Constructing a modelling-based learning environment for the enhancement of learner performance in Grade 6 Mathematics classrooms: A design study

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DEDICATION

This work is dedicated to the children of the Northern Cape. May this Gazebo-Model assist in tapping their potential, today, tomorrow and always.

And

My Mother, Joey Van Schalkwyk, My late Father, John Martin van Schalkwyk, and late Spiritual Brother Glen Richard Phike

DECLARATIONS

A: Language Editor

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For: Frank van Schalkwyk

Title: Constructing a modelling-based learning environment for the enhancement of learner performance in Grade 6 Mathematics classrooms: A design study.

This took place during October/November 2013.

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CHECKING OF BIBLIOGRAPHY

Hereby I declare that I have checked the technical correctness of the Bibliography of the PhD-thesis of Mr Frank van Schalkwyk according to the prescribed format of the Senate of the North-West University.

Yours sincerely

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Hiermee word bevestig dat Statistiese Konsultasiediens die data verwerk het en ook betrokke was by die interpretasie van die resultate. Enige opinie, bevinding of aanbeveling uitgespreek in die dokument is egter die van die outeur en Statistiese Konsultasiediens van NWU (Potchefstroomkampus) neem nie verantwoordelikheid vir die statisties korrektheid van die gerapporteerde data nie.

Vriendelike groete

Prof. H.S.Steyn (Pr. Sci. Nat.)
Statistiese Konsultant

ABSTRACT

The purpose of this study is to focus on constructing a modelling-based learning environment to improve learner performance in grade 6 mathematics classrooms. The purpose emanates from the continued poor performance of learners in mathematics at different school levels, especially grade 6. The teaching and learning of mathematics is explained from an ontological point of departure, focussing on constructivist paradigms. Different types of constructivism are discussed with special attention to the school mathematics domain. The learning, problem based learning, problem solving and learning environment are key components in the discussion. A theoretical perspective on the design of modelling as a powerful learning environment in primary schools mathematics classrooms is provided. Focus is placed on the applicability of the modelling-based learning environment on the South African mathematics curriculum and on study orientation as a key component to help develop an understanding of why learners perform or do not perform in mathematics.

A mixed method research design, in which quantitative and qualitative are combined to achieve the outcomes of the research problem, is chosen for this research study project to provide a purposeful research framework. The findings of the research include not only learners' improvement in dealing with non-routine, mathematical word problems but also in general-routine, mathematical word problems. A second finding shows that the overall SOM pre/post/retention showed good reliability, acceptable construct validity, good practical significance, and large effect but had low to medium effect in individual fields. The univariate analysis for the Crossover design used indicated that the problem solving field had statistical significance and practical significance, and the study milieu and mathematical confidence field might have statistical significance and practical significance. The third finding provided evidence concerning teacher administration, teacher and learner interaction, assessment and homework. The findings from the quantitative and qualitative data-analysis and interpretations, and literature review, guided the researcher in proposing a construct for a modelling-based learning environment as a means to improve learners' mathematics performance in grade 6 mathematics classes in the John Toalo Gaetswe (JTG) District.

The contribution that this study makes is to propose a construct for a modelling-based learning environment to improve learner performance in grade 6 mathematics.

KEY WORDS

Constructivism;

Learning;

Problem solving;

Problem based learning;

Powerful learning environment;

Modelling;

Modelling-based learning environment;

Study orientation in Mathematics;

Mixed method research; and

Crossover design

OPSOMMING

Konstruksie van ‘n modelleringsbaseerde leeromgewing vir die verbetering van leerderprestasie in Graad 6-Wiskundeklasse: ‘n ontwerpstudie

Die doel van die studie is om te fokus op die konstruksie van ‘n modelleringsbaseerde leeromgewing ten einde leerderprestasie van graad 6 wiskundeklaskamers te verbeter. Die doel spruit uit die volgehoue swak prestasie van leerders in wiskunde op verskillende skoolvlakke, met spesifieke verwysing na graad 6. Die onderrig en leer van wiskunde word verduidelik vanuit ‘n ontologiese vertrekpunt met die fokus op ‘n konstruktivistiese paradigmas. Verskeie tipes konstruktivisme word bespreek met spesifieke verwysing na die veld van skoolwiskunde. Die leer, probleemgebaseerde leer en leeromgewing is hoofkomponente van die bespreking. ‘n Teoretiese perspektief op die ontwerp van modellering as ‘n kragtige leeromgewing in primêre skoolwiskundeklaskamers word voorsien. Die fokus word geplaas op die toepaslikheid van die modelleringsbaseerde leeromgewing op die Suid-Afrikaanse wiskundekurrikulum en op studieoriëntering as ‘n sluitelkomponent ten einde by te dra tot die ontwikkeling van ‘n begrip waarom leerders presteer al dan nie in wiskunde.

‘n Gemengde navorsingsmetode, waarin kwantitatiewe en kwalitatiewe metodes gekombineer is ten einde die doelwitte van die navorsingsprobleem te bereik, is gekies vir hierdie navorsingstudieprojek ten einde ‘n doelmatige navorsingsraamwerk te voorsien. Die navorsingsbevindinge sluit die verbetering in leerders se hantering van nie-roetine wiskundige woordprobleme sowel as algemene-roetine wiskundige woordprobleme in. ‘n Tweede bevinding toon aan dat die algehele SOM voor/na/retensie goeie betroubaarheid, ‘n aanvaarbare konstrukgeldigheid, ‘n goeie praktiese beduidenheid en groot effek toon, maar dat dit lae tot medium effek in individuele sfere getoon het. Die eenveranderlike analiese van die kruisingsontwerp wat gebruik was het getoon dat problemoplossingsveld statistiese beduidend is en ook praktiese beduidend is, en dat die studie milieu en wiskundige houdings moontlik statisties and prakties beduidend kan wees. Die derde bevinding verskaf bewyse van onderwyseradministrasie, onderwyser- en leerderinteraksie, assessering en tuiswerk. Bevindinge van die kwantitatiewe en kwalitatiewe data-analises en –interpretasies, sowel as literatuuroorsigte, het die navorser gelei in die voorstel van ‘n konstruk vir ‘n modelleringsbaseerde leeromgewing as ‘n wyse waarop leerders se wiskundeprestasie in graad 6 wiskundeklasse in die JTG-distrik verbeter kan word.

Die bydrae wat hierdie studie lewer is om ‘n konstruk voor te stel vir ‘n modelleringsbaseerde leeromgewing ten einde leerderprestasie in graad 6 wiskunde te verbeter.

SLEUTELWOORDE

Konstruktivisme;

Leer;

Probleemoplossing;

Probleem-gebaseerde leer;

Kragtige leeromgewing;

Modellering;

Modelleringsbaseerde leeromgewing;

Studie Orientasie in Wiskunde;

Gekombineerde metode navorsing; en

Kruisingsontwerp

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CHAPTER 1:

INTRODUCTION, PROBLEM STATEMENT, AIMS AND PLAN OF RESEARCH

The first Chapter provides an introduction to the research study. Figure 1 gives an outline of the Chapter 1

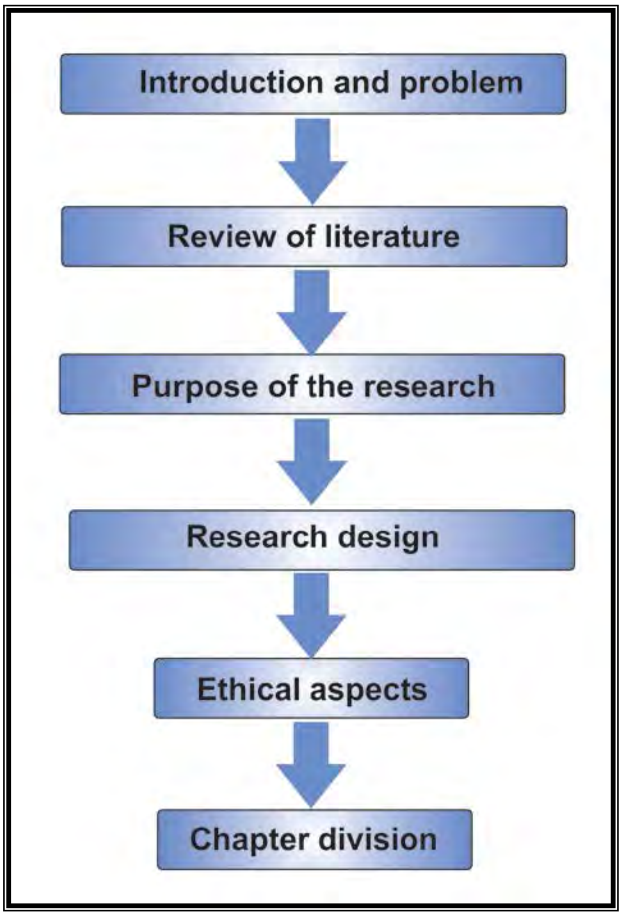


Figure 1: Outline of Chapter 1

1.1 INTRODUCTION AND THE PROBLEM STATEMENT

For the logical development of the introduction, the researcher initiated the discussion with a question, followed by a statement of the importance of mathematics (i.e. maths) as a key contributor to knowledge and skills development. The reasons for embarking on this study are highlighted through the background to the study. The background includes reflections on both the personal experience as a mathematics educator and trainer of educators, and location of gaps in previous research. Based on the background discussion, the researcher formulated a problem statement, highlighting the research questions, and the aims of this study. A literature review and methodology follows the introduction and problem statement providing a proper theoretical discussion framework and the research design for the study (Mouton, 2001).

"What would a world devoid of mathematics be?"

The response to such a question posed would possibly elicit an infinite number of discussions. A probable common response might be equivalent to the answer of "division by zero." These responses and discussions will depend on the person's ontology. Mathematics is a discipline, a way of life, a human right, which by its nature is part of most sectors and all societies across the globe (NDE, 2002:4; NDE, 2003:19). Most researchers agree that school mathematics forms a central component of the future of any country (Christensen, Stentoft & Valero, 2008:1; Du Toit, 1993:241; Ralston, 1988:33; Strauss, 1990:1; Schoenfeld, 1990: 1).

Background to the study

The opinions from mathematics teachers, past and present, seem to place the blame for poor learners performance on "lower grade teachers, who don't teach the basics right." These basics, according to those mathematics teachers, refer to "learners' ability to add and subtract integers, to work with fractions, and to illustrate an understanding of measurement, thereby to master different cognitive levels, concepts" (Recorded with the permission of involved teachers by this researcher).

Research evidence of poor performance seems to ingrain such views. Poor performing schools do spend time but not enough time on classroom lesson activities such as: practicing numerical operations without a calculator; working on fractions and decimals; interpreting data in tables, charts, or graphs and writing equations and functions to represent relationships, have been poor (Mullis *et al*., 2004). The activities are mainly in the knowing cognitive domain. When problems such as: 7 + 9 = ___ + 8, were given to learners at different times and different schools as an informal assessment, the response of most learners was 16. When asked how they arrived at the answer of 16, their reaction was; “7 + 9 = 16, look I can show you on the number line, and using my fingers, an equal sign means giving an answer, isn’t it?”

The above responses agree with the finding by researchers such as Verschaffel, Greer, De Corte, Carpenter, Romberg and Cobb, inter alia, which shows that the teaching process is failing learners to help them understand the mathematics that they engage in (Verschaffel *et al*., 2000:ix ; Carpenter & Romberg, 2004).

A common belief held by educators informally questioned by this researcher, highlights; a)that learners do not understand the steps to solve problems, b)that learners can’t link their knowledge from the classroom to their outside environment and c)that some can do some routine problems but struggle with word problems. Reports such as the Trends in International Mathematics and Science Study (TIMSS) (Mullis *et al.,* 2004) and the systemic evaluations (Hindle, 2005:2) seem to concur with the perceptions of teachers which suggest that mathematics in South African schools is in a terrible situation. Learners are not tuned in to learning (Mahlomaholo, 2012a:3). South Africa’s grade 3/4 and grade 8/9 learners have the lowest testing scores of all the TIMSS participating countries for the period1999 to 2003 (Blain, 2007). At provincial level, focusing on the Northern Cape, in the systemic evaluations the average for mathematics was below 33% in the Intermediate Phase for 2004 and 2005 (NCDE, 2006: 13). The systemic evaluation study shows that the implementation and organization of classroom environment plays a key role in the performance of learners (Hindle, 2005).

The lack of learners relating what they have learnt to their daily lives, lack of learners explaining their answers and lack of learners deciding on procedures for solving complex problems were highlighted as problems in the TIMSS (Mullis *et al.,* 2004). Comments such as “these outcome based education (OBE) learners don’t know their tables and they can’t work with negative numbers, etc." are still very much alive in hallways of schools. TIMSS reported on the performance of learners on the different cognitive levels. Knowing, applying and reasoning are the three cognitive domains in maths achievement. The knowing domain involves learners' ability to know facts, procedures and concepts (Mullis *et al*., 2004). The lack of a specific domain of knowledge and skills (e.g. concepts, formulas, algorithms, problem solving) and shortcomings in the heuristic, meta-cognitive and affective aspects of mathematical competence are contributing factors in the handling of mathematical content application (Verschaffel *et al*., 1999:196; Altun & Arslan, 2007:1). During lesson times, little to no emphasis is spent on mathematical, problem solving activities in poor, performing countries (Mullis *et al.,* 2004). In most of the lessons, teachers spend a greater percentage of time to practice adding, subtracting, multiplying, or dividing (Mullis *et al.,* 2004).

The response of learners to non-routine questions such as "How old is the Shepherd question?" provide evidence of a lack of understanding mathematics in context (Verschaffel *et al.,* 2000:3; Van Dooren *et al.,* 2007; Verschaffel *et al.,* 1999; Chavez, 2007). Verschaffel, De Corte and Lasure, cited in to Little and Jones (2007:48), found that children fail to take cognisance of a realistic context in answering word problems. The word problem section of the Intermediate Phase mathematics assessment (national and international), were answered extremely poorly in the public school domain (Mayatula, 2008; Khoele, 2008:4; NCDE, 2006:13; Mullis *et al.,* 2004). South Africa attained 32% which is poor compared to the international average of 61% (Mullis *et al.,* 2004). In 2007, the numeracy scores for the systemic evaluation was 34% (Khoele, 2008:4), which shows that performance of learning in numeracy has not improved since. The Systematic Evaluation Report highlighted that learners were not able to: solve routine problems involving whole numbers and decimals in context, do calculations that involve measurement, read and interpret data presented in graphs, do basic operations nor identify geometric shapes and space (Hindle, 2005:2; NCDE, 2006; Carnoy & Chisholm, 2008). Lack of mathematical performance seems to be strongly correlated to the skill shortage in South Africa (Pandor, 2007:3; SA JIPSA, 2006; MacGregor, 2008; Ronis, 2008:vii). Learners' poor performance seems to be a result of the traditional education system's inability to cope with technological advancements. The HSRC report findings highlighted that most learners could not: do whole number story problems using basic operations, order any set of three or more whole numbers, write and solve number decimal problems nor solve simple linear equations (Carnoy & Chisholm, 2008).

"Mathematics is only for the clever learners." This common misconception is a view held by many mathematics teachers, parents, and members of society at large. Carpenter asserts that there is a misconception that mathematics can only be done by a select group of learners with special abilities (Carpenter & Romberg, 2004:3). The TIMSS study also refers to the influence of attitude towards mathematics. The performance of learners is influenced by their attitudes and beliefs towards mathematics (Altun & Arslan, 2007:2; Mullis *et al.,* 2004). The HSRC report found that the most significant factors which influenced learner performance in mathematics were; the class environment, factors concerning feedback from teachers, students' prior cognitive ability, the instructional quality, students' disposition to learning, and how questions were managed (Carnoy & Chisholm, 2008).

The purpose of this study is to construct a modelling-based learning environment for the enhancement of learner performance in grade 6 mathematics classrooms.

1.2 REVIEW OF RELATED LITERATURE

Learner continued poor performance in mathematics, necessitates a deep-rooted investigation into teaching and the learning of school mathematics. The overwhelming educational research in the recent years proposed a constructivist paradigm as a means to address the lack of functioning on cognitive levels required to perform (Cobb, 1988:87; Verschaffel, *et al*., 2000:3; Van Dooren *et al.,* 2007; Verschaffel *et al.,* 1999; Carpenter & Romberg, 2004; Carpenter *et al.,* 2004). The constructivist classroom is signified by engagement in higher cognitive thinking, learning and teaching (Cobb, 1988: 90; Yager, 1991: 55; Verschaffel *et al.,* 2000). Learning is seen as a "constructive, cumulative, self regulatory, goal-oriented, situated, collaborative, and individually different process of knowledge building and meaning construction" (De Corte, 2000:8; De Corte, 2004). The learners partake actively in their mathematical development. Teachers in constructivist classrooms must engage learners inside and outside classrooms, optimising the real life context problems (Cobb, 1988:87). The learning environment forms a key to effective teaching and learning (Howie,1997:19).

The interventions must be aimed at creating socio-mathematical norms, which will result in a classroom learning environment conducive to the development of learners' appropriate beliefs about mathematics and mathematics learning and teaching (Verschaffel *et al.,* 2000). These norms include learners being actively involved and taking ownership and accountability for their learning, while the teacher takes the responsibility of creating a modelling, learning environment. The essential elements for a modelling-based learning environment focuses research on the understanding of learning, the understanding of problem solving, the understanding of a learning environment and the understanding of modelling. Learners need to be given problems which would provide them with the opportunity to develop in learning with understanding, thereby functioning on a higher cognitive level. Problem-based learning provides such an opportunity (Uden & Beaumont, 2006).

For learning to be effective, there needs to be a constructively interactive learning environment for learners (Verschaffel *et al.,* 2000). Problem-based learning has 'problems' at its core. Problem solving as proposed by Polya is the essential for solving (messy problems) which they face in real life. Powerful learning environments function within the constructivist paradigm (Van Petegem *et al.,* 2005:2; De Corte *et al*., 1996). The modeller must, in stage one of the five stage model, build a mental representation of the problem (Verschaffel *et al.,* 2000). The learning processes need to be realised in a learning environment which is the address of the culture of meaningful contexts (De Corte, 2000:8). The teacher also needs to realise the external regulation of knowledge and skills gradually.

With the introduction of mathematical modelling, self regulatory learning activities such as analysing the problem, monitoring the solution process and evaluating its outcome, are strongly developed (Verschaffel *et al*., 1999). There are different types of models,namely; *Look-alike models, Function-alike models, Descriptive models, and Explanatory models (*Carpenter & Romberg, 2004:6). The cyclic modelling process, involves understanding the event, model construct, evaluate, interpretation and model revision (Carpenter & Romberg, 2004:16). A modelling-based learning environment will enable learners to be actively involved in constructing solutions to non-routine and routine problems. Learners who engage in modelling practices, develop modelling and critical skill, and become more socially inclined (Verschaffel, *et al.,* 2000: 172; Romberg, 2001).

In South Africa, the mathematics learners must acquire certain knowledge, skills, values and attitudes as captured in the National Curriculum Statement (NDE, 2002: 4). Mathematics should enable learners to ascertain the connection between mathematics as a discipline and the use of maths to solve real-world context problems (DoE, 2008:11). The learners' performance in maths is directly related to the study orientation of learners towards the subject (Steyn & Maree, 2003: 50). According to Kapetanas and Theodosios (2007: 98) researchers confirm that a relationship between attitude, belief, mathematical learning and mathematical performance does exist. The study orientation in mathematics (SOM) questionnaire is, according to Steyn and Maree (2003: 50), a significant predictor for mathematics conceptual understanding and performance.

The above mentioned problems prompted this researcher to ask the following critical research question:

"What are the essential elements of a modelling-based learning environment which enhance learner performance in grade 6 mathematics classrooms?"

This critical research question suggests that the following secondary questions should be addressed:

- What are the critical constitutes of the modelling-based learning environment?
- What are the building blocks for implementing a modelling-based learning environment?
- How does a modelling-based learning environment influence the learners' solving of non-routine problems in grade 6 mathematics classrooms?
- How does a modelling-based learning environment influence learners' study orientation in grade 6 mathematics classrooms?

1.3 RESEARCH AIM

The study primarily aims to construct a modelling-based learning environment for the enhancement of learner performance in grade 6 mathematics classrooms.

1.3.1 Research objective

The following objectives have been identified in order to realise the primary aim:

a. To determine what the critical constitutes of the modelling-based learning environment are.
b. To determine what the building blocks for implementing a modelling-based learning environment are.
c. To determine how a modelling-based learning environment influences the learners' solving of non-routine problems in grade 6 mathematics classrooms.
d. To determine how a modelling-based learning environment influences learners' study orientation in grade 6 mathematics classrooms.

1.4 METHOD OF RESEARCH

1.4.1 Literature review

A comprehensive literature review precedes the planning meetings with all relevant stakeholders in the research project and implementation of the phase model.

The following key words did inform the search process:

Learning; Constructivist; Modelling; Problem solving; Problem-based learning; Powerful learning environment; study orientation in mathematics (SOM)

This researcher explored the following databases available to obtain the most relevant and recent literature regarding the research project:

- North West University Libraries in Potchefstroom campus

- ERIC (Published by SilverPlatter)
- International ERIC (The Dialog Corporation)
- Wilson Education Abstracts and Education Index (Published by SilverPlatter)
- Internet service using Google Advanced and Google Scholar
- CompuMath Citation Index (Published by: Institute for Scientific Information)
- Social sciences index (SSI)
- EBSCOhost (Premier Search).

1.4.2 The experimental design

The pragmatic paradigm enables research to be both quantitative and qualitative (i.e. research follows a mixed method design). Mixed method research enables the use of quantitative and qualitative methods, to address research questions posed (see par 4.31). A quantitative-qualitative-method approach was used in a multi-phased process. The quantitative part includes summative pre- and post-retention test and pre- and post-retention SOM, and the qualitative part includes video-recordings and systematic observations of the classroom. This research is divided into five phases, in a crossover design model (See Table 4.1).

1.4.3 Population and sample

The teacher and learner population is from the grade six mathematics classrooms in the John Taole Gaewetsi (JTG) district in the Northern Cape Province. The sample of learners consists of three experimental sixth grade classes and three comparable control classes. The selected classes belong to different gender mixed (boys and girls) elementary schools in the JTG district. These schools are poverty-stricken rural schools. The six teacher participants assisted this researcher with dissemination of the questionnaires and tests in their respective schools. This researcher, with the help of the statistical consultation service (SCS) of NWU, utilised descriptive statistics and inferential statistics to arrive at an answer to the research question.

1.4.4 Measurement instruments

This researcher used different instruments to evaluate the implementation and effects of the experimental learning environment.

*Instrument 1 (*Pre-test*,* Post-test *& Retention test)*

A summative pre- and post-retention test (adapted) consisting of 8 matched pairs of word problems. The sixteen word problems given to learners containing eight standard items which are routine questions and eight problem items, which are non-routine questions (see par 4.4.5). The primary aim of the test is to answer the research aim 1.3.1c (see par1.3.1c). The classroom teacher administered the test.

Instrument 2 (pre-and post- retention SOM questionnaire)

Instrument 2 used is the study orientation in mathematics (SOM) questionnaire (Steyn & Maree, 2003). The SOM questionnaire consists of seventy six questions or items. These items address the five fields, namely; study attitude (SA), mathematics confidence (MC), study habits (SH), problem solving behaviour (PSB), and study environment (SE) of the study orientation in mathematics for the Intermediate phase (see par 4.4.5). The primary aim of using the SOM questionnaire is to answer the research aim 1.3.1d (see par 1.3). The SOM questionnaire is set up to fit the South African context.

Instrument 3 (video recording and observation notes)

This researcher made video recordings and took observation notes on a continuous base. Reflection meetings to highlight achievements and challenges will be held at the end of each day when classes are concluded. Records of reflection meetings, the classroom notes and the video recording will be kept. The video recording and taking of observation notes occurs in phase 2 and phase 4.

1.4.5 Data analysis

With the help of NWU SCS descriptive and inferential statistical techniques were used to organise, analyse, and interpret the quantitative data for both the pre-and post-retention test instrument and the pre-and post-retention SOM questionnaire (see par 4.4.11). For the analysis of the qualitative section of the research, this researcher used transcripts from video recording and field notes of class visits conducted (see par 4.4.11).

1.5 RESEARCH ETHICS OBSERVED

School principal, teacher and learners were informed about the purpose of the research. All participants were given the assurance of anonymity of the participation in research and the confidentiality of the results. This researcher followed the guidelines as given by the Association of Social Anthropologists (ASA), to ensure that proper consent and no deception of probable participants is recorded (Bailey, 2007:15; Flick, 2006:45). Researcher gave special attention to the control group explaining their role in the whole project and asking for their consent.

Permission needed

Researcher acquired permission from DoE institution, at different levels. Permission were requested from the teachers and from SGB of the participating schools and parents of learners involved in the study.

1.6 CHAPTER FRAMEWORK

This part will summarise the structure and content of the report.

Chapter 1:

Introduction, problem statement, aims and plan of research

In Chapter 1, the research is introduced, and description of the problem statement is followed by a brief literature review. The research aims and key questions are given and the brief description of the methodology is given as a means to provide possible answers to research questions.

Chapter 2:

Mathematical learning within the constructivist paradigm

In Chapter 2, the focus is placed on understanding learning in the constructivist paradigm, problem solving, and learning environments.

Chapter 3:

A modelling-based learning environment in mathematics classrooms

In Chapter 3, the focus is, mainly, on understanding the design for modelling as a powerful learning environment in primary schools mathematics classrooms. Special attention is given to study orientation as a key component to help develop an understanding of why learners perform or do not perform in mathematics.

Chapter 4:

Research design and methodology

Chapter 4 provides the research design and a methodological perspective on achieving the objective set out for this study. This chapter look into the methodology used in gathering data for this study.

Chapter 5:

Results, Data Analysis and Interpretation

This Chapter 5 presents the results, discussion on the data analysis of this study.

Chapter 6:

Summary, findings, recommendations and limitations

This Chapter 6 present a summary of the research, findings, recommendations for further research and limitations encountered during this study.

CHAPTER 2:

MATHEMATICAL LEARNING WITHIN THE CONSTRUCTIVIST PARADIGM

This Chapter 2 aims to provide literature background to acquire a better understanding of the ontological foundation of the research project. The main focus is placed on the learning and powerful learning environments in the constructivist paradigm. The Figure 2 below provides a schematic outline of the key areas addressing mathematical learning within the constructivist paradigm.

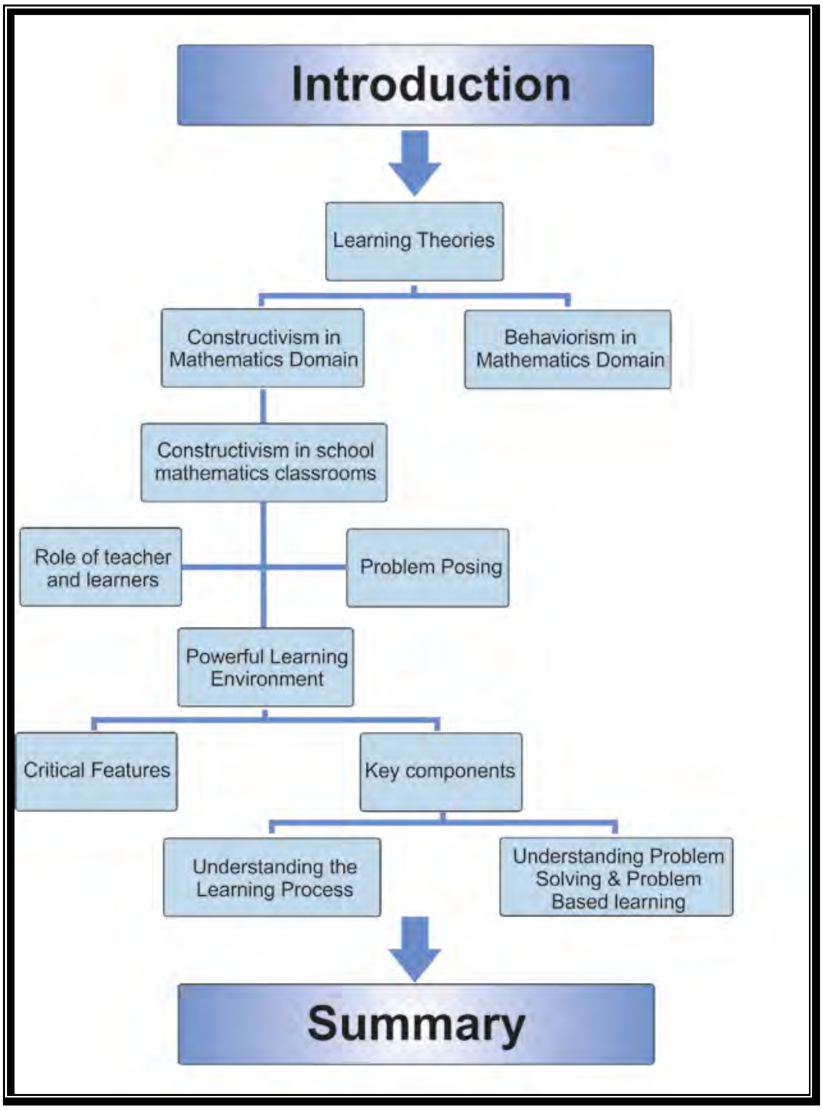


Figure 2: Outline of Chapter 2

2.1 INTRODUCTION

The continued poor performance of learners, as highlighted in the problem statement (see par 1.2), necessitates a focal look at leaning, learning environments and problem solving to assist in addressing the poor performance of learners in school mathematics.

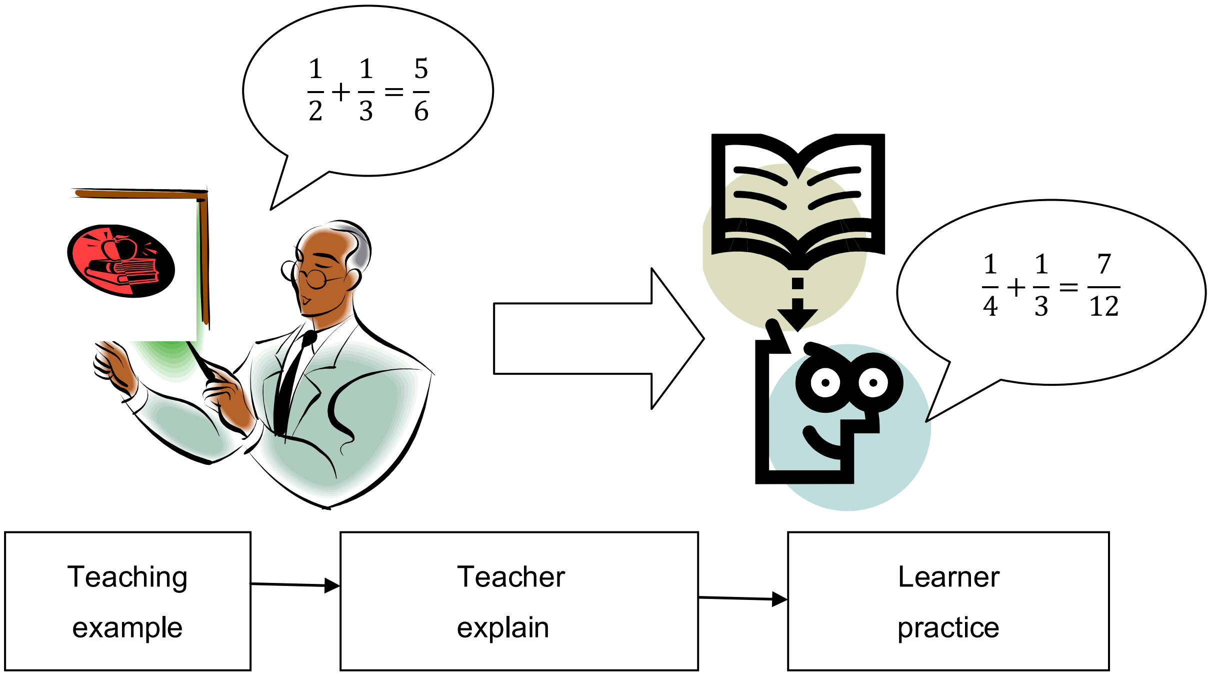
In this Chapter, the researcher engages with learning theories, with a specific focus on constructivist learning within the mathematics domain. This literature review provides a theoretical perspective on learning from the constructivist perspective, the learning environment and its critical features and building blocks. Focus will be placed on building block for a powerful learning environment, which involves the learning process, problem based learning and problem solving.

2.2 LEARNING THEORIES

What learning is, how it occurs, what processes and products are involved in learning, and other issues pertaining to learning have for centuries fascinated the research fraternity, seen from the great amount of literature that has been published (Huetinck & Munshin, 2004:38; Van Rooyen & De Beer, 2007:47; Cobb, 2007:3). Various researchers focused in the area of learning, which led to the development of many theories regarding learning (Scales, 2008:57). Notwithstanding the numerous theories regarding learning, focus will only be placed on two very prominent theories, which include the behaviourist, and the constructivist theory of learning (Huetinck & Munshin, 2004:38; Van Rooyen & De Beer, 2007:47; Cobb, 2007:3). The reason for focusing mainly on the two mentioned theories is because South Africa before 1994 followed a traditional education system which was teacher centered and examination focused (Pretorius, 1998:1). After 1994, the education system transformed from the Traditional Education System, which was teacher centered and examination focused to an outcomes based education (OBE) system which is more learner centered and continues an assessment focus implemented in 1998 (Van der Horst & McDonald, 1998:5).

2.2.1 Behaviourist learning in the mathematics domain

In the behaviourist approach, the teacher provides a set of stimuli and reinforcements that are likely to get students to emit an appropriate response (Scales, 2007:59; Huetinck & Munshin, 2004:39; Kiviet & Du Toit, 2007:47; Van Rooyen & De Beer, 2007:47; Nieuwoudt, 2003:18).



(Adapted from Nieuwoudt, 1998: 55, 87)

Figure 2.1: Linear one dimensional process

The above Figure 2.1 illustrates the teachers' and learners' actions in a behaviourist classroom setting. The teacher gives his class an example of addition of fractions same numerator and different denominators. The teacher explains the algorithm of adding fractions with the same numerator and different denominators to his class. Learners are then given similar problems with the same numerator and different denominators to practice. The child's mind is seen as a blank slate (Huetinck & Munshin, 2004:39). The reinforcement of the observable stimuli–response connection, through timely reward, was taken as the sole mechanism through which all learning including mathematical learning occurs (Goldin, 2002:193). Behavioural learning according to Uden and Beaumont (2006:3) can be described as a change in the observable behaviour and performance of the learner. Scales (2007:59) agrees with this stating that learning occurs in response to external stimuli. A stimulus can be regarded as an external or internal factor which stimulates an organism, which results in action (Scales, 2007:59). The approach is very effective when the teacher wants learners to replicate certain behaviour which the teacher feels is essential for the learners' development (Kiviet & Du Toit, 2007). The teacher sits with the assumption that they can transfer knowledge and understanding to their learners (Libman, 2010: 2-3).

Learning is defined as a persistent change in human performance potential (Uden & Beaumont, 2006:3). The effectiveness of learning depends on the amount of change that occurred (Dossey *et al.,* 2002). In the behavioural environment the learner is reactive to the environmental conditions rather than taking an active role in discovering the environment (Uden & Beaumont, 2006:5; Huetinck & Munshin, 2004:39). The teaching strategy mainly followed is the explanation followed by practice worksheets (Nashon, 2006:2). The teacher presents the material, works a few examples, gives learners time to try a few examples, then gives learners similar problems as homework (Nieuwoudt, 1998:87). Learners were not asked to reason the development of the concept (Dossey *et al.,* 2002:37). The reward system, called positive reinforcement as introduced by Skinner (Scales, 2007:60), the giving of tokens for achievement is central in the behaviourist classroom (Dossey *et al.,* 2002:37). The performance of learners in achievement tests defined the learners' learning success (Huetinck & Munshin, 2004:40).

The pedagogical principles on which much of behaviourist instruction such as competition, management, group aptitudes, fixed content body of knowledge, are based (Schoenfeld, 1988:4), seems inadequate, and assumes that learners absorb what has been taught. Not surprisingly, Romberg and Carpenter according to Schoenfeld, (1988:4) called the traditional model an absorption theory of learning. Learners in the behaviourist paradigm struggle to find the correct algorithm to apply to solve real-life problems (Taylor, 2003:2). This approach does have limitations if the teacher wants to engage learners in understanding, problem solving, modelling, synthesis, generalisation, justification, the application and use and the ability to use information in a new situation (Yager, 1991:55).

Learning theories have undergone a radical shift due to the progress made in the cognitive sciences (Schoenfeld, 1988:4). For the purpose of this research, the researcher will place greater focus on the constructivist learning theory within the mathematics domain. Constructivist learning is based on the assumption that learners construct knowledge as they make sense of their experiences (Uden & Beaumont, 2006: 18; Ronis, 2008: 28). Each learner constructs their own ontology – their own way in which they view the reality.

2.2.2 Constructivist learning within the mathematics domain

Although there are various types of constructivism, it appears to be divided into two main types. Constructivism can be divided into cognitive constructivism and the social constructivism. The cognitive constructivists as seen by Piaget (Sjober, 2007:2) focus on how the individual learner understands constructs, in terms of developmental stages and learning styles while social constructivism as seen by Vygotsky (Sjoberg, 2007:2) focus on how meaning and understanding grows out of social interactions (Atherton, 2011:1; Cobb & Yakel, 1996:209; Carpenter, Dossey & Koehler, 2004). The social constructivist approach has evolved from an initial psychological constructivist position (Gupta, 2008: 382; Cobb & Yakel, 1996:209; Carpenter, Dossey & Koehler, 2004; Ernest, 1993). Vygotsky viewed social interaction as a key role-player in cognitive development (Gallaway, 2001:1).

Constructivism describes the 'how' activity of learning mathematics, by making the active involvement and participation central to the theoretical framework (Confrey & Kazak, 2006:306; Huetinck & Munshin, 2004:43). Learning is a result of mental construction of new information connected to what is already known (Scales, 2008:61). Piaget's genetic structure, according to Steffe and Kieren (1994:69), has shown that concrete operational stage children are able to learn the fundamental structures of mathematics. Research also suggests that the Piaget studies were devoted to investigating the readiness of learners to learn mathematics (Can, 2009; Gupta, 2008: 382; Libman, 2010: 2; Steffe & Kieren, 1994:74; Nieuwoudt, 2003:30). There seems to be different definitions of constructivism. These definitions depend also on the form of constructivism. Constructivism is a "philosophy of learning founded on the premise that, by reflecting on our experiences, we construct our own understanding of the world we live in" (Uden & Beaumont, 2006:10; Kiviet & Du Toit, 2007:49).

Social constructivists focus on the geneses of mathematics rather than its justification (Ernest, 1993:43). With the social constructivist, the focus is placed on knowledge dissemination amongst individuals, the tools, artifacts, and books that they use and among the communities and practices in which they participate (Planche, 2009; Gupta, 2008: 383; Sjoberg, 2007:3; Verschaffel *et al.,* 1999:6). Mathematics is seen as a social construction (Ernest, 1993:42), where subjective and objective knowledge is linked in a cycle with a symbiotic nature, leading to the renewal of the other (Ernest, 1993:43). Subjective knowledge refers to the personal creations of the individual and objective knowledge to published mathematical knowledge, which undergoes inter-subjective scrutiny, reformulation and acceptance (Ernest, 1993:43).

Von Glaserfeld presented a more radical view to Piaget's cognitive constructivism (Carpenter, Dossey, & Koehler, 2004; Cobb, 2007:10; Steffe & Kieren, 1994:74, Nieuwoudt; 1998:108). Mathematical truths in the radical social constructivism are seen as a social consensus (Goldin, 2002:205). This view on constructivism asserts that children gradually build up their cognitive structures while also maintaining that these cognitive structures are reflections of an ontological reality. Conceptual analysis (now known as the teaching experiment) as proposed by Von Glaserfeld were seen as the new influence in mathematics education (Steffe & Kieren, 1994:74; Gupta, 2008: 382).

Within constructivism, a person is not studying reality, but the process of how the reality is constructed. The learner plays an active role in the learning process (Scales, 2008:7; Geiger, 2008:2; Sjoberg, 2007:3). Cognitive processes are at the core of interaction with environment (Steffe & Kieren, 1994:75). Central to the constructivist model is engagement, which would assist the learner to understand the problem, exploration where learners draw up a mathematical model, explanation where learners articulate on the model, elaboration where learners make generalizations and, finally, the evaluation where learners justify their model (Romberg, Carpenter & Kwako, 2005:20; Carpenter *et.al*., 2004:3; Dossey *et.al.,* 2002:vii; Dossey, 1999:238).

2.2.3 Constructivism within the school mathematics domain

Mathematics classroom teachers are being bombarded with the challenge to ensure effective teaching and learning of mathematics (Howie, 1997: 19). This outcry for better mathematics and science education is coming from different spheres, most vocally from politicians. To address the continued poor performance of learners in mathematics, overwhelming educational research in the recent years proposed a constructivist paradigm. This paradigm is seen as "best fit" to deal with the lack of functioning on cognitive levels required to perform the practices of mathematicians (Cobb, 1988:87; Verschaffel *et al.,* 2000:3; Van Dooren *et al.,* 2007; Verschaffel *et al.,* 1999; Carpenter & Romberg, 2004; Carpenter *et al.,* 2004). There is an increase attention on understanding mathematics, exploring and communicating in favour of memorising and rote learning (Kristinsdóttir, 2003). There is, according to Verschaffel *et al*. (1999: 3) and De Corte (2000: 4), a strong focus internationally on the learning of mathematical problem solving skills, reasoning skills, attitudes and the ability to apply the acquired knowledge to real life situations.

The aim of mathematics teaching should be to build complex, powerful, and abstract structures (Carpenter & Romberg, 2004). The mathematics teacher must according to De Corte and Weinert (1996:26) have “the required knowledge and insight on how learners learn, and the processes that leads to learning.” The mathematics teacher needs to know and be able to employ the skills and knowledge of the practicing mathematician as a key part of their teaching of maths. The mathematics teacher should create an environment where learners can do what the mathematician do (Carpenter & Romberg, 2004:3).

