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ADVANCED NETWORK MODELLING TECHNIQUES FOR POWER SYSTEM STATE ESTIMATION

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Abstract

Title: Advanced Network Modelling Techniques for Power System State Estimation

Keywords: Power Systems, State Estimation, Load Flow, Measurements, State Variables, Topology, Observability, Pseudo-measurements, Quadratic-Tangent Objective Function

State estimation is the process to calculate estimates for unknown state variables of a power network by minimising an objective function.

Classical state estimators take into account that network parameters e.g. impedances and admittances, and topology of the power network are accurately known for state estimation. This assumption can cause state variables to be incorrectly estimated. Network impedances can be incorrect or even unknown when a state estimation process needs to be done. The topology of the power network can also be incorrect or even unknown. A solution for these problems is to use a generalised state estimator. A generalised state estimator uses advanced network modelling techniques. These techniques can be used to give estimates for network parameters and topology.

The following advanced network modelling techniques are introduced:

- Modelling of circuit breakers
- Modelling of zero impedance branches
- Modelling of network parameters
- Modelling of transformer parameters

These advanced modelling techniques are used in a state estimation process which minimised a Weighted Least Squares objective function. These advanced modelling techniques are also used in a state estimation process which minimised a Quadratic-Tangent objective function.

Measurements obtained from a network are fraught with errors/bad data. Bad data is used in a state estimation process which minimised a Weighted Least Squares objective function. Bad data

is also used in a state estimation process which minimised a Quadratic-Tangent objective function. Advanced network modelling techniques are used in both these estimation processes.

From the results obtained, it is shown that in the presence of bad data, advanced network modelling techniques can be used in a state estimation process which minimises a Quadratic-Tangent objective function.

Uittreksel

Titel: Gevorderde Netwerk Modellering Tegnieke vir Kragstelsel Toestandsberaming

Sleutelterme: Kragstelsels, Toestandsberaming, Las Vloei, Metings, Toestandsveranderlikes, Topologie, Waarneembaarheid, Skynmetings, Kwadratiese-Raaklyn Doelfunksie

Toestandsberaming is die proses om beramings vir onbekende toestandsveranderlikes van 'n kragnetwerk te bereken deur 'n doelfunksie te minimaliseer.

Klassieke toestandsberamers neem in aanmerking dat netwerk parameters bv. impedansies en admittansies, en topologie van die kragnetwerk, akkuraat bekend is vir toestandsberaming. Hierdie aanname kan veroorsaak dat toestandsveranderlikes verkeerdelik beraam word. Netwerk impedansies kan verkeerd of selfs onbekend wees wanneer 'n toestandsberaming proses gedoen moet word. Die topologie van die kragnetwerk kan ook verkeerd of selfs onbekend wees. 'n Oplossing vir hierdie probleme is om 'n veralgemeende toestandsberamer te gebruik. 'n Veralgemeende toestandsberamer gebruik gevorderde netwerk modelleringstegnieke. Hierdie tegnieke kan gebruik word om beramings te gee vir netwerk parameters en topologie.

Die volgende gevorderde netwerk modelleringstegnieke is bekend gestel:

- Modellering van stroombaanbrekers
- Modellering van nul impedansie takke
- Modellering van netwerk parameters
- Modellering van transformator parameters

Hierdie gevorderde modelleringstegnieke is gebruik in 'n toestandsberaming proses wat 'n Geweegde Kleinste Kwadrate doelfunksie geminimaliseer het. Hierdie gevorderde modelleringstegnieke is ook gebruik in 'n toestandsberaming proses wat 'n Kwadratiese-Raaklyn doelfunksie geminimaliseer het.

Metings verkry van 'n netwerk is belaai met foute/foutiewe data. Foutiewe data is gebruik in 'n toestandsberaming proses wat 'n Geweegde Kleinste Kwadrate doelfunksie geminimaliseer het. Foutiewe data is ook gebruik in 'n toestandsberaming proses wat 'n Kwadratiese-Raaklyn doelfunksie geminimaliseer het. Gevorderde netwerk modelleringstegnieke is in beide hierdie beramingsprosesse gebruik.

Vanuit die resultate verkry, is dit getoon dat in die teenwoordigheid van foutiewe data, gevorderde netwerk modelleringstegnieke gebruik kan word in 'n toestandsberaming proses wat 'n Kwadratiese-Raaklyn doelfunksie minimaliseer.

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John 3:16 "For God so loved the world, that He gave His only begotten Son, that whosoever believeth in Him should not perish, but have everlasting life."

Contents

List of Figures vii

List of Tables viii

List of Symbols ix

1 Introduction 1

2 An Introduction to State Estimation in Power Systems 4

2.1 Introduction 4

2.2 Control Structure of a Power Network 4

2.3 SCADA and EMS 6

2.3.1 Supervisory control and data acquisition 6

2.3.1.1 Data acquisition 7

2.3.1.2 Monitoring and event processing 7

2.3.1.3 Control functions 7

2.3.1.4 Man (human)-machine communication 8

2.3.2 Energy management system 8

2.4 An Introduction to State Estimation 12

2.4.1 The load flow problem 13

2.4.1.1 Load flow solution by Newton-Raphson 14

2.4.1.2 A comparison between load flow and state estimation 15

2.5 Conclusion 16

3 State Estimation in Power Systems **18**

3.1 Introduction 18

3.2 Measurements in a Power Network 19

3.3 State Estimation 20

3.3.1 State variables 20

3.3.2 Classical measurement equations 21

3.3.2.1 Injection measurement equations 22

3.3.2.2 Power flow measurement equations for transmission lines 23

3.3.2.3 Power flow measurement equations for transformers 23

3.3.2.4 Voltage measurement and phase angle measurement equations . . . 24

3.3.2.5 Current measurement equations 24

3.3.2.6 Zero-injection measurement equations 24

3.4 Least Squares 25

3.5 State Estimation Using WLS Criterion 26

3.6 Numerical Solution of the State Estimation Problem 28

3.7 A Numerical Example 29

3.7.1 A load flow approach using meters with no errors 29

3.7.2 A load flow approach using meters with errors 32

3.7.3 State estimation 34

3.8 Conclusion 38

4 Generalised State Estimation **40**

4.1 Introduction 40

4.2 Generalised State Estimation 40

4.2.1 Augmented state variable vector 41

4.2.2 Augmented measurement vector 41

4.3 Advanced Network Modelling Techniques 42

4.3.1 Additional measurement equations 42

4.3.1.1 Circuit breaker modelling 43

4.3.1.2 Modelling of zero impedance branches 44

4.3.1.3 Modelling of network parameters 46

4.3.1.4 Modelling of ULTC transformer parameters 49
4.3.1.5 Modelling of power injection buses 50
4.4 Conclusion 53

5 Functions Supporting State Estimation **54**
5.1 Introduction 54
5.2 Topology Processing and Parameter Estimation 55
5.2.1 Topology Processing 55
5.2.2 Parameter Estimation 55
5.3 Observability and Pseudo-Measurements 56
5.3.1 Observability 56
5.3.2 Pseudo-Measurements 57
5.4 Bad Data Detection and Identification 58
5.4.1 Bad Data Detection 58
5.4.2 Bad Data Identification 59
5.5 Conclusion 60

6 Results **62**
6.1 Introduction 62
6.2 IEEE 14-bus Network 63
6.2.1 Modelling of zero impedance branches 63
6.2.1.1 Non-Augmented method 64
6.2.1.2 Augmented method 65
6.2.2 Circuit breaker modelling 67
6.2.2.1 Circuit breakers in series with a zero impedance branch 68
6.2.2.2 Circuit breakers in series with a non-zero impedance branch 69
6.2.3 Modelling of network parameters 70
6.2.3.1 The series branch 71
6.2.3.2 The shunt branch 72
6.2.4 Modelling of ULTC transformer parameters 73
6.3 IEEE 30-bus Network 75

6.3.1 Modelling of zero impedance branches 75
6.3.1.1 Non-Augmented method 75
6.3.1.2 Augmented method 76
6.3.2 Circuit breaker modelling 78
6.3.2.1 Circuit breakers in series with a zero or low impedance branch . . . 78
6.3.2.2 Circuit breakers in series with a non-zero impedance branch 79
6.3.3 Modelling of network parameters 81
6.3.3.1 The series branch 81
6.3.3.2 The shunt branch 82
6.3.4 Modelling of ULTC transformer parameters 83
6.4 IEEE 57-bus Network 85
6.4.1 Modelling of zero impedance branches 85
6.4.1.1 Non-Augmented method 85
6.4.1.2 Augmented method 86
6.4.2 Circuit breaker modelling 87
6.4.2.1 Circuit breakers in series with a zero or low impedance branch . . . 87
6.4.2.2 Circuit breakers in series with a non-zero impedance branch 89
6.4.3 Modelling of network parameters 90
6.4.3.1 The series branch 90
6.4.3.2 The shunt branch 92
6.4.4 Modelling of ULTC transformer parameters 93
6.5 Articles Published In Conference Proceedings 95
6.6 Conclusion 95

7 Conclusion **97**

Appendix **102**

A Power Flow Equations for Transmission Lines **102**

B Power Flow Equations for Transformers **104**

C Modelling of Network Parameters **107**

C.1 The Series Branch 107

C.2 The Shunt Branch 109

C.3 Modelling of ULTC Transformer Parameters 110

D Bad Data Detection and Identification **112**

E A derivation of Newton methods **113**

E.1 QT objective function 114

E.2 Gradient vector 115

E.3 Hessian 116

E.4 Alternative Gauss-Newton Method 118

F State Estimation Process **120**

G Jacobian Derivatives in Polar Format **122**

G.1 Power Flows 122

G.2 Power Injections 122

G.3 Modelling of zero impedance branches 123

G.4 Modelling of circuit breakers 123

G.4.1 Circuit breakers in series with a non-zero impedance branch 123

G.4.2 Circuit breakers in series with a zero impedance branch 124

G.5 Modelling of unknown series and shunt parameters 124

G.5.1 Series parameters 124

G.5.2 Shunt parameters 125

G.6 Modelling of transformer impedance when tap position is known 125

G.7 Jacobian Derivatives 126

G.7.1 Power Injections 126

G.7.1.1 Active Power 126

G.7.1.2 Reactive Power 127

G.7.2 Power Flows 128

G.7.2.1 Active Power 128

G.7.2.2 Reactive Power 129
G.7.3 Voltage Phase Angles 130
G.7.4 Voltage Magnitude 130
G.7.5 Equations modelling Zero Impedance branches 130
G.7.6 Circuit breakers in series with a non-zero impedance branch 130
G.7.7 Unknown Series parameters 130
G.7.8 Unknown Shunt parameters 132
G.7.9 Unknown transformer impedance 133

H Jacobian Derivatives in Rectangular Format **134**
H.1 Power Flows 134
H.2 Power Injections 135
H.3 Modelling of zero impedance branches 135
H.4 Modelling of circuit breakers 136
H.4.1 Circuit breakers in series with a non-zero impedance branch 136
H.4.2 Circuit breakers in series with a zero impedance branch 136
H.5 Modelling of unknown series and shunt parameters 137
H.5.1 Series parameters 137
H.5.2 Shunt parameters 137
H.6 Modelling of transformer impedance when tap position is known 138
H.7 Jacobian Derivatives 138
H.7.1 Power Injections 139
H.7.1.1 Active Power 139
H.7.1.2 Reactive Power 140
H.7.2 Power Flows 141
H.7.2.1 Active Power 141
H.7.2.2 Reactive Power 142
H.7.3 Voltage Magnitude 142
H.7.4 Equations modelling Zero Impedance branches 143
H.7.5 Circuit breakers in series with a non-zero impedance branch 143
H.7.6 Unknown Series parameters 143
H.7.7 Unknown Shunt parameters 145
H.7.8 Unknown transformer impedance 146

I IEEE Data for Test Cases **147**

I.1 IEEE 14-bus Network 147

I.2 IEEE 30-bus Network 151

I.3 IEEE 57-bus Network 156

J Generation of Measurements **163**

K Measurements for Test Cases **164**

L Augmented method **171**

List of Figures

2.1 Subsystems of a control structure of a power network 5

2.2 A RTU in the local system 6

2.3 An Energy Management System 8

2.4 Power system control using power application software 10

2.5 State Estimation in EMS 11

3.1 A measurement system for a state estimator 20

3.2 Pi - equivalent transmission line model 22

3.3 Transformer pi - equivalent network 23

3.4 A three bus system 30

3.5 Three MW power flow meters 30

3.6 Three meters with errors 32

3.7 Power flows resulting from measurement errors 33

3.8 Results for using meters M_{12} and M_{32} 34

3.9 Results of the State Estimator 38

4.1 Unknown series branch 47

4.2 Unknown shunt branch 48

4.3 Unknown transformer parameters 49

4.4 Power injection bus 51

5.1 Various functions needed to support a state estimator 54

6.1 Zero impedance branch 64

A.1 Transmission line model 102

B.1 Per-unit equivalent circuit 104

B.2 Transformer pi - equivalent network 105

C.1 Unknown series branch 107

C.2 Unknown shunt branch 109

C.3 ULTC transformer model 111

D.1 Hypothesis testing for the identification of a single bad data point 112

E.1 A graph of the Quadratic-Tangent objective function 115

F.1 A flow diagram indicating a general state estimation process 121

I.1 IEEE 14-bus network 148

I.2 IEEE 30-bus network 152

I.3 IEEE 57-bus network 157

J.1 Flow diagram indicating the generation of measurements 163

List of Tables

2.1 State variables for each bus 14

6.1 Non-Augmented method 65

6.2 Non-Augmented method, bad data 66

6.3 Augmented method 67

6.4 Augmented method, bad data 67

6.5 Circuit breakers in series with a zero impedance branch 69

6.6 Circuit breakers in series with a zero impedance branch, bad data 69

6.7 Circuit breakers in series with a non-zero impedance branch 70

6.8 Circuit breakers in series with a non-zero impedance branch, bad data 71

6.9 The unknown series parameter 72

6.10 The unknown series parameter, bad data 72

6.11 The unknown shunt parameter 73

6.12 The unknown shunt parameter, bad data 74

6.13 The unknown transformer parameter 74

6.14 The unknown transformer parameter, bad data 75

6.15 Non-Augmented method 76

6.16 Non-Augmented method, bad data 76

6.17 Augmented method 77

6.18 Augmented method, bad data 78

6.19 Circuit breakers in series with a zero impedance branch 79

6.20 Circuit breakers in series with a zero impedance branch, bad data 79

6.21 Circuit breakers in series with a non-zero impedance branch 80

6.22 Circuit breakers in series with a non-zero impedance branch, bad data 81

6.23 The unknown series parameter 82
6.24 The unknown series parameter, bad data 82
6.25 The unknown shunt parameter 83
6.26 The unknown shunt parameter, bad data 83
6.27 The unknown transformer parameter 84
6.28 The unknown transformer parameter, bad data 84
6.29 Non-Augmented method 85
6.30 Non-Augmented method, bad data 86
6.31 Augmented method 87
6.32 Augmented method, bad data 88
6.33 Circuit breakers in series with a zero impedance branch 88
6.34 Circuit breakers in series with a zero impedance branch, bad data 89
6.35 Circuit breakers in series with a non-zero impedance branch 90
6.36 Circuit breakers in series with a non-zero impedance branch, bad data 91
6.37 The unknown series parameter 91
6.38 The unknown series parameter, bad data 92
6.39 The unknown shunt parameter 93
6.40 The unknown shunt parameter, bad data 93
6.41 The unknown transformer parameter 94
6.42 The unknown transformer parameter, bad data 94

I.1 Branch data for the 14-bus network 147
I.2 Operating conditions for the 14-bus network 149
I.3 Regulated bus data for the 14-bus network 149
I.4 Transformer data for the 14-bus network 150
I.5 Branch data for the 30-bus network 153
I.6 Operating conditions for the 30-bus network 154
I.7 Regulated bus data for the 30-bus network 155
I.8 Transformer data for the 30-bus network 155
I.9 Branch data for the 57-bus network 158
I.10 Branch data for the 57-bus network (Continued) 159

I.11 Operating conditions for the 57-bus network 160

I.12 Operating conditions for the 57-bus network (Continued) 161

I.13 Regulated bus data for the 57-bus network 162

I.14 Transformer data for the 57-bus network 162

K.1 Measurements used for the IEEE 14-bus in polar format 165

K.2 Measurements used for the IEEE 30-bus in polar format 166

K.3 Measurements used for the IEEE 30-bus in polar format (Continued) 167

K.4 Measurements used for the IEEE 57-bus in polar format 168

K.5 Measurements used for the IEEE 57-bus in polar format (Continued) 169

K.6 Measurements used for the IEEE 57-bus in polar format (Continued) 170

List of Symbols

The following notations will be used:

- Matrices and vectors will be shown in **bold**.
- Matrices will be shown in upper case and vectors in lower case.
- Complex numbers will be underlined.

$\mathbf{z}$	Measurement vector
$\mathbf{h}$	Non-linear measurement vector function
r_i	Residual for the i- th measurement
$\widehat{\mathbf{x}}$	Vector containing the solution of state variables
v	Random measurement error
$\mathbf{v}$	Vector of random measurement errors
$\underline{V}_i$	Phasor voltage at node i
V_i	Voltage magnitude at node i
θ_i	Voltage phase angle at node i
f_i	Imaginary part of phasor voltage at node i
e_i	Real part of phasor voltage at node i
P_i	Active power injection at node i
P_i^m	Active power injection measurement
P_{ij}	Active power flow from node i to node j
P_{ij}^m	Active power flow measurement
Q_i	Reactive power injection at node i
Q_i^m	Reactive power injection measurement
Q_{ij}	Reactive power flow from node i to node j
Q_{ij}^m	Reactive power flow measurement
G_{ij}	Conductance of a transmission line between node i and node j
B_{ij}	Susceptance of a transmission line between node i and node j
m	Number of measurements
n	Number of state variables
N	Number of nodes in the power system
$\mathbf{R}$	Covariance matrix
σ	Standard deviation
$\mathbf{H}$	Jacobian matrix
$\mathbf{I}$	Identity matrix
J	Objective function
$\mathbf{J}_x$	Gradient vector of objective function
$\mathbf{J}_{xx}$	Hessian matrix of objective function
$\Delta\mathbf{x}$	State correction vector

Chapter 1

Introduction

An overall control structure of a power system network consists of several subsystems. Local systems include the parts of the control structure installed at e.g. substations and power plants for remote data acquisition and control. A communication system connects the geographically distributed local system with a central system. The central system gathers the information received from the communication system. It provides the operators with information in real-time through the basic Supervisory Control And Data Acquisition (SCADA) functions. The SCADA system constitutes a basis for implementing more advanced functions of the Energy Management System (EMS), e.g. the state estimator.

The load flow problem is the computation of voltage magnitude and phase angle at each bus in a power system network. A system of non-linear equations is solved by means of procedures involving iterative techniques to obtain the voltage magnitude and phase angle at each bus. The load flow problem serves as an introduction to the numerical solution and concepts of the state estimation problem.

State estimation solves a non-linear optimisation problem whose solution for the unknown state variables, yields the state estimate for the entire system. State variables are defined as a non-redundant set of variables which completely characterise the condition of a system. Once the state is estimated, estimates for all other quantities of interest such as line flows, can be computed. Measurement equations and their corresponding measurements are used in the state estimation problem. These measurements are obtained from measurement devices in a power system network. Residuals are defined as the difference between a measurement from the network and the computed value of its corresponding measurement equation. A state estimation process minimises criteria to obtain estimates for unknown state variables. These criteria are functions of residuals. Several criteria can be used in the state estimation process e.g. Weighted Least Squares criterion, Quadratic-Tangent criterion etc. .

Classical state estimators take into account that the network parameters e.g. impedances and admittances, and topology of the power network are accurately known for state estimation. This assumption can cause state variables to be incorrectly estimated. A generalised state estimator

can be used instead to prevent state variables from being incorrectly estimated. A generalised state estimator uses advanced network modelling techniques. These techniques can be used to give estimates for network parameters and topology.

The following advanced network modelling techniques will be introduced:

- Modelling of circuit breakers
- Modelling of zero impedance branches
- Modelling of network parameters
- Modelling of ULTC transformer parameters

Some of these advanced network modelling techniques used in a state estimation process which minimises the Weighted Least Squares criterion, are discussed in [19]. In this dissertation, exhaustive testing will be done on a state estimator which uses these advanced network modelling techniques. The criteria which will be used are the Weighted Least Squares criterion and the Quadratic-Tangent criterion. A comparison will be made between the estimates obtained for using each of these two criteria and the different advanced network modelling techniques. Measurements obtained from a network are fraught with errors/bad data. Exhaustive testing will be done on a state estimator which uses these advanced network modelling techniques in the presence of bad data. The criteria which will be used are the Weighted Least Squares criterion and the Quadratic-Tangent criterion. A comparison will be made between the estimates obtained in the presence of bad data for using each of these two criteria and the different advanced network modelling techniques.

The layout for this dissertation will be as follows:

- Chapter two will give an introduction to state estimation in power systems. An overall control structure of a power network will be discussed. An overview of the Supervisory Control And Data Acquisition (SCADA) functions and the Energy Management System (EMS) will be given. The load flow problem will be explained and discussed.
- Several aspects of state estimation will be discussed in chapter three. The role of measurements, state variables and measurement equations in the state estimation problem will be discussed. Several examples of classical measurement equations will be given. A short overview on Least Squares will be given which serves as an introduction to state estimation using the Weighted Least Squares criterion. A numerical solution of the state estimation problem will be given.
- Chapter four will discuss generalised state estimation. The state variable vector, measurement equations vector and measurement vector will be augmented. The various advanced modelling techniques will be discussed in detail. These techniques are used in the generalised state estimator.

- In chapter five, several functions which support a state estimator, are mentioned and discussed. The role of topology processing and parameter estimation are mentioned. The importance of observability and pseudo-measurements are discussed. This chapter concludes with an overview on bad data detection and identification.

- Results will be given in chapter six. Exhaustive testing will be done on a state estimator which uses advanced network modelling techniques. The criteria which will be used are the Weighted Least Squares criterion and the Quadratic-Tangent criterion. Exhaustive testing will also be done on a state estimator which uses advanced network modelling techniques in the presence of bad data.

- Finally, a conclusion is given in chapter seven.

Chapter 2

An Introduction to State Estimation in Power Systems

2.1 Introduction

An introduction to state estimation in power systems will be given in this chapter.

In section 2.2, an overall control structure of a power system network will be given. The control structure of a power network consists of several subsystems. The interconnections between these subsystems will be discussed. Section 2.2 concludes with the role and purpose of Remote Terminal Units (RTU's) in local subsystems.

The basic requirements for a control system of a power system network always include the Supervisory Control And Data Acquisition (SCADA) functions. Section 2.3 will explain the SCADA functions in more detail as well as the role of an Energy Management System (EMS) in a power system network. The SCADA system consists of several main functions which will be briefly described. The object of the EMS is to ensure secure and economic operation of power systems. Section 2.3 concludes with basic features of a modern Energy Management System.

The last section of this chapter, section 2.4, will give an introduction to state estimation in power systems. To obtain a better understanding of the concept of state estimation, the load flow problem will be introduced. The role of a state estimator in an EMS will be explained as well and this chapter concludes with several functions performed by a state estimator in a power system network.

2.2 Control Structure of a Power Network

An overall control structure of a power network consists of several subsystems as shown in Figure 2.1. According to Figure 2.1, the basic control structure of a power system network consists of a local system, a communication system and a central system.

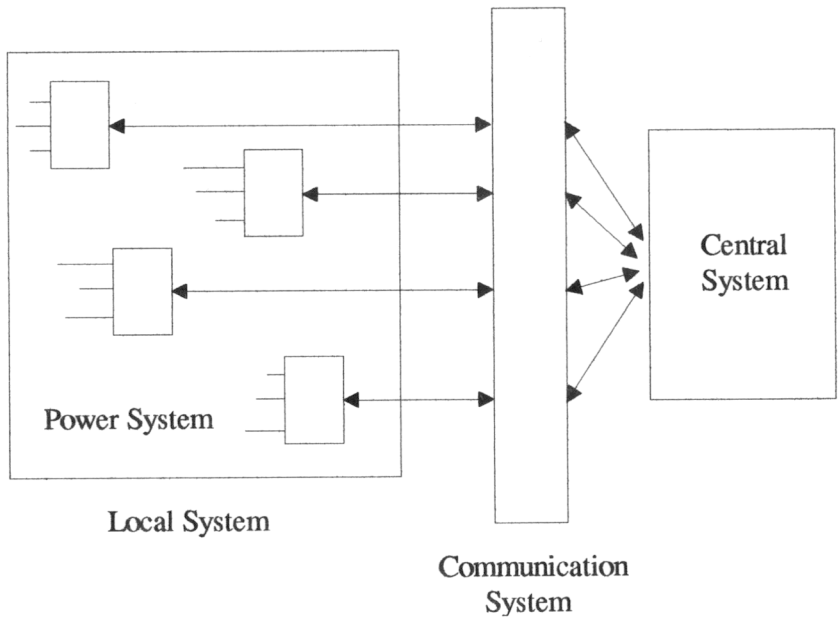

Figure 2.1: Subsystems of a control structure of a power network

Local systems include the parts of the control structure installed at e.g. substations and power plants for remote data acquisition and control. The physical connection to the power system is made through the local system. A local system makes it possible to collect data and to execute control commands from a central system. A local system reports also status and status changes to the central system. It is also possible to perform control functions via the local system which will be of a more local control nature.

The local system communicates with a central system through communication links. These communication links form a communication system which connects the geographically distributed local system with a central system. As mentioned before, the communication system must be able to handle mutual communication. Thus, it must handle communication from the local system to the central system and also communication from the central system to the local system.

The central system gathers the information received from the communication system. It provides the operators with information in real-time through the basic Supervisory Control And Data Acquisition (SCADA) functions. A central system comprises the equipment in the control centre e.g. the centrally located control room with the man (human)-machine equipment and the central computer system together with its database.

The database in the control centre needs to be updated with the most up to date data available from the local system. The local system consists of a number of Remote Terminal Units (RTU's) connected to a central system via a communication system as shown in Figure 2.2. The basic feature of the RTU is storage of data. In Figure 2.2, a signal is collected via a transducer and carried in an analog form over cables to an analog meter and the data of the meter is stored in the RTU. Similarly, from the RTU an operation of a breaker can be performed via the maneuver switch and the analog signal triggers the breaker mechanism to open or close the breaker. Local control instructions given

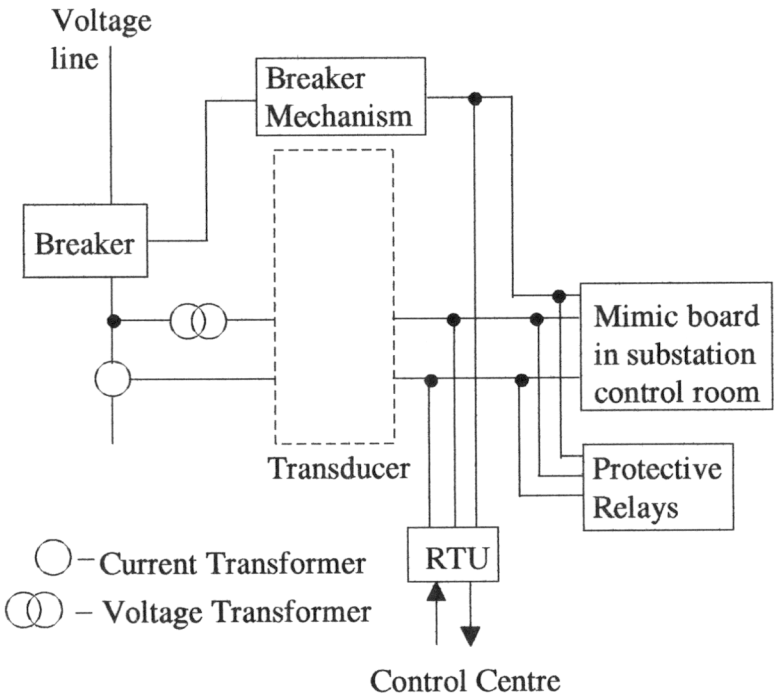

Figure 2.2: A RTU in the local system

by the control centre can be done via the RTU. For more information on the control structure of a power network, see [1].

It is possible to store data of the local system in a RTU as mentioned before. The basic mechanism for acquiring and processing collected data is thus obtained by storing items of data, for example measured values. This is a very important function of the SCADA system which will be discussed in the next section.

2.3 SCADA and EMS

2.3.1 Supervisory control and data acquisition

The basic requirements for a control system of a power system network always include the Supervisory Control And Data Acquisition (SCADA) functions. The SCADA system is a general hardware and software concept providing a flexible set of functions. The SCADA system constitutes a basis for implementing more advanced functions of the Energy Management System (EMS), for example the state estimator. SCADA also provides for further development of the control system. The main functions of a simple SCADA system are:

- Data acquisition
- Monitoring and event processing

- Control functions
- Man (Human)-Machine communication

2.3.1.1 Data acquisition

As can be seen from Figure 2.2, the distributed control system equipment in the local system enables remote data acquisition. The basic information with regard to the power system e.g. voltage values, current values, breaker states etc., are collected by equipment in the various substations and power plants and stored in the database at the control centre. Data may also be entered manually into the database or it may be calculated and stored in the database. This data is treated exactly like the automatically collected data by the equipment in the various substations and power plants. New data can also be calculated and stored in the database using the automatically collected values.

2.3.1.2 Monitoring and event processing

The acquired power system data as described in section 2.3.1.1, is automatically monitored to ensure that measured and calculated values lie within the permissible limits. The status of circuit breakers, alarm signals and other signals are represented by means of status indications. These status indications are monitored with regard to changes and recorded in the correct time sequence.

Measured values of various kinds, e.g. voltage levels, active and reactive power flows, temperature, tap changer positions etc. are collected by the control system. The measured values are monitored with regard to rate-of-change and for continuous trend recording.

If the monitoring process gives rise to violation of limits and changes of status indications, then event processing is required. This processing implies grouping and classification of events on the mimic board, the Visual Display Units (VDU's), the console keyboards and on various kinds of reports, so that an appropriate alarm presentation to the operators can be achieved. Thus, event processing results in certain statuses and classes being assigned to events that are reported to the operators.

2.3.1.3 Control functions

The control functions of the SCADA system can be grouped into four subclasses. A very brief discussion of each subclass will be given:

- *Individual device control* - Individual device control functions are the basic commands that can be directed towards the power system e.g. on/off commands for circuit breakers and disconnectors, start/stop commands towards generators etc.

- *Transmission of control messages to regulating equipment* - Applications for these functions include raise/lower regulation and set point regulation. The entered values are limit-checked against predetermined limits, to prevent very abnormal values from being entered.
- *Sequential control schemes* - The sequential control facility permits a sequence of control commands to be executed automatically in a predefined order, including suitable security checks and time delays.
- *Automatic control schemes* - There are also automatically initiated commands e.g. the Automatic Generation Control (AGC) function, which will adjust generator output to meet the objectives of keeping the frequency at the scheduled value.

2.3.1.4 Man (human)-machine communication

Human-machine communication via a console includes the use of a VDU, a keyboard, some cursor positioning devices and auxiliary communication equipment. All VDU's are used for simultaneous presentation of different displays and for entering commands to the control system. For more information on supervisory control and data acquisition functions, see [1].

2.3.2 Energy management system

SCADA forms the basis for EMS. A modern energy management system (EMS) of a power system network is shown in Figure 2.3. Figure 2.3 also indicates the transfer of real-time data from the

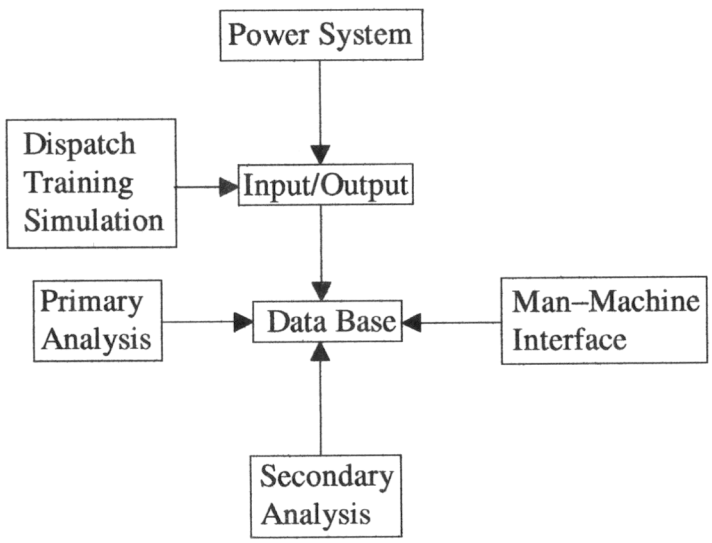

Figure 2.3: An Energy Management System

power system to the database. The dispatch training simulator is used to give the operators the necessary experience and training on how to control the power system network. The function of the man-machine or human machine interface is as described in section 2.3.1.4. The primary analysis in Figure 2.3 is responsible for the following tasks:

- Control of the information display systems
- Recording of status and events
- Information processing for telecommand and control.

The secondary analysis in Figure 2.3 consists of advanced decision support and optimisation tools which include:

- State Estimation
- Optimal power flow
- Network security analysis
- Load prediction
- Long-, medium- and short-term optimisation.

The purpose of the EMS is to ensure secure and economic operation of power systems. The basic features of a modern EMS are as follows:

- Monitoring the system state by means of analog and digital information
- Interaction with the power system in a preventive or corrective manner
- Regulation of frequency and voltage
- Reliable and complete system information despite wrong or missing data
- Evaluation of operational risks for the present or future state
- Actions to improve power system security
- Economic power system operation
- Optimal primary energy usage observing environmental aspects [2].

Power application software are also available to help the operators in controlling the power system network. These software are used to ensure the secure and economic operation of a power system network. Power application software can include state estimation, on-line load flow, security assessment, stability analysis and short-circuit analysis. Figure 2.4 shows the use of power application software as part of the EMS [2].

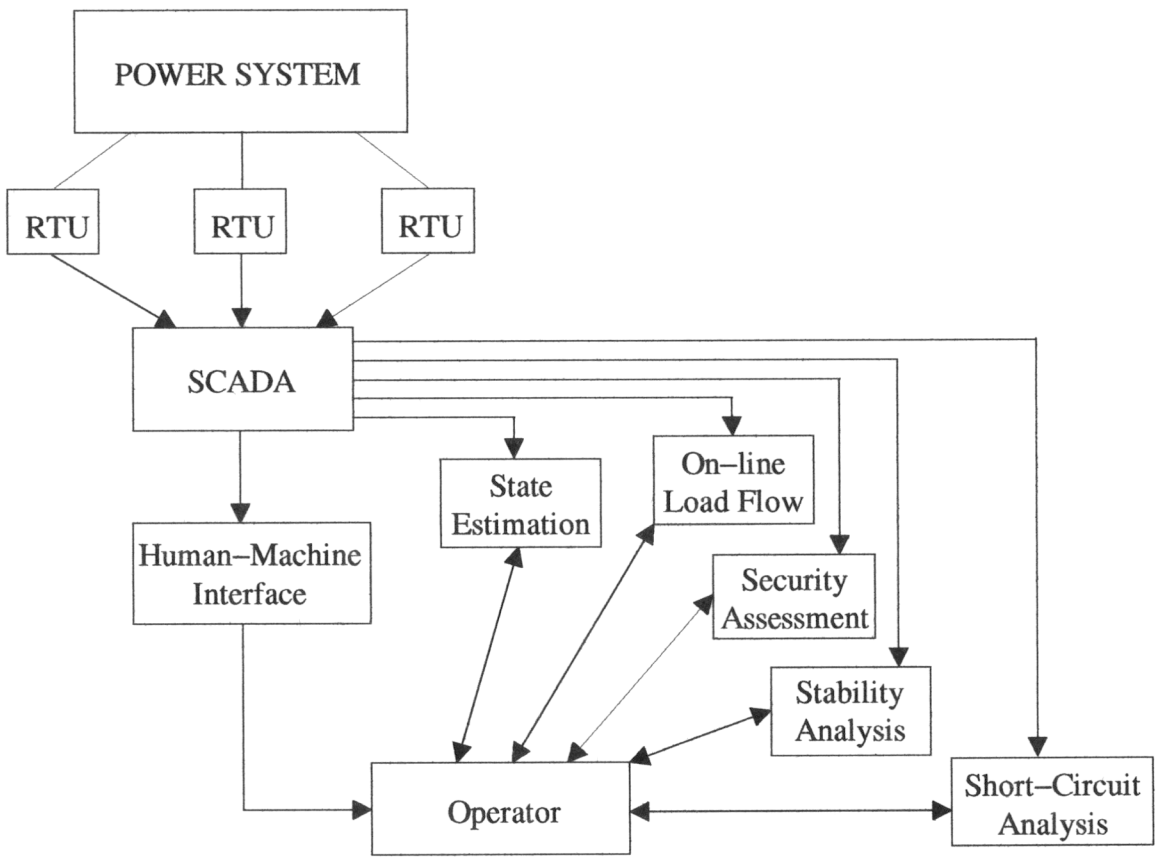

Figure 2.4: Power system control using power application software

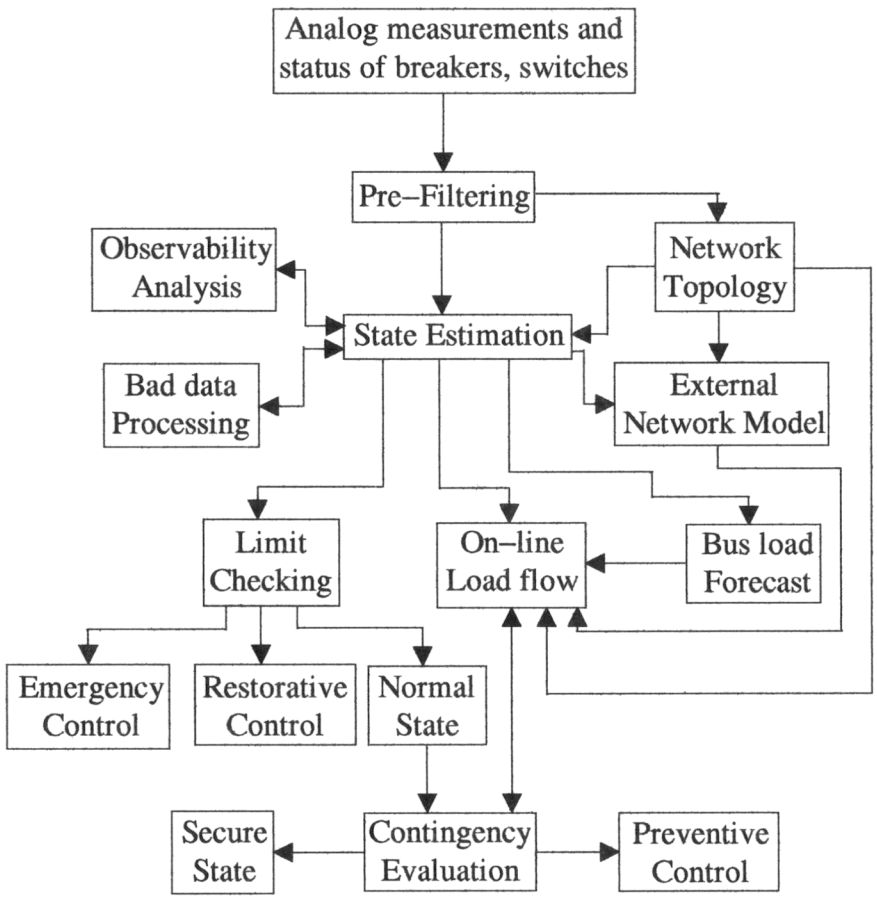

Figure 2.5: State Estimation in EMS

2.4 An Introduction to State Estimation

As mentioned in section 2.3.2, the secondary analysis of a modern Energy Management System (EMS) consists of advanced decision support and optimisation tools which include state estimation. Figure 2.5 shows the role of the state estimator in the Energy Management System.

Some of the functions which the state estimator can perform, according to Figure 2.5, are the following:

- Observability analysis - Tests whether or not the available measurements are sufficient to estimate the entire system state. If the test fails, then observable parts of the system will be identified and pseudo-measurements will be added to enlarge the observable islands/parts of the system. The concept of pseudo-measurements will be discussed in section 5.3.2.
- Topology preprocessing - Obtains an one-line model of the system based on the information of the circuit breaker / switch states. Topology processing will be discussed in section 5.2.
- State estimation - Solves a non-linear optimisation problem whose solution yields the state estimate for the entire system. Once the state is estimated, estimates for all other quantities of interest such as line flows, can be computed.
- Bad data processing - Checks the measurements for the existence of possible bad data. If any of the measurements are flagged as bad data, they will be removed or corrected. Bad data processing will be discussed in section 5.4.
- Gives a real-time model of the power system which is mathematically complete and consistent.
- Provides a reliable snap-shot of the current operating conditions of the power system.
- Gives the fundamental base, which is necessary for advanced control of the power system with other application functions e.g.bus load forecasting, optimal power flow etc.
- Gives the missing pieces or measurements of e.g. analog measurements in the raw measurement data set.
- Provides estimates of hard to measure (or impossible to measure) quantities or state variables e.g. bus voltage, phase angles etc.
- Can identify incorrect parameters in the database, wrong device settings e.g. transformer taps, topology errors etc. . For more information on functions which the state estimator can perform, see [1] and [8].

The next section gives an overview of the load flow problem. The load flow problem serves as an introduction to the numerical solution and concepts of the state estimation problem. A comparison between the load flow problem and the state estimation problem is also presented.

2.4.1 The load flow problem

The basic load flow problem can be defined as the situation where, for given load demands at the various load buses and for assumed generation levels at specified supply voltages, one is interested in the overall voltage profile of the power network, real and reactive flows on transmission lines and transformers, bus voltage magnitudes and phase angles, line currents, line losses and other related steady state variables [3].

Computer programs are used to solve load flow problems and are divided into two types e.g. static or off-line and dynamic or on-line/real-time. Most load flow studies for system analysis are based on static network models. Real time load flows that incorporate data input from the actual networks are typically used by utilities with Supervisory Control And Data Acquisition (SCADA) systems. A load flow calculation determines the state of the power system for a given load and generation distribution.

The load flow problem uses typically three bus types in its mathematical solution which are:

1. *Load bus* - A load bus is any bus that does not have a generator. A load bus does not need to have load, it can also be an interconnection point for two or more lines. Most buses in a typical load flow problem are load buses.

2. *Voltage controlled bus or generator bus* - Examples of this type of bus are buses to which generators, switched shunt capacitors or static var systems are connected. Another example is a bus to which a tap-changing transformer is connected. The load flow program then computes the tap setting.

3. *Swing bus or slack bus*- There is only one swing bus in the network. The swing bus is a reference bus. This type of bus is also a special type of generator bus that is needed by the solution process of the load flow problem. All the generators in the system supply MW and MVAR losses that are not otherwise accounted for. Generation at the buses can not be independently specified, so one generator bus is chosen as the slack bus.

The load flow problem is the computation of voltage magnitude and phase angle at each bus in a power system network. This computation is under balanced three-phase steady-state conditions. A by-product of this computation will be to calculate real and reactive power flows. Each of above mentioned types of buses is represented with four state variables in the load flow problem. The following four state variables can be associated with each bus in the power network: voltage magnitude (V), phase angle of bus voltage referred to a common reference (θ), net real power into the transmission network (P) and reactive power into the transmission network (Q). At each bus, two of these state variables are specified as input data while the other two state variables are unknowns to be computed by the load flow program. Table 2.1 shows the different state variables of each bus as input data for the load flow program as well as the unknown state variables for each bus that needs to be calculated by the load flow program.

Table 2.1: State variables for each bus

Type of bus	**Known state variables**	**Unknown state variables**
Load bus	P , Q	V , θ
Voltage controlled bus	P , V	Q , θ
Voltage controlled bus to which a tap-changing transformer is connected	P , Q , V	θ , *tap setting*
Swing bus or slack bus	V , θ	P , Q

The load flow program solves a system of non-linear equations by means of procedures involving iterative techniques. The Gauss-Seidel and Newton-Raphson techniques are mostly used to solve the unknown state variables of each bus. Section 2.4.1.1 will give a brief overview on how the load flow problem is solved by using the Newton-Raphson method. For more detail on load flow solutions by means of Newton-Raphson and Gauss-Seidel techniques, refer to [4] and [5].

2.4.1.1 Load flow solution by Newton-Raphson

The following formulation uses a load bus only. The known state variables are P and Q. The unknown state variables are V and θ. Suppose the system of non-linear equations is represented by the following non-linear equation

$$\mathbf{y} = \mathbf{f}(\mathbf{x}), \tag{2.1}$$

with

$$\mathbf{x} = \begin{bmatrix} \theta \\ \mathbf{V} \end{bmatrix} = \begin{bmatrix} \theta_2 \\ \vdots \\ \theta_N \\ V_2 \\ \vdots \\ V_N \end{bmatrix} ; \quad \mathbf{y} = \begin{bmatrix} \mathbf{P} \\ \mathbf{Q} \end{bmatrix} = \begin{bmatrix} P_2 \\ \vdots \\ P_N \\ Q_2 \\ \vdots \\ Q_N \end{bmatrix} ; \quad \mathbf{f}(\mathbf{x}) = \begin{bmatrix} \mathbf{P}(\mathbf{x}) \\ \mathbf{Q}(\mathbf{x}) \end{bmatrix} = \begin{bmatrix} P_2(x) \\ \vdots \\ P_N(x) \\ Q_2(x) \\ \vdots \\ Q_N(x) \end{bmatrix}, \tag{2.2}$$

where N is the number of buses in the network.

