills in Grade 8 in Mathematics classrooms.

A literature study was undertaken to highlight the importance and nature of the development of critical thinking skills in the Mathematics classroom, and to establish how critical thinking could be nurtured during the teaching, learning and assessment of Mathematics. Various teaching methods and assessment strategies, types of learning material, a variety of classroom activities and how to create a classroom conducive to the development of critical thinking skills were explored. The literature review provided the framework to design a questionnaire that was utilized to obtain the perceptions of Grade 8 Mathematics teachers and learners regarding the opportunities provided for the development of critical thinking skills in Mathematics classrooms.

By means of quantitative, non-experimental descriptive research, the self-constructed, closed-ended questionnaire was administered to a convenient sample of a purposively selected group of Mathematics teachers (n = 92) and learners (n = 204) in the Ekurhuleni District of Gauteng, South Africa.

The triangulation of learner and teacher data revealed that teachers do have an understanding of the importance of critical thinking in the Mathematics classroom, but that their understanding is not always fully translated into practical opportunities for the development of critical thinking skills. It was revealed that teachers do make use of questioning and allow learners to communicate during problem-solving, which are important strategies for the development of critical thinking. However, it was evident that teachers appear to be inhibiting the development of critical thinking skills by relying heavily on the use of textbooks and transmission of knowledge during teaching, and seem not to acknowledge the merits of cooperative learning and real life experiences during the teaching and learning of Mathematics.

The study is concluded with recommendations on how to nurture and improve critical thinking in the Mathematics classroom.

Key words: critical thinking, critical thinking in Mathematics, classroom climate, higher-order thinking, teaching methods, assessment approaches

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CHAPTER ONE

INTRODUCTION AND STATEMENT OF THE PROBLEM

1.1 INTRODUCTION

One of the challenges of education transformation in South Africa is to ensure that South Africans have the knowledge, values and skills required to build democracy, establish a system of lifelong learning and promote social development and growth in the 21st century (Odora Hoppers, 2001:1). To fulfil the need for this kind of transformation in education in South Africa, the National Department of Education identified Critical Outcomes that would assist learners in achieving the above-mentioned ideals (South African Qualifications Authority, 1997:7). One prominent element that emanates from the Critical Outcomes is an emphasis on the development of critical thinking skills and an emphasis that learners should no longer be treated "...as empty vessels that have to be filled with knowledge..." (Department of Education, 1997:30). This implies that teachers have to, among other things, base their teaching on constructivist principles that will provide learners with the opportunity to develop as thinkers (Green, 2006:310-327). Specifically with regard to the teaching of Mathematics, the development of critical thinking skills is important. The unique features of learning and teaching Mathematics include, among other things, problem-solving and analysing patterns and relationships, which require a critical awareness of mathematical relationships (Department of Education, 2002:5).

The *Third International Mathematics and Science Study* (TIMSS) as well as the *Third International Mathematics and Science Study – Repeat* (TIMSS-R) indicated shocking results for the achievement of South African learners in subjects that rely on higher-order thinking skills such as Mathematics and Natural Science. In a comparison of the results for more than forty countries, it was revealed that South African learners achieved the poorest results (Maree, Louw & Millard, 2004:25-34; Howie, 2007). Furthermore, research conducted by Lombard and Grosser (2004:212) and Brodie (2007:3) indicate that teachers do not possess adequate skills and knowledge to nurture the development of critical thinking skills among learners, or understand the importance of this (Bataineh & Zghoul, 2006:33).

1.2 PROBLEM STATEMENT

Against the background of the above-mentioned discussion that highlights the importance of learners having to develop the skill to think critically, and the fact that there appears to be problems with the development of higher-order thinking skills of learners in the Mathematics classroom, the problem that this study wished to address focused on determining what opportunities teachers create for the development of critical thinking skills in the Mathematics classroom.

1.3 LITERATURE REVIEW

The Department of Education describes Mathematics as a human activity practised by all cultures. According to the Department of Education, Mathematics is developed and contested over a period of time through both language and symbols after the observation of a variety of patterns and formulation of theories by the use of thorough logical thinking (Department of Education, 2004:2). According to Ernest (2002), by implementing critical thinking in the Mathematics classroom, learners will be able to make more use of their Mathematical knowledge and skills in their daily lives. It will also broaden their perspectives on Mathematics and they will appreciate Mathematics more in the contemporary world.

The nurturing of critical thinking in Mathematics will enable learners to become more competent in the use of Mathematical process skills in the context of Numbers and Algebra, Geometry (including the techniques of trigonometry and transformational geometry), Measurement, Data Handling and Probability. Critical thinking skills will enable learners to understand the uses of Mathematics in society and to interpret, evaluate and critique the Mathematics used in social, commercial and political systems (Ernest, 2002). In order to develop critical thinking skills in the classroom, teachers need to create opportunities for voicing conflicting opinions and views, allow learners to explore and solve problems by themselves. Learners, who are able to think critically in Mathematics, find it easier to overcome barriers like higher education and employment and thereby increase economic self-determination (Ernest, 2002).

Specifically with regard to the development of geometric thinking, the development of critical thinking skills is important. According to Van de Walle (2001:309), this importance is highlighted by the Van Hiele theory. The Van Hiele theory is one of the most influential factors in the geometry curriculum. Van Hiele described five reasoning levels that learners need to acquire, but later indicated that it will be easier to use three levels for teaching mathematics at school level. The Van Hiele model presents a hierarchy for understanding spatial ideas. The levels describe the types of thinking in a geometric context and focus on the types of ideas that are thought about in a geometric context. A very noticeable difference between the different levels is the objects of thought – what we are able to think about geometrically. Attention must be paid to the development of skills needed to master each of these levels. For the visual level learners should be able to recognize, draw, manipulate and interpret figures. The descriptive level expects of learners to be able to discover and describe properties and their relationships, as well as recognize relationships in figures. For the theoretical level, reasoning skills have to be well developed.

Several teaching perspectives are discernible in South African Mathematics syllabi, textbooks and education programmes. A perspective that has left its mark on the Mathematics curriculum in South Africa and has had an influence in shaping curriculum development is Behaviourism (Vithal & Volmink, 2005). Behaviourism in the Mathematics classroom influences teachers to specify objectives and to measure observable behaviour. Teachers show a behaviouristic approach to teaching when they plan their lessons thoroughly, down to every last detail as well as the responses expected from the learners. Use is made of tests or worksheets to see whether the specific skills, knowledge and behaviours have been learned and understood.

With the implementation of the new Mathematics curriculum since 1994, a paradigm shift has occurred in the Mathematics classroom in the direction of

Constructivism. The constructivist perspective that is also known as the problem-solving approach came to South Africa as a new orthodoxy (Vithal & Volmink, 2005). In classrooms where teachers make use of the constructivist approach to teaching and learning, learners are more involved and active during problem-solving and communicate more openly and freely with the teacher. They also interact intellectually with both subject content and with one another. Learners have to construct their own knowledge and understanding and should critically analyse arguments and generate insight into interpretations in ways that display critical thinking (Churach & Fisher, 2001:223). By constructing meaning, and understanding towards the problem, learners should apply personal judgements and interpretations, recognizing that there is an element of uncertainly and self-regulation in critical thinking. According to the constructivist perspective, the teacher is seen as a facilitator who directs the learners to discover certain knowledge on their own and helps them to identify logical flaws, methodological flaws, and unwarranted inferences in arguments presented to them (Churach & Fisher, 2001:223).

Based on the preceding discussion, it appears as if the constructivist perspective seems to be ideal in the development of critical thinking in the Mathematics classrooms in South Africa. Therefore this study will be approached from a constructivist perspective *(cf.* 2.5).

It is well known that Mathematics has an image of being cold, abstract and difficult. What many learners and teachers do not always realize, is the importance of Mathematics in our daily lives. Learners often ask teachers why they should do Mathematics, if they don't need it to be successful in life. The Association for Mathematics Education of South Africa (AMESA), works hard to change learners' attitudes towards Mathematics, and tries to make learners realize that Mathematics in South Africa is socially good and that it should be perceived to be a source of status and power. Mathematics should be regarded as the key to higher education and better paid jobs (Setati, 2002).

The first step that needs to be taken to improve the learners' attitude towards Mathematics is to give learners the message that Mathematics is doable and fun. In support of the constructivist principles to teaching and learning, learners should be given the opportunity to solve problems, interact with the study material, think creatively and critically and be involved in the Mathematics classroom (Setati, 2002). By developing critical thinking skills in the Mathematics classroom, learners will have a better understanding of the knowledge presented to them and be able to solve problems more successfully (Setati, 2002). According to Oleinik (2002) and Skovsmose and Valero (2002:385), problem-solving is a key issue in the development of critical thinking in Mathematics. Problem-solving involves the ability to explore, think through an issue, reason logically and solve routine as well as non routine problems.

Critical thinking has two main features, firstly it refers to reasonable thinking that leads to deductions and the making of sound decisions that are justified and supported by acceptable proofs. Secondly, critical thinking also refers to reflective thinking that shows a complete awareness of the thinking steps that lead to the making of deductions and conclusions. Both of these critical thinking aspects are relevant to all the steps of planning a Mathematics lesson (Pang, 2003:34).

In South Africa, we need to change people's perceptions and learners' performance in Mathematics. To do so, we need high quality Mathematics teachers and high quality Mathematics teaching in every classroom. High quality Mathematics teachers are well-motivated, professional, highly qualified in Mathematics, helpful and care for their learners (Setati, 2004). They consider critical thinking as an aspect of their teaching and plan lessons in a clear, focused and balanced way (Innabi, 2003). One of the problems concerning critical thinking in the South African Mathematics classroom is that it appears that teachers do not have enough knowledge about the nature of critical thinking in Mathematics or the teaching strategies needed to stimulate critical thinking. They are not always able to understand the methods and different approaches learners use to solve problems critically (Mc Peck, 1990, Sonn, 2000:257-265; Schraw & Olafson, 2003:178-239).

Paul, Binker, Jensen and Kreklau (1990:91) assert that Mathematics teachers should be authentic individuals who strive to improve their practice through the use of critical thought. Teachers should feel the need to improve critical thinking in their classroom and analyse their own thinking processes and classroom practices and provide reasons for what they do. They should be open-minded and encourage learners to follow their own thinking and not simply repeat what the teacher has said. It is important that they change their own positions when the evidence warrants, being willing to admit a mistake and consistently provide opportunities for learners to select activities and assignments from a range of appropriate choices. They should also allow for learner participation in rule setting and decision-making related to all aspects of learning, including assessment and evaluation (Ferrando, 2001).

High quality Mathematics teachers will strive to improve critical thinking in their classrooms and involve the will and desire of the learners to go beyond what is given and to make an attempt to understand the self and question the motives of others (Paul *et al.*, 1990:90). Teachers should not only tell learners that their answers are incorrect, but also make sure that the learners understand why their answers are incorrect so that they do not make the same mistakes in the future.

According to Setati (2004), a teaching method that still seems to be active in the Mathematics classroom in South Africa is the setting of meaningless rules and procedures that the learners have to memorize and use whenever needed. It is difficult to develop critical thinking skills in these classrooms where a behaviouristic approach is still the dominant approach to teaching and learning.

In addition to this, Gough (1991:1) asserts that not much attention is given to learner involvement and participation in the Mathematics classrooms. Learners should feel free to explore and express opinions, to examine alternative positions on interesting topics, and to justify beliefs about what is true and good, while participating in a classroom conversation. It should be acknowledged that there is no single correct way to understand and evaluate arguments and that all attempts are not necessarily successful (Mayer & Goodchild, 1990:4).

One of the most recognized problems in the Mathematics classroom is the presentations of Mathematics by means of a textbook (Pang, 2003). In South Africa, one third of the Mathematics teachers use the textbook as the primary basis for their lessons. For the remaining group, the textbook is a supplementary resource (Reddy, 2006:105). According to Reddy (2006:105), there are a few problems that occur with the use of the Mathematics textbook as primary teaching method. Firstly, learners read a Mathematics text book with a highlighter and as a result they neither understand what they read, nor do they have a chance to engage with the ideas and concepts critically. Secondly, about one third of Mathematics teachers reported that a shortage of textbooks for learners was one of the factors that limit the teaching in the classroom. Some learners apparently share textbooks because of the high cost involved in purchasing books and other material. Teachers should rather choose real objects and experiences over workbooks and textbooks in developing understanding whenever possible (Cluster in Oleinik, 2002).

Cluster (in Oleinik, 2002) is of the opinion that, in Mathematics, attention should be paid to the introduction of metacognitive processes, i.e. the mastering of thinking strategies. If the implementation rules for cognitive actions are ignored, it will lead to the formation of false and formal views and conceptions (Oleinik, 2002). It is important that learners listen to the different opinions of their peers and understand the importance of joint discovery and the development of argumentation for or against ideas. Critical thinking skills in the Mathematics classroom will improve by creating an environment in the classroom that provides opportunities for argumentative accepting or rejecting of the views and ideas of others.

Against the background of the importance of the development of critical thinking skills in the new South African school curriculum, specifically in Mathematics, the central question this research set out to answer was: **What opportunities do teachers create to develop critical thinking skills in the Mathematics classroom?**

Within this central question, the following sub-questions arose:

- What does the development of critical thinking skills imply?
- How can critical thinking skills be developed during the teaching and learning of Mathematics?
- How do teachers perceive the development of critical thinking in the Mathematics classroom?
- What types of teaching methods and assessment strategies do teachers utilize in the Mathematics classroom to develop critical thinking skills?
- What types of learning material do teachers use during Mathematics teaching to develop critical thinking skills?
- What types of leaming activities do teachers structure in the Mathematics classroom to develop critical thinking skills?
- How do teachers create a classroom climate conducive to the development of critical thinking skills in the Mathematics classroom?

Flowing from the research questions, an overall aim and a number of objectives were identified.

1.4 AIM AND OBJECTIVES OF THE STUDY

The overall aim of this study was to determine the opportunities that teachers create for the development of critical thinking skills in the Mathematics classroom.

The overall aim was operationalized as follows:

- by delineating the meaning of the development of critical thinking skills through a literature review;
- by determining how critical thinking skills can be developed during the teaching and learning of Mathematics through a literature review;

- by scrutinizing teachers' perceptions regarding ways in which critical thinking skills can be developed in the Mathematics classroom, by means of an empirical study;
- by establishing what types of teaching methods and assessment strategies teachers utilize in the Mathematics classroom to develop critical thinking skills by means of an empirical study;
- by determining the different types of learning material that teachers use during Mathematics teaching to develop critical thinking skills by means of an empirical study;
- by establishing the types of learning activities that teachers structure in the Mathematics classroom to develop critical thinking skills by means of an empirical study; and
- by examining how teachers in the Mathematics classroom create a climate conducive to the development of critical thinking skills by means of an empirical study

1.5 EMPIRICAL RESEARCH DESIGN

1.5.1 Literature study

A thorough study was made of primary and secondary literature sources to determine what critical thinking is, how it applies to the Mathematics classroom and how it can be developed during teaching and learning. Furthermore, literature sources that provided the researcher with a clear perspective on issues related to research methodology were also consulted. The following national and international databases were consulted to identify resources: EBSCOHost, Google Scholar, JSTOR, ERIC, NEXUS and SABINET. The following key words and phrases were used to identify sources from the data bases: *cognition, critical thinking, Mathematics, secondary school level Mathematics, critical reflection, teaching Mathematics, critical thinking in Mathematics, classroom climate, teaching methods, learning material, assessment strategies, higher-order thinking, problem-*

solving, classroom climate, quantitative research, descriptive research, questionnaire design, population and sampling, reliability and validity, descriptive statistics, inferential statistics and *ethical considerations.*

Table 1.1 provides an overview of the variety of literature sources (internet articles, journal articles and books) that were utilized to delineate the conceptual framework of the study, namely critical thinking and Mathematics.

Table 1.1: Summary of literature consulted

	Internet articles	Journal articles	Books
Introduction and statement of problem: critical thinking in Mathematics			
	Ernest, 2002 Howie, 2007 Innabi, 2003 Oleinik, 2002 Setati, 2002 Setati, 2004 Vithal & Volmink, 2005	Brodie, 2007 Churach & Fisher, 2001 Gough, 1991 Green, 2006 Lombard & Grosser, 2004 Maharaj, 2007 Maree *et al.*, 2004 Mayer & Goodchild, 1990. Odora Hoppers, 2001 Schraw & Olafson, 2003. Sonn, 2000	Department of Education, 1997 Department of Education, 2002 Department of Education, 2004 Mc Peck, 1990 Pang, 2003 Paul *et al.*, 1987 Reddy, 2006 Skovmose & Valero, 2002 South African Qualifications authority, 1997 Van de Walle, 2001
Critical thinking in the Mathematics classroom			
Introduction cognition and critical thinking	Van Schalkwyk, 2002 Udall & Daniels, 1991	Berthold, Nuckles & Renkl, 2007 Brodie, 2007	Department of Education, 2002 Department of Education,

		Carr & Jessup, 1995 Van der Walt & Maree, 2007	2003
	Cheung *et al.*, 2002 Department of Education, 2007a Glazer, 2001 Lockwood, 2003 Patrick, 1986	Seng & Kong, 2006	Adams, 2002a Adams, 2002b Eggen & Kauchak, 2004 Grosser, 1999 Halpern, 2007 Israel, Block & Kinnucan-Welch, 2005 Kincheloe & Horn, 2006 Kok, 2007 Liljedahl, 2007 Monteith, 2002 Ormrod, 2008 Taylor, 2005 Thornton, 2002 Woolfolk, 2004
Critical thinking: A general concept clarification	Cheung *et al.*, 2002 Dowden, 2002 Facione, 2009 Fisher, 2001	Alazzi, 2008 Atkinson, 1997 Barnes, 2005 Bataineh & Zghoul, 2006	Dewey, 1933 Glaser, 1941 Halpern, 2007 Paul, 1990

	Muirhead, 2002 Oak, 2008 Tsui, 2008	Bayou & Reinstein, 1997 Elder & Paul, 1994 Graven, 2002 Halpern, 1998 Halx & Reybold, 2005 Oliver & Utermohlen, 1995 Pithers & Soden, 2000 Tsui, 2001 Vandermensbrugghe, 2004	Tempelaar, 2008 Woolfolk, 2004
Critical thinking in the Mathematics classroom	Adler *et al.*, 2000 Berns & Erickson, 2001 Bopape, 1998 Chen, Cai & Zheng, 2009 Damji *et al.*, 2003 Department of Education, 2007b Dowden, 2002 Duatepe & Ubuz, 2004 Ellis, 2000 Erwin, 2000 Fromboluti & Rinck, 1999	Beyer, 1985 Brodie, 2007 Chisolm, 2005 Colucciello, 1997 Curcio, 1987 Elder & Paul, 2002 Ennis,1993 Graven, 2002 Innabi & ElSheikh, 2006 Lombard & Grosser, 2004 Macintyre, 2006	Bernstein, 2000 Bishop, 1988 Cangelosi, 2003 Department of Education, 1997 Department of Education, 2002 Department of Education, 2003 Department of Education, 2005 Department of Education, 2007a

	Gallagher, 1975 Glazer, 2001 Kollars, 2008 Moloi, 2005 Naik, 2009 Oak, 2008 Pratt, 2005 Schafersman, 1991 Simic-Muller, 2007	Msila, 2007 Norris, 1985 Sezer, 2008 Shaughnessy & Zawojewski, 1999 Singh *et al.*, 2002 Suliman, 2006 Treffinger, 1994 Wedekind *et al.*, 1996 Winicki-Landman, 2001 Winstead, 2004	Khuzwayo, 1997 King, 2007 Kok, 2007 Mahaye & Jacobs, 2007 Maker & Nielson,1996 National Council of Teachers of Mathematics, 1995 Pratt, 2005 Schoënfeld, 1994 Vakalisa, 2007 Van de Walle, 2007 Winch, 2006
Developing critical thinking in the Mathematics classroom	Ash, 2005 Bellis, 1999 Boston, 2002 Byers, 2004 Cantrell, 2000 Crotty, 2002 Elder, 2007 Ellis, 2000 Ferrando, 2001	Black & William, 1998 Black *et al.*, 2004 Briggs & Sommerfeldt, 2002 Bullen, 1998 Carter, 2005 Delandshere & Arens, 2003 Facione *et al.*, 2000 Facione, 2009 Gokhale, 1995	Appelbaum, 2004 Arends, 2009 Beyer, 1985 Borich, 2004 Cangelosi, 2003 Ennis, 1992 Gawe, 2007 Halpern, 2007 Jacobs *et al.*, 2004

	Gallagher, 1975	Gough, 1991	Lake, 2009
	Gupta, 2001	Grabe & Grabe, 2004	Lidz & Gindis, 2003
	Hida *et al.*, 2005	Halpern, 1999	Mahaye and Jacobs, 2007
	Keefe & Walberg, 1992	Horton & Ryba, 1986	Maree & Fraser, 2004
	Kestell, 2006	Jonassen, 1997	Matutu, 2006
	Monteith, 1999	Klein & Orr, 1991	McMillan, 2001
	Morris, 2007	Leader & Middleton, 2004	McMillan, 2007
	Muirhead, 2002	Lehman & Hayes, 1985	Myren, 1995
	Olivares, 2005	Marcut, 2005	National Council of Teachers of Mathematics, 1989
	Porch, 2002	Middleton & Roodhart, 1997	National Council of Teachers of Mathematics, 1995
	Potts, 1994	Niedringhaus, 2001	National Council of Teachers of Mathematics, 2000
	Searls, 2006	Polya, 1973	Niess & Garofalo, 2006
	Spache & Spache, 1986	Sezer, 2008	Niss, 1998
	Stein *et al.*, 2006	Staples, 2007	Schoënfeld, 1994
		Sternberg & Martin, 1988	Smith, Smith & LeLisi, 2001
		Stiggins, 2002	Van de Walle, 2001
		Suurtamm, 2004	Van der Horst & McDonald, 2003
		Van der Walt & Maree, 2007	Volmink, 1994

			Winch, 2006
Research methodology			
Research paradigms Research method Research design Population and sampling Data collection instruments Data analysis Ethical considerations	Garson, 2008 Simon, 2008 Van Deventer, 2007	Akbaba, 2006 Eldabi *et al.*, 2002 Van Teijlingen & Hundley, 2001	Cohen, Manion & Morrison, 2006 Leedy & Ormrod, 2005 Maree & Pietersen, 2007b Maree & Van der Westhuizen, 2007 Maree & Pietersen, 2007a Maree & Pietersen, 2007b Maree & Pietersen, 2007c Maree & Pietersen, 2007d Maree & Pietersen, 2007e McMillan & Schumacher, 2006 Pietersen & Maree, 2007 Sekaran, 2000 Saslow, 1982

The next section provides a brief overview of the empirical research design utilized in the context of this study.

1.5.2 Empirical Research

1.5.2.1 Research paradigm

The study focused on a positivist paradigm. It was the researcher's intention to act as an objective observer during the collection of data (Maree & van der Westhuizen, 2007:33). A quantitative research method was therefore utilized.

1.5.2.2 Research method

As it was the researcher's intention to construct a rich and meaningful picture of the teaching and learning situation in Mathematics classrooms in South Africa, a quantitative research was conducted to gather information about the development of critical thinking in the Mathematics classroom. A quantitative research method was suitable for this study as the researcher wanted to establish and confirm a given situation (Leedy & Ormrod, 2005:94-95).

1.5.2.3 Research design

A non-experimental, descriptive survey research design was utilized in this research. This design was suitable for this research as this study entailed a first investigation and the researcher simply wanted to provide a summary of an existing phenomenon, and assess the nature of existing conditions. No intervention took place (McMillan & Schumacher, 2006:24, 215). Survey research is used to describe attitudes, beliefs and opinions (McMillan & Schumacher (2006:25). In the context of the study it was the researcher's intention to gather data related to the opinions of teachers and learners on the nurturing of critical thinking, and therefore survey research was seen as suitable.

1.5.2.4 Population and sample

The population of the study involved all teachers and learners of Mathematics. It was not possible to conduct research with the entire population. Therefore, by means of purposive sampling, the focus of the study was placed on Grade 8 Mathematics teachers and learners. Purposive sampling relies on the judgement of the researcher who selects subjects that will provide the best information to address the purpose of the research (McMillan & Schumacher, 2006:126). The researcher also teaches Mathematics to Grade 8, is knowledgeable on the content of Grade 8 Mathematics and has experimented with the development of critical thinking skills related to Grade 8 Mathematics content. The researcher was therefore of the opinion that she would be able to understand the opinions of teachers and learners regarding the development of critical thinking skills related to the context of Grade 8 Mathematics.

Due to time and logistical constraints the researcher also decided to make use of convenient sampling. As the researcher works in the Ekurhuleni District of the Gauteng Department of Education and had easy access to and contact with school principals and Mathematics teachers in fourteen schools in this District, it was decided to conduct the study in this District. The researcher approached all fourteen schools to take part in the research. The sample comprised the following schools:

- two township schools where the learners' home language is an African language, but the medium of instruction is English;
- one Afrikaans school where Afrikaans is used as the medium of instruction;
- two parallel medium schools where the medium of instruction is both English and Afrikaans;
- eight English schools where the medium of instruction is English; and
- one private school with English as medium of instruction.

All the teachers in the 14 identified schools who taught Mathematics at Grade 8 level were requested to take part in the research. Ultimately, a heterogeneous group of 92 teachers from the fourteen schools, comprising different age groups, genders, ethnic groups, years of experience in Mathematics teaching and qualification levels, took part in the research (*cf.* 4.3.2).

In each of the identified schools, Grade 8 learners who were willing to participate were requested to take part in the research. In total, a heterogeneous group of 204 learners comprising different genders, cultures and home languages took part in the research (*cf.* 4.3.1).

1.6 DATA COLLECTION INSTRUMENT

Two closed-ended questionnaires, for teachers and learners respectively, were constructed by the researcher in accordance with the literature study and aims and objectives of the study. As the researcher wanted to learn more about the opinions and experiences of a large population, a questionnaire was a suitable instrument to survey a sample of the population (Leedy & Ormrod, 2005:183).

Information gathered from the literature study was used to develop and design two structured questionnaires with closed questions, for teachers and learners respectively, to gather information regarding the opportunities provided in Mathematics classrooms for the development of critical thinking skills. Group administration of the questionnaires, by the various Heads of Departments Mathematics, was applied to the learners (Maree & Pietersen, 2007b:157). The teachers were requested to complete the questionnaires in their own time. The perceptions and views of the participants were measured by using a Likert scale. This provided an ordinal measure of the participants' viewpoints (Maree & Pietersen, 2007a, b:148,167) (*cf.* Annexure A). The learner responses were compared to the responses of the teachers to determine differences and similarities in perceptions in order to support or refute the responses received by the teachers.

The questionnaires aimed at collecting data to determine the perceptions of the sampled teachers and learners regarding critical thinking, the application of teaching methods and assessment strategies in order to promote critical thinking, the choice of learning material during the teaching of Mathematics, the choice of learning activities to promote critical thinking, the role of the teacher during the teaching of Mathematics and how teachers create classroom climates conducive to critical thinking. A pilot study was conducted with a group of Grade 8 teachers (n= 50) and learners (n = 50) who were not part of the sample in order to determine the reliability and validity of the measuring instruments. Cronbach alpha coefficients were calculated to determine the reliability of the questionnaires (*cf.* 3.5.1; 4.2). Validity was determined by considering face, content, criterion and construct validity (*cf.* 3.5.2). Inter-item correlations were also determined for the various questionnaire items (*cf.* 3.5.1; 4.2).

1.7 DATA ANALYSIS

By means of descriptive statistics the data analysis for the teacher and learner responses to the questionnaire was interpreted. The responses to the questionnaires were summarized with frequency counts, percentages and means and inferences were drawn. Inferential statistics were utilized to determine if differences that occurred between teacher and learner responses were statistically significant or not. For this purpose, t-tests were utilized (*cf.* 3.7). If statistical significant differences occurred, Cohen's D was calculated to determine the practical effect of the differences (Steyn, 2005:20) (*cf.* 4.5.1).

In order to determine the effects of the various biographic variables on the development of critical thinking skills in the Mathematics classroom, an ANOVA was run (*cf.* 3.7). If the ANOVA indicated significant differences between the various groupings of biographic variables, post hoc tests were run (Tukey Honestly Significant Difference (HSD) Tests) to determine which of the groupings displayed the differences (McMillan & Schumacher, 2006: 301, 302).

1.8 ETHICAL ASPECTS

A full account of how ethical issues were dealt with in the context of the study is provided in Chapter three (*cf.* 3.8).

1.9 CONCEPTUAL FRAMEWORK OF THE STUDY

The concepts central to the study, namely **critical thinking** and **Mathematics**, are elucidated in Chapter two. The following section provides a brief definition of the concepts as they will be conceptualized in the context of the study.

Literature bears evidence of various conceptualizations that emphasize the multi-dimensional nature of critical thinking. For the purpose of this research, critical thinking was conceptualized according to the viewpoints of Pithers and Soden (2000:239), Cheung *et al.* (2002), Dowden (2002); Vandermensbrugghe (2004:419), Barnes (2005:46), Halx and Reybold (2005:296); Halpern (2007:10-12) and Maharaj (2007:34) who view critical thinking as the development of interrelated **cognitive skills** such as problem-solving, formulating inferences, decision-making, logical and cohesive reasoning, analysis, questioning, identifying assumptions, interpretation, evaluation, creating and comparing arguments, application, identifying assumptions and reasoning.

A number of definitions for the subject Mathematics were also identified in the literature (Glazer, 2001; Graven, 2002:24, Department of Education, 2003:7; Van de Walle, 2007:13). In the context of the study that focused on teaching and learning in the context of the Learning Area Mathematics Grade 8 in the NCS, the definition of the Department of Education (2003:21) and Van de Walle (2003:7) guided the understanding of critical thinking in Mathematics. According to this definition, Mathematics *"involves representing and investigating patterns and quantitative relationships in physical and social phenomena and between mathematical objects."* The definition of Van de Walle emphasizes the importance of problem-solving, logical thinking and analysis as being prominent features of Mathematics. According to the researcher, both definitions require the application of interrelated cognitive skills such as analysis and evaluation, which link well with the aforementioned conceptualization of critical thinking.

1.10 CHAPTER DIVISION

The study unfolded according to the following chapter division:

Chapter 1: Introduction and statement of problem

Chapter 2: Literature review: Critical thinking in the Mathematics classroom

Chapter 3: Empirical research design

Chapter 4: Data analysis and interpretation

Chapter 5: Summary, findings and recommendations

1.11 CHAPTER SUMMARY

This research deals with the opportunities that teachers create for the development of critical thinking skills in the Mathematics classroom. Chapter one explored the background to this study. This chapter provided a short introduction to the problem and aim of this study that focuses on determining what opportunities teachers create for the development of critical thinking skills in the Mathematics classroom. It also gives an overview of the procedures according to which data was collected, analysed and interpreted. For the purpose of this study quantitative research was conducted to gather information. Two closed-ended questionnaires, for teachers and learners respectively were used to gather information. The following chapter provides a more comprehensive overview of the concepts central to the study, namely critical thinking and Mathematics teaching and learning.

CHAPTER TWO

CRITICAL THINKING IN THE MATHEMATICS CLASSROOM

2.1 INTRODUCTION

The development of critical thinking is one of the cornerstones of the National Curriculum Statement (NCS) new curriculum in South Africa (Department of Education, 2003:2). According to the NCS, all teachers need to plan teaching and learning experiences around the Critical Cross-Field Outcomes. The Critical Cross-Field Outcomes contribute to the full development of each learner and the social development of the nation at large. These are outcomes that are essential to all learning and include, *inter alia*, skills such as being able to think creatively and critically, to solve problems, to collect information, to organize information, to analyse information, to work in a group as well as independently, to communicate effectively, and to make responsible decisions (Van Schalkwyk, 2002). A key principle of the NCS is also to ensure that the educational imbalances of the past are readdressed, and that equal educational opportunities are provided for the entire population (Department of Education, 2003:2).

The main aim of this chapter is to provide a description of how the development of critical thinking skills applies to teaching Grade 8 Mathematics. The literature review concentrates on the following topics:

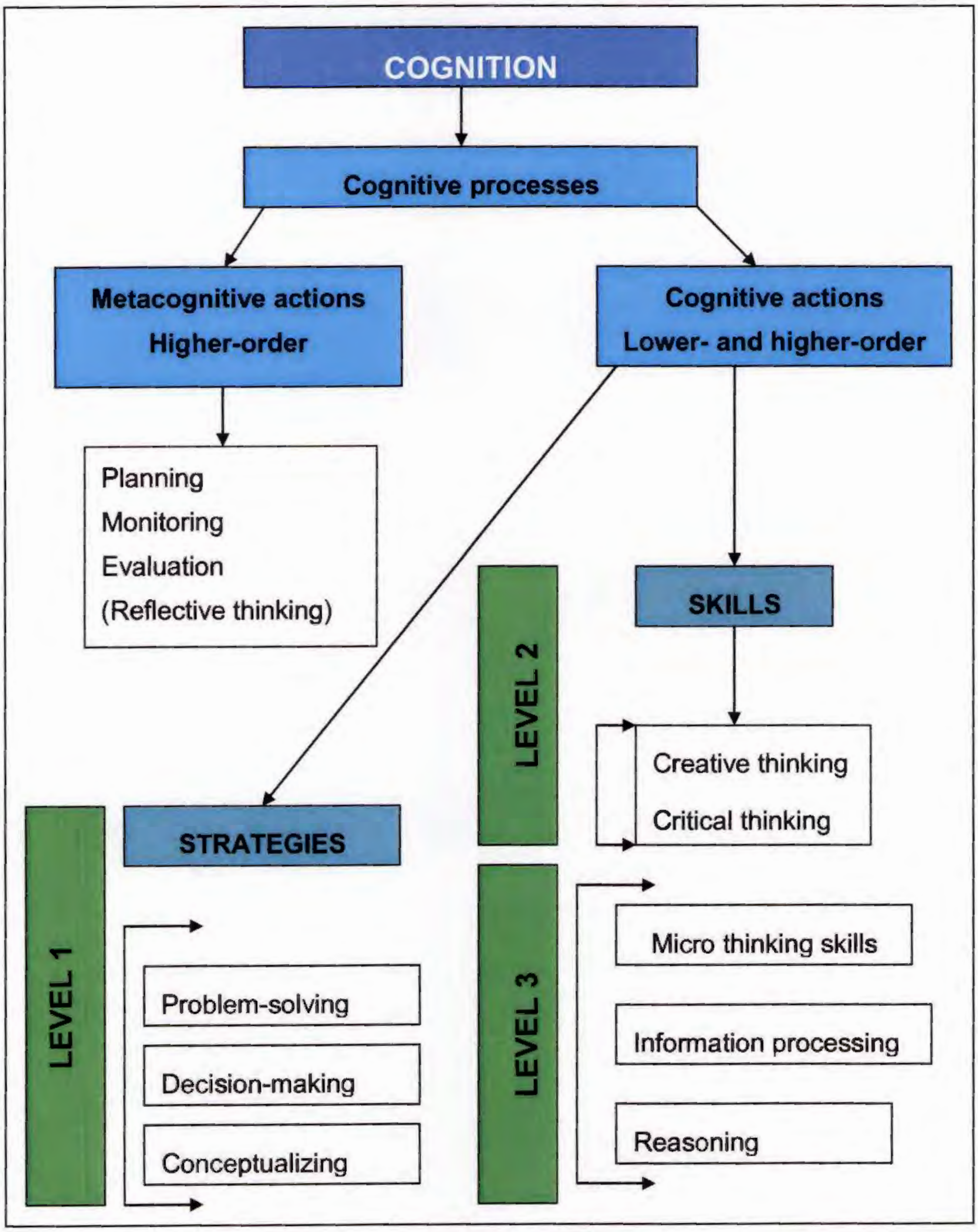
- Cognition and critical thinking
- Critical thinking: a general concept clarification
- Critical thinking in the Mathematics classroom
- Developing critical thinking skills in the Mathematics classroom

In order to determine the role of critical thinking skills during the teaching and learning of Mathematics, the concept cognition is first elucidated and the place of critical thinking within the context of cognition is highlighted.

2.2 COGNITION AND CRITICAL THINKING

The word cognition can be defined as "*the mental action or process of acquiring knowledge through thought, experience and the senses*" (Taylor, 2005:2). Cognition deals with how people think, reason and decide and can be considered as a complex procedure that contains complex thinking (Weiten, 2004:303; Taylor, 2005:2). Complex thinking is a type of cognition that requires basic thinking and is characterized by multiple possible answers, judgment on the part of the person participating and the imposition of meaning on a situation (Adams, 2002b:153). Types of complex thinking include critical thinking, creative thinking and problem-solving (Udall & Daniels, 1991). The following conceptualization of cognition (*in* Grosser, 1999:56) in Figure 2.1 provides a framework of the dimensions and levels of cognition and the place of critical thinking within this framework. The discussion that follows will explain the different levels of cognition and then link the conceptual structure to the teaching and learning of Mathematics.

Figure 2.1: Conceptualizing cognition.



As indicated by Figure 2.1 above, cognition comprises three difficulty levels of lower and higher-order cognitive processes, as well as a metacognitive component comprising the reflective thinking skills of planning, monitoring and evaluation.

The **cognitive processes** can be executed on three levels of difficulty. Level 1 refers to the cognitive strategies which are the most difficult to acquire as they involve the complex execution of sequenced actions, for example problem-solving and decision-making (Grosser, 1999:54; Monteith, 2002:97). Decision-making is part of problem-solving and involves complex strategies such as defining goals, reformulating decisions and searching for alternative ways to solve problems (Grosser, 1999:57; Halpern, 2007:8). Conceptualization refers to the identification of the characteristics of categories of concepts with the aim of arriving at a general idea that would benefit the organization of information (Thornton, 2002:102).

Creative and critical thinking function at level 2 and are not complex strategies that require the execution of sequenced actions. Creative and critical thinking are multi-dimensional in nature and require the integrated application of a number of interrelated cognitive skills such as: problem-solving, decision-making, reasoning, identifying assumptions, making conclusions, value judgements, analysis, evaluation and application (Woolfolk, 2004:337; Seng & Kong, 2006:54, 55; Berthold, Nuckles & Renkl, 2007:565; Ormrod, 2008:283-292). Critical thinking is an essential element of general cognitive processes, such as problem-solving, elaboration, decision-making, organizing, analysing, synthesizing and evaluation, but is not synonymous with them (Patrick, 1986; Eggen & Kauchak, 2004:335).

Level 3 comprises the non-complicated cognitive skills. According to Grosser (1999:58) and Ormrod (2008:202), information processing skills refer to skills that enable learners to choose information selectively for memorization purposes and skills to recall memorized knowledge. Reasoning is a very important skill in Mathematics. In order to interpret, analyse, synthesize and evaluate, well developed reasoning skills which require the ability to think critically, is necessary (Liljedahl, 2007:65).

It is clear from the above explanation that the ability to think critically is essential for the execution of cognitive actions on all three levels indicated in Figure 2.1.

Besides cognitive strategies, **metacognitive skills** are also an important component of cognition. Metacognition refers to the knowledge and awareness of one's own cognitive processes and the ability to control and manage those processes actively (Berthold *et al.,* 2007:565). The more learners are aware of their cognitive processes and skills, the more they will be able to regulate them.

According to Adams (2002b:154) and Brown and Palinscar (*in* Van der Walt & Maree, 2007:225), metacognition also refers to an individual's ability to adapt cognitive actions in order to improve understanding. For example, learners who ask themselves if they understand the meaning of the content under study (self-monitoring) and generate appropriate strategies to eliminate confusions and/or seek additional information (self-regulation), will be able to achieve metacognitive thinking (Israel, Block & Kinnucan-Welch, 2005:317). With regard to critical thinking in Mathematics, metacognitive learning refers to learners' growing understanding of why and when Mathematics strategies are best used and the monitoring of problem-solving activities (Carr & Jessup, 1995:236). According to Carr and Jessup (1995:236), metacognitive monitoring is essential for successful mathematical problem-solving. Metacognition is also known as the awareness of one's thinking processes; how one constructs questions, solves problems, makes decisions, organizes daily activities, and all of the other cognitive activities that mediate our desires and actions (Kincheloe & Horn, 2006:829). Studies have shown that there is a relationship between metacognition and good Mathematics performances. Learners' failure in Mathematical problem-solving can frequently be attributed to their lack of reflection on their cognitive processes, either before or during problem-solving. It is therefore important that teachers teach learners to make use of elaborative, integrative or specific strategies and related metacognitive knowledge because it will help to improve Mathematics achievement and retention (Carr & Jessup, 1995:236).

Halpern (2007:10) asserts that a learner's ability to execute critical thinking skills is closely allied to metacognition. Critical thinking involves the metacognitive skill of reflecting on the personal understanding of a task, focussing on relevant instead of irrelevant information and trying out new strategies to solve a problem or complete a task. Critical thinking skills and metacognition can be strongly linked to active participation of learners during the construction of knowledge. The application of critical thinking skills refer to logical and reflective thinking that involves the processes of actively questioning and analyzing information to gain knowledge. Metacognition is looking at the results along the way. Metacognition implies the evaluation of results obtained for completed work. It enables learners to be more aware of what they know or don't know. It also involves being aware of strategies to control and improve learning (Lockwood, 2003).

