



**An application of Box-Jenkins Transfer Function
analysis to consumption-income relationship in
South Africa**

by

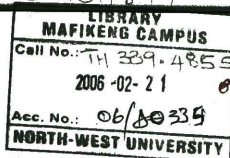
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Abstract

Using a simple linear regression model for estimation could give misleading results about the relationship between Y_t and X_t . Possible problems involve (1) feedback from the output series to the inputs, (2) omitted time-lagged input terms, (3) an autocorrelated disturbance series and, (4) common autocorrelation patterns shared by Y and X that can produce spurious correlations. The primary aim of this study was therefore to use the Box-Jenkins Transfer Function analysis to fit a model that related petroleum consumption to disposable income. The final Transfer Function Model $z_t = \frac{C(1-w_1B)}{(1-\delta_1B)} B^s z_t^{(x)} + (1-\theta_1B)a_t$ significantly described the data. Forecasts generated from this model show that petroleum consumption will hit a record of up to 4.8636 in 2014 if disposable income is augmented. There is 95% confidence that the forecasted value of petroleum consumption will lie between 4.5276 and 5.1997 in 2014.

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CHAPTER 1

INTRODUCTION

1.1 Background

Using a simple linear regression model for estimation could give misleading results about the relationship between Y_t and X_t . Possible problems involve (1) feedback from the output series to the inputs, (2) omitted time-lagged input terms, (3) an autocorrelated disturbance series and, (4) common autocorrelation patterns shared by Y and X that can produce spurious correlations.

This study tries to set out a practical model identification procedure designed to handle problems (1), (2), (3) and (4). The details for model identification used in this study differ from those proposed by Box and Jenkins (1976), but the ideas are the same. Autoregressive Moving Average (ARMA) models have been studied extensively by Box and Jenkins (ibid). The authors have effectively put together, in a comprehensive manner, the relevant information required to understand the use of univariate time series ARMA models which forecast future values of an output series based on its past values.

The primary objective of this investigation was to build a model that is intended to show a relationship between the output time series, $\{Y_t\}_{t \in \mathbb{Z}}$, and the input time

series, $\{X_t\}_{t \in \mathbb{Z}}$ using Box-Jenkins Transfer Function Analysis. It is hoped that any model chosen might be interesting in its own right but the interest will also be in using it to predict future values of Y_t from future values of X_t . When both time series fall into the class of ARMA type models then it is appropriate to use 'Transfer Function Models' (Priestley, 1981). However, the researcher's modeling methodology is intended for series that exhibit patches of stationarity.

The Box-Jenkins Transfer Function model combines the characteristics of both the univariate process and those of simple linear regression. Thus, it combines the time-series approach with the causal approach to forecast. Makridakis and Wheelwright (1978) prescribe the use of these models as a powerful tool when appropriate conditions exist for their use. As recommended by Box and Jenkins (1976), the three phases; (1) identification, (2) estimation and checking and (3) application of the best model are considered. Pankratz (1991), Bowerman and O'Connell (1993), and Abraham and Ledolter (1983) amongst others suggest prewhitening of the model and the use of cross-correlations at the identification stage. This was considered in this study.

Bowerman and O'Connell (1993) and Abraham and Ledolter (1983), also recommended the use of the Residual Sample Autocorrelations (RSAC),

Residual Sample Partial Autocorrelations (RSPAC) and the Residual Sample Cross Correlations (RSCC) in the identification of the final model. Therefore, these techniques have also been considered in this study.

The following discussion focuses on the objectives of the study, the research methodology, the importance of the study and the limitations of the study.

1.2 Objectives of the study

The objectives of this study were:

- To build a Box-Jenkins Transfer Function model $y_t = v(B)x_t + \eta_t$, that describes a dynamic relationship between two variables namely; petroleum consumption and disposable income of households.
- To use the model to forecast an output, petroleum consumption.

1.3 Research methodology

This section discusses the manner in which data were collected, the methods that were used for data analysis and the type of data required for Box-Jenkins Transfer Function Analysis.

1.3.1 Data collection

Since the Box-Jenkins Transfer Function Analysis has more to do with time series analysis than anything else, secondary data were used. Yearly petroleum consumption and household income figures were sourced from the South African Reserve Bank. The data covered the period from 1968 to 2004. A total of 37 observations for each variable was used.

1.3.2 Data analysis

The statistical technique that was used to analyze the data was the Box-Jenkins Transfer Function Analysis. Changes within each variable over time were looked at with a graphic display of time series plots for each variable. It was assumed that the series were stationary with zero a mean and a constant variance.

The Phillips-Perron (PP) test introduced by Phillips and Perron (1988) and the Augmented Dickey-Fuller (ADF) test by Dickey and Fuller (1979) were used to test the variables for the presence of unit roots. In addition, the portmanteau tests introduced by Box and Pierce (1970) were carried out to check the residuals for autocorrelation. The two information criteria, Akaike's Information Criterion (AIC) introduced by Akaike (1973) and Bayesian's Information Criterion (BIC) introduced by Schwarz (1978) were also used for the selection of the final

Transfer Function Model. The model that has a less AIC or SBC value was selected as the best model. However, as indicated in Babias and Nazem (1994), variability in observed time series data is usually measured by the standard deviation of the series. Then, the standard deviations were also used in the selection of the best model. The Systems Analysis Software (SAS) was employed to generate all the results.

1.3.3 Data

The first question to be dealt with was the choice of variable to be used as an indicator of the output series. Presumably researchers are guided by some theory or experience in deciding which input(s) to include in a Transfer Function model, Pankratz (1991).

It is reasonable to assume that past values of input series (X_t) influence future values of an output series (Y_t), but not vice versa. If past Y_t values also influence future X_t values, then the Transfer Function Analysis would not be appropriate. It would be biased and inconsistent since X_t and Y_t would be correlated. In this study, it is assumed that disposable income (X_t) influences petroleum consumption (Y_t) but not vice versa.

1.4 Importance of the study

The study will inform researchers about a Box-Jenkins Transfer Function Model that describes a dynamic relationship between petroleum consumption and disposable income of households in South Africa. The model might contribute to the body of knowledge that has to do with forecasts of an output series based on its past values as well as the future values of an input-series without encountering the following problems; (1) feedback from the output series to the inputs, (2) omitted time-lagged input terms, (3) an autocorrelated disturbance series and, (4) common autocorrelation patterns shared by Y and X that can produce spurious correlations.

1.5 Study limitations

The study only attempted to build a Transfer Function Model using Box-Jenkins techniques. One can also use Multivariate Autoregressive Moving Average (MARMA) Transfer Function Analysis to develop the forecasts, but this was not considered for this study. A single input series was used in building the model and making the forecasts.

1.6 Research outline

The study is organised into four chapters. Sections 1.1 of Chapter 1 reflect the background of the study, 1.2 objectives of the study, 1.3 research methodology, 1.4 importance of the study and 1.5 study limitations. In Chapter 2, section 2.1 reflects an introduction of Box-Jenkins Transfer Function Analysis and section 2.2 reflects the review of statistical procedures for the Transfer Function Model. In section 2.3, the unit root tests are discussed.

The presentation of data and the discussion of the results are given in Chapter 3. Chapter 4 concludes the study and makes recommendations. Bibliography, appendices and a list of exhibits are given after the final chapter.

CHAPTER 2

THEORY AND LITERATURE REVIEW

2.1 Introduction

The introduction of the method of Box-Jenkins for univariate time-series model building has raised a significant controversy about the value of integrated autoregressive-moving average models (ARIMA) versus econometric models. In the context of forecasting, ARIMA models sometimes outperform econometric models (Nelson 1972). This is a surprising finding, because ARIMA models use the information of only one time-series for forecasting. However, it is currently known that ARIMA and econometric models are related to each other (Anderson 1971). This relationship removes much of the original controversy, but, more importantly, it creates some unique opportunities for empirical causal model building and testing on longitudinal data. In Trivedi (1975)'s words, "... time series and structural models need not be thought of as competing approaches."

The methodology for the investigation of relationships between time series has been severely criticized. Since Box and Newbold (1971) pointed out that many of the relationships established by Coen, Gomme and Kendall (1971) were improperly identified because insufficient care was taken over the appropriate formulation for the autocorrelation structure of the errors from regression

equations, some serious shortcomings of the classical methodology have been recognised. These shortcomings are lack of model identification and the poor treatment of residuals.

One of the major goals in time series correlation studies is to show or identify causality among economic variables. Granger (1973) explain cases where a variable Y_t causes X_t , if one is better able to predict X_t using all the available information than if the information apart from Y_t has been used. However, interactions among economic variables do not always have a "one-way character". Feedback is said to occur when the variable X_t causes Y_t and also Y_t causes X_t .

Many applied researchers (Pindyk and Rubinfeldt (1991) and Wonnacott (1986)) have reported very strong relationships between econometric variables and have shown high R^2 values. However it is suggested by Pindyk and Rubinfeldt (1991) that most of the causality that is thought to be measured can be considered as "spurious correlation", because not all available information was included in the model and/or the errors were not free of autocorrelation, Anderson (1971). Spurious correlation is a situation in which measures of two or more variables are statistically related (they cover) but are not in fact causally linked—usually

because the statistical relation is caused by a third variable. When the effects of the third variable are removed, they are said to have been partialled out and autocorrelation refers to the correlation of a time series with its own past and future values.

Bowerman and O'Connell (1993) formulated a three-step procedure for building a Transfer Function Model. These steps are (i) identification of a model describing the input series, (ii) identifying a preliminary Transfer Function Model describing an output series, and (iii) using residuals for the preliminary model to identify a model describing the error structure of the preliminary model and to form a Transfer Function Model. The details of identification procedure found in Bowerman and O'Connell (1993), Pankratz (1991), Makridakis and Wheelwright (1978) and Wei (1990) are identical. However they differ from those in Box and Jenkins (1976), though the main ideas are the same. The chief advantage of the procedure given by Box-Jenkins (ibid) is that they can handle multiple inputs with relative ease, unlike the procedure of Pankratz (1991).

The identification or specification stage of the Transfer Function Analysis poses various difficulties such as identifying the parameters to be included in the model, in particular for multiple-input models and for seasonal models. The preliminary

Transfer Function model is $v(B) = \frac{\omega_s(B)B^b}{\delta_r(B)}$ and the coefficients are ω_s and δ_r .

The identification of this model from data requires specifying the degrees of polynomials and the non-zero coefficients of the polynomials. Usually, this process consists of two steps; firstly, the degrees of the polynomials are selected; secondly, the vector of the unknown coefficients of the polynomials is estimated. In case of a single input series, a widely used selection procedure (cf. Box and Jenkins (1976)) requires examining cross-correlation functions between (prewhitened) input and output. By definition, prewhitening the two series before computing cross-correlations means transforming these series to white noise by application of suitable filters.

When dealing with multivariate time series, Haugh and Box (1977) introduced a double prewhitening method. Liu and Hanssens (1982) used ordinary least squares method for estimating $v(B)$, which is a preliminary Transfer Function model, and a modification of the corner method for Transfer Function Model identification. Therefore, there is an increasing interest towards the definition of automatic selection procedures (Muller & Wei 1997), which could reduce subjectivity of model construction. Multi-input modeling represents one time series as dependent on several input time series.

2.2 Review of statistical procedures

2.2.1 Single-input Transfer Function models

This study presents the identification method emphasized by Box-Jenkins (1976) which involves a procedure called prewhitening to make the series stationary and cross correlation functions (CCF). This method is mentioned often in the literature on Dynamic Regression models by authors such as Abraham and Ledolter (1983), Granger (1973), Pankratz (1991) and Wei (1990).

Consider the case of a single-input dynamic regression model. Assume x_t and y_t are properly transformed series so that they are both strongly stationary. In a single-input, single-output linear system, the output series y_t and the input series x_t are related through a linear filter as

$$y_t = v(B)x_t + \eta_t \quad \dots\dots\dots(2.1)$$

where $v(B) = \sum_{j=-\infty}^{\infty} v_j B^j$ is referred to as the Transfer Function filter by Box and Jenkins (1976), v_j is an impulse response weights and B is a coefficient of x_t . η_t is the noise series of the system that is independent of the input series x_t . Box-Jenkins call equation (2.1) the Transfer Function Model. When x_t and η_t

are assumed to follow some ARMA models, equation (2.1) is also known as ARMAX model.

The coefficients in the Transfer Function Model (2.1) are the impulse response weights. As a function of j , v_j is also called the impulse response function. The Transfer Function Model is said to be stable if the sequence of these impulse response weights is absolutely summable.

i.e., if $\sum |v_j| < \infty$. Thus, in a stable system, a bounded input always produces a bounded output. The Transfer Function Model is said to be causal if $v_j = 0$ for $j < 0$. The study confines itself to the stable and causal Transfer Function model:

$$y_t = v_0 x_t + v_1 x_{t-1} + v_2 x_{t-2} + \dots + \eta_t$$

$$= v(B)x_t + \eta_t \quad \dots\dots\dots (2.2)$$

where $v(B) = \sum_{j=0}^{\infty} v_j B^j$, $\sum_{j=0}^{\infty} |v_j| < \infty$, B is a coefficient of x_t and x_t and η_t are independent. The process x_t is stationary. Sometimes it is not stationary, some differencing may be necessary to make it so.

The purposes of the Transfer Function modeling are to identify and estimate the transfer function $v(B)$ and the noise model for η_t based on the available information of the input series x_t and the output series y_t . The difficulties are that the information on x_t and y_t is finite and the Transfer Function $v(B)$ in equation (2.2) may contain an infinite number of coefficients.

To alleviate these difficulties, the Transfer Function $v(B)$ is represented in the following form:

$$v(B) = \frac{\omega_s(B)B^b}{\delta_r(B)} \dots\dots\dots(2.3)$$

The operators $\omega_s(B) = \omega_0 - \omega_1 B - \dots - \omega_s B^s$ and $\delta_r(B) = 1 - \delta_1 B - \dots - \delta_r B^r$ are polynomials in B , where B is a parameter representing the delay between the variables. Furthermore, it is assumed that the roots of $\delta(B) = 0$ are not outside the unit circle.

