

SOLAR AND SOLAR-LIKE OSCILLATIONS

By

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of the North-West University

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Declaration

I, Oyirwoth Patrick Abedigamba, declare that the work presented in this thesis is my original work and has not been presented for any awards at this or any other university. Where other sources of information have been used, they have been acknowledged.

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CERTIFICATE OF ACCEPTANCE FOR EXAMINATION

This thesis entitled “SOLAR AND SOLAR-LIKE OSCILLATIONS”, submitted by Oyirwoth Patrick Abedigamba (student number 23271124) of the Department of Physics in the Faculty of Agriculture, Science and Technology is hereby recommended for acceptance for examination.

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Dedication

Dedicated to my late grandfathers, *Valente Thinu Owachi (Abaa pa Alak)*, *Marcelino Ochiba (Abaa Lino)* and my late young brother *Abedigamba Walter Ukurboth (Ukur)* who both passed on during the time when I started writing this thesis in 2013. May their souls Rest In Peace.

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This thesis makes use of (i) the irradiance data from the InterPlanetary Helioseismology by Irradiance (IPHIR) instruments on the PHOBOS 2 space craft and velocity data obtained from Birmingham Instrument at Tenerife. (ii) data collected by the Kepler mission. Funding for the Kepler mission is provided by the NASA Science Mission directorate. We wish to thank the Kepler team for their generosity in allowing the data to be released and for their outstanding efforts which have made these results possible. The data were obtained from the Mikulski Archive for Space Telescopes (MAST). STScI is operated by the Association of Universities for Research in Astronomy, Inc., under NASA contract NAS5-26555. Support for MAST for non-HST data is provided by the NASA Office of Space Science via grant NNX09AF08G and by other grants and contracts.

Abstract

In this thesis we study aspects of solar-like oscillations in the Sun and Red Giant stars. In the first part of the thesis, we re-calculate theoretical amplitude ratios and phase differences and compare with existing data in the Sun. Previous work to do the same was performed by Houdek (1996) where he used a pulsation code that includes treatment of non local convection and Eddington's approximation to radiative transfer (old code). Phorah (2007) improved on this code by replacing Eddington's approximation with radiative transfer by using the same non local convection theory (new code). Both codes show peaks in the luminosity amplitudes that correspond to depression in the damping rates. These were explained by Houdek (1996) as artifacts created by time dependent mixing length formalism and incomplete treatment of the non adiabatic effects. We also get similar value of the mean amplitude ratio of $0.2 \text{ ppm s cm}^{-1}$ with both codes in the frequency range of 2.5 - 4.0 mHz. Comparisons of the theoretical mean amplitude ratios obtained with the two codes to the observed data show agreement in the frequency range of 2.5 - 4.0 mHz. We conclude that there are no significant differences between the codes when theoretical results are compared with the observational data in a given frequency range.

In the second part of the thesis we use the median gravity mode period separation to search for Red Giant Clump (RGC) stars from a list of Red Giant (RG) stars in the *Kepler* field. The *Kepler* data used spans a period of 4 years starting in 2009. We construct echelle diagrams (plot of frequency versus frequency modulo large frequency separation) for some of the RG stars in NGC 6819, however, we are only able to identify 10 RGC single member (SM) stars in the *Kepler* open cluster NGC 6819. We measure the large frequency separation, $\Delta\nu$ and the frequency of maximum amplitude, ν_{max} for all the 10 RGC stars. We derive luminosities, radii, masses and distance moduli for each individual RGC star, from which we get the mean distance modulus of $\mu_0 = 11.520 \pm 0.105 \text{ mag}$ for the cluster when we use all the 10 RGC

stars with reddening from the KIC. A value of $\mu_0 = 11.747 \pm 0.086$ mag is obtained when uniform reddening value $E(B-V) = 0.15$ is used for the cluster. The values of μ_0 obtained are roughly in agreement with the values in the literature. A comparison of the observations with an isochrone of Age = 2.5 Gyr, $Z = 0.017$ with no mass loss using a statistical technique is made. A fractional mass loss of 7 ± 3 percent is obtained if we assume that no correction to $\Delta\nu$ between RC and red-giant branch (RGB) is necessary. However, models suggest that an effective correction of about 1.9 percent in $\Delta\nu$ is required to obtain the correct mass of RC stars owing to the different internal structure of stars in the two evolutionary stages. In this case we find that the mass loss in the red giant branch is not significantly different from zero. This finding is in agreement with the result of Miglio et al. (2012). It is clear that the mass estimate obtained by asteroseismology is not sufficient to deduce the mass loss on the red giant branch.

The same approach of using median gravity mode period separation was also applied to another open cluster NGC 6866. We have found that based on the value of median gravity mode period separation, ΔP , KIC 8263801 is a Secondary Red Clump (SRC) star. In literature, no classification for this star has been provided.

Publications from this thesis

Several papers have been published during the time the research for this thesis was undertaken. Listed below are journal papers coming directly from the thesis. The contents of these papers are incorporated in this thesis.

1. **OP Abedigamba**, LA Balona, R Medupe. (2016). **Distance moduli of open cluster NGC 6819 from Red Giant Clump stars**. New Astronomy NewAst. 46, 90-93 DOI: 10.1016/j.newast.2016.01.001.
2. **OP Abedigamba**. (2016). KIC 8263801: **A clump star in the Kepler open cluster NGC 6866 field?**. New Astronomy NewAst. 46, 21-24 DOI:10.1016/j.newast.2015.12.001.
3. **OP Abedigamba**, LA Balona, TR Medupe. (2015). **Red Clump stars in Kepler open cluster NGC 6819**. EPJ Web of Conferences, 101, 06001. DOI: 10.1051/epj-conf/201510106001.
4. LA Balona, T Medupe, **OP Abedigamba**, G Ayane, L Keeley, M Matsididi, G Mekonnen, MD Nhlapo, N Sithole. (2013). **Kepler Observations of the open cluster NGC 6819**. Monthly Notices of Royal Astronomical Society MNRAS. 430, 3472 - 3482. DOI: 10.1093/mnras/stt148.

List of Abbreviations

IPHIR	InterPlanetary Helioseismology by Irradiance
RG	Red Giant
RGC	Red Giant Clump
NGC	New General Catalogue
RGB	Red Giant Branch
RC	Red Clump
KIC	<i>Kepler</i> Input Catalogue
SRC	Secondary Red Clump
KASOC	Kepler Asteroseismic Science Operations Center
HR	Hertzsprung Russell
USSR	Union of Soviet Socialist Republics
BiSON	Birmingham Solar Oscillations Network
CCD	Charge coupled device
SDSS	Sloan Digital Sky Survey
UBVRI	Ultraviolet Blue Visible Red Infrared
2MASS	Two Micron All Sky Survey
ZAMS	Zero Age Main Sequence
MAST	Mikulski Archive for Space Telescopes
NASA	The National Aeronautics and Space Administration
FOV	Field of View
SAP	Simple Aperture Photometry
PDC	Presearch Data Conditioning
COROT	COncvection ROTation and planetary Transits
YREC	Yale Rotation and Evolution Code
WOCS	Wryn Open Cluster Study

List of Figures

1.1	<i>Surface distortion resulting from non-radial oscillations for modes with spherical harmonics (l, m) indicated on the left column. The $(0, 0)$ mode is a periodic contraction and expansion with time. The $(1, 0)$ mode is a wave moving from North to South and back again. The $(1, 1)$ mode is a wave rotating around the equator; the $(1, -1)$ mode is the same, but rotating in the opposite direction. The $(2, 0)$ mode consists of two waves moving from North to South and back in opposite directions. The $(2, 1)$ mode is a wave moving from North to South and another wave moving around the equator. The $(2, 2)$ mode consists of two diametrically opposed waves rotating round the star. Kindly illustrated and provided by L. A. Balona (private communication).</i>	5
1.2	<i>Propagation of sound waves in (a) and gravity waves in (b) in a cross section of a Sun-like star. This figure shows that the g modes are sensitive to the conditions in the very core of the star. Taken from Cunha et al. (2007).</i>	7
1.3	<i>The HR diagram showing instability strips for various pulsating stars, indicated by hatched areas. Taken from Aerts et al. (2010).</i>	8
1.4	<i>Schematic figure showing the different layers of the structure of the Sun.</i>	11
1.5	<i>A periodogram for the Sun showing localized comb-like structure with amplitudes which decrease sharply from a central maximum. Taken from Aerts et al. (2010).</i>	13
1.6	<i>In the top panel: Observational data of phase difference between irradiance and velocity as a function of pulsation frequency. Filled circle symbol is for coherence exceeding 0.7, open square for a coherence between 0.55 and 0.7 and + for a coherence between 0.5 and 0.55. In the bottom panel: The thick solid line is the running mean of the observational data both at low and high frequencies. For the dashed, triangle and other symbols, see Schrijver et al. (1991). Figure taken from Schrijver et al. (1991).</i>	17

1.7	<i>Observational data of amplitude ratios between irradiance and velocity as a function of frequency. In the insert, the thick solid line is the running mean of the observational data in the frequency range 2.5 - 4.5 mHz. Taken from Schrijver et al. (1991). Filled circle is for coherence exceeding 0.7, open square for a coherence between 0.55 and 0.7 and + for a coherence between 0.5 and 0.55.</i>	18
1.8	<i>Theoretical amplitude ratio between surface luminosity and velocity calculated at various heights in solar atmosphere as a function of frequency compared to observational data from Schrijver et al. (1991). The model and observations did not fit well in moderate frequency ranges, larger optical depths, and higher in the atmosphere. The thick, solid line indicate a running-mean average of the observational data. Taken from Houdek et al. (1995).</i>	19
1.9	<i>Theoretical phase shifts between surface luminosity and velocity as a function of frequency fitted with observational data from Schrijver et al. (1991) The model and observations did not fit well in moderate frequency ranges, larger optical depths and higher in the atmosphere. The thick, solid line indicate a running-mean average of the observational data. Taken from Houdek et al. (1995).</i>	20
1.10	<i>The different stages of post main sequence evolution of $1 M_{\odot}$ star in the HR diagram. Adapted from Carroll & Ostlie (2006).</i>	23
1.11	<i>An observational color - magnitude diagram showing greater density of clump stars in Kepler open cluster NGC 6819. The clump stars are marked in a circle, the purple dots are red giants with solar-like oscillations, the dark shadings are cluster member stars from Hole et al. (2009) and the lines are the theoretical isochrones. More information about Kepler open cluster NGC 6819 see chapter 4. Taken from Stello et al. (2011b).</i>	24
1.12	<i>A periodogram for a giant star KIC 6779699 showing localized comb-like structure (green dotted line in the Gaussian form) with amplitudes which decrease sharply from a central maximum. Vertical green dotted line indicates the location of the central maximum, $\nu_{\max}$.</i>	26

1.13	<i>A periodogram showing large frequency separation $\Delta\nu$ and small frequency separation, $\delta\nu$ for solar data. Taken from Christensen-Dalsgaard (2002b). The (n, l) are indicated for each frequency peak.</i>	26
1.14	<i>Theoretical mass-loss rates fitted with observations. Top left: points are the observational data while solid line is a fit using Reimers (1975) formula. Top right: points are the observational data while solid line is a fit using Goldberg (1979) formula. Bottom left: points are the observational data while solid line is a fit using Mullan (1978) formula. Bottom right: points are the observational data while solid line is a fit using Judge & Stencel (1991) formula. Adopted from Origlia et al. (2002).</i>	29
2.1	<i>The real part of $\delta r/r$ vs $\log p$ for (a) new code, (b) old code.</i>	51
2.2	<i>The real part of $\delta T/T$ vs $\log p$ for (a) new code, (b) old code. We observe numerical instabilities in the plot of $\delta T/T$ vs $\log p$ with both pulsation modeling codes in the range of $\log p = 5.0 - 6.5$.</i>	52
2.3	<i>The real part of $\delta P/P$ vs $\log p$ for (a) new code, (b) old code. Notice the little ‘bump’ seen in (a) and not visible in (b). The arrow indicates the position of photospheric layer.</i>	53
2.4	<i>Left panel: Real part of the surface luminosity eigenfunction versus pulsation frequency evaluated at the outer mesh point. The dashed line corresponds to the new code while solid line corresponds to the old code. Right panel: Imaginary part of the surface luminosity eigenfunction versus pulsation frequency evaluated at the outer mesh point. The dotted line corresponds to the new code while solid line corresponds to the old code. The arrows show the depressions.</i>	55
2.5	<i>Left panel: The norm (magnitude) of the surface luminosity eigenfunction versus pulsation frequency evaluated at the outer mesh point ($\tau \sim 10^{-4}$). The dashed line corresponds to the new code while solid line corresponds to the old code. There are peaks at frequencies 2.5 and 4.5 mHz in the left panel. Right panel: The velocity evaluated at a height of 200 km above the photosphere.</i>	56

2.6	Left panel: amplitude ratio at height $h = 200$ km above the photosphere. The dotted line corresponds to the new code while solid line corresponds to the old code. Right panel: phase shift at height $h = 200$ km above the photosphere. The dotted line corresponds to the new code while solid line corresponds to the old code.	57
2.7	The ratio of the amplitude ratio between the surface luminosity and velocity with the new code to the amplitude ratio between the surface luminosity and velocity with the old code, that is to say, A_{L-V} (Radiation)/ A_{L-V} (Eddington Approximation). Results for frequency range from 2.5 - 4.5 mHz (5 minutes range).	58
2.8	The phase shifts between the surface luminosity and velocity with the new code to the phase shifts between the surface luminosity and velocity with the old code, that is to say, φ_{L-V} (Radiation)/ φ_{L-V} (Eddington Approximation). Results for frequency range from 2.5 - 4.5 mHz (5 minutes range).	59
2.9	The theoretical damping rate as a function of frequency showing a depression at $\nu = 2.6$ mHz. The filled circles are data obtained from Libbrecht (1988). There is agreement between the old and new codes in the damping rate - frequency plot. . . .	60
2.10	Comparison of the running mean of the observational data of amplitude ratios between irradiance and velocity as a function of frequency with our model constructed with mixing length $\alpha = 2.0$, non-local mixing length parameters, $a = b = \sqrt{300}$ calculated at the height $h = 200$ km in the atmosphere. Solid line (Eddington approximation), dotted line (Consistent radiation treatment) and dash line is the running mean of the observational data.	62
2.11	Top: Theoretical amplitude ratio at height $h = 200$ km above the photosphere shown with the observational data of the amplitude ratios (Schrijver et al., 1991) in the 5 minutes range. The dotted line corresponds to the new code while solid line corresponds to the old code. Bottom panel: theoretical phase shift at height $h = 200$ km above the photosphere estimated shown with the observational data of the phase shifts (Schrijver et al., 1991) in the 5 minutes range. The dotted line corresponds to the new code while solid line corresponds to the old code.	64

2.12	<i>Comparison of the running mean of the observational data of phase shifts between irradiance and velocity as a function of frequency with our model constructed with mixing length $\alpha = 2.0$, non-local mixing length parameters, $a = b = \sqrt{300}$. Solid line (old code), dotted line (new code) and dash line is the running mean of the observational data.</i>	65
3.1	<i>Comparison of filter transmission in Johnson/Cousins (BVRI), the SDSS (ugriz) and the 2MASS (JHK) systems (left panel). Adapted from Bessell (2005). Right panel: The transmission response of the Kepler photometer. Taken from http://keplergo.arc.nasa.gov/CalibrationR</i>	
3.2	<i>The map showing the Kepler data for all the stars in the field of NGC 6819 (open circles) while filled circles are the RG stars with solar-like oscillations discovered by visual inspections of the periodogram and light curves (see Balona et al. (2013b) and this work).</i>	70
3.3	<i>The Kepler space craft with the photometer and the detailed field of view (FOV) on the right showing the position of all the CCD in the field. Taken from http://kepler.nasa.gov/</i>	71
3.4	<i>The distinction between RGC and RGB stars based on period separation for field stars, ΔP as shown by Bedding et al. (2011). The points with $\Delta P > 100$ s (red and yellow) are the RGC while points with $\Delta P < 100$ s (blue points) are the RGB stars. The solid lines are the theoretical lines calculated with the models indicating various masses.</i>	73
3.5	<i>The raw (uncorrected) light curve of stars KIC 6779699, 4902641 and 6928997 observed by Kepler space mission.</i>	77
3.6	<i>The corrected light curve of stars KIC 6779699, 4902641 and 6928997 observed by Kepler space mission.</i>	78
3.7	<i>The periodogram of KIC 6779699, KIC 4902641 and KIC 6928997 obtained after correcting for the drifts and jumps in the the raw (uncorrected) light curves of the stars. Comb-like structures are clearly seen in all the three periodogram, which are typical characteristics of stars with solar-like oscillations. The location of the frequency of the maximum amplitude is indicated in each plot as a vertical dash line.</i>	79

3.8	<i>Smoothed periodograms for the three stars we used to test our methods (each star name is indicated in each panel). The Figure shows the observed frequencies extracted from the periodograms in Figure 3.7 (the vertical dashed - green lines indicate the extracted frequencies). We used running mean approach to smooth the periodogram. The horizontal lines are the noise level limit.</i>	80
3.9	<i>Echelle diagram for the three stars constructed using the extracted observed frequencies and the large frequency separations. The modes $l = 0, 1$ and 2 are marked. The star names are also indicated.</i>	81
3.10	<i>Autocorrelation function for KIC 6928997 between $80 < f < 155 \mu\text{Hz}$, KIC 4902641 between $60 < f < 140 \mu\text{Hz}$ and KIC 6779699 between $60 < f < 120 \mu\text{Hz}$. The frequency ranges are the expanded view of relevant regions of interest. The y axis is $A(\delta f)$. Note that the plot is symmetrical around zero Hz.</i>	82
3.11	<i>Left-hand panel: a periodogram fitted with a Gaussian for the six identified clump stars in NGC 6819 cluster. The peaks in the periodogram are broad and messy as a result of oscillations which are stochastic. Right-hand panel: smoothed periodograms showing observed frequencies extracted from the periodograms from the left-hand panel. We used running mean approach to smoothen the periodogram. The horizontal lines are the noise level limit.</i>	85
3.12	<i>The distribution of gravity-mode period spacings, ΔP for the stars in Table 3.1. The vertical arrows indicate the ΔP chosen.</i>	87
3.13	<i>Comparison of ΔP obtained by Corsaro (2012) and this work with a slope of 0.934 ± 0.076. The data plotted are in Table 3.2.</i>	89
3.14	<i>Echelle diagram for some of the red giant stars in NGC 6819 constructed using the extracted observed frequencies and the large frequency separations taken from Table 3.2. The modes $l = 0, l = 1$ and $l = 2$ are clearly marked. In each plot the name of the star is indicated. In calculating the median gravity-mode period spacings, we used only points for $l = 1$ for which gravity modes dominate. To distinguish between $l = 0$ and $l = 2$, one needs to rely on the fact that $l = 0$ modes generally have higher amplitudes. Symbol sizes are proportional to the amplitude.</i>	91

3.15	<i>Echelle diagram for some of the red giant stars in NGC 6819 constructed using the extracted observed frequencies and the large frequency separations taken from Table 3.2. The modes $l = 0$, $l = 1$ and $l = 2$ are clearly marked. In each plot the name of the star is indicated. In calculating the median gravity-mode period spacings, we used only points for $l = 1$ for which gravity modes dominate. To distinguish between $l = 0$ and $l = 2$, one needs to rely on the fact that $l = 0$ modes generally have higher amplitudes. Symbol sizes are proportional to the amplitude.</i>	92
3.16	<i>Echelle diagram for some of the red giant stars in NGC 6819 constructed using the extracted observed frequencies and the large frequency separations taken from Table 3.2. The modes $l = 0$, $l = 1$ and $l = 2$ are clearly marked. In each plot the name of the star is indicated. In calculating the median gravity-mode period spacings, we used only points for $l = 1$ for which gravity modes dominate. To distinguish between $l = 0$ and $l = 2$, one needs to rely on the fact that $l = 0$ modes generally have higher amplitudes. Symbol sizes are proportional to the amplitude.</i>	93
3.17	<i>Echelle diagram for some of the red giant stars in NGC 6819 constructed using the extracted observed frequencies and the large frequency separations taken from Table 3.2 . The modes $l = 0$, $l = 1$ and $l = 2$ are clearly marked. In each plot the name of the star is indicated. In calculating the median gravity-mode period spacings, we used only points for $l = 1$ for which gravity modes dominate. To distinguish between $l = 0$ and $l = 2$, one needs to rely on the fact that $l = 0$ modes generally have higher amplitudes. Symbol sizes are proportional to the amplitude.</i>	94
3.18	<i>The distribution of gravity-mode period spacings, ΔP for some of the stars identified in Table 3.2. The vertical arrows indicate the ΔP chosen.</i>	95
3.19	<i>The distribution of gravity-mode period spacings, ΔP for some of the stars identified in Table 3.2. The two vertical arrows indicate the ΔP with the same number. In this case, it is difficult to specify the ΔP for such stars.</i>	96
3.20	<i>Theoretical isochrone for $\log t = 9.4$ and $Z = 0.017$, $Y = 0.30$ with mass loss rate free parameter η set to 0.0 calculated with Padova Evolution code (Marigo et al., 2008). The filled circles are the RGC stars while the open circles are the RGB stars. .</i>	104

3.21	<i>Comparison of $\nu_{\max}$ obtained by Corsaro (2012) and this work. The data plotted are in Table 3.2.</i>	111
3.22	<i>Comparison of $\Delta\nu$ obtained by Corsaro (2012) and this work. The data plotted are in Table 3.2.</i>	111
3.23	<i>Comparison of T_{eff} obtained by Basu et al. (2011) and this work. The data plotted are found in Table 3.12.</i>	112
4.1	<i>The map of the field showing the location occupied by the RG stars with solar-like oscillations in NGC 6866. Open circles are the 23 RG stars discovered by visual inspections of the periodogram and light curves Balona et al. (2013a). Filled circles are the clump and RGB stars identified in this work.</i>	118
4.2	<i>The periodograms of KIC 8263801, KIC 8329820 and KIC 8264074 obtained after correcting for the drifts and jumps in the the raw (uncorrected) light curves of the stars. Comb-like structures which are typical characteristics of stars with solar-like oscillations are clearly seen in all the three periodograms. The location of the frequency of the maximum amplitude gives what is known as the $\nu_{\max}$.</i>	119
4.3	<i>The periodograms of KIC 8196817 and KIC 8264079 obtained after correcting for the drifts and jumps in the the raw (uncorrected) light curves of the stars. Comb-like structures which are typical characteristics of stars with solar-like oscillations are clearly seen in the two periodograms. The location of the frequency of the maximum amplitude gives what is known as the $\nu_{\max}$.</i>	120
4.4	<i>Smoothed periodograms showing observed frequencies extracted from the periodograms in Fig. 4.2. We used running mean approach to smoothen the periodogram. The horizontal lines are the noise level limit.</i>	121
4.5	<i>Echelle diagram for the three stars constructed using the extracted observed frequencies and the large frequency separations. Vertical points running parallel and closer together are the $l = 0$ and $l = 2$ modes while points scattered and far away from the two parallel lines are the $l = 1$ mixed modes. In calculating the median gravity-mode period spacings, we used only points for $l = 1$ for which gravity modes dominate. Symbol sizes are proportional to the amplitude.</i>	122

4.6	Top panel: HR diagram for the RG stars identified by Balona et al. (2013a), their Table 7. The open circles are the RGB stars and filled circle is the RC star identified in this thesis while the crosses are the stars which we could not construct echelle diagram for and therefore could not assign them as RGB or RGC using median gravity mode period separations, ΔP . Middle panel: Calculated radius versus mass. Bottom panel: $\nu_{\max}/\Delta\nu$ versus $\nu_{\max}$ (symbols as in the top panel).	123
4.7	The distribution of gravity-mode period spacings, ΔP . Left panel: Histogram for KIC 8263801 which we have classified as SRC star. Right panel: Histogram for KIC 8264074 which we have classified as RGB star. The vertical arrows indicate the ΔP chosen.	124

List of Tables

2.1	<i>The input parameters used in the calculations of the solutions to pulsation equations. The columns are: (1) Mass, (2) Effective temperature, (3) Radius, (4 - 5) Non-local mixing length parameters a & b, (6) Mixing length parameter, α, (7) logarithm of surface gravity, $\log g$.</i>	49
3.1	<i>Results analysis and comparison with Bedding et al. (2011) work. The 2nd and 3rd columns are the results for $\Delta\nu$ and ΔP obtained in this thesis while the 4th and 5th columns are the results obtained by Bedding et al. (2011).</i>	86
3.2	<i>A list of RG stars in NGC 6819 in which we were able to construct echelle diagrams and there after calculate the median gravity-mode period spacings. Comparison is made with the values of the frequency of maximum amplitude, $\nu_{\max}$, the large frequency separation, $\Delta\nu$, median gravity-mode period spacings with the work of Corsaro (2012). The columns are: the Kepler Input Catalogue number - KIC WIYN OPEN CLUSTER STUDY - WOCS, Membership - Mem (Single Member - SM, Binary Likely Member - BLM, Binary Member - BM); 4th - 6th columns are the results in this work while 7th - 9th columns are from Corsaro (2012). The radial velocity memberships were obtained from Hole et al. (2009).</i>	90
3.3	<i>A list of RG stars in the field of NGC 6819. The columns are: the Kepler Input Catalogue number - KIC, WIYN OPEN CLUSTER STUDY - WOCS, Membership - Mem, the frequency of maximum amplitude, $\nu_{\max}$, the large frequency separation, $\Delta\nu$, estimated effective temperature, T_{eff}, the mass, radius and luminosity calculated from the solar-like oscillations, $M/M_{\odot}$, $R/R_{\odot}$, and $(\log L/L_{\odot})$. The majority of the RGB stars were those previously identified by Basu et al. (2011) as those on the RGB and by Stello et al. (2011a) as single member & not clump stars.</i>	98