Vector $\mathbf{x}$ contains the unknown state variables to be calculated by the Newton-Raphson iterative method. Vector $\mathbf{y}$ contains the known P and Q injection values into the network. Vector $\mathbf{f}(\mathbf{x})$ contains the equations formulated for each bus. Applying the Newton-Raphson steps to the load flow problem and starting with

$$\mathbf{x}(i) = \begin{bmatrix} \theta(i) \\ \mathbf{V}(i) \end{bmatrix}, \tag{2.3}$$

for the ith iteration, will give

Step 1: Compute

$$\triangle \mathbf{y}(i) = \mathbf{y} - \mathbf{f}[\mathbf{x}(i)] = \begin{bmatrix} \triangle \mathbf{P}(i) \\ \triangle \mathbf{Q}(i) \end{bmatrix} = \begin{bmatrix} \mathbf{P} - \mathbf{P}[\mathbf{x}(i)] \\ \mathbf{Q} - \mathbf{Q}[\mathbf{x}(i)] \end{bmatrix}. \tag{2.4}$$

Step 2: Calculate the Jacobian matrix

$$\mathbf{J}(i) = \left.\frac{d\mathbf{f}}{d\mathbf{x}}\right|_{\mathbf{x}=\mathbf{x}(i)} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_N} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_N} \\ \vdots & \vdots & \cdots & \vdots \\ \frac{\partial f_N}{\partial x_1} & \frac{\partial f_N}{\partial x_2} & \cdots & \frac{\partial f_N}{\partial x_N} \end{bmatrix}_{\mathbf{x}=\mathbf{x}(i)}. \tag{2.5}$$

Step 3: Use Gauss elimination and back substitution to solve

$$\begin{bmatrix} \frac{\partial \mathbf{P}_2}{\partial \theta_2} & \cdots & \frac{\partial \mathbf{P}_2}{\partial \theta_N} & \frac{\partial \mathbf{P}_2}{\partial \mathbf{V}_2} & \cdots & \frac{\partial \mathbf{P}_2}{\partial \mathbf{V}_N} \\ \vdots & & \vdots & \vdots & & \vdots \\ \frac{\partial \mathbf{P}_N}{\partial \theta_2} & \cdots & \frac{\partial \mathbf{P}_N}{\partial \theta_N} & \frac{\partial \mathbf{P}_N}{\partial \mathbf{V}_2} & \cdots & \frac{\partial \mathbf{P}_N}{\partial \mathbf{V}_N} \\ \frac{\partial \mathbf{Q}_2}{\partial \theta_2} & \cdots & \frac{\partial \mathbf{Q}_2}{\partial \theta_N} & \frac{\partial \mathbf{Q}_2}{\partial \mathbf{V}_2} & \cdots & \frac{\partial \mathbf{Q}_2}{\partial \mathbf{V}_N} \\ \vdots & & \vdots & \vdots & & \vdots \\ \frac{\partial \mathbf{Q}_N}{\partial \theta_2} & \cdots & \frac{\partial \mathbf{Q}_N}{\partial \theta_N} & \frac{\partial \mathbf{Q}_N}{\partial \mathbf{V}_2} & \cdots & \frac{\partial \mathbf{Q}_N}{\partial \mathbf{V}_N} \end{bmatrix} \begin{bmatrix} \triangle \theta(i) \\ \triangle \mathbf{V}(i) \end{bmatrix} = \begin{bmatrix} \triangle \mathbf{P}(i) \\ \triangle \mathbf{Q}(i) \end{bmatrix}. \tag{2.6}$$

Step 4: Compute

$$\mathbf{x}(i+1) = \begin{bmatrix} \theta(i+1) \\ \mathbf{V}(i+1) \end{bmatrix} = \begin{bmatrix} \theta(i) \\ \mathbf{V}(i) \end{bmatrix} + \begin{bmatrix} \triangle \theta(i) \\ \triangle \mathbf{V}(i) \end{bmatrix}. \tag{2.7}$$

The Newton-Raphson iterative procedure continues until convergence is obtained or until the number of iterations exceeds a specified maximum. The unknown state variables for each bus are thus solved [5].

2.4.1.2 A comparison between load flow and state estimation

The state estimation problem can be seen as a load flow problem. State estimation is based on the same mathematical procedures and concepts as described above. Why is state estimation then needed for? The next discussion will give a short comparison between load flow and state estimation:

- The voltage magnitude (V) must be known at the slack bus for load flow while any bus in the system can be used as a reference bus for state estimation. In state estimation, a reference bus whose phase angle is fixed at some arbitrary value is used.
- For load flow, all the injection values used in the calculation process should at least have mismatches within a certain tolerance. The injection values can almost be seen as a clean

input for the load flow problem. For state estimation, none of the measurements used will have mismatches within a certain tolerance due to big measurement errors. The input to the state estimator is thus fraught with errors.

- Load flow uses a deterministic set of injection values while state estimation uses a redundant set of measurements in the calculation process.
- The Jacobian matrix used in the load flow calculations is an $n \times n$ matrix as in equation (2.5). The Jacobian matrix used in the state estimation calculations is an $m \times n$ matrix as will be shown in equation (3.18). An overdetermined system of non-linear equations needs to be solved by the state estimator.
- Load flow gives an output vector $\mathbf{x}$ as in equation (2.7), which is a vector of n state variables. These state variables can be seen as a more accurate solution for the load flow problem. The load flow problem is used in an ideal/clean environment. State estimation also gives an output vector of n state variables which is an estimated solution for the state estimation problem. State estimation is used in a real-time environment.
- Load flow is a circuit solution. State estimation is a statistical estimation based on redundant observations of the system quantities.

An on-line situation deals with measurements that are fraught with errors. A tool is needed to filter these measurements. Therefore, a state estimator can be used. As from above discussion, state estimation is based on redundancy and the state estimator is able to solve an overdetermined system of non-linear equations. State estimation processes more equations than unknowns and is able to filter these measurements containing errors.

2.5 Conclusion

A control structure of a power network consists of several subsystems. Local systems include the parts of the control structure installed at e.g. substations and power plants for remote data acquisition and control. The local system communicates with a central system via communication links. The central system gathers information received from the communication system. The central system provides the operators with information in real-time through the basic Supervisory Control And Data Acquisition (SCADA) functions.

The main functions of a simple SCADA system are mentioned and discussed. Different types of data are collected by equipment in the various substations and power plants of the local system. These types of data are stored in the database at the control centre. These different types of data are also monitored to ensure that measured and calculated values lie within the permissible limits. SCADA forms the basis for the Energy Management System (EMS). The secondary analysis of an EMS consists of advanced decision support and optimisation tools which include the state estimator.

Several functions which can be performed by the state estimator, are mentioned. An overview of the load flow problem is given. Three types of buses are used in the mathematical process of the load flow problem. A load flow program solves a system of non-linear equations by means of procedures involving iterative techniques.

Finally, a comparison between load flow and state estimation is given.

The next chapter will discuss state estimation in power systems in much more detail.

Chapter 3

State Estimation in Power Systems

3.1 Introduction

This chapter will discuss state estimation in power systems.

Section 3.2 will give a description of the different measurements available in a power system. A state estimator uses measurements, which are obtained from the power system, in its estimation process. This section will also mention the fact that measurements which are obtained from a power system, are fraught with errors. These errors are not known, but a statistical measure is associated with each error. This section concludes with a typical measurement system which can be used for a state estimator.

Section 3.3 will mention several concepts which will be used in a state estimation process. Different types of state variables will be given. This section will discuss how classical measurement equations e.g. injection equations, power flow equations, voltage and phase angle equations, current equations and zero-injection equations form part of the state estimation process.

The next section, section 3.4, will give an overview of Least Squares. This section will show how an overdetermined system can be solved by means of Least Squares. This idea will be used as an introduction to Weighted Least Squares criterion.

Section 3.5 will explain how to minimise residuals through the use of Weighted Least Squares criterion. The WLS criterion is based on Least Squares, except that WLS uses weights or variances for each measurement error in the state estimation process. This section concludes with the difference between constrained and unconstrained Weighted Least Squares.

After knowing the criteria being used by the state estimator, it is important to know how the state estimation problem will be numerically solved to obtain estimates for the unknown state variables. A numerical solution of the state estimation problem will be given in section 3.6. This section will show how to minimise the unconstrained Weighted Least Squares objective function.

Section 3.7 will give a numerical example which will explain the discussion of section 3.6. This example will also show the difference between state estimation and load flow.

3.2 Measurements in a Power Network

The state estimator uses measurements which are obtained from measurement devices in a power system network. These measurements are used in the mathematical procedures to solve the state estimation problem. The following types of measurements can be used by the classical state estimator:

- Active power branch flow P_{km} in branch km .
- Active power branch group flow ΣP_{km} in a designated group of branches.
- Active power bus injections P_k into bus k .
- Reactive power branch flow Q_{km} in branch km .
- Reactive power branch group flow ΣQ_{km} in a designated group of branches.
- Reactive power bus injections Q_k into bus k .
- Voltage magnitude V_k at bus k .
- Voltage angle θ_k at bus k obtained from synchronised phasor measurements.
- Current magnitude flow I_{km} in branch km and current injection magnitude I_k at bus k [6].

The above measurements can be represented as vector $\mathbf{z}$ in equation (3.4) of section 3.3.2. The above measurements are also fraught with measurement errors. The value obtained from the measurement device is close to the true value of the parameter being measured but differs with an unknown small error. The following mathematical model can be used to describe this situation:

$$z^{meas} = z^{true} + v\,, \tag{3.1}$$

where z^{meas} is the measured value as received from a measurement device e.g. voltage, current, active and reactive power, voltage angle, etc. , z^{true} is the true value of the quantity being measured, v is the random measurement error e.g. high frequency noise, calibration errors etc. .

The error v is obviously not known. Statistical properties of the random measurement error can be assumed. The statistical measure is given in terms of a standard deviation of error. Statistically,

$$E\left[v_i\right] = 0 \quad \text{and} \quad E\left[v_i^2\right] = \sigma_i^2\,. \tag{3.2}$$

This means that the error v_i is zero on the average and that its standard deviation is σ_i. For more information on the statistical properties of the state estimator, see [3]. The random measurement errors are assumed to be normally distributed with a zero mean. The probability density function of v is an assumed function and can be as follows:

$$PDF(\mathbf{v}) = \frac{1}{\sigma\sqrt{2\pi}} \exp(-\mathbf{v}^2/2\sigma^2), \tag{3.3}$$

where σ is called the standard deviation and σ^2 is called the variance of the random measurement error. The standard deviation gives an indication of the seriousness of the random measurement error. When σ is large, the measurement is relatively inaccurate, thus a poor quality measurement device. When σ is small, a small error spread occurs, thus a higher quality measurement device [7].

Figure 3.1 shows a typical measurement system which can be used for a state estimator.

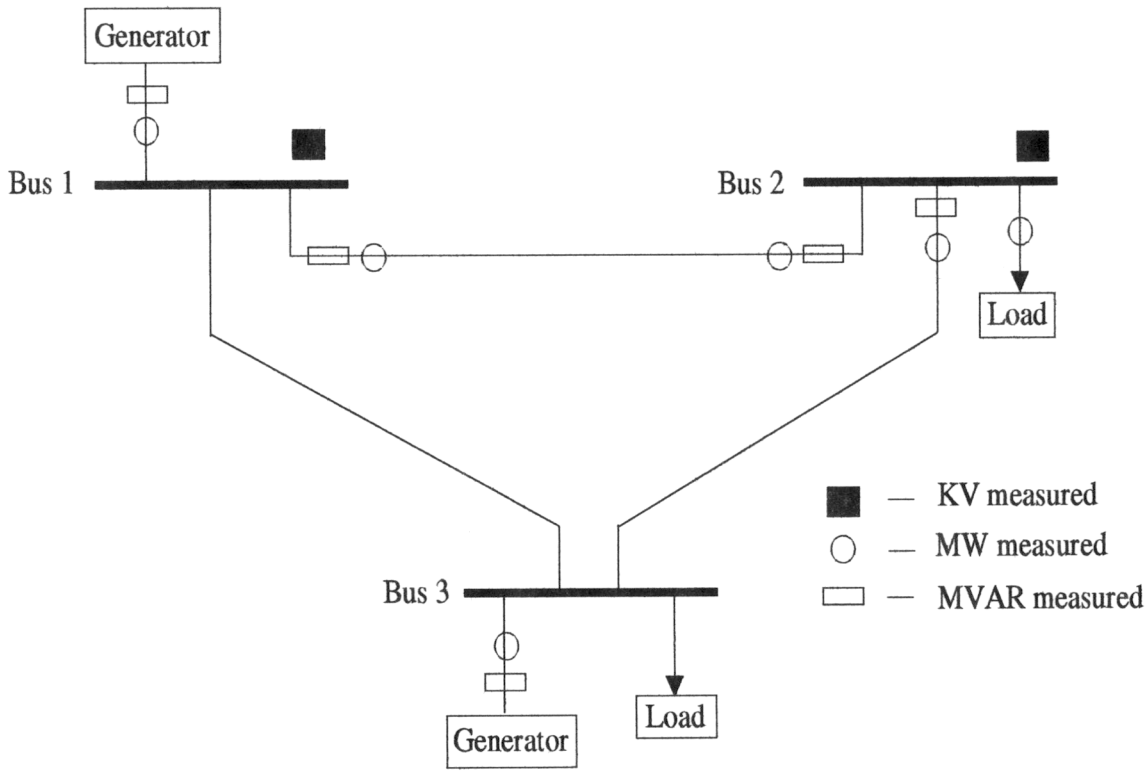

Figure 3.1: A measurement system for a state estimator

Suppose the state variables of the 3-bus network in Figure 3.1 consist of all the bus voltages and angles. Thus there are six state variables (V_1, V_2, V_3, θ_1, θ_2, θ_3). One of the buses is a reference bus, say bus 3. Thus $\theta_3 = 0$. There will be 5 unknown state variables to estimate. There are 13 individual measurements as shown in Figure 3.1. A careful selection of only five measurements will be needed for estimating the five unknown state variables through the state estimation process.

3.3 State Estimation

3.3.1 State variables

A conventional or classical state estimator normally uses the following set of state variables in its estimation process:

- Voltage magnitude V_k at bus k .
- Voltage angle θ_k at bus k .

State variables are defined as a non-redundant set of variables which completely characterise the condition of a system. A state estimator provides estimates for the unknown state variables. After the above unknown state variables are solved, it will be possible to know the state or condition of the whole system or network. It is also possible to calculate power flows from one bus to another by means of the estimates of above mentioned state variables.

3.3.2 Classical measurement equations

Consider a power system with measurements available from SCADA as in Figure 3.1 of section 3.2. Suppose there are not only 13 measurements, but m measurements. Suppose there are not only six state variables (V_1, V_2, V_3, θ_1, θ_2, θ_3), but n state variables. The state variables can be written as an n - dimensional state vector $\mathbf{x}$. For a network with N buses, the state vector contains N bus voltage magnitudes and $N-1$ bus voltage angles. One of the buses is chosen as the reference bus or swing bus and is assigned a voltage angle of zero radians. The state vector, vector $\mathbf{x}$ has dimension $n = 2N - 1$ [8]. In vector notation, the state vector will look like the following:

$$\mathbf{x} = \begin{bmatrix} \theta_2 \\ \vdots \\ \theta_n \\ V_1 \\ \vdots \\ V_n \end{bmatrix},$$

assuming that the reference bus is the first bus. The measurements can be written as an m - dimensional measurement vector $\mathbf{z}$, which is related to the state vector by the following measurement equation

$$\mathbf{z} = \mathbf{h}(\mathbf{x}) + \mathbf{v}\,, \tag{3.4}$$

where $\mathbf{h}(\mathbf{x})$ is the non-linear vector function relating the measurement vector to the state vector and $\mathbf{v}$ is a vector of measurement errors. In section 2.4.1.1, the vector function $\mathbf{f}(\mathbf{x})$ is used to indicate the measurement equations formulated for each bus and vector $\mathbf{y}$ contained the known P and Q injection values. For the purpose of state estimation, the vector function $\mathbf{h}(\mathbf{x})$ will be used to indicate the non-linear measurement equations and vector $\mathbf{z}$ will contain the known measurements of the system as mentioned before. The vector $\mathbf{f}(\mathbf{x})$ contained the injection equations formulated for each bus, while the vector $\mathbf{h}(\mathbf{x})$ is not restricted to only injection equations. Section 4.3 will show that different types of measurement equations can be used in advanced modelling techniques.

The following sections, section 3.3.2.1 to section 3.3.2.6, will give examples of conventional or classical measurement equations. Detail derivations for some of the measurement equations are given in the Appendix. Figure 3.2 will help to understand some of the mathematical terms that will be used in the different measurement equations. In Figure 3.2, $G_{ij} + jB_{ij}$ is the series admittance

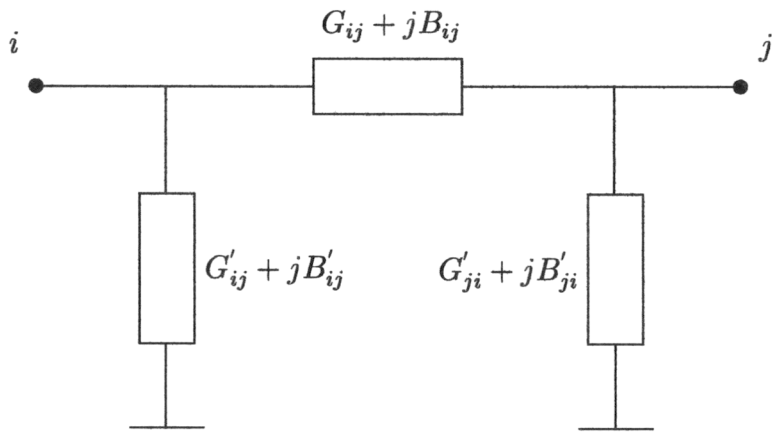

Figure 3.2: Pi - equivalent transmission line model

of the line, $G'_{ij} + jB'_{ij}$ is one half of the total shunt line admittance, $G'_{ji} + jB'_{ji}$ is the other half of the total shunt line admittance.

3.3.2.1 Injection measurement equations

Power injection measurements are calculated as the summation of power flows in adjacent branches. The following two equations are injection measurement equations for a node i of a π - equivalent transmission line model:

$$P_i^m = G_{ii}V_i^2 - V_i \sum_{\substack{j=1 \\ j\neq i}}^{N} V_j(G_{ij}\cos\theta_{ij} + B_{ij}\sin\theta_{ij}), \tag{3.5}$$

and

$$Q_i^m = -B_{ii}V_i^2 - V_i \sum_{\substack{j=1 \\ j\neq i}}^{N} V_j(G_{ij}\sin\theta_{ij} - B_{ij}\cos\theta_{ij}), \tag{3.6}$$

where

$$G_{ii} = \sum_{j\neq i} (G_{ij} + G'_{ij}),$$

and

$$B_{ii} = \sum_{j\neq i} (B_{ij} + B'_{ij}).$$

P_i^m is the active power injection measurement at node i, Q_i^m is the reactive power injection measurement at node i, G_{ij} is the conductance of a transmission line between bus i and bus j, B_{ij} is the susceptance of a transmission line between bus i and bus j, and $\theta_{ij} = \theta_i - \theta_j$.

3.3.2.2 Power flow measurement equations for transmission lines

A power flow, is the flow of active power and reactive power from one bus to the next bus. The following two equations are power flow measurement equations from a bus i to a bus j:

$$P_{ij}^m = (G_{ij}^{'} + G_{ij})V_i^2 - V_iV_j(G_{ij}\cos\theta_{ij} + B_{ij}\sin\theta_{ij})\,, \tag{3.7}$$

and

$$Q_{ij}^m = -(B_{ij}^{'} + B_{ij})V_i^2 - V_iV_j(G_{ij}\sin\theta_{ij} - B_{ij}\cos\theta_{ij})\,, \tag{3.8}$$

where P_{ij}^m is the active power flow measurement, Q_{ij}^m is the reactive power flow measurement, G_{ij} is the conductance of a transmission line between bus i and bus j , B_{ij} is the susceptance of a transmission line between bus i and bus j and $\theta_{ij} = \theta_i - \theta_j$. Equation (3.7) and equation (3.8) are derived in Appendix A.

3.3.2.3 Power flow measurement equations for transformers

Figure 3.3 shows a transformer π - equivalent network. The power flow measurement equations are:

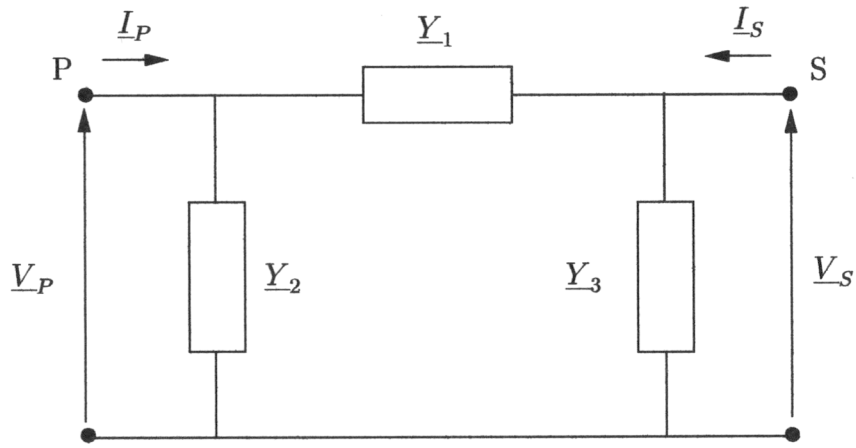

Figure 3.3: Transformer pi - equivalent network

a) The active power flow for the transformer from node P towards node S is:

$$P_{PS}^m = c^2G_TV_P^2 - cV_PV_S(G_T\cos\theta_{PS} + B_T\sin\theta_{PS})\,, \tag{3.9}$$

where P_{PS}^m is the active power flow measurement.

b) The reactive power flow for the transformer from node P towards node S is:

$$Q_{PS}^m = -c^2 B_T V_P^2 - cV_P V_S(G_T \sin\theta_{PS} - B_T \cos\theta_{PS}), \tag{3.10}$$

where Q_{PS}^m is the reactive power flow measurement.

c) The active power flow for the transformer from node S towards node P is:

$$P_{SP}^m = G_T V_S^2 - cV_S V_P(G_T \cos\theta_{SP} + B_T \sin\theta_{SP}), \tag{3.11}$$

where P_{SP}^m is the active power flow measurement.

d) The reactive power flow for the transformer from node S towards node P is:

$$Q_{SP}^m = -B_T V_S^2 - cV_S V_P(G_T \sin\theta_{SP} - B_T \cos\theta_{SP}), \tag{3.12}$$

where Q_{SP}^m is the reactive power flow measurement.

Equations (3.9), (3.10), (3.11) and (3.12) are derived in Appendix B.

3.3.2.4 Voltage measurement and phase angle measurement equations

Equations for a voltage measurement and phase angle measurement are:

$$V_i^m = V_i \text{ and } \theta_i^m = \theta_i, \tag{3.13}$$

or

$$V_{ij}^m = V_i - V_j \text{ and } \theta_{ij}^m = \theta_i - \theta_j, \tag{3.14}$$

where V_i^m is the voltage measurement at bus i and θ_i^m is the angle measurement at bus i.

3.3.2.5 Current measurement equations

For short lines with negligible line charging, the current measurement equation will be:

$$I_{ij}^2 = (V_i^2 + V_j^2 - 2V_i V_j \cos\theta_{ij})(G_{ij}^2 + B_{ij}^2). \tag{3.15}$$

3.3.2.6 Zero-injection measurement equations

Equation (3.5) and equation (3.6) are called zero-injection measurement equations if there are no active power injections and reactive power injections into a bus i, thus $P_i^m = 0$ and $Q_i^m = 0$. For example:

$$0 = G_{ii}V_i^2 - V_i \sum_{\substack{j=1 \\ j\neq i}}^{N} V_j(G_{ij}\cos\theta_{ij} + B_{ij}\sin\theta_{ij})\,, \tag{3.16}$$

and

$$0 = -B_{ii}V_i^2 - V_i \sum_{\substack{j=1 \\ j\neq i}}^{N} V_j(G_{ij}\sin\theta_{ij} - B_{ij}\cos\theta_{ij})\,. \tag{3.17}$$

All above mentioned equations from section 3.3.2.1 to section 3.3.2.6, are represented as $\mathbf{h(x)}$ in equation (3.4). The measurement equations discussed in this section, are known as "classical measurement equations" and they are used in all commercial state estimators. For more information on classical measurement equations, see [6].

With state estimation, any meaningful set of system quantities can be measured. These measurements can be used together with any system of equations $\mathbf{h(x)}$, to yield the wanted state variables of vector $\mathbf{x}$. It can therefore be said that state estimation is the process of assigning a value to an unknown system state variable based on measurements from that system according to some criteria. Section 3.5 will give more detail about the criterion being used by the state estimation process.

3.4 Least Squares

As mentioned in section 2.4.1.2, state estimation solves an overdetermined system of non-linear equations. Thus, a system consisting of more measurements than unknown state variables.

Consider an overdetermined system of linear equations, thus a system with more equations than unknowns of the following type

$$\mathbf{Ax} = \mathbf{b}\,, \tag{3.18}$$

where vector $\mathbf{x}$ is n - dimensional and vector $\mathbf{b}$ is m - dimensional, with $n < m$. The matrix $\mathbf{A}$ is an m x n matrix. The Least Squares solution provides one way of dealing with a system as described in equation (3.18). The Least Squares problem for the overdetermined system in equation (3.18), is to find the n - dimensional vector $\mathbf{x}$ for which the index $J(\mathbf{x})$, defined by

$$J(\mathbf{x}) = \frac{1}{2}(\mathbf{b} - \mathbf{Ax})^T(\mathbf{b} - \mathbf{Ax})\,, \tag{3.19}$$

is minimised. $J(\mathbf{x})$ can also be called a performance index or objective function. The residual vector is defined as

$$\mathbf{r} = \mathbf{b} - \mathbf{Ax}\,. \tag{3.20}$$

The Least Squares solution $\widehat{\mathbf{x}}$ is such that

$$\min_{\mathbf{x}} J(\widehat{\mathbf{x}}) = \frac{1}{2}(\mathbf{b} - \mathbf{A}\widehat{\mathbf{x}})^T(\mathbf{b} - \mathbf{A}\widehat{\mathbf{x}})\,. \tag{3.21}$$

From the above discussion, it is clear that the Least Squares problem provides a solution or estimate $\widehat{\mathbf{x}}$ for the vector $\mathbf{x}$ of an overdetermined system. For more information on Least Squares, see [6] and [9]. Valuable information on Least Squares solution methods, is given by [10] and [11].

3.5 State Estimation Using WLS Criterion

As mentioned in section 3.3.2, measurements can be written as an m - dimensional measurement vector $\mathbf{z}$, which is related to the state vector through equation (3.4). The elements of equation (3.4) in vector notation are:

$$\mathbf{z} = \begin{bmatrix} z_1 \\ \vdots \\ z_m \end{bmatrix}, \quad \mathbf{h(x)} = \begin{bmatrix} h_1(\mathbf{x}) \\ \vdots \\ h_m(\mathbf{x}) \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} \theta \\ \mathbf{V} \end{bmatrix}, \quad \mathbf{V} = \begin{bmatrix} v_1 \\ \vdots \\ v_m \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} \theta_2 \\ \vdots \\ \theta_n \\ V_1 \\ \vdots \\ V_n \end{bmatrix}.$$

From above, the residuals are defined as

$$r_i = z_i - h_i(x)\ ,\ i = 1, ..., m\,, \tag{3.22}$$

and it is the difference between the i-th measurement from the network z_i and the computed value of the corresponding measured quantity $h_i(x)$. The residuals can be minimised through the use of the Weighted Least Squares criterion as explained in the next paragraphs.

For estimating a single unknown state variable x using m measurements, the minimisation will be:

$$\min_{x} J(x) = \sum_{i=1}^{m} \frac{[z_i^{meas} - h_i(x)]^2}{\sigma_i^2}\,, \tag{3.23}$$

where

$J(x)$ - objective function,

z_i^{meas} - i-th measured quantity,

m - number of measurements,

$h_i(x)$ - analytical expression or measurement equation for z_i^{meas}; a function of the state variables which is used to calculate the value being measured,

σ_i^2 - variance for measurement error of the i-th measurement.

For estimating n unknown state variables using m measurements, the minimisation will be:

$$\min_{x_1,x_2,\ldots,x_n} J(x_1,x_2,\ldots,x_n) = \sum_{i=1}^{m} \frac{[z_i^{meas} - h_i(x_1,x_2,\ldots,x_n)]^2}{\sigma_i^2}\,, \tag{3.24}$$

or in matrix form

$$\min_{\mathbf{x}} J(\mathbf{x}) = [\mathbf{z}^{meas} - \mathbf{h}(\mathbf{x})]^T \mathbf{R}^{-1} [\mathbf{z}^{meas} - \mathbf{h}(\mathbf{x})]\,, \tag{3.25}$$

where

$$\mathbf{R} = \begin{bmatrix} \sigma_1^2 & 0 & \cdots & 0 \\ 0 & \sigma_2^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \cdots & \sigma_m^2 \end{bmatrix},$$

is called the covariance matrix of measurement errors [7]. The covariance matrix is used as a weighting matrix. Each measurement used in the state estimation process, has a variance or weight for the measurement error as discussed in section 3.2.

As with the Least Squares problem in section 3.4, the solution or estimate $\widehat{\mathbf{x}}$ of the unknown state variables $\mathbf{x}$ is obtained by minimising the Weighted Least Squares performance index $J(\mathbf{x})$ where $J(\mathbf{x})$ is given by equation (3.24). Weighted Least Squares (WLS) minimises the sum of the squares of the weighted deviations of the estimated measurements from the actual measurements. It is easy to show that when the variance of each measurement error is equal to one, equation (3.21) of section 3.4 is obtained [3].

This type of WLS state estimation problem is also called the unconstrained WLS state estimation problem. Buses with neither load nor generation are referred to as zero injection buses. Suppose there are p constraints imposed by zero injection buses in the network and let $\mathbf{g}(\mathbf{x})$ be a p - dimensional vector of constraint equations, then the constrained WLS formulation may be expressed as a minimisation of the following objective function

$$J(\mathbf{x}) = \frac{1}{2}[\mathbf{z} - \mathbf{h}(\mathbf{x})]^T \mathbf{R}^{-1} [\mathbf{z} - \mathbf{h}(\mathbf{x})]\,, \tag{3.26}$$

subject to

$$\mathbf{g}(\mathbf{x}) = 0\,. \tag{3.27}$$

For a more detail discussion on the unconstrained Weighted Least Squares formulation and the constrained Weighted Least Squares formulation, see [8] and [12].

This section discussed the Weighted Least Squares objective function. There are also other types of objective functions $J(\mathbf{x})$ which can be used e.g. Quadratic-Tangent objective function, Quadratic-Squares-Root objective function etc. , see [13].

3.6 Numerical Solution of the State Estimation Problem

As mentioned in section 3.5, state estimation is the process to calculate the wanted state vector $\widehat{\mathbf{x}}$ by minimising the Weighted Least Squares objective function $J(\mathbf{x})$ as in equation (3.24).

There are several methods to minimise the unconstrained Weighted Least Squares objective function $J(\mathbf{x})$ as in equation (3.24). The Gradient methods can be used to minimise $J(\mathbf{x})$. Examples of these methods are the Conjugate-Gradient method, Steepest-Descent method etc. . There are also Newton methods that can be used to minimise $J(\mathbf{x})$. The Gauss-Newton method will be used in this section to explain the numerical solution of a state estimator. This method requires that the gradient of $J(\mathbf{x})$, needs to be equal to zero if $J(\mathbf{x})$ want to be a minimum:

$$\nabla \mathbf{J} = \mathbf{J}_x\left(\widehat{\mathbf{x}}\right) = 0\,,$$

or

$$\mathbf{J}_x(\widehat{\mathbf{x}}) = \left.\frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}\right|_{\mathbf{x}=\widehat{\mathbf{x}}} = -H(\widehat{\mathbf{x}})^T \mathbf{R}^{-1}[\mathbf{z} - \mathbf{h}(\widehat{\mathbf{x}})] = 0\,, \tag{3.28}$$

where $H(\widehat{\mathbf{x}})$ is the measurement Jacobian matrix:

$$H(\widehat{\mathbf{x}}) = \left.\frac{\partial \mathbf{h}(\mathbf{x})}{\partial \mathbf{x}}\right|_{\mathbf{x}=\widehat{\mathbf{x}}} = \begin{bmatrix} \frac{\partial h_1(\mathbf{x})}{\partial x_1} & \frac{\partial h_1(\mathbf{x})}{\partial x_2} & \cdots & \frac{\partial h_1(\mathbf{x})}{\partial x_n} \\ \frac{\partial h_2(\mathbf{x})}{\partial x_1} & \frac{\partial h_2(\mathbf{x})}{\partial x_2} & \cdots & \frac{\partial h_2(\mathbf{x})}{\partial x_n} \\ \vdots & \vdots & \cdots & \vdots \\ \frac{\partial h_m(\mathbf{x})}{\partial x_1} & \frac{\partial h_m(\mathbf{x})}{\partial x_2} & \cdots & \frac{\partial h_m(\mathbf{x})}{\partial x_n} \end{bmatrix}.$$

$H(\widehat{\mathbf{x}})$ is an $m \times n$ Jacobian matrix containing the first order partial derivatives of the measurement equations $\mathbf{h}(\mathbf{x})$ with respect to the state variables $\mathbf{x}$.

Equation (3.28) is a set of non-linear equations and its solution requires the application of an iterative method. An iterative method e.g. Gauss-Newton can be used to solve equation (3.28). To solve the set of non-linear equations, it must be linearised first. A linearisation around some working point $\mathbf{x}_k$, by using a truncated Taylor series expansion, of this equation will give:

$$\begin{aligned} \mathbf{J}_{xx}\left(\mathbf{x}_k\right)\Delta\mathbf{x}_k + \mathbf{J}_x\left(\mathbf{x}_k\right) &= 0\,, \text{ or} \\ \mathbf{J}_{xx}\left(\mathbf{x}_k\right)\Delta\mathbf{x}_k &= -\mathbf{J}_x\left(\mathbf{x}_k\right)\,, \end{aligned} \tag{3.29}$$

where $\mathbf{J}_{xx}(\mathbf{x}_k)$ is the Hessian matrix (the matrix containing all the second order derivatives) of J evaluated at point $\mathbf{x}_k$. $\mathbf{J}_{xx}(\mathbf{x}_k)$ is also called the gain matrix. The state correction vector, $\Delta\mathbf{x}_k = \left[\Delta x_1^{(k)}, \Delta x_2^{(k)}, ..., \Delta x_n^{(k)}\right]^T$, can then be calculated from this system of linear equations. An iterative procedure can be used and better iterates of the state vector can be calculated by

$$\mathbf{x}_{k+1} = \mathbf{x}_k + \Delta\mathbf{x}_k \,, \tag{3.30}$$

until the solution is found. Convergence is obtained when $\|\Delta\mathbf{x}_k\| < \varepsilon$, where ε is the convergence criterion. Thus, convergence is obtained if during every iteration the current $\mathbf{x}_k$ comes closer to $\widehat{\mathbf{x}}$ (the estimated solution of the wanted state variables) than in the previous iteration.

Fast decoupled solution algorithms can also be used to solve the state estimation problem, see [6] and [14].

For more information on the numerical solution of a state estimation problem, see [8] and [9]. [10], [11], and [15] give valuable information on numerical iterative solutions and least squares problems. The next section will give a simple numerical example to explain some of the concepts mentioned in this chapter.

3.7 A Numerical Example

The numerical example of this section is based on the example used by A. J. Wood and B. F. Wollenberg in [7]. But the example of this section, uses different measurement readings as well as different generator and load values. The example of this section also gives more mathematical detail during the solution process.

The following example will show a state estimation solution obtained by minimising a Weighted Least Squares objective function. It will also illustrate the difference between a load flow solution and a state estimation solution using a three bus network. This example will illustrate why a load flow approach can not determine the state of the power system in the presence of measurement errors while state estimation can determine the state of the power system in the presence of errors.

Three sections will be considered. In section 3.7.1, a load flow approach will be used where meters are considered with no errors. Section 3.7.2 will show a load flow approach using meters with errors. The last section, section 3.7.3 will consider the state estimation approach.

3.7.1 A load flow approach using meters with no errors

Figure 3.4 shows a three bus system.

The parameters for the three bus system are as follows:

$S_{Base} = 100$ MVA ,

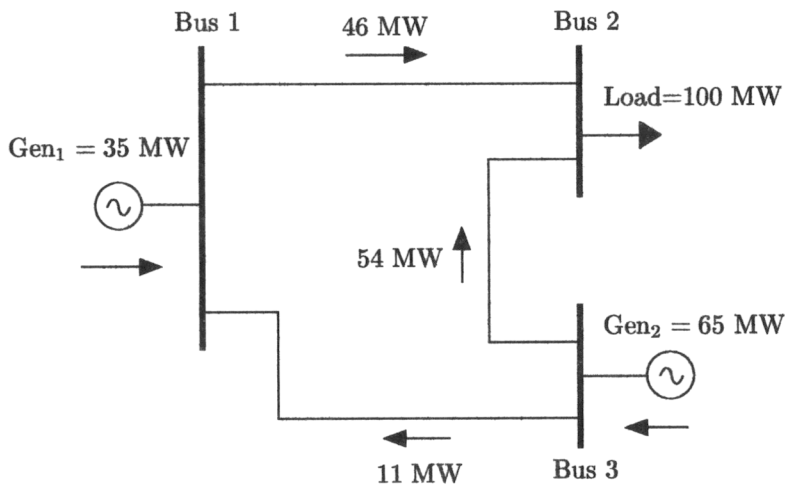

Figure 3.4: A three bus system

$X_{12} = 0.2$ pu ,

$X_{13} = 0.4$ pu and

$X_{23} = 0.25$ pu .

This example will follow a DC or linearised load flow approach. Choose bus 3 to be the slack bus or the reference bus, therefore $\theta_3 = 0$ rad. For the three bus network, there are only two unknowns to be calculated which are θ_1 and θ_2. System information are only provided by three power flow meters M_{12}, M_{31} and M_{32} as shown in Figure 3.5. These measurements are:

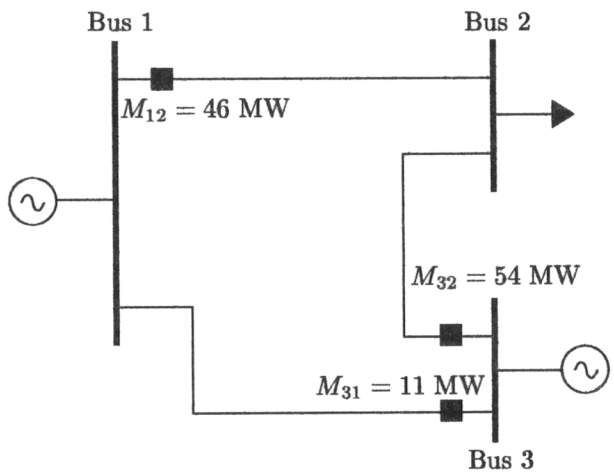

Figure 3.5: Three MW power flow meters

$$M_{12} = 46 \text{ MW} = 0.46 \text{ pu},$$
$$M_{31} = 11 \text{ MW} = 0.11 \text{ pu},$$
$$M_{32} = 54 \text{ MW} = 0.54 \text{ pu}.$$

Assume that M_{12}, M_{31} and M_{32} are measurement readings with no measurement errors. The state

variables are:

$$\left.\begin{array}{l}\theta_3 \text{ -known} = 0 \text{ rad}, \\ \left.\begin{array}{l}\theta_1 \\ \theta_2\end{array}\right\} \text{-unknown} \ .\end{array}\right.$$

Only two quantities or measurements are required in order to calculate θ_1 and θ_2. Assume for a lossless network that $X_{ij} >> R_{ij}$ and $\theta_i - \theta_j$ is small, therefore $\sin(\theta_i - \theta_j) \cong \theta_i - \theta_j$. These assumptions result in the formula:

$$P_{ij} = \frac{\theta_i - \theta_j}{X_{ij}} \text{ or in matrix form } [\mathbf{P}] = [\mathbf{B}]\,[\theta] \ . \tag{3.31}$$

Suppose M_{31} and M_{12} are used:

For the measurement M_{31}, the following analytical expression can be written:

$$M_{31} = f_{31}(\theta_1, \theta_2, \theta_3) = \frac{\theta_3 - \theta_1}{X_{31}} = 0.11 \text{ pu} \ . \tag{3.32}$$

For the measurement M_{12}, the following analytical expression can be written:

$$M_{12} = f_{12}(\theta_1, \theta_2, \theta_3) = \frac{\theta_1 - \theta_2}{X_{12}} = 0.46 \text{ pu} \ . \tag{3.33}$$

Since $\theta_3 = 0$ rad (reference bus), it is possible to solve equation (3.32) for θ_1 and equation (3.33) for θ_2:

$$\theta_1 = -M_{31}X_{31} = -0.11 \times 0.4 = -0.044 \text{ rad} \ , \tag{3.34}$$

and

$$\theta_2 = \theta_1 - M_{12}X_{12} = -0.044 - 0.46 \times 0.2 = -0.136 \text{ rad} \ . \tag{3.35}$$

A solution for the state vector is obtained:

$$\begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} = \begin{bmatrix} -0.044 \\ -0.136 \\ 0 \end{bmatrix} .$$

The same results will be obtained if the other two combinations M_{31} and M_{32} or M_{12} and M_{32} are used because perfect or without error readings were used. To proof this statement, use M_{12} and

M_{32}:

$$M_{32} = \frac{\theta_3 - \theta_2}{X_{32}} = 0.54 \text{ pu} \quad \text{and} \quad \theta_2 = -M_{32}X_{32} = -0.135 \text{ rad},$$

$$M_{12} = \frac{\theta_1 - \theta_2}{X_{12}} = 0.46 \text{ pu} \quad \text{and} \quad \theta_1 = \theta_2 + M_{12}X_{12} = -0.043 \text{ rad}.$$

A solution for the state vector is obtained:

$$\begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} = \begin{bmatrix} -0.043 \\ -0.135 \\ 0 \end{bmatrix}.$$

3.7.2 A load flow approach using meters with errors

This section will investigate the case where all three meter readings have slight errors. Suppose the readings obtained from the measurements in the three bus system are:

$$M_{12} = 44 \text{ MW} = 0.44 \text{ pu},$$
$$M_{31} = 10 \text{ MW} = 0.10 \text{ pu},$$
$$M_{32} = 57 \text{ MW} = 0.57 \text{ pu}.$$

Figure 3.6 shows the three meter readings with slight errors. As in section 3.7.1, the state variables

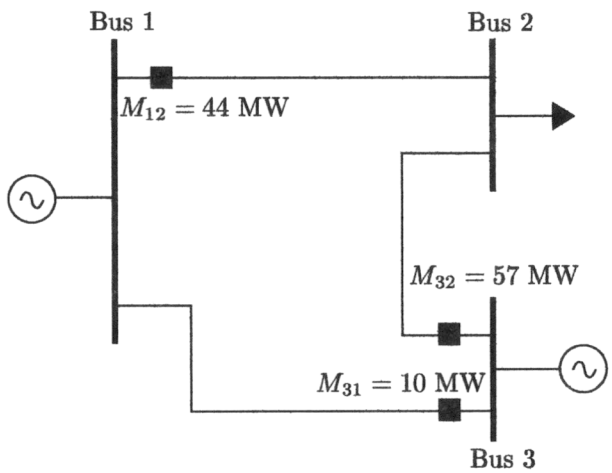

Figure 3.6: Three meters with errors

θ_1, θ_2 and θ_3 are obtained by means of the following combinations: M_{31} and M_{12}, or M_{31} and M_{32}, or M_{12} and M_{32}. Using the same mathematical procedure as in section 3.7.1 and suppose M_{31} and M_{12} are used:

$$M_{31} = \frac{\theta_3 - \theta_1}{X_{31}} = 0.10 \text{ pu} \quad \text{and} \quad \theta_1 = -0.04 \text{ rad}, \tag{3.36}$$

$$M_{12} = \frac{\theta_1 - \theta_2}{X_{12}} = 0.44 \text{ pu} \quad \text{and} \quad \theta_2 = -0.128 \text{ rad}. \tag{3.37}$$

A solution for the state vector is obtained:

$$\begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} = \begin{bmatrix} -0.04 \\ -0.128 \\ 0 \end{bmatrix}.$$

The power flows can now be calculated and are:

$$P_{12} = \frac{\theta_1 - \theta_2}{X_{12}} = 0.44 \text{ pu} \ , \tag{3.38}$$

$$P_{31} = \frac{\theta_3 - \theta_1}{X_{31}} = 0.10 \text{ pu} \ , \tag{3.39}$$

$$P_{32} = \frac{\theta_3 - \theta_2}{X_{32}} = 0.512 \text{ pu} \ . \tag{3.40}$$

Equation (3.38) corresponds with $M_{12} = 0.44$ pu, equation (3.39) corresponds with $M_{31} = 0.10$ pu and equation (3.40) does not correspond with $M_{32} = 0.57$ pu.

Figure 3.7 shows the power flows as a result of using measurements with errors.

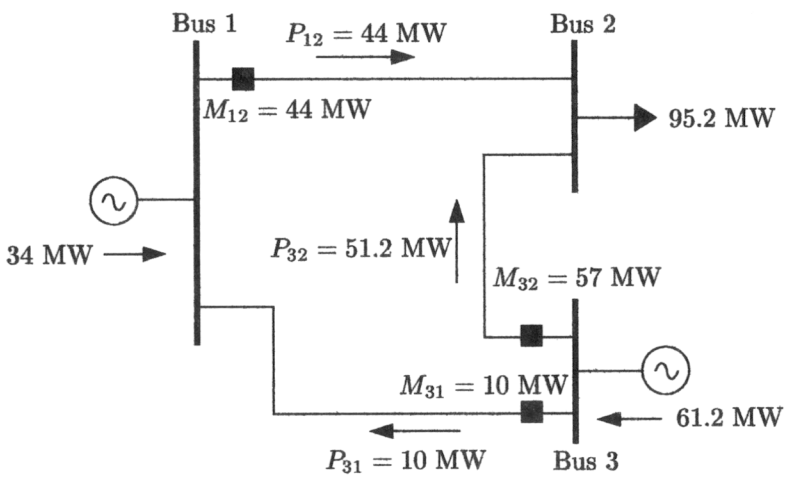

Figure 3.7: Power flows resulting from measurement errors

If the same mathematical procedure as above is done while using meters M_{12} and M_{32}, the following will be obtained:

$$M_{32} = \frac{\theta_3 - \theta_2}{X_{32}} = 0.57 \text{ pu} \quad \text{and} \quad \theta_2 = -0.1425 \text{ rad}, \tag{3.41}$$

$$M_{12} = \frac{\theta_1 - \theta_2}{X_{12}} = 0.44 \text{ pu} \quad \text{and} \quad \theta_1 = -0.0545 \text{ rad}. \tag{3.42}$$

A solution for the state vector is obtained:

$$\begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} = \begin{bmatrix} -0.0545 \\ -0.1425 \\ 0 \end{bmatrix}.$$

The power flows can now be calculated and are:

$$P_{12} = \frac{\theta_1 - \theta_2}{X_{12}} = 0.44 \text{ pu} \ , \tag{3.43}$$

$$P_{31} = \frac{\theta_3 - \theta_1}{X_{31}} = 0.136 \text{ pu} \ , \tag{3.44}$$

$$P_{32} = \frac{\theta_3 - \theta_2}{X_{32}} = 0.57 \text{ pu} \ . \tag{3.45}$$

Equation (3.43) corresponds with $M_{12} = 0.44$ pu, equation (3.44) does not correspond with $M_{31} = 0.10$ pu and equation (3.45) corresponds with $M_{32} = 0.57$ pu.