2.2.2 The cross-correlation function and transfer function models

(a) The cross-correlation function

This is a measure of strength and direction between two random variables. Given two stochastic processes x_t and y_t for $t = 0, \pm 1, \pm 2, \dots$, it is said that x_t and y_t are jointly bivariate stationary if x_t and y_t are both univariate stationary processes and the cross-covariance between x_t and y_s , $Cov(x_t, y_s)$, is a function of the time difference $(s - t)$ only. In such cases, the following cross-covariance between x_t and y_t exists:

$$\gamma_{xy}(k) = E[(x_t - \mu_x)(y_{t+k} - \mu_y)] \text{ for } k = 0, \pm 1 \pm 2, \dots \quad (2.4)$$

Upon standardization, the following CCF results:

$$\rho_{xy}(k) = \frac{\gamma_{xy}(k)}{\sigma_x \sigma_y} \text{ for } k = 0, \pm 1 \pm 2, \dots, \quad (2.5)$$

where σ_x and σ_y are standard deviations of x_t and y_t .

It is important to note that the cross-covariance function (CCF) $\gamma_{xy}(k)$ and cross-correlation functions $\rho_{xy}(k)$ are generalizations of autocovariances and autocorrelation functions because $\gamma_{xx}(k) = \gamma_x(k)$ and $\rho_{xx}(k) = \rho_x(k)$.

The autocorrelation function $\rho_x(k)$, is symmetric around the origin, i.e.,

$$\rho_x(k) = \rho_x(-k), \text{ but the CCF is not necessarily symmetric, i.e., } \rho_x(k) \neq \rho_x(-k).$$

Instead,

$$\gamma_{xy}(k) = E[(x_t - \mu_x)(y_{t+k} - \mu_y)] = \gamma_{xy}(k) = E[(y_{t+k} - \mu_y)(x_t - \mu_x)] = \gamma_{yx}(-k).$$

Therefore $\rho_{xy}(k) = \rho_{yx}(-k)$. Thus the cross-correlation function measures not only the strength of an association but also its direction. To see the full picture of the relationship between the series x_t and the series y_t , it is important to examine the CCF, $\rho_{xy}(k)$, for both positive lags $k > 0$ and negative lags $k < 0$.

(b) The relationship between the CCF and the Transfer Function

For time $t+k$, the Transfer Function Model (2.2) can be written as

$$y_{t+k} = \nu_0 x_{t+k} + \nu_1 x_{t+k-1} + \nu_2 x_{t+k-2} + \dots + \eta_{t+k}. \quad \dots\dots\dots(2.6)$$

Without loss of generality it is assumed that $\mu_x = 0$ and $\mu_y = 0$. Multiplying by x_t on both sides of the equation (2.6) and taking expectations, this result:

$$\gamma_{xy}(k) = \nu_0 \gamma_{xx}(k) + \nu_1 \gamma_{xx}(k-1) + \nu_2 \gamma_{xx}(k-2) + \dots \quad \dots\dots\dots(2.7)$$

where $\gamma_{xx}(k) = 0$ for all k . Hence,

$$\rho_{xy}(k) = \frac{\sigma_x}{\sigma_y} [v_0 \rho_x(k) + v_1 \rho_x(k-1) + v_2 \rho_x(k-2) + \dots] \quad \dots\dots\dots(2.8)$$

The relationship between the CCF, $\rho_{xy}(k)$ and the impulse response function v_j is evidently contaminated by the autocorrelation structure of the input series x_t .

If $r = 0$ in (2.3) the Transfer Function $v(B)$ contains only a finite number of impulse response weights. In that case of equation (2.8) to form a system of equations to solve for v_j as a function of $\rho_{xy}(k)$ and $\rho_x(k)$ is difficult.

If the input series is white noise, i.e., $\rho_x(k) = 0$ for $k \neq 0$, equation (2.8) reduces to

$$v_0 = \frac{\sigma_y}{\sigma_x} * \frac{\rho_{xy}^{(0)}}{\rho_x^{(0)}} \quad \dots\dots\dots(2.9)$$

Thus the impulse response function v_k is directly proportional to the cross-correlation function $\rho_{xy}(k)$.

Conditions imposed on the process in this study

- Unless mentioned otherwise, the process x_t and y_t are assumed to be jointly stationary in this study.

- In the general Transfer Function Model (2.2), it is assumed that the input series x_t follows an ARMA process

$$\phi_x(B)x_t = \theta_x(B)\alpha_t \text{ where } \alpha_t \text{ is white noise.}$$

The series α_t , given by

$$\alpha_t = \frac{\phi_x(B)}{\theta_x(B)} x_t \dots\dots\dots(2.10)$$

is a prewhitened input series.

Applying the same prewhitening transformation to the output series y_t , a filtered output series is obtained:

$$\beta_t = \frac{\phi_x(B)}{\theta_x(B)} y_t \dots\dots\dots(2.11)$$

Let $\varepsilon_t = \theta_x^{-1}(B)\phi_x(B)\eta_t$, where $\theta_x^{-1}(B)\phi_x(B)$ is a the linear filter.

Transforming the y_t series using the same filter generates the following series

$$\beta_t = \theta_x^{-1}(B)\phi_x(B)y_t \dots\dots\dots(2.12)$$

This implies that the output series becomes

$$y_t = v(B)\alpha_t + \eta_t \quad \dots\dots\dots(2.13)$$

The Transfer Function Model (2.2) becomes

$$\beta_t = v(B)\alpha_t + \eta_t^* \quad \dots\dots\dots(2.14)$$

where $\eta_t^* = \theta_x^{-1}(B)\phi_x(B)\eta_t$. This implies that the transfer function between X and Y is the same as that between α and β . Multiplying both sides of (2.14) by

α_{t-k} and taking expectations, the resulting equation becomes

$$E(\alpha_{t-k}\beta_t) = [\nu_0 E(\alpha_{t-k}\alpha_t) + \dots + \nu_1 E(\alpha_{t-k}^2) + \dots] + E(\alpha_{t-k}\eta_t^*)$$

which yields

$$\rho_{\alpha\beta}(k) = \nu_k \sigma_\alpha^2.$$

All other terms disappear from the equation, since $E(\alpha_{t-k}\alpha_{t-j}) = 0$ for $j \neq k$,

and $E(\alpha_{t-k}\eta_t^*) = 0$ for all k (since x_t and η_t are uncorrelated). Thus,

$$\nu_k = \frac{\rho_{\alpha\beta}(k)}{\sigma_\alpha^2} = \frac{\sigma_\beta}{\sigma_\alpha} \frac{\rho_{\alpha\beta}(k)}{\sigma_\alpha \sigma_\beta} = \frac{\sigma_\beta}{\sigma_\alpha} \gamma_{\alpha\beta}(k) \quad k=0,1,2,\dots \quad \dots\dots\dots(2.15)$$

where σ_β^2 is the variance of the β_t process. The coefficient ν_k is proportional to the cross-correlation $\rho_{\alpha\beta}(k)$ between α_t and β_t at lag k. Hence, the impulse response weights ν_k for the Transfer Function can therefore be found as

$$v_k = \rho_{\alpha\beta}(k) \frac{\sigma_\beta}{\sigma_\alpha} \dots\dots\dots(2.16)$$

where σ_β and σ_α denote the estimated standard deviations of the α_t and β_t series.

This leads to the fundamental steps of the identification of the Transfer Function Models to be discussed in the next section.

2.3 Construction of Transfer Function models

2.3.1 Sample cross-correlation function

For a given set of time series data x_t and y_t , $1 \leq t \leq n$, the cross-correlation function (2.5) is estimated by the following sample cross-correlation statistic:

$$\hat{\rho}_{xy}(k) = \frac{\gamma_{xy}(k)}{S_x S_y} \quad k = 0, \pm 1 \pm 2, \dots \dots\dots(2.14)$$

where

$$\hat{\gamma}_{xy}(k) = \begin{cases} \frac{1}{n} \sum_{t=1}^{n-k} (x_t - \bar{x})(y_{t+k} - \bar{y}), & k \geq 0, \\ \frac{1}{n} \sum_{t=1-k}^k (x_t - \bar{x})(y_{t+k} - \bar{y}), & k < 0, \end{cases} \dots\dots\dots(2.15)$$

$$S_x = \sqrt{\hat{\gamma}_{xx}(0)}, \dots\dots\dots(2.16)$$

$$S_y = \sqrt{\hat{\gamma}_{yy}(0)}, \dots\dots\dots(2.17)$$

and $\bar{x}$ and $\bar{y}$ are the sample means of the x_t and y_t observations, respectively.

To test whether certain values of the CCF $\rho_{xy}(k)$ are zero, the Sample Cross Correlation Function (SCCF) $\hat{\rho}_{xy}(k)$ is compared with their standard errors. Under the normal assumption, Bartlett (1955) derives the approximate variance and covariance between two sample cross-correlations $\hat{\rho}_{xy}(k)$ and $\hat{\rho}_{xy}(k+j)$. The covariance under the hypothesis that the two series x_t and y_t are uncorrelated and the x_t series is white noise is

$$Cov[\hat{\rho}_{xy}(k), \hat{\rho}_{xy}(k+j)] \approx (n-k)^{-1} \rho_{yy}(j). \quad \dots\dots\dots(2.18)$$

And the variance is

$$Var[\hat{\rho}_{xy}(k)] \approx (n-k)^{-1}. \quad \dots\dots\dots(2.19)$$

Thus, when the x_t series is white noise, the hypothesis that the two series x_t and y_t are not cross-correlated can be tested by comparing the SCCF, $\hat{\rho}_{xy}(k)$ with their approximate standard errors $1/\sqrt{(n-k)}$. In practice the x_t series is usually not white noise, and it has to be prewhitened and the output series also have to be filtered.

2.3.2 Identification of Transfer Function Models

Based on the above illustrations, the Transfer Function $\nu(B)$ is obtained from the following steps:

- Prewhiten the input series using (2.10)
- Calculate the filtered output series. That is, transform the output series y_t using the above prewhitening model to generate the (2.11).
- Calculate the sample CCF $\hat{\rho}_{\alpha\beta}(k)$ between α_t and β_t to estimate ν_k ,

$$\hat{\nu}_k = \hat{\rho}_{\alpha\beta}(k) \frac{\hat{\sigma}_\beta}{\hat{\sigma}_\alpha} \quad \dots\dots\dots(2.20)$$

The significance of the CCF can be tested by comparing it with its standard error $(n-k)^{-1/2}$.

- Identify $b, \quad \delta_r(B) = 1 - \delta_1 B - \dots - \delta_r B^r,$ and $\omega_s(B) = \omega_0 - \omega_1 B - \dots - \omega_s B^s.$

Thus the preliminary Transfer Function $\nu(B)$ is

$$\hat{\nu}(B) = \frac{\hat{\omega}_s(B)B^b}{\hat{\delta}_r(B)} \quad \dots\dots\dots(2.21)$$

2.3.3 The noise model identification

Once the preliminary Transfer Function is obtained, the noise model is calculated as

$$\hat{\eta}_t = y_t - \hat{\nu}(B)x_t = y_t - \frac{\hat{\omega}_s(B)}{\hat{\delta}_r(B)} B^b x_t. \quad (2.22)$$

The appropriate model for the noise can then be identified by observing the manner in which its autocorrelation function (ACF) and partial autocorrelation function (PACF) giving

$$\phi(B)\hat{\eta}_t = \theta(B). \quad (2.23)$$

2.4 Estimation of a Transfer Function Model

Combining the Transfer Function Model and the noise model leads to

$$y_t = \frac{\omega_s(B)}{\delta_r(B)} x_{t-b} + \frac{\theta_q(B)}{\phi_p(B)} \eta_t \quad (2.24)$$

The parameters $\beta' = (\omega', \delta', \phi', \theta') = (\omega_0, \omega_1, \dots, \omega_s, \delta_1, \dots, \delta_r; \phi_1, \dots, \phi_p; \theta_1, \dots, \theta_q)$ in (2.24) have to be estimated from past data $(x_t, y_t), t = 1, 2, \dots, n$.

2.5. Diagnostic checks

After estimating the model diagnostic checks are performed to determine its adequacy. Certain test statistics are compared with relevant critical values to check if the model is valid and parameters contribute significantly to the model. The fitted model can be inadequate if either (i) the noise model, or (ii) the Transfer Function, or (iii) both noise and Transfer Function Models are incorrect.

If the residuals $\hat{\eta}_t$ are uncorrelated, but the cross-correlations $\rho_{\alpha\eta}(k)$ ($k = 0, \pm 1, \pm 2, \dots$) are negligible compared to their standard errors $(n - k)^{-1/2}$, then the noise model needs to be modified to meet the requirements for Transfer Function Analysis (Abraham and Ledolter 1983). If, in addition, significant cross-correlations $\rho_{\alpha\alpha}(k)$ ($k = 0, \pm 1, \pm 2, \dots$) are observed, then the preliminary Transfer Function Model and possibly the noise model have to be changed by adjusting the parameters.

In addition, portmanteau tests formulated by Box-Pierce (1970) can be carried out to test whether the noise model is adequate. The statistic

$$Q_1 = m(m+2) \sum_{k=1}^K \frac{\rho_{\eta}^2(k)}{m-k} \dots\dots\dots (2.25)$$

can be used, where $\rho_i(k)$ ($k=1,2,\dots,K$) denotes the first K sample autocorrelations from m available residuals. Under the hypothesis of model adequacy, Q_1 follows approximately a χ^2 distribution with $K-p-q$ degrees of freedom. If the test statistics is greater compared to the critical value obtained from a chi-square distribution, the null hypothesis of no autocorrelation is rejected.

By referring this statistic to a table of χ^2 percentage points, a test of model adequacy is obtained. To test if the input and the residuals are correlated, the following statistic is used:

$$Q_1 = m(m+2) \sum_{k=0}^K \frac{\rho^2(k)}{m-k} \dots\dots\dots(2.26)$$

This statistic is compared to the percentage points from a χ^2 distribution with $K+1-m$ degrees of freedom, where $m=r+s+1$ which is the number of residuals calculated and K is the number of parameters δ_r and ω_s estimated in the Transfer Function model $v(B) = \frac{\omega_s(B)B^b}{\delta_r(B)}$. If the test statistic (2.26) exceeds the critical value, the null hypothesis of no autocorrelation is rejected and thus it is concluded that the noise model is adequate.

2.6 Forecasting

Once a Transfer Function Model is found to be adequate, it can be used to improve the forecast of the output series y_t by using the past history of both the output series y_t and the associated input series x_t . This is particularly true if the input series is a leading indicator.

