

Mathematical knowledge for secondary school teaching: exploring the perspectives of South African research mathematicians

SE Labuschagne
22982485

Dissertation submitted in fulfilment of the requirements for the degree Magister Educationis in Mathematics Education at the Potchefstroom Campus of the North-West University

Supervisor: Prof. HD Nieuwoudt
SEPTEMBER 2016

ACKNOWLEDGEMENTS

I dedicate this research study to my husband, Louis, for his continuous support and motivation throughout this study. I would not have been able to complete this study without his love, understanding and help.

In particular, I would like to thank my supervisor, Prof. H Nieuwoudt for his invaluable support and guidance to me as a student. He did not allow me to give up when I wanted to and was always encouraging, always directing me in what I should look at and study.

I wish to express my gratitude towards my colleagues at the Potchefstroom Boys High School for all their support and encouragement. I hope the boys see an improvement in the teaching.

My gratitude is extended to my children, family members and friends for your continuous support and encouragement throughout this study.

A special word of thanks and appreciation goes to the research mathematicians, who voluntarily participated in this study. Without your co-operation, this study would not have been possible.

ABSTRACT

School mathematics education in South Africa is not doing very well at present, and has not done so for some time. There are many problems, historically and socially. However, amidst such a problematic situation, there are mathematicians who completed their school education in South Africa and have become excellent in their field. As a mathematics teacher, I explored the stories of such research mathematicians, specifically examining their perspectives of mathematical knowledge for teaching. Avoiding all abbreviations and acronyms, in an attempt to make this study more accessible to teachers, I used an interpretive narrative research design and email-interviewed a number of these mathematicians, analysed the generated data in three ways and then compared the results and findings with those of leading researchers and mathematics educationists. All of them consider a teachers' mathematical content knowledge to be of utmost importance. Not one of the mathematicians worked alone to gain their knowledge and understanding of mathematics, but collaborated in some way with teachers, parents or fellow learners. In addition, according to them, the use of different types of knowledge is instrumental for teachers, to realise the goal of meaningful learning of mathematical concepts. To this end, effective continuous professional development is essential.

Key words for indexing: Mathematical knowledge for teaching; teaching proficiency in mathematics; school mathematics; mathematics teacher; mathematics learner; continuous professional development; research mathematician.

OPSOMMING

Skoolwiskundeonderwys vaar tans nie baie goed in Suid Afrika nie, en het vir 'n lang ruk nie goed gevaar nie. Daar is verskeie probleme, beide geskiedkundig en maatskaplik. Te midde van die sodanige problematiese situasie, is daar tog navorsingswiskundiges wat hulle skoolonderwys in Suid-Afrika voltooi het en uitnemend in hulle veld presteer. As wiskundeonderwyseres het ek die verhare van die wiskundiges verken en in die besonder die verskynsel van wiskundige kennis vir onderrig vanuit hulle ervaring aan die hand van 'n interpretivistiese narratiewe navorsingsopset ondersoek. Ten einde die studie ook vir onderwysers toeganklik te maak, het ek die gebruik van afkortings en akronieme vermy. Ek het e-posonderhoude met 'n aantal van die wiskundiges gevoer, die ingesamelde data op drie maniere ontleed, en die resultate en bevindings met dié van leidende wiskunde-onderrigvakkundiges en -navorsers vergelyk. Al die wiskundiges beskou onderwysers se wiskundige inhoudskennis as uiters belangrik. Verder het nie een van hierdie wiskundiges alleen gewerk om hulle kennis en begrip van wiskunde te verwerf nie, maar het op een of ander wyse saam met onderwysers, ouers of medeleerders gewerk, om dit te bereik. Volgens hulle is die gebruik van verskillende tipes onderrigkennis instrumenteel vir die onderwyser om die doel van betekenisvolle leer van wiskundige konsepte te verwesenlik. In hierdie verband is effektiewe deurlopende professionele ontwikkeling noodsaaklik.

Sleutelwoorde vir indeksering: Wiskundige kennis vir onderrig; onderrigbevoegdheid in wiskunde; skoolwiskunde; wiskunde-onderwyser; wiskundeleerder; deurlopende professionele ontwikkeling; navorsingswiskundige.

TABLE OF CONTENTS

Acknowledgements.....	i
Abstract	ii
Opsomming	iii
CHAPTER ONE: PROBLEM STATEMENT AND PROGRAMME OF INVESTIGATION	1
1.1 Discussion of research problem and motivation for study	1
1.2 Background: South African context.....	1
1.3 Importance of the teacher and continuous professional development.....	2
1.4 Different aspects of teachers' knowledge.....	3
1.5 Mathematicians	4
1.6 Gaps in literature	4
1.7 Research question.....	6
1.8 Aims of this study	6
1.9 Research design, methodology and methods	6
1.10 Sampling strategy.....	9
1.11 Methods of data generation or collection	10
1.12 Methods of data analysis	11
1.13 Ethical considerations.....	12
1.14 Conclusion.....	13
CHAPTER 2: KNOWLEDGE FOR PROFICIENT TEACHING OF MATHEMATICS	14
2.1 Conceptual Framework.....	15
2.2 Activity theory	15

2.3	Beginnings of the research of mathematical knowledge	18
2.4	Different frameworks.....	19
2.5	Fragmentation of research.....	20
2.6	Teachers' content knowledge for teaching.....	20
2.7	Teachers' pedagogical knowledge.....	25
2.8	Pedagogical content knowledge	27
2.9	Learning mathematics	29
2.10	Teaching mathematics.....	32
2.11	Technological knowledge	34
2.12	Actual mathematics teaching.....	35
2.13	Conclusion.....	37
CHAPTER THREE: RESEARCH DESIGN AND METHODOLOGY		39
3.1	Research Paradigm	40
3.2	Research Design	42
3.3	Crystallisation	43
3.4	Participants involved in the research	44
3.5	Data generation – Interviews	45
3.6	Analysis of qualitative data	45
3.7	Activity theory framework.....	47
3.8	Writing of the narrative.....	48
3.9	Summary	49

CHAPTER FOUR: ANALYSIS OF DATA	50
4.1 Coding or constant comparison analysis.....	52
4.2 Word count	60
4.3 Content Analysis.....	66
4.4 Results in table form	77
4.5 Linking themes to Cultural historical activity theory	77
4.6 Conclusion.....	81
 CHAPTER 5 SUMMARY, CONCLUSIONS AND RECOMMENDATIONS	 82
5.1 Gaps in literature	83
5.2 Summary of findings	85
5.3 Limitations of this study	90
5.4 Recommendations and future research	91
5.5 Contribution of this study	92
5.6 Final analysis.....	92
 APPENDICES	
Appendix A: Interviews	107
Appendix B: Letter and Declaration of Consent	130
Appendix C: Email survey	133
Appendix D: Ethics Clearance	135
Appendix E: Editing Certification	136

LIST OF FIGURES

Figure 2-1:	First generation activity theory.....	16
Figure 2-2:	Third generation activity theory	16
Figure 4-1:	My findings aligned to third generation activity theory.....	78

CHAPTER ONE: PROBLEM STATEMENT AND PROGRAMME OF INVESTIGATION

1.1 Discussion of research problem and motivation for study

South Africa is a country with many problems in its education system (Mouton, Louw, & Strydom, 2013:8) and more specifically with the teaching and learning of mathematics (Adler, 2005:10). Teachers are central in the teaching and learning of mathematics and not many learners at school achieve the results that are acceptable for them to pursue mathematics further. I studied the lived experiences of South African research mathematicians in order to identify from their perspective, aspects of their teachers' mathematical knowledge which contributed to their success in mathematics. Their insights could be used to challenge and change the knowledge that South African teachers need to teach mathematics successfully.

A mathematician is someone who studies and practices mathematics and believes in that system, and that truth is absolute under the axioms or conditions within which they work. A research mathematician is a mathematician who is actively involved in researching and discovering new theorems and new mathematics by means of rigorous deductive logic applied to an axiomatic framework (McKnight, Magid, Murphy, & McKnight, 2000:1). Mathematics education research is inquiry by carefully developed research methods, aimed at providing evidence about the nature and relationships of many mathematics learning and teaching phenomena (McKnight *et al.*, 2000:1). In South Africa, there are research mathematicians, as well as mathematical educationists, who are world leaders in their field of study.

1.2 Background: South African context

In my honours project (Labuschagne, 2013), I investigated the mandatory in-service training that secondary school mathematics teachers receive. Although research (Darling-Hammond, Wei, Andree, Richardson, & Orphanos, 2009: 43; Adler & Davies, 2006) indicates the importance of these programmes, the conclusions from the project was that mandatory in-service training does not lead to the type of professional development that is necessary. Teachers who have participated in these programmes, found them a waste of money and the training had no effect on the learning in their classrooms. In South Africa at present, mathematics teachers are not receiving the kind of training which leads to sustainable or effective professional development (Mouton *et al.*, 2013: 33).

South African learners underachieve at mathematics, as can be seen in the results from many local and international assessments of mathematical achievement (Spaull & Kotze, 2015:13). A variety of studies indicated that numeracy rates amongst learners in schools in South Africa are

among the lowest worldwide. Situations in classrooms are difficult for the learners, with problems of poor discipline, overcrowding in classrooms, complications with communication and a non-existence of resources (Mouton *et al.*, 2013:32; Setati, 2008:106; Carnoy, Jacks, Chisholm, & Chilisa, 2012). In addition to these problems, multilingual learners face the further challenge of trying to learn mathematics at the same time as learning the language of instruction (Setati, 2008:108). To compound the problem, the language of instruction is also often different to the teacher's mother tongue.

Teachers are often blamed for the poor performance of learners (Taylor, 2008:6). The role of the teacher is, on the other hand, a direct outcome of the education system that is in place (Lelliott, Mwakapenda, Doidge, Du Plessis, Mhlolo, Msimanga, & Bowie, 2009:49). Teachers are just one of the many stakeholders involved. The influential policy makers, government, teachers, learners, parents and educational officials, who decide on curriculum, are therefore all responsible for the poor performance of learners (Long, 2007:14). Because of the educational injustices of the past, it was critical to have a comprehensive overhaul of the national curriculum. Sincere, committed and constructive interactions between all stakeholders involved in education are essential for any curriculum change to be effective (Mouton *et al.*, 2013:36).

1.3 Importance of the teacher and continuous professional development

I agree with Taylor (2008:11) that "the only way to improve outcomes is to improve instruction." The teacher is central to the solution in improving education, but there is a need to invest in an alternative strategy of developing the teaching and learning of mathematics, and not merely alternative methods of instruction or simply changing materials. Pre-service training is not enough to teach teachers how and what to teach for their whole lifetime of their teaching (Zakaria, Daud, & Meerah, 2009:226).

In-service training can be defined as structured activities designed exclusively, or primarily to improve professional performance (Darling-Hammond *et al.*, 2009:36). In-service training thus has an important role to play in professional development. Mathematics teachers should be seen as whole human beings and not just quasi-mathematicians. Teaching is a small part of a complex social experience for teachers and their learning experiences come from a multitude of formal and informal activities; in their own classrooms and schools, at a university course, or through on-line activities. Professional development is noticeable when there are changes in a teacher's knowledge and practice (Desimone, 2009:182). Teachers also develop during the actual teaching of mathematics. The development is not only in mathematical content knowledge, but their beliefs and perceptions about mathematics are also constantly changing and developing.

Teachers need to be life-long learners; however, there is a general dissatisfaction with in-service training programmes, which are perceived to be ineffective (Taylor, 2008:22). Any teacher can find opportunities for in-service training and continuous professional development, yet for it to be effective; they need to be central in their own development. In the South African context, teachers are seen as being actively uninvolved in their own professional development. There are complaints that the in-service training is unrelated to the needs that the teachers face (du Plessis & Webb, 2012:48).

1.4 Different aspects of teachers' knowledge

Without knowledge, teaching cannot take place, though this knowledge is more than just knowing the mathematics in the curriculum (Schoenfeld & Kilpatrick, 2008:322). A teacher's knowledge has many different facets. For example, content of the subject, mathematics, pedagogical knowledge, pedagogical content knowledge and technological pedagogical knowledge, to mention just a few. A teacher who understands the importance of knowledge will provide opportunities for the learning that learners need.

New technologies such as, computers, educational games and the internet, have changed the nature of the classroom or have the potential to do so (Mishra & Koehler, 2006:1023). Many South African teachers have access to computer programs. These could be used to assist with the teaching of topics, such as graphs, statistics and geometry but teachers either do not use these technologies (Stols & Kriek, 2011:137) or else teach using them but with substandard results (Schoenfeld, 1989:338).

The mathematical content demands of teaching are substantial (Ball, Thames, & Phelps, 2008:6). Teachers need to be able to go beyond merely the calculations assigned to the learners. According to Taylor (2008: 12), a significant number of teachers in South Africa have very low levels of subject expertise. A reason for the problems found in subject knowledge could be that the training received by many teachers was in a different era to the one in which they teach. Subsequently, teachers have not received adequate training, to teach what they are currently teaching (Rakumako & Laugksch, 2010:139).

A teacher's own mathematical knowledge needs to improve through reflection on their new mathematical knowledge in addition to how it is understood (Plotz, Froneman, & Nieuwoudt, 2012:80). Teachers must be capable of teaching mathematics using the multidimensional nature of knowledge and understanding. Knowing the subject of mathematics in breadth and depth, as well as knowing the learners as learners, and thinkers, needs to be included in the framework for proficiency in teaching (Schoenfeld & Kilpatrick, 2008:108).

Teachers not making efficient use of the time available to teach will result in learners not having sufficient opportunity to learn (Stols, 2013:16). Opportunity to learn is the amount of time learners are exposed to the curriculum, in a given academic year (Reeves, Carnoy, & Addy, 2013:426). Hence lack of learning is a consequence of teachers not using teaching time in specific teaching situations.

Professional development, union meetings and department meetings, all compete for a teachers' time (Taylor, 2015:16). Teachers find it difficult to balance time for various assessments in the time allocated to teaching (Mouton *et al.*, 2013:8). Time issues are exacerbated by absenteeism in the teaching staff of schools. If in South Africa, the teachers taught more mathematics lessons during the course of the academic year, most of the learners' mathematical ability would improve (Reeves *et al.*, 2013:434).

1.5 Mathematicians

Any person who is an expert or a student, in the study of mathematics, can be called a mathematician. There are different categories of mathematicians and different types of research that these mathematicians are involved in, but they all have a deep knowledge of mathematics and a love for mathematics. Mathematics education research is not the same as research in mathematics. As Adler (2005:2) puts it: "Mathematics advances through the increasing solution of outstanding problems. Mathematics education is a very different research field. Problems are not well defined, nor are they solved once and for all." There is a clear divide between mathematics educationists and research mathematicians, which some have referred to as "math wars" (Schoenfeld, 2004:254).

There is a similar divide between mathematics teachers and mathematics educationists. This manifests when theory and practice do not support each other. There is not a transfer from teacher education into school practice (Kieran, Krainer, & Shaughnessy, 2013:361). Educationists make recommendations in all their papers, but teachers do not read or apply the recommendations. The issue of teacher education, which includes professional development, is a much-investigated area of research (Venkat, Adler, Rollnick, Setati, & Vhurumuku, 2009:15). This is predicated by the fact that if teachers are not developing continuously, teaching will not improve.

1.6 Gaps in literature

In my reading of literature on mathematics education in South Africa and the problems that the education system are facing, I discovered four gaps that I thought could be filled. My research study adds to the body of scholarship on the topic of mathematics teacher education and

continuous professional development; by contributing to four gaps identified in South African literature on Mathematics education that have not been adequately researched.

Mouton *et al.* (2013:40) have recommendations for an improvement in education in South Africa. They recommend an emphasis on the importance of continuous professional development; any training that teachers receive should be relevant and high quality training, which is the first gap that I identified and addressed in this research. I added to the discussion around the mathematical knowledge for teaching that it should be included in this continuous professional development and training. As a mathematics teacher, I realised that this study is part of my own continuous professional development and I think other teachers should be encouraged to do the same.

Moremedi (2007:28) calls on SAMS, the South African Mathematics Society, to help with the deficiencies in the content knowledge of the majority of teachers in South Africa, as in his judgement, they are best placed to identify problem areas which need attention. Even though this call was over nine years ago, in my opinion, it has not been adequately addressed. My research is part of the response to this call. This is the second gap, as I approach research mathematicians who are part of the SAMS organisation for their views and suggestions on how teachers' mathematical knowledge needs can be developed in order to address the insufficiencies.

Vhurumuku and Mokeleche (2009:109) recommend future research focusing on developing teachers' beliefs, knowledge and pedagogical practices with regard to indigenous knowledge systems. Pais (2011:219) expanded the common definition of ethno-mathematics as only indigenous mathematics, to include learners' social, historic, political and economic background in the meaning of indigenous knowledge of mathematics. I concur with this expansion and use this as the definition with which I work. Research mathematicians cannot be separated from their own indigenous knowledge systems and beliefs. This is not the focus of this research but I do believe that this research adds to this discussion. This is the third gap my research covers. South Africa has its own specific indigenous knowledge systems. The narratives from research mathematicians answer questions about conflict between indigenous knowledge and mathematics, or finding ways of enhancing mathematical teaching by indigenous knowledge.

According to Adler (2005:2), mathematicians and mathematics educationists are very different and often it is difficult for them to understand each other. By listening to the voices of research mathematicians, I contribute to filling this fourth gap in research. In the current state of mathematics education in South Africa, some learners have developed to the point of being research mathematicians, and even world leaders in their field of research. Those are the

people, whose voices I investigate, and whose stories I tell of how they think the knowledge of teachers should be improved.

1.7 Research question

As a mathematics teacher, I explored the stories of research mathematicians, specifically examining the mathematical knowledge for teaching from their experiences. My central question is: What insights on the content of mathematical knowledge for secondary school teaching in South Africa are in their stories?

Adding to the central question, subordinate questions were:

- What mathematical knowledge did teachers have that was helpful, and what areas needed improving?
- What influence, if any, did indigenous knowledge systems have on mathematical knowledge?
- Should teachers use technology in teaching mathematics?
- What is the importance of having strong subject knowledge?
- What pedagogical knowledge has a positive effect?

1.8 Aims of this study

In this study, I explored the stories of South African research mathematicians, who managed to excel in mathematics, notwithstanding the severe problems in its education system. From their perspectives, insights were collected which were used to develop the importance of content of mathematical knowledge for teaching secondary school, which in time can be used for continuous professional development. This mathematical knowledge for teaching includes all aspects of knowledge that a teacher needs, including indigenous knowledge relevant to the South African context. In the process of this study, I minimise the gaps between research mathematicians, mathematics education researchers and mathematics teachers.

1.9 Research design, methodology and methods

1.9.1 Research design

This is an interpretive narrative study that used purposeful sampling to select a group of research mathematicians who have gone through the South African schooling system. Data was generated from interviews. Biographical questionnaires were given to members of SAMS, to help with the selection process. Data was analysed using content analysis, as well as other methods (Merriam, as quoted in Creswell, 2009:199). As an inductive method, it will allow the

research findings to emerge. The research project is somewhat open-ended, which is suited to an inductive method.

1.9.2 Methodology

Narrative enquirers do not begin with a definite problem, but with an interest in a phenomenon, which could be understood narratively (Clandinin & Connelly, 1994:14). According to Clandinin and Connelly (2000:2), the study of a narrative is a study of the ways humans experience the world. A qualitative narrative methodology is specifically appropriate for this research task because it focuses on investigating the way people experience the world and particularly the mathematical knowledge for teaching that South African teachers need, in the mathematics classroom.

Narrative methodology is not just a story, because a story could be fictional and therefore have falsifications. A definition of narratives, according to Clandinin and Connelly (2000:20), is “discourses with clear sequential order that connect events in a meaningful way for a definite audience, which offers insights about the world and people’s experiences.” A narrative methodology helped me understand how these specific people experienced their mathematics education. The particular audience of this narrative is mathematicians in its broad inclusive meaning, teachers and researchers. The life experiences are continuous from schooling to their current position as research mathematician and keeping the continuous sense of time is associated with a narrative study (Clandinin & Connelly, 1994:6).

According to Creswell (2009:190), the steps in a qualitative narrative study are as follows:

- Researcher gathers information to understand the life experience.
- Purposefully selected participants answer open-ended questions in interviews.
- Recorded data is analysed to form themes.
- A process of sequencing and organising is used to find broad patterns.
- Collaboration must exist between the researcher and the participants.
- The narrative outcome is compared with theories and general literature on the topic for crystallisation.

1.9.3 Philosophical orientation

Any research project needs theory to block the reproduction of the obvious (MacLure, 2010:280). Research gives rise to language and tools, which shape what we see and say about the world around us (Pais, 2012:3). In my investigation, I use a constructivist-interpretive, qualitative focus, which focuses on experience and the qualities of life and education (Clandinin & Connelly, 2000:3).

An interpretivist paradigm uses qualitative methods, which is a “softer, more subjective, participatory role” (Cohen *et al.* as quoted in Maree, 2007:32). It is a process of understanding, where the researcher develops a complex picture and analyses it in its natural setting (Creswell, 2009:258). The key goal of this approach is to explore and understand the central phenomenon that is being revealed, and to interpret it in its context, taking all factors that influence into consideration (Creswell, 2009:259).

According to Maree (2007:59), an interpretivist perspective is based on the following assumptions:

- Human life can only be understood from within a subjective study of phenomenon.
- Social life is a distinctly human activity, and understood in this social context.
- The human mind is the purposive source and through exploring phenomena, we understand meanings.
- Human behaviour is affected by knowledge of the social world. There are multiple perceived realities.
- The social world does not exist apart from human knowledge. Our own knowledge and understanding constantly influences us.

In a qualitative narrative approach, a reader must understand the situation that the author was in, to understand a text. Perceived reality in any situation of social life is subjective (Creswell, 2009:232), and all situations are unique. Behaviour is affected by the social world, by how we know and understand situations, and therefore I explain my personal interest in this research. I have been teaching secondary school mathematics for about twenty years. I do not aim to turn all the learners in my classes into research mathematicians, but I do value mathematics and want all the learners in my classes to see the significance and importance of mathematics. I do not want to be accused of teaching anything that would place a stumbling block in the path of their pursuit of mathematics at higher cognitive levels. Knowledge must be transferred or else it will be lost, (Ball *et al.*, 2008:398) and narrative is a powerful way of transferring knowledge.

There was a clash of paradigms between me as researcher and the participants. Most mathematics research has positivism as its paradigm. In positivism, social behaviour is explained only by observable entities (Creswell, 2009:6). The reaction to this is interpretivist, where human behaviour is explained, by referring to the subjective states of the people acting in it (Maree, 2007:49). Knowledge can be viewed in one of two ways, it can be either hard, real or objective or a more subjective, participatory role as can be found in an interpretive, non-positivist stance (Maree, 2007:31). This clash was noticed in the answering of questions and the analysis of the data.

Another research challenge was the personal nature of this study. The social context involved was purposefully explored to establish meanings applicable to improving knowledge that teachers in South Africa need. I realise that there are multiple possible realities and hope this can be used to help teachers who teach a class filled with learners who themselves also have a variety of realities and understandings.

1.10 Sampling strategy

The group that I have named 'research mathematicians', was my study population from which my sample group was purposefully selected. I distributed approximately one hundred biographical questionnaires via email, to research mathematicians from the organization South African Mathematical Society (SAMS) and invited them to participate voluntarily in this study.

These questionnaires used in the selection of the sample group had definite criteria:

- The research mathematicians must have been schooled in South African school system.
- Specifically, I would like participants to be from both urban and rural areas, as well as private and government schools.
- They must be willing, and feel able to give their own perspectives in a narrative procedure, on the following topics:
 - The different aspects of a teachers' mathematical knowledge for teaching.
 - Positive aspects of their secondary school teachers' mathematical knowledge.
 - Negative aspects of their secondary school teachers' mathematical knowledge.
 - Indigenous knowledge systems that was relevant to their knowledge.

I interviewed ten participants who, as close as possible, fulfilled the criteria. The participants were selected on convenience criteria to facilitate the interviews. These interviews were conducted via email after I had attended the annual conference of the South African Mathematical Society and had spoken to mathematicians who had volunteered to participate. I was able to take an interactive role in the social context. These interviews covered the topics that were in the original email, on which they agreed to give their perspectives. Data, generated on improving a teachers' mathematical knowledge for teaching, was collected.

1.11 Methods of data generation or collection

The three most commonly used methods of data collection for descriptive studies of teachers and teaching is observation, interviews, and surveys or questionnaires (Desimone, 2009:188). People were the source of information. The method I used for the generation of data was unstructured individual interviews. Interviews were conducted via email, between the participants and me. The interview was used to aid in understanding the ways in which the experience had shaped the participants (Clandinin & Connelly, 2000:6). The interviews were focused on the mathematical knowledge needed by teachers, from the participants' perspectives.

The questions in the interview were (see Appendix C):

1. Reflecting on your own experience as a learner from grade 10 to matric in a South African school, comment on different teaching styles in your school learning experiences and the effect of these styles.
2. Explain your experiences of getting 'stuck' and what strategies were used for resolving these types of problems.
3. What was the role of the teachers' knowledge with the topics that you found easy or hard?
4. How would you compare mathematics with other subjects, in terms of the kind of work expected and how you approached the subject matter and tasks?
5. What were your reasons for choosing mathematics at university?
6. As an accomplished research mathematician in order to help teachers trying to teach mathematics, please answer the following:
 - 6.1 What are your perspectives on the forms of knowledge that are involved in teaching mathematics? I am referring to content knowledge, pedagogical knowledge and technological knowledge.
 - 6.2 Explain your perspectives on whether these different forms of knowledge are of equal importance in teaching mathematics or is there a hierarchical order to the forms of knowledge?
 - 6.3 What is strong subject knowledge? What is the importance of strong subject knowledge? What role do you think this strong subject knowledge plays in teaching?
 - 6.4 From your experiences, is the traditional model of teaching, where the teacher is central source of knowledge, effective?
 - 6.5 Indigenous knowledge, a knowledge that learners and teachers have from their background, is part of the pedagogical knowledge of a teacher. Should teachers have

knowledge of culture and habits unique to South Africa or is teaching more effective when the teacher is aware of the culture of the learners?

6.6 Classroom discipline or control: is this essential to effective mathematics teaching?

6.7 In respect to technological knowledge, which includes the use of calculators, computers, as well as black boards and overhead projectors, etc., what are your perspectives on this topic at High school level, in terms of the teachers' knowledge?

7. Any other views on what kind of approach from the teachers would lead to success in school mathematics in the South African school.

1.12 Methods of data analysis

The process of data analysis is making sense of the data that has been generated (Creswell, 2009:183). An interpretive method was followed in the analysis. The steps I followed are well described in Creswell (2009: 185). The data from the interviews was organised, and transcribed. After making sure that all information was understood, by a thorough reading of all the transcribed data, it was coded into various themes. The meaning of these themes was then interpreted. The interpretations cannot be separated from me, as the researcher, and various views can be formed from this approach.

There are two ways of interpreting these themes: deductively - where themes existed before the research was done, or inductively - where themes are discovered from the narratives. I used both an inductive analysis and a deductive analysis. I started with an idea of what questions I wanted to ask and I saw what emerged from the respondents. This is part of the qualitative approach (Creswell, 2009:259; Lodico, Spaulding, & Voegtler, 2010: 5). Categories, themes, and interrelationships with the focal point being, to understand the complexities of the situation that the participants were schooled in, and specifically how this is associated with the teachers' mathematical knowledge for teaching.

According to Lodico *et al.* (2010:5) inductive reasoning is used when a researcher proceeds from specific observations to general statements. Deductive reasoning is used when a general statement is made and specific evidence is used to support or disconfirm that statement. Specific practical insights from the data were used to explore this mathematical knowledge for teaching. There were numerous perspectives that could have been discussed but the themes of mathematical knowledge for teaching are what was analysed.

Generalisation cannot be made with this narrative enquiry. Any results from this research were validated by comparing any outcomes with existing research and literature. This is explained as

a crystallisation, which provides a deeper, complex, understanding of the topic (Tobin & Begley, 2004:388). A rich description of the participants, interactions, culture and background support the transferability of this data (Lodico *et al.*, 2010: 274). Trustworthy issues were addressed, using the different data sources on mathematical knowledge for teaching and a comparison with the results from this study. Member checks, where the participants have a copy of the transcribed interviews as well as summaries, were used to insure that my own bias does not influence how the perspectives have been portrayed.

1.13 Ethical considerations

Following Clandinin (2006:52), ethical considerations imply more than filling out forms with a narrative inquiry. It is about co-operation, respect, support, and openness to multiple voices. This enquiry was with adults. No identities were revealed throughout the report. All references to place names, people or schools were omitted. Participants received feedback on the report, to show their contribution to education in South Africa.

As researcher, I have a participatory role. My twenty years of teaching experience influenced how I asked questions and interpreted responses. My own knowledge of the history of South African education also added to me being involved in this project. I checked all conclusions around the different themes with the participants, and with my supervisor, to avoid bias.

Ethical clearance was obtained from the NWU ethics committee. All necessary procedures were followed, such as the data will be kept for a minimum period of five years. All interview schedules and consent letters were attached to the ethics application form.

All participants completed an informed consent form. Sarantakos, as quoted in Creswell (2009:89), states elements that must be in such a form:

- Identification of the researcher.
- Identification of the institution involved.
- Identification of the purpose of the research.
- Guarantee of confidentiality.
- Participants participate on a voluntary basis.
- Without stating their reasons, participants can withdraw at any time from the research and without fear of any form of penalty for doing so.

1.14 Conclusion

As mentioned, South African mathematics education is not flourishing according to many reports. Many teachers feel discouraged by this situation. There have been changes made to the syllabus by the Department of Education, and teachers' knowledge is falling between the cracks in this system. Teachers need a continuous development as a teacher. A four year degree will not suffice for a life time of teaching. There are different aspects to the knowledge of mathematics' teacher as well as different beliefs about mathematics.

I found four significant gaps in literature, which I contribute to with this project. The division between research mathematicians, educational mathematics researchers as well as the gap between researchers and teachers - all do not focus on effective mathematics education. Continuous professional development of teachers is something that should not be ignored.

My research project is a qualitative narrative study of stories from the research mathematicians and what knowledge from their perspective, is important for mathematics teachers. I interviewed research mathematicians who are part of the South African Mathematical Society. All ethical considerations were taken into account. These results are part of my own life-long learning as a mathematics teacher and part of my own continuous professional development.

The motivation for this study is similar to what McKnight *et al.* (2000:5), who posited that just as a physician consults literature for the evidence of effectiveness to treat diseases, this should lead to more success. This is my evidence-based study: as a teacher I consult the mathematics education research literature for evidence of effectiveness and I select and evaluate pedagogical method from this evidence. McKnight *et al.* (2000:5) goes on to say that there are philosophical problems in the theory of knowledge, most people are probably intuitive. I try to use my intuitiveness in study.

CHAPTER 2: KNOWLEDGE FOR PROFICIENT TEACHING OF MATHEMATICS

Shulman, (1986) and Ball, Thames & Phelps (2008), developed a widely accepted theoretical model of “mathematical knowledge for teaching”. It is the philosophy of mathematical and pedagogical knowledge required for teachers to be successful in teaching for meaningful learning of mathematics. Since the development of the theory of mathematical knowledge for teaching, there has been an increased emphasis on the nature of knowledge needed to teach mathematics. Schoenfeld and Kilpatrick (2008:108) proposed a theoretical framework for teaching proficiency in mathematics that added to the understanding from an educational perspective of the phenomenon of proficient school mathematics teaching.

The theoretical model incorporates the fields of mathematical content knowledge and pedagogical content knowledge, specifically aimed at the knowledge of mathematics, in the curriculum, that is taught. With the increase in available technology in the teaching environment, the model has been amended to also incorporate technological, pedagogical and content knowledge (Mishra & Koehler, 2006).

