A COMPARISON OF THE RESPONSE MODEL/ COMBINED ARRAY METHOD AND THE TRADITIONAL METHODS FOR ROBUST DESIGN

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In memory of my wife and son, Tilly and Ndidi, with whom all this began.

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ABSTRACT

Genichi Taguchi alerted statisticians to the importance of considering variation as well as target when designing experiments. The Box-Taguchi method (a variation of his classical method) has placed little emphasis on C-N interactions. In this study, an attempt is made to understand the C-N interactions. To achieve this, simulated examples are used to demonstrate a modern day technique called Response Model/Combined Array. Also, Taguchi's designs call for a complete cross between a (possibly) fractionated so-called control array and a (possibly) fractionated so-called noise array. It is sought to explore the possibility of cutting down on the total number of experimental runs by fractionating a design consisting of control and noise factors put together in one design. The conclusion drawn is that the Response Model/Combined Array method offers a better possibility of saving in the total number of experimental runs than the Box-Taguchi method.

Titel

'n Vergelyking tussen die Responsmodel / Gekombineerde Matriksmetood en die tradisionele metodes van Robuuste Ontwerp.

Opsomming

Genichi Taguchi het statistici bewus gemaak van die belang daarvan om variasie asook doelwit in ag te neem in die ontwerp van eksperimente. Die Box-Taguchi metood ('n variasie van die klassieke Taguchi metood) lê min klem op die interaksies tussen ruisfaktore en ontwerpfaktore. In hierdie studie word genoemde interaksies ondersoek. Gesimuleerde voorbeelde word gebruik om 'n onlangs ontwikkelde tegniek, die Responsmodel/ Gekombineerde Matriksmetood te illustreer. Verder behels Taguchi ontwerpe 'n volledige kruising tussen 'n (moontlik) gefraksioneerde ruismatriks en 'n (moontlik) gefraksioneerde ontwerpmatriks. Die moontlikheid om op die totale aantaal eksperimentele lopies te spaar deur die fraksionering van 'n ontwerp, word uit ontwerp- en ruisfaktore gekombineer in een ontwerp, word dus ondersoek. Die gevolgtrekking is dat die Responsmodel/Gekombineerde Matriksmetood 'n beter kans op die besparing van totale eksperimentele lopies bied as die Box-Taguchi metode.

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CHAPTER ONE

INTRODUCTION

1.1 PREAMBLE

1.1.1 ROBUST DESIGN

Robust Design is a methodology for finding the optimum settings of control factors to make a product or process insensitive to noise factors (Shoemaker *et al*, 1991). Control factors are factors whose values are set by the designer and noise factors are uncontrolled factors that can cause deviations from the target values in a product's functional characteristics. Robust Design is the most economically and technologically sensible way of designing high quality into product and processes without eliminating the causes of variation by manipulating control factors that have an impact on noise. This is in contrast with trying in an expensive way to eliminate noise by, for example, buying new technology. The method ensures quality by (Phadke, 1989:xv):

(i) making performance insensitive to raw material variation, thus allowing for low grade material and components in most cases,

(ii) making designs robust against manufacturing variation, thus reducing labour and material cost for re-work and scrap,

(iii) making the design least sensitive to variation in operating environment, thus improving reliability and reducing operating cost, and

(iv) using the new structured development process so that engineering time is used most productively.

1.1.2 TAGUCHI'S CLASSICAL APPROACH AND THE BOX-TAGUCHI APPROACH

The traditional Robust design ideas were first formulated by Genichi Taguchi (Taguchi, 1986). His proposed method also known as Parameter Design calls for choosing the levels of control factors to minimise the loss caused by noise factors.

Taguchi recommended a two-part designed experiment which can be represented by a control array (the combination of all control factors to be studied in a designed experiment) and the noise array (the combination of all noise factors). The columns of a control array will represent the control factors and each row a specific combination of levels of control factors. The same applies for the noise array. A combination of control and noise arrays as indicated in **Fig 1.1**, will constitute a complete traditional Taguchi design experiment. For constructing noise and control arrays, Taguchi recommended the use of orthogonal arrays. **(see Chapter Two)**

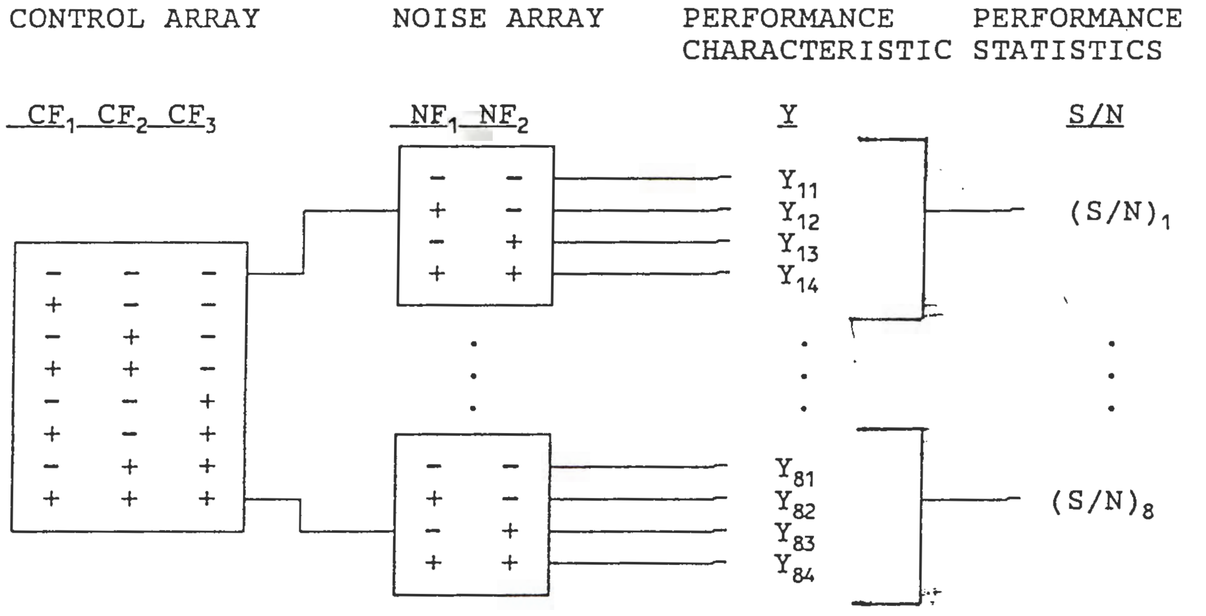

Fig 1.1 An example of Taguchi's Two-part Plan.

Each row in the control array is run for all rows in the noise array. Such a set-up is known as product array design. For a given row in the control array, the replicated observations generated by the noise array are used to compute a performance statistic called the signal-to-noise ratio (S/N). The objective is to find a combination of levels of the control factors that optimises the performance statistic.

In this study a variation of the classical Taguchi's method, which will be called the Box-Taguchi method, will be adopted as a method of analysis in Chapter Two. Its optimising procedure is different from Taguchi's method in that the logarithm of standard deviation is minimised instead of optimising S/N and then the mean is driven to its target value by choosing the level combination of the other factors that will optimise the mean performance. If all control factors affecting the mean are used up in adjusting the variance, then a compromise of using some of the factors affecting variance has to be made to adjust the mean.

1.1.3 DISADVANTAGES OF TAGUCHI'S CLASSICAL APPROACH

The problem with the Box-Taguchi method is that many runs have to be made (since a full cross of control and noise factors is made) and hence it is uneconomical. Moreover, no flexibility is possible as far as the degrees of freedom is concerned for the simplification of the problem because many degrees of freedom are used to estimate interactions that could be judiciously neglected, such as higher order interactions.

Since the focus is on the modelling of loss R, the non-linearity of this function, R may be a problem and this may hide some of the relationships between individual control factors and noise factors.

The remedy to this was first proposed by Welch (Welch *et al*,

1990) when he proposed that only responses must be modelled instead. To do this, Shoemaker *et al* (1991) suggested that the control and the noise factors should be combined to form a single matrix which they called a combined array, also named a standard setup by Gosh and Derderian (1993). The reponse values from the design are used to identify level combinations of control factors which which may reduce variability. The mean response is subsequently adjusted to its target value.

With such an approach it is possible to informally interpret control-to-noise interactions. As such it is easy to identify the control factors that dampen the individual noise factors. In addition the run-size is remarkably reduced. Hence the method looks effective and highly economic.

1.2 PROBLEM STATEMENT

The purpose of this study is to:

(i) combine into one work some important ideas on Robust Design scattered in literature,

(ii) survey the traditional Taguchi's approach to Robust Design which leads to the Box-Taguchi method, and Combined Array method,

(iii) show that the new Combined Array method works well by showing that it gives the same results as the Box-Taguchi method for same problems,

(iv) compare significant effects of both the mean and variance models of the Box-Taguchi method with the corresponding effects of the simulated models (simulation procedure will be described in the **Appendix**) and to

(v) show that provided engineering knowledge is used, run savings can be made when Combined Array is used.

1.3 RELEVANCY

There is a world-wide recognition of the fact that pre-production experiments, properly designed and analysed can significantly contribute to the efforts of quality improvement of products and processes and reduction of cost and waste (Logothetis,1990). Such experiments could be Combined Array and Taguchi's approach to Robust Design.

Apart from saving money and time these experiments may be used to acquire dependable information about industrial setup (Deming,1988). The potential of Taguchi's experimental methods was first realised by the Japanese industry and it is now embraced by the West (Box and Meyer, 1986).

This study will provide an introduction to the classical

Taguchi's approach to Robust Design and its variant version, the Box-Taguchi method, and to the specialised topic of Combined Array. The new method (Combined Array) offers the possibility of saving in total runs and the easy identification of the factors dampening the individual noise factors. Thus the method is attractive because it can save time and money.

The study is therefore not only of academic importance, but it also offers practical benefits.

1.4 RESEARCH METHODOLOGY

A careful integration of literature on Robust Design is made. The literature survey will cover the traditional Taguchi method, the Box-Taguchi method and the new Combined Arrays method.

The Box-Taguchi method and Combined Array method will be used to analyse some simulated examples. Simulated examples are preferred because the aspects that are investigated are known *apriori*. It must be noted, however, that simulation is used in this study to learn about models and not to investigate the validity of the simulation procedure.

A simulated design will be fractionated and analysed. It will be first fractionated by a method advocated by Taguchi but analysed by the Box-Taguchi method. A different fractionation procedure popularised by Box *et al* (1978) will be used twice to fractionate this design. The Combined Array method will be employed to analyse both fractional designs.

1.5 OUTLINE

In Chapter Two, Taguchi's methods for Robust Design are introduced. These are his traditional Parameter Design and the Box-Taguchi method. The Parameter Design is reviewed as a background to the Box-Taguchi method which will be employed to analyse simulated examples.

Chapter Three gives an overview of the Combined Array method and its application to the simulated examples of Chapter Two. The results realised are compared with those obtained in Chapter Two. This method adds a new dimension to Robust Design - only the noise-to-control interactions are interpreted to determine control factors that potentially have effect on noise.

Finally, an assesment of Box-Taguchi and Combined Array methods is made in **Chapter Four.**

CHAPTER TWO

TAGUCHI'S METHODS

2.1 INTRODUCTION

Taguchi's philosophy of quality plays an important role in the understanding of Robust Design (Phadke, 1989: 4). In this chapter Taguchi's approach to quality improvement is discussed.

Section 2.1 gives a brief background of the Taguchi's traditional approach to Robust Design. The importance of categorising factors into control and noise factors is emphasised by two practical examples in **Section 2.2.** It is subsequently seen that to effectively and economically find settings of control factors such that variability due to noise factors about the target value is reduced, properly designed experiments are required.

The "loss to society" idea of Taguchi is at the heart of what is called the loss function. Taguchi contends that the "loss to society" increases as the value of a quality characteristic departs from is optimal value. **Section 2.4** focuses on quality and **Section 2.5** on the loss function.

Section 2.6 describes a designed experiment, its components and the relationship of these components to noise and control factors. The importance of orthogonal arrays is highlighted in **Section 2.7**. **Section 2.8** discusses Parameter Design. Because of criticism levelled against its optimisation procedure (Ryan, 1989: 357; Schmidt and Launsby; 1989: 6-22), a variation of this method called the Box-Taguchi method is introduced. **Section 2.9** discusses the analysis by Box-Taguchi method of four representative simulated examples. The important points of this chapter are summarised in **Section 2.10**.

2.2 TAGUCHI'S PHILOSOPHY OF ROBUST DESIGN

Taguchi's Robust Design technique, also known as Parameter Design (Phadke, 1989: 6) rests on the importance of economically achieving high quality, low variability and consistency of a functional characteristic. By making effective use of experimental design, the method concentrates on minimising performance variation about a target caused by hard-to-control or uncontrollable factors which he calls noise factors. Noise factors could be external or internal (Logothetis & Wynn, 1989: 243). Operating environmental variables such as temperature and humidity are examples of external noise factors. There are two types of internal noise factors. They are unit-to-unit variation (e.g. cause by different machine settings) and deterioration (due to loss of spring resilience) (Phadke,1989: 23).

The effect of noise factors can change with the levels (settings) of the easy-to-control factors called design factors (Kackar, 1985: 23; Logothetis & Wynn, 1989: 4). Design factors are factors under the control of the manufacturer whereas noise factors are not.

The design factors are divided into two main groups (Logothetis, 1989: 244; Taguchi *et al*, 1989: 3):

(i) those that affect variability of the quality characteristic (are known as control factors) and

(ii) those that affect only the mean level of quality characteristic. These are known as target control factors or signal factors.

The main aim of Parameter Design / Taguchi's traditional technique is to reduce variability by changing control factors while maintaining the required average performance through appropriate adjustment of signal factors. In so doing quality will be designed into a product or process. This is in contrast with Tolerance Design where narrow tolerances are specified for deviations of control factors in relation with ideal levels.

The following are simple applications that exist in the literature. With these examples, it is intended to illustrate the idea of categorising factors in the formulation of Parameter Design.

2.3 PRACTICAL EXAMPLES

2.3.1 CAKE MIXTURE (BOX, BISGAARD & FUNG, 1987)

Background

This experiment was done to evaluate a ready-made cake mixture to be supplied to retailers for selling to customers. In an effort to evaluate this mixture and determine which factors have the most influence on taste, a 2^5 factorial design (denoted by $2^2 \times 2^3$) was devised using a 2^3 control design and a 2^2 noise design.

Objective

To detect which of the control factors are significant in maximising the mean Hedonic Index (the units for measure of taste) with minimum variation, induced by noise factors.

Noise factors (At 2 levels : High (+) and Low (-))

FACTOR	LEVEL	
Oven Temperature	-	+
Oven Time	-	+

Control factors (At 2 levels : High (+) and Low (-))

FACTOR	LEVEL	
Amount of Flour	-	+
Amount of Butter	-	+
Number of Eggs	-	+

Response variable

Hedonic Index

In the above example the oven temperature and the baking time are factors that can be controlled by the consumer. These factors will consequently vary from consumer to consumer and are out of scope of the manufacturer. Even if the manufacturer can recommend operation limits to consumers there is no way that he/she can enforce this. Therefore, the two factors are clearly noise factors in this situation. On the other hand, in designing the mixture, the manufacturer is able to decide what measurements of flour, butter and eggs may be used. The three factors are therefore control factors.