3.4	Unreddened distance moduli, μ_0 , for RGC single members of NGC 6819 (Hole et al., 2009). Values obtained with A_V from KIC. Mean distance moduli of $\mu_0 = 11.520 \pm 0.105$ mag is obtained.	101
3.5	Unreddened distance moduli, μ_0 , for RGC single members of NGC 6819 (Hole et al., 2009). Values obtained with uniform reddening for all stars using the mean value $E(B - V) = 0.15$. Mean distance moduli of $\mu_0 = 11.747 \pm 0.086$ mag is obtained.	102
3.6	A list of NGC 6819 cluster age estimate by different authors. The columns are: the age (Gyr), methods used for determining the age, and the last column: authors.	105
3.7	Mass loss results by varying age with constant metallicity $Z = 0.017$, $Y = 0.30$. No correction to the $\Delta\nu$ scaling was applied to clump stars.	108
3.8	Mass loss results by varying Z , Y with constant age of 2.5 Gyr. No correction to the $\Delta\nu$ scaling was applied to clump stars.	109
3.9	Mass loss results by varying age with constant metallicity $Z = 0.017$, $Y = 0.30$. Correction to the $\Delta\nu$ scaling was applied to clump stars.	109
3.10	Mass loss results by varying Z , Y with constant age of 2.5 Gyr. Correction to the $\Delta\nu$ scaling was applied to clump stars	109
3.11	Summary of the results of mass-loss in red giants from asteroseismology of RGB and RGC stars using different ages and metallicities for NGC 6819. The fourth column shows mass loss derived directly from equation (1.3) and (1.5) while the last column shows the mass loss when a correction of 1.9 % is applied to $\Delta\nu$ for RGC stars.	109
3.12	Stars that were studied in this thesis as well as in previous studies by Basu et al. (2011) and Corsaro (2012). Column 4th - 5th (this work), 6th - 7th (Basu et al., 2011), 8th - 9th (Corsaro, 2012).	114

4.1	<i>A list of stars in NGC 6866 for which we were able to construct echelle diagrams and there after calculated the median gravity-mode period spacings. The columns are: the Kepler Input Catalogue number - KIC, Membership, the logarithm of the corrected effective temperature, the corrected effective temperature, the KIC luminosity, the calculated luminosity $(\log L/L_{\odot})_{\nu}$, the frequency of maximum amplitude, $\nu_{\max}$, the large frequency separation, $\Delta\nu$, the mass, radius and the period spacing calculated from the solar-like oscillations, $M/M_{\odot}$, $R/R_{\odot}$. M means the star which is a member while N is for non member.</i>	126
A.1	<i>Global list of solar parameters. Taken from Zombeck (2007), and Kivelson & Russell (1995).</i>	141

Table of Contents

Declaration	i
Certificate of acceptance for examination	ii
Dedication	iii
Acknowledgments	iv
Abstract	vi
List of Abbreviations	ix
List of Figures	x
List of Tables	xviii
1 Introduction	1
1.1 Introduction	1
1.2 Problem Statement	3
1.3 Stellar Pulsation	3
1.3.1 Driving mechanisms	7
1.3.2 Asymptotic Relations	9
1.4 Solar and solar-like oscillations	11
1.4.1 The structure of the Sun	11
1.4.2 Oscillations in the Sun	12
1.4.2.1 Some of the important results of helioseismology	13
1.4.2.2 Historical background of amplitude ratios and phase shifts	14
1.4.2.3 Brief description of the theoretical calculation approach	20

1.5	Solar-like oscillations in Red Giants	21
1.5.1	Red Clump Stars	21
1.5.2	Oscillations in red giants	24
1.5.2.1	Mass, luminosity and radius determination	25
1.5.2.2	Solar-like oscillations & mass loss at the tip of RGB	27
2	Modeling amplitude ratios and phase differences and comparison with data	31
2.1	Introduction	31
2.2	Equilibrium Model	32
2.2.1	Local and non-local description of convection	33
2.2.2	Radiative transfer in the atmosphere of the Sun	34
2.3	Pulsation Equations	35
2.3.1	Radial Oscillations in the Solar Atmosphere	36
2.3.2	Energy equation in the Solar atmosphere	40
2.3.3	Boundary conditions for consistent radiation treatment	45
2.4	Solutions to the pulsation equations	48
2.4.1	Explanation of the pulsation modeling codes	49
2.4.2	Running of the modeling codes, input and output parameters	50
2.4.3	Displacement eigenfunction	50
2.4.4	Temperature perturbations	51
2.4.5	Perturbations in pressure	53
2.4.6	Amplitude ratios (solar luminosity and velocity amplitudes)	54
2.4.7	Data and theory	58
2.4.7.1	Amplitude ratios and phase shifts data descriptions	58
2.4.7.2	Comparison of the theoretical results with the data	61
2.4.7.3	Limitations in the study of amplitude ratios and phase shifts	64
3	Red Clump stars in <i>Kepler</i> open cluster NGC 6819	67
3.1	The <i>Kepler</i> Input Catalogue (KIC)	67
3.2	Distance modulus	68
3.3	NGC 6819 Open Cluster	69

3.4	The data	70
3.5	Searching for clump stars amongst the red giant stars	72
3.6	Testing the software	75
3.6.1	Methods in the reduction of the <i>Kepler</i> raw data	75
3.7	Autocorrelation	82
3.8	Using echelle diagram to separate out different modes	84
3.9	Measuring period spacing (ΔP)	86
3.10	Using ΔP to distinguish between RGB and RGC stars	86
3.11	Special case - The distribution of gravity-mode period spacings	96
3.12	Selection of RGB and RGC stars	97
3.12.1	Distance determination and reddening	99
3.13	Mass loss estimation	103
3.13.1	Introduction	103
3.13.2	Summary of age estimate of NGC 6819 in the literature	103
3.13.3	Statistical techniques to study mass loss	104
3.14	Results and discussions	110
4	Search for RGB and RGC stars amongst solar-like stars in the open cluster	
	NGC 6866	116
4.1	Introduction	116
4.2	The data	117
4.3	Results and Discussions	125
5	Main conclusions	127
5.1	The amplitude ratios and phase shifts	127
5.2	<i>Kepler</i> open cluster NGC 6819 study	128
5.3	<i>Kepler</i> open cluster NGC 6866 study	129
	References	130
	Appendix A	141
	Appendix A1: The Global Parameters of the Sun	141

Appendix A2: Propagation of errors in Mass, Radius and Luminosity	142
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Chapter 1

Introduction

1.1 Introduction

The study of solar oscillations is important because it allows us to access the interior of the Sun in a way that is not normally possible because most parts of a star are opaque. Oscillations in the Sun are excited stochastically by convection. A study of oscillations in the Sun (also called helio-seismology) involves a comparison between theoretical models of the Sun and observational data. It is the study of seismic waves inside the Sun to infer the interior structure of the Sun. Most of the inferences are from low-degree modes in which global properties of the Sun like radius, mass and age have been determined. The models typically include fluid mechanical equations where energy transport mechanisms are included. By including the input physics of the models in such a way that one can match observations with the theoretical results, one can learn more about the physics of the Sun.

Some of the important results of helioseismology are its contribution towards solving the Solar Neutrino Problem - SNP (Bahcall & Peña-Garay, 2004), the determination of the depth of the Solar convection zone (Basu, 1998), and detailed determination of the Solar rotation rate profile in 3D (Schou et al., 1998) as further explained in detail in section 1.4.2.1.

Solar-like oscillations have been found in other class of stars. Stars in which the oscillations are excited in a similar way to that of the Sun are known as solar-like stars. For a star to show solar-like oscillations it must be cool enough to have a surface convective zone (Bedding, 2014). Red giants, cool subgiants, and stars on the lower main sequence (cooler than the red edge of the classical instability strip), show solar-like oscillations. The study of

solar-like stars falls under what is popularly known as Asteroseismology. Asteroseismology is the study of oscillations within pulsating intrinsic variable stars. In intrinsic variables the luminosity variations are caused by changes in the stellar interior. Chaplin & Miglio (2013) give the definition of asteroseismology as the study of stars by the observation of natural, resonant oscillations. Asteroseismology of solar-like stars has grown tremendously as a result of both ground and space-based (photometric) data being released and have been useful in determining mass, radius, luminosity and age of solar-like stars (Basu et al., 2011; Miglio et al., 2012; Balona et al., 2013b).

There are two parts to this thesis. The first part (chapter 2) contributes to helioseismology by matching stellar pulsation models with improved radiation transport mechanism to observed amplitude and phase differences. Previous attempts to do this by Houdek et al. (1995) and Houdek (1996) were met with limited success because of the limitation in the models used.

In the second part of the thesis we present a study of post main sequence stars whose oscillation properties are similar to the Sun (solar-like stars). We use data from the *Kepler* space mission to study solar-like stars in the field of the open clusters NGC 6819 and NGC 6866. In particular, using asteroseismic techniques, we search for Red Giant Clump (RGC) and Red Giant Branch (RGB) stars amongst Red Giant (RG) stars with solar-like oscillations.

This thesis started as an attempt to fit theoretical model to the data. We found our results to be very similar to the results available in the literature and this prompted us to study solar-like oscillations in red giant stars with *Kepler* data as in the second part of this thesis. Particularly, the study of mass loss in red giant stars is presented.

The outline of the structure of part I of the thesis are as follows: Chapter 1 focuses on the introduction and literature reviews on the amplitude ratios and phase shifts and solar-like oscillations. Chapter 2 is the theoretical modeling of the radial pulsations in the solar atmosphere, amplitude ratios and phase shifts together with the results and discussions on part I of the thesis.

The outline of the structure of part II of the thesis are as follows: Chapter 3 is on the Red

Clump stars in the open cluster NGC 6819. Chapter 4 is on the search of RGB and RGC stars in the open cluster NGC 6866 amongst solar-like stars while Chapter 5 is the main conclusion on the whole thesis.

1.2 Problem Statement

There are two main problems we seek to address in this thesis. The first one is to improve the matching of theoretical luminosity - velocity amplitude ratios and phase differences to solar data. This is a continuation of the work started by Houdek et al. (1995) and Houdek et al. (1995) where they found that they could reasonably fit upper atmosphere data, but failed to fit the lower atmosphere data. This poor matching to data was ascribed to inadequate modelling of radiation transport in the solar atmosphere. In this thesis we address this problem by using a pulsation code developed by Phorah (2007) where he used radiative transfer equation to model transport of radiation in solar atmosphere. The results of this and comparison to earlier models of Houdek are presented in chapter 2.

The second major problem we wish to address in chapter 3 of this thesis is mass-loss problem in RGB stars at the tip of RGB phase in open cluster NGC 6819. This is a problem because Red Giant stars are expected to lose mass on the RGB phase nearly all at the tip of the RGB phase. However, how much is lost remains an unsolved problem. We aim to address this problem by using statistical techniques where we define a goodness-of-fit criterion by applying χ^2 minimization to the observational data and stellar models. In chapter 4, we present a survey of RG (RGB & RGC) stars in the field of open cluster NGC 6866, since there have been no study in literature on the search of RG (RGB & RGC) stars in the field of open cluster NGC 6866.

1.3 Stellar Pulsation

A pulsation mode is a possible way in which a star can pulsate. About 10^7 distinct pulsation modes are thought to be excited in the Sun; of those over 250,000 have been identified¹. The

¹<http://www.stat.berkeley.edu/stark/Seminars/Aaas/helio.htm>

main reason for studying stellar pulsations is to learn about stellar physics by comparing the observed pulsation parameters such as frequencies, etc, with those calculated from a model. This is done by comparing the deviations between the observed and calculated frequencies. If the deviation is large, it tells us that our model is incorrect. If we believe the physics is correct, we can modify the model to match the observed frequencies as closely as possible, which gives us information on stellar parameters such as the mass, chemical composition profile, rotational profile, etc. This is called model fitting. A simple way of studying stellar pulsations is by introducing a small perturbation in the static equilibrium model and calculating how the physical quantities from point to point in the star vary with time. Pulsations are treated as linear perturbation to fluid dynamic equations that describes a star in equilibrium. A further simplification of the equations of fluid dynamics can be made by neglecting heat gains and losses. This is the adiabatic approximation which still allows pulsation frequencies to be calculated, but does not give any information about mode stability.

If magnetic fields are neglected and rotation is absent or is very slow (that is to say, the frequency of rotation is much smaller than the pulsation frequency), an approximation may be made which allows the pulsation solution to be separated into a radial part and an angular part. The angular part is represented by a single spherical harmonic, $Y_m^l(\theta, \phi)$, where the integer $l = 0, 1, 2, \dots$ is the spherical harmonic degree describing the variation of physical quantities in the co-latitude, θ , and m ($-l \leq m \leq +l$) is the azimuthal order which describes the variation of these quantities in longitude, ϕ . Physically, l (angular degree) describes the number of surface nodes that are present while m (azimuthal order) describes the number of the surface nodes that are parallel to lines of longitude. The angular variation of all physical quantities are therefore completely described by the two numbers (l, m) as demonstrated in Figure 1.1. In this figure, the changes in the surface as a function of time for different modes are shown. The radial variation of any physical quantity such as the perturbed pressure, $\delta P(r)$, is a solution of the linearized equations (the eigenfunctions) as will be explained in detail in chapter 2. The number of nodes in the radial direction are denoted as n (overtone or radial order).

Modes where the restoring force is pressure are called pressure modes (p modes) or acoustic modes. They are characterized by high frequencies and predominantly radial dis-

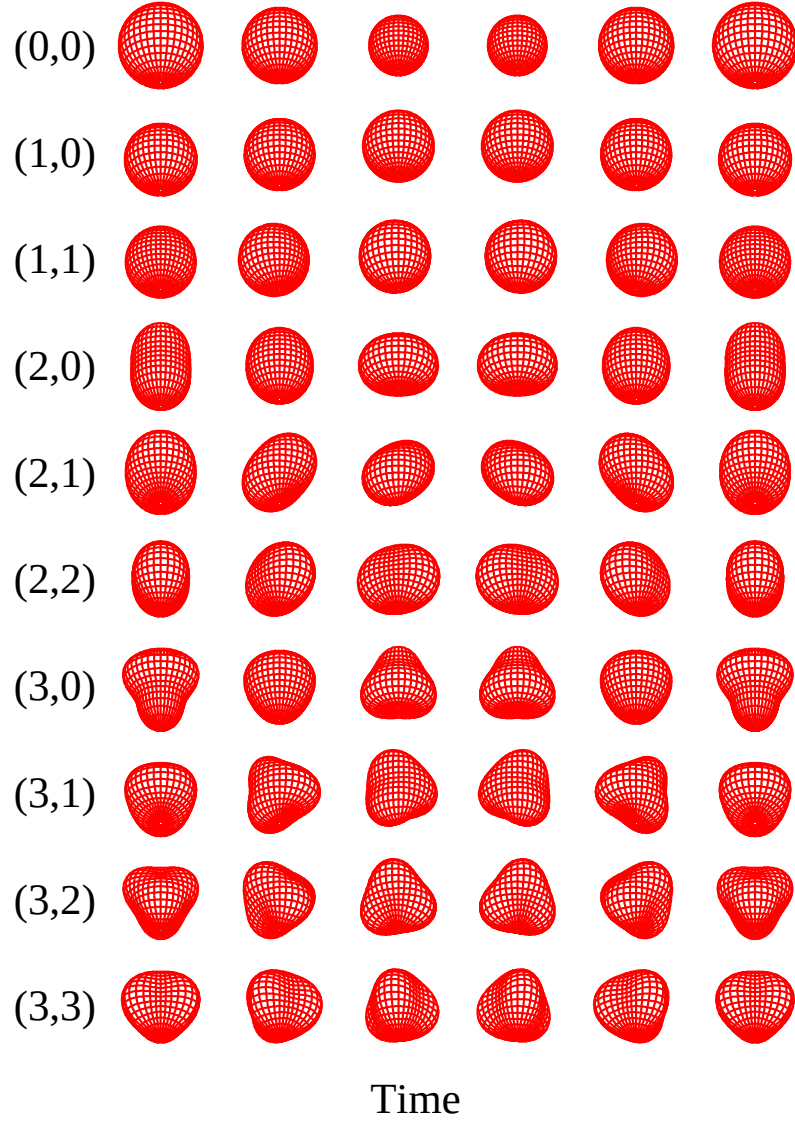


Figure 1.1: Surface distortion resulting from non-radial oscillations for modes with spherical harmonics (l, m) indicated on the left column. The $(0, 0)$ mode is a periodic contraction and expansion with time. The $(1, 0)$ mode is a wave moving from North to South and back again. The $(1, 1)$ mode is a wave rotating around the equator; the $(1, -1)$ mode is the same, but rotating in the opposite direction. The $(2, 0)$ mode consists of two waves moving from North to South and back in opposite directions. The $(2, 1)$ mode is a wave moving from North to South and another wave moving around the equator. The $(2, 2)$ mode consists of two diametrically opposed waves rotating round the star. Kindly illustrated and provided by L. A. Balona (private communication).

placements. Modes where the restoring force is mainly due to negative buoyancy are called gravity modes (g -modes) and are characterized by low frequencies (only for main sequence stars) and predominantly horizontal displacements. The f -modes are the horizontal surface waves and are very similar to ocean waves. There are special cases where a mode is never a pure p mode or a pure g mode but contains a mixture of p and g characteristics (Scuflaire, 1974; Deheuvels & Michel, 2010; Bedding et al., 2011). The reason why there is mixed modes in some stars is that the interior of a star is not uniform but contains discontinuities (such as the boundary between convective and radiative zones or a boundary between different chemical abundances).

For stars having an internal structure similar to that of the Sun, the p modes are most sensitive to conditions in the outer part of the star, whereas g modes are most sensitive to conditions in the deep interior of the star. In white dwarfs the g modes are sensitive mainly to conditions in the stellar envelope. According to Gough (1993), both p and g modes of high order can be described in terms of propagation of rays. An example is shown in Figure 1.2 (a) for a Sun-like star where p mode of various l values can travel in different parts of the stars and they create shallow and deep acoustic cavities. A wave travelling into the star is refracted as it travels through regions of high temperature and hence increasing wave speed. They refract back towards the stellar surface. The acoustic ray paths in Figure 1.2 (a) are bent by the increase in sound speed with depth until they reach the inner turning point (indicated by the dotted circles) where they undergo total internal refraction. At the surface the acoustic waves are reflected by the rapid decrease in density. Shown are rays corresponding to modes of frequency $3000 \mu\text{Hz}$ and degrees (in order of increasing penetration depth) = 75, 25, 20 and 2; the line passing through the centre schematically illustrates the behavior of a radial mode. The g -mode ray path (panel b) corresponds to a mode of frequency $190 \mu\text{Hz}$ and angular degree 5 and is trapped in the interior. In this illustration, it does not propagate in the convective outer part. The g modes are observed at the surface of other types of pulsators. In Figure 1.2 (b), g modes in solar-like stars are trapped beneath the convective envelope, when they are looked/viewed at as rays. The g modes can trace in the interior of the star and are mostly confined to the regions below the convective zone.

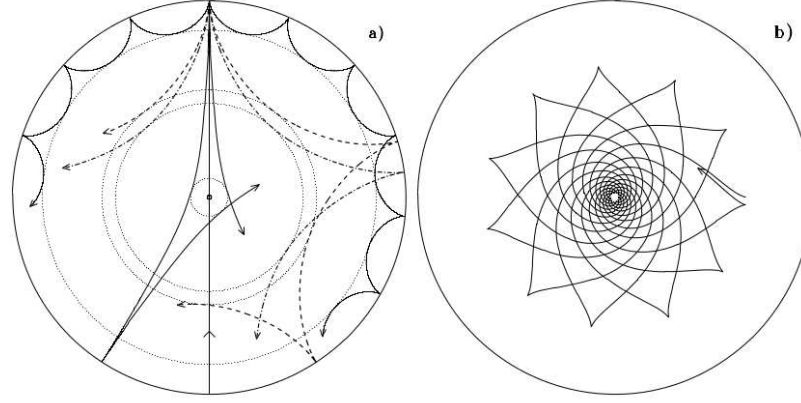


Figure 1.2: *Propagation of sound waves in (a) and gravity waves in (b) in a cross section of a Sun-like star. This figure shows that the g modes are sensitive to the conditions in the very core of the star. Taken from Cunha et al. (2007).*

1.3.1 Driving mechanisms

The first physical mechanism behind pulsation was suggested in 1926 by Eddington (1926) who called it the ‘valve’ mechanism. The idea was to see if a layer in the atmosphere releases heat during the compression stage or retains it. If the atmosphere retains it then the layer will contribute to the instability of the structure.

The opacity, kappa (κ) is the key factor which determines how radiation diffuses from the interior outwards. It depends on many parameters like the atoms involved, density, the wavelength of the radiation. At some depth into the star there is a zone, above which hydrogen is neutral and below which it is ionized (partial ionization zone) while at some depth, there is a zone where helium is singly ionized and, deeper, a zone where it is doubly ionized. Outside the partial ionization zone, if a star is compressed, it heats up, the radiation flow increases and the opacity actually decreases. The opacity scales as $\kappa = \rho/T^{3.5}$ (Kramer’s Law), where ρ and T are density and temperature respectively. This therefore means that for a given shell, more energy is lost at the upper level than is received at the lower part. This radiative damping quickly damps out pulsation. Stars excited by this kind of mechanism (κ) are Cepheids, RR Lyrae, and δ Scuti (Chevalier, 1971).

In the partial ionization zone, if a star is pulsating, as the star is compressed, the energy which

would normally heat the zone mostly goes into increasing the ionization. This increases the opacity of the zone, trapping more radiation and resulting in outward pressure. This gives rise to γ mechanism. The zone is then driven outwards, and cooling at the same time. Further cooling of the zone gives rise in recombination of the ionized material. If the zone is too deep in the star, it cannot drive against the overlying layers and if the zone is high, it has got nothing above it to drive. The location of the zone is critical in determining whether pulsation occurs and this explains why there are ‘instability strips’ in the HR diagram as shown in Figure 1.3. The instability strips are areas in the HR diagram where the stellar temperature is such that the driving zone is well located. Both γ and κ mechanisms are collectively called ‘heat mechanism’. Figure 1.3 is a HR diagram showing location of instability strips for various pulsating stars.

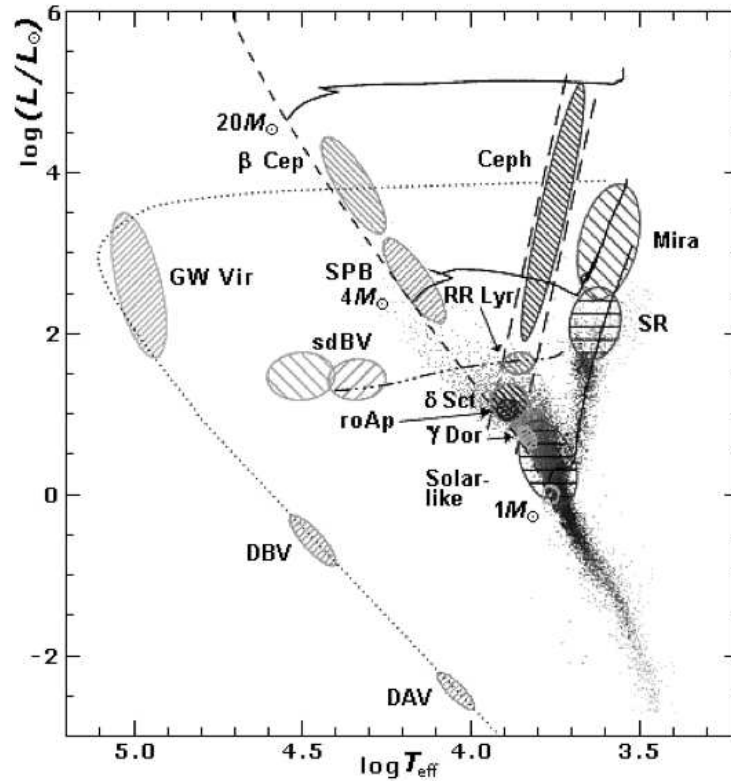


Figure 1.3: The HR diagram showing instability strips for various pulsating stars, indicated by hatched areas. Taken from Aerts et al. (2010).