Minimum mean square error forecasts

Suppose that Y_t and X_t are stationary process in the following stable Transfer Function Model:

$$y_t = \frac{\omega_s(B)}{\delta_r(B)} x_{t-b} + \frac{\theta_q(B)}{\phi_p(B)} a_t \quad \dots\dots\dots(2.27)$$

and

$$\phi_q(B)X_t = \theta_p(B)a_t \quad \dots\dots\dots(2.28)$$

where $\omega_s(B), \delta_r(B), \theta_p(B)$ and $\phi_q(B)$ are finite order of polynomials of B ;
the roots of $\omega_s(B)=0$, $\delta_r(B)=0$, $\theta_p(B)=0$ and $\phi_q(B)=0$ are all

outside the unit circle; and a_t and α_t are independent zero mean white noise series with variances σ_a^2 and σ_α^2 , respectively.

Let

$$u(B) = \frac{\omega_s(B)B^b\theta_p(B)}{\delta_r(B)\phi_q(B)} = u_0 + u_1B + u_2B + \dots \quad (2.29)$$

and

$$\psi(B) = \frac{\theta_p(B)}{\phi_q(B)} = 1 + \psi_0 + \psi_1B + \psi_2B + \dots \quad (2.30)$$

Equation (2.31) can be written as

$$\begin{aligned} Y_t &= u(B)\alpha_t + \psi(B)\eta_t \\ &= \sum_{j=0}^{\infty} u_j\alpha_{t-j} + \sum_{j=0}^{\infty} \psi_j\eta_{t-j} \end{aligned} \quad (2.31)$$

where $\psi_0 = 1$. Thus $Y_{t+l} = \sum_{j=0}^{\infty} u_j\alpha_{t+l-j} + \sum_{j=0}^{\infty} \psi_j\eta_{t+l-j}$. Let

$\hat{Y}_t(l) = \sum_{j=0}^{\infty} u_j^* \alpha_{t+l-j} + \sum_{j=0}^{\infty} \psi_j^* \eta_{t+l-j}$ be the l -step-ahead optimal forecast of Y_{t+l} .

The forecast error becomes

$$\begin{aligned} Y_{t+l} - \hat{Y}_t(l) &= \sum_{j=0}^{l-1} [u_j\alpha_{t+l-j} + \psi_j\eta_{t+l-j}] \\ &\quad - \sum_{j=0}^{l-1} [u_j^*\alpha_{t+l-j} + \psi_j^*\eta_{t+l-j}] \end{aligned} \quad (2.32)$$

The mean square of the forecast error $E[Y_{t+1} - \hat{Y}_t(l)]^2$ obtained by Wei (1990) is

$$E[Y_{t+1} - \hat{Y}_t(l)]^2 = \sum_{j=0}^{l-1} (\sigma_a^2 u_j^2 + \sigma_\eta^2 \psi_j^2) + \sum_{j=0}^{\infty} \sigma_a^2 (u_{l+j}^* - \psi_{l+j})^2 + \sum_{j=0}^{\infty} \sigma_\eta^2 (\psi_{l+j}^* - \psi_{l+j})^2 \quad \dots\dots\dots (2.33)$$

which is minimised when $u_{l+j}^* = u_{l+j}$ and $\psi_{l+j}^* = \psi_{l+j}$. In other words, the minimum mean square error forecasts $\hat{Y}_t(l)$ of Y_{t+1} at time origin t is given by the conditional expectation of Y_{t+1} at time t . Because $E(Y_{t+1} - \hat{Y}_t(l)) = 0$, the forecast is unbiased and the variance of the forecast found by Wei (1990) is

$$V(l) = E[Y_{t+1} - \hat{Y}_t(l)]^2 = \sigma_a^2 \sum_{j=0}^{\infty} u_j^2 + \sigma_\eta^2 \sum_{j=0}^{\infty} \psi_j^2 \quad \dots\dots\dots (2.34)$$

2.7. Stationarity test

2.7.1. Introduction

This section deals with some of the tests for stationarity. Many time series data are non-stationary in nature. This means that the mean, variance and autocovariance depend on time t . A time series $\{X_t : t = 1, 2, 3, \dots\}$ is stationary if its mean, variance and outocovariance are independent of time. In Box-Jenkins

setting, Box and Jenkins (1976), if the mean of the series is less than its corresponding standard deviation, it is representable as

$$X_t = \sum_{i=1}^p \phi_i X_{t-i} + \varepsilon_t - \sum_{j=1}^q \theta_j \varepsilon_{t-j}, \quad \dots\dots\dots(2.7.1a)$$

where $\{\phi_i : i = 1, 2, \dots, p\}$ are the autoregressive parameters of order p , and $\{\theta_j : j = 1, 2, \dots, q\}$ are the moving average parameters of order q . Equation (2.7.1a) can further be adjusted to be

$$X_t = c + \sum_{i=1}^p \phi_i X_{t-i} + \varepsilon_t - \sum_{j=1}^q \theta_j \varepsilon_{t-j}, \quad \dots\dots\dots(2.7.1b)$$

where c is a constant term.

If the series has a polynomial trend, further adjustments made to (2.7.1b) yield

$$X_t = \sum_{h=0}^l \alpha_h t^h + \sum_{i=1}^p \phi_i X_{t-i} + \varepsilon_t - \sum_{j=1}^q \theta_j \varepsilon_{t-j}, \quad \dots\dots\dots(2.7.1c)$$

In the above three equations, ε_t follows a white noise process with zero mean and σ^2 , i.e.

$$\varepsilon_t \sim i.i.d.N(0, \sigma^2). \quad \dots\dots\dots(2.7.1d)$$

X_t is non-stationary in levels, but the differenced series

$$\Delta X_t = X_t - X_{t-1} \quad \dots\dots\dots(2.7.1e)$$

is stationary, and thus X_t is said to contain a unit root . Suppose $q=1$, then
AR(1) process

$$X_t = \phi_1 X_{t-1} + \varepsilon_t \quad \dots\dots\dots(2.7.1f)$$

If $|\phi_1| < 1$, stationarity condition is satisfied, and thus

$$X_t = \phi_1 X_{t-1} + \varepsilon_t$$

Inserting a backshift operator B, the new equation is

$$(1 - \phi_1 B)X_{t-1} = \varepsilon_t$$

$$X_t = \varepsilon_t + \phi_1 \varepsilon_{t-1} \quad \dots\dots\dots(2.7.1g)$$

It follows that

$$E(X_t) = E(\mu^{(t)}) + E(\varepsilon_t) = \mu_t = 0 \quad \dots\dots\dots(2.7.1h)$$

$$\begin{aligned} \text{var}(X_t) &= E(X_t - \mu)^2 \\ &= E(\varepsilon_t + \phi \varepsilon_{t-1} + \phi^2 \varepsilon_{t-2} + \phi^3 \varepsilon_{t-3} + \dots)^2 \\ &= (1 + \phi^2 + \phi^4 + \phi^6 + \dots) \cdot \sigma^2 \quad \dots\dots\dots(2.7.1i) \\ &= \frac{\sigma^2}{1 - \phi^2} \end{aligned}$$

$$\begin{aligned}
\text{cov}(X_t, X_{t-k}) &= E(X_t - \mu)(X_{t-k} - \mu) \\
&= E[\varepsilon_t + \phi\varepsilon_{t-1} + \phi^2\varepsilon_{t-2} + \dots + \phi^j\varepsilon_{t-j} + \phi^{j+1}\varepsilon_{t-j-1} \\
&\quad + \phi^{j+2}\varepsilon_{t-j-2} + \dots] * [\varepsilon_{t-j} + \phi\varepsilon_{t-j-1} + \phi^2\varepsilon_{t-j-2} + \dots] \\
&= [\phi^j + \phi^{j+2} + \phi^{j+4} + \dots]\sigma^2 \dots\dots\dots(2.7.1j) \\
&= \phi^j[1 + \phi^2 + \phi^4 + \dots]\sigma^2 \\
&= \frac{\phi^{2j}}{1 - \phi^2}\sigma^2, \quad k = 1, 2, \dots,
\end{aligned}$$

X_t have a unit root if $|\phi| < 1$.

In the above case (2.7.1f) becomes

$$X_t = X_{t-1} + \varepsilon_t = X_0 + \varepsilon_t + \varepsilon_{t-1} + \varepsilon_{t-2} + \dots \dots\dots(2.7.1k)$$

The study relied heavily on a statistical methodology called Box-Jenkins Transfer Function Analysis. However, a common norm of testing for a unit root in a macroeconomic time series allows, as a starting point, to investigate the presence of unit root in each of the time series variables (Pindyk and Rubinfeld 1991). The unit root test employed by Dickey-Fuller (1979) and Phillips-Perron (1988) are engaged. In this section, the two unit root test procedures are presented and their applications are presented in Chapter 3. Section 2.7.2 presents the Augmented Dickey-Fuller (ADF) test while Section 2.7.3 presents the Phillips-Perron (PP) test.

2.7.2 Augmented Dickey-Fuller (ADF) test

Assuming that the series $\{y_t\}_{t=1}^T$ follows the AR(p) process, Hamilton (1994) shows that the rejection or acceptance of the null hypothesis of a unit root, $H_0 : \phi_1 = 1$ is based on running the regression

$$z_t = \mu + (\phi_1 - 1)y_{t-1} + \sum_{j=1}^{p-1} c_j z_{t-j} + \varepsilon_t \quad (2.7.2a)$$

where $z_{t-j} = y_{t-j} - y_{t-j-1}$ for $j = 0, 1, 2, \dots, p-1$ and ε_t is a white noise process.

$H_0 : \phi_1 = 1$ is rejected if the test statistic

$$\tau = \frac{\hat{\phi}_1 - 1}{se(\hat{\phi}_1 - 1)} \quad (2.7.2b)$$

is less than an appropriate critical value of t for the Dickey-Fuller Test. $se(\hat{\phi}_1 - 1)$

is the standard error of $(\hat{\phi}_1 - 1)$. Alternatively, the null hypothesis is rejected if the prob-value of the test statistic is less than some conventional level of significance, say 0.05.

2.7.3 Phillips-Perron (PP) test

Phillips (1989) developed a more appropriate unit root test regression equation

$$z_t = a_0 + a_1 t + a_2 D_t + (\phi_1 - 1)y_{t-1} + \sum_{j=1}^{p-1} c_j z_{t-j} + \varepsilon_t \dots\dots\dots(2.7.3a)$$

where D_t ($t=1,2$) dummy variable defined below. In this case, the null and alternative hypotheses to consider are

$$H_0 : \phi_1 = 1 \quad \text{and} \quad a_1 = 0 \quad \dots\dots\dots(2.7.3b)$$

$$\text{where } D_1 = \begin{cases} 0, & t < t_0 \\ 1, & t \geq t_0 \end{cases},$$

$$H_1 : \phi_1 \neq 1 \quad \text{and} \quad a_1 \neq a_3 \quad \dots\dots\dots(2.7.3c)$$

$$\text{where } D_2 = \begin{cases} 1, & t = t_0 + 1 \\ 0, & \text{otherwise.} \end{cases}$$

Under $H_0 : \phi_1 = 1$ and $a_1 = 0$ is a unit root process with a one-time jump in the level of the sequence period $t = t_0 + 1$, $H_1 : \phi_1 \neq 1$ and $a_1 \neq 0$.

is stationary in the intercept.

The *t*-statistic for the formulated null hypothesis $H_0 : \phi_1 = 1$ is then compared to the appropriate critical value. At some conventional level α , H_0 is rejected in favour of the alternative if the *t*-statistic is less than the critical value.

2.8 Summary

Some literature based on the Box-Jenkins Transfer Function Analysis was presented in this chapter. It was stated that for a Box-Jenkins Transfer Function Model to be valid, an input series should influence an output series but not vice versa. The discussion also states that the series should be stationary. In Chapter 3, unit root tests are performed after the series is differenced. ADF and PP tests are used to test for the presence of unit root. A preliminary Transfer Function Model, a noise model and a final Transfer Function Model are built using the statistical procedures discussed in Section 2.2. Forecasts are developed from the derived Transfer Function Model.

CHAPTER 3

RESULTS

3.1. Introduction

A step in analysing time series data may be to plot the observations against time. The resulting plot is hereafter called the time plot. The time plot usually reveals important features such as trend, seasonalities, discontinuities and outliers. It may also indicate whether or not it is necessary to transform the values of the variables. If variational components exist in the time series, the next step would be to remove them by using different transformations such as cosine, exponentials, antilogs, and so on, leaving a series of residuals that may or may not be random (Bowerman and O'Connell 1993).

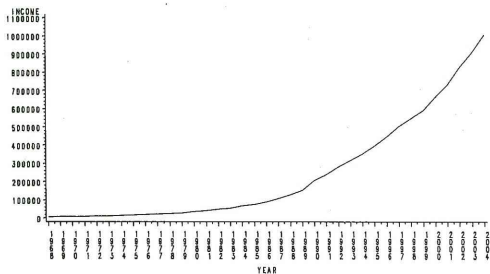
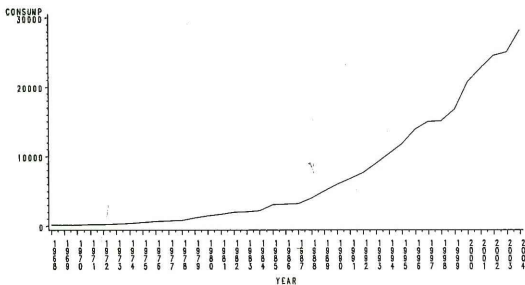
In the discussions that follow, Section 3.2 shows the univariate analysis for each variable and in Section 3.3 the Box-Jenkins Transfer Function analysis is employed for the two variables combined.

3.2 Empirical Results and discussions

3.2.1 Univariate analysis of each variable

Exhibits 3.2.1 (a) and (b) are plots of petroleum consumption and disposable income data respectively.

Original time plots



Variance-stabilising transformations have been applied to the data since the original data reveal curvilinear plots. This transformed data will therefore be referred to as the log-transformed series in the following analysis unless

mentioned otherwise. The following exhibits 3.2.1(c) and (d) are the nonstationary plots of log transformed series.