Figure 3.8 shows the power flows as a result of using measurements with errors.

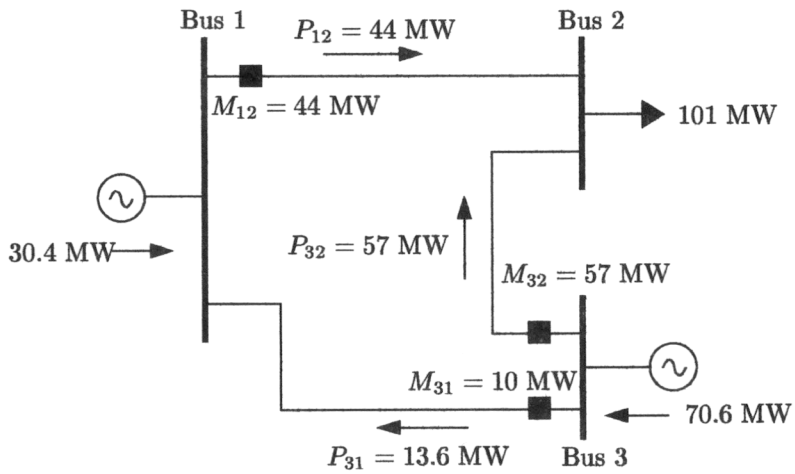

Figure 3.8: Results for using meters M_{12} and M_{32}

This section proved that when two measurements are used in the calculation procedures, only two power flows matched the measurement readings. A procedure is needed that uses all the measurements available in order to produce the best estimate of the actual angles, line flows, bus load and bus generations. Therefore, state estimation must be used.

3.7.3 State estimation

The state variables of the three bus example will be estimated by using the Weighted Least Squares (WLS) criterion. To estimate n unknown state variables using m measurements, the WLS objective function, equation (3.24) of section 3.5, must be minimised.

In this example there are three measurements $m = 3$, that can be used to determine two state variables $n = 2$, which are θ_1 and θ_2. The three measurement readings with measurement error are:

$$\begin{aligned} M_{12} &= 44 \text{ MW} = 0.44 \text{ pu}, \\ M_{31} &= 10 \text{ MW} = 0.10 \text{ pu}, \\ M_{32} &= 57 \text{ MW} = 0.57 \text{ pu}. \end{aligned}$$

These two state variables can be estimated by minimising the WLS objective function $J(\theta_1, \theta_2)$. Assume that all three meters have the same characteristics:

- meter full scale value: 100 MW
- meter accuracy: $\pm$ 3 MW

which means that the meters will give a reading within $\pm$ 3 MW of the true value for approximately 99% of the time. It can be shown that $\pm$ 3 MW corresponds to a metering standard deviation of $\sigma = 1$ MW $\equiv 0.01$ pu.

Equation (3.25) of section 3.5 must be minimised. The minimum of $J(\mathbf{x})$ is found when the gradient of $J(\mathbf{x})$, $\nabla J(\mathbf{x})$ is exactly zero. Then $\nabla J(\mathbf{x}) = 0$ gives

$$\mathbf{x}^{est} = \begin{bmatrix} \theta_1^{est} \\ \theta_2^{est} \end{bmatrix} = \left[\mathbf{H}^T \mathbf{R}^{-1} \mathbf{H}\right]^{-1} \mathbf{H}^T \mathbf{R}^{-1} \mathbf{z}^{meas}, \tag{3.46}$$

where $\mathbf{H}$ is the measurement function coefficient matrix, also called the Jacobian matrix, $\mathbf{x}^{est}$ is the vector of estimated state variables, $\mathbf{R}$ is the measurement covariance matrix, $\mathbf{z}^{meas}$ is the vector of measurement values. Equation (3.46) is derived in [7].

To derive the $\mathbf{H}$ matrix, the measurements must be written as a function of the state variables θ_1 and θ_2. These functions in per unit are:

$$z_1 \equiv M_{12} = f_1(\mathbf{x}) = \frac{\theta_1 - \theta_2}{X_{12}} = \frac{1}{0.2}(\theta_1 - \theta_2) = 5\theta_1 - 5\theta_2, \tag{3.47}$$

and

$$z_2 \equiv M_{31} = f_2(\mathbf{x}) = \frac{\theta_3 - \theta_1}{X_{31}} = \frac{1}{0.4}(\theta_3 - \theta_1) = -2.5\theta_1, \tag{3.48}$$

and

$$z_3 \equiv M_{32} = f_3(\mathbf{x}) = \frac{\theta_3 - \theta_2}{X_{32}} = \frac{1}{0.25}(-\theta_2) = -4\theta_2 \,. \tag{3.49}$$

The elements of the Jacobian matrix $\mathbf{H}$ are partial derivatives which are formed from the derivatives of the measurement functions with respect to the state variables.

$$\mathbf{H} = \begin{bmatrix} \dfrac{\partial f_1(\mathbf{x})}{\partial x_1} & \dfrac{\partial f_1(\mathbf{x})}{\partial x_2} \\ \dfrac{\partial f_2(\mathbf{x})}{\partial x_1} & \dfrac{\partial f_2(\mathbf{x})}{\partial x_2} \\ \dfrac{\partial f_3(\mathbf{x})}{\partial x_1} & \dfrac{\partial f_3(\mathbf{x})}{\partial x_2} \end{bmatrix},$$

where

$$\begin{aligned}
\frac{\partial f_1(\mathbf{x})}{\partial x_1} &= \frac{\partial}{\partial \theta_1}(5\theta_1 - 5\theta_2) = 5\,, \\
\frac{\partial f_1(\mathbf{x})}{\partial x_2} &= \frac{\partial}{\partial \theta_2}(5\theta_1 - 5\theta_2) = -5\,, \\
\frac{\partial f_2(\mathbf{x})}{\partial x_1} &= \frac{\partial}{\partial \theta_1}(-2.5\theta_1) = -2.5\,, \\
\frac{\partial f_2(\mathbf{x})}{\partial x_2} &= \frac{\partial}{\partial \theta_2}(-2.5\theta_1) = 0\,, \\
\frac{\partial f_3(\mathbf{x})}{\partial x_1} &= \frac{\partial}{\partial \theta_1}(-4\theta_2) = 0\,, \\
\frac{\partial f_3(\mathbf{x})}{\partial x_2} &= \frac{\partial}{\partial \theta_2}(-4\theta_2) = -4\,.
\end{aligned}$$

The elements of $\mathbf{H} = \begin{bmatrix} 5 & -5 \\ -2.5 & 0 \\ 0 & -4 \end{bmatrix}$.

For the covariance matrix $\mathbf{R}$:

$$\mathbf{R} = \begin{bmatrix} \sigma^2_{M_{12}} & & \\ & \sigma^2_{M_{31}} & \\ & & \sigma^2_{M_{32}} \end{bmatrix} = \begin{bmatrix} 0.0001 & & \\ & 0.0001 & \\ & & 0.0001 \end{bmatrix}.$$

The Weighted Least Squares estimate for θ_1 and θ_2 is:

$$
\begin{aligned}
\begin{bmatrix} \theta_1^{est} \\ \theta_2^{est} \end{bmatrix} &= \left[\mathbf{H}^T\mathbf{R}^{-1}\mathbf{H}\right]^{-1}\mathbf{H}^T\mathbf{R}^{-1}\mathbf{z}^{meas} \\
&= \left[\begin{bmatrix} 5 & -2.5 & 0 \\ -5 & 0 & -4 \end{bmatrix}\begin{bmatrix} 0.0001 & & \\ & 0.0001 & \\ & & 0.0001 \end{bmatrix}^{-1}\begin{bmatrix} 5 & -5 \\ -2.5 & 0 \\ 0 & -4 \end{bmatrix}\right]^{-1} \times \\
&\quad \begin{bmatrix} 5 & -2.5 & 0 \\ -5 & 0 & -4 \end{bmatrix}\begin{bmatrix} 0.0001 & & \\ & 0.0001 & \\ & & 0.0001 \end{bmatrix}^{-1}\begin{bmatrix} 0.44 \\ 0.10 \\ 0.57 \end{bmatrix} \\
&= \begin{bmatrix} 312500 & -250000 \\ -250000 & 410000 \end{bmatrix}^{-1} \times \begin{bmatrix} 19500 \\ -44800 \end{bmatrix} \\
&= \begin{bmatrix} -0.048838 \\ -0.139048 \end{bmatrix}.
\end{aligned}
$$

From these estimated angles θ_1 and θ_2, the power flows in each transmission line and the net generation or load at each bus can be calculated. The power flows are:

$$P_{12} = \frac{\theta_1 - \theta_2}{X_{12}} = 0.451 \text{ pu} = 45.1\,\text{MW},$$

$$P_{31} = \frac{\theta_3 - \theta_1}{X_{31}} = 0.122 \text{ pu} = 12.2\,\text{MW},$$

$$P_{32} = \frac{\theta_3 - \theta_2}{X_{32}} = 0.556 \text{ pu} = 55.6\,\text{MW}.$$

The final results are shown in Figure 3.9. From the above estimated state variables, it is clear that state estimation will give the best estimates for unknown state variables, while using measurement readings containing slight errors. If the residual is calculated, the value of the objective function is:

$$
\begin{aligned}
J(\theta_1,\theta_2) &= \frac{(z_{12} - f_{12}(\theta_1,\theta_2))^2}{\sigma_{12}^2} + \frac{(z_{31} - f_{31}(\theta_1,\theta_2))^2}{\sigma_{31}^2} + \frac{(z_{32} - f_{32}(\theta_1,\theta_2))^2}{\sigma_{32}^2} \\
&= \frac{(0.44 - (5\theta_1 - 5\theta_2))^2}{0.0001} + \frac{(0.10 - (-2.5\theta_1))^2}{0.0001} + \frac{(0.57 - (-4\theta_2))^2}{0.0001} \\
&= 8.01.
\end{aligned}
$$

The objective function is a measure of how big the deviation or error is between the measured values and the measurement functions using the estimated state variables.

For an AC network, iterative solvers are used to solve the non-linear system. The objective function

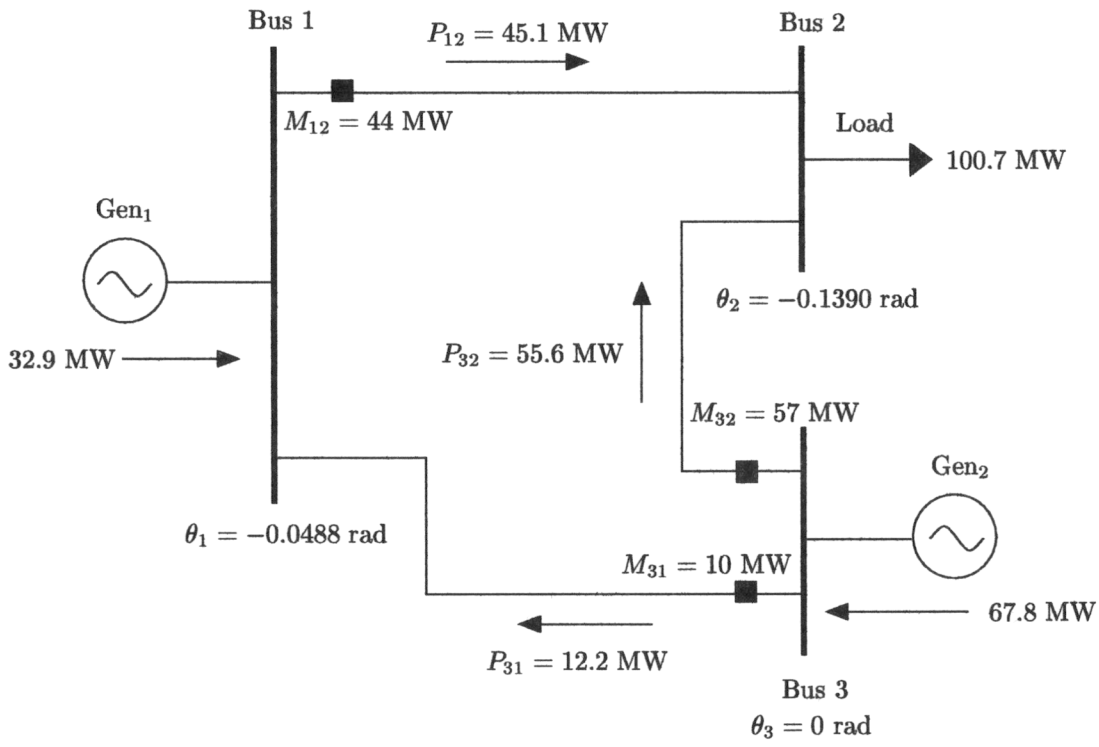

Figure 3.9: Results of the State Estimator

will decrease in value as the estimated state variables approach the true values. The iterative process will stop when convergence is obtained.

3.8 Conclusion

A state estimator uses measurements and measurement equations in its mathematical process. Measurements are obtained from measurement devices in a power system network via the SCADA system. Different examples of classical measurement equations are given and discussed.

An overview of Least Squares is given. It is mentioned that state estimation solves an overdetermined system of non-linear equations. Least Squares provided a solution or estimate $\widehat{\mathbf{x}}$ for the vector $\mathbf{x}$ of an overdetermined system.

The Weighted Least Squares (WLS) criterion is used in a state estimation process. The WLS criterion is based on Least Squares, except that WLS uses weights or variances for each measurement error in the state estimation process. As with the Least Squares problem, the solution or estimate $\widehat{\mathbf{x}}$ of the unknown state variables $\mathbf{x}$ is obtained by minimising the Weighted Least Squares objective function $J(\mathbf{x})$.

Several methods can be used to minimise the unconstrained Weighted Least Squares objective function $J(\mathbf{x})$. The Gauss-Newton method is used in the numerical solution which is explained in this chapter. This method required that the gradient of $J(\mathbf{x})$, needs to be equal to zero if $J(\mathbf{x})$

want to be a minimum. A set of non-linear equations is obtained which is linearised and solved for the unknown state variables.

A numerical example is given to illustrate the difference between a load flow solution and a state estimation solution using a three bus network. This example illustrated why a load flow approach could not determine the state of the power system in the presence of measurement errors while state estimation could determine the state of the power system in the presence of errors.

The next chapter will discuss the advantages and use of a generalised state estimator.

Chapter 4

Generalised State Estimation

4.1 Introduction

Classical state estimators take into account that network parameters and topology of the network are accurately known. Section 4.2 will discuss the generalised state estimator. The generalised state estimator takes into account that network parameters and topology are not accurately known. The generalised state estimator will be used to estimate unknown network parameters and unknown states of circuit breakers. Section 4.2 will show the effect of the generalised state estimator on the measurement vector and state variable vector.

The generalised state estimator can estimate unknown impedances etc. . To be able to do this type of estimation, the generalised state estimator will make use of advanced network modelling techniques. Section 4.3 will discuss advanced network modelling techniques. The chapter concludes with modelling of power injection buses using advanced network modelling techniques.

4.2 Generalised State Estimation

Over the years, the focus of power system state estimation has shifted towards a more holistic approach, meaning to utilise all available data. In order to construct the best load flow model, all data sources must be regarded as subject to statistical error, including device states and impedance values. Because of the statistical error, generalised state estimation is used. Generalised state estimation is a generalised approach which takes into account that network parameters and topology are not accurately known.

Generalised state estimation is performed on a model where parts of the network can be represented at the physical level, i.e. modelled in terms of bus sections and switching devices. Thus, it allows to include also measurements made on zero or low impedance branches and measurements made on switching devices. The idea of zero impedance branches or short lines will be discussed in section 4.3.1.2. The flow in each switch and the voltage at each bus section can now be calculated, subject

to observability. The explicit modelling of switches enormously facilitates switch status bad data analysis. For more information on generalised state estimation, see [6].

4.2.1 Augmented state variable vector

Generalised state estimators make use of advanced network modelling techniques. This is why section 4.3 is included and discussed. By means of a generalised state estimator, it is now possible to model and estimate unknown impedances of a network, to estimate the states of circuit breakers etc., which were not possible by using classical state estimators. The conventional state variables used by the classical state estimator as in section 3.3.1, are augmented with new state variables. These new state variables can include the following types:

- A state variable κ to model circuit breakers in series with a non-zero impedance.
- Active power flows and reactive power flows in unknown series and shunt parameters of a transmission line.
- Active power flows and reactive power flows in unknown admittances of an ULTC transformer.
- Active power flows and reactive power flows for circuit breaker modelling.
- Active power flows and reactive power flows for zero impedance branch modelling.

The above mentioned state variables will be used in the advanced modelling/estimation techniques. The state vector, vector $\mathbf{x}$ in $\mathbf{z} = \mathbf{h}(\mathbf{x}) + \mathbf{v}$, of section 3.3.2, will be augmented with above mentioned state variables.

4.2.2 Augmented measurement vector

The set of measurements needed for the advanced modelling techniques will also augment. The measurement vector $\mathbf{z}$ in $\mathbf{z} = \mathbf{h}(\mathbf{x}) + \mathbf{v}$, of section 3.3.2, will be augmented with the following types of measurements:

Measurements for -

- A state variable κ to model circuit breakers in series with a non-zero impedance.
- Active power flows and reactive power flows in unknown series and shunt parameters of a transmission line.
- Active power flows and reactive power flows in unknown admittances of an ULTC transformer.
- Active power flows and reactive power flows for circuit breaker modelling.
- Active power flows and reactive power flows for zero impedance branch modelling.

The next section will explain how to use above mentioned state variables and measurements in the advanced modelling techniques.

4.3 Advanced Network Modelling Techniques

It is possible that network impedances are not accurately known if a state estimation process needs to be done. It is also possible that the status of a circuit breaker is not accurately known if a state estimation process needs to be done. To overcome these types of problems, advanced network modelling techniques can be used. It will then be possible to estimate an unknown impedance of a network, the status of a circuit breaker etc. . The advanced network modelling techniques which will be explained, are used by the generalised state estimator. Each technique will be explained in section 4.3.1. The results obtained by using these modelling techniques, will be discussed in chapter 6. Section 4.3.1 will explain how the measurement equations vector will be effected by advanced modelling techniques.

4.3.1 Additional measurement equations

Advanced network modelling techniques which will be discussed in this section, will include the modelling of:

- Circuit breakers
- Zero impedance branches
- Network parameters
- Under-load tap changer (ULTC) transformer parameters

The above mentioned modelling techniques will use additional measurement equations to model circuit breakers, zero impedance branches, network parameters and ULTC transformer parameters. When using additional measurement equations in the state estimation problem, the existing set of measurement equations is extended as will be shown in this section. Equation (3.22) of section 3.5 will become:

$$r_i = z_i - h(x, s, b, y, u)\,, \tag{4.1}$$

where the voltages of modelled bus sections are added to x, the flows through modelled switches are represented by s, the flows through zero impedance branches are represented by b, the flows through network parameters of uncertain value are represented by y and the flows through ULTC parameters of uncertain value are represented by u.

4.3.1.1 Circuit breaker modelling

Circuit breaker modelling is used for solving state estimation problems where circuit breakers of a network can be either close, open or unknown. Although circuit breakers are intuitively related to zero-impedance branches, actually, two different cases can be identified, namely:

- Circuit breakers intended for line, transformer and shunt element switching which are, consequently, in series with a non-zero impedance.
- Circuit breakers which, within a substation, link busbar sections and determine the actual electrical nodes and are in series with a zero-impedance.

Circuit breakers in series with a zero or low impedance branch:

These type of circuit breakers can be divided into breakers with unknown status, circuit breakers which are closed and circuit breakers which are open. The concept of zero impedance branches will be discussed in section 4.3.1.2.

a) *Modelling of breakers with unknown status:*

The basic idea behind the modelling of an unknown breaker status is that active and reactive power flows P_{ij} and Q_{ij}, are added as state variables. Therefore, the power flow measurements P_{ij}^m and Q_{ij}^m, that form part of the measurement vector $\mathbf{z}$ as in equation (3.4), are expressed in terms of P_{ij} and Q_{ij}. The power flow measurements P_{ij}^m and Q_{ij}^m, are not expressed in terms of angles and voltages as mentioned in equation (3.7) and equation (3.8) of section 3.3.2.2. This assures that the series impedance of the device is not included in the model. These modifications can be summarised as follows:

$$P_{ij}^m = P_{ij} + v\,, \tag{4.2}$$

and

$$Q_{ij}^m = Q_{ij} + v\,, \tag{4.3}$$

where P_{ij}^m and Q_{ij}^m are the measured values, P_{ij} and Q_{ij} are the estimated values and v represent the measurement errors. The power injection measurements at node i and node j are given by the summation of the power flows in the adjacent branches. But, the difference is that the power flow across branch $i-j$, whose impedance will be unknown, is directly expressed in terms of state variables P_{ij} and Q_{ij}. The flows across the other branches are given in terms of the state variables θ and V as usual. The above additional measurement equations, equation (4.2) and equation (4.3), can be used in the estimation process to estimate an unknown circuit breaker status [16].

b) *Modelling of closed breakers:*

Closed breakers are modelled as short circuit branches as in section 4.3.1.2. The angle difference and voltage drop of the branch containing the closed breaker, are introduced as zero measurements and are as follows:

$$\theta_{ij}^{m} = \theta_i - \theta_j = 0\,, \tag{4.4}$$

and

$$V_{ij}^{m} = V_i - V_j = 0\,. \tag{4.5}$$

These pseudo-measurements can also be represented as equality constraints. The above pseudo-measurement equations, equation (4.4) and equation (4.5), can be used in the estimation process to model a closed breaker [16].

c) *Modelling of open breakers:*

The trivial case for modelling open breakers will be to ignore the open breaker and its connected devices from the state estimation problem. An alternative is to make the active and reactive power flows, P_{ij} and Q_{ij}, as state variables. This type of modelling will be similar, as in the case with the modelling of closed breakers and modelling of circuit breakers with an unknown status. But, the active and reactive power flows are also introduced as zero measurements and are as follows:

$$P_{ij}^{m} = P_{ij} = 0\,, \tag{4.6}$$

and

$$Q_{ij}^{m} = Q_{ij} = 0\,. \tag{4.7}$$

The measurement equations, equation (4.6) and equation (4.7), are used in the state estimation process to model an open breaker [16].

Circuit breakers in series with non-zero impedance branch:

These circuit breakers can be modelled by adding an extra state variable κ to the state vector and substituting the branch parameters G, B and B-shunt with κG , κB and κB - shunt. If the estimated value of κ is close to one, then the circuit breaker is closed. A value approaching zero implies a disconnected element. In practice, the estimated value of κ may differ from zero or one due to measurement errors. Also, the presence of bad data may result in an estimated value for κ to be around 0.5. An equality constraint $\kappa(1-\kappa) = 0$ is used to enforce κ to take either of the two discrete values. By means of this modelling technique, it will be possible to model circuit breakers in series with non-zero impedances [17].

4.3.1.2 Modelling of zero impedance branches

The main causes for numerical ill-conditioning of the gain matrix in power system state estimation, are the presence of injection measurements and the existence of circuits or lines with very small series impedances. These type of lines or branches are usually called zero impedance branches. Branches with zero (low) impedances are encountered in certain station arrangements:

- In some cases short lines need to be modelled due to power flow measurements which can not be associated with a single facility e.g. line, transformer, generator etc. This situation can either be permanent e.g. a current transformer in series with a breaker followed by several other power system elements, or temporary e.g. a by-pass of a breaker in a substation with short lines.

- Zero impedance branches are found when sub-transmission lines are connected to step-down transformers and the sub-transmission network is modelled. Very small impedances could also result in the equivalent circuits for the three-winding transformers e.g. when two windings are identical.

- The modelled network has to include fictitious zero impedance lines due to network data base requirements or to accommodate for limitations in the network applications software:

    1. A common data base restriction is for transformers to be contained in a station. Two stations can not be connected by transformers. Since different RTU's (Remote Terminal Units) might include only a certain voltage level, often stations are defined as substations (same voltage level) and a fictitious line is needed to connect them.
    2. In order to study a bus split outage when the contingencies are described in terms of branches (and not breakers), such a zero impedance line could be defined to connect two ring buses in a station.
    3. A shunt reactor is connected to a line and the current transformer provides the total current (line plus reactor). In this case, as an alternative to modelling the line as an asymmetrical-pi circuit, a fictitious metered zero impedance line could be defined between the bus and the reactor.

It is not always possible to model these branches by means of the classical state estimator because:

- If the zero impedance or short circuit branch is represented with a zero, the state estimator gain matrix may become singular and it will not be possible to continue the state estimation process and solve for the unknown state variables.

- If the terminal nodes of a short circuit are merged to form a single bus, useful measurement information will be lost, for example, a power flow meter placed between the two merging terminal nodes will be lost.

One possible way out of above mentioned dilemma is to represent the short circuit as a small impedance which must be small enough to not effect accuracy and at the same time, big enough to yield a well conditioned gain matrix. An other alternative is to use a more flexible modelling approach, which allows for an exact and reliable representation of zero impedance branches. By using this modelling technique, one can eliminate one of the most serious causes of ill-conditioning: the presence of circuits or lines with very small series impedances connected to buses to which

circuits with normal impedance values are also connected. In this section, modelling of a zero impedance series branch is illustrated by using two formulations of a state estimation problem:

- Non-Augmented method. This method uses pseudo-measurements as part of vector $\mathbf{z}$ in an estimation process.
- Augmented method. This method uses pseudo-measurements as equality constraints in an estimation process.

Non-Augmented method:

This method uses the measurement equations as part of $\mathbf{h(x)}$ in $\mathbf{z} = \mathbf{h(x)}+\mathbf{v}$. The objective function is minimised without any constraints. The basic idea behind the modelling of zero impedance series branches is to avoid using the device impedance as part of the model given by equation (3.4) of section 3.3.2. In order to achieve this, consider the active and reactive power flows P_{ij} and Q_{ij} as state variables. As a consequence of this modification, the power flow measurements P_{ij}^m and Q_{ij}^m, which form part of the measurement vector $\mathbf{z}$ as in equation (3.4), are expressed in terms of the previous mentioned state variables P_{ij} and Q_{ij}, as in equation (4.2) and equation (4.3). The power flow measurements P_{ij}^m and Q_{ij}^m are therefore not expressed in terms of the regular state variables θ and V. This is also valid for power injection measurements at node i and node j. The injections are still given by the summation of the power flows in the adjacent branches. The only difference is that the power flow across branch $i-j$ is directly expressed in terms of state variables P_{ij} and Q_{ij}, while the flows across the remainder branches are given in terms of the state variables θ and V. The second modification consists in considering the angle difference and voltage drop as zero measurements as in equation (4.4) and equation (4.5). The above modifications can be used in the state estimation process to model a zero impedance branch [18].

Augmented method:

In the previous type of modelling technique, the angle difference and the voltage drop across the zero impedance branches are treated as pseudo-measurements. For this type of modelling method, the angle difference and voltage drop as in equation (4.4) and equation (4.5), are treated as equality constraints. These equations are added to the set of constraints $g(x) = 0$ as in section 3.5. The active and reactive power flows P_{ij} and Q_{ij} for a branch $i-j$, are added to the set of state variables as explained in the previous paragraph. The power flow measurements P_{ij}^m and Q_{ij}^m, which form part of the measurement vector $\mathbf{z}$, are expressed in terms of the previous mentioned state variables P_{ij} and Q_{ij} and do not involve the regular state variables θ and V. The injection equations are handled in the same way as mentioned in the previous paragraph. The above modifications can be used in the state estimation process to model a zero impedance branch [18].

4.3.1.3 Modelling of network parameters

The same idea for modelling switches and zero impedance branches are used in this section. If line parameters (series or shunt) want to be estimated, they are removed from the network model and are

replaced by state variables representing its terminal power flows. As in the case of switches and zero impedance branches, this avoids the explicit use of branch parameters during the estimation process. Once the solved state variables are obtained, the device impedances can be trivially calculated.

The series branch:

Figure 4.1 shows an unknown series parameter of a transmission line. For the estimation of an

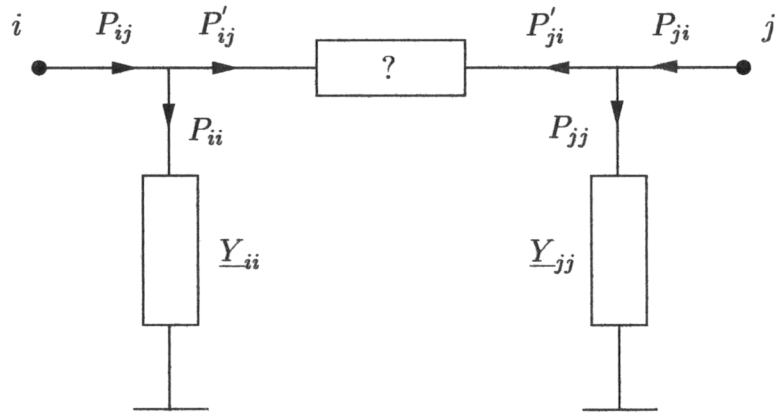

Figure 4.1: Unknown series branch

unknown series impedance, a pseudo-measurement requiring the current flows at each end of the series impedance to be the same, is used. For example:

$$\underline{I}'_{ij} + \underline{I}'_{ji} = 0\,, \tag{4.8}$$

or alternatively derived from equation 4.8:

$$P'_{ij}V_j + (P'_{ji}\cos\theta_{ij} - Q'_{ji}\sin\theta_{ij})V_i = 0\,, \tag{4.9}$$

and

$$Q'_{ij}V_j + (P'_{ji}\sin\theta_{ij} + Q'_{ji}\cos\theta_{ij})V_i = 0\,. \tag{4.10}$$

Equation (4.9) and equation (4.10) are derived in Appendix C. The above measurement equations are used in the state estimator to model an unknown series impedance of a transmission line. As mentioned in section 3.3.2, the m - dimensional vector $\mathbf{z}$ is related to the state vector $\mathbf{x}$ through the measurement equation as in equation (3.4), where $\mathbf{h}(\mathbf{x})$ is the non-linear vector function of measurement equations. Above mentioned equations, equation (4.9) and equation (4.10), are represented as $\mathbf{h}(\mathbf{x})$ in equation (3.4). The state variables representing the terminal power flows are: P'_{ij} , Q'_{ij} , P'_{ji} and Q'_{ji}. Once the solved state variables are obtained by the state estimation process, the unknown series device parameter is calculated as follows:

$$\underline{Y}_{ij} = \frac{P'_{ij} - jQ'_{ij}}{\underline{V}_i^*(\underline{V}_i - \underline{V}_j)}\,, \tag{4.11}$$

For more information on series branch modelling, see [6] and [19].

The shunt branch:

Figure 4.2 shows an unknown shunt parameter of a transmission line. For the estimation of an

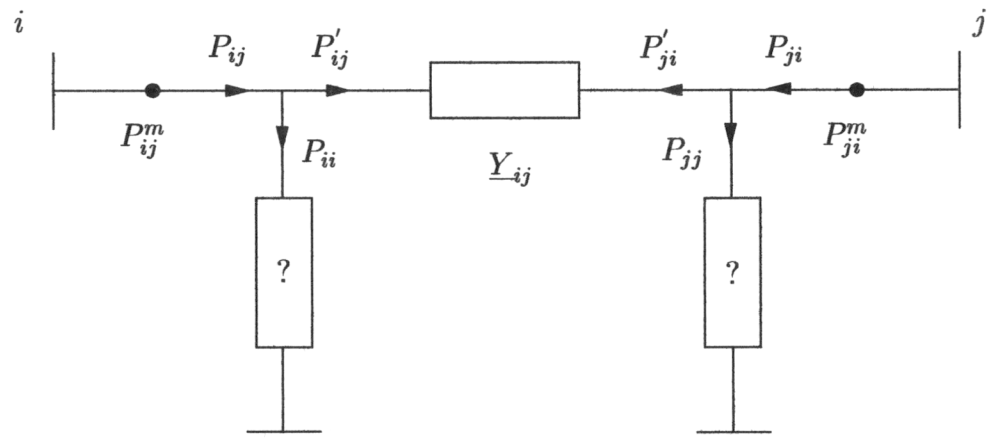

Figure 4.2: Unknown shunt branch

unknown shunt admittance of a symmetrical-pi transmission line, a pseudo-measurement requiring the legs at each end to be the same, is used. For example:

$$\underline{Y}_{ii} = \underline{Y}_{jj}\,, \tag{4.12}$$

or alternatively derived from equation (4.12):

$$P_{ii}V_j^2 - P_{jj}V_i^2 = 0\,, \tag{4.13}$$

and

$$Q_{ii}V_j^2 - Q_{jj}V_i^2 = 0\,. \tag{4.14}$$

Equation (4.13) and equation (4.14) are derived in Appendix C. The above measurement equations are also represented as $\mathbf{h(x)}$ and are used in the state estimation process to model an unknown shunt admittance of a transmission line. The state variables representing the terminal power flows are: P_{ii} , Q_{ii} , P_{jj} and Q_{jj}. Once the solved state variables are obtained, the shunt parameter is calculated as follows:

$$\underline{Y}_{ii} = \underline{Y}_{jj} = \frac{P_{ii} - jQ_{ii}}{V_i^2}\,. \tag{4.15}$$

These above modelling techniques can be used to estimate unknown series and shunt parameters of a transmission line. See also [6] and [19].

4.3.1.4 Modelling of ULTC transformer parameters

A model of an ULTC transformer is shown in Figure 4.3. This modelling technique is used to

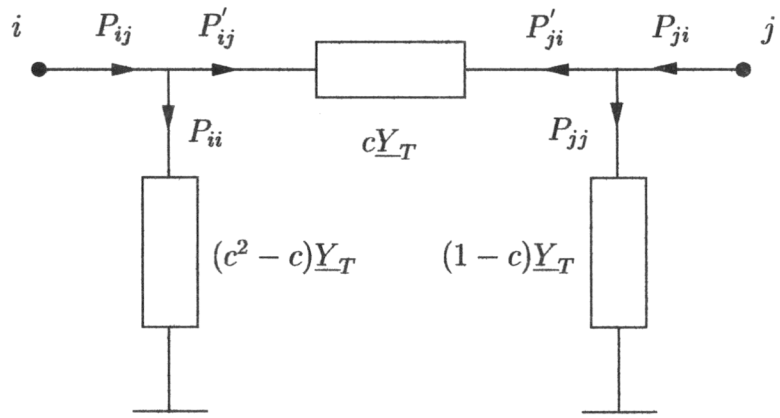

Figure 4.3: Unknown transformer parameters

estimate the unknown admittance of an ULTC transformer when the tap position is known. The same procedures used in the previous sections can be used to estimate $\underline{Y}t$ when c is known. This modelling technique can be seen as a combination of the modelling techniques from section 4.3.1.3. For example:

For estimating the unknown series impedance of the transformer, a pseudo-measurement requiring the current flows at each end of the series impedance to be the same, is used. For example:

$$\underline{I}'_{ij} + \underline{I}'_{ji} = 0\,, \tag{4.16}$$

or alternatively derived from equation (4.16):

$$P'_{ij}V_j + (P'_{ji}\cos\theta_{ij} - Q'_{ji}\sin\theta_{ij})V_i = 0\,, \tag{4.17}$$

and

$$Q'_{ij}V_j + (P'_{ji}\sin\theta_{ij} + Q'_{ji}\cos\theta_{ij})V_i = 0\,, \tag{4.18}$$

and for estimating the unknown shunt admittance, a modified zero admittance difference equation will be:

$$\underline{Y}_{ii} = -c\underline{Y}_{jj}\,, \tag{4.19}$$

or alternatively derived from equation (4.19):

$$P_{ii}V_j^2 + cP_{jj}V_i^2 = 0\,, \tag{4.20}$$

and

$$Q_{ii}V_j^2 + cQ_{jj}V_i^2 = 0\,. \tag{4.21}$$

Equations (4.17), (4.18), (4.20) and (4.21) are derived in Appendix C. The above measurement equations form part of $\mathbf{h(x)}$ and are used in the state estimator to model an unknown admittance $\underline{Y}t$. The state variables for this type of modelling technique will be: P_{ii} , Q_{ii} , P'_{ij} , Q'_{ij} , P'_{ji} , Q'_{ji} , P_{jj} and Q_{jj}. When the solved state variables are obtained by the state estimation process, $\underline{Y}t$ can be calculated as follows:

$$\underline{Y}t = \frac{1}{c}\frac{P'_{ij} - jQ'_{ij}}{\underline{V}_i^*(\underline{V}_i - \underline{V}_j)}\,. \tag{4.22}$$

For more information on ULTC transformer modelling, see [6] and [19].

A more effective modelling technique for modelling tap-changer transformers, is shown in [20].

4.3.1.5 Modelling of power injection buses

Power injection measurements are calculated as the summation of the power flows in the adjacent branches:

$$P_i^m = \sum_{\substack{j=1 \\ j\neq i}}^{N} P_{ij} + v_{P_i}\,, \tag{4.23}$$

and

$$Q_i^m = \sum_{\substack{j=1 \\ j\neq i}}^{N} Q_{ij} + v_{Q_i}\,. \tag{4.24}$$

The following example will show how a power injection bus is modelled by means of the advanced modelling techniques, as described in this chapter.

Figure 4.4 shows a power injection at node i. According to equation (4.23) and equation (4.24), the power injection will be the sum of, the power flow across the branch with the closed switch (from node i to node j), plus the power flow on the transmission line (from node i to node k), plus the power flow on the transmission line with the unknown series parameter (from node i to node s).

Equation (4.23) will become:

$$P_i^m = P_{ij} + P_{ik} + P_{is} + v_{P_i}\,, \tag{4.25}$$

therefore

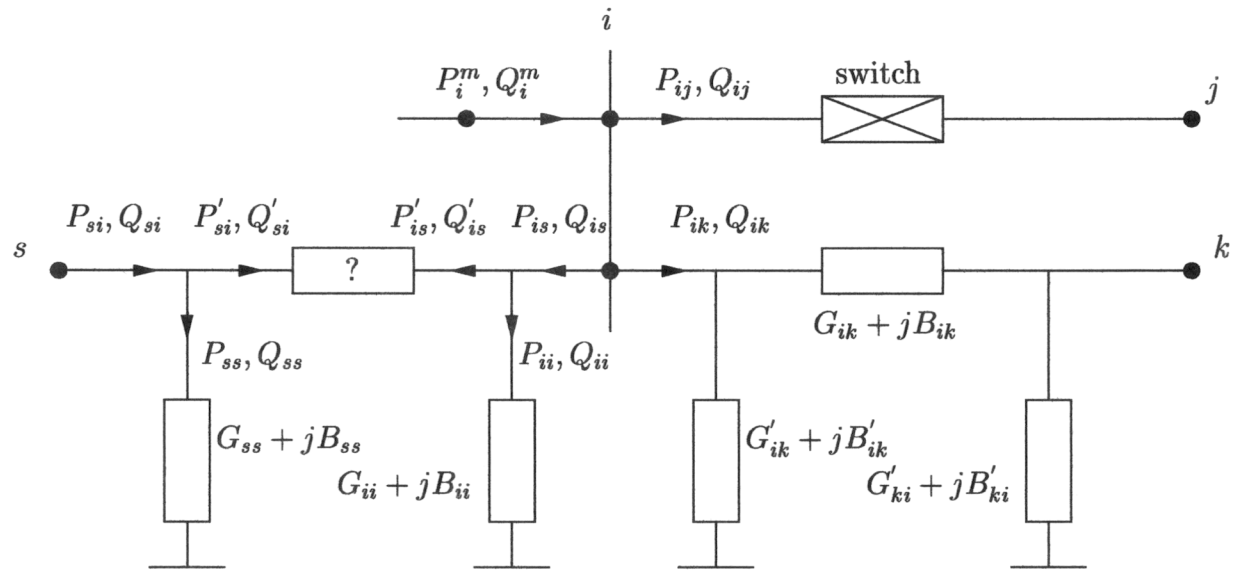

Figure 4.4: Power injection bus

$$P_i^m = P_{ij} + [(G'_{ik} + G_{ik})V_i^2 - V_iV_k(G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik})] + P_{is} + v_{P_i}\,,$$

and equation (4.24) will become:

$$Q_i^m = Q_{ij} + Q_{ik} + Q_{is} + v_{Q_i}\,, \tag{4.26}$$

therefore

$$Q_i^m = Q_{ij} + [-(B'_{ik} + B_{ik})V_i^2 - V_iV_k(G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik})] + Q_{is} + v_{Q_i}\,.$$

For the branch with the closed switch:

Assume the closed switch is in series with a low or zero impedance branch. The closed switch will be modelled as mentioned in section 4.3.1.1. The power flow in this branch will thus not be expressed in terms of θ and V as usual. Equation (4.4) and equation (4.5) will be used as well. The following measurement equations will be used:

$$P_{ij} = P_{ij} + v\,, \tag{4.27}$$

and

$$Q_{ij} = Q_{ij} + v\,. \tag{4.28}$$

The following pseudo-measurements are needed as well:

$$\theta_{ij}^m = \theta_i - \theta_j = 0\,, \tag{4.29}$$

and

$$V_{ij}^{m} = V_i - V_j = 0\,. \tag{4.30}$$

Equations (4.27), (4.28), (4.29) and (4.30) will be part of the measurement equations vector $\mathbf{h(x)}$ of $\mathbf{z} = \mathbf{h(x)} + \mathbf{v}$. The state variables to be estimated will be P_{ij} and Q_{ij}. These state variables will also be part of vector $\mathbf{x}$. P_{ij} of equation (4.25) can be calculated and Q_{ij} of equation (4.26) can be calculated.

For the transmission line:

The following measurement equations will be used:

$$P_{ik} = (G_{ik}^{'} + G_{ik})V_i^2 - V_iV_k(G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik})\,, \tag{4.31}$$

and

$$Q_{ik} = -(B_{ik}^{'} + B_{ik})V_i^2 - V_iV_k(G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik})\,. \tag{4.32}$$

Equation (4.31) and equation (4.32) will be part of the measurement equations vector $\mathbf{h(x)}$ of $\mathbf{z} = \mathbf{h(x)} + \mathbf{v}$. The state variables to be estimated will be V_i, V_k, θ_i and θ_k. These state variables will also be part of vector $\mathbf{x}$. P_{ik} of equation (4.25) can be calculated and Q_{ik} of equation (4.26) can be calculated.

For the transmission line with an unknown series parameter:

The power flow from node i to node s is:

$$P_{is} = P_{is}^{'} + P_{ii}\,, \tag{4.33}$$

and

$$Q_{is} = Q_{is}^{'} + Q_{ii}\,. \tag{4.34}$$

The power flows for P_{ii} and Q_{ii} are:

$$P_{ii} = G_{ii}V_i^2\,,$$

and

$$Q_{ii} = -B_{ii}V_i^2\,.$$

The unknown series parameter will be modelled as mentioned in section 4.3.1.3. The following measurement equations will be used:

$$P_{is}^{'}V_s + (P_{si}^{'}\cos\theta_{is} - Q_{si}^{'}\sin\theta_{is})V_i = 0\,, \tag{4.35}$$

and

$$Q_{is}^{'}V_s + (P_{si}^{'}\sin\theta_{is} + Q_{si}^{'}\cos\theta_{is})V_i = 0\,. \tag{4.36}$$

Equation (4.35) and equation (4.36) will be part of the measurement equations vector $\mathbf{h}(\mathbf{x})$ as in $\mathbf{z} = \mathbf{h}(\mathbf{x}) + \mathbf{v}$. The state variables to be estimated will be V_i, V_s, θ_i, θ_s, P'_{is}, Q'_{is}, P'_{si} and Q'_{si}. These state variables will also be part of vector $\mathbf{x}$. The state estimator will estimate P'_{is} of equation (4.33) and Q'_{is} of equation (4.34). Therefore, P_{is} and Q_{is} will be known in equation (4.25) and equation (4.26).

For another type of modelling technique on multiple injection buses, see [21].

4.4 Conclusion

The concept of generalised state estimation is introduced.

Generalised state estimators make use of advanced network modelling techniques. Several examples of advanced network modelling techniques are given.

By making use of advanced network modelling techniques, it was possible to:

- Estimate the status of unknown circuit breakers in series with a zero or low impedance branch.
- Model open circuit breakers in series with a zero or low impedance branch.
- Model closed circuit breakers in series with a zero or low impedance branch.
- Estimate unknown circuit breakers in series with a non-zero impedance branch.
- Model open circuit breakers in series with a non-zero impedance branch.
- Model closed circuit breakers in series with a non-zero impedance branch.
- Model zero impedance branches using the Augmented method.
- Model zero impedance branches using the Non-Augmented method.
- Estimate an unknown series parameter of a transmission line.
- Estimate an unknown shunt parameter of a transmission line.
- Estimate an unknown admittance of an ULTC transformer when the tap position is known.
- Modelling of power injection buses by means of advanced network modelling techniques.

The next chapter will discuss several functions which support a state estimator.

Chapter 5

Functions Supporting State Estimation

5.1 Introduction

In this chapter, a short overview will be given on several functions supporting a state estimator.

A state estimator uses real-time/on-line data or measurements obtained from a power system network. These measurements are used in an estimation process to estimate unknown state variables. Before a state estimator can estimate unknown state variables, several processes must be completed first. There are also functions which must be done after a state estimation process is completed. These processes/functions support a state estimator. Figure 5.1 shows the various processes needed to support a state estimator.