Cognitive and metacognitive skills can be characterized as **lower- or higher-order**. Lower-order thinking involves the memorization of knowledge and facts (Kok, 2007:28). In the context of Mathematics teaching, the researcher argues that lower-order thinking is important for learning rules, formulas, definitions and algorithms. Higher-order thinking refers to the execution of complex cognitive processes such as analysing, synthesizing and evaluation and plays an important role in Mathematics (Adams, 2002a:154, Brodie, 2007:3; Kok, 2007:28-30; Van der Walt & Maree, 2007:223-238).

The next section will elucidate the concept critical thinking.

2.3 CRITICAL THINKING: A GENERAL CONCEPT CLARIFICATION

In recent years, the concept critical thinking has become a household name in many classrooms although not many teachers and learners understand the connotation or the importance of it (Bataineh & Zghoul, 2006:33). The thought of critical thinking in education has been pulled in many different directions and is seldom clearly or comprehensively defined (Atkinson, 1997:71-94; Fisher, 2001; Alazzi, 2008:244). This section is devoted to an examination of the concept of critical thinking.

The American philosopher, psychologist and educator, John Dewey, defined critical thinking as an active, persistent, and careful consideration of a belief or supposed form of knowledge in the light of the grounds which support it and the further conclusions to which it tends. He called it reflective thinking, and a process in which metacognitive thinking skills are applied to think things through, raise questions, find relevant information yourself instead of learning in a passive way and evaluating the thinking process during the making of a decision (Dewey, 1933:9). Glaser (1941:5) and Alazzi (2008:245) expanded on Dewey's definition of critical thinking by defining it as a persistent effort to examine any belief or supposed form of knowledge in light of the evidence that supports it and the further conclusions to which it tends. In this regard Elder and Paul (1994:34) state that: "*critical thinking is best understood as the ability of thinkers to take charge of their own thinking.*" This requires that learners develop sound **metacognitive skills** for analysing and assessing their own thinking and routinely use the same criteria to improve on the quality of thinking. In this regard Tempelaar (2008:175) refers to learners' abilities to predict their performances on various tasks and monitoring their understanding (Tempelaar, 2008:175).

Glazer (2001) defines critical thinking in Mathematics as follows: *"Critical thinking in Mathematics is the ability and disposition to incorporate prior knowledge, mathematical reasoning and cognitive strategies to generalize, prove, or evaluate unfamiliar mathematical situations in a reflective manner."* According to this definition, Glazer (2001) considers critical thinking to be reflective and reasonable thinking that focuses on deciding what to believe or do. Beyer (1985:271) on the other hand, views critical thinking as the process of determining the authenticity, accuracy and worth of information or knowledge. He claims that critical thinking has two important dimensions. It is both a frame of mind and a number of specific mental operations. Both Beyer (1985:276) and Glazer (2001) define critical thinking in an evaluative sense.

According to Dewey (in Fisher, 2001), critical thinking also implies the possession of an **attitude** or **disposition** to use critical thinking skills, which is just as important as possessing the skills (in Fisher, 2001). These attitudes and dispositions imply *inter alia* the following: "a spirit of inquiry", "open-mindedness", "fair-mindedness", "respect for reasons and truth", "inquisitiveness", "truth-seeking", "independent-mindedness", "respect for legitimate intellectual authority and intellectual work-ethic" and "scepticism"

(Cheung *et al.,* 2002; Seng & Kong, 2006:58; Halpern, 2007:10; Facione, 2009)

In addition to attitudes and dispositions, the development of critical thinking skills also involves the development of **behavioural critical thinking habits**. These habits refer to *inter alia* the following: "making comparisons", "argumentation", "non-compliance", "responsible deliberation", "generating original approaches", "identifying alternative perspectives", "scrutinizing knowledge before consumption", "assessment of reasons and arguments" and "imagining consequences" (Cheung *et al.,* 2002; Tsui, 2002:748).

Critical thinking is disciplined, self-directed thinking which exemplifies the perfections of thinking appropriate to a particular mode or domain of thought (Paul, 1990:52). This implies that a thorough **knowledge base** is a prerequisite for executing critical thinking.

Critical thinking can and must be used to describe thinking that is multidimensional purposeful, reasoned and goal directed. It is the kind of thinking that involves the interrelated development and application of the following **cognitive skills**: "problem-solving", "formulating inferences", "decision-making", "logical and cohesive reasoning", "analysis", "questioning", "interpretation"," evaluation", "application", "identifying assumptions" and "inductive and deductive reasoning", “formulating inferences”, “calculating likelihoods”, and “making decisions” (Bayou & Reinstein, 1997:339; Pithers & Soden, 2000:239; Cheung, *et al.*, 2002; Vandermensbrugghe, 2004:412; Barnes, 2005:42-46; Halx & Reybold, 2005:296; Seng & Kong, 2006:53; Halpern, 2007:10-12; Oak, 2008; Tempelaar, 2008:175-177).

Critical thinking is the ability to engage in reasoned discourse with intellectual standards such as clarity, accuracy, precision and logic, and to use analytic skills with a fundamental value orientation that emphasizes intellectual humility, intellectual integrity, and fair-mindedness (Dowden, 2002). Critical thinking can be described in the broader term as reasoning in an open-ended manner, with unlimited numbers of solutions. The critical thinking process involves constructing a problem situation, finding solutions to solve the problem and supporting the reasoning behind a solution. Although critical thinking skills are important, critical thinking is more than just a set of skills. Critical thinkers must be able to provide justifications for their actions (Tsui, 2002:748).

Critical thinking skills can be described as the ability to think critically through the use of cognitive skills to resolve a problem, attain a conclusion, form an inference and make a decision through reasoned, logical and goal-directed processes (Bayou & Reinstein, 1997:339). It refers to an ability of weighing substantial and insubstantial as well as concrete and abstract factors central to a situation, in order to obtain the best possible solution to a problem (Oak, 2008). According to Woolfolk (2004:338), the development of critical thinking in the classroom should focus on the following three aspects:

Defining and clarifying problems: the learners should be able to:

- identify central issues or problems;
- compare similarities and differences;
- determine which information is relevant; and
- formulate appropriate questions.

Judging information related to a problem: the learners should be able to:

- distinguish between fact, opinion and reasoned judgment;
- check consistency;
- identify unstated assumptions;
- recognize stereotypes and clichés;
- recognize bias, emotional factors, propaganda and semantic slanting; and
- recognize different value systems and ideologies.

Solving problems/drawing conclusions: the learners should be able to:

- recognize the adequacy of data; and
- predict probable consequences.

Critical thinking skills can either be directly related to the outcomes of the learning process or to the variables that measure the quality of learning (Tempelaar, 2008:175). Teachers can develop critical thinking skills in the classroom, by making use of relevant assignments that encourage creativity, reflective thinking, and self-directed learning (Muirhead, 2002). These types of assignments will enable learners to apply critical thinking skills to their academic studies, to the complex problems that they will face and the critical choices they will be forced to make as a result of the information explosion and other rapid technological changes (Oliver & Utermohlen, 1995:1). It is clear that the development of critical thinking skills in the Mathematics classroom depend, among others, on the teachers' choice of teaching methods and assessment practices (*cf.* 2.5.1; 2.5.2).

After having read all these different definitions and explanations about critical thinking, the researcher noticed five major trends among the definitions and explanations offered by pioneers in the field of critical thinking. These trends can be briefly summarized as follows:

- The development of critical thinking skills involves the development of dispositions for effortful thinking (Cheung *et al.*, 2002; Halpern, 2007:10; Facione, 2009).
- The development of critical thinking skills involves the development and application of interrelated cognitive skills which demand high levels of abstract and logical thinking (Pithers & Soden, 2000:239; Cheung *et al.*, 2002; Vandermensbrugghe, 2004:412; Barnes, 2005:42-57; Halx & Reybold, 2005:296; Halpern, 2007:10-12).
- The development of critical thinking skills involves the development of behavioural critical thinking habits (Cheung *et al.*, 2002; Tsui, 2002:748).

- The development of critical thinking skills involves the development of meta-cognitive skills such as reflection, so that learners learn to monitor and evaluate their own thinking processes (Halpern, 2007:10).

- The development of critical thinking skills requires the development of a sound knowledge base (Tempelaar, 2008:175-178). This includes the depth of knowledge, understanding and experience a person is able to demonstrate in a particular area, which determines the degree to which such a person will be capable of thinking critically in that area.

In the context of the study, the researcher focused attention specifically on the opportunities provided for the development of interrelated cognitive skills during teaching and learning.

In the next section, critical thinking will be conceptualized in terms of its role and importance in the Mathematics classroom.

2.4 CRITICAL THINKING IN THE MATHEMATICS CLASSROOM

2.4.1 Changes experienced in the Mathematics classroom

The South African Mathematics syllabus has experienced a radical change since 1994. According to Moloi (2005), the Mathematics curriculum used to be heavily content laden, encouraged rote learning of Mathematical strategies and algorithms and lent itself to a very limited application in everyday experiences of learners. The locus of the underlying pedagogy was on teaching rather than learning.

Msila (2007:146) asserts that during the Apartheid years, schools were divided according to race, and education enhanced the divisions in society. By separating the Black learners from the White learners, the government maintained the Black South African learners in a permanent state of political and economic subordination. According to Kallaway (in Msila, 2007:149), the Black learners made use the Bantu education system that restricted the learners' development by distorting school knowledge to ensure control over the intellect of the learners and teachers and promoting state propaganda (Wedekind, Lubisi, Harley & Gulltig, 1996:422).

The teachers had the duty to implement a ready-made curriculum that was teacher-centred (Wedekind *et al.*, 1996:422) in their classrooms and to make use of certain prescribed strategies to test the learners (Khuzwayo, 1997:323). The Mathematics teachers received a syllabus describing the goals and aims, some guidelines on methodological aspects, the content to be covered per year and the evaluation procedure to be followed. The curriculum was designed elsewhere and was just handed over to the teachers to implement. The origin of the curriculum as well as the processes followed and the underlying motivation of the curriculum were unknown (Khuzwayo, 1997:323)

The Mathematics curriculum during the Apartheid years was largely a technique-driven curriculum (Khuzwayo, 1997:324). According to Bopape (1998), rote learning constituted one of the main approaches to Mathematics teaching and the teachers utilized specific strategies which drilled content into the minds of the learners. In order for learners to be successful in their learning of Mathematics, they needed to master these strategies of learning Mathematics (Khuzwayo, 2997:324). Bishop (1988:8-9) is critical of a technique driven curriculum and argues that it did not help understanding, develop meaning and enable learners to think critically inside and outside the Mathematics classroom.

Bishop (1988:8-9) argues that the weaknesses that the Mathematics curriculum had during the Apartheid years was that it focused on helping learners to learn to carry out a number of different types of pure Mathematical problems, using some combination of mental and written knowledge and skills. It took learners a long period of time to develop a reasonable level of speed and accuracy in performing addition, subtraction, multiplication and division on integers, decimal fractions and fractions. Speed and accuracy declined relatively rapidly without continued practice of the aforementioned skills, and the system only produced modest results (Bishop, 1988:8-9).

In 1994, in the immediate aftermath of the election, a curriculum revision was undertaken which involved the cleansing of the curriculum from any racist and sexist elements (Chisolm, 2005:193). The constitutional democracy identified national Critical and Developmental Outcomes as a basis for the curriculum (Bernstein, 2000:65). On 26 February 1997 the Council of Education Ministers (CEM) decided to replace the Apartheid Education system with the Outcomes-Based Education system (OBE) in the General Education and Training bands (Grades R-9). This decision was extended and envisaged the phasing in of OBE into both the GET and FET bands (Grades 10-12) by 2005, and therefore it was decided to name the curriculum, Curriculum 2005 (C2005) (Department of Education, 2003:5). The new curriculum adopted *inter alia a* constructivist approach to teaching and learning which is more learner-centred, relevant and assessable in terms of a hierarchy of demonstrable and measurable competencies. The curriculum reform aimed to readdress the educational imbalances and inequalities of the "old" South Africa with the focus to construct new pedagogic identities in teachers and learners (Bernstein, 2000:65). The Department of Education also decided to formulate Specific Outcomes that refer to the specification of what learners should be able to do at the end of their learning experience in a particular Learning Area. The Specific Outcomes included skills, knowledge and values, which informed the demonstration of the achievement of an outcome or a set of outcomes (Department of Education, 1997:17).

In 2002, the Department of Education decided to revise Curriculum 2005 and develop a simpler and stronger curriculum based on the best of Curriculum 2005. This curriculum is now called the National Curriculum Statement (NCS) and is still based on the principles of Outcomes-Based Education and the promotion of constructivist principles during teaching and learning. Constructivist principles focus on learners being involved in their own construction of knowledge, taking ownership of their own learning, learners given opportunities to explain what they think, argue and solve problems (Pratt, 2005; Mahaye & Jacobs, 2007:174-176; Vakalisa, 2007:5) The NCS envisages creating a new South African identity that encompasses critical consciousness, the transformation of the South African society, the promotion of democracy and the magnifying of learner involvement in education (Msila, 2007:151).

One of the many aims of OBE in the subject Mathematics is that the teaching and learning of Mathematics should work towards the attainment of the Critical and Developmental Outcomes (Department of Education, 2005:8). The OBE framework outlines aspects of culture, which Mathematics teachers have to keep in mind when planning for teaching Mathematics in the classroom. By including these cultural artefacts or products during Mathematics teaching, it will contribute towards the restoration of self-pride among communities whose cultures were not recognized before, such as the geometric strategies used in the design and the building of thatched roof houses by people who hardly or never had an opportunity to attend any formal Mathematics education (Bopape, 1998). Teachers also need a better subject knowledge and a far better understanding of the Mathematical content to be able to make a success of the curriculum (Adler, Brombacher & Shan, 2000).

Regarding the subject Mathematics, the NCS defines the Learning Area Mathematics as follows: *"Mathematics involves representing and investigating patterns and quantitative relationships in physical and social phenomena and between mathematical objects. Mathematical symbols and notation form a specialised language."* (Department of Education, 2003:21). As this study focused on the teaching of Mathematics in Grade 8, the Learning Outcomes and Assessment Standards that Grade 8 learners need to achieve, were investigated in order to determine to what extent critical thinking skills are imbedded in the Learning Outcomes and Assessment Standards. In the table below, Table 2.1, the researcher provides an overview of the Learning Outcomes and Assessment Standards, and underlines the critical thinking skills required in order for learners to achieve the Assessment Standards.

Table 2.1: Critical thinking skills imbedded in the Learning Outcomes and Assessment Standards for Grade 8

Learning Outcomes	Summary of the Assessment Standards
1. Numbers, operations and relationships The learner will be able to recognise, describe and represent numbers and their relationships, and to count, estimate, calculate and check with competence and confidence in solving problems.	• Describes, respects, appreciates and illustrates the historical and cultural development of real numbers • Recognises, classifies and represents these real numbers to describe and compare them • Recognises and uses equivalent forms of the rational numbers • Solves problems in contexts, including contexts that may be used to build awareness of other learning areas or economic and environmental issues, e.g. financial profit and loss • Estimates and calculates by selecting and using operations appropriate to solving problems that involve rounding off; multiple operations with real numbers (excluding irrational numbers); exponents • Uses a range of techniques to perform calculations including using the commutative, associative and distributive properties and using a calculator • Uses a range of strategies to check solutions and judges reasonableness of solutions • Recognises, describes and uses e.g. the 4 basic operations with real numbers and multiplication and division of exponents

Learning Outcomes	Summary of the Assessment Standards
2. Patterns, Functions and Algebra The learner will be able to recognise, describe and represent patterns and relationships, as well as to solve problems using algebraic language and skills.	• Investigates and extends numeric and geometric patterns while moving between physical, diagrammatic, tabular and algebraic representations • Explains and justifies the rules that generate patterns using algebra • Uses these representations: words; flow diagram; tables; formulae, equations and expressions; graphs • Recognises and expresses equivalence by transforming between a given representation and another more appropriate to solving problems • Solves equations by inspection, trial-and- improvement and algebraic processes • Analyses and interprets the equivalence of different representations of a problem and selects the most useful representation for a given situation • Describes a situation by drawing and interpreting a graph of the situation, with a special focus on trends and features • Uses conventions of algebra to transform between equivalent forms of algebraic expressions and equations to solve problems

Learning Outcomes	Summary of the Assessment Standards
3. Space and Shape The learner will be able to describe and represent characteristics and relationships between two-dimensional shapes and three-dimensional objects in a variety of orientations and positions.	• Recognises, visualises and names shapes and objects in natural and cultural forms and geometric settings • Describes and classifies geometric shapes and objects in terms of properties including • Uses vocabulary and are able to describe e.g. parallel lines cut by transversal, perpendicular lines • Uses a pair of compasses, ruler and protractor to construct shapes accurately for investigation of their properties and design of nets • Designs and uses nets to make models of geometric objects studied up to and including this grade • Uses transformations and symmetry to investigate properties of geometric shapes • Uses proportion to describe the effect of similar enlargements and reduction on properties of geometric shapes • Draws and interprets sketches of objects from different perspectives with attention to the presentation of properties • Locates positions on co-ordinate systems(ordered grids); cartesian plane (1st quadrant); maps; and describes how to move between positions

Learning Outcomes	Summary of the Assessment Standards
4. **Measurement** The learner will be able to use appropriate measuring units, instruments and formulae in a variety of contexts.	• Solves problems involving time, including: relating time, distance and speed • Solves length, perimeter, area, mass and capacity/volume problems involving: estimating, selecting, calculating and converting between appropriate S.I units to at least 2 decimal places • Calculates by selecting and using appropriate formulae, e.g. the perimeter of polygons • Describes the meaning of and uses π in calculations involving circles and discusses its historical development in measurement • Estimates, compares, measures and draws, accurate to 1 degree, angles using rotograms and protractors • Investigates the relationship between the sides of a right-angled triangle to develop the Theorem of Pythagoras • Uses the Theorem of Pythagoras to calculate a missing length of a right angled triangle, leaving answers in surd form (✓) • Describes and illustrates ways of measuring in different cultures throughout history including: Determining right angles using knotted string (leading to the Theorem of Pythagoras)

Learning Outcomes	Summary of the Assessment Standards
5. **Data handling** The learner will be able to collect, summarise, display and critically analyse data in order to draw conclusions and make predictions, and to interpret and determine chance variation.	• Poses questions relating to issues in his/her environment • Selects appropriate sources for the collection of data e.g. peers • Designs and uses questionnaires with a variety of possible responses in order to collect data to answer questions • Performs simple experiments to collect data, using random number generators, coins, spinners and dice • Organises and records data, using tables, tallies and stem-and-leaf graphs • Summarises grouped and ungrouped numerical data by determining e.g. the mean, median and mode as measures of central tendency, and distinguishes between them • Determines measures of distribution, including range and extremes • Draws a variety of graphs by hand/technology to display and interpret data • Critically reads and interprets data presented in a variety of ways so as to draw conclusions and make predictions sensitive to the role of context, categories within the data, data manipulation and the role of outliers on data distribution and any other human rights and inclusivity issues • Considers a simple situation (with equally likely outcomes) that can be described using probability and lists the possible outcomes

In order to achieve these Learning Outcomes (Table 2.1) in Mathematics, the learners have to acquire the knowledge, skills, values and attitudes highlighted in the Assessment Standards. Learners have to make use of Mathematical process skills to identify, pose and solve problems creatively and critically (Winicki-Landman, 2001:30; Singh, Granville & Dika, 2002:324; Winstead, 2004:44, Department of Education, 2007a:4; Sezer, 2008:351). Learners must also be able to collect, analyse, organize and critically evaluate information given to them. It is important that both the teacher and learners communicate appropriately, using descriptions in words, graphs, symbols, tables and diagrams. It is also important to develop the learners' language skills in Mathematics, in order for them to gain confidence to investigate and explain Mathematical terms and definitions critically. Teachers should also allow learners to work with others as members of a team, group, organization and community. Collaborative work in teams and groups are believed to enhance Mathematical understanding (Department of Education, 2007a). The overall aims of the National Curriculum Statement Grade R- 9, as well as for FET Grade 10-12, is to develop a critical awareness of how Mathematical relationships in a social, environment-cultural setting can be used in an economic context and build up the necessary self-confidence and competence to deal with any Mathematical situation without fear of being impeded by Mathematics. Learners should be able to develop in-depth understanding of concepts in order to be successful in Mathematics (Department of Education, 2002:4-5).

The National Council of Teachers of Mathematics (1995:15) defines Mathematics as: a "*human activity practised by all cultures*" that enables creative and critical thinking and logical reasoning. Knowledge is constructed by observing patterns with rigorous logical thinking. Mathematical problem-solving is seen as a key element which enables learners to understand the world and make use of that understanding in their daily lives (Department of Education, 2003:9).

The consequences of the above mentioned requirements for training the teaching force are a major concern, particularly since most teachers are unfamiliar with the approaches needed to develop the necessary skills of argumentation and debate in their learners (Lombard & Grosser, 2004:212; Innabi & Sheikh, 2006:66; Brodie, 2007:3). According to Ellis (2000), the present school Mathematics has not brought much of an improvement as it is still failing to inspire and motivate learners to take up Mathematical careers. Instead, learners dislike Mathematics, which in turn blocks the possibility for them to enter into a Mathematical or technical career. This negative scenario is supported by the pass rate figures for Mathematics Higher and Standard Grade over the period of 2004 – 2007. The pass rate for Mathematics Higher Grade has decreased from 70.6% to 68.7% and the pass rate for Mathematics Standard Grade has also decreased from 51.3% to 44.4% (Department of Education, 2007a). Maharaj (2007:34) highlights the following as being possible causes for the poor performance in Mathematics: a poor understanding of terminology and concepts, an inability to recall and apply formulas and algorithms, an inability to interpret and apply skills to do calculations and solve equations and a poor knowledge base. Linking the highlighted causes to what critical thinking in general implies, it is evident to the researcher that the following critical thinking skills appear to be absent among learners in Mathematics classrooms, namely the ability to identify, interpret, distinguish relevant information from irrelevant information, and make deductions and conclusions (*cf.* 2.3). In addition to this, it could imply and/or carefully be assumed that a lack of critical thinking skills attributes to the poor performance of learners in Mathematics.

Flowing from the above discussion, the importance of critical thinking in the Mathematics classroom can be summarized as follows. Critical thinking in the Mathematics classroom will enable learners to:

- be more involved in education;
- identify, pose and solve problems creatively and critically;
- collect, analyse, organize and critically evaluate information given to them;

- communicate appropriately, using descriptions in words, graphs, symbols, tables and diagrams;
- critically investigate and explain mathematical terms and definitions;
- work with other learners as members of a team or group;
- observe patterns with rigorous logical thinking;
- apply formulas and algorithms;
- interpret and apply skills to do calculations and solve equations; and
- be equipped with knowledge, skills and values that will enable meaningful participation and offer benefits for society.

The next section will investigate in greater detail the specific role and importance of critical thinking in Mathematics.

2.4.2 The specific role and importance of critical thinking in Mathematics

Critical thinking in the field of Mathematics can be defined in many ways. According to Ennis (1993a:179), the upper three levels of Bloom's Taxonomy of educational objectives, namely analysis, synthesis and evaluations, might be offered as a definition for critical thinking. There is, however, one problem: these three concepts are too vague to guide us in developing and assessing critical thinking.

According to Graven (2002:24) Mathematics is "*a subject that is relevant and applicable to different aspects in our everyday life and local context. It involves conventions, skills and algorithms that are needed to gain access to further studies. Mathematics is the construction of knowledge that deals with qualitative and quantitative relationships of space and time.*" Van de Walle (2007:13) adds that Mathematics deals with patterns, problem-solving, logical thinking, analysing and critical thinking. Learners need to make use of Mathematics to understand the new modern world and express their understanding through language, symbols and social interaction. According to Graven (2002:24), Mathematics allows learners to critique Mathematical applications in various social, political and economical contexts.

Concerning the explanation of Mathematics above, it is clear that critical thinking in the subject of Mathematics can be considered as an important skill for it provides the learners the opportunity to view the world through a Mathematical lens and also allow them to think critically during problem-solving. Learners that think more critically in the Mathematics classroom will ask appropriate questions, gather relevant information efficiently and creatively sort through information given, reason logically from this information, and come to reliable and trustworthy conclusions (Schafersman, 1991).

Critical thinking helps a person to ask questions, gather relevant information, reason and come to a reliable conclusion (Naik, 2009). Nobody possesses these skills right from birth or develops them from basic day-to-day thinking. These skills need to be taught and developed by teachers (Naik, 2009). In the context of this study the research supports the view of Maker and Nielson (1996:69) and Dowden (2002) regarding the critical thinking skills that are important to develop in the Mathematics classroom. These skills are:

- determining fact and opinion;
- choosing relevant from irrelevant information;
- determining the accuracy of a statement;
- determining the credibility of a source;
- recognizing ambiguities, identifying underlying assumptions;
- determining external and internal bias;
- recognizing valid and fallacious arguments;
- locating an argument in a passage;

- detecting errors of reasoning and explaining how the reasoning is in error;
- evaluating evidence;
- distinguishing whether argument's conclusions follow with certainty or with probability;
- explaining similarities among problems;
- identifying and solving a variety of problems;
- creating and comparing arguments;
- identifying implicit assumptions as well as the issue in a disagreement; and
- detecting logical inconsistency and removing vagueness and ambiguity.

These skills also correspond well with the researcher's conceptualization of critical thinking in general terms, namely that it could *inter alia* be viewed as a set of interrelated cognitive skills (*cf.* 1.9; 2.3).

The absence of the above-mentioned skills among learners, as mentioned by Maker and Nielson (1996:69) and Dowden (2002), is also supported by Schafersman (1991) who asserts that *"Many 17-year-olds do not possess the higher-order intellectual skills we should expect of them. Nearly 40 percent cannot draw inferences from written material, only one-fifth can write a persuasive essay, and only one-third can solve a Mathematics problem requiring several steps."* The need for the development of critical thinking skills is thus evident from the above assertion.

According to Schafersman (1991) learners struggle to master critical thinking skills in the Mathematics classroom because of the following two reasons:

- teachers focus all their energies and efforts on the task of transmitting knowledge and learners acquiring basic knowledge;
- learners and teachers don't have the time in the Mathematics classroom to spend on the development of critical thinking skills; and

- critical thinking skills are often so subtle that teachers fail to recognize them and learners fail to realize its absence (Schafersman, 1991).

It is important for a critical thinker in a Mathematics classroom to have a critical spirit about all the aspects of life and to think critically about your own thinking. This implies the acquisition of attitudes and dispositions to get involved in critical thinking (*cf.* 2.3). Critical thinking skills could be developed by inculcating a habit of analytical and strategic thinking (*cf.* 2.3). If learners can get used to the habit of analysing every situation critically, be given time to evaluate information before reaching a conclusion, they will gradually acquire critical thinking skills (Cangelosi, 2003:125; Oak, 2008). It is therefore important that the development of the learners' disposition and attitude to critical thinking should not be ignored (Halpern, 2007:10; Facione, 2009). Critical thinking is more than just the successful use of the right skill in an appropriate context. It is also an attitude or disposition to recognize when a skill is needed and the willingness to exert the mental effort to apply it (Halpern, 2007:10).

According to Damji, Dell'Anno, McGrath and Warden (2003), critical thinking skills are nothing more than problem-solving skills that result in reliable knowledge. Critical thinking skills are needed for the practice of processing information in the most skilful, accurate and rigorous manner possible, in such a way that it leads to the most reliable, logical and trustworthy conclusions upon which one can make responsible decisions about one's life, behaviour and actions with full knowledge of assumptions and consequences of those decisions.

Now that the meaning of the concept critical thinking in Mathematics has been delineated, critical thinking has to be linked to the teaching and learning of Mathematics in more practical terms. Mathematics helps learners make sense of the world around them. Through Mathematics they will understand the world around them in terms of numbers and shapes. It is important to remember that Mathematics is not only about all the rules, operations, routine work and memorizing the different steps to solve problems. It is about the connections that learners are able to make and seeing relationships in everything that they do. Learners should develop their own knowledge and understanding of Mathematics through practical activities, explorations and discussion, learning to talk about their methods and to explain their reasoning (Fromboluti & Rinck, 1999).

Critical thinking plays an important role in Mathematics during problem-solving, algebraic reasoning, interpreting graphs and geometrical thinking. The role of critical thinking with regard to each of these areas is elucidated in the following sections.

2.4.2.1 Critical thinking and problem-solving in Mathematics

Critical thinking plays an important role during problem-solving (Fromboluti, & Rinck, 1999). Treffinger (1994:308) suggests that there are two main kinds of problems in Mathematics, **real problems** and **realistic problems**. Real problems are problems that occur and need a solution that will be implemented. Realistic problems are problems that are plausible. Both these problems provide practice in decision-making for which learners need critical thinking skills. Traditionally, critical thinking and problem-solving have been associated with different fields: critical thinking is rooted in the behavioural sciences, whereas problem-solving is associated with Mathematics and science disciplines (Erwin, 2000). Although a distinction is made between the two concepts, in real life situations the terms critical thinking and problem-solving are often used interchangeably. The goal of problem-solving is to find and implement a solution, usually to a well-defined and well-structured problem. Learners learn through problem-solving that there are many different ways to solve a problem and that there is more than one possible answer. It gives the learners the chance to explore, think through a problem, and reason logically to solve routine as well as non-routine problems (Fromboluti & Rinck, 1999). Fromboluti and Rinck (1999) argue that learners who develop the skill to think critically in Mathematics enjoy doing Mathematics more and are able to solve problems in their own way and in their own time. They understand what they are doing and why something does or doesn't work (Fromboluti & Rinck, 1999).

Critical thinking is a skill that is important during problem-solving, therefore it is important that teachers prepare the learners to think critically in the Mathematics classroom (Winch, 2006:74). In the present South African Mathematics classroom, teachers encourage rote learning (King, 2007:121). Their teaching style is one-directional and they feed information to the learners by telling them what to do. There is not much interaction between the learner and the teacher, therefore the learner finds it difficult to retain and apply new information given to him/her. According to Suliman (2006:77), teachers are likely to teach in the way they themselves were taught. Van de Walle (2001:1) asserts that teachers need to appreciate the discipline of Mathematics, understand how the learners learn and construct their own ideas and be able to design and select a variety of tasks that enable the learner to learn Mathematics in a problem-solving environment if they want to promote the development of critical thinking.

Problem-solving in Mathematics requires from learners to be able to separate relevant from irrelevant information. A critical thinker is one that is open-minded and diligent in seeking relevant information, clear about issues and orderly in complex matters. They are reasonable in identifying and selecting the most relevant information during problem-solving (Colucciello, 1997:239). The information that the learner has selected to assist with the solution to a problem needs to be analysed and integrated during the solution of the problem.

It is important that learners acquire steps for solving problems. Polya (in Macintyre, (2006:8-11) and Van de Walle (2007:37) suggest the following steps that reflect a number of critical thinking skills, to guide problem-solving. Learners need to:

- solve problems on their cognitive developmental level;
- understand the problem;
- construct a plan to solve the problem;
- motivate and explain the steps involved to obtain an answer;

- execute the plan; and
- reflect on the entire process.

In conclusion, problem-solving involves critical thinking; therefore the role of critical thinking in the Mathematics classroom can be seen as a helping hand to obtain the solutions to a variety of Mathematical problems (Schafersman, 1991).

2.4.2.2 Critical thinking and Algebra

Algebra is considered as a key area for developing critical thinking skills, for it provides the language and foundation for numerous fields in the private sector (Kollars, 2008). Critical thinking skills can be developed by allowing learners to pose their own questions and be given assignments in which they reflect on the relation between the different concepts that they have learned (matrices, vector spaces, linear transformations) (Simic-Muller, 2007). Critical thinking skills are also important when teaching algebraic reasoning, for it involves representing and formalizing patterns and regularity in all aspects of Mathematics. It is difficult to find an area in Mathematics that does not involve generalizing and formalizing in some way (Van de Walle, 2001:384). Simic-Muller (2007) argues that teachers who would like to develop critical thinking skills in the Mathematics classroom, should make use of a learner-centred classroom, be more flexible with the curriculum and need to be knowledgeable in the subject as a whole (Simic-Muller, 2007).

2.4.2.3 Critical thinking and interpreting graphs

Critical thinking skills are also important in Mathematics in order to help learners to interpret numerical relationships in graphs (Curcio, 1987:387). The interpretation of graphs is a major challenge for learners. Learners, who are at primary school level, have trouble making the transition from concrete "object graphs" to abstract representations of data. Later in elementary and middle school, they often fail to develop the deep conceptual understanding necessary to perform higher-level data analysis skills. Many learners have difficulty drawing inferences or making predictions, identifying appropriate ways to display data, or recognizing when a graph's scale has been manipulated to mislead them (Shaughnessy & Zawojewski, 1999:713). A critical aim of the Mathematics curriculum is to teach learners to interpret and analyse data from a graph, to solve problems and make decisions. According to Curcio (1987:387), all learners should master the following graph interpretation skills: extracting specific values from the graph, integrate information from different parts of the graph and understand the data set as a whole, making predictions or noticing trends.

2.4.2.4 Critical thinking and Geometry

Geometry is also an important area in the Mathematics curriculum where critical thinking is required. Many studies have shown that learners have difficulties in learning geometry and thinking geometrically (Duatepe & Ubuz, 2004). Geometry is abstract, it is about critical thinking, learning concepts and principles and knowing when and how to apply them (Chen, Chai & Zheng, 2009). When engaged with geometry problems, teachers can develop critical thinking skills by promoting the learners' imagination through improvising a concept or an event. Furthermore, teachers need to help learners to experience all aspects of a problem. Duatepe and Ubuz (2004) comment that by integrating cognitive and critical thinking skills into the geometry lesson, the learners become more used to expressing themselves and their fear of making mistakes decreases.

Now that the importance of critical thinking in the Mathematics classroom has been highlighted, ways in which critical thinking can be developed during teaching and learning will be explored.

2.5 DEVELOPING CRITICAL THINKING IN THE MATHEMATICS CLASSROOM

Based on the strong cognitive focus that is evident in the Learning Outcomes and Assessment Standards of Grade 8 Mathematics, it appears that, *inter alia*, a constructivist approach to teaching and learning should characterize the teaching and learning in Mathematics classrooms. In order to enhance cognitive development among learners, a constructivist approach advocates:

- learner participation;
- learners who create ideas on their own;
- learners who interact with others during the construction of knowledge;
- learners who seek information and solve problems independently;
- learners who can communicate and justify their thoughts; and
- learners who are involved in authentic and relevant teaching and learning activities (Eggen & Kauchak, 2004:258, 291, 301; Woolfolk, 2004:321, 332; Bjorklund, 2005:81; Ormrod, 2008:29, 196).

Based on the above mentioned characteristics, the following sections will view the opportunities that teachers can create in the Mathematics classroom for the development of critical thinking skills from a constructivist perspective.

Teachers can promote critical thinking skills through the development of problem-solving and reasoning skills (Schoënfeld, 1994:59; Elder & Paul, 2002:85; Department of Education, 2007b:4; Oak, 2008). Learners must be able to process and evaluate information given to them. Teachers, who want to integrate critical thinking in the Mathematics classroom, must plan, implement, reflect upon, revise lessons, be competent, dedicated, caring and qualified in their classrooms (Department of Education, 2003:5). Such lesson plans are based on critical thinking principles and approaches that require teachers to serve in the following roles: facilitator, organizer of the teaching/ learning/ assessment process, role model, learning mentor, content specialist and knowledge dispenser. Teachers need to use critical thinking more effectively and be prepared to understand its various aspects (Berns & Erickson, 2001).

Mathematics is a subject that requires logical and critical thinking skills as well as the ability to see alternative possible solutions to a problem (Winch, 2006:74; Sezer, 2008:351). It is therefore important that the teaching and assessment practices of teachers develop critical thinking and encourage learners to solve problems and create Mathematical ideas (Schoënfeld, 1994:59).

The aforementioned introduction made the researcher aware that opportunities for the development of critical thinking can be created by teachers through the choice of teaching methods, assessment strategies, learning material, providing a variety of activities that involve learners actively during teaching and the creation of a classroom climate that invites thinking. The following section will thus explore the development of critical thinking in the Mathematics classroom under the following headings:

- Teaching methods and strategies to develop critical thinking skills in Mathematics
- Assessment methods and strategies to develop critical thinking skills in Mathematics
- Learning material to develop critical thinking skills in Mathematics
- Learner involvement in activities to develop critical thinking skills in Mathematics
- Creating a classroom climate to develop critical thinking skills in Mathematics

2.5.1 Teaching methods and strategies to develop critical thinking skills in Mathematics

Teaching styles refer to the overarching characteristics, manner or way in which teachers' present learning material or learning activities to learners and teaching methods and strategies refer to the planning of teaching in a specific and structured way to achieve a certain outcome (Van der Horst and McDonald (2003:121).

The teaching style of a teacher can either be teacher-centred or learner-centred. The first style implies that the teacher plays the central role during teaching and instruction, and relies heavily on transmission and reception or direct teaching. Within the direct way of teaching, a teacher can make use of a number of teaching strategies such as demonstrations, video presentations and the completion of work sheets. The latter style focuses on the teacher as a facilitator of learning and relies on indirect, independent and interactive methods of teaching, where the learner plays a central role during teaching and learning. Each of the mentioned methods also comprise *inter alia* a variety of teaching strategies such as: role plays, debates, case studies, projects, investigations, experiments, discussions, problem-solving, questioning, discoveries and cooperative learning. The researcher is of the opinion that the facilitation style which focuses on teaching methods that promotes independent and interactive learning will provide learners with opportunities to develop their critical thinking skills. Therefore teaching strategies that promote independent and interactive learning will be discussed in the section that follows.

The new emphasis on critical thinking instruction has fundamentally altered what and how teachers teach in the Mathematics classroom (Halpern, 1999:71). According to the NCS Grade R-9, as well as for Grade 10-12, the teaching and learning of Mathematics aims to develop:

- critical awareness of how Mathematical relationships in a social environment-cultural setting can be used in an economic context; and
- the necessary self-confidence and competence to deal with any Mathematical situation without fear of being impeded by Mathematics.

It is therefore important for teachers to implement teaching methods and strategies that will develop metacognitive skills in order to promote critical thinking in their classrooms (Facione, Facione & Giancarlo, 2000:63; Van der Walt & Maree, 2007:227). When Mathematics teachers plan and design learning activities to support the development of critical and analytical thought in the Mathematics classroom, they should also identify and plan for the development of the cognitive skills needed to master the content that will be taught (Facione *et al.*, 2000:63; Appelbaum, 2004:308; Van der Walt & Maree, 2007:237).

Teachers need to make use of teaching strategies that motivate learners to participate in classroom activities and feel free to explore and express opinion. All learners should be given opportunities to examine alternative positions on controversial topics and to justify their beliefs about what they think is true and good, while they participate in orderly classroom discussions (Gough, 1991:3). Learners must also be given the opportunity to look at different ideas and concepts from a variety of perspectives and analyse the ideas and concepts before making a decision (Ferrando, 2001). Some of the strategies that Horton and Ryba (1986:24) identified for the development of critical thinking skills are: decision-making, problem-solving, classification and generating hypotheses. These strategies will help to foster a climate that motivates the development of critical thinking and create warmth that will improve both teaching and learning in the Mathematics classroom (*cf.* 2.5.5). It is important that teachers make use of teaching methods and strategies to motivate the learners to be more interested in Mathematics as it can improve the development and understanding of Mathematics concepts (Ferrando, 2001).

According to Cangelosi (2003:4), teachers should utilize teaching strategies and methods that focus on meaningful learning objectives that are consistent with their understanding of how learners learn. For example: the teacher designs a lesson that leads the learners to discover the arithmetic sequences by themselves. By making use of this strategy, the learners will have the opportunity to develop critical thinking skills during problem-solving, as the learner will be able to make Mathematical discoveries by themselves.

The **inquiry instruction strategy** also promotes the development of critical thinking in Mathematics as this strategy focuses on independent learning and provides learners the opportunity to analyse and create ideas on their own (Hida, Laidi & Alamrani, 2005). In order for learners to become more independent, they need the necessary critical thinking skills that will encourage them to become more actively involved in their own learning. These skills will enable them to determine their own goals, make efforts to achieve the goals and establish priorities. It will enhance learners' self-reliance, self-confidence and critical thinking skills (Hida *et al.*, 2005). The inquiry instruction strategy also helps learners to gauge how they are thinking about and learning Mathematics by providing opportunities to communicate mathematically (Ash, 2005).

Gokhale (1995:22) and Myren (1995:5) indicate that the **collaborative strategy** works very well with all learners to develop critical thinking skills. Collaborative learning refers to an instruction strategy in which learners at various levels work together and actively exchange ideas in small groups towards a common goal (Gokhale, 1995:22; Gawe, 2007:208-227). Learners are randomly selected for group work so that they have the opportunity to interact with different individuals. During the group process, the learners are allowed to make use of a variety of materials like whiteboards, papers and books, to create and draw diagrams or flowcharts to express their thinking and then present them to the other group members (Myren, 1995:5). Collaborative learning can be used in the Mathematics classroom to increase the learners' interest as well as to promote critical thinking skills (Gokhale, 1995:22). Collaborative learning provides learners the opportunity to think creatively with others, communicate thoughts to others, solve problems and make decisions as a team and reflect on the solution for a problem (Gokhale, 1995:28; Marcut, 2005:63). According to Olivares (2005), collaborative critical thinking could be defined as a relatively unstructured social process that results in judgments being made or problems solved through the process of conversation and through the use of evidence, inference, interpretation, logic and reflection. By making use of critical thinking skills during small group collaborative learning, learners will develop the ability to discuss, clarify and evaluate ideas and problems that they may have encountered (Gokhale, 1995:28). As with the inquiry instructional strategy, collaborative learning offers learners the opportunity to freely express their ideas in Mathematical terms which in turn could improve their Mathematical language. It is important that learners learn how to communicate their Mathematical thinking coherently and more clearly to their other classmates and teachers to improve their reasoning skills. Whatever teaching strategy the teacher selects to use in the Mathematics classroom, it should allow the learners to analyse, assess and communicate their Mathematical thinking to others while making use of the

Mathematical language to express their Mathematical ideas (Appelbaum, 2004:309)..

