

Modelling globular clusters as multi-wavelength diffuse emitters

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Declaration of Authorship

I, Hambeleleni Davids, declare that this thesis titled “Modelling globular clusters as multi-wavelength diffuse emitters” and the work presented in it has been generated by me. I confirm that:

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- Where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated.
- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.
- I have acknowledged all main sources of help.
- Where the thesis is based on work done by myself jointly with others, I have made clear exactly what I have contributed and what was done by others.

Signed: *Davids*

Date: **6 April 2021**

“There’s an indigenous tribe in Africa that tells of life coming from the second star in Alpha Centauri, the one no one can see without a high-powered observatory telescope. Come to think of it, the Greeks, the Aboriginals, and the Plains Indians all lived continents apart and all, independently, looked at the same septuplet knot of the Pleiades and believed them to be seven young girls running away from something that threatened to hurt them.

Make of it what you will.”

Jodi Picoult, *My Sister’s Keeper*

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“I will give thanks to you, Lord, with all my heart; I will tell of all your wonderful deeds.” (Psalm 28:7, New International Version (NIV) Bible).

Abstract

The future ground-based Cherenkov Telescope Array (CTA) is expected to detect more than half of the known Galactic globular cluster (GC) population. In this thesis I present results from an emission code that assumes millisecond pulsars (MSPs) to be sources of relativistic particles in GCs that will give broad-band radiation due to their interaction with the cluster magnetic and soft-photon fields. I perform a parameter study to investigate the GC model's behaviour and study the detectability of Galactic GCs for the High Energy Stereoscopic System (H.E.S.S.) and CTA. H.E.S.S. has plausibly¹ detected only one GC (Terzan 5) at 5.3σ significance that has an unexpectedly asymmetric VHE morphology, offset from the optical source position. I therefore present new *Fermi* Large Area Telescope (LAT) data on this source and model its updated broadband spectral energy distribution (SED). I furthermore derive constraints on the embedded MSPs' luminosity function. Stacking upper limits by H.E.S.S. on the γ -ray flux of a population of 15 Galactic GCs are very constraining for leptonic emission models. I therefore show that uncertainty in model parameters leads to a large spread in the predicted flux, and there are indeed regions in parameter space for which the stacking upper limits are satisfied. It is thus important to increase measurement accuracy on key model parameters in order to improve predictions of cluster fluxes so as to better guide the observational strategy of CTA. The various applications considered in this thesis indicate that continued multi-wavelength observations of GCs will facilitate model refinement, and this will represent a substantial contribution to the Galactic science of CTA and that of satellites operating in lower-energy bands.

Keywords: globular clusters; millisecond pulsars; gamma rays; non-thermal emission; multi-wavelength diffuse emission.

¹The probability for a chance detection in this direction is $\sim 10^{-4}$.

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Table of abbreviations

A table containing a list of abbreviations that will be used throughout text. Abbreviations that appear only once are not listed.

BH	Black Hole
BS	Blue Straggler
CMB	Cosmic Microwave Background
CMD	Colour Magnitude Diagram
CR	Curvature Radiation
CTA	Cherenkov Telescope Array
CV	Cataclysmic Variables
DM	Dark Matter
FoV	Field of View
GC	Globular Cluster
HB	Horizontal Branch
HE	High Energy
H.E.S.S.	High Energy Stereoscopic System
HST	Hubble Space Telescope
IACT	Imaging Atmospheric Cherenkov Telescope
IC	Inverse Compton
ICS	Inverse Compton Scattering
IMBH	Intermediate Mass Black Hole
IOF	Inertial Observer's Frame
K-N	Klein-Nishina
LAT	Large Area Telescope
LESR	Low-Energy Synchrotron Radiation
LMXB	Low Mass X-ray Binary
LOS	Line of Sight
MAGIC	Major Atmospheric Gamma Imaging Cherenkov
MSP	Millisecond Pulsar
MS	Main Sequence
MSTO	Main Sequence Turn Off
OC	Open Cluster
OoI	Oosterhoff type I

OoII	Oosterhoff type II
OoIII	Oosterhoff type III
PC	Polar Cap
PSF	Point Spread Function
ROI	Region of Interest
SED	Spectral Energy Distribution
SG	Slot Gap
SR	Synchrotron Radiation
VHE	Very High Energy
VLA	Very Large Array
WD	White Dwarf

Chapter 1

Introduction and Motivation

Humankind’s curiosity led to the observation of the Universe and since then, our knowledge and comprehension of physics is steadily growing. Observational astrophysics helps us to understand nature and also brings us closer to the foundation of the world. γ -rays are part of the highest energy radiation of the electromagnetic spectrum. They are generated by non-thermal processes in the most extreme conditions and environments of the Universe. The field of γ -ray astrophysics has been growing rapidly in recent years as the detection of γ -rays constitutes an important probe for understanding the nature of the extreme environments they are produced in. The astronomical community is driven to venture into high-energy (HE) astrophysics due to the mystery of the exact nature of the acceleration mechanisms and understanding of the interactions of particles. There are several impressive instruments around the world committed to γ -ray astrophysics. This includes three major Imaging Atmospheric Cherenkov Telescopes (IACTs) and a space-based instrument that are currently operational.

The *Fermi* Large Area Telescope (LAT) is a γ -ray space-based instrument operating at 20 MeV – 300 GeV and has led to many discoveries, including the detection of GeV γ -rays from globular clusters (GCs). GCs are approximately spherically shaped objects with stars that are gravitationally bound to each other. About two dozen GCs have been detected by the *Fermi* LAT ([Abdollahi et al., 2020](#)). However, an X-ray and γ -ray instrument, *AstroRivelatore Gamma a Immagini Leggero* (*AGILE*)¹, with a low sensitivity did not detect any GC to date. A single GC, Terzan 5 within the Milky Way, has been plausibly detected at very-high energies (VHEs) by the ground-based Cherenkov Telescope, H.E.S.S., which is operated in a pointing mode ([Abramowski et al., 2011](#)). In addition to the above-mentioned instruments, other Cherenkov telescopes could only produce upper limits on GC TeV emission (e.g., [Anderhub et al., 2009](#)). [Ndiyavala et al. \(2018\)](#) predicted that TeV emission is expected to be seen from more than half of the known Galactic GCs by

¹It has a γ -ray imager (sensitive in the energy range 30 MeV – 50 GeV), a hard X-ray imager (sensitive in the range 18 – 60 keV) and a calorimeter (sensitive in the range 350 keV – 100 MeV).

the future Cherenkov Telescope Array (CTA) that will be 10 times more sensitive than H.E.S.S. and operating in energy range of 20 GeV to 300 TeV. Apart from GCs having been detected in the γ -ray band, they have also been detected in radio (Clapson et al., 2011) and as diffuse X-ray sources (Eger et al., 2010, Eger and Domainko, 2012, Wu et al., 2014). They are therefore referred to as multi-wavelength objects i.e., visible from radio to γ -ray wavelengths, and are exciting targets for deeper future observations with more sensitive telescopes.

GCs host many different stellar types, including millisecond pulsars (MSPs). The high stellar densities in the cores of GCs also enhance stellar encounter rates, facilitating the formation of objects such as low-mass X-ray binaries (LMXBs) that are believed to be the progenitors of MSPs (Venter and Kopp, 2015). It is difficult to resolve individual γ -ray MSPs within clusters, since they are compact sources and may be faint emitters. However, their cumulative pulsed emission may be detectable as diffuse HE emission. That is, individual pulsars add to the photon seeds e.g., cumulative pulsed curvature radiation (CR) coming from many sources and or injection of particles via the pulsar winds that eventually leads to unpulsed synchrotron radiation (SR) as well as inverse Compton (IC) radiation. Typically, γ -ray sources represent violent and extreme conditions, associated with acceleration of particles to relativistic energies. On the other hand, GCs are comprised of about one million old stars, and are thus not expected to be visible in γ rays. Yet, they also contain pulsars due to high core densities that facilitate formation of LMXBs via stellar interactions, leading to relatively large populations of MSPs in GCs. The latter represent extreme compact objects that can inject relativistic particles into GCs. These particles cool efficiently and yield broadband spectra reaching to γ -ray energies. Indeed, one finds that there is a link between the γ -ray luminosity of GCs detected by *Fermi* and the number of MSPs they contain (Abdo et al., 2010); moreover, the only GC detected in the TeV range is the one with the largest number of MSPs, solidifying this link. Other factors may also influence GC broadband visibility, e.g., cluster photon and magnetic fields (chapter 2).

Several models attempt to explain the multi-wavelength emission of GCs. Bednarek and Sitarek (2007) considered a scenario where millisecond pulsars (MSPs) accelerate leptons at relativistic shocks produced during collisions of pulsar winds within the clusters' cores. The cumulative GeV flux was furthermore modelled by assuming a pair-starved polar cap electric field (Harding et al., 2005, Venter and De Jager, 2005). Venter and De Jager (2008b) modelled the cumulative CR from 100 pulsars by randomising over MSP geometry as well as period and period time derivative. This approach was then refined by Venter and De Jager (2010) while Venter et al. (2009a) predicted the GeV spectrum of 47 Tucanae (NGC 104) within a factor of two in both energy and flux level, prior to its detection by *Fermi* LAT. An alternative scenario was explored to produce GeV and TeV emission by calculating IC radiation from electrons and positrons upscattering the cosmic microwave background (CMB), stellar photons, and the Galactic background (Cheng et al., 2010).

There was also an attempt to explain the observed TeV emission from Terzan 5 using hadronic models (for more details see [Domainko, 2011](#)). In addition to models invoking MSPs, there are also those invoking GRBs, WDs, or DM. See Chapter 3 for details.

1.1 Problem Statement

Several unanswered questions remain about GCs. For example, knowing that Terzan 5 might be the only Galactic GC detected in VHE band, is it special? Is it possible to detect more GCs in the VHE band, i.e., will more Galactic GCs be visible to H.E.S.S. and CTA? Also, since CTA will be 10 times more sensitive than H.E.S.S., can one predict the number of Galactic GCs that might be visible by CTA? The leptonic model of [Kopp et al. \(2013\)](#) will provide a framework to address the above questions. One would also like to know how the model output changes as one changes parameters in multi-dimensional phase space; and to be able to physically explain the changes in the outputs. Since there are some new multi-wavelength data, e.g., for Terzan 5, I will model the updated broadband spectral energy distribution (SED), attributing this to cumulative pulsed emission from a population of embedded MSPs, as well as unpulsed emission from the interaction of their leptonic winds with the ambient magnetic and soft-photon fields. Also can one constrain the properties of the embedded MSPs? The use of a parametric luminosity function and constrain its parameters using X-ray data will address this question. Another unanswered question is whether one will ever be able to discriminate between competing scenarios (such as dark matter annihilation, white dwarf (WD) winds, or hadronic interactions) proposed for the broadband emission, as well as constraining degenerate model parameters. In this thesis, we will apply a mature leptonic model in the MSP scenario to see if we can constrain this model using available data. Lastly but not least, if there are uncertainties regarding the predicted fluxes that have been done, is it worthwhile to keep the models in the case that they overshoot the available data or upper limits or does one just discard them? Assessing uncertainties associated with flux predictions will answer this question.

1.2 Research Goals

The goals of this research are to perform a parameter study and also constrain model parameters for three GCs using γ -ray and X-ray data. I will also study the detectability of 16 Galactic GCs for H.E.S.S. and CTA, ranking them according to their predicted TeV flux. To do so I will accumulate the necessary parameters for 16 Galactic GCs, taking into account errors on these quantities, and then run a leptonic model to make predictions for the γ -ray flux expected from each GC for H.E.S.S. and CTA. Next, I will present new data from the *Fermi* LAT and model the updated broadband SED of Terzan 5, attributing this to cumulative pulsed emission from a population of embedded MSPs, as well as unpulsed

emission from the interaction of their leptonic winds with the ambient magnetic and soft-photon fields. I will also derive constraints on the MSP luminosity function using the diffuse X-ray data and the *Chandra* sensitivity. Lastly, I will demonstrate that uncertainty in model parameters leads to a large spread in the predicted flux, and there are indeed regions in parameter space for which the stringent stacking upper limits by H.E.S.S. on 15 GCs are satisfied. I will lastly show that it is significant to increase measurement accuracy on key model parameters in order to improve predictions of cluster fluxes so as to better guide the observational strategy of the CTA.

1.3 Thesis Outline

This thesis is structured as follows:

In **Chapter 2**, the reader is presented with a relevant background on GCs and MSPs. I will summarise the stellar populations in GCs and discuss the non-thermal and thermal radiation mechanisms that take place in GCs. I will also briefly discuss multi-wavelength observations of GCs.

In **Chapter 3**, I will review the existing multi-wavelength spectral models (i.e., GC and pulsar models) used in this study.

In **Chapter 4**, I present a parameter study to further investigate the model's behaviour, list the best candidate sources as predicted by the model for CTA (in terms of predicted VHE flux) and constrain model parameters using multi-wavelength data.

In **Chapter 5**, I discuss the estimation of the optical upper limit of Terzan 5 and fit the multi-wavelength data of this source.

In **Chapter 6**, I assess uncertainties on the predicted VHE γ -ray flux of GCs for a given leptonic model and give theoretical guidance to CTA's observational strategy.

In **Chapter 7**, I simulate the broad-band emission to predict the spectral shapes of the top five Galactic GCs for CTA using ctools.

Chapter 8 provides a summary of the thesis and future offer prospects.

1.4 Publications that have Resulted from this Thesis

The following is a list of peer-reviewed journal publications and a conference proceedings article that resulted from this research study.

- Ndiyavala-Davids, H., Venter, C., Kopp, A., and Backes, M. (2020). Assessing uncertainties in the predicted very high energy flux of globular clusters in the Cherenkov Telescope Array era. *MNRAS*, 500(4):4827–4836².
- Ndiyavala, H., Venter, C., Johnson, T. J., Harding, A. K., Smith, D. A., Eger, P., Kopp, A., and Van der Walt, D. J. (2019). Probing the Pulsar Population of Terzan 5 via Spectral Modeling. *ApJ*, 880(1):53³.
- Ndiyavala, H., Krüger, P. P., and Venter, C. (2018). Identifying the brightest Galactic globular clusters for future observations by H.E.S.S. and CTA. *MNRAS*, 473:897–908⁴.
- Ndiyavala, H., Krüger, P. P., and Venter, C. (2017). Investigating Gamma-ray Fluxes from Globular Clusters. Ed. Meintjes, P., et al., *Proceedings of SAIP2016*. ISBN: 978-0-620-77094-1⁵.

²<https://doi.org/10.1093/mnras/staa3588>

³<https://doi.org/10.3847/1538-4357/ab24ca>

⁴<https://doi.org/10.1093/mnras/stx2336>

⁵<http://events.saip.org.za/internalPage.py?pageId=10&confId=86>

Chapter 2

Background on Globular Clusters

The peculiar properties of GCs have been useful in diverse astrophysical disciplines such as galaxy formation, stellar evolution, cosmology, dynamics, as well as binary and variable stars (Harris, 1996, 2010). GCs are multi-wavelength objects since they are visible from radio to γ -rays (see Section 2.10). In this Chapter, I will discuss some background on GCs and millisecond pulsars (MSPs) embedded in them. I will give a definition of a GC and summarise its characteristics and evolution. As mentioned in Chapter 1, MSPs in GCs inject relativistic particles into the GC, and thus they emit both cumulatively pulsed (magnetospheric) and unpulsed (diffuse) radiation. I will therefore discuss relevant non-thermal emission mechanisms, i.e., synchrotron radiation (SR), inverse Compton (IC), and curvature radiation (CR) expected to operate in GCs, as well as (thermal) blackbody (BB) radiation. Lastly, I will broadly discuss multi-wavelength observations of Galactic GCs.

2.1 Introduction

GCs are gravitationally bound ancient stellar systems consisting of $10^4 - 10^6$ stars (see Figure 2.5), with ages ranging between 11 and 13 Gyr (Krauss and Chaboyer, 2003, Marín-Franch et al., 2009b, Forbes and Bridges, 2010). They are associated with a host galaxy that is built up via hierarchical growth (White and Frenk, 1991, Springel et al., 2005). The first known GC, M 22, was discovered by a German amateur astronomer Abraham Ihle in 1665 (Lynn, 1886). However, individual stars within a GC were not resolved until in 1764 when a French astronomer, Charles Messier, observed a GC known as M 4. In 1782, William Herschel began a sky survey and discovered 36 GCs. He then went further and resolved their member stars (Herschel, 1789).

GCs differ significantly from open clusters (OCs), which are groups of stars that were

formed from the same giant molecular cloud and have roughly the same age. OCs are located in the Plane or Disc of the Galaxy and they contain no more than 10^4 members (Bajkova and Bobylev, 2019). These clusters dissipate slowly through the ages, possibly due to interactions between their members, and because of close encounters with external stars and molecular clouds (Carraro, 2006). Kharchenko et al. (2013), Liu and Pang (2019) showed that most of the studied OCs are actually younger than 10 Gyr. That is, they are quite younger than GCs.



Figure 2.1: *Hubble Space Telescope* image of the brightest Galactic GC in the night sky, i.e., Terzan 5 (https://commons.wikimedia.org/wiki/File:Terzan_5_HST_9799_R814GB606.png).

Most galaxies, including the Milky Way, are penetrated and surrounded by a system of GCs. An example, a famous Galactic cluster, Terzan 5 (discussed in Chapter 5), can be seen in Figure 2.1. There are about 160 currently known GCs in the Milky Way (Harris, 2010), distributed roughly spherically about the Galactic centre (see Figure 2.2), lying at an average radius of 12 kpc. Figure 2.3 shows the spatial distribution of GCs, with most GCs seen toward the Galactic Centre.

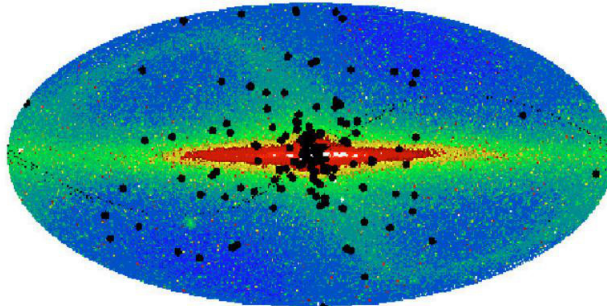


Figure 2.2: This figure shows how GCs are distributed around the Galaxy. The black circles indicate the positions of GCs (Harris, 1996) on top of the COBE FIRAS 2.2 micron map of the Galaxy, using a Mollweide projection (Benacquista, 2002).

Their distribution near the Galaxy’s “heart” might have been caused by the fact that there are so much materials (molecular clouds and gas) at the centre of the Galaxy which might have contributed to the formation of GCs. Currently, there are two families of models that describe the formation and evolution of GC systems. One family of models (e.g., [Peebles and Dicke, 1968](#), [Katz and Ricotti, 2014](#), [Trenti et al., 2015](#), [Kimm et al., 2016](#)) describes the formation of GCs as being associated with special conditions in low-mass dark matter (DM) halos during or before the epoch of reionisation. The other family of models (e.g., [Elmegreen, 2010](#), [Shapiro et al., 2010](#), [Kruijssen, 2015](#)) considers formation of GCs as a natural byproduct of the active star formation process, which is thought to be linked to high gas pressures seen at high redshift. In the latter family of models, there is an ongoing debate between studies finding that most of GC formation proceeds in galaxy discs ([Kruijssen, 2015](#), [Pfeffer et al., 2018](#)) and those finding GCs to be a product of galaxy mergers ([Li et al., 2017](#), [Renaud et al., 2017](#), [Kim et al., 2018](#)).

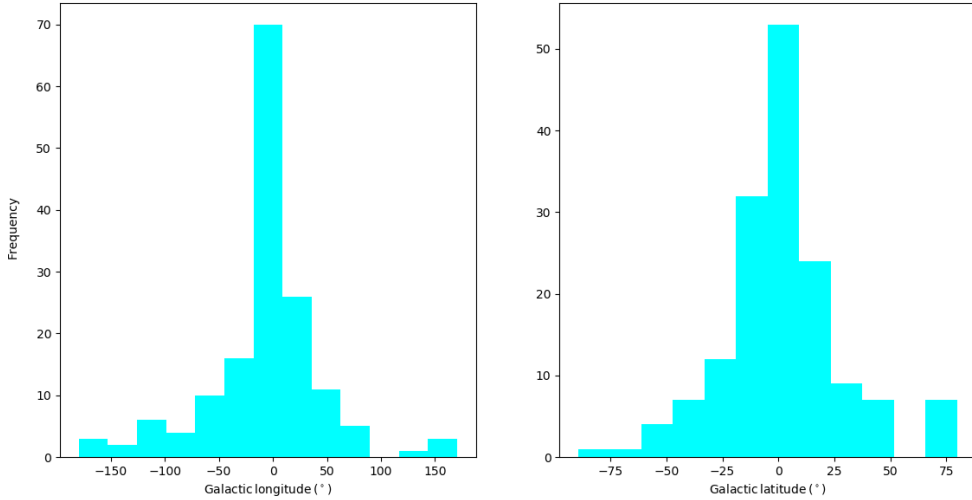


Figure 2.3: The histograms of Galactic longitudes and latitudes of Galactic GCs. Most Galactic GCs are found near the centre of the Galaxy (data taken from [Harris, 1996](#)).

2.2 General Stellar Populations in GCs

Individual GCs are assumed to have stars that have been formed roughly at the same time, sharing the same chemical abundances, and lying at the same distance from us. Hence, these systems were originally thought to consist of homogeneous stellar populations.

A typical colour-magnitude diagram (CMD) of a GC indicates luminosity and the surface colour, as well as summarising their evolution. The CMD shows the embedded stellar population forming a distinct main sequence, main sequence turnoff (MSTO), a horizontal and red-giant branch, RR Lyrae variable stars, and WDs (see [Figure 2.4](#) for a CMD of a

typical GC).

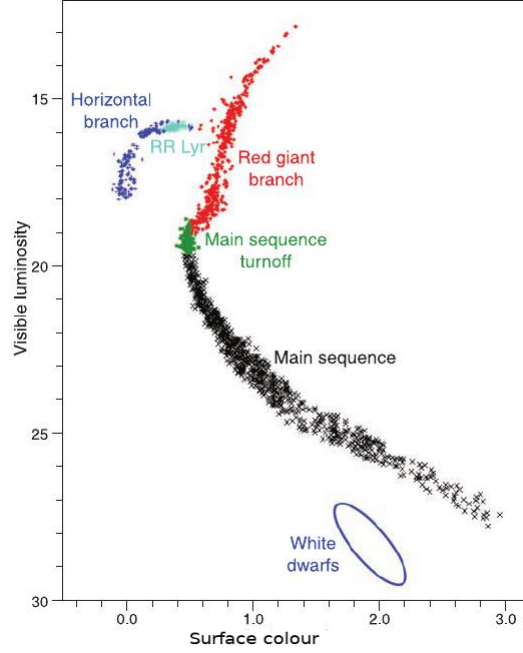


Figure 2.4: Luminosity versus the surface colour of the stars for a typical GC (Krauss and Chaboyer, 2003).

There exists a large variety of exotic objects that populate the dense environment of GCs (Dalessandro and Cosmic-Lab Team, 2016) like blue straggler (BS) stars, red giants (RGs), as well as binary and ternary systems. There are numerous BSs in GCs (Ferraro et al., 1997, 1999, 2003, Piotto et al., 2004, Lanzoni et al., 2007b,a, Dalessandro et al., 2009, 2013, Sanna et al., 2014).

BSs are main sequence (MS) stars found in OCs or GCs and are brighter and bluer than the MSTO, thus mimicking a population significantly younger than normal cluster stars. They were observed for the first time in the outer regions of the M3 (Sandage, 1953). Since then, BSs have been detected in any properly observed GC. Observations of BSs demonstrated that they have larger masses than MSTO stars ($m = 1.2 - 1.7M_{\odot}$; Shara et al., 1997, Fiorentino et al., 2014, Dalessandro and Cosmic-Lab Team, 2016). The origin of BSs cannot be explained within the framework of normal, single-star evolution (i.e., in stellar systems with no evidence of recent star formation; Dalessandro and Cosmic-Lab Team, 2016). There are two main formation channels that are favoured: (a) mass transfer in binary systems (McCrea, 1964), and (b) stellar collisions (Hills and Day, 1976). According to Lombardi et al. (2002), both of these processes can bring new hydrogen into the stellar core and rejuvenate a star to its MS stage. BSs are ideal tools to probe the dynamical evolution of stellar systems, since they are the brightest among the most massive stars within their host cluster.

GCs also contain a significant number of binary stars. Observations across the broadband

electromagnetic spectrum have detected a number of binaries and their tracers in both Galactic and extra-galactic GCs (Benacquista and Downing, 2013). Observations of BSs in 13 GCs showed a correlation between the specific frequency of BSs and the binary fraction in the GC (Sollima et al., 2008), indicating a link between these two stellar populations: according to Leigh et al. (2007), binary coalescences are the governing formation mechanism for BSs in GCs.

Dieball et al. (2010) found about 30 WDs, 60 CVs as well as WD binaries in M80. There are also low-mass X-ray binaries (LMXBs), progenitors of MSPs, which are very bright and are seen anywhere in the Galactic GC system (Rappaport et al., 1987, Deutsch et al., 2000, Rasio et al., 2000, Grindlay et al., 2001, Heinke et al., 2003, Verbunt and Lewin, 2006, Ivanova et al., 2008, Guillot et al., 2009, 2011). Fifteen bright LMXBs have been detected in 12 GCs, of which seven are transients showing bright outbursts and longer epochs in which they are faint (Verbunt and Lewin, 2006, Heinke et al., 2010).

2.3 Age Estimates

Since Galactic GCs are so old, most of the stars with masses above $\sim 0.8M_{\odot}$ have already left the MS, and are on the RG branch. The more massive stars have evolved past the RG and horizontal branches (HBs) and have become compact remnants (e.g., WDs and MSPs) with low luminosity. GCs in our Galaxy have a total integrated magnitude of -12.9 (Harris, 1996). For a typical mass-to-light ratio of $\gamma_V \approx 1.5$ (McLaughlin, 2000, Strader et al., 2011), this corresponds to a total mass of about $1.2 \times 10^7 M_{\odot}$. The stellar halo is estimated to have the total mass of $(4 - 7) \times 10^8 M_{\odot}$ for $1 < R < 40$ kpc (Bland-Hawthorn and Gerhard, 2016), thus the Milky Way GC system currently accounts for about 2 – 3% of the stellar halo mass. However, GCs are expected to lose mass over time and some GCs may have completely disrupted because of dynamical evolution. Therefore, the initial mass of the GC population is most likely higher than the current observed value (Forbes et al., 2018). The distance, age, and chemical composition, as well as other properties of GCs, are estimated by measuring the magnitude of the MSTO. This is then compared to stellar evolution models.

The key parameter to understand the GCs' relationship to galaxy formation and testing model predictions is knowing the ages of clusters and their formation redshift for a given cosmology. A number of GCs in the Milky Way have age measurements based on deep CMDs observed with the *Hubble Space Telescope* (HST). GCs are old systems (≈ 13.0 Gyr for the VandenBerg et al., 2013 GC sample) and thus the Universe must be older ($\approx 13.8 \text{ Gyr} \pm 0.1 \text{ Gyr}$; Komatsu et al., 2011, Ade et al., 2013). This fact puts one constraint on the value of the cosmological parameters (Hartle, 2003). One panel of Figure 2.5 shows the histogram of the ages in giga years of Galactic GCs. It is seen that most GCs have an age of ≈ 13 Gyr. The very old, metal-poor subpopulation GCs are found to be

somewhat older than the metal-rich subpopulation, i.e., ~ 12.5 Gyr versus ~ 11.5 Gyr, respectively (Marín-Franch et al., 2009a, Forbes and Bridges, 2010, Leaman et al., 2013, Vandenberg et al., 2013). Forbes et al. (2018) noted that a handful of Galactic GCs have their ages determined from the WD cooling sequence, which gives ages consistent with those measured from MS fitting. Krauss and Chaboyer (2003) determined the age of the oldest GC with a Monte Carlo simulation, using estimates for parameter ranges for the variables associated with stellar evolution (see Chaboyer et al., 1996, 1998). They obtained a lower bound on this maximum age of 10.4 Gyr, which is more stringent than the previous estimates, and an upper bound ~ 18 Gyr. The best-fit age was found to be 12.6 Gyr. Iben and Renzini (1984) estimated the age of the GC M9 to be 16 ± 3.5 Gyr, and the uncertainty was based on the variability in visual magnitude $\Delta M(V)$. However, the estimated age dropped to $\sim 14 \pm 3.5$ Gyr when they chose other values for the abundance of helium. The age of a GC cannot be more accurately determined than about ± 4 Gyr, if one only takes the HB and the absolute luminosities of the MSTO into account. The uncertainties in determining the age of the oldest GCs were discussed by Van den Berg et al. (1996). The stars having lower metallicity (oldest stars) have ages ~ 15 Gyr. When helium diffusion is taken into account, their estimated age decreases by 7%. Jimenez et al. (1996) estimated the age of the oldest GCs using the HB morphology method in their study of M22, M5, M68, M107, M72, M92, M3, as well as 47 Tucanae, and found the oldest GC to be 13.5 ± 2.0 Gyr. Jimenez (1998) used a particular luminosity function method and obtained an age of M55 of 12.0 ± 0.5 Gyr. Strader et al. (2005) mention that GCs have ages of ~ 10 Gyr, with an uncertainty of ~ 2 Gyr. Isochrone fitting, HB morphology, luminosity-function (Jimenez, 1998), and Bayesian-estimation (Jørgensen and Lindegren, 2005) methods are also used to compute the ages of GCs.

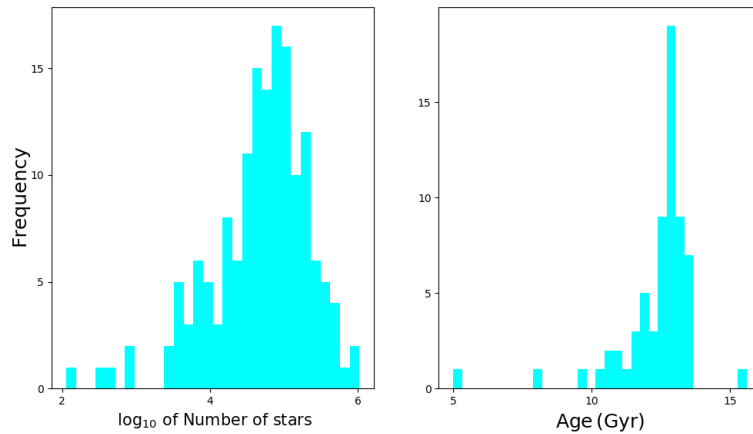


Figure 2.5: This figure shows the total number of stars in all the known Galactic GCs and the ages of 65 selected Galactic GCs (data taken from Buonanno et al., 1987, Borissova and Spassova, 1995, Côté et al., 2002, Martell et al., 2002, Carraro, 2005, Lewis et al., 2006, Carraro et al., 2007, Sarajedini et al., 2007, Dotter et al., 2010, Küpper et al., 2011, Van den Bergh, 2011).

2.4 Distances

Determining the intrinsic luminosity of a GC's turnoff stars can be achieved by knowing the distance to the GC. There are different methods that can be employed to estimate the distance. The methods depend on various assumptions and calibration steps. The most common method for estimating the distance to the cluster involves “standard candles”, whereby one makes an assumption that some class of stars have the same intrinsic luminosity. The observation of the flux (apparent luminosity on the sky) of this class is then made and the distance to the GC is obtained by comparing these two quantities. That is, one can determine the luminosity of a nearby star from the apparent brightness and distances determined by triangulation using the inverse square law:

$$L = 4\pi d^2 f, \quad (2.1)$$

where f is the apparent brightness of a star, L is the luminosity, and d is the distance (Hartle, 2003). Astronomers use absolute magnitude M , which is related to the total luminosity L by

$$M = -2.5 \log_{10}(L/L_{\odot}) + 4.74. \quad (2.2)$$

Here $L_{\odot} = 3.85 \times 10^{33}$ erg/s is the solar luminosity. They also use the apparent magnitude m , which is given by

$$m = -2.5 \log_{10}(f/f_{\odot \text{at} 10 \text{pc}}) + 4.74, \quad (2.3)$$

where $f_{\odot \text{at} 10 \text{pc}}$ is the flux of the sun at the earth if it were 10 pc away. The apparent magnitude can be equated to the absolute magnitude if it is 10 pc away. The logarithmic measure $m - M$ of f/L is referred to as the distance modulus, and space flat, $m - M = 5 \log_{10}(d/10 \text{pc})$. This method has been employed by using HB stars (Gratton, 1998), which are stars burning helium into carbon in their cores, RR Lyrae stars (Benedict et al., 2002), which are a subclass of HB stars, MS stars (Chaboyer, 1999), and WDs (Renzini et al., 1996). It was also recognised that Cepheid variable stars that are massive, bright, yellow stars could be used to determine distances through their period-luminosity relation, since they brighten and dim periodically. Their period is regular with a uniform function of their brightness (Braga et al., 2020, Bhardwaj et al., 2020, Riess et al., 2018). Because of their intrinsic brightness, these stars can be distinguished at great distances and make excellent markers for determining distances through their period-luminosity relationship (Shapley, 1918). The distances that are determined by a previous step in the distance ladder can be used to find the luminosities of Cepheid variables in sufficiently nearby clusters. Thus, Cepheid variables are considered to be standard candles via the period-luminosity relationship. These member stars can therefore be used to determine distances to more distant clusters (Hartle, 2003).

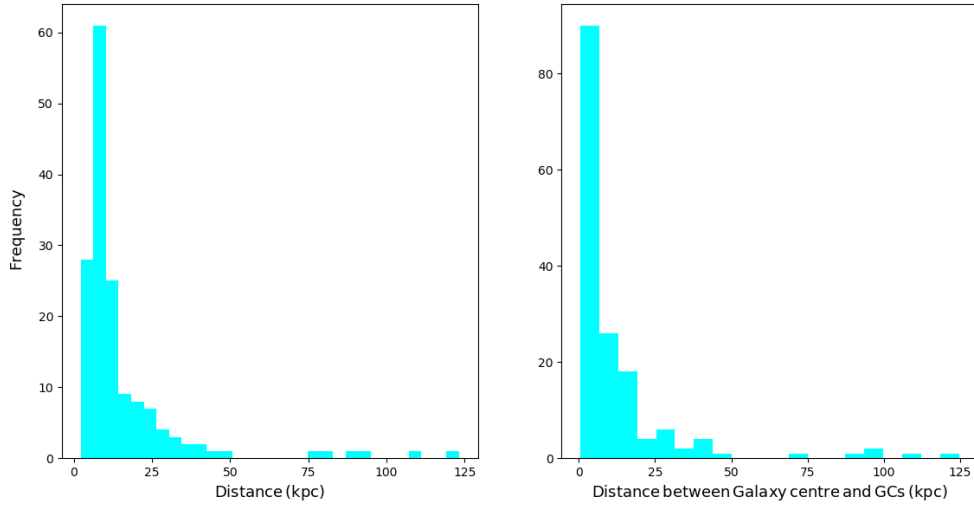


Figure 2.6: Histograms of distances of GCs measured from the sun and from the Galactic centre. The x -axes show the total distances to the GCs in kpc, while the y -axes show the frequency (data taken from [Harris, 1996](#)).

An independent method for determining the distance to a GC is provided through studies of the internal dynamics of the stars within a cluster. The method of dynamical distance estimation compares the relative motion of GC stars in the plane of the sky to their motions along the line of sight (LOS) to the star (radial velocities). These velocities are independent of distance, while the measured proper motions are smaller for more distant objects. The distance to a GC can then be estimated by using the comparison of the two observations ([Krauss and Chaboyer, 2003](#)).

[Hartle \(2003\)](#) notes that individual stars of a given colour are not good standard candles, however the statistical properties of the relation between their colours and luminosities can be used as a standard candle. Suppose the colours and apparent brightness of local stars are measured within a cluster that is too far away to obtain its distance by triangulation (parallax measurements). Next assume that all the stars within the cluster are approximately the same distance from us. The distance at which the inferred MS of the cluster matches the MS of local stars whose distances have been determined by triangulation measurement, is the actual distance to the cluster. [Figure 2.6](#) shows the distances of GCs as measured from the sun. The distances have been obtained from the [Harris \(1996\)](#) catalog. I note that most known Galactic GCs are at ~ 10 kpc from the sun. [Figure 2.6](#) also indicates the distribution of distances of Galactic GCs with respect to the Galactic Centre.

2.5 Metallicities

Oosterhoff (1939) showed that Galactic GCs may be divided into two distinct types according to the properties of their RR Lyrae variables. There is an argument that a sharp division exists between Oosterhoff type I (OoI, with average periods of the ab-type RR variables $\langle P_{ab} \rangle \approx 0.55$ day) and Oosterhoff type II (OoII, with $\langle P_{ab} \rangle \approx 0.65$ day) GCs, with very few clusters in the range $0.58 \leq \langle P_{ab}(d) \rangle \leq 0.62$, i.e., the Oosterhoff gap (see Figure 2.7). On the other hand, the dwarfs spheroidal (dSph) satellite galaxies of our Galaxy and their respective GCs fall in the Oosterhoff gap region (Catelan, 2004, 2009). Not too many GCs satisfy the above selection criteria involving P_{ab} . That is, within this selection criteria, Catelan (2004) found that 19 GCs were OoI, 9 were OoII, and two were Oosterhoff intermediate. Pritzl et al. (2000, 2001, 2002, 2003) assigned two additional GCs (NGC 6388 and NGC 6441) to a new class OoIII. Van den Bergh (2011) noted that Galactic GCs with ages ≥ 13 Gyr belong to the OoII class, whereas almost all GCs with < 13 Gyr belong to the OoI class.

The distribution of clusters of OoI and OoII have been inspected and it was shown that “metallicity” $[\text{Fe}/\text{H}]$ seems to be the only parameter that appears to correlate with the Oosterhoff type. GCs of type OoI have $[\text{Fe}/\text{H}] > -1.6$, whereas those of OoII have $[\text{Fe}/\text{H}] < -1.6$. That is, the OoI GCs are relatively metal-rich while group II GCs are more metal-poor (Jang and Lee (2015); see Figure 2.7).

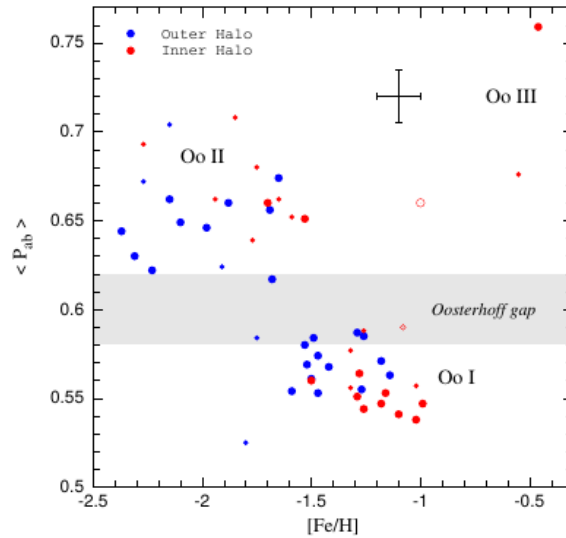


Figure 2.7: The Oosterhoff period groups among GCs within the Milky Way. GCs are divided into three groups Oo I, Oo II, and Oo III according to period ab-type (P_{ab}) and metallicity. The inner and outer halo GCs are represented by red and blue circles, respectively (Jang and Lee, 2015). The two red open circles are bulge GCs recently reported by Alonso-García et al. (2015).

2.6 Surface Brightness Profiles

[Jeans \(1916\)](#) derived the density law of GCs where he theoretically showed that the outer parts of a GC should have a star density that varies as r^{-4} , where r is the distance from the centre of the cluster. Unfortunately his result was incorrect, even though he supported his conclusion by citing star counts of [Bailey \(1915\)](#). The result of [Jeans \(1916\)](#) depends on two assumptions that were unjustified. The first assumption was that the velocity distribution is isotropic at every point in the GC. Secondly, he assumed that the potential U at large r can be represented by the power series:

$$-U = (GM/r)(1 + c_1/r + c_2/r^2 + \dots), \quad (2.4)$$

with $c_1 \neq 0$. Jeans' r^{-4} law follows from the non-vanishing of c_1 , for which no proof was given.

Bailey's counts showed that within the outer parts of a cluster so few stars are included that the densities are quite uncertain. Even so, the counts, when plotted on logarithmic scales, suggested that the exponent of r increases with increasing r . Furthermore, Jeans acknowledged that two of Bailey's ten clusters did not fit his law. [King \(1962\)](#) corrected the background density using wide angle Schmidt plates (see [Figure 2.8](#)). The surface density f drops to zero at value $r = r_t$, the tidal radius where the gravitational influence of the cluster is equal to that of the Galaxy. This led to the following empirical law:

$$f = f_1 \left(\frac{1}{r} - \frac{1}{r_t} \right)^2, \quad r \leq r_t. \quad (2.5)$$

Here f_1 is a constant and r_t is the value of r at which f reaches zero. [Von Hoerner \(1957\)](#) gave the limiting radius of a cluster as

$$r_t = d(M/2M_g)^{1/3}, \quad (2.6)$$

where d is the distance of the cluster from the Galactic Centre and M and M_g are the masses of cluster and Galaxy, respectively.

[Equation 2.5](#) predicts an infinite density at the centre of the cluster and therefore the central parts cannot be expected to fit by this equation. [King \(1962\)](#) stated that [Equation 2.5](#) is not sufficient for modelling the central part of the GC. He found the surface density in the inner parts of a concentrated cluster to be represented by

$$f = \frac{f_0}{1 + (r/r_c)^2}, \quad (2.7)$$

where f_0 is the central surface density and r_c is a core radius (scale factor). GCs are structurally characterised by core, tidal, and half-mass radii. The core radius is the dis-

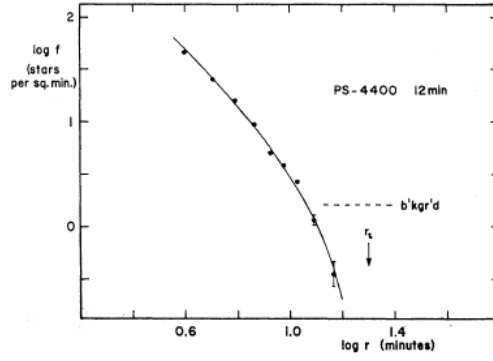


Figure 2.8: The result of a count made on a photograph of M15 taken with the 48-inch Schmidt camera of the Palomar Observatory. Mean errors are indicated above and below last two points. Arrow indicates value of r_t ; the dotted line indicates background count, and the solid line is a model fit using Equation 2.5 (King, 1962).

tance from the centre of the cluster, at which the apparent surface brightness of the cluster reduces by half. The tidal radius is the distance from the cluster core beyond which the gravitational influence of the Galaxy is larger than that of the GC, and the half-mass radius is the radius from the core including half of the total mass of the cluster (Janes, 2000).

A single formula that embodies the characteristics of both Equation 2.5 and Equation 2.7 is

$$f = k \left\{ \frac{1}{[1 + (r/r_c)^2]^{\frac{1}{2}}} - \frac{1}{[1 + (r_t/r_c)^2]^{\frac{1}{2}}} \right\}^2. \quad (2.8)$$

In a typical GC, r_t/r_c is of the order of 30, so that for small to moderate values of r/r_c , Equation 2.8 differs only slightly from Equation 2.7, with

$$f_0 = k \left\{ 1 - \frac{1}{[1 + (r_t/r_c)^2]^{\frac{1}{2}}} \right\}^2. \quad (2.9)$$

Equation 2.8 comes very close to Equation 2.5 for $r \gg r_c$ with

$$f_1 = k r_c^2. \quad (2.10)$$

The second term in brackets in Equation 2.8 could be replaced by a single constant; it is written in this more complicated form in order to show the role of r_t . Equation 2.8 can be compared directly with observations. This is done by computing standard curves of $\log f$ against $\log r$, so that scale factors can be removed by sliding the curves horizontally and vertically.

The individual curves are labeled by values of the logarithm of the parameter

$$c = r_t/r_c, \quad (2.11)$$

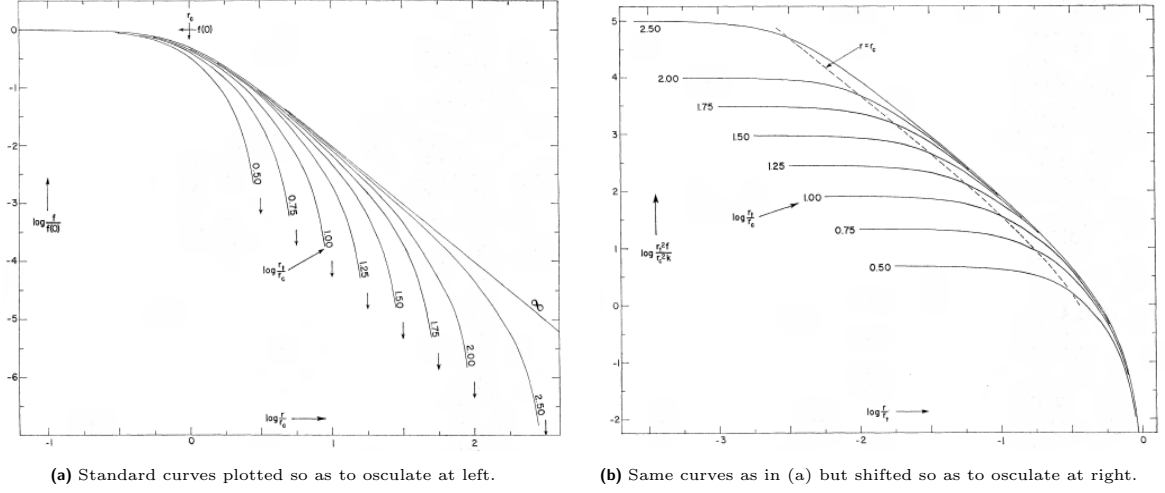


Figure 2.9: Standard curves calculated from Equation 2.8.

which may be referred to simply as the concentration ratio. Interpolation between the plotted curves is greatly simplified by using Figure 2.9a for the central region of a cluster and Figure 2.9b for the outer parts. If an observed curve is fitted to the two figures alternately, the process of determining c converges rapidly.

