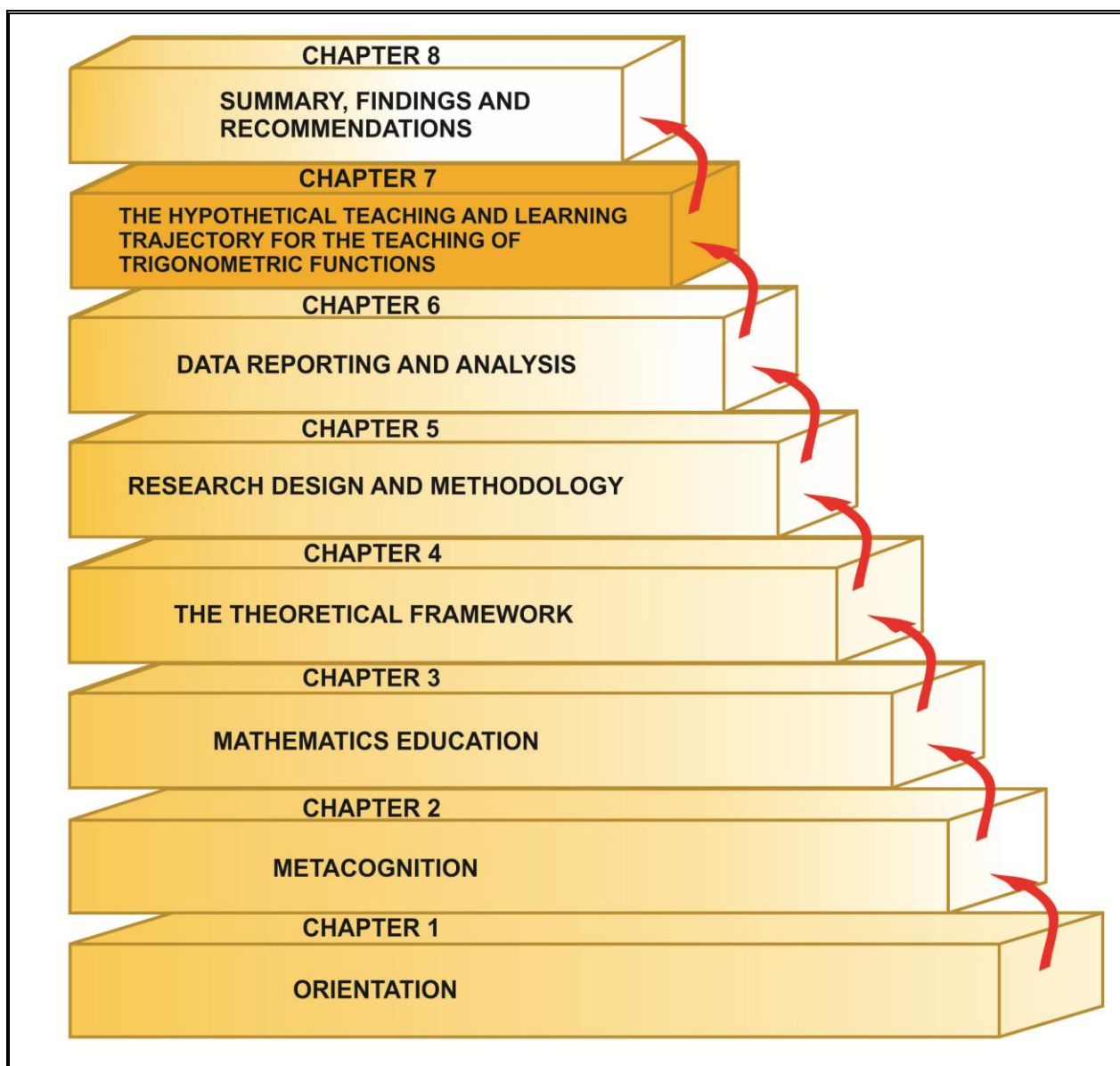
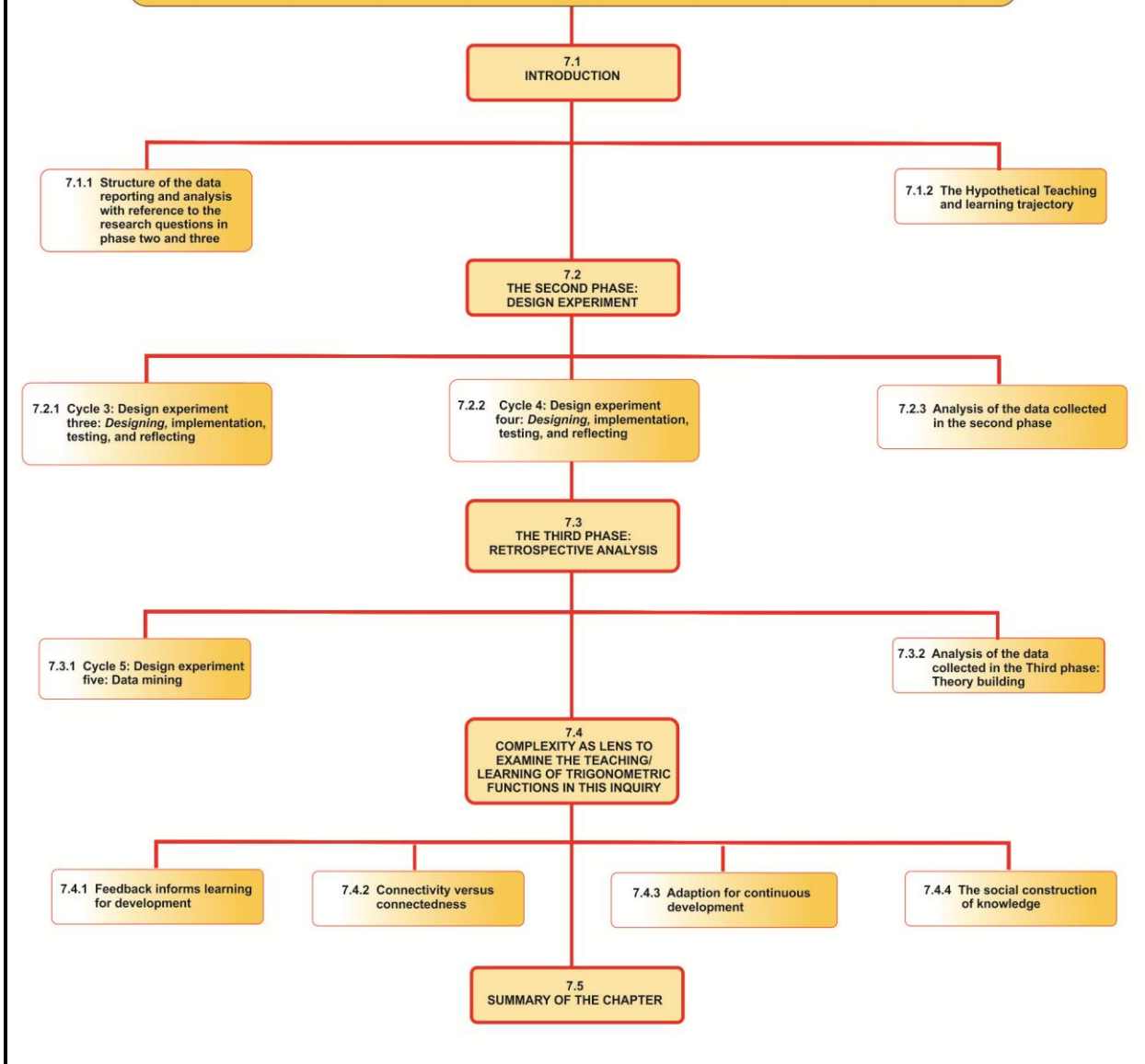


CHAPTER 7: THE HYPOTHETICAL TEACHING AND LEARNING TRAJECTORY FOR THE TEACHING OF TRIGONOMETRIC FUNCTIONS



CHAPTER 7: THE HYPOTHETICAL TEACHING AND LEARNING TRAJECTORY FOR THE TEACHING OF TRIGONOMETRIC FUNCTIONS



7.1 INTRODUCTION

In the previous chapter the data that were collected from the first phase in this inquiry were reported on and analyzed, which subsequently informed an metacognitive performance profile for each of the two grade 10 teachers. This data reporting and analysis also informed the development of the hypothetical teaching and learning trajectory for the teaching of trigonometric functions from the first two cycles of the adapted lesson study. The metacognitive performance profiles of the two teachers and the hypothetical teaching and learning trajectory developed in the previous chapter constitute the first three themes in the data analysis. In this chapter the trajectory is implemented in three more cycles of adapted lesson study within the next two phases, phases two and three, and subsequently this chapter provides an analytic view on this implementation in order to fine-tune the trajectory for the teaching of trigonometric functions. This implementation and fine-tuning of the trajectory gave birth to the next three themes in the inquiry, and this chapter gives effect to the third sub-question *viz. How can the teaching of trigonometric functions be improved focusing on the metacognitive skills and mathematical language used by mathematics teachers?* The chapter starts with the structure of the data analysis with reference to the research questions (§7.1.1). In the next section (§7.2) data within the second phase is reported which includes the reporting of design experiment three (§7.2.1), followed by design experiment four (§7.2.2). Hereafter the analysis of the data collected in phase two (§7.2.3) is provided. In the same way the reporting of the final phase, the third phase, is provided (§7.3). Lastly the analysis of the data collected in this final phase is addressed before a summary of the chapter is provided. Figure 6.1 from the previous chapter has relevance in this chapter as well and is therefore provided again.

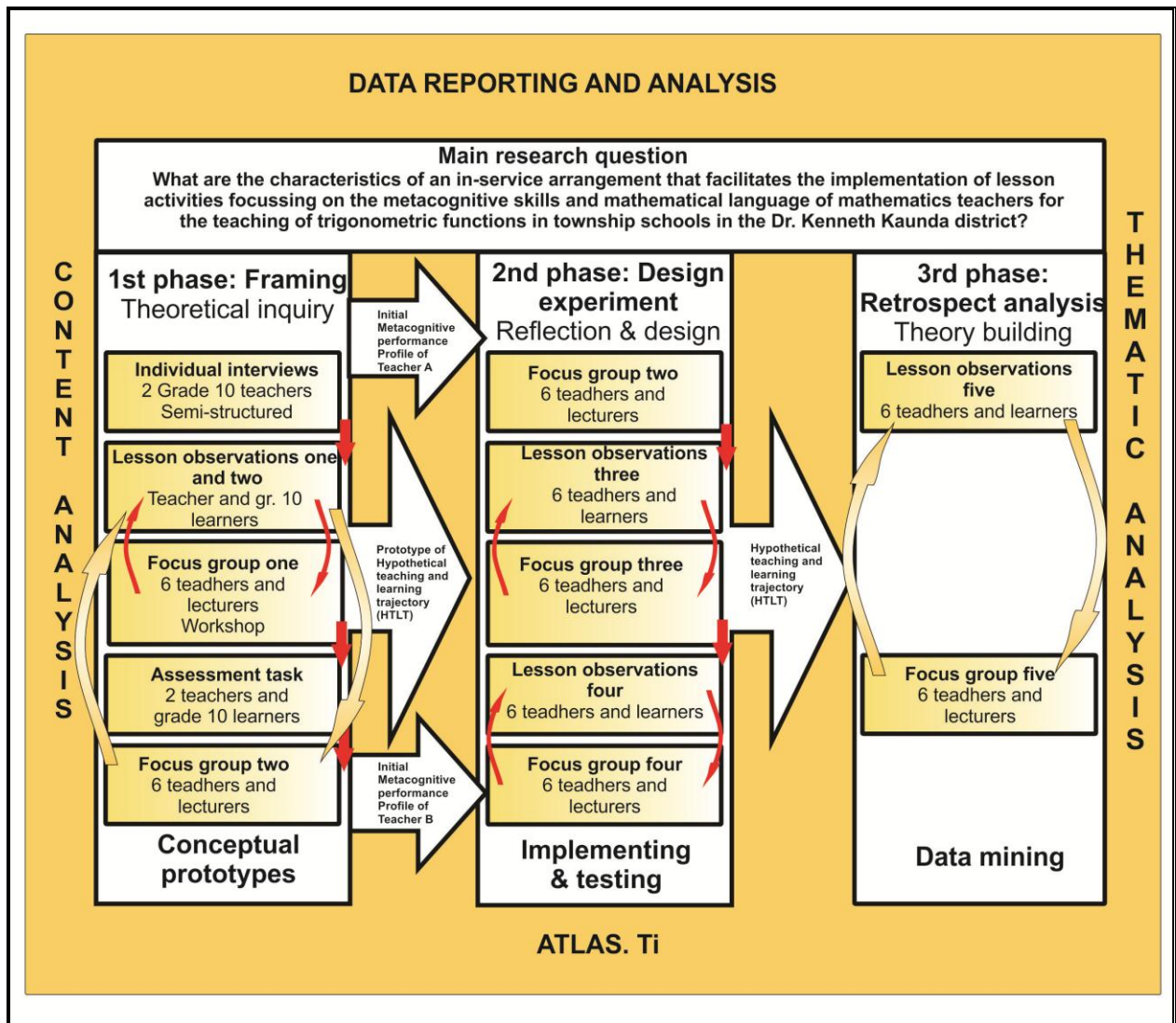


Figure 7.1: Data reporting and analysis in the three phases within this study

7.1.1 Structure of the data reporting and analysis with reference to the research questions in phase two and three

Table 7.1 provides an outline of the research questions with an indication of how and where data are reported and analysed in this chapter in order to address the third sub-question.