The teacher should, therefore, not only convey mathematical knowledge, facilitate profound cognitive restructuring and apply conceptual reorganizing. The teacher and the learner need to be on equivalent structures to ensure and enable effective teaching and learning (Carpenter & Romberg, 2004). According to Verschaffel *et al*. (1999: 3) and De Corte (2000: 4) strong emphasis is placed in the international arena on the acquisition of mathematical problem solving skills, reasoning skills, attitudes and the ability to apply acquired knowledge and skills in a real life context, which is crucial in the constructivist paradigm.

Developing understanding in mathematics is an important goal of a mathematics teacher. Learners learn new mathematical concepts and procedures by building on what they already know (Cei, 2001). Learning with understanding involves building relationships within existing knowledge and between existing knowledge and new knowledge (Romberg *et al.,* 2005:23; Carpenter *et al.,* 2004:3; Carpenter & Romberg, 2004). Learners have an intuitive understanding of different concepts in mathematics (Cei, 2001). Teachers need to design a learning environment that helps students to move to higher cognitive levels (Yetkin, 2003:2). The errors learners make are mostly systematic and rule based. These errors occur due to rote memorization. If learners have to construct conceptual knowledge they need to identify the characteristics of the concepts, recognize similarities and differences amongst concepts and constructing relationships. The use of appropriate representations is also an important feature in learning mathematics with understanding. The use of appropriate representations will help learners to construct different characteristics of concepts (Yetkin, 2000:2). The teacher and learner communication should facilitate cognitive restructuring and conceptual reorganizing of the learners.

The constructions children make must correspond with those the teacher assumes they have made (Cobb, 1988:96). The teacher, therefore, needs to be aware of the fallibility of his/her inferences. The instructional environment and variations in the cognitive constructions of learners influences teachers' actions. The teacher needs to be aware that he/she cannot inevitably lead the child to the correct construction (Cobb, 1988:96). The child's own participation in the learning process is paramount to the construction of his/her knowledge. Constructivists have, however, identified developmental levels and have developed viable models of learners' conceptual operation at each level (Nashon, 2006:4). Teachers use concrete materials and manipulate these without connecting the learners' construction and algorithms with the learners' own culture (Nashon, 2006:4).

2.2.4 Problem posing in the constructivist approach

Problem posing is essential in extrapolating from the constructivist model. Problem posing can be defined as a process where learners construct personal interpretations of concrete situations to formulate meaningful problems relying on their mathematical experience (Bonotto, 2009:300). Problem posing has a profound effect on classroom environment (Whitin, 2004: 129). Problem posing lies at the heart of a mathematical activity (Bonotto, 209:299). It positively impacts on a learner's mathematical thinking, problem solving skills, attitude and confidence in the mathematics and mathematical problem solving (Bonotto, 209:299). Problem posing supports the learners in observing, describing, predicting, hypothesizing, conjecturing, in doing what mathematicians do (Whitin, 2004:1); effective problem posing is very valuable, in developing and improving learners' understanding of mathematical concepts and ideas (NCTM, 2000: 256). According to Bonotto (2009:300),learners must be involved in the problem posing process in order to: make sure that they distinguish between the significant data and irrelevant data; ascertain relationships between or given data; decide whether the information at their disposal is sufficient to solve the problem; investigate whether the numerical data is numerical and/or contextual coherent.

2.2.5 Mathematical classroom within the constructivist paradigm

School mathematics should, at its core, have problem posing and problem solving (Ernest, 1993:283). Problem solving, reasoning, critical thinking and active use of knowledge constitute the goals of constructivist teaching (Discroll, 2005; Nashon, 2006:1). In a constructivist classroom, learners will be taught with concrete materials, then pictures, followed by numbers (Nashon, 2006:2). Within the applicable, real-life context learners will look for patterns, raise questions, build models, strategies and approaches, in the process of constructing meaning by and for themselves (Libman, 2010: 3). The teacher will, as a facilitator, according to Nashon (2006:1), spend less time at the beginning of the class on lecturing the proposed outcomes to create more time for learner-active involvement.

Contemporary approaches to learning hold the view that knowledge and cognitive strategies are actively constructed by the learner (Can, 2009; Geiger, 2008:3; Cobb, 1988:87). The constructivist model might work under the following conditions:

- Involve learning in identification of problems in a real-life context (Can, 2009:63; Libman, 2010:3)
- Use local human and material resources as original sources of information to solve problems
- Create problems that can challenge learners' mathematical interpretation and capture their imagination (Taylor, 2003:2)
- Involve learners in seeking information that might be applied in solving real-life problems (Can, 2009:63; Libman, 2010:3)
- Extend learning beyond the class period, classroom and school (Can, 2009:63)
- Focus on the impact of mathematics on an individual learner
- Emphasise the skills (modelling, problem solving) mathematicians use when they do mathematics
- Provide opportunities for learners to perform in citizenship roles in order to resolve issues they have identified (Yager, 1991:55)
- A powerful, realistic, relevant learning environment where the teacher's teaching augments the learners' active involvement in their learning (Can, 2009: 63; Geiger, 2008:3)

- Learners and teachers act as mathematicians in their classrooms (Carpenter & Romberg, 2004), supporting multiple perspectives and modes of representation (Can, 2009:63)
- Learners and teachers are involved in practices of modelling, generalization, and justification (Carpenter & Romberg, 2004).

2.2.6 The role of learners in the constructivist classroom

Within the constructivist paradigm, learning is seen as a "constructive, cumulative, self regulatory, goal-oriented, situated, collaborative, and individually different process of knowledge building and meaning construction" (De Corte, 2000:8; De Corte, 2004). Critical dialogue and collaboration in a learning setup plays an important part in any learning process. It develops the learners' self-confidence and control (Johnson & Dye, 2005:2). The learners are active participants in their mathematical development. Learners, according to Gupta (2008:383), will start to learn arithmetic with simple addition and subtraction, followed by more complex addition and subtraction algorithms. Learners will learn by understanding algorithms, taking cognisance of the context and the meaning and role of operations, concepts, symbols, and variables (Carpenter & Romberg, 2004; Carpenter *et al.,* 2004:5). Learners will play an active role in construction of their unique and personal knowledge through the development of symbolic representations used to make sense of the world (Geiger. 2008:20). Learners will grow to understand the meaning of an equal, equivalence and inequality signs in mathematics (Carpenter & Romberg, 2004). Learners need to be able to conduct internal reflective dialogue (Cobb, 1988: 95). The learners are active participants in their mathematical development.

Learners need to be created the opportunity to actively construct knowledge (Cobb, 1988: 95).

2.2.7 Teachers' roles in a constructivist classroom

The teacher's presence and participation is seen as a precondition active learner involvement (Mahlomaholo, 2012a:4). The learning activities must be posed such that the learners can reorganize their conceptual understanding, thereby become more actively involved in the problem solving process. These activities must arise from genuine mathematical problems (Cobb, 1988: 95). The teacher needs to create opportunities for learners to understand the meaning of an equal, equivalence and inequality signs in mathematics (Carpenter & Romberg, 2004). Teachers need to be able to connect the knowledge learners are learning to what they already know (Carpenter e*t al.,* 2004:5). The teacher also needs to be involved in the discussion process which might allude to possible limitations in learners' algorithm, thus suggesting alternatives. Teachers need to engage learners in inquiry and problem solving (Carpenter *et al.,* 2004:5). The involvement of the teacher in the learners' activity needs to be balanced towards a facilitating role, while taking responsibility for validating the learners' ideas and procedures (Carpenter *et al.,* 2004:5).

The teachers need to be aware of his/her oblivious intervention which might overshadow the learners' possible meaning (Cobb, 1988: 96). The learners might fall within a hole of teachers' methods as the only correct method, which reduces powerful conceptual structures. The teacher needs to be aware that he/she cannot inevitably lead the child to the correct construction (Cobb, 1988: 96). The construction of increasingly powerful conceptual structures and the development of intellectual autonomy are crucial within the constructivist perspective (Cobb, 1988: 99). The teacher is a facilitator, co-participant, who challenges, and guides learners thinking towards more complete understanding (Huetinck & Munshin, 2004:45). Teachers should select and design appropriate problems which will elicit active participation in the solution process (Cia, 2003:7). These problems would solidify, extend and stimulate learners learning (Cia, 2003:7).

Teachers in constructivist classrooms must extend learning beyond the class period, classroom, and school, utilizing real-life, contextual, non-routine problems (Cobb, 1988:87). This will allow teachers to challenge themselves and their learners with activities which will include operations, fractions, tables, equations, and functions applied in a real-life context. Focus must be placed on the impact of mathematics on an individual learner. Each learner needs to be involved in modelling mathematical problems to develop higher order skills (Carpenter & Romberg, 2004; Verschaffel *et al.,* 2000).

In the constructivist classroom, opportunities must be provided for learners to perform in citizenship roles in order to resolve issues they have identified (Yager, 1991:55). Learners, as part of a community, must be given problem activities that are realistic and society-problem-based. The teacher should play an important role in organising his/her classroom discourse. A teacher also needs to have knowledge on when, where and how the use the correct problems in optimizing the learning process (Cia, 2003:8). One of the aims of the SA Curriculum is to encourage the development of positive citizenship in learners who are active and critical and have high knowledge and skills (NDE, 2002, DoE, 2011:4). Teachers must involve learning in identification of problems in the local context and seek information that might be applied in solving real-life problems and use local human and material resources, as original sources of information to solve problems (Libman, 2010:3; Cobb, 1988:87). The key factor in the teaching and learning process is the learning environment (Van Rooyen & De Beer, 2007:43). The constructivist, by nature, provides the opportunity for teachers to construct a powerful learning environment whereby the teacher, teaching augments the learners' active involvement in their learning, to actively construct their own knowledge (De Corte *et al.*, 1996; De Corte, 2004). A learner's action within the educational environment is the result of the interaction between the learner's personality and his/her immediate environment (Weinert, 1996:3). Different researchers agree that the learning environment can influence learning positively or negatively (Baxter, 1995:1). The learner's interaction with his/her environment will bring about change in a learner's performance (Uden & Beaumont, 2006:3).

2.3 LEARNING ENVIRONMENTS

2.3.1 Defining a learning environment

A learning environment can be defined as a broadcast to the external environment of everything that has been generated via a sphere of experiences, thereby allowing teachers and learners to have shared perceptions (Von Glasersfeld, 1991:31; Fraser & Walberg, 1991:1). A learning environment is an environment where learners respect each other and are able to perform well academically and otherwise (Mahlomaholo, 2012b:1). A classroom environment can be defined as any shared perception of learners and teachers in that environment (Fraser & Walberg, 1991:1; Margianati *et al.*, 2003:477). Steffe and Wood (1990:10) assert that a learning environment can be created using available conceptual operations. The actions of learners are seen as a function of the learner and his/her environment (Margianati *et al*., 2003:477). A learner's learning depends on the interaction between the personal needs of the learner and his/her external environment. A positive learning environment increases the learner's active participation and motivation (Van Rooyen & De Beer, 2007:45). The 'learning environment learner' can be described as a function of the concepts and operations being used to interpret situations (Cobb, 1994:105).

2.3.2 Critical features of the learning environment

Verschaffel *et al.* (2000:3) assert that a learning environment has certain critical features. The mathematics teacher must develop or have a varied set of carefully designed activities which have complex, realistic context based, challenging and open problems that force the modeller to apply intended, heuristics and meta-cognitive skills. The presentation of each problem must be focused such that it facilitates developing a learner's ability to perform the modelling process, taking cognisance of the realistic context (Verschaffel *et al.,* 2000:3). Problems can be presented in a purely textual format, in the form of a story told by the teacher or the print media or electronic media (newspaper article, a flyer, comic strip, a table), or a combination of two or more of these presentational formats.

Howie (1997:19) states that teachers are challenged to create a powerful learning environment in order to ensure effective teaching and learning; hence, facilitate learners constructing their own ideas (Yetkin, 2003:2). Carnoy and Chisholm (2008:96) agree, stating that a pivotal role of the mathematics teacher is to create a supportive environment for learning mathematics. Teaching, learning and assessment must be part of the lesson plan. The lesson plan must have a variety of activities for both teachers and learners that will be conducted. Each lesson consists of one or two small-group assignments solved in fixed heterogeneous groups of three to four learners, each of which was followed by a whole–class discussion, and an individual assignment which was also followed by a whole-class discussion. The teacher must play a facilitating role during all these activities. The teacher needs to encourage and build up learners to engage in, and to reflect upon, the kinds of cognitive and meta-cognitive activities involved in the modelling activities (Carpenter & Romberg, 2004). These encouragements and 'scaffolds' must be gradually withdrawn as the learners become more competent and confident, and take responsibility for their own learning and modelling (Verschaffel *et al.,* 2000).

The interventions must be aimed at creating socio-mathematical norms, which will result in a classroom learning environment conducive to the development of learners' appropriate beliefs about mathematics and mathematics learning and teaching (Verschaffel *et al.,* 2000). These norms include learners being actively involved and taking ownership and accountability for their learning, while the teacher takes the responsibility of creating a modelling, learning environment. For example, the teacher is not the only evaluator of the correctness realistic nature of generated solution, but rather the whole class after looking at the pros and cons of the proposed solution. The teacher needs to take cognisance that a rough estimate can be a better answer then an exact number and a solution procedure might include flow-charts and a sketch or a diagram (Verschaffel *et al.,* 2000).

2.4 THE KEY COMPONENTS OF A POWERFUL LEARNING ENVIRONMENT IN A CONSTRUCTIVIST PARADIGM

The key components of a powerful learning environment in the constructivist paradigm, focuses research on the understanding of learning, the understanding of problem solving, the understanding of a learning environment and the understanding of modelling.

2.4.1 The learning process

The concept of learning is one of those concepts that have been thoroughly researched. International literature shows that the central focus of teaching is to develop in learners the necessary knowledge and skills while motivating new mathematical tasks and problems in a purposeful way (Verschaffel & De Corte, 1995:64). Although a clear-cut definition for learning, which is shared by all, does not currently exist, the researcher here focuses and uses the definition as stated by De Corte and Verschaffel and highlighted in 2.2.6. Learners need to be given problems which would provide them with the opportunity to engage in modelling and develop in learning with understanding, thereby function on a higher cognitive level. Problem based learning provides such an opportunity (Uden & Beaumont, 2006).

2.4.1.1 Learning is a constructive process

Research since 1980 does corroborate that learners construct their own knowledge and skills (De Corte, Verschaffel & Masui, 2004:369; Verschaffel & De Corte, 1995:68; Geiger 2008: 2; Verschaffel & De Corte, 1995:65; Marray, Olivier & Human, 1998:170; Can, 2009: 62; Planche, 2009:7). A reorganization of cognitive structure occurs during the interaction with the learning environment (De Corte, 1996:37). The traditional approach which involves knowledge transmission is still the dominant approach used in schools and educational institutions. Some researchers, especially the constructivists qualify this information transmission as a poor learning environment. Learners, according to these researchers, are in an active process and constructive process (Geiger 2008: 2; Verschaffel & De Corte, 1995:65). Learners are not empty vessels who can be filled up with knowledge. The learners actively participate in their learning process and reorganizing the cognitive and meta-cognitive structures, such that new knowledge is constructed (Can, 2009: 63; Sjoberg, 2007: 3; Taylor, 2003: 3) and used within a mathematically applicable context (Libman, 2010:2; Can, 2009: 61). Learners can apply their knowledge in real life (Can, 2009: 63; Taylor, 2003:5).

The learning of new knowledge and skills should be an active process of cognitive assimilation and intellectual harnessing for the learner. The teacher plays a very important role in the construction of knowledge as a facilitator (Taylor, 2003: 3; Jansen, 2010). The constructive nature of learning realises in solving the elementary addition and subtraction problems (Verschaffel & De Corte, 1995:65). The learning environment which teachers should construct must provide learners the opportunity to construct knowledge.

2.4.1.2 Learning is cumulative

Learners sometimes have misconceptions and faulty procedures which are negative and inhibitory, instead of a higher order understanding and problem solving ability (De Corte, 1996:38). According to Verschaffel and De Corte (1995:65), research shows that children in an informal setup, such as streets demonstrate sharp, accurate, handy negotiating, and reasoning skills. These children can easily determine without the use of a calculator the profit, buying and selling prices and amount to return to customer within a spaza shop. If the same learners were given a similar problem in the formal classroom setup, they seem to perform less effectively. The learners in the classroom setup in contrast with an informal street setup, therefore, took their recourse to the formulae and procedures given to them by the teachers. No link between the street strategies and the algorithms in class were drawn (Verschaffel & De Corte, 1995:65).

Serious consideration should be given to the cumulative nature of learning in the mathematics teaching process (De Corte, Verschaffel & Masui, 2004:369; Verschaffel & De Corte, 1995:65). The formal and informal and prior knowledge should be used in constructing new knowledge (De Corte & Masui, 2009:176; De Corte, Verschaffel & Masui, 2004:369). A powerful learning environment where consent to cumulative learning is essential for constructive effective learning to take place must be encouraged.

2.4.1.3 Learning is self-regulatory

One of the strategies of meta-cognition is self-regulation (Masui & De Corte, 2005:352). Meta-cognition, according to Masui and De Corte (2005:352), is knowledge about one's own cognitive functioning. Self-regulation embraces the learners' management and monitoring of their learning process (De Corte, 1996:37; Masui & De Corte, 2005:364; Eynde, De Corte & Verschaffel, 2006:85; Schoenfeld, 1992:57). The focus is placed on the ability of the learner to control and regulate his/her own learning process (Planche, 2009:10; De Corte, Verschaffel & Masui, 2004:370; De Corte, 1996:37; Eynde, De Corte & Verschaffel, 2006:85). Cognitive self-regulating skills, according to De Corte and Masui (2009:175), include orienting, planning, self-checking, and reflecting. Orienting involves examining the characteristics of the learning task. Planning deals with decisions on how to approach the learning task. Self-checking is vital in determining whether the intermediate outcomes match the intended learning goals, while refection focuses on a backwards look at the learning process to draw conclusions about the factors that influence it (De Corte & Masui, 2009:175). In the traditional education approach, the teacher is the central figure, the sole provider of knowledge and skills (Nashon, 2006:2). The learners follow and mirror the teacher's teaching. The self-regulatory aspect of learning is placed on the learners in a constructivist approach (Gupta, 2008; Planche, 2009:10). Learning is a self-regulatory process of struggling between the existing personal models of the real world and new insight in constructing models of reality (De Corte & Masui, 2009:174; Gupta, 2008:382). The learner should be stimulated and encouraged from an early age. The learners should also be aware of the directions and able to direct the learning process (De Corte, Verschaffel & Masui, 2004:372; De Corte, 1996:38). Self-regulation should be promoted in a powerful learning environment (De Corte & Masui, 2009:173; De Corte, Verschaffel & Masui, 2004:368). The teacher should take cognizance of self regulation when constructing a powerful learning environment.

2.4.1.4 Learning is goal-orientated

The acquisition of new knowledge and skills would be more successful if learners are being allowed to determine and follow their own goals (Planche, 2009: 4; De Corte, Verschaffel & Masui, 2004:369; Verschaffel & De Corte, 1995:64). Learning is most productive when learners choose and determine their own goals (De Corte, Verschaffel & Masui, 2004: 369). The learners should be actively involved in determining their own goals. The identification and setting of outcomes, which need to be achieved by teachers and learners will be realised in an appropriate powerful learning environment. Amos and Archer, 1995 (cited by Verschaffel & De Corte (1995:64), have shown that goal-orientation has a direct effect on the mathematical performance of the learners. Learners with a high self-efficacy will have higher effort and interest in solving the mathematical problems (Jansen, 2010). The teacher needs to take cognisance of goal-orientation when constructing a learning environment.

2.4.1.5 Learning is situated

In the cognitive psychology domain, learning and thinking are viewed as processes which occur in the brain. Knowledge is regarded as the pure cognitive assimilation of facts, comprehension, ideas, relations, principles, and procedures (Verschaffel & De Corte, 1995:68). The important influence of social and cultural factors on learning seems to be ignored by many researchers. Learning must be in a familiar context which is relevant and meaningful to the learner (Scales, 2008:82). Learning does not only occur in the brain but also in a continued interaction between social and cultural context environment (De Corte & Masui, 2009:174; De Corte, Verschaffel & Masui, 2004:368; Verschaffel & De Corte, 1995:68). The active involvement in situated and culture based activities and practice influence learning (De Corte, Verschaffel & Masui, 2004:369; Verschaffel & De Corte, 1995:68).

One of the important implications of situated learning and situated knowledge is that mathematics should be practically applicable in work contextual problems where learners can apply the skills and knowledge (De Corte, Verschaffel & Masui, 2004: 369). According to Verschaffel and De Corte (1995:68), mathematical knowledge and skills are taught and learnt divorced from the social and physical context. Ignoring of the social and physical context might lead to disjuncture of effective teaching and learning of mathematics. The teacher needs to take cognisance of the fact that learning is contextual when constructing a powerful learning environment.

2.4.1.6 Learning is interactive

Verschaffel and De Corte (1995:71) viewed learning as the interactive reconstruction of knowledge. Individual knowledge construction comes to realization during the processes of interaction, and community involvement whereby learners become part of the mathematical community and culture (Murray, Olivier & Human, 1998:170; Verschaffel & De Corte, 1995). This, of course, does not rule out the fact that learners discover and develop their own new knowledge and skills (Verschaffel & De Corte, 1995:71). The interaction between student and student and between teacher and student is an essential constitute of learning (Uden & Beaumont, 2006:11). Social interaction forms a key ingredient of the problem solving activity (Dossey *et al.,* 2002:39)

Within the cognitive domain the value of social interaction and communication (Can, 2009: 61), as part of mathematical tasks and problems, might lead to understanding and applying modelling. The acquisition of cognitive and meta-cognitive strategies can be improved. Learning within a small group set-up might result in effective cognitive social mathematical outcomes of learning (Verschaffel & De Corte, 1995:68). The nature and quality of the group work activity and facilitation thereof plays an important role in the success of the learning process. For learning to be effective, there needs to be a constructively interactive learning environment for learners. The ideal learning environment must create an opportunity for contextualizing the learning area.

2.4.2 PROBLEM SOLVING

Mathematics, according to social constructivists, is a social institution which results from problem posing and problem solving. Most problems faced by individuals are usually very complicated and non-routine, requiring problem solving to an exact solution or a reasonable estimate (Dossey, 1999:232). Teaching maths through problem solving starts with a problem (Cia, 2003:3). A problem involves known and unknown information. Naidoo *et al.* (1995: 20-21) identified two types of problems: routine and non-routine problems. Routine problems are one step or multi-step problems that can be solved by applying an algorithm. Non-routine problems are problems that cannot be solved by straightforward application of algorithm; rather by heuristics. Problem solving is central to inquiry and applications (NCTM, 200:257). Problem solving according to Van der Horst and Macdonald (1997: 139) is the process of applying existing knowledge to new or unfamiliar situations in order to gain new knowledge. Through problem solving, learners are provided a chance to experience the power and usefulness of mathematics (NCTM, 2000:256) There has been significant progress made in the understanding of problem-solving as a focus of teaching mathematics (Cia, 2003:2). Learners play an active role in finding their own solutions by exploring the problem situation through problem solving (Cia, 2003:3).

2.4.2.1 Approaches to teaching problem solving

According to Naidoo *et al.* (1995: 24) teachers have three approaches to teaching problem solving based on Polya's model. These approaches are:

- **Teaching about problem solving**

In this approach the focus of the teacher is to teach his/her learners Polya's four-phased model. The learners gain knowledge about the processes involved in problem solving.

- **Teaching for problem solving**

In this approach the focus of the teacher is on ways of applying the mathematics learnt in solving the posed problems. Opportunity is given to learners to apply acquired knowledge to solution of problems.

- **Teaching via problem solving**

The teacher's teaching strategy involves teaching a content topic by starting with a problem situation. The solution methods are developed during the lesson. This method will change non-routine problems into routine problems.

The method of teaching by problem solving is more applicable to the transformational OBE system undertaken by South Africa.

2.4.2.2 Problem solving as developed by Polya

The problem-solving model developed by Polya forms a basis used by some learners, to develop their own models of problem solving. Polya's model has four distinct phases (Polya, 1988: xvi; Confrey & Kazak, 2006:307; Leung, 2009:220; Peter-Koop, 2003:456; Schoenfeld, 1992:16; Nieuwoudt, 2003:35; Nieuwoudt, 1998:102). The four phases are:

- Understanding the problem
- Devising a plan
- Carrying out the plan
- Looking back.

For problem solving to be effective it is essential to follow all the phases and to answer the questions in each phase. In each of the phases it is necessary to pose questions and answer questions (Nieuwoudt, 2003:36; Polya, 1988: xvi). Askew and William (1983:16) identified two types of questions, namely; lower-level and higher-level questions. Lower level questions test the ability to recall facts and procedures while higher level questions focus on the ability to apply, synthesize or explain information. These questions can be integrated to achieve a desired outcome as seen in Polya's method. Each phase has its set of unique questions. The problem solver is confronted with leading questions that could steer him/her on the correct solution path (Polya, 1988: xvi; Steward, 2003:58-59).

- *Phase 1: Understanding the problem (Problem understanding phase)*

When a problem solver is faced with an unknown problem, he/she needs to first get a clear understanding of the problem. The problem solver thus needs to follow certain strategies proposed by Polya to obtain a solution. The problem solver must look critically at the unknown, the given data and the conditions stated in the problem. To understand the problem, the problem solver needs to get answers to the following questions: "What is the unknown? What is the given data? What is the condition? What is the question?"

The problem solver needs to draw a figure and introduce suitable notation (Polya, 1988: xvi; Steward, 2003:58). When choosing symbols for unknown quantities, the problem solver can use letters such as a, b, c, x, y, etc. It is advisable to use the initials such as A for area, V for Volume, and t for time (Steward, 2003:58). The critical answering of these questions is imperative for progress in the solution process and indicates that the problem solver has a clear understanding of the problem. The problem solver can now enter phase two in the solution process (Polya, 1988: xvi).

- *Phase 2: Devising a plan (Plan-construction phase)*

A plan is devised after the problem is understood. In this phase the problem solver must find a relationship between the data and unknown. Here the questioning is mainly aimed at devising a plan. The following questions must be posed (Polya, 1988: xvi; Steward, 2003:58):

- Have you previously encountered the same type of problem?
- Look at the unknown and think of a similar problem having the same unknown. Look critically at whether you can use any of the following information from the similar problem, which may include the result, method, etc.
- Is it possible to restate the problem?

At the end of phase 2 the problem solver needs to have answered the following questions (Polya, 1988: xvi):

- Was all the data used? If not, what sense can be drawn from the data not used?
- Did the problem solver employ the whole condition?
- Were all the essential notions involved in the problem taken into account?

Answers on the above mentioned questions are crucial before the problem solver can start with phase 3.

- *Phase 3: Carrying out the plan (Implementing phase)*

The constructed solution plan needs to be implemented in this phase. The problem solver needs to critically check whether each step is correct. The problem solver must be able to prove that each stated step is correct (Polya, 1988: xvi; Steward, 2003:59).

- *Phase 4: Looking back (Evaluation phase)*

In this phase the focus is mainly on evaluation. The answer arrived at by the problem solver must be assessed in this phase. The result and argument must be checked. The focal questions in this phase are as follows (Polya, 1988; Steward, 2003:59):

- Could the problem solver have arrived at a different result?
- Could the result or method be used in other problems?

Each step must be dealt with critically. If each step is not dealt with critically, the problem solver might try to obtain a result while devising a plan in phase 2, thereby mixing the construction of a solution plan with the implementation of the plan.

In problem solving, more than one solution for the problem is being investigated. The problem solver does not only focus on one answer for the problem. Problem solving is one of the critical outcomes stated in the OBE curriculum implemented in SA. Problem solving as both a teaching strategy for teachers and a learning strategy for learners has value in creating effective and functional communities. The use of problem solving as a teaching strategy will help the teaching and learning process as follows:

- Learners will gain a better understanding of the content area, as opposed to just recalling information
- Learners will develop the ability to analyse situations by applying existing knowledge to new situations, and to recognize the differences between facts and opinions, to draw conclusions, and to make objective judgments
- Learners will be able to assess and respond to new situations
- Learners will be intellectually challenged

- Learners will be encouraged to be active participants in their learning process (Van der Horst & Macdonald, 1997: 139).

Each phase has its set of unique questions. The problem solver is confronted with leading questions that could steer him/her onto the correct solution path (Polya, 1988: xvi). In Phase one, the learner should understand the problem, by answering the answering the following questions:. what is the unknown, what are the data, what is the condition? Draw a figure, introduce suitable notation and separate the various parts of the condition. In phase 2 the learner is expected to devise a plan. In this phase the learner needs to find the connection between data and the unknown, look for familiarity, and see whether the learner knows of a related problem. In Phase 3 the learner must carry out the plan. The learner needs to check each step, verify that each step is correct, and prove that each step is correct. In the last phase, which involves looking back, the learners will examine the solution obtained. The learner must check the result, and argument. The learner needs to check whether a different result could have been obtained, and can the result or method be used for another problem (Polya, 1988: xvi; Posamentier *et al.,* 2010:108). Problem solving elements emphasised that mathematics is more than just a set of formal definitions, theorems, or proofs. It acknowledges the important role of problems in generating new possible solutions (Confrey & Kazak, 2006:307). The mathematics teacher must create a learning environment which exposes learners to problems (non-routine problems) which they face in real life. Real life, complex, non-routine problems with no structure mostly cannot be solved in a few minutes. Learners, according to research, show that they have the capability to invent their own solution strategies to solve problems through problem solving (Cia, 2003:4) A learning environment, according to Uden and Beaumont (2006:13) should provide learners with an experience of the full complexity and authenticity of the real life problems.

2.4.2.3 Teachers role in problem solving classroom

Teachers must build problem solving skills in all their learners, which will contribute positively to cognitive development (Posamentier *et al.,* 2010:107). The teacher poses the question, and leaves the method of the solution open to the learner. The learner is then expected to find his/her own way to solve the problem (Ernest, 1993:286). Teachers need to consider the following strategies to assist their learners developing problem solving skills: Working backwards, finding patterns, adopting different viewpoints, solving simpler corresponding problems, consider extreme cases, making visual representations, intelligent guessing and testing, approximation, accounting for all possibilities, organising data, and logical reasoning

(Posamentier *et al.,* 2010:113). The effective usage of problem solving is dependent on a person's knowledge base, problem solving strategies, meta-cognitive activities, and beliefs and practices (Schoenfeld & Kilpatrick, 2008: 1).

2.4.2.4 Teaching strategies in problem solving

Schoenfeld (1985: 15) highlighted that resources, heuristics, control and belief systems are crucial necessities for teachers to consider in problem solving performance. The teaching material needs to be organised such that it introduces four key cognitive processes such as the ability to read the problem statement with care, making sense of the problem situation, evaluation and checking answers and clearly distinguishing between relevant and irrelevant information of data (Wu *et al.,* 2002: 5). The minimum requirements for teachers should, according to (Schoenfeld & Kilpatrick, 2008:2), be to know the school mathematics to be taught in depth and breadth, know his/her learners as thinkers, know learners as individuals, crafting and managing learning environments, developing classroom norms and supporting classroom discourse as an essential part of teaching for understanding, constructing relationships that support learning, and reflecting on one's practice.

Teachers need to be aware of misconceptions that the learner might have and correct them. Examples of misconceptions are; that processes in the formal maths arena are divorced from discovery or inventions, that mathematics problems must be solved in a few minutes, that only geniuses can discover, create and really understand mathematics and also that success in maths is directly related to the teacher as the only source of mathematics (Schoenfeld, 1988:7). A teacher's dependency on textbooks as the expert in the classroom, also contribute to a lack of ownership in teaching (Schoenfeld, 1988:7). The teacher needs to understand and 'buy-in' to the belief that the major purpose of mathematics instruction is to help learners think mathematically. Thinking mathematically involves the mastery of various facts and procedures and connecting them, applying formal mathematical knowledge flexibly in real-life context appropriately. Learners are active interpreters of the world around them, and are able to connect the formal and informal knowledge with the real world (Schoenfeld, 1988:7).

2.4.3 PROBLEM BASED LEARNING

Both the cognitive constructivist (see par. 2.) and social constructivist (see par. 2.1.) theories provide insights into problem based learning (Hmelo & Evensen, 2000:4). Problems and tasks play a central role in the construction of learners' mathematical understanding (Dossey *et al.,* 2002:39). Problem based learning is compatible with the constructivist learning model (Lesh & Caylor, 2009: 338) that underpins most of the identified principles for developing cognitive and meta-cognitive skills (Uden & Beaumont, 2006:29). Knowlton (2003:5) views problem based learning as a pedagogical approach which requires learners to solve an unknown problem. Within problem based learning the structuring of a messy real-life problem is essential (Whitin, 2004: 129). In SA the focus was place in problem centred learning which is also compatable with the constructivist learning model (Nieuwoudt, 1998:112). Problem centered learning according to Nieuwoudt (2003:105) was implemented in the previous TED-controlled primary schools. Problem centered learning was seen in some schools as the alternative to the traditional approach (Nieuwoudt, 1998:112). The focus of problem centered learning was to develop in learners the ability to solve routine and non-routine real-life problems (Nieuwoudt, 2003:105).

Central in problem based learning is teaching which is learner-centred and which provides opportunity for free inquiry from learners (Knowlton, 2003:6; Lesh & Caylor, 2009:338). Problem based learning emphasises higher order thinking in learners (Hung, Bailey & Jonassen, 2003:15). It requires learners' meta-cognitive awareness of the efficacy of the process. Problem based learning inherently involves self-regulation (Hmelo & Evensen, 2000:4). Problem based learning, which originated form Dewey in 1916 (Uden & Beaumont, 2006:32) focuses on experiential, hands-on, mind-on, direct learning. In problem based classrooms the teacher will open class with a brief introduction of the problem under investigation, followed by learners engaging in small group activity. The teacher will circulate, guide, assess, and motivate learners in their small groups (Lesh & Caylor, 2009; Dossey *et al.,* 2002:41). The teacher will continuously focus on sharpening learners' problem solving abilities and their ability to reason, and articulate, their connection of ideas, and move between representations of mathematical concepts and ideas (Dossey *et al.,* 2002:41). Problem based learning will empower learners with problem solving skills, information skills, computing skills, thinking skills, communication skills, team working skills, management skills, and learning to learn skills (Uden & Beaumont, 2006: 34) which will enable them to solve non-routine problems with modelling. For learning to be effective, there needs to be a constructively interactive learning environment for learners (Verschaffel *et al.,* 2000). The teacher needs to design a task and learning environment to support and challenge learners' thinking (Lesh & Caylor, 2009:338).

The characteristics of problem based learning are learners who collaborate, formulate learning issues which in solution process, relevant content identification, learner self evaluation, self directed learning, learner active participation and increased motivation for learning (Knowlton, 2003:6; Hmelo & Evensen, 2000:3). Problem based learning can promote an active and collaborative environment (Knowlton, 2003:6; Speck, 2003:59).

2.4.3.1 Setting up appropriate problems for problem based learning

All mathematics learning in problem based learning starts with a problem (Schmidt & Moust, 2000:21). Designing an appropriate problem to be solved, which is non-routine and contextual based, forms a crucial part of problem based learning (Weiss, 2003:25; Schmidt & Moust, 2000:19). The appropriate design problem will elicit higher-order thinking in learners (Weiss, 2003:25). When designing the problem, teachers need to be clear on the purpose of the problem, which should enhance and promote the mathematics content. The purpose of the problem can vary from guiding, testing, illustrating a principle, fostering a concept, or providing stimulus for an activity (Weiss, 2003:26).

Problems that foster higher order thinking must be appropriate for the learners, well structured, collaborative, authentic, and promote lifelong and self directed learning (Weiss, 2003:28). Solving appropriate well structured problems involve problem solving (Speck, 2003:59).

Problem based learning addresses the construction of knowledge, developing reasoning, self-directed learning, and increases motivation for learning and collaboration.

2.5 Summary

The Chapter provided an understanding of the two most prominent theories of learning. The reason for highlighting only two theories was attributed to the South African education which transformed from traditional to outcomes based education (OBE). In the behaviourist learning the teacher provides a set of stimuli to elicit a appropriate response. Greater attention was given to the constructivist learning, as this research project fits within the constructivist paradigm. Different types of constructivism were discussed followed by constructivism in the maths domain then in the school mathematics domain. The role of the learners and the teacher in the constructivist classrooms highlighted the importance of a learning environment. Attention was given to the critical features and building blocks of the learning environment, followed by the unpacking of the definition of learning in a constructivist paradigm. Great attention was given to the theory of problem solving as proposed by Polya and problem posing as essential in mathematics learning. Special focus was placed on problem based learning.

In the following Chapter, focus is placed on establishing a theoretical perspective on designing a modelling-based learning environment in the South African context.

CHAPTER 3:

A MODELLING-BASED LEARNING ENVIRONMENT IN MATHEMATICS CLASSROOMS

In Chapter 3, the literature review focuses mainly on modelling as a powerful learning environment. The Figure 3 below provides a schematic outline of the key areas addressing modelling-based learning environment in mathematics classrooms.

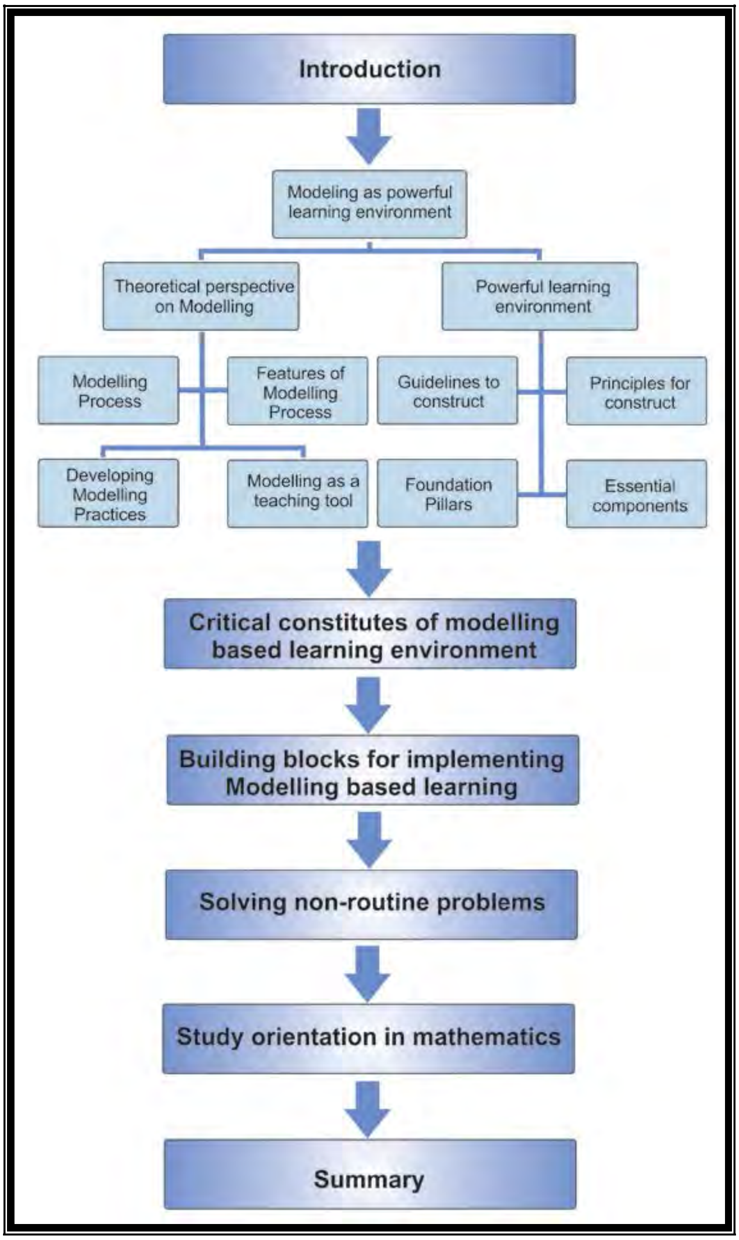


Figure 3: Outline of Chapter 3

3.1 Introduction

Learners should start at primary schools with mathematical modelling (English & Watters, 2004:60). Elementary school children do have the capability to develop their own models and sense making system for dealing with complex problem situations (English, 2007: 121). Learning which occurs via making models is an active process, which allows the construction of mathematics. Learners express their ideas. Learners do pose their own questions and provide their own possible solution (Schwartz, 2007:168). The learners' ability to develop powerful models is one of the most significant goals of mathematics education (English, 2007:121).

In Chapter 3, focus is being placed on what literature says regarding design for modelling as a powerful learning environment in grade 6 mathematics classrooms. The review will provide a theoretical perspective on the modelling, its process and its features in school maths. Focus will be placed on modelling as a teaching tool, a powerful learning environment (PLE), the guidelines for constructing a PLE, the principles of PLE, the foundation pillars, and identifying essential components for a PLE, critical constitutes of a modelling learning environment, the building blocks for implementing the modelling, the expected influence of modelling a learning environment on learners solving non-routine problems and the expected influence of modelling learning environments on learners' study orientation in mathematics (SOM).

Modelling plays a key role in developing an understanding of mathematical and scientific constructs and can be a critical tool for any learner to use coping with their milieu (English, 2007:121).

3.2 MODELLING

3.2.1 Defining modelling and models in the mathematics arena

Applications of mathematics in the everyday world involve a tentative and intuitive process (Dossey, 1999:233; Peter-Koop, 2003:455). The process of model construction, model evaluation, and model revision is central to understanding the domain and fundamental practices of mathematicians (Romberg et al., 2005:10; Dossey, 1999:233; English & Watters, 2004:59). Blomhøj (2004:145) defines modelling in mathematics as a teaching and learning practice, which centralizes the relation between real life and the mathematics being taught and learnt. A mathematical model is the relationship between a mathematical object (the source) and a situation or phenomenon of a non-mathematical nature (Blomhøj, 2004:146). A mathematical model can be defined as a mathematical construct which is designed to study a real life problem system or event or phenomena (Dossey *et al.,* 2002:115). A model is the theoretical world that describes a natural process, which can be used to describe a natural phenomenon (Romberg *et al.,* 2005:11). Models are not only representations which are descriptive but also explanatory, and predictive (Romberg *et al.,* 2005:13), but also involve justifying, conjecturing, quantifying, coordinating and organising data (English & Watters, 2004:59). A model can, according to Dossey (1999:234), be described as a parallel world showcasing real-world systems in terms of mathematical concepts relationships and principles. Models assist in making explicit critical features and the relationship between the features. Models provide a basis for communicating ideas and the phenomenon or system been investigated (Romberg & Kaput, 1999:10). Models focus interactions on relevant ideas (Carpenter & Romberg, 2004:6). Steward (2003:25) defines a model as not only a mathematical description, such as a function or an equation of the real world phenomenon, but also warn that it cannot be an absolutely, accurate representation of the physical situation. Steward (2003:25) calls the mathematical model an idealization.