The experimental objective is then to determine the amount of flour, eggs and butter to be used in the mixture such that, irrespective of what oven temperature and baking time, the taste should remain best with minimum variation caused by noise and other factors.

2.3.2 THE FORK LIFT (BRELIN & SEKSARIA, 1985)

Background

Unlike a car, the fork-lift has no isolating suspension system to absorb the vibrations as perceived by an operator through his various points of contact with the vehicle. The only operator isolation to the engine induced vibrations comes from the cushion in the seat and the rubber in the engine mounts and the tyres. It is therefore crucial that operator comfort,being an ergonomic factor, be an important consideration in the design of a fork-lift.

Objectives

To identify various vehicle design factors and to evaluate their influence on the vibrational response of the vehicle.

Noise factors

FACTOR	LEVEL 1	LEVEL 2
Tyre Type	Pneumatic	Cushion
Vehicle Weight	Maximum Capacity	3/4 of Maximum
Operator Weight	175lb	220lb
Seat Position	Midpoint	Full Rear
Type of Mast	Triple Stage	Standard
Engine Type	Petrol	Diesel
Payload	Forks Empty	3500lb

Control factors

FACTOR	LEVEL 1	LEVEL 2	LEVEL3
Engine Mount Loc	Present	Forward	Backward
Engi Mount Stiff	Present	4.5 times	
Steer Wheel Loc	Present	Lower	
DOG Stiff	Present	0.3 times	
Engine Stiff	Present	0.2 times	
Frame Stiff	Present	0.3 times	
Pivot Loc	Present	Forward	
CNT Weight Stiff	Present	Max	
Firewall Stiff	Present	Max	

(**Brelin & Seksaria**, *ibid.*)

Response variable

Vibration Severity Index

In designing a fork-lift, the manufacturer has no way of knowing whether a consumer will, for example, use (out of preference) pneumatic (soft) tyres or solid (hard) tyres. Hence tyre type is an example of a noise factor. It can, however, be decided during the design stage, at what height should the steering wheel be placed. Since the choice of the location of the steering wheel is determined by the designer, such a factor may be regarded as a control factor. Similar argument for all other factors in both categories may be presented. But such arguments may tend to be technical and this will not serve the purpose of this study.

It is therefore easy for the designer to experiment with the steering wheel location rather than with the tyre type in order that the operator's comfort may be improved. Because once the vehicle is sold, tyre type is out of his control. Thus the experimenter will endeavour to find levels of control factors such as steering wheel location, that will give the best possible comfort to the driver with minimum variation caused by noise (*e.g.*, tyre type).

2.4 QUALITY

Quality may be defined as the loss a product causes to society after being shipped, other than any losses caused by its intrinsic function (Logothetis & Wynn, 1989: 5). This implies that quality is determined by the desirability of a product as determined by societal loss it generates from the time the product is shipped to the customer. Societal loss can, amongst others, be poor and varied performance of a product, harmful side effects caused by the product and failure to meet the customer's requirement of fitness for use on prompt delivery. However, the loss that a product may inflict on society through its intrinsic function is not regarded as quality loss. As an example to support this statement (Logothetis & Wynn, 1989: 5) it is argued that it is nonsensical to manufacture non-intoxicating liquor (as intoxication is the intrinsic function) because of fights and accidents that may result while under the influence of alcohol. On the other hand better quality liquor is the one which gives minimum hangover, least side effect caused by preservatives or which does not spoil easily or vary because of absence of preservatives.

Also, one of the main reasons for manufacturing vehicles is transportation.
It is for this reason that transportation may be regarded the intrinsic function of a car. Societal loss due to, for example, accidents on the road caused by negligence, should not be regarded as poor quality of the product (car) and blamed on the manufacturer. But breakdowns in transit indicate poor quality.

Following Phadke (1989: 11), the ideal quality a customer may expect is that every product delivers the target performance each time the product is used, under all intended operating conditions and throughout its intended life, with no harmful effects. This ideal quality is used as a reference point for measuring quality level.

According to Taguchi (Taguchi *et al*, 1989: 2), we measure the quality of a product in terms of the total loss to the society due to functional variation and harmful side-effects. We can therefore conclude that the greater the loss, the lower the quality. This loss is modelled according to the so-called loss function.

2.5 THE LOSS FUNCTION

The simplest and the most useful loss function is the quadratic loss function (Kackar, 1985; Logothetis and Wynn, 1989: 6-7; Phadke, 1989: 18; Ryan, 1989: 349; Taguchi *et al*, 1989: 18-20). This function is based on the second order Taylor's expansion of the functional characteristic Y about a target value τ. The value Y can deviate from τ both during the product's life span and across different units of the product. These deviations cause losses to the product's user. This is why a quadratic function is used.

The expression for the quadratic loss function is found as follows:

Let L(Y) represent the monetary loss suffered by an arbitrary user of a product due to deviation of Y from τ. Then the quadratic loss function is given by

$$L(Y) = k\ (Y-\tau)^2 \qquad (2.1)$$

where k is some constant.

The unknown constant k, can be determined if L(Y) is known for any value of Y. However, k can be expressed in terms of a tolerance limit: Suppose $(\tau-\Delta, \tau+\Delta)$ is the customer's tolerance interval. If a product performs unsatisfactorily when Y slips out of this interval, and the cost to the customer for repairing or discarding the product is A rands, then from the expression of the quadratic loss function above

$$k = A/\Delta^2 \quad (2.2)$$

The manufacturer's tolerance interval $(\tau-\delta,\tau+\delta)$ can also be obtained from the loss function. Suppose before a product is shipped, the cost to the manufacturer of repairing an item that exceeds the customer's tolerance limit is B rands. Then

$$B = (A/\Delta^2)(Y-\tau)^2 \quad (2.3)$$

$$Y = \tau \pm (B/A)^{1/2}\Delta \quad (2.4)$$

and

$$\delta = (B/A)^{1/2}\Delta \quad (2.5)$$

Because B is usually much smaller than A, the manufacturer's tolerance interval will be narrower than the customer's tolerance interval (see **Figure 2.1**).

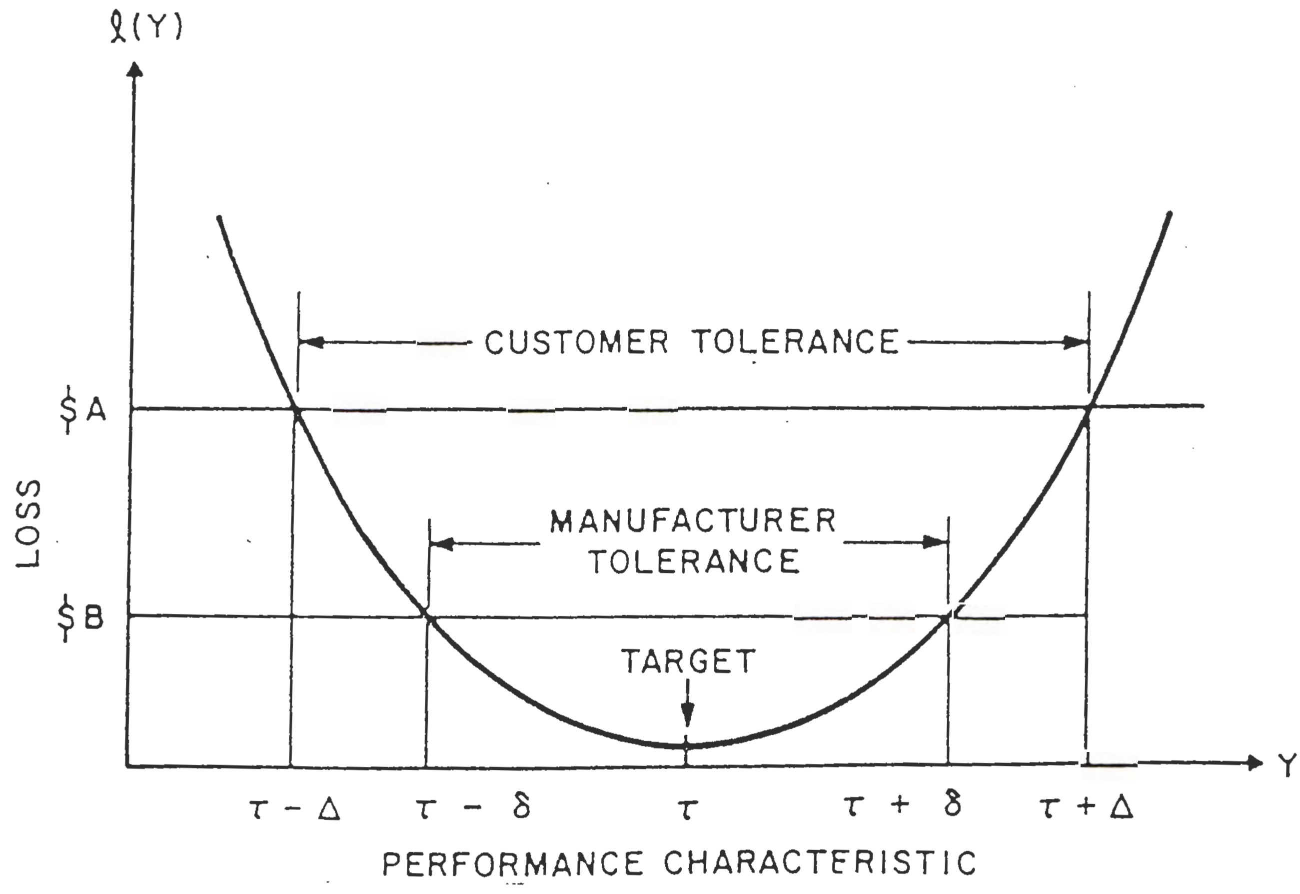

Fig 2.1 **Loss Function.**

For the quadratic loss function given by equation (2.1), the mean square error is given by:

$$E(L) = E(Y-\tau)^2 \tag{2.6}$$

Equation (2.6) can be written as:

$$E(L) = \sigma_y^2 + [E(Y)-\tau]^2 \tag{2.7}$$

It is clear from equation (2.7) that if we had $E(Y) = \tau$, then $E(L)$ could be thought as the average of the square deviation $(Y-\tau)^2$. The term
$[E(Y)-\tau]^2$ is generally known as the square bias and σ_y^2 is called the variability. Hence the reduction in the expected loss resulting from a reduction of variance is one reason why manufacturers should continually strive to reduce variance for all of their processes.

It should be noted that there are situations where the loss function is not symmetric about the target value as in the case considered above. The asymmetric loss function will arise in a situation where, for example, the performance characteristic Y has a non-negative distribution and the target value of Y is zero (Kackar, 1985).

An asymmetric quadratic loss function has the form (Kackar, 1985; Phadke, 1989: 21)

$$L(Y) = \begin{cases} k_1(Y-\tau)^2 & \text{if } Y \leq \tau \\ k_2(Y-\tau)^2 & \text{if } Y \geq \tau \end{cases} \tag{2.8}$$

The unknown constants k_1 and k_2 can be determined if L(Y) is known for a value of Y below τ and for a value of Y above τ.

$$L(Y) = \frac{A_0}{(\Delta_0)^2}(Y-\tau)^2 \qquad (2.9)$$

2.6 DESIGNED EXPERIMENT

A Taguchi Robust Design consists of two parts : a design matrix and the noise matrix. The columns of a design matrix represent run levels of the factors, and each row of the matrix a product or process design. The columns of a noise matrix represent noise factors, and rows of the matrix represent different combinations of levels of noise factors.

A complete Robust Design consists of a combination of design and noise matrices as previously indicated in Fig. 1.1. If the design matrix has m rows and the noise matrix has n rows, the total number of runs for the experiment is m x n. For each m rows of the design matrix, the n rows of the noise matrix provide n repeat observations on the performance characteristic. The levels of the noise factors are chosen so that these repeat observations are representative of the effects of all possible levels of noise factors. The repeat observations on the performance characteristic from each test run in the design matrix are then used to compute performance statistics.

The m values of the performance statistic associated with the m test runs in the design matrix are then used to predict levels of control factors that will minimise the expected loss.

The performance statistic estimates the effect of noise on the response variable. According to Kackar (1985) an efficient performance statistic takes advantage of prior engineering knowledge about the product, the loss function and the distribution of the response variable.

The expected loss as a performance statistic has a disadvantage. The statistic is sometimes more complicated than necessary because it does not take advantage of engineering knowledge (Kackar, 1985). This has lead to a development of a more desirable performance statistic called the signal-to-noise ratio (S/N).

According to Kackar (*ibid.*), Taguchi has defined more than sixty S/N for various engineering applications. However,it has been argued that in most applications these ratios can be categorised into three groups for static problems (Kackar, 1985; Phadke, 1989 :108-112; Schmidt & Launsby, 1988: 6-12).

The cases are as follows:

(i) When a specified target is the criterion, then

$$(S/N) = 10 \log (Y^2/s^2).$$

(ii) In the case of the target being zero and

(iii) when the target is infinite the above formula reduces to

$$(S/N) = -10 \log MSD$$

where

$$MSD = (1/n)\ \Sigma y_i^2 \text{ , for a zero target}$$

or

$$MSD = (1/n)\ \Sigma 1/y_i^2 \text{ , for an infinite target.}$$

2.7 ORTHOGONAL ARRAYS

A random search of the parameter space (the set of all possible levels of the design factors) for a set which will optimise the performance statistic is not cost-effective. An efficient way to study several factors simultaneously is to statistically plan an experiment using the matrix experiment method constructed from orthogonal arrays (Phadke, 1989: 149-156; Kackar, 1985).

A design is said to be orthogonal if the sum of every possible dot product for all possible variable pairs is zero (Schmidt & Launsby, 1989: 2-3). Such a design has a pairwise balancing property; that every test setting of a design parameter occurs with every test setting of all other design parameters the same number of times.

An orthogonal array for a particular matrix experiment can be constructed from the knowledge of the number of control factors, their levels, and the desire to study specific interactions. A detailed account on how orthogonal arrays are used by beginners, intermediate and advanced practitioners is given by Phadke (1989: 171-174). The process of fitting an orthogonal array to a specific project is done by employing graphic tools called the linear graphs.

In the Taguchi parameter design methodology, one experiment or design is selected for the control factors and another design is selected for the noise factors. Taguchi, according to Kackar (1985) recommended that both designs be orthogonal arrays. The matrix containing control factors is called the inner array and that containing the noise factors is called the outer array (Montgomery, 1991: 414-433).

The use of orthogonal arrays in matrix experiment offers some advantages (Phadke, 1989: 94-95). Firstly, the conclusion arrived at from such experiments are valid over the entire experimental region spanned by the control factors and their settings. Secondly, there is a large saving in the experimental effort. Thirdly, the data analysis is relatively easy. Finally, it can detect departure from the additive model (a model where the total effect of several factors is equal to the sum of the individual factors).

2.8 METHODS OF ANALYSIS

A typical Taguchi strategy for problems where the target is fixed (called static problems) involves optimising one of the S/N ratios (this study is restricted to only such problems).

The use of S/N ratios has, nevertheless, received considerable criticism. Pignatiello and Ramberg (1985) point out that the use of S/N ratio implies somewhat that a unit increase in $\log(y^2)$ is of equal importance as the unit decrease in $\log(s^2)$. They conclude that it would be better to simply study the variability s, by itself. Leon *et al* (1987) on the other hand, proposed an alternative to the use of S/N because for certain models and appropriate loss function, the use of S/N will not minimise the expected loss. They call this alternative the performance measure independent of adjustment.