Table 7.1: References to sections that address the sub-question in this chapter

Sub-questions	Individual interviews	Trigonometry Assessment task	Lesson observations	Focus group Discussions
3. How can the teaching of trigonometric functions be improved focusing on the metacognitive skills and mathematical language used by mathematics teachers?			(§7.2)	(§7.2)
			(§7.3)	(§7.3)
			(§7.4)	(§7.4)

7.1.2 The Hypothetical Teaching and learning trajectory

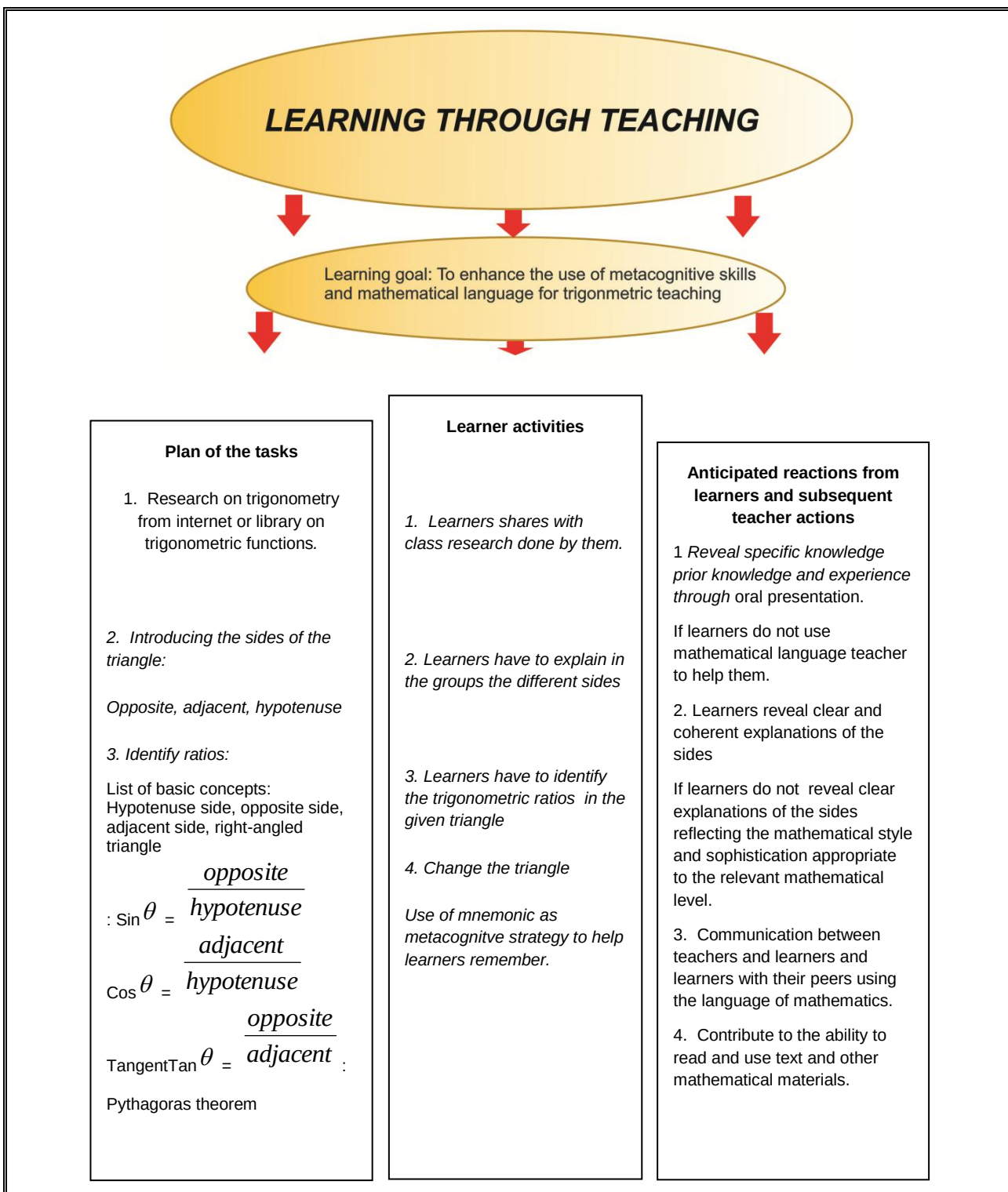


Figure 7.2: The hypothetical teaching and learning trajectory

The *prototype* of the hypothetical teaching and learning trajectory developed in Chapter Three (Figure 3.6) was adjusted to form the hypothetical teaching and learning trajectory (Figure 7.2) that was used as basis for the implementation and testing, reflecting, designing, data mining and theory building which characterize the next two phases. The inquiry endeavored to implement the four adjusted guiding principles (Quinlan, 2004: 20) in the trajectory while zooming in on how the teachers are using their metacognitive skills and mathematical language.

7.2 THE SECOND PHASE: DESIGN EXPERIMENT

The first two cycles were done in the previous chapter. This second phase of the data collection consisted of the next two cycles and involved implementation and testing (lesson observations) and reflecting and designing (focus group discussions) of the hypothetical teaching and learning trajectory.

7.2.1 Cycle three: Design experiment three: *Designing*, implementation, testing, and reflecting

This cycle started with a discussion by the research team, the focus group discussion (§7.2.1.1), then the implementation of what was discussed by way of the lesson (§7.2.1.2), followed by another focus group discussion (§7.2.1.3). Although focus group discussion two was used in the first phase as the reflection of what transpired in lesson one and two, it is used here again focusing on the planning of lesson three. Focus group two can therefore be regarded as the planning phase within cycle three while focus group three would in this cycle be the reflection on lesson three.

7.2.1.1 Focus group discussion two: Designing

The main aim with the questions for focus group two (see Addendum D4) was to discuss the thinking, engagement and behavior of the learners. Only question four from the questions for focus group discussion two centered on the planning of lesson three:

4. *How can this lesson be adjusted to increase the use of metacognitive skills by teacher and learners as well as their mathematical language in terms of the following?*
 - The outcome of the lesson

- The activities done by learners (What do we expect from them)?
- The opportunities for using metacognitive skills by learners created by the teacher
- Mathematical language usage within the lesson.

These considerations are subsequently reported on (Table 7.2) in terms of the design principles, metacognitive skills and mathematical language from the hypothetical teaching and learning trajectory (HTLT) for trigonometric functions (Figure 7.2).

7.2.1.2 Lesson observation three: Implementation and testing

Lesson on 8 March 2013 at the Blue School

Presenter: Teacher C

This lesson started with the teacher giving the learners cardboard and koki-pens. Learners worked in groups and were much more active in comparison with the learners in the first two lessons. The teacher had given learners some research work to do prior to the lesson. The research work included reading on the use of trigonometry in daily life as well as what trigonometry actually is. The teacher successfully extracted the information that learners had to access through Internet regarding the uses of trigonometry in daily life and also elaborated some more on the answers of the learners. Teacher C, like the other teachers who presented lesson one and lesson two, held the perception that trigonometry deals with the right-angled triangle only. Learners were sitting in groups of four in order for group work. Table 7.2 reports on this third lesson in terms of the principles from the HTLT.

7.2.1.3 Focus group discussion three: Reflecting

Questions set up for focus group three included, as discussed earlier on (§7.2.1.1), questions for reflection, but also questions for the planning of the next lesson (see Addendum D4). The first part aimed to elicit reflection from the teacher who presented the lesson while the next two questions were aimed towards reflective input from the group:

To teacher who presented the lesson: Please reflect on the lesson in terms of the following:

Did you follow the lesson 100% as it was planned by the group? If no, please explain reasons for not following the lesson and indicate the places where the lesson was adjusted. If yes, give reasons for not adjusting it.

To Group:

Look at the lesson and discuss:

1. Do you think we have reached our aims? Motivate.
2. In your opinion: What did work? Why did it work? What did not work? Why did it not work?

This focus group discussion was attended by three teachers, two lecturers and the researcher. The general impressions at the discussion of the third lesson were characterized by an overall notion of improvement. Table 7.2 shows what transpired in this third focus group discussion.

Table 7.2: Table reporting on focus group discussions two and three

Categories (Quinlan, 2004:20 adjusted)	Focus group discussion two (P5) (Before lesson three)	Lesson observation three (P12)	Focus group discussion three (P6) (After lesson three)
<i>Go from the concrete to the abstract, while explicitly mentioning the outcome/s of the lesson. Avoid starting with definitions</i>	It is important to be prepared, to plan and to anticipate responses that the teacher might get from the learners when he/she moves from the concrete to the abstract: <i>Be prepared, imagining yourself giving that lesson. How will they understand you? How will they be feeling?</i> (P5:087) One member of the research team was of the opinion that the teacher in lesson one was spoon- feeding the learners. There needs to be more interaction between the learners and the teacher and also amongst the learners themselves: <i>There was no interaction. So the teacher was, let me just say this in another way, just been feeding, giving all the answers.</i> (P5: 013)	Although the teacher started the lesson with requesting learners to tell him what trigonometry is, he was still starting with the definition of trigonometry: <i>Right, when you look at this concept, firstly we want to define, we want to define that particular concept, say, what is Trigonometry? Now in terms of the definition, I throw it back to you because I to give you the chance to, uh, in fact, uh, research, like, how do we define Trigonometry? How do we define it?</i> (P12:003)	This time around, the teacher did do some pre-planning: <i>Uhm, prior to the lesson I, uhm, I informed them about the topic that I was to discuss. So, I asked them to go and do the research. Look at where we use trigonometry, what is Trigonometry. Those are the two aspects that I wanted them to go and do.</i> (P6:012) The learners were much more active and more interaction could be observed by the research team: <i>I remember from last time, just the clips that we saw, these were very, right from the start, I think they were very involved.</i> P6:010)

Categories (Quinlan, 2004:20 adjusted)	Focus group discussion two (P5) (Before lesson three)	Lesson observation three (P12)	Focus group discussion three (P6) (After lesson three)
<i>Go from particular to general.</i>	One member of the research team wanted to see learners who are involved in the particular right from the start: <i>Being a new topic that you are introducing today I would also like students there to, I mean this is now completely new, at least they must look with “wow”, there was something new today, it was peace from there, I came right, I’m going to hear something new today and then of course, ask questions, right from the start. That’s also the kind of learner I would like. (P5:047)</i>	Teacher C asked the learners to define first a particular side and also to write up their thinking: Right, so what we have to do for now, in our books, I want you to be able to define this adjacent side in relationship to angular. The relationship between this adjacent side and this angle. Just write something down. You are not going to say it out loud, I will just ask you to display your answer. So if somebody has a bigger hand writing. Try to define that adjacent side.(P12:079)	Reflecting on lesson three, one member of the research team confirmed that the teacher focused on a particular side and then let learners discover the other information in general. <i>Because I think that is where the teacher said learners must define what is the adjacent side, in terms of that, whereas the learners will be able to discover the information. (P6:020)</i>
<i>Context of any new concept before technicalities, intricacies and mathematical jargon.</i>	The teacher who presented lesson one reflected about the issue of putting the concept firstly into context by saying that he did not want to use the Cartesian plane as context because learners would then not grasp the fact that the hypotenuse is not always fixed: <i>That’s why my thinking was, if I take this smaller triangle and put it there aside without using the Cartesian plane, then they are going to see in other words, that the hypotenuse is not always fixed. You can</i>	Although the teacher requested from the learners to do some research about trigonometry and by so doing sketch the context, he was not really introducing a real life situation in which the trigonometric functions could have been applied.	One member of the research team commended teacher on letting learners grasp the fundamental knowledge first: <i>But once the learners got involved to self-discovery, I thought that was the best part of it... to spend that much time to make sure that, when I talk about basic stuff, that they see the fundamental in there, from there I can then knew, which was the opposite side, which was the adjacent side and then only he brought them to the</i>

Categories (Quinlan, 2004:20 adjusted)	Focus group discussion two (P5) (Before lesson three)	Lesson observation three (P12)	Focus group discussion three (P6) (After lesson three)
	<i>draw your triangle this way, your putting's would be that side, draw it sideways, or you can just move it around. Then the learners can see that this thing is something different than that one you are used to because that one is fixed and this one you can change it around depending on your position of your triangle. (P5:113)</i>		<i>Trigonometry' part. I really think that that is a good way for them to begin.(P6:022)</i>
<i>The actions of the teacher should elicit favourable reactions from the learners.</i>	Actions by teacher should elicit interaction between learners and teacher: <i>There must be the interacting between the teacher and the learners.(P5:028)</i> Actions from the teacher should elicit questions from the learners that challenge the teacher: <i>So I want learners that will ask questions, they must challenge me. (P:045)</i>	Questioning by the teacher invited a deeper level of thinking and not only Yes/No answers or some completion of a sentence was elicited, although these types of questions were also used by the teacher.	Time given to learners by teacher elicited the use of metacognitive skills: <i>We gave them time to discuss in the groups and then also to write down and let them hold up the poster. I think that should help them with metacognitive skills also. Even if one, I mean I'm thinking this and he's saying something else, it changes my thinking also and I agree to that more. It's sort of, isn't that metacognitive? I listen to what he said, in my mind I say, uhu, I need a bit more, so that helps me to think in that way. (P6:075)</i>

Discussion

The discussions in focus groups two and three were geared towards reflection by the teacher who presented lesson three, but also towards the analysis of the lesson by the research team as a whole. The focus was on the use of metacognitive skills and mathematical language when teaching trigonometric functions.

From the lesson observation: After first explaining the different sides and the relationship of the reference angle to a particular side, teacher C asked the learners to discuss within the group how they would know what the adjacent side was, decide on the relevant side and then write down their answers on the cardboard to display their answers. This was the idea that was discussed in the previous focus group discussions. Teacher C implemented this idea (that learners needed to make visible their thinking) very well in the lesson. Learners discussed what their ideas of each side were amongst the group. Teacher C moved around between the groups while they were busy.

From the focus group discussion: Another very important activity done by Teacher C was that the thinking of the learners was acknowledged by applause after each group answered. Teacher C controlled the answers during the feedback that each group gave. Not only did he evaluate their answers, but he also explained why other answers were incorrect, comparing the different answers with one another. He used the diagram continuously. Teacher C not only made attempts to understand the answers by the group, but he also tried to make visible their thinking to the rest of the class. Teacher C evaluated the position of the class in terms of their understanding of the adjacent side before he moved to explaining the opposite side. Teacher C referred to the theorem of Pythagoras as well as the Pythagorean triplets. Another implementation of what was discussed is the avoidance of choir answering, which the teacher was sidestepping by writing down the answers. He also justified why he was using writing (to prevent choir answering). He acknowledged correct answers: "I'm very impressed" and monitor the learners' thinking: "Are we agreeing?" "We are in consensus" "Any problem with the Sine ratio?" Teacher C succeeded in involving all his learners: "Somebody who said nothing yet" "Let me try to change the situation here". This is metacognitive skills usage as teacher changes the situation to make learners apply their acquired knowledge to the adjusted situation.

- In comparison with the first two lessons, this lesson seemed to be much more interactive.
- The thinking of the learners was more visible and made possible by the use of charts to show answers.
- The thinking of teacher C was also much clearer when the teacher was constantly aware of the thinking of his learners, keeping track of their learning; “Are you following?” “It was clear that...” “Everyone can follow their thinking?”

It appeared as if the teacher taught *with* metacognition as well as *for* metacognition. The group agreed that the teacher monitored as well as regulated not only the thinking of the learners, but also his own thinking.

7.2.1.4 Analysis of cycle three

Guiding principle one: *Go from the concrete to the abstract, while explicitly mentioning the outcome/s of the lesson. Avoid starting with definitions*

Pre-planning in going from the concrete to the abstract while explicitly stating the outcomes is important. As far as concrete examples are concerned, the teacher also did not go from the concrete to the abstract in the sense that learners could see the trigonometric functions at work in a real life mathematics problem first before abstracting the different trigonometric functions and their ratios. The pre-planning of the lesson made it possible for the teacher to involve the learners much more than it would have been without their prior research. Introducing the trigonometric functions using the unit circle seemed to enhance learning more than not using it, though in all the lessons until now, the unit circle had been avoided by the teachers. This phenomenon does not support the notion of Kendal and Stacey (1997) who found that learners who had learnt trigonometric functions in the context of a right triangle model performed better on a post-test than those who had learnt about the subject using a unit circle model. It was apparent that learners teachers needed to start future lessons with a real life situation (concrete) in which the application of trigonometric functions are required to solve the problem.

Guiding principle two: Go from particular to general, allowing learners also to concentrate on a particular side and then let them discover the more general information themselves.

Starting with a particular real life situation in which a trigonometric function is applied in order to solve the problem, will also adhere to this principle. Corral (2009) warns against approaching trigonometric functions with too much of an analytic emphasis because it confuses learners and makes much of the material appear less motivating. The subject knowledge of the teacher seems to be critical as the teacher has to distinguish between the particular and the general in the material that needs to be taught. Although Alexander, Rose and Woodhead (1992: 77) are of the opinion that subject knowledge is a critical factor at every point in the teaching process, it is in this very first stage of the lesson that teachers need to make sure that the attention of the learners is caught and captured.

Guiding principle three: Immerse learners in the context of any new concept before technicalities, intricacies and mathematical jargon.

