

Appendices

Appendix A

Linear curve fit results with latitude excluded as variable

The first figure in each case is a plot of the actual pCO_2 values, together with the mean values of the estimated pCO_2 from the 100 curve fits. The 95% confidence interval is also shown for the 100 curve fits in each case. The second figure in each case is the actual pCO_2 values plotted against the estimated pCO_2 values. The 10% error lines indicate the number of points that fall outside the 10% error. The third figure in each case is a histogram of the errors on the estimation of the pCO_2 , showing the distribution of the errors for each case.

It can be seen from the figures that as the number of points sampled from the data set increases, the fit of the data is improved and less points falls outside the 10% error interval. Also, from the figures it can be seen that the D-optimal sampling yields a smaller 95% confidence interval than the random sampling. The histograms generally show that the error on the estimation has a normal distribution around zero.

The mean coefficients for the linear equation for all 100 curve fits are given in Table A.1. The standard deviation for the coefficients are given in Table A.2. It can be seen from the table of the standard deviations that the coefficients for the D-optimal sampling have a smaller standard deviation than the coefficients for the random sampling.

Table A.1: Coefficients of linear curve fitting

Sampling	Random	Random	Random	Random	D-optim	D-optim	D-optim	D-optim
No of points k	10	50	100	200	10	50	100	200
$\beta_1(\times 10^2)$	4.047	4.055	4.065	4.054	3.987	3.898	3.881	3.908
$\beta_2(\times 10^0)$	-2.115	-2.316	-2.314	-2.288	-1.545	-1.210	-1.116	-1.205
$\beta_3(\times 10^{-2})$	5.227	5.794	4.835	5.448	4.252	8.970	9.751	8.005
$\beta_4(\times 10^1)$	-3.128	-2.958	-3.010	-2.950	-2.501	-2.362	-2.321	-2.361

Table A.2: Standard deviation of coefficients of linear curve fitting

Sampling	Random	Random	Random	Random	D-optim	D-optim	D-optim	D-optim
No of points k	10	50	100	200	10	50	100	200
$\beta_1(\times 10^1)$	3.774	1.456	1.062	0.6111	3.528	1.826	1.183	0.7855
$\beta_2(\times 10^0)$	1.499	0.4221	0.3099	0.1862	1.073	0.5695	0.4112	0.2514
$\beta_3(\times 10^{-1})$	4.156	1.381	1.015	0.6179	2.450	1.176	0.7197	0.5076
$\beta_4(\times 10^1)$	1.750	0.3990	0.2916	0.1726	0.6322	0.3036	0.2066	0.1350

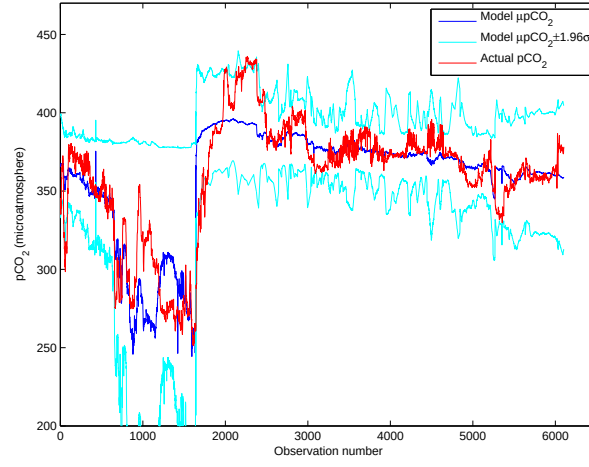


Figure A.1: Linear curve fit results. 10 Random points sampled.

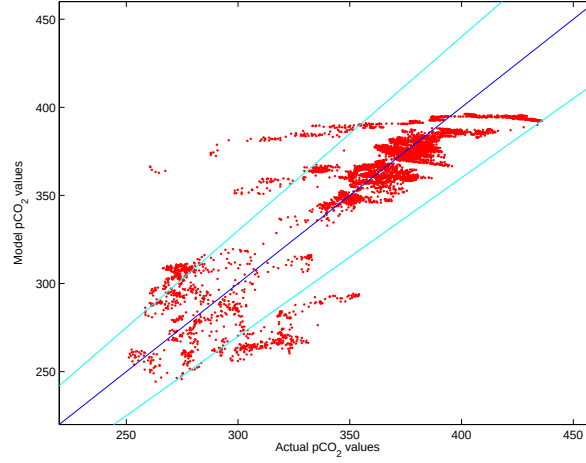


Figure A.2: Linear curve fit results with 10 random points sampled. The degree to which the modelled points fall in the 10 % error margin is shown. The mean pCO_2 estimation for all 100 runs is shown here.

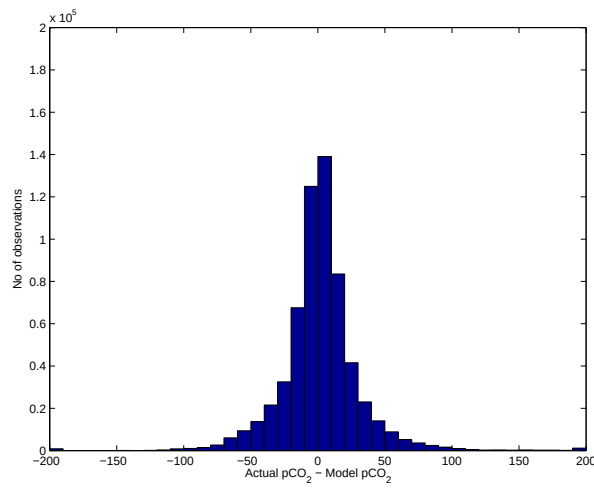


Figure A.3: Linear curve fit results. Histogram of errors for all 100 runs and 10 random points sampled.

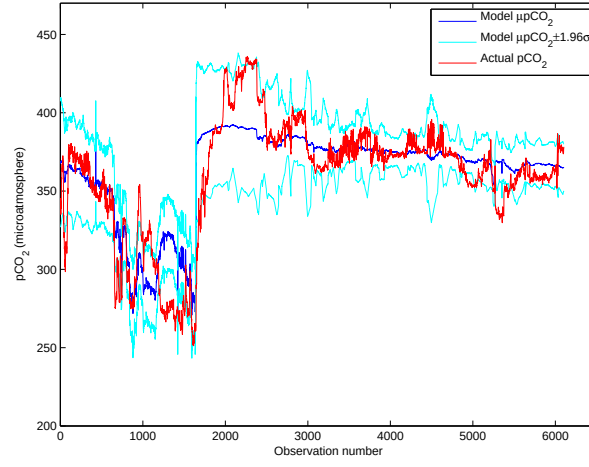


Figure A.4: Linear curve fit results. 10 D-optimal sampled points.

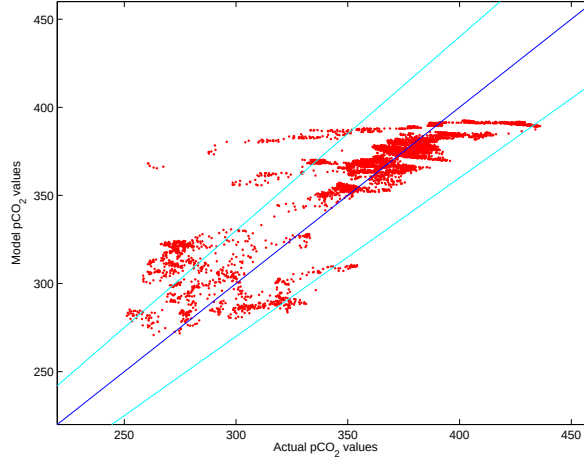


Figure A.5: Linear curve fit results with 10 D-optimal sampled points. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

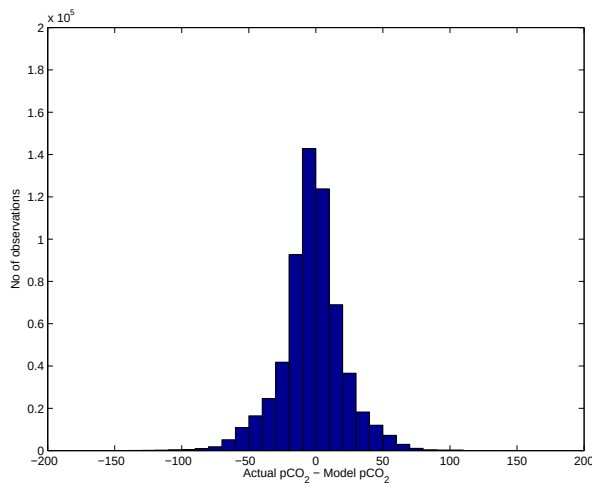


Figure A.6: Linear curve fit results. Histogram of errors for all 100 runs and 10 D-optimal sampled points.

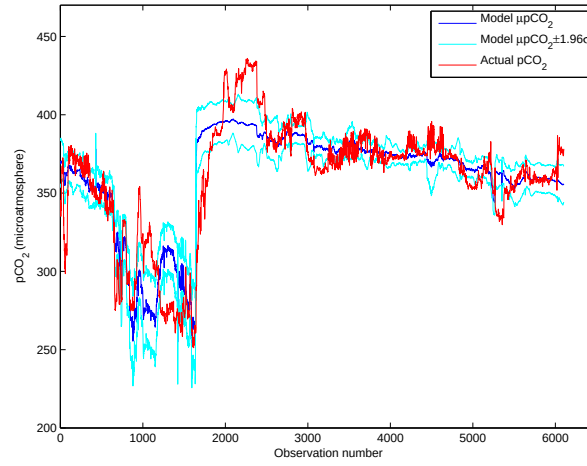


Figure A.7: Linear curve fit results. 50 Random points sampled.

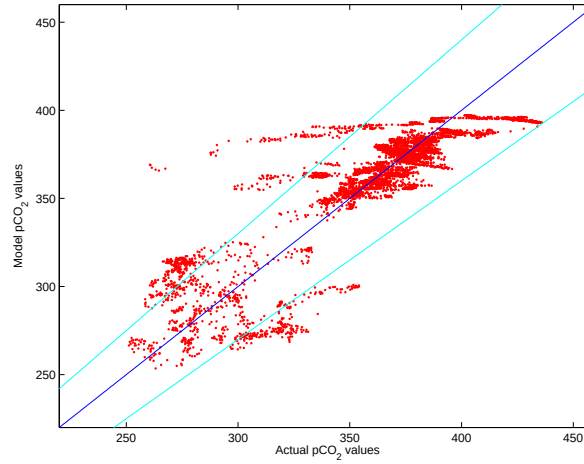


Figure A.8: Linear curve fit results with 50 random points sampled. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

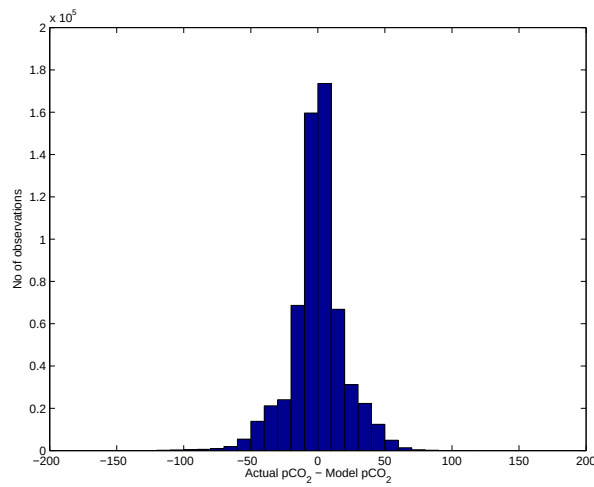


Figure A.9: Linear curve fit results. Histogram of error for all 100 runs and 50 random points sampled.

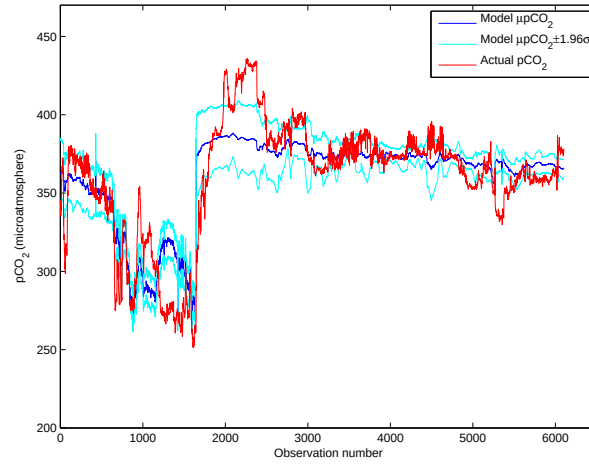


Figure A.10: Linear curve fit results. 50 D-optimal sampled points.

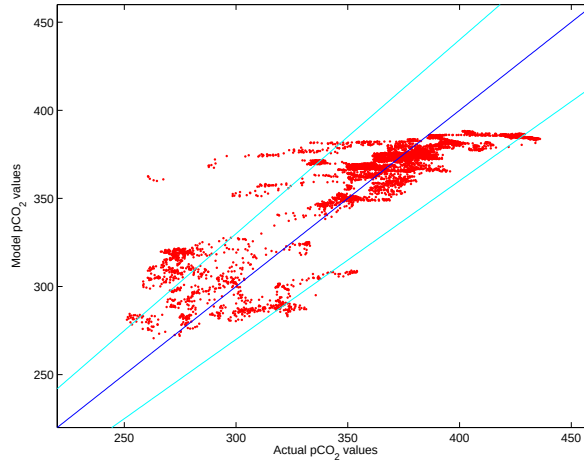


Figure A.11: Linear curve fit results with 50 D-optimal sampled points. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

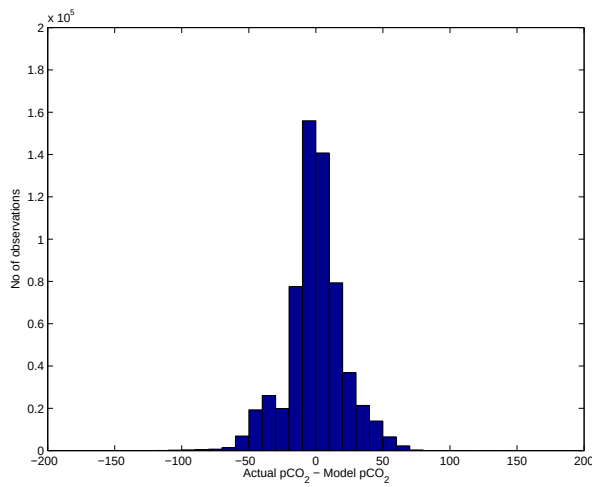


Figure A.12: Linear curve fit results. Histogram of errors for all 100 runs and 50 D-optimal sampled points.

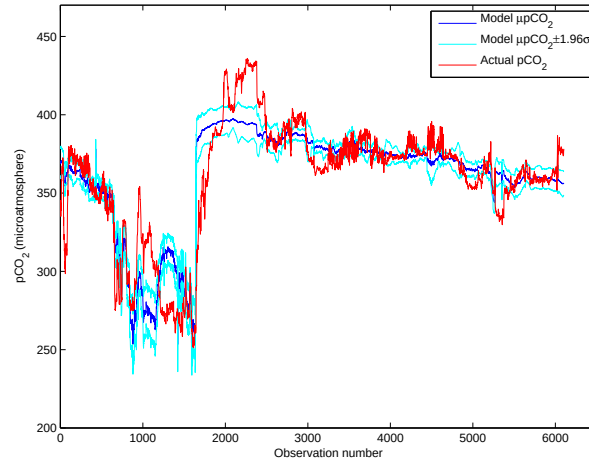


Figure A.13: Linear curve fit results. 100 Random points sampled.

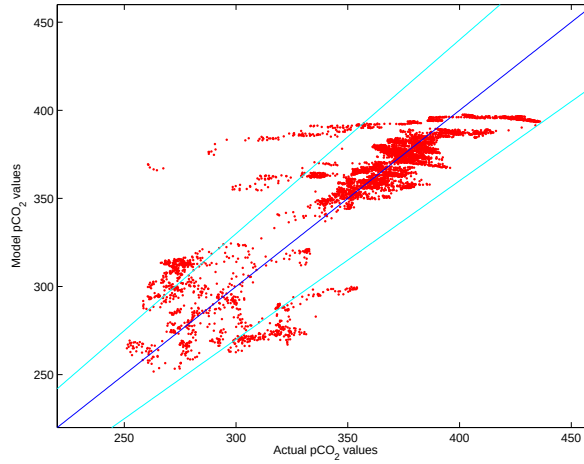


Figure A.14: Linear curve fit results with 100 random points sampled. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

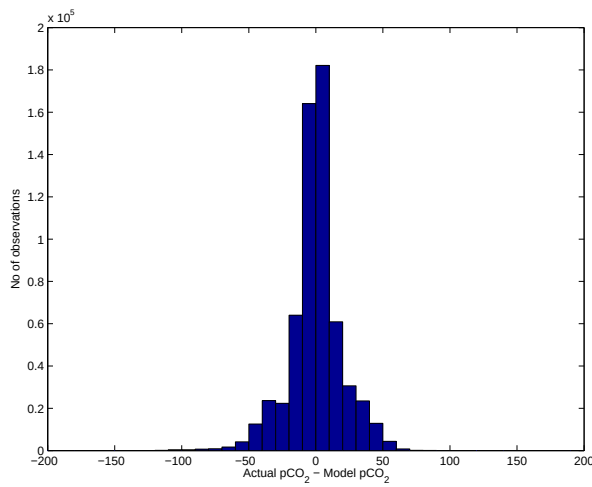


Figure A.15: Linear curve fit results. Histogram of error for all 100 runs and 100 random points sampled.

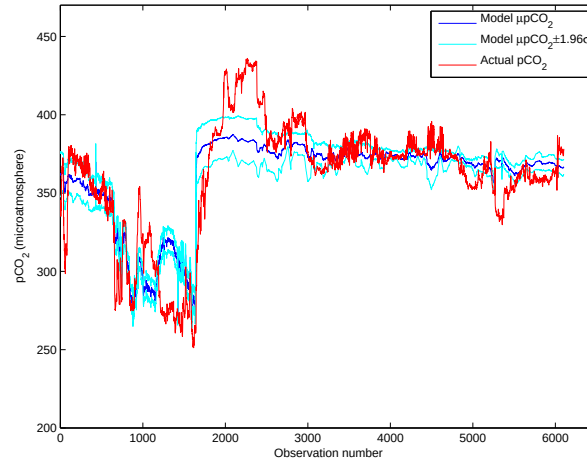


Figure A.16: Linear curve fit results. 100 D-optimal sampled points.

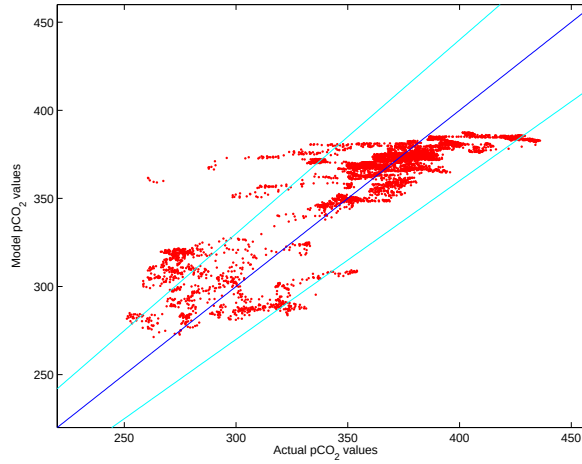


Figure A.17: Linear curve fit results with 100 D-optimal sampled points. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

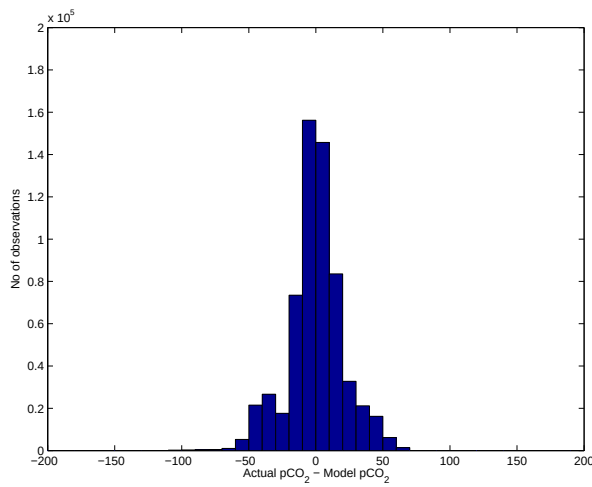


Figure A.18: Linear curve fit results. Histogram of errors for all 100 runs and 100 D-optimal sampled points.

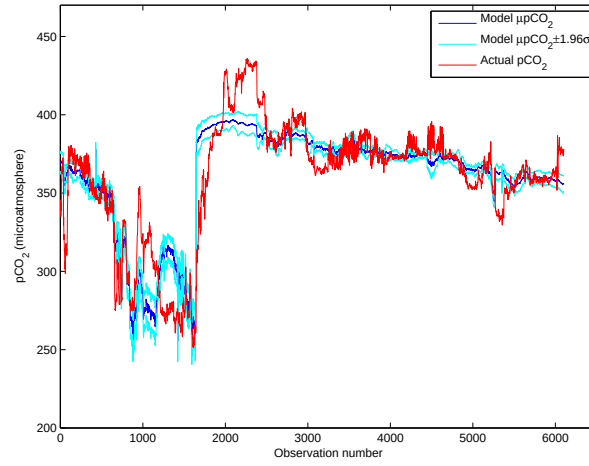


Figure A.19: Linear curve fit results. 200 Random points sampled.

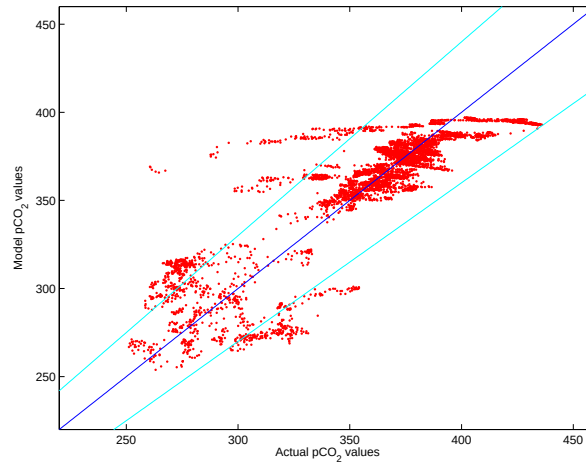


Figure A.20: Linear curve fit results with 200 random points sampled. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

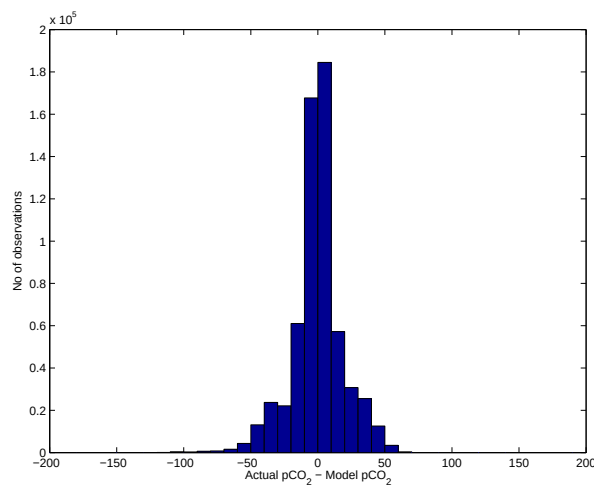


Figure A.21: Linear curve fit results. Histogram of error for all 100 runs and 200 random points sampled.

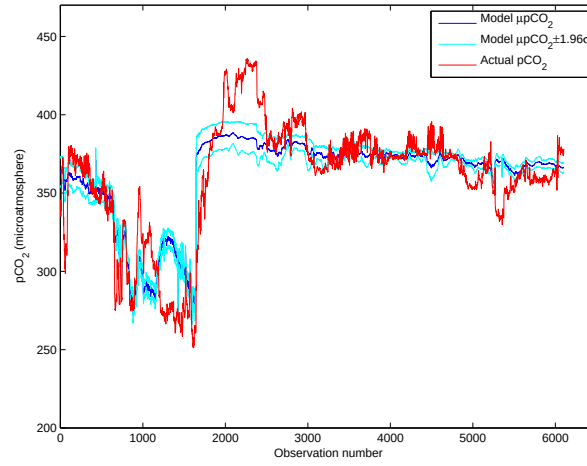


Figure A.22: Linear curve fit results. 200 D-optimal sampled points.

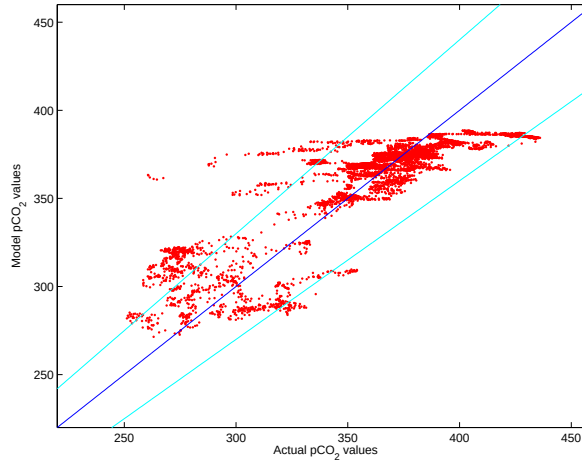


Figure A.23: Linear curve fit results with 200 D-optimal sampled points. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

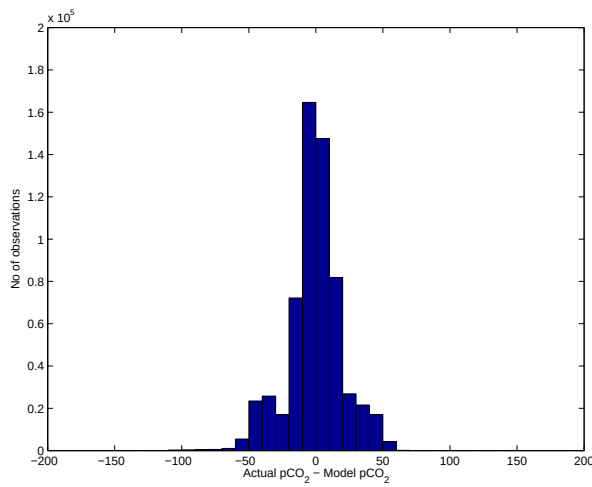


Figure A.24: Linear curve fit results. Histogram of errors for all 100 runs and 200 D-optimal sampled points.

Appendix B

Quadratic curve fit results with latitude excluded as variable

The first figure in each case is a plot of the actual pCO_2 values, together with the mean values of the estimated pCO_2 from the 100 curve fits. The 95% confidence interval is also shown for the 100 curve fits in each case. The second figure in each case is the actual pCO_2 values plotted against the estimated pCO_2 values. The 10% error lines indicate the number of points that fall outside the 10% error. The third figure in each case is a histogram of the errors on the estimation of the pCO_2 , showing the distribution of the errors for each case.

It can be seen from the figures that as the number of points sampled from the data set increases, the fit of the data is improved and less points falls outside the 10% error interval. Also, from the figures it can be seen that the D-optimal sampling yields a smaller 95% confidence interval than the random sampling. The histograms generally show that the error on the estimation has a normal distribution around zero.

The mean coefficients for the quadratic equation for all 100 curve fits are given in Table B.1. The standard deviation for the coefficients are given in Table B.2. It can be seen from the table of the standard deviations that the coefficients for the D-optimal sampling have a smaller standard deviation than the coefficients for the random sampling.

Table B.1: Coefficients for quadratic curve fitting

Sam-pling	Random	Random	Random	Random	D-optim	D-optim	D-optim	D-optim
No of Points k	25	50	100	200	25	50	100	200
$\beta_1(\times 10^2)$	4.049	4.020	4.094	4.038	3.621	3.501	3.469	3.489
$\beta_2(\times 10^0)$	0.4693	1.235	-0.2634	0.1997	1.661	3.170	3.653	3.244
$\beta_3(\times 10^0)$	0.4697	0.4988	0.4101	0.5037	1.217	1.306	1.339	1.334
$\beta_4(\times 10^1)$	-7.827	-7.053	-7.168	-6.967	-4.353	-3.943	-3.721	-3.811
$\beta_5(\times 10^{-2})$	-9.673	-7.606	-3.196	-3.604	-2.262	-6.140	-7.604	-6.447
$\beta_6(\times 10^{-3})$	-3.973	-3.257	-2.796	-3.148	-5.142	-5.333	-5.450	-5.495
$\beta_7(\times 10^0)$	9.757	7.704	7.462	7.290	4.047	3.512	3.330	3.443
$\beta_8(\times 10^{-2})$	-1.326	-3.073	-2.597	-3.077	-5.572	-6.144	-6.272	-6.125
$\beta_9(\times 10^0)$	-1.420	-1.433	-0.6603	-0.7012	0.4747	0.3131	0.1749	0.2396
$\beta_{10}(\times 10^{-1})$	4.176	3.241	3.060	2.964	0.01082	-0.03852	-0.2300	-0.2318

Table B.2: Standard deviation of coefficients for quadratic curve fitting

Sam-pling	Random	Random	Random	Random	D-optim	D-optim	D-optim	D-optim
No of Points k	25	50	100	200	25	50	100	200
$\beta_1(\times 10^1)$	8.202	4.978	3.647	2.352	6.317	3.727	2.628	1.637
$\beta_2(\times 10^1)$	1.047	0.5636	0.3478	0.2492	0.5322	0.3366	0.2238	0.1481
$\beta_3(\times 10^0)$	1.829	1.122	0.8478	0.5389	1.179	0.6322	0.4834	0.2800
$\beta_4(\times 10^1)$	6.042	2.8461	1.960	1.168	2.583	1.5041	1.164	0.7535
$\beta_5(\times 10^{-1})$	3.854	1.887	1.165	0.8020	1.470	0.9291	0.6079	0.4123
$\beta_6(\times 10^{-2})$	1.080	0.6229	0.4588	0.2804	0.5689	0.2985	0.2350	0.1355
$\beta_7(\times 10^0)$	10.14	3.593	2.023	1.218	2.900	1.760	1.315	0.8837
$\beta_8(\times 10^{-2})$	5.621	2.876	1.853	1.186	2.799	1.738	1.212	0.7297
$\beta_9(\times 10^0)$	6.913	3.365	2.189	1.425	1.397	0.7856	0.5409	0.3487
$\beta_{10}(\times 10^{-1})$	8.368	3.778	2.714	1.706	2.751	1.426	1.174	0.6920

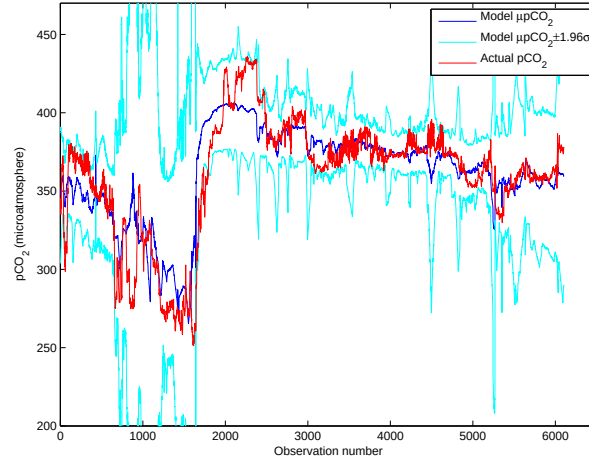


Figure B.1: Quadratic curve fit results. 25 Random points sampled.

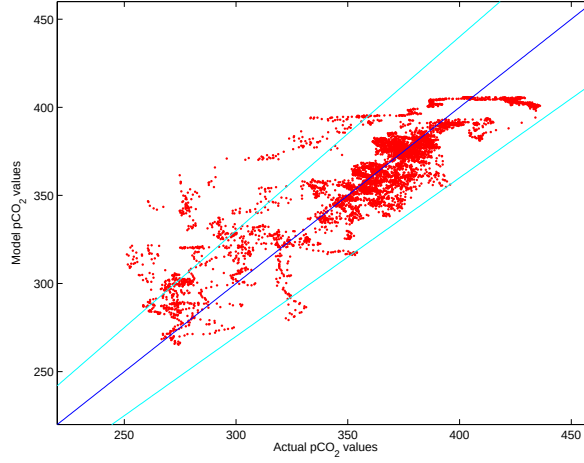


Figure B.2: Quadratic curve fit results with 25 random points sampled. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

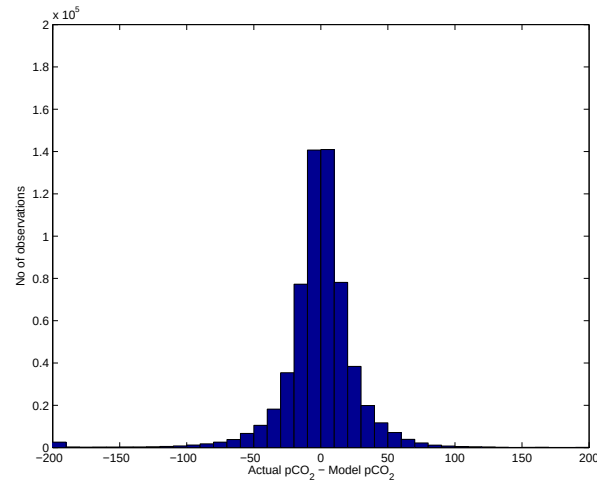


Figure B.3: Quadratic curve fit results. Histogram of errors for all 100 runs and 25 random points sampled.

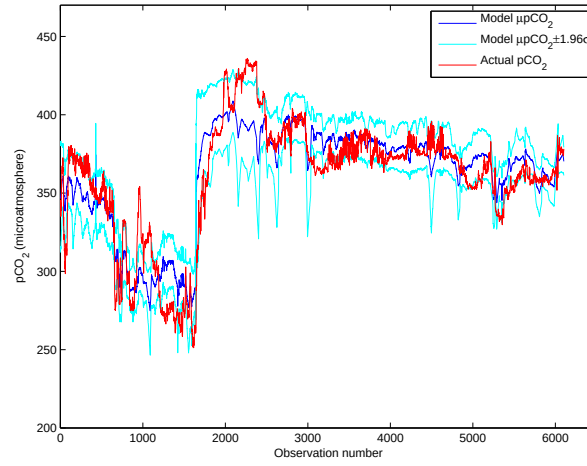


Figure B.4: Quadratic curve fit results. 25 D-optimal sampled points.

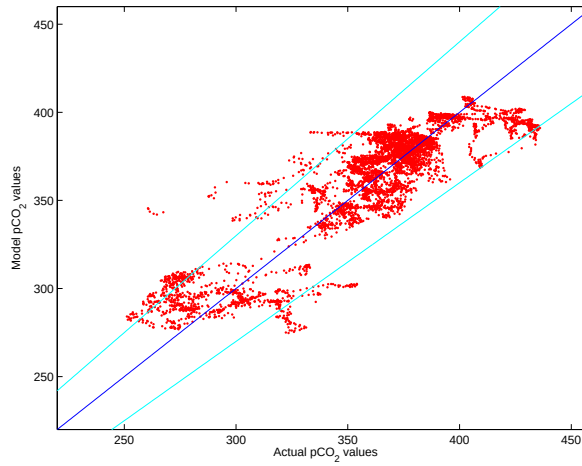


Figure B.5: Quadratic curve fit results with 25 D-optimal sampled points. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

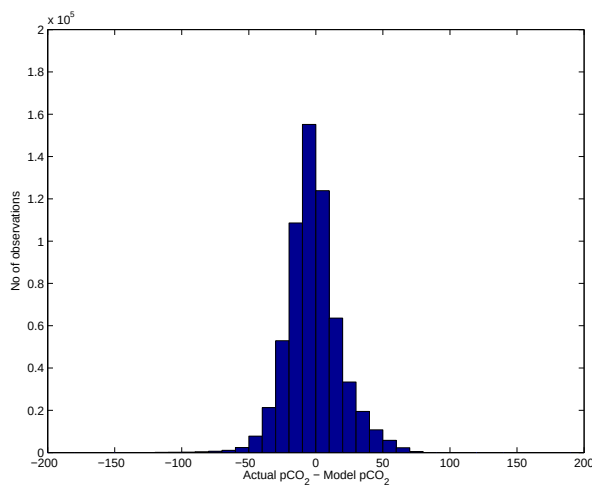


Figure B.6: Quadratic curve fit results. Histogram of errors for all 100 runs and 25 D-optimal sampled points.

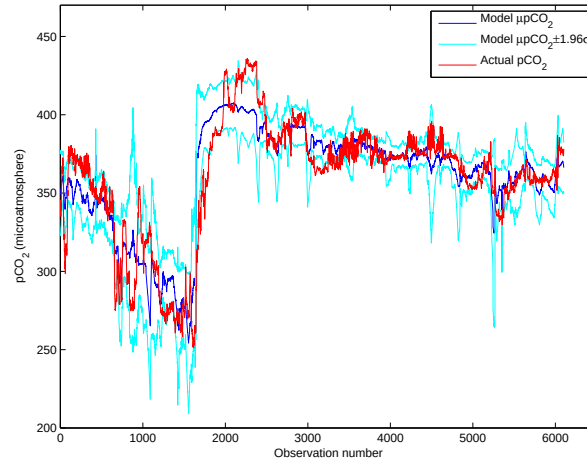


Figure B.7: Quadratic curve fit results. 50 Random points sampled.