It is for the reasons mentioned above that the Box-Taguchi method, is employed in the analyses of designs later. The method differs with the classical Taguchi method in the optimisation procedure. Instead of minimising the S/N ratio, the logarithm of the standard deviation (log s) is minimised and then the mean is driven to its target value. The method is also recommended by Schmidt and Launsby (1989: 6-22).

The Box-Taguchi method involves the following steps :

(i) Set up the control factor array (CFA) and the noise factor array (NFA).

(ii) Each level combination of control factors in the CFA is run for the NFA to obtain responses Y.

(iii) The performance statistics log (s) and $\bar{Y}$ are calculated.

(iv) The normal probability plot of log (s) is used to select factors that have a significant effect on noise while the normal probability plot of $\bar{Y}$ will determine factors that have impact on mean performance.

(v) A diagnostic check of both models is made by means of normal probability plot of residuals. The normal probability plot or the half normal probability plot is used, whichever is the more illustrative.

(vi) The interaction plots of significant effects in the variance model are used to select levels of control factors that may minimise the effect of noise.

(vii) The remaining control factors are then manipulated to drive the mean response to its target value.

It should be noted that steps (iv) - (vi) are executed automatically with Design-Ease (1987) package.

2.9 SIMULATED EXAMPLES

In this section (and in **Chapter Three**) simulated examples are analysed. Simulated examples are used because the models that are investigated are already known. This is helpful since the Box-Taguchi method of analysis will be evaluated in two ways:

(i) Can the method screen out control factors that have effect on noise?

(ii) Are the results obtained with Box-Taguchi method the same as those achieved through Combined Arrays method ?

In all the examples that follow, it is important to note that emphasis will be put on analysing C-N interactions. Examples 2.9.1 and 2.9.4 are typical examples where simulated models have C-N interactions of different magnitudes. The existence of such interactions affords (in general) an opportunity to manipulate control factors in order that robustness may be improved. Whereas in Example 2.9.2 also, the model has C-N interactions, because these cancel out, different results should be expected. Another situation to investigate is where there are no C-N interactions.

2.9.1 THE SIMULATED 2^2 X 2^3: TYPICAL DESIGN

The simulated model is

$$Y = 30+6A+4B+2C-3D-2E+4AC-5BD+5.5DE+N(0.1).$$

The noise factors are A and B and control factors are C, D, and E. This is a typical model because it contains the important interactions, the noise-to control interactions (AC and BD).

Objectives

(i) To identify the best combination of control factors which may minimise variability about target response.

(ii) To screen out factors which may be used to drive the mean response to its target.

(iii) To compare factors in (ii) with factors of the simulated model.

The Design

There are 3 control factors (C, D and E) at 2 levels crossed with 2 noise factors (A and B) also at 2 levels. The experimental scheme may be represented as follows:

RUN	DESIGN MATRIX					RESPONSE	LOG(s)	$\overline{Y}$
1	-	-	-	-	-	27.034		
2	-	-	-	+	-	31.750	1.05828	39.024
3	-	-	-	-	+	46.635		
4	-	-	-	+	+	50.723		
5	+	-	-	-	-	21.660		
6	+	-	-	+	-	43.540	1.21175	41.802
7	+	-	-	-	+	40.578		
8	+	-	-	+	+	61.430		
9	-	+	-	-	-	21.494		
10	-	+	-	+	-	22.758	0.9672	21.415
11	-	+	-	-	+	19.739		
12	-	+	-	+	+	21.669		
13	+	+	-	-	-	16.263		
14	+	+	-	+	-	36.463	1.0581	25.296
15	+	+	-	-	+	14.648		
16	+	+	-	+	+	33.811		
17	-	-	+	-	-	13.672		
18	-	-	+	+	-	16.524	0.9965	23.621
19	-	-	+	-	+	31.537		
20	-	-	+	+	+	32.747		
21	+	-	+	-	-	8.718		
22	+	-	+	+	-	27.691	1.18684	27.281
23	+	-	+	-	+	26.365		
24	+	-	+	+	+	46.351		

25	-	+	+	-	-	28.478		
26	-	+	+	+	-	29.397	0.24601	28.565
27	-	+	+	-	+	26.150		
28	-	+	+	+	+	30.328		
29	+	+	+	-	-	23.349		
30	+	+	+	+	-	42.078	1.0590	31.872
31	+	+	+	-	+	20.574		
32	+	+	+	+	+	41.398		

Another way of displaying this data is :

		A	-	+	-	+		
		B	-	-	+	+		
C	D	E					LOG(s)	$\bar{Y}$
-	-	-	27.034	31.750	46.350	50.723	1.058	39.024
+	-	-	21.660	43.540	40.578	61.430	1.212	41.802
-	+	-	21.494	22.758	19.739	21.669	0.967	21.415
+	+	-	16.263	36.463	14.648	33.811	1.058	25.296
-	-	+	13.672	16.524	31.537	32.747	0.997	23.621
+	-	+	8.718	27.691	26.365	46.351	1.187	27.281
-	+	+	28.478	29.397	26.150	30.328	0.246	28.565
+	+	+	23.349	42.078	20.574	41.398	1.059	31.872

Subsequently, this format of data presentation will be adopted for the Box-Taguchi method of analysis.

Results

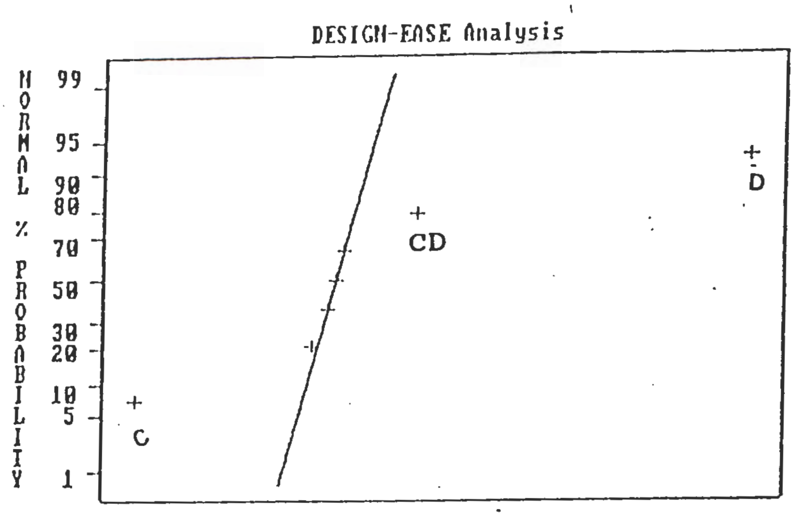

Fig 2.2(a) Normal Probability Plot of Effects for the Variance Model: C, D and CD appear significant.

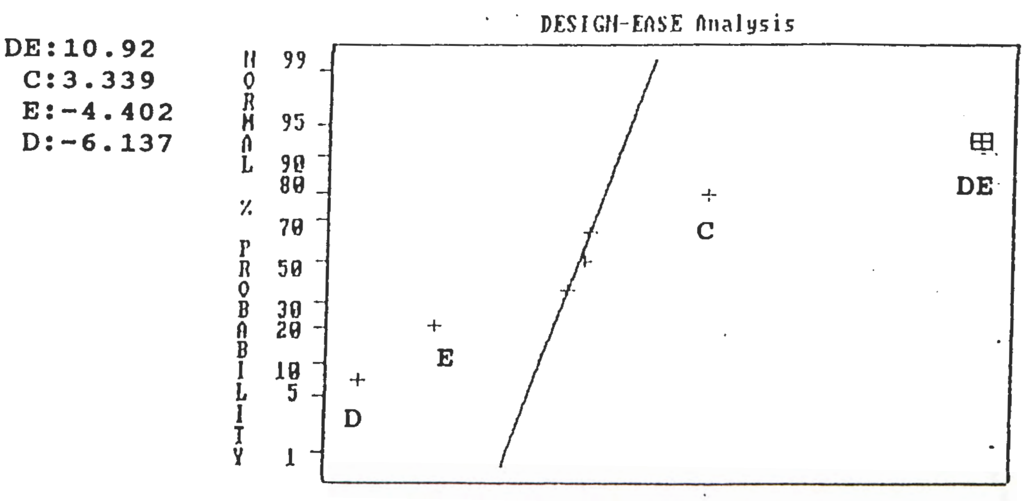

Fig 2.2(b) Normal Probability Plot of Effects for the Mean Model: C, D, E and DE are significant

Fig 2.2(a) is the normal probability plot of effects for the variance model. It is evident from this figure that the main effects C and D are significant because they lie significantly off the line. Since both C and D are control factors, the designer is afforded an opportunity to manipulate in order to minimise variance.

Similarly, C, D, E and DE may have significant effect on the mean response as can be witnessed in **Fig 2.2(b)**. Also, these factors compare well with those of the simulated model, because the maximum proportion error is 0.15 (see **Table 2.1**)

TABLE 2.1

Comparison of the significant effects for the Mean and Simulated Models

EFFECT	SIMULATED	MEAN	ERROR
C	2	1.699	**0.1505**
D	-3	-3.068	0.023
E	-2	-2.021	0.0105
DE	5.5	5.46	0.0073

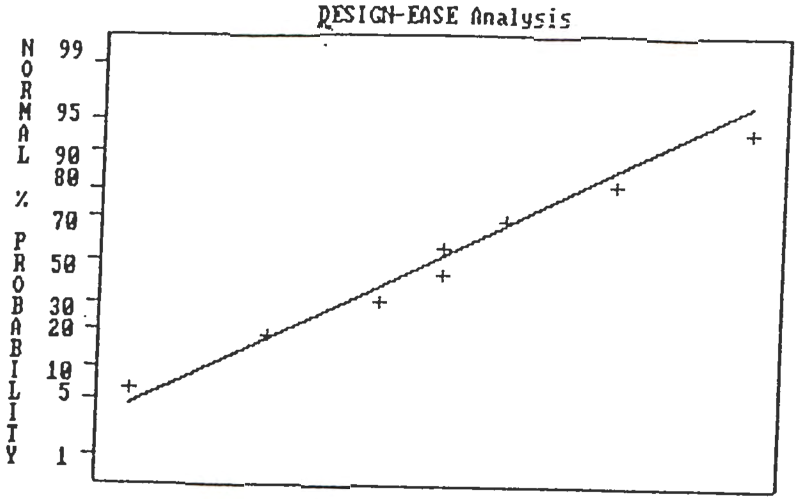

Fig 2.3(a) **The Normal Probability Plot of Residuals for the Variance Model.**

From the normal probability plot of residuals (**Fig 2.3(a)**) there is no evidence that a model with C and D main effects significant is not to be accepted. This is because all points generally lie on a straight line.

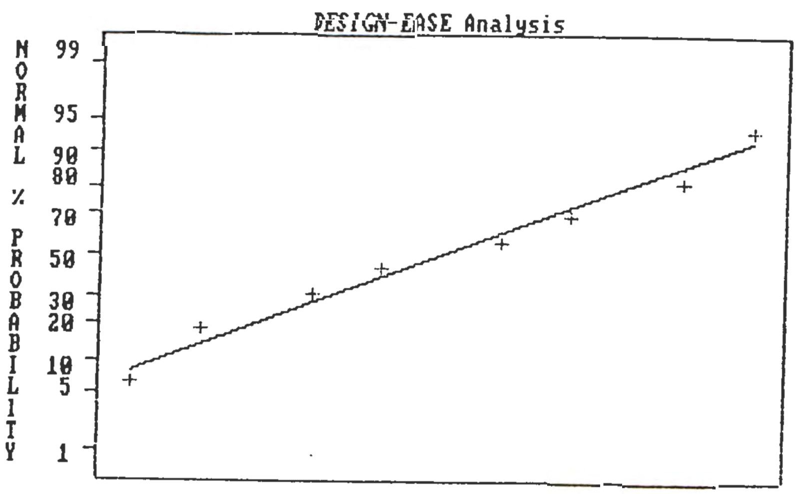

Fig 2.3(b) **The Normal Probability Plot of Residuals for the Mean Model.**

It can be similarly argued that from **Fig 2.3(b)** that the mean model with C, D, E and DE is adequate.

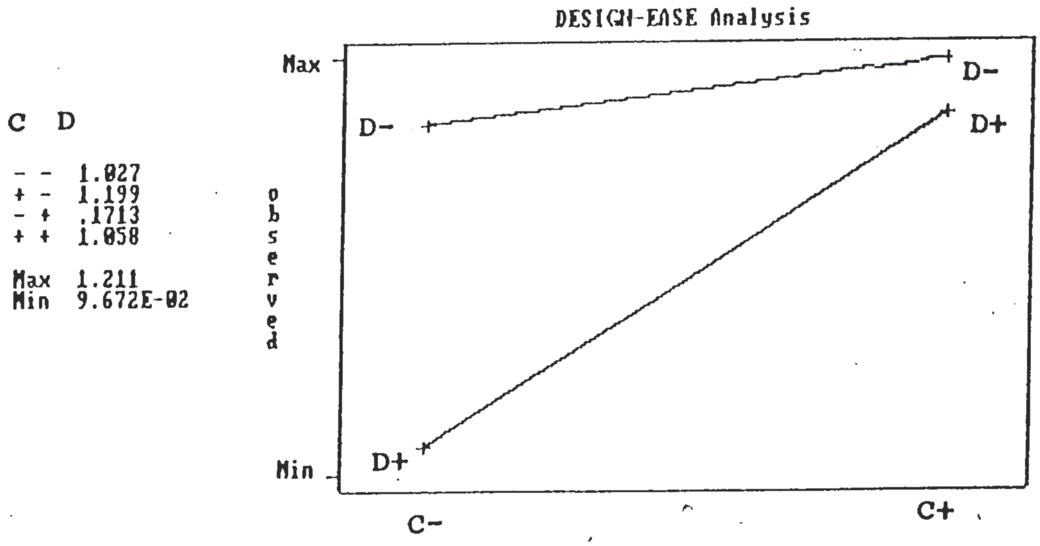

Fig 2.4 **CD Interaction Plot:(C- D+) may minimise variance.**

From **Fig 2.4** (the interaction plot of effect) it is seen that D at + level and C at - level may minimise variance caused by noise.

Therefore the possible solution for minimisation of variability caused by noise is (C- ,D+). Since C and D have impact on dispersion, the only factor that can be manipulated to drive mean to its target is E.

2.9.2 THE 2^2 X 2^3 : EFFECTS CANCEL OUT

The simulated model is

$$Y = 40+4D+5E+10BC-10AC+N(0.1)$$

Where A and B are noise factors and C, D, and E are control factors. Thus AC and BC are the noise-to-control interactions that cancel out.

Objectives

(i) To show that there will be no significant effects showing up in the Box- Taguchi model, when the noise-to-control interactions cancel out.

(ii) To compare the significant effects of the mean model with those of the simulated model.