Together with the context of any new concept, the fundamental knowledge should also be addressed when introducing the new concept. This notion of required fundamental or prior knowledge when introducing trigonometry in grade 10 for the first time (see 3.9.5), is maintained by Blackett and Tall (1991) and Weber (2005) amongst others. Hand in hand with this, teachers need to know mathematical procedures. It is exactly these special ways that Ball and Bass (2003), Hill and Ball (2004) and Hill, Rowan and Ball (2005) refer to which a teacher needs to know in order to interact productively with learners in the context of mathematics teaching. This endeavour also requires a good command of what is called “the register of mathematics” by Halliday (1978) in order not to confuse learners more with unnecessary jargon. The effective use of a teacher’s metacognitive skills cannot be downplayed in this process.

Guiding principle four: The actions of the teacher should elicit favourable reactions from the learners.

If a teacher needs specific reactions from the learners he/she should know what other actions can be expected from the learners. The teacher therefore needs to anticipate these actions from the learners. In this regard, prediction plays a critical role in this process which Shraw (2001:4) classifies as metacognitive control, together with strategies such as comprehension monitoring, planning of learning activities and revision. In the same vein, Hartman (2002b:44) mentions the expectations of the teacher as a critical element in effective functioning of his/her metacognitive skills. It is these considerations from the literature and the analysis of what transpired from focus group discussions two and three and lesson three (see Table 7.2) that this last guiding principle changed to the following principle:

The actions of the teacher should elicit favourable reactions from the learners, in terms of the use of metacognitive skills.

These guiding principles were adjusted after cycle three and gave birth to the new Hypothetical teaching and learning trajectory projected in Figure 7.3.

THE HYPOTHETICAL TEACHING AND LEARNING TRAJECTORY FOR TRIGONOMETRIC FUNCTIONS

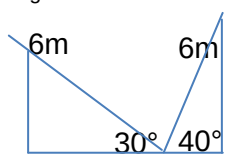
Learning goal: To enhance the use of metacognitive skills and mathematical language for trigonometric teaching

Guiding principles (Quinlan, 2004:20 adjusted)

1. *Go from the concrete to the abstract, while explicitly mentioning the outcome/s of the lesson. Avoid starting with definitions*
2. *Go from particular to general.*
3. *Immerse students in the context of any new concept before explicating its technicalities and intricacies and mathematical jargon.*
4. *The actions of the teacher should elicit favourable reactions from the learners, in terms of the use of metacognitive skills.*

Outcomes for the learners:

(i)	To know the basic concepts of triangle trigonometry
(ii)	To use trigonometric ratios to solve mathematical real-world problems

Content	Teacher Activities	Learner activities
Introduction Real world application A 6m long ladder with its foot in the street makes an angle of 30° with the surface of the street when its top rests on a building on one side of the street. The same ladder makes an angle of 40° with the street when its top rests on a building on the other side of the street while the foot of the ladder remains in the same position. How wide is the street (to the nearest metre)?	Teacher gave the problem to learners after they were handed charts and markers to make a drawing to show their understanding of the problem. Instructions: • Work in pairs • Make a drawing • Show your drawing • Solve the problem Teacher moved between the pairs and assisted where needed. Teacher guided learners toward correct drawing and correct calculations When pairs had finished, teacher asked one pair to come to the board and explain their solution to the class.	Anticipated actions: Learners might make the correct drawing  and use the correct ratio: Cosine theta: $\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ Learners understood that in order to answer the question which was that the width of the street needed to be calculated, two distances had to be found: First distance: $\cos 30^\circ = \frac{\text{distance}}{6\text{m}}$ $\text{Distance} = \cos 30^\circ \times 6\text{m} = 5.96152423$ Second distance: $\cos 40^\circ = \frac{\text{distance}}{6\text{m}}$ $\text{Distance} = \cos 40^\circ \times 6\text{m} = 4.5962666659$ Learners understood the two distances should be added: $5.96152423 + 4.5962666659 = 10.55779089$ Therefore the street is about 11m wide. Learners might have the drawing

		incorrect and use correct ratio Learners might have drawing incorrect and use incorrect ratios.
List of basic concepts: Hypotenuse side, opposite side, adjacent side, right-angled triangle $\text{Sine theta: } \sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$ $\text{Cosine theta: } \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ $\text{Tangent theta: } \tan \theta = \frac{\text{opposite}}{\text{adjacent}} ;$ Pythagoras theorem Plan of the tasks 1. Research on trigonometry from internet or library on trigonometric functions. 2. Introducing the sides of the triangle: <i>Opposite, adjacent, hypotenuse</i> 3. Identify ratios	Teacher revised the basic terminology by question and answer method Learner activities 1. Learners shares with class research done by them. 2. Learners have to explain in the groups the different sides 3. Learners have to identify the trigonometric ratios in the given triangle 4. Change the triangle Use of mnemonic as metacognitive strategy to help learners remember.	Learners raised their hands and responded individually Anticipated reactions from learners and subsequent teacher actions 1. Reveal specific knowledge prior knowledge and experience through oral presentation. If learners do not use mathematical language teacher to help them. 2. Learners reveal clear and coherent explanations of the sides. If learners do not reveal clear explanations of the sides reflecting the mathematical style and sophistication appropriate to the relevant mathematical level. 3. Communication between teachers and learners and learners with their peers using the language of mathematics. 4. Contribute to the ability to read and use text and other mathematical materials.

Figure 7.3: The Hypothetical teaching and learning trajectory for trigonometric functions

The adjusted hypothetical teaching and learning trajectory was then implemented and tested in cycle four, and the next section reports and analyses the implementation thereof.

7.2.2 Cycle four: Design experiment four: *Designing*, implementation and testing, and reflecting

The fourth cycle started with a discussion by the research team, the focus group discussion (§7.2.2.1), then the implementation of what was discussed by way of the lesson (§7.2.2.2), followed by another focus group discussion (§7.2.2.3). Although focus group discussion three was used earlier in the phase as the reflection of what had transpired in lesson three, it is used here again because it also focused on the planning of lesson four. Focus group discussion three can therefore be regarded as the planning within cycle four while focus group discussion four would be the reflection of lesson four in this cycle.

7.2.2.1 Focus group discussion three: Designing

The last question from the questions for focus group discussion three (see Addendum D4) centred on the planning of lesson four:

3. *Now indicate how we can improve this lesson in order for learners to learn, focusing on the use of metacognitive skills and mathematical language.*

These suggestions by the members in the research team from focus group discussion three (Primary document six) are subsequently reported on (Table 7.4) in terms of the design principles, metacognitive skills, and mathematical language informing the design principles within the adjusted hypothetical teaching and learning trajectory for trigonometry teaching (Figure 7.3).

7.2.2.2 Lesson observation four: Implementation and testing

Lesson four on 20 May 2013 at Red School

Presenter: Teacher F

The lesson (Primary document 13 in the HU) started with the teacher who revised the basic concepts in trigonometry. Learners seemed quite confident with the concepts, which was in sharp contrast with their struggling just moments after this when they attempted to solve the mathematical problem in the real life situation to which these basic concepts and trigonometric ratios was supposed to be applied; they could hardly draw the diagram. This observation might confirm the perception that trigonometry is taught in a manner in which learners are reciting the concepts, using mnemonics without really understanding (Brown, 2005). Quite a lot of learners raised their hands to answer the questions, especially when the question required from them to give the particular trigonometric ratio. The teacher explicitly focused on individual learners answering, and avoided choir answering which was also discussed in our previous collaborate sessions. Table 7.2 reports on what transpired in lesson four.

7.2.2.3 Focus group discussion four: Reflecting

Focus group discussion four at the University on 29 May 2013

This fourth focus group discussion (Primary document 14) took place after lesson four which was presented by Teacher F at the Red school, and was held at the university again. Four teachers attended this discussion. As this discussion was held just before examination was about to start, the other teachers were busy teaching extra classes to their learners in an attempt to cover the material that would be asked in the examination papers. One of the teachers was responsible for the photocopying of all the examination papers for the examination at the Blue school and could not attend. This session was particularly insightful because an international scholar who visited the university, specifically the Self-Directed Learning focus area, was also invited to attend the focus group because of his specialized knowledge of problem-based learning and metacognition. The questions for this focus group discussion were exactly the same as for focus group discussion three (§ 7.2.1.3) where the first part aimed to elicit reflection from the teacher who presented the lesson while the next two questions were aimed towards reflective input from the group. Table 7.3 shows what transpired in this fourth focus group discussion.

Table 7.3: Table reporting on focus group discussions three and four and lesson observation four

Categories (Quinlan, 2004:20 adjusted)	Focus group discussion three (P6) (Before lesson four)	Lesson observation four (P13)	Focus group discussion four (P14) (After lesson four)
<i>Go from the concrete to the abstract, while explicitly mentioning the outcome/s of the lesson. Avoid starting with definitions</i>	The members planned to start with the application of the trigonometric functions in a real life situation in the next lesson. <i>...this is a very practical sort of part of maths, you know. They must already in the first lesson, get that interest into this. This is something we are going to do out there. I would even say, they know Pythagoras by then and that we use a lot of Pythagoras in trigonometry. And even given in the in the prior thing, but make it something in real life. (P6:065)</i>	However, the outcomes of the lesson were explicitly stated by the teacher: <i>There are our outcomes. The first one it is to understand the basic concepts of triangle trigonometry. I belief most of you know the concepts, but we will come to that one. And then number B, apply trigonometric ratios to solve mathematical problems in the real world, like I said. (P13:0016)</i>	The members of the research team commented on the thinking which the real life situation caused in the following way: <i>What I really liked is the way that they were thinking. You could really see that they were thinking very hard. And being not in a school for quite some time now, that is quite amazing to see that they are trying so hard to come up with a plan, where...(P14:0117)</i>
<i>Go from particular to general.</i>	One of the members of the research team anticipated that learners would have wrong answers if the function is not connected to the angle when going from the particular to the general. <i>If you don't connect that, because sine cause there are functions and they are acting on something. If you don't make it explicit to the learners these things are kept on something, then they would have wrong values for these functions because they are making the wrong connection to the function. (P:086)</i>	The lesson started with a particular real life situation and then went on to the revision of the more general trigonometric functions: <i>So we need to start with revision. Now let us right away get into revision. Just to see how much we know about trigonometry. Now, if you remember very well, which type of triangle is usually used to solve trigonometry ratios? (P13:0028)</i>	Teacher reflected on his teaching saying that he needed to adjust the planning of the lesson to include other sides as well, and concretize the work more. <i>But we concentrated only on solving it, maybe looking for the Cosine of that side and then thereafter getting the answer and that's finished. But while I was seated and thinking about it, I thought of maybe it will be better if we can even show the learners, even if we are not in a hurry to go to the correct answer, showing them that even there are other sides that we can still calculate. But that does not make the problem... that... that solution correct. So that they should learn from the wrong and get to the right solution...okay...of the problem. (P14:0009)</i>

Categories (Quinlan, 2004:20 adjusted)	Focus group discussion three (P6) (Before lesson four)	Lesson observation four (P13)	Focus group discussion four (P14) (After lesson four)
<i>Context of any new concept before technicalities, intricacies and mathematical jargon.</i>	The research team seemed to think that the research idea worked effectively in the previous lesson in order to create a context for trigonometric functions and should be done again. <i>Give those particulars, do some research. Then show you, they have, they are able to recognise the language of trigonometry. But using those guides, I would eventually contribute every day and so forth. (P6:083)</i>	The real life situation, in which the new concepts were embedded, created the context for the trigonometric functions. <i>We want to apply in our day to day life; how can we apply it? How can trigonometry help us in our day to day life? (P13:0020)</i>	One member of the research team was impressed with the way giving the context first got the learners thinking: <i>What I really liked is the way that they were thinking. You could really see that they were thinking very hard. And being not in a school for quite some time now, that is quite amazing to see that they are trying so hard to come up with a plan, where...(P14:0117)</i>
<i>The actions of the teacher should elicit favourable reactions from the learners.</i>	Think-time given to the learners by the teacher is anticipated by the research team to elicit constructive thinking from them. <i>Ja, I also want to say that during the lesson if the teacher gives the learner's time to think, he must still be, or managing the time, so that it is sort of constructive time then. And he must still be prompt and in that way I mean in that way he can give them guidelines or do your thing to think this. If he just say, like LD said, I'm going to give you 50 minutes to think. By the end of 50 minutes the period is over and maybe some of them were thinking about something else. So I think, that you just make sure, ja. (P6:085)</i>	Teacher F's action of giving his learners problem solving to do, elicited drawings from them that show their thinking: <i>The other thing that I mentioned to them, I explicitly mentioned that your drawing will show whether you understand the problem or not. So make sure that you understand the problem, then your drawing will also indicate that. So they started doing that. And then ultimately they came up with the correct answer. (P13:0125)</i>	Teacher F reflected that he did give them time to think first. <i>You see, in your class you ask a question on the learners. And then you have those who are... who already have the answer... who can get it faster. And normally what I've learned is that I must not be in a hurry to say yes. You must give them a chance to think. (P14:0165)</i>

Discussion

From the lesson observation: This fourth lesson in the adapted lesson study (see Addendum D4 for the lesson plan) differed slightly from the previous three lessons in that it started with a real life situation in which the trigonometric functions had to be applied. The teacher wrote the real life problem on a chart in an attempt to save time. With the task right in front of them, the teacher asked one learner to read from the chart to the rest of the class. Another learner was asked to read the task. Learners were given A4 typing paper and markers per pair to make the drawings to show. Not only was the real life problem on the poster in front of them, but the instruction was also included for them to know exactly what should be done. In spite of the teacher asking them explicitly to read with understanding, it seemed that learners had a difficult time comprehending the real life problem. Almost the whole class drew only one triangle and did not manage to visualize the street with the two buildings on opposite sides. It was only when the teacher explained to some of the groups that they could grasp the picture of the street and two buildings. This activity took more time than we had anticipated. Language seemed to be the biggest challenge here as learners found it very hard to visualize the problem. Physical demonstration of the situation by using his hands and a ruler by the teacher resulted in some learners only grasping the picture then. Only then could learners think of the actual calculations within the problem. Two learners seemed to have solved the problem, and the teacher after a while asked them to explain their thinking on the blackboard to the rest of the class. The period came to an end with most of the learners not finished with this activity.

From the focus group discussion: From the table it can be deduced that the focus group discussions this time concentrated a lot more on what the members thought of metacognition. It appears as if the teachers had a fairly good understanding of what metacognition is which was important for metacognitive instruction. When the teachers discussed metacognition in focus group discussion four, one of the lecturers commented that conversation hinders thinking:

conversation is 'n steering vir... thinking." (Translation: ..is a hindrance for ...) (P6:035)

I think, metacognitive conversation is an end product of thinking. Like if you talk to somebody continuously, they are actually taking out the process of thinking that is taking place inside. (P6:038)

This was a very interesting comment as it raised questions on the usefulness of conversations in the classroom if metacognition has to be enhanced in the mathematics classroom.