For cooler stars located to the right hand side of the instability strip, the major energy

transporting mechanism is convection instead of radiation (this depends on the mass and the stage of the evolution of a star). Such kind of stars include the Sun, red giants and solar - type stars and the mechanism responsible for driving pulsation in them is *stochastic excitation* (Goldreich & Keeley, 1977; Samadi et al., 2008).

In γ Dor stars, the convective envelope is deep and extends well beyond the region of partial ionization of He II where the κ mechanism operates. Thus, it has been suggested by Guzik et al. (2000) that a different mechanism is responsible for pulsation driving in γ Dor stars. This mechanism which is known as ‘*convective blocking*’, extra heat is trapped at the base of the convective zone at maximum compression because there is not enough time for the heat to be transported to the top of the convective zone during one pulsation period.

In the white dwarf ZZ ceti and V777 Her stars, nearly all the flux in the envelope is carried by convection. In these stars, the convective eddies have a turn-over timescale which is much shorter than the pulsational period. Therefore at maximum compression heat is immediately absorbed by the convection zone. In these mechanisms heat is absorbed during compression and released during expansion, causing an instability in specific regions (ionization zones) of the star. Brickhill (1991) termed such a driving mechanism in ZZ ceti stars as ‘*convective driving*’.

1.3.2 Asymptotic Relations

The asymptotic relations are very important in pulsating stars where $n \gg l$. One can also use the relationship for interpreting results about the stars. The relations are applicable to both p and g modes. The following are the properties of p modes;

- (i) as the number of radial nodes increases, the frequencies of p modes increase.
- (ii) The p modes are most sensitive to conditions in the outer parts of the star.
- (iii) For overtones $n \gg l$, there is an asymptotic relation for p modes which states that they are approximately equally spaced in frequency (Tassoul, 1980, 1990).

In the first order asymptotic approximation, Tassoul (1980, 1990) showed that for p modes, the frequencies are approximately given by:

$$\nu_{nl} = \Delta\nu \left(n + \frac{l}{2} + \tilde{\alpha} \right) + \epsilon_{nl},$$

where $\Delta\nu = \nu_{nl} - \nu_{n-1,l}$ is the large frequency separation, n and l are the overtone and degree of the mode respectively, $\tilde{\alpha}$ is a constant of order unity and ϵ_{nl} is a small offset. The large separation, $\Delta\nu$, (constant frequency spacing) is the inverse of the sound travel time from the surface of the star to the core and back again. It is related to mass and radius of a star by:

$$\Delta\nu = \left(2 \int_0^R \frac{dr}{c(r)} \right)^{-1} \propto (M/R^3)^{1/2},$$

where $c(r)$ is the sound speed (Tassoul, 1980, 1990; Aerts et al., 2008). The large separation is sensitive to radius of the star, and near the main sequence, it is a good measure of the mass of the star. ϵ_{nl} gives rise to the small separation, $\delta\nu$, written as:

$$\delta\nu \simeq -(4l + 6) \frac{\Delta\nu}{4\pi^2\nu_{nl}} \int_0^R \frac{dc}{dr} \frac{dr}{r},$$

which is sensitive to the sound-speed gradient in the core and hence age of the star. The small separation is a very useful diagnostics of stellar evolution.

The following are the properties of g modes;

- (i) as the number of radial nodes increases, the frequencies of the g mode decrease.
- (ii) the g modes are most sensitive to conditions in the deep interior of the star except in white dwarfs where the g modes are sensitive to conditions in the stellar envelope.
- (iii) for overtone $n \gg l$, there is an asymptotic relation for g modes which says that they are approximately equally spaced in period.

The period of g modes is given asymptotically by

$$P_{nl} \approx \frac{(n + \frac{l}{2} + \epsilon)P_0}{\sqrt{l(l+1)}},$$

where $P_0 = 2\pi^2 \left(\int \frac{N}{r} dr \right)^{-1}$, N is the Brunt-Väisälä frequency weighted by the inverse of the radius and ϵ is a small constant. This means that for a given l , modes of consecutive radial orders, n , are equally spaced in period by an amount $\Delta P_g = \sqrt{l(l+1)}P_0$. If we could measure the pure g modes in the core of a red giant stars (Bedding et al., 2011; Corsaro et al., 2012), they would be approximately equally spaced by ΔP_g , just as the p modes are approximately equally spaced in frequency by $\Delta\nu$. To successfully use p -modes and g -modes for seismology, one needs to detect as many frequencies in a star as possible. This requires

observation with low noise and this is best achieved with space - based observatories like K2, *Kepler*, and MOST².

1.4 Solar and solar-like oscillations

1.4.1 The structure of the Sun

The basic structure of the Sun can be divided up into three different regions (layers). The inner most region around the center of the Sun is known as the *core*, where energy is produced by nuclear fusion. The radius of the core is about 30 % of the full solar radius. Above the core, we have the *envelope* and above the envelope is the *solar photosphere* (see Figure 1.4 for detail). Above the photosphere is the chromosphere which is about 10^4 km thick. The outermost layer of the Sun is the corona, which extends far into space. It is very faint, and is only observable during total solar eclipse. The Sun is the source of energy for Earth

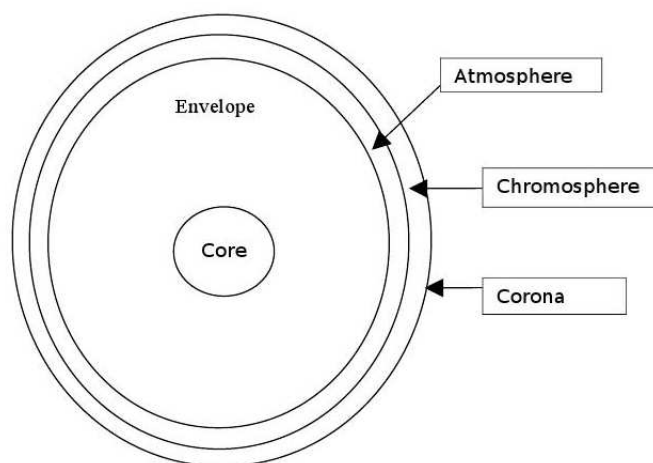


Figure 1.4: Schematic figure showing the different layers of the structure of the Sun.

controlling the Earth's environment, humans would not exist without the Sun. It is therefore important that we understand the physics going on in the Sun. In astrophysical sense, it is a

²Microvariability and Oscillation of STars

perfect laboratory for stellar physics due to its proximity to us. Helioseismology helps us to accurately determine the structure and dynamics of the Solar interior. This allows us to test and develop our ideas, theories and models of solar evolution. Homer Lane (1869) was the first person to attempt to model the Sun. In the present Solar model, energy is known to be carried by radiation in the inner 72 % of the radius and as you approach the last 28 % of the radius, energy is transported largely by convection. This means that in the present models of the Sun, energy is carried out in part by radiation and convection (Roxburgh, 1996).

1.4.2 Oscillations in the Sun

Convective motions in the outermost layers of a star have characteristic turn-over time scale. A turn-over time scale is the time scale over which turbulent motions occur. If the pulsation period of such a star coincides with the turn-over time scale, there will be transfer of energy from the motion of the convective material to drive the global pulsation mode at that particular period, for example, (Houdek et al., 1999). Random convective noise is generated which is transformed into distinct p -modes with a wide range of spherical harmonics (Houdek et al., 1999; Samadi & Goupil, 2001; Houdek, 2006).

There is a balance between the amount of energy pumped into a particular mode and the natural tendency of the mode to decay. This balance determines the mean amplitude of the mode, and the decay produces a characteristic Lorentzian profile in the frequency spectrum as shown in Figure 1.5. Any oscillation driven in such a manner is called a solar-like oscillation (stochastic oscillation).

According to Christensen-Dalsgaard (2002a), it is possible that the first indications of solar oscillations were detected by Plaskett (1916), who observed fluctuations in the solar surface Doppler velocity in measurements of the solar rotation rate. It was not clear whether the fluctuations detected by Plaskett (1916) were truly solar or they were induced by effects in the Earth's atmosphere. The solar origin of these fluctuations were established by Hart (1954). Observations of oscillations of the solar surface were made by Leighton et al. (1962) who detected roughly periodic oscillations in Doppler velocity with period around 300 s.

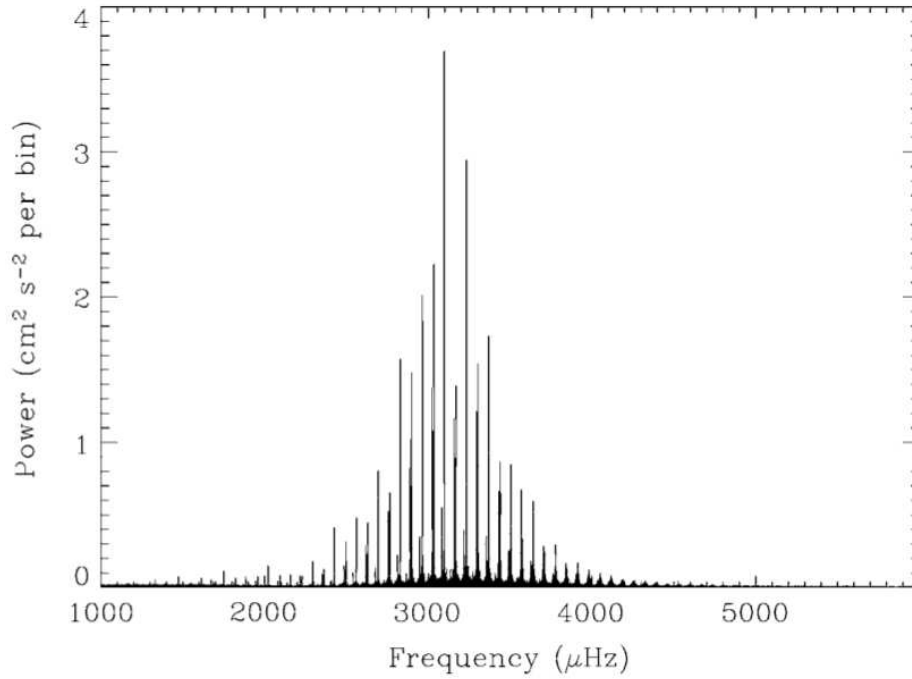


Figure 1.5: A periodogram for the Sun showing localized comb-like structure with amplitudes which decrease sharply from a central maximum. Taken from Aerts et al. (2010).

1.4.2.1 Some of the important results of helioseismology

The Solar Neutrino Problem (SNP) was first identified from the Neutrino experiments where only one-third of Solar Neutrino were detected compared to theoretical prediction. Solar physicists came up with various explanations for SNP (Bahcall, 1964; Domogatsky et al., 1965; Reines, 1964) such as if Neutrinos have non-negligible mass, then they can change from one type to another (Neutrino oscillations), then the electron-type Neutrinos will transform into other types on their way from the Sun. Upon reaching the Earth, the Neutrino flux will contain a mixture of all types of Neutrinos and the total sum will add to the original flux that left the solar core. Since at that time it was not known that a Neutrino can transform from one type to another, there was a suspicion that the inadequate knowledge of solar physics was responsible for the SNP. Astronomers/solar physicists/astrophysicists came up with various possible explanations of low observed Neutrino fluxes (Roxburgh, 1996). When helioseismology came into being, it was used to show that the models of the Sun were correct suggesting that SNP had more to do with particle physics than solar physics. However, ex-

periments that were designed to detect all three types of Neutrinos (Electron, Muon and Tau) eventually showed that the SNP was due to Neutrino Oscillations (Poon, 2002).

The solar convection zone is in the outer-most 30 % of the solar radius. The history of the determination of the depth of the extent of the solar convection zone goes back to the early days of helioseismology. Gough (1977) was able to determine that the convection zone extends down to 0.3 of the solar radius from the surface. He used indirect method in a way that it was based on the observation of high degree modes. These modes are trapped in the outer parts of the convection zone, but they allow a determination of adiabat (a line on a graph relating the pressure and temperature of a substance undergoing an adiabatic change) in the adiabatically stratified part of the convection zone. Later on, a more direct method of determining the depth of the base of the convection zone was suggested by Gough (1986) from noting that the sound speed gradient was closely related to the temperature gradient. He noted that the base of the convection zone was marked by a transition from adiabatic temperature gradient (inside the convection zone) to radiative one (beneath the convection zone). By inferring sound speed from Helioseismic techniques, Christensen-Dalsgaard et al. (1991) were able to determine the base of the convection zone as $(0.287 \pm 0.003)R_{\odot}$.

The solar rotation profile (rate) was first determined by helioseismology. It has been shown that the Sun's surface behaves differently, i.e., the equator rotates faster than the poles as has been confirmed by Thompson et al. (1996) and Kosovichev et al. (1997).

1.4.2.2 Historical background of amplitude ratios and phase shifts

Stars are massive hot balls of ionized gas known as plasma. The released energy is produced at the core by the burning of hydrogen into helium for the main sequence stars. However for the evolved stars which are massive enough, it is other elements (Carbon, Nitrogen and Oxygen) which are involved in the nuclear fusion. Low mass stars ($\sim 1 M_{\odot}$) end up as White dwarfs. The Sun is the nearest star to us, because of this, we can study it in greater detail than other stars and thereafter use the information obtained to help us study other stars which have similar oscillation characteristics like the Sun. We also focus on the Sun because the observational data of amplitude ratios (the ratio of surface luminosity to velocity) and phase shifts (the inverse of tangent of amplitude ratios) do exist and have not been adequately un-

derstood. We expect to provide theoretical estimate of amplitude ratio and phase shifts in the atmosphere of the Sun. The aim of the first part of the thesis is an in-depth investigation of some of the observable parameters in the atmosphere of the Sun. This is done by solving the non - adiabatic pulsation equations with consistent treatment of radiation and a non - local mixing length theory of convection (Phorah, 2007; Houdek et al., 1995; Houdek, 1996; Balmforth, 1992; Baker & Gough, 1979). The research aims at improving our current understanding of how consistent treatment of radiation affects the amplitude ratios and phase shifts in the atmosphere of the Sun. Amplitude ratios and phase shifts are important because they help to determine the degree of non - adiabaticity in the solar atmosphere.

Many different authors have studied the effects of radiation and convection on the oscillations in stars. For example, Baker & Gough (1979) used linear non - adiabatic theory with local mixing length theory to study the model of RR Lyrae stars. Later on, Balmforth (1992) improved the Baker & Gough (1979) theory by using the non - local mixing length theory of convection and including the Eddington's approximation for radiation instead of the diffusion approximation for a model of the Sun. Houdek (1996) improved Balmforth's work by including the Eddington's approximation to correct for the thermal stratification of the optically thin layers and used it to study amplitude ratios and phase shifts for the model of the Sun. Houdek (1996) results for the amplitude ratios and phase shifts were in good agreement with the data in the lower layers in the solar atmosphere. However, higher in the solar atmosphere, the fits of theoretical models to the data were less satisfactory. Phorah (2007) improved on Houdek (1996) results by replacing the Eddington approximation in Houdek's computer code with a consistent treatment of radiation. Phorah (2007) did not calculate amplitude ratios and phase shifts for the solar model in the atmosphere of the Sun but rather used his code to study the effects of consistent treatment of radiation on pulsation eigenfunctions of rapidly oscillating Ap (roAp) stars. In chapter 2 of this thesis, Phorah (2007) pulsation modeling code is used to study amplitude ratio & phase shifts in the atmosphere of the Sun and later a comparison with the observations is made.

Schrijver et al. (1991) presented amplitude ratios and phase shifts measurements obtained from IPHIR³ instruments on the PHOBOS 2 spacecraft (one of the space mission launched by the Soviet Union to study Mars and its moons Phobos) and BiSON⁴ for $l = 0, 1$, and 2 modes in the Sun within frequency range of 1.8 - 4.6 mHz. Schrijver et al. (1991) data showed that the phase shifts (differences) between irradiance and velocity at wavelength 500 nm in the frequency range 2.5 - 4.3 mHz were roughly constant ($-119 \pm 3^\circ$). Observations at 500 nm correspond to height of 200 km above photosphere. The phase shift increased to $-68 \pm 6^\circ$ at around 4.3 - 4.5 mHz. The average value of the amplitude ratio between irradiance and velocity of p-modes at 500 nm was (0.235 ± 0.018) ppm s cm⁻¹ between 2.5 and 4 mHz. Their phase difference is shown in Figure 1.6. The top panel of Figure 1.6 shows the observational data of phase difference between irradiance and velocity as a function of frequency. The coherence values were calculated using bivariate spectral analysis as presented by Schrijver et al. (1991). The techniques uses the power-spectral density of two time series of a given length, and amplitudes. It makes use of the power-spectral density to define a co-spectral density & quadrature spectral density. Therefore, coherence can be defined as the linear correlation coefficient between two time series in linear-regression analysis. It is a measure of the degree that one series can be represented as the output of a linear filter with input of another series (Schrijver et al., 1991). The bottom panel of Figure 1.6 shows the running mean of the observational data both at low and high frequencies (thick solid line).

Figure 1.7 shows the amplitude ratio data obtained by Schrijver et al. (1991). In the insert, the thick solid line is the running mean of the observational data in the frequency range 2.5 - 4.5 mHz. According to Schrijver et al. (1991), the data below 2.5 mHz are uncertain as a result of large effects of noise. Schrijver et al. (1991) suggested in their paper that a good theoretical model for the computation of the phase shifts between irradiance and velocity in the deep photosphere should include convective dynamics and radiative transfer. Indeed this was first attempted by Houdek et al. (1995) and later Houdek (1996) where they modelled the amplitude ratios and phase shifts in the solar atmosphere by including non - local theory of convection in their pulsation equations. In Houdek et al. (1995) and Houdek (1996), radiative

³InterPlanetary Helioseismology by Irradiance

⁴Birmingham Solar Oscillations Network

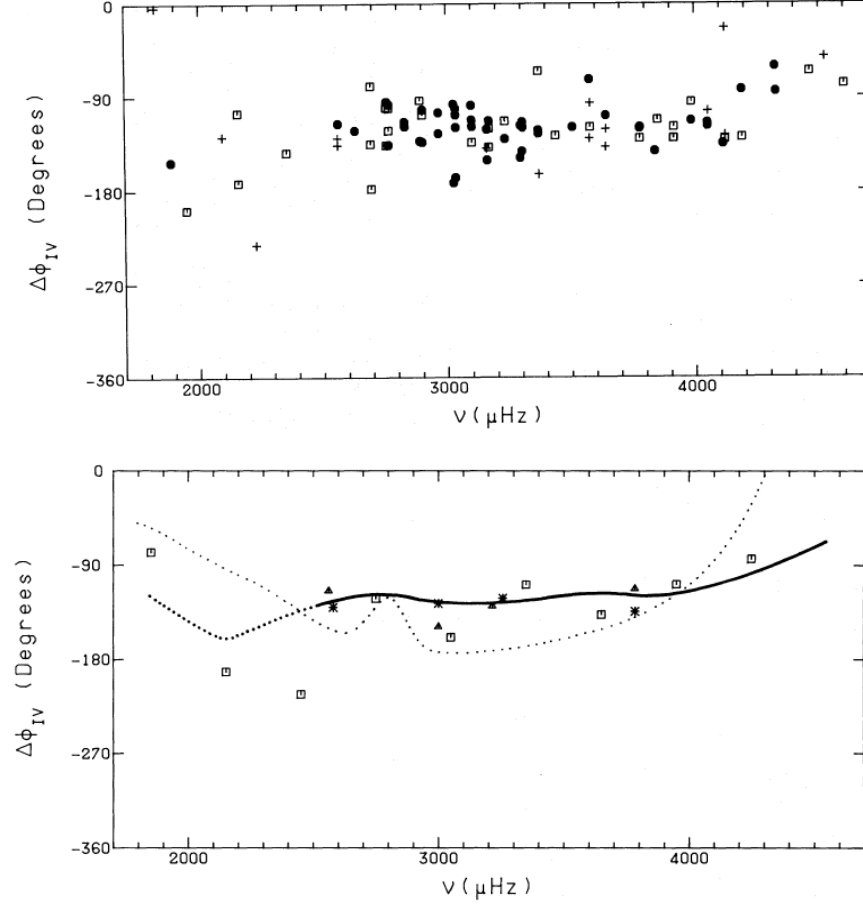


Figure 1.6: In the top panel: Observational data of phase difference between irradiance and velocity as a function of pulsation frequency. Filled circle symbol is for coherence exceeding 0.7, open square for a coherence between 0.55 and 0.7 and + for a coherence between 0.5 and 0.55. In the bottom panel: The thick solid line is the running mean of the observational data both at low and high frequencies. For the dashed, triangle and other symbols, see Schrijver et al. (1991). Figure taken from Schrijver et al. (1991).

transfer was treated with the Eddington approximation in both the equilibrium and the pulsation models. Figure 1.8 shows the theoretical amplitude ratio between surface luminosity and velocity as a function of frequency calculated by Houdek et al. (1995) at different heights above the photosphere shown with observational data from Schrijver et al. (1991). The thick, solid line indicates a running-mean average of the observational data. Houdek et al. (1995) showed in Figure 1.8 that their models did not fit the observations well, especially in the

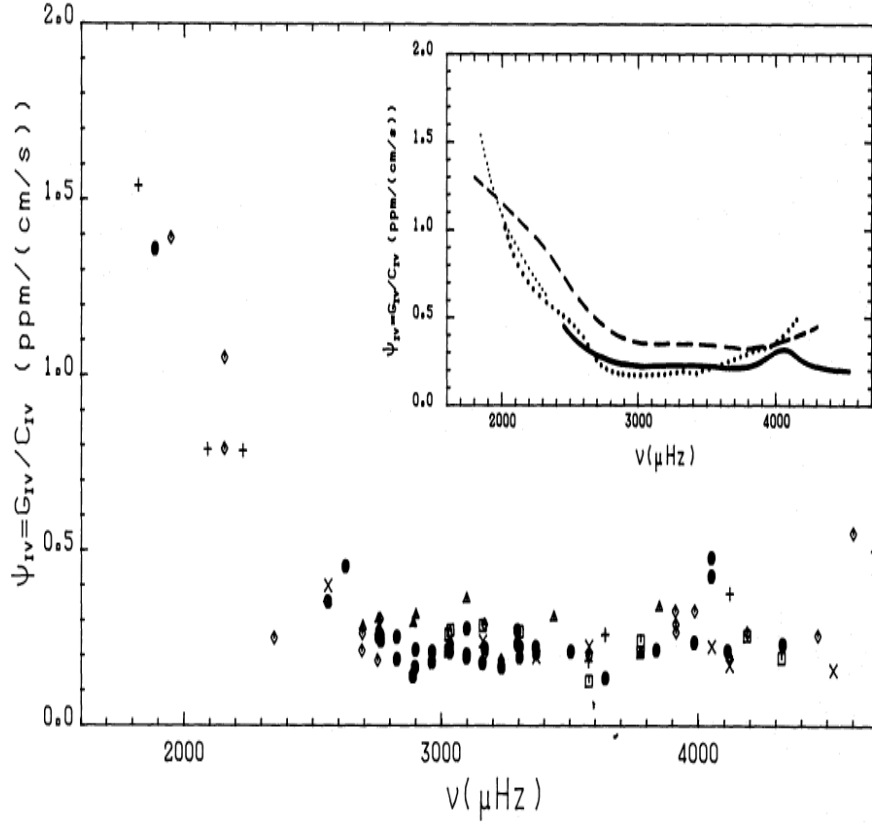


Figure 1.7: Observational data of amplitude ratios between irradiance and velocity as a function of frequency. In the insert, the thick solid line is the running mean of the observational data in the frequency range 2.5 - 4.5 mHz. Taken from Schrijver et al. (1991). Filled circle is for coherence exceeding 0.7, open square for a coherence between 0.55 and 0.7 and + for a coherence between 0.5 and 0.55.

frequency range 2.5 - 2.7 mHz and 3.7 - 4.2 mHz.

Figure 1.9 shows theoretical phase shifts between surface luminosity and velocity as a function of frequency fitted with observational data from Schrijver et al. (1991). The model calculated by Houdek et al. (1995) and observations did not fit well in moderate frequency ranges, larger optical depths and higher in the atmosphere. The thick, solid line indicate a running-mean average of the observational data.