Interactive learning is a well used strategy to help learners to think more critically in any classroom. Teachers can make use of this strategy by demonstrating critical thought through asking questions such as "*why*", "*what about*", "*tell me more*" *or* "*explain*" to encourage learners to make their own comparisons and conclusions related to the problems given to them (Searls, 2006; Arends, 2009:345-452). Teachers can create more interactive teaching and learning by demonstrating to the learners how they should study and explore the new material at the beginning of a lesson (Searls, 2006). After the work has been demonstrated and explained, the learners should be given the responsibility and necessary resources to assist them to seek information on their own or to complete the task or problem given to them independently (Searls, 2006). Teachers who provide more opportunities for interaction with learners are more able to identify those learners who need to know more about a topic before requesting them to make decisions or draw conclusions (Morris, 2007).

Teachers then need to ask the learners questions that trigger them to think more critically (Kok, 2007:11; Morris, 2007). For example, when teachers present a specific theorem to their learners, such as the Pythagorean Theorem, teachers could ask the learners what they might think of the Pythagoras Theorem before they introduce the learners to the steps proving this theorem. Questioning can facilitate a discussion that will encourage learners to ask questions to get them involved. Answering questions and formulating questions could assist learners in making their own conclusions about Mathematics, and encourage them to start thinking in a logical way. It could also enable them to see applications for themselves, and to avoid them relying on the teacher to apply the material for them. Learners' active involvement during teaching and learning reinforces the development of critical thinking, and contributes to the learners' quest to become independent learners. It is important for the development of critical thinking that learners experience a dialogical style of teaching (Bullen, 1998:23). Teachers, who make use of a dialogical style in teaching, enhance ongoing interaction between the learners and themselves through discussion, inquiry, and the free exchange of ideas (Sternberg & Martin, 1988:535-578). A dialogical style of teaching is non-directive and focuses more on process rather than content (Bullen, 1998:23). It is important that the dialogue between the teacher and learners are focused and relevant to the topic and involves all the learners to participate and interact in the classroom (Muirhead, 2002).

Van de Walle (2001:58) introduced the notion of a **pattern strategy** that can be used during many teaching and learning activities, especially in the algebraic reasoning strand. The learners need to look for different patterns in numbers as it helps the learners to learn and master basic facts. This strategy is very important in Mathematics during the learners' middle and high school years. Van de Walle (2001:58) also suggests that the teachers should present the learners first with problems that are easy and simple before giving them more complex problems to solve. By making use of this mathematical strategy, the learner will understand and analyse a task better and be more motivated to solve the more difficult problems. Teachers who abandon these methods and strategies in the Mathematics classroom make learners passive recipients of information rather than adopting those strategies that transform them into active participants in their own intellectual growth (Marcut, 2005:63).

According to Mahaye and Jacobs (2007:200), a teacher can use **experiential learning strategies** as it promotes the development of critical thinking in any classroom. Experiential learning is constructivist learning, because experimental learning involves learners to be more active during teaching and learning. Learners get the chance to construct their own knowledge, rather than observing the demonstrative behaviour of a teacher. Experiential learning could involve one or more of the following instructional strategies: experiments, field trips, games, model building, role plays, simulations and surveys. Because experiential learning is active learning, learners more readily understand what they are learning and thus retain the knowledge to a greater degree than when merely having information presented to them by another. The hands-on nature of experiential learning is highly motivating for learners (Mahaye & Jacobs, 2007:200).

The following teaching strategies that teachers could use to promote critical thinking skills in their classrooms focus primarily on encouraging the learners to think logically during problem-solving while they analyse data (Spache & Spache, 1986).

Two strategies identified by Leader and Middleton (2004:65), namely the **Jasper Problem Series** and **Decision-making**, can be used to promote critical thinking in the Mathematics classroom. Both of them involve ill-structured problem-solving and promote repeated expression of critical thinking through the give and take of possible alternatives for solving problems and developing support for arguments (Leader & Middleton, 2004:65). The Jasper Problem Series provides the learners with richly detailed ill- structured problems that are designed to create interesting and realistic contexts that encourage the active construction of knowledge by learners (Leader & Middleton, 2004:61). The use of ill-structured problems helps the learners to define what the problem is actually about and determine what information and critical skills are needed to help solve the problem on their own (Jonassen, 1997:68). Learners who have been exposed to this learning method seem to be less anxious about Mathematics, more likely to view Mathematics as relevant to everyday life, more likely to describe it as useful and more likely to appreciate complex challenges (Leader & Middleton, 2004: 62). The Decision- making strategy involves the learners in the application of linear algebra to difficult real-world problems with multiple solutions (Leader & Middleton, 2004:62). These are effective methods for they allow the learners to construct their own ideas before the final answers are given (Middleton & Roodhart, 1997:46).

According to Beyer (1985:273) and Borich (2004:214-256), teachers need to recognize several general aspects when they develop a teaching method to teach critical thinking in any subject, including Mathematics. Firstly, teachers need to promote **interaction** among the learners as they learn. Interaction during teaching and learning makes learners more relaxed and willing to ask questions. Secondly, teachers need to make use of **open-ended questions** as a **strategy** because these questions do not presume that there is only one right answer. Open-ended questions encourage learners to think and respond creatively, without the fear of giving a wrong answer. It also allows the learners to solve the problem more critically. Thirdly, teachers need to allow the learners sufficient **time to reflect** on the questions asked or the problems that have been given to them. Teachers who allocate the necessary time to the learners to solve problems in the Mathematics classroom will obtain a better response from the learners. Finally, teachers should make sure that the **learners know what is expected** from them and that they have the necessary skills to think critically while they solve problems.

The above mentioned methods and strategies are not only important to promote critical thinking in the Mathematics classroom, but also necessary to inspire the learners to think about Mathematics more creatively (Ferrando, 2001). According to Appelbaum (2004:310), in any Mathematics classroom, teachers need to design activities and use strategies throughout the year that enable learners to organize and consolidate their Mathematical thinking and foster critical thinking.

In the context of this study, the researcher aimed to determine whether the teachers who took part in the study made use of the variety of teaching strategy and methods as discussed above in order to create opportunities for the development of critical thinking skills.

The following discussion focuses on the different assessment strategies and methods that teachers could use in the Mathematics classroom to develop critical thinking skills.

2.5.2 Assessment methods and strategies to develop critical thinking skills in Mathematics

Just as important as the use of a variety of teaching methods and strategies is for the development of critical thinking, so is the use of a variety of assessment strategies to assess each learner's performance.

According to Kotze in Maree and Fraser (2004:51), assessment refers to the use of different methods, strategies/techniques and tools teachers can use to assess the learners. Assessment strategies are the approaches taken to assess a learners' performance, using a number of assessment forms appropriate to the task and the level of the learners understanding. Some examples of assessment strategies include oral presentation, practical activities, assignments, projects, research and tests (Maree & Fraser, 2004:51). Assessment methods on the other hand can be used to collect evidence of learners' performance. Some assessment methods that teachers may use are self assessment, peer assessment and teacher assessment. Teachers then usually make use of criteria to assess individual learners for different purposes to determine their level of performances (Niedringhaus, 2001:10; Maree & Fraser, 2004:68).

The term assessment in Mathematics refers to the identification and appraisal of learner knowledge, insight, understanding, skills, achievement, performance and capability in Mathematics (Niss, 1998:263). Van de Walle (2001:62) defines assessment as *"the process of gathering evidence about a learner's knowledge of, ability to use, and disposition toward Mathematics and of making inferences from the evidence for a variety of purposes".* In essence, the purposes of assessment in Mathematics are to promote academic and cognitive growth, improve instructions, modify programmes and recognize accomplishments by the learners (Van de Walle, 2001: 64).

Teachers who teach Mathematics need to make use of a variety of valid assessment methods and strategies which will make Mathematics accessible to the broader population and promote equity between the learners (Niess & Garofalo, 2006:3798). Equity in assessment sets high Mathematical standards for all learners, including those with special needs (National Council of Teachers of Mathematics, 1995:15) and will help increase the pass rate (Matutu, 2006:2). Presently in Mathematics classrooms, teachers most often only make use of exams, projects, assignments and term tests to assess the learner's performances (Elder, 2007). These assessment methods are not enough to provide evidence that the learners understand and know how to solve problems. According to the NCS, teachers also need to use criterion-referenced assessment and peers assessment methods, which will ensure transparency. Teachers, who apply a wider range of assessment methods and make use of contextual problem-solving during a Mathematics lesson, will be more likely to nurture the development of critical thinking skills and accommodate different learner preferences (Matutu, 2006:3). Assessment should reflect the Mathematics that all learners need to know and be able to do in order to enhance Mathematics learning (National Council of Teachers of Mathematics, 1995:11-13). It is also important that teachers encourage critical thinking by assessing the reasons for and against doing something and then base their decisions on the basis of a fair assessment (Lidz & Gindis, 2003:100; Le Grange & Reddy (in Jacobs, Vakalisa, Gawe, 2004:270).

Critical thinking in Mathematics focuses primarily on problem-solving; therefore it is important that teachers should assess the following aspects:

- learners' ability to separate relevant from irrelevant information;
- learners' ability to integrate information to solve problems;
- learners' ability to use Mathematical skills to solve real-world problems; and
- learners' ability to learn and apply new information (Stein, Haynes & Redding, 2006).

Teachers need to make use of well planned assessment methods that meet these afore-mentioned aspects to determine how learners' performances reflect their achievements of the curriculum outcomes, for example self assessment, peer assessment and educators assessment (Niedringhaus, 2001:10). Stein *et al.* (2006) identified three important characteristics of assessment that can help to improve the quality of assessment of critical thinking in Mathematics:

- the assessment method that the teacher uses must be an authentic and a valid measure of progress toward the underlying goal;

- the assessment should promote the motivation from within to change; and
- the assessment should evaluate skills that are considered important within the framework of contemporary learning sciences.

Teachers need to assess critical thinking in Mathematics by making use of assessment strategies that will help to provide more accurate feedback on the learners' performances and also assess the learners' ability to perform various tasks (Ellis, 2000). It is also important that the teachers assess the extent to which critical thinking is being fostered in the Mathematics classroom. They can make use of random samples of learners' work and then assess the work for evidence of critical thinking (Elder, 2007).

Teachers do not always have to make use of the prescribed assessment methods that the Department of Education has given to them. They do have the choice to develop and design their own assessment methods for a specific task and lesson (Stein *et al.*, 2006). Before teachers do design and develop an assessment method, they need to take the following two aspects into consideration, namely the need to identify what the focus of the lesson is and what they want to assess afterwards. According to Byers (2004), teachers who don't make use of a variety of assessment methods and strategies to assess learners as problem-solvers on algorithmic and routine memory computations, may find that the learners perform poorly. The learners mostly lose interest and are less involved in the Mathematics classroom. It is important that the Mathematics teacher designs and uses a variety of assessment methods as the value of independent, critical thinkers will be lost in a static, skills-based curriculum that is limited in context (Department of Education, 2007c:7).

The new NCS curriculum for Mathematics is problem-centred; therefore new assessment strategies that support the classroom activities, assist learners' development and seek to assess the learners' understanding of Mathematics concepts, are required (Suurtamm, 2004:499; Department of Education, 2007d:7). Two assessment approaches that can be used in the Mathematics classroom are assessment of learning (summative assessment) and assessment for learning (formative assessment). Teachers use assessment of learning to gather and provide evidence of the learners' achievements while assessment for learning focuses on utilizing assessment to promote the holistic growth and development of learners (Stiggins, 2002:759). Assessment for learning refers to all the activities undertaken by the teachers and by the learners in assessing themselves, which provides information that teachers can use as feedback to modify the teaching and learning activities in which learners engage (Black & William, 1998:139; Black, Harrison, Lee, Marshall & William, 2004:10). Assessment of learning is about feedback on teaching and learning and using that feedback to shape the instructional process and improve learning further (Carter, 2005:10).

Assessment of learning generally takes place after a period of instruction and requires making a judgement about the learning that has occurred (Boston, 2002). Assessment for learning should mostly be used in the Mathematics classroom to identify the range of the learners' understanding and to allow access to all learners to use their informal knowledge to solve Mathematical problems (Kestell, 2006). Assessment for learning encourages the development of critical thinking skills as this type of assessment expects of learners to explain their thinking and reasoning within a secure classroom ethos.

Performance assessment and **authentic assessment** are two very well known approaches to assessment that are used in the Mathematics classroom. Unlike the traditional assessment strategies that mainly focus on the answer or the use of a suitable algorithm to get to the answer, authentic assessment and performance assessment are more capable to provide the teacher and learners with a broader range of measures to assess the learners' learning and cognitive skills (Suurtamm, 2004:499; McMillan, 2001:9-10; Smith, Smith, & DeLisi, 2001:46). Authentic assessment, also known as alternative, performance-based, or outcomes-based assessment includes the use of a variety of strategies such as: written products, solutions to problems, experiments, exhibitions, performance, portfolios of work and teachers' observations, checklists and inventories, and cooperative group projects (Hart, 1994:30; Stiggins, 2002:761; Department of Education, 2007a). In Mathematics, authentic assessment can be used by asking open-ended questions. Open-ended questions have no single right answer because the question is designed to see how a learner thinks through a problem, thereby indicating the ability to use Mathematics. The learners are then asked to write their own responses to the problem. This method of assessment is very useful to gather information about the learners' critical thinking skills in a variety of contexts (Neufeld, 1994:111; McMillan, 2001:9-10; Smith *et al.*, 2001:46).

Performance assessment, on the other hand, contains tasks that are more complex than those normally associated with traditional assessment instruments, and are rich in contextual detail. It is designed to assess the learners' critical thinking skills (Neufeld, 1994:98). Performance assessment in Mathematics measures higher-order thinking skills that reflect ideal instructional practices, involving the teacher as participant or observer and allowing the learners to do more investigation. It also requires collaboration during problem-solving, allows multiple strategies for solving problems, prefers multiple solutions to problems, integrates knowledge and processes, is relevant to real life experiences, pitched at an appropriate level of difficulty and is cost effective (Jorgensen, 1994:48). This assessment method does not focus on neat, tidy kinds of problems often found in textbooks or standardized tests. It can be used to assess various types of issues of learning and is therefore a very good measuring tool to assess critical thinking in the Mathematics classroom (Neufeld, 1994:111; McMillan, 2001:9-10).

Bolte (1999:22) listed a few creative **formative assessment** methods that should be present during Mathematics teaching, namely peer-assessment, self-assessment and learners coming up with their own questions which could be considered for summative assessment purposes. These assessment tasks, along with concept maps and interpretive essays, assess learners' organization of mathematical knowledge. Formative assessment tools should be used to assess the mathematical content knowledge and the multiple mathematical cognitive processes that the learners use in Mathematics like reasoning, communicating, problem-solving and making connections. According to McIntosh (1997:93), formative assessment can be utilized to assess the learners' mathematical disposition such as attitudes, persistence, confidence and cooperative skills. Assessment that is a routine part of ongoing classroom activity, rather than an interruption, helps learners in setting goals, assuming responsibility for their own learning and becoming more independent learners. These skills named above are very important for solving mathematical problems and improving the learners' academic performances (Delandshere & Arens, 2003:59; McMillan, 2007:4).

In the context of the study the researcher investigated whether teachers provide opportunities for assessment of learning in order to develop critical thinking skills.

2.5.3 Learning support material to develop critical thinking skills in Mathematics

According to Volmink (1994:61) and Ellis (2000), South African Mathematics teachers rely mainly on Mathematics textbooks that include routine exercises that emphasize drill and practice learning. Although a textbook can be the starting point for developing critical thinking skills, Lehman and Hayes (1985:166) believe a teacher needs to identify a variety of resources that learners may use for comparing and synthesizing their ideas. Many of the textbooks that the teachers and learners use in the Mathematics classroom focus mainly on providing learners with exercises. The textbooks also illustrate different Mathematical strategies that the learners may use to solve problems (Volmink, 1994:61). Ellis (2000) argues that these textbooks limit the learning process by only illustrating a few selected learning methods and strategies that the learners may use to solve problems. It is therefore important for teachers, apart from the textbook, to make use of a variety of learning material depending on whether the aim is to develop analytical understanding or a problem-solving ability (Ellis, 2000). In the context of the study I argued that excessive reliance on textbooks would inhibit the development of critical thinking in the Mathematics classroom.

Teachers may also make use of children's literature to create a variety of problems drawn from real-life experiences. It will help the teachers move from direct instruction to inquiry instruction where the learners discover properties of numbers and relationships on their own. Teachers who make use of Mathematics related literature allow the learners to bring their own background of experiences to the text and extend the learners' understanding of concepts and skills in a problem-solving environment (Lake, 2009:14).

Critical thinking in the classroom can also be promoted by making use of technology. Calculators, computers, software programmes, power point, computer Mathematics games, on-line assessments and active white boards are well used materials that teachers may use to enhance critical thinking in the Mathematics classroom (Ash, 2005). Visual aids, such as posters, also encourage ongoing attention to critical thinking. Teachers may ask a variety of questions or make suggestions that remind the learners what they should or could do to answer the questions posed to them during the presentation of a poster.

2.5.4 Learner involvement in activities to develop critical thinking skills in Mathematics

In order to make Mathematics more interesting for the learners, teachers need to give the learners the opportunity to organize their ideas, support value judgements and apply and analyse new information that they have gained on their own during the learning experience. Not only will the learners be more active in the Mathematics classroom, but the teacher will also promote critical thinking skills (Gallagher, 1975). While thinking about what critical thinking skills the teacher wants to implement into their classroom, it is important that he/she is well grounded in the knowledge, skills, values, principles, methods and procedures applicable to South African conditions. The critical thinking skills should also be relevant to the certain topic and learning material that will be used. Teachers need to teach diverse groups of learners in the Mathematics classroom, therefore it is important that, when teachers plan their lessons, they take into consideration what the different needs and abilities of the learners are (Van de Walle, 2001:59). Teachers also need to consider the critical thinking skills that learners bring with them to the Mathematics classroom, for it will support the learners' involvement during group and class discussions (Appelbaum, 2004:308).

One way to involve learners more in the Mathematics classroom is by starting a discussion. Discussions provide learners with the opportunity to formulate their thoughts and motivate them to be more willing to participate during the lesson. It is also important to ask for explanations to accompany all answers because most incorrect answers are the result of small errors in the context of otherwise excellent thinking (Van de Walle, 2001:50). Van de Walle (2001:51) argues that the learners will develop critical thinking in the Mathematics classroom easier when they are given the chance to explore ways for solving problems on their own and then explain all the methods and strategies that they have used during problem-solving. In addition to this, Marcut (2005:62-63) argues that learners who are more involved in the Mathematics classroom are able to organize and consolidate their mathematical thinking through communicating clearly with peers, teachers and others. They are also able to analyse and evaluate the mathematical thinking and strategies of others and use the Mathematics language to express their mathematical ideas precisely.

By improving the ability to use Mathematics language efficiently, it is important that teachers don't only make use of formal mathematical language, but also recognize Mathematics as a group experience that requires reading, writing, listening, speaking and the use of various models of representations (Appelbaum, 2004: 308). The National Council of Teachers of Mathematics (2000:60) asserts that "*the students who have opportunities, encouragement, and support for speaking, writing, reading, and listening in Mathematics classes reap dual benefits: they communicate to learn Mathematics, and they learn to communicate mathematically*". Communicating ideas in the classroom will enable learners to develop critical thinking skills (Marcut, 2005:62-63). In support of Marcut (2005:62-62) Applebaum (2004:308) argues that learners who participate actively in the experiences of the Mathematics classroom should be able to demonstrate critical thinking by:

- organizing and consolidating their mathematical thinking through communication;
- communicating their mathematical thinking coherently and clearly to peers, teachers and others;
- analysing and evaluating the mathematical thinking and strategies of others; and
- using the language of Mathematics to express mathematical ideas precisely.

Learners who are critical thinkers are able to identify the essential elements in a problem as well as interact between those elements. They can assign relative values to essential elements of a problem and use those values to rank elements in meaningful ways. Learners should be able to assess all the similarities and differences between elements during problem-solving and then construct relationships between the essential elements (Ennis, 1992:88). Learners need to extract implications and conclusions from facts or data gathered to determine the strengths, limitations, value of information and solutions in productive ways. Learners then need to build new solutions through novel combinations of existing information (Ennis, 1992:88).

Learners need to be fair and open-minded while thinking carefully about what to do or what to believe (Facione, 2009; Halpern, 2007:10). Teachers should encourage learners to think critically in the Mathematics classroom by making lessons relevant to the learners' lives (Ash, 2005). They should respond to the learner's interest and needs, incorporate a variety of learning activities to accommodate strengths in learning styles, and encourage cooperative learning. Learners will then feel more willing to participate in the classroom and make a bigger effort to understand and solve the Mathematical problems (Cantrell, 2000).

In the context of the study, the researcher looked for evidence of a variety of learning activities to support the fact that opportunities for the development of critical thinking skills are provided during the teaching of Mathematics.

2.5.5 Creating a classroom climate to develop critical thinking skills in Mathematics

The National Council of Teaching of Mathematics (1989:29) indicates that, *"A climate should be established in the classroom that places critical thinking at the heart of instruction. To give learners' access to Mathematics as a powerful way of making sense of the world, it is essential that an emphasis on reasoning pervades all Mathematical activity."*

Recent Mathematics education reform efforts call for creating Mathematics classroom environments where the learners have opportunities to reason and construct their own understanding (Staples, 2007:167). Mathematics is a subject that should require from learners to take risks, explore, reason, ask questions and motivate their problem-solving methods. This may sound like fun, but learners are frightened when the teachers don't tell them exactly what to do while they are busy solving a problem. It is therefore important for the learners to experience a classroom climate where doing Mathematics and problem-solving are not threatening to them. The learners should feel comfortable making use of a variety of problem-solving methods and be willing to take risks knowing that they will not be ridiculed (Van de Walle, 2001:17).

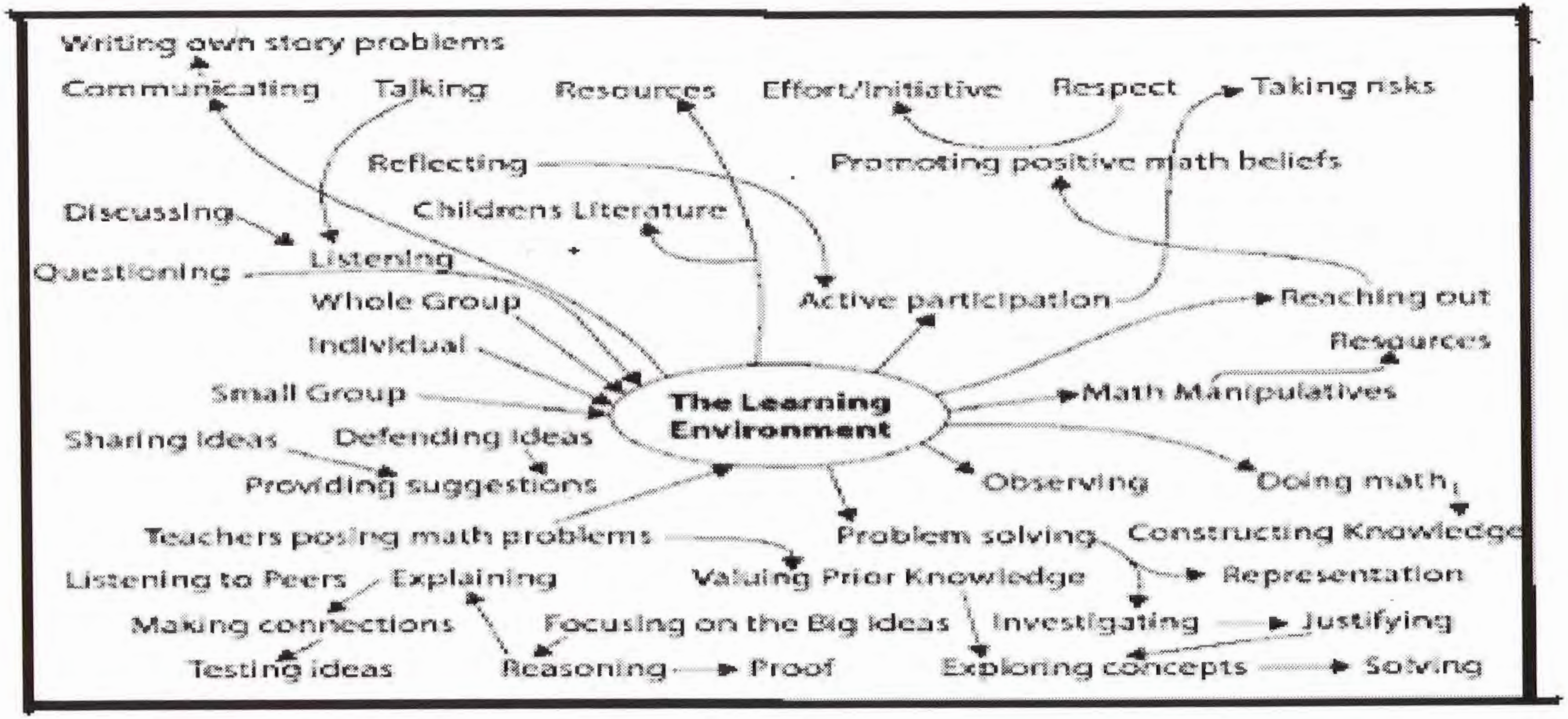
Developing critical thinking is a kind of shorthand for a wide range of activities designed to promote cognitive development. Monteith (1999) is of the opinion that it is important to note, however, that classroom activities for teaching thinking skills are unlikely to be successful unless the teacher establishes an appropriate classroom climate. Learners will not feel safe to engage in the process and motivated to do so.

Although the development of critical thinking skills depends to an extent on brain maturation, it relies just as much on opportunities to learn how to think critically (Monteith, 1999). Almost everyone can become better at thinking and learning. Some people manage this for themselves in the course of their development. Many others can do so with the right kind of assistance. This means that teachers have a responsibility to teach learners how to use their minds as well as to impart subject knowledge. This is not simply a matter of impersonal instruction in "thinking skills". It also involves feelings, values, attitudes and motivation. Learners can only take ownership of critical thinking skills if they are motivated to do so (Monteith, 1999).

According to Monteith (1999) and Crotty (2002), a safe learning environment has to be created by the teacher through the acceptance of diversity in terms of opinions, learning styles and ways of thinking. What may be new in this type of approach is the explicit emphasis on drawing learners' attention to how they are thinking and learning and on showing learners effective and recognized ways of using their minds instead of just hoping that these will develop.

The characteristics embedded in a learning environment mindscape conducive to the development of critical thinking, as indicated in Figure 2.2, demonstrates a shift in the culture of the Mathematics classroom. In an environment that focuses on nurturing the development of critical thinking, learners and teachers listen carefully to each other, respond to each other and pose questions to each other (Lake, 2009:14).

Figure 2.2: Developing a problem-solving environment in Mathematics (Lake, 2009:14)



The above figure supports the argument of Keefe and Walberg (1992) that critical thinking in the classroom is facilitated by a physical and intellectual environment that encourages a spirit of discovery. Critical thinking in Mathematics can be promoted by creating the correct motivational climate, and is facilitated by a physical and intellectual environment that inspires learners to explore on their own (Potts, 1994). To help promote this perfect classroom climate, all learners need to experience a positive classroom climate that is characterized by high expectations, teachers' warmth and encouragement, and pleasant physical surroundings that enhance all kinds of learning. All the needs of the learners should be considered and the teachers should evaluate the general culture of their classroom systematically as the climate will have an effect on the opportunities for developing critical reasoning among learners (Klein & Orr, 1991:131). Critical thinking is something that should live and grow with the learners over the years and make a deep and lasting imprint in their minds (Elder, 2007).

It is the role of the teacher to create a climate for the nurturing of critical thinking skills in the Mathematics classroom as it will produce learners who are more productive in the learning process. It will also create a broader base of participation and capitalize on a wider range of learner contributions (Staples, 2007:201). Teachers, who want to create a suitable climate for critical thinking in the Mathematics classroom, need to know what the foundations and the necessary conditions for critical thinking are, as both play a primary role in learning (Elder, 2007; Porch, 2002). In the Mathematics classroom, teachers need to develop the understanding of Mathematics concepts. Teachers need to design a classroom climate that brings meaning to Mathematics and gives the learners the opportunity to think, explain, compare and explore. Teachers need to foster a climate of trust, inquiry and expectation where the learners feel comfortable trying out new ideas, sharing insight, challenging each other and explaining their methods for problem-solving. A climate that has few or all of these characteristics can help the learners to believe that they can do Mathematics (Elder, 2007).

A mathematical classroom climate requires both the teacher and learner to respect one another, listen attentively to what the other one has to say and learn to disagree without offending each other (Van de Walle, 2001:36). Teachers need to create classrooms that invite intellectual openness and a stimulating and supportive climate for the development of critical thinking skills (Porch, 2002). According to Thacker (quoted by Gough 1991:3), the following teacher behaviours will help in fostering a climate that encourages the development of critical thinking skills in any classroom. Teachers need to:

- set ground rules well in advance. Learners must know from the beginning what they may and may not do and the type of behaviour expected from them;
- provide well planned activities that are not threatening to the learners. Learners must find these activities interesting and enjoy doing them. As soon as learners feel that activities are boring or too difficult, they will lose interest and won't bother doing the activities at all;
- be flexible in the subject and use a wide variety of teaching methods. It is important that teachers don't only make use of one teaching method, for this will create a climate of boredom;
- allow the learners to be active participants. A social climate in a classroom contributes to independent learning and increases the motivation to learn;
- show respect for the learners. Teachers should respect the learners' differences and acknowledge everyone in their classroom. Learners will then feel more comfortable in the classroom and feel free to inquire and ask questions; and
- exhibit a positive attitude towards the class as well as the work. This will create warmth and an affirming classroom climate that will maximize learner performances.

It is important that teachers see critical thinking as the heart of teaching and learning and create a warm and comfortable learning environment for learners to develop these thinking skills. Critical thinking must become the defining concept for the Mathematics classroom (Elder, 2007).

2.6 CHAPTER SUMMARY

From the literature review, the researcher concludes that critical thinking is not only important to education, but exceptionally necessary for the teaching and learning of Mathematics (*cf.* 2.4.2). To develop critical thinking in the Mathematics classroom, learners need to become more active during the learning process and not remain passive as is currently experienced (*cf.* 2.5.1). Learners should be given the chance to take their learning into their own hands by analysing, questioning and coming to their own conclusions (*cf.* 2.4.2). Through the process of critical thinking, the learners can develop the necessary skills needed in Mathematics for the following areas: problem-solving, algebraic reasoning, communication, making interpretations of numerical relationships, thinking geometrically (*cf.* 4.2) and coming to conclusions (*cf.* 2.5.2). It is important that learners need to be more involved in the Mathematics classroom as it is important to develop critical thinking more successfully. If they are given the opportunity to explore and solve problems by themselves, they will do better in Mathematics (*cf.* 2.4.2). Learning becomes more personal and meaningful to the learners when they are able to investigate, discover, reason, argue, justify, prove, and apply what they have learned. Developing critical thinking in the classroom is a process that takes time and experience from both the teachers and the learners (*cf.* 2.4.1). A strong emphasis on the use of the textbook during teaching should be avoided. Learners should be encouraged to bring their own information to the classroom and real life experiences should also be included during the teaching of Mathematics (*cf.* 2.5.3). Teachers need to prepare their lessons very thoroughly (*cf.* 2.2) by utilizing a variety of teaching methods and strategies that promote learner participation, independent learning, collaborative learning and experiential learning (*cf.* 2.5.2). Furthermore, teachers should utilize assessment for learning approaches where learners are involved in self-assessment, peer-assessment, performance assessment and authentic assessment activities (*cf.* 2.5.3). Learning tasks that involve learners in activities where they need to discuss, communicate with their peers, make conclusions and evaluate different viewpoints are essential for the development of critical thinking skills (*cf.* 2.5.4). Classroom climates conducive to the development of critical thinking skills should bear evidence of promoting productive learning, a spirit of inquiry and discovery, encouraging exploration, teachers and learners listening to each other, responding to each other and posing questions to each other (*cf.* 2.5.5). Teachers need to know what the foundations of critical thinking are (*cf.* 2.3), as it will enable teachers to provide opportunities in the classroom that will produce learners who are more productive. Teachers must bring meaning to Mathematics and create opportunities for learners to enjoy the subject to its fullest.

Teachers need to break away from teaching methods that only allow the learners to memorize facts and assess them with multiple choice tests and papers (Gallagher, 1975). Mathematics teachers need to become personally involved in the learning process. Learning is not passive, but an active process requiring a personal commitment from both the teachers and learners (Gupta, 2001).

This chapter explored the concepts which central to the study, namely critical thinking and how it could be developed in a Mathematics classroom.

CHAPTER THREE

EMPIRICAL RESEARCH DESIGN

3.1 INTRODUCTION

The previous chapter explained the concept critical thinking: its nature, role and importance in the Mathematics classroom. In this chapter, the research method used to investigate the research problem, namely to determine the opportunities that teachers provide for the development of critical thinking skills in Mathematics, is elucidated.

This chapter is devoted to a description of the empirical research resulting from a literature study. It was necessary for the researcher to get acquainted with the theoretical background regarding research frameworks, quantitative research, the construction and administering of questionnaires, analysing data by means of descriptive and inferential statistical procedures and the ethical procedures that have to be adhered to when conducting research. The main purpose of this chapter is to provide a comprehensive explanation of the following:

- Aim and objectives of the study
- Empirical research
- Data gathering instruments
- Pilot study
- Population and sample
- Data analysis
- Ethical considerations

3.2 AIM AND OBJECTIVES OF THE STUDY

As indicated in the opening chapter of this study, the overall aim of this study was to determine the opportunities that teachers create for the development of critical thinking skills in the Mathematics classroom.

The overall aim was operationalized as follows:

- by delineating the meaning of the development of critical thinking skills through a literature review;
- by determining how critical thinking skills can be developed during the teaching and learning of Mathematics through a literature review;
- by scrutinizing teachers' perceptions regarding ways in which critical thinking can be developed in the Mathematics classroom by means of an empirical study;
- by establishing what types of teaching strategies and methods and assessment strategies and methods teachers utilize in the Mathematics classroom to develop critical thinking skills by means of an empirical study;
- by determining the different types of learning material that teachers use during Mathematics teaching to develop critical thinking skills by means of an empirical study;
- by establishing the types of learning activities that teachers structure in the Mathematics classroom to develop critical thinking skills by means of an empirical study; and
- by examining how teachers in the Mathematics classroom create a climate conducive to the development of critical thinking skills by means of an empirical study.

Before the research was conducted, the researcher had to determine from which lens the phenomenon critical thinking as it relates to teaching and learning in the Mathematics classroom, would be viewed. For this purpose, the research first of all had to be located within a research paradigm.

3.3 EMPIRICAL RESEARCH

3.3.1 Research paradigm

According to Cohen, Manion and Morrison (in Maree & Van der Westhuizen, 2007:31), research is about understanding the world/phenomena and how you see the world/phenomena. This implies that a researcher should:

- determine whether phenomena will be understood from an external, objective (realist) view, or through the words created by individuals (nominalist view) (Cohen *et al.* in Maree & Van der Westhuizen, 2007:31);
- determine whether knowledge gathered can be viewed as objective (positivist stance) or interpretive (anti-positivist stance); and
- adopt a stance regarding human nature, that is: whether humans respond mechanically to their environment, whether they initiate their own actions or whether they fall somewhere between the two mentioned extremes.

In the context of this study, the researcher wanted to understand the phenomena critical thinking and the teaching of Mathematics from an external and objective stance. This stance directed the study towards a quantitative study. The researcher thus operated from a positivistic paradigm and envisaged objectively obtaining information to achieve the aim and objectives of the study.

The following sections will highlight the choice of the research method, research design, data collection instrument and the procedure for data analysis as related to the positivistic research paradigm.

3.3.2 Research method

When choosing what research methodology the researcher wants to make use of, the researcher needs to establish what kind of data needs to be gathered and what the role of the researcher is. In support of the positivistic paradigm, the researcher decided to make use of data in the form of numbers and to be an external observer whose intent it was to establish and confirm relationships and to develop generalizations that contribute to theory. A quantitative approach was therefore chosen (Leedy & Ormrod, 2005:95). The researcher wanted to establish how the present application of teaching methods, assessment strategies, the types of learning material and learning activities used, as well as the classroom climate that is created during the teaching of Mathematics in order to provide opportunities for the development of critical thinking skills.

Quantitative research can be defined as a process that is systematic and objective in its ways of using numerical data from only a selected subgroup of a universe (or population) to generalize the findings to the universe that is being studied (Maree & Pietersen, 2007a:145). According to Eldabi, Irani, Paul and Love (2002:65), quantitative research is *"a logical linear structure, in which hypotheses take the form of expectations about likely causal links between the constituent concepts identified in the hypotheses".* It is used to answer questions about relationships among measured variables with the purpose of explaining, predicting and controlling phenomena (Leedy & Ormrod, 2005:94). As it was the intention of the researcher to examine and explain existing conditions in Mathematics classrooms, quantitative research was chosen. Quantitative research relies on the measurement and analysis of statistical data that determines the relationship between one set of data to another. It was also important for the researcher to establish whether quantitative research could be regarded as a valid way to conduct research that set out to determine the opportunities that teachers create for the development of critical thinking skills in the Mathematics classroom.

3.3.2.1 Validity of quantitative research for this study

Validity is a measure or instrument that is valid if it measures what it is supposed to measure (Maree & Pietersen, 2007a:147). To Leedy and Ormrod (2005:97), validity means accuracy, meaningfulness and credibility of a study as a whole (Leedy & Ormrod, 2005:97). McMillan and Schumacher (2006:134) refer to validity as the truthfulness of findings and conclusions.

For the results of a study to be trustworthy, the research must have a high degree of both internal as well as external validity. It was necessary to determine whether the research conducted complied with the following validity criteria identified by Leedy and Ormrod (2005:97-99) and McMillan and Schumacher (2006:134-142).

A high degree of **internal validity** of the research design means that there was sufficient control of the variables and that the design allowed the researcher to draw accurate conclusions (Leedy & Ormrod, 2005: 97). Researchers often use triangulation, which is the cross-validation measure among data sources, data collection strategies, time periods and theoretical schemes to increase validity (McMillan & Schumacher, 2006:374). In this regard, Maree and Van der Westhuizen (2007:40) refer to crystallization. Lincoln and Guba (in Maree & Van der Westhuizen, 2007:40) refer to crystallization as "*attending to voices that differ from your own to enable you to study multiple constructed realities*". The researcher gathered data from teachers and learners to find regularities in the data and to check whether the same pattern/s kept recurring in order to strengthen the conclusions made (McMillan & Schumacher, 2006:374). In this way, the study could be regarded as compliant with criteria for internal validity.

External validity, on the other hand, refers to the degree to which results can be generalized to the entire population and to other contexts (Leedy & Ormrod, 2005:99; McMillan & Schumacher, 2006:134; Maree & Pietersen, 2007a:151). Although the sample used in the study was small and geographically bound and results could not be generalized to the population, the external validity of the study was enhanced by the fact that the study was conducted in a real life setting (Leedy & Ormrod, 2005:99).

Construct validity concerns the efficiency of using a particular data collection instrument for the research purpose (McMillan & Schumacher, 2006:140). The researcher was convinced that the use of a questionnaire to determine the teacher perceptions regarding the nurturing of critical thinking in the Mathematics classroom was appropriate, as this study was only an initial investigation, descriptive in nature.

Statistical conclusion validity refers to the appropriate use of statistical tests and procedures to determine whether purported relationships are a reflection of actual relationships (McMillan & Schumacher, 2006:135). In this regard, the researcher sought the assistance of the Statistical Consultation Services at the North-West University, Vaal Triangle Campus. The researcher is therefore convinced that the statistical procedures utilized in this study (*cf.* 4.4; 4.5) were appropriate for the study to determine whether opportunities are provided in the Mathematics classroom to develop critical thinking.

3.3.3 Research design

For the purpose of the quantitative study, a non-experimental, descriptive survey research design was chosen (Leedy & Ormrod, 2005:179-185; McMillan & Schumacher, 2006:23). Descriptive research provides a summary of an existing phenomenon by using numbers to characterize individuals or a group. Leedy and Ormrod (2005:179) indicate that descriptive research involves either identifying the characteristics of an observed phenomenon or exploring possible correlations among two or more phenomena. Descriptive research assesses the nature of existing conditions and aims to characterize something as it is (McMillan & Schumacher, 2006:24). In the context of the study, the researcher aimed at determining whether opportunities are provided for the development of critical thinking skills in Mathematics classrooms.