7.2.2.4 Analysis of cycle four

Guiding principle one: *Go from the concrete to the abstract, while explicitly mentioning the outcome/s of the lesson. Avoid starting with definitions.*

This principle was planned in the focus group discussion prior to the lesson, to be adhered to by the group for the first time in the adapted lesson study. Although the group decided that the lesson should start with the real life problem, that is, really going from the concrete to the abstract, the teacher still started with revising the trigonometric functions.(P13:0028–P13:0060):

T *So we need to start with revision. Now let us right away get into revision. Just to see how much we know about trigonometry. Now, if you remember very well, which type of triangle is usually used to solve trigonometry ratios?*

L *Right-angled triangle.*

T *A right-angled triangle.*

L *Yes.*

T *Let me see anyone who wants to draw for us a right-angled triangle. Okay, V...?*

[student drew on board]

T *Okay, thank you very much, V... Is it right?*

Class *Yes.*

T *It's the right-angled triangle. Who can define for us a right-angled triangle? If somebody comes to you and ask you what is a right-angled triangle? What would you say?*

He justified his decision as follows:

You know what I wanted to do in the classroom? I wanted to start with the problem right away, you know. The real life problem right away. And then we deal with it. I was worried if they would really remember how to make the subjects of the room, all those things. Then I decided to do the ratio-part, although it took long. (P14:0994)

In the focus group discussion following the lesson in which lesson four was discussed the group members were of the opinion that it enhanced the thinking within the classroom.

Guiding principle two: Go from particular to general, allowing learners also to concentrate on a particular side and then let them discover the more general information themselves.

This second principle would have been adhered to if Teacher F had started with the application of the trigonometric functions in the real life situation. By starting with the revision of the trigonometric functions, teacher F moved the other way around, from the general to the particular.

Guiding principle three: Immerse learners in the context of any new concept before technicalities, intricacies and mathematical jargon.

Once again this principle could also not be applied in the fourth lesson as Teacher F did not succeed in creating the context in which he had to immerse the learners first before he could go into the mathematical jargon. Language in particular was a big problem because learners could not understand the problem for the most part of the lesson. It is not easy to create a context when learners are struggling with the language in the first place:

Maybe also the word problem was they had to read and try to understand. Analysing it could be a problem. As they draw the ladder, do they know it's supposed to be that way, because someone just drew it straight. So I think it was just the language problem, besides English. (P14:0069)

Guiding principle four: The actions of the teacher should elicit favourable reactions from the learners.

The international scholar (IS) commented on the reactions from the learners that he observed as follows.

That was something I noticed as well, especially I was focused on, I think it was mostly the women on the left side that were not raising their hands. But then when they were solving the problems they were very actively engaged. So clearly they were keeping up and they were thinking and they were motivated, but they somehow seemed like they weren't in the direct class. They were free and they felt they didn't have to, or maybe they were more shy. Do you have any hypotheses of why they were less...(P14: 0161)

Teacher F responded as follows to this question:

You see, in your class you ask a question on the learners. And then you have those who are... who already have the answer... who can get it faster. And normally what I've learned is that I must not be in a hurry to say yes. You must give them a chance to think. (P14: 0165)

In conclusion, the teacher was very aware of his thinking and of the thinking of the learners. He constantly tried to better the situation by thinking of other ways of making learners see the picture. It was clear that learners in this lesson did not read with understanding and the assumption can be made that they would never have correctly visualized the situation without the teacher guiding them.

As far as the learning of the learners is concerned, the aim in the next phase is to concentrate more on the understanding of the mathematical language within the word problem, and therefore the hypothetical teaching and learning trajectory was adjusted (Figure 7.4).

THE HYPOTHETICAL TEACHING AND LEARNING TRAJECTORY FOR TRIGONOMETRIC FUNCTIONS

Learning goal: To enhance the use of metacognitive skills and mathematical language for trigonometric teaching

Guiding principles (Quinlan, 2004:20 adjusted)

1. *Go from the concrete to the abstract, while explicitly mentioning the outcome/s of the lesson. Avoid starting with definitions*
2. *Go from particular to general.*
3. *Immerse students in the context of any new concept before explicating its technicalities and intricacies and mathematical jargon.*
4. *The actions of the teacher should elicit favourable reactions from the learners.*

Outcomes for the learners:

(i)	To know the basic concepts of triangle trigonometry
(ii)	To use trigonometric ratios to solve mathematical real-world problems

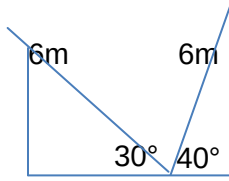
Content	Teacher Activities	Learner activities
Introduction Real world application A 6m long ladder with its foot in the street makes an angle of 30° with the surface of the street when its top rests on a building on one side of the street. The same ladder makes an angle of 40° with the street when its top rests on a building on the other side of the street while the foot of the ladder remains in the same position. How wide is the street (to the nearest meter)?	Teacher gave the problem to learners after they were handed charts and markers to make a drawing to show their understanding of the problem. Instructions: • Work in pairs • Make a drawing • Show your drawing • Solve the problem Teacher had to move between the pairs and assist where needed. Teacher guided learners toward correct drawing and correct calculations When pairs had finished, teacher asked one pair to come to the board and explain their solution to the class.	Anticipated actions: Learners might make the correct drawing  and use the correct ratio: Cosine theta: $\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ Learners understood that in order to answer the question which was that the width of the street needed to be calculated, two distances had to be found: First distance: $\cos 30^\circ = \frac{\text{distance}}{6m}$ $\text{Distance} = \cos 30^\circ \times 6m = 5.96152423$ Second distance: $\cos 40^\circ = \frac{\text{distance}}{6m}$ $\text{Distance} = \cos 40^\circ \times 6m = 4.596266659$ Learners understood the two distances should be added: $5.96152423 + 4.596266659 = 10.55779089$ Therefore the street is about 11m wide. Learners might have the drawing incorrect and use correct ratio
List of basic concepts: Hypotenuse side, opposite side, adjacent side, right-angled triangle $\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$ $\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ $\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$ Pythagoras theorem	Teacher revised the basic terminology by question and answer method Learner activities 1. Learners shares with class research done by them. 2. Learners have to explain in the groups the different sides 3. Learners have to identify the trigonometric ratios in the given triangle 4. Change the triangle Use of mnemonic as metacognitive strategy to help learners remember.	Learners raised their hands and respond individually Anticipated reactions from learners and subsequent teacher actions 1. Reveal specific knowledge prior knowledge and experience through oral presentation. If learners do not use mathematical language teacher to help them. 2. Learners reveal clear and coherent explanations of the sides. If learners do not reveal clear explanations of the sides reflecting the mathematical style and sophistication appropriate to the relevant mathematical level.

Figure 7.4: The Hypothetical teaching and learning trajectory for trigonometric functions

7.2.3 Analysis of the data collected in the second phase

The use of metacognitive skills and mathematical language within the conceptual frameworks for metacognitive instruction (Figures 2.5 and 2.6) is subsequently addressed.

7.2.3.1 Theme four: The Metacognitive Teaching for Metacognition Framework (MTMF)

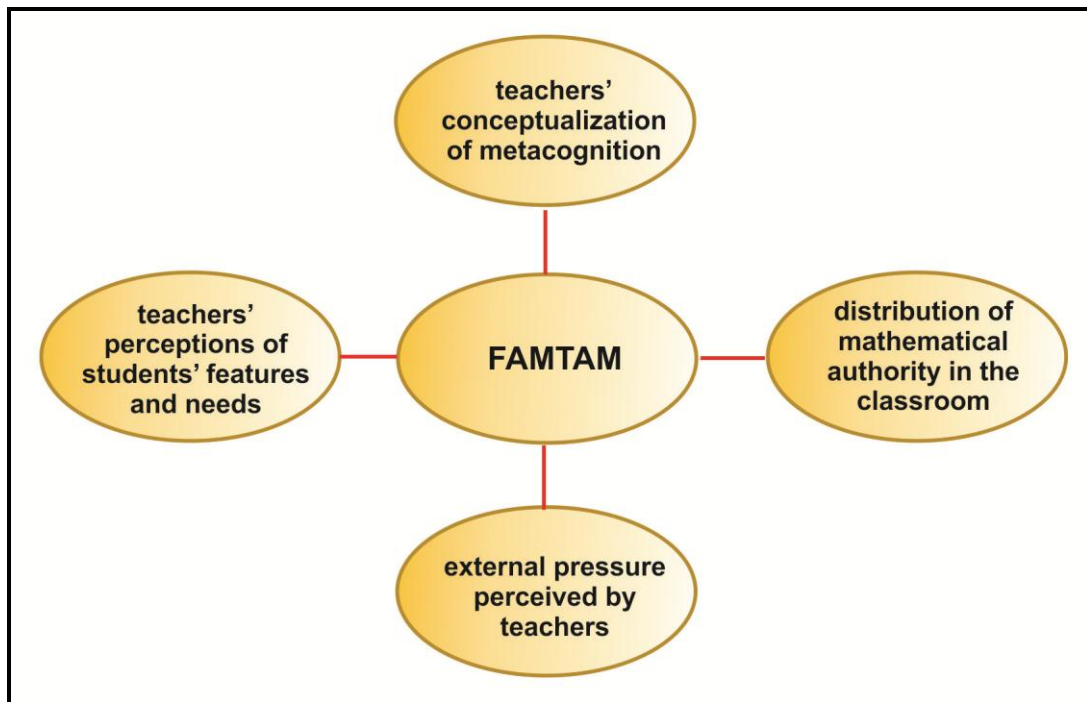
The overall impression of the two lessons presented in the second phase appeared to be that the thinking of the learners as well as that of the teachers was much more visible than in the previous two lessons: Teaching *with* metacognition goes hand in hand with teaching *for* metacognition.

I think in this one, learners are given more time to think, and to answer and to participate. Whereas the other clip, nê, part of the lesson was answering in chorus form, so they were more, given space of time for the learners to reflect and then to start asking questions. That is what I see. (P6:014)

The next section illustrates that all the components reflected by the framework representing Mathematics teaching *for* metacognition (FANTAM) as well as the components in the framework representing teaching *with* metacognition were present in each of the two lessons.

Teaching *for* metacognition

The framework for teaching for metacognition (Figure 7.5) was used to conceptualize whether Teacher C and F did teach for metacognition in lesson three and four.



Source: Ader, 2013: 24.

Figure 7.5: A Framework for Analysing Mathematics Teaching for the Advancement of Metacognition (FAMTAM)

Teacher's conceptualization of metacognition

According to McElvany (2009) the teacher should not only know the definition of metacognition, but should also have an understanding of the construct and the main characteristics of metacognitive processes, as well as have a good repertoire of metacognitive strategies. Teacher C displayed a fair amount of metacognitive strategies: For example, the teachers made use of charts to make visible the thinking of the learners; made the learners explain their thinking and gave them time to think. It appears as if Teacher C and Teacher F knew what metacognition was as they focused on the thinking of the learners: Teacher C used the word “thinking” about four times during lesson three while Teacher F used the word “think” no fewer than six times in the lesson:

“Lets acknowledge their thinking.” (P12:083);

“Right, I like, uh, I like, uh, in fact that way of thinking.” (P12:091);

“I just want to see whether we will be able to also adjust our thinking.” (P12:168)

You have to picture it first. You have to picture what is happening. Two buildings, street, ladder this side, ladder that side. Where’s the street? There’s the building, there’s the other building. Same ladder makes thirty degrees. Think about those angles. (P13:1012)

Distribution of mathematical authority in the classroom

For Ader (2013) mathematical authority refers to the responsibility for the evaluation of the mathematical work in the classroom. In traditional classrooms the mathematical authority usually lies with the teacher, but in the lessons of both Teacher C and Teacher F, the mathematical authority was embedded in the metacognition of the learners who were using knowledge, planning, monitoring and evaluating their own cognition while they (the learners) were listening to each other’s definitions of the adjacent, opposite and hypotenuse sides of the triangle. Teacher C commented once during the lesson:

“According to your definition which I didn’t give you was the one that give me that definition.” (P12:153)

Teacher’s perception of learners’ features and needs

Both Teacher C and Teacher F appeared to be well aware of the features and needs of their respective learners and displayed a certain amount of “sensitivity” (Ader, 2013) to their learners. During the focus group, for example, where his lesson was discussed, Teacher C commented as follows about his learners:

“Generally, uh, with the class, they are very determined and uh, I actually enjoy teaching them. They are not a difficult class.” (P6:096)

External pressures perceived by teacher

According to Duffy (2005), there are a myriad of factors that can influence the teaching that is done daily. The two lessons presented in this second phase are no exception. Teachers C and F were pressured in the first place by time and secondly by the lack of understanding of the language of their learners, although Teacher C did manage to use the time fairly well. Teacher F used a lot of time on the revision of the basic concepts and trigonometric ratios:

I think it's a good connection that you make. But maybe it just took too much time. But, I mean, you must come up with a better way to... and a shorter way of giving... attend the basics that they going to work with; the mathematics. (P14:0149)

Time can be a factor which can prevent the teacher from thinking well while teaching:

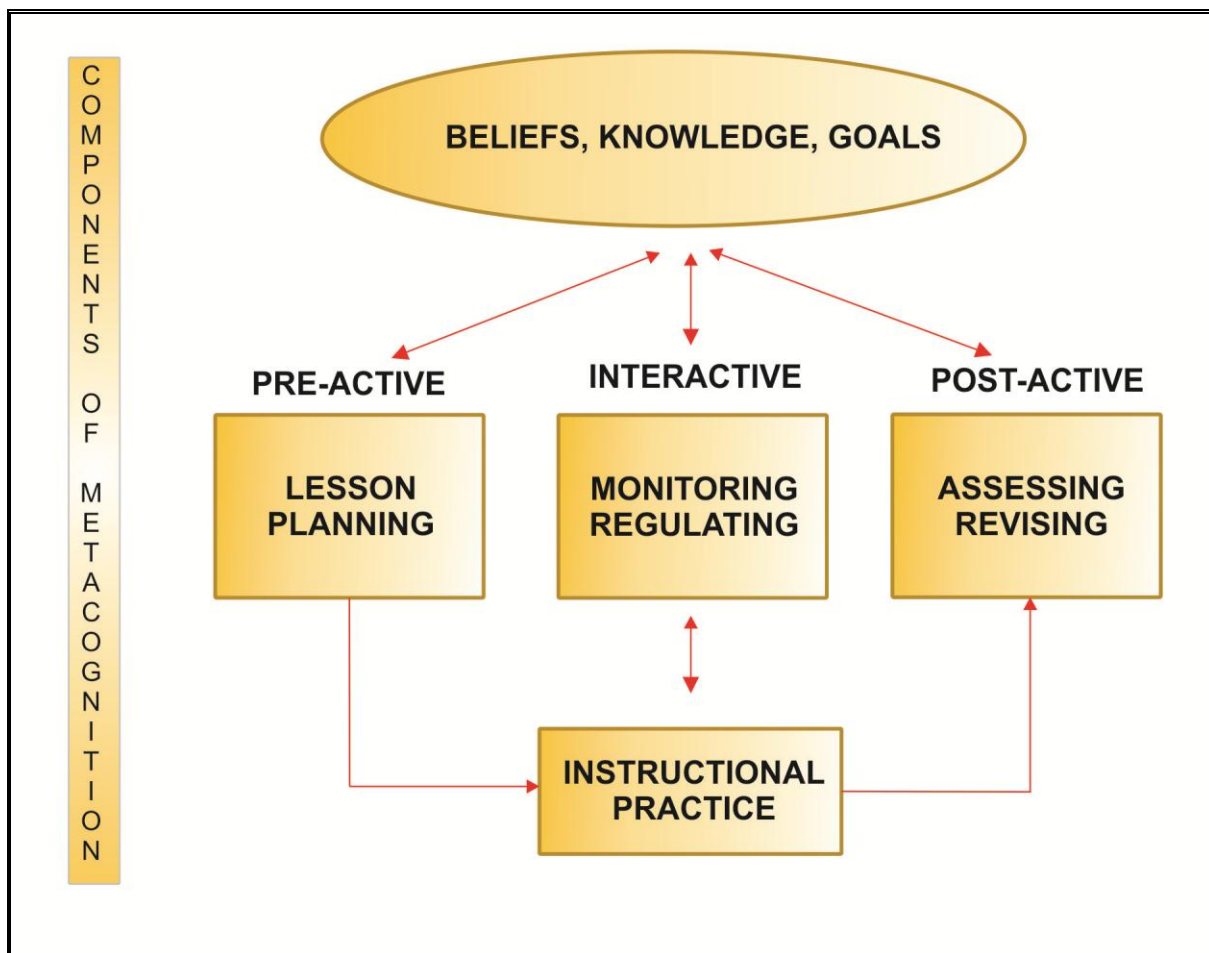
At that time, you as the teacher don't even think, because you want to push, your time is also up, against you, you want to cover, but when you look at them it is mostly yes. (P5:032)

Teacher C elicited good mathematical language from his learners, but they did not have the word problem that was given to the learners of Teacher F. The learners in Teacher F's class struggled with conceptualizing the problem. Although the problem was read to them twice, most of the learners drew only one building which illustrates their lack of understanding the language. Teacher F asked his learners to make a drawing of the problem which can be seen as a strategy of Teacher F to find out what the thinking of his learners was. The language however, made it difficult for the learners to understand the problem and only few of the learners reached the stage where they had to apply the trigonometric functions. The application of the trigonometric functions in the real life situation was therefore hindered by time as and language as external pressures which prevented the use of metacognition for the learners, which influenced metacognitive instruction negatively (Ader, 2013).