The Design

The designed experiment is as follows :

		A	-	+	-	+		
		B	-	-	+	+		
							LOG(s)	$\bar{Y}$
C	D	E						
-	-	-	31.42	51.39	11.77	31.08	1.210	33.415
+	-	-	33.12	12.10	50.51	50.05	1.197	31.441
-	+	-	40.45	58.52	18.54	38.26	1.214	38.943
+	+	-	39.24	23.23	59.67	37.68	1.208	38.951
-	-	+	41.36	61.18	20.75	38.91	1.219	40.549

+	-	+	40.54	20.38	61.83	41.58	1.228	41.082
-	+	+	47.33	69.27	28.99	49.88	1.217	48.691
+	+	+	49.94	28.15	68.06	48.60	1.213	48.687

Results

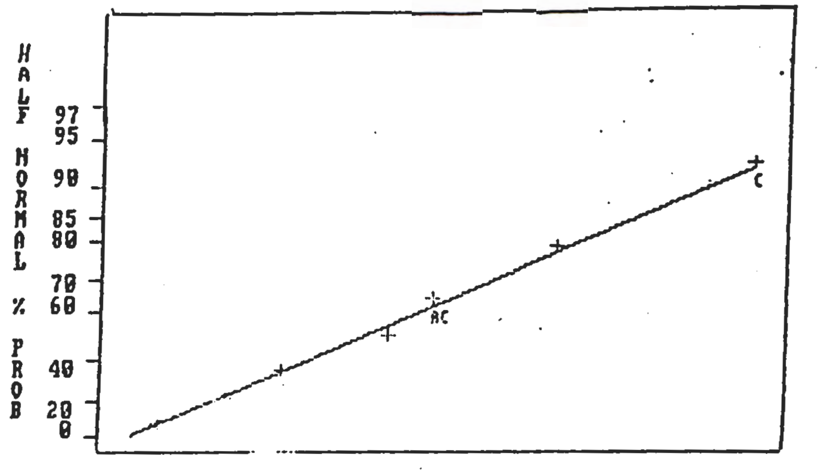

Fig 2.5(a) The Half Normal Probability Plot of Effects for the Variance Model: No effects are significant.

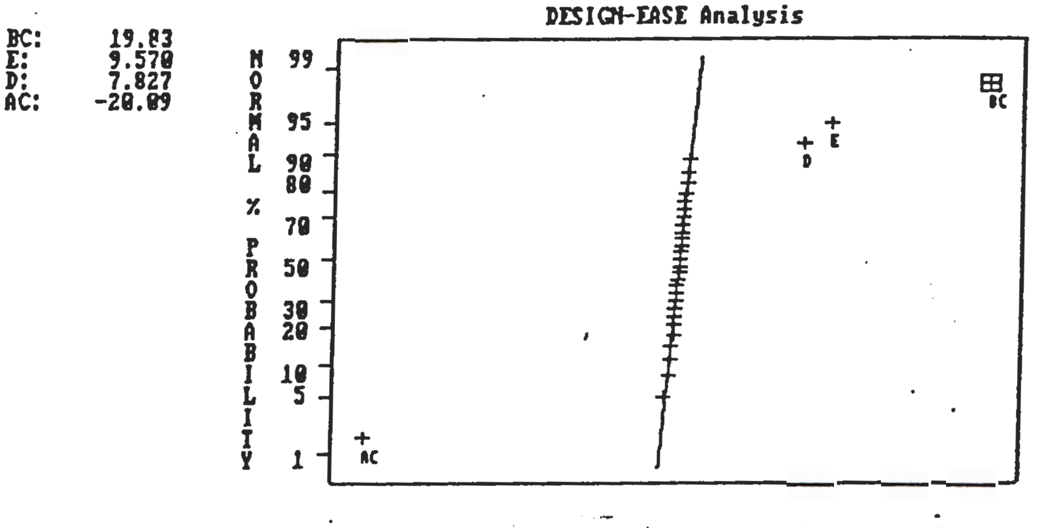

Fig 2.5(b) Normal Probability Plot of Effects for the Mean Model: D, E, BC and AC are significant.

The normal probability plot of in **Fig 2.5(a)** has points generally on a straight line. This may be interpreted as an indication that there are no significant effects in the Box-Taguchi variance model. That is there are no control factors that can be adjusted to improve robustness. However, D and E may have significant effect on the mean response because these factors lie significantly off the line in the normal probability plot of effect (see **Fig 2.5(b)**). Also, there is a 0.1 maximum proportion error when comparing significant effects of the mean and the simulated models (**Table 2.2**)

TABLE 2.2

Comparison of the significant effects for the Mean and Simulated Models

EFFECT	SIMULATED	MEAN	ERROR
D	4	3.633	**0.0919**
E	5	4.567	0.0866

2.9.3 THE MODEL WITH NO CONTROL-TO-NOISE INTERACTIONS

The simulated model is

$$Y = 40+4A+5B+2C+6D-3D+8AB-7DE+N(0.1).$$

Where A and B are noise factors and C, D, and E are control factors. The only interaction occurring are noise-to-noise (AB) and control-to-control interactions.

Objectives

(i) To show that there will be no significant effects in the Box- Taguchi variance model.

(ii) To compare significant effects of the mean model with those of the simulated model.

The Design

The experimental design will take the form:

			A	-	+	-	+		
			B	-	-	+	+	LOG(s)	$\bar{Y}$
C	D	E							
-	-	-		17.24	7.73	13.68	34.29	1.057	18.237
+	-	-		19.65	11.19	14.75	40.51	1.118	21.526
-	+	-		42.60	37.03	37.54	61.57	1.062	44.686
+	+	-		45.46	36.51	41.75	64.12	1.079	46.959
-	-	+		24.79	17.18	19.39	44.83	1.100	26.593
+	-	+		28.79	20.55	23.87	45.46	1.044	29.713
-	+	+		21.86	14.37	17.02	40.88	1.078	23.533
+	+	+		27.31	18.31	20.91	44.20	1.076	27.637

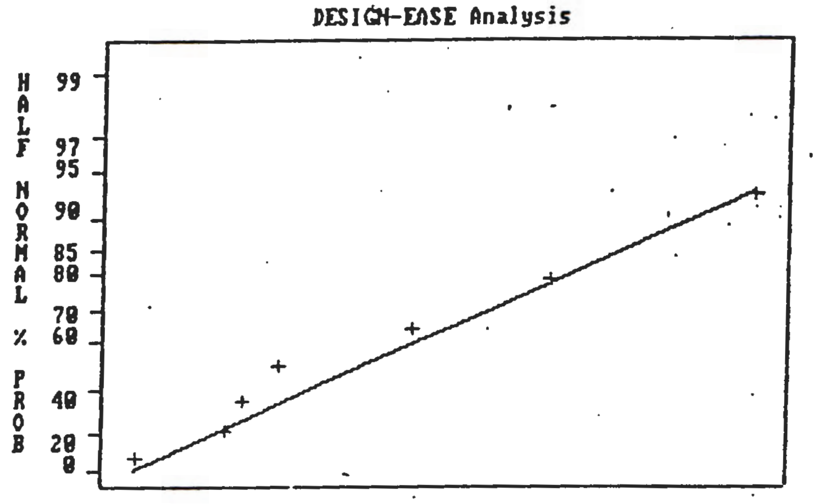

Fig 2.6(a) The Half Normal Probability Plot of Effects for the Variance Model: No effects are significant.

D:11.68
C:3.196
E:-5.983
DE:-14.25

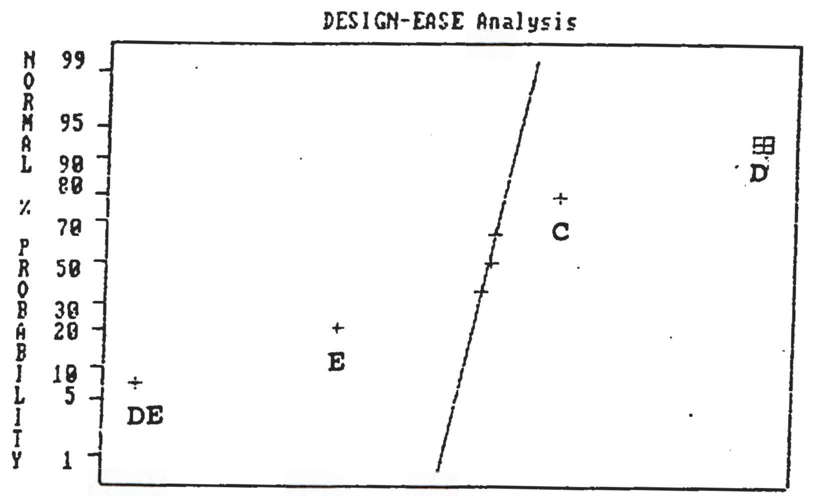

Fig 2.6(b) The Normal Probability Plot of Effects for the Mean Model: D, C, E and DE are significant.

Points on the normal probability plot of effects in **Fig 2.6(a)** lie largely on a straight line. This indicates that there is no significant effects in the Box-Taguchi variance model. Hence no control factor could be manipulated to improve robustness.

The mean model has C, D, E and DE as significant effects because the corresponding points in the normal probability plot of effects do not lie on the line through zero (**Fig 2.6(b)**). The maximum proportion error is 0.201 if these effects are compared with those in the simulated model (**Table 2.3**).

TABLE 2.3

Comparison of significant effects for the Mean and Simulated Models

EFFECT	SIMULATED	MEAN	ERROR
C	2	1.598	**0.201**
D	6	5.890	0.0183
E	-3	-2.9915	0.0028
ED	-7	-7.125	0.0036

2.9.4 THE 2^{3-1} X 2^{4-1} PRODUCT ARRAY DESIGN

The simulated model is

$$Y = 30+6A+4B+2C-3D-2E+4F+5G+7AE-8CD-6FG+N(0.1).$$

The design for this model is a $2^3 \times 2^4$ design composed of 3 noise factors (A, B and C) and 4 control factors (D, E, F and G).

Objectives

(i) To find levels of control factors which may minimise variability caused by noise.

(ii) To find factors that can be used to drive mean response to its target.

(iii) To compare factors in (ii) with those of the simulated model.

Since AE and CD are the only control-to-noise interaction effects in the simulated model it is expected that only D and E will be significant in the Box-Taguchi variance model.

The Design

The $2^3 \times 2^4$ design is fractionated into a $2^{3-1} \times 2^{4-1}$ design in such a way that

$$C = AB \text{ and } G = DEF$$

are the defining contrasts. This is the way that Taguchi would have fractionated.
The experimental design will be:

			A	-	+	-	+		
			B	-	-	+	+		
D	E	F						LOG(s)	$\bar{Y}$
-	-	-		25.4	3.41	15.0	32.0	1.097	18.95
+	-	-		26.9	35.5	46.4	33.8	0.907	35.65
-	+	-		31.5	37.8	20.1	66.0	1.291	38.85
+	+	-		-13.0	22.9	5.86	22.5	1.231	6.565
-	-	+		43.8	22.7	32.3	50.6	1.092	37.35
+	-	+		24.2	34.1	45.2	31.3	0.941	33.70
-	+	+		29.8	33.8	16.6	64.3	1.305	36.13
+	+	+		4.62	42.1	25.5	38.8	1.231	27.76

Results

E:0.2575
DE:5.400E-02
D:-0.1210

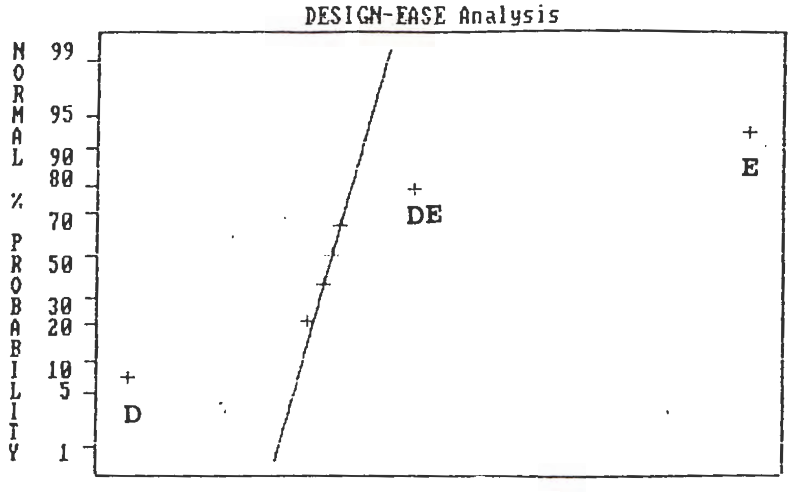

Fig 2.7(a) **The Normal Probability Plot of Effect for the Variance Model: D, E and DE appear significant.**

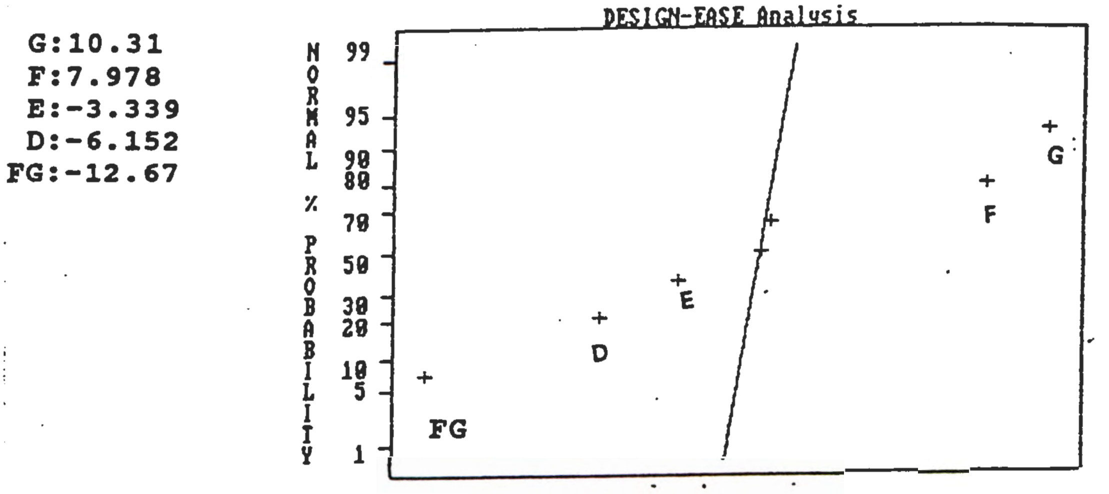

Fig 2.7(b) **The Normal Probability Plot of Effects for the Mean Model: D, E, F, G and FG appear significant.**

From **Fig 2.7(a)** it is evident that factors E and D are the significant factors, that is, they are factors that can be manipulated in order that variability caused by noise could be minimised since points corresponding to these factors lie off the line spanning zero. Similarly from **Fig 2.7(b)**, D, E, F, G and FG are the significant factors in the mean model.

When effects D, E, F, G and FG of the mean model are compared with the same of the simulated model, it is seen from **Table 2.4** that the maximum proportion error is 0.2.