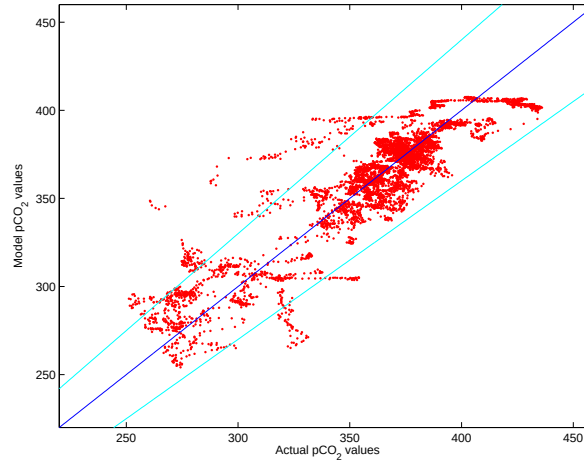


Figure B.8: Quadratic curve fit results with 50 random points sampled. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

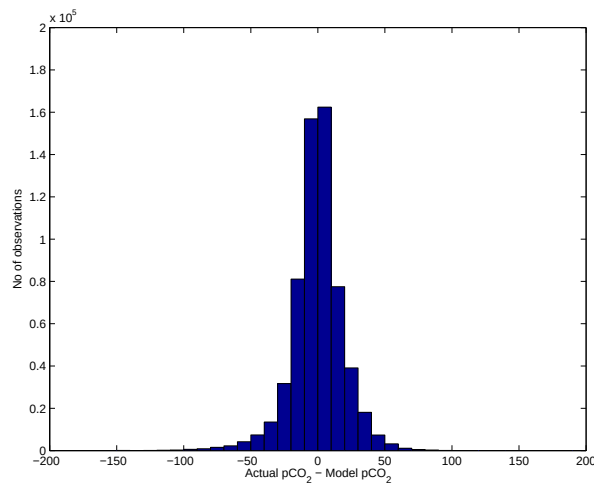


Figure B.9: Quadratic curve fit results. Histogram of errors for all 100 runs and 50 random points sampled.

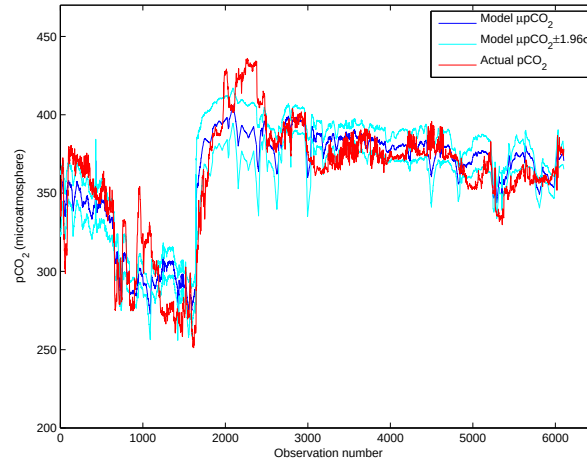


Figure B.10: Quadratic curve fit results. 50 D-optimal sampled points.

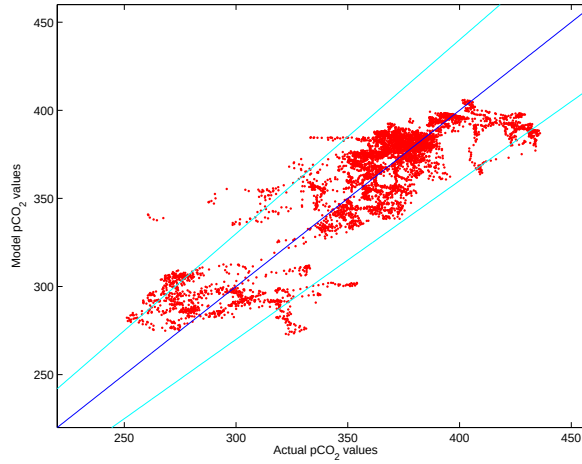


Figure B.11: Quadratic curve fit results with 50 D-optimal sampled points. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

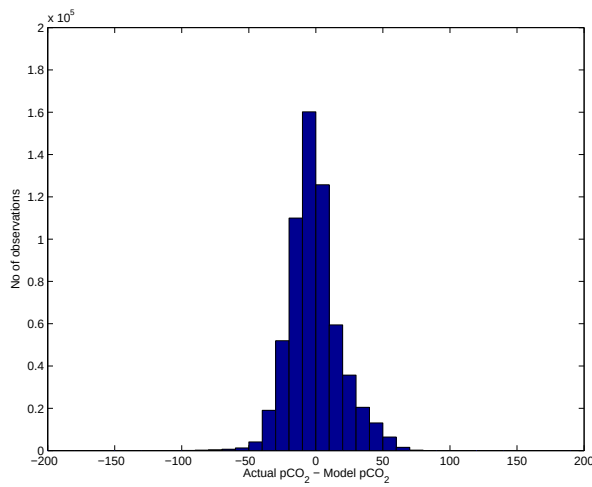


Figure B.12: Quadratic curve fit results. Histogram of errors for all 100 runs and 50 D-optimal sampled points.

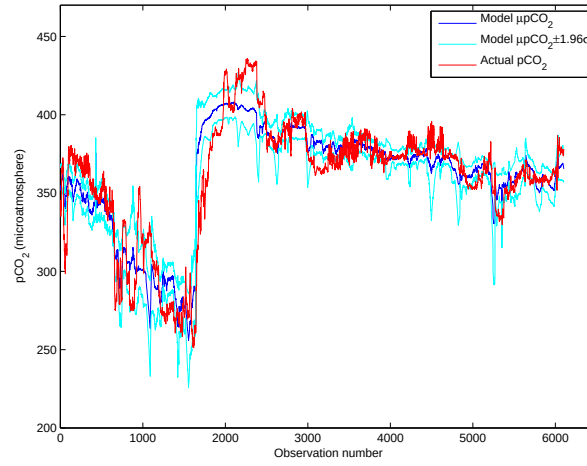


Figure B.13: Quadratic curve fit results. 100 Random points sampled.

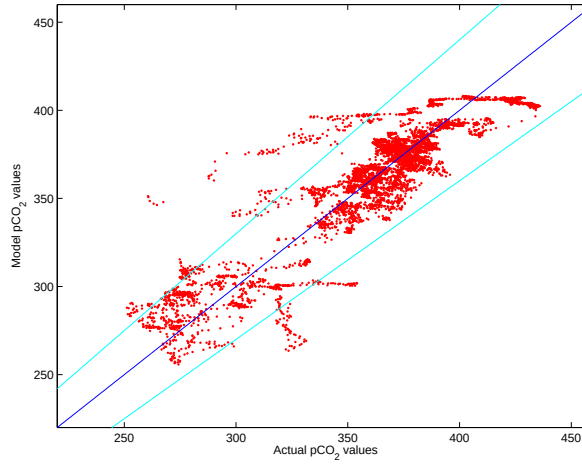


Figure B.14: Quadratic curve fit results with 100 random points sampled. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

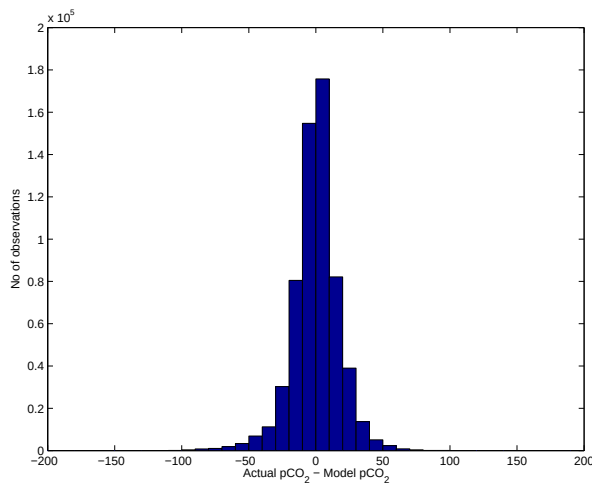


Figure B.15: Quadratic curve fit results. Histogram of errors for all 100 runs and 100 random points sampled.

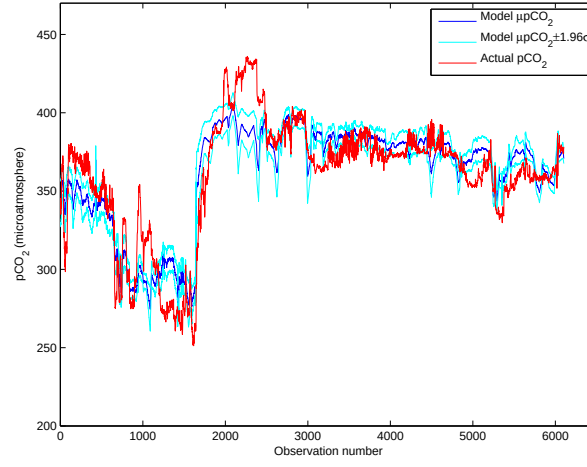


Figure B.16: Quadratic curve fit results. 100 D-optimal sampled points.

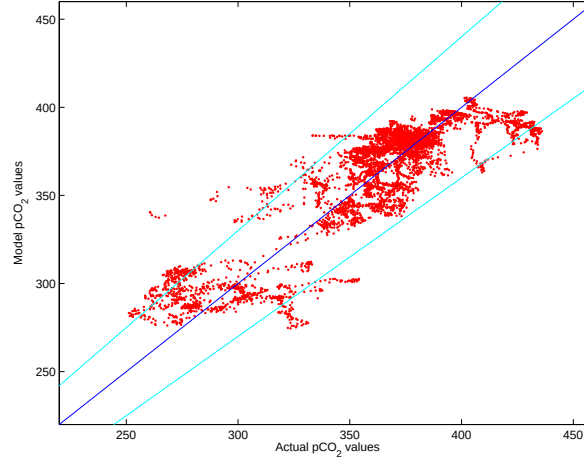


Figure B.17: Quadratic curve fit results with 100 D-optimal sampled points. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

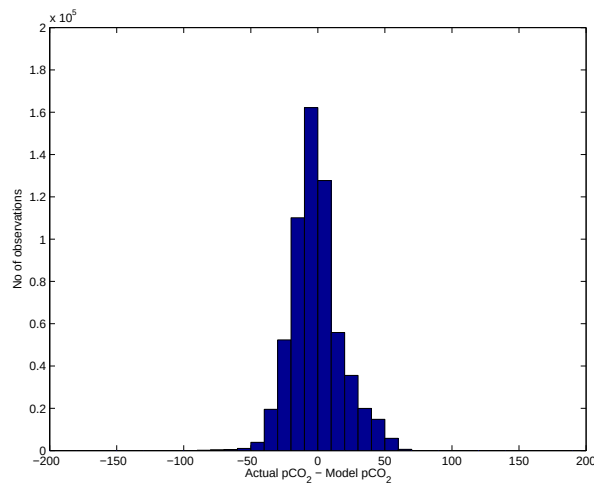


Figure B.18: Quadratic curve fit results. Histogram of errors for all 100 runs and 100 D-optimal sampled points.

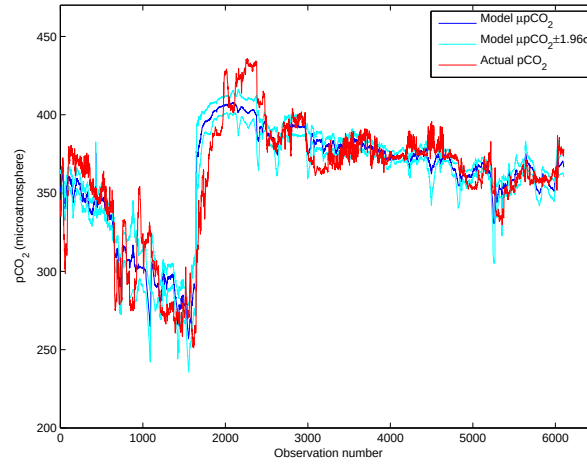


Figure B.19: Quadratic curve fit results. 200 Random points sampled.

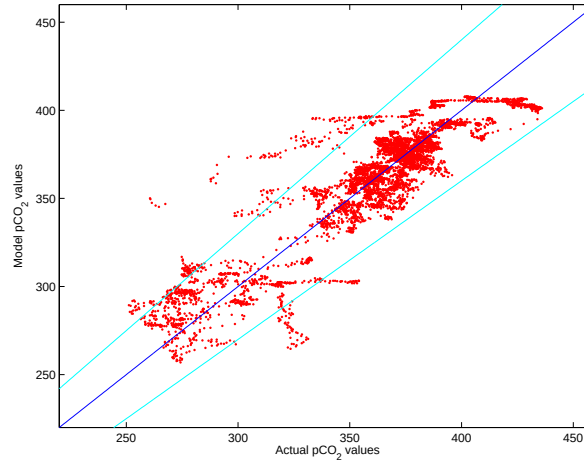


Figure B.20: Quadratic curve fit results with 200 randomly sampled points. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

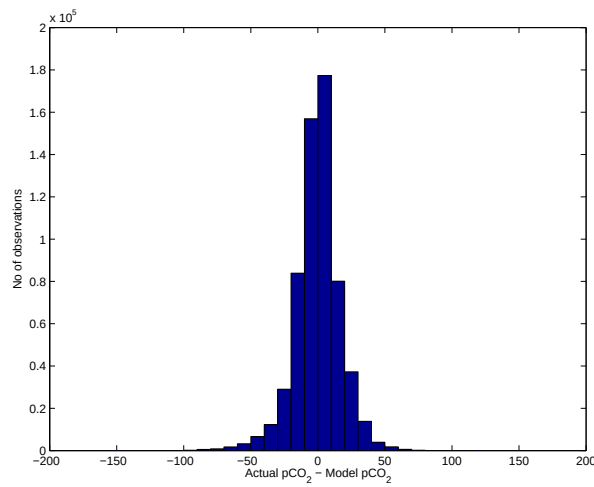


Figure B.21: Quadratic curve fit results. Histogram of errors for all 100 runs and 200 random points sampled.

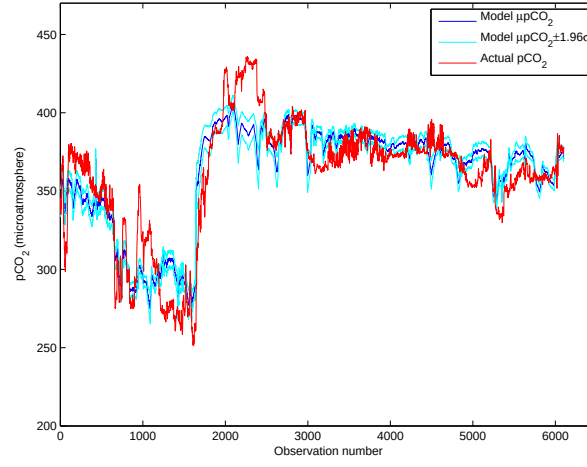


Figure B.22: Quadratic curve fit results. 200 D-optimal sampled points.

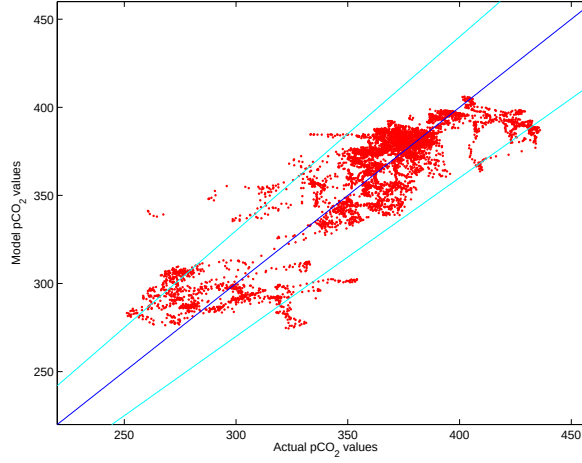


Figure B.23: Quadratic curve fit results with 300 D-optimal sampled points. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

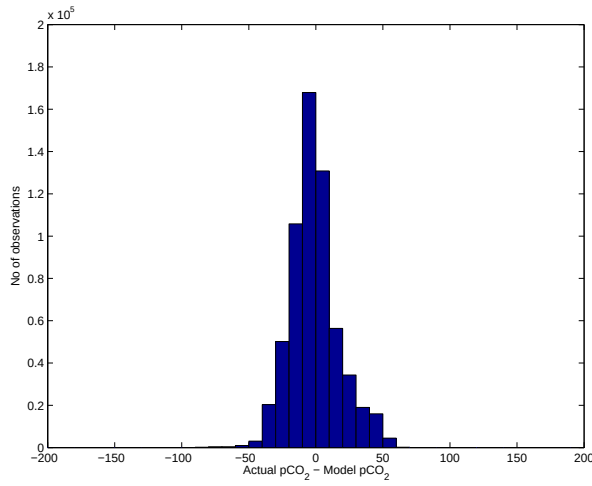


Figure B.24: Quadratic curve fit results. Histogram of errors for all 100 runs and 200 D-optimal sampled points.

Appendix C

Cubic curve fit results with latitude excluded as variable

The first figure in each case is a plot of the actual pCO_2 values, together with the mean values of the estimated pCO_2 from the 100 curve fits. The 95% confidence interval is also shown for the 100 curve fits in each case. The second figure in each case is the actual pCO_2 values plotted against the estimated pCO_2 values. The 10% error lines indicate the number of points that fall outside the 10% error. The third figure in each case is a histogram of the errors on the estimation of the pCO_2 , showing the distribution of the errors for each case.

It can be seen from the figures that as the number of points sampled from the data set increases, the fit of the data is improved and less points falls outside the 10% error interval. Also, from the figures it can be seen that the D-optimal sampling yields a smaller 95% confidence interval than the random sampling. The histograms generally show that the error on the estimation has a normal distribution around zero.

The mean coefficients for the cubic equation for all 100 curve fits are given in Table C.1. The standard deviation for the coefficients are given in Table C.2. It can be seen from the table of the standard deviations that the coefficients for the D-optimal sampling have a smaller standard deviation than the coefficients for the random sampling.

Table C.1: Coefficients of Cubic curve fitting

Sam-pling	Random	Random	Random	Random	D-optim	D-optim	D-optim	D-optim
No of Points k	50	55	100	200	50	55	100	200
$\beta_1(\times 10^2)$	2.189	2.543	2.819	2.588	2.060	1.960	2.100	2.086
$\beta_2(\times 10^1)$	7.762	6.153	5.338	5.404	6.931	6.935	6.875	6.888
$\beta_3(\times 10^0)$	2.543	2.448	2.119	2.921	4.855	5.127	4.666	4.786
$\beta_4(\times 10^1)$	4.629	-0.03724	-2.289	-0.3086	-5.991	-3.832	-5.511	-5.695
$\beta_5(\times 10^0)$	-4.796	-4.234	-3.800	-3.663	-4.674	-4.698	-4.712	-4.734
$\beta_6(\times 10^{-2})$	-2.071	-3.014	-2.850	-3.322	-4.748	-5.052	-4.624	-4.833
$\beta_7(\times 10^1)$	-1.733	-0.7125	0.01918	-0.1074	3.824	3.058	3.596	3.690
$\beta_8(\times 10^{-1})$	-5.569	-3.390	-2.838	-3.472	-5.884	-5.762	-5.573	-5.576
$\beta_9(\times 10^1)$	-6.989	-5.606	-4.734	-4.493	-3.613	-3.685	-3.625	-3.592
$\beta_{10}(\times 10^0)$	0.6692	1.445	1.400	0.4969	-0.7155	-1.014	-0.7351	-0.7432
$\beta_{11}(\times 10^{-2})$	8.768	8.877	7.893	7.294	8.843	8.942	9.047	9.079
$\beta_{12}(\times 10^{-4})$	0.5948	1.198	1.155	1.083	1.154	1.273	1.165	1.275
$\beta_{13}(\times 10^0)$	1.112	0.8320	0.3650	0.1812	-4.456	-3.784	-4.266	-4.385
$\beta_{14}(\times 10^{-2})$	1.187	0.5292	0.5801	0.7319	1.633	1.593	1.557	1.577
$\beta_{15}(\times 10^0)$	2.288	2.021	1.748	1.614	1.441	1.472	1.491	1.491
$\beta_{16}(\times 10^{-1})$	2.384	1.490	1.082	1.111	0.7274	0.7067	0.6056	0.5758
$\beta_{17}(\times 10^{-3})$	1.593	1.218	0.9052	1.116	1.600	1.568	1.501	1.482
$\beta_{18}(\times 10^{-2})$	-0.4556	-0.8714	-0.5361	0.1690	1.921	2.014	1.889	1.896
$\beta_{19}(\times 10^1)$	1.157	0.8970	0.7568	0.6975	0.4350	0.4562	0.4415	0.4349
$\beta_{20}(\times 10^{-1})$	-0.9076	-1.919	-2.389	-1.535	-1.645	-1.103	-1.461	-1.444

Table C.2: Standard deviation of coefficients of Cubic curve fitting

Sam-pling	Random	Random	Random	Random	D-optim	D-optim	D-optim	D-optim
No of Points k	50	55	100	200	50	55	100	200
$\beta_1(\times 10^2)$	2.026	1.970	1.258	0.7352	0.3343	0.2695	0.1526	0.1165
$\beta_2(\times 10^1)$	5.815	4.506	2.599	1.565	0.9255	0.7558	0.4868	0.3612
$\beta_3(\times 10^0)$	5.194	5.410	3.534	2.102	0.9504	0.8018	0.4948	0.3729
$\beta_4(\times 10^2)$	2.896	2.601	1.676	1.058	0.5205	0.5594	0.3571	0.2219
$\beta_5(\times 10^0)$	4.023	2.937	1.636	0.9815	0.7418	0.6186	0.3916	0.2834
$\beta_6(\times 10^{-2})$	6.003	5.700	3.908	2.245	1.062	0.9600	0.5704	0.4454
$\beta_7(\times 10^1)$	7.693	7.374	4.276	2.534	1.759	1.891	1.206	0.7917
$\beta_8(\times 10^{-1})$	5.947	4.988	2.953	1.860	0.9570	0.7550	0.5143	0.3216
$\beta_9(\times 10^1)$	5.499	4.244	2.529	1.424	0.7094	0.5996	0.3883	0.2286
$\beta_{10}(\times 10^0)$	6.751	6.131	3.650	2.577	0.8925	0.9127	0.6042	0.3617
$\beta_{11}(\times 10^{-2})$	9.457	6.306	3.635	2.135	1.819	1.510	0.9436	0.6633
$\beta_{12}(\times 10^{-4})$	2.544	2.160	1.511	0.8653	0.4722	0.4695	0.2523	0.1980
$\beta_{13}(\times 10^0)$	6.582	6.001	3.349	1.868	1.600	1.680	1.081	0.7292
$\beta_{14}(\times 10^{-2})$	1.929	1.558	0.8180	0.5672	0.2817	0.2018	0.1516	0.1016
$\beta_{15}(\times 10^0)$	2.126	1.549	0.8774	0.5081	0.3678	0.2849	0.1914	0.1166
$\beta_{16}(\times 10^{-1})$	3.984	3.353	1.950	1.172	0.5026	0.4157	0.2714	0.1499
$\beta_{17}(\times 10^{-3})$	1.548	1.595	1.080	0.5338	0.2959	0.2565	0.1697	0.09930
$\beta_{18}(\times 10^{-2})$	4.690	4.128	2.304	1.623	0.5319	0.4905	0.3078	0.2041
$\beta_{19}(\times 10^0)$	9.136	6.852	4.141	2.062	1.166	1.008	0.6320	0.3627
$\beta_{20}(\times 10^{-1})$	9.491	9.688	4.849	3.360	1.230	1.325	0.8681	0.5316

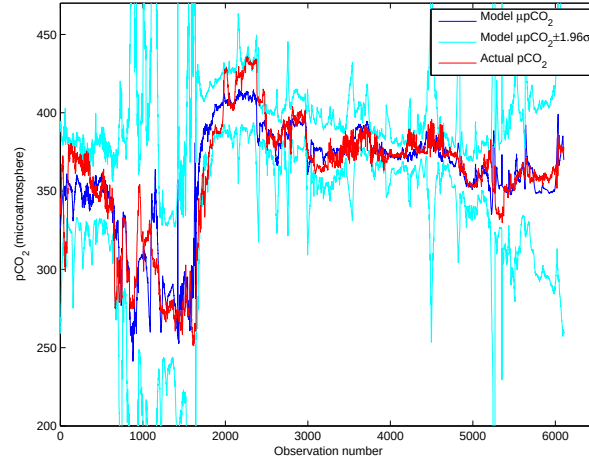


Figure C.1: Cubic curve fit results. 50 Random points sampled.

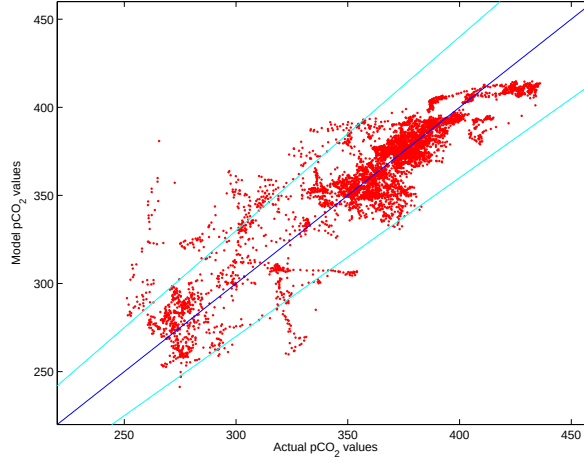


Figure C.2: Cubic curve fit results with 50 random points sampled. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

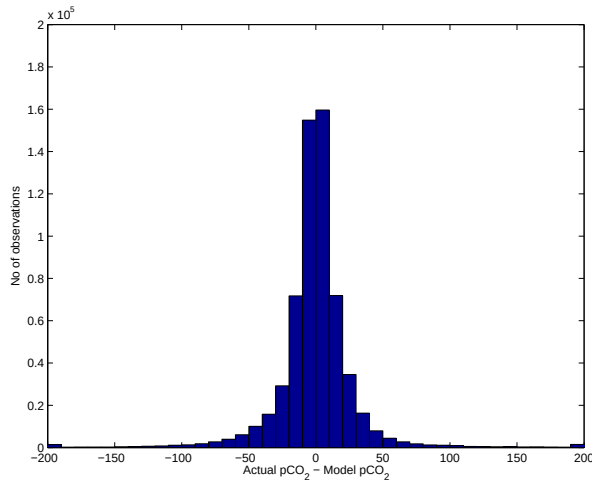


Figure C.3: Cubic curve fit results. Histogram of errors for all 100 runs and 50 random points sampled.

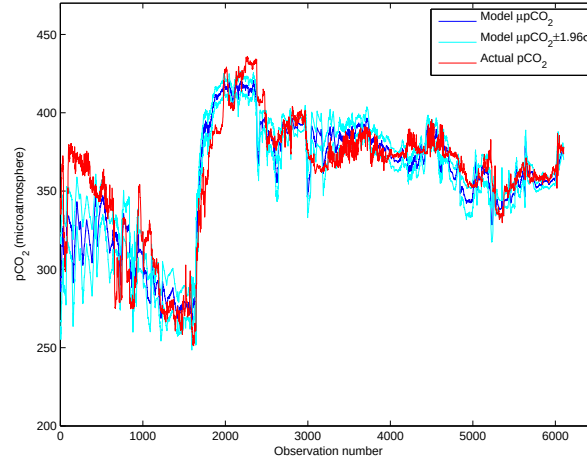


Figure C.4: Cubic curve fit results. 50 D-optimal sampled points.

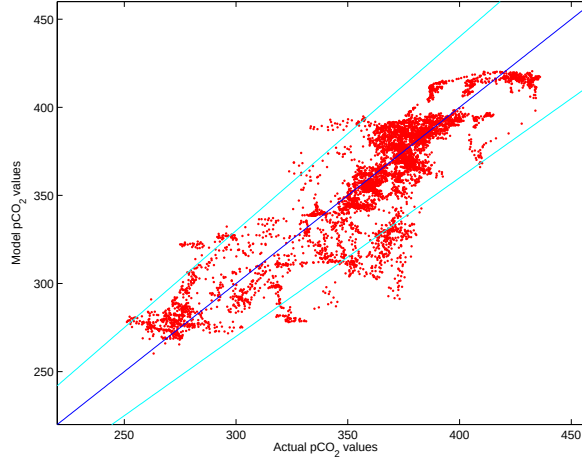


Figure C.5: Cubic curve fit results with 50 D-optimal sampled points. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

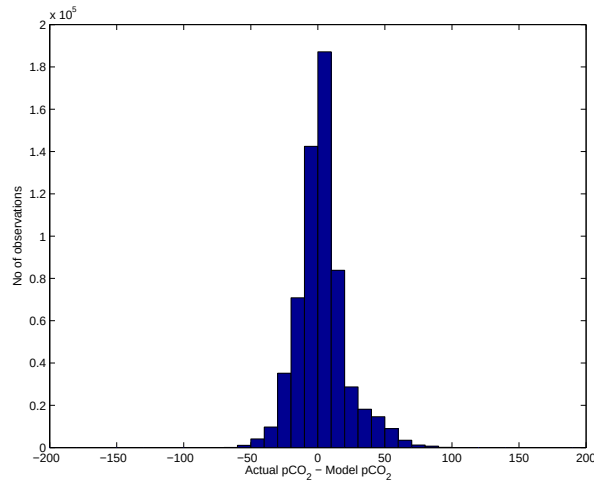


Figure C.6: Cubic curve fit results. Histogram of errors for all 100 runs and 50 D-optimal sampled points.

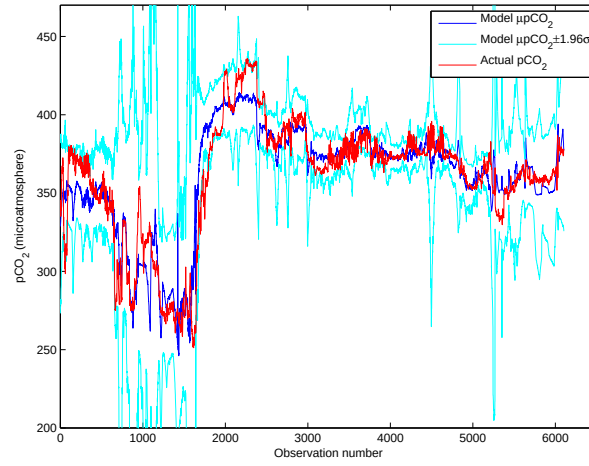


Figure C.7: Cubic curve fit results. 55 Random points sampled.

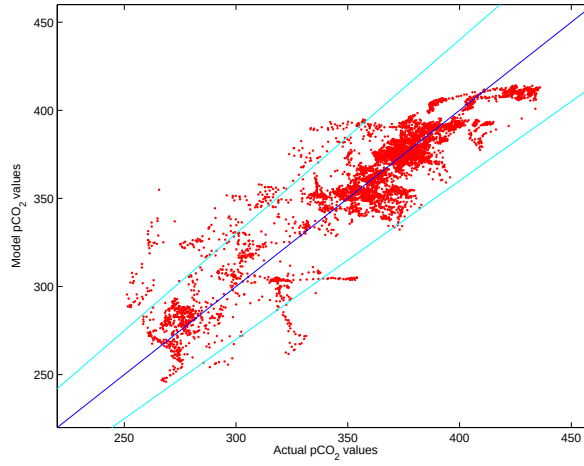


Figure C.8: Cubic curve fit results with 55 random points sampled. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

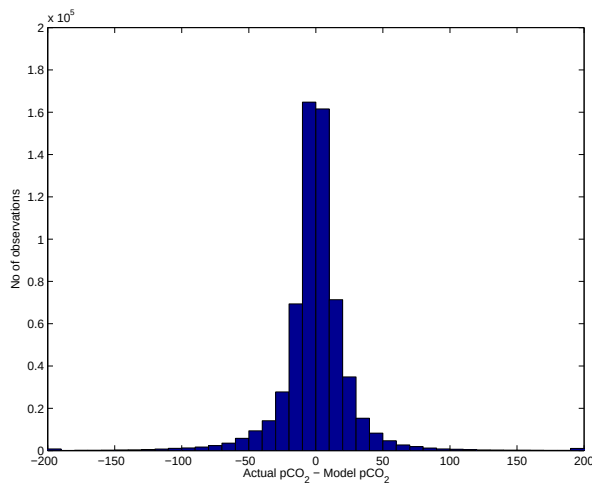


Figure C.9: Cubic curve fit results. Histogram of errors for all 100 runs and 55 random points sampled.

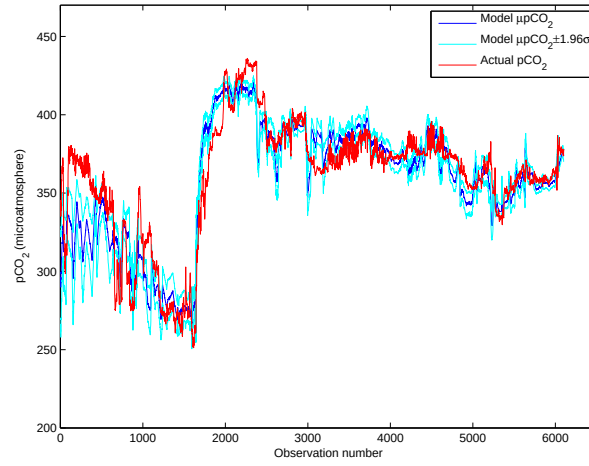


Figure C.10: Cubic curve fit results. 55 D-optimal sampled points.

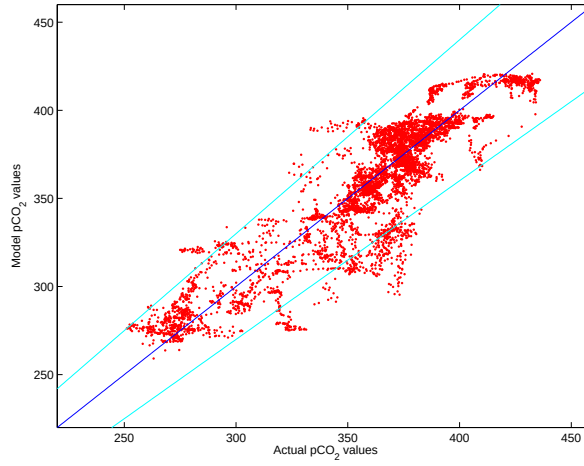


Figure C.11: Cubic curve fit results with 55 D-optimal sampled points. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

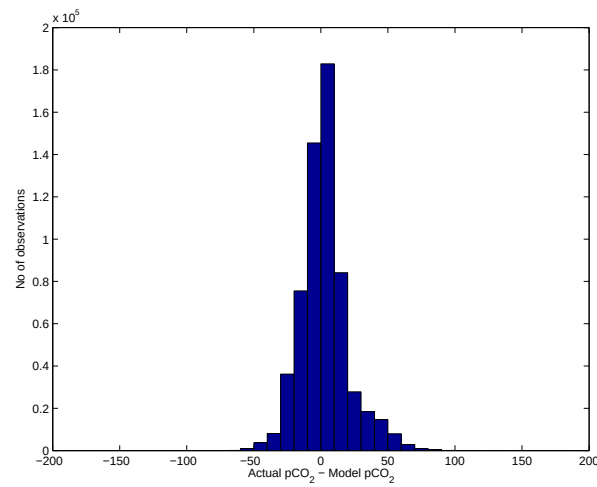


Figure C.12: Cubic curve fit results. Histogram of errors for all 100 runs and 55 D-optimal sampled points.

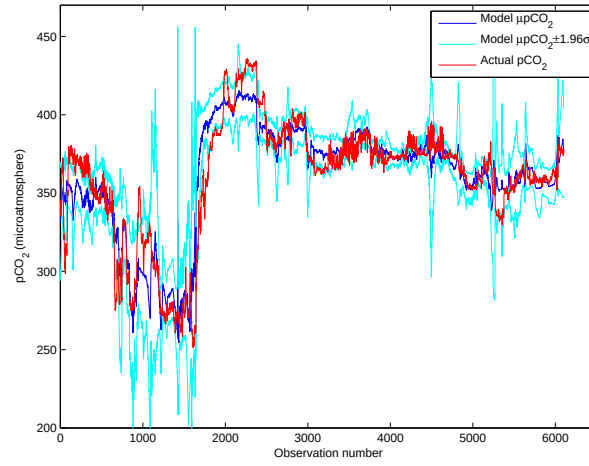


Figure C.13: Cubic curve fit results. 100 Random points sampled.