In conclusion, regarding, teaching for metacognition, all the above conditions in teaching *for* metacognition were adhered to by Teacher C and Teacher F. Therefore it can be concluded that Teacher C and Teacher F were indeed teaching the particular lessons *for* metacognition.

Teaching *with* metacognition

The framework for teaching with metacognition (Figure 7.6) was used to conceptualize whether Teacher C and F taught *with* metacognition as well.



Source: Artzt & Armour-Thomas, 2002: 130

Figure 7.6: A Framework for the examination of teacher metacognition related to instructional practice in mathematics

Beliefs, Knowledge and Goals

During the second focus group discussion, Teacher C verbalised his beliefs and his expectations of his learners when he commented as follows:

“I would love to see a learner who, when the introduction is given, they must be able to relate it to real life situations to see where can this, uh, this function, uh, in real life. (P5:036)

“As long as the communication is two way, uhm, of course the way the course is structured, from the teacher, from the learner to the educator and vice versa. Uhm, I also in fact, uhm, I also agree with the other speaker who said that the learners must be able to ask questions, they must also ...” (P5:063)

From his goals for the lesson and his participation in the focus group discussions, it appears as if Teacher C as well as Teacher F believed in what Davis and Sumara (2008: 171) refer to as coherence theories of learning while teacher C seemed to have participatory conceptions of teaching, which include synonyms for teaching like improvising, occasioning, conversing, caring and engaging minds. Teacher C and Teacher F both displayed good knowledge of the mathematics subject knowledge during the lessons presented by each of them.

Instructional practice

Pre-active: Lesson planning

Teacher C provided a lesson plan (see Addendum E18) that bore evidence of thorough planning of his lesson. It appeared as if teacher C had his lesson well-planned, as the lesson itself went smoothly; with each learning situation was dealt with quite professionally. Teacher F also provided the lesson plan (see Addendum E18) for the lesson and constituted what the group planned in collaboration for the lesson. The pre-lesson planning for Teacher C's lesson included a request to his learners to do research which shows boldness and confidence in his own knowledge. Other teachers avoid the Internet because they feel threatened by the extra knowledge sources that are available to learners. It appears as if Teacher C facilitated this session in which the learners brought knowledge not provided by him to the classroom very well, and made good use of it in order to create a context for the new area of mathematics he started with his learners. Teacher F, although he changed his initial planning, can be evaluated as using

his metacognitive skills in adjusting a lesson when he thought about his thinking again before finally presenting the lesson.

Inter-active: Monitoring and regulating

The time given to learners to actually think contributed greatly to the fact that the thinking was more visible in the lessons presented by Teacher C and F. It appeared that all members of the research team agreed that the learners in lesson three were encouraged to explain the reasons for their thinking and strategies to one another (Kriewaldt, 2009:5), first to the group and then to the whole class. During focus group three (P6) the lecturers were able to share their expertise and experiences with the teachers:

I still remember when I was still at school, many many years ago, I think we only started in grade 10, grade 12, standard 12 at that time. We used a graph paper, A4 page only, we all had to draw, I think let's say it was the 30-degree angle, that line. Then, we had on the graph paper it's now 1cm, 2cm and then let's say, at 1cm's we had to draw the particular line up there and we attach the other determine there, and that's now B1. And at 2cm's we get B2 and a certain X and Y every time there, and at 3cm's, 4cm's. So we actually had different sizes triangles but with the same 30-degree square. And every time we changed the, let's say the adjacent side, the opposite side also became longer and hypotenuse of course became longer. We had to, for all those different sizes of the adjacent side, we had to find the different ratios. And also it would show that everybody's is the same. And also for the same triangle. For the same angle, you get the same ratios, if it's opposite over they hypotenuse, doesn't matter how long it is, it is the same. (P6:073).

What the lecturer alluded to in the quotation confirms what Cavanagh (2008) tried out by drawing one diagram with various lengths of the different sides, as was discussed in Chapter Three (Figure 3.6). Further discussions resulted in teachers developing what Silverman and Thompson (2008) referred to as a *key developmental understanding* (KDU) as a powerful springboard for learning, which is a key element in the development of Mathematical knowledge for teachers (MKT). The forming of the concept of how changing the different lengths of the sides, brought about a new *opportunity for conceptualization* for the teachers as well as the lecturers.

There is something else that I was thinking of now. Also, even if it is a flat surface like either a tomography or smaller rectangle or whatever, so that they can draw diagonal. But everybody even maybe in a different size also, and then measure the sides, measure the angles, and then once they come to class with their data, then let every group then, ok there's their triangle that they are working with as their rectangle, and now they are going to find ratios. Because there, having different sizes of rectangles, of course the angle let's say the angle here is going to be different than everyone else's. And then you let them calculate the ratio from this side to that side. And then everybody's are going to be more or less the same. So you are going to ask them, all right, lets now give this thing then a name. This is where trigonometry starts. (P6:071)

Another contributing factor to the visibility of the thinking in lesson three and four was the use of paper, cardboard and koki's with learners writing up their thinking for everyone to see, and making a drawing in the case of lesson four. The teachers' facilitation of the group interaction enabled the learners to think critically. In this way learners', written thinking and spoken reasons were made known to each other.

Post-active: Assessing and Revising

Teacher C was **assessing** and **revising** the thinking of his learners, as well as his own thinking, constantly as can be gathered from the following quotation:

Right, lets look at the next definition. Right, the relationship between the adjacent angle, right, the adjacent, this one here you are talking about the side. Right, the relationship between the uh, the adjacent side and $\emptyset$. So, the group must correct that. The adjacent, that one is a side not a triangle. Right, the adjacent side is the base side of the and is next to angular theta. Right, and then we are told it is opposite to the hypotenuse. Lets look at that and see whether indeed they are right. Right, I only have sub reservations on the use of the base. Right, my other reservation is also on the fact that they say that this side is opposite to this. They are not necessarily opposite. Right, you are following? (P12:085)

Assessment and evaluation of his lesson occurred for Teacher F in the way he adjusted his strategy when he observed that learners, as he evaluated from going around amongst the groups, could not comprehend the problem. He then guided the learners, showing them using his hands and a ruler what the problem is about.

7.2.3.2: Theme Five: Chorus answering and metacognition

In the analysis of the transcripts of the five lessons, the total times the grade 10 learners answered questions posed by their respective teachers in a chorus was 267 against the 193 individual responses. “Chorus answering” in the context of this study, is understood as any non-individual response to any question from the teacher, which is characterized by the simultaneous verbal response by more than one learner in a particular class. This total indicates that chorus answering was the kind of response that the five teachers elicited from their learners more than half of the time while they were teaching. This frequency, together with the fact that literature on chorus answering is almost non-existent, probed me to look a little bit deeper into this phenomenon. In searching for literature on chorus-answering, I used the following terms: chorus answering, group answering, learner responses, student responses, and discourse in the classroom. Having been an educator for the greatest part of my life I am of the understanding that choir-answering has a negative ring to it. From table 7.2 and table 7.3 this issue was also discussed as one of the reactions elicited from the learners and was observed by one of the lecturers first as early as the second focus group discussion:

And sort of in a group also they answered. (P5:016)

Chorus answering enhances learner participation but misleads the teacher: Learners were described by the research team as participative, because of chorus-answering:

There were some questions that were asked from them. The teacher asked the learners a question and they, um, also participated. (P5:018)

The majority of the members within the research team were of the opinion that chorus answering sent a wrong message to the teacher:

What I wanted to say, sometimes when the teacher is asking questions, just check along there are others who don't know, if they answer in a group form, some of them would think that they know what is going on there, but if you look at the individual learners, you would find that some of the learners, they would not know. (P5:026)

...with the answer actually we are busy, we think everyone knows, because we are busy, we are looking at overall picture. (P5:029)

The other thing about group answer, is that it is, um, misleading the teacher, to think that they are following, whereas maybe it is one, two, three and then everybody has followed in a chorus. (P5:030)

Teachers allow chorus answering because their time to complete the syllabus is limited:

If chorus answering is misleading the teacher, the question remains why the teacher allows it, because although chorus answering was discussed in the focus group discussion, it continued to happen in the subsequent lessons. It appears as if the answer to this question is provided by the following quote:

At that time, you as the teacher don't even think, because you want to push, your time is also up, against you, you want to cover, but when you look at them it is mostly yes. (P5:032)

Teachers are most of the time pressured by the need to finish their syllabi, which forces them to work at a faster pace than they really should work.

Chorus answering obstructs the teacher's ability to evaluate the thinking of the learners.

With chorus answering, the teacher finds it hard to find out what the learners are thinking:

"I wanted to say with the It is difficult for the teacher to read their thoughts, and really, actually get what they are thinking." (P5:034)

The research group agreed on rather giving learners group work to do and getting one of the group members to share their answer with the rest of the class.

Let them discuss amongst themselves and then each group they can give their answer. It is better that way. (P5:229)

It must be discouraged. (P5:234)

The use of mathematical language in chorus answering

The fact that learners are indeed using mathematical language during chorus answering cannot be ignored. However, chorus answering makes it hard for the teacher to always know which learners do actually understand what they are saying, as some learners are merely “joining in” in what other learners, who know the answer, are saying. In their endeavours to make visible the thinking of the learners both Teacher C and Teacher F provided charts and paper for their learners to write up their thinking using mathematical language. Not only is Teacher C using mathematical language himself, but he is also giving his learners the opportunity to use mathematical language on the charts he provided for them. The groups first had to discuss their definition of what an adjacent side means; then they had to write it up on the chart and lastly one of the group members had to explain the group’s thinking to the whole class:

Right, I want to see your definitions. So, you must display your charts facing the chalk board. Right, the first group I can see here they are say the adjacent side it is next to the $\emptyset$ sine theta. Or it is next to angle theta. Next, right, lets try to analyze that. There is theta. Right, everybody is watching there. Right, this is your theta and then the, uh, if fact, the side that is next to this theta is the adjacent side. Right, can we agree with that group? (P12:081)

In sharp contrast with the first two lessons, where Teacher A and B made mostly use of content language which was more informative of nature, it appears as if Teacher A and B elicited chorus answering more often than the other three teachers. Teacher C and F were using more academic language (See Primary document 12 and 13 in the HU). The actions of Teacher C and F during their lessons were also more facilitative than the talk and chalk approach followed by Teacher A and B. Learners attending lesson three and four also had more opportunities to use mathematical language than the learners who had to listen all the time in lesson one and two.

Table 7.6 and Figures 7.7 and 7.8 show the main codes, sub-categories, categories used and themes which crystallized from the analysis of the data in this second phase, using ATLAS.Ti.

Table 7.4: Data analysis in the second phase

Codes	Sub-categories	Categories	Themes
Metacognitive action, conceptualization, conceptualization of metacognition. concept	Teacher's conceptualization of metacognition	Category six: Teaching <i>for</i> metacognition	Theme four: The Metacognitive Teaching for Metacognition Framework (MTMF)
Teacher expectation, incorrect solution,	Distribution of mathematical authority		
Knowledge of learners, knowledge of other,	Teacher's perception of learners' features and needs		
Time, language	External pressures perceived by teacher		
Explaining concepts, symbolic language, academic language, content language, mnemonic	Knowledge, beliefs and goals	Category seven: Teaching <i>with</i> metacognition	
Lesson, planning	Pre-active		
Monitoring, regulating	Inter-active		
Assessing, evaluating	Post-active		
Meta-memory, declarative knowledge, mnemonic, repetition, readong off from board		Category seven: Chorus answering enhances learner participation, but misleads the teacher.	Theme five: Chorus answering obstructs the use of metacognitive skills
Absence of outcome, talk and chalk approach, challenge, external pressure		Category eight: Teachers allow chorus answering because their time to complete the syllabus is limited.	
Chorus answering, individual response, drilling of concepts,		Category nine: Chorus answering obstructs the teacher's ability to evaluate the thinking of the learners.	
Content language, academic language		Category ten: The use of mathematical language in chorus answering.	