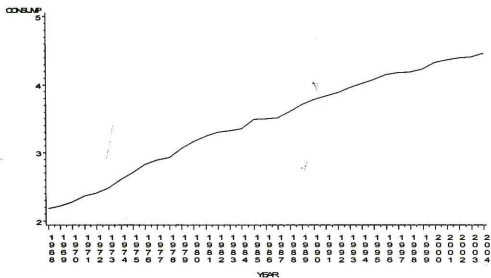


Exhibit 3.2.1(c) Plot of consumption logarithmic series

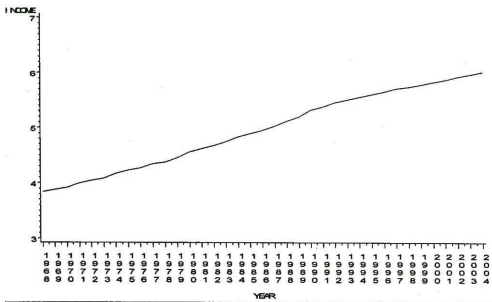


Exhibit 3.2.1(d) Plot of income logarithmic series

Plots of sample autocorrelation function (ACF) and partial autocorrelation function (PACF) both for consumption expenditure and disposable income are shown respectively in exhibit 3.2.1 (e) and (f).

Name of variable = CONSUMP.
Mean of working series = 3.440064
Standard deviation = 0.701511
Number of observations = 37

Autocorrelations																								
Lag	Covariance	Correlation	-1	9	8	7	6	5	4	3	2	1	0	1	2	3	4	5	6	7	8	9	1	Std
0	0.492117	1.00000																						0
1	0.454473	0.92350																						0.164399
2	0.415454	0.84422																						0.270421
3	0.375068	0.76215																						0.334144
4	0.334916	0.68056																						0.378221
5	0.293441	0.59628																						0.409984
6	0.253196	0.51450																						0.432788
7	0.214795	0.43647																						0.449015
8	0.177044	0.35976																						0.460339
9	0.141161	0.28684																						0.467876

"." marks two standard errors

"." marks two standard errors

Partial Autocorrelations

Lag	Correlation	-1	9	8	7	6	5	4	3	2	1	0	1	2	3	4	5	6	7	8	9	1
1	0.92350																					
2	-0.05874																					
3	-0.06163																					
4	-0.04272																					
5	-0.06680																					
6	-0.03519																					
7	-0.02806																					
8	-0.04671																					
9	-0.03092																					

Exhibit 3.2 .1(e) ACF and PACF plots of Petroleum Consumption

The ACF shows a nonstationary series, as the lags are dying down slowly with no oscillation. The plot of PACF have autocorrelations at lag 1 in the unit root.

The mean and standard deviation of petroleum consumption are 3.440 and 0.702 respectively. Since the mean is greater than the standard deviation, and the ACF and PACF have autocorrelations at lag 1, the three competing models AR (1), MA (1) and ARMA (1,1) unit root autoregressions with constants included in each model are considered. $X_t = c + \phi_1 X_{t-1} + \varepsilon_t$, $X_t = c - \theta_1 X_{t-1} + \varepsilon_t$ and $X_t = c + \phi_1 X_{t-1} + \varepsilon_t - \theta_1 y_{t-1}$, where $\varepsilon_t \sim i.i.d.N(0, \sigma_\varepsilon^2)$.

The series is then checked for white noise, which is an approximate statistical test for the hypothesis that none of the autocorrelations of the series up to a given lag are significantly different from zero. The autocorrelations are checked in groups of six lags. Exhibits 3.2.1(f) give the results for autocorrelation checks for white noise.

Autocorrelation Check for White Noise (consump)

To	Chi	Autocorrelations					
Lag	Square	DF	Prob				
6	136.83	6	0.000	0.924	0.844	0.762	0.681 0.596 0.515

Exhibit 3.2.1(f) Autocorrelation Check for White Noise

In this case, the white noise hypothesis is rejected very strongly because the prob-value is extremely small, which is expected since the series is nonstationary. The prob-value for the test of the first six autocorrelations is

printed as 0.000 for petroleum consumption. These prob-value is less than the significance level of 5%.

ACF and PACF plots of Disposable Income

Name of variable = INCOME.
Mean of working series = 4.954903
Standard deviation = 0.689469
Number of observations = 37

Autocorrelations

Lag	Covariance	Correlation	-1	9	8	7	6	5	4	3	2	1	0	1	2	3	4	5	6	7	8	9	1	Std
0	0.475368	1.00000												*****										0
1	0.441873	0.92954												*****										0.164399
2	0.407045	0.85627												*****										0.271537
3	0.371231	0.78093												*****										0.336697
4	0.335716	0.70622												*****										0.382531
5	0.299564	0.63017												*****										0.416281
6	0.262815	0.55287												*****										0.441311
7	0.226275	0.47600												*****										0.459650
8	0.189535	0.39871												*****										0.472784
9	0.152530	0.32087												*****										0.481786

"," marks two standard errors

Partial Autocorrelations

Lag	Correlation	-1	9	8	7	6	5	4	3	2	1	0	1	2	3	4	5	6	7	8	9	1
1	0.92954												*****									
2	-0.05715										*											
3	-0.05439										*											
4	-0.03711										*											
5	-0.05370										*											
6	-0.05579										*											
7	-0.04593										*											
8	-0.05524										*											
9	-0.05945										*											

Exhibit 3.2.1(g) ACF and PACF for petroleum consumption and disposable income of the series

Like in Petroleum Consumption, the ACF shows a nonstationary series as the lags are dying down slowly with no oscillation. The plots of PACF in the above exhibits have autocorrelations at lag 1 in the unit root. The mean and standard deviations are 4.954 and 0.689 respectively. Since the mean is greater than the standard deviation, and the ACF and PACF have autocorrelation at lag 1, the three competing models AR(1), MA(1) and ARMA(1,1) unit root autoregressions with constants included in each model are considered. $X_t = c + \phi_1 X_{t-1} + \varepsilon_t$, $X_t = c + \theta_1 X_{t-1} + \varepsilon_t$ and $X_t = c + \phi_1 x_{t-1} + \varepsilon_t + \theta_1 y_{t-1}$, where $\varepsilon_t \sim i.i.d.N(0, \sigma_\varepsilon^2)$.

The series is then checked for white noise, to see if the autocorrelations of the series up to a given lag are significantly different from zero or not. The autocorrelations are again checked in groups of 6 lags. Exhibits 3.2.1(h) give the results for autocorrelation checks for white noise.

Autocorrelation Check for White Noise (income)

To	Chi	Autocorrelations							
Lag	Square	DF	Prob						
6	144.69	6	0.000	0.930	0.856	0.781	0.706	0.630	0.553

Exhibit 3.2.1(h) Autocorrelation Check for White Noise


```
Lag Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
1 0.15400 | . . . . . . . . . . . . . . . . . . . .
2 -.25426 | . . . . . . . . . . . . . . . . . . . .
3 0.24690 | . . . . . . . . . . . . . . . . . . . .
4 -.00943 | . . . . . . . . . . . . . . . . . . . .
5 0.34332 | . . . . . . . . . . . . . . . . . . . .
6 0.01156 | . . . . . . . . . . . . . . . . . . . .
7 -.24835 | . . . . . . . . . . . . . . . . . . . .
8 0.02853 | . . . . . . . . . . . . . . . . . . . .
9 -.21046 | . . . . . . . . . . . . . . . . . . . .
```

Both the SAC (sample autocorrelation) and SPAC (sample partial autocorrelation) function of these two series die down quickly with oscillation, implying that the two series are now stationary. The study is led to tentatively consider AR (1), MA (1) and ARMA (1,1) as the three competing models using the guidelines of nonseasonal operator of order 1.

Autocorrelation Check for White Noise (consump.)

To	Chi				Autocorrelations				
Lag	Square	DF	Prob						
6	7.78	6	0.255	0.154	-0.225	0.143	0.134	0.228	0.141

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Differenced Disposable Income series

Name of variable = INCOME.
 Period(s) of Differencing = 1.
 Mean of working series = 0.059985
 Standard deviation = 0.02047
 Number of observations = 36

NOTE: The first observation was eliminated by differencing.

Sample Autocorrelations													
Lag	Covariance	Correlation	-1	9	8	7	6	5	4	3	2	1	Std
0	0.00041902	1.00000											0
1	0.0000997	0.23794										*****	0.166667
2	0.00009308	0.22213										****	0.175850
3	0.00014084	0.33612										*****	0.183479
4	0.00003763	0.06980										**	0.199852
5	0.00004627	0.11043										**	0.200970
6	0.00004233	0.10103										**	0.202649
7	-6.1721E-6	-0.01473										.	0.204043
8	-3.7404E-6	-0.00893										.	0.204072
9	-0.0000228	-0.05450									*	.	0.204083

Partial Autocorrelations

Lag	Correlation	-1	9	8	7	6	5	4	3	2	1	0	1	2	3	4	5	6	7	8	9	1
1	0.23794												*****	.								
2	0.17545												****	.								
3	0.27428												*****	.								
4	-0.06180											*	.									
5	0.00469											.	.									
6	-0.01305											.	.									
7	-0.05835										.	*	.									
8	-0.04386										.	*	.									
9	-0.06694										.	*	.									

Exhibit 3.2.2 (c) Differenced Disposable Income series

Autocorrelation Check for White Noise (income)

To	Chi	Autocorrelations									
Lag	Square	DF	Prob								
6	10.23	6	0.115	0.236	0.222	0.336	0.090	0.110	0.101		

Exhibit 3.2.2(d) Autocorrelation Check for White Noise

Both the SAC and SPAC in exhibit 3.2.2(c) die down quickly with exponential decay, implying that the two series are stationary. The three models considered AR(1), MA(1) and ARMA(1,1), using the guidelines of nonseasonal operator.

In exhibit 3.2.2(d) above, the prob-value is printed as 0.115. To check if the series is white noise or not, this prob-value is compared with a significance level of 5%. The null hypothesis of no white noise is accepted since the prob-value is greater than the level of significance, and therefore it can be concluded that the series follows a white noise process.

3.2.3 Unit root test

ADF and PP unit root tests for each series are then performed. The hypothesis to test is $H_0 : \phi_1 = 1$ and is rejected if the test statistic is less than the critical value or if the prob-value associated with the test statistic is less than some conventional level of significance. The following are the results for PP and ADF unit root test for petroleum consumption and disposable income respectively.

		ADF	PP
Consumption	T	8.4195	8.4195
	Prob<T	0.9999	0.9999
Income	T	15.0024	15.0024
	Prob<T	0.9999	0.9999

Exhibit 3.2.3 Unit root results

The prob-values are 0.9999 both for petroleum consumption and disposable income. These prob values are greater than 0.05 probability. In conclusion, the null hypothesis of no unit root is accepted. Therefore, the series does not have a unit root.

3.2.4 Estimation and diagnostic checking

The three competing models are estimated and each model is checked for adequacy. Exhibit 3.2.4(a) and Exhibit 3.2.4(b) show the results of AR(1), MA(1) and ARMA(1,1) for an output series (Petroleum Consumption) and an input series (Disposable Income) respectively.

Petroleum consumption

AR (1) process

Conditional Least Squares Estimation

Parameter	Estimate	Approx. Std Error	T Ratio	Lag
MU	0.06272	0.0071114	8.82	0
AR1,1	0.15458	0.16966	0.91	1

Constant Estimate = 0:05302156

Variance Estimate = 0.00131587

Std Error Estimate = 0.03627493

AIC = -134.69138*

SBC = -131.52434*

Number of Residuals= 36

* Does not include log determinant.

Correlations of the Estimates

Parameter	MU	AR1,1
MU	1.000	-0.014
AR1,1	-0.014	1.000

Autocorrelation Check of Residuals

To Lag	Chi Square	DF	Prob	Autocorrelations									
6	7.46	5	0.189	0.039	-0.280	0.165	0.084	0.195	0.157				
12	14.43	11	0.210	-0.308	-0.011	0.099	-0.124	0.140	-0.042				
18	23.46	17	0.135	-0.247	0.184	0.162	-0.091	0.048	-0.106				
24	30.17	23	0.145	-0.043	0.075	0.002	-0.143	-0.145	-0.128				

MA (1) process

Conditional Least Squares Estimation

Parameter	Estimate	Approx. Std Error	T Ratio	Lag
MU	0.06269	0.0083488	7.51	0
MA1,1	-0.42268	0.15729	-2.69	1

Constant Estimate = 0.06268636

Variance Estimate = 0.00126832

Std Error Estimate = 0.03561352

AIC = -136.01629*

SBC = -132.84925*

Number of Residuals= 36

* Does not include log determinant.

Correlations of the Estimates

Parameter	MU	MA1,1
MU	1.000	-0.008
MA1,1	-0.008	1.000

Autocorrelation Check of Residuals

To Lag	Chi Square	DF	Prob	Autocorrelations					
6	7.48	5	0.187	-0.137	-0.234	0.213	-0.006	0.139	0.200
12	17.44	11	0.095	-0.343	0.039	0.122	-0.189	0.175	-0.019
18	28.88	17	0.036	-0.262	0.195	0.136	-0.141	0.122	-0.134
24	33.21	23	0.077	-0.027	0.073	0.003	-0.128	-0.080	-0.11

ARMA (1,1) process

Conditional Least Squares Estimation

Parameter	Estimate	Approx. Std Error	T Ratio	Lag
MU	0.06316	0.0071670	8.81	0
MA1,1	-0.78468	0.22518	-3.48	1
AR1,1	-0.43752	0.31289	-1.40	1

Constant Estimate = 0.09079693

Variance Estimate = 0.00121325

Std Error Estimate = 0.03483177

AIC = -136.68907*

SBC = -131.93852*

Number of Residuals= 36

Does not include log determinant.

Correlations of the Estimates

Parameter	MU	MA1,1	AR1,1
MU	1.000	-0.009	0.002
MA1,1	-0.009	1.000	0.861
AR1,1	0.002	0.861	1.000

Autocorrelation Check of Residuals

To	Chi	Autocorrelations							
Lag	Square	DF	Prob						
6	3.93	4	0.415	-0.062	-0.124	0.089	0.114	0.140	0.176
12	10.72	10	0.380	-0.309	0.006	0.080	-0.124	0.134	-0.051
18	19.60	16	0.239	-0.194	0.137	0.172	-0.112	0.103	-0.159
24	25.62	22	0.268	-0.008	0.051	-0.004	-0.155	-0.115	-0.135

Exhibit 3.2.4(a) Estimation and diagnostic checking of the three models (consumption expenditure)

The Conditional Least Squares Estimation tables reveal the parameter estimates for each of the three models. These estimates satisfy both the stationarity and invertibility conditions, because for ARMA (1,1) the stationarity condition is $|\phi_1| < 1$ and the invertibility condition is $|\theta_1| < 1$. Hence, the estimates are all less than 1. Stationarity conditions for AR (1) and MA (1) are also met, because the parameter estimates for these models are less than 1.