Michie (1963a) obtained a distribution function for spherical stellar systems. His analysis included a range of stellar masses for the individual stars, a velocity space anisotropy that produces increasingly elongated orbits at greater distances from the centre of the system, and a smoothed-out potential consistent with the distribution function for all values of energy and angular momentum (i.e., potential obtained from Poisson's equation). Michie (1963c) derived the distribution function that describes the distribution in the phase space of stars with mass m encountering stars with an average mass m_0 . The following distribution function obtained is

$$f(r, v, \mu, t) = A e^{-m/m_0(\alpha E + \beta J^2)} \left[\frac{I - e^{-kx_e^2(0)(1-E/E_e)}}{I - e^{-kx_e^2(0)}} \right], \quad (2.12)$$

where E and J^2 are the total energy and angular momentum squared per unit mass, and are integrals of the motion in the absence of encounters:

$$E = \frac{1}{2}v^2 + \phi(r), \quad (2.13)$$

$$J^2 = r^2 v^2 (I - \mu^2). \quad (2.14)$$

An extensive set of models for spherical stellar systems using a distribution function whereby the inner regions are obtained using the Boltzmann equation with the encounters. In this case, it was assumed that encounters and orbital mixing played an important role in determining the observed structure during early stages of the formation of clusters. This is because at that stage the systems are considered to be near equilibrium (i.e., the

systems have not changed much). Therefore, this assumption permits an extension of the initial distribution function to describe the whole system at later times (Michie and Bodenheimer, 1963). Furthermore, Michie (1963b) calculated the relative rate of stellar mass loss from spherical systems using a distribution function that includes a velocity-space anisotropy and which satisfies the Boltzmann equation for both large and small energies. He found that stars with a mass of about $1/4$ of the average stellar mass may actually leave the system due to distant encounters. Michie (1963c) made a comparison between the Galactic cluster surface densities resulting from star counts down to faint magnitudes with theoretical surface densities and found that theoretical density distributions that include an anisotropic distribution of velocities and spatial truncation are in agreement with all the observed Galactic cluster density distributions studies. He also found that the faint end of the luminosity function of all observed Galactic clusters at that time either remain constant or decreases. This implied depopulation of low-mass stars is not so strong for systems in which the velocity distribution is very anisotropic, while clusters with more isotropic orbits exhibit a wide range in the slope of the luminosity function at the faint end.

The early evolution of GCs is also modelled using an N-body simulation. For example, Fukushige and Heggie (1995) modelled GCs using N-body code and compared their simulations with the Fokker-Planck calculations of Chernoff and Weinberg (1990). The results of both simulations were found to be similar. Heggie (1995) noted that N-body simulations are only of use as statistical tests of a system's behaviour, and thus the evolution of any cluster must be tested over a statistically significant number of simulations before any robust conclusions can be drawn about its properties. Goodwin (1996) also carried out some N-body simulations of GCs. The simulations included residual gas that is expelled from the GC by the actions of massive ($> 8M_{\odot}$) stars. A comparison between the indicated simulation results and the observations of eight Large Magellanic Clouds (LMC) GCs were made and it was found that the observations can be accounted for in a model where the GCs form from proto-cluster clouds.

2.7 Special Stellar Members (BHs, CVs, WDs) and DM in GCs

There have been a number of theoretical and observational efforts to prove the existence of BHs in GCs (e.g., Miller and Hamilton, 2002, Portegies Zwart and McMillan, 2002, Baumgardt et al., 2005, Gebhardt et al., 2005, Van der Marel and Anderson, 2010). The Laser interferometer Gravitational-Wave Observatory (LIGO) and the Virgo Interferometer have detected a number of BH binaries (LIGO Scientific Collaboration, 2015, The LIGO Scientific Collaboration, 2020). Although the astrophysical origin of these sources is still unknown, there is a large possibility that they are formed in the dense cores of

GCs through dynamical three-body interactions (Sigurdsson and Hernquist, 1993, Downing et al., 2011). Several studies have confirmed the role of internal dynamics in the formation, as well as evolution of BH binaries as gravitational wave sources (Morscher et al., 2015, Askar et al., 2017, Hong et al., 2018). Askar et al., 2017, Kremer et al., 2020 found that BH binary mergers are produced at a local rate of $\approx 5 \text{ Gpc}^{-3} \text{ yr}^{-1}$. There is an empirical relation between binary BH production and the host GC's properties (Hong et al., 2018): there is a close correlation between the number of binary BH mergers and a cluster's mass, half-mass radius, and binary fraction. This may probably give us a clue as to the link between the gravitational wave detection in this epoch and the formation of GCs in the early Universe (see also Fujii et al., 2017, Choksi et al., 2019, Mapelli et al., 2019).

GCs contain binaries, and some of these binaries are CVs, which are consisting of a WD (primary) accreting matter from a roughly MS star (secondary) (Knigge, 2012). They are expected to exist in non-negligible numbers in GCs. CVs in GCs are difficult to identify due to high stellar crowding in GCs. However, there are GCs that have been studied with respect to CV populations (e.g., Cohn et al., 2010, Cool et al., 2013, Lugger et al., 2017, Henleywillis et al., 2018, Rivera Sandoval et al., 2018). CVs are found to be divided into two populations, a bright and faint one (Cohn et al., 2010, Lugger et al., 2017). The bright CVs lie close to the MS while the faint ones lie close to the WD cooling sequence on the CMDs. Only one population of CV (i.e., faint CVs) has been observed in the non-core collapsed clusters ω Cen and 47 Tucanae (Belloni et al., 2019). CV progenitors might be destroyed in the cores of dense clusters, but would likely survive in the outskirts. The number of such primordial CVs in a single massive cluster is estimated to be $N_{\text{CV,p}} \sim 100$ (Davies, 1997, Knigge, 2012). Di Stefano and Rappaport (1994) explored the role of two-body encounters in producing CVs in GCs and found that the number of CVs produced via tidal capture of a main sequence star by a WD is likely to be ~ 100 (Fabian et al., 1975). Finally, three-body interactions were considered and it was found that this channel again might contribute $N_{\text{CV,3b}} \sim 100$ CVs (Davies, 1995).

The first clear identification of WDs in GCs was made in 1995 (Elson et al., 1995, Paresce et al., 1995a,b, Richer et al., 1995), even though the search for these systems in Galactic GCs has already begun in 1978 (see Richer, 1978). Cho et al. (2015) searched for hot and luminous WDs in several nearby Galactic GCs, which resulted in the identification of luminous WD candidates in the GCs M13 (NGC 6205) and M22 (NGC 6656). Several interesting binary objects were discovered based on the SED and spectroscopy by the Far-Ultra-Violet (FUV) of 48 blue objects in 47 Tucanae observed by HST. These include one BS-WD star binary in the cluster (Knigge et al., 2008). This was the first BS-WD to be detected in 47 Tucanae. A second one, which consisted of a hot WD companion to a BS was discovered in the GC NGC 5466 (Sahu et al., 2019).

A primordial scenario of GCs within individual DM halos was first proposed by Peebles

and Dicke (1968). But, until today, the existence of DM halos in GC formation has been debated (Heggie and Hut, 1996, Ibata et al., 2012). The existence of DM in GCs could be inferred from the kinematics of stars within the clusters. GCs are expected to form with their own DM halos in the standard cold DM cosmology. However, there are other scenarios proposing that GCs were initially embedded in massive DM halos that evolved during the cluster lifetime (Fortes et al., 2020). These authors studied the effect of DM on halos surrounding an intermediate mass black hole (IMBH) in a GC using a DM spike profile. The spike structure of this profile was motivated by the strong evidence of DM mass in 47 Tucanae. Brown et al. (2018) attempted to find an explanation of the γ -ray excess seen in the direction of 47 Tucanae by analysing 9 years of *Fermi* LAT data. They noted that the γ -ray flux of 47 Tucanae cannot be explained by MSPs alone (see Section 2.8) because of the systematic discrepancy between the observed and predicted flux. Hence the spectrum of 47 Tucanae described a two-source population model, considering both MSPs and annihilating DM. Brown et al. (2019) found that ω Cen contains a component of DM by measuring the distribution of mass inside this cluster. Their result was inferred from the observed velocities of stars within the cluster, with an assumption that the stars are in dynamic equilibrium with the gravitational potential. Wirth et al. (2020) demonstrated that massive GCs like ω Cen may contain a significant amount of DM, if they originate from nucleated dwarfs orbiting the ancient Galaxy. These authors predicted GCs to have central DM densities ranging from 0.1 to several $M_{\odot} \text{ Mpc}^{-3}$, depending on the orbits and the masses of their host dwarf galaxies. However, they have also noted that GCs formed outside the central regions of their host dwarfs may not have DM after they are stripped from their host, even if they are massive at birth. They therefore suggested that the formation sites of GCs in their hosts, rather than their original masses, can determine whether they can contain DM.

2.8 MSPs in GCs

In this Section, I will focus on MSPs because they are relevant to the models discussed in Chapter 3. MSPs are types of pulsar that have small rotational periods (on the order of a few milliseconds). Pulsars are highly magnetised, spherical, compact stars that emit beams of electromagnetic radiation out of their magnetic poles. LMXBs, which are thought to be the progenitors of MSPs, are one stellar-type member occurring in GCs. According to the standard evolutionary (recycling) scenario of Alpar et al. (1982), the MSP is spun up via particle and angular momentum transfer during accretion, and the companion may be ablated by the MSP wind and radiation later on. Stars in GCs form LMXBs via dynamical encounters, and the number of LMXBs correlates with the stellar encounter rate (Gendre et al., 2003). The abundance of LMXBs in GCs implies the presence of a large number of GC MSPs. A linear correlation between the inferred number of γ -ray MSPs within a particular GC and the stellar encounter rate was found by Abdo et al.

(2010). Using their correlation they predicted that 2 600 – 4 700 MSPs are detectable in γ -rays in Galactic GCs. There is also a correlation between the number of radio GC MSPs and encounter rate as well as metallicity (see [Hui et al., 2010](#)). Furthermore, according to [Hui et al. \(2011\)](#), the MSP γ -ray luminosity correlates with encounter rate as well as with metallicity, which establishes a link between the GC γ -ray emission and their embedded population of MSPs (also validating the idea that MSPs are the progeny of LMXBs). The first discovery of MSPs was made by [Backer et al. \(1982\)](#) when they reported the detection of a MSP in 4C 21.53, which had a period of 0.001557708 s. Subsequently, in 1987, the first MSP in the core of a GC, M28, was found ([Lyne et al., 1987](#)) and within the next few years a total of 34 MSPs were found in GCs. There are currently 157 radio MSPs known in 30 GCs¹, with 25 MSPs being detected in 47 Tucanae ([Pan et al., 2016](#)) and 37 MSPs in Terzan 5 ([Cadelano et al., 2018](#)). GC MSPs have been detected in radio, X-ray, and γ -ray bands of the electromagnetic spectrum. For example, [Pan et al. \(2020\)](#) reported an eclipsing binary MSP in the cluster M92 (NGC 6341) detected using the Five-hundred-meter Aperture Spherical radio Telescope (FAST). Furthermore, [Dai et al. \(2020\)](#) reported the discovery of five MSPs in ω Cen following observations with the Parkes radio telescope. Four of the discovered pulsars are found to be isolated with spin periods of 4.1, 4.2, 4.6 and 6.8 ms. The fifth pulsar has a spin period of 4.8 ms and is in an eclipsing binary system with an orbital period of 2.1 hours.

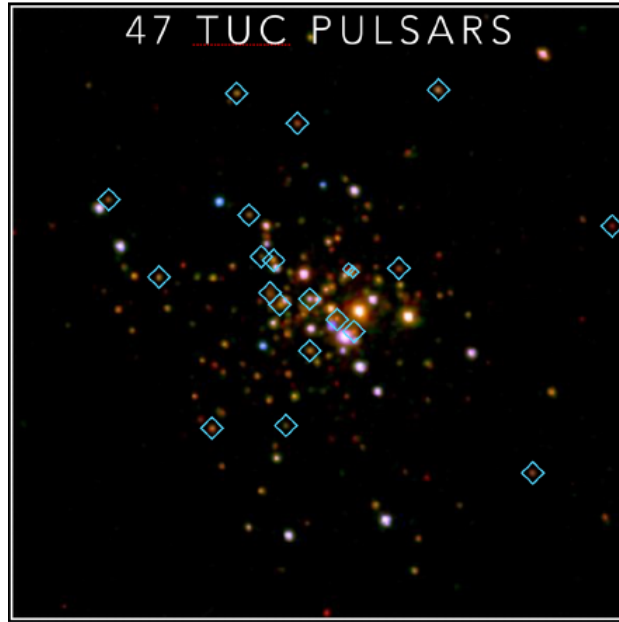


Figure 2.10: True colour image for the pulsars in an X-ray map of 47 Tuc (<https://www3.mpifr-bonn.mpg.de/staff/pfreire/47Tuc/>; credit: C.O. Heinke).

On the other hand, [Bhattacharya et al. \(2017\)](#) obtained X-ray spectra from MSPs 47 Tucanae F, 47 Tucanae S, 47 Tuc aa, 47 Tuc ab and 47 Tuc Z (compare [Figure 2.10](#)). A search for the the X-ray, optical and γ -ray counterparts of the radio MSP M22A was per-

¹<http://www.naic.edu/~pfreire/GCpsr.html>

formed by Amato et al. (2019). The researchers found persistent X-ray emission in two *Chandra* observations.

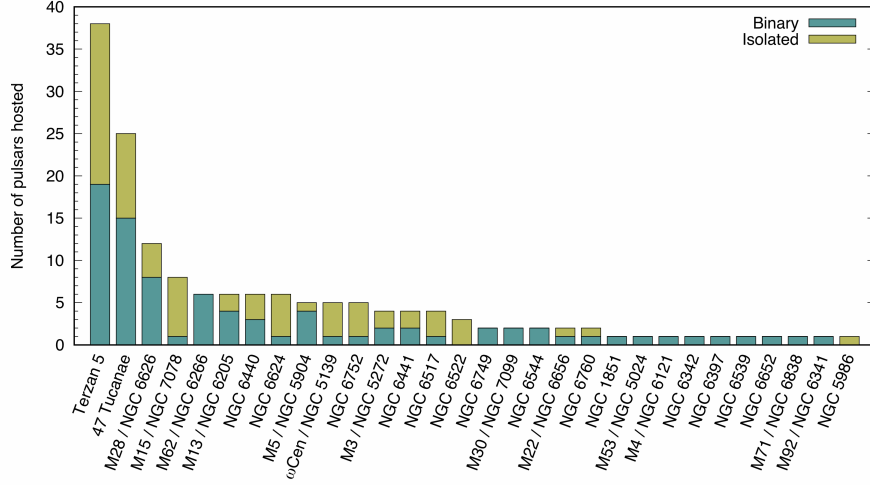


Figure 2.11: The number of binary and isolated MSPs detected in several Galactic GCs (<http://www.naic.edu/~pfreire/GCpsr.html>).

Figure 2.11 shows the distribution of the number of binary and isolated pulsars detected in 30 GCs. The abundance of MSPs in GCs provides the basis for our spectral GC model (Chapter 3), which assumes that MSPs provide relativistic particles that can radiate via SR and IC (see Section 2.9). Our model scales linearly with N_{MSP} . If there are no MSPs, then other processes such as DM annihilation should be invoked.

2.9 Radiation Mechanisms Relevant to GCs

The mechanisms responsible for radiation from MSPs in GCs are thought to be CR, SR, and IC. Pulsed CR is a direct (primary) emission component from the pulsars' magnetospheres (see Section 3.3.4). Additionally, there may also be pulsed SR by pairs within the MSP magnetospheres (Section 3.3.5). Unpulsed SR and IC are indirect (secondary) emissions that occur when pulsars inject particles into the GCs and these radiate (diffusively) as they move through the cluster (see Section 3.2.4). In this work, I consider both pulsed and unpulsed emissions. These three non-thermal processes are described below. I also discuss BB emission in Section 2.9.4, since hot stellar surfaces yield a soft-photon target field that may be upscattered by relativistic particles, yielding photons in the VHE γ -ray band.

2.9.1 Curvature Radiation (CR)

CR occurs when relativistic, charged particles are constrained to move along curved field lines, for instance, along magnetic field lines above a pulsar's stellar surface. The critical frequency ω_{CR} and thus the spectral cut-off of single-particle emission spectrum (e.g., [Harding, 1981](#), [Daugherty and Harding, 1982](#), [Cheng and Zhang, 1996](#), [Story et al., 2007](#), [Venter et al., 2009b](#)) is defined as

$$\omega_{\text{CR}} \equiv \frac{3\gamma^3 c}{2\rho_c} = \frac{\omega_{\text{CR}}^{\text{J}}}{2}, \quad (2.15)$$

with ρ_c being the curvature radius of the particle trajectory and the boosted $\omega_{\text{CR}}^{\text{J}}$ is due to the Doppler effect resulting in a contraction of the pulse width of the emission by a relativistic particle ([Longair, 2011](#)).

$$E_{\text{CR}} = \hbar\omega_{\text{CR}} = \frac{3\hbar c\gamma^3}{2\rho_c} = \frac{3\lambda_c\gamma^3}{2\rho_c} m_e c^2, \quad (2.16)$$

where E_{CR} is the high-energy cutoff, $\lambda_c = \hbar/(m_e c)$ is the Compton wavelength. The instantaneous power spectrum in this case (in units of $\text{erg s}^{-1} \text{erg}^{-1}$) is given by

$$\left(\frac{dP}{dE_\gamma} \right)_{\text{CR}} = \frac{\sqrt{3}\alpha_f \gamma c}{2\pi\rho_c} \kappa \left(\frac{E_\gamma}{E_{\text{CR}}} \right), \quad (2.17)$$

where $\alpha_f = e^2/\hbar c$ is the fine-structure constant, and ([Erber, 1966](#))

$$\kappa(x) \equiv x \int_x^\infty K_{5/3}(x') dx' \quad (2.18)$$

$$\approx \begin{cases} 2.149 x^{1/3} & x \ll 1 \\ 1.253 x^{1/2} e^{-x} & x \gg 1. \end{cases} \quad (2.19)$$

where $K_{5/3}(x')$ is a modified Bessel function of order $5/3$.

The total power radiated is obtained by integrating [Equation 2.17](#) over photon energy and is equal to the total CR loss rate of the electrons:

$$\int \left(\frac{dP}{dE_\gamma} \right)_{\text{CR}} dE_\gamma = \dot{E}_{\text{CR}} = \frac{-2e^2 c \gamma^4}{3\rho_c^2}. \quad (2.20)$$

[Equation 2.19](#) may be used to approximate the low-energy part of the CR photon spectrum ([Story et al., 2007](#))

$$\left(\frac{dN}{dE_\gamma} \right)_{\text{low}} = \frac{1}{dA E} \left(\frac{dP}{dE} \right)_{\text{CR}} \approx \frac{0.518\alpha_f}{dA \hbar^{1/3}} \left(\frac{c}{\rho_c} \right)^{2/3} E_\gamma^{-2/3}. \quad (2.21)$$

where $dA = 1/(4\pi d^2)$. Similarly, the high-energy CR photon spectrum becomes

$$\left(\frac{dN}{dE_\gamma}\right)_{\text{high}} \approx \frac{0.282\alpha_f}{dA} \left(\frac{c}{\gamma\rho_c\hbar}\right)^{1/2} E_\gamma^{1/2} e^{-E_\gamma/E_{\text{CR}}}. \quad (2.22)$$

2.9.2 Inverse Compton (IC) Scattering

IC is a process in which ultra-relativistic electrons upscatter low-energy photons to high energies such that energy and momentum are transferred from the electron to the photon. There exist two IC regimes: the Thomson and the Klein-Nishina regime.

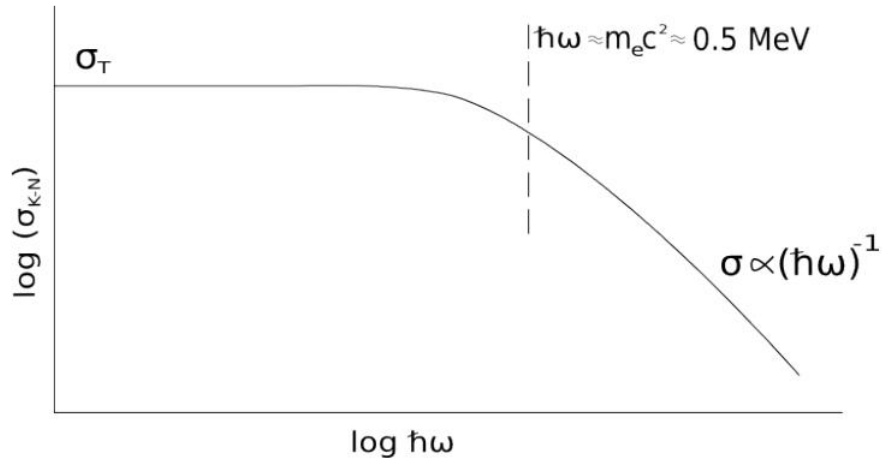


Figure 2.12: A schematic diagram showing the dependence of the IC cross section upon photon energy, with σ_T the Thomson cross section and σ_{KN} the Klein-Nishina cross section (Longair, 1992).

Consider the electron's rest frame and denote the energy of the incoming photon by ϵ' . If

$$\epsilon' \ll m_e c^2, \quad (2.23)$$

with m_e the mass of the electron and c the speed of light, then we are in the Thomson regime. However, if

$$\epsilon' \gg m_e c^2, \quad (2.24)$$

we are in the Klein-Nishina (K-N) regime. The average scattered photon energy $\langle\epsilon_1\rangle$ in the laboratory frame (in the Thomson regime) for an isotropic distribution of incident photons is

$$\langle\epsilon_1\rangle = \frac{4}{3}\gamma_e^2\epsilon. \quad (2.25)$$

Here ϵ is the energy of the soft (incident) photons in the lab frame and γ_e is the electron's Lorentz factor (Blumenthal and Gould, 1970). In the K-N, regime the average scattered

photon energy ϵ_1 is (Blumenthal and Gould, 1970)

$$\langle \epsilon_1 \rangle \sim \gamma_e m_e c^2. \quad (2.26)$$

In the Thomson regime the total energy loss rate of a single electron after averaging over electron direction is (Blumenthal and Gould, 1970)

$$P_{\text{ICS}} = - \left(\frac{dE_e}{dt} \right)_T = \frac{4}{3} \sigma_T c \beta^2 \gamma_e^2 U_{\text{ph}}, \quad (2.27)$$

where $\sigma_T = (8\pi/3)r_0^2 = 6.65 \times 10^{-25} \text{cm}^2$ is the Thomson cross section (Rybicki and Lightman, 1979), with r_0 the classical electron radius $r_0 = e^2/m_e c^2$, e the charge of the electron, $\beta = v/c$, and U_{ph} is the target photon energy density. The scattered photon distribution in the Thomson limit as derived by Blumenthal and Gould (1970) is

$$\frac{dN_{\gamma_e, \epsilon}}{dt d\epsilon_1} = \frac{\pi r_0^2 c}{2\gamma_e^4} \frac{n(\epsilon) d\epsilon}{\epsilon^2} \left(2\epsilon_1 \ln \frac{\epsilon_1}{4\gamma_e^2 \epsilon} + \epsilon_1 + 4\gamma_e^2 \epsilon - \frac{\epsilon_1^2}{2\gamma_e^2 \epsilon} \right). \quad (2.28)$$

Here, $n(\epsilon)$ is the number density of photons assuming an isotropic target photon field. The scattered photon energy can be expressed in terms of its maximum value $\hat{\epsilon}_1 \equiv 4\epsilon\gamma_e^2\epsilon_1^2$. Hence

$$\frac{dN_{\gamma_e, \epsilon}}{dt d\hat{\epsilon}_1} = 8\pi r_0^2 c n(\epsilon) d\epsilon f(\hat{\epsilon}_1), \quad (2.29)$$

where

$$f(\hat{\epsilon}_1) = 2\hat{\epsilon}_1 \ln \hat{\epsilon}_1 + \hat{\epsilon}_1 + 1 - 2\hat{\epsilon}_1^2 \quad (2.30)$$

is the distribution function.

According to Rybicki and Lightman (1979) the total K-N cross-section is

$$\sigma_{\text{KN}} = \frac{3}{4} \sigma_T \left[\frac{1+x}{x^3} \left(\frac{2x(1+x)}{1+2x} - \ln(1+2x) \right) + \frac{1}{2x} \ln(1+2x) - \frac{1+3x}{(1+2x)^2} \right], \quad (2.31)$$

where $x \equiv h\nu/m_e c^2$. Blumenthal and Gould (1970) derived the energy loss rate of a single electron for a BB photon distribution with temperature T in the K-N regime as

$$- \left(\frac{dE}{dt} \right)_{\text{KN}} = \frac{1}{6} \pi r_0^2 \frac{(m_e c k_B T)^2}{\hbar^3} \left(\ln \frac{4\gamma_e k_B T}{m_e c^2} - \frac{5}{6} - C_E - C_l \right), \quad (2.32)$$

where k_B is the Boltzmann constant, $C_E = 0.5772$, and

$$C_l = \frac{6}{\pi^2} \sum_{k=2}^{\infty} \frac{\ln k}{k^2} = 0.5700.$$

The general equation (including both regimes) for the scattered photon spectrum per

electron is

$$\frac{dN_{\gamma,\epsilon}}{dtdE_\gamma} = (\gamma_e m_e c^2) \frac{dN_{\gamma,\epsilon}}{dtd\epsilon_1} = \frac{2\pi r_0^2 m_e c^3 n(\epsilon) d\epsilon}{\gamma_e \epsilon} \left[2q \ln q + (1+2q)(1-q) + \frac{1}{2} \frac{(\Gamma_e q)^2}{1+\Gamma_e q} (1-q) \right], \quad (2.33)$$

where $E_\gamma = \epsilon_1/\gamma_e m_e c^2$, $\Gamma_e = 4\gamma_e \epsilon/m_e c^2$, and $q = E_\gamma/\Gamma_e(1-E_\gamma)$ is a dimensionless parameter that determines the scattering regime (Jones, 1968). To obtain the spectrum of Compton-scattered photons by an electron distribution N_e , one has to integrate the product of the electron distribution and Equation 2.33 over the electron Lorentz factor γ_e and soft-photon energy ϵ , i.e.,

$$\left(\frac{dN_\gamma}{dtd\epsilon_1} \right)_{\text{tot}} = \iint N_e(\gamma_e) d\gamma_e \left(\frac{dN_{\gamma,\epsilon}}{dtd\epsilon_1} \right), \quad (2.34)$$

where $dN_e = N_e(\gamma_e) d\gamma_e$ is the differential number of electrons per γ -interval (Blumenthal and Gould, 1970) and the last factor in the integrand is defined in Equation 2.33. The spectrum of the scattered photons is a power law if and only if the electron spectrum is described by a power-law distribution function, i.e., $N_e(\gamma_e) \propto \gamma_e^{-p}$. In the Thomson limit $\left(\frac{dN_\gamma}{d\epsilon_1} \right)_{\text{tot}} \propto \epsilon_1^{-(p+1)/2}$ and in the extreme K-N limit $\left(\frac{dN_\gamma}{d\epsilon_1} \right)_{\text{tot}} \propto \epsilon_1^{-(p+1)}$ (Rybicki and Lightman, 1979).

2.9.3 Synchrotron Radiation (SR)

When particles move through a magnetised region with magnetic field B , they will follow a spiral trajectory. As this is an accelerated motion, classical electrodynamics predict that radiation is emitted as a consequence. In the case of relativistic particles, one obtains a broad spectrum of SR. Thus, SR is produced when relativistic and ultrarelativistic charges (electrons) spiral in a magnetic field. Generally, the particle will emit synchro-curvature emission (Cheng and Zhang, 1996). The two limits to this process is SR (large perpendicular particle momentum) and CR (vanishing perpendicular particle momentum); see Viganò et al. (2015). Many astronomical objects emit SR at radio to optical and X-ray wavelengths. SR is therefore thought to be responsible for the low-energy component of the observed SED of a GC (Chapter 4).

Whereas CR is concerned with longitudinal particle motion (Section 2.9.1), SR is concerned with perpendicular particle motion. Figure 2.13 shows an electron of energy $E_e = \gamma_e m_e c^2$ and velocity $\vec{v}$ spiralling in a magnetic field $\vec{B}$. The pitch angle (i.e., the constant angle) between its velocity and $\vec{B}$ is α' . The electron spirals with gyration frequency of rotation $\omega_r = eB/\gamma_e m_e c$ independent of the pitch angle (α' ; e.g., Blumenthal and Gould (1970), Longair (2011)). To obtain the SR spectrum, one can actually calculate radiation emitted by an electron always moving perpendicular to the B -fields. In the *nonrelativistic limit*, a single particle spiralling in a magnetic field will emit (cyclotron) power according

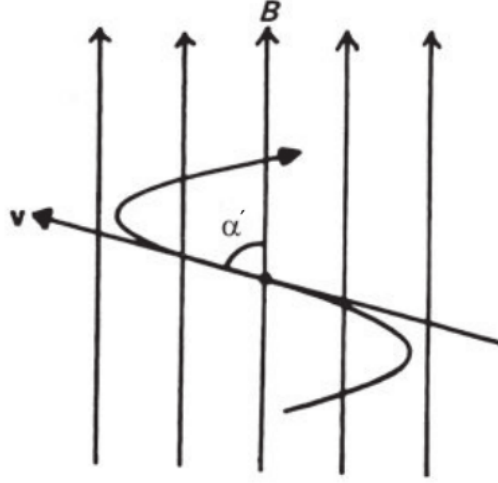


Figure 2.13: An electron spiralling in magnetic field lines emits SR due to the acceleration (change in direction) of the electron (Longair, 2011).

to the Larmor formula

$$P_{\text{emitted}} = - \left(\frac{dE}{dt} \right)_{\text{SR}} = \frac{2e\mathbf{a}^2}{3c^3}, \quad (2.35)$$

where $\mathbf{a}$ is the acceleration and e is the charge of the particle ($q = e$ for an electron) (Blumenthal and Gould, 1970). The total emitted radiation in the relativistic case when we set $d\vec{v}_{\parallel}/dt = 0$ (since we assume a zero E -field and the B -field cannot change the parallel component of the velocity) is

$$P_{\text{SR}} = \frac{2e^4}{3c^5} \gamma_e^2 \frac{B^2}{m_e^2} v_{\perp}^2, \quad (2.36)$$

where $v_{\perp}$ is the speed of the electron perpendicular to the B -field (Rybicki and Lightman, 1979) or

$$P_{\text{SR}} = \frac{2}{3} r_0^2 c \beta_{\perp}^2 \gamma^2 B^2 \quad (2.37)$$

with $r_0 = e^2/m_e c^2$ (Blumenthal and Gould, 1970). Blumenthal and Gould (1970) derived the SR energy loss rate as

$$\dot{E}_{\text{SR}} = - \frac{2r_0^2}{3c} \gamma_e^2 B^2 v^2 \sin^2 \alpha'. \quad (2.38)$$

The SR power emitted by a single electron of a given pitch angle α' may be rewritten as

$$\dot{E}_{\text{SR}} = P = 2\sigma_{\text{T}} c U_B \gamma^2 \beta^2 \sin^2 \alpha'. \quad (2.39)$$

In an isotropic distribution of pitch angles, averaging the term $\sin^2 \alpha'$ over the solid angle yields

$$P_{\text{tot}} = \dot{E}_{\text{SR}} = \frac{4}{3} \sigma_{\text{T}} c U_B \gamma_e^2 \beta^2, \quad (2.40)$$

which is of the same form as Equation 2.27, where σ_T is the Thomson scattering cross section and $U_B = B^2/8\pi$ is the magnetic energy density (Blumenthal and Gould, 1970). A critical frequency characterises the SR spectrum of a single electron. This cut-off frequency is given by

$$\omega_c = \frac{3}{2} \left(\frac{c}{v} \right) \gamma_e^3 \omega_r \sin \alpha'. \quad (2.41)$$

If $v \rightarrow c$ we get

$$\omega_c = \frac{3}{2} \gamma^3 \omega_r \sin \alpha' = \frac{3eB}{2m_e c} \gamma^2 \sin \alpha', \quad (2.42)$$

According to Rybicki and Lightman (1979) the emitted power per frequency radiated by a single electron is given as

$$\left(\frac{dP}{d\omega} \right)_{\text{SR}} (\gamma_e, \alpha', \omega) = \frac{\sqrt{3}e^3}{2\pi m_e c^2} B \sin \alpha' \tilde{\kappa}(x), \quad (2.43)$$

where $x = \omega/\omega_c$ and

$$\kappa(x) = x \int_x^\infty K_{5/3}(\xi) d\xi. \quad (2.44)$$

The total SR power emitted by particles is

$$\left(\frac{dP}{d\omega} \right)_{\text{tot}} (\alpha', \omega) \propto \omega^{-s}, \quad (2.45)$$

with the spectral index $s = (p-1)/2$ when the electrons are described by a power-law distribution function $N_e(\gamma_e) \propto \gamma_e^{-p}$ (Rybicki and Lightman, 1979). Thus,

$$\frac{dN_{\gamma, \text{SR}}}{dE_\gamma} \propto E_\gamma^{\frac{-(p+1)}{2}} \quad (2.46)$$

and

$$E_\gamma^2 \frac{dN_{\gamma, \text{SR}}}{dE_\gamma} \propto E_\gamma^{\frac{-(p-3)}{2}}. \quad (2.47)$$

2.9.4 Blackbody (BB) Radiation

A BB is an object that is idealised to absorb and emit all frequencies of radiation. BB radiation is the thermal electromagnetic radiation that a BB gives off at a given temperature. The spectral distribution of the thermal energy radiated by a BB depends only on its temperature. We use a BB because there are millions of stars in a GC and the photons create a photon field that I will be using as a target photon field being upscattered by the particles from MSPs. Several laws can describe the characteristics of BB radiation:

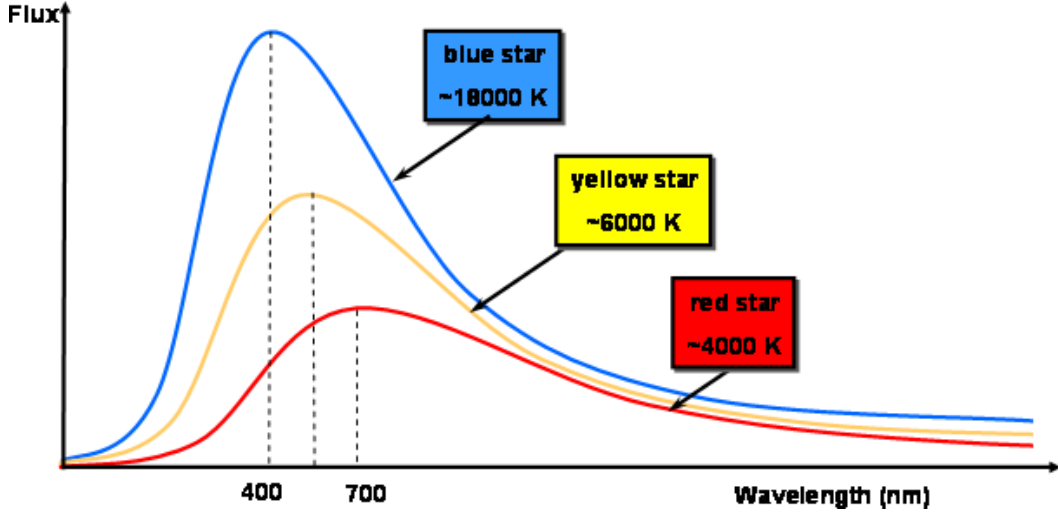


Figure 2.14: BB spectra for three different temperatures, e.g., for $\sim 4\,000$ K (red), $\sim 6\,000$ K (yellow) and $\sim 18\,000$ K (blue) stars. The wavelength and intensity of the peak of each curve moves as the temperature changes. *Credit: Swinburne.*

1. **The Stefan-Boltzmann law** relates the total emitted energy per area per unit time to the absolute temperature, i.e.,

$$R(T) = \sigma T^4, \quad (2.48)$$

where $\sigma = 5.67 \times 10^{-5} \text{ erg cm}^{-2} \text{ K}^{-4} \text{ s}^{-1}$ is the Stefan-Boltzmann constant (Rybicki and Lightman, 1979). Figure 2.14 shows the spectra for three different BBs. The total energy being radiated increases rapidly as the temperature increases. Although the intensity may be very low at very short or long wavelengths, at any temperature above absolute zero, energy is theoretically emitted at all wavelengths (i.e., the BB radiation curve does not reach zero). In other words, the normalisation of the curve changes as a function of temperature, as does the wavelength at which the peak occurs, moving to shorter wavelengths for higher temperatures (see Wien's displacement law discussed below).

2. **Planck's Law:** The Planck distribution describes the energy density of BB radiation, per frequency interval, as

$$u_\nu = \frac{8\pi h \nu^3}{c^3} \frac{1}{e^{h\nu/k_B T} - 1}, \quad (2.49)$$

where ν is the frequency and h is Planck's constant (Rybicki and Lightman, 1979). In terms of specific intensity I_ν , the BB intensity can be written in the Rayleigh-Jeans limit in cgs units ($\text{erg}^{-1} \text{ s}^{-1} \text{ cm}^{-2} \text{ Hz}^{-1} \text{ steradian}^{-1}$) as:

$$I_\nu = \frac{2h\nu^3}{c^2} (e^{h\nu/k_B T} - 1)^{-1} \equiv B_\nu = \frac{c}{4\pi} u_\nu. \quad (2.50)$$

Transforming Equation 2.50 using $\lambda = c/\nu$ and $B_\lambda d\lambda = B_\nu d\nu$, yields

$$B_\lambda = B_\nu \left| \frac{d\nu}{d\lambda} \right| = B_\nu \frac{c}{\lambda^2} = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda k_B T} - 1}. \quad (2.51)$$

To calculate the frequency or wavelength of the peak of a BB spectrum, one should take its derivative and equate it to zero, i.e.,

$$\frac{dB_\nu}{d\nu} = 0$$

or

$$\frac{dB_\lambda}{d\lambda} = 0.$$

From Figure 2.14 it is seen that the BB radiation curves have a complex shape described by Planck's Law. The curve at a specific temperature corresponds to a specific peak wavelength, and vice versa.

3. **Wien's Displacement Law:** As noted above, as the temperature of the BB increases, the peak wavelength decreases. The flux at all wavelengths increases as the temperature of the BB increases. This leads to Wien's law in two forms: $\lambda_{\max} T = 0.29 \text{ cm K}$ and $h\nu_{\max} = 2.8k_B T$ (Rybicki and Lightman, 1979). Therefore, the frequency of the peak emission ($\nu_{\max}$) is linearly proportional to absolute temperature (T).

2.10 Multi-wavelength Observations of GCs and their Embedded MSPs

GCs are visible from radio to γ -rays and are thus multi-wavelength objects. As mentioned before, within the MSP scenario, these emission components are thought to originate from cumulative magnetospheric (pulsed) and diffuse (unpulsed) emission by particles accelerated either within the pulsar magnetospheres or in interstellar shocks.

Several radio observations have been conducted on GCs. Lehto et al. (1990) reported the detection of radio emission from the position of the luminous X-ray source A1850–08 in the cluster NGC 6712. Sun et al. (2002) performed an analysis of the Very Large Array (VLA) radio survey data of the NVSS and FIRST, and found 13 GCs to have radio sources within their half-mass radii. They have identified sources detected in NGC 7078 and NGC 6440, however, they could not detect pulsars in NGC 6121, NGC 6440 and NGC 7078 due to insufficient survey sensitivity and resolution. Ulvestad et al. (2007) searched for radio emission from the putative Intermediate-Mass Black Hole (IMBH) in the GC G1 (Mayall-II) in M31, and detected emission within an arcsecond of the cluster centre with an 8.4 GHz power of $2 \times 10^{15} \text{ W Hz}^{-1}$.

An example of the detection of point sources in GCs is provided by the 25 radio MSPs found in 47 Tucanae (Pan et al., 2016, Ridolfi et al., 2016, Freire et al., 2017), 19 of which have been identified at X-ray energies (Heinke et al., 2005, Bogdanov, 2006, see also Section 2.8). Figure 2.15 shows pulse profiles for all known pulsars (at that time) in 47 Tuc at 20 cm. The horizontal errorbars shown in Ridolfi et al. (2016) indicate the time resolution of the data. Freire et al. (2017) reported the results of long-term timing of the MSPs in 47 Tucanae with the Parkes 64-m radio telescope. They obtained improved timing parameters that provide additional information for studies of the cluster dynamics such as the pulsar proper motions yielding an estimate of the proper motion of the cluster as a whole as well as the motion of the pulsars relative to each other. They measured the second spin-period derivatives caused by the change of the pulsar line-of-sight accelerations; 47 Tuc H, U and possibly J are being affected by nearby objects. For 10 binary systems, they measured changes in the orbital period caused by their acceleration in the gravitational field of the cluster. From all these measurements, they derived a cluster distance no smaller than 4.69 kpc and showed that the characteristics of these MSPs are very similar to their counterparts in the Galactic Disc. Figure 2.16 shows angular offsets² in right ascension (θ_α) and declination (θ_δ) relative to the centre of 47 Tucanae of 22 of its 25 known pulsars and proper motions of the pulsars in 47 Tucanae.

Some authors detected diffuse radio (e.g., Clapson et al., 2011) and diffuse X-ray emission (e.g., Eger et al., 2010, Eger and Domainko, 2012, Wu et al., 2014) from some GCs. Additionally, a bright X-ray GC pulsar B1821–24 has been detected (Hui et al., 2009) in M28 using the periodicity obtained via radio observations (Lyne et al., 1987). Bogdanov et al. (2011) detected X-ray pulsations as well as a non-thermal spectrum from PSR B1824–21, which may be due to magnetospheric emission, an unresolved pulsar wind nebula or small-angle scattering of the pulsed X-rays by interstellar dust grains. X-rays from four faint radio MSPs in NGC 6752, consistent with thermal emission from the neutron star surfaces using *Chandra* data, have been detected (Forestell et al., 2014). Also, Biggs et al. (1994) discovered a young, energetic MSP PSR J1823–3021A with a period of 5.44 ms and a relatively high inferred surface magnetic field of 4×10^9 G in a survey of GCs, and later detected by *Fermi* in NGC 6624 (Freire et al., 2011). See Ransom (2008), Hui et al. (2009), Freire (2013), Hessels et al. (2015) for an overview of radio and X-ray pulsars in GCs.

Unfortunately, pulsars are not detectable at optical wavelengths among the stellar emission in the cluster, however, some of their companions (see Pallanca et al., 2010, Mucciarelli et al., 2013, Pallanca et al., 2013, Cadelano et al., 2015, Rivera-Sandoval et al., 2015, Pallanca et al., 2016) and also types of exotic binaries such as CVs, active binaries, and BSs (e.g., Geffert et al., 1997, Heinke et al., 2003, 2005) are detectable in that band.

²In Chapter 3 we make the simplification that all MSPs are located in the GC centre, for mathematical expediency. This assumption may have to be reconsidered in future emission models.

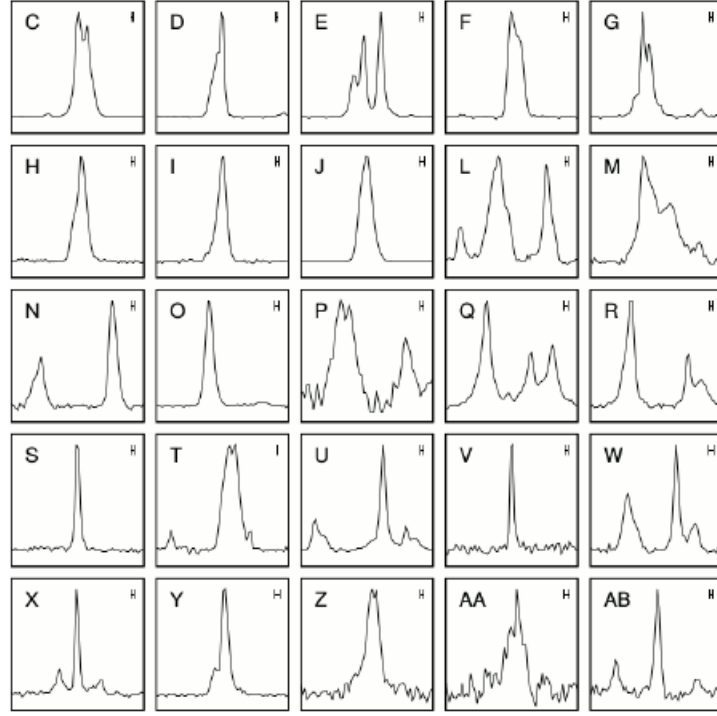


Figure 2.15: Pulse profiles for all known pulsars in 47 Tuc at 20 cm (Ridolfi et al., 2016).