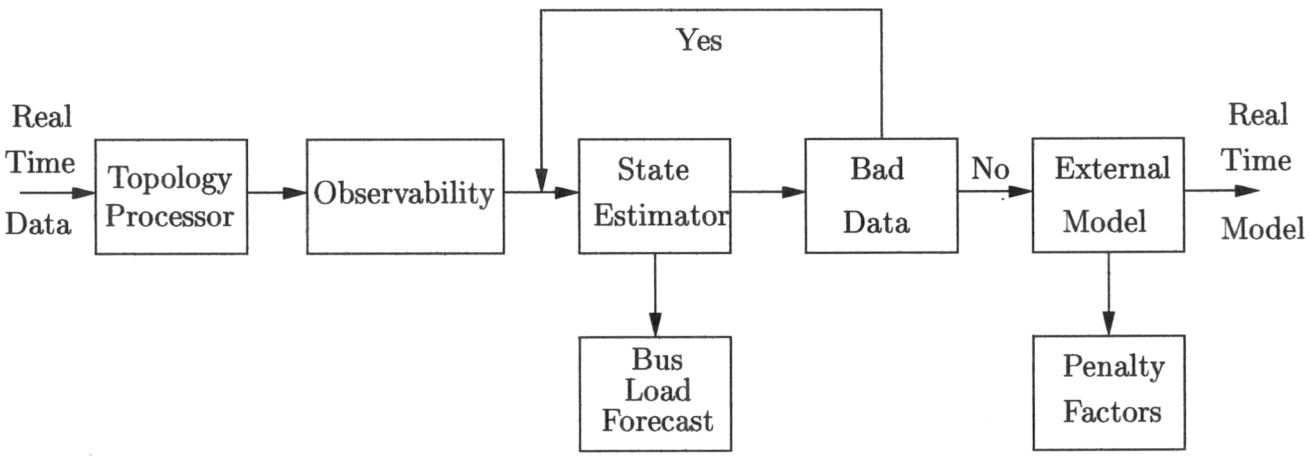

Figure 5.1: Various functions needed to support a state estimator

The topology processor receives real-time data from the power network via SCADA. The real-time data is stored in a database. The database contains the description of the network in terms of its parameters such as branch impedances and connectivity. The function of the topology processor in Figure 5.1, is to determine the present topology/connectivity of a power network. Figure 5.1 shows

that conventional topology processing is performed before state estimation. Topology processing and parameter estimation will be discussed in section 5.2. Once the network topology is known, the state estimator assumes that this topology is correct and proceeds to estimate the unknown state variables. The state estimator can only proceeds if the network is observable as will be discussed in section 5.3. During the state estimation process, bad data can be identified as will be explained in section 5.4.

For more information on bus load forecast factors, external modelling and penalty factors, see [8].

5.2 Topology Processing and Parameter Estimation

5.2.1 Topology Processing

The database describes the connectivity of the power network in terms of bus-sections and circuit breakers (a physical level representation). The status data for the circuit breakers can either be telemetered or entered manually by system operators. The state estimator uses a bus-branch model of the network topology. One of the functions of the network topology processor, is to transform the bus-circuit breaker model into a bus-branch model, which is required by the state estimator. There are several methods to convert bus-circuit breaker topology into bus-branch topology. Some methods include the use of logic tables, incidence matrices, tree search algorithms etc. . For more detail on the use of a tree search algorithm to convert bus-circuit breaker topology into bus-branch topology, see [8].

Processing topology errors using the conventional bus/branch network model, is not always effective. Especially, during the implementation phase of a state estimator. The number and frequency of errors can be high. The generalised topology processor forms an important part of the generalised state estimator, which can overcome above mentioned problems. A problem with the conventional topology processor, is that an incorrect status can cause many more islands to be identified, where there is actually only one island. A generalised topology processor can estimate this incorrect status to prevent islands of being identified incorrectly. For a more detail explanation on generalised topology processing, see [6].

5.2.2 Parameter Estimation

Parameter errors can also appear in a power network. Parameter errors will influence the output of the state estimator in a negative way. Parameter errors normally include the calculated values of transmission lines and transformer admittances. Examples of these parameter errors for transmission lines are:

- Occasional gross human errors due to manual data handling at the initial input phase.

- Mathematical approximations used in calculations, e.g. truncation of Taylor series expansion, round off errors etc. .
- Weather effects which modify conductor temperatures, causing different levels of sagging. These effects can modify line resistances and inductances as well.
- Assumptions made to simplify modelling techniques, e.g. completely transposed lines etc. [3].

A solution for these problems will be to estimate the parameters. Several methods can be used to estimate the parameters. Advanced network modelling techniques, from section 4.3, can be used to estimate the parameters of transmission lines and the parameters of transformers. Section 4.3.1.3 shows how the series parameter and the shunt parameter of a transmission line can be estimated. Section 4.3.1.4 shows how the admittance of an ULTC transformer can be modelled when the tap position is known. The generalised state estimator can thus be used for parameter estimation.

Another method which can be used to estimate parameters of a network, is a real time recursive parameter estimation method. This method uses a Kalman filter and is able to estimate impedance parameters of network branches. This method provides accurate estimation of branch parameters in the presence of noise in the measurements. It has also the ability to identify and reject gross measurement errors. For more information on this method, see [22].

5.3 Observability and Pseudo-Measurements

5.3.1 Observability

Section 5.2.1 mentions that the network topology processor is used to reduce the physical level model of the network to a model formed of buses and branches. The state estimator uses this type of model to estimate the state variables. Observability analysis is performed prior to state estimation to determine whether enough real-time measurements are available to make state estimation possible. If it happens that fewer and fewer measurements are available for using in the estimation process, the gain matrix will become singular.

A power system network will be observable when the rank of the gain matrix equals the number of unknown state variables n. Availability of the measurements and network topology may vary with time. It is therefore important to perform an observability test every time there is a change in the set of available measurements or the network topology. If the network is observable, state estimation can be done as indicated in Figure 5.1.

If a network is not observable, it can be made observable by using several techniques. A basic topological algorithm and a numerical algorithm is presented in [6], which can be used to build observable islands in networks modelled at the bus-branch level. A Hybrid algorithm is also discussed in [6]. These algorithms are used to ensure observability for networks expressed as a bus-branch model. There are also methods available for observability of networks expressed as a bus-circuit

breaker model. The extended numerical observability algorithm is discussed in [6]. For more information on observability, see also [23].

5.3.2 Pseudo-Measurements

Pseudo-measurements are used to improve the observability of a network. Pseudo-measurements can be used to make an unobservable network, observable. The pseudo-measurement is used in the state estimator as if it was an actual measured value from SCADA [7]. Pseudo-measurements are part of vector $\mathbf{z}$ and the pseudo-measurement equations are part of $\mathbf{h(x)}$. Pseudo-measurements can also be part of a constraint vector. The state estimation problem is solved subject to these constraints, see Appendix L. Examples of pseudo-measurements are:

- Zero-injection measurements, as described in section 3.3.2.6.
- It is mentioned in section 4.3.1.1, that a closed breaker in series with a zero impedance branch, can be modelled by making use of pseudo-measurements. The angle difference and voltage drop of a branch containing the closed breaker, are introduced as zero measurements and are as follows:

$$\theta_{ij}^{m} = \theta_i - \theta_j = 0\,,$$

and

$$V_{ij}^{m} = V_i - V_j = 0\,.$$

- The Non-Augmented method can be used to model a zero impedance branch as explained in section 4.3.1.2. The angle difference and voltage drop of the zero impedance branch are introduced as zero measurements and are as follows:

$$\theta_{ij}^{m} = \theta_i - \theta_j = 0\,,$$

and

$$V_{ij}^{m} = V_i - V_j = 0\,.$$

- Section 4.3.1.3 explains the modelling of network parameters. For the estimation of an unknown series impedance of a transmission line, a pseudo-measurement requiring the current flows at each end of the series impedance to be the same, is used. The following pseudo-measurements are used:

$$P_{ij}^{'}V_j + (P_{ji}^{'}\cos\theta_{ij} - Q_{ji}^{'}\sin\theta_{ij})V_i = 0\,,$$

and

$$Q_{ij}^{'}V_j + (P_{ji}^{'}\sin\theta_{ij} + Q_{ji}^{'}\cos\theta_{ij})V_i = 0\,.$$

For the estimation of an unknown shunt admittance of a symmetrical-pi transmission line, a pseudo-measurement requiring the legs at each end to be the same, is used. The following pseudo-measurements are used:

$$P_{ii}V_j^2 - P_{jj}V_i^2 = 0\,,$$

and

$$Q_{ii}V_j^2 - Q_{jj}V_i^2 = 0\,.$$

- Section 4.3.1.4 explains the modelling of an ULTC transformer when the tap position is known. For estimating the unknown series impedance of the transformer, a pseudo-measurement requiring the current flows at each end of the series impedance to be the same, is used. The following pseudo-measurements are used:

$$P'_{ij}V_j + (P'_{ji}\cos\theta_{ij} - Q'_{ji}\sin\theta_{ij})V_i = 0\,,$$

and

$$Q'_{ij}V_j + (P'_{ji}\sin\theta_{ij} + Q'_{ji}\cos\theta_{ij})V_i = 0\,,$$

and for estimating the unknown shunt admittance, a modified zero admittance difference equation is used. The following pseudo-measurements are used:

$$P_{ii}V_j^2 + cP_{jj}V_i^2 = 0\,,$$

and

$$Q_{ii}V_j^2 + cQ_{jj}V_i^2 = 0\,.$$

5.4 Bad Data Detection and Identification

5.4.1 Bad Data Detection

The state estimator uses real-time data, available from the power network via SCADA as mentioned in Chapter 1. These data can be bad. Major sources of bad data can be:

- Model approximations as well as statistical error modelling inadequacies.
- Rounding errors which will effect metered values.
- Gross measurement errors due to device failures, incorrect wiring, non-instantaneous meter scan, bad calibration or transducer bias drift with the weather, time or temperature etc. .
- Parameter errors which corrupt the values of the π - equivalent circuit elements.

- Topology errors which are induced by status data errors [24].

The degree to which bad data can be detected and identified, depends on the degree of redundancy in the measurement set. Normally, m (number of measurements in the system) is two to three times the value of n (number of unknown state variables in the system). This relationship will allow a considerable amount of redundancy in the set of measurement information. If $m = n$, there is no redundancy available in the set of measurement information. In this case, the state estimates will fit the measurements perfectly and none of the erroneous measurements will be filtered out. A measurement error is said to be detectable if an error in the measurement shows up in the measurement residual vector $r_i = z_i - h_i(x)$, see equation (3.22) of section 3.5.

There are several types of tests which can be used for bad data detection. The objective function $J(x)$, calculated after the state estimation process has converged, will be the smallest if there were not bad measurements in the estimation process. The presence of bad data will cause the converged value of $J(x)$ to be larger than expected. Under certain assumptions, it can be shown that the probability function of $J(x)$ has a chi-squared distribution $\chi^2(K)$. The quantity K is the degrees of freedom of the chi-squared distribution:

$$K = N_m - N_s\,, \tag{5.1}$$

where N_m is the number of measurements and N_s is the number of state variables. For more information on K, see [7]. A threshold can be set up for $J(x)$:

$$J(x) > T_{J(x)}\,, \tag{5.2}$$

which means that bad measurements are present if equation (5.2) is valid. This threshold can cause many false alarms if $T_{J(x)}$ is too small or too large. If it is too small, the test might indicate the presence of bad measurements when there were none. If it is too large, the test might indicate that no bad measurements were present, when there were actually bad measurements. The following solution for above mentioned can be used:

$$\text{prob}[J(x) > T_{J(x)} \mid J(x) \text{ is chi-squared with } K \text{ degrees of freedom}] = \alpha\,. \tag{5.3}$$

Equation (5.3) implies that the probability that $J(x)$ is greater than $T_{J(x)}$, is equal to α, given that the probability density for $J(x)$ is chi-squared with K degrees of freedom. The term α is called the significance level of the test. This type of testing is called hypothesis testing [7].

For more information on bad data detection, see also [6].

5.4.2 Bad Data Identification

The test described in section 5.4.1, is used to detect bad data. When bad data is detected, it must be identified.

The most common way to identify bad data is to use the normalised measurement residual approach. The state estimation process will give an estimated solution $\widehat{\mathbf{x}}$ for the unknown state variables $\mathbf{x}$ as mentioned in section 3.6. It is possible to calculate the estimated values for the measurement equations $\mathbf{h}(\widehat{\mathbf{x}})$ by using the estimated solution $\widehat{\mathbf{x}}$. The measurement residual can be calculated and it is the difference between the estimated measurement equations $\mathbf{h}(\widehat{\mathbf{x}})$ and the measurements $\mathbf{z}$. Therefore,

$$r_i = z_i - h_i(\widehat{x}), \tag{5.4}$$

for the i - th measurement. If the measurement residual is divided by its standard deviation, it is called the normalised measurement residual r_i^N. To obtain the standard deviation of a residual, see [7].

The usual process for identifying bad measurements is as follows: Once $\widehat{\mathbf{x}}$ is available from the state estimator, equation (5.4) can be calculated for each measurement used in the state estimation process. The normalised measurement residual r_i^N can also be calculated for each measurement, as mentioned above. The absolute value of the normalised measurement residual $\left|r_i^N\right|$ is calculated for each measurement. The measurement with the largest $\left|r_i^N\right|$ is suspect and is removed from the state estimation process. The state estimation process is calculated again to obtain a new set of $\widehat{\mathbf{x}}$. A new $\widehat{\mathbf{x}}$ will also mean a new $J(\widehat{\mathbf{x}})$ value. The detection step of equation (5.3) in section 5.4.1 must be calculated again. Assume the same significance level. If the new $J(\widehat{\mathbf{x}})$ is less than the new value for $T_{J(x)}$, the measurement which was taken out from the state estimation process, is identified as a bad measurement. If the new $J(\widehat{\mathbf{x}})$ is greater than the new value for $T_{J(x)}$, $\left|r_i^N\right|$ is calculated for the rest of the measurements. The measurement with the largest $\left|r_i^N\right|$ is removed and the whole process is repeated until $J(\widehat{\mathbf{x}})$ is less than $T_{J(x)}$ [7].

A flow chart for bad data detection and identification using the hypothesis detection test is shown in Appendix D.

For more information on identification of bad data, see [24].

More valuable information on the normalised residual approach is given in [25]. This reference gives also valuable information on bad data detection and identification techniques for multiple noninteracting bad data and multiple interacting bad data.

5.5 Conclusion

There are several functions supporting a state estimator. Real-time data is received via SCADA and stored in a database. The topology processor receives the real-time data. The database describes the connectivity of the power network in terms of bus-sections and circuit breakers. The state estimator uses a bus-branch model of the network topology. One of the functions of the network topology processor, is to transform the bus-circuit breaker model into a bus-branch model, which is required by the state estimator.

Parameter errors can also appear in a power network. A generalised state estimator can be used to estimate parameters of a network. A generalised state estimator uses advanced network modelling techniques for parameter estimation.

Observability analysis is performed prior to state estimation to determine whether enough real-time measurements are available to make state estimation possible. A power system network will be observable when the rank of the gain matrix equals the number of unknown state variables n. If a network is not observable, it can be made observable by using several techniques.

Pseudo-measurements are used to improve the observability of a network. Pseudo-measurements can be used to make an unobservable network, observable. A pseudo-measurement is used in the state estimator as if it was an actual measured value from SCADA. Several examples of pseudo-measurements are given.

The state estimator uses real-time data, which can be bad. The degree to which bad data can be detected and identified, depends on the degree of redundancy in the measurement set. The hypothesis testing can be used to detect bad data. One way to identify bad data is to use the normalised measurement residual approach.

The next chapter will give the results obtained by making use of the WLS objective function and the QT objective function in a state estimation process.

Chapter 6

Results

6.1 Introduction

As mentioned in section 3.5, the Weighted Least Squares objective function is minimised during a state estimation process. Several other objective functions can also be used in a state estimation process for example the Quadratic-Tangent (QT) objective function. For more information on the Quadratic-Tangent objective function, see Appendix E and [13]. The QT objective function can also be minimised by taking the gradient of the function and set it equal to zero. A detail derivation for the gradient of the QT function is given in Appendix E.

Results for the test cases are obtained by making use of advanced network modelling techniques of section 4.3. Most of these advanced network modelling techniques have been used in a state estimation problem which minimised a WLS objective function, see [19]. [27] shows the results obtained for using advanced network modelling techniques in a state estimation process which minimised the QT objective function. In this chapter, a comparison will be made between advanced network modelling techniques used in a state estimation process which minimises a WLS objective function and advanced network modelling techniques used in a state estimation process which minimises a QT objective function. The unknown state variables are estimated through the state estimation process as shown in Appendix F. This state estimation process is programmed and used in MATLAB. The results are obtained in the following way:

The numerical solution of the state estimation problem is obtained by making use of WLS and QT objective functions and the Gauss-Newton iterative method. A derivation for the Gauss-Newton method is given in Appendix E. The elements of the Jacobian matrix can be expressed in polar format or rectangular format. The Jacobian derivatives in polar format are given in Appendix G. The Jacobian derivatives in rectangular format are given in Appendix H. It is possible that less number of steps will be required to obtain convergence, if Jacobian derivatives in rectangular format are used. Only Jacobian derivatives in polar format are used for the test cases of this chapter.

The measurement vector, vector $\mathbf{z}$, which is needed for the state estimation process, is calculated by solving the load flow problem. The load flow problem is discussed in section 2.4.1. A load flow

problem is solved for the IEEE 14-bus network (Standard of the Institute of Electrical and Electronic Engineers), IEEE 30-bus network and IEEE 57-bus network from [26], to obtain the measurements needed for the state estimation process of each test case. Data for these IEEE networks are given in Appendix I. A flow diagram, indicating the generation process of these measurements used in the test cases, is given in Appendix J. Bus 1 is used for the slack bus in all the test cases.

Only a small, but representative part of exhaustive results obtained, will be given in this chapter. All the estimated values of the state variables are given in per unit value. The following groups of results will be given:

- Section 6.2 shows the results obtained for an IEEE 14-bus network.
- Section 6.3 shows the results obtained for an IEEE 30-bus network.
- Section 6.4 shows the results obtained for an IEEE 57-bus network.

The chapter concludes with an overview on published papers.

6.2 IEEE 14-bus Network

6.2.1 Modelling of zero impedance branches

The IEEE data used in all the test cases, do not contain zero impedance branches. Zero impedance branches need to be created. The following technique are used to create zero impedance branches for all the test cases:

A non-zero impedance branch, e.g. branch 1 - 5, can be split and represented as a double line as shown in Figure 6.1 a). The double line can also be represented by introducing an extra node as shown in Figure 6.1 b). Branch 1 - 5 and branch 5 - 15 are the two double lines in part a). Branch 1 - 15 is the zero impedance branch.

Zero impedance branches are not included in the load flow problem. Power flow measurements are obtained for the zero impedance branches by dividing the power flow of the non-zero impedance branch, e.g. branch 1 - 5 (power flow obtained from load flow), into a power flow for branch 1 - 5 (in Figure 6.1 b)) and a power flow for branch 5 - 15 (in Figure 6.1 b)). The power flow of the zero impedance branch will be equal to the power flow in branch 5 - 15.

Modelling of a zero impedance series branch is illustrated by using two formulations of a state estimation problem:

- Non-Augmented method. This method uses the pseudo-measurements as part of vector $\mathbf{z}$ in an estimation process.

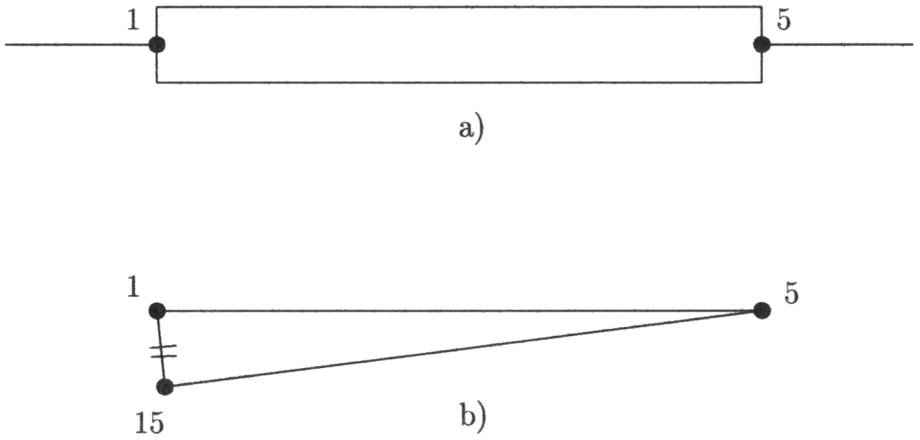

Figure 6.1: Zero impedance branch

- Augmented method. This method uses the pseudo-measurements as equality constraints in an estimation process. See Appendix L.

The above mentioned formulations are used to model zero impedance branches for all the test cases.

6.2.1.1 Non-Augmented method

Table 6.1 shows the results obtained by making use of the following procedure:

- Branches 1 - 5, 3 - 4, 6 - 11 and 9 - 14 of the IEEE 14-bus network are used for this modelling technique.
- The above non-zero impedance branches are split to obtain zero impedance branches 1 - 15, 3 - 16, 6 - 17 and 9 - 18 as explained in Figure 6.1.
- The power flow measurements for the zero impedance branches are shown in the third column.
- Measurements for the IEEE 14-bus of Appendix K are used. No errors are added to the measurements.
- The WLS and QT objective functions are used in the estimation process.
- The Non-Augmented method is used to model the zero impedance branches.
- A convergence criterion of 10^{-13} is used for the linear solver.
- The estimated values of the power flows are shown in the fourth column.
- WLS required 5 steps to converge.
- QT required 5 steps to converge.

The estimated power flows are correct estimated as shown in Table 6.1.

Table 6.1: Non-Augmented method

Objective function	Branch	Measured values of P_{ij} and Q_{ij}	Estimated values of P_{ij} and Q_{ij}
WLS	1 - 15	$P_{1,15} = 0.2464$ $Q_{1,15} = 0.0244$	$P_{1,15} = 0.2464$ $Q_{1,15} = 0.0244$
	3 - 16	$P_{3,16} = -0.1616$ $Q_{3,16} = -0.0003$	$P_{3,16} = -0.1616$ $Q_{3,16} = -0.0003$
	6 - 17	$P_{6,17} = 0.0089$ $Q_{6,17} = 0.0204$	$P_{6,17} = 0.0089$ $Q_{6,17} = 0.0204$
	9 - 18	$P_{9,18} = 0.0198$ $Q_{9,18} = 0.0109$	$P_{9,18} = 0.0198$ $Q_{9,18} = 0.0109$
QT	1 - 15	$P_{1,15} = 0.2464$ $Q_{1,15} = 0.0244$	$P_{1,15} = 0.2464$ $Q_{1,15} = 0.0244$
	3 - 16	$P_{3,16} = -0.1616$ $Q_{3,16} = -0.0003$	$P_{3,16} = -0.1616$ $Q_{3,16} = -0.0003$
	6 - 17	$P_{6,17} = 0.0089$ $Q_{6,17} = 0.0204$	$P_{6,17} = 0.0089$ $Q_{6,17} = 0.0204$
	9 - 18	$P_{9,18} = 0.0198$ $Q_{9,18} = 0.0109$	$P_{9,18} = 0.0198$ $Q_{9,18} = 0.0109$

Zero impedance branches are also modelled in the presence of bad data. Measurement errors (bad data) are added to the measurement vector $\mathbf{z}$ after solving the load flow problem. None random errors are used. Only gross errors are used. The measurement errors are calculated by adding gross errors to several arbitrary entries of the measurement vector $\mathbf{z}$. The following measurement errors are created:

$$ZPF(2) = ZPF(2) + 1\,, \tag{6.1}$$

$$ZQF(3) = ZQF(3) + 3\,, \tag{6.2}$$

$$ZV(2) = ZV(2) + 1\,, \tag{6.3}$$

where $ZPF(2)$ is the second active power flow measurement in vector $\mathbf{z}$, $ZQF(3)$ is the third reactive power flow measurement in vector $\mathbf{z}$, $ZV(2)$ is the second voltage measurement in vector $\mathbf{z}$. The errors $(1, 3, 1)$, are added respectively to each of above mentioned measurements.

Table 6.2 shows the results obtained when gross errors are present in the estimation process. The same procedure for estimating the results of Table 6.1 is used, except that the measurement errors, equations (6.1), (6.2) and (6.3), are used together with the measurements for the IEEE 14-bus of Appendix K.

- WLS required 14 steps to converge.
- QT required 10 steps to converge.

The estimated power flows are not correct estimated if the WLS objective function is used in the estimation process with bad data.

6.2.1.2 Augmented method

Table 6.3 shows the results obtained by making use of the following procedure:

Table 6.2: Non-Augmented method, bad data

Objective function	Branch	Measured values of P_{ij} and Q_{ij}	Estimated values of P_{ij} and Q_{ij}
WLS	1 - 15	$P_{1,15} = 0.2464$ $Q_{1,15} = 0.0244$	$P_{1,15} = 0.2295$ $Q_{1,15} = -0.0458$
	3 - 16	$P_{3,16} = -0.1616$ $Q_{3,16} = -0.0003$	$P_{3,16} = 0.0034$ $Q_{3,16} = -0.1923$
	6 - 17	$P_{6,17} = 0.0089$ $Q_{6,17} = 0.0204$	$P_{6,17} = 0.0081$ $Q_{6,17} = 0.0209$
	9 - 18	$P_{9,18} = 0.0198$ $Q_{9,18} = 0.0109$	$P_{9,18} = 0.0332$ $Q_{9,18} = 0.0372$
QT	1 - 15	$P_{1,15} = 0.2464$ $Q_{1,15} = 0.0244$	$P_{1,15} = 0.2464$ $Q_{1,15} = 0.0244$
	3 - 16	$P_{3,16} = -0.1616$ $Q_{3,16} = -0.0003$	$P_{3,16} = -0.1616$ $Q_{3,16} = -0.0003$
	6 - 17	$P_{6,17} = 0.0089$ $Q_{6,17} = 0.0204$	$P_{6,17} = 0.0089$ $Q_{6,17} = 0.0204$
	9 - 18	$P_{9,18} = 0.0198$ $Q_{9,18} = 0.0109$	$P_{9,18} = 0.0198$ $Q_{9,18} = 0.0109$

- Branches 1 - 5, 3 - 4, 6 - 11 and 9 - 14 of the IEEE 14-bus network are used for this modelling technique.
- The above non-zero impedance branches are split to obtain zero impedance branches 1 - 15, 3 - 16, 6 - 17 and 9 - 18 as explained in Figure 6.1.
- The power flow measurements for the zero impedance branches are shown in the third column.
- Measurements for the IEEE 14-bus of Appendix K are used. No errors are added to the measurements.
- The WLS and QT objective functions are used in the estimation process.
- The Augmented method is used to model the zero impedance branches.
- A convergence criterion of 10^{-13} is used for the linear solver.
- The estimated values of the power flows are shown in the fourth column.
- WLS required 4 steps to converge.
- QT required 4 steps to converge.

The estimated power flows are correct estimated as shown in Table 6.3.

Table 6.4 shows the results obtained when gross errors are present in the estimation process. The same procedure for estimating the results of Table 6.3 is used, except that the measurement errors, equations (6.1), (6.2) and (6.3), are used together with the measurements for the IEEE 14-bus of Appendix K.

- WLS did not converge.
- QT required 10 steps to converge.

Table 6.3: Augmented method

Objective function	Branch	Measured values of P_{ij} and Q_{ij}	Estimated values of P_{ij} and Q_{ij}
WLS	1 - 15	$P_{1,15} = 0.2464$ $Q_{1,15} = 0.0244$	$P_{1,15} = 0.2464$ $Q_{1,15} = 0.0244$
	3 - 16	$P_{3,16} = -0.1616$ $Q_{3,16} = -0.0003$	$P_{3,16} = -0.1616$ $Q_{3,16} = -0.0003$
	6 - 17	$P_{6,17} = 0.0089$ $Q_{6,17} = 0.0204$	$P_{6,17} = 0.0089$ $Q_{6,17} = 0.0204$
	9 - 18	$P_{9,18} = 0.0198$ $Q_{9,18} = 0.0109$	$P_{9,18} = 0.0198$ $Q_{9,18} = 0.0109$
QT	1 - 15	$P_{1,15} = 0.2464$ $Q_{1,15} = 0.0244$	$P_{1,15} = 0.2464$ $Q_{1,15} = 0.0244$
	3 - 16	$P_{3,16} = -0.1616$ $Q_{3,16} = -0.0003$	$P_{3,16} = -0.1616$ $Q_{3,16} = -0.0003$
	6 - 17	$P_{6,17} = 0.0089$ $Q_{6,17} = 0.0204$	$P_{6,17} = 0.0089$ $Q_{6,17} = 0.0204$
	9 - 18	$P_{9,18} = 0.0198$ $Q_{9,18} = 0.0109$	$P_{9,18} = 0.0198$ $Q_{9,18} = 0.0109$

The estimated power flows are not correct estimated if the WLS objective function is used in the estimation process with bad data. The estimation process did not converge if the WLS objective function is used.

Table 6.4: Augmented method, bad data

Objective function	Branch	Measured values of P_{ij} and Q_{ij}	Estimated values of P_{ij} and Q_{ij}
WLS	1 - 15	$P_{1,15} = 0.2464$ $Q_{1,15} = 0.0244$	-
	3 - 16	$P_{3,16} = -0.1616$ $Q_{3,16} = -0.0003$	-
	6 - 17	$P_{6,17} = 0.0089$ $Q_{6,17} = 0.0204$	-
	9 - 18	$P_{9,18} = 0.0198$ $Q_{9,18} = 0.0109$	-
QT	1 - 15	$P_{1,15} = 0.2464$ $Q_{1,15} = 0.0244$	$P_{1,15} = 0.2464$ $Q_{1,15} = 0.0244$
	3 - 16	$P_{3,16} = -0.1616$ $Q_{3,16} = -0.0003$	$P_{3,16} = -0.1616$ $Q_{3,16} = -0.0003$
	6 - 17	$P_{6,17} = 0.0089$ $Q_{6,17} = 0.0204$	$P_{6,17} = 0.0089$ $Q_{6,17} = 0.0204$
	9 - 18	$P_{9,18} = 0.0198$ $Q_{9,18} = 0.0109$	$P_{9,18} = 0.0198$ $Q_{9,18} = 0.0109$

6.2.2 Circuit breaker modelling

There are two different types of classes for circuit breaker modelling. Circuit breaker modelling is used for:

- Circuit breakers intended for line, transformer and shunt element switching which are in series with a non-zero impedance branch.
- Circuit breakers which, within a substation, link busbar sections and determine the actual electrical nodes and are in series with a zero-impedance.

These types of circuit breaker modelling are used for all the circuit breakers in the test cases.

6.2.2.1 Circuit breakers in series with a zero impedance branch

Two states of circuit breakers are considered:

- Modelling of closed breakers, in series with a zero impedance branch.
- Modelling of open breakers, in series with a zero impedance branch.

Table 6.5 shows the results obtained by making use of the following procedure:

- Branches 1 - 5 and 3 - 4 of the IEEE 14-bus network are used for this modelling technique.
- The above non-zero impedance branches are split to obtain zero impedance branches 1 - 15 and 3 - 16 as explained in Figure 6.1.
- The correct states of the circuit breakers are shown in the third column.
- Measurements for the IEEE 14-bus of Appendix K are used. No errors are added to the measurements.
- The WLS and QT objective functions are used in the estimation process.
- A convergence criterion of 10^{-13} is used for the linear solver.
- The estimated states of the circuit breakers are shown in the fourth column.
- WLS required 5 steps to converge when the circuit breakers were closed.
- WLS required 5 steps to converge when the circuit breakers were open.
- QT required 5 steps to converge when the circuit breakers were closed.
- QT required 5 steps to converge when the circuit breakers were open.

The estimated states of the circuit breakers are correct estimated as shown in Table 6.5.

Table 6.6 shows the results obtained when gross errors are present in the estimation process. The same procedure for estimating the results of Table 6.5 is used, except that the measurement errors, equations (6.1), (6.2) and (6.3), are used together with the measurements for the IEEE 14-bus of Appendix K. Only a closed status is considered for the circuit breakers of branches 1 - 15 and 3 - 16.

- WLS required 16 steps to converge.
- QT required 15 steps to converge.

The estimated states of the circuit breakers are not correct estimated if the WLS objective function is used in the estimation process with bad data.

Table 6.5: Circuit breakers in series with a zero impedance branch

Objective function	Breaker connected between nodes	Status of circuit breaker	Estimated status of circuit breaker
WLS	1 - 15	Closed	Closed
	3 - 16	Closed	Closed
	1 - 15	Open	Open
	3 - 16	Open	Open
QT	1 - 15	Closed	Closed
	3 - 16	Closed	Closed
	1 - 15	Open	Open
	3 - 16	Open	Open

Table 6.6: Circuit breakers in series with a zero impedance branch, bad data

Objective function	Breaker connected between nodes	Status of circuit breaker	Estimated status of circuit breaker
WLS	1 - 15	Closed	Open
	3 - 16	Closed	Open
QT	1 - 15	Closed	Closed
	3 - 16	Closed	Closed

6.2.2.2 Circuit breakers in series with a non-zero impedance branch

Two states of circuit breakers are considered:

- Modelling of closed breakers, in series with a non-zero impedance branch.
- Modelling of open breakers, in series with a non-zero impedance branch.

Table 6.7 shows the results obtained by making use of the following procedure:

- Branches 2 - 3 and 4 - 5 of the IEEE 14-bus network are used for this modelling technique.
- The correct states of the circuit breakers are shown in the third column.
- Measurements for the IEEE 14-bus of Appendix K are used. No errors are added to the measurements.
- The WLS and QT objective functions are used in the estimation process.
- A convergence criterion of 10^{-13} is used for the linear solver.
- The estimated states of the circuit breakers are shown in the fifth column.

- WLS required 7 steps to converge when the circuit breakers were closed.
- WLS required 8 steps to converge when the circuit breakers were open.
- QT required 7 steps to converge when the circuit breakers were closed.
- QT required 8 steps to converge when the circuit breakers were open.

The estimated states of the circuit breakers are correct estimated as shown in Table 6.7.

Table 6.7: Circuit breakers in series with a non-zero impedance branch

Objective function	Breaker connected between nodes	Status of circuit breaker	Estimated value of κ:	Estimated status ($\kappa = 0$: open) ($\kappa = 1$: closed)
WLS	2 - 3	Closed	1	Closed
	4 - 5	Closed	1	Closed
	2 - 3	Open	0.0000	Open
	4 - 5	Open	$-0.4764e{-}18$	Open
QT	2 - 3	Closed	1	Closed
	4 - 5	Closed	1	Closed
	2 - 3	Open	$-0.0195e{-}19$	Open
	4 - 5	Open	$-0.9254e{-}19$	Open

Table 6.8 shows the results obtained when gross errors are present in the estimation process. The same procedure for estimating the results of Table 6.7 is used, except that the measurement errors, equations (6.1), (6.2) and (6.3), are used together with the measurements for the IEEE 14-bus of Appendix K.

- WLS required 18 steps to converge when the circuit breakers were closed.
- WLS required 60 steps to converge when the circuit breakers were open.
- QT required 10 steps to converge when the circuit breakers were closed.
- QT required 12 steps to converge when the circuit breakers were open.

The estimated states of the circuit breakers are not all correct estimated if the WLS objective function is used in the estimation process with bad data.

6.2.3 Modelling of network parameters

The modelling of network parameters is divided into two types of modelling:

- Modelling of the unknown series branch of a π - equivalent transmission line model.
- Modelling of the unknown shunt branch of a π - equivalent transmission line model. Assume $G = 0$ in $Y = G + jB$.

Table 6.8: Circuit breakers in series with a non-zero impedance branch, bad data

Objective function	Breaker connected between nodes	Status of circuit breaker	Estimated value of κ:	Estimated status ($\kappa = 0$: open) ($\kappa = 1$: closed)
WLS	2 - 3	Closed	1	Closed
	4 - 5	Closed	0	Open
	2 - 3	Open	$0.3054e-3$	Open
	4 - 5	Open	$-0.0866e-3$	Open
QT	2 - 3	Closed	1	Closed
	4 - 5	Closed	1	Closed
	2 - 3	Open	$0.1636e-11$	Open
	4 - 5	Open	$-0.1002e-11$	Open

6.2.3.1 The series branch

Table 6.9 shows the results obtained by making use of the following procedure:

- Branches 1 - 2, 1 - 5, 6 - 13 and 10 - 11 of the IEEE 14-bus network are used for this modelling technique.
- The correct impedances of the series parameters are shown in the third column.
- Measurements for the IEEE 14-bus of Appendix K are used. No errors are added to the measurements.
- The WLS and QT objective functions are used in the estimation process.
- A convergence criterion of 10^{-13} is used for the linear solver.
- The estimated impedances of the series parameters are shown in the fourth column.
- WLS required 5 steps to converge.
- QT required 5 steps to converge.

The estimated impedances are correct estimated as shown in Table 6.9.

Table 6.9: The unknown series parameter

Objective function	Branch	True values for $Z = R + jX$	Estimated values for $Z = R + jX$
WLS	1 - 2	$R = 0.01938 \quad X = 0.05917$	$R = 0.01938 \quad X = 0.05917$
	1 - 5	$R = 0.05403 \quad X = 0.22304$	$R = 0.05403 \quad X = 0.22304$
	6 - 13	$R = 0.06615 \quad X = 0.13027$	$R = 0.06615 \quad X = 0.13027$
	10 - 11	$R = 0.08205 \quad X = 0.19207$	$R = 0.08205 \quad X = 0.19207$
QT	1 - 2	$R = 0.01938 \quad X = 0.05917$	$R = 0.01938 \quad X = 0.05917$
	1 - 5	$R = 0.05403 \quad X = 0.22304$	$R = 0.05403 \quad X = 0.22304$
	6 - 13	$R = 0.06615 \quad X = 0.13027$	$R = 0.06615 \quad X = 0.13027$
	10 - 11	$R = 0.08205 \quad X = 0.19207$	$R = 0.08205 \quad X = 0.19207$

Table 6.10 shows the results obtained when gross errors are present in the estimation process. The same procedure for estimating the results of Table 6.9 is used, except that the measurement errors, equations (6.1), (6.2) and (6.3), are used together with the measurements for the IEEE 14-bus of Appendix K.

- WLS required 10 steps to converge.
- QT required 9 steps to converge.

The estimated values for the series parameters are not correct estimated if the WLS objective function is used in the estimation process with bad data.

Table 6.10: The unknown series parameter, bad data

Objective function	Branch	True values for $Z = R + jX$	Estimated values for $Z = R + jX$
WLS	1 - 2	$R = 0.01938 \quad X = 0.05917$	$R = -0.08116 \quad X = -0.04917$
	1 - 5	$R = 0.05403 \quad X = 0.22304$	$R = -0.09079 \quad X = 0.01652$
	6 - 13	$R = 0.06615 \quad X = 0.13027$	$R = 0.16812 \quad X = 0.19998$
	10 - 11	$R = 0.08205 \quad X = 0.19207$	$R = 0.79336 \quad X = 0.16569$
QT	1 - 2	$R = 0.01938 \quad X = 0.05917$	$R = 0.01938 \quad X = 0.05917$
	1 - 5	$R = 0.05403 \quad X = 0.22304$	$R = 0.05403 \quad X = 0.22304$
	6 - 13	$R = 0.06615 \quad X = 0.13027$	$R = 0.06615 \quad X = 0.13027$
	10 - 11	$R = 0.08205 \quad X = 0.19207$	$R = 0.08205 \quad X = 0.19207$

6.2.3.2 The shunt branch

Table 6.11 shows the results obtained by making use of the following procedure:

- Branches 1 - 2, 1 - 5, 2 - 3 and 4 - 5 of the IEEE 14-bus network are used for this modelling technique.

- The correct admittances of the shunt parameters are shown in the third column.
- Measurements for the IEEE 14-bus of Appendix K are used. No errors are added to the measurements.
- The WLS and QT objective functions are used in the estimation process.
- A convergence criterion of 10^{-13} is used for the linear solver.
- The estimated admittances of the shunt parameters are shown in the fourth column.
- WLS required 5 steps to converge.
- QT required 5 steps to converge.

The estimated admittances are correct estimated as shown in Table 6.11.

Table 6.11: The unknown shunt parameter

Objective function	Branch	True values for $Y = G + jB$	Estimated values for $Y = G + jB$
WLS	1 - 2	$B = 0.02640$	$B = 0.02640$
	1 - 5	$B = 0.02460$	$B = 0.02460$
	2 - 3	$B = 0.02190$	$B = 0.02190$
	4 - 5	$B = 0.00640$	$B = 0.00640$
QT	1 - 2	$B = 0.02640$	$B = 0.02640$
	1 - 5	$B = 0.02460$	$B = 0.02460$
	2 - 3	$B = 0.02190$	$B = 0.02190$
	4 - 5	$B = 0.00640$	$B = 0.00640$

Table 6.12 shows the results obtained when gross errors are present in the estimation process. The same procedure for estimating the results of Table 6.11 is used, except that the measurement errors, equations (6.1), (6.2) and (6.3), are used together with the measurements for the IEEE 14-bus of Appendix K. Only branches 1 - 2 and 1 - 5 are used for this testing with bad data.

- WLS required 8 steps to converge.
- QT required 9 steps to converge.

The estimated values for the shunt parameters are not correct estimated if the WLS objective function is used in the estimation process with bad data.

6.2.4 Modelling of ULTC transformer parameters

This modelling technique is used to estimate the unknown admittance of an ULTC transformer when the tap position is known. Assume $G = 0$ in $Y = G + jB$.

Table 6.13 shows the results obtained by making use of the following procedure:

Table 6.12: The unknown shunt parameter, bad data

Objective function	Branch	True values for $Y = G + jB$	Estimated values for $Y = G + jB$
WLS	1 - 2	$B = 0.02640$	$B = 0.16344$
	1 - 5	$B = 0.02460$	$B = -0.01065$
QT	1 - 2	$B = 0.02640$	$B = 0.02640$
	1 - 5	$B = 0.02460$	$B = 0.02460$

- Branches 4 - 7, 4 - 9, and 5 - 6 of the IEEE 14-bus network are used for this modelling technique.
- The correct admittances of the transformers are shown in the third column.
- Measurements for the IEEE 14-bus of Appendix K are used. No errors are added to the measurements.
- The WLS and QT objective functions are used in the estimation process.
- A convergence criterion of 10^{-13} is used for the linear solver.
- The estimated admittances of the transformers are shown in the fourth column.
- WLS required 5 steps to converge.
- QT required 5 steps to converge.

The estimated admittances are correct estimated as shown in Table 6.13.

Table 6.13: The unknown transformer parameter

Objective function	Branch	True values for $Y_T = G + jB$	Estimated values for $Y_T = G + jB$
WLS	4 - 7	$B = -4.88951$	$B = -4.88951$
	4 - 9	$B = -1.85550$	$B = -1.85550$
	5 - 6	$B = -4.25745$	$B = -4.25745$
QT	4 - 7	$B = -4.88951$	$B = -4.88951$
	4 - 9	$B = -1.85550$	$B = -1.85550$
	5 - 6	$B = -4.25745$	$B = -4.25745$

Table 6.14 shows the results obtained when gross errors are present in the estimation process. The same procedure for estimating the results of Table 6.13 is used, except that the measurement errors, equations (6.1), (6.2) and (6.3), are used together with the measurements for the IEEE 14-bus of Appendix K.

- WLS required 20 steps to converge.

- QT required 13 steps to converge.

The estimated values for the transformer parameters are not correct estimated if the WLS objective function is used in the estimation process with bad data.

Table 6.14: The unknown transformer parameter, bad data

Objective function	Branch	True values for $Y_T = G + jB$	Estimated values for $Y_T = G + jB$
WLS	4 - 7	$B = -4.88951$	$B = -73.42625$
	4 - 9	$B = -1.85550$	$B = -51.88394$
	5 - 6	$B = -4.25745$	$B = -18.79761$
QT	4 - 7	$B = -4.88951$	$B = -4.88951$
	4 - 9	$B = -1.85550$	$B = -1.85550$
	5 - 6	$B = -4.25745$	$B = -4.25745$

6.3 IEEE 30-bus Network

6.3.1 Modelling of zero impedance branches

6.3.1.1 Non-Augmented method

Table 6.15 shows the results obtained by making use of the following procedure:

- Branches 3 - 4, 9 - 11, 18 - 19 and 25 - 26 of the IEEE 30-bus network are used for this modelling technique.
- The above non-zero impedance branches are split to obtain zero impedance branches 3 - 31, 9 - 32, 18 - 33 and 25 - 34 as explained in Figure 6.1.
- The power flow measurements for the zero impedance branches are shown in the third column.
- Measurements for the IEEE 30-bus of Appendix K are used. No errors are added to the measurements.
- The WLS and QT objective functions are used in the estimation process.
- The Non-Augmented method is used to model the zero impedance branches.
- A convergence criterion of 10^{-13} is used for the linear solver.
- The estimated values of the power flows are shown in the fourth column.
- WLS required 5 steps to converge.

Table 6.15: Non-Augmented method

Objective function	Branch	Measured values of P_{ij} and Q_{ij}	Estimated values of P_{ij} and Q_{ij}
WLS	3 - 31	$P_{3,31} = 0.6895 \quad Q_{3,31} = -0.0817$	$P_{3,31} = 0.6895 \quad Q_{3,31} = -0.0817$
	9 - 32	$P_{9,32} = 0.0302 \quad Q_{9,32} = -0.2236$	$P_{9,32} = 0.0302 \quad Q_{9,32} = -0.2236$
	18 - 33	$P_{18,33} = 0.0116 \quad Q_{18,33} = 0.0075$	$P_{18,33} = 0.0116 \quad Q_{18,33} = 0.0075$
	25 - 34	$P_{25,34} = 0.0085 \quad Q_{25,34} = 0.0049$	$P_{25,34} = 0.0085 \quad Q_{25,34} = 0.0049$
QT	3 - 31	$P_{3,31} = 0.6895 \quad Q_{3,31} = -0.0817$	$P_{3,31} = 0.6895 \quad Q_{3,31} = -0.0817$
	9 - 32	$P_{9,32} = 0.0302 \quad Q_{9,32} = -0.2236$	$P_{9,32} = 0.0302 \quad Q_{9,32} = -0.2236$
	18 - 33	$P_{18,33} = 0.0116 \quad Q_{18,33} = 0.0075$	$P_{18,33} = 0.0116 \quad Q_{18,33} = 0.0075$
	25 - 34	$P_{25,34} = 0.0085 \quad Q_{25,34} = 0.0049$	$P_{25,34} = 0.0085 \quad Q_{25,34} = 0.0049$

- QT required 5 steps to converge.

The estimated power flows are correct estimated as shown in Table 6.15.

Table 6.16 shows the results obtained when gross errors are present in the estimation process. The same procedure for estimating the results of Table 6.15 is used, except that the measurement errors, equations (6.1), (6.2) and (6.3), are used together with the measurements for the IEEE 30-bus of Appendix K.

- WLS required 8 steps to converge.
- QT required 7 steps to converge.

The estimated power flows are not correct estimated if the WLS objective function is used in the estimation process with bad data.