Survey research refers to almost any form of descriptive, quantitative research (Gay & Airasian in Leedy & Ormrod, 2005:183). Survey research involves acquiring information about one or more groups of people about their characteristics, opinions, attitudes or previous experiences (McMillan & Schumacher, 2006:183). The ultimate goal is to learn about a large population by surveying a sample of the population. Inferences about a particular population are drawn from the responses obtained from the sample. As it was the intention of the researcher to explore the opinions, attitudes and experiences of the research participants with regard to the opportunities for the development of critical thinking skills during Mathematics teaching, survey research was deemed suitable (Leedy & Ormrod, 2005:183-184). Survey research poses a series of questions to willing participants; summarizes their responses with percentages, frequency counts, or more sophisticated indexes; and then draws inferences about a particular population from the responses of the sample (Leedy & Ormrod, 2005:184). In the context of the study the participant's responses were reported as frequencies, means and percentages before inferences were drawn.

3.4 DATA-GATHERING INSTRUMENTS

There are a number of data-gathering and measuring instruments that researchers can make use of for conducting quantitative research. A literature study and the aims and objectives of the study were used to guide the researcher in constructing two closed-ended questionnaires for teachers and learners respectively, to gather information regarding the opportunities for the development of critical thinking skills in the Mathematics classroom. The reasons why the researcher made use of questionnaires are:

- It was more cost-effective and time-saving to deliver questionnaires to and collect them from the sample schools instead of phoning participants for interviews and scheduling times for interviews (Leedy & Ormrod, 2005:185); and
- The survey can be done relatively quickly (Maree & Pietersen, 2007b:161).
- A questionnaire is suitable to use when the characteristics of a phenomenon needs to be determined. In the context of the study the researcher wanted to characterize classrooms in terms of the opportunities they provide for the development of critical thinking.

The reasons why the researcher made use of only closed-ended questionnaires are:

- the data obtained from the administration of the closed-ended questionnaire were easier to analyse (Maree & Pietersen, 2007b:161); and

- the participants could respond to the questions with assurance that their responses would be anonymous, ensuring that their responses might have been more truthful than they would have been in a personal interview (Leedy & Ormrod, 2005:185).

Before the questionnaires were constructed, the researcher had to acquaint herself with the research literature to determine important aspects that impact on the design of a questionnaire. The following sections provide an overview of the literature that the researcher took into consideration for the design of the questionnaire.

3.4.1 Questionnaires

A questionnaire is one of the most commonly and reliable instruments to collect data and is often used to make data collection more efficient and standardized. Questionnaires are frequently used to provide the main source of data in primary research. They provide invaluable descriptive data about individuals or a group. They are concerned with description and measurement (Saslow, 1982:13).

As indicated by Cohen *et al.* (2007:318), the purpose of the questionnaire was informed by the aim and objectives of the study. In this study, the researcher wanted to explore the opinions of teachers and learners regarding the opportunities created by teachers for the development of critical thinking skills in the Mathematics classroom. In order to achieve this, various sections linked to the central aim and the objectives of the study, were constructed (*cf.* Annexure A) Thereafter iterns were identified to elicit information regarding each of the sections/constructs.

Maree and Pietersen (2007b:157) identify different types of questionnaires that researchers could make use of during their studies, namely the group-administered, postal surveys, telephone surveys and face-to-face surveys. For the purpose of this study, the group-administered questionnaire was chosen. The researcher visited the different schools that took part in the study over a period of two weeks. Each school that participated in the research received learner and teacher questionnaires that were distributed and administered by the Mathematics Heads of Departments in the respective schools to the various participants. The Heads of Department distributed the questionnaires to the teachers in their respective schools who completed the questionnaires in their own time and returned them to the Heads of Departments. The learner questionnaire was distributed by the Heads of Departments to the group of learners who participated in their respective schools at a specific arranged time. Thirty minutes were allocated for the completion of the questionnaire in the presence of the Heads of Departments, after which the completed questionnaires were again collected. The researcher explained to the Heads of Departments exactly what the expectancies of the questionnaires were, so that they would be able to answer questions if any should arise from the learners or teachers. The completed questionnaires were collected by the researcher a week after distribution. This gave the schools enough time to complete the questionnaires.

Cohen *et al.* (270:320) also distinguish between structured, semi-structured and unstructured questionnaire with closed and/or open-ended questions. Cohen *et al.* (2003:45) indicates that a questionnaire is an instrument with open or closed questions or statements to which a respondent must respond. It is also known as the most widely used instrument for collecting survey information and then providing structured and numerical data (Cohen *et al.*, 2003:245). As it was the purpose of the researcher to determine patterns and make comparisons through the frequencies of responses, a closed and structured questionnaire was used (Cohen *et al.*, 2007:321). A structured questionnaire refers to a series of questions set in a clear structure and sequence that are presented to participants who have to respond by choosing one of the options supplied to answer the question. Participants do not respond or comment in a way they think best (Cohen *et al.*, 2007:321). When designing a questionnaire, the researcher has to keep in mind what data will be generated by the questions and the statistical techniques that will by used to analyse it (Maree & Pietersen, 2007a:158). The type of data that the researcher wishes to gather will determine whether open (structured) or closed (unstructured) questions will be used (Bell in Maree & Pietersen, 2007b:161).

3.4.1.1 Open-ended questions

Open-ended questions are questions that are asked and space is provided for a word or sentence. For example: How do you feel about nurturing critical thinking in Mathematics? (Maree & Pietersen, 2007b:160). Open-ended questions do not suggest any response to the set question. They do not force participants to choose rigidly between limited responses. Instead they permit participants to answer in their own frame of reference (Bell in Maree & Pietersen, 2007b:160). Maree and Pietersen (2007b:160) listed a few advantages and disadvantages of open-ended questions. These advantages and disadvantages are summarized in Table 3.1.

Table 3.1: Advantages and disadvantages of open-ended questions

ADVANTAGES	DISADVANTAGES
• Participants can give honest answers in detail. • The participant's thinking process is revealed. • Complex questions can be adequately answered. • Thematic analysis of the responses will yield extremely interesting information, categories and subcategories.	• The amount of detail may differ among participants. • Coding of answers may be difficult. • Participants may need time to think and write their response. • They are difficult for illiterate people to answer. • Statistical analysis is difficult.

3.4.1.2 Closed questions

Closed questions, on the other hand, provide participants with a set of questions with possible answers from which they have to choose. The data obtained from the administration of closed questions is easier to analyse than data obtained from open questions. The following advantages and disadvantages are noted for closed questions (Maree and Pietersen, 2007b:160). These advantages and disadvantages are summarized in Table 3.2.

Table 3.2: Advantages and disadvantages of closed questions

ADVANTAGES	DISADVANTAGES
• They are easy and quick to answer. • Coding and statistical analysis are easy. • Sensitive questions are more easily answered.	• The given responses may suggest an answer that the respondent would not have thought of. • The answer of the participant wants to give may not be one of the options. • It is easy to answer any question, even those that are misunderstood. • Answers are very simple with no detail. • A participant can answer even if she or he has no opinion or knowledge.

During this study the closed-structured questionnaire option was used as it was the intention of the researcher to determine patterns and make comparisons through the frequencies obtained for the various responses. In addition to this, as data obtained from closed questions is easier to analyse than data obtained from open questions, the researcher opted for closed questions.

3.4.2 Questionnaire design

Questionnaire design is a very important part of the research process since this is where data is generated (Maree & Pietersen, 2007b:158). Attention was paid to the following aspects indicated by Maree and Pietersen (2007b:159): appearance of the questionnaire, question sequence, wording of questions and response category.

Appearance of the questionnaire

The researcher incorporated the following aspects as suggested by Maree and Pietersen (2007b:159) to make the questionnaire user-friendly. The printing was done neatly, the font was not too small, clear instructions were given and the purpose for compiling the questionnaire was indicated.

Completion of the questionnaire

According to Maree and Pietersen (2007b:159), learners should preferably be able to complete a questionnaire in less than thirty minutes and adults in less than twenty minutes. The questionnaire should not contain more than 100 to 120 items. In the context of the research, the learners were allocated thirty minutes to complete the thirty seven questions and the teachers were given a week to complete their thirty nine questions

Question sequence

Questions were ordered into sections to prevent the participants from becoming confused. In each section, the questions were linked to a specific construct related to the opportunities for developing critical thinking skills (Maree & Pietersen, 2007b:160). Special attention was paid to the wording of the questions. Some of the guidelines suggested by Maree and Pietersen (2007b:160) and Cohen *et al.* (2007:334-337) were applied, namely:

- clear and unambiguous language was used;
- double-barrelled, complex questions were avoided;
- hypothetical questions were avoided;

- items were formulated as statements and not as questions;
- leading questions that guided participants in a certain way were avoided and
- double-negative questions were avoided.

The pilot study (cf. 1.6; 3.5) also revealed reliable results and the researcher therefore assumed that the questionnaire complied with the criteria for language clarity.

3.4.3 Aims of the questionnaires

The aims of the questionnaire for this particular research were to identify the opportunities that teachers presently create for the development of critical thinking skills in Grade 8 Mathematics classrooms by examining teacher and learner opinions. It was important to determine teachers' understanding of critical thinking, and whether their choice of teaching and assessment practices and learning material develop critical thinking skills among the learners during the teaching of Mathematics. Furthermore, the questionnaire also wanted to determine the role that the teacher plays in the creation of a classroom climate conducive to the development of critical thinking skills.

3.4.4 Types of questions

Bell (in Maree & Pietersen, 2007b:161) and Cohen *et al.* (2007:324-334) distinguish between different types of closed questions: list, ranking, rating, category, quantity, grid and scale. For the purpose of the research, scale questions were used. According to Bell (in Maree & Pietersen, 2007b:167), scale questions help researchers to discover strength of feeling or attitude. The response options are set up so that the variables measured can be expressed as a numerical score that are of either an ordinal, interval or ratio type (Maree & Pietersen, 2007b:167). In this study, nominal scales that consisted of two or more classes (eg. male/female) were utilized in the section of the questionnaire that focused on biographic information. Ordinal scales were used to determine knowledge and perceptions regarding the development of critical thinking skills (eg. strongly agree, agree, disagree, strongly disagree and almost always, often, sometimes and very seldom). These questions are also referred to as Likert scale questions (Maree & Pietersen, 2007b:167).

3.4.5 Structure of the questionnaires

Based on the literature review, the questionnaires were developed to determine teachers' understanding of critical thinking in Mathematics and the opportunities that they provide for the development of critical thinking skills.

Two questionnaires were developed by the researcher, for teachers and learners respectively. The same questions put to the teachers were modified to suit the learners. This was done to counter-check teachers and learners' responses for accuracy and credibility.

Both the questionnaires had a short covering letter to explain the purpose of the questionnaire to the participants. The questionnaires to teachers were categorized into seven sections (*cf.* Appendix A) and the learner questionnaire into six sections (*cf.* Appendix B).

Based on the literature review, the teacher questionnaire was structured as follows:

- Section A: biographical information (*cf.* 4.3.2).
- Section B: Questions 1 to 5.10 focused on the teachers' understanding of the meaning of critical thinking in the Mathematics classroom (*cf.* 2.4).
- Section C: Questions 6 to 16 focused on the general principles for the development of critical thinking during teaching and learning, and questions 17.1 to 17.9 investigated the use of specific teaching and assessment methods and strategies in the Mathematics classroom (*cf.* 2.5.1; 2.5.2; 2.5.3).
- Section D: Questions 18 to 22 focused on the learning support material used in the Mathematics classroom (*cf.* 2.5.3).

- Section E: Questions 23-28 focused on learner involvement and participation in the Mathematics classroom (*cf.* 2.5; 2.5.4).

- Section F: Linked to learner involvement, questions 29 to 30.12 focused on the role of the teacher in the Mathematics classroom (*cf.* 2.5.1; 2.5.4).

- Section G: Questions 31 to 39 focused on the classroom climate in Mathematics (*cf.* 2.5.5).

Two types of four-point scale questions were used from which the participants chose the answer that suited them the best. The participants could choose from these four options:

Sections B and C comprised Likert scale questions and participants had to agree or disagree with the statements on the following four-point scale:

1 → Strongly agree

2 → Agree

3 → Disagree

4 → Strongly disagree

A part of section C as well as sections D to G also comprised Likert scale questions and participants had to agree or disagree with the statements on the following four-point scale:

1 → Almost always

2 → Often

3 → Sometimes

4 → Very seldom

The learner questionnaire was structured in accordance with the teacher questionnaire as follows:

- Section A: biographical information (*cf.* 4.3.1).

- Section B: Questions 4 to 14 focused on the general principles for the development of critical thinking during teaching and learning, and questions 15.1 to 15.9 investigated the use of specific teaching methods and assessment strategies in the Mathematics classroom (*cf.* 2.5.3).

- Section C: Questions 16 to 20 focused on the learning support material used in the Mathematics classroom (*cf.* 2.5.3).

- Section D: Questions 21-26 focused on learner involvement and participation in the Mathematics classroom (*cf.* 2.5; 2.5.4).

- Section E: Linked to the questions on learner involvement, questions 27 to 28.12 focused on the role of the teacher in the Mathematics classroom (*cf.* 2.4.2; 2.5.4).

- Section F: Questions 29 to 37 focused on the classroom climate in Mathematics (*cf.* 2.5.5).

Two types of four-point Likert scale questions were used from which the participants chose the answer that suited them the best.

Section B comprised Likert scale questions and participants had to agree or disagree with the statements on the following four-point scale:

1 → Strongly agree

2 → Agree

3 → Disagree

4 → Strongly disagree

A part of section B and sections C to F also comprised Likert scale questions and participants had to agree or disagree with the statements on the following four-point scale:

1 → Almost always

2 → Often

3 → Sometimes

4 → Very seldom

In order to determine the reliability of the questionnaire, a pilot study was conducted prior to the distribution of the questionnaire to the research participants.

3.5 PILOT STUDY

A pilot study is a small or trial run done in preparation for a major study. It is a mini version of a full scale study or a trial run done in preparation of the complete study. It can also be a specific pre-testing of research instruments, including questionnaires or interviews schedules (Van Teijlingen & Hundley, 2001:1). According to Van Teijlingen and Hundley (2001:1), a pilot study is conducted for the following important reasons:

- to assess whether the research protocol is realistic and workable;
- to establish whether the sampling frame and techniques are effective;
- to identify logistical problems;
- to collect preliminary data;
- to estimate variability in outcomes to help determine sample size;
- to assess the feasibility of a full-scale study or survey; and
- to determine the reliability and validity of the data collection instruments.

A sample of fifty teachers and fifty learners from the target population, who were not part of the actual research sample, were identified for the pilot administration regarding the qualities of the measurement and appropriateness of the questionnaire. The outcome of the pilot study was an indicator of whether the questionnaire was a reliable and valid instrument to use. The Statistical Consulting Services of the North-West University, Vaal Triangle Campus was utilized in determining the validity and reliability of the data collection instrument. The results of the pilot study are reported in Chapter four (*cf.* 4.2).

3.5.1 Reliability of the questionnaire

Reliability has to do with the consistency or repeatability of a measure or data collection instrument such as a questionnaire. High reliability is obtained when the measure or instrument will give the same result if the research is repeated on the same sample (Maree & Pietersen, 2007a:147). For this purpose, the Cronbach alpha coefficient is used.

The researcher also had to assess the validity of the questionnaire that was utilized in the context of the study.

3.5.2 Validity of the data collection instrument

Validity was determined by making use of both content and construct validity. **Content validity** refers to the extent to which the instrument covers the complete content of the particular construct that it set out to measure (Maree & Pietersen, 2007d: 217). Content validity indicates how well the various parts of the content domain under investigation are represented in appropriate proportions in the questionnaire (Leedy & Ormrod, 2005:92). In this research, the content validity for the data collection instrument was supported by the fact that the questionnaires were constructed according to the essential components of the content domain (developing critical thinking skills in the Mathematics classroom) as highlighted by the literature. This implied having a closer look at how critical thinking skills can be developed through the choice of teaching methods and assessment strategies (*cf.* 2.5.1; 2.5.2), learning material (*cf.* 2.5.3), learning activities (*cf.* 2.5; 2.5.4), the role of the learner and teacher (*cf.* 2.5.4) and the classroom climate (*cf.* 2.5.5).

Construct validity is the type of validity that is needed for standardization and has to do with how well the construct covered by the instrument is measured by different groups of related items (Maree & Pietersen, 2007d: 217). According to Leedy and Ormrod (2005:92), it is the extent to which an instrument measures a characteristic that cannot be directly observed, but must be inferred. The researcher needs to obtain some kind of evidence that the questionnaire items measure the construct in question. The data collection instrument used in the study measured the construct in question, namely, critical thinking in Mathematics, because the questionnaire sections focused on a variety of components related to the development of the construct, namely teaching methods (*cf.* 2.5.1), assessment strategies (*cf.* 2.5.2); learning material (*cf.* 2.5.3), learner involvement (*cf.* 2.5.4), the role of the teacher (*cf.* 2.5.4) and classroom climate (*cf. 2.5.5*), as indicated in the literature review.

The questionnaire also complied with the criteria for **face validity** as the instrument truly measured what the researcher wanted to measure (Leedy & Ormrod, 2005:92). All the sections in the questionnaire only focused on obtaining information regarding the opportunities that can be provided in the classroom for the development of critical thinking.

The questionnaire thus complied validity criteria and adequately covered the nature of the construct to avoid inaccurate inferences being made (McMillan & Schumacher, 2006:141).

3.6 THE POPULATION AND SAMPLE

Sampling is defined as taking a portion of a population or universe and considering it as representative of the population or universe (Strydom & Venter, 2002:209). The universe refers to all potential subjects who possess the attributes in which the researcher is interested. Population refers to individuals in the universe who possess specific characteristics of interest to the researcher (Strydom & Venter, 2002:209). Due to time, financial and logical constraints, it was not possible to conduct research with the entire population, and therefore a sample was selected.

According to Maree and Pietersen (2007c:172), two major classes of sampling can be distinguished, namely probability sampling and non-probability sampling. Probability sampling is based on principles of randomness and non-probability sampling not. Consequently, probability sampling satisfies the requirements to generalize to the population, while this is not the case with non-probability sampling. Probability sampling is, for example, when researchers choose a sample that they assume have the same characteristics as the total population. Non-probability sampling is when the researcher makes no pretence of identifying a representative subset of a population and then takes people that are readily available (Leedy & Ormrod, 2005:199, 206).

The population of the study involved all teachers and learners of Mathematics. As it was not possible to conduct research with the entire population, the focus of the study was purposively placed on Grade 8 Mathematics teachers and learners. Purposive sampling relies on the judgement of the researcher who selects subjects that will provide the best information to address the purpose of the research (McMillan & Schumacher, 2006:126). The researcher also teaches Mathematics to Grade 8, is knowledgeable on the content of Grade 8 Mathematics and has herself experimented with the development of critical thinking skills related to Grade 8 content. The researcher was therefore of the opinion that she would specifically be able to understand the perceptions of teachers and learners regarding the development of critical thinking skills related to the context of Grade 8 Mathematics.

Due to time and logistical constraints, and because the researcher works in the Ekurhuleni District of the Gauteng Department of Education and had easy access to and contact with school principals and Mathematics teachers in fourteen schools in this District, it was decided to conduct the study in this District. The sample comprised the following schools:

- two township schools where the learners' home language is an African language, but the medium of instruction is English;
- one Afrikaans school where Afrikaans is used as the medium of instruction;
- two parallel medium schools where the medium of instruction is both English and Afrikaans;
- eight English schools where the medium of instruction is English; and
- one private school with English as medium of instruction.

Non-probability convenience sampling was utilized as the researcher did not include any type of random selection from the population, but used participants who were willing and accessible (McMillan & Schumacher, 2006:125).

All the teachers in the 14 identified schools who taught Mathematics at Grade 8 level were requested to take part in the research. Ultimately a heterogeneous group of 92 teachers of different ages, genders, cultures, years of experience in Mathematics teaching and qualification levels, took part in the research (*cf.* 4.3.2).

In each of the identified schools, Grade 8 learners who were willing were requested to take part in the research. In total, a heterogeneous group of 204 learners comprising different genders, ethnic groups and home languages took part in the research (*cf.* 4.3.1).

3.7 DATA ANALYSIS

3.7.1 Questionnaires

The Statistical Consultancy Service of the North West University: Vaal Triangle Campus was consulted for assistance in the capturing, analysis and interpretation of the data collected. The collected data was captured, analysed and interpreted, using descriptive statistics. Descriptive statistics were used to organize and summarize data meaningfully in order to promote an understanding of the data characteristics (Maree & Pietersen, 2007e:185). Frequencies, percentages and means were calculated for the various responses to the questionnaire items.

As the research wanted to go beyond summarizing and describing data, inferential statistics were also utilized to interpret differences between the teachers and learners' responses in order to determine significance (Maree & Pietersen, 2007e:198). T-tests were utilized for this purpose. P-values smaller than 0.5 were regarded as significant and values larger than 0.5 as non-significant (Maree& Pietersen, 2007b:230). In order to determine if the statistical significant differences had a practical effect, Cohen's D was calculated (Steyn, 2005:20) (*cf.* 4.5.1).

An ANOVA was conducted to summarize data on a single biographic variable in relation to the development of critical thinking. According to McMillan and Schumacher (2006:301) ANOVA is an extension of the t-test. Rather than using multiple t-tests to compare all possible means in a study of two or more groups, ANOVA allows the researcher to test for differences between all groups and make more accurate probability statements than when using separate t-tests. If statistical significant results were revealed between the different groupings of the biographic variables by the ANOVA, subsequent post hoc tests (Tukey Honestly Significant Difference (HSD) Tests) were run to determine which of the biographic groupings displayed the differences (McMillan & Schumacher, 2006:302).

3.8 ETHICAL CONSIDERATIONS

It is important that a researcher protects the rights of the participants of a research study and those of the institution in which the study is conducted. The research process may influence individuals or groups of people directly or indirectly, therefore it is important that the researcher should consider the ethical considerations while designing or structuring the research process.

Leedy and Ormrod (2005:101-103), McMillan and Schumacher (2006:142-146) and Van Deventer (2007) identified ethical principles that researchers have to observe, particularly where human participants are involved. In the context of this study, the researcher complied with ethical principles in the following ways:

- Permission was obtained from the Gauteng Department of Education to conduct the research (*cf.* Addendum C). School principals, the Heads of Departments of Mathematics, Mathematics teachers, learners and their parents were also approached to obtain permission to continue with the research.

- The purpose of the research was explained to all participants. Participants were provided with a description of what their participation would involve, as well as a statement including that participation was voluntary and could be terminated at any time.

- Participants gave written consent to participate in the research (*cf.* Addendum D).
- Participants were assured that responses would be treated confidentially, as only the researcher, the study leader and the Statistical Consultation Services of the North-West University: Vaal Triangle Campus had access to data.
- Questionnaires were completed anonymously and participants were identified by means of codes.
- Participants were informed that the results would be made known to them after the research.
- The researcher also submitted an application to the Ethics committee of the North-West University in order to obtain approval to conduct the research. Ethical clearance was obtained with the issuing of a project number by the North-West University:NWU-0039-08-S2

3.9 CHAPTER SUMMARY

The main purpose of this chapter was to outline and motivate the choice of the empirical design utilized in this study. In the following chapter, the researcher will report on the data gathered with the questionnaire and analyse and interpret the data obtained for learners and teachers.

CHAPTER FOUR

DATA ANALYSIS AND INTERPRETATION

4.1 INTRODUCTION

This chapter presents the statistical analyses and the interpretations of the responses obtained from the teacher and learner participants in order to determine the opportunities that teachers provide for the development of critical thinking skills in Grade 8 Mathematics classrooms.

The data analyses and interpretations will be dealt with in the following sequence:

- Reliability of the questionnaire for the pilot study and the actual study
- Biographic information of the participants
- Data analysis and interpretation: learner responses
- Data analysis and interpretation: teacher responses
- Data analysis and interpretation: a comparison between the learner and teacher responses
- Triangulation of the teacher and learner responses

The next section reports on the reliability of the questionnaire for the pilot study and the actual study.

4.2 RELIABILITY OF THE QUESTIONNAIRE

Table 4.1 indicates the Cronbach alpha coefficients that were calculated for the various constructs in the teacher and learner questionnaires for the pilot study.

Table 4.1: Cronbach alpha coefficients: pilot study

	Pilot study : Learners	Pilot study: Teachers
Questionnaire constructs	**Cronbach alpha**	**Cronbach alpha**
Understanding critical thinking	-	0.70
Teaching methods and strategies (General principles)	0.59	0.82
Teaching methods and assessment strategies	0.71	0.78
Learning material	0.69	0.68
Learner involvement	0.66	0.81
Role of the teacher	0.78	0.88
Classroom climate	0.81	0.72

Table 4.2 reflects the Cronbach alpha coefficients for the actual study.

Table 4.2: Cronbach alpha coefficients: actual study

	Actual study : learners	Actual study: teachers
Questionnaire constructs	**Cronbach alpha**	**Cronbach alpha**
Understanding critical thinking	-	0.91
Teaching methods and strategies (General principles)	0.67	0.74
Teaching methods and assessment strategies	0.66	0.78
Learning material	0.55	0.66
Learner involvement	0.65	0.82
Role of the teacher	0.81	0.92
Classroom climate	0.78	0.85

The Cronbach alpha coefficient was calculated to determine the internal consistency of the various questionnaire sections. The Cronbach alpha is a reliability coefficient that calculates the extent to which items, such as found in a questionnaire, are correlated positively to one another (Akbaba, 2006:183). Cronbach alpha measures consistency among individual items in a scale

(Simon, 2008). Sekaran (2000) points out that the internal consistency reliability becomes higher as the Cronbach alpha moves closer to 1.

In most Social Sciences, a Cronbach alpha coefficient between 0.7 and 0.8 is yielded as acceptable when working with a set of items to be considered on a scale, but some use 0.75 or 0.80 while others are lenient and accept 0.60 (Simon, 2008). According to Simon (2008) and Garson (2008), 0.60 could be seen as in order for an exploratory study. As this study was a first exploration of the development of critical thinking skills in Grade 8 Mathematics classroom, it is clear from Table 4.2 that the questionnaire for learners and teachers complied with reliability criteria.

Inter-item correlations were also determined for the various items listed in the various sections of the questionnaire. An inter-item correlation is used to judge the reliability of the instrument by estimating how well the items that reflect the same construct yield similar results (Trochim, 2006). The following results were revealed for the various questionnaire constructs. Table 4.3 reports the inter-item correlations for the pilot study.

Table 4.3: Inter-item correlations for the pilot study

Questionnaire constructs	Inter-item correlations: learners	Inter-item correlations: teachers
Understanding critical thinking	-	0.42
Teaching methods and strategies (General principles)	0.28	0.22
Teaching methods and assessment strategies	0.23	0.26
Learning material	0.27	0.28
Learner involvement	0.22	0.37
Role of the teacher	0.27	0.44
Classroom climate	0.23	0.33

Table 4.4 reflects the results for the inter-item correlations for the actual study.

Table 4.4: Inter-item correlations for the actual study

Questionnaire constructs	Inter-item correlations: learners	Inter-item correlations: teachers
Understanding critical thinking	-	0.44
Teaching methods and strategies (General principles)	0.31	0.21
Teaching methods and assessment strategies	0.18	0.29
Learning material	0.19	0.28
Learner involvement	0.24	0.44
Role of the teacher	0.25	0.47
Classroom climate	0.29	0.38

According to Trochim (2006), an inter-item correlation of between 0.15 and 0.5 yields an acceptable value. Both questionnaires complied with criteria for acceptable inter-item correlations for the pilot study and the actual study.

The next section focuses on the biographic information of the participants who took part in the study. Data is displayed in tables and graphs. The data in the graphs were rounded off to the nearest integer.

4.3 BIOGRAPHIC INFORMATION OF THE PARTICIPANTS

4.3.1 Biographic information of the learners

In Table 4.5 the biographic information of the learners related to the ethnic groups that they represented, is indicated.

Table 4.5: Ethnic groups of learners

Ethnic group	Frequency	Percentage
Asian	2	1.0
Coloured	3	1.5
Black	90	44.1
White	106	52.0
Other	3	1.5
Total	204	100.0

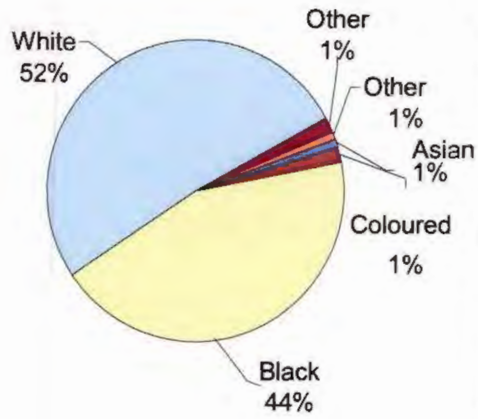


Table 4.5 indicates that the group of learners who took part in the study comprised a mixed ethnic group. The majority of the learners who participated in the study were White (n=106) followed by the Black learners (n=90).

Table 4.6 presents the biographic information of the learners related to their gender.

Table 4.6: Gender of learners

Gender	Frequency	Percentage
Female	101	49.5
Male	102	50.0
Missing	1	0.5
Total	204	100.0

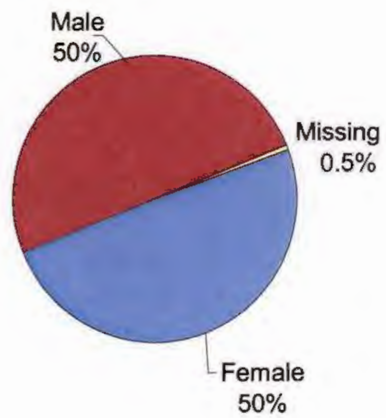


Table 4.6 indicates that more or less an equal number of female learners (n=101) and male learners (n=102) participated in the research.

Table 4.7 presents the biographic information of the learners related to their home language.

Table 4.7: Home language of learners

Ethnic group	Frequency	Percentage
English	38	18.6
Afrikaans	72	35.3
Sesotho	78	38.2
IsiZulu	9	4.4
Other	5	2.5
Missing	2	1.0
Total	204	100.0

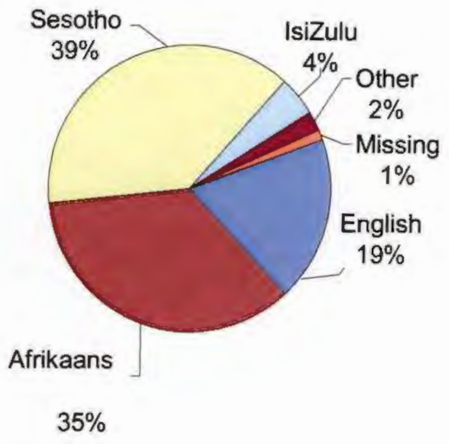


Table 4.7 indicates that the group of learners who took part in the study comprised different home languages. The majority of the learners who took part in the study were Sesotho-speaking (n=78), followed by Afrikaans-speaking learners (n=72) and English-speaking learners (n=38). This means that the sample population consisted of learners from different home language backgrounds.

The next section presents the biographic information of the teachers who took part in the research.

4.3.2 Biographic information of the teachers

In Table 4.8 the biographic information of the teachers related to their different age groups, is indicated.

Table 4.8: Age of teachers

Age	Frequency	Percentage
21 – 25	3	3.3
26 – 30	9	9.8
31 – 35	22	23.9
36 – 40	26	28.3
40 +	31	33.7
Missing	1	1.1
Total	92	100.0

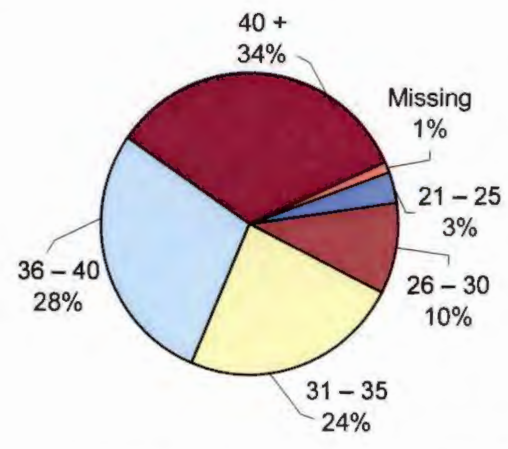


Table 4.8 indicates that the majority of the teachers who participated in the study were older that 40 years (n=31), followed by those teachers between 36 and 40 years of age (n=26). The teacher sample comprised mainly mature teachers.

In Table 4.9 the biographic information of the teachers related to their different positions is indicated.

Table 4.9: Position of teachers

Position	Frequency	Percentag e
Deputy Principal	2	2.2
Head of Department	6	6.5
Teacher	84	91.3
Total	92	100.0

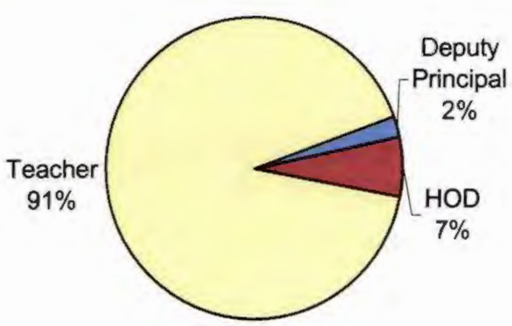


Table 4.9 indicates that the majority of the participants who took part in the study were teachers (n=84) who, on a daily basis, teach Mathematics to Grade 8 learners. Based on their daily teaching experience of Mathematics to

Grade 8 learners, they were in a position to provide the researcher with relevant information pertaining to teaching and learning in Grade 8 Mathematics classrooms. The Heads of Department (n=6) and Deputy principals (n=2) were also involved in the teaching of Mathematics to Grade 8 learners.

In Table 4.10 the biographic information of the teachers related to the various ethnic groups to which they belonged, is indicated.

Table 4.10: Ethnic groups of teachers

Ethnic Group	Frequency	Percentage
Asian	1	1.1
Black	72	78.3
White	19	20.7
Total	92	100.0

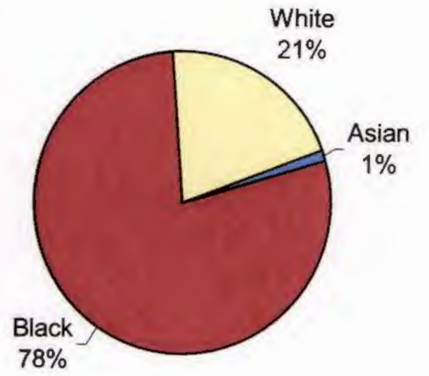
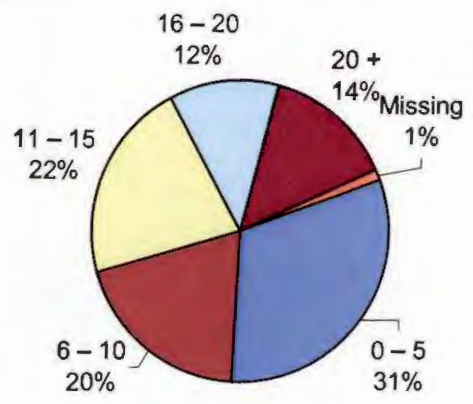


Table 4.10 indicates that the majority of teachers who took part in the study were Black teachers (n=72), followed by the White teachers (n=19) and one Asian teacher.

In Table 4.11 the biographic information of the teachers related to their experience in the teaching of Mathematics, is indicated.

Table 4.11: Experience of teachers

Years experience	Frequency	Percentage
0 – 5	29	31.5
6 – 10	18	19.6
11 – 15	20	21.7
16 – 20	11	12.0
20 +	13	14.1
Missing	1	1.1
Total	92	100.0



It is clear from the above table that the majority of the teachers who participated in the study only had between 0 and 5 years experience (n=29) in teaching Mathematics, followed by those with 11 to 15 years experience (n=20) and those with 6 to 10 years experience (n=18). Only thirteen educators had more than 20 years experience in the teaching of Mathematics. The lack of experience could have serious implications for the quality of teaching and learning in the Mathematics classroom.

The following table, Table 4.12, indicates the biographic information of the teachers related to their level of education.

Table 4.12: Teachers' level of education

Years experience	Frequency	Percentage
3 year diploma	43	46.7
4 year diploma	15	16.3
Degree + diploma	20	21.7
BEd degree	12	13.0
Masters	1	1.1
Phd	1	1.1
Total	92	100.0

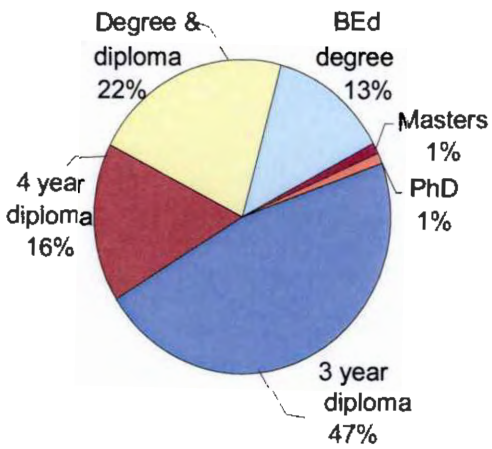


Table 4.12 above indicates that the majority of the teachers who participated in the study only had a 3 year diploma (n=43). This is disconcerting, because it implied that the majority of the teachers were not well qualified to teach Mathematics, which could adversely affect the quality of the teaching and learning in the Mathematics classroom.

Although it was not explicitly stated as part of the objectives of the study, the researcher utilized the biographic information of both the learners and the teachers as independent variables in the study to determine whether they had any impact on the way in which the provision of opportunities for the development of critical thinking skills were perceived (*cf.* 4.5.2).

The next section reports the data analysis and interpretation of the learner and teacher responses for the various sections of the questionnaire. For the purpose of this study the researcher was particularly interested in the "almost always" and "strongly agree" responses as it was important for the researcher that teachers create opportunities for the development of critical thinking skills on a constant basis and not only often. The Statistical Consultation Services also indicated that results for Cohen's D should only be reported if statistical significant results were obtained for the difference between means.

4.4 DATA ANALYSIS: DESCRIPTIVE STATISTICS

4.4.1 Learner responses for the questionnaire

In the interpretation of the teacher and learner responses the researcher was guided by the importance of critical thinking skills in Grade 8 Mathematics as indicated by the Learning Outcomes and Assessment Standards in Table 2.1. The researcher was of the opinion that teachers have to nurture critical thinking skills on a **daily basis** as the achievement of all the Learning Outcomes and Assessment Standards require the application of critical thinking skills. It was therefore important to obtain evidence from the learners and teacher responses to the questionnaire items that critical thinking skills are **always** nurtured during the teaching, learning and assessment of Mathematics.

The next section presents the responses obtained from the learners for each of the sections in the questionnaire. Each section focused on a specific construct in relation to the development of critical thinking. Although the learner and teacher questionnaires focused on the same issues, the questions were phrased differently and some of the sections did not contain the same number of questions. Therefore the learner and teacher results are not reported together. In section 4.6 the learner and teacher responses are triangulated in order to arrive at final conclusions. The data in the tables are presented as frequencies and percentages for each of the questionnaire statements in the various questionnaire sections.

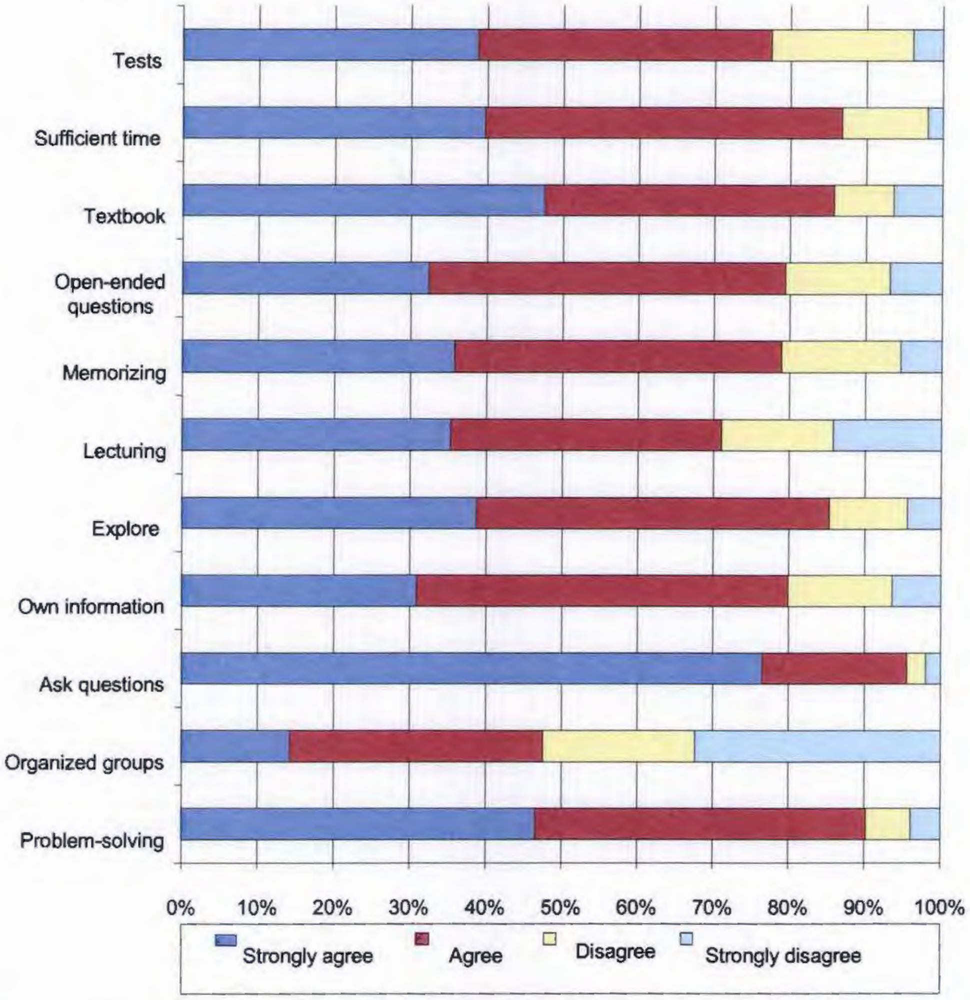
4.4.1.1 Learner responses: teaching methods and assessment strategies used in the Mathematics classroom: general principles

This section enquired from the learners to indicate to what extent the general principles for the application of teaching methods and assessment strategies to develop critical thinking skills were utilized by their teachers during Mathematics teaching. Question 15 specifically focused on the application of certain methods for the development of critical thinking skills. The learner responses to the questions appear in Table 4.13 below.