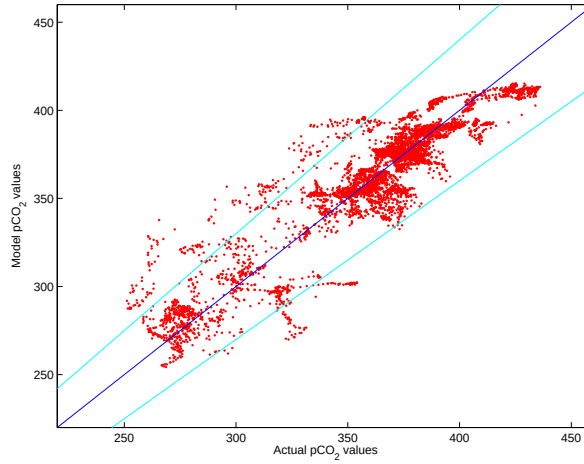


Figure C.14: Cubic curve fit results with 100 random points sampled. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

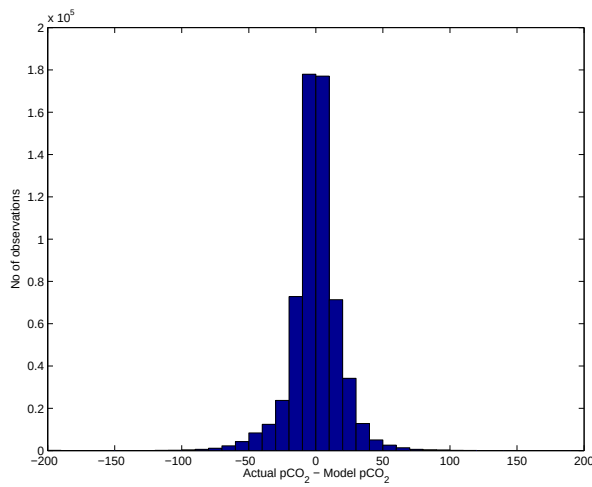


Figure C.15: Cubic curve fit results. Histogram of errors for all 100 runs and 100 random points sampled.

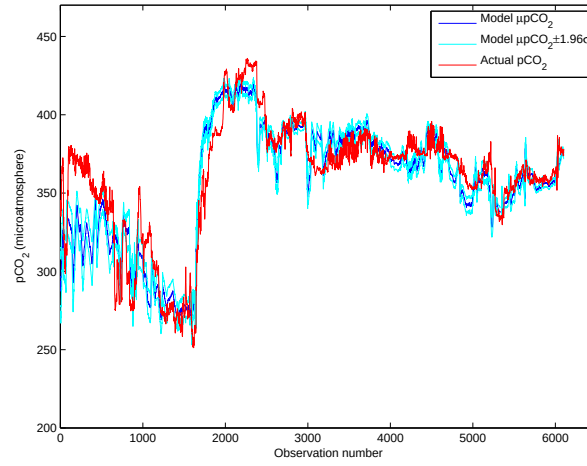


Figure C.16: Cubic curve fit results. 100 D-optimal sampled points.

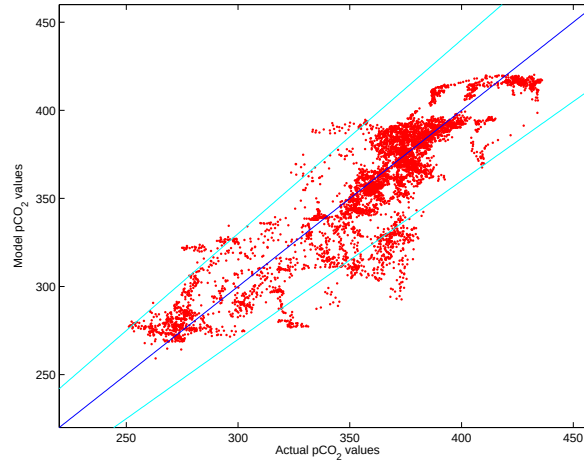


Figure C.17: Cubic curve fit results with 100 D-optimal sampled points. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

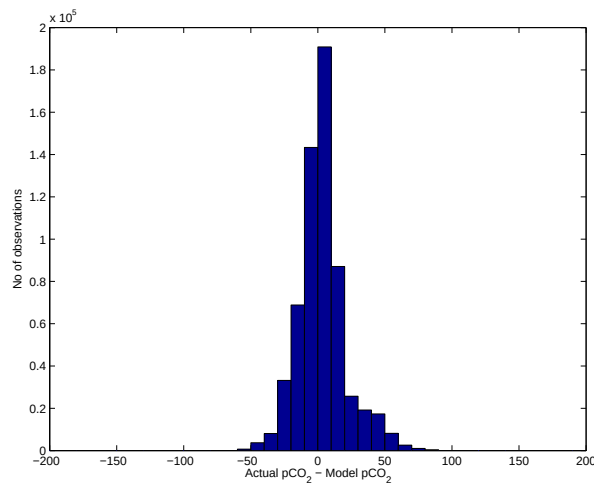


Figure C.18: Cubic curve fit results. Histogram of errors for all 100 runs and 100 D-optimal sampled points.

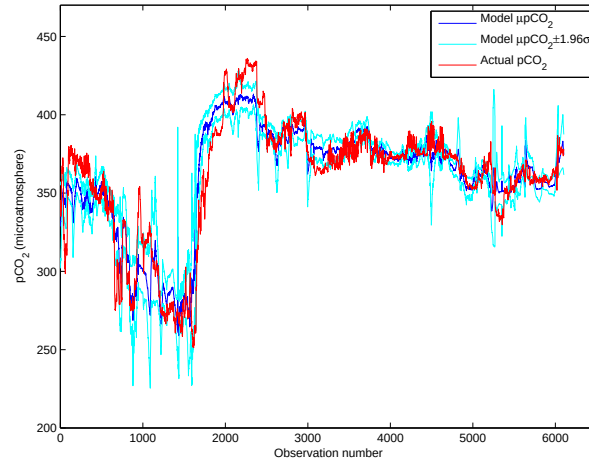


Figure C.19: Cubic curve fit results. 200 Random points sampled.

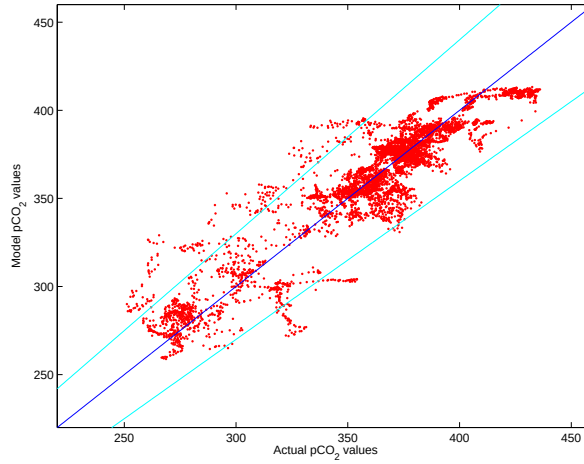


Figure C.20: Cubic curve fit results with 200 random points sampled. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

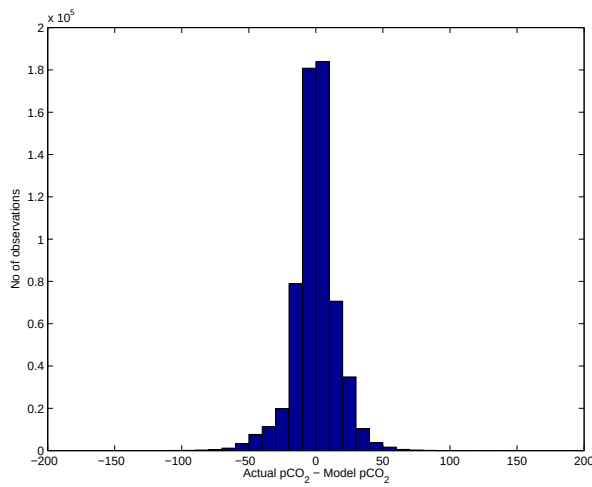


Figure C.21: Cubic curve fit results. Histogram of errors for all 100 runs and 200 random points sampled.

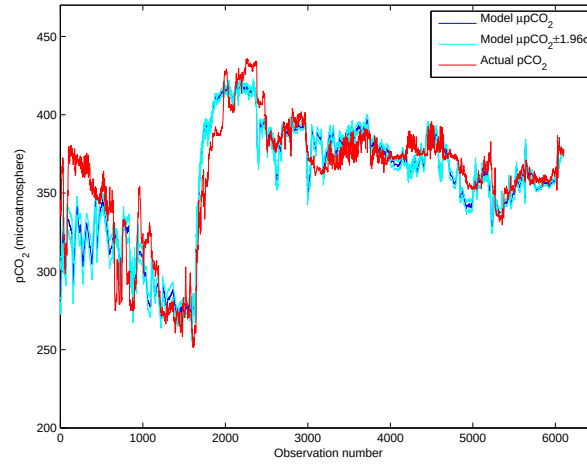


Figure C.22: Cubic curve fit results. 200 D-optimal sampled points.

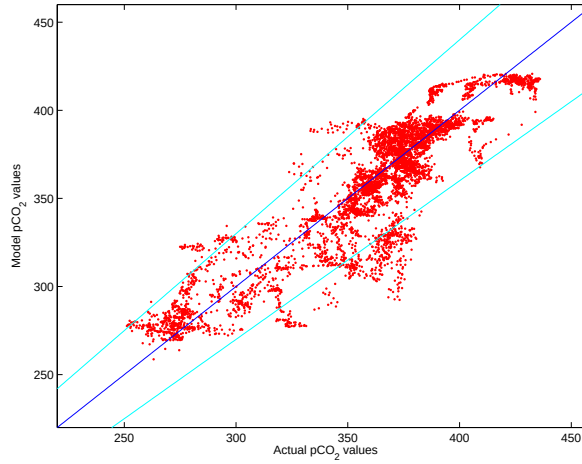


Figure C.23: Cubic curve fit results with 200 D-optimal sampled points. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

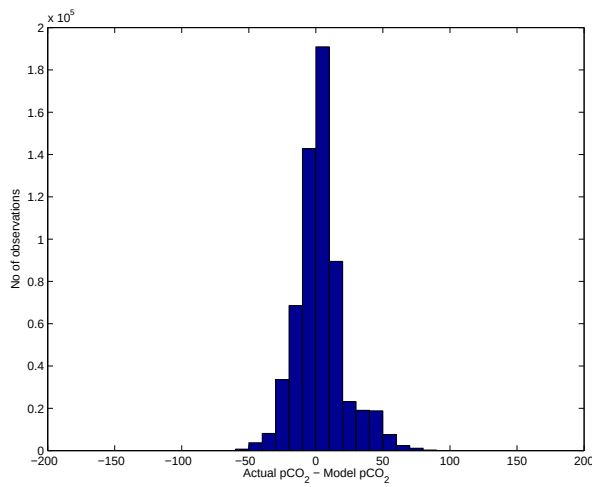


Figure C.24: Cubic curve fit results. Histogram of errors for all 100 runs and 200 D-optimal sampled points.

Appendix D

Fourth order curve fit with latitude excluded as variable

The first figure in each case is a plot of the actual pCO_2 values, together with the mean values of the estimated pCO_2 from the 100 curve fits. The 95% confidence interval is also shown for the 100 curve fits in each case. The second figure in each case is the actual pCO_2 values plotted against the estimated pCO_2 values. The 10% error lines indicate the number of points that fall outside the 10% error. The third figure in each case is a histogram of the errors on the estimation of the pCO_2 , showing the distribution of the errors for each case.

It can be seen from the figures that as the number of points sampled from the data set increases, the fit of the data is improved and less points falls outside the 10% error interval. Also, from the figures it can be seen that the D-optimal sampling yields a smaller 95% confidence interval than the random sampling. The histograms generally show that the error on the estimation has a normal distribution around zero.

The mean coefficients for the fourth order equation for all 100 curve fits are given in Table D.1. The standard deviation for the coefficients are given in Table D.2. It can be seen from the table of the standard deviations that the coefficients for the D-optimal sampling have a smaller standard deviation than the coefficients for the random sampling.

Table D.1: Coefficients of Fourth Order curve fitting

Sampling No of Points	Random	Random	Random	Random	D-optim	D-optim	D-optim	D-optim
	100	120	200	400	100	120	200	400
$\beta_1(\times 10^2)$	-4.116	-4.422	-4.517	-3.835	0.4143	0.5537	0.42111	0.5590
$\beta_2(\times 10^2)$	3.464	3.524	3.417	3.266	1.643	1.629	1.646	1.615
$\beta_3(\times 10^1)$	1.504	1.595	1.754	1.535	1.088	1.041	1.091	1.042
$\beta_4(\times 10^2)$	8.600	9.220	9.528	8.353	0.3807	0.1579	0.4305	0.2182
$\beta_5(\times 10^1)$	-2.605	-2.760	-2.690	-2.634	-1.128	-1.123	-1.150	-1.118
$\beta_6(\times 10^{-1})$	-1.440	-1.498	-1.872	-1.622	-1.366	-1.283	-1.363	-1.304
$\beta_7(\times 10^2)$	-3.626	-3.906	-3.984	-3.509	0.3477	0.4680	0.3566	0.4393
$\beta_8(\times 10^0)$	-4.874	-4.848	-4.667	-4.318	-3.111	-3.107	-3.105	-3.053
$\beta_9(\times 10^2)$	-4.034	-4.151	-4.055	-3.860	-1.499	-1.470	-1.499	-1.464
$\beta_{10}(\times 10^1)$	-0.8107	-1.057	-0.1293	-0.9682	-0.2467	-0.2279	-0.3032	-0.2416
$\beta_{11}(\times 10^{-1})$	6.165	7.281	7.375	7.494	2.481	2.523	2.708	2.544
$\beta_{12}(\times 10^{-3})$	1.020	0.9444	1.178	1.047	0.7732	0.6980	0.7503	0.7315
$\beta_{13}(\times 10^1)$	7.130	7.324	7.577	6.770	-0.3617	-0.6089	-0.4356	-0.5699
$\beta_{14}(\times 10^{-1})$	2.793	2.771	2.517	2.337	1.377	1.367	1.355	1.342
$\beta_{15}(\times 10^1)$	2.030	2.203	2.240	2.217	0.8357	0.8225	0.8512	0.8233
$\beta_{16}(\times 10^0)$	3.972	4.107	3.956	3.613	2.049	2.052	2.058	2.033
$\beta_{17}(\times 10^{-2})$	1.973	1.911	1.972	1.811	1.747	1.757	1.767	1.713
$\beta_{18}(\times 10^{-1})$	0.3071	0.6059	1.052	0.7381	0.6909	0.6998	0.7695	0.7000
$\beta_{19}(\times 10^2)$	1.164	1.213	1.167	1.107	0.33971	0.3280	0.3366	0.3257
$\beta_{20}(\times 10^0)$	0.6154	1.605	1.964	1.207	-1.323	-1.388	-1.192	-1.330
$\beta_{21}(\times 10^0)$	-4.164	-4.255	-4.711	-4.294	-0.1492	0.007981	-0.08673	-0.01166
$\beta_{22}(\times 10^{-1})$	-2.766	-3.100	-3.023	-2.265	1.096	1.204	1.079	1.170
$\beta_{23}(\times 10^1)$	-1.103	-1.101	-1.041	-0.9772	-0.2176	-0.2029	-0.2137	-0.2014
$\beta_{24}(\times 10^{-2})$	2.684	1.885	1.193	1.353	0.8578	0.8248	0.7069	0.7927
$\beta_{25}(\times 10^{-1})$	-4.668	-5.685	-5.010	-4.552	-2.208	-2.225	-2.194	-2.192
$\beta_{26}(\times 10^0)$	-2.762	-3.129	-3.445	-3.456	-1.451	-1.429	-1.450	-1.411
$\beta_{27}(\times 10^{-4})$	-5.550	-6.740	-8.138	-6.686	-5.774	-5.851	-5.919	-5.698
$\beta_{28}(\times 10^{-3})$	-6.446	-5.431	-7.319	-6.573	-6.957	-7.058	-7.299	-7.131

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Table D.1 – continued from previous page

Sampling	Random	Random	Random	Random	D-optim	D-optim	D-optim	D-optim
No of Points k	100	120	200	400	100	120	200	400
$\beta_{29}(\times 10^{-1})$	-1.374	-1.469	-1.249	-1.143	-0.4154	-0.4125	-0.4019	-0.3986
$\beta_{30}(\times 10^{-1})$	-1.892	-2.399	-3.053	-3.260	-1.437	-1.426	-1.540	-1.464
$\beta_{31}(\times 10^{-6})$	-2.092	-1.365	-1.988	-1.884	-1.295	-1.049	-1.226	-1.238
$\beta_{32}(\times 10^{-5})$	-1.821	-2.184	-1.798	-1.469	-0.5274	-0.5221	-0.4857	-0.3468
$\beta_{33}(\times 10^{-4})$	-7.476	-6.936	-7.117	-6.758	-7.127	-7.152	-7.179	-7.095
$\beta_{34}(\times 10^{-3})$	-3.518	-3.536	-2.882	-2.625	-0.3155	-0.2856	-0.2517	-0.2407
$\beta_{35}(\times 10^{-3})$	-4.189	-6.559	-7.118	-7.605	-2.470	-2.637	-3.033	-2.732

Table D.2: Standard deviation for coefficients of fourth order curve fitting

Sampling	Random	Random	Random	Random	D-optim	D-optim	D-optim	D-optim
No of Points k	100	120	200	400	100	120	200	400
$\beta_1(\times 10^2)$	5.235	4.651	2.712	1.574	0.8560	0.7495	0.5609	0.3497
$\beta_2(\times 10^2)$	2.097	1.641	0.8740	0.5178	0.3641	0.2873	0.2474	0.1468
$\beta_3(\times 10^1)$	1.482	1.488	0.9499	0.5141	0.3173	0.3021	0.2172	0.1720
$\beta_4(\times 10^2)$	9.035	7.952	4.815	2.883	1.405	1.255	0.9287	0.6137
$\beta_5(\times 10^1)$	1.967	1.609	0.7950	0.5039	0.4120	0.3363	0.2955	0.1841
$\beta_6(\times 10^{-1})$	2.326	2.246	1.509	0.7513	0.5589	0.5129	0.3710	0.2950
$\beta_7(\times 10^2)$	4.111	3.228	1.805	1.128	0.6252	0.5259	0.4045	0.2575
$\beta_8(\times 10^0)$	3.903	2.951	1.542	0.9661	0.4172	0.3596	0.2781	0.1649
$\beta_9(\times 10^2)$	2.594	2.051	1.137	0.6518	0.3972	0.2932	0.2615	0.1567
$\beta_{10}(\times 10^1)$	2.615	2.511	1.548	0.9467	0.4839	0.4473	0.3028	0.2382
$\beta_{11}(\times 10^{-1})$	9.104	7.216	3.543	2.474	1.951	1.606	1.435	0.9162
$\beta_{12}(\times 10^{-3})$	1.877	1.620	1.029	0.5262	0.4076	0.3761	0.2595	0.2071
$\beta_{13}(\times 10^1)$	7.542	5.205	2.726	1.779	1.123	0.8980	0.7066	0.4331
$\beta_{14}(\times 10^{-1})$	2.198	1.684	0.8996	0.5180	0.3029	0.2548	0.1922	0.1139
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Table D.2 – continued from previous page

Sampling	Random	Random	Random	Random	D-optim	D-optim	D-optim	D-optim
No of Points k	100	120	200	400	100	120	200	400
$\beta_{15}(\times 10^1)$	1.807	1.458	0.6970	0.4493	0.2461	0.1832	0.1593	0.1072
$\beta_{16}(\times 10^0)$	3.953	3.240	1.687	1.012	0.44581	0.3731	0.3478	0.2006
$\beta_{17}(\times 10^{-2})$	2.143	1.498	0.8535	0.5114	0.2493	0.2306	0.1591	0.1200
$\beta_{18}(\times 10^{-1})$	3.203	3.162	1.971	1.146	0.6633	0.5770	0.3876	0.3001
$\beta_{19}(\times 10^1)$	8.108	6.313	3.335	1.896	1.230	0.8788	0.7779	0.4765
$\beta_{20}(\times 10^0)$	8.364	7.079	3.872	2.544	1.202	1.094	0.7518	0.5898
$\beta_{21}(\times 10^0)$	5.349	3.590	1.658	1.156	0.8114	0.6238	0.5025	0.3008
$\beta_{22}(\times 10^{-1})$	6.996	5.149	2.809	1.933	0.8630	0.7332	0.5427	0.3934
$\beta_{23}(\times 10^0)$	7.317	5.743	2.983	1.686	1.009	0.7334	0.6006	0.3934
$\beta_{24}(\times 10^{-2})$	5.715	5.154	2.823	1.629	0.7883	0.7310	0.4585	0.3790
$\beta_{25}(\times 10^{-1})$	7.709	5.656	2.554	1.637	0.8513	0.6893	0.6413	0.3774
$\beta_{26}(\times 10^0)$	3.947	3.135	1.286	0.8207	0.4243	0.3013	0.2740	0.1766
$\beta_{27}(\times 10^{-3})$	1.390	1.372	0.8153	0.4913	0.3081	0.2432	0.1753	0.1286
$\beta_{28}(\times 10^{-3})$	15.56	13.82	6.487	3.915	1.510	1.331	1.181	0.6622
$\beta_{29}(\times 10^{-1})$	1.134	0.8720	0.4762	0.2666	0.15732	0.1220	0.09276	0.06428
$\beta_{30}(\times 10^{-1})$	5.313	3.953	1.873	1.318	0.5978	0.4533	0.3851	0.2792
$\beta_{31}(\times 10^{-6})$	6.183	4.653	2.641	1.559	1.347	1.243	0.8111	0.6296
$\beta_{32}(\times 10^{-5})$	5.304	3.986	2.375	1.193	0.8086	0.8057	0.5696	0.4289
$\beta_{33}(\times 10^{-4})$	6.914	4.882	2.526	1.695	0.6464	0.6391	0.4631	0.2528
$\beta_{34}(\times 10^{-3})$	3.974	3.349	1.703	0.8663	0.6886	0.5659	0.4152	0.2607
$\beta_{35}(\times 10^{-2})$	1.638	1.210	0.6569	0.4604	0.3431	0.2774	0.2504	0.1608

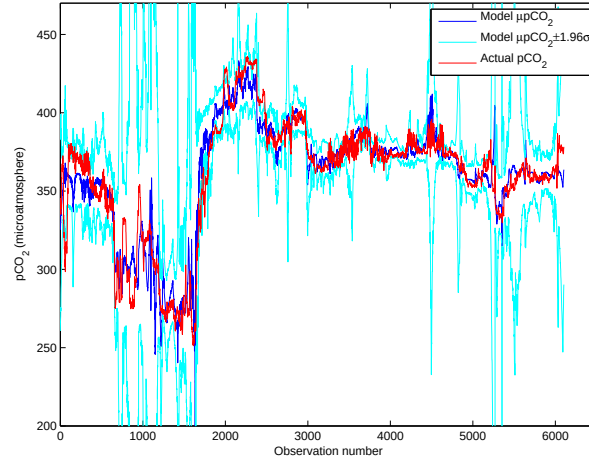


Figure D.1: Fourth order curve fit results. 100 Random points sampled.

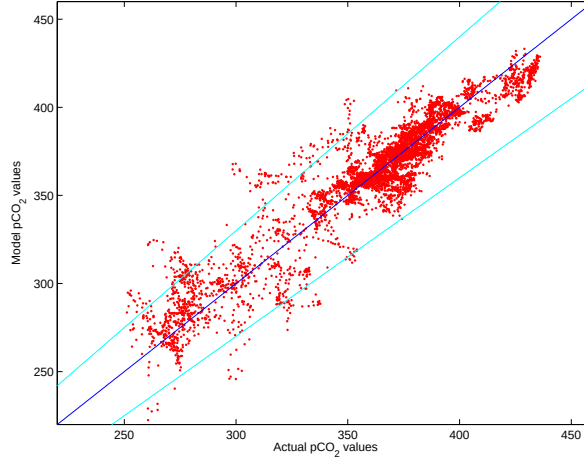


Figure D.2: Fourth order curve fit results with 100 random points sampled. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

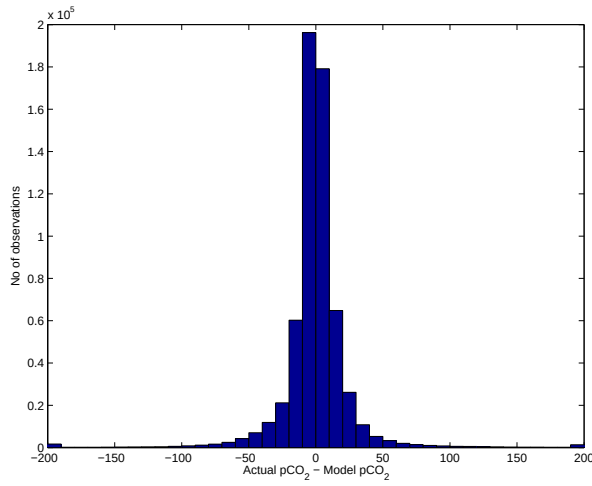


Figure D.3: Fourth order curve fit results. Histogram of errors for all 100 runs and 100 random points sampled.

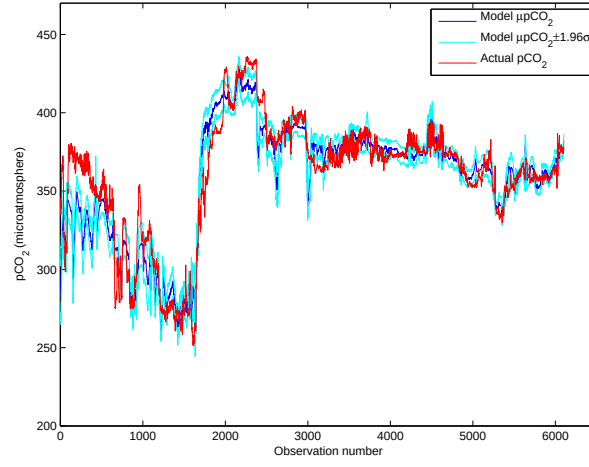


Figure D.4: Fourth order curve fit results. 100 D-optimal sampled points.

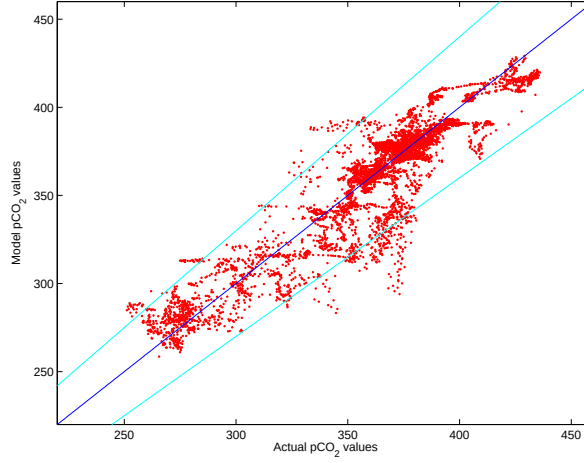


Figure D.5: Fourth order curve fit results with 100 D-optimal sampled points. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

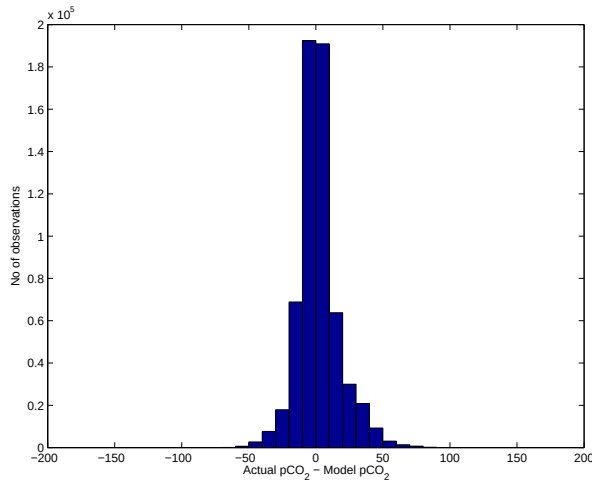


Figure D.6: Fourth order curve fit results. Histogram of errors for all 100 runs and 100 D-optimal sampled points.

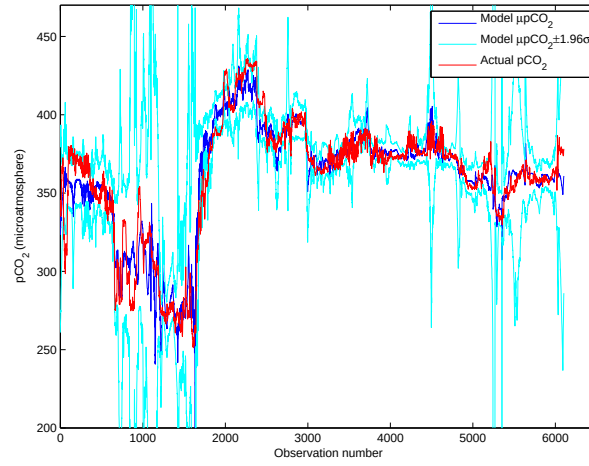


Figure D.7: Fourth order curve fit results. 120 Random points sampled.

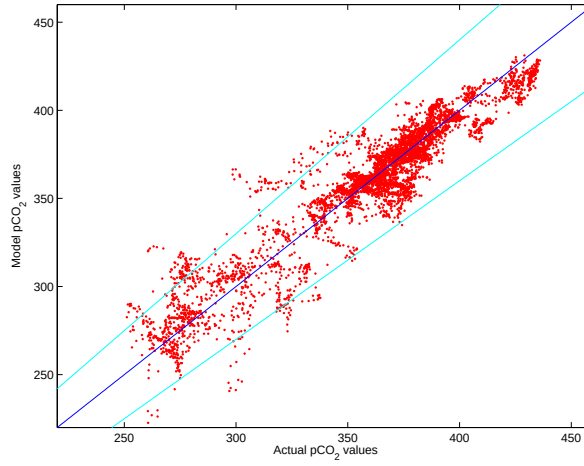


Figure D.8: Fourth order curve fit results with 120 random points sampled. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

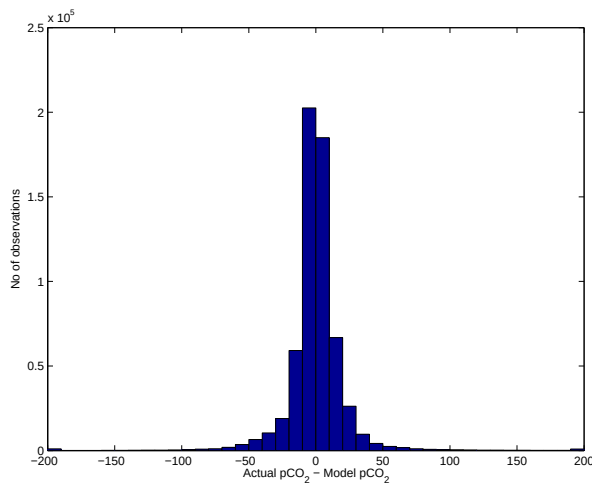


Figure D.9: Fourth order curve fit results. Histogram of errors for all 100 runs and 120 random points sampled.

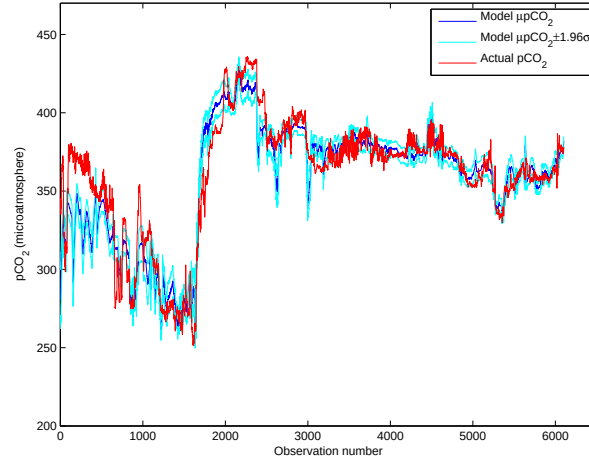


Figure D.10: Fourth order curve fit results. 120 D-optimal sampled points.

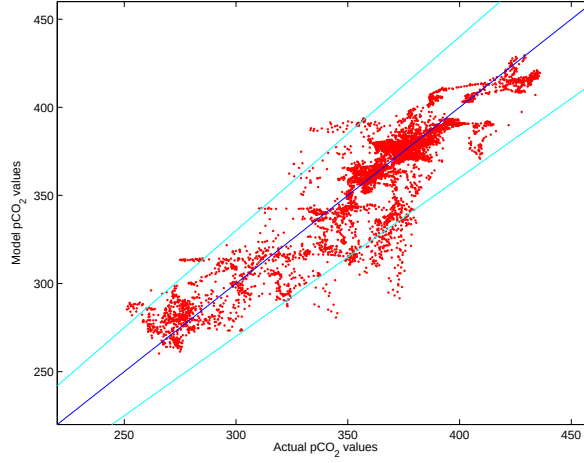


Figure D.11: Fourth order curve fit results with 120 D-optimal sampled points. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

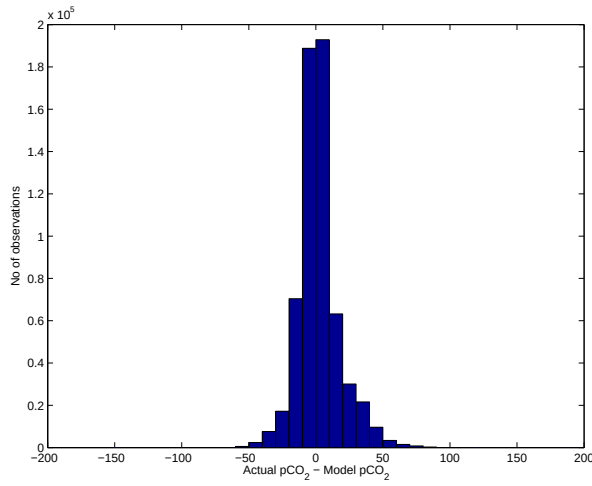


Figure D.12: Fourth order curve fit results. Histogram of errors for all 100 runs and 120 D-optimal sampled points.

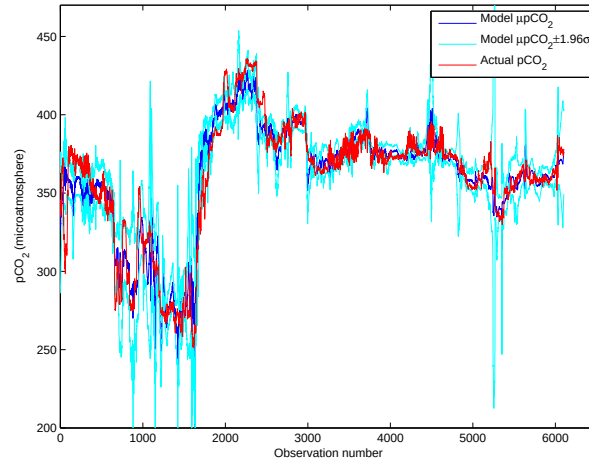


Figure D.13: Fourth order curve fit results. 200 random points sampled.

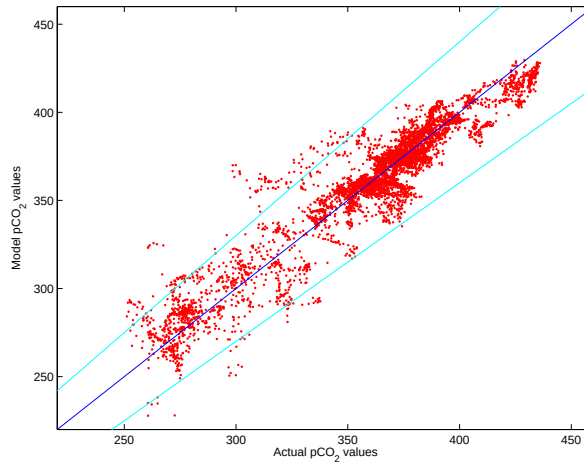


Figure D.14: Fourth order curve fit results with 200 Random points sampled. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

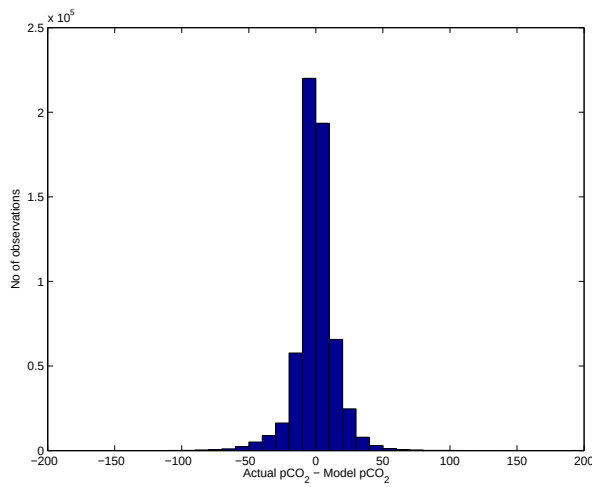


Figure D.15: Fourth order curve fit results. Histogram of errors for all 100 runs and 200 random points sampled.