In this thesis we use the radial pulsation (old and new code) to calculate amplitude ratios and phase shifts. The amplitude ratios were estimated by taking the value of luminosity

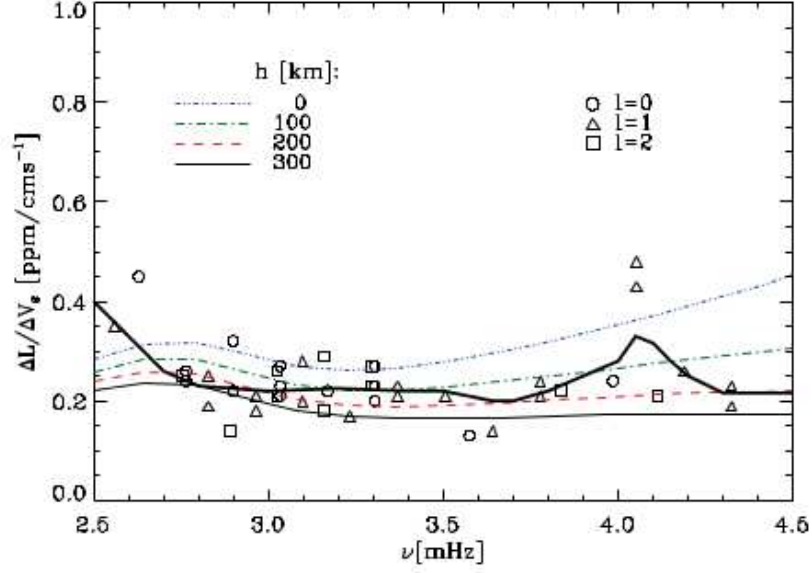


Figure 1.8: Theoretical amplitude ratio between surface luminosity and velocity calculated at various heights in solar atmosphere as a function of frequency compared to observational data from Schrijver et al. (1991). The model and observations did not fit well in moderate frequency ranges, larger optical depths, and higher in the atmosphere. The thick, solid line indicate a running-mean average of the observational data. Taken from Houdek et al. (1995).

eigenfunctions divided by the norm of the complex eigenfunctions evaluated at a height of 200 km above the photosphere (defined as a layer with temperature equal to T_{eff}). This (200 km) is the height at which potassium line forms and is indicative of the physical parameters of a star such as T_{eff}). The specific aims and objectives of the first part of the thesis were; (i) Calculation of theoretical intensity-velocity amplitude ratios $\frac{\Delta L_s}{\Delta V} = \frac{\delta L/L}{\omega_r r \delta r/r}$ of radial pressure (p) mode using new code, where $\delta L/L$ is the surface luminosity perturbation, ω_r is the eigenfrequency (real part) and δr is the displacement perturbation. (ii) Calculation of phase shifts in the atmosphere of the Sun, (iii) Comparison of the theoretical amplitude ratios and phases with data such as the one presented in Schrijver et al. (1991) and Jiménez et al. (1999) as was done by Houdek et al. (1995).

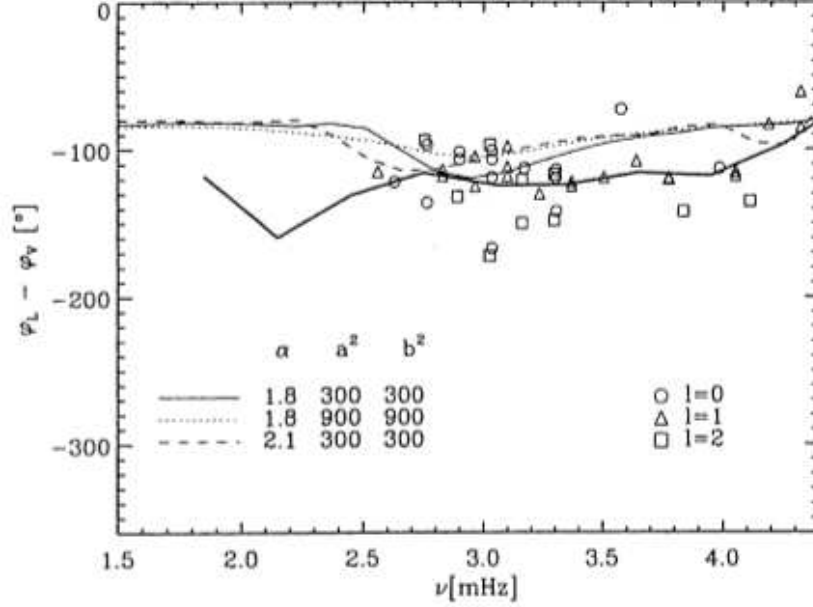


Figure 1.9: *Theoretical phase shifts between surface luminosity and velocity as a function of frequency fitted with observational data from Schrijver et al. (1991) The model and observations did not fit well in moderate frequency ranges, larger optical depths and higher in the atmosphere. The thick, solid line indicate a running-mean average of the observational data. Taken from Houdek et al. (1995).*

1.4.2.3 Brief description of the theoretical calculation approach

Through improving observation techniques, accurate data now exists of amplitude ratios and phase shifts between different types of oscillation measurements (data from IPHIR instruments and BiSON). The amplitude ratios and phase shifts measurements depend basically on the properties of the oscillations just below and in the solar atmosphere (Houdek et al., 1995).

Houdek et al. (1995) modelled the turbulent transfer of heat and momentum by means of nonlocal, time dependent mixing-length theory, and used the Eddington approximation to radiative transfer. They also considered a grey radiation approximations and later compared calculated amplitudes and phases of theoretical eigenfunctions with various observations. In particular, Houdek et al. (1995) compared the amplitude ratios and phase shifts with irra-

diancance measurements from IPHIR instruments on the PHOBOS 2 spacecraft and velocity measurements obtained from BiSON at Tenerife (Schrijver et al. 1991). Houdek et al. (1995) model compared tolerably with observation, especially in moderate frequency ranges and at larger optical depths (see Figure 1.8). The thesis does not make use of data from other solar observatories such as the Solar and Heliospheric Observatory (SoHO) & the Global Oscillation Network Group (GONG) because (i) PHOBOS 2 data are readily available and easy to obtain, (ii) need to compare this thesis work with previous work using the same data.

Phorah's code (see Phorah 2007, also called new code) solves the radial non-adiabatic pulsation equations that includes non-local theory of convection with consistent treatment of radiation instead of Eddington approximation. Phorah (2007) substituted Eddington approximation with full radiation treatment with the subroutine for full radiation taken from Medupe (2002) code. Medupe (2002) code solves non-adiabatic radial pulsation equations that includes only radiation without convection while Houdek (1996) code solves the radial non-adiabatic pulsation equations that includes non-local theory of convection with Eddington approximation. The contribution that I have made is in using the new code to calculate amplitude ratios and phase differences. Phorah (2007) only used the code for purposes other than calculating amplitude ratios and phase differences.

1.5 Solar-like oscillations in Red Giants

1.5.1 Red Clump Stars

A star is formed from a cloud of gas and dust. The cloud contracts under its own gravitation and condenses into many hundreds of smaller pieces. As a cloud collapses under its own gravitation, the gas becomes compressed and therefore heats up (gravitational energy converts into thermal energy). At some point the temperature at the center of the cloud will rise so high that nuclear reactions can begin. When this happens, the star has reached the end of its contraction phase and is on the zero-age main sequence (ZAMS). Since the masses of the stars are different, more massive stars reach the ZAMS sooner than less massive stars. However, after a short time, practically all stars have reached the ZAMS. One can see pre-ZAMS stars in only a few very young clusters in star-forming regions.

Because the more massive stars burn nuclear energy much faster than less massive stars, the most massive stars start moving off the main sequence. After several million years the less massive, cooler, stars are still on the ZAMS (having burned very little nuclear fuel), but the most massive, hottest, stars have already burned most of the fuel in the core of the star and have undergone changes in effective temperature and luminosity, which causes them to move off the ZAMS. The more massive, hotter, stars leave the ZAMS, become red giants and then white dwarfs. In the end, only low mass cool stars still survive on the ZAMS because they burn their fuel so slowly.

When stars exhaust hydrogen in the core, they start burning hydrogen in a shell, causing the star to rapidly increase in radius and become cooler, entering the red giant phase. Due to the rapid evolution during hydrogen shell burning, there is a gap in the region between the main sequence and the red giant phase. The evolution in the red giant phase is quite complex and depends on the mass of the star as well as its chemical composition. Figure 1.10 shows the evolutionary track for a star similar to the Sun. The star spends a long time in the red clump region where it is burning helium in the core.

Red clumps (RGC) originate from red giants that undergo the helium flash at the tip of the RGB while still having a total mass of more than $\sim 1 M_{\odot}$ as shown in Figure 1.10. The fact that He burning cannot start until the core mass has reached $0.45 M_{\odot}$ (for only stars with degenerate cores), it follows that by the time He starts to burn in the core, practically every star will have this core mass. Hence the red clump stars all have nearly the same luminosity. This is a useful feature, because this standard luminosity can be used as a distance indicator.

As briefly mentioned before, the RGC is formed by the evolution of low-mass stars that have degenerate He cores when evolving up along the RGB. A more massive stars do not have a degenerate He core. Stars just massive enough to burn He in non-degenerate conditions define a secondary red clump (SRC) in the HR diagram which is less luminous and hotter than the main RC. Evolution along the red giant branch depends critically on the mass of the star. This is due to the fact that for $M < 1.8 M_{\odot}$ the core of the star becomes degenerate (RGC), while for $M \geq 1.8 M_{\odot}$ the core of the star never becomes degenerate forming SRC instead.

Cannon (1970) studied red giants in old open clusters and found a prominent feature

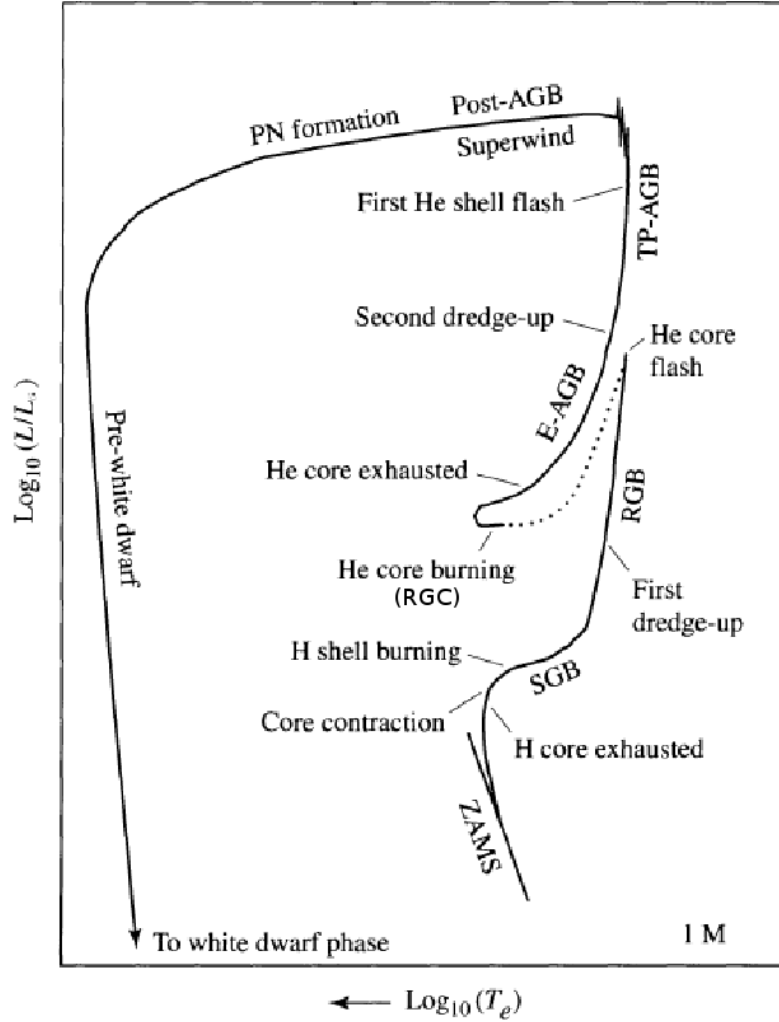


Figure 1.10: The different stages of post main sequence evolution of $1 M_{\odot}$ star in the HR diagram. Adapted from Carroll & Ostlie (2006).

that all the clusters having a ‘clump’ of red giants centered near $M_V = +1$, $B - V = +1.0$. This indicated that the stars in those regions have a helium - burning core surrounded by a hydrogen shell and these were called RGC stars. At the core helium burning stage, there is a slow change in color and the luminosity of such a star remains almost constant. Thus, there is a greater density of these stars in the color - magnitude diagram. In fact, this is why they look like a ‘clump’ in the color - magnitude diagram (see Figure 1.11).

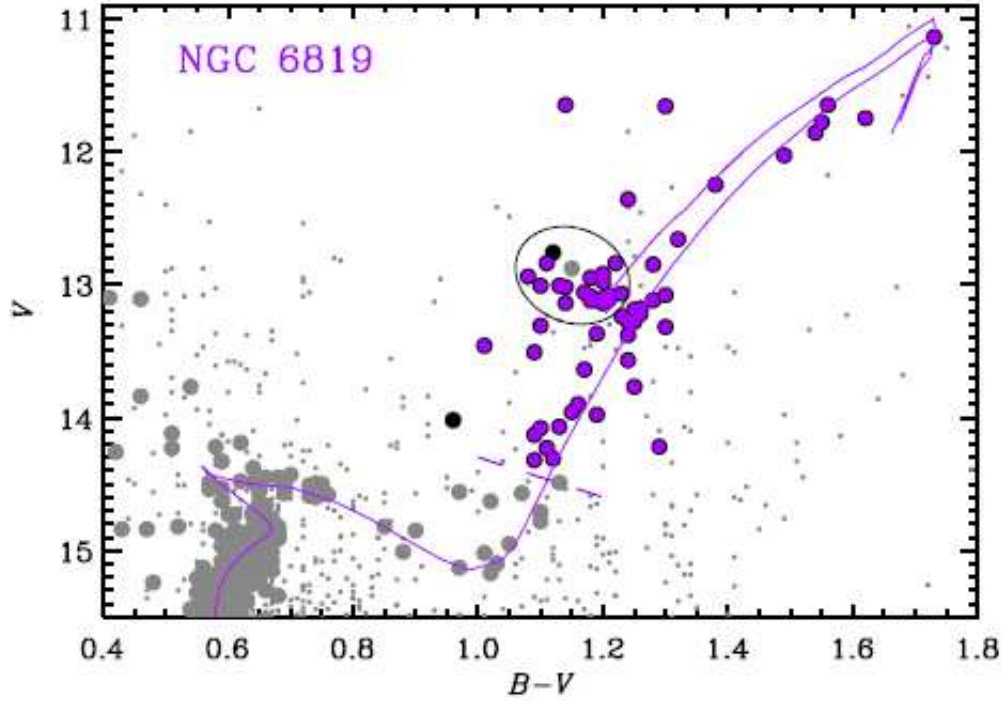


Figure 1.11: An observational color - magnitude diagram showing greater density of clump stars in Kepler open cluster NGC 6819. The clump stars are marked in a circle, the purple dots are red giants with solar-like oscillations, the dark shadings are cluster member stars from Hole et al. (2009) and the lines are the theoretical isochrones. More information about Kepler open cluster NGC 6819 see chapter 4. Taken from Stello et al. (2011b).

1.5.2 Oscillations in red giants

The search for stars which show oscillations similar to that of the Sun started by Smith et al. (1987) who were the first to report on the short period variability in a giant star (Arcturus) with period of hours. Innis et al. (1988) also found such variability in the star α BOO (Arcturus) on the basis of radial velocity measurements. Edmonds & Gilliland (1996) found photometric variation in K giants in the Globular cluster 47 Tuc which appeared to be consistent with solar - like oscillations. Another long term radial velocity monitoring of Arcturus by Merline (1999) revealed solar-like oscillations with period ranging from 1.7 to 8.8 d. The first firm discovery of solar-like oscillations in a giant was for G7 III star ξ Hya by Frandsen et al. (2002) using radial velocity measurements.

Some G - type and red - giant stars show rich spectra of solar - like oscillation, which are ex-

cited and intrinsically damped by turbulence in the outer layers of their convective envelopes (Chaplin & Miglio, 2013). A necessary condition for stars to show solar - like oscillations is the presence of near - surface convection. Solar-like oscillations have higher amplitudes in red giants than in main sequence and sub giants stars because of more vigorous surface convection (Christensen-Dalsgaard & Frandsen, 1983c). Red giants stars typically have radii in the range 2 - 100 $R_{\odot}$, stellar masses ranging from 1 - 3 $M_{\odot}$ (Hatzes & Cochran, 1998) and luminosities of a few tens to a couple of thousand times that of the Sun, $\log g \sim 1.5 - 3.0$, $v \sin i \sim 1 - 10$ km/s and $T_{\text{eff}} \sim 3500 - 5000$ K (Allen, 1973; Pasquini et al., 2000). Red giants show oscillations of very long period, corresponding to the large dynamic time-scale resulting from their large radii. In the periodogram, solar-like oscillations are easily identified because of the localized comb-like structure with amplitudes that decrease sharply from a central maximum as shown in Figure 1.12. One of the global oscillation properties that we can characterize in different solar-like stars is the frequency of maximum power, ν_{max} , that changes as a function of basic stellar parameters mass, radius and luminosity. Two other global oscillation characteristics are the large frequency separation, $\Delta\nu$ and small frequency separation, $\delta\nu$ as shown in Figure 1.13 and both described in detail in section 1.3.2. The large frequency separation, $\Delta\nu$ is a good proxy for the density of the star while small frequency separation, $\delta\nu$ is a good proxy of sound speed gradient in the core (Ulrich, 1986; Chaplin & Miglio, 2013).

1.5.2.1 Mass, luminosity and radius determination

The mass and radius of a star with solar-like oscillation can be determined from a measurement of the frequency of maximum amplitude ν_{max} and the large separation if the effective temperature is known. We call this the direct method. Other methods do exist, for example, the grid-based methods, where a grid of model in radius and mass is made and the characteristics of stars are determined by searching among the models to get a ‘best fit’ for a given observed set of $\Delta\nu$, ν_{max} , T_{eff} , and [Fe/H]. The ν_{max} is related to the physical parameters (Kjeldsen & Bedding, 1995):

$$\nu_{\text{max}} \approx \nu_{\text{max}\odot} \frac{M/M_{\odot}}{(R/R_{\odot})^2 \sqrt{T_{\text{eff}}/T_{\text{eff}\odot}}}, \quad (1.1)$$

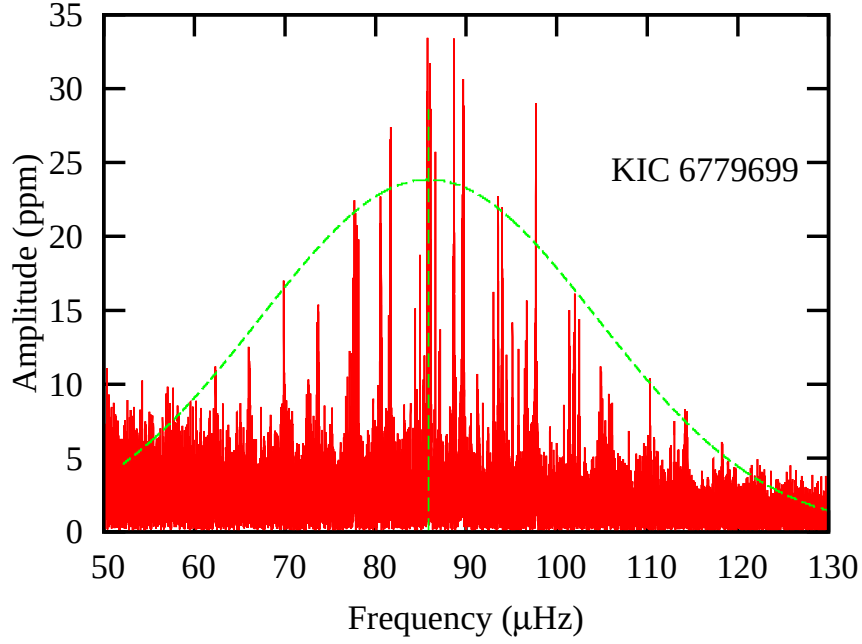


Figure 1.12: A periodogram for a giant star KIC 6779699 showing localized comb-like structure (green dotted line in the Gaussian form) with amplitudes which decrease sharply from a central maximum. Vertical green dotted line indicates the location of the central maximum, $\nu_{\max}$.

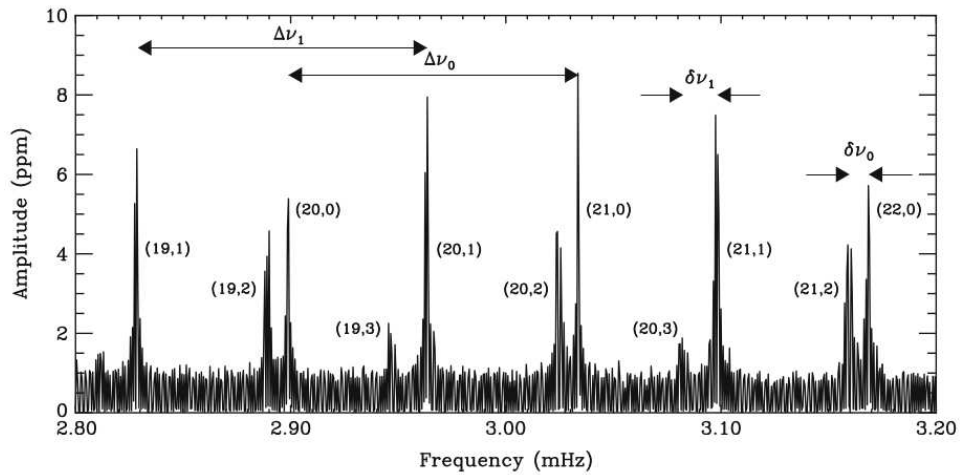


Figure 1.13: A periodogram showing large frequency separation $\Delta\nu$ and small frequency separation, $\delta\nu$ for solar data. Taken from Christensen-Dalsgaard (2002b). The (n, l) are indicated for each frequency peak.

where the solar value for the frequency of maximum amplitude are $\nu_{\max\odot} = 3120 \mu\text{Hz}$ (Kallinger et al., 2010), $M/M_\odot$, $R/R_\odot$ and $T_{\text{eff}}/T_{\text{eff}\odot}$ are the stellar mass, radius and effective temperature relative to solar values respectively.

In section (1.3.2 we discussed how acoustic modes for high overtone pulsators tend to form equally-spaced frequency structures consisting of modes of the same spherical harmonic degree, l , but of successive overtones, n . The large frequency separation, $\Delta\nu$, is related to stellar mass and radius according to the following equation (Kjeldsen & Bedding, 1995):

$$\Delta\nu \approx \Delta\nu_\odot \sqrt{\frac{M/M_\odot}{(R/R_\odot)^3}}, \quad (1.2)$$

where $\Delta\nu_\odot = 134.88 \mu\text{Hz}$ (Kallinger et al., 2010). With $\nu_{\max}$ and $\Delta\nu$ in μHz , Balona et al. (2013b) re-wrote equations (1.1) and (1.2) in the following way:

$$\log M/M_\odot = -7.6056 + \frac{3}{2} \log T_{\text{eff}} + 3 \log \nu_{\max} - 4 \log \Delta\nu, \quad (1.3)$$

$$\log R/R_\odot = -1.1151 + \frac{1}{2} \log T_{\text{eff}} + \log \nu_{\max} - 2 \log \Delta\nu, \quad (1.4)$$

$$\log L/L_\odot = -17.274 + 5 \log T_{\text{eff}} + 2 \log \nu_{\max} - 4 \log \Delta\nu. \quad (1.5)$$

Thus from $\nu_{\max}$, $\Delta\nu$ and T_{eff} one can determine M, R and L. In chapter 3 and 4, we show how to apply these equations to determine M, R, L for solar-like stars in the field of open cluster NGC 6819 and NGC 6866.

1.5.2.2 Solar-like oscillations & mass loss at the tip of RGB

The Red Giant (RG) stars are expected to lose mass on the RGB phase of their evolution. Various authors have studied different empirical formulae to describe the mass loss of RG stars, for example, Reimers (1975) used the following model for mass loss rate:

$$\frac{dM}{dt} = \eta \times 4 \times 10^{-13} \frac{L}{gR} [M_\odot/\text{yr}], \eta = 0.3, \quad (1.6)$$

where η is the mass loss rate free parameter, g is gravitational acceleration, L is the luminosity and R is the radius. Mullan (1978) used the following formula in his studies of mass loss

$$\frac{dM}{dt} = 1.6 \times 10^{-19} M R^{\frac{1}{2}} \frac{L}{g R} [M_{\odot}/yr]. \quad (1.7)$$

Goldberg (1979) used the following model that does not depend on luminosity:

$$\frac{dM}{dt} = 2.6 \times 10^{-12} R^2 [M_{\odot}/yr]. \quad (1.8)$$

Finally, Judge & Stencel (1991) proposed a model of $\dot{M}$ that depends on surface gravity only:

$$\frac{dM}{dt} = 4.9 \times 10^{-8} g^{-1.4} [M_{\odot}/yr]. \quad (1.9)$$

Origlia et al. (2002) showed through Figure 1.14 that none of the analytical formulae described in equations (1.6), (1.7), (1.8), and (1.9) are able to reproduce the measured mass-loss rate.

The mass-loss rates of evolved stars are of primary importance for stellar and galactic evolution model (Miglio et al., 2012). The mass loss from asymptotic giant branch (AGB) stars is well understood in terms of global pulsations lifting gas out to distances above the photosphere where dust forms (Bowen, 1988) and the action of radiation pressure on dust that further drives dust and gas away (Höfner, 2009). However for the case of Red Giant Branch stars, there is no reliable theory to properly explain the mass-loss. Attempts have been made in the past to come up with the empirical laws to try to explain and describe the mass-loss along the RGB (Reimers, 1975; Mullan, 1978; Goldberg, 1979; Judge & Stencel, 1991). On the observational sides, some work have been carried out, for example, Origlia et al. (2002) detected the circumstellar matter around Globular Cluster red giants using the ISOCAM camera on-board the ISO satellite and later derived mass-loss rates for a few RGB stars. Further observational work of mass-loss of red giant stars was carried by Wood (2007) who explored how mass loss rates depended on luminosity and mass.