Modelling according to English and Watters (2004: 59) focuses on structural characteristics like patterns, interactions and relationships amongst elements of a phenomenon. These constructs may include graphical, symbolic, simulation, and experimental constructs (Dossey *et al.,* 2002:115). Modelling consists of three fundamental facets, namely; creative and empirical model construction, model analysis and model research (Dossey *et al.,* 2002: vii). To experience the mathematical model, a clear distinction between the situation modelled and the role that mathematics plays as two separate but interrelated entities, is crucial.

Modelling provides its user with access to some unattainable information about a system or phenomena at hand (Thompson & Yoon, 2007:200). Modelling is viewed as a constructive process (Carpenter, Dossey & Koehler, 2004:75).

3.2.2 Types of models

For a learner to understand mathematics in its complexity, the learner needs to involve him-/her-self with the practices of mathematicians. These practices are taken for granted and put on the periphery of teaching and learning of mathematics at schools (Carpenter & Romberg, 2004:3). Word problems are best answered using mathematical modelling (Martin & Bassok, 2005: 471; English, 2007). Schools do not seem to capture an explicit focused attention on the practices of mathematicians. These critical practices, neglected according to Carpenter and Romberg (2004:3), involve modelling, generalization, and justifications.

The term 'model' is often used to describe a replica. Examples of these are aeroplane models, car models, etc. In the mathematics and science paradigm, the notion of a model is much more complicated than just a mere replica of some object. Modelling can be seen as an artificial environment which refers to an external reality (Schwartz, 2007:162). Carpenter and Romberg (2004:6) have identified four different types of models such as:

- *The look alike models*

The physical characteristics of the object been modelled is central. An example of the model is the miniature Ferrari car packaged in a transparent cube (Romberg *et al.,* 2005:15; Lehrer & Schauble, 2005:31). Another example is the miniature cell phone made of plastic. .

- *Function alike models*

The focus of these models is on functionality rather than appearance. The models replicate some of the functions of the object or phenomena been modelled. Examples of the models are levers being used in the construction of a model to illustrate knee or elbow actions (Romberg *et al.,* 2005:15; Lehrer & Schauble, 2005:31). .

- *Descriptive models*

The focus of these models is on representation of the features of the object, system, or phenomena without any physical resemblance. A key feature of the descriptive models is its representation using tables, graphs, or formulae. Descriptive models identify and describe patterns and assist in informed predictions (Romberg *et al.,* 2005:15).

- *Explanatory models*

The explanatory models are used to understand and portray the mechanism underlying a situation or a phenomenon. Explanation models involve mechanisms which are not directive or observable (Romberg *et al.,* 2005:13).

A descriptive model is less complicated, then the explanatory model. The explanatory model builds on assumptions related to similarities between different situations. The validation of the prediction needs a realization of the predicted activity (Romberg *et al.,* 2005). Proven research and statistical information support descriptive models (Blomhøj, 2004:146). Modelling allows learners to apply mathematics into real life contexts, therefore, also considering the relationship between mathematical concepts, representations, and their meaning in the context. Displaying the model, allows for scrutiny of the model and its uses.

Other viewpoints on models is according to Schwartz (2007:164) based on structure which are also called constrain models and a model based on function which is also called a process model. Function based models refer to a set of equations with an dependent and independent variable, also referred to as computer based models (Schwartz, 2007:164). The structure based models are focused on the structure, which allows its users to explore the explicit relationship among a set of related variables (Schwartz, 2007:167).

3.3 MODELLING PROCESS

Different researchers have highlighted different viewpoints of what the modelling process entails. One of these viewpoints is illustrated by Dossey, *et al.* (2002: vii) as problem identification, model construction or selection, identification or collection of date, model validation, calculation of solutions to the model and model implementation. Carpenter and Romberg, (2004:16) appear to have similar viewpoint using the following phases which involve understanding the event, model construct (situation and followed by the mathematical model), evaluate, interpretation and model revision.

Within the modelling cycle (Lehrer, & Schauble, 2005:38), a modeller's way of thinking undergoes different shifts in thinking and interpretation. The modeller will use prior experiences to interpret the real situation. The modeller will then express their interpretation of the situation in a mathematical model. The model provides the modeller with a lens through which the situation can be re-interpreted. The model will then be revised based on findings from the new interpretation (Thompson & Yoon, 2007:207).

3.3.1 The steps in a modelling process

The modelling process which is cyclic in nature, involves understanding the event, model construct (situation and followed by the mathematical model), evaluation, interpretation and model revision (Carpenter & Romberg, 2004:16; Romberg *et al.,* 2005:12; Bonotto, 2009:298; Verschaffel *et al.,* 2000:168-172; Steward, 2003:25). For example, the event: John has 4 bananas, Thandeka has 8 bananas. How many bananas do they have all together? The modeller needs to get an understanding of the event informed by the given information. The situation model will be a combination of the sets. The mathematical model is 4 + 8, taking cognisance of the real-life context. The derivations from the model are 4 + 8 = 12. The interpreted results are 12 bananas; then evaluating the results using the situation model which was a combination of sets. The correct response will be reported: they have 12 bananas altogether (Verschaffel *et al.,* 2000:134; Martin & Bassok, 2005: 471). During the problem phase, the specific question or phenomena under examination are identified. The spectrum of features of a phenomenon (obvious and hidden) must be taken into account. The essential knowledge and data are included, while the non-essential parts are stripped away (Verschaffel *et al.,* 2000). The teacher's instruction needs to take into account the following elements; investigating using concrete materials, making a prediction about the possible answer, executing the task, articulating the steps followed, evaluating the initial prediction and writing down the answer (Verschaffel *et al.,* 2000:49; Romberg, 2001).

There exist different illustrations of the modelling process. This researcher will utilise the elaborated view of the modelling process as proposed by Verschaffel which is illustrated below:

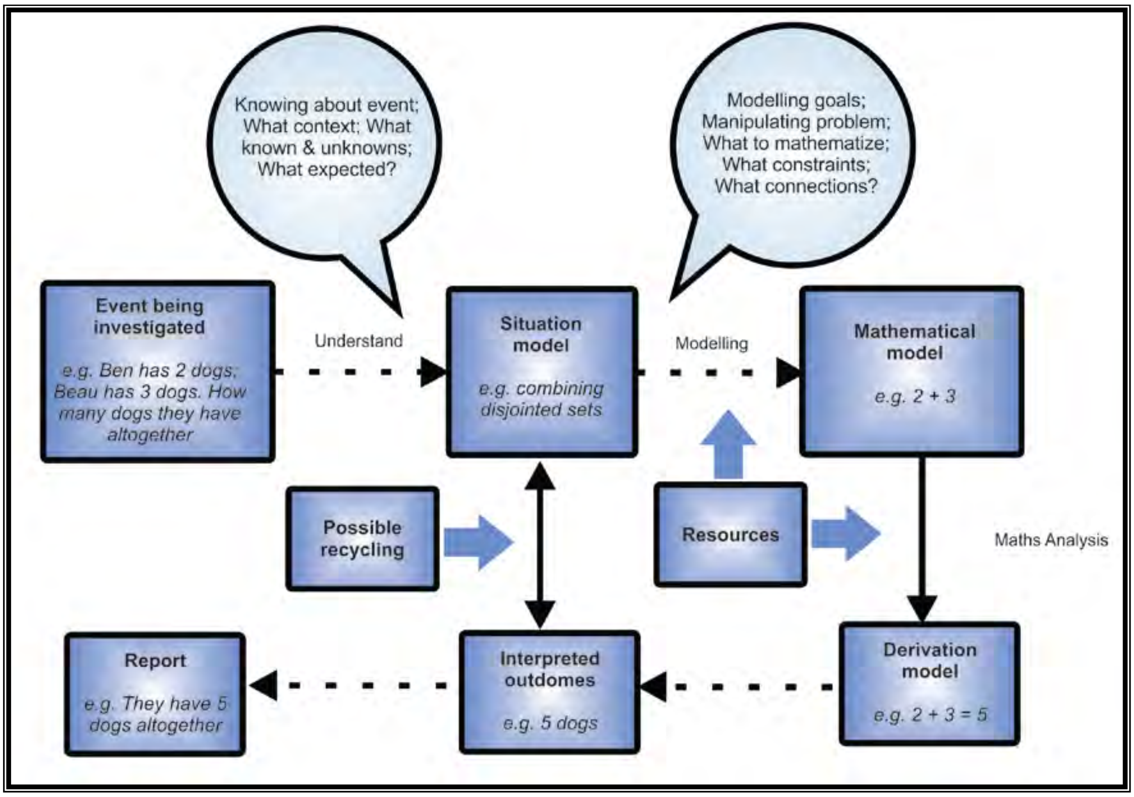


Figure 3.1: Elaborative view of modelling process (Verschaffel et al, 2000: xiii, 13, 168 &134)

The first stage involves obtaining an understanding of the event being investigated. The novice modeller can utilise different resources to further his/her understanding of the situation and develop a situation model. The transformation of the situation model into a mathematical model involves identifying key quantities and relationships between them and expressing them in the form of a mathematical equation (Bonotto, 2009:298). Essential is the knowledge base of mathematical concepts, relationships, expressions, and equations. If challenges exist with a knowledge base, modeller can utilise other resources like the internet to extend his/her understanding and knowledge base, and utilising the five stage problem solving which includes the eight heuristic strategies. The mathematical analysis will allow the modeller to draw derivations form the mathematical model. These derivations will inform the modeller to interpret the results, based on the situation model. If the model does not answer to the situation model, the recycling of the process restarts. If the results satisfy the situation model a report will be drawn up and reported on (Verschaffel *et al.,* 2000:168-172; Bonotto, 2009:298/299).

Verschaffel *et al.,* (1999:202) proposes five steps modelling process with strong heuristic presence in the first two steps. The competent problem solving model underlying the learning environment according to Verschaffel *et al.,* (1999:202) is:

Step 1: Build a mental representation of the problem

Heuristics
Draw a picture
Make a list, scheme or a table
Distinguish relevant from irrelevant data
Use your real–world knowledge.

Step 2: Decide how to solve problem

Heuristics
Make a flowchart
Guess and check
Look for a pattern
Simplify the numbers.

Step 3: Execute the necessary calculations.

Step 4: Interpret the outcome and formulate an answer.

Step 5: Evaluate the solution.

Schoenfeld (1980:796) highlighted the heuristics: Draw a diagram if possible, if there is an integer parameter, look for an inductive argument; consider a logical alternative; consider a similar problem and try to establish sub-goals. Dossey (1999:238) showed similar reasoning highlighting only four steps, namely; problem identification, making assumptions, develop a functional relationship, and verifying the model. In the last step which is to verify the model specific questions need to be answered in Dossey's model. These questions are essential to verify applicability of the model and its predictability on the real world. Dossey's revised model will be discussed later.

3.3.2 Features of the modelling process

A good model simplifies reality enough to allow mathematical calculation (Steward, 2003:25). The modelling process is not a linear process rather a cyclic one as discussed in 3.3.1. The modeller must make use of theory and data presented (Verschaffel *et al.,* 2000:168-172). The bases of all sub-processes are theoretical knowledge and explicit or implicit available data. The theoretical knowledge varies from expert mathematization to ad hoc assumptions. The modelling process is not static and unchangeable. This process is not a perfect representation of mathematical modelling but aims to provide possible guidelines for implementing the classroom setup (Verschaffel *et al.,* 2000:168-172).

Mathematical problem solving according to Blomhøj (2004:150) forms an integral part of the modelling process. The real life problem should be guided by the modelling process and problem solving is subordinate. Mathematical modelling forms an important aspect of the need for the exploitation in the technological sphere. Modelling activities according to Blomhøj (2004:145) might motivate the learning process and develop cognitive roots. Cognitive roots play an important role in construction of mathematics concepts. Modelling elicits activities which require the learners to interpret realistic and complex situations mathematically, by applying, modifying, or extending constructs, thereby developing a mathematical model (Gainsburg, 2007: 37).

The enhancement of strategies for and attitudes about mathematical modelling and problem solving have the following common characteristics:. the use of more realistic and challenging tasks; the use of a variety of teaching methods and learner activities, which include expert modelling of strategic aspects of competent solution process; small group work and whole group discussions; the creation of a climate that is conducive to developing learners' views of mathematical modelling and beliefs and attitudes (Verschaffel *et al.,* 2006:61). Mathematical modelling seems to challenge learners to use mathematics to describe and analyse everyday contexts in a mathematical way using basic mathematical concepts. Mathematics can by mathematical modelling be experienced as a means to describe, analyse, draw conclusions, and make predictions into a broader understanding of every-day life (Blomhøj, 2004:145).

3.3.3 Mathematical modelling as a teaching tool

The teacher needs to create an environment where the student can work with real life contextual problems which are familiar and which allow them to apply their mathematical knowledge in a modelling process (Carpenter & Romberg, 2004; Romberg, Carpenter & Kwako, 2005:21). To introduce learners to modelling, the teacher according to Lehrer and Schauble (2005:32) and Romberg and Kaput (1999:10) needs to develop the learners' meta-representational capabilities which include representations such as diagrams, maps, drawings graphs, text, etc.

The teacher also needs to differentiate in his/her class the level of the learners' mathematical ability thus to fit the real life problem to the level of the learner (Carpenter & Romberg, 2004; Romberg *et al.,* 2005:21).

Teachers who place high value on learners self-invented procedures, who place an emphasis on the connections within mathematical knowledge in skills, who understand and know the content knowledge to be taught in depth and breadth (Schoenfeld & Kilpatrick, 2008:2), who provide opportunity for learners to enhance mathematical modelling ability, prove to be very effective teachers according to Verschaffel, Greer and Torbeyns (2006:55). The use of more realistic and challenging tasks than what is offered in the traditional textbooks, the use of a variety of teaching methods and learner activities which include expert modelling strategies, and the creation of a classroom climate conducive to the learners' elaborative views of mathematical modelling, which is accompanied by beliefs and attitude, is essential for learners becoming mathematicians (Verschaffel *et al.,* 2006:61).

The tasks given to learners must be relevant, thus motivating learners to seek information making the more sense of the problem situation (Romberg & Kaput, 1999:12). The components of the modelling process with its sub- competencies and the argument of justification can be used as tools for planning teaching and learning activities. The justification of mathematical modelling as an element of mathematics teaching has been an issue in mathematical educational research for many years Blomhøj (2004:156). There is an increase in the importance of modelling as an object of mathematical teaching and learning (Goldstein & Hall, 2007:57)

Modelling eliciting tasks which are open-ended will provide learners with an opportunity to find relevant problems in complex situation (non-routine problems), develop representational tools to describe and analyse the problem structure, and to compare different approaches to the solution (Goldstein & Hall, 2007:57). Modelling–eliciting activities are the vehicle for developing understanding of mathematical concepts (Verschaffel *et al.,* 2006:61). Modelling activities might create suitable cognitive roots for learners' construction of mathematical concepts. Mathematical knowledge which includes concepts and procedures according to Blomhøj (2004:150) is not a pre-requisite for modelling activities. Knowledge of the process of mathematical modelling is important, structuring learners' mathematical experiences.

The key element in any powerful practice in the mathematic classroom is mathematical modelling, the construction of an argument to justify and make generalizations (Carpenter & Romberg, 2004:3). Modelling practices provide the unique learning environment, which engage learners' interest in the real-world (Goldstein & Hall, 2007:57). The focus of most schools' curricula is on concepts and procedures (Carpenter & Romberg, 2004:3). For learners to fully understand and assimilate with mathematics they need to do what mathematicians do. Learners need to participate in the practices of mathematicians. Mathematicians are mainly, according to Carpenter and Romberg (2004:3), busy with practices of modelling, justifications and generalizations. These identified practices did not receive explicit recognition in traditional instruction.

The engagement of learners in practices which involve the construction, evaluation, and revision of mathematical models is a primary element of mathematical behaviour (Little & Jones, 2007:50; Little, 2008:75). These key elements are not learnt in abstract isolation. The acquisition of mathematical content, according to Carpenter and Romberg (2004:4), runs concurrently with the critical practices of modelling, justification, and generalization. Understanding mathematics involves actively participating in the practice of modelling, justification, and generalization. An excellent mathematical modeller has an excellent understanding of mathematics content in depth and breadth (Carpenter & Romberg, 2004:4; Blomhøj, 2004:150; Schoenfeld & Kilpatrick, 2008:2; Verschaffel *et al.,* 2006:55).

3.3.4 Developing modelling practices

For learners to act as mathematicians they need do what mathematicians do. The learners need to involve themselves with the practices of mathematicians. The learners need to be involved in constructing different types of models especially explanatory models. The explanatory modelling process is regarded by Carpenter and Romberg (2004:6) as the highest level of mathematical modelling. Explanatory modelling needs to be a long term goal of teachers teaching mathematics and needs to start in the lower grades. Learners also need to be involved in representational models in class, which involves the development of carefully structured physical tasks which guide the need to conceptualize, test, and refine ideas about a given task (Romberg *et al.,* 2005:15). The activities learners do need to involve modelling practices which include constructing, evaluating, and revising models (Carpenter & Romberg, 2004:18).

A teacher who wants to start introducing modelling needs to rather start with the look alike model. During this process the teacher and the learners gain valuable information of the most basic kind of modelling practices. The teacher might start with physical models of familiar situations. The next step is when the teacher applies the “function alike" type of modelling practice. Having acquired the basic understanding of basic modelling the learners can be introduced to the more abstract forms of modelling. The descriptive modelling process can then be used. The learners in grade 1 to 3 will, according to Carpenter and Romberg (2004:10), be able to deal with descriptive modelling practices. The practices of observation, data recording, data organising, and communication from the descriptive modelling inform and set the basis for engaging in explanatory modelling practices. The learners who are participating in explanatory modelling, justification, and generalizations do what mathematicians do. Teachers need to develop or identify appropriate model-eliciting tasks (Lehrer & Schauble, 2005:37). Good modelling problems provoke in learners variability in thinking, conceptual understanding, provide feedback for model testing and model revision (Lehrer, & Schauble, 2005:38).

3.4 POWERFUL LEARNING ENVIRONMENTS

3.4.1 Defining a powerful learning environment

Powerful learning environments function within the constructivist paradigm (Van Petegem *et al.,* 2005:2; De Corte *et al.,* 1996) and enable learners to actively construct their own knowledge (De Corte *et al.,* 1996; Weinert, 1996:3). A powerful learning environment evokes a constructive modelling-based learning process. Powerful learning environments are based on the theories of constructivism (Van Petegem *et al.,* 2005:2). Within constructivism, learners are viewed as active participants of their learning process who construct their own meaning (See par 2.2.4). Learners are not merely passive recipients of a knowledge transfer (see par 2.2.1). Powerful learning environments enable learners to construct new knowledge and then put it in to pragmatic practice (Van Petegem *et al.,* 2005:2). The reproduction of knowledge appears not be enough in this new age of knowledge explosion (Van Petegem *et al.,* 2005:2). Learners must also be able to apply their knowledge to changing and real world contexts.

Powerful learning environments include the following components: a presentation on content that delivers the basic knowledge; collaboration and coaching opportunities, and problem based tasks. All the components in the learning environment must be designed to work towards achieving the objective and personalized learning goals. The environment is learner centered; the collaboration and coaching opportunities promote a deeper comprehension of the concepts learned. Knowledge is made applicable by the problem based tasks (Van Petegem *et al.,* 2005:3). Learning environments must elicit active learner engagement by means of encouraging learners to: construct and restructure their own meaning and knowledge; monitor their own learning processes, and apply their new skills to solve real world phenomena. Powerful learning, according to Van Petegem *et al*. (2005:9) is an active process regulated by learners' goals that are social in nature.

The aims of the learning environment are learners' acquisition of strategy (competent five stage problem solving model utilizing eight heuristic steps) to solve mathematical application problems and the acquisition of appropriate beliefs and positive attitudes towards mathematics learning (Verschaffel *et al.,* 2000).

The modeller must, in stage one of the five stage model, build a mental representation of the problem (Verschaffel *et al.,* 2000). The learner must be able to draw a picture, make a list, a scheme, or a table; distinguish relevant from irrelevant data and use their real-world knowledge in step one of the heuristics.

The second stage in the model is to decide how to solve the problem. The heuristic strategies involved are the construction of a flowchart; guess and check; look for a pattern and to simplify the numbers.

For the third stage, the necessary operations must be carried out, followed by an interpretation of the result/s and formulation of the answer.

The last stage involves an evaluation of the solution (Verschaffel *et al.,* 2000).

3.4.2 Framework for designing powerful learning environments

De Corte, Verschaffel and Masui (2004:368) have proposed the CLIA-model as a framework for designing powerful learning environments (De Corte & Masui, 2009:173). This model consists of interconnected components, namely; competence, learning, intervention and assessment (CLIA) (De Corte & Masui, 2009:173; De Corte, Verschaffel & Masui, 2004:368). The competence component refers to a learner acquiring a sufficient domain specific knowledge base, heuristics methods, high level of meta-cognitive knowledge, self-regulatory skills, and positive believe about him-/her-self (De Corte & Masui, 2009:173; De Corte, Verschaffel & Masui, 2004:368). The learning component has the characteristics as highlighted in the definition of learning in 2.2.6 (De Corte & Masui, 2009:173; De Corte, 2000:8; De Corte, 2004). According to De Corte and Masui (2009:173) refers to the intervention component as the guiding principles as highlighted in 3.4.3/3.4.4 for designing powerful learning environments. The last component, assessment, is integral as it aligns the preceding components of the CLIA-framework. The assessment component, according to De Corte and Masui (2009:174), should:

- Monitor learner progress in acquisition of competence component
- Provide diagnostic feedback, understanding, mastery and usage of their learning and thinking skills
- Contain assignments offering opportunities for self-regulation and collaboration

- Help learners to develop skills in individual and group self-assessment.

3.4.3 Guidelines for constructing a powerful learning environment

Each and every learner has the capability to learn the necessary knowledge skills and values (Verschaffel & De Corte, 1995:64; De Corte, 1996:37; De Corte, 2000:8). The constructive, accumulative and goal orientated acquisition processes should be induced by the learning environment. Acquisition processes should be embedded within an authentic context to which the student can relate (Little, 2008:75; Little & Jones, 2007:50). The learner must become actively involved in the learning process and take responsibility for his/her own learning. Self-regulation of the learning processes should be fostered by the learning environment. An increase in the learner's self-belief, confidence, and competence needs to be followed up with an external regulation of the acquisition of knowledge and skills (Verschaffel & De Corte, 1995:64; De Corte, 1996:37; De Corte, 2000:8). The learning processes need to be realised in a learning environment which is trusting and original within meaningful contexts (De Corte, 2000:8). The teacher also needs to realise the external regulation of knowledge and skills gradually.

Resources and learning material need to be present in the learning environment (Verschaffel *et al.,* 2000). Learners must be given sufficient opportunities for teamwork. The instructional support should be flexibly adaptable to cater for the differentiation of cognitive, affective and motivational characteristics (De Corte, 2000:8). The individuality of the learner needs to be considered. The acquisition of meta-cognitive skills should be integrated within the subject matter domain. Both the cognitive and the meta-cognitive knowledge and skills, play a complementary role in the learning and thinking process (De Corte, 1996:37; De Corte, 2000:8). With the introduction of mathematical modelling, self-regulatory learning activities such as analysing the problem, monitoring the solution process and evaluating its outcome, are strongly developed (Verschaffel *et al.,* 1999).

A critical factor of the problem solving learning environment is that the learners need to be asked to describe and explain their strategies being used to solve any given problem (Carpenter *et al.,* 1999:59). These discussions of alternative strategies have a dual function in that it is reflective and articulates opportunities and also constructs relationships amongst different strategies (Carpenter *et al.,* 1999:59).

3.4.4 Principles of a powerful learning environment

To guide teachers who want to construct powerful learning environments, certain principles were established. De Corte (1996:38) has provided the following principles:

- The learning environment should be supportive. Constructive, collaborative, and self-regulatory processes should be initiated. Mathematics is a constructive activity, therefore, learners need to be provided opportunities to gather discover and create their own mathematical knowledge and skills.

- The learning environment needs to be flexibly adaptable.

- Taking the collaborative nature of effective learning the powerful learning environment should evoke in all learners meaningful learning with understanding. Progressing towards higher levels of cognitive development, abstraction and formulation, thus bridging the gap between informal strategies and formal mathematics (Verschaffel *et al.,* 2006:55).

- The learning environment should be embedded with non-routine word problems (De Corte & Masui, 2009:174; De Corte, 1996:38; English & Watters, 2004:60; Verschaffel, Greer & De Corte, 2000:132). The use of meaningful or realistic context problems as anchoring developmental points, mathematics must not be divorced from reality (Verschaffel *et al.,* 2006:55).

- The learning environment should provide opportunities for attaining general skills embedded in the subject matter (De Corte & Masui, 2009:174; De Corte, 1996:38).

- The learning environment should create a classroom culture that encourages learners' explication of and reflection on learning and cognitive activities (De Corte & Masui, 2009:174). Learning should focus on social interaction and cooperation which intends to develop learners' reflective skills, and interconnecting the various learning strands within mathematical teaching (Verschaffel *et al.,* 2006:55).

Learners need to become progressive agents of their own learning. The process of becoming agents of one's own learning should happen with a lot of care changing from transmission to a constructive cumulative self-regulated, goal orientated, situated, collaborative participant (De Corte, 1996:38; De Corte, 2000:12). The learners' tasks should be visionary based and meaningful. The teaching and learning process should be holistic.

Learners react differently to different situations which might face them. One the most intrinsic tasks of a teacher is his/her ability to construct a learning environment which is powerful. A powerful learning environment, according to Weinert (1996:3), can be described as an environment in which the learners effectively learn required, applicable knowledge, skills and values. The creation of a powerful learning environment presupposes that the teacher has the necessary knowledge and skills regarding learners' learning processes (Weinert, 1996:26). The teacher should have a clear understanding of how his/her learners learn.

3.4.5 The starting points for a powerful learning environment

A powerful learning environment must not only be constructed for the mere construction of a learning environment. Within a powerful learning environment, learning is experienced as a constructive process. According to Verschaffel and De Corte (1995:74) these are starting points for a powerful learning environment:

- The skill of learning, thinking, and problem solving is important
- In the acquisition of a mathematical disposition it is important that the learner's learning orientation develops
- Learning is an active process. The constructive acquisition of knowledge and skills can be guided through collaborative and tailor made interventions which include all the stakeholders.

It is of the utmost importance that both the teacher and the learner see the learning environment as an essential part of teaching and learning. The learner must be actively involved in the teaching and learning process.

3.4.6 The major foundation pillars of the powerful learning environment

Verschaffel, *et al.* (1999:202) have identified three major pillars which take into account the strong relationship between the meta-cognitive strategies and the domain related beliefs and attitudes. These major pillars shape the foundation of the powerful learning environment.

- Problem design

The use of a varied set of carefully designed realistic complex and open problems is the first major pillar. The nature of the problem designed is such that it falls within the learners' contextual sphere and age, yet it also should challenge learners to apply their meta-cognitive and heuristic strategies (Verschaffel *et al.,* 1999:202). The problem also should be stated such that it is open to be answered in different ways using modelling, justification, and generalization.

- Instructional techniques

The use of a varied set of instructional techniques is the second pillar. The exploratory teaching method which involves a short introduction, group assignment, followed by whole class discussion, an individual assignment also followed by whole class discussion should be used. The group activities involve scaffolding, jigsaw, and other collaborative activities (Verschaffel *et al.,* 1999:202).

- Classroom culture creation

The creation of a classroom culture aimed at the development of new social and socio-mathematical norms about teaching and learning mathematical problem solving. The learners are stimulated to reflect upon their beliefs, misconceptions, and the modelling process. The learners are also involved in debating new norms, good responses, and solution processes (Verschaffel *et al.,* 1999:202).

From the above discussion on pillars it is evident the role of the teacher is imperative in each of the pillars. The teacher's role is creating a powerful learning environment in which learners are stimulated.

3.4.7 Essential components of a powerful learning environment

The essential components of a powerful learning environment, according to De Corte (2000:15) are:

- Teaching and learning content

The acquisition of cognitive and meta-cognitive strategies plays a very important role in solving contextual mathematic problems. The solution process involves different phases which include heuristic strategies. The teacher should facilitate the learning of meta-cognitive and cognitive strategies within the subject-matter domain (De Corte, 2000:15).

- The nature of problems

A varied set of realistic, complex, and open problems should be used in the teaching and learning process (Little, 2008:75). These problems differ from the problems found in the traditional textbooks and study guides. Different sources such as newspaper articles, a brochure, a comic strip, or a combination of different sources, can be used when setting or designing the problems (De Corte, 2000:15).

- Teaching strategies

Killen (2000: v) refers to six important teaching strategies which can play an important role in the creation of a powerful learning environment. These six strategies according to Killen (2000:1; 31; 72; 99; 128; 169) are: direct teaching; discussions; small group work; cooperative learning; problem solving and research.

Some of the strategies such as direct teaching can be closely associated with the traditional teacher centred approach (Killen, 2000: xiii). Kindsvatter *et al*. (1988) holds the view that different teaching strategies which are flexibly used might boost the learner's interest and performance. The teaching strategy should equip learners to demonstrate the required outcomes (Killen, 2000: xiii).

From the above discussion (Killen and other researchers) it seems that the use of a single strategy would not always suffice for effective learning. The challenge to teachers is to make informed decisions on the correct teaching strategy in a powerful learning environment to evoke in learners their meta-cognitive and cognitive skills

- Classroom culture

An innovative classroom culture should be designed through establishing new socio-mathematical norms. Focus needs to be placed on positive attitudes and values of both teachers and their learners. The learner should be given a platform and stimulated to articulate and reflect the solution process they are involved with (De Corte, 2000:15).

3.5 MODELLING-BASED LEARNING ENVIRONMENT

The mathematics education system must, according to English (2007:121), give greater attention to developing mathematical modelling abilities in young children, especially because of its applicability outside the school classroom. Within a constructivist classroom (Cobb, 1988:87), the learners and teachers are involved in practices of modelling, generalization, and justification (Carpenter & Romberg, 2004:3). The construction of relationships, extending and applying mathematical justifying and explaining generalizations and procedures and taking responsibility for making sense of mathematical knowledge will yield improved understanding and optimize functioning on the higher cognitive levels (Cobb, 1988:90; Yager, 1991:55; Verschaffel *et al.,* 2000).

In a modelling-based learning environment, learners will be actively involved in constructing solutions to real life problems, which have been identified as a key area, contributing to poor performance (See par problem statement in 1). Learners will be applying existing knowledge to generate new knowledge. The instructional tools are used to present the constructed models and the generalization will provide the context and focus for the classroom discussion (Carpenter & Romberg, 2004:32). In classes where learners are involved in modelling practices, the learners' reasoning becomes logical.

Learners reason on alternative strategies and different ways of interpreting mathematical ideas. The learners develop a habit of validating an explanatory model and making generalizations. Mathematics becomes a language of thought rather than just a collection of facts and doing exercises applying learnt algorithms. In the modelling learning environment, the opportunity for learners to share models, arguments, ideas, and generalizations is created. The learners in these classes generate and justify acquired knowledge and skills. The teacher is not the sole provider and arbiter of what knowledge is important and what is not. In this modelling learning environment the teachers and learners both develop predispositions to understand and engage in practices of modelling (Carpenter & Romberg, 2004:32). The teacher and learners strive to understand and engage in these practices which include modelling, generalizations and justifications. The teacher and the learners must be reflective about the modelling process they are engaged in. Both teachers and learners look for relationships, critical evaluations, and new production relationships. The learners are viewed as mathematical modellers who have problem solving skills and can construct and validate models and make generalizations, thereby extending their knowledge base.

3.6 MODELLING AS A MEANS TO SOLVE NON-ROUTINE WORD PROBLEMS

Research has shown that learners from the primary school do struggle with non-routine problems such as: A ship carrying a cargo of five sheep, ten bulls, and seven pigs. How old is the captain? Most learners gave: the captain is 22 years old, which is not a realistic response (see par 1.1; Xin, 2009:161; Verschaffel *et al.,* 2000:3). The none-realsitic responses to non-routine wordproblems (see par 1.1), might be addressed by mathematical modelling (Verschaffel *et al.,* 2000:3) Mathematical modelling, according to DoE (2008:11), provides learners with the powerful and creative means to mathematically analyse and describe their real life situation. The key elements in any powerful practice in the mathematic classroom are mathematical modelling, the construction of an argument to justify and making generalizations (Carpenter & Romberg, 2004:3), which is analogous to the expectations from DoE. The modelling process seems to provide the elements of the required guidelines identified. Mathematics can, by mathematical modelling, be experienced as a means to describe, analyse, draw conclusions and predictions into broader understanding of everyday-life (Blomhøj, 2004:145).

Research evidence shows that learners who engage in modelling practices develop modelling and critical skill, greater change to develop mathematical inventions, develop a mathematical disposition, become more socially inclined and develop a deeper understanding of the nature of maths (Verschaffel *et al.,* 2000: 172; Romberg, 2001), which has been identified as a key area contributing to poor performance (See par problem statement in 1). Learners understanding of equality has proven to be problematic seen from research (Carpenter, et al., 2005:83).

Using modelling strategies (Carpenter, et al., 2005: 84-87), learners develop a better understanding of the value of the equal sign and its role in understanding relationships. Providing learners with the correct context for discussion on a range of different true and false statements can be given (Carpenter *et al.,* 2005: 84-87) and by providing learners with practical illustrations assisted in guiding learners to develop better understanding of the equal sign and its role in relationships (Mathematics in context development team, 1998). Possible activities include the comparing quantities e.g. battering activity, banana activity, carrot activity and thirst quencher activity (Mathematics in context development team, 1998). These activities provide learners with a reasoning opportunity from real life context.

A learner with mathematical modelling ability, who is confronted with a non-routine situation will identify the problem, make assumptions, collect data, propose a model, test the assumptions, and refine the model if need be, fit the appropriate model to data, analyse the underlying mathematical structure of the model, and evaluate and be sensitive to the conclusions (Dossey *et al.,* 2002: vi). If a learner with mathematical modelling ability is given a model, the learner will work backwards, to uncover implicit assumptions, critical evaluate the appropriateness of the assumptions, and be sensitive to the conclusions (Dossey *et al.,* 2002: vii). Within the model research facet the learner will investigate a specific phenomenon to gain deeper insight into some behaviour and the learner will use that which has already been created or discovered (Dossey *et al.,* 2002: vii). From the above definition, it seems fair to assume that the application of mathematics in the real life context, constituted mathematical modelling implicitly or explicitly.

3.7 MODEL ELICITING TASKS AS A VEHICLE TO DEVELOP MODELLING SKILLS

To develop in learners the ability to construct a mathematical model, the teacher needs to provide learners with modelling eliciting activities (MEA) (Thompson & Yoon, 2007:193; Lesh & Caylor, 2009:339; Lehrer & Schauble, 2005:37) as a key component of a modelling-based environment. Model eliciting tasks provide learners an opportunity to create physical or mathematical descriptions of nature (Lehrer & Schauble, 2005:37). Model eliciting activities require learners to explicitly develop a model of the problem situation/phenomenon they are faced with (Lesh & Caylor, 2009:339). Problem situations are given to learners via word problems, which can be a textual description with a mathematical question in context (Palm, 2009:3; Frankenstein, 2009: 112). Word problems, according to Van Dooren, *et al.* (2005: 266), may trigger in learners a set of implicit rules and expectations established by socio-mathematical norms of the classroom setting. Word problems according to Palm (2009:3) provide the possible link between the abstract nature of pure mathematics and its application to real life contexts. Palm (2009:5) and English and Watters (2004:60) propose authentic word problems to replace normal school mathematical word problems which are devoid from authentic real life contexts. Many traditional word problems are referred to by researchers as pseudo-realistic word problems (Palm, 2009:4; Verschaffel, Greer, & De Corte, 2001: 32).

The mathematics teacher needs to pose questions like (Palm, 2009: 7/8; Little, 2008:75; Little & Jones, 2007:50):

"225 learners go by bus on a school trip. Each bus can hold 52 learners. How many busses are required?" (Peter-Koop, 2003:454; Verschaffel *et al.,* 1999: 207; Verschaffel, Greer, De Corte, 2000:88)

The lift at a shopping mall has the following sign: "The lift can only carry 11 people. In the morning of a clearance sale 253 people are in a rush to get to the sale on the top floor. How many times must the lift go up?" (Peter-Koop, 2003:454; Verschaffel *et al.,* 1999: 207; Verschaffel, Greer & De Corte, 2000:88)

All the learners of a local school will for the spring 10day holiday go on a school trip together in busses. The organising team have decided that all the learners will be transported by bus.

As part of the organizing team you are requested to order the buses. The name lists show that the total of learners is 360. Your teacher said to you that you can order the buses from Xholi-bus and that each bus can hold 48 learners. Fill in the note below, which you are going to send to Xholi-bus to order the buses.

Xholi-Bus Order Your Name:... School:... Date of trip:... Number of busses to order...

The teacher also needs to, according to Verschaffel *et al.,* (2009:145), provide learners with problems which allow them to create or pose mathematical realistic problems (Little & Jones, 2007:50) by providing them with the problem posing task such as:

- Invent stories belonging to the numerical problem 100 divided by 8, such that the result is, respectively: 13; 12; and 12 remainder 4; & 12.5.

These type of problems provide learners with an opportunity to bring their own problems statements in to being (Veschaffel *et al.,* 2009: 146;). The non-routine nature of the problem posing tasks can reveal to the teacher the difficulties experienced during the phases of a modelling cycle (Veschaffel *et al.,* 2009: 146).

The teacher can also provide learners with problem solving tasks (Selter, 2009:318; Veschaffel *et al.,* 2000:6/88/89) such as:

- Solve the following three problems and write down how you arrived at the answer:
    - o 100 sheep are been transported by a small lorry to be sold at an auction. Each small lorry can hold 8 sheep. How many small Lorries are needed? (Answer: 13 small lorries) This type of problem can be used to interpret the outcome and formulate the answer (Verschaffel *et al.,* 1999:207)

- Grandmother gives her six grandchildren a box containing 78 balloons, which they share equally. How many balloons does each grandchild get? (answer: 13 Balloons)
- A tailor bought a large piece of cloth with the length 75m. He wants to cut the cloth into 6 pieces of the same length. How long is each piece? (Answer: 12.5m)

- Thabo saves money regularly each week. In the first week, he saves R7. Each week after that, he saves R3 more than the previous week. In which week does he save R73? Find his total savings after 30 weeks? (Little, 2008:72; Little & Jones, 2007:50). This type of problem can be used to assist learners to look for a pattern (Verschaffel *et al.,* 1999:2006).

A teacher needs to provide learners with problems which will assist in the development of heuristics e.g.

- Peter would like to make a swing at a branch of an old tree. The branch has a height of 5meters. The wooden seat has already been made. Peter only needs to purchase a piece of rope for the swing. How many meters of rope will Peter need. This type of problem will assist learners to use real world knowledge (Verschaffel *et al.,* 1999:205)
- John and Susan have constructed a miniature city with cardboard, for them to play in. They do not yet have parking for their cars. The available ground space between the church and the city-hall is been earmarked for the parking space. The space available is $50cm^2$. A piece of paper with measurement $50cm^2$ is given to you. Please assist John and Susan to:

  1. Decide how many of the auto mobiles can be parked.
  2. Sketch the possible parkings on the provided paper.
  3. Describe how you arrived at your answer.

  (Verschaffel *et al.,* 1999:209; Verschaffel, Greer & De Corte, 2000:101)

Mathematical modelling improvement happens when learners are being engaged in modelling constantly (Thompson & Yoon, 2007:201). Modelling eliciting tasks need to provoke variability in learners' thinking (Lehrer & Schauble, 2005:38). The misconception of many, according to Thompson and Yoon (2007:202) is that mathematical modelling is only the act of applying traditional mathematics e.g. geometry, algebra, etc. to solve a real life problem. The model already exists and must be placed on the problem situation. Modelling involves creative thinking, use of existing models to ascertain its usefulness. The cultivation of modelling abilities is to teach specific modelling skills (Thompson & Yoon, 2007:203). These skills involve listing factors, making approximations and simplifications, making assumptions, choosing mathematical structures, translating situation model into mathematics, Isolating variables, and reducing a number of parameters (Thompson & Yoon, 2007:203).

The teachers must supplement non-routine realistic problems in daily teaching and examinations, and establish learning environments which cultivate ability to solve such problems (Xin, 2009:174). Learners need to construct own word problems which provide the correct context for the given answers e.g. If given the following rational number: $\frac{37}{5}$, make up a word problem where the answer derived from the rational number ($\frac{37}{5}$) is 7; or where the answer is 8; or where the answer is 7.4 (Fennema, Sower & Carpenter, 1999:193).

Authentic activities, such as investigating what the temperature of water in a container will be if you pour 500ml of water at 80°C and 500ml of water at 40°C into it, according to Wu, *et al.* (2002:4) will assist learners to make sense of problem situations, by confronting their routine understanding. According to Verschaffel, *et al.* (2000:48) the teacher's instruction must consist of the following key elements: concrete materials must accompany the problem being investigated; learners must be created an opportunity to make predictions; learners must execute task and comment on what they are deciding whether they will stick to initial prediction or change it; write down their initial answer.

3.8 THE APPLICABILITY OF THE MODELLING LEARNING ENVIRONMENT TO THE SOUTH AFRICAN MATHEMATICS CURRICULUM

South Africa has been following an outcomes-based education (OBE) system since 1998 (see par. 2.2), which appears to be framed in the constructivist paradigm (DoE, 2002:5). The curriculum has undergone significant changes with the most recent accepted curriculum being curriculum and assessment policy statements (CAPS) (DBE, 2011). The combination of two national curriculum statements, grade R to 9 and grade 10 to 12 respectively into a single comprehensive document to be known as the national curriculum statements (NCS) grade R – 12 (DBE, 2011). The NCS grade R – 12 consists of:

- Curriculum and Assessment Policy Statements (CAPS) for approved subjects
- The National policy for programmes and promotion requirements
- And lastly a national protocol for Assessment Grades R – 12 (DBE, 2011).