Milone et al. (2017) used high-precision photometry of RGB stars in 57 Galactic GCs, mostly from the HST UV Legacy Survey of Galactic GCs, to identify and characterise their multiple stellar populations. They presented a pseudo-two-colour diagram (or ‘chromosome map’) for each GC. In the chromosome map of most GCs, stars separate in two distinct groups that they identified with the first (1G) and the second generation (2G). The fraction of 1G stars ranges from $\sim 8\%$ to $\sim 67\%$ and anti-correlates with the cluster mass, indicating that incidence and complexity of the multiple population phenomenon both increase with cluster mass. Pallanca et al. (2019) used optical images acquired with the UVIS channel of the Wide Field Camera 3 on board the HST to construct the first high-resolution extinction map in the direction of NGC 6440. The absorption clouds showed patchy and filamentary substructures with extinction variations as large as $\delta E(B - V) \sim 0.5$ mag.

An X-ray (sensitive in the 18 - 60 keV energy range) and γ -ray (sensitive in the 30 MeV - 50 GeV energy range) instrument, *Astro Rivelatore Gamma a Immagini Leggero (AGILE)*, has not detected any GC to date due to its relatively low sensitivity. On the other hand, about two dozen GCs have been detected by *Fermi* LAT, which detect photos in the energy range of 20 MeV to 300 GeV (Abdollahi et al., 2020). Recently, De Menezes et al. (2019) investigated the γ -ray emission from all GCs in the Galaxy using 9 years *Fermi* LAT data. Twenty three GCs (containing many MSPs, some of which were found to pulsate in the γ -ray band) were found as γ -ray bright, with two of them never having been reported before. The authors also found that magnetic braking probably has a smaller impact on

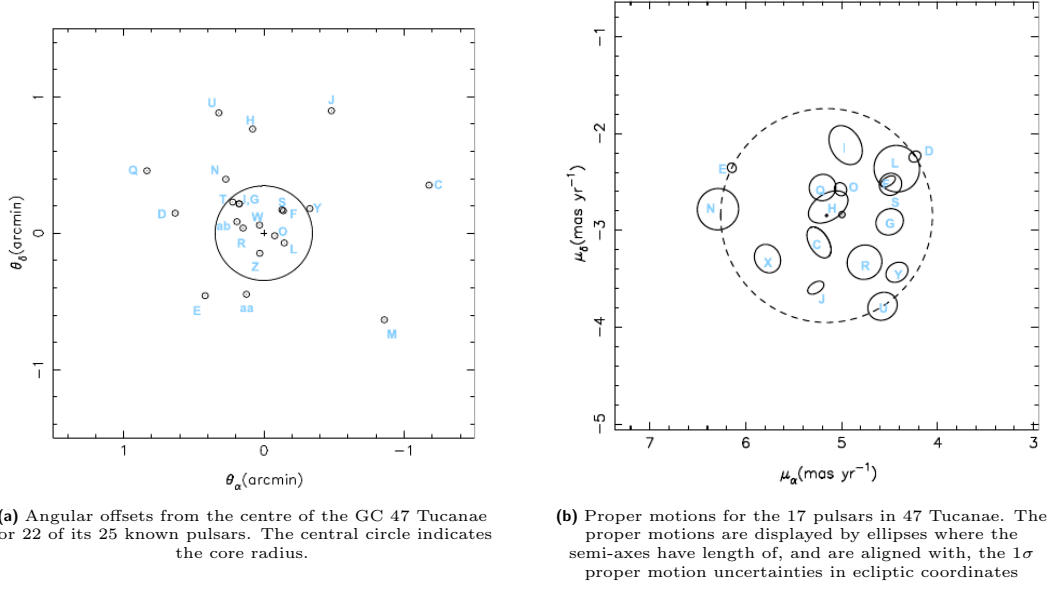


Figure 2.16: Angular offsets in right ascension (θ_α) and declination (θ_δ) relative to the centre of 47 Tucanae of 22 of its 25 known pulsars and proper motions of these pulsars (Freire et al., 2017)

the formation rate of binary systems in metal-rich GCs than previously suggested, while a large value for the two-body encounter rate seems to be a necessary condition. They did not find any extended emission even for clusters whose optical extent is $\approx 0.5^\circ$; all of them are point-like sources spatially in agreement with the optical cores of the GCs, supporting previous X-ray results of heavier objects sinking into the clusters' cores via dynamical friction.

H.E.S.S., which is a ground-based Cherenkov telescope operated in a pointing mode, has plausibly³ detected only a single Galactic cluster, which is Terzan 5 (which is a special GC and stands out among other Galactic GCs since it hosts the largest number of MSPS; see details in Chapter 5.) at VHEs (> 100 GeV) (Abramowski et al., 2011). Other Cherenkov telescopes could only produce upper limits (e.g., Anderhub et al., 2009). Observations of the GC 47 Tucanae (NGC 104) were performed with H.E.S.S. leading to an upper limit on the integral γ -ray flux of $F(E > 800 \text{ GeV}) < 6.7 \times 10^{-13} \text{ cm}^{-2} \text{ s}^{-1}$ (Aharonian et al., 2009). Observations of the GC M15 by the Major Atmospheric Gamma Imaging Cherenkov Telescopes (MAGIC) provided a deep upper limit of $F(E > 300 \text{ GeV})$, i.e., $< 0.26\%$ of the Crab Nebula flux. Also, stringent differential flux upper limits were obtained on this source (Acciari et al., 2019). CTA will be about 10 times more sensitive than H.E.S.S. (Wagner et al., 2010), and is expected to detect more than half of the known Galactic GC population, depending on observation time and model parameters (Ndiyavala et al., 2018, Chapter 4). Therefore, GCs form an important class of Galactic emitters of broad-band radiation, both point-like and diffuse in morphology, and are prime targets for deeper future observations by more sensitive telescopes.

³The probability for a chance detection in this direction is $\sim 10^{-4}$.

2.11 Summary

In this Chapter, I discussed the general stellar populations in GCs as well as the estimation of their distances, ages, and metallicities. I have also discussed special stellar members such as BHs, CVs, WDs, and MSPs in GCs. Furthermore, I discussed the radiation mechanisms that take place in these systems. Finally, I briefly discussed multi-wavelength observations of GCs. Next, I will discuss the numeric emission models used in Chapters [4](#), [5](#), and [6](#).

Chapter 3

Model Description

In the previous Chapter, I have discussed GCs in general terms, including general stellar populations in GCs, distances, age estimates, and metallicities of GCs. I have also discussed surface brightness profiles, and special stellar members in GCs such as BHs, CVs, WDs, and MSPs. I furthermore discussed the radiation processes that take place in GCs. Lastly, I briefly discussed the multi-wavelength observations of GCs. In this Chapter, I will first give an overview of models of broadband emission from GCs, particularly in the HE band, and also discuss the two models I used: 1) A 1D spherically-symmetric model used to calculate particle transport (including diffusion and radiation losses), as well as the SED emitted by GCs for a very broad energy range by considering SR as well as IC emission (Kopp et al., 2013); 2) A model of Harding and Kalapotharakos (2015) that calculates the pulsed CR by primaries and pulsed SR by electron-positron pairs generated within the magnetosphere of the host MSPs to radiate pulsed SR. I have applied the model of Kopp et al. (2013) (discussed in Section 3.2) to different Galactic GCs (to conduct parameter studies and also for spectral fitting) and also obtained outputs from the Harding and Kalapotharakos (2015) code for several different parameters (discussed in Section 3.3), fitting these to X-ray spectral data.

3.1 Overview of Previous GC Models

Chen (1991) was first to estimate the γ -ray luminosity of GCs, assuming that these systems may contain large numbers of MSPs. A later prediction of the L_γ was performed by Wei et al. (1996) using an outer gap model. Harding et al. (2005) modelled the pulsed CR γ -ray flux using a pair-starved polar cap model expected from a population of MSPs in 47 Tucanae. They summed up individual predicted pulsed CR spectra from members of an ensemble of MSPs to estimate the total contribution from the GC MSPs in the GeV band. They noted that the γ -rays are produced in the magnetosphere of MSPs via

accelerated electron primaries that are moving along curved magnetic field lines. [Bednarek and Sitarek \(2007\)](#) calculated the GeV - TeV γ -ray spectra for various models where leptons are injected by MSPs and accelerated to a power-law spectrum at the relativistic shocks that are produced inside the cores of GCs during the collisions of pulsar winds. These particles radiate unpulsed IC radiation and SR. The authors therefore concluded that accelerated leptons in the GCs should produce well-localised VHE γ -ray sources that are coincident with and centred on the optical GC sources. [Venter and De Jager \(2008a\)](#) modelled the pulsed γ -ray flux expected from a population of GC MSPs (with $N_{\text{MSP}} = 100$) by randomising over pulsar geometry as well as period P and period derivative $\dot{P}$ of the individual pulsars. [Harding et al. \(2005\)](#) as well as [Venter et al. \(2009b\)](#) estimated the GeV GC flux by assuming a pair-starved polar cap (PSPC) electric field while [Venter and De Jager \(2005\)](#) stated that the HE flux scales linearly with the N_{MSPs} . Hence, one needs to use observations to constrain the average accelerating electric field and polar cap (PC) current. [Venter and De Jager \(2010\)](#) improved their procedure of estimating the cumulative CR flux and they obtained results corresponding to the measured spectrum of 47 Tucanae by *Fermi* LAT to within a factor of two in both energy and flux ([Venter et al., 2009a](#)).

[Cheng et al. \(2010\)](#) explored an alternative model to calculate IC radiation by accelerated electron-positron pairs in the pulsar wind of MSPs in the GCs as they upscatter the CMB, stellar photons, as well as the Galactic background radiation. They assumed a two-valued step-like spatial dependence for their diffusion coefficient and constrained the coefficient using spatial γ -ray data from *Fermi* LAT. [Büsching et al. \(2011\)](#) and [Kopp et al. \(2013\)](#) performed a line-of-sight (LOS) calculation to model diffuse X-rays for Terzan 5. They found similar results to [Cheng et al. \(2010\)](#), who noted that the IC emission of HE photons should extend to beyond the cores with core radii > 10 pc. Their model also predicted VHE (> 100 GeV) components in some cases. [Hui et al. \(2011\)](#) reported that L_γ correlated positively with the stellar encounter rate Γ_e and the metallicity $[\text{Fe}/\text{H}]$, linking the γ -ray emission to the MSP population. They also found that L_γ increases with the Galactic background soft-photon energy densities in different wavebands, which actually supports the idea of [Cheng et al. \(2010\)](#) that the GeV flux is due to IC scattering. [Freire et al. \(2011\)](#) and [Johnson et al. \(2013\)](#) reported the detection of energetic γ -ray pulsars in GCs M 28 and NGC 6624, dominating the GeV emission, and thus pointing to a CR origin of the GeV emission. The GeV emission may therefore be a mixture of IC scattering and CR.

[Venter et al. \(2009a\)](#) presented pulsed CR and unpulsed IC calculations for Terzan 5 and 47 Tucanae's MSP population and constrained N_{MSP} and cluster B -field using both *Fermi* and H.E.S.S. sensitivity curves and focusing on the IC flux components. They performed particle transport involving IC and SR losses plus diffusive escape, with power-law injection spectra and assumed a steady-state scenario, Bohm diffusion, calculating IC scattering on the CMB, and stellar photons using a two-zone model (having two spherical regions of

different stellar photon energy densities). They furthermore performed an SR calculation by assuming a homogeneous cluster B -field. The H.E.S.S. data for Terzan 5 imply that particle reacceleration may happen within the GC, because a power-law spectrum was measured in the TeV band, and this implies a power-law particle injection spectrum. The model of [Venter et al. \(2009b\)](#) was extended by [Prinsloo et al. \(2013\)](#), where they calculated the soft-photon energy density profile for a large number of zones $u(r_s)$ (where r_s is the radial distance from the centre) from first principles, to reproduce the result of [Bednarek and Sitarek \(2007\)](#). They stated that it is significant to add a third larger outer spatial zone containing a soft-photon target field because the emission does not drop off within the first two zones as assumed by [Venter et al. \(2009a\)](#). [Zajczyk et al. \(2013\)](#) modelled γ -ray emission from synthetic MSP using PSPC. In addition to pulsar emission characteristics, they obtained spectra of electrons escaping pulsar magnetosphere.

[Bednarek \(2012\)](#) investigated the existence of HE γ -ray sources produced by relativistic electrons injected by the population of fast-rotating, magnetised WDs within the GC. He concluded that, depending on the model parameters, if one includes the decay of the WD B -fields and different scenarios for formation and evolution of WDs then there is a possibility of producing γ -ray emission at a flux level detectable by the future CTA.

On the other hand, [Domainko \(2011\)](#) explored a model based on γ -ray burst remnants as sources of energetic leptons and hadrons. When one considers a short burst that is caused by two merging neutron stars, some fraction of the shock wave energy might be tapped and accelerate hadrons following the explosion. [Domainko \(2011\)](#) constrained the spatial diffusion coefficient κ_0 using the observed VHE extension of HESS J1747–248 (the γ -ray source associated with Terzan 5). He then compared the burst age with the short burst occurrence rate and stated that the detected source extension is compatible with a relatively slow diffusion of hadrons in the GC. [Domainko \(2011\)](#) also noted that there might be many multi-wavelength signatures that may provide support for this scenario, for example, diffuse thermal X-rays arising from hot plasma and diffuse X-rays from IC by electrons accelerated by the blast wave. See [Bednarek \(2013\)](#) and [Tam et al. \(2016\)](#) for a review of γ -ray observations and modelling of GCs. [Brown et al. \(2018\)](#) stated that a combination of cumulative CR from MSPs and dark matter annihilation may explain the GeV emission detected by *Fermi* LAT from 47 Tucanae. Also, [Brown et al. \(2019\)](#) argued that ω Cen may contain DM with density as high as compact dwarf galaxies and that it emits γ -rays (in the 0.1 – 100 GeV energy range) with an energy spectrum matching that expected from annihilation of DM particles. They do not expect TeV emission from this source but we do expect some TeV emission within the framework of our model. All models predict an SED with various levels of success, some give spatial predictions (source sizes). In Chapter 5, we model Terzan 5 invoking four spectral components. Our model can in principle also predict the source size in different wavebands (see [Kopp et al. \(2013\)](#)).

3.2 Spherically-Symmetric Leptonic GC Model

A large number of MSPs in GCs is expected to radiate several spectral components in the radio through γ -ray wavebands (Ndiyavala et al., 2019). We expect pulsed emission via SR and CR, and also possibly even via IC scattering initiated by electron-positron pairs from within the magnetospheres of MSP embedded in the cluster (Harding et al., 2005, Venter and De Jager, 2005). The unpulsed emission is also expected to transpire through the continuous injection of relativistic leptons by the MSPs into the ambient GC region (Bednarek and Sitarek, 2007), which in turn produce unpulsed SR and IC emission when they encounter the cluster magnetic field as well as background (stellar and CMB) photon components.

The model of Kopp et al. (2013) (in what follows, we indicate spectral component predictions from this model using blue lines) is a multi-zone, steady-state, spherically symmetric model that calculates the particle transport (1D: radial spatial coordinate) and observed spectrum for GCs. The model predicts the SED from GCs by considering unpulsed SR as well as IC emission, attributing it to the interaction of collective leptonic winds with the ambient magnetic and soft-photon fields. The model includes the following improvements with respect to earlier models of Cheng et al. (2010), Tam et al. (2011), Venter et al. (2009a), Eger and Domainko (2012): a line-of-sight calculation of the X-ray surface brightness (see X-ray surface brightness profile of Terzan 5 in Chapter 5) to constrain the diffusion coefficient and multiple spatial zones. This model was calibrated against the earlier models by Venter and De Jager (2008a), Venter et al. (2009a). That is, in Venter and De Jager (2008a), Venter et al. (2009a), they invoked simplified particle transport and only 2 or 3 spatial zones; still, we could calibrate our results against their (spatially-integrated) SEDs and found that they were very similar, given the different approaches. The new code by Kopp et al. (2013) enabled spatially-dependent flux predictions, moving beyond these earlier models.

The main free parameters of this model are: cluster magnetic field (B), power law index (Γ) of the injected particle spectrum (the particles are assumed to have undergone acceleration in inter-pulsar shocks where colliding winds interact), number of GC stars visible in the optical band ($N_\star$), distance to the cluster (d), the conversion efficiency of spin-down luminosity into particle acceleration (η), the average spin-down luminosity per pulsar ($\dot{E}$), and number of MSPs in the GC (N_{MSP}).

3.2.1 Particle Transport Equation

The following transport equation of relativistic electrons and positrons in GCs is solved numerically (Kopp et al., 2013):

$$\frac{\partial n_e}{\partial t} = \vec{\nabla} \cdot (\vec{\kappa} \cdot \vec{\nabla} n_e) - \frac{\partial}{\partial E_e} (\dot{E}_e n_e) + Q, \quad (3.1)$$

where n_e is the electron density per energy and volume, which is a function of central GC radius $r_s = |\vec{r}_s|$, E_e the electron energy, $\vec{\kappa}$ is the diffusion tensor, $\dot{E}_e$ the radiation losses, and Q the electron source term. However, if one considers scalar diffusion, and time independence i.e., $\frac{\partial}{\partial t} = 0$, n_e depends only on $\vec{r}_s$ and E_e , and Equation 3.1 becomes

$$0 = \vec{\nabla} \cdot (\vec{\kappa} \cdot \vec{\nabla} n_e) - \frac{\partial}{\partial E_e} (\dot{E}_e n_e) + Q. \quad (3.2)$$

Setting $\vec{\nabla} n_e = \frac{\partial n_e}{\partial r_s} \vec{e}_r$ and $\frac{\partial}{\partial \theta} = \frac{\partial}{\partial \phi} = 0$, with $\vec{e}_r$ the radial unit vector, one finds

$$0 = \frac{1}{r_s^2} \frac{\partial}{\partial r_s} \left(r_s^2 \kappa \frac{\partial n_e}{\partial r_s} \right) - \frac{\partial}{\partial E_e} (\dot{E}_e n_e) + Q. \quad (3.3)$$

Equation 3.3 can be expressed as follows:

$$\frac{\partial}{\partial E_e} (\dot{E}_e n_e) = \frac{1}{r_s^2} \frac{\partial}{\partial r_s} \left(r_s^2 \kappa \frac{\partial n_e}{\partial r_s} \right) + Q \quad (3.4)$$

$$\dot{E}_e \frac{\partial n_e}{\partial E_e} + n_e \frac{\partial \dot{E}_e}{\partial E_e} = \frac{1}{r_s^2} \frac{\partial}{\partial r_s} \left(r_s^2 \kappa \frac{\partial n_e}{\partial r_s} \right) + Q \quad (3.5)$$

$$\frac{\partial n_e}{\partial E_e} = \frac{1}{\dot{E}_e} \left(\frac{1}{r_s^2} \frac{\partial}{\partial r_s} \left(r_s^2 \kappa \frac{\partial n_e}{\partial r_s} \right) - n_e \frac{\partial \dot{E}_e}{\partial E_e} + Q \right). \quad (3.6)$$

Equation 3.6 can be solved numerically using a Crank-Nicolson algorithm.

Assuming a source term Q located at $\vec{r}_s = \vec{r}_{\text{inj}}$ (r_{inj} is at the GC centre), that is in the form

$$Q = Q_0 E_e^{-\Gamma} \delta(\vec{r}_s - \vec{r}_{\text{inj}}), \quad (3.7)$$

with the spectral index Γ and using the Gauss theorem, plus assuming $\dot{E} = 0$ for the innermost zone. Thus, as a simplification, we assume that all MSPs are located at the GC centre. In future, we will study the impact of a distribution of sources throughout the cluster, which may require a Monte-Carlo approach (Bednarek et al., 2016). The source term Q can be replaced by a boundary condition:

$$0 = \vec{\nabla} \cdot (\kappa \vec{\nabla} n_e) + Q_0 \frac{\delta(\vec{r}_s - \vec{r}_{\text{inj}})}{E_e^\Gamma} \quad (3.8)$$

$$-\frac{Q_0}{E_e^\Gamma} = \oint \kappa(\vec{\nabla} n_e) \cdot d\vec{A} \quad (3.9)$$

$$\left. \frac{\partial n_e}{\partial r_s} \right|_{r_{s,\min}} = -\frac{Q_0}{4\pi r_{s,\min}^2 \kappa(E_e, r_{s,\min}) E_e^\Gamma}, \quad (3.10)$$

where $r_{\min}$ bounds the spherical region containing all particle sources that are assumed to be concentrated near the GC centre.

3.2.2 Spatial Diffusion

Two different diffusion process coefficients are considered: 1) Bohm diffusion:

$$\kappa(r_s, E_e) = \frac{cE_e}{3eB(r_s)}, \quad (3.11)$$

where B is the GC B-field, c the speed of light, and e the electron charge. 2) A diffusion coefficient (a parametric description of the particle flow) inspired by Galactic cosmic-ray propagation studies

$$\kappa(r_s, E_e) = \kappa_0 \left(\frac{E_e}{E_0} \right)^\alpha, \quad (3.12)$$

where $E_0 = 1$ TeV and $\alpha = 0.6$, e.g., [Moskalenko and Strong \(1998\)](#).

3.2.3 Particle Injection Spectrum

It is assumed that the particle injection spectrum is a power law (see [Equation 3.7](#)):

$$Q(E_e) = Q_0 E_e^{-\Gamma}, \quad (3.13)$$

with units of $\text{erg}^{-1}\text{s}^{-1}$, between energies $E_{e,\min}$ and $E_{e,\max}$. This is motivated by the fact that H.E.S.S. detected a power-law spectrum for the VHE source associated with Terzan 5. It is possible that this spectral shape is due to particle acceleration at the relativistic shocks that are formed during collisions of pulsar winds inside the cores of GCs ([Bednarek and Sitarek, 2007](#)). If this is true, and the process is of a stochastic Fermi-type, then one should expect a power-law shape (see, e.g., [Longair, 2011](#)). The normalisation of the injection spectrum requires

$$L_e \equiv \int_{E_{e,\min}}^{E_{e,\max}} E_e Q dE_e = N_{\text{MSP}} \eta \langle \dot{E}_{\text{rot}} \rangle = L_{\text{eff}}, \quad (3.14)$$

with η the fraction of the average spin-down power $\langle \dot{E}_{\text{rot}} \rangle$ converted into particle power. That is, the normalisation of the injection spectrum is

$$Q_0 = \begin{cases} \frac{L_{\text{eff}}}{\ln(E_{\text{e,max}}/E_{\text{e,min}})} & \text{if } \Gamma = 2.0, \\ \frac{L_{\text{eff}}(2-\Gamma)}{E_{\text{e,max}}^{2-\Gamma} - E_{\text{e,min}}^{2-\Gamma}} & \text{if } \Gamma \neq 2.0. \end{cases} \quad (3.15)$$

3.2.4 Radiation losses: Energy-Loss Terms

One needs to specify the energy loss ($\dot{E}$) per particle for IC scattering and SR¹ discussed in Section 2.9. For simplicity, one can assume isotropic photon and electron distributions. In the general case (Thomson and Klein-Nishina limits; [Blumenthal and Gould, 1970](#)) the IC losses are given by

$$\dot{E}_{\text{IC}}(E_{\text{e}}, r_{\text{s}}, T) = - \sum_{j=0}^{k-1} \int \chi(E_{\gamma}, E_{\text{e}}, u_{\text{rad},j}, T_j) dE_{\gamma}, \quad (3.16)$$

where k is the total number of (BB) soft-photon components, E_{γ} represents the final energy of the upscattered photons, $u_{\text{rad},j}$ is the energy density of the soft-photons, T_j is the photon temperature of component j , and h is Planck's constant. The function χ is expressed as

$$\chi(E_{\gamma}, E_{\text{e}}, u_{\text{rad},j}, T_j) = \int n_{\varepsilon,j}(r_{\text{s}}, \varepsilon, T_j) \frac{E_{\gamma}}{E_0} \zeta(E_{\text{e}}, E_{\gamma}, \varepsilon) d\varepsilon, \quad (3.17)$$

with $n_{\varepsilon,j}$ the photon density of the BB emitter, ε the soft-photon energy, and $E_0 = m_{\text{e}}c^2$ the electron rest energy assuming isotropic photon and electron distributions. The collision rate ζ is given by

$$\zeta(E_{\text{e}}, E_{\gamma}, \varepsilon) = \frac{2\pi e^4 E_0 c}{\varepsilon E_{\text{e}}^2} \hat{\zeta}(E_{\text{e}}, E_{\gamma}, \varepsilon), \quad (3.18)$$

where ([Jones, 1968](#))

$$\hat{\zeta}(E_{\text{e}}, E_{\gamma}, \varepsilon) = \begin{cases} 0 & \text{if } E_{\gamma} \leq \frac{\varepsilon E_0^2}{4E_{\text{e}}^2}, \\ \frac{E_{\gamma}}{\varepsilon} - \frac{E_0^2}{4E_{\text{e}}^2} & \text{if } \frac{\varepsilon E_0^2}{4E_{\text{e}}^2} \leq E_{\gamma} \leq \varepsilon, \\ f(q, g_0) & \text{if } \varepsilon \leq E_{\gamma} \leq \frac{4\varepsilon E_{\text{e}}^2}{E_0^2 + 4\varepsilon E_{\text{e}}}, \\ 0 & \text{if } E_{\gamma} \geq \frac{4\varepsilon E_{\text{e}}^2}{E_0^2 + 4\varepsilon E_{\text{e}}}. \end{cases}$$

¹In this model we are only considering SR and IC. The CR is relevant for the model of Harding (Section 3.3.4)

The function $f(q, g_0)$ is defined as:

$$f(q, g_0) = 2q \ln q + (1 - q)(1 + (2 + g_0)q),$$

with $q = E_0^2 E_\gamma / (4\varepsilon E_e (E_e - E_\gamma))$ and $g_0(\varepsilon, E_\gamma) = 2\varepsilon E_\gamma / E_0$. All terms put together gives (with $g_{IC} = 2\pi e^4 c$):

$$\dot{E}_{IC}(E_e, r_s, T_j) = -\frac{g_{IC}}{E_e^2} \sum_{j=0}^{k-1} \iint n_{\varepsilon,j}(r_s, \varepsilon, T_j) \frac{E_\gamma}{\varepsilon} \hat{\zeta}(E_e, E_\gamma, \varepsilon) d\varepsilon dE_\gamma. \quad (3.19)$$

In the SR case, the loss rate (averaged over all pitch angles) is given by (Blumenthal and Gould, 1970)

$$\dot{E}_{SR}(E_e, r_s) = -\frac{g_{SR}}{8\pi} E_e^2 B^2(r_s), \quad (3.20)$$

with $g_{SR} = 4\sigma_T c / 3E_0^2 = 32\pi(e/E_0)^4 c / 9$, where σ_T denotes the Thomson cross section.

3.2.5 Target Soft-photon Energy Densities

For one to calculate $\dot{E}_{IC}$, the soft-photon densities should be specified. Kopp et al. (2013) used a photon density for a BB (see also Zhang et al., 2008):

$$n_{\varepsilon,j}(r_s, \varepsilon, T_j) = \frac{15u_{\text{rad},j}(r_s, T_j)}{(\pi k_B T_j)^4} \frac{\varepsilon^2}{e^{\frac{\varepsilon}{k_B T_j}} - 1}, \quad (3.21)$$

where $u_{\text{rad},j}$ is the photon energy density of the j^{th} component of soft-photon field and k_B is the Boltzmann's constant. The following were included at the position of the GC: the CMB, stellar photons, and the Galactic background radiation field (Kopp et al., 2013). They used a line-of-sight (LOS) integration for the soft-photon component (see Prinsloo et al., 2013):

$$n_{\varepsilon,j}(r_s, \varepsilon, T_1) = \frac{8\pi}{h^3 c^3} \frac{\varepsilon^2}{e^{\frac{\varepsilon}{k_B T_1}} - 1} \left(\frac{1}{2} \frac{N_{\text{tot}} R_\star^2}{r_c^2 \tilde{R}} \right) \int_{r'=0}^{r'=r_t} \hat{\rho}(r') \frac{r'}{r_s} \ln \left(\frac{|r' + r_s|}{|r' - r_s|} \right) dr' \quad (3.22)$$

where N_{tot} represents the total number of cluster stars that can be written as $N_{\text{tot}} = M_{\text{tot}}/m$, with M_{tot} the total mass of the cluster, and m the average stellar mass. Here, $R_\star$ is the average stellar radius, r_c indicates the core radius of the cluster, and $\tilde{R} = 2r_h - 2r_c/3 - r_h^2/r_t$, with r_h the half-mass radius and r_t the tidal radius of the cluster. Solar values for $R_\star$ and m are used and it is assumed that all stars in the simulation have a solar temperature $T = 4\,500\text{ K}$ (Kopp et al., 2013).

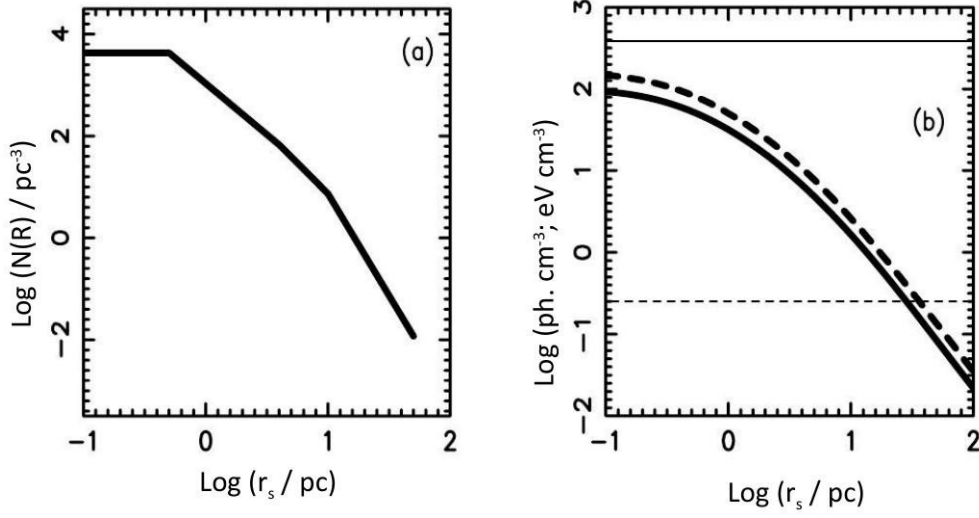


Figure 3.1: The assumed density of stars (a), and photon and energy density of stellar photons (b) as a function of distance r_s from the centre of a typical GC. The energy density and photon density are illustrated by thick dashed and solid curves respectively. The thin horizontal lines show the corresponding values for the CMB. From [Bednarek and Sitarek \(2007\)](#).

3.2.6 Calculation of the Photon Spectra

The following expression was used to calculate the IC scattering emissivities ([Kopp et al., 2013](#)):

$$X_{\text{IC}}(E_\gamma, r_s) = \left(\frac{dN_\gamma}{dE_\gamma} \right)_{\text{IC}} \quad (3.23)$$

$$= \frac{g_{\text{IC}}}{A} \sum_{j=0}^{k-1} \iint n_{\varepsilon,j}(r_s, \varepsilon, T_j) \frac{N_e(E_e, r_s)}{\varepsilon E_e^2} \hat{\zeta}(E_e, E_\gamma, \varepsilon) d\varepsilon dE_e, \quad (3.24)$$

with $A = 4\pi d^2$, where d is the distance to the source, and N_e is the number of electrons per energy interval in a spherical shell around r_s . To calculate the SR emissivity, [Kopp et al. \(2013\)](#) used

$$X_{\text{SR}}(E_\gamma, r_s) = \frac{1}{A} \frac{1}{hE_\gamma} \int P_\nu(E_e, \nu, r_s) N_e(E_e) dE_e, \quad (3.25)$$

where

$$P_\nu(E_e, \nu, r_s) = \sqrt{3} \frac{B(r_s)}{E_0} \int_0^{\pi/2} \tilde{\kappa} \left(\frac{\nu}{\nu_{\text{cr}}(E_e, \vartheta, r_s)} \right) \sin^2 \vartheta d\vartheta$$

and the critical frequency is

$$\nu_{\text{cr}}(E_e, \vartheta, r_s) = \frac{3ec}{4\pi E_0^3} E_e^2 B(r_s) \sin \vartheta. \quad (3.26)$$

with n_r being the number of radiation bins and

$$\chi = \frac{A}{V_{\text{shell}}}(X_{\text{SR}} + X_{\text{IC}}), \quad (3.30)$$

and the volume element $V(r_i, l, b)$, where V_{shell} is the volume of spherical shell. [Kopp et al. \(2013\)](#) calculated the total flux in a given annulus of the projected image to convert Φ into a surface flux. The referred annulus is represented by the grey ring with radius ρ_a in [Figure 3.2](#). A grid in l and b is used whereby they denoted the grid points by (r_i, l_j, b_k) to have the following:

$$\Phi_{\text{tot}}(E_\gamma, \rho_a) = \sum_{jk} \Phi_{jk}(E_\gamma) \delta_{jk}^{(a)}, \quad (3.31)$$

where $\delta_{jk}^{(a)} = 1$ for grid points belonging to annulus ρ_a , otherwise it is zero (i.e., used to select the relevant emission). The surface brightness ($B_\star$) is given by

$$B_\star(E_\gamma, \rho_a) = \frac{\Phi_{\text{tot}}(E_\gamma)}{\Omega_{\text{tot}}^{(a)}}, \quad (3.32)$$

where $\Omega_{\text{tot}}^{(a)}$ is the total solid angle subtended by the annulus of radius ρ_a . Lastly, they obtained the surface brightness profile by multiplying $B_\star$ with E_γ and integrating over the photon energy, yielding

$$S(\rho_a) = \int_{E_{\gamma, \text{min}}}^{E_{\gamma, \text{max}}} E_\gamma B_\star(E_\gamma, \rho_a) dE_\gamma. \quad (3.33)$$

3.3 The Pulsar Emission Model

We also used the pulsar emission model by [Harding and Kalapotharakos \(2015\)](#) (indicating predictions of pulsed emission by red lines in what follows). The model predicts the broad-band X-ray to VHE γ -ray pulsed spectra of pulsars. Primaries are injected at the stellar surface; they produce CR γ -rays that are converted into pairs via magnetic pair creation. The electron-positron pairs lose their momentum perpendicular to the magnetic field to SR near the PCs and then coast into the outer magnetosphere where they may encounter radio photons that may increase their pitch angles, leading to further emission of SR. The model also includes CR radiation of primary particles accelerated by electron fields in a slot gap (SG or separatrix (extending into the current sheet)) geometry. The model assumes a force-free magnetosphere configuration as an improvement over the retarded vacuum dipole.

3.3.1 3D Magnetosphere Geometry

The magnetic field's structure is a critical component to emission models. This is because the radiation of relativistic particles is beamed along their direction of momentum, and particle trajectories in pulsar magnetosphere follow closely the magnetic field lines (in the corotating frame). A static dipole is not a good approximation to a pulsar magnetic field, even near the neutron star, since the rotation causes a sweepback of the field lines near and beyond the light cylinder. Many radiation models (Romani and Yadigaroglu, 1995, Cheng et al., 2000, Harding and Others, 2008) adopt the vacuum retarded dipole solution (Deutsch, 1955), which includes the sweepback due to retardation of the field and produces a distortion and shift of the open field volume toward its trailing edge (Dyks and Harding, 2004). In force-free (plasma-filled) magnetospheres, there is a larger degree of magnetic field sweepback and also straightening of poloidal field lines due to currents, as shown by the numerical solutions of force-free magnetospheres (Spitkovsky, 2006, Timokhin, 2006). Dissipative pulsar magnetosphere simulations (Kalapotharakos et al., 2012, Li et al., 2012) show that the magnetic field's structure stays close to force-free limit for the high plasma conductivity, which is needed to match the γ -ray light curves and spectra of young pulsars (Kalapotharakos et al., 2014, Brambilla et al., 2015).

The model therefore adopts the force-free magnetosphere structure to model the trajectories of particles and radiation. All calculations are done in the non-rotating inertial observer's frame (IOF). The pairs are assumed to be produced at the PCs and to screen the accelerating E -field (parallel to the local B -field) $E_{\parallel}$ at higher altitudes (and smaller co-latitudes), except in the SG. The acceleration of primary electrons are assumed to be in the SG only, with a constant $E_{\parallel}$ that is taken as a free parameter. The pairs and primary electrons are injected at the surface of the neutron star at footpoints of open field lines that are labelled by open volume coordinates (Dyks et al., 2004). The coordinates map the open field region bounded by field lines that close within the light cylinder, $R_{LC} = c/\Omega$, with Ω the angular rotation frequency of the pulsar (i.e., at the boundary or separatrix of the closed field-line region).

The model performs fourth-order Runge-Kutta integrations along the field lines of the numerical solution to determine the open field footpoints and computes the rim of the PC on the surface of the neutron star. The open volume coordinates $(r_{\text{ovc}}, l_{\text{ovc}})$ are defined to identify footpoints at the surface of the neutron star of field lines along which the particles are traced. The radial coordinate r_{ovc} is 0 at the magnetic pole and 1 at the rim of the PC. The arc-length along each of the distorted rings with fixed r_{ovc} is measured using the azimuthal coordinate l_{ovc} .

A set of rings between $r_{\text{ovc}}^{\text{min}}$ and $r_{\text{ovc}}^{\text{max}}$ with equal spacing d_{ovc} have been used to model an injection of particles that is uniform over the PC. The footpoints are then spaced uniformly in each of the rings with $N_l = \Delta_{\text{azm}} \times 360$ equal divisions. The contribution

from each different ring is weighted by $l_{\text{ring}}/l_{\text{rim}}$, where l_{ring} is the length of a certain ring and l_{rim} is the length of the total PC rim. The weighting of ring contribution was done because the rings have varying circumferences and each ring has the same number of footpoints. The total particle velocity (sum of a drift component) and a component parallel to the magnetic field (Kalapotharakos et al., 2014) are required to determine the particle trajectories in the IOF, i.e.,

$$v = \left(\frac{\mathbf{E} \times \mathbf{B}}{B^2 + E_0^2} + f \frac{\mathbf{B}}{B} \right) c, \quad (3.34)$$

where

$$E_0^2 = B_0^2 - \mathbf{B}^2 + E^2 \quad (3.35)$$

$$B_0^2 = (\mathbf{B}^2 - \mathbf{E}^2 + \sqrt{(\mathbf{B}^2 - \mathbf{E}^2)^2 + 4(\mathbf{B} \cdot \mathbf{E})^2})/2 \quad (3.36)$$

and

$$B_0 = \text{sign}(\mathbf{B} \cdot \mathbf{E}) \sqrt{B_0^2} \quad (3.37)$$

(Gruzinov, 2008, Li et al., 2012). The scalar quantity f at each point along the trajectory can be obtained by assuming $v \simeq c$ and that the motion is outward. For a force-free magnetosphere case, $\mathbf{v}$ remains parallel to B in the corotating frame.

3.3.2 Pair Cascade Spectra

The pulsar emission model incorporates results from a sub-model that calculates a pair spectra. The pulsar model thus assumes that the SR is radiated by the pairs in the outer magnetosphere coming from the cascades above the PC, which are initiated in the strong B -fields near the surface of the neutron star. The rotation of the magnetic field induces an electric field which in turn accelerates primary particles and they emit CR as they are constrained to move along curved field lines. Magnetic pair attenuation then absorbs the highest-energy curvature photons (Erber, 1966, Daugherty and Harding, 1983) to produce a spectrum of first-generation electron-positron pairs. The pairs radiate synchrotron photons that spawn generations of pairs over a longer distance and time. The total cascade multiplicity M_+ , which is the average number of pairs produced by each primary particle, depends on the period P of the pulsar and surface of the magnetic field strength B_s . High M_+ cascades are produced by young pulsars while the older pulsars have much lower M_+ . However, the magnetic PCs can be offset from the dipole axis due to the sweepback of magnetic field lines near the light cylinder and the possible field distortions near the neutron star (Dyks and Harding, 2004). Increasing the offset of the PC will enhance pair multiplicity when a perturbed dipole field structure is introduced (Harding and Muslimov, 2011b). The PC cascade sub-model includes this magnetic field structure, which incorpo-

rates configurations for the dipole offset for the symmetric or asymmetric magnetic field. A symmetric offset of the PC hotspot is required for modelling the thermal X-ray light curves of several MSPs (Bogdanov, 2013). In order to simulate the spectra of pair particles (which is an input to the larger pulsar emission model), a symmetric distorted field is adopted in the PC cascade. The adopted magnetic field in spherical polar coordinates (η, θ, ϕ) is (Harding and Muslimov, 2011b)

$$\mathbf{B} \approx \frac{B_s}{\eta^3} \left[\hat{\mathbf{r}} \cos \theta + \frac{1}{2} \hat{\boldsymbol{\theta}} (1 + a) \sin \theta - \hat{\boldsymbol{\phi}} \epsilon \sin \theta \cos \theta \sin(\phi - \phi_0) \right], \quad (3.38)$$

with $\eta = r/R$ being the dimensionless radial coordinate in units of the stellar radius R , the surface field strength at the magnetic pole B_s , $a = \epsilon \cos(\phi - \phi_0)$ being a parameter defining the azimuthal distortion of the polar field lines from a dipole, and ϕ_0 the azimuthal magnetic angle defining the plane of the PC offset. The parallel component of the electric field, $E_{\parallel}$ is derived using this field structure and used to accelerate electrons from different starting points over the surface of the PC. This component is also used to simulate the pair cascades over the whole PC for a range of P , $\dot{P}$, and offset parameter ϵ (Harding and Muslimov, 2011b). The PC cascade sub-model assumes a non-zero offset for the MSPs.

3.3.3 Emission Simulation

In our work, we focus on pulsed primary CR and pair SR spectral predictions from the pulsar emission model. The emission of radiation over the entire spectrum, i.e., from optical to VHE γ -ray wavelength, has been modelled for a generic MSP. The simulation of emission from both pairs and primary particles have been modelled as they move along their trajectories with step size Δl , starting from the surface of the neutron star to a maximum radius r_m . It is assumed that primary particles undergo continuous acceleration by a constant electric field in a narrow gap along the boundary or separatrix of the open field region, between $r_{\text{ovc}} = 0.95 - 0.99$. The pair spectrum is solved by an independent code modelling the cascading process; particles then coast outward along field lines, but with no further acceleration, as they are moving in a plasma-filled region interior to the gap, between $r_{\text{ovc}} = 0.91 - 0.95$ where the E -field is screened. The primary electrons are injected at the surface of the neutron star with Lorentz factors of $\gamma = 2$, whereas the pairs are injected with the pair cascade spectra. The pairs lose their perpendicular momentum to SR close to the surface of the neutron star in the pair cascades, and thus it is assumed that all particles initially have momentum parallel to the magnetic fields. Single primary electrons are injected at the start of each primary trajectory and their radiation is normalised to the primary particle flux

$$\dot{n}_p = n_{\text{GJ}} c \pi R^2 \theta_{\text{PC}}^2 \left[(r_{\text{ovc}}^{\text{max}})^2 - (r_{\text{ovc}}^{\text{min}})^2 \right] \quad (3.39)$$

with $n_{\text{GJ}} = B_0 \Omega / 2\pi e c$ the Goldreich-Julian density and the half-angle $\theta_{\text{PC}} = (\Omega R / c)^{1/2}$.

The radiation emitted via CR and SR are simulated for all particles and are described below.

3.3.4 Pulsed CR Calculation

The CR energy spectrum from a single electron with Lorentz factor γ is defined as

$$N_{\text{CR}}(\varepsilon) = \sqrt{3} \frac{e^2}{c} \gamma \kappa\left(\frac{\varepsilon}{\varepsilon_{\text{cr}}}\right) \quad (3.40)$$

where ε is the emitted photon energy in units of $m_e c^2$, and the spectral cut-off is defined as

$$\varepsilon_{\text{cr}} = \frac{3}{2} \frac{\lambda_c}{\rho_c} \gamma^3. \quad (3.41)$$

The function $\kappa(x)$ is defined as in [Equation 2.18](#) in Chapter 2

$$\kappa(x) \equiv x \int_x^\infty K_{5/3}(x') dx'. \quad (3.42)$$

The radius of curvature of the particle trajectory ρ_c is determined by calculating the inverse of the trajectory's curvature using the second order derivative at the particle position:

$$\rho_c = \left| \frac{d^2 \mathbf{r}}{dl^2} \right|^{-1}. \quad (3.43)$$

The approximated form of the photon spectrum is defined as a power law with an exponential cutoff at ε_{cr} ,

$$\frac{\dot{N}_{\text{CR}}(\varepsilon)}{d\varepsilon} = \frac{\alpha}{(\lambda m_e c)^{1/3}} \left(\frac{c}{\rho_c} \right)^{2/3} \varepsilon^{-2/3} \exp(-\varepsilon/\varepsilon_{\text{cr}}). \quad (3.44)$$

For more details, see [Section 2.9.1](#).