With the launch of *Kepler* satellite, more RGB stars have been observed in the *Kepler* clusters. As a result of the wealth of data available, Miglio et al. (2012) studied mass-loss in NGC 6791 and NGC 6819 by comparing the average mass in the red clump with that of

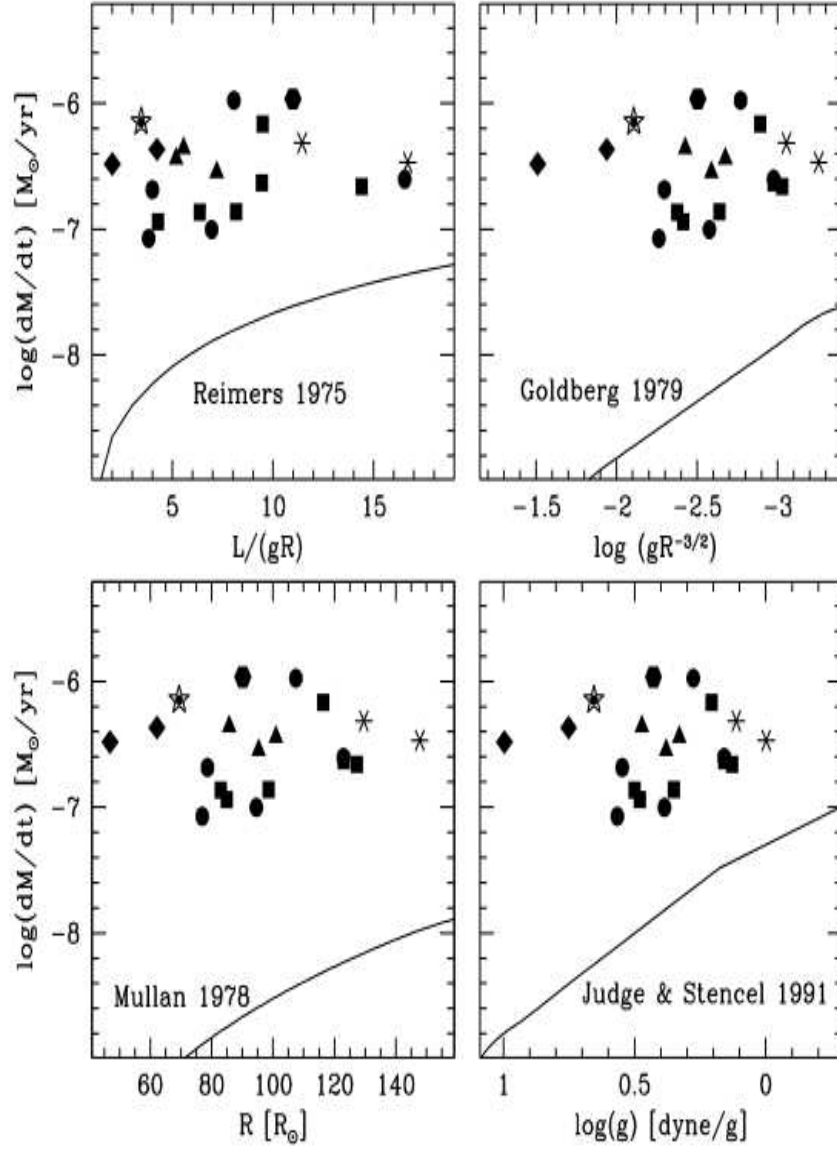


Figure 1.14: Theoretical mass-loss rates fitted with observations. Top left: points are the observational data while solid line is a fit using Reimers (1975) formula. Top right: points are the observational data while solid line is a fit using Goldberg (1979) formula. Bottom left: points are the observational data while solid line is a fit using Mullan (1978) formula. Bottom right: points are the observational data while solid line is a fit using Judge & Stencel (1991) formula. Adopted from Origlia et al. (2002).

stars in the low-luminosity portion of the RGB. They found less robust result for mass-loss of RGB stars in NGC 6819. Using a model they found that a correction factor of 1.9 % is

needed to be applied to $\Delta\nu$ of RGC for the mass of the RGC to be the same as the mass of the RGB stars. In other words, they found out that for two stars of the same mass (that is to say, RGC and RGB stars), the RGC star has a 1.9 % larger value of $\Delta\nu$. That means that one needs to reduce the observed $\Delta\nu$ in RGC stars by 1.9 % to give the correct mass using the pulsation analysis.

In this thesis, we have developed an alternative approach of studying mass-loss at the tip of the RGB in NGC 6819 using statistical techniques as is explained in details in chapter 3.

Chapter 2

Modeling amplitude ratios and phase differences and comparison with data

2.1 Introduction

We will make use of two pulsations models to calculate the amplitude ratios and phase shifts. The difference between the two is in how they treat radiation transport. Both models include non-local time dependent mixing length theory of convection. The older model by Houdek (1996) approximates radiation transport in the atmospheres of stars using Eddington approximation. We call a FORTRAN code that implements this model ‘old code’. The second model treats radiation transport fully by making use of radiative transfer equation as described in Christensen-Dalsgaard & Frandsen (1983a) and Medupe (2002). The FORTRAN code that implements this was put together by Phorah (2007) and here it is referred to as ‘new code’. In this chapter, I will therefore discuss and derive some pulsation equations used in both codes. I will then present some of the solutions of the pulsation equations obtained from the old and new codes and show how they differ. Finally I will present comparison of amplitude ratios and phase differences with those obtained from the data. The equations are solved in the radial, grey situation (where the absorption coefficients are assumed to be frequency independent). A proper way of modeling Solar atmosphere is by taking into account the interaction between the hydrodynamics of the motion and the radiative energy transfer. Auer & Mihalas (1970) give a detailed description of the equations of radial oscillation in the Solar atmosphere and the use of the variable Eddington factors to treat the directional dependence of the radiation.

2.2 Equilibrium Model

The physical structure of a star is described by equations of fluid dynamics. This is because a star is considered to be mostly made of gas. These equations of hydrodynamics are;

- (1) continuity equation that describes the fact that the mass of a star is conserved. In mathematical form it is written as

$$\frac{\partial \rho}{\partial t} + \text{div}(\rho \vec{v}) = 0, \quad (2.1)$$

where ρ is density, $\vec{v}$ is velocity of stellar gas and t is time.

- (2) equation of motion which uses Newton's second law of motion to balance all forces that act on a star. It is written as

$$\rho \frac{\partial^2 \vec{r}}{\partial t^2} = -\vec{\nabla} P + \rho \vec{f}, \quad (2.2)$$

where $\vec{f}$ is the body force per unit mass, P is the pressure and ρ is density

- (3) Energy equation that accounts for all energy sources and sinks in a star is given by

$$\rho \frac{dq}{dt} = \rho \epsilon - \text{div} \vec{F}, \quad (2.3)$$

where dq/dt is the rate of heat loss or gain, ϵ is the energy generation rate per unit mass and $\vec{F}$ is the energy flux (includes all form of energy transport such as radiation and convection).

The equilibrium model of a star is necessary since pulsations are treated as perturbation of an equilibrium star. Therefore the equilibrium model is used to calculate coefficients for pulsation equations. The equilibrium model was kindly provided by Guenter Houdek (private communication). Convection was treated with time - dependent, non - local, mixing length which included convective heat flux and turbulent pressure. Balmforth (1992) was the first to give detailed explanation of the way the model used in this thesis was constructed. Houdek et al. (1995) also used similar models in their studies. For the purpose of clarity, I shall base my descriptions of the equilibrium model on Balmforth (1992) and Houdek et al. (1995).

The equilibrium model was constructed using a computational mesh with 2574 points. For the treatment of opacities in the envelope, OPAL 95 tables (Iglesias et al., 1992), and the Kurucz tables (Kurucz, 1991) were used for low temperature. The abundances used are $X = 0.7$, $Y = 0.28$ and $Z = 0.02$, where X is the hydrogen mass fraction, Y is the helium mass fraction and Z is the mass fraction of metals. The equation of state included treatment of the ionization of C, N, & O and pressure ionization. Diffusion approximation to radiative transfer was used to complete the trial solution. In the final model the entire envelope was re-integrated as a boundary-value problem using non-local mixing length theory and Eddington approximation to radiative transfer. The atmosphere was treated as grey and plane parallel & the temperature gradient was corrected using variable Eddington factor (Auer & Mihalas, 1970) to match model C of Vernazza et al. (1981).

2.2.1 Local and non-local description of convection

Heat transfer processes are classified into mainly three types; the first is conduction which is defined as transfer of heat occurring through intervening matter without bulk motion of the matter. The second heat transfer process is convection or heat transfer due to a flowing fluid. The fluid can be a gas or a liquid. In convection heat transfer, the heat is transported through bulk transfer of a fluid. The third is radiation where energy is transported by photons through the stars.

A general equation that describes how energy is transported by radiation, and in particular how the photons interact with stellar material as they travel from the core (where energy is generated) to the surface is called the radiative transfer equation. It is an integro-differential equation that in most cases require numerical techniques to be solved. However, it can be approximated at optically thin regimes by the Eddington approximation. At larger optical depths, the radiative transfer equation can be approximated by the diffusion approximation where the energy flux is directly proportional to the temperature gradient (Christensen-Dalsgaard & Frandsen, 1983b). The theory that is used to describe turbulent convection is the mixing length theory which has been proposed initially by two different authors (Vitense, 1953; Spiegel, 1963). The first one is the local mixing length theory which is characterized by mixing length scale l which must be shorter than any length scale associated with the struc-

ture of the star (Vitense, 1953). However, this condition is violated when modeling solar-like & red giant stars and thus making it to have drawbacks. The second theory is the non local theory which takes into account the finite size of convective element. Non local theory was first proposed by Spiegel (1963) and was based on the idea of the eddy phase space and convective flux equation. It was as a result of the major drawbacks of a local theory approach. The convection model used is a mixing length (α) with parameters a and b. The physical interpretation of parameter, a, is that it describes the spatial coherence of the ensemble of eddies contributing to the total heat and momentum fluxes while that of b is to describe the degree to which the turbulent fluxes are coupled to the local stratification. Both a and b are called non-local mixing length parameters. In this thesis, we selected a model with $a^2 = b^2 = 300$ and $\alpha = 2.0$. The mixing length (α) and non-local (a and b) mixing length parameters were chosen because of the values were found to give better agreement between equilibrium model and data. In other words, the model was in better agreement with the observational data of line width of the Sun, that is to say, the values of a and b were calibrated with the observational data.

2.2.2 Radiative transfer in the atmosphere of the Sun

Here, we present the full treatment of radiative transfer equation in the atmosphere. The radiative transfer equation for a grey plane parallel atmosphere with the scattering is given according to Mihalas (1978) as

$$\mu \frac{dI}{dr} = \rho \kappa_a B + \rho \kappa_s J - \rho (\kappa_a + \kappa_s) I \quad (2.4)$$

with κ_a as the cross section for all the absorption processes, κ_s for all scattering, I is the radiation intensity, J is the mean intensity and B is the Planck function. The grey models are independent of frequency of radiation. With the substitution $dm = \rho dr$ into equation (2.4) we get

$$\mu \frac{dI}{dm} = \kappa_a B + \kappa_s J - (\kappa_a + \kappa_s) I \quad (2.5)$$

which when integrated over μ between -1 and 1 gives

$$\frac{dH}{dm} = \kappa_a(B - J). \quad (2.6)$$

Here we have neglected scattering terms. The symbol H is the **Eddington flux** and is defined as

$$H(\tau) = \frac{F}{4\pi} \quad (2.7)$$

where F is the stellar flux.

2.3 Pulsation Equations

Stellar pulsation equations are derived as linear perturbations to equations that describe a star in static equilibrium. Adiabatic processes are ones in which no heat is transferred or there is no heat exchange with the environment while non adiabatic processes are ones in which there is heat exchange with the surrounding. Pulsations are treated as linear perturbations to the equilibrium structure of a star. Linear non-adiabatic descriptions and calculations for oscillation of various model of stars have been presented (see Woltjer (1936), Lal & Bhatnagar (1956), Cox (1957), Cox (1958)). For example, Cugier et al. (1994) using linear non adiabatic approach calculated non adiabatic parameters (observables) in β Cephei stars. They used the non adiabatic parameters to determine low harmonic degree, l and the radial order (n) of the observed modes in β Cephei stars.

To obtain eigenfunctions, we consider the harmonic time dependence $e^{i\omega t}$ and spherical harmonic horizontal dependence $Y_l^m(\theta, \phi)$ for first order perturbed quantities. The eigenfrequency is made up of two parts, the real and imaginary, that is to say, $\omega = \omega_r + i\omega_i$. As given by Cugier et al. (1994), the period of oscillation is defined by

$$\Pi = \frac{2\pi}{Re(\omega)}, \quad (2.8)$$

where $Re(\omega)$ indicates the real part of the eigenfrequency. If we ignore the effects of rotation, the displacement of a mass element for a single mode of oscillation may be written in the form

$$\xi(r, \theta, \phi, t) = \xi_r \vec{a}_r + \xi_\theta \vec{a}_\theta + \xi_\phi \vec{a}_\phi, \quad (2.9)$$

where:

$$\xi_r(r, \theta, \phi, t) = a(r)Y_l^m(\theta, \phi)e^{i\omega t} \quad (2.10)$$

$$\xi_\theta(r, \theta, \phi, t) = b(r)\frac{\partial}{\partial\theta}Y_l^m(\theta, \phi)e^{i\omega t} \quad (2.11)$$

$$\xi_\phi(r, \theta, \phi, t) = \frac{b(r)}{\sin\theta}\frac{\partial}{\partial\phi}Y_l^m(\theta, \phi)e^{i\omega t}. \quad (2.12)$$

Where ξ_r , ξ_θ , ξ_ϕ are the displacement of the oscillations in the radial, latitude and longitudinal direction and ω is related to the oscillation frequency ν by $\omega = 2\pi\nu$. The spherical harmonic $Y_l^m(\theta, \phi)$ is defined as

$$Y_l^m(\theta, \phi) = \sqrt{\frac{(2l+1)}{4\pi} \frac{(l-m)!}{(l+m)!}} P_l^m(\cos\theta) e^{im\phi} \quad (2.13)$$

and the Legendre Polynomials

$$P_l^m(\cos\theta) = \frac{(-1)^m}{2^l l!} (1 - \cos^2\theta)^{\frac{m}{2}} \frac{d^{l+m}}{d\cos^{(l+m)}\theta} (\cos^2\theta - 1)^l, \quad (2.14)$$

with θ being measured from the pulsation pole.

For example the relative Lagrangian pressure perturbation is expressed as

$$\frac{\delta P}{P} = p(r)Y_l^m(\theta, \phi)e^{i\omega t}. \quad (2.15)$$

2.3.1 Radial Oscillations in the Solar Atmosphere

In the following subsections we derive the pulsation equations solved in Phorah (2007) code.

Using Cartesian coordinates and applying linear perturbation on equation (2.1) we obtain:

$$\frac{\partial(\delta\rho)}{\partial t} + \rho \frac{\partial(\delta v)}{\partial r} = 0 \quad (2.16)$$

where $\delta v = \frac{\partial\delta r}{\partial t}$.

This implies that from equation (2.16):

$$\frac{\partial}{\partial r} \frac{\partial(\delta r)}{\partial t} = -\frac{1}{\rho} \frac{\partial(\delta \rho)}{\partial t}. \quad (2.17)$$

Integrating equation (2.17) with respect to time for equilibrium star which is assumed static we obtain:

$$\frac{d\delta r}{dr} = -\frac{\delta \rho}{\rho}. \quad (2.18)$$

Equation (2.18) is the perturbed continuity equation with spherical terms neglected. Where r is a vertical coordinate or radial distance at equilibrium, δr is the displacement, ρ is the density and δ is the amplitude of the Lagrangian perturbation (perturbation following the motion; see Christensen-Dalsgaard & Frandsen (1983b) for example).

We however want to include spherical terms by considering a shell of mass dM_r , a distance r from the center of a star. Its volume is given by:

$$dV = 4\pi r^2 dr \quad (2.19)$$

and therefore its mass dM_r is given as:

$$\frac{dr}{dM_r} = \frac{1}{4\pi r^2 \rho}. \quad (2.20)$$

We then perturb equation (2.20) in such away that radius and density are expressed as

$$r = r_o + \delta r, \rho = \rho_o + \delta \rho \quad (2.21)$$

where r_o and ρ_o represent the equilibrium model quantities. δr and $\delta \rho$ are the Lagrangian perturbation in radius and density respectively. Putting equation (2.21) into equation (2.20), we obtain:

$$\frac{d\delta r}{dm} = -\frac{\delta \rho}{\rho_o} \frac{dr_o}{dm} - 2\frac{\delta r}{r_o} \frac{dr_o}{dm}. \quad (2.22)$$

Applying differentiation, we know that

$$\frac{d}{dm} \left(\frac{\delta r}{r_o} \right) = \frac{1}{r_o} \frac{d\delta r}{dm} - \frac{\delta r}{r_o^2} \frac{dr_o}{dm}. \quad (2.23)$$

Substituting equation (2.22) into equation (2.23) we obtain

$$\frac{d}{dm} \left(\frac{\delta r}{r_o} \right) = \frac{1}{r_o} \left[-\frac{\delta \rho}{\rho} \frac{dr_o}{dm} - 2 \frac{\delta r}{r_o} \frac{dr_o}{dm} \right] - \frac{\delta r}{r_o} \frac{dr_o}{dm}. \quad (2.24)$$

Equation (2.24) can further be simplified as

$$\frac{d}{dm} \left(\frac{\delta r}{r_o} \right) = -\frac{1}{r_o} \frac{dr_o}{dm} \left(\frac{\delta \rho}{\rho_o} + 3 \frac{\delta r}{r_o} \right). \quad (2.25)$$

But

$$\frac{dr_o}{dm} = \frac{1}{4\pi r_o^2 \rho_o}$$

and

$$\frac{1}{r_o} \frac{dr_o}{dm} = \frac{1}{4\pi r_o^3 \rho_o}.$$

The perturbed density is related to perturbed temperature and pressure by;

$$\frac{\delta \rho}{\rho_o} = \alpha_o \frac{\delta p_g}{p_{go}} - \delta_o \frac{\delta T}{T_o} \quad (2.26)$$

where

$$\alpha_o = \left(\frac{\partial \ln \rho_o}{\partial \ln p_{go}} \right)_{T_o}.$$

$$\delta_o = - \left(\frac{\partial \ln \rho_o}{\partial \ln T_o} \right)_{p_o}.$$

The gas pressure and perturbed turbulent pressure are related by

$$\frac{\delta p_g}{p_{go}} = \frac{1}{1 - \nu_1} \left(\frac{\delta p}{p_o} - \frac{\delta p_t}{p_o} \right). \quad (2.27)$$

Where p_g is gas pressure, p_{go} is gas pressure before applying perturbation and the perturbed gas pressure is related to perturbed turbulent pressure by $\nu_1 = \frac{p_t}{p_o}$ which is the contribution of turbulent pressure to the total pressure. Thus if there is no turbulence $\frac{\delta p}{p_o} = \frac{\delta p_t}{p_o}$. Substituting equation (2.26) into equation (2.25), we obtain

$$\frac{d}{dm} \left(\frac{\delta r}{r_o} \right) = -\frac{1}{4\pi r_o^3 \rho_o} \left[\left(\alpha_o \frac{\delta p_g}{p_{go}} - \delta_o \frac{\delta T}{T_o} \right) + 3 \frac{\delta r}{r_o} \right]. \quad (2.28)$$

And substituting equation (2.27) in equation (2.28) we get:

$$\frac{d}{dm} \left(\frac{\delta r}{r_o} \right) = -\frac{1}{4\pi r_o^3 \rho_o} \left[3 \frac{\delta r}{r_o} + \frac{\alpha_o}{1 - \nu_1} \left(\frac{\delta p}{p_o} - \frac{\delta p_t}{p_o} \right) - \delta_o \frac{\delta T}{T_o} \right]. \quad (2.29)$$

We need to make logarithm of radial pressure $\ln p_o$ the independent variable instead of radial mass m in equation (2.29). This is because the pulsation modeling codes are in the form of $\ln p_o$ not radial mass m . To do this, we need to start from the hydrostatic equilibrium equation and the mass conservation equation. The hydrostatic equilibrium equation can be expressed as

$$\frac{dp_o}{dr_o} = -\frac{GM\rho_o}{r_o^2} \quad (2.30)$$

and the mass equation is

$$\frac{dm}{dr_o} = 4\pi r_o^2 \rho \mu \quad (2.31)$$

where $\mu = \left[1 + (3 + \Phi) \frac{p_t}{\rho_o g_o r_o} \right]^{-1}$ describes the dynamical correction to the total pressure gradient due to the anisotropy of Reynolds stresses (the net rate of momentum in a fluid as a result of turbulence is not identical in all directions). Φ is a measure of the anisotropy of the turbulence (the eddy shape parameter) and is defined as

$$\Phi = \frac{k^2}{k_h^2} = 1 + \frac{k_v^2}{k_h^2} \quad (2.32)$$

where k_h and k_v are the horizontal and vertical wavenumbers with $k^2 = k_h^2 + k_v^2$. If $\Phi = 1$, the convective cell is long and thin while if $\Phi = 2$, the horizontal and vertical dimensions of the cell are equal. Thus Φ gives the shape of the convective cell. In this thesis, a constant value of eddy shape parameter, $\Phi = 5/3$ has been used since the source function (S) is considered to be isotropic.

Using the chain rule and making use of equation (2.30) and (2.31) we have;

$$\frac{dp_o}{dm} = \frac{dp_o}{dr_o} \times \frac{dr_o}{dm}. \quad (2.33)$$

Combining equation (2.30) and (2.31) and using (2.33), equation (2.33) becomes

$$\frac{dp_o}{dm} = -\frac{GM\rho_o}{4\pi r_o^4\mu}. \quad (2.34)$$

Multiplying both sides of equation (2.29) by $\frac{dm}{dp_o}$, we get;

$$\frac{dm}{dp_o} \times \frac{d}{dm} \left(\frac{\delta r}{r_o} \right) = \frac{dm}{dp_o} \times -\frac{1}{4\pi r_o^3\rho_o} \left[3\frac{\delta r}{r_o} + \frac{\alpha_o}{1-\nu_1} \left(\frac{\delta p}{p_o} - \frac{\delta p_t}{p_o} \right) - \delta_o \frac{\delta T}{T_o} \right] \quad (2.35)$$

and making use of equation (2.34), we can write equation (2.35) as

$$\frac{d}{dp_o} \left(\frac{\delta r}{r_o} \right) = -\frac{r_o\mu}{GM\rho_o} \left[3\frac{\delta r}{r_o} + \frac{\alpha_o}{1-\nu_1} \left(\frac{\delta p}{p_o} - \frac{\delta p_t}{p_o} \right) - \delta_o \frac{\delta T}{T_o} \right] \quad (2.36)$$

which can also be written as

$$\frac{d}{d\ln p_o} \left(\frac{\delta r}{r_o} \right) = -\frac{r_o\mu p_o}{GM\rho_o} \left[3\frac{\delta r}{r_o} + \frac{\alpha_o}{1-\nu_1} \left(\frac{\delta p}{p_o} - \frac{\delta p_t}{p_o} \right) - \delta_o \frac{\delta T}{T_o} \right]. \quad (2.37)$$

Equation (2.37) is the perturbed continuity equation that takes into consideration sphericity, turbulent pressure and the shape of the convective bubble. It is one of the equations used in both Houdek et al. (1995) and Phorah (2007) codes.

2.3.2 Energy equation in the Solar atmosphere

Apply perturbation to equation (2.4) and getting

$$\begin{aligned} \mu \frac{d\delta I}{dm} &= \kappa_s \delta J + \delta \kappa_s J + \kappa_a \delta B + \delta \kappa_a B \\ &\quad - (\kappa_s + \kappa_a) \delta I - (\delta \kappa_s + \delta \kappa_a) I \end{aligned} \quad (2.38)$$

according to Christensen-Dalsgaard & Frandsen (1983c), which becomes

$$\mu \frac{d\delta I}{dm} = \kappa \delta B + \delta \kappa B - \kappa \delta I - \delta \kappa I, \quad (2.39)$$

when scattering opacity is ignored.

The moments of radiative transfer for grey atmosphere are given as

$$\delta J(r) = \frac{1}{2} \int_{-1}^1 \delta I(\mu, r) d\mu \quad (2.40)$$

$$\delta H(r) = \frac{1}{2} \int_{-1}^1 \delta I(\mu, r) \mu d\mu \quad (2.41)$$

$$\delta K(r) = \frac{1}{2} \int_{-1}^1 \delta I(\mu, r) \mu^2 d\mu. \quad (2.42)$$

(Christensen-Dalsgaard & Frandsen, 1983c) Equation (2.39) can also be integrated over μ within the range $[-1, 1]$.