TABLE 2.4

Comparison of the significant effects

for the Mean and Simulated Models

EFFECT	SIMULATED	MEAN	% ERROR
E	-2	-1.6695	**0.16525**
D	-3	-3.0750	0.025
F	4	3.9890	0.00275
G	5	5.1550	0.031
FG	-6	-6.3350	0.0558

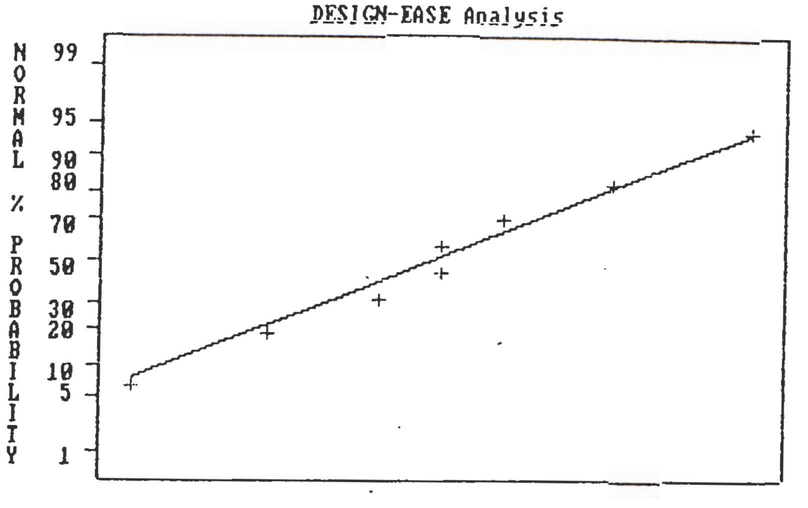

Fig 2.8(a) **The Normal Probability Plot of Residuals for the Variance Model: The model looks adequate.**

The probability plot of residuals in **Fig 2.8(a)** shows that the Box-Taguchi model with only E and D significant is looks adequate as points lie largely on the line. Similarly from **Fig 2.8(b)** the mean model with D, E, F, G and FG significant appears adequate.

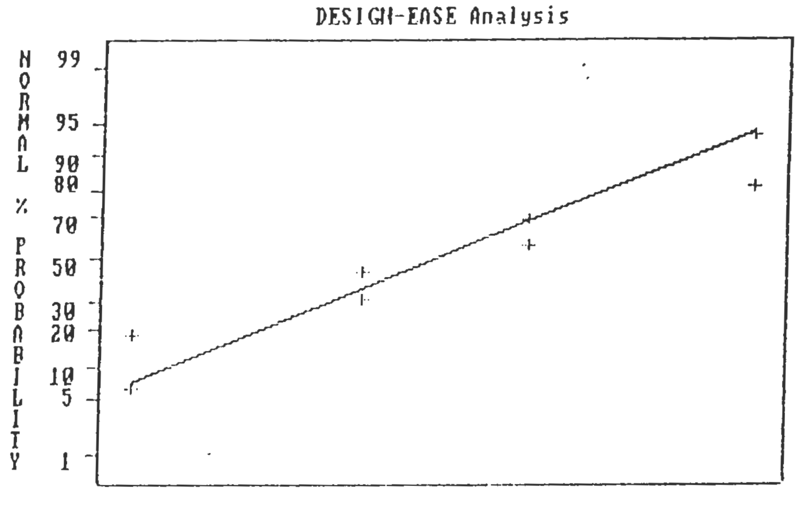

Fig 2.8(b) **The Normal Probability Plot of Residuals for the Mean Model: The model looks adequate.**

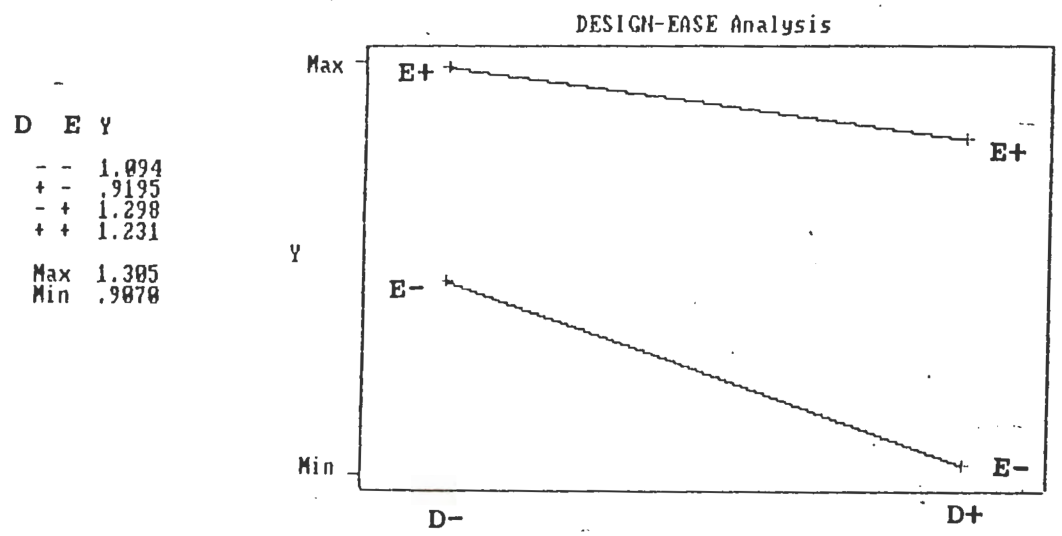

Fig 2.9 **DE Interaction Plot: (D+ E-) may minimise variance.**

The interaction plot of effects in **Fig 2.9** indicates that E at - level and D at + level is level combination of control factors that may minimise variability due to noise.

The possible solution from the Box-Taguchi variance model is therefore
(D+, E-). Since D and E were used to adjust variability, only F and G could be employed to adjust the mean.

2.10 CONCLUSIONS

It was shown that distinguishing between the noise and control factors is fundamental in the formulation of Taguchi's traditional method of Robust Design. The importance of categorisation of factors was made clear when the Cake-Mixture and Fork-Lift examples were considered.

Also demonstrated is the fact that quality as defined by Taguchi leads to the use of signal-to-noise ratios as the performance statistics. But in line with the argument presented by Box (1986) it was decided that logarithm of variance (log s) and mean $\bar{Y}$ should be used instead as the performance statistics. This modification of Taguchi's traditional approach was termed the Box-Taguchi method.

The Box-Taguchi method was illustrated on simulated models. The method was successful in screening out potential dispersion reduction factors. In particular, from the $2^2 \times 2^3$

design it was established that factor C at - level and factor D at + level were the best candidates for minimisation of variation induced by noise. Also, from the $2^{3-1} \times 2^{4-1}$ it was shown that D at + level and E at - level is a level combination that could minimise variability.

In both cases, control factors that have a dampening effect on noise were identified but not the particular noise factors being dampened. For example, although it was known from simulation that AE and CD interactions were responsible for E and D showing up as significant effects in Example 2.9.4, it was not evident that A and C were the corresponding noise factors as it will be seen later in **Chapter Three.**

It was also demonstrated that the Box-Taguchi method is also effective in identifying factors that have impact on the mean response. The significant effects so obtained were compared with those in the four simulated examples. The effects corresponded fairly well.

There were two cases where factors which had impact on variability also had effect on mean. In both cases, there was (were) factor(s) which could be adjusted without disturbing optimality on variability. These were taken as adjustment factors for the mean.

CHAPTER THREE

COMBINED ARRAYS / RESPONSE MODEL

3.1 INTRODUCTION

The Box-Taguchi method discussed in **Chapter Two** proved to be effective in making products and processes insensitive to noise. In some applications, however, this approach leads to unnecessarily expensive experiments (Shoemaker *et al*, 1991). As an alternative, Welch *et al*. (1990) suggested that a combined experiment be constructed to predict the response Y as a function of the noise and control factors and select an appropriate model for Y. The technique was called Response Model or Combined Array approach. This chapter is concerned with the discussion and application of this approach.

Section 3.2 gives the background of the method and an outline of the rest of the chapter. Section 3.2 contains two sub-sections:

- Section 3.2.1 is concerned with motivation for Combined Array method by, firstly giving an overview of Taguchi's loss model approach. Shortcomings of this method are then described. The benefits of using the alternative Combined Array method are subsequently presented.
- Section 3.2.2 discusses Combined Array method in

detail. It describes how noise and control arrays are combined into a single array called combined array and thereby lending itself to aggressive but sensible fractionation.

Section 3.3 focuses on analysing the four representative examples of **Chapter Two** using Combined Array method. The analyses of these examples demonstrate the ease with which the control factors that have a dampening effect on individual noise factors could be identified. It is also shown how the difference in slopes in the interaction plots is used to determine potential robustness. Further presented is the fact that the fractionation method used by Box et al (1978:410) has more scope than Taguchi's way of fractionation. The results obtained for each of the simulated examples are compared with the corresponding results in **Chapter Two**.

Concluding remarks are made in Section 3.4.

3.2 RESPONSE MODEL / COMBINED ARRAY

3.2.1 MOTIVATION

In order to appreciate the nature and advantages of Combined Arrays approach, it is important to outline the classical Taguchi's approach to Robust Design.

Taguchi's method employs two experimental designs : one for control factors called control array (CA) and the other for noise factors called noise array (NA). Observations on a quality characteristic are taken for every combination of control factors in the CA and the noise variables in the NA.

For a given level combinations of control factors in the control array, the replicated observations generated by the noise array are reduced to a loss statistic or a relevant S/N ratio. The loss statistic averages a measure of loss over the distribution of the noise factors. The objective is to find a design that minimises the expected loss as indicated by the loss statistic.

Shoemaker *et al* (1991) proved that the effects that are estimable in the product array, CA x NA correspond to:

(i) those that are estimable in CA,

(ii) those that are estimable in NA and

(iii) the generalised interaction of the effects in CA and NA.

There are two inherent deficiencies of the traditional Taguchi's method emanating from this result and they are:

(i) If CA has n runs and NA has m runs, then out of mn-1 degrees of freedom in CA x NA, only (n-1)(m-1) are used for estimating C-N and higher order interactions.

(ii) Because in practice only a small number of C-N interactions can be important, it seems wasteful to keep all estimable factors in the model. Further, there is no flexibility in reassigning the degrees of freedom to estimate C-C interaction of control factors that may be important.

It is because of the structure of the Taguchi's approach and the corresponding product array designs that the following additional disadvantages are noticed (Welch *et al*, 1990; Shoemaker *et al*, 1991) :

(i) A very large number of runs may be required because the noise array is repeated for every row in the control array.

(ii) The focus is on modelling the loss statistic, which is often a non-linear, many-to-one transformation of the response. Even when the response follows a linear model in both the control and noise factors, it is less likely that it can be modelled well by a low order linear model even if data transformations are employed.

(iii) Modelling loss may also hide some of the relationships between individual control and noise factors.

3.2.2 THE METHOD

As mentioned earlier, Welch *et al* (1990) proposed the Combined Array approach as a remedy to the above situation. The approach models the response instead of the loss and uses the response model to discover the control factor level combination that has a dampening effect on noise. The method requires that the control array and the noise array be combined into a single design matrix (hence the name Combined Array). In other words, a single factorial experiment is performed with level combinations of both control and noise factors.

Welch *et al* (1990) suggested that the combined array design should be used to obtain a model relating response to both control and noise factors. From this model estimates of the expected loss should be obtained. These estimates are then used to predict control factor levels that minimises the loss statistic.

It can be shown (Shoemaker *et al*, 1991) that the Combined Array approach can be profitably augmented with informal interpretation of the C-N interactions in the response model. From the examination of the probability plots of the C-N interactions, it is fairly easy to identify control factors that have a dampening effect on individual noise factors.

It should be noted that the size of a C-N interaction effect alone is not necessarily a measure of the potential for improving robustness (Shoemaker *et al*, 1991). It is only the difference in slopes in the interaction plot that determines robustness potential.

The Combined Array format has the advantage that it is more economical because it lends itself to aggressive (and sensible) fractionation. Also, the use of product array (Taguchi's designs) necessitates estimates of some effects regardless of their importance as apposed to combined arrays which allow for selection of estimable effects.

3.3.1 THE 2^5 DESIGN

The simulated model is

$$Y = 30+6A+4B+2C-3D-2E+4AC-5BD+5.5DE+N(0.1),$$

used in Chapter Two. Here also, A and B are noise factors whereas C, D, and E are control factors.

Objectives

(i) To determine whether the same results are obtained with combined Array method as with Box-Taguchi method used $2^2 \times 2^3$ design in **Chapter Two**.

(ii) To compare significant effects so obtained with those of the simulated model.

(iii) To show that the Combined Array method re-establishes the above model.

(iv) To obtain the same results as those of the Box-Taguchi method used on $2^2 \times 2^3$ design in Chapter Two.

The Design

The Combined Arrays experimental design takes form :

	NOISE FACTORS		CONTROL FACTORS			RESPONSE
Run	A	B	C	D	E	$\bar{Y}$
1	-	-	-	-	-	27.043
2	+	-	-	-	-	31.750
3	-	+	-	-	-	46.635
4	+	+	-	-	-	50.723
5	-	-	+	-	-	21.660
6	+	-	+	-	-	43.540
7	-	+	+	-	-	40.578
8	+	+	+	-	-	61.430
9	-	-	-	+	-	21.494
10	+	-	-	+	-	22.758
11	-	+	-	+	-	19.739
12	+	+	-	+	-	21.669
13	-	-	+	+	-	16.263
14	+	-	+	+	-	36.463
15	-	+	+	+	-	14.648

16	+	+	+	+	-	33.811
17	-	-	-	-	+	13.677
18	+	-	-	-	+	16.524
19	+	+	-	-	+	31.537
20	+	+	-	-	+	32.747
21	-	-	+	-	+	8.718
22	+	-	+	-	+	27.691
23	-	+	+	-	+	26.365
24	+	+	+	-	+	46.351
25	-	-	-	+	+	28.478
26	+	-	-	+	+	29.397
27	-	+	-	+	+	26.150
28	+	+	-	+	+	30.238
29	-	-	+	+	+	23.349
30	+	-	+	+	+	42.078
31	-	+	+	+	+	20.574
32	+	+	+	+	+	41.398

Results

A: 11.34
DE: 10.91
AC: 8.719
B: 8.354
C: 3.406
E: -4.049
D: -6.145
BD: -9.872

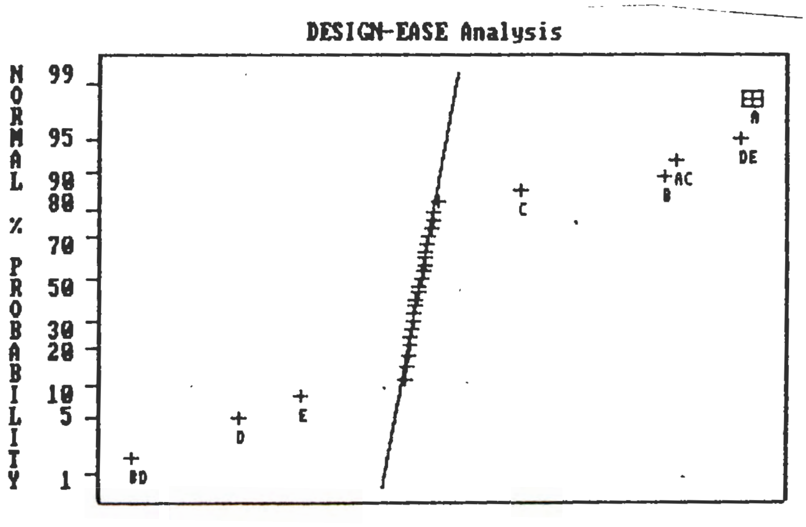

Fig 3.1 The Normal Probability Plot of Effects: **Same effects as in the simulated model are significant**

From the normal probability plot of effects in **Fig 3.1** it is clearly shown that the marked effects are significant since they do not lie on the line through zero. On comparing these effects with the corresponding in the simulated model, it is seen in **Table 3.1** that the maximum proportion error is no more than 0.15.