Table 4.13: Learner responses to the questions on teaching methods and assessment strategies used in the Mathematics classroom

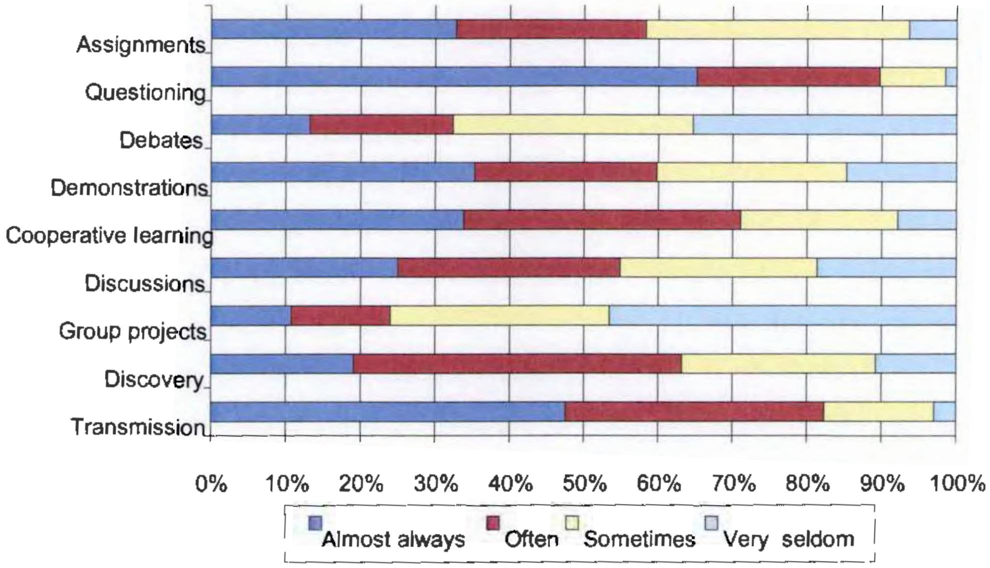
	Statement	Strongly agree	Agree	Disagree	Strongly disagree
4	Problem-solving is the main method used in the Mathematics classroom.	95 46.6%	89 43.6%	12 5.9%	8 3.9%
5	My teacher allows us to work in organized group activities.	29 14.2%	68 33.3%	41 20.1%	66 32.4%
6	My teacher gives us the opportunity to ask questions.	156 76.5%	39 19.0%	5 2.5%	4 2.0%
7	My teacher allows us to find information ourselves.	63 30.9%	100 49.0%	28 13.7%	13 6.4%
8	My teacher allows us to explore different alternative solutions to problems.	79 38.7%	95 46.6%	21 10.3%	9 4.4%
9	My teacher makes use of lecturing in the Mathematics classroom.	72 35.3%	73 35.8%	30 14.7%	29 14.2%
10	My teacher ask us to memorize information.	73 35.8%	88 43.1%	32 15.7%	11 5.4%
11	My teacher asks open-ended questions.	66 32.4%	96 47.0%	28 13.7%	14 6.9%
12	My teacher encourages us to study	97	78	16	13

	Mathematics from a textbook.	47.5%	38.2%	7.8%	6.4%
13	My teacher allows the learners sufficient time to think before answering.	81 39.7%	96 47.0%	23 11.3%	4 2.0%
14	My teacher views tests as the most appropriate way to assess Mathematics.	79 38.7%	79 38.8%	38 18.6%	8 3.9%



	Statement	Almost always	Often	Some-times	Very seldom
15	Teaching methods and assessment strategies used				
15.1	Transmission of knowledge	97 47.5%	71 34.9%	30 14.7%	6 2.9%
15.2	Discovery	39	90	53	22

	19.1%	44.1%	26.0%	10.8%
15.3 Group projects	22 10.8%	27 13.2%	60 29.4%	95 46.6%
15.4 Discussions	51 25.0%	61 29.9%	54 26.5%	38 18.6%
15.5 Cooperative learning	69 33.8%	76 37.3%	43 21.1%	16 7.8%
15.6 Demonstrations	72 35.3%	50 24.5%	52 25.5%	30 14.7%
15.7 Debates	27 13.2%	39 19.1%	66 32.4%	72 35.3%
15.8 Questioning	133 65.2%	50 24.5%	18 8.8%	3 1.5%
15.9 Assignments	67 32.8%	52 25.5%	72 35.3%	13 6.4%



The literature review strongly emphasizes that learners need to be exposed to a variety of teaching methods and assessment strategies in the Mathematics classroom as this will enable them to develop critical thinking skills easier, which in turn will promote the ability to do independent exploration and then explain the work on their own (Ferrando, 2001; Van de Walle, 2001:17, 38; Cangelosi, 2003:4; Lake, 2009:14) (*cf.* 2.5.1; 2.5.2; 2.5.4). According to Oliver and Utermohlen, (1995:1) (*cf.* 2.3), learners will then also be able to apply the acquired critical thinking skills to their academic studies and when doing problem-solving in the Mathematics classroom. From the data obtained, it appeared that transmission of knowledge (47.5%); asking questions (76.5%), textbooks (47.5%) and questioning (65.2%) are the most popular ways in which teaching and assessment are conducted.

From the learners' response as indicated in Table 4.13 it also seems that not many teachers make use of cooperative learning (33.8%), class discussions (25.0%), assignments (32.8%), debates (13.2%), demonstrations (35.3%), group projects (10.8%), to find information on their own (30.9%) and discovery (19.1%). Only 14.2% of the learners have indicated that organized group activities can improve their critical thinking abilities and 46.6% strongly agreed and 43.6% agreed that problem solving is the main method used in their mathematics classroom. It is especially disconcerting that it appears as if cooperative learning is underutilized in the Mathematics classrooms of the learners who took part in the study. Cooperative learning is very important in the Mathematics classroom, for the interactive nature of the method enhances better mathematical understanding (Department of Education, 2007a:4) (*cf.* 2.4.1) and promotes cognitive development through the exchanging of ideas, as well as explaining and motivating answers (Borich, 2004:214-256; Marcut, 2005:63; Searls, 2006; Gawe, 2007:208-227) (*cf.* 2.5.1). Class discussions and debates are also strategies that promote the development of critical thinking. From the responses of the learners it appeared that these two strategies are not frequently used. According to Van de Walle (2003:51) and Marcut (2005:62-63), discussions and debates provide learners with the opportunity to reason about their thinking and argue for or against certain viewpoints, which promote the development of critical thinking skills (*cf.* 2.5.4). In support of the use of discussions and debate, Appelbaum (2004:309), Ash (2005) and Morris (2007) support the idea of communication during the teaching of Mathematics, and indicate that a dialogical style supports the development of critical thinking. This style of teaching focuses more on equipping learners with processes to make sense of the subject content than the content itself (*cf.* 2.5.1).

It also appeared that not many of the learners strongly agreed to experiencing opportunities to answer open-ended questions in their Mathematics classroom (32.4%), exploring different alternative solutions to problems given to them (38.7%) or are allowed sufficient time to think before they answer questions (39.7%). Many of the learners strongly agreed (47.5%) and agreed (38.2%) to the fact that their teachers ask them to study Mathematics from a textbook. Literature indicates that an over reliance on textbooks will inhibit the development of the critical thinking skills of the learners. The learners should rather be encouraged to develop their own knowledge and understanding of Mathematics through interactive learning, practical activities, explorations and discussions (Fromboluti, & Rinck, 1999; Gupta, 2001; Marcut, 2005:62-63; Searls, 2006) (*cf.* 2.4.2; 2.5.1; 2.5.3; 2.5.4). Teachers should therefore not rely on textbooks that limit the learning process (Ellis, 2000) (*cf.* 2.5.3) and provide very little opportunity for assessing learners' creative and critical thinking skills (Ferrando, 2001) (*cf.* 2.5.1). The use of textbooks that was highlighted above could also be seen against the fact that 35.3% strongly agreed and 35.8% agreed that lecturing is still frequently used in the classroom. This also corroborates the response to question 15.1 where 47.5% and 34.9% respectively indicated that they are almost always or often exposed to the transmission of knowledge. In addition to this, 35.8% strongly agreed and 43.1% agreed that they are expected to memorize information.

What is encouraging from the responses is that it appears that questioning is frequently used during teaching and learning. A number of learners, 76.5%, strongly agreed that their teachers allow them to answer and ask questions in the Mathematics classroom. This is supported by the learners' response to question 15.8 where they strongly agreed (65.2%) that questioning is utilized as a teaching strategy. According to Searls (2006) (*cf.* 2.5.1), Mathematics is a subject that requires the learners to ask questions in order to develop critical thinking skills and then to use the information obtained, to formulate their own understanding.

It also appears that testing is a popular method of assessment utilized in the Mathematics classrooms. A large number of learners (38.7%) strongly agreed and agreed (38.8%) that tests are viewed as the most appropriate way to assess Mathematics.

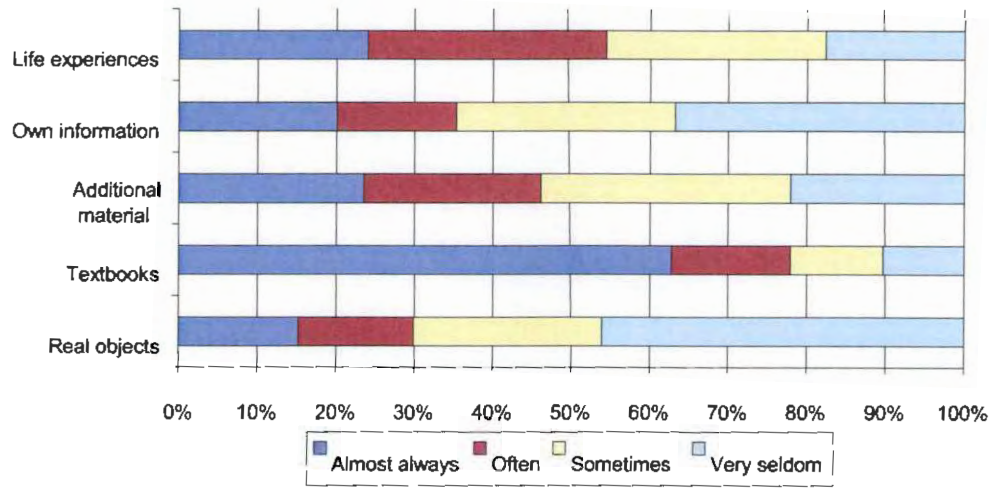
The next section reports on the results obtained from the learner data regarding the use of learning material in the Mathematics classroom.

4.4.1.2 Learner responses: the learning support material used in the Mathematics classroom

The type of learning support material that teachers use in the Mathematics classroom is very important for the development of critical thinking (Lehman & Hayes, 1985:166; Ash, 2005) (*cf.* 2.5.3). The learners' responses to the questions regarding the learning support material used in the Mathematics classroom appear in Table 4.14 below.

Table 4.14: Learner responses to the questions on the learning support material used in the Mathematics classroom

	Statement	Almost always	Often	Some-times	Very seldom
16	My teacher makes use of real objects during the teaching of Mathematics (eg models, pictures etc.)	31 15.2%	30 14.7%	49 24.0%	94 46.1%
17	My teacher mainly uses prescribed textbooks when teaching Mathematics.	128 62.7%	31 15.2%	24 11.8%	21 10.3%
18	My teacher makes use of additional material when teaching Mathematics.	48 23.5%	46 22.5%	65 31.9%	45 22.1%
19	My teacher requests us to bring our own information to the Mathematics classroom.	41 20.1%	31 15.2%	57 27.9%	75 36.8%
20	My teacher incorporates real life experiences into Mathematics teaching.	49 24.0%	62 30.5%	57 27.9%	36 17.6%



In order to nurture critical thinking, the literature review strongly emphasizes that the learners must be given the necessary resources they need to assist them to seek information (Searls, 2006; Van der Walt & Maree, 2007:237; Facione *et al.*, 2000:63) (*cf.* 2.5.1), synthesize their ideas (Lehman & Hayes, 1985:166) (*cf.* 2.5), apply new information (Maharaj, 2007:34) (*cf.* 2.4.1) and make new discoveries on their own (Hida *et al.*, 2005) (*cf.* 2.5.1). The learners' responses to the question whether their teachers make use of a variety of learning material are not convincing. It seems that the learners still experience transmission and rote learning in the Mathematics classroom and that their teachers rely mainly on Mathematics textbooks that include routine exercises to teach Mathematics. It is indicated that the use of textbooks (62.7%) almost always dominates classroom instruction. This response corresponds with the response received for Question 12 and 15.1 (*cf.* Table 4.13) where it was also indicated by the majority of the learners that teachers encourage the use of textbooks and favour a transfer of knowledge approach to teaching.

It is disturbing that real life objects (15.2%); additional teaching material (23.5%) and real life experiences (24.0%) are not always used in the classrooms of the learners who took part in the research. Only 20.1% of the learners have indicated that their teachers almost always request from them to bring their own information to the Mathematics classroom. According to Mahaye and Jacobs (2007:200), real world problems and experiential learning are crucial for the development of critical thinking (*cf.* 2.5.1). It also appears that learners rely on the information that they get from the teacher, as only 20.1% indicated that they are almost always allowed to bring their own information to the classroom. This response corroborates the response to question 7 (*cf.* Table 4.13) where only 30.9% indicated that they are always allowed to find information on their own.

The next section investigates learner involvement in the Mathematics classroom.

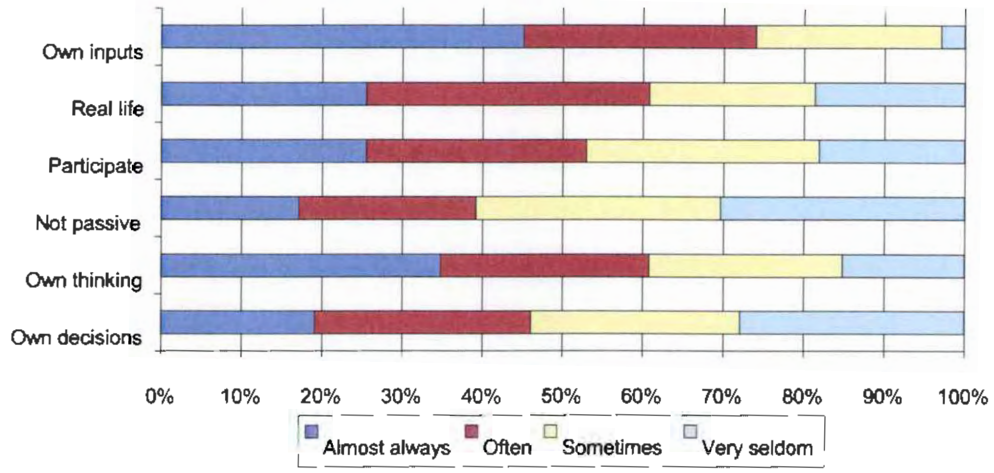
4.4.1.3 Learner responses: learner involvement in the Mathematics classroom

This section enquired what learning activities are provided for the development of critical thinking skills in the Mathematics classroom. The responses to the questions appear in Table 4.15 below.

Table 4.15: Learner responses to the questions on learner involvement in the Mathematics classroom

	Statement	Almost always	Often	Some-times	Very seldom
21	My teacher allows learners to make their own decisions in the Mathematics classroom.	39 19.1%	55 27.0%	53 26.0%	57 27.9%
22	My teacher allows learners to follow their own thinking in the Mathematics classroom.	71 34.8%	53 26.0%	49 24.0%	31 15.2%
23	My teacher indicates to us that we should not passively accept what he/she says in the Mathematics classroom.	35 17.1%	45 22.1%	62 30.4%	62 30.4%

24	My teacher allows learners to participate in decision-making in the Mathematics classroom.	52 25.5%	56 27.5%	59 28.9%	37 18.1%
25	My teacher relates the teaching of Mathematics to real life experience.	52 25.5%	72 35.3%	42 20.6%	38 18.6%
26	My teacher allows us to give our own inputs in solving mathematical problems.	92 45.2%	59 28.9%	47 23.0%	6 2.9%



The literature review strongly emphasizes that, in order to nurture critical thinking, learner involvement during teaching and learning needs to be magnified (Msila, 2007:151) (*cf.* 2.4.1). This is supported by Ferrando (2001), Simic-Muller (2007) and Lake (2009:14) (*cf.* 2.4.1; 2.4.2.2; 2.5.1) who argue that learners need to be actively involved in their own thinking and intellectual growth. Teachers therefore need to involve learners in activities that will stimulate the development of critical thinking skills and involve them in group and class discussions where learners can give their own input and take part in decision-making (Appelbaum, 2004:308) (*cf.* 2.5.4). It seems from Table 4.15, that only a few learners are almost always allowed to make their own decisions (19.1%), to participate during decision-making (25.5%), give their own input during problem-solving (45.2%), follow their own thinking (34.8%) and involve them actively (17.1%) in the Mathematics classroom. Only 25.5%

of the learners are almost always relate the teaching of Mathematics to real life experiences.

Bearing in mind that critical thinking skills are not easily developed as it involves the acquisition of a set of interrelated skills for which a lot of practice and opportunities for development are required (*cf.* 2.3), one would have expected to see that learners are almost always involved in decision-making, giving their input during teaching and learning and actively involved in problem-solving. The responses obtained for these questions could point in the direction of a more passive approach to teaching and learning which does not purposefully address the development of critical thinking skills. This gives reason for concern, as active involvement in decision-making and problem-solving is regarded as the most important way in which critical thinking can be developed in the Mathematics classroom (Schoënfeld, 1994:59; Fromboluti & Rinck, 1999; Erwin, 2000; Winicki-Landamn, 2001:30; Singh *et al.*, 2002:324; Winstead, 2004:44; Polya (in Macintyre, 2006:8-11); Van de Walle, 2007:37; Winch, 2006:74; Sezer, 2008:351) (*cf.* 2.4.1; 2.4.2.1; 2.4.2.4; 2.5).

Once again, as in Question 20 (*cf.* Table 4.14); the learners indicated that they are not always involved (24.0%) in real life experiences during the teaching of Mathematics. According to the responses of the learners, it appears as if the Mathematics classrooms that participated in the study are not yet fully learner-centred (*cf.* 2.5.1). According to Simic-Muller (2007) (*cf.* 2.4.2.2), learner-centred classrooms are conducive to the development of critical thinking skills. It also seems as if teachers are not moving away from direct to inquiry-based teaching (Cangelosi, 2003:4; Hida *et al.*, 2005) (*cf.* 2.5.1). Against this background, a concern is raised as to how well critical thinking is nurtured in the Mathematics classrooms of the learners who took part in the study.

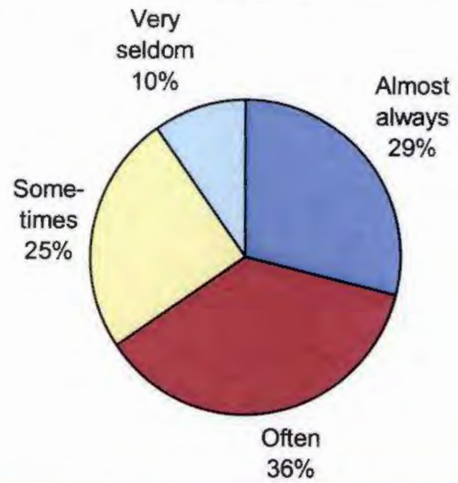
The next section focuses on the role of the teacher during the teaching of Mathematics.

4.4.1.4 Learner responses: the role of the teacher during the teaching of Mathematics

The purpose of the following questions asked to the learners, was to gather information regarding the role that their teachers play in the Mathematics classroom. Their responses to the questions appear in Table 4.16 below.

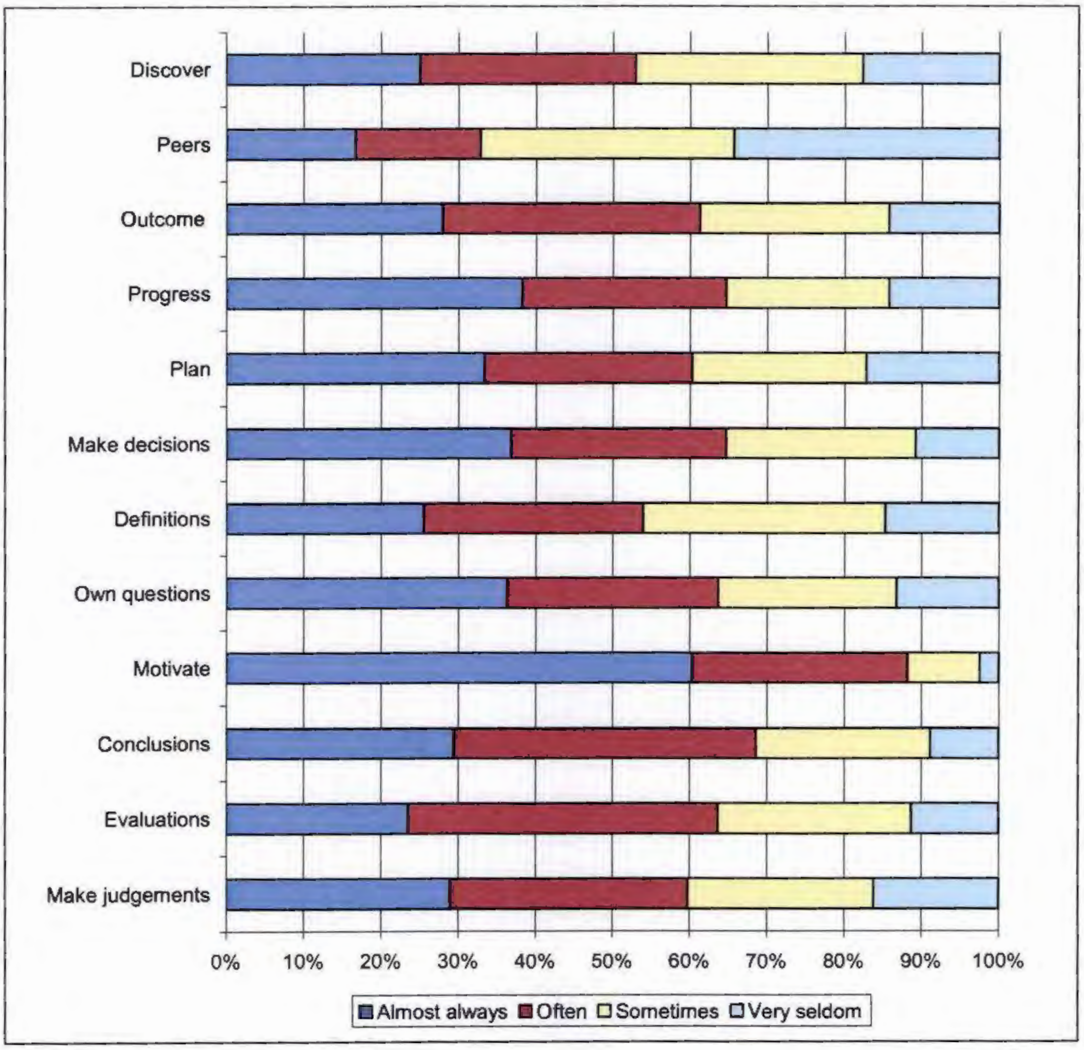
Table 4.16: Learner responses to the questions on the role of the teacher in the Mathematics classroom

Statement		Almost always	Often	Some-times	Very seldom
27	My teacher acknowledges that there is no single correct way to solve problems.	59 28.9%	75 36.8%	50 24.5%	20 9.8%



Statement		Almost always	Often	Some-times	Very seldom
28	My teacher nurtures the following skills among learners when teaching Mathematics:				
28.1	Make judgements	59 28.9%	63 30.9%	49 24.0%	33 16.2%
28.2	Do evaluations	48 23.5%	82 40.2%	51 25.0%	23 11.3%
28.3	Come to conclusions	60 29.4%	80 39.3%	46 22.5%	18 8.8%

28.4	Motivate our answers	123 60.3%	57 27.9%	19 9.3%	5 2.5%
28.5	Formulate our own questions	74 36.3%	56 27.5%	47 23.0%	27 13.2%
28.6	Formulate definitions	52 25.5%	58 28.4%	64 31.4%	30 14.7%
28.7	Make decisions	75 36.8%	57 27.9%	50 24.5%	22 10.8%
28.8	Plan our work before they start.	68 33.3%	55 27.0%	46 22.5%	35 17.2%
28.9	Monitor our own progress.	78 38.2%	54 26.5%	43 21.1%	29 14.2%
28.10	Evaluate the outcome of our own work.	57 27.9%	68 33.3%	50 24.5%	29 14.3%
28.11	Work with our peers.	34 16.7%	33 16.2%	67 32.8%	70 34.3%
28.12	Discover on our own.	51 25.0%	57 27.9%	60 29.4%	36 17.7%



The literature review strongly emphasizes the fact that when teachers want to develop critical thinking skills, the learners should be more productive during the learning process (Van de Walle, 2001:17; Crotty, 2002, Appelbaum, 2004:308; Searls, 2006; Staples, 2007:201; Elder, 2007) (*cf.* 2.5.3; 2.5.4; 2.5.5) and teachers should make use of a variety of teaching methods and strategies to enable learners to take part in discussions, reflect on their own work, make conclusions, communicate their thoughts in discussions and take part in cooperative learning activities with their peers (Beyer, 1985:283; Cantrell, 2000; Crotty, 2002; Appelbaum, 2004:308; Marcut, 2005:62, 63; Morris, 2007) (*cf.* 2.5.3; 2.5.4; 2.5.5). Table 4.16 indicates that the learners appear to feel that their teachers don't always acknowledge multiple ways to solve problems (28.9%), allow them to make judgments (28.9%), do evaluations (23.5%), come to conclusions (29.4%), formulate definitions (25.5%) and own questions (36.3%), plan, monitor and evaluate the outcomes of their own work (33.3%, 38.2% & 27.9% respectively), make decisions (36.8%), work with their peers (16.7%) and make discoveries on their own (25.0%). The low percentage obtained for working with peers is supported by the low percentage obtained for question 5 (*cf.* Table 4.13) where only 14.2% indicated that they are almost always allowed to work in groups. According to these responses, it appears as if the development of critical thinking skills is not adequately addressed during the teaching of Mathematics in the Grade 8 classrooms that took part in the study.

According to the literature review, the development of metacognitive skills (planning, monitoring, evaluation) are important for the development of critical thinking skills (Facione *et al.,* 2000:63; Van der Walt & Maree, 2007:227) (*cf.* 2.5.1). The learner responses, however, indicated that they are not always involved in reflecting on their own work in terms of planning before the execution of a task, monitoring the completion of a task and evaluating the outcome of a task.

The response to question 28.5 appears that only 36.3% of the learners indicated that they were almost always allowed to formulate questions. Table 4.16 also indicates that the learners are almost always allowed to motivate their answers (60.3%). This is important for the development of critical thinking skills in Mathematics. Motivating answers nurtures various skills that are important for critical thinking, namely the skill of evaluating different options before an answer is given, making decisions about answers and providing evidence for answers (Monteith, 1999; Van de Walle, 2001:17; Van de Walle, 2003:51) (*cf.* 2.5.4).

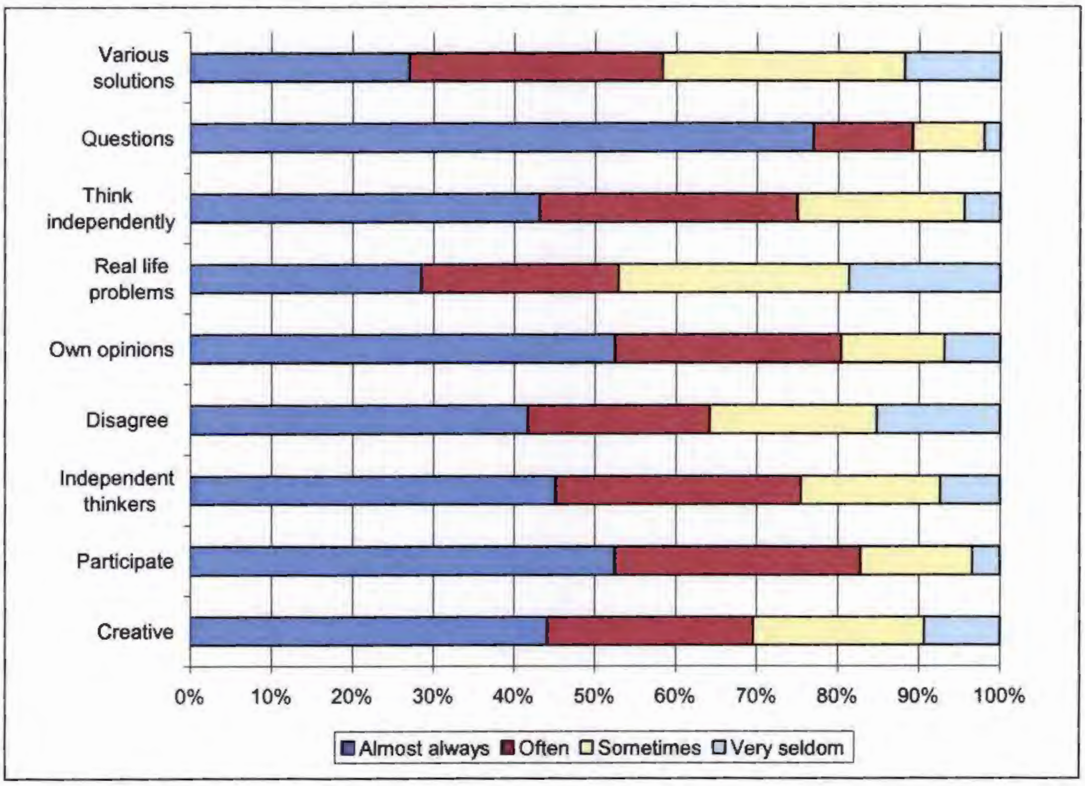
The next section focuses on the type of classroom climate that is created during the teaching of Mathematics.

4.4.1.5 Learner responses: classroom climate in Mathematics

The following questions focused on how the learners experienced the climate in their Mathematics classrooms. The learners' responses to these questions appear in Table 4.17 below.

Table: 4.17: Learner responses to the questions on classroom climate

	Statement	Almost always	Often	Some-times	Very seldom
29	We are allowed to be creative.	90 44.1%	52 25.5%	43 21.1%	19 9.3%
30	We are allowed to participate in problem-solving.	107 52.5%	62 30.4%	28 13.7%	7 3.4%
31	We are encouraged to become independent thinkers.	92 45.1%	62 30.4%	35 17.2%	15 7.3%
32	We are given the freedom to disagree with the teacher.	85 41.7%	46 22.5%	42 20.6%	31 15.2%
33	We are welcomed to give our own opinions.	107 52.5%	57 27.9%	26 12.7%	14 6.9%
34	We have the opportunity to solve real life problems.	58 28.4%	50 24.5%	58 28.4%	38 18.7%
35	We are encouraged to think independently in the classroom.	88 43.1%	65 31.9%	42 20.6%	9 4.4%
36	We may ask questions in the class.	157 77.0%	25 12.3%	18 8.7%	4 2.0%
37	We do not have to solve problems in the same way.	55 27.0%	64 31.4%	61 29.8%	24 11.8%



The literature study strongly emphasizes that in order to nurture critical thinking, the climate in the Mathematics classroom should provide intellectual openness that encourages learners to reason and construct their own understanding (Crotty, 2002; Porch, 2002; Staples, 2007:201; Lake, 2009:14) *cf.* 2.5.4; 2.5.5). It is important that the classroom climate should make learners feel comfortable to get involved in problem-solving (Van de Walle, 2001:17; Oleinik, 2002; Skovsmose & Valero, 2002:385) (*cf.* 1.1; 2.5.5) in order to promote the development of critical thinking skills (Klein & Orr, 1991:131) (*cf.* 2.5.5). Table 4.17 indicates that learners are apparently not always given enough opportunity to solve real life problems (28.4%), have the freedom to disagree with their teachers (41.70%) or to be creative (44.1%). Only 27.0% indicated that they are almost always allowed to solve problems in different ways and to become independent thinkers (45.1%) and think independently (43.1%). According to Monteith (1999), Ferrando (2001) and Van de Walle (2001:17) (*cf.* 2.5.1; 2.5.5), learners should be allowed to disagree with their teachers without offending them. Learners also need to experience a climate in the Mathematics classroom that will enable them to solve problems in their own way (Hida *et al.*, 2005) (*cf.* 2.4.2; 2.5.1). Learners need to be given the opportunity to analyse and be creative as it will promote independent learning and create a climate for the development of critical thinking (Gallagher, 1975; Appelbaum, 2004:308) (*cf.* 2.5.4). It also appears from the responses obtained for question 37 that creative problem-solving is not high on the agenda in the classrooms where the research was conducted, as only 27% of the learners indicated that they are almost always allowed to solve problems in different ways.

Once again it appears that the learners are allowed to ask questions in their Mathematics classrooms most of the time (77.0%). This response corroborates the response to question 15.8 (*cf.* Table 4.13), where 65.2% of the learners indicated that they are almost always allowed to ask questions. It appears as if the teachers create some opportunities for learners to always (52.5%) or often (30.4%) participate in problem-solving and to give their own opinions (52.5%). According to the literature, problem-solving gives learners the chance to explore and think logically and critically (Fromboluti & Rinck, 1999; Winch, 2006:74) (*cf.* 2. 4.2.1)

The following section summarises the averages obtained for each of the different questionnaire sections for the learner responses.

4.4.1.6 Summary: learner responses

Means were calculated for the various sections in the questionnaire according to the ordinal scales utilized for classifying the questionnaire responses, namely 1 = strongly agree/almost always, 2 = agree/often, 3 = disagree/sometimes, 4 =strongly disagree/very seldom. Table 4.18 summarizes the means obtained for the various questionnaire sections. In interpreting the results it is important to note that the lower the mean, the more favourable the response as it is closer to 1.

Table 4.18: Learner responses: means for the various questionnaire sections

Questionnaire sections	N	Mean	Std dev
Teaching methods and assessment strategies (general principles) (Questions 4-14)	204	1.88	0.39
Teaching methods and assessment strategies (Questions 15.1-15.9)	204	2.25	0.49
Learning material (Questions 16-20)	204	2.48	0.64
Learner involvement (Questions 21-26)	204	2.35	0.62
Role of the teacher (Questions 27-28.12)	204	2.21	0.56
Classroom climate (Questions 29-37)	204	1.91	0.58

Table 4.18 indicates that the general principles for the application of teaching methods and assessment strategies to develop critical thinking, as well as the classroom climate created for the development of critical thinking, were the two aspects that learners judged the most favourable for the development of critical thinking. It was indicated in the interpretations of each of the questionnaire sections that the asking of questions by learners (*cf.* 4.4.1.1), and learners being able to motivate their answers (*cf.* 4.4.1.4) were dominant teaching strategies utilized by the teachers. This is encouraging, as Searls (2006) and Arends (2009:345-452) indicate that questioning is an important teaching strategy for the development of critical thinking (*cf.* 2.5.1) as it encourages interaction, discussion and inquiry among learners. The frequent use of questioning is also an indication of a classroom climate that invites intellectual openness (Porch, 2002; Elder, 2007; Staples, 2007:167) (*cf.* 2.5.5). Although it appeared as if classrooms that create intellectual openness are created, the learner responses did not always convincingly indicate that they are always stimulated to think critically through the use of learning material and their involvement in the classroom. This is supported by the fact that the learner responses indicated a strong focus on the textbook approach (*cf.* 4.4.1.2), the dominant role of the teacher during teaching (*cf.*

4.4.1.3; 4.4.1.4), the limited involvement of learners in decision-making, making evaluations, making judgements, coming to conclusions, formulating definitions, planning, monitoring and evaluating their own work, having the opportunity to work with their peers and that teaching and learning is not related to real life experiences *(cf. 4.4.1.1;* 4.4.1.2; 4.4.1.3; 4.4.1.5). A dominant teacher will not create enough opportunities for learner input and participation during teaching and learning. According to Dowden (2002) (*cf.* 2.4.2), learner input and participation are crucial for the development of critical thinking. Although it appears that the learners are more involved during problem-solving (*cf.* 4.4.1.5), the learners seems to have limited opportunities to give their own input during problem-solving (*cf.* 4.4.1.3). It appears as if more opportunities should be provided to enhance the development of critical thinking through learner participation and learner input during teaching and learning in the classrooms of the learners who took part in the research.

From the above discussion it appears that teachers do provide some opportunities for the development of critical thinking through their choice of teaching methods and assessment strategies. However, more purposeful efforts are needed to develop critical thinking skills on a more frequent basis. It appears that the teaching in the Mathematics classrooms in which the study was conducted, should become more active and learner-centred than they presently appear to be (Gupta, 2001).

The following section focuses on the responses obtained from the teachers for the various sections of the questionnaire.

4.4.2 Teacher responses for the questionnaire

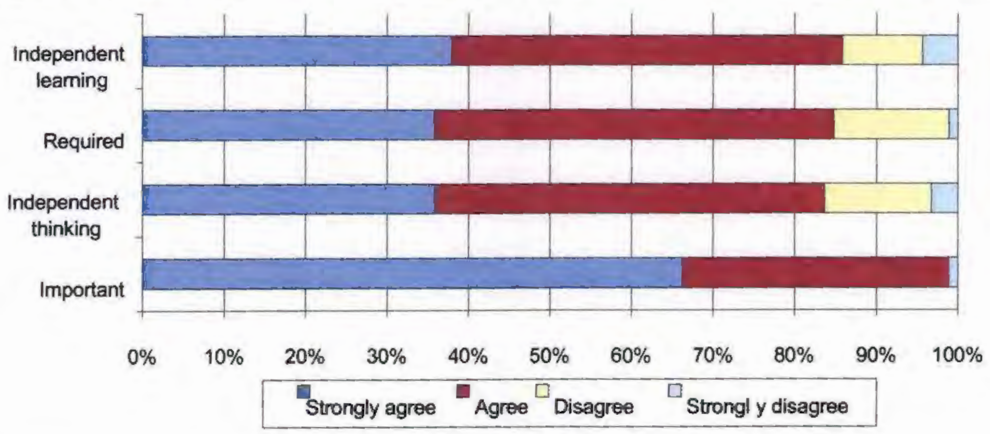
This section presents the responses obtained from the teachers for each of the sections in the questionnaire. Each section focused on a specific construct in relation to the development of critical thinking.

4.4.2.1 Understanding the meaning of critical thinking in the Mathematics classroom

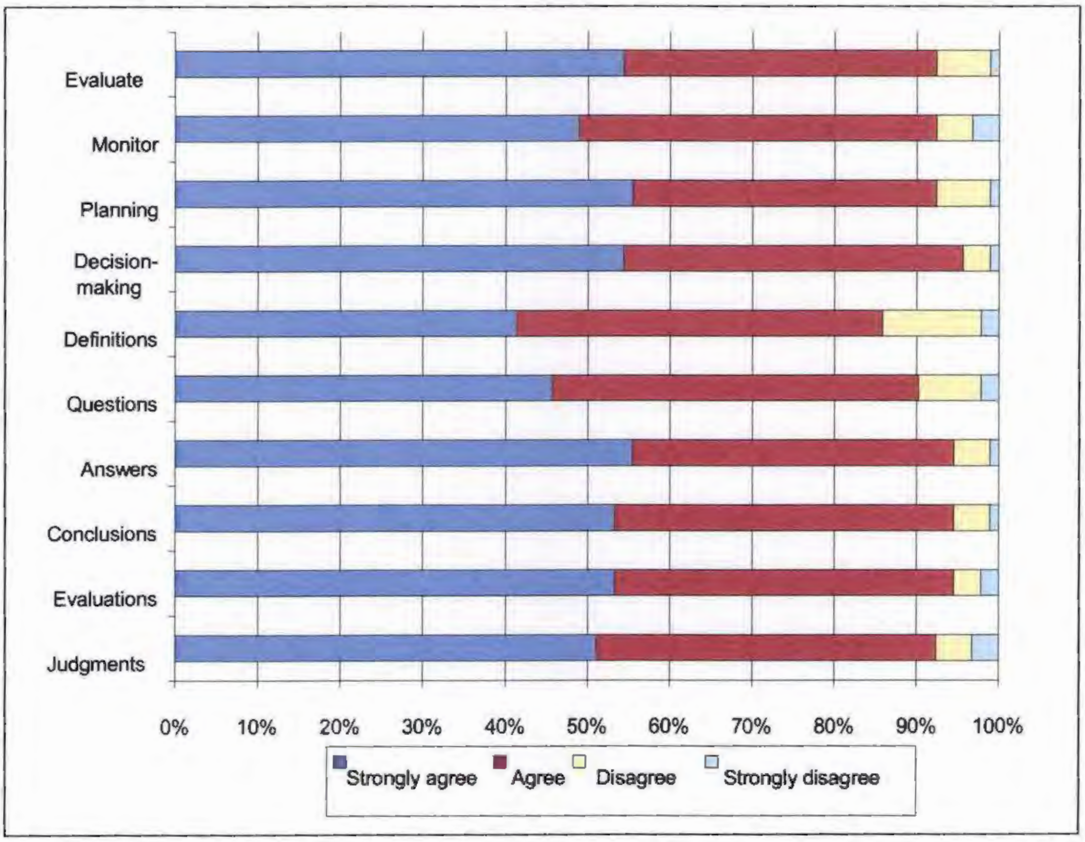
The following questions of the questionnaire were asked to determine whether the teachers understand the meaning of critical thinking in the Mathematics classroom. The researcher is of the opinion that if teachers understand the nature and importance of critical thinking skills, they would be better prepared to purposefully provide opportunities for the development of these skills. The teachers responses to the questions asked appear in Table 4.19 below.