Thus

$$\frac{\partial \delta H}{\partial m} = \delta \kappa(B - J) + \kappa(\delta B - \delta J). \quad (2.43)$$

For the case of a plane parallel atmosphere the mass variable $dm = \rho dr$. If we substitute ρdr for dm into equation (2.43), then

$$\frac{\partial \delta H}{\partial r} = \rho[\delta \kappa(B - J) + \kappa(\delta B - \delta J)]. \quad (2.44)$$

If we multiply equation (2.39) by μ and integrate it over μ between the limits 1 and -1 we get

$$\frac{\partial}{\partial m} \left(\frac{1}{2} \int_{-1}^1 \mu^2 \delta I d\mu \right) = -\kappa \int_{-1}^1 \delta I \mu d\mu - \delta \kappa \int_{-1}^1 I \mu d\mu \quad (2.45)$$

since

$$\int_{-1}^1 \mu d\mu = 0. \quad (2.46)$$

We finally obtain:

$$\frac{d\delta K}{dm} = -(\kappa \delta H + \delta \kappa H) \quad (2.47)$$

Phorah's code uses equation (2.44) and (2.47) to describe perturbed energy transport in the model atmospheres.

From radiative transfer equation, we can also write

$$\varpi^2 \frac{dI}{d\tau} = \varpi(I_\nu - S_\nu), \quad (2.48)$$

where $\varpi = \cos\theta$, I_ν is Intensity, S_ν is source function and τ is the optical depth. Integrating equation (2.48) over all solid angles, Ω , to obtain

$$\frac{d}{d\tau} \int_{\Omega} \varpi^2 I d\Omega = \int_{\Omega} \varpi I d\Omega - \int_{\Omega} \varpi S d\Omega. \quad (2.49)$$

The left-hand integral in equation (2.49) is generally abbreviated as K (the second moment). The last integral on the right-hand side in equation (2.49) is zero, since S can be taken to be isotropic while the first integral on the right-hand side of equation (2.49) is generally written as H (Eddington flux). The optical depth $d\tau = -\kappa\rho dr$, where κ is the opacity. The Eddington flux (H) is related to radiative flux (F) as $H = F/4\pi$ and $L_r = 4\pi r^2 F$, where L_r is the radiative luminosity. Hence equation (2.49) can be written as,

$$\frac{dK}{\rho dr} = -\frac{\kappa L}{(4\pi)^2 r^2}, \quad (2.50)$$

but $dM = 4\pi r^2 \rho dr$. Thus, we obtain equation (2.51)

$$\frac{dK}{dm} = -\frac{\kappa L_r}{64\pi^3 r^4}. \quad (2.51)$$

Perturbing equation (2.51) and writing $K = K_o + \delta K$, $L_r = L_{ro} + \delta L_r$, $r = r_o + \delta r$, $\kappa = \kappa_o + \delta\kappa$, we obtain:

$$\frac{d}{dm}(K_o + \delta K) = -\frac{[(\kappa_o + \delta\kappa)(L_{ro} + \delta L_r)]}{64\pi^3 (r_o + \delta r)^4}. \quad (2.52)$$

Upon ignoring higher order perturbations, we get:

$$\frac{d\delta K}{dm} = -\frac{\kappa_o L_{ro}}{64\pi^3 r_o^4} \left[-4\frac{\delta r}{r_o} + \frac{\delta L_r}{L_{ro}} + \frac{\delta\kappa}{\kappa_o} \right]. \quad (2.53)$$

The opacity derivatives κ_T and κ_p are given as $\kappa_T = \left(\frac{\partial \ln \kappa_o}{\partial \ln T_o} \right)_{p_{go}}$ and $\kappa_p = \left(\frac{\partial \ln \kappa_o}{\partial \ln p_{go}} \right)_{T_o}$. The perturbed opacity is given by

$$\frac{\delta\kappa}{\kappa_o} = \kappa_T \frac{\delta T}{T_o} + \kappa_p \frac{\delta p_g}{p_{go}} \quad (2.54)$$

The perturbed opacity is given by equation (2.54), but we also know that

$$\frac{d}{dm} \left(\frac{\delta K}{K_o} \right) = \frac{1}{K_o} \frac{d\delta K}{dm} - \frac{\delta K}{K_o^2} \frac{dK_o}{dm}. \quad (2.55)$$

Then substituting equation (2.53) into equation (2.55), we get

$$\frac{d}{dm} \left(\frac{\delta K}{K_o} \right) = \frac{1}{K_o} \left\{ -\frac{\kappa_o L_{ro}}{64\pi^3 r_o^4} \left[-4\frac{\delta r}{r_o} + \frac{\delta L_r}{L_{ro}} + \frac{\delta \kappa}{\kappa_o} \right] \right\} - \frac{\delta K}{K_o^2} \frac{dK_o}{dm}. \quad (2.56)$$

Using

$$\frac{dK_o}{dm} = -\frac{\kappa_o L_{ro}}{64\pi^3 r_o^4}$$

then equation (2.56) can be written as

$$\frac{d}{dm} \left(\frac{\delta K}{K_o} \right) = -\frac{\kappa_o L_{ro}}{64\pi^3 r_o^4 K_o} \left[-4\frac{\delta r}{r_o} + \frac{\delta L_r}{L_{ro}} + \frac{\delta \kappa}{\kappa_o} \right] - \frac{\delta K}{K_o^2} \times \left[-\frac{\kappa_o L_{ro}}{64\pi^3 r_o^4} \right]. \quad (2.57)$$

Equation (2.57) can also be simplified to

$$\frac{d}{dm} \left(\frac{\delta K}{K_o} \right) = -\frac{\kappa_o L_{ro}}{64\pi^3 r_o^4 K_o} \left[-4\frac{\delta r}{r_o} + \frac{\delta L_r}{L_{ro}} + \frac{\delta \kappa}{\kappa_o} - \frac{\delta K}{K_o} \right]. \quad (2.58)$$

Substituting for the perturbed opacity (equation (2.54)), then equation (2.58) becomes

$$\frac{d}{dm} \left(\frac{\delta K}{K_o} \right) = -\frac{\kappa_o L_{ro}}{64\pi^3 r_o^4 K_o} \left[-4\frac{\delta r}{r_o} + \frac{\delta L_r}{L_{ro}} + \kappa_T \frac{\delta T}{T_o} + \kappa_p \frac{\delta p_g}{p_{g_o}} - \frac{\delta K}{K_o} \right] \quad (2.59)$$

multiplying both sides of equation (2.59) by $\frac{dm}{dp_o}$,

and using

$$\frac{dm}{dp_o} = \frac{-4\pi r_o^4 \mu}{GM}$$

we obtain:

$$\frac{d}{dp_o} \left(\frac{\delta K}{K_o} \right) = -\frac{\mu \kappa_o L_{ro}}{16\pi^2 GM K_o} \left[-4\frac{\delta r}{r_o} + \frac{\delta L_r}{L_{ro}} + \kappa_T \frac{\delta T}{T_o} + \kappa_p \frac{\delta p_g}{p_{g_o}} - \frac{\delta K}{K_o} \right] \quad (2.60)$$

which can be written as:

$$\frac{d}{d \ln p_o} \left(\frac{\delta K}{K_o} \right) = -\frac{\mu \kappa_o L_{ro} p_o}{16\pi^2 G M K_o} \left[-4 \frac{\delta r}{r_o} + \frac{\delta L_r}{L_{ro}} + \kappa_T \frac{\delta T}{T_o} + \kappa_p \frac{\delta p_g}{p_{g_o}} - \frac{\delta K}{K_o} \right]. \quad (2.61)$$

Equation (2.61) becomes

$$\begin{aligned} \frac{d}{d \ln p_o} \left(\frac{\delta K}{K_o} \right) = & -\frac{\mu \kappa_o L_{ro} p_o}{16\pi^2 G M K_o} \left[-4 \frac{\delta r}{r_o} + \frac{\delta L_r}{L_{ro}} + \kappa_T \frac{\delta T}{T_o} + \frac{\kappa_p}{1 - \nu_1} \right. \\ & \left. \left(\frac{\delta p}{p_o} - \frac{\delta p_t}{p_o} \right) - \frac{\delta K}{K_o} \right] \end{aligned} \quad (2.62)$$

using the fact that $\delta L = \delta L_r + \delta L_c$; $\delta L - \delta L_c = \delta L_r$ and writing $L_{ro} = (1 - f)L_o$, where L_r is radiative luminosity, L_c is convective luminosity, L_{ro} is radiative luminosity at the equilibrium before perturbation, L_o is total luminosity at the equilibrium before perturbation, f is the contribution of convective luminosity to the total luminosity ($f = \frac{L_{co}}{L_o}$), then equation (2.62) can be written as

$$\begin{aligned} \frac{d}{d \ln p_o} \left(\frac{\delta K}{K_o} \right) = & -\frac{\mu \kappa_o (1 - f) L_o p_o}{16\pi^2 G M K_o} \left[-4 \frac{\delta r}{r_o} + \frac{\delta L - \delta L_c}{(1 - f) L_o} + \kappa_T \frac{\delta T}{T_o} + \right. \\ & \left. \frac{\kappa_p}{1 - \nu_1} \left(\frac{\delta p}{p_o} - \frac{\delta p_t}{p_o} \right) - \frac{\delta K}{K_o} \right] \end{aligned} \quad (2.63)$$

and finally

$$\begin{aligned} \frac{d}{d \ln p_o} \left(\frac{\delta K}{K_o} \right) = & -\frac{\mu \kappa_o (1 - f) L_o p_o}{16\pi^2 G M K_o} \left[-4 \frac{\delta r}{r_o} + \frac{1}{(1 - f)} \left(\frac{\delta L}{L_o} - \frac{\delta L_c}{L_o} \right) + \right. \\ & \left. \kappa_T \frac{\delta T}{T_o} + \frac{\kappa_p}{1 - \nu_1} \left(\frac{\delta p}{p_o} - \frac{\delta p_t}{p_o} \right) - \frac{\delta K}{K_o} \right] \end{aligned} \quad (2.64)$$

where $K_o = f_{eq} J_o$, J_o is zeroth order moment and f_{eq} is the equilibrium Eddington factor defined as $f_{eq} = K/J$.

$$\begin{aligned} \frac{d}{d \ln p_o} \left(\frac{\delta K}{K_o} \right) = & -\frac{\mu \kappa_o (1 - f) L_o p_o}{16\pi^2 G M (f_{eq} J_o)} \left[-4 \frac{\delta r}{r_o} + \frac{1}{(1 - f)} \left(\frac{\delta L}{L_o} - \frac{\delta L_c}{L_o} \right) + \right. \\ & \left. \kappa_T \frac{\delta T}{T_o} + \frac{\kappa_p}{1 - \nu_1} \left(\frac{\delta p}{p_o} - \frac{\delta p_t}{p_o} \right) - \frac{\delta K}{K_o} \right]. \end{aligned} \quad (2.65)$$

Equation (2.65) shows the perturbed second order moment of transfer equation. Equation (2.65 is the same as Houdek's one (Houdek, 1996) given by

$$\begin{aligned} \frac{d}{d \ln p_o} \left(\frac{\delta J}{J_o} \right) = & -\frac{\mu \kappa_o (1-f) L_o p_o}{16 \pi^2 G M (f_{eq} J_o)} \left[-4 \frac{\delta r}{r_o} + \frac{1}{(1-f)} \left(\frac{\delta L}{L_o} - \frac{\delta L_c}{L_o} \right) + \right. \\ & \left. \kappa_T \frac{\delta T}{T_o} + \frac{\kappa_p}{1-\nu_1} \left(\frac{\delta p}{p_o} - \frac{\delta p_t}{p_o} \right) - \frac{\delta J}{J_o} \right] \end{aligned} \quad (2.66)$$

Medupe (2002) later investigated the relationship between $\frac{\delta T}{T_o}$ and $\frac{\delta J}{J_o}$ and found out that $4 \frac{\delta T}{T_o} = \frac{\delta J}{J_o}$ for large optical depths for purely radiative case.

Equation (2.65) was first derived by Phorah (2007) and used in his pulsation code. Houdek (1996) did not make use of $\frac{\delta K}{K_o}$ since he used Eddington approximation (see (equation 2.66)). In the following paragraph, I shall present other pulsation equations for non local mixing length theory of convection and consistent treatment of radiation as used in the code by Phorah (2007). From the perturbed equation of motion, one obtains

$$\begin{aligned} \frac{d}{d \ln p_o} \left(\frac{\delta p}{p_o} \right) = & -\frac{\delta p}{p_o} - \left(3 + \mu \left[1 + \frac{\omega^2}{\Omega^2} \right] \right) \frac{\delta r}{r_o} + \mu \nu_1 \frac{p_o r_o}{G m \rho_o} [(3 - \Phi_o) \times \\ & \left[\delta_o \frac{\delta T}{T_o} - \frac{\alpha_o}{1-\nu_1} \left(\frac{\delta p}{p_o} - \frac{\delta p_t}{p_o} \right) + \frac{1}{\nu_1} \frac{\delta p_t}{p_o} \right] + \delta \Phi], \end{aligned} \quad (2.67)$$

while the perturbed luminosity is given by equation (2.68)

$$\frac{d}{d \ln p_o} \left(\frac{\delta L}{L_o} \right) = i \omega \mu \frac{4 \pi r_o^4 p_o^2 \delta_o}{G m \rho_o L_o} \left[\frac{\delta p_t}{\delta p_o} - \frac{\delta p}{p_o} + \frac{1-\nu_1}{\nabla_{ad}} \frac{\delta T}{T_o} \right] \quad (2.68)$$

The pulsation modeling codes produce the output $\frac{\delta r}{r_o}$, $\frac{\delta p}{p_o}$, $\frac{\delta T}{T_o}$ and $\frac{\delta L}{L_o}$ for both the real and imaginary parts.

The new code uses equations ((2.37), (2.65), (2.67) and (2.68) while the old code uses equations (2.37), (2.66), (2.67) and (2.68).

2.3.3 Boundary conditions for consistent radiation treatment

The boundary conditions presented here are those previously used by Houdek et al. (1995), Houdek (1996) and Phorah (2007).

- At the surface:

- The first boundary condition at the outer layer of the atmosphere is to set the displacement to 1;

$$\frac{\delta r}{r_o} = 1, \quad (2.69)$$

at $\tau = \tau_{atmosphere}$.

- Furthermore, at surface ($\tau = \tau_{atmosphere}$, at $\tau \sim 10^{-4}$) we impose this boundary condition:

$$\frac{\delta p}{p_o} = - \left(4 + \frac{\omega^2}{\Omega^2} \right) \frac{\delta r}{r_o}, \quad (2.70)$$

where $\Omega^2 = \frac{Gm}{r_o^3}$. This boundary condition is applied at a suitable point in the atmosphere, for example at a temperature minimum. It takes into account the possibility that waves can propagate out (Baker & Kippenhahn, 1965).

- The surface mechanical boundary condition in equation (2.70) is modified when solving the non-adiabatic pulsation equation with local mixing length theory of convection such that it is of the form given by equation (2.76).

$$\frac{\delta p}{p_o} = \Gamma_1 \lambda_- \delta \tilde{r} \quad (2.71)$$

where $\delta \tilde{r} = \delta r / H_p$. This boundary condition applies only at the surface of a star where

$$\lambda_- = \frac{1}{2} - \frac{1}{2} \left(1 - \frac{\omega^2}{\omega_{ac}^2} \right)^{1/2}. \quad (2.72)$$

ω_{ac} is acoustic cut-off frequency and at the surface $\delta \tilde{r} = 1$. Waves with frequencies below ω_{ac} are trapped whereas those with higher frequencies dissipates. For $\omega < \omega_{ac}$, the wave is reflected back inwards while if $\omega > \omega_{ac}$, the wave leaks through the boundary.

- At the base of the envelope (at a radius fraction $r/R_\odot = 0.2$):
 - The displacement is set to zero at the base of the envelope where the equilibrium model is truncated. This has the effect of creating a node in the displacement and makes the whole problem eigenvalue problem.

$$\frac{\delta r}{r_o} = 0. \quad (2.73)$$

- Also we require the oscillations to be adiabatic at the base of the envelope, hence we have;

$$\frac{\delta T}{T_o} = \nabla_{ad} \frac{\delta p}{p_o}. \quad (2.74)$$

Where p_o is the equilibrium pressure.

- Additional boundary conditions to solve non-adiabatic pulsation equations are given by the thermal outer boundary condition given by (2.75).

$$\frac{\delta L}{L_o} = 2 \left[1 + \frac{\tau_o}{\tau_o + \frac{2}{3}} \right] \frac{\delta r}{r_o} + \left[4 - \frac{\tau_o \kappa_T}{\tau_o + \frac{2}{3}} \right] \frac{\delta T}{T_o} - \left[\frac{\tau_o}{\tau_o + \frac{2}{3}} \right] \kappa_p \frac{\delta p}{p_o} \quad (2.75)$$

This is a thermal boundary condition which is obtained by perturbing the temperature - optical depth relation. However, because one needs to look at the stellar atmosphere in detail, the boundary condition that there is no incoming radiation at the surface has to be used (Medupe, 2002). This is the reason for setting up this boundary condition.

- Convective flux and turbulent pressure conditions
 - In addition to the boundary conditions given by equation (2.69), (2.73), (2.70) and (2.75), one needs other boundary conditions in order to obtain solution for the linear non - adiabatic pulsation equation using non - local mixing length theory of convection. The additional boundary conditions are given by equation (2.76), and (2.77). This is because for the bubble to continue rising, the temperature difference between the bubble and the surrounding should be positive. To accomplish

this, the boundary conditions arising from non-local mixing-length theory must be imposed above (+) and below (-) the convective instability region. This is only possible by setting the convective flux and turbulent pressure as in equation (2.76) and (2.77) respectively.

$$\frac{a}{\alpha_c} \frac{\delta L_c}{L_o} = \pm \frac{d}{d \ln p_o} \left(\frac{\delta L_c}{L_o} \right), \quad (2.76)$$

$$\frac{a}{\alpha_c} \frac{\delta p_t}{p_o} = \pm \frac{d}{d \ln p_o} \left(\frac{\delta p_c}{p_o} \right). \quad (2.77)$$

– Super-adiabatic lapse rate condition

- * Furthermore, at the upper (+) and lower (-) boundaries of the convection zone, the average superadiabatic temperature gradient must be the same as the local superadiabatic temperature gradient. For this to happen, another boundary condition is needed by setting:

$$\delta \bar{\beta} = \delta \beta \quad (2.78)$$

above and below the convective instability region. In the equation (2.76), (2.77), (2.78), a is the non-local mixing length parameter defined in section (2.2.1), α_c is the mixing length parameter, β is the super-adiabatic lapse rate and $\bar{\beta}$ is the average super-adiabatic lapse rate.

2.4 Solutions to the pulsation equations

In this section I present solutions to the pulsation equations according to the old and new codes as discussed in section 2.3. The input parameters used in the calculations are given in table 2.1.

Table 2.1: *The input parameters used in the calculations of the solutions to pulsation equations. The columns are: (1) Mass, (2) Effective temperature, (3) Radius, (4 - 5) Non-local mixing length parameters a & b , (6) Mixing length parameter, α , (7) logarithm of surface gravity, $\log g$.*

Mass ($M_{\odot}$)	T_{eff} (K)	Radius ($R_{\odot}$)	a^2	b^2	α	$\log g$
1.00	5778	1.00	300	300	2.0	4.437

The eigenfunctions considered are: displacement eigenfunction $\frac{\delta r}{r}$, temperature eigenfunction $\frac{\delta T}{T}$ and pressure eigenfunction $\frac{\delta P}{P}$. The energy transport mechanisms (processes) are: (a) Full radiation treatment + convection (new code) and (b) Convection + Eddington's approximation (old code). The first attempt was to ensure that all the equilibrium models and the physical parameters used in the two approaches were the same for easy and proper comparison. The boundary conditions and testing of the codes were done. The output ($\frac{\delta r}{r}$, $\frac{\delta T}{T}$) of the two pulsation codes were both function of depth ($\log p$). It is important to note the range that corresponds to the Sun's atmosphere. It ranges from round about $2 < \log p < 4.5$ (Phorah, 2007). The results of the effects of the two energy transport mechanism are as shown in the form of graphs and discussed hereafter.

2.4.1 Explanation of the pulsation modeling codes

The two pulsation modeling codes which include convection treatment have been kindly provided to me by Phorah Motee William. One of the code includes non local mixing length convection and Eddington's approximation to radiation while the other one includes non local mixing length convection and full radiation treatment. The codes solve the non - adiabatic pulsation equations presented in section 2.3 and were used by Phorah (2007). In this subsection, I shall give a brief method how the two codes work. Three main steps are carried out;

- (a). The linear adiabatic pulsation equations are solved to provide trial solution for the next step
- (b). This is followed by obtaining solution of linear non - adiabatic pulsation equations using local mixing - length theory of convection.
- (c). Finally the non - adiabatic pulsation equations are solved using non - local mixing

- length theory of convection and results in terms of eigenfunctions are obtained.

2.4.2 Running of the modeling codes, input and output parameters

The steps taken in solving the pulsation equations are those used by Phorah (2007):

- (i). Make initial guess of $f_{osc} = f_{eq}$ (f_{eq} is the equilibrium Eddington factor).
- (ii). The pulsation equations are solved by replacing $\delta J/J_o$ by $\frac{f_{eq}}{f_o} \frac{\delta K}{K_o}$ for the first iteration. In the case of the old code, we retain $\delta J/J_o$.
- (iii). We obtain $\delta\kappa$ and δB from $\delta T/T_o$ and $\delta P/P_o$ calculated in step (ii).
- (iv). Compute δJ and δK and then get f_{osc} .
- (v). If f_{osc} has not converged, the process goes back to step (ii) until f_{osc} converges.
- (vii). The final output will be amongst others, both the imaginary and real parts of $\frac{\delta T}{T_o}$, $\frac{\delta P}{P_o}$, $\frac{\delta L}{L_o}$. We are interested in the norm of $\frac{\delta L}{L_o}$.

2.4.3 Displacement eigenfunction

The displacement eigenfunction as a function of $\log P$ is shown in Figure 2.1 for the two codes. As is expected for p-modes, the $\delta r/r$ is largest near the surface. The two codes give identical solutions. This solution is used to determine velocity amplitude, $\Delta V = \omega_r \delta r$.

Figure 2.1 is a plot of displacement eigenfunction $\frac{\delta r}{r}$ vs $\log p$ (depth). In Figure 2.1 (a) we observe a clear ‘bump’ at round about $\log p = 5$ for all the n values (indicator of frequency). The higher the n values the higher the frequency and vice versa. The ‘bump’ becomes clearer as the n value increases. In Figure 2.1 (b), the ‘bumps’ are still seen but not as pronounced as in (a). The most significant observation in addition to the above is that the higher the n values, the more clearer the bump becomes and the steeper the gradient in the atmosphere. The ‘bump’ in Figure 2.1 seems to occur at roughly the same region as the one seen in the temperature eigenfunction in Figure 2.2. It appears that it is the region corresponding to the hydrogen ionization zone and perhaps it is due to opacity fluctuations and radiation being treated fully and partially

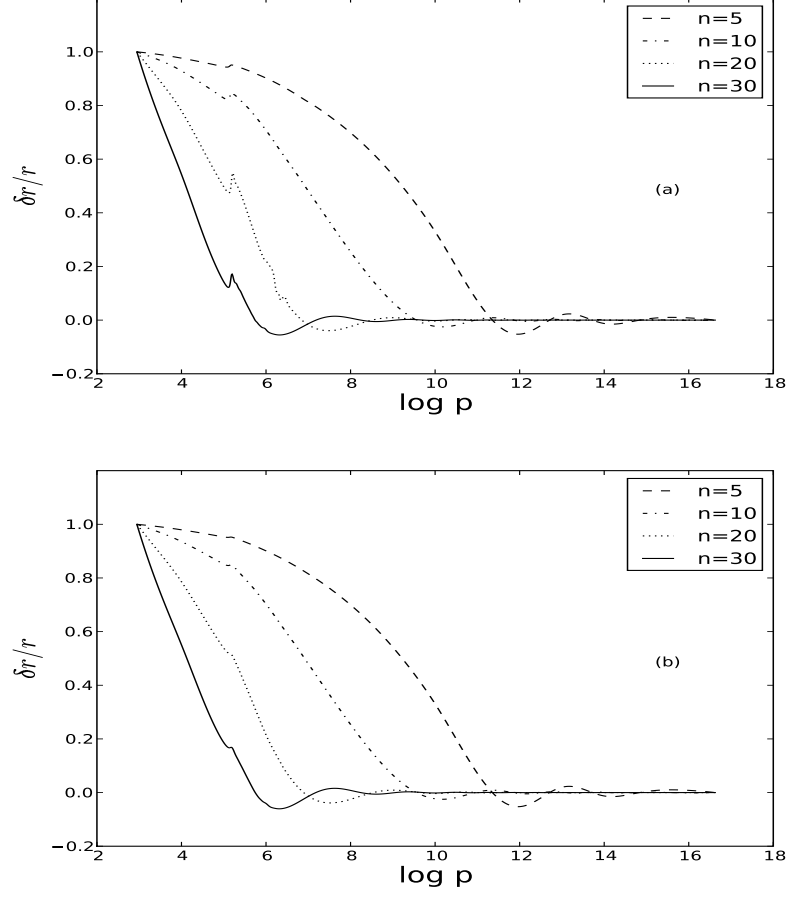


Figure 2.1: The real part of $\delta r/r$ vs $\log p$ for (a) new code, (b) old code.

in Figure 2.1 (a) and (b) respectively.