Table 6.16: Non-Augmented method, bad data

Objective function	Branch	Measured values of P_{ij} and Q_{ij}	Estimated values of P_{ij} and Q_{ij}
WLS	3 - 31	$P_{3,31} = 0.6895 \quad Q_{3,31} = -0.0817$	$P_{3,31} = 0.6269 \quad Q_{3,31} = -0.5170$
	9 - 32	$P_{9,32} = 0.0302 \quad Q_{9,32} = -0.2236$	$P_{9,32} = 0.0303 \quad Q_{9,32} = -0.1945$
	18 - 33	$P_{18,33} = 0.0116 \quad Q_{18,33} = 0.0075$	$P_{18,33} = 0.0124 \quad Q_{18,33} = 0.0090$
	25 - 34	$P_{25,34} = 0.0085 \quad Q_{25,34} = 0.0049$	$P_{25,34} = 0.0063 \quad Q_{25,34} = 0.0015$
QT	3 - 31	$P_{3,31} = 0.6895 \quad Q_{3,31} = -0.0817$	$P_{3,31} = 0.6895 \quad Q_{3,31} = -0.0817$
	9 - 32	$P_{9,32} = 0.0302 \quad Q_{9,32} = -0.2236$	$P_{9,32} = 0.0302 \quad Q_{9,32} = -0.2236$
	18 - 33	$P_{18,33} = 0.0116 \quad Q_{18,33} = 0.0075$	$P_{18,33} = 0.0116 \quad Q_{18,33} = 0.0075$
	25 - 34	$P_{25,34} = 0.0085 \quad Q_{25,34} = 0.0049$	$P_{25,34} = 0.0085 \quad Q_{25,34} = 0.0049$

6.3.1.2 Augmented method

Table 6.17 shows the results obtained by making use of the following procedure:

- Branches 3 - 4, 9 - 11, 18 - 19 and 25 - 26 of the IEEE 30-bus network are used for this modelling technique.
- The above non-zero impedance branches are split to obtain zero impedance branches 3 - 31, 9 - 32, 18 - 33 and 25 - 34 as explained in Figure 6.1.
- The power flow measurements for the zero impedance branches are shown in the third column.
- Measurements for the IEEE 30-bus of Appendix K are used. No errors are added to the measurements.
- The WLS and QT objective functions are used in the estimation process.
- The Augmented method is used to model the zero impedance branches.
- A convergence criterion of 10^{-13} is used for the linear solver.
- The estimated values of the power flows are shown in the fourth column.
- WLS required 5 steps to converge.
- QT required 4 steps to converge.

The estimated power flows are correct estimated as shown in Table 6.17.

Table 6.17: Augmented method

Objective function	Branch	Measured values of P_{ij} and Q_{ij}	Estimated values of P_{ij} and Q_{ij}
WLS	3 - 31	$P_{3,31} = 0.6895$ $Q_{3,31} = -0.0817$	$P_{3,31} = 0.6895$ $Q_{3,31} = -0.0817$
	9 - 32	$P_{9,32} = 0.0302$ $Q_{9,32} = -0.2236$	$P_{9,32} = 0.0302$ $Q_{9,32} = -0.2236$
	18 - 33	$P_{18,33} = 0.0116$ $Q_{18,33} = 0.0075$	$P_{18,33} = 0.0116$ $Q_{18,33} = 0.0075$
	25 - 34	$P_{25,34} = 0.0085$ $Q_{25,34} = 0.0049$	$P_{25,34} = 0.0085$ $Q_{25,34} = 0.0049$
QT	3 - 31	$P_{3,31} = 0.6895$ $Q_{3,31} = -0.0817$	$P_{3,31} = 0.6895$ $Q_{3,31} = -0.0817$
	9 - 32	$P_{9,32} = 0.0302$ $Q_{9,32} = -0.2236$	$P_{9,32} = 0.0302$ $Q_{9,32} = -0.2236$
	18 - 33	$P_{18,33} = 0.0116$ $Q_{18,33} = 0.0075$	$P_{18,33} = 0.0116$ $Q_{18,33} = 0.0075$
	25 - 34	$P_{25,34} = 0.0085$ $Q_{25,34} = 0.0049$	$P_{25,34} = 0.0085$ $Q_{25,34} = 0.0049$

Table 6.18 shows the results obtained when gross errors are present in the estimation process. The same procedure for estimating the results of Table 6.17 is used, except that the measurement errors, equations (6.1), (6.2) and (6.3), are used together with the measurements for the IEEE 30-bus of Appendix K.

- WLS did not converge.
- QT required 7 steps to converge.

The estimated power flows are not correct estimated if the WLS objective function is used in the estimation process with bad data. The estimation process did not converge if the WLS objective function is used.

Table 6.18: Augmented method, bad data

Objective function	Branch	Measured values of P_{ij} and Q_{ij}	Estimated values of P_{ij} and Q_{ij}
WLS	3 - 31	$P_{3,31} = 0.6895 \quad Q_{3,31} = -0.0817$	-
	9 - 32	$P_{9,32} = 0.0302 \quad Q_{9,32} = -0.2236$	-
	18 - 33	$P_{18,33} = 0.0116 \quad Q_{18,33} = 0.0075$	-
	25 - 34	$P_{25,34} = 0.0085 \quad Q_{25,34} = 0.0049$	-
QT	3 - 31	$P_{3,31} = 0.6895 \quad Q_{3,31} = -0.0817$	$P_{3,31} = 0.6895 \quad Q_{3,31} = -0.0817$
	9 - 32	$P_{9,32} = 0.0302 \quad Q_{9,32} = -0.2236$	$P_{9,32} = 0.0302 \quad Q_{9,32} = -0.2236$
	18 - 33	$P_{18,33} = 0.0116 \quad Q_{18,33} = 0.0075$	$P_{18,33} = 0.0116 \quad Q_{18,33} = 0.0075$
	25 - 34	$P_{25,34} = 0.0085 \quad Q_{25,34} = 0.0049$	$P_{25,34} = 0.0085 \quad Q_{25,34} = 0.0049$

6.3.2 Circuit breaker modelling

6.3.2.1 Circuit breakers in series with a zero or low impedance branch

Table 6.19 shows the results obtained by making use of the following procedure:

- Branches 3 - 4 and 9 - 11 of the IEEE 30-bus network are used for this modelling technique.
- The above non-zero impedance branches are split to obtain zero impedance branches 3 - 31 and 9 - 32 as explained in Figure 6.1.
- The correct states of the circuit breakers are shown in the third column.
- Measurements for the IEEE 30-bus of Appendix K are used. No errors are added to the measurements.
- The WLS and QT objective functions are used in the estimation process.
- A convergence criterion of 10^{-13} is used for the linear solver.
- The estimated states of the circuit breakers are shown in the fourth column.
- WLS required 5 steps to converge when the circuit breakers were closed.
- WLS required 5 steps to converge when the circuit breakers were open.
- QT required 5 steps to converge when the circuit breakers were closed.
- QT required 5 steps to converge when the circuit breakers were open.

Table 6.19: Circuit breakers in series with a zero impedance branch

Objective function	Breaker connected between nodes	Status of circuit breaker	Estimated status of circuit breaker
WLS	3 - 31	Closed	Closed
	9 - 32	Closed	Closed
	3 - 31	Open	Open
	9 - 32	Open	Open
QT	3 - 31	Closed	Closed
	9 - 32	Closed	Closed
	3 - 31	Open	Open
	9 - 32	Open	Open

The estimated states of the circuit breakers are correct estimated as shown in Table 6.19.

Table 6.20 shows the results obtained when gross errors are present in the estimation process. The same procedure for estimating the results of Table 6.19 is used, except that the measurement errors, equations (6.1), (6.2) and (6.3), are used together with the measurements for the IEEE 30-bus of Appendix K. Only a closed status is considered for the circuit breakers of branches 3 - 31 and 9 - 32.

- WLS required 8 steps to converge.
- QT required 8 steps to converge.

The estimated states of the circuit breakers are not correct estimated if the WLS objective function is used in the estimation process with bad data.

Table 6.20: Circuit breakers in series with a zero impedance branch, bad data

Objective function	Breaker connected between nodes	Status of circuit breaker	Estimated status of circuit breaker
WLS	3 - 31	Closed	Open
	9 - 32	Closed	Open
QT	3 - 31	Closed	Closed
	9 - 32	Closed	Closed

6.3.2.2 Circuit breakers in series with a non-zero impedance branch

Table 6.21 shows the results obtained by making use of the following procedure:

- Branches 2 - 5 and 6 - 8 of the IEEE 30-bus network are used for this modelling technique.

- The correct states of the circuit breakers are shown in the third column.
- Measurements for the IEEE 30-bus of Appendix K are used. No errors are added to the measurements.
- The WLS and QT objective functions are used in the estimation process.
- A convergence criterion of 10^{-13} is used for the linear solver.
- The estimated states of the circuit breakers are shown in the fifth column.
- WLS required 9 steps to converge when the circuit breakers were closed.
- WLS required 11 steps to converge when the circuit breakers were open.
- QT required 8 steps to converge when the circuit breakers were closed.
- QT required 9 steps to converge when the circuit breakers were open.

The estimated states of the circuit breakers are correct estimated as shown in Table 6.21.

Table 6.21: Circuit breakers in series with a non-zero impedance branch

Objective function	Breaker connected between nodes	Status of circuit breaker	Estimated value of κ:	Estimated status ($\kappa = 0$: open) ($\kappa = 1$: closed)
WLS	2 - 5	Closed	1	Closed
	6 - 8	Closed	1	Closed
	2 - 5	Open	$0.1277e-15$	Open
	6 - 8	Open	$0.4045e-15$	Open
QT	2 - 5	Closed	1	Closed
	6 - 8	Closed	1	Closed
	2 - 5	Open	$0.1388e-18$	Open
	6 - 8	Open	$-0.0267e-18$	Open

Table 6.22 shows the results obtained when gross errors are present in the estimation process. The same procedure for estimating the results of Table 6.21 is used, except that the measurement errors, equations (6.1), (6.2) and (6.3), are used together with the measurements for the IEEE 30-bus of Appendix K.

- WLS required 10 steps to converge when the circuit breakers were closed.
- WLS required 10 steps to converge when the circuit breakers were open.
- QT required 11 steps to converge when the circuit breakers were closed.
- QT required 12 steps to converge when the circuit breakers were open.

The estimated states of the circuit breakers are correct estimated if the WLS and QT objective functions are used in the estimation process with bad data.

Table 6.22: Circuit breakers in series with a non-zero impedance branch, bad data

Objective function	Breaker connected between nodes	Status of circuit breaker	Estimated value of κ:	Estimated status ($\kappa = 0$: open) ($\kappa = 1$: closed)
WLS	2 - 5	Closed	1	Closed
	6 - 8	Closed	1	Closed
	2 - 5	Open	$-0.5406\,e-4$	Open
	6 - 8	Open	$-0.0020\,e-4$	Open
QT	2 - 5	Closed	1	Closed
	6 - 8	Closed	1	Closed
	2 - 5	Open	$-0.2191\,e{-}12$	Open
	6 - 8	Open	$0.0671\,e-12$	Open

6.3.3 Modelling of network parameters

6.3.3.1 The series branch

Table 6.23 shows the results obtained by making use of the following procedure:

- Branches 1 - 2, 9 - 11, 16 - 17 and 23 - 24 of the IEEE 30-bus network are used for this modelling technique.
- The correct impedances of the series parameters are shown in the third column.
- Measurements for the IEEE 30-bus of Appendix K are used. No errors are added to the measurements.
- The WLS and QT objective functions are used in the estimation process.
- A convergence criterion of 10^{-13} is used for the linear solver.
- The estimated impedances of the series parameters are shown in the fourth column.
- WLS required 6 steps to converge.
- QT required 5 steps to converge.

The estimated impedances are correct estimated as shown in Table 6.23.

Table 6.24 shows the results obtained when gross errors are present in the estimation process. The same procedure for estimating the results of Table 6.23 is used, except that the measurement errors, equations (6.1), (6.2) and (6.3), are used together with the measurements for the IEEE 30-bus of Appendix K.

- WLS required 8 steps to converge.

Table 6.23: The unknown series parameter

Objective function	Branch	True values for $Z = R + jX$	Estimated values for $Z = R + jX$
WLS	1 - 2	$R = 0.01920 \quad X = 0.05750$	$R = 0.01920 \quad X = 0.05750$
	9 - 11	$X = 0.20800$	$X = 0.20800$
	16 - 17	$R = 0.08240 \quad X = 0.19230$	$R = 0.08240 \quad X = 0.19230$
	23 - 24	$R = 0.13200 \quad X = 0.27000$	$R = 0.13200 \quad X = 0.27000$
QT	1 - 2	$R = 0.01920 \quad X = 0.05750$	$R = 0.01920 \quad X = 0.05750$
	9 - 11	$X = 0.20800$	$X = 0.20800$
	16 - 17	$R = 0.08240 \quad X = 0.19230$	$R = 0.08240 \quad X = 0.19230$
	23 - 24	$R = 0.13200 \quad X = 0.27000$	$R = 0.13200 \quad X = 0.27000$

- QT required 12 steps to converge.

The estimated values for the series parameters are not correct estimated if the WLS objective function is used in the estimation process with bad data.

Table 6.24: The unknown series parameter, bad data

Objective function	Branch	True values for $Z = R + jX$	Estimated values for $Z = R + jX$
WLS	1 - 2	$R = 0.01920 \quad X = 0.05750$	$R = 0.11988 \quad X = -0.08295$
	9 - 11	$X = 0.20800$	$X = 2.33377$
	16 - 17	$R = 0.08240 \quad X = 0.19230$	$R = -0.30329 \quad X = -0.60339$
	23 - 24	$R = 0.13200 \quad X = 0.27000$	$R = -0.71693 \quad X = -1.40188$
QT	1 - 2	$R = 0.01920 \quad X = 0.05750$	$R = 0.01920 \quad X = 0.05750$
	9 - 11	$X = 0.20800$	$X = 0.20800$
	16 - 17	$R = 0.08240 \quad X = 0.19230$	$R = 0.08240 \quad X = 0.19230$
	23 - 24	$R = 0.13200 \quad X = 0.27000$	$R = 0.13200 \quad X = 0.27000$

6.3.3.2 The shunt branch

Table 6.25 shows the results obtained by making use of the following procedure:

- Branch 1 - 2 of the IEEE 30-bus network is used for this modelling technique.
- The correct admittances of the shunt parameters are shown in the third column.
- Measurements for the IEEE 30-bus of Appendix K are used. No errors are added to the measurements.
- The WLS and QT objective functions are used in the estimation process.
- A convergence criterion of 10^{-13} is used for the linear solver.

- The estimated admittances of the shunt parameters are shown in the fourth column.
- WLS required 5 steps to converge.
- QT required 5 steps to converge.

The estimated admittances are correct estimated as shown in Table 6.25.

Table 6.25: The unknown shunt parameter

Objective function	Branch	True values for $Y = G + jB$	Estimated values for $Y = G + jB$
WLS	1 - 2	$B = 0.02640$	$B = 0.02640$
QT	1 - 2	$B = 0.02640$	$B = 0.02640$

Table 6.26 shows the results obtained when gross errors are present in the estimation process. The same procedure for estimating the results of Table 6.25 is used, except that the measurement errors, equations (6.1), (6.2) and (6.3), are used together with the measurements for the IEEE 30-bus of Appendix K.

- WLS required 9 steps to converge.
- QT required 10 steps to converge.

The estimated values for the shunt parameters are not correct estimated if the WLS objective function is used in the estimation process with bad data.

Table 6.26: The unknown shunt parameter, bad data

Objective function	Branch	True values for $Y = G + jB$	Estimated values for $Y = G + jB$
WLS	1 - 2	$B = 0.02640$	$B = 0.06668$
QT	1 - 2	$B = 0.02640$	$B = 0.02640$

6.3.4 Modelling of ULTC transformer parameters

This modelling technique is used to estimate the unknown admittance of an ULTC transformer when the tap position is known. Assume $G = 0$ in $Y = G + jB$.

Table 6.27 shows the results obtained by making use of the following procedure:

- Branches 4 - 12 and 6 - 9 of the IEEE 30-bus network are used for this modelling technique.
- The correct admittances of the transformers are shown in the third column.

- Measurements for the IEEE 30-bus of Appendix K are used. No errors are added to the measurements.
- The WLS and QT objective functions are used in the estimation process.
- A convergence criterion of 10^{-13} is used for the linear solver.
- The estimated admittances of the transformers are shown in the fourth column.
- WLS required 5 steps to converge.
- QT required 5 steps to converge.

The estimated admittances are correct estimated as shown in Table 6.27.

Table 6.27: The unknown transformer parameter

Objective function	Branch	True values for $Y_T = G + jB$	Estimated values for $Y_T = G + jB$
WLS	4 - 12	$B = -4.19126$	$B = -4.19126$
	6 - 9	$B = -4.91584$	$B = -4.91584$
QT	4 - 12	$B = -4.19126$	$B = -4.19126$
	6 - 9	$B = -4.91584$	$B = -4.91584$

Table 6.28 shows the results obtained when gross errors are present in the estimation process. The same procedure for estimating the results of Table 6.27 is used, except that the measurement errors, equations (6.1), (6.2) and (6.3), are used together with the measurements for the IEEE 30-bus of Appendix K.

- WLS required 9 steps to converge.
- QT required 8 steps to converge.

The estimated values for the transformer parameters are not correct estimated if the WLS objective function is used in the estimation process with bad data.

Table 6.28: The unknown transformer parameter, bad data

Objective function	Branch	True values for $Y_T = G + jB$	Estimated values for $Y_T = G + jB$
WLS	4 - 12	$B = -4.19126$	$B = 1.48022$
	6 - 9	$B = -4.91584$	$B = 1.19043$
QT	4 - 12	$B = -4.19126$	$B = -4.19126$
	6 - 9	$B = -4.91584$	$B = -4.91584$

6.4 IEEE 57-bus Network

6.4.1 Modelling of zero impedance branches

6.4.1.1 Non-Augmented method

Table 6.29 shows the results obtained by making use of the following procedure:

- Branches 13 - 14, 21 - 22, 34 - 35 and 57 - 56 of the IEEE 57-bus network are used for this modelling technique.
- The above non-zero impedance branches are split to obtain zero impedance branches 13 - 58, 21 - 59, 34 - 60 and 57 - 61 as explained in Figure 6.1.
- The power flow measurements for the zero impedance branches are shown in the third column.
- Measurements for the IEEE 57-bus of Appendix K are used. No errors are added to the measurements.
- The WLS and QT objective functions are used in the estimation process.
- The Non-Augmented method is used to model the zero impedance branches.
- A convergence criterion of 10^{-13} is used for the linear solver.
- The estimated values of the power flows are shown in the fourth column.
- WLS required 5 steps to converge.
- QT required 5 steps to converge.

The estimated power flows are correct estimated as shown in Table 6.29.

Table 6.29: Non-Augmented method

Objective function	Branch	Measured values of P_{ij} and Q_{ij}	Estimated values of P_{ij} and Q_{ij}
WLS	13 - 58	$P_{13,58} = -0.1180 \quad Q_{13,58} = 0.1873$	$P_{13,58} = -0.1180 \quad Q_{13,58} = 0.1873$
	21 - 59	$P_{21,59} = -0.0020 \quad Q_{21,59} = 0.0069$	$P_{21,59} = -0.0020 \quad Q_{21,59} = 0.0069$
	34 - 60	$P_{34,60} = -0.0570 \quad Q_{34,60} = -0.0379$	$P_{34,60} = -0.0570 \quad Q_{34,60} = -0.0379$
	57 - 61	$P_{57,61} = -0.0150 \quad Q_{57,61} = -0.0077$	$P_{57,61} = -0.0150 \quad Q_{57,61} = -0.0077$
QT	13 - 58	$P_{13,58} = -0.1180 \quad Q_{13,58} = 0.1873$	$P_{13,58} = -0.1180 \quad Q_{13,58} = 0.1873$
	21 - 59	$P_{21,59} = -0.0020 \quad Q_{21,59} = 0.0069$	$P_{21,59} = -0.0020 \quad Q_{21,59} = 0.0069$
	34 - 60	$P_{34,60} = -0.0570 \quad Q_{34,60} = -0.0379$	$P_{34,60} = -0.0570 \quad Q_{34,60} = -0.0379$
	57 - 61	$P_{57,61} = -0.0150 \quad Q_{57,61} = -0.0077$	$P_{57,61} = -0.0150 \quad Q_{57,61} = -0.0077$

Table 6.30 shows the results obtained when gross errors are present in the estimation process. The same procedure for estimating the results of Table 6.29 is used, except that the measurement errors, equations (6.1), (6.2) and (6.3), are used together with the measurements for the IEEE 57-bus of Appendix K.

- WLS required 8 steps to converge.
- QT required 10 steps to converge.

The estimated power flows are not correct estimated if the WLS objective function is used in the estimation process with bad data.

Table 6.30: Non-Augmented method, bad data

Objective function	Branch	Measured values of P_{ij} and Q_{ij}	Estimated values of P_{ij} and Q_{ij}
WLS	13 - 58	$P_{13,58} = -0.1180 \quad Q_{13,58} = 0.1873$	$P_{13,58} = -0.0968 \quad Q_{13,58} = 0.0046$
	21 - 59	$P_{21,59} = -0.0020 \quad Q_{21,59} = 0.0069$	$P_{21,59} = -0.0159 \quad Q_{21,59} = -0.0379$
	34 - 60	$P_{34,60} = -0.0570 \quad Q_{34,60} = -0.0379$	$P_{34,60} = -0.0605 \quad Q_{34,60} = -0.0574$
	57 - 61	$P_{57,61} = -0.0150 \quad Q_{57,61} = -0.0077$	$P_{57,61} = -0.0168 \quad Q_{57,61} = -0.0090$
QT	13 - 58	$P_{13,58} = -0.1180 \quad Q_{13,58} = 0.1873$	$P_{13,58} = -0.1180 \quad Q_{13,58} = 0.1873$
	21 - 59	$P_{21,59} = -0.0020 \quad Q_{21,59} = 0.0069$	$P_{21,59} = -0.0020 \quad Q_{21,59} = 0.0069$
	34 - 60	$P_{34,60} = -0.0570 \quad Q_{34,60} = -0.0379$	$P_{34,60} = -0.0570 \quad Q_{34,60} = -0.0379$
	57 - 61	$P_{57,61} = -0.0150 \quad Q_{57,61} = -0.0077$	$P_{57,61} = -0.0150 \quad Q_{57,61} = -0.0077$

6.4.1.2 Augmented method

Table 6.31 shows the results obtained by making use of the following procedure:

- Branches 13 - 14, 21 - 22, 34 - 35 and 57 - 56 of the IEEE 57-bus network are used for this modelling technique.
- The above non-zero impedance branches are split to obtain zero impedance branches 13 - 58, 21 - 59, 34 - 60 and 57 - 61 as explained in Figure 6.1.
- The power flow measurements for the zero impedance branches are shown in the third column.
- Measurements for the IEEE 57-bus of Appendix K are used. No errors are added to the measurements.
- The WLS and QT objective functions are used in the estimation process.
- The Augmented method is used to model the zero impedance branches.
- A convergence criterion of 10^{-13} is used for the linear solver.

- The estimated values of the power flows are shown in the fourth column.
- WLS required 5 steps to converge.
- QT required 5 steps to converge.

The estimated power flows are correct estimated as shown in Table 6.31.

Table 6.31: Augmented method

Objective function	Branch	Measured values of P_{ij} and Q_{ij}	Estimated values of P_{ij} and Q_{ij}
WLS	13 - 58	$P_{13,58} = -0.1180 \quad Q_{13,58} = 0.1873$	$P_{13,58} = -0.1180 \quad Q_{13,58} = 0.1873$
	21 - 59	$P_{21,59} = -0.0020 \quad Q_{21,59} = 0.0069$	$P_{21,59} = -0.0020 \quad Q_{21,59} = 0.0069$
	34 - 60	$P_{34,60} = -0.0570 \quad Q_{34,60} = -0.0379$	$P_{34,60} = -0.0570 \quad Q_{34,60} = -0.0379$
	57 - 61	$P_{57,61} = -0.0150 \quad Q_{57,61} = -0.0077$	$P_{57,61} = -0.0150 \quad Q_{57,61} = -0.0077$
QT	13 - 58	$P_{13,58} = -0.1180 \quad Q_{13,58} = 0.1873$	$P_{13,58} = -0.1180 \quad Q_{13,58} = 0.1873$
	21 - 59	$P_{21,59} = -0.0020 \quad Q_{21,59} = 0.0069$	$P_{21,59} = -0.0020 \quad Q_{21,59} = 0.0069$
	34 - 60	$P_{34,60} = -0.0570 \quad Q_{34,60} = -0.0379$	$P_{34,60} = -0.0570 \quad Q_{34,60} = -0.0379$
	57 - 61	$P_{57,61} = -0.0150 \quad Q_{57,61} = -0.0077$	$P_{57,61} = -0.0150 \quad Q_{57,61} = -0.0077$

Table 6.32 shows the results obtained when gross errors are present in the estimation process. The same procedure for estimating the results of Table 6.31 is used, except that the measurement errors, equations (6.1), (6.2) and (6.3), are used together with the measurements for the IEEE 57-bus of Appendix K.

- WLS did not converge.
- QT required 11 steps to converge.

The estimated power flows are not correct estimated if the WLS objective function is used in the estimation process with bad data. The estimation process did not converge if the WLS objective function is used.

6.4.2 Circuit breaker modelling

6.4.2.1 Circuit breakers in series with a zero or low impedance branch

Table 6.33 shows the results obtained by making use of the following procedure:

- Branches 13 - 14 and 21 - 22 of the IEEE 57-bus network are used for this modelling technique.
- The above non-zero impedance branches are split to obtain zero impedance branches 13 - 58 and 21 - 59 as explained in Figure 6.1.

Table 6.32: Augmented method, bad data

Objective function	Branch	Measured values of P_{ij} and Q_{ij}	Estimated values of P_{ij} and Q_{ij}
WLS	13 - 58	$P_{13,58} = -0.1180 \quad Q_{13,58} = 0.1873$	-
	21 - 59	$P_{21,59} = -0.0020 \quad Q_{21,59} = 0.0069$	-
	34 - 60	$P_{34,60} = -0.0570 \quad Q_{34,60} = -0.0379$	-
	57 - 61	$P_{57,61} = -0.0150 \quad Q_{57,61} = -0.0077$	-
QT	13 - 58	$P_{13,58} = -0.1180 \quad Q_{13,58} = 0.1873$	$P_{13,58} = -0.1180 \quad Q_{13,58} = 0.1873$
	21 - 59	$P_{21,59} = -0.0020 \quad Q_{21,59} = 0.0069$	$P_{21,59} = -0.0020 \quad Q_{21,59} = 0.0069$
	34 - 60	$P_{34,60} = -0.0570 \quad Q_{34,60} = -0.0379$	$P_{34,60} = -0.0570 \quad Q_{34,60} = -0.0379$
	57 - 61	$P_{57,61} = -0.0150 \quad Q_{57,61} = -0.0077$	$P_{57,61} = -0.0150 \quad Q_{57,61} = -0.0077$

- The correct states of the circuit breakers are shown in the third column.
- Measurements for the IEEE 57-bus of Appendix K are used. No errors are added to the measurements.
- The WLS and QT objective functions are used in the estimation process.
- A convergence criterion of 10^{-13} is used for the linear solver.
- The estimated states of the circuit breakers are shown in the fourth column.
- WLS required 4 steps to converge when the circuit breakers were closed.
- WLS required 4 steps to converge when the circuit breakers were open.
- QT required 4 steps to converge when the circuit breakers were closed.
- QT required 4 steps to converge when the circuit breakers were open.

The estimated states of the circuit breakers are correct estimated as shown in Table 6.33.

Table 6.33: Circuit breakers in series with a zero impedance branch

Objective function	Breaker connected between nodes	Status of circuit breaker	Estimated status of circuit breaker
WLS	13 - 58	Closed	Closed
	21 - 59	Closed	Closed
	13 - 58	Open	Open
	21 - 59	Open	Open
QT	13 - 58	Closed	Closed
	21 - 59	Closed	Closed
	13 - 58	Open	Open
	21 - 59	Open	Open

Table 6.34 shows the results obtained when gross errors are present in the estimation process. The same procedure for estimating the results of Table 6.33 is used, except that the measurement errors, equations (6.1), (6.2) and (6.3), are used together with the measurements for the IEEE 57-bus of Appendix K. Only a closed status is considered for the circuit breakers of branches 13 - 58 and 21 - 59.

- WLS required 7 steps to converge.
- QT required 9 steps to converge.

The estimated states of the circuit breakers are not correct estimated if the WLS objective function is used in the estimation process with bad data.

Table 6.34: Circuit breakers in series with a zero impedance branch, bad data

Objective function	Breaker connected between nodes	Status of circuit breaker	Estimated status of circuit breaker
WLS	13 - 58	Closed	Open
	21 - 59	Closed	Open
QT	13 - 58	Closed	Closed
	21 - 59	Closed	Closed

6.4.2.2 Circuit breakers in series with a non-zero impedance branch

Table 6.35 shows the results obtained by making use of the following procedure:

- Branches 9 - 12 and 1 - 17 of the IEEE 57-bus network are used for this modelling technique.
- The correct states of the circuit breakers are shown in the third column.
- Measurements for the IEEE 57-bus of Appendix K are used. No errors are added to the measurements.
- The WLS and QT objective functions are used in the estimation process.
- A convergence criterion of 10^{-13} is used for the linear solver.
- The estimated states of the circuit breakers are shown in the fifth column.
- WLS required 10 steps to converge when the circuit breakers were closed.
- WLS required 11 steps to converge when the circuit breakers were open.
- QT required 9 steps to converge when the circuit breakers were closed.

Table 6.35: Circuit breakers in series with a non-zero impedance branch

Objective function	Breaker connected between nodes	Status of circuit breaker	Estimated value of κ:	Estimated status ($\kappa = 0$: open) ($\kappa = 1$: closed)
WLS	9 - 12	Closed	1	Closed
	1 - 17	Closed	1	Closed
	9 - 12	Open	0	Open
	1 - 17	Open	0	Open
QT	9 - 12	Closed	1	Closed
	1 - 17	Closed	1	Closed
	9 - 12	Open	0	Open
	1 - 17	Open	0	Open

- QT required 10 steps to converge when the circuit breakers were open.

The estimated states of the circuit breakers are correct estimated as shown in Table 6.35.

Table 6.36 shows the results obtained when gross errors are present in the estimation process. The same procedure for estimating the results of Table 6.35 is used, except that the measurement errors, equations (6.1), (6.2) and (6.3), are used together with the measurements for the IEEE 57-bus of Appendix K.

- WLS required 11 steps to converge when the circuit breakers were closed.
- WLS required 11 steps to converge when the circuit breakers were open.
- QT required 9 steps to converge when the circuit breakers were closed.
- QT required 12 steps to converge when the circuit breakers were open.

Not all the estimated states of the circuit breakers are correct estimated if the WLS objective function is used in the estimation process with bad data.

6.4.3 Modelling of network parameters

6.4.3.1 The series branch

Table 6.37 shows the results obtained by making use of the following procedure:

- Branches 1 - 2, 10 - 12, 30 - 31 and 56 - 41 of the IEEE 57-bus network are used for this modelling technique.
- The correct impedances of the series parameters are shown in the third column.

Table 6.36: Circuit breakers in series with a non-zero impedance branch, bad data

Objective function	Breaker connected between nodes	Status of circuit breaker	Estimated value of κ:	Estimated status ($\kappa = 0$: open) ($\kappa = 1$: closed)
WLS	9 - 12	Closed	0	Open
	1 - 17	Closed	0	Open
	9 - 12	Open	0	Open
	1 - 17	Open	0	Open
QT	9 - 12	Closed	1	Closed
	1 - 17	Closed	1	Closed
	9 - 12	Open	0	Open
	1 - 17	Open	0	Open

- Measurements for the IEEE 57-bus of Appendix K are used. No errors are added to the measurements.
- The WLS and QT objective functions are used in the estimation process.
- A convergence criterion of 10^{-13} is used for the linear solver.
- The estimated impedances of the series parameters are shown in the fourth column.
- WLS required 5 steps to converge.
- QT required 5 steps to converge.

The estimated impedances are correct estimated as shown in Table 6.37.

Table 6.37: The unknown series parameter

Objective function	Branch	True values for $Z = R + jX$	Estimated values for $Z = R + jX$
WLS	1 - 2	$R = 0.00830 \quad X = 0.02800$	$R = 0.00830 \quad X = 0.02800$
	10 - 12	$R = 0.02770 \quad X = 0.12620$	$R = 0.02770 \quad X = 0.12620$
	30 - 31	$R = 0.32600 \quad X = 0.49700$	$R = 0.32600 \quad X = 0.49700$
	56 - 41	$R = 0.55300 \quad X = 0.54900$	$R = 0.55300 \quad X = 0.54900$
QT	1 - 2	$R = 0.00830 \quad X = 0.02800$	$R = 0.00830 \quad X = 0.02800$
	10 - 12	$R = 0.02770 \quad X = 0.12620$	$R = 0.02770 \quad X = 0.12620$
	30 - 31	$R = 0.32600 \quad X = 0.49700$	$R = 0.32600 \quad X = 0.49700$
	56 - 41	$R = 0.55300 \quad X = 0.54900$	$R = 0.55300 \quad X = 0.54900$

Table 6.38 shows the results obtained when gross errors are present in the estimation process. The same procedure for estimating the results of Table 6.37 is used, except that the measurement errors, equations (6.1), (6.2) and (6.3), are used together with the measurements for the IEEE 57-bus of Appendix K.

- WLS required 7 steps to converge.
- QT required 12 steps to converge.

The estimated values for the series parameters are not correct estimated if the WLS objective function is used in the estimation process with bad data.

Table 6.38: The unknown series parameter, bad data

Objective function	Branch	True values for $Z = R + jX$	Estimated values for $Z = R + jX$
WLS	1 - 2	$R = 0.00830 \quad X = 0.02800$	$R = 0.02439 \quad X = -0.02739$
	10 - 12	$R = 0.02770 \quad X = 0.12620$	$R = 0.21637 \quad X = 0.22309$
	30 - 31	$R = 0.32600 \quad X = 0.49700$	$R = 0.29453 \quad X = 0.49756$
	56 - 41	$R = 0.55300 \quad X = 0.54900$	$R = 1.02721 \quad X = 0.62424$
QT	1 - 2	$R = 0.00830 \quad X = 0.02800$	$R = 0.00830 \quad X = 0.02800$
	10 - 12	$R = 0.02770 \quad X = 0.12620$	$R = 0.02770 \quad X = 0.12620$
	30 - 31	$R = 0.32600 \quad X = 0.49700$	$R = 0.32600 \quad X = 0.49700$
	56 - 41	$R = 0.55300 \quad X = 0.54900$	$R = 0.55300 \quad X = 0.54900$

6.4.3.2 The shunt branch

Table 6.39 shows the results obtained by making use of the following procedure:

- Branches 1 - 2, 10 - 12 and 38 - 44 of the IEEE 57-bus network are used for this modelling technique.
- The correct admittances of the shunt parameters are shown in the third column.
- Measurements for the IEEE 57-bus of Appendix K are used. No errors are added to the measurements.
- The WLS and QT objective functions are used in the estimation process.
- A convergence criterion of 10^{-13} is used for the linear solver.
- The estimated admittances of the shunt parameters are shown in the fourth column.
- WLS required 4 steps to converge.
- QT required 4 steps to converge.

The estimated admittances are correct estimated as shown in Table 6.39.

Table 6.40 shows the results obtained when gross errors are present in the estimation process. The same procedure for estimating the results of Table 6.39 is used, except that the measurement errors, equations (6.1), (6.2) and (6.3), are used together with the measurements for the IEEE 57-bus of Appendix K.

Table 6.39: The unknown shunt parameter

Objective function	Branch	True values for $Y = G + jB$	Estimated values for $Y = G + jB$
WLS	1 - 2	$B = 0.06450$	$B = 0.06450$
	10 - 12	$B = 0.01640$	$B = 0.01640$
	38 - 44	$B = 0.00100$	$B = 0.00100$
QT	1 - 2	$B = 0.06450$	$B = 0.06450$
	10 - 12	$B = 0.01640$	$B = 0.01640$
	38 - 44	$B = 0.00100$	$B = 0.00100$

- WLS required 7 steps to converge.
- QT required 9 steps to converge.

The estimated values for the shunt parameters are not correct estimated if the WLS objective function is used in the estimation process with bad data.

Table 6.40: The unknown shunt parameter, bad data

Objective function	Branch	True values for $Y = G + jB$	Estimated values for $Y = G + jB$
WLS	1 - 2	$B = 0.06450$	$B = 0.19447$
	10 - 12	$B = 0.01640$	$B = 0.05834$
	38 - 44	$B = 0.00100$	$B = 0.02196$
QT	1 - 2	$B = 0.06450$	$B = 0.06450$
	10 - 12	$B = 0.01640$	$B = 0.01640$
	38 - 44	$B = 0.00100$	$B = 0.00100$

6.4.4 Modelling of ULTC transformer parameters

This modelling technique is used to estimate the unknown admittance of an ULTC transformer when the tap position is known. Assume $G = 0$ in $Y = G + jB$.

Table 6.41 shows the results obtained by making use of the following procedure:

- Branches 7 - 29 and 14 - 46 of the IEEE 57-bus network are used for this modelling technique.
- The correct admittances of the transformers are shown in the third column.
- Measurements for the IEEE 57-bus of Appendix K are used. No errors are added to the measurements.
- The WLS and QT objective functions are used in the estimation process.

- A convergence criterion of 10^{-13} is used for the linear solver.
- The estimated admittances of the transformers are shown in the fourth column.
- WLS required 5 steps to converge.
- QT required 5 steps to converge.

The estimated admittances are correct estimated as shown in Table 6.41.

Table 6.41: The unknown transformer parameter

Objective function	Branch	True values for $Y_T = G + jB$	Estimated values for $Y_T = G + jB$
WLS	7 - 29	$B = -15.95874$	$B = -15.95874$
	14 - 46	$B = -15.11716$	$B = -15.11716$
QT	7 - 29	$B = -15.95874$	$B = -15.95874$
	14 - 46	$B = -15.11716$	$B = -15.11716$

Table 6.42 shows the results obtained when gross errors are present in the estimation process. The same procedure for estimating the results of Table 6.41 is used, except that the measurement errors, equations (6.1), (6.2) and (6.3), are used together with the measurements for the IEEE 57-bus of Appendix K.

- WLS required 7 steps to converge.
- QT required 11 steps to converge.

The estimated values for the transformer parameters are not correct estimated if the WLS objective function is used in the estimation process with bad data.

Table 6.42: The unknown transformer parameter, bad data

Objective function	Branch	True values for $Y_T = G + jB$	Estimated values for $Y_T = G + jB$
WLS	7 - 29	$B = -15.95874$	$B = -11.33252$
	14 - 46	$B = -15.11716$	$B = 0.49608$
QT	7 - 29	$B = -15.95874$	$B = -15.95874$
	14 - 46	$B = -15.11716$	$B = -15.11716$

6.5 Articles Published In Conference Proceedings

The following articles on state estimation were presented and taken up in conference proceedings during the research period:

- S.L. Els and R. Zivanovic, "Advance Network Modelling for State Estimation", SAUPEC 99, Proceedings of the 8th Southern African Universities Power Engineering Conference 1999, January 19-20, 1999, pp. 52-55.
- S. L. Els, A. D. Els, J. A. Jordaan and R. Zivanovic, "Projection Statistics for Power System State Estimation", IEEE Region 8, 5th Africon Conference, Vol.2, September 1999, pp. 783-786.
- S. L. Els and R. Zivanovic, "Advance Network Modelling Techniques for Power System State Estimation", SAUPEC 2000, Proceedings of the 9th Southern African Universities Power Engineering Conference 2000, January 27-28, 2000, pp. 20-23.
- S.L. Els and R. Zivanovic, "Robust Generalised State Estimation", SAUPEC 2001, Proceedings of the 10th Southern African Universities Power Engineering Conference 2001, January 2001.

6.6 Conclusion

Exhaustive testing is done on advanced network modelling techniques used in a state estimator. The Weighted Least Squares objective function and the Quadratic-Tangent objective function are used. Results for several test cases are shown. The Gauss-Newton iterative method is used in each estimation process. IEEE 14-bus, IEEE 30-bus and IEEE 57-bus networks are used as data for the test cases. Measurements are generated through solving a load flow problem for each IEEE network.

A comparison is made between the estimates obtained for using advanced network modelling techniques in an estimation process which minimised the WLS criterion and advanced network modelling techniques in an estimation process which minimised the QT criterion. It is shown that a generalised state estimator using a WLS objective function, could estimate all the unknown state variables correct. The same is true for the generalised state estimator using a QT objective function. All the unknown state variables are also correct estimated.

Exhaustive testing is also done on advanced network modelling techniques used in a state estimation process in the presence of bad data. A comparison is made between a generalised state estimation process using a WLS objective function and a generalised state estimation process using a QT objective function. It is shown that a generalised state estimator using a WLS objective function, could not estimate all the unknown state variables correct in the presence of bad data. The opposite is true for the generalised state estimator using a QT objective function. All the unknown state

variables are correct estimated in the presence of bad data. An explanation for these results obtained can be:

Weighted Least Squares criterion uses a quadratic type of objective function as shown in equation (3.24) of section 3.5. As mentioned before, residuals are defined as the difference between a measurement from the network and the computed value of its corresponding measurement equation. Residuals are squared in the WLS objective function. A residual of a bad measurement, will have a negative influence on the objective function. This negative influence, will also be "squared" and its effect on the objective function will be even worse.

Quadratic-Tangent criterion uses an objective function which is both of a quadratic type and of a linear type. The QT objective function is shown in equation (E.8) of Appendix E. Residuals are squared if the absolute value of a residual is less or equal than β. Residuals are not squared if the absolute value of a residual is greater than β. A residual of a bad measurement will have a bigger absolute value. If this absolute value of the residual is greater than β, it will not be squared. The opposite is true for WLS. All the residuals are squared in the WLS objective function. It does not matter if residuals of bad measurements are used or not.

Chapter 7

Conclusion

Network impedances can be incorrect or even unknown when a state estimation process is carried out. The topology of the power network can also be incorrect or even unknown. A generalised state estimator is used to overcome these problems. A generalised state estimator uses advanced network modelling techniques. Several types of advanced network modelling techniques are given and completely discussed. The following advanced network modelling techniques are introduced:

- Modelling of circuit breakers
- Modelling of zero impedance branches
- Modelling of network parameters
- Modelling of ULTC transformer parameters

Exhaustive testing is done on a state estimator which uses these advanced network modelling techniques. Different criteria are used in the estimation process. The Weighted Least Squares criterion and the Quadratic-Tangent criterion are used. A comparison is made between the estimates obtained for the state variables of these two generalised state estimation processes. Exhaustive testing is also done on a state estimator which uses the different types of advanced network modelling techniques in the presence of bad data.

The correct estimates for the unknown state variables are obtained, even in the presence of bad data, if the Quadratic-Tangent objective function is used. It is shown that the advanced network modelling techniques can be used with a statistically more robust criterion, to obtain better estimates for network parameters and topology.

The goal of this dissertation was to investigate the use of advanced network modelling techniques in estimation processes with different criteria. The Quadratic-Tangent criterion and Weighted Least Squares criterion are introduced and used in the estimation processes. The goal was not to investigate the short comings of the WLS and QT criteria, but rather to investigate the use of

advanced network modelling techniques. The goal of this dissertation was not to obtain the most robust criterion.

The current solution tool does not use sparsity techniques. To be able to solve larger systems in the estimation processes, sparsity techniques need to be incorporated. The use of sparsity techniques in the estimation process will speed up the solution process and thus will it be possible to solve larger systems.

Contribution of this dissertation:

- A generalised state estimator is programmed and tested on IEEE 14-bus, IEEE 30-bus and IEEE 57-bus networks.
- Advanced network modelling techniques are used in a state estimation process which minimised the Quadratic-Tangent objective function. Accurate estimates for network parameters and topology are obtained. Only the Weighted Least Squares criterion was used in the past in estimation processes using advanced network modelling techniques.

Proposal for future work:

- Use advanced network modelling techniques in a state estimation process which minimises the Quadratic-Squares-Root objective function. This estimation process must be done in the presence of bad data.

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Appendix A

Power Flow Equations for Transmission Lines

A transmission line is modelled as a π - circuit as shown in Figure A.1. The complex current flow

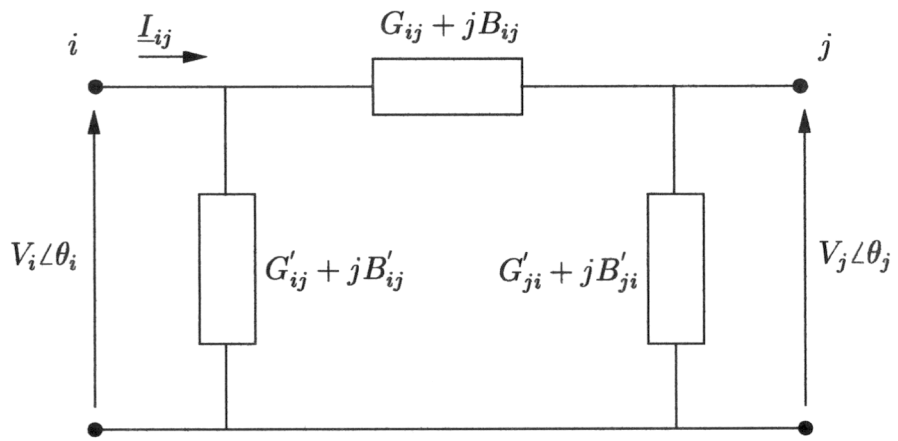


Figure A.1: Transmission line model

$\underline{I}_{ij}$ from node i towards node j in Figure A.1 is given by

$$\begin{aligned} \underline{I}_{ij} &= Y_{ii}(V_i\angle\theta_i) + Y_{ij}(V_i\angle\theta_i - V_j\angle\theta_j) \qquad \text{(A.1)} \\ &= (G'_{ij} + jB'_{ij})(V_i\cos\theta_i + jV_i\sin\theta_i) + (G_{ij} + jB_{ij})(V_i\cos\theta_i + jV_i\sin\theta_i - V_j\cos\theta_j - jV_j\sin\theta_j)\,, \end{aligned}$$

where

V_i , θ_i - voltage magnitude and phase angle at node i,

V_j , θ_j - voltage magnitude and phase angle at node j,

$G_{ij} + jB_{ij}$ - series admittance of the line,

$G'_{ij} + jB'_{ij}$ - parallel (shunt) line admittance.