3.3.5 Pulsed SR Calculation

Particles will not radiate any SR until they gain finite pitch angles. This is because particles start their trajectories with momentum parallel to the magnetic field. [Harding and Kalapotharakos \(2015\)](#) assume that pairs acquire pitch angles by resonant absorption of radio photons (see [Shklovsky, 1970](#), [Lyubarskii and Petrova, 1998](#)). The relativistic particles absorb photons that have energies at the cyclotron resonant frequency in their rest frame, which result in an increase in pitch angle. If the particle's momentum perpendicular to the magnetic field is non-relativistic in the frame in which the parallel momentum vanishes, the particle will then emit spontaneous cyclotron radiation. If the perpendicular

momentum is relativistic, it will emit SR. The resonant absorption condition is defined as

$$B' = \gamma \varepsilon_0 (1 - \beta \mu_0) \quad (3.45)$$

with γ the Lorentz factor of the particle, ε_0 the lab frame energy of the radio photon in units of $m_e c^2$, $\beta = (1 - 1/\gamma^2)^{1/2}$, $B' = B/B_{\text{cr}}$ the local magnetic field in units of the critical field strength $B_{\text{cr}} = 4.4 \times 10^{13}$ G, $\mu_0 = \cos \theta_0$, and θ_0 the angle between the direction of the photon and the momentum of the particle in the lab frame. In a pulsar magnetosphere, particles with large Lorentz factors will see radio photons at the cyclotron resonance at high altitude above the surface of the neutron star, when the local magnetic field satisfies the resonant condition from Equation 3.45, at critical energies of

$$\gamma_{\text{R}} = 2.8 \times 10^5 \frac{B_8}{\varepsilon_{0,\text{GHz}}(1 - \beta \mu_0)}, \quad (3.46)$$

where μ_0 is the incident absorption angle. The particle undergoes absorption in a low Landau state. The particle Landau state (and pitch angle) will then increase continuously until the increase in pitch angle through resonant absorption and the decrease in pitch angle by synchrotron emission reach an equilibrium. However, the balance is not reached until the particle occupies a high Landau state, where it will radiate synchrotron emission.

Harding and Kalapotharakos (2015) simulate the resonant cyclotron absorption and subsequent synchrotron emission from both pairs and primary electrons. The resonant absorption rate is

$$\left(\frac{dp_{\perp}}{dt} \right)^{\text{abs}} = D \frac{\gamma}{p_{\perp}} + \frac{p_{\perp} \gamma}{\gamma - 1} \left(\frac{d\gamma}{dt} \right)^{\text{abs}}, \gamma < \gamma_{\text{R}} \quad (3.47)$$

where

$$D = 5.7 \times 10^9 \text{ s}^{-1} \gamma_{\text{R}}^{-v} \left(\frac{d_{\text{kpc}}}{\eta} \right)^2 \phi_0(\text{mJy})(1 - \beta \mu). \quad (3.48)$$

The $d\gamma/dt$ from acceleration, curvature, and synchrotron losses are much larger, and thus they neglected $(d\gamma/dt)^{\text{abs}}$ from Equation 3.47.

An instantaneous synchrotron spectrum is radiated at each step along the particle's trajectory (Tademaru, 1973),

$$\dot{N}_{\text{SR}} = \frac{2^{2/3}}{\Gamma\left(\frac{1}{3}\right)} \alpha B' \sin \psi \varepsilon^{-2/3} \varepsilon_{\text{SR}}^{-1/3} \exp(-\varepsilon/\varepsilon_{\text{SR}}), \quad (3.49)$$

where $\sin \psi = p_{\perp}/p$, $p^2 = \gamma^2 - 1$, and $\varepsilon_{\text{SR}} = (3/2)\gamma^2 B' \sin \psi$ is the synchrotron critical energy.

3.4 Summary

In this Chapter, I described the models that I will apply in Chapter 4, 5, and 6 to the Galactic GCs to produce low-energy synchrotron radiation (LESR), high-energy synchrotron radiation (HESR), CR and IC spectra for some clusters. The GC model includes a transport equation, and predicts the SED emitted by GCs for a very broad energy range by considering unpulsed SR as well as IC emission. The pulsar emission model predicts the broad-band pulsed X-ray to VHE γ -ray spectra of pulsars. In the next Chapter, I will use the GC model to study the effect of changing the parameters of the GC model, constrain cluster parameters using γ -ray and X-ray data of three GCs, and study the detectability of GCs for H.E.S.S. and CTA, ranking them according to their fluxes.

Chapter 4

Parameter Study of Globular Cluster Model and Ranking According to Their TeV Fluxes

In the previous Chapter, I have discussed two models that I will apply to Galactic GCs. This Chapter is based on our previous work initiated during my master's project ([Ndiyavala, 2016](#)), and culminating in a publication: [Ndiyavala et al., 2018](#) where I performed a parameter study to investigate the GC model's behaviour and discussed the parameters that I have constrained using multi-wavelength data for chosen Galactic GCs. I will also list the five most promising Galactic GCs for both H.E.S.S. and CTA in terms of their predicted VHE flux.

4.1 Timescales and Reference Model

As a first step, I consider the dominant physical effects that are at play in our model. I will study these transport effects as a function of radius r_s (with the origin being at the cluster centre) and electron energy E_e using several different timescales. The diffusion timescale is given by:

$$t_{\text{diff}} = \frac{r_s^2}{2\kappa}, \quad \kappa(E_e) = \kappa_B \frac{E_e}{B}, \quad \kappa_B = \frac{c}{3e}, \quad (4.1)$$

where c is the speed of light and e is the elementary charge. The SR cooling timescale is given by

$$t_{\text{SR}} = \frac{E_e}{\dot{E}_{\text{SR}}}, \quad (4.2)$$

where

$$\dot{E}_{\text{SR}} = -g_{\text{SR}} E_e^2 U_{\text{mag}} = \frac{-g_{\text{SR}}}{8\pi} E_e^2 B^2, \quad (4.3)$$

with

$$g_{\text{SR}} = \frac{4}{3} \frac{\sigma_T c}{m_e^2 c^2} = \frac{32\pi}{9} \left(\frac{e}{m_e c^2} \right)^4 c. \quad (4.4)$$

The IC cooling timescale is given by $t_{\text{IC}} = E_e / \dot{E}_{\text{IC}}$, where

$$\dot{E}_{\text{IC}}(E_e, r_s, T) = - \sum_{j=1}^{k-1} \int \chi(E_\gamma, E_e, u_{\text{rad},j}, T_j) \frac{1}{h} dE_\gamma. \quad (4.5)$$

and the function χ is

$$\chi(E_\gamma, E_e, u_{\text{rad},j}, T_j) = \int n_{\varepsilon,j}(r_s, \varepsilon, T_j) h \frac{E_\gamma}{E_0} \zeta(E_e, E_\gamma, \varepsilon) d\varepsilon \quad (4.6)$$

with $E_0 = m_{e,0} c^2$ being the electron rest energy (where $m_{e,0}$ is the electron rest mass) and the collision rate ζ as given in Equation 3.18.

Finally, the total radiation timescale is then given by (e.g., [Venter and De Jager, 2008b](#))

$$t_{\text{rad}} = \frac{E_e}{\dot{E}_{\text{SR}} + \dot{E}_{\text{IC}}}. \quad (4.7)$$

The effective timescale¹ ([Zhang et al., 2008](#)) is given by:

$$t_{\text{eff}}^{-1} \approx t_{\text{esc}}^{-1} + t_{\text{rad}}^{-1}, \quad (4.8)$$

where $t_{\text{esc}} = t_{\text{diff}}$.

[Table 4.1](#) shows a list of parameters used to calculate a reference model for the timescales, particle spectrum, and SED graphs.

Table 4.1: In this Table I list the reference model parameter values that resemble those of Terzan 5.

Parameters	Parameter values
Diffusion coefficient (κ)	Bohm diffusion
Magnetic field (B)	$5 \mu\text{G}$
Injection spectral index (Γ)	2.0
Number of stars (N_{tot})	4.6×10^5
Injection spectral normalisation (Q_0)	$6.33 \times 10^{33} \text{erg}^{-1} \text{s}^{-1}$
Distance (d)	5.9 kpc
Core radius (R_c)	0.21 pc
Half-mass radius (R_{hm})	0.94 pc
Tidal radius (R_t)	17.4 pc

¹This can be defined using the diffusion-loss transport equation for a stationary case, and linearising the terms. It basically interpolates the minima of all time scales versus energy to yield the time scale of the dominant process for each particle energy.

In what follows, for the reference model, I change one parameter at a time as indicated in the subsequent sections while fixing all other parameters. I use the fraction of spin-down power that is converted to particle power $\eta = 10\%$ for all graphs.

I plot the timescales of the reference model for diffusion (dash-dotted lines), radiation losses (dashed lines), and the effective timescale (solid lines) as a function of E_e for different values of r_s in Figure 4.1. The larger values of r_s are indicated by thicker lines. SR dominates IC at the highest energies since the IC cross section drops as one goes from the Thomson regime at low energies to the Klein-Nishina regime at high energies. I note that both t_{diff} and $t_{\text{SR}} \equiv E_e/\dot{E}_{\text{SR}}$ scale as E_e^{-1} as seen in Figure 4.1. Diffusion dominates close to the core and this implies that particles will escape before radiating. However, SR dominate over diffusion at larger radii (since the diffusion timescale scales as r_s^2). At intermediate radii, one can see the change in regime: for $r_s = 0.1R_t$, with R_t the tidal radius, the SR timescale is only slightly lower than the diffusion timescale at the highest particle energies, while SR dominates diffusion at $r_s = 1.2R_t$.

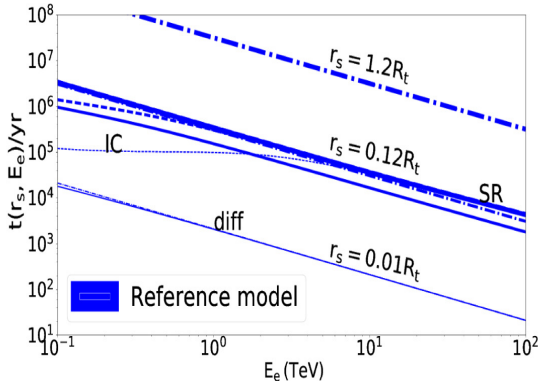


Figure 4.1: Timescale graph (diffusion indicated by dash-dotted lines, radiation losses by dashed lines and the effective scale by solid lines) for the reference model as a function of r_s and E_e . Thicker lines indicate larger radii. The labels ‘IC’, ‘SR’ and ‘diff’ indicate where IC, SR and diffusion dominates, respectively.

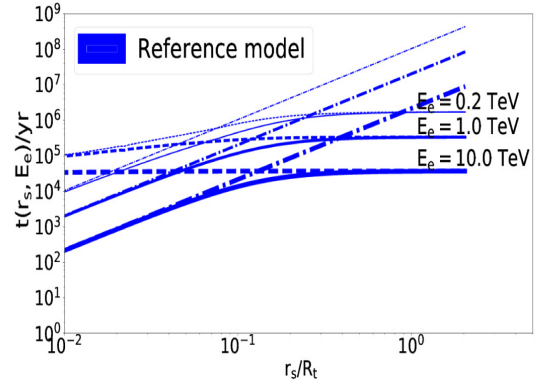


Figure 4.2: Timescale graph (diffusion indicated by dash-dotted lines, radiation losses by dashed lines and the effective scale by solid lines) for the reference model as a function of E_e and r_s . Thicker lines indicate larger energies.

I plot the timescales as a function of radius for diffusion (dash-dotted lines), radiation losses (dashed lines), and the effective scale (solid lines) for different energies (see Figure 4.2). For diffusion, the graph of t_{diff} versus r_s has a slope of 2. Also, t_{diff} is higher for lower energies, since such particles diffuse slower (this is evident at smaller r_s , where diffusion dominates). SR dominates at higher particle energies (i.e., t_{SR} is lower). The graph of t_{SR} versus r_s will be flat for constant E_e if B -field is not a function of r_s . However, t_{SR} is larger for lower particle energies since $t_{\text{SR}} \propto E_e^{-1}$. From the graph, it is clear that the effective timescale ‘takes the minimum’ between t_{diff} and t_{rad} , always being the lowest timescale. One can again see that diffusion dominates radiation processes at low radii. The inverse is true at larger radii.

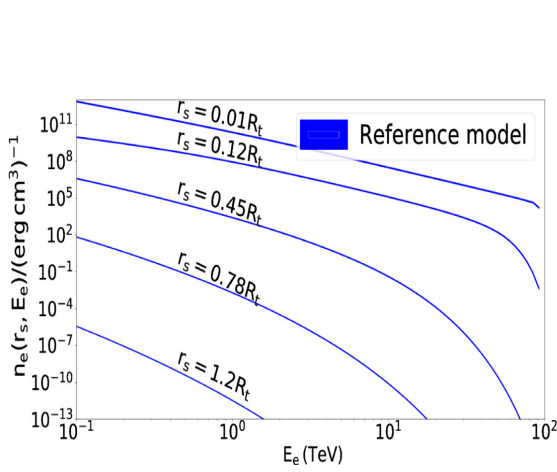


Figure 4.3: The steady-state particle spectrum as a function of energy E_e at different r_s .

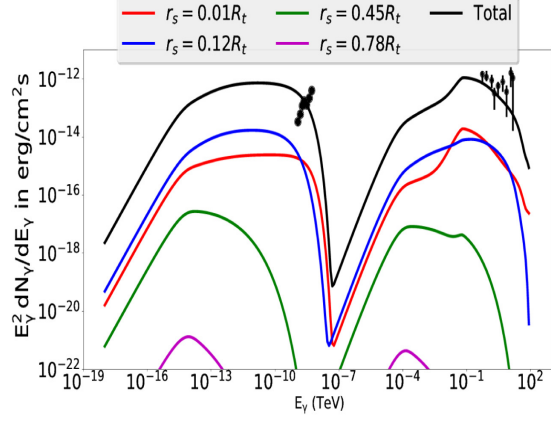


Figure 4.4: The SED for the reference model (black line) for Terzan 5, as well as *Chandra* (Eger et al., 2010) and H.E.S.S. (Abramowski et al., 2011) data. The relative contributions for a number of representative radii (zones) are also shown using different colours.

In Figure 4.3 I plot the steady-state particle spectrum as a function of energy E_e at different r_s for the reference model. At a fixed radius, n_e is higher at low energies and becomes low at higher energies due to assumed injection spectrum (a power law injected at $r_s = 0$). At small radius, the particle spectrum follows the injection spectrum ($\Gamma = 2.0$), but it becomes softer with distance because high-energy particles diffuse faster than low-energy particles. One can see that there is a cut-off introduced at higher energies due to SR. At $r_s = 0.12R_t$, the slope of the mid-part of the spectrum is still close to $\Gamma = 2$; however, at low energies, the density drops, given that low-energy particles diffuse much slower, and that IC operates there. An energy cutoff also starts to develop at high energies. This is due to SR losses. However, the cutoff developing at $r_s = 0.01$ is due to diffusive escape of particles, not SR, given the low timescale of the first compared to the second there. The cut-off becomes increasingly lower at larger radii since high-energy particles continue to lose energy due to SR, i.e., the SR losses give rise to the cut-off at high energy.

Figure 4.4 shows the SED predicted by the reference model (black curve). Two sub-components associated with the two soft-photon target densities are clearly observed in the IC component. I also plot *Chandra* (Eger et al., 2010) and H.E.S.S. (Abramowski et al., 2011) data for Terzan 5. I plot the relative contributions from a number of zones at representative radii r_s indicated by different colours. Initially the contribution grows with radius (due to the increasing volume of the zones) but farther out drops significantly due to a decline in soft-photon and particle number densities. The high-energy part of the IC component dominates the SR component at very small distances where the stellar soft-photon background is larger, while the SR one starts to slightly dominate the IC component at larger distances. The data do not fit in X-ray band; this will be the subject of Chapter 5, where we will invoke cumulative pair SR to explain this.

4.1.1 Changing the Number of Stars (N_{tot})

The number density of soft photons n_ε scales linearly with the total number of cluster stars N_{tot} (see Equation 9 in [Ndiyavala et al., 2018](#)). Equation 4.5 shows that the IC loss rate scales linearly with N_{tot} . I plot the timescales for diffusion (dash-dotted lines), radiation losses (dashed lines), and the effective scale (solid lines) in [Figure 4.5](#). The colours represent two different values of N_{tot} , as shown in the legend. The IC loss rate is lower for smaller number of stars since the photon number density is lower. It therefore takes a longer time for particles to lose energy in this case. This effect can be seen clearly at lower particle energies and smaller radii where IC dominates over SR due to the high value of n_ε there. Thicker lines represent larger radii. The photon number density rapidly declines (leading to smaller $\dot{E}_{\text{IC}}$ and longer t_{IC}); thus SR dominates over IC at larger radii.

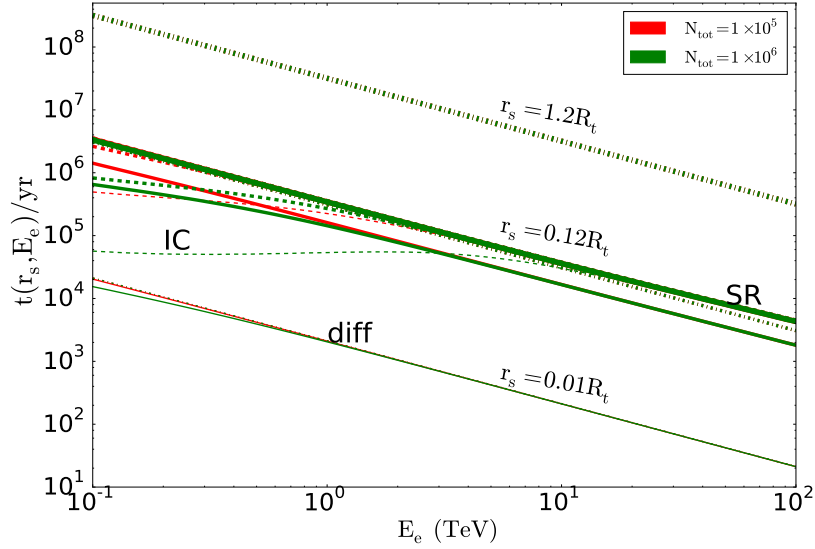


Figure 4.5: Timescales as a function of energy for diffusion (dash-dotted lines), radiation losses (dashed lines), and the effective scale (solid lines). Thicker lines represent larger radii. Dominating processes at different radii and particle energies are indicated in the graph. The colours represent different values of N_{tot} as noted in the legend.

I plot n_e versus E_e in [Figure 4.6](#). It is seen that the steady-state particle spectrum n_e is higher for a smaller value of N_{tot} (at low energies) at a fixed radius. This is because $\dot{E}_{\text{IC}}$ is lower in this case and more particles survive. This effect is larger for the zone at $r_s = 0.12R_t$ compared to that around $r_s = 0.01R_t$, since the first is associated with a larger volume. This effect vanishes at larger energies since SR dominates IC and SR is not a function of N_{tot} . The effect of changing N_{tot} on the value of n_e is smaller at larger radii and this is because n_ε and $\dot{E}_{\text{IC}}$ decrease rapidly with distance. Also, the overall level of n_e decreases with radius since it represents a particle density, and the volume scales as r_s^3 . The same effect is seen in [Figure 4.7](#), for n_e versus r_s : n_e decreases with radius due to an increase in volume; n_e decreases with energy due to the injection of fewer particles at

higher energy; the effect of changing N_{tot} is only apparent at low particle energies, where IC dominates SR. The exact particle density depends, however, on both the diffusive and IC loss timescales at low energy. As t_{IC} decreases for larger N_{tot} , the effective timescale changes in a non-linear way.

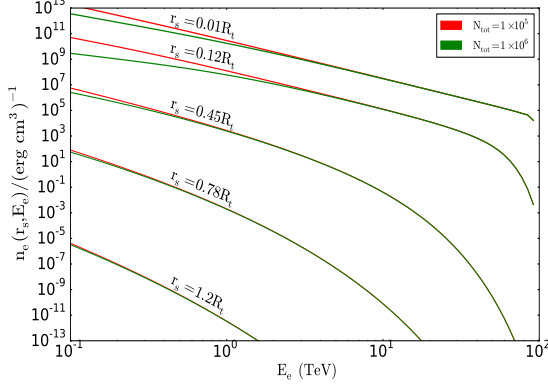


Figure 4.6: The steady-state particle spectrum as a function of energy E_e at different r_s assuming Bohm diffusion, $B = 5.0 \mu\text{G}$, $\Gamma = 2.0$, $Q_0 = 6.33 \times 10^{33} \text{ erg s}^{-1}$, $d = 5.9 \text{ kpc}$ and $N_{\text{tot}} = 10^5$ or 10^6 .

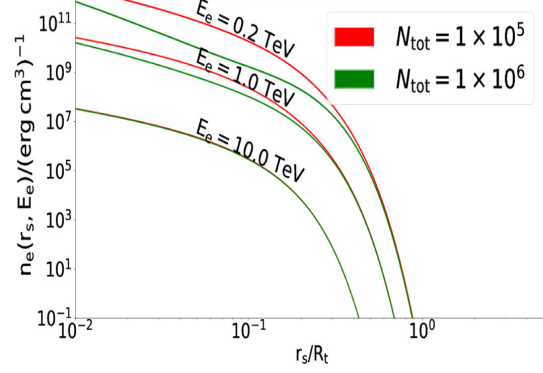


Figure 4.7: The steady-state particle spectrum as a function of radius r_s at different particle energies E_e assuming Bohm diffusion, $B = 5.0 \mu\text{G}$, $\Gamma = 2.0$, $Q_0 = 6.33 \times 10^{33} \text{ erg s}^{-1}$, $d = 5.9 \text{ kpc}$ and $N_{\text{tot}} = 10^5$ or 10^6 .

Figure 4.8 shows the SED components in the case of changing N_{tot} . It is understood that when one increases N_{tot} , there will be more optical photons, which boosts the IC emission and loss rate. Therefore there will be less energy available for SR and IC on the CMB, hence suppressing the low-energy IC and SR spectral fluxes. It is interesting how IC spectrum changes because there are a superposition of two effects that are operating here: 1) at low energies, particles lose energy faster for more stars due to increased IC losses, and hence there are fewer particles and the flux is lower, even though the target photon energy density for IC is higher. 2) At higher energies, SR losses dominate, so only the energy density of photons is relevant, and hence the model predicts a higher IC flux for the case of a larger number of stars.

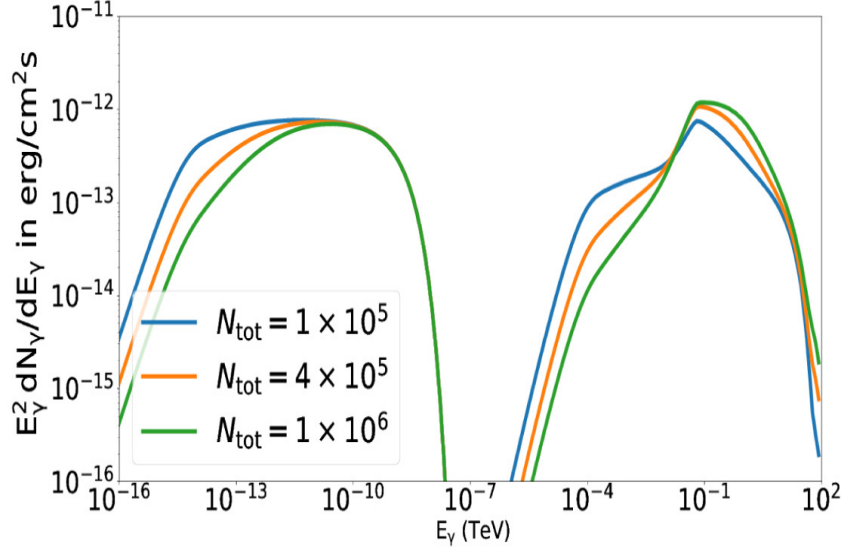


Figure 4.8: SED plot indicating the effect of changing the number of stars in the cluster on SR and IC components, assuming Bohm diffusion, $\Gamma = 2.0$ and $B = 5 \mu\text{G}$.

4.1.2 Changing the B -field

The magnetisation state of plasma in GCS is not known and thus one only use a reasonable values of $B \sim 1 - 10 \mu\text{G}$. This estimation of GC magnetisation is from [Bednarek and Sitarek \(2007\)](#) where they have used typical values for the pulsar wind shock radius and magnetisation parameter σ . These B -field values also yield reasonable SR and IC spectra. Two main effects are expected when changing B -field: A smaller diffusion coefficient and increased SR losses due to a larger B -field. In [Figure 4.9](#) I plot different timescales versus particle energy. The colour indicates different magnetic fields in the cluster, and I use the following parameters: Bohm diffusion, $\Gamma = 2.0$, $Q_0 = 6.33 \times 10^{33} \text{ erg}^{-1} \text{ s}^{-1}$, $d = 5.9 \text{ kpc}$, $N_{\text{tot}} = 4.6 \times 10^5$, and $B = 3 \mu\text{G}$, $10 \mu\text{G}$. It can be seen that for a lower B -field, the SR timescale is longer (i.e., $t_{\text{SR}} = E_e / \dot{E}_{\text{SR}} \propto E_e^{-1} B^{-2}$). It therefore takes a longer time for particles to lose energy due to SR. Also, the diffusion timescale is relatively shorter and hence particles will escape faster from a particular zone in this case (i.e., the energy dependence of the escape time is quite steep (i.e., $t_{\text{esc}} \propto E_e^{-1}$). I also note that IC dominates SR at low energies whilst radiation dominates diffusion at large r_s . An interesting regime change occurs between IC and SR domination around $\lesssim 1 \text{ TeV}$ for these two B -field strengths: for $B = 10 \mu\text{G}$, SR dominates over IC for $E_e \gtrsim \text{TeV}$, while this change occurs around $E_e \gtrsim 10 \text{ TeV}$ for $B = 3 \mu\text{G}$.

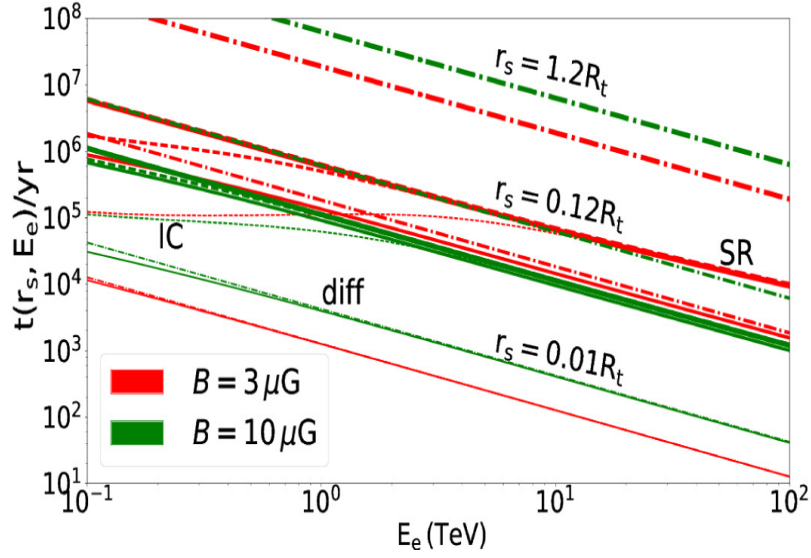


Figure 4.9: Loss timescales as a function of energy. The solid lines indicate effective timescales, the dashed lines represent radiation losses, and diffusion is shown by dash-dotted lines. The colours correspond to the magnetic fields shown in the legend.

In [Figure 4.10](#), at fixed radius, n_e is higher beyond $r_s \gtrsim 0.05 R_t$ for a lower magnetic field since $\dot{E}_{\text{SR}}$ is lower and more particles survive in this case. There is a cut-off at higher energies due to SR, which becomes increasingly lower with an increase in radii. This cutoff is more severe for a larger B -field. For $B = 10 \mu\text{G}$, the particle spectrum is very small at larger radii (i.e., $r_s > 0.05 R_t$, thus these lines are not visible). In [Figure 4.11](#), the effect of changing the magnetic field on the value of n_e is higher for higher B -field because of slower diffusion, since this implies a smaller source size, and therefore the spectra for higher B cut off at a lower radius r_s . However, at large r_s , it is seen that a larger B -field leads to a substantially lower cut-off energy due to increased $\dot{E}_{\text{SR}}$.

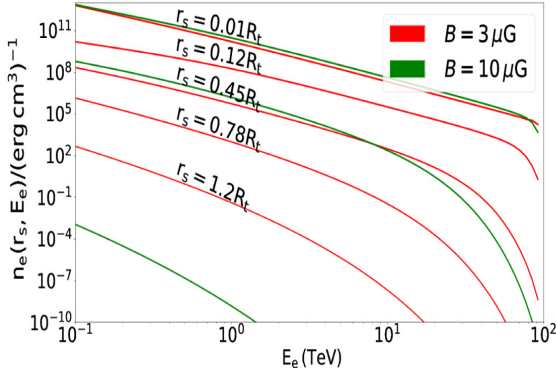


Figure 4.10: Steady-state particle spectrum as a function of energy at different magnetic fields $B = 3 \mu G$, and $10 \mu G$, assuming Bohm diffusion, $\Gamma = 2.0$, $Q_0 = 6.33 \times 10^{33} \text{ erg}^{-1} \text{ s}^{-1}$, $d = 5.9 \text{ kpc}$, and $N_{\text{tot}} = 4.6 \times 10^5$. The colours correspond to the magnetic field in the cluster, as shown in the legend.

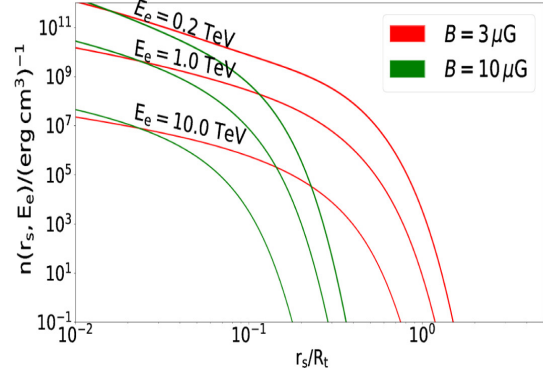


Figure 4.11: Steady-state particle spectrum for different magnetic fields $B = 3 \mu G$, and $10 \mu G$, assuming Bohm diffusion, $\Gamma = 2.0$, $Q_0 = 6.33 \times 10^{33} \text{ erg}^{-1} \text{ s}^{-1}$, $d = 5.9 \text{ kpc}$, and $N_{\text{tot}} = 4.6 \times 10^5$. The colours correspond to the magnetic field in the cluster, as shown in the legend.

In [Figure 4.12](#) I plot the SED plot in the case of different B -fields. The SR loss rate strongly depends on the magnetic field strength. One can see that if the magnetic field is increased from $1 \mu G$ to $5 \mu G$, the SR increases rapidly, because $\dot{E}_{\text{SR}} \propto E_e^2 B^2$ leads to a higher SR flux. Therefore, higher-energy particles lose more energy leaving fewer particles available at higher energies to radiate IC. Thus the IC flux is lower for higher magnetic fields. I study the radial dependence of the cluster radiation in [Figure 4.13](#). The colours indicate different radii, and the higher B -field is shown by dashed lines whilst the lower B -field is shown by solid lines. At small radii, both SR and IC emission increase with an increase in B -field because of slower diffusion in this case, leading to particles spending a longer time there. For different diffusion coefficients, one has different source sizes. If particles move faster then the source becomes bigger. If they move slower, the source becomes smaller, thus a higher B -field leads to more losses at small r_s , and a compact source is predicted. This explains the line swap in the graph, i.e., the non-monotonic behaviour of lines as r_s is increased (cf. Figure 6 from [Kopp et al., 2013](#)). It can be seen that both IC and SR fluxes decrease with an increase in B -field at large radii.

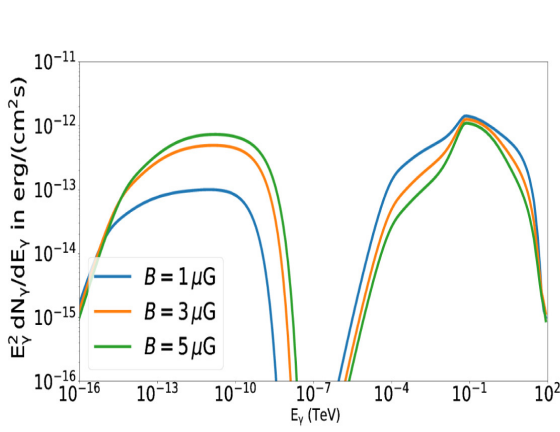


Figure 4.12: SED plot showing the effect of changing the magnetic field in the cluster on SR and IC components, assuming Bohm diffusion, and $\Gamma = 2.0$, $Q_0 = 6.33 \times 10^{33} \text{ erg}^{-1} \text{ s}^{-1}$, $d = 5.9 \text{ kpc}$, and $N_{\text{tot}} = 4.6 \times 10^5$.

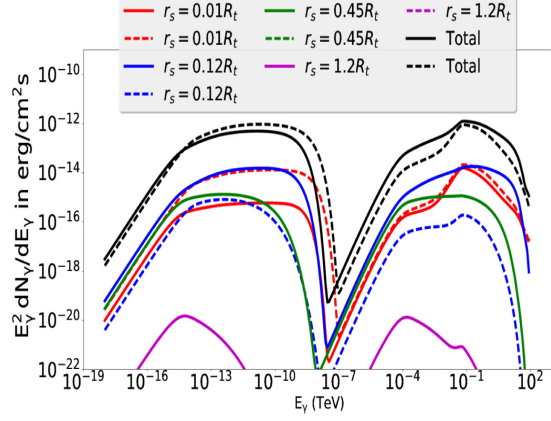


Figure 4.13: Spectra for different magnetic fields in the cluster for the case of Bohm diffusion, $\Gamma = 2.0$, $Q_0 = 6.33 \times 10^{33} \text{ erg}^{-1} \text{ s}^{-1}$, $d = 5.9 \text{ kpc}$, and $N_{\text{tot}} = 4.6 \times 10^5$, and $B = 10 \mu\text{G}$ indicated by dashed lines and $B = 3 \mu\text{G}$ represented by solid lines.

4.1.3 Changing Electron Source Term: Spectral Index Γ

I note that changing Γ does not change the timescales versus E_e or r_s , however, it does change the source term. Figure 4.14 shows the steady-state particle spectrum as a function of energy for different spectral indices Γ as shown by the colours in the legend. I note that, at a fixed radius, the steady-state particle spectrum n_e is higher for a hard spectrum at large radii and energies.

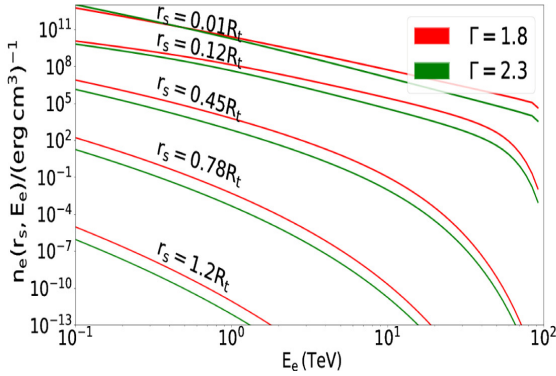


Figure 4.14: Steady-state particle spectrum as a function of energy at different radii for two different Γ .

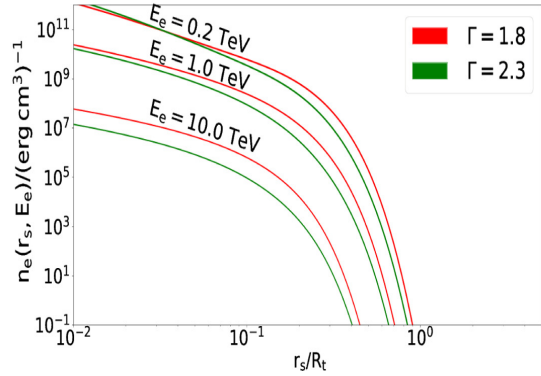


Figure 4.15: Steady-state particle spectrum as a function of radius at different r , for two different Γ .

However, close to the GC core the low-energy tail of the spectrum is slightly higher for a softer spectral index because the L_e is kept constant (same total power in particles). At low energies and smaller radii the effect of changing the Γ is smaller. This is also seen in Figure 4.15. I also chart the SED plot for different spectral indices in Figure 4.16.

Both SR and IC decline for a soft Γ and increase for a harder injection spectrum (more high-energy particles are injected into the GC).

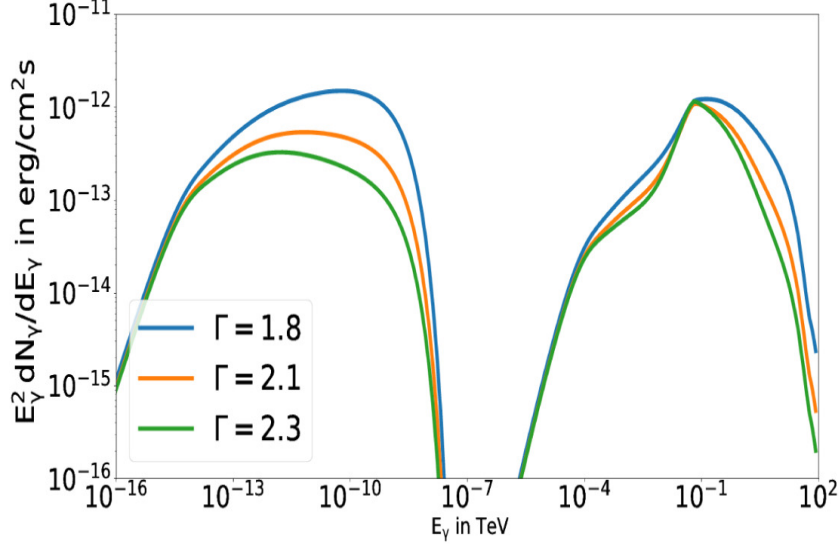


Figure 4.16: An SED plot indicating the effect of changing the spectral index on the SR and IC components.

4.1.4 Changing the Electron Source Term: Normalisation Q_0

Changing the source term increases or reduces the number of particles injected into the cluster which in turn affect the level of radiation received from the cluster. It has no effect on the timescales versus E_e or r_s . Therefore the SED will scale linearly with Q_0 .

4.1.5 Changing the Source Distance d

Changing the distance to the cluster changes the physical size of the cluster, since we keep the angular size constant (e.g., the core radius $r_c = \theta_c d$, with θ_c the ‘angular radius’ of the core). For a smaller distance the IC timescale is lower (i.e., the volume is smaller and thus the energy density becomes larger, leading to an increased IC loss rate). Furthermore, since $t_{\text{diff}} \propto r_s^2 \propto d^2$, when the distance decreases particles escape faster from a particular zone, leading to lower diffusion timescales. However, the SR timescale is not influenced by a change in distance, because we assumed a constant B -field everywhere in the cluster.

In Figure 4.17 one can see that the steady-state particle spectrum (per volume) is higher for a smaller distance due to a decrease in volume $\propto d^3$.

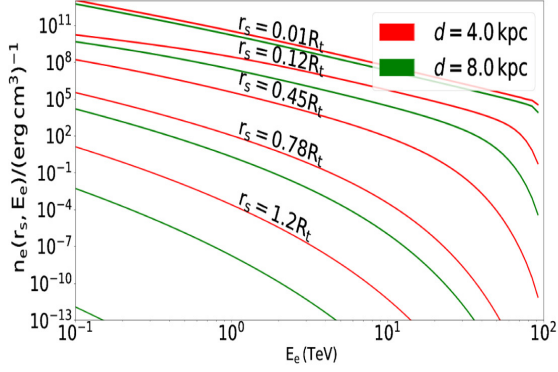


Figure 4.17: Steady-state particle spectrum as a function of energy for different distances (see legend).

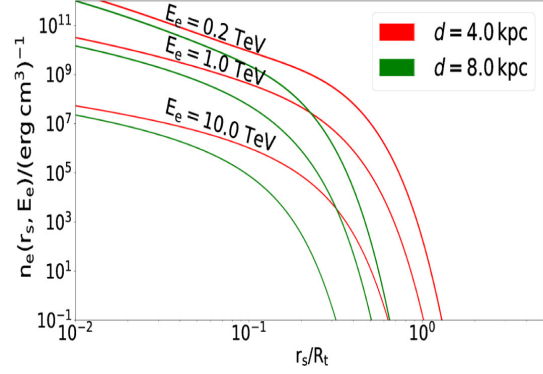


Figure 4.18: Steady-state particle spectrum as a function of radius for different distances (see legend).

The effect of changing the distance to the cluster is lower at small radii (see also [Figure 4.18](#)). I used the same number of zones for these two cases of d , but the actual $R_t \propto d$ changes. Thus the relative radius r_s/R_t changes. Physically, the green lines are associated with a larger cluster than the red lines. Thus particles take longer to escape (having to traverse a longer actual path) and therefore lose more energy via SR losses while diffusing through the cluster. This explains the lower spectral cutoff as a function of r_s/R_t for a larger d .

[Figure 4.19](#) shows that as the distance to the cluster increases from 2 kpc to 8 kpc both the SR and IC fluxes decrease. Particles diffusing through clusters that are more distant (larger physical size source) tend to lose more energy than those that are nearer in both SR and IC. Fluxes furthermore scale as $1/d^2$.

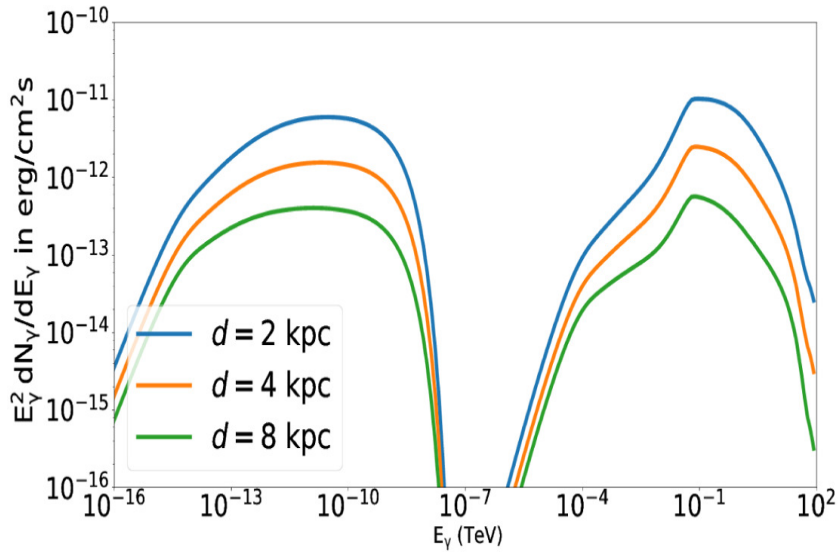


Figure 4.19: The effect of changing the distance to the cluster on the SR and IC components.

4.1.6 Changing the Spatial Diffusion Coefficient $\kappa(E)$

In Figure 4.20 I plot the timescale graph for diffusion (dash-dotted lines), radiation losses (dashed lines), and effective timescale (solid lines). The radiation timescales does not change. The colours represent the following two diffusion coefficients as shown in the legend: Bohm diffusion at $\sim 6 \times 10^{24} \text{ cm}^2/\text{s} \sim 2 \times 10^{-5} \text{ kpc}^2/\text{Myr}$ at 1 TeV that goes like $\kappa \propto E_e^1$, and an illustrative value of $\kappa_0 \sim 2 \times 10^{25} \text{ cm}^2/\text{s} \sim 7 \times 10^{-5} \text{ kpc}^2/\text{Myr}$ at 1 TeV that goes like $\kappa \propto E_e^{0.6}$.

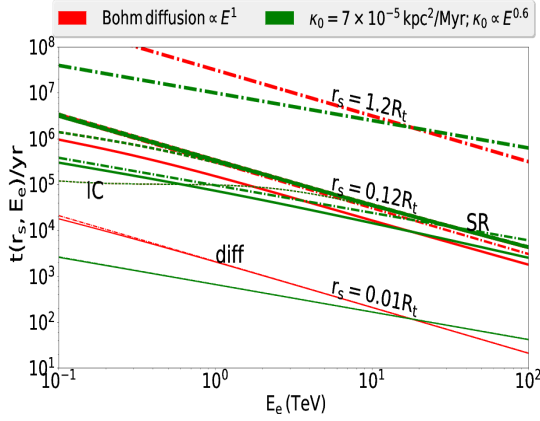


Figure 4.20: Timescales as a function of energy for different diffusion coefficients.

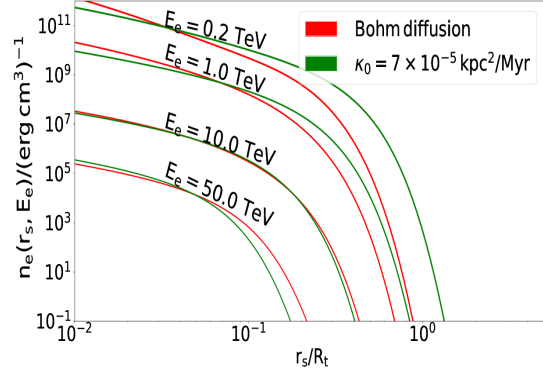


Figure 4.21: Steady-state particle spectrum as a function of radius for different diffusion coefficients.

I note that IC still dominates at low energies and SR dominates at higher energies. High-energy particles diffuse faster than low-energy particles. The Bohm diffusion timescale has a slope of -1 whilst in the second case, τ_{diff} has a slope of -0.6 . The Bohm diffusion timescale is larger at $E_e \lesssim 20 \text{ TeV}$ (low energies) and smaller at $E_e \gtrsim 20 \text{ TeV}$ (higher energies).

In Figure 4.21, n_e is slightly higher for the Bohm case at the lowest radii and low energies, and drops below the value of the second case at larger radii, but exceeds that value again for the highest energies. The particles lose energy, e.g., 20 TeV particles become 10 TeV particles as they diffuse out from the centre of the cluster, which results in the cross-over point where the two values of n_e coincide and moves toward increasingly lower energies (i.e., this is a cooling effect). One can observe the same effect in Figure 4.22. That is, at a fixed radius, n_e is higher for the $\kappa \propto E_e^{0.6}$ value of the diffusion coefficient at low energies at $r_s \gtrsim 0.45 R_t$ (at the lowest radii diffusion dominates and n_e is slightly higher for the Bohm case). In this case, more particles reach larger radii (outer zones) because of faster diffusion. The Bohm diffusion is faster at VHEs, and hence there are slightly fewer particles in the centre of the cluster. The n_e is thus higher at larger radii and energies in this case.