All stakeholders in education will agree that without knowledge teaching cannot take place. Knowledge is multifaceted, so a general view of knowledge is important. The different aspects of the knowledge that are needed for teaching are interconnected in a way that makes classifying knowledge, difficult. There is a specificity about the mathematical knowledge needed for teaching that is unique to teachers (Adler & Davies, 2006) but an efficient teacher will rarely, consciously be able to identify which category of knowledge is being used in the classroom (Shulman, 1987:6).

All research needs a conceptual framework and the conceptual framework which guided this research is the interpretivist–constructivist approach, as described by Stinson and Bullock (2013:12). No researcher is a blank slate without any framework. It is through a certain framework that the literature has been reviewed. In this chapter, I start with analysing the framework I used in this research, before looking at the literature on the mathematical knowledge needed for teaching.

I investigate this mathematical knowledge from teaching theories, firstly with a brief overview of their history, and putting the theories in the context of the state of mathematical education in South Africa, both as a learner and as a teacher. I applied the theories of the knowledge a teacher needs. The chapter ends with a summary of the knowledge that South African secondary school teachers’ need in order for effective mathematical learning to take place.

2.1 Conceptual Framework

I have not done this research without my own beliefs, assumptions and values influencing the way I think. According to Creswell (2009:15), the theoretical framework, which is guiding the researcher, is based on personal beliefs, assumptions and values. My research is based on the interpretivist–constructivist framework.

In the interpretivist–constructivist moment, the aim of the researcher is no longer to predict social phenomena but rather to understand it (Stinson & Bullock, 2013:12). The purpose of my analysis is not to build a new theory of good or effective mathematics teaching and learning, but rather to interpret the research mathematicians' understandings of effective teaching and to determine if their understandings were congruent with theories on knowledge needed for effective teaching. Mathematics educational research does not work with strict logical deduction but seeks instead to carefully build up evidence relevant to answering questions about the phenomena with which it is concerned (McKnight *et al.*, 2000:7).

The constructivist researcher understands meaning as something constructed through experience. The focus of my research is on understanding and identifying the processes of how people acquire or construct different meaning over time (Stinson & Bullock, 2013:13). The broad framework of this research is the interpretivist approach but then to narrow it down, I use a narrative approach, which is a study of the ways humans experience the world (Clandinin & Connelly, 2000:2).

A cultural-historical activity theory is used to contextualise and support the data. According to Engeström (2001:134), cultural-historical activity theory was initiated by Lev Vygotsky in the 1920s and early 1930s. It was further developed by Vygotsky's colleague and disciple Alexei Leont'ev. I apply Engeström's modification of the original theory. It is particularly applicable in the South African context with its diverse historical problems.

2.2 Activity theory

Hashim and Jones (2007:6) explain activity theory as a theoretical framework for the analysis and understanding of human interaction through their use of tools and artefacts. It offers a holistic and contextual method of discovery that can be used to support qualitative and interpretive research. It is particularly relevant in situations that have a significant historical and cultural context and where the participants, their purposes and their tools are in a process of rapid and constant change.

My project has examined a constructivist learning environment. The knowledge of teachers, needed to teach effectively, has been analysed and examined, as to how that knowledge has taught the research mathematicians fundamental mathematics at school level. According to Jonassen and Rohrer-Murphy (1999:62), the most powerful structure for analysing needs, tasks and outcomes in this type of environment, is using activity theory as a framework. I used this framework to evaluate the collected data.

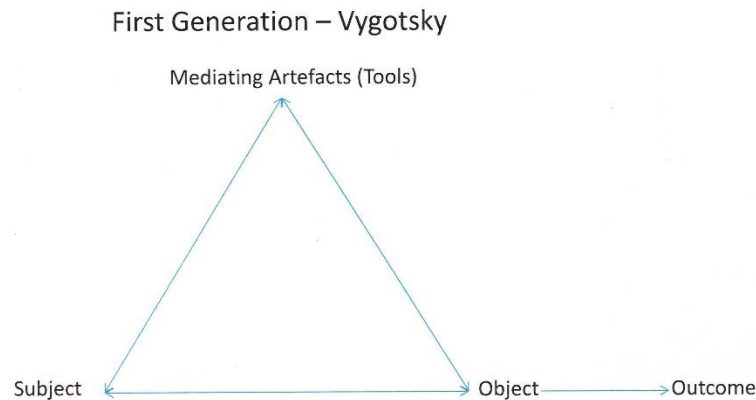


Figure 2-1: The first generation activity theory [Source: Engeström (1987:40)]

The activity theory framework was chosen, as it examines the data from a contextual perspective. Figure 2-1 clearly demonstrates the first generation activity theory with subject, artefacts or tools and object. I applied third generation cultural historical activity theory to the data in my project (see figure 2-2). According to Hashim and Jones (2007:5), Engeström's modification of Vygotsky's original theory provides for two additional units of analysis, which have an implicit effect on work activities. The original theory had three units of analysis, namely the subject, the object and the mediating artefact or tools. The additional units are rules, community and labour. Rules are sets of conditions that help to determine how and why individuals may act, which are a result of social conditioning. Division of labour provides for the distribution of actions and operations among a community of workers, which is the third unit that was added.

According to Engeström (2001:21), learning is usually depicted as a vertical process aimed at elevating learners upward. He suggests that learning should have a complementary perspective, namely that of horizontal or sideways. I chose this framework to view the bigger picture of teacher's knowledge within a South African community. The teachers in this narrative are not in isolation because each of them comes from a different cultural, social and school background. Cultural-historical activity theory is therefore appropriate since it looks at the teacher in a general approach.

I have taken this basic description of Cultural-historical activity theory from Jonassen and Rohrer-Murphy (1999:61). Activity theory framework is a socio-cultural and socio-historical lens through which human activity is analysed. It focuses on activity in a context. The activity structures are placed in a triangle diagram, focusing on the activity.

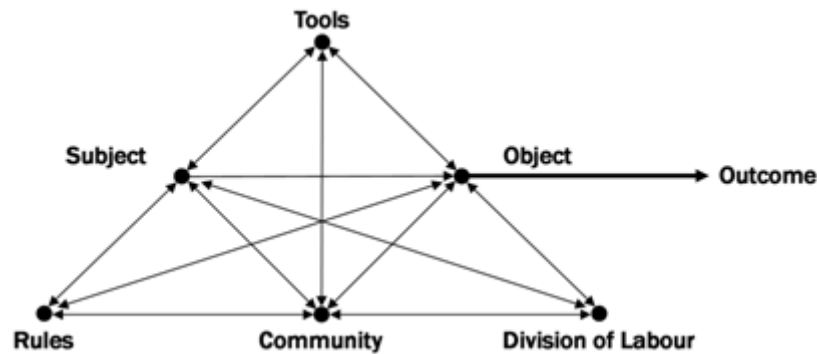


Figure 2-2: The third generation activity theory [Source: Engeström, (1987:78)]

The different headings in the triangle diagram are matched to different aspects of data from the research project. The relationship between the different aspects is in both directions. For example, the tools affect the subject as well as the subject affecting the tools. Activity theory focuses on this dynamic relationship rather than the process of knowledge transmission. Activity theory focuses on the purposeful activities, recognised through conscious intentions.

Basically, it is saying that learning and doing are inseparable, and initiated through an intention. All this happens in a community and not in isolation. The tools in the system alter that human activity. Meaningful activity is not accomplished individually, but in a group. Jonassen and Rohrer-Murphy (1999:62) describe the subject as the individual or group of people engaged in an activity. This is the central driving character. The object is the physical or mental product that is sought and the tools can be anything used in the transformation process.

In my research, the following meanings were applied to the different headings:

- *Tools* are the resources available to the teacher, such as textbooks, exercises, technological resources, technological skills, etc. and the teachers' technological knowledge of these tools.
- The *subject* is the learners who are learning mathematics and the teachers' content knowledge of the subject.
- The *object* is effective mathematical teaching and learning.

- *Rules* refer to teachers' pedagogical knowledge, the discipline structures in the classroom.
- The *community* is all the stakeholders in the education. This includes the Department of Education, teachers, principals, learners as well as parents.
- *Division of labour* refers to the work that must be done by learners and teachers.
- The *outcome* is effective mathematical learning that will give learners the opportunity to be effective as mathematical learners.

Activity theory is the holistic, contextual lens through which I studied mathematical knowledge. To understand mathematical knowledge for in depth teaching, I looked at literature on the topic. I found a lot of really interesting and helpful information from many different parts of the world. I have put this information together in a narrative manner.

2.3 Beginnings of the research of mathematical knowledge

Mathematics has been taught and learnt for millennia in many different continents of the world but it was not until the past century that the nature and quality of teaching and learning mathematics was studied (Kilpatrick, 2014:267). Teachers used to be self-taught and textbooks were the sole form of knowledge, which was seen to be static (Schubring & Karp, 2014:263). Many of the early researchers in mathematics education were mathematics teachers who had become interested in how mathematics is taught and learnt and questioned classroom practice (Kilpatrick, 2014:267).

Subsequently studies have been conducted on communication, the development of norms, and how teachers and learners build relationships in classrooms (Fennema, Carpenter, Franke, Levi, Jacobs, & Empson, 1996:404). Ball (1990) called for an analysis of practice, for better understanding how teachers both use what they know and know what is needful to use in practice. Research on the knowledge that is required for teaching has become a major aspect of this current research in mathematics education (Kilpatrick, 2014:269).

2.3.1 Recent educational research frameworks

In his seminal work, Shulman (1986) outlined categories of teacher knowledge that support teachers' practices, including pedagogical content knowledge and subject matter knowledge. The interest and research that followed formed an important knowledge base of understanding and improved subject-specific teaching (Venkat & Adler, 2014:477). Numerous different categorisation schemes and frameworks have been developed to define, and embody the types

of knowledge and beliefs that appear relevant to the teaching of mathematics (Kilpatrick, 2014:270).

2.3.2 Research around the world

Researchers have done studies in different countries around the world comparing the learners and the teachers' knowledge, for example, China and USA (Ma, 1999:145). Ball *et al.*, (2008) have researched why teachers in the USA lack sound mathematical understanding and skill. South Africa and Botswana have also done studies comparing learning and teaching in these two countries (Carnoy *et al.*, 2012) and found that South Africa has a low level of performance. This is just a few of numerous studies between countries.

Studies from preschool to tertiary instruction on teaching and teachers in these diverse countries have become a major strand of current research in mathematics education (Kilpatrick, 2014:268). Professional development and teacher training has been changed and adapted because of these studies (Venkat & Adler, 2014:479).

2.3.3 Comparing primary and secondary school studies

Most of these studies of a teachers' mathematical knowledge, for example, Ma (1999), Ball (1990) and Fennema *et al.*, (1996) have focussed on an elementary level of teaching, where basic arithmetic is taught. Much of the knowledge needed to teach mathematics in the primary school is the same type of knowledge needed for secondary school.

On the other hand, Speer, King, & Howell, (2015:107) researched the difference between mathematical knowledge for teaching in the primary school compared to the secondary school. They concluded that overgeneralising the current theoretical framework from primary school to secondary school is missing both an opportunity for better understanding the nature of mathematical knowledge for teaching more generally and is likely to develop unproductive interventions for improvement of teacher learning that support their learners.

2.4 Mathematical knowledge frameworks

The mathematical knowledge for teaching model is the foremost model used in the USA and its influence has changed research internationally (Venkat & Adler, 2014:478). Other frameworks use it as a basis for comparison or validation of their own structures, even where those philosophies may be different. The Knowledge Quartet framework was developed by Turner and Rowland (2011), as mentioned in Speer *et al.*, (2015:108). The United Kingdom uses this framework to examine mathematical knowledge in teaching.

Cognitively Guided Instruction is an approach that is used by Carpenter, Fennema, and Franke (Fennema *et al.*, 1996). The approach provides teachers with a framework to construct a coherent, organised knowledge base that they can draw from to solve complex pedagogical problems which are encountered in teaching primary school mathematics. It has a focus on the learners thinking and understanding. It shows that knowledge is not static but is constantly developing and changing (Fennema *et al.*, 1996:416).

In the frameworks of Ball, Thames & Phelps (2008) and other researchers, the concept of mathematical knowledge for teaching is used to describe the many different components of knowledge used in the work of teaching and learning mathematics. In Shulman's (1986) definitions of teacher knowledge, there is a distinction between knowledge that is purely mathematical and the knowledge that applies content to the tasks of teaching.

2.5 Fragmentation of research

Mathematics education is becoming fragmented by the diversity of theoretical approaches used in research (Bergsten, 2014:379). There are a wide variety of aspects from around the world that have been studied under the topic of teachers' knowledge and more specifically a teacher of mathematics' knowledge. In the reading of research articles, I noticed many acronyms which mathematics educationists use prolifically. Venkat and Adler (2014:479) also mention the differences in nomenclature.

Reviews of Shulman's work (1986) and categorisations argue that mathematical knowledge for teaching categorised in separate components is unhelpful because of the collaborative and active nature teaching. The knowledge for teaching should rather be seen as a characteristic of pedagogic practices in a specific context and related to specific mathematical ideas; than a generalised aspect of the teacher and teaching (Venkat & Adler, 2014:478).

To some extent, my research's origin is similar to the beginning of mathematical education research - I am a mathematics teacher who wants to improve teaching. Therefore to avoid confusion and in an attempt to make this project readable to a wider audience, I prefer to avoid all the abbreviations and acronyms.

2.6 Teachers content knowledge for teaching

It is obvious to all involved in education, that mathematics teachers need knowledge of mathematical content but researchers have found it challenging to establish definitive relationships between measures of teachers' content knowledge and learners' achievements (Ball, Lubienski, & Mewborn, 2001:434). Qualitative and quantitative studies on learners' achievements and teachers' knowledge have failed to find how to teach teachers the knowledge

that is needed (Adler & Davis, 2006:279). The question rises whether this knowledge is teachable (Adler, 2005:5), or do teachers learn their own methods in the actual practice of teaching. According to Speer *et al.* (2014:120), the answer to such questions is currently based on the wisdom of practice, rather than in theoretical or empirical results.

2.6.1 Knowledge or skills?

There is research, which finds that teachers' knowledge is not an important factor in teaching, but their beliefs and practices have an influence on learners' achievements. An example of this is Eisenberg (1977:222), who claims that there is no correlation between teacher knowledge and learner achievement, and that other factors appear to be responsible for learner success. Spaul and Kotze (2015:23) state that knowledge, skills and values are imparted to the learner via the teacher and the schooling system.

Shulman (1987:20) warned that we must be careful that the knowledge-based approach does not produce an overly technical notion of teaching. Teaching and learning is about the people in the classrooms. We need to have a proper understanding of the sources of knowledge as well as the pedagogical intricacies involved.

Carnoy *et al.* (2012:157) concluded, after their research comparing teaching practices in Botswana and South Africa that more knowledgeable teachers teach what they are trained to teach, teaching effectively and covering more material. Learners with more knowledgeable teachers learn more effectively. Included in their research report was the fact that being a more knowledgeable teacher will obviously have the conclusion of more effective teaching.

2.6.2 Beliefs about mathematics

A teacher's beliefs or understanding about what mathematics is, profoundly influence the form of knowledge that they think is important. Mathematics is not just a list of rules to be learnt and memorised, it is not just the science of computation (Radford, 2014:105). As the context of this research is the South African schooling system, an explanation of what mathematics is, has been defined in terms of the South African Department of Basic Education National Curriculum Statement.

"Mathematics is a language that makes use of symbols and notations for describing numerical, geometric and graphical relationships. It is a human activity, which involves observing, representing and investigating patterns and qualitative relationships in physical and social phenomena and between mathematical objects themselves. It helps to develop mental processes that enhance logical and critical thinking, accuracy and problem solving that will

contribute to decision-making. Mathematical problem solving enables us to understand the world around us, and most of all, to teach us to think creatively “(DoE, 2003:4).

2.6.3 Mathematics as a search for truth

Mathematics is a subject that makes sense and is based upon a creative search for truth. Justification of concepts is an important belief in mathematics, but its use has been shifted out of school mathematics and replaced by production of technical answers (Radford, 2014:105). According to Schoenfeld (2012:318), with mathematics everything fits in place beautifully.

Mathematics exists independently of the creator or the consensus of the community (Rowlands, 2007:100), for example, the angle property of the plane triangle is independent of any one person and appears in textbooks because it accurately depicts the construction. Mathematics is also independent of ideological persuasion. It is not masculine, Eurocentric or oppressive but mathematics can only make sense within the context of the history of cultures and it operates within the forces of society (Radford, 2014:106).

2.6.4 A vertically integrated subject

Mathematics is a vertically integrated subject (Spaull & Kotze, 2015:11), in that it requires getting hold of higher order knowledge and intellectual skills. To accomplish this first subordinate, skills need to be mastered and a clear foundational mathematical knowledge, and arithmetic knowledge, needs to be in place. There is a difference between being able to work with mathematical symbols and understanding the underlying mathematical ideas in that area (Schoenfeld, 1989:3).

Learning proceeds along definite paths. Tasks that teachers chose for a group of learners should fit logically into the path of the learner’s skills at the specific point along his learning, which is the trajectory theory of learning (Simon, 2014:237). The logic of the learner becomes more sophisticated over time and the teachers’ knowledge should include identifying this learning curve (Sztajn, Confrey, Wilson, & Edgington, 2012:150).

2.6.5 Mathematical thinking

Schoenfeld (1994:55) explains mathematics by describing characteristics of mathematical thinking. Mathematical thinking is the process of valuing mathematics and abstraction, and being able to apply mathematical procedures. Mathematical thinking is sometimes concealed by the procedures and calculations that characterise mathematics in the classrooms of today (Radford, 2014:107).

According to Ball (1990:462), the teachers' knowledge, ways of thinking, beliefs and feelings, jointly affected their practice. All are important and cannot be isolated. The teachers' knowledge, ways of thinking, beliefs and feelings, all play a role in the knowledge that the teacher brings to the teaching and learning.

2.6.6 Mathematics as a set of rules

In Ball's study (1990:458) of student teachers' understanding of division, fewer than half of the secondary school mathematics teachers could provide a meaningful explanation for why division by 0 is undefined, although they could produce the correct rule. Student teachers, who professed to be good at mathematics, were only good at listing and using the rules. Many children and adults perform mathematical calculations without understanding the underlying principles or meaning, but see mathematics as a set of rules.

2.6.7 Components of content knowledge

Shulman (1987:20) suggests that the first source of knowledge is content knowledge. Content knowledge rests on two foundations: accumulated literature and studies in the content. The teacher has special responsibility as the primary source of content knowledge for the learners. Shulman (1986:14) states that the ultimate test of understanding, rests on the ability to transform one's knowledge into teaching. Teaching cannot be assessed without reference to the content being taught. Teaching is more than high quality instruction. It requires a sophisticated professional knowledge that goes beyond simple rules.

Content knowledge can be divided into common content knowledge, the mathematical knowledge that is not unique to teaching (Ball *et al.*, 2008:406), and specialised content knowledge, the mathematical knowledge, expected of a teacher (Ball *et al.*, 2008:389). Hill, Ball, & Schilling, (2008:395) also conclude from their research that teachers have skills, insights and wisdom beyond that of other well educated adults.

The mathematical way of thinking and teachers' mathematical content knowledge, is strongly related to the mathematical quality of their teaching (Speer *et al.*, 2015:109). A teacher makes decisions about the method used to explain mathematical concepts and representations. A teacher responds to learners' mathematical ideas, and needs to have the ability to avoid mathematical mistakes (Hill *et al.*, 2008:389).

A teachers' mathematics is different from the mathematics that an engineer or salesman would need. They have a different nature of mathematical understanding to that needed to research mathematics. A teachers' talk and the language used to teach students demonstrates this understanding (Schoenfeld, 1989:335). The capacity to see the content from another's

perspective and to understand what another person is doing entails mathematical reasoning and skill that are not needed to research mathematics.

2.6.7.1 Depth and breadth of knowledge

Content knowledge is more than just reading a mathematics textbook, it is knowing school mathematics in depth and breadth (Schoenfeld & Kilpatrick, 2008:2). It is broad because a teacher should have many ways of making the content understandable. The key aspects should be obvious in explanations and any connections to other concepts in the learners' grade, should be seen. The depth is measured by teachers knowing the origins and relevant history, as well as to where the mathematics will lead.

2.6.7.2 Profound understanding of mathematics

Ma (1999:145) studied teachers and teaching mathematics in US and China. These teachers gave answers to mathematical tasks in the context of teaching. She described the flexibility, depth, and coherence of the knowledge displayed by the Chinese teachers in their answers, which showed their understanding of mathematics. She called this a "profound understanding of mathematics". Four essential components are connectedness, multiple perspectives, basic ideas and longitudinal coherence. These "packages" of knowledge reveal the difference between mathematical knowledge any adult has and knowledge that is effective in teaching.

Teachers claim to learn subject matter from teaching it, according to Ball (1990:464), but they do not change their beliefs or ideas about mathematics. They cannot learn from teaching unless they first have this profound understanding. Hill *et al.*, (2008:396) conclude that content knowledge for teaching is multidimensional in that it is put into operation with learners, in conjunction with content.

2.6.7.3 Big ideas in content

The teacher must organise and prioritise the work done in the lessons guided by the curriculum (Spaull & Kotze, 2015:23). Big ideas or major themes should be introduced to learners and not get lost in the everyday teaching and focusing on passing the next exam. The content knowledge needs to be grounded in the big ideas of mathematics, as these continue throughout school mathematics (Van de Walle, Karp, & Bay-Williams, 2007:258). Big ideas are part of the vertically integrated component of school mathematics.

2.6.7.4 Horizon content knowledge

Speer *et al.* (2015:108) refer to horizon content knowledge as essential knowledge preparing learners for future mathematical studies. Sztajn *et al.* (2012:147) classify this knowledge as the

most sophisticated understanding that sits at the top of a particular trajectory, representing the ultimate mathematical goal of a learning progression. Horizon content knowledge is the knowledge that is essential for research mathematicians.

According to Ball (1990:457), there are three goals in teaching mathematics, which rely on the teachers' content knowledge. Firstly, teachers' knowledge of concepts and procedures should be correct, secondly, they must understand the underlying principles and meanings. Finally, teachers must appreciate and understand the connections among mathematical ideas, fundamental ideas and for future studies. For example, the connections between functions and real-life situation.

2.6.7.5 Conceptual and procedural knowledge

A deep understanding of mathematics is the conceptual knowledge. According to Ma (1999:182), a solid knowledge of what to represent in teaching is essential. It cannot be replaced by knowledge of the lives of the learners. It cannot be replaced by motivation or passion for the subject. Only an inflexible, solid conceptual knowledge can produce a conceptually correct representation.

Learners need strong knowledge of concepts to build connections to new ideas and new procedures (Van de Walle *et al.*, 2007:29). Simple mathematical concepts and procedures go before more complex concepts, and ways of thinking (Spaull & Kotze, 2015:14). For example, before a student can factorise complex algebraic expressions, they should be able to understand the concept of being able to factorise natural numbers. Procedural knowledge is knowledge of the procedures involved in performing common tasks (Hill *et al.*, 2008). A mathematician may have knowledge of the definitions involved, but a mathematics teacher should have the procedural knowledge. A teacher needs an understanding about the nature of mathematical knowledge and how procedures are based on logic and convention and not merely a set of rules (Ball, 1990:458). A teacher should be able to distinguish between procedural and conceptual knowledge.

2.7 Teachers pedagogical knowledge

A teachers' pedagogical knowledge is also referred to as a teachers' professional knowledge, as it is the knowledge needed in the profession of teaching. Shulman's description of a teachers' pedagogical knowledge is "General pedagogical knowledge, with special reference to those broad principles and strategies of classroom management and organization that appear to transcend subject matter" (Shulman, 1987:8). Pedagogical knowledge is the general use of instructional techniques, beyond lecture style. It includes classroom management skills and the quality of teaching material (Shulman, 1986:10).

2.7.1 Pedagogy

Pedagogy is the process and practice, or methods of teaching and learning. It is the application of the teachers' content knowledge to the task of teaching. It is the understanding and skill needed to teach mathematics (Blömeke, Buchholtz, Suhl, & Kaiser, 2014:133). A connection between the assessment of learners' knowledge, the teacher's knowledge and the learning, is known as the mathematics teaching cycle. The cycle is meant to capture a progression in the learning of mathematics. Learners' knowledge is observed throughout and this monitoring leads to a new understanding of learner thinking and learning (Kilpatrick, 2014:273).

The knowledge of pedagogical practices is understudied and not well understood, according to Hill *et al.* (2008). They go on to explain why it is not understood, because of its multidimensional nature. The knowledge is put into operation in unique situations, with a specific subject and with particular learners.

2.7.2 Beliefs about pedagogy

A teacher's beliefs about pedagogy, changes the teaching situation. For example, if a teacher has a constructivist belief about teaching and learning, the learners will be able to solve complex problems. Constructivism is a theory that learners can construct meaning, from their experiences (Van de Walle *et al.*, 2007:21). A consequence of constructivism is that mathematics, including the basics such as arithmetical computation, should be taught through problem solving (Cobb, Wood, & Yackel, 1991:394). In contrast to the constructivist belief, a teacher could be a traditional teacher where knowledge is transmitted from a teacher to the learners (Cobb *et al.*, 1991:598).

2.7.3 Relationships part of teaching

Teaching is relational. Whichever belief is followed, teachers, subject matter and learners are all in relation to each other, and can only be understood in this manner (Fennema *et al.*, 1996:412). Part of pedagogical knowledge is knowledge about these relationships. Teachers and learners are engaged in a process which has a personal and moral aspect as well as the study of mathematics (Grootenboer, 2013:324). These relationships are appreciated in actual classroom practice, for example, discipline and teaching style. The teachers' preparations for lessons and contact with learners' parents, colleagues, and departmental officials are examples of relations outside the classroom.

Mouton *et al.* (2013:1) mention that lack of discipline in classrooms is a major concern in classrooms in South Africa. Carnoy *et al.* (2012) in their comparison between schools in Botswana and South Africa also mention classroom discipline as a concern. Classroom

discipline and control is part of the pedagogical knowledge of teachers. Positive reinforcement and recognition in learner's development cannot be overemphasised and teachers should endeavour to exercise this relationship, far more often than control as a form of discipline (Mouton *et al.*, 2013:9).

2.8 Pedagogical content knowledge

Research has found that pedagogical content knowledge is the most predictive of learner achievement (Blömeke *et al.*, 2014:131). Venkat and Adler (2014:477) explain that this knowledge forms a critical knowledge base for understanding and improving subject-specific teaching. Therefore, to understand this knowledge, I compared many different explanations in literature, of what this knowledge entails. All these descriptions are clarifying the same type of knowledge, but from different points of view.

2.8.1 Understanding the complexities of the subject

The commencement of the discussion on pedagogical content knowledge is attributed to Shulman's 1985 speech. Shulman's (1987:6) definition of pedagogical content knowledge includes "an understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons" (Shulman, 1987:8). He goes on to explain that this is the knowledge that separates the teachers from the content specialist.

2.8.2 How learners think

Hill *et al.* (2008:375) refer to content knowledge being intertwined with knowledge of how learners think. It provides a context in which teachers can interpret and apply general pedagogical knowledge. Teachers construct their own understandings of learners' thinking (Fennema *et al.*, 1996:405). Learners have their own ideas, which could differ from traditional methods. They need to be shown if their thinking is correct, (Rowlands, 2007:96) and not merely told precise answers. Different opinions should also be acknowledged.

Understanding a learner's typical understanding, and where it originated in mathematics, provides a framework for teachers to develop understanding of other sides of learners' thinking (Blömeke *et al.*, 2014:135). Knowledge not only provides a basis for understanding learners' thinking; it can also provide an outline for teachers' knowledge of mathematics and curriculum. The teacher communicates truth, consciously or not, to the learners, which influences their understanding (Shulman, 1987:9).

2.8.3 Practical application of subject matter to teaching

Pedagogical content knowledge is that kind of understanding where knowledge of the discipline and knowledge of its particular range of teachings intersect, to create a special kind of knowledge (Shulman, 2011:5). Pedagogical content knowledge focuses more on knowing learners, than on teaching or curriculum (Speer *et al.*, 2015:115). A teacher develops a knowledge that applies content of mathematics to the tasks of teaching (Speer *et al.*, 2015:108). Pedagogical content knowledge is a bridge between content knowledge and the actual practice of teaching (Ball *et al.*, 2008:403). It is a knowledge that is unique to teachers, with discussions on topics relevant to teaching and learning.

Pedagogical content knowledge describes the difference between knowing something and coming to know it well enough to teach it to someone else (Shulman, 2011: 5). Consequently, content knowledge and pedagogical content knowledge issues are generally interconnected (Speer *et al.*, 2015:115). A gifted mathematician does not necessarily have a deep understanding of the teaching and learning of mathematics. In the same way, an expert in general principles of pedagogy is unlikely to be expert in the teaching and learning of non-Euclidean geometries (Shulman, 2011:5).

A teacher with only content knowledge is likely to be as useless to teaching as teaching skill without content. The teacher needs to be able to see the mathematical potential of any particular learning situation (Grootenboer, 2013:340). The two aspects of a teacher's capacities require that we pay as much attention to the content aspects of teaching as we have recently devoted to the elements of the teaching process (Shulman, 1986:8).

2.8.4 Opportunity to learn

Carnoy *et al.* (2012:114) found in their research that opportunity to learn was a key factor contributing to learner achievement in mathematics education. The teachers did not spend enough time covering the syllabus, and the pace of work was very slow. Opportunity to learn is part of pedagogical content knowledge, as it is the best explanation of the relationship between teaching and learning (Stols, 2013:1). The opportunity to learn has also been used to explain differences between countries in international comparative studies, in mathematics learning (Hiebert & Grouws, 2007:371).

The teacher in the classroom is the only person who can directly create and shape the basic elements of time-on-task, curriculum coverage, coherence, and cognitive demand (Stols, 2013:14). Learning effectively requires a teacher whose focus is learners' understanding and performance. The knowledge of a teacher's successful interpretation of the mathematical ideas

and a learner's contribution to a class discussion is more influential for learning opportunities than the knowledge derived from other kinds of analyses (Speer *et al.*, 2015:109).

2.8.5 Knowledge from experience

According to Hill *et al.* (2008:373), there is a knowledge of content and learners that is important in teaching, and this knowledge is influenced by the experience of teacher. From this experience, a teacher has knowledge of miscalculations that learners make and why they make those errors. Although experience is important, Wolhuter (2014:17) mentions research that recognises that teachers' knowledge is a much better predictor of learners' achievement than the experience of teachers.

Although most stakeholders and researchers in education agree that knowledge from experience exists and that it is essential in the teaching of mathematics, there is not much evidence of what this knowledge is and the effect it has on learning. Many teachers have learned how to solve the mathematical problems in their classrooms; but teachers do not usually conceptualise and teach this knowledge to student teachers (Adler, 2005:10).

Adler (2005:9) explains how this knowledge is not properly understood. Even though there is evidence that this knowledge is essential for teaching, it is not evident in any of the teacher training courses or professional development courses. Maybe, knowledge from experience cannot be taught to student teachers.

2.9 Learning mathematics

Learning and teaching mathematics are two sides of the same coin. You cannot have one without the other. Even though this seems obvious, most research focuses on either learning or teaching (Kilpatrick, 2014:270). After explaining the aspects of pedagogical content knowledge, I look at how this learning should actually take place. What does literature say about the learning of mathematics? I look at learning mathematics before concentrating on the teaching side of the coin.

2.9.1 Learners' informal mathematical knowledge

Children enter school with an informal or intuitive knowledge of mathematics and numbers, which is a basis that needs to be built on, developed and not ignored (Kilpatrick, 2001:107). Pedagogical knowledge about different situations is necessary to overcome and transform those initial conceptions, and misconceptions, to develop an important component of the understanding of subject matter (Shulman, 1986:10).

Children are taught this basic arithmetic knowledge which in time develops into the formal mathematics of the primary school curriculum. The development of abstract symbolic procedures is learnt in the secondary school curriculum (Fennema *et al.*, 1996:403). A learner needs to be able to take the mathematical content knowledge at the secondary school level and extend it to further studies in mathematics. There is often damage done in mathematics classrooms as many learners leave school without an understanding of mathematics and this limits their life opportunities (Grootenboer, 2013:340).