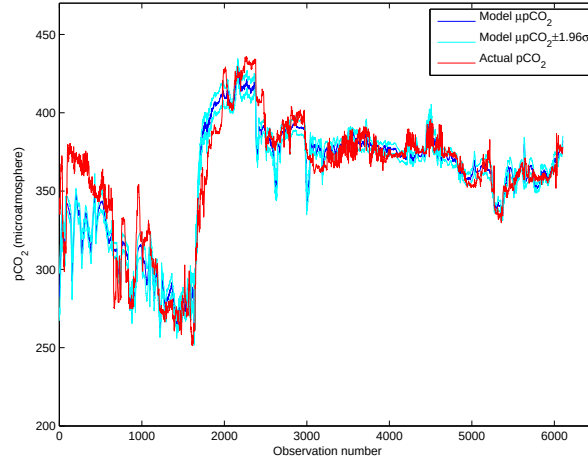


Figure D.16: Fourth order curve fit results. 200 D-optimal sampled points.

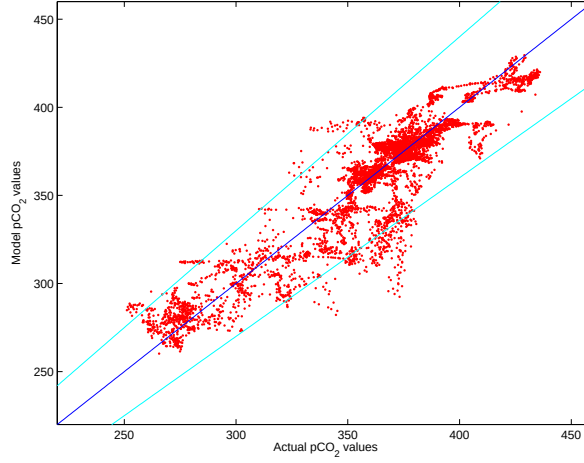


Figure D.17: Fourth order curve fit results with 200 D-optimal sampled points. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

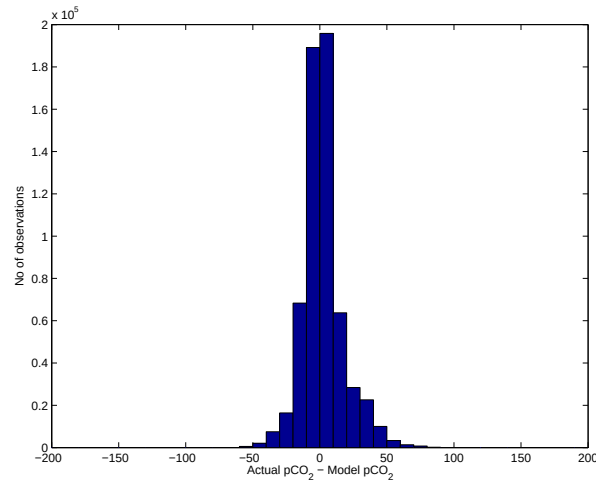


Figure D.18: Fourth order curve fit results. Histogram of errors for all 100 runs and 200 D-optimal sampled points.

Appendix E

Linear curve fit results with latitude included as variable

The first figure in each case is a plot of the actual pCO_2 values, together with the mean values of the estimated pCO_2 from the 100 curve fits. The 95% confidence interval is also shown for the 100 curve fits in each case. The second figure in each case is the actual pCO_2 values plotted against the estimated pCO_2 values. The 10% error lines indicate the number of points that fall outside the 10% error. The third figure in each case is a histogram of the errors on the estimation of the pCO_2 , showing the distribution of the errors for each case.

It can be seen from the figures that as the number of points sampled from the data set increases, the fit of the data is improved and less points falls outside the 10% error interval. Also, from the figures it can be seen that the D-optimal sampling yields a smaller 95% confidence interval than the random sampling. The histograms generally show that the error on the estimation has a normal distribution around zero.

The mean coefficients for the linear equation for all 100 curve fits are given in Table E.1. The standard deviation for the coefficients are given in Table E.2. It can be seen from the table of the standard deviations that the coefficients for the D-optimal sampling have a smaller standard deviation than the coefficients for the random sampling.

Table E.1: Coefficients of linear curve fitting with latitude included as variable

Sampling	Random	Random	Random	Random	D-optim	D-optim	D-optim	D-optim
No of points k	10	50	100	200	10	50	100	200
Terms								
$\beta_1(\times 10^2)$	4.954	4.341	4.298	4.348	4.221	4.267	4.257	4.259
$\beta_2(\times 10^0)$	-4.265	-2.888	-2.772	-2.868	-1.548	-1.388	-1.347	-1.359
$\beta_3(\times 10^{-2})$	-4.431	2.531	2.111	2.013	-4.793	-5.723	-5.991	-5.878
$\beta_4(\times 10^1)$	-3.133	-2.890	-2.957	-2.884	-2.685	-2.547	-2.531	-2.531
$\beta_5(\times 10^0)$	-1.369	-0.4470	-0.3621	-0.4567	-0.3081	-0.4430	-0.4327	-0.4337

Table E.2: Standard deviation of coefficients of linear curve fitting with latitude included as variable

Sampling	Random	Random	Random	Random	D-optim	D-optim	D-optim	D-optim
No of points k	10	50	100	200	10	50	100	200
Terms								
$\beta_1(\times 10^2)$	2.449	0.4709	0.3053	0.1845	0.5043	0.1460	0.1083	0.07597
$\beta_2(\times 10^0)$	7.490	0.9395	0.6515	0.4049	0.9783	0.2.735	0.1933	0.1470
$\beta_3(\times 10^{-1})$	4.630	1.485	1.093	0.6039	0.9878	0.3094	0.1869	0.1346
$\beta_4(\times 10^1)$	2.318	0.4355	0.3014	0.1888	0.3214	0.09490	0.06795	0.04377
$\beta_5(\times 10^0)$	3.781	0.7480	0.4379	0.3018	0.8356	0.2429	0.1898	0.1277

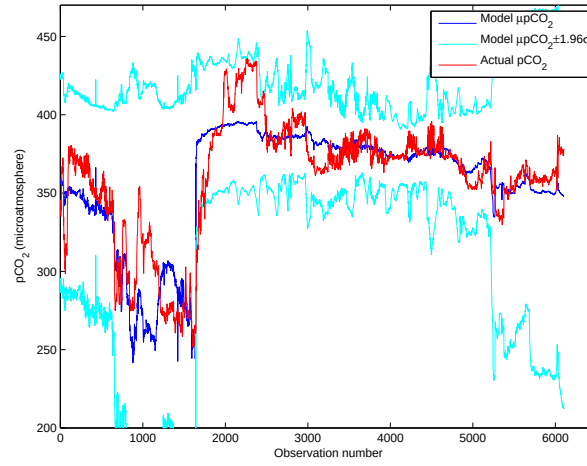


Figure E.1: Linear curve fit results. 10 Random points sampled.

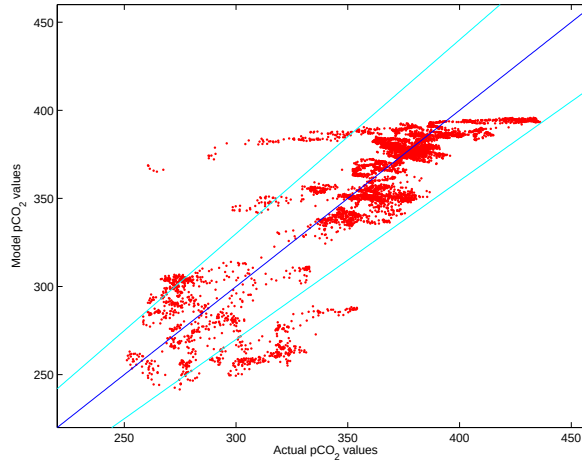


Figure E.2: Linear curve fit results with 10 random points sampled. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

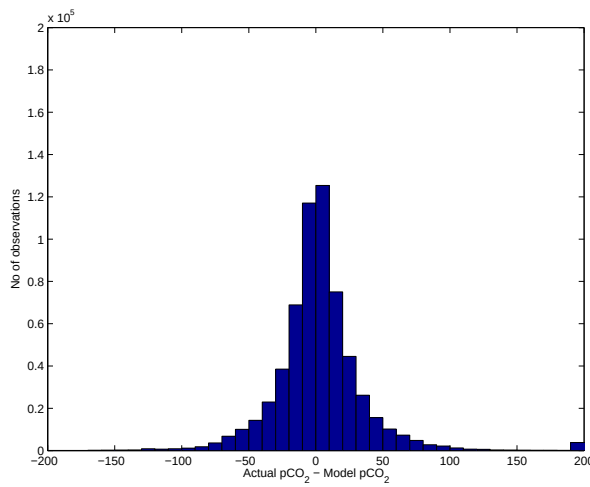


Figure E.3: Linear curve fit results. Histogram of errors for all 100 runs and 10 random points sampled.

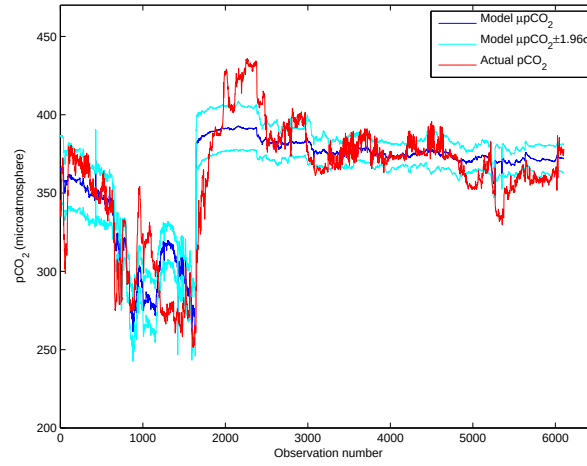


Figure E.4: Linear curve fit results. 10 D-optimal sampled points.

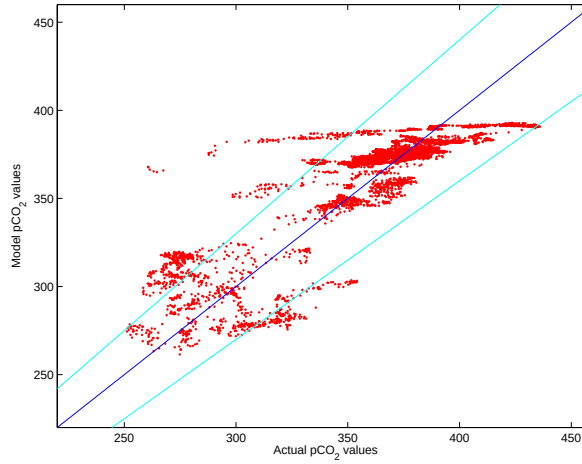


Figure E.5: Linear curve fit results with 10 D-optimal sampled points. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

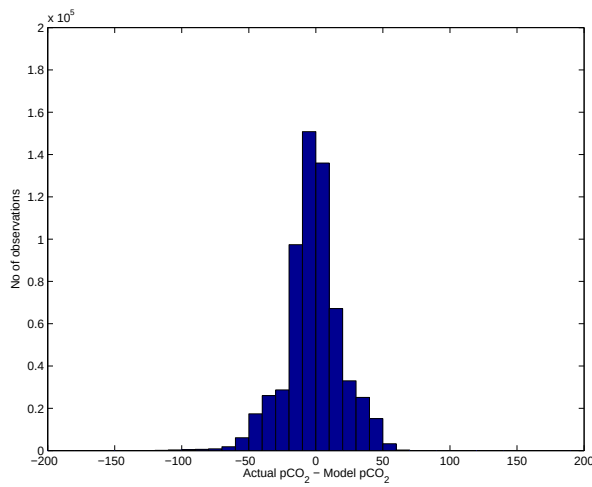


Figure E.6: Linear curve fit results. Histogram of errors for all 100 runs and 10 D-optimal sampled points.

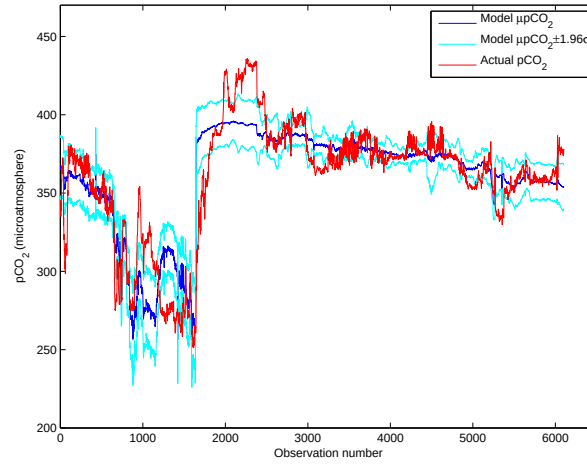


Figure E.7: Linear curve fit results. 50 Random points sampled.

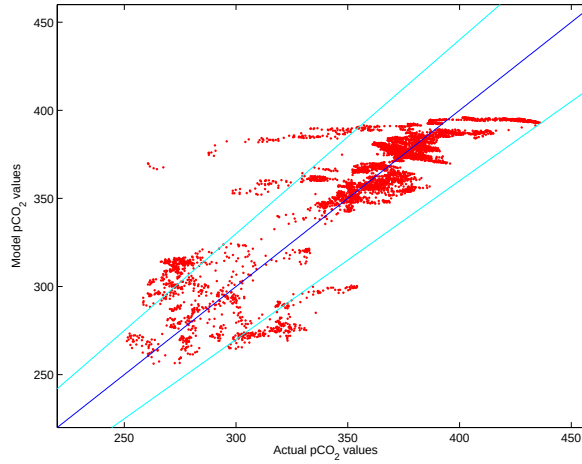


Figure E.8: Linear curve fit results with 50 random points sampled. The degree to which the modelled points fall in the 10 % error margin is shown. The mean pCO₂ estimation for all 100 runs is shown here.

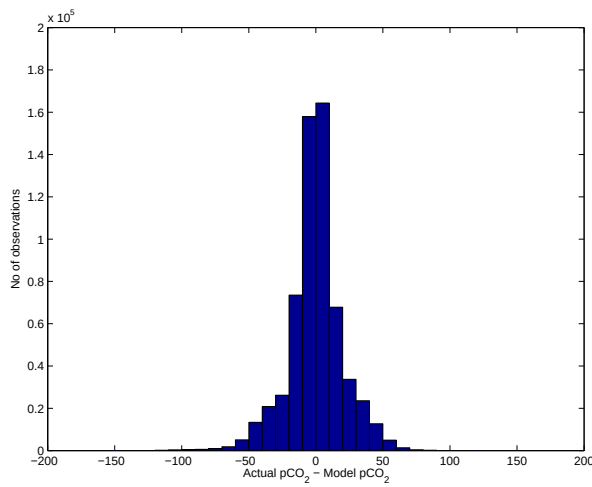


Figure E.9: Linear curve fit results. Histogram of error for all 100 runs and 50 random points sampled.

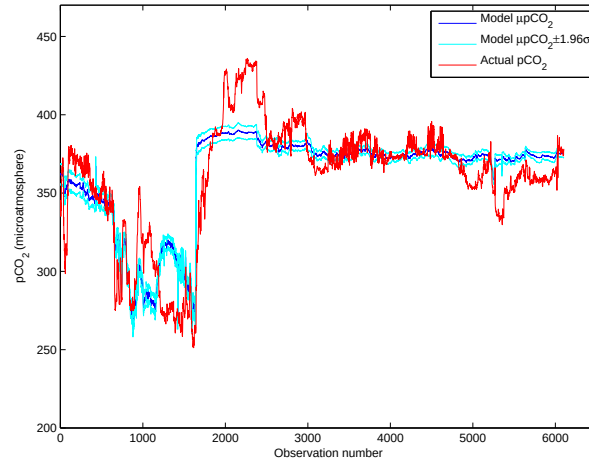


Figure E.10: Linear curve fit results. 50 D-optimal sampled points.

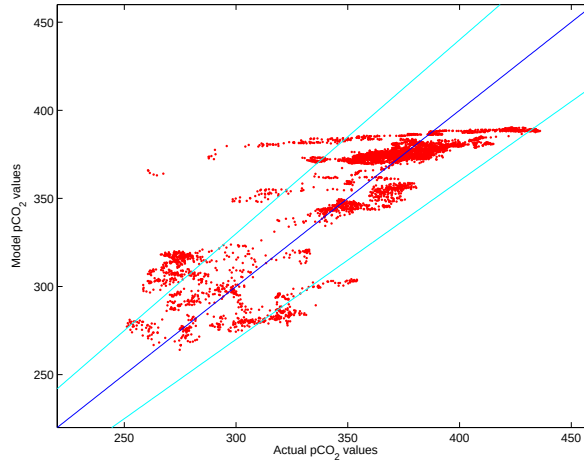


Figure E.11: Linear curve fit results with 50 D-optimal sampled points. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

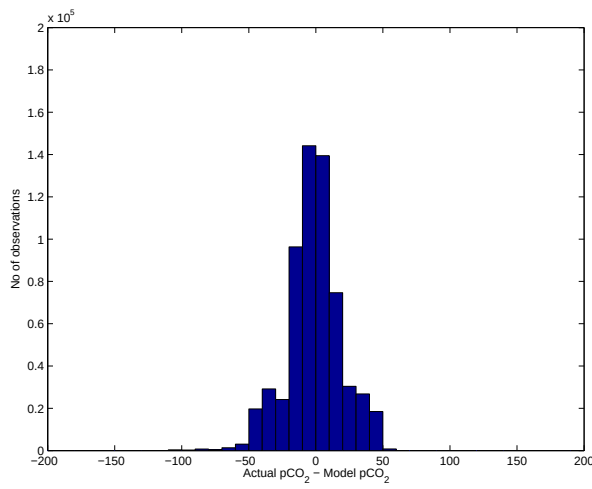


Figure E.12: Linear curve fit results. Histogram of errors for all 100 runs and 50 D-optimal sampled points.

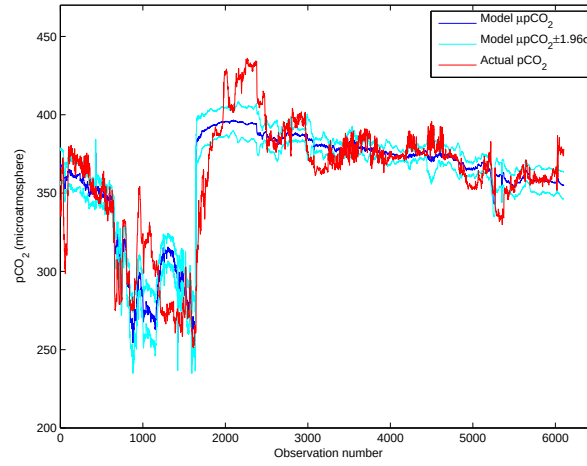


Figure E.13: Linear curve fit results. 100 Random points sampled.

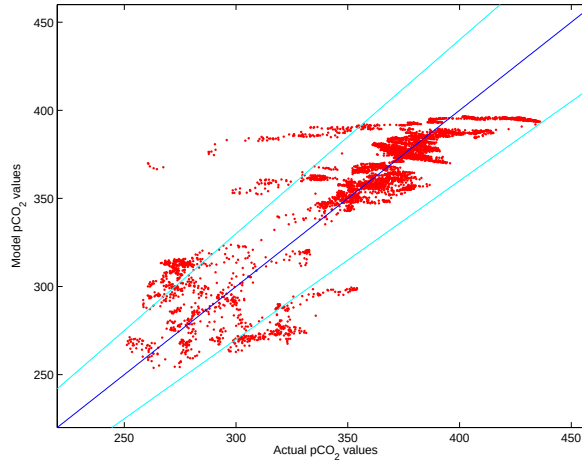


Figure E.14: Linear curve fit results with 100 random points sampled. The degree to which the modelled points fall in the 10 % error margin is shown. The mean pCO₂ estimation for all 100 runs is shown here.

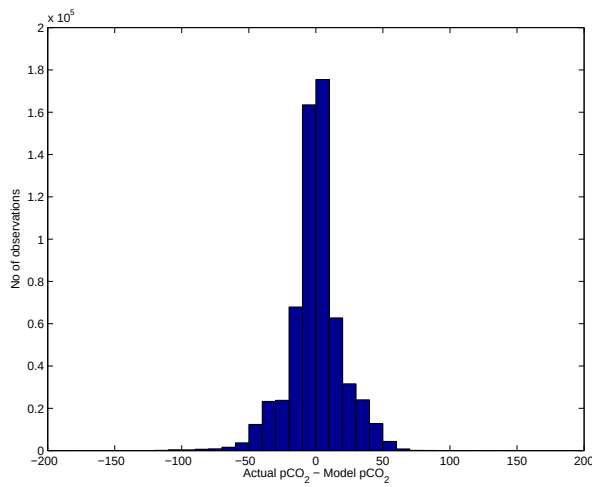


Figure E.15: Linear curve fit results. Histogram of error for all 100 runs and 100 random points sampled.

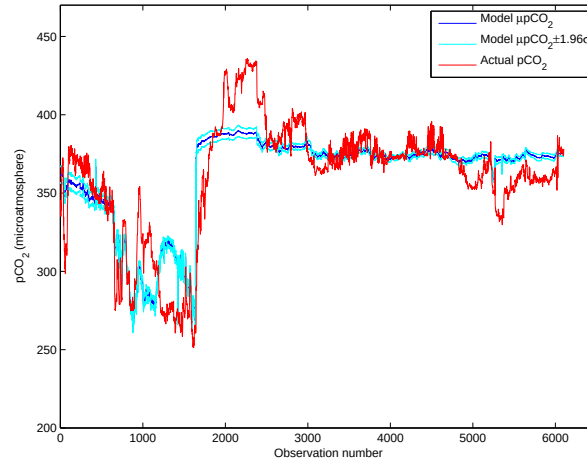


Figure E.16: Linear curve fit results. 100 D-optimal sampled points.

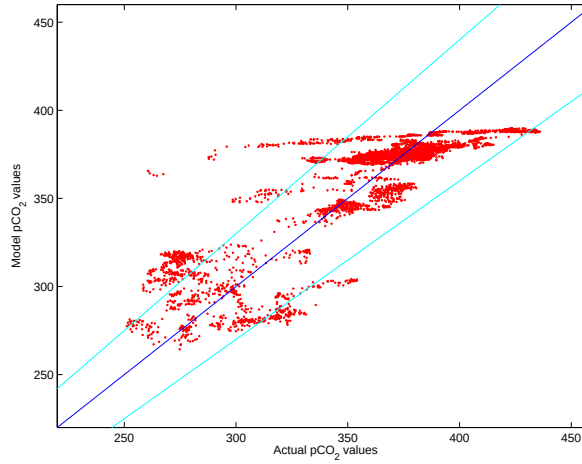


Figure E.17: Linear curve fit results with 100 D-optimal sampled points. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

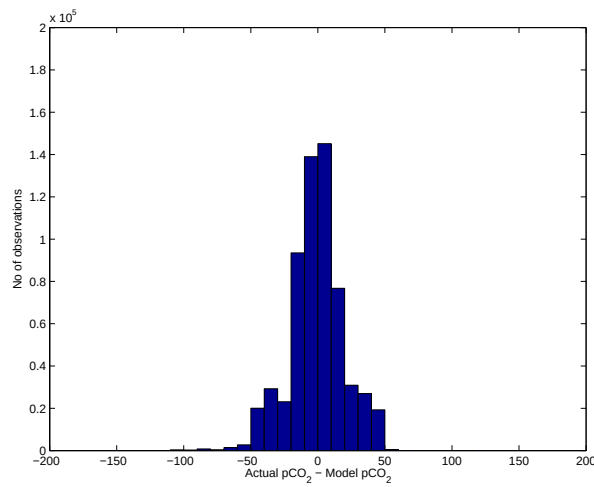


Figure E.18: Linear curve fit results. Histogram of errors for all 100 runs and 100 D-optimal sampled points.

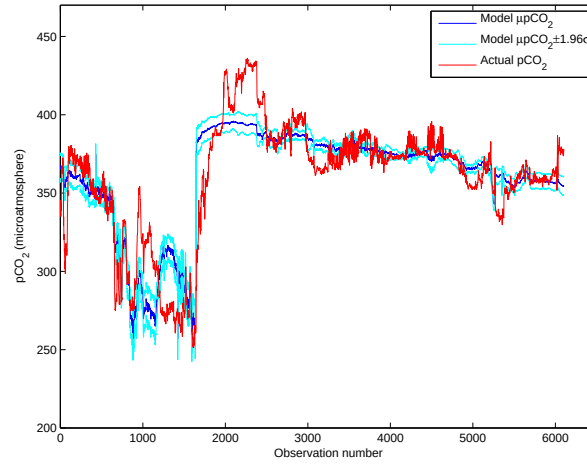


Figure E.19: Linear curve fit results. 200 random points sampled.

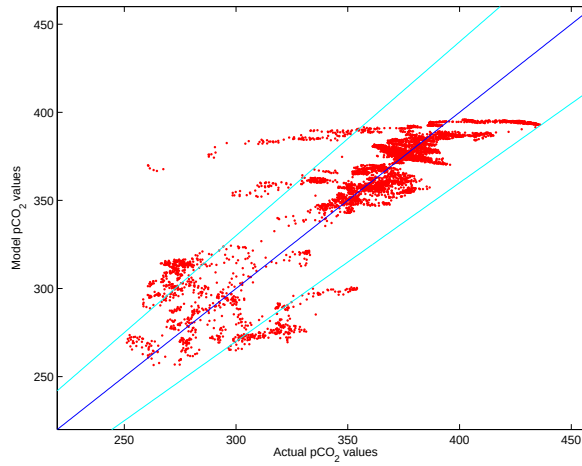


Figure E.20: Linear curve fit results with 200 Random points sampled. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

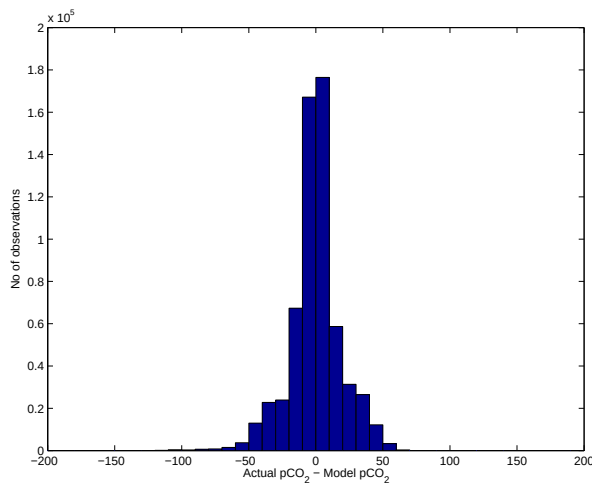


Figure E.21: Linear curve fit results. Histogram of errors for all 100 runs and 200 random points sampled.

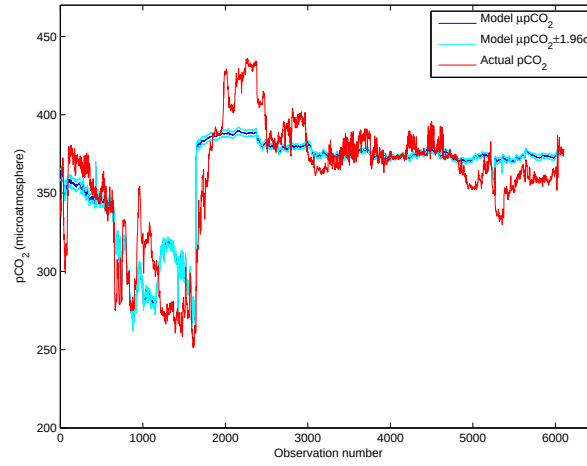


Figure E.22: Linear curve fit results. 200 D-optimal sampled points.

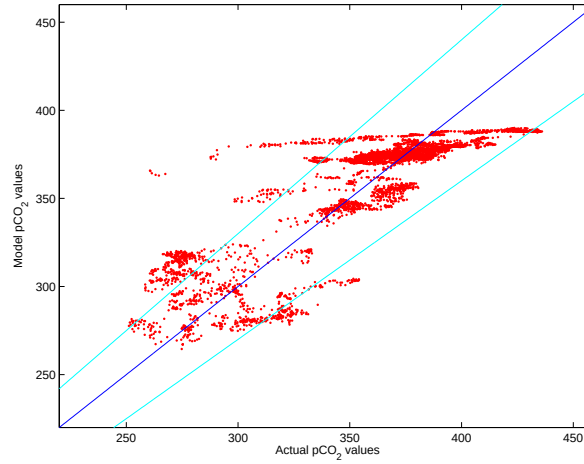


Figure E.23: Linear curve fit results with 200 D-optimal sampled points. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

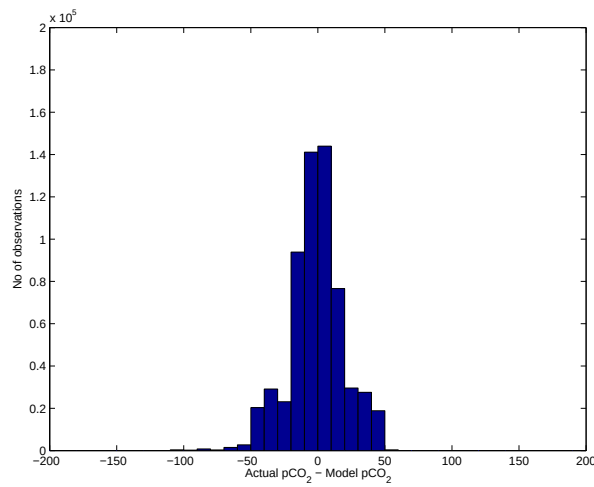


Figure E.24: Linear curve fit results. Histogram of errors for all 100 runs and 200 D-optimal sampled points.

Appendix F

Quadratic curve fit results with latitude included as variable

The first figure in each case is a plot of the actual pCO_2 values, together with the mean values of the estimated pCO_2 from the 100 curve fits. The 95% confidence interval is also shown for the 100 curve fits in each case. The second figure in each case is the actual pCO_2 values plotted against the estimated pCO_2 values. The 10% error lines indicate the number of points that fall outside the 10% error. The third figure in each case is a histogram of the errors on the estimation of the pCO_2 , showing the distribution of the errors for each case.

It can be seen from the figures that as the number of points sampled from the data set increases, the fit of the data is improved and less points falls outside the 10% error interval. Also, from the figures it can be seen that the D-optimal sampling yields a smaller 95% confidence interval than the random sampling. The histograms generally show that the error on the estimation has a normal distribution around zero.

The mean coefficients for the quadratic equation for all 100 curve fits are given in Table F.1. The standard deviation for the coefficients are given in Table F.2. It can be seen from the table of the standard deviations that the coefficients for the D-optimal sampling have a smaller standard deviation than the coefficients for the random sampling.

Table F.1: Coefficients for quadratic curve fitting with Latitude included as variable

Sam-pling	Random	Random	Random	Random	D-optim	D-optim	D-optim	D-optim
No of Points k	25	50	100	200	25	50	100	200
$\beta_1(\times 10^3)$	2.453	1.690	1.573	1.493	2.537	2.452	2.452	2.453
$\beta_2(\times 10^1)$	-7.769	-4.180	-3.550	-3.030	-7.333	-7.273	-7.273	-7.769
$\beta_3(\times 10^0)$	-4.569	-3.434	-3.174	-3.184	-5.036	-4.866	-4.866	-4.569
$\beta_4(\times 10^2)$	-1.752	-5.372	-4.288	-4.099	-2.092	-1.897	-1.897	-1.752
$\beta_5(\times 10^1)$	-6.150	-3.177	-3.004	-2.772	-6.314	-6.111	-6.111	-6.150
$\beta_6(\times 10^{-1})$	7.170	2.962	2.486	1.939	7.245	6.705	6.705	7.170
$\beta_7(\times 10^{-4})$	-0.6947	10.93	6.611	2.064	5.051	0.9055	0.9055	-0.6947
$\beta_8(\times 10^0)$	2.447	9.244	9.637	9.125	4.606	3.125	3.125	2.447
$\beta_9(\times 10^{-1})$	4.364	1.810	1.836	1.652	4.469	4.324	4.324	4.364
$\beta_{10}(\times 10^{-2})$	4.069	3.659	3.371	3.215	5.391	4.763	4.763	4.069
$\beta_{11}(\times 10^0)$	2.695	8.887	7.328	7.064	4.558	3.461	3.461	2.695
$\beta_{12}(\times 10^0)$	1.199	0.4821	0.3985	0.3126	1.021	1.078	1.078	1.199
$\beta_{13}(\times 10^{-1})$	-2.221	1.299	0.8102	0.3002	-2.492	-2.342	-2.342	-2.221
$\beta_{14}(\times 10^{-2})$	9.169	6.033	5.755	5.987	9.876	9.660	9.660	9.169
$\beta_{15}(\times 10^0)$	2.384	7.056	5.387	5.175	2.688	2.535	2.535	2.384

Table F.2: Coefficients for quadratic curve fitting with Latitude included as variable

Sam-pling	Random	Random	Random	Random	D-optim	D-optim	D-optim	D-optim
No of Points k	25	50	100	200	25	50	100	200
$\beta_1(\times 10^3)$	2.453	1.690	1.573	1.493	2.537	2.452	2.452	2.453
$\beta_2(\times 10^1)$	-7.769	-4.180	-3.550	-3.030	-7.333	-7.273	-7.273	-7.769
$\beta_3(\times 10^0)$	-4.569	-3.434	-3.174	-3.184	-5.036	-4.866	-4.866	-4.569
$\beta_4(\times 10^2)$	-1.752	-5.372	-4.288	-4.099	-2.092	-1.897	-1.897	-1.752
$\beta_5(\times 10^1)$	-6.150	-3.177	-3.004	-2.772	-6.314	-6.111	-6.111	-6.150
$\beta_6(\times 10^{-1})$	7.170	2.962	2.486	1.939	7.245	6.705	6.705	7.170
$\beta_7(\times 10^{-4})$	-0.6947	10.93	6.611	2.064	5.051	0.9055	0.9055	-0.6947
$\beta_8(\times 10^0)$	2.447	9.244	9.637	9.125	4.606	3.125	3.125	2.447
$\beta_9(\times 10^{-1})$	4.364	1.810	1.836	1.652	4.469	4.324	4.324	4.364
$\beta_{10}(\times 10^{-2})$	4.069	3.659	3.371	3.215	5.391	4.763	4.763	4.069
$\beta_{11}(\times 10^0)$	2.695	8.887	7.328	7.064	4.558	3.461	3.461	2.695
$\beta_{12}(\times 10^0)$	1.199	0.4821	0.3985	0.3126	1.021	1.078	1.078	1.199
$\beta_{13}(\times 10^{-1})$	-2.221	1.299	0.8102	0.3002	-2.492	-2.342	-2.342	-2.221
$\beta_{14}(\times 10^{-2})$	9.169	6.033	5.755	5.987	9.876	9.660	9.660	9.169
$\beta_{15}(\times 10^0)$	2.384	7.056	5.387	5.175	2.688	2.535	2.535	2.384

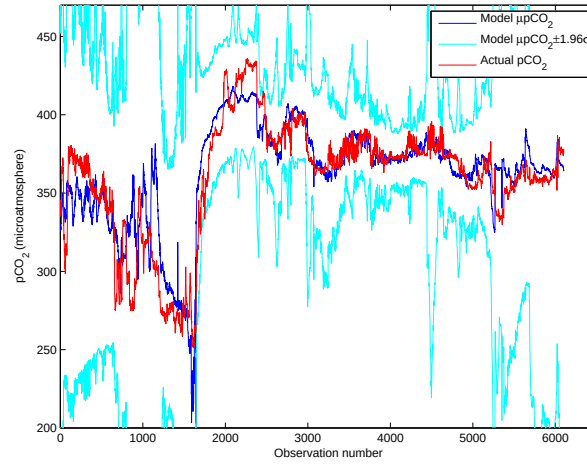


Figure F.1: Quadratic curve fit results. 25 Random points sampled.

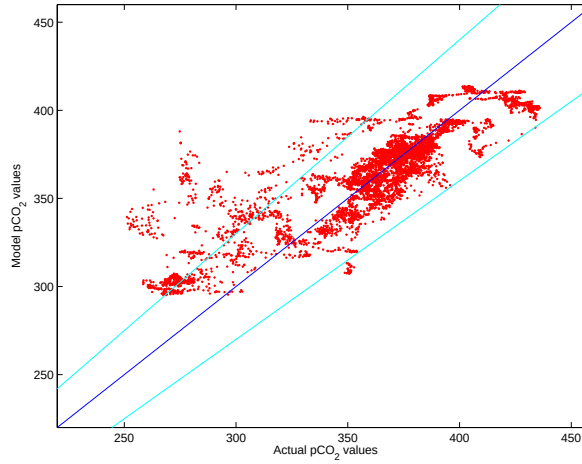


Figure F.2: Quadratic curve fit results 25 random points sampled. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

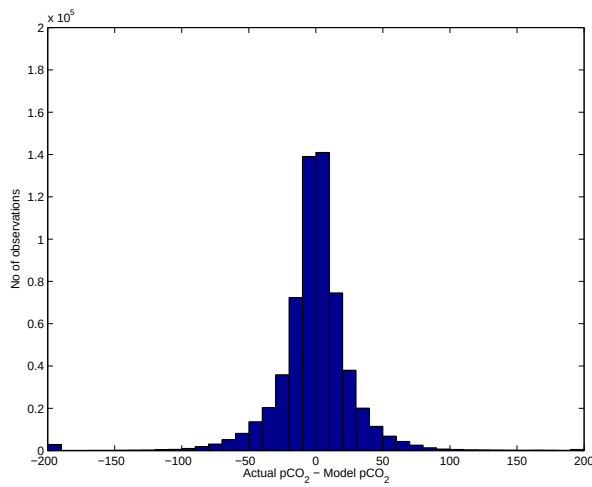


Figure F.3: Quadratic curve fit results. Histogram of errors for all 100 runs and 25 random points sampled.