Table 3.1

Comparison of significant effects for Combined Array and simulated Models

EFFECT	SIMULATED	COMBINED ARRAY	ERROR
A	6	5.64	0.06
B	4	4.177	0.0442
C	2	1.703	**0.1485**
D	-3	-3.0725	0.0241
E	-2	-2.0245	0.01225
AC	4	4.3595	0.0898
BD	-5	-4.936	0.0128
DE	5.5	5.454	0.009

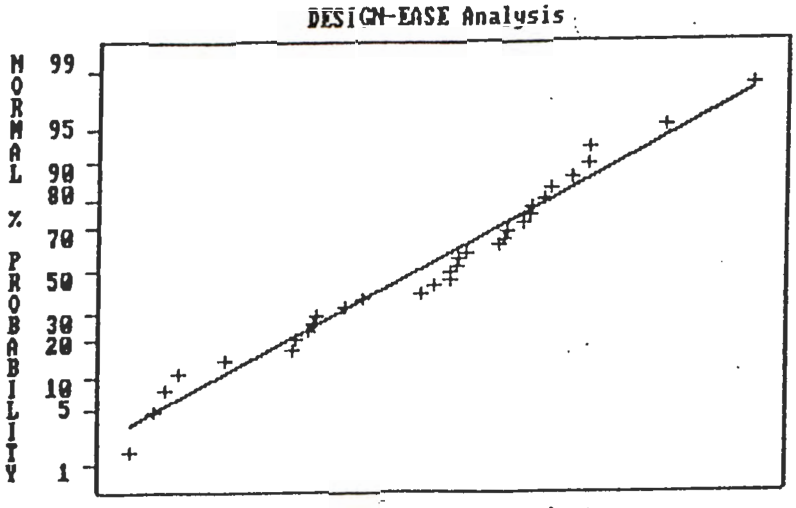

Fig 3.2 **The Normal Probability Plot of Residuals: The Model appears adequate**

The normal probability plot of residuals in **Fig 3.2** indicates that there is no evidence for not accepting Combined Array model with significant effect as shown in **Fig 3.1**. This is because points lie largely on a line.

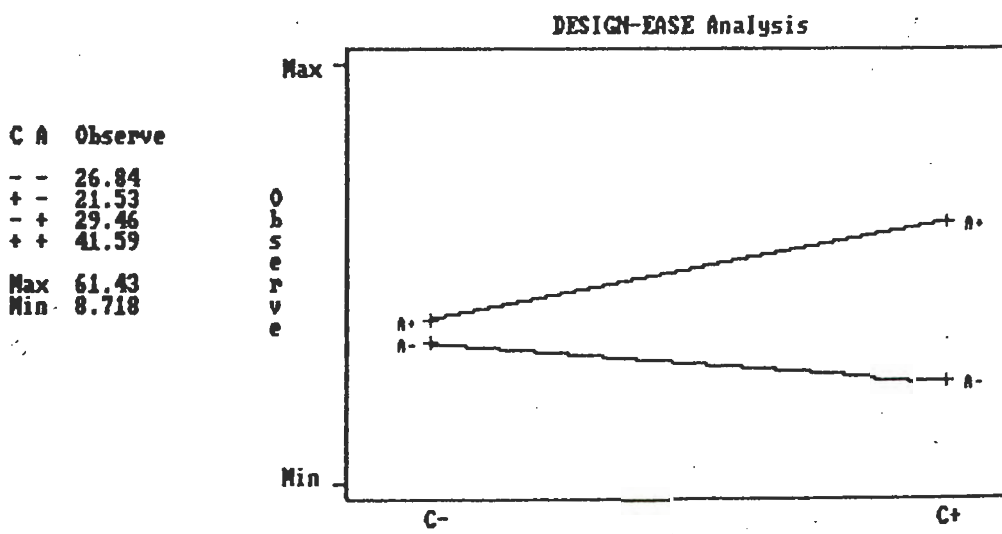

Fig 3.3 **The AC Interaction Plot: C- minimises variance**

From the AC interaction plot in Fig 3.3 it is evident that C at - level has a dampening effect noise factor A since the difference in mean values at C- is small as compared at C+.

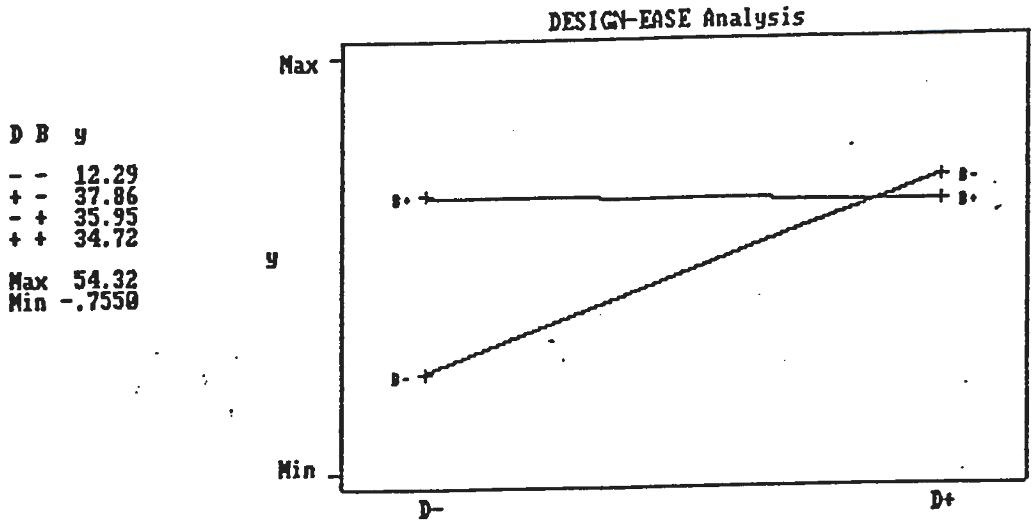

FIG 3.4 **The BD Interaction Plot: D+ may minimise variance**

The BD interaction plot of Fig 3.4 indicates that control factor D at + level has a dampening effect on noise factor B since the differrence in mean values at D+ is smaller as compared at D.

Thus the possible solution is (C-, D+) which is the same solution obtained for the Box-Taguchi variance model.

3.3.2 2^5 SIMULATED DESIGN: EFFECTS CANCEL OUT

The simulated model is given by

$$Y = 40+4D+5E+10BC-10AC+N(0.1).$$

Control factors are C, D, and E with A and B as noise factors.

Objectives

To show that if the model is such that the C-N interactions cancel out, then there is no way of manipulating factor C to minimise variance.

The Design

The designed experiment is as follows:

	NOISE FACTORS		CONTROL FACTORS			RESPONSES
Run	A	B	C	D	E	Y
1	-	-	-	-	-	31.4207
2	+	-	-	-	-	51.3942
3	-	+	-	-	-	11.7679
4	+	+	-	-	-	31.0783
5	-	-	+	-	-	33.1155
6	+	-	+	-	-	12.1043
7	-	+	+	-	-	50.5098
8	+	+	+	-	-	30.0503
9	-	-	-	+	-	40.4527
10	+	-	-	+	-	58.5194
11	-	+	-	+	-	18.5386
12	+	+	-	+	-	38.2614

13	-	-	+	+	-	39.2364
14	+	-	+	+	-	20.2298
15	-	+	+	+	-	59.6662
16	+	+	+	+	-	37.6730
17	-	-	-	-	+	41.3626
18	+	-	-	-	+	61.7164
19	-	+	-	-	+	20.7506
20	+	+	-	-	+	38.9069
21	-	-	+	-	+	40.5422
22	+	-	+	-	+	20.3762
23	-	+	+	-	+	61.8257
24	+	+	+	-	+	41.5846
25	-	-	-	+	+	47.7335
26	+	-	-	+	+	69.2707
27	-	+	-	+	+	28.9971
28	+	+	-	+	+	49.8782
29	-	-	+	+	+	49.9428
30	+	-	+	+	+	28.1456
31	-	+	+	+	+	68.0608
32	+	+	+	+	+	48.6006

Results

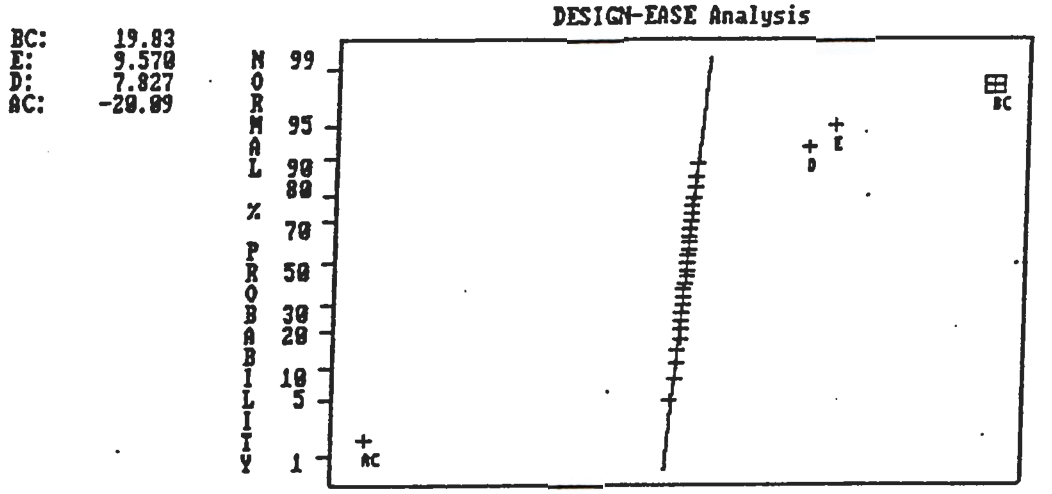

Fig 3.5 The Normal Probability Plot of Effects: The same effects as in the simulated model are significant

The normal probability plot in **Fig 3.5** shows that the simulated model has been fairly re-established. This is evident in Table 3.2 where coefficients of models are compared. The maximum proportion error is less than 0.05.

TABLE 3.2

Comparison of significant effects for the Combined Array and Simulated Models

EFFECT	SIMULATED	COMBINED ARRAY	ERROR
D	4	3.9135	0.0216
E	5	4.785	**0.043**

BC	10	9.915	0.0085
AC	-10	-10.04	0.0045

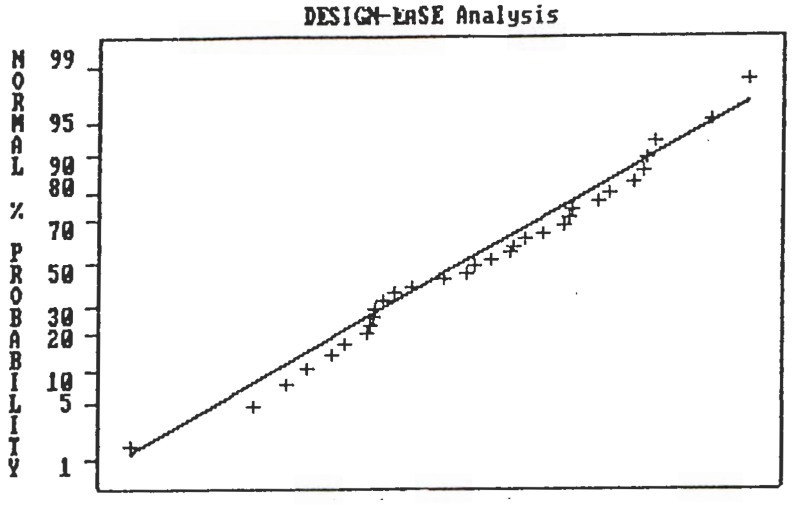

Fig 3.6 **The Normal Probability Plot of Residuals: The model appears adequate**

From the probability plot of residuals in **Fig 3.6**, there is no evidence that the model with indicated significant effects is not acceptable.

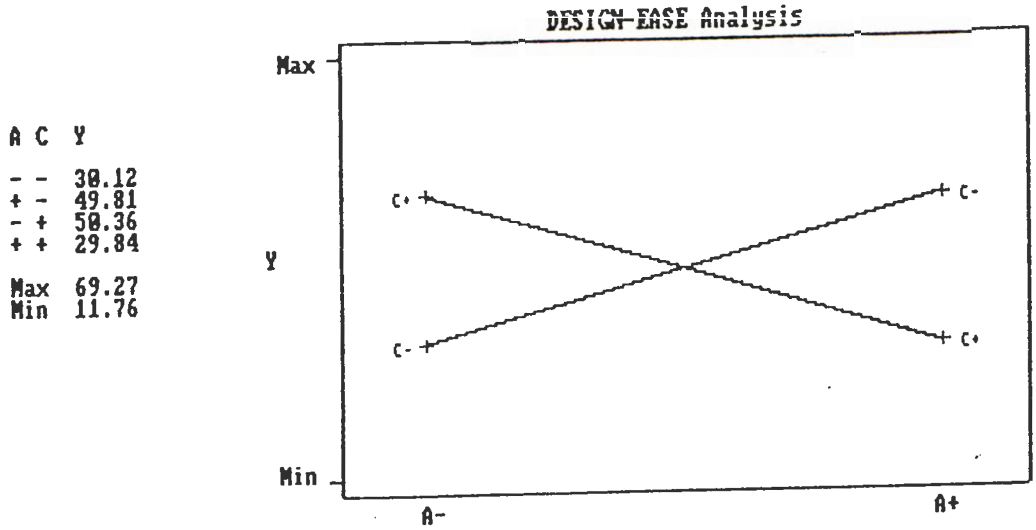

Fig 3.7 **The AC Interaction Plot: C cannot be manipulated**

The AC interaction plot of **Fig 3.7** indicates that control factor C cannot be manipulated in order that noise caused by noise factor A could be dampened since slopes at C- and C+ are practically of equal magnitude.

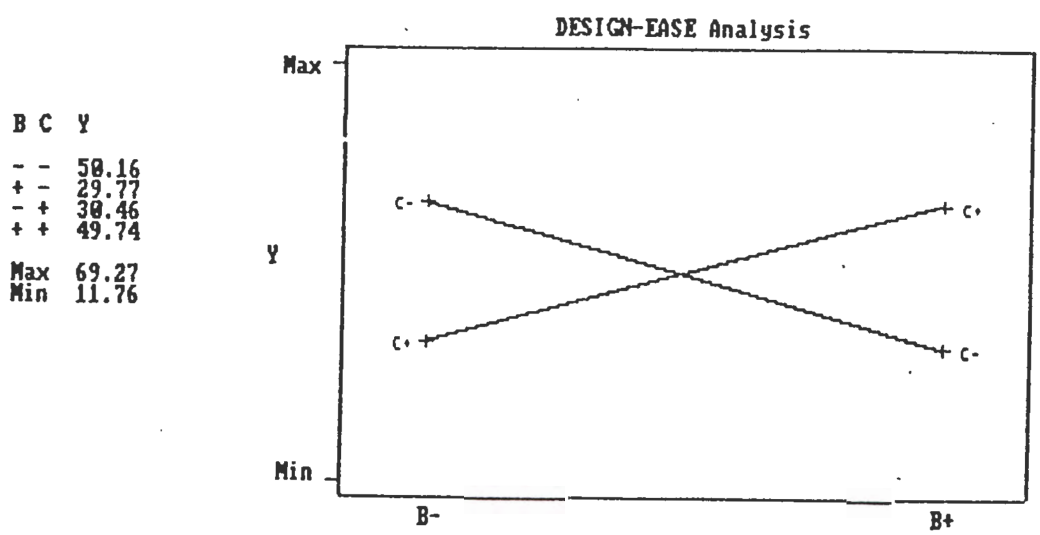

Fig 3.8 **The BC Interaction Plot: C cannot be manipulated**

Also from **Fig 3.8** it is realised that the BC interaction does not give an opportunity to manipulate C in order that robustness can be improved. This is because the slopes at C- and at C+ are practically equal in magnitude.