The hypotheses $H_0 : \theta = 0$ and $\phi = 0$ versus $H_1 : \theta \neq 0$ and $\phi \neq 0$ is tested. The null hypothesis is rejected in favour of the alternative by setting the probability of type one error equal to α , if the following equivalent conditions hold;

$$(1) |t| > t_{[n-np]}^{(\alpha/2)}$$

$$(2) \text{prob-value} < \alpha$$

From the output the absolute t-statistics for MA(1) is $\theta = 2.69$, AR(1) is $\phi = 0.91$ and for ARMA(1,1) is $\theta_1 = 3.48$ and $\phi_1 = 1.40$ in exhibit 3.2.4(a). The parameters for AR(1), $\phi = 0.91$ and ARMA(1,1), $\phi_1 = 1.40$ not significant when compared with 1.96 at the significance level of 5%. This accepts the null hypothesis and it is concluded that the parameters are not significant in the model. The t-statistics for MA(1) $\theta = 2.69$ and ARMA(1,1) $\theta_1 = 3.48$ together with the means 8.82, 7.51 and 8.81 for AR(1), MA(1) and ARMA(1,1) respectively, are significant in the model.

To check the correlation of residuals, reference is made to the table of autocorrelation check for residuals in exhibit 3.2.4(a). The prob-values of the three models show that the residuals are uncorrelated at lags when compared with the significance level of 5% except for residuals of MA(1) at lag 18. The values of standard errors and AIC or SBC help to choose the best model. The smaller the standard error, AIC or SBC, the best the model. Since the standard error (0.0348) and AIC (-136.689) of ARMA (1,1) are less than those of AR (1)

and MA (1), ARMA (1,1), $(1 - \phi_1^{(x)}B)z_t^{(x)} = (1 - \theta_1^{(x)}B)a_t^{(x)}$ where $z_t^{(x)} = x_t - x_{t-1}$

is chosen as the best model.

Disposable Income

AR (1) process

Conditional Least Squares Estimation

Parameter	Estimate	Std Error	T Ratio	Lag
MU	0.05953	0.0044633	13.34	0
AR1,1	0.24371	0.16800	1.45	1

Constant Estimate = 0.04502454

Variance Estimate = 0.00041797

Std Error Estimate = 0.02044422

AIC = -175.97811*

SBC = -172.81107*

Number of Residuals= 36

- Does not include log determinant.

Correlations of the Estimates

Parameter	MU	AR1,1
MU	1.000	-0.044
AR1,1	-0.044	1.000

Autocorrelation Check of Residuals

To Lag	Chi Square	DF	Prob	Autocorrelations						
6	4.94	5	0.423	-0.044	0.108	0.299	-0.008	0.078	0.096	
12	9.42	11	0.583	-0.031	0.013	-0.052	0.071	-0.188	-0.192	
18	19.68	17	0.291	0.071	-0.335	-0.098	0.041	-0.142	-0.101	
24	22.78	23	0.473	-0.093	-0.076	-0.117	-0.009	-0.003	-0.076	

MA (1) process

Conditional Least Squares Estimation

Parameter	Estimate	Approx. Std Error	T Ratio	Lag
MU	0.05976	0.0040904	14.61	0
MA1,1	-0.19819	0.16866	-1.18	1

Constant Estimate = 0.05975843

Variance Estimate = 0.00042376
 Std Error Estimate = 0.02058538
 AIC = -175.48268*
 SBC = -172.31564*
 Number of Residuals= 36

* Does not include log determinant.

Correlations of the Estimates

Parameter	MU	MA1,1
MU	1.000	0.017
MA1,1	0.017	1.000

Autocorrelation Check of Residuals

To Lag	Chi Square	DF	Prob	Autocorrelations								
6	5.81	5	0.325	0.021	0.166	0.306	0.019	0.093	0.096			
12	10.86	11	0.455	-0.029	0.012	-0.062	0.046	-0.196	-0.213			
18	21.75	17	0.195	0.030	-0.345	-0.112	0.010	-0.153	-0.112			
24	25.47	23	0.327	-0.112	-0.092	-0.124	-0.019	-0.008	-0.070			

ARMA (1,1) process

Conditional Least Squares Estimation

Parameter	Estimate	Approx. Std Error	T Ratio	Lag
MU	0.05651	0.0080086	7.06	0
MA1,1	0.63513	0.30489	2.08	1
AR1,1	0.85723	0.21670	3.96	1

Constant Estimate = 0.00806864

Variance Estimate = 0.00040147
 Std Error Estimate = 0.02003666
 AIC = -176.50263*
 SBC = -171.75208*
 Number of Residuals= 36

* Does not include log determinant.

Autocorrelation Check of Residuals									
Lag	Chi Square			Autocorrelations					
	To	Chi	DF	Prob					
6	2.42	4	0.659	-0.065	-0.010	0.207	-0.078	0.017	0.066
12	5.65	10	0.844	-0.036	0.024	-0.002	0.127	-0.148	-0.139
18	13.36	16	0.646	0.125	-0.275	-0.053	0.114	-0.085	-0.080
24	15.53	22	0.838	-0.071	-0.056	-0.100	-0.001	0.003	-0.071

Exhibit 3.2.4(b) Estimation and diagnostic checking for the three models (disposable income)

The parameters of the ARMA (1,1) model contribute significantly to the model at 5% significance level, with a critical value of 1.96. The t-values are greater than the critical value. For AR (1) and MA (1), the t-statistics are less than 1.96 making the parameters to be insignificant. The residuals for the three models are not correlated with one another as the prob-values are greater than a conventional level of 5%. In this study the choice of the best model also depend on the values of standard errors, AIC and SBC. ARMA(1,1) is the best model because it has less standard error and AIC values.

3.3 Transfer Function Analysis

Box and Jenkins (1979) suggest that once an appropriate model describing x_t has been obtained, it has to be used to prewhiten the values of x_t and y_t . This prewhitening is a preliminary step to determining the relationship between the x_t and y_t .

The model

$$(1 - \phi_1^{(x)} B) z_t^{(x)} = (1 - \theta_1^{(x)} B) a_t^{(x)} \text{ where } z_t^{(x)} = x_t - x_{t-1}$$

gives the equation that calculates the prewhitened series as

$$a_t^{(x)} = c + \frac{(1 - \phi_1^{(x)} B)}{(1 - \theta_1^{(x)} B)} z_t^{(x)},$$

The respective least squares point estimates of c , $\phi_1^{(x)}$ and $\theta_1^{(x)}$ are $\hat{c} = 0.057$,

$\hat{\phi}_1^{(x)} = 0.857$ and $\hat{\theta}_1^{(x)} = 0.635$, from ARMA(1,1) exhibit 3.2.4(b) and the

prewhitened x_t values are calculated by the equation

$$\begin{aligned} \alpha_t &= c + \frac{(1 - \phi_1^{(x)} B)}{(1 - \theta_1^{(x)} B)} z_t^{(x)} \\ &= 0.057 + \frac{(1 - 0.857 B)}{(1 - 0.635 B)} z_t^{(x)} \end{aligned}$$

The prewhitened y_t values are calculated by the equation

$$\begin{aligned} \beta_t &= c + \frac{(1 - \phi_1^{(x)} B)}{(1 - \theta_1^{(x)} B)} z_t^{(x)} \\ &= 0.057 + \frac{(1 - 0.857 B)}{(1 - 0.635 B)} z_t^{(x)} \end{aligned}$$

where α_t represents a prewhitened input series and β_t a prewhitened output series.

3.3.1. The SCC function between the prewhitened values

To identify a preliminary Transfer Function Model describing the relationship between disposable income and consumption expenditure, this study used the sample cross-correlation (SCC) function between the α_t values and the β_t values. In Exhibit 3.3.1 below, the study presents the SCC between the prewhitened values of disposable income and the prewhitened values of petroleum consumption expenditure. One aspect noticed is that there are no spikes in the SCC at all lags less than zero, a prerequisite for using Transfer Function Models. Therefore the sample cross-correlation function $r_{-1}(\alpha_t, \beta_t)$ is not statistically different from zero, meaning the values of α_t are not related to the values of β_{t-1} . Since the present values of disposable income are not related to past values of consumption expenditure, the Transfer Function Model would then use be used.

This means that these models assume that present values of x_t are not related to past values of y_t . Exhibit 3.3.1 shows that there is no spike in the SCC at lag 0 and, therefore, implies that $r_0(\alpha_t, \beta_t)$ is not statistically different from zero. This means that the present values of consumption expenditure are not related to present values of disposable income in the same year. Similarly, the fact that there is no spike in the SCC at lag 1 implies that $r_1(\alpha_t, \beta_t)$ is not statistically

different from zero, meaning that consumption expenditures in the present year are not related to disposable income one year ago. However, since there is a spike in the SCC at lag 7, it follows that $r_7(\alpha_t, \beta_t)$ is statistically different from zero, which says that consumption expenditures in the present year are related to disposable income seven years ago. To summarise, since the first spike in the SCC, occurs a lag 7, it is concluded that it takes seven years for disposable income to affect consumption expenditure.

Correlation of CONSUMP and INCOME
Variable INCOME has been differenced.
Period(s) of Differencing = 1.
Both series have been prewhitened.
Variance of transformed series = 0.001309 and 0.00037
Number of observations = 36
NOTE: The first observation was eliminated by differencing.

			Crosscorrelations																							
Lag	Covariance	Correlation	-1	9	8	7	6	5	4	3	2	1	0	1	2	3	4	5	6	7	8	9	1			
-12	7.34312E-6	0.01055																								
-11	0.00009646	0.13862														***										
-10	0.00001036	0.01488																								
-9	0.00005491	0.07891														**										
-8	0.00004975	0.07149														*										
-7	-0.0000437	-0.06273														*										
-6	-0.0000244	-0.03502														*										
-5	0.00012163	0.17479																***								
-4	-0.0000729	-0.10483														**										
-3	-0.000021	-0.03014														*										
-2	-0.0000529	-0.07598														**										
-1	0.00010655	0.15312																***								
0	0.00019386	0.27860																*****								
1	5.48146E-6	0.00788																								
2	3.4289E-6	0.00493																								
3	-0.0000775	-0.11136														**										
4	-0.000064	-0.09197														**										
5	0.00024471	0.35167																	*****							
6	-2.3365E-6	-0.00336																								
7	-0.0002961	-0.42557																	*****							
8	0.00010798	0.15517																	***							
9	0.00002984	0.04289																	*							
10	0.00002566	0.03687																		*						
11	7.9944E-6	0.01149																								
12	-0.0001238	-0.17788																	****							

, marks two standard errors

, marks two standard errors

Crosscorrelation Check Between Series

To	Chi	CROSSCORRELATIONS									
Lag	Square	DF	Prob								
5	8.00	6	0.238	0.279	0.008	0.005	-0.111	-0.092	0.352		
11	15.51	12	0.215	-0.003	-0.426	0.155	0.043	0.037	0.011		

Exhibit 3.3.1 Cross-correlation between the two prewhitened series

3.3.2 Identification of a preliminary Transfer Function Model

After the examination of SCC between two prewhitened series, the identification of a preliminary Transfer Function Model is discussed. The general preliminary Transfer Function Model is of the form

$$z_t = \mu + \frac{Cw(B)}{\delta(B)} B^b z_t^{(x)} + \eta_t$$

Here

1. z_t represents the stationary y_t values and $z_t^{(x)}$ represents the stationary x_t values.
2. C is an unknown scale parameter.
3. η_t represents the error component.
4. b is the number of periods before the input series (x_t) begins to affect the output series (y_t) and equals the lag where the first spike in the SCC was encountered. In examining Exhibit 3.3.1, $b=7$, which implies that it takes seven years for disposable income to affect consumption expenditure as previously discussed.
5. $w(B) = (1 - w_1 B - w_2 B^2 - \dots - w_s B^s)$

is called the $z_t^{(x)}$ operator of order s , where s represents the number of past $z_t^{(x)}$ values influencing z_t . From the results in exhibit 3.3.1, the first spike is followed by a clear dying-down pattern. The value s is set equal to the number of lags that reside between the first spike in the SCC and the beginning of a clear

dying-down pattern. In this study, if Exhibit 3.3.1 is examined, the value of s is not obvious. After the first spike in the SCC (at lag 7), a sinusoidal dying-down pattern appears to begin. The study will (somewhat arbitrarily) set $s=7$, implying that the operator $w(B) = (1 - w_1 B)$ should be used.

$$6. \delta(B) = (1 - \delta_1 B - \delta_2 B^2 - \dots - \delta_r B^r)$$

is called the z_t operator of order r , where r intuitively represents the number of its own past values to which z_t is related. It is reasonable to determine r by examining the manner in which the SCC die down at lags after lag $(b+s)$. In Exhibit 3.3.1 the SCC dies down in a damped exponential fashion at lags after lag $(b+s)=(7+1)=8$. Then $r=6$, the spike before the input series begins to affect the output series. This implies that the suitable operator is

$$\delta(B) = (1 - \delta_1 B - \delta_2 B^2 - \delta_3 B^3 - \delta_4 B^4 - \delta_5 B^5 - \delta_6 B^6).$$

7. μ is a constant term that should be included if, when included in the model, the absolute t -value associated with the least squares point estimate of μ is greater than 2.