The time allocation for mathematics is with first additional language (6hours) more than the other subjects, which is evidence of the high importance placed on mathematics in the intermediate phase (DBE, 2011:6). The five learning outcomes - now called content areas in the intermediate phase are:

- Numbers, operations and relationships
- Patterns, functions and algebra
- Space and shape (geometry)
- Measurement
- Data handling (DBE, 2011:9).

The numbers operation and relationships contribute to 50% of the mathematics in the intermediate phase. Each of these areas has a general content focus and a specific content focus. These general content areas focus on numbers, operations and relationships, which have at its core the development of a number sense. This includes the meaning of and relationship between and relative size of different kinds of numbers, representation of numbers, the effect of operating with numbers, the ability to estimate and check solutions (DBE, 2011:11). The view of mathematics within the South African school context seems to also be framed within the constructivist paradigm (see par 2.2.3).

In the mathematics education field, mathematics is defined differently by different researchers. Some researchers see mathematics as a human activity that deals with patterns, problem solving, and logical thinking in an attempt to understand the world (Menon & Gyan, 2012; Romberg & Kaput, 1999:5; Nieuwoudt, 2003; De Corte, Greer & Verschaffel, 1996). Pretorius (1998: 32) defines mathematics as the construction of knowledge that deals with qualitative and quantitative relationships of time and space. There is an increased attention given to understanding, exploring, and communicating mathematics as opposed to memorizing and rote learning (Kristinsdóttir, 2003). Freudenthal viewed mathematics as a human activity of making sense rather than a subject that can be transmitted (Menon & Gyan, 2012; Nieuwoudt, 2003:13; Freudenthal, 1973), and that reality forms the source and domain of applications in mathematics (Julie, 2012:16). Freudenthal's realistic mathematics education (RME), according to Julie (2012:16), influenced both the development of school mathematics and mathematics educational research internationally. In South Africa, the RME programme came to light via the RME in South Africa (REMESA) movement (Julie, 2012:16). The South African DoE seems to share the viewpoint stating that mathematics is a human activity that deals with patterns, problem solving, critical thinking, and logical thinking (DBE, 2011:4; DoE, 2002:5; NDE, 2002: 4). Mathematics is used to get a better understanding of the world (Pretorius, 1998: 32). According to Ralston (1988: 33), mathematics is the bedrock on which all science and technology rests. Mathematics has both utilitarian and inherent value (Pretorius, 1998: 32). Mathematics, according to Skovsmose (1998: 196), is interpreted as a descriptive tool and as a source of decision-making and action. Mathematics plays a very important role in many areas in real life (Strauss, 1990: 1). According to DBE (2011:4) mathematics is a language that makes use of symbols and notations to describe numerical, geometric and graphical relationships. Schools should, therefore, equip learners with the ability to learn valuable new things (Naidoo *et al.,* 1995:20). For the purpose of this study this researcher uses the definitions as given by DBE (DBE, 2011:4).

Learners' ability to solve problems depends on their ability to draw on the fundamental properties of addition, subtraction, multiplication, and division (Carpenter *et al.,* 2005:83), which is key to the mathematics curriculum in South Africa (NDE, 2002). The development of algebraic reasoning and constructing the appropriate context to develop the problem solving skills are essential in mathematics teaching (Carpenter *et al.,* 2005:83). Mathematics should enable learners to ascertain the connection between mathematics as a discipline and the use of maths to solve real world context problems (DoE, 2008: 11).

3.9 THE TEACHING AND LEARNING OF MATHEMATICS IN SOUTH AFRICAN PRIMARY SCHOOLS

The teaching and learning of mathematics in the South African school context transpires via the mathematics learning area (DoE, 2002), now called the mathematics subject (DBE, 2011). In South Africa, the mathematics learners must acquire certain knowledge and skills as captured in the mathematics learning outcomes, now called content areas in CAPS. The knowledge referred to is knowledge of numbers, operations and relationships; patterns, functions and algebra; space and shape; measurement and data handling (DBE, 2011:4; DoE, 2002:5; NDE, 2002:4). The process skills that the learners need to acquire are: representation and interpretation; conjecturing, estimation and calculation; reasoning and communication; problem posing; problem solving and investigation; modelling; and organizing, describing and analyzing (DoE, 2002:5; NDE, 2002: 4; DoE 2008: 11). Mathematics has both a utilitarian and an inherent value (Pretorius, 1998: 32). Mathematics according to DoE (2008:11) is a concept and process driven subject.

The introduction of CAPS places a more profound focus on mathematical modelling as one specific aim of the mathematics subject in further education and training band. *"Mathematical modelling is an important focal point of the curriculum. Real-life problems should be incorporated into all sections whenever appropriate. Examples used should be realistic and not contrived. Contextual problems should include issues relating to health, social, economic, cultural, scientific, political and environmental issues whenever possible." and "To provide the opportunity to develop in learners the ability to be methodical, to generalize, make conjectures and try to justify or prove them"* (*DBE, 2011:8*). Learners in grade 6 form the part of the intermediate phase group which consists of the grades 4 to 6 (DoE, 2002:31).

The mathematics content to be taught to learners in the intermediate phase is captured in the five Content Areas: Content area 1, with title: numbers operations and relationship; Content area 2, patterns, functions, and algebra; Content area 3, space and shape; Content area 4, measurement; and Content area 5, data handling (DoE, 2002, NDE, 2002). The NDE

(2002:4) also suggests that the teaching and learning of mathematics should focus on developing in learners a critical awareness of how mathematical interactions in social, environmental, cultural, and economical sphere are being applied. An appreciation for the exquisiteness and sophistication of mathematics are also instilled in learners to confront the phobia for mathematics (NDE, 2002:4). Both teacher and learners need to understand what mathematics is. A feeling of inquisitiveness and love for mathematics should be developed in learners. The study of mathematics and the right to study mathematics is a human right (NDE, 2002:4; NDE, 2003:19).

Mathematics is an active constructive based entity fuelled by processes of modelling, justification, and generalization (Carpenter & Romberg, 2004:3). The learning of mathematics is a constructive process. There is an increase in attention on understanding, exploring, and communicating mathematics as opposed to memorising and rote learning (Kristinsdóttir, 2003). Mathematical proficiency is based on learners' ability for:

- *conceptual understanding* – comprehension of mathematical concepts, operations, and relations
- *procedural smoothness* – skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- *strategic capability* – ability to formulate, represent, and solve mathematical problems
- *adaptive reasoning* – capacity for logical thought, reflection, explanation, and justification
- *productive disposition* – habitual inclination to see mathematics as sensible, useful and worthwhile, coupled with a belief in diligence and one's own efficacy (Schoenfeld, 2006:15).

The skill of problem solving is one of the unique features of the teaching and learning of mathematics (NDE, 2002: 4). Having learnt mathematics, the learner will be able will use mathematical language and notation in describing applicable situations and the application of mathematics in a variety of different contexts. The learner will display cognitive, algorithmic, and technological freedom and accuracy. The learners will also be able to integrate mathematical knowledge and skills with other learning areas (NDE, 2002: 5). The objective of the South African mathematics curriculum aims to develop in learners:

- A critical awareness of how mathematical relationships are used in social, environmental, cultural and economic relations
- An appreciation for the beauty and elegance of mathematics
- The necessary confidence and competence to deal with any mathematical situation without being hindered by a fear of mathematics
- A spirit of curiosity
- A love for mathematics. (DBE, 2011: 4; NDE, 2002:5)
- Recognising mathematics as a creative part of human activity
- Deep conceptual understanding in order to make sense of mathematics and an acquisition of specific knowledge and skills (DBE, 2011:4).

Mathematics, according to NDE (2002:5) and DBE (2011:4), will equip learners with the necessary knowledge and skills that will enable learners to:

- Participate equitably and meaningfully (with an awareness of rights) in political, social, environmental, and economic activities by being mathematically literate
- Contribute responsibly to the reconstructions and development of society by using mathematical tools to expose inequality and assess environmental problems and skills
- Display critical and insightful reasoning and interpretative and communicative skills when dealing with mathematical and contextualized problems
- Describe suitable situations using mathematical notation and language
- Apply mathematics in a variety of contexts
- Transfer mathematical knowledge and skills between learning areas and within mathematics
- Display mental, algorithmic and technological confidence and accuracy in working with numbers, data, space, shape, Investigating patterns and relationships

- Problem posing, problem solving
- Constructing new insights and meaning
- The correct use of the language of mathematics.

From the above mentioned knowledge and skills it can be deducted that the learners as practicing “mathematicians” would become meaningful role-players in social, environmental, cultural, and economical spheres.

Modelling begins according to Mudaly and de Villiers (2004:4), with a real life problem situation which may or may not be relatively controlled. Model application in the South African context can be divided into three categories, namely; direct application (model can immediately be used), analogical application (developmental model is similar to existing model) and creative application (creating a complete new model using new techniques and concepts) (Mudaly & de Villiers, 2004:4). At the evaluation phase of the modelling process, the modeller needs to ask and attempt to answer the following question: Why does the result hold true/or not true (Mudaly, & de Villiers, 2004:5). The learner’s mathematical confidence, attitude towards maths, and problem solving behaviour, and learning environment seems to directly impact on the learner’s performance in maths.

3.10 STUDY ORIENTATION AS A CONTRIBUTORY CONDITION IN MATHEMATICS PERFORMANCE

Many learners’, according to Maree, *et al*. (1997:2), study orientation in mathematics is negative. The study orientation of learners towards mathematics (SOM) closely relates to their performance in mathematics (Steyn & Maree, 2003: 50). Study orientation in mathematics was developed for high schools learners in the mid-1990s, where it had numerous adaptations to also focus on the senior phase. Schoenfeld and other researchers have validated that a relationship between attitude, belief, mathematical learning and mathematical performance does exist (Kapetanas & Theodosios, 2007:98). The learner’s level of mathematics confidence is an important success indicator of mathematical performance (Nashon, 2006:1). The SOM questionnaire is, according to Steyn and Maree (2003: 50), a significant predictor for mathematics conceptual understanding and performance. The study orientation in maths (SOM) consists of 5 fields for the intermediate phase. The five fields are: study attitude (SA), mathematics confidence (MC), study habits (SH), problem solving behaviour (PSB), and study environment (SE).

Learners do show a specific study attitude towards mathematics (Maree, Prinsloo & Claassen, 1997:3). Study Attitude deals with feelings such as beliefs about mathematics which can be subjective but also objective experiences and mind sets or attitudes towards mathematics (lack of confidence in own ability) and that affects a learner's drive, expectation and curiosity with regard to mathematics. Attitudes include various factors such as subject enjoyment, self-confidence, usefulness of the subject and the challenges the subject offers (Maree *et al.,* 1997:4). According to Steward (cited in Maree *et al.,* 1997:3), motivation and expectations about mathematics influence a learner's interest in mathematics. The possible reason for low motivation is that the learning content in mathematics is disjointed from the learner's level of knowledge and thinking, which increases frustration and inhibits motivation to perform in mathematics (Maree *et al.,* 1997:3).

Mathematical confidence deals with the feelings of comfort of learners towards mathematics. When learners feel 'uncomfortable' toward mathematics, researchers associate the feeling with anxiety. Examples of mathematical anxiety are excessive sweating, scrapping of correct answers and an inability to formulate mathematical concepts. Research has shown that maths anxiety decreases significantly within constructivist classrooms (Nashon, 2006:2). If mathematical content does not make sense to learners they become more anxious, uncertain and frustrated (Maree *et al.,* 1997:3). The presentation of mathematics in a too abstract or theoretical manner, contributes to the increase in anxiety, hence, a decrease in mathematical confidence (Maree *et al.,* 1997:3). Self-confidence is on the opposite side of anxiety (Maree *et al.,* 1997:3). A mathematically confident learner showcases a high level of understanding of maths and a high level of maths performance.

Study habits are an acquired skill. Study habits deal with the utilization of study strategies which are favourable for effective learning. Effective study habits include attending class regularly and spending sufficient time doing mathematics. Pintrich and Johnson agree (cited in Maree *et al.,* 1997:3), stating that consistent and effective study methods improve study orientation in maths. The study attitude in mathematics culminates in study habits (Steyn & Maree, 2003:50; Maree *et al.,* 1997:4/9).

Problem solving behaviour deals with cognitive and meta-cognitive strategies which include planning, self-monitoring, self-evaluation, self-regulation and decision making during the problem solving process. Learners acquire investigative strategies like searching for patterns, and relationships in mathematics, continuous testing, estimating, and approximating of answers, applying Polya's four steps during problem solving, an ability to abandon strategies which do not fit the solution process and then try alternative strategies to arrive at a solution (see par. 2.4.2; Maree *et al.,* 1997:8). The learner's attitude towards solving of problems contributes significantly to achievement in mathematics. Teachers need to implement meta-cognitive learning strategies (see par. 3.4.2, 3.4.3, 3.4.6, 2.4.1.3; Steyn & Maree, 2003:50; Maree *et al.,* 1997:4/9). Incomplete conceptualization, is a lack of understanding of the relationship between concepts, inhibit problem solving behaviour (Maree *et al.,* 1997:4). Learners' abilities to solve routine and non-routine problems are attributed to positive problem solving behaviour (see par. 2.4.2).

Study environment deals with different aspects relating to the social, physical and perceived environment. Learners from the non–stimulating environment often lag behind. Non-stimulating environments inhibit the mathematical performance of learners (Steyn & Maree, 2003:50; Maree *et al.,* 1997:4, 8, 9). The study (learning) environment impacts significantly on learners' performance in mathematics (see par. 2.3, 2.4, 3.4, 3.5)

A learner's study orientation in mathematics addresses the above-mentioned crucial elements, which play a role in improvement of mathematical performance. A positive study orientation in mathematics implies that the learner has a positive attitude towards mathematics, that the learner is confident in dealing with mathematics, that the learner portrays positive study habits, that the learner shows positive, and effective problem solving behaviour and that the learner is in a positive study environment (Steyn & Maree, 2003; Maree *et al.,* 1997). The SOM questionnaire will provide the researcher with a summary of the above-mentioned matters and a measure of the learner's study orientation in mathematics. According to Dossey *et al.* (2002:56) the belief learners hold about maths plays an important role in the learning process. The study orientation in mathematics does influence learners' problem solving behaviour and, therefore, their performance in mathematics (Maree *et al.,* 1997:4).

3.11 SUMMARY

This Chapter mainly focused on design for modelling as a powerful learning environment in primary schools mathematics classrooms. A theoretical perspective on modelling was given focussing on different viewpoints of modelling and models and highlighting the different types of modelling as a process within the mathematics education domain (see par. 3.2). Modelling must form part of every school's mathematics education (English, 2007). The literature also alluded to the major steps in the cyclic modelling process which involve understanding the phenomenon, model construction, model implementation, model revision (see par. 3.3). Focus was given to developing heuristics using modelling eliciting tasks as a vehicle which mainly consists of non-routine word problems (see par. 3.5). The important role the teachers play in developing modelling practices is key in teaching and learning. Powerful learning environments are discussed in great detail as an essential component of the modelling practices in the above Chapter. The Chapter alluded to a framework for designing, guidelines to construct, principles of, major foundations of, and essential components of powerful learning environments. A theoretical perspective of modelling-based learning environment is given to the reader. The Chapter also addresses the modelling as a vehicle to solving non-routine word problems. An important focus is placed on the applicability of the modelling-based learning environment on the South African mathematics curriculum. The Chapter ends of with teaching and learning of mathematics in the primary schools of South Africa and the study orientation as a key component to help develop an understanding of why learners perform or do not perform in mathematics.

The following Chapter will focus on the quantitative investigation undertaken to further research the objective of the study. In achieving the objectives, this research is divided into five phases (Collins, Joseph & Bielaczye, 1992: 32). A pre-test, interim intervention (with experimental group), post-test, final intervention (with control group), and retention test with an experimental group and a control group is the overall research design.

CHAPTER 4:

RESEARCH DESIGN AND METHODOLOGY

This Chapter initiates the investigation of the research project by providing a detailed description of the research design and methodology used. Figure 4 provides an outline of Chapter 4.

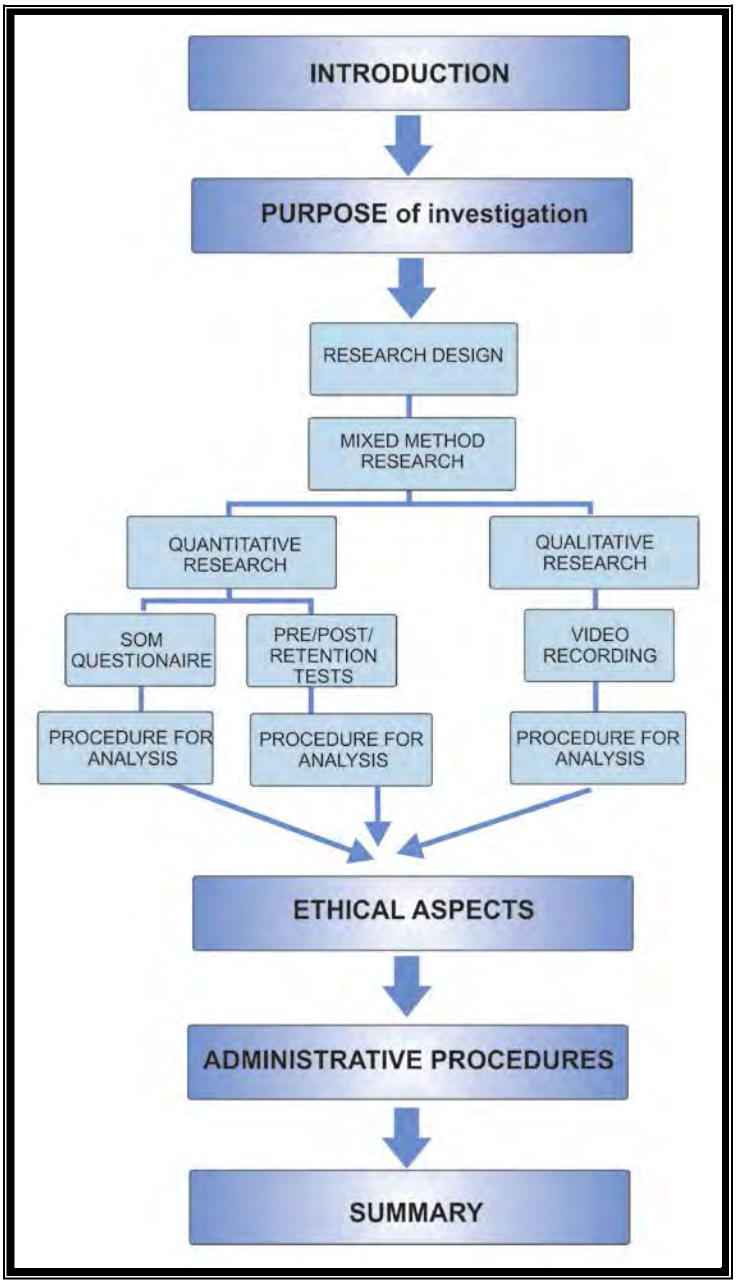


Figure 4: Outline of Chapter 4

4.1 INTRODUCTION

In Chapters 2 and 3, the literature review provided a theoretical perspective regarding the key research questions on constructing a modelling-based learning environment to improve mathematics performance of learners in grade 6. These previous chapters formed the contextual and theoretical framework for the research report.

The questions of *what to investigate* and *how to do the investigation* are been addressed in the descriptive detail of the research design (Van Vuuren, 2008: 179). This chapter describes the systematic and focused investigation approach according to the following topics: purpose of empirical section, mixed method research approach, quantitative research and qualitative research, ethical aspects and administrative procedures.

4.2 THE PURPOSE OF INVESTIGATION

The purpose of the investigation section of this research report is to describe an applicable research design as a scientific process to obtain valid and reliable quantitative and qualitative data concerning the research problem and accompanying research questions guiding this study (Van Vuuren, 2008:179). The research problem involves constructing a modelling-based learning environment for the enhancement of learner performance in grade 6 mathematics classrooms.

The research aim comprises determining the critical constitutes of the modelling-based learning environment; determining what the building blocks for implementing a modelling-based learning environment are; determining how a modelling-based learning environment influences the learners' solving of non-routine problems in grade 6 mathematics classrooms and, determining how a modelling-based learning environment influences learners' study orientation in grade 6 mathematics classrooms (see par. 4.2).

4.3 RESEARCH DESIGN

4.3.1 Mixed method research

Creswell and Garret (2008:322) refer to mixed method as the third movement in the evolution of research methodology. According to Leech, (2010:255), Julia Brannen, Alan Bryman, John Creswell and other researchers played a key role in contributing to mixed method research field. There are numerous definitions of mixed method research (Creswell, 2010:51). Creswell held the viewpoint that a person's method is closely related to that person's philosophy (Leech, 2010:256). Mixed method research is defined as the class of research where the researcher or team of researchers mixes or combines quantitative and qualitative research techniques, methods, approaches, concepts or language into a single study (Creswell & Garret, 2008; Ary *et al.,* 2010:23,559; Tashakkori, & Teddlie, 2010:19; Creswell, 2010:51). Mixed method research is any study that combines both quantitative and qualitative research methods (Moghaddam *et al.,* 2003:113). Mixed method approach does not replace quantitative and qualitative approaches, rather this approach tries to optimise the strengths of both approaches (Ary *et al.,* 2010:559). The incorporation of various quantitative and qualitative strategies into a single project may have either a quantitative or qualitative drive (Morse, 2003: 190). The mixed method approach stands in favour of a pragmatic approach (Ary *et al.,* 2010:559; Biesta, 2010:113), the approach selected in this study to investigate a mathematical modelling approach to practically enhance the quality of learning in school mathematics classes (see par 4.3.2).

4.3.2 The purpose for conducting mixed method research

Researchers tend to base knowledge claims on pragmatic grounds (Basit, 2010:17). The pragmatic approach as a research paradigm allows the researcher/s to work in both the quantitative and qualitative domains. Pragmatism can be defined as the meaning of an idea or a proposition lies in its observable practical consequences, a conduct that emphasises practicality. The founders of pragmatism, such as Pierce, Dewey and others, believed that humans have the capacity to change their conditions through intelligent actions (Maxcy, 2003:51). Human thought is intrinsically linked to their actions (Maxcy, 2003:51). Mixed method research enables the use of quantitative and qualitative methods (Basit, 2010: 17), to address research questions posed. A Quantitative-Qualitative-method approach was used. The researcher undertook a multi-phased study (Ary *et al.,* 2010:561).

There are five general purposes for conducting mixed-method research. These purposes are: triangulation; complementarity; development; initiation and expansion (Ary *et al.,* 2010:561).

- Triangulation purpose:

The purpose of triangulation is to collect, analyse and merge the data from the research to better understand the research problem. Converging evidence from different research methods of the same study or corroborating the findings of one method examining the findings of a different method is essential in triangulation. With triangulation, the researcher can test whether the findings are converging, inconsistent or contradictory.

- Complementarity purpose:

Provides the researcher with an opportunity to use the result of one method to elaborate, illustrate, enhance or clarify the results of the other method in the same study.

- Development purpose:

The focus of this purpose is that the use of results of the one approach will inform or develop the other approach.

- Initiation purpose:

The purpose of initiation is to focus on discovering contradictions in the findings which may in turn lead to reframing a theory

- Expansion purpose:

This purpose of mixed method research focuses on expanding the range and breadth of the research by using different method for different components of the study. Qualitative observations focus on teachers' actions in class, while quantitative tests focus on measuring programme effectiveness (Ary *et al.,* 2010:562).

From the above mentioned purposes highlighted, the complementary purpose is crucial to this researcher to address the objective of his research.

4.3.3 Specific mixed method designs

There are six different types of designs in mixed method research (Ary *et al.,* 2010:563; Creswell *et al.,* 2003: 223). These designs vary according to the value priority, which refers to the level of importance of the quantitative and qualitative component (Ary *et al.,* 2010:562). These designs seem to be mainly sequential or concurrent. Figure 4.1 below provides an overview of the types of mixed-method designs Creswell, *et al.* (2003: 223-236).

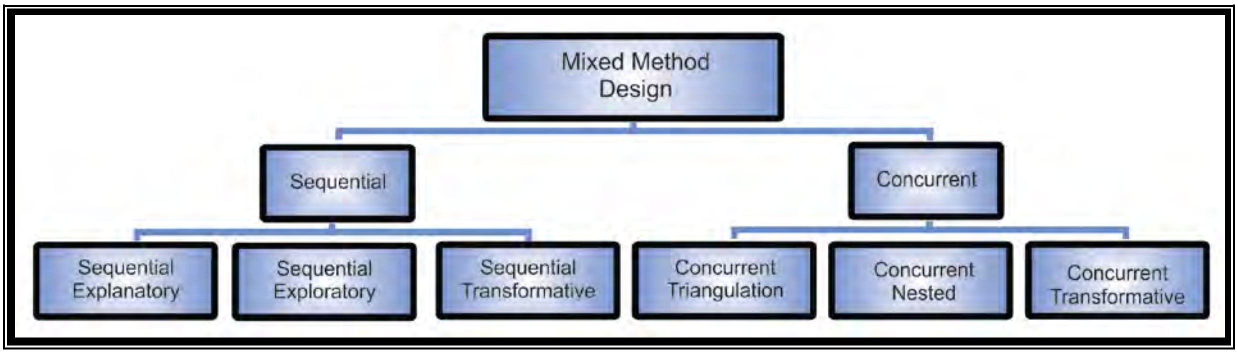


Figure 4.1: Mixed method designs

Sequential designs (Creswell *et al.,* 2003: 228; Nastasi *et al.,* 2010:316):

- Sequential explanatory design (Creswell *et al.,* 2003: 228)

The sequential explanatory design focuses on the collection and analysis of quantitative data followed by collection and analysis of qualitative data. Quantitative data is given a priority, and integration occurs during the interpretation of entire analysis. The design occurs usually over phases

- Sequential exploratory design (Creswell *et al.,* 2003: 227)

This design focuses on the collection and analysis of qualitative data followed by collection and analysis of quantitative data. The main purpose of the design is to use quantitative data and results to assist in interpreting qualitative findings. The design occurs usually over phases.

- Sequential transformative design (Creswell *et al.,* 2003: 228; Mertens *et al.,* 2010:200)

In this design, the priority may be given to either the qualitative or quantitative method or both may be used first. Key in the design is the theoretical perspective which will guide the study. The main purpose of transformative sequential design is to employ methods which will best serve the theoretical perspective that guides the study.

Concurrent design (Creswell *et al.,* 2003: 229; Nastasi *et al.,* 2010:316):

- Concurrent triangulation design

In this design, the researcher is allowed the usage of different methods to confirm, cross-validate or corroborate findings. The data collection for both qualitative and quantitative happens in one phase. The integration of results of the two methods used occur during the interpretation phase.

- Concurrent nested design (Creswell *et al.,* 2003: 229)

Like the concurrent triangulation design, the concurrent nested design also allows the usage of different methods to confirm, cross-validate or corroborate findings. The design may or may not have a theoretical perspective guiding the study.

- Concurrent transformative design (Creswell *et al.,* 2003: 230; Mertens *et al.,* 2010:204).

In both the sequential transformative design and the concurrent transformative design, priority may be given to either the qualitative or quantitative method or both may be used first. Key in the design is the theoretical perspective which will guide the study. Features of the triangulation or nested designs may be used in this design. Two types of data are collect during the data collection phase. The integration of the data may occur during the analysis phase.

From the above six designs the best fitted mixed-method design for this study is the sequential explanatory design to answer secondary questions posed (see par 1.2). The research process involves five phases (see par 4.4). Quantitative and qualitative methods were used sequential explanatory design to realise the research objectives by answering the secondary research questions. The investigation occurred in different phases (see par. 4.4). In Phase 1, researcher used the quantitative approach to address the research aim c and d (see par 1). Learners completed the pre-Math test and pre-SOM questionnaire. In Phase 2 the treatment was given to experimental group (see par.4.4) Qualitative approach were used to gather information (see par. 4.4).

4.3.4 Strengths of mixed method research

Although mixed method research enables the researcher to optimise the good from quantitative research and good from qualitative research approach, it is not 100% cleared from weaknesses. All approaches of researchers have strengths and weaknesses, as is the case for mixed method research (Ary *et al.,* 2010:567).

The strength of mixed method research is in that researcher can take advantage of the combined strengths of qualitative and quantitative approaches. The strength of one method can also be used to overcome the weakness of another method (Ary *et al.,* 2010:23,567). Strong evidence for a conclusion can be drawn through corroboration of findings. Mixed method provide researcher with an opportunity to have better insights which might have been lost using only one approach. Combining the approaches might produce a complete understanding of the phenomenon being investigate (Ary *et al.,* 2010:567).

4.4 METHODOLOGY

A literature study will precede the planning meetings with all relevant stakeholders in the research project and implementation of the phase model. Research follows a mixed method design. The researcher will use a variety of evaluation techniques as proposed by Collins, Joseph and Bielaczye (1992: 9), which include pre-/post-/retention test, pre-/post-/retention SOM, video-recording and, systematic observations of the classroom. Both qualitative and quantitative evaluations are essential for the research design.

In achieving the objectives, this research is divided into five phases (Collins, Joseph & Bielaczye, 1992: 32) as delineated in Table 4.1. A pre-test, interim intervention (with experimental group), post-test, final intervention (with control group), and retention test with an experimental group and a control group is the overall research design (see par Table 4.1). The crossover design will be used. Crossover research design study is a study in which subjects receive a sequence of different treatments (Senn, 1996:2; Berggren, 2012:10). Crossover designs have 2 groups of subjects that switch or crossover. The first group receives the treatment while the second group have no treatment (Senn, 1996:2). A switch over implies that the second group will start with the treatment while first group receive no treatment (Mills *et al.,* 2009). Dangers for the crossover design are the chances of carryover between the phases (Mills *et al.,* 2009; Burnelle, 2000).

The Table 4.1 below provides an overview of the crossover design model where the intervention is first done with the experimental group and then with the control group.

Table 4.1: Phases of crossover design research model to be implemented

Phases	Activity	Experimental Group School 1, School 2 & School 4	Control Group School 3, School 5 & School 6
Phase 1- Introductory	1) Pre-Test	Learners complete a Pre-test focused on mathematical problems solving/ Modelling	Learners complete a Pre-test focused on mathematical problems solving/ Modelling
	2) Questionnaire on study orientation in mathematics (SOM)	Learners also complete SOM questionnaire	Learners also complete SOM questionnaire
Phase 2- Interim Intervention	Interim Intervention with experimental group	The learners were treated with a modelling-based learning environment which consists out of 20 lessons. Professional development to teachers. For evaluation purposes video-recording and field notes were made during classroom support visits.	Learners underwent their normal regular mathematics programme.
Phase 3 - Experimental closure	1) Post-test	Learners complete a Post-test. Post-test focused on mathematical problems solving/ Modelling	Learners complete a Post-test. Post-test focused on mathematical problems solving/ Modelling
	2) Questionnaire on study orientation in mathematics (SOM)	Learners also complete SOM questionnaire	Learners also complete SOM questionnaire
Phase 4- Final Intervention	Final Intervention with control group	Teachers continue treatment utilising newly acquired modelling learning environment without help of researcher	The learners were treated with a modelling-based learning environment which consists out of 20 lessons. Professional development to teachers. For evaluation purposes video-recording and field notes were made during classroom support visits.
Phase 5 - Control Closure	1) Retention Test	Learners complete a Retention test focused on mathematical problems solving/ Modelling	Learners complete a retention test focused on mathematical problems solving/ Modelling
	2) Questionnaire on study orientation in mathematics.	Learners also complete SOM questionnaire	Learners also complete SOM questionnaire

Phase 1

A pre-test will be conducted here during phase 1 which is the introductory phase, with both the control group and the experimental group. The pre-test has been adapted from Verschaffel, *et al.* (2000:214); Verschaffel and De Corte, (1997: 584); Yoshida, Verschaffel and De Corte, (1997: 334) to fit with-in the South African context. The adapted pre-test will be coded and made reliable and valid by the statistical unit of NWU. The invigilators (teacher participants) will conduct both the pre-test and the SOM questionnaire separately at different times. The invigilators of the pre-test will be trained on how to conduct the pre-test, using the pre-test manual as the guideline. The SOM questionnaires will be given to the learners of both control group and the experimental group (see par Table 4.1).

Phase 2

The completion of activities in phase one leads to activities of phase two (interim intervention phase). Both control and experimental classes focus on word problems. The researcher will replace the stereotyped saturated diet of word problems with realistic non-routine problem situations, which are designed to stimulate learners. The problem situations alert learners to the complexities of realistic mathematical modelling and also allow learners to distinguish between realistic and non-realistic solutions of mathematical application problems.

Learning environment organization

The construction process of modelling learning environment will constitute 20 lessons of 1h to 1.5 hours. This researcher, in consultation with his supervisor, cooperated with regular class teachers; design the modelling-based learning environment. The 20 lessons have been adapted by the researcher from the work of Carpenter and Romberg, and works of Verschaffel and De Corte (refer to literature review). The experimental programme consists of five teaching/learning units (20 lessons). The experimental lessons will run over a time of five to six weeks. During these weeks, there will be no other lessons in solving mathematical application problems in the experimental classes. The target group learners have an average age 12 years, a good attendance at school and mathematics classes. These learners are from a poor, socio-economic status. This researcher requested and utilizes, with support from the supervisor, the teaching learning material, SOM questionnaire, pre-test, post-test and retention test, participants' full support. The NCDoE and principals provided administrative support, and the Statistical Consultation Services of the North-West University provided statistical support.

Parallel to the lessons of the experimental group, learners from the control classes will continue to follow the regular mathematics curriculum, which also involves a considerable number of lessons in normal diet word problems as captured in the curriculum outlined by the Standard Assessment. Due to the difficulty following the same analytical systematic approach to the control groups, the researcher intends to involve instead of two classes, three classes. Five classes were stratified randomly chosen, which are representative on how teaching and learning of word problems are done in the JTG-District of the Northern Cape.

Phase 3

Phase 3 will follow phase 2. A post–test will be conducted during phase 3 (interim conclusion phase) with both the control group and the experimental group. Learners from both the control group and the experimental group will complete the SOM questionnaire.

Phase 4

Phase 4 follows phase 3. The activities in phase 2 would be repeated for the control group. The model developed during phase 2 will be applied to phase 4 with the control group only. The experimental group will continue with the programme having acquired the new strategies, without intervention from the researcher.

Phase 5

Phase 5 will follow phase 4. A final retention test will be conducted during phase 5 (conclusion phase) with both the control group and the experimental group. Learners from both the control group and the experimental group will complete the SOM questionnaire.

4.4. QUANTITATIVE RESEARCH

Quantitative research follows a positivist approach, which focuses on facts, excluding speculation (Brazelle, 2003: 5; Ary *et al.,* 2010:23). Positivist paradigm subscribe to quantitative methodology (Basit, 2010:15, Ary *et al.,* 2010:23; Carr, 1994:716). According to Van Tonder (2000: 358) aims quantitative research to deduct social rules which intend to explain or describe certain social phenomena. Social worlds and physical worlds can be best understood in-terms of objective procedures, underlined by scientific method. Inferences are drawn from statistically valid research (Ary *et al.,* 2010:23). Quantification can be described as a numerical method used to make observations and draw inferences on concrete material according to Best (in Brazelle, 2003: 5). Although absolute objectivism is impossible, researchers agree that quantitative research is the most objective research in the research paradigm (Niemann, 2003:5; Basit, 2010:15). Within the quantitative paradigm, the researchers attempt not to be judgmental. Highly regarded structured methods on data-collection, where answers provided noted as anonymous and logical deductions and findings are made in terms of numbers (Brazelle, 2003:5). Quantitative research methods deals with numeric research data (Gorard, 2010:xxii). The pre-/post-/retention test and pre-/post-/retention SOM questionnaire will be quantitatively analysed with the help of the NWU statistical unit.

4.4.1 Rationale and purpose of quantitative research to be undertaken

The performance of learners before and after treatment (see par 1.3 c) and their study orientation in mathematics before and after treatment (see par. 1) will be measured quantitatively. The purpose of quantitative approach will be used pre-/post-/retention-test as a means to measure effectiveness of the learning environment treatment for the grade 6 mathematics learners. The test will potentially provide valuable information regarding how a modelling-based learning environment influences the learners' solving of non-routine problems in grade 6 mathematics classrooms.

The SOM questionnaire provides research with valuable information regarding how a modelling-based learning environment influences learners' study orientation in grade 6 mathematics classrooms

4.4.2 Population and sample

The population will be from the grade 6 mathematics classrooms in a district in Northern Cape. The participants (the sample) consist of both teachers and learners. Learners in grade 6 have already progressed into Piaget's concrete operational phase (Ojose, 2008: 27; Huitt & Hummel, 2003:2; Huitt, 1997: 2) which allows them not to struggle with reading and writing challenges in dealing with word problems. The sample of learners who will participate in the study will consist of three experimental sixth grade classes (31, 33, and 46 learners, respectively) and three comparable control classes (31, 52, and 32 learners, respectively). The selected classes belong to different gender mixed elementary schools in a district of the Northern Cape Province. The schools are poverty-stricken rural schools. The schools are located in a sparsely populated area. All six teachers of the selected classes will assist the researcher with dissemination of the questionnaires and tests in their respective schools.

The schools have been numbered School 1 to School 6. School 6 that forms part of the control group, has been excluded from the project due to the large number of learners not completing the pre-/post-/retention test and pre-/post-retention SOM questionnaire. The data from school 6 will, therefore, not be relevant to findings.

4.4.3 Role of researcher and teachers in quantivative research

The teachers must work closely with the researcher and be willing to be receptive to innovation and experiment with unproven methods. The teachers can contribute significantly to the research. The teachers must be visionaries who are attracted to innovative practices and not early adopters. Bereiter (2002) warns against early adopters because they are more likely to seize during the project to go for a new, next project. The researcher gets buy-in from teachers and must be patient (Devers & Frankel, 2000:264). The researcher needs to develop and maintain a good relationship with participants for effective sampling and credibility of results (Devers & Frankel, 2000:264).

4.4.4 Variables

Variables, in a narrow sense, are classified as dependent and independent. Dependent variables refer to e.g. behaviour being predicted, while independent refers to conditions being manipulated (Kilpatrick, 2004; Collins, Joseph & Bielaczye, 1992) The dependent variables for this study will include Pre SOM, Post SOM and Retention SOM, also the different SOM fields which include, study attitude (SA), mathematics confidence (MC), study habits (SH), problem solving behaviour (PSB), and study environment (SE).

4.4.5 Measurement instruments

Different instruments will be used by the researcher to evaluate the implementation and effects of the experimental learning environment.

Instrument 1 (Pre-test, Post-test & Retention-test)

A pen and paper test (Adapted from Verschaffel, et al. 2000:214; Verschaffel & De Corte, 1997: 584; Yoshida, Verschaffel & De Corte, 1997: 334) consisting of eight matched pairs of word problems. The eight pairs of word problems given to learners containing S-version of eight problems mixed with P-version of the other eight problems. The same test was used as pre-/post-/retention test. The classroom teacher administered the test. Instructions were kept to a minimum for each problem. Learners wrote answers in the answer box and computations/ other workings in the comment box. Learners stated any problems they experience in the comments box. Each problem pair consists of two parts a standard problem (S-Problem) and a parallel problem (P-Problem). The focus of the S-Problems is on straightforward application of basic algorithm with given numbers. The focus of the parallel P-Problem is on mathematical modelling where assumptions are more difficult. The participants took serious account of the real life context within the problem statement. Problems are mostly chosen from the test constructed by Verschaffel and others from Romberg (Verschaffel, et. al. 2000:214; Verschaffel & De Corte, 1997: 584; Yoshida, Verschaffel & De Corte, 1997: 334). The standard test items consist of the knowledge questions (31.25%) and the comprehension & application (18.75%) of the total number of test items, while the problem test items constituted 50% of analytical and problem solving questions. The test items lend themselves to efficient application of problem solving skills in development of mathematical models.

The answers of learners on each item of the test will be categorized and scored according to Realistic response (RR), Non- Realistic Response (NR) (Verschaffel, et. al. 2000:214; Verschaffel & De Corte, 1997: 584; Yoshida, Verschaffel & De Corte, 1997: 334). The learning environment focused on mathematical modelling and not on computational accuracy, purely technical errors will be considered a correct response. Researcher will use chi-square analysis for practical significance and phi to measure the effect size. The responses from the pre-test, the post-test and the retention test, were scrutinized for evidence of mathematical modelling practices using heuristic strategies.

Instrument 2 which will be used was the Study Orientation in Mathematics (SOM) questionnaire (Maree, Prinsloo, & Claassen, 1997). The primary aim of using the SOM questionnaire is to provide researcher with a possible answer regarding the secondary research question regarding the study orientation in Mathematics (see par 1.2). The SOM questionnaire is standardised for South African learners from grade 7 to 12 (Maree *et al*., 1997:5) and has been tested for reliability using reliability coefficient (r_{tt}) (Maree *et al*., 1997:25) and construct validity (Maree *et al*., 1997: 27). The construct validity was established by identifying and clearly defining the constructs that the SOM questionnaire intends to measure (Maree *et al*., 1997: 27).

The SOM questionnaire has a response rating of five, as per a 5-point Likert scale, where learners will be expected to estimate their response rating as illustrate in Table 4.2 and Annexure B by shading the block of their choice. A Likert scale is used to show differentiation between responses in a numerical form (Basit, 2010:83; Van Aardt & Steyn, 1991:44).

Table 4.2: Response rating of SOM Questionnaire

Possible choice	Rarely	Sometimes	Frequently	Generally	Almost always
Key on Questionnaire	1	2	3	4	5

The SOM questionnaire comprises seventy six questions or items. These items address the five fields of the study orientation in mathematics for Intermediate phase (see par 3.10). The five fields are: Study attitude (SA), Mathematics confidence (MC), Study habits (SH), Problem solving behaviour (PSB), and Study Milieu (environment) (SE).