2.9.2 Learners thinking style

Mathematical thinking styles, the way an individual prefers to understand and think through mathematical facts, has an effect on whether learners understand mathematics (Ferri, 2012:2). One way of classifying thinking styles is that they are either visual, analytic or integrated and if a learner has a different learning style to the teacher there will be problems in understanding (Ferri, 2012:10). According to Simon (2014:273), even though learning takes place in personal ways, there is unity in their ways of learning that can be the basis for instruction.

A learners understanding of mathematics does not only depend on ability. The way a learner thinks and the way the teacher teaches all influence the outcome. A learner's learning is significantly affected by the opportunities and constraints that are provided by the structure and content of the mathematics lessons (Simon, 2014:273). Classrooms today contain a range of learner ability. An important work of the teacher is to plan for this difference. The problem-based approach to teaching is the best way to teach mathematics and attend to this range of abilities (Van de Walle *et al.*, 2007: 64).

2.9.3 Learners assimilate knowledge

According to Fennema *et al.* (1996:404) learners construct rather than assimilate knowledge. Learners are not blank slates with the teacher hammering knowledge into them. The constructivist idea of learning explains how new knowledge is always built on preceding knowledge (Van de Walle *et al.*, 2007:20). Engeström (1987) explains that learning is more than a top down experience. Learning is built within the learner, and the classroom culture contributes to the learner as well as the learner contributing to the classroom culture (Cobb *et al.*, 1991:390).

Learners are seen as the best judges of what they find problematic and they should construct solutions that they find satisfactory (Cobb *et al.*, 1991:395). In doing so, they change their ways of knowing and new knowledge is integrated into their thinking, so that when obstacles or contradictions arise they can make sense of the situations.

New knowledge that is constructed is called a learning trajectory (Simon, 2014:272), which consists of three components, a learning goal, a set of learning tasks and a learning process. The learning is used as the basis for instructional decisions (Sztajn *et al.*, 2012:147). These trajectories can support growth in mathematical knowledge. Teachers need to make sense of the learners' thinking and keep that at the centre of the teaching that takes place.

2.9.4 Learning connections

At school, learners' mathematical thinking should include; a knowledge of mathematical concepts and procedures, the relationships among them and why they work (Shulman, 1987:9). To learn mathematics with understanding, connections between different representations is important. Knowledge gained, interconnects with previous knowledge. Meaningful learning takes place if it is learning with understanding. Cobb *et al.* (1991:598) views mathematical learning as an interactive, constructivist and problem-centred process. Learners should learn about mathematical ways of knowing, so that they would be able to participate in mathematical arguments (Ball, 1990:457). Mathematical arguments depend on an understanding of the connections.

2.9.5 Cooperative learning

Cobb *et al.* (1991:406) describe three positive outcomes of learning opportunities when learners work together on mathematical activities. Firstly, learners can use aspects of each other's solutions. Secondly, they have the opportunity to alter and rethink a problem in order that miscalculations can be analysed. Thirdly, the learners have an opportunity to extend their own conceptual framework, to make sense of another learner's solution in order to reach consensus. These opportunities do not occur in traditional classrooms.

Van de Walle *et al.* (2007:67) explain in detail the importance of cooperative learning. It is beneficial to all the learners, but be carefully planned by the teacher. A teacher should maintain high expectations in the learning process.

2.9.6 Mathematical proficiency

A learner should have developed mathematical proficiency during their time at school. Kilpatrick (2001:106) explained mathematical proficiency as five interwoven and interdependent strands. Mathematical proficiency cannot be attained by only focussing on one strand, but for students to progress in mathematical proficiency all strands need to be developed. The five strands of mathematical proficiency are conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition.

2.10 Teaching mathematics

2.10.1 Schoenfeld's view of teaching mathematics

The goal of mathematics instruction depends on a teachers' view of what mathematics is, mastering facts or pattern seeking (Schoenfeld, 1992:334). Schoenfeld goes on to explain that merely mastering facts belittles mathematics. It is important to develop learners who think mathematically. Mathematics is a social activity where practitioners engage in the science of patterns. Teachers need to educate and help learners to think mathematically. Mathematical thinking is developing a mathematical point of view, valuing the processes of mathematics and abstraction, as well as applying these processes (Schoenfeld, 1992:334). The learners need to learn the tools of mathematics to make sense of the structures.

Schoenfeld (1992:334) stresses the importance of knowing the learners as thinkers. A teachers' goal should be to pose relevant problems, which learners have to solve (Schoenfeld, 1994:334). A perfect textbook or a faultless lesson plan will not help the teacher who is confronted with learners who want to know why: $2^5 \times 2^8 = 2^{13}$. Teachers must understand mathematics well themselves, to be able to respond adequately to learners' questions. Teachers could answer the question directly, or change the question so that students can figure it out for themselves (Ball, 1990:458).

Teachers need an extensive knowledge of various interventions to help learners in areas where problems are seen to be systemic. Learners are disadvantaged if they struggle or make mistakes, without being corrected, as this can build up and develop into bad habits (Schoenfeld, 1989:13). Knowledge that the learners acquire comes in small pieces that can be grasped individually; and these small pieces of knowledge are then built up and connected to other small pieces of knowledge. The outcome of teaching does not depend exclusively on the teaching aspects, but also on the input of the learners (Wolhuter, 2014:18).

2.10.2 Shulman's view of teaching

A view of teaching by Shulman (1987:7), is that teaching starts with the teacher understanding a topic and it ends with new comprehension by both the learner and the teacher. Teaching mathematics is more of an art than a science, according to Taylor (2015:1). The practice of teaching is guided by a formal knowledge base and a learner attempts to formalise this knowledge, challenging the teacher's own ideas and identities. The act of teaching always has to be closely related to the idea of learning (Shulman, 2011:2).

2.10.3 Ball's ideas on teaching

Ball (1990:447) mentions a comparable goal of mathematics teaching: it is for learners to develop mathematical understanding. This infers that learners should acquire knowledge of mathematical concepts and procedures, the relationships between them and why they work. Understanding equally implies learning about mathematical ways of knowing, as well as about mathematical substance.

2.10.4 Adler's view on teaching

Adler (2005:3) explains mathematics teaching as a special kind of mathematics problem solving. For example, when dealing with the learners' misconceptions and errors, the teacher needs to be able to analyse the errors. Unpacking or decompressing is a description of the distinctiveness of the mathematical work that teachers do. Unpacking is working with the mathematics as it is learnt and the opposite is to compress mathematics into abstract forms. Teaching should always use the learners' misconceptions and as well as accurate perceptions to influence the ensuing learning. Misconceptions need to be transformed into positive learning. Teachers need to try to find new forms of thinking mathematically that is relevant to the culture of the learners in their classes (Valero, 2004:11).

2.10.5 Transformation of knowledge

Mathematics teaching is more than knowledge and procedures that have been studied by the teacher, a learner's beliefs and attitudes toward the subject are also being developed. A teacher is continually making pedagogical decisions that are morally informed and value-laden. All learning occurs in relationship with others (Grootenboer, 2013:322). A teacher transforms what he or she already knows and has learnt into new representations that can help learners to make sense of the world.

Mathematics teachers have their own individual style of thinking and beliefs about the mathematics, which will influence how they teach, and all explanations that are given (Ferri, 2012:10). It is important for the teacher to realise that they have their own thinking style and beliefs, in order for them to give each learner an equal opportunity to excel.

2.10.6 Problem solving approach

A problem solving approach to teaching requires mathematical questions for which the solution or method are unknown in advance. For Schoenfeld (1994:33), problem solving is a problem, which has no immediate answer and no algorithm that can be used directly to solve it. Teaching problem solving is difficult, because teachers need to know many different methods

as well as the mathematical content involved. Learners need to analyse the question and use all knowledge that they have, to work out a strategy and find a solution. For Cobb *et al.*, (1991:399) an important aspect is that teachers accept right or wrong answers in a non-evaluative way.

Teaching mathematics requires teachers to be flexible, answer questions and sort out confusions as they occur in the classroom, which are not covered in training but learnt as a teacher teaches. The mathematical identity of the learner is adapted and changed according to these situations (Grootenboer, 2013:340). Therefore, the relations between teachers and learners, and the relation between teacher educators and teachers should be included in any training (Adler, Ball, Krainer, Lin, & Novotna, 2005:369).

Venkat and Spaul (2015:128) describe how, in the South African context, content needs to be presented in ways that are well adjusted to the nature of mathematical working. The problem solving approach, where given information is used to find unknown information, is an appropriate approach. It emphasises the development of the teachers' capacity for mathematical explanations.

2.11 Technological knowledge

Knowledge of technology is separate from knowledge of pedagogy and content. Technology is inclusive of basic properties common to all schools, such as chalkboards; and cutting-edge, such as digital computers and smart boards. The use of these in teaching is complex. Technologies often come with their own problems that affect the content that has to be taught. Deciding what to use in teaching could be helpful or confusing to the students, so the knowledge of technology is not isolated from knowledge of pedagogy and content (Mishra & Koehler, 2006:1020).

2.11.1 Benefits of using technology in teaching

Social and contextual factors also complicate the relationships between teaching and technology. Recently, many technological tools have been designed to facilitate learning mathematics. These new tools enable new different practices to enrich learning. Some projects incorporate dynamic mathematics applications, such as GeoGebra for group work on math problems (Stols & Kriek, 2011:137). Groups of learners can share their mathematical explorations and co-construct geometric figures online. The possibilities opened by new technologies challenge the beliefs of mathematics teaching. Learners can work together for long periods without a teacher guiding them and this challenges educational approaches for which adult guidance is central for development (Schwarz, 2014:508).

2.11.2 Challenges with technology

Teachers do not consider themselves sufficiently prepared to use all this new technology in the classroom. They often do not realise its value to teaching and learning of mathematics. Teachers would have to obtain new skills and new knowledge. According to Mishra and Koehler (2006), the core of good teaching with technology are three components: content, pedagogy, and technology, plus the relationships among and between them.

There have been discussions about providing technology in the form of laptops for every learner in South Africa. However, Lavinias and Veiga (2013), in their article on the Brazil's one laptop per child program, found that despite providing universal access and seemingly providing equal opportunities for all learners, the non-poor benefit the most from this intervention. Their research focused on the results derived from learners in both municipal and state public schools in the five municipalities, and it showed that the project is underperforming, with operating standards that vary considerably from site to site.

The teacher remains central to the teaching. Online interactive systems offer a new way of learning, where the teacher is able to give individualised teaching. Teachers would have to change their work process in order to integrate technology into their classroom in order to get the best work out of each learner (Lavinias & Veiga, 2013:546). Technologies help the learners focus on life-long learning and allow teachers to experience professional development, as enjoyable and useful (du Plessis & Webb, 2012:46).

2.12 Actual mathematics teaching

According to Kilpatrick (2001:107), proficient teaching of mathematics is in many respects the same as mathematical proficiency, but includes learners as well as mathematics. It requires conceptual understanding of mathematics, learners, and teaching. Basic instructions need to be well carried out. Skill is needed in planning effective instruction and solving practical mathematical problems. All the different aspects of mathematical knowledge are the backbone of mathematics teaching proficiency, (Schoenfeld & Kilpatrick, 2008:7).

Problem solving is an essential requirement of effective mathematical teaching, as solving problems is the work of a mathematician (Schoenfeld, 1994: 339). Teachers should also be able to give the learners further work around their questions, which would develop their mathematical understanding. Cobb *et al.* (1991:397) suggested that problem solving is not just to solve specific problems, but the result of the activity is mathematical understanding.

2.12.1 Actual practical classroom teaching in South Africa

Worldwide educational system, changed in the 1960s. Industrialised countries made “Mathematics for all learners” a goal for secondary school education. Social changes and technological advances in the professions have caused these changes (Schubring & Karp, 2014:216). There is an enormous challenge of providing adequately trained mathematics teachers in this situation, where all in society are expected to be mathematically literate (Adler, 2005:3).

Mathematics needs to be attuned to cultural practices (Adler & Davis, 2006), and South Africa has specific cultural practices. In the practice of teaching, knowledge about the cultures cannot be ignored. Effectiveness in teaching is not the knowledge that a teacher has, but how this knowledge is used in the classroom (Hill, Rowan, & Ball, 2005:375), and the culture that is represented in the classroom.

In teaching mathematics in South Africa, the history and cultures of this country cannot be ignored. South Africa is a developing country where education performs poorly and lags far behind systems of much poorer neighbouring African countries (Letseka, 2014:4868). A serious problem is that teachers do not provide learners with the opportunity to learn, which includes covering the syllabus (Carnoy *et al.*, 2012:153).

The fundamental changes for assessment proposed by Outcomes Based Education also contributed substantially to teachers’ negative reactions and resistance, as they found it difficult to maintain a balance between teaching and assessment time (Mouton *et al.*, 2013:8). Recently, the South African curriculum has been changed with the implementation of CAPS (Curriculum and Assessment Policy Statement) being implemented in Grades 1, 2, 3 and 10 in 2012. In 2013, it was implemented in Grades 4, 5, 6, and 11. According to Taylor (2015:2) the new CAPS, is again, being implemented without teachers being trained adequately.

Learners in developing countries have large learning deficits (Venkat & Spaull, 2015:13). They have not acquired basic skills, needed to learn mathematics with understanding, and so fall further and further behind. Intervention to this situation in the secondary school is often seen as impossible.

In South Africa, problems with teachers’ mathematical knowledge were noticed from studies of classroom practice (e.g. Taylor & Vinjevold, 1999). Learner performance continues to be low, and has not been addressed by curriculum reform. There are three aspects of teachers’ knowledge: disciplinary knowledge, subject knowledge for teaching and classroom competence, which need attention (Venkat & Spaull, 2015:206).

2.12.2 Lifelong learners

A teacher needs to be able to reflect on her teaching practice and be able to explain how to improve it (Kieran *et al.*, 2013:2). Knowledge of teaching is not static. A teacher's outlook towards mathematics, teaching, learning and the improvement of practice must be positive. Teachers need to be life-long learners, to learn from real-life situations (Kieran *et al.*, 2013:5) and not be left behind with curriculum changes. Mathematics teachers in South Africa are expected to teach on topics which do not match their prior learning - in school or in teacher education (Taylor, 2013:28). There are new directions to teaching mathematics and there are new focuses in the curriculum (Spaull & Kotze, 2015:23). These topics have not been learnt by the teachers currently in practice e.g. data handling, probability and Euclidean geometry (Adler, 2005:4).

According to Taylor (2015:3), important basics in the training of teachers are to produce knowing, caring and committed 'reflective practitioners'. Strong subject content knowledge is central to this conception of teachers and teaching, together with a nurturing attitude and ethical behaviour. Teacher training has developed and changed over the years. In many countries, secondary school teachers are specialised as mathematics teachers. Research in teacher education is often more complex. It deals not only with the beliefs, knowledge and practices of teachers, but also the learners' beliefs and knowledge (Schubring & Karp, 2014:262).

Mathematics teaching is extremely varied (Wolhuter, 2014:17). The scientific community produces much empirical and theoretically based knowledge; and much knowledge is produced from the experiences of thousands of teachers (Kieran *et al.*, 2013:3). It is clear that the communication and possible collaboration between teachers and researchers is very important.

2.13 Conclusion

After looking at many different research articles on the teaching of mathematics, I agree with Kilpatrick (2001:112) that everyone has evidence to support his or her position on a given topic, and almost everyone has evidence to contest an opposing position. Circumstantial evidence that is utterly convincing, and even confirming research evidence, is often there to back up any claim. Research evidence that ought to be rejected because it is flawed, or simply unconvincing, is also well known and stories are disconfirming because they are only circumstantial. The difficulty is that the evidence that convinces one person often fails completely to convince another.

Research has a major role to play in understanding the mathematical knowledge needed for teaching, but that will not happen if existing research is ignored or misinterpreted. New research must build on existing available knowledge. If mathematics education research

endeavours to enhance the understanding of mathematics teaching and learning including its social, political, and economic conditions and consequences, only a productive interaction of research approaches is likely to move the field forward (Bergsten, 2014:379).

In the research on secondary school teaching of mathematics, there are many different frameworks and no consensus in the field about the structure of secondary mathematical knowledge for teaching. Educational research has all the answers to the questions asked in classrooms, by teachers and learners but they are not looking to educational research for answers. According to Hill *et al.* (2005:316), “mathematical knowledge for teaching is still emergent. Without better theoretical mapping of this domain, no instrument can hope to fully capture the knowledge and reasoning skills teachers possess”.

CHAPTER THREE: RESEARCH DESIGN AND METHODOLOGY

The actual practice of teaching mathematics at many schools in South Africa is very disheartening. A quantitative analysis of a research conducted by Matoti (2010) indicates that the majority of teachers are uncertain about their own futures in education, as well as the future of education in South Africa. Some of their fears and concerns include the political and economic climate in the country, changes in policies and changes in the curriculum, high rates of teacher attrition, unsafe school environments, unsatisfactory working conditions, the declining quality of education, role conflict, poor teacher morale, unprofessional conduct of educators, lack of co-ordinated departmental workshops, poor management and leadership in schools, as well as a lack of accountability (Matoti, 2010:568).

Every new innovative method tried only seems to result in worse behaviour and lower results from the learners. In this country, which is documented as having one of the worst school results for mathematics (Mouton *et al.*, 2013:35), there are some people who have done extremely well. They not only achieved well at school level, but became world leading research mathematicians. It is my contention that the grounding at school must have had a substantial and positive influence. The question arises in this regard about the knowledge that their teachers had and what knowledge current teachers should have.

I would like to find out more about their stories, what it was that helped them in their learning of mathematics. A general topic about their stories is too broad and it would take a lifetime to study one person. I narrowed the topic down to what they think about the knowledge that a mathematics teacher must have. All teachers have knowledge but what knowledge did the teachers have, that contributed to the learners' achievement? Are the results of educationists who study Mathematical Knowledge for teaching, valid in the context that I teach?

Any study should be grounded in theory or else it can be discarded as a teacher's "mad ramblings" similar to Ophelia in William Shakespeare's Hamlet (Act v scene iv). Without rigor, research is worthless and becomes fiction, and loses its utility (Morse, Barrett, Mayan, Olson, & Spiers, 2008:14). Scientific educational research is defined by Lodico *et al.* (2010:4) as the application of systematic methods and techniques which help researchers and practitioners to understand and enhance the teaching and learning process. Through this research, I would like to enhance the teaching and learning process in my own classroom.

As a mathematics teacher, I would like to explore the stories of research mathematicians. What practical insights can be gleaned on the content of mathematical knowledge for teaching in South Africa, specifically at further education and training (FET) level?

3.1 Research Paradigm

3.1.1 Qualitative study

In this study the stories of research mathematicians are explored. This study is qualitative in nature. The aim is to understand the context of the research mathematicians through a qualitative approach where human beings construct meanings as they engage with the world they are interpreting (Maree, 2007:56). Qualitative researchers tend to use open-ended questions so that participants can express their views (Creswell, 2009:9). As to this study, the focus is on the mathematicians and their views on mathematics teachers' knowledge.

According to Lodico *et al.* (2010:34), qualitative researchers believe that full understanding of phenomena is dependent on the context. The stories and the context of South African research mathematicians need to be understood. The qualitative approach incorporates much more of a literary form of writing, and data is gathered from experiences by conducting open-ended interviews (Creswell, 2009:22).

A qualitative approach is one in which the inquirer often makes knowledge claims based on interpretive perspectives. Interpretive perspectives examine the multiple meanings of individual experiences with intent of developing theory (Creswell, 2009:18). The paradigm, or perspective of this study, must be an interpretive perspective. The participants' experiences were complex. Theory can only be developed after the perspectives have been examined (Maree, 2007:62).

According to Lodico *et al.* (2010:263), the goal of qualitative research, is to provide an in-depth understanding of a limited setting, group or person. An understanding may be cultivated through thorough descriptions. Educational processes and the achievements of outcomes are analysed on a deep level. The goal of my research question is to do the same; seek an in-depth understanding of any link between the research mathematicians' mathematics experience at school and their mathematics teachers' knowledge.

As a qualitative researcher, similarity and uniformity between design and implementation as well as between the research question, the literature, the participants and the analysis must be addressed. The focus is maintained while the information is thoroughly checked. The fit of

data, conceptual work of analysis and interpretation are monitored, and constantly confirmed (Morse *et al.*, 2008:17).

3.1.2 Interpretive framework

Qualitative research is done by persons who hold an interpretive or naturalistic framework (Lodico *et al.*, 2010:7). The assumption that reality can be reduced to its components will not work for this study. People are too complex to take their experiences and reduce the experiences to a list of formula.

The interpretive framework started as a move away from the scientific realist assumption that reality can be reduced to its components (Maree, 2007:57). This study endeavours to understand what is happening in the social situation in a mathematics classroom between the teacher and the learners. Specifically, the importance of the role of different aspects of mathematical knowledge that teachers use.

According to Stinson and Bullock (2013:1257), Mathematics education research can be understood as operating in four distinct, yet overlapping and simultaneously operating, historical moments: the process–product moment, the interpretivist–constructivist moment, the social-turn moment and the socio-political-turn movement. Each movement has a unique theoretical perspective. Methodology is linked to theory. Each movement has different methodological possibilities. In an interpretive study, researchers observe participants and there is an informative quality involved. Interpretations exist, because one of the main functions of research is reflection and retelling participants' stories (Clandinin & Connelly, 1998:2).

Maree (2007:59) gives five assumptions that are the base of the interpretivist perspective and consequently relevant to my study:

- Human life can only be understood from within.
- Social life is a distinctly human product.
- The human mind is the purposive source or origin of meaning.
- Human behaviour is affected by knowledge of the social world.
- The social world does not exist independently of human knowledge.

One criticism against the interpretivist is that the researcher is subjective and the approach cannot generalise beyond the situation studied (Maree, 2007:60). I realise this problem of subjectivity. A study of relevant literature on the topic of mathematics teachers' knowledge will clarify all possible applicable and valid interpretations. Analysis techniques which are not

restricted to only one type of analysis, should help to promote rigor in qualitative research (Leech & Onwuegbuzie, 2007:564).

Interpretivist researchers should continually monitor their own subjective perspectives and biases by reflecting on their thoughts (Lodico *et al.*, 2010:274). An experienced supervisor who regularly checks and notes subjectivity is invaluable in the process - to approach the generated data in a way that does not weaken the relevant results.

The failure to be able to generalise (Maree, 2007:60) is another criticism of the interpretive paradigm. There may be patterns that we can learn from, but a sweeping statement cannot be established with this paradigm. The researchers' primary concern is to explore individuals in their natural context and they have little interest in generalising the results beyond the participants in the study (Lodico *et al.*, 2010:140). My key concern is to explore these research mathematicians' stories and not to generalise beyond the inherent limitations set by this study.

3.2 Research Design

A research design is a plan, which moves from philosophical assumptions to selection of participants, data gathering techniques and data analysis (Maree, 2007:70). Qualitative studies use strategies of inquiry such as narratives, phenomenologies, ethnographies, grounded theory studies, or case studies (Creswell, 2009:18).

A narrative analysis is a research project that is designed to collect and analyse the stories of participants (Maree, 2007: 103). After examining all the strategies in an interpretive qualitative study, the best fitting strategy would be a narrative. I examined the narrative research design in more detail to explain why it is the best applicable design of research to follow.

3.2.1 Narrative research

Narrative research is a qualitative research perspective, where data is collected and summarised using narrative or verbal methods (Lodico *et al.*, 2010:5). The general concept of a narrative is developed into the view that education and educational research is the construction and reconstruction of personal and social stories (Connelly & Clandinin, 1990:2). All participants in my research have personal and social stories which should be heard. I studied how the mathematicians experienced their world of schooling. The storytellers are the mathematicians.

Teachers' and learners' stories are central to teacher education and to the improvement of schools (Bignold & Su, 2013:403). All stakeholders in education research: learners, teachers,

and researchers are storytellers and characters in their own and other's stories. A sense of equality between participants is particularly important in narrative inquiry (Connelly & Clandinin, 1990:2). This sense of equality is upheld in my study by ensuring that all participants are members of the same Mathematical society and treated equally. There is also a standardisation to the questions asked in the interview.

Narrative writers frequently move back and forward in time in a single document, as various threads are narrated (Connelly & Clandinin, 1990:7). A narrative study sees changes and improvements within a period of time (Andrews, Squire, & Tamboukou, 2013:13). The stories of the research mathematicians in my study focus on the Further Education and Training phase of schooling. These are the final years of a learners' time at school. The participants are not all the same age, so obviously their experiences are from different time frames. Their experiences have passed, but the recollections and stories are relevant to any situation. The stories of the researchers are continuous.

Unlike many qualitative frameworks, narrative research offers no automatic starting or finishing points (Andrews *et al.*, 2013:1). Narrative research offers no overall rules about suitable materials or modes of investigation, or the best level at which to study stories (Andrews *et al.*, 2013:1). Because of the focus in my research, on experience and the qualities of life and education, the stories fit into the narrative research in this respect and all other aspects.

3.3 Crystallisation

Narrative relies on criteria other than validity, reliability, and generalisability (Connelly & Clandinin, 1990:5). Criteria, that best apply to the narrative, such as appearance, credibility and transferability, is important. Guba and Lincoln (1981:198) substituted the concept of reliability and validity, which applies to quantitative research, with the parallel concept of trustworthiness - containing four aspects: credibility, transferability, dependability and confirmability.

Guba (1981: 88) sets out strategies to establish trustworthiness in qualitative research, namely audit trail; member checks when coding; categorising or confirming results with participants and peer debriefing. It is important to focus on these strategies during the study, and not just at the end (Morse *et al.*, 2008:14). In my study, the audit trail is available. All interview transcripts are attached in the Appendix A for any checking that needs to be done.

Three different methods of analysis were used to establish trustworthiness of the results. The results from the different methods were compared in table form, to establish the soundness of the results. The process of verification was checking, confirming, making sure and being certain

of the conclusions made in the analysis. The cultural historical activity theory contextualised and verified these results further. In qualitative research, verification refers to the mechanisms used during the process of research that contribute to ensuring reliability and validity. These mechanisms merge into every step of the inquiry, to construct a solid product (Creswell, 2009:297).

The settings are naturalistic and allow me to ask questions about routine behaviours (Maree 2007:80). These observations could produce findings that could be triangulated with the interview data, increasing the validity of data, findings and conclusions (Lodico *et al.*, 2010:270). Crystallisation is another theory suitable for this study as this study does not have fixed points of reference to verify my work.

Richardson's (2000:13) description of crystallisation is that it combines symmetry, with an infinite variety of shapes, transmutations and angles of approach. There is not a single truth but the texts validate themselves. Using this crystallisation approach, as opposed to a triangulation approach, has given me a more complex and deeper understanding of the phenomenon (Maree, 2007:81).

3.4 Participants involved in the research

In a qualitative research, the researchers' subjectivity is part of the process (Maree, 2007:79). The challenge of a qualitative researcher is to deliver a non-biased study (Marshall & Rossman, 2014:28). I am a mathematics teacher who has been teaching mathematics at a diverse range of schools in South Africa.

Participants in this data gathering were purposefully chosen I wanted to interview South African research mathematicians who were schooled in South Africa. The South African Mathematical Society, as an organisation, arranges conferences in South Africa for research mathematicians who wish to participate. Included in the conference is a section on mathematical education research. Even though the conference proved to be a challenge, I approached research mathematicians to get their approval to participate in this study. I emailed the organisations leadership and sent a request to all of their members to participate in this research.

Despite their willingness to participate, arranging an interview was not successful. I resorted to non-verbal, email interviews. Participants answered questions quite willingly. Questions were open-ended and there was space for many interpretations of the questions; but even with this process, the number of respondents was not quite enough to research the topic adequately. Maree (2007:87) warns a researcher not to base a study on too few participants.

I therefore approached individual research mathematicians that I knew, and personally asked them to participate. The number of participants reached ten. I felt I could work with this group and get what Guba (1981:83) calls thick descriptive data. The email interview was emailed to all participants. Copies were made, which become part of the on-going narrative record (Connelly & Clandinin, 1990:5). (Copies of all these interviews are attached in Appendix A).

3.5 Data generation – Interviews

The participants were interviewed via email to determine how they have personally experienced their schooling and specifically the knowledge that their mathematics teacher had and used. The method of data collection in this study was the unstructured interview. An unstructured interview is open-ended to collect detailed views from participants (Creswell, 2009:21).

Social stories or social patterns can be developed in a narrative research from conversations between people or email exchanges (Andrews *et al.*, 2013:6). The interview questions were open-ended and covered different aspects of teachers' knowledge. The focus was trying to find out from them which type of knowledge they thought was the most important in their schooling. As research mathematicians they perhaps did not see the need to have a teacher, or on the other hand, their teachers could have been instrumental in their success.

3.6 Analysis of qualitative data

According to Leech and Onwuegbuzie (2007:557), one of the most important steps in the qualitative research process is the analysis of data. Data analysis is the systematic search for meaning imbedded in the data (Hatch, 2002:148). Analysis of this generated is based on an interpretive philosophy. It aims at examining meaningful and symbolic content. The way Maree (2007:99) puts it is that the analysis tries to establish how participants make meaning of a specific phenomenon by analysing their perceptions, attitudes, understanding, knowledge, values, feelings and experiences in an attempt to approximate their creation of the phenomenon.

3.6.1 Methods of analysis

There are many different ways of analysing qualitative data. To get a more complete understanding of the data it is important to follow more than one method of analysis (Leech & Onwuegbuzie, 2011:562). Before starting any analysis, it is important to collect and review literature (Lodico *et al.*, 2010:211). Literature was reviewed, in order that recent theories, which are relevant to mathematical knowledge for teaching, may be identified. A study on the literature

on the teaching and learning of mathematics was made, particularly the various understandings around knowledge that teachers need for teaching mathematics effectively.

After reviewing the literature three different data analysis tools were followed. All data analysis tools are similar, in that they involve the three simultaneous activities: data reduction, data display and conclusion-drawing verification (Miles & Huberman, 1994:160). I took numerous small pieces of data that I had collected and gradually combined or related the information to form broader, more general descriptions and conclusions, as explained by Lodico *et al.* (2010:300).

There are two ways of interpreting these themes: deductively, where themes existed before research, or inductively where themes are discovered from the narratives. I use both types of analysis, as this is part of the qualitative approach (Creswell, 2009:259; Lodico *et al.*, 2010: 295).

The first method of analysis that I used was constant comparison analysis, or coding. Codes are identified inductively (Leech & Onwuegbuzie, 2011:77) and emerge from a thorough reading of the data. Identifying codes deductively is working from the general to the specific, while an inductive method is the other way around from a specific observation to a broader generalisation. I first read through all the data that I had gathered. I then grouped the data into smaller meaningful parts. I labelled each part with a descriptive title or a “code.” Each code was compared, so similar pieces were labelled with the same code. After all the data had been coded, the codes were grouped by similarity and a theme was identified and documented based on each grouping.

The second method of analysis that I used to analyse the data was the word count method. According to Leech and Onwuegbuzie (2007:566), this form of analysis is to count the number of times a specific word is used. Word counts are based on the belief that all people have distinctive vocabulary and word usage patterns. I used this method on the entire data set. It is also a basis for the audit trail that is essential for verifying the trustworthiness of qualitative data.

The final method of analysis that was used was content analysis. Themes were created and coding was developed deductively. The number of times each code was applied was counted. I identified which codes were used most and which might be the most important concepts for the participants. Codes were included as descriptive information about the data (Leech & Onwuegbuzie, 2007:569).

3.6.2 Summarising the themes

The collected data was examined from the various viewpoints. According to Lodico *et al.* (2010:267), the researcher summarises and explains the themes and patterns in narrative form. Interpretation also involves discussion of how the findings from this study relate to findings from the review of literature. Findings can then be shared through reports, journals and informal meetings.