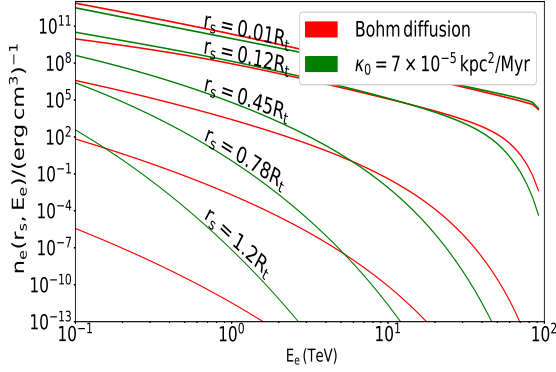


Figure 4.22: Steady-state particle spectrum as a function of radius for different diffusion coefficients.

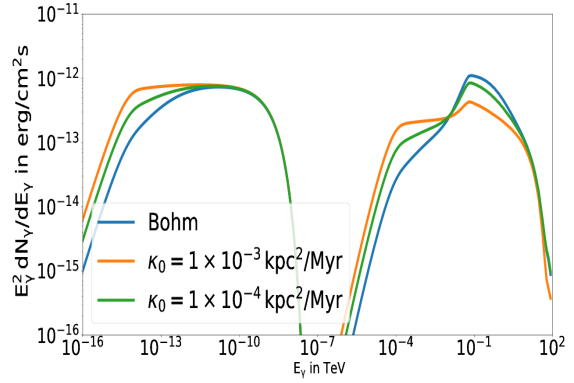


Figure 4.23: SED plot for a cluster with a change in κ_0 .

From Figure 4.23 it can be seen that the HE IC component is higher for Bohm diffusion. This is because this component originates mostly at the GC centre, and there are more particles in the Bohm case. Since particles lose more energy due to IC for slower diffusion, there is less energy available for SR, and thus, the SR flux scales with the spatial diffusion coefficient at low energies (since B is not a function of r_s).

In Figure 4.24 I plot the spectra for changing diffusion coefficients in the cluster for the case of $B = 5 \mu\text{G}$, $\Gamma = 2.0$, Bohm diffusion indicated by dashed lines and $\kappa_0 = 7 \times 10^{-5} \text{kpc}^2/\text{Myr}$ represented by solid lines. Again, coloured lines show different values of cluster radii. Disentangling the effect of using different diffusion coefficients is quite a challenge, since we are dealing with multiple influences: first, the system changes from being diffusion-dominated to being radiation dominated as radius is increased; second, as energy is increased, Bohm diffusion becomes faster than the alternative case given its energy dependence; third, the stellar photon density drops rapidly with radius; fourth, there are two IC components: CMB and stellar; fifth, the electron density shown in Figure 4.22 also plays a role.

At low radii and energies, the SR flux is higher in the Bohm case; however, this changes as radius is increased. The effect of different diffusion coefficients is less pronounced at higher SR energies. At low energies, the IC flux is higher for Bohm diffusion at low radii, but this changes as radius is increased. The high-energy IC flux is higher at low radii, given the higher stellar photon flux there. However, this behaviour is swapped for larger radii, where the stellar photon density is decreased.

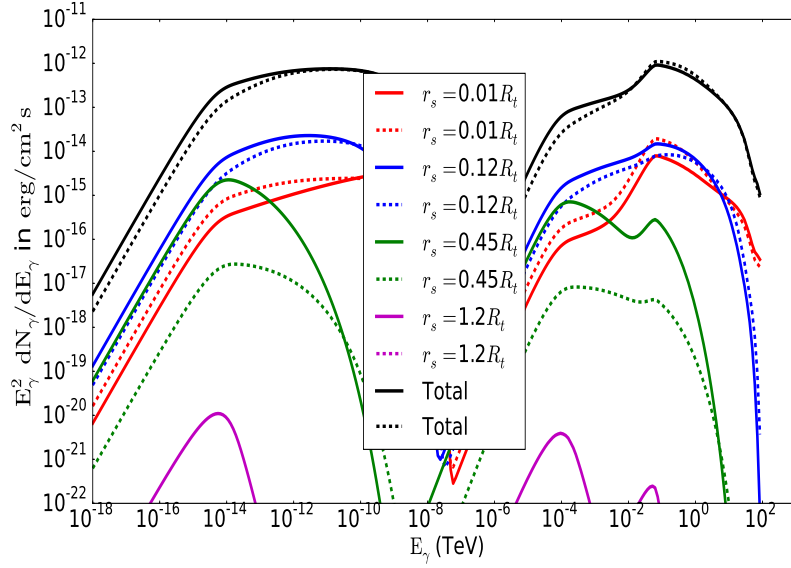


Figure 4.24: Spectra for changing diffusion coefficients in the cluster for the case of $B = 5 \mu\text{G}$, $\Gamma = 2.0$, Bohm diffusion indicated by dashed lines and $\kappa_0 = 7 \times 10^{-5} \text{ kpc}^2/\text{Myr}$ represented by solid lines.

4.2 Spectral Modelling of Selected GCs

Abramowski et al. (2013) analysed H.E.S.S. data from 15 Galactic GCs to search for VHE emission, but no detection from any of the cluster was made, neither could they obtain a significant cumulative signal from their stacking analysis. Therefore, Terzan 5 remains the only Galactic GC to be plausibly detected at VHEs (Abramowski et al., 2011). I list some structural parameters of GCs in Table 4.2. In what follows, I use the VHE upper limits of the 15 GCs and a measured spectrum for Terzan 5. In addition, diffuse X-ray emission has been detected from two of these GCs: Terzan 5 (Eger et al., 2010) and 47 Tucanae (Wu et al., 2014); an upper limit has been obtained for NGC 6388 (Eger and Domainko, 2012). The spectra of the measured X-rays for Terzan 5 seem to be much harder than what is predicted by the GC model. That is, the X-ray data are not well modelled. I therefore postulate that these points are to be explained by a “new” spectral component, as discussed in Chapter 5. I will therefore treat the X-ray data here as upper limits for the SR component predicted by the current model (Section 4.2.2). I finally apply the GC model to 16 GCs, for fixed parameters and obtain a ranking according to the predicted VHE flux for both H.E.S.S. and CTA. I also list the five most promising GCs according to their predicted VHE fluxes.

4.2.1 Structural and other Characteristics of Selected GCs

I list some structural parameters that I used to model the SED of the 16 GCs (Figure 4.28) in Table 4.2, where the columns indicate the core (R_c), tidal (R_t) and half-mass (R_h) radius (Harris, 1996, 2010), the number of stars (N_{tot}) estimated from $N_{\text{tot}} = 10^{0.4(4.79-M_V)}$ hosted by each cluster, where M_V is the integrated absolute magnitude (Lang, 1992). I also list the estimated number of MSPs in each GC (Abdo et al., 2010, Venter and Kopp, 2015), the GC's distances from the Sun (Harris, 2010) and the assumed values of Q_0 based on the assumption of $\langle \dot{E} \rangle = 2 \times 10^{34} \text{ erg/s}$, $\eta = 0.01$, $E_{\text{min}} = 0.1 \text{ TeV}$ and $E_{\text{max}} = 100 \text{ TeV}$, and N_{MSP} as given in the Table 4.2. Furthermore, I use the updated value of $R_c = 5.94 \text{ arcsec} = 0.1 \text{ arcmin}$ for Terzan 5 as given by Prager et al. (2017).

Table 4.2: Structural and other parameters of selected 15 clusters and of Terzan 5 used in this study. The columns are: cluster identification; core radius in parsec; half-mass radius in parsec; tidal radius in parsec; estimated number of stars; estimated number of MSPs; distances in kiloparsec; and the assumed values for Q_0 .

GC name	R_c (pc)	R_{hm} (pc)	R_t (pc)	Estimated $N_{\text{tot}}/10^5$	N_{MSP}	d (kpc)	Q_0 ($\times 10^{32} \text{ erg}^{-1} \text{ s}^{-1}$)
NGC 104	0.47	4.1	56.1	4.57	> 33	4.5	9.55
NGC 6388	0.16	0.68	8.13	5.81	100	9.9	52.1
NGC 7078	0.18	1.31	28.1	4.13	25	10.4	7.24
Terzan 6	0.07	0.58	22.8	0.29	25	6.8	7.24
Terzan 10	1.18	2.03	6.62	0.30	25	5.8	7.24
NGC 6715	0.12	1.07	9.78	4.79	25	26.5	7.24
NGC 362	0.24	1.07	21.1	1.58	25	8.6	7.24
Pal 6	0.86	1.57	10.9	0.31	25	5.8	7.24
NGC 6256	0.03	1.13	9.94	0.21	25	10.3	7.24
Djorg 2	0.43	1.31	13.8	0.51	25	6.3	7.24
NGC 6749	0.81	1.44	6.82	1.78	25	7.9	7.24
NGC 6144	1.23	2.13	43.5	0.48	25	8.9	7.24
NGC 288	1.77	2.92	16.9	0.32	25	8.9	7.24
HP 1	0.04	4.06	10.8	0.48	25	8.2	7.24
Terzan 9	0.04	1.02	10.8	0.02	25	7.1	7.24
Terzan 5	0.21	0.94	17.4	0.77	>34	6.9	20.0

4.2.2 Constraining Parameters via X-ray and γ -ray Data

I constrain cluster parameters for three sources (i.e., Terzan 5, 47 Tucanae, and NGC 6388) using diffuse X-ray and VHE γ -ray observations so as not to violate the data. I first present the results for Terzan 5. The γ -ray (Abramowski et al., 2011) and X-ray (Eger et al., 2010) data are shown in Figure 4.25.

Table 4.3: Parameter combinations for each of the line colours appearing in Figure 4.25 for Terzan 5.

Line colours	κ (kpc^2/Myr)	B (μG)	Γ	Q_0 ($\text{erg}^{-1} \text{ s}^{-1}$)	d (kpc)	N_{tot}	$E_{\text{e,max}}$ (TeV)
Blue	Bohm diffusion	5	1.8	1.16×10^{34}	5.9	7.7×10^4	100
Red	Bohm diffusion	1	1.8	6.33×10^{33}	5.9	7.7×10^4	20
Green	0.7×10^{-4}	2	2.0	9.84×10^{33}	5.9	7.7×10^4	50

Unfortunately, the GC model cannot reproduce the flat slope of the X-ray data. I therefore postulate a new radiation component (see Chapter 5) to explain these data. I therefore treat the X-ray data as upper limits and thus the predicted SR component must be below these. I try to find sample parameters so that the predicted SR component is not in conflict with the X-ray “upper limits”, while at the same time fitting the γ -ray data, rather than fitting the data using rigorous statistical techniques.

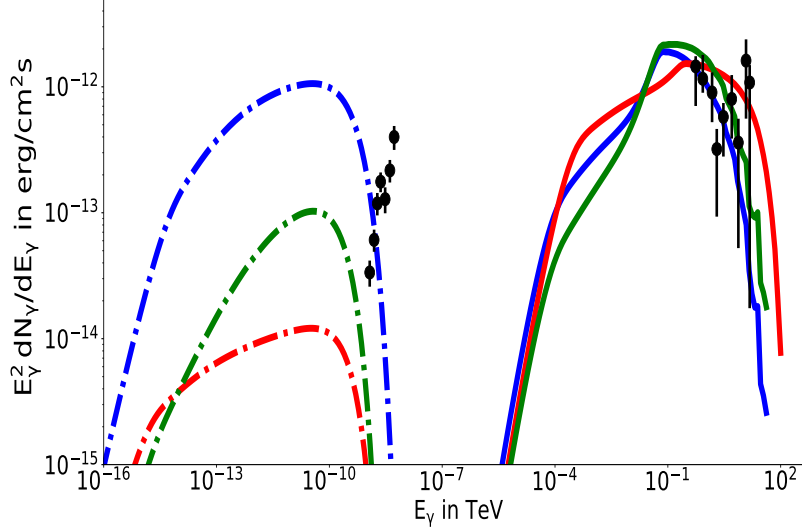


Figure 4.25: The predicted SED for Terzan 5 indicating the SR (integrated between $55' < r_s < 174'$, the dash-dotted lines) for the inner part of the source and IC (integrated over all r_s , solid lines) components for different combinations of parameters (Table 4.3), as well as *Chandra* (Eger et al., 2010) and H.E.S.S. (Abramowski et al., 2011) data.

Figure 4.25 shows the predicted differential SED components for Terzan 5 indicating the predicted SR (integrated between $55' < r_s < 174'$ ² corresponding to the X-ray data; Eger et al., 2010) and IC (integrated over all r_s , solid lines) components using a combination of parameters (i.e., same extraction in model as in data). The red and green component satisfy the data. We do not consider *Fermi* LAT data, as these are usually modelled as a cumulative CR spectrum, not considered in this Chapter. Table 4.3 list the parameters corresponding to these lines. However, one can for example allow $E_{e,max}$ to be $\sim 30 - 100$ TeV and $B \lesssim 5\mu G$. Other combinations of parameters also exist that can give the similar fits. One should be able to break this degeneracy by adding more data in future.

I similarly present results for 47 Tucanae using diffuse X-ray data (Wu et al., 2014) and H.E.S.S. upper limits (Abramowski et al., 2013) to constrain parameters of the model. Figure 4.26 shows the predicted SED for 47 Tucanae showing the predicted SR (integrated

²In Figures 4.25, 4.26, and 4.27 the dash-dotted lines represent the inner part of the source, and the solid lines indicate the whole source visible in γ -rays. The FoV of the γ -ray telescope is large, and therefore one can see the whole source, whilst only a small part of the source is seen in X-rays since the FoV of *Chandra* is small.

between $0.72' < r_s < 3.835'$) and IC (integrated over all r_s) components using several combinations of parameters. From the plot, it can be seen that the blue line violates the X-ray data (treated as “upper limits”). Table 4.4 gives a summary of parameter combinations for 47 Tucanae. I find that $E_{e,\max} \lesssim 100$ TeV and $B \lesssim 5 \mu\text{G}$ satisfy the measurements. Additionally, one could also invoke another spectral component to explain the hard X-ray spectrum, similar to what is done for Terzan 5 in Chapter 5.

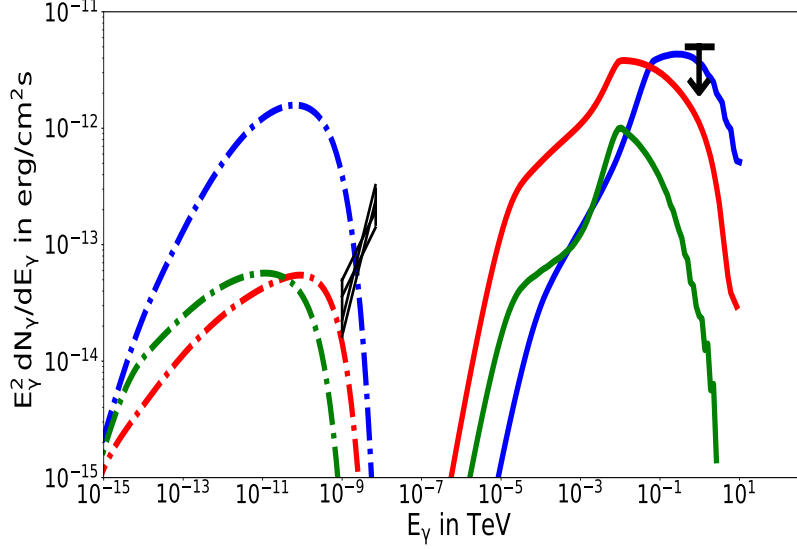


Figure 4.26: The predicted SED for 47 Tucanae indicating the SR (integrated between $0.72' < r_s < 3.835'$, the dash-dotted lines), and IC (integrated over all r_s , solid lines) components for different combinations of parameters (Table 4.4), as well as *Chandra* data (Wu et al., 2014) and a H.E.S.S. upper limit (Abramowski et al., 2013).

In Figure 4.27 I show the predicted SED for NGC 6388. The combinations of parameters used for this cluster are listed in Table 4.5. I once again note that there are different combinations of parameters that satisfy the the data constraints. For example, one requires a small B and $E_{e,\max}$, or a small Q_0 and soft Γ to satisfy the *Chandra* and H.E.S.S. upper limits.

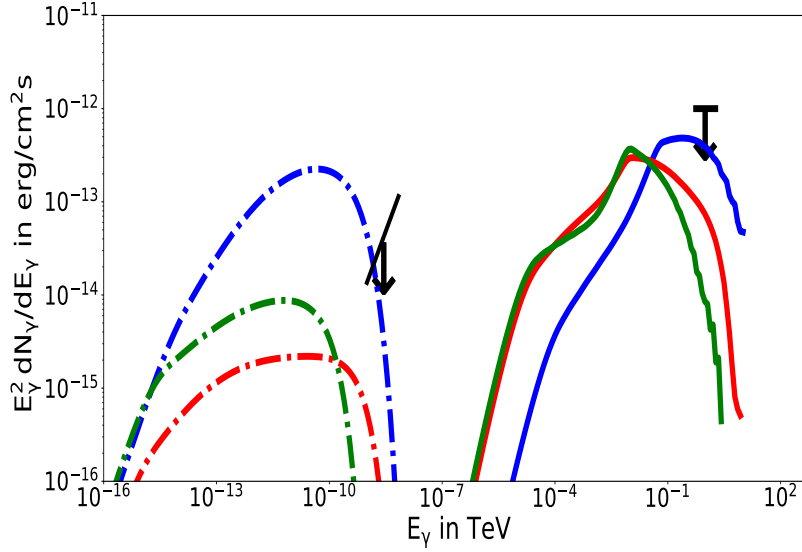


Figure 4.27: The predicted SED for NGC 6388 indicating the SR (integrated between $25'' < r_s < 139''$, the dash-dotted lines) and IC (integrated over all r_s , solid lines) components for different combinations of parameters (Table 4.5). The power law and arrow represent the X-ray upper limit (Eger and Domainko, 2012) whilst the line-arrow represents the H.E.S.S. upper limit (Abramowski et al., 2013).

Table 4.4: Parameter combinations for each of the line colours appearing in Figure 4.26 for 47 Tucanae.

Line colours	κ (kpc ² /Myr)	B (μ G)	Γ	Q_0 (erg ⁻¹ s ⁻¹)	d (kpc)	N_{tot}	$E_{\text{e,max}}$ (TeV)
Blue	Bohm diffusion	5	1.8	1.16×10^{34}	4.5	4.6×10^5	10
Red	1.1×10^{-4}	1	2.0	9.55×10^{33}	4.5	4.6×10^5	100
Green	1.1×10^{-4}	4	2.3	3.18×10^{33}	4.5	4.6×10^5	30

Table 4.5: Parameter combinations for each of the line colours appearing in Figure 4.27 for NGC 6388.

Line colours	κ (kpc ² /Myr)	B (μ G)	Γ	Q_0 (erg ⁻¹ s ⁻¹)	d (kpc)	N_{tot}	$E_{\text{e,max}}$ (TeV)
Blue	Bohm diffusion	5	1.8	6.33×10^{33}	9.9	5.8×10^5	10
Red	Bohm diffusion	1	2.0	3.47×10^{33}	9.9	5.8×10^5	100
Green	1.1×10^{-4}	2	2.3	5.21×10^{33}	9.9	5.8×10^5	30

4.2.3 Ranking the GCs According to Predicted VHE Flux

I apply the GC model to 15 non-detected GCs and to Terzan 5 using the values given in Table 4.2. For reference SR and IC spectra components of each GC, I assume Bohm diffusion, $\Gamma = 2.0$, $B = 5 \mu\text{G}$, and $E_{\text{e,max}} = 100 \text{ TeV}$. Our predictions show that H.E.S.S. may detect two more Galactic GCs (i.e., 47 Tucanae represented by the blue line and NGC 6388 indicated by the green line) in addition to Terzan 5 (indicated by the orange line) if the GCs are observed for 100 h (see Figure 4.28). NGC 6388 and 47 Tucanae have not been detected by H.E.S.S. yet, since they have only been observed only for about 20 h each. However, the flux prediction and the ranking are very sensitive to the choice of parameters (i.e., there is therefore significant error bars on the predicted fluxes - see Chapter 6).

Also, if there are X-ray data that can be treated as an upper limit, this may reduce the TeV flux somewhat; however, the available X-ray data on GCs are typically very scarce and unconstraining, and we do not expect this to impact the TeV predictions substantially. The CTA may detect more GCs since it will be 10 times more sensitive than H.E.S.S. I found that depending on observation time and parameters of the model, more than half of the known Galactic population may be detectable by CTA. However, if there is a non-detection by CTA, this will constrain the energetics of the embedded MSP population in a particular cluster. The top five promising GCs (see Figure 4.28) for CTA that I found are NGC 6388, 47 Tucanae, Terzan 5, Djorg 2, and Terzan 10.

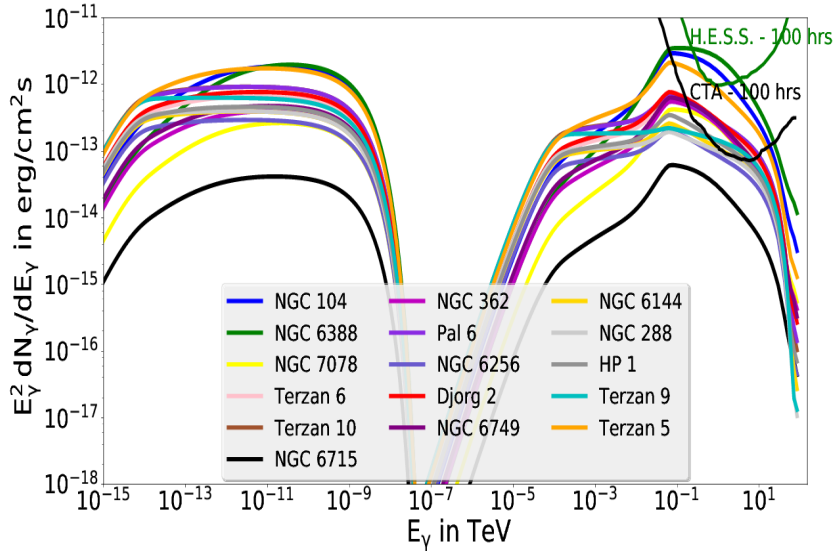


Figure 4.28: SED showing GCs ranked according to their predicted fluxes. The two components represent the SR and IC spectra. The H.E.S.S. and CTA sensitivities (for 100 hours) are also shown.

4.3 Summary

In this Chapter, I discussed the effects of changing the GC model parameters: number of stars N_{tot} in the cluster, B -field in the cluster, the spectral index (Γ) of the injected spectrum, the electron source term (Q_0), the distance to the cluster/the size of the cluster (d), and the spatial diffusion coefficient (κ_0 and δ) on both the diffusion timescales and energy losses to show where radiation dominates, and in which zones particle escape dominates. I also showed the effect on the radiated SED as well as the steady-state particle spectrum. Below, I summarise the effect of increasing the model parameters on the particle spectrum (see Table 4.6) and also the effect of increasing the model Table 4.7).

The parameters of each GC are uncertain and I noted that there are different parameter combinations that satisfy the observational constraints for the three GCs (i.e., Terzan 5, NGC 6388, and 47 Tucanae) and thus making parameter fitting degenerate. I also noted

Table 4.6: In this table I describe the effect of increasing model parameters on the steady-state particle spectrum.

Model parameters	Close to the core		$r \approx R_t$	
	Low energies	High energies	Low energies	High energies
Number of stars (N_{tot})	-	=	=	=
Magnetic field (B)	=	$\approx$	-	-
Spectral index (Γ)	$\approx$	-	-	-
Electron source term (Q_0)	+	+	+	+
Distance (d)	-	-	-	-
Diffusion coefficient (κ_0)	$\approx$	=	-	-

that CTA may detect many more GCs, since it will be 10 times more sensitive than the current VHE ground-based Cherenkov telescopes (e.g., H.E.S.S. and MAGIC). In the next Chapter, I will study multi-wavelength data on Terzan 5 and also constrain its model parameters.

Table 4.7: In this table I describe the effect of increasing model parameters on the SED plot.

Model parameters	Close to the core			$r \approx R_t$		
	Radio	X-rays	γ -rays	Radio	X-rays	γ -rays
Number of stars (N_{tot})	-	=	+	$\approx$	$\approx$	$\approx$
Magnetic field (B)	+	+	$\approx$	-	-	-
Spectral index (Γ)	$\approx$	-	-	-	-	-
Electron source term (Q_0)	+	+	+	+	+	+
Distance (d)	-	-	-	-	-	-
Diffusion coefficient (κ_0)	+	+	$\approx$	-	-	$\approx$

Chapter 5

Spectral Modelling of Terzan 5

In the previous Chapter I discussed my population study of the brightest VHE clusters for future observations by H.E.S.S. and CTA (Ndiyavala et al., 2018). There are a lot of data available for Terzan 5 and we realise that we can model this source and also constrain the underlying pulsar luminosity function. I will therefore discuss this work that is based on our published paper (Ndiyavala et al., 2019) in this Chapter. I have specifically been involved in estimation of an optical upper limit and running of the models (fitting SED data). I will discuss some background on Terzan 5 and use this cluster as a case study, since it has multi-wavelength (radio, X-rays, HE, and VHE) data that makes it unique. I will also discuss the estimation of the optical upper limit of Terzan 5. I will discuss the models I used to fit the multi-wavelength data of this source to model SED data. Lastly, I will discuss how we used the X-ray data to constrain the luminosity function of the embedded pulsars in Terzan 5.

5.1 Introduction

Terzan 5 is a Galactic GC located in the inner Galaxy lying at a distance $d = 5.9 \pm 0.5$ kpc (Valenti et al., 2007) and exhibiting prominent X-ray and γ -ray emission. This object hosts the largest number of identified MSPs (i.e., 37¹ MSPs) (Cadelano et al., 2018) detected so far and exhibits the highest central stellar density, and with the brightest GeV range flux as measured by *Fermi* LAT, Terzan 5 stands out among Galactic GCs. It is also the only GC that has plausibly been detected at VHE (Abramowski et al., 2011) and is expected to contain many binary systems, including low-mass X-ray binaries (which are progenitors of MSPs) due to its high core density. Terzan 5 has the highest stellar interaction rate compared to all other Galactic GCs (Verbunt and Hut, 1987) and high metallicity. More

¹<http://www.naic.edu/~pfreire/GCpsr.html>

MSPs are expected to be found in Terzan 5 than the current 37 known ones² (Lanzoni et al., 2010, Freire et al., 2017, Cadelano et al., 2018). Therefore this cluster has been an attractive source to model and observe, since MSPs are known not only to radiate pulsed emission in multiple wavebands, but also to generate relativistic particles that may in turn upscatter ambient photons into the VHE band (Bednarek and Sitarek, 2007), or interact with the cluster magnetic field that may result in diffuse SR (Venter and De Jager, 2008b). The *Fermi* LAT (Atwood et al., 2009) has detected a GeV source that seems to be associated with Terzan 5. Diffuse X-rays have also been detected from this source (Eger et al., 2010), while some radio observations have revealed several extended structures in the vicinity of Terzan 5 (Clapson et al., 2011).

5.2 Existing Multi-wavelength Data and Spectral Upper Limits

5.2.1 Radio Emission

This object was detected by the Very Large Array (VLA) of the National Radio Astronomy Observatory (NRAO) Sky Survey (NVSS) at 21 cm as a single source with a flux of 5 mJy. Effelsberg 11 cm and 21 cm archival data have been analysed and several radio structures were detected in the direction of Terzan 5 (Clapson et al., 2011). In Figure 5.1 a radio structure to the north-west of Terzan 5 could be due to the large population of MSPs in this object. Also a smaller compact radio structure (labelled 10) may be associated with ionised material produced by an O star.

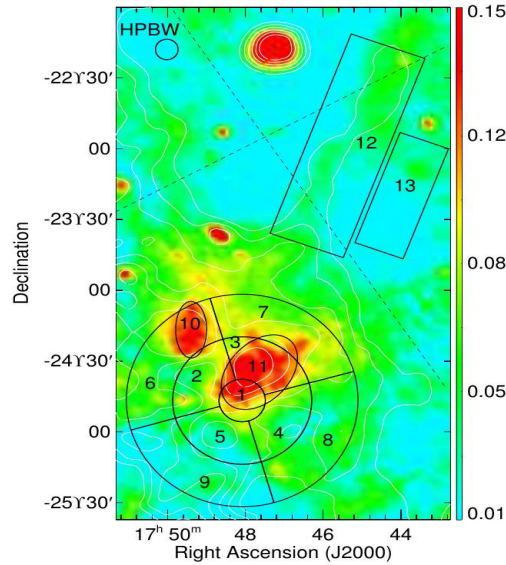


Figure 5.1: Radio map of Terzan 5 from the Effelsberg Galactic plane survey at 11 cm. Region 1 is centred on Terzan 5. The dashed lines show the radio scan direction at 11 cm (Clapson et al., 2011).

²<http://www.naic.edu>

5.2.2 Estimation of the Optical Background Level

I have especially concentrated on this part of the work to establish whether we will be able to see the SR emission from Terzan 5. [Kopp et al. \(2013\)](#) estimated the low-energy synchrotron radiation (LESR) component to be at a low level, and since there are many stars ($\sim 10^5$) that contribute a high level of BB radiation that may swamp any diffuse non-thermal emission SR, it will therefore be difficult to directly observe this component. One can, however, obtain an estimate background level of optical emission by estimating the νF_ν flux from stars near the tidal radius of the GC. I take the surface-density profile of Terzan 5 from [Cohn et al. \(2002\)](#) that was constructed using counts of stars from the central and offset near-infrared camera 2 (NIC2) field. I obtained an estimate of the area of an annulus

$$A_{\text{ann}} = \pi(r_2^2 - r_1^2), \quad (5.1)$$

with r_1 and r_2 the edges (angular radii) of a particular annulus. I found the average number of stars in such an annulus by interpolating the surface brightness f and calculating $N_{\text{ann}} = f A_{\text{ann}}$.

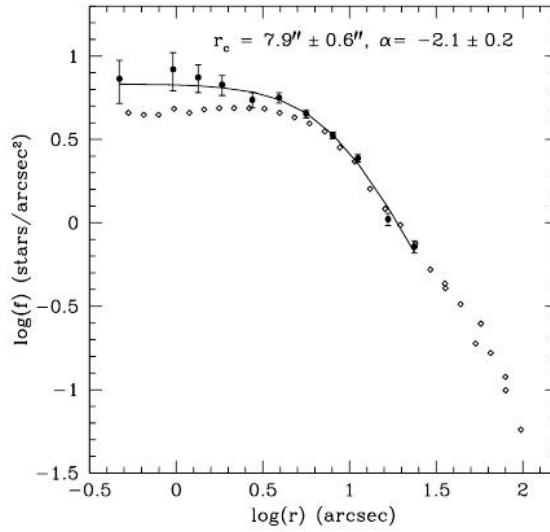


Figure 5.2: Surface-density profile from [Cohn et al. \(2002\)](#) for F187W star counts compared with surface brightness profile from [Trager et al. \(1995\)](#) (open diamonds).

I approximate the emitted spectrum of each star by a BB spectrum at an average frequency of

$$\langle \nu \rangle = \frac{2.7k_B T}{h} = 2.5 \times 10^{14} \text{ Hz}, \quad (5.2)$$

where k_B is the Boltzmann constant and h the Planck constant, assuming a constant stellar surface temperature of $T = 4500 \text{ K}$. I then multiply the surface area by the Planck spectrum B_ν and obtain the following expression:

$$\langle B_\nu \rangle A_\star = \frac{2h \langle \nu \rangle^3}{c^2} \frac{4\pi R^2}{e^{h\langle \nu \rangle / k_B T} - 1}, \quad (5.3)$$

where the angular brackets indicate average values of quantities. I lastly find the thermal νF_ν flux in one annulus to be

$$\begin{aligned} \frac{B_\nu < \nu > A_\star}{d^2} &= \frac{8\pi R^2 h < \nu >^4 N_{\text{ann}}}{d^2 c^2} \frac{1}{e^{h\nu/k_B T} - 1} \\ &\sim 1.7 \times 10^{-14} R_{10}^2 N_{\text{ann}} \text{ erg cm}^{-2} \text{ s}^{-1}, \end{aligned} \quad (5.4)$$

assuming $R_{10} = R/10^{10} \text{ cm}$, $d = 5.9 \text{ kpc}$, and c the speed of light. I find the predicted BB flux of $\sim 6.2 \times 10^{-8} \text{ erg cm}^2 \text{ s}^{-1}$ for $R_{10} = 7 = R_\odot$ when applying Equation (5.4) to the whole cluster (i.e., choosing $N_\star = N_{\text{ann}} = 7.7 \times 10^4$; Lang, 1992), while the predicted νF_ν flux for the LESR at 1 eV is only $\sim 7.2 \times 10^{-13} \text{ erg cm}^{-2} \text{ s}^{-1}$ (Section 5.3), which is a factor $\sim 10^5$ lower than the estimated thermal flux level. One can only detect the LESR component if the stellar flux falls faster with radius than does the LESR flux. One can find a smaller ratio between thermal and non-thermal fluxes by focusing on different annuli. I find the BB to LESR flux ratio for all annuli in Cohn et al. (2002) to be dropping from $\sim 10^5$ to $\sim 10^3$ with increasing radius, out to $\sim 0.35 R_t$, with R_t the tidal radius (see Figure 5.2). The annular area increases as r^2 while the surface brightness falls as $r^{-2.1}$ (as fit in by Cohn et al., 2002) for larger radii, thus the flux remains almost constant for annuli of similar area (with $N_{\text{ann}} \sim 10^3$) out to R_t . The BB flux therefore still exceeds the LESR flux by a factor $\sim 10^3$, even in the outer annuli where the stellar density has significantly dropped. The thermal emission seems to provide a very unconstraining upper limit to the LESR emission under the assumption that the latter will not exceed the thermal flux. However, there might be hope to detect the LESR flux outside of the optical range (e.g., the millimeter or ultraviolet to low-X-ray range) where the BB component dominates, since the BB spectrum covers a much narrower energy range than the LESR does.

5.2.3 Diffuse X-ray Emission

Terzan 5 has also been searched for diffuse X-ray emission, which is expected by several models. Eger et al. (2010) followed up on the detection of extended TeV γ -ray emission by H.E.S.S., and discovered the presence of hard and diffuse X-ray emission above the particle background level.

This was done by extracting spectra from eight concentric annular regions centered on the cluster core to measure the level of diffuse X-ray emission around it as shown in Figure 5.3. It is seen in Figure 5.3 that the diffuse X-ray signal extended well beyond the half-mass radius ($R_{\text{hm}} \sim 30''$) of the cluster and the surface brightness appears to be peaked at the GC centre and decrease smoothly outwards (see Eger et al., 2010 for more details). I therefore fit the diffuse X-ray data observed from Terzan 5 by invoking a new high-energy synchrotron radiation (HESR) component that maybe due to the cumulative pulsed magnetospheric SR from pairs (see Figure 5.8).

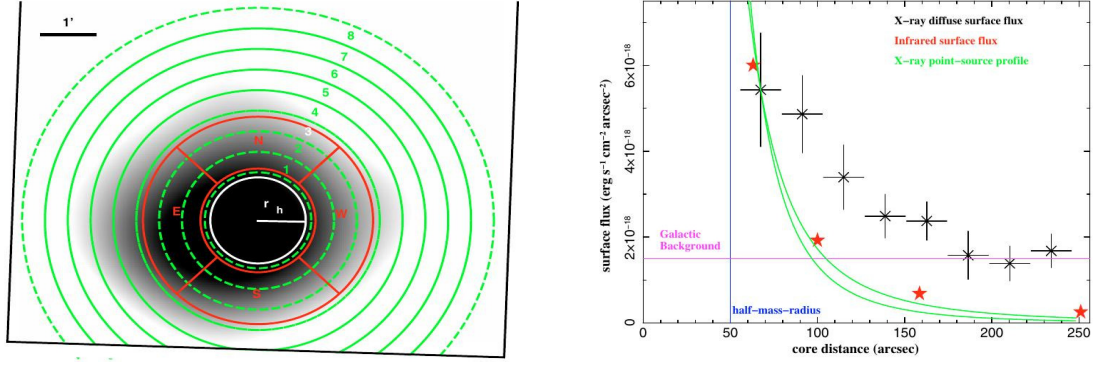


Figure 5.3: *Chandra* image of diffuse X-ray emission around Terzan 5 (left) and the radial dependence of the diffuse X-ray surface flux above the particle background (right) (Eger et al., 2010).

5.2.4 High-Energy Emission *Fermi* LAT Data Analysis

Terzan 5 is the second γ -ray-emitting GC to be associated with a *Fermi* LAT source after 47 Tucanae. Kong et al. (2010) compared Terzan 5 to 47 Tucanae and suggested that γ -ray emission from Terzan 5 is probably associated with MSPs and might be from the MSP magnetospheres or IC scattering by relativistic particles (i.e., electrons or positrons) in the pulsar wind of the background soft photons from the Galactic plane. The authors analysed 1.4 years of Pass 6 (P6) LAT data from the region around Terzan 5 and detected a significant point source coinciding with the optical centre of Terzan 5, with pulsar-like spectrum having a photon index $\Gamma = 1.9 \pm 0.2$, a cutoff energy $E_c = 3.8 \pm 1.2$ GeV, integrated photon flux of $(3.4 \pm 1.1) \times 10^{-8} \text{ cm}^{-2} \text{ s}^{-1}$, and energy flux of $(6.8 \pm 2.0) \times 10^{-11} \text{ erg cm}^{-2} \text{ s}^{-1}$. Abdo et al. (2010) also analysed the region around Terzan 5 using 1.5 years of P6 LAT data and found a point source, located $2.4'$ from the centre of the cluster (see Figure 5.4). Their best-fit simple exponentially-cutoff power-law spectrum had $\Gamma = 1.4^{+0.2,+0.4}_{-0.2,-0.3}$, $E_c = 2.6^{+0.7,+1.2}_{-0.5,-0.7}$ GeV, integrated photon flux of $(7.6^{+1.7,+3.4}_{-1.5,-2.2}) \times 10^{-8} \text{ cm}^{-2} \text{ s}^{-1}$, and energy flux of $(7.1^{+0.6,+1.0}_{-0.5,-0.5}) \times 10^{-11} \text{ erg cm}^{-2} \text{ s}^{-1}$.

These authors estimated the number of MSPs in Terzan 5 to be $N_{\text{MSP}} = 180^{+100}_{-90}$ using an estimate of the average MSP spin-down power ($\langle \dot{E} \rangle$) of Galactic MSPs known at the time) and the measured γ -ray flux. Here, we considered seven years of Pass 8 (P8) LAT data with *evclass* = 128, and *evtype* = 3 within 15° of the best-fit position of 3FGL J1748.0–2247. Our region of interest (ROI) included all 3FGL sources within 25° of the ROI centre. The spectrum of 3FGL J1748.0–2247 was modelled using both a simple power law and an exponentially-cutoff power law. We obtained new *Fermi* data that we fit using a model for the cumulative CR from a population of MSPs embedded within Terzan 5 (see Figure 5.6). These data proved to be constraining for the low-energy tail of the unpulsed IC component. Our best-fit energy flux agrees well with the newer analysis by De Menezes et al. (2018) who recorded a value of $G_{100} = (7.441 \pm 0.271) \times 10^{-11} \text{ erg cm}^{-2} \text{ s}^{-1}$ using nine years of P8 data.

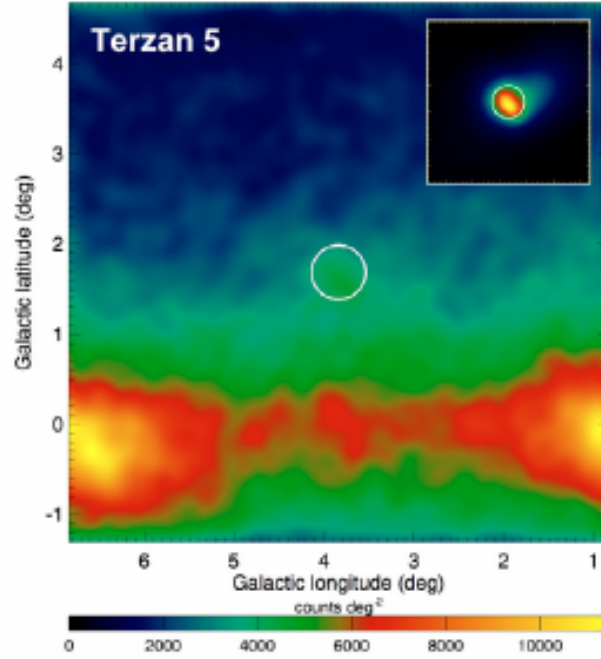


Figure 5.4: Gaussian kernel smoothed ($\sigma = 0.1^\circ$) counts map of the ROIs for Terzan 5. Circles indicate the nominal locations of the source (Abdo et al., 2010).

5.2.5 Very-High-Energy (VHE) Emission: H.E.S.S. Data

H.E.S.S. discovered a VHE γ -ray source HESS J1747–248, located in the close vicinity of Terzan 5. The source extends well beyond the H.E.S.S. (see Figure 5.5) PSF. The spectrum of the source was described by a power law with index of $2.5 \pm 0.3_{\text{stat}} \pm 0.2_{\text{sys}}$, and its integral flux above 440 GeV of the source was measured as $(1.2 \pm 0.3) \times 10^{-12} \text{ cm}^{-2} \text{ s}^{-1}$ (Abramowski et al., 2011). The probability of a chance coincidence between an unrelated VHE γ -ray source and Terzan 5 is low ($\sim 10^{-4}$). I have done independent analysis (Ndiyavala et al., 2018) on this source and detect the source with a significance of 6σ , which is similar to the detection found by Abramowski et al. (2011).

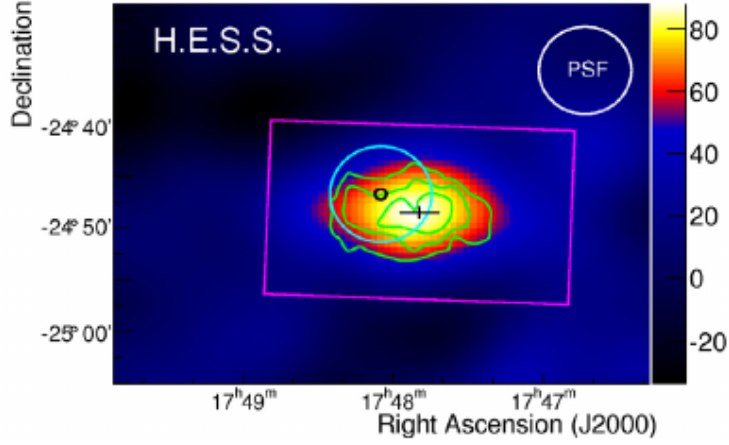


Figure 5.5: Exposure-corrected excess image from the H.E.S.S. data, smoothed with a Gaussian function of width 0.1° and overlaid with significance contours ($4 - 6\sigma$) in RA-Dec J2000 coordinates. The circles represent the half-mass radius (in black) and the larger tidal radius (in cyan) of the source. The cross indicates the best-fit source position of HESS J1747–248, the rectangle represents the integration region used for the full-source spectral analysis, and the upper-right corner circle illustrates the instrumental PSF.