Table 4.19: Teachers' understanding of critical thinking in the Mathematics classroom

Statement	Strongly agree	Agree	Disagree	Strongly disagree
1 Critical thinking in Mathematics important.	61 66.3%	30 32.6%	- -	1 1.1%
2 Critical thinking can be seen as thinking that is independent.	33 35.9%	44 47.8%	12 13.0%	3 3.3%
3 Most of the thinking in the Mathematics classroom requires critical thinking.	33 35.9%	45 48.9%	13 14.1%	1 1.1%
4 Critical thinking in Mathematics is needed for independent learning.	35 38.0%	44 47.8%	9 9.8%	4 4.3%



Statement	Strongly agree	Agree	Disagree	Strongly disagree
5. Critical thinking in Mathematics implies the following:				
5.1 Making judgements	47 51.1%	38 41.3%	4 4.3%	3 3.3%
5.2 Making evaluations	49 53.3%	38 41.3%	3 3.3%	2 2.2%
5.3 Formulating conclusions	49 53.3%	38 41.3%	4 4.3%	1 1.1%
5.4 Motivating answers	51 55.4%	36 39.1%	4 4.3%	1 1.1%
5.5 Formulating questions	42 45.7%	41 44.6%	7 7.6%	2 2.2%
5.6 Formulating definitions	38 41.3%	41 44.6%	11 12.0%	2 2.2%
5.7 Making decisions	50 54.3%	38 41.3%	3 3.3%	1 1.1%
5.8 Planning work before starting	51 55.4%	34 37.0%	6 6.5%	1 1.1%
5.9 Monitoring the own progress	45 48.9%	40 43.5%	4 4.3%	3 3.3%
5.10 Evaluating the outcomes of your own work	50 54.3%	35 38.1%	6 6.5%	1 1.1%



It appears as if teachers acknowledge the importance of critical thinking in Mathematics as a number of participants strongly agreed (66.3%) and agreed (32.6%) that critical thinking in Mathematics is important. Against the importance of critical thinking for Mathematics that was highlighted in the literature review (*cf.* 2.4.2); the researcher carefully assumed that because all teachers did not strongly agree to the preceding statements regarding critical thinking, it appears as if there might still be gaps in their understanding of the nature and importance of critical thinking skills. Teachers need to understand that the development of critical thinking skills in Mathematics will not only help learners to develop the ability to do problem-solving, but will also prepare them to succeed in the modern world (Fromboluti & Rinck, 1999; Erwin, 2000; Graven, 2002:24; Damji *et al.,* 2003; Winch, 2006:7) (*cf.* 2.4.2). Table 4.19 indicates that many of the teachers who participated in the study did not think that critical thinking in the Mathematics classroom can always be seen as thinking that is independent (35.9%). Only a few teachers also indicated that most of the thinking in the Mathematics classroom requires critical thinking (35.9%) and that critical thinking in Mathematics is needed for independent learning (38%). These responses support the learners' perceptions that teachers still rely on transmission of knowledge during teaching and a textbook approach (*cf.* 4.4.1.1; 4.4.1.2; 4.4.1.4), and that they (the learners) are involved to a lesser extent in the making of judgements, doing evaluations and coming to conclusions (*cf.* 4.4.1.3; 4.4.1.4). According to Winch (2006:74) (*cf.* 2.5), Mathematics is a subject that requires critical thinking skills in order to help the learners to see alternative solutions to a problem. It is also important that teachers make use of independent learning, as it will give the learners an opportunity to analyse and create ideas on their own, while seeking alternative solutions during problem-solving (Hida *et al.*, 2005) (*cf.* 2.5.1).

Approximately half of the participants strongly agreed that critical thinking implies making judgements (51.1%), making evaluations (53.3%), motivating answers (55.4%), making decisions (54.3%), planning (55.4%), monitoring (48.9%) and evaluating work (54.3%) as well as formulating conclusions, questions and definitions (53.3%, 45.7% & 41.3% respectively). To the researcher this is disturbing, as one would have expected to see all the teachers strongly agreeing that critical thinking always implies all of the mentioned skills. The teacher responses therefore not convincingly indicate that the teachers who took part in the study fully understand that critical thinking is a set of interrelated cognitive skills that always have to be developed and nurtured during the teaching and learning of Mathematics (Pithers & Soden, 2000:239; Cheung *et al.*, 2002; Barnes, 2005:42; Seng & Kong, 2006:54; Halpern, 2007:10-12) (*cf.* 2.3).

The next section reports on the teacher responses obtained for the use of teaching methods and assessment strategies in the classroom.

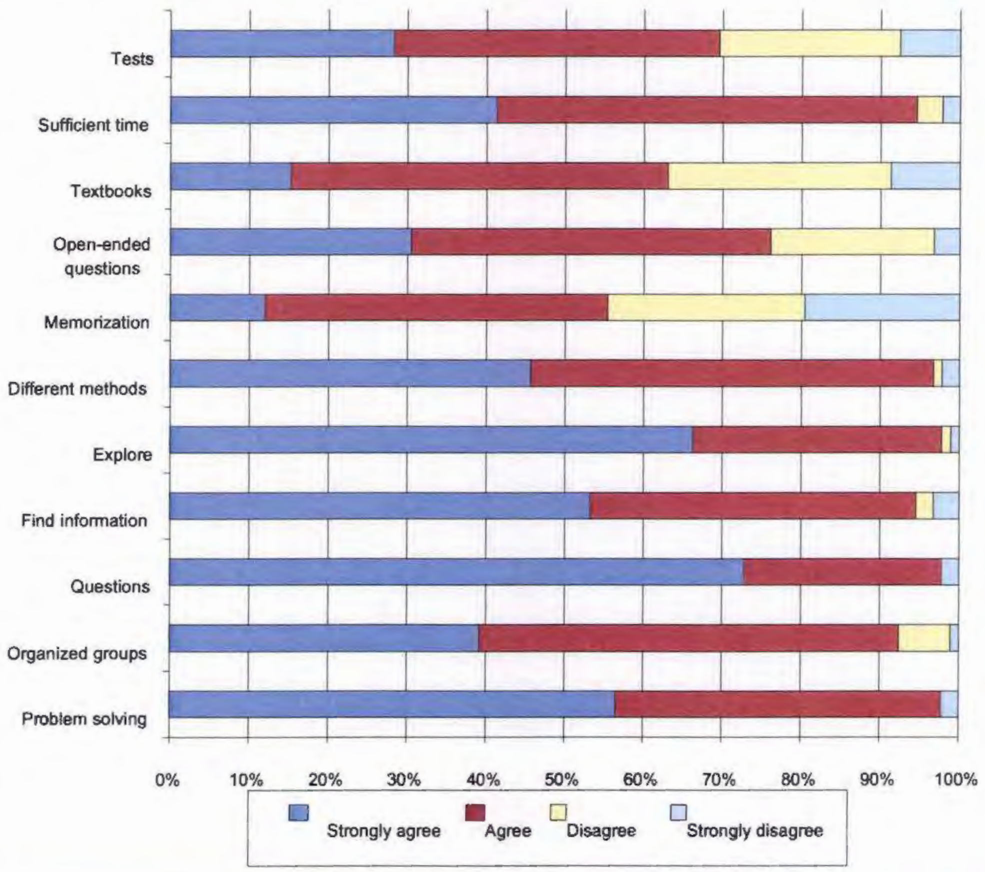
4.4.2.2 Teacher responses: teaching methods and assessment strategies in the Mathematics classroom

The main aims with the following questions were firstly to determine to what extent teachers agree with the application of general principles underpinning the development of critical thinking, and secondly to establish specifically what teaching methods and assessment strategies teachers use for the development of critical thinking. The teacher responses to the questions are indicated in Table 4.20 below.

Table 4.20: Teacher responses to the questions on teaching methods and assessment strategies used in the Mathematics classroom

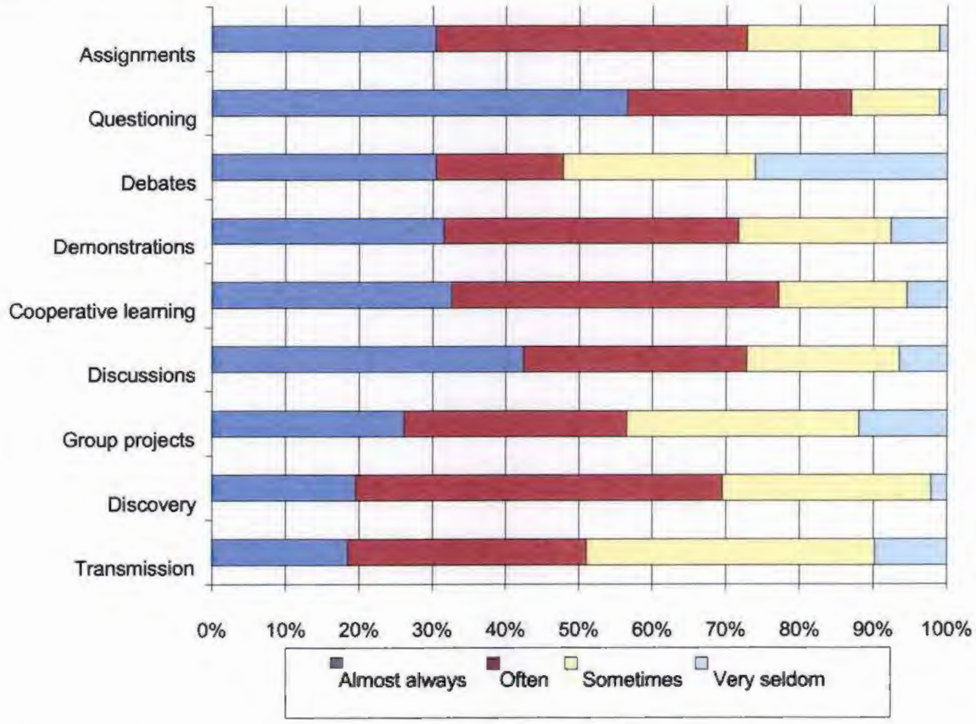
Statement	Strongly agree	Agree	Disagree	Strongly disagree
6 Problem-solving is a key issue in the development of critical thinking in Mathematics.	52 56.5%	38 41.3%	- -	2 2.2%
7 Working in organized group activities can improve the critical thinking abilities of learners in Mathematics.	36 39.1%	49 53.3%	6 6.5%	1 1.1%
8 Learners should be given the opportunity to ask questions.	67 72.8%	23 25.0%	-	2 2.2%
9 Learners should be allowed to find information themselves.	49 53.3%	38 41.3%	2 2.2%	3 3.3%
10 Learners should be allowed to explore different alternative solutions to problems.	61 66.3%	29 31.5%	1 1.1%	1 1.1%
11 Critical thinking in Mathematics is developed by making use of different teaching methods.	42 45.7%	47 51.1%	1 1.1%	2 2.2%
12 Memorization of information is still appropriate in the Mathematics classroom.	11 12.0%	40 43.5%	23 25.0%	18 19.6%
13 Teachers need to ask open-ended questions.	28 30.4%	42 45.7%	19 20.7%	3 3.3%
14 Learners should be encouraged to study Mathematics from the textbooks.	14 15.2%	44 47.8%	26 28.3%	8 8.7%
15 Teachers need to allow the learners sufficient time to think before they answer questions.	38 41.3%	49 53.3%	3 3.3%	2 2.2%
16 Tests are the most appropriate way to assess learners in the Mathematics	26	38	21	7

classroom.	28.3%	41.3%	22.8%	7.6%



Statement	Almost always	Often	Some-times	Very seldom
17 Teaching methods and assessment strategies used				
17.1 Transmission of knowledge	17 18.5%	30 32.6%	36 39.1%	9 9.8%
17.2 Discovery	18 19.6%	46 50.0%	26 28.3%	2 2.2%
17.3 Group projects	24 26.1%	28 30.4%	29 31.5%	11 12.0%
17.4 Discussions	39 42.2%	28 30.4%	19 20.7%	6 6.5%
17.5 Cooperative learning	30 32.6%	41 44.6%	16 17.4%	5 5.4%
17.6 Demonstrations	29 31.5%	37 40.2%	19 20.7%	7 7.6%

17.7	Debates	28 30.4%	16 17.4%	24 26.1%	24 26.1%
17.8	Questioning	52 56.5%	28 30.4%	11 12.0%	1 1.1%
17.9	Assignments	28 30.4%	39 42.4%	24 26.1%	1 1.1%



In the literature study, researchers such as Ferrando (2001), Cangelosi (2003:4), Appelbaum (2004:38) and Van der Walt and Maree (2007:237) (*cf.* 2.5.1) strongly recommend that teachers need to make use of a variety of teaching methods and assessment strategies to promote critical thinking in the Mathematics classroom. According to Table 4.20, it seems that not many teachers almost always allow their learners to discover (19,6%), to get involved in group projects (26,1%), to take part in discussions (42.2%), to experience cooperative learning (32.6%), to watch demonstrations (31,5%), to take part in debates (30.4%) and to complete assignments (30.4%). In the context of nurturing critical thinking, these strategies are vital for the development of critical thinking and have to be utilized on a more frequent basis as currently indicated by the responses (Leader & Middleton, 2004:65; Ash, 2005; Hida *et al.*, 2005:17; Gawe, 2007:208-227) (*cf.* 2.5.1). It is however encouraging to note that a number of teachers agreed to the mentioned principles and methods for nurturing critical thinking skills.

According to Gokhale (1995:22) group work and cooperative learning are very effective teaching strategies to use because they enhance mathematical understanding and increase the learners' interests in the Mathematics classroom. They also provide learners the opportunity to discuss, clarify and evaluate ideas and reflect together on the solutions to problems (Gokhale, 1995:28; Marcut, 2005:63) (*cf.* 2.5.1). Open-ended questions are another strategy that Searls (2006) and Arends (2009:345-452) (*cf.* 2.5.1) strongly recommend for they encourage the learners to think creatively and allow them to solve problems in a novel way. A number of teachers, 72.8% strongly agreed to the fact that learners should be given opportunities to ask questions. This response corroborates the learners' responses which also supported the frequent use of questioning (*cf.* 4.4.1.1).

It is important that teachers allow the learners to explore and discover new information on their own. Learners must be given the opportunity to demonstrate and explain their work and make use of relevant assignments that reflect their thinking and self-directed learning (Searls, 2006; Mahaye & Jacobs, 2007:200) (*cf.* 2.5.1). In this regard, 66.3% indicated that they strongly agreed that learners have to be allowed to explore alternative solutions to problems and 53.3% indicated that learners should be allowed to find information themselves. This response however, does not support the learners' responses (*cf.* 4.4.1.3) that indicated that they are not always allowed to give their own input during problem-solving.

It is important that teachers make use of a variety of teaching methods and assessment strategies when teaching Mathematics in order to encourage the learners to solve problems and create mathematical ideas (Schoënfeld, 1994:59; Winch, 2006:74; Sezer, 2008:351) (*cf.* 2.5). It is encouraging that a number of teachers (45.7%) strongly agreed and agreed (51.1%) to this point, as well as to the fact that problem-solving is a key issue for the development of critical thinking (56.5%). The latter corresponds with the learners' responses (*cf.* 4.4.1.5) that indicated that they are frequently involved in problem-solving.

It appears that involving learners in organized group activities and allowing learners to ask open-ended questions are underutilized as only 39.1% and 30.4% of the teachers respectively, strongly agreed to utilizing group activities and open-ended questions during teaching. Only 41.3% of the teachers strongly agreed that they allow the learners sufficient time to think before they answer questions. This response is very similar to the learners' response to question 13 (*cf.* Table 4.13) were 39.7% of the learners strongly agreed that they are allowed sufficient time to think.

The responses of the teachers to questions 12, 14 and 17.1 do not corroborate the responses of the learners to similar questions. Teachers indicated that they utilize textbooks (15.2%), memorization of information (12.0%) and transmission of knowledge (lecturing) (18.5%) to a lesser extent than what the learners indicated (*cf.* Table 4.13).

The next section reports the results obtained for the learning material used in the Mathematics classroom.

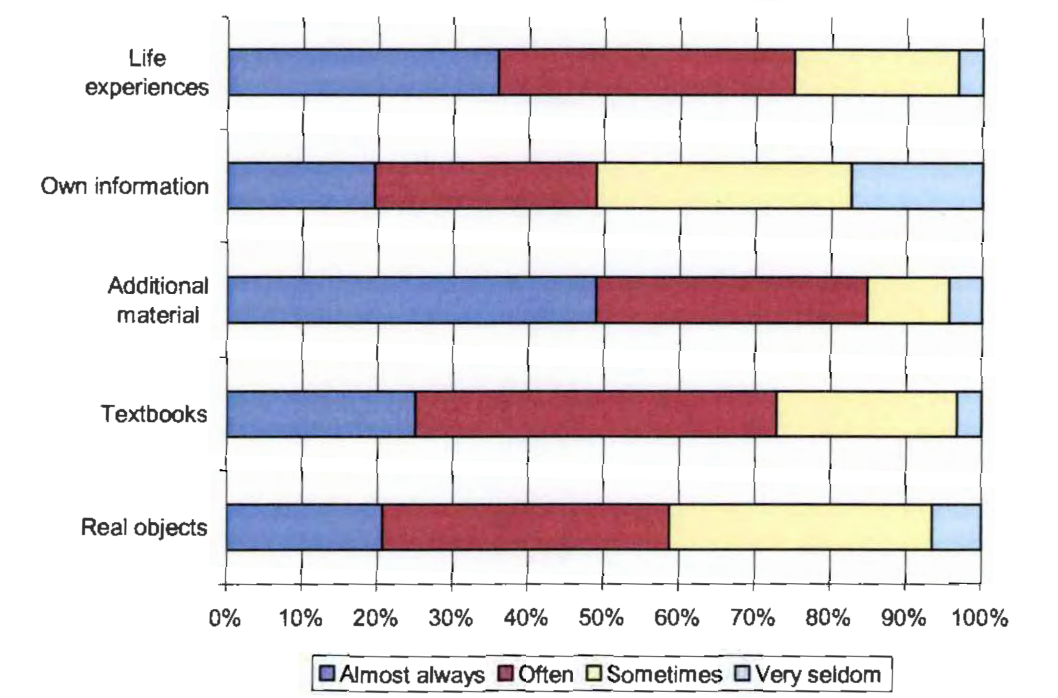
4.4.2.3 Teacher responses: the learning material used in the Mathematics classroom

In this section, the teachers were asked to reflect on the type of learning material they use during teaching. Their responses to these questions are reported in Table 4.21 below.

Table 4.21: Teacher responses to the questions on the learning material used in the Mathematics classroom

	Statement	Almost always	Often	Some-times	Very seldom
18	I make use of real objects during the teaching of Mathematics (eg models, pictures etc.)	19 20.7%	35 38.0%	32 34.8%	6 6.5%

19	I mainly use prescribed textbooks when teaching Mathematics.	23 25.0%	44 47.8%	22 23.9%	3 3.3%
20	I make use of additional material when teaching Mathematics.	45 48.9%	33 35.9%	10 10.9%	4 4.3%
21	I request learners to bring their own information to the Mathematics classroom.	18 19.6%	27 29.3%	31 33.7%	16 17.4%
22	I incorporate real life experiences into Mathematics teaching.	33 35.9%	36 39.1%	20 21.7%	3 3.3%



In order to nurture critical thinking through the choice of learning material, the literature strongly emphasizes that teachers need to encourage learners to seek information and construct knowledge on their own during problem-solving (Leader & Middleton, 2004:61; Mahaye & Jacobs, 2007:200) (*cf.* 2.5.1). Table 4.21 indicates that only a few teachers always request learners to bring their own information to the classroom (19.6%). In addition to this, only a small number of teachers always make use of real objects (20.7%) and additional material during the teaching of Mathematics (48.9%), and incorporate real-life experiences into Mathematics teaching (35.9%). The low percentages obtained for the various items corroborates the responses of the learners (*cf.* Table 4.14) where it was indicated that they were not frequently involved in teaching where real objects were used (15.2%), additional material were used (23.5%), they were allowed to bring their own information to the classroom (20.1%) or exposed to teaching that involved real life experiences (24.0%). The teacher and learner responses do not correspond for the use of textbooks. Once again, the teachers indicated that they rely on textbooks to a lesser extent (25.0%) than what the learners indicated (62.7%) (*cf.* Table 4.14).

According to Ash (2005) and Lake (2009) (*cf.* 2.5.3; 2.5.4), teachers should allow learners to bring their own background and experiences to the Mathematics classroom and also make use of real-life experiences as well as additional material, for it will enable the learners to develop the skill to draw appropriate real life conclusions, which in turn is important for the development of critical thinking. The use of additional learning material and real life experiences during teaching will provide opportunities to learners to communicate their ideas and provide logical arguments for their choices during decision-making, as well as a chance to discover things for themselves (Cangelosi, 2003:4) (*cf.* 2.5.1; 2.5.3) (*cf.* Table 4.14).

The next section focuses on how teachers deal with learner involvement in the Mathematics classroom.

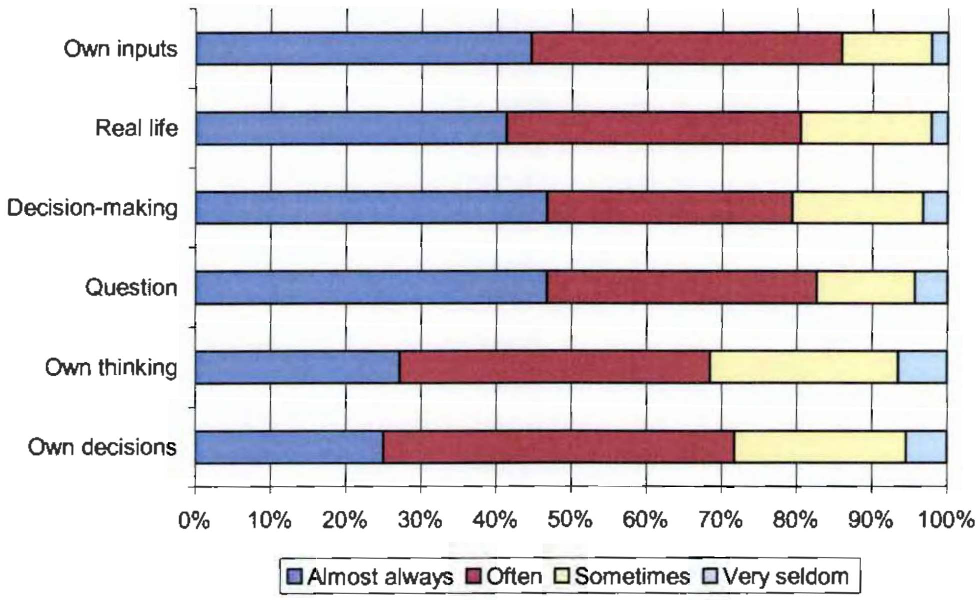
4.4.2.4 Learner involvement in the Mathematics classroom

The following questions focus on how the teachers viewed the learners' involvement and participation in the Mathematics classroom. Their responses to the questions asked, is given in Table 4.22 below.

Table 4.22: Teacher responses to the questions on learner involvement in the Mathematics classroom

Statement		Almost always	Often	Some-times	Very seldom
23	I allow learners to make their own decisions in the Mathematics classroom.	23 25.0%	43 46.7%	21 22.8%	5 5.4%

24	I allow learners to follow their own thinking in the Mathematics classroom.	25 27.2%	38 41.3%	23 25.0%	6 6.5%
25	I allow learners to question what I say in the Mathematics classroom.	43 46.7%	33 35.9%	12 13.0%	4 4.3%
26	I allow learners to participate in decision-making in the Mathematics classroom.	43 46.7%	30 32.6%	16 17.4%	3 3.3%
27	I relate the teaching of Mathematics to real life experience.	38 41.3%	36 39.1%	16 17.4%	2 2.2%
28	I allow learners to give their own inputs in solving mathematical problems.	41 44.6%	38 41.3%	11 12.0%	2 2.2%



One of the aims of the NCS is to create a new South African identity that encompasses critical consciousness and to promote learner involvement in education (Msila, 2007:151) (*cf.* 2.4.1). According to Table 4.22, not many of the Mathematics teachers who participated in the study always allow their learners to: make decisions on their own (25.0%), follow their own thinking (27.2%), question what the teacher says (46.7%), participate in decision-making (46.7%), relate their teaching to real-life experiences (41.3%) or allow learner input in problem-solving (44.6%). These results indicated that the teachers who participated in the study might perhaps still focus too strongly on a passive approach to teaching and learning which does not purposefully develop critical thinking skills. The responses above corroborate the learners' responses (*cf.* 4.4.1.3) that indicated that teachers favour a transfer of knowledge approach. The responses obtained in Table 4.15 also support a more passive approach to teaching, as the learners indicated that they are not frequently involved in opportunities: to make their own decisions (19.1%), to follow their own thinking (34.8%), not to passively accept what the teacher says (17.1%), to participate in decision-making (25.5%), involved in real life experiences (25.5%) and allowed input in solving problems (45.2%). It appears as if the focus in the classrooms that took part in the study, is not as Moloi (2005) indicates, on learning but rather on teaching content (*cf.* 2.4.1).

It is important that the learners in the Mathematics classroom become active participants (Ferrando, 2001; Appelbaum, 2004:308; Searls, 2006) (*cf.* 2.5.1; 2.5.4) who are given the freedom to explore and express their own opinions during classroom discussions (Gough, 1991:1) (*cf.* 1.3; 2.5.5). Active learning will enable learners to integrate knowledge and processes and provide them with opportunities to organize information, communicate ideas and opinions and analyse and evaluate their mathematical thinking (Appelbaum, 2004:308) (*cf.* 2.5.4). These skills are all important for intellectual growth and the development of critical thinking in the Mathematics classroom (Ferrando, 2001; Appelbaum, 2004:310) (*cf.* 2.5.1).

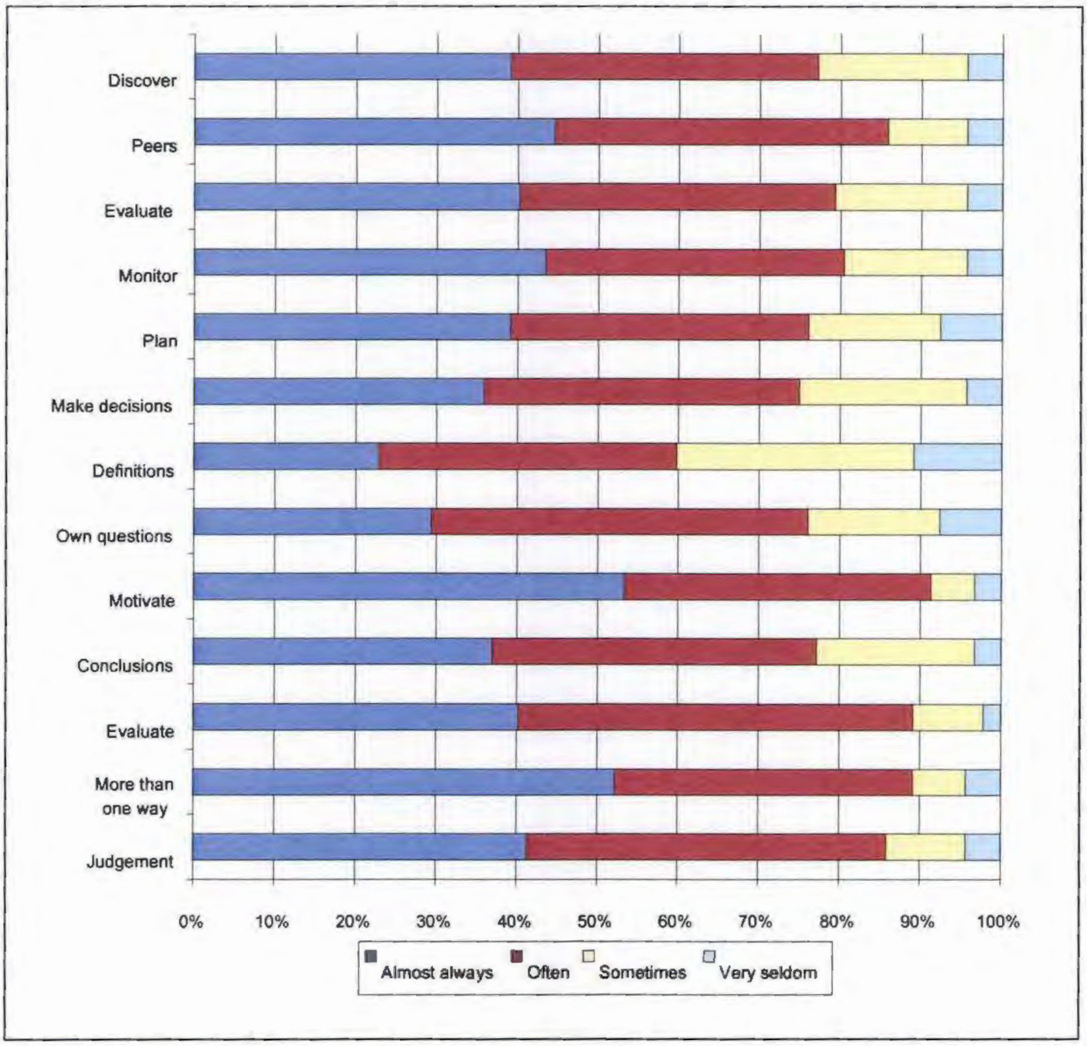
In the next section, the data obtained for the role of the teacher in the Mathematics classroom is explored.

4.4.2.5 Teacher responses: the role of the teacher in the Mathematics classroom

The purpose of the questions asked in this section was to determine the perceptions of the teachers on the roles that they play during the teaching of Mathematics. Their responses to the questions appear in Table 4.23 below.

Table 4.23: Teacher responses to the questions on the role of the teacher in the Mathematics classroom

	Statement	Almost always	Often	Some-times	Very seldom
29.	I acknowledge that there is no single correct way to solve problems.	48 52.2%	34 37.0%	6 6.5%	4 4.3%
30.	I nurture the following skills among learners when teaching Mathematics:				
30.1	Make judgements	38 41.3%	40 43.5%	9 9.8%	5 5.4%
30.2	Make evaluations	37 40.2%	45 48.9%	8 8.7%	2 2.2%
30.3	Formulate conclusions	34 37.0%	37 40.2%	18 19.6%	3 3.3%
30.4	Motivate their answers	49 53.3%	35 38.0%	5 5.4%	3 3.3%
30.5	Formulate their own questions	27 29.3%	43 46.7%	15 16.3%	7 7.6%
30.6	Formulate definitions	21 22.8%	34 37.0%	27 29.3%	10 10.9%
30.7	Make decisions	33 35.9%	36 39.1%	19 20.7%	4 4.3%
30.8	Plan their work before they start	36 39.1%	34 37.0%	15 16.3%	7 7.6%
30.9	Monitor their own progress	40 43.5%	34 37.0%	14 15.2%	4 4.3%
30.10	Evaluate the outcome of their own work	37 40.2%	36 39.1%	15 16.3%	4 4.3%
30.11	Work with their peers	41 44.6%	38 41.3%	9 9.8%	4 4.3%
30.12	Discover on their own	36 39.1%	35 38.0%	17 18.5%	4 4.3%



In the context of developing critical thinking skills, the literature review strongly emphasizes the role of the teacher as facilitator, organizer, role model, learning mentor, content specialist, knowledge dispenser, creator of a climate that nurtures critical thinking and a producer of learners who are productive in the Mathematics classroom (Maker & Nielson, 1996:69; Adler *et al.,* 2000; Dowden, 2002; Maharaj, 2007:3) (*cf.* 2.4.1; 2.4.2). Table 4.23 indicates that not many of the teachers who participated in the study almost always nurture the development of the following skills among learners when teaching Mathematics, namely making judgements (41.3%), making evaluations (40.2%), formulating conclusions (37.0%), motivate their answers (53.3%) formulating own questions (29.3%), formulating definitions (22.8%), making decisions (35.9%), planning work before starting (39.1%), monitoring the execution of work (43.5%), evaluating the outcomes of work (40.2%), working with peers (44.6%) and self-discovery (39.1%) (*cf.* 2.4.1; 2.4.2). All the preceding activities are important for promoting the development of critical thinking skills. Literature specifically emphasizes the importance of the development of higher-order and metacognitive skills (planning, monitoring and evaluation) for the development of critical thinking (Glazer, 2001; Muirhead, 2002; Berthold *et al.*, 2007:564-577; Kok, 2007:28-30; Van der Walt & Maree, 2007:227) (*cf.* 2.2; 2.3; 2.5.1). Carr and Jessup (1995:236) (*cf.* 2.2) emphasize the importance of metacognitive skills in Mathematics by indicating that performance in Mathematics depends on the execution of metacognitive skills. The learner responses in Table 4.16 indicated the same tendency, namely that they are not always involved in opportunities to make judgements (28.9%), to do evaluations (23.5%), to come to conclusions (29.4%), to formulate own questions (36.3%) to formulate definitions (25.5%), to make decisions (36.8%), to plan (33.3%), monitor (38.2%) and evaluate their own work (27.9%), to work with their peers (16.7%) and to discover on their own (25.0%).

Only 52.2% of the teachers who participated in this study, acknowledged that there is no single correct way to solve problems. Mathematics requires from learners to ask questions and to motivate their choices of methods for problem-solving (Van de Walle, 2003:17) (*cf.* 2.5.5). Teachers who promote the use of metacognitive strategies motivate learners more in the Mathematics classroom by improving the development and understanding of Mathematics concepts (Ferrando, 2001) (*cf.* 2.5.1) and help the learners to gain more confidence to justify their answers (Gough, 1991:5) (*cf.* 2.5). Teachers also need to motivate the learners to solve problems in their own way. Teachers who abandon independent problem-solving in the Mathematics classroom make learners passive recipients of information (Leader & Middleton, 2004:61) (*cf.* 2.5.1). Teachers can generate more activity in the Mathematics classroom by making use of strategies that allow the learners to collect, analyse, organize and critically evaluate the information given to them. It is then also important that the teacher gives the learners the opportunity to demonstrate their findings (Department of Education, 2007a) (*cf.* 2.4.1). Collaborative learning in teams and peers are believed to be a good strategy that teachers can use to enhance active participation and better mathematical understanding in the Mathematics classroom (Department of Education, 2007:4) (*cf.* 2.4.1). It appears that the teachers who took part in the study do not nurture the development of the aforementioned strategies to the full.

It is important that the teachers who took part in the study integrate critical thinking into the Mathematics classroom by creating more opportunities for learners to formulate conclusions, make their own decisions, evaluate unfamiliar mathematical situations (Glazer, 2001; Liljedahl, 2007:65; Oak, 2008; Tempelaar, 2008:175; Naik, 2009) (*cf.* 2.2; 2.3; 2.4.1; 2.4.2), construct and formulate appropriate questions (Carr & Jessup, 1995:236) (*cf.* 2.2), make predictions (Tempelaar, 2008:175) (*cf.* 2.3), monitor their understanding (Tempelaar, 2008:175) (*cf.* 2.3), support value judgements (Gallager, 1975; Dowden, 2002) (*cf.* 2.4.2; 2.4.2.1), process, analyse and evaluate information (Dowden, 2002) (*cf.* 2.4.2) and formulate conclusions (Cangelosi, 2003:125; Naik, 2009) (*cf.* 2.4.2; 2.4.2.1). Learners, who are able to analyse and evaluate information before they reach a conclusion, have developed the aptitude for the use of critical thinking skills. (Singh *et al.*, 2002; Oak, 2008; Sezer, 2008:351) (*cf.* 2.4.1; 2.4.2.1). Creating activities to develop the above-mentioned skills are important in the Mathematics classroom as they will not only enhance learner activity during the learning process, but also promote the use of critical thinking skills.

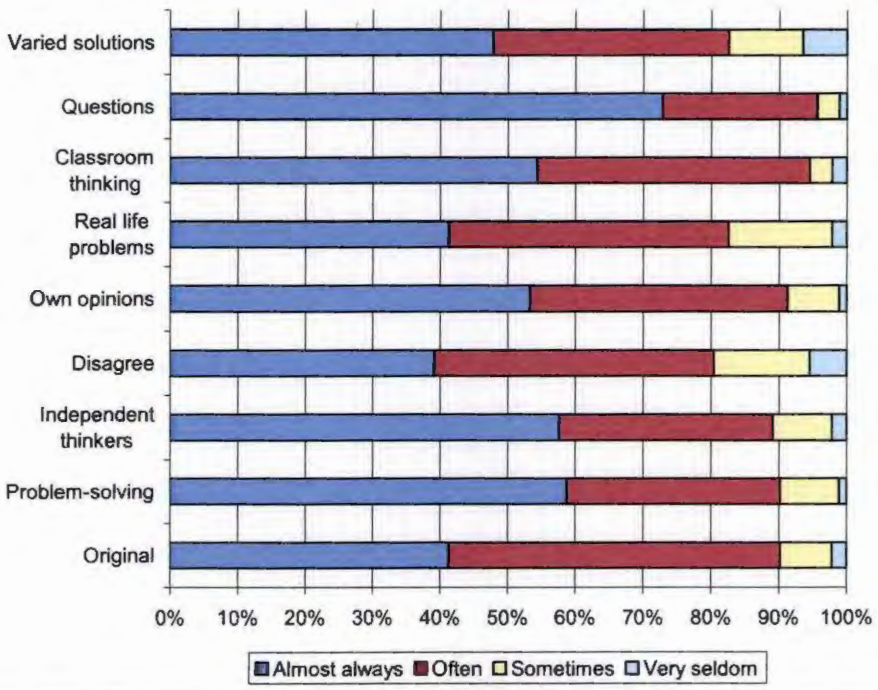
The next section reports on the results obtained for the responses received on the type of classroom climate that teachers create during the teaching of Mathematics.

4.4.2.6 Teacher responses: classroom climate in Mathematics

The following questions were asked to determine what type of classroom climate the teachers who took part in the study create during the teaching of Mathematics. Their responses to the questions appear in Table 4.24 below.

Table 4.24: Teachers responses to the questions on classroom climate

Statement	Almost always	Often	Some-times	Very seldom
31. My learners are allowed to be original.	38 41.3%	45 48.9%	7 7.6%	2 2.2%
32. My learners are allowed to participate in problem-solving.	54 58.7%	29 31.5%	8 8.7%	1 1.1%
33. My learners are encouraged to become independent thinkers.	53 57.6%	29 31.5%	8 8.7%	2 2.2%
34. My learners are given the freedom to disagree with me.	36 39.1%	38 41.3%	13 14.1%	5 5.4%
35. My learners are welcomed to give their own opinions.	49 53.3%	35 38.0%	7 7.6%	1 1.1%
36. My learners have the opportunity to solve real life problems.	38 41.3%	38 41.3%	14 15.2%	2 2.2%
37. My learners are encouraged to become independent thinkers in the classroom.	50 54.3%	37 40.2%	3 3.3%	2 2.2%
38. My learners may ask me questions in the class.	67 72.8%	21 22.8%	3 3.3%	1 1.1%
39. My learners do not have to solve problems in the same way.	44 47.8%	32 34.8%	10 10.9%	6 6.5%



The literature study strongly emphasizes the fact that teachers need to create a classroom climate in the Mathematics classroom that promotes critical thinking skills (Staples, 2007:167; Van de Walle, 2001:17; Lake, 2009:14) (*cf.* 2.5.5). Table 4.24 indicates that not many of the teachers who participated in the study always allow their learners to be original (41.3%), give their learners the freedom to disagree with them (39.1%), give them the opportunity to solve real life problems (41.3%) or allow learners to solve problems in their own way (47.8%). Similar responses were obtained from the learners (*cf.* Table 4.17) where they indicated that they are not always allowed to disagree with the teacher (41.7%), have the opportunity to solve real life problems (28.4%) or are allowed to solve problems in different ways (27.0%). The mentioned activities are all very important for creating a climate for the development of critical thinking skills in the Mathematics classroom. A classroom climate that promotes the development of cognitive skills will help produce learners who are more productive in the Mathematics classroom and in the learning process (Staples, 2007) (*cf.* 2.5.5). Teachers need to develop a classroom climate that gives the learners an abundance of opportunities to think on their own, to explain and share their own findings, to disagree with their teacher and to solve real life problems (Van de Walle, 2001:17; Porch, 2002; Elder, 2007; Staples, 2007:201) (*cf.* 2.5.5).