In the analysis, track was kept of the sequences, chronology, stories or processes keeping the nature of a narrative in mind; that stories move forward and backward in time. The search for narrative strings were undertaken. A narrative was used because one was able to see different and sometimes inconsistent layers of meaning. It is my contention that this report brings them into useful dialogue with each other, and an understanding of the individual and social changes that occurred, is seen, (also see Andrews *et al.*, 2013:2).

3.7 Activity theory framework

Jonassen and Rohrer-Murphy (1999:63) explain the steps in using activity theory to contextualise the data as follows:

1. Clarify the purpose of the activity. Understand the context in which the activities take place, as well as understand the motivation for the activities.
2. Analyse the activity system. Who is the learner who drives the system? The participants are the ones who determine the goal of the activity.
3. Analyse the activity structures. Actions are complex and can be analysed further as a chain of reactions in the activity structure.
4. Analyse the tools and mediators. Examine the role of permanent structures involved in the system.
5. Analyse the context. The context and the bigger picture: seeing all the support structures that are essential in the process.
6. Analyse the dynamical system. Step back and examine the system, how do the different members work together?

I followed these six steps with the data that had been analysed. I used the basic triangle diagram and gave meaning to the different aspects from my analysed data, then broke it down further into themes. The activity theory framework gave further significance to the collected results. The teaching of mathematics is activity based learning.

3.8 Writing of the narrative

There are pitfalls to be aware of in the writing of a narrative. I was aware of these difficulties. The writing of the narrative study does not have an automatic starting or finishing point (Andrews *et al.*, 2013:2). It is a continuous story of teaching. When writing the narrative and in the whole process of using the narrative approach for this research, there are challenges that should not be ignored.

According to Bignold and Su (2013:412), there are three challenging areas for consideration when writing in the narrative approach: contributor opinion, contextual complexities and researcher positionality. It is important to present research contributor's experiences in their own language, ensuring that their own opinions are heard alongside the opinion of the researcher. The contextual complexities in South Africa will include the history of apartheid and the poor teacher knowledge in this situation. The researcher's positionality is my standing as an experienced mathematics teacher in South Africa.

Connelly and Clandinin mention a few more problems to be avoided in writing a narrative. The narrative should not stress the individual over the social context (Connelly & Clandinin, 1990:2). A balance to this problem is to focus on 'small stories' which tends to prioritise socially-orientated over individually-oriented narrative research (Andrews *et al.*, 2013:8). Each mathematical researcher will have their own small story which will all be joined together to form a bigger story.

Another risk in narrative is what is called, "the Hollywood plot," where everything works out well in the end (Connelly & Clandinin, 1990:10). This could be a risk in this research as all of these participants were school children who became research mathematicians. This study specifically focussed on the teachers' knowledge. In this reduced focus this pitfall of a happy ending to the narrative is avoided.

Similarly, a narrative may be falsified. The data may be used dishonestly as easily as a truthfully. To avoid this deception, according to Connelly and Clandinin (1990:10), listen closely to critics. The plan for the research is to be read and approved by the participants themselves. Narrative research is more than description and a development of themes but involves a

complex process of analysis steps based on the central feature of retelling a story from the original raw data (Lodico *et al.*, 2010:265).

3.9 Summary

I have explained the details of the research design and methodology in this chapter and how the intricacies involved were tackled. As researcher, I had the choice to predetermine the histories that are being researched from my own background and develop themes from what emerged from the data. I collected open-ended, emerging data with the primary intent of developing themes from the data (Creswell, 2009:18). As researcher it is necessary to demonstrate that the findings emerging from the data are not my own predispositions (Shenton, 2004:63), but part of the mathematics researchers' stories.

An interpretive method was followed in the analysis, using three different tools of analysis. The steps followed are well described in Creswell (2009: 185), and Leech and Onwuegbuzie, (2007:569). The data from the interviews was organised, and documented. After making sure that all information was understood, by a thorough reading of all the data, it was coded into various themes, by various methods. The meaning of these themes was then interpreted using activity theory as a framework. The interpretations cannot be separated from me, as the researcher, and multiple views can be formed with this approach. Categories, themes, and interrelationships were examined with the focal point being, to understand the complexities of the situation that the participants were schooled in, and specifically how this is associated with the teachers' mathematical knowledge for teaching.

I attempted to represent the frustrations of my own problems in teaching mathematics in a secondary school in South Africa, into a researchable educational setting. I looked for appropriate research and design that will help find answers, and not reinforce my own ideas and theories. I tried to avoid jargon and subject-specific abstractions, which end up confusing the situation. In this process, I realised that in education research there is a large amount of knowledge, which will answer many of the questions and problems, which teachers face. I hope that this enquiry will be of benefit to others who similarly look for answers and that other teachers will be challenged to do their own continuous professional development in a similar manner.

CHAPTER FOUR: ANALYSIS OF DATA

The analysis of the interview data is presented in this chapter. Furthermore, the results were interpreted in terms of the reviewed literature. The report is based on the narrative criteria of telling the stories of these research mathematicians, and it is focused on mathematical knowledge that teachers need to teach. In the analysis of the data three different methods were used namely, the word count method, a deductive method and an inductive method. The results were explained using the different methods and examine similarities and differences between the results.

Studying and interpreting the interviews was very exciting. I could see from the stories told, how a teacher should teach to encourage learners to study mathematics further, or a teacher could discourage and damage the learners' ability to do mathematics. In analysing the interviews, I could see a correspondence with the literature that had been reviewed; and specifically, in the South African context, I could see emphases which were different from the literature.

In presenting the findings, I list examples from what was said in the interviews, so that the conclusions can be clear to all involved. All examples from the interviews are referenced, stating which interview it was from and giving the appropriate line. All discussion in this section is from the data that I collected, or from the literature, and not my own conclusions.

The first method I report on is the coding method. Here I read and reread the interview script to see what themes were inductively found in the responses. There are only four main themes that I report on, namely:

- Personal stories
- Teaching styles
- Learners attitude to mathematics
- Theories of teachers' knowledge.

The second method of word count: I basically counted the number of times a specific word was used in the interviews by the respondents. Obviously words such as "teach" and "knowledge" have to be ignored because even though they are used many times, the context in which they are used changes their meaning. To understand what I mean, the word "teach" was used 130 times, but each time the context was different. It referred to good and bad teaching.

The criteria that I used to choose words that I refer to as significant words, was because the specific words had been repeated by eighty percent of the respondents and were repeated approximately fifteen times. These words were chosen because of the number of times they

were used by participants. In examining these words there was an obvious connection with themes. Even though words used less frequently may be significant, according to the method of analysis as explained by Leech and Onwuegbuzie (2007:567), one way to understand the perspective of a person is to count the words used. In counting the words, I say it is an approximate number. The reason is that there are times when the word was used out of context. If the context changes the meaning of the word is also changed. I found this method really interesting as it does not depend on my personal opinion or interpretation, but on numbers. Here I also mention words which I call significant words:

- Understanding
- Work
- Discipline or control
- Concepts
- Content
- Approach

The last method of analysis was a deductive method of coding, content analysis. I asked essentially the same questions to each participant. There were therefore themes around which I asked questions. These themes are deductive because I had chosen them from the literature that was reviewed, before the data was collected. The answers were in the themes that I had used, but answers varied considerably. These themes are:

- Pedagogical knowledge needed for teaching
- Technological knowledge needed for teaching
- Content knowledge needed for teaching
- Experiences from own schooling.

In all this analysis, the methodology is a narrative. The most interesting results were from the stories that the research mathematicians told about their own mathematics school education. In looking at their experiences, one must remember that these are people who have been successful in their education. They are at present research mathematicians, who could not have reached their present position without solid basic mathematical education.

4.1 Coding or constant comparison analysis

4.1.1 Personal stories

Examples of personal stories are fascinating, as the South African history and apartheid system of education (Wolhuter, 2014:4) played a part in these stories. There are many interesting stories that were told, but I include only three examples of these personal stories, to show the differences in South African schools. All these examples are from research mathematicians who were schooled under the apartheid system. The first example is from someone who was privileged to be in a “white” school, and the second and third examples are people who were in “black” schools.

Example one: (cf. Interview 1, line 5)

“I had “good” mathematics teachers throughout my senior high school years in the sense that they had strong subject knowledge and were able to explain concepts clearly. They were all extremely traditional in their style and not one of them stands out as particularly enthusiastic. In fact, I cannot remember any one of them “going the extra mile” by doing anything particularly interesting or different.”

Example two: (cf. Interview 5, line 5)

“I had serious challenges due to lack of qualified mathematics teachers. I was forced to change schools after Grade 10 because of this. I came across a proper mathematics teacher in Grade 11. I struggled a lot because I did not have the required background from the earlier grades. Also my grade 12 was unfortunately interrupted by the political changes that were going through the country. We only managed to finish half of the work in most subjects. We had to work hard to try and finish the syllabus on our own. Some of our teachers used weekends to assist us.”

Example three: (cf. Interview 2 line 6)

“In Grade 11 we had also problems as were trying to adjust from the science teachers that resigned; this resulted in us having a new teacher almost every six months, the teachers were the so called private teachers (unqualified teachers), who have just completed grade 12 and did not manage to go to University and some were those that did not do well at university because of the semester system. For Grade 12, the remarkable thing that I can still remember from my teacher, who was also a private teacher, was: there were frequent assessments, immediate feedback and prizes throughout the year, certainly there were certain concepts that he could not

teach us, he confessed. We had to teach ourselves. These encouraged us to work even harder, because, for one thing we had to redo the Grade 11 and some grade 10 concepts. That is when we introduced evening classes; I was one of the tutors in that year. For one thing, one started being disciplined and independent.”

4.1.2 Conclusions from personal stories

The problems with education in South Africa, as a result of the history of apartheid in this country, were also noted in the literature. Letseka (2014: 4868) talks about South Africa as a developing country and that the education performs poorly and lags far behind systems of much poorer neighbouring African countries. Wolhuter (2014:1) also points out how the history of apartheid was detrimental to the education of learners.

4.1.2.1 Qualified teachers

From the personal stories, there are many ideas of good teaching methods that can be concluded. One is the importance of qualified teachers, those who know the content of what has to be taught. Spaull and Kotze (2015:22) also noted the importance of the quality of a teacher as well as the support that the teacher receives while teaching.

4.1.2.2 Collaborative learning

Another idea is that of learners working together in groups to help each other. One of the participants said “we had to teach ourselves” (c.f. interview 2 line 13), he didn’t work alone. Schwarz, (2014:508) found that it is important for learners to work together in groups. Another of the many examples is Cobb *et al.* (1991:390) who explains the benefits of collaborative learning. Learners who are involved in teaching and working with their peers also benefit from this experience.

4.1.2.3 Stakeholders in education

The main stakeholder participants in education are all mentioned in these three short examples. The teacher, the learner, the syllabus, as well as the political or government system. These participants are mentioned in literature. Long (2007:14) explains how the policy makers, government, teachers, learners, parents and educational officials who decide on curriculum, are all responsible for the poor performance. Mouton *et al.* (2013:36) describes how sincere, committed and constructivist interactions between all stakeholders involved in education, is essential for any curriculum change to be effective.

4.1.3 Teaching styles

In the answers to the questions, many participants referred to the different teaching styles of the teachers. It was mentioned that all the teachers are individuals and have their own manner of teaching. The styles of teaching can generally be subdivided into two broad groups: a traditional lecture and collaborate learning. As learners, they also mentioned that at different stages of their schooling, they appreciated different teaching styles.

4.1.3.1 Traditional lecture style

The most common method of teaching they encountered at school was the traditional lecture style. This is “where teacher was king” (cf. Interview 10 line 22). There were times when this style was appreciated, as one respondent said, “At the beginning of my school mathematics, I appreciated the teacher insisting on the answers to questions being in a very formal structure. I am thinking about the beginnings of school geometry and for the answers to problems, we had to split the page in two with a vertical line. The left column was headed: “statement”; and the right: “reason”. This had the unconscious effect of forcing one to think in a very logical manner.” (cf. Interview 3 line 5).

A problem that was named with this style is that “There are teachers who wants to act as if they know everything and that leads to misguidance.” (cf. Interview 6 line 36).

Another problem with this style of teaching is “Lack of application (how can we use mathematics to better the world, it is not easy to see the power behind this subject) at low level makes mathematics less interesting and other subject learners can pick easily their application (e.g. biology for anyone who wants to be a doctor).” (cf. Interview 6 line 39).

4.1.3.2 Cooperative learning

The second broad group of teaching styles was the cooperative learning. It was also called a “loose structure” of teaching (cf. Interview 3 line 11). This is a more participatory, group work style of teaching. One respondent mentioned that this “discussing with your peers create a good learning environment” (cf. Interview 6 line 13).

Another enjoyed the freedom of a loose style of teaching. This particular participant’s words were “For my matric year, I appreciated a very loose structure where I was allowed to wander around the class room assisting class mates with their problems. This had the effect of building my self-confidence and I guess was an affirmation of my mathematical maturity.” (cf. Interview 3 line 11).

Participants mentioned that they changed their own views as they developed. There were times in their schooling when a lecture style was appreciated, but there were definitely times when less control was beneficial. The style of teaching was not seen as the defining element of whether the teaching was effective or not. As one participant said, “In the hands of a good teacher, blackboard and chalk are sufficiently effective to teach maths.” (cf. Interview 10 line 83).

4.1.4 Conclusions from teaching styles:

It should be noted that most of the participants received their mathematical education at a time where “teacher centred” styles were dominant and generally accepted. In the theme of teaching styles the conclusions that I would like to draw about teaching styles are the following:

- Teachers should be aware of the style of teaching that they are using and be prepared to change when needed.
- It is important to adapt the teaching to the learning that needs to take place.
- Teachers should not just use the style that is most common, but there should be times with group work.

Ferri (2012:4) indicated that mathematics teachers have their own teaching styles and this influences the way they teach. He also concluded that teachers should be aware of their own style of teaching to give each learner an equal opportunity. The different styles that he referred to were how a learner understands and thinks through the mathematics. This conclusion is similar to what I saw in my research.

According to Venkat and Adler (2014:478), a criticism of Ball *et al.*, (2008), is that the style of teaching is not included in the knowledge needed for teaching. Venkat and Adler (2014:479) place the style of teaching in the category of pedagogical content knowledge for mathematics teaching. There is much literature on the topic of this pedagogical content knowledge and how it is indispensable. (cf. 2.8).

4.1.5 Learners attitude to mathematics

I noticed a theme of the importance of learners’ attitude, in the responses. In all topics that were taught, it was mentioned that the learners’ thinking and feelings were very important, but this is not independent of the teachers. The teachers can encourage or discourage the learners love for mathematics.

As one participant said, “The reason I chose Mathematics is mainly because we were all discouraged by our teachers from following Maths & Science because some of the “best learners” had failed to get top marks due to Mathematics & Science. So I decided to defy the teachers” (cf. Interview 5 line 31).

From the replies, the following were identified as examples of important learners’ attitudes which help them with mathematics:

- “...the love of problem solving and to seek for deep knowledge of the subject.” (cf. Interview 6 line 44).
- “...study the subject almost on a daily basis and do as many tasks as possible on each topic, more than what is actually prescribed.” (cf. Interview 10 line 30).
- “...and have to work through many textbook exercises to get to grips with what we were being taught.” (cf. Interview 8 line 20).
- “...learners are taught to pass examinations and not to understand the subject matter” (cf. Interview 5 line 85).
- “It was my favourite subject and the one I enjoyed most studying for.” (cf. Interview 7 line 19).

4.1.6 Beliefs about mathematics

The mathematicians that I interviewed often referred back to the nature of what mathematics is, in answering the questions on teachers and teaching. The topic of the meaning of mathematics is most relevant to themselves as research mathematicians. They could not adequately respond without referring to their own definition of mathematics. They had developed these perspectives of mathematics from their own teachers and they think it is important for teachers to convey a good theory of what mathematics is to their learners.

Some examples of the beliefs about mathematics are:

- “Actually, mathematics is exactly the opposite of authority. By this I mean received authority (such as a central source of knowledge might convey). Because mathematics in my opinion is the most radical form of understanding. In other words, every single concept and argument has to be made the individual learner’s own. By this I mean that each individual that claims to know the subject must be able to explain from the agreed starting point how

any argument is developed; or from their own understanding what is the meaning of any defined concept.” (cf. Interview 3 line 70).

- “Mathematics has abstraction as an essential component. It is probably the most essential part of the subject. Although, there is also an essential component required which is an understanding of the structures (situations, issues, etc.) from which you are abstracting. Thus a pre-abstraction component is also essential in mathematics.” (cf. Interview 3 line 30).
- “Mathematics requires a lot of hard work from the learners. One needs to learn to solve the problems on their own. Also one needs to work consistently to ensure that one doesn't forget the building blocks.” (cf. Interview 5 line 25).

4.1.7 Beliefs influence the teaching of mathematics

Teaching mathematics is more than just teaching to pass the exam. Many life skills are being taught alongside the subject matter. They felt that it is not the same as other subjects. Here I would like to quote one respondent “I would believe that other subjects have less requirement for abstraction to varying degrees depending on whether they are sciences or not. However, in any other subject, I believe that mathematics may present a model for proceeding, even though the model is likely to form one extreme of the possible approaches.” (cf. Interview 3 line 34).

One reply concluded the following “Teachers need to be given enough space to focus more on ensuring learners understand and are competent enough to apply their analytical skills to tackle mathematical and life problems on their own.” (cf. Interview 5 line 88).

4.1.8 Conclusions on learners' attitude to mathematics

The theme that I identified adds to literature on the importance of beliefs about mathematics. The literature that I reviewed had a similar emphasis on the meaning of mathematics and how the different views and beliefs of mathematics influence the teaching and learning of mathematics. (cf. 2.3.2).

The beliefs and ideas of teaching and pedagogy also influence the teaching. This understanding of mathematics has also been well documented in literature. (cf. 2.6.2).

In literature, knowledge about learners' attitude to the mathematics is classified as pedagogical content knowledge. This is knowledge that is pedagogical, but specific to the content of mathematics. This theme of the learners' attitude to mathematics reemphasises the importance of this section of a teachers' knowledge. Hill *et al.*, (2008) mentions this classification of knowledge and how it should not be overlooked. Research confirms the importance of pedagogical content knowledge.

Schoenfeld (1994:334) said that a teachers' goal should be to pose relevant problems which learners have to solve. Ball (1990) also elaborates on the importance of the connections between mathematics and real-life situations. (cf. 2.10.1).

Grootenboer (2013:340) makes the point that there is often damage done in mathematics classrooms as many learners leave school without an understanding of mathematics and this limits their life opportunities. This study confirms this idea. The reason that they studied mathematics was in some cases linked in a negative or a positive way to their teacher.

Schoenfeld and Kilpatrick (2008:2) stress the importance of a teacher knowing the learners as mathematical thinkers and mathematical learners. (cf. 2.10.1). Teachers must take their responsibilities seriously.

4.1.9 Theories of teachers' knowledge

There was unquestionably a topic about the theories of teachers' knowledge for teaching, from the responses. The mathematicians responded to questions around theories of teachers' knowledge but a few of them felt that they were unable to adequately respond, as they didn't know the different theories. Their refusal to answer questions about theories of mathematical knowledge for teaching is part of this theme.

There was mentioned that they were grateful that this is a narrative study, as they felt their knowledge about the theories of teachers' knowledge, wasn't sufficient. (cf. interview 10 line 1).

4.1.9.1 Classifications of mathematical knowledge for teaching

Without knowing, or using the terminology that has been used in literature, there were still many references to knowledge that teachers need. All respondents felt that knowledge of the content of mathematics is vital to teaching mathematics. Teachers must know more than just what is required for the level that is being taught. One mentioned that "the challenge, though, is how to ensure that the teacher's knowledge is passed to the learners in an effective way." (cf. Interview 5 line 62).

All respondents felt that discipline or control is vital in a mathematics classroom. A respondent said that "authoritarian control may facilitate sitting still and not being disruptive but won't facilitate learning". (cf. interview 8 line 59). This is clearly part of the theme of a teachers' pedagogical knowledge without using traditional terminology.

All mentioned that calculators, computers and other technological equipment should be used in the mathematics classroom. For example, "it is important that teachers have knowledge of most

of these things, in particular, calculators” (cf. interview 1 line 94). This is an example of technological knowledge.

The three examples, show that the theories of teachers’ knowledge are important, and mentioned by the participants, even if the terminology specific to mathematic education was not used.

4.1.9.2 Criticisms about mathematical knowledge for teaching

Negative comments about classifications of a teachers’ knowledge for teaching is part of this theme. For example:

- “Perhaps what is the greatest of these is an over-analysing of what is essentially human interaction. The danger of over-analysing is that it leads to increased bureaucratisation. So now you have 3 categories which teachers must account for usually by filling in some mindless and time consuming form, instead of humanly interacting with the learners.” (cf. interview 3 line 51)

Another reaction to theories of teachers’ knowledge was that

- “Most of the theories were, in my opinion, just a lot of theoretical nonsense”. (cf. interview 1 line 62)

In the knowledge of technology it was mentioned that

- “I have heard horror stories of teachers having to use calculators because they can’t work out simple arithmetic in their heads!” (cf. interview 7 line 25)

Some respondents felt that they could not to answer questions on theories of mathematical knowledge:

- “Unfortunately I am not *au fait* with the various teaching methods.” (cf. interview 9 line 23).
- “I have no experience in teaching with the technology now being used, so can’t comment in this regard.” (cf. interview 8 line 33).

4.1.10 Conclusions on comments about a teachers' knowledge

There was agreement that content knowledge is the basic cornerstone for teaching mathematics. Literature on teachers' mathematical knowledge for teaching, was started with the work of Shulman (1986) and Ball (1990), as the knowledge required for teachers to be successful in teaching for meaningful learning of mathematics.

My project confirms the importance of the literature, but the respondents emphasised that content knowledge should be stressed above pedagogical and technological knowledge. The respondents felt that a teacher with strong content knowledge would be better than a teacher with strong discipline or pedagogical knowledge. Schoenfeld and Kilpatrick (2008:5) give a similar emphasis when they reflect on how content knowledge must be, knowing school mathematics in depth and breadth. (cf. 2.6.2).

Content or knowledge of the subject must not be relevant only to the matter that is being taught, but it must be at a higher level. Knowing mathematics at a higher level is, according to Speer *et al.* (2015:108), as horizon content knowledge, and he also concludes that it is important.

Mishra and Koehler (2006) acknowledge that the knowledge of technology is important and not isolated from knowledge of pedagogy and content. My research confirms this significant research. (cf. 2.11).

The interviews showed that the focus of any teacher training and teacher development should be on the content. Other aspects of a teachers' knowledge cannot be ignored completely, but a teacher should not be ignorant of dangers in pedagogical and technological knowledge. Venkat and Adler (2014:479) talk about the importance and relevance of teacher training in the different components of knowledge.

In literature there have been criticisms and questions around the theories of mathematical knowledge for teaching, but not in as much detail or as confrontational as in these interviews. Adler and Davies (2006:279) state that studies on learners' achievements and teachers' knowledge have failed to find how to teach teachers, the necessary knowledge.

4.2 Word count

I found the word count method of analysing most interesting and inspiring, because the method does not rely on my interpretation of what was said. I thought the word "logical" would be used many times but it was only used five times. I was surprised at how many times the word "concepts" was used.

The process that was used in counting the words was time consuming. I used the Word document and counted the number of times each noun, adjective, or verb was used from each participants replies. From the list I looked at which words were used significantly more times than other words. The theory as explained in Leech and Onwuegbuzie (2007:568) is that word counts are based on the belief that all people have distinctive vocabulary and word usage patterns. From the list of words, I took the significant words and explain the context in which they were used. Conclusions are taken from the responses around those significant words. Conclusions are made at the end of this section on the words which were used. The conclusions are partly why those specific words were used. The explanations given are from the responses, and not my own conclusions.

4.2.1 Understanding or understand

The word 'understanding' or 'understand' was used twenty four times in total and by almost all respondents. It was used in context with teachers, learners and mathematics as a subject. In context, the mathematicians were obviously emphasising the importance of understanding in the teaching and learning of mathematics.

Examples of what was said about the understanding that teachers need as well as the importance of the learner understanding the mathematics are listed below.

- "How can you teach something of which you do not possess an in-depth knowledge and understanding yourself "(cf. interview 1 line 73)
- "...an essential component required which is an understanding of the structures" (cf. interview 3 line 31)
- "Teachers need to be given enough space to focus more on ensuring learners understand." (cf. interview 5 line 86)
- "It is useless to mislead the teacher by coping something from Google search with no understanding." (cf. interview 6 line 86)
- "A teacher must understand their student in order to teach effectively." (cf. interview 7 line 43).
- "Understanding why learners make the mistakes they do, and how to help them understand, is also essential" (cf. interview 8 line 30)
- "So teachers were not wasting time with pupils who did not understand simple concepts" (cf. interview 9 line 18).

- “How can you teach something of which you do not possess an in-depth knowledge and understanding yourself?” (cf. interview 10 line 74)

4.2.2 Work

The word “work” was used fifty-three times, by all respondents. It was obvious that the mathematicians value work which is done by learners and teachers. The word was used as a noun in worksheets, homework or meaning content. “Work” was used as a verb with methods that work, or to work on solving problems as well as hard work that is required. Hard work was referred to as work which took a lot of time to understand as well as having a good work ethic. Basically mathematics cannot be done without work.

In these fifty-three times, references to working as a group, was mentioned ten times. The frequent referral to the importance of working in a group shows that, that method is important. There were also ten references to working on exercises, which emphasises how learners need to practice mathematics in order to do well and be taught efficiently.

I include just two examples of the use of the word “work” from the interviews in the three different contexts:

Firstly, referring to the content:

- “...the work was always structured well in the sense that complexity of concepts increased gradually and care was taken that each previous step was understood properly before the next concept was introduced.” (cf. interview 1 line 15).
- “All the work was summarised really well during these extra-maths classes.” (interview 1 line 38)

Secondly, referring to teachers:

- “Being hard-working and dedicated. Having a good work ethic so that learners will learn by your example.” (cf. interview 1 line 99)
- “Adequate feedback is also important and this takes hard work and dedication.” (cf. interview 1 line 103).

Lastly, as learners:

- “These encouraged us to work even harder, because, for one thing we had to redo the Grade 11 and some grade 10 concepts.” (cf. interview 2 line 14)

- “Invariably we got homework every day and I would first do my maths homework before any other subjects to ensure that I was up to date. In fact, I was afraid of getting behind since I did not think that I could catch up by myself.” (cf. interview 10 line 11).
- “Mathematics requires a lot of hard work from the learners. One needs to learn to solve the problems on their own. Also one needs to work consistently I prefer to take my learning a step further and work as a learner to add on top of what a teacher taught me.” (cf. interview 5 line 23).

4.2.3 Discipline or control

The word “discipline” was used twenty-two times, by most of the participants. The word “discipline” has two meanings, one in reference to mathematics as a subject and two to the actual classroom practice and control. I only took the words that referred to discipline and control in this section.

The word “discipline” was applied to the studying subject of mathematics. Two examples of this usage are:

- “mathematics is a disciplined subject to study - takes dedication and focus” (cf. Interview 7 line 45)
- “Since Maths requires a lot of independent thinking/problem solving, discipline and consistency are very crucial.” (cf. interview 2 line 80)

The word “discipline” was also associated with the pedagogical knowledge of the teacher. Two further sub-divisions are; why there should be discipline and what is meant by discipline. Examples of these are:

- “Discipline is part of respect. Where there is mutual respect it should be possible to allow for cooperation, collaboration, being considerate of others ... all of which fall under the umbrella of classroom control and discipline. In teaching mathematics there must be discipline and control.” (cf. interview 8 line 59)
- “It is necessary to maintain some degree of discipline, especially for the benefit of those pupils who do not quickly grasp the intricacies of mathematics.” (cf. interview 9 line 46)

It was mentioned that it is not only in mathematics that this is important.

- “I would say discipline is essential in any subject, not for its own sake but as a facilitator of the ability to concentrate” (cf. interview 3 line 85).

- “Discipline maps individual in right direction and nothing will out shine it.” (cf. interview 6 line 80)

4.2.4 Concepts

I chose “concepts” as a significant word as it was used fifteen times. According to the Dictionary, a concept is defined as “an abstract idea”. I find it interesting that the word “concepts” is used so many times when talking about their experience of teachers and teaching. Evidently, they feel that concepts cannot be ignored, but must be emphasised.

A few examples of the context where the word “concept” or “concepts” were used are listed below:

- “...every single concept and argument has to be made the individual learner’s own.” (cf. interview 3 line 72).
- “...they had strong subject knowledge and were able to explain concepts clearly teachers with a strong subject knowledge and were able to explain concepts clearly.” (cf. interview 1 line 6)
- “Matlab is a very useful tool for both tedious calculations as well as improved understanding of concepts through improved visualisation.” (cf. interview 10 line 51).

4.2.5 Content

“Content” was a significant word as it was mentioned twenty-five times. The context was describing the type of knowledge in most cases and linked to the school curriculum. It emphasised the need for teachers to understand the content that is being taught.

A few examples of the use of the word in “context”:

- “A strong subject knowledge and are confronted with content that you may not have seen before, you can easily master the new content.” (cf. interview 1 line 74)
- “Content knowledge will always stand out, since then you can easily adapt your teaching and support it with technological knowledge not the other way round.” (cf. interview 2 line 49)
- “I would say that if you have the content knowledge you would have the confidence to pursue the pedagogical and technological skills necessary to convey the content to your students.” (cf. interview 4 line 25).

4.2.6 Approach

I was surprised that the word “approach” appeared so many times. There were twenty-nine times that this word surfaced. It is links to teaching because the teachers choose the way to approach the learners and the content that needs to be taught. It was also used in the context of being approachable as a teacher, so learners were not afraid to ask questions.

Examples of the use of the word “approach”:

- “You turn to make things difficult for yourself at times if you don't approach getting stuck in the right way.” (cf. interview 6 line 24).
- “You could approach math problems from different angles, e.g. geometry and still be ‘right’.” (cf. interview 10 line 32).
- “The teacher was approachable on both easy and hard topics, in addition he was honest when challenged by students.” (cf. interview 2 line 24).

4.2.7 Conclusions from word count method

Even though the research mathematicians said that they did not know the correct terminology, used by mathematics education researchers, they referred to the same types of knowledge. These six significant words clearly highlight important aspects of knowledge in the teaching of mathematics. (cf. 2.13)

4.2.7.1 Understanding

Ball (1990) agreed with the emphasis on understanding. She said that the goal of mathematics teaching is for students to develop mathematical understanding. Shulman (1987:7) said teaching starts with a teacher understanding a topic. Ma (1999:145) suggested that secondary school teachers do not have a sufficient understanding of mathematics. Cobb *et al.* (1991:598) also emphasises the importance of mathematical understanding. There are many other examples from literature about the importance of understanding. My research just adds to this research list.

4.2.7.2 Work

Mathematics is a subject that needs a lot of work from all who are involved. Teaching is also a profession that cannot take place if there is not a lot of work. Spaul and Kotze (2015:20) mention the work that is required by teachers and learners for effective teaching and learning to take place. Schwarz (2014:505) talks about learners working together and the importance of this collaboration and learning from each other. .

4.2.7.3 Discipline or control

Pedagogical knowledge includes how to discipline a class. This is something that the respondents found important and something that they said is missing in many South African classrooms. Mouton *et al.* (2013:1) emphasises the importance of discipline in the classroom and notes that it is one of the problems in South African education today.

4.2.7.4 Concepts

The significant use of the word “concepts” highlights the importance of understanding of concepts and procedures. Shulman (1987:9) stated that at school, learners’ mathematical thinking should include knowledge of mathematical concepts and procedures, the relationships among them and why they work. In our South African context, we should also emphasise this understanding.

4.2.7.5 Content

Van de Walle *et al.* (2007:258) explains that content knowledge needs to be grounded in the big ideas of mathematics, which is the same result as in my interviews. Without these big ideas as the basic building blocks, Mathematics teaching and learning will not take place.