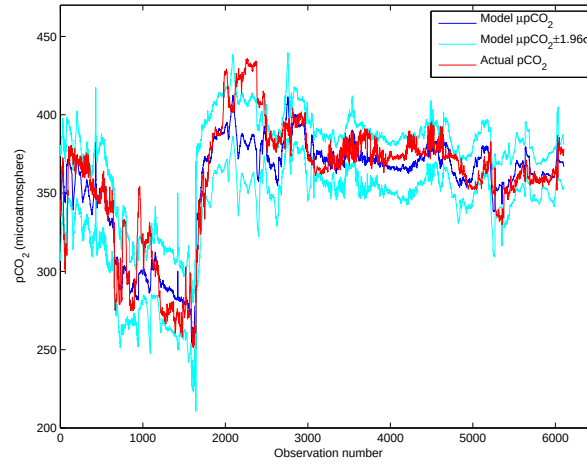


Figure F.4: Quadratic curve fit results. 25 D-optimal sampled points.

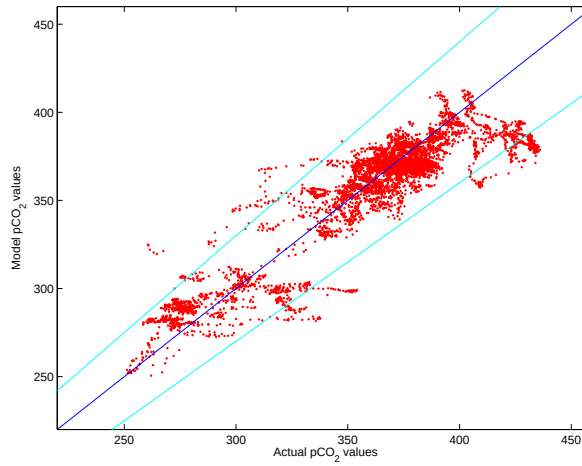


Figure F.5: Quadratic curve fit results 25 D-optimal sampled points. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

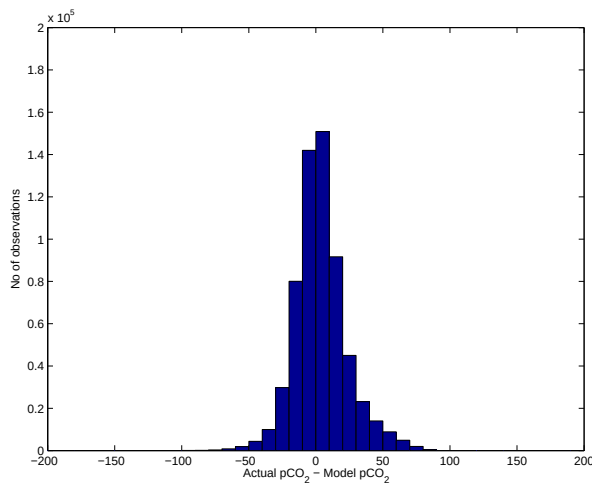


Figure F.6: Quadratic curve fit results. Histogram of errors for all 100 runs and 25 D-optimal sampled points.

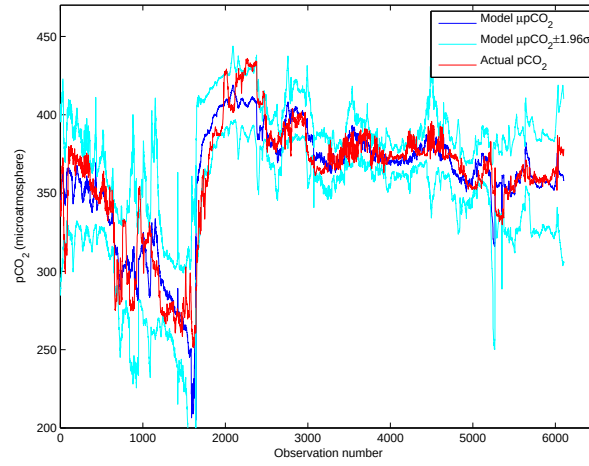


Figure F.7: Quadratic curve fit results. 50 Random points sampled.

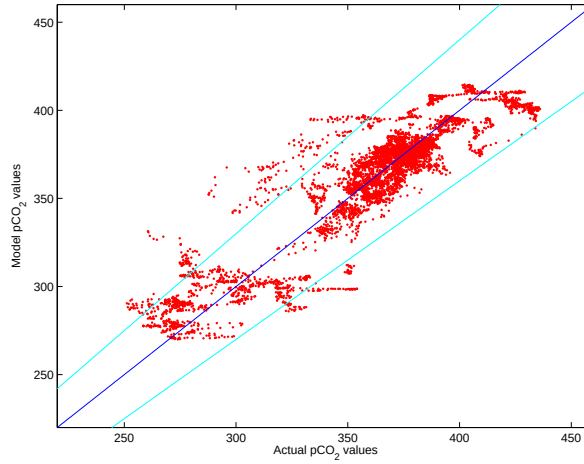


Figure F.8: Quadratic curve fit results 50 random points sampled. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

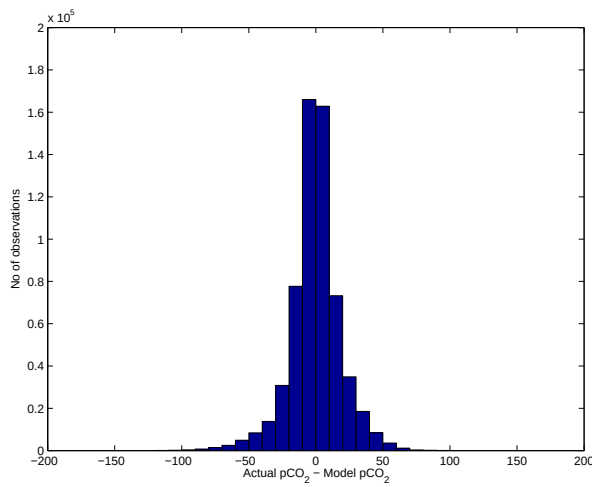


Figure F.9: Quadratic curve fit results. Histogram of errors for all 100 runs and 50 random points sampled.

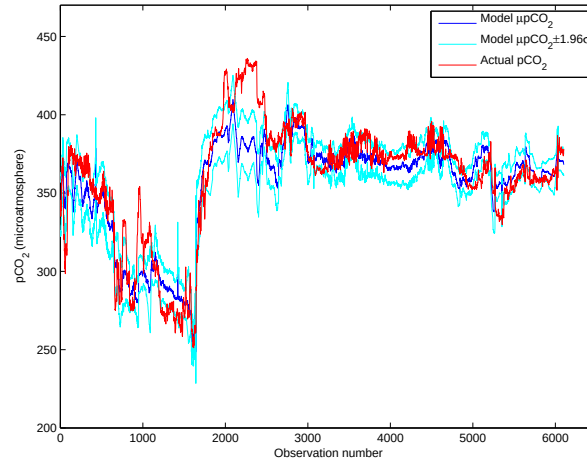


Figure F.10: Quadratic curve fit results. 50 D-optimal sampled points.

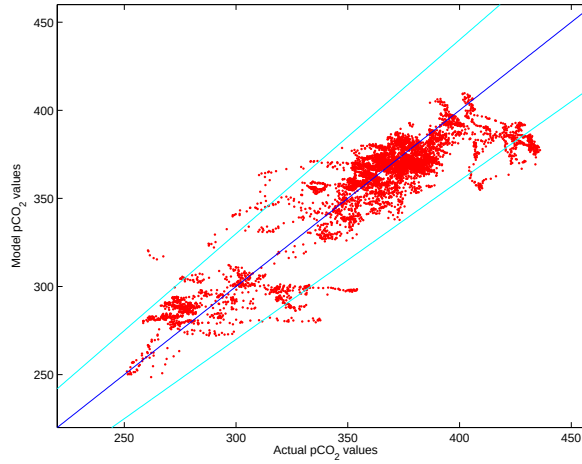


Figure F.11: Quadratic curve fit results 50 D-optimal sampled points. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

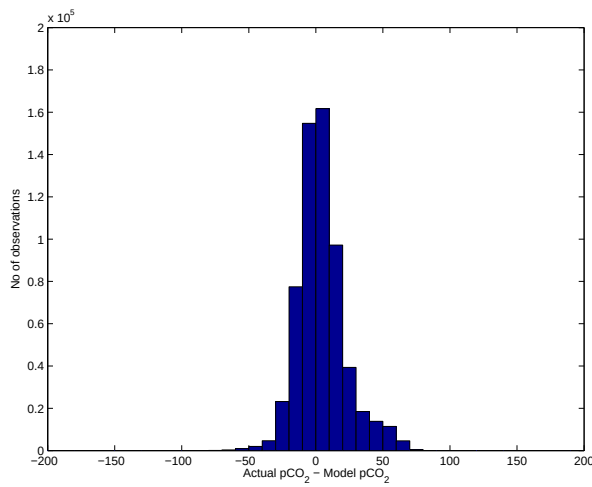


Figure F.12: Quadratic curve fit results. Histogram of errors for all 100 runs and 50 D-optimal sampled points.

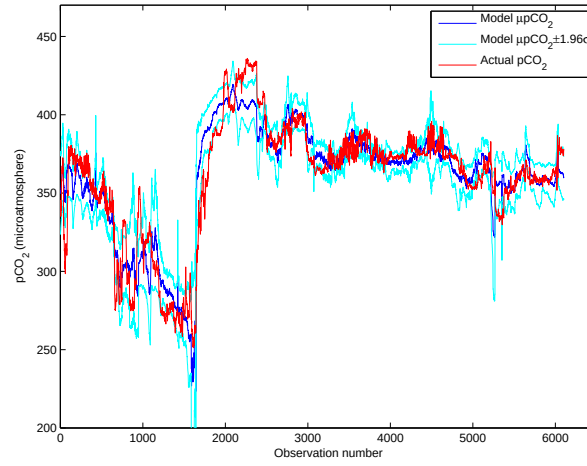


Figure F.13: Quadratic curve fit results. 100 Random points sampled.

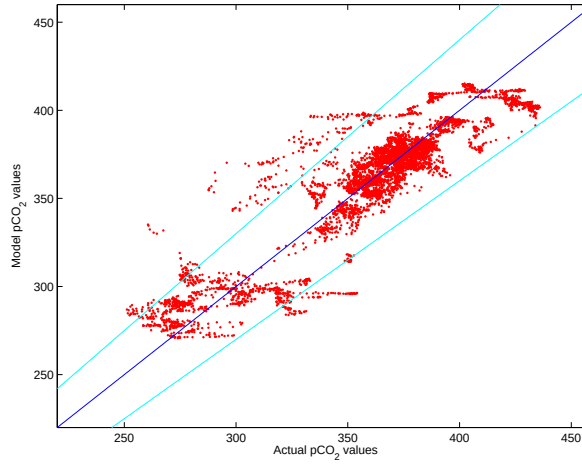


Figure F.14: Quadratic curve fit results 100 random points sampled. The degree to which the modelled points fall in the 10 % error margin is shown. The mean pCO₂ estimation for all 100 runs is shown here.

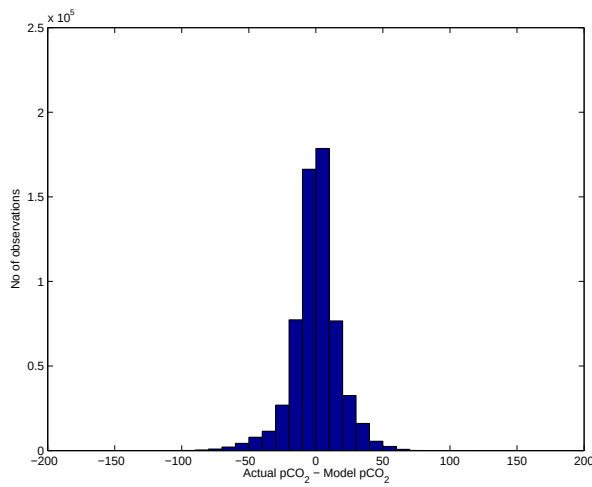


Figure F.15: Quadratic curve fit results. Histogram of errors for all 100 runs and 100 random points sampled.

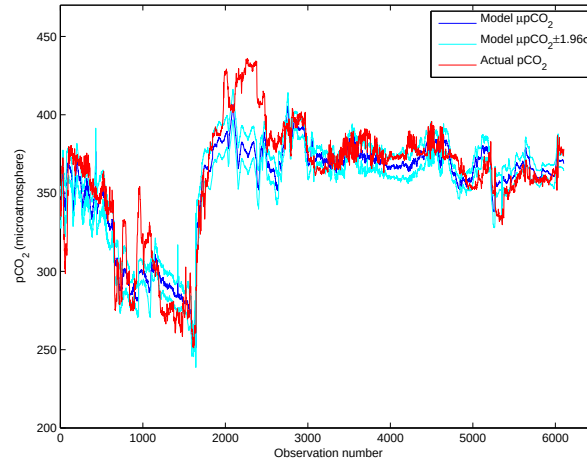


Figure F.16: Quadratic curve fit results. 100 D-optimal sampled points.

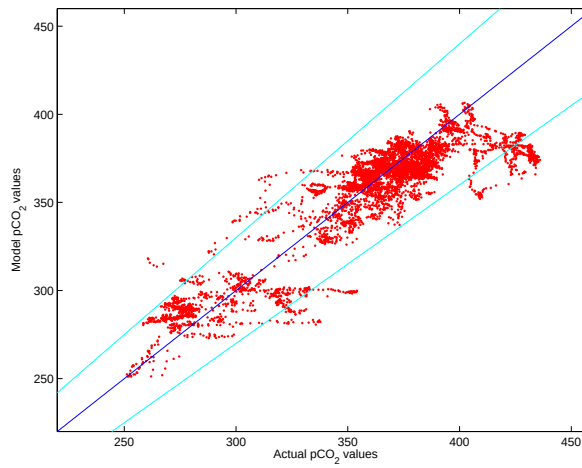


Figure F.17: Quadratic curve fit results with 100 D-optimal sampled points. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

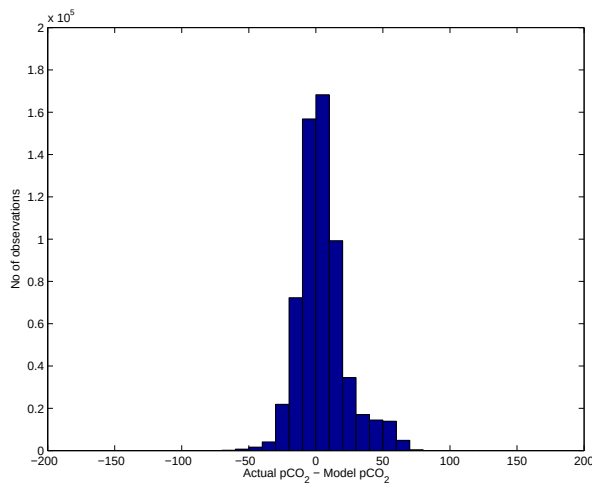


Figure F.18: Quadratic curve fit results. Histogram of errors for all 100 runs and 100 D-optimal sampled points.

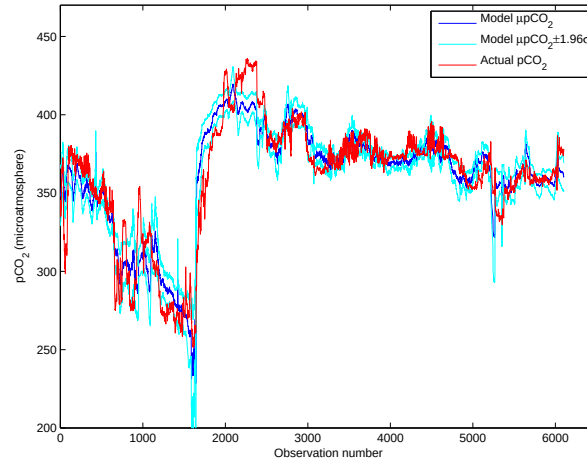


Figure F.19: Quadratic curve fit results. 200 Random points sampled.

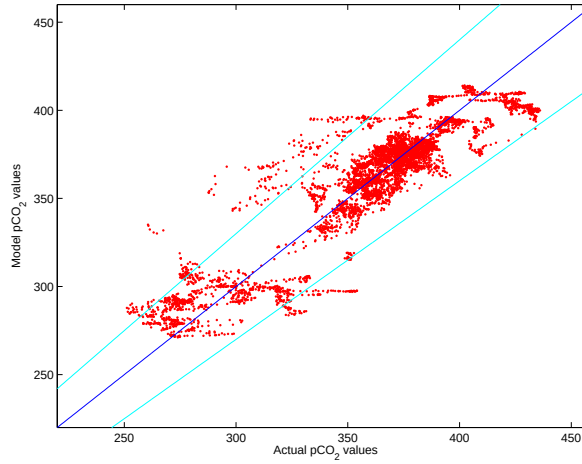


Figure F.20: Quadratic curve fit results with 300 randomly sampled points. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

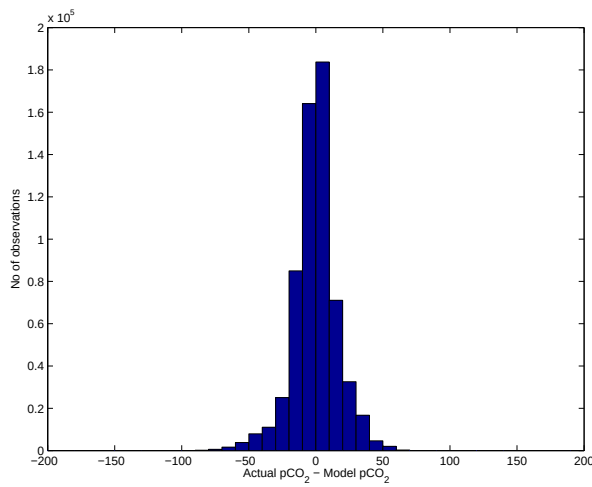


Figure F.21: Quadratic curve fit results. Histogram of errors for all 100 runs and 200 random points sampled.

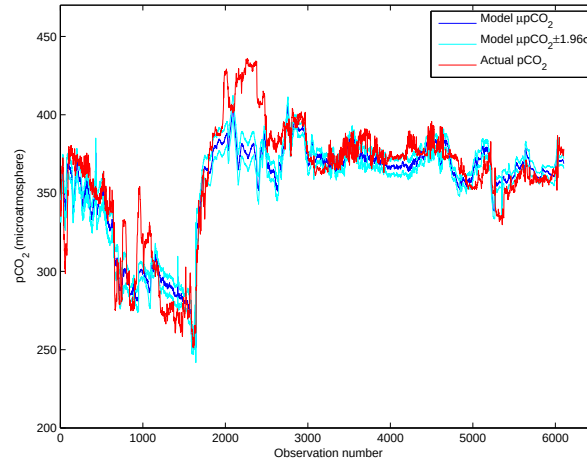


Figure F.22: Quadratic curve fit results. 200 D-optimal sampled points.

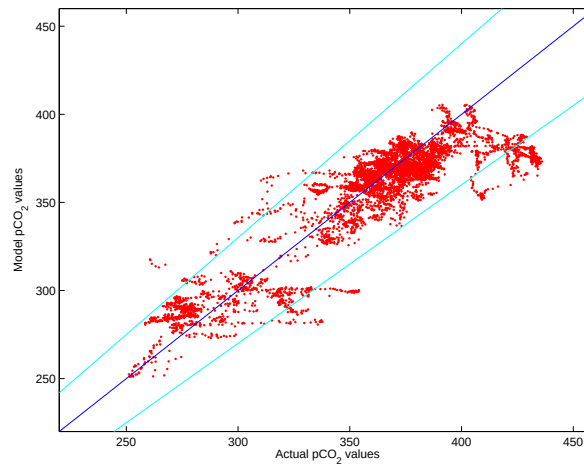


Figure F.23: Quadratic curve fit results with 200 D-optimal samples points. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

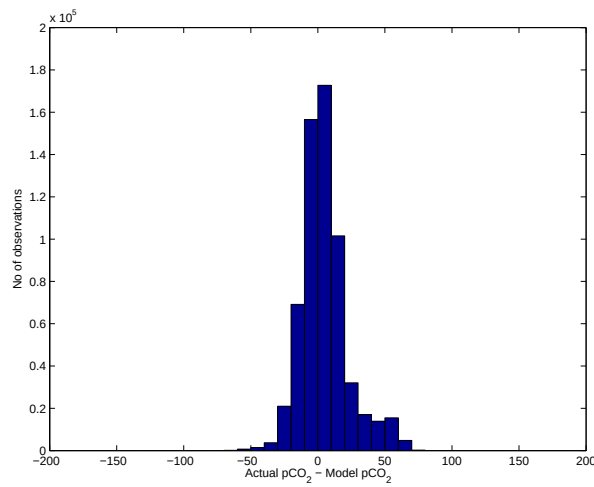


Figure F.24: Quadratic curve fit results. Histogram of errors for all 100 runs and 200 D-optimal sampled points.

Appendix G

Cubic curve fit results with latitude included as variable

The first figure in each case is a plot of the actual pCO_2 values, together with the mean values of the estimated pCO_2 from the 100 curve fits. The 95% confidence interval is also shown for the 100 curve fits in each case. The second figure in each case is the actual pCO_2 values plotted against the estimated pCO_2 values. The 10% error lines indicate the number of points that fall outside the 10% error. The third figure in each case is a histogram of the errors on the estimation of the pCO_2 , showing the distribution of the errors for each case.

It can be seen from the figures that as the number of points sampled from the data set increases, the fit of the data is improved and less points falls outside the 10% error interval. Also, from the figures it can be seen that the D-optimal sampling yields a smaller 95% confidence interval than the random sampling. The histograms generally show that the error on the estimation has a normal distribution around zero.

The mean coefficients for the cubic equation for all 100 curve fits are given in Table G.1. The standard deviation for the coefficients are given in Table G.2. It can be seen from the table of the standard deviations that the coefficients for the D-optimal sampling have a smaller standard deviation than the coefficients for the random sampling.

Table G.1: Coefficients of cubic curve fitting with Latitude included as variable

Sampling No of Points k	Random	Random	Random	Random	D-optim	D-optim	D-optim	D-optim
	100	120	200	400	100	120	200	400
$\beta_1(\times 10^4)$	-3.020	-2.357	-1.958	-2.226	-2.728	-2.575	-2.509	-2.501
$\beta_2(\times 10^3)$	2.602	1.888	1.546	1.681	1.759	1.686	1.623	1.613
$\beta_3(\times 10^1)$	-2.036	-0.1402	-1.983	-1.789	1.648	-0.9363	-1.412	-1.451
$\beta_4(\times 10^3)$	6.131	-0.7016	-3.359	-3.116	0.2843	0.2116	0.3449	0.4868
$\beta_5(\times 10^3)$	1.370	1.188	1.047	1.165	1.356	1.297	1.268	1.260
$\beta_6(\times 10^1)$	-7.056	-4.594	-3.790	-4.074	-3.816	-3.829	-3.672	-3.662
$\beta_7(\times 10^{-1})$	1.496	0.7582	0.9766	0.9678	-0.1176	0.1269	0.1565	0.1595
$\beta_8(\times 10^2)$	7.619	5.814	3.013	2.438	1.098	1.480	1.489	1.557
$\beta_9(\times 10^1)$	-1.970	-1.874	-1.747	-1.931	-2.186	-2.114	-2.074	-2.054
$\beta_{10}(\times 10^{-1})$	0.3137	-0.9622	3.648	0.3694	-9.241	2.481	4.920	4.759
$\beta_{11}(\times 10^1)$	-36.02	0.7025	12.91	12.36	0.2393	-0.2469	-0.6701	-1.744
$\beta_{12}(\times 10^1)$	-7.643	-6.211	-5.249	-5.577	-5.633	-5.423	-5.233	-5.163
$\beta_{13}(\times 10^1)$	2.248	3.349	2.430	1.814	0.7035	0.7070	0.6284	0.6902
$\beta_{14}(\times 10^{-1})$	0.5666	-6.731	0.5548	1.266	-7.384	0.8856	2.531	2.529
$\beta_{15}(\times 10^2)$	-2.416	-0.3444	0.6303	0.6108	-0.2518	-0.2521	-0.2863	-0.3367
$\beta_{16}(\times 10^{-1})$	6.562	4.117	3.499	3.639	3.209	3.322	3.217	3.245
$\beta_{17}(\times 10^{-5})$	-7.032	0.3499	-3.265	-3.037	-0.1909	-0.04219	-0.2685	-0.5965
$\beta_{18}(\times 10^0)$	5.620	2.547	0.6630	-1.446	-6.219	-6.887	-7.068	-6.959
$\beta_{19}(\times 10^{-1})$	0.8959	0.9388	0.9323	1.034	1.161	1.133	1.116	1.102
$\beta_{20}(\times 10^{-2})$	0.6496	-0.3039	-0.3433	0.3721	1.012	-0.1427	-0.4433	-0.4780
$\beta_{21}(\times 10^0)$	6.019	1.642	-0.1329	-0.4318	0.5371	0.6645	0.7392	0.9172
$\beta_{22}(\times 10^{-1})$	9.900	7.021	5.887	6.274	5.638	5.701	5.453	5.373
$\beta_{23}(\times 10^{-1})$	-2.674	-7.039	-4.582	-3.335	-1.754	-1.972	-1.881	-2.146
$\beta_{24}(\times 10^{-2})$	0.8171	1.936	0.5845	0.7807	2.125	0.1880	-0.2267	-0.1148
$\beta_{25}(\times 10^{-1})$	57.19	2.752	-18.24	-16.78	-0.6841	0.7977	1.223	3.212
$\beta_{26}(\times 10^{-3})$	-2.373	-1.831	-1.487	-1.390	-0.2551	-0.6927	-0.7307	-0.7600
$\beta_{27}(\times 10^{-2})$	-2.992	-2.841	-1.363	-0.7752	0.2181	0.8464	0.8534	0.9286
$\beta_{28}(\times 10^{-3})$	-2.004	-0.9704	-1.459	-1.558	0.2443	-0.2595	-0.3092	-0.3043

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Table G.1 – continued from previous page

Sampling	Random	Random	Random	Random	D-optim	D-optim	D-optim	D-optim
No of Points k	100	120	200	400	100	120	200	400
$\beta_{29}(\times 10^{-1})$	-2.983	-4.656	-3.376	-2.447	-0.8403	-0.9257	-0.8092	-0.9206
$\beta_{30}(\times 10^0)$	-5.958	-3.721	0.9690	0.08012	0.9690	0.2623	0.3506	0.1141
$\beta_{31}(\times 10^{-1})$	-0.3367	-1.092	-2.386	-3.429	-3.802	-3.053	-2.922	-2.699
$\beta_{32}(\times 10^1)$	-1.221	-0.9128	-0.4563	-0.3324	-0.06533	-0.1188	-0.1192	-0.1317
$\beta_{33}(\times 10^{-1})$	5.577	5.037	4.401	4.576	4.526	4.362	4.226	4.132
$\beta_{34}(\times 10^{-3})$	3.860	10.58	3.962	2.487	7.356	0.8967	-0.5007	-0.4244
$\beta_{35}(\times 10^0)$	2.267	0.6887	-0.1896	-0.2310	0.2754	0.2915	0.3125	0.3569

Table G.2: Standard deviation of coefficients of Cubic curve fitting with Latitude included as variable

Sampling	Random	Random	Random	Random	D-optim	D-optim	D-optim	D-optim
No of Points k	100	120	200	400	100	120	200	400
$\beta_1(\times 10^4)$	6.198	4.970	2.287	1.084	2.905	0.7080	0.4901	0.3198
$\beta_2(\times 10^3)$	4.734	3.648	1.627	0.7978	1.990	0.4667	0.3258	0.2104
$\beta_3(\times 10^2)$	2.315	2.312	0.9270	0.5026	1.196	0.2739	0.1802	0.1113
$\beta_4(\times 10^3)$	14.97	15.09	7.626	3.513	4.534	1.181	0.7344	0.5133
$\beta_5(\times 10^3)$	2.857	2.172	0.9994	0.4627	1.354	0.3289	0.2279	0.1468
$\beta_6(\times 10^1)$	12.15	9.151	4.183	2.167	4.863	1.129	0.7640	0.5176
$\beta_7(\times 10^{-1})$	4.752	4.784	1.881	0.8868	2.358	0.3999	0.2508	0.1614
$\beta_8(\times 10^2)$	7.883	6.964	2.200	0.9632	1.567	0.3799	0.2492	0.1823
$\beta_9(\times 10^1)$	4.438	3.222	1.478	0.6720	2.109	0.5077	0.3519	0.2232
$\beta_{10}(\times 10^0)$	10.99	10.56	3.915	2.109	4.835	1.081	0.7032	0.4376
$\beta_{11}(\times 10^2)$	8.259	8.029	3.906	1.906	2.067	0.6006	0.2980	0.2232
$\beta_{12}(\times 10^1)$	14.64	10.75	4.767	2.304	6.298	1.452	1.021	0.6321
$\beta_{13}(\times 10^1)$	5.246	5.543	2.508	1.353	1.458	0.2760	0.2096	0.1602
$\beta_{14}(\times 10^0)$	7.035	6.786	2.777	1.478	3.757	0.8696	0.5794	0.3607
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Table G.2 – continued from previous page

Sampling	Random	Random	Random	Random	D-optim	D-optim	D-optim	D-optim
No of Points k	100	120	200	400	100	120	200	400
$\beta_{15}(\times 10^2)$	4.479	4.423	2.269	1.022	1.434	0.3787	0.2399	0.1674
$\beta_{16}(\times 10^{-1})$	10.43	7.900	3.599	1.981	3.946	0.9321	0.6056	0.4296
$\beta_{17}(\times 10^{-4})$	3.528	3.604	1.289	0.5709	1.915	0.4030	0.2755	0.1705
$\beta_{18}(\times 10^0)$	17.22	11.33	5.247	2.910	4.873	1.413	0.8099	0.5894
$\beta_{19}(\times 10^{-1})$	2.322	1.627	0.7415	0.3343	1.099	0.2611	0.1807	0.1129
$\beta_{20}(\times 10^{-2})$	13.11	12.42	4.323	2.423	5.144	1.223	0.7500	0.4582
$\beta_{21}(\times 10^0)$	12.02	11.12	5.121	2.777	3.040	0.9635	0.4212	0.2986
$\beta_{22}(\times 10^{-1})$	18.80	13.45	6.133	3.128	7.842	1.775	1.206	0.7850
$\beta_{23}(\times 10^{-1})$	12.48	14.46	5.591	3.087	3.363	0.9094	0.6022	0.4188
$\beta_{24}(\times 10^{-2})$	16.56	16.03	5.935	3.037	7.689	1.643	1.117	0.7116
$\beta_{25}(\times 10^0)$	12.85	11.84	6.010	2.849	3.467	0.9959	0.5095	0.3844
$\beta_{26}(\times 10^{-3})$	9.518	8.968	3.628	1.726	3.979	7.935	0.5042	0.3456
$\beta_{27}(\times 10^{-2})$	8.823	8.267	3.479	1.778	2.741	0.9340	0.4485	0.3245
$\beta_{28}(\times 10^{-3})$	7.282	7.015	2.874	1.360	4.055	0.6723	0.4498	0.2915
$\beta_{29}(\times 10^{-1})$	7.349	7.570	3.447	1.867	2.163	0.4415	0.3110	0.2272
$\beta_{30}(\times 10^0)$	22.66	22.91	7.462	3.951	6.765	1.409	0.9310	0.6568
$\beta_{31}(\times 10^{-1})$	19.88	15.35	7.026	3.118	5.355	1.729	0.9654	0.6336
$\beta_{32}(\times 10^0)$	12.82	10.66	3.613	1.588	2.418	0.6836	0.4392	0.3159
$\beta_{33}(\times 10^{-1})$	11.50	8.183	3.589	1.733	5.036	1.133	0.8047	0.4717
$\beta_{34}(\times 10^{-2})$	5.430	5.079	2.145	1.123	2.964	0.6886	0.4626	0.2888
$\beta_{35}(\times 10^0)$	3.389	3.299	1.706	0.7508	1.144	0.3081	0.1985	0.1387

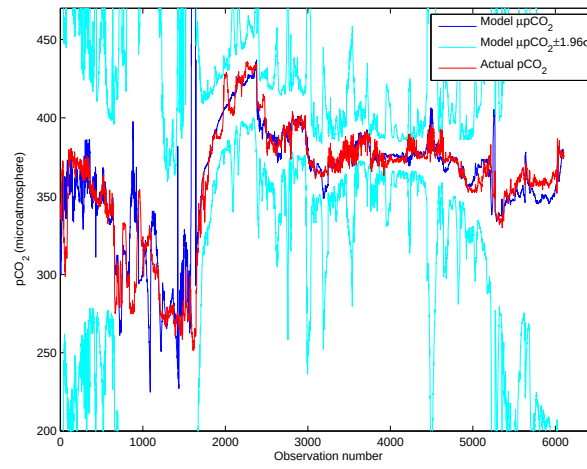


Figure G.1: Cubic curve fit results. 50 Random points sampled.

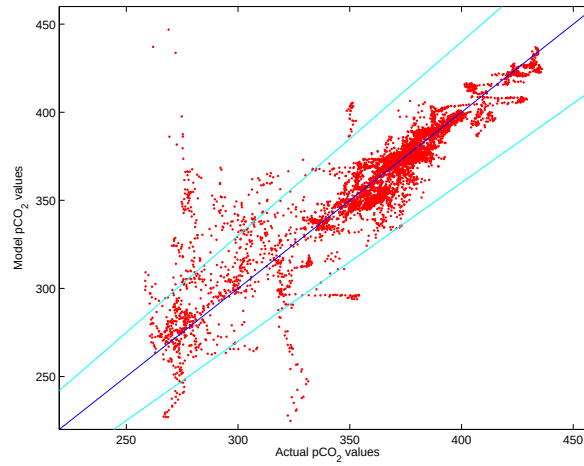


Figure G.2: Cubic curve fit results with 50 random points sampled. Indication of the degree to which the modelled points fall in the 10 % error margin .

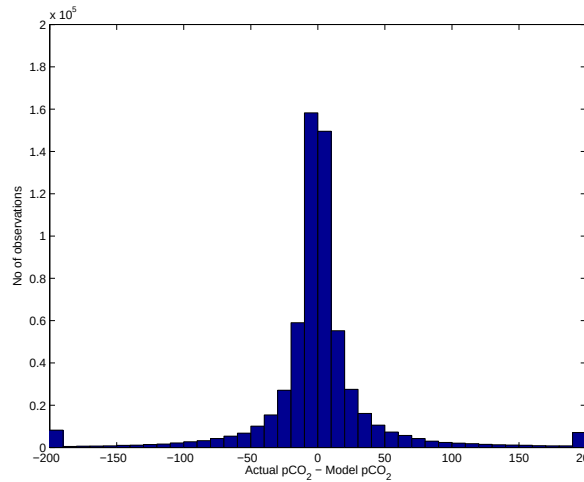


Figure G.3: Cubic curve fit results. Histogram of errors for all 100 runs and 50 random points sampled.

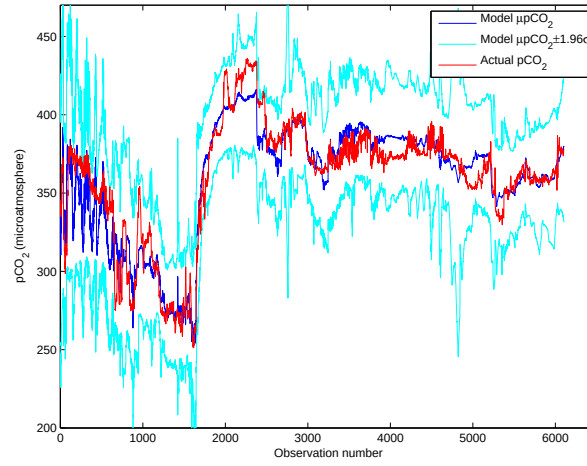


Figure G.4: Cubic curve fit results. 50 D-optimal sampled points.