Inserting the appropriate operators into the general model

$$z_t = \mu + \frac{Cw(B)}{\delta(B)} B^b z_t^{(s)} + \eta_t$$

the researcher is led to tentatively consider the model

$$z_t = \mu + \frac{C(1 - w_1 B)}{(1 - \delta_1 B - \delta_2 B^2 - \delta_3 B^3 - \delta_4 B^4 - \delta_5 B^5 - \delta_6 B^6)} B^7 z_t^{(x)} + \eta_t$$

where

$$z_t^{(x)} = x_t - x_{t-1} \text{ and } z_t = y_t - y_{t-1}$$

ARIMA Estimation Optimization Summary

```

Estimation Method:                Conditional Least Squares
Parameters Estimated:              9
Termination Criteria:             Maximum Relative Change in Estimates
Iteration Stopping Value:         0.001
Criteria Value:                   0.00076974
Alternate Criteria:               Relative Change in Objective Function
Alternate Criteria Value:         2.57065E-7
Maximum Absolute Value of Gradient: 0.00002241
R-Square (Relative Change in Regression SSE) from Last Iteration
Step:                             0.00060782
Objective Function:               Sum of Squared Residuals
Objective Function Value:         0.00768038
Marquardt's Lambda Coefficient:   1E-9
Numerical Derivative Perturbation Delta: 0.001
Iterations:                       13

```

Conditional Least Squares Estimation

Parameter	Estimate	Approx. Std Error	T Ratio	Lag	Variable	Shift
MU	0.20097	0.24959	0.81	0	CONSUMP	0
SCALE1	-0.74749	0.29063	-2.57	0	INCOME	7
NUM1,1	0.28240	0.57275	0.49	1	INCOME	7
DEN1,1	-0.36008	0.16992	-2.12	1	INCOME	7
DEN1,2	-0.14334	0.11136	-1.29	2	INCOME	7
DEN1,3	-0.22747	0.09486	-2.40	3	INCOME	7
DEN1,4	0.09512	0.11140	0.85	4	INCOME	7
DEN1,5	0.39961	0.14480	2.76	5	INCOME	7
DEN1,6	1.07083	0.15976	6.70	6	INCOME	7

Constant Estimate = 0.20096726

Variance Estimate = 0.00069822
 Std Error Estimate = 0.02642378
 AIC = -82.538838*
 SBC = -73.577248*
 Number of Residuals= 20

Autocorrelation Check of Residuals

To	Chi	Autocorrelations							
Lag	Square	DF	Prob						
6	16.89	6	0.010	-0.068	-0.495	0.044	0.351	-0.061	-0.480
12	23.54	12	0.023	0.242	0.122	-0.228	-0.131	0.080	0.133
18	24.08	18	0.153	-0.031	0.024	0.045	-0.037	-0.022	0.013

Crosscorrelation Check of Residuals with Input INCOME

To	Chi	Crosscorrelations							
Lag	Square	DF	Prob						
5	0.00	0	0.000	0.239	-0.344	0.005	-0.018	-0.356	-0.089
11	4.37	5	0.497	0.148	-0.027	0.020	0.050	-0.006	-0.015

Exhibit 3.3.2 Identification of a noise and a final transfer function model

The results above reveal nine estimated parameters. These estimates are $\hat{\mu} = 0.201$, $\hat{C} = -0.745$, $\hat{\omega} = 0.282$, $\hat{\delta}_1 = -0.360$, $\hat{\delta}_2 = -0.143$, $\hat{\delta}_3 = -0.227$, $\hat{\delta}_4 = 0.095$, $\hat{\delta}_5 = 0.399$ and $\hat{\delta}_6 = 1.071$. The absolute t-ratio of the mean associated with the least squares point estimate is 0.81 and is not greater than 2. This t-value should be greater than 2 and since it is less than 2, the preliminary transfer function model is not valid. The residuals are also correlated at lags 6 and 12 when compared with a significance level of 5%. Comparing the chi-square statistics with 6, 12 and 18 degrees of freedom at 5% level of significance, the corresponding critical values are 12.52, 21.026 and 28.869 respectively and the corresponding test statistics are 16.89, 23.54 and 24.08 respectively.

Since the test statistics are greater than their critical values at lags 6 and 12, the hypothesis that the residuals are not statistically independent is rejected and it is concluded that the residuals are statistically dependent on one another. This also invalidates the transfer function model. The constant term is now removed from the model and the model that describes an input series without a constant is fitted. The following are the results of the final model that describe x_t .

Conditional Least Squares Estimation

Parameter	Estimate	Approx. Std Error	T Ratio	Lag
MA1,1	0.51894	0.15417	3.37	1
AR1,1	0.99216	0.03157	31.43	1

Variance Estimate = 0.00051121

Std Error Estimate = 0.02260995

AIC = -168.72842*

SBC = -165.56138*

Number of Residuals= 36

* Does not include log determinant.

Correlations of the Estimates

Parameter	MA1,1	AR1,1
MA1,1	1.000	0.311
AR1,1	0.311	1.000

Autocorrelation Check of Residuals

To Lag	Chi Square	DF	Prob	Autocorrelations					
6	1.80	4	0.772	-0.051	0.038	0.143	-0.104	0.082	0.030
12	8.54	10	0.577	-0.083	0.078	-0.029	0.290	-0.032	-0.167
18	13.92	16	0.605	0.178	-0.194	0.027	0.113	-0.009	0.054
24	19.89	22	0.590	0.027	0.019	0.013	-0.140	-0.071	-0.174

Autocorrelation Check for White Noise

To	Chi			Autocorrelations					
Lag	Square	DF	Prob						
6	7.78	6	0.255	0.154	-0.225	0.143	0.134	0.228	0.141

Exhibit 3.3.3 Estimation and diagnostic checking of modified model that describe x_t

The parameter estimates of this model are all less than 1 and satisfies both the stationarity and invertibility conditions. The estimates have t-values of 3.37 and 31.47 for θ_1 and ϕ_1 respectively. These values are greater than the critical value of 1.96 at 5% significance level. The parameters are therefore contributing significantly to the model. The estimates are also less correlated, with a correlation coefficient of 0.311 and the residuals are also not correlated. Their prob-values 0.772, 0.577, 0.605 and 0.590 are greater than 0.05 level of significance. The significance of these results can also be achieved by comparing the chi-square statistics with their corresponding critical values.

The statistics are 1.80, 8.54, 13.92 and 19.89 and their critical values with degree of freedom of 6, 12, 18 and 24 are 12.592, 21.026, 28.869 and 36.415 respectively. Comparisons of these values substantiate the researcher's decision that the residuals are not correlated as the test statistics are less than their corresponding critical values. Lastly, the hypothesis that the residuals are not white noise are tested by looking at the last portion of autocorrelation check for white noise. The probability value of 0.255 is greater than $\alpha = 0.05$. The null hypothesis that the residuals are not white noise is rejected. Since the model is

valid, it is now used to prewhiten the values of both an input and output series.

The equation for a prewhitened input series is

$$\alpha_t = \frac{(1 - \phi_1^{(x)} B)}{(1 - \theta_1^{(x)} B)} z_t^{(x)}$$

$$= \frac{(1 - .992B)}{(1 - .519B)} z_t^{(x)}$$

and the prewhitened output series can be calculated using the equation

$$\beta_t = \frac{(1 - \phi_1^{(x)} B)}{(1 - \theta_1^{(x)} B)} z_t^{(x)}$$

$$= \frac{(1 - .992B)}{(1 - .519B)} z_t^{(x)}$$

Including a constant in the model invalidates the preliminary Transfer Function Model. Since the model above is valid, it will be used in identification of a preliminary Transfer Function Model by examining the following SCC.

Correlation of CONSUMP and INCOME
Variable INCOME has been differenced.
Period(s) of Differencing = 1.
Both series have been prewhitened.
Variance of transformed series = 0.001551 and 0.000415
Number of observations = 36
NOTE: The first observation was eliminated by differencing.

Crosscorrelations

Lag	Covariance	Correlation	-1	9	8	7	6	5	4	3	2	1	0	1	2	3	4	5	6	7	8	9	1
-8	0.00004588	0.05720												*									
-7	-0.000074	-0.09227												**									
-6	-0.000058	-0.07226												*									
-5	0.00013731	0.17116													***								
-4	-0.0001035	-0.12908												***									
-3	-0.0000358	-0.04466												*									
-2	-0.0000942	-0.11748												**									
-1	0.00008985	0.11201													**								
0	0.00020271	0.25270													*****								
1	-0.0000318	-0.03965												*									
2	-0.0000121	-0.01507																					
3	-0.0001119	-0.13952												***									
4	-0.0000993	-0.12382												**									
5	0.00030265	0.37728													*****								
6	-2.315E-6	-0.00289																					
7	-0.0003698	-0.46094												*****									
8	0.00015651	0.19511													****								

, marks two standard errors

Crosscorrelation Check Between Series

To	Chi				Crosscorrelations							
Lag	Square	DF	Prob									
5	8.74	6	0.189	0.253	-0.040	-0.015	-0.140	-0.124	0.377			

Exhibit 3.3.4 RSCC

The cross correlation function reveals that there are no spikes at all lags less than zero, which is a prerequisite for using Transfer Function Model. The first spike occurs at lag 5, meaning that $r_5(\alpha, \beta_1)$ is statistically different from zero. This implies that consumption expenditures in the present year are related to

disposable income five years ago meaning that that it takes five years for disposable income to affect consumption expenditure.

After the examination of SCC between two prewhitened series, the identification of a preliminary Transfer Function Model is discussed. The general preliminary Transfer Function Model is of the form

$$z_t = \mu + \frac{Cw(B)}{\delta(B)} B^b z_t^{(s)} + \eta_t$$

and an examination of a cross correlation function identifies a preliminary

$$\text{Transfer Function Model } z_t = \frac{C(1-w_1B)}{(1-\delta_1B-\delta_sB^s)} B^s z_t^{(s)} + \eta_t$$

where $z_t^{(s)} = x_t - x_{t-1}$ and $z_t = y_t - y_{t-1}$, $b = 5$, $r = 5$ and $s = 1$. The SCC in exhibit 3.3.4 dies down in a damped exponential fashion at lags after lag $b + s = 5 + 1 = 6$.

After the preliminary Transfer Function Model has been identified, it is estimated and checked for adequacy. The following are the results of a preliminary Transfer Function Model.

ARIMA Estimation Optimization Summary

```

Estimation Method:                Conditional Least Squares
Parameters Estimated:              4
Termination Criteria:             Maximum Relative Change in Estimates
Iteration Stopping Value:         0.001
Criteria Value:                   0.00078422
Alternate Criteria:               Relative Change in Objective Function
Alternate Criteria Value:         1.54823E-8
Maximum Absolute Value of Gradient: 1.40492E-6
R-Square (Relative Change in Regression SSE) from Last Iteration
Step:                             0.00009214
Objective Function:               Sum of Squared Residuals
Objective Function Value:         0.01753537
Marquardt's Lambda Coefficient:   1E-12
Numerical Derivative Perturbation Delta: 0.001
Iterations:                       9
  
```

Conditional Least Squares Estimation Approx.

Parameter	Estimate	Std Error	T Ratio	Lag	Variable	Shift
SCALE1	0.66010	0.14709	4.49	0	INCOME	5
NUM1,1	-1.37060	0.37698	-3.64	1	INCOME	5
DEN1,1	-0.83961	0.22680	-3.70	1	INCOME	5
DEN1,2	-0.10825	0.24279	-0.45	5	INCOME	5

```

Variance Estimate = 0.00092291
Std Error Estimate = 0.0303795
AIC                = -91.846505*
SBC                = -87.304528*
Number of Residuals= 23
  
```

- Does not include log determinant.

(a) Estimation

Autocorrelation Check of Residuals

To Lag	Chi Square	DF	Prob	Autocorrelations							
6	5.92	6	0.432	0.009	-0.429	0.121	-0.023	-0.082	-0.083		
12	13.87	12	0.309	-0.160	0.004	-0.007	-0.095	0.330	0.165		
18	23.88	18	0.159	-0.336	-0.038	0.174	0.038	-0.101	-0.038		

Autocorrelation Plot of Residuals

Lag	Covariance	Correlation	-1	9	8	7	6	5	4	3	2	1	0	1	2	3	4	5	6	7	8	9	1	Std
0	0.00092291	1.00000												*****										0
1	8.38902E-6	0.00909																						0.208514
2	-0.0003958	-0.42885																						0.208532
3	0.00011135	0.12066												**										0.243880
4	-0.0000208	-0.02250																						0.246462
5	-0.0000758	-0.08213												**										0.246551
6	-0.0000765	-0.08289												**										0.247738
7	-0.0001475	-0.15983												***										0.248941
8	3.64974E-6	0.00395																						0.253363
9	-6.3649E-6	-0.00690																						0.253366
10	-0.0000872	-0.09453												**										0.253374
11	0.00030443	0.32986													*****									0.254903
12	0.0001519	0.16459													***									0.272831
13	-0.00031	-0.33589												*****										0.277115
14	-0.0000353	-0.03820												*										0.294285
15	0.00016042	0.17382												***										0.294500

"," marks two standard errors

(b) The RSAC

Partial Autocorrelations

Lag	Correlation	-1	9	8	7	6	5	4	3	2	1	0	1	2	3	4	5	6	7	8	9	1
1	0.00909																					
2	-0.42896																					
3	0.15948																					
4	-0.27494																					
5	0.09181																					
6	-0.32239																					
7	-0.11084																					
8	-0.23549																					
9	-0.19099																					
10	-0.29816																					
11	0.29894																					
12	-0.20577																					
13	-0.07024																					
14	-0.33726																					
15	0.02342																					

(c) The RSPAC

Crosscorrelation Check of Residuals with Input INCOME

To	Chi	Crosscorrelations									
Lag	Square	DF	Prob								
5	6.50	2	0.039	-0.106	-0.072	0.000	-0.546	0.060	0.208		
11	7.30	8	0.505	-0.109	0.074	0.084	-0.071	-0.088	0.083		
17	7.51	14	0.913	0.077	-0.032	-0.057	0.042	0.001	-0.006		

(d) The RSCC

Exhibit 3.3.5 Estimation and diagnostic checking of the model

$$z_t = \frac{C(1 - w_1 B)}{(1 - \delta_1 B - \delta_5 B^5)} B^5 z_t^{(x)} + \eta_t$$

The SAS output of estimation and diagnostic checking for the tentative model is given above. In addition to the RSAC and RSPAC, this output gives the sample cross-correlation function of the residuals with the values of the input disposable income (denoted RSCC). The fitted model is

$$\begin{aligned} z_t &= \frac{C(1 - w_1 B)}{(1 - \delta_1 B - \delta_5 B^5)} B^5 z_t^{(x)} + \eta_t \\ &= \frac{0.660(1 + 0.371B)}{(1 + 0.840B + 0.108B^5)} B^5 z_t^{(x)} + \eta_t \end{aligned}$$

The t-ratios of parameters are significant at the significance level of 5% except for $\delta_5 = -0.108$ which is not significant. To check if the input series and the residuals are cross-correlated, the researcher compared the chi-square statistics of RSCC with their critical values at 5% level of significance. If the test statistic falls within the rejection area as shown in the figure, the null hypothesis needs to be accepted. The critical values with 5, 11 and 17 degrees of freedom are 11.071, 19.675 and 27.587 respectively. Their corresponding test statistics are 6.50, 7.30 and 7.51 respectively. Since the test statistics are less than their critical values, the hypothesis of no autocorrelation is rejected and a conclusion

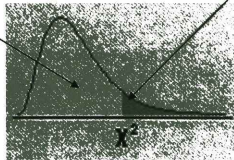
reached is that the residuals are not correlated with the prewhitened input series.