5.3 Modelling the Broadband SED

5.3.1 Parameter Constraints from General Considerations

In this Section, we derive general constraints on the spatial diffusion coefficient κ (for simplicity, we assume that this coefficient is only a function of particle energy, not of space) and the cluster B -field. As a first approach, the Bohm value has been used in the past to model the particle diffusion (Bednarek and Sitarek, 2007, Venter and De Jager, 2008b),

$$\kappa_{\text{Bohm}} = \frac{cE_e}{3eB} = 3.3 \times 10^{25} E_{\text{TeV}} B_{-6}^{-1} \text{ cm}^2 \text{ s}^{-1}, \quad (5.5)$$

with $E_{\text{TeV}} = E_e/1 \text{ TeV}$ the particle energy and $B_{-6} = B/1 \mu\text{G}$. By invoking a containment argument, one may obtain a constraint on this coefficient: since we observe VHE γ -ray emission up to $E_\gamma \sim 10 \text{ TeV}$, one can write that the diffusion time (in the limit that it exceeds the escape time, taken to be equal to the diffusion time) should exceed the typical timescale for IC emission:

$$\tau_{\text{esc}} > \tau_{\text{IC}}. \quad (5.6)$$

This leads to

$$\frac{R^2}{6\langle\kappa\rangle} > \frac{E_e}{\dot{E}_{\text{IC}}}. \quad (5.7)$$

Let us concentrate on the optical soft-photon background with photons at $T \sim 4500 \text{ K}$ (i.e., having an average energy $\langle\epsilon\rangle \sim 1 \text{ eV}$). For very energetic leptons, we have to take Klein–Nishina effects into account when calculating the IC loss rate. We thus use the

expression of [Schlickeiser and Ruppel \(2010\)](#),

$$\dot{E}_{\text{IC}} \approx \frac{4\sigma_{\text{T}}cu}{3} \frac{\gamma_{\text{e}}^2 \gamma_{\text{KN}}^2}{\gamma_{\text{e}}^2 + \gamma_{\text{KN}}^2}, \quad (5.8)$$

with $\sigma_{\text{T}} = 6.65 \times 10^{-25} \text{ cm}^2$ the Thomson cross section, u the average soft-photon energy density, and

$$\gamma_{\text{KN}} \equiv \frac{3\sqrt{5}}{8\pi} \frac{m_{\text{e}}c^2}{k_{\text{B}}T} \quad (5.9)$$

the critical Klein-Nishina Lorentz factor. If the particle Lorentz factor $\gamma_{\text{e}}^2 \gg \gamma_{\text{KN}}^2$, the IC loss rate reduces to

$$\dot{E}_{\text{IC}} \approx \frac{4\sigma_{\text{T}}cu\gamma_{\text{KN}}^2}{3}, \quad (5.10)$$

yielding

$$\tau_{\text{IC}} \approx 6 \times 10^{12} \left(\frac{E_{\text{TeV}} T_{4500}^2}{u_{50}} \right) \text{ s} \approx 2 \times 10^5 \left(\frac{E_{\text{TeV}} T_{4500}^2}{u_{50}} \right) \text{ yr}, \quad (5.11)$$

with $u_{50} \equiv u/(50 \text{ eV cm}^{-3})$ and $T_{4500} = T/4500 \text{ K}$. We use u_{50} to scale our results, since this value reflects a spatially averaged value for the energy density; see Figure 1 of [Bednarek and Sitarek \(2007\)](#) and [Prinsloo et al. \(2013\)](#). If we set $R \sim R_{\text{t}} \sim 10 \text{ pc}$, we find

$$\langle \kappa \rangle < 2.6 \times 10^{25} \left(\frac{R_{10}^2 u_{50}}{E_{\text{TeV}} T_{4500}^2} \right) \text{ cm}^2 \text{ s}^{-1}, \quad (5.12)$$

with $R_{10} \equiv R/10 \text{ pc}$ and R_{t} the tidal radius. This upper limit is similar to the value of the Bohm coefficient at $E_{\text{e}} = 1 \text{ TeV}$. [Kopp et al. \(2013\)](#) inferred values for κ that are slightly larger at 1 TeV than the Bohm value (for $B_{-6} \sim 5$) when fitting the X-ray surface brightness profile, although they assumed an energy dependence $\kappa \propto E_{\text{e}}^{0.6}$. They also noted that by assuming Bohm diffusion, they could fit the X-ray surface brightness data, and that the degeneracy in diffusion index and normalisation may be broken by using spatial data in a different waveband, as well as more spectral data. The caveat is that both the spatial and spectral fit should be reasonable. While [Kopp et al. \(2013\)](#) could fit the X-ray surface brightness profile, their predicted SED did not match the data. We thus update their calculation so as to fit both these quantities (Section 5.3.2).

Additionally, one may argue that since we observe IC emission up to $E_{\gamma} \sim 10 \text{ TeV}$, we must have

$$\tau_{\text{SR}} \gtrsim \tau_{\text{IC}}. \quad (5.13)$$

This implies (at those high energies) that

$$\dot{E}_{\text{SR}} \lesssim \dot{E}_{\text{IC}}, \quad (5.14)$$

which yields a limit on the magnetic field

$$B_{-6} \lesssim 8 \left(\frac{\sqrt{u_{50}}}{T_{4500} E_{\text{TeV}}} \right). \quad (5.15)$$

Therefore, from the simple arguments above, we find typical values of $B_{-6} \sim 10$ and $\langle \kappa \rangle \sim 5 \times 10^{25} \text{ cm}^2 \text{ s}^{-1}$ around $E_{\text{TeV}} \sim 1$, similar to what was found by Kopp et al. (2013). At these typical cluster B -fields, the LESR spectrum should peak around

$$E_\gamma = 0.29 h \nu_{\text{crit}} \approx 2 \times 10^{-5} B_{-6} E_{\text{TeV}}^2 \text{ keV}, \quad (5.16)$$

and for $B_\perp \sim 5 \mu\text{G}$ and $E_e \sim 10 \text{ TeV}$, this component should peak around $\sim 0.01 \text{ keV}$, with $B_\perp = B \sin \alpha'$ and α' the pitch angle. This is consistent with our findings in the next section.

5.3.2 Leptonic Modelling of the Broadband SED of Terzan 5

LESR and IC Components

We present new spectral fits³ to the SED of Terzan 5 using the model of Kopp et al. (2013), as shown in Figure 5.6 (blue dashed lines). The model includes a spatial dimension, refined stellar soft-photon energy density profile, and full particle transport, taking diffusion and radiation losses into account with the assumptions of spherical symmetry and a steady-state regime.

In Figure 5.6, I indicate radio data (labeled “Effelsberg”) associated with Region 11 (a prominent, large-scale, asymmetric feature offset from the centre) as defined by Clapson et al. (2011). I fit these points with our predicted LESR component, in keeping with the suggestion by Clapson et al. (2011) that the flux from this region may be due to unpulsed SR from leptons that were injected by the MSPs into the GC and diffused throughout the cluster. Our predicted LESR component is much below the estimated BB flux level in the optical band (see Section 5.2.2). As mentioned in Equation 5.16, we expect this component to peak around 1 keV for particle energies $E_e \sim 100 \text{ TeV}$ and $B \sim 10 \text{ G}$. This led Kopp et al. (2013) to fit the X-ray surface brightness profile measured by *Chandra* in order to constrain the diffusion coefficient to $\kappa \sim 3.3 \times 10^{25} \text{ cm}^2 \text{ s}^{-1}$ at $1 \sim \text{TeV}$, similar to Equation 5.5 and Equation 5.12. However, although their model prediction reproduced the flux level at a few keV, they could not fit the spectral slope of the observed data. This implies that the observed diffuse X-ray emission may be due to a different spectral component. We therefore now choose the maximum particle energy $E_{e,\text{max}} = 10 \text{ TeV}$ and a slightly lower B -field of $B = 4 \mu\text{G}$ so that our new LESR component peaks around

³The main aim of this Chapter is to ascertain whether we can elucidate the broadband spectral emission properties of Terzan 5, as well as those of the underlying sources that inject particles into Terzan 5. However, we realise that the energy-dependent morphology of this cluster is quite complex, so much so that it challenges the idea of a single (collective) particle population injected by the MSPs being responsible for all spectral emission components originating from partially overlapping spatial regions of different extents. Yet, to facilitate usable conclusions to be drawn from the current data, we do invoke a single population and study the source energetics, while deferring a study of the spatial properties of Terzan 5 to future work.

$E_\gamma \sim 0.01$ keV and thus cuts off below the *Chandra* data. Furthermore, the low-energy tail of our predicted IC component satisfies the new *Fermi* data and upper limits, and we also reproduce the H.E.S.S. data.

Primary CR and Pair HESR Components

I use the model of [Harding and Kalapotharakos \(2015\)](#) to fit the GeV and keV data. Similar to previous studies (e.g., [Harding et al., 2005](#), [Venter et al., 2009a](#), [Zajczyk et al., 2013](#)), I fit the *Fermi* LAT data using the cumulative primary CR component of pulsed γ -ray emission originating in the MSP magnetospheres (GeV component indicated by a solid red line and labeled “CR” in [Figure 5.6](#)). This has been a standard interpretation for the GeV spectrum measured by the LAT for several GCs ([Abdo et al., 2010](#)).

Following the idea of [Kopp et al. \(2013\)](#), we propose that the *Chandra* data indicate the presence of a “new” HE SR component that has not been modelled in detail in this context⁴ before: the cumulative pulsed SR from pairs generated within the magnetospheres of the host MSPs, radiating at altitudes that are a substantial fraction of the light cylinder (at a radius $R_{LC} = c/\Omega$, with Ω the angular speed, where the corotational speed equals c) through cyclotron resonant absorption of radio photons (see [Harding and Others, 2008](#)). Given the much higher local B -field (e.g., the magnetospheric field at the MSPs’ light cylinder may reach $B_{\text{HESR}} \sim 10^6$ G for the most energetic ones⁵ versus the much lower GC field $B_{\text{LESR}} \sim 10^{-5}$ G) and the much smaller average pitch angle ($\alpha_{\text{HESR}} \sim 0.1$ versus $\alpha_{\text{LESR}} \sim \pi/2$ rad), as well as different particle energies, the cutoff energy of this new component is much higher than that of the LESR spectrum:

$$\frac{E_{\text{HESR, cut}}}{E_{\text{LESR, cut}}} \sim \left(\frac{\gamma_{\text{SR}}}{\gamma_{\text{LESR}}} \right)^2 \frac{B_{\text{HESR}} \sin \alpha_{\text{HESR}}}{B_{\text{LESR}} \sin \alpha_{\text{LESR}}} \quad (5.17)$$

$$\sim \left(\frac{10^4}{10^7} \right)^2 \frac{10^6 \text{ G} \times 10^{-1}}{10^{-5} \text{ G}} \sim 10^4. \quad (5.18)$$

This simple scaling predicts a cut-off $E_{\text{HESR, cut}} \lesssim 100$ keV and thus provides us with a low-energy tail that might fit the X-ray data. This idea is also supported by observations of sources embedded in 47 Tucanae: [Bogdanov et al. \(2008\)](#) noted that even though most of the observed MSPs exhibit soft thermal spectra, three of them manifest hard power-law components. These components may plausibly be attributed to binary shock emission or magnetospheric SR. It is furthermore supported by detection of hard non-thermal X-ray emission from a number of field MSPs.

⁴As a practical measure, we attribute the *Chandra* data solely to collective, pulsed, non-thermal, magnetospheric pulsar emission. There could be contributions by other sources, but we do not know *a priori* the properties of unresolved stellar members hosted by Terzan 5.

⁵The average B -field at the light cylinder may be closer to $\sim 10^4$ G; however, the pair SR component is likely dominated by the MSPs with the highest spin-down luminosities and B -fields.

As a proof of principle, I now calculate model spectra invoking a cumulative pulsed HESR component originating in the MSP magnetospheres to fit the *Chandra* data (keV component indicated by a solid red line and labelled as “HESR” in Figure 5.6). I use a force-free B -field in the inertial observer frame, choosing a slot gap width of $0.03\Theta_{\text{PC}}$, with Θ_{PC} the polar-cap angle (the inner and outer angular boundaries of the gap were set at open-volume coordinates $r_{\text{ovc}} \in (0.90, 0.93)$, where the r_{ovc} coordinate labels self-similar rings, $r_{\text{ovc}} = 0$ being the magnetic pole and $r_{\text{ovc}} = 1$ being the polar-cap rim; see Dyks and Harding, 2004), and a constant E -field from the MSP surface to $2R_{\text{LC}}$, set by an inverse acceleration length scale of $R_{\text{acc}} = d\gamma_e/dl = 2\text{ cm}^{-1}$ (i.e., $E_{\parallel} = R_{\text{acc}}m_e c^2/e$, with γ_e the particle Lorentz factor and dl the step length along the particle trajectory, m_e and e the electron mass and charge). I divide the fraction of stellar surface covered by B -field line foot points that are within the gap using four self-similar rings and 72 azimuthal divisions. I choose an average pulsar period of $\langle P \rangle = 7.7\text{ ms}$, and, by fixing the average surface B -field to $\langle B_s \rangle = 5.8 \times 10^9\text{ G}$ and moment of inertia $\langle I \rangle = 1.56 \times 10^{45}\text{ g cm}^2$, I obtain $\langle \dot{P} \rangle \sim 7 \times 10^{-19}\text{ ss}^{-1}$ and $\langle \dot{E} \rangle \sim 9.08 \times 10^{34}\text{ erg s}^{-1}$. The latter may represent significant contributions from more energetic MSPs. Nonetheless, we take these values as representative⁶ of the pulsars in Terzan 5. I furthermore use a PC pair spectrum calculated for an offset PC B -field (Harding and Muslimov, 2011a,b, Barnard et al., 2016) with an offset parameter of $\epsilon = 0.6$. I choose an average magnetic inclination angle of $\langle \alpha \rangle = 45^\circ$ and average observer angle of $\langle \zeta \rangle = 60^\circ$ (for both HESR and CR components). See Harding and Kalapotharakos (2015) for details.

The number of visible γ -ray pulsars (N_{vis}^γ) is constrained by the primary CR flux level for a given set of model parameters. Alternatively, if we fix $N_{\text{vis}}^\gamma = N_{\text{vis}}^{\text{rad}} = 37$ to the number of visible radio pulsars (since nearly all currently detected γ -ray MSPs by *Fermi* are radio-loud), we may constrain other parameters, such as the gap width and average pulsar geometry α and ζ , or $\langle P \rangle$ and $\langle \dot{P} \rangle$. Unfortunately, it is difficult to break this degeneracy using X-ray data, since one may expect that $N_{\text{vis}}^X \lesssim N_{\text{vis}}^\gamma$ if their X-ray beams are slightly narrower than the γ -ray ones, and equality may not hold exactly. One may additionally write that $N_{\text{tot}}^X = N_{\text{vis}}^X + N_{\text{invis}}^X \geq N_{\text{vis}}^{\text{rad}} = 37$ and $N_{\text{vis}}^\gamma \geq 37$. The product $M_{\text{avg}}N_{\text{vis}}^X$ is being set by the LESR flux level, so these two parameters are degenerate. Using the HESR (*Chandra*) flux level, we constrain the product $N_{\text{vis}}^X \langle M_{\text{avg}} \rangle \sim 1.9 \times 10^4$, with $\langle M_{\text{avg}} \rangle$ the average number of pairs produced per primary extracted from the PC per pulsar (the average electron–positron pair multiplicity). If we take $N_{\text{vis}}^X \approx 35$, we obtain $\langle M_{\text{avg}} \rangle \approx 540$. However, this value crucially depends on the assumptions of the magnetospheric model: more optimistic assumptions about the electrodynamics (e.g., a higher B -field or current that will influence the particle transport) may lead to a larger

⁶Unfortunately, there is a large uncertainty in the MSP population’s properties. While a full Monte Carlo investigation of the SED may be preferable from a first-principles point of view, this will introduce many more uncertainties and a large range for the SED components’ shapes and levels, so this will probably not lead to any conclusive answers. We therefore deem the approach of studying the behaviour of an “average MSP” as the most practical, although we are cognizant of the fact that a particularly powerful MSP may skew the results.

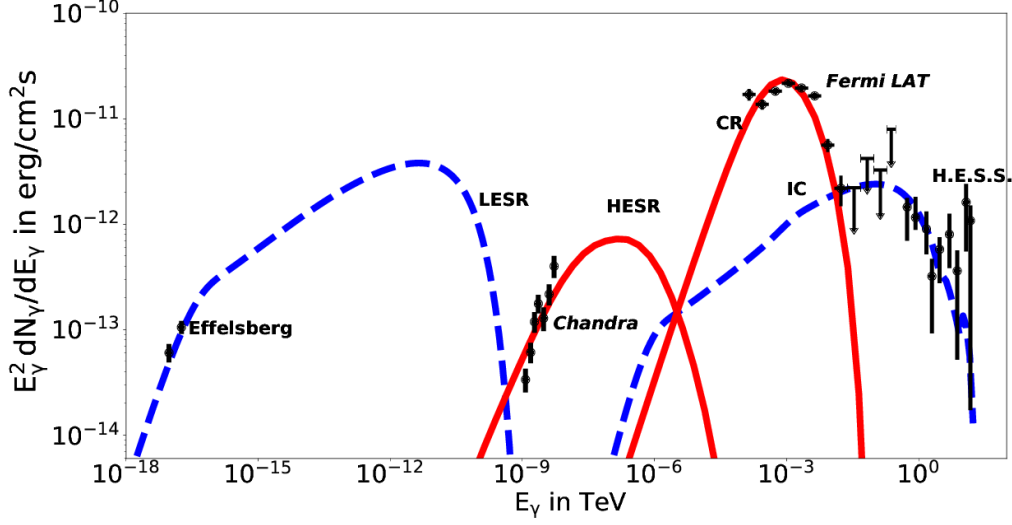


Figure 5.6: Different spectral components for Terzan 5 predicted by the leptonic models of Kopp et al. (2013) and Harding and Others (2008), Harding and Kalapotharakos (2015). Using the first model, we calculate the low-energy SR (LESR) and VHE IC components (integrated over all r_s ; dashed blue lines). We assumed $E_{e,\min} = 9 \times 10^{-3}$ TeV, $E_{e,\max} = 10$ TeV, $Q_0 = 1.4 \times 10^{34}$ erg $^{-1}$ s $^{-1}$, $B = 4.0$ μ G, $\Gamma = 1.5$, and $\kappa = 7 \times 10^{-5}$ kpc ^{2} Myr $^{-1} \approx 2 \times 10^{25}$ cm ^{2} s $^{-1}$. We used a distance of $d = 5.9$ kpc, core radius $R_c = 0.15' = 0.26$ pc, half-mass radius $R_{hm} = 0.52' = 0.89$ pc, and tidal radius $R_t = 4.6' = 7.9$ pc. The HESR and CR components (red lines) are predictions using the model of Harding and Others (2008), Harding and Kalapotharakos (2015) for $\alpha = 45^\circ$, $\zeta = 60^\circ$, $\langle P \rangle = 7.7 \times 10^{-3}$ s, and $B_s = 5.8 \times 10^9$ G. We also indicate Chandra (Eger et al., 2010), H.E.S.S. Abramowski et al. (2011), and radio data (“Region 11” as defined by Clapson et al., 2011). The uncertainties in our LAT points do not reflect possible systematic errors on the Galactic diffuse emission model.

single-MSP spectrum and yield a lower constant (value for the product of $N_{\text{vis}}^X \langle M_{\text{avg}} \rangle$), thus lowering the value for $\langle M_{\text{avg}} \rangle$. Previously, Kopp et al. (2013) found an optimal source strength of $Q_0 \sim 6 \times 10^{33}$ erg $^{-1}$ s $^{-1}$ when fitting the LESR and IC components. The value of Q_0 is usually constrained by assuming a parametric form for the particle injection spectrum

$$Q(E_e) = Q_0 E_e^{-\Gamma} \quad (5.19)$$

and using conservation of charge and energy per unit time (i.e., conservation of current and luminosity; Büsching et al., 2008, Venter et al., 2015),

$$\int_{E_{e,\min}}^{E_{e,\max}} Q(E_e) dE_e = N_{\text{MSP,tot}} (\langle M_{\text{avg}} \rangle + 1) \langle \dot{n}_{\text{GJ}} \rangle, \quad (5.20)$$

$$\int_{E_{e,\min}}^{E_{e,\max}} E_e Q(E_e) dE_e = N_{\text{MSP,tot}} \eta_p \langle \dot{E} \rangle, \quad (5.21)$$

with $\langle \dot{n}_{\text{GJ}} \rangle = 4\pi^2 B_s R^3 / ceP^2 \propto \langle \dot{E} \rangle^{1/2}$ the average Goldreich-Julian rate of particles injected per second for a pulsar period P , surface magnetic field B_s , and stellar radius R (Goldreich and Julian, 1969), and η_p the efficiency of converting the average spin-down luminosity to particle power. The “+1” in the equation above represents the contribu-

tion from primary particles. The above system of equations may have up to 10 free parameters, implying a large degeneracy of parameters. I found an optimal value of $Q_0 \sim 1.4 \times 10^{34} \text{ erg}^{-1} \text{ s}^{-1}$ (Figure 5.6) by fitting the unpulsed spectral components (for particular choices of other free parameters, e.g., κ and B , and using $\langle \dot{E} \rangle = 9.08 \times 10^{34} \text{ erg s}^{-1}$, $\eta_p = 3\%$, and $N_{\text{MSP,tot}} \sim 40$). This leads to a constraint on the average multiplicity,

$$\langle M_{\text{avg}} \rangle = \frac{Q_0(E_{\text{e,max}}^{1-\Gamma} - E_{\text{e,min}}^{1-\Gamma})}{(1-\Gamma)N_{\text{MSP,tot}}\langle \dot{n}_{\text{GJ}} \rangle} - 1 \quad (5.22)$$

$$\approx 20 \left(\frac{\eta_p}{3\%} \right) \left(\frac{2.7 \times 10^{32} \text{ s}^{-1}}{\langle \dot{n}_{\text{GJ}} \rangle} \right) \left(\frac{\langle \dot{E} \rangle}{9 \times 10^{34} \text{ erg s}^{-1}} \right) \propto \langle \dot{E} \rangle^{1/2}, \quad (5.23)$$

with the value of $\langle \dot{n}_{\text{GJ}} \rangle$ reflecting the choice for the average $\langle P \rangle$ and $\langle B_s \rangle$ of the MSPs, as mentioned earlier, for consistency. This estimate of $\langle M_{\text{avg}} \rangle \approx 20$ is quite a bit lower than the previous one of $\langle M_{\text{avg}} \rangle \approx 540$, as inferred from the HESR component. There are ways to mitigate this difference, given the uncertainty and degeneracy in several model parameters. The estimate of $\langle M_{\text{avg}} \rangle$ using the unpulsed spectral components may be raised to $\langle M_{\text{avg}} \rangle \approx 60$ by using $E_{\text{min}} \sim 40 \text{ GeV}$, $E_{\text{max}} \sim 7 \text{ TeV}$, and $\Gamma = 1.6$ without significantly changing the SED. Next, the discrepancy can be lowered to a factor of ~ 4.5 by increasing $\langle I \rangle$ by a factor of ~ 2 , since $\langle M_{\text{avg}} \rangle \propto \langle \dot{E} \rangle^{1/2}$ for the unpulsed case, while $\langle M_{\text{avg}} \rangle \propto \langle \dot{E} \rangle^{-1/2}$ for the pulsed case. This, however, raises $Q_0 \propto \langle \dot{E} \rangle$ by a factor of ~ 2 so that LESR overshoots the data slightly, but this effect can then be mitigated by choosing $B \approx 1 \mu\text{G}$. The remaining discrepancy of a factor of ~ 4.5 can be ameliorated by assuming a larger value for $\epsilon \sim 0.7$ (implying more pairs) and a larger gap width (say, increasing the upper boundary to $r_{\text{ovc}} \sim 0.96$, implying a larger active area on the stellar surface and thus lowering the demand on $\langle M_{\text{avg}} \rangle$). There are also uncertainties in the angles $\langle \alpha \rangle$ and $\langle \zeta \rangle$ that may have a significant effect on the HESR flux. Lastly, using average values $\langle B_s \rangle$ and $\langle P \rangle$ leads to average values for $\langle \dot{n}_{\text{GJ}} \rangle$ and $\langle \dot{E} \rangle$, and this introduces further uncertainty. It is thus possible to pick (non-unique combinations of) values for some model parameters that would make the two estimates of $\langle M_{\text{avg}} \rangle$ (using the pulsed and unpulsed SED components) consistent with each other without violating the observed SED. The actual value of M_{avg} for MSPs is quite uncertain. Polar-cap pair cascades in a pure dipole field give very low values of M_{avg} for the bulk of MSPs, which prompted the suggestion that distortions of the B -field near the neutron star could increase M_{avg} (Harding and Muslimov, 2011b), but the magnitude and structure of such distortions are not known. Comparing the results of particle-in-cell simulations (Kalapotharakos et al., 2018) with the γ -ray spectral cut-offs seen in *Fermi* pulsars can give the estimates of MSP M_{avg} needed to screen the global electric fields. This study indicates that the estimated MSP M_{avg} span a large range from 1 to 10^3 . We also note that one can get different SEDs which may fit the data, and this basically imply a range of M_{avg} . I have done some runs which imply a lot of different M_{avg} but they fit the spectrum well (this is not shown in Ndiyavala et al., 2019).

5.4 Correcting the Predicted Spectrum of Terzan 5

In [Ndiyavala et al. \(2019\)](#), we predicted the broadband spectrum of Terzan 5, invoking four spectral components: unpulsed low-energy synchrotron radiation (LESR) spanning the radio and low-energy X-ray band, pulsed high-energy synchrotron radiation (HESR) spanning the keV to MeV band, pulsed curvature radiation (CR) in the GeV band and unpulsed inverse Compton (IC) emission in the TeV band. Here, “pulsed” refers to cumulative emission from the magnetospheres of embedded pulsars in the cluster, and “unpulsed” refers to emission by escaping particles from the magnetospheres, as they traverse the cluster and encounter soft target photons and cluster B -field. The unpulsed LESR and IC components were calculated using a code described in [Kopp et al. \(2013\)](#), while the pulsed HESR and CR components were calculated using a code by [Harding and Kalapotharakos \(2015\)](#). After publication of our paper ([Ndiyavala et al., 2019](#)), I discovered an error in the script that calculates the *pulsed* spectra, pertaining to angular units. When calculating the phase-averaged pulsed spectra from first principles, one needs to perform an integral over observer phase ϕ . We were inadvertently using degrees instead of radians for ϕ when performing this calculation. Thus, after correcting the units, the predicted HESR and CR pulsed fluxes, given our previous best-fit parameters, now underpredict the data by a factor $180/\pi \sim 57$. However, the unpulsed components were calculated correctly. I now adjust parameters of both pulsed and unpulsed models to consistently fit the data for a given choice of pair multiplicity (the number of particles spawned by a single primary via the process of magnetic pair production) to counter this error in angular units.

5.5 Scaling the Primary CR Component

After fixing the units for ϕ , our corrected CR spectrum dropped and thus needed to be increased by a factor ~ 57 . We previously used a slot gap situated between open volume coordinates ([Dyks and Harding, 2004](#)) $r_{\text{ovc, min}} = 0.91$ and $r_{\text{ovc, max}} = 0.92$. If I increase the lateral gap size for the CR emission region such that $r_{\text{ovc, min}} = 0.90$ and $r_{\text{ovc, max}} = 0.93$, the cross-sectional surface area A_{gap} of the gap will increase by a factor of 3. This increases the particle injection rate from the PC surface and therefore the CR flux by this same factor. I next increased the primary flux by another factor $f_{\text{CR}} \sim 20$ (as in [Harding et al. \(2018\)](#) for Vela), which is quite reasonable, given the uncertainty in the actual PC current in pulsar magnetospheres. Symbolically, I have

$$L_{\text{CR}} \propto \dot{N} \propto f_{\text{CR}} \dot{N}_{\text{GJ}} \propto f_{\text{CR}} A_{\text{gap}},$$

with L_{CR} the CR luminosity and $\dot{N}$ the injection rate of primary particles from the stellar surface.

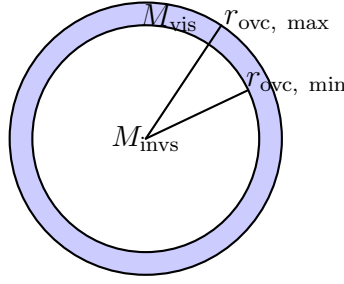


Figure 5.7: Illustration for the calculation of M_{avg} involving M_{invis} and M_{vis} .

5.6 Scaling Both the Unpulsed and Pulsed SR Components

The correction to angular units lowered our pulsed HESR spectrum by a factor of ~ 57 , and it thus needs to be increased by this same factor to fit the data. In [Ndiyavala et al. \(2019\)](#), we used the same value for M_{pair} to fit both the radio data with the unpulsed LESR component and the X-ray data using the pulsed HESR component. However, the LESR flux was not impacted by the unit error, creating tension in the value for M_{pair} implied by the LESR vs the corrected HESR component (they should be the same). To ameliorate this, I now decouple the pulsed and unpulsed SR components somewhat by using different effective pair multiplicities associated with each component, but still obtaining the same average value for pairs from the full PC. The particles responsible for the unpulsed SR come from the two whole PCs (characterised by an average multiplicity M_{avg} , averaged with respect to surface area where the particles originate), while for the pulsed SR, we only detect the visible emission, probably coming from close to the foot points of field lines leading into the current sheet, where the multiplicity (M_{vis}) may locally be much higher than M_{avg} . The idea is that there is an inner (interior) region where the M_{invis} is quite low, and a high M_{vis} in the gap, while the total multiplicity of particles (weighted by the areas of the interior PC region and the gap region near the rim) coming from the whole PC M_{avg} that is equal to the value implied by the fitting of the unpulsed LESR to the data (See [Figure 5.7](#)).

5.6.1 Unpulsed SR

In order to facilitate obtaining a consistent value for M_{avg} between the LESR and HESR components, I first increase M_{avg} connected to the LESR component, which will make subsequent adjustment of M_{vis} easier. Previously, we obtained a source strength (particle injection spectrum normalization; cf. [Kopp et al. 2013](#)) of $Q_0 \sim 1.35 \times 10^{34} \text{ erg}^{-1} \text{ s}^{-1}$

(Ndiyavala et al., 2019), which implied

$$\langle M_{\text{avg}} \rangle = \frac{Q_0(E_{\text{e,max}}^{1-\Gamma} - E_{\text{e,min}}^{1-\Gamma})}{(1-\Gamma)N_{\text{MSP}}\langle \dot{n}_{\text{GJ}} \rangle} - 1 \quad (5.24)$$

$$\approx 20 \left(\frac{\eta_{\text{p}}}{3\%} \right) \left(\frac{2.7 \times 10^{32} \text{ s}^{-1}}{\langle \dot{n}_{\text{GJ}} \rangle} \right) \left(\frac{\langle \dot{E} \rangle}{9 \times 10^{34} \text{ erg s}^{-1}} \right) \propto \langle \dot{E} \rangle^{1/2}, \quad (5.25)$$

with E_{min} and E_{max} the minimum and maximum particle energies, Γ the injection spectral slope, N_{MSP} number of MSPs in the GC, η_{p} the conversion efficiency of MSP spin-down luminosity $\dot{E}$ to particle luminosity and $\langle \dot{n}_{\text{GJ}} \rangle$ the Goldreich-Julian rate, reflecting the choice for the average $\langle P \rangle$ and $\langle B_{\text{s}} \rangle$ of the MSPs. We adapt the expression for the Goldreich-Julian (GJ) injection rate from 2 PCs by multiplying this by a factor $\cos \alpha$, with α the average magnetic inclination angle (Muslimov and Harding, 1997). For a population average $\alpha = 45^\circ$, this decreases the particle injection rate from the PC surface by ~ 0.7 , implying $M_{\text{avg}} \sim 30$. Next, the value for M_{avg} is particularly sensitive to the assumption of minimum particle energy $E_{\text{e,low}}$, but also to choices for maximum particle energy $E_{\text{e,max}}$, injection spectrum slope Γ , N_{MSP} , and conversion efficiency η_{part} (fraction of the spin-down luminosity converted to particle acceleration). For a parameter combination of $E_{\text{e,min}} = 3.06 \text{ GeV}$, $E_{\text{e,max}} = 19.3 \text{ TeV}$, $\Gamma = 1.78$, $B = 5.0 \mu\text{G}$, $\kappa_0 = 1.3 \times 10^{-5} \text{ kpc}^2 \text{ Myr}^{-1}$, $N_{\text{MSP}} = 37$, and $\eta_{\text{part}} = 3.4\%$, I now find $M_{\text{avg}} = 158$.

5.6.2 Pulsed Pair SR

Previously, we fit the data with our HESR component using $N_{\text{MSP}} = 37$ pulsars and a multiplicity of $M_{\text{vis}} = 510$. However, the angular unit error implies that $M_{\text{vis}} \approx 29000$. By assuming a slot-gap-like emission region for the pulsed SR demarcated by $r_{\text{ovc,min}} = 0.91$ and $r_{\text{ovc,max}} = 0.92$ (implying a ratio of the emission gap to PC cross-sectional areas of $a_{\text{vis}} = 0.018$), and setting $M_{\text{avg}} = 550$, one finds

$$M_{\text{invis}} = [M_{\text{avg}} - M_{\text{vis}}a_{\text{vis}}] [1 - a_{\text{vis}}]^{-1} = 20. \quad (5.26)$$

Lastly, by changing the average radio emission height r_{em} , one can make the pulsed X-ray emission intrinsically brighter (using $r_{\text{em}} = 0.4R_{\text{lc}}$ instead of $r_{\text{em}} = 0.2R_{\text{lc}}$), where the light cylinder radius is defined as $R_{\text{lc}} = cP/2\pi$, and P the pulsar period. This finally implies that $M_{\text{vis}} = 2.5 \times 10^3$, $M_{\text{invis}} = 115$, $M_{\text{avg}} = 158$. The latter value is the same as that implied by the new LESR fit.

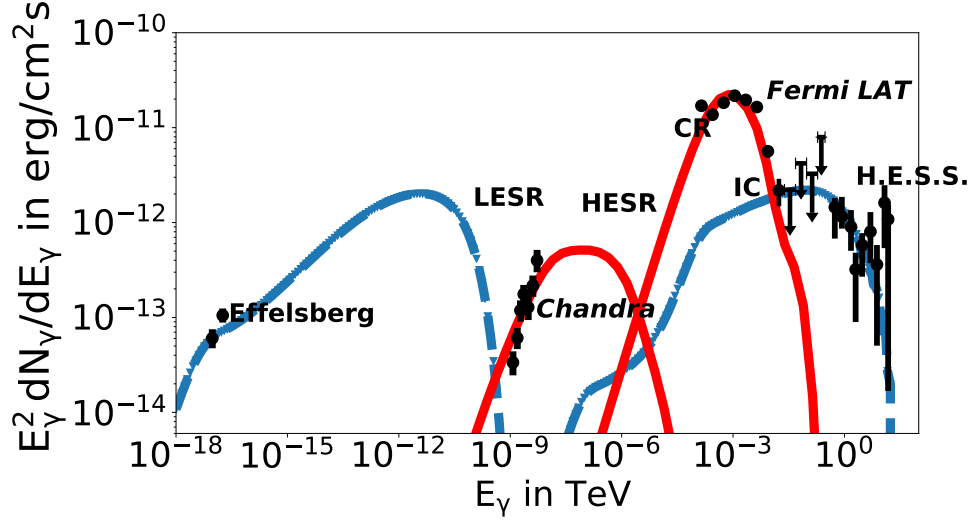


Figure 5.8: New scaled spectral components for Terzan 5 predicted by the leptonic models of Kopp et al. (2013) and Harding and Others (2008), Harding and Kalapotharakos (2015). For the LESR and IC components (integrated over all r_s ; dashed blue lines), I now invoke $E_{e,\min} = 3.06 \times 10^{-3}$ TeV, $E_{e,\max} = 19.3$ TeV, $Q_0 = 1.38 \times 10^{34}$ erg $^{-1}$ s $^{-1}$, $B = 5.0$ μ G, $\Gamma = 1.78$, and $\kappa_0 = 1.3 \times 10^{-5}$ kpc ^{2} Myr $^{-1}$. The HESR and CR components (red lines) are new predictions using the model of Harding and Others (2008), Harding and Kalapotharakos (2015) for $\alpha = 45^\circ$, $\zeta = 60^\circ$, $\langle P \rangle = 7.7 \times 10^{-3}$ s, and $\langle B_s \rangle = 5.8 \times 10^9$ G, linked to the new values of $M_{\text{vis}} \approx 29,000$ and $r_{\text{em}} = 0.4 R_{\text{lc}}$. I also indicate *Chandra* (Eger et al., 2010), H.E.S.S. Abramowski et al. (2011), and radio data (“Region 11” as defined by Clapson et al., 2011). This version of the figure should be considered as our final and corrected spectral fit.

5.7 Balancing the Energetics of the MSP Population

Even though the model of Harding and Kalapotharakos (2015) gives reasonable fits to the *Chandra* and *Fermi* data, one has to also consider whether this scenario is plausible in terms of the energetics and the sensitivity of *Chandra*, i.e., would *Chandra* have seen these “unresolved MSPs” postulated by the model to explain the diffuse X-ray flux seen by Eger et al. (2010), or can one explain the observed SR flux by a reasonable number of visible and unresolved (invisible) MSPs? The answer to this question lies in the (uncertain) population properties and emission energetics of the MSPs. We investigate this question by taking two approaches below.

We can obtain three constraints from the *Chandra* data analysis. A point-source sensitivity of $\sim 2 \times 10^{-15}$ erg s $^{-1}$ cm $^{-2}$ in the 0.5 – 7.0 keV band was assumed by Eger et al. (2010). This results to the first constraint of the minimum detectable luminosity of (i) $L_{\text{X},\text{Chandra}} \sim 7 \times 10^{30}$ erg s $^{-1}$ for their assumed distance of $d = 5.5$ kpc. This value is similar to the value of Heinke et al. (2006) who obtained $L_{\text{X},\text{Chandra}} \sim (1-3) \times 10^{31}$ erg s $^{-1}$ for an assumed distance of $d = 8.7$ kpc. Let us adopt the first value. Second, the total observed unabsorbed diffuse excess luminosity⁷ is $L_{\text{X,tot}} = 2 \times 10^{33}$ erg s $^{-1}$, the contribution

⁷We note that the power-law fit to the data implies that the visible non-thermal luminosity is $L_{\text{X,vis}} = 8.52 \times 10^{32}$ erg s $^{-1}$ (Eger et al., 2010), using data from annuli lying between $55''$ and $174''$. By integrating our predicted $E_\gamma dN/dE_\gamma$ HESR spectrum in the 1–7 keV band, we find $L_{\text{X,HESR}} \sim 5.5 \times 10^{32}$ erg s $^{-1}$. This

of unresolved point sources⁸ in the $1' - 3'$ region is estimated to be $= 7 \times 10^{31} \text{ erg s}^{-1}$ (Eger et al., 2010). We therefore set (ii) $L_{X,\text{vis}} = 2 \times 10^{33} \text{ erg s}^{-1} - 7 \times 10^{31} \text{ erg s}^{-1} = 1.93 \times 10^{33} \text{ erg s}^{-1}$ and (iii) $L_{X,\text{invis}}^X = 7 \times 10^{31} \text{ erg s}^{-1}$. One needs efficiency factor η_X in order to convert X-ray luminosities to pulsar spin-down values:

$$L_{X,\text{vis}} = \eta_{\text{vis}}^X N_{\text{vis}}^X \langle \dot{E} \rangle_{\text{vis}} \quad (5.27)$$

$$L_{X,\text{invis}} = \eta_{\text{invis}}^X N_{\text{invis}}^X \langle \dot{E} \rangle_{\text{invis}}, \quad (5.28)$$

with the total number of MSPs $N_{\text{tot}}^X = N_{\text{vis}}^X + N_{\text{invis}}^X \geq N_{\text{vis}}^{\text{rad}} = 37$. This is a very unconstrained system of equations, but one may, however, simplify this by assuming a single X-ray efficiency factor $\eta_X = \eta_{\text{vis}}^X = \eta_{\text{invis}}^X$. Division of the former equation by the latter and fixing $\langle \dot{E} \rangle_{\text{vis}}$ then yields the following constraint:

$$N_{\text{vis}}^X \langle \dot{E} \rangle_{\text{vis}} = k_L N_{\text{invis}}^X \langle \dot{E} \rangle_{\text{invis}}, \quad (5.29)$$

with $k_L = L_{X,\text{vis}}/L_{X,\text{invis}}$ being a constant. There is some degeneracy, but there are also reasonable parameters that may satisfy this constraint. For example $\langle \dot{E} \rangle_{\text{vis}} = 9.08 \times 10^{34} \text{ erg s}^{-1}$, $\langle \dot{E} \rangle_{\text{invis}} = 8 \times 10^{33} \text{ erg s}^{-1}$, $N_{\text{vis}}^X = 41$ and $N_{\text{invis}}^X = 17$ (implying $\eta_X = 0.05\%$). If we adopt $\langle \dot{E} \rangle_{\text{vis}} = 1.8 \times 10^{34} \text{ erg s}^{-1}$ (e.g., Abdo et al., 2010), the following values satisfy the constraint above: $\langle \dot{E} \rangle_{\text{invis}} = 10^{33} \text{ erg s}^{-1}$, $N_{\text{vis}}^X = 23$ and $N_{\text{invis}}^X = 15$ (implying $\eta_X = 0.5\%$); alternatively, we can set $\langle \dot{E} \rangle_{\text{vis}} = 10^{34} \text{ erg s}^{-1}$, obtaining $\langle \dot{E} \rangle_{\text{invis}} = 2.8 \times 10^{32} \text{ erg s}^{-1}$, $N_{\text{vis}}^X = 19$ and $N_{\text{invis}}^X = 25$ (implying $\eta_X = 1\%$). These numbers seem reasonable and adhere to the constraint that $N_{\text{tot}}^X = N_{\text{vis}}^X + N_{\text{invis}}^X \geq N_{\text{vis}}^{\text{rad}} = 37$. We therefore consider our assumption of attributing the X-ray emission to the cumulative pair SR as justified.

We next consider a parameterised pulsar spin-down luminosity function $N_{\text{MSP}}(>\dot{E}) \propto \dot{E}^{-\gamma_L}$ (Johnston and Verbunt, 1996) to obtain stronger constraints on the MSP population and break some parameter degeneracies. We will use this to balance the required X-ray energetics by assuming that $\dot{E}_{\text{vis}} \propto L_{X,\text{vis}}$ and $\dot{E}_{\text{invis}} \propto L_{X,\text{invis}}$. This implies $dN/d\dot{E} = N'_0 (\dot{E}/\dot{E}_0)^{-(\gamma_L+1)}$, with N'_0 a normalisation constant. A GC value of $\gamma_L \sim 0.5$ was inferred by Johnston and Verbunt (1996), while Heinke et al. (2006) find $\gamma_L \sim 0.4 - 0.7$ for Terzan 5, depending on the energy band. One can next recover the following quantities by defining

is close to this luminosity, with the discrepancy explained by the fact that the model does not perfectly match the data in terms of the spectral slope. However, this power-law luminosity is a factor of ~ 2 lower than the total observed luminosity as noted in the main text, which is also the number quoted by Eger et al. (2010) in their interpretation section. We decided to use the higher value, following this usage by Eger et al. (2010), and note that if we use the lower value, the solutions in Table 5.1 have similar best-fit parameters but with lower MSP numbers reflecting the lower value of $L_{X,\text{vis}}$ in this case.

⁸We use the label “invisible” in what follows to refer to those pulsars that have too low a spin-down luminosity to be detectable as single point sources by *Chandra* but that may contribute to the cumulative unresolved point-source luminosity as a population of low-energetic pulsars. As before, we discard the contribution of other source classes to this unresolved luminosity.

$$\dot{E}_b = L_{\text{X,Chandra}}/\eta_{\text{X}}:$$

$$N_{\text{tot}}^{\text{X}} = \int_{\dot{E}_{\text{min}}}^{\dot{E}_{\text{max}}} \left(\frac{dN}{d\dot{E}} \right) d\dot{E}, \quad (5.30)$$

$$N_{\text{vis}}^{\text{X}} = \int_{\dot{E}_b}^{\dot{E}_{\text{max}}} \left(\frac{dN}{d\dot{E}} \right) d\dot{E}, \quad (5.31)$$

$$N_{\text{invis}}^{\text{X}} = \int_{\dot{E}_{\text{min}}}^{\dot{E}_b} \left(\frac{dN}{d\dot{E}} \right) d\dot{E}, \quad (5.32)$$

$$\langle \dot{E} \rangle_{\text{vis}} = \frac{1}{N_{\text{vis}}^{\text{X}}} \int_{\dot{E}_b}^{\dot{E}_{\text{max}}} \dot{E} \left(\frac{dN}{d\dot{E}} \right) d\dot{E}, \quad (5.33)$$

$$\langle \dot{E} \rangle_{\text{invis}} = \frac{1}{N_{\text{invis}}^{\text{X}}} \int_{\dot{E}_{\text{min}}}^{\dot{E}_b} \dot{E} \left(\frac{dN}{d\dot{E}} \right) d\dot{E}. \quad (5.34)$$

We have four quantities that we want to solve: $\dot{E}_{\text{min}}$, $\dot{E}_{\text{max}}$, N_0' (or equivalently $N_{\text{tot}}^{\text{X}}$), and γ_{L} ; once these are fixed, we can infer the MSP population properties through the above equations. We note, however, that we are using this luminosity function to fit X-ray luminosities, which are integral quantities. We therefore expect to find degenerate solutions as different combinations might give the same integral luminosities. We therefore need four constraints or measurements. We can use the same three constraints as before. Crucially, one needs to specify a fourth parameter η_{X} to convert from spin-down luminosities to X-ray luminosities. We can implicitly fix the product $N_{\text{vis}}^{\text{X}} \langle \dot{E} \rangle_{\text{vis}}$ by fixing η_{X} . As a first attempt, we assume $\eta_{\text{X}} = 0.05\%$ (e.g., for $N_{\text{vis}}^{\text{X}} = 41$ and $\langle \dot{E} \rangle_{\text{vis}} = 9.08 \times 10^{34} \text{ erg s}^{-1}$ to make the calculation consistent with the previous estimate). It is difficult to obtain the actual value for $\langle \dot{E} \rangle_{\text{vis}}$, given the effect of the GC potential on the $\dot{P}$ of each MSP (e.g., [Bogdanov et al., 2008](#)). If we could, this would further constrain the system via [Equation 5.33](#). [Heinke et al. \(2006\)](#) noted that while they did not detect an X-ray MSP explicitly, one X-ray source could plausibly be an MSP based on the proximity to a radio MSP position; they also noted that more detections of X-ray MSPs could be made as radio positions become available. We thus have additional constraints $N_{\text{vis}}^{\text{X}} \gtrsim 1$ and $N_{\text{tot}}^{\text{X}} \geq N_{\text{vis}}^{\text{rad}}$ (which may be used as checks on the consistency of the solutions we obtain). We obtain a non-unique solution for each fixed value of η_{X} . However, η_{X} is not known and the parameters that satisfy the other three constraints are quite degenerate, as expected. [Table 5.1](#) indicates a number of parameter combinations that satisfy the observational constraints⁹. It is clear that a different choice of η_{X} will favour a different solution that will imply a different value of $\langle \dot{E} \rangle_{\text{vis}}$ (which also depends on the average moment of inertia $\langle I \rangle$, $\langle P \rangle$, and $\langle \dot{P} \rangle$). For example, a higher value of η_{X} will yield a lower value of $\langle \dot{E} \rangle_{\text{vis}}$ or $\langle I \rangle$ for a given value of $N_{\text{vis}}^{\text{X}}$ and keeping other parameters fixed. If we require $\langle \dot{E} \rangle_{\text{vis}}$ to be the same as assumed in

⁹A preliminary Markov chain Monte Carlo investigation ([Foreman-Mackey et al., 2013](#)) confirmed the degenerate nature of the free parameters (some are correlated), as well as their being quite unconstrained (reflected by asymmetrical and flat probability distributions, as well as elongated confidence contours). Best-fit values furthermore depend on the choice of priors/parameter bounds. The median values are, however, similar to those in [Table 5.1](#).

the model used to predict the pulsed emission, this may lead to unrealistic values for γ_L for a given η_X . Relaxing this requirement (which may easily be done, given other parameter uncertainties) implies more suitable values for the other parameters. It is therefore clear that the system of equations is very coupled and the parameters are degenerate, given the lack of suitable constraints. One may think to constrain the solution space by requiring $N_{\text{tot}}^X = N_{\text{tot}}^\gamma \approx N_{\text{vis}}^\gamma = 180_{-100}^{+120}$, the latter being the estimated total number of MSPs in Terzan 5 as inferred from the *Fermi*-measured GeV energy flux (Abdo et al., 2010). However, this estimate is uncertain and does not contain uncertainties in distance (the square of which determines the γ -ray luminosity L_γ) and conversion efficiency of $\dot{E}_{\text{vis}}$ to L_γ , so this does not seem to be a strong constraint. Likewise, we chose $N_{\text{tot}}^\gamma = 37$ when fitting the CR component, but this value is also subject to other model assumptions, such as MSP geometry and gap width. Finally, it seems that the last few entries in Table 5.1 might be the more plausible combinations in view of the independent constraints on $\gamma_L \sim 0.4 - 0.7$ and $N_{\text{vis}}^X \gtrsim 1$ (i.e., probably relatively small numbers of visible X-ray pulsars) gleaned from the analysis of Heinke et al. (2006). Thus, the uncertainty in several parameters, particularly η_X , as well as parameter degeneracies preclude us from making definite statements about the MSP population properties. Yet we see that there are several plausible solutions that characterise and constrain the MSP population’s energetics, implying that the scenario of MSPs being responsible for the broadband SED may be justified and thus plausible.