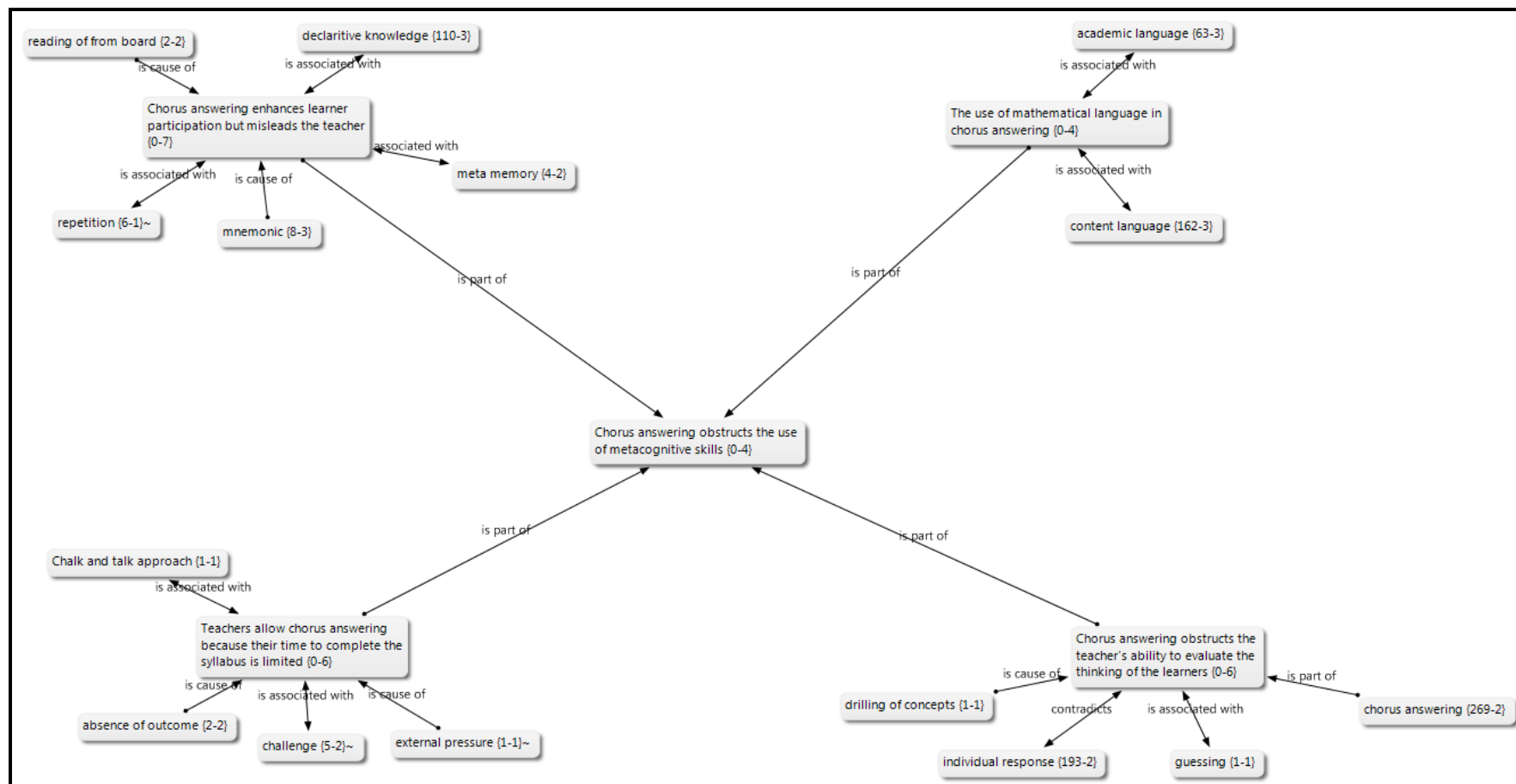


Figure 7.8: Chorus answering obstructs the use of metacognitive skills

7.3 THE THIRD PHASE: RETROSPECTIVE ANALYSIS

Only one final cycle was done in this last phase. Although theory building and data mining were tentatively done in the previous cycles, it was only in this last cycle that the hypothetical teaching and learning trajectory could be finalized. Having said that, it is acknowledged that this trajectory can be refined in consecutive cycles in future research endeavors.

7.3.1 Cycle five: Design experiment five: Data mining

This fifth cycle started with a discussion by the research team in focus group discussion four (§7.3.1.1), then the implementation of what was discussed by way of the lesson (§7.3.1.2), followed by focus group discussion five (§7.3.1.3). Although focus group discussion four was used in the previous phase as the reflection of what had transpired in lesson four, it is used here again because it also focused on the planning of lesson five. Focus group discussion four can therefore be regarded as the planning within cycle five while focus group discussion five would be the reflection on lesson five in this cycle.

7.3.1.1 Focus group discussion four: Designing

The last question from the questions for focus group discussion four (see Addendum D4) was developed towards the planning of lesson five:

3. *Now indicate how we can improve this lesson in order for learners to learn, focusing on the use of metacognitive skills and mathematical language.*

The suggestions by the members in the research team from focus group discussion four (Primary document 14) are subsequently reported on (Table 7.5) in terms of the design principles, metacognitive skills and mathematical language informing the design principles within the adjusted hypothetical teaching and learning trajectory for trigonometry teaching (Figure 7.4).

7.3.1.2 Lesson five: Data mining

Lesson 5 on Tuesday 30th July 2013 at the Red School

Presenter: Teacher D

Twenty-four (24) learners were sitting in single rows. Teacher D introduced the lesson by writing the two outcomes on the board. He then made a drawing on the board while asking the learners whether they recognized the drawing. It was the drawing of the well-known Eiffel tower in Paris. The learners initially did not seem to know, but Teacher D gave the clue that this tower is situated in one of the big cities in the world. This seemed to trigger the right answer as one could then hear some learners saying that it was the Eiffel tower. Teacher D then explained to the learners the angle of depression and angle of elevation using the tower which the learners seemed to understand. Teacher D then drew another sketch on the board. This time he explained about the wall and the ladder being the hypotenuse in the real life problem. He then asked which trigonometric function to use in order to find the angle. One learner answered wrongly and the teacher explained why the answer was wrong. After careful thinking, the same learner then answered correctly and Teacher D completed the sum on the board with the input from the learners. He warned repeatedly against chorus answering. After explaining the whole sum, the learners were handed a new real life situation problem of buildings on both sides of the street. Teacher D went through the problem with the learners. Then the learners were left to solve the problem.

Before starting, one learner needed to know where the work should be done. Teacher D explained that the sum should be answered on the given paper. Some learners underlined concepts while reading again the problem. Except for one learner, all learners were drawing pictures of only one building. Teacher D then decided to show concretely the buildings, using two learners for the two buildings and a board ruler as the ladder. Teacher D demonstrated how the ladder was leaning against one building and then again against the second building. After this demonstration most learners were drawing two buildings. Learners then struggled again with which trigonometric ratio to be used. One learner in front seemed to have been doing the problem correctly. Teacher D asked him to explain his thinking and how he got to the answer. The learner explained but decided wrongly that the ladder was in the middle of the street. This made Teacher D correct him, and he explained why we could not assume that the ladder was in the middle of the street. Table 7.5 reports on what transpired in the lesson in terms of the design principles and the use of metacognitive skills and mathematical language.

7.3.1.3 Focus group discussion five: Theory building

Focus group five at University: August 2013

This last focus group discussion was characterized by a sense of conclusiveness because the participants knew that this was the last time they would come together for this study. It was therefore filled with some emotion and a sense of gratitude and acknowledgement for one another's competency and professionalism that manifested out during the previous sessions of discussion. The questions for the focus group were therefore focused on reflection on the presented lesson as was in the case of previous focus group discussions, but also on how this professional collaboration was experienced by the teachers as well as the lecturers:

To the teacher who presented the lesson:

Please reflect on the lesson in terms of the following:

- Did you follow the lesson 100% as it was planned by the group?
- If no, please explain reasons for not following the lesson.
- Indicate the places where you have adjusted the lesson.

To the Group:

1. The planned outcomes of the lesson were:

At the end of the lesson learners will be able to:

- (i) Understand the basic concepts of triangle trigonometry;
- (ii) Apply trigonometric functions to solve mathematical and real-world problems.

Do you think the outcomes have been reached? Motivate.

2. Metacognition in its simplest form refers to **thinking about thinking**. Comment on the use of metacognitive skills used by:
 - (i) the teacher; and
 - (ii) the learners in the lesson.

3. Comment on the use of **mathematical language used** in the lesson. How can the use of mathematical language be stimulated in follow-up lessons?
4. You have been taking part in discussions with other mathematics educators around the teaching and learning of trigonometry. Please reflect on your experiences in this collaboration. Table 7.5 reports now on what transpired in this last focus group discussion.

Table 7.5: Reporting on focus group discussions four and five and lesson observation five

Categories (Quinlan, 2004:20 adjusted)	Focus group discussion four (P14) (Before lesson five)	Lesson observation five (P15)	Focus group discussion five (P16) (After lesson five)
<i>Go from the concrete to the abstract, while explicitly mentioning the outcome/s of the lesson. Avoid starting with definitions</i>	Members of the research team seemed to agree on the fact that the learners need to see the purpose of doing trigonometry: <i>...they actually have to see a purpose.</i> (P14:0257) <i>Ja, they must visualise this...</i> (P14:0269)	The teacher presenting this lesson started with the outcomes, and then focused on definitions: <i>And at the end of the lesson I am able to reach these outcomes. The first one is to understand... you must understand the basic concepts of triangle trigonometry. And then the second one: you have to apply the trig ratios and solve mathematical problems in real world. Right. Before we carry on with our lesson, let's try and define angles of elevation; you must try to find the angle of elevation and the angle of depression.</i> (P15:002)	On the request on assigning a percentage to rate the extent to which the outcomes have been reached in this last lesson, members of the research team agreed that the outcomes were reached to a great extent after the demonstration from the teacher in which the word problem was concretized. <i>I would say 50 per cent. And after the demonstration it went up to say 80 per cent, P16:029)</i> <i>From after the demonstration I can see that for that learner I would give 90 per cent. (P16:045)</i> <i>In terms of the diagram itself, it also presented a lot of problems to the learners. But I must also say that the educator was able to concretise everything and then from thereon learners were able to understand the nature of the problem. (P16:109)</i>
<i>Go from particular to general.</i>	Members of the research team were of the opinion that trigonometry, apart from going from the particular to the general, should be more practical. <i>Yeah, and where is the angle and what is the amount of the incline. To have it very practical, and now they have...They realise that if I have a protractor on the desk they can calculate the angle with only a ruler. And then you can abstract it to the ladder against the wall which they can't actually do. But to first... I'm a very practical person; I like to hold things and...</i> (P14:0257) <i>Do it, and draw it, and then...</i> (P14:0273) <i>I want to emphasise here that it should not be only the teacher who does who represents the real thing; it should be the students. Find something that you can... If</i>	Teacher D demonstrated practically the real life situation, using two learners, in order to make visible the situation for the learners. <i>Let me just give you a clue. [Calls student to the front] Let's ask you to come here. Just stand here. These are the two buildings. Are we clear?</i> (P15:434)	This lesson was more practical than the previous lesson: <i>I'm aware that the other class mates were also doing some calculations; measuring with rulers and protractors and all that. And, I'm not aware... I'm not so... The view of that one learner is the way the whole class did. But for that learner, basically, I would say he did well. (P16:053)</i>

Categories (Quinlan, 2004:20 adjusted)	Focus group discussion four (P14) (Before lesson five)	Lesson observation five (P15)	Focus group discussion five (P16) (After lesson five)
	<i>you don't have a ruler, then you put your pen there. (P14:0277)</i>		
<i>Context of any new concept before technicalities, intricacies and mathematical jargon.</i>	It appeared that the members of the research team suggested that the context of trigonometry be given first before the real life problem: <i>Given that they are busy... they have been busy with revision, why not give them first the context before the problem, and let them see now what... Okay, this problem looks familiar. Maybe I should use trigonometry. And then let them struggle with the modeling, drawing the situation. It doesn't need to be right, correct drawing. They can also explain via role playing so that you can see but where did they go about. (P14:0245)</i> <i>Without telling them first then okay now we are doing revision around trigonometry. Let them first realise, ah, this is trigonometry. (P14:0245)</i>	Teacher D gave the context of doing trigonometry by using the Eiffel tower: <i>This tower. Haven't you seen this tower? It's the first time you've seen this? Hmm? (P15:018)</i>	Teacher D admitted that it was hard for him to give the context first as learners made a lot of mistakes: <i>Uhm, I tried to follow this lesson 100 per cent but it was not easy, because the learners were just making mistakes. From what they write they only came up with one side of the diagram. So that's why I had to go and to explain to them by giving them... by just taking them to the class and then taking the ruler... [inaudible 00:01:39:28] ... tried to make some sense out of this. After that it was very... [inaudible 00:01:54:68]... trying to find it very easy; it was difficult for them. (P16:005)</i>
<i>The actions of the teacher should elicit favourable reactions from the learners.</i>	Teachers should give appropriate examples without spoon feeding the learners <i>I'm very much of the older type also, that I like... I don't want to spoon-feed them, but I like to give them like an appropriate example, you know. Because, as T... said, this one is quite difficult. And then.. so they start there. In the end before they solve the problem, everyone must understand, they must be able to draw that, or visualise that figure of that diagram. (P14:0297)</i>	True to the discussions in the previous focus group, Teacher D gave the learners an appropriate example first: <i>Right. Let's have an example. Let's come now to the main business. Example. Right, supposing we've got a ladder which is leaning against the wall. Let's take this as a wall. Are we together? (P15:090)</i>	The members of the research team were of the opinion that the actions from the teacher elicited favourable reactions from the learners. It elicited thinking: <i>I also liked the methodology that was employed by the educator where each individual was given the chance to think. They were not collaborating, so almost everybody was actually thinking. (P16:201)</i>

Discussion

From the lesson observation: From Table 7.7 it can be deduced that this last lesson appeared to have gone better than the previous lesson in terms of time management. More learners arrived at applying the trigonometric function to solve the problems than in the previous lesson. The mathematical language used by the teacher and some learners was also noticeably good. Giving examples first seemed to have worked to sketch the context of using trigonometry in daily life and provided a purpose of the teaching of trigonometry. Another feature that distinguished this last lesson from the previous lessons was the individual problem solving in which learners had ample time to think on their own while they were trying to solve the problem.

From the focus group discussions: Table 7.7 depicts the use of metacognitive skills as a prominent feature in the lesson during the focus group discussions. Focus group discussion five in which members of the research team had to evaluate to what extent the learning outcomes had been reached, illustrated that members were of the opinion that the outcomes were met to a reasonable extent with percentages ranging from 50% to about 90%. In both the discussions, members were in agreement that **the mathematical problem in the real life situation elicited good interaction and thinking from the learners:**

I like the idea with the real-life thing, you know, what you said. (P14:1222)

It's a beautiful problem, I love the problem. (P14:0329)

And you get a good response from your learner, you really get a good response from them. (P14:0830)

I think the whole time while they were correcting their work, and struggling and failing, and trying to... before giving up the teacher coming with this and they could draw it, and then after them thinking about which functions should they use, even though they thought wrong, but thinking about which function to use. And at least they remembered the functions. (P16:189)

I also liked the methodology that was employed by the educator where each individual was given the chance to think. They were not collaborating, so almost everybody was actually thinking. (P16:201)

Not only should the prior knowledge, sometimes referred to as the pre-knowledge of the learners, be revisited, but **the missing knowledge and misconceptions that the learners might have are of great importance and should be thoroughly addressed:**

And you started... You began with checking for prior knowledge and missing assumptions. That is also good. That was delightful to see. (P14:0345)

I wonder by the pre-knowledge, or the by-knowledge, if there one also can say if it was now Sine Theta, is AB over AC. You just emphasise there Sine, Cos and tan is always taken of an angle. Sine, Cos and tan on its own doesn't mean a thing, alright. So it has to go with an angle. The ratio that I get there – the AB over AC – remember, these are sides. So it's distances, it's lengths. It's not angles. (P14:0794)

7.3.1.4 Analysis of cycle five

The hypothetical teaching and learning trajectory for trigonometric functions (Figure 7.4) in this inquiry were developed and adjusted following the four principles (Quinlan, 2004:20) in five cycles of adapted lesson study.

Guiding principle one: *Go from the concrete to the abstract. Avoid starting with definitions.*

It was evident within this cycle that the purpose of doing trigonometry was a prominent discussion point and something to consider thoroughly when introducing trigonometric functions to learners. The outcomes of the lesson then also become critical because there should be a good relationship between the outcomes of the lesson and the purpose of doing trigonometry. Although Quinlan (2004) warned against the use of definitions in the start of the lesson, it sometimes becomes inevitable that definitions be used as the outcomes might consist of terminology which needs clarification. The use of Internet by learners, although it might be challenging for some communities, can be a good source of introducing this part of mathematics. The fact that trigonometry, and in particular trigonometric functions, lends itself so well to application in real life situations should be capitalized on in order for the learners to see the value of learning about trigonometry.