The literature also emphasises the depth of content knowledge needed for teaching (Schoenfeld & Kilpatrick, 2008:2). This content knowledge includes understanding and this understanding is about the concepts in the mathematics.

4.2.7.6 Approach

The approach that the teacher will have to teaching shows the relational side of teaching as well as the teaching style. This can be linked to literature, about opportunity to learn. It requires a teacher whose focus is learners’ understanding and performance. It is the best explanation of the relationship between teaching and learning (Stols, 2013:1).

4.3 Content Analysis

I think this final method of analysis contains data is most logical and understandable. It is the longest section. The themes are from the interview questions. They are the themes that are deductive, as I decided on these themes and chose questions, which fitted into these themes. The responses were answers to the questions, but varied considerably, as each individual has their own background and insights. I give examples from each theme and analyse the data that was in each of these themes. I also compare these results to current literature on the topic.

4.3.1 Pedagogical knowledge needed for teaching

There was much data that could fit into this theme. I would like to subdivide it into two subheadings: strategies used for solving problems and knowledge of culture and habits unique to South Africa. Both of these subheadings are part of the complicated topic of pedagogical knowledge for teaching. The strategies for solving problems are more specifically pedagogical content knowledge, as it is the knowledge that is particular to teachers of mathematics.

4.3.1.1 Strategies for solving problems

The respondents developed strategies for solving mathematical problems while they were learners. Alternatively the strategies were taught by the teachers or they saw them being used by teachers. To get to their current position as research mathematicians, they had to have well-developed strategies to overcome mathematical problems. Mathematics cannot be done without problem solving, as solving problems is the work of a mathematician (Schoenfeld, 1994: 339).

In solving problems, some said they did not get stuck at school level, but most worked out a problem solving strategy. Personally, I found the list of strategies very helpful - if they could overcome problems, then so could learners that I am teaching.

If I ignore the responses which said they did not get stuck in mathematics, then there are basically three different strategies that are mentioned. They worked alone, seeing problems as a challenge, using resources such as textbook or teacher and there were learners who worked in groups. The importance of the teacher in helping the learners to understand the mathematics, stood out above the other resources that were used. Examples follow:

- “Luckily I never got stuck on any topic in mathematics.” (cf. interview 9 line 8)
- “One needs to learn to solve the problems on their own.” (cf. interview 5 line 23).
- “Not being able to do a problem initially was experienced by me as a challenge and I might spend hours solving a problem.” (cf. interview 3 line 17).
- “One needs to take a break and take one step back and rethink the approach again.” (cf. interview 6 line 27).
- “I had to go back and read Grade 9/10 textbooks on my own in order to resolve most of the problems I had in Grades 11 & 12.” (cf. interview 5 line 14).
- “If we were not sure about how to handle a particular problem, we could go up to her desk to ask and then we received individual attention.” (cf. interview 1 line 27).

- “When I did get stuck, I looked forward to the teacher's explanation.” (cf. interview 10 line 19).
- “Meeting mathematics students from other schools, during winter holidays helped to overcome problems.” (cf. interview 2 line 21).

I found these problem solving strategies similar to those given in literature by Cobb *et al.* (1991:394). They say problem solving results in mathematical understanding and reasoning. Schoenfeld (1989:338) says that solving a problem is an essential part of effective teaching. Adler (2005:2) notes that teachers must be involved in a problem solving activity.

The strategies for overcoming problems are really useful to any teacher. It emphasises the content knowledge of a teacher and how this content knowledge supports pedagogical knowledge. They do not exist independently. It is referring to what literature calls pedagogical content knowledge. Speer *et al.* (2015:108) explains how teachers develop a knowledge specific to teaching. The examples actually agree with his findings but show that learners have also developed this knowledge of applying content to learning.

Pedagogical content knowledge is the intersection between the knowledge of the discipline, mathematics and knowledge of its particular range of teachings (Shulman, 2011:5). Literature has emphasised its significance, for example, Speer *et al.* (2015) explains its importance and how it focuses on the learner and not just the content. Hill *et al.* (2008) and Adler (2005) are two more examples of research, which emphasises this pedagogical content knowledge needed for teaching mathematics (cf. 2.8).

These strategies can add to the literature on learners solving problems in a classroom. In the South African classroom, learners need to take responsibility for their studies and be taught how to solve the mathematics problems.

4.3.1.2 Knowledge of culture and habits of learners

Part of pedagogical knowledge is knowledge of the learner. In South Africa, our specific learners have their own culture and habits. The responses to these questions varied. Some felt that teachers should be very involved and even speak the same language as the learners, whereas, others felt culture had nothing to do with mathematics. The following are examples on this topic that I picked out from their responses:

- “The teacher should certainly have knowledge of the cultural background of the learners. This will assist them to provide more constructive feedback and to identify misconceptions or learning barriers.” (cf. interview 4 line 46).

- “Teachers must understand the culture as it will help them to connect and create a friendly and easy going approach for both teachers and learner. Learners give more to their studies when they like the teacher cause and work hard to make the teacher happy.” (cf. interview 6 line 78).
- “If teachers get to know their learners, and win their respect (which means understanding and respecting them as real people, with real emotions, problems, etc.) they will be more effective as teachers. Knowing a learner’s culture is a small part of this” (cf. interview 8 line 55).
- “It is very important for a teacher to know his learners as to what level I do not have an idea, also, I think this will be at an individual level in most of the case” (cf. interview 2 line 77).
- “I think teaching is more effective when the teacher is aware of the culture of the learners and can even understand and perhaps speak their home language” (cf. interview 10 line 75).
- “In my view culture has nothing to do with understanding (or otherwise) of mathematics.” (cf. interview 9 line 44).
- “If this can assist in the teaching and learning of mathematics, it should be used. However caution must be exercised because not all methods that work well in other subjects can be wholly employed in mathematics without doing damage to the teaching and learning of mathematics.”(cf. interview 5 line 69).

4.3.1.3 Conclusions on pedagogical knowledge

According to Fennema *et al.* (1996:406), teaching is relational. My research project definitely agrees with this statement. The respondents linked the knowledge of culture to the relationship between the learner and the teacher. They felt that this relationship has to be good for learning to take place.

Apart from the relational part of knowledge of culture, there was no consensus about the knowledge of culture in the interviews. Some respondents think it is valuable, while others do not see the relevance of this knowledge in a mathematics classroom. The ideas around knowledge of culture are not specific to a mathematics classroom, but any classroom. Pais (2011:219) explains the importance of a teacher using the learners’ social, historic, political and economic background in the teaching of mathematics. My project on these mathematicians does not replicate the same conclusion.

Valero (2004:11) said that teachers need to try to find new forms of thinking mathematically that is relevant to the culture of the learners in their classes. Even though the cultures of my respondents differed to their teachers in many cases, they did not mention this as important. The mathematicians felt that knowledge of the subject was more important than making it relevant to their culture.

Thinking that mathematics has nothing to do with culture corresponds closely to the claim made by Radford (2014:106) namely that mathematics is independent of ideological persuasion. Rowlands (2007:100) also stated that mathematics exists independently of the creator or the consensus of the community.

4.3.2 Technological knowledge needed for teaching

The responses to these questions also surprised me. Even though there were concerns around the use of calculators, no research mathematician was completely against the use of calculators and computers. They felt that overhead projectors were old fashioned and teachers should keep up with developing technology. I would like to express my uncertainty as to if the participants realise that many classrooms in South Africa do not have computers and use overhead projectors extensively.

Listed below are examples of what was narrated:

- “I think that teachers should use technology, especially computers as a means of challenging learners.” (cf. interview 3 line 90).
- “All Mathematics teachers should be well prepared to use technology to enhance the learning and teaching in their classroom practice.” (cf. interview 4 line 60).
- “Teachers should be trained to use all these necessary tools to enhance the teaching and learning environment.” (cf. interview 5 line 80).
- “Learners must use calculators and computers to learn not to copy. It is useless to mislead the teacher by coping something from google search with no understanding. We must use this technologies to learn more” (cf. interview 6 line 85).
- “Calculators are important if they are managed well. Answers should be questioned in the light of reasonableness, which requires a certain amount of content knowledge.” (cf. interview 8 line 66).

- “Overhead projectors are now largely obsolete and time-consuming. Whether a teacher uses a blackboard or a whiteboard is probably a matter of choice, provided that the mathematics being presented is logically sound.” (cf. interview 8 line 69).
- “Chalk and black board seem sufficient from the way I see it. Calculators are ok; I don’t think they hinder the understating of mathematics by pupils. Yes it’s good to know that $6 \times 7 = 42$, but there is no harm in using a calculator to find the square root of 7.” (cf. interview 9 line 52).
- “In the modern world, it is important that teachers have knowledge of most of these things, in particular, calculators and black boards. It is however totally possible to teach mathematics even if you have never used a computer before.” (cf. interview 1 line 95).
- “Technology knowledge and its benefits in teaching are amazing.” (cf. interview 2 line 85).
- “Technological knowledge can greatly improve the teaching of spatial concepts such as in geometry and graphs, for instance Matlab is a very useful tool for both tedious calculations as well as improved understanding of concepts through improved visualisation.” (cf. interview 10 line 51).
- “There are a variety of sources of knowledge and this include in the subject mathematics, however, this sources cannot replace the role of a teacher. So in my opinion the teacher is still central,” (cf. interview 2 line 70).

4.3.2 Conclusions on technical knowledge

As can be seen from the examples, it was said that teachers should use technology in the mathematics classroom, even though it is not absolutely necessary. This is definitely the same as what was noted by Mishra and Koehler (2006:1021) - the core of good teaching with technology is three components: content, pedagogy, and technology.

There are many advantages to the use of computers, which should be considered and not ignored. Basic arithmetic is still important, but this importance does not negate the use of the calculator. Stols and Kriek (2011:137) explain how technology can be efficiently used in the classroom as learners can work together in groups to solve problems. This has the same results as with my research project. Technology should be used, but cannot replace the importance of basic arithmetic.

Classrooms should keep up to date with technology as it advances. In spite of the advances in technology, the teacher cannot be replaced. Lavinias and Veiga (2013:545) have similar

conclusions from their research, how even with the use of computers, the teacher is still central in the teaching and learning process.

The use of a computer can actually in some places, replace textbooks. Teachers need to find resources which enhance teaching and not stay with old ideas. Exercises and notes can be downloaded from mathematical resources on line, but be careful that it is correct. Schoenfeld (1994:334) explains how a perfect textbook cannot replace the teacher. A teacher must first see if the information is true. It cannot be used without solid content knowledge.

The result from my research around a teacher using technology actually confirms related work of Lavinias and Veiga (2013:546) that technologies help the learners focus on life-long learning. Kieran *et al.* (2013:5) also stress the importance of teachers being life-long learners.

Stols and Kriek (2011:138) did a research project around the use of the computer software called GeoGebra. One of my respondents also recommended its use. They suggested “packages like Geogebra or Geometer’s Sketchpad to teach Geometry effectively.” (cf. interview 4 line 63). Van de Walle *et al.* (2007:258) explains in detail that geometry is a difficult section to teach and learn. The question therefore arises, why computer software programs are not used. It is my contention that this fact shows the lack of communication between researchers and educators.

4.3.3 Content knowledge needed for teaching

One theme from the questions was, the content knowledge needed for teaching, whether it was important and what is the role of this knowledge. The responses show that without exception, everyone thought this knowledge was vitally important, and teaching mathematics cannot take place without it.

Listed below are examples to illustrate their emphasis on content knowledge:

- “Content knowledge is the most important since if it is missing, the teacher simply becomes a conveyer of gobbledy gook without the least insight into the philosophical meaning of what is being taught as an illustration of human progress (or its opposite).” (cf. interview 3 line 60).
- “My view point is that content knowledge is very important, as it provides the teacher with confidence in the subject and also to instil confidence in the learners on the subject. Especially with the important role that the subject plays in our everyday lives.” (cf. interview 2 line 44).

- “How can you teach something of which you do not possess an in-depth knowledge and understanding yourself? If you have a strong subject knowledge and are confronted with content that you may not have seen before, you can easily master the new content. Strong subject knowledge is of critical importance in teaching.” (cf. interview 1 line 76).
- “It allows the teacher to be part of an enlightenment as to the possible role of mathematics in the human world. This is a philosophical deepening. Otherwise, the danger is that mathematics becomes completely misunderstood and mythologised (as appears to be the case for the most part in SA).” (cf. interview 3 line 65).
- “Strong subject knowledge gives you the confidence to prepare your lessons well, integrating suitable pedagogical skills, using technology as a resource and developing good questioning skills. Strong subject knowledge enables good classroom and assessment practice.” (cf. interview 4 line 36).
- “Strong subject knowledge means the teacher should be matured enough to move into new areas on their within mathematics to cope with syllabus changes. Without the ability to learn new areas, one becomes rather useless easily.” (cf. interview 5 line 55).
- “Teachers need to be well equipped in order to facilitate the art of “thinking” required in the subject mathematics, and this requires some deeper knowledge of the subject.” (cf. interview 2 line 101).
- “I also think that the interest and value attached to the subject can help and this can be instilled by looking at immediate applications of the subject in other disciplines. We do not really want every learner to be a mathematician but want to create an awareness on how important maths is in our lives, society, and country and beyond.” (cf. interview 2 line 103).

4.3.4 Conclusions on content knowledge

Content knowledge is the most important knowledge that a mathematics teacher can possess to teach mathematics, this is clearly shown in these examples. Ball *et al.* (2001:434) also conclude that content knowledge is the most important knowledge for a mathematics teacher. The content knowledge is a basis for all other knowledge needed for teaching. A strong content knowledge enables the teacher to have knowledge beyond what is needed at school. Content knowledge makes the mathematics relevant and useful in real world applications.

Schoenfeld and Kilpatrick (2008:322) stress the importance of this content knowledge. The depth, breadth of knowledge as well as flexibility is clearly seen in my research. My research project confirms all that has been said in literature by Ma (1999:145), Ball *et al.* (2001) and

Schoenfeld and Kilpatrick (2008:322), on the importance of content knowledge in effective mathematical teaching. (cf. 2.10.4).

4.3.5 Experiences from own schooling

In answering questions about their own school recollections, there is much that can be learnt about good and bad teaching practices. In the stories of their experiences from school, I think the most significant data was obtained. Note that these accounts are from research mathematicians, who are currently very good at their mathematics. The basic building blocks of their mathematics must have had value, as mathematics is a vertically integrated subject (Spaull & Kotze, 2015:11). From these stories, inferences can be made about the characteristics of an effective teacher.

I selected these examples to show different aspects of teaching that they had learnt in their experience.

- “On the occasional problem that I couldn’t do, I would ask the teacher who usually couldn’t do it either.” (cf. interview 3 line 10).
- “The teacher was approachable on both easy and hard topics; in addition he was honest when challenged by students. Especially regarding hard topics, he was willing to learn like us.” (cf. interview 2 line 25).

In the first two examples, as a learner, they had realised that there were times that the teacher did not know everything, and they had to find help outside the classroom. In mathematics, there are times when the teacher does not have all the answers. It might seem strange to put this in the category of an effective teacher, but the learners must feel free to find answers for themselves, from a different source if necessary. Again, it shows the importance of a teacher being a life-long learner. (cf. 2.12.2).

Grootenboer (2013:340) came to a similar conclusion. He reported that teaching mathematics requires teachers to be flexible and answer questions, sort out confusions as they occur in the classroom, which is not covered in training but learnt as a teacher teaches. Teachers should encourage the learner to think for themselves and they should be encouraged to test the validity of statements (Schoenfeld 1992:334).

- “My teacher was more systematic and focussed.” (cf. interview 7 line 8).
- “They were effective and authoritarian in their approach. This means we were to be quiet and listen and then try to imitate them in doing the problems on our own in silence.” (cf. interview 10 line 10).

- “The teacher was “king”. I was amazed that the teacher was good at everything; meaning algebra, graphs, trigonometry and geometry. From my previous experiences with math teachers, I almost expected them to fall flat on their faces when a certain topic had to be dealt with.” (cf. interview 10 line 24).

The third and fourth example shows teachers who they had considered to be good teachers, they were systematic and focused and knew the answers. This describes traditional education, which is defined as teacher-centred delivery of instruction to classes of learners who are the receivers of information. In contrast to this belief of education, constructivism is popular in modern education. Constructivism is popular due to the idea that the learner is not a passive recipient of knowledge but that knowledge is ‘constructed’ by the learner in some way (Rowlands & Carsons, 2001:1). These respondents learnt well in the traditional method.

Shulman (1987:20) explains how teaching starts with a teacher understanding a topic and it ends with new comprehension by both the learner and the teacher. I think this is the ideal. A teacher who does not fall on their face, but knows all topics. Unfortunately, this is not true in many situations.

- “What you get from a teacher is not enough. That made me to engage in groups.” (cf. interview 6 line 9).
- “I am a reserved person and it was difficult for me to ask question in class, but I manage to keep up with my high school work. Reason being: I had peers who were willing to work in groups to discuss and share knowledge.” (cf. interview 6 line 11).

The fifth and sixth examples show a classroom situation which is quiet and disciplined, in the traditional way, but a reserved child might feel intimidated to ask for help. Kieran *et al.* (2014:2) explains the importance of a teacher reflecting on her teaching practice. In the above examples, the teacher should have reflected on her practice and adapted to the needs of the learners.

I think the last two examples also show the importance of collaborative learning in a classroom. Van de Walle *et al.* (2007:67) explains the importance of collaboration in building mathematical understanding.

- “There was never any question that mathematics would take lots of time that we would get homework always, and have to work through many textbook exercises to get to grips with what we were being taught.” (cf. interview 8 line 20).

- “Perseverance: The 21st Century embraces a culture of instant gratification, instant success, and instant satisfaction. Mathematical success requires time, perseverance, patience, diligence.” (cf. interview 8 line 82).

The final examples that I have quoted, mentioned how it takes time to be an efficient teacher, and to learn mathematics efficiently. There are no shortcuts. Learners must spend time on the exercises and teacher must spend time on preparations. Reeves, Carnoy and Addy (2013:426), explain the concept of opportunity to learn as the amount of time learners are exposed to the curriculum in a given academic year. My research has the same conclusion. There was a definite emphasis on the importance of spending time to learn the mathematics. Stols (2013:2) explains that “opportunity to learn” factors are under the control of the teachers in their own classroom, maybe I can add that there are “opportunity to learn” factors in the control of the learners as well.

- “So in my opinion the teacher is still central, but his role might not necessarily be the same as in the traditional set up.” (cf. interview 2 line 70).
- “Currently due to political and other pressures, learners are taught to pass examinations and not to understand the subject matter. The “matric pass rate” business is also affecting the way teachers are approaching their tasks.” (cf. interview 5 line 85).
- “I didn’t understand the teacher I had in Grade 10; my dad helped me persevere until eventually the penny dropped.” (cf. interview 8 line 12).

The final three examples that I chose clearly show three important stakeholders in any learners’ education - the parents, the teacher and the department of education. From the data, it is clear that these stakeholders need to acknowledge their responsibility. Without all three working together, our education system cannot improve. Mouton *et al.* (2013:36) have the same conclusion that sincere, committed and constructive interactions between all stakeholders involved in education, is essential for any effective teaching to take place.

I have ended with the experiences, because that is the information that I think is most authoritative. It shows actual examples of where learning has taken place. It is not always perfect situations and settings, but real life. Real life is sometimes full of mistakes and problems. The advice that was given by Hill *et al.* (2008:389), of how a teacher, when deciding on how to explain mathematical concepts and representations, needs to have the ability to avoid mathematical mistakes, is similar to conclusion from my research. Schoenfeld (1989) also said that learners are disadvantaged if they struggle or make mistakes, because that can build up bad habits.

4.4 Results in table form

From the three methods of analysing the results, I have only given brief summaries of the results. I would like to point out common themes in all three methods. The eight themes from all three methods are my findings from this research. I think that these themes are the most important and reflect all the results.

I have taken the findings from the three different methods of analysis and placed them in a table comparing the results and demonstrating how the eight themes were acquired.

Table 4.1 Summarised findings from the data analysis

Findings	Coding or constant comparison	Word count method	Content analysis
Importance of content knowledge	Teachers must be well qualified	Concepts	Content knowledge is important
All stake holders involved	Parents, teachers and syllabus significant in personal stories	Approach	
Teacher development	Teachers must be qualified	Understand	Calculators should be used Computer programs are useful
Mathematical understanding	Beliefs of mathematics	Understand	Deep knowledge of content
Collaboration	Collaborate with each other when learning mathematics	Work in groups	Pedagogical knowledge
Time and effort	Personal stories point to the importance of much time and effort spent on Mathematics.	Work	Problem solving strategies
Culture	Historically disadvantaged	Discipline or classroom control	Knowledge of culture and habits
Theories of teaching	Teaching styles	Content	Pedagogical knowledge Technological knowledge Content knowledge

4.5 Linking themes to Cultural historical activity theory

There are eight themes that I refer to as 'my findings', from this collection of data. These themes are constant in all the analysis that was done and correspond to the literature that was reviewed. These findings can be seen in the results in the table above.

1. Importance of content knowledge. Content knowledge is most important in teaching secondary school. Subject knowledge of the content must be deep and broad.
2. All stakeholders involved. For education to be successful, all stakeholders must be involved. The list of stakeholders is teachers, learners, parents, as well as departmental officials.
3. Teacher development. Teachers must be life-long learners, always willing to adjust their teaching methods and practices.
4. Mathematical understanding. Mathematical understanding is most important in the learning of mathematics, for both the learner and teacher.
5. Collaboration. Learners and teachers should be encouraged to collaborate with each other in the learning process. Teachers should also be willing to collaborate to learn.
6. Time and effort. Learning mathematics takes a lot time and effort. There are no quick answers to problems that arise.
7. Culture. Culture is a contributing factor in the teaching, but not to the same extent as content knowledge.
8. Theories of teaching. The research into mathematical knowledge for teaching is useful, but do not neglect relationships in the actual classroom.

Venkat and Spaul (2015:206) summarised that there are three aspects of teachers' knowledge: disciplinary knowledge, subject knowledge for teaching, and classroom competence, which need attention. My research shows the same conclusion.

Mouton *et al.* (2013:41) summarised the position of education in South Africa as facing two fundamental challenges. Firstly, moral values within individuals, families, schools and communities should be realised and instilled; and secondly, teachers should also take pride in their professions and realise that teaching should return to basic concepts. If these two fundamental challenges were met, then the findings from my results would be realised.

In this section, the themes from the findings are linked to the Cultural-historical activity theory, using the triangle as described in chapter two (cf. 2.2). After analysing the data, the initial themes in chapter two have changed.

Firstly, I would like to point out how the three main divisions of knowledge for teaching mathematics are clearly indicated at the top of the triangle (figure 4-1): Initially, I thought that tools corresponded to technological knowledge for teaching, subject corresponds with content

knowledge for teaching and rules correspond to pedagogical knowledge for teaching. As was seen in this project, there is constant interaction, backwards and forwards between this important knowledge for teaching. After the data analysis, I realise that all the forms of knowledge could be considered tools that a teacher needs.

The community, which includes the Department of Education and the syllabus, is another key factor in the triangle and in the research project. The textbooks that are used in the classroom, is another factor in this “community”. The community includes parents, learners, school structures and institutions where the teacher is taught how to teach. The community again links all aspects in this learning process.

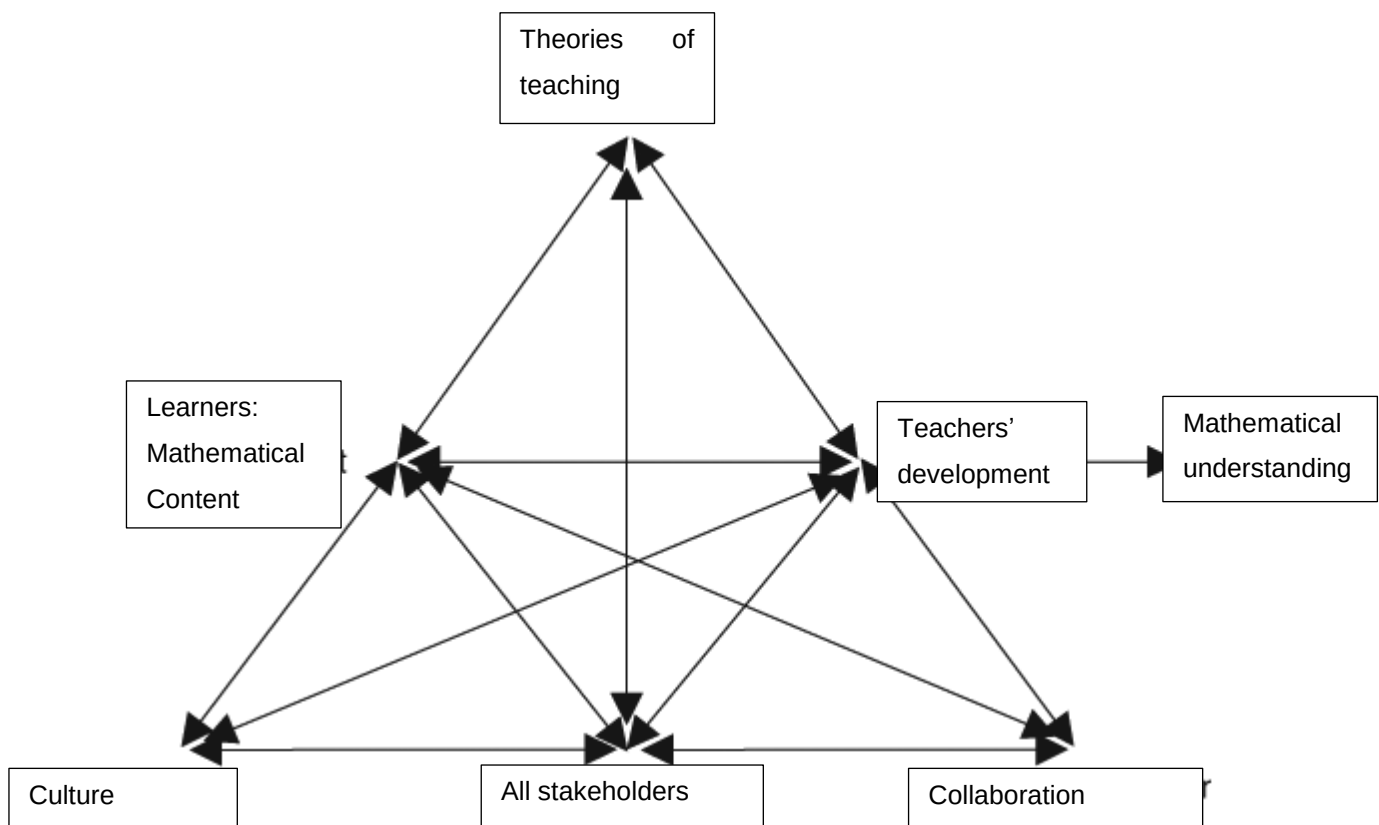


Figure 4-1: My findings aligned to activity theory

The subheadings on figure 4-1 are explained below:

4.5.1 The tools: Theories of teaching

The tools in the teaching of mathematics include all the theories of knowledge that are needed to teach effectively. All forms of knowledge can be used as a tool to teach effectively. Theories of teaching are inclined to focus on the different forms of knowledge, but in the stories, the tools

that a teacher needs are all forms of knowledge, with content knowledge being the most important.

In the initial discussion, the overhead projector was mentioned as a tool, but this is seen as old-fashioned. Teachers should move on to newer equipment. Tools that should be used include computer programs such as GeoGebra. Tools also include textbooks, as well as exercises that teachers choose to work with. The data showed the importance of these tools.

4.5.2 The subject: Learners of mathematics

The subject is the learners who are learning mathematics, but after the research project, I realise that there is a depth to Mathematics that should not be overlooked. Mathematical content knowledge is essential in teaching. The connections between the learners and mathematical content knowledge can be seen from the diagram. Aspects of this content knowledge, affects all other aspects in the effective teaching of mathematics.

Content knowledge is important for teaching in secondary schools. Subject knowledge of the content must be deep and broad.

4.5.3 The rules: Culture

Classroom structures, discipline and control are aspects of pedagogical content knowledge that are essential to effective teaching of mathematics. All respondents stressed the importance of these rules in effective teaching. The rules are linked to culture as it was seen in the data, that teachers must know their learners, and without taking culture into consideration, a teacher will not know them.

4.5.4 The community: All stakeholders involved

All stakeholders in education are important. The learners' parents, the education authorities, school leadership, the teachers, as well as the learners, must all work together for effective learning of mathematics. For Mathematics teaching to be successful, all stakeholders must be involved. The list of stakeholders includes teachers, learners, parents, as well as departmental officials.

4.5.5 The division of labour: Collaboration

Learners should collaborate with each other to obtain a better understanding of concepts involved in mathematics. Teachers should also collaborate with each other in the quest for life-long learning. Collaboration is important between all the stakeholders in the learning process. Teaching is relational.

4.5.6 The object: Teacher development

Teachers must be life-long learners, always willing to adjust their teaching methods and practices. All the different aspects move towards the teacher developing into an effective teacher. Technology and collaboration with others are tools to help with teacher development.

4.5.7 The outcome: Mathematical understanding

Mathematical understanding is most important in the learning of mathematics, for both the learner and teacher.

4.5.8 Connecting arrows: Time and effort

The theme of time and effort is clearly indicated by the strong connections between the different headings. Without time and effort, the whole structure would fall apart. Learning mathematics takes a lot time and effort. There are no quick answers to problems that arise.

4.6 Conclusion

I have used three methods of analysis to search for truth in the interviews that were conducted. The conclusions were tabulated and activity theory was used as a lens to extract meaning of the data that was analysed. The personal stories in the interviews were the most significant in the data analysis. Those stories showed the true background and history of mathematical learning of these South African mathematicians.

Analysing the data is according to Leech and Onwuegbuzie (2007:557), one of the most important steps in the qualitative research process. I have tried to do thorough, honest and reflective analyses of the interviews. This is a qualitative research, so no generalisations can be made but it has been worthwhile looking at these specific successful mathematicians' views on mathematical knowledge for teaching.

The analysis has taken more time than any other section of this research project. I have been careful to include all the interviews in the findings. There were many contradictions in the respondents' replies. I have spent time showing all the different views. My analysis of the data has left me with a similar view as Mouton *et al.* (2013:32), that there are pockets of excellence, but unfortunately these are in the minority and are sustained by capable individuals and not by the system itself.

CHAPTER 5 SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

The aim of this research project was to explore the stories of research mathematicians, and find practical insights in the content of mathematical knowledge for secondary school teaching in South Africa. To this end, the following research questions were investigated:

- What mathematical knowledge did teachers have that was helpful and what areas needed improvement?
- What influence, if any, did indigenous knowledge systems have on mathematical knowledge?
- Should teachers use technology in teaching mathematics?
- What is the importance of strong subject knowledge?
- What pedagogical knowledge had a positive effect?

A review of the literature on the topic of mathematical knowledge for teaching was conducted, using the lenses of Schoenfeld and Kilpatrick (2008), as well as Shulman (1986) and Ball *et al.*, (2008) amongst others. Research mathematicians were approached and interviewed through e-mails. The style of this project was a narrative and the theoretical framework was interpretivist–constructivist. I explored the stories that the research mathematicians have from their own school careers. The conceptual framework of Cultural-historical activity theory was used, as a lens to contextualise the data after the collected results had been examined using different methods, for the narrative (Engeström, 2000). I tried divergent methods of investigation in order to reach results that were not based on my own opinion. I then compared the results with the literature and made conclusions.

This summary explains how the research has contributed to the gaps that I had identified in the commencement of the project. The gaps of the divide between educationist and teacher, as well as between research mathematicians, teacher and educationist. The summary also answers my initial question on the knowledge that is needed to teach mathematics efficiently in South Africa. From the outset, I studied this topic with one aim - to improve my own teaching. I teach mathematics at a secondary school in South Africa. This research project has influenced my own teaching.

This chapter contains the main findings from the data. The findings are the themes and subthemes, obtained by the data collection and analysis through three methods of analysis.