It is now possible to calculate the complex power flow from node i towards node j by using equation (A.1) as follows:

$$\begin{aligned} P_{ij} - jQ_{ij} &= \underline{V}_i^* \underline{I}_{ij} \\ &= (G'_{ij} + jB'_{ij})\underline{V}_i^* \underline{V}_i + (G_{ij} + jB_{ij})\underline{V}_i^*(\underline{V}_i - \underline{V}_j) \\ &= \left[(G'_{ij} + G_{ij}) + j(B'_{ij} + B_{ij})\right] V_i^2 - (G_{ij} + jB_{ij})(V_i V_j \cos\theta_{ij} - jV_i V_j \sin\theta_{ij}) , \end{aligned} \tag{A.2}$$

where

$$\theta_{ij} = \theta_i - \theta_j . \tag{A.3}$$

Active power flow for a transmission line from node i to node j is:

$$P_{ij} = (G'_{ij} + G_{ij})V_i^2 - V_i V_j (G_{ij} \cos\theta_{ij} + B_{ij} \sin\theta_{ij}) . \tag{A.4}$$

Reactive power flow for a transmission line from node i to node j is:

$$Q_{ij} = -(B'_{ij} + B_{ij})V_i^2 - V_i V_j (G_{ij} \sin\theta_{ij} - B_{ij} \cos\theta_{ij}) . \tag{A.5}$$

Appendix B

Power Flow Equations for Transformers

The per-unit equivalent circuit of Figure B.1 can be used to represent a transformer with a fixed (or off-load) tap on one side and an under-load tap changer (ULTC) on the other side. The off-nominal

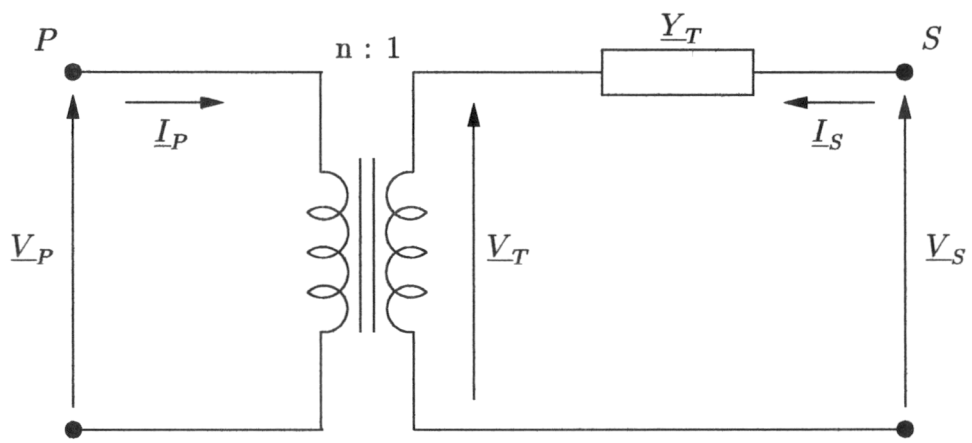


Figure B.1: Per-unit equivalent circuit

turns ratio is assigned to the side with the ULTC and $\underline{Z}_T$ has a value corresponding to the fixed-tap position of the other side. From Figure B.1, the terminal current at node P is:

$$\begin{aligned} \underline{I}_P &= \frac{\underline{Y}_T(\underline{V}_T - \underline{V}_S)}{n} \\ &= \frac{\underline{Y}_T}{n}\left(\frac{\underline{V}_P}{n} - \underline{V}_S\right) \\ &= \frac{\underline{Y}_T}{n^2}(\underline{V}_P - n\underline{V}_S)\,. \end{aligned} \tag{B.1}$$

Similarly, the terminal current at node S is:

$$\underline{I}_S = (n\underline{V}_S - \underline{V}_P)\frac{\underline{Y}_T}{n}\,. \tag{B.2}$$

It is not suitable to use the representation of Figure B.1 in the state estimation problem. Figure B.1 is transformed to the π - equivalent network as shown in Figure B.2. The corresponding terminal

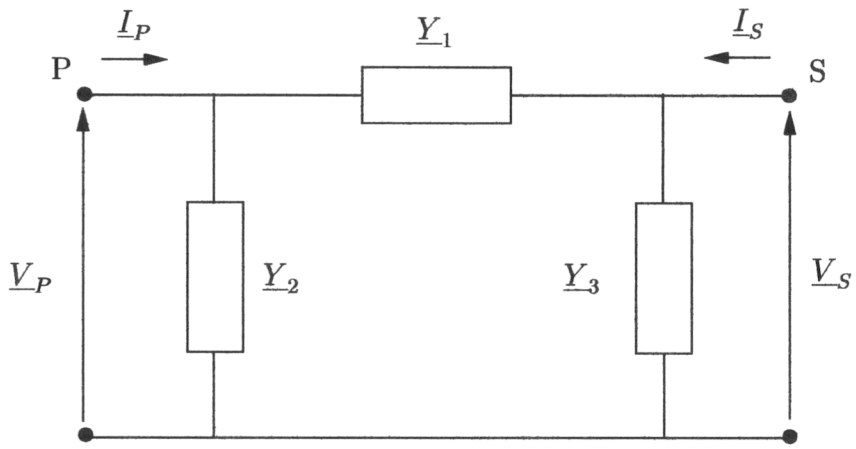


Figure B.2: Transformer pi - equivalent network

currents for the π - network as shown in Figure B.2, are:

$$\underline{I}_P = \underline{Y}_1 (\underline{V}_P - \underline{V}_S) + \underline{Y}_2 \underline{V}_P , \tag{B.3}$$

and

$$\underline{I}_S = \underline{Y}_1 (\underline{V}_S - \underline{V}_P) + \underline{Y}_3 \underline{V}_S , \tag{B.4}$$

where

$$\underline{Y}_1 = \frac{\underline{Y}_T}{n} = c\underline{Y}_T , \tag{B.5}$$

and

$$\begin{aligned} \underline{Y}_2 &= \left(\frac{1}{n^2} - \frac{1}{n}\right) \underline{Y}_T \\ &= (c^2 - c)\, \underline{Y}_T , \end{aligned} \tag{B.6}$$

and

$$\underline{Y}_3 = (1 - c)\, \underline{Y}_T . \tag{B.7}$$

The complex power flow from node P towards node S can be derived and is as follows:

$$\begin{aligned} P_{PS} - jQ_{PS} &= \underline{V}_P^* \underline{I}_P \\ &= \underline{V}_P^* \underline{Y}_1 (\underline{V}_P - \underline{V}_S) + \underline{V}_P^* \underline{Y}_2 \underline{V}_P \\ &= (\underline{Y}_1 + \underline{Y}_2) \underline{V}_P^2 - \underline{Y}_1 \underline{V}_P^* \underline{V}_S \\ &= c^2 (G_T + jB_T) V_P^2 - c(G_T + jB_T) V_P V_S (\cos\theta_{PS} - j \sin\theta_{PS}) \ . \end{aligned} \tag{B.8}$$

The active power flow for the transformer from node P towards node S is:

$$P_{PS} = c^2 G_T V_P^2 - c V_P V_S (G_T \cos\theta_{PS} + B_T \sin\theta_{PS}) \ . \tag{B.9}$$

The reactive power flow for the transformer from node P towards node S is:

$$Q_{PS} = -c^2 B_T V_P^2 - c V_P V_S (G_T \sin\theta_{PS} - B_T \cos\theta_{PS}) \ . \tag{B.10}$$

The complex power flow from node S towards node P can also be derived and is as follows:

$$\begin{aligned} P_{SP} - jQ_{SP} &= \underline{V}_S^* \underline{I}_S \\ &= \underline{V}_S^* \underline{Y}_1 (\underline{V}_S - \underline{V}_P) + \underline{V}_S^* \underline{Y}_3 \underline{V}_S \\ &= (\underline{Y}_1 + \underline{Y}_3) \underline{V}_S^2 - \underline{Y}_1 \underline{V}_S^* \underline{V}_P \\ &= (G_T + jB_T) V_S^2 - c(G_T + jB_T) V_S V_P (\cos\theta_{SP} - j \sin\theta_{SP}) \ . \end{aligned} \tag{B.11}$$

The active power flow from equation (B.11) for the transformer from node S towards node P is:

$$P_{SP} = G_T V_S^2 - c V_S V_P (G_T \cos\theta_{SP} + B_T \sin\theta_{SP}) \ . \tag{B.12}$$

The reactive power flow from equation (B.11) for the transformer from node S towards node P is:

$$Q_{SP} = -B_T V_S^2 - c V_S V_P (G_T \sin\theta_{SP} - B_T \cos\theta_{SP}) \ . \tag{B.13}$$

Appendix C

Modelling of Network Parameters

Each series or shunt parameter to be estimated is removed from the network model and is replaced by state variables representing its terminal power flows.

C.1 The Series Branch

Figure C.1 shows an unknown series branch. For an active power flow measurement from node i

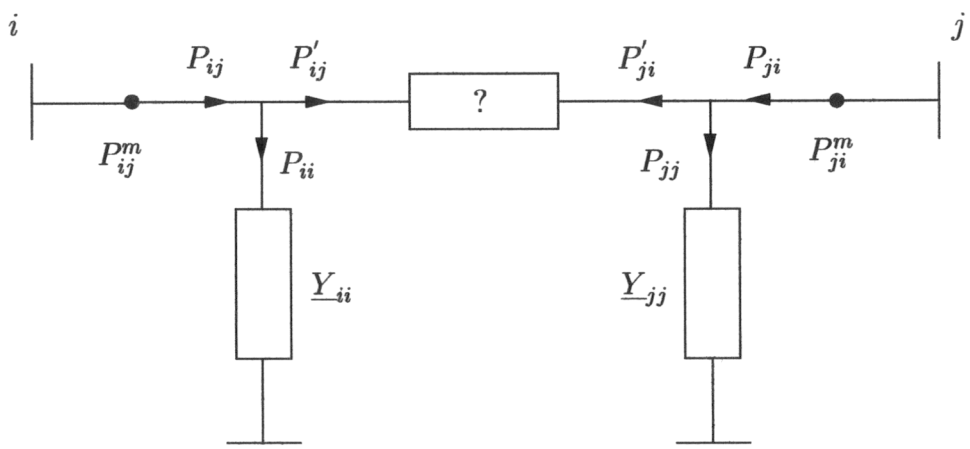


Figure C.1: Unknown series branch

towards node j, the following is valid:

$$P^m_{ij} = P_{ij} + v = P_{ii} + P'_{ij} + v\,, \tag{C.1}$$

and for a reactive power flow measurement from node i towards node j , the following is valid:

$$Q^m_{ij} = Q_{ij} + v = Q_{ii} + Q'_{ij} + v\,, \tag{C.2}$$

where v is a measurement error. In equation (C.1) and equation (C.2), the terms P_{ii} and Q_{ii} can be calculated as:

$$\begin{aligned} P_{ii} - jQ_{ii} &= \underline{V}_i^* \underline{Y}_{ii} \underline{V}_i \\ &= \underline{Y}_{ii} V_i^2 , \end{aligned}$$

thus

$$P_{ii} = G_{ii} V_i^2 , \tag{C.3}$$

and

$$Q_{ii} = -B_{ii} V_i^2 . \tag{C.4}$$

The terms that are needed to be calculated in equation (C.1) and equation (C.2), are P'_{ij} and Q'_{ij}. For an unknown series impedance, a pseudo-measurement requiring the current flows at each end of the line to be the same is needed, therefore:

$$\underline{I}'_{ij} + \underline{I}'_{ji} = 0 , \tag{C.5}$$

and therefore

$$\frac{P'_{ij} - jQ'_{ij}}{\underline{V}_i^*} + \frac{P'_{ji} - jQ'_{ji}}{\underline{V}_j^*} = 0 , \tag{C.6}$$

which will give:

$$P'_{ij} V_j + (P'_{ji} \cos\theta_{ij} - Q'_{ji} \sin\theta_{ij}) V_i = 0 , \tag{C.7}$$

and

$$Q'_{ij} V_j + (P'_{ji} \sin\theta_{ij} + Q'_{ji} \cos\theta_{ij}) V_i = 0 . \tag{C.8}$$

By using this modelling technique, the state variables of vector $\mathbf{x}$ in equation (3.4) of section 3.3.2 are increased with P'_{ij} , Q'_{ij} , P'_{ji} and Q'_{ji}. The number of measurement equations of vector $\mathbf{h}(\mathbf{x})$ in equation (3.4) of section 3.3.2, are also increased with equation (C.7) and equation (C.8). Once the solved state variables are obtained, the unknown series impedance can be calculated as:

$$\underline{Y}_{ij} = \frac{P'_{ij} - jQ'_{ij}}{\underline{V}_i^* \left(\underline{V}_i - \underline{V}_j\right)} . \tag{C.9}$$

C.2 The Shunt Branch

Figure C.2 shows an unknown shunt branch. For an active power flow measurement from node i

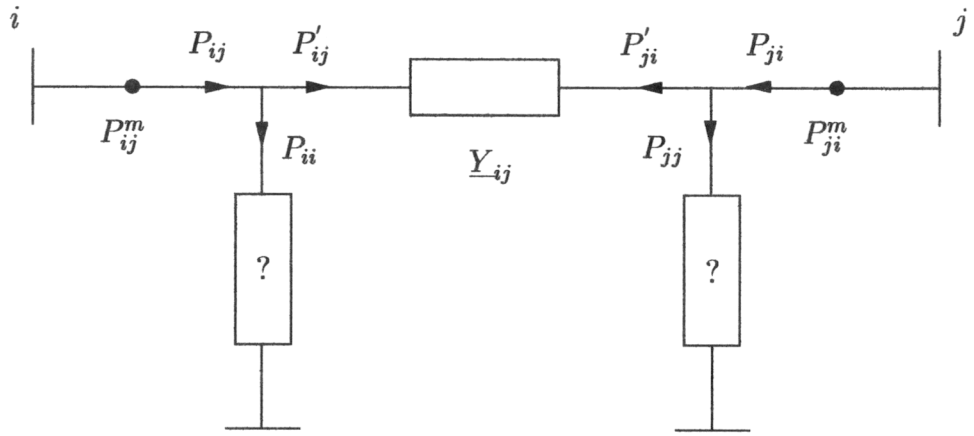


Figure C.2: Unknown shunt branch

towards node j , the following is valid:

$$P^m_{ij} = P_{ij} + v = P_{ii} + P'_{ij} + v\,, \tag{C.10}$$

and for a reactive power flow measurement from node i towards node j , the following is valid:

$$Q^m_{ij} = Q_{ij} + v = Q_{ii} + Q'_{ij} + v\,, \tag{C.11}$$

where v is a measurement error. In equation (C.10) and equation (C.11), the terms P'_{ij} and Q'_{ij} can be calculated as:

$$\begin{aligned} P'_{ij} - jQ'_{ij} &= \underline{V}^*_i \underline{Y}_{ij} \left(\underline{V}_i - \underline{V}_j\right) \\ &= \underline{Y}_{ij} V_i^2 - \underline{Y}_{ij} \underline{V}^*_i \underline{V}_j\,, \end{aligned} \tag{C.12}$$

thus

$$P'_{ij} = G_{ij} V_i^2 - V_i V_j \left(G_{ij} \cos\theta_{ij} + B_{ij} \sin\theta_{ij}\right)\,, \tag{C.13}$$

and

$$Q'_{ij} = -B_{ij} V_i^2 - V_i V_j \left(G_{ij} \sin\theta_{ij} - B_{ij} \cos\theta_{ij}\right)\,. \tag{C.14}$$

The terms that are needed to be calculated in equation (C.10) and equation (C.11), are P_{ii} and Q_{ii}. For an unknown shunt admittance of a symmetrical-pi transmission line, a pseudo-measurement requiring the legs at each end of the line to be the same is needed, thus

$$\underline{Y}_{ii} = \underline{Y}_{jj}\,, \tag{C.15}$$

and

$$\frac{P_{ii} - jQ_{ii}}{V_i^2} = \frac{P_{jj} - jQ_{jj}}{V_j^2}\,, \tag{C.16}$$

which will give:

$$P_{ii}V_j^2 - P_{jj}V_i^2 = 0\,, \tag{C.17}$$

and

$$Q_{ii}V_j^2 - Q_{jj}V_i^2 = 0\,. \tag{C.18}$$

By using this modelling technique, the state variables of vector $\mathbf{x}$ in equation (3.4) of section 3.3.2 are increased with P_{ii} , Q_{ii} , P_{jj} and Q_{jj}. The number of measurement equations of vector $\mathbf{h}(\mathbf{x})$ in equation (3.4) of section 3.3.2, are also increased with equation (C.17) and equation (C.18). Once the solved state variables are obtained, the unknown shunt admittance can be calculated as:

$$\underline{Y}_{ii} = \underline{Y}_{jj} = \frac{P_{ii} - jQ_{ii}}{V_i^2}\,. \tag{C.19}$$

C.3 Modelling of ULTC Transformer Parameters

A model of an ULTC transformer is shown in Figure C.3. It has two parameters which are c and $\underline{Y}_T$. The same procedure for modelling an unknown series parameter and an unknown shunt parameter, as in the previous sections, can be used to estimate $\underline{Y}_T$ when c is known. For an active power flow measurement from node i towards node j , the following is valid:

$$P_{ij}^m = P_{ij} + v = P_{ii} + P_{ij}^{'} + v\,, \tag{C.20}$$

and for a reactive power flow measurement from node i towards node j , the following is valid:

$$Q_{ij}^m = Q_{ij} + v = Q_{ii} + Q_{ij}^{'} + v\,, \tag{C.21}$$

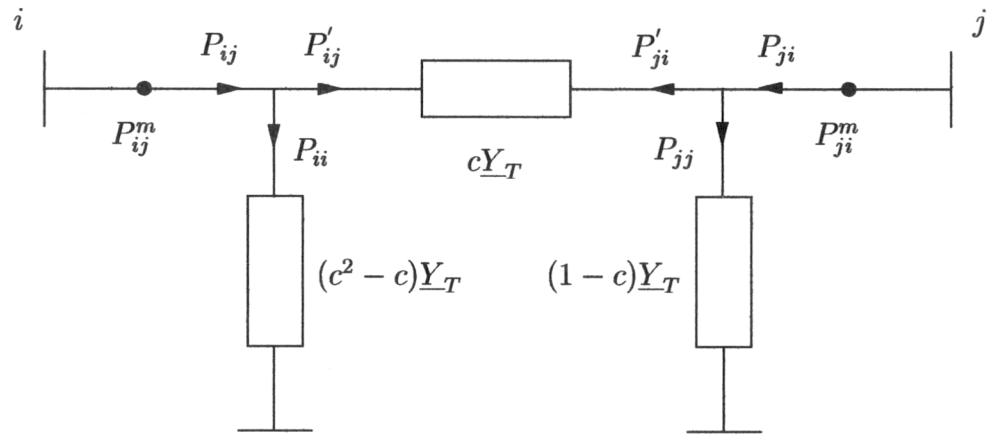


Figure C.3: ULTC transformer model

where v is a measurement error. In this modelling technique, the state variables will be P_{ii} , Q_{ii} , P'_{ij} , Q'_{ij} , P'_{ji} , Q'_{ji} , P_{jj} and Q_{jj}. The following pseudo-measurement equations are used:

a) a pseudo-measurement requiring the current flows at each end of the line to be the same, therefore equation (C.7) and equation (C.8) are needed.

b) a pseudo-measurement requiring the legs at each end of the line to be the same is needed, thus

$$\underline{Y}_{ii} = -c\underline{Y}_{jj}\,, \tag{C.22}$$

and

$$\frac{P_{ii} - jQ_{ii}}{V_i^2} = -c\frac{P_{jj} - jQ_{jj}}{V_j^2}\,, \tag{C.23}$$

which will give:

$$P_{ii}V_j^2 + cP_{jj}V_i^2 = 0\,, \tag{C.24}$$

and

$$Q_{ii}V_j^2 + cQ_{jj}V_i^2 = 0\,. \tag{C.25}$$

Once all the state variables are solved, $\underline{Y}_T$ can be calculated as follows:

$$\underline{Y}_T = \frac{1}{c}\frac{P'_{ij} - jQ'_{ij}}{\underline{V}_i^*\left(\underline{V}_i - \underline{V}_j\right)}\,. \tag{C.26}$$

Appendix D

Bad Data Detection and Identification

The following flow chart shows bad data identification using the hypothesis detection test.

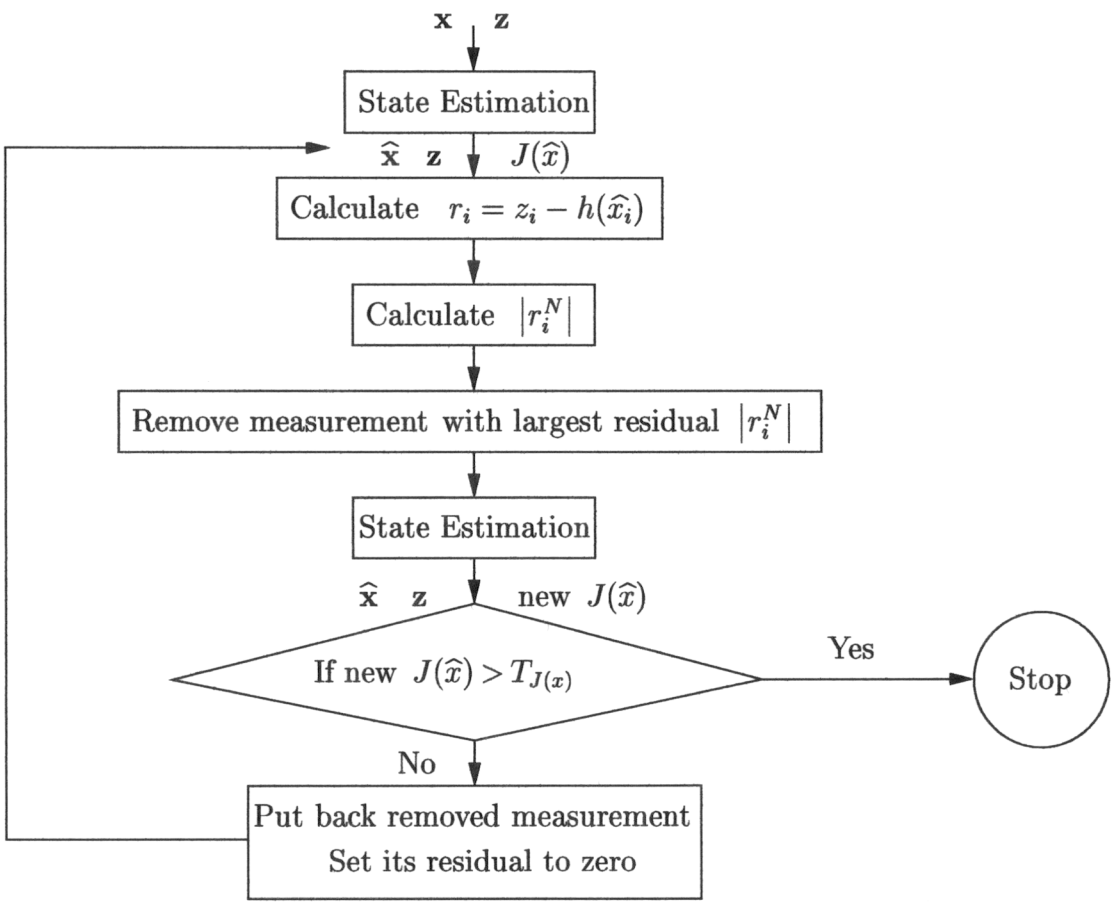


Figure D.1: Hypothesis testing for the identification of a single bad data point

Appendix E

A derivation of Newton methods

Consider the objective function $J(\mathbf{x})$. To minimise the function, get its gradient and set it equal to zero:

$$\nabla \mathbf{J} = \mathbf{J}_x(\mathbf{x}) = 0\,. \tag{E.1}$$

This gives a system of non-linear equations. To solve this system of equations, it must be linearised first. A linearisation around some working point $\mathbf{x}_k$, by using a truncated Taylor series expansion, of this equation gives

$$\begin{aligned} \mathbf{J}_{xx}(\mathbf{x}_k)\,\Delta\mathbf{x}_k + \mathbf{J}_x(\mathbf{x}_k) &= 0\,, \text{ or} \\ \mathbf{J}_{xx}(\mathbf{x}_k)\,\Delta\mathbf{x}_k &= -\mathbf{J}_x(\mathbf{x}_k)\,, \end{aligned} \tag{E.2}$$

where $\mathbf{J}_{xx}(\mathbf{x}_k)$ is the Hessian matrix (the matrix containing all the second order derivatives) of J evaluated at point $\mathbf{x}_k$. The state correction vector, $\Delta\mathbf{x}_k = \left[\Delta x_1^{(k)}, \Delta x_2^{(k)}, ..., \Delta x_n^{(k)}\right]^T$, can be calculated from this system of linear equations. An iterative procedure can be used and better iterates of the state vector can be calculated by

$$\mathbf{x}_{k+1} = \mathbf{x}_k + \Delta\mathbf{x}_k\,, \tag{E.3}$$

until the solution is found. Convergence is obtained when $\|\Delta\mathbf{x}_k\| < \varepsilon$, where ε is the convergence criterion.

The Quadratic-Tangent (QT) objective function will be discussed in the next section. The gradient vector and Hessian will be derived for this objective function as well.

E.1 QT objective function

The QT objective function is a function of residuals, where the residuals are defined as follows:

$$r_i = z_i - h_i(\mathbf{x}) \,, \tag{E.4}$$

$$r_{s_i} = \frac{r_i}{\sigma_i \omega_i} \,, \tag{E.5}$$

$$E_s = \underset{i}{median} \; |r_{s_i}| \,, \tag{E.6}$$

$$\widetilde{r}_{s_i} = \frac{r_{s_i}}{E_s} \,, \tag{E.7}$$

where the residual r_i is the difference between the $i-$th measurement taken from the network (z_i) and the computed value of the corresponding measured quantity $h_i(\mathbf{x})$, r_{s_i} is the standardised residual and E_s is a scaling factor whose purpose is to implement robust scaling of the standardised residual. The weight σ_i, is the standard deviation of measurement errors and ω_i is an additional weight for measurements that are classified as leverage points. Leverage points are created by those measurements whose parameters are outliers in the parameter space. This means that leverage points are positional outliers compared to the bulk of parameters.

The QT objective function is defined as follows:

$$\begin{aligned} J(\mathbf{x}) &= \omega^T \rho(\widetilde{\mathbf{r}}_s)\, \omega \\ &= \omega_1^2 \rho(\widetilde{r}_{s_1}) + \omega_2^2 \rho(\widetilde{r}_{s_2}) + \cdots + \omega_m^2 \rho(\widetilde{r}_{s_m}) \,, \end{aligned} \tag{E.8}$$

where

$$\rho(\widetilde{r}_{s_i}) = \begin{cases} \dfrac{\widetilde{r}_{s_i}^2}{2} & |\widetilde{r}_{s_i}| \le \beta \\ \beta\,|\widetilde{r}_{s_i}| - \dfrac{\beta^2}{2} & |\widetilde{r}_{s_i}| > \beta \end{cases} \,, \tag{E.9}$$

$$\rho(\widetilde{\mathbf{r}}_s) = \begin{bmatrix} \rho(\widetilde{r}_{s_1}) & 0 & \cdots & 0 \\ 0 & \rho(\widetilde{r}_{s_2}) & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & \rho(\widetilde{r}_{s_m}) \end{bmatrix} \,, \tag{E.10}$$

β is the break-even point of the non-quadratic function $\rho(\widetilde{r}_{s_i})$ and m is the number of measurements taken from the system. Figure E.1 shows a graph of the QT objective function. A $\beta = 1.345$ is used.

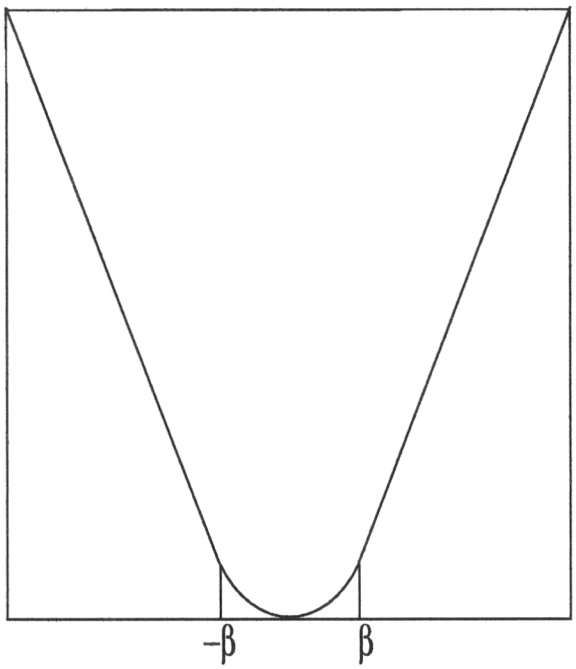


Figure E.1: A graph of the Quadratic-Tangent objective function

E.2 Gradient vector

Let the number of state variables be n. The gradient of J is:

$$\mathbf{J}_x = \begin{bmatrix} \frac{\partial J}{\partial x_1} \\ \frac{\partial J}{\partial x_2} \\ \vdots \\ \frac{\partial J}{\partial x_n} \end{bmatrix} = \begin{bmatrix} \omega_1^2 \frac{\partial \widetilde{r}_{s_1}}{\partial x_1} \cdot \frac{\partial \rho}{\partial \widetilde{r}_{s_1}} + \omega_2^2 \frac{\partial \widetilde{r}_{s_2}}{\partial x_1} \cdot \frac{\partial \rho}{\partial \widetilde{r}_{s_2}} + \cdots + \omega_m^2 \frac{\partial \widetilde{r}_{s_m}}{\partial x_1} \cdot \frac{\partial \rho}{\partial \widetilde{r}_{s_m}} \\ \omega_1^2 \frac{\partial \widetilde{r}_{s_1}}{\partial x_2} \cdot \frac{\partial \rho}{\partial \widetilde{r}_{s_1}} + \omega_2^2 \frac{\partial \widetilde{r}_{s_2}}{\partial x_2} \cdot \frac{\partial \rho}{\partial \widetilde{r}_{s_2}} + \cdots + \omega_m^2 \frac{\partial \widetilde{r}_{s_m}}{\partial x_2} \cdot \frac{\partial \rho}{\partial \widetilde{r}_{s_m}} \\ \vdots \\ \omega_1^2 \frac{\partial \widetilde{r}_{s_1}}{\partial x_n} \cdot \frac{\partial \rho}{\partial \widetilde{r}_{s_1}} + \omega_2^2 \frac{\partial \widetilde{r}_{s_2}}{\partial x_n} \cdot \frac{\partial \rho}{\partial \widetilde{r}_{s_2}} + \cdots + \omega_m^2 \frac{\partial \widetilde{r}_{s_m}}{\partial x_n} \cdot \frac{\partial \rho}{\partial \widetilde{r}_{s_m}} \end{bmatrix}$$

$$= \begin{bmatrix} \sum_{i=1}^{m} \omega_i^2 \frac{\partial \widetilde{r}_{s_i}}{\partial x_1} \cdot \frac{\partial \rho}{\partial \widetilde{r}_{s_i}} \\ \sum_{i=1}^{m} \omega_i^2 \frac{\partial \widetilde{r}_{s_i}}{\partial x_2} \cdot \frac{\partial \rho}{\partial \widetilde{r}_{s_i}} \\ \vdots \\ \sum_{i=1}^{m} \omega_i^2 \frac{\partial \widetilde{r}_{s_i}}{\partial x_n} \cdot \frac{\partial \rho}{\partial \widetilde{r}_{s_i}} \end{bmatrix}, \tag{E.11}$$

where for $i = 1, 2, ..., m$ and $j = 1, 2, ..., n$ follows

$$\frac{\partial \widetilde{r}_{s_i}}{\partial x_j} = \frac{\partial \left\{ \dfrac{z_i - h_i(\mathbf{x})}{\sigma_i \omega_i E_s} \right\}}{\partial x_j} = -\frac{1}{\sigma_i \omega_i E_s} \cdot \frac{\partial h_i(\mathbf{x})}{\partial x_j},$$

and

$$\frac{\partial \rho}{\partial \widetilde{r}_{s_i}} = \begin{cases} \widetilde{r}_{s_i} & |\widetilde{r}_{s_i}| \leq \beta \\ sign(\widetilde{r}_{s_i}) \cdot \beta & |\widetilde{r}_{s_i}| > \beta \end{cases}.$$

In matrix formulation $\mathbf{J}_x$ is:

$$\mathbf{J}_x = -E_s^{-1} \mathbf{H}^T \mathbf{R}^{-\frac{1}{2}} \mathbf{W} \mathbf{D} \widehat{\mathbf{e}}, \tag{E.12}$$

where the elements of the Jacobian matrix $\mathbf{H}$ are

$$[\mathbf{H}]_{ij} = \frac{\partial h_i(\mathbf{x})}{\partial x_j}, \tag{E.13}$$

and the diagonal matrices are

$$\mathbf{R} = diag\left(\sigma_1^2, \ldots, \sigma_m^2\right), \tag{E.14}$$

$$\mathbf{W} = diag\left(\omega_1, \ldots, \omega_m\right), \tag{E.15}$$

$$\mathbf{D} = diag\left(\frac{\partial \rho}{\partial \widetilde{r}_{s_1}}, \ldots, \frac{\partial \rho}{\partial \widetilde{r}_{s_m}}\right), \tag{E.16}$$

and $\widehat{\mathbf{e}}$ is a m-dimensional column vector with all its entries equal to one.

E.3 Hessian

The Hessian of J will be constructed as follows:

$$\mathbf{J}_{xx} = \begin{bmatrix} \dfrac{\partial^2 J}{\partial x_1 \partial x_1} & \dfrac{\partial^2 J}{\partial x_1 \partial x_2} & \cdots & \dfrac{\partial^2 J}{\partial x_1 \partial x_n} \\ \dfrac{\partial^2 J}{\partial x_2 \partial x_1} & \dfrac{\partial^2 J}{\partial x_2 \partial x_2} & \cdots & \dfrac{\partial^2 J}{\partial x_2 \partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \dfrac{\partial^2 J}{\partial x_n \partial x_1} & \dfrac{\partial^2 J}{\partial x_n \partial x_2} & \cdots & \dfrac{\partial^2 J}{\partial x_n \partial x_n} \end{bmatrix}. \tag{E.17}$$

For $i = 1, 2, ..., m$ and $j, k = 1, 2, ..., n$ consider one element in the Hessian matrix:

$$\begin{aligned}
\frac{\partial^2 J}{\partial x_j \partial x_k} &= \frac{\partial \left\{ \dfrac{\partial J}{\partial x_j} \right\}}{\partial x_k} \\
&= \omega_1^2 \left(\frac{\partial^2 \widetilde{r}_{s_1}}{\partial x_j \partial x_k} \cdot \frac{\partial \rho}{\partial \widetilde{r}_{s_1}} + \frac{\partial \widetilde{r}_{s_1}}{\partial x_j} \cdot \frac{\partial^2 \rho}{\partial \widetilde{r}_{s_1} \partial x_k} \right) + \\
&\quad \omega_2^2 \left(\frac{\partial^2 \widetilde{r}_{s_2}}{\partial x_j \partial x_k} \cdot \frac{\partial \rho}{\partial \widetilde{r}_{s_2}} + \frac{\partial \widetilde{r}_{s_2}}{\partial x_j} \cdot \frac{\partial^2 \rho}{\partial \widetilde{r}_{s_2} \partial x_k} \right) + \cdots + \\
&\quad \omega_m^2 \left(\frac{\partial^2 \widetilde{r}_{s_m}}{\partial x_j \partial x_k} \cdot \frac{\partial \rho}{\partial \widetilde{r}_{s_m}} + \frac{\partial \widetilde{r}_{s_m}}{\partial x_j} \cdot \frac{\partial^2 \rho}{\partial \widetilde{r}_{s_m} \partial x_k} \right) .
\end{aligned} \tag{E.18}$$

The last term of each bracket will be:

$$\begin{aligned}
\frac{\partial^2 \rho}{\partial \widetilde{r}_{s_i} \partial x_k} &= \frac{\partial^2 \rho}{\partial x_k \partial \widetilde{r}_{s_i}} \\
&= \frac{\partial \left\{ \dfrac{\partial \rho}{\partial x_k} \right\}}{\partial \widetilde{r}_{s_i}} \\
&= \frac{\partial \left\{ \dfrac{\partial \widetilde{r}_{s_i}}{\partial x_k} \cdot \dfrac{\partial \rho}{\partial \widetilde{r}_{s_i}} \right\}}{\partial \widetilde{r}_{s_i}} \\
&= \frac{\partial \widetilde{r}_{s_i}}{\partial x_k \partial \widetilde{r}_{s_i}} \cdot \frac{\partial \rho}{\partial \widetilde{r}_{s_i}} + \frac{\partial \widetilde{r}_{s_i}}{\partial x_k} \cdot \frac{\partial^2 \rho}{\partial \widetilde{r}_{s_i}^2} \\
&= 0 + \frac{\partial \widetilde{r}_{s_i}}{\partial x_k} \cdot \frac{\partial^2 \rho}{\partial \widetilde{r}_{s_i}^2} \\
&= \frac{\partial \widetilde{r}_{s_i}}{\partial x_k} \cdot \frac{\partial^2 \rho}{\partial \widetilde{r}_{s_i}^2} .
\end{aligned} \tag{E.19}$$

Therefore

$$\frac{\partial^2 J}{\partial x_j \partial x_k} = \sum_{i=1}^{m} \omega_i^2 \left(\frac{\partial^2 \widetilde{r}_{s_i}}{\partial x_j \partial x_k} \cdot \frac{\partial \rho}{\partial \widetilde{r}_{s_i}} + \frac{\partial \widetilde{r}_{s_i}}{\partial x_j} \cdot \frac{\partial \widetilde{r}_{s_i}}{\partial x_k} \cdot \frac{\partial^2 \rho}{\partial \widetilde{r}_{s_i}^2} \right) , \tag{E.20}$$

where

$$\frac{\partial^2 \widetilde{r}_{s_i}}{\partial x_j \partial x_k} = \frac{\partial^2 \left\{ \dfrac{z_i - h_i(\mathbf{x})}{\sigma_i \omega_i E_s} \right\}}{\partial x_j \partial x_k} - \frac{1}{\sigma_i \omega_i E_s} \cdot \frac{\partial^2 h_i(\mathbf{x})}{\partial x_j \partial x_k} ,$$

$$\frac{\partial^2 \rho}{\partial \widetilde{r}_{s_i}^2} = \begin{cases} 1 & |\widetilde{r}_{s_i}| \le \beta \\ 0 & |\widetilde{r}_{s_i}| > \beta \end{cases} .$$

The complete Hessian of J can also be written in matrix notation:

$$\mathbf{J}_{xx} = \sum_{i=1}^{m} \left(-\frac{\omega_i}{\sigma_i E_s} \cdot \frac{\partial \rho}{\partial \widetilde{r}_{s_i}} \cdot \mathbf{G}_i \right) + E_s^{-2} \mathbf{H}^T \mathbf{R}^{-1} \widetilde{\mathbf{D}} \mathbf{H} , \tag{E.21}$$

where each element in the matrix $\mathbf{G}_i$ is given by

$$[\mathbf{G}_i(\mathbf{x})]_{jk} = \frac{\partial^2 h_i(\mathbf{x})}{\partial x_j \partial x_k}, \tag{E.22}$$

and the diagonal matrix $\widetilde{\mathbf{D}}$ is given by

$$\widetilde{\mathbf{D}} = diag\left(\frac{\partial^2 \rho}{\partial \widetilde{r}_{s_1}^2}, \ldots, \frac{\partial^2 \rho}{\partial \widetilde{r}_{s_m}^2}\right). \tag{E.23}$$

Note that each matrix $\mathbf{G}_i$ is the Hessian matrix of corresponding measurement equation $h_i(\mathbf{x})$, and that $\mathbf{J}_{xx}$ is the Hessian of the objective function. The Hessian of the objective function consists of a combination of all the Hessians of the measurement equations and of a term containing the Jacobian matrix. The above derivation is used for the Newton method.

The Gauss-Newton method approximates the Hessian matrix by setting the second order derivative term $\mathbf{G}_i(\mathbf{x})$ equal to zero . The Hessian then becomes:

$$\mathbf{J}_{xx} = E_s^{-2}\mathbf{H}^T\mathbf{R}^{-1}\widetilde{\mathbf{D}}\mathbf{H}. \tag{E.24}$$

E.4 Alternative Gauss-Newton Method

There is another way of deriving the Gauss-Newton method. It gives a slightly different result as in the case where the matrix $\mathbf{G}_i$ in equation (E.21) is ignored, and then substituting equation (E.21) and equation (E.12) in equation (E.2), and solving

$$\left(E_s^{-2}\mathbf{H}^T\mathbf{R}^{-1}\widetilde{\mathbf{D}}\mathbf{H}\right)\Delta\mathbf{x}_k = E_s^{-1}\mathbf{H}^T\mathbf{R}^{-\frac{1}{2}}\mathbf{W}\mathbf{D}\widehat{\mathbf{e}}. \tag{E.25}$$

The alternative derivation can be done by substituting equation (E.12) in equation (E.1) to obtain

$$-E_s^{-1}\mathbf{H}^T\mathbf{R}^{-\frac{1}{2}}\mathbf{W}\mathbf{D}\widehat{\mathbf{e}} = 0. \tag{E.26}$$

Define

$$\widetilde{\mathbf{R}}_s = diag\left(\widetilde{r}_{s_1}, \widetilde{r}_{s_2}, \ldots, \widetilde{r}_{s_m}\right), \tag{E.27}$$

$$\mathbf{Q} = \mathbf{D}\widetilde{\mathbf{R}}_s^{-1}. \tag{E.28}$$

Multiply equation (E.26) by $-E_s$ and substitute equation (E.28) in equation (E.26):

$$\mathbf{H}^T\mathbf{R}^{-\frac{1}{2}}\mathbf{W}\mathbf{Q}\widetilde{\mathbf{R}}_s\widehat{\mathbf{e}} = 0. \tag{E.29}$$

Using the fact that $\widetilde{\mathbf{r}}_s = \widetilde{\mathbf{R}}_s\widehat{\mathbf{e}}$, and that $\widetilde{\mathbf{r}}_s = E_s^{-1}\mathbf{W}^{-1}\mathbf{R}^{-\frac{1}{2}}(\mathbf{z} - \mathbf{h}(\mathbf{x}))$, and the commutative

property of diagonal matrices, rewrite equation (E.29) to obtain:

$$\mathbf{H}^T\mathbf{R}^{-1}\mathbf{Q}\left(\mathbf{z}-\mathbf{h}\left(\mathbf{x}\right)\right)=0\,. \tag{E.30}$$

Use a truncated Taylor series approximation of $\mathbf{h}\left(\mathbf{x}\right)$ around a point $\mathbf{x}_k$ to obtain:

$$\mathbf{h}\left(\mathbf{x}\right)\approx\mathbf{h}\left(\mathbf{x}_k\right)+\mathbf{H}\left(\mathbf{x}_k\right)\Delta\mathbf{x}_k\,. \tag{E.31}$$

Substitute equation (E.31) in equation (E.30), to obtain:

$$\mathbf{H}\left(\mathbf{x}_k\right)^T\mathbf{R}^{-1}\mathbf{Q}\left(\mathbf{x}_k\right)\mathbf{H}\left(\mathbf{x}_k\right)\Delta\mathbf{x}_k=\mathbf{H}\left(\mathbf{x}_k\right)^T\mathbf{R}^{-1}\mathbf{Q}\left(\mathbf{x}_k\right)\Delta\mathbf{z}\,, \tag{E.32}$$

where $\Delta\mathbf{z}=\mathbf{z}-\mathbf{h}\left(\mathbf{x}_k\right)$. If equation (E.3) is used, better iterates of the state vector can be calculated iteratively until the solution is found. Equation (E.32) is equivalent to obtaining the solution of the following system of redundant equations:

$$\mathbf{A}\Delta\mathbf{x}\approx\mathbf{y}\,, \tag{E.33}$$

where

$$\mathbf{A}=\mathbf{R}^{-\frac{1}{2}}\mathbf{Q}^{\frac{1}{2}}\mathbf{H}\,, \tag{E.34}$$

$$\mathbf{y}=\mathbf{R}^{-\frac{1}{2}}\mathbf{Q}^{\frac{1}{2}}\Delta\mathbf{z}\,. \tag{E.35}$$

Equations (E.33), (E.34) and (E.35) are used by the alternative Gauss-Newton method. This Gauss-Newton method is more robust than the first derivative for Gauss-Newton, because the Hessian of equation (E.24) tends to be ill-conditioned. This Gauss-Newton method is used to obtain the results.

Appendix F

State Estimation Process

Figure F.1 illustrates an overview of a general state estimation process. The output of the state estimation process will be the estimated state variables of the power network.