Gonthier et al. (2018) derive a luminosity function for MSPs in 47 Tucanae through a population synthesis that fits the *Fermi* LAT spectrum, presumed to be the combined emission from all MSPs in the cluster. They found a γ -ray luminosity distribution peaking at $\sim 10^{33} \text{ erg s}^{-1}$ and a spin-down power distribution peaking at $\sim 2 \times 10^{34} \text{ erg s}^{-1}$, extending down to $\sim 10^{30} \text{ erg s}^{-1}$, with a γ -ray efficiency around 0.1. Their peak $\dot{E}$ value is similar to the $\langle \dot{E}_{\text{vis}} \rangle$ we have derived for Terzan 5.

Table 5.1: Sample parameter combinations that lead to a balance of the X-ray-implied energetics.

η_X	$\dot{E}_{\text{invis}}$	$\dot{E}_{\text{vis}}$	$\dot{E}_{\text{min}}$	$\dot{E}_{\text{max}}$	γ_L	N_{vis}^X	N_{invis}^X	N_{tot}^X
0.05%	3.0×10^{33}	9.2×10^{34}	10^{31}	2.4×10^{35}	-0.19	43	45	88
0.05%	3.5×10^{32}	1.8×10^{35}	10^{29}	10^{36}	0.21	22	399	421
0.5%	1.2×10^{32}	3.8×10^{34}	10^{31}	10^{36}	0.50	10	116	126
0.5%	1.5×10^{32}	2.8×10^{34}	10^{31}	3.6×10^{35}	0.40	14	96	110
1%	7.4×10^{31}	2.5×10^{34}	10^{31}	2.0×10^{36}	0.60	8	95	103
1%	1.3×10^{32}	2.4×10^{34}	3.0×10^{31}	2.9×10^{36}	0.64	8	53	61
1%	2.5×10^{32}	2.0×10^{34}	10^{32}	3.0×10^{36}	0.69	10	27	37

The bulk of visible radio MSPs occur in the core of the cluster, and one expects the majority of pulsars here to be due to the deep potential well of the GC. However, one may expect to find a small number of MSPs farther out, depending on their birth and evolutionary history. The fact that the diffuse X-ray flux profile measured by Eger et al. (2010) drops off slightly slower than the generalised King profile fit (e.g., King, 1962) to the detected X-ray point source distribution (Heinke et al., 2006), as well as the infrared

surface brightness profile measured by [Trager et al. \(1995\)](#), may support this idea, and a (slowly) decreasing MSP density with radius may plausibly correlate with the observed decreasing X-ray flux profile. [Eger et al. \(2010\)](#) detected non-thermal X-ray emission beyond the half-mass radius of Terzan 5. If we take the above energetics argument as plausible, this would imply possibly tens of unresolved MSPs and a handful MSPs that are in principle visible in X-rays in this region. This would also imply that even more MSPs should be visible in X-rays at the core versus outer reaches of the GC, but we do not have constraints on the diffuse X-ray emission at the GC centre at this stage, and source confusion in this dense region may complicate the matter. Future constraints on the central diffuse X-ray emission, the spatial distribution of the MSP population, and the average expected multiplicity and spin-down power will thus more deeply probe our hypothesis that the HESR component is due to magnetospheric, pulsed SR from pairs.

5.8 Summary

The main focus of this Chapter has been to gather more data on Terzan 5 and to scrutinise ideas about the particle sources and emission processes responsible for the broadband emission spectrum observed from this cluster. The models I used postulated four spectral components (LESR, HESR, CR and IC). I also attempted to constrain the MSP population's distribution of spin-down luminosity using the observed X-ray diffuse emission.

I obtained new *Fermi* data that I could fit using a model for the cumulative CR from a population of MSPs embedded within Terzan 5. These data also proved to be constraining for the low-energy tail of the unpulsed IC component.

I fit the radio spectral points by invoking an LESR component that might extend into the optical range, and argued that our predicted LESR flux is far below the expected thermal optical flux level. Thus, obtaining an upper limit on the non-thermal flux in the optical band would be very difficult, since there roughly $N_* \sim 10^{5-6}$ point sources that have to be subtracted from an optical map of the GC. Even when performing and subtracting a King model fit to the surface brightness profile, the uncertainty on the remaining diffuse flux would be very large. However, since the BB spectrum occurs over a much narrower energy range than the LESR, there might be hope of detecting the non-thermal flux outside of the optical range.

[Venter and De Jager \(2008b\)](#), [Venter et al. \(2009a\)](#), [Kopp et al. \(2013\)](#) predicted that GCs should typically have SR components that peak in the optical/ultraviolet range, and [Bednarek et al. \(2016\)](#) also points out the problem of the dominating radiation field produced by the large population of GC stars. They also mentioned that typical parameters would be needed to produce an observable level of X-ray flux, and it would be problematic to compare optical and X-ray brightness profiles, since the underlying source distributions

have different spatial and emission characteristics. Thus, to fit the observed data, we invoked a new component (HESR) to explain the hard *Chandra* spectrum: cumulative SR from pair plasma in MSP magnetospheres. The low-energy tail of this HESR component reproduces the spectral slope of the X-ray data quite well. We argued that the required energetics and numbers of the MSP source population needed to reproduce the detected diffuse X-ray emission are plausible. The MSP scenario to explain the broadband SED of Terzan 5 should therefore be scrutinised further by future constraints on the properties (e.g., number of visible X-ray pulsars and their average spin-down luminosity) of the MSPs embedded in this GC.

There were no new data available for the VHE band. The high-energy tail of our predicted unpulsed IC component produced a good fit for the current H.E.S.S. data. More data by H.E.S.S. or new data from the CTA may better constrain the shape and cut-offs of the IC component, owing to the lower energy threshold as well as increased sensitivity of the CTA.

We have modelled the pulsed SR and CR (in red) components using a magnetospheric pulsar model, while we have modelled the LESR and IC components using an independent transport and emission model. While we have attempted to apply both these codes simultaneously for consistent parameter choices, a unified approach may lead to even deeper constraints on the cluster environment and stellar members.

I corrected an error in units that impacted the predictions of the pulsed spectra. Using a two-step multiplicity, I could fit the data for a consistent value of M_{avg} as implied by the LESR and HESR components.

Using Terzan 5 as a case study, we could constrain our leptonic model for broadband emission from GCs. CTA is expected to detect many more VHE GCs ([Ndiyavala et al., 2018](#)), while multi-wavelength data on these sources should also continue to improve in both quantity and quality. This will allow us to scrutinise competing emission models, as well as developing new, more complete and comprehensive ones that might explain the spatial *and* spectral properties of Galactic GCs.

Chapter 6

Assessing Uncertainties in the Predicted Very-High Energy Flux of Galactic GCs in the Cherenkov Telescope Array Era

In the previous Chapter, I estimated optical upper limits to fit the multi-wavelength data of Terzan 5. I also fit the broadband SED using four spectral components from two models. This Chapter is based on our recent paper accepted to Monthly Notices of the Royal Astronomical Society (MNRAS). In this Chapter, I will assess uncertainties in the predicted VHE γ -ray flux of GCs for the given leptonic model introduced in Section 3.2 and give theoretical guidance to CTA's observational strategy.

6.1 Introduction

[Ndiyavala et al. \(2018\)](#) predict that tens of Galactic GCs may be detectable within a reasonable amount of observation time for CTA. However, emission models predict quite a wide range of fluxes depending on how well constrained the model parameters are. Several leptonic and hadronic models exist that predict the radiated spectrum by GCs (for more see details Chapter 3). It is therefore important to assess the uncertainties on the predicted flux by each model when assessing their viability.

H.E.S.S. searched for VHE γ -ray emission from 15 other Galactic GCs and did not detect any of those; neither did they detect any significant signal using a stacking analysis (involving a live-time-weighted GC flux; [Abramowski et al., 2013](#)). Therefore, they derived integral flux upper limits from their single-GC and stacking analyses. Interpreting these upper

limits, they noted that a simple scaling of existing theoretical predictions invoking IC from relativistic leptons produces a VHE flux that violates these limits. Observations by other Cherenkov telescopes could only produce upper limits (e.g., [Anderhub et al., 2009](#), [McCutcheon, 2009](#)). Observations of the GC 47 Tucanae were performed with H.E.S.S. leading to an upper limit on the integral γ -ray flux of $F(E > 800 \text{ GeV}) < 6.7 \times 10^{-13} \text{ cm}^{-2}\text{s}^{-1}$ ([Aharonian et al., 2009](#)). Observations of the GC M15 by MAGIC telescopes provided a deep upper limit on $F(E > 300 \text{ GeV})$, i.e., $< 0.26\%$ of the Crab Nebula flux. Also, stringent differential flux upper limits were obtained on this source ([Acciari et al., 2019](#)). Although we focus on γ -rays, we note that GCs are broadband emitters, i.e., they are also detected in X-rays, optical, and radio and these data are typically modelled using an SR spectrum ([Eger et al., 2010](#), [Abdo et al., 2010](#), [Clapson et al., 2011](#)) as explained in Chapter 2.

In this Chapter, I model the IC γ -ray flux expected from several GCs to see whether the predicted flux exceeds the CTA sensitivity and whether we can satisfy the H.E.S.S. upper limits with a range of predicted fluxes (due to parameter uncertainties). We argue that one cannot simply scale the model output among different GCs, since the range of predicted fluxes depends crucially (and non-linearly in some cases) on the unique input parameters plus their uncertainties per source. The use of a population of clusters versus a single cluster should furthermore reduce the uncertainty in the predicted stacked flux, which can be used to make a better informed decision of whether a particular model should indeed be discarded in light of measurements or not; conversely, such an approach may yield more stringent constraints on the model parameters, if a given model remains viable. I will specifically (1) model a population of GCs and compare the predicted *integral* fluxes with H.E.S.S. upper limits, (2) model both the Northern-hemisphere cluster, M15, and Southern-hemisphere one, Omega Cen (ω Cen), and compare our results with *differential* flux upper limits. We chose these two GCs given the deep MAGIC upper limits found for M15, and given the recent radio pulsar detections in the case of the latter. I will illustrate that there are indeed regions in parameter space for which the stringent stacking upper limits by H.E.S.S. are satisfied, for some subset of free parameters, given their uncertainty. Yet larger parameter uncertainties and more free model parameters will only increase the uncertainty on the predicted flux, making it even easier to satisfy these observational constraints.

6.2 The Model and its Free Parameters

A large number of spectral components is expected from GCs ([Ndiyavala et al., 2019](#)). The unpulsed emission should transpire through the continuous injection of relativistic leptons by the MSPs into the ambient GC region ([Bednarek and Sitarek, 2007](#)), which in turn produce unpulsed SR and IC emission when they encounter the cluster magnetic field

as well as background (stellar and CMB) photon components. We also expect cumulative pulsed SR and CR components initiated by primaries and electron-positron pairs from within the magnetospheres of MSPs embedded in the cluster (Harding et al., 2005, Venter and De Jager, 2005). The model by Kopp et al. (2013) that I use here calculates the particle transport (including diffusion and radiation losses) in a spherically symmetric, stationary approach and predicts the SED from GCs for a very broad energy range by considering unpulsed SR and IC. The main free parameters of this model are (cf. Ndiyavala et al., 2018, 2019): cluster magnetic field (B), power-law index (Γ) of the injected particle spectrum (the particles are assumed to have undergone acceleration in inter-pulsar shocks; Bednarek and Sitarek, 2007), number of GC stars (N_*), distance to the cluster (d), the average spin-down luminosity per pulsar ($\langle \dot{E} \rangle$), the conversion efficiency of spin-down luminosity into particle acceleration (η), and number of MSPs in the GC (N_{MSP}). For more details see Chapter 3.

One can obtain the integral flux via

$$F(E_\gamma \geq E_{\text{th}}) = \int_{E_{\text{th}}}^{E_{\gamma, \text{max}}} E_\gamma \frac{dN_\gamma}{dE} dE_\gamma \quad (6.1)$$

where E_{th} is the threshold energy for each cluster. I kept the spatial diffusion coefficient (κ), the core radius (r_c), the half-mass radius (r_h), and the tidal radius (r_t) as fixed parameters.

6.3 Monte Carlo Method

6.3.1 Integral Flux Upper Limits: the H.E.S.S. GC Population

Venter and Kopp (2015) applied a leptonic GC model to the population of clusters mentioned in Abramowski et al. (2013) to predict the live-time-averaged (stacked) flux and compare it to the H.E.S.S. stacking upper limits. They used fixed (best-guess) model parameters as noted in their Table 1, and used the inferred values from Abdo et al. (2010) for N_{MSP} where possible, and N_* values from Lang (1993), and obtained distances d and structural parameters¹ r_c , r_{hm} , and r_t from Harris (1996). Typical values of $\eta \sim 0.01$ (Venter and De Jager, 2005), $N_{\text{MSP}} \sim 25$, and $\langle \dot{E} \rangle \sim 2 \times 10^{34} \text{ erg s}^{-1}$ led to typical values of $Q_0 \sim 10^{33-34} \text{ erg}^{-1} \text{ s}^{-1}$ for the source strength (see Equation 3.15). They also used $B = 5 \mu\text{G}$ and $\Gamma = 2.0$ throughout, and assumed optical photons from the stellar population embedded in the GC plus CMB photons as target fields for their IC calculation (assuming an average stellar temperature of $\langle T \rangle = 4500 \text{ K}$). They found that none of the predicted single-cluster spectra violated the H.E.S.S. TeV upper limits for individual sources. Their averaged flux furthermore satisfied the stringent stacking upper limits.

¹<http://gclusters.altervista.org/index.php>

However, they noted that different choices of parameters led to average fluxes that in some cases may exceed these limits. That is why I refine their calculation in this Chapter, now also including an estimation of uncertainties on the predicted VHE fluxes.

In this Section, I describe the method I employ for modelling the IC γ -ray flux expected from the 15 Galactic GCs observed by H.E.S.S. I follow the method of [Venter and De Jager \(2008a\)](#) where they used a Monte Carlo technique to model the pulsed CR γ -ray flux and associated uncertainties expected from 100 MSPs hosted by a GC. Their calculated CR flux was randomised over the magnetic inclination (α) and observer (ζ) angles that are usually estimated from radio polarisation measurements involving the rotating vector model ([Manchester and Johnston, 1995](#)), as well as taking into account the fixed, pulsar-specific parameters such as period P and period derivative $\dot{P}$ (or $\dot{E}$) of each MSP member. In our case, I use the same method to calculate the single-GC and stacked integral VHE flux of 15 GCs ([Abramowski et al., 2013](#)), with uncertainties due to the uncertainty in model parameters. I do this by randomising over seven free parameters as mentioned in Section 6.2: B , Γ , N_* , d , η , $\dot{E}$, and N_{MSP} , in addition to fixing the structural and diffusion coefficient parameters as noted above.

I first apply the model of [Kopp et al. \(2013\)](#) to the population of clusters to obtain the differential spectrum (Section 6.2) for different parameter combinations, initially setting $Q_0 = 1$ and only using four free parameters (B , Γ , N_* , and d). I then calculate the integral flux using Equation 6.1 and obtain a spread of fluxes, based on the range of values we picked for these four parameters. To indicate the build-up of a smooth flux distribution, I use the four free parameters for illustration (for the full modeling, we use seven). Figure 6.1 thus indicates how a distribution of $\log_{10}$ of the integral flux becomes smooth as we increase the resolution at which each of four free parameters is sampled. A finer grid thus lead to a smoother distribution of fluxes. Second, I indicate convergence of the flux distribution with an increase of the number of trials, again for four free model parameters. That is, I calculate the integral flux for N_t trials (each trial corresponding to a different parameter combination on a pre-defined grid), obtaining convergence as N_t gets close to or exceeds the number of unique combinations (N_{comb}) of the four free parameters. Thus, I reproduce the last panel of Figure 6.1 to indicate convergence in Figure 6.2. We note that undersampling will not give a smooth flux distribution, but oversampling will lead to convergence.

I thirdly check for the convergence of the source strength parameter Q_0 (see Figure 6.3) via two methods, using 47 Tucanae as an example. For the first method, I pick values of four relevant (different) parameters on a specified grid: $\eta \in [0.005, 0.08]$ in steps of 0.005, $\log_{10}\langle\dot{E}\rangle \in [33.7, 34.7]$ in steps of 0.05, $N_{\text{MSP}} \in [5, 150]$ in steps of 1, and $\Gamma \in [1.7, 2.9]$ in steps of 0.1. For the second method, I randomise over the same four parameters η , $\log \dot{E}$, N_{MSP} , and Γ , for fixed illustrative values of $E_{\text{e,min}} = 0.1 \text{ TeV}$ and $E_{\text{e,max}} = 100 \text{ TeV}$, within the parameter ranges given above (but not on a fixed grid). I conduct $N_t = 10^6$

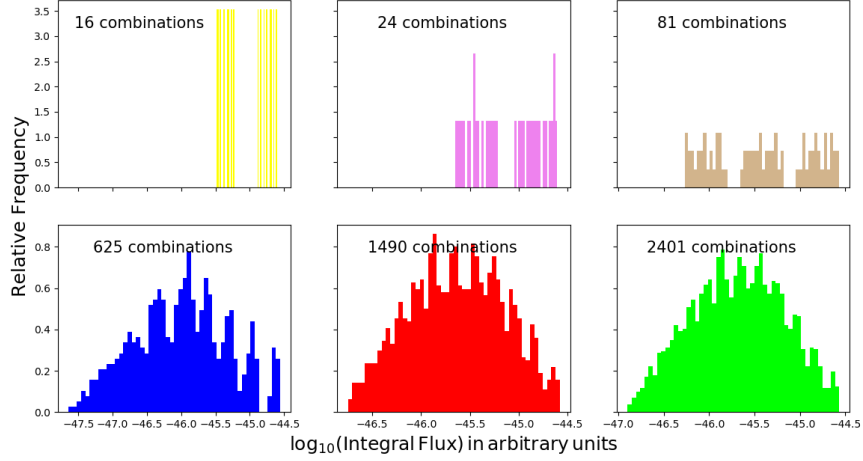


Figure 6.1: Histograms of $\log_{10}(F(E > 0.72 \text{ TeV}))$ in arbitrary units, indicating how we build up statistics for the flux distribution for a finer grid in parameter space. The x -axis shows the integral fluxes while the y -axis shows the normalised frequency. The label indicates the number of unique parameter combinations in each case.

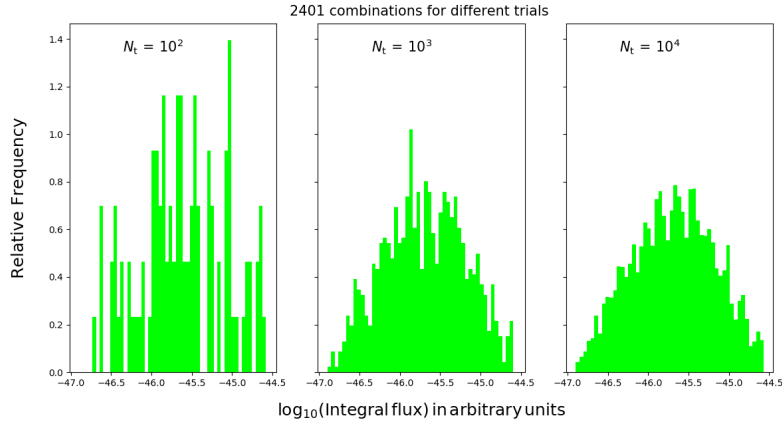


Figure 6.2: Histograms of $\log_{10}(F(E > 0.72 \text{ TeV}))$ in arbitrary units, indicating convergence for N_t getting close to number of combinations (N_{comb}) and $N_t > N_{\text{comb}}$.

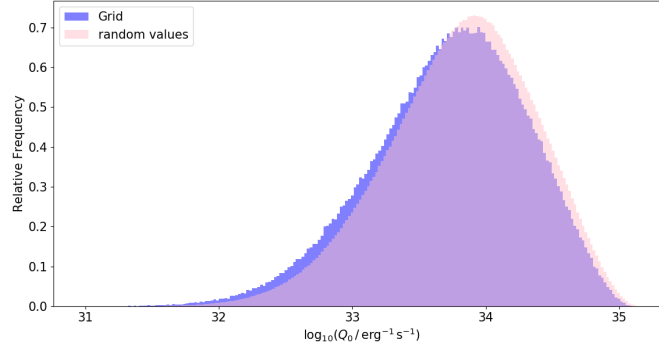


Figure 6.3: Histogram of $\log_{10}(Q_0)$ obtained via two different methods: sampling from a grid or randomly sampling for a set range for four free parameters. The pink histogram has a median $Q_{0,\mu} = 7.0 \times 10^{33} \text{ erg}^{-1} \text{ s}^{-1}$, 16th percentile of $1.8 \times 10^{33} \text{ erg}^{-1} \text{ s}^{-1}$, and 84th percentile of $2.3 \times 10^{34} \text{ erg}^{-1} \text{ s}^{-1}$ whilst the blue histogram has $Q_{0,\mu} = 5.9 \times 10^{33} \text{ erg}^{-1} \text{ s}^{-1}$, 16th percentile of $1.4 \times 10^{33} \text{ erg}^{-1} \text{ s}^{-1}$, and 84th percentile of $2.0 \times 10^{34} \text{ erg}^{-1} \text{ s}^{-1}$.

trials in both cases. The two different approaches yield very similar results, which also correspond to the values found by [Kopp et al. \(2013\)](#) as well as to the calculation by [Venter and Kopp \(2015\)](#), with the two peaks in the histogram only slightly shifted with respect to each other. Thus, sampling on a grid vs. randomly picking parameter values does not make too much of a difference in terms of the distribution of Q_0 .

I lastly perform nested loops over the seven free model parameters to calculate the source strength $Q_0(\Gamma, \eta, \langle \dot{E} \rangle, N_{\text{MSP}}) \neq 1$, and multiply the correct flux, pre-calculated for $Q_0 = 1$ and depending only on B, Γ, d , and N_* , by the actual value of Q_0 to obtain the final differential and integral fluxes in observational units (see [Equation 3.15](#) and the final panel of [Figure 6.4](#) for the case of 47 Tucanae). The spread in flux reflects the uncertainty in model parameters. I use the observational threshold energy E_{th} associated with a particular cluster to calculate $F(> E_{\text{th}})$. In [Figure 6.4](#), I randomly sample integral fluxes for a different total number of trials N_t from the grid of pre-calculated fluxes to show convergence as $N_t \rightarrow N_{\text{comb}}$.

6.3.2 Differential Flux Predictions: M15 and ω Cen

MAGIC has recently published differential flux upper limits for M15 ([Acciari et al., 2019](#); see [Section 6.4.2](#) for details on the source and VHE data). I apply the model of [Kopp et al. \(2013\)](#) to M15 to obtain the *differential* fluxes for each combination of seven free model parameters (see [Section 6.2](#)). Next, I calculate the final differential flux as described in [Section 6.3.1](#).

I again use the method of [Venter and De Jager \(2008a\)](#) to estimate the uncertainty in the differential flux of M15 at 13 different observer-defined energy bins ([Acciari et al., 2019](#)). Specifically, I logarithmically interpolate model fluxes that were calculated on a prescribed

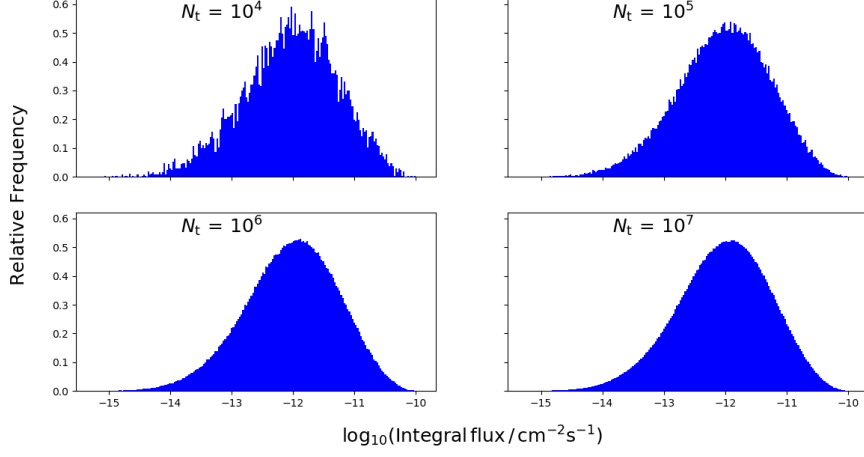


Figure 6.4: Histograms of $\log_{10}(F(E > 0.72 \text{ TeV}) / \text{cm}^{-2}\text{s}^{-1})$ for different N_t as indicated in each panel, for 47 Tucanae, and for 7 free parameters.

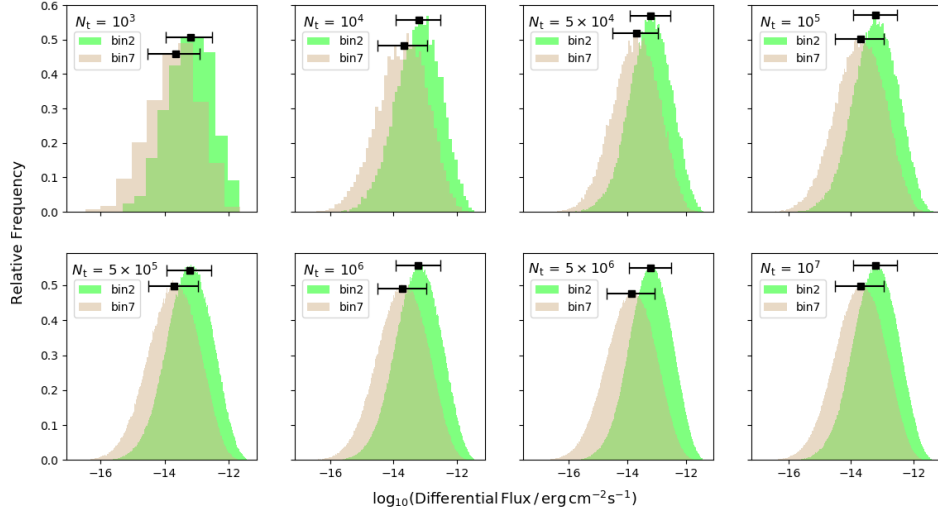


Figure 6.5: Distribution plots of $\log_{10}$ of the differential flux for MAGIC bin 2 ($E = 62 \text{ GeV}$) and bin 7 ($E = 954 \text{ GeV}$), with the median and 1σ uncertainties indicated by the black square and error bars. The x -axis shows the $\log_{10}$ of the νF_ν flux while the y -axis shows the normalised frequency. The number of trials N_t in each case is indicated in each panel.

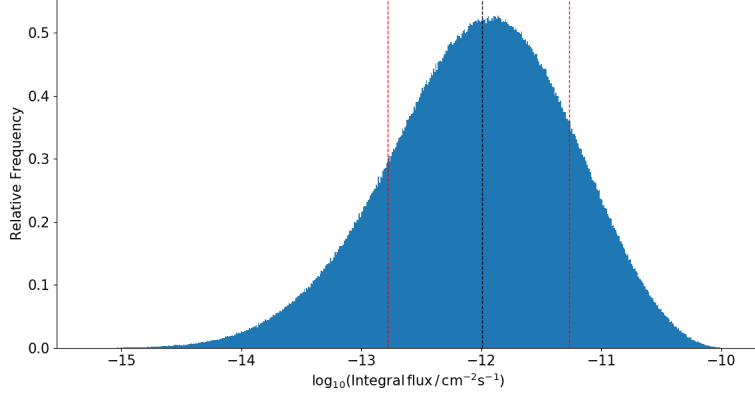


Figure 6.6: Histogram of $\log_{10}(F(E > 0.72 \text{ TeV}) / \text{cm}^{-2}\text{s}^{-1})$. The x -axis shows the $\log_{10}$ of integral flux while the y -axis shows the normalised frequency. The red dashed lines represent the 16th and 84th percentiles, and black dashed line shows the median of the integral flux distribution for 47 Tucanae.

energy grid (see Equation 6.2 to obtain predictions at the specified observational energies associated with the upper limits. The interpolated differential flux is

$$DF = e^A \quad (6.2)$$

where DF is the interpolated differential flux and

$$A = (1 - F)y_1 + Fy_2, \quad (6.3)$$

$$F = \frac{\log E_\star - \log E_1}{\log E_2 - \log E_1}, \quad (6.4)$$

$$y_1 = \log(DF_1), \quad (6.5)$$

and

$$y_2 = \log(DF_2). \quad (6.6)$$

Our parameter grid for M15 was chosen as follows: $B \in [1, 3, 5, 7, 9] \mu G$, injection spectral index $\Gamma \in [1.7, 2.0, 2.3, 2.6, 2.9]$, $N_\star \in [2.05, 2.90, 4.10, 5.80, 8.20] \times 10^5$, the distance $d \in [5.20, 7.35, 10.4, 14.7, 20.8] \text{ kpc}$, $\eta \in [0.003, 0.063]$ with steps of 0.005, $\log_{10}\langle\dot{E}\rangle \in [33.7, 34.7]$ with steps of 0.05, and the number of MSPs $N_{\text{MSP}} \in [8, 24]$, in steps of 1. In the case of ω Cen, I used the same B and Γ range as for M15, but $N_\star \in [0.5, 0.7, 1.0, 1.4, 2.0] \times 10^6$, $d \in [2.6, 3.7, 5.2, 7.6, 10.5] \text{ kpc}$, $\eta \in [0.005, 0.055]$, $\log_{10}\langle\dot{E}\rangle \in [33.9, 34.9]$ in steps of 0.05, and $N_{\text{MSP}} \in [5, 299]$ in steps of 1. I conducted a maximum of $N_t = 10^7$ trials, with $N_{\text{comb}} = 2.9 \times 10^6$ for M15 and $N_{\text{comb}} = 7.4 \times 10^6$ for ω Cen, respectively (cf. Figure 6.5, where the value of N_t is indicated in each panel).

6.4 Results

6.4.1 The H.E.S.S. GC Population

To obtain a sample of GCs suited for the study by H.E.S.S., [Abramowski et al. \(2013\)](#) selected a population of clusters based on the GC catalog of [Harris \(2010\)](#) from which all the basic parameters such as position, d , r_c , and central luminosity density ρ_0 have been taken. [Abramowski et al. \(2013\)](#) furthermore applied different cuts on the target and observation run lists:

1. GC $|\text{Galactic latitude}| \geq 1.0^\circ$. This cut was done to conservatively exclude GCs towards the direction of the Galactic Plane to avoid faint emission from unresolved sources, diffuse emission, and also chance coincidences with unrelated sources.
2. Standard quality selection criteria were applied, with the telescopes pointed $< 2.0^\circ$ offset from the GC position.
3. A selected GC had to have at least 20 available runs passing the cut described in point 2 to ensure a reasonably long exposure on these potentially faint sources.

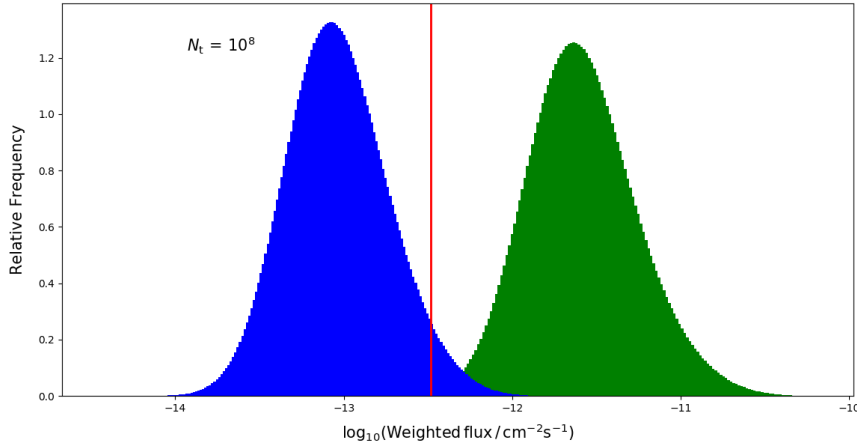


Figure 6.7: Distribution of $\log_{10}(F_w(E > E_{th}) / \text{cm}^{-2}\text{s}^{-1})$ for all GCs. The x -axis shows the $\log_{10}$ of total weighted flux while the y -axis shows the normalised frequency. The green histograms are associated with the first parameter combination, while the blue ones are for the second parameter combination. The red line represents the stacked upper limit as noted in [Table 6.1](#). We used $N_t = 10^8$.

In order to compare our model predictions with the H.E.S.S. flux upper limits F_{UL} , I have to characterise the mean predicted flux plus its uncertainty² First, I obtain the median

²We note that it is common in certain statistical contexts to use the 2nd quartile (median) as an indication of an average value, and the 1st and 3rd quartiles (25th and 75th percentiles) as indications of uncertainty or the spread of values around the median, as is done in box plots. However, we opt to use the median and the 16th and 84th percentiles to estimate the average and uncertainty in model flux (in analogy to what is typical for a normal distribution of values), as is commonly done in certain physics contexts, given the fact that our flux distributions appear to be approximately log-normal. Our main conclusion,

Table 6.1. H.E.S.S. flux upper limits from Abramowski et al. (2013) as well as model predictions corresponding to a first set of parameter combinations (associated with the green histograms in Figure 6.7). The columns in the Table are: the GC name; energy threshold of the analysis, defined as the location of the peak in the distribution of reconstructed photon energies; integral photon flux upper limits (99% confidence level following Feldman and Cousins, 1998) assuming a power law with an index of 2.5 (Abramowski et al., 2013) $F_{\text{UL}}(E > E_{\text{th}})$; median of $\log_{10}$ of integral flux distribution F_{μ} ; ratio between the σ_{16} and the median; ratio between σ_{84} and the median; ratio between the median and the flux upper limit; geometric mean to characterise distributions with a significant variation across many orders of magnitude; ratio between standard deviation σ and the median; and the ratio between the geometric mean and median. The bottom row represents results for a stacking scenario, i.e., $F_{\text{UL}}(E > E_{\text{th}})$ represents the live-time-weighted flux of 15 GCs, while the modelling results (cf. Eq. 6.7) are for the same 15 GCs, and weighted similarly.

GC name	E_{th} (TeV)	$F_{\text{UL}}(E > E_{\text{th}})$ $\times 10^{-13}$ ($\text{ph cm}^{-2} \text{s}^{-1}$)	F_{μ} $\times 10^{-13}$ ($\text{ph cm}^{-2} \text{s}^{-1}$)	σ_{16}/F_{μ} (%)	σ_{84}/F_{μ} (%)	F_{μ}/F_{UL}	F_G $\times 10^{-13}$ ($\text{ph cm}^{-2} \text{s}^{-1}$)	σ/F_{μ} (%)	F_G/F_{μ}
NGC 104	0.72	19	10.0	84	427	0.53	9.4	611	0.92
NGC 6388	0.28	15	5.9	85	461	0.39	5.5	706	0.93
NGC 7078	0.40	7.2	1.7	84	465	0.24	1.5	673	0.94
Terzan 6	0.28	21	6.9	86	518	0.33	6.3	890	0.91
Terzan 10	0.23	29	11.0	87	530	0.38	10.0	903	0.91
NGC 6715	0.19	9.3	1.2	85	478	0.13	1.1	722	0.92
NGC 362	0.59	24	1.5	88	572	0.06	1.4	1081	0.93
Pal 6	0.23	12	11.0	87	534	0.92	9.9	919	0.90
NGC 6256	0.23	32	3.2	87	534	0.10	2.9	941	0.91
Djorg 2	0.28	8.4	10.0	86	508	1.19	9.2	831	0.92
NGC 6749	0.19	14	6.9	87	534	0.49	6.3	913	0.91
NGC 6144	0.23	14	4.3	87	554	0.31	3.9	972	0.91
NGC 288	0.16	5.3	6.1	87	553	1.15	5.6	945	0.92
HP 1	0.23	15	5.4	87	539	0.36	5.0	925	0.93
Terzan 9	0.33	45	2.8	89	644	0.06	2.6	1341	0.93
Stacking analysis	0.23	3.3	25	50	119	7.6	26.0	135	1.04

flux F_μ (50th percentile) for each cluster, as well as the 16th and 84th percentile, as shown in Figure 6.6 and listed in Table 6.1. I calculate $\sigma_{16} = \mu - F_{16}$ and $\sigma_{84} = F_{84} - \mu$ and also express them as percentages: $100\sigma_{16}/\mu$ and $100\sigma_{84}/\mu$, with F_{16} and F_{84} being the 16th and 84th percentiles. I also calculate the geometric mean of the flux $\bar{F}_G$ (since we want to be insensitive to outliers), as well as the standard deviation σ of this flux, noting that the geometric mean is equal to the antilog of the arithmetic mean of the log of fluxes. I finally list the ratios F_μ/F_{UL} as well as $\bar{F}_G/F_\mu$, indicating that the median is usually below the upper limit, except for Djorg 2 and the stacking upper limit, and that $F_\mu \approx \bar{F}_G$.

In Figure 6.7 I show the distribution of the $\log_{10}$ of weighted (stacked) flux for all GCs. I plot the weighted flux as a function of N_t for all 15 GCs. I use $E_{th} = 0.23$ TeV and calculate the weighted flux as

$$F_w = \sum_{i=1}^{15} \frac{\tau_i}{\tau_{tot}} \times F_i(> 0.23 \text{ TeV}), \quad (6.7)$$

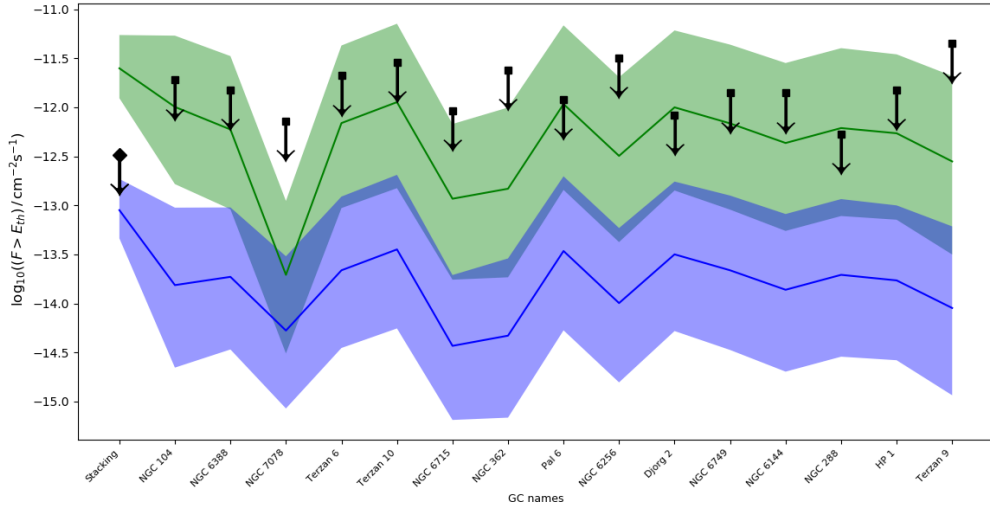


Figure 6.8: Flux upper limits (black diamond and squares with arrows) on the observed γ -ray flux from the population of GCs, as well as the predicted flux medians plus 1σ uncertainties (green and blue) for each of the two parameter combinations. The first entry indicates the stacked (live-time-weighted) flux with 1σ uncertainties, as indicated in Table 6.1.

where i represents a specific GC, τ_i is the live time (acceptance-corrected observational time passing quality cuts) for each GC, and τ_{tot} is the total live time. The median and geometric mean of this quantity are listed in the bottom row of Table 6.1, in column 4 and 8, respectively, for comparison to the stacked upper limit published by H.E.S.S. as listed in column 3. The first set (range) of parameter combinations used for the green histograms in Figure 6.7 is as follows: $d_{cluster} \in [\frac{d}{2}, \frac{d}{\sqrt{2}}, d, \sqrt{2}d, 2d]$, $N_* \in [\frac{N}{2}, \frac{N_*}{\sqrt{2}}, N_*, \sqrt{2}N_*, 2N_*]$ (see

that there are large uncertainties on the median model fluxes due to uncertainty on model parameters, is not affected by this choice of convention.

Table 6.2: H.E.S.S. flux upper limits and model predictions for the second parameter combination, associated with the blue histograms in Figure 6.7.

GC name	E_{th} (TeV)	$F_{\text{UL}}(E > E_{\text{th}})$ $\times 10^{-13}$ (ph cm $^{-2}$ s $^{-1}$)	F_{μ} $\times 10^{-13}$ (ph cm $^{-2}$ s $^{-1}$)	σ_{16}/F_{μ} (%)	σ_{84}/F_{μ} (%)	F_{μ}/F_{UL}	F_G $\times 10^{-13}$ (ph cm $^{-2}$ s $^{-1}$)	σ/F_{μ} (%)	F_G/F_{μ}
NGC 104	0.72	19	0.154	86	614	0.0081	0.145	929	0.94
NGC 6388	0.28	15	0.187	82	410	0.0125	0.179	610	0.96
NGC 7078	0.40	7.2	0.053	84	573	0.0074	0.051	737	0.95
Terzan 6	0.28	21	0.218	84	463	0.0104	0.208	766	0.92
Terzan 10	0.23	29	0.356	84	475	0.0123	0.337	778	0.95
NGC 6715	0.19	9.3	0.037	82	429	0.0040	0.035	622	0.96
NGC 362	0.59	24	0.047	85	513	0.0020	0.044	923	0.94
Pal 6	0.23	12	0.343	85	578	0.0286	0.324	787	0.94
NGC 6256	0.23	32	0.101	84	482	0.0032	0.096	810	0.95
Djorg 2	0.28	8.4	0.317	84	451	0.0377	0.302	713	0.95
NGC 6749	0.19	14	0.218	84	577	0.0156	0.205	780	0.94
NGC 6144	0.23	14	0.138	86	492	0.0097	0.129	819	0.93
NGC 288	0.16	5.3	0.196	85	492	0.0370	0.183	806	0.93
HP 1	0.23	15	0.172	85	481	0.0115	0.162	797	0.95
Terzan 9	0.33	45	0.090	87	580	0.0020	0.084	1147	0.93
Stacking analysis	0.23	3.3	0.895	48	108	0.271	0.923	113	1.03

Venter and Kopp, 2015 for values of d and N_*). I used $B \in [1, 9]$ in steps of 2, $\Gamma \in [1.7, 2.9]$ with steps of 0.3, $\eta \in [0.005, 0.08]$ in steps of 0.0075, $\log_{10}\langle\dot{E}\rangle \in [33.7, 34.7]$ in steps of 0.1, and $N_{\text{MSP}} \in [5, 150]$ in steps of 10. Although somewhat arbitrary, these were chosen to be an example set of reasonable (best-guess) parameter values. However, we see from Table 6.1 that for this initial choice, the median stacked flux exceeds the upper limit by a factor of 7.6. I therefore consider a second parameter set, associated with the blue histograms of weighted flux for 15 GCs in Figure 6.7: $\eta \in [0.003, 0.03]$ in steps of 0.003, $\log_{10}\langle\dot{E}\rangle \in [33, 34]$ in steps of 0.1, and $N_{\text{MSP}} \in [5, 50]$ in steps of 5. Again, this choice might seem arbitrary, but given the freedom in model parameters, there is no unique way of reducing the GC flux. Thus, I reduce the three (degenerate) main parameters determining the source strength Q_0 , i.e., η , $\dot{E}$, and N_{MSP} by a factor of a ~ 3 to illustrate that by lowering Q_0 , the flux upper limits may be satisfied. While we do not claim this second set as a final answer, we deem these ranges to still be realistic, given the uncertainty in their intrinsic values. The fluxes obtained for the second choice of parameter grid are indicated in Table 6.2. In this case, the predicted median flux is a factor of ~ 4 below the H.E.S.S. upper limit (indicated by the red solid line in Figure 6.7). This is one example (i.e., changing some parameters so as to yield a smaller Q_0) of how one can use the H.E.S.S. data to constrain the MSP population’s characteristics (see also Ndiyavala et al., 2019), although there may be some degeneracy with other model parameters such as κ , Γ , d and to a lesser extent B (this will impact the SR component more) and N_* , although the latter is relatively well constrained.