This is a necessary condition for the validity of Transfer Function Modeling.

Chi-square statistics are also compared with their critical values to check if the error terms (residuals) are not autocorrelated. The critical values at 5% level of significance with 6, 12 and 18 degrees of freedom are 12.52, 21.026 and 28.869 respectively. Their corresponding test statistics are 5.92, 13.87 and 23.88 respectively. Since the test statistics are less than their critical values, the null hypothesis that the residuals are not autocorrelated is accepted. Therefore the residuals are not correlated. This is also a necessary condition for the validity of Transfer Function Modeling. The prob-values are also large compared to 0.05 significance level.

Acceptance region

Rejection region



3.3.3 Identification of noise and final transfer function model

To find the model describing η_t , (which represents the residuals), the behaviour of RSAC and RSPAC is examined. At nonseasonal level, both RSAC and RSPAC die down quickly according with a mixture of damped exponentials and damped sine waves. Using Box-Jenkins guidelines of nonseasonal operators, both the nonseasonal autoregressive operator of order 1

$$\phi_1(B) = (1 - \phi_1 B)$$

and the nonseasonal moving average operator of order 1

$$\theta_1(B) = (1 - \theta_1 B)$$

might be employed. This choice implies that the model

$$(1 - \phi_1 B)\eta_t = (1 - \theta_1 B)a_t$$

describes η_t . Thus

$$\eta_t = \frac{(1 - \theta_1 B)}{(1 - \phi_1 B)} a_t$$

and an appropriate Transfer Function Model would be

$$\text{Model 1: } z_t = \frac{C(1 - w_1 B)}{(1 - \delta_1 B - \delta_2 B^2)} B^s z_t^{(s)} + \frac{(1 - \theta_1 B)}{(1 - \phi_1 B)} a_t$$

where $z_t^{(s)} = x_t - x_{t-1}$ and $z_t = y_t - y_{t-1}$

ARIMA Estimation Optimization Summary

```

Estimation Method:                Conditional Least Squares
Parameters Estimated:              6
Termination Criteria:             Maximum Relative Change in Estimates
Iteration Stopping Value:         0.001
Criteria Value:                   0.43959686
Maximum Absolute Value of Gradient: 0.10153625
R-Square (Relative Change in Regression SSE) from Last Iteration
    Step:                         0.57210103
Objective Function:               Sum of Squared Residuals
Objective Function Value:         0.01731995
Marquardt's Lambda Coefficient:   0.001
Numerical Derivative Perturbation Delta: 0.001
Iterations:                       14
    
```

Conditional Least Squares Estimation

Parameter	Estimate	Approx. Std Error	T Ratio	Lag	Variable	Shift
MA1,1	1.00000	0.23063	4.34	1	CONSUMP	0
AR1,1	0.64713	0.30045	2.15	1	CONSUMP	0
SCALE1	0.80320	0.33569	2.39	0	INCOME	5
NUM1,1	0.42714	1.83750	0.23	1	INCOME	5
DEN1,1	0.28609	1.92261	0.15	1	INCOME	5
DEN1,2	0.13060	0.29600	0.44	5	INCOME	5

```

Variance Estimate = 0.00101882
Std Error Estimate = 0.03191897
AIC                = -88.130805*
SBC                = -81.317839*
Number of Residuals= 23
* Does not include log determinant.
    
```

(a) Estimation

Autocorrelation Check of Residuals

Lag	To	Chi	Square	DF	Prob	Autocorrelations				
6	2.61	4	0.626	0.015	-0.081	0.262	0.108	0.041	0.029	
12	10.98	10	0.359	-0.084	0.184	-0.063	-0.045	0.367	0.046	
18	13.92	16	0.605	-0.137	0.048	0.123	0.003	-0.066	-0.041	

Autocorrelation Plot of Residuals																								
Lag	Covariance	Correlation	-1	9	8	7	6	5	4	3	2	1	0	1	2	3	4	5	6	7	8	9	1	Std
0	0.0010188	1.00000																						0
1	0.00001564	0.01535																						0.208514
2	-0.0000826	-0.08110																						0.208564
3	0.00026742	0.26248																						0.209930
4	0.00011043	0.10839																						0.223745
5	0.00004194	0.04116																						0.226016
6	0.00002998	0.02943																						0.226342
7	-0.0000859	-0.08433																						0.226508
8	0.00018764	0.18418																						0.227869
9	-0.0000646	-0.06344																						0.234252
10	-0.0000456	-0.04473																						0.234998
11	0.00037397	0.36706																						0.235368
12	0.0000469	0.04604																						0.259064
13	-0.0001399	-0.13727																						0.259420
14	0.00004889	0.04798																						0.262559
15	0.00012537	0.12306																						0.262940

"," marks two standard errors

, marks two standard errors

(b) RSAC

Partial Autocorrelations

Lag	Correlation	-1	9	8	7	6	5	4	3	2	1	0	1	2	3	4	5	6	7	8	9	1
1	0.01535																					
2	-0.08135											**										
3	0.26691													*****								
4	0.09463												**									
5	0.08755												**									
6	-0.02701										*											
7	-0.13996										***											
8	0.15937												***									
9	-0.11701											**										
10	0.05468												*									
11	0.32998													*****								
12	0.05046												*									
13	-0.09378										**											
14	-0.16389										***											
15	0.05258												*									

(c) RSPAC

Crosscorrelation Check of Residuals with Input INCOME

To	Chi			Crosscorrelations						
Lag	Square	DF	Prob							
5	3.51	2	0.173	-0.154	0.230	-0.048	-0.276	-0.066	0.189	
11	5.12	8	0.745	-0.205	0.159	0.006	0.081	-0.094	0.081	
17	5.34	14	0.981	0.070	-0.005	-0.069	0.049	-0.001	-0.008	

(d) RSCC

Exhibit 3.3.6 Estimation, diagnostic checking and forecasting of the model

$$z_t = \frac{C(1-w_1B)}{(1-\delta_1B-\delta_5B^5)} B^5 z_t^{(x)} + \frac{(1-\theta_1B)}{(1-\phi_1B)} a_t$$

The output above shows that the absolute t-ratios of AR (1), MA (1) and a constant are highly significant at 5% level of significance as they are greater than a critical value of 1.96 and the other parameters are not. The model is valid since the residuals are not correlated. The prob-values of the residuals 0.626, 0.359 and 0.605 are greater than 0.05 level of significance and thus the model is valid. The chi square statistics are also greater than their critical values. The study also compares the chi-square statistics with their corresponding critical values at a significance level of 5% to check if the prewhitened input series is not correlated with the residuals. The chi square-statistics are 3.51, 5.12 and 5.34 and the critical values with the 5, 11, and 17 degrees of freedom are 11.071, 10.675 and 27.587 respectively. Since, the test statistics are less than their corresponding critical values at the significance level of 5%, the null hypothesis that the residuals are not correlated with the input series is accepted.

Alternatively, the prob-values can be compared with the significance level of 5%.

These prob-values are greater than the significance level of 5%. Since some parameters of this model are not significant a preliminary Transfer Function Model needs to be modified. The modified final preliminary Transfer Function

Model will then be $z_t = \frac{C(1-w_1B)}{(1-\delta_1B)} B^s z_t^{(s)} + \eta_t$. The following are the results of a

final preliminary Transfer Function Model.

Conditional Least Squares Estimation							
Parameter	Estimate	Approx. Std Error	T Ratio	Lag	Variable	Shift	
SCALE1	0.71567	0.15801	4.53	0	INCOME	5	
NUM1,1	-1.28904	0.34944	-3.69	1	INCOME	5	
DEN1,1	-0.82755	0.22187	-3.73	1	INCOME	5	
Variance Estimate = 0.00107719							
Std Error Estimate = 0.03282054							
AIC = -105.0593*							
SBC = -101.17178*							
Number of Residuals= 27							
* Does not include log determinant.							

(a) Estimation

Autocorrelation Check of Residuals

To Lag	Chi Square	DF	Prob	Autocorrelations					
6	2.96	6	0.813	0.212	-0.165	0.081	-0.047	-0.083	-0.077
12	10.19	12	0.599	-0.225	0.017	0.172	0.050	0.270	0.071
18	20.68	18	0.296	-0.283	-0.104	0.005	-0.098	-0.200	-0.134
24	22.98	24	0.521	-0.017	0.091	0.045	-0.043	-0.059	-0.021

Autocorrelation Plot of Residuals

Lag	Covariance	Correlation	-1	9	8	7	6	5	4	3	2	1	0	1	2	3	4	5	6	7	8	9	1	Std
0	0.0010772	1.00000												*****										0
1	0.00022819	0.21184										***												0.192450
2	-0.0001781	-0.16538									***													0.200901
3	0.00008703	0.08080									**													0.205882
4	-0.000051	-0.04732									*													0.207053
5	-0.0000897	-0.08325									**													0.207453
6	-0.0000828	-0.07684									**													0.208687
7	-0.0002423	-0.22491									****													0.209732
8	0.00001788	0.01660																						0.218482
9	0.00018562	0.17232											***											0.218529
10	0.0000536	0.04976										*												0.223505
11	0.00029058	0.26975										*****												0.223915
12	0.00007678	0.07128										*												0.235644
13	-0.0003044	-0.28258									*****													0.236441
14	-0.0001119	-0.10384									**													0.248635
15	5.0254E-6	0.00467																						0.250236

(b) RSAC

Partial Autocorrelations

Lag	Correlation	-1	9	8	7	6	5	4	3	2	1	0	1	2	3	4	5	6	7	8	9	1
1	0.21184										*		****									
2	-0.22014									****												
3	0.18725										****											
4	-0.17744									****												
5	0.04168									*												
6	-0.14731									***												
7	-0.17878									****												
8	0.11941										**											
9	0.05104									*												
10	0.07630									**												
11	0.29684									*****												
12	-0.16787									***												
13	-0.18700									****												
14	-0.11484									**												
15	0.01324																					

(C) RSPAC

Crosscorrelation Check of Residuals with Input INCOME

To	Chi				Crosscorrelations							
Lag	Square	DF	Prob									
5	7.97	3	0.047	0.100	0.059	-0.365	-0.329	0.299	0.135			
11	8.94	9	0.442	-0.047	-0.054	-0.049	0.038	-0.182	-0.046			
17	14.05	15	0.522	0.206	-0.232	-0.166	0.301	0.086	-0.103			

(d) RSCC

Exhibit 3.3.7 Modified preliminary transfer function model

The diagnostic checking of the model above shows that the model is adequate. All the parameters are highly significant at $\alpha = 0.05$. The chi-square statistics of autocorrelation check for the residuals are less than their critical values and hence it is concluded that the residuals are statistically independent. The RSCC also shows that the residuals are not correlated with a prewhitened input series since the chi-square statistics are less than the critical values at 5% probability.

To build a noise model, the manner in which the RSAC and the RSPAC behave is observed again. The RSAC and RSPAC both die down according to a mixture of damped exponentials and sine waves. The RSAC has a spike at lag 1 and the RSPAC does not have a spike at all lags thus AR(0) and MA(1) are used. Then the noise model will be $\eta_t = (1 - \theta_1 B)a_t$ and the final Transfer Function Model is

$$z_t = \frac{C(1 - w_1 B)}{(1 - \delta_1 B)} B^s z_t^{(s)} + (1 - \theta_1 B)a_t .$$

The model is then checked for adequacy

and if it is adequate, forecasts are generated from it. The following are the results of a final Transfer Function Model.

Conditional Least Squares Estimation

Parameter	Estimate	Approx.		T Ratio	Lag	Variable	Shift
		Std Error					
MA1,1	-0.48947	0.18668		-2.62	1	CONSUMP	0
SCALE1	0.69071	0.14063		4.91	0	INCOME	5
NUM1,1	-1.31349	0.23718		-5.54	1	INCOME	5
DEN1,1	-0.77058	0.15148		-5.09	1	INCOME	5

Variance Estimate = 0.00101332
Std Error Estimate = 0.03183264
AIC = -105.85878*
SBC = -100.67543*
Number of Residuals= 27
* Does not include log determinant.

(a) Estimation

Autocorrelation Check of Residuals									
To	Chi	Autocorrelations							
Lag	Square	DF	Prob						
6	2.81	5	0.730	-0.123	-0.173	0.162	-0.076	-0.061	0.079
12	11.74	11	0.384	-0.279	0.041	0.189	-0.139	0.239	0.104
18	19.10	17	0.323	-0.315	0.015	0.029	-0.046	-0.110	-0.086
24	21.34	23	0.561	-0.046	0.112	0.004	-0.048	-0.017	-0.028

Autocorrelation Plot of Residuals																								
Lag	Covariance	Correlation	-1	9	8	7	6	5	4	3	2	1	0	1	2	3	4	5	6	7	8	9	1	Std
0	0.0010133	1.00000												*****										0
1	-0.0001246	-0.12295										**												0.192450
2	-0.0001751	-0.17281										***												0.195338
3	0.00016431	0.16215											***											0.200920
4	-0.0000772	-0.07614										**												0.205710
5	-0.0000614	-0.06057										*												0.206751
6	0.00007993	0.07888											**											0.207407
7	-0.0002831	-0.27936										*****												0.208515
8	0.00004133	0.04079											*											0.221944

(b) RSAC

Partial Autocorrelations

Lag	Correlation	-1	9	8	7	6	5	4	3	2	1	0	1	2	3	4	5	6	7	8	9	1
1	-0.12295										**											
2	-0.19081										***											
3	0.11948											**										
4	-0.07628										**											
5	-0.03161										*											
6	0.02506											*										
7	-0.28335										*****											
8	0.00391																					

(c) RSPAC

Crosscorrelation Check of Residuals with Input INCOME

To Lag	Chi Square	DF	Prob	Crosscorrelations							
5	6.50	3	0.089	0.087	-0.024	-0.365	-0.150	0.362	-0.028		
11	7.29	9	0.607	-0.069	-0.029	-0.064	0.042	-0.154	0.023		
17	12.61	15	0.632	0.201	-0.300	-0.039	0.317	-0.055	-0.079		

(d) RSCC

Exhibit 3.3.8 Estimation and diagnostic checking of a final transfer function model

The results above do not show any evidence of model inadequacy.