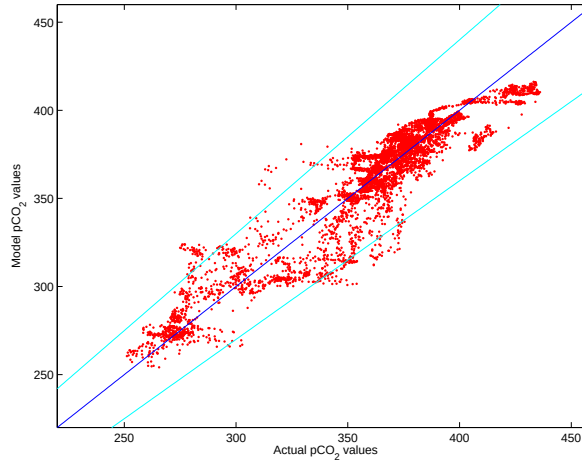


Figure G.5: Cubic curve fit results with 50 D-optimal sampled points. The degree to which the modelled points fall in the 10 % error margin is shown. The mean pCO_2 estimation for all 100 runs is shown here.

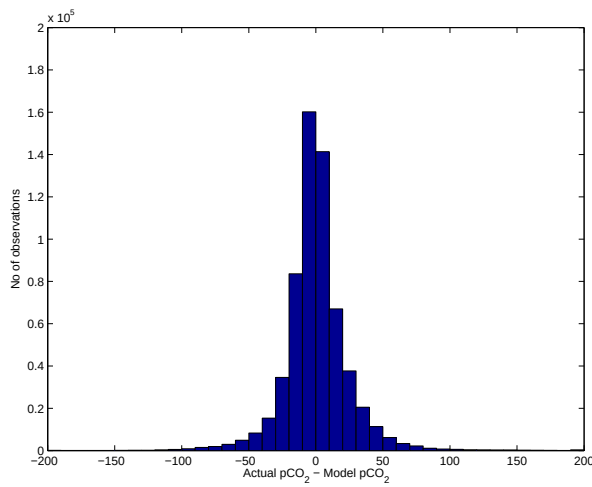


Figure G.6: Cubic curve fit results. Histogram of errors for all 100 runs and 50 D-optimal sampled points.

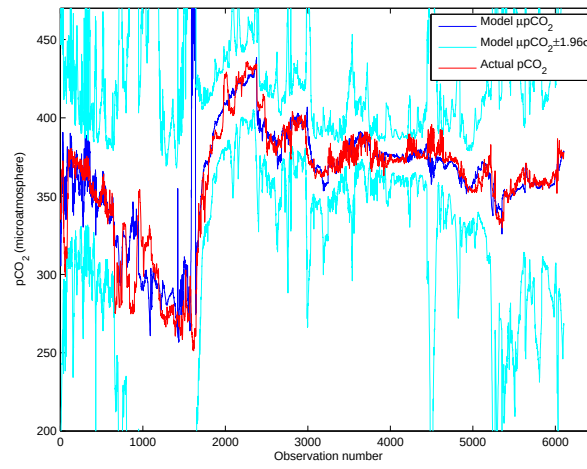


Figure G.7: Cubic curve fit results. 55 Random points sampled.

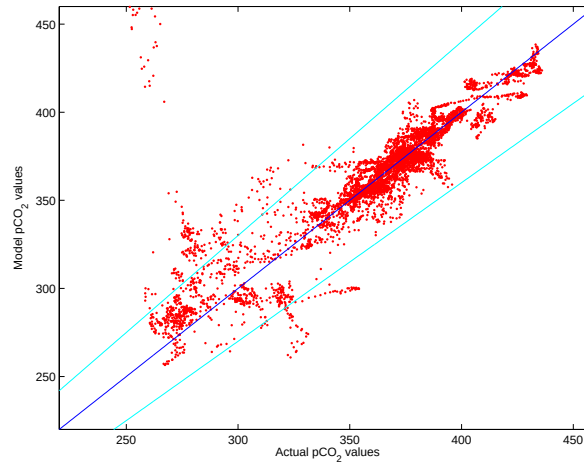


Figure G.8: Cubic curve fit results with 55 random points sampled. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

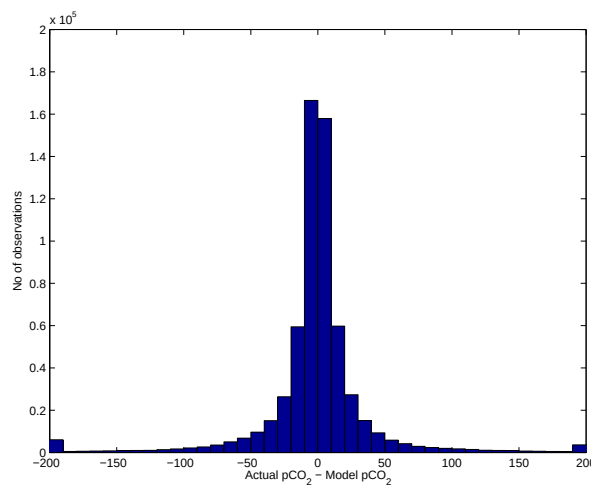


Figure G.9: Cubic curve fit results. Histogram of errors for all 100 runs and 55 random points sampled.

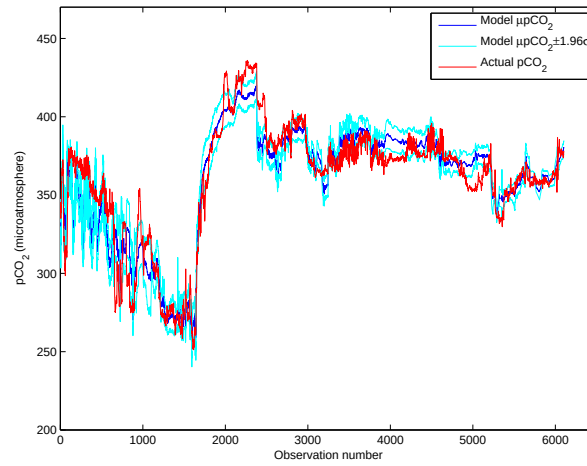


Figure G.10: Cubic curve fit results. 55 D-optimal sampled points.

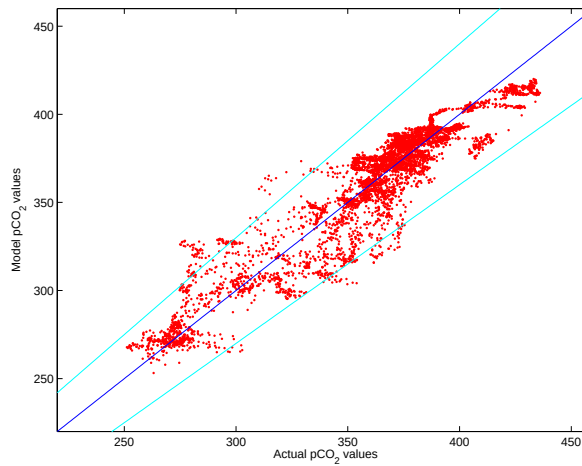


Figure G.11: Cubic curve fit results with 55 D-optimal sampled points. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

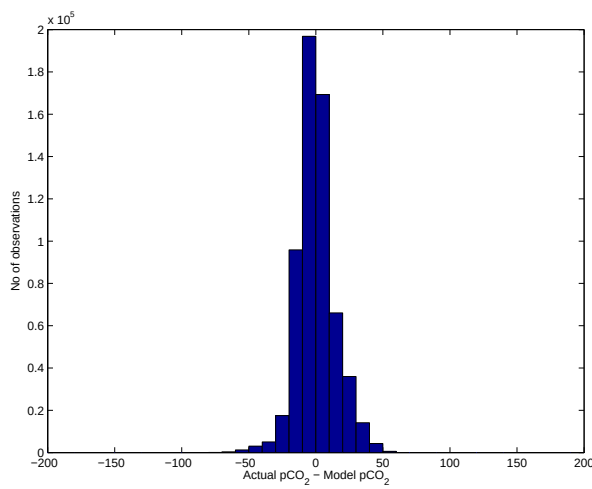


Figure G.12: Cubic curve fit results. Histogram of errors for all 100 runs and 55 D-optimal sampled points.

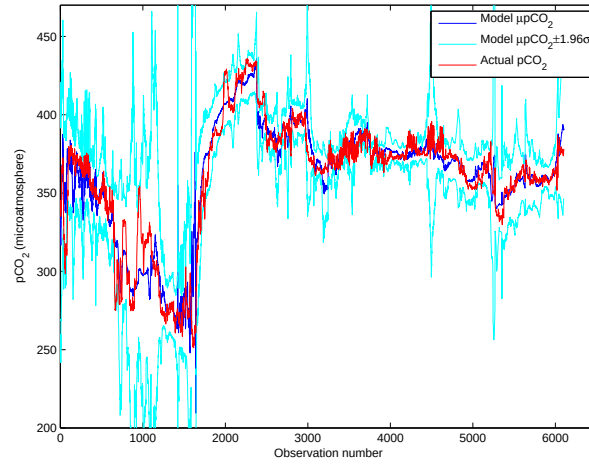


Figure G.13: Cubic curve fit results. 100 Random points sampled.

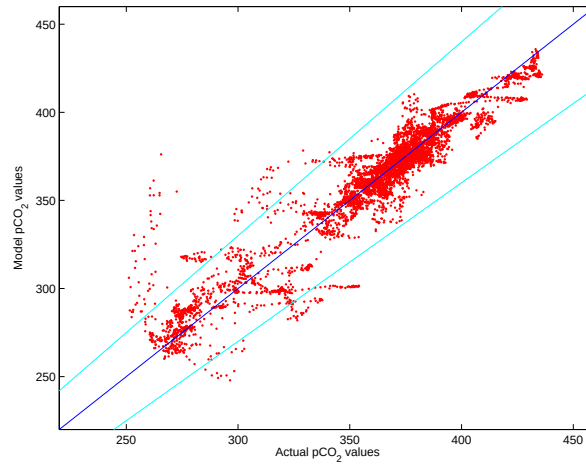


Figure G.14: Cubic curve fit results with 100 random points sampled. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

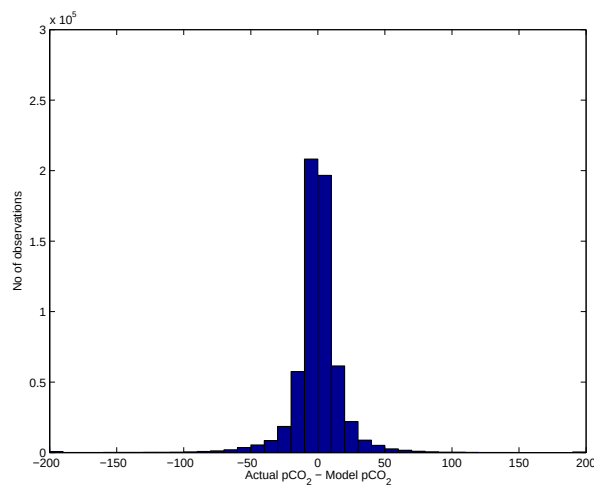


Figure G.15: Cubic curve fit results. Histogram of errors for all 100 runs and 100 random points sampled.

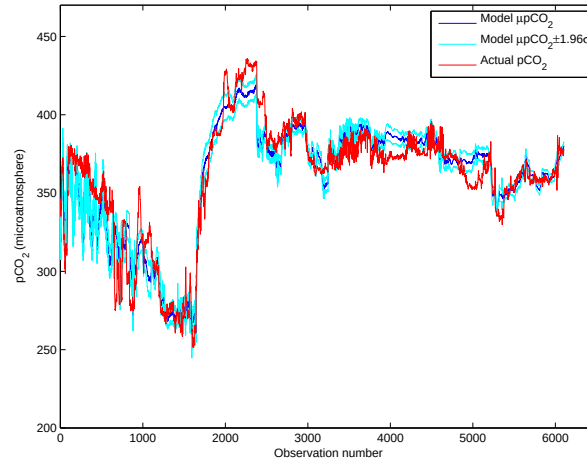


Figure G.16: Cubic curve fit results. 100 D-optimal sampled points.

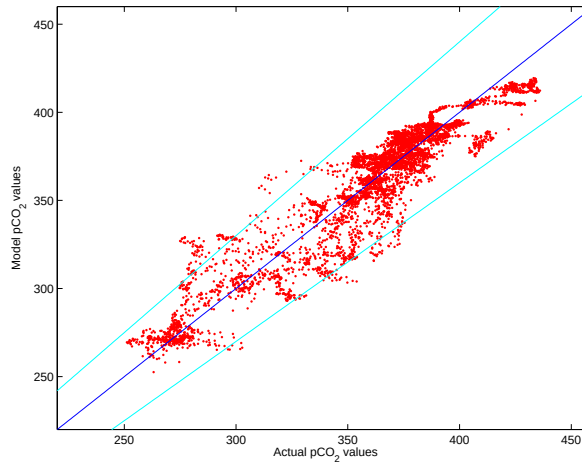


Figure G.17: Cubic curve fit results with 100 D-optimal sampled points. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

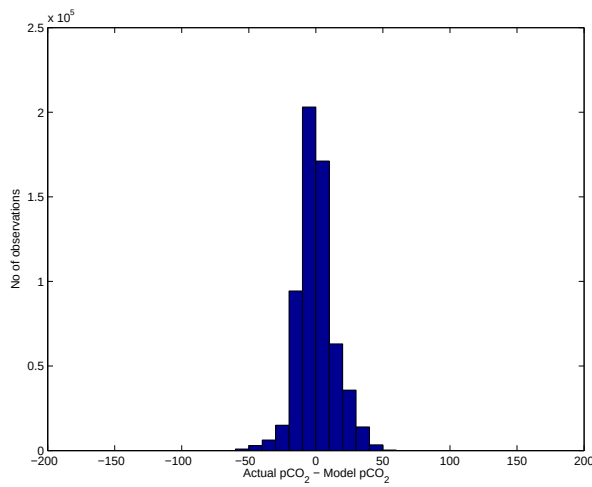


Figure G.18: Cubic curve fit results. Histogram of errors for all 100 runs and 100 D-optimal sampled points.

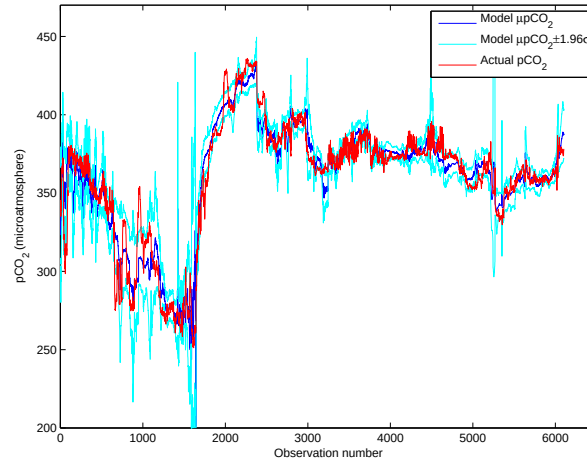


Figure G.19: Cubic curve fit results. 200 Random points sampled.

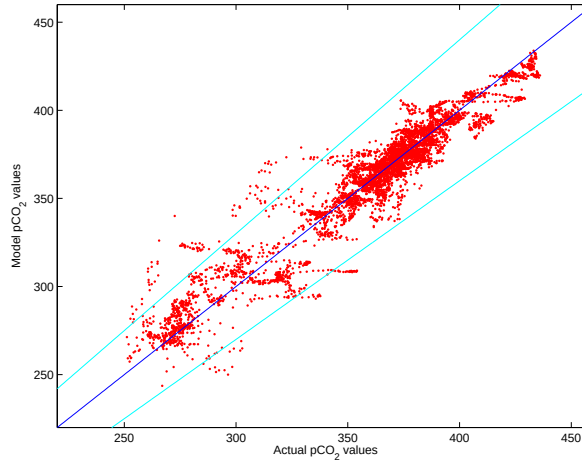


Figure G.20: Cubic curve fit results with 200 random points sampled. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

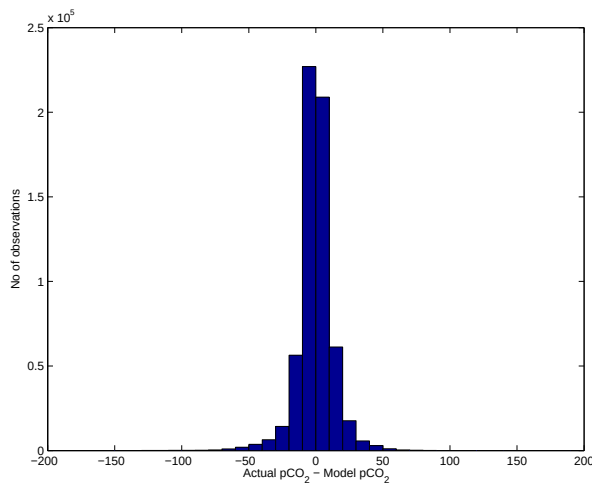


Figure G.21: Cubic curve fit results. Histogram of errors for all 100 runs and 200 random points sampled.

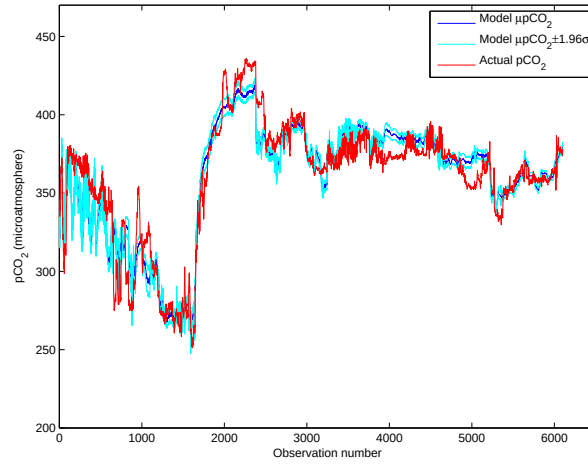


Figure G.22: Cubic curve fit results. 200 D-optimal sampled points.

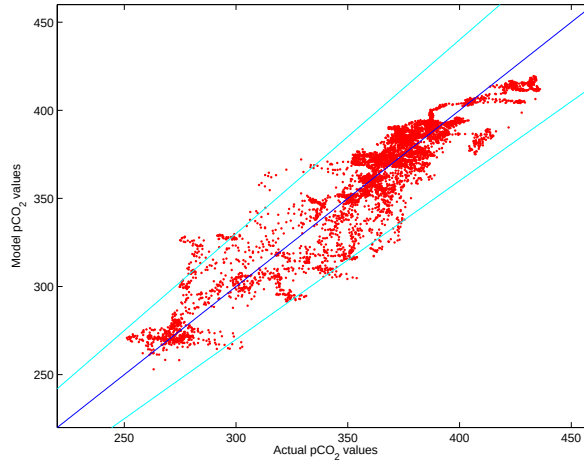


Figure G.23: Cubic curve fit results with 200 D-optimal sampled points. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

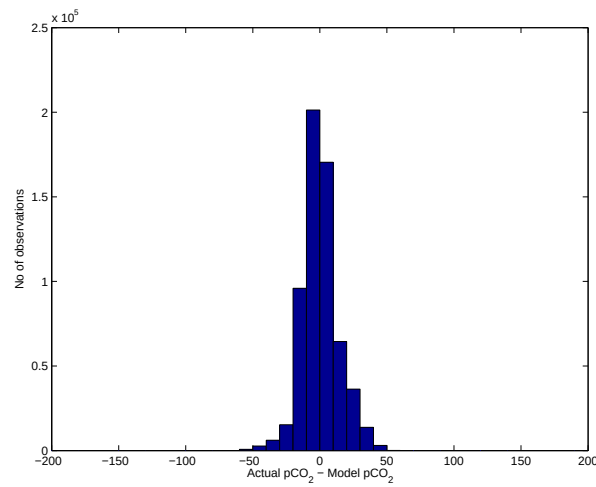


Figure G.24: Cubic curve fit results. Histogram of errors for all 100 runs and 200 D-optimal sampled points.

Appendix H

Fourth order curve fit results with latitude included as variable

The first figure in each case is a plot of the actual pCO_2 values, together with the mean values of the estimated pCO_2 from the 100 curve fits. The 95% confidence interval is also shown for the 100 curve fits in each case. The second figure in each case is the actual pCO_2 values plotted against the estimated pCO_2 values. The 10% error lines indicate the number of points that fall outside the 10% error. The third figure in each case is a histogram of the errors on the estimation of the pCO_2 , showing the distribution of the errors for each case.

It can be seen from the figures that as the number of points sampled from the data set increases, the fit of the data is improved and less points falls outside the 10% error interval. Also, from the figures it can be seen that the D-optimal sampling yields a smaller 95% confidence interval than the random sampling. The histograms generally show that the error on the estimation has a normal distribution around zero.

The mean coefficients for the fourth order equation for all 100 curve fits are given in Table H.1. The standard deviation for the coefficients are given in Table H.2. It can be seen from the table of the standard deviations that the coefficients for the D-optimal sampling have a smaller standard deviation than the coefficients for the random sampling.

Table H.1: Coefficients of Fourth Order curve fitting

Sampling No of Points	Random	Random	Random	Random	D-optim	D-optim	D-optim	D-optim
	100	120	200	400	100	120	200	400
$\beta_1(\times 10^5)$	-1.818	-0.5631	-0.1294	-0.6057	-0.7246	-0.7429	-0.5065	-0.4664
$\beta_2(\times 10^4)$	1.270	-0.06796	0.4893	-0.04899	0.4126	0.4403	0.2290	0.1661
$\beta_3(\times 10^2)$	18.14	10.37	13.43	7.308	-1.730	-0.7802	-1.448	-1.323
$\beta_4(\times 10^4)$	8.5094	5.7154	8.5224	7.2634	6.830	7.945	6.982	7.026
$\beta_5(\times 10^3)$	9.109	2.140	6.065	2.401	3.996	3.893	2.622	2.354
$\beta_6(\times 10^2)$	-4.364	0.7402	-1.483	0.1188	-2.946	-3.043	-2.262	-1.972
$\beta_7(\times 10^0)$	-4.597	-2.169	-3.892	-2.640	-1.147	-1.448	-1.418	-1.576
$\beta_8(\times 10^3)$	-5.195	-4.647	-2.104	-2.288	-3.150	-3.229	-3.230	-3.132
$\beta_9(\times 10^2)$	-1.633	-0.2278	-0.9863	-0.2643	-0.8066	-0.7333	-0.4827	-0.4160
$\beta_{10}(\times 10^1)$	-8.465	-3.308	-4.199	-0.08657	7.236	6.849	7.347	7.372
$\beta_{11}(\times 10^3)$	-2.333	1.367	-2.031	-1.270	-0.8164	-1.487	-0.7763	-0.8221
$\beta_{12}(\times 10^2)$	-5.001	0.4009	-1.404	0.6109	-2.419	-2.454	-1.625	-1.315
$\beta_{13}(\times 10^2)$	-3.538	-5.572	-4.396	-3.702	-2.502	-2.854	-2.627	-2.560
$\beta_{14}(\times 10^1)$	-7.953	-4.163	-5.622	-2.939	1.008	0.6207	0.8935	0.8463
$\beta_{15}(\times 10^3)$	-3.500	-2.084	-3.538	-3.039	-3.004	-3.497	-3.059	-3.088
$\beta_{16}(\times 10^0)$	7.031	-2.845	2.710	0.3760	6.671	6.846	5.587	5.087
$\beta_{17}(\times 10^{-3})$	6.896	5.727	6.637	4.645	3.195	3.488	3.400	3.549
$\beta_{18}(\times 10^2)$	-3.427	-3.333	-1.835	-1.730	-0.8348	-0.8112	-0.8789	-0.8673
$\beta_{19}(\times 10^{-1})$	12.17	0.1311	6.192	-0.01557	7.065	5.803	3.672	2.930
$\beta_{20}(\times 10^0)$	1.157	0.2887	0.1754	-0.7331	-2.409	-2.384	-2.507	-2.519
$\beta_{21}(\times 10^2)$	0.1939	-1.180	0.04919	-0.1556	-0.3897	-0.2495	-0.4114	-0.4021
$\beta_{22}(\times 10^0)$	13.00	0.07764	4.115	0.5203	13.07	13.24	11.12	10.15
$\beta_{23}(\times 10^1)$	0.8536	1.835	1.574	1.285	0.9953	1.085	0.9838	0.9831
$\beta_{24}(\times 10^0)$	2.490	0.7938	1.007	-0.2148	-2.482	-2.379	-2.514	-2.534
$\beta_{25}(\times 10^1)$	4.657	-7.950	2.664	1.025	0.4530	2.495	0.2030	0.3663
$\beta_{26}(\times 10^{-2})$	15.21	2.792	10.89	6.304	-0.7698	0.3668	0.1563	0.7072
$\beta_{27}(\times 10^{-1})$	8.264	7.593	6.190	4.243	2.130	2.998	2.693	2.548
$\beta_{28}(\times 10^{-1})$	1.287	0.5020	1.083	0.7389	0.3105	0.3957	0.3926	0.4448

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Table H.1 – continued from previous page

Sampling	Random	Random	Random	Random	D-optim	D-optim	D-optim	D-optim
No of Points	100	120	200	400	100	120	200	400
$\beta_{29}(\times 10^1)$	0.9789	1.678	1.276	1.116	0.8282	0.9157	0.8498	0.8260
$\beta_{30}(\times 10^3)$	2.835	3.955	3.294	2.788	2.423	2.387	2.580	2.543
$\beta_{31}(\times 10^0)$	3.200	-6.786	4.494	1.494	-10.60	-9.155	-9.239	-8.759
$\beta_{32}(\times 10^2)$	1.914	1.774	0.7832	0.8662	1.127	1.144	1.146	1.110
$\beta_{33}(\times 10^0)$	6.5450	-0.3674	1.230	-1.194	4.788	4.699	3.669	3.169
$\beta_{34}(\times 10^0)$	1.165	0.5495	0.7798	0.3851	-0.1897	-0.1367	-0.1734	-0.1677
$\beta_{35}(\times 10^1)$	4.677	2.279	4.833	4.158	4.324	5.058	4.395	4.457
$\beta_{36}(\times 10^0)$	-4.337	-4.769	-5.711	-4.968	-2.209	-1.979	-2.060	-1.925
$\beta_{37}(\times 10^0)$	6.078	5.992	3.901	3.604	1.680	1.598	1.722	1.680
$\beta_{38}(\times 10^{-1})$	-1.147	-1.178	-1.243	-0.8474	0.5495	0.5770	0.4882	0.4671
$\beta_{39}(\times 10^0)$	2.843	3.556	-1.158	-1.094	-0.7698	-0.6562	-0.4567	-0.4060
$\beta_{40}(\times 10^0)$	-1.780	-1.693	-0.7933	-0.8645	-1.017	-1.021	-1.025	-0.9924
$\beta_{41}(\times 10^{-2})$	-0.02792	13.57	-1.978	1.040	16.18	14.14	14.30	13.71
$\beta_{42}(\times 10^0)$	-4.253	-6.126	-4.346	-3.687	-3.460	-3.422	-3.734	-3.685
$\beta_{43}(\times 10^{-2})$	-2.672	-1.678	-2.560	-1.819	-0.9674	-1.131	-1.124	-1.232
$\beta_{44}(\times 10^{-1})$	-4.841	-0.2714	-3.968	-2.746	1.118	0.8425	1.119	1.125
$\beta_{45}(\times 10^0)$	-2.192	-5.595	-6.224	-5.260	-4.653	-4.550	-5.102	-5.070
$\beta_{46}(\times 10^{-1})$	-1.989	-0.6341	-2.133	-1.821	-2.018	-2.386	-2.053	-2.096
$\beta_{47}(\times 10^{-2})$	-7.021	-12.87	-9.673	-8.713	-6.918	-7.450	-6.972	-6.775
$\beta_{48}(\times 10^{-1})$	-1.854	8.824	0.1974	1.008	0.9986	-0.5525	1.289	1.150
$\beta_{49}(\times 10^{-3})$	-9.148	-8.797	-5.270	-3.527	-1.503	-2.619	-2.195	-1.883
$\beta_{50}(\times 10^{-1})$	-0.9659	-2.739	-2.050	-1.731	-1.517	-1.619	-1.484	-1.484
$\beta_{51}(\times 10^{-1})$	-0.6081	23.38	5.143	7.271	10.29	8.059	10.97	10.82
$\beta_{52}(\times 10^{-3})$	-1.209	-1.062	-1.273	-0.8817	-0.5509	-0.6003	-0.5573	-0.5607
$\beta_{53}(\times 10^{-3})$	-9.015	-5.521	-9.412	-7.948	-8.526	-9.995	-9.332	-9.082
$\beta_{54}(\times 10^{-1})$	-0.6832	-1.919	-1.776	-1.306	-0.8254	-0.8669	-0.7485	-0.7699
$\beta_{55}(\times 10^{-1})$	1.212	15.580	1.396	3.107	4.883	3.875	4.749	4.709
$\beta_{56}(\times 10^{-3})$	-3.044	0.5513	-1.029	0.9181	-2.272	-1.588	-0.9340	-0.6223
$\beta_{57}(\times 10^{-3})$	-5.699	-2.369	-3.555	-1.607	1.172	0.9267	1.091	1.071
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Sampling	Random	Random	Random	Random	D-optim	D-optim	D-optim	D-optim
No of Points	100	120	200	400	100	120	200	400
$\beta_{58}(\times 10^{-2})$	-2.827	-0.1006	-0.2890	0.6370	-3.078	-2.960	-2.566	-2.304
$\beta_{59}(\times 10^{-4})$	-8.917	-2.595	-7.550	-5.141	-1.986	-2.599	-2.637	-3.073
$\beta_{60}(\times 10^{-2})$	-1.864	-0.4612	-0.5979	0.3073	2.065	1.998	2.087	2.114
$\beta_{61}(\times 10^{-1})$	-1.005	-0.2750	-0.3762	-0.2029	-1.414	-1.416	-1.280	-1.200
$\beta_{62}(\times 10^{-4})$	-1.094	-0.8786	-1.030	-0.7636	-0.6086	-0.6420	-0.6260	-0.6478
$\beta_{63}(\times 10^{-4})$	-20.89	-0.7119	-13.85	-6.751	5.180	3.289	3.526	2.625
$\beta_{64}(\times 10^{-2})$	-1.634	-0.2247	0.1512	1.513	4.149	4.131	4.297	4.338
$\beta_{65}(\times 10^{-1})$	-1.182	0.09378	-0.4730	-0.2204	-1.490	-1.517	-1.335	-1.248
$\beta_{66}(\times 10^{-6})$	0.1908	-0.4665	-0.5702	0.3108	1.692	1.324	1.261	1.143
$\beta_{67}(\times 10^{-4})$	-1.472	-1.120	-1.142	-0.8367	-0.6050	-0.6030	-0.6012	-0.6243
$\beta_{68}(\times 10^{-4})$	-11.10	2.343	-8.366	-4.917	1.064	-0.02961	0.4033	-0.09652
$\beta_{69}(\times 10^{-2})$	-0.4937	-0.07378	0.2434	0.9222	2.166	2.184	2.281	2.283
$\beta_{70}(\times 10^{-2})$	-3.584	3.840	-1.537	-0.2267	-4.321	-4.416	-3.716	-3.423

Table H.2: Standard deviation of coefficients of Fourth Order curve fitting

Sampling No of Points	Random	Random	Random	Random	D-optim	D-optim	D-optim	D-optim
	100	120	200	400	100	120	200	400
$\beta_1(\times 10^5)$	10.36	5.460	3.123	1.695	1.303	0.9558	0.6241	0.4299
$\beta_2(\times 10^4)$	10.28	5.531	2.904	1.638	1.368	0.9809	0.6713	0.4146
$\beta_3(\times 10^3)$	4.013	2.678	1.265	0.7124	0.4728	0.3815	0.2579	0.1482
$\beta_4(\times 10^5)$	2.883	1.951	0.9297	0.5622	0.4077	0.3168	0.2033	0.1422
$\beta_5(\times 10^4)$	6.248	3.235	1.869	1.001	0.7959	0.5765	0.3781	0.2580
$\beta_6(\times 10^3)$	3.856	2.141	1.068	0.6264	0.5532	0.3958	0.2717	0.1563
$\beta_7(\times 10^0)$	11.52	7.143	3.282	1.564	1.141	0.9478	0.5615	0.4123
$\beta_8(\times 10^4)$	2.986	1.673	0.6833	0.3290	0.1199	0.09647	0.07063	0.04778
$\beta_9(\times 10^2)$	14.24	7.245	4.212	2.229	1.841	1.318	0.8675	0.5848
$\beta_{10}(\times 10^2)$	2.799	1.853	0.8364	0.4980	0.3531	0.2711	0.1963	0.1072
$\beta_{11}(\times 10^4)$	2.3424	1.5674	0.7355	0.4744	0.3040	0.2275	0.1466	0.1024
$\beta_{12}(\times 10^3)$	4.6463	2.4743	1.3053	0.7156	0.6377	0.4503	0.3112	0.1868
$\beta_{13}(\times 10^2)$	13.73	7.618	4.142	1.926	2.189	1.586	0.9384	0.6443
$\beta_{14}(\times 10^1)$	17.48	12.11	5.672	3.243	2.126	1.700	1.176	0.6571
$\beta_{15}(\times 10^3)$	12.84	8.745	4.214	2.565	1.805	1.413	0.9172	0.6406
$\beta_{16}(\times 10^1)$	6.446	3.704	1.789	1.088	0.9386	0.6872	0.4620	0.2582
$\beta_{17}(\times 10^{-3})$	17.16	10.33	4.547	2.634	2.229	1.848	1.032	0.6536
$\beta_{18}(\times 10^2)$	7.013	4.384	1.644	0.8264	0.5538	0.3990	0.2290	0.1542
$\beta_{19}(\times 10^0)$	14.54	7.278	4.239	2.215	1.911	1.355	0.8956	0.5943
$\beta_{20}(\times 10^0)$	6.423	4.474	1.870	1.181	0.8387	0.6305	0.4637	0.2541
$\beta_{21}(\times 10^2)$	6.207	4.254	1.956	1.334	0.8092	0.5830	0.3826	0.2590
$\beta_{22}(\times 10^1)$	11.68	6.453	3.224	1.821	1.788	1.255	0.8752	0.4832
$\beta_{23}(\times 10^1)$	6.310	3.716	1.825	0.9506	0.9014	0.6212	0.3922	0.2626
$\beta_{24}(\times 10^0)$	8.140	5.506	2.497	1.526	1.099	0.8321	0.6084	0.3271
$\beta_{25}(\times 10^2)$	7.342	4.828	2.288	1.463	0.9310	0.6959	0.4514	0.3143
$\beta_{26}(\times 10^{-1})$	5.070	3.170	1.349	0.5797	0.4240	0.3476	0.2276	0.1562
$\beta_{27}(\times 10^0)$	3.199	1.696	0.8913	0.4386	0.4379	0.3677	0.2085	0.1489
$\beta_{28}(\times 10^{-1})$	3.379	2.163	0.9792	0.4637	0.3347	0.2677	0.1654	0.1226

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Table H.2 – continued from previous page

Sampling	Random	Random	Random	Random	D-optim	D-optim	D-optim	D-optim
No of Points	100	120	200	400	100	120	200	400
$\beta_{29}(\times 10^1)$	3.980	2.237	1.221	0.5631	0.6637	0.4732	0.2827	0.1905
$\beta_{30}(\times 10^2)$	16.45	8.048	3.428	1.781	0.5996	0.5486	0.3666	0.2216
$\beta_{31}(\times 10^1)$	6.531	4.152	1.888	1.093	0.6447	0.4766	0.3220	0.1934
$\beta_{32}(\times 10^2)$	9.038	5.056	2.043	0.9833	0.3483	0.2918	0.2199	0.1442
$\beta_{33}(\times 10^1)$	7.044	3.740	1.977	1.053	0.9979	0.6954	0.4843	0.2826
$\beta_{34}(\times 10^0)$	2.590	1.854	0.8661	0.4988	0.3303	0.2608	0.1835	0.1003
$\beta_{35}(\times 10^1)$	19.13	13.12	6.386	3.907	2.678	2.119	1.394	0.9682
$\beta_{36}(\times 10^0)$	8.660	7.248	2.859	1.377	1.495	1.122	0.7214	0.4344
$\beta_{37}(\times 10^0)$	10.99	6.735	2.726	1.337	1.060	0.7680	0.4343	0.2858
$\beta_{38}(\times 10^{-1})$	13.78	9.622	3.217	2.003	1.600	1.344	0.7265	0.4212
$\beta_{39}(\times 10^0)$	20.83	12.76	5.342	2.701	1.466	0.9585	0.6105	0.3988
$\beta_{40}(\times 10^0)$	6.927	3.856	1.534	0.7435	0.2750	0.2373	0.1816	0.1142
$\beta_{41}(\times 10^{-1})$	9.904	6.186	2.800	1.740	1.067	0.7937	0.5256	0.3014
$\beta_{42}(\times 10^0)$	24.98	12.25	5.029	2.625	0.8508	0.8036	0.5552	0.3369
$\beta_{43}(\times 10^{-2})$	10.57	6.973	2.879	1.413	1.077	0.8738	0.5421	0.4058
$\beta_{44}(\times 10^{-1})$	24.96	14.50	6.706	3.667	1.868	1.315	0.9445	0.5941
$\beta_{45}(\times 10^0)$	26.25	13.02e	5.962	3.133	1.863	1.399	0.8071	0.5161
$\beta_{46}(\times 10^{-1})$	9.537	6.596	3.239	1.987	1.338	1.072	0.7150	0.4918
$\beta_{47}(\times 10^{-1})$	2.900	1.671	0.9073	0.4222	0.5171	0.3647	0.2195	0.1450
$\beta_{48}(\times 10^0)$	5.769	3.739	1.782	1.126	0.7125	0.5335	0.3498	0.2421
$\beta_{49}(\times 10^{-2})$	4.478	2.384	1.223	0.5827	0.5745	0.4856	0.2789	0.1942
$\beta_{50}(\times 10^{-1})$	9.443	5.492	2.760	1.440	1.402	0.9448	0.6043	0.4018
$\beta_{51}(\times 10^0)$	10.09	6.741	3.155	2.086	1.336	0.94751	0.6183	0.4137
$\beta_{52}(\times 10^{-3})$	3.223	2.121	1.121	0.6436	0.6140	0.4882	0.2989	0.2060
$\beta_{53}(\times 10^{-2})$	7.121	4.360	1.912	0.9064	0.7573	0.6361	0.3542	0.2538
$\beta_{54}(\times 10^{-1})$	7.534	4.843	2.164	1.223	1.021	0.6841	0.4672	0.2977
$\beta_{55}(\times 10^0)$	5.554	3.839	1.700	1.226	0.6572	0.4694	0.3145	0.2126
$\beta_{56}(\times 10^{-2})$	5.612	2.770	1.607	0.8309	0.7529	0.5302	0.3519	0.2290
$\beta_{57}(\times 10^{-3})$	13.09	9.595	4.498	2.591	1.767	1.378	0.9804	0.5279
Continued on next page								

Table H.2 – continued from previous page

Sampling	Random	Random	Random	Random	D-optim	D-optim	D-optim	D-optim
No of Points	100	120	200	400	100	120	200	400
$\beta_{58}(\times 10^{-1})$	3.587	1.914	1.010	0.5241	0.5244	0.3616	0.2530	0.1440
$\beta_{59}(\times 10^{-4})$	25.46	16.74	7.524	3.549	2.652	2.037	1.317	0.9715
$\beta_{60}(\times 10^{-2})$	6.056	4.197	1.928	1.198	0.8873	0.6583	0.4827	0.2580
$\beta_{61}(\times 10^{-1})$	8.922	4.965	2.484	1.357	1.463	1.008	0.7131	0.3802
$\beta_{62}(\times 10^{-5})$	25.72	15.81	6.938	4.059	3.155	2.613	1.393	0.9260
$\beta_{63}(\times 10^{-3})$	7.538	4.799	2.005	0.8885	0.6792	0.5425	0.3735	0.2529
$\beta_{64}(\times 10^{-2})$	9.314	6.527	2.752	1.792	1.292	0.9702	0.7177	0.3882
$\beta_{65}(\times 10^{-1})$	9.800	5.596	2.686	1.570	1.561	1.115	0.7657	0.4076
$\beta_{66}(\times 10^{-6})$	9.335	5.839	2.024	1.029	1.313	0.9879	0.7161	0.3984
$\beta_{67}(\times 10^{-4})$	3.613	2.141	0.8864	0.5091	0.3965	0.3389	0.1844	0.1162
$\beta_{68}(\times 10^{-3})$	5.665	3.654	1.474	0.6299	0.4589	0.3600	0.2460	0.1599
$\beta_{69}(\times 10^{-2})$	4.961	3.784	1.446	0.9369	0.6593	0.4853	0.3580	0.2003
$\beta_{70}(\times 10^{-1})$	4.086	2.415	1.152	0.7179	0.5595	0.4273	0.2782	0.1559

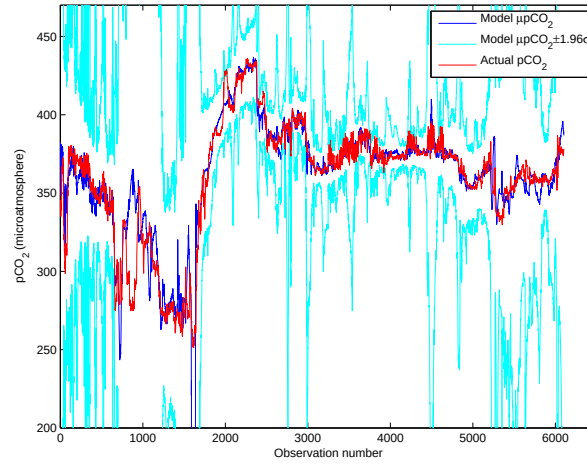


Figure H.1: Fourth order curve fit results. 100 Random points sampled.