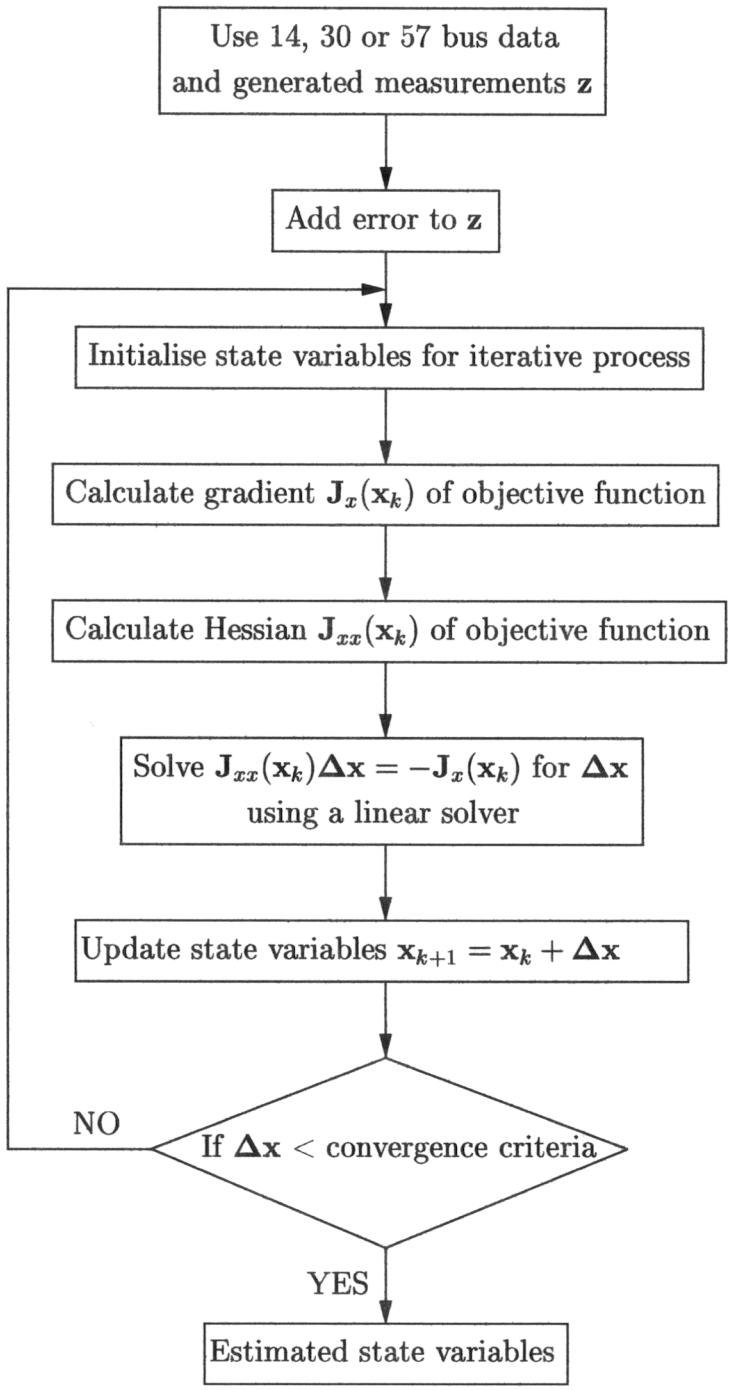


Figure F.1: A flow diagram indicating a general state estimation process

Appendix G

Jacobian Derivatives in Polar Format

This chapter gives the Jacobian derivatives of the measurement equations $\mathbf{h}(\mathbf{x})$, used in the numerical process for the state estimator. These derivatives will be given in polar co-ordinates format. A short overview of the different measurement equations used, will be given. The Jacobian derivatives will be given after that.

G.1 Power Flows

Let the voltage at any node i be $\underline{V}_i = V_i \angle \theta_i$. Let N be the number of nodes. The power flow from node i to node j will be:

$$P_{ij} = (G'_{ij} + G_{ij})V_i^2 - V_i V_j (G_{ij} \cos\theta_{ij} + B_{ij} \sin\theta_{ij}) ,$$

and

$$Q_{ij} = -(B'_{ij} + B_{ij})V_i^2 - V_i V_j (G_{ij} \sin\theta_{ij} - B_{ij} \cos\theta_{ij}) .$$

G.2 Power Injections

The power injections at node i are:

$$P_i = G_{ii}V_i^2 - V_i \sum_{\substack{j=1 \\ j \neq i}}^{N} V_j (G_{ij} \cos\theta_{ij} + B_{ij} \sin\theta_{ij}) = \sum_{\substack{j=1 \\ j \neq i}}^{N} P_{ij} ,$$

and

$$Q_i = -B_{ii}V_i^2 - V_i \sum_{\substack{j=1 \\ j \neq i}}^{N} V_j (G_{ij} \sin\theta_{ij} - B_{ij} \cos\theta_{ij}) = \sum_{\substack{j=1 \\ j \neq i}}^{N} Q_{ij} ,$$

where $G_{ii} = \sum_{j \neq i} (G_{ij} + G'_{ij})$, $B_{ii} = \sum_{j \neq i} (B_{ij} + B'_{ij})$ and $\theta_{ij} = \theta_i - \theta_j$.

G.3 Modelling of zero impedance branches

If there is a zero impedance branch between node i and node j , the power flows P_{ij} and Q_{ij} for this particular branch are used as state variables. The injection measurement equations for node i and node j will be

$$P_i = \sum_{\substack{s=1 \\ s \neq j}}^{N} P_{is} + P_{ij} \ , \ Q_i^M = \sum_{\substack{s=1 \\ s \neq j}}^{N} Q_{is} + Q_{ij} \, ,$$

and

$$P_j = \sum_{\substack{s=1 \\ s \neq i}}^{N} P_{js} + P_{ji} \ , \ Q_j^M = \sum_{\substack{s=1 \\ s \neq i}}^{N} Q_{js} + Q_{ji} \, ,$$

where for the power flows between nodes i and j, the values of the state variables P_{ij} and Q_{ij} are used instead of calculating the power flows by using the power flow measurement equations. Since branch ij is a zero impedance branch, $P_{ji} = P_{ij}$ and $Q_{ji} = Q_{ij}$. Because of a zero voltage drop, also add the following pseudo-measurements as constraints:

$$V_{ij} = V_i - V_j = 0 \, ,$$

and

$$\theta_{ij} = \theta_i - \theta_j = 0 \, .$$

G.4 Modelling of circuit breakers

G.4.1 Circuit breakers in series with a non-zero impedance branch

Introduce the circuit breaker variable κ, which indicates the position of the switch. If $\kappa = 0$, the breaker is open and if $\kappa = 1$, the breaker is closed. Locate the branch where the circuit breaker is installed, and substitute the branch parameters G, B, G' and B' with κG, κB, $\kappa G'$ and $\kappa B'$. To force the variable κ to either the value of 0 or 1, introduce the following pseudo-measurement as a constraint:

$$\mathsf{K}_{ij} = \kappa_{ij}(1 - \kappa_{ij}) = 0 \, .$$

The power flow equations for this particular branch ij become:

$$P_{ij}(\kappa) = \kappa_{ij} P_{ij}\,,$$

and

$$Q_{ij}(\kappa) = \kappa_{ij} Q_{ij}\,,$$

where P_{ij} and Q_{ij} are the conventional power flows calculated when $\kappa = 1$. The indices i and j indicate that the circuit breaker is connected between nodes i and j, meaning that $\kappa_{ij} = \kappa_{ji}$, and also that κ_{ij} and κ_{ji} are the same variable. The power injections at node i are:

$$P_i = \sum_{\substack{s=1 \\ s \neq j}}^{N} P_{is} + \kappa_{ij} P_{ij}\,,$$

and

$$Q_i = \sum_{\substack{s=1 \\ s \neq j}}^{N} Q_{is} + \kappa_{ij} Q_{ij}\,.$$

G.4.2 Circuit breakers in series with a zero impedance branch

Model the zero impedance branch as discussed earlier, and then use the power flows through and the voltage over the circuit breaker to determine the position of the breaker. For an open breaker the following equations hold:

$$P_{ij} = 0 \text{ and } Q_{ij} = 0\,,$$

and for a closed breaker the following:

$$V_i - V_j = 0 \text{ and } \theta_i - \theta_j = 0\,.$$

G.5 Modelling of unknown series and shunt parameters

G.5.1 Series parameters

The following additional equations will be used:

$$Sel_{ij} = P'_{ij} V_j + (P'_{ji} \cos\theta_{ij} - Q'_{ji} \sin\theta_{ij}) V_i = 0\,,$$

and

$$Se2_{ij} = Q'_{ij}V_j + (P'_{ji}\sin\theta_{ij} + Q'_{ji}\cos\theta_{ij})V_i = 0\,,$$

where P'_{ij}, Q'_{ij}, P'_{ji}, and Q'_{ji} will be the state variables. If the state variables are calculated, the series parameter $\underline{Y}_{ij}$ can be calculated as follows:

$$\underline{Y}_{ij} = \frac{P'_{ij} - jQ'_{ij}}{\underline{V}_i^*(\underline{V}_i - \underline{V}_j)}\,.$$

G.5.2 Shunt parameters

The following additional equations will be used:

$$Sh1_{ij} = P_{ii}V_j^2 - P_{jj}V_i^2 = 0\,,$$

and

$$Sh2_{ij} = Q_{ii}V_j^2 - Q_{jj}V_i^2 = 0\,,$$

where P_{ii}, Q_{ii}, P_{jj}, and Q_{jj} will be the state variables. If the state variables are calculated, the shunt parameters can be calculated as follows:

$$\underline{Y}_{ii} = \underline{Y}_{jj} = \frac{P_{ii} - jQ_{ii}}{V_i^2}\,.$$

G.6 Modelling of transformer impedance when tap position is known

This modelling technique is a combination of series and shunt parameter modelling. The same equations as for the modelling of series parameters are used. But the shunt parameter equations need to be modified and they are given as:

$$Tr1_{ij} = P_{ii}V_j^2 + cP_{jj}V_i^2 = 0\,,$$

and

$$Tr2_{ij} = Q_{ii}V_j^2 + cQ_{jj}V_i^2 = 0\,,$$

where c is the known tap position and P'_{ij}, Q'_{ij}, P'_{ji}, Q'_{ji}, P_{ii}, Q_{ii}, P_{jj}, and Q_{jj} will be the state variables. If the state variables are calculated, the unknown parameter $\underline{Y}_T$ can be calculated as follows:.

$$\underline{Y}_T = \frac{1}{c}\frac{P'_{ij} - jQ'_{ij}}{\underline{V}_i^*(\underline{V}_i - \underline{V}_j)},$$

or

$$\underline{Y}_T = \frac{1}{c(c-1)}\frac{P_{ii} - jQ_{ii}}{V_i^2}.$$

G.7 Jacobian Derivatives

This section will show the Jacobian derivatives for all the previous mentioned equations in this chapter.

The circuit breaker parameter κ is not shown in all the equations where circuit breakers are modelled. It is only shown where the parameters G, B, G' and B' for a particular branch are replaced with κG, κB, $\kappa G'$ and $\kappa B'$. If the original values of G, B, G' and B' are used as a result for calculating a derivative with respect to κ, it will be indicated by $^{(\dagger\dagger)}$. κ will only be shown where necessary. Also, only the non-zero derivatives will be shown. This means that if a particular derivative is zero for all possible situations, it will not be shown in the following derivations.

G.7.1 Power Injections

G.7.1.1 Active Power

$$\frac{\partial P_i}{\partial \theta_m} = \begin{cases} -V_i V_m(G_{im}\sin\theta_{im} - B_{im}\cos\theta_{im}) & m \neq i \\ V_i \sum\limits_{\substack{s=1 \\ s\neq i}}^{N} V_s(G_{is}\sin\theta_{is} - B_{is}\cos\theta_{is}) & m = i \\ 0 & m \neq i, \text{ where } im \text{ is a zero impedance branch} \end{cases}$$

$$\frac{\partial P_i}{\partial V_m} = \begin{cases} -V_i(G_{im}\cos\theta_{im} + B_{im}\sin\theta_{im}) & m \neq i \\ 2G_{ii}V_i - \sum\limits_{\substack{s=1 \\ s\neq i}}^{N} V_s(G_{is}\cos\theta_{is} + B_{is}\sin\theta_{is}) & m = i \\ 0 & m \neq i, \text{ where } im \text{ is a zero impedance branch} \end{cases}$$

$$\frac{\partial P_i}{\partial \kappa_{im}} = \begin{cases} (G'_{im} + G_{im})V_i^2 - V_i V_m (G_{im}\cos\theta_{im} + B_{im}\sin\theta_{im})^{(\dagger\dagger)} & \text{if there is a circuit breaker on line } im \\ 0 & \text{if there is no circuit breaker on line } im \end{cases}$$

$$\frac{\partial P_i}{\partial P_{mk}} = \begin{cases} 1 & m = i \\ -1 & k = i \quad \text{(if } mk \text{ is a zero impedance branch)} \\ 0 & m, k \neq i \end{cases}$$

$$\frac{\partial P_i}{\partial P'_{ij}} = 1 \text{ (if } ij \text{ is a line where the series impedance is modelled, else 0)}$$

$$\frac{\partial P_i}{\partial P_{ii}} = 1 \text{ (if } P_{ii} \text{ is on a line where the shunt impedance is modelled, else 0)}$$

G.7.1.2 Reactive Power

$$\frac{\partial Q_i}{\partial \theta_m} = \begin{cases} V_i V_m (G_{im}\cos\theta_{im} + B_{im}\sin\theta_{im}) & m \neq i \\ -V_i \sum\limits_{\substack{s=1 \\ s\neq i}}^{N} V_s (G_{is}\cos\theta_{is} + B_{is}\sin\theta_{is}) & m = i \\ 0 & m \neq i, \text{ where } im \text{ is a zero impedance branch} \end{cases}$$

$$\frac{\partial Q_i}{\partial V_m} = \begin{cases} -V_i (G_{im}\sin\theta_{im} - B_{im}\cos\theta_{im}) & m \neq i \\ -2B_{ii}V_i - \sum\limits_{\substack{s=1 \\ s\neq i}}^{N} V_s (G_{is}\sin\theta_{is} - B_{is}\cos\theta_{is}) & m = i \\ 0 & m \neq i, \text{ where } im \text{ is a zero impedance branch} \end{cases}$$

$$\frac{\partial Q_i}{\partial \kappa_{im}} = \begin{cases} -(B'_{im} + B_{im})V_i^2 - V_i V_m (G_{im}\sin\theta_{im} - B_{im}\cos\theta_{im})^{(\dagger\dagger)} & \text{if there is a circuit breaker on line } im \\ 0 & \text{if there is no circuit breaker on line } im \end{cases}$$

$$\frac{\partial Q_i}{\partial Q_{mk}} = \begin{cases} 1 & m = i \\ -1 & k = i \quad \text{(if } mk \text{ is a zero impedance branch)} \\ 0 & m, k \neq i \end{cases}$$

$$\frac{\partial Q_i}{\partial Q'_{ij}} = 1 \text{ (if } ij \text{ is a line where the series impedance is modelled, else 0)}$$

$$\frac{\partial Q_i}{\partial Q_{ii}} = 1 \text{ (if } Q_{ii} \text{ is on a line where the shunt impedance is modelled, else 0)}$$

G.7.2 Power Flows

G.7.2.1 Active Power

$$\frac{\partial P_{ij}}{\partial \theta_m} = \begin{cases} V_i V_j (G_{ij} \sin\theta_{ij} - B_{ij} \cos\theta_{ij}) & m = i \\ -V_i V_j (G_{ij} \sin\theta_{ij} - B_{ij} \cos\theta_{ij}) & m = j \\ 0 & m \neq i, j \\ 0 & \text{if } ij \text{ is a zero impedance branch} \end{cases}$$

$$\frac{\partial P_{ij}}{\partial V_m} = \begin{cases} 2(G'_{ij} + G_{ij})V_i - V_j (G_{ij} \cos\theta_{ij} + B_{ij} \sin\theta_{ij}) & m = i \\ -V_i (G_{ij} \cos\theta_{ij} + B_{ij} \sin\theta_{ij}) & m = j \\ 0 & m \neq i, j \\ 0 & \text{if } ij \text{ is a zero impedance branch} \end{cases}$$

$$\frac{\partial P_{ij}}{\partial \kappa_{ij}} = \begin{cases} (G'_{ij} + G_{ij})V_i^2 - V_i V_j (G_{ij} \cos\theta_{ij} + B_{ij} \sin\theta_{ij}) \text{ (††)} & \text{if there is a circuit breaker on line } ij \\ 0 & \text{if there is no circuit breaker on line } ij \end{cases}$$

$$\frac{\partial P_{ij}}{\partial P_{mk}} = \begin{cases} 1 & m = i, k = j \\ -1 & m = j, k = i \quad \text{(if } mk \text{ is a zero impedance branch)} \\ 0 & m, k \neq i, j \end{cases}$$

$$\frac{\partial P_{ij}}{\partial P'_{ij}} = 1 \text{ (if } ij \text{ is a line where the series impedance is modelled, else 0)}$$

$$\frac{\partial P_{ij}}{\partial P_{ii}} = 1 \text{ (if } P_{ii} \text{ is on a line where the shunt impedance is modelled, else 0)}$$

G.7.2.2 Reactive Power

$$\frac{\partial Q_{ij}}{\partial \theta_m} = \begin{cases} -V_i V_j (G_{ij} \cos\theta_{ij} + B_{ij} \sin\theta_{ij}) & m = i \\ V_i V_j (G_{ij} \cos\theta_{ij} + B_{ij} \sin\theta_{ij}) & m = j \\ 0 & m \neq i, j \\ 0 & \text{if } ij \text{ is a zero impedance branch} \end{cases}$$

$$\frac{\partial Q_{ij}}{\partial V_m} = \begin{cases} -2(B'_{ij} + B_{ij})V_i - V_j (G_{ij} \sin\theta_{ij} - B_{ij} \cos\theta_{ij}) & m = i \\ -V_i (G_{ij} \sin\theta_{ij} - B_{ij} \cos\theta_{ij}) & m = j \\ 0 & m \neq i, j \\ 0 & \text{if } ij \text{ is a zero impedance branch} \end{cases}$$

$$\frac{\partial Q_{ij}}{\partial \kappa_{ij}} = \begin{cases} -(B'_{ij} + B_{ij})V_i^2 - V_i V_j (G_{ij} \sin\theta_{ij} - B_{ij} \cos\theta_{ij})^{(\dagger\dagger)} & \text{if there is a circuit breaker on line } ij \\ 0 & \text{if there is no circuit breaker on line } ij \end{cases}$$

$$\frac{\partial Q_{ij}}{\partial Q_{mk}} = \begin{cases} 1 & m = i, k = j \\ -1 & m = j, k = i \quad (\text{if } mk \text{ is a zero impedance branch}) \\ 0 & m, k \neq i, j \end{cases}$$

$$\frac{\partial Q_{ij}}{\partial Q'_{ij}} = 1 \text{ (if } ij \text{ is a line where the series impedance is modelled, else 0)}$$

$\frac{\partial Q_{ij}}{\partial Q_{ii}} = 1$ (if Q_{ii} is on a line where the shunt impedance is modelled, else 0)

G.7.3 Voltage Phase Angles

$$\frac{\partial \theta_{ij}}{\partial \theta_m} = \begin{cases} 1 & m = i \\ -1 & m = j \\ 0 & m \neq i, j \end{cases}$$

G.7.4 Voltage Magnitude

$$\frac{\partial V_i}{\partial V_m} = \begin{cases} 1 & m = i \\ 0 & m \neq i \end{cases}$$

G.7.5 Equations modelling Zero Impedance branches

$$\frac{\partial V_{ij}}{\partial V_m} = \begin{cases} 1 & m = i \\ -1 & m = j \\ 0 & m \neq i, j \end{cases}$$

$$\frac{\partial \theta_{ij}}{\partial \theta_m} = \begin{cases} 1 & m = i \\ -1 & m = j \\ 0 & m \neq i, j \end{cases}$$

G.7.6 Circuit breakers in series with a non-zero impedance branch

$$\frac{\partial \mathsf{K}_{ij}}{\partial \kappa_{mk}} = \begin{cases} 1 - 2\kappa_{ij} & m, k = i, j \\ 0 & m, k \neq i, j \end{cases}$$

G.7.7 Unknown Series parameters

$$\frac{\partial Se1_{ij}}{\partial \theta_m} = \begin{cases} (-P'_{ji} \sin\theta_{ij} - Q'_{ji} \cos\theta_{ij}) V_i & m = i \\ (P'_{ji} \sin\theta_{ij} + Q'_{ji} \cos\theta_{ij}) V_i & m = j \\ 0 & m \neq i, j \end{cases}$$

$$\frac{\partial Se1_{ij}}{\partial V_m} = \begin{cases} P'_{ji}\cos\theta_{ij} - Q'_{ji}\sin\theta_{ij} & m = i \\ P'_{ij} & m = j \\ 0 & m \neq i, j \end{cases}$$

$$\frac{\partial Se1_{ij}}{\partial P'_{mk}} = \begin{cases} V_j & m = i, k = j \\ V_i\cos\theta_{ij} & m = j, k = i \\ 0 & m, k \neq i, j \end{cases}$$

$$\frac{\partial Se1_{ij}}{\partial Q'_{mk}} = \begin{cases} 0 & m = i, k = j \\ -V_i\sin\theta_{ij} & m = j, k = i \\ 0 & m, k \neq i, j \end{cases}$$

$$\frac{\partial Se2_{ij}}{\partial \theta_m} = \begin{cases} (P'_{ji}\cos\theta_{ij} - Q'_{ji}\sin\theta_{ij})V_i & m = i \\ (-P'_{ji}\cos\theta_{ij} + Q'_{ji}\sin\theta_{ij})V_i & m = j \\ 0 & m \neq i, j \end{cases}$$

$$\frac{\partial Se2_{ij}}{\partial V_m} = \begin{cases} P'_{ji}\sin\theta_{ij} + Q'_{ji}\cos\theta_{ij} & m = i \\ Q'_{ij} & m = j \\ 0 & m \neq i, j \end{cases}$$

$$\frac{\partial Se2_{ij}}{\partial P'_{mk}} = \begin{cases} 0 & m = i, k = j \\ V_i\sin\theta_{ij} & m = j, k = i \\ 0 & m, k \neq i, j \end{cases}$$

$$\frac{\partial Se2_{ij}}{\partial Q'_{mk}} = \begin{cases} V_j & m = i, k = j \\ V_i \cos\theta_{ij} & m = j, k = i \\ 0 & m, k \neq i, j \end{cases}$$

G.7.8 Unknown Shunt parameters

$$\frac{\partial Sh1_{ij}}{\partial V_m} = \begin{cases} -2P_{jj}V_i & m = i \\ 2P_{ii}V_j & m = j \\ 0 & m \neq i, j \end{cases}$$

$$\frac{\partial Sh1_{ij}}{\partial P_{mm}} = \begin{cases} V_j^2 & m = i \\ -V_i^2 & m = j \\ 0 & m \neq i, j \end{cases}$$

$$\frac{\partial Sh2_{ij}}{\partial V_m} = \begin{cases} -2Q_{jj}V_i & m = i \\ 2Q_{ii}V_j & m = j \\ 0 & m \neq i, j \end{cases}$$

$$\frac{\partial Sh2_{ij}}{\partial Q_{mm}} = \begin{cases} V_j^2 & m = i \\ -V_i^2 & m = j \\ 0 & m \neq i, j \end{cases}$$

G.7.9 Unknown transformer impedance

$$\frac{\partial Tr1_{ij}}{\partial V_m} = \begin{cases} 2cP_{jj}V_i & m = i \\ 2P_{ii}V_j & m = j \\ 0 & m \neq i, j \end{cases}$$

$$\frac{\partial Tr1_{ij}}{\partial P_{mm}} = \begin{cases} V_j^2 & m = i \\ cV_i^2 & m = j \\ 0 & m \neq i, j \end{cases}$$

$$\frac{\partial Tr2_{ij}}{\partial V_m} = \begin{cases} 2cQ_{jj}V_i & m = i \\ 2Q_{ii}V_j & m = j \\ 0 & m \neq i, j \end{cases}$$

$$\frac{\partial Tr2_{ij}}{\partial Q_{mm}} = \begin{cases} V_j^2 & m = i \\ cV_i^2 & m = j \\ 0 & m \neq i, j \end{cases}$$

Appendix H

Jacobian Derivatives in Rectangular Format

This chapter gives the Jacobian derivatives of the measurement equations $\mathbf{h}(\mathbf{x})$, used in the numerical process for the state estimator. These derivatives will be given in rectangular co-ordinates format. A short overview of the different measurement equations used, will be given. The Jacobian derivatives will be given after that.

H.1 Power Flows

Let the voltage at any node i be $\underline{V}_i = e_i + jf_i$. Use the following notations:

$$\begin{aligned} \sin\theta_{ij} &= \sin(\theta_i - \theta_j) \\ &= \sin\theta_i \cos\theta_j - \cos\theta_i \sin\theta_j \,, \end{aligned}$$

and

$$\begin{aligned} \cos\theta_{ij} &= \cos(\theta_i - \theta_j) \\ &= \cos\theta_i \cos\theta_j + \sin\theta_i \sin\theta_j \,. \end{aligned}$$

The power flow equations in polar co-ordinates format can be rewritten in rectangular co-ordinates format by using $e_i = V_i \cos\theta_i$ and $f_i = V_i \sin\theta_i$ for any node i:

$$P_{ij} = (G'_{ij} + G_{ij})(e_i^2 + f_i^2) - f_i(f_j G_{ij} + e_j B_{ij}) - e_i(e_j G_{ij} - f_j B_{ij}) \,,$$

and

$$Q_{ij} = -(B'_{ij} + B_{ij})(e_i^2 + f_i^2) - f_i(e_j G_{ij} - f_j B_{ij}) + e_i(f_j G_{ij} + e_j B_{ij}) \,.$$

H.2 Power Injections

The power injections at node i are:

$$P_i = G_{ii}(e_i^2 + f_i^2) - e_i \sum_{\substack{j=1 \\ j\neq i}}^{N} (e_j G_{ij} - f_j B_{ij}) - f_i \sum_{\substack{j=1 \\ j\neq i}}^{N} (f_j G_{ij} + e_j B_{ij}) ,$$

and

$$Q_i = -B_{ii}(e_i^2 + f_i^2) - f_i \sum_{\substack{j=1 \\ j\neq i}}^{N} (e_j G_{ij} - f_j B_{ij}) + e_i \sum_{\substack{j=1 \\ j\neq i}}^{N} (f_j G_{ij} + e_j B_{ij}) ,$$

where $G_{ii} = \sum_{j\neq i} (G_{ij} + G'_{ij})$ and $B_{ii} = \sum_{j\neq i} (B_{ij} + B'_{ij})$.

H.3 Modelling of zero impedance branches

If there is a zero impedance branch between nodes i and j, the power flows P_{ij} and Q_{ij} for this particular branch are used as state variables. The injection measurement equations for nodes i and j will be:

$$P_i = \sum_{\substack{s=1 \\ s\neq j}}^{N} P_{is} + P_{ij} \ , \ Q_i^M = \sum_{\substack{s=1 \\ s\neq j}}^{N} Q_{is} + Q_{ij} ,$$

and

$$P_j = \sum_{\substack{s=1 \\ s\neq i}}^{N} P_{js} + P_{ji} \ , \ Q_j^M = \sum_{\substack{s=1 \\ s\neq i}}^{N} Q_{js} + Q_{ji} ,$$

where for the power flows between nodes i and j, the values of the state variables P_{ij} and Q_{ij} are used instead of calculating the power flows by using the power flow measurement equations. Since branch ij is a zero impedance branch, $P_{ji} = P_{ij}$ and $Q_{ji} = Q_{ij}$. Because of a zero voltage drop, also add the following pseudo-measurements as constraints:

$$e_{ij} = e_i - e_j = 0 ,$$

and

$$f_{ij} = f_i - f_j = 0 .$$

H.4 Modelling of circuit breakers

H.4.1 Circuit breakers in series with a non-zero impedance branch

Introduce the circuit breaker variable κ, which indicates the position of the switch. If $\kappa = 0$, the breaker is open and if $\kappa = 1$, the breaker is closed. Locate the branch where the circuit breaker is installed, and substitute the branch parameters G, B, G' and B' with κG, κB, $\kappa G'$ and $\kappa B'$. To force the variable κ to either the value of 0 or 1, introduce the following pseudo-measurement as a constraint:

$$\mathsf{K}_{ij} = \kappa_{ij}(1 - \kappa_{ij}) = 0\,.$$

The power flow equations for this particular branch ij will be:

$$P_{ij}(\kappa) = \kappa_{ij} P_{ij}\,,$$

and

$$Q_{ij}(\kappa) = \kappa_{ij} Q_{ij}\,,$$

where P_{ij} and Q_{ij} are the conventional power flows calculated when $\kappa = 1$. The indices i and j indicate that the circuit breaker is connected between nodes i and j, meaning that $\kappa_{ij} = \kappa_{ji}$, and also that κ_{ij} and κ_{ji} are the same variable. The power injections at node i are:

$$P_i = \sum_{\substack{s=1 \\ s \neq j}}^{N} P_{is} + \kappa_{ij} P_{ij}\,,$$

and

$$Q_i = \sum_{\substack{s=1 \\ s \neq j}}^{N} Q_{is} + \kappa_{ij} Q_{ij}\,.$$

H.4.2 Circuit breakers in series with a zero impedance branch

Model the zero impedance branch as discussed earlier, and then use the power flows through and the voltage over the circuit breaker to determine the position of the breaker. For an open breaker the following equations hold:

$$P_{ij} = 0 \text{ and } Q_{ij} = 0\,,$$

and for a closed breaker the following:

$$V_i - V_j = 0 \text{ and } \theta_i - \theta_j = 0\,.$$

H.5 Modelling of unknown series and shunt parameters

H.5.1 Series parameters

The following additional equations will be used:

$$Se1_{ij} = P'_{ij}(e_j^2 + f_j^2) + P'_{ji}(e_i e_j + f_i f_j) - Q'_{ji}(f_i e_j - e_i f_j) = 0\,,$$

and

$$Se2_{ij} = Q'_{ij}(e_j^2 + f_j^2) + P'_{ji}(f_i e_j - e_i f_j) + Q'_{ji}(e_i e_j + f_i f_j) = 0\,,$$

where P'_{ij}, Q'_{ij}, P'_{ji}, and Q'_{ji} will be the state variables. If the state variables are calculated, the series parameter $\underline{Y}_{ij}$ can be calculated as follows:

$$\underline{Y}_{ij} = \frac{P'_{ij} - jQ'_{ij}}{\underline{V}_i^* \left(\underline{V}_i - \underline{V}_j\right)}\,.$$

H.5.2 Shunt parameters

The following additional equations will be used:

$$Sh1_{ij} = P_{ii}(e_j^2 + f_j^2) - P_{jj}(e_i^2 + f_i^2) = 0\,,$$

and

$$Sh2_{ij} = Q_{ii}(e_j^2 + f_j^2) - Q_{jj}(e_i^2 + f_i^2) = 0\,,$$

where P_{ii}, Q_{ii}, P_{jj}, and Q_{jj} will be the state variables. If the state variables are calculated, the shunt parameters can be calculated as follows:

$$\underline{Y}_{ii} = \underline{Y}_{jj} = \frac{P_{ii} - jQ_{ii}}{e_i^2 + f_i^2}\,.$$

H.6 Modelling of transformer impedance when tap position is known

This modelling technique is a combination of series and shunt parameter modelling. The same equations as for the modelling of series parameters are used. But the shunt parameter equations need to be modified and they are given as:

$$Tr1_{ij} = P_{ii}(e_j^2 + f_j^2) + cP_{jj}(e_i^2 + f_i^2) = 0\,,$$

and

$$Tr2_{ij} = Q_{ii}(e_j^2 + f_j^2) + cQ_{jj}(e_i^2 + f_i^2) = 0\,,$$

where c is the known tap position and P'_{ij}, Q'_{ij}, P'_{ji}, Q'_{ji}, P_{ii}, Q_{ii}, P_{jj}, and Q_{jj} will be the state variables. If the state variables are calculated, the unknown parameter $\underline{Y}_T$ can be calculated as follows:

$$\underline{Y}_T = \frac{1}{c}\frac{P'_{ij} - jQ'_{ij}}{\underline{V}_i^* \left(\underline{V}_i - \underline{V}_j\right)}\,,$$

or

$$\underline{Y}_T = \frac{1}{c\,(c-1)}\frac{P_{ii} - jQ_{ii}}{e_i^2 + f_i^2}\,.$$

H.7 Jacobian Derivatives

This section will show the Jacobian derivatives for all the previous mentioned equations in this chapter.

The circuit breaker parameter κ is not shown in all the equations where circuit breakers are modelled. It is only shown where the parameters G, B, G' and B' for a particular branch are replaced with κG, κB, $\kappa G'$ and $\kappa B'$. If the original values of G, B, G' and B' are used as a result for calculating a derivative with respect to κ, it will be indicated by $^{(\dagger\dagger)}$. κ will only be shown where necessary. Also, only the non-zero derivatives will be shown. This means that if a particular derivative is zero for all possible situations, it will not be shown in the following derivations.

H.7.1 Power Injections

H.7.1.1 Active Power

$$\frac{\partial P_i}{\partial e_m} = \begin{cases} -e_i G_{im} - f_i B_{im} & m \neq i \\ 2e_i G_{ii} - \sum\limits_{\substack{s=1 \\ s \neq i}}^{N} (e_s G_{is} - f_s B_{is}) & m = i \\ 0 & m \neq i, \text{ where } im \text{ is a zero impedance branch} \end{cases}$$

$$\frac{\partial P_i}{\partial f_m} = \begin{cases} e_i B_{im} - f_i G_{im} & m \neq i \\ 2f_i G_{ii} - \sum\limits_{\substack{s=1 \\ s \neq i}}^{N} (f_s G_{is} + e_s B_{is}) & m = i \\ 0 & m \neq i, \text{ where } im \text{ is a zero impedance branch} \end{cases}$$

$$\frac{\partial P_i}{\partial \kappa_{im}} = \begin{cases} (G'_{im} + G_{im})(e_i^2 + f_i^2) - f_i(f_m G_{im} + e_m B_{im}) & \text{if there is a circuit} \\ -e_i(e_m G_{im} - f_m B_{im})\ (\dagger\dagger) & \text{breaker on line } im \\ 0 & \text{if there is no circuit} \\ & \text{breaker on line } im \end{cases}$$

$$\frac{\partial P_i}{\partial P_{mk}} = \begin{cases} 1 & m = i \\ -1 & k = i \quad (\text{if } mk \text{ is a zero impedance branch}) \\ 0 & m, k \neq i \end{cases}$$

$$\frac{\partial P_i}{\partial P'_{ij}} = 1 \text{ (if } ij \text{ is a line where the series impedance is modelled, else 0)}$$

$$\frac{\partial P_i}{\partial P_{ii}} = 1 \text{ (if } P_{ii} \text{ is on a line where the shunt impedance is modelled, else 0)}$$

H.7.1.2 Reactive Power

$$\frac{\partial Q_i}{\partial e_m} = \begin{cases} e_i B_{im} - f_i G_{im} & m \neq i \\ -2e_i B_{ii} + \sum\limits_{\substack{s=1 \\ s \neq i}}^{N} (f_s G_{is} + e_s B_{is}) & m = i \\ 0 & m \neq i, \text{ where } im \text{ is a zero impedance branch} \end{cases}$$

$$\frac{\partial Q_i}{\partial f_m} = \begin{cases} f_i B_{im} + e_i G_{im} & m \neq i \\ -2f_i B_{ii} - \sum\limits_{\substack{s=1 \\ s \neq i}}^{N} (e_s G_{is} - f_s B_{is}) & m = i \\ 0 & m \neq i, \text{ where } im \text{ is a zero impedance branch} \end{cases}$$

$$\frac{\partial Q_i}{\partial \kappa_{im}} = \begin{cases} -(B'_{im} + B_{im})(e_i^2 + f_i^2) - f_i(e_m G_{im} - f_m B_{im}) \\ +e_i(f_m G_{im} + e_m B_{im}) \text{ (††)} & \text{if there is a circuit breaker on line } im \\ 0 & \text{if there is no circuit breaker on line } im \end{cases}$$

$$\frac{\partial Q_i}{\partial Q_{mk}} = \begin{cases} 1 & m = i \\ -1 & k = i \\ 0 & m, k \neq i \end{cases} \quad \text{(if } mk \text{ is a zero impedance branch)}$$

$$\frac{\partial Q_i}{\partial Q'_{ij}} = 1 \text{ (if } ij \text{ is a line where the series impedance is modelled, else 0)}$$

$$\frac{\partial Q_i}{\partial Q_{ii}} = 1 \text{ (if } Q_{ii} \text{ is on a line where the shunt impedance is modelled, else 0)}$$

H.7.2 Power Flows

H.7.2.1 Active Power

$$\frac{\partial P_{ij}}{\partial e_m} = \begin{cases} 2e_i G'_{ij} + (2e_i - e_j)G_{ij} + f_j B_{ij} & m = i \\ -f_i B_{ij} - e_i G_{ij} & m = j \\ 0 & m \neq i, j \\ 0 & \text{if } ij \text{ is a zero impedance branch} \end{cases}$$

$$\frac{\partial P_{ij}}{\partial f_m} = \begin{cases} 2f_i G'_{ij} + (2f_i - f_j)G_{ij} - e_j B_{ij} & m = i \\ e_i B_{ij} - f_i G_{ij} & m = j \\ 0 & m \neq i, j \\ 0 & \text{if } ij \text{ is a zero impedance branch} \end{cases}$$

$$\frac{\partial P_{ij}}{\partial \kappa_{ij}} = \begin{cases} (G'_{ij} + G_{ij})(e_i^2 + f_i^2) - f_i(f_j G_{ij} + e_j B_{ij}) - e_i(e_j G_{ij} - f_j B_{ij}) \text{ (††)} & \text{if there is a circuit breaker on line } ij \\ 0 & \text{if there is no circuit breaker on line } ij \end{cases}$$

$$\frac{\partial P_{ij}}{\partial P_{mk}} = \begin{cases} 1 & m = i, k = j \\ -1 & m = j, k = i \quad \text{(if } mk \text{ is a zero impedance branch)} \\ 0 & m, k \neq i, j \end{cases}$$

$\frac{\partial P_{ij}}{\partial P'_{ij}} = 1$ (if ij is a line where the series impedance is modelled, else 0)

$\frac{\partial P_{ij}}{\partial P_{ii}} = 1$ (if P_{ii} is on a line where the shunt impedance is modelled, else 0)

H.7.2.2 Reactive Power

$$\frac{\partial Q_{ij}}{\partial e_m} = \begin{cases} -2e_i B'_{ij} + (e_j - 2e_i)B_{ij} + f_j G_{ij} & m = i \\ e_i B_{ij} - f_i G_{ij} & m = j \\ 0 & m \neq i, j \\ 0 & \text{if } ij \text{ is a zero impedance branch} \end{cases}$$

$$\frac{\partial Q_{ij}}{\partial f_m} = \begin{cases} -2f_i B'_{ij} + (f_j - 2f_i)B_{ij} - e_j G_{ij} & m = i \\ e_i G_{ij} + f_i B_{ij} & m = j \\ 0 & m \neq i, j \\ 0 & \text{if } ij \text{ is a zero impedance branch} \end{cases}$$

$$\frac{\partial Q_{ij}}{\partial \kappa_{ij}} = \begin{cases} -(B'_{ij} + B_{ij})(e_i^2 + f_i^2) - f_i(e_j G_{ij} - f_j B_{ij}) + e_i(f_j G_{ij} + e_j B_{ij})^{(\dagger\dagger)} & \text{if there is a circuit breaker on line } ij \\ 0 & \text{if there is no circuit breaker on line } ij \end{cases}$$

$$\frac{\partial Q_{ij}}{\partial Q_{mk}} = \begin{cases} 1 & m = i, k = j \\ -1 & m = j, k = i \quad \text{(if } mk \text{ is a zero impedance branch)} \\ 0 & m, k \neq i, j \end{cases}$$

$\frac{\partial Q_{ij}}{\partial Q'_{ij}} = 1$ (if ij is a line where the series impedance is modelled, else 0)

$\frac{\partial Q_{ij}}{\partial Q_{ii}} = 1$ (if Q_{ii} is on a line where the shunt impedance is modelled, else 0)

H.7.3 Voltage Magnitude

Square the voltage magnitude:

$$V_i^2 = e_i^2 + f_i^2.$$

The derivatives of V_i^2 are:

$$\frac{\partial V_i^2}{\partial e_m} = \begin{cases} 2e_i & m = i \\ 0 & m \neq i \end{cases}$$

$$\frac{\partial V_i^2}{\partial f_m} = \begin{cases} 2f_i & m = i \\ 0 & m \neq i \end{cases}$$

H.7.4 Equations modelling Zero Impedance branches

$$\frac{\partial e_{ij}}{\partial e_m} = \begin{cases} 1 & m = i \\ -1 & m = j \\ 0 & m \neq i, j \end{cases}$$

$$\frac{\partial f_{ij}}{\partial f_m} = \begin{cases} 1 & m = i \\ -1 & m = j \\ 0 & m \neq i, j \end{cases}$$

H.7.5 Circuit breakers in series with a non-zero impedance branch

$$\frac{\partial \mathsf{K}_{ij}}{\partial \kappa_{mk}} = \begin{cases} 1 - 2\kappa_{ij} & m, k = i, j \\ 0 & m, k \neq i, j \end{cases}$$

H.7.6 Unknown Series parameters

$$\frac{\partial Se1_{ij}}{\partial e_m} = \begin{cases} P'_{ji} e_j + Q'_{ji} f_j & m = i \\ 2P'_{ij} e_j + P'_{ji} e_i - Q'_{ji} f_i & m = j \\ 0 & m \neq i, j \end{cases}$$

$$\frac{\partial Se1_{ij}}{\partial f_m} = \begin{cases} P'_{ji} f_j - Q'_{ji} e_j & m = i \\ 2P'_{ij} f_j + P'_{ji} f_i + Q'_{ji} e_i & m = j \\ 0 & m \neq i, j \end{cases}$$

$$\frac{\partial Se1_{ij}}{\partial P'_{mk}} = \begin{cases} e_j^2 + f_j^2 & m = i, k = j \\ e_i e_j + f_i f_j & m = j, k = i \\ 0 & m, k \neq i, j \end{cases}$$

$$\frac{\partial Se1_{ij}}{\partial Q'_{mk}} = \begin{cases} 0 & m = i, k = j \\ -f_i e_j + e_i f_j & m = j, k = i \\ 0 & m, k \neq i, j \end{cases}$$

$$\frac{\partial Se2_{ij}}{\partial e_m} = \begin{cases} -P'_{ji} f_j + Q'_{ji} e_j & m = i \\ 2Q'_{ij} e_j + P'_{ji} f_i + Q'_{ji} e_i & m = j \\ 0 & m \neq i, j \end{cases}$$

$$\frac{\partial Se2_{ij}}{\partial f_m} = \begin{cases} P'_{ji} e_j + Q'_{ji} f_j & m = i \\ 2Q'_{ij} f_j - P'_{ji} e_i + Q'_{ji} f_i & m = j \\ 0 & m \neq i, j \end{cases}$$

$$\frac{\partial Se2_{ij}}{\partial P'_{mk}} = \begin{cases} 0 & m = i, k = j \\ f_i e_j - e_i f_j & m = j, k = i \\ 0 & m, k \neq i, j \end{cases}$$

$$\frac{\partial Se2_{ij}}{\partial Q'_{mk}} = \begin{cases} e_j^2 + f_j^2 & m = i, k = j \\ e_i e_j + f_i f_j & m = j, k = i \\ 0 & m, k \neq i, j \end{cases}$$

H.7.7 Unknown Shunt parameters

$$\frac{\partial Sh1_{ij}}{\partial e_m} = \begin{cases} -2P_{jj}e_i & m = i \\ 2P_{ii}e_j & m = j \\ 0 & m \neq i, j \end{cases}$$

$$\frac{\partial Sh1_{ij}}{\partial f_m} = \begin{cases} -2P_{jj}f_i & m = i \\ 2P_{ii}f_j & m = j \\ 0 & m \neq i, j \end{cases}$$

$$\frac{\partial Sh1_{ij}}{\partial P_{mm}} = \begin{cases} e_j^2 + f_j^2 & m = i \\ -e_i^2 - f_i^2 & m = j \\ 0 & m \neq i, j \end{cases}$$

$$\frac{\partial Sh2_{ij}}{\partial e_m} = \begin{cases} -2Q_{jj}e_i & m = i \\ 2Q_{ii}e_j & m = j \\ 0 & m \neq i, j \end{cases}$$

$$\frac{\partial Sh2_{ij}}{\partial f_m} = \begin{cases} -2Q_{jj}f_i & m = i \\ 2Q_{ii}f_j & m = j \\ 0 & m \neq i, j \end{cases}$$

$$\frac{\partial Sh2_{ij}}{\partial Q_{mm}} = \begin{cases} e_j^2 + f_j^2 & m = i \\ -e_i^2 - f_i^2 & m = j \\ 0 & m \neq i, j \end{cases}$$

H.7.8 Unknown transformer impedance

$$\frac{\partial Tr1_{ij}}{\partial e_m} = \begin{cases} 2cP_{jj}e_i & m = i \\ 2P_{ii}e_j & m = j \\ 0 & m \neq i,j \end{cases}$$

$$\frac{\partial Tr1_{ij}}{\partial f_m} = \begin{cases} 2cP_{jj}f_i & m = i \\ 2P_{ii}f_j & m = j \\ 0 & m \neq i,j \end{cases}$$

$$\frac{\partial Tr1_{ij}}{\partial P_{mm}} = \begin{cases} e_j^2 + f_j^2 & m = i \\ c(e_i^2 + f_i^2) & m = j \\ 0 & m \neq i,j \end{cases}$$

$$\frac{\partial Tr2_{ij}}{\partial e_m} = \begin{cases} 2cQ_{jj}e_i & m = i \\ 2Q_{ii}e_j & m = j \\ 0 & m \neq i,j \end{cases}$$

$$\frac{\partial Tr2_{ij}}{\partial f_m} = \begin{cases} 2cQ_{jj}f_i & m = i \\ 2Q_{ii}f_j & m = j \\ 0 & m \neq i,j \end{cases}$$

$$\frac{\partial Tr2_{ij}}{\partial Q_{mm}} = \begin{cases} e_j^2 + f_j^2 & m = i \\ c(e_i^2 + f_i^2) & m = j \\ 0 & m \neq i,j \end{cases}$$

Appendix I

IEEE Data for Test Cases

I.1 IEEE 14-bus Network

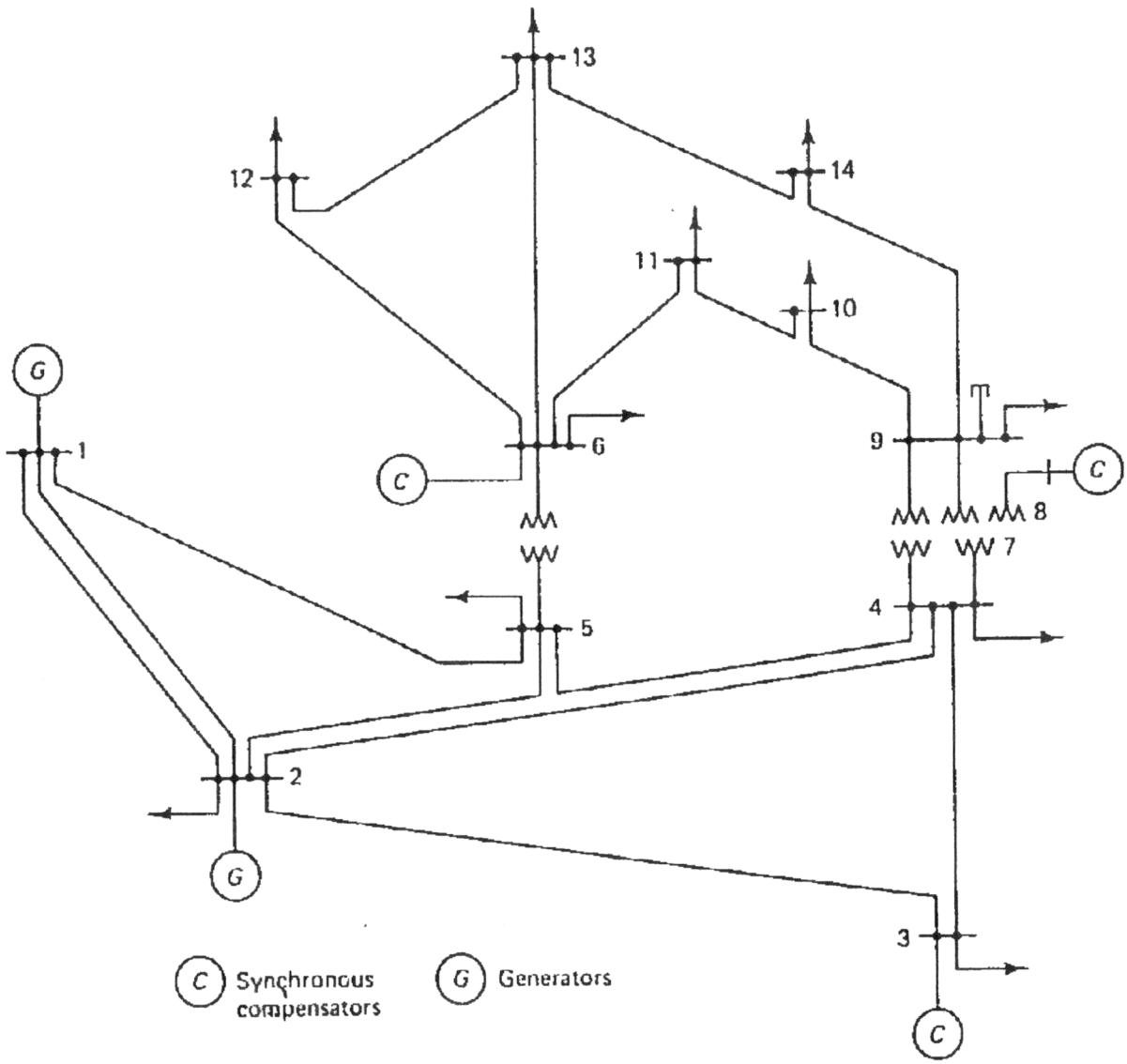
Figure I.1 shows the IEEE 14-bus network.