Guiding principle two: *Go from particular to general, allowing learners also to concentrate on a particular side and then let them discover the more general information themselves.*

The notion that the teaching of trigonometric functions should be more practical came very strongly to the foreground in this last cycle. The use of rulers and protractors to physically measure the sides and angles seemed to be of equal importance as the notion of going from the

particular to the general. Corral (2009) warns in this regard that approaching trigonometric functions too analytically is confusing for learners and makes much of the material appear unmotivated. The suggestion in the focus group discussion (§7.2.3.1) prior to this last lesson to use the triangle and measure different lengths (Cavanagh, 2008) was not applied by Teacher D.

Guiding principle three: Immerse learners in the context of any new concept before technicalities, intricacies and mathematical jargon.

Teacher D used the Eiffel tower as context for teaching the trigonometric functions. He admitted that he found it difficult to engage the learners as they were making a lot of mistakes. The ability to connect trigonometry, and for that matter trigonometric functions, viz. Sine theta, Cos theta and Tan theta to meaningful context is not that easy because of the abstract nature thereof. This is the reason why teachers are using mnemonics such as SOHCAHTOA which Mitchelmore (2000) refers to as the ABC approach where abstract definitions are taught to learners before they are given concrete examples.

Guiding principle four: The actions of the teacher should elicit favourable reactions from the learners.

The actions of Teacher D, which included letting learners work individually, indeed elicited favourable reactions in that learners were thinking. This thinking of the learners could be observed by the way they were engaged with trying to solve the mathematical problem of how wide the street would be if a ladder made two different angles with the street when placed against the wall of two different buildings at the opposite sides of the street.

I think that works; working alone it helps them to think, unlike working as a group one person getting all the credit. But as... They were all working and everyone was just busy. (P16:213)

The only disadvantage of working individually is that learners do not get ample opportunities to use mathematical language:

But of course that mathematical language is now problematic if they are working alone because they don't get the time to actually talk. (P:16:221)

In conclusion, Teacher D, as was the case with Teacher F in the previous lesson, constantly tried to adjust his teaching to enable learners to think more and harder in order to solve the

problem. It appeared as if Teacher D exerted metacognitive control during his lesson in line with what Pressley *et al.* (1998:347) advocate that teachers for instance should ascertain whether a specific learning opportunity has successfully been presented. Teacher D's monitoring informed him about the extent to which the outcomes of the lesson had been achieved and whether his chosen teaching strategies were suitable for the learning opportunity (§2.2.5.2).

7.3.2 Analysis of the data collected in the Third phase: Theory building

Although data in the previous two phases in this enquiry were analysed each time, formative analysis was employed. This last phase employed summative evaluation in line with what Van den Akker (2010) advocates. Van den Akker (2010) also suggests that this evaluation not only highlights the weak points of the intervention, but should make suggestions for improvement. The next section is therefore structured to include firstly the weak points of the Hypothetical teaching and learning trajectory, followed by suggestions for improvement (Table 7.6).

Table 7.6: Weak points and suggestions for improvement of the Hypothetical Teaching and Learning Trajectory for Trigonometric functions

Weak points	Suggestions for improvement
The mathematical language within the trajectory was not sufficiently addressed.	Mathematical language needs to be made thoroughly clear and exhibited in the classroom to include all possible terms and concepts that are anticipated during the presentation of the lesson
The use of other languages than English was not catered for.	The improved trajectory should include translations for the most prominent terminology that would be used in the trajectory.

7.3.2.1 The adapted lesson study

The following criteria were used to analyse the data generated by the five lesson observations in the adapted lesson study: Nature of the mathematical content, questioning, motivation and teaching strategies, homework, use of class time, verbal behaviour of learners, task orientation of learners, assessment, teacher expectations and stereotyping and lastly culminating observation which includes all the other aspects into one overall observation. These were all criteria that were used to find out how the teachers are using their metacognitive skills and the mathematical language when they were teaching trigonometric functions. These criteria are now

presented in Table 7.7 to illustrate how each one played out in the lesson observations done in this study:

Table 7.7: Observations in the five lessons for the adapted lesson study

Aspect	Lesson 1 Teacher A	Lesson 2 Teacher B	Lesson 3 Teacher C	Lesson 4 Teacher F	Lesson 5 Teacher D
Nature of the mathematical content	Trigonometric functions Finding missing angles and sides	Introduction to trigonometric functions	Introduction to trigonometric functions	Application of trigonometric functions into real life problems	Application of trigonometric functions into real life problems
Questioning	Yes or no questions that invited choir answering	Completing the question with less choir answering	Questioning invited some deeper level of thinking Learners were also given opportunities to ask questions at the end of the lesson which some learners used to ask teachers to explain some concepts again.	Wrong answers were explained by teacher	Learners explained their thinking
Motivation and teaching strategies	No real motivation and teaching in traditional way of chalkboard and talk. Whole class teaching Did a lot of examples but no independent work was required from the learners other than finding answers on their calculators.	Very much the same as in lesson one	Group work using charts to show the thinking of the groups	Peers teaming up; needed to show their thinking using a diagram	Much like in lesson four with the difference that individual learners working by themselves had to draw a diagram to show their thinking.
Homework	Homework was given at the end of the lesson when the bell announced the end of the period.	Similar to lesson one	Similar to lesson one	Similar to lesson one	Similar to lesson one
Use of class time	Used all the time with on-task behaviour	Same as in lesson one.	Same as in lesson one.	Same as in lesson one.	Same as in lesson one.

Aspect	Lesson 1 Teacher A	Lesson 2 Teacher B	Lesson 3 Teacher C	Lesson 4 Teacher F	Lesson 5 Teacher D
Verbal behaviour of learners	Chorus answering... Learners had very little opportunity to actually verbalise their thinking. Very hard to assess the mathematical language used by the learners	Chorus answering... Only opportunity was when they completed the sentences started by the teacher From the little opportunity learners got to verbalise their thinking it seemed that learners do not really know the mathematical terms and concepts that were used by the teacher.	Individual learners explains on behalf of all the members in his/her group. Group thinking required group leaders to explain the thinking done by the group. Assessing the mathematical language used by the learners was less challenging as learners got ample opportunity to voice the thinking done in groups. They appeared to be comfortable using mathematical language, although one learner's use of the concept <i>sublimation line</i> of which the actual concept in mathematical language is doubtful.	Learners were required to draw a diagram that to a great extent showed their thinking,	Individual learners were required to explain their thinking
Task orientation of learners	Listened passively to teacher Used their calculators when it is requested from them.	Similar to situation in lesson one, but with more involvement from learners required by teacher	Learners did research prior to lesson Learners shared their research with rest of the group. Very task-oriented,	Learners actively involved in solving the real life problem by drawing the diagram.	Similar to situation in lesson four
Assessment	No real assessment of understanding other than homework given at the end of the lesson	Similar to situation in lesson one.	Teacher assessed understanding right from the start when he is asking individual learners to share their understanding of what trigonometry is.	Understanding is assessed by a diagram	Similar to situation in lesson four but here learners also had to explain their thinking

Aspect	Lesson 1 Teacher A	Lesson 2 Teacher B	Lesson 3 Teacher C	Lesson 4 Teacher F	Lesson 5 Teacher D
Teacher expectations and stereotyping	Not much expectations by teacher from learners other than that learners need to listen to him and provide the answers from their calculators	Similar to situation in lesson one with little more teacher expectations that learners are able to remember the angles and the sides that were done already	Lot of teacher expectations. Learners were expected to do research prior to the lesson on their own. Learners were expected to work in groups and a member of the group reported back to class the thinking of the groups.	Much teacher expectations as learners were expected to work in pairs and come with correct diagram to illustrate the solution to the real life problem. Teacher expected from the learners to convert the words into a diagram.	Similar to the situation in lesson four with the exception that teacher expectations were now directed at individual performances.
Culminating observation	Teacher centred. Positivist-based Lesson was taught in the traditional way of talk and chalk.	Teacher centred Similar to lesson one but a bit more interaction from learners.	Learner centred This lesson took a much more inter-active approach.	Learner centred Still more action in pairs by learners	Learner centred Much more interaction from individual learners.

Discussion

From Table 7.7 it can be concluded that teachers A and B in lessons one and two were following the traditional approach of talk and chalk which can be classified as the correspondence learning approach (§3.3.1), according to Davis and Sumara (2008), as these teachers were explaining, telling, informing and setting right which are all about the act of training or conditioning learners to respond in a certain way. The teachers in lessons three, four and five, on the other hand, seemed to have been using the coherence learning approach (§3.3.2) which allowed their learners to explore and investigate. Teachers C, D and F displayed a kind of teaching that showed traces of interpretivistic and participatory conceptions of teaching which hold that learning is not determined by teaching, but conditioned by and subsequently dependent on teaching (Davis & Sumara, 2008).

7.3.2.2 Theme six: Learning happened within The Mathematics Educators' Reflective Inquiry (ME'RI) group

Table 7.9 and Figure 7.9 show the codes forming categories and subsequently forming theme six in this inquiry.

Table 7.8: Data analysis in the third phase

Codes	Categories	Themes
Reflection-on-action, instructional practice, knowledge of learners	The teachers and lecturers shared the same goal	Theme six: The Mathematics Educators' Reflective Inquiry group (ME'RI)
Reflection-in-action, improving as teacher	The metacognitive skills of the teachers were enhanced	
Need, want	There is a need for continuing professional collaboration	

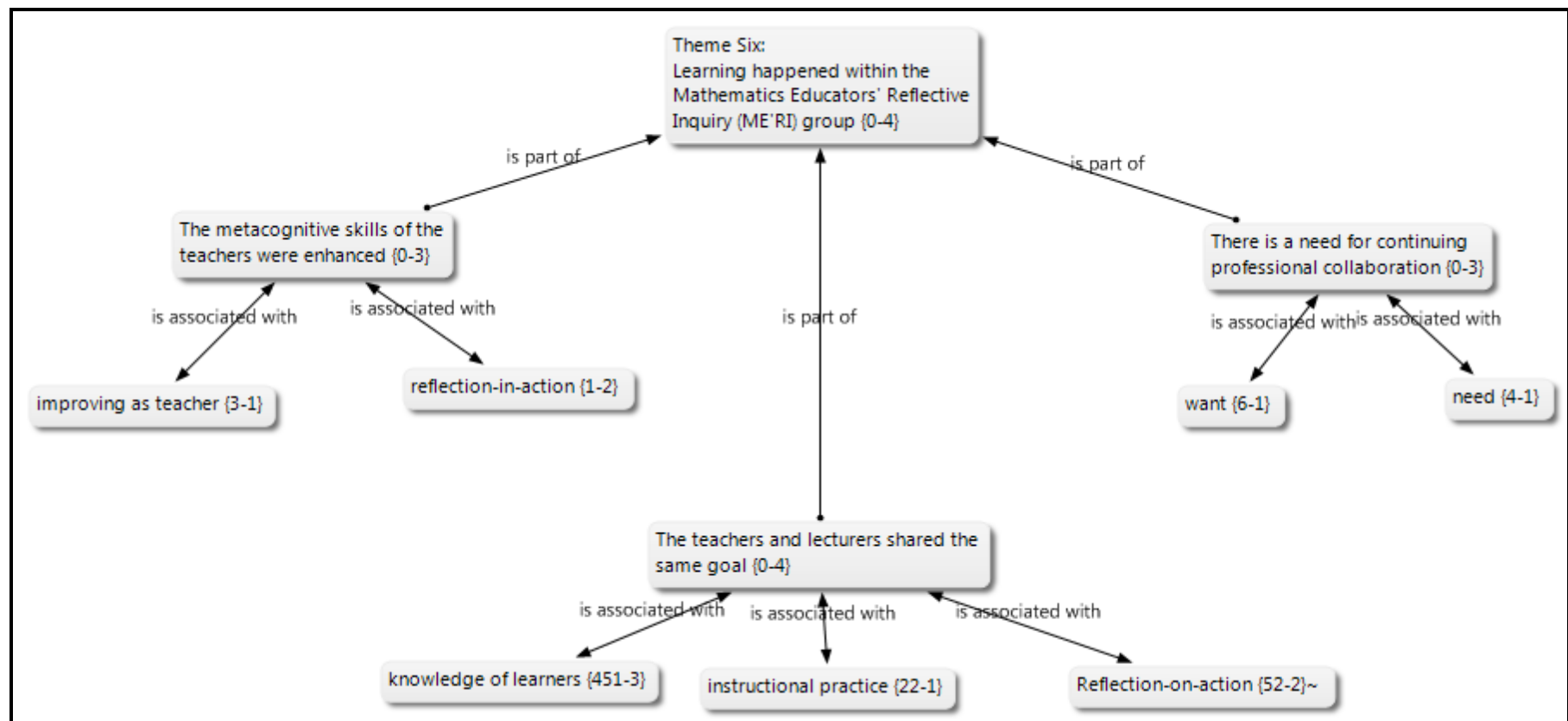


Figure 7.9: Learning happened within the Mathematics Educators' Reflective Inquiry (ME'RI) group

Synthesis:

The teachers and lecturers shared the same goal: The mathematics teachers and the lecturers has been working together for two years, pooling their competencies, experiences, knowledge and skills together towards one goal which was **to enhance the use of metacognitive skills and mathematical language for the teaching of trigonometric functions**. They shared ideas and experiences, tips of what works and what does not work, discussed amongst each other, debated and agreed with one another, laughed at and with one another and they even ate together. This collaboration was only possible because they were heading in the same direction, sharing the same goal, even when they had different backgrounds, different cultures, different perceptions, different training opportunities and different competencies (5.4.4). For Davis et al. (2008, p. 33) although perceptions are “constrained by culture”, perceptions are at the same time also “enabled by culture”. It was in this diversity that opportunities for learning lay.