Recommendations, limitations and suggestions for future research are also included in this chapter.

5.1 Gaps in literature

In the first chapter, I stated that there were four gaps in literature that I had identified and that could be filled, using this research project. This project was the first step in closing the gaps. There is much more research needed on the division between researchers and teaching.

5.1.1 Discussion around mathematical knowledge for teaching

I have added to the discussion around the mathematical knowledge for teaching that should be included in this continuous professional development and training. My own self-initiated research has added to the discourse on mathematical knowledge for teaching. This whole project is an example of continuous professional development.

My research added to the discussion. Many of the philosophies of mathematical knowledge for teaching were recognised by this research project. This research project was in the South African context, and it clearly shows that the same knowledge that has been researched around the world is relevant here. All the different classifications of knowledge, pedagogical knowledge, technological knowledge as well as content knowledge, which the literature stated as being important, were reflected in this project. The importance of a strong content or subject knowledge was confirmed. The emphasis on pedagogical content knowledge was also substantiated in this project. Pedagogical content knowledge being a determining factor in learner achievement was also seen in the stories of the respondents. Caution with over analysis and bureaucratic tendencies, was further included to the discussion around theories of mathematical knowledge for teaching.

The importance of teachers being life-long learners, which is part of continuous professional development, was clear in the outcome of the data collection. The results of this research project could be used further to inform stakeholders how to apply the results to continued professional development. The need for life-long learning of the teachers was clear and that this life-long learning should be included in knowledge for teaching mathematics. Answers are available; they just need to be accessed.

5.1.2 Research mathematicians

The second gap was approaching research mathematicians. The discourse was with research mathematicians who are part of the South African Mathematical Society (SAMS) organisation. I

explored their stories, for their perspectives and suggestions on how teachers' mathematical knowledge needs development.

My research started to cover this gap. The SAMS organisation and the research mathematicians have a vast pool of knowledge that can be used more extensively. The research project showed that content knowledge is the area where the most improvement and development is needed. The content or subject level of the teachers of the respondents is below what it should be. Learners have to study textbooks for themselves to cover gaps in their teachers' mathematical knowledge. Technology with computer programmes and the internet can help with the problems of mathematical knowledge.

The views of these research mathematicians were shown through the whole project and their narratives of the political problems and examples of why they excelled. The narrative approach was very important in covering this gap, as they felt that they could not answer questions using the correct terminology. The stories that they told had many views and suggestions even though they felt inadequate to broach the subject. The research project was an attempt at moving out of our traditional reference frame and working together for the sake of better mathematic education.

Their view, as was seen in the data collection, was clearly that teachers' mathematical content knowledge should improve. The knowledge can improve by a better use of technology, as it is the source of much knowledge. Teachers' pedagogical content knowledge emphasises that learners also have their part to play. Teachers must teach mathematics with understanding. All stakeholders, not only teachers, need to be involved in the education.

5.1.3 Conflict between indigenous knowledge and mathematics

The narratives from research mathematicians answer questions about conflict between indigenous knowledge and mathematics, or finding ways that mathematical understanding can be enhanced by indigenous knowledge.

The research project found that content knowledge is more important than indigenous knowledge in the mathematical educational process. There was no evidence of contention between indigenous knowledge and mathematics. There were conflicts between political systems and mathematical learning, but not indigenous knowledge. Cultural or indigenous knowledge can help with the teaching and pedagogical knowledge in running a classroom, but not in the mathematics itself.

The indigenous knowledge and culture that the research mathematicians had did not hinder or help their mathematics. The self-discipline to be dedicated and work consistently was beneficial

to the learning of the mathematics. No specific culture showed to have more dedication or self-discipline. No special adjustment was made by teachers, to account for different cultures.

Belonging to a culture is less important than the practice of self-discipline.

5.1.4 Bridge the gap between research mathematicians and educationists

According to Adler (2005:2), mathematicians and mathematics educationists are very different and often it is difficult for them to understand each other. By listening to the voices of research mathematicians, I contribute to filling this fourth gap in research.

This gap still exists. There are similar gaps between teachers and educationist researchers. The respondents were reluctant to talk to a mathematical educationist, but this research started to build the bridge. Even though they did not use the same terminology, their thoughts on the education of mathematics was the same. The emphasis was not on the constructivist approach to teaching, but it is not excluded. A teacher, who knows the subject that is being taught, should pass the understanding on to the learners.

Mathematical educationists are known to overemphasise the constructivist approach to teaching, where problem solving is the basis of all new concepts (McKnight *et al.*, 2000:5). The mathematicians definitely emphasised collaboration between learners, but a teacher who has a strong content knowledge and explains the concepts of the content adequately is just as essential. One result of this research project, from the stories, is that there is an intermediate way of teaching, between radical constructivism and the blank slate approach. The teacher uses the strong content knowledge to teach for understanding, the learners work together in groups to develop this understanding further.

5.2 Summary of findings

The findings cannot be generalised but they can be used as they are, as points of reference. I summarise the findings from chapter four as presented in the themes that were identified. All of these are in line with my initial question, as well as my secondary questions (see paragraph 1.7 and 1.8).

The initial question was: as a mathematics teacher, I would like to explore the stories of research mathematicians, what practical insights can be gleaned from the content of mathematical knowledge for secondary school teaching in South Africa?

In this research project, I explored the stories of research mathematicians. Practical insights that I gleaned on the content of mathematical knowledge for secondary school teaching can be

divided into three broad topics: content knowledge, pedagogical knowledge and technological knowledge.

5.2.1 Importance of content knowledge

The first and most distinct finding is the importance of a strong content knowledge of teachers of mathematics. The most important conclusion from the study of the narratives and literature is that a secondary school teacher of mathematics in South Africa must have a sound content or subject knowledge. Content knowledge must go beyond what is required by the syllabus and must be deeper than knowledge that is gained from textbooks. The content knowledge of mathematics teachers would help the teachers in all other aspects of knowledge needed for teaching as well as give teachers more self-confidence as they teach.

This knowledge of the content, which is broad and deep, (Schoenfeld & Kirkpatrick, 2008:108) must be used to teach the concepts with understanding. The learners could then collaborate with each other to form an even deeper understanding of the mathematics for themselves. This will assist them to solve practical mathematical problems. The teaching that is the most successful according to this research project is a mixture of a lecture style and a discovery style. Both sides of this coin must be used to teach effectively. Culture could be seen as part of the pedagogical knowledge of the teachers and not part of the pedagogical content knowledge.

Findings about content knowledge refer to the subheadings, depth of knowledge and breadth of knowledge.

5.2.1.1 Depth of knowledge

The respondents in this project appreciated the depth of knowledge of teachers. The finding shows that teacher of mathematics at the FET level should have a content knowledge beyond school level, and beyond what needs to be taught. It was said that a degree in mathematics was the minimum requirement for an effective teacher of mathematics. This depth of knowledge influences a teacher's self-confidence as well as understanding.

5.2.1.2 Breadth of knowledge

The breadth of content knowledge is another factor that is important in effective mathematics teaching. This is knowledge, which helps a teacher to put mathematics into a context, which is understandable and appropriate for the learners that are being taught. Context is vital in showing mathematics as a useful, relevant subject.

5.2.2 All stakeholders in education must be involved

For effective teaching to take place, all stakeholders, which include teachers, learners, parents, department of education as well as other relevant officials in the education system, must be involved.

5.2.2.1 Teachers

A teacher being involved in the education process is a tautology, but in the data there were teachers who were seemingly uninterested in the actual teaching. To be effectively involved, teachers must have the relevant knowledge, which includes knowledge of the learners and their culture. They should have confidence to teach the topics with understanding. They must be willing to be life-long learners. They should be up to date with knowledge of technology that is relevant to the teaching. Teachers should be willing to collaborate with other teachers in order to improve their teaching.

Teachers should also be approachable. Learners should not be afraid to ask questions when they need to understand the topic. Learners should have assurance that their teachers can answer their questions correctly. Teachers should respect the culture of the learners and this should not cause a barrier to the learning.

5.2.2.2 Learners

Learners being involved in their own education - is a needless repetition of the same idea, but all learners who are learning mathematics must be willing to work hard at their work. Homework and exercises should all be completed. Collaboration with fellow learners is important in the learning process. Learners need to understand the urgency of being involved in their own education and find someone to collaborate with in the process.

5.2.2.3 Parents

Parents need not know the work that is being taught, but their involvement is important. Their role is to encourage and support the teacher and the learner in the learning process. They also play a role in reinforcing the discipline and control in the learning environment. Learning cannot take place without a controlled atmosphere. Positive reinforcement of the teacher and school discipline structure should come from the parents.

5.2.2.4 Department of education

The syllabus and explanations of the meaning of mathematics, is initially set out by the relevant authorities. The respondents mentioned the meaning of mathematics and its importance. The

development of the syllabus should be carefully considered to include and support this meaning. The basic definition of what mathematics is, should continuously be adhered to. Textbooks should be relevant and a source that teachers and learners can rely on without fearing that the answers are incorrect.

5.2.3 Teachers development

Teachers must be willing to adapt their teaching practices to changes in syllabus and changes in learners' ability. Teachers should not be unalterable in how they approach the teaching of mathematics. As life-long learners, new methods of teaching should be explored. Collaboration with other teachers is key to this development. New or different methods from other countries should be explored.

A teacher who has a strong content knowledge is more likely to have the self-confidence to try new challenges. Technology is an excellent resource for teacher development. There are many resources that can be used to make topics more understandable, for example, GeoGebra (Stols & Kriek, 2011:137). Teachers could also work with the educational researcher to the advantage of both (Kieran *et al.*, 2013). The noticeable gap between the work of teachers and educationists is short-sighted and irrational. In a situation where mathematics education is weak, all stakeholders should use every resource available.

5.2.4 Mathematical understanding

In the teaching of mathematics, understanding is crucial. This applies to learner and teacher. The teacher must have the understanding of the topic before attempting to teach, and it should be taught in a manner which focuses on understanding. Understanding leads to the mathematics being relevant to practical situations. Understanding of both the procedures and the concepts that are involved in the mathematics (Ma, 1999:145).

Teachers have to have the conceptual understanding to pass the knowledge to the learners. The findings did not show that learners had to discover the mathematics themselves, but teachers should lead them towards conceptual understanding. All mathematics makes sense, (Schoenfeld, 1992:334) but many learners do not see this as they have not seen the connections between that mathematics and the real world. Problem situations can lead towards a deeper understanding of the mathematics that is needed. The emphasis in school should not be to pass exams, but develop understanding.

5.2.5 Collaboration

All the research mathematicians who were interviewed, had searched for, and found, someone to work in partnership with to help them understand the mathematics. A privileged few could ask their parents for help. Some had approachable mathematics teachers who helped them. One participant mentioned that they had to change schools to find a teacher to work alongside, with the content knowledge sufficient for them. Many participants worked with fellow learners, but no one worked alone at their mathematics and reached the necessary understanding. They all worked with someone who supported them to understand.

A finding from the data was that joint effort between learners is important. Learners who excel in their learning of mathematics had collaborators with whom they worked. Learners, who had been disadvantaged by a tragic education system, were able to excel through collaborations that they established themselves. The findings showed that learners chose their own collaborators and learnt from each other. Likewise, teamwork between fellow teachers is important. Knowledge that is gained should be shared.

5.2.6 Theories of teaching

Research into the teaching of mathematics is important, and is a tool in the teaching of mathematics, but this tool can never take the place of actual relationships amongst teachers and learners. The theories of teaching should not be seen as the outcome of the mathematics classroom, but a tool that is used in the classroom. Teachers should be made aware of these tools and learn how to use them effectively.

Included in the theories of teaching is the knowledge of technology. This can be unequivocally helpful in the teaching of mathematics. Cognisance should be taken that technology should not be used without the content knowledge as a solid foundation.

Pedagogical knowledge is vital, but again it is useless without content knowledge. Pedagogical knowledge that includes knowledge of the culture of the learners is essential. A learner is more likely to work for a teacher that they feel confident with, than someone who makes them feel unwelcome and an irritation.

5.2.7 Culture

Indigenous knowledge was equated to culture of learners. There was no mention of any other indigenous knowledge that helped or hindered any learner mathematically. All learners and teachers have their own cultures, their own historical background that they bring to the classroom. The histories have had an effect on the educational systems in South Africa.

Mathematics is seen as separate from this background, without ignoring the actual people involved. Any person can attempt to do mathematics - it is not dependant on their culture. Respect of cultural background is imperative and part of a teachers' pedagogical knowledge.

Understanding of culture will lead to a better understanding of the learners. With this knowledge, the teacher can identify misconceptions and understand how the learners are thinking. Concepts and content are more effectively explained. Contextualising the mathematics is more relevant if the learners' culture is considered. If the teacher can speak the learners' language, the teacher will seem more approachable.

5.2.8 Time and effort

All the aspects of teaching and learning are held together with time and effort. Passivity and inactivity would not accomplish mathematical understanding. Each theme that has been mentioned would not exist without time and effort. A teacher cannot develop a deep or broad content knowledge without time spent on studies. All stakeholders being involved, does not materialise without the collaboration being initialised and stakeholders being intentional about their involvement.

There are many opportunities for teachers to develop and be life-long learners, but again it will not come about without time and effort. Mathematical understanding, for learners and teachers, only occurs with time and effort. A teacher learning about and respecting a learners' culture, only transpires after time and effort is found in the situation.

5.3 Limitations of this study

The following factors limited the scope of this project:

5.3.1 Generalisation

The findings from this study cannot be generalised. This was a qualitative study and no sweeping statements can be construed from the findings. The findings are what these mathematicians said in response to the questions that were asked. In a different setting and at different times, the findings could possibly change.

5.3.2 Time and budget constraints

I changed my interview process from verbal to non-verbal, as neither I, nor the participants, could find the time or money to travel to conduct face-to-face interviews. There was a research mathematician from Cape Town who volunteered, but did not want to participate via email only.

The email interviews added a dimension where the participants could spend more time thinking about their responses.

5.3.3 Personal limitations

As a mathematics teacher I have felt my limitations in undertaking to complete this project. I moreover, felt my limitations in attending the conference of research mathematicians. Teaching takes a lot of time and effort, as has been mentioned and I have had to fit this project in without disadvantaging my own learners.

5.4 Recommendations and future research

Further research can be done for each of the topics in the recommendations. The quest for answers has made me realise that there are many more questions that could be researched. The recommendations are the ways in which I can use the findings in a practical way to teach the learners in my classroom. Further research in the stories of people who were successful at school: this research was limited to a small number of successful learners. Many more people have received an efficacious education. Additional stories could be explored and further conclusions drawn around the teachers' knowledge.

5.4.1 Knowledge can be considered a tool

The realisation is that all forms of teachers' knowledge can be considered to be tools that the teacher must use to teach mathematics with understanding. Concepts should be carefully presented and explained in teaching. The tool of mathematical knowledge for teaching is in order for the learners to obtain an understanding of mathematics. Mathematical concepts should not be neglected and understanding of mathematical concepts should be highlighted. One of the most important focus for a mathematics teacher is the content knowledge.

5.4.2 All Stakeholders

Discussions should take place with all stakeholders in research and teaching. An intentional effort should be placed on involving all possible stakeholders in mathematics education in South Africa at the present time. Encourage learners to collaborate with each other as fellow learners. Teachers should communicate the concepts in an understandable way. Teamwork is best, done as a spontaneous outworking of a problem. Teamwork again should be between all stakeholders. Collaboration was how some of these research mathematicians managed to excel in a deprived education system.

5.4.3 Continuous professional development

Continuous professional development should be researched further, specifically, how teachers in South Africa can practically apply education research to their teaching. There is a plethora of information available. Teachers should be life-long learners. Practical teaching problems, such as geometry or any other new topic could be answered with this information. It begs the question, why is it not used by teachers.

5.4.4 Culture in education

After realising that the participants in this research project think it is important that teachers understand the culture of the learners to facilitate the learning of mathematics, I would like to research this further. More questions around how to accept and involve all cultures in this multicultural country could be explored. The link between discipline and control of learners in the classroom is also linked to knowledge of culture.

5.5 Contribution of this study

The aim of this study was to explore the stories of South African research mathematicians, who managed to excel in mathematics notwithstanding the severe problems in its education system. From their perspectives, insights were collected which can be used to develop the content of mathematical knowledge for teaching secondary school, which in turn can be used for continuous professional development. This mathematical knowledge for teaching should include all aspects of knowledge that a teacher needs, including indigenous knowledge relevant to the South African context.

This study contributes to the body of scholarship on mathematical knowledge for teaching for teachers in secondary school in South Africa, from a qualitative viewpoint. Indigenous knowledge was connected to culture, and the importance of the practical use of knowledge as a tool in teaching and learning.

Continuous professional development is an area that needs much more research. Its importance has been established, but details of how it should happen, needs to be instituted. Continuous professional development should obviously increase a teachers' content knowledge, but how to do this is a question that could be researched further. Life-long learning is part of a teacher's obligations, but implication is not so simple.

5.6 Final analysis

What do the research mathematicians in South Africa say is important about mathematical knowledge for secondary school teaching? Without any doubt, they would like secondary

school teachers to possess a solid content knowledge. This content knowledge should exceed the knowledge required for the learners to pass secondary school. Knowledge of the learners and culture is not specifically helpful to the mathematics education, but to add to a teachers' pedagogical knowledge. Technology, such as computer programs, should not be neglected, but used to enhance the content that is taught.

I have spent many months working on this project. I had not realised before this, how much information on the teaching of mathematics was available, as well as information and research on the problems in South Africa's education system. In this plethora of information, I found many abbreviations and acronyms. This form of writing did not make it easier to study the research, but more complicated. In an attempt to make my study readable to anyone, I have avoided this form of writing.

Interviewing the research mathematicians was intimidating, but very fascinating. Several of them have been in academia for many years and have strong opinions on teaching and mathematics. It made me realise again, my role as a teacher in the lives of the learners in my classroom. I sincerely hope some of them further their study of mathematics and that when they do, they will remember the correct understandings of mathematics from the classroom.

As a teacher, I am not alone. All stakeholders should be involved, and all stakeholders are responsible for the learning that takes place in the classrooms. It took some time to accustom myself to reading research papers, but once I understood the importance and place for the theory, I realised that there is much research that teachers could and should use. Educational research and its different paradigms was another stumbling block in my own study of literature. I have realised its absolute importance and relevance. I found methods that I could make sense of and use to answer my questions. The analysis of all the interviews was daunting, but I appreciated the approach of Leech and Onwuegbuzie (2011:562), where different methods of analysis were used so as not to miss any interpretation, and for a more accurate and definitive understanding of the data.

I have realised that this research project covers a broad spectrum. Teaching is broad. In certain circumstances the different aspects of teaching are researched and analysed, but as a teacher I think it is important to look at all aspects together. This project was not trying to solve all problems, as no generalisation can take place in a qualitative study. I tried to look at all aspects of teaching and the knowledge that is involved in a practical South African classroom setting.

This project has added to my own life-long learning. I have more questions about my teaching that I would like to have answered, and I would like to research further. I would like to encourage other teachers to try their own projects.

REFERENCE LIST

- Adler, J. 2005. Mathematics for teaching: what is it and why is it important that we talk about it. *Pythagoras*, 62: 2-11.
- Adler, J. & Davis, Z. 2006. Opening another black box: researching mathematics for teaching in mathematics teacher education. *Journal for research in mathematics education*, 37(4):270-296.
- Adler, J., Ball, D., Krainer, K., Lin, F.L. & Novotna, J. 2005. Reflections on an emerging field: researching mathematics teacher education. *Educational studies in mathematics*, 60(3):359-381.
- Andrews, M., Squire, C., & Tamboukou, M., eds. 2013. Doing narrative research. Los Angeles, CA: Sage.
- Ball, D.L. 1990. The mathematical understandings that prospective teachers bring to teacher education. *Elementary school journal*, 90(4):449-466.
- Ball, D.L., Lubienski, S.T., & Mewborn, D.S. 2001. Research on teaching mathematics: the unsolved problem of teachers' mathematical knowledge. *Handbook of research on teaching*, 4:433-456
- Ball, D.L., Thames, M.H. & Phelps, G. 2008. Content knowledge for teaching what makes it special? *Journal of teacher education*, 59(5):389-407.
- Bergsten, C. 2014. Mathematical approaches. (In Encyclopedia of mathematics education, 376-383).
- Bignold, W. & Su, F. 2013. The role of the narrator in narrative inquiry in education: construction and co-construction in two case studies. *International journal of research & method in education*, 36(4):400-414.
- Blömeke, S., Buchholtz, N., Suhl, U. & Kaiser, G. 2014. Resolving the chicken-or-egg causality dilemma: the longitudinal interplay of teacher knowledge and teacher beliefs. *Teaching and teacher education*, 37:130-139.

Carnoy, M., Jacks, V., Chisholm, L. & Chilisa, B. 2012. The low-achievement trap: changing the culture of inefficiency in teaching. <http://www.hsrc.ac.za/en/review/june-2012/low-achievement-trap> Date of access: 6 Nov. 2015.

Clandinin, D.J. 2006. Narrative inquiry: a methodology for studying lived experience. *Research studies in music education*, 27(1):44-54.

Clandinin, D.J. & Connelly, F.M. 1994. Personal experience methods. (In Denzin, N.K. & Lincoln, Y.S., eds. *Handbook of qualitative research*. Thousand Oaks, CA: Sage. p. 413-427).

Clandinin, D.J. & Connelly, F.M. 2000. Narrative inquiry: experience and story in qualitative research. San Francisco, CA: Jossey-Bass.

Cobb, P., Wood, T. & Yackel, E. 1991. 'A constructivist approach to second grade mathematics'. (In Von Glaser Field, E., ed. *Radical constructivism in mathematics education*. Dordrecht: Kluwer Academic. p. 157-176).

Creswell, J.W. 2009. *Research design: qualitative, quantitative, and mixed methods approaches*. Thousand Oaks, CA: Sage.

Darling-Hammond, L., Wei, R.C., Andree, A., Richardson, N. & Orphanos, S. 2009. *Professional learning in the learning profession*. Washington, DC: National Staff Development Council.

South Africa. Department of Education. 2003. *National curriculum statements for mathematics for FET: grade 10 – 12*. Pretoria.

Desimone, L.M. 2009. Improving impact studies of teachers' professional development: toward better conceptualizations and measures. *Educational researcher*, 38(3):181-199.

Du Plessis, A. & Webb, P. 2012. A teacher proposed heuristic for ICT professional teacher development and implementation in the South African context. *The Turkish online journal of educational technology*, 11(4):46-55.

Eisenberg, T. 1977. Begle revisited: teacher knowledge and student achievement in algebra. *Journal for research in mathematics education*, 8:216–222.

Engeström, Y. 1987. *Learning by expanding*. Helsinki: Orienta-Konsultit Oy.

Engeström, Y. 2000. Activity theory as a framework for analyzing and redesigning work. *Ergonomics*, 43(7):960-974.

Engeström, Y. 2001. Expansive learning at work: toward an activity theoretical reconceptualization. *Journal of education and work*, 14(1):133-156.

Fennema, E., Carpenter, T.P., Franke, M.L., Levi, L., Jacobs, V.R. & Empson, S.B. 1996. A longitudinal study of learning to use children's thinking in mathematics instruction. *Journal for research in mathematics education*, 27(4):403-434.

Ferri, R.B. 2012. Mathematical thinking styles and their influence on teaching and learning mathematics. Paper presented at the 12th International Congress on Mathematical Education, Seoul, Korea, 8 July. http://www.icme12.org/upload/submission/1905_f.pdf Date of access: 6 Jun. 2015.

Grootenboer, P. 2013. The praxis of mathematics teaching: developing mathematical identities. *Pedagogy, culture & society*, 21(2):321-342.

Guba, E.G. 1981. Criteria for assessing the trustworthiness of naturalistic inquiries. *Educational communication and technology journal*, 29(2):75-91.

Guba, E.G. & Lincoln, Y.S. 1981. Effective evaluation: improving the usefulness of evaluation results through responsive and naturalistic approaches. San Francisco, CA: Jossey-Bass.

Hashim, N.H. & Jones, M.L. 2007. Activity theory: a framework for qualitative analysis. Paper presented at the 4th International Qualitative Research Convention, Selangor, Malaysia, 3 September. <http://ro.uow.edu.au/cgi/viewcontent.cgi?article=1434&context=commpapers> Date of access: 6 Jun. 2015.

Hatch, J.A. 2002. Doing qualitative research in education settings. Albany, NY: Suny.

Hiebert, J. & Grouws, D.A. 2007. The effects of classroom mathematics teaching on students' learning. (In Lester, F.K., ed. Second handbook of research on mathematics teaching and learning. Charlotte, NC: Information age. p. 371-404).

Hill, H.C., Rowan, B., & Ball, D.L. 2005. Effects of teachers' mathematical knowledge for teaching on student achievement. *American educational research journal*, 42(2):371-406.

Hill, H.C., Ball, D.L. & Schilling, S.G. 2008. Unpacking pedagogical content knowledge: conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for research in mathematics education*, 39(4):372-400.

Jonassen, D.H. & Rohrer-Murphy, L. 1999. Activity theory as a framework for designing constructivist learning environments. *Educational technology research and development*, 47(1):61-79.

Kieran, C., Krainer, K., & Shaughnessy, J.M. 2013. Linking research to practice: teachers as key stakeholders in mathematics education research. (In Clements, M.A., Bishop, A.J., Keitel, C., Kilpatrick, J. & Leung, F.K.S., eds. Third international handbook of mathematics education. New York, NY: Springer. p. 361-392).

Kilpatrick, J. 2001. Understanding mathematical literacy: the contribution of research. *Educational studies in mathematics*, 47(1):101-116.

Kilpatrick, J. 2014. History of research in mathematics education. (In Encyclopedia of mathematics education, 267-272).

Labuschagne, S.E. 2013. FET mathematics teachers' perceptions of in-service training. Potchefstroom: NWU. (Unpublished BEd Hons project report).

Lavinas, L. & Veiga, A. 2013. Desafios do modelo brasileiro de inclusão digital pela escola. *Cadernos de Pesquisa*, 43(149):542-569.

Leech, N.L. & Onwuegbuzie, A.J. 2007. An array of qualitative data analysis tools: a call for data analysis triangulation. *School psychology quarterly*, 22(4):557-584.

Leech, N.L. & Onwuegbuzie, A.J. 2011. Beyond constant comparison qualitative data analysis: using nvivo. *School psychology quarterly*, 26(1):70-84.

Lelliott, A., Mwakapenda, W., Doidge, M., Du Plessis, J., Mhlolo, M., Msimanga, A. & Bowie, L. 2009. Issues of teaching & learning in South Africa: a disjunction between curriculum policy and implementation. *African journal of research in mathematics, science and technology education*, 13(1):47-64.

Letseka, M. 2014. The illusion of education in South Africa. *Procedia-social and behavioral sciences*, 116:4864-4869.

Lodico, M.G., Spaulding, D.T. & Voegtle, K.H. 2010. *Methods in educational research: from theory to practice*. 2nd ed. San Francisco, CA: John Wiley.

Long, C. 2007. What can we learn from TIMSS 2003? Proceedings of the 13th Annual Congress of the Association for Mathematics Education of South Africa, Nelspruit, South Africa, 5 Apr. <http://www.amesa.org.za/AMESA2007/Volumes/Volume1.pdf#page=2> Date of access: 1 Aug. 2015.

Ma, L. 1999. *Knowing and teaching elementary mathematics: teachers' understandings of fundamental mathematics in China and the United States*. Mahwah: Lawrence Erlbaum.

MacLure, M. 2010. The offence of theory. *Journal of education policy*, 25(2):277-286.

Maree, K. 2007. *First steps in research*. Pretoria: Van Schaik.

Marshall, C. & Rossman, G.B. 2014. *Designing qualitative research*. Los Angeles, CA: Sage.

Matoti, S.N. 2010. The unheard voices of educators: perceptions of educators about the state of education in South Africa. *South African journal of higher education*, 24(4):568-584.

McKnight, C.C., Magid, A., Murphy, T. & McKnight, M. 2000. *Mathematics education research: a guide for the research mathematician*. Norman, OK: American Mathematical Society.

Miles, M.B. & Huberman, A.M. 1994. *Qualitative data analysis: an expanded sourcebook*. Thousand Oaks, CA: Sage.

Mishra, P. & Koehler, M. 2006. Technological pedagogical content knowledge: a framework for teacher knowledge. *The teachers college record*, 108(6):1017-1054.

Moremedi, G.M. 2007. Challenges in implementing the new mathematics curriculum. Proceedings of the 13th Annual Congress of the Association for Mathematics Education of South Africa, Nelspruit, South Africa, 5 Apr. <http://www.amesa.org.za/AMESA2007/Volumes/Volume1.pdf#page=2> Date of access: 1 Aug. 2015.

Morse, J.M., Barrett, M., Mayan, M., Olson, K., & Spiers, J. 2008. Verification strategies for establishing reliability and validity in qualitative research. *International journal of qualitative methods*, 1(2):13-22.

Mouton, N., Louw, G.P. & Strydom, G. 2013. Critical challenges of the South African school system. *The international business & economics research journal*, 12(1):31-44.

Pais, A. 2011. Criticisms and contradictions of ethnomathematics. *Educational studies in mathematics*, 76(2):209-230.

Pais, A. 2012. Mathematics education and subjectivity—cultures and cultural renewal by Tony Brown. *ZMD mathematics education*, 51:815-818.

Plotz, M., Froneman, S., & Nieuwoudt, H. 2012. A model for the development and transformation of teachers' mathematical content knowledge. *African journal of research in mathematics, science and technology education*, 16(1): 69-81.

Radford, L. 2014. Reflections on history of mathematics. (In Fried, M.N. & Dreyfus, T., eds. *Mathematics & mathematics education: searching for common ground*. Netherlands: Springer. p. 89-109).

Rakumako, A. & Laugksch, R. 2010. Demographic profile and perceived INSET needs of secondary mathematics teachers in Limpopo province. *South African journal of education*, 30(1):139-152.

Reeves, C., Carnoy, M. & Addy, N. 2013. Comparing opportunity to learn and student achievement gains in Southern African primary schools: a new approach. *International journal of educational development*, 33(5):426-435.

Richardson, L. 2000. New writing practices in qualitative research. *Sociology of sport journal*, 17(1):5-20.

Rowlands, S. 2007. Some reflections on the philosophy of mathematics education. *Proceedings of the British Society for Research into learning mathematics*, 27(3):49-54.

Rowlands, S. & Carson, R. 2001. The contradictions in the constructivist discourse. *Philosophy of mathematics education journal*, 14:1-15.

Schoenfeld, A.H. 1989. Explorations of students' mathematical beliefs and behavior. *Journal for research in mathematics education*, 20(4):338-355.

Schoenfeld, A.H. 1992. Learning to think mathematically: problem solving, metacognition, and sense making in mathematics. (*In* Grouws, D.A., ed. *Handbook of research on mathematics teaching and learning*. Ann Arbor, MI: University of Michigan. p. 334-370).

Schoenfeld, A.H. 1994. Reflections on doing and teaching mathematics. (*In* Schoenfeld, A., ed. *Mathematical thinking and problem solving*. Hillsdale, NJ: Lawrence Erlbaum. p. 53-69).

Schoenfeld, A.H. 2004. The math wars. *Educational policy*, 18(1):253-286.

Schoenfeld, A.H. 2012. A modest proposal. *Notices of the American mathematical society*, 59(2):317-319.

Schoenfeld, A.H. & Kilpatrick, J. 2008. Toward a theory of proficiency in teaching mathematics. *International handbook of mathematics teacher education*, 2:321-354.

Schubring, G. & Karp, A.P. 2014. History of mathematics teaching and learning. (*In* *Encyclopedia of mathematics education*, 260-267).

Schwarz, B.B. 2014. Psychological approaches in mathematics education. (*In* *Encyclopedia of Mathematics Education*, 505-510).

Setati, M. 2008. Access to mathematics versus access to the language of power: the struggle in multilingual mathematics classrooms. *South African journal of education*, 28(1):103-116.