4.4.6 The dissemination and collection process of the instruments

The six selected schools will all be visited according to a appointment schedule confirmed by schools. The researcher will first meet with the principals off all six schools and the maths teachers of grade 6 classes providing them with a clear description of the research, its aim and objectives timeframe. Schools will hopefully give this researcher permission to conduct research and provide research with all the grade 6 class lists. The researcher will randomly select one class per school to participate in the research. The reason for choosing one class is that it would not disrupt the schools normal programme, which would have been the case if learners from different classes were chosen. During the meeting, the research project with its different phases (see Table 4.1), was explained to teachers and principal. A schedule will be given to schools to clearly indicate when both instruments, the pre-test (Annexure C) and the SOM questionnaire (see Annexure B), will be delivered, when it must be conducted and when it will be collected. Specific dates for the pre-test and the SOM questionnaire will be given to schools. Storage of the pre-test and SOM questionnaire needs to be in the strong room until such time that the learners will be expected to complete the SOM and pre-test. An instruction manual for the pre-test and for the SOM questionnaire will be shared with the teachers, as they will be expected to disseminate, invigilate and collect the instruments from the learners. The researcher will collect the completed SOM and pre-test at the agreed time. All the questionnaires and pre-tests given to schools will be checked and returned, thus, a 100% return rate. Absent learners and incomplete questionnaires and pre-tests are included in the 100% return rate. A favourable return rate of both the questionnaire and the pre-tests assures a representative database of responses from the study population for quantitative analysis and interpretation to contribute to construction of the modelling-based learning environment.

4.4.7 Reliability

Reliability refers to the repeatability of the research process at another time on similar participants in a similar context with similar results (Basit, 2010:69). The way the research is carried out and the data-collection must not be influenced by the difference in sample and setting (Basit, 2010:69). Reliability, according to (Ary *et al.,* (2010:237), refers to the degree of consistency in, for example, the SOM questionnaire as an instrument measures. Reliability can be achieved by standardisation of instruments been used in the research and by cross-checking data. According to Basit (2010:69) reliability is a prerequisite for validity. The Cronbach α Coefficient will be used to test for reliability for the SOM questionnaire used.

Cronbach α Coefficient

The SOM questionnaire is one of the key instruments in the research process (see par 4.5). The Cronbach α coefficient provides the best possible reliability measure when using a Likert scale in questionnaires (Ary *et al.,* 1990: 279; Fraenkel & Wallen, 1990: 136). The Cronbach α coefficient is based on the average correlation of items within the instrument or scale, and is an indicator for internal consistency (Van der Merwe, 2010:300).

This researcher will use the Cronbach α coefficient to establish the reliability of the questionnaire used. The value of α ranges from 0 which means zero reliability, to 1 which means perfect reliability (Ary *et al*., 1990: 279; Fraenkel & Wallen, 1990: 136; Van Aardt & Steyn, 1991:44). A Cronbach α coefficient of greater than 0.5 is acceptable, and larger than 0.6 is substantial (Van Aardt & Steyn, 1991:44; Smith, 2009:284), while a value between 0.7 and 0.9 indicates a very good reliability of the instrument or section of the instrument measured (Van Vuuren, 2008).

4.4.8 Validity

Validity implies that the research conducted measures or describes the phenomenon it intended to measure or describes (Basit, 2010:62; Ary *et al.,* 2010:23; Onwuegbuzie & Teddlie, 2003:354). Validity is seen as the most important consideration in developing and evaluating measuring instruments (Ary *et al.,* 2010:225). According to Basit (2010:64), validity can be directly linked to the confidence of the researcher to make sound deductions or generalizations. Research which does not have sufficient validity to it is worthless (Basit, 2010:64). Basit (2010:64) further concedes that no research can be 100% valid as the threats to validity cannot be totally removed. The instrument's validity does not travel with it, therefore, validity in one population tested does not immediately imply validity in another population (Ary *et al.,* 2010:225). It is, therefore, important to always test the validity of the instrument used in the conditions it is used.

Construct validity of the SOM questionnaire

Construct validity is the validity of inferences made about the construct based on measures, treatment subjects and settings used in the study (Ary *et al*., 2010:291). Construct validity of a test is the extent to which the test is measuring the psychological construct it intended to measure (*Ary et al.,* 2010:291). The constructs of the measuring instrument which are going to be measured have been clearly identified and defined (see par 3.10; Maree *et al*., 1997:27). Construct validity will be ascertained by factor analysis and final communalities (Smit, 2009:284) on the different fields of the SOM. Communalities indicate the percentage of variance of every item that was explained by removed or isolated factors, thus how much information is retained from every item (Van Aardt & Steyn, 1991:44; Hair *et al.,* 1998:233). According to Van Aardt and Steyn (1991:47), is a number of factors, accounting for a large percentage of variance, and high communalities an indication of construct validity. A final communality value >0.3 implies (Van Aardt & Steyn, 1991:44; Van Der Walt, *et al*., 2008: 294) acceptable construct validity (Smit, 2009:338). With the help of the statistical consultation service (SCS) of the NWU, the researcher will ensure validity and reliability of the scores using Cronbach α Coefficient and Factor Analysis (for construct validity), frequency tables, and ANOVA, which will be utilised so as to arrive at an answer to the research question. Statistical analyses were carried out using the Statistica and SAS software tools (SAS Institute Inc. 2011; StatSoft, Inc. 2013).

4.4.9 Data analysis

Descriptive and inferential statistical techniques will be used to organise, analyse, and interpret the quantitative data for both the pre/post/retention test instrument and the pre/post/retention SOM questionnaire.

Descriptive analysis:

Descriptive analysis assists the researcher to describe information contained in the scores using the mean and/or the median. The range as a measure of variability, represent the distance between the lowest and highest scores. The mean as a measure of central tendency and standard deviation as a measure of variability provides useful information the numerical description of the data (Fraenkel & Wallen, 2008:195).

Inferential analysis

Inferential analysis will assist the researcher to make inferences about the population based on the data obtained from the sample. The ANOVA (Analysis Of Variance) will be used as an inferential analysis technique.

Crossover Analysis Of Variance (ANOVA)

The analysis of variance, ANOVA refers to a technique the researcher will use to test for significance in the difference of the two or more sample means (Smit, 2009:339; King & Minium, 2008:336; Fraenkel & Wallen, 2008:195). The research project has two subpopulations - a control group and experimental group. A small p-value ($p<0.05$) will show sufficient evidence of statistical significance (Ellis & Steyn, 2003:51; Field, 2005:371, 405, Steyn, 2012). A small p-value ($p<0.05$) is an indication that the null hypothesis can be rejected (Field, 2005:371, 405). A p-value below 0.05 is called significant, p-values below 0.01 are called highly significant, and p-values below 0.001 are called very highly significant (Garth, 2008:53).

For the analysis in crossover designs, this researcher focusses on the effect tests: The test for carryover effect, the test for period effect and the test for treatment effect (Mills *et al.,* 2009; Brunelle, 2000:5; Berggren, 2012:13). Crossover design yields efficient comparisons on treatments (Piantadosi, 2005). According to Berggren, (2012:13) will the first period only be used, if the interaction between the treatment and the period is significant ($p < 0.05$), thus a carryover effect. A $p > 0.05$ implies no carryover effect. The period effect is significant if the p-value is less than or equal to 0.0500 (Berggren, 2012:13). The partial eta^2-values are the percentage of variance in each of the effects and its associated error that is accounted for by that effect (Brown, 2008:42). The following eta^2-value provides a clear indicator of the effect sizes:

Eta-sq = 0.01: small effect

Eta-sq = 0.06: medium effect

Eta-sq = 0.14: large effect.

The greater a differential treatment effect the more the F-value tends to exceed unity. If the ratio is considerably larger than unity, then the null hypothesis is not true and can be rejected. When comparing two groups the t-statistic, which is the square root of F, can be used. The alternative hypothesis will thus be true, which implies that there exists a significant difference in the means of the two subpopulations (Ellis & Steyn, 2003:51).

Chi-square statistics and effect sizes

Chi-square statistics are used for 2x2-frequency tables and provides a statistic to test the relationship between two dichotomous variables. Chi-square can be calculated by (Spatz, 2011:308):

$X^2 = \frac{n(AD-BC)^2}{(A+B)(C+D)(A+C)(B+D)}$, where A, B, C, D refers to the frequencies of the four cells in the 2x2-table, and *n* is the number of observations.

Determining effect sizes using Phi for 2x2-tables

Phi (φ) is a measure used to determine the strength of association between two binary variables (Wright & London, 2009:182; Steyn, 2012). *Phi* can be calculated by:

$\varphi = \frac{(AD-BC)}{\sqrt{(A+B)(C+D)(A+C)(B+D)}}$ where A, B, C, D refers to the four cells in the 2x2-table, or

$\varphi = \sqrt{\frac{X^2}{n}}$, where X^2 is the X^2-value from a 2x2-table. (Spatz, 2011:308; Steyn, 2012).

The value near zero implies there is no relationship, and a value near to 1 implies there is an almost perfect relationship. The following Phi-values give a clear indication of effect size (Spatz, 2011:308; Steyn, 2012).

$\varphi = 0.10$: Small effect

$\varphi = 0.30$: Medium effect

$\varphi = 0.50$: Large effect

In short, the following techniques will be used in this study to analyse the data:

- Cronbach α coefficient (see par 4.2) for reliability
- Factor analysis for construct validity (see par 4.4.9)
- Eta^2-values for effect size of crossover design statistics
- Chi-square for testing relationships between dichotomous variables
- Phi for effect size of 2x2-frequency tables
- Analysis of variance (ANOVA) to determine the total amount of variability

Report on the Factor analysis will not be needed as the instruments showed a high reliability described by Cronbach α coefficient (see par 4.2) and acceptable construct validity described by final estimate communalities (see par 4.4.8).

4.5 QUALITATIVE RESEARCH

There exist different definitions for qualitative research (Niemann, 2003: 5). Qualitative research can be viewed as a method of data capturing and recording which is open and compliant. Within the qualitative framework the presentation of data is not based on numerical mathematical systems (Carr, 1994:716). Hitchcock and Hughes, according to Niemann (2003: 6), define quantitative research as an approach which gives researchers first-hand experience of the social world of the respondents as part of a population. Qualitative research procedures allow the researcher to derive greater depth of understanding of the research problem (Berg, 2007:2). The interpretive paradigm subscribes to the qualitative methodology (Basit, 2010:15, Ary *et al.,* 2010:29). This researcher has considered the appropriateness of the methods and theories to be used in this research, the perspectives of the participants (Flick, 2006: 14). Through qualitative research, the researcher searches for proper answers to questions by examining social settings and the individuals who inhabit these settings (Berg, 2007:3). According to Berg (2007:3) qualitative research techniques allow researchers to share understanding and perceptions of others and to explore how people structure and give meaning to their daily lives. Qualitative methodology includes direct observation, an overview of the different documents and artefacts, participative observation and unstructured interviews (Van Tonder, 2000: 359), which is essential for the researcher in addressing the objectives of this study. Qualitative research seems to search for answers to questions through examining various social settings and the individuals who reside in these setting Berg (2004:7). The strength of qualitative research is that it provides a framework to investigate social processes (Torrance, 2010: xxviii).

The use of video tapes has become one of the most useful and complete running records available to especially the archival researchers (Berg, 2007:217). Researchers' in the educational field have for long time used the utility video-taping in classroom settings (Berg, 2007:217).

4.5.1 The purpose of qualitative research

Anthropologists and sociologists are seen as the original pioneers who have developed and applied qualitative research methodology. Van Tonder (2000: 358) believes that qualitative research brings humans towards having a closer insight of his/her situation. Berg (2007:7) seems to agree stating that qualitative techniques how people learn about and make sense of self and of others. Qualitative research aims to understand and describe the world as seen through the eyes of the involved participants giving their interpretation of the world. Qualitative research according to (Niemann, 2003:6) try to explain the world as it is experience by the individual. Qualitative research can be used to describe the world in laymen language without the use of numbers, quantitative structures. Qualitative research provides a means of accessing non-quantifiable facts about the actual human participants being observed and talked to (Berg, 2007:7). Concrete measurements are not needed as evidence to validate understanding of a situation. A key task of qualitative research is to identify ways in which definitions of particular situations are created and sustained while multiple perspectives of different participants are simultaneously in play (Torrance, 2010:xxix).

4.5.2 The rationale of qualitative research

The rationale for this part of the investigation in this study was to qualitatively get an understanding of the learners' reasoning when they solved the non-routine problems. This researcher conducted observation using video recordings of few classes and field notes of visits conducted. This researcher made use of indicators to realise the objectives of the study. The following three key indicators (Teacher administration, Teacher & learner activity, and Assessment & Homework) were explored in the classroom observation and field notes of class visits was taken. The indicators focused on the common items which are key in the teaching and learning of mathematics in a modelling-based learning environment

4.5.3 Participants

The population will be from the grade 6 mathematics classrooms in the JTG district in the Northern Cape which is the same as for the quantitative research method. The participants will consist of both teachers and learners. The grade 6 learners' ideal for the research as they already progressed in the concrete operational phase (Ojose, 2008: 27; Huitt & Hummel, 2003:2; Huitt, 1997: 2). For the qualitative research, researcher used the sample of learners from three experimental sixth grade classes (31, 33, and 46 learners, respectively). This researcher video captured two schools and took field notes in the other school. The selected classes still belong to different gender mixed (boys and girls) elementary schools in the JTG district of the Northern Cape Province as mentioned in the quantitative part. The schools are poverty-stricken rural schools. The schools are located in sparsely populated area. The three teachers gave permission to research to visit their classes, to video-capture the treatment lessons, and to take notes of teacher activity and learner activity.

The schools have been numbered school 1 to school 6. School 1, 2, 4 forms part of the experimental schools and schools 3, 5, and 6 forms part of the control group. Grade 6 math classes of School 1 and School 2 have been video captured and notes taken in schools 1, 2, and 4 which form the experimental schools. The video capture provided research with valuable information regarding the actual teacher activity and learner activity with regards to the learning environment treatment.

4.5.4 Role of researcher and teachers in qualitative reseach

The teachers must work closely with the researcher and be willing to be receptive to innovation and experiment with unproven methods. The teachers will contribute significantly to the research. Bereiter (2002) warns against early adopters because they are more likely to seize during the project to go for a new, next project. The researcher will get buy-in from teachers and must be patient (Devers & Frankel, 2000:264). The researcher needs to constantly improve on the design and, therefore, frequently crosses the boundaries between observer and active participant and works in close collaboration with the teachers (Bereiter, 2002). The researcher will try to develop and maintain a good relationship with participants for effective sampling and credibility of results (Devers & Frankel, 2000:264).

4.5.5 Data generation

This researcher used different instruments to evaluate the implementation and effects of the experimental learning environment.

Instrument 3 (video recording and observation and field notes)

This researcher made video recordings of four lessons of the experimental group lessons conducted by the teachers. Observation notes were taken during classes on a continuous base. Reflection meetings to highlight achievements and challenges were being held at the end of each day when class has concluded. Records of reflection meetings were kepted. The records of the three video recorded lesson reflections will be focused on for analysis. The self-made observation instrument takes cognisance of the principles for powerful learning environments, the pillars of powerful learning environments and essential components of a powerful learning environment (Verschaffel *et al.,* 1999:215). This researcher did build ongoing positive relationships with teachers through negotiations of the joint venture.

4.5.6 Data analysis of qualitative research

The classroom observations, field notes and the video tape transcript of the experimental lessons were analysed following Verschaffel *et al.* (2000: 221) and using indicators, namely; (1) Teacher administration, (2) Teacher and learner activity, and (3) Assessment and homework. The analysis and interpretation of these indicators did provide this researcher with valuable information regarding implementation of modelling-based learning environment.

Rigour is a means to demonstrate legitimacy of the research process (Tobin & Begley, 2004:390). Rigour helps to demonstrate integrity, competence, authenticity and trustworthiness (Tobin & Begley, 2004:390). Emphasis will be placed on rigour to ensure reliability and validity.

4.5.7 Trustworthiness

The research is trustworthy if the research procedure can be repeated and similar finding can be attained (Basit, 2010). For the trustworthiness of the research, this researcher focussed on (Tobin & Begley, 2003:391):

Credibility – The data (observation notes, transcripts and records) were thoroughly interrogated to demonstrate clear link between data and interpretation. The use of verbatim examples of participants' responses reflects the range and tone of the responses. Regular discussions were held and adjustments made according to suggestions and recommendations.

Dependability - Researcher ensures that the research process was logical, traceable and well documented in a reflective way by giving detailed account of the research process.

Confirmability – An audit process (Video recordings and observation notes, etc. available at any time) were undertaken looking forward and backward in the research process to ensure that the findings are not figments of the researcher's imagination.

Authenticity- The development of the observation schedule, learning environment treatment lessons, training of reflective discussions with participating teachers, were based on a substantial theoretical bases as described in Chapters 2 and 3.

4.6 ETHICAL ASPECTS OF THE RESEARCH

Ethical issues play a major role in design research framework. The relationship between the researcher and the participants is crucial for the success of the research. This researcher did ensure that the research is conducted in an ethical manner (Basit, 2010: 56). Adherence to the ethical principles in the conducting of research forms is a key part the research process (Berg, 2007:57). The formulation of a code of ethics is to regulate the relations of the researcher to the people and the field they intend to study (Flick, 2006: 45). Bailey (2007: 15) notes three major ethical issues to be considered by the researcher, namely; informed consent, deception, and confidentiality. The research study were conducted with sensitivity, with focus on preserving dignity of the participants, and not harm or hurt them in any way or form during the research process (Basit, 2010:56). This researcher demonstrated an acute sensitivity to the gender, race/ethnicity, and social class disability as highlighted by Basit (2010:57). Flick (2006: 45) agrees with Bailey (2007) noting that research should be based on informed consent. The cross-over design ensures that both the experimental and control group benefitted from this researcher as both groups did undergo the treatment.

Permission needed

This researcher acquired permission from DoE institution, at provincial level, district level and school level. Permission has been requested from the teachers and from SGB of the participating schools and parents of learners involved in the study.

4.7 CONSTRAINTS IN THE RESEARCH PROJECT

4.7.1 Limitations to mixed method research

As a weakness, a single researcher might find it difficult to carry-out both qualitative and quantitative research. A mixed method approach might need great time and financial commitment by a researcher (Ary *et al.,* 2010:568). An understanding of both approaches is very important for the researcher to have. Quantifying and qualitising data can have its own difficulty (Ary *et al.,* 2010:568; Tashakkori & Teddlie, 2003:689). The researcher of this research project did realize that the claims to be made were based on the researcher's influence on the context and understands that these claims may not be generalized to other contexts where research does not directly influence the context.

4.7.2 Limitations in sample

The sample to be used in the project includes only learners from two classes for the experimental group and three classes for the control group which is very small in a quantitative framework. The sample, therefore, cannot be merely considered random and representative of the South African mathematical learner population; hence, the results cannot be merely generalized or extrapolated to the South African mathematical learner population.

4.8 ADMINISTRATIVE PROCEDURES

The researcher was granted permission from the NWU, to conduct the research. The researcher will be accompanied by a letter from the promoter stating the NWU's permission to conduct research. The researcher first met with the JTG District Director (Northern Cape Department of Education), explaining the research and gaining access to the selected schools to conduct the research. Permission to conduct research in the sample of the schools catering for intermediate phase learners in JTG district was obtained from the director of curriculum and the district director of JTG district (see Annexure A). A certified copy of the letter was provided to selected schools. The completion and filling-in of questionnaires and pre-tests, video recording and field notes will be done according an organised administration system. A pre-test booklet and a cover letter for the SOM questionnaire will be given to teachers. The researcher will collect all the questionnaire and pre-test forms from the principals.

Arrangements will also be finalised with the statistical consultative service of the North West University for statistically processing the gathered data.

4.9 SUMMARY

In this methodological Chapter, this researcher describes and explains in detail the empirical study to be undertaken. Focused will be placed on mixed method research as a methodological framework guided by pragmatism. Mixed method design, the type of design, the rational and purpose of the design, its strengths and weaknesses were described and explained. The triangular concurrent type of mixed method design best fitted the research project to address the key research aims.

Both quantitative and qualitative approaches were describe and explained according to rational and purpose, population and sample, validity and reliability. The methodological process was described (see table 4.1). The Chapter ends of with important ethical and administrative processes, which research needs to adhere to.

The next Chapter in the report will provide a detailed description of the statistical data analysis and interpretation and content analysis and interpretation.

CHAPTER 5:

RESULTS, DATA ANALYSIS AND INTERPRETATION

The following schematic figure provides an overview to the course of this Chapter, namely; results, data analysis and interpretation of the results. Figure 5 provide an outline of the Chapter 5.

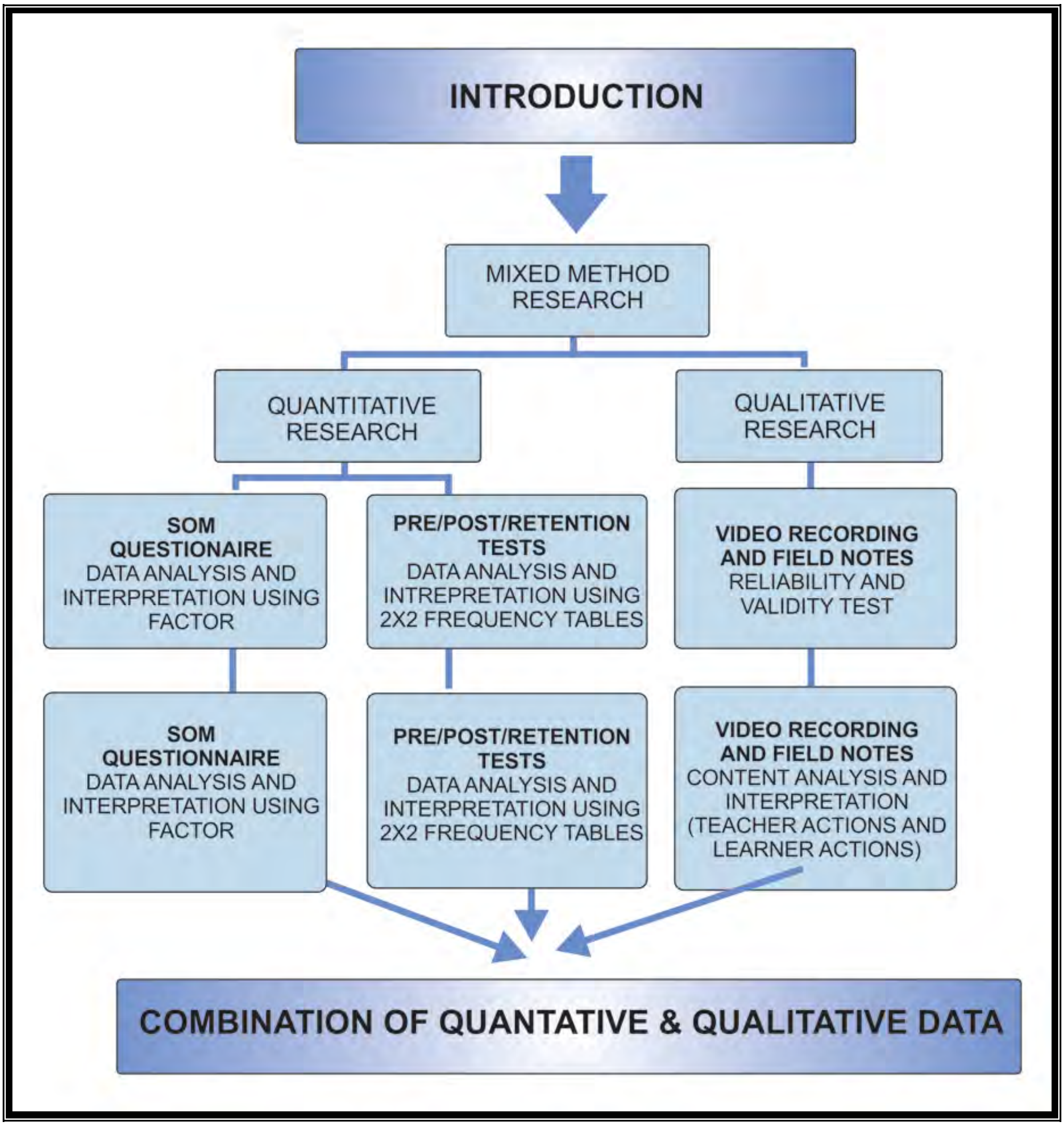


Figure 5: Outline of Chapter 5

5.1 INTRODUCTION

In Chapter 4, the methodological process to be undertaken was discussed in detail. The measuring instruments were put under evaluative statistical techniques to establish reliability and validity (see par 4.4.8 - 4.4.9). The pre-test and pre-SOM were conducted during Phase 1 of the methodological process. The post-test and post-SOM were conducted during Phase 3 and the retention test and retention-SOM were conducted during Phase 5 (see Table 4.1). The video recording and classroom observation field notes will provide qualitative evidence to assist in addressing the objectives of the research project.

The secondary research questions of the project include, on the theoretical side, determining what the critical constitutes of the modelling-based learning environment (see par 1.2); determining what the building blocks for implementing a modelling-based learning environment (see par 1.2); and on the practical side, determining how a modelling-based learning environment influences the learners' solving of non-routine problems in grade 6 mathematics classrooms (see par 1.2), and determining how a modelling-based learning environment influences learners' study orientation in grade 6 mathematics classrooms (see par 1.2).

5.2 QUANTITATIVE DATA ANALYSIS AND INTERPRETATION

5.2.1 Reliability of the SOM questionnaire

The study orientation in maths (SOM) is a significant predictor for mathematics conceptual understanding that will support performance (see par. 3.10). The SOM questionnaire measures the learners' study orientation in maths according to various fields of the SOM (see par. 3.10). The Table 5.1 below *inter alia* provides the reliability measured by the Cronbach α coefficient for the pre-phase SOM fields. The SOM questionnaire was again given to both the experimental (group 1) and control groups (group 2) in phase one, phase three and phase five. The total number of observations was 209 of which 183 was used for analysis. The reason for this was that some responses were spoiled (see par 4.4.6. The resulting Cronbach α values are shown in Table 5.1 below.

Table 5.1: Cronbach α coefficient values for pre-, post- and retention phase SOM-fields

SOM Field	Cronbach α Coefficient		
	Pre SOM	Post SOM	Retention SOM
Study attitude	0.727	0.695	0.692
Mathematics Confidence	0.730	0.798	0.787
Study Habits	0.833	0.783	0.781
Problem Solving Behaviour	0.782	0.765	0.824
Study Environment	0.646	0.747	0.719

From Table 5.1 it is evident that the SOM questionnaire is reliable for the sample used in all three phases and in all the different SOM fields. The Cronbach α coefficient values in the pre SOM range between a minimum of 0.646 for the study environment and maximum of 0.833 for study habits, while for the Post SOM it ranges from 0.695 (Study Attitude) to 0.798 for mathematical confidence. In the retention phase, the Cronbach α values range from 0.692 (study attitude) to 0.824 (Problem solving behaviour), and the Cronbach α value for SOM field Study environment in pre phase and study attitude Post and retention phase is substantial. For all the other SOM fields, the Cronbach α value indicates a good reliability.

5.2.2 Construct validity of the SOM questionnaire

The SOM questionnaire consists of a total of 76 items. These items split into five different fields, namely; Study attitude, Mathematical confidence, Study habits, Problem Solving and Study environment (see par 3.10). The final communality for each of the fields and each of the phases are given below in Table 5.2.

Table 5.2: Table on factor analysis and final communalities

SOM Field	Phase	Number of factors extracted	Total Variance explained by extracted factors)	Range of final Communalities
Study Attitude	Pre Phase	4	52.22%	0.31 - 0.69
	Post Phase	4	51.37%	0.31 - 0.65
	Retention Phase	5	59.36%	0.41 - 0.79
Mathematical Confidence	Pre Phase	5	61.65%	0.41 - 0.80
	Post Phase	5	62.08%	0.40 - 0.81
	Retention Phase	5	60.45%	0.38 - 0.68
Study Habits	Pre Phase	5	57.89%	0.46 - 0.70
	Post Phase	6	61.24%	0.37 – 0.80
	Retention Phase	6	62.24%	0.39 – 0.78
Problem Solving	Pre Phase	6	62.15%	0.47 – 0.70
	Post Phase	6	59.35%	0.44 – 0.75
	Retention Phase	6	55.28%	0.41 – 0.69
Environment	Pre Phase	4	53.11%	0.30 – 0.75
	Post Phase	4	54.73%	0.36 – 0.83
	Retention Phase	4	55.26%	0.31 – 0.81

SOM field: Study attitude.

In the SOM questionnaire, the field Study Attitude comprises 14 items (see Annexure B), which have a bearing on feelings and attitudes with regard to mathematics and the learning of it (see par 3.10). 100% of the items were responded on. From the Table 5.2 it is evident that the number of factors extracted for Study attitude 4 to 5 across the pre-/post and retention phase, which indicate more than one underlying construct throughout. To be construct valid the ideal would be uni-dimensionality (i.e. only one factor) of this field. The total percentages of variance explained by the extracted factors were 52.22%, 51.37% and 59.36% which were not too low. The final communality values for SOM field Study attitude across the 3 phases range from 0.31 to 0.79. The final communality value is larger than 0.3, which implies that all items were part of the factor structure.

SOM field: Mathematical confidence

The mathematical confidence field consists of 14 items (see Annexure B), which have a bearing on panic, anxiety, repetitive behaviour when doing or learning mathematics (see par. 3.10). In the mathematical confidence field, 5 factors have been extracted for each of the pre-/post and retention phase, indicating more than one underlying construct. The total percentages of variance explained are 61.65%, 62.08% and 60.45% for each of the pre-/post and retention phase respectively, which indicated an acceptable construct. From Table 5.2 it is evident that the final communality for the pre-SOM field Mathematical confidence range from 0.40 to 0.79. The final communality for the post-phase Mathematical confidence field ranges from 0.40 to 0.80, while it ranged from 0.38 to a value of 0.66 for the retention phase. The final communality value is larger than 0.3 for all the phase, which implies acceptable communality and acceptable construct validity for the Mathematical confidence field.

SOM field: Study habits

There are 17 items in the SOM questionnaire which addresses the Study habits field (See Annexure B), which includes displaying effective study methods and habits (like planning time, preparation, etc.), and willingness to do mathematics (see par. 3.10). From the Table 5.2 above it is evident that for the pre-phase 5 factors extracted while the post and retention phase had 6 factors extracted. The total percentage variance explained by the extract factors were 57.89% (pre-phase), 61.24% (post-phase), and 62.24% (retention phase), which were not too low. The final communality range from 0.47 to 0.70 for the pre-phase SOM; 0.37 to 0.80 for post and 0.38 to 0.78 for the retention phase. The number of factor is high but for all three phases the final communality value is greater than 0.3 which indicates acceptable communality, therefore, an acceptable construct validity for the Study habits field.

SOM field: Problem solving

The problem solving field comprises 17 items (see Annexure B), which cognitive and meta-cognitive learning strategies (see par. 3.4, 3.10, 2.4). It is evident from the above Table 5.2 that the number of factors extracted for pre/post and retention phase is 6. The total percentage of variance explained by the extracted factors was 62.15%, 59.35% and 55.28%, for each of the pre-/post and retention phase respectively, which was acceptable. The final communality of the pre-phase SOM field Problem solving, ranges from 0.46 to 0.76, for the post-phase ranges from 0.43 to 0.76 and for the retention phase it ranges from 0.41 to 0.69. The number of factors for problem solving is high, but for all three phases the final communality value is greater than 0.3 which indicates acceptable communality thus an acceptable construct validity for the problem solving field.

SOM field: Study milieu (environment)

There are 12 items in the SOM questionnaire which addressed the Study Milieu field (see Annexure B), which includes social, physical and experience environment (see par. 3.10). In the Study milieu field, 4 factors have been extracted for each of the pre-/post and retention phase. The total percentages of variance explained by the extracted factors were 53.11% (pre phase), 54.73% (post phase) and 55.26% (retention phase) which were not too low. From the above Table 5.2 it is evident that the final communality ranges from 0.3 to 0.81 for all the phases in the field Study Milieu. The final communality value is greater than 0.3 which indicates an acceptable communality thus an acceptable construct validity for the study milieu field for retention SOM.

From the above discussion it is evident that all the SOM fields indicate an acceptable construct validity.

5.2.3 The pre–, post- and retention test data analysis and interpretation

One of the key aims of this research project is to determine how a modelling-based learning environment influences the learners' solving of non-routine problems in grade 6 mathematics classrooms (see par 1.4 c). This researcher has identified two sub-questions derived from the secondary question (see par 1.2), that will guide the pre-/post-/retention-test analysis.

The test consists of 16 items which are divided into eight standard routine word problems and eight problem non-routine word problems (see par 4.4). The test was given to both experimental and control group learners (see par 4.4). The test items addressed different cognitive levels e.g. basic knowledge questions, comprehension & application questions and the analysis & problems solving questions.

5.2.3.1 Comparison of the control group and experimental group in pre - / post - and retention test for non-routine problems

The following analysis will attempt to provide answers to the sub-question stating: "How does the mathematics performance of grade 6 control group learners in the pre-, post-, and retention test compare to the experimental group with regard to non-routine word problem items?" The Table 5.3 below highlights chi-square statistics and effect size

Table 5.3: Chi-square statistics and effect size for Problem Test Items

Test Items		df	Chi-Square Value	p-value	Phi-value	Effect size	Conclusions on Chi-square
P1	Pre	1	1.16	0.28	-0.1	Small	Non- Practical Significant
	Post	1	5.15	0.02	0.2	Small to medium	Significant
	Retention	1	7.34	0.007	0.2	Small to medium	Significant
P2	Pre	1	0.74	0.39	-0.1	Small	Non- Practical significant
	Post	1	11.03	0.0009	0.37	Medium	Significant
	Retention	1	6.81	0.009	0.2	Small to medium	Significant

Test Items		df	Chi-Square Value	p-value	Phi-value	Effect size	Conclusions on Chi-square
P3	Pre	1	1.14	0.28	-0.1	Small	Non- Practical significant
	Post	1	6.38	0.01	0.2	Small to medium	Significant
	Retention	1	3.08	0.07	0.1	Small	Non- Practical significant
P4	Pre	1	2.3	0.13	-0.11	Small	Non- Practical significant
	Post	1	7.46	0.005	0.2	Small to medium	Significant
	Retention	1	0.11	0.74	0.03	Small	Non- Practical significant
P5	Pre	1	2.3	0.13	-0.1	Small	Non- Practical significant
	Post	1	1.09	0.29	0.1	Small	Non- Practical significant
	Retention	1	1.48	0.22	0.19	Small	Non- Practical significant
P6	Pre	1	1.14	0.28	-0.18	Small	Non- Practical significant
	Post	1	3.95	0.04	0.2	Small to medium	Significant
	Retention	1	3.08	0.08	0.1	Small to medium	Significant
P7	Pre	1	5.88	0.02	-0.2	Small to medium	Significant
	Post	1	1.65	0.19	0.1	Small	Non- Practical significant
	Retention	1	1.77	0.18	0.1	Small	Non- Practical significant
P8	Pre	1	1.44	0.3	-0.1	Small	Non- Practical significant
	Post	1	15.28	<0.0001	0.3	Medium	Practical significant
	Retention	1	8.91	0.003	-0.2	Small to medium	Significant

From the above Table 5.3 it is evident that the pre-test Item P1 – P6 and P8 all have a $p > 0.05$, which implies that there is no statistical significant ($p > 0.05$) relationship between experimental and control group. The non-statistical significance in most of the problem test items for the pre-phase is acceptable since no intervention were at the pre-phase instituted. Pre test item P7 have a $p = 0.02$ which implies statistical significance. For the post-test for test Item P1 have a chi-square value (X^2) = 5.15 and $p < 0.05$ which imply a statistically significant difference in favour of the experimental group. The Phi-value (φ) = 0.2 is between small to medium effect which may imply practical significance in-favour of the experimental group. The retention-test has X^2= 7.34, $p = 0.007$, which implies statistical significance ($p < 0.05$). The φ = 0.2 is also between small to medium size effect which may indicate practical significance.

The P2 item for the post-test a X^2 = 11.03, $p = 0.0009$, which implies the there exists a statistically significant ($p < 0.05$) difference in favour of the experimental group. The post-test showed a φ = 0.3, thus medium effect size which may indicate practical significant. The retention-test a X^2 = 6.81and $p < 0.05$, which implies the there exists a statistically significant difference in favour of the experimental group. The retention-test φ = 0.2 which is between small and medium size effect, thus may indicate practical significance.

The post-test item P3, X^2 = 6.38 and $p = 0.011$, which implies the there exists a statistically significant difference ($p < 0.05$) in favour of the experimental group. The post-test φ= 0.21 shows a small to medium size effect, which may indicate practical significance. The retention-test X^2= 3.08, $0.05 < p < 0.1$, which may indicate statistical significance. φ = 0.13 which is between small and medium size effect, therefore, may indicate practical significance.

The post-test a X^2 = 7.64 and $p = 0.005$, which implies that there exists a statistically significant difference in favour of the experimental group. The post-test P4 test item has φ = 0.2 showing a small to medium size effect, which may indicate practical significance.

The post-test P6 item X^2 = 3.9 and p = 0.04, which implies that there exists a statistically significant ($p < 0.05$) difference in favour of the experimental group. The retention-test P6 item X^2 = 3.7 and $0.05 < p < 0.1$, implying that there may exist a statistically significant difference in favour of the experimental group. Post-test and retention-test have φ= 0.15 and 0.13 which implies small to medium size effect which may indicate practical significance.

The post-test item P8 have X^2 = 15.3, and a $p < 0.001$ which implies a statistical significance. The φ=0.31, which implies a medium to large effect size, thus is practically significant. The retention-test a X^2 = 8.9 and p = 0.002, implying that there exists a statistically significant difference in favour of the experimental group. The retention-test φ = 0.22 which is a small to medium size effect, which may indicate practical significance.

5.2.3.2 Comparing experimental group with control group in pre-test non-routine problem (P) items

Following are examples of actual learner responses:

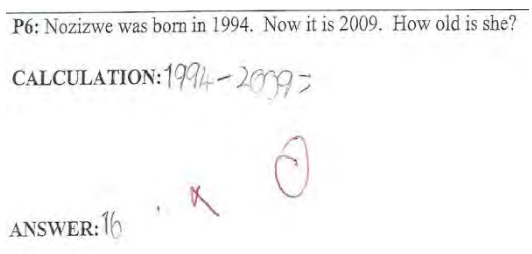


Figure 5.1: **Pre-test Experimental group learner (P6)**

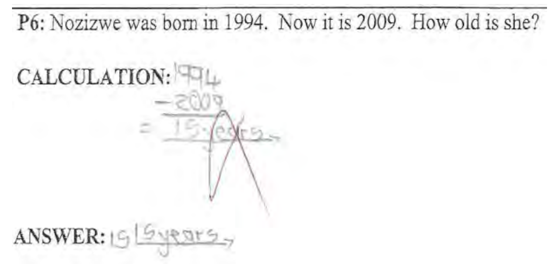


Figure 5.2: **Pre-test Control group Learner (P6)**

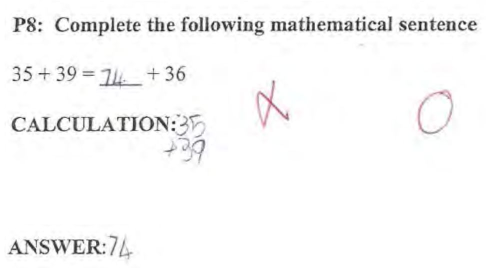


Figure 5.3: **Pre-test Experimental group learner (P8)**

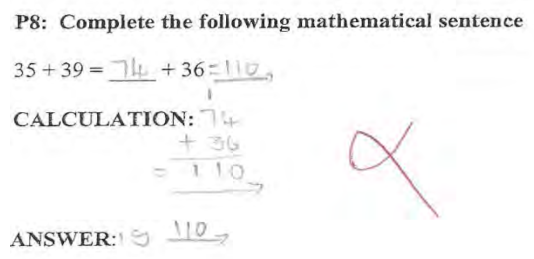


Figure 5.4: **Pre-test Control group Learner (P8)**

From the above Figures 5.1- 5.4 it is clear that learners provided non-realistic responses (for example of realistic response see Annexure C). The response

in Figure 5.1 shows that the learner made a mistake in the answer of the subtraction algorithm. In both Figure 5.1 and Figure 5.2 the learners seem to get confused with which quantity is to be subtracted from which quantity, e.g. 1994 – 2009 = 15. Learners seem not to understand the context of the problem. Learners also did not evaluate whether 'the solution fits the question.'

From Figure 5.3 and Figure 5.4 it is evident that learners from both the experimental group and the control group saw the equal sign as giving an answer with both writing 74 just after the equal sign. In the calculation section of Figure 5.4, the learner from the control group continued to add the answer 74 to another quantity 36 giving a final answer of 110. The learners from both experimental group and control group did not understand the value of the equal sign.