2.4.4 Temperature perturbations

Figure 2.2 is a plot of temperature eigenfunction $\frac{\delta T}{T}$ vs depth ($\log p$). Again there are some striking observations which are noticed in Figure 2.2 (a) and (b). At about $\log p = 5$, we observe a sharp deep spike ('bump') in (a) for the plot of $\frac{\delta T}{T}$ vs $\log p$ while the spike is shorter in (b) compared to that in (a). We attribute the deep sharp spike ('bump') in (a) due to full radiation treatment while the shallower 'bump' in (b) is due to the fact that radiation was not fully treated. It was approximated with the Eddington's approximation. Medupe et al. (2009) explained the 'bump' often seen in

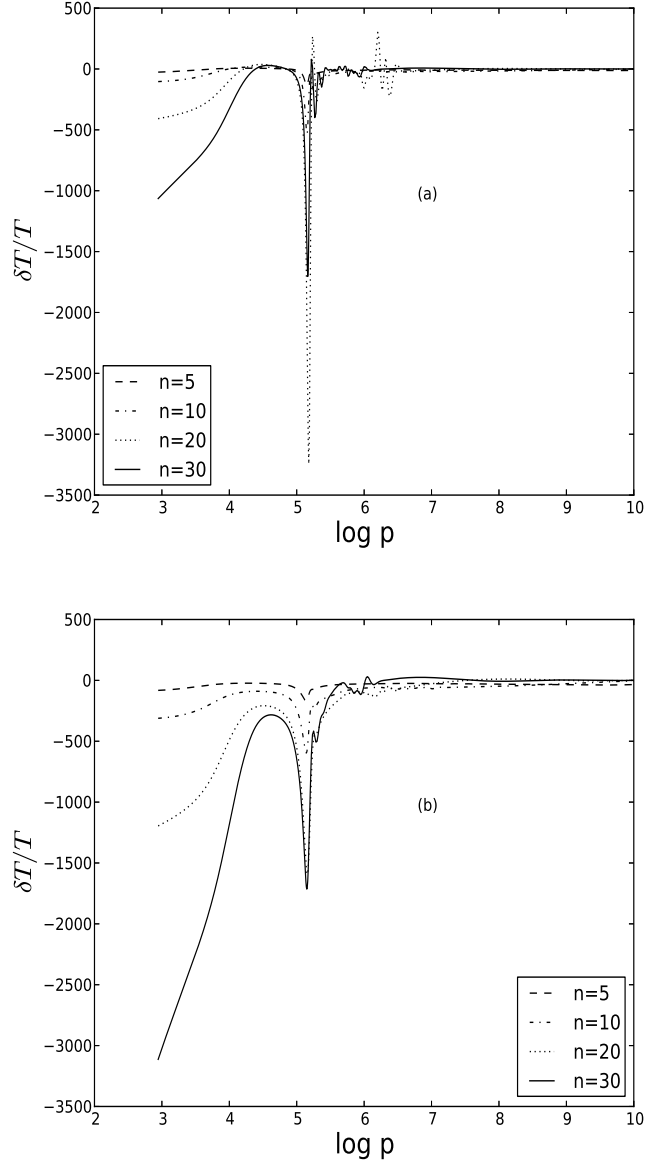


Figure 2.2: The real part of $\delta T/T$ vs $\log p$ for (a) new code, (b) old code. We observe numerical instabilities in the plot of $\delta T/T$ vs $\log p$ with both pulsation modeling codes in the range of $\log p = 5.0 - 6.5$.

the temperature eigenfunction in the region of the hydrogen ionization zone as being due to opacity fluctuations in the atmosphere of the Sun.

2.4.5 Perturbations in pressure

Figure 2.3 is a plot of pressure eigenfunction $\frac{\delta P}{P}$ vs depth ($\log p$). At about $\log p = 6$, we observe a plateau in both (a) and (b) for higher order modes. The plateau disappears for lower degree modes.

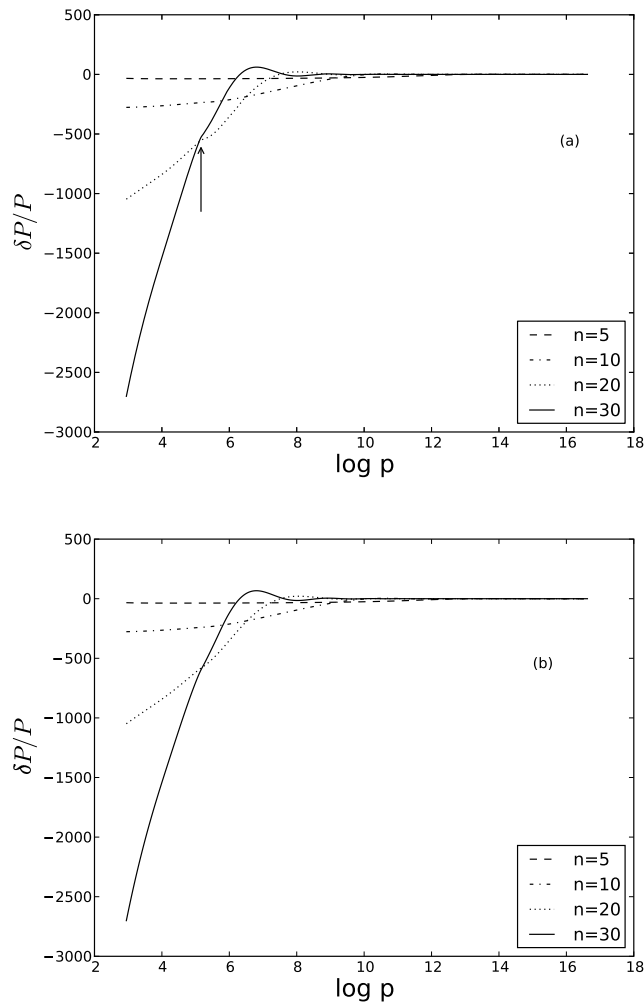


Figure 2.3: The real part of $\delta P/P$ vs $\log p$ for (a) new code, (b) old code. Notice the little ‘bump’ seen in (a) and not visible in (b). The arrow indicates the position of photospheric layer.

2.4.6 Amplitude ratios (solar luminosity and velocity amplitudes)

Balmforth (1992) first presented the linearized pulsation equations for non-adiabatic radial oscillations that include non local turbulent convection. Later on, Houdek et al. (1999) used the pulsation equations presented by Balmforth (1992) to derive and define theoretical luminosity - velocity amplitude ratios as

$$\frac{\Delta L_s}{\Delta V} := \frac{\delta L/L}{\omega_r r \frac{\delta r}{r}}, \quad (2.79)$$

where ω_r is the (real) pulsation frequency, $\delta r/r$ is the displacement eigenfunction, ΔV is the velocity amplitude and ΔL_s is the theoretical intensity amplitude. The phase difference is expressed according to Houdek et al. (1999) as

$$\varphi_L - \varphi_V = -\tan^{-1} \left(\frac{\Delta L_s}{\Delta V} \right). \quad (2.80)$$

From equation (2.79), the theoretical intensity amplitude is given as

$$\Delta L_s = \delta L/L \quad (2.81)$$

and the velocity amplitude

$$\Delta V = \omega_r r \frac{\delta r}{r}, \quad (2.82)$$

I used equations (2.79) and (2.80) to compute theoretical amplitude ratios and phase shifts respectively because the modeling codes provide us with the opportunity of obtaining $\delta L/L$ without the need of relating it to flux perturbation $\delta F/F$ and displacement eigenfunction $\delta r/r$. The importance (advantage) of equation (2.79) is that it allows direct comparison with observations without the need of a specific excitation model and all the uncertainties associated in describing the turbulence spectrum. In fact, equations (2.79) and (2.80) provide a useful test of pulsation theory independent of excitation model (Houdek et al., 1995; Houdek, 2010). In this thesis, I did not delve into the stochastic excitation treatment for the reason mentioned.

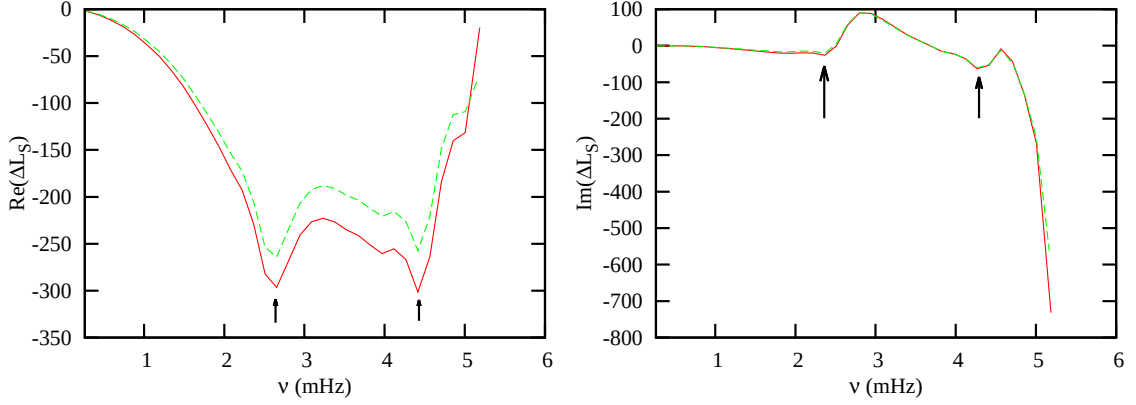


Figure 2.4: *Left panel: Real part of the surface luminosity eigenfunction versus pulsation frequency evaluated at the outer mesh point. The dashed line corresponds to the new code while solid line corresponds to the old code. Right panel: Imaginary part of the surface luminosity eigenfunction versus pulsation frequency evaluated at the outer mesh point. The dotted line corresponds to the new code while solid line corresponds to the old code. The arrows show the depressions.*

Figure 2.4 is a plot of the real part of the surface luminosity eigenfunction versus pulsation frequency evaluated at the outer mesh point (left panel). The dashed line corresponds to the new code while solid line corresponds to the old code. The right panel of Figure 2.4 is a plot of the imaginary part of the surface luminosity eigenfunction versus pulsation frequency evaluated at the outer mesh point (the dotted line corresponds to the new code while solid line corresponds to the old code). There are depressions being seen in both the plot of real and imaginary parts of the surface luminosity eigenfunction versus pulsation frequency indicated by the arrows.

Figure 2.5 is a plot of the norm (magnitude) of surface luminosity eigenfunction versus pulsation frequency evaluated at the outer mesh point ($\tau = 10^{-4}$). There are peaks in $\delta L/L$ at frequencies 2.5 and 4.5 mHz (Figure 2.5 left panel). The peaks in the amplitudes ($\delta L/L$) resulting from the depression in the damping rates, which may be an

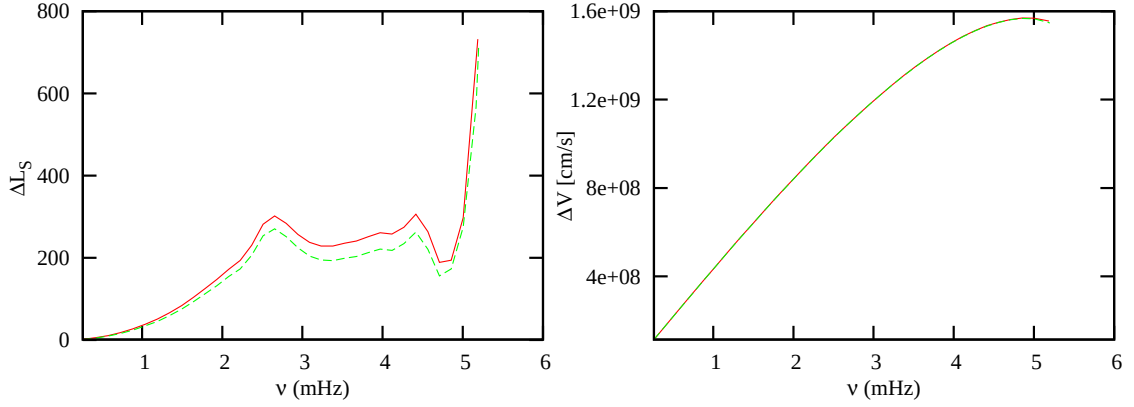


Figure 2.5: *Left panel: The norm (magnitude) of the surface luminosity eigenfunction versus pulsation frequency evaluated at the outer mesh point ($\tau \sim 10^{-4}$). The dashed line corresponds to the new code while solid line corresponds to the old code. There are peaks at frequencies 2.5 and 4.5 mHz in the left panel. Right panel: The velocity evaluated at a height of 200 km above the photosphere.*

artifact of the time dependent mixing length formalism and from incomplete treatment of the non adiabatic effects. Such peaks were first noticed by Houdek (1996) in the study of η Boo.

Figure 2.6 shows the theoretical results for the amplitude ratios and phase shifts for frequency range 0 - 6 mHz calculated at height of 200 km (this is the height of formation of the potassium line in Solar atmosphere). The amplitude ratios and phase shifts calculations in Figure 2.6 are 1:1 anti-correlated because the phase differences are being measured with respect to the downward velocity instead of the upward velocity, otherwise if the phase differences were being measured with respect to the upward velocity, the plot in Figure 2.6 would be 1:1 correlated (Houdek et al., 1999; Schrijver et al., 1991). In addition, for small value of the amplitude ratios, i.e., $\tan(x) \sim x$ for small x , thus $x = \Delta L_s / \Delta V$ is $\ll 1$. Therefore, according to equation (2.80); $\varphi_L - \varphi_V \simeq -\frac{\Delta L_s}{\Delta V}$. This explains why the right hand panel of Figure 2.6 is negative of the left hand panel.

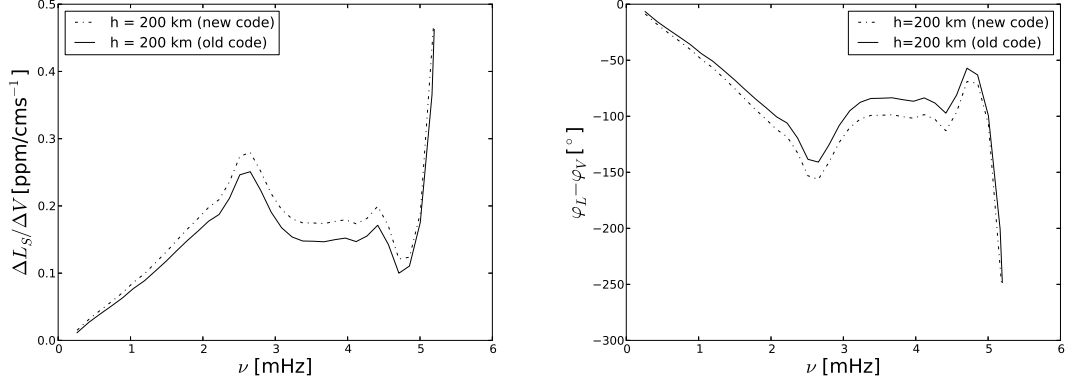


Figure 2.6: Left panel: amplitude ratio at height $h = 200$ km above the photosphere. The dotted line corresponds to the new code while solid line corresponds to the old code. Right panel: phase shift at height $h = 200$ km above the photosphere. The dotted line corresponds to the new code while solid line corresponds to the old code.

In Figures 2.7 and 2.8 the old and new codes are compared. In order to determine by how much the new code differs with the old code, I calculated the ratio of the amplitude ratio determined using the new code to that calculated using the old code. This ratio, which I call $A_{L-V}(\text{Radiation})/A_{L-V}(\text{Eddington})$ is shown in Figure 2.7 and differs by up to 18 % between the two pulsation codes. The same results apply for the phase differences as shown in Figure 2.8. Thus, using the old pulsation codes which approximates radiation transport in stellar atmosphere under-estimates the amplitude ratios and phase differences by approximately 18 %.

It is also noted that Figures 2.7 and 2.8 are nearly identical as a result of the way the phase differences are measured. It should be clearly pointed out that the importance of Figures 2.7 and 2.8 are to only compare the results of the theoretical calculations of amplitude ratios and phase shifts with the new and old codes.

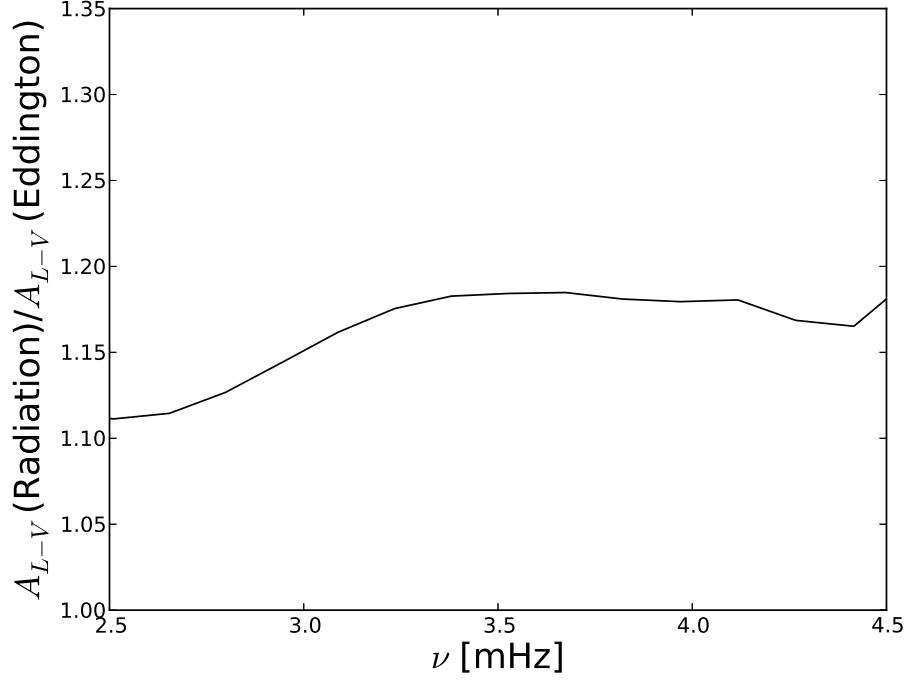


Figure 2.7: The ratio of the amplitude ratio between the surface luminosity and velocity with the new code to the amplitude ratio between the surface luminosity and velocity with the old code, that is to say, $A_{L-V}(\text{Radiation})/A_{L-V}(\text{Eddington Approximation})$. Results for frequency range from 2.5 - 4.5 mHz (5 minutes range).

2.4.7 Data and theory

2.4.7.1 Amplitude ratios and phase shifts data descriptions

In this section, the data which Houdek et al. (1995) and Houdek (1996) used and the one that will be used in this thesis is presented. The data comprise of irradiance (luminosity) measurements from the InterPlanetary Helioseismology by Irradiance (IPHIR) instrument flown on the PHOBOS 2 mission to Mars. The PHOBOS 2 spacecraft was a satellite of the Soviet Union which was launched in mid 1988 after PHOBOS 1. The irradiance data were obtained over 155 days in the same year when the PHOBOS 2 was launch. The IPHIR instrument had 3 Sunphotometers which observed the solar irradiance at three different wavelengths. The irradiance data obtained from the IPHIR in-

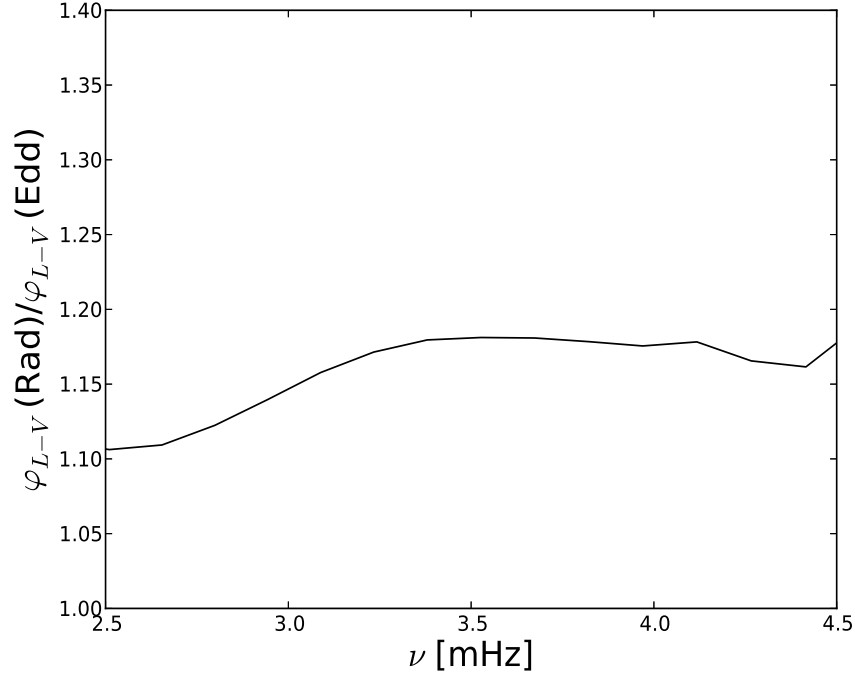


Figure 2.8: The phase shifts between the surface luminosity and velocity with the new code to the phase shifts between the surface luminosity and velocity with the old code, that is to say, ϕ_{L-V} (Radiation)/ ϕ_{L-V} (Eddington Approximation). Results for frequency range from 2.5 - 4.5 mHz (5 minutes range).

strument is easily accessible and available in the literature. Velocity data were obtained at Tenerife during four intervals within 155 days period of observation of PHOBOS 2. Based on the irradiance and velocity data, phase difference (phase shift) between irradiance and velocity were obtained & the ratio of amplitude between irradiance and velocity were also obtained by Schrijver et al. (1991). The intensity - velocity phase difference is defined as: $\Delta\phi_{IV} = \phi_I - \phi_V$, where ϕ_I is the intensity phase and ϕ_V is the velocity phase. The negative phase difference (shift) means that intensity trails downward velocity (Schrijver 1991). The data extracted was for coherence equal or more than 0.7. This was because any data below 0.7 had poor signal to noise ratio and the uncertainties, scatter in the data were large.

In 1988, the Union of Soviet Socialist Republics (USSR) mission to the Martian satel-

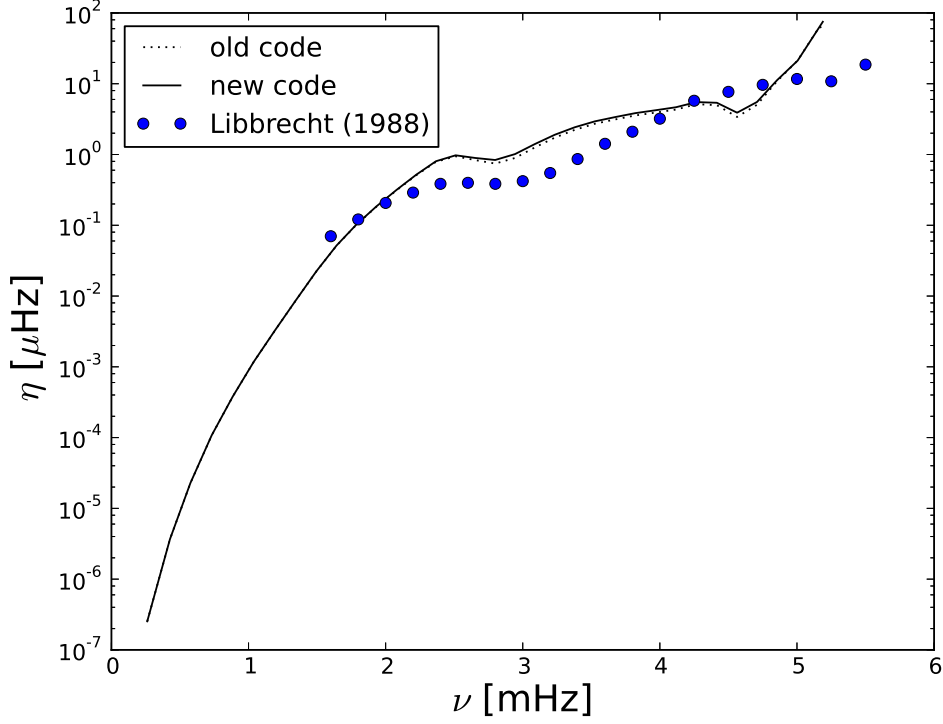


Figure 2.9: *The theoretical damping rate as a function of frequency showing a depression at $\nu = 2.6$ mHz. The filled circles are data obtained from Libbrecht (1988). There is agreement between the old and new codes in the damping rate - frequency plot.*

lite PHOBOS was launched. The primary objective of the mission was to explore Mars by looking at its two moons, Phobos (Phobos 1 and 2). In addition to exploring the Martian satellites, Phobos 2 mission also carried instruments to study the Sun, the interplanetary medium, and gamma-ray burst sources (Schrijver et al., 1991). The most remarkable use of PHOBOS 2 mission for the Sun was that it offered opportunity to measure amplitude ratios ($\Delta L_s / \Delta V$) and phase shifts ($\Delta \phi$) using space intensity data of the IPHIR instruments. However, there was a need to model the data theoretically in order to provide proper understanding of the solar physics.