It is also apparent and encouraging that more than half of the teachers who participated in the study almost always allow their learners to participate (58.7%) and to demonstrate independent thinking during problem-solving (57.6%). They also encourage learners to ask questions in the class (72.8%). The aforementioned are important teaching and learning practices that will develop a more positive attitude towards Mathematics, create a classroom climate conducive to intellectual openness and increase learner performance (Porch, 2002) (*cf.* 2.5.5).

The following section summarizes the averages obtained for each of the different questionnaire sections for the teacher responses.

4.4.2.7 Summary: teacher responses

Means were calculated for the various sections in the questionnaire according to the ordinal scales utilized for classifying the questionnaire responses, namely 1=strongly agree/almost always, 2=agree/often, 3= disagree/sometimes, 4=strongly disagree/very seldom). Table 4.25 reports the means obtained for each of the sections in the questionnaire. In interpreting the results it is important to note that the lower the mean, the more favourable the response, as it is closer to 1.

Table 4.25: Teacher responses: means for the various questionnaire sections

Questionnaire sections	N	Mean	Std dev
Understanding critical thinking (Questions 1-5.10)	92	1.61	0.47
Teaching methods and strategies (general principles) (Questions 6-16)	92	1.77	0.37
Teaching methods and assessment strategies (Questions 17.1-17.9	92	2.05	0.34
Learning material (Questions 18-22)	92	2.08	0.57
Learner involvement (Questions 23-28)	92	1.87	0.60
Role of the teacher (Questions 29-30.12)	92	1.85	0.60
Classroom climate (Questions 31-39)	92	1.62	0.49

Table 4.25 indicates that the teachers who took part in the study had a fair understanding of what critical thinking entails. This understanding is also reflected in the favourable results obtained for the sections on the general principles for the application of teaching methods and strategies, learner involvement, the role of the teacher and the classroom climate for the development of critical thinking skills. To the researcher it appears that the choice of learning material and the specific teaching and assessment methods

and strategies need more attention. This assumption is supported by the responses of the teachers which revealed that teachers need to promote the use of real life experiences in the class and request learners to bring their own information to class (*cf.* 4.4.1.2; 4.4.2.3). In support of the learner responses the teachers indicated that they use questioning on a frequent basis and allow learners to participate in problem-solving (*cf.* 4.4.1.1; 4.4.1.5; 4.4.2.2; 4.4.2.6). However, it was evident that the nurturing of cognitive skills such as making judgments, evaluating information, making conclusions, making decisions, working with peers and discovering information were not developed to the full (*cf.* 4.4.1.3; 4.4.1.4; 4.4.2.5). To the researcher it appears that the teachers' understanding of critical thinking skills and their development does not always translate into practice of a daily basis.

The researcher acknowledges the fact that the sample used in the study was too small and geographically bound, to generalize the findings of the study to the larger population. Therefore inferential statistics were utilized only for the purpose to determine differences between the responses of the learners and teachers who took part in the study, in order to come to final conclusions regarding the sample with which the research was conducted.

4.5 DATA ANALYSIS: INFERENTIAL STATISTICS

4.5.1 Comparison: Learner and teacher responses

In order to determine whether there were any statistical significant differences between the teacher and learner responses obtained for the questionnaire, the responses were compared based on the mean scores for each of the questionnaire sections. T-tests were utilized to determine whether differences that occurred were statistically significant or not. P-values lower than 0.05 were regarded as statistical significant (Pietersen & Maree, 2007:230). To determine the effect size of the statistical significant difference, Cohen's D was calculated and the effect sizes were interpreted as follows:

- 0.2: small effect size
- 0.5: medium effect size
- 0.8: large effect size (Steyn 2005:20)

Table 4.26 reports the means, standard deviations, statistical significance of the differences between the means and the effect sizes of the differences between means for the learner and teacher responses for the various sections of the questionnaire.

Table 4.26: Differences between learner and teacher responses

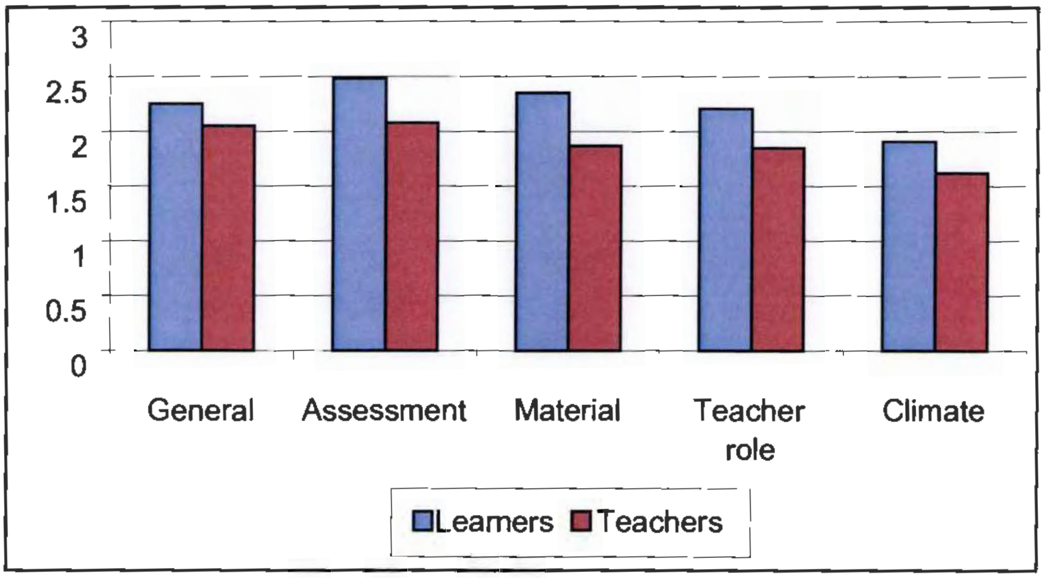
Questionnaire section		Mean	Std Dev	Sig.	Cohen's D	Effect size
Teaching methods and strategies (General principles)	Learners Teachers	1.88 1.77	0.40 0.37	0.204	-	
Teaching methods and assessment strategies	Learners Teachers	2.25 2.05	0.50 0.55	0.129	-	
Learning material	Learners Teachers	2.48 2.08	0.65 0.57	0.042 *	0.615	Medium
Learner involvement	Learners Teachers	2.35 1.87	0.63 0.61	0.686	-	
Role of the teachers	Learners Teachers	2.21 1.85	0.56 0.60	0.771	-	
Classroom climate	Learners Teachers	1.91 1.62	0.58 0.50	0.049 *	0.499	Medium

* Significance: $p<0.05$

In interpreting the results, it is important to note that the lower the mean, the more favourable the response, as it is closer to 1. The data in the table indicates that statistical significant differences were noted between the learners' and teachers' responses for the section on learning material and the role of the teachers. The teachers appeared to be more of the opinion than the learners that their choice of learning material and the role that they play in the classroom support the development of critical thinking skills. For each of these statistical significant differences a medium effect size was also noted.

The means of the different sections are presented in graphical form in Figure 4.1.

Figure 4.1: Comparison between the means of the teacher and learner responses



The X-axis in the graph indicates the different constructs for development of critical thinking in the Mathematics classroom. The Y-axis represents the means obtained by the teachers and learners for each of the constructs.

Based on the data in Table 4.26 and Figure 4.1, it is interesting to note that for all of the sections the teachers were more of the opinion that they created the necessary opportunities for the development of critical thinking skills, than the learners were.

Although it was not one of the objectives of the study to determine the impact of various demographic variables on the development of critical thinking, the researcher felt that this could perhaps add an interesting dimension to the study, and therefore decided to make use of an ANOVA (Analysis of variance).

4.5.2 Analysis of variance related to the development of critical thinking

In the following section, the impact of the biographic variables on the learner responses will be investigated. The following variables were considered:

- Ethnic group (Black / White)
- Gender (Male / Female)
- Home language (Afrikaans / English / Sesotho)

4.5.2.1 Analysis of variance: learner responses

An ANOVA was conducted to summarize data on the single biographic variables in relation to the various sections of the questionnaire (constructs for the development of critical thinking). In the table below, Table 4.27, the data for the biographic variable, **ethnic group**, in relation to the development of critical thinking skills, is reported. The following scale guides the interpretation of the table with reference to the ethnic groups represented in the study:

- Group 1 = Black
- Group 2 = White

In interpreting the results, it is important to note that the lower the mean, the more favourable the response, as it is closer to 1.

Table 4.27: ANOVA - Learner variable: ethnic group and the development of critical thinking skills

Questionnaire section	Ethnic group	N	Mean	Std dev	Sig.	Cohen's D	Effect size
Teaching methods and strategies (General principles)	Black White	95 109	1.81 1.95	0.46 0.33	0.013*	0.30	Small
Teaching methods and assessment strategies	Black White	95 109	2.20 2.28	0..50 0.50	0.238	-	
Learning material	Black White	95 109	2.25 2.69	0.70 0.51	0.000*	0.63	Medium
Learner involvement	Black White	95 109	2.35 2.34	0.63 0.62	0.901	-	
Role of the teacher	Black White	95 109	2.14 2.27	0.54 0.57	0.095	-	
Classroom climate	Black White	95 109	1.90 1.91	0.52 0.62	0.901	-	

*Significance: $p<0.05$

Table 4.27 indicates that there was a statistical significant difference with a small effect size between the opinions of the Black and White learners regarding the general principles for utilizing teaching methods and strategies to develop critical thinking skills. A statistical significant difference with a medium effect size was also noted between the perceptions of the Black and White learners regarding the learning material that their teachers utilize for the development of critical thinking skills.

It seems that the Black learners were more of the opinion that their teachers make use of a variety of teaching methods to enhance critical thinking in the Mathematics classroom, than the White learners were. Furthermore, it appeared that the Black learners judged their teachers' use of learning material to develop critical thinking more favourably than the White learners judged their teachers' use of learning material for the development of critical thinking skills.

Some of the questions that could be raised as to why the Black learners were more convinced that their teachers nurture the development of critical thinking skills, are the following:

- Could it be that the White learners are mainly exposed to the use of prescribed textbooks in the Mathematics classroom? During the research, a large percentage of the learners (62.7%) indicated that their teachers rely mostly on textbooks while teaching Mathematics (*cf.* 4.4.1.2).

- Could it be that the White learners are not allowed to participate actively in the Mathematics classrooms, and that their teachers do most of the talking? This assumption could be supported by the results obtained for learner involvement which reflected that some of the learners are not always involved in independent thinking, discoveries and participation during decision-making. Only 52.5% of the learners indicated that they are allowed to participate during problem-solving in the Mathematics classroom (*cf.* 4.4.1.1; 4.4.1.2; 4.4.1.3; 4.4.1.4).

- Could it be that the teachers of the White learners feel more at ease teaching Mathematics the old fashioned way, by means of transmission and the textbook, or the way they were taught? (Suliman, 2006:77) (*cf.* 2.4.2.1). This assumption could be supported by the fact that a number of learner responses revealed that teachers use transmission of knowledge 47.5% of the time and textbooks 62.7% of the time. The implication of this type of scenario is obviously that important skills such as decision-making, evaluation, making conclusions, inquiry and independent thinking will not be at the order of the day in such classrooms (*cf.* 4.4.1.1; 4.4.1.2).

As the ethnic group variable only comprised two groups, no post hoc tests Tukey HSD Tests) could be conducted.

The following section reports the results for the biographic variable **gende**r in relation to the development of critical thinking skills. The following key guides the interpretation of Table 4.28 with reference to the gender groups represented in the study: Female and Male

In interpreting the results, it is important to note that the lower the mean, the more favourable the response, as it is closer to 1.

Table 4.28: ANOVA – Learner variable: gender and the development of critical thinking skills

Questionnaire sections	Gender groups	N	Mean	Sig.
Teaching methods and strategies (General principles)	Female Male	101 102	1.84 1.92	0.170
Teaching methods and assessment strategies	Female Male	101 102	2.27 2.23	0.571
Learning material	Female Male	101 102	2.51 2.46	0.541
Learner involvement	Female Male	101 102	2.38 2.31	0.392
Role of the teacher	Female Male	101 102	2.19 2.23	0.584
Classroom climate	Female Male	101 102	1.83 1.98	0.061

*Significance: $p<0.05$

Table 4.28 indicates that there were no statistical significant differences in opinion between the male and female learners regarding the development of critical thinking in the Mathematics classroom. It appears as if male and female learners had similar perceptions regarding the ways in which teachers develop critical thinking in the Mathematics classroom. As the ANOVA did not indicate any statistical significant differences between the perceptions of females and males, and the gender variable also comprised only two groups, no further post hoc tests were conducted.

The following section reports the results for the biographic variable **home language** in relation to the development of critical thinking skills. The following key guides the interpretation of Table 4.29 with reference to the home language groups represented in the study:

- Group 1 = Afrikaans
- Group 2 = English

- Group 3 = Sesotho

In interpreting the results, it is important to note that the lower the mean, the more favourable the response, as it is closer to 1.

Table 4.29: ANOVA and Tukey HSD: learner variable: home language and the development of critical thinking skills

Key: 1 = Afrikaans 2 = English 3 = Sesotho

Questionnaire section	ANOVA sig	Language	N	Mean	Language Comparison	Sig.	Cohen's D	Effect size
Teaching methods and strategies (General principles)	0.046*	1	38	1.98	1→2 1→3	0.731 0.064	- -	- -
		2	72	1.92	2→1 2→3	0.731 0.169	- -	- -
		3	92	1.81	3→1 3→2	0.064 0.169	- -	- -
Teaching methods and assessment strategies	0.004*	1	38	2.49	1→2 1→3	0.009* 0.004*	0.93 0.96	Large Large
		2	72	2.20	2→1 2→3	0.009* 0.989	0.93 -	Large -
		3	92	2.19	3→1 3→2	0.004* 0.989	0.96 -	Large -

Questionnaire section	ANOVA sig	Language	N	Mean	Language Comparison	Sig.	Cohen's D	Effect size
Learning material	0.000*	1	38	2.74	1→2 ↘3	0.935 0.000*	- 1.47	- Large
		2	72	2.70	2→1 ↘3	0.935 0.000*	- 1.50	- Large
		3	92	2.23	3→1 ↘2	0.000* 0.000*	1.47 1.50	Large Large
Learner involvement	0.025*	1	38	2.58	1→2 ↘3	0.018* 0.152	0.96 -	Large -
		2	72	2.24	2→1 ↘3	0.018* 0.452	0.96 -	Large -
		3	92	2.36	3→1 ↘2	0.152 0.452	- -	- -

Questionnaire section	ANOVA sig	Language	N	Mean	Language Comparison	Sig.	Cohen's D	Effect size
Role of the teacher	0.007*	1	38	2.47	1→2 ↘3	0.010* 0.012*	0.96 0.90	Large Large
		2	72	2.15	2→1 ↘3	0.010* 0.917	0.96 -	Large -
		3	92	2.17	3→1 ↘2	0.012* 0.917	0.90 -	Large -
Classroom climate	0.003*	1	38	2.18	1→2 ↘3	0.002* 0.026*	1.15 0.86	Large Large
		2	72	1.79	2→1 ↘3	0.002* 0.476	1.15 -	Large -
		3	92	1.89	3→1 ↘2	0.026* 0.476	0.86 -	Large -

*Significance: $p<0.05$

The above table highlights that the ANOVA indicated that home language had a statistical significant influence on the perceptions of learners regarding all the sections in the questionnaire.

Consequently, a post hoc test was conducted to determine which language groups displayed these differences. A Tukey HSD (Honesty Significant Difference) test was utilized for this purpose (McMillan & Schumacher, 2006:302). This test investigated whether there were differences in the perceptions regarding the development of critical thinking skills between the different home language groups.

Statistical significant differences with large effect sizes were noted between the responses of the Afrikaans-speaking and English-speaking, as well as between the Afrikaans and Sesotho-speaking learners with regard to the teachers' use of teaching methods and assessment strategies. It appeared as if the Sesotho-speaking learners and the English-speaking learners perceived their teachers' application of teaching methods and assessment strategies for the development of critical thinking different and more favourable for the development of critical thinking than the learners of the Afrikaans-speaking group perceived their teachers' teaching and assessment practices.

With regard to the choice of learning material, a statistical significant difference with a large effect size was noted between the responses of the Afrikaans and Sesotho-speaking learners, as well as between the responses of the English and Sesotho-speaking learners. The Sesotho-speaking learners apparently perceived their teachers' choice of learning material to develop critical thinking more conducive than the Afrikaans and English-speaking learners perceived their teachers' choice of learning material for the development of critical thinking.

When coming to learner involvement in the classroom, a statistical significant difference with a large effect size was noted between the Afrikaans and English-speaking learners. The English-speaking learners were of the opinion that their teachers did more for learner involvement than the Afrikaans-speaking learners indicated for their teachers. No statistical significant differences were noted between the responses of the English and Sesotho-speaking, or the Afrikaans and Sesotho-speaking learners.

With regard to the role of the teacher during Mathematics teaching, it appeared as if the Sesotho-speaking learners and the English-speaking learners perceived their teachers' role in the classroom for the development of critical thinking significantly different and more favourable for the development of critical thinking skills than the learners of the Afrikaans-speaking group perceived their teachers' role. Statistical significant differences with large effect sizes were noted between the responses of the Afrikaans and Sesotho-speaking, as well as between the Afrikaans and English-speaking learners. It also appeared that the English-speaking learners were more convinced that their teachers' role in the classroom supports the development of critical thinking skills, than what the responses of the Afrikaans-speaking learners indicated for their teachers.

With regard to the classroom climate that teachers create during Mathematics teaching it was revealed that the Sesotho-speaking learners and the English-speaking learners perceived their teachers' role as more conducive in creating a climate for the development of critical thinking skills, than what the Afrikaans-speaking learners perceived their teachers' role in creating a classroom climate that stimulates critical thinking. Statistical significant differences with large effect sizes were noted between the responses of the Afrikaans and English-speaking learners, as well as between the Afrikaans and Sesotho-speaking learners. No statistical significant difference was noted between the perceptions of the English and Sesotho-speaking learners.

Overall, it appeared as if the Sesotho and English-speaking learners were more convinced than what the Afrikaans-speaking learners were about their teachers' efforts to nurture the development of critical thinking in the Mathematics classroom.

Some of the reasons why the Afrikaans-speaking learners might not feel the same as their Sesotho and English-speaking peers about the development of critical thinking skills during Mathematics teaching could possibly be attributed to the following factors:

- The teachers of the Afrikaans-speaking learners might still be teaching in the old fashioned they were taught (Suliman, 2006:73-79) (*cf.* 2.4.2.1) and expect of their learners to be passive in the classroom. The teachers of the Afrikaans-speaking learners appear to prefer a transmission and content-based approach to teaching (Moloi, 2005) (*cf.* 2.4.1).
- The teachers of the Sesotho and English-speaking learners might be more open to the use of new teaching methods and assessment strategies that focus on learner involvement during teaching and learning (Gupta, 2001; Cangelosi, 2003:4; Maharaj, 2007:34; Msila, 2007:151; Simic-Muller, 2007) (*cf.* 2.4.1; 2.4.2.2; 2.5.1).
- The teachers of the Afrikaans-speaking learners might prefer a prescribed textbook that they can trust (Volmink, 1994:61; Ellis, 2000) (*cf.* 2.5).

The following section focuses on the analysis of variance for the various biographic variables of the teachers in relation to the development of critical thinking skills.

4.5.2.2 Analysis of variance: teacher responses

In order to add a deeper dimension to the data obtained from the teacher responses, the biographic variables were examined to determine their influence on the participants' perceptions regarding the development of critical thinking in the Mathematics classroom.

It is important to mention that some of the original groupings of variables as they appeared in the questionnaire were reshuffled and clustered together to obtain sufficient participant numbers to do the statistical calculations. The following groupings were utilized for the purpose of the ANOVA:

The following four groupings were made for age of the teachers:

- 21 – 30

- 31 – 35
- 36 – 40
- 40+

The various groupings of the types of schools were not considered as a variable for the purposes of an ANOVA, as some of the groups did not have enough participants, and could therefore not be included in the statistical calculations.

The following groupings were made with regard to the experience in teaching Mathematics:

- 0 – 5 years
- 6 – 10 years
- 11 – 15 years
- 16+ years

The following groupings were made regarding the education qualification levels of the teachers:

- 3 year diploma
- 4 year diploma
- Degree and diploma
- Honours, Masters and PhD (Postgraduate qualification)

The following grouping were made regarding the ethical group of the teachers:

- Asian
- Black
- White

With regard to **teaching position**, the majority of the participants were teachers (n = 84). The principals and Heads of Department did not have enough participants and therefore teaching position was not considered in the statistical calculations of the ANOVA.

An ANOVA was conducted to summarize data on single independent variables in relation to the various sections of the questionnaire. The ANOVA did not yield statistical significant results for the impact of **age, ethnic group** and **experience in teaching Mathematics** on the perceptions of teachers regarding the nurturing of critical thinking in the Mathematics classroom. Therefore no post hoc tests (Tukey HSD Tests) were conducted with these variables.

However, with regard to teachers' **qualification levels**, statistical significant differences in perceptions regarding the development of critical thinking were obtained for the various qualification groupings. The ANOVA indicated that statistical significant differences occurred between the perceptions of teachers from the different qualification levels regarding:

- Learning material
- Role of the teacher (*cf.* Table 4.30).

Consequently, a post hoc test was conducted to determine which qualification groups displayed the differences. A Tukey HSD test was utilized for the following groupings of the qualification levels:

- Group 1: 3 year diploma
- Group 2: 4 year diploma
- Group 3: Degree and diploma
- Group 4: Honours, Masters and PhD (Postgraduate education)

Table 4.30 reports the results for the ANOVA and the Tukey HSD test regarding the qualification levels of the teachers and their perceptions regarding the development of critical thinking.

Table 4.30: ANOVA and Tukey HSD test: teacher qualification level and the development of critical thinking skills

Key: 1 = 3 year diploma 2 = 4 year diploma 3 = Degree & diploma 4= Post graduate

Dependent Variable	Anova Sig	Educa-tion level	N	Mean	Education level Comparison	Std. Error	Sig.	Cohen's D	Effect size
The learning material used in the Mathematics classroom	0.006*	1	43	1.94	2 3 4	0.16 0.14 0.16	0.003* 0.937 0.635	0.87	Large
		2	15	2.53	3 4	0.18 0.20	0.039* 0.220	0.53	Medium
		3	20	2.03	4	0.18	0.933		
		4	14	2.14					
The role of the teacher in the Mathematics classroom	0.023*	1	43	1.67	2 3 4	0.17 0.15 0.17	0.038* 0.566 0.125	0.52	Medium
		2	15	2.14	3 4	0.19 0.21	0.524 0.984		
		3	20	1.87	4	0.20	0.776		
		4	14	2.07					

*Significance: p<0.05

According to the above table, the Tukey HSD tests revealed teachers in possession of a three year teachers' diploma were more of the opinion that their choice of learning material supports the development of critical thinking, than the responses of the teachers in possession of a four year teachers' diploma were. A statistical significant difference with a large effect size was noted between these two groups.

It was also interesting that a statistical significant difference with a medium effect size was noted between the responses of the teachers in possession of a degree and a diploma and the teachers with a four year diploma. It appeared as if the teachers in possession of a three year qualification, and those with a degree and a diploma had a better understanding of the use of different learning material for the development of critical thinking skills than their colleagues who were in possession of a four year diploma and those with a postgraduate qualification.

In addition to this, the teachers in possession of a three year teachers' diploma responded more positively with regard to the role of the teacher in the nurturing of critical thinking in the Mathematics classroom, than the teachers in possession of a four year teachers' diploma did. A statistical significant difference with a medium effect size was noticed between the responses of the two groups.

4.6 TRIANGULATION OF TEACHER AND LEARNER DATA

In order to make final conclusions, the data obtained from the learner and teacher responses were triangulated.

The questionnaire section on the general **teaching and learning principles for the application of teaching methods and strategies** indicated that both learners and teachers agreed to the importance attached to questioning during the teaching of Mathematics. Learners and teachers indicated with 65.2 % and 56.5% respectively that questioning was very frequently used (*cf.* 4.4.1.1; 4.4.2.2). In particular, the learners were allowed to ask their own questions. Although questioning is important for the development of critical thinking, it appears that questioning is the main method utilized for assessing learners' knowledge and skills. This is not in line with the literature that highlights the use of a variety of assessment methods and strategies for the development of critical thinking skills (Ellis, 2000; Byers, 2004; Suurtamm, 2004:497; Stein *et al.*, 2006) (*cf.* 2.5.2). In the section on classroom climate, learners indicated that they were allowed to ask their own questions 77.0% of the time. The teachers confirmed this by indicating that they allow learners 72.8% of the time to ask their own questions (*cf.* 4.4.1.5; 4.4.2.6).

With reference to the **learning material** that teachers make use of in the Mathematics classroom, the responses to the following questions indicated a difference between the learner and the teacher responses.

- According to the learners, the teachers mainly make use of prescribed textbooks (62.7%), whereas the teachers indicated that they utilize textbooks 25.0% of the time (*cf.* 4.4.1.1; 4.4.1.2; 4.4.2.2; 4.4.2.3).

- The learners indicated that the teachers make use of additional material when teaching Mathematics about 23.5% of the time, whereas the teachers indicated that they utilize additional material 48.9% of the time (*cf.* 4.4.1.2; 4.4.2.3).

- The incorporation of real life experiences into the Mathematics classroom was evaluated by learners and teachers with 24.0% and 35.9% of the time respectively (*cf.* 4.4.1.2; 4.4.2.3).

In light of the fact that the teacher responses to the questions which examined their understanding of the nature and importance of the development of critical thinking revealed apparent gaps in their understanding (*cf.* 4.4.2.1), the researcher carefully assumes that the learner responses could be regarded as more trustworthy in this regard.

Responses to the following questions did not reveal large differences in opinion between the learners and teachers. The frequency, with which real objects an models were utilized during the teaching of Mathematics, was judged by learners and teachers with 15.2% and 20.7% respectively (*cf.*

4.4.1.2; 4.4.2.3). Regarding the question whether teachers request learners to bring their own information to the classroom, learners and teachers responded that this happens 20.1% and 19.6% of the time, respectively (*cf.* 4.4.1.2; 4.4.2.3).

With regard to **learner involvement** and the **role of the teacher** during the application of **teaching methods and assessment strategies,** it appeared from both the learner and teacher responses that the teacher still plays a dominant role in the classroom, as important skills such as evaluation, making conclusions, formulating decisions, reflecting and discovering were not promoted on a frequent basis during teaching, learning and assessment (*cf.* 4.4.1.3; 4.4.1.4; 4.4.2.4; 4.4.2.5). Teaching methods and strategies such as the utilization of debates, group activities, group projects and cooperative learning also appeared not to be implemented on a frequent basis (*cf.* 4.4.1.1; 4.4.1.4; 4.4.2.2; 4.4.2.5).

With reference to the **classroom climate** created for the development of critical thinking in the Mathematics classroom, it was found that the teachers and learners had different opinions with regard to the following questions:

- Learners indicated that they were only given the opportunity to solve real life problems 28.4% of the time, whereas the teachers indicated that this happens about 41.3% of the time (*cf.* 4.4.1.5; 4.4.2.6).

- Learners perceived the opportunities for thinking independently in the classroom as 43.1% of the time, whereas the teachers indicated that independent thinking is nurtured for about 54.3% of the time. This response corroborates the response related to the encouragement of independent thinking. Once again learners indicated that this happens 45.1% of the time whereas the teachers indicated that it happens about 57.6% of the time (*cf.* 4.4.1.5; 4.4.2.6).

- Learner and teacher perceptions were also divergent regarding the way in which problems are solved. Learners perceived that they were not supposed to solve problems in the same way for 27.0% of the time, whereas teacher indicated that they allowed more freedom to solve problems in different ways by indicating that they allow learners more or less 47.8 % of the time to solve problems in different ways (*cf.* 4.4.1.5; 4.4.2.6).

The teachers and learners almost gave similar responses to the following questions:

- The extent to which creative thought is allowed: the teachers responded with 41.3% and the learners with 44.1% (*cf.* 4.4.1.5; 4.4.2.6).
- The extent to which learners are allowed to participate in problem-solving: the teachers responded with 58.7% and the learners with 52.5% (*cf.* 4.4.1.5; 4.4.2.6).
- The extent to which learners are given the freedom to disagree with the teacher: the teachers responded with 39.1% and the learners with 41.7% (*cf.* 4.4.1.5; 4.4.2.6).
- The extent to which learners may give their own opinions: the teachers responded with 53.3% and the learners with 52.5% (*cf.* 4.4.1.5; 4.4.2.6).

4.7 CHAPTER SUMMARY

In summary, it appears that teachers are aware of the importance of the development of critical thinking skills in the Mathematics classroom (*cf.* 4.4.2.1), and that they are making an effort to develop these skills among learners by utilizing questioning and learner participation in problem-solving. Both questioning and problem-solving are regarded as important for the development of critical thinking (Erwin, 2000; Van de Walle, 2001:17; Graven, 2002:24; Winch, 2004:74, Searls, 2006; Suliman, 2006:77) (*cf.* 2.4.2.1; 2.5.1). However, the researcher argues that he reality of the classrooms that took part in the study, do not yet meet the ideals set out by the NCS for the development of critical thinking skills. More can to be done to nurture the development of critical thinking in the Grade 8 Mathematics classrooms that took part in the study. Against the background of the Learning Outcomes and Assessment Standards for Grade 8 Mathematics (*cf.* Table 2.1) it is clear critical thinking skills are imbedded in almost all the Assessment Standards that the learners have to achieve. The researcher therefore argues that teachers need to constantly, **on a daily basis**, provide opportunities for the nurturing of critical thinking skills. Recommendations for improving the development of critical thinking skills are made in Chapter five (*cf.* 5.7).

CHAPTER FIVE

SUMMARY, FINDINGS AND RECOMMENDATIONS

5.1 INTRODUCTION

This study was conducted with the purpose to determine whether Grade 8 Mathematics teachers provide opportunities for the development of critical thinking skills in their classrooms during the teaching, learning and assessment of Mathematics. The objectives formulated at the onset of the study are revisited in this chapter, to determine whether they were achieved or not.

The researcher's task is to make sure whether the literature review and the data collected by means of the questionnaires that were completed by learners and teachers, contributed to answering the problem question on which the study was based and assisted the researcher to achieve the overall aim and objectives of the study. This chapter provides information regarding the following:

- An overview of the study
- Findings from the literature review
- Findings from the empirical research
- Findings in relation to the aim and objectives of the study
- Limitations of the study
- Recommendations
- Suggestions for further research

5.2 AN OVERVIEW OF THE STUDY

The overview of the study intends to provide a brief summary of the gist of the preceding chapters of the study.

Chapter 1

The purpose of this chapter was to orientate the reader regarding the problem statement, the aims and objectives of the study and the empirical research design utilized in the study (*cf.* 1.2; 1.4; 1.5).

The problem statement, which was translated into the main aim of the study, focused on determining the opportunities that teachers create for the development of critical thinking skills in the Grade 8 Mathematics classroom (*cf.* 1.1; 1.2). A quantitative research method with a descriptive research design was utilized to gather data by means of a questionnaire from Grade 8 learners (n=204) and teachers of Mathematics (n=92) in the Ekurhuleni South District in Gauteng. The main objectives of the questionnaire were to determine the perceptions of learners and teachers regarding the following:

- teachers' understanding of the development of critical thinking in the Mathematics classroom;
- the teaching methods and assessment strategies utilized by teachers to develop critical thinking skills;
- the learning material that teachers utilize for the development of critical thinking skills;
- the learning activities that teachers structure for the development of critical thinking;
- learner involvement in the Mathematics classroom;
- the role that the teacher plays during the teaching and learning of Mathematics; and
- the type of classroom climate that teachers create during the teaching of Mathematics.

Chapter 2

This chapter focused specifically on providing an insight into the development of critical thinking in the subject Mathematics in South African classrooms. Firstly, the concept critical thinking was delineated in general terms (*cf.* 2.2; 2.3) and then the nature, role and importance of critical thinking in the subject Mathematics was explored from a national and an international perspective (*cf.* 2.4; 2.4.2). The researcher also explored national and international literature to determine how critical thinking can be developed in the Mathematics classroom by investigating different teaching and assessment methods (*cf.* 2.5.1; *cf.* 2.5.2), the learning material that should be utilized for the development of critical thinking (*cf.* 2.5.1; 2.5.3); the learning activities that should be structured for the development of critical thinking (*cf.* 2.5.4), the role of the teacher in developing critical thinking skills (*cf.* 2.5.1; 2.5.2; 2.5.3; 2.5.4), learner involvement in the Mathematics classroom (*cf.* 2.5.4) and the type of classroom climate that promotes the development of critical thinking skills (*cf.* 2.5.5).

Chapter 3

Chapter three elaborated on the empirical research design used to investigate the opportunities that teachers provide for the development of critical thinking skills in Grade 8 Mathematics classrooms. The research method, research design and data collection instrument were discussed in detail and the choice of a quantitative descriptive research design by means of questionnaires was motivated. The quantitative descriptive research design was suitable for the study as the researcher did a first exploration to establish a given situation through the opinions of learners and teachers regarding the opportunities that are presently provided to develop critical thinking skills in Grade 8 Mathematics classrooms.

Chapter 4

The data obtained from the questionnaires were analysed and interpreted in this chapter by means of descriptive and inferential statistics.

In general, it appeared that the teachers who took part in the study were aware of the importance of the development of critical thinking skills in the Mathematics classroom (*cf.* 4.4.2.1), and are trying to create opportunities to develop these skills among learners mainly by utilizing questioning and learner participation in problem-solving to some extent (*cf.* 4.4.1.1; 4.4.1.5; 4.4.2.2; 4.4.2.6). However, the results obtained from the questionnaires revealed that the ideals of the NCS for the development of critical thinking have not yet fully become a reality in the classrooms on which the research focused.

The learner responses confirmed that teachers do not utilize a variety of teaching methods and assessment strategies, and that the classrooms still focus on transmission of knowledge and the use of textbooks (*cf.* 4.4.1.1; 4.4.1.2; 4.4.2.2; 4.4.2.3)

5.3 FINDINGS FROM THE LITERATURE REVIEW

A literature review was conducted in order to obtain information on the nurturing of critical thinking in the Mathematics classroom. The information obtained from the literature review was utilized for the formulation of questions for the questionnaire.

From the literature review, the following conclusions were made regarding critical thinking, its nature, role and importance in the subject Mathematics.

- The development of critical thinking abilities involves the development of dispositions for effortful thinking (Cheung *et al.,* 2002; Halpern, 2007:10; Facione, 2009), the application of interrelated cognitive skills (Pithers & Soden, 2000:239; Cheung *et al.,* 2002; Vandermensbrugghe, 2004:422; Barnes, 2005:46; Halx & Reybold, 2005:296; Halpern, 2007:10-12), the development of behavioural critical thinking habits (Cheung *et al.,* 2002; Tsui, 2002:748) and the development of metacognitive skills such as reflection, so that learners learn how to monitor and evaluate their own thinking processes (Halpern, 2007:10) (*cf.* 2.3).

- In order to develop critical thinking, learners first need to acquire a sound and thorough knowledge base in the subject area where the critical thinking skills need to be applied (Tempelaar, 2008:175) (*cf.* 2.3).

- The ability to think critically is embedded in the Learning Outcomes and Assessment Standards of Grade 8 Mathematics (*cf.* 2.4.1), and has to be developed and nurtured by teachers (Department of Education, 2002:4-5; Naik, 2009) (*cf.* 2.4.2).

- Critical thinking is an essential skill to do problem-solving in Mathematics, as problem-solving requires the application of a set of interrelated cognitive skills that are necessary for the execution of critical thinking such as: decision-making, formulating inferences, logical reasoning, analysis, questioning, evaluation, application, making conclusions and organizing and analyzing information (Winicki-Landman, 2001:30; Singh *et al.*, 2002:324; Innabi & El Sheikh, 2006:66; Brodie, 2007:3; Maharaj, 2007:34; Sezer, 2008:351; Naik, 2009) (*cf.* 2.2; 2.3; 2.4.1; 2.4.2).

- Critical thinking is important in the subject Mathematics because it enables the learners to become more involved in their own learning through the opportunities that arise for critically investigating and explaining Mathematical terms and definitions (Appelbaum, 2004:309; Ash, 2005; Morris, 2007) (*cf.* 2.2; 2.3; 2.4.2; 2.5.1).

- The literature review suggested that critical thinking can be developed in the Mathematics classroom by teachers who become personally involved in the learning process. Teachers need to make use of a variety of teaching and assessment methods and strategies that motivate the learners to participate in the classroom activities. It is important for teachers to create a warm climate for learning that motivates the learners to explore and participate positively during a Mathematics lesson (Ferrando, 2001; Van de Walle, 2001:17; Searls, 2006) (*cf.* 2.5; 2.5.1; 2.5.2; 2.5.5).

- Critical thinking skills are required for algebraic reasoning (Simic-Muller, 2007; Kollars, 2008), interpretation of numerical relationships in graphs (Curcio, 1987:387), learning geometry and thinking geometrically (Duatepe & Ubuz, 2004; Chen *et al.*, 2009) (*cf.* 2.4.2.2; 2.4.2.3; 2.4.2.4).

- Critical thinking skills can be developed in the classroom if teachers focus on a learner-centred approach to teaching (Simic-Muller, 2007) with an emphasis on inquiry (Cangelosi, 2003:4; Hida *et al.*,2005), learner participation (Ferrando, 2001), independent learning (Hida *et al.*, 2005), collaborative learning (Marcut, 2005:63), interactive learning (Searls, 2006), encouraging learners to identify alternative solutions to problems (Winch, 2006:74), Sezer, 2008:351) and providing problem-solving and decision-making activities (Leader & Middleton, 2004:61-65; Winch, 2006:74) (*cf.* 2.5.1).

- Critical thinking skills can be developed by utilizing assessment strategies that do not focus only on tests and exams, but also include projects, assignments, peer assessment, performance assessment and authentic assessment (Boston, 2002; Suurtamm, 2004:499; Carter, 2005:10, Kestell, 2006) (*cf.* 2.5.2).

- Critical thinking in the Mathematics classroom should be at the heart of teaching and teachers need to know what the foundations of critical thinking are. It is important that the learners experience problem-solving in the Mathematics classroom more positively for it will make the teaching and learning process more productive (Elder, 2007) (*cf.* 2.3; 2.4).

- In order for critical thinking skills to develop, learners should be given the opportunity during assessment and in the execution of learning activities to pose and solve problems (Winicki-Landman, 2001:30; Winstead, 2004:44), acquire skills of argumentation and debate (Innabi & El Sheikh, 2006:66) and learn to interpret and apply information (Maharaj, 2007:34) (*cf.* 2.4.1).

- The textbook approach should not dominate instruction in the Mathematics classroom, and real life experiences should be integrated into the teaching, learning and assessment of Mathematics (Ellis, 2000; Suurtamm, 2004:499; Ash, 2005) (*cf.* 2.5; 2.5.1; 2.5.2).
- The classroom climate created by teachers should invite intellectual openness (Van de Walle, 2001:36; Crotty, 2002, Elder, 2007, Lake, 2009:14) (*cf.* 2.5.5).

5.4 FINDINGS FROM THE EMPIRICAL RESEARCH

Although only exploratory in nature, a number of important findings from this study corroborated the findings from the literature review.

Although more than half of the teacher participants strongly agreed that critical thinking in Mathematics is important, it is disturbing that not all of the participants shared this view (*cf.* 2.4.2). This was further supported by the fact that the data revealed that the development of the cognitive skills important for the execution of critical thinking was not receiving enough attention during teaching and learning (*cf.* 4.4.1.; 4.4.1.4; 4.4.2.4). It could be argued that if teachers do not acknowledge and/or fully understand the importance of critical thinking in Mathematics, how they will be able to provide opportunities for the development thereof. On the other had, teachers might acknowledge the importance of critical thinking, but lack the skills to translate this knowledge into practice. This, in turn, could attribute to the poor results obtained by learners in Mathematics, as argued by Maharaj (2007:34) (*cf.* 2.4.1), or corroborate the research findings of Lombard and Grosser (2004:212) and Brodie (2007:4) who indicate that teachers do not possess adequate skills and knowledge to develop critical thinking skills among learners (*cf.* 1.1).

It appeared from both the learner and teacher responses that the teacher still plays a dominant role in the classroom (*cf.* 4.4.1.3; 4.4.1.4; 4.4.2.4; 4.4.2.5) and that teaching methods and strategies such as the utilization of debates, group activities, group projects and cooperative learning also appear not to be implemented on a frequent basis (*cf.* 4.4.1.1; 4.4.1.4; 4.4.2.2; 4.4.2.5). This finding does not support the literature review that emphasizes the fact that learner-centred classrooms in which learners participate tend to be more conducive to the development of critical thinking skills (Ferrando, 2001; Simic-Muller, 2007) (*cf.* 2.5.1). The finding rather supports the view of King (2007:121) that South African teachers presently still encourage rote learning in the Mathematics classroom (*cf.* 2.4.2.1).