3.4 Performance analysis

In this section, the focus will be on the results from the two Transfer Function Model;

$$\text{Model 1: } z_t = \frac{C(1-w_1B)}{(1-\delta_1B-\delta_2B^2)} B^s z_t^{(x)} + \frac{(1-\theta_1B)}{(1-\phi_1B)} a_t$$

$$\text{Model 2: } z_t = \frac{C(1-w_1B)}{(1-\delta_1B)} B^s z_t^{(x)} + (1-\theta_1B)a_t$$

In order to determine the best model for describing the consumption-income data, the performance of the 2 models is compared. A comparison of key indicators from the 2 models is summarised in the table below. Both AIC and SBC criteria select model 2 as the best model. For model 1 AIC=-88.131 and SBC=-81.318, and for model 1 the AIC=-105.859 and SBC=-100.675. The model with less AIC and SBC is regarded as the best model. The standard errors for model 1 and model 2 are respectively as follows, 0.03191897 and 0.03183264. The standard errors also select model 2 as the best model.

However, as indicated in Babias and Nazem (1994), variability in observed time series data is usually measured by the standard deviation of the series.

	Model 1	Model 2
t-ratios	4.34 2.15 2.39 0.23 0.15	-2.62 4.91 -5.54 -5.09
Autocorrelation	0.626 0.359 0.605	0.730 0.384 0.323 0.561
Crosscorrelation	0.173 0.745 0.981	0.089 0.607 0.632
Standard error	0.03191897	0.03183264
Variance	0.00101882	0.00101332
AIC	-88.130805	-105.85878
SBC	-81.317839	-100.67543

Exhibit 3.4.1 Relative performance of the two models

3.5 Forecasting petroleum consumption

Using model 2, the forecasts for the period 2005-2014 are reported in Exhibit

3.5.1 below.

Year	Forecasts of petroleum consumption	95% Confidence Interval	
		Lower 95%	Upper 95%
2005	4.5036	4.4412	4.5660
2006	4.5456	4.4336	4.6575
2007	4.5882	4.4427	4.7337
2008	4.6328	4.4602	4.8055
2009	4.6663	4.4703	4.8624
2010	4.7102	4.4911	4.9293
2011	4.7459	4.5000	4.9918
2012	4.7874	4.5151	5.0597
2013	4.8238	4.5202	5.1275
2014	4.8636	4.5276	5.1997

Exhibit 3.5.1 Forecasts for petroleum consumption expenditure using model 2

The analysis based on this model reveals a continuing increase in petroleum consumption for the next ten years. The forecasts show that petroleum consumption will hit a record of up to 4.864 in 2014 if disposable income is augmented. There is 95% confidence that petroleum consumption will be at least 4.528 and at most 5.199.

3.6 Summary

In this chapter, the Box-Jenkins Transfer Function Analysis was used to model the relationship between petroleum consumption and disposable income. The univariate analysis of each variable was firstly performed and the series were nonstationary. Differencing was applied to each series to induce stationarity. Stationarity and invertibility conditions were met. ARMA (1,1) was chosen as the best model for both variables through the help of standard errors, AIC and SBC. A constant term was included in the model since the mean value of first differenced petroleum consumption and disposable income series is greater than standard deviation.

ARMA(1,1) for first differenced disposable income series was then used in prewhitening the values of an input and an output series. It was also seen that the two series satisfied the validity of a Transfer Function Analysis, as the input series depended on the output series but not vice versa. This was shown by the RSCC plot, which did not have spikes at negative lags. After examination of the RSCC, the study identified the preliminary Transfer Function Model,

$$z_t = \mu + \frac{C(1 - w_1 B)}{(1 - \delta_1 B - \delta_2 B^2 - \delta_3 B^3 - \delta_4 B^4 - \delta_5 B^5 - \delta_6 B^6)} B^7 z_t^{(x)} + \eta_t,$$

where $z_t^{(x)} = x_t - x_{t-1}$ and $z_t = y_t - y_{t-1}$. The mean of this model was found to be insignificant because it was not greater than 2. The residuals were also

correlated at some lags and the study did proceed with the analysis because the model was invalid. A new model without a constant was fitted and it was found to be suitable. The model is $(1 - \phi_1^{(x)}B)z_t^{(x)} = (1 - \theta_1^{(x)}B)a_t^{(x)}$ where $z_t^{(x)} = x_t - x_{t-1}$.

The new equation that calculates the prewhitened input and output series was formed from this model. This equation was $\eta_t = \frac{(1 - \phi_1^{(x)}B)}{(1 - \theta_1^{(x)}B)} z_t^{(x)}$. The SCC

function cut at lag 5 unlike in the case where a constant was included. After examining the SCC again, the study fitted to fit a preliminary Transfer Function Model, $z_t = \frac{C(1 - w_1B)}{(1 - \delta_1B - \delta_3B^3)} B^3 z_t^{(x)} + \eta_t$. Some of the parameters of this model

were not significant and hence the researcher could also not continue with the model. The model was modified again and the following preliminary Transfer Function Model $z_t = \frac{C(1 - w_1B)}{(1 - \delta_1B)} B^3 z_t^{(x)} + \eta_t$ was obtained. This model was found

to be valid. The RSAC and RSPAC of this model were examined and the following noise model was generated, $\eta_t = \frac{(1 - \theta_1B)}{(1 - \phi_1B)} a_t$. A Transfer Function

Model was then $z_t = \frac{C(1 - w_1B)}{(1 - \delta_1B - \delta_3B^3)} B^3 z_t^{(x)} + \frac{(1 - \theta_1B)}{(1 - \phi_1B)} a_t$. This was regarded

as model one in this study. The model was adequate in terms of autocorrelations and crosscorrelations, but some parameters were still not significant. The

preliminary Transfer Function as well as the noise models were further modified and a final Transfer Function Model which is model two was

$$z_t = \frac{C(1-w_1B)}{(1-\delta_1B)} B^s z_t^{(s)} + (1-\theta_1B)a_t \quad \text{where} \quad z_t^{(s)} = x_t - x_{t-1} \quad \text{and} \quad z_t = y_t - y_{t-1}.$$

Forecasts were generated from this model.

CHAPTER 4

CONCLUSIONS AND RECOMMENDATIONS

4.1 Summary

The main purpose of this study was to use Box-Jenkins techniques to build a Transfer Function Model with a single input series disposable income. The model was used to carry out forecasts of the output series, petroleum consumption using future values of the input series. Time series data values ranging from 1968 to 2004 were used for analysis. The data was obtained from the South African Reserve Bank. SAS was used to analyse the data and to obtain the results.

4.2 Conclusion

In this study, two Transfer Function Models using a single-input disposable income were built. The selected model satisfied all the conditions of a Transfer Function Model and it was checked for adequacy. It revealed no correlation between residuals. The input series was also not correlated with the residuals. This was exposed through the portmanteau test statistics and the *prob-values*. The *t-statistics* also showed that the parameters were significant at the significance level of 5%. It was found that it takes five years for disposable income to affect petroleum consumption since the first spike in the SCC occurred

at lag five. The forecasted 2014 value of the output series was found to be 4.8636 with confidence bounds of 4.5276 and 5.1997. The built final Transfer Function Model is $z_t = \frac{C(1-w_1B)}{(1-\delta_1B)}B^s z_t^{(x)} + (1-\theta_1B)a_t$ where $z_t^{(x)} = x_t - x_{t-1}$ and $z_t = y_t - y_{t-1}$.

4.3 Recommendations and further study

A single input Transfer Function Analysis was used to develop forecasts of petroleum consumption, using disposable incomes as an input. At face value it is tempting to use this technique for forecasting the trends of petroleum consumption into the future reasonably well. The researcher is of the view that seemingly the use of the Box-Jenkins Transfer Function Model to forecast petroleum consumption works well. For further study, research that would apply *MARMA* Transfer Function Analysis and other competing time series models is recommended for comparison purposes.

Appendices

Appendix A1: Data

Log transformed and first differences of disposable income and petroleum consumption data sets

Year	Income	Consump	Log income	Log consump	dif1 income	dif1 consump
1968	6935	152.4	3.84107	2.18307	*	*
1969	7490	165.4	3.87448	2.21848	0.033418	0.035413
1970	8309	188.3	3.91957	2.27485	0.045083	0.056367
1971	9862	234.1	3.99395	2.36946	0.074384	0.094607
1972	11050	258	4.04336	2.41169	0.049408	0.04223
1973	12148	304.9	4.08451	2.4841	0.04115	0.072413
1974	14564	403.5	4.16328	2.60584	0.078771	0.121743
1975	16854	511.1	4.22671	2.70851	0.063426	0.102662
1976	18544	670.3	4.26821	2.82629	0.041505	0.117783
1977	22008	774.6	4.34257	2.88907	0.07436	0.062783
1978	23600	844	4.37292	2.92632	0.030346	0.03725
1979	28404	1146.3	4.45338	3.05931	0.080466	0.132992
1980	35857	1453.8	4.55458	3.16251	0.101196	0.103197
1981	40940	1724.2	4.61215	3.23659	0.05757	0.07408
1982	47618	2004.8	4.67777	3.30207	0.065621	0.065481
1983	55332	2090.6	4.74297	3.32026	0.065204	0.018192
1984	67088	2235.2	4.82664	3.34932	0.083667	0.029056
1985	76212	3086.7	4.88202	3.48949	0.055379	0.14017
1986	87774	3146.5	4.94337	3.49783	0.061345	0.008344
1987	106471	3225	5.02723	3.50853	0.083866	0.010698
1988	129506	4042.8	5.11229	3.60668	0.085058	0.098151
1989	153591	5056.3	5.18636	3.70383	0.074074	0.097151
1990	205747	6020.8	5.31333	3.77965	0.126968	0.075821
1991	238266	6765	5.37706	3.83027	0.063729	0.050614
1992	281503	7641	5.44948	3.88315	0.072421	0.052882
1993	314765	8944	5.49799	3.95153	0.048504	0.068382
1994	349731	10326	5.54373	4.01393	0.045748	0.0624
1995	394606	11754	5.59616	4.07019	0.05243	0.056254
1996	443600	13878	5.64699	4.14233	0.050828	0.072141
1997	501130	14984	5.69995	4.17563	0.052959	0.033301
1998	541559	15059	5.73365	4.1778	0.033695	0.002168
1999	587979	16665	5.76936	4.22181	0.035716	0.044009
2000	660477	20659	5.81986	4.31511	0.050496	0.093304

2001	727160	22621	5.86163	4.35451	0.041772	0.039403
2002	822853	24397	5.91532	4.38734	0.053692	0.032825
2003	905698	24909	5.95698	4.39636	0.041661	0.00902
2004	1001242	28097	6.00054	4.44866	0.043556	0.052304

Source: South African Reserve Bank

Appendix A2: SAS program

SAS program that produces time series plots for disposable income and petroleum consumption

```
proc gplot;  
plot income * year = 1/haxis =1968 to 2004;  
plot consump * year = 1/haxis =1968 to 2004;  
symbol i =join;  
run;
```

Program that compute unit root tests, the SAC and SPAC and the transformations for disposable income and petroleum consumption

```
proc ARIMA data = transfe;  
identify var = income nlag=8;  
identify var = income;  
identify var = income(1);  
IDENTIFY var = income stationarity = (ADF =1);  
identify var = income stationarity = (PP =1);  
proc ARIMA data = transfe;  
identify var = consump nlag=8;  
identify var = consump;  
identify var = consump(1);  
IDENTIFY var = consump stationarity = (ADF =1);  
identify var = consump stationarity = (PP =1);  
run;
```

Program that fits the model describing x_t and y_t

```
proc ARIMA;  
identify var = income(1);  
estimate p =1 q =1 noconstant printall plot;  
identify var = consump(1);  
estimate p =1 q =1 NOCONSTANT printall plot;  
run;
```

Program that computes the prewhitened α_i and β_i values and also computes the SCC

```
proc ARIMA;
identify var = income(1) noprint;
estimate p =1 NOCONSTANT;
identify var = consump(1) crosscor = (income(1)) nlag = 8;
run;
```

Program that fits the preliminary Transfer Function Model

```
proc ARIMA;
identify var = income(1) noprint;
estimate p =1 q=1 noconstant;
identify var = consump(1) crosscor = (income(1)) nlag = 8;
ESTIMATE INPUT = (5$(1)/(1)income) noconstant PRINTALL ALTPARM MAXIT = 30
BACKLIM =-3 PLOT;
RUN;
```

Program that fits and forecasts using each of the two tentative models

```
proc ARIMA;
identify var = income(1) noprint;
estimate p =1 q=1 noconstant;
identify var = consump(1) crosscor = (income(1)) nlag = 8;
estimate p=(0) q=(1) input = (5$(1)/(1)income) noconstant PRINTALL ALTPARM MAXIT
= 30 BACKLIM =-3 PLOT;
forecast lead =10;
run;
```

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Exhibits

- 3.2.1(a) Plot of original consumption series
- 3.1.2(b) Plot of original income series
- 3.2.1(c) Plot of consumption logarithmic series
- 3.1.2(d) Plot of income logarithmic series
- 3.1.2(e) ACF and PACF for petroleum consumption
- 3.2.1(f) Autocorrelation Check for White Noise (Consumption)
- 3.2.1(g) ACF and PACF for disposable income
- 3.2.1(h) Autocorrelation Check for White Noise (Income)
- 3.2.2(a) Differenced Petroleum consumption series
- 3.2.2(b) Autocorrelation Check for White Noise
- 3.2.2 (c) Differenced Disposable Income series
- 3.2.2(d) Autocorrelation Check for White Noise
- 3.2.3 Unit root results
- 3.2.4(a) Estimation and diagnostic checking of the three models (consumption expenditure)
- 3.2.4(b) Estimation and diagnostic checking for the three models (disposable income)
- 3.3.1 Cross-correlation between the two prewhitened series
- 3.3.2 Identification of a noise and a final transfer function model

3.3.3 Estimation and diagnostic checking of modified model that describe x_t

3.3.4 RSCC

3.3.5 Estimation and diagnostic checking of the model

3.3.6 Estimation, diagnostic checking and forecasting of the model

3.3.7 Modified preliminary transfer function model

3.3.8 Estimation and diagnostic checking of a final transfer function model

3.4.1 Relative performance of the two models

3.5.1 Forecasts for petroleum consumption expenditure using model 2