And the other very important part that you said, for learners, I think, what is the main aim of teaching someone? It's for him to learn. I mean, I can teach and teach and teach the whole time; if he doesn't learn a thing, I wasted my time. And also, I've been doing it this way is really... Learners, I think, in the end they actually want to engage in this. And that's fun for them so that they learn without even knowing that they learn. So for me also it was a pleasure to have been part and also it was a very good experience for me. Thank you. (P16:358)

In this teamwork it taught me that after presenting a lesson one has to go back and sit and reflect, and ask yourself whether the intended lesson is being delivered – what I wanted the learners to know, did they capture it correctly? So, for me I've seen that I thought I was doing my very best, but after reflecting back I can see that I have done one, two, three, of which it is not covered. Then it is very much important every time for one to prepare thoroughly before you go and present a lesson so that you can close all the loopholes, all the gaps that might come up. (P16:338)

We learning from each other. (P11:054)

I mean we are learning from each other, sharpening our strategies on how to give our lessons. (P11:055)

The metacognitive skills of the teachers were enhanced: Teachers were of the opinion that they had become better metacognitive teachers and were now also able to teach in a way that elicited thinking from the learners. That means that the collaboration helped teachers to teach *with* metacognition and *for* metacognition:

I, for one, I also want to confess that from the ideas that I gained from the group I'm a better teacher now. And one thing that is very important is how

we were able to come up with a... in fact, the methods where the learner is actually being given a chance to think where he is actually learning rather than teaching. (P16:354)

From what we have been doing with this team here, it has learnt me always to reflect back to what I've been doing, especially after presenting the lesson. Whether or not I achieved the outcome of the lesson. Sometimes there is no time to reflect back, but we have to make provision for reflection on the lesson. (P16:346)

Since I started with this team, working together with our colleagues here on the issue of trigonometry, from the discussions that we've had, I want to just confess that, really, it was a wonderful experience. I've gained a lot and it was an eye-opener. The way I viewed things have changed altogether. That really shows that we really need each other, especially when it comes to, you know, different methods of teaching. You know, it's true when they say iron sharpens iron; one man sharpens another. I want to believe that in this present moment I am sharpened when it comes to trigonometry. Why? It's all of the views, the strategies, the methodologies that I really learned from my colleagues. I gained a lot, really, and I'm very much thankful for that. (P16:330)

There is a need for continuing professional collaboration: Teachers and lecturers within the ME'RI group experienced the group positively and expressed the desire and need to continue with this collaboration:

Okay, I also want to say I think, I'm definitely excited about having been part of this. And also, I learnt a lot. I mean, I actually can't wait to go back and teach again so that I can also teach this again. But I think also, exactly what you said, is I also learnt that if I... whatever I prepare to present, I actually have to reflect on that again. And the other thing I think is very important to also take from here, that even though we from different schools, I mean, us sitting at the university, we now all focused on teaching trigonometry for grade 10s, but there are many other concepts we are also going to teach now, and we can still, sort of as a team, you know, if you have to start to teach let's say linear programming or something, and that has always been a challenge to you or even if it's not a challenge to you, then you can share with the others. So I really think we must not have gained only on the trigono... triangle trigonometry; we must try and sort of also feel free then to contact another person, 'cause I mean, it's difficult to get together as a group again, but to get from other teachers and from us and us from you: how do we teach that then? You know? And maybe... how did you find it? How's it best to teach that? (P16:358)

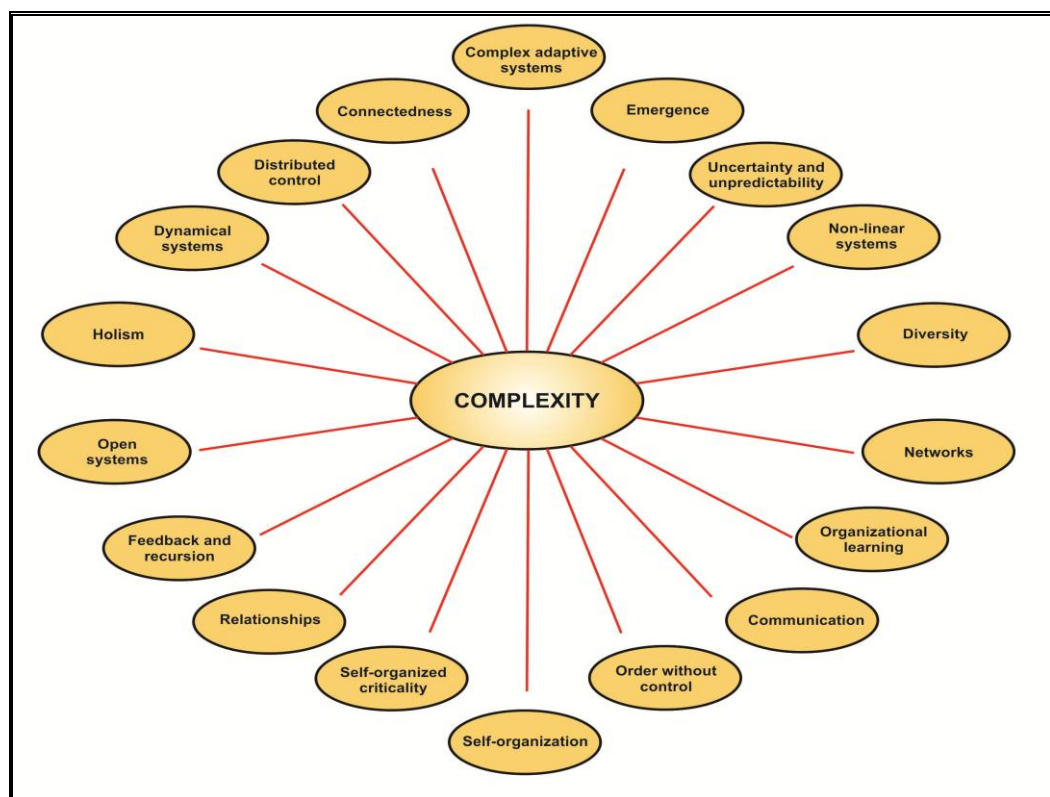
In conclusion it seems that the ME'RI group holds promise for productive interaction between teachers and lecturers in a way that could benefit the learner to a great extent.

7.4 COMPLEXITY AS LENS TO EXAMINE THE TEACHING/LEARNING OF TRIGONOMETRIC FUNCTIONS IN THIS INQUIRY

The complexity of teaching (Hiebert et al., 2002; Davis et al., 2008) was once again confirmed by the different lessons presented in this inquiry (Figure 7. 10) and brought to the fore by members of the research team during the focus group discussions:

Here teaching is another business. (P14:0646)

And teaching maths is not an easy task. (P14:0830)



Source: Cohen, Manion and Morrison, 2007: 2

Figure 7.10: Elements of complexity theory

Various complexivists, for example Ricca (2012), Davis and Sumara (2006) and Alhadeff-Jones (2008), amongst others, promulgate complexity theory (also referred to as complexity thinking or complexity science) as a new lens through which teaching methods can be examined. As was explained in Chapter Four (§4.5), the learning of the teacher as a complex learning system with several agents, viz. learners, researcher, lecturers and other teachers, and the ideas they created in this inquiry, was parsed using four elements of complexity theory (§4.5) in the next section. The order in which the elements are discussed is in no way indicative of the importance of one element over another. The elements must be seen as “all at once” (Ricca, 2012, p. 32) which is characteristic of a complex system, according to Davis and Sumara (2006). Like Ricca (2012), I am making no claim that these elements are the best ones or the only ones that can be used in a complexity approach; I acknowledge that other approaches, using other elements, are also possible. For me these four elements sufficiently describe complex systems in this study. The four elements (Figure 7.11) were selected to use in this inquiry because of their applicability in a social but professional collective, in which learning and change happen by the mutual influence of the one on the other.

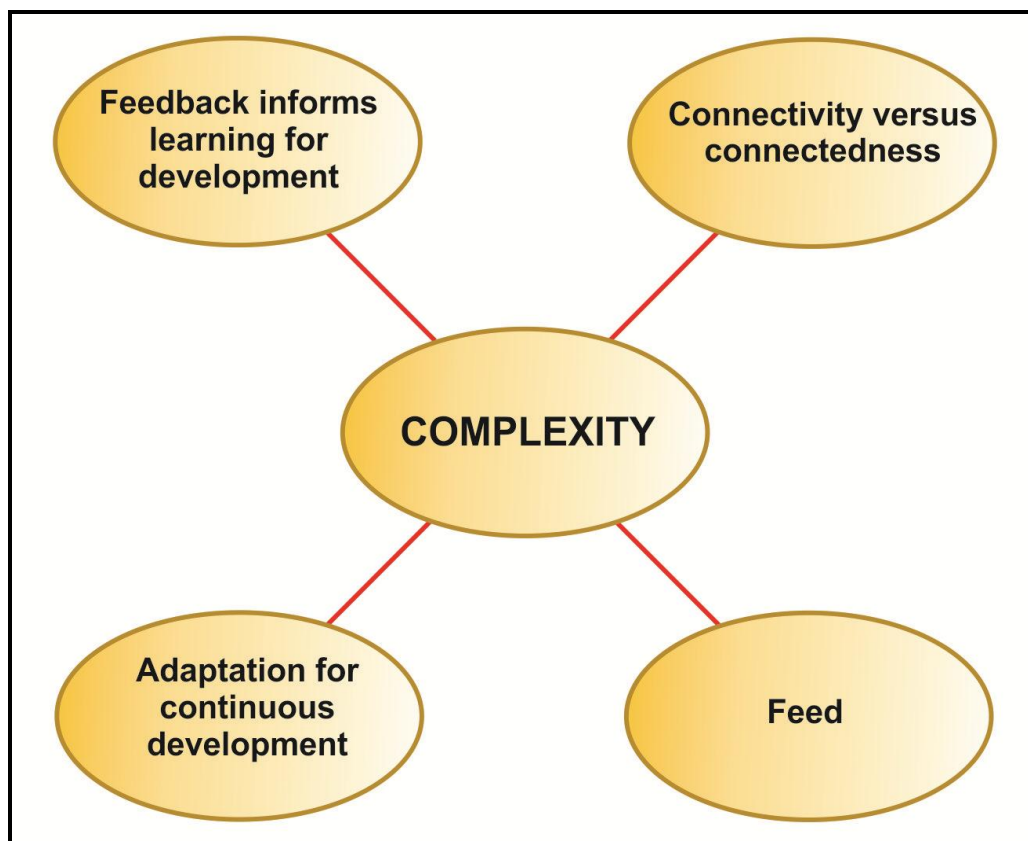


Figure 7.11: Complexity theory

7.4.1 Feedback informs learning for development

For Morrison (2006), feedback constitutes one of the key terms of reference for complexity theory. Feedback in the context of this study refers to the reflective action after a lesson in which the research team engaged into. Feedback and learning for development amongst the teachers in the research team can be seen in terms of proactivity (the planning in the focus group discussions), activity (implementation during the lessons) and reactivity (reflections in the focus group discussions) that happened within the research team.

Proactivity in this inquiry refers to all the thinking and subsequent actions, including the considerations that are taken into account by the teacher prior to the lessons. This thinking was informed by the metacognitive knowledge, metacognitive skills and metacognitive experiences of the presenting teacher, which were challenged by the feedback in the focus group discussions. The feedback from the teacher as well as reflection from the members of the group caused a disequilibrium (Davis et al., 2008) of existing knowledge which in most cases came about as a result of how the teacher was taught mathematics at school level, the training received by the teacher as well as the experiences as mathematics teacher accumulated over the years. It is this feedback that can be seen as catalyst that transforms the thinking of the teacher and brings learning to the mathematics teacher.

Activity in this inquiry refers to all the actions and thinking that the teacher exercised during the presentation of the lesson. Reflection-in-action is critical in the identification of the capacity to monitor own learning (Kriewaldt, 2009). Reflection-in-action indicates control of the learning process through the monitoring and regulation while the learning process is taking place (Minott, 2008). The monitoring and regulating of not only the teacher's thinking, but also that of his/her learners happens at the spur of the moment and are influenced by the uniqueness of the situation. This situation and what transpires during it cannot be replicated. The thinking of the teacher and the learners at specific moments is influenced by the complexities of the situation and learning is dependent on how the complexities play out during certain teaching episodes.

Reactivity refers to reflection-on-action which indicates an active process which helps the individual make sense of previous experiences with the objective of improving current and future thinking and actions (Kriewaldt, 2009; Minott, 2008). Reactivity in this inquiry includes all the actions and reflections of the teacher together with the reflections by the members of the research team which took place in the focus group discussions following the lesson.

7.4.2 Connectivity versus connectedness

Connectivity: The meaning of connectivity has a rather mechanical connotation: “Connectivity is the ability of a computing device to connect to other computers or to the Internet”. In this regard Davis *et al.* (2008) explain that complexity thinking prompts attention to the idea of a class as a *collective learner* rather than merely a *collection of learners*. A very popular feature of complexity theory is that the whole is the sum of the parts (Davis *et al.*, 2008). The teachers owned the lessons as *their* lessons. Although only one teacher presented the lesson, they were experiencing a connection with the teacher: This collegiality can be seen as ‘connectivity’, the quality of relationships which agents within a complex system possess (Ricca, 2012). For Davis *et al.* (2008, p. 11) “it seems that one’s immune system is related to oneself in the same way that the individual is related to the collective”. Ricca (2012, p. 37) explains this feature of connectivity further by stating:

It is true that individuals make up classrooms, which make up schools, which make up society; and so the individual can be found within the society. However, it is also true that society can be found within the individual.”

Connectedness: If one thing is **connected with** another, there is a link or relationship between them. The agents within the system are interconnected and exist in a special relationship with one another. This feature found exposition in the focus group discussions during which all members within the research team participated with the intention to embrace the inherent complexities of diverse forms in an acknowledgment that they cannot be reduced to one another.

7.4.3 Adaptation for continuous development

The notion of adaptation as the “zone of creative adaptability” (Lewin & Regine, 2000, p. 28) can be compared to Vygotsky’s (1978) “zone of proximal development”. Adaptability as the ability of teachers to change their ideas or behaviour in order to deal with new situations is intrinsically linked to their continuous development. It is a well-known fact that it is nearly impossible for a teacher to be taught everything that he needs to know during post school training. Therefore mathematics teachers need to engage in professional mathematics communities of learning if mathematics teachers want to become specialists in mathematics. Mathematics teachers need to reflect, investigate and improve their teaching practice by establishing a tradition of reflective and critical practice and collaborative growth with other mathematics educators. Continuous development happened in this inquiry in the collaborations between researchers and teachers which resulted in an academic inquiry group which I refer to in this study as the ME’RI (Mathematics Educators’ Reflective Inquiry)

group. Teachers were discussing ideas for instructional practice after reflecting on video recordings of lessons which they had been watching together, and as a collective of mathematics educators they shared and adapted ideas for improved practices.

7.4.4 The social construction of knowledge

Knowledge can simply be seen as that which a person might know, but unfortunately it is not that straightforward. According to Davis *et al.* (2008, p. 66), knowledge is not some sort of object locked in one's head. For Davis *et al.* (2008, p. 66) "knowledge is *embodied* (there must be an actor) and *situated* (there must be a context for the action)". Cohen *et al.* (2007, p. 34) maintain that the social construction of knowledge "argues for participatory, collaborative and multi-perspectival approaches to educational research". This collaboration happened in the activities of the ME'RI group in which teachers learn together, but the most important is that they have learnt for the benefit of the learner. Even though they were from different cultures, they connected with one another and constructed knowledge together. Davis *et al.* (2008, p. 33) maintain in this regard that:

Language and other cultural tools might be understood as more than means to select some aspects and ignore others; they are also strategies to connect experiences and to compress information so that we can cope with more. In this way language and other cultural tools enable our capacities for thought and perception even while shutting out most perceptual possibilities.

This social construction of knowledge within the ME'RI group included not only the construction of teacher content knowledge and pedagogical content knowledge (§3.5.1), but it also encompassed the construction of mathematical knowledge for teaching (§3.5.2). It appears as if the outcomes of this professional collaboration between teachers and lecturers benefitted both the individual teacher and the group (Davis *et al.*, 2008, p. 67) and holds promise for professional development of the mathematics teacher teaching in township schools. It also encompassed **conceptualization** as teachers were challenging each others' understanding of different concepts all the time. Conceptualizations skills were therefore challenged and enhanced.

7.5 SUMMARY OF THE CHAPTER

In this chapter the third sub-question was addressed. It was addressed using the HTLT in three more cycles during adapted lesson study in the second phase (§7.2) and the third phase (§7.3) of the inquiry. In the analysis of the second phase the last two themes which emerged from the data were discussed (§7.2.3), in which the conceptual frameworks was applied to form a new framework, the Metacognitive Teaching for Metacognition Framework (§7.2.3.1). This was followed by a discussion on chorus answering as obstacle for the use of metacognitive skills (§7.2.3.2) before an analysis of all five of the lessons in the adapted lesson study (§7.3.2.1) was provided. Lastly the teaching of trigonometric functions during the lessons and reflections thereof within this inquiry were examined through the lens of complexity theory (§7.4). The next chapter summarizes all the preceding chapters and provides the findings as well as the contributions and recommendations for further research endeavours in this field.