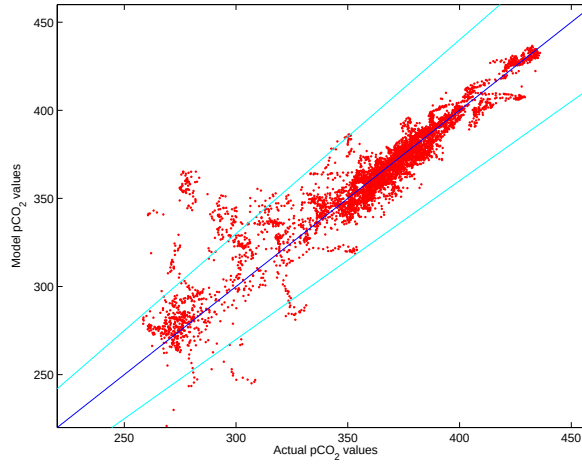


Figure H.2: Fourth order curve fit results with 100 random points sampled. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

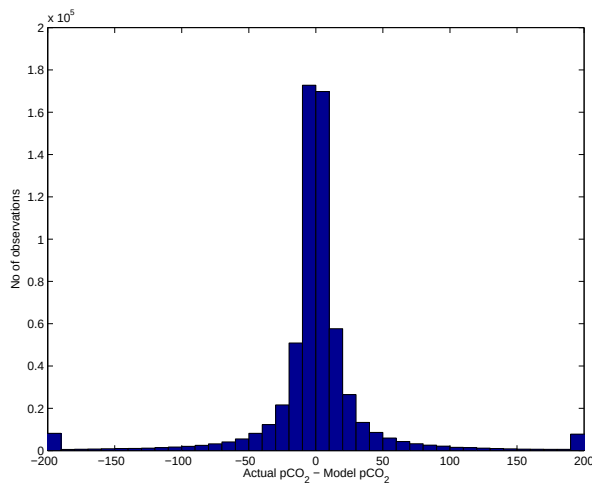


Figure H.3: Fourth order curve fit results. Histogram of errors for all 100 runs and 100 random points sampled.

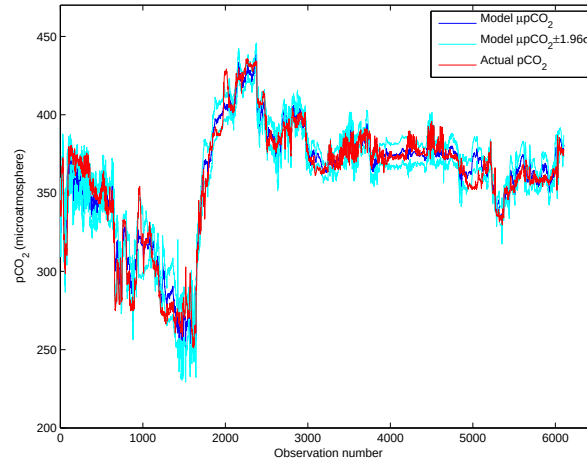


Figure H.4: Fourth order curve fit results. 100 D-optimal sampled points.

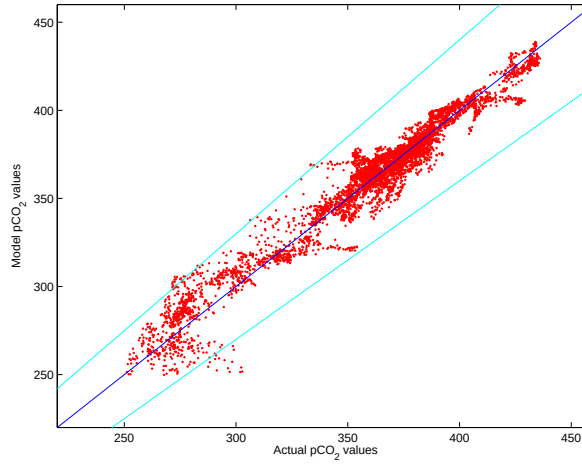


Figure H.5: Fourth order curve fit results with 100 D-optimal sampled points. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

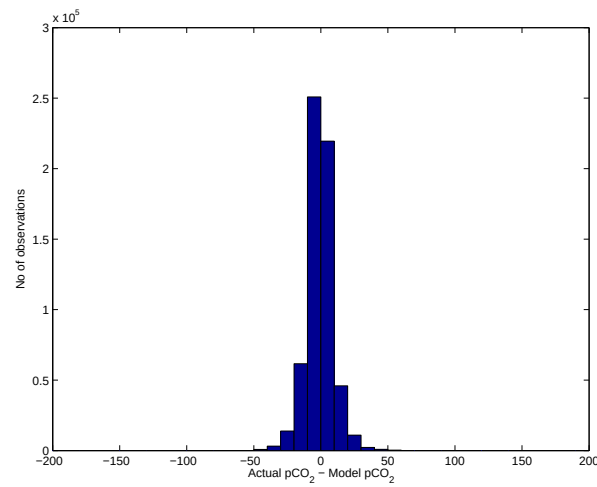


Figure H.6: Fourth order curve fit results. Histogram of errors for all 100 runs and 100 D-optimal sampled points.

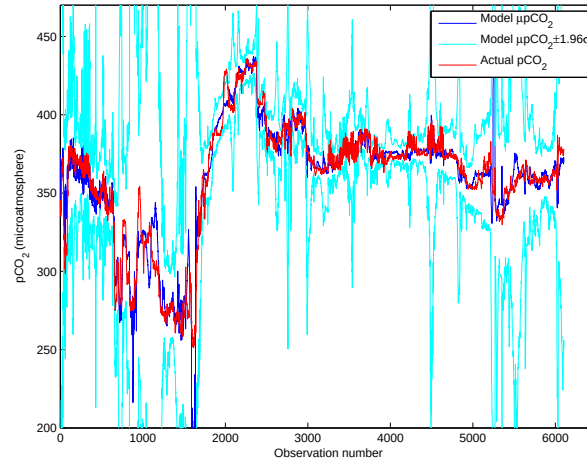


Figure H.7: Fourth order curve fit results. 120 Random points sampled.

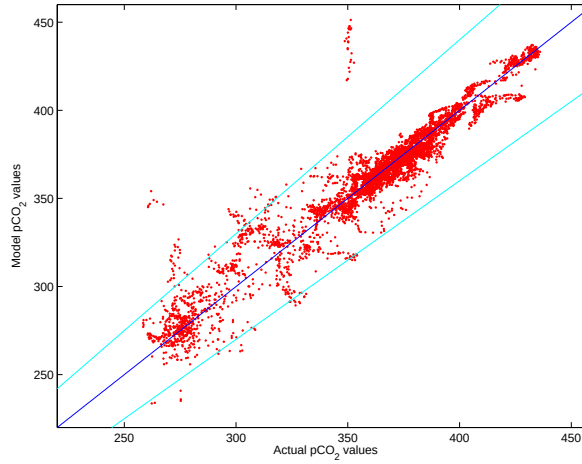


Figure H.8: Fourth order curve fit results with 120 random points sampled. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

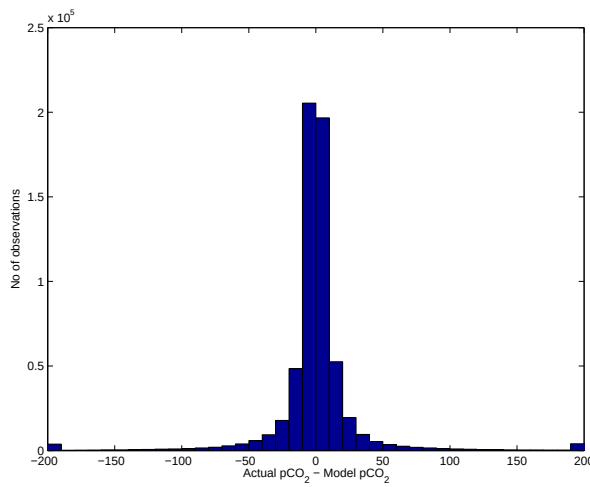


Figure H.9: Fourth order curve fit results. Histogram of errors for all 100 runs and 120 random points sampled.

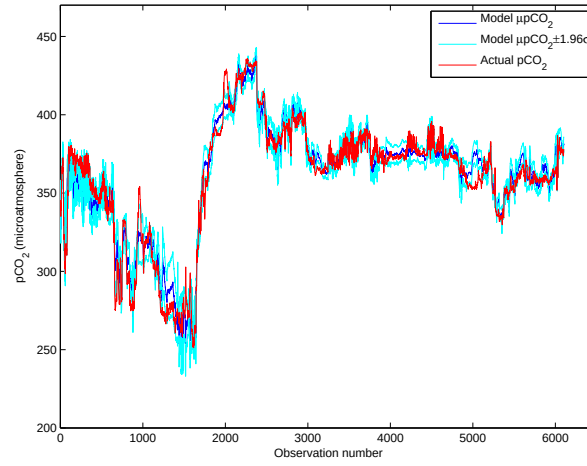


Figure H.10: Fourth order curve fit results. 120 D-optimal sampled points.

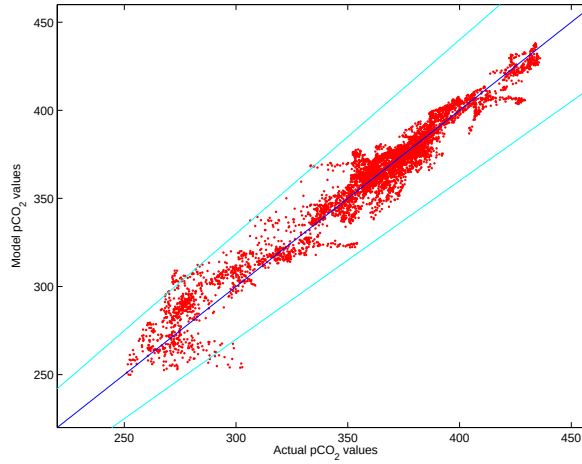


Figure H.11: Fourth order curve fit results with 120 D-optimal sampled points. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

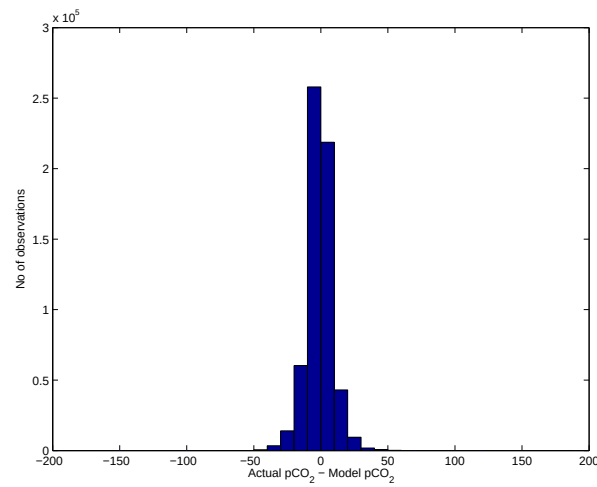


Figure H.12: Fourth order curve fit results. Histogram of errors for all 100 runs and 120 D-optimal sampled points.

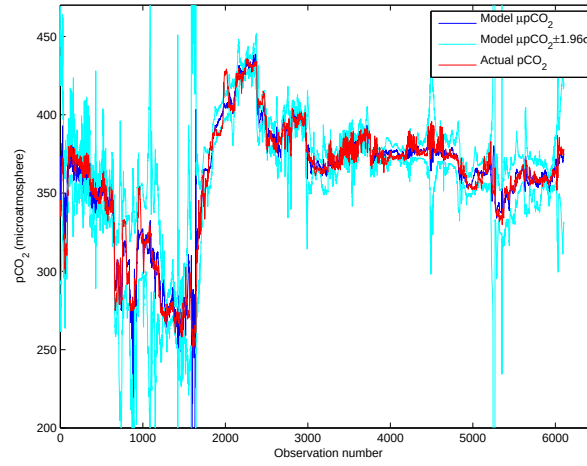


Figure H.13: Fourth order curve fit results. 200 Random points sampled.

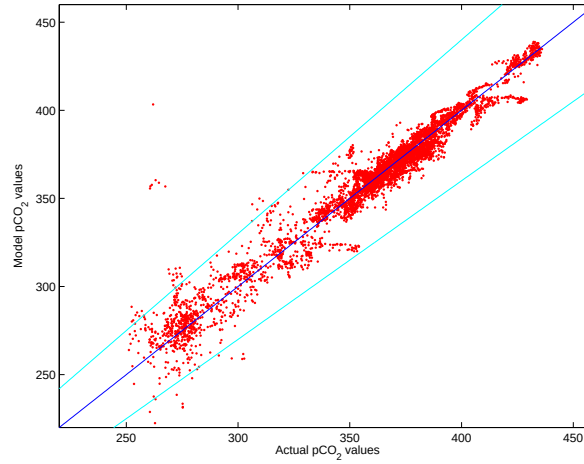


Figure H.14: Fourth order curve fit results with 200 random points sampled. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

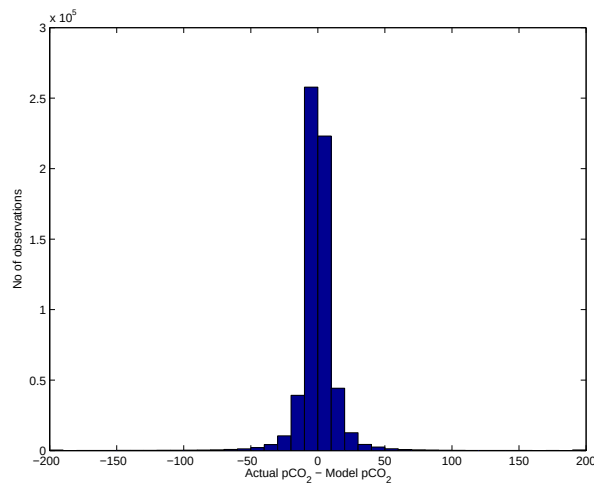


Figure H.15: Fourth order curve fit results. Histogram of errors for all 100 runs and 200 random points sampled.

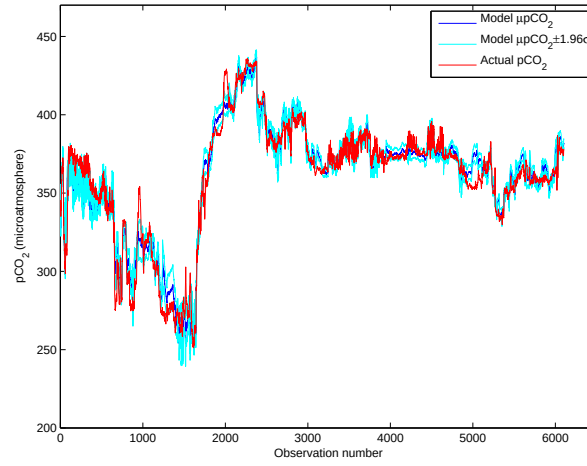


Figure H.16: Fourth order curve fit results. 200 D-optimal sampled points.

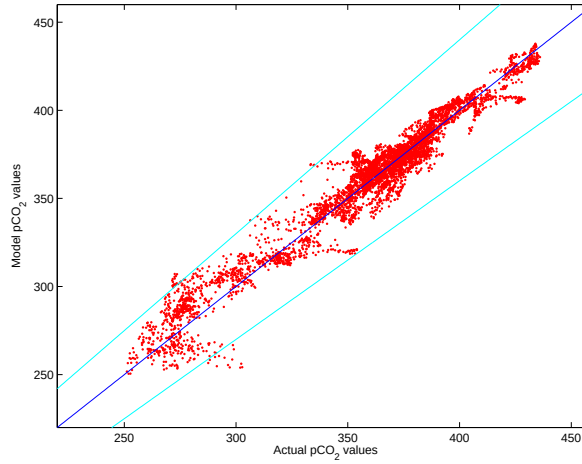


Figure H.17: Fourth order curve fit results with 200 D-optimal sampled points. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

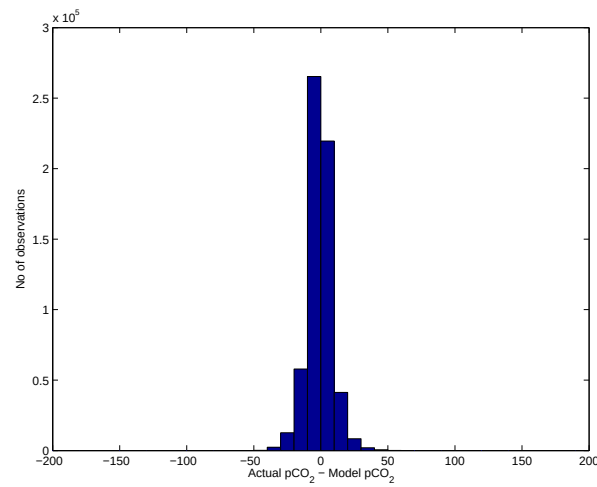


Figure H.18: Fourth order curve fit results. Histogram of errors for all 100 runs and 200 D-optimal sampled points.

Appendix I

Radial basis function interpolation with latitude excluded as variable

The first figure in each case is a plot of the actual pCO_2 values, together with the mean values of the estimated pCO_2 from the 100 curve fits. The 95% confidence interval is also shown for the 100 curve fits in each case. The second figure in each case is the actual pCO_2 values plotted against the estimated pCO_2 values. The 10% error lines indicate the number of points that fall outside the 10% error. The third figure in each case is a histogram of the errors on the estimation of the pCO_2 , showing the distribution of the errors for each case.

It can be seen from the figures that as the number of points sampled from the data set increases, the fit of the data is improved and less points falls outside the 10% error interval. Also, from the figures it can be seen that the D-optimal sampling yields a smaller 95% confidence interval than the random sampling. The histograms generally show that the error on the estimation has a normal distribution around zero.

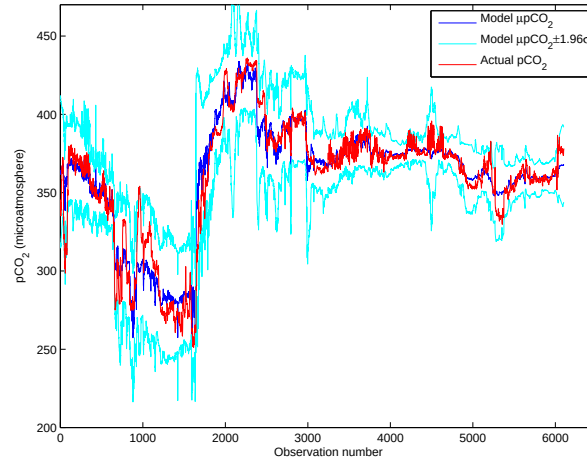


Figure I.1: RBF interpolation results. 50 Random points sampled with the 0-2 scaled data set.

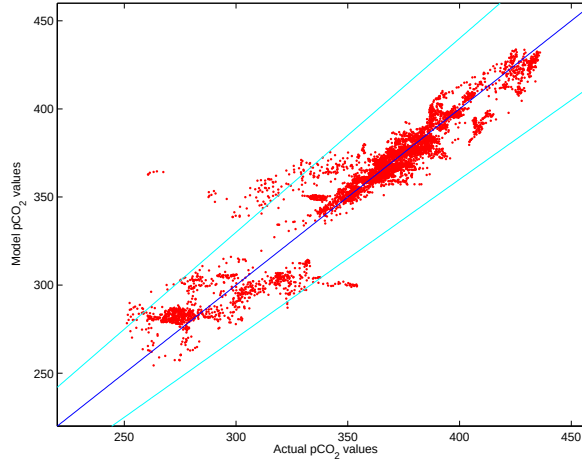


Figure I.2: RBF interpolation results with the 0-2 scaled data set and 50 randomly sampled points. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

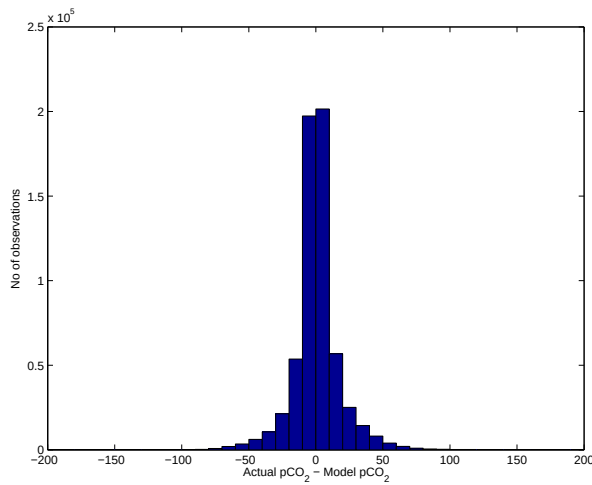


Figure I.3: RBF interpolation results. Histogram of errors for all 100 runs and 50 Random points sampled with the 0-2 scaled data set.

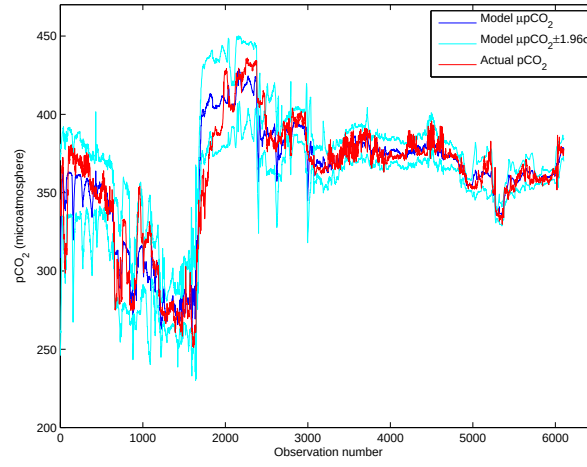


Figure I.4: RBF interpolation results. 50 D-optimal sampled points with the 0-2 scaled data set.

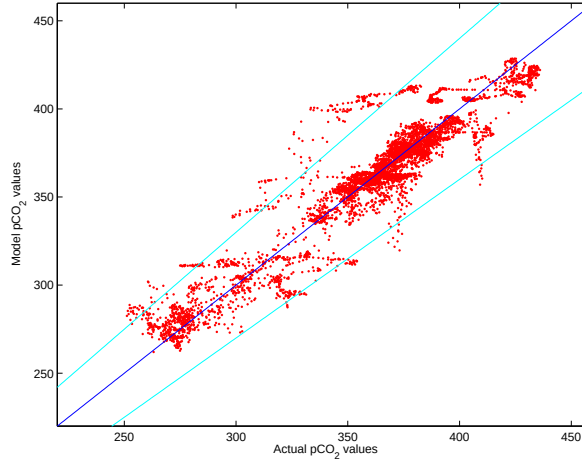


Figure I.5: RBF interpolation results with D-optimal sampled points with the 0-2 scaled data set. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

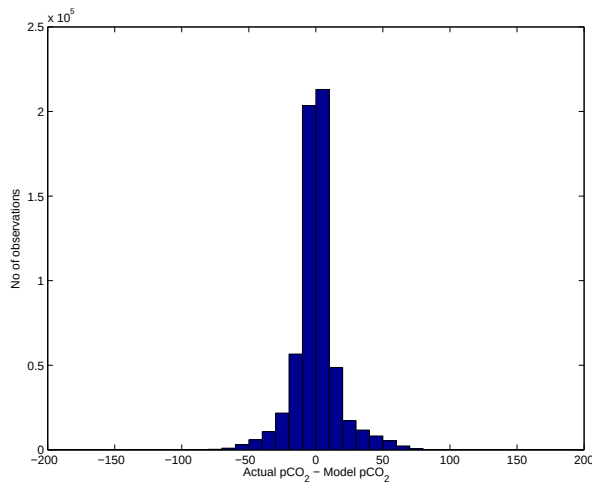


Figure I.6: RBF interpolation results. Histogram of errors for all 100 runs and 50 D-optimal sampled points with the 0-2 scaled data set.

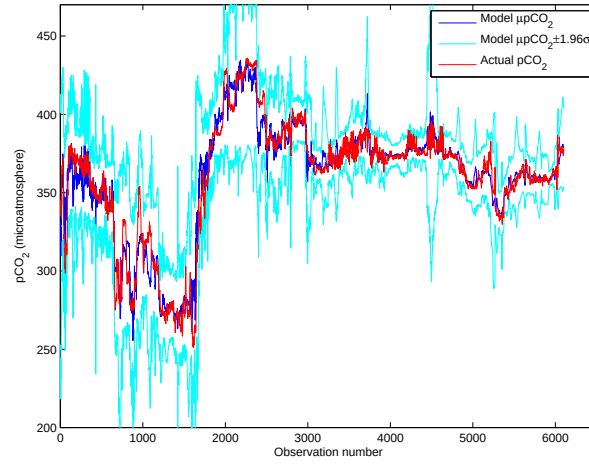


Figure I.7: RBF interpolation results. 100 Random points sampled with the 0-2 scaled data set.

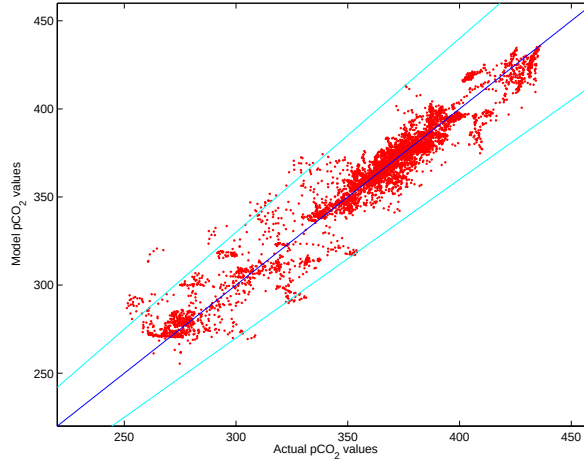


Figure I.8: RBF interpolation results with the 0-2 scaled data set and 100 randomly sampled points. The degree to which the modelled points fall in the 10 % error margin is shown. The mean pCO_2 estimation for all 100 runs is shown here.

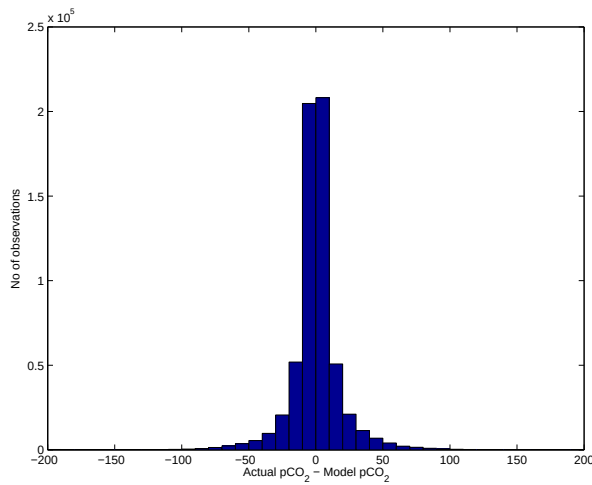


Figure I.9: RBF interpolation results. Histogram of errors for all 100 runs and 100 Random points sampled with the 0-2 scaled data set.

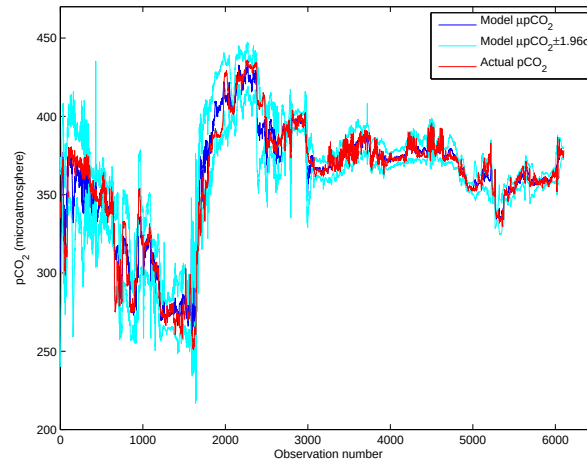


Figure I.10: RBF interpolation results. 100 D-optimal sampled points with the 0-2 scaled data set.

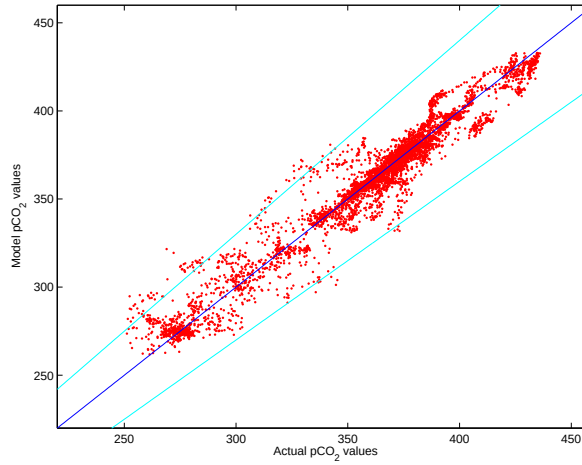


Figure I.11: RBF interpolation results with D-optimal sampled points with the 0-2 scaled data set. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

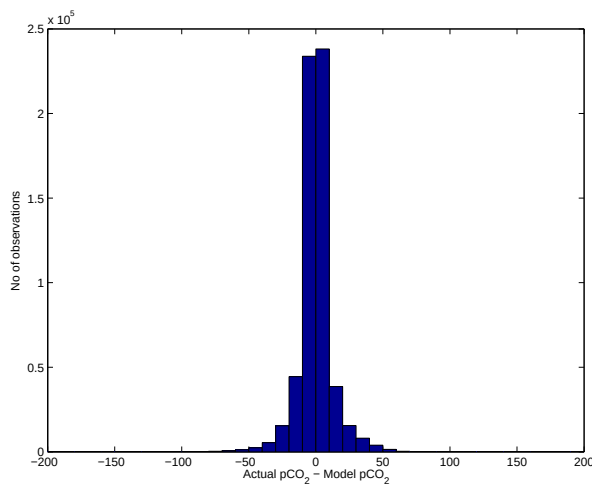


Figure I.12: RBF interpolation results. Histogram of errors for all 100 runs and 100 D-optimal sampled points with the 0-2 scaled data set.

Appendix J

Radial basis function interpolation with latitude excluded as variable and optimal scaling of variables

The first figure in each case is a plot of the actual pCO_2 values, together with the mean values of the estimated pCO_2 from the 100 curve fits. The 95% confidence interval is also shown for the 100 curve fits in each case. The second figure in each case is the actual pCO_2 values plotted against the estimated pCO_2 values. The 10% error lines indicate the number of points that fall outside the 10% error. The third figure in each case is a histogram of the errors on the estimation of the pCO_2 , showing the distribution of the errors for each case.

It can be seen from the figures that as the number of points sampled from the data set increases, the fit of the data is improved and less points falls outside the 10% error interval. Also, from the figures it can be seen that the D-optimal sampling yields a smaller 95% confidence interval than the random sampling. The histograms generally show that the error on the estimation has a normal distribution around zero.

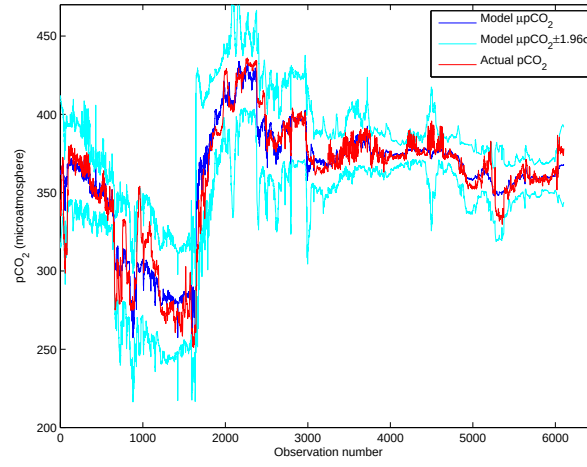


Figure J.1: RBF interpolation results. 50 Random points sampled with the optimally scaled variables.