Shenton, A.K. 2004. Strategies for ensuring trustworthiness in qualitative research projects. *Education for information*, 22(2):63-75.

Shulman, L. 1986. Those who understand: knowledge growth in teaching. *Educational researcher*, 15(2):4-14.

Shulman, L. 1987. Knowledge and teaching: foundations of the new reform. *Harvard educational review*, 57(1):1-23.

Shulman, L. 2011. Feature essays: the scholarship of teaching and learning: a personal account and reflection. *International journal for the scholarship of teaching and learning*, 5(1):1-7.

Simon, M. 2014. Hypothetical learning trajectories in mathematics education. (In *Encyclopedia of Mathematics Education*, 272-275).

Spaull, N. & Kotze, J. 2015. Starting behind and staying behind in South Africa: the case of insurmountable learning deficits in mathematics. *International journal of educational development*, 41:13-24.

Speer, N.M., King, K.D. & Howell, H. 2015. Definitions of mathematical knowledge for teaching: using these constructs in research on secondary and college mathematics teachers. *Journal of mathematics teacher education*, 18(2):105-122.

Stinson, D.W. & Bullock, E.C. 2013. Diversity in methodology: different possibilities for data collection, analysis, and representation. (In Martinez, M. & Castro-Superfine, A., eds. *Proceedings of the 35th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education organized by University of Illinois, Chicago, Illinois*. Chicago, IL: University of Illinois. p. 1256–1259).

Stols, G. & Kriek, J. 2011. Why don't all maths teachers use dynamic geometry software in their classrooms? *Australasian journal of educational technology*, 27(1):137-151.

Stols, G. 2013. An investigation into the opportunity to learn that is available to grade 12 mathematics learners. *South African journal of education*, 33(1):1-18.

Sztajn, P., Confrey, J., Wilson, P.H. & Edgington, C. 2012. Learning trajectory based instruction toward a theory of teaching. *Educational researcher*, 41(5):147-156.

Taylor, N. 2008. What's wrong with South African schools? What's working in school Development Conference, JET Education Services, Cape Town, South Africa, 29 February. http://www.jet.org.za/publications/jet-education-services-conferences/copy_of_What%20works%20in%20school%20development/Papers/Taylor%20Wha%20wrong%20with%20SA%20schools%20JET%20Schools%20Conf%20final.pdf Date of access: 1 Aug. 2015.

Taylor, N. & Vinjevold, P., eds. 1999. *Getting learning right: report of the President's education initiative research project*. Johannesburg: Joint Education Trust.

Taylor, N. 2011. *Priorities for addressing South Africa's education and training crisis*. Pretoria: Jet Education.

- Taylor, N. 2015. JET bulletin. How can teaching become more professional? <http://www.jet.org.za/publications/bulletins/bulletin-february-2015> Date of access: 1 Aug. 2015.
- Tobin, G.A. & Begley, C.M. 2004. Methodological rigour within a qualitative framework. *Journal of advanced nursing*, 48(4):388-396.
- Valero, P. 2004. Socio-political perspectives on mathematics education. (In Valero, P. & Zevenbergen, R., eds. *Researching the socio-political dimensions of mathematics education: issues of power in theory and methodology*. New York, NY: Springer. p. 5-23).
- Van de Walle, J.A., Karp, K.S. & Bay-Williams, J.M. 2007. *Elementary and middle school mathematics: teaching developmentally*. Upper Saddle River, NJ: Pearson.
- Venkat, H., Adler, J., Rollnick, M., Setati, M., & Vhurumuku, E. 2009. Mathematics and science education research, policy and practice in South Africa: what are the relationships? *African journal of research in mathematics, science and technology education*, 13(1):5-27.
- Venkat, H. & Adler, J. 2014. Pedagogical content knowledge in mathematics education. (In *Encyclopedia of Mathematics Education*, 477-480).
- Venkat, H. & Spaul, N. 2015. What do we know about primary teachers' mathematical content knowledge in South Africa? An analysis of SACMEQ 2007. *International journal of educational development*, 41:121-130.
- Vhurumuku, E. & Mokeleche, M. 2009. The nature of science and indigenous knowledge systems in South Africa, 2000-2007: a critical review of the research in science education. *African journal of research in mathematics, science and technology education*, 13(1):96-114.
- Wolhuter, C.C. 2014. Weaknesses of South African education in the mirror image of international educational development. *South African journal of education*, 34(2):1-25.
- Zakaria, E., Daud, M.Y. & Meerah, S.M. 2009. Perceived needs of urban and rural mathematics majors teaching science in Malaysian secondary schools. *International education studies*, 2(2):82-89.

Appendix A:

Interviews

	INTERVIEW 1
1	1. Reflecting on your own experience as a learner in grade 10 to matric in the South African
2	school, please answer the following:
3	Comment on different teaching styles in your school learning experiences and the effect of
4	these styles.
5	I had "good" mathematics teachers throughout my senior high school years in the sense
6	that they had strong subject knowledge and were able to explain concepts clearly. They
7	were all extremely traditional in their style and not one of them stands out as particularly
8	enthusiastic. In fact, I cannot remember any one of them "going the extra mile" by doing
9	anything particularly interesting or different. Their typical style was to teach the new
10	concept(s) by explaining verbally and on the blackboard and following this up with exercises
11	and homework. I usually managed to finish the work in class. These exercises were usually
12	marked at the beginning of the next class, mostly in a class set-up with the teacher giving us
13	the solutions and addressing any problems if several pupils struggled with a particular
14	question. Sometimes our books were taken in and all the work checked.
15	In my recollection, the work was always structured well in the sense that complexity of
16	concepts increased gradually and care was taken that each previous step was understood
17	properly before the next concept was introduced.
18	In my high school education, classes were streamed so teachers were not wasting time with
19	pupils who did not understand simple concepts. They explained, did one or two worked
20	examples with us, gave us exercises and responded to questions if we were getting stuck. In
21	fact, I only realised that some children really struggled with mathematics when I became a
22	teacher myself and had to deal with non-streamed classes.
23	Explain your experiences of getting 'stuck' and what strategies were used for resolving
24	these type of problems.
25	Usually my teachers would be sitting at her desk doing her own work after giving us
26	exercises to work on. If we were not sure about how to handle a particular problem, we
27	could go up to her desk to ask and then we received individual attention. This did not
28	happen very often with me or my classmates.
29	What was the role of the teachers' knowledge with the topics that you found easy or hard?
30	I really cannot remember. I think that if the teacher noticed that we were all struggling with
31	a topic, she would explain it in a different way.
32	How would you compare mathematics with other subjects, in terms of the kind of work
33	expected and how you approached the subject matter and tasks?
34	I had less homework because I usually managed to finish in class time. To get a distinction
35	though in matric math it was not enough to learn the material really well as was the case
36	for learning subjects. I understood everything very well but needed extra-maths classes (I
37	went to.....) to help me with an overview of everything. All the work was summarised
38	really well during these extra-maths classes.
39	What were your reasons for choosing mathematics at university?
40	It was quite accidental. I wanted to study psychology but needed a bursary so I applied for a
41	teaching bursary from the TED and then had to choose two teaching subjects. I picked two
42	subjects I enjoyed at school, namely Mathematics and Geography. I ended up really
43	enjoying the Mathematics at university and ended up dropping Psychology (which I did not
44	enjoy) and changing my degree from a BA to a BSc without losing a year. Nowadays this
45	would be completely impossible.
46	As a girl I never considered continuing my studies in Mathematics because girls became
47	teachers and nowhere in my undergraduate training was I ever encouraged to think
48	differently. Once I had started teaching, I went to discuss the fact that I could not see
49	myself being a teacher for the rest of my life with an ex-lecturer mine and he encouraged
50	me to study further and become a lecturer.
51	2. As an accomplished research mathematician in order to help teachers trying to teach
52	mathematics, please answer the following:

53	What are your perspectives on the forms of knowledge that are involved in teaching
54	mathematics? I am referring to content knowledge, pedagogical knowledge and
55	technological knowledge.
56	Content knowledge is by far the most important. Pedagogical knowledge is relatively
57	unimportant unless it is specifically on mathematics as a subject and "how best to teach
58	particular content matter". I did an HDipEd as well as an Honours in Mathematics Education
59	and never felt that I learnt something critically important to my success as a teacher and
60	lecturer in the pedagogy subjects. As you can see, I cannot even remember the correct
61	terminology and have to revert to inverted commas. Most of it was, in my opinion, just a lot
62	of theoretical nonsense. I assume that technical knowledge refers to things like how to use
63	a black board, overhead projector, data projector etc? If so, yes, it is of practical use but
64	should be quick and easy to learn by simply giving sample lessons.
65	Explain your perspectives on whether these different forms of knowledge are of equal
66	importance in teaching mathematics or is there a hierarchical order to the forms of
67	knowledge?
68	Hierarchical: Content knowledge, then "how to..", then technical and lastly general
69	pedagogical theory.
70	What is a strong subject knowledge? What is the importance of a strong subject
71	knowledge? What role do you think this strong subject knowledge plays in teaching?
72	How can you teach something of which you do not possess an in-depth knowledge and
73	understanding yourself? If you have a strong subject knowledge and are confronted with
74	content that you may not have seen before, you can easily master the new content. Strong
75	subject knowledge is of critical importance in teaching.
76	From your experiences is the traditional model of teaching, where the teacher is central
77	source of knowledge effective? Please elaborate on your answer.
78	Yes, I do, provided that the teacher has adequate knowledge. It is however not the only
79	model that can work.
80	Indigenous knowledge, a knowledge that learners and teachers have from their
81	background, is part of the pedagogical knowledge of a teacher. Should teachers have
82	knowledge of culture and habits unique to South Africa or is teaching more effective when
83	the teacher is aware of the culture of the learners?
84	Yes, it is important to a certain extent. The teacher has to understand and know his/her
85	learners. In senior secondary school however, it should be of far less importance than in
86	primary school, especially if learners are being prepared for tertiary education.
87	Classroom discipline, or control is this essential to effective mathematics teaching?
88	Without the slightest doubt. It is critically important. I cannot see how teaching can in any
89	way be effective without it.
90	In respect to technological knowledge, which includes the use of calculators, computers, as
91	well as black boards and overhead projectors, etc. What are your perspectives on this topic
92	at High school level in terms of the teachers' knowledge?
93	In the modern world, it is important that teachers have knowledge of most of these things,
94	in particular, calculators and black boards. It is however totally possible to teach
95	mathematics even if you have never used a computer before.
96	3. Any other views on what kind of approach from the teachers would lead to success in
97	school mathematics in the South African school.
98	Being prepared to walk the extra mile. Being hard working and dedicated. Having a good
99	work ethic so that learners will learn by your example. How they teach (talk and chalk,
100	group learning etc) is far less important than what they teach and the structured way
101	in which they teach it. Adequate feedback is also important and this takes hard work and
102	dedication.
103	

	INTERVIEW 2
1	1. Reflecting on your own experience as a learner in grade 10 to matric in the South
2	African school, please answer the following:
3	Comment on different teaching styles in your school learning experiences and the effect of
4	these styles.
5	In Grade 10 and 11, there is not much to tell, since in the middle of year we had disruptions,
6	where science teachers resigned from school. In Grade 11 we had also problems as we
7	trying to adjust from the science teachers that resigned, this resulted in us having a new
8	teacher almost every six months, the teachers were the so called private teachers
9	(unqualified teachers), who have just completed grade 12 and did not manage to go to
10	University and some were those that did not do well at university because of the semester
11	system. For Grade 12, the remarkable thing that I can still remember from my teacher, who
12	was also a private teacher was: there were frequent assessments, immediate feedback and
13	prizes throughout the year, certainly there were certain concepts that he could not teach
14	us, he confessed. We had to teach ourselves. These encouraged us to work even harder,
15	because, for one thing we had to redo the Grade 11 and some grade 10 concepts. That is
16	when we introduced evening classes, I was one of the tutors in that year. For one thing, one
17	started being disciplined and independent.
18	Explain your experiences of getting 'stuck' and what strategies were used for resolving
19	these type of problems.
20	☐ Consultation to different text books
21	☐ Meeting mathematics students from other schools, during winter holidays
22	What was the role of the teachers' knowledge with the topics that you found easy or
23	hard?
24	☐ The teacher was approachable on both easy and hard topics, in addition he was
25	honest when challenged by students. Especially regarding hard topics, he was willing to
26	learn like us.
27	How would you compare mathematics with other subjects, in terms of the kind of work
28	expected and how you approached the subject matter and tasks?
29	☐ Mathematics is a hierarchical subject, unlike other subjects. I found that I had to
30	study the subject almost on a daily basis and do as many tasks as possible on each topic,
31	more than what is actually prescribed.
32	What were your reasons for choosing mathematics at university?
33	☐ I enjoy solving problems and spending time on problems. Also during, the mid-
34	eighties, there were very few teachers who enrolled for mathematics courses at university
35	level. For me that was interesting, and thus was encouraged to take the subject at
36	university level. I was a high school teacher then. I enrolled with Unisa.
37	2. As an accomplished research mathematician in order to help teachers trying to teach
38	mathematics, please answer the following:
39	What are your perspectives on the forms of knowledge that are involved in teaching
40	mathematics? I am referring to content knowledge, pedagogical knowledge and
41	technological knowledge.
42	☐ My view point is that content knowledge is very important, as it provides the teacher
43	with confidence in the subject and also to instil confidence in the learners on the subject.
44	Especially with the important role that the subject plays in our everyday lives.
45	Explain your perspectives on whether these different forms of knowledge are of equal
46	importance in teaching mathematics or is there a hierarchical order to the forms of
47	knowledge?
48	☐ Teaching is dynamic, I therefore think that each situation will dictate how the three
49	forms of knowledge will be applicable. However, content knowledge will always stand out,
50	since then, you can easily adapt your teaching and support it with technological knowledge

51	not the other way round. My view point is that the pedagogy knowledge and technology
52	knowledge should support a teacher with an enough content knowledge to do his job easily
53	and not the other way round and also to use technology in teaching you first need the
54	pedagogy knowledge. Thus in the core of this we should have content knowledge,
55	Pedagogical knowledge, and at the periphery we should have technological knowledge.
56	What is a strong subject knowledge? What is the importance of a strong subject
57	knowledge? What role do you think this strong subject knowledge plays in teaching?
58	☐ This would depend on the level of the learners, my suggestion is that a teacher
59	should have passed at least two levels higher on the subject he/she is teaching beyond
60	grade 12. The strong subject knowledge helps in the building of confidence (correct
61	definitions and examples are provided from the onset) in our learners and also in providing
62	an overview of the course/subject without controversy. It helps the teacher to
63	think/explore beyond what is prescribed is a realistic/possible way and provide appropriate
64	guidance to the learners.
65	From your experiences is the traditional model of teaching, where the teacher is central
66	source of knowledge effective? Please elaborate on your answer.
67	
68	☐ In modern times there are a variety of sources of knowledge and this include in the
69	subject mathematics, however, this sources cannot replace the role of a teacher. So in my
70	opinion the teacher is still central, but his role might not necessarily be the same as in the
71	traditional set up.
72	
73	Indigenous knowledge, a knowledge that learners and teachers have from their
74	background, is part of the pedagogical knowledge of a teacher. Should teachers have
75	knowledge of culture and habits unique to South Africa or is teaching more effective when
76	the teacher is aware of the culture of the learners?
77	☐ It is very important for a teacher to know his learners as to what level I do not have
78	an idea, also, I think this will be at an individual level in most of the cases
79	Classroom discipline, or control is this essential to effective mathematics teaching?
80	☐ Since Maths requires a lot of independent thinking/problem solving, discipline and
81	consistency are very crucial.
82	In respect to technological knowledge, which includes the use of calculators, computers, as
83	well as black boards and overhead projectors, etc. What are your perspectives on this topic
84	at High school level in terms of the teachers' knowledge?
85	☐ Technology knowledge and its benefits in teaching are amazing. The challenge though
86	is whether there is an alignment of this with the other two kinds of knowledge mentioned
87	above. Because if there is some tension at a particular level, the results will not be
88	appealing and we might even experience resentment. I suggest that such a change of the
89	role of a teacher be done is a carefully managed way, especially to the so called" older
90	generation of teachers" Anyway we do not have a choice, but to use technology (which is
91	rapidly changing) on a daily basis. In mathematics, the use of technology should not replace
92	the art of problem solving. For example, given a word problem, before you solve it you have
93	to derive some sort of an equation. To derive an equation, you might need to think hard,
94	and in certain instances, the computer/calculator might not assist but these tools can assist
95	in getting the solution of the word problem quickly and accurately once some "thinking"
96	has been done.
97	
98	3. Any other views on what kind of approach from the teachers would lead to success
99	in school mathematics in the South African school.
100	☐ Teachers need to be well equipped in order to facilitate the art of "thinking" required
101	in the subject mathematics, and this requires some deeper knowledge of the subject. I also
102	think that the interest and value attached to the subject can help and this can be instilled by
103	looking at immediate applications of the subject in other disciplines. We do not really want

104	every learner to be a mathematician but want to create an awareness on how important maths is in our lives, society, country and beyond.
105	

	INTERVIEW 3	
1	1. <i>Reflecting on your own experience as a learner in grade 10 to matric in the South</i>	
2	<i>African school, please answer the following:</i>	
3	<i>Comment on different teaching styles in your school learning experiences and the effect</i>	
4	<i>of these styles.</i>	
5	At the beginning of my school mathematics, I appreciated the teacher insisting on the	
6	answers to questions being in a very formal structure. I am thinking about the	
7	beginnings of school geometry and for the answers to problems, we had to split the	
8	page in two with a vertical line. The left column was headed: "statement"; and the right:	
9	"reason". This had the unconscious effect of forcing one to think in a very logical	
10	manner.	
11	For my matric year, I appreciated on the other hand, a very loose structure where I was	
12	allowed to wander around the class room assisting class mates with their problems. This	
13	had the effect of building my self-confidence and I guess was an affirmation of my	
14	mathematical maturity.	
15	<i>Explain your experiences of getting 'stuck' and what strategies were used for resolving</i>	
16	<i>these type of problems.</i>	
17	I very rarely got stuck. Not being able to do a problem initially was experienced by me as	
18	a challenge and I might spend hours solving a problem. On the occasional problem that I	
19	couldn't do, I would ask the teacher who usually couldn't do it either. These were	
20	however the most interesting of the challenges and usually involved resorting to	
21	suspecting that they might be outside of the axiom system within which we were	
22	working and occasionally called forth arguments which involved out of the box thinking.	
23	I vaguely recall once using a type of argument that required a reference to an infinite	
24	number of possibilities when the thinking framework was strictly finite in nature.	
25	What was the role of the teachers' knowledge with the topics that you found easy or	
26	hard?	
27	Usually, simply setting up a structure which gave me maximum freedom to explore.	
28	How would you compare mathematics with other subjects, in terms of the kind of	
29	work expected and how you approached the subject matter and tasks?	
30	Mathematics has abstraction as an essential component. It is probably the most	
31	essential part of the subject. Although, there is also an essential component required	
32	which is an understanding of the structures (situations, issues, etc) from which you are	
33	abstracting. Thus a pre-abstraction component is also essential in mathematics.	
34	I would believe that other subjects have less requirement for abstraction to varying	
35	degrees depending on whether they are sciences or not. However, in any other subject, I	
36	believe that mathematics may present a model for proceeding, even though the model	
37	is likely to form one extreme of the possible approaches.	
38	What were your reasons for choosing mathematics at university?	
39	Curiosity: wanting to increase my own understanding. Also, perhaps some emotional	
40	reasons: I felt secure with the fact that in mathematics, things were provable; whereas	
41	in questions of politics, aesthetics or morality, one had to put oneself on the line in	
42	defending or propagating one's views. Growing older, I am less scared of taking a stand	
43	(usually unpopular).	
44		
45	2. As an accomplished research mathematician in order to help teachers trying to	
46	teach mathematics, please answer the following:	
47	What are your perspectives on the forms of knowledge that are involved in teaching	
48	mathematics? I am referring to content knowledge, pedagogical knowledge and	
49	technological knowledge.	
50	In a fast evolving world, I guess they are all necessary. Although I think your question	

51	itself has many dangers. Perhaps what is the greatest of these is an over-analysing of	
52	what is essentially human interaction. The danger of over-analysing is that it leads to	
53	increased bureaucratisation. So now you have 3 categories which teachers must account	
54	for usually by filling in some mindless and time consuming form, instead of humanly	
55	interacting with the learners.	
56	Explain your perspectives on whether these different forms of knowledge are of equal	
57	importance in teaching mathematics or is there a hierarchical order to the forms of	
58	knowledge?	
59	Content knowledge is the most important since if it is missing, the teacher simply	
60	becomes a conveyer of gobbledy gook without the least insight into the philosophical	
61	meaning of what is being taught as an illustration of human progress (or its opposite).	
62	What is a strong subject knowledge? What is the importance of a strong subject	
63	knowledge? What role do you think this strong subject knowledge plays in teaching?	
64	It allows the teacher to be part of an enlightenment as to the possible role of	
65	mathematics in the human world. This is a philosophical deepening. Otherwise, the	
66	danger is that mathematics becomes completely misunderstood and mythologised (as	
67	appears to be the case for the most part in SA).	
68	From your experiences is the traditional model of teaching, where the teacher is central	
69	source of knowledge effective? Please elaborate on your answer.	
70	No. Actually, mathematics is exactly the opposite of authority. By this I mean received	
71	authority (such as a central source of knowledge might convey). Because mathematics in	
72	my opinion is the most radical form of understanding. In other words, every single	
73	concept and argument has to be made the individual learner's own. By this I mean that	
74	each individual that claims to know the subject must be able to explain from the agreed	
75	starting point how any argument is developed; or from their own understanding what is	
76	the meaning of any defined concept.	
77	Indigenous knowledge, a knowledge that learners and teachers have from their	
78	background, is part of the pedagogical knowledge of a teacher. Should teachers have	
79	knowledge of culture and habits unique to South Africa or is teaching more effective	
80	when the teacher is aware of the culture of the learners?	
81	Yes. But only in order to convey the power of the abstract methods of mathematics so	
82	that this becomes the central tool of understanding. In discussing the philosophy of	
83	mathematics there is a place for discussion of its limitations as a human endeavour.	
84	Classroom discipline, or control is this essential to effective mathematics teaching?	
85	Yes. But I would say discipline is essential in any subject, not for its own sake but as a	
86	facilitator of the ability to concentrate.	
87	In respect to technological knowledge, which includes the use of calculators, computers,	
88	as well as black boards and overhead projectors, etc. What are your perspectives on this	
89	topic at High school level in terms of the teachers' knowledge?	
90	I think that teachers should use technology, especially computers as a means of	
91	challenging learners.	
92	Any other views on what kind of approach from the teachers would lead to success in	
93	school mathematics in the South African school.	
94	No.	

	INTERVIEW 4	
1	1. Reflecting on your own experience as a learner in grade 10 to matric in the South	
2	African school, please answer the following:	
3	Comment on different teaching styles in your school learning experiences and the effect of	
4	these styles.	
5	Explain your experiences of getting 'stuck' and what strategies were used for resolving	
6	these type of problems.	
7	What was the role of the teachers' knowledge with the topics that you found easy or	
8	hard?	
9	How would you compare mathematics with other subjects, in terms of the kind of work	
10	expected and how you approached the subject matter and tasks?	
11	What were your reasons for choosing mathematics at university?	
12		
13	2. As an accomplished research mathematician in order to help teachers trying to teach	
14	mathematics, please answer the following:	
15	What are your perspectives on the forms of knowledge that are involved in teaching	
16	mathematics? I am referring to content knowledge, pedagogical knowledge and	
17	technological knowledge.	
18	In teaching mathematics successfully, Content knowledge, pedagogical and technological	
19	knowledge are important as well as a knowledge of the content of the next level is required,	
20	i.e. if you are teaching Grade 10, you must be familiar with the Grade 11 curriculum as well.	
21	Explain your perspectives on whether these different forms of knowledge are of equal	
22	importance in teaching mathematics or is there a hierarchical order to the forms of	
23	knowledge?	
24	Content/ subject knowledge only doesn't make you a better teacher but in terms of	
25	hierarchy, I would say that if you have the content knowledge you would have the	
26	confidence to pursue the pedagogical and technological skills necessary to convey the	
27	content to your students.	
28	What is a strong subject knowledge? What is the importance of a strong subject	
29	knowledge? What role do you think this strong subject knowledge plays in teaching?	
30	Strong subject knowledge is when you are familiar and competent with the content of the	
31	curriculum that you are teaching, in addition to the content of the curriculum grade levels	
32	higher. It is also important to have more subject knowledge than what is required to teach	
33	for that specific grade. Strong subject knowledge gives you the confidence to prepare your	
34	lessons well, integrating suitable pedagogical skills, using technology as a resource and	
35	developing good questioning skills. Strong subject knowledge enables good classroom and	
36	assessment practice.	
37	From your experiences is the traditional model of teaching, where the teacher is central	
38	source of knowledge effective? Please elaborate on your answer.	
39	It can be effective if the teacher also gives opportunities for the learners to interact and	
40	discover through activities. However, in mathematics, the most effective learning takes	
41	place when the learner is actively engaged in activities to promote critical thinking.	
42	Indigenous knowledge, a knowledge that learners and teachers have from their	
43	background, is part of the pedagogical knowledge of a teacher. Should teachers have	
44	knowledge of culture and habits unique to South Africa or is teaching more effective when	
45	the teacher is aware of the culture of the learners?	
46	The teacher should certainly have a knowledge of the cultural background of the learners.	
47	This will assist them to provide more constructive feedback and to identify misconceptions	
48	or learning barriers.	
49	Classroom discipline, or control is this essential to effective mathematics teaching?	
50	Classroom discipline is essential to effective maths teaching.	
51	Learners need to be encouraged to think and problem solve and this will only happen if all	
52	the learners in the class are actively engaged in the same activity. Talking or fooling around	

53	will distract them from their tasks and the objective of the lesson will not be met. The	
54	teacher will spend all her time disciplining, rather than teaching or facilitating group	
55	discussions. Effective discipline rules for the classroom can be agreed upon by the learners	
56	who should have an input of what they find acceptable too.	
57	In respect to technological knowledge, which includes the use of calculators, computers, as	
58	well as black boards and overhead projectors, etc. What are your perspectives on this topic	
59	at High school level in terms of the teachers' knowledge?	
60	All Mathematics teachers should be well prepared to use technology to enhance the	
61	learning and teaching in their classroom practice.	
62	In terms of the technology, the teacher should also have the skills to use computer	
63	application packages like Geogebra or Geometer's Sketchpad to teach Geometry	
64	effectively. Overhead projectors are out of date. Maths teachers should be able to use data	
65	projectors and be familiar with maths packages that will enhance their teaching	
66	methodology.	
67		
68	3. Any other views on what kind of approach from the teachers would lead to success	
69	in school mathematics in the South African school.	
70	Maths teachers should be lifelong learners. They should attend cluster meetings,	
71	conferences etc to share their assessments and teaching ideas. They must be good	
72	researchers so that they can produce their own worksheets based on the level or grade	
73	which they teach and not have to rely on textbooks only. They need to learn how to ask the	
74	right questions and to promote critical thinking and problem solving skills in their learners.	
75	Maths teachers should promote Mathematics as a subject that will equip their learners with	
76	the necessary skills required for life, and not just as a number crunching subject. The	
77	emphasis must be on the process rather than on the product.	

	INTERVIEW 5
1	1. Reflecting on your own experience as a learner in grade 10 to matric in the South African
2	school, please answer the following:
3	Comment on different teaching styles in your school learning experiences and the effect of
4	these styles.
5	I had serious challenges due to lack of qualified mathematics teachers. I was forced to
6	change schools after Grade 10 because of this. I came across a proper mathematics teacher
7	in Grade 11. I struggled a lot because I did not have the required background from the
8	earlier grades. Also my grade 12 was unfortunately interrupted by the political changes that
9	were going through the country. We only managed to finish half of the work in most
10	subjects. We had to work hard to try and finish the syllabus on our own. Some of our
11	teachers used weekends to assist us.
12	Explain your experiences of getting 'stuck' and what strategies were used for resolving
13	these type of problems.
14	I had to go back and read Grade 9/10 textbooks on my own in order to resolve most of the
15	problems I had in Grades 11 & 12. Grade 12 was the most challenging year due to political
16	events as mentioned above. Also the politics at the school prevented the more qualified
17	mathematics from going to matric with us.
18	What was the role of the teachers' knowledge with the topics that you found easy or hard?
19	The teachers' knowledge was central to our learning. Our matric teacher treated topics that
20	he was more comfortable with first and the rest were never properly treated in class.
21	How would you compare mathematics with other subjects, in terms of the kind of work
22	expected and how you approached the subject matter and tasks?
23	Mathematics requires a lot of hard work from the learners. One needs to learn to solve the
24	problems on their own. Also one needs to work consistently to ensure that one doesn't
25	forget the building blocks.
26	What were your reasons for choosing mathematics at university?
27	When I was in Grade 10, I wanted to do History because it was one of my favourite subjects.
28	I was asked to use the Grade 9 marks as a guide. This was not useful to me because my
29	marks in General Science, Geography, History and Mathematics were all high. The reason I
30	chose Mathematics is mainly because we were all discouraged by our teachers from
31	following Maths & Science because some of the "best learners" in then Standard 8 (JC) had
32	failed to get top marks due to Mathematics & Science. So I decided to defy the teachers.
33	When I got to University I followed a generic BSc degree and then did an Honours in
34	Mathematics Education. Half of the modules were Pure Maths topics and the other half
35	were Teaching & Learning Theories. I did very well in the Pure Maths topics and this
36	prompted to recruit me to join the Pure Maths stream. This is how I ended up in
37	Mathematics!
38	2. As an accomplished research mathematician in order to help teachers trying to teach
39	mathematics, please answer the following:
40	What are your perspectives on the forms of knowledge that are involved in teaching
41	mathematics? I am referring to content knowledge, pedagogical knowledge and
42	technological knowledge.
43	All three are important, but the first two, content and pedagogical knowledge are central in
44	the teaching and learning of mathematics. Technology can be used in most instances but
45	learners need to learn to do mathematics. For this we still need a piece of chalk and the
46	black/green board.
47	Explain your perspectives on whether these different forms of knowledge are of equal
48	importance in teaching mathematics or is there a hierarchical order to the forms of
49	knowledge?
50	Content knowledge followed by pedagogical knowledge. Technology can be a nice to have if
51	it can add value.
52	What is a strong subject knowledge? What is the importance of a strong subject

53	knowledge? What role do you think this strong subject knowledge plays in teaching?
54	Strong subject knowledge means the teacher should be matured enough to move into new
55	areas on their within mathematics to cope with syllabus changes. Without the ability to
56	learn new areas, one becomes rather useless easily. I saw it with some of my teachers who
57	had been trained in colleges. They couldn't handle the introduction of Calculus & Analytical
58	Geometry, for example. They were used to seeing the same stuff for many years.
59	From your experiences is the traditional model of teaching, where the teacher is central
60	source of knowledge effective? Please elaborate on your answer.
61	In mathematics, this is almost indispensable. The challenge, though, is how to ensure that
62	the teacher's knowledge is passed to the learners in an effective way. Interaction with
63	learners is also very important.
64	Indigenous knowledge, a knowledge that learners and teachers have from their
65	background, is part of the pedagogical knowledge of a teacher. Should teachers have
66	knowledge of culture and habits unique to South Africa or is teaching more effective when
67	the teacher is aware of the culture of the learners?
68	If this can assist in the teaching and learning of mathematics, it should be used. However
69	caution must be exercised because not all methods that work well in other subjects can be
70	wholly employed in mathematics without doing damage to the teaching and learning of
71	mathematics.
72	Classroom discipline, or control is this essential to effective mathematics teaching?
73	This is central for effective teaching and learning of mathematics. The teacher should be
74	always be in control of the classroom. Otherwise the overall learning and teaching
75	environment is severely compromised.
76	In respect to technological knowledge, which includes the use of calculators, computers, as
77	well as black boards and overhead projectors, etc. What are your perspectives on this topic
78	at High school level in terms of the teachers' knowledge?
79	Teachers should be trained to use all these necessary tools to enhance the teaching and
80	learning environment.
81	3. Any other views on what kind of approach from the teachers would lead to success in
82	school mathematics in the South African school.
83	
84	Currently due to political and other pressures, learners are taught to pass examinations and
85	not to understand the subject matter. Teachers need to be given enough space to focus
86	more on ensuring learners understand and are competent enough to apply their analytical
87	skills to tackle mathematical and life problems on their own. The "matric pass rate"
88	business is also affecting the way teachers are approaching their tasks.
89	

	INTERVIEW SIX
1	1. Reflecting on your own experience as a learner in grade 10 to matric in the South
2	African school, please answer the following:
3	Comment on different teaching styles in your school learning experiences and the effect
4	of these styles.
5	lecture style
6	It is effective since it give an over view of what one should learn and you get a chance to
7	ask questions, but personally I prefer to take my learning a step further and work as a
8	learner to add on top of what a teacher taught me. What you get from a teacher is not
9	enough. That made me to engage in groups.
10	group style
11	I am a reserved person and it was difficult for me to ask question in class, but I manage
12	to keep up with my high school work. Reason being: I had peers who were willing to work
13	in groups to discuss and share knowledge. It was so effective because we were discovering
14	a lot that one could not learn in class because of the time frame the teacher has to finish
15	everything for that level or grade. What I have realized, is an educator can do his/her bit
16	and learners must to their part... Working alone is not beneficial because two minds or
17	more are better than one and working in groups gives lot of courage. When you work
18	alone you turn to give up easily. Discussing with your peers create a good learning
19	environment and I saw friends growing and becoming free to express their view and
20	contribute whereas they do the opposite in class. Everyone from the group improved a
21	lot.
22	Explain your experiences of getting 'stuck' and what strategies were used for resolving
23	these type of problems.
24	You turn to make things difficult for yourself at times if you don't approach getting stuck
25	in the right way. One get stuck because different reasons, but from my point of view is
26	when you over work yourself. One needs to take a break and take one step back and
27	rethink the approach again. Two when you use wrong resources with lack of deep
28	knowledge of subject. Lastly, seek help from people with experience. My point is when
29	you work on a problem one turn to overlook somethings that other person can point to.
30	Collaboration is key.
31	What was the role of the teachers' knowledge with the topics that you found easy or
32	hard?
33	
34	Easy: Guidance which comes from the teacher's knowledge made things easier and
35	misguidance things hard. There are teachers who wants to act as if they know everything
36	and that leads to misguidance.
37	How would you compare mathematics with other subjects, in terms of the kind of work
38	expected and how you approached the subject matter and tasks?
39	Lack of application (how can we use mathematics to better the world, it is not easy to see
40	the power behind this subject) at low level makes mathematics less interesting and other
41	subject learners can pick easily their application (e.g. biology for anyone who wants to be
42	a doctor).
43	What were your reasons for choosing mathematics at university?
44	Passion, the love of problem solving and to seek for deep knowledge of the subject , but
45	overall after completing my first degree I felt that this world needs analytical
46	thinkers and we as mathematicians we can do the world justice by becoming analyst.
47	
48	2. As an accomplished research mathematician in order to help teachers trying to teach
49	mathematics, please answer the following:
50	What are your perspectives on the forms of knowledge that are involved in teaching
51	mathematics? I am referring to content knowledge, pedagogical knowledge and
52	technological knowledge.