Table I.1 shows the branch data used for the 14-bus network.

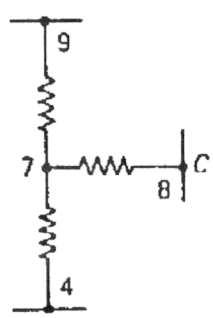
Table I.1: Branch data for the 14-bus network

Node i	Node j	Resistance p.u.	Reactance p.u.	Line charging p.u.
1	2	0.01938	0.05917	0.0264
1	5	0.05403	0.22304	0.0246
2	3	0.04699	0.19797	0.0219
2	4	0.05811	0.17632	0.0187
2	5	0.05695	0.17388	0.0170
3	4	0.06701	0.17103	0.0173
4	5	0.01335	0.04211	0.0064
4	7	0.00000	0.20912	0.0000
4	9	0.00000	0.55618	0.0000
5	6	0.00000	0.25202	0.0000
6	11	0.09498	0.19890	0.0000
6	12	0.12291	0.25581	0.0000
6	13	0.06615	0.13027	0.0000
7	8	0.00000	0.17615	0.0000
7	9	0.00000	0.11001	0.0000
9	10	0.03181	0.08450	0.0000
9	14	0.12711	0.27038	0.0000
10	11	0.08205	0.19207	0.0000
12	13	0.22092	0.19988	0.0000
13	14	0.17093	0.34802	0.0000

Table I.2 shows the operating conditions for the 14-bus network.



(a) Bus-code diagram



(b) 3-winding transformer equivalent

Figure I.1: IEEE 14-bus network

Table I.2: Operating conditions for the 14-bus network

Bus number	Starting bus voltage		Generation		Load	
	Magnitude p.u.	Phase angle	MW	MVAr	MW	MVAr
1	1.06	0.00	0.00	0.00	0.00	0.00
2	1.00	0.00	40.00	0.00	21.70	12.70
3	1.00	0.00	0.00	0.00	94.20	19.00
4	1.00	0.00	0.00	0.00	47.80	3.90
5	1.00	0.00	0.00	0.00	7.60	1.80
6	1.00	0.00	0.00	0.00	11.20	7.50
7	1.00	0.00	0.00	0.00	0.00	0.00
8	1.00	0.00	0.00	0.00	0.00	0.00
9	1.00	0.00	0.00	0.00	29.50	16.60
10	1.00	0.00	0.00	0.00	9.00	5.80
11	1.00	0.00	0.00	0.00	3.50	1.80
12	1.00	0.00	0.00	0.00	6.10	1.60
13	1.00	0.00	0.00	0.00	13.50	5.80
14	1.00	0.00	0.00	0.00	14.90	5.60

Table I.3 shows the regulated bus data for the 14-bus network.

Table I.3: Regulated bus data for the 14-bus network

Bus Number	Voltage Magnitude p.u.	Minimum MVAr capability	Maximum MVAr capability
2	1.045	–40.0	50.0
3	1.010	0.0	40.0
6	1.070	–6.0	24.0
8	1.090	–6.0	24.0

Table I.4 shows the transformer data for the 14-bus network.

Table I.4: Transformer data for the 14-bus network

Transformer designation		Tap setting
Node i	Node j	
4	7	0.978
4	9	0.969
5	6	0.932

I.2 IEEE 30-bus Network

Figure I.2 shows the IEEE 30-bus network.

Table I.5 shows the branch data used for the 30-bus network.

Table I.6 shows the operating conditions for the 30-bus network.

Table I.7 shows the regulated bus data for the 30-bus network.

Table I.8 shows the transformer data for the 30-bus network.

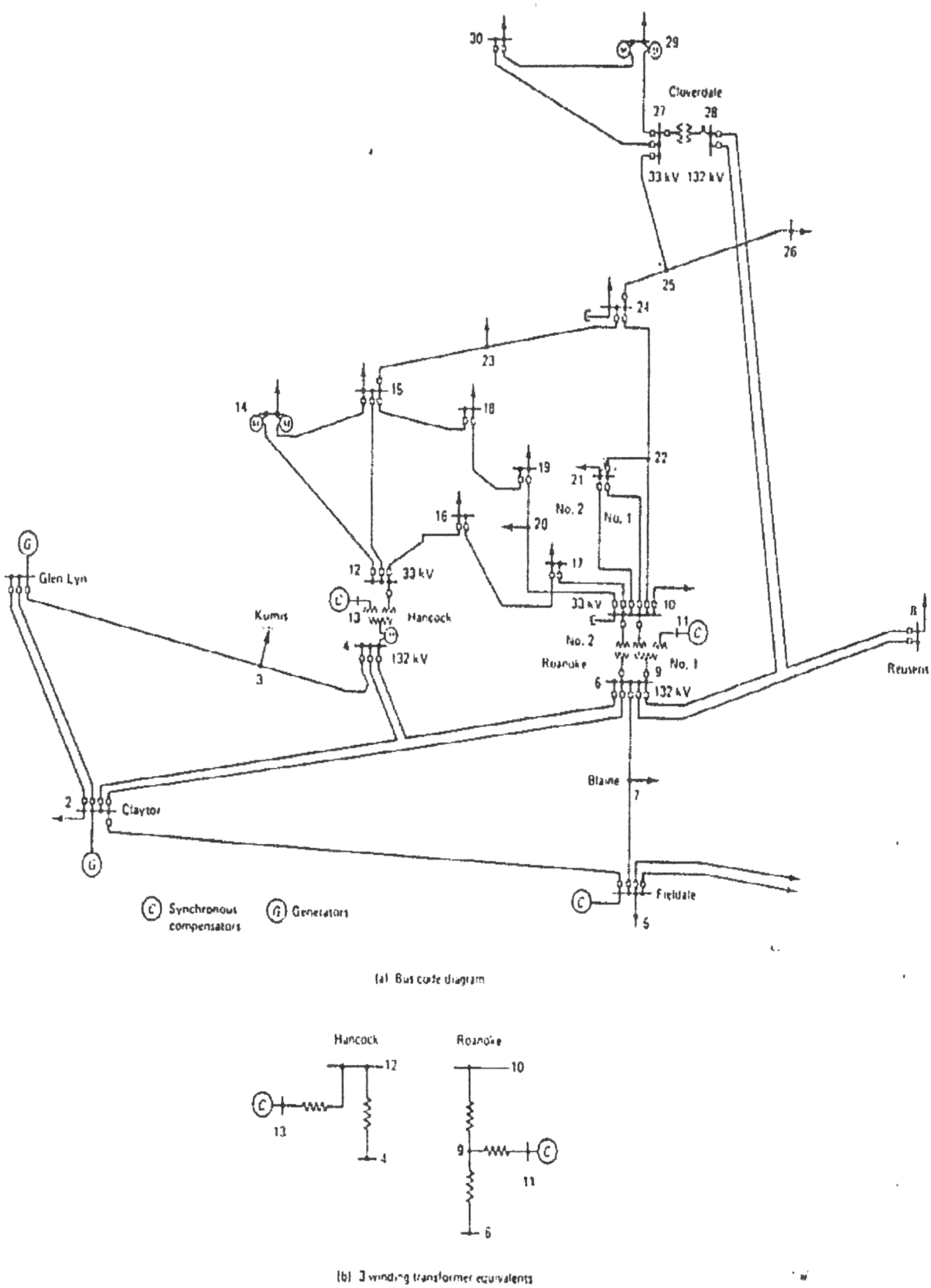


Figure I.2: IEEE 30-bus network

Table I.5: Branch data for the 30-bus network

Node i	Node j	Resistance p.u.	Reactance p.u.	Line charging p.u.
1	2	0.0192	0.0575	0.0264
1	3	0.0452	0.1852	0.0204
2	4	0.0570	0.1737	0.0184
3	4	0.0132	0.0379	0.0042
2	5	0.0472	0.1983	0.0209
2	6	0.0581	0.1763	0.0187
4	6	0.0119	0.0414	0.0045
5	7	0.0460	0.1160	0.0102
6	7	0.0267	0.0820	0.0085
6	8	0.0120	0.0420	0.0045
6	9	0.0000	0.2080	0.0000
6	10	0.0000	0.5560	0.0000
9	11	0.0000	0.2080	0.0000
9	10	0.0000	0.1100	0.0000
4	12	0.0000	0.2560	0.0000
12	13	0.0000	0.1400	0.0000
12	14	0.1231	0.2559	0.0000
12	15	0.0662	0.1304	0.0000
12	16	0.0945	0.1987	0.0000
14	15	0.2210	0.1997	0.0000
16	17	0.0824	0.1923	0.0000
15	18	0.1073	0.2185	0.0000
18	19	0.0639	0.1292	0.0000
19	20	0.0340	0.0680	0.0000
10	20	0.0936	0.2090	0.0000
10	17	0.0324	0.0845	0.0000
10	21	0.0348	0.0749	0.0000
10	22	0.0727	0.1499	0.0000
21	22	0.0116	0.0236	0.0000
15	23	0.1000	0.2020	0.0000
22	24	0.1150	0.1790	0.0000
23	24	0.1320	0.2700	0.0000
24	25	0.1885	0.3292	0.0000
25	26	0.2544	0.3800	0.0000
25	27	0.1093	0.2087	0.0000
28	27	0.0000	0.3960	0.0000
27	29	0.2198	0.4153	0.0000
27	30	0.3202	0.6027	0.0000
29	30	0.2399	0.4533	0.0000
8	28	0.0636	0.2000	0.0214
6	28	0.0169	0.0599	0.0065

Table I.6: Operating conditions for the 30-bus network

Bus number	Starting bus voltage		Generation		Load	
	Magnitude p.u.	Phase angle	MW	MVAr	MW	MVAr
1	1.06	0.00	0.00	0.00	0.00	0.00
2	1.00	0.00	40.00	0.00	21.70	12.70
3	1.00	0.00	0.00	0.00	2.40	1.20
4	1.00	0.00	0.00	0.00	7.60	1.60
5	1.00	0.00	0.00	0.00	94.20	19.00
6	1.00	0.00	0.00	0.00	0.00	0.00
7	1.00	0.00	0.00	0.00	22.80	10.90
8	1.00	0.00	0.00	0.00	30.00	30.00
9	1.00	0.00	0.00	0.00	0.00	0.00
10	1.00	0.00	0.00	0.00	5.80	2.00
11	1.00	0.00	0.00	0.00	0.00	0.00
12	1.00	0.00	0.00	0.00	11.20	7.50
13	1.00	0.00	0.00	0.00	0.00	0.00
14	1.00	0.00	0.00	0.00	6.20	1.60
15	1.00	0.00	0.00	0.00	8.20	2.50
16	1.00	0.00	0.00	0.00	3.50	1.80
17	1.00	0.00	0.00	0.00	9.00	5.80
18	1.00	0.00	0.00	0.00	3.20	0.90
19	1.00	0.00	0.00	0.00	9.50	3.40
20	1.00	0.00	0.00	0.00	2.20	0.70
21	1.00	0.00	0.00	0.00	17.50	11.20
22	1.00	0.00	0.00	0.00	0.00	0.00
23	1.00	0.00	0.00	0.00	3.20	1.60
24	1.00	0.00	0.00	0.00	8.70	6.70
25	1.00	0.00	0.00	0.00	0.00	0.00
26	1.00	0.00	0.00	0.00	3.50	2.30
27	1.00	0.00	0.00	0.00	0.00	0.00
28	1.00	0.00	0.00	0.00	0.00	0.00
29	1.00	0.00	0.00	0.00	2.40	0.90
30	1.00	0.00	0.00	0.00	10.60	1.90

Table I.7: Regulated bus data for the 30-bus network

Bus Number	Voltage Magnitude p.u.	Minimum MVAr capability	Maximum MVAr capability
2	1.045	–40.0	50.0
5	1.010	–40.0	40.0
8	1.010	–10.0	40.0
11	1.082	–6.0	24.0
13	1.071	–6.0	24.0

Table I.8: Transformer data for the 30-bus network

Transformer designation		Tap setting
Node *i*	Node *j*	
4	12	0.932
6	9	0.978
6	10	0.969
28	27	0.968

I.3 IEEE 57-bus Network

Figure I.3 shows the IEEE 57-bus network.

Table I.9 and Table I.10 show the branch data used for the 57-bus network.

Table I.11 and Table I.12 show the operating conditions for the 57-bus network.

Table I.13 shows the regulated bus data for the 57-bus network.

Table I.14 shows the transformer data for the 57-bus network.

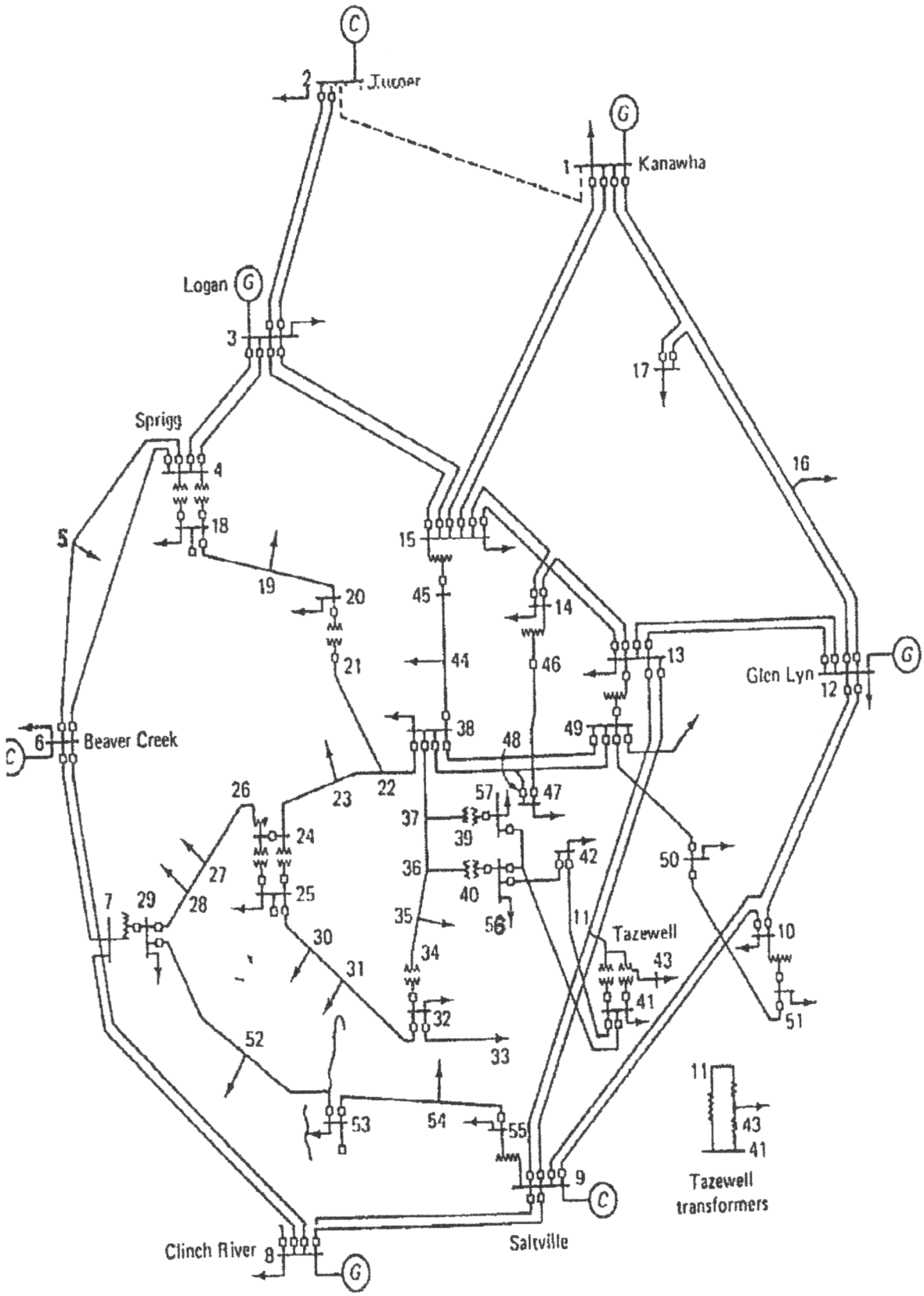


Figure I.3: IEEE 57-bus network

Table I.9: Branch data for the 57-bus network

Node i	Node j	Resistance p.u.	Reactance p.u.	Line charging p.u.
1	2	0.0083	0.0280	0.0645
2	3	0.0298	0.0850	0.0409
3	4	0.0112	0.0366	0.0190
4	5	0.0625	0.1320	0.0129
4	6	0.0430	0.1480	0.0174
6	7	0.0200	0.1020	0.0138
6	8	0.0339	0.1730	0.0235
8	9	0.0099	0.0505	0.0274
9	10	0.0369	0.1679	0.0220
9	11	0.0258	0.0848	0.0109
9	12	0.0648	0.2950	0.0386
9	13	0.0481	0.1580	0.0203
13	14	0.0132	0.0434	0.0055
13	15	0.0269	0.0869	0.0115
1	15	0.0178	0.0910	0.0494
1	16	0.0454	0.2060	0.0273
1	17	0.0238	0.1080	0.0143
3	15	0.0162	0.0530	0.0272
4	18	0.0000	0.5550	0.0000
4	18	0.0000	0.4300	0.0000
5	6	0.0302	0.0641	0.0062
7	8	0.0139	0.0712	0.0097
10	12	0.0277	0.1262	0.0164
11	13	0.0223	0.0732	0.0094
12	13	0.0178	0.0580	0.0302
12	16	0.0180	0.0813	0.0108
12	17	0.0397	0.1790	0.0238
14	15	0.0171	0.0547	0.0074
18	19	0.4610	0.6850	0.0000
19	20	0.2830	0.4340	0.0000
21	20	0.0000	0.7767	0.0000
21	22	0.0736	0.1170	0.0000
22	23	0.0099	0.0152	0.0000
23	24	0.1660	0.2560	0.0042
24	25	0.0000	1.1820	0.0000
24	25	0.0000	1.2300	0.0000
24	26	0.0000	0.0473	0.0000
26	27	0.1650	0.2540	0.0000
27	28	0.0618	0.0954	0.0000

Table I.10: Branch data for the 57-bus network (Continued)

Node i	Node j	Resistance p.u.	Reactance p.u.	Line charging p.u.
28	29	0.0418	0.0587	0.0000
7	29	0.0000	0.0648	0.0000
25	30	0.1350	0.2020	0.0000
30	31	0.3260	0.4970	0.0000
31	32	0.5070	0.7550	0.0000
32	33	0.0392	0.0360	0.0000
34	32	0.0000	0.9530	0.0000
34	35	0.0520	0.0780	0.0016
35	36	0.0430	0.0537	0.0008
36	37	0.0290	0.0366	0.0000
37	38	0.0651	0.1009	0.0010
37	39	0.0239	0.0379	0.0000
36	40	0.0300	0.0466	0.0000
22	38	0.0192	0.0295	0.0000
11	41	0.0000	0.7490	0.0000
41	42	0.2070	0.3520	0.0000
41	43	0.0000	0.4120	0.0000
38	44	0.0289	0.0585	0.0010
15	45	0.0000	0.1042	0.0000
14	46	0.0000	0.0735	0.0000
46	47	0.0230	0.0680	0.0016
47	48	0.0182	0.0233	0.0000
48	49	0.0834	0.1290	0.0024
49	50	0.0801	0.1280	0.0000
50	51	0.1386	0.2200	0.0000
10	51	0.0000	0.0712	0.0000
13	49	0.0000	0.1910	0.0000
29	52	0.1442	0.1870	0.0000
52	53	0.0762	0.0984	0.0000
53	54	0.1878	0.2320	0.0000
54	55	0.1732	0.2265	0.0000
11	43	0.0000	0.1530	0.0000
44	45	0.0624	0.1242	0.0020
40	56	0.0000	1.1950	0.0000
56	41	0.5530	0.5490	0.0000
56	42	0.2125	0.3540	0.0000
39	57	0.0000	1.3550	0.0000
57	56	0.1740	0.2600	0.0000
38	49	0.1150	0.1770	0.0030
38	48	0.0312	0.0482	0.0000
9	55	0.0000	0.1205	0.0000

Table I.11: Operating conditions for the 57-bus network

Bus number	Starting bus voltage		Generation		Load	
	Magnitude p.u.	Phase angle	MW	MVAr	MW	MVAr
1	1.04	0.00	0.00	0.00	55.00	17.00
2	1.00	0.00	0.00	0.00	3.00	88.00
3	1.00	0.00	40.00	0.00	41.00	21.00
4	1.00	0.00	0.00	0.00	0.00	0.00
5	1.00	0.00	0.00	0.00	13.00	4.00
6	1.00	0.00	0.00	0.00	75.00	2.00
7	1.00	0.00	0.00	0.00	0.00	0.00
8	1.00	0.00	450.00	0.00	150.00	22.00
9	1.00	0.00	0.00	0.00	121.00	26.00
10	1.00	0.00	0.00	0.00	5.00	2.00
11	1.00	0.00	0.00	0.00	0.00	0.00
12	1.00	0.00	310.00	0.00	377.00	24.00
13	1.00	0.00	0.00	0.00	18.00	2.30
14	1.00	0.00	0.00	0.00	10.50	5.30
15	1.00	0.00	0.00	0.00	22.00	5.00
16	1.00	0.00	0.00	0.00	43.00	3.00
17	1.00	0.00	0.00	0.00	42.00	8.00
18	1.00	0.00	0.00	0.00	27.20	9.80
19	1.00	0.00	0.00	0.00	3.30	0.60
20	1.00	0.00	0.00	0.00	2.30	1.00
21	1.00	0.00	0.00	0.00	0.00	0.00
22	1.00	0.00	0.00	0.00	0.00	0.00
23	1.00	0.00	0.00	0.00	6.30	2.10
24	1.00	0.00	0.00	0.00	0.00	0.00
25	1.00	0.00	0.00	0.00	6.30	3.20
26	1.00	0.00	0.00	0.00	0.00	0.00
27	1.00	0.00	0.00	0.00	9.30	0.50
28	1.00	0.00	0.00	0.00	4.60	2.30

Table I.12: Operating conditions for the 57-bus network (Continued)

Bus number	Starting bus voltage		Generation		Load	
	Magnitude p.u.	Phase angle	MW	MVAr	MW	MVAr
29	1.00	0.00	0.00	0.00	17.00	2.60
30	1.00	0.00	0.00	0.00	3.60	1.80
31	1.00	0.00	0.00	0.00	5.80	2.90
32	1.00	0.00	0.00	0.00	1.60	0.80
33	1.00	0.00	0.00	0.00	3.80	1.90
34	1.00	0.00	0.00	0.00	0.00	0.00
35	1.00	0.00	0.00	0.00	6.00	3.00
36	1.00	0.00	0.00	0.00	0.00	0.00
37	1.00	0.00	0.00	0.00	0.00	0.00
38	1.00	0.00	0.00	0.00	14.00	7.00
39	1.00	0.00	0.00	0.00	0.00	0.00
40	1.00	0.00	0.00	0.00	0.00	0.00
41	1.00	0.00	0.00	0.00	6.30	3.00
42	1.00	0.00	0.00	0.00	7.10	4.40
43	1.00	0.00	0.00	0.00	2.00	1.00
44	1.00	0.00	0.00	0.00	12.00	1.80
45	1.00	0.00	0.00	0.00	0.00	0.00
46	1.00	0.00	0.00	0.00	0.00	0.00
47	1.00	0.00	0.00	0.00	29.70	11.60
48	1.00	0.00	0.00	0.00	0.00	0.00
49	1.00	0.00	0.00	0.00	18.00	8.50
50	1.00	0.00	0.00	0.00	21.00	10.50
51	1.00	0.00	0.00	0.00	18.00	5.30
52	1.00	0.00	0.00	0.00	4.90	2.20
53	1.00	0.00	0.00	0.00	20.00	10.00
54	1.00	0.00	0.00	0.00	4.10	1.40
55	1.00	0.00	0.00	0.00	6.80	3.40
56	1.00	0.00	0.00	0.00	7.60	2.20
57	1.00	0.00	0.00	0.00	6.70	2.00

Table I.13: Regulated bus data for the 57-bus network

Bus Number	Voltage Magnitude p.u.	Minimum MVAr capability	Maximum MVAr capability
2	1.010	-17.0	50.0
3	0.985	-10.0	60.0
6	0.980	-8.0	25.0
8	1.005	-140.0	200.0
9	0.980	-3.0	9.0
12	1.015	-50.0	155.0

Table I.14: Transformer data for the 57-bus network

Transformer designation		Tap setting
Node i	Node j	
4	18	0.970
4	18	0.978
7	29	0.967
9	55	0.940
10	51	0.930
11	41	0.955
11	43	0.958
13	49	0.895
14	46	0.900
15	45	0.955
21	20	1.043
24	25	1.000
24	25	1.000
24	26	1.043
34	32	0.975
39	57	0.980
40	56	0.958

Appendix J

Generation of Measurements

Figure J.1 illustrates the generation of the measurements for the state estimation process. These measurements are expressed as vector $\mathbf{z}$ in $\mathbf{z} = \mathbf{h(x)} + \mathbf{error}$.

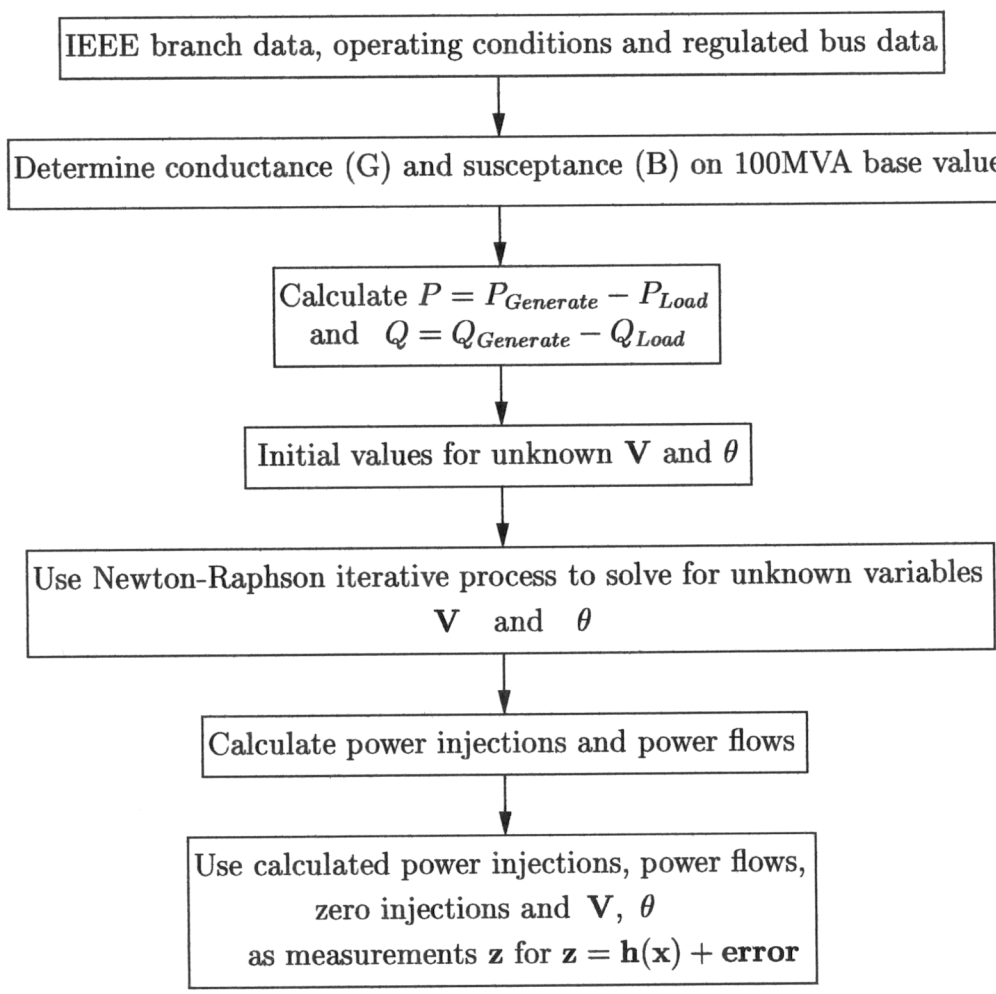


Figure J.1: Flow diagram indicating the generation of measurements

Appendix K

Measurements for Test Cases

The following tables will show the measurements used for the test cases.

The entries of the *Number* - column in each table, show the position of the measurement in the $\mathbf{z}$ vector.

The following different types of measurements are used for the test cases:

- Pi - Active power injection measurement at node i
- Pij - Active power flow measurement from node i to node j
- THETA - Phase angle difference between node i and node j
- Qi - Reactive power injection measurement at node i
- Qij - Reactive power flow measurement from node i to node j
- V - Voltage measurement at node i

The entries of the *Node(s)* - column illustrate the node(s) of the network used for the different measurements.

The IEEE 14-bus, IEEE 30-bus and IEEE 57-bus in rectangular formats, use the same measurements as for the polar formats, except that no THETA - measurements are used for the rectangular formats.

Table K.1: Measurements used for the IEEE 14-bus in polar format

Number	Type	Node(s)	Number	Type	Node(s)
1	Pi	1	38	Qi	4
2	Pi	3	39	Qi	7
3	Pi	4	40	Qi	9
4	Pi	7	41	Qi	11
5	Pi	8	42	Qi	14
6	Pi	9	43	Qij	1–2
7	Pi	11	44	Qij	2–3
8	Pi	14	45	Qij	2–4
9	Pij	1–2	46	Qij	3–4
10	Pij	2–3	47	Qij	4–2
11	Pij	2–4	48	Qij	4–5
12	Pij	3–4	49	Qij	4–7
13	Pij	4–2	50	Qij	5–1
14	Pij	4–5	51	Qij	5–2
15	Pij	4–7	52	Qij	5–6
16	Pij	5–1	53	Qij	6–5
17	Pij	5–2	54	Qij	6–11
18	Pij	5–6	55	Qij	6–13
19	Pij	6–5	56	Qij	8–7
20	Pij	6–11	57	Qij	9–4
21	Pij	6–13	58	Qij	9–7
22	Pij	8–7	59	Qij	9–10
23	Pij	9–4	60	Qij	9–14
24	Pij	9–7	61	Qij	10–11
25	Pij	9–10	62	Qij	12–6
26	Pij	9–14	63	Qij	12–13
27	Pij	10–11	64	Qij	13–14
28	Pij	12–6	65	Qij	14–9
29	Pij	12–13	66	Qij	14–13
30	Pij	13–14	67	V	1
31	Pij	14–9	68	V	2
32	Pij	14–13	69	V	3
33	THETA	1–2	70	V	6
34	THETA	4–9	71	V	8
35	THETA	5–6	72	V	10
36	Qi	1	73	V	12
37	Qi	3			

Table K.2: Measurements used for the IEEE 30-bus in polar format

Number	Type	Node(s)	Number	Type	Node(s)
1	Pi	1	36	Pij	14–15
2	Pi	2	37	Pij	16–17
3	Pi	3	38	Pij	18–19
4	Pi	4	39	Pij	19–20
5	Pi	5	40	Pij	21–22
6	Pi	6	41	Pij	15–23
7	Pi	7	42	Pij	22–24
8	Pi	8	43	Pij	23–24
9	Pi	9	44	Pij	24–25
10	Pi	10	45	Pij	25–26
11	Pi	11	46	Pij	25–27
12	Pi	13	47	Pij	27–28
13	Pi	14	48	Pij	27–29
14	Pi	18	49	Pij	29–30
15	Pi	22	50	THETA	1–2
16	Pi	25	51	THETA	5–7
17	Pi	27	52	THETA	6–10
18	Pi	28	53	THETA	9–11
19	Pij	1–2	54	THETA	4–12
20	Pij	2–4	55	THETA	12–13
21	Pij	3–4	56	THETA	15–18
22	Pij	2–5	57	THETA	22–24
23	Pij	4–6	58	THETA	27–30
24	Pij	5–7	59	Qi	1
25	Pij	6–7	60	Qi	2
26	Pij	6–8	61	Qi	3
27	Pij	6–9	62	Qi	4
28	Pij	6–10	63	Qi	5
29	Pij	9–11	64	Qi	6
30	Pij	11–9	65	Qi	7
31	Pij	12–13	66	Qi	8
32	Pij	13–12	67	Qi	9
33	Pij	12–14	68	Qi	10
34	Pij	12–15	69	Qi	11
35	Pij	12–16	70	Qi	13

Table K.3: Measurements used for the IEEE 30-bus in polar format (Continued)

Number	Type	Node(s)	Number	Type	Node(s)
71	Qi	14	99	Qij	15–23
72	Qi	18	100	Qij	22–24
73	Qi	22	101	Qij	23–24
74	Qi	25	102	Qij	24–25
75	Qi	27	103	Qij	25–26
76	Qi	28	104	Qij	25–27
77	Qij	1–2	105	Qij	27–28
78	Qij	2–4	106	Qij	27–29
79	Qij	3–4	107	Qij	29–30
80	Qij	2–5	108	V	1
81	Qij	4–6	109	V	2
82	Qij	5–7	110	V	3
83	Qij	6–7	111	V	6
84	Qij	6–8	112	V	8
85	Qij	6–9	113	V	10
86	Qij	6–10	114	V	11
87	Qij	9–11	115	V	12
88	Qij	11–9	116	V	13
89	Qij	12–13	117	V	18
90	Qij	13–12	118	V	20
91	Qij	12–14	119	V	21
92	Qij	12–15	120	V	22
93	Qij	12–16	121	V	23
94	Qij	14–15	122	V	24
95	Qij	16–17	123	V	25
96	Qij	18–19	124	V	26
97	Qij	19–20	125	V	27
98	Qij	21–22			

Table K.4: Measurements used for the IEEE 57-bus in polar format

Number	Type	Node(s)	Number	Type	Node(s)
1	Pi	1	43	Pi	56
2	Pi	3	44	Pi	57
3	Pi	4	45	Pij	1–2
4	Pi	5	46	Pij	2–3
5	Pi	7	47	Pij	3–4
6	Pi	8	48	Pij	4–5
7	Pi	10	49	Pij	6–7
8	Pi	11	50	Pij	8–9
9	Pi	12	51	Pij	9–10
10	Pi	13	52	Pij	9–11
11	Pi	15	53	Pij	13–14
12	Pi	17	54	Pij	13–15
13	Pi	19	55	Pij	1–16
14	Pi	21	56	Pij	4–18
15	Pi	22	57	Pij	7–8
16	Pi	23	58	Pij	10–12
17	Pi	24	59	Pij	11–13
18	Pi	25	60	Pij	12–13
19	Pi	26	61	Pij	12–16
20	Pi	27	62	Pij	12–17
21	Pi	29	63	Pij	18–19
22	Pi	30	64	Pij	19–20
23	Pi	31	65	Pij	20–21
24	Pi	33	66	Pij	21–22
25	Pi	34	67	Pij	22–23
26	Pi	35	68	Pij	23–24
27	Pi	36	69	Pij	24–25
28	Pi	37	70	Pij	24–26
29	Pi	38	71	Pij	26–27
30	Pi	39	72	Pij	27–28
31	Pi	40	73	Pij	28–29
32	Pi	41	74	Pij	7–29
33	Pi	43	75	Pij	25–30
34	Pi	44	76	Pij	30–31
35	Pi	45	77	Pij	31–32
36	Pi	46	78	Pij	32–33
37	Pi	47	79	Pij	32–34
38	Pi	48	80	Pij	34–35
39	Pi	49	81	Pij	36–37
40	Pi	51	82	Pij	37–38
41	Pi	53	83	Pij	37–39
42	Pi	55	84	Pij	36–40

Table K.5: Measurements used for the IEEE 57-bus in polar format (Continued)

Number	Type	Node(s)	Number	Type	Node(s)
85	Pij	11–41	127	Qi	15
86	Pij	41–42	128	Qi	17
87	Pij	41–43	129	Qi	19
88	Pij	38–44	130	Qi	21
89	Pij	15–45	131	Qi	22
90	Pij	14–46	132	Qi	23
91	Pij	46–47	133	Qi	24
92	Pij	48–49	134	Qi	25
93	Pij	49–50	135	Qi	26
94	Pij	10–51	136	Qi	27
95	Pij	13–49	137	Qi	29
96	Pij	52–53	138	Qi	30
97	Pij	54–55	139	Qi	31
98	Pij	11–43	140	Qi	33
99	Pij	44–45	141	Qi	34
100	Pij	40–56	142	Qi	35
101	Pij	56–41	143	Qi	36
102	Pij	39–57	144	Qi	37
103	Pij	57–56	145	Qi	38
104	Pij	9–55	146	Qi	39
105	THETA	1–2	147	Qi	40
106	THETA	4–18	148	Qi	41
107	THETA	9–13	149	Qi	43
108	THETA	11–43	150	Qi	44
109	THETA	12–13	151	Qi	45
110	THETA	20–21	152	Qi	46
111	THETA	24–25	153	Qi	47
112	THETA	30–31	154	Qi	48
113	THETA	34–35	155	Qi	49
114	THETA	41–43	156	Qi	51
115	THETA	53–54	157	Qi	53
116	THETA	39–57	158	Qi	55
117	Qi	1	159	Qi	56
118	Qi	3	160	Qi	57
119	Qi	4	161	Qij	1–2
120	Qi	5	162	Qij	2–3
121	Qi	7	163	Qij	3–4
122	Qi	8	164	Qij	4–5
123	Qi	10	165	Qij	6–7
124	Qi	11	166	Qij	8–9
125	Qi	12	167	Qij	9–10
126	Qi	13	168	Qij	9–11

Table K.6: Measurements used for the IEEE 57-bus in polar format (Continued)

Number	Type	Node(s)	Number	Type	Node(s)
169	Qij	13–14	210	Qij	10–51
170	Qij	13–15	211	Qij	13–49
171	Qij	1–16	212	Qij	52–53
172	Qij	4–18	213	Qij	54–55
173	Qij	7–8	214	Qij	11–43
174	Qij	10–12	215	Qij	44–45
175	Qij	11–13	216	Qij	40–56
176	Qij	12–13	217	Qij	56–41
177	Qij	12–16	218	Qij	39–57
178	Qij	12–17	219	Qij	57–56
179	Qij	18–19	220	Qij	9–55
180	Qij	19–20	221	V	1
181	Qij	20–21	222	V	2
182	Qij	21–22	223	V	3
183	Qij	22–23	224	V	6
184	Qij	23–24	225	V	8
185	Qij	24–25	226	V	9
186	Qij	24–26	227	V	12
187	Qij	26–27	228	V	15
188	Qij	27–28	229	V	17
189	Qij	28–29	230	V	18
190	Qij	7–29	231	V	20
191	Qij	25–30	232	V	23
192	Qij	30–31	233	V	25
193	Qij	31–32	234	V	28
194	Qij	32–33	235	V	30
195	Qij	32–34	236	V	31
196	Qij	34–35	237	V	33
197	Qij	36–37	238	V	35
198	Qij	37–38	239	V	38
199	Qij	37–39	240	V	40
200	Qij	36–40	241	V	42
201	Qij	11–41	242	V	44
202	Qij	41–42	243	V	45
203	Qij	41–43	244	V	47
204	Qij	38–44	245	V	51
205	Qij	15–45	246	V	53
206	Qij	14–46	247	V	54
207	Qij	46–47	248	V	55
208	Qij	48–49	249	V	56
209	Qij	49–50	250	V	57

Appendix L

Augmented method

As mentioned in section 3.5, there can be buses with zero injections. These zero injections or pseudo-measurements become constraints of the type

$$\mathbf{c}(\mathbf{x}) = [c_1(\mathbf{x}), c_2(\mathbf{x}), \ldots, c_s(\mathbf{x})]^T = 0, \tag{L.1}$$

where $\mathbf{c}$ is a vector function with dimension s, and s is the number of pseudo-measurements. There are also other types of pseudo-measurements as discussed in section 5.3.2. These pseudo-measurements can be included in the objective function of equation (E.8). Equation (L.1) can also be treated as constraints in the minimisation of equation (E.8).

Let n be the number of state variables. Assign a vector λ of Lagrange multipliers to equation (L.1) and obtain the Lagrangian

$$\begin{aligned} L &= J(\mathbf{x}) - \lambda^T \mathbf{c}(\mathbf{x}) \\ &= \sum_i^m \left(\omega_i^2 \rho(\widetilde{r}_{s_i})\right) - \sum_i^s \lambda_i c_i(\mathbf{x}) . \end{aligned} \tag{L.2}$$

At the minimum, seek

$$\mathbf{L}_\lambda = \frac{\partial L}{\partial \lambda} = \mathbf{c}(\mathbf{x}) = 0, \tag{L.3}$$

which is the same as equation (L.1), and seek

$$\mathbf{L}_x = \frac{\partial L}{\partial \mathbf{x}} = 0, \tag{L.4}$$

which gives

$$\mathbf{L}_x = \begin{bmatrix} \dfrac{\partial L}{\partial x_1} \\ \dfrac{\partial L}{\partial x_2} \\ \vdots \\ \dfrac{\partial L}{\partial x_n} \end{bmatrix} = \begin{bmatrix} \displaystyle\sum_{i=1}^{m} \left(\omega_i^2 \frac{\partial \widetilde{r}_{s_i}}{\partial x_1} \cdot \frac{\partial \rho}{\partial \widetilde{r}_{s_i}} \right) + \sum_{i}^{s} \lambda_i \frac{\partial c_i}{\partial x_1} \\ \displaystyle\sum_{i=1}^{m} \left(\omega_i^2 \frac{\partial \widetilde{r}_{s_i}}{\partial x_2} \cdot \frac{\partial \rho}{\partial \widetilde{r}_{s_i}} \right) + \sum_{i}^{s} \lambda \frac{\partial c_i}{\partial x_2} \\ \vdots \\ \displaystyle\sum_{i=1}^{m} \left(\omega_i^2 \frac{\partial \widetilde{r}_{s_i}}{\partial x_2} \cdot \frac{\partial \rho}{\partial \widetilde{r}_{s_i}} \right) + \sum_{i}^{s} \lambda_i \frac{\partial c_i}{\partial x_n} \end{bmatrix} . \tag{L.5}$$

In matrix notation:

$$\mathbf{L}_x = \mathbf{J}_x(\mathbf{x}) + \mathbf{C}^T \lambda = 0 , \tag{L.6}$$

where $\mathbf{J}_x$ is the same as equation (E.12) and for $i = 1, 2, \ldots, s$ and $j = 1, 2, \ldots, n$

$$[\mathbf{C}]_{ij} = \frac{\partial c_i}{\partial x_j} . \tag{L.7}$$

By using a truncated Taylor series expansion, linearise $\mathbf{L}_x$ at a working point $\mathbf{x}_k$, and solve for $\mathbf{x}$, by iteratively calculating better values for $\mathbf{x}$. Substitute the linearisation in equation (L.6):

$$\mathbf{J}_{xx}(\mathbf{x}_k) \Delta \mathbf{x}_k + \mathbf{C}^T(\mathbf{x}_k) \lambda = -\mathbf{J}_x(\mathbf{x}_k) , \tag{L.8}$$

where $\mathbf{J}_{xx}$ can be any one of equation (E.21) or equation (E.24). Linearise the constraints, equation (L.3):

$$\mathbf{C}(\mathbf{x}_k) \Delta \mathbf{x}_k = -\mathbf{c}(\mathbf{x}_k) . \tag{L.9}$$

Equation (L.8) and equation (L.9) can be used to solve for $\mathbf{x}$. Write it in matrix formulation:

$$\begin{bmatrix} \mathbf{J}_{xx}(\mathbf{x}_k) & \mathbf{C}^T(\mathbf{x}_k) \\ \mathbf{C}(\mathbf{x}_k) & 0 \end{bmatrix} \begin{bmatrix} \Delta \mathbf{x}_k \\ \lambda \end{bmatrix} = \begin{bmatrix} -\mathbf{J}_x(\mathbf{x}_k) \\ -\mathbf{c}(\mathbf{x}_k) \end{bmatrix} . \tag{L.10}$$

This is a system of linear equations which can be solved to obtain the state correction vector [8] and [28].