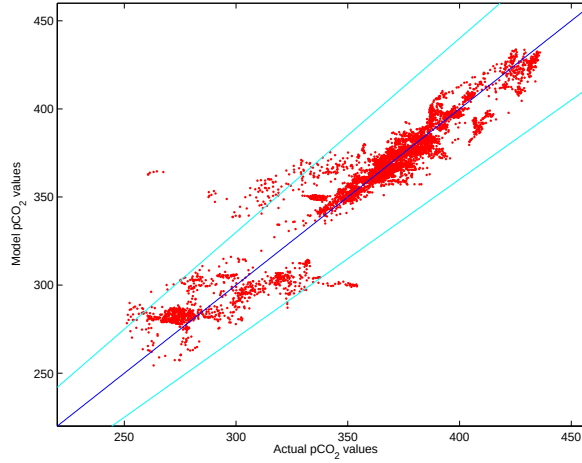


Figure J.2: RBF interpolation results with 50 random points sampled and optimally scaled variables. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

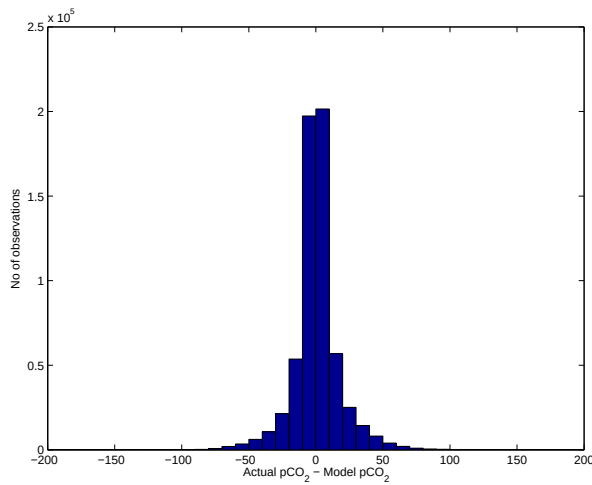


Figure J.3: RBF interpolation results. Histogram of errors for all 100 runs and 50 Randomly sampled points with optimally scaled variables

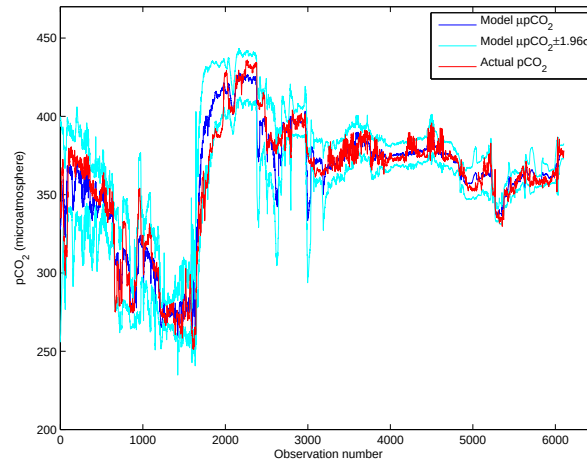


Figure J.4: RBF interpolation results. 50 D-optimal sampled points with the optimally scaled variables.

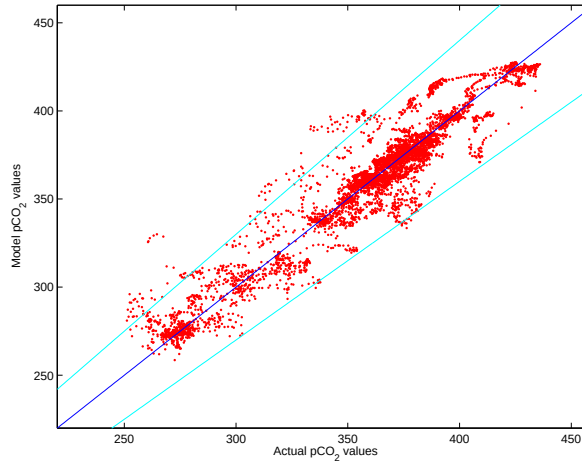


Figure J.5: RBF interpolation results with 50 D-optimal sampled points and optimally scaled variables. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

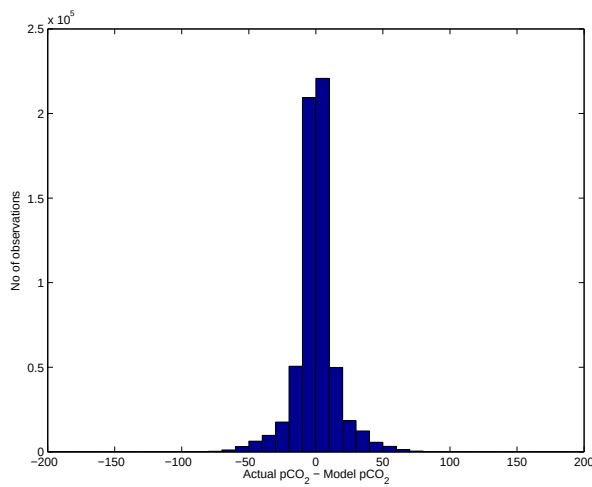


Figure J.6: RBF interpolation results. Histogram of errors for all 100 runs and 50 Randomly sampled points with optimally scaled variables

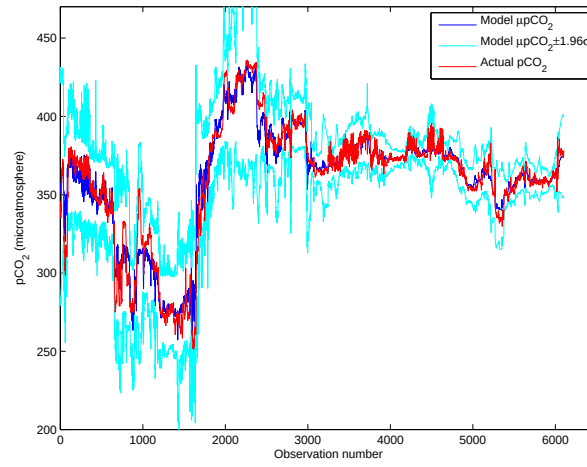


Figure J.7: RBF interpolation results. 100 Random points sampled with the optimally scaled variables.

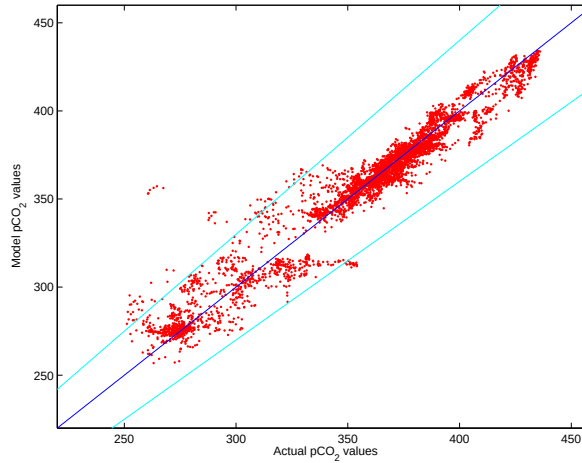


Figure J.8: RBF interpolation results with 100 random points sampled and optimally scaled variables. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

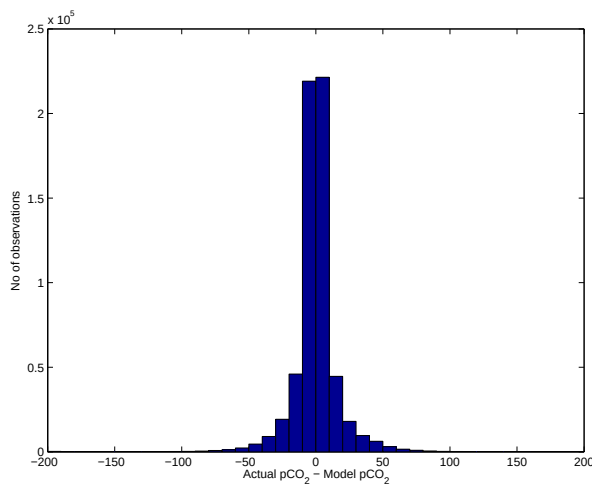


Figure J.9: RBF interpolation results. Histogram of errors for all 100 runs and 100 Randomly sampled points with optimally scaled variables

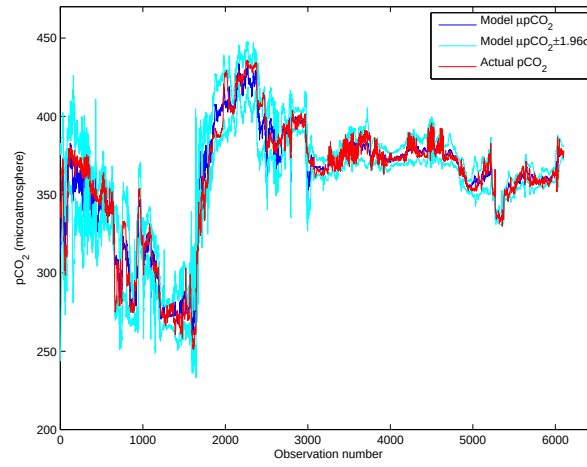


Figure J.10: RBF interpolation results. 100 D-optimal sampled points with the optimally scaled variables.

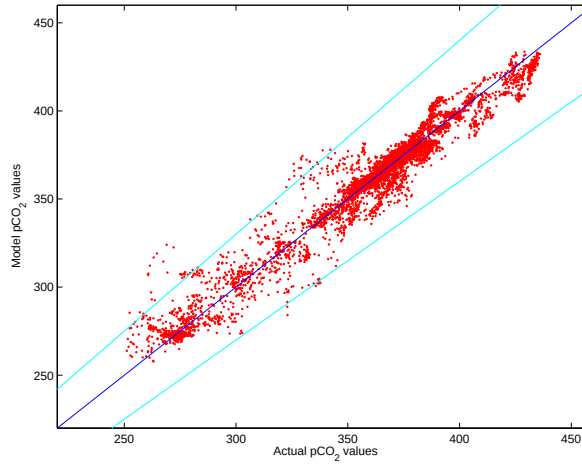


Figure J.11: RBF interpolation results with 100 D-optimal sampled points and optimally scaled variables. The degree to which the modelled points fall in the 10 % error margin is shown. The mean pCO_2 estimation for all 100 runs is shown here.

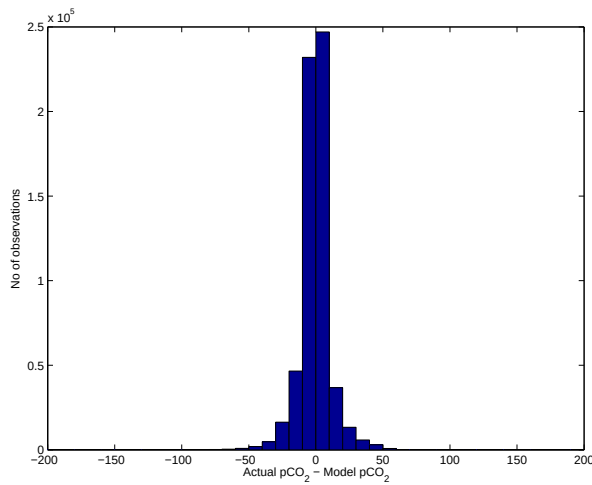


Figure J.12: RBF interpolation results. Histogram of errors for all 100 runs and 100 Randomly sampled points with optimally scaled variables

Appendix K

Radial basis function interpolation with latitude included as variable

The first figure in each case is a plot of the actual pCO_2 values, together with the mean values of the estimated pCO_2 from the 100 curve fits. The 95% confidence interval is also shown for the 100 curve fits in each case. The second figure in each case is the actual pCO_2 values plotted against the estimated pCO_2 values. The 10% error lines indicate the number of points that fall outside the 10% error. The third figure in each case is a histogram of the errors on the estimation of the pCO_2 , showing the distribution of the errors for each case.

It can be seen from the figures that as the number of points sampled from the data set increases, the fit of the data is improved and less points falls outside the 10% error interval. Also, from the figures it can be seen that the D-optimal sampling yields a smaller 95% confidence interval than the random sampling. The histograms generally show that the error on the estimation has a normal distribution around zero.

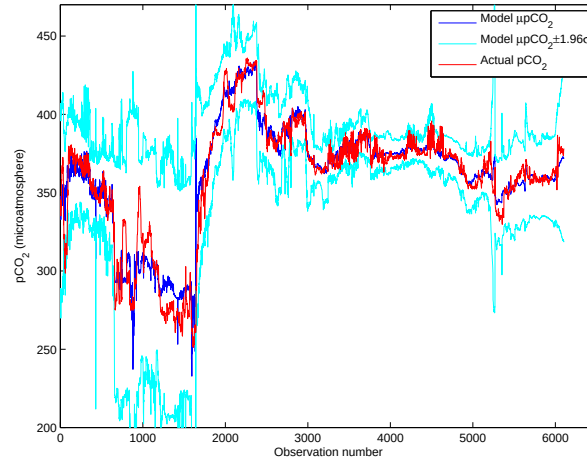


Figure K.1: RBF interpolation results. 50 Random points sampled with the 0-2 scaled data set.

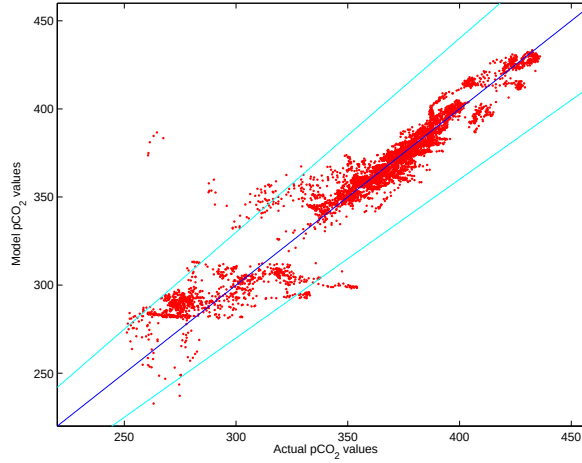


Figure K.2: RBF interpolation results with the 0-2 scaled data set and 50 randomly sampled points. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

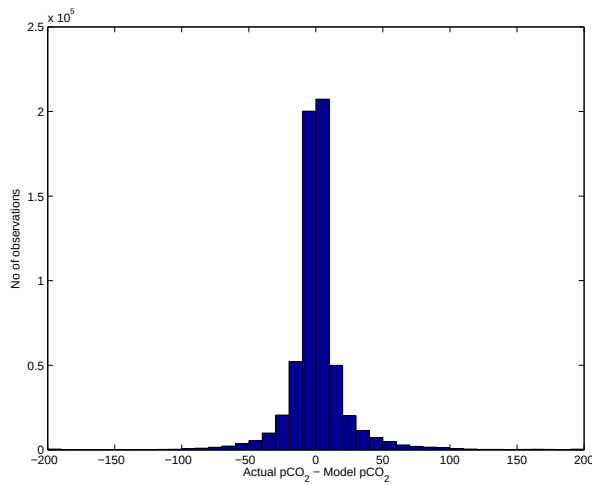


Figure K.3: RBF interpolation results. Histogram of errors for all 100 runs and 50 Random points sampled with the 0-2 scaled data set.

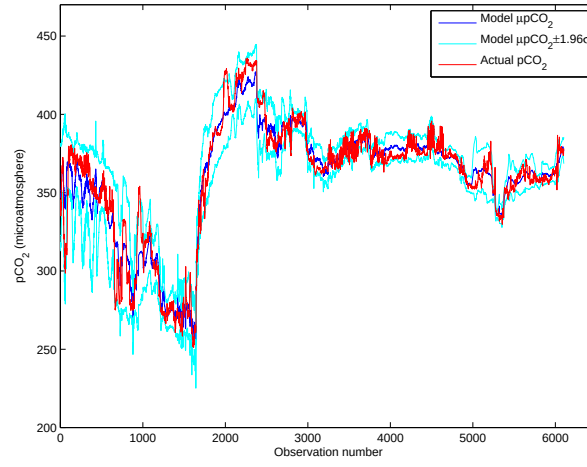


Figure K.4: RBF interpolation results. 50 D-optimal sampled points with the 0-2 scaled data set.

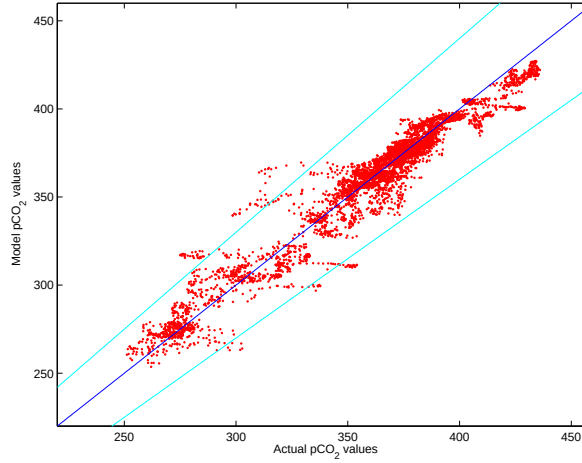


Figure K.5: RBF interpolation results with D-optimal sampled points with the 0-2 scaled data set. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

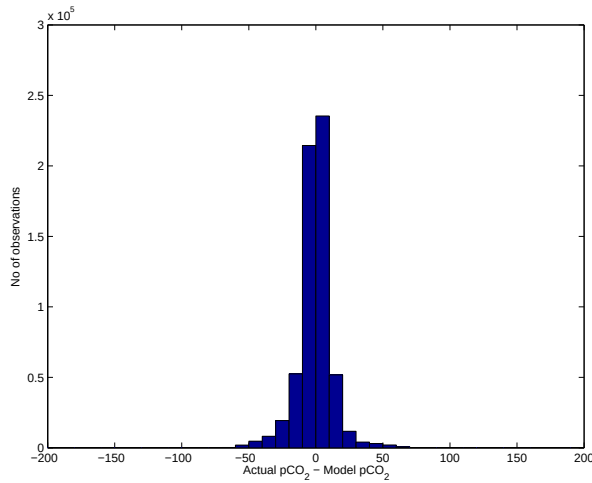


Figure K.6: RBF interpolation results. Histogram of errors for all 100 runs and 50 D-optimal sampled points with the 0-2 scaled data set.

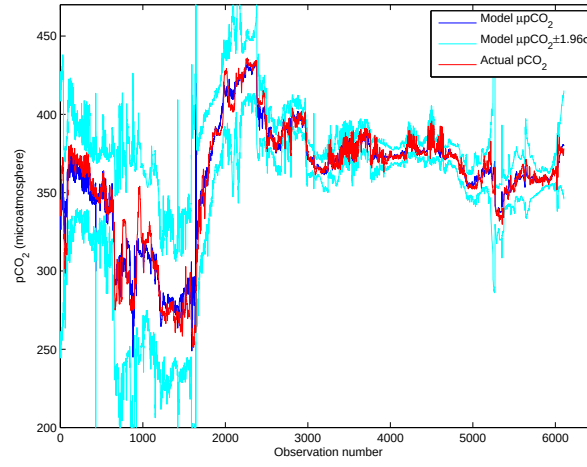


Figure K.7: RBF interpolation results. 100 Random points sampled with the 0-2 scaled data set.

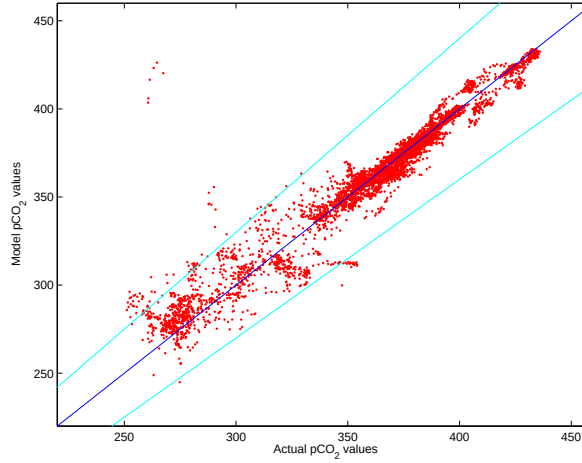


Figure K.8: RBF interpolation results with the 0-2 scaled data set and 100 randomly sampled points. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

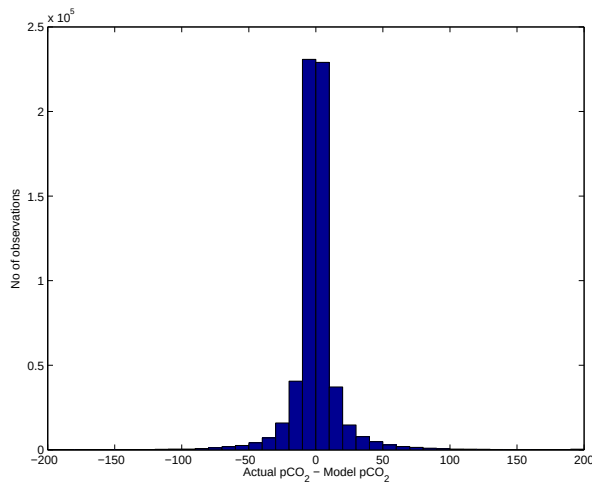


Figure K.9: RBF interpolation results. Histogram of errors for all 100 runs and 100 Random points sampled with the 0-2 scaled data set.

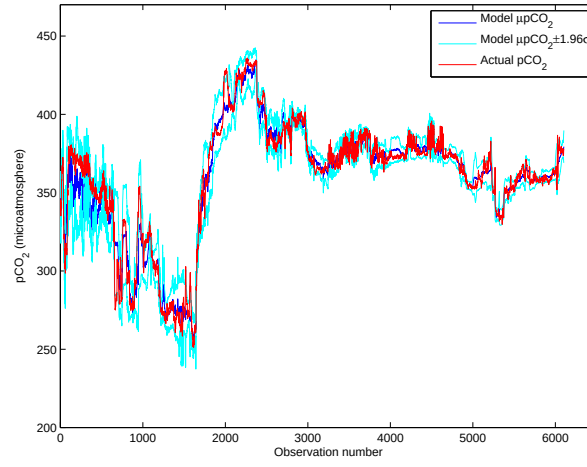


Figure K.10: RBF interpolation results. 100 D-optimal sampled points with the 0-2 scaled data set.

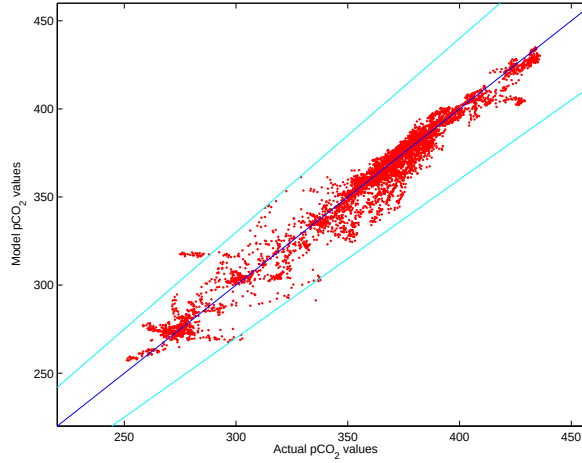


Figure K.11: RBF interpolation results with D-optimal sampled points with the 0-2 scaled data set. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

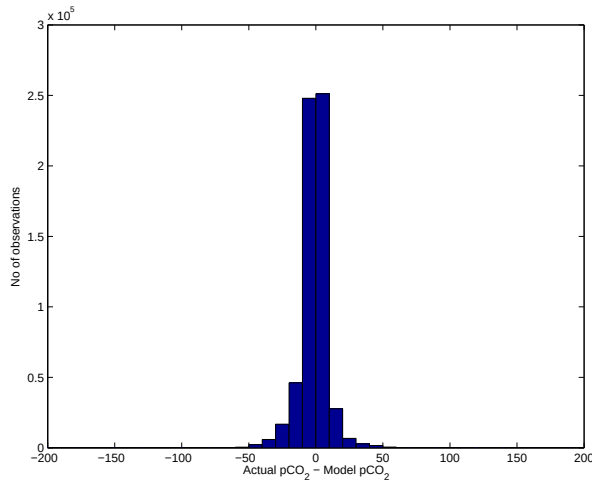


Figure K.12: RBF interpolation results. Histogram of errors for all 100 runs and 100 D-optimal sampled points with the 0-2 scaled data set.

Appendix L

Radial basis function interpolation with latitude included as variabel and optimal scaling of variables

The first figure in each case is a plot of the actual pCO_2 values, together with the mean values of the estimated pCO_2 from the 100 curve fits. The 95% confidence interval is also shown for the 100 curve fits in each case. The second figure in each case is the actual pCO_2 values plotted against the estimated pCO_2 values. The 10% error lines indicate the number of points that fall outside the 10% error. The third figure in each case is a histogram of the errors on the estimation of the pCO_2 , showing the distribution of the errors for each case.

It can be seen from the figures that as the number of points sampled from the data set increases, the fit of the data is improved and less points falls outside the 10% error interval. Also, from the figures it can be seen that the D-optimal sampling yields a smaller 95% confidence interval than the random sampling. The histograms generally show that the error on the estimation has a normal distribution around zero.

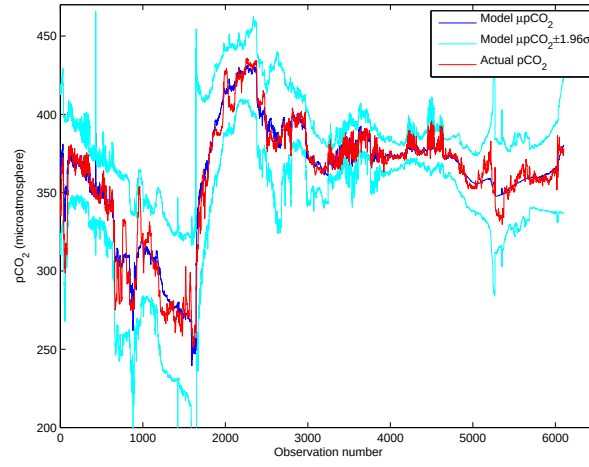


Figure L.1: RBF interpolation results. 50 Random points sampled with the optimally scaled variables.

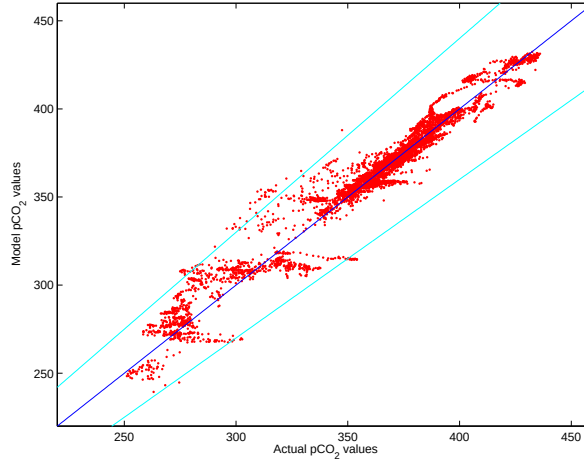


Figure L.2: RBF interpolation results with 50 random points sampled and optimally scaled variables. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

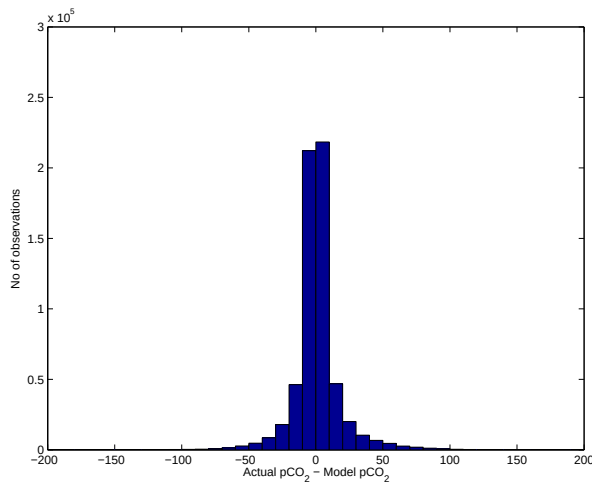


Figure L.3: RBF interpolation results. Histogram of errors for all 100 runs and 50 Randomly sampled points with optimally scaled variables

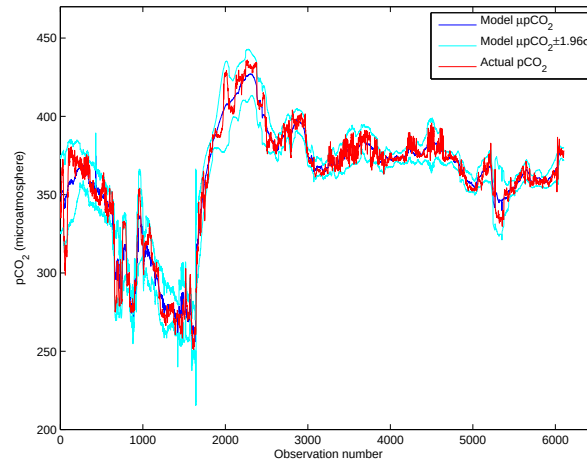


Figure L.4: RBF interpolation results. 50 D-optimal sampled points with the optimally scaled variables.

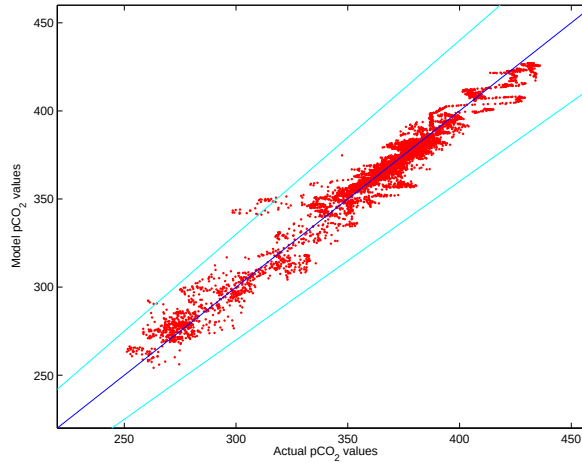


Figure L.5: RBF interpolation results with 50 D-optimal sampled points and optimally scaled variables. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

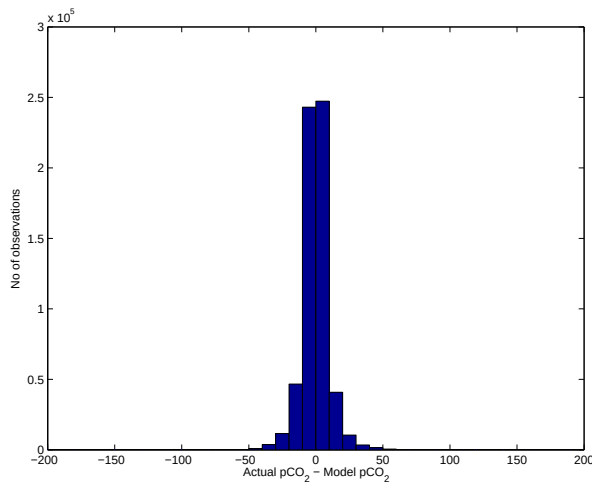


Figure L.6: RBF interpolation results. Histogram of errors for all 100 runs and 50 Randomly sampled points with optimally scaled variables

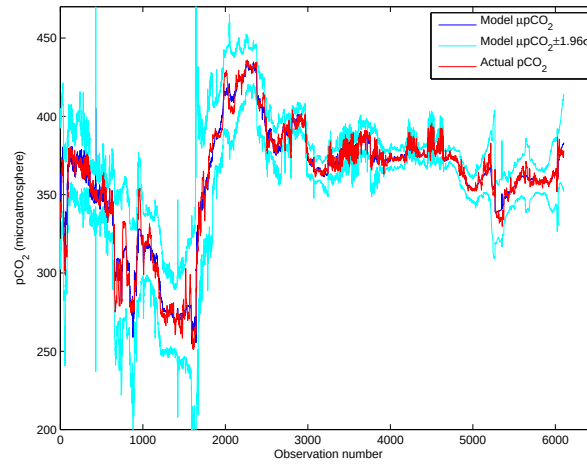


Figure L.7: RBF interpolation results. 100 Random points sampled with the optimally scaled variables.

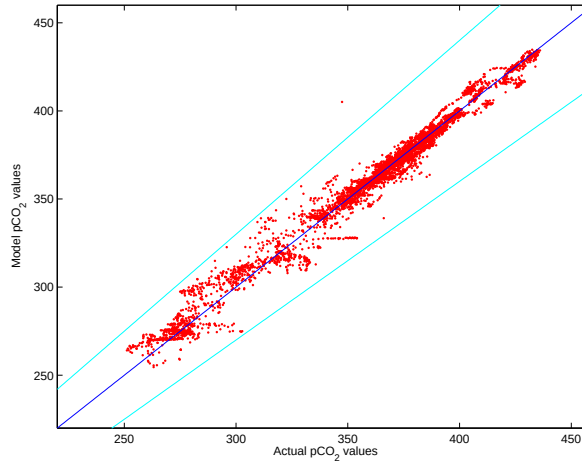


Figure L.8: RBF interpolation results with 100 random points sampled and optimally scaled variables. The degree to which the modelled points fall in the 10 % error margin is shown. The mean $p\text{CO}_2$ estimation for all 100 runs is shown here.

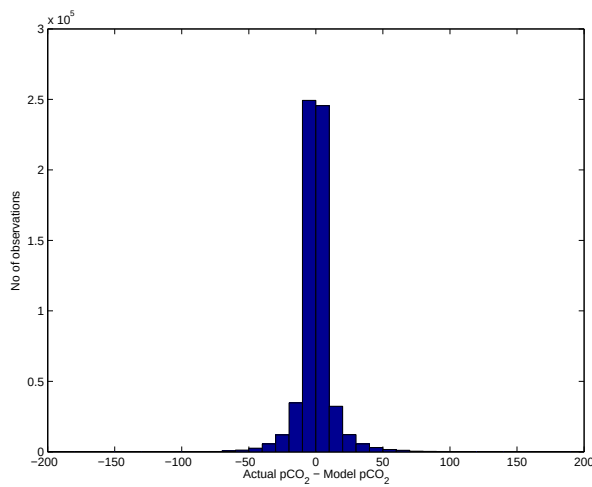


Figure L.9: RBF interpolation results. Histogram of errors for all 100 runs and 100 Randomly sampled points with optimally scaled variables

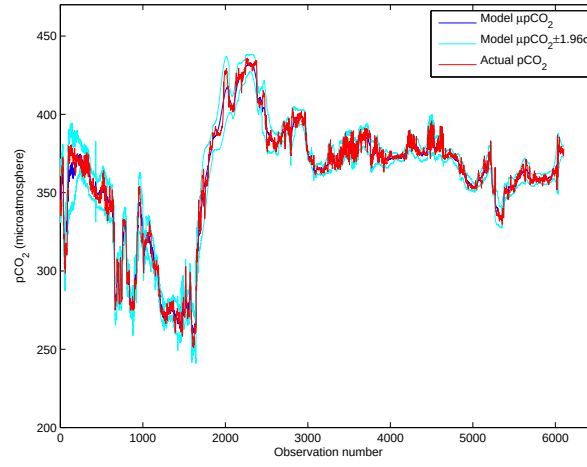


Figure L.10: RBF interpolation results. 100 D-optimal sampled points with the optimally scaled variables.

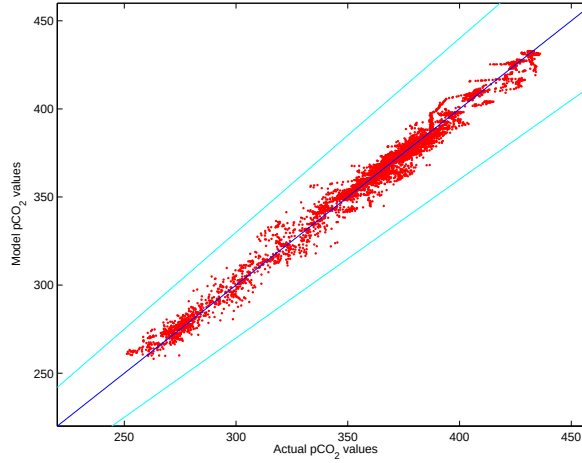


Figure L.11: RBF interpolation results with 100 D-optimal sampled points and optimally scaled variables. The degree to which the modelled points fall in the 10 % error margin is shown. The mean pCO_2 estimation for all 100 runs is shown here.

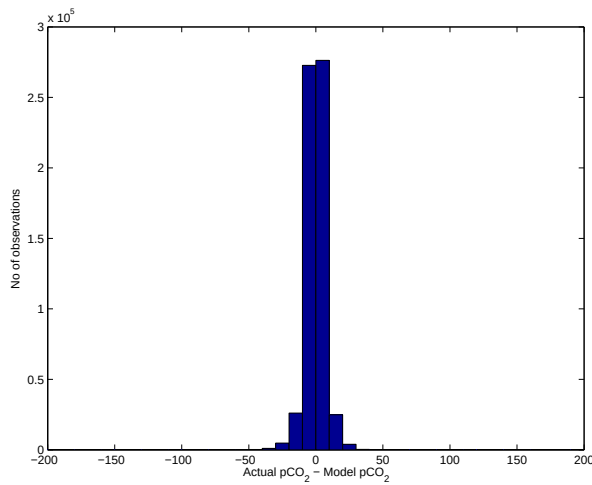


Figure L.12: RBF interpolation results. Histogram of errors for all 100 runs and 100 Randomly sampled points with optimally scaled variables