5.2.3.3 Comparing experimental group with control group in post-test problem (P) Items

The following are examples of the actual learner response in the post test

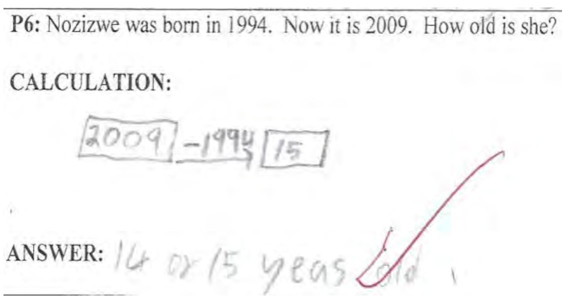


Figure 5.5: Post-test: Experimental group learner (P6)

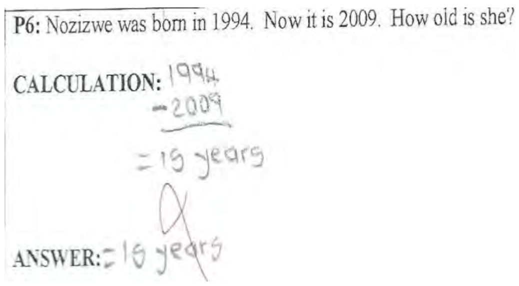


Figure 5.6: Post-test: Control group Learner (P6)

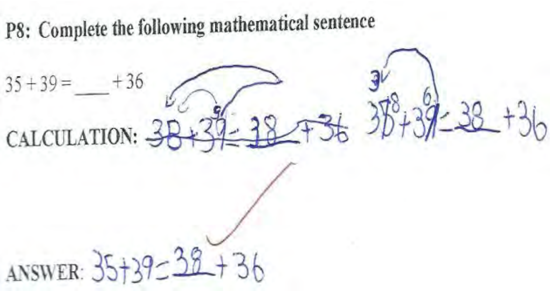


Figure 5.7: Post-test: Experimental group learner (P8)

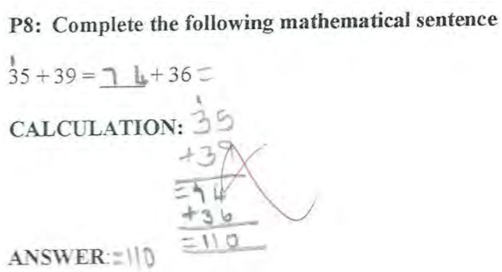


Figure 5.8: Post-test: Control group learner

From the above Figure 5.5 and Figure 5.7 it is evident that the learner from the experimental group provided realistic responses. In Figure 5.5 the learner used the “arrow language” which is a new skill acquired during the learning environment treatment. The answer 14 or 15 years old shows that the learner did understand the problem in its context and could learner deduce a realistic response. The learner from the control group did apply subtraction of the two quantities others did get 15 years which is an indication of a non realistic response. The learner from the control group (Figure 5.6) seems still, as in the pre test, to get confused with which quantity is the subtrahend, the number to be subtracted and which is the minuend, the number from which another number is to be subtracted, putting the subtraction sign with the bigger number 2009. The learner from the experimental group (Figure 5.5) did the correct application and also gave a response which shows that the learner took cognisance of the question posed and its context.

From the reasoning seen in Figure 5.7 it is clear the learner from the experimental group did understand the problem in its context and could Learner deduce a realistic response. Learners do understand the equal sign showing the quantity on left must be the same as the quantity on the right. The learner's method of answering in the experimental group is different from the control group learner. The learner from the experimental group showed the equal sign does not mean providing an answer as given in the case of the control group (see Figure 5.7). The learner from the control group just added the two quantities, others did get 74 and added again to get 110 which is an indication of a non realistic response (see Figure 5.8). The control group learners had no modelling learning environment treatment. The learner from the experimental group did undergo a modelling learning environment treatment.

5.2.3.4 Comparing experimental group with control group in retention-test problem items

The following Figures are examples of actual response of learners from both groups

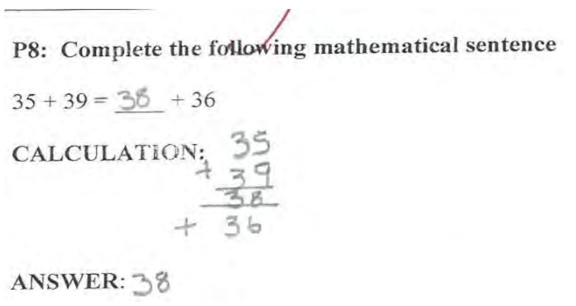


Figure 5.9: **Retention-test: Control group learner (P8)**

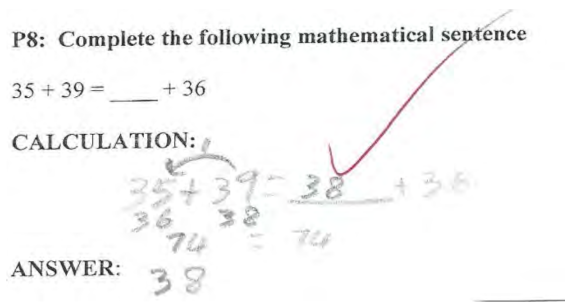


Figure 5.10: **Retention-test: Experimental group learner (P8)**

The above responses seen in Figure 5.9 and Figure 5.10 gave a clear indication that learners from the both the control group and the experimental group did understand the equal sign. The difference is in showing the method used to derive at a realistic response. The learner from the experiment group (see Figure 5.9) continues to use strategy as in post-test. The learner understood the problem and deduced a realistic response. The learner from the both the experimental group (in phase two) and control group (in phase four) did undergo a modelling learning environment treatment.

5.2.3.5 Comparison of the control group and experimental group in pre-/ post- and retention test for standard (S) items

The following analysis will attempt to provide answers to the sub-question stating: "How does the mathematics performance of grade 6 control group learners in the pre-, post-, and retention test compare to the experimental group with regard to routine word problem items?" The Table 5.4 below highlights chi-square statistics and effect size measurement for the problem test items.

Table 5.4: Chi-square statistics and effect size for Standard Test Items

Test Items		df	Chi-Square Value	p-value	Phi-value	Effect size	Conclusions on Chi-square
S1	Pre	1	0.41	0.52	-0.04	Small	Non-Practical Significant
	Post	1	0.13	0.71	-0.03	Small	Non-Practical Significant
	Retention	1	0.35	0.55	0.04	Small	Non-Practical Significant
S2	Pre	1	0.01	0.93	-0.01	Small	Non-Practical Significant
	Post	1	3.12	0.08	0.14	Small to medium	Significant
	Retention	1	6.36	0.01	0.19	Small to medium	Significant
S3	Pre	1	1.41	0.23	-0.09	Small	Non-Practical Significant
	Post	1	0.00	0.99	-0.004	Small	Non-Practical Significant
	Retention	1	4.5	0.03	0.16	Small to medium	Significant

Test Items		df	Chi-Square Value	p-value	Phi-value	Effect size	Conclusions on Chi-square
S4	Pre	1	0.48	0.49	-0.05	Small	Non-Practical Significant
	Post	1	6.39	0.01	0.20	Small to medium	Significant
	Retention	1	53.75	<0.0001	0.43	Medium to Large	Practical Significant
S5	Pre	1	2.95	0.08	0.13	Small to medium	Significant
	Post	1	1.54	0.21	0.09	Small	Non-Practical Significant
	Retention	1	1.46	0.23	0.09	Small	Non-Practical Significant
S6	Pre	1	1.38	0.24	-0.09	Small	Non-Practical Significant
	Post	1	0.95	0.33	0.08	Small	Non-Practical Significant
	Retention	1	12.43	0.0004	0.3	Medium	Practical Significant
S7	Pre	1	2.35	0.13	-0.11	Small	Non-Practical Significant
	Post	1	3.36	0.06	0.15	Small to medium	Significant
	Retention	1	0.004	0.95	0.01	Small	Non-Practical Significant
S8	Pre	1	1.20	0.27	0.08	Small	Non-Practical Significant
	Post	1	0.002	0.96	-0.04	Small	Non-Practical Significant
	Retention	1	3.71	0.05	0.14	Small to medium	Significant

From the above Table 5.4 it is evident that the pre-test item S1 – S8 all have a $p > 0.05$, which implies that there is no statistical significant ($p > 0.05$) relationship between experimental and control group. The non-statistical significance in most of the standard test items for the pre-phase is acceptable since no intervention was instituted at the pre-phase, also that the standard items formed part of the normal diet of routine word problems, usually part of teaching and learning (see par 3.7).

The post- and retention-test Item S1 shows $p > 0.5$ which implies that there is no statistical significant relationship between experimental and control group. The S1 test items were mostly answered correctly by both experimental group and control group in pre-/ post and retention- test.

The S2 item for the post-test $X^2 = 3.12$, $0.05 < p < 0.1$, therefore implying that there may exist a statistically significant difference in favour of the experimental group. The post-test showed a $\varphi = 0.14$ which implies a small to medium effect size, which may indicate practical significance. The retention test $X^2 = 6.04$ and $p = 0.001$, implies that there exists a statistically significant ($p < 0.05$) difference. The retention-test $\varphi = 0.2$ which is a small to medium size effect, thus might imply practical significance.

The post-test S3 item has $p = 0.99$, thus there is no statistical significant relationship. The retention test $X^2 = 4.5$ and $p = 0.03$, which implies that there exists a statistically significant ($p < 0.05$) difference in favour of the experimental group. The retention-test $\varphi = 0.16$ showing a small to medium size effect, which might imply practical significance.

The post test S4 $X^2 = 6.38$ and $p = 0.01$, which implies that there exists a statistically significant difference in favour of the experimental group. The post-test S4 test item have $\varphi = 0.22$ showing a small to medium size effect which might imply practical significance. The retention-test have a $X^2 = 33.35$, and $p < 0.0001$, which implies there is a statistically significant difference in favour of the experimental group. The $\varphi = 0.42$ which implies a medium to large size effect, thus practically significant.

The retention-test S6 item X^2 = 12.4, and p = 0.0004, which implies a statistically significant difference in favour of the experimental group. The φ =0.26, having a small to medium size effect, thus might be practically significant.

The retention-test for S8 has a X^2=3.7, and p = 0.05, which implies there is a statistically significant difference in favour of the experimental group The S8 item for the retention-test φ = 0.24 which is between small to medium size effect, hence might be practically significant.

5.2.3.6 Comparing experimental group with control group in pre-test standard (S) items

The examples of the learners' response per group:

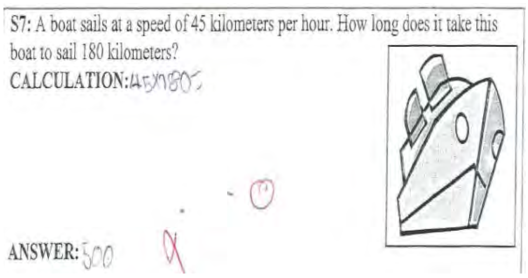


Figure 5. 11: Pre–test: Experimental group learner (S7)

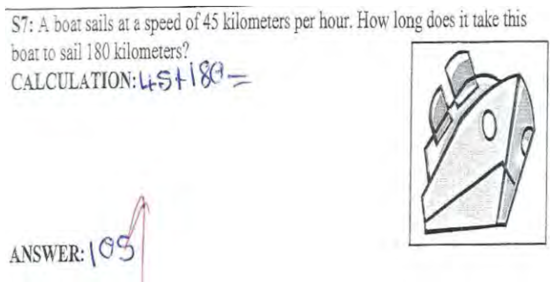


Figure 5. 12: Pre-test: Control group learner (S7)

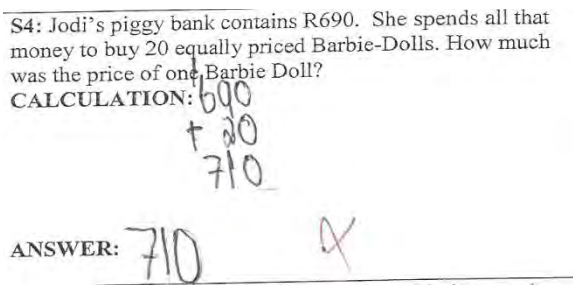


Figure 5.13: Pre–test: Experimental group learner (S4)

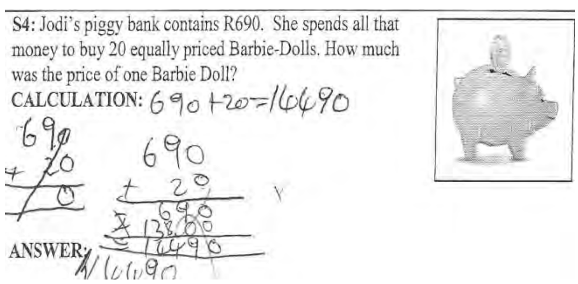


Figure 5.14: Pre-test: Control group learner (S4)

The Figure 5.11 and Figure 5.12 indicate that the learners did not know how to solve the test item S7. The learner from the experimental group used multiplication (45x180=500), while the learner from the control group used addition (45+180 = 105). In Figure 5.13 and Figure 5.14 also indicate that the learners struggled with the test item S4. This time the learner from the experimental group used addition (690+20 = 710), while the learner from the control group used multiplication (690x 20 = 14490). None of the groups at phase one when the pre test was written underwent a modelling learning environment treatment (see par 4.4).

5.2.3.7 Comparing experimental group with control group in post-test standard items

The following are examples of the actual learner response in the post-test

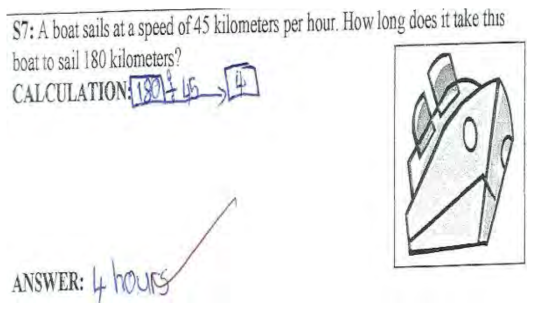


Figure 5.15: Post-test: Experimental group learner (S7)

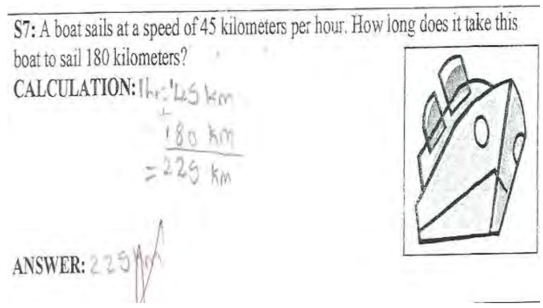


Figure 5.16: Post-test: Control group learner (S7)

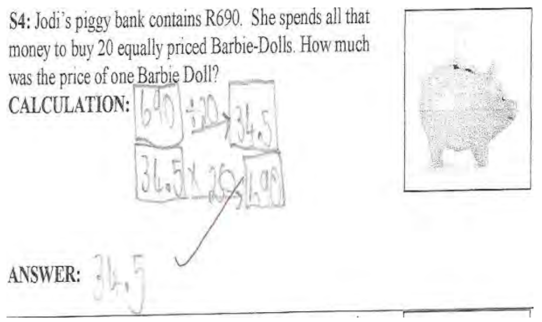


Figure 5.17: Post-test:- Experimental group learner (S4)

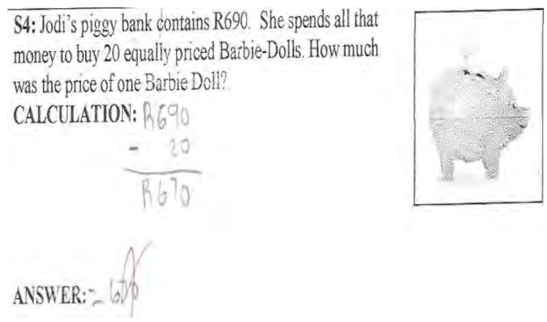


Figure 5.18: Post-test: Control group learner (S4)

Figure 5.16 and Figure 5.18 are a clear indication that the learner from the control group continued to struggle answering the standard test item S7 and S4. The learner from the control group (figure 5.16) did apply addition of the two quantities, getting a response of 225km, which is an indication of non realistic response. Figure 5.15 and Figure 5.17 on the other hand give a clear indication that the learner from the experimental group improved on their answering of the standard test item S4 and S7, with realistic response used the arrow language to solve the problem. In both cases the learner the division strategy. The learner from the experimental group did the correct application. The learner used the division as the inverse operation of multiplication, knowing the dividend is divided by the divisor, resulting in the quotient. The learner also tested the response, which shows showed that the learner took cognisance of the question posed and its context.

The control group learners had no modelling learning environment treatment. The learner from the experimental group did undergo a modelling learning environment treatment (see par 4.4).

5.2.3.8 Comparing experimental group with control group in retention-test problem items

The following Figures are examples of actual responses of learners from both groups

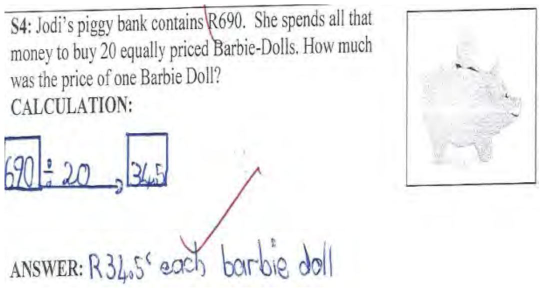


Figure 5.19: Retention-test: Experimental group learner (S4)

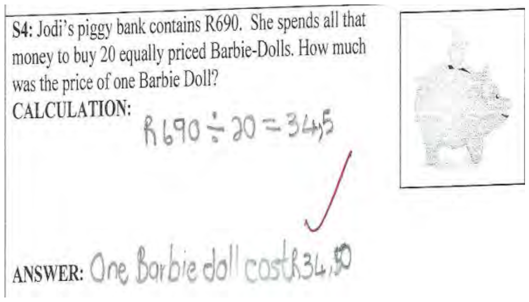


Figure 5.20: Retention-test: Control group learner (S4)

From Figure 5.19 it is evident the learners from the experimental group continued to use the division strategy with the "arrow language" to solve the standard test item S4. In the answer, the learner not only wrote R34.5 but also wrote each Barbie doll indicating the learner attempted to answer the question posed. Figure 5.20 indicates that the control group learner used another strategy, and also wrote the cost per doll as answer to question posed. The learners from both the experimental group (in phase two) and control group (in phase four) did undergo a modelling learning environment treatment.

5.2.3.9 Comparative analysis of the pre-test and post-test for the control group and the experimental group in problem items

Example of control group learner response to pre-test and post- test for non-routine test items

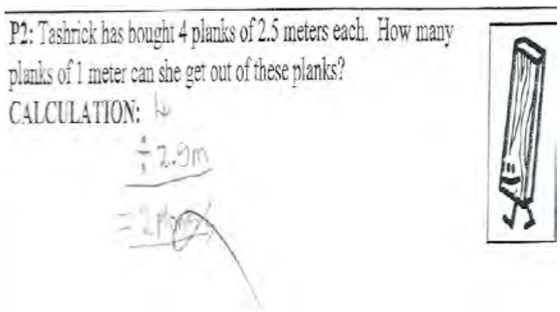


Figure 5.21: Pre-test: Control group learner (P2)

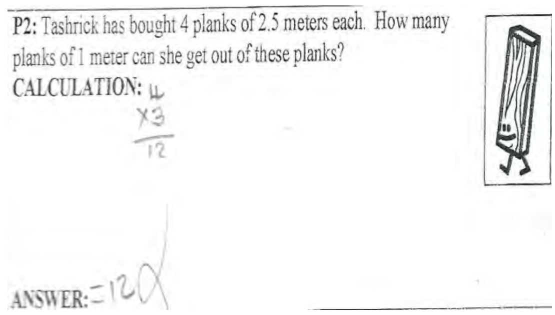


Figure 5.22: Post-test: Control group learner (P2)

From the above Figure 5.21 and Figure 5.22 it is evident that the learner did not understand the question. The non-realistic responses in both the pre-test and the post-test bare evidence of inability to solve non-routine word problems. The learners from the control group had no modelling-based learning environment treatment.

Example of an experimental group learner response to pre-test and post-test for non-routine test items

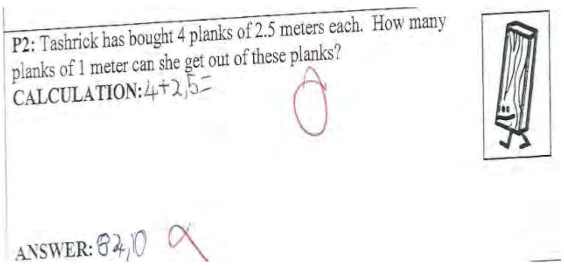


Figure 5.23: Pre-test: Experimental group learner (P2)

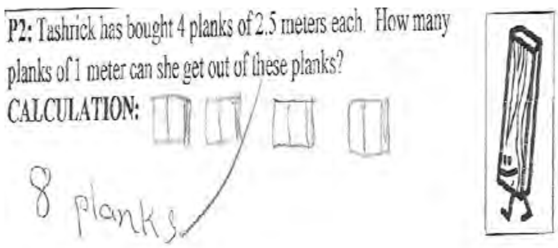
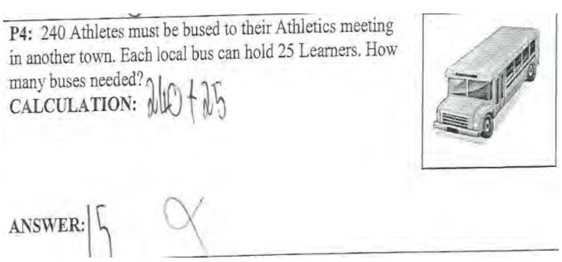


Figure 5.24: Post-test: Experimental group learner (P2)



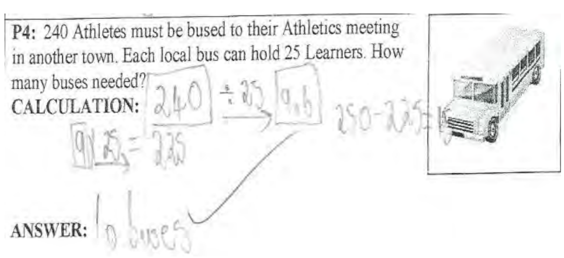
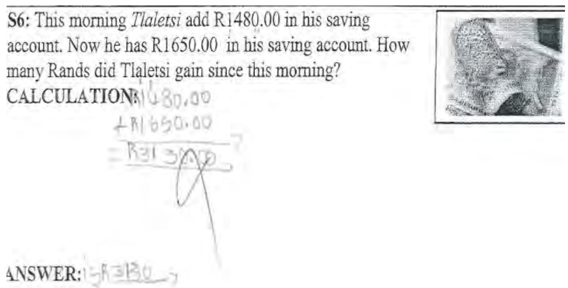


Figure 5.25: Pre-test: Experimental group learner (P4)	**Figure 5.26: Post-test: Experimental group learner (P4)**

From the Figures 5.23 and 5.25 it is evident the learner from the experimental did not understand the question (applying the addition algorithm) and struggled with answering non-routine problem questions. At the first phase, the learner from the experiment group also did not undergo the learning environment treatment. In Figure 5.24, the learner drew planks, divided each plank into 1m planks and scratched out the 0.5 of the plans. The learner could deduce the correct number of planks giving a realistic response. There was a noticeable change in the learners' approach to the problem item. In Figure 5.26 the learner employed a newly learnt strategy called the "arrow language" as a method for answering the question posed. After completion of calculations the learner deduced that 10 busses were needed which is the realistic response. The learner from the experimental group did undergo modelling-based learning environment treatment.

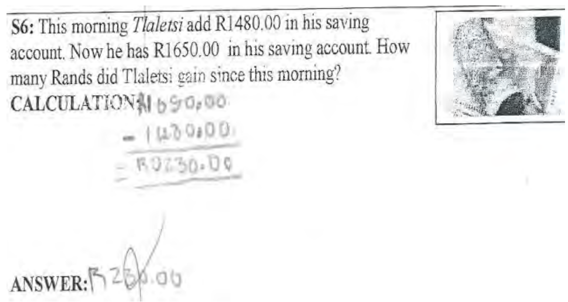
Example of control group learner response to pre-test and post-test for routine test items


S6: This morning *Tlaletsi* add R1480.00 in his saving account. Now he has R1650.00 in his saving account. How many Rands did Tlaletsi gain since this morning?

CALCULATION:

ANSWER:

Figure 5.27: Pre-test: Control group learner (S6)


S6: This morning *Tlaletsi* add R1480.00 in his saving account. Now he has R1650.00 in his saving account. How many Rands did Tlaletsi gain since this morning?

CALCULATION:

ANSWER:

Figure 5.28: Post-test: Control group learner (S6)

From the figures 5.27 and figure 5.28 it is clear that the learner from the control group provided non-realistic responses in both the pre-test and the post-test. The learner seems not to understand the question.

Example of Experimental group learner response to pre test and Post test for Routine Test items

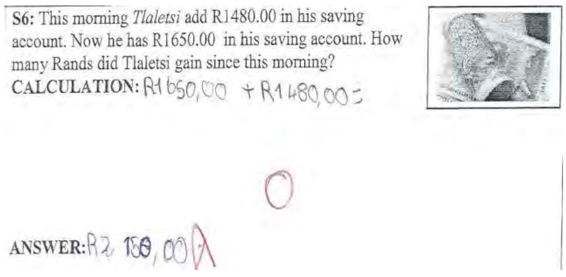


Figure 5.29: Pre-test: Experimental group learner (S6)

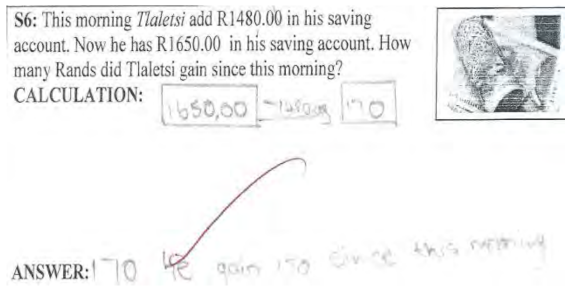


Figure 5.30: Post-test: Experimental group learner (S6)

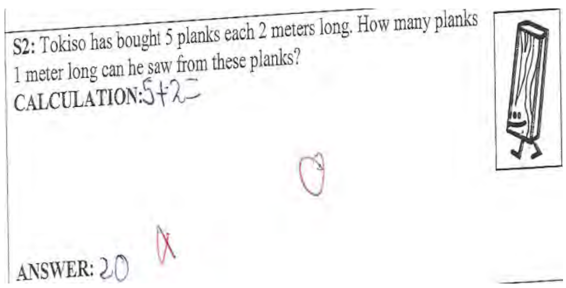


Figure 5.31: Pre-test: Experimental group learner (S2)

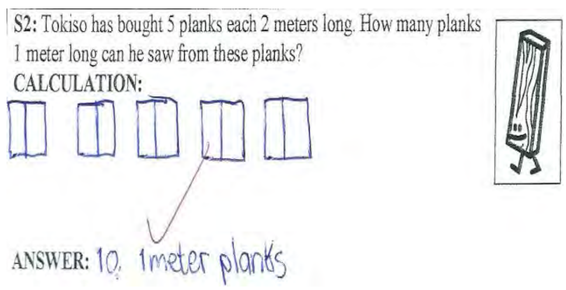


Figure 5.32: Post-test: Experimental group learner (S2)

From the Figure 5.29 and 5.31 it is evident the learner from the experimental did not understand the question (applying the addition algorithm) and struggled with answering routine problem questions. At the first phase, the learner from the experiment group also did not undergo the learning environment treatment. In Figure 5.30, the learner employed the newly learnt strategy called the “arrow language” as a method to answer the question posed. The learner applied the subtraction algorithm correctly. After completion of calculations the learner deduced the answer of R170 as the realistic response. In Figure 5.32, the learner drew planks and divided each plank into 1m planks. The learner could deduce the correct number of planks as giving a realistic response. There was a noticeable change in the learner’s approach to the standard test item (See Figures 5.29 to 5.32). The learner from the experimental group did undergo modelling-based learning environment treatment.

5.2.3.10 Comparative analysis of the pre-test and retention-test for the control group and the experimental group in problem items

Example of control group learner response to pre-test and retention test for non-routine test items

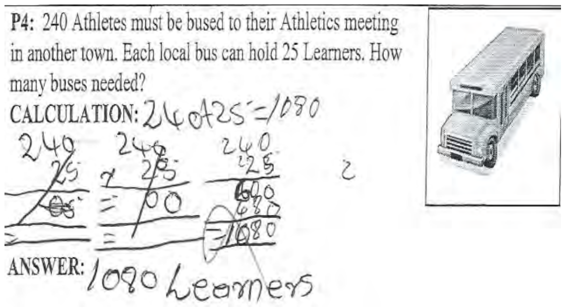


Figure 5.33: **Pre-test: Control group learner (P4)**

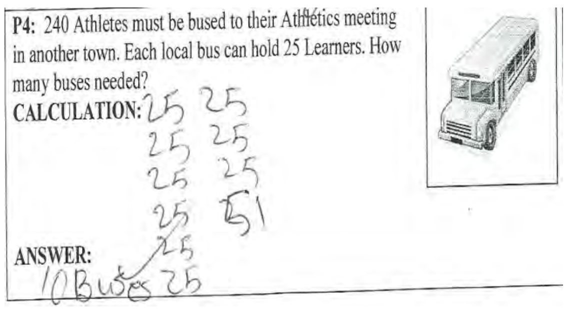


Figure 5.34: Retention-test: Control group learner (P4)

From the Figure 5.33 it is evident the learner from the control group did not understand the question (applying the addition algorithm) and struggled with answering non-routine problem questions. At the first phase, the learner from the control group also did not undergo the learning environment treatment. Figure 5.34 shows the learner's realistic response in the retention test. The learner did understand the question and deduced the realistic response. The learner did undergo the modelling learning environment treatment in phase 4 of the implementation process (see par 4.4).

Example of experimental group learner response to pre-test and retention-test for non-routine test items

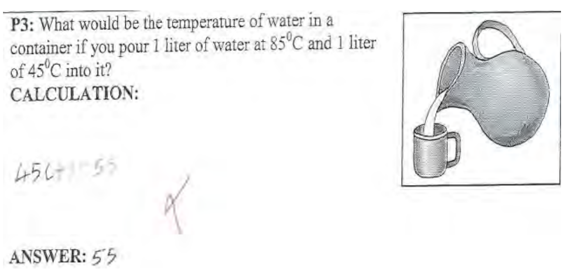


Figure 5.35: Pre-test - Experimental group learner (P3)

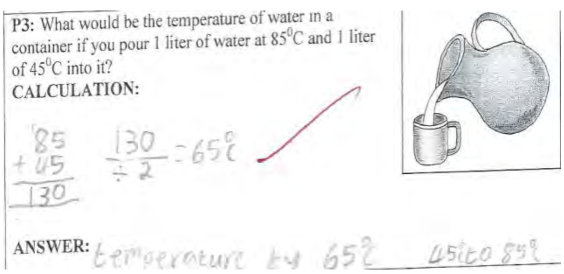


Figure 5.36: Retention -test- Experimental group learner (P3)

The Figure 5.35 shows the learner did not understand the non-routine question applying the wrong algorithm and deducing a non-realistic response. In Figure 5.36 the learner applied the correct algorithm to solve the problem. Important is the learner's response after the calculation given as 65°C; 45°C to 85°C. This type of response can be an indication that the learner put thought into the answer, thereby indicating that the learner evaluated the answer.

The learner did undergo the modelling learning environment treatment in phase 2. The test was written in phase 5 (see par 4.4).

Example of control group learner response to pre-test and retention- test for routine test items

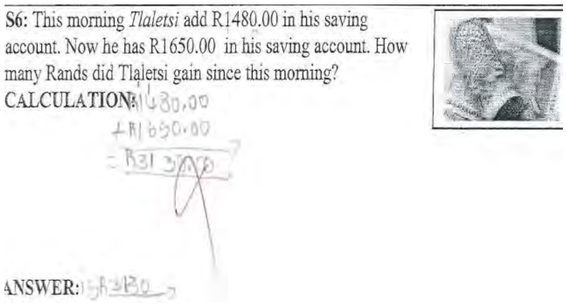


Figure 5.37: Pre–test: Control group learner (S6)

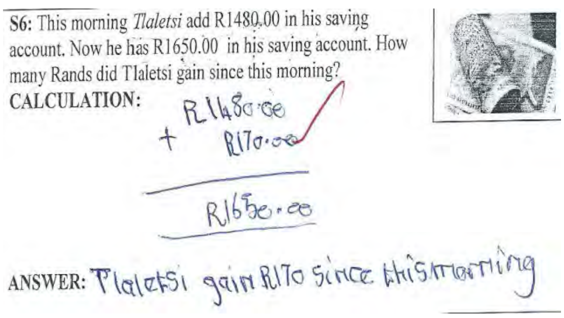
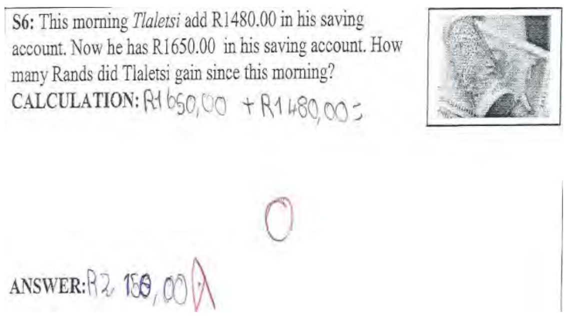


Figure 5.38: Retention-test: Control group learner (S6)

From the Figure 5.37 it is evident that the learner from the control group did not understand the question (applying the addition algorithm) and struggled with answering non-routine problem questions. At the first phase, the learner from the control group also did not undergo the learning environment treatment. Figure 5.38 shows the learner's realistic response in the retention test. The learner did understand the question and deduced the realistic response. The learner did undergo the modelling learning environment treatment in phase 4 of the implementation process (see par 4.4).

Example of experimental group learner response to pre-test and retention-test for routine

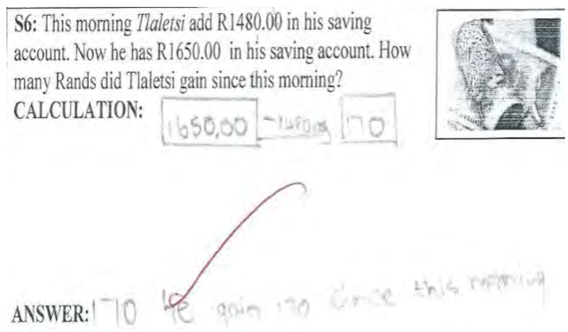


S6: This morning *Tlaletsi* add R1480.00 in his saving account. Now he has R1650.00 in his saving account. How many Rands did Tlaletsi gain since this morning?

CALCULATION: R1 650,00 + R1 480,00 =

ANSWER: R2 180,00

Figure 5.39: Pre-test: Experimental group learner (S6)



S6: This morning *Tlaletsi* add R1480.00 in his saving account. Now he has R1650.00 in his saving account. How many Rands did Tlaletsi gain since this morning?

CALCULATION: 1650,00 – 1480,00 170

ANSWER: 170 he gain 170 since this morning

Figure 5.40: Retention -test: Experimental group learner (S6)

The above Figures 5.39 and 5.40 bare evidence of learners' improved response to standard item S6. The learner used the strategy ("arrow language") learnt in phase 2 (see par 4.4) to solve the given problem. Learner was able to understand the problem, and deduce a realistic response.

5.2.4 The pre-, post-, retention SOM data analysis and interpretation

Another key aim of this research project is determining how a modelling-based learning environment influences learners' study orientation in grade 6 mathematics classrooms. Answering the secondary question (see par 1.2) will help address the objective regarding study orientation in mathematics (see par 1.4d)

5.2.4.1 The "Crossover" analysis of variance (ANOVA)

The analysis of variance, ANOVA refers to a technique researchers can use to test for significance in the difference of the two or more sample means (Smit, 2009:339: King & Minium, 2008:336). The cross over design was used (see par 4.4.9). SOM for the experimental group (group 1) compared to the control group (group 2) for pre-phase, post-phase and retention phase.

5.2.4.1 Univariate analysis for crossover design model per SOM field

study attitude

Table 5.5: Univariate tests of significance, effect sizes for study attitude

	Degr. of (Freedom)	MS	F	p	Partial eta-squared
Intercept	1	156.97	1.60	0.21	0.01
Learner_nr	194	40.10	0.41	1.00	0.312
Intervention	1	24.15	0.25	0.62	0.001
Period	1	778.45	7.92	0.01	0.043
Error	174	98.23			

From the above Table 5.5 it is evident that the phase (period) effect with $F(1,174) = 7.92$ ($p = 0.01$) indicates statistical significance ($p < 0.05$, at 1 degree of freedom and error of 174). From a 2-way-ANOVA with group and subjects-nested within-groups factors, the group effect $F(1, 176) = 0.15$ ($p = 0.69$) indicates no statistical significance ($p > 0.05$, at 1 degree of freedom and error of 176) therefore, there is no statistically significant carry-over effect (Burnelle, 2000). The pooled date from both period 1 and period 2 can be used to estimate the intervention effect.

The intervention effect for the study attitude field has a $F(1, 174) = 0.25$ ($p = 0.62$) which implies that the intervention effect was not statistically significant ($p > 0.05$, at 1 degree of freedom and error of 174) for study attitude. The partial eta-squared = 0.001 for intervention, which is small size effect.

Table 5.6: Descriptive statistics and 95% confidence limits of the intervention effect and period effect for study attitude

		Mean	Standard. Error	95% Lower limit	95% Upper limit	N
Intervention	1	0.40	0.75	-0.35	1.16	184.00
	2	0.93	0.74	0.19	1.67	187.00
Period	1	2.17	0.75	1.42	2.91	186
	2	-0.84	0.75	-1.59	-0.08	185

The Table 5.6 shows a no difference in the means in the intervention effect between group 1 (mean = 0.40) and group 2 (mean = 0.93) for the study attitude field. The period effect shows a difference between the means of group 1 (mean = 2.17) and group 2(mean = -0.84). There was no significant difference in means for the intervention effect and p > 0.05, which implies no statistical significance.

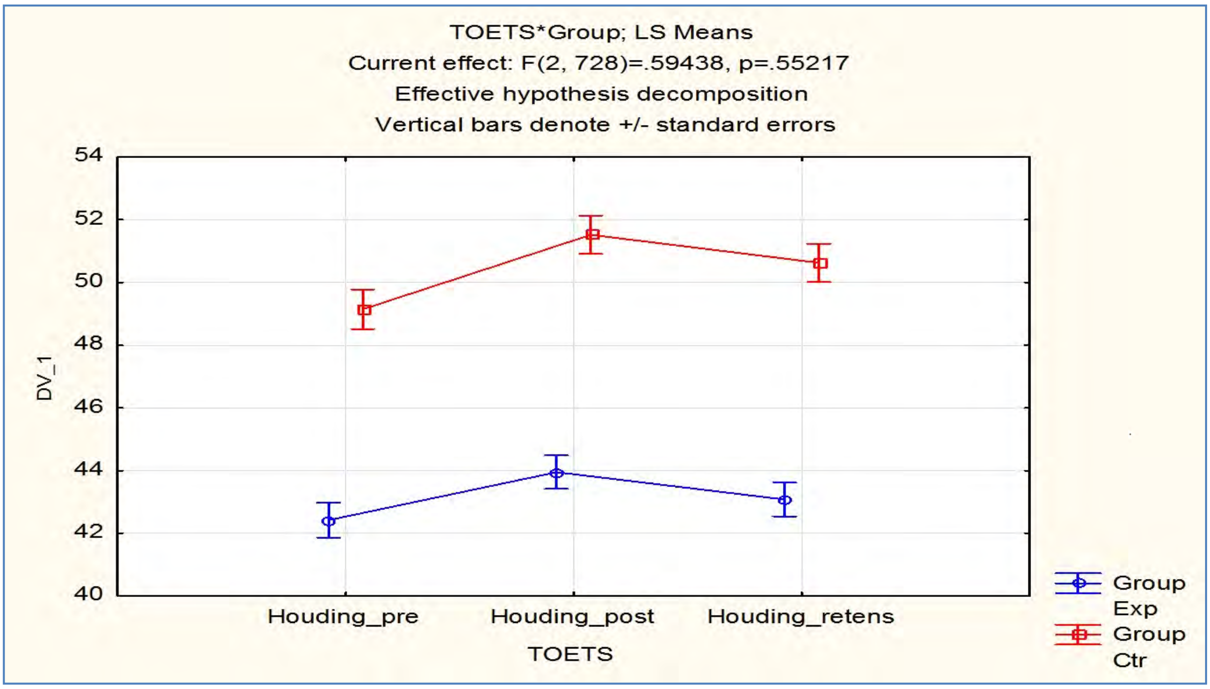


Figure 5.41: Means (with standard errors) of the 3 different tests for the two groups in the study attitude field

Figure 5.41 shows that the learners in group 1 (experimental group – intervention only between pre- and post-measurements [Phase 1]), and the group 2 (Control group – intervention occurred only between the post- and retention measurements [Phase 2]), showed similar increases and decreases. From the above Figure 5.41 it is evident that F(2, 728)= 0.594, p = 0.552, indicate that the means are different at 2 degree freedom and p > 0.1, confirming no statistical significance.

Mathematical confidence

Table 5.7: Univariate tests of significance, effect sizes for mathematical confidence

	Degr. of (Freedom)	MS	F	p	Partial eta-squared
Intercept	1.00	46.07	0.30	0.59	0.00
Period	1.00	115.88	0.75	0.39	0.00
Intervention	1.00	333.58	2.16	0.14	0.01
Learner nr	195.00	54.67	0.35	1.00	0.29
Error	169.00	154.41			

From the above Table 5.7 it is evident that the Phase (Period) effect with F(1,169) = 0.75 (p = 0.39) indicates no statistical significance (p > 0.05, at 1 degree of freedom and error of 169). From a 2-way-ANOVA with group and subjects-nested within-groups factors, the group effect F(1,171) = 0.04 (p = 0.84) indicates no statistical significance (p > 0.05, at 1 degree of freedom and error of 171) therefore, there is no statistically significant carry-over effect (Burnelle, 2000). The pooled date from both period 1 and period 2 can be used to estimate the intervention effect.

The intervention effect for the mathematical confidence field have a F(1,169) = 2.16 (p = 0.14) which implies that the intervention effect was not statistically significant (p > 0.05, at 1 degree of freedom and error of 169) for mathematical confidence. The partial eta-squared = 0.01 for intervention, which is small size effect.

Table 5.8: Descriptive statistics and 95% confidence limits of the intervention effect and period effect for mathematical confidence

		Mean	Standard. Error	95% Lower limit	95% Upper limit	N
Inter-vention	1	-1.36	0.97	-2.33	-0.39	180.00
	2	0.63	0.93	-0.30	1.56	187.00
Period	1	0.22	0.96	-0.73	1.18	182
	2	-0.95	0.95	-1.90	-0.01	185

The Table 5.8 shows a no significant difference in the means in the intervention effect between group 1 (mean = 1.8) and group 2 (mean = -1.14) and a $p > 0.05$ for the mathematical confidence field implies no statistical significance. The period effect shows no significant difference ($p > 0.05$) between the means of group 1 and group 2.