Remarks

With the Combined Array method it was possible to show that if the noise-to-control interactions were such that they cancel out, then there is no way of manipulating the control variable in question such that noise can be minimised.

It is also seen from **Fig 3.7** and **Fig 3.8** that it is not the size of AC and BC interaction effect alone that determines the potential for improving robustness, but rather the difference in slopes in the interaction plots.

3.3.3 2^5 SIMULATED DESIGN: NO NOISE-TO-CONTROL INTERACTIONS

The simulated model is given by

$$Y = 40+4A+5B+2C+6D-3E+8AB+7DE+N(0.1).$$

A and B are noise factors and C, D, and E are control factors.

Objective

To show that if the model is such that there are no control-to-noise interactions, then there is no way of finding a control factor to robustify the experiment.

The Design

The design experiment for the Combined Array Method is:

	NOISE FACTORS		CONTROL FACTORS			RESPONSES
Run	A	B	C	D	E	$\bar{Y}$
1	-	-	-	-	-	17.241
2	+	-	-	-	-	7.733
3	-	+	-	-	-	13.677
4	+	+	-	-	-	34.299
5	-	-	+	-	-	19.645
6	+	-	+	-	-	11.197
7	-	+	+	-	-	14.749
8	+	+	+	-	-	40.513
9	-	-	-	+	-	42.601
10	+	-	-	+	-	37.034
11	-	+	-	+	-	37.543
12	+	+	-	+	-	61.567
13	-	-	+	+	-	45.456
14	+	-	+	+	-	36.505
15	-	+	+	+	-	41.751
16	+	+	+	+	-	64.123
17	-	-	-	-	+	24.974

18	+	-	-	-	+	17.180
19	-	+	-	-	+	19.385
20	+	+	-	-	+	44.833
21	-	-	+	-	+	28.972
22	+	-	+	-	+	20.548
23	-	+	+	-	+	23.874
24	+	+	+	-	+	45.458
25	-	-	-	+	+	21.864
26	+	-	-	+	+	14.365
27	-	+	-	+	+	17.017
28	+	+	-	+	+	40.885
29	-	-	+	+	+	27.307
30	+	-	+	+	+	18.130
31	-	+	+	+	+	20.907
32	+	+	+	+	+	44.202

Results

AB:15.77
D:11.68
B:10.87
A:7.6
C:3.196
E:-5.983
DE:-14.25

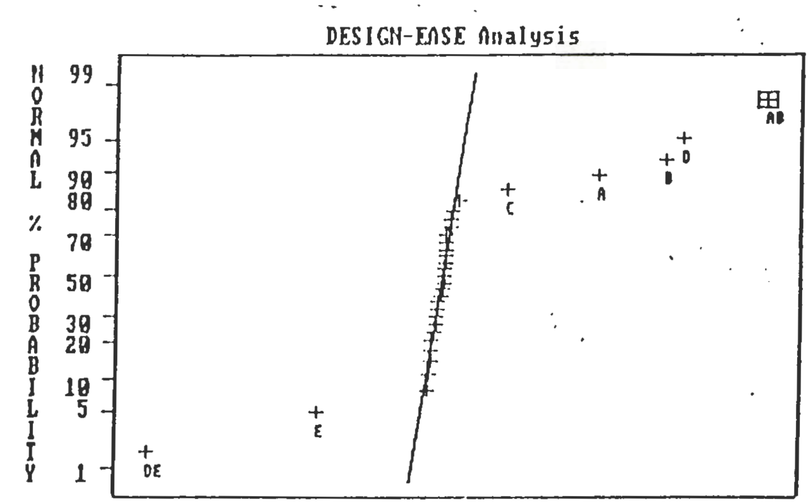

Fig 3.9 The Normal Probability Plot of Effects: The same effects as in the simulated model are significant.

From the normal probability plot of effects in **Fig 3.9** it is seen that only marked effects are significant because they lie off the straight line spanning zero. Further these effects do not differ much with those of the simulated model as can be noticed in **Table 3.3** that the maximum proportion error is at most 0.2.

TABLE 3.3

Comparison of significant effects for the Combined Array and Simulated Models

EFFECT	SIMULATED	COMBINED ARRAY	ERROR
A	4	3.8	0.05
B	5	5.435	0.087
C	2	1.598	**0.201**
D	6	5.84	0.0183
E	-3	-2.990	0.003
AB	8	7.885	0.0144
DE	-7	-7.125	0.0179

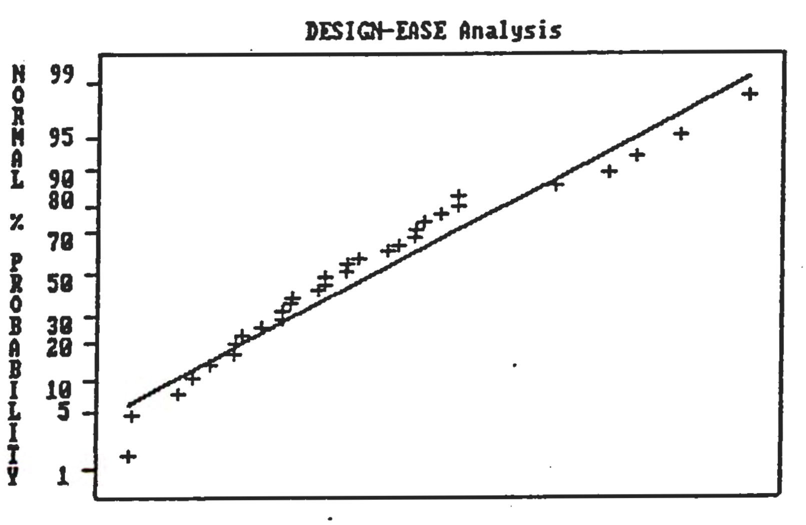

Fig 3.10 The Normal Probability Plot of Residuals: The model looks adequate

The probability plot of residuals in **Fig 3.10** indicates that the model determined in **Fig 3.9** may not be rejected since points in the graph almost lie on a straight line.

Since AB is a noise-to-noise interaction and DE is a control-to-control interaction, their interaction plot cannot be interpreted. Therefore there is no way of manipulating control factors in order that robustness may be improved.

3.3.4 THE 2^{3-1} X 2^{4-1} COMBINED ARRAY DESIGN

This is the same simulated model of **Chapter Two**. The model is repeated here for clarity:

$$Y = 30+6A+4B+2C-3D-2E+4F+5G+7AE-8CD-6FG+N(0.1),$$

where A, B, and C are noise factors while D, E, F, and G are control factors.

Objective

To show that the above-mentioned design gives the same results, when analysed by combined Array method, as the $2^{3-1}x2^{4-1}$ product array design of **Chapter 2**, analysed by Taguchi's method. Moreover, the dampened noise factors are readily identifiable.

The Design

The design is a Combined Array experiment such that

$$C = AB \text{ and } G = DEF$$

and it has the following form:

	NOISE FACTORS			CONTROL FACTORS				RESPONSES
Run	A	B	C	D	E	F	G	$\bar{Y}$
1	-	-	+	-	-	-	-	25.4
2	+	-	-	-	-	-	-	3.41
3	-	+	-	-	-	-	-	15.0
4	+	+	+	-	-	-	-	32.0
5	-	-	+	+	-	-	+	26.9
6	+	-	-	+	-	-	+	35.5
7	-	+	-	+	-	-	+	46.4
8	+	+	+	+	-	-	+	33.8
9	-	-	+	-	+	-	+	31.5
10	+	-	-	-	+	-	+	37.8
11	-	+	-	-	+	-	+	20.1
12	+	+	+	-	+	-	+	66.0
13	-	-	+	+	+	-	-	-13.0
14	+	-	-	+	+	-	-	22.9
15	-	+	-	+	+	-	-	5.86
16	+	+	+	+	+	-	-	22.5
17	-	-	+	-	-	+	+	43.8
18	-	-	+	-	-	+	+	22.7
19	-	+	-	-	-	+	+	32.3
20	+	+	+	-	-	+	+	50.6

21	-	-	+	+	-	+	-	24.7
22	+	-	-	+	-	+	-	34.1
23	-	+	-	+	-	+	-	45.2
24	+	+	+	+	-	+	-	31.3
25	-	-	+	-	+	+	-	29.8
26	+	-	-	-	+	+	-	33.8
27	-	+	-	-	+	+	-	16.6
28	+	+	+	-	+	+	-	64.3
29	-	-	+	+	+	+	+	4.62
30	+	-	-	+	+	+	+	42.1
31	-	+	-	+	+	+	+	25.5
32	+	+	+	+	+	+	+	38.8

Results

AE:13.96
A:11.96
G:10.28
F:8.009
B:8.764
C:4.609
E:-3.37
D:-6.12
FG:-12.7-
CD:-15.6

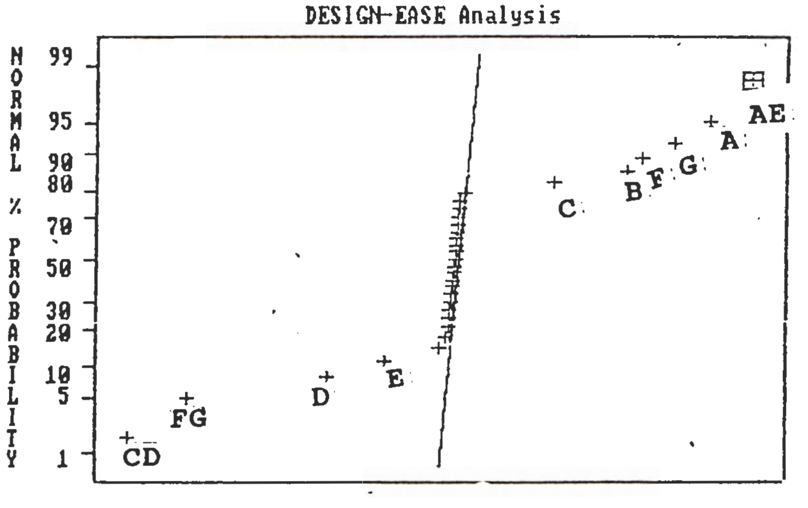

Fig 3.11 **The Normal Probability of Effects: The same effects as in the simulated model are significant**

It can be seen in Fig 3.11 that the marked effects are significant since they do not lie on the straight line of the normal probability plot spanning zero. Moreover, these do not differ much with those of the simulated model since the maximum proportion error is 0.16. (see Table 3.4).

TABLE 3.4

EFFECT	SIMULATED	COMBINED ARRAY	ERROR
A	6	5.98	0.003
B	4	4.382	0.0955
C	2	2.305	0.152
D	-3	-3.06	0.02
E	-2	-1.685	0.157
F	4	4.005	0.0011
G	5	5.14	0.028
AE	7	6.98	0.025
CD	-8	-7.80	0.0028
FG	-6	-6.35	0.058

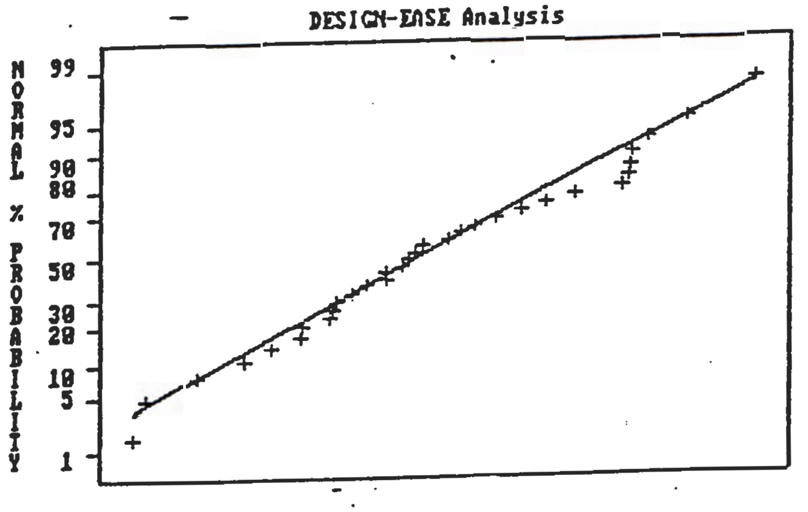

Fig 3.12 The Normal Probability Plot of Residuals: The model appears adequate

Points in the normal probability plot of residuals (**Fig 3.12**) lie almost on a straight line. Thus there is no evidence for rejecting the model with significant effects indicated in **Fig 3.11**.

E	A	Y
-	-	32.46
+	-	15.12
-	+	30.42
+	+	41.02
Max		66.00
Min		-13.00

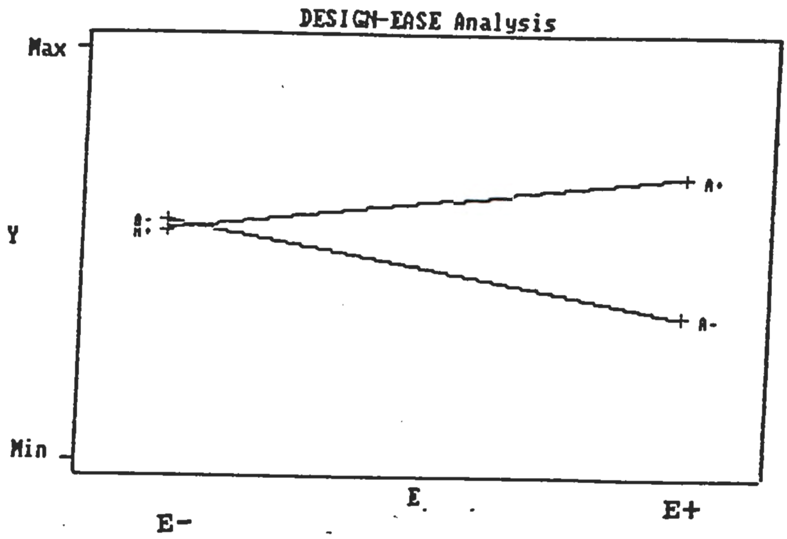

Fig 3.13 **The AE Interaction Plot: E- may minimise variance**

In **Fig 3.13** (the AE interaction plot) it is seen that the difference in mean values at E- is smaller as compared with mean values at E+. This implies that E- has a dampening effect on A, that is, E at level - may improve robustness. Similarly **Fig 3.14** may be used to argue that D at + level may minimise noise caused by factor C.

C	D	Y
-	-	22.71
+	-	42.93
-	+	32.2
+	+	21.2

Max	66
Min	-13

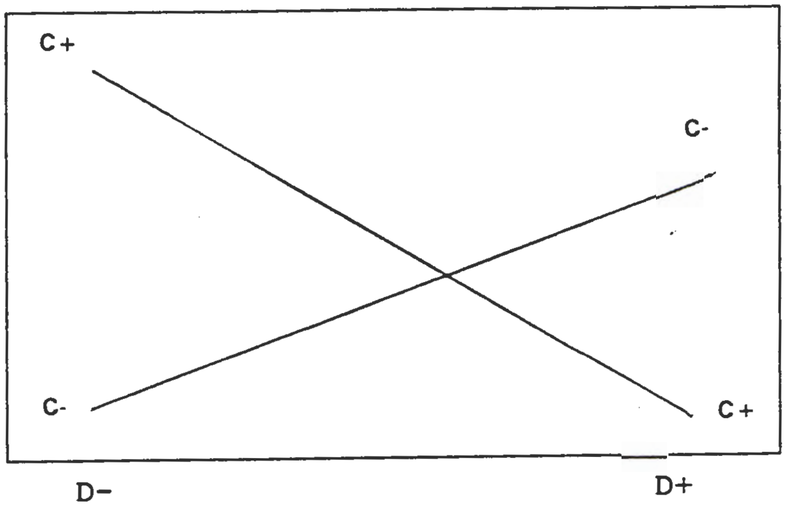

Fig 3.14 **The CD Interaction Plot: D+ may minimise variance**

Hence the solution could be (E-, D+) as before.