Learner and teacher responses differed with regard to the use of learning material in the classroom. According to the learners, the teachers mainly made use of prescribed textbooks, and both learner and teacher responses indicated that real life experiences and additional material when teaching Mathematics, are seldom incorporated (*cf.* 4.4.1.2; 4.4.2.3). This finding does not corroborate the findings from the literature review that advocates for the frequent use of real life experiences and the involvement of learners in bringing their own information to the classroom (Leader & Middleton, 2004:61-65; Ash, 2005) (*cf.* 1.1; 2.5.1).

The dominant role that the teachers, who took part in the research, still play in the classrooms was emphasized by the fact that important skills, such as evaluation, making conclusions, formulating decisions, reflecting and discovering information, were not promoted on a frequent basis during teaching, learning and assessment (*cf.* 4.4.1.3; 4.4.1.4; 4.4.2.4; 4.4.2.5). This finding is also not in line with the literature review, as the literature review clearly indicates that all the aforementioned skills need to be enhanced and developed frequently during the teaching of Mathematics (Dowden, 2002; Maharaj, 2007:34; Oak, 2008) (*cf.* 2.4.1; 2.4.2; 2.4.2.1).

With reference to the classroom climate created for the development of critical thinking it was found that the teachers and learners also had different opinions. Learners indicated that they are not frequently given the opportunity to solve real life problems, allowed to think independently or given the opportunity to solve problems in different ways. The teachers, on the other hand, were of the opinion that they provided more opportunities for solving problems, thinking independently and solving problems in different ways, than the learners indicated (*cf.* 4.4.1.5; 4.4.2.6). It appears as if this finding also does not support the literature review that indicates that intellectual openness should be regarded as the cornerstone for the development of critical thinking skills (Van de Walle, 2001:17; Staples, 2007:167; Lake, 2009:14) (*cf.* 2.5.5). Furthermore, it is indicated that identifying, posing and solving problems are key outcomes in the Mathematics curriculum (Winicki-Landman, 2001:30; Singh *et al.*, 2002:234; Winch, 2006:74; Department of Education, 2007; Sezer, 2008:351) (*cf.* 2.4.1; 2.4.2).

It was evident that teachers do allow their learners to ask questions in the Mathematics classroom (*cf.* 4.4.1.1; 4.4.1.5; 4.4.2.2; 4.4.2.7), motivate their answers (*cf.* 4.4.1.4; 4.4.2.5) and participate during problem-solving (*cf.* 4.4.1.5; 4.4.2.6). These are important activities for the development of critical thinking, as they allow the learners to formulate their own understanding. This finding supports the literature which indicates that teachers should introduce the idea of questioning in their classrooms, as questioning could assist learners in making conclusions, encourage logical thinking, enable learners to start seeing applications for themselves and not rely on the teacher. Questioning reinforces active and independent learning (Bullen, 1998:23) (*cf.* 2.5.1). Problem-solving on the other hand, provides practice in decision-making and opportunities to evaluate information and motivate answers which are important for the development of critical thinking skills (Erwin, 2000, Van de Walle, 2007:37) (*cf.* 2.4.2.1).

Not many of the learners who participated in the research study indicated that they are allowed to make their own decisions or to participate during decision-making in the Mathematics classroom (*cf.* 4.4.1.3; 4.4.1.4; 4.4.2.4; 4.4.2.5). This finding is also not in line with the literature which argues for the use of decision-making to promote critical thinking in the Mathematics classroom as decision-making allows learners to construct their own ideas before final answers are given (Leader & Middleton, 2004:62) (*cf.* 2.5.1).

Additional findings

From the empirical research, additional findings that were not directly related to the literature review were also derived. Statistical significant differences in opinion regarding the development of critical thinking skills were noted between the different learner ethnic and home language groups, as well as between the different qualification groupings of the teachers. The following additional findings are noted:

- There were statistical significant differences in opinion between the different ethnic groupings of learners who participated in the research study. The Black learners indicated that their teachers make more use of a variety of learning material while teaching Mathematics to develop critical thinking skills than the White learners indicated for their teachers (*cf.* 4.5.2.1).

- It appeared as if the Sesotho-speaking learners and the English-speaking learners perceived their teachers' application of teaching methods and assessment strategies for the development of critical thinking different and more favourable for the development of critical thinking than the learners of the Afrikaans-speaking group perceived their teachers' teaching and assessment practices (*cf.* 4.5.2.1).

- The English-speaking learners were of the opinion that their teachers did more for learner involvement that the Afrikaans-speaking learners indicated for their teachers (*cf.* 4.5.2.1).

- There were statistical significant differences between the responses of the Sesotho and Afrikaans-speaking, as well as between the Afrikaans and English-speaking learners. The Sesotho-speaking learners and the English-speaking learners perceived their teachers' role in the classroom for the development of critical thinking different and more favourable than the learners of the Afrikaans-speaking group perceived their teachers' role (*cf.* 4.5.2.1). The English and Sesotho-speaking learners were also more of the opinion than the Afrikaans-speaking learners that their teachers put in an effort to develop critical thinking skills (*cf.* 4.5.2.1).

- The Sesotho-speaking learners and the English-speaking learners perceived their teachers' role in creating a climate more conducive to the development of critical thinking than the Afrikaans-speaking learners experienced the climate created by their teachers (*cf.* 4.5.2.1).

- The teachers in possession of a three year teachers' diploma indicated a more favourable approach to the choice of learning material for the development of critical thinking than the responses of the teachers in possession of a four year teachers' diploma (*cf.* 4.5.2.2).

- It appeared as if the teachers in possession of a three year qualification and those with a degree and a diploma had a better understanding of the use of different learning material for the development of critical thinking skills than their colleagues in possession of a four year diploma and those with a postgraduate qualification (*cf.* 4.5.2.2).

- The teachers in possession of a three year teachers' diploma also responded more positively than the teachers in possession of a four year teachers' diploma with regard to their role in the nurturing of critical thinking in the Mathematics classroom (*cf.* 4.5.2.2).

- It appeared that many of the teachers who took part in the research were not well qualified and actually inexperienced to teach Mathematics (*cf.* Table 4.10; Table 4.11). This could, perhaps, be regarded as a contributing factor to their inadequate understanding regarding the development of critical thinking in the Mathematics classroom.

5.5 FINDINGS IN RELATION TO THE AIM AND OBJECTIVES OF THE STUDY

This study aimed at obtaining information to achieve the overall aim and objectives identified at the onset of the study (*cf.* 1.2). The researcher endeavours to revisit the aim and objectives of the study in order to ascertain whether they have been achieved.

Objective 1: delineating the meaning of the development of critical thinking skills

This objective was achieved through a literature review that, firstly, highlighted the general nature of critical thinking. The literature review revealed that the development of critical thinking skills involves the development and application of interrelated cognitive skills (Pithers & Soden, 2000:239; Cheung *et al.,* 2002; Vandermensbrugghe, 2004:417-422; Barnes, 2005:42-46; Halx & Reybold, 2005:296; Halpern, 2007:10-12), the development of behavioural critical thinking habits (Cheung *et al.,* 2002; Tsui, 2002:748) and the development of meta-cognitive skills such as reflection, so that learners learn to monitor and evaluate their own thinking processes (Halpern, 2007:10) (*cf.* 2.2; 2.3).

Secondly, in the context of Mathematics teaching a literature review of national and international resources indicated that the development of critical thinking implies *inter alia* to the following: Learners need to:

- be more involved in education;
- determine fact and opinion;
- create and compare arguments;
- identify and solve a variety of problems;
- identify, pose and solve problems creatively and critically;
- collect, analyse, organize and critically evaluate information given to them;
- communicate appropriately using descriptions in words, graphs, symbols, tables and diagrams;
- argue and debate;
- reflect and execute reasonable thinking;
- critically investigate and explain Mathematical terms and definitions;

- work with other learners as members of a team or group;
- observe patterns with rigorous logical thinking;
- apply formulas and algorithms;
- interpret and apply skills to do calculations and solve equations; and
- be equipped with knowledge, skills and values that will enable meaningful participation and offer benefits for society (Dowden, 2002; Glazer, 2001; Muirhead, 2002; Brodie, 2007:3; Maharaj, 2007:34; Tempelaar, 2008:175) (*cf.* 2.4.1; 2.4.2).

Objective 2: determining how critical thinking skills can be developed during the teaching and learning of Mathematics

This objective was also achieved by means of a literature review comprising national and international resources. The literature review revealed that inquiry-based teaching, experiential learning, interactive learning, collaborative learning, problem-solving and decision-making should be utilized during teaching in order to nurture the development of critical thinking skills (Van de Walle, 2001:58; Muirhead, 2002; Appelbaum, 2004:309; Byers, 2004; Ash, 2005; Hida *et al.*, 2005; Olivares, 2005, Searls, 2006; Elder, 2007; Morris, 2007) (*cf.* 2.5.1). Furthermore, it was highlighted by the literature that formative assessment, performance assessment, authentic assessment and peer assessment hold merits for the development of critical thinking skills (Ellis, 2000; Niedringhaus, 2001:10; Boston, 2002; Byers, 2004, Suurtamm, 2004:499) (*cf.* 2.5.2). A strong focus should also be placed on learner involvement in activities where they have to organize their thinking, communicate their thoughts, take part in discussions, communicate with their peers; analyse and evaluate strategies for solving problems, separate relevant from irrelevant information and apply new information (Cantrell, 2000; Appelbaum, 2004:308; Marcut, 2005:57-66; Stein *et al.*, 2006) (*cf.* 2.5.2; 2.5.4). The learning material utilized in the Mathematics classroom should include a variety and not focus on the mere use of a textbook. Learners should be requested to bring their own information to the classroom and real objects and real life experiences should also be included during teaching (Ellis, 2000; Ash, 2005; Pang, 2003:42) (*cf.* 1.3; 2.5.3). For the development of critical thinking skills it is also important to create a climate that promotes intellectual openness where learners can reason and explore, take risks and ask questions without feeling threatened or uncomfortable (Van de Walle, 2001:17; Porch, 2002; Staples, 2007:167) (*cf.* 2.5.5).

The aforementioned information informed the design of the questionnaires that were utilized in the context of the study.

Objective 3: scrutinizing teachers' perceptions regarding ways in which critical thinking skills can be developed in the Mathematics classroom

This objective was achieved by analysing the data obtained from the teacher responses to the questionnaire items (questions 1-5.10) that focused on the meaning teacher attach to the development of critical thinking in the Mathematics classroom.

More than half of the participants strongly agreed that critical thinking in Mathematics is important (*cf.* 2.4.2). However, this research revealed that important skills, such as evaluation, making conclusions, formulating decisions, reflecting and discovering were not promoted on a frequent basis during teaching, learning and assessment (*cf.* 4.4.1.3; 4.4.1.4; 4.4.2.4; 4.4.2.5). This implies that teachers might not be fully knowledgeable on what the development of critical thinking skills entails, which in turn could impact on the quality of the opportunities that they provide for the development of critical thinking skills.

Objective 4: establishing what types of teaching methods and assessment strategies teachers utilize in the Mathematics classroom to develop critical thinking skills

This objective was achieved through the questionnaire responses obtained from learners and teachers. The data revealed that the teachers apparently do make use of the questioning technique and allow the learners to ask questions in the Mathematics classroom. The teachers, however, don't really seem to make use of organized group activities and cooperative learning to nurture critical thinking in the Mathematics classroom (*cf.* 4.4.1.1; 4.4.2.2). Collaborative learning is important in the Mathematics classroom for it could increase the learners' interest as well as promote critical thinking skills (*cf.* 2.5.1)

Objective 5: determining the different types of learning material that teachers use during Mathematics teaching to develop critical thinking skills

According to the findings from the questionnaires responses obtained from the learners and teachers, it appeared that the teachers don't allow the learners to bring their own information to the Mathematics classroom or make use of real objects while teaching Mathematics. According to the learners' responses, it appeared that their teachers rely mainly on prescribed textbooks (*cf.* 4.4.1.2; 4.4.2.3). The literature review strongly suggested that teachers should make use of real objects and real life experiences rather than workbooks and textbooks to develop better understanding in the Mathematics classroom (*cf.* 2.5.3).

Objective 6: establishing the types of learning activities that teachers structure in the Mathematics classroom to develop critical thinking skills

The empirical study revealed that there is a need to make teachers more aware of how to involve their learners during problem-solving in the Mathematics classroom. It could be deducted from the learner and teacher responses to the questionnaire, that the teachers don't allow their learners to follow their own thinking, to make their own decisions or to give their own inputs when solving Mathematical problems (*cf.* 4.4.1.4; 4.4.2.5). Furthermore, the involvements of learners in activities that expect of them to evaluate, judge, plan, monitor, define and make conclusions are also underutilized (*cf.* 4.4.1.3; 4.4.1.4).

Objective 7: examining how teachers in the Mathematics classroom create a climate conducive to the development of critical thinking skills

According to the findings of the empirical research, the learners experienced the classroom climate as open and inviting for asking questions. However they did not experience the climate conducive to their becoming independent thinkers who have the freedom to disagree with their teachers (*cf.* 4.4.1.5; 4.4.2.6).

Based on the aforementioned explanation, the researcher is of the opinion that the overall aim of the study was achieved due to the fact that all the objectives, which were sub-aims located in the main aim, were achieved. In relation to the main aim of the study, the researcher therefore concludes that the Mathematics classrooms which took part in the research bear some evidence of opportunities for the development of critical thinking, but more purposeful efforts are needed to enhance and develop the critical thinking skills of the learners who took part in the research on a frequent basis.

5.6 LIMITATIONS OF THE STUDY

The following limitations of the research should be taken into consideration:

- Only a quantitative research approach was utilized. This means that the researcher had only a limited amount of data to work with. If the researcher integrated some qualitative strategies, namely interviews and/or observations, into the research approach, more comprehensive data could have been gathered regarding the instructional practices in the Mathematics classroom.

- The current sample was small and geographically bound. The interpretation of results should therefore be done with caution. This study was conducted in only one District in one of the nine provinces of South Africa which limits the generalization of the findings.

- The literature review highlighted the importance of the use of technology during the teaching of Mathematics to enhance the development of critical thinking skills (Ash, 2005) (*cf.* 2.5.1). This dimension was not explored during the study.

- The fact that the researcher used purposive sampling increased the likelihood of error due to experiment or subject bias (McMillan & Schumacher, 2006:128). In order to avoid bias, the researcher made use of triangulation of learner and teacher data.

- Due to time and convenience, the limited number of participating schools did not make it possible for the researcher to make comparisons between different types of schools.

- The fact that the learners questionnaire was in English only, could have had an impact on the Afrikaans and Sesotho-speaking learners' understanding of the questionnaire items.

- The researcher did not clearly establish the teaching experience of the teachers with specific reference to Grade 8 Mathematics. Inexperience linked to teaching Mathematics to Grade 8 in specific, could have had an impact on the quality of the teaching.

5.7 RECOMMENDATIONS

In light of the findings and to assist the teachers in dealing with critical thinking in the Mathematics classroom, the researcher recommends the following: in order to nurture the development of critical thinking skills effectively, the teachers who took part in the study have to acknowledge the following:

- The Mathematics classrooms that were involved in the research have to become more learner-centred (Simic-Muller, 2007) (*cf.* 2.4.2.2).

- Teachers need to move away from direct teaching to inquiry-based teaching (Cangelosi 2003:4; Hida *et al.*, 2005) (*cf.* 2.5.1).

- Learners need to participate during teaching and learning and look at things from different perspectives (Ferrando, 2001) (*cf.* 2.5.1).

- Independent learning and the creation of own ideas should be promoted (Hida *et al.*, 2005) (*cf.* 2.5.1).

- Collaborative and interactive teaching, learning and assessment have to be used on a frequent basis (McMillan 2001:9-10; Marcut, 2005:57-66; Searls, 2006; Gawe, 2007:208-227; McMillan, 2007:4) (*cf.* 2.5.1; 2.5.2).

- Mathematical communication has to be promoted through a dialogical style of teaching (Appelbaum, 2004:309; Ash, 2005, Morris, 2007) (*cf.* 2.5.1).

- A stronger focus has to be placed on assessment for learning approaches and on the assessment of learners by means of performance and authentic assessment tasks (Stiggins, 2002:759; Boston, 2002; Suurtamm, 2004:499; Carter, 2005:10; Kestell, 2006) (*cf.* 2.5.2).

- A stronger emphasis has to be placed on the importance of problem-solving (Schoënfeld, 1994:53-70; Elder & Paul, 2002:34-35, Cangelosi, 2003:125; Polya (in Macintyre, 2006:8-11), as problem-solving provides learners with the opportunity to develop the interrelated cognitive skills required for critical thinking, namely argumentation, interpretation, application, evaluation, making decisions and making conclusions (Winicki-Landman , 2001:30-38, Singh *et al.*, 2002:323-323; Winstead, 2004:34-50; Sezer, 2008:349-362; Naik, 2009) (*cf.* 2.4.1).

- Critical thinking skills are important for representing and formalizing patterns in Algebra, algebraic reasoning, interpreting numerical relationships in graphs and thinking geometrically (Shaughnessy & Zawojewski, 1999:713; Van de Walle, 2001:384; Duatepe & Ubuz, 2004, Simic-Muller, 2007, Kollars, 2008, Chen *et al.*, 2009) (*cf.* 2.4.2.2; 2.4.2.3; 2.4.2.4).

Furthermore, the following recommendations are made:

- Through in-service training, the Department of Education should make the Mathematics teachers more aware of the role and importance of critical thinking skills and how to develop these skills during the teaching of Mathematics.
- Teachers should engage in discussions with colleagues in their respective Mathematics departments in order to share their knowledge and understanding regarding the nurturing of critical thinking.
- The Department of Education should introduce extensive and comprehensive workshops and conferences where topics related to the development of critical thinking are discussed. These workshop programmes should provide the teachers with the necessary information to help them to develop and enhance their knowledge and understanding of the nurturing of critical thinking in the Mathematics classroom.

5.8 SUGGESTIONS FOR FURTHER RESEARCH

This study has made the researcher more aware of related topics that require further research. The following suggestions are made for further research:

- Further research can be done in other provinces in order to understand different experiences of teachers regarding the development of critical thinking in the Mathematics classroom from combined quantitative and qualitative perspectives.
- A study can be conducted to determine how factors such as time, overcrowded classes and lack of resources impact on the effective development of critical thinking in the Mathematics classroom.
- Further research is recommended on the teaching and learning material that teachers use in the Mathematics classroom due to the differences in opinion between the teacher and the learner responses revealed in this study.

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ADDENDUM A

TEACHER QUESTIONNAIRE

Teacher questionnaire on critical thinking in the Mathematics classroom

Dear teacher

I am currently busy with a Masters Degree at the North-West University, Vaal Triangle Campus. My research focuses on the development of critical thinking in the Mathematics classroom. I will appreciate it if you can complete the questionnaire. You will complete the questionnaire anonymously and all information will be handled with the utmost confidentiality. The results will be used for research purposes only. Thank you, your time and cooperation are valued.

Mrs A Deuchar

Please answer the following questions by marking your choice with an X in the appropriate block.

SECTION A: BIOGRAPHIC INFORMATION

1. Your age	21-25	26-30	31-35	36-40	40+	
2. Your current position	Principal	Deputy principal	Head of department	Educator		
3. Type of school	Township	Ex-Model C	Private school	Farm school		
4. Your ethnic group	Asian	Coloured	Black	White	Other	
5. Your experience in teaching Mathematics	0-5 yrs	6-10 yrs	11-15 yrs	16-20 yrs	20+ yrs	
6. Your highest education level	3 yr Diploma	4 yr Diploma	Degree and Diploma	B Ed degree	Masters degree	Ph D degree
Other (Please specify)						

SECTION B: UNDERSTANDING THE MEANING OF CRITICAL THINKING IN THE MATHEMATICS CLASSROOM: GENERAL PRINCIPLES

Please indicate the extent to which you would agree or disagree with the following statements by marking with an X in the appropriate block.

	Statement	Strongly agree	Agree	Disagree	Strongly disagree
1.	Critical thinking in Mathematics is important.	1	2	3	4
2.	Critical thinking can be seen as thinking that is independent.	1	2	3	4

	Question	Strongly agree	Agree	Disagree	Strongly disagree
3.	Most of the thinking in the Mathematics classroom requires critical thinking.	1	2	3	4
4.	Critical thinking in Mathematics is needed for independent learning.	1	2	3	4
5.	Critical thinking in Mathematics implies the following:				
5.1.	Making judgements	1	2	3	4
5.2.	Making evaluations	1	2	3	4
5.3.	Formulating conclusions	1	2	3	4
5.4.	Motivating answers	1	2	3	4
5.5.	Formulating questions	1	2	3	4
5.6.	Formulating definitions	1	2	3	4
5.7.	Making decisions	1	2	3	4
5.8.	Planning work before starting	1	2	3	4
5.9.	Monitoring the own progress	1	2	3	4
5.10.	Evaluating the outcome of your own work	1	2	3	4

SECTION C: TEACHING METHODS AND ASSESSMENT STRATEGIES IN THE MATHEMATICS CLASSROOM

Please indicate the extent to which you would agree or disagree with the following statements by marking with an X in the appropriate block.

	Teaching methods and strategies: general principles	Strongly agree	Agree	Disagree	Strongly disagree
6.	Problem solving is a key issue in the development of critical thinking in Mathematics.	1	2	3	4
7.	Working in organized group activities can improve the critical thinking abilities of learners in Mathematics.	1	2	3	4
8.	Learners should be given the opportunity to ask questions.	1	2	3	4
9.	Learners should be allowed to find information themselves.	1	2	3	4
10.	Learners should be allowed to explore different alternative solutions to problems.	1	2	3	4

11.	Critical thinking in Mathematics is developed by making use of different teaching methods.	1	2	3	4
12.	Memorization of information is still appropriate in the Mathematics classroom.	1	2	3	4
13.	Teachers need to ask open-ended questions.(there is more than one answer to a question)	1	2	3	4
14.	Learners should be encouraged to study Mathematics from a textbook.	1	2	3	4
15.	Teachers need to allow the learners sufficient time to think before they answer questions.	1	2	3	4
16.	Tests are the most appropriate way to assess learners in the Mathematics classroom.	1	2	3	4

17. Please indicate how often you utilize the following teaching methods and assessment strategies in your classroom:

Teaching methods and assessment strategies		Almost always	Often	Some-times	Very seldom
17.1.	Transmission of knowledge (Lecturing)	1	2	3	4
17.2.	Discovery	1	2	3	4
17.3	Group projects	1	2	3	4
17.4	Discussions	1	2	3	4
17.5	Cooperative learning	1	2	3	4
17.6	Demonstrations	1	2	3	4
17.7	Debates	1	2	3	4
17.8	Questioning	1	2	3	4
17.9	Assignments	1	2	3	4

SECTION D: THE LEARNING MATERIAL USED IN THE MATHEMATICS CLASSROOM

Please indicate the extent to which you utilize the following during Mathematics teaching by marking with an X in the appropriate block.

Learning material		Almost always	Often	Some-times	Very seldom
18.	I make use of real objects during the teaching of Mathematics (eg models, pictures etc).	1	2	3	4

19.	I mainly use prescribed textbooks when teaching Mathematics.	1	2	3	4
20.	I make use of additional material when teaching Mathematics.	1	2	3	4
21.	I request learners to bring their own information to the Mathematics classroom.	1	2	3	4
22.	I incorporate real life experiences into Mathematics teaching.	1	2	3	4

SECTION E: LEARNER INVOLVEMENT IN THE MATHEMATICS CLASSROOM

Please indicate the extent to which you allow and make room for the following during Mathematics teaching by marking with an X in the appropriate block.

	Learner involvement	**Almost always**	**Often**	**Some-times**	**Very seldom**
23.	I allow learners to make their own decisions in the Mathematics classroom.	1	2	3	4
24.	I allow learners to follow their own thinking in the Mathematics classroom.	1	2	3	4
25.	I allow learners to question what I say in the Mathematics classroom.	1	2	3	4
26.	I allow learners to participate in decision-making in the Mathematics classroom.	1	2	3	4
27.	I relate the teaching of Mathematics to real life experience.	1	2	3	4
28.	I allow learners to give their own inputs in solving mathematical problems.	1	2	3	4

SECTION F: THE ROLE OF THE TEACHER IN THE MATHEMATICS CLASSROOM

Please indicate how often you fulfill the following roles in the Mathematics classroom by marking with an X in the appropriate block.

	Role of the teacher	**Almost always**	**Often**	**Some-times**	**Very seldom**
29.	I acknowledge that there is no single correct way to solve problems.	1	2	3	4
30.	I nurture the following skills among learners when teaching Mathematics. **I allow learners to:**				
30.1.	Make judgements	1	2	3	4
30.2.	Make evaluations	1	2	3	4

30.3. Formulate conclusions	1	2	3	4
30.4. Motivate their answers	1	2	3	4
30.5. Formulate their own questions	1	2	3	4
30.6. Formulate definitions	1	2	3	4
30.7. Make decisions	1	2	3	4
30.8. Plan their work before they start	1	2	3	4
30.9. Monitor their own progress	1	2	3	4
30.10. Evaluate the outcome of their own work	1	2	3	4
30.11. Work with their peers	1	2	3	4
30.12. Discover on their own	1	2	3	4

SECTION G: CLASSROOM CLIMATE IN MATHEMATICS

Please indicate the extent to which you allow the following in order to create a classroom climate that is conducive to the development of critical thinking during the teaching of Mathematics by marking with an X in the appropriate block.

	Classroom climate	Almost always	Often	Some-times	Very seldom
31.	My learners are allowed to be original.	1	2	3	4
32.	My learners are allowed to participate in problem-solving.	1	2	3	4
33.	My learners are encouraged to become independent thinkers.	1	2	3	4
34.	My learners are given the freedom to disagree with me.	1	2	3	4
35.	My learners are welcomed to give their own opinions.	1	2	3	4
36.	My learners have the opportunity to solve real life problems.	1	2	3	4
37.	My learners are encouraged to become independent thinkers in the classroom.	1	2	3	4
38.	My learners may ask me questions in the class.	1	2	3	4
39.	My learners do not have to solve problems in the same way.	1	2	3	4

Thank you for your participation in this survey.

ADDENDUM B

LEARNER QUESTIONNAIRE

Learner questionnaire on critical thinking in the Mathematics classroom

Dear learner

I am currently busy with a Masters Degree at the North-West University, Vaal Triangle Campus. My research focuses on the development of critical thinking in the Mathematics classroom. I will appreciate it if you can complete the questionnaire. You will complete the questionnaire anonymously and all information will be handled with the utmost confidentiality. The results will be used for research purposes only. Thank you, your time and cooperation are valued.

Mrs A Deuchar

Please answer the following questions by marking your choice with an X in the appropriate block.

SECTION A: BIOGRAPHIC INFORMATION

1. Your ethnic group	Asian	Coloured	Black	White	Other
2. Your gender	Female			Male	
3. Your home language	English	Afrikaans	SeSotho	IsiZulu	Other

SECTION B: TEACHING METHODS AND ASSESSMENT STRATEGIES IN THE MATHEMATICS CLASSROOM

Please indicate the extent to which your teacher utilizes the following methods and strategies in the Mathematics classroom by marking with an X in the appropriate block.

Teaching methods and assessment strategies:general principles		**Strongly agree**	**Agree**	**Disagree**	**Strongly disagree**
4.	Problem solving is the main method used in the Mathematics classroom.	1	2	3	4
5.	My teacher allows us to work in organized group activities.	1	2	3	4
6.	My teacher gives us the opportunity to ask questions.	1	2	3	4
7.	My teacher allows us to find information ourselves.	1	2	3	4
8.	My teacher allows us to explore different alternative solutions to problems.	1	2	3	4
9.	My teacher makes use of lecturing in the Mathematics classroom.	1	2	3	4
10.	My teacher ask us to memorize information.	1	2	3	4
11.	My teacher asks open-ended questions (there is more than one answer to a question).	1	2	3	4

	Strongly agree	Agree	Disagree	Strongly disagree
12. My teacher encourages us to study Mathematics from a textbook.	1	2	3	4
13. My teacher allows the learners sufficient time to think before answering.	1	2	3	4
14. My teacher views tests as the most appropriate way to assess Mathematics.	1	2	3	4

15. My teacher utilizes the following teaching methods and assessment strategies in the Mathematics classroom:

Teaching methods and assessment strategies	**Almost always**	**Often**	**Some-times**	**Very seldom**
15.1. Transmission of knowledge (Lecturing)	1	2	3	4
15.2. Finding information on you own	1	2	3	4
15.3 Group projects	1	2	3	4
15.4 Discussions	1	2	3	4
15.5 Cooperative learning (working in groups with peers)	1	2	3	4
15.6 Demonstrations	1	2	3	4
15.7 Debates	1	2	3	4
15.8 Questioning	1	2	3	4
15.9 Assignments	1	2	3	4

SECTION C: THE LEARNING MATERIAL USED IN THE MATHEMATICS CLASSROOM

Please indicate the extent to which your teacher utilizes the following during Mathematics teaching by marking with an X in the appropriate block.

Learning material	**Almost always**	**Often**	**Some-times**	**Very seldom**
16. My teacher makes use of real objects during the teaching of Mathematics (e.g. models, pictures etc).	1	2	3	4
17. My teacher mainly uses prescribed textbooks when teaching Mathematics.	1	2	3	4
18. My teacher makes use of additional material when teaching Mathematics.	1	2	3	4

19. My teacher requests us to bring our own information to the Mathematics classroom.	1	2	3	4
20. My teacher incorporates real life experiences into Mathematics teaching.	1	2	3	4

SECTION D: LEARNER INVOLVEMENT IN THE MATHEMATICS CLASSROOM

Please indicate the extent to which you are involved during Mathematics teaching by marking with an X in the appropriate block.

Learner involvement	**Almost always**	**Often**	**Some-times**	**Very seldom**
21. My teacher allows learners to make their own decisions in the Mathematics classroom.	1	2	3	4
22. My teacher allows learners to follow their own thinking in the Mathematics classroom.	1	2	3	4
23. My teacher indicates to us that we should not passively accept what he/she says in the Mathematics classroom.	1	2	3	4
24. My teacher allows learners to participate in decision-making in the Mathematics classroom.	1	2	3	4
25. My teacher relates the teaching of Mathematics to real life experience.	1	2	3	4
26. My teacher allows learners to give their own inputs in solving mathematical problems.	1	2	3	4

SECTION E: THE ROLE OF THE TEACHER IN THE MATHEMATICS CLASSROOM

Please indicate how often your teacher fulfills the following roles in the Mathematics classroom by marking with an X in the appropriate box.

Role of the teacher	**Almost always**	**Often**	**Some-times**	**Very seldom**
27. My teacher acknowledges that there is no single correct way to solve problems.	1	2	3	4
28. My teacher nurtures the following skills among learners when teaching Mathematics. **We are allowed to:**				
28.1. Make judgements	1	2	3	4
28.2. Do evaluations	1	2	3	4
28.3. Come to conclusions	1	2	3	4

We are allowed to:	Almost always	Often	Some-times	Very seldom
28.4. Motivate our answers	1	2	3	4
28.5. Formulate our own questions	1	2	3	4
28.6. Formulate definitions	1	2	3	4
28.7. Make decisions	1	2	3	4
28.8. Plan our work before we start	1	2	3	4
28.9. Monitor our own progress	1	2	3	4
28.10. Evaluate the outcome of our own work	1	2	3	4
28.11. Work with our peers	1	2	3	4
28.12. Discover on our own	1	2	3	4

SECTION F: CLASSROOM CLIMATE IN MATHEMATICS

Please indicate the extent to which the following is allowed in your Mathematics class in order to create a classroom climate that is conducive to the development of critical thinking by marking with an X in the appropriate block.

Classroom climate	Almost always	Often	Some-times	Very seldom
29. We are allowed to be creative.	1	2	3	4
30. We are allowed to participate in problem-solving.	1	2	3	4
31. We are encouraged to become independent thinkers.	1	2	3	4
32. We are given the freedom to disagree with the teacher.	1	2	3	4
33. We are welcomed to give our own opinions.	1	2	3	4
34. We have the opportunity to solve real life problems.	1	2	3	4
35. We are encouraged to think independently in the classroom.	1	2	3	4
36. We may ask questions in the class.	1	2	3	4
37. We do not have to solve problems in the same way.	1	2	3	4

Thank you for your participation in this survey.

ADDENDUM C

CONSENT: DEPARTMENT OF EDUCATION

UMnyango WezeMfundo
Department of Education

Lofapha la Thuto
Departement van Onderwys

Date:	05 March 2007
Name of Researcher:	Deuchar Annelize
Address of Researcher:	13 Villa Granada
	St Aubyn Street
	New Redruth, Alberton
Telephone Number:	0837830540
Fax Number:	0116172212
Research Topic:	The development of critical thinking in the Mathematics classroom
Number and type of schools:	37 Primary Schools & 3 Technical Schools
District/s/HO	Johannesburg East, South & Ekurhuleni East

Re: Approval in Respect of Request to Conduct Research

This letter serves to indicate that approval is hereby granted to the above-mentioned researcher to proceed with research in respect of the study indicated above. The onus rests with the researcher to negotiate appropriate and relevant time schedules with the school/s and/or offices involved to conduct the research. A separate copy of this letter must be presented to both the School (both Principal and SGB) and the District/Head Office Senior Manager confirming that permission has been granted for the research to be conducted.

Permission has been granted to proceed with the above study subject to the conditions listed below being met, and may be withdrawn should any of these conditions be flouted:

1. *The District/Head Office Senior Manager/s concerned must be presented with a copy of this letter that would indicate that the said researcher/s has/have been granted permission from the Gauteng Department of Education to conduct the research study.*
2. *The District/Head Office Senior Manager/s must be approached separately, and in writing, for permission to involve District/Head Office Officials in the project.*
3. *A copy of this letter must be forwarded to the school principal and the chairperson of the School Governing Body (SGB) that would indicate that the researcher/s have been granted permission from the Gauteng Department of Education to conduct the research study.*

Director: Knowledge Management and Research
Room 525, 111 Commissioner Street, Johannesburg, 2001 P.O.Box 7710, Johannesburg, 2000
Tel: (011) 355-0488 Fax: (011) 355-0286

4. *A letter / document that outlines the purpose of the research and the anticipated outcomes of such research must be made available to the principals, SGBs and District/Head Office Senior Managers of the schools and districts/offices concerned, respectively.*
5. *The Researcher will make every effort obtain the goodwill and co-operation of all the GDE officials, principals, and chairpersons of the SGBs, teachers and learners involved. Persons who offer their co-operation will not receive additional remuneration from the Department while those that opt not to participate will not be penalised in any way.*
6. *Research may only be conducted after school hours so that the normal school programme is not interrupted. The Principal (if at a school) and/or Senior Manager (if at a district/head office) must be consulted about an appropriate time when the researcher/s may carry out their research at the sites that they manage.*
7. *Research may only commence from the second week of February and must be concluded before the beginning of the last quarter of the academic year.*
8. *Items 6 and 7 will not apply to any research effort being undertaken on behalf of the GDE. Such research will have been commissioned and be paid for by the Gauteng Department of Education.*
9. *It is the researcher's responsibility to obtain written parental consent of all learners that are expected to participate in the study.*
10. *The researcher is responsible for supplying and utilising his/her own research resources, such as stationery, photocopies, transport, faxes and telephones and should not depend on the goodwill of the institutions and/or the offices visited for supplying such resources.*
11. *The names of the GDE officials, schools, principals, parents, teachers and learners that participate in the study may not appear in the research report without the written consent of each of these individuals and/or organisations.*
12. *On completion of the study the researcher must supply the Senior Manager: Strategic Policy Development, Management & Research Coordination with one Hard Cover bound and one Ring bound copy of the final, approved research report. The researcher would also provide the said manager with an electronic copy of the research abstract/summary and/or annotation.*
13. *The researcher may be expected to provide short presentations on the purpose, findings and recommendations of his/her research to both GDE officials and the schools concerned.*
14. *Should the researcher have been involved with research at a school and/or a district/head office level, the Senior Manager concerned must also be supplied with a brief summary of the purpose, findings and recommendations of the research study.*

The Gauteng Department of Education wishes you well in this important undertaking and looks forward to examining the findings of your research study.

Kind regards

ACTING CHIEF DIRECTOR: OFSTED

The contents of this letter has been read and understood by the researcher.	
Signature of Researcher:	[signature]
Date:	5/3/2007

ADDENDUM D

CONSENT: LEARNERS, TEACHERS AND PARENTS

NORTH-WEST UNIVERSITY
YUNIBESITI YA BOKONE-BOPHIRIMA
NOORDWES-UNIVERSITEIT
VAAL TRIANGLE CAMPUS

PO Box 1174, Vanderbijlpark
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Fax: (016) 910-3116
Web: http://www.nwu.ac.za

Tel: (016) 016 910 3063
Fax (016) 016 901 3078
EMail School of Education Sciences@nwu.ac.za

Informed consent (Learner)

20 February 2008

Dear Learner

I am busy with a research study for my M.Ed-degree. I need your assistance to provide me with information to complete the study. This document will provide you with information regarding the project and what your involvement will entail. If you feel comfortable with the contents of the explanation I will appreciate it if you could sign the section indicating your consent to take part in the study.

With the assistance of the Heads of Department Mathematics I will identify Grade 8 learners to participate in the research study. Participation will not be compulsory and you may withdraw at any time should you feel uncomfortable. I would like you to complete a questionnaire which will take approximately 30 minutes of your time. Th questionnaire will be administered by the Heads of Department Mathematics at your school at a scheduled time which will not interfere with your classes. The aim of the questionnaire is to determine how you experience the development of critical thinking skills in the Mathematics classroom. No possible risks are envisaged. There are no direct benefits for taking part in the study.

CONFIDENTIALITY: you will complete the questionnaire anonymous, and the information that you disclose during the completion of the questionnaire will be kept confidential by the researcher. When reporting on the questionnaires information no names of individuals or the schools will be revealed.

The research is conducted by a Masters student, Annelize Deuchar, under the supervision of Prof. M.M. Grosser from the School of Educational Sciences, North-West University (Vaal Triangle Campus). If you have any questions or queries you can contact Prof. Grosser at 016 910 3063 (work).

CONSENT:

I ....(full name) have read and understand the nature of my participation in the project and agree to participate.

Signature:... Date:...

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20 February 2008

Informed consent (Teacher)

Dear Teacher

I am busy with a research study for my M.Ed-degree. I need your assistance to provide me with information to complete the study. This document will provide you with information regarding the project and what your involvement will entail. If you feel comfortable with the contents of the explanation I will appreciate it if you could sign the section indicating your consent to take part in the study.

Participation will not be compulsory and you may withdraw at any time should you feel uncomfortable. I would like you to complete a questionnaire in your own time, which should take you approximately 30 minutes. The aim of the questionnaire is to determine your understanding of the development of critical thinking skills during Grade 8 Mathematics teaching, and which opportunities you provide for the development of critical thinking in Mathematics. No possible risks are envisaged. There are no direct benefits for taking part in the study.

CONFIDENTIALITY: questionnaires will be completed anonymous, and the information that you disclose during the completion of the questionnaire will be kept confidential by the researcher. When reporting on the questionnaires information no names of individuals or the schools will be revealed.

The research is conducted by a Masters student, Annelize Deuchar, under the supervision of Prof. M.M. Grosser from the School of Educational Sciences, North-West University (Vaal Triangle Campus). If you have any questions or queries you can contact Prof. Grosser at 016 910 3063 (work).

CONSENT:

I ....(full name) have read and understand the nature of my participation in the project and agree to participate.

Signature:... Date:...

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Informed consent (Parents/Guardians)

Dear Parent/Guardian

I am busy with a research study for my M.Ed-degree. I need your permission to involve .... (Name of learner) as a participant in my study. The study aims to obtain information from Grade 8 learners as well as teachers by means of the completion of a questionnaire. This letter provides you with information regarding the project and what the involvement of your son/daughter will entail. If you feel comfortable with the contents of the explanation I will appreciate it if you could sign the section indicating your consent that your son/daughter may take part in the study.

Participation will not be compulsory and your son/daughter may withdraw at any time should they feel uncomfortable. I would like your son/daughter to complete a questionnaire at a time scheduled by the Heads of Department Mathematics which will not interfere with their classes. It will take your son/daughter approximately 30 minutes to complete the questionnaire. The aim of the questionnaire is to determine the perception of your son/daughter regarding the opportunities provided in the Mathematics classroom for the development of critical thinking skills. No possible risks are envisaged. There are no direct benefits for taking part in the study.

CONFIDENTIALITY: the questionnaire will be completed anonymous, and the information that your son/daughter will disclose during the completion of the questionnaire will be kept confidential by the researcher. When reporting on the questionnaires information no individual names or the name of the school will be revealed.

The research is conducted by a Masters student, Annelize Deuchar, under the supervision of Prof. M.M. Grosser from the School of Educational Sciences, North-West University (Vaal Triangle Campus). If you have any questions or queries you can contact Prof. Grosser at 016 910 3063 (work).

CONSENT:

I .... (full name parent/guardian) have read and understand the nature of the participation in the project and agree that .... (name of learner) may participate.

Signature:.. Date:..