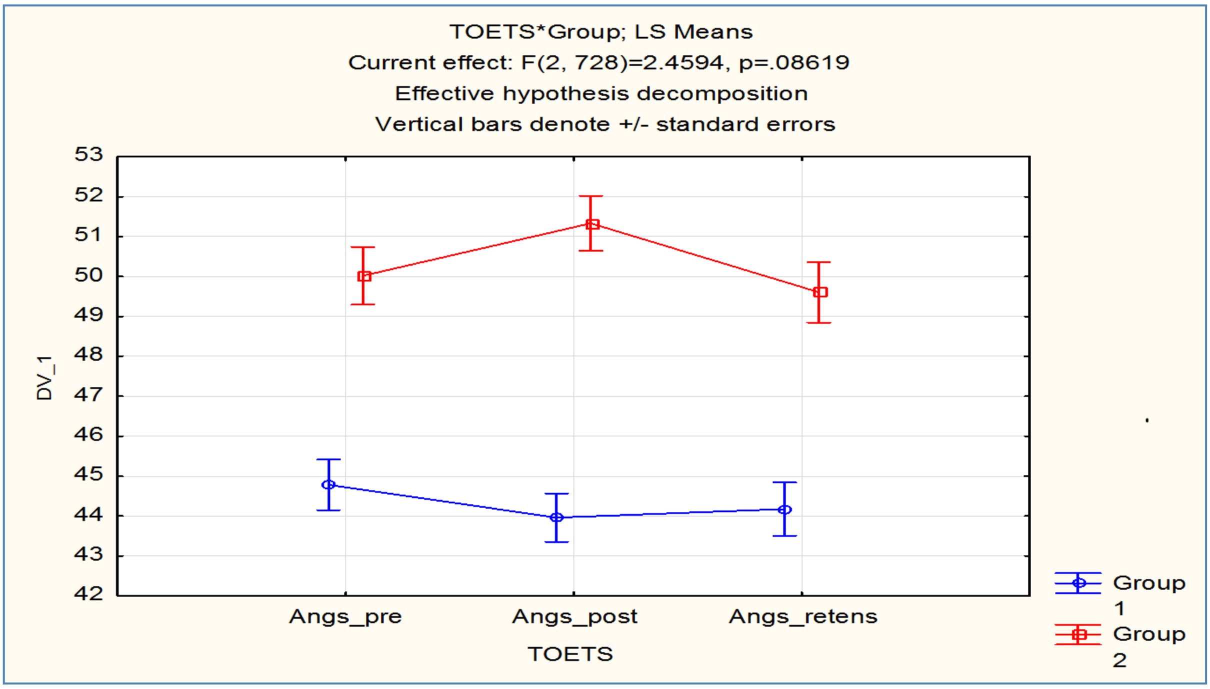


Figure 5.42: Means (with standard errors) of the 3 different tests for the two groups in the mathematical confidence field

Figure 5.42 shows that the learners in group 1 (experimental group – intervention only between pre and post phase) showed a decrease between pre and post and a minimal increase between post and retention. The group 2 (Control group – intervention only between post phase and retention phase) showed an increase between pre and post and decrease post and retention phase. It is evident from the above Figure 5.42 that F(2, 728)= 2.459, p = 0.086, indicate that the means are different at 2 degree freedom and p < 0.1, implying at a confidence level 90%, statistical significance.

Study Habits

Table 5.9: Univariate tests of significance, effect sizes for study habits

	Degr. of (Freedom)	MS	F	P	Partial eta-squared
Intercept	1.00	72.22	0.54	0.46	0.00
Learner nr	196.00	57.05	0.43	1.00	0.32
Intervention	1.00	33.52	0.25	0.62	0.00
Period	1.00	390.25	2.92	0.09	0.02
Error	174.00	133.86			

From the above Table 5.9 it is evident that the phase (period) effect with F(1,174) = 2.92 (p = 0.09) indicates there might be statistical significance (p < 01, at 1 degree of freedom and error of 174, at 90% confidence level). From a 2-way-ANOVA with group and subjects-nested within-groups factors, the group effect F(1, 176) = 0.85 (p = 0.36) indicates no statistical significance (p > 0.05, at 1 degree of freedom and error of 176) therefore, there is no statistically significant carry-over effect (Burnelle, 2000). The pooled date from both period 1 and period 2 can be used to estimate the intervention effect.

The intervention effect for the study attitude field have a F(1, 174) = 0.25 (p = 0.62) which implies that the intervention effect was not statistically significant (p> 0.05, at 1 degree of freedom and error of 169) for study habits. The partial eta-squared < 0.01 for intervention, implies small size effect.

Table 5.10: Descriptive statistics and 95% confidence limits of the intervention effect and period effect for study habits

		Mean	Standard. Error	95% Lower limit	95% Upper limit	N
Intervention	1	0.76	0.87	-0.11	1.64	186
	2	0.14	0.87	-0.73	1.01	187
Period	1	1.51	0.87	0.64	2.38	187
	2	-0.61	0.87	-1.48	0.27	186

The Table 5.10 shows a 'no difference' in the means in the intervention effect between group 1 (mean = 0.76) and group 2 (mean = 0.14) for the study habits field. The period effect shows a difference between the means of group 1 (mean = 2.17) and group 2(mean = -0.84). There was no significant difference in means for the intervention effect and $p > 0.05$, which implies no statistical significance.

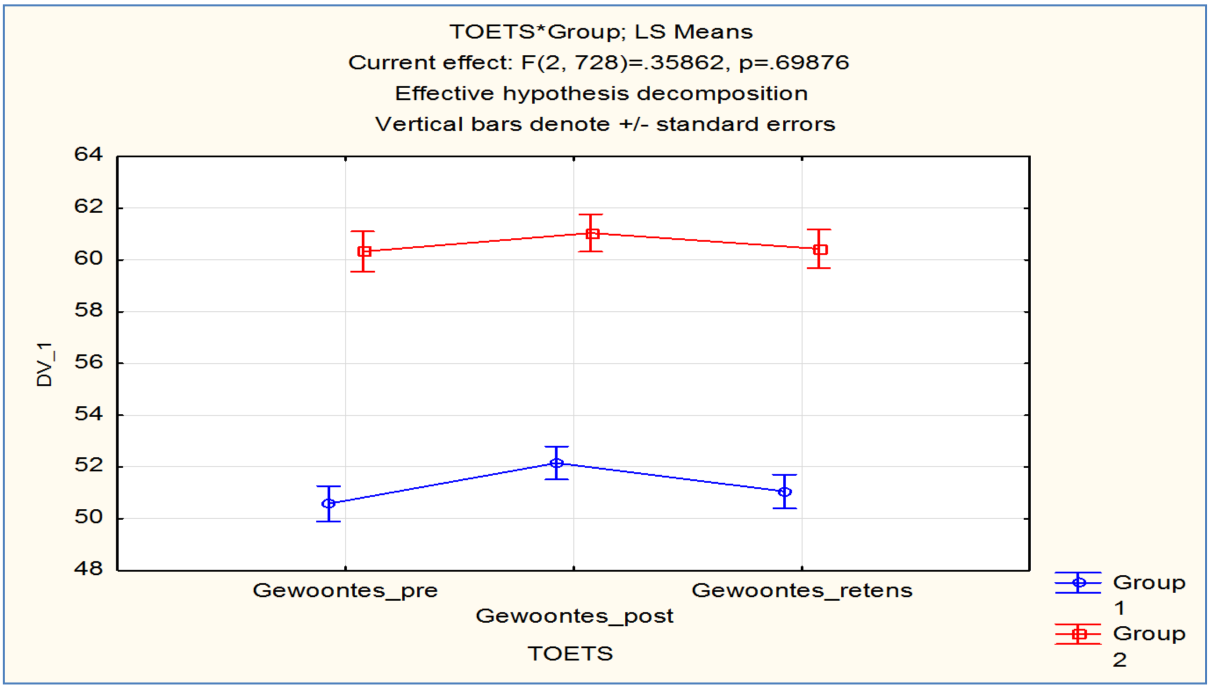


Figure 5.43: Means (with standard errors) of the 3 different tests for the two groups in study habits field

Figure 5.43 shows that the learners in group 1 (experimental group – intervention only between pre- and post-measurements [Phase 1]), and the group 2 (Control group – intervention occurred only between the post- and retention measurements [Phase 2]), showed similar increases and decreases. From the above Figure 5.43 it is evident that $F(2, 728) = 0.359$, $p = 0.698$, indicate that the means are different at 2 degree freedom and $p > 0.1$, confirming no statistical significance.

Problem Solving

Table 5.11: Univariate tests of significance, effect sizes for problem solving

	Degr. of (Freedom)	MS	F	P	Partial eta-squared
Intercept	1	44.72	0.23	0.63	0.00
Learner nr	194	74.27	0.38	1.00	0.30
Intervention	1	762.03	3.91	0.04	0.02
Period	1	48.01	0.25	0.62	0.00
Error	170	195.03			

From the above Table 5.11 it is evident that the phase (period) effect with $F(1,170) = 0.25$ ($p = 0.62$) indicates no statistical significance ($p > 0.05$, at 1 degree of freedom and error of 170). From a 2-way-ANOVA with group and subjects-nested within-groups factors, the group effect $F(1, 172) = 0.03$ ($p = 0.87$) indicates no statistical significance ($p > 0.05$, at 1 degree of freedom and error of 172) therefore, there is no statistically significant carry-over effect (Burnelle, 2000). The pooled date from both period 1 and period 2 can be used to estimate the intervention effect.

The intervention effect for the problem solving field have a $F(1, 170) = 3.91$ ($p = 0.04$) which implies that the intervention effect is statistically significant ($p < 0.05$, at 1 degree of freedom and error of 170) for problem solving. The Partial eta-squared = 0.02 for intervention, which is between small and medium size effect.

Table 5.12: Descriptive statistics and 95% confidence limits of the intervention effect and period effect for problem solving

		Mean	Standard. Error	95% Lower limit	95% Upper limit	N
Intervention	1	1.86	1.06	0.80	2.92	185
	2	-1.14	1.07	-2.22	-0.07	182
Period	1	-0.02	1.06	-1.08	1.05	184
	2	0.73	1.07	-0.33	1.80	183

The Table 5.12 indicates a difference in the means in the intervention effect between intervention (mean = 1.8) and no intervention (mean = -1.14) for the problem solving field. The phase effect shows no significant difference between the means of phase 1 and phase 2. The difference in means for the intervention effect implies statistical significance while the 'no difference' in the mean for the group effect implies non-statistical significance.

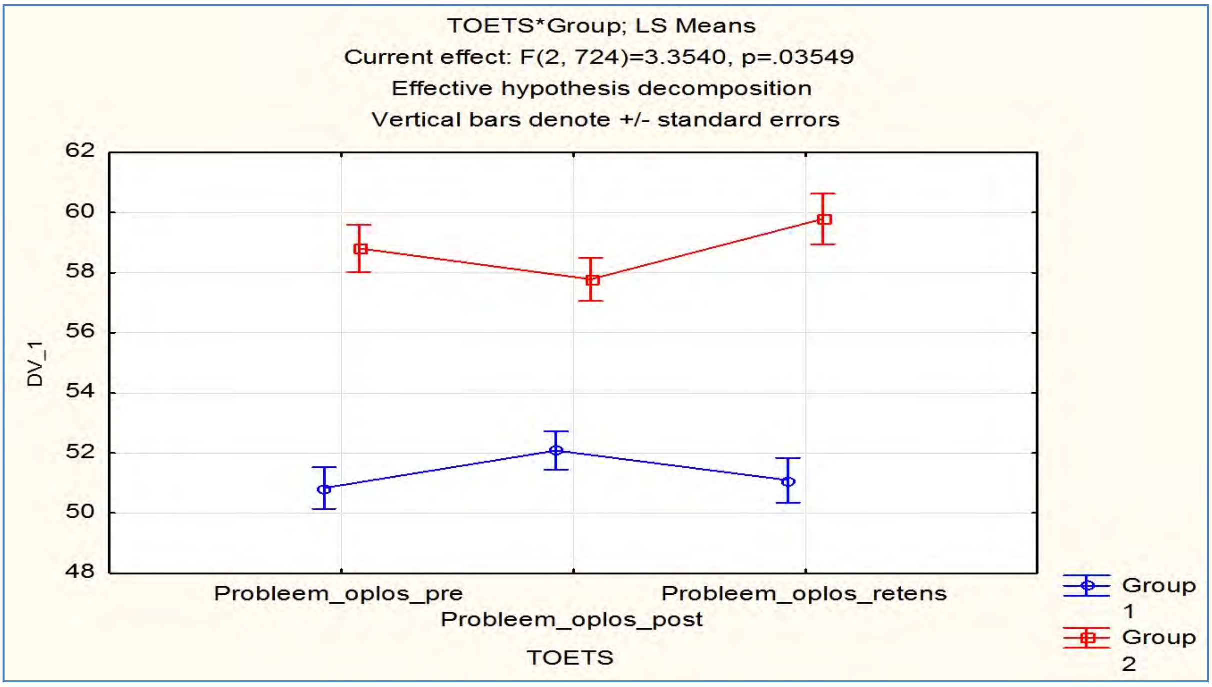


Figure 5.44: Means (with standard errors) of the 3 different tests for the two groups in problem solving field

Figure 5.44 shows that the learners in group 1 (experimental group – intervention only between pre- and post-measurements Phase 1) showed a significant improvement between pre- and post- but decreased between post- and retention. There was no intervention in group 1 between the post phase and the retention phase. The group 2 (Control group – intervention occurred only between the post- and retention measurements (Phase 2) showed a decrease between pre- and post- and an improvement between post- and retention phase.

Evident from the above Figure 5.44 is that F(2, 724)= 3.354, p = 0.035, which indicates that the means are different at 2 degree freedom and p < 0.5, confirming statistical significance. The intervention had a positive effect on the problem solving field for the learners in both group 1 and group 2. The decrease in the retention phase indicated that the intervention by the teacher needs to be continued to have a continued, improved effect. The statistical significance of the intervention effect does not imply practical significance. Percentage of variance due to effect sizes and to between group variation (R^2) in the problem solving field for overall pre/ post and retention SOM provide information of practical significance (see par 5.2.4.3).

Study Milieu

Table 5.13: Univariate tests of significance, effect sizes for study milieu

	Degr. of (Freedom)	MS	F	P	Partial eta-squared
Intercept	1.00	2.20	0.02	0.89	0.00
Learner nr	194.00	38.80	0.35	1.00	0.29
Intervention	1.00	165.84	1.50	0.22	0.01
Period	1.00	385.37	3.49	0.06	0.02
Error	164.00	110.49			

From the above Table 5.13 it is evident that the phase (period) effect with F(1,164) = 3.49 (p = 0.06) indicates there might be statistical significance (p < 01, at 1 degree of freedom and error of 164, at 90% confidence level). From a 2-way-ANOVA with group and subjects-nested within-groups factors, the group effect F(1,166) = 1.00 (p = 0.32) indicates no statistical significance (p > 0.05, at 1 degree of freedom and error of 166) therefore, there is no statistically significant carry-over effect (Burnelle, 2000). The pooled data from both period 1 and period 2 can be used to estimate the intervention effect.

The intervention effect for the study milieu field have a F(1, 164) = 1.50 (p = 0.22) which implies no statistical significance (p> 0.05). The partial eta-squared = 0.01 for intervention is a small effect.

Table 5.14: Descriptive statistics and 95% confidence limits of the intervention effect and period effect for the study milieu

		Mean	Standard. Error	95% Lower limit	95% Upper limit	N
Intervention	1	-0.63	0.84	-1.47	0.21	175
	2	0.79	0.79	0.00	1.58	186
Period	1	1.16	0.83	0.34	1.99	177
	2	-1.00	0.80	-1.80	-0.20	184

The Table 5.14 shows a ‘no significant difference’ in the means in the intervention effect between group 1 (mean = -0.63) and group 2 (mean = 0.79) for the problem solving field. The period effect shows no significant difference between the means of group 1 (mean = 1.16) and group 2 (mean = -1.00). The p > 0.05 for both intervention and period effect implies no statistical significance.

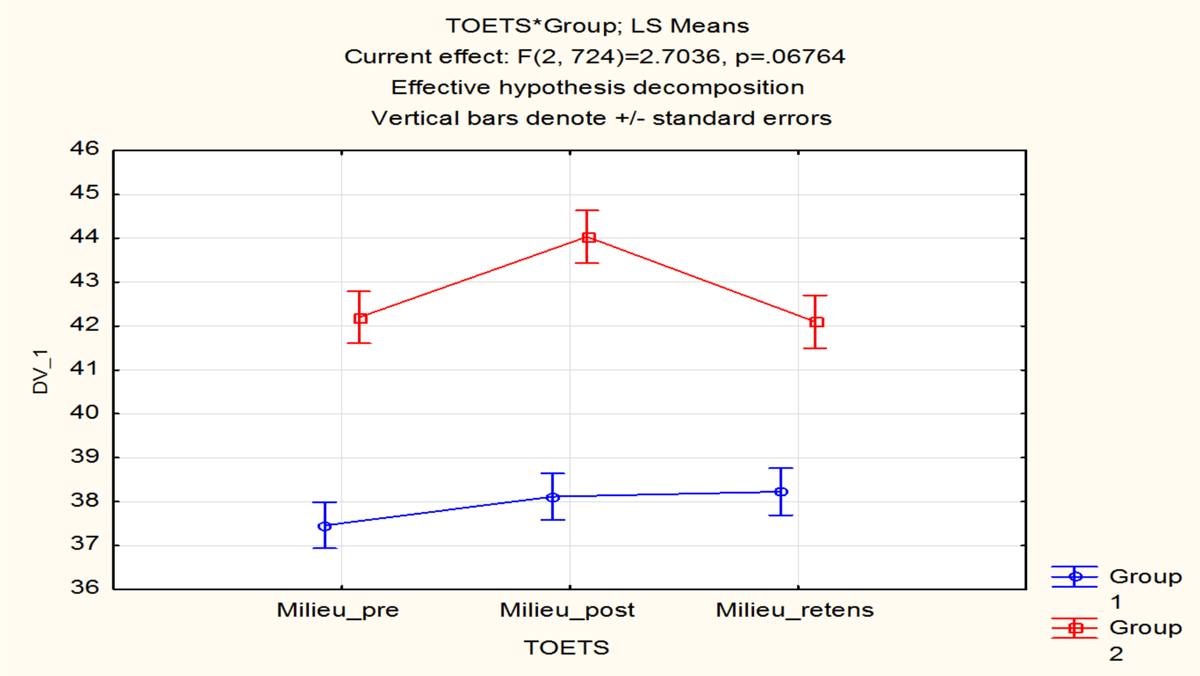


Figure 5.45: Means (with standard errors) of the 3 different tests for the two groups in study milieu field

Figure 5.45 shows that the learners in group 1 (experimental group – intervention only between pre and post phase) showed an improvement between pre- and post- but and a further improvement between post and retention. There was no intervention in group 1 between the post phase and the retention phase. The group 2 (Control group – intervention only between post phase and retention phase) showed an increase between pre and post but a decrease between post and retention phase. The above Figure 5.45 shows that F(2, 724)= 2.703, p = 0.068, which indicates that the means are different at 2 degree freedom and p < 0.1, implying confidence level 90%.

5.3 QUALITATIVE RESEARCH DATA ANALYSIS AND INTERPRETATION

5.3.1 The analysis of the field notes and video recording captured during classroom visits.

The researcher used indicators to realise the objectives of the study (see par 1.3 & 4.5.2). The indicators which researcher identified as common items in the class room observation and field note during the class visits conducted to be used, are:

Indicator 1: Teacher preparation and planning

Indicator 2: Teacher and learner activity

Indicator 3: Assessment and homework

The analysis and interpretation of above mentioned indicators will provide researcher with valuable information regarding implementation of modelling-based learning environment. All the findings will be measured against the findings from literature review conducted in Chapter 2 and Chapter 3.

Indicator 1: Planning and preparation

Lesson Plan: Teacher had a lesson plan in the teacher file, for all the lessons respectively. Lesson plan had authentic modelling eliciting tasks captured in the teacher and learner activities.

Teacher preparation: Teacher seems well prepared, did not refer to lesson plan frequently, in fact, very few times. Teacher knew the number of learners and their names of her class and did prepare for learners. Teacher had worksheets. For each of the lessons employed an instructional technique which fit the lesson.

Indicator 2: Teacher and learner activity

The Indicator 2 highlights two examples of actual class room teacher and learner interactions.

Lesson 2

Teacher X: "Good morning class."

Class: "Good morning teacher."

Teacher X: "This will be just revision of yesterday's class, yesterday we started with the models and then we went over to the equal sign. What is the meaning of the equal sign?"

Learners: (Listening without any response given)

Teacher X: "As I explained Equal means that what is on your left hand side must be the same as that on your right hand side. Now take that first one there : example 4 + 7".

(Teacher X moves to front table and takes out bucket with sweets).

Teacher X: "If I have four sweets here, throwing the sweets one by one in the bucket, while counting 1, 2, 3, 4. Plus"

(Teacher then took another bucket with sweets, some learners arrived late, teacher first pause, ask learners why they were late and also requested them to take their seats and follow what she's doing, then show the other bucket)

Teacher X: "throw the sweet again into another bucket, one by one saying one, two, three, four, five, six seven. So it is four Plus seven, so 4 + 7 is equals to something I don't know + plus (Counting sweets in one by one until six) 6."

(Learners put up their hands indicating they have an answer)

Leaner: "Five mam."

Teacher X: "Can u please explain how you got the answer five?"

Learner (showing with hand to chalkboard): "You take the one from the seven which leave you with six, and place it by the four, which will become five

4 + 7 = ... + 6; 4+ 7 = 5 + 6"

(Teacher then consolidated by doing the problem on chalkboard so that the whole class could understand the learners' reasoning).

(Teacher showed learners on the chalkboard the algorithm, indicating you take a look at the problem).

Teacher X: “You take one from the seven and give it to the five, so what is the answer to make both side equal?”

Learners: “Five.”

Teacher write five in empty space, and asks: “Am I right?”

Learners: “Yes mam.”

Teacher X: “This is how she got the answer”

(Learners clap hands to acknowledge correct working).

(Teacher then calls seven learners to the front to again explain the equal sign:

(Teacher then counted with learners one, until seven

And asks a learner to write on the chalkboard the number seven in symbolic format)

(Learner wrote 7)

(Teacher then thanks learner and called another two learners to the front).

Teacher X: “So this is two plus seven, how many are they all together?”

Learners: “Eleven, mam.”

Other learner: “No mam they are ten.”

(Teacher: “Please come forward and count them.”).

Learner: “One, two, etc to nine”

Teacher: “How many did you count?”

Learner: “Oh! Ok, mam I counted nine.”

Teacher X: “Do all agree they are nine?”

Learners: “Yes mam.”

Teacher X: “So you have nine learners all together.”

(Teacher writes on chalkboard, 7 + 2 = ... + 4 , “So how many learners do I need if on the other side I have a 4 learners already?” Teacher then moved the four learners away from the nine. The learners were split into three and two standing a little further away.)

Learners: “I count 3 mam.”

Teacher X: “So if I take three and place them with the four, is it nine?”

Learners: “No Mam.”

(Teacher so how many do i need to have nine)

Learner: “We need five learners to make total of nine.”

Teacher X: “Now I have the whole nine learners, can you see?” (showing to the learners)

Teacher on chalkboard: “So 7 + 2 = 5 + 4.”

Teacher X: “Ok you can sit down.”

Teacher X: “Now give me a sum, your own one, give me a number. Come on, don’t be shy, give me a number any number.”

(Learners said number in Setswana)

Teacher: “Can someone say it in English please?”

Learners: “Five + three.”

Teacher X: “Writing on chalkboard, 5 + 3 = ___ + 4”

Teacher X: “Now what number must we write in ______?”

Learner: “Four.”

Teacher X: “She says four and wrote four on the chalkboard.”

Teacher X: “5 + 3 = 4 + 4, now let us check? Because what we have here must be the same as what we have there” (Teacher showing to the problem on chalkboard)

Teacher X: “5 + 3 = ...”

Learner: “8”

(Teacher wrote 8 under the five Plus three)

Teacher X: “4 + 4”

Learner: “8”

(Teacher wrote 8 under 4 + 4)

Teacher: so eight is equal to eight

Teacher X: “Now I come back to you (showing to the learner who said 4), how did you get that four?”

Teacher X: “You can’t just write a number, you must have a reason”

(Learner responded in Setswana)

Teacher X: “Can someone say that in English?”

Teacher then said in Setswana what learners said: “arsiti wa eka ...”

Teacher X: “Is it only Learner x who knows the answers?”

Class: “No mam.”

(Learner in Setswana)

Teacher X: “You take one from the five (using an arrow to show) and take it to the three, now it become four.”

Teacher X: “So we will have four plus four, then you get your eight,

Thank you class, so equal means that the things on the right must be the same as the things on the left”

(Teacher then gave a class assessment and homework)

Lesson 5

State outcome of lesson: “T*oday you will learn new ways of mathematical writing, using the arrow language, mathematical trees and parentheses. This will help you to understand the use of formulas and other mathematical expressions.”*

Teacher activity: Teacher handed the Bus activity to learners. Teacher asked if all learners received the bus Riddle activity. Learners answered “Yes mam.” Teacher then asked one learner to read, a few put up their hands and teacher selected one girl learner to read the question. Learner read “Pretend that you are bus driver. Early one morning, you leave the garage with no passengers. At the first stop, 10 people get on the bus. At the next stop, six people, get on. At the next stop, four people get off the bus, and seven get on. After that, five people get on, and two people get off. At the next stop, four people get off, and no one gets on”. After learners read question one which state “How old is the bus driver?” The first question seems to put a change in facial expression. The question elicited a buzz in the class with hands going up. Some learners answered 18 while others said 38. On the question why learner gave answer of 18, learners response “I added the different numbers.” Teacher did not correct learners but asked them to first leave the answer. On the second question “Did you expect the first question to be about the number of passengers on the bus?” Learners immediately answered “Yes mam.” The third question stated “How could you determine the number of passengers on the bus after the last stop mentioned above?" With guidance of teacher learners indicated that they will they need to understand the meaning of the words in the introduction. They started by adding and subtracting as they change the words into mathematical text. With guidance of teacher they manage to get 10+6 =16-4= 12+7=19+5= 24-2= 22-4= 18+0=18

The teacher then asked do the learners see anything confusing, the learners replied yes mam to many equal signs. Teacher said "Can I propose another way of solving the buss riddle?" Learners: "Yes please mam."

Teacher: We use the arrow to help us with the different stages of the solution process. Please follow me: Teacher asks learner to read first part of the problem.

0 → 10 → 16 → 12 → 19 → 24 → 22 → 18 → 18

+10 +6 -4 +7 +5 -2 -4 +0

Teacher used other examples from homework activity to consolidate understanding.

Indicator 3: Assessment and homework

Assessment: Learner did complete assessment activity as per lesson plan. The assessment activity attempted to confirm learners' achievement of lesson outcome. An example of assessment activity given was: 24 + 43 = ... (1) ; 53 + ... = 55 + 35 (1) (See lesson plans available on request). Another example was "At the beginning of the year, lake eye held about 30,000 fish. During the year, 12,000 new fish hatched, and 5,000 others died naturally. Another 6,000 were caught by fishermen. Write an Arrow string to show the changes in the number of fish (2) What is the total number of fish (1)? (See Lesson plans available on request).

Learners did complete the Assessment activity within the specified time. Homework was given to ensure consolidation of concept taught.

Homework: Each lesson had homework activity, which was reflected on the next day. An example of a homework activity given was: "From a dot pattern you can see immediately if a number is even or odd. Make dot patterns for 6, 13, 16, and 17" (see lesson plan available on request)

5.3.2 Interpretation of the key indicators

The teacher was also well prepared and had routine and non-routine modelling eliciting problem centred activities which could challenge learners thinking (see par 2.2.5). The activities arose from genuine mathematical situations (see par 2.2.7). Teachers selected the appropriate designed problems from lesson plan which seem to elicit active participation in the solution process. Modelling eliciting tasks which are open-ended did provide learners with an opportunity to find relevant problems in complex situations (non-routine problems), develop representational tools to describe and analyse the problem structure, and to compare different approaches to the solution (see par 3.3.3). Modelling eliciting activities are the vehicle for developing understanding of mathematical concepts (see par 3.3.3). The problems seemed to solidify, extend and stimulate learners learning (see par 2.2.7).

Teacher was supportive, flexible, provided learners with opportunities to learn, allowing learners to collaborate and reflect. Teacher stating the lesson outcome for the lesson to be conducted provided learners with a clear view of what learning is to be expected. For lesson 1 teacher activities assisted learners to build an understanding of modelling at different levels, and used their real world knowledge of sharing (see par 2.2.6 and 2.4.2.4). In lesson 1, learners were actively involved in their learning. Learners engaged in critical dialogue with teacher (see par learner activity and 2.2.6). Teacher was involved in the discussion process and involved learners (see par 2.2.7). The key finding in this lesson 1 was that learners were able to recognise and differentiate between different types of models. The key finding is that the learners did seem to understand the value of the equal sign in a mathematical sentence. The teacher activities assisted learners to build mental representations of the problem providing; learners draw pictures distinguished between relevant and irrelevant information, used their real world knowledge of sharing (see par 2.2.6 and 2.4.2.4), learners then had to decide how to solve the problem. Learners mostly guess and check and looked for patterns. Learners were activity involve in their learning. Learners grew to understand the meaning of an equal, equivalence and inequality signs in mathematics. The key finding is that the learners did seem to understand the value of the equal sign in a mathematical sentence. Evident from the improvement in the experimental group from pre-test to post-test for item S8 and P8 (see par 5.2.3)

Learners need to be able to conduct internal reflective dialogue. The learners are active participants in their mathematical development. Learners learnt by understanding algorithms, taking cognisance of the context and the meaning and role operations, concepts, symbols, variables. Learners were provided with the opportunity to actively construct knowledge (See par 2.2.6). The teacher was clear on the purpose of the problem, which should enhance and promote the mathematics content. The purpose of the problem can vary from guiding, testing, illustrating a principle, fostering a concept, or provide stimulus for activity (see par 2.4.3.1). Learners then had to decide how to solve the problem. Learners mostly guess and check and looked for patterns. Learners were actively involved in their learning. Learners engaged in critical dialogue with the teacher (see par learner activity and 2.2.6). Learners did learn by understanding algorithms, taking cognisance of the meaning and role of operations, concepts, and symbols (see par learner activity and 2.2.6).

Teacher was involved in the discussion process and involved learners (see par 2.2.7). Teacher was aware of not to provide hastily the answers; giving learners a chance to interrogate problem stated (see par 2.2.7). Learners implemented the solution strategies. Learners then interpreted the outcome and formulated the answer. They then, with teacher, checked whether the answer was correct.

In lesson 5, the teacher activities assisted learners' to build mental representations of the problem. For example, the first question posed provided teacher with a bases to elicit discussion on maths context. The learners changed the problem statement to a mathematical situation and the model using mathematical symbols. Learners were involved in symbol representation (refer to 2.2.7). Learners did learn by understanding algorithms, taking cognisance of the context and the meaning and role of operations, concepts, symbols, variables (see par 2.2.6). Learners distinguished between relevant and irrelevant information, and used their real world knowledge of sharing (see par 2.2.6 & 2.4.2.4). The activities developed a new solution strategy (called arrow language), which seemed to improve learners ability to deal with word problems. Learners then had to decide how to solve the problem. Learners mostly guess and check and looked for patterns.

Assessment and homework provided teacher with an opportunity to assess formative and summative whether the outcome was achieved. The homework provided teacher with opportunity to ensure that learners consolidate understanding. Learners were given an opportunity to actively construct their knowledge (see par 2.2.6).

5.4 MERGING OF QUANTITATIVE AND QUALITATIVE DATA ANALYSIS

An essential part of the mixed method research is that the qualitative and quantitative date must be merged during the research process (see par. 4.4). A multi-phased crossover model was implemented.

The pre-test, post-test and retention test data analysis provided the researcher with a clear understanding of the possible impact of a modelling-based learning environment intervention in grade 6 mathematics classes. The analyses provided evidence that both the control group and experimental group learners before the treatment performed extremely poor in the non-routine problems. A surprising finding was the both groups performed also very poorly in the routine problems. The control group performed slightly better than the experimental group in problem test items. The results confirm the reason for doing the research, as highlighted in the problem statement. The tests occurred in phases one, three and five. The SOM questionnaire provided was to provide information regarding the learners study orientation in mathematics. The SOM questionnaire was also given in phase one, three and five. The SOM questionnaire was reliable, but did not show good construct validity in all the phases. There was a significant difference in the mean of the control group and experimental group. The learning environment organization consisted out of 20 lessons of 1,5hours per lesson. From the treatment analysis of the lesson notes and observation, three indicators were focused on, namely; teacher administration, teacher and learner activity, and assessment & homework. Teachers did prepare for lessons, and were clear on the outcome of the lessons. Achievement of the outcome was demonstrated via the assessment. Consolidation of the lesson content was demonstrated by homework activity. The lessons focused on developing in learners a heuristic set of skills which will enable learners to engage in mathematical modelling. The impact of what the modelling-based learning environment treatment for the experimental group did, was highlighted by the post-test learners' analysis. The experimental group performed better then the control group in all the problems test items and significantly better in the standard test items. The Chi-square, degree of freedom and p-value guided the researcher on the statistical significance, while phi-value guided the practical significance. The pre-test for both the control group and experimental group had no significant difference, which implies that learners' performances showed no difference. The post-test and retention test for some of the items did show statistical significance and the effect size ranged between small to large. Some items had no statistical significance and no practical significance. Examples of learners' answers showed a difference in answering indicating new strategies learnt and applied. For the SOM, evidence suggests that the tool was reliable and was construct valid in pre-/post- and retention phase. All the fields showed no carry-over effect. The results confirm that only the problem solving field showed statistical significance and a small to medium size effect, therefore, practically significant. For the fields, namely; study attitude, study habits, the intervention effect registered showed no statistical significance and no practical significance. For the fields, namely; mathematical confidence, study milieu, the intervention effect might have statistical significance.

5.5 SUMMARY

The Pre/Post/Retention test showed clear evidence of learners' improvement in dealing with non-routine mathematical word problems but also in general routine, mathematical word problems, showing for some items statistical significance and medium to large effect size, hence practical significance. The overall SOM pre/post/retention showed good reliability, and practical significance, large effect but had low to medium effect in individual fields. The qualitative indicators provide clear evidence of teacher administration, teacher & learner interaction and assessment & homework. The findings from the quantitative and qualitative data analysis and interpretations and literature review will provide this researcher with a construct for a modelling-based learning environment. In the next Chapter a construct of modelling-based learning environment will be proposed as a means to improve learners' mathematics performance in grade 6 mathematics classes in the JTG district.

CHAPTER 6:

SUMMARY, FINDINGS, RECOMMENDATIONS AND LIMITATIONS

The final Chapter of this study presents a summary of the core of the previous Chapters, describe the findings according to the research aims and concludes with some recommendations. The Figure 6 gives an outline of the Chapter 6

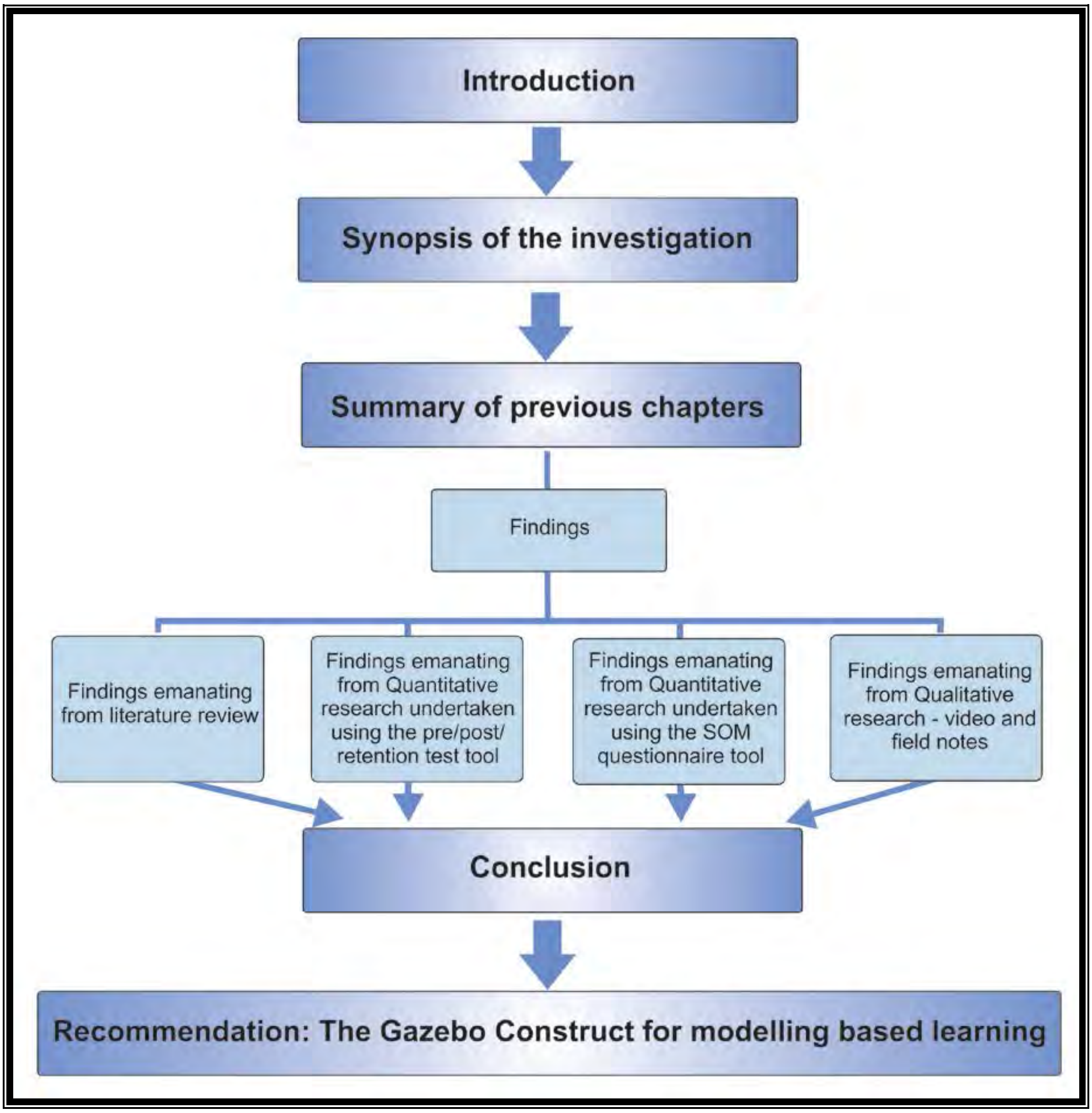


Figure 6: Outline of Chapter 6

6.1 INTRODUCTION

The final Chapter, provides a synopsis of the investigation conducted. The summary of the research findings will be given to indicate that the research objectives as stated in 1.3 have been addressed. The summary will highlight findings emanating from the literature review, from quantitative research and from qualitative research. This is followed by conclusions and recommendations that are drawn from the findings of this study. The designed Gazebo construct emanating from analysis, discussions and findings will be part of the main recommendations. Finally, the problem areas for further research are identified followed by limitations of this study, which conclude this Chapter 6.

6.2 SYNOPSIS OF THE INVESTIGATION

This section presents a summary of the study. A general overview of the contents of each Chapter is given, followed by major themes that emerge from the literature review and empirical findings in order to suggest a way to address the challenges.

6.2.1 Summary of the research

In Chapter 1 the background of the research project was set out. This includes the purpose of the study, problem statement, the research question and secondary questions, the objectives of the study and, clarification of the concepts, the research methodology, and the research framework. The project deals with constructing a modelling-based learning environment for the enhancement of learner performance in grade 6 mathematics classrooms. The specific objectives of the research project are:

a. To determine what the critical constitutes of the modelling-based learning environment are
b. To determine what the building blocks for implementing a modelling-based learning environment are
c. To determine how a modelling-based learning environment influences the learners solving of non-routine problems in grade 6 mathematics classrooms
d. To determine how a modelling-based learning environment influences learners' study orientation in grade 6 mathematics classrooms.

Chapter 2 gives a comprehensive look into learning theories, with a specific focus on the constructivist learning within the mathematics domain. This literature review will provide a theoretical perspective on learning from a constructivist perspective, the learning environment and its critical features and building blocks. Focus will be placed on building blocks for a powerful learning environment, which involves the learning process, problem based learning and problem solving.

Chapter 3 focuses on what literature says regarding design for modelling as a powerful learning environment in grade 6 mathematics classrooms. The literature review will provide a theoretical perspective on the modelling, its process and its features in school maths. At the centre of the discussion is modelling as a teaching tool, a powerful learning environment, the guidelines for constructing a powerful learning environment, the principles of powerful learning environments, the foundation pillars, and identifying essential components for a powerful learning environment, critical constitutes of a modelling learning environment, the building blocks for implementing such a model, the expected influence of a modelling learning environment on learners solving non-routine problems and the expected influence of a modelling learning environment on learners' study orientation in maths.

Chapter 4 initiates the empirical section of the research project. This Chapter focused on a description of the systematic and focused approach according to the following topics; purpose of empirical section, mixed method research approach, quantitative research and qualitative research, ethical aspects and administrative procedures.

Chapter 5 provides the analysis of the quantitative and qualitative data gathered during the phased implementation of the research project. The analysed data are reported on in Chapter 6 and a probable construct is suggested.

6.3 FINDINGS:

6.3.1 Summary of findings emanating from the literature review

The literature review provided research with certain set points to guide the construction of a modelling-based learning environment to improve learner performance in grade 6 mathematics classrooms in the JTG district of the Northern Cape.

The first theoretical set point is learning in a constructivist paradigm

It is evident that the two most prominent theories of learning do provide an understanding framework for the research that has been undertaken (see par 2.2). The behaviourist theory seems not to be able to cater for the needs of the modelling-based learning environment which forms a DNA of the research (see par 2.2.1). The constructivist theory as applied in the school domain, specifically in the mathematics classrooms (see par 2.2.2; 2.2.3), seems to best fit the focus of the research providing a solid framework for understanding learning (see par 2.2.4; 2.2.5). The constructivist learning definition (see par 2.2.6) best fits the modelling-based learning environment. Learners learn new mathematical concepts and procedures by building on what they already know, which involves building relationships within existing knowledge and between existing knowledge and new knowledge (see par 2.2.3). Learners and teachers have certain proposed key roles in the constructivist maths classroom (see par 2.2.6; 2.2.7)

The second theoretical set point is the learning environment

A learning environment in the constructivist paradigm has some critical features (see par 2.3.2) which teachers need to take cognisance of. The building blocks of a modelling-based learning environment are the learning process (See par 2.4.1), problem solving (See par 2.4.2) as describe by Polya and problem based learning (See par 2.4.3). Teachers need to be good problem posers as it lies at the heart of a mathematical activity (see par 2.2.4). Focus was placed on the teachers' role in a problem solving classroom and the strategies to develop and enhance problem solving (see par 2.2.7).