Finally, I graphically summarise the fluxes I obtained for both the stacked as well as single-GC cases in Figure 6.8. The black diamond and squares and arrows correspond to the observational upper limits, while the green and blue squares and error bars are for the median fluxes and 1σ uncertainty intervals, for the two different parameter combinations as discussed above. We note that there are indeed parameter combinations that yield fluxes below all of these observational upper limits. One example is afforded by combinations of degenerate parameters yielding a lower Q_0 ; there may also be other parameter combinations that can lower the flux, as indicated before. Further constraints derived from future observations may aid in breaking such degeneracies.

6.4.2 M15

The northern GC M15 (also known as NGC 7078) is one of the densest³ GCs ever discovered in the Milky Way Galaxy. It has undergone a core collapse contraction and thus belongs to the class of core-collapsed brightest GCs (Acciari et al., 2019). M15 hosts at least eight MSPs detected so far (Freire, 2015). It is located at a distance of $d = 10.4$ kpc (Harris, 1996) at RA (J2000) $21^{\text{h}}29^{\text{m}}58^{\text{s}}.33$ and Dec $+12^{\circ}10'01''.2$ (Galactic coordinates: $l = 65.01^{\circ}$, $b = -27.31^{\circ}$) and exhibits $r_c = 0'.14$ and a half-light radius $r_h = 1'.00$ (Harris,

³<https://www.nasa.gov/feature/goddard/2017/messier-15>

1996).

Acciari et al. (2019) observed M15 with the MAGIC telescopes, since it is the only GC discovered by *Fermi* LAT at GeV γ -ray energies that they could observe at low zenith angles. Its γ -ray luminosity measured above 0.1 GeV is $L_{\gamma}^{\text{M15}} = (5.26^{+1.31}_{-1.16}) \times 10^{34} \text{ erg s}^{-1}$ and the spectrum is similar to the power-law type with spectral index 2.84 ± 0.18 when observed up to ~ 5 GeV (Zhang et al., 2016). The estimated number of MSPs in M15 is 37^{+9}_{-8} , for a conversion efficiency of the MSP's spin-down power into GeV γ -rays of $\eta_{\gamma} \approx 0.08$ and an average luminosity of $\langle L_{\gamma}^{\text{MSP}} \rangle = 1.44 \times 10^{33} \text{ erg s}^{-1}$ per MSP. However, no detection from the direction of M15 has been made in the TeV band (Acciari et al., 2019). These authors therefore followed Rolke et al. (2005) using a 95% confidence level and assuming a 30% total systematic uncertainty on the collection area and spectral index of -2.6 to calculate the upper limits on the VHE flux from the source. They found an upper limit of $F(E > 0.3 \text{ TeV}) = 3.2 \times 10^{-13} \text{ cm}^{-2} \text{ s}^{-1}$, similar to the $F(E > 0.44 \text{ TeV}) = 7.2 \times 10^{-13} \text{ cm}^{-2} \text{ s}^{-1}$ as published earlier by Abramowski et al. (2013).

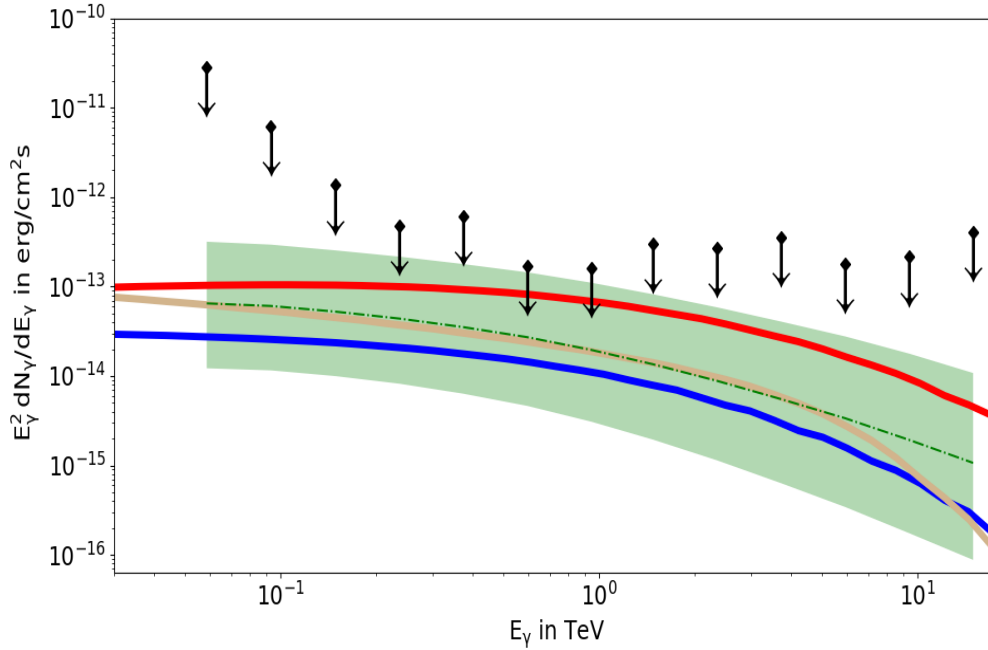


Figure 6.9: Differential flux upper limits (black diamonds with arrows) on the γ -ray flux from M15 plus a few typical model spectra (solid lines) for typical parameter combinations, as well as predicted medians and 1σ error bars (green error band). For the example fits, we used lepton $E_{\min} = 0.01 \text{ TeV}$, $E_{\max} = 100 \text{ TeV}$, assumed Bohm diffusion, $B = 5\mu\text{G}$, $\Gamma = 1.9$, and $Q_0 = 1.81 \times 10^{33} \text{ erg}^{-1} \text{ s}^{-1}$ (red line); $E_{\min} = 0.01 \text{ TeV}$, $E_{\max} = 30 \text{ TeV}$, $k_0 = 7 \times 10^{-5} \text{ kpc}^2/\text{Myr}$, $B = 3\mu\text{G}$, $\Gamma = 2.1$, $Q_0 = 4.75 \times 10^{32} \text{ erg}^{-1} \text{ s}^{-1}$ (blue line) and $E_{\min} = 0.01 \text{ TeV}$, $E_{\max} = 30 \text{ TeV}$, $k_0 = 1.0 \times 10^{-3} \text{ kpc}^2/\text{Myr}$, $B = 5\mu\text{G}$, $\Gamma = 1.9$, and $Q_0 = 1.81 \times 10^{33} \text{ erg}^{-1} \text{ s}^{-1}$ (golden line).

I create a distribution of differential fluxes (see Figure 6.5 for energy bins 2 and 7, centred on energies of $E = 62 \text{ GeV}$ and $E = 955 \text{ GeV}$, respectively) and calculate the median and 1σ uncertainties of the $\log_{10}$ of these fluxes by finding the 16th, 50th and 84th percentiles.

As before, the fluxes were obtained by randomising over seven free parameters many times (see the different N_t as indicated in each panel) to study the convergence.

Figure 6.9 shows the upper limits from 165 hrs of MAGIC observations on the differential flux, the predicted spectrum for a few different combinations of model parameters, and the median points with 1σ error. We note that the differential data are much more constraining than the integral data since there are 13 points of comparison instead of 1. For each bin, we are able to tell whether the model exceeds the flux at that energy bin or not, thus constraining the spectral shape. For our choice of parameters, we satisfy the MAGIC flux upper limits.

6.4.3 ω Cen

ω Centauri (NGC 5139), which is located at RA 201.7° , Dec -47.56° , stands out among all the known GCs in our Galaxy. Its origin and history are not fully understood yet, but its properties have led to suggestions that it may be the remnant core of a dwarf galaxy that has been disrupted (e.g., Bekki and Freeman, 2003). ω Cen is the most massive, complex, brightest, and has the largest core and half-light radius of all GCs in the Galaxy (Harris, 1996). It is thought to have been the nuclear star cluster of either the Sequoia or Gaia-Enceladus Galaxy discussed in Myeong et al. (2019). The cluster also shows characteristic features such as its incredible multiplicity in stellar populations (e.g., Pancino et al., 2000, Bellini et al., 2017) and its broad metallicity distribution (e.g., Freeman and Rodgers, 1975, Magurno et al., 2019). ω Cen is furthermore suspected to harbour a BH with total mass $\sim 10^5 M_\odot$ at its centre, which makes this GC an interesting candidate to model for possible future CTA observations (Zocchi et al., 2019).

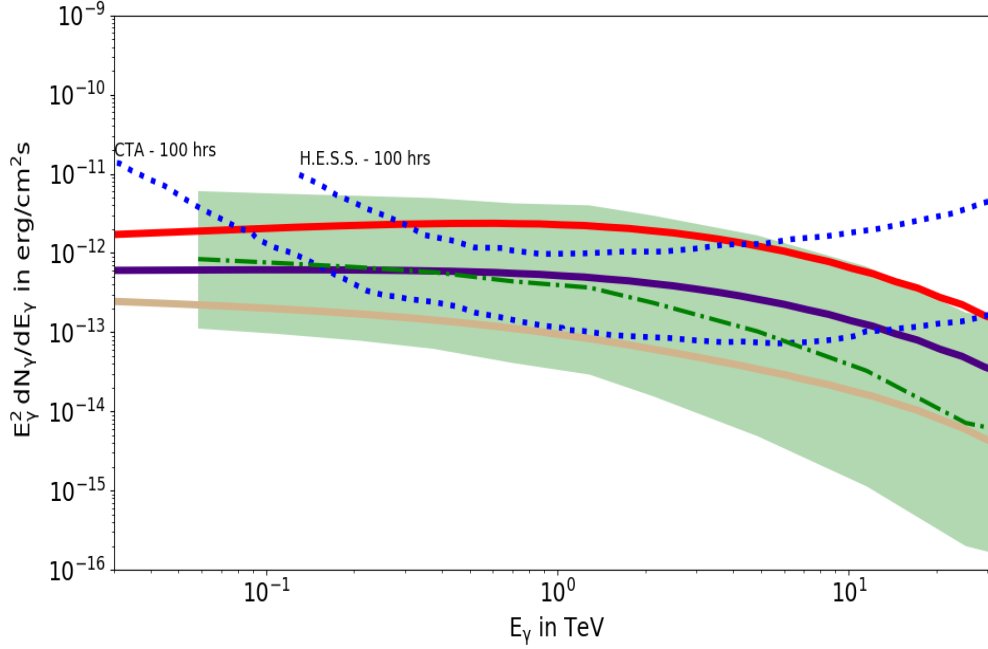


Figure 6.10: Differential flux predictions (green error band) for ω Cen, the solid lines indicating example predictions. The blue dashed lines represent the H.E.S.S. and CTA sensitivity curves for 100 hours of observations (Funk et al., 2013). For the example spectra, we chose $E_{\min} = 0.001$ TeV and $E_{\max} = 100$ TeV for all cases. We then assume Bohm diffusion, $B = 5\mu G$, $\Gamma = 1.9$, and $Q_0 = 8.17 \times 10^{33} \text{ erg}^{-1} \text{ s}^{-1}$ (red line); $\kappa_0 = 7 \times 10^{-5} \text{ kpc}^2/\text{Myr}$, $B = 3\mu G$, $\Gamma = 2.1$, and $Q_0 = 2.40 \times 10^{33} \text{ erg}^{-1} \text{ s}^{-1}$ (violet line) and $k_0 = 1.0 \times 10^{-3} \text{ kpc}^2/\text{Myr}$, $B = 1\mu G$, $\Gamma = 2.3$, and $Q_0 = 7.57 \times 10^{32} \text{ erg}^{-1} \text{ s}^{-1}$ (golden line).

Observations by *Fermi* LAT has indicated that ω Cen’s spectrum was reminiscent of a typical MSP spectrum. However, it was a mystery that no radio or X-ray MSPs had been detected (Abdo et al., 2010) at that time. The Parkes Radio Telescope has now detected five radio MSPs in this cluster, four of which are isolated and a fifth one is in an eclipsing binary system. This confirms the scenario that the cumulative MSP γ -ray emission may explain the *Fermi* detection of this cluster. However, no pulsations in the GeV data have been found to date (Dai et al., 2020).

I apply the same method as in Section 6.3.2 on ω Cen as an example of a Southern source. In Figure 6.10 I compare the predicted differential fluxes with the H.E.S.S. and CTA sensitivity curves for 100 hours of observations. I also indicate typical predicted γ -ray spectra for a few choices of parameter combinations. We note that our current IC predictions are higher than predicted by Ndiyavala et al. (2018). This is because Ndiyavala et al. (2018) assumed relatively low numbers of stars within this cluster, while we now use $N_* = 10^7$ (Miller and Walker, 2019), creating a much higher ambient target photon density. We do not fit *Fermi* LAT data, as these are commonly fit using CR spectra (not produced by the Kopp et al., 2013 model).

6.5 Discussion and Conclusions

A number of models attempt to explain the multi-wavelength spectra detected from several Galactic GCs. In addition to the broad leptonic and hadronic scenarios mentioned in Section 6.1, there are other processes or refinements that we did not consider in this Chapter, but that can significantly influence the level of the predicted TeV emission from GCs. For example, [Bednarek and Sobczak \(2014\)](#) performed numerical calculations of the asymmetric TeV γ -ray emission morphology produced by the interaction of stellar winds leaving the GC and the Galactic medium, which should create a bow shock nebula around GC. Furthermore, [Bednarek et al. \(2016\)](#) considered the role of advection of leptons within the GC by the mixture of winds from the embedded populations of MSPs and other stellar members, as well as the effect of a non-central location of a dominating, energetic MSP, on the predicted γ -ray flux. In order to constrain or discard some of these models, careful testing of them is needed.

This Chapter focused on assessing uncertainties in the predicted VHE γ -ray flux of GCs within the context of the leptonic GC model of [Kopp et al. \(2013\)](#), with the main aim to give theoretical guidance to CTA’s observational strategy. I demonstrated that uncertainty in model parameters leads to a large spread in the predicted flux. I modelled the IC γ -ray flux expected from 15 GCs that have been observed, but not detected, by H.E.S.S. in the VHE band. I confirmed that a finer grid in parameter space leads to a smoother flux distribution, as expected. Furthermore, a larger number of trials led to convergence of the flux distribution, as $N_t \rightarrow N_{\text{comb}}$. Also, the eventual predicted range of fluxes depends on the number of free model parameters, as well as their respective ranges. Interestingly, we could demonstrate the power of using a stacking approach: the relative errors on the predicted live-time-weighted flux is nearly one order of magnitude lower than that for the individual GCs. This well-known effect is also apparent in the observational upper limits, where the weighted-flux upper limit is lower and therefore more constraining than the single-GC ones ([Figure 6.8](#)). We also note that the median flux and geometric mean flux were very close. The different methods of estimating the error on the mean flux yielded slightly different, yet broadly consistent results, but one can appreciate the large spread (uncertainty) in predicted fluxes based on these error estimates.

We found that while none of our predicted individual cluster median fluxes plus uncertainties violated the respective upper limits ([Figure 6.8](#)), our total integral weighted flux violated the H.E.S.S. upper limit for our first set of parameter combinations ([Table 6.1](#)). However, one has to be careful to summarily discard models if calculations for best-guess parameters exceed observational limits, noting the considerable uncertainty in predictions that may stem from uncertain parameter values. Our second parameter combination satisfied the stacked upper limits ([Table 6.2](#)). Thus we could use this stringent upper limit to constrain the source properties of the MSPs embedded within the GC. For M15, we

found that the differential upper limits were more constraining than the H.E.S.S. integral flux upper limits. Yet, we could satisfy these upper limits for typical parameters. As an example of a Southern-hemisphere source, I calculated the TeV flux for ω Cen, indicating that this source may be a possible candidate to be observed by H.E.S.S. or CTA.

In conclusion, we note that increasing measurement accuracy on model parameters will improve predictions of GC fluxes, and this will provide better guidance to CTA's observations. Future refined models will have to undergo continued scrutiny, also taking into account the effect of parameter uncertainty on their predictions, as they are confronted with new data.

6.6 Summary

In this Chapter, I assessed the uncertainties in the predicted VHE γ -ray flux of GCs within the context of a particular leptonic GC model. I demonstrated that uncertainty in model parameters leads to a large spread in the predicted flux. This makes it difficult to discriminate between models, emphasising the need for improved measurements. In the next Chapter, I will simulate the observed spectrum by CTA for five most promising GC candidates using our predicted SED as input. I will specifically take into account the CTA instrument response functions (IRFs) to study the impact thereof on the simulated observed spectrum.

Chapter 7

Simulating Future Cherenkov Telescope Array Detections of Globular Clusters

In the previous Chapter, I assessed uncertainties in the predicted VHE γ -ray flux of several GCs and gave theoretical guidance to CTA's observational strategy. In this Chapter, I will simulate the spectral shape of the top five promising GCs (Terzan 5, 47 Tucanae, Djorg 2, NGC 6388, and Terzan 10; [Ndiyavala et al., 2018](#)) to be detected by CTA using the relevant model inputs and instrument response functions (IRFs). To do so, a software package developed for CTA will be used. I will give a step-by-step description of data simulation and present the spectral results of these GCs¹.

7.1 CTA Instrument Response Functions (IRFs)

IRFs for CTA are needed to translate the collective reconstructed variables such as particle nature (ID'), direction (p'), and Energy (E') from all detected events into an observed photon flux ([Ward et al., 2015](#)). They are the parameterised representations of the instrument performance and are factorised into three parts: the effective area $A_{\text{eff}}(p, E, t)$, the point-spread function or PSF($p'|p, E, t$), and the energy dispersion function $E_{\text{disp}}(E'|p, E, t)$ following²:

$$R(p', E', t'|p, E, t) = A_{\text{eff}}(p, E, t) \times \text{PSF}(p'|p, E, t) \times E_{\text{disp}}(E'|p, E, t). \quad (7.1)$$

¹This work was done during a visit to the University of Oxford with the goal to calculate the expected observability of GCs with the next-generation gamma-ray observatory.

²<http://cta.irap.omp.eu/ctools/users/glossary.html#glossary-irf>

where $R(p', E', t' | p, E, t)$ is the instrument response, p and p' are the “real” and “reconstructed” photon directions, E is the true photon energy, and t is the temporal length of the shower.

7.2 Event Data Simulation

To simulate event data for the GCs, I create a model definition XML file. This file takes as the intrinsic spectrum the predicted SED using the (Kopp et al., 2013) model. Other inputs include the source position and the background model, which is parameterised as a power law. The general structure of a model definition file (using Terzan 5 as an example) is as follows:

```
<source_library title="source library">
  <source name="Terzan 5" type="PointSource">
    <!-- point source units are cm^-2 s^-1 MeV^-1 -->
    <spectrum file="Terzan5.dat" type="FileFunction">
      <parameter free="1" max="1e5" min="1e-5" name="Normalization" scale="1" value="1.0"/>
    </spectrum>
    <spatialModel type="SkyDirFunction">
      <parameter free="0" max="360." min="-360." name="RA" scale="1.0" value="3.84"/>
      <parameter free="0" max="90." min="-90." name="DEC" scale="1.0" value="1.69"/>
    </spatialModel>
  </source>
  <source name="CTABackgroundModel" type="CTAIrfBackground" instrument="CTA">
    <spectrum type="PowerLaw">
      <parameter name="Prefactor" scale="1.0" value="1.0" min="1e-3" max="1e+3" free="1"/>
      <parameter name="Index" scale="1.0" value="0.0" min="-5.0" max="+5.0" free="1"/>
      <parameter name="PivotEnergy" scale="1e6" value="1.0" min="0.01" max="1000.0" free="0"/>
    </spectrum>
  </source>
</source_library>
```

Next, I simulate event data using the `ctobssim` that simulates a list of events using the characteristics of the instrument specified by the IRFs and an input model. The following was done on a command line where `ctobssim` queries for several parameters that define the characteristics of the simulation:

```
$ ctobssim
RA of pointing (degrees) (0-360) [83.63] 3.84
Dec of pointing (degrees) (-90-90) [22.51] 1.69
Radius of FOV (degrees) (0-180) [5.0]
Start time (UTC string, JD, MJD or MET in seconds) [2020-01-01T00:00:00] 2020-01-01T00:10:00
Stop time (UTC string, JD, MJD or MET in seconds) [2020-01-01T00:30:00] 2020-01-06T00:50:00
Lower energy limit (TeV) [0.1] 0.03
Upper energy limit (TeV) [100.0] 150.0
Calibration database [prod2]
Instrument response function [South_0.5h]
Input model definition XML file [$CTOOLS/share/models/crab.xml] Terzan5.xml
Output event data file or observation definition XML file [events.fits] Terzan5_events.fits
```

Each line represents a query for a particular parameter value or input file. The line starts with a short description of the parameter, followed by the default value proposed by `ctobssim` in squared brackets. The default value is used if no parameter value is entered.

In the example above the RA, Dec, length of the observation, energy range, input model file, and output event data file were specified. For the remaining parameters, the default values were used. The events are written to the file `Terzan5_events.fits`.

As a next step, I use the `ctselect` tool to select some events from the simulated event data. I restrict the events region to 3.0° around the pointing direction, and extract a time slice of ~ 100 h out of a total of 120 h from the data while limiting the energy range of the events to $0.03 - 100$ TeV. See the details in the box below:

```
$ ctselect
Input event list or observation definition XML file [events.fits] Terzan5_events.fits
Radius of ROI around pointing or specified RA/DEC (0-180) [3.0]
Start time (UTC string, JD, MJD or MET in seconds) [NONE] 2020-01-01T00:00:00
Stop time (UTC string, JD, MJD or MET in seconds) [NONE] 2020-01-05T04:00:00
Lower energy limit (TeV) [0.1]
Upper energy limit (TeV) [100.0]
Output event list or observation definition XML file [selected_events.fits] Terzan5_selected_events.fits
```

The selected events are written to `Terzan5_selected_events.fits`. I next binned the selected event data into a counts cube as follows:

```
$ ctbin
Input event list or observation definition XML file [events.fits] Terzan5_selected_events.fits
Coordinate system (CEL - celestial, GAL - galactic) (CEL|GAL) [CEL]
Projection method (AIT|AZP|CAR|GLS|MER|MOL|SFL|SIN|STG|TAN) [CAR]
First coordinate of image center in degrees (RA or galactic l) (0-360) [83.63] 3.84
Second coordinate of image center in degrees (DEC or galactic b) (-90-90) [22.51] 1.69
Image scale (in degrees/pixel) [0.02]
Size of the X axis in pixels [200]
Size of the Y axis in pixels [200]
Algorithm for defining energy bins (FILE|LIN|LOG|POW) [LOG]
Lower energy limit (TeV) [0.1]
Upper energy limit (TeV) [100.0]
Number of energy bins (1-200) [20]
Output counts cube file or observation definition XML file [cntcube.fits] Terzan5_cntcube.fits
```

A counts cube is a 3-dimensional data cube that is spanned by Galactic longitude, Galactic latitude, and photon energy. The events from file `Terzan5_selected_events.fits` are binned into a counts cube that is stored into the file `Terzan5_cntcube.fits`. The counts cube is centred on the pointing direction (Galactic longitude 3.84° , Galactic latitude 1.69°). A cartesian projection aligned in celestial coordinates is used and the counts cube has 200×200 spatial pixels of $0.02^\circ \times 0.02^\circ$ in size, using 20 logarithmically-spaced energy bins to cover energy range from 0.03 TeV to 100 TeV.

To speed up the model fitting, I pre-compute the instrument response (exposure cube, a PSF cube, and a background cube) for the counts cube. This was done as follows:

```

$ ctexpcube
Input event list or observation definition XML file [NONE] Terzan5_selected_events.fits
Calibration database [prod2]
Instrument response function [South_0.5h]
Input counts cube file to extract exposure cube definition [NONE] Terzan5_cntcube.fits
Output exposure cube file [expcube.fits] Terzan5_expcube.fits

$ ctpsfcube
Input event list or observation definition XML file [NONE] Terzan5_selected_events.fits
Calibration database [prod2]
Instrument response function [South_0.5h]
Input counts cube file to extract PSF cube definition [NONE]
Coordinate system (CEL – celestial, GAL – galactic) (CEL|GAL) [CEL]
Projection method (AIT|AZP|CAR|GLS|MER|MOL|SFL|SIN|STG|TAN) [CAR]
First coordinate of image center in degrees (RA or galactic l) (0–360) [83.63] 3.84
Second coordinate of image center in degrees (DEC or galactic b) (–90–90) [22.51] 1.69
Image scale (in degrees/pixel) [1.0]
Size of the X axis in pixels [10]
Size of the Y axis in pixels [10]
Algorithm for defining energy bins (FILE|LIN|LOG|POW) [LOG]
Lower energy limit (TeV) [0.1]
Upper energy limit (TeV) [100.0]
Number of energy bins (1–1000) [20]
Output PSF cube file [psfcube.fits] Terzan5_psfcube.fits

$ ctbkgcube
Input event list or observation definition XML file [NONE] Terzan5_selected_events.fits
Calibration database [prod2]
Instrument response function [South_0.5h]
Input counts cube file to extract background cube definition [NONE] Terzan5_cntcube.fits
Input model definition XML file [NONE] $CTOOLS/share/models/Terzan5.xml
Output background cube file [bkgcube.fits] Terzan5_bkgcube.fits
Output model definition XML file [NONE] Terzan5_models.xml

```

The `ctexpcube` tool is used to compute the exposure cube and provides the effective area multiplied by the observation livetime. The PSF cube is computed using the `ctpsfcube` tool and is a 4-dimensional cube spanned by Galactic longitude, Galactic latitude, true photon energy, and offset angle between true and reconstructed arrival direction of a photon. The calculation of the PSF cube³ is given as

$$PSF(\delta|p, E) = \frac{\sum_i PSF_i(p'|p, E) \times A_{\text{eff},i}(p, E) \times \tau_i}{\sum_i A_{\text{eff},i}(p, E) \times \tau_i} \quad (7.2)$$

where δ is the angular separation between the true and measured real and reconstructed photon directions p and p' , and τ_i is the livetime of observation i with the sum taken over all observations. The `ctbkgcube` tool is used to compute the background cube, which is a 3-dimensional cube spanned by reconstructed Galactic longitude, reconstructed Galactic latitude, and reconstructed photon energy. The background cube $B(p', E')$ ⁴ is computed using

$$B(p', E') = \frac{\sum_i B_i(p', E') \times \tau_i}{\sum_i \tau_i}, \quad (7.3)$$

where $B_i(p', E')$ is the background rate estimate for observation i , p' is the reconstruction photon arrival direction, E' is the reconstruction photon energy, and τ_i is the livetime of observation i . Again, the sum is taken over all observations.

³<http://cta.irap.omp.eu/ctools/users/glossary.html#glossary-psfcube>

⁴<http://cta.irap.omp.eu/ctools/users/glossary.html#glossary-bkgcube>

Next, I fit the simulated data with a model using the `ctlike` tool as shown below.

```
$ clike
Input event list , counts cube or observation definition XML file [events.fits] Terzan5_cntcube.fits
Input exposure cube file [NONE] Terzan5_expcube.fits
Input PSF cube file [NONE] Terzan5_psfcube.fits
Input background cube file [NONE] Terzan5_bkgcube.fits
Input model definition XML file [$CTOOLS/share/models/crab.xml] Terzan5_models.xml
Output model definition XML file [crab_results.xml] Terzan5_results.xml
```

The data are fitted in binned mode, which means that the events have been binned into a 3-dimensional counts cube and the fit calculates the log-likelihood function by summing over all $200 \times 200 \times 20$ bins of the counts cube. The output (produced via `ctlike` tool) of this process is saved to `Terzan5_results.xml`, which is a copy of the model definition XML input file `Terzan5_models.xml` with the parameter values replaced by the fit results. Additionally, the statistical uncertainties were added for each fitted parameter.

```
$ csspec
Input event list , counts cube, or observation definition XML file [events.fits] Terzan5_cntcube.fits
Input exposure cube file [NONE] Terzan5_expcube.fits
Input PSF cube file [NONE] Terzan5_psfcube.fits
Input background cube file [NONE] Terzan5_bkgcube.fits
Input model definition XML file [$CTOOLS/share/models/crab.xml] Terzan5_results.xml
Source name [Crab] Terzan5
Spectrum generation method (SLICE|NODES|AUTO) [AUTO]
Binning algorithm (FILE|LIN|LOG|POW) [LOG]
Lower energy limit (TeV) [0.1]
Upper energy limit (TeV) [100.0]
Number of energy bins [20] 10
Output spectrum file [spectrum.fits] Terzan5_spectrum.fits
```

Lastly, I determine the spectrum of Terzan 5 using the `csspec` script below. The script divides the energy range into a number of independent energy bins and performs a maximum likelihood fit of the source model in each of the energy bins.

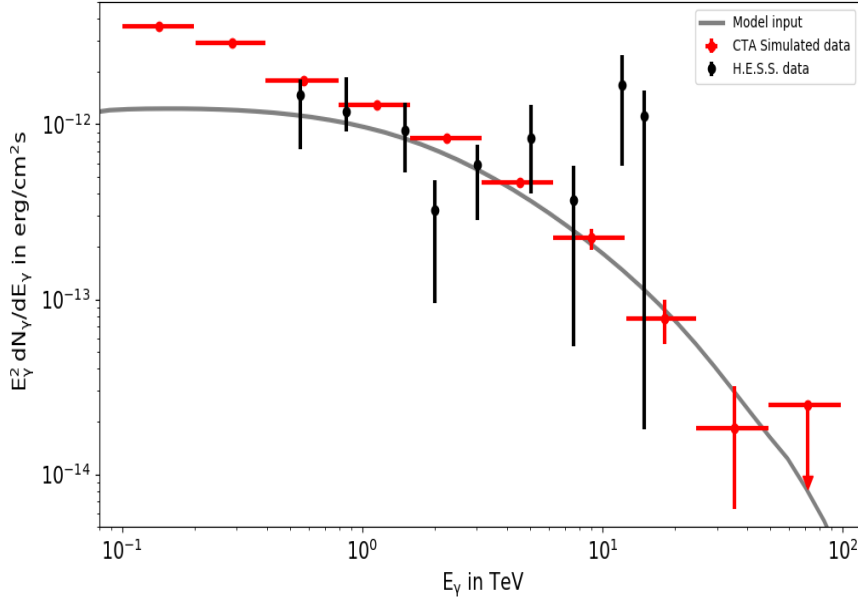
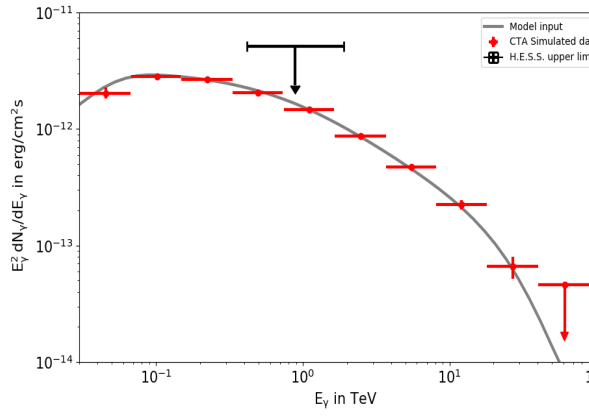


Figure 7.1: Simulated data, model input, and H.E.S.S. data points for Terzan 5.

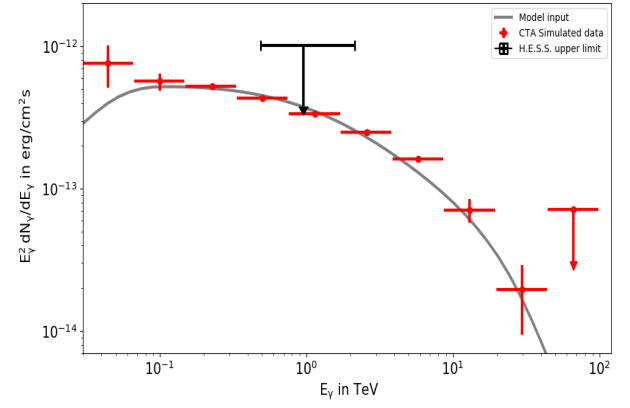
The spectral shape is constrained poorly within a narrow energy bin and thus all spectral parameters are fixed for the fitting (except parameters of the scaling factor of the source model). The fit result is then converted into a flux value per energy bin. The upper flux limits and the test statistic value of the source in each of the bins are also obtained via the `csspec` script. Figure 7.1 shows the simulated and reconstructed data of Terzan 5 for 10 energy bins between $E_{\text{low}} = 0.1$ TeV and $E_{\text{high}} 100$ TeV. It also shows how the simulated data compare with the H.E.S.S. data as well as with the model used. I note that the simulated data are similar to the input model.

7.3 Summary

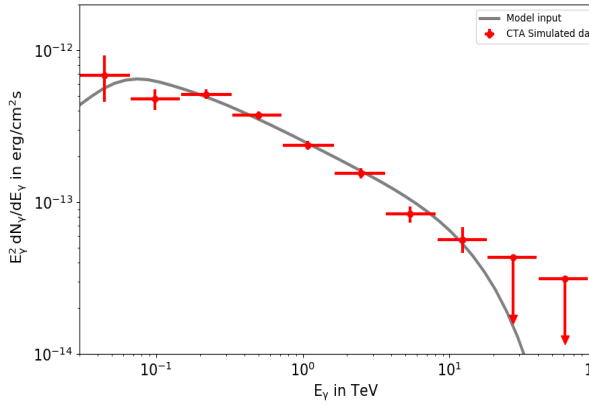
In this Chapter, I simulated the spectral shapes of the top five most promising GCs as it would appear to CTA by applying the CTA IRFs to the GC emission's prediction using a GC leptonic model in Chapter 3. I note that the spectra of the selected GCs are very close to the simulated data. Figure 7.2 shows how the simulated data compare with the model used for 47 Tucanae, NGC 6388, Djorg 2, and Terzan 10. I note that CTA should detect more GCs than H.E.S.S. as predicted in Chapter 4.



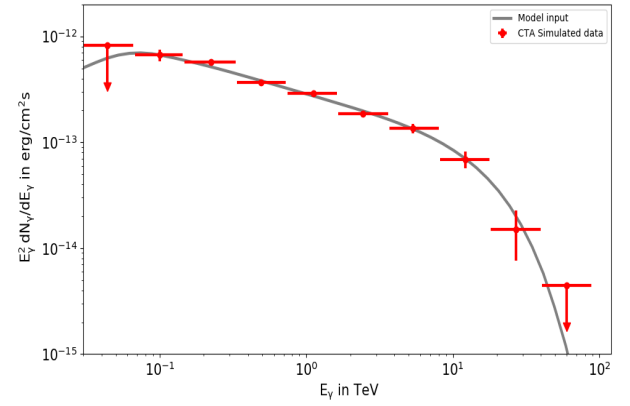
(a) Simulated SED and model input of 47 Tucanae



(b) Simulated SED and model input of NGC 6388



(c) Simulated SED and model input of Djorg 2



(d) Simulated SED and model input of Terzan 10

Figure 7.2: Simulated SED of 47 Tucanae, NGC 6388, Djorg 2, and Terzan 10 for 10 energy bins between 0.03 TeV and 100 TeV for 100 hours.

Chapter 8

Conclusions and Future Work

This study focused on modelling the SEDs of Galactic GCs, gathering more data on Terzan 5, and scrutinising ideas about the particle sources and emission processes responsible for the broadband emission spectrum we observe from this cluster. Also to show that uncertainty in model parameters leads to a large spread in the predicted flux, and to demonstrate that there are indeed regions in parameter space for which the stringent stacking upper limits are satisfied. In this study, I also simulated the observed spectrum by CTA for selected GCs.

8.1 Conclusions

- I used a leptonic emission code to make flux predictions and performed a parameter study, varying six model parameters. For Terzan 5, I found that $E_{e,\max}$ could be varied between 30 and 100 TeV, and the B -field between 1 and 4 μG while still fitting the SED by fixing the other parameters. Similarly, I found that for 47 Tucanae, $E_{e,\max} \leq 100$ TeV, and $B < 5$ μG , and one requires a small B and $E_{e,\max}$ or a small Q_0 and large Γ for NGC 6388 in order to satisfy the upper limits. I therefore found that the parameters of the individual GCs were uncertain and quite unconstrained by the available data, and I noted that there were different combinations of parameters that satisfied the observational constraints. I also found that the predicted IC component for the majority of the 16 GCs I studied were below the H.E.S.S. sensitivity limit. However, H.E.S.S. may detect two more GCs, i.e. 47 Tucanae and NGC 6388, if the clusters are observed for 100 h, in addition to Terzan 5. The five most promising GCs for future TeV observations are NGC 6388, 47 Tucanae, Terzan 5, Djorg 2, and Terzan 10. Therefore, future multi-wavelength studies should allow us to constrain some parameters better as well as discriminate between competing radiation models.

- I obtained new *Fermi* data that I could fit using a model for the cumulative CR from a population of MSPs embedded within Terzan 5. These data also proved to be constraining for the low-energy tail of the unpulsed IC component, yielding a particle efficiency of $\eta_p \sim 3\%$, depending on the choice of several parameters, notably $\langle \dot{E}_{\text{vis}} \rangle$ and $N_{\text{MSP,tot}}$. I showed that I could fit the radio spectral points by invoking an LESR component that might extend into the optical range. We furthermore argued that our predicted LESR flux is far below the expected thermal optical flux level. Thus, obtaining an upper limit on the non-thermal flux in the optical band would be very difficult, given the roughly $N_* \sim 10^{5-6}$ point sources that have to be subtracted from an optical map of the GC. Even when performing and subtracting a King model fit to the surface brightness profile, the uncertainty on the remaining diffuse flux would be very large. However, since the BB spectrum occurs over a much narrower energy range than the LESR, there may yet be hope of detecting the latter outside of the optical range.
- [Bednarek et al. \(2016\)](#) concurred with our prior predictions ([Venter and De Jager, 2008b](#), [Venter et al., 2009a](#), [Kopp et al., 2013](#)) that GCs should typically have SR components that peak in the optical/ultraviolet range, but also pointed out the problem of the dominating radiation field produced by the large population of GC stars. They furthermore noted that quite atypical parameters would be needed in order to produce an observable level of X-ray flux. Lastly, it would be problematic to compare optical and X-ray brightness profiles, since the underlying source distributions have different spatial and emission characteristics. The respective telescope PSFs also differ, compounding the problem. Also, the observed *Chandra* spectrum is not well fit by a single LESR component. To solve these problems and still fit the observed data, we invoked a new spectral component to explain the hard *Chandra* spectrum: cumulative SR from pair plasma in MSP magnetospheres. The low-energy tail of this HESR component reproduces the spectral slope of the X-ray data quite well. We argued that the required energetics and numbers of the MSP source population needed to reproduce the detected diffuse X-ray emission are plausible, even though they are not very well constrained. The MSP scenario to explain the broadband SED of Terzan 5 should thus be further scrutinised by future constraints on the properties of the MSPs embedded in this GC.
- There were no new data available on Terzan 5 for the VHE band. The HE tail of our predicted unpulsed IC component yields a good fit to the current H.E.S.S. data. More data obtained by H.E.S.S. or new data from the CTA may better constrain the shape and cut-offs of the IC component owing to the lower energy threshold as well as increased sensitivity of the latter. This may limit the particle minimum and maximum energies, source strength, and average multiplicity, as well as the conversion efficiency of spin-down luminosity to particle power.
- In Chapter 5, I considered a magnetospheric pulsar model to model the pulsed SR

and CR components, while I reviewed an independent transport and emission model to model the LESR and IC components. While I have attempted to apply both these codes simultaneously for consistent parameter choices, a unified approach may lead to even deeper constraints on the cluster environment and stellar members. I discovered an error in the script that calculates the pulsed spectra (i.e., I overestimated the spectra by a factor $180/\pi$ by using degrees instead of radians for $d\phi$). I therefore wrote a Matlab script in order to adjust parameters of both pulsed and unpulsed models to consistently fit the data for a given choice of pair multiplicity to counter the error in units.

- Using Terzan 5 as a case study, we thus constrained parameters of our leptonic model for broadband emission from GCs.
- In addition to the work above, I also assessed uncertainties in the predicted VHE γ -ray flux of GCs for a given leptonic model due to uncertainties in model parameters, and gave theoretical guidance to CTA's observational strategy. To do so, I modelled the IC γ -ray flux expected from 15 Galactic GCs to check whether the predicted flux exceeds the CTA sensitivity and whether one can satisfy the H.E.S.S. upper limits with a range of predicted fluxes. I noted that a finer grid in parameter space produces a smooth distribution of fluxes, and that a larger number of trials lead to a convergence of the flux distribution.
- I also considered a stacking approach and found that the relative errors on the predicted live-time-weighted flux is almost one order of magnitude lower than that for the individual GCs. I found that the total integral weighted flux violated the H.E.S.S. upper limits for our first combination of parameters, but the second parameter combination satisfied the stacking upper limits. Thus, one has to be careful to summarily discard models if calculations for best-guess parameters exceed observational limits, noting the considerable uncertainty in predictions that may stem from uncertain parameter values.
- I also did a case study on two GCs, M15 and ω Cen to assess their predicted fluxes plus errors, and found that the differential upper limits were more constraining for M15 than the H.E.S.S. integral flux upper limits. I calculated the TeV flux for ω Cen and showed that this GC maybe a possible candidate to be observed by H.E.S.S. or CTA.
- I therefore noted that one can improve the precision of predictions of GC fluxes by increasing measurement accuracy on model parameters and that models should undergo scrutiny, taking into account the effect of parameter uncertainty on their predictions as they are confronted with new data. This will allow better guidance for the observations by CTA.
- Simulation of GCs' spectra by CTA indicates that we will be able to measure the spectra much better (higher quality) than H.E.S.S., given the increase in sensitivity.

8.2 Future Recommendations

One of the models I used assumed embedded pulsars within the GCs to be the sources of relativistic particles that are injected at the core of the cluster. The stars within the clusters were assumed to have the same mass. As a next step, I plan to improve the current GC model (discussed in Chapter 3) in several respects:

- Modelling the stellar wind effects from red giant population (Bednarek et al., 2016). This will impact the convective properties of the emitting particles injected by the pulsars, leading to sources with X-ray and γ -ray morphologies that depart from spherical symmetry.
- Investigating the effect of different locations of pulsars within the cluster, departing from the idea that they are located at the centre of the cluster. This may lead to morphological changes in the emission from the GCs (Bednarek et al., 2016).
- The morphology of structures associated with Terzan 5 differs significantly in extent and position for the different energy bands, challenging the idea that a single particle population is responsible for all spectral components. Higher-resolution images of the GC will aid in elucidating the spatial properties of the different emission structures, possibly constraining the diffusion coefficient and cluster B -field profile.
- Investigating the effect of stars with different masses and surface temperatures. This will be done consistently to reproduce the measured Hertzsprung-Russell (HR) diagram and also the surface-brightness (King) profile in the optical band. The refinement of the model will influence the background soft-photon field, since individual stars radiate as BBs, and the surface temperature of each star should be a function of its mass. This will lead to an improved treatment of particle transport and the IC spectral calculations. A Monte Carlo approach may be used to solve the particle transport equation.
- Model refinement as well as latest multi-wavelength data will help in constraining several model quantities, which will increase our understanding of the GC environment, e.g., constraining the cluster magnetic field and particle diffusion coefficient. Confronting the model with detailed spectral data will help in constraining the properties of the particle sources, e.g., the number of MSPs as well as their average rotational spin-down and the average efficiency of converting their rotational energy into particle acceleration. This will highly contribute to the Galactic science of *Fermi* LAT, H.E.S.S., the future CTA, as well as that of satellites operating in lower-energy bands.
- The CTA will probably detect many more VHE GCs (Ndiyavala et al., 2018), while multi-wavelength data on these sources should also continue to improve in both

quantity and quality. This will allow us to further scrutinize competing emission models, as well as developing new, more complete and comprehensive ones that might explain the spatial and spectral properties of Galactic GCs at an ever-increasing level of detail.

- Obtaining data or upper limits at low energies will help in constraining the shape of the diffuse synchrotron spectrum. This should probe the lowest particle energies, as well as the injection spectral slope, and even the particle injection spectrum's normalisation, constraining aspects of the MSP population size, energetics, and particle acceleration. Additionally, the model I plan to refine can predict source extension (although it assumes spherical symmetry) in different energy wavebands. Using multi-wavelength data in conjunction with MeerKAT data / limits, we plan to use predictions of spectra in conjunction with source extension and brightness profiles constrain aspects of the particle transport in the cluster, e.g., the diffusion coefficient properties as well as particle reacceleration beyond the MSP magnetospheres.
- When more X-ray data or upper limits become available, this can be used to first constrain the SR spectrum, and then one can have more robust TeV predictions.
- In future one can also consider the effect of $\gamma\gamma$ absorption in GCs.

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