53	We need both since one compliment another, but most institutions choose one
54	over another.
55	Explain your perspectives on whether these different forms of knowledge are of equal
56	importance in teaching mathematics or is there a hierarchical order to the forms of
57	knowledge?
58	As I have mentioned above pedagogical gives as the edge and technological give us
59	unlimited access of knowledge.
60	What is a strong subject knowledge? What is the importance of a strong subject
61	knowledge? What role do you think this strong subject knowledge plays in teaching?
62	Know your basics. It is important to know your basics because those are you tools to learn
63	more. They are your knowledge base and from them you can extend your knowledge
64	and become better.
65	From your experiences is the traditional model of teaching, where the teacher is central
66	source of knowledge effective? Please elaborate on your answer.
67	No, a learning environment is about everyone sharing what they know. If we want the
68	teacher to be the only one sharing we have only one source and being one dimensional it
69	limits the learning environment. Involve learner and you will be surprised to find out what
70	knowledge they have about the subject.
71	Indigenous knowledge, a knowledge that learners and teachers have from their
72	background, is part of the pedagogical knowledge of a teacher. Should teachers have
73	knowledge of culture and habits unique to South Africa or is teaching more effective when
74	the teacher is aware of the culture of the learners?
75	Yes the must. It will help them to connect and create a friendly and easy going approach
76	for both teachers and learner. Learners give more to their studies when they like the
77	teacher cause and work hard to make the teacher happy.
78	Classroom discipline, or control is this essential to effective mathematics teaching?
79	Yes, I am a better person because of that. It maps individual in right direction and nothing
80	will out shine it.
81	In respect to technological knowledge, which includes the use of calculators, computers,
82	as well as black boards and overhead projectors, etc. What are your perspectives on this
83	topic at High school level in terms of the teachers' knowledge?
84	Learners must use calculators and computers to learn not to copy. It is useless to mislead
85	the teacher by coping something from google search with no understanding. We must use
86	this technologies to learn more.
87	
88	3. Any other views on what kind of approach from the teachers would lead to success in
89	school mathematics in the South African school?
90	
91	Teacher's own development: Teachers must start their own support group and share
92	knowledge within themselves. School must invite ex-students to come and share with
93	other learners what is that they have benefited from the teacher and school or subject
94	and why it is important to know what they are studying. They must thank the school
95	(teachers) for giving them the edge to carry themselves out and that will motivate current
96	learners.
97	

	INTERVIEW 7
1	1. Reflecting on your own experience as a learner in grade 10 to matric in the South
2	African school, please answer the following:
3	Comment on different teaching styles in your school learning experiences and the effect
4	of these styles.
5	I was lucky to have the same teacher from Grade 10 to Grade 12. There was another
6	teacher allocated in Grade 12 but I immediately realised I could not learn in her manner –
7	she simply gave many exercises and then wrote the answers on the board. My teacher
8	was more systematic and focussed.
9	Explain your experiences of getting 'stuck' and what strategies were used for resolving
10	these type of problems.
11	Did many exercises to overcome the hurdle.
12	What was the role of the teachers' knowledge with the topics that you found easy or
13	hard?
14	First step towards realising why I was having trouble, then I fixed it myself.
15	How would you compare mathematics with other subjects, in terms of the kind of work
16	expected and how you approached the subject matter and tasks?
17	Similar in all aspects except that many exercises need to be done to master mathematics.
18	What were your reasons for choosing mathematics at university?
19	It was my favourite subject and the one I enjoyed most studying for.
20	2. As an accomplished research mathematician in order to help teachers trying to teach
21	mathematics, please answer the following:
22	What are your perspectives on the forms of knowledge that are involved in teaching
23	mathematics? I am referring to content knowledge, pedagogical knowledge and
24	technological knowledge. Currently I believe our teachers do not have adequate
25	mathematics knowledge at all – I have heard horror stories of teachers having to use
26	calculators because they can't work out simple arithmetic in their heads! They need to
27	know mathematics up to third year at least. All three knowledge forms above are
28	important.
29	Explain your perspectives on whether these different forms of knowledge are of equal
30	importance in teaching mathematics or is there a hierarchical order to the forms of
31	knowledge? All equally important.
32	What is a strong subject knowledge? Knowing higher maths than what you are teaching in
33	order to have the bigger perspective. What is the importance of a strong subject
34	knowledge? To provide the insight of the bigger picture. What role do you think this
35	strong subject knowledge plays in teaching? As before.
36	From your experiences is the traditional model of teaching, where the teacher is central
37	source of knowledge effective? Please elaborate on your answer. Yes a student needs to
38	have a role model to learn from and be guided.
39	Indigenous knowledge, a knowledge that learners and teachers have from their
40	background, is part of the pedagogical knowledge of a teacher. Should teachers have
41	knowledge of culture and habits unique to South Africa or is teaching more effective when
42	the teacher is aware of the culture of the learners? Definitely – a teacher must understand
43	their student in order to teach effectively.
44	Classroom discipline, or control is this essential to effective mathematics teaching? Yes,
45	mathematics is a disciplined subject to study - takes dedication and focus.
46	In respect to technological knowledge, which includes the use of calculators, computers,
47	as well as black boards and overhead projectors, etc. What are your perspectives on this
48	topic at High school level in terms of the teachers' knowledge? Could be done without in
49	the big picture but essential in the end.
50	3. Any other views on what kind of approach from the teachers would lead to success in
51	school mathematics in the South African school.
52	Ultimately better qualified teachers who are paid better and encouraged to go into

53 54	teaching because they are better qualified, better teachers.
----------	--

	INTERVIEW 8
1	1. Reflecting on your own experience as a learner in grade 10 to matric in the South
2	African school, please answer the following:
3	Comment on different teaching styles in your school learning experiences and the effect
4	of these styles. I related very well to my mathematics teacher in Grade 11 and 12, but in
5	retrospect although he knew his subject he was not a good teacher – would bring
6	newspaper advertisements to school for plumbers, electricians, etc and tell those who
7	were battling to give up school and go and get a job, since they'd never be able to cope.
8	All other teachers used more or less only chalk-and-talk, but there seemed to be time for
9	discussion as well.
10	Explain your experiences of getting 'stuck' and what strategies were used for resolving
11	these type of problems. My only experience of being 'stuck' (it's a long time ago!) was in
12	finding factorisation difficult. I didn't understand the teacher I had in Grade 10; my dad
13	helped me persevere until eventually the penny dropped.
14	What was the role of the teachers' knowledge with the topics that you found easy or
15	hard? There were no teachers whose knowledge appeared to be inadequate, from my
16	limited learner perspective
17	How would you compare mathematics with other subjects, in terms of the kind of
18	work expected and how you approached the subject matter and tasks? There was never
19	any question that mathematics would take lots of time, that we would get homework
20	always, and have to work through many textbook exercises to get to grips with what we
21	were being taught.
22	What were your reasons for choosing mathematics at university? I thought it was
23	easier than many other subjects; I had originally intended doing biochemistry and
24	mathematics was a prerequisite.
25	2. As an accomplished research mathematician in order to help teachers trying to
26	teach mathematics, please answer the following:
27	What are your perspectives on the forms of knowledge that are involved in teaching
28	mathematics? I am referring to content knowledge, pedagogical knowledge and
29	technological knowledge. Content knowledge way beyond the level being taught, is
30	critical. Pedagogical content knowledge i.e. understanding why learners make the
31	mistakes they do, and how to help them understand, is also essential. This really comes
32	more from experience than from learning, although both are necessary. I have no
33	experience in teaching with the technology now being used, so can't comment in this
34	regard.
35	Explain your perspectives on whether these different forms of knowledge are of equal
36	importance in teaching mathematics or is there a hierarchical order to the forms of
37	knowledge? Content knowledge would rank highest, since all other forms are dependent
38	on that.
39	What is strong subject knowledge? What is the importance of strong subject
40	knowledge? What role do you think this strong subject knowledge plays in teaching? I
41	think Finland is a good example, where teachers are required to have master's degrees in
42	their subjects. There is no question then that they 'know' their subject, and make
43	connections between different aspects they're teaching. This also provides background as
44	to why learning certain apparently irrelevant things is important.
45	From your experiences is the traditional model of teaching, where the teacher is central
46	source of knowledge effective? Please elaborate on your answer. When I was at school
47	this seemed to be fine. Now I think there is much more valuable material on the internet
48	and elsewhere – it would be foolish not to teach learners how to evaluate its worth,
49	usefulness etc.
50	Indigenous knowledge, a knowledge that learners and teachers have from their
51	background, is part of the pedagogical knowledge of a teacher. Should teachers have
52	knowledge of culture and habits unique to South Africa or is teaching more effective when

53	the teacher is aware of the culture of the learners? This question is really part of a bigger
54	question: if teachers get to know their learners, and win their respect (which means
55	understanding and respecting them as real people, with real emotions, problems, etc.)
56	they will be more effective as teachers. Knowing a learner's culture is a small part of this
57	Classroom discipline or control is this essential to effective mathematics teaching?
58	Discipline is part of respect. Authoritarian control may facilitate sitting still and not being
59	disruptive but won't facilitate learning. Where there is mutual respect it should be
60	possible to allow for cooperation, collaboration, being considerate of others ... all of which
61	fall under the umbrella of classroom control and discipline.
62	In respect to technological knowledge, which includes the use of calculators, computers,
63	as well as black boards and overhead projectors, etc? What are your perspectives on this
64	topic at High school level in terms of the teachers' knowledge? Calculators are important
65	if they are managed well. Answers should be questioned in the light of reasonableness,
66	which requires a certain amount of content knowledge. They are useful to avoid drudgery,
67	provided learners know that they would still be able to find an answer if there was no
68	calculator. Overhead projectors are now largely obsolete and time-consuming. Whether a
69	teacher uses a blackboard or a whiteboard is probably a matter of choice, provided that
70	the mathematics being presented is logically sound. It's very helpful for learners to
71	understand the 'messy' nature of trying to solve a problem – one approach tried, didn't
72	work, try something else, see where it leads, etc ... but in all this proper symbols, proper
73	notation and terminology should be used.
74	3. Any other views on what kind of approach from the teachers would lead to success
75	in school mathematics in the South African school.
76	Confidence: Teachers need strong, broad and deep mathematical knowledge. This enables
77	them to be confident and allow room for questions. Teachers lacking in confidence don't
78	encourage questions and learners become mentally passive. Learners who are
79	encouraged to ask questions, are not in any way ridiculed regardless of the question, start
80	becoming more confident themselves.
81	Perseverance: The 21st Century embraces a culture of instant gratification, instant
82	success, instant satisfaction. Mathematical success requires time, perseverance, patience,
83	diligence. These characteristics aren't necessarily modelled at school or at home, and
84	without them learners cannot be mathematically successful.
85	

	INTERVIEW 9
1	1. Reflecting on your own experience as a learner in grade 10 to matric in the South
2	African school, please answer the following:
3	Comment on different teaching styles in your school learning experiences and the effect
4	of these styles.
5	There was basically one style when I was at school.
6	Explain your experiences of getting 'stuck' and what strategies were used for resolving
7	these type of problems.
8	Luckily I never got stuck on any topic in mathematics. Actually, in all subjects.
9	What was the role of the teachers' knowledge with the topics that you found easy or
10	hard? Our teachers were all well qualified and dedicated.
11	
12	How would you compare mathematics with other subjects, in terms of the kind of
13	work expected and how you approached the subject matter and tasks?
14	Because I enjoyed mathematics, I did it every day. In other subjects I simply studied when
15	it was necessary to do so, such as when exams/test were near.
16	What were your reasons for choosing mathematics at university?
17	It was because I liked it and I found that I was capable of doing it.
18	2. As an accomplished research mathematician in order to help teachers trying to
19	teach mathematics, please answer the following:
20	What are your perspectives on the forms of knowledge that are involved in teaching
21	mathematics? I am referring to content knowledge, pedagogical knowledge and
22	technological knowledge.
23	Unfortunately I am not au fait with the various teaching methods.
24	Explain your perspectives on whether these different forms of knowledge are of equal
25	importance in teaching mathematics or is there a hierarchical order to the forms of
26	knowledge?
27	I can sadly not make a well-informed response to this question. It appears to require
28	knowledge of mathematics education as a discipline.
29	What is strong subject knowledge? What is the importance of strong subject
30	knowledge? What role do you think this strong subject knowledge plays in teaching?
31	A teacher who has a sound grasp of a significantly higher level of mathematics than what
32	he/she teaches is in my view much sounder teacher, provided of course he/she is also well
33	grounded on teaching methodology.
34	From your experiences is the traditional model of teaching, where the teacher is central
35	source of knowledge effective? Please elaborate on your answer.
36	Yes. The elaboration is just that the traditional method worked, as many people who went
37	through it are testimony to that. There were of course those who did not make it, but I
38	think that has to do with their innate abilities, and that they would also not make it under
39	the so-called modern methods of teaching of mathematics.
40	Indigenous knowledge, a knowledge that learners and teachers have from their
41	background, is part of the pedagogical knowledge of a teacher. Should teachers have
42	knowledge of culture and habits unique to South Africa or is teaching more effective when
43	the teacher is aware of the culture of the learners?
44	In my view culture has nothing to do with understanding (or otherwise) of mathematics.
45	Classroom discipline or control is this essential to effective mathematics teaching?
46	It is necessary to maintain some degree of discipline, especially for the benefit of those
47	pupils who do not quickly grasp the intricacies of mathematics.
48	In respect to technological knowledge, which includes the use of calculators, computers,
49	as well as black boards and overhead projectors, etc? What are your perspectives on this
50	topic at High school level in terms of the teachers' knowledge?
51	I can't give an informed answer. Chalk and black board seem sufficient from the way I see
52	it. Calculators are ok; I don't think they hinder the understating of mathematics by pupils.

53	Yes it's good to know that $6 \times 7 = 42$, but there is no harm in using a calculator to find the
54	square root of 7.
55	3. Any other views on what kind of approach from the teachers would lead to success
56	in school mathematics in the South African school.
57	Teachers must identify the highly capable pupils and ensure they are not bored by the
58	slow pace required to cater for the slower pupils.

	INTERVIEW 10
1	I am grateful that this is a narrative study, as my knowledge about the theories of
2	teachers' knowledge, isn't sufficient
3	1. Reflecting on your own experience as a learner in grade 10 to matric in the South
4	African school, please answer the following:
5	Comment on different teaching styles in your school learning experiences and the effect
6	of these styles.
7	I can only recall two teachers of mathematics at school level; one was a spinster M..... at
8	Standard 2/Grade 4 and the other was M..... at Standard 9 and 10/Grade 11 and 12.
9	The reason I only remember them is because they were effective and authoritarian in
10	their approach. This means we were to be quiet and listen and then try to imitate them in
11	doing the problems on our own in silence. Invariably we got homework every day and I
12	would first do my maths homework before any other subjects to ensure that I was up to
13	date. In fact, I was afraid of getting behind since I did not think that I could catch up by
14	myself.
15	Explain your experiences of getting 'stuck' and what strategies were used for resolving
16	these type of problems.
17	I usually did not get stuck. Maybe because the problems were carefully selected from
18	"easy" to "hard" to "harder" so that one builds step-by-step on previous success. When I
19	did get stuck, I looked forward to the teacher's explanation. If that was not forthcoming, I
20	would stress a bit and try those problems again later. As a last resort, I would try to read
21	the textbook or given set of notes to see if I missed anything.
22	What was the role of the teachers' knowledge with the topics that you found easy or
23	hard?
24	The teacher was "king". I was amazed that M..... was good at everything; meaning
25	algebra, graphs, trigonometry and geometry. From my previous experiences with math
26	teachers, I almost expected them to fall flat on their faces when a certain topic had to be
27	dealt with.
28	How would you compare mathematics with other subjects, in terms of the kind of work
29	expected and how you approached the subject matter and tasks?
30	I actually enjoyed geography and biology a lot more at that stage. Again, because of good
31	teachers in these fields. I always felt that I could do the homework in these other subjects
32	and even be absent and still catch-up by myself if needed. You could approach math
33	problems from different angles, e.g. geometry and still be "right". In other subjects, e.g.
34	geography and biology certain theories and facts had to be explained in particular
35	preferred ways.
36	What were your reasons for choosing mathematics at university?
37	By trial and error. I first tried physics, botany, zoology and chemistry as a basic year in
38	Dentistry. Then tried geology, maths and applied maths because geography and biology
39	were not available as options. I had some success with maths and then decided to keep
40	going.
41	2. As an accomplished research mathematician in order to help teachers trying to teach
42	mathematics, please answer the following:
43	What are your perspectives on the forms of knowledge that are involved in teaching
44	mathematics? I am referring to content knowledge, pedagogical knowledge and
45	technological knowledge.
46	Sound content knowledge is indispensable in order to be a good maths teacher.
47	Pedagogical knowledge may certainly enhance/improve the abilities of the teacher to
48	effectively explain subtle concepts and harder concepts. Technological knowledge can
49	greatly improve the teaching of spatial concepts such as in geometry and graphs, for
50	instance Matlab is a very useful tool for both tedious calculations as well as improved
51	understanding of concepts through improved visualisation.
52	Explain your perspectives on whether these different forms of knowledge are of equal

53	importance in teaching mathematics or is there a hierarchical order to the forms of
54	knowledge?
55	My experience leads me to think that there is a hierarchical order and specifically: first
56	content knowledge; then pedagogical knowledge and finally technological knowledge,
57	although things may be a-changing.
58	What is a strong subject knowledge? What is the importance of a strong subject
59	knowledge? What role do you think this strong subject knowledge plays in teaching?
60	Strong subject knowledge is understanding all of the relevant concepts, derivations and
61	techniques or methods. Its importance is that it translates into occupations using maths,
62	everyday applications as well as modern technology without most people even realising
63	that maths is being used, for, e.g. mobile telecommunications. Students learn to trust that
64	the teacher knows their subject and are willing to participate in learning.
65	From your experiences is the traditional model of teaching, where the teacher is central
66	source of knowledge effective? Please elaborate on your answer.
67	Certainly at Grade 4 and at Grade 11 and 12 level the teachers were the central source of
68	knowledge and therefore I think they were effective. My maths marks were probably the
69	"best" in those grades.
70	Indigenous knowledge, a knowledge that learners and teachers have from their
71	background, is part of the pedagogical knowledge of a teacher. Should teachers have
72	knowledge of culture and habits unique to South Africa or is teaching more effective when
73	the teacher is aware of the culture of the learners?
74	I think teaching is more effective when the teacher is aware of the culture of the learners
75	and can even understand and perhaps speak their home language.
76	Classroom discipline, or control is this essential to effective mathematics teaching?
77	I believe classroom discipline or control is essential to effective maths teaching. This is
78	based on my own experience both as a student and a lecturer.
79	In respect to technological knowledge, which includes the use of calculators, computers,
80	as well as black boards and overhead projectors, etc. What are your perspectives on this
81	topic at High school level in terms of the teachers' knowledge?
82	In the hands of a good teacher, blackboard and chalk are sufficiently effective to teach
83	maths. Those teachers who have technology available to them should harness it
84	effectively as well.
85	
86	3. Any other views on what kind of approach from the teachers would lead to success in
87	school mathematics in the South African school.
88	

Appendix B:
Letter and Declaration of Consent



NORTH-WEST UNIVERSITY
YUNIBESITHI YA BOKONE-BOPHIRIMA
NOORDWES-UNIVERSITEIT
POTCHEFSTROOMKAMPUS

Private Bag X6001, Potchefstroom
South Africa, 2520

Tel: 018 299-1111/2222
Web: <http://www.nwu.ac.za>

**School of Natural Sciences and Technology for
Education**

Tel: 018 2991875
Fax: 018 2994238
E-mail: hercules.nieuwoudt@nwu.ac.za

To Whom It May Concern

January 2015

Dear Sir/Madam

MED STUDY IN MATHEMATICS EDUCATION: MRS SE LABUSCHAGNE (22982485)

The undersigned hereby declares that *Mrs Sheila Labuschagne* is a registered M.Ed. student in Mathematics Education at the North West University (NWU), working under my supervision. Her research proposal has already been approved Masters' and Doctoral Programme Committee and the Ethics Committee of the NWU has already granted her ethical clearance for conducting the research. For the purpose of her project, Mrs Labuschagne undertakes an explorative investigation titled: *Mathematical knowledge for secondary school teaching: exploring the perspectives of South African research mathematicians*. In this respect she will inform you in more detail and as required by you. Mrs Labuschagne is fully acquainted with the ethical guidelines that apply to research, in general, and to her project, in particular, and she will you inform you in this respect as well. Your participation in the project is appreciated and is voluntary; you can end your participation at any stage without any implication to you. All information will be treated confidentially. The identity of no person or institution will be revealed.

As supervisor of Mrs Labuschagne's project, I invite you to contact me should you any need more information regarding her study.

Kind regards

Prof HD Nieuwoudt (MSc, PhD) Supervisor

Informed Consent

Herewith I, the undersigned, declare that I am fully informed about the purpose of Mrs SE Labuschagne's investigation titled: *Mathematical knowledge for secondary school teaching: exploring the perspectives of South African research mathematicians*

I further declare that I am fully informed about the following ethical guidelines according to which the project will be executed:

- That my participation is voluntary
- That I can end my participation at any stage without any implication for me
- That all information I provide will be treated confidentially
- That my identity or the identity of the school/institution will not be revealed
- That I will receive oral feedback about the findings of the investigation.

I also declare that, in view of the above, I consent to participate voluntarily in the project and that I am willing to as best as possible share the information required from me by Mrs. S. Labuschagne

Participants initial and surname:

Participant's signature:

Date:

Appendix C:
Email Survey: Interview questions

I am a Mathematics teacher at the FET level, grade 10 to 12, and working on this Masters project part time. I am concerned about teaching of mathematics in South Africa and wondered how you, as a research mathematician, managed to do well in mathematics in this country.

The title of my dissertation is:

Mathematical knowledge for secondary school teaching: exploring the perspectives of South African research mathematicians

For practical reasons this interview is going to be non-verbal. I would appreciate it if you could answer the following questions in this light. I will ask more questions if it is deemed necessary.

There is a consent letter attached, your participation is voluntary and you can withdraw at any time.

In answering the questions focus on your perspectives regarding the following issues so that practical insights can be gleaned on the content of mathematical knowledge for secondary school teaching in South Africa.

1. Reflecting on your own experience as a learner in grade 10 to matric in the South African school, please answer the following:

Comment on different teaching styles in your school learning experiences and the effect of these styles.

Explain your experiences of getting 'stuck' and what strategies were used for resolving these type of problems.

What was the role of the teachers' knowledge with the topics that you found easy or hard?

How would you compare mathematics with other subjects, in terms of the kind of work expected and how you approached the subject matter and tasks?

What were your reasons for choosing mathematics at university?

2. As an accomplished research mathematician in order to help teachers trying to teach mathematics, please answer the following:

What are your perspectives on the forms of knowledge that are involved in teaching mathematics? I am referring to content knowledge, pedagogical knowledge and technological knowledge.

Explain your perspectives on whether these different forms of knowledge are of equal importance in teaching mathematics or is there a hierarchical order to the forms of knowledge?

What is strong subject knowledge? What is the importance of strong subject knowledge? What role do you think this strong subject knowledge plays in teaching?

From your experiences is the traditional model of teaching, where the teacher is central source of knowledge effective? Please elaborate on your answer.

Indigenous knowledge, a knowledge that learners and teachers have from their background, is part of the pedagogical knowledge of a teacher. Should teachers have knowledge of culture and habits unique to South Africa or is teaching more effective when the teacher is aware of the culture of the learners?

Classroom discipline or control is this essential to effective mathematics teaching?

In respect to technological knowledge, which includes the use of calculators, computers, as well as black boards and overhead projectors, etc.? What are your perspectives on this topic at High school level in terms of the teachers' knowledge?

3. Any other views on what kind of approach from the teachers would lead to success in school mathematics in the South African school.

Appendix D:

Ethics Clearance



NORTH-WEST UNIVERSITY
YUNIBESITHI YA BOKONE-BOPHIRIMA
NOORDWES-UNIVERSITEIT

Private Bag X6001, Potchefstroom
South Africa 2520

Tel: (018) 299-4900
Faks: (018) 299-4910
Web: <http://www.nwu.ac.za>

**Institutional Research Ethics Regulatory
Committee**

Tel +27 18 299 4849
Email Ethics@nwu.ac.za

ETHICS APPROVAL CERTIFICATE OF PROJECT

Based on approval by Ethics Committee of the Faculty of Education Sciences, the North-West University Institutional Research Ethics Regulatory Committee (NWU-IRERC) hereby approves your project as indicated below. This implies that the NWU-IRERC grants its permission that, provided the special conditions specified below are met and pending any other authorisation that may be necessary, the project may be initiated, using the ethics number below.

Project title: Mathematical knowledge for secondary school teaching: exploring the perspectives of South African research mathematicians.																															
Project Leader:	Dr HD Nieuwoudt																														
Student:	SE Labuschagne																														
Ethics number:	<table border="1"> <tr> <td>N</td><td>W</td><td>U</td><td>-</td><td>0</td><td>0</td><td>1</td><td>9</td><td>3</td><td>-</td><td>1</td><td>4</td><td>-</td><td>A</td><td>2</td> </tr> <tr> <td colspan="3">Institution</td> <td colspan="6">Project Number</td> <td colspan="2">Year</td> <td colspan="4">Status</td> </tr> </table>	N	W	U	-	0	0	1	9	3	-	1	4	-	A	2	Institution			Project Number						Year		Status			
N	W	U	-	0	0	1	9	3	-	1	4	-	A	2																	
Institution			Project Number						Year		Status																				
Status: S = Submission; R = Re-Submission; P = Provisional Authorisation; A = Authorisation																															
Approval date: 2015-02-26	Expiry date: 2020-02-25																														
Category:	N/A																														

Special conditions of the approval (if any): None

General conditions: While this ethics approval is subject to all declarations, undertakings and agreements incorporated and signed in the application form, please note the following: - The project leader (principle investigator) must report in the prescribed format to the NWU-IRERC: - annually (or as otherwise requested) on the progress of the project, - without any delay in case of any adverse event (or any matter that interrupts sound ethical principles) during the course of the project. - The approval applies strictly to the protocol as stipulated in the application form. Would any changes to the protocol be deemed necessary during the course of the project, the project leader must apply for approval of these changes at the NWU-IRERC. Would there be deviation from the project protocol without the necessary approval of such changes, the ethics approval is immediately and automatically forfeited. - The date of approval indicates the first date that the project may be started. Would the project have to continue after the expiry date, a new application must be made to the NWU-IRERC and new approval received before or on the expiry date. - In the interest of ethical responsibility the NWU-IRERC retains the right to: - request access to any information or data at any time during the course or after completion of the project; - withdraw or postpone approval if: - any unethical principles or practices of the project are revealed or suspected, - it becomes apparent that any relevant information was withheld from the NWU-IRERC or that information has been false or misrepresented, - the required annual report and reporting of adverse events was not done timely and accurately, - new institutional rules, national legislation or international conventions deem it necessary.
--

The IRERC would like to remain at your service as scientist and researcher, and wishes you well with your project. Please do not hesitate to contact the IRERC for any further enquiries or requests for assistance.

Yours sincerely

Linda du Plessis

Digitally signed by Linda du Plessis
DN: cn=Linda du Plessis, o=NWU,
ou=Vaal Triangle Campus,
email=Linda.duplessis@nwu.ac.za,
c=ZA
Date: 2015.07.13 10:47:45 +0200

Prof Linda du Plessis

Chair NWU Institutional Research Ethics Regulatory Committee (IRERC)

Mr. J. MOGOROSI

17696 Extension 11
Ikageng
Potchefstroom
2531

Cell: 084 293 5759

Date: 4 June 2016

Technical editing of bibliography: Dissertation

I hereby declare that I have edited the bibliography of the manuscript of Mrs S E Labuschagne. The completeness and accuracy of the manuscript remains the responsibility of the candidate.

A handwritten signature in black ink, appearing to read 'J Mogorosi', written over a dotted line.

J Mogorosi

This is to certify that the degree,
Magister Educationis in Mathematics Education
of
Sheila Labuschagne
has been edited by
Valerie Viljoen, Editing Excellence.

The complete dissertation has been edited and includes the following:

Pages i - iii

Chapter 1 – Problem statement and programme of investigation

Chapter 2 – Knowledge for proficient teaching of Mathematics

Chapter 3 – Research design and methodology

Chapter 4 – Analysis of data

Chapter 5 – Summary, conclusions and recommendations

Appendix A – E.

Date: 9 June 2016