3.3.5 THE 2^{7-3} DESIGN

The design of **Example 3.3.4** above may be viewed as a 2^{7-2} combined array design. Further fractionation results into a 2^{7-3} design. This example is a half fraction of the design in Example 3.3.4.

Objectives

To show that even if further fractionation of the 2^{2-1} x 2^{3-1} design the way Taguchi could have done it (fractionating the noise and control arrays separately) is not possible, viewed as a combined array design, further fractionation in a manner used by Box *et al* (1978 :410) is possible. Also to show that the resultant fraction (2^{7-3}) gives the same qualitative result as above. (**Example 3.3.4**)

The Design

To obtain the design the generators

$$I = ABCE = BCDF = ACDG$$

were employed. It should be noted that the fraction where all generators are positive was chosen for this example. There are 7 other posibilities which were also investigated. The design for the chosen fraction takes the form :

	NOISE FACTORS			CONTROL FACTORS				RESPONSE
Run	A	B	C	D	E	F	G	Y
1	-	-	-	-	-	-	-	6.42
2	+	-	-	-	+	-	+	37.8
3	-	+	-	-	+	+	-	16.6
4	+	+	-	-	-	+	+	30.5
5	-	-	+	-	+	+	+	27.6
6	+	-	+	-	-	+	-	44.3
7	-	+	+	-	-	-	+	56.8
8	+	+	+	-	+	-	-	41.9

9	-	-	-	+	-	+	+	34.1
10	+	-	-	+	+	+	-	44.7
11	-	+	-	+	+	-	+	29.4
12	+	+	-	+	-	-	-	22.2
13	-	-	+	+	+	-	-	-13
14	+	-	+	+	-	-	+	25.3
15	-	+	+	+	-	+	-	34.1
16	+	+	+	+	+	+	+	38.8

Results

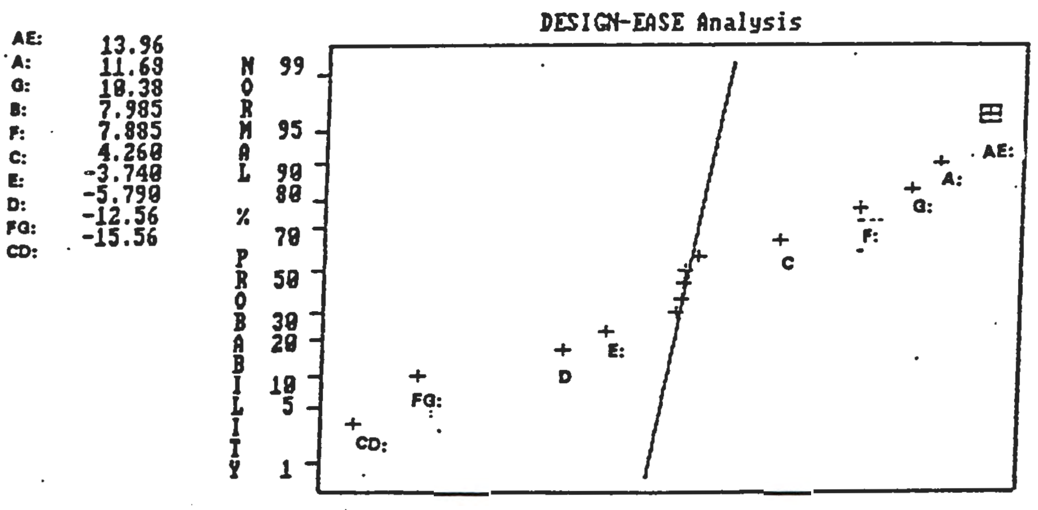

Fig 3.15 The Normal Probability of Effects: The same effects as in the simulated model are significant

Significant effects are those marked in the normal probabilty plot of effects in **Fig 3.15** (because they lie off the line). They compare well in value with those in the simulated model since the maximum proportion error is 0.065

(see **Table 3.5**)

TABLE 3.5

Comparison of significant effects for the Combined Array and Simulated Models

EFFECT	SIMULATED	COMBINED ARRAY	% ERROR
A	6	5.84	0.0267
B	4	3.942	0.015
C	2	2.13	**0.065**
D	-3	-2.895	0.035
E	-2	-1.87	0.0625
F	4	3.994	0.00175
G	5	5.19	0.038
CD	-8	-7.78	0.0275
AE	7	6.98	0.0029
FG	-6	-6.28	0.046

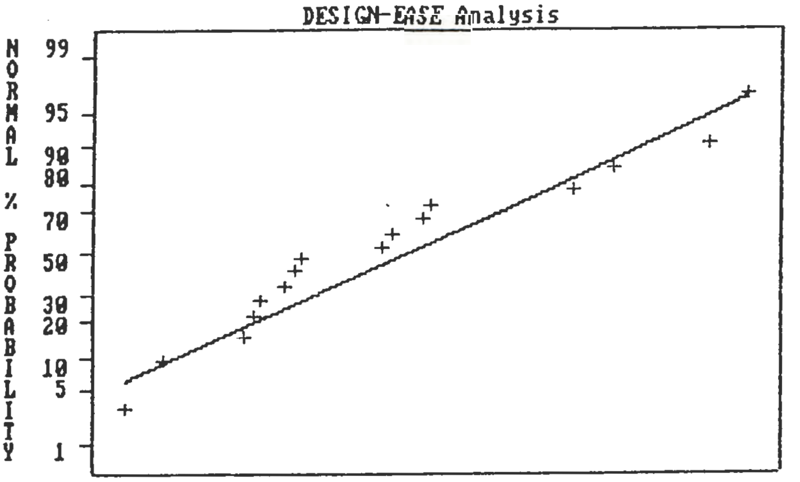

Fig 3.16 **The Normal Probability Plot of Residuals: The model looks adequate**

There are no points lying significantly off the straight line in the normal probability plot of residuals in **Fig**

3.16. Hence there is not enough evidence that the model chosen above should be rejected.

C	D	Y
-	+	22.8
+	-	43.15
-	+	32.85
+	+	21.3

Max	56.8
Min	-13

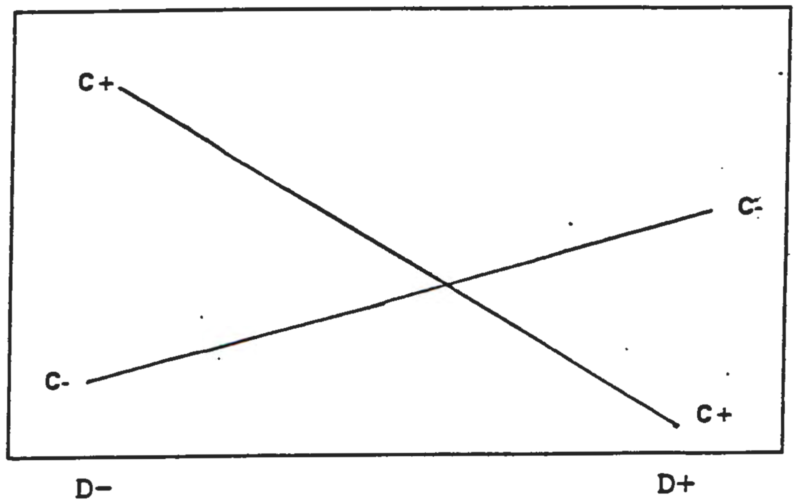

Fig 3.17 The CD Interaction Plot: D+ may minimise variance

Since the difference in mean values at D+ is smaller than at D- in **Fig 3.17** (CD interaction plot), D+ may have a dampening effect on noise factor C. By same argument it is evident from the AE interaction plot in **Fig 3.18** that E- can have a dampening effect on A.

A	E	Y
-	-	32.86
+	-	30.58
-	+	15.15
+	+	40.8

Max	56.8
Min	-13

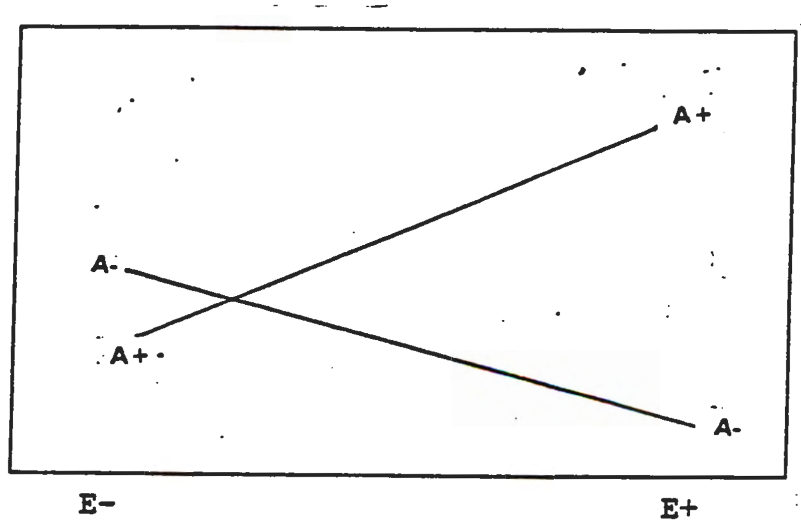

Fig 3.18 The AE Interaction Plot: E- may minimise the variance

Hence the solution could be (D+, E-). These are the same results obtained for Examples 2.9.4 and 3.3.4. It follows that one could save on experimental runs by the strategy used in this example. The other fractions that were investigated also gave the same solution.

3.4 CONCLUSIONS

Motivation for the use of Combined Arrays as an alternative method to the traditional Taguchi's approach was made. It was seen that the biggest gain made from this method was the saving on experimental runs. This was demonstrated when it was shown that a 2^{7-3} design gives qualitatively the same results as the 2^{7-2} design. This would not have been possible if Box-Taguchi method is used because a $2^{3-1} \times 2^{4-1}$ cannot be further fractionated. The Combined Array results could not be achieved without a price - assumption based on engineering knowledge had to be made.

The reliability of Combined Array method was demonstrated by considering two simulated examples, namely, examples where

(i) there are no noise-to-control interactions, and

(ii) the noise-to-control interaction effects cancel out.

Both examples gave expected results in that there was no way of manipulating control factor in an attempt to minimise variance caused by noise.

CHAPTER FOUR

CONCLUDING REMARKS

The study has integrated into one document results on Robust Design scattered throughout literature. Shoemaker *et al* (1991) have considered such work with Integrated-Circuit Fabrication, but this study may be the first attempt to do so with simulated models.

The Box-Taguchi method was illustrated with simulated examples in **Chapter Two**. It was seen that the method was effective in screening out both the location and dispersion effects. When focusing on variability, it was established that control-to-noise (C-N) interactions were important. In cases where the simulated models had the C-N interactions either cancelling out or not present, the Box-Taguchi method gave a flat model (no control factors were significant). It was therefore not possible to tell one variance model from the other because the method could not identify control factors that could have dampening effect on individual noise factors. This was not the case with the Combined Array method.

In **Chapter Three**, it was observed that the Combined Arrays method yielded qualitatively the same results for the same problems of **Chapter Two**. In particular, for the 2^{3-1} x 2^{4-1} product design and the same 2^{7-2} combined array design the potential dispersion reduction level combination of control factors was D+ and E-.

It is clear that the 2^{3-1} x 2^{4-1} product design cannot be further fractionated in the way Taguchi would have done it (fractionating the noise and the control array separately). Viewed as a combined array (2^{7-2}) this design can be further fractionated by the method used by Box *et al* (1978:410). Fractionated in this manner, the resultant 2^{7-3} design is of Resolution IV. This implies that second order interactions are confounded with other second order interactions. To resolve this, additional knowledge (possibly engineering knowledge) is needed. If this could be achieved, the fractionated 2^{7-2} design would substantially reduce the cost of the experiment. Example 3.3.5 demonstrated this since like the 2^{7-3}, the 2^{7-3} D+ and E- could minimise variance due to noise. If engineering knowledge could not clear up the ambiguity (caused by confounding patterns), additional runs would be needed to de-alias the effects (see Box *et al*, 1978 :413).

As far as the mean target is concerned, the Box-Taguchi method can only recommend the best control factors level combinations. In addition to this, the Combined Array method could be used to do sensitivity analysis around the target.

Despite the many advantages that the Combined Array method has, Shoemaker *et al* (1991) maintain that there are practical situations in which Taguchi's formulation may be more suitable. Also, Combined Array method depends more critically than Taguchi's approach on how the model fits. If this method is employed, an ommision of a noise factor that has a strong interaction with a control factor , from the model could lead to unacceptable results.

In a short study such as this one, it was not possible to cover everything on Robust Design. Chen *et al* (1993) have indicated that the resolution criterion as proposed by Box and Hunter (1961) is not the best criterion for fractional designs because designs of same resolution may not be equally good. They (Chen *et al*) constructed a catalogue of designs which are judged to be good by minimum abberation criterion. Also, Ma's (1993) Minimum Main Confounding Design criterion for best fractions may give improved results. However, Resolution IV designs such as the one used to construct the 2^{7-3} design of Example 3.3.5, are still the most efficient compromise in the use of fractional factorial designs (Hurley,1994).

APPENDIX

Let **x** = $(x_1, x_2, \ldots, x_n)$ denote the noise factors and **z** = $(z_1, z_2, \ldots, z_r)$ denote the control factors. If f(**x**,**z**) is a function of **x** and **z**, then a general statistical model is given by (Phadke, 1989 :48)

$$Y = \mu + f(\mathbf{x},\mathbf{z}) + N(0.1) \qquad \text{(A1)}$$

where N(0.1) are normally distributed random errors with mean 0 and variance 1 and μ is the mean value of Y for the experimental region.

For a 2^k design, the term f(**x**,**z**) would consist of 2^k-1 effects which would include k main effects, ${}^kC_2$ two factor interactions, ${}^kC_3$ three factor interactions ..., and one k-factor interaction (Montgomery,1991 :288).

To generate these errors, the following theorem due to Box and Muller (1958) was used :

Theorem A1

If U_1, U_2 are independent random variables, each uniformly distributed on the interval 0 to 1, then the random variables

$$X_1 = (-2\ln U_1)^{1/2} \cos 2\pi U_2 \qquad \text{(A2)}$$

$$X_2 = (-2\ln U_1)^{1/2} \sin 2\pi U_2 \qquad \text{(A3)}$$

are independent random variables, each normally distributed with mean 0 and variance 1.

Using random numbers tables such as Table XV in Hines and Montgomery (1980 :628) the random errors N(0.1) were generated with the aid of transformation equations A2 and A3.

The mean μ for each model was chosen such that very few simulated responses became negative to avoid working with negative numbers (for convenience).

The coefficients of significant effects were chosen so that a model should have

(i) at least two C-N interactions, or

(ii) the C-N interactions cancelling out, or

(iii) no C-N interactions.

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