The third theoretical set point is models and modelling

The theoretical perspective on modelling is given, focussing on different viewpoints of modelling and models and highlighting the different types of modelling as a process within the mathematics education domain (see par 3.2). Modelling must be integrated into schools' mathematics education (see par 3.2). The major steps in the cyclic modelling process were focused on (see par 3.3.1). Encapsulated in the first two steps of the modelling process is the heuristic (see par 3.3.1). Developing heuristically formed an essential part of the lessons of modelling learning environment treatment using modelling eliciting tasks (see par 3.3.1). Teachers play an important role in developing modelling practices (see par 3.3.2). The modelling process has certain features such as mathematical problem solving which forms an integral part of the modelling process (see par 3.3.2). Using modelling strategies, learners develop a better understanding of the value of the equal sign and its role in understanding relationships (see par 3.6; 5.3). A learner who engages in mathematical modelling will, when confronted with a non-routine situation, identify the problem, make assumptions, collect data, propose a model, test the assumptions, and refine the model if need-be, fit the appreciate model to data, analyse the underlying mathematical structure of the model, and evaluate and be sensitive to the conclusions (see par 3.6). Teachers can use mathematical modelling as a teaching tool, providing learners with modelling eliciting tasks. Developing modelling practices occurs effectively in a powerful learning environment (see par 3.3.3).

The fourth theoretical set point is the modelling-based environment as a powerful learning environment

A powerful learning environment evokes a modelling-based learning process (see par 3.4.1). The competence, learning, intervention and assessment can be a framework for designing powerful learning environments (see par 3.4.2). The teacher also needs to consider the guidelines and principles for constructing powerful learning environments (see par 3.4.3; 3.4.4). The major pillars such as problem design, instructional techniques and classroom culture creation and the essential components identified such as teaching and learning content, nature of problems, teaching strategies and classroom culture need to be considered (see par 3.4.6; 3.4.7). Learners' active involvement in constructing solutions to real-life problems are realised in a modelling-based environment (see par 3.5). Learners can apply their mathematical knowledge to solve problems and generate new knowledge in a modelling-based learning environment (see par 3.5). Learners can become mathematical modellers who have problem solving skills and can construct and validate models and make generalizations, thereby extending their knowledge base (see par 3.5). The learner works backwards, to uncover implicit assumptions, critical evaluate the appropriateness of the assumptions, and be sensitive to the conclusions if given a model (see par 3.6). The teacher must have a good resources bank of modelling eliciting activities (see par 3.7).

The fifth theoretical set point is the South African curriculum accessibility to a modelling-based environment

The New Curriculum and assessment policy statement is accessible and promotes modelling and problem solving as essential in mathematics teaching and learning (see par 3.8). The teaching and learning of mathematics in primary schools shows a great need for a modelling-based learning environment to provide realisation of the specific aims and expected knowledge and skills to be acquired (see par 3.9). Study orientation in mathematics which includes; study attitude, mathematical confidence, study habits, problem solving and study environment provide a good indicator for school mathematics fluency (see par 2.10). Study orientation in mathematics (SOM) is a clear, measurable indicator of required behaviour of learners in their mathematics performance (see par 2.10). A good study orientation implies good mathematics conceptual understanding and performance (see par 2.10).

6.3.2 Summary of findings emanating from quantitative research undertaken using the pre-/post-/retention-test tool

To determine how a modelling-based learning environment influences the learners' solving of non-routine problems in grade 6 mathematics classrooms.

In addressing the above mentioned aim, the researcher conducted a pre-test/post-test and retention-test (See par 4.4). The findings are as follows:

6.3.2.1 Finding on pre-test non-routine problem (P) items and routine standard (S) items comparing experimental and control groups

Almost all of the problem (P1, P2, P3, P4, P5, P6, P8) test Items, for the pre-test showed no statistical significant ($p > 0.2$) difference between the control group and the experimental group and small size effect with $\varphi \leq 0.1$, which implies non-practical significance (see par 5.2.3.1). The results confirm the findings from Verschaffel, Greer, De Corte, Carpenter, Romberg and Cobb, on the performance on learners in the non-routine problems (see par

1.2). This poor performance can be attributed to a lack of mathematical modelling ability (see par 1.2).

All the pre-test standard items (S1 – S8) had a $p > 0.1$, which implies no statistical significance (see par 5.2.3.5). The $\varphi \leq 0.1$, for most of the pre-test Standard test items, which implies small size effect, therefore, of no practical significance. Pre-test item S5, had a small to medium effect size (see par 5.2.3.5).

6.3.2.2 Findings on the post-test non-routine problem Items and routine standard items comparing experimental and control groups

The post-test did show statistical significance ($p < 0.05$) and small to medium effect ($0.1 \leq \varphi < 0.3$) for most of the problem test items except P5 and P7. There was a statistical significant difference in favour of the experimental group (See par 5.2.3.1). For the post-test items P2 and P8 the effect size was medium to large ($0.3 \leq \varphi < 0.5$), which may imply practical significance (See par 5.2.3.1). The learners from the experimental group made use of new strategies (see figures 5.5 and 5.7) learnt during the modelling learning environment treatment. From the reasoning, it is clear learners did understand the problem in its context and could deduce a realistic response (see par 5.2.3.3). Learners with realistic responses did understand the problem in its context, applying heuristically -drawn sketches (see par 3.3.1; 5.2.3.9), using the modelling method to solve problems (see par 3.6; 5.2.3.9). Some of the learners did not only keep to one answer but did indicate there were different possible answers to the question posed (see par 3.6; 5.2.3.3).

The post-test did show for some standard test items (S2, S4, S7) statistical significance ($p < 0.5$) and the effect size ranging ($0.1 \leq \varphi < 0.3$) from small to medium which implies practical significance (See par 5.2.3.5). There was a statistically significant difference between the control group and the experimental group (See par 5.2.3.5). Most (S1, S3, S5, S6, S8) of the post- test items showed $p > 0.1$, which Implies no statistical significance. The $\varphi < 0.1$ implies no practical significance. The possible reason for this is that the test items were routine, which is part of the normal diet of teaching.

The response method differed in that learners from the experimental group had undergone the modelling learning environment treatment and the learners from the control group did not up to phase 3 of the implementation process (see par 4.4). It is evident that learners from the experimental group with realistic responses used the arrow language to solve a problem (see par 5.2.3.7).

6.3.2.3 Findings on the retention test non-routine problem test items and routine standard test items comparing experimental and control groups

The retention-test did show for the problem (P1, P2, P3, P6, P7, P8) test and (S2, S3, S4, S5, S6, S8) items statistical significance ($p < 0.05$) and an effect size ranging from small to medium to large, which implies practical significance (See par 5.4.1).

Both groups of learners underwent the modelling learner environment treatment. The experimental learners who underwent the modelling learning environment treatment in phase two continued with the treatment without help from the researcher, while the learners from the control group underwent treatment in phase four for the first time (see par 4.4).

The results for the retention test confirm that the learners did improve in their responses to the standard test items posed. In most of the realistic responses applied new strategy (arrow language) and drawing sketches to assist in solving the problem (See par 5.4; 5.2.3.10), learnt during modelling learning environment treatment. Both the experimental group and the control group improved in their way of responding to questions posed (see par 5.2.3.10).

From the above highlighted findings of the different sub questions delineated for the main aim, this researcher can provide a positive answer to the aim which states: “How does the modelling-based learning environment improve grade 6 learner performance in JTG schools?” Learners seem to have gathered that their modelling ability improved, applying the heuristic they learnt (see par 2.3.2; 2.4.2; 3.3; 3.4 & 3.6). Learners do understand the importance of the equal sign, learner do take cognisance of the context of the real-life problems (see par 3.5).

6.3.3 Summary of findings emanating from the quantitative research undertaken using the SOM questionnaire tool

6.3.3.1 Findings on the control group learner SOM fields compare to the experimental group for pre-phase, post-phase and retention phase in crossover design model.

The results confirmed that for all the SOM fields, $p > 0.05$ at 1 degree freedom, which implies no statistical significant carry-over effect. The collective data from both period 1 and period 2 can be used to estimate the intervention effect (see par 5.2.4.1).

The results confirm that only the problem solving field showed an intervention effect ($p < 0.05$), which implies statistical significance and a partial eta-squared = 0.02, which is a small to medium size effect, therefore, practically significant (see par 5.2.4.1).

For the fields, namely; study attitude, study habits, the intervention effect registered a $p > 0.1$, which implies no statistical significance and a partial eta-squared < 0.1, which implies no practical significance (see par 5.2.4.1)

For the fields, namely; mathematical confidence, study milieu, the intervention effect registered $0.05 < p < 0.1$, which may imply statistical significance (see par 5.2.4.1).

6.3.3.2 Findings on the F-ratio statistics and percentage of variance due to between group variation (R-squared) and effect sizes for overall SOM in pre-/post- and retention phase

The findings in all the three phases, the pre-phase, post-phase and retention phase the SOM show clearly that the F-ratio is big and $p < 0.0001$, therefore, a statistically significant difference between experimental group and control group (see par 5.2.4.2). The R-squared > 0.25, which implies large effect, is therefore, of high practical importance (see par 5.2.4.2).

6.3.3.3 Findings on the percentage of variance due to between group variation (R-squared) and effect sizes for SOM per field namely study attitude field, mathematical confidence, study habits, problem solving and learning environment

The results confirm that the SOM field study attitude has a medium effect and practical significance in the pre/post and retention SOM (see par 5.2.4.3).

The field mathematical confidence has very low R-square, small effect thus non-significant for pre-end retention phase. In the mathematical confidence field only the post-phase SOM medium effect, which implies practical significance (see par 5.2.4.3).

The study habits field showed a medium effect only in the post-phase, which implies practical significance (see par 5.2.4.3).

The problem solving in mathematics field showed a medium effect, which implies practical significance (see par 5.2.4.3).

The study environment field has medium effect, therefore, practical significance in the post-phase (see par 5.2.4.3)

6.3.4 Summary of findings emanating from qualitative research undertaken using the video data analysis and classroom visit field notes analysis

Permission was given by the principal and teacher for the classroom visit and video recording. For the classroom video recording, the teacher did ask learners if researcher can capture them and the classroom action on video. Learners gave permission. For the classroom video recording, the researcher focused on lesson planning, teacher preparation and preparedness, lesson outcome, teacher activity/learner activity, assessment activity and homework.

6.3.4.1 Interpretation of the above observation

From the field notes on the observed lessons and video recorded lessons the teacher did bring a lesson plan. Teachers were also prepared and had non-routine problem centred activities which could challenge learners thinking (see par 2.2.5). Opportunity needs to be created to actively construct their knowledge (see par 2.2.6). Teachers selected the appropriate designed problems from the lesson plan which seemed to elicit active participation in the solution process. The problems seemed to solidify, extend and stimulate learners learning (see par 2.2.7). The activities arose from genuine mathematical situations (see par 2.2.7). Modelling eliciting tasks which are open-ended will provide learners with an opportunity to find relevant problems in complex situations (non-routine problems), develop representational tools to describe and analyse the problem structure, and to compare different approaches to the solution (see par 3.3.3). Modelling eliciting activities are the vehicle for developing an understanding of mathematical concepts (see par 3.3.3). The learner content did elicit internal dialogue in some of the learners, which they confirmed with their peers before giving the answer (see par 2.2.6).

Teacher did state the lesson outcome which provided learners with a clear view of what learning was to be expected.

From the teacher and learner activities it is evident that the teachers were clear on the purpose of the problem, which seemed to enhance and promote the understanding of mathematics content. The purpose of the problem can vary from guiding, testing, illustrating a principle, fostering a concept, or providing stimulus for activity (see par 2.6.1). Teacher was involved in the discussion process and involved learners (see par 2.2.7). Teachers were aware of not to provide too hastily the answers, giving learners a chance to interrogate the problem stated (see par 2.2.7).

The teacher activities assisted learners to build mental representations of the problem. Learners drew pictures which distinguished between relevant and irrelevant information, and used their real world knowledge of sharing (see par 2.2.6 and 2.4.2.4). Learners grew to understand the meaning of an equal, equivalence and inequality signs in mathematics. Learners need to be able to conduct internal reflective dialogue. The learners are active participants in their mathematical development. Learners learnt by understanding algorithms, taking cognisance of the context and the meaning and role operations, concepts, symbols, variables. Learners were provided with the opportunity to actively construct knowledge (See par 2.2.6).

Learners then had to decide how to solve the problem. Learners mostly guessed and checked and looked for patterns. Learners were actively involved in their learning. Learners engaged in critical dialogue with teacher (see par learner activity and 2.2.6). Learners were involved in symbol representation (see par 2.2.7). Learners' understood algorithms, taking cognisance of the context and the meaning and role of operations, concepts, symbols, and variables (see par learner activity and 2.2.6). Learners implemented the solution strategies.

Learners then interpreted the outcome and formulated the answer. They then, with teacher, checked whether the answer was correct.

Assessment and homework provided teacher with the opportunity to assess formatively and summatively whether the outcome was achieved. The homework provided teacher with an opportunity to ensure that learners consolidate their understanding.

6.4 CONCLUSION

The specific objective of this research project is:

To determine what the critical constitutes of the modelling-based learning environment are.

From the findings of the literature review, the quantitative and qualitative investigations, which led to the proposed construct, the following critical constitutes for a modelling-based learning environment have been identified:

- Ontological foundation is a constructivist paradigm and constructivist view of learning (see par 6.3)
- Competence, learning, intervention and assessment as a framework for designing modelling-based learning environments (see par 3.4.2)
- Major pillars of modelling-based learning environment, such as problem design, instructional techniques and classroom culture creation (see par 3.4.6). Researcher added meta-cognitive and heuristic development as a fourth pillar (see par 6.3)
- The classroom components identified such as teacher administration (lesson plan & preparation), teaching and learning content, the assessment and homework (see par 5.4)
- The essential components are: modelling eliciting teaching and learning content; nature of the problems; platter of instructional techniques classroom culture (see par 3.4.6, 3.4.7)
- Understanding of models and process of modelling (3.2.1; 3.2.2; 3.2.3).

The second objective of the research project is:

To determine what the building blocks for implementing a modelling-based learning environment are.

The following building blocks for implementing a modelling-based learning environment have been identified:

- The teacher needs to know and understand the guidelines for implementing a modelling-based learning environment (see par 3.4.3)
- The teacher needs to know and understand the principles for constructing modelling-based learning environments (see par 3.4.3; 3.4.4)
- The teacher needs to also have knowledge of the starting point for constructing a modelling-based learning environment (see par 3.4.5)
- Nature of the problems used in the lessons (non-routine modelling eliciting tasks – using carefully designed realistic open complex problems – see par 3.6; 3.7; 2.4.3)
- Teacher being well prepared and a facilitator with a set of teaching techniques (2.2.7; 2.3.2; 2.4.2.3; 2.4.3; 3.7)
- The teachers' and learners' activities; way of dealing with non-routine word problems (meta-cognitive, strategy training which includes problem-solving and heuristics, awareness training, knowing and articulating and reflecting on the modelling process and self-regulation training (i.e. monitoring the modelling process) – see par 3.3.1; 3.4)
- Creation of a classroom culture which is aimed at setting new norms based on mathematical modelling e.g. stimulating learner confidence competence and active involvement within small-group activities and whole class discussions to articulate, debate, and reflect on concepts freely within a mathematical sphere (see par 3.3.4; 3.4.6)
- Lesson plan with key areas lesson outcome, teacher activities, learner activities, assessment and homework (see par 5.4).

The third objective is:

To determine how a modelling-based learning environment influences the learners' solving of non-routine problems in grade 6 mathematics classrooms.

Effect size ranged from small to medium to large, implying practical and statistical significance between the pre-/post-/retention test performance for the non-routine problem (P) and routine standard (S) test items (see par 5.2). The findings confirm that a modelling-based learning environment influences the grade 6 learners solving non-routine problems positively (see par 5.2). The modelling learning environment also improved the learners' ability to solve routine problems significantly (see par 5.2). Learners improved in their general ability to answer word problems; routine and non-routine (see par 5.2), practical significance and statistical significance, and evidence (see figures 5.1-5.40) form learners answer scripts.

One of the key findings is that the learners did seem to understand the value of the equal sign in a mathematical sentence. Evident from the improvement in the experimental group from pre-test to post-test for item S8 and P8 en even better in the post- to retention for both control and experimental group, having medium to large size effect (see par 5.2).

The fourth and last objective of the research project is:

To determine how a modelling-based learning environment influences learners' study orientation in grade 6 mathematics classrooms

The findings confirm that the SOM showed a high cronbach α which indicates that the test is reliable, and construct valid seen from factor analysis and final communality. Although the SOM questionnaire has shown reliability, and constructs validity, measurement of the specific sample in the JTG district seems not to have responded as expected. Possible reasons are that English is the second language. Learners speak mostly Setswana. Evidence of this conclusion is that the high number of spoilt answers and papers, seen from the recorded data and used data.

The overall SOM showed statistical significance ($p < 0.0001$) and a medium effect, therefore, practically significant. This significant difference implies that the results are reflective of the pattern that exists. The crossover design analysis of the SOM fields show that the problem solving field was statistically significant ($p < 0.05$) and small to medium size effect, thus practically significant. The study habit and study attitude fields showed no statistical significance ($p > 0.1$), small size effect, therefore, of no-practical significance. The mathematical confidence and study milieu, showed possible statistical significance for $p < 0.1$.

6.5 RECOMMENDATIONS

South African Mathematics Education System have specific aims (see par 3.9) for the learners in the mathematics field. It also expects learners to acquire knowledge and skill as highlighted in 3.9. The modelling-based learning environment seems to be the best vehicle to address these identified specific aims and will equip learners with the knowledge and skill as given by CAPS. The essence of all five content areas, are underlined by specific aims and specific skills of which a modelling-based learning environment will contribute positively (see par 3.8). The development of modelling practices, through a modelling-based learning environment should not emanate from a specific part of the curriculum but rather the entire curriculum. The Gazebo construct below is proposed to improve the learner performance in mathematics grade 6 classes.

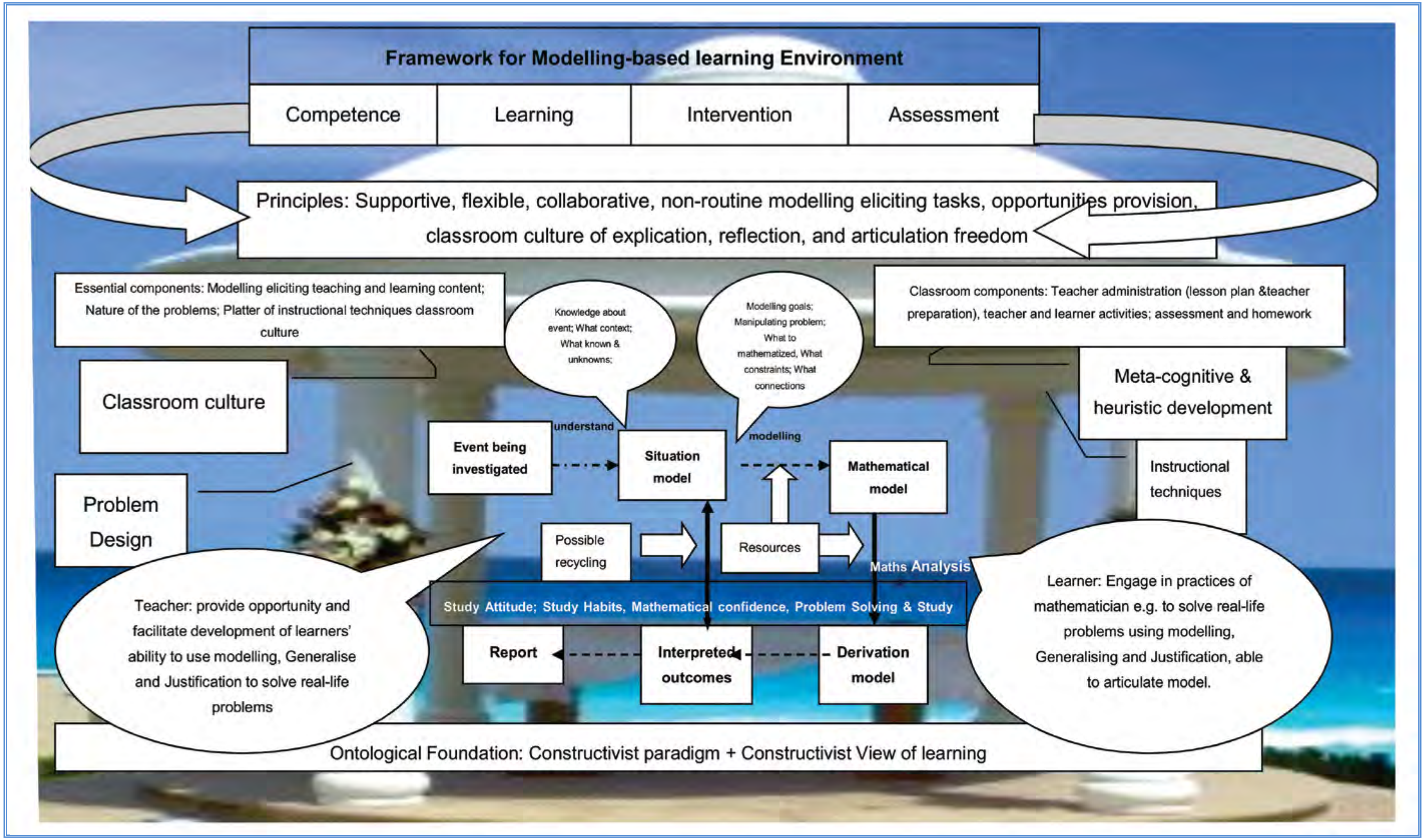


Figure 6.1: Proposed Gazebo construct of a modelling-based learning environment

From the above proposed construct Figure 6.1 it is evident that the ontological foundation is the constructivist paradigm and its view of learning (see par 2.2.5; 2.2.6; 2.4.1-2.4.3).

Teachers need to take cognisance of the framework for designing a modelling-based environment. The framework included is competence, learning, intervention and assessment (see par 3.4.2).

The pillars are problem design, teaching techniques and classroom culture (see par 3.4.6). This researcher added meta-cognitive and heuristic development as the fourth pillar of the modelling-based learning environment (see par 2.3.2). The principles that guide a modelling-based environment are: supportiveness, flexibility, collaborative, non-routine modelling eliciting tasks, opportunity provision, classroom culture of explication+ reflection, and articulation freedom (see par 3.4.4). The essential components of the modelling-based learning environment are: modelling eliciting teaching and learning content; the nature of the problems; a platter of instructional techniques and classroom culture (see par 3.4.7).The classroom components as identified by key indicators: teacher administration (lesson plan, teacher preparation), teacher and learner activities, assessment and homework (see par 7.3.3). The modelling process needs to form part of normal teaching practice and learners' learning. The teacher needs to provide the opportunity to and facilitate the development of learners' ability to solve real-life problems using modelling, justifying and generalizing (see par 2.2.7; 2.3.2; 2.4.2.3; 2.4.3). The learner engages actively in practices of mathematicians to solve real-life problems using modelling, generalizing and justification (2.2.6; 2.4.1-2.4.3). The modelling learning environment impacts on the learners' study attitude, study habits, mathematical confidence, problem solving, and study environment (see par 3.10)

An example of realising the above construct is the twenty lessons modelling learning environment organization. The modelling learning environment organization consists of 20 lessons of 1h to 1.5 hours. Each of the lessons is captured in a lesson plan, to guide teachers in the development of learners to become active mathematical modellers at the intermediate phase. The 20 lessons have been adapted by the research team from the work of Carpenter and Romberg, and works of Verschaffel and De Corte (see par Chapter 2 & Chapter 3). The average age of the target group learners should be 12 years and have a good attendance at school and mathematics classes. Modelling abilities of young learners is essential (See par 3.5). The lessons provide learners with an opportunity to develop in the construction of relationships, extending and applying mathematical justifying and explaining generalizations and procedures and taking responsibility for making sense of mathematical knowledge. This will yield improved understanding and optimize functioning on the higher cognitive levels (see par 3.5). The lessons are a vehicle for mathematics to become a language of thought rather than just a collection of facts and doing exercises applying learnt algorithms (see par 3.5). Lessons provide learners with opportunity to be mathematical modellers who have problem solving skills and can construct and validate models and make generalizations, thereby extending their knowledge base (see par 3.5).

Learners develop problem posing, problem solving skills and solve non-routine and routine real-life problems using mathematical modelling. Each of the 20 lesson plans have the teacher preparation, lesson outcome, the resources needed, the teacher activity, the learner activity, the assessment and the homework. Each lesson is geared to develop the necessary skills and knowledge of learners to be active in mathematical modelling to solve real life problems (see par 3.5). The lesson content captured in the teaching learners assessment and homework activities are geared towards developing in learners the ability to construct mathematical models (see par 3.7). This researcher made use of modelling eliciting activities as the lesson content (see par 3.7). The engagement in these lesson activities provides learners the vehicle to develop modelling skills. The modelling eliciting activities provide the possible link between the abstract nature of pure mathematics and its application to a real-life context (see par 3.8). These authentic word problems have replaced normal school mathematical word problems which are devoid from authentic real-life context (see par 3.7).

6.6 VALUE OF THE RESEARCH

6.6.1 Subject area

Mathematics teaching and learning continues to be in a pitiable state. This research will contribute in providing a designed model for implementation of modelling as a powerful learning environment in the grade 6 mathematics classrooms, and create opportunities for developing new theory regarding teaching and learning of mathematics. The research has confirmed that a modelling-based learning environment does improve learners' problem solving ability, with no statistical significance and practical significance for study attitude, and study habits. The mathematical confidence and study milieu fields may have statistical significance and practical significance.

6.6.2 Research focus area

Broaden the field to design research in mathematics education. This research will epistemologically contribute to the theoretical arena and methodologically to the practical arena. Based on the findings, recommendations will be made on a design for modelling as a powerful learning environment in the grade 6 mathematics classrooms.

6.7 LIMITATIONS

6.7.1 Limitations to design experiments

A challenge of design research is that the role of the researcher is dual; being both a researcher and a designer which, according to some researchers, is conflicting in roles (Cobb *et al.,* 2001). The researcher does realize that the claims to be made are based on the researcher's influence on the context and understands that these claims might not be generalized to other contexts where research does not directly influence the context.

6.7.2 Limitations in sample

The sample used in the project included only learners from two classes for the experimental group and three classes for the control group which is very small in a quantitative framework. The sample, therefore, cannot be merely considered random and representative of the South African mathematical learner population; hence, the results cannot be merely generalized/extrapolated to the South African mathematical learner population. A possible short coming in the teacher implementation of the learning environment may also explain why no strong effects for most of those been treated were obtained. The study did highlight that effective realisation of the modelling-based learning environment placed a very high demand on the teacher. This researcher did support teachers and they were prepared within the constraints of the research project, and the focus of this research was to design a modelling-based learning environment, not to test a scientifically based system of teacher and learner development and support.

The 20 lesson treatment was given separately from the normal mathematic curriculum. The probability for better results seems higher if the available instructional time and the learning environment were integrated into regular lessons.

6.8 AREAS FOR FURTHER RESEARCH

The study orientation in mathematics which focuses on the study attitude, mathematical confidence, problem solving, study habits, and study environment and its relationship to modelling-based learning environment can provide new findings. A much broader study can be done to understand why the SOM provided a high acceptable Cronbach α value and low acceptable construct validity for the groups investigated (See par 5.2.1), but only problem solving showed a clear statistical significance and practical significance (see par 5.2.4.1).

From a theoretical and a practical view point, future research regarding the modelling-based learning environment is extremely important.

6.9 FINAL ANALYSIS

The purpose of the study, which emanated from poor learner performance for different grades in mathematics, was to construct a modelling-based learning environment to improve learner performance in grade 6 mathematics classrooms. The research showed that the modelling-based learning environment has statistical significance and practical significance for the problem-solving field, and may have statistical and practical significance for the study milieu and mathematical confidence fields. The study attitude and study habits field showed no statistical and practical significance. This researcher can conclude that the modelling-based learning environment impacts positively on problem solving but not on all the fields of study orientation in mathematics.

The study proposes the GAZEBO construct for a modelling-based learning environment to improve learner performance in grade 6 mathematics.

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ANNEXURE A

DEPARTMENT OF EDUCATION
DEPARTEMENT VAN ONDERWYS
LEFAPHA LA THUTO
ISEBE LEZEMFUNDO

John Taolo Gaetsewe
District Office
Private Bag X115
MOTHIBISTAD
8474
Republic of South Africa
Tel. (053773 9600
Fax (053) 773 1807

Enquiries / Dipatlisiso / Imibuzo / Navrae : **V J TEISE**

Reference / Tshupelo / Isalathiso / Verwysings :

Date / Leshupelo / Umhla / Datum : **21 FEBRUARY 2011**

TO: Mr Frank van Schalkwyk

FROM: Mr Vuyani Joy Teise
DISTRICT DIRECTOR

SUBJECT: <u>PERMISSION TO CONDUCT RESEARCH</u>

Permission is hereby granted for you to conduct research in schools in John Taolo Gaetsewe District. The identified schools are as follows.

1. Sesheng Intermediate
2. Bankhara combined
3. Isagontle Primary
4. Moraladi Primary
5. Gakgatsana Primary
6. T.T Lekalake Primary.

The District will like to take this opportunity to wish all the best in your studies.

Yours sincerely

....
VJ Teise
DISTRICT DIRECTOR
JOHN TAOLO GAETSEWE DISTRICT

NOORD-KAAP ONDERWYS DEPT
JOHN TAOLO GAETSEWE DISTRIK KANTOOR
Privaatsak X115, Mothibistad 8474
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21 FEB 2011
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JOHN TAOLO GAETSEWE DISTRICT OFFICE
Private Bag X115, Mothibistad 8474
NORTHERN CAPE EDUCATION DEPT

HIV/AIDS is everyone's concern.

Page 1 of 1

ANNEXURE B

STUDY ORIENTATION: MATHS QUESTIONNAIRE

S.O.M

1- Rarely	2 - Sometimes	3 - Frequently	4 - Generally	5 - Always

1	I enjoy solving Maths Problems
2	While answering tests or exams in Maths, I panic
3	I catch up lost work in Maths
4	I explain Maths to my friends, parents or other persons
5	My teacher uses words that I do not know and that confuse me
6	I believe that I can do well in Maths
7	When I do Maths in Class, I become anxious
8	I test myself in writing as well as orally on Maths that I learn.
9	I can think of examples where I use Maths outside the class.
10	I like the place where I do my homework and prepare for tests and exams in Maths
11	If the Maths lesson is boring, I write letters or do something else which interests me more.
12	I cannot speak clearly when I suddenly have to answer a question in a Maths class.
13	I postpone my Maths homework and do something I enjoy more.
14	I know what to do when I have to solve Maths problems.
15	Unhappiness or frustration prevents me from working as hard as I can in Maths.
16	I think the Maths topics are useful.
17	I lose marks in Maths tests and exams because I cross out correct answers.
18	I repeatedly read through a longer problem until I fully understand what is going on.
19	I try to find connections between different sections of Maths.

20	I am too frighten of my Maths teacher to ask questions about Maths
21	It is more important to me to know how to solve a Maths problem than just to find the answer.
22	When my friends talk about a sum or way of solving a problem in the Maths class, I chew on my fingernails, pencil or other objects
23	I keep my Maths homework up to date by completing every day's work properly.
24	As far as I can, look for a less difficult example of Maths problem to try to solve it (to see how work that I know well is connected to new work).
25	I ask my teacher to help me understand Maths assignments that are not clear to me
26	I try to carry out the following four steps for problem solving in Maths: find out what is given and asked; make a plan; carry out the plan and verify the solution.
27	When I do Maths with other pupils, I make sure that I also try to work out the answers.
28	During the Maths lesson I pay attention to the work.
29	I play nervously with my pen, ruler or something else when I have to solve difficult Maths problems.
30	While doing my homework in Maths, it is important to me to find out which concepts (ideas) I do not understand.
31	I find it easy to say what I want to say in Maths tests and exams.
32	It is my parents' or teacher's fault that I do not work hard at Maths.
33	When I start doing Maths I become sleepy, tired or bored.
34	I lose marks in Maths tests and exams because I work too quickly or too slowly.
35	I postpone my Maths homework (or to learn for a Maths test) by doing something else first.
36	I try to apply the Maths that I learn in class in everyday life.

37	I cannot see or hear well in the Maths class, but I hesitate to mention it to my teacher.
38	After having worked for a while I find that I cannot concentrate on Maths any more.
39	While writing Maths tests or exams, I become worried when I see how quickly the other children work while I make slow progress because I battle.
40	I make sure that I follow up my Maths tests and exams and that I understand why I have made mistakes.
41	I first read through all the work quickly to get a complete picture of the test or exam work in Maths on which I shall be tested before I begin preparing.
42	I get poor marks in Maths because of the situation at home.
43	I talk enthusiastically to my parents and friends about Maths.
44	Even though I know that certain sums are incorrect, I mark them correct.
45	I learn formulas and theorems until I know then by heart.
46	When I find that I do not understand a sum, I look at it from another angle or read it in another way.
47	I do not understand words in Maths.
48	I believe it is important to use Maths to help to make the world a better place
49	in Maths class, I find that I have to visit the toilet
50	I make sure that my sketches in geometry are big and clear, I use colour pencils to make the sketches more clear to me.
51	While doing long sum, I stop in between to make sure that I understand what I have already done.
52	I am unsure whether my work is correct, but I hesitate to ask my teacher questions in this regard.
53	Maths is a subject in which problem solving is more important that memorisation.

54	I try to understand why the rules in Maths work.
55	I continue to work in Maths, even if I find it uninteresting or boring.
56	In the Maths class I perspire more than in other classes.
57	When I get totally stuck while trying to solve a problem in Maths, I go back to the beginning of it.
58	It is important to me to estimate my answers in Maths before doing actual calculations.
59	The names and examples that are used in "word problems" are unfamiliar to me.
60	I try to be interested in Maths even if it seems more enjoyable to do something else.
61	I struggle with certain sums because I have not read them carefully.
62	When preparing for Maths tests and exams, I work out a number of new problems and not only well -known problems
63	I talk to my friends about Maths and we discuss mathematical terms and concepts (ideas).
64	Personal problems are why I cannot do my best in Maths.
65	I look for help when I have problems in Maths.
66	I move my feet when my Maths teacher asks me a question.
67	I work out previous Maths tests and exam papers.
68	I ask questions and take part in discussions during the Maths lesson.
69	I read slowly, therefore I do not completed my test and exam papers.
70	I make sure that I understand formulas and theorems before I memorise them.
71	I am afraid to discuss my personal problems with my Maths teacher.
72	I make sure that I know how much time I need for revision before Maths tests and exams and I plan my time accordingly.

73	I know which subsections of Maths I cannot do.
74	When I come across a new word or symbol in Maths, I make sure that I understand it.
75	I try to find patterns when I do Maths.
76	I look smaller details when I study Maths and I cannot see the whole picture.

Study-Orientation in Maths (**Answer sheet**)

Instructions:

1. Don't make any marks on questionnaire booklet
2. Please complete on the answer sheet provided to you
3. Please Shape the Corresponding letter as per examples stated in the Questionnaire booklet

1- Rarely; 2 – Sometimes; 3 – Frequently; 4 – Generally; 5 - Always

Remember: There is no right or wrong answer. Be honest!

1	1	2	3	4	5	39	1	2	3	4	5
2	1	2	3	4	5	40	1	2	3	4	5
3	1	2	3	4	5	41	1	2	3	4	5
4	1	2	3	4	5	42	1	2	3	4	5
5	1	2	3	4	5	43	1	2	3	4	5
6	1	2	3	4	5	44	1	2	3	4	5
7	1	2	3	4	5	45	1	2	3	4	5
8	1	2	3	4	5	46	1	2	3	4	5
9	1	2	3	4	5	47	1	2	3	4	5
10	1	2	3	4	5	48	1	2	3	4	5
11	1	2	3	4	5	49	1	2	3	4	5
12	1	2	3	4	5	50	1	2	3	4	5
13	1	2	3	4	5	51	1	2	3	4	5
14	1	2	3	4	5	52	1	2	3	4	5
15	1	2	3	4	5	53	1	2	3	4	5
16	1	2	3	4	5	54	1	2	3	4	5
17	1	2	3	4	5	55	1	2	3	4	5
18	1	2	3	4	5	56	1	2	3	4	5
19	1	2	3	4	5	57	1	2	3	4	5
20	1	2	3	4	5	58	1	2	3	4	5
21	1	2	3	4	5	59	1	2	3	4	5
22	1	2	3	4	5	60	1	2	3	4	5

23	1	2	3	4	5	61	1	2	3	4	5
24	1	2	3	4	5	62	1	2	3	4	5
25	1	2	3	4	5	63	1	2	3	4	5
26	1	2	3	4	5	64	1	2	3	4	5
27	1	2	3	4	5	65	1	2	3	4	5
28	1	2	3	4	5	66	1	2	3	4	5
29	1	2	3	4	5	67	1	2	3	4	5
30	1	2	3	4	5	68	1	2	3	4	5
31	1	2	3	4	5	69	1	2	3	4	5
32	1	2	3	4	5	70	1	2	3	4	5
33	1	2	3	4	5	71	1	2	3	4	5
34	1	2	3	4	5	72	1	2	3	4	5
35	1	2	3	4	5	73	1	2	3	4	5
36	1	2	3	4	5	74	1	2	3	4	5
37	1	2	3	4	5	75	1	2	3	4	5
38	1	2	3	4	5	76	1	2	3	4	5

MATHEMATICS

Grade 6 Pre-Test

Grade 6 Pre-Test

Time: 1hour

2011

Age of Learner: ___________________ (Optional)

Gender of Learner: Male ____________ Female__________ (Optional)

Please complete the following pre-test. Answer all the question to the best of your ability in the pace provided, under each question. *Your answers will be treated strictly confidentially*

S1: Rorisang organized a birthday party for his tenth birthday. He invited 8 boy friends and 4 girl friends.

How many friends did Rorisang invite for his birthday party?

CALCULATION:

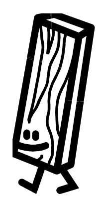
ANSWER:

P1: Shaieda has 5 friends and Kiana has 6 friends, Shaieda and Kiana decide to give a party together. They invite all their friends. All friends are present. How many friends are there at the party?

CALCULATION:

ANSWER:

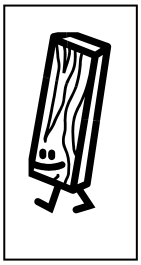
S2: Tokiso has bought 5 planks each 2 meters long. How many planks 1 meter long can he saw from these planks?



CALCULATION:

ANSWER:

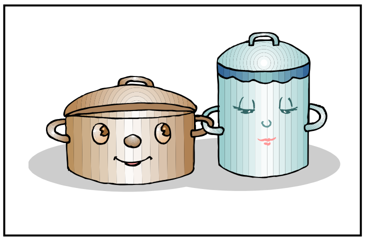
P2: Tashrick has bought 4 planks of 2.5 meters each. How many planks of 1 meter can she get out of these planks?



CALCULATION:

ANSWER:

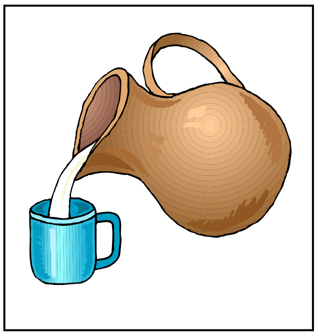
S3: A shopkeeper (Elainne) has two containers for apples. The first container contains 60 apples and the other 90 apples. She puts all the apples into a new, bigger container. How many apples are there in the new container?



CALCULATION:

ANSWER:

P3: What would be the temperature of water in a container if you pour 1 liter of water at 85^0C and 1 liter of 45^0C into it?



CALCULATION:

ANSWER:

S4: Jodi's piggy bank contains R690. She spends all that money to buy 20 equally priced Barbie-Dolls. How much was the price of one Barbie Doll?

CALCULATION:

ANSWER:

P4: 240 Athletes must be bused to their Athletics meeting in another town. Each local bus can hold 25 Learners. How many buses needed?

CALCULATION:

ANSWER:

S5: Ghita made a walking tour. In the morning he walked 8 kilometers and in the afternoon he walked 15 kilometers. How many kilometers did Ghita walk?

CALCULATION:

ANSWER:

P5: Naledi and Jemima go to the same school. Naledi lives at a distance of 10 kilometres from the school and Jemima at 4 kilometres. How far do Naledi and Jemima live from each other?

CALCULATION:

ANSWER:

S6: This morning *Tlaletsi* add R1480.00 in his saving account. Now he has R1650.00 in his saving account. How many Rands did Tlaletsi gain since this morning?

CALCULATION:

ANSWER:

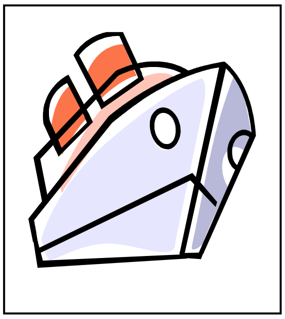
P6: Nozizwe was born in 1994. Now it is 2009. How old is she?

CALCULATION:

ANSWER:

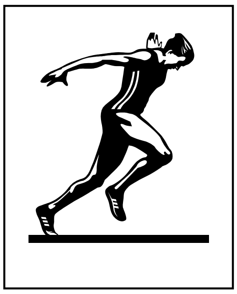
S7: A boat sails at a speed of 45 kilometers per hour. How long does it take this boat to sail 180 kilometers?

CALCULATION:



ANSWER:

P7: Richley's best time to run 100 metres is 16 seconds. How long will it take to run 1 kilometre?



CALCULATION:

ANSWER:

S8: Complete the following mathematical sentence

37 + 39 = _____

CALCULATION:

ANSWER:

P8: Complete the following mathematical sentence

35 + 39 = ____ + 36

CALCULATION:

ANSWER: