THE EFFECT OF USING GEOMETER SKETCHPAD ON GRADE 10 LEARNERS' UNDERSTANDING OF GEOMETRY: A CASE OF A SCHOOL IN A VILLAGE IN BOJANALA DISTRICT.

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Dissertation submitted in fulfilment of the requirements for the degree *Master of Education*in *Mathematics/Science Education* at the North-West University

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October 2017

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DECLARATION BY CANDIDATE

I, Aletta Mmantwa Kgatshe, hereby declare that this thesis, submitted for the qualification of Master's degree in Education in the Faculty of Education at the North West University (Mahikeng) has not previously been submitted to this or any other university. I further declare that it is my own work and that, as far as is known, all material used has been acknowledged and referenced.

.... 14 November 2016

Duly signed Date

DEDICATION

This thesis is dedicated to my late parents

ROSINA MMALEBAKANG MPHOMANE

AND

PETRUS PUNI RAPOO MPHOMANE

May their souls rest in peace!

ACKNOWLEDGEMENTS

I give God all the glory and thank Him for providing me with the wisdom, health, and opportunity to successfully complete this project. It is not possible for me to mention all who contributed to this research project else the acknowledgements will be thicker than the thesis, but I would like to express my heartfelt gratitude to some people that contributed in one way or the other to the success of the project:

My special appreciation goes to the forerunner, Professor JP Sepeng for his meticulous and professional supervision of this research project. It was indeed a privilege to have worked with him. His guidance, support, constructive criticism, quick responses and for asking the questions that helped me clarify the direction I wanted this study to take led to successful completion of this project in good time. Thank you Prof.

I would like to thank my language editor, Genevieve Wood for prompt response and timeous completion of this study, always checking when are the other chapters ready for editing, and I appreciated that. To Mr. Mosime, subject education specialist for Geography in Area Project Office of Rustenburg for his untiring encouragements, I thank you sir.

I would like to thank, our two precious treasures, Aobakwe and Omphemetse for being there for me during this painstaking journey by making sure that internet is always ready for the sources used in this study. My siblings who always encouraged me when I feel like quitting, the whole Mphomane and Kgatshe's families, I thank you all.

The principal of the secondary school at which this study was conducted, teachers, especially mathematics teachers, grade 10 mathematics learners; I would like you to read this piece of work as a token of appreciation. Special thanks to colleagues and sisters, who motivated me to make this study a success, boMme Mogomotsi LR le Ntshabele CS, and statistician, Mr. Diale T, I thank you sir.

Finally, I appreciated what my late parents believed in; "thuto ga e golelwe, ebile go ithuta ga go fele". Your values and integrity are still observed and honoured. This is a special gratitude to you and the way you brought me up and taught me to gear forth unflinchingly. You taught me simple mathematical equation: *Humility + Submission + Respect=Success*. Rest in Peace, I made it!

ABSTRACT

The National Department of Education in South Africa for matric Examination Analysis and moderators` reports (2014 and 2015) revealed that learners performance in Mathematics in general and geometry in particular was generally unbecoming. Only a few number of candidates who sat for the final National Senior Certificate Examination passed.

This study employed the van Hiele`s levels of mental development in geometry learning to investigate the effects of using GSP on grade 10 learners` understanding of geometry learning in a rural secondary school. Both pre-and post-tests were written by both control and experimental groups and interviews for experimental group after the intervention was administered, were used to solicit information regarding learners` feelings about the teaching styles used in their classroom before the post-test was written. This information was collected from 80 learners from the Secondary School in Bojanala district in Rustenburg.

Findings from the study revealed that learners had difficulties in identifying properties and naming geometric figures and/or concepts. Giving the reason why is the square a rectangle and the relationships among squares, rhombi, rectangles and parallelograms. Also, they had greater difficulties when using geometric terminologies to do proofs of theorems, for example, congruency, opposite sites, diagonals, parallel lines etc.

The analysis showed that students mostly had difficulties at the level of Abstraction and Deduction. This gave an indication that the vast majority of the learners in grade 10 are reasoning at the lowest two levels of the van Hiele's model which are Visualization and Description. For these learners' difficulties to be curbed, the analysis demonstrated amongst others that teachers needed to use Information Communication Technology (ICT) during the process of teaching and learning. Manipulative materials, like GSP loaded computers provide experience in which learners can transfer their understanding smoothly from one concept to another.

The significant mean difference in post-test for both control and experimental groups showed that GSP used as an instructional tool yielded good results in the performance of geometry.

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CHAPTER 1: INTRODUCTION AND OVERVIEW

1. INTRODUCTION

The integration of Information Communication Technology (ICT) in the teaching and learning of school mathematics in the 21st century seems to hold potential for the South African basic education system (DBE, 2010:95). The Department of Basic Education (GDBE).MEC Panyaza Lesufi, 15 January 2016, Gauteng Province has rolled out a paperless teaching project by equipping schools with mobile gadgets such as laptops, tablets, data projector, interactive smart-board and white board technology. Such a move is seen by many as a way to use technological devices to improve the quality of teaching technical subjects such as Mathematics, Science and Technology.

This is one of the reasons why is necessary to change the way mathematics is taught. We are now able to help our students analyse, visualise and make informal conjectures by using many types of manipulation that include the geometric and algebraic computer software. In 1989, the Curriculum and Evaluation Standards for School Mathematics (The Standards) was published by the National Council of Teachers of Mathematics, which recommended the change in the way we teach mathematics.

Christen(2009:28) contends that schools need to develop curricula that address the soft skills required in today's global, information-driven workforce, integrate technology and pedagogy, and look for diverse partners that can add to their pedagogical strengths and help shore up their weaknesses.

Christen(2009:28-31) maintains that conventional teaching methods of lecture and note-taking seem not to be an intuitive process for students who are, instead, accustomed to text messaging, social media and online data retrieval. Christen (2009:31) concludes that ICT may act as a trigger that transforms the classroom into an interactive learning environment. Such a learning environment possesses the power to make the teacher a better facilitator or coach, and bring greater resources to bear in the classroom, adjusting the instruction to fit the individual.

The purpose of this study is to outline the nature and extent of teaching and learning geometry using GSP in Grade 10 mathematics classroom at secondary school, and to suggest ways in which student attainment can be improved and achieved.

2. LITERATURE REVIEW

Hiebert and Grouws (2007:253) defined struggle as an intellectual effort students expend to make sense of mathematical concepts that are challenging but fall within the students' reasonable capabilities.

Learning theories provide a lens to explain and understand teaching and learning in greater depth. However, lenses draw certain areas closer to the eye, while ignoring other aspects and hence it is not likely to be possible to have a theory that encompasses every aspect of learning. For this reason, my study draws from Van Hiele's theory (1986:310) of Geometric Thinking and is presented below.

Making a case for what has been missing in school mathematics is a noble task but not an easy task. In recent years, in particular since the early 1980s, only a few technological developments have been introduced and made available for teachers and students. This marked the beginning of a major change in the way we teach geometry. An instructional software known as the Geometric Supposers for Apple II computers has been developed in 1985 that enabled teachers and students to use computers as teaching and learning tools. The software helped in creating an environment in which students explore geometric figures and make conjectures about their properties.

Learning geometry would then be turned into a sequence of part-to-part, part-to-whole, and the discovery of whole-to-whole interrelationships of geometric figures. This has been viewed as a process that would open the door wide for concrete reasoning of proofs. This approach reflects the research done by the Dutch mathematics scholars Pierre van Hiele and Dina van Hiele-Geldof (1984:215). Based on their research finding in the classrooms, the van Hieles observed that students pass through a sequence of thought development levels in geometry: visualization, analysis, informal deduction, formal deduction, and rigor.

Most of the literatures assume that students are able to employ formal deduction right away. Little, if any research has been done to enable students to visualise, to analyse and to make conjectures about geometric shape. The three levels have been skipped at once.

The main goal, therefore, is to bring students through the first three levels: visualization, analysis, and informal reasoning. The Geometer's Sketchpad has been created to bring students through these three levels. It fosters a process that encourages observation, discovery, and making conjectures; a process that closely reflects how mathematics is normally created.

The Van Hiele levels are not age-dependent, since learners are at different stages of development. However, this study makes an assumption that a good and well-planned geometry lesson is accessible to all learners, in the process, allowing them to work at their own level of development and cognition. It is further argued in this study that any form of teaching intended to foster development from one level to the next should include sequences of activities, beginning with an exploratory phase, gradually building concepts and related language, and culminating in a summary of activities that assist students to integrate what they have learned into what they already know (1984:233). In addition "*A Model for Teaching with Technology*" (1984:233) as a lens through which teaching using technology was analysed.

3. CONTEXT AND BACKGROUND

Mathematics teaching, geometry teaching in particular, has been reshaped due to certain innovative developments in recent years. For centuries geometry has been taught through deductive reasoning approach. Although the deductive reasoning approach is otherwise acceptable, it fails in reaching the majority of students. In 1989, the National Council of Teachers of Mathematics (NCTM) called for substantial changes in the way mathematics is taught (The Standards, 1989). In teaching geometry, the Standards called for an increase in exploration, conjecture making, and use of geometric transformations. The Standards recognised the impact of technology on the teaching and learning mathematics through freeing students from lengthy, time-consuming tasks and taking them into the world of observation, and exploration of mathematical ideas.

McGlynn (2005:12) concluded that, digital generations understand the language of technology and have little context for life without various technology tools. When they were born cell phones, MP3 players, digital cameras, Wii and Nintendo game sets, laptop computers, and iPods were already part of the normal function in society. Educating the millennium generation, viz. those children born after 1992, is quite a challenge compared to the way it was in the past, and requires an appropriate adjustment in teaching techniques.
Furthermore, Clements (2003:110) stated that "appropriately designed geometric software is designed to have a high level of interaction". He believed that by using geometric software

students are unable to "hide" what they do not know, that is, it is easy for a teacher to reach each learner in his/her mathematics classroom when an appropriately selected software is used as an instructional tool.

It is therefore against this background and taking into account the contexts discussed above, that the study sought to investigate the relationship between the use of GSP and its effectiveness in the teaching and learning of geometry in Grade 10 classrooms. In addition, the study is of significance because it attempts to change perceptions about the complex nature of Geometry as a topic in mathematics classrooms of South Africa.

4. STATEMENT OF THE PROBLEM

The purpose of the study is to explore the effect of using Geometer Sketchpad on the Grade 10 students' understanding of geometry in a village in Bojanala District. The study is embedded in the assumption that the use of technology in the teaching of mathematics is an evolutionary event that creates interactive space which appears to make the learning of mathematics easy, real and interesting for students (Humphrey, 1999:105). However, it is argued in the study that in all mathematics classrooms, and technology, as a crucial tool, enhances learning, and promotes mathematical understanding, in particular. Here, conceptual understanding is a guiding principle. As Nieuwoudt (2010:175-6) noted, if used inappropriately technology can hamper learning instead of enhancing it.

It is therefore against this background that the study seeks to investigate whether the use of GSP in classroom will help improve Grade10 students' academic achievement in Euclidian Geometry. In addition, the use of technology and the inclusion or accessibility of mathematics software for higher-order learning in mathematics classrooms is likely to result in an increased mathematical achievement (Usun, 2007:231). Following a study by Symington & Stanger (2000:125), which agreed with Stanger and Khalsa (1998:164) that technology encourages some students to work harder than they had before. To find out whether this may be a case for the context(s) in the research site, the study responds to the following three research questions:

The main research question:

i. *What is the effect of using Geometer Sketchpad on Grade 10 students' understanding of geometry?*

Subsidiary questions:

ii. *What are the benefits of using Geometer Sketchpad in the teaching and learning of Euclidean geometry in Grade 10 mathematics classroom?*
iii. *How does the use of Geometer Sketchpad impact Grade 10 students' academic achievement in Euclidean geometry?*

5. THE AIMS OF THE STUDY

The main aim of this study is to investigate the effects of using GSP on the students' understanding of geometry.

In order to address the research questions, the following sub- aims were identified:

I. to identify the benefits of using Geometer Sketchpad in the teaching and learning of Euclidean geometry in Grade 10 mathematics classrooms; and
II. to establish the effects of using Geometer Sketchpad on Grade 10 leaners' academic achievement in Euclidean geometry.

6. RESEARCH DESIGN AND METHODOLOGY

Methodology is a research strategy that translates ontological and epistemological principles into guidelines that show how research is to be conducted (Sarantakos, 2005:208), and principles, procedures, and practices that govern research (Kazdin, 1992, 2003a, cited in Marczyk et al, 2005: 106).

The study is focused on rural disadvantaged areas where learners have never been introduced or exposed to a computer. The participants (students) were categorised as the experimental and comparison groups. The experimental group was taught using computers that was loaded with the GSP software, thereby opening access to the GSP technology to all experimental group students regardless of SES. The comparison will be taught in a traditional, lecture method.

6.1 Research Design

A research design is a plan that indicates how the researcher intends to investigate the research problem (Denzin & Lincoln, 2006:309; Huysamen, 2001:219, Mouton, 2002:67). In the study, the researcher uses a pre-test – intervention – post-test design.

The study type of research strategies followed in this study is quantitative experiments used to answer the cause - and - effect research question. The participants, experimental and control groups were subjected to first being measured on the dependent variable (pre-test). Thereafter, only the experimental group was taught using GSP and the control group taught by pencil and paper method, and then both groups were measured on the dependent variable again (post-test).

The answer to the main question as to whether the treatment had an effect was obtained by comparing the two groups on the post-test. In this design randomisation ensured that the two groups were equivalent on statistical grounds. Pre-test helped the researcher to establish how learners perform in geometry. Quantitative data were gathered from pre-testing and post-testing of geometric abilities of students. Multilingual semi- rural secondary school was a convenient sample. Sampling refers to the selection of people to participate in a study, usually with the goal of being able to use these people to make inferences about a larger group of individuals (Creswell, 2009:212).

The experiment was planned to take place over two months in the form of workshops on the use of GSP in teaching and learning of geometry. The sole purpose of this workshop was to improve or develop students' technological content knowledge in the teaching and learning of geometry, using GSP.

Technology, pedagogical and content knowledge (TPCK) describes the infusion of technology to this mix (Mishra & Koehler. 2006:166 & Niess, 2008:245). The NCTM Position Statement on the Role of Technology in the Teaching and Learning of Mathematics (March, 2008:10) suggests that teachers consider technology as a conscious component of each lesson, as well as each strategy for enhancing student learning. Therefore, the teacher training on the use of GSP will be for the purpose as outlined in NCTM.

A sampled group of Grade 10 students (n=8) participated in focus group discussion immediately after the pre-test. Data obtained from the focus group assisted the researcher to understand how students solve problems, and why they solved them the way they did. In addition, the focus group discussions were informed by the results of the pre-test.

The baseline observation was done before the beginning of the intervention, with the intention of understanding the nature of instruction in the geometry classroom of the experimental group. The observation was undertaken during and after an intervention, with the intention of observing possible benefits of using the GSP (if any) in teaching and learning geometry.

The data collection phase ended when a post-test was administered to both experimental and focus groups, respectively. The post-test data were used to measure the effect of teaching Geometry using GSP, as compared to traditional way of teaching the subject.

a. Research Site

The study was conducted in one of the secondary school in the village in Bojanala District. The secondary school was chosen as a convenient site, because it is characterised as the suitable site in its approach of teaching and learning context, and is a public and previously marginalised. The school draws students from poor economic backgrounds. The school is characterised as public, multilingual, village, under-resourced, no- fee paying, classified as Quintile 3. The researcher collaborated with all stakeholders involved, at the secondary school before the study was conducted by means of a letter seeking their permission.

b. Participants

According to Howell (2004:240) a sample is a subset of the population. A population is the entire collection of events or objects in which the researcher is interested (Howell, 2004:240). The intention of sampling in quantitative study is to select individuals that are representative of a population, so as to ensure that the results can be generalised to a population and that inferences can easily be drawn (Creswell & Plano Clark, 2007:209-240).

The main aim of the sampling in this study was, among other things, to select possible research participants, because they possess characteristics, roles, opinions, knowledge, ideas or experiences that may be particularly relevant to this study (Gibson & Brown, 2009:172).

For the purpose of this study, the researcher collected data from the researcher's current Grade10 Mathematics class. The Mathematics class consists of 56 girls and 24 boys. This makes a total of 80 learners who are possible participants in the study. The actual number of participants depends on parental informed consent being granted to the researcher. The student body is wholly composed of black students, with all levels of performance represented (Level 1-7), that is, comprising of mixed ability learners. The learners' group was made up of all 80 students with Setswana as their home language and 19 with different African home languages. The respondents were divided into two equal numbers depending on the returned consent forms for participation.

c. Data Collection Strategies

As discussed earlier, for the purpose of the study, data collection strategies include tests (pre- and post-tests) administered to students, the test items were CAPS inclined. The test consisted of items on Euclidian Geometry. Learners were allowed space to comment on how and why they solved items the way they did. Students' written work was marked and analysed in a way that identified errors emerging from their solution processes. Learners' responses were then classified according to Van Hiele's theory, discussed earlier.

Data Analysis

Creswell (2009:136) has argued that, the process of data analysis involves making sense out of the data, which requires the skill to depict the understanding of the data in writing. While Henning (2004:84) stated that the aim of the data analysis is to seek an in-depth understanding of the phenomenon under investigation, which involves preparing the data for analysis, conducting different analyses, moving deeper and deeper into understanding the data, representing the data, and making interpretation of the larger meaning of the data.

Quantitative data analysis

Quantitative data were examined and organised according to categories, using an elaboration of the classification schema developed by Verschaffel, et al. (1994 cited in Sepeng, 2010:78) in order to obtain descriptive statistics of the mean, mediation, mode, and standard deviation. Minimum and maximum values and graphs were presented in this part of statistics participants involved in this study. All the data obtained from the test were captured in a Microsoft Office Excel spreadsheet and subjected to analysis of variance (ANOVA) techniques to provide both descriptive and inferential statistics.

d. Validity and Reliability

Quantitative and qualitative methods rely on different degrees of validity and are subject to different threats. Creswell (2005:) defines threats as the problems that threaten our ability to draw correct cause and effect inferences that arise due to the experimental procedures or the experiences of participants.

For the results of an experiment to be trustworthy, the experiment should have a high degree of both internal and external validity. If an experiment has a high degree of internal validity it means that there was a sufficient control over variables other than the treatment, and consequently it can be concluded that the treatment alone was the causal factor that produced a change in the dependent variable (Maree, 2010:151).External validity, on the other hand, refers to the degree to which results can be generalised to the entire population (Schumacher, 2001 in Maree, 2010:151). A high degree of external validity means that the experimental findings should not only be true in similar experiments, but also in real life.

In this experimental study, the researcher seeks to describe the errors that learners make when solving geometry problems involving representations, and to determine the reasons for these errors. Due to the nature of the study, generalisability and evaluative validity was researcher's focus.

Ethical issues

Informed consent from participants was requested and obtained after prior permission to conduct this research, as part of the ethical clearance processes at the North-West University, Mahikeng Campus. Permission was sought from the Department of Basic Education and Sports, the Principal, teachers and School Governing Body before approaching the target class of data collection.

Prior to data collection, participants were given an oral explanation and written outline information sheet of the research project's aims, nature and data collection methods. In particular, participants were informed that ethical requirements were adhered to. Both the oral explanation and the written information sheet stressed that participation in the research project is voluntary and that all reporting kept participants' details anonymous (Creswell, 2014:135).

The written information sheet contained a separate cut-off section for learners to sign giving their informed consent to participation in various sections of the research. The informed consent forms contained a section seeking parents'/guardians' informed consent. If parents/guardians decided not to provide consent for their charges to participate in various parts of the research, then the learners were not allowed to participate. In particular the research was conducted after school so that those students to whom consent was not granted were not unduly prejudiced. All raw data were kept under lock and key during the study and researcher did not discuss the performance of one learner with other learners or other colleagues. After that, the raw data were locked away in the office of the supervisor.

Both teachers and students were assured of confidentiality and anonymity, that participation was voluntary, and given a guarantee that they could withdraw from the study at any time and that no personal details would be disclosed. Confidentiality of information collected in the school was ensured, and participants were further reassured that no portion of the data collected would be used for any purpose other than that of the research study.

Researchers' role

The researcher will be an insider conducting a case study research in usual contexts. As an insider-researcher, the researcher assumed the role of an objective individual who partly utilise qualitative methodology and work towards making research credible. As Bonner and Tolhurst (2002:251) noted, there are three key advantages of being an insider-researcher as given below:

i. having a greater understanding of the culture being studied;

ii. not altering the flow of social interaction unnaturally; and

iii. having an established intimacy which promotes both the telling and the judging of truth.

The benefits of assuming a role of an insider-researcher is that the researcher knows and understands the politics and contexts of the research site, not only the formal hierarchy but also how it really works.

7. SIGNIFICANCE OF THE STUDY

As we know, Mathematics is long known as a dull subject, due to the memorisation of formulae and monotonous computation. Most of the time, the tools to manipulate numbers are the pencil and paper. However, according to Heid (1997:106), mathematics usually deals with logic and reasoning, problem-solving, number sense and a search of relationships. Therefore, to enhance the teaching and learning process, we look at technology as a tool. Graphing software like Geometers' Sketchpad can help inject excitement and enthusiasm in the teaching and learning of mathematics.

The Grade 10 students of a secondary school in a village have over the years experienced varieties of difficulties in the learning of geometry. From the past years of teaching mathematics, mathematics' results analysis showed that underperformance is on the questions of geometry. If learning difficulties experienced by learners are investigated and explained by using the Van Hiele's levels for geometry learning in this study, then the result of the study and its recommendation provided useful information for teachers, school administrators and curriculum developers, as well as society more broadly, on how to ease learners' difficulties in the teaching and learning of geometry in the classrooms.

Research conducted in South African schools suggests that teachers who lack experience in confidence teaching and general pedagogical content knowledge resort to methods of expository teaching, rote learning, and the avoidance of classroom situations, where something might go wrong(Taylor &Vinjevold,1999:301).

Primarily, the study is intended to provide a deeper understanding of when, why and how to use technology with students in Grade 10 mathematics classroom and the benefits it can provide. Secondly, this study is focused on the teachers' use of the technology-GSP, which provides insight to other mathematics teachers, such that it might curtail the under-performing or poor-performing tendency found in geometry, specifically.

At the secondary school level, the use of a dynamic geometry environment is an ideal setting for the examination of a constructivist technology tool. Using an emergent perspective this study advises for the integration of such suitable technology in the secondary school geometry curriculum. A teacher development (Simon, 2000:197-210) conducted with the goal of exploring the pedagogical, technological, and mathematical development of mathematics teachers. This methodological approach also could be classified as "learning technology by design" (Mishra & Koehler, 2006:233), since technology is learnt by the classroom teacher through the cyclic act of designing or retooling the geometry course.

It is therefore hoped that findings from this proposed study will inform the development of effective instruction in geometry, contribute towards development of topic-specific technological pedagogical content knowledge as well as develop researcher's own teaching practice as a teacher. In addition, the study is of significance because it will attempt to change perceptions about the complex nature of Geometry as a topic in mathematics classrooms of South Africa.

8. CHAPTER SUMMARY AND REPORT OUTLINE

The study was divided into five chapters as follows:

Chapter 1

The background to the research problems and the research statement are discussed in this chapter. The purpose of the study, the research questions and the significance of the study are highlighted.

Chapter 2

This chapter reviews the relevant literature on the effect of using GSP on Grade 10 learners' understanding of geometry with special focus on how students learn geometry and the difficulties they have during learning.

The researcher presented theories that framed this study as well as appropriate literature reviewed.

Chapter 3

This chapter presents research methodological aspects of the study and the overview of data collection in quantitative approach. In this chapter, a step-by-step approach is followed in order to answer the research questions and achieve research aims.

Chapter 4

This chapter reflects the empirical findings of the study with specific reference to the benefits (if any) of the effective use of GSP on the teaching and learning of geometry on Grade 10 learners in the secondary school. In addition, the findings are described using the theoretical underpinnings of the study in relation to existing literature used in the study.

Chapter 5

This chapter consists of conclusions of the major findings and the results of the study, the recommendations for the improvement of the current performance of Grade 10 mathematics learners and the approach to be followed by the role- players and the community of the village in which the secondary school is situated.

CHAPTER 2: THEORIES AND LITERATURE REVIEW

2.1 INTRODUCTION

The purpose of this study is to devise the activities based on Van Hiele levels of geometric thought using computer software, Geometer's Sketchpad (GSP) as a tool. The most challenging task facing teachers of geometry is the development of student facility for understanding geometric concepts and properties. The National Council of Teachers of Mathematics (1989; 1991; 2000) and the National Research Council (Hill, Griffiths, Bucy, et al. 1989) have supported the development of exploring and conjecturing ability for helping students to learn mathematical properties better. Examples of the activities built in GSP for students are designed to illustrate the ways in which Van Hiele's model can be implemented into classroom practice.

According to Hoffer (1981:18) traditionally, geometry in textbooks has centered on fostering deductive reasoning abilities of students. Most of geometry instruction is on geometric proof, which, in many respects, seems to be beyond the grasp of a large portion of students. Students copy by memorising theorems and proofs, and come away from these experiences with no under-standing and appreciation of either geometry or deductive reasoning and proof. Hoffer (1981:18-28) claimed that some geometry courses do not develop understanding but rather encourage memorisation.

The availability of computers in mathematics provides a unique opportunity to develop useful methods for attacking problems with geometry. By exploring and conjecturing geometric ideas, students will become more engaged in subject matter, and will become more skilled at inductive and deductive reasoning. With the infusion of a tool such as the Geometer's Sketchpad into geometry such an approach is feasible.

Since students differ in their respective abilities, teachers should therefore, present instructions in a manner that takes this into account during teaching and learning. Furthermore, in a geometry class, gifted students rely on symbolic thinking, while those less gifted should visualise the problem in problem solving situation. Certainly visualisation does not harm the gifted students, but the argument forwarded here is that, if left out of the curriculum, it limits the chance of success in geometry problem solving of the less gifted child. (Kirby, 1991:109-125). In the teaching and learning of geometry in schools, there are some views and theories expressed by researchers in the field of education, such as Piaget, Freudenthal and Van Hiele.

2.2 Piaget Theory of learning

The view of Piaget is that individuals learn in a unique way through their own dynamic construction of information in their mind. According to Piaget, the way in which this takes place depends mainly on biological growth. Piaget was one of the most influential people in the sphere of education, particularly in the area of Mathematics and Science, proposing that children pass through a series of stages of thought as they progress from infancy to adolescence. He employed the biological thesis of adaptation, whereby through the twin process of assimilation and adaptation, the individual adapted to the environment and there is a pressure to organise structures of thinking. These stages of thought are qualitatively different from each other, so that the child at one stage of thought reasons quite differently than does a child at a different stage of thinking (Piaget and Inhelder, 1971:331).

Piaget defined intelligence as the individual's ability to cope with the changing world. According to Piaget, this can be achieved through constructing and reconstructing the experience, which the child has been exposed to. Piaget's development is highlighted by four stages, namely, the sensory motor stage, the operational stage, concrete operational stage and the formal operational stage.

In the field of education, Piaget's work was not adopted not until the early 1960's. Before this time, much of the content of school curriculum was taught in a rote fashion, in which students were expected to do a considerable amount of their work in a pencil and paper format. The introduction of Piaget's work was embraced in most western countries, and subsequently new mathematics pedagogy was developed and implemented.

Piaget proposed four stages of development which seem to correlate with certain ages, although there is to be an expected range in the ages. He further described these stages of cognitive development as:

Sensory motor (0 to 2 years)

According to Piaget (1973:209), a child at this stage of development may be able to co-ordinate senses and perceptions with movement and action. This stage is also characterized by limited capacity of the child to participate consequences of action and the child sees only the permanent nature of an object.

Preoperational stage (two to seven years)

At this stage, children view themselves as the centre of a personalise universe, and they also perceive inanimate object as having innate qualities, they are unable to mentally reverse actions, but they begin to use language and mental images to generalise. The form of the idea at this stage may sometimes be unreasonable. The child's idea is connected but not reliable at this stage of development (Piaget, 1973:213).

Concrete operational (seven to 12 years)

At this stage of the child development, the child is able to consider the perspective of others, conserve numbers, mass, volume, area and length, and operate on more than one aspect of a problem. The child can also play games with rules and can mentally reverse action.

Formal operational stage (12+)

This stage for Piaget coincides with adolescence. The child at this stage is able to work with abstract object, and can employ deductive, hypothesis testing, and verbal proposition (not concrete) in problem solving. For example, if A is greater than B and B is greater than C, it implies that A is greater than C, thus ($a> b$, and $b > c$ then and $a> c$). Piaget's views above are extremely relevant to the teaching and learning of mathematics in schools. The significance of these views is briefly discussed in the next section.

2.2.1 Mapping Piaget concept onto the teaching and learning of mathematics

The above stages of a child's development as described by Piaget have a significant implication in the teaching and learning of mathematics in the classroom. For example, at the preoperational stage (between the ages of two and seven) this is characterised by the child's "perceptual or intuitive thought". In mathematics teaching and learning, it requires that the child be given lots of free play and the use of concrete materials by the teacher (Piaget, 1973:215).

At the concrete operational stage (between the ages of 7 and 12) or the middle to upper primary school, the child is able to play games with rules and sees the reversibility of an entity. In mathematics, therefore, this suggests that concrete materials begin to give way to numerical symbols. Lastly, at the formal operational stage at age 12+ (secondary school) when the child abstract thought, deductive reasoning and hypothesis testing developed. It implies that in mathematics, numerical symbols give way to algebraic symbols, and algebraic logic as explained earlier at the formal operational stage above (Piaget and Inhelder, 1971).

From my personal perspective, I am of the opinion that Piaget's theory has other noticeable implications in the teaching and learning of mathematics in general. For example, prior to the 1960's a quick look through the textbook used by schools, suggests that there was a strong emphasis on rote learning and learning and the repetition of standard algorithm for students. There was a strong emphasis on pencil and paper work and students were expected to complete the work, revealing that they fell short of what is been described as current practice in the learning and teaching of mathematics.

Taking Piaget's key ideas which are applicable to school age children and mapping them against current pedagogical practice in mathematics education in general, it appears that many of the Piagetian ideas have been incorporated into current practices in schools. One of the biggest changes in mathematics education came in infant grades where Piaget proposed that young students need to construct meaning for them through direct interaction with the environment. For this aim to be achieved, it calls for teachers to adapt their teaching and learning environment such that students are able to engage in learning activities and play with a variety of concrete experiences. The more experiences students have, the more likely they are to construct new schema.

The National Council of Mathematics Teachers (NCTM, 1986) has also suggested that students should be given access to variety of concrete building materials and construction sets, so that in their free play, they will have the opportunity to develop judgement of length and manipulative skills (NCTM, 1986).

This statement indicates the importance of play and the use of concrete materials in the development of mathematical concepts. According to research conducted some countries like Nigeria, Lesotho, Zimbabwe, Botswana and South Africa now uses Piagetian theory in mathematics teaching and learning (Ada & Kurtulus, 2010: 901-909).This manifestation could be seen in most current mathematics textbooks and the mathematics curriculum where there seem to be a link made between the various stages of development and what is expected of students at the introductory chapters.

2.2.2 Hans Freudenthal descriptions of learning

Freudenthal is the founder of the so called realistic mathematics education in the Netherlands. In realistic mathematics education, realities do not only serve as an area of applying mathematical concepts but is also the source of learning. For Freudenthal, mathematics does not only mean mathematising realities, meaning transforming a problem field in reality into a mathematical problem. It is also mathematising mathematics itself (Freudenthal, 1973:376).

Freudenthal wrote on the theory of discontinuity in the learning process devised by his student and colleague, Van Hiele. Freudenthal and Van Hiele both discovered similar levels of child development. This is how Freudenthal described the reasoning of a child at the third level. At the third level, if a child knows what a rhombus and parallelogram is, he/she can visually discover the properties of these shapes. For example, in a parallelogram, opposite sides are parallel and equal, opposite angles are equal, adjacent angle sum up to 180°, and the diagonals bisect each other. The parallelogram has a centre of symmetry, it can be divided into congruent triangle and the plane can be paved with congruent parallelogram.

This is a collection of visual properties, which ask for organisation (Freudenthal, 1973:398). Freudenthal explained that deductive reasoning starts at this point. It is not imposed. It unfolds itself from its local germs. The properties of the parallelogram are connected with each other. One among them, can become the same from which the others spring. In this manner, a definition arises, and knowing its related properties, it becomes clear why a square shall be understood as a rhombus, and a rhombus a parallelogram, respectively (Freudenthal, 1973:409). Freudenthal explained further that the following level of thought may contribute to a more precise understanding of the level of thought.

According to Freudenthal (1973:413), at the first level, figures were in fact just as determined by their properties, but a student who is thinking at this level is not conscious of these properties. Each level has its own linguistic symbols and its own network of relation uniting these signs. A relation that is 'exact' on one level can be revealed to be 'inexact' on another level.

Two people who are reasoning on two different levels cannot understand one another. To Freudenthal, this is what often happens with the teacher and the students, where neither of them succeeds in grasping the progress of the others' thought, and their discussion can be continued only because the teacher tries to get an idea of the students thought process and to conform to it (Freudenthal, 1973:413).

Freudenthal further explained that certain teachers gave an explanation at their own level, and that they invite the students to answer questions on that level. Freudenthal advised that for a meaningful teaching and learning to take place, the teachers must dialogue with and operate on the students' level. In this case, the teachers must often after the class, question themselves about the students' meaning, and strive to understand them.

The maturation process that leads to a higher level unfolds in a characteristic way, where one can distinguish several phases. This maturation must be considered principally as a process of apprenticeship and not as a ripening on the biological order (Freudenthal, 1973:415). Freudenthal advised further that it is then possible and desirable for the teacher to encourage and hasten the maturation process of the child and it is the goal of didactics to ask the question about how these phases are passed through by the child and about how to furnish effective help to the students (Freudenthal, 1973:435). In the same light, Van Hiele also expressed his views on these phases.

2.2.3 Van Hiele description of learning

Van Hiele (1986:163) gave a description of how children learn geometry. According to him, students progress through levels of thought in geometry, with specific characteristics, as explained by Van Niekerk (1997:323). Van Hiele proposed that learning is a discontinuous process implying that there are quantitatively different levels of thinking that these levels are sequential, and that Van Hiele's description of learning is hierarchical. A student cannot function adequately at one level, without having mastered most of the previous levels.

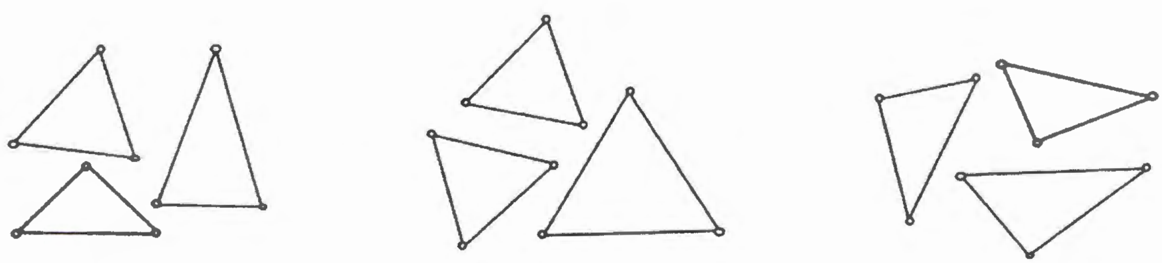
The progress from one level to the next is more dependent upon instruction than on age or biological maturation. According to Van Hiele, concepts that are implicitly understood at one level become understood implicitly at another level and each level has its own language. During teaching and learning, two people that reason at different levels cannot understand each other, or follow the thought processes of the other. Language is a critical factor in the movement through these levels Van Niekerk (1997).Van Hiele (1986:313) distinguished five levels of geometry thought.

These levels of thought can be summarised as follows:

Level1-Recognition

Students recognise a figure by its appearance (or shape/form). It is the appearance of the shape that defines it for the student. A square is a square, "because it looks like a square". And a child recognises a rectangle by its form and a rectangle seems different to him than a square (Van Hiele 1999:311), or, "it is a rectangle because it looks like a door" (Van der Sand and Nieuwoudt 2005:109). Since the appearance is dominant at this level, appearances can overpower properties of a shape, and where students learn to recognize the field of investigation by means of the materials presented to them

Figure 1



(a) **(b)** **(c)**

Level 2- Analysis

Students at this level are able to consider all shapes within a class rather than a single shape. By focusing on a class of shapes, students are able to think about "what makes a rectangle a rectangle" (Van de Walle 2001:309). Students at this level identify a figure by its properties, which are seen as independent of one another (Pegg& Davey, 1998). Class inclusion is not yet understood. students are provided with opportunities- to measure, colour, fold, cut, model, and tile in order to identify properties of Figures(Figure 2.1) and other geometric relationships and property cards for someone who has never seen one (Figure 2.2).

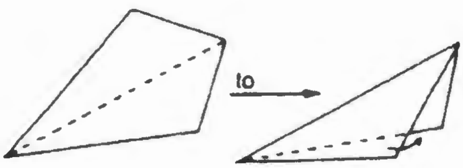


Figure 2.1

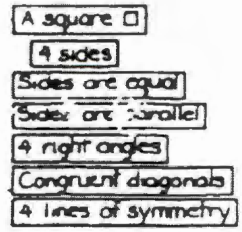


Figure 2.2

Level 3-Informal Deduction

Students at this level discover and formulate generalisations about previously studied properties and rules and develop informal arguments to show those generalisations to be true (Malloy, 2002:176). They no longer see properties of figures as independent. They recognise that a property proceeds or follows from other properties. They also understand relationship between different figures (Pegg& Davey, 1998:132). But the role and importance of formal deduction, however, is not yet understood (Mason, 1998:143). Using property cards: to identify minimum sets of properties that describes a figure.

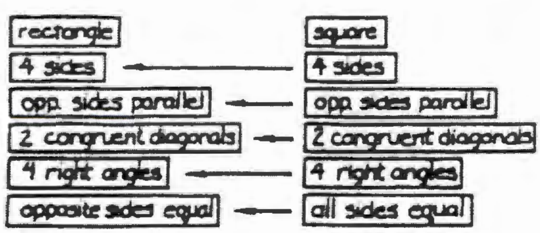


Figure 3.1

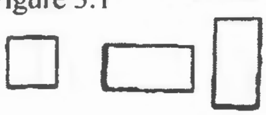


Figure 3.2

Level 4-Deduction:

Students at this level prove theorems deductively and understand the structure of the geometric system (Fuys et al. 1988; Malloy, 2002). The students reason formally within the context of a mathematical system, complete with undefined terms, axioms and underlying logical system, definitions, and theorems (Burger & Shaughnessy, 1986). They use the concept of necessary and sufficient conditions and can develop proofs rather than learning by rote. They can devise definitions (Pegg & Davey, 1998), and are able to make conjectures and prove them (De Villiers, 2003:133). The nature of deduction is understood as providing students opportunities to identify what is given and what is to be proved in a problem.

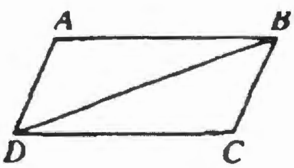


Figure 4.1

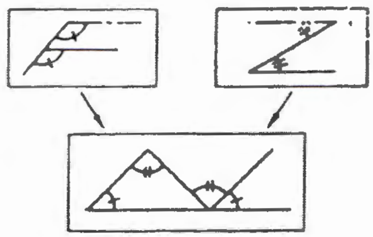


Figure 4.2

Level 5- Rigour

This is the highest level in the Van Hiele hierarchy (Teppo, 1991). Students at this level can establish theorems in different systems of postulates and compare and analyse deductive systems (Fuys et al. 1988; Malloy, 2002). Clements and Battista (1992:429) argued that "children initially perceive geometric shapes, but may attend to only a subset of a shape's visual characteristics and they are unable to identify many common shapes".

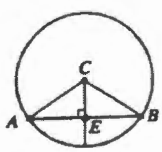


Figure5.1

figure 5.2

In Figure 5.1, learners are asked to follow deductive arguments, perhaps supplying a few missing steps. For example, C is the centre of the circle, why is —

A. $AC = BC$

B. $< CAB = < CBA$

C. $\Delta ACE \equiv \Delta BCE$

D. $AE = EB$

Students are able to devise a formal geometric proof and to understand the process employed (Van Hiele, 1986: 86). For this level, the following activities can be introduced during the instruction for developing formal deductive reasoning and for applying what they understand about the centroid to more open-ended problems.

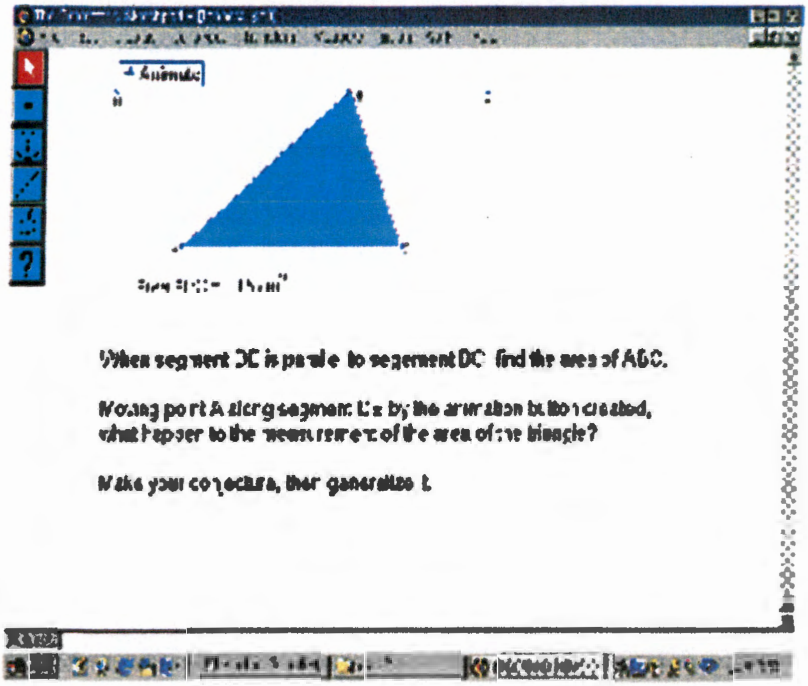


Figure 6

2.2.4 Properties of the Model

In addition to furnishing insights into the thinking that is specific to each level of geometric thought, Van Hieles identified some generalities that characterize the model. These properties are particularly significant to educators because they provide guidance for making instructional decisions.

Characteristics of the Levels

There are five characteristics of the levels:

1. Sequential.

As with most developmental theories, a person must proceed through the levels in order. To function successfully at a particular level, a learner must have acquired the strategies of the preceding levels.

2. Advancement.

Progress (or lack of it) from level to level depends more on the content and methods of instruction received than on age, where no method of instruction allows a student to skip a level: some methods enhance progress, whereas others retard or even prevent movement between levels. Van Hiele points out that it is possible to teach "a skilful pupil abilities above his actual level, like one can train young children in the arithmetic of fractions without telling them what fractions mean, or older children in differentiating and integrating though they do not know what differential quotients and integrals are" (Freudenthal, 1973: 25). Geometric examples include the memorisation of an area formula or relationships like "a square is a rectangle." In situations like these, what has actually happened is that the subject matter has been reduced to a lower level and understanding has not occurred.

3. Intrinsic and extrinsic.

The inherent objects at one level become the objects of study at the next level. For example, at Level 1, only the form of a figure is perceived. The figure is of course, determined by its properties. But it is not until Level 2 that the figure is analysed, and its components and properties discovered.

4. Linguistic.

Van Hiele noted that "each level has its own linguistic symbols and its own systems of relations connecting these symbols" (1984a: 246). Thus a relation that is correct at one level may be modified at another level. For example, a figure may have more than one name (class inclusion) a square is also a rectangle (and a parallelogram). A student at Level 1 does not conceive of the fact that this kind of nesting can occur. This type of notion and its accompanying language is however, fundamental at Level 2.

5. Mismatch.

If the student is at one level and instruction is at a different level, the desired learning and progress may not occur. In particular, if the teacher, instructional materials, content, vocabulary and so on are at a higher level than the student's, they will not be able to follow the thought processes being used.

2.2.5 Phases of Learning

In proceeding to a new level of thought the students must study relationships between the object of their study at the current level and attempt to discover the relationships of the network which will be the object of study at the new level. To move from one level to another, a student must sequentially experience five phases. These five phases are: information, directed orientation, free orientation, and integration. After the completion of these five phases, the student arrives at the next level. Forcing a particular student to a level when they are not ready is cautioned against by Van Hiele, because, any student forced to a level which they are not ready for will only involve the imitation of their teacher's. work with no meaning (Van Hiele, 1986:310).

The study was further framed by the Van Hiele's (1959) phases of learning. To help students progress from one level to the next, the Van Hiele proposed a sequence of five phases of learning, or "phase-based instruction" (Hoffer & Hoffer, 1992:11-13; Van Hiele, 1959/1984, 1986; Van Hiele-Geldof, 1959/1984):

Phase 1: Enquiry /Information

At this stage, the teacher and students engage in conversation and activity about the objects of study for this level. Observations are made, questions are raised, and level-specific vocabulary is introduced (Hoffer, 1983: 208). For example, the teacher asks students, "what is a rhombus: a square? a parallelogram: in what way are they alike, or different?" How are they alike or different?" To gauge whether learners understand that a square could be a rhombus or a rhombus a square? The purpose of these activities is twofold: (1) the teacher learns what prior knowledge the students have about the topic: and (2) the students learn what direction further study will take.

As the teacher engages the students in conversation about the topic of study, evaluates their responses, learns how they interpret the words used and gives them some awareness of why they are studying the topic, so as to set the stage for further study.

Phase 2: Guided/directed orientation.

The students explore the topic of study through materials that the teacher has carefully sequenced. These activities should gradually reveal to the students the structures characteristic of this level. Thus, much of the material will be short tasks, designed to elicit specific responses. For example, the teacher might ask students to use a GSP to construct a rhombus with equal diagonals, to construct another that is larger, to construct another that is smaller. Another activity would be to build a rhombus with four right angles, then three right angles, two right angles, one right angle (Van Hiele, 1999:316).

Next, students actively explore the topic of study by doing short (often one-step) tasks designed to elicit specific responses. These steps help students acquaint themselves with the objects from which geometric ideas are abstracted.

Phase 3: Explication.

Building on their previous experiences, students express and exchange their emerging views about the structures that have been observed. Other than to assist students in using accurate and appropriate language, the teacher's role is minimal. It is during this phase that the level's system of relations begins to become apparent. Continuing the rhombus example, students would discuss with each other as well as the teacher what figures and properties emerged in the activities above.

In this phase, students learn to express their opinions about the structures observed during class discussions. The teacher leads students' discussion of the objects of study in their own words, so that students become explicitly aware of the objects of study. Then, the teacher introduces the relevant vocabulary.

Phase 4: Free orientation.

The student encounters more complex tasks, with many steps, tasks that can be completed in several ways, along with open-ended tasks. Hoffer notes that, "they gain experience in finding their own way or resolving the tasks by orienting themselves in the field of investigation, many relations between the objects of study become explicit to the students" (Hoffer, 1983: 208). For example, students would complete an activity such as the following:

"Fold a piece of paper in half, then in half again, as shown here (Fig.7). Try to imagine what kind of figure you would get if you cut off the corner made by the folds (Fig.7). Justify your answer before you cut. What type(s) of figures do you get if you cut the corner at a 30^0 angle? at a 45^0 angle? Describe the angles at the point of intersection of the diagonals. The point of intersection is at what point on the diagonals? Why is the area of a rhombus described by one-half the product of the two diagonals?"

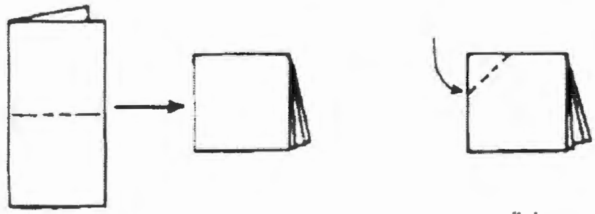


Figure 7

Next, the researcher challenged students with more complex tasks that can be completed in different ways. The researcher encourages students to solve and elaborate on these problems and their solution strategies.

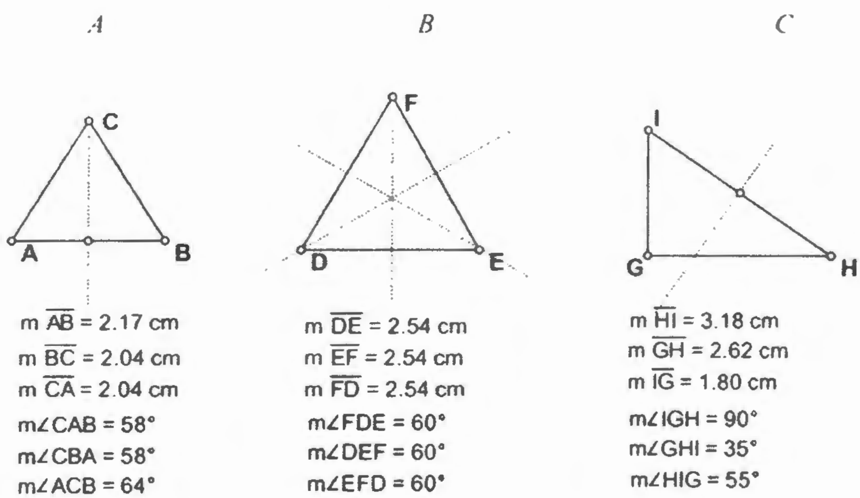


Figure 8

Phase 5: Integration.

The students review and summarise what they have learned with the goal of forming an overview of the new network of objects and relations. The researcher can assist in this synthesis "by furnishing global surveys" (Van Hide, 1984a: 247) of what the students have learned. It is important, however, that these summaries not present anything new. The properties of the rhombus that have emerged would be summarised, and their origins reviewed. At the end of the fifth phase, students have attained a new level of thought. The new domain of thinking replaces the old, and students are ready to repeat the phases of learning at the next level.

The bad results of the teaching of geometry must almost entirely be attributed to the disregard of the levels. The learning process in geometry covers many levels, but appreciation of these levels still need to be emphasised during teaching in the classroom. It is through the disregard of the hierarchical nature of the levels with the teacher and the students operating at different levels that account for much of the difficulties students have in the process of learning geometry. Pierre van Hiele observed that two persons who are reasoning at the different levels will not understand each other. The teacher and the other students who progressed to a higher level seem to speak the same language, which cannot be understood by the student who has not yet reached that level (Van Hiele, 1984:247).

They students might accept the explanation of the teacher, but the concept taught will not sink into their minds. The students themselves will feel perhaps that they can imitate certain action, but they have no view of their own activity until they have reached the new levels (Van Hiele, 1986:215). Van Hiele further explain that if a student is at the zero level and the teacher speaks on the first or even second level, the student does not understand the teacher. The teacher would think s/he had made it very simple and plain, but the student acts as though the teacher was talking abstract. At this point in time, the teacher may feel helpless, and if so, the teaching process can come to a standstill (Van Hiele, 1986:233).

The above Van Hiele findings clearly indicated that the teacher and the students have a problem in communicating, because they are on different levels. This may result in frustrations and discouragement on the part of the teacher and the students. As a way forward, the researcher thinks the frustration experienced by the teacher and the students' resulting from communication breakdown, pointing to the necessity in teacher education to prepare teachers to understand these levels, and to equip prospective teachers with all necessary skills needed to communicate effectively with students at their own level. These skills are essential for teachers because one of the reasons for communication breakdown is the difference in the language used for the different levels. Each level has its own set of language and symbols, and its own network of relationships connecting the symbols.

Van Hiele explained further that another reason why students have difficulties in geometry is that most of them are not adequately prepared for high school geometry. High school geometry is targeted at Level 3, and most students were at lower levels. Van Hiele's theory differs from Piaget's, in that Van Hiele advocated that the progress from one level to the next is not biologically determined, but can be accelerated by appropriate pedagogical intervention. However, Van Hiele believes that learning should shift from being merely teacher directed and students must be encouraged to work independently in a problem-solving situation (Van Hiele, 1986:97).

2.2.6. STUDIES RELATED TO THE VAN HIELE'S MODEL

Studies related to the Van Hiele's model have been carried out currently and in the past by researchers such as Soon (1992), Pegg (1991), Corley, Ted (1990), Mayberry (1981), Fuys, Geddes and Tischler (1985:310). These studies used the Van Hiele's levels of learning geometry to investigate, and evaluate students' understanding of geometry. These researches revealed that the Van Hiele model of development in geometry serves as a useful frame of reference when analysing student's reasoning processes in geometry tasks. Details of some of these Van Hiele's related research are described below.

Soon (1992: 347) investigated Van Hiele's levels of achievements in geometry of secondary school students and the existence of the hierarchy of Van Hiele level of understanding of geometry. An interview and observation technique was used to collect data from a group of about 20 students within the age range of 15 to16. The result of the investigation indicated that the levels as exemplified by the task did form scales, which seemed to support the existence of a hierarchy of the Van Hiele's level for transformation geometry. The study further revealed that students could recognize transformations easily, but they had problems in describing figure.

According to the findings, in terms of tasks for each of the concept strand, students were more successful for tasks in reflection and least for enlargement. Students in the study generally did not know the rigor of proofs. Analyses from the interview indicated that students did proofs by giving particular examples. This suggested to the researcher that students' response to the interview reveal rote learning (Soon, 1992:347). Similarly, the Chicago Project was fashioned to test the ability of the Van Hiele's theory to describe and predict the performance of students in secondary school geometry (Usiskin, 1982:109).

Approximately 2900 students from six different states in the USA were involved in this study. Four tests were administered in this project, they included:

- a multiple-choice test that was used to test prerequisites of high school geometry administered as pre-test and post-tests:
- multiple choice test associated with the Van Hiele levels, was also administered as pre-and post-test:
- a proof writing ability test was administered after a year of high school geometry and finally
- a commercial standardised geometry test on geometry achievement was given as a post-test.

Mayberry (1981:250) studied the Van Hiele's level of geometry thought of undergraduate pre-service teachers. He looked at the hierarchical nature of the Van Hiele levels. The study developed test items corresponding to the Van Hiele model on seven concepts in geometry which include, 'square, circle, isosceles triangle, right triangle, parallel lines, similar figures and congruent figures'. The items were validated by thirteen (13) mathematician and mathematics teachers. They were then revised and administered to nineteen (19) pre-service elementary teachers through interviews.

The result of the study confirmed that the Van Hiele's levels formed a hierarchy, and that her students could be assigned a level. However, there was no consensus across concepts, implying that students could be at different levels for various concepts (Mayberry, 1981: 259). In a similar study, Denis (1987:280) also investigated the relationships between Piagetian stage of development and Van Hiele level of geometry thought among Puerto Rican adolescents. His study showed that Van Hiele levels are hierarchical among subjects in the formal operational stage of development. Denis also found no consensus across concepts in the Van Hiele levels.

Denis (1987:309) and Mayberry's (1987:58-59) studies greatly favoured the Van Hiele model in the study of geometry. The hierarchical nature of the Van Hiele's levels exists and the levels appear to be useful in explaining student's thinking processes in geometry. The Van Hiele's theory explains the behaviour of students in learning and provides guidelines to diagnose the difficulties experienced by students in solving geometry problems (Denis, 1987:347). However, Burger (1983:31-34) recommended using the model for the investigation of students' responses on other mathematics topics, and suggested its use in the study of geometry.

2.2.7. The Effectiveness of Van Hiele's Phases on Learning Geometry using GSP

There are few past studies that tested the effectiveness of the activities based on Van Hiele's phases of learning geometry in a technological environment. Choi-Koh (2000:301) had developed activities based on Van Hiele's phases of learning geometry using GSP software. Those activities were done by the student with the aid of Geometer's Sketchpad software (GSP) and they covered the topic of types of triangles. Penelope (2008:196), on the other hand, has executed a project that used these phases of learning approach by inserting the elements of technology to assist the teaching and learning geometry process in Mathematics class.

The topic involved in the study was Space and Geometry where the subtopic being stressed was "classifying, building and identifying traits of triangles and quadrilaterals" and "proving the traits of quadrilaterals". According to the results of her studies, Van Hiele's phases of learning were an effective work frame in organising activities that used dynamic geometry software.

The students also showed continuous active participation in doing their tasks and they also interacted among themselves, where this situation caused the change of language register from informal to formal. Besides that, other software such as the spreadsheet that was used for recording purposes also played a crucial part in making the role of the dynamic geometry software more effective.

However, there are few factors that affect the usage of information technology in the education field. Factors like personality, attitude and environment are known to have positive relation with ICT usage in the classroom. On the other hand, low level of knowledge and skills coupled with limited resources were known to be the deterrent factor for successful ICT usage in the classroom (Norizan & Mohamed Amin, 2007:91-97). The deterrent mentioned above can be overcome through training or participating in professional development programme (Frey & Fisher, 2009:674-680). However, this was not the case as educators in the secondary school do not face obstacles in their teaching.

According to a study by Effandi et al. (2007a: 1-14), two factors have been identified as the main factors in the application of technology in the teaching and learning of mathematics. The first factor is the teachers' perception that the use of technology is not able to help in the teaching and learning of mathematics. This was further worsened by the fact that teachers always claim that they do not have sufficient time to prepare for ICT integrated lessons.

2.2.8 CONCLUSIONS

The model of geometric thought and the phases of learning developed by the Van Hieles proposed a means for identifying a student's level of geometric maturity, and suggest ways to help students to progress through the respective levels. Instruction rather than maturation is highlighted as the most significant factor contributing to this development. Research has supported the accuracy of the model for assessing students' understanding of geometry (Burger, 1985; Burger and Shaughnessy 1986; Geddes et al. 1982; Geddes, Fuys, and Tischler, 1985; Mayberry, 1981; Shaughnessy and Burger 1985; Usiskin, 1982).

It has also shown that materials and methodology can be designed to match levels and promote growth through the levels (Burger, 1985; Burger and Shaughnessy, 1986; Geddes et al.1982; Geddes, Fuys, and Tischler 1985; Shaughnessy and Burger, 1985). The present task is for mathematics' teachers and researchers to refine the phases of learning, develop Van Hiele-based materials, and implement those materials and philosophies in the classroom setting. Geometric thinking can indeed be accessible to everyone.

Although Piaget's theory was based on maturation, Van Hiele's levels are not age-dependent. In addition, the students will be at different stages of development. However, the study makes an assumption that a good and well-planned geometry lesson need to be accessible to all students, and in the process allowing them to work at their own level of development and cognition. It is further argued in the study that any form of teaching intended to foster development from one level to the next should include sequences of activities, beginning with an exploratory phase, gradually building concepts and related language, and culminating summary activities that assist learners in integrating what they have learned into what they already know.

CHAPTER 3: RESEARCH DESIGN AND METHODOLOGY

3.1 Introduction

In the previous chapter, the literature review of this study was discussed. This chapter describes the research design and the data collection procedures that were followed in this investigation. It specifically focuses on data collection, data processing, measures to ensure validity and reliability, and ethical measures.

3.2 Research paradigms

Research is a systematic process of collecting and logically analysing data for some purpose. This inquiry of making informed decisions can either adopt one of the following methodological paradigms, the qualitative approach or quantitative approach. The word "paradigm" as used in this context is, according to Maree (2010), referring to a set of assumptions or beliefs about aspects of reality which gives rise to how we view reality.

3.2.1 Logical Positivist and Post positivist paradigms

Logical positivism emphasised that there is a single reality within known probability, objectivity, empiricism, and numbers (MacMillian & Suhumacher 2014:13; Creswell, 2009). He (Creswell) further argued that, post positivists hold a deterministic philosophy in which causes probably determines effects or outcomes. Thus, the problems studied by post positivists reflect the need to identify and assess the causes that influence outcomes such as found in experiments. It is also reductionistic in that the intent is to reduce the ideas into a small, discrete set of ideas to test, such as the variables that comprise hypotheses and research questions: The knowledge that develops through a post positivist lens is based on careful observation and measurement of the objective reality that exists "out there" in the world. Thus, developing numeric measures of observations and studying the behaviour of individuals becomes paramount for a post- positivist. Finally, there are laws or theories that govern the world, and these need to be tested or verified and refined so that we can understand the world. Thus, in the scientific method the accepted approach to research by post positivists, an individual begins with a theory, collects data that either supports or refutes the theory, and then makes necessary revisions before additional tests are made. The following are the assumptions made by Phillips and Burbules (2000) about post positivists, adapted from (Creswell 2009):

1. Knowledge is conjectural (and antifoundational) absolute truth can never be found. Thus, evidence established in research is always imperfect and fallible. It is for this reason that researchers state that they do not prove a hypothesis; instead, they indicate a failure to reject the hypothesis.

2. Research is the process of making claims and then refining or abandoning some of them for other claims more strongly warranted. Most quantitative research, for example, starts with the test of a theory.

3. Data, evidence, and rational considerations shape knowledge. In practice, the researcher collects information on instruments based on measures completed by the participants or by observations recorded by the researcher.

4. Research seeks to develop relevant, true statements, ones that can serve to explain the situation of concern or that describe the causal relationships of interest. In quantitative studies, researchers advance the relationship among variables and pose this in terms of questions or hypotheses.

5. Being objective is an essential aspect of competent inquiry; researchers must examine methods and conclusions for bias. For example, standard of validity and reliability are important in quantitative research.

3.2.2 Interpretivists/constructivists paradigm

Interpretive/constructivist researchers use systematic procedures but maintain that there are multiple socially constructed realities (unlike post positivism, which postulates a single reality). Rather than trying to be objective, the researcher's professional judgments and perspective are considered in the interpretation of data (MacMillan & Suhumacher, 2014; Sepeng, 2010). For the Interpretivists, there is more emphasis on values and contexts than on data (Creswell, 2009; MacMillian & Suhumacher, 2014).

3.2.3 Transformative paradigm

In this paradigm, emphasis is on social, political, cultural, gender and ethnic factors as the significant contribution to the design and interpretation of studies (Mertens, 2005). In this worldview, humans engage with their world and make sense of it based on their historical and social perspectives-we are all born into a world of meaning bestowed upon us by our culture. According to (Crotty, 1998), qualitative researchers seek to understand the context or setting of the participants through visiting this context and gathering information personally. They also interpret what they find, an interpretation shaped by the researcher's own experiences and background. The basic generation of meaning is always social, arising in and out of interaction with a human community. The process of qualitative research is largely inductive, with the inquirer generating meaning from the data collected in the field Creswell 2009, 2012).

3.2.4 Pragmatic paradigm

There is a belief that the scientific method by itself is insufficient rather, common sense and practical thinking are used to determine the best approach (e.g., quantitative, qualitative), depending on the purpose of the study and contextual factors (MacMillian & Suhumacher, 2010, 2014). According to (Creswell, 2009), Pragmatists do not see the world as an absolute unity. In a similar way, mixed methods researchers look to many approaches for collecting and analysing data rather than subscribing to only one way (e.g., quantitative or qualitative). The pragmatic researchers are against a positivist position, namely that the truth about the real world can be accessed by a single scientific method (Mertens, 2005). The pragmatist researchers look for the 'what' and 'how' to research, based on the intended consequences- where they want to go with it (MacMillan & Suhumacher 2014). Mixed methods researchers need to establish a purpose for their mixing, a rationale for the reasons why quantitative and qualitative data need to be mixed in the first place.

The pragmatic research worldview is embraced by many researchers as the worldview or paradigm of mixed methods research (Creswell & Plano Clark 2011:41-43). To this effect, the researcher identified with a pragmatic worldview for this study. This means that instead of focusing on the research methods, the researcher emphasised the consequences of the research, used multiple methods of data collection to answer the research questions, at the same time abiding by ethical considerations and practical standards. The pragmatic worldview is inherent in this study because a problem-centred teaching and learning environment was created and the intent was to answer the research questions by whatever ethical or practical means available. Therefore, it is the aim of this study to explore the benefits of using problem-centred approach in the teaching and learning of geometry in grade10 classroom using mixed method research. Hence, both positivist and interpretivist paradigms seem to be appropriate framework within which to show the intent, motivation and expectation of the study. The use of several approaches and methods leads to a better understanding of the issue under investigation (Cohen, Marion & Morrison, 2003:31-34).

3.3 Qualitative methods

Qualitative research begins with assumption, a worldview, the possible use of a theoretical lens, and the study of research problems inquiring into individual or groups ascribe to a social or human problem (MacMillian & Suhumacher, 2014:344). Assumptions and worldview that used in qualitative studies are different from those of quantitative research but also different among variations of qualitative studies. Qualitative research is based more on constructionism, which assumes that multiple realities are socially constructed through individual and collective perceptions of the same situation (Creswell, 2009). According to (MacMillian & Suhumacher, 2014; Creswell, 2009, 2013), the main characteristics of qualitative research are as follows:

- Natural setting: Study of behaviour as it occurred naturally.
- Context sensitivity: Consideration of situation factors
- Direct data collection: Researcher collects data directly from the source.
- Rich narrative description: Detailed narrative that provide in-depth understanding of behaviour.
- Process orientation: Focus on why and how behaviour occurs.
- Inductive data analysis: Generalisations are induced from synthesizing gathered information.
- Participant perspective: Focus on participants' understanding, descriptions, labels and meanings.
- Emergent design: The design evolves and changes as the study takes place.

In this study, the information gathered was by actually engaging or interacting with, talking directly to the participants, and seeing them behave and act within their context, which Creswell (2009) refers to as a major characteristic of qualitative research. In the natural setting, the researcher had face-to-face interaction with the participants over time.

In the research literature (Cohen, Manion & Morrison, 2000; Leedy & Ormrod, 2005 and Creswell, 2007, 2009 & 2012) six types of qualitative research designs are often discussed: Conceptual Studies, Historical Research, Action Research, Case Study Research, Ethnography and Grounded Theory.

Conceptual research is largely based on secondary sources and it critically engages with understanding of concepts and it aims to add to the existing body of knowledge, whereas historical research is a systematic process of describing, analysing and interpreting the past based on information selected from sources as they relate to the topic under review.

Case study design is, according to Bromley (1990), a systematic inquiry into an event or a set of related events which aims to describe and explain the phenomenon of interest and it does not differ much from Action Research which draws attention to collaborative or participative dimensions and to the focus on practical problem experienced by participants for which the practical solution is sought.

Ethnography, as a term, has traditionally been associated with anthropology and more specifically social and cultural anthropology. In the field of anthropology, ethnography means the description of a community or a group that focuses on social systems and cultural heritage.

Action Research draws attention to collaborative or participative dimensions and to the focus on practical problem experienced by participants for which the practical solution is sought. As a research design, it often utilises both quantitative and qualitative data, but they focus more on procedures useful in addressing practical problems in schools and in classrooms (Creswell, 2012). It is a systematic procedure done by teachers or other individuals in the education setting to gather information about and subsequently improve the ways their particular educational setting operates, their teaching and their student learning (Mills, 2011)

Action Research is typically cyclical in terms of data collection and analysis (Maree, 2010). It starts with identifying the problem, collecting data (through the use of a variety of data gathering techniques), analysing data, taking action to resolve the problem and assessing/evaluating the outcome of the intervention.

Grounded theory is a strategy of inquiry in which the researcher derives a general abstract theory of a process action or interaction grounded in the views of participants (Creswell, 2007, 2009 & 2012). This process involves using multiple stages of data collection and the refinement and interrelationship or categories of information (Channaz, 2006: Strauss and Corbin, 1990, 1998). Two primary characteristics of this design are the constant comparison of data with emerging categories and theoretical sampling of different groups to maximise the similarities and the differences of information.

Phenomenological research is a strategy of inquiry in which the researcher identifies the essence of human experiences about a phenomenon as described by participants. Understanding the lived experiences marks phenomenology as a philosophy as well as a method, and the procedure involves studying a small number of subjects through extensive and prolonged engagement to develop patterns and relationships of meaning (Moustakas, 1994). In this process, the researcher brackets or sets aside his or her own experiences in order to understand those of the participants in the study (Nieswiadomy, 1993).

3.4. Quantitative Methods

Quantitative research designs emphasize objectivity in measuring and describing phenomena (Johnson & Christensen, 2012; MacMillian & Suhumacher, 2014). In quantitative research, statistics are used to determine whether a relationship exists between two or more variables (MacMillian & Suhumacher, 2014; Creswell, 2009, 2013). Quantitative approaches follow a positivist paradigm that science quantitatively measures independent facts about a single apprehensible reality (Healy & Perry, 2000; Tashakkori & Teddlie, 2010). The objective of the quantitative aspects of this study is to provide data for triangulation with the qualitative data generated in order to attempt to answer the main research-question:

"What are the benefits of using problem-centred approach in teaching and learning of quadratic equations in Grade 10 classroom?"

3.5 Mixed Methods

3.5.1 Sequential design

According to (Creswell, 2009:207; MacMillan & Suhumacher, 2014:431) the procedures are those in which the researcher seeks to elaborate on or expand on the findings of one method with another method. This may involve beginning with a qualitative interview for exploratory purposes and following up with a quantitative, survey method with a large sample so that the researcher can generalize results to a population. Alternatively, the study may begin with a quantitative method in which a theory or concept is tested, followed by a qualitative method involving detailed exploration with a few cases or individuals. The Fig 9.1 and Fig 9.2 below shows both the explanatory and exploratory design respectively.

Fig 9.1 sequential Explanatory design

Fig 9.2 Sequential Exploratory design

3.5.2 Triangulation design

This is a procedure in which the researcher converges or merges quantitative and qualitative data in order to provide a comprehensive analysis of the research problem. In this design, the investigator collects both forms of data at the same time and then integrates the information in the interpretation of the overall results (Creswell, 2009:207; MacMillan & Suhumacher, 2014:431). Also, in this design, the researcher may embed one smaller form of data to another larger data collection in order to analyse different types of questions (the qualitative addresses the process while the quantitative the outcomes). The figure below shows the triangulation design layout, Quan and Qual stands for (quantitative data and qualitative data respectively)

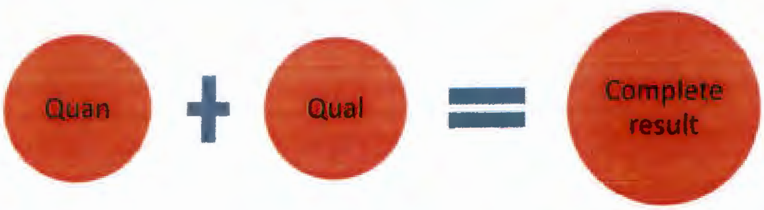


Fig 9.3 Triangulation design

3.6 Research Design

Research design is the plan and procedures for the study, providing the overall framework for collecting the data. It outlines the detailed steps of the study and provides guidelines for systematic sampling techniques, the sample size, instruments and data gathering decisions from broad assumptions to detailed methods of data analysis (Creswell, 2009:116-118). Further, the research design encompasses all the structural aspects of a study (Gay & Airasian, 2000:109-117). In this study a mixed methods approach is followed. Mixed methods research intentionally engages a multiple set of approaches; all approaches are valuable and have something to contribute to understanding. The use of several approaches and methods leads to a better understanding of the issue under investigation (Cohen, Marion & Morrison, 2003:31-34). Hence, a mixed-method approach using both quantitative and qualitative investigation is employed. The two approaches are seen as complementary and the limitations of one approach can be offset by the advantages of another (Creswell, 2009:204-226). According to Cohen, Marion and Morrison (2003:24-28), "use of both forms of data allows researchers to simultaneously make generalizations about a population from the results of a sample and to gain a deeper understanding of the phenomena of interest".

Creswell and Plano Clark (2011) further stress that the use of quantitative and qualitative approaches combined gives more evidence and a better understanding of the research problem than either approach by itself. The mixed methods research approach provides a comprehensive account to the research questions of the study since it draws on the strengths of both methods; therefore this study used the mixed methods design in order to capitalise on the strengths of each approach so as to sharpen the research findings. The researcher realised that one data source may be insufficient to outline the benefit of using problem-centred teaching and learning approach in teaching Euclidean geometry in Grade 10 classroom.

In this study, the mixed methods research approach assisted the researcher in answering questions that could not be answered by qualitative or quantitative research alone. The other reason for combining both quantitative and qualitative data is to better understand this research problem by converging both quantitative (broad numeric trends) and qualitative (detailed views) data Creswell, 2009:123).

For example, students journals afforded the researcher the opportunity to gain insight into the reasoning and cognitive processes of learners, for which quantitative research alone would have been insufficient. Lastly, the researcher used mixed methods design to enhance the credibility of the findings. Creswell and Plano Clark (2011:62) advocate that a research design that integrates different research methods is more likely to produce better results in terms of quality and scope. Creswell and Plano Clark (2011:13-16) point out the challenges of using the mixed methods research design. One such challenge is that it requires extensive time, resources and effort on the part of researchers. For this study, enough resources were made available and problems with time were factored into the programme. Another challenge is the question of skills for doing the mixed methods research design. To overcome this particular challenge, the researcher extensively familiarised himself with both quantitative and qualitative research methods separately before undertaking the mixed methods research design.

Creswell and Plano Clark (2011:68-104) state six major types of mixed methods research design: The explanatory sequential design, exploratory sequential design, convergent design, embedded design, transformative design and multiphase design. The convergent research design was employed for this study and it is discussed in the next section.

3.6.1 Design type

3.6.1.1 Convergent design:

This design was originally conceptualised as a "triangulation" design (Creswell & Plano Clark 2011:77). Creswell and Plano Clark (2011), define the convergent design as the process in which the researcher collects and analyses both quantitative and qualitative data during the same phase of the research process and then merges the two sets of results into an overall interpretation. This view concurs with that of McMillan and Schumacher (2006:404), who state that in a triangulation design (convergent research design), the researcher simultaneously gathers both quantitative and qualitative data, merges them using both quantitative and qualitative analysis and then interprets the findings together to provide a better understanding of the research problem. In this design, both quantitative and qualitative data are collected and given equal emphasis thereby allowing the researcher to combine the strengths of both methods. For this study the quantitative research methods and qualitative research methods occurred concurrently in all phases and the researcher equally prioritised both methods. The researcher kept the data collection and analysis independent and then mixed the findings during the overall interpretation.

The convergent notation for this study is QUANTITATIVE + QUALITATIVE = complete understanding (Creswell & Plano Clark 2011:77-78).

3.6.1.2 Why the convergent design?

The aim of the convergent design is "to obtain different but complementary data on the same topic" (Morse, 1991:122) in order to best understand the research problem. The researcher also felt that there was equal value in collecting and analysing both quantitative and qualitative data at the same time in order to explore the benefits of using PCTL approach in teaching Euclidean geometry in Grade 10 classroom(s). The researcher opted for the convergent design because it makes "intuitive sense" to him and it is an "efficient design"

(Creswell & Plano Clark 2011:78) with which he was able to collect both types of data simultaneously. During lessons, the researcher was able to carry out video observation during the lessons and questioning during the discussion with each group in the class (qualitative research) while the students were solving mathematical tasks (quantitative research). The convergent design also helped the researcher "to directly compare and contrast quantitative statistical results with qualitative findings" (Creswell & Plano Clark 2011:77) in order to elaborate well-substantiated conclusions about the benefits of using PCTL approach to teach geometry in Grade 10 classroom(s). The Figure below shows the detailed convergent design.

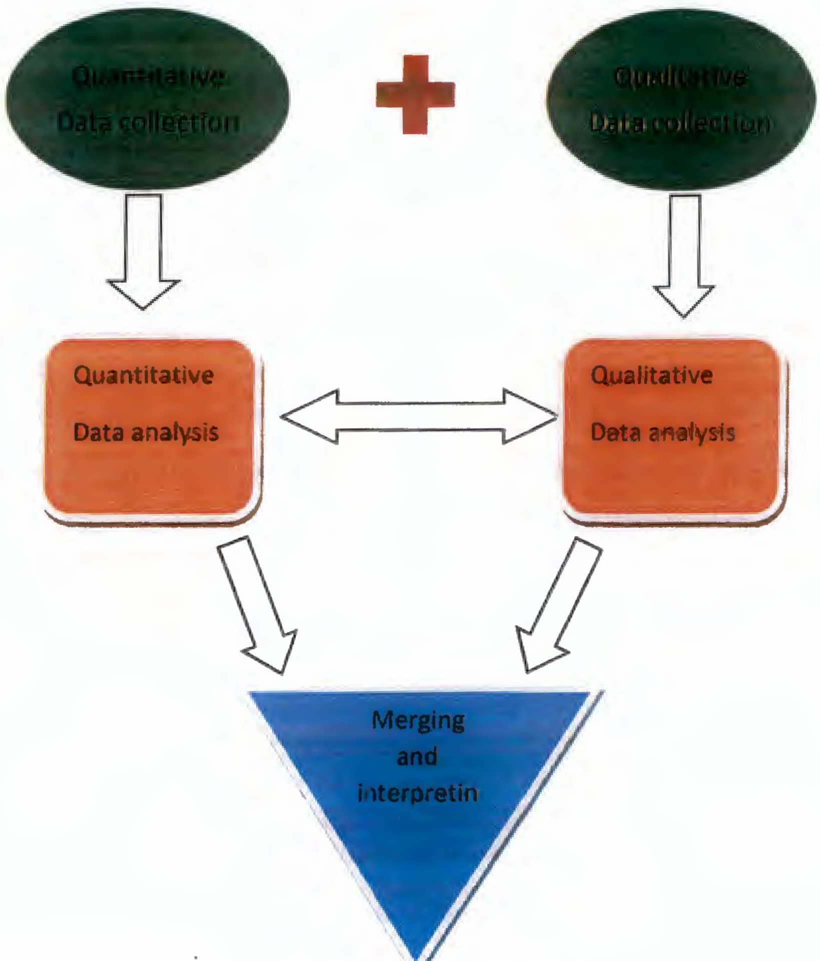


Fig 9.4 convergent design

3.6.1.3 The programme of intervention

It is important to note that in this study the researcher acted both as a researcher and teacher of the grade-11 mathematics class. This is because many teachers in South Africa do not know yet how to effectively implement the PCTLA. To ensure that a PCTL environment was created for the learners, the researcher had to teach the learners himself. The role of the researcher as teacher was that of facilitator. During class or group discussions, the researcher intervenes with questions to probe for reasoning and explanations. The researcher initiated and moderated class discussions and maintained a spirit of enquiry and critical reasoning during problem solving sessions. The questionnaire, the pre- and post-tests were administered to all the participants at the beginning and at the end of the intervention. The researcher started the intervention programme by explaining to them what the Problem-Centred Teaching and Learning Approach (PCTL) entails. As presented by the grade 10 mathematics CAPS (Curriculum Assessment Policy Statement) documents. The researcher created a classroom environment in which social interaction was highly valued. This is an environment in which learners believed that what was important was the effort they spent looking for solutions and that they would have learnt something even if they did not find the correct solution to the given problem.

Polya's four-phase problem solving process used to solve the problems and tasks for this intervention.

1. Problem posing
2. Understanding the problem
3. Looking back Making a plan
4. Carrying out the plan

There were no formal lessons during the problem solving. After grappling with unfamiliar problems, students were required to place their solutions on the board and to fully explain their work to the class. Other students in the class were encouraged to critique the solution and at the same time try to provide alternative solutions to the problem. As the class discussed the solution to the problem, the researcher guided the discussions as needed by asking questions to ensure that students understood the solution before moving on to the next problem.

The researcher also employed teacher-led and student-led groups in his teaching. This was done by forming five groups of four students in each group, for example group A, B, C, D and E. On each day, the researcher would work with each group for about five minutes, questioning them in the process. After the lesson, the researcher would assign follow-up work to each group for discussion on the following day. Decoding of the video recording takes place after the lesson for each day in order to record and analyse the observation in the observation comment cards.

During the intervention programme, the researcher implemented the strategies for developing mathematical problem solving skills as explained in (section 2.2.3) by encouraging students to always look out for patterns, perform thought experiments, explain the solution methods to the teacher or their peers and develop conjectures. The researcher structured learning situations to develop the experimental group's mathematical problem solving skills by implementing the following:

1. When given a new problem, students were encouraged to identify how it was similar or different from previous problems and how this could influence their approach to solving the problem.

2. Students were required to use different problem solving strategies to solve given problems and were encouraged to compare the effectiveness of the different strategies. Discussions were held towards the end of lessons to compare all the different strategies and valid solutions generated by all students.

3. The researcher encouraged students to value the problem solving process by implementing the techniques stated in (section 2.2.3).

4. Students were encouraged to generate their own problems. The action of having students generate their own questions transforms their relationship with authority and tests (Holt, 1968) and at the same time affords them the opportunity to develop mathematical problem solving skills.

5. Students were given enough time to think before responding to questions. Providing students with waiting time before answering questions helps to develop their mathematical problem solving skills since they have the opportunity to think deeply about the problem at hand.

3.6.2 Participants

The participants in this study were 80 Grade 10 students from a public high school in a rural part of Bojanala district, about 25km from Rustenburg, North West Province. The participants' ages range from 16 to 21. Furthermore, all of the participating learners are Black (African) 24 males, and 56 females in mathematics class. However, the participants of the study cannot be considered representative of all Grade 10 learners in Bojanala district, as certain Secondary Schools are resourcefully well equipped than others. Therefore, the results of this study should be interpreted accordingly.

3.6.3 Data generating instruments.

The English language was chosen as the research language because it is the languages of teaching and learning (LoLT) in South African schools. At the time of data collection. the participants had already studied geometry in about 15 lessons at the same Grade 10. The researcher was expected to teach the subject through Van Hiele Geometric Thinking. the recognition. analysis and deduction methods. In addition. the curriculum emphasised teaching symbolic and tabular representations. as well as the graphical representations of quadratic equations with two unknown. Moreover. although the curriculum guidelines explicitly emphasise problem-solving with the use of geometry. they suggest using word problems to find shapes. properties and solution sets of theorems given as riders. Furthermore, the mathematics textbooks used in schools emphasise visualisation and recognition approaches regarding geometry, such as, identifying the figure by its properties and shapes, and to use properties to draw the matching figures.

3.6.3.1 Quantitative data collection (Instrumentation):

1. Pre-test and Post-test.

For this study, data were collected through tools such as, pre- and post-test on geometry (VHGT). A test was administered to all the participants (Grade 10 mathematics classes in the school) at the same time of the same day before the intervention (pre-test) and after the intervention (post-test). The researcher monitored invigilation sessions by availing herself to the classroom that participated in the study. Learners' works were marked using a marking criteria. (See Table1 below) The researcher used 25 VHGT questions adapted from Van Hiele and was developed by Cognitive Development and Achievement in Secondary School Geometry (CDASSG) group from the University of Chicago was given to the supervisor for approval. The instrument with 25 items has five Van Hiele's level of geometric thinking: 1-5, visualisation. 6-10. analysis. 11-15. informal deduction, 16-20, deduction and 22-25. rigour.

Table1. Marking criteria in VHGT

Mark	Criteria of the items to be fulfilled	Van Hiele's levels of geometric thinking
1	1-5	1
2	6-10	2
4	11-15	3
8	16-20	4
16	21-25	5

Source: Usiskin, 1982

3.6.3.2 Qualitative data collection (instrumentation):

To evaluate the development of their mathematical problem solving skills, the researcher questioned students whilst they grappled with problems during several problem- solving sessions. The researcher recorded the responses and findings on the spot using the problem solving comment card. (See appendices O and P respectively).

To gain insight into individual students' development of mathematical problem solving skills, the researcher required students to complete questionnaire, to write a report in their journals about every problem solving experience they completed, and video recording for later analysis.

3.6.3.3 The recording techniques.

As the researcher observed the participants through the recorded video while they solved problems on geometry after each intervention, the findings are briefly and objectively recorded. The recordings included the students' actions and mathematical problem solving skills and the researcher's interpretations of these. The recording techniques used were the problem solving comment card and the problem solving observation rating scale (see appendices O and P). The recording scales were developed by the researcher based on (section 2.2.3 of chapter 2), visualizing/recognizing, focusing, and information-gathering, organising, evaluating, analysing, deducing, postulating and integrating skills that learners should develop. The researcher did not complete a recording scale for every student on a daily basis as this was impractical and unnecessary. The researcher tried to complete one scale sheet for every student at least once a week. A problem solving folder was kept for each student's problem solving comment cards and problem solving rating scales. Summary data from the comment cards, rating scales for every learner were kept in a problem solving evaluation notebook. The data in the problem solving evaluation notebook was of crucial importance to the researcher in making decisions about each student's development of mathematical problem solving skills.

3.6.3.4 Pilot testing the instruments.

Pilot testing of the questionnaire, pre- and post-tests was conducted with 26 grade 10 mathematics students at a neighbouring school. The students were well informed that it was a pilot test and were assured of confidentiality and anonymity. Babbie (2010:98 & 233) points out that piloting of instruments is essential because it improves reliability in that people understand the items or statements in the same way as each other. For this study, piloting tested the wording, language use, the length, clarity and appropriateness of the statements and instructions of the questionnaire, pre- and post-tests. The pilot test also checked if the data that would be obtained from the questionnaire, pre- and post-tests would reflect real understanding of the participants.

The respondents of the pilot test were required to provide feedback on individual items and the whole questionnaire, pre- and post-test. The feedback was used to amend, simplify and clarify some of the items. Adaptations and amendments were made to the questionnaire, pre- and post- tests to make them fully understandable to participants. The researcher was present when the instruments were piloted and responded to any uncertainties the respondents may have had.

3.6.4 Data Analysis

Data analysis is the process of making sense out of data, which involves interpreting, consolidating and reducing what participants have said, how they have responded and what the researcher has seen and read in order to derive or make meaning out of the process. Mouton (2001:108) sees data analysis as "breaking up" data into manageable themes, trends, patterns and relationships. The purpose of data analysis in this study was to provide answers to the research questions through understanding of various constitutive elements of the data. In the next section, procedures of quantitative and qualitative data analysis are discussed.

Researchers go through similar steps for both qualitative and quantitative data analysis. Creswell and Plano Clark (2011:204) list the data analysis steps as follows:

- preparing the data for analysis
- exploring the data
- analysing the data
- representing the analysis
- interpreting the analysis
- validating the data and interpretations

3.6.4.1 Quantitative data analysis:

All participants in this study were administered the pre-test and the post-test by their mathematics teacher and the researcher during regular class time and were given 1hour 45 minutes to complete it. The quantitative data analysis consisted of descriptive statistical analysis (McMillan & Schumacher 2006:153). Univariate analysis techniques include analysis of measures of central tendency (mean), standard deviation, range, and overall test scores. Descriptive statistics transform a set of observations into indices that characterise the data and thus are used to summarise and organise observations (McMillan & Schumacher 2006:150), so that readers can have a mental picture of how the data relates to the phenomena under study. For this study, data from the pre- and post-test were analysed using descriptive statistics including means, standard deviations and independent sample T- tests.

The researcher also categorise learners performance in solving geometry with respect to their use of methods (i.e., identifying shapes, properties, similarities, relationships, theorems and their proofs, riders that include solving and proving a variety of axioms, congruency) to solve geometry problems. Students' answers were coded and categorised with respect to their comprehension of the problem statement, setting up the correct relationship and formulating geometric equations, and solving the formulated geometric equation. Correct, incorrect, incomplete, and blank solutions were some major codes that evolved from the data.

3.6.4.2 Qualitative data analysis:

According to McMillan and Schumacher (2006:364), qualitative data analysis is an inductive process of organising data into categories and identifying patterns among the categories. Creswell and Plano Clark (2011:208) state that qualitative data analysis involves "coding the data, dividing the text into small units, assigning a label to each unit and then grouping the codes into themes." In this study, qualitative data analysis started as soon as data collection began and it was an ongoing process. Qualitative data analysis involved analysing findings from the video recording and questioning that were recorded on the problem solving comment card and problem solving rating scale. Students' journals were analysed by categorising the mathematical problem solving skills that could be identified behind students' learning conceptions.

- **Learner journal**

During the intervention, learners were regularly requested to write a report in their journals on a problem solving experience they had completed. The learners were requested to write briefly on how they solved the geometry problems given to them during each intervention. The researcher used learner journals because they provide information on "individual learners' use of problem solving skills and strategies" (Wheatley 1991:23).

- **Video recording and questioning**

The Grade10 mathematics teacher (the researcher) was responsible for the video recordings; this is done purposely to make sure that the learners do not feel threatened because using someone unfamiliar may cause anxiety. The researcher regularly moved unobtrusively around the classroom while questioning learners as they solved problems in their groups of four learners. The purpose of using video recording and questioning in the qualitative strand of this mixed methods research design was to try and understand how learners in a problem solving situation make sense of the problem solving process. Video recording and questioning also helped the researcher to establish what learners required as a prerequisite for mathematical problem solving skills to develop in solving geometry. The researcher used the video recording and questioning method as a form of measurement because with it, he obtained learners' perceptions of the problem solving processes that were expressed in their actions, feelings, thoughts and beliefs (McMillan & Schumacher 2006:347). McMillan and Schumacher (2006) further point out that the observational method is important because it relies on the researcher's seeing and hearing things and recording these observations, instead of relying on a participant's self-report responses to questions or statements. The researcher asked learners stimulating questions that helped to evaluate each learner's development of mathematical problem solving skills. Below are examples of some questions that the researcher asked.

- What did you do first when you started to solve the problem?

- What do you think is the most important thing in trying to understand the problem?
- Have you used any strategies in solving the problem? Which ones?
- If your chosen strategy failed, what did you do when your strategy failed?
- Are you sure this is the answer to the question?
- Why do you think this is the correct answer?
- Can you describe your solution to the problem?
- How do you feel about your experience with this problem?

In this study, questioning was one of the most useful technique for establishing what grade 10 learners need as a prerequisite for mathematical problem solving skills as this is very important in a problem-centred teaching classroom, to develop and for evaluating their development of these skills, their willingness to try new problems and perseverance in solving quadratic equations. This technique was flexible, allowed each learner to be evaluated at a time and afforded the researcher the opportunity to evaluate mathematical problem solving skills in a natural classroom setting.

- The recording techniques.

As the researcher observed the participants through the recorded video while they solved problems on geometric problems after each intervention, the findings are briefly and objectively recorded. The recordings included the learners' actions and mathematical problem solving skills and the researcher's interpretations of these. The recording techniques used were the problem solving comment card and the problem solving observation rating scale (see appendices O and P). The recording scales were developed by the researcher based on (section 2.2.3 of chapter 2), visualizing/recognizing, focusing, and information-gathering, organising, evaluating, analysing, deducing, postulating and integrating skills that learners should develop. The researcher did not complete a recording scale for every learner on a daily basis as this was impractical and unnecessary. The researcher tried to complete one scale sheet for every learner at least once a week. A problem solving folder was kept for each learner's problem solving comment cards and problem solving rating scales. Summary data from the comment cards, rating scales for every learner were kept in a problem solving evaluation notebook. The data in the problem solving evaluation notebook was of crucial importance to the researcher in making decisions about each learner's development of mathematical problem solving skills.

- **Baseline observations**

One classroom observation was carried out in an experiment, before the researcher introduced the intervention strategy. Through the use of the baseline observation, the type of interactions that exist during classroom practice, the choice, and use of GSP software and how students' answer the geometry problems in the experimental and control classrooms were observed. The observations done revealed the following.

- **Researcher's use of language in the classroom**

The researcher used English, the official language of learning and teaching (LoLT) for both teaching and assessment of activities developed for teaching during a lesson. The LoLT was broadly used for explanation of geometrical terms, clarification of geometric terminologies, and to ask questions and provide feedback to the learners; however, learners were free to communicate with each other using any of the languages known to them during peer-to-peer interactions as described below.

- **Students' use of language in the classroom**

Students used their home languages (Setswana and/or Sesotho) when they solved problems or tasks given in pairs or individually. Students found it difficult to pose questions and build upon previous responses using language that is not their home language. They were instructed to take out their activity books and write; they appeared as if they did not understand what the researcher was saying. They waited for the researcher to code switch from English to their Home language.

- **Teaching methods and learning styles**

The classroom was characterised by the use of textbook and narration methods forming a fundamental approach to the teaching and learning of geometry in this classroom. The researcher employed the chalk-and-talk method, with learners, receiving top-down information. The lessons were dominated by researcher's talk and learners' roles took that of spectators in the learning process.

- **Classroom interactions**

The researcher's teaching approaches and strategies did not promote discussion and argumentation in the classroom. The classroom atmosphere did not provide opportunities for learners to engage in dialogue, where they could agree to disagree in order to reach a common understanding. Forms of interactions in this classroom followed a narration and one-way question and answer approach.

- **Semi-structured Interviews**

The audio tape, pen and paper, verbatim quotes from the students and researcher's notes used during the interviews were also analysed for difficulties experienced by individual students to a particular question or cluster of questions. The interview questions and its analysis focused on the extent to which students can visualize, describe, analyse, abstract relation, and deduction. A student was considered having difficulties in a particular level if he/she fails to meet the performance indicator as described in Table 1.

Consequently, the researcher used the multiple data analysis arising from written test, interviews with reference to students' verbatim quote and notes from paper and pencil. These were compared to find whether a common pattern existed with regards to the difficulties which students had.

3.6.5 Mixed methods data analysis

Mixed methods data analysis includes analysing separately the quantitative data by quantitative methods and the qualitative data using qualitative methods and then merging the two databases. According to Creswell and Plano Clark (2011:212), mixed methods data analysis is when analytic techniques are applied to both quantitative and qualitative data, as well as to the integration of the two forms of data concurrently and sequentially in a single project or multiphase project. In this convergent research design, quantitative and qualitative data were collected concurrently and the researcher analysed the findings separately and then merged the two databases in the results, interpretation and conclusion phase. As suggested by Creswell and Plano Clark (2011:215-216), the convergent research design data analysis took the following steps:

- Quantitative and qualitative data were collected concurrently.
- Separately analysing the quantitative data using quantitative methods and the qualitative data using qualitative methods.
- The quantitative data together with the qualitative data were analysed using a side-by-side comparison for the merged data (Creswell & Plano Clark 2011:223).
- An interpretation was given of how the merged results answered the research questions.

3.6. Pilot testing the instruments.

Pilot testing of the questionnaire, pre- and post-tests was conducted with 26 grade 10 mathematics learners at a neighbouring school. The learners were well informed that it was a pilot test and were assured of confidentiality and anonymity. Babbie (2010:98 & 233) points out that piloting of instruments is essential because it improves reliability in that people understand the items or statements in the same way as each other. For this study, piloting tested the wording, language use, the length, clarity and appropriateness of the statements and instructions of the questionnaire, pre- and post-tests. The pilot test also checked if the data that would be obtained from the questionnaire, pre- and post-tests would reflect real understanding of the participants.

The respondents of the pilot test were required to provide feedback on individual items and the whole questionnaire, pre- and post-test. The feedback was used to amend, simplify and clarify some of the items. Adaptations and amendments were made to the questionnaire, pre- and post- tests to make them fully understandable to participants. The researcher was present when the instruments were piloted and responded to any uncertainties the respondents may have had.

3.7 Test for validity and reliability of measuring instruments

The critical questions that required validating were: Were all the aspects of geometric thinking covered in the tasks (the pre-test and the post-test)? Do the assessment tasks measure the aspects that were assumed central to the learning of geometry?

3.7.1 Reliability

Reliability refers to the degree of consistency with which a data collection tool measures whatever it is supposed to measure. This is the extent to which the data collection tool gives similar results and conclusions if it is administered to a different group of participants under different conditions such as time and venue. This also implies that if the same research is done again under similar conditions, the researcher will obtain the same results and not inconsistent results.

Qualitative reliability means that observations from participants are consistent and stable over time (Creswell & Plano Clark 2011:211). To address this issue, the questionnaire used in this study was used in similar study (Chirinda, 2013) and the reliability coefficients was 0.82 (see Table 3.1) which is excellent for these instruments (McMillan & Schumacher 2006:183; 186-

187). McMillan and Schumacher (2006) go on to state that the Cronbach alpha is generally the most appropriate type of reliability for questionnaires in which there is a range of possible answers for each item and this was the case for the questionnaire used in this study.

The Spearman-Brown formula (McMillan & Schumacher 2006:185) was used to calculate the reliability coefficients of the, pre- and post- tests. The Spearman-Brown coefficients for these instruments were generally above 0.70 which is acceptable for these kinds of instruments.

Instrument	Recording technique	Reliability instrument	Value of reliability instrument
Questionnaire	Likert scale	Cronbach alpha	0.82
Participants observation and questioning	Comment card, checklist and rating scale	Prolonged data collection period	
Pre-test	Analytic scoring scale	Spearman-Brown	0.77
Post-test	Analytic scoring scale	Spearman-Brown	0.77

Table2 Reliability of instruments.

3.7.2 Validity

The definition of validity was provided in the first chapter and its purpose is "to check the quality of the data, the results and the interpretations" (Creswell & Plano Clark 2011:210). Validity determines whether the research truly measures that which it was intended to measure or how trustworthy the research results and findings are. McMillan and Schumacher (2006:134) concur with this when they state that validity refers to the truthfulness of findings and conclusions. It is essential that procedures to ensure the validity of the data, results and their interpretations are utilised. In the next section, quantitative validity and qualitative validity are explored.

3.7.2.1 Quantitative validity

Quantitative validity looks at the quality of the scores that one obtains from the data collection tools and the quality of the conclusions that can be drawn from the results of the quantitative analyses (Creswell & Plano Clark 2011:210). Quantitative validity involves content validity, criterion-related validity, construct validity and external validity.

Content validity is concerned with whether the data collection tools are representative of all possible items. For this study, regarding content validity, the mathematical problem solving skills and other factors explored in the literature review were all represented by the items in the different sections of the questionnaire and the learner journal focus questions.

Criterion-related validity refers to whether the scores relate to some external standard such as scores on a similar instrument. For this study, criteria set in the teachers' guide for Grade 10 mathematics were used as a reference for criterion-related validity.

Construct validity is concerned with whether data collection tools measure what they intend to measure. To strengthen construct validity of this study various methods of assessment, that is, written work and observation and questioning, were used to test learners' achievement and performance in quadratic equations.

In quantitative research, internal validity is the extent to which the researcher can reach a conclusion that there is a "cause and effect relationship among variables" (Creswell & Plano Clark 2011:211). For this study, the selection threat to internal validity was not a factor as there is no comparison group. There were no extraneous events that occurred during the data collection period and this led the researcher to conclude that there was no known history threat to internal validity for this study.

External validity refers to the generalisability of the results, that is, the results and findings can be generalised to a large population. This might be a threat because the chosen school is a rural school with little resources compared to other urban schools in South Africa.

3.7.2.2 Qualitative validity

Qualitative validity is concerned with whether the account given by the researcher and the responses given by the participants are accurate, trustworthy and credible. Internal validity in qualitative research checks whether researchers observe what they think they observe and actually hear what they think they hear. In this research, it was essential that the researcher understood students' responses during the analysis of video observation and also, questioning. To enhance qualitative validity the researcher employed triangulation and "member checks"

(Creswell & Plano Clark 2011:211; McMillan & Schumacher 2006:324). Member checks imply that the researcher went back to participants after the completion of the study to ask participants if the findings were truly what they experienced during the data collection process. However, no changes were provided by the participants to the data that were presented to them during the member checks process.

3.8 Ethical considerations

Ethics deals with the beliefs or guidelines about what is right or wrong, proper or improper, good or bad from a moral perspective (McMillan & Schumacher 2006:142). McMillan and Schumacher (2006) go on to stress that the researcher has a moral obligation and is ethically responsible for protecting participants' rights and welfare, including physical and mental discomfort, harm and danger. For this research, all precautions were taken before the data collection process in order to adhere to the ethical measures to respect the integrity, confidentiality, anonymity, privacy, caring, consent and humanity of the participants. The next section looks at the ethical considerations that were important for this study.

3.8.1 Obtaining informed consent

Informed consent means that the participants have a choice of either participating or not participating in the research. Wiersma and Jurs (2009:456) explain that when human subjects participate in a research study, they should be informed of their role, the procedures, the purpose of the research, the possible risks of the research and they should give their written consent for participation. Concerning obtaining informed consent to collect data, the researcher wrote letters to the school principal, School Management Team (SMT) and School Governing Board (SGB) (see appendix A) seeking permission to collect research data from their school. The school principal and the school governing board willingly gave the researcher approval. However, the letters were not included in the appendices for the sake of anonymity and confidentiality. The researcher received a research ethical clearance certificate which is granted to her by the North West University ethics committee at the time of writing this chapter.

At the onset of the study, the researcher wrote letters to the participants and participants' parents or guardians (see appendices B & C) that clearly explained among other issues, the purpose of the study, their role and voluntary participation. After understanding the content of the letters, the participants and their parents or guardians agreed to participate in the study and gave their written consent by signing the letters.

3.8.2 Voluntary participation

The researcher made sure that the participants were well informed about the purpose of the research, the procedures of the data collection process and the research's possible impact on them. The researcher clearly explained to participants that they were free to decide whether they wanted to participate in the research or not and had freedom to withdraw from the research at any time without incurring any negative consequences. Participants were not deceived in any way and the researcher was open and honest about all aspects of the study.

3.8.3 Confidentiality and anonymity

Confidentiality as explained by Wiersma and Jurs (2009:458) is the act of not disclosing the identity of participants in a research or study and anonymity implies that the names of the participants where data is obtained are unknown. For this study, participants were assured that their confidentiality and privacy would be respected and that their responses would be used for the purpose of the study only. All reasonable efforts and necessary precautions to maintain complete participant confidentiality and anonymity were in place and were enforced. School name, principal name, mathematics teacher name and participant names were eliminated from all reports and pseudonyms were assigned. McMillan and Schumacher (2006:334) stress that the settings and participants should not be identified in print and no-one should have access to participant names except the researcher.

3.9 Summary

The methodology and the research design that were used for this study were explained in this chapter. The research method used for this study was indicated as the mixed methods research design. The type of mixed methods design that was adopted for this study was the convergent research design. The reasons, advantages and challenges of adopting the mixed methods and convergent design were explained. Various data collection tools were used in this study to enhance reliability and validity of the findings. In the next chapter, the researcher analyses and interprets the data and presents the findings.

CHAPTER 4: FINDINGS

4.1 INTRODUCTION

This chapter presents the analysis and interpretation of data obtained from both the pre- and post-tests administered to both experimental and control groups. The aim of the study was to investigate the effective use of GSP in the teaching and learning of geometry in order to address difficulties faced by students in the learning. A detailed analysis and discussion of findings of the tests were analysed in this section of the study.

The quantitative data generated in this study used the methodological tools described in chapter three. Quantitative data generated from pre- and post-tests administered in both experimental (n=40) and control (n=40) groups were described. These data were analysed and subjected to analysis using Wilcoxon t-test techniques to provide descriptive and inferential statistics. As such, the study has made use of quantitative method, where data was analysed in this chapter, and discussed within the theoretical framework provided by the literature review and methodology in prior chapters.

It was evident in the experimental group that the learners had begun to display signs of confidence in and understanding of key aspects of the intervention. They managed to incorporate the strategy learned by GSP from the researcher in their learning styles to mathematics geometry problems. In fact, the researcher engaged learners in new innovative pedagogies that created an atmosphere conducive for the learners to participate actively in open discussion.

4.2 QUALITATIVE RESULTS

4.2.1. Observations during implementation

Classroom observations were done during and after the intervention to reveal the ways in which the use GSP was used by both the researcher and students to implement the intervention strategy used in this study. Teachers and students were observed using a four point scale classroom observation.

The activity of classroom observation was carried out in the experimental group during the implementation of intervention strategy. One of the objectives of this study was to measure the effective use of GSP in Grade 10 students' learning of geometry. In this study, the focus of the researcher was also to observe how the students respond during the implementation of the intervention, and whether the researcher allowed enough opportunities available for learners to engage in the handling of the software during the process of intervention. The data gathered through implementation attempted to respond to the following objectives of this study:

i. Establish the effects of using Geometer Sketchpad on Grade 10 students' academic achievement in Euclidean geometry.

ii. Identify the benefits of using Geometer Sketchpad in the teaching and learning of Euclidean geometry in Grade 10 mathematics classrooms;

iii. Understand the impact of using Geometer Sketchpad on Grade 10 learners` academic achievement in Euclidean geometry.

The data based on classroom observation and handling of software during implementation was analysed as part of the findings of the study but the researcher`s interest.

4.2.2. Students interviews

Learners` interviews were conducted to probe their experiences and perceptions of learners towards the use of GSP in the geometry classroom when they learned and solved problems in geometry. As noted earlier, English appeared to be the students` preferred language for classroom discourse, for example, when they communicate with the researcher, and in cases where they had to present their feedback to the entire classroom.

The students` interview, which was unstructured, was conducted to investigate which method of learning geometry was preferred to use during classroom disposition. The experimental group consisted of eight (n=8) learners, who were interviewed after the intervention. The data collected during unstructured interview were used as part of findings.

4.2.3. The administration of VHGT

As noted earlier, students (n=80) wrote the Van Hiele Geometric Test (VHGT) following a particular order. The tests were administered in an experimental group (40) and control group (n=40) at the same time, to avoid unfair results. In this section, both quantitative and qualitative pre- and post-test results gathered from the two groups are presented.

The Van Hiele Geometry Test is a 25-question multiple choice test. There are two cases to choose from, as well as two criteria to choose from, to determine learners understanding at each level. The Van Hiele Test is organised in blocks of five questions that were created using behaviours identified from the nine writings published by the Van Hiele about their theory.

The questions are arranged sequentially, in blocks of five questions each, such that questions 1-5 measure students' understanding at Level 1: Questions 6-10 measure students' understanding at Level 2: Questions 11-15 measure understanding at Level 3: Questions 16-20 measure understanding at Level 4: and Questions 21-25 measure understanding at Level 5. Usiskin (1982) found that the behaviour provided by Van Hieles (1957) that described the first three Van Hiele Levels were of sufficient number and detail so that questions that test these levels were easy to develop.

Usiskin (1982:149) also concluded that there was enough behaviour provided by the Van Hieles (1957:310) so that questions that test for understanding at Level 4 could be developed given additional effort on the part of the test developers. However, because the behaviours identified by the Van Hieles (1957:310) for the fifth level seemed quite vague and open to interpretation, the authors felt that the testability of students` understanding at this level was questionable. It is because of this questionability that the creators of the Van Hiele Test identified two cases that can be used to measure learners` understanding.

Each of the cases that can be used in determining understanding is based (1) on the students' highest consecutive Level attained: and (2) that the student does not skip levels. In other words, students are identified as being at level 2 if they correctly answer the allotted number of Questions in each of the first two blocks, Questions 1-5 and questions 6-10, but do not correctly answer the allotted number of questions in any of the remaining blocks (questions 11-15, Questions 16-20, and Questions 21-25). As noted in Chapter 3 of this study, students' interviews after the pre-test were based on the interview protocol developed by Inoue (2005) as follows:

✓ Is a square a rectangle? Choose NO/YES and give reasons for your answer.

Here are two statements.

Statement 1: Figure F is a rectangle.

Statement 2: Figure F is a triangle.

Which ONE is correct?

A. If 1 is true, and then 2 is true. B. If 1 is false, then 2 is true.

C. 1 and 2 cannot both be true. D. 1 and 2 cannot both be false.

E. None of the above.

✓ *Your solution may not work in real life because of realistic considerations. Why did you answer that way?*

Statement S: ΔABC has three sides of the same length

Statement T: In ΔABC, ∠B and ∠C have the same measure.

Which statement is correct?

A. Statement S and T cannot both be true. B. If S is true, then T is true.

C. If T is true, then S is true. D. If S is false, then T is false.

E. None of the above.

✓ *Please think about on what condition your answer could become realistic. Could you come up with any assumptions or explanations that can make your answers justifiable?*

What do all rectangles have that some parallelograms do not have?

A. Opposite sides equal B. Diagonals equal

C. Opposite sides parallel D. Opposite angle equal

E. None of the above.

✓ *If the same thing happens to you, would you respond in the same way in real life as you did in solving the problem? Why?*

Here are three properties of a figure.

Property D: It has diagonals of equal length.

Property S: It is a square.

Property R: It is a rectangle.

Which is true?

A. D implies S, which implies R. B. D implies R, which implies S.

C. S implies R, which implies D. D. R implies D, which implies S.

E. R implies S, which implies D.

- ✓ *Do you think the contexts used in the content of tasks (or problems) that you solved in the pre-test are familiar and/or relevant to your everyday life experiences? Why?*

4.3 Results of Van Hiele Geometry Test

Students' responses (or answers) to the adapted Van Hiele's Geometry Test questions were coded into five Van Hiele's Geometric thinking levels and phase-based learning. As noted earlier in Chapter Three, Van Hiele's Geometry Test consisted of 25 questions, grouped into five sections as illustrated in Table 4. See Appendix A

Van Hiele's level of geometric thinking	Question number
Level 1: Visualisation	1-5
Level 2: Analysis	6-10
Level 3: Informal deduction	11-15
Level 4: Deduction	16-20
Level 5: Rigour	21-25

Table3. Distribution of questions in VHGT (Source: Usiskin, 1982)

The following Table represents marking criteria in VHGT which was adopted and used by the researcher, and its operational mode explained below (see Table1, p.46).

Mark	Criteria of the Items to be fulfilled	Van Hiele's levels of geometric thinking
1	1-5	1
2	6-10	2
4	11-15	3
8	16-20	4
16	21-25	5

Table 3: Source: Usiskin, 1982

If a student answers at least four questions correctly for Level 1 and Level 2, but not for the remaining levels, then the student would be awarded a weighted score of 3 (1 + 2 + 0 + 0 + 0). However, if the student also answered at least four questions correctly for Level 4 or Level 5, then the students would be identified as not fitting the Van Hiele Theory model.

Using the M4 case/criterion, if a student answers at least four questions correctly for Level 1 and Level 2, but not for the remaining levels, then the student would be awarded a weighted score of 3 (1 + 2 + 0 + 0 + 0). If the student answered at least four questions correctly for Level 1, Level 2 and Level 5 then the student would be awarded a weighted score of 19 (1 + 2 + 0 + 0 + 16).

However, if the learner also answered at least four questions correctly for Level 4 regardless of whether or not they answered at least four questions correctly for Level 5, then the learner would be identified as not fitting the Van Hiele Theory model. Based on this system, and using the Modified case, the identification of a learner's level of understanding is shown in Table 4 below.

If assigned a Weighted Sum of:	The learner is identified at level:
0 or 16	0
1 or 17	1
3 or 19	2
7 or 23	3
15 or 31	4

Table4. Identification of Level Attained by Weighted Sum

Initially, all students were measured using all four case/criterion scenarios. There were total of 80 subjects who took the pre-test from both groups.

4.3.1 PRE- AND POST-TEST: QUANTITATIVE RESULTS

Descriptive statistics generated from pre- and post-test data are discussed below in light of research objectives of the study. The analysis done on students' VHGT (Van Hiele Geometry Test) items adapted from Usiskin's 1982 (CDASSG) focused on computational (or mathematical) correctness of learners' answers, together with situational experimental and comparison groups were discussed before and after the intervention.

Table5. Wilcoxon-T statistics

	Treatment group – Control group
Z	-0.775(a)
Asymp. Sig. (2-tailed)	.439

Based on Table 5, the significant value .439 is more than .05. The result of Wilcoxon-T-test is significant (T=48.00, $p > 0.05$), which shows that there is no significant difference between the initial levels of geometric thinking of the two groups. Students did not attain Van Hiele levels of understanding geometry expected of students in Grade 10. This led to an investigation into whether or not there is any level of attainment, and what level a student should be at to realise a higher level of attainment. One of the properties of the Van Hiele Theory is the separation property, viz. the principle that two people who understand and reason about geometry at different levels cannot understand each other. Usiskin (1982:259) concluded that this property is why students who have not attained a level of understanding of Van Hiele's levels upon entering secondary school will have a 60% or less chance of success in a secondary school geometry scores.

The descriptive statistics of initial levels of geometric thinking for control and experimental groups are shown in Table 6.

Table6. Descriptive statistics of initial levels of Geometric thinking for control and experimental groups

	N	Mean	Std. Deviation	Minimum	Maximum
Control group	40	1.17	.524	0	2
Experimental group	40	1.23	.428	1	2

Eight students were randomly selected, with five students from both the control group and experimental group. Student A, B, C and D were in control group, while student E, F, G and H were in treatment group. Based on Tables 5 and 6, it can be seen that both groups are balanced for the acquisition of geometric thinking. The majority of the learners attained a complete acquisition of the first level of geometric thinking, which is visualisation. Almost all the students in both groups showed a low acquisition of second level, while almost all failed to reach the third level of informal deduction.

4.3.2 Final levels of students' geometric thinking/ attainment

To test the main research aim: Establish the effects of using Geometer Sketchpad on Grade 10 students' academic achievement in Euclidean geometry, the Wilcoxon-T- test for the design of matching samples in two different situations was used.

Table7. Wilcoxon-T- test

	Treatment group – Control group
Z	-4.388(a)
Asymp. Sig. (2-tailed)	.000

Based on Table 7, the significant value .000 is less than .05. The result of Wilcoxon-T- test is significant ($T = 34.50$, $p < 0.05$), which shows that there is a significant difference between the final levels of geometric thinking of the two groups. The other factor that can be attributed to the significant difference might be brought about by the use of GSP software in the experimental group which appeared to have some level of benefits in terms of enhancing students' geometric thinking.

The descriptive statistics of final levels of control group and experimental groups are shown in Table 7. This result was supported by Boxplot graph median value for both the ordinal scores of the two groups (Table 7), which clearly shows the experimental group's Van Hiele's final levels of geometric thinking.

Level1-recognition: explicit in the data collected from question 1-5. Students visually recognise geometric shapes by their global appearance.

Level 2-Analysis: from questions 6-10 in VHGT, students' scores showed that they managed to analyse properties of shapes and acquired the appropriate technical terminology.

Level 3- Informal deduction: from questions 11-15, there was a vast progress from being able to logically order the properties of shapes by short chains of deductions and now understood their interrelationships between shapes, like class inclusions, but they did not understand the role and importance of formal deductions.

Level4-Deduction: from questions 16-20, data collected in this level were analysed as students developed longer sequence of statements deducing one from the other to justify observations and to understand the significance of deduction, the role of axioms, theorems and proofs. They did not yet recognise the need for rigor.

Level 5-Rigour: from questions 21-25, VHGT's answer scripts (raw data) revealed that learners can reason formally about mathematical system, understand the necessity for rigour, and are able to make abstract deduction.

The unstructured students' interviews results showed an improvement in a significant difference between the final levels of geometric thinking of the two groups. The descriptive statistics of final levels of control group and experimental groups are shown in Table 7. This result was supported by histogram graph median value for both the ordinal scores of the two groups (Table 5), which clearly shows the experimental group's final levels of geometric thinking.

Based on Figures 10 and 11, it can be seen that there is a significant difference in the final levels of geometric thinking between the two groups. Students in the control group showed improvement in the first and second levels, although there were two students who showed degradation from a complete acquisition after first level. However, all the learners showed improvement in the second level, analysis, at which they improved from a low and an intermediate acquisition level to an intermediate and a high acquisition level. None of the students in the control group attained the third level, which is informal deduction.

On the other hand, the students in the experimental group showed improvement in all the three levels, with all of them attaining complete acquisition of visualisation level. One student attained an intermediate acquisition level, while another one scored a high acquisition of second level. Three other students attained a complete second level. As for the third, only one student did not manage to score that level. The rest of the learners managed to attain a complete and a high acquisition rate for the third level of geometric thinking.

4.4. Analysis of students' achievements in pre-test

Inferential statistics of independent Wilcoxon T-tests were used to analyse the data obtained from the pre-and post- Van Hiele Geometry Tests.

4.4.1 Control group

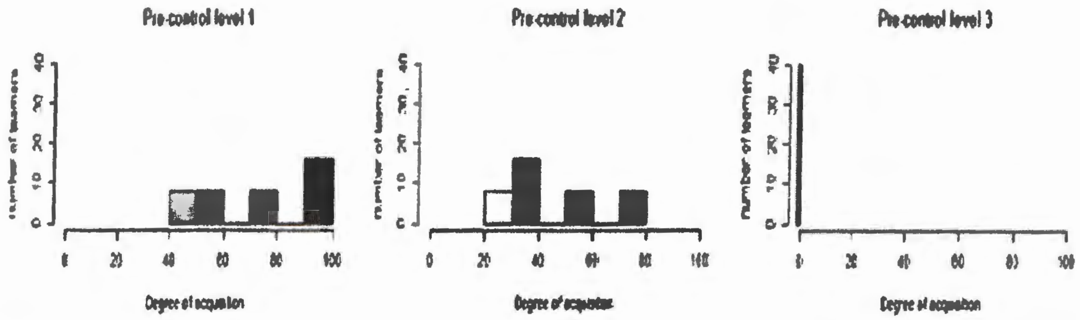


Figure10. Number of students and the degree of acquisition of geometric thinking level for the students in the control group.

To determine the level of geometric thinking of the control group, qualitative data can be summarised that 16 students out of 40 students attained a complete acquisition on the visualisation level. However, they showed low acquisition on the analysis level, and they did not reach the informal deduction level. Eight learners attained an intermediate acquisition level for the first level, low level for the second level and did not score on the third, 8 students showed complete acquisition level, an intermediate acquisition level for the second and did not score on the third. Another last group of eight learners attained a high acquisition rating for the first level, a low rating for second level and did not reach the third level.

Since students differ in abilities, teachers should, therefore, present instructions in a manner that takes this into account during teaching and learning. Furthermore, in a geometry class, gifted students rely on symbolic thinking, while those less gifted should visualise the problem in problem solving situation. Certainly visualisation does not harm the gifted students, but if left out of the curriculum, it limits the chance of success in geometry problem solving of the less gifted child (Kirby, 1991:109-125). In the teaching and learning of geometry in schools, there are some views and theories expressed by researchers in the field of education, such as Piaget, Freudenthal and Van Hieles as reflected in the research questions.

For students to attain low acquisition in pre-test can be attributed to the reason of students not ready in terms of geometrical concepts as Piaget reiterated, namely that students in a particular grade or age are expected to have reached a perceptual or intuitive thought, but if they showed a low acquisition on the post-test analysis, this reveals that other factors can be responsible for this, for example the rote learning from previous grade and/or as Freudenthal(1973:87) alluded, deductive reasoning is not imposed, but unfolds itself from its local germs (Chapter 2). To this study such behaviour can be caused by students' readiness and/or language barrier as LoLT, English in this regard is the students' First Additional Language (EFAL).

This was followed by an analysis of difficulties. At this stage of analysis, all students' area of difficulties as identified in the concept of geometry, resulting from the test (pen and paper) was analysed and described according to the difficulties that learners had in finding solutions to questions asked. This was done in accordance with Van Hiele's geometric thinking level.

Three overarching questions guided the study:

1. What is the effect of using Geometer Sketchpad on Grade 10 students' understanding of geometry?

2. What are the benefits of using Geometer Sketchpad in the teaching and learning of Euclidean geometry in Grade 10 mathematics classroom?

3. How does the use of Geometer Sketchpad impact Grade 10 students' academic achievement in Euclidean Geometry?

GSP provides a medium for the learning of Euclidean geometry in which students are able, through "direct manipulation" (Laborde &Laborde, 2008:31) to operate on theoretical geometric objects in a "physical sense (Laborde, 1995:242) as diagrammes on a computer screen. GSP in the simplest sense is used as a means to create constructions which replace the pencil and paper method of construction (Guven, 2011:327).

In contrast, the notion of "situated abstractions" was introduced by Noss and Hoyles (1996:125), where computer environments provide an acceptable model for learning, in which the original mathematical concepts are "preserve and extended by the learners".

Although diagrammes are seen as an integral part of geometry teaching, they do have their drawbacks. A theory of internal representation argues that students combine all mental pictures and properties associated with the concept to create a 'concept image' (Vinner & Herschkowitz, 1980). Further to this, it was argued that concept images are the staple of student's reasoning, rather than definitions of these concepts (Clements, 2001). In this context, diagrammes posed significant difficulty for learners, who failed to understand that drawings do not necessarily represent all the information about its representation (Parzysz, 1988).

The results of the analysis emanating from both the written test and the interview indicated that students from control group are mostly functioning at the Van Hiele's Level One and Two, which are Visualisation and Description.

4.4.2 Experimental Group

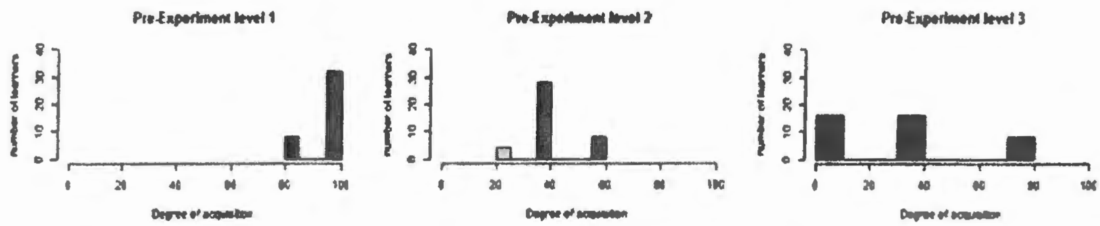


Figure11. Number of students and the degree of acquisition of the geometric thinking level for the students in the experimental group.

For the 40 students in the experimental group, it was found that for a complete acquisition of visualisation level, 32 students managed to achieve and eight learners showed a high acquisition on the first level. Eight students attained intermediate acquisition for the analysis level. The other 32 students were low on the analysis level. However, only eight students managed to show an intermediate acquisition rating at the second level. For Level Three and eight students, attained high acquisition, where 16 students were at low rating on the deduction level, while the last 16 did not reach the third level.

The difference in means gain indicates phase-based instruction using GSP software loaded computer enhanced the students' achievement in geometry. All the students advanced from lower to higher Van Hiele levels after the intervention, consistent with the findings of the previous studies on phase-based instruction using static or dynamic geometry computer software packages(Baynes,1999;Bobango,1987), and phase-based instruction using GSP(Choi;1996).

4.4.3 Analysis of students' achievements on post –test

After the administration of pre-test and analysis of its results, the researcher engaged students in the following activities:

- ✓ Instruction using the traditional/conventional method
- ✓ Post Van Hiele Geometry Test

4.4.4 Control Group

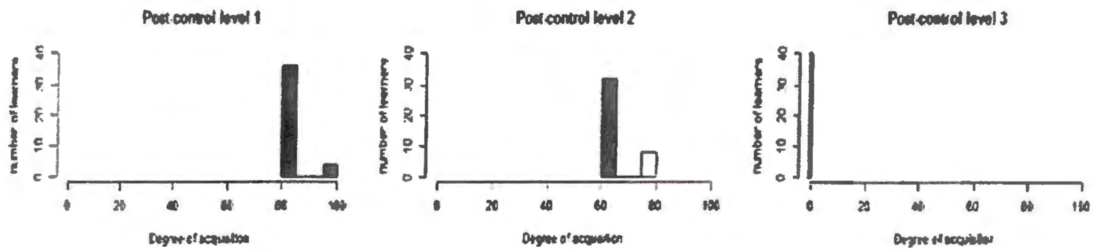


Figure12. Number of students and the degree of acquisition in the post-test of the geometric thinking level for the students in the control group.

The above graph depicts the geometric thinking levels of the control group, qualitative data were analysed. It can be summarised that from 40 students, 36 students attained a high acquisition for the first level thinking, with only four learners attaining complete acquisition of first level. Eight learners showed a high acquisition rating for second level, and 32 students showed a high acquisition for second level. However, none of the learners in the control group scored on the third level.

Regardless of the grading method, the fact remains that in both the current study and that of Usiskin (1982), students obtained very low mean scores in the VHGT. This indicates that the majority of the students in this study (as in Usiskin's) were at low Van Hiele levels, possibly Levels One or Two.

Van Hiele,(1986, as cited in Usiskin, 1982:6),emphasised that it takes about "20 lessons" to raise students' thought from Level One to Level Two, and "50 lessons" to get them from Level Two to Level Three. Van Hiele seems to claim that in Grade 10 students, there exists a wider cognitive gap between Van Hiele Levels Two and Three than there is between Levels One and Two. The data in this study (like that in earlier studies, e.g. Siyepu, 2005) tends to support this claim, since the difference in the mean scores of these students between the Levels Two and Three subtests was much wider than the difference in their mean scores between the Levels One and Two subtests. The above information is an indication that students in the control group were taught in a conventional method, after pre-test results analysis, which means if GSP was used, the mean gain was likely to improve.

These results can be argued that in experimental group, either there were more students at higher Van Hiele levels than students in the control group. Given the rather low means obtained by control group for the VHGT, the latter case appears more probable. The results indicate that Grade 10 students from experimental group demonstrated a better understanding of geometric ideas/concepts than their peers from control group. In contrast with what Freudenthal has advised, it is then possible and desirable for the teacher to encourage and hasten the maturation process of the child, and it is the goal of didactics to ask the question as to how these phases are passed through by the child and how to furnish effective help to the students (Freudenthal, 1973).

In item 12 which are true?

A. All properties of rectangles are properties of all parallelograms.

B. All properties of squares are properties of all rectangles.

C. All properties of squares are properties of all parallelograms.

D. All properties of rectangles are properties of all squares.

E. None of the above.

The learners' responses to this item, and indeed to all the items in the VHGT that exemplify Van Hiele Level Three questions, tend to indicate that learners generally have difficulty with the ordering of the properties of simple geometric shapes. The learners' difficulty was clearly outlined by Freudenthal (1973; 149). At the first level, figures were in fact just as determined by their properties, but a learner who is thinking at this level is not conscious of these properties. Each level has its own linguistic, symbols and its own network of relation uniting these signs. A relation which is "exact" on one level, can be revealed to be "inexact" on another level (Chapter 2.2.2). This implies that the key elements of this property are that understanding depends on the content and methods of instructions received, more than it does on age (Crowley, 1987:69).

The implication of the above contention is that, a student who is not ready in terms of Van Hiele's geometric level, cannot be able to answer this question and supply a reason.

Lack of progress to Level 3 might be attributed to several factors: students might have low achievement in mathematics, with reference to Piaget's theory on deductive level reached at 12+ years, this level could have not been reached and yet learners progressed to Grade 10. Contrary to what Piaget theorised, namely that students in Grade 10 are expected to work with abstract object and employ deductive hypothesis, learners seemed to have difficulties remembering geometric terminologies such as opposite, congruent, parallel and parallelogram. This problem can also be attributed to lack of proficiency in the English Language, which reinforces the conclusions made by Fuys et al. (1988:63) that progress into higher levels was also influenced by instruction and ability, particularly language ability (LoLT).

4.4.5 Experimental Group

Informed by the results of pre-test, the researcher engaged students in this group with the following activities:

1. Phase-based instruction using the Geometer's Sketchpad software
2. Post Van Hiele Geometry Test

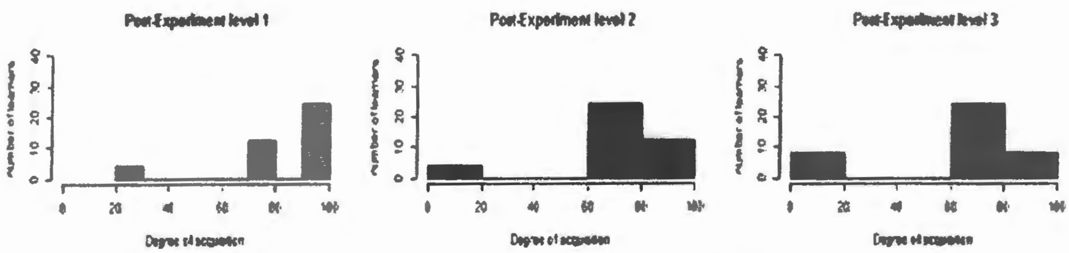


Figure13. Number of students and the degree of acquisition in the post-test of the geometric thinking level for the students in the experimental group.

The following observation was concluded, in detail, for the 40 students in experimental group, where thirty six managed to reach the visualisation level. Of the thirty six, twenty four students managed to reach the complete acquisition of visualisation level, twelve showed high acquisition on the first level, and only four students did not make it to Level One. Twelve students showed a complete acquisition of second level, twenty four reached high acquisition on the second (analysis) level and only four did not make it to this level. On the third level, eight students managed to attain complete acquisition, 24 acquired high acquisitions of Level Three and eight students did not manage to acquire any degree.

For the four students who did not reach Van Hiele Level One of understanding, there might be reason(s) attributed to this performance, one can be that the students did not attain the level before taking a secondary school course, or alternatively had a level of understanding too low to insure success. Therefore, the expectation of the successful completion of a course in formal geometry at the secondary school level can only be realised if the students have attained the simple deduction level of understanding geometry upon completion of elementary school.

Another contributory factor could be, if the teacher's own level of understanding is low, the students will resemble the teacher's own level, for example, the teacher will be unable to provide the scaffolding needed for the students' level of understanding to advance or if the teacher's level of understanding geometry is only a Level One or level Two, then the students will probably be provided with scaffolding only up to the teacher's level.

This being the case, it is reasonable to assume that for learners to be prepared for success in secondary school geometry they must achieve the level of understanding identified as simple deduction (Usiskin, 1982:69), abstraction (Burger & Shaughnessy, 1986:199), or informal deduction (Crowley,1987:27), so that they can mature to the level of understanding identified as deduction (Usiskin,1982; Burger & Shaughnessy, 1986;Crowley,1987) upon completion of secondary school geometry course.

The results indicate that most of the students were at Level One for geometrical concepts prior to the intervention. This implies that all learners could recognise and name figures, discriminate squares from rhombi and parallelograms before the intervention.

At a comparative level, more of the experimental group was successful in this task (identifying and naming shapes) than students from the control group. The experimental group showed improvement in their achievement after the intervention which concurs with the study, conducted by Tay (2003:152).

The researcher's guidance might also contribute to the students' progress; during information, the researcher engaged the learners in conversations to learn what they already knew. During Guided Orientation, the researcher carefully sequenced the instructional activities for students to investigate, and for explication, students were encouraged to share their findings using their own words, and to introduce new and relevant vocabulary when appropriate, because unlike Piaget, Van Hiele believed that progression from one level to the next is more dependent upon instruction than it is on age or maturation. One of Van Hiele's properties of Geometry Understanding viz. which is distinction, which is based on the ability to use and understand the vocabulary associated with levels, was displayed.

As reaffirmed by Crowley (1987:157), attainment outlines the learning process that leads to complete understanding at the next level. The key elements of this property are that understanding depends on the content and methods of instructions received, more than it does on age. This finding suggests a strong emphasis on pedagogies that promotes procedural understanding, rather than conceptual understanding of geometry problems in mathematics classrooms. The findings as reported by Crowley (1987:157).

An example of this is teaching a student to memorise a formula or property, such as "a square is a rectangle" where, unless the learner can reason why the formula works or which properties of a square are also the properties of rectangle, the level of understanding has not been increased but the complexity of material has been reduced (Chapter 2.2.2).

4.4.6 Comparative tables of results showing pre-test performances of experimental and control groups

The following section highlights a summary of performances in the pre-and post-tests, followed by a brief narration on the quantitative data obtained from both experimental and control groups, respectively. The data presented here should also be seen within the context of performances before and after the intervention (or experiment) as discussed in the previous chapter.

Table8. Descriptive statistics of the experimental and control groups for the pre-test

	Mean	Standard deviation	Calculated T	Critical T at 0.05 level of significance and 60 degrees of freedom
Experimental	32.88	14.58	0.6691	2.000
Control	28.29	12.21	0.6691	2.000

In this instance, p=0.05 level of significance. It could be rejected if T-calculated > T-critical and accepted if T-calculated < T-critical. The learners' T-test was used because of the sample which was not large, and it was manageable. According to Best (1977:283), when a small sample is involved, the learners' T-test proves to be an appropriate test to determine the significance of the difference between the means of two independent groups. Table 9 below shows the results of the T-test application on pre-test score.

STATISTICS	**Experimental Group**	**Control Group**

Minimum	13	15
Maximum	38	35
Range	25	20
Mean	24.5	23
Standard deviation	7.595	5.485

Table9. Results of students`-test application on the pre-test scores

Table 9 depicts a slight significant difference between the mean scores of the experimental and control groups of students in the pre-test. The two groups of learners were comparable at the pre-test stage.

STATISTICS	**Experimental Group**	**Control Group**
Minimum	12	13
Maximum	52.5	51
Range	40.5	38
Mean	32.875	28.2875
Standard deviation	14.58	12.21

Table10. Descriptive statistics of the experimental and control groups for the post-test

The performances of students in both experimental and control groups improved significantly in the post-test, with the experimental group scoring slightly higher than the control group. A better average performance observed in the experimental group might be attributed to the intervention done for this group. The statistical analysis that shows the results of the T-test application on the post-test scores is given in following table.

GROUP	Mean	Standard deviation	Calculated T	Critical T at 0.05 level of significance and 60 degrees of freedom

EXPERIMENTAL	25.857	7.595	1.0135	2.000
CONTROL	23.871	5.485	1.0135	2.000

Table11. Results of T-test application on the post-test scores

According to Table11, the results cannot be rejected because the calculated T-value is less than the critical T-value, at 0.05 level of significance. This shows that there is no difference between the experimental and control groups. From this it can be observed that the mean score of the experimental group was higher than that of the control group. In conclusion, the mean scores shows that there was a significant difference between the experimental group and the control group, indicating that there was a significant difference between the use of GSP loaded computer and traditional pen and paper to performance and achievements in geometry.

4.5 ANSWERING THE RESEARCH QUESTIONS

As mentioned before, the study aimed to answer the following research questions:

What is the effect of using Geometer Sketchpad on Grade 10 learners' understanding of geometry?

In responding to the research question above, the sub-questions, which emanate from the objectives of the study, and the research question, are answered in the next sections.

4.5.1 Do the students' academic achievement in Euclidean geometry established when using GSP?

The quantitative and qualitative data collected after pre-tests on both experimental and control groups showed that learners thought that the only mathematics teaching methods available is when the teachers use textbook, chalkboard and chalk or whiteboard and marker as teaching and learning aids. The traditional method involves learners as passive listeners to one-way teacher talk, where the teacher merely completes mathematical problems on the chalkboard, and a written class activity.

The overall interpretation of these results is that the majority of the students who wrote the VHGT were at Level Zero on the Van Hiele geometric scale, which means that their knowledge of school geometry was poor. The near absence of learners at Levels Three and Four implies that most of them did not possess the experience necessary for the formal study of high school geometry.

During intervention, the experimental group was taught using pre-constructed GSP models while the control group was taught using traditional methods.

After intervention there were more students in the experimental group who had a weak knowledge of geometric concepts than there were in the control group. This offers a plausible explanation of why experimental group learners outperformed their control group counterparts in the tests (post-tests). The intervention was done with the use of GSP software. However, post-test results indicated that experimental learners improved significantly better when using GSP.

4.5.2 Are the geometric skills and errors that the students' exhibit related to teaching styles (methodologies) or are they generic?

The analyses of classroom observations and student interviews illustrated that the geometry problems whose solutions require and involve interpretation of geometrical terminologies and figures, focus on using different kinds and arrangements of properties. As a result, understanding the geometrical rules and acquisition of geometrical skills appeared to be necessary to understand the meaning of contexts or situations that illuminate the manner in which the reality of the world is structured. On the other hand, quantitative data suggest that geometrical errors that learners make, particularly the conceptual skills, seem to stem from the inability to use language, home (different African languages spoken by learners in the class) and/or LoLT (English) effectively in order to solve geometry problems. In fact, formal written mathematical language appeared to take precedence over the informal spoken mathematical language that learners exhibit during geometrical problem-solving and meaning making. In particular, this finding could possibly be attributed to the disconnection between classroom mathematics activities and learners' everyday life knowledge and experiences, and/or out-of-school mathematics.

4.5.3 Does the introduction of GSP impact on the students' academic achievement?

Analysis of the data generated as triangulation of the data obtained from pre- and post-tests, interviews and classroom observation schedule, revealed that students' problem solving abilities in post-tests improved over time (in favour of the group that used GSP as noted before) after the intervention. In fact, statistical results illustrate that there was a statistically significant difference ($p < 0.005$) between the experimental and control groups before the intervention (pre-tests). However, after the intervention the experimental group performed statistical significantly ($p < 0.05$) better in geometry achievement compared to control group.

4.6 DISCUSSION OF FINDINGS

Qualitative results

As noted earlier, qualitative data were generated from classroom observations and interviews at both learner and teacher levels in the experimental group during the intervention. Another set of data were gathered via four focus group discussions involving eight learners from the experimental group.

4.6.1. Classroom observations

The classroom observations in this study were used to provide insight into and explanations of the use of GSP impact the learning of geometry in multilingual mathematics classroom (first objective of this study). The classroom observations were also used to track the experimental group's progress and judge their ability to answer geometry questions using GSP strategies learned during the intervention session.

4.6.2Baseline observations

The results of the classroom observations appear to substantiate that what the researcher do serves as a fundamental component to raising learner outcomes (Douglas, 2009). Overall, the pre-intervention classroom observations in the experimental group revealed four important findings:

- Very little discussion took place in the classrooms before the intervention, and in cases where discussion took place, it was characterised by, as identified and stated by Lemke (1990), talk and/or arguments that were high in quantity and low in quality;

- There were no opportunities created for learners in writing to learn geometry problems before the intervention and as such, very little evidence of writing was available; and

- Although experimental geometry learners struggled to promote the use of geometry terminology, particularly learners' home language, as a visible and/or invisible resource in maximising learner participation in mathematics discourse, they seemed to improve over time during the intervention;

- After the intervention, lessons involved a set of group interactions and communication that occurred through some order of turn taking, where each party to the interaction made their talk comprehensible to all (Heap, 1990).

Data that were generated through classroom observations during the intervention responded: to the two objectives of this study:

to identify the use of language by both the teacher and students, when teaching and learning in multilingual mathematics classrooms; and to check whether the introduction of discussion and argumentation into classroom practice has an influence on students' sense-making and problem solving abilities.

The data generated via the classroom observation schedule presents possible explanations to researcher's classroom practices and learners' behaviour during and after the intervention and are presented in the following sections (Sepeng, 2010:169).

4.6.3. Use of language in the classroom

Lerman (2001) reiterates the importance of accounting for alignment and power in analysing language in mathematics classroom, suggesting that the official language of the classroom can give certain groups power and privilege. Although experimental learners were afforded opportunities to use the language they preferred for discussion and problem-solving in their small groups, the use of English by the researcher suggested the researcher as a figure of a powerful authority, which had an effect on the language used by the students in the classrooms.

Reports by researchers (Adler, 2001; Kaphesi, 2003 Moschkovich, 2002; Setati, 2005a) indicate that teaching and learning mathematics in neither a language that is not the learners' nor teachers' home language is complex and can create dilemmas for teachers. As Setati and Adler (2001) argued, the movement from informal spoken language to formal written language is complicated by the fact that the learners' informal spoken language is typically not the LoLT. Mathematics teachers in multilingual classrooms are faced with yet another dilemma, that is, of encouraging learners to participate actively in mathematical discourse, and classroom talk in general. Baseline observations revealed that only a few learners in the experimental group participated in the discourse because they are not confident and competent in linguistic exchanges (Zevenbergen, 2000).

The baseline observations suggest that most of the classroom talk was researcher dominated and in the process, that students' roles were relegated to that of passive spectators in the teaching and learning of mathematics (Alexander, 2004). In so doing, teaching mathematics, particularly using Van Hiele's geometric levels and understanding was not attempted and/or achieved in these classrooms.

In an analysis of lessons observed in the experimental group, English emerged as the language of teaching, and thus the language of mathematics, and assessment (Setati, 2002).

Data generated from observations revealed that, although most of the learners in the experimental group were found to be largely using English as the language of mathematics, authority and assessment (Setati, 2005), there were very few instances, contrary to findings by Setati, where the students' home language (an African language, dominantly Setswana), functioned mainly as the language of consolidation. In fact, students' home languages functioned mainly as the language connecting classroom mathematics activities with students' everyday-life knowledge during small group discussions. As such, it appeared that the majority of the students in the experimental group preferred to use their home languages when discussing and solving problems in small groups.

In rare cases where the researcher would use English throughout the lesson, communication and utterances were the domain of the researcher only. Only few students responded to the researcher's questions in English, which possibly signalled their linguistic incompetence in this regard (Mayaba, 2009).

4.6.4 Classroom interactions

The baseline observations revealed that classroom interactions in the experimental group took the form of teacher initiated talk (Mercer, 1995), characterised by teachers' regular use of inauthentic initiation turns. In cases where the teacher asked questions, students responded in chorus (Mayaba, 2009). Moreover, these classrooms were embedded with social discourses that reflected students' socio-cultural backgrounds (Lemke, 1990). There were only few occasions that resulted in students' engagement in dialogue, which occurred between the teacher and a few individual students. As such, there were no understanding and agreement of rules of engagement between the teacher and students in these classrooms to actively engage with mathematical discourse in order to contribute positively in problem solving initiatives.

The tendency by students to be passive may be attributed to the classroom linguistic structures that were restricted to English, characterised by researcher's inability to attend to gestures, representations, and everyday descriptions that second language students draw on to create and communicate meaning in mathematics classrooms (Nasir, Hand, & Taylor, 2008). In doing so, teachers inadvertently missed the multiple, rich resources that students bring to the classroom. However, data obtained from observations during and after the intervention illustrate that teachers demonstrated the abilities to allow students to actively engage in mathematical discourses that paved way for the learners to effectively interact with the mathematics contexts in content via classroom discussion.

4.6.5. Implementation of the intervention strategy of this study

It was evident in all the experimental groups that students had begun to display signs of confidence in and understanding of key aspects of the intervention (teaching using GSP). They managed to incorporate the strategy learned from the intervention in their learning styles of geometry. In fact, they engaged students in new and innovative pedagogies that created an atmosphere conducive for the students to participate actively in open discussion

4.7 Van Hiele's geometric levels and phases of learning

4.7.1 First Learning Session

In this session, learning activities are provided to help students advance from Level One of visualisation, to Level Two analysis. Students went through all phases; visualisation, guided orientation, explicitation, free orientation and integration to move from the first level to the second level. The objective of the activities is to help students identify quadrilaterals and to understand their properties. For example, learners come to understand that a parallelogram has equal and parallel opposite sides, equal opposite angles and its diagonals bisect each other. In Phase 1, information, learners become acquainted with the activity, where the researcher presented a new idea, and allowed students to begin working on the concept. In the example given, shapes such as rhombus are introduced in this phase. Students are then introduced to other geometrical shapes and asked if the shapes are rhombus.

The researcher showed students few figures of various shapes and asked them to identify triangles and other shapes.

In the information phase, students were able to identify triangles and other shapes. They were able to identify the type of triangle, be it equilateral triangle, isosceles triangle, or right triangle.

In the topic of circles, learners used their own description to name the sides in a circle in the information phase. They most probably named the sides based on their external properties. In this study, the activities were designed to help students to develop and recognise the variety of the quadrilaterals. Learners recognised (a) rectangle: (b) square; (c) parallelogram: (d) rhombus; and (e) kite (see Figure 1). By using the GSP, the students were able to conduct quadrilaterals and then identify the properties they possessed.

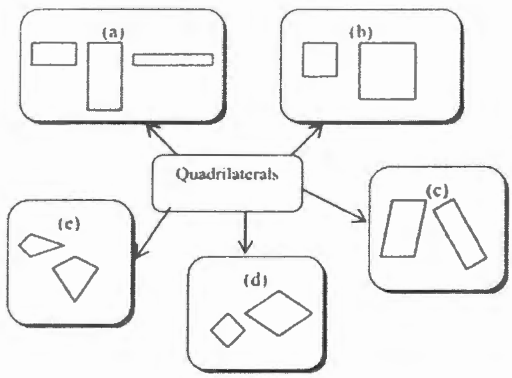


Figure 14

The guided orientation phase

Learners were given activities that allow them to become familiar with many properties of the new geometric concept. Students explore the objects used in the instruction. In this phase, students explore the properties of rhombus by folding a rhombus at its axial symmetry and by observing the diagonals and sides. In this phase for triangles, the researcher discovered that students in this group were asked to observe figures of triangles and non-triangles. They were then asked to classify the figures into triangles and non-triangles. After that, they were asked to cut figures of triangles and draw the figures again in various sizes. The purpose of this activity was to help students explore the properties of the various types of triangles. In this study, the students used the GSP software to explore the properties of equilateral triangle, isosceles triangle, and right triangle.

In this study, in the topic of Circles, students were asked to measure the angles and state the relationship between the two angles. In this study, the activities gave students opportunity to explore the properties possessed by any quadrilaterals by using GSP. The processes of constructing quadrilaterals and exploring their properties can be done easily and effectively because the dragging capability of the GSP allowed students to manipulate and reshape the geometrical objects with the use of the mouse.

The explicitation phase

Students expressed in their own words what they had discovered in the previous phase. The role of the researcher here was to introduce relevant geometrical terms. In this phase, students exchange their opinions about the properties of rhombus. In the topic of Triangles, students explain their experience with their classmates and teachers on the properties of each type of triangle by using their own words. In the topic of Circles, learners discuss the relationship of the angles that they have explored in front of the class.

The researcher then introduced the exact terminologies to the students. The students then explained their observations from the activities carried out earlier. With reference to the data derived from exploration using GSP, learners can now explain the properties possessed by a square, rectangle, parallelogram, rhombus and kite. In Phase 4, learners carried out more complex tasks that are more open-ended than in the guided orientation phase.

The problems were more complex and required more free exploration to find solutions. In this phase, a few edges and sides of rhombus were given in various positions and learners were asked to build the whole figure of a rhombus. In the free orientation phase, learners were given a triangle with two sides. They were then asked to put another side to make equilateral triangle, isosceles triangle, or right triangle. In this study, students were asked to connect the assigned dots to produce specific quadrilaterals. They (learners) can build a particular shape correctly if they understood the properties possessed by quadrilaterals.

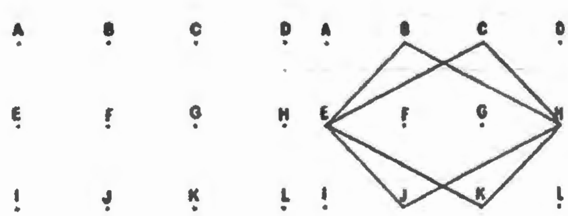


Figure15. In the fourth phase, students connect the assigned dots to produce kites.

The figure above shows kites constructed by connecting the points, a form of learning by play.

The final phase: Integration

Students summarise and integrate what they have learnt and develop a new network of objects and relations. This was achieved in the form of discussions and activities. Students were allowed to summarise the properties of rhombus in this phase. In the topic of Triangles, students were asked to summarise the various properties of triangles besides being able to differentiate the types of triangles based on their properties. In this study, the researcher helped the students to summarise the concepts that they have explored and come to understand in the learning session. The students were able to describe the properties possessed by the forms of the four sides of a square, rectangle, parallelogram, rhombus and kite.

4.7.2 Second Learning Session

The objective of this session was to assist learners in increasing their geometric thinking from Level Two to Level Three. Therefore, as shown in Figure 14, the activities in this session were designed to help students strengthen their understanding on the properties of quadrilaterals and the relationships among them. Students were able to verify these relationships by using non-formal deduction. In this learning session, students went through all the phases in order to assist their movement from Level Two analysis to Level Three informal deduction.

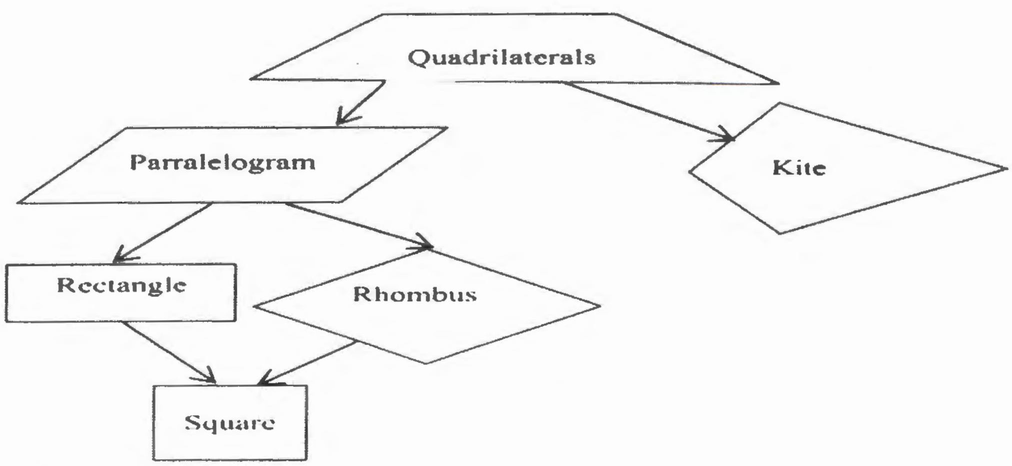


Figure 16

Phase1: Information. Students reflected on the properties possessed by the quadrilaterals that they were produced in the previous session (first learning session). They were asked to build quadrilaterals using the GSP. In the guided orientation phase, the purpose of the activities was to help learners identify the relationships among the quadrilaterals. Firstly, notes concerning the properties of quadrilaterals are provided in the GSP, and learners understood their properties in detail by clicking on the buttons provided. After analysing the quadrilaterals, they were then asked to classify the quadrilaterals in terms of sides, angles and diagonals in the table.

According to Figure 16, students were then asked to establish relationships among the quadrilaterals. Students and the researcher discussed why a particular quadrilateral is distinct from other quadrilaterals in the explicitation phase. In Phase 4, free orientation, learners were given a particular quadrilateral (for example, a rectangle). They were asked to find the value of its properties. Secondly, they were asked to determine by dragging any vertices of the rectangle by using the GSP, why other quadrilateral (for example, a square) was a special case of the original quadrilateral (a rectangle). Next, they were asked to find the common properties possessed by these quadrilaterals. Finally, upon completion of the second learning session, in the integration phase, learners were able to summarise all the relationships among quadrilaterals. At this stage, learners were able to distinguish the quadrilaterals by their definitions and classification.

After the intervention, lessons involved a set of group interactions and communication that occurred through some order of turn taking, where each party to the interaction made their talk comprehensible to all (Heap, 1990).

Data generated through classroom observations, interview, pre-and post-tests before and after intervention responded to the three research objectives of this study.

4.7.3 Interviews

As pointed out earlier, the students were interviewed. In this section the results of the semi-structured interviews conducted with the eight purposefully selected samples of students from experimental group were discussed. This was followed by face-to-face semi-structured interviews held with each student in the predicted and emerged general categories related to the theoretical underpinnings presented in prior chapters of this study, and used to present a discussion of the interview results. The purpose or value of conducting these interviews was to interrogate three objectives of the study.

4.7.4. Student interviews

Student interviews were conducted to probe the experiences and perceptions of student towards the use of GSP in the teaching and learning of mathematics when they interact and solve geometry problems. As noted earlier, English appeared to be students' preferred language for classroom discourse, for example, when they communicate with the researcher and in cases where they had to present their feedback to the entire classroom.

Concerning direct feelings towards activities developed by means of GSP software, the majority of the students felt that the GSP activities were enjoyable, interesting, and motivating. Their feedback was as follows:

S1: *Very interesting, I want more mathematics activities with the computer;*

S2: *A fantastic activity, from a piece of square paper I am able to form seven different sizes and shapes by means of moving a mouse. When those corners are "dragged", it will form a bigger but same shape.*

S3: *I enjoyed learning geometry using GSP software; an interesting way to learn geometry that used to be quite boring.*

Another direct feeling towards the activities is flexibility in learning. They mentioned that:

S4: *I can explore and create more/new shapes freely using the same software; by using the software, I am able to form different shapes easily.*

The GSP software activities have also fostered learners' cooperation in solving problems. As learners said:

S5: *I was engaged to compare my work (drawing of rectangles, triangles etc) with my classmates and discuss how to play around the computer mouse in sketching our figures.*

Students generally learned from peer assessment and felt 'ownership' over their outcomes. As some students from the interviewed group remarked:

S6: *Since this activity will be marked by our friends, I have tried my best to produce the new shapes from the computer. Even though it is very challenging for me, I am satisfied from my efforts to arrange and draw the shapes using my own creativity; I learn something when I mark my friend's worksheets. I was able to see other shapes which are different from mine, so actually we learn from each other.*

Concerning perceived learning outcomes in creativity development, learners felt that by playing and manipulating GSP software, the activities help them to enhance creativity and imagination. Related responses are as follows:

S7: *I gained a better understanding of geometric shapes after rotating and moving the software. It enhanced my mind to get more shapes; it helped to stimulate my imagination by creating new geometric shapes.*

Concerning perceived learning outcomes in geometry learning, nearly all the students in experimental group, felt that they have experienced the three levels of Van Hiele Model and therefore, that their skills in describing and interrelating properties of groups in geometric shapes were enhanced. As they reported:

S7: *I learn a new way of relating different shapes using GSP software; the activities allow me to explore and describe the properties of 2-D shape in detail.*

S8: *It is easy for me to identify the properties of geometric shapes with the help of GSP software; the activities help me to compare contrast shapes easily. The GSP activities help to consolidate ideas and further my understanding of geometrical concept.*

Another perceived learning outcome was that appreciation towards geometry was enhanced by the use of GSP in geometry classroom.

As one of the students mentioned:

S4: *Wow! I began to appreciate geometry shapes in my surroundings, the activities eliminate my previous feelings that geometry is a tough and boring subject, I love to explore the field of geometry."*

The above perceptions uttered by the learners from the experimental group are in line with the strategies of the Department of Education to roll out paperless learning and teaching materials in view of improving public basic education.

Results suggest that most students in the study have moved one or two Van Hiele levels, and the majority of the students' levels of geometry thinking are at Van Hiele Levels Two or Three after their participation in a geometry intervention lessons. However, when comparing students' written response in the Van Hiele Geometry Test with his/her interview response, it appears that the van Hiele level of a student determined by the written test is not always coherent with expected geometric discourse at the given level described in the model of the Development of Geometric Discourse. For example, after being assigned to Van Hiele Level Three based on his/her written response in the Van Hiele Geometry Test, the student was interviewed.

Analysis of the students' geometric discourse with respect to their word use, routines, visual mediators and endorsed narratives shows that the students' Van Hiele levels are at Level Two instead of Level Three. This result does not indicate that the Van Hiele Geometry Test is inaccurate in determining learners' Van Hiele levels, but rather suggests that using a discursive lens to analyse students' geometric discourses at each Van Hiele level provides additional information about the students' levels of geometric thinking, and detects information which has been missed in the Van Hiele Geometry Test.

4.8 The control group

In the class assigned for the control group, the lessons were imparted in what is referred to as the traditional way, with the researcher transmitting all the knowledge and the students were passively accepting it without question (Ijeh, 2003:35). In the traditional mathematics classroom, where the researcher showed how and what was to be done, there was little discussion. Students were seldom given the chance to ask questions if they did not understand. No meaningful teaching and learning take place, and no communication between the researcher and the students or even among the students themselves. In contrast, the traditional mathematics classroom is ironically a place where students' opinions are never heard (Ijeh, 2003:36).

4.8.1 *Teaching methods and learning styles*

The teaching and learning was centred and planned within a question-and-answer approach. The researcher provided limited opportunities for observing students and listening carefully to their ideas and alternative conceptions. More emphasis was put on procedural understanding, with low levels of comprehension of mathematical concepts and relations that were taught. The teacher did not show the ability to teach students on how to reflect, explain, and justify their own claims.

Without the use of any dynamic geometry software, students find difficulty in constructing the shapes and getting the right values for their widths, lengths and angles. This was due to the weaknesses in construction and exploration when using paper, pencil and compass. In Figure3, students were asked to explore the properties possessed by a square; the data obtained were filled into the table for the purpose of discussion in the next phase. Question like: *Is a square a rectangle? Choose YES/NO and give reasons for your answer.*

A response like "Yes, it looks like a rectangle" would be classified as Level One, as this reasoning reflects visual considerations. However, a response like "Yes, a square is a rectangle with equal side" would be classified as Level Three, as this entails hierarchical thinking of the student, which is not the case for this group.

4.8.2 *Classroom interactions*

Interactions in this classroom took the form of teacher initiated discussions, typified by researchers' frequent use of inauthentic initiating question turns. The follow-up turns by either the teacher or students did not happen during classroom discourse. The researcher asked questions and students responded mostly in chorus. The interactions that took place within this classroom were found to have highly ritualised components that are not explicitly taught, but are embedded within the classroom culture.

4.9. SUMMARY OF MAIN FINDINGS

Based on the pre-test results analyses, most of the students' levels of geometric thinking were at Level One, i.e., visualisation. This is consistent with the findings of studies reported elsewhere (see for example Razananadiah, 2006:27; Chong, 2001:86; Noraini, 2008:78). In this study, such observations were highly probable, given that the visualisation level of geometric thinking, and does not involve the argumentative ability in students, but is more concerned with their perspective (Mason, 1998: 163). In addition, and similar to reports made elsewhere (Crowley,1987:16; Halat, 2008:14; Noraini,2006:78), students' performances in the pre-test were found to be at the first level, although some struggled to recognise and identify certain geometric shapes on the overall entity of the objects.

However, the post-intervention data revealed that the post-test performances of students in the experimental group were significantly higher than those in the control group. In actual fact, the experimental group demonstrated better levels of geometric thinking when compared to the control group. This finding may attributed to the use of GSP software as a technique in the teaching and learning of geometry in the experimental group.

It thus appeared that the implementation this means that the implementation of Van Hiele's phases of learning geometry with GSP software as a medium, assisted students in achieving better levels of geometric achievement as compared to those students who learned the same topics conventionally. The findings of this study are in accordance with previous studies that were conducted by Teppo (1991:210-221), Matthews (2005:179), Wu (1994:1665) and Craft (2000:77).

Current teaching and learning practice in the classroom does not reflect the importance of geometry in the lives of students, and the emphasis that is placed on it in the curriculum. Teacher training is still bound to a traditional approach that is teacher-centred(Mullis et al,2000,2004 &2008 and Noraini,2006:66).According to Wan (1998:69) in terms of teacher training and attitude, more often teachers who teach mathematics use the blackboard to explain theorems, definitions and concepts, and to show the solutions for the related problems. Students are commonly shown methods and algorithms, which they memorized without actually understanding the concepts (Craft, 2000:178).

In other countries, such as Malaysia, these findings are in line with the studies for example conducted by Tay(2003:363), who focused on manipulative materials in the phase- based activities and Chew(2009:31-48) who focused on other geometric topics. This study has also showed that improvement from one level of geometric thinking to a higher level of geometric thinking depends on the lesson taken by the students and not on their maturity (Van Hiele, 1986:243-252).

4.10. CHAPTER SUMMARY

The discussion of results in this chapter focused on quantitative data generated from the study. Both descriptive and inferential statistical analysis of pre- and post-tests were examined within the theoretical underpinnings presented in Chapters Two and Three.

Qualitative data indicated that English, the official language of learning and teaching in South African public schools, is the preferred language in the teaching and learning of mathematics in multilingual classrooms of the experimental group. However, analysis of students' interviews suggests that, although English is the preferred LoLT, they proposed a dual-use and/or parallel-use of English and African Languages for teaching and learning for students to understand geometric terminologies with ease.

CHAPTER 5: CONCLUSION AND RECOMMENDATIONS

5.1 INTRODUCTION

For many decades, mathematics teachers have been calling for more relevance and meaning in terms of learners' classroom mathematical activities (Verschaffel et al. 2000, 2009; Brownell, 1945:323-331; Freudenthal, 1991:23), while researchers have been reporting on the role of technology in the teaching and learning of geometry, in particular, GSP software in teaching and learning of geometry in multilingual classrooms (see Adler, 2001:47-64; Arthur, 1994:63-78; Chitera, 2009; Freitag, 2005:16-21; Setati, 2005a).

In this final chapter the rationale and design of the study is reflected upon and thereafter conclusions drawn and inferences made as to the extent to which the intervention of this study has impacted students' geometric thinking abilities, as well as their ability to make realistic considerations of geometry problems.

The aim of this study was to establish the effects of using Geometer Sketchpad on Grade 10 students' academic achievement in Euclidean geometry. The researcher used Van Hiele's phases of learning geometry in Grade 10's geometry in order to assist the students in enhancing their levels of thinking to higher levels. The phases involved were information, guided orientation, Explicitation, free orientation and integration. Geometer's Sketchpad (GSP) software was used as a medium to implement the activities. The students in the treatment group learned the geometry topics based on the activities developed from Van Hiele's phases of learning geometry by using the GSP software.

Meanwhile, the students in the control group learned the same topics using the conventional approach. The students' pre and post level (initial and final levels) of geometric thinking in both groups were identified quantitatively and qualitatively. It was found that learners in the treatment group showed a better increment in geometric thinking levels compared to students in the control group. Therefore, in accordance with the Department of Basic Education and Sports Development, Euclidean geometry concepts are examined in Paper 2, since the inception of CAPS (Curriculum Assessment Policy Statement) in 2012 for grades 1, 3, 6 and 10, 2013 grades 9 and 11 and 2014 grade 12 where previously, the concept was examined in Paper 3 during the NCS (National Curriculum Statement) era. In this study, Geometers' Sketchpad was used as a tool in teaching and learning of these geometrical concepts.

5.2. RATIONALE AND DESIGN

As presented, pre-test—intervention—post-test design was used in the study. The pre-tests investigated geometric concepts abilities of Grade 10 Second Language Mathematics students, after which follow-up interviews were conducted to clarify what problems they may have encountered, and why they solved the problems the way in which they did. An observation schedule was used to track students' attempts at discussion and argumentation in the classroom, and post-tests investigated any changes and possible reasons for them.

5.3 MAIN FINDINGS

The results of the study show the impact of how the usage of Geometer's Sketchpad is useful in the teaching and learning of the geometry in the tenth grade.

The implications for teachers and curriculum planners from the findings of this investigation are: Firstly, that the majority of the students in treatment group demonstrated that they are able to identify and visualise a figure and its shapes, but have difficulties in arriving at the properties of figures. Students provided different explanations on the properties of figures, and solve problems related to geometric figures, as they grew in geometric understanding, and their views of a figure and its determining properties changes (Pegg, 1991:13). This evolution was observed by treatment group after the intervention was administered. It takes time for the growth from one level to the other to be encouraged, but it cannot be rushed through by the researcher. It takes time for learners to play the game of geometry by clicking with the mouse, while learning and this does not start until students were able to see the properties of geometric figures as important aspects of their description.

Level two and three concepts appeared to be difficult for many students. These levels represent new and important way of organising thinking, which usually does not come naturally to students. However, it represented an important and often overlooked link in the chain of events moving to formal deduction in geometry learning. Students' ability to link in terms of geometric properties at level three concepts represents a further development in understanding beyond the concept of class enclosure. For such activities (finding the properties) to be viable, students will need to have extensive familiarity with the properties of figures and the relationships between them (Pegg, 1991:10-13).

From the results obtained, a number of implications can be put forward in improving mathematics teaching and learning. Firstly, the significant difference of the pretest and posttest (VHGT) indicates that Geometers' Sketchpad is possibly contributing in the learning of geometry.

The findings of this study suggest that the objectives of the research were achieved, namely:

i. establishing the effects of using Geometer Sketchpad on Grade 10 leaners' academic achievement in Euclidean geometry;

ii. identifying the benefits of using Geometer Sketchpad in the teaching and learning of Euclidean geometry in Grade 10 mathematics classrooms; and

iii. understanding the impact of using Geometer Sketchpad on Grade 10 students' academic achievement in Euclidean geometry.

The above objectives of the study were successfully achieved by the use of adapted VHGT used to address the following research questions:

1. What is the effect of using Geometer Sketchpad on Grade 10 learners' understanding of geometry?

The data to answer this research question were obtained by the researcher by using VHGT administered before and after the intervention in the treatment group.

The result of this study also supports the findings of Lester (1996:306), which mentioned that Geometer's Sketchpad provides intelligent capabilities for improving teaching and learning. Noraini, I (2001:287) also conducted a quasi-experimental research on the effects of Van-Hiele based instructional activities with Geometer's Sketchpad on Van Hiele levels. The result she obtained indicated a significant difference between the treatment and control groups in rank on van Hiele levels from pre-test to post-test. The researcher concluded that the significant improvement of geometry achievement using the specially prepared Van Hiele based instructions with Geometer's Sketchpad, indicated the need to provide more interactive and hands on learning activities for geometry learning in Grade 10's understanding of geometry.

2. What are the benefits of using Geometer Sketchpad in the teaching and learning of Euclidean geometry in Grade 10 Mathematics classroom?

In addition, the increase in scores from the pretest and posttest also indicates that the learners' usage of Geometer's Sketchpad does help in solving geometry problems. Geometers' Sketchpad will be a tool in improving learners' understanding in geometric concepts in relevant topics. According to NCTM (1999), "calculators don't think, students do." This also applies to the Sketchpad. Students need to understand the mathematics problem they are solving. With that information, only then they can decide what operations to use, and take the next action. Therefore, software like Geometer's Sketchpad does cause students to think and explore to find the solutions. Purdy (2000:224-228) also discovered that in maximum-volume problems, Geometer's Sketchpad helps in the practical exploration of the problem. Furthermore, he discovered that his students have been lead to a deeper understanding of the problem and its solution as a result of their exploration.

Secondly, the significantly better results in the VHGT achieved by the experimental group of students implied that the learning of geometry with the Geometer's Sketchpad had been beneficial and useful for the learners. The learners seemed to have a more positive attitude in the drawing of triangles, rectangles, parallelograms etc. while using Geometer's Sketchpad. Learners are enjoying the lessons of drawing figure and also able to interpret the properties (angles, sides, angles etc.) of geometrical sketches/figures better with Sketchpad.

3. How does the use of Geometer Sketchpad impact Grade 10 leaners' academic achievement in Euclidean geometry?

These findings support the results of Groman (1996:264) that learners' reaction is overwhelmingly positive on using Geometer's Sketchpad in Mathematics class. Furthermore, the usage of Geometer's Sketchpad indicated a more positive reaction from both the students and the researcher in developing conjectures and constructions. Garofalo &Bell (2004:155) showed how Geometer's Sketchpad sketches could be extended and expanded to different levels to enrich the teaching and learning of geometric concepts.

In the light of the above research question, the utterances made by the students from the treatment group after the intervention bear witness that the software impacted positively in students' academic achievement in geometry:

L3: *I enjoyed learning geometry using GSP software; an interesting way to learn geometry that used to be quite boring*, and

L7:*It is easy for me to identify the properties of geometric shapes with the help of GSP software; the activities help me to compare contrast shapes easily; The GSP activities help to consolidate ideas and further my understanding of geometrical concept.*

In conclusion, this study suggests that the use of Geometers' Sketchpad in the Mathematics classroom is useful in helping students perform better in geometrical sketches. Furthermore, they have a positive attitude towards learning geometrical concepts with the usage of Geometer's Sketchpad. Consequently, the use of software encouraged learners to take part in the learning of the geometrical concepts in a more enjoyable and interesting way.

The study on this topic seemed to be quite clear. Gningue (2003:207-224) reminded us that we need to use technology, because students are used to it and Yelland (1999:39-59) talked about how learning by doing is now the rule, whereby it used to be the exception. Their research shows that we need to use not only technology, but more importantly, dynamic software, to bring out the best in the students. Edwards (Spring 2005:32-40) also agreed when he said that dynamic geometry software(DGS) enables us in that the questions are as important as answers and in which explaining one's ideas rigorously is as important as writing two-column proofs.

Finally, Santos-Trigo (2004:399-413) said that with dynamic software, students are able to discover new theorems or relationships and are able to engage in a way of thinking that goes beyond reaching a particular solution or response to a particular problem. So, we have learned that dynamic software can improve student learning because it allows us to learn by doing and to be able to establish new theorems or relationships. Qualitatively and quantitatively students in an experimental group showed an improvement in their performance after they have been exposed or introduced to intervention, by being taught geometry using GSP.

A descriptive statistical mean of both groups on pre and posttest showed a significant difference. The mean value of control group is less than that of treatment group (see Table 6) and Figures 3 and 4 indicate the growth after intervention.

5.4 LIMITATIONS OF THE STUDY

The findings of the study should be viewed in light of the following limitations. The selection of students for focus group was made on the basis of convenience sampling, rather than on statistical considerations. In addition, only a limited number of students participated in the study and as such, students of this study do not represent the population of schools in the North West, or even of the Bojanala district nor Rustenburg Area Project Office (APO), but of the secondary school where the study was conducted. The limitation of this study is the number of subjects.

Due to the time constraints in completing this study, only two Grade 10 classes in the secondary school were able to be tested. This data should be collected for several years and a reinvestigation into the findings with a larger sample size ought to be considered.

The time of the day the study was conducted might be a factor in how the students learnt and answered questions on the concept been taught in geometry. This study was conducted after school hours in the afternoon, and as a result, some of the students looked tired during the intervention and writing sessions, where this might have affected the outcome of the investigation.

English as LoLT was used in the teaching and learning, as a result, understanding of terminologies used in imparting geometrical concepts posed a challenge as the students in the sampled tenth grade were Setswana and/Sesotho/isiTsonga/isiXhosa speaking as home language. Mother tongue is a barrier as students participating in the study were not confident enough to express themselves effectively using LoLT, the researcher took a decision and concluded on code switching, though, on the other hand, the researcher did not have more knowledge on isiTsonga and was left with no option, but consulted with a colleague, who speaks isiTsonga as her home language. The researcher's consultation was based on the issues of transcriptions to be used and consistent interpretation of students' responses and, as such, these issues did not significantly threaten the validity of the study.

5.5 RECOMMENDATIONS

Furthermore, students' needs to possess the necessary language skills associated with geometrical concepts that will enable them to use expressions such as opposite angles are equal, congruent and line segment in the right context. A teacher's decision not to introduce the correct mathematical language eliminates any opportunity for the students to choose to learn that language (Pickreign et al., 2000:89). Teachers should therefore encourage students to talk about geometric concepts relating to triangles, rectangles, rhombi etc., and discover the properties themselves, so as to develop expressive language. In a classroom situation, teachers should ask students to describe a figure, rather than just to select a name for it from the list. Students' understanding of key concepts, such triangles, parallelograms, similarity, congruency, and circles should be emphasised.

Teachers should also bond to the use of manipulative material, as discussed earlier in this study. They also provide experience in which students can transfer their understanding smoothly from one concept to another. One way of letting the lower achieving students concentrate on the learning of geometry is to use information communication technology (ICT). For example, students could be given a number of geometric shapes placed at different positions, and the task would be to find a way through geometry. This could be a game for students, where one learner instructs the computer to perform a figure, and the other has to find out which one it was. If repeated, this would help students to get a feel for what the image looks like. For example, if it is four-sided it has to be rectangle.

The secondary school geometry curriculum should be appropriate for the various geometric levels. It should guide students to learn about significant and interesting concepts. It should permit students to use visual justification and empirical thinking, because such thinking is the foundation for higher level of geometric thought (Pickreign et.al. 2000:106).

The curriculum should require learners to explain and justify their ideas. It should also encourage students to refine their thinking. In conclusion, the present study adds the following to the field of geometry education:

- ✓ It has employed the Van Hiele's theory of geometry learning to describe and analyse students' difficulties in geometry within the context of Euclidean geometry.
- ✓ It has also suggested guidelines for classroom practice that can contribute to improved teaching and learning of geometry.

With the findings emanating from this study, it is hoped that the recommendations will assist teachers in their perception of their students' difficulties in learning geometry.

This could have changed through teachers increased familiarity with recommendation based on Van Hiele's model. Teachers through this study's recommendation would recognise that they need to discuss the importance of the Van Hiele's model with their students. This should be done because students need to be assured that their readiness for geometry is related to their previous experience and instruction and that lack of readiness is not a reflection of their intelligence.

Although this study did not directly investigate the teacher and textbooks used in the classroom. The teachers do rely heavily on texts for their daily instructions. To bring about these changes, textbook and teachers' education that is focused on the Van Hiele's model are recommended.

The results of the investigation are an indication that the Van Hiele's model of development in geometry can serve as a useful frame of reference when analysing student's thinking processes in geometry tasks.

5.6 CONCLUSIONS

The conclusions are also synonymous with some conclusions reached by some cited researcher such as Piaget (1973:215), Van Hiele and Freudenthal (1973:149) in the literature review. However, the researcher believes that the result obtained and the differences noted among the different kinds of learners indicate that the proposed method of evaluation of the Van Hiele levels is coherent and should be researched further.

Based on the results and discussion, it can be concluded that Van Hiele's phases of learning geometry namely information, guided orientation, explicitation; free orientation and integration are a referable and implementable alternative learning strategy for geometry topics. Van Hiele's (1999:310-316) phases of learning make students' geometry activities more organised and systematic. In the first phase, that is information, new ideas will be introduced to the student to give an early description to the student about the concepts of geometry that they will explore and learn. In phase two directed orientation, the prepared activities give the student opportunities to explore concepts of geometry themselves. Based on the observation from the finding obtained in the second phase, the students will solve more complex problems, where usually, the questions can be solved in many ways.

In the last phase of integration, the students will make a summary of what they have learnt for the purpose of drawing a new general picture of a network of objects and correlation between them. The students will present the results of their work and the teacher will then correct their findings. Meanwhile, the Geometer's Sketchpad (GSP) software was used as a medium to make the students' learning activities smoother and more enjoyable for them. In addition to that, students can explore a concept accurately, quickly and effectively with the use of technology, especially by means of GSP software, as compared to the traditional approach, which consumes more of the students' learning time.

For example, a student measures the length of a side of the square. Using the traditional approach, the student uses a pencil and a ruler to build and measure each side of the square. The length of the sides may not be exactly equal for each side of the square. However, that problem will not occur if GSP software (constructivist method) is used to build and measure each side of the geometrical figuring/drawings. Hence, the use of Van Hiele's phases of learning geometry is very much encouraged to be applied in learning geometrical topics. Based on past studies that had proven that the use of Van Hiele's Geometric phases of learning with the aid of dynamic geometry software, like the GSP software, can have a positive impact on learners, such as increasing their achievement in geometry, their understanding in geometrical concepts and terminologies can enhance level of confidence while learning geometry.

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APPENDIX A: Permission letter

NORTH-WEST UNIVERSITY
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Tel 018 389-2111
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Education Leadership Development
Tel 018 3892500 (Secretary)
Email eliza.senne@nwu.ac.za

The Principal

....

Bojanala (District) Area Office

RUSTENBURG

Date: 28 May 2016

APPENDIX A: Permission to conduct research (principal, SMT and SGB letter)

Dear Sir/Madam,

REQUEST FOR PERMISSION TO CONDUCT RESEARCH

This is to confirm that **Mrs AM. Kgatshe (Student No: 23349425)** is a **M Ed** student registered at the North-West University, Mafikeng Campus. The title of the dissertation is **The Effect of Using Geometer Sketchpad on Grade 10 learners' Understanding of Geometry: A case of a school in a village in Bojanala District.**

Permission is hereby kindly requested to enter your to collect data from the teachers and heads of departments. Data collection will be by way of interviews.

Collection of data will occur outside school contact time so as not to interfere with teaching and assessment processes or office duties. The dates and times of the collections are to be agreed upon by the heads of departments and all other participants.

Participants will participate voluntarily in the data collection. The identity of the participants and the school and district will be kept confidential and anonymous. The information collected therefore cannot and will not be used to evaluate the school in terms of its performance in comparison with others, because the information collected will not be about academic results or teachers' teaching performance in specific schools.

Should you enquire more information about the project, kindly contact the supervisor for this project: **Prof. JP. Sepeng at 018-389-2887.**

Herewith permission is kindly requested to perform this research in your area. It would be appreciated if you would kindly grant **written** permission to this student. Any assistance given to the student to perform the research will be appreciated.

Yours sincerely

Prof N. Diko

Acting Director: School for Education Leadership Development (School in which the Masters and PhD programme is registered)

Mafikeng Campus

APPENDIX B: Letter to parents or guardians of participants

19 July 2015

Dear Parent/ Guardian

My name is Aletta Kgatshe. I am currently enrolled in a Master's degree in Education specialising mathematics education at the University of North West. and I am in the process of collecting empirical data for my thesis. I hereby invite you to give consent for your child to participate in a study on the development of mathematical problem solving skills of Grade 10 learners in a problem-centred teaching and learning environment.

The aim of this project is to evaluate the effect of developing problem solving skills on learners' performance and achievement in mathematics. If you would want your child to participate in this study, kindly sign this form and your child should return it to the researcher at the onset of the data collection process. Your participation in this study is voluntary and confidential. At no time will your name, the name of your child's school or your child's name be identified. While this study may be published, you are guaranteed that neither your name nor your child's name will be identified in any report of the results of the study. No costs will be incurred by either your child's school or you as the parent or guardian of the participant.

If you give consent for your child to participate, your child will be involved in this research during his/her usual mathematics lessons, that is, 4.5 hours a week for 10 weeks during the 3rd term of the 2015 academic year. In total, during these 10 weeks your child is expected to attend the intervention programme for a minimum of 4.5 x 10 weeks = 45 hours.

Kindly note, that your child does have a choice of not taking part in this research. If your child is not taking part in this project s/he will be automatically put in the control group where lessons will continue as normal with the current mathematics teacher (the researcher). Your child will neither be required to fill in the questionnaire nor write the pre- and post-tests that will be written by other learners in the experimental group.

If you would like more information about this research study, you can contact me on the following number: 082 404 9299. If you would like the results of the study kindly supply a postal address where I can forward the results to:

....

Parent's/guardian signature: __________ Date: ________

Researcher's signature: __________ Date: ________

Kindest Regards

Ms Aletta Kgatshe

APPENDIX C: Letter to participants

16 July 2015

Dear Participant

My name is Aletta Kgatshe. I am currently enrolled in a Master's degree in Education specialising in mathematics education at the University of North West, and I am in the process of collecting empirical data for my thesis. I hereby invite you to participate in a study on the development of mathematics problem solving skills of Grade 10 learners in a problem-centred teaching and learning environment. The aim of this project is to evaluate the effect of developing problem solving skills on learners' performance and achievement in mathematics.

If you are willing to participate in this study, you will be given a consent form to be signed by either your parent or guardian and you should return it to the researcher at the onset of the data collection process. Your participation in this study is voluntary and confidential. At no time will the name of your school or your name be identified. While this study may be published, you are guaranteed that neither your school nor your name will be identified in any report of the results of the study. No costs will be incurred by either your school or you as a participant.

If you are willing to participate, you will be involved in this research during your usual mathematics lessons, that is, 4.5 hours a week for 10 weeks during the 3rd term of the 2012 academic year. In total, during these 10 weeks you are expected to attend the intervention programme for a minimum of 4.5 x 10 weeks = 45 hours.

Kindly note, that you do have a choice of not taking part in this research. If you are not taking part in this project you will be automatically put in the control group where lessons will continue as normal with your current mathematics teacher. You will neither be required to fill in the questionnaire nor write the pre- and post- tests that will be written by other learners in the control group.

If you would like more information about this research study, you can contact me on the following number: 082 404 9299. If you would like the results of the study kindly supply a postal address where I can forward the results:

Participant's signature: ____________ Date: ________

Researcher's signature: ____________ Date: _________

Kindest Regards

Ms Aletta Kgatshe

APPENDIX D

Descriptions of terms in this study

Multilingual: Refers to an individual who is proficient in two or more languages respectively.

Multilingual classroom: Refers to a situation where learners bring into a class a range of home languages. This does not imply that all learners and/or teachers in the class are themselves necessarily multilingual.

Code-switching: Means shifting from one code (i.e. language, dialect or language variety) to another between utterances or for a section of an utterance that is at least of sentence length. All forms of code-switching presuppose a speaker's sensitivity to different social contexts and conventions.

Discourse: This term refers to ways of using words, including the purpose to which the language is put.

Language of Learning and Teaching: Is the term that refers to language(s) used for both learning and teaching across the curriculum and gives equal importance to both learning and teaching. These terms can also be referred as —language of instructionl or —medium of instructionl. Thus, in this thesis, these two terms are used interchangeably.

Student: Is a learner, or someone who attends an educational institution. In some nations, the English term (or its cognate in another language) is reserved for those who attend university, while a schoolchild under the age of eighteen is called a pupil.

APPENDIX E

Pre-post-test

Grade 10 Mathematics pre-post-test: VHGT

Date: ____________________

(Please provide the following information about you by completing spaces below)

Name of Leaner: ____________________________

Name of School: ____________________________

Grade: ____________________________

Age of Learner: ____________________________

Home Language: ____________________________

Gender: ____________________________

Ethnicity: ____________________________

Instructions:

1. Read each question carefully.

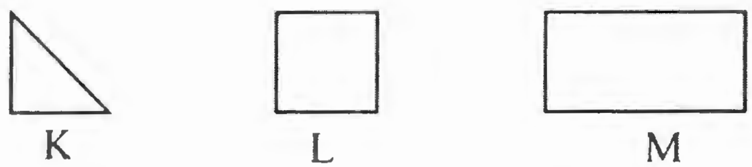
2. Decide upon the answer you think is correct. There is only one correct answer to each question. Cross out the letter corresponding to your answer on the answer sheet.

3. Use space provided on the answer sheet for figuring or drawing. Do not mark on this test booklet.

4. You will have 35 minutes for this test

VAN HIELE GEOMETRY TEST

1. Which of these are squares?



A. K only

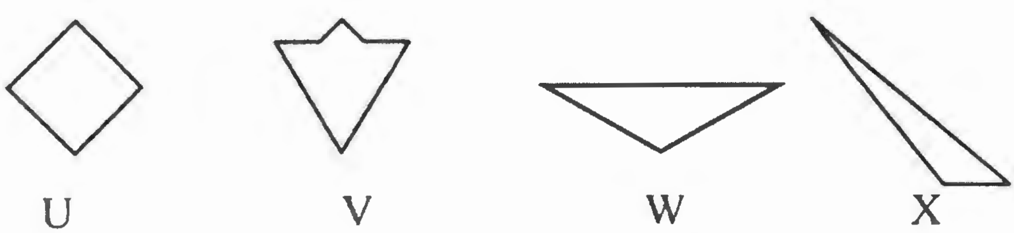
B. L only

C. M only

D. L and M only

E. All are squares

2. Which of these are triangles?



A. None of these are triangles.

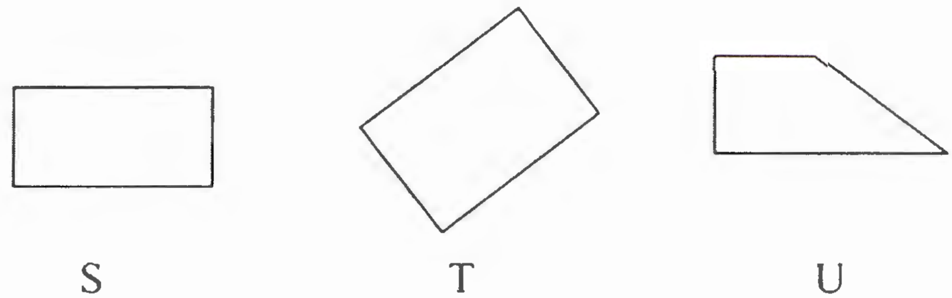
B. V only

C. W only

D. W and X only

E. V and W only

3. Which of these are rectangles?



A. S only

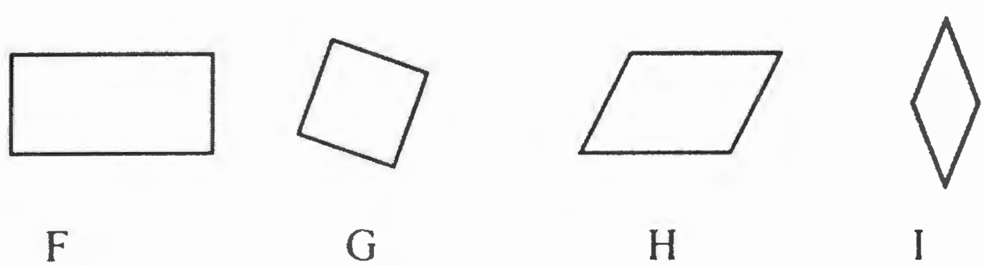
B. T only

C. S and T only

D. S and U only

E. All are rectangles.

4. Which of these are squares?



A. None of these are squares.

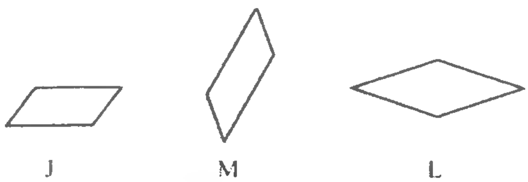
B. G only

C. F and G only

D. G and I only

E. All are squares.

5. Which of these are parallelograms?



A. J only

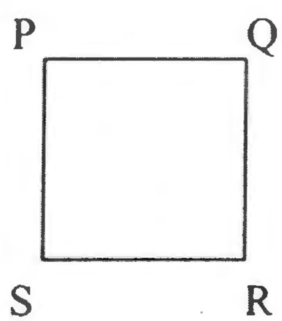
B. L only

C. J and M only

D. None of these are parallelograms.

E. All of the above are parallelograms.

6. PQRS is a square.



Which relationship is true in all squares?

A. PR and RS have the same length.

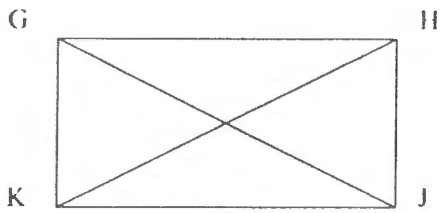
B. QS and PR are perpendicular.

C. PS and QR are perpendicular.

D. PS and QS have the same length.

E. Angle Q is larger than angle R.

7. In the rectangle GHJK, GJ and HK are the diagonals.



Which of (A)-(E) is not true in every rectangle?

A. There are four right angles.

B. There are four sides.

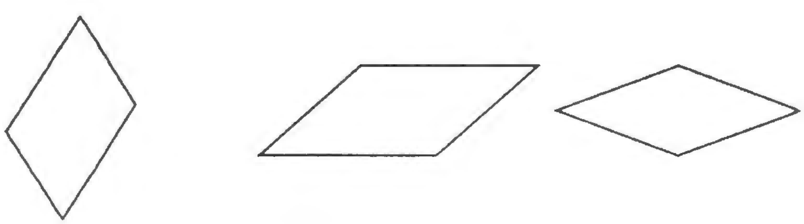
C. The diagonals have the same length.

D. The opposite sides have the same length.

E. All of the above are true in every rectangle.

8. A rhombus is a 4-sided figure with all sides of the same length.

Here are three examples.



Which of (A)-(E) is not true in every rhombus?

A. The two diagonals have the same length.

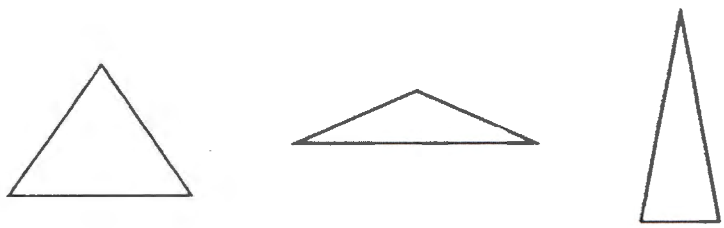
B. Each diagonal bisects two angles of the rhombus.

C. The two diagonals are perpendicular.

D. The opposite angles have the same measure.

E. All of the above are true in every rhombus.

9. An isosceles triangle is a triangle with two sides of equal length. Here are three examples.



Which of (A)-(E) is true in every isosceles triangle?

A. The three sides must have the same length.

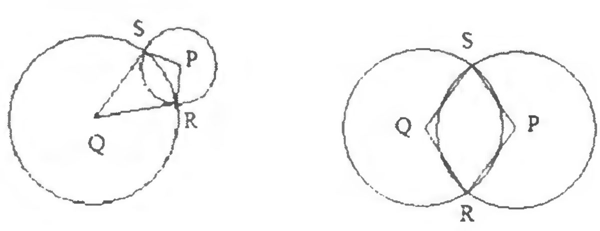
B. One side must have twice the length of another side.

C. There must be at least two angles with the same measure.

D. The three angles must have the same measure.

E. None of the above is true in every isosceles triangle.

10. Two circles with centres P and Q intersect at R and S to form a 4-sided figure PRQS. Here are two examples.



Which of (A)-(E) is not always true?

A. PRQS will have two pairs of sides of equal length.

B. PRQS will have at least two angles of equal measure.

C. The lines PQ and RS will be perpendicular.

D. Angles P and Q will have the same measure.

E. All of the above are true.

11. Here are two statements.

Statement 1: Figure F is a rectangle.

Statement 2: Figure F is a triangle.

Which is correct?

A. If 1 is true, then 2 is true.

B. If 1 is false, then 2 is true.

C. 1 and 2 cannot both be true.

D. 1 and 2 cannot both be false.

E. None of the above.

12. Here are two statements.

Statement S: ΔABC has three sides of the same length

Statement T: In ΔABC, $\angle B$ and $\angle C$ have the same measure.

Which is correct?

A. Statement S and T cannot both be true.

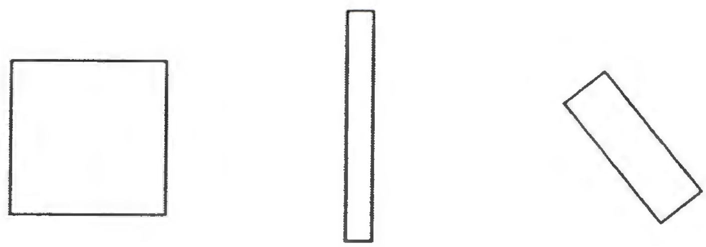
B. If S is true, then T is true.

C. If T is true, then S is true.

D. If S is false, then T is false.

E. None of (A)-(D) is correct.

13. Which of these can be called rectangles?



P Q R

A. All can.

B. Q only

C. R only

D. P and Q only

E. Q and R only

14. Which is true?

A. All properties of rectangles are properties of all squares.

B. All properties of squares are properties of rectangles.

C. All properties of rectangles are properties of all parallelograms.

D. All properties of squares are properties of all parallelograms.

E. None of (A)-(D) is true.

15. What do all rectangles have that some parallelograms do not have?

A. Opposite sides equal

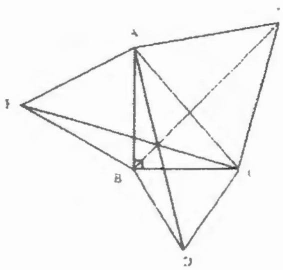
B. Diagonals equal

C. Opposite sides parallel

D. Opposite angles equal

E. None of (A)-(D)

16. Here is a right triangle ABC. Equilateral triangles ACE, ABF, and BCD have been constructed on the sides of ABC.



From this information, one can prove that AD, BE, and CF have a point in common. What would this proof tell you?

A. Only in this triangle drawn can we be sure that AD, BE and CF have a point in common.

B. In some but not all right triangles, AD, BE and CF have a point in common.

C. In any right triangle, AD, BE and CF have a point in common.

D. In any triangle, AD, BE and CF have a point in common.

E. In any equilateral triangle, AD, BE and CF have a point in common.

17. Here are three properties of a figure.

Property D: It has diagonals of equal length.

Property S: It is a square.

Property R: It is a rectangle.

Which is true?

A. D implies S which implies R.

B. D implies R which implies S.

C. S implies R which implies D.

D. R implies D which implies S.

E. R implies S which implies D.

18. Here are two statements.

I: If a figure is a rectangle, then its diagonals bisect each other.

II: If the diagonals of a figure bisect each other, the figure is a rectangle.

Which is correct?

A. To prove I is true, it is enough to prove that II is true.

B. To prove II is true, it is enough to prove that I is true.

C. To prove II is true, it is enough to find one rectangle whose diagonal bisect each other.

D. To prove II is false, it is enough to find one non-rectangle whose diagonals bisect each other.

E. None of the above.

19. in geometry:

A. Every term can be defined and every true statement can be proved true.

B. Every term can be defined but it is necessary to assume that certain statements are true.

C. Some terms must be left undefined but every true statement can be proved true.

D. Some terms must be left undefined and it is necessary to have some statements which are assumed true.

E. None of the above

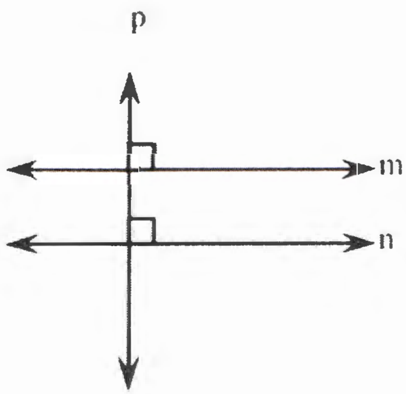
20. Examine these three sentences.

1. Two lines perpendicular to the same line are parallel.

2. A line that is perpendicular to one of two parallel lines is perpendicular to the other

3. If two lines are equidistant, then they are parallel.

In the figure below, it is given that lines m and p are perpendicular and lines n and p are perpendicular. Which of the above sentences could be the reason that line m is parallel to line n?



A. (1) only

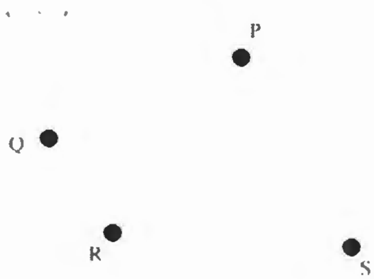
B. (2) only

C. (3) only

D. Either (1) or (2)

E. Either (2) or (3)

21. In F-geometry, one that is different from the one you are used to, there are exactly four points and six lines. Every line contains exactly two points. If the points are P, Q, R and S, and the lines are {P,Q}, {P,R}, {P,S}, {Q,R}, {Q,S}, and {R,S}.



Here are how the words "intersect" and "parallel" are used in F-geometry.

The lines {P,Q} and {P,R} intersect at P because {P,Q} and {P,R} have P in common.

The lines {P,Q} and {R,S} are parallel because they have no points in common.

From this information, which is correct?

A. {P, R} and {Q, S} intersect.

B. {P, R} and {Q, S} are parallel.

C. {Q, R} and {R, S} are parallel.

D. {P, S} and {Q, R} intersect.

E. None of the above.

22. To trisect an angle means to divide it into three parts of equal measure. In 1847, P.L. Wantzel proved that, in general, it is impossible to trisect angles using only a compass and an unmarked ruler. From his proof, what can you conclude?

A. In general, it is impossible to bisect angles using only a compass and an unmarked ruler.

B. In general, it is impossible to trisect angles using only a compass and a marked ruler.

C. In general, it is impossible to trisect angles using any drawing instruments.

D. It is still possible that in the future someone may find a general way to trisect angles using only a compass and an unmarked ruler.

E. No one will ever be able to find a general method for trisecting angles using only a compass and an unmarked ruler.

23. There is a geometry invented by a mathematician J in which the following is true:

The sum of the measures of the angles of a triangle is less than 180°.

Which is correct?

A. J made a mistake in measuring the angles of the triangle.

B. J made a mistake in logical reasoning.

C. J has a wrong idea of what is meant by "true".

D. J started with different assumptions than those in the usual geometry.

E. None of the above.

24. The geometry books define the word rectangle in different ways.

Which is true?

A. One of the books has an error.

B. One of the definitions is wrong. There cannot be two different definitions for rectangle.

C. The rectangles in one of the books must have different properties from those in the other book.

D. The rectangles in one of the books must have the same properties as those in the other book.

E. The properties of rectangles in the two books might be different.

25. Suppose you have proved statements I and II.

I: If p, then q.

II: If s, then not q.

Which statement follows from statements I and II?

A. If p, then s.

B. If not p, then not q.

C. If p or q, then s.

D. If s, then not p.

E. If not s, then p.

APPENDIX F

Van Hiele Geometry Test Answer Sheet

VAN HIELE GEOMETRY TEST

Cross out the correct answer Space for drawing or figuring

1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E

13	A	B	C	D	E
14	A	B	C	D	E
15	A	B	C	D	E
16	A	B	C	D	E
17	A	B	C	D	E
18	A	B	C	D	E
19	A	B	C	D	E
20	A	B	C	D	E
21	A	B	C	D	E
22	A	B	C	D	E
23	A	B	C	D	E
24	A	B	C	D	E
25	A	B	C	D	E

APPENDIX G

Summary Table of Answers to Items

Scores of initial and final geometric levels of the experimental group

Table G.1 Item Summary: for Pre-test (Pe) and Post-test (Po) (learners)

Level		A	B	C	D	E	Blank
1	1 Pe:	0.0%	86.4%	3.4%	10.2%	0.0%	0.0%
	1 Po:	0.0%	80.4%	2.0%	17.6%	0.0%	0.0%
	2 Pe:	0.0%	1.7%	5.1%	91.5%	1.7%	0.0%
	2 Po:	0%	0%	4%	96%	0%	0%
	3 Pe:	1.7%	0.0%	94.9%	0.0%	3.4%	0.0%
	3 Po:	0.0%	0.0%	100.0%	0.0%	0.0%	0.0%
	4 Pe:	0.0%	78.0%	8.5%	5.1%	6.8%	1.7%
	4 Po:	2.0%	80.4%	9.8%	2.0%	5.9%	0.0%
	5 Pe:	1.7%	1.7%	15.3%	0.0%	81.4%	0.0%
	5 Po:	0.0%	0.0%	17.6%	0.0%	82.4%	0.0%
2	6 Pe:	6.8%	67.8%	20.3%	5.1%	0.0%	0.0%
	6 Po:	2.0%	80.4%	13.7%	3.9%	0.0%	0.0%
	7 Pe:	6.8%	0.0%	1.7%	1.7%	89.8%	0.0%
	7 Po:	2.0%	0.0%	0.0%	2.0%	96.1%	0.0%
	8 Pe:	66.1%	6.8%	13.6%	5.1%	6.8%	1.7%
	8 Po:	74.5%	2.0%	7.8%	9.8%	5.9%	0.0%

	9 Pe:	3.4%	0.0%	94.9%	0.0%	1.7%	0.0%
	9 Po:	3.9%	2.0%	82.4%	0.0%	11.8%	0.0%
	10 Pe:	3.4%	5.1%	8.5%	69.5%	6.8%	6.8%
	10 Po:	2.0%	2.0%	3.9%	78.4%	11.8%	2.0%
3	11 Pe:	3.4%	8.5%	76.3%	1.7%	8.5%	1.7%
	11 Po:	0.0%	9.8%	86.3%	2.0%	2.0%	0.0%
	12 Pe:	8.5%	79.7%	5.1%	5.1%	1.7%	0.0%
	12 Po:	2.0%	88.2%	3.9%	2.0%	2.0%	2.0%
	13 Pe:	76.3%	0.0%	0.0%	0.0%	23.7%	0.0%
	13 Po:	88.2%	0.0%	0.0%	0.0%	11.8%	0.0%
	14 Pe:	44.1%	10.2%	10.2%	1.7%	32.2%	1.7%
	14 Po:	58.8%	21.6%	0.0%	0.0%	15.7%	3.9%
	15 Pe:	3.4%	67.8%	3.4%	10.2%	11.9%	3.4%
	15 Po:	3.9%	62.7%	3.9%	13.7%	13.7%	2.0%
4	16 Pe:	20.3%	8.5%	44.1%	5.1%	8.5%	13.6%
	16 Po:	17.6%	3.9%	47.1%	11.8%	15.7%	3.9%
	17 Pe:	23.7%	15.3%	42.4%	8.5%	10.2%	0.0%
	17 Po:	23.5%	23.5%	41.2%	5.9%	3.9%	2.0%
	18 Pe:	10.2%	16.9%	5.1%	50.8%	11.9%	5.1%
	18 Po:	21.6%	19.6%	5.9%	35.3%	13.7%	3.9%
	19 Pe:	45.8%	25.4%	10.2%	15.3%	1.7%	1.7%

19 Po: 49.0% 23.5% 5.9% 11.8% 9.8% 0.0%

20 Pe: 32.2% 5.1% 8.5% 44.1% 5.1% 5.1%

20 Po: 41.2% 2.0% 3.9% 45.1% 7.8% 0.0%

21 Pe: 39.0% 33.9% 0.0% 5.1% 6.8% 15.3%

21 Po: 35.3% 41.2% 5.9% 3.9% 9.8% 3.9%

22 Pe: 6.8% 27.1% 5.1% 23.7% 16.9% 20.3%

22 Po: 17.6% 21.6% 3.9% 41.2% 13.7% 2.0%

23 Pe: 18.6% 10.2% 3.4% 33.9% 11.9% 22.0%

23 Po: 31.4% 3.9% 2.0% 51.0% 9.8% 2.0%

24 Pe: 3.4% 5.1% 5.1% 18.6% 44.1% 23.7%

24 Po: 5.9% 3.9% 21.6% 39.2% 29.4% 0.0%

25 Pe: 1.7% 22.0% 6.8% 39.0% 6.8% 23.7%

25 Po: 3.9% 21.6% 5.9% 58.8% 5.9% 3.9%

Summary Table of Answers to Items

Scores of initial and final geometric levels of the control group

Table G.2 Item Summary: for Pre-test (Pe) and Post-test (Po) (learners)

Level	Item	A	B	C	D	E	Blank
1	1 Pe:	0.0%	81.6%	2.6%	15.8%	0.0%	0.0%
	1 Po:	0.0%	78.9%	0.0%	21.1%	0.0%	0.0%
	2 Pe:	0.0%	2.6%	5.3%	89.5%	2.6%	0.0%
	2 Po:	0.0%	0.0%	2.6%	97.4%	0.0%	0.0%
	3 Pe:	2.6%	0.0%	94.7%	0.0%	2.6%	0.0%
	3 Po:	0.0%	0.0%	100.0%	0.0%	0.0%	0.0%
	4 Pe:	0.0%	68.4%	10.5%	7.9%	10.5%	2.6%
	4 Po:	2.6%	73.7%	13.2%	2.6%	7.9%	0.0%
	5 Pe:	0.0%	2.6%	21.1%	0.0%	76.3%	0.0%
	5 Po:	0.0%	0.0%	23.7%	0.0%	76.3%	0.0%
2	6 Pe:	10.5%	52.6%	28.9%	7.9%	0.0%	0.0%
	6 Po:	2.6%	73.7%	18.4%	5.3%	0.0%	0.0%
	7 Pe:	7.9%	0.0%	2.6%	0.0%	89.5%	0.0%
	7 Po:	2.6%	0.0%	0.0%	2.6%	94.7%	0.0%
	8 Pe:	71.1%	5.3%	10.5%	7.9%	2.6%	2.6%
	8 Po:	76.3%	0.0%	5.3%	10.5%	7.9%	0.0%
	9 Pe:	5.3%	0.0%	94.7%	0.0%	0.0%	0.0%

9 Po: 5.3% 2.6% 76.3% 0.0% 15.8% 0.0%

10 Pe: 5.3% 7.9% 13.2% 55.3% 10.5% 7.9%

10 Po: 0.0% 2.6% 5.3% 76.3% 13.2% 2.6%

3 11 Pe: 5.3% 10.5% 68.4% 2.6% 10.5% 2.6%

11 Po: 0.0% 13.2% 84.2% 0.0% 2.6% 0.0%

12 Pe: 10.5% 73.7% 5.3% 7.9% 2.6% 0.0%

12 Po: 0.0% 86.8% 5.3% 2.6% 2.6% 2.6%

13 Pe: 63.2% 0.0% 0.0% 0.0% 36.8% 0.0%

13 Po: 89.5% 0.0% 0.0% 0.0% 10.5% 0.0%

14 Pe: 36.8% 7.9% 15.8% 2.6% 34.2% 2.6%

14 Po: 57.9% 23.7% 0.0% 0.0% 13.2% 5.3%

15 Pe: 5.3% 55.3% 5.3% 13.2% 15.8% 5.3%

15 Po: 5.3% 55.3% 5.3% 15.8% 15.8% 2.6%

4 16 Pe: 28.9% 7.9% 34.2% 5.3% 10.5% 13.2%

16 Po: 18.4% 5.3% 42.1% 10.5% 18.4% 5.3%

17 Pe: 34.2% 13.2% 26.3% 10.5% 15.8% 0.0%

17 Po: 28.9% 23.7% 31.6% 7.9% 5.3% 2.6%

18 Pe: 10.5% 21.1% 7.9% 44.7% 13.2% 2.6%

18 Po: 18.4% 21.1% 7.9% 31.6% 15.8% 5.3%

19 Pe: 50.0% 10.5% 15.8% 18.4% 2.6% 2.6%

19 Po: 55.3% 18.4% 7.9% 10.5% 7.9% 0.0%

20 Pe:	28.9%	7.9%	13.2%	42.1%	2.6%	5.3%
20 Po:	31.6%	2.6%	5.3%	50.0%	10.5%	0.0%
21 Pe:	50.0%	18.4%	0.0%	5.3%	7.9%	18.4%
21 Po:	44.7%	26.3%	7.9%	5.3%	10.5%	5.3%
22 Pe:	7.9%	26.3%	7.9%	23.7%	10.5%	23.7%
22 Po:	15.8%	23.7%	5.3%	44.7%	7.9%	2.6%
23 Pe:	26.3%	15.8%	0.0%	18.4%	15.8%	23.7%
23 Po:	42.1%	2.6%	2.6%	39.5%	10.5%	2.6%
24 Pe:	5.3%	5.3%	5.3%	15.8%	42.1%	26.3%
24 Po:	7.9%	2.6%	15.8%	39.5%	34.2%	0.0%
25 Pe:	2.6%	21.1%	7.9%	34.2%	7.9%	26.3%
25 Po:	5.3%	21.1%	7.9%	55.3%	5.3%	5.3%

APPENDIX H: Classroom observation

CLASSROOM OBSERVATION SCHEDULE

School Name: ....Topic: ....

Researcher Name: ....Gender: ....Qualifications: ....

Grade Level: .... Number of learners...

Observer Name: .... Date of observation: ....

NB: The researcher is the teacher and the observer at the same time

Component 1: Uses of language by the teacher (researcher) (asking questions. teaching. giving feedback and answering questions. explanations of terms/concepts.			
4	3	2	1
Teacher uses English only	Teacher uses English and switches to Home language when necessary	Teacher encourages the use of home language even if learners do not want to use it	Teacher uses home language only
Description..			

Component 2: Uses of language by the learners in general classroom discussion(seek clarification, elaborate, answering questions. solve problems, pose questions, build upon the previous response, etc.			
4	3	2	1
Learners use English only	Learners use English and switch to home language	Learners seldom use English	Learners use home language only
Description:			

Component 3: Learners' use of language for discussion and peer group (parallel learning and collaboration)			
4	3	2	1
Learners use English only	Learners use English but switch to home language	Learners seldom use English	Learners use home language only

Description..			
Component 4:Learners writing (use of writing frames, comprehensive writing, etc)			
4 Learners write effectively to record	3 Learners write to record their answers	2 Learners write ineffectively	1 Learners do not write at all

Component 5: Teacher Promoting discussion (collaborative tasks – paired activities, group presentation, arguments, etc.)			
4 Clear expectations set about behaviour	3 Prompts/sentence starters	2 Unclear expectations set about behaviour	1 No discussion, talk is irrelevant,
Description:			
Component 6: Learner Responses (individual, group, paired, hands-up, at the board, verbal, in writing, negotiation of meaning, etc.)			
4 Responses are valued,	3 Some responses are valued;	2 Learners struggle to initiate	1 Learners do not respond at all,
Description:			
Component 7: Learner Work in Groups			
4 Groups of learners discuss problems, questions and activities by themselves	3 Only two or three learners in a large group interact	2 Groups of learners with limited interaction/interact when teacher motivates	1 Learners sit in groups but work as individuals.
Description:			

APPENDIX I: Statistical Significance (Pre-and Post-test scores)

Group	Mean	Standard deviation	Calculated T	Critical t at 0.05 level of significance and 60 degrees of freedom
Experimental	32.88	14.58	0.6691	2.000
Control	28.29	12.21	0.6691	2.000

Statistical Significance based on Mean Difference (post-test)

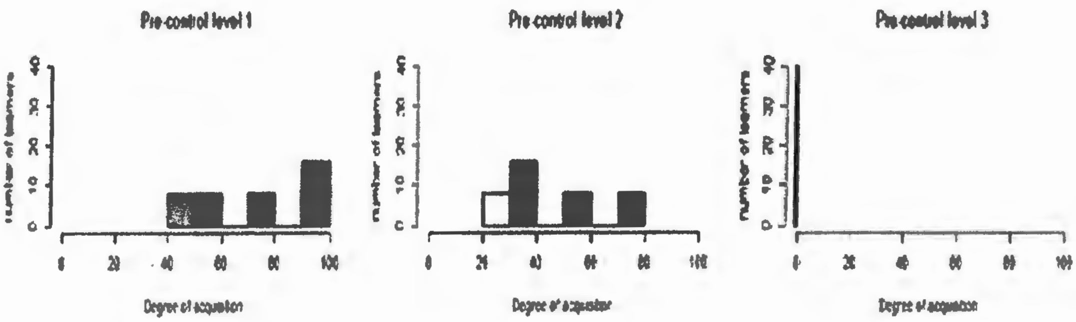
GROUP	Mean	Standard deviation	Calculated T	Critical t at 0.05 level of significance and 60 degrees of freedom
EXPERIMENTAL	25.857	7.595	1.0135	2.000
CONTROL	23.871	5.485	1.0135	2.000

Statistical Significance based on Mean Difference (pre-test)

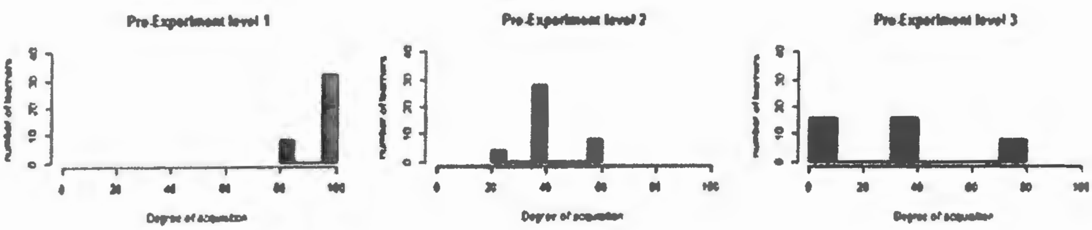
APPENDIX J

ANALYSIS OF VARIANCE

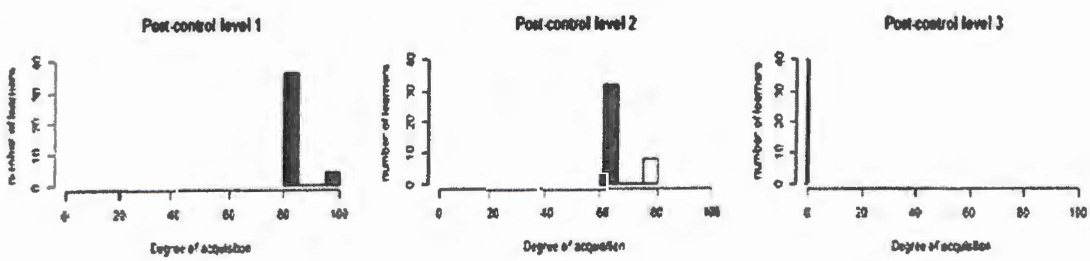
4.4.1 Control group



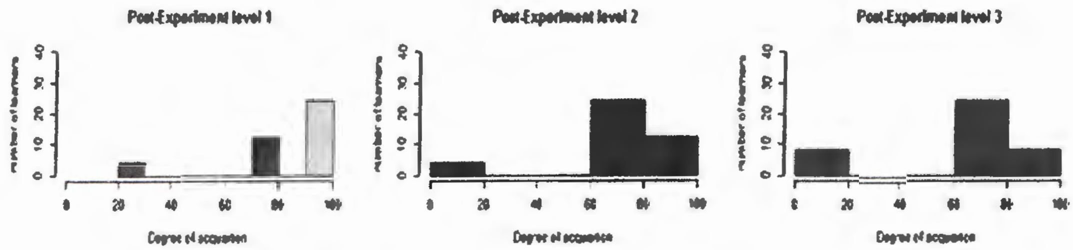
4.4.2 Experimental group



4.4.3 Control group



4.4.4 Experimental group



Comparative tables of results showing pre-test performances of experimental and control groups

STATISTICS	Experimental Group	Control Group
Minimum	12	13
Maximum	52.5	51
Range	40.5	38
Mean	32.875	28.2875
Standard deviation	14.58	12.21

Table10. Descriptive statistics of the experimental and control groups for the pre-test

APPENDIX K

Wilcoxon T-test (Experimental group and Control groups) for pre-test

	Treatment group - Control group
Z	-0.775(a)
Asymp. Sig. (2-tailed)	.439

Wilcoxon T-test (Experimental and Control groups) for post-test

	Treatment group - Control group
Z	-4.388(a)
Asymp. Sig. (2-tailed)	.000

APPENDIX L: The questionnaire

SECTION A: Write answers in the spaces provided

1. Name of school? ________________________

2. What is your gender? ________________________

3. How old are you? ________________________

Section B, C, D, and E, are about how you feel about learning and studying mathematics and how you feel about problems solving in mathematics

Read the statements carefully and mark one of the most appropriate choices for you for each item on the answer sheet:

5. Completely Agree

4. Agree

3. Undecided

2. Disagree

1. Completely Disagree

	STATEMENT	1	2	3	4	5
	SECTION B: Attitude towards Mathematics					
1	Mathematics is the subject that I like.					
2	I look forward to my mathematics lessons.					
3	I do mathematics because I enjoy it.					
4	I am interested in the things that I learn in mathematics.					
5	If there are no mathematics classes, being a student will be more enjoyable.					
6	I like discussing mathematics with my friends.					
7	I wish there were more mathematics classes a week.					
8	Time passes so slowly during mathematics classes.					
9	I would not get bored if I study mathematics for years.					
10	I have always believed that mathematics is one of my best subjects.					
11	Among all the lessons, mathematics is most unlikable.					
12	I learn mathematics quickly.					
13	Making an effort in mathematics is worth it because it will help in the work that I want to do later.					
14	Mathematics is an important subject for me because I need it for what I want to study later on.					
15	I will learn many things in mathematics that will help me get a job.					
	SECTION C: Willingness to engage in problem solving activities.					
1	I can solve most mathematical problems if I invest the necessary effort.					
2	I will try almost any mathematics problem.					
3	It is no fun to try and solve problems.					
4	I like to try challenging problems.					
5	There are some problems I will just not try.					
6	I do not like to try problems that are hard to understand.					
7	I like to try to solve problems.					
8	One learns mathematics best by memorizing facts and procedures.					
9	I try to understand the problem solving process instead of just getting answers to the problems.					
10	I solve the problems the way the teacher shows me and do not think up of my own ways.					
11	I try to find different ways to solve problems.					
12	Mathematics is about inventing new ideas.					
13	When I am confronted with mathematics problems, I can usually find several solutions.					
14	If I am engaged with a difficult mathematics problem, I can usually think of a strategy to use.					
15	If I am engaged with a difficult mathematics problem, I can usually think of a strategy to use.					
16	I am willing to try a different problem solving approach when my first attempt fails.					
17	I am willing to try a different problem solving approach when my first attempt fails.					
18	When I have finished working on a problem, I look back to see whether my answer makes sense.					

19	With my level of resourcefulness, I can solve mathematics problems that I am not familiar with					
20	I try to explain my ideas to other learners					
	SECTION D: Perseverance during the problem solving process					
1	With perseverance and determination. I can solve challenging mathematics problems.					
2	I do not stop working on a problem until I get a solution.					
3	I put down any answer just to finish a problem.					
4	When I do not get the right answer right away, I give up.					
5	I work for a long time on a problem.					
6	I keep on working on a problem until I get it right.					
7	I give up on challenging problems right away.					
	SECTION E: Self-confidence with respect to problem solving.					
1	I get nervous doing mathematics problems.					
2	My ideas about how to solve problems are not as good as other students' ideas.					
3	I can only do problems everyone else can do.					
4	Problems solving makes me feel uncomfortable.					
5	I am sure I can solve most mathematics problems.					
6	I am better than many students in solving mathematics problems.					
7	I need someone to help me work on mathematics problems.					
8	I can solve most hard mathematics problems.					
9	Most mathematics problems are too hard for me to solve.					
10	I am a good problem solver.					

APPENDIX M: The mathematical problem solving skills inventory

Where1=strongly disagree and 10=strongly agree

Problem solving skill	**rating of skill**
1. Understanding or formulating the question in a problem	1 2 3 4 5 6 7 8 9 10
2. Understanding the conditions and variables in the problem	1 2 3 4 5 6 7 8 9 10
3. Selecting or finding the data needed to solve the problem	1 2 3 4 5 6 7 8 9 10
4. Formulating sub-problems and selecting appropriate solution strategies to pursue	1 2 3 4 5 6 7 8 9 10
5. Correctly implementing the solution strategy or strategies and solve sub- problems	1 2 3 4 5 6 7 8 9 10
6. Giving an answer in terms of the data in the problem	1 2 3 4 5 6 7 8 9 10
7. Evaluating the reasonableness of the answer	1 2 3 4 5 6 7 8 9 10

APPENDIX N: The semi-structured interview plan

This interview plan depends on an individual learner.

1. The researcher will firstly establish rapport to make the learner feel comfortable.

2. The researcher will ask the learner to "talk about what he/she will be doing or thinking", while solving the problem.

3. After this a problem will be handed out to the learner

4. As the learner attempts to understand the problem, question and conditions, the researcher will observe the learner and ask questions such as the following, if appropriate:

a. What did you do first when you were given the problem? Next?

b. Can you verbalise this problem?

c. What question is asked in the problem? Can you visualise the problem? What are the important facts and conditions in the problem? Do you need any information not given in the problem? If learner does not understand the researcher can take different entry points.

d. Is there anything you don't understand about the problem?

5. As the learner works on the problem, the researcher will remind him/her to talk about it, and ask questions such as the following, if appropriate:

a. What plan are you using? Do you think this plan will lead to a solution? Have you thought about using other strategies? Which ones?

b. Where are you having difficulties? What are your ideas about where to go from here? What is wrong with your plan?

6. As the learner finds an answer to the problem, the researcher will observe the ways, if any, in which he/she checks the answer and its reasonableness as a solution. Asking questions such as:

a. Are you sure this is the correct answer to the problem? Why?

b. Do you think it is important to check your answer? Why?

7. After the learner has solved the problem, the researcher will ask questions such as:

a. Can you describe the solution to the problem & how you found it?

b. Is this problem like any other problem you've solved? How?

c. Do you think this problem could be solved in another way? What are your ideas?

d. How did you feel while you were solving this problem? How do you feel now that you have found a solution?

APPENDIX O: The problem solving observation comment card.

The problem solving observation comment card

Learner __________ DATE _____________

Comments: (Examples of what was written by the researcher when observing and questioning learners)

- Knows how and when to look for a pattern.
- Knows that a table will help him find a pattern.
- Keeps trying even when he has trouble finding a solution.
- Needs to be reminded to check his solutions.
- He is able to explain his solution to other learners.

APPENDIX P: The problem solving observation rating scale

The problem solving observation rating scale

Learner ____________________ **Date** ________

	Frequently	**Sometimes**	**Never**
1. Understands the given problem	___	___	___
2. Verbalisms the problem	___	___	___
3. Understands the conditions and variables in the problem	___	___	___
4. Selects the data needed to solve the problem	___	___	___
5. Extracts information from the problem	___	___	___
6. Formulates sub-problems	___	___	___
7. Selects appropriate solution strategies	___	___	___
8. Accurately implements solution strategies	___	___	___
9. Tries a different solution strategy when stuck (Without help from the teacher)	___	___	___
10. Approaches problems in a systematic manner (clarifies the question, identifies needed data, plans, solves and checks)	___	___	___
11. Uses various modelling techniques	___	___	___
12. Gives an answer in terms of the data in the problem	___	___	___
13. Reflect on the reasonableness of the answer	___	___	___
14. Shows willingness to engage in problem solving activities	___	___	___
15. Demonstrates self-confidence	___	___	___
16. Perseveres during the problem solving process	___	___	___

APPENDIX Q: The problem solving observation checklist

The problem solving observation checklist

Learner................................ Date..

___ 1 .Likes to solve problems.

___ 2. Works cooperatively with others in the group.

___ 3. Contributes ideas to group problem solving.

___ 4. Perseveres-sticks with a problem.

___ 5. Tries to verbalise what a problem is about.

___6. Can understand the conditions and variables in a problem.

___ 7. Can identify relevant data needed to solve a problem.

___ 8. Thinks about which strategy might be useful.

___ 9. Is flexible - tries different strategies if needed.

___ 10. Can correctly implement a solution strategy and solve sub-problems

___ 11. Can give an answer in terms of the data in the problem

___ 12. Checks and evaluates the reasonableness of the solution to the problem.

___ 13. Can describe or analyse a solution to the problem.

APPENDIX R: LIST OF JOURNAL ENTRIES

Journal entry 1:19/07/2016

I do not think I will ever pass maths, how can a teacher expect me to find my own way of solving a question without first working out on example on the board!

Journal entry 2:20/07/2016

23 today I felt confused, how can the new teacher just give us a problem with no showing us an Example what method was I supposed to used? I was blank .....

Journal entry 3:21/07/2016

I no longer jump into conclusions quickly. I now first read and understand question and most of all think carefully about it

Journal entry 4:22/07/2016

To-day was great! After we were given a task, I was able to ask myself what I already ~~knew~~ knew about it before solving the task, I realised that I knew a lot about it and this helped me to solve the questions quickly.

Journal entry 5:25/07/2016

It is now easy for me ~~fof~~ to contribute to the group.
I was nervous to verbalise given problems at the beginning of this term but now it is easy like drinking water to verbalise most problems.

Journal entry 6:26/07/2016

WHEN WE STARTED THIS TERM LEARNING IN THIS DIFFERENT WAY I DID NOT KNOW WHAT THE SUBPROBLEM MEANT NOW I KNOW WHAT IT MEANS & I HAVE LEARNT A LOT. I NOW KNOW TO FORMULATE A SUB.PROM WHEN GIVE A PROBLEM BY THE TEACHER

Journal entry 7:27/07/2016

today I found solutions to all the problems that we were given. I could check the solutions too. I feel like if we get a test on financial mathematics I will get 100%.

Journal entry 8:28/07/2016

I used to be scared of using different strategies to solve problems. Now I am able to use several methods to solve a given problem.

Journal entry 9: 29/07/2016

ABBREVIATIONS

ANOVA	Analysis of Variance
CAPS	Curriculum and Assessment Policy Statement
CDASSG	Cognitive Development and Achievement in Secondary School Geometry
ENGFAL	English First Additional Language
GSP	Geometer Sketchpad
ICT	Information Communication Technology
LoLT	Language of Learning and Teaching
LRC	Learner Representative Council
NCTM	National Council of Teachers of Mathematics
PCTLA	Problem Centred Teaching and Learning Approach
PCTL	Problem Centred Teaching and Learning
SGB	School Governing Body
TPCK	Technology Pedagogy and Content Knowledge
SMT	School Management Team
TIMSS	Third International Mathematics and Science Study
VHGT	Van Hiele Geometry Test