2.4.7.2 Comparison of the theoretical results with the data

In section 2.4.6 I discussed how the theoretical amplitude ratios and phase differences are calculated from pulsation codes. In this subsection I seek to compare theoretical amplitude ratios and phase differences with observed data. I have described how the data was obtained in section 2.4.7.1. Figure 2.9 is a plot of the theoretical damping rate as a function of frequency showing a depression at around $\nu = 2.5$ and 4.5 mHz. The filled circles are data obtained from Libbrecht (1988). There is agreement between the old and new codes in the damping rate - frequency plot. There is an agreement between the observational data and the theoretical calculation at low frequency ($\sim 1.5 - 2$ mHz). However, at high and intermediate frequency ranges, there seems to be disagreement between observational data and the theoretical calculations.

Figure 2.10 shows the running mean amplitude ratios of the data together with our theoretical calculation obtained with old and new codes. Below 2.5 mHz, the theoretical results and the observation totally disagree. However, according to Schrijver et al. (1991), the data below 2.5 mHz are not reliable since the gain was affected by noise. Comparisons are made for 2.5 - 4 mHz (5 mins range). Constant amplitude ratios are obtained between 3 - 4 mHz.

Mean value of amplitude ratios of $0.207 \text{ ppm s cm}^{-1}$ with the new code for frequency between 2.5 and 4.0 mHz was obtained while a mean value of amplitude ratios of $0.235 \text{ ppm s cm}^{-1}$ was obtained from the observational data of Schrijver et al. (1991) in the same frequency range (Figure 2.10, 2.11 and 2.12). The mean value of amplitude ratios of $0.180 \text{ ppm s cm}^{-1}$ was obtained with the old code for frequency between 2.5 and 4.0 mHz while a mean value of amplitude ratios of $0.235 \text{ ppm s cm}^{-1}$ was obtained from the observational data of Schrijver et al. (1991) in the same frequency range. These discussions are limited by the fact that it is difficult to estimate errors in our models.

In the top panel of Figure 2.11 I plot the theoretical amplitude ratios using both old and new codes against the data of the Sun. It is difficult to determine which of the two

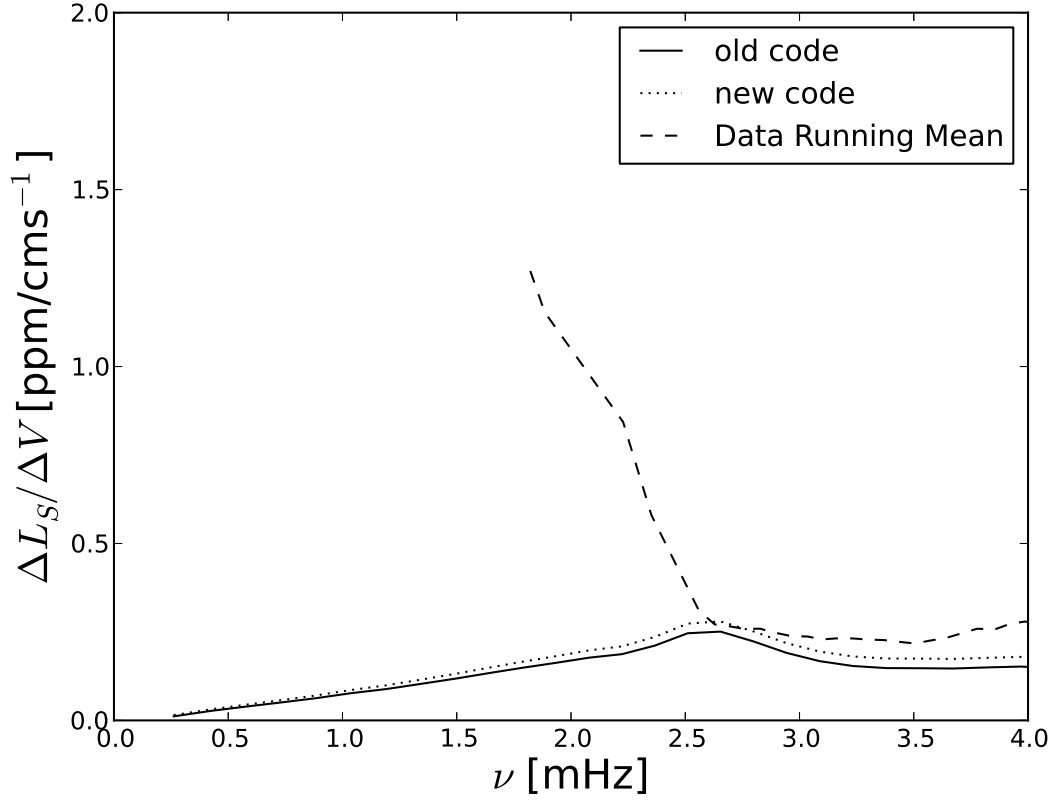


Figure 2.10: Comparison of the running mean of the observational data of amplitude ratios between irradiance and velocity as a function of frequency with our model constructed with mixing length $\alpha = 2.0$, non-local mixing length parameters, $a = b = \sqrt{300}$ calculated at the height $h = 200$ km in the atmosphere. Solid line (Eddington approximation), dotted line (Consistent radiation treatment) and dash line is the running mean of the observational data.

pulsation codes matches the data well because of the large scatter in the data. However, the running mean of the data shows similar trend as both pulsation codes in the frequency range 2.5 - 4.0 mHz as shown in Figure 2.10. The new code appears closest to the running mean than the old code. This is shown in Figure 2.10. A comparison of theoretical phase differences with observed data is shown in the bottom panel Figure 2.11 while the running mean of the data is in Figure 2.12. Again both codes show the same trend. Both codes predict a depression near $\nu = 2.5$ mHz whereas there is a peak in the running mean data. We however, have to be cautious as the errors in the data

have not been taken into consideration when making this comparison. Furthermore, it is difficult to estimate the error in the calculations. We therefore conclude that unless the errors in measurements are much less than 18 %, we cannot distinguish which of the pulsation codes better describes the data.

A mean value of phase shifts of -114° with consistent radiation pulsation and convection modelling code for frequency between 2.4 and 4.3 mHz compared to a mean value of phase shifts of -119° obtained from the observational data of Schrijver et al. (1991) in the same frequency range as shown in Figure 2.12. The errors quoted on the theoretical values are merely internal errors obtained by taking the standard deviation of the mean and do not reflect the (unknown) real error. The frequency ranges for comparison were selected such that the frequencies were close to 5 min oscillations of the Sun, and also because the data below 2.5 mHz and above 4.5 mHz had large effects of noise thus making them not reliable (uncertain).

Another important observation made from the calculation is that in Figure 2.12, for frequencies greater than 2.5 mHz, the theoretical phases bend towards larger values for both the codes with Eddington approximation and consistent radiation treatment. Evidently, the Eddington approximation bends towards higher values than consistent radiation treatment results. Such observations were also noticed by Houdek (1996) in his Eddington's approximation results for the phase shifts calculation and the explanation for the bending towards higher values of the phase is because of the mechanism arising from the interaction of the pulsation with convection.

A mean value of phase shifts of -99° was obtained with the old code for frequency between 2.4 and 4.3 mHz compared to a mean value of phase shifts of -119° between 2.4 and 4.3 mHz obtained from the observational data of Schrijver et al. (1991) in the same frequency range as shown in Figure 2.12. It is clearly evident that the mean value of phase shifts obtained with the new code is the same as the mean value of the phase shifts of the observational data of Schrijver et al. (1991) in the frequency range of 2.5 - 4.0 mHz compared to that obtained with the old code.

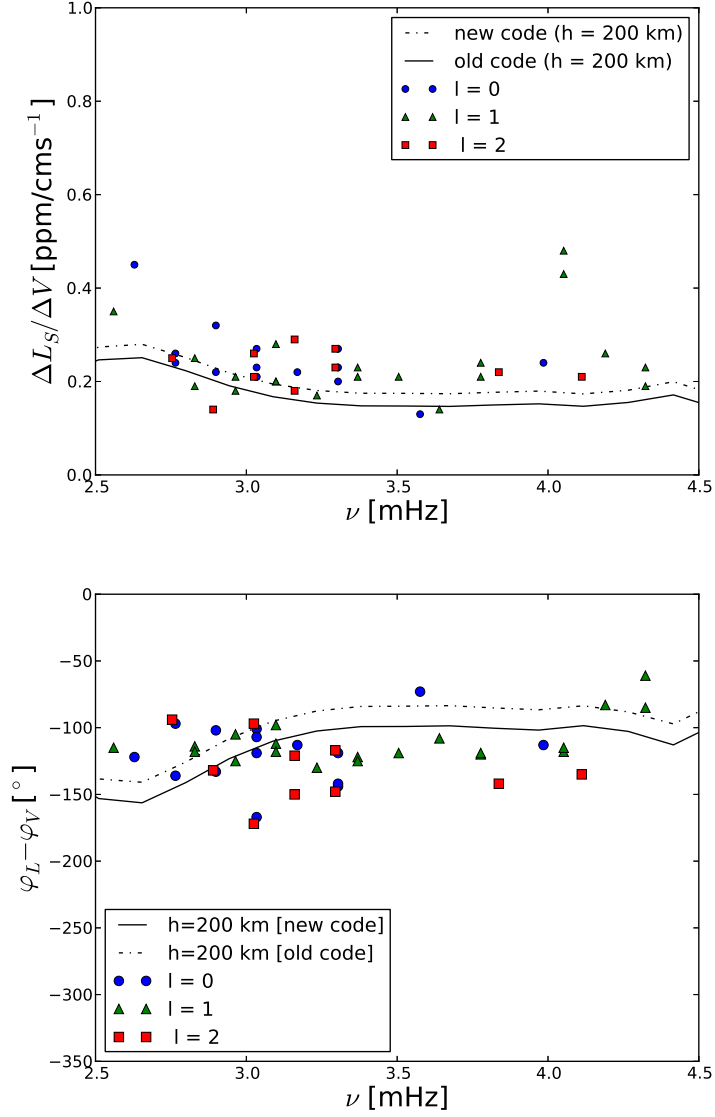


Figure 2.11: Top: Theoretical amplitude ratio at height $h = 200$ km above the photosphere shown with the observational data of the amplitude ratios (Schrijver et al., 1991) in the 5 minutes range. The dotted line corresponds to the new code while solid line corresponds to the old code. Bottom panel: theoretical phase shift at height $h = 200$ km above the photosphere estimated shown with the observational data of the phase shifts (Schrijver et al., 1991) in the 5 minutes range. The dotted line corresponds to the new code while solid line corresponds to the old code.

2.4.7.3 Limitations in the study of amplitude ratios and phase shifts

When carrying out study in this thesis on the estimate of amplitude ratios and phase shifts, I encountered some limitations, hence when looking at the results that I have so far obtained,

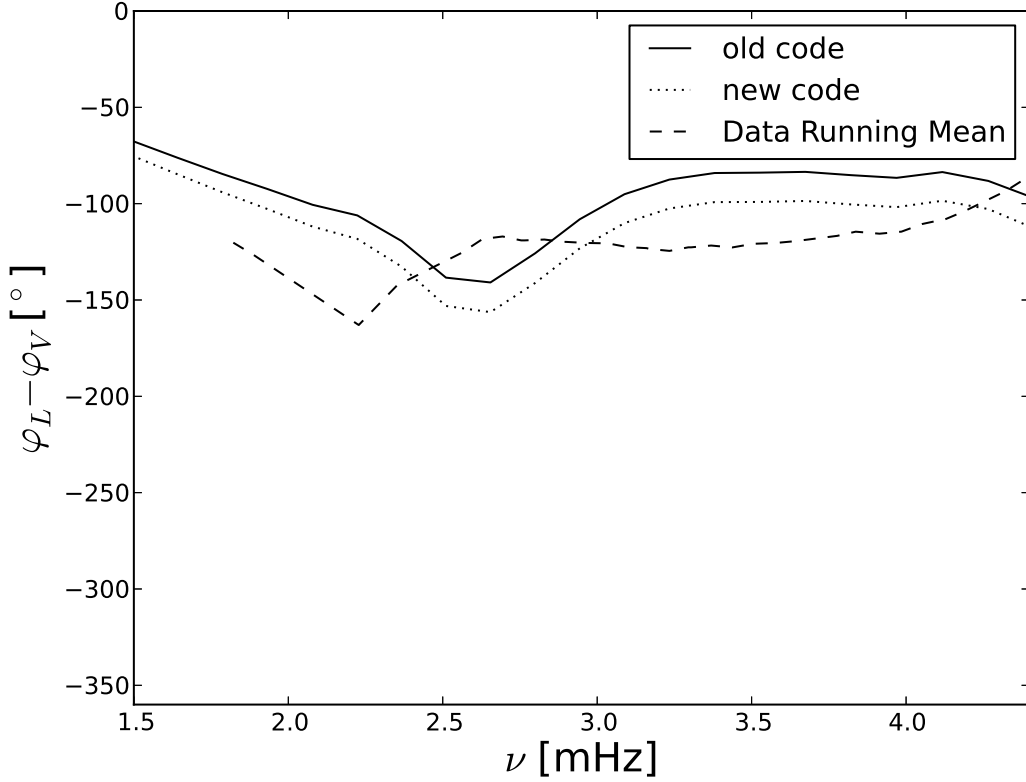


Figure 2.12: Comparison of the running mean of the observational data of phase shifts between irradiance and velocity as a function of frequency with our model constructed with mixing length $\alpha = 2.0$, non-local mixing length parameters, $a = b = \sqrt{300}$. Solid line (old code), dotted line (new code) and dash line is the running mean of the observational data.

one MUST take note of the following;

- Only one equilibrium model of the Sun was used in this study. This was because at the moment our stellar pulsation research group at North West University (Mafikeng) has to rely on the calculations of equilibrium models from other groups. There was only one equilibrium model of the Sun available at the time I was carrying out the study presented in this thesis. There is need to perform calculations of amplitude ratios and phase shifts using different equilibrium models with different mixing - length and non mixing - length parameters in order to make proper comparisons with the results presented in this thesis. Future work should use the following equilibrium models in

the calculations of amplitude ratios and phase shifts, (i). Equilibrium model of the Sun with Mass $M = 1.0 M_{\odot}$, Temperature $T_{eff} = 5778$, Luminosity $L = 1.0 L_{\odot}$, non - local convection parameter $a^2 = 600$, $b^2 = 300$ and mixing - length parameter $\alpha = 1.8$, (ii). Equilibrium model of the Sun with Mass $M = 1.0 M_{\odot}$, Temperature $T_{eff} = 5778$, Luminosity $L = 1.0 L_{\odot}$, non - local convection parameter $a^2 = b^2 = 600$ and mixing - length parameter $\alpha = 1.8$, (iii). Equilibrium model of the Sun with Mass $M = 1.0 M_{\odot}$, Temperature $T_{eff} = 5778$, Luminosity $L = 1.0 L_{\odot}$, non - local convection parameter $a^2 = 900$, $b^2 = 900$ and mixing - length parameter $\alpha = 1.8$, (iv). Equilibrium model of the Sun with Mass $M = 1.0 M_{\odot}$, Temperature $T_{eff} = 5778$, Luminosity $L = 1.0 L_{\odot}$, non - local convection parameter $a^2 = 300$, $b^2 = 300$ and mixing - length parameter $\alpha = 1.8$, (v). Equilibrium model of the Sun with Mass $M = 1.0 M_{\odot}$, Temperature $T_{eff} = 5778$, Luminosity $L = 1.0 L_{\odot}$, non - local convection parameter $a^2 = 300$, $b^2 = 300$ and mixing - length parameter $\alpha = 2.1$, (vi). Equilibrium model of the Sun with Mass $M = 1.0 M_{\odot}$, Temperature $T_{eff} = 5778$, Luminosity $L = 1.0 L_{\odot}$, non - local convection parameter $a^2 = 600$, $b^2 = 2000$ and mixing - length parameter $\alpha = 2.0$.

- The equilibrium model that I have used in this thesis is based on the semi - empirical $T(\tau)$ relation in which the atmosphere is not properly consistent with the radiative transfer. Future models should include $T(\tau)$ relation in which the atmosphere is consistent with the radiative transfer.
- The approach of calculating the amplitude ratios and phase shifts in this thesis does not include turbulent treatment and stochastic excitation mechanism. There is a need to include excitation mechanism so as to be able to make proper comparisons with the approach that I have used in this study. Therefore, future study MUST include stochastic excitation even though it is well known that there are a lot of uncertainties associated in performing calculation when turbulence is included as pointed out in this thesis.

Chapter 3

Red Clump stars in *Kepler* open cluster NGC 6819

3.1 The *Kepler* Input Catalogue (KIC)

Before the launch of *Kepler* space mission, there was an intensive ground based program to observe the *Kepler* field with multicolor CCD photometry in order to characterize the 150 000 stars. The photometry was made using the filter system used in the Sloan Digital Sky Survey (SDSS), the *ugriz* system. The filter response function for different data is shown in Figure 3.1. The Johnson BVRI system is also shown in that plot for comparison. We see that *ugriz* roughly corresponds to BVRI. In Figure 3.1 (left panel) we also show the 2MASS JHK bands in the infrared and on the right panel is the transmission response of the *Kepler* photometer¹.

Many stars in the *Kepler* field also have 2MASS photometry. The results of the ground based photometric survey of star in the *Kepler* field are given in the Kepler Input Catalogue². The KIC information gives one a clue about the location of the different stars in the HR diagram, although it was never made for such purpose (Brown et al., 2011). For example, the KIC luminosity and temperature will indicate the different stages of evolution of the various types or class of stars in the *Kepler* public archives or *Kepler* open clusters.

¹<http://keplergo.arc.nasa.gov/CalibrationResponse.shtml>

²<http://archive.stsci.edu/pub/kepler/catalogs/kic.txt.gz>

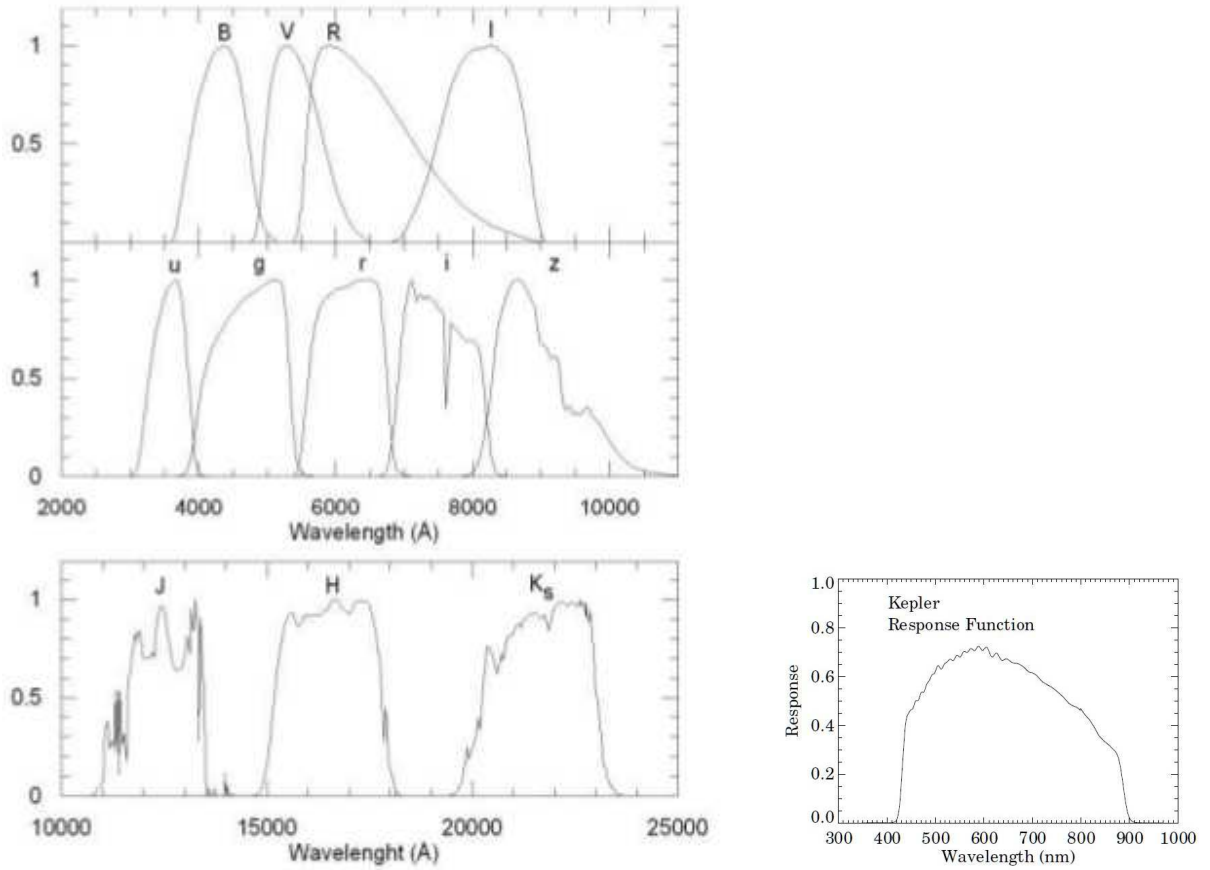


Figure 3.1: Comparison of filter transmission in Johnson/Cousins (BVRI), the SDSS (ugriz) and the 2MASS (JHK) systems (left panel). Adapted from Bessell (2005). Right panel: The transmission response of the Kepler photometer. Taken from <http://keplergo.arc.nasa.gov/CalibrationResponse.shtml>.

3.2 Distance modulus

Stars located in the field of a cluster can be used for in the determination of a distance modulus of a cluster. Distance modulus, μ , is the difference between the apparent magnitude (m) and the absolute magnitude (M) of a star: $\mu = m - M$. Where absolute magnitude is the apparent magnitude that a star would have if it were 10 parsecs away from Earth. Given the distance, d , in parsecs, the absolute magnitude, M , is given by $M = m - 5(\log_{10} d - 1)$. Due to the fact that the apparent magnitude in most cases has to be corrected for reddening, it is commonly written as $\mu_0 = m_0 - M$ as is shown under section 3.3.

3.3 NGC 6819 Open Cluster

NGC 6819 is an open cluster centered at RA = 19:41:18, Dec = 40:11:12. It is a moderately old cluster with age of ≈ 2.5 Gyr (Rosvick & Vandenberg, 1998; Kalirai et al., 2001; Basu et al., 2011; Balona et al., 2013b) and near-solar or slightly super-solar metallicity ([Fe/H] = $+0.09 \pm 0.03$; Bragaglia et al. (2001)). Rosvick & Vandenberg (1998) obtained a distance modulus of $\mu_0 = V_0 - M_V = 12.36$ mag and a reddening of $E(B - V) = 0.16$ from *BV* photometry and main-sequence fitting. Kalirai et al. (2001) used *BV* photometry and derived a distance modulus $\mu_0 = 12.30 \pm 0.12$ mag and a reddening of $E(B - V) = 0.10$. Balona et al. (2013b) used asteroseismic method to calculate the cluster distance modulus $(m - M)_0 \approx 12.2 \pm 0.06$ mag with reddening of $E(B - V) = 0.15$ and metallicity of [Fe/H] = $+0.09$ dex from the RG stars showing solar-like oscillations. Hole et al. (2009) obtained photometry and radial velocities for 1207 stars in the field of the cluster from which they determined 480 cluster members. They also obtained twenty four red giant clump candidates in NGC 6819. Red giant clump stars are important in NGC 6819 because they have roughly the same luminosities which means that they are useful in the determination of its distance and reddening.

In this thesis, the field containing the open cluster NGC 6819 was studied using the Sloan *griz* photometry of the stars from the *Kepler* Input Catalogue - KIC (Brown et al., 2011) and time-series photometry from the *Kepler* public archives available at the MAST³. The effective temperature for each star was estimated using the color-temperature calibrations following the method of Ramírez & Meléndez (2005). We used the (V - K) color with the V values obtained from Hole et al. (2009) and the K values from the 2MASS catalog (Skrutskie et al., 2006). The adopted metallicity used in obtaining T_{eff} in this work are those in Bragaglia et al. (2001). The RGC stars were searched for in all the RG stars showing solar-like oscillations in NGC 6819 that were identified by Balona et al. (2013b). The aim was to find and study RGC stars and use them to determine the distance and age of NGC 6819. We further used RGC and RGB stars to study mass loss of RG stars from the tip of the RGB to the RGC region (horizontal branch).

³<http://archive.stsci.edu/kepler/>

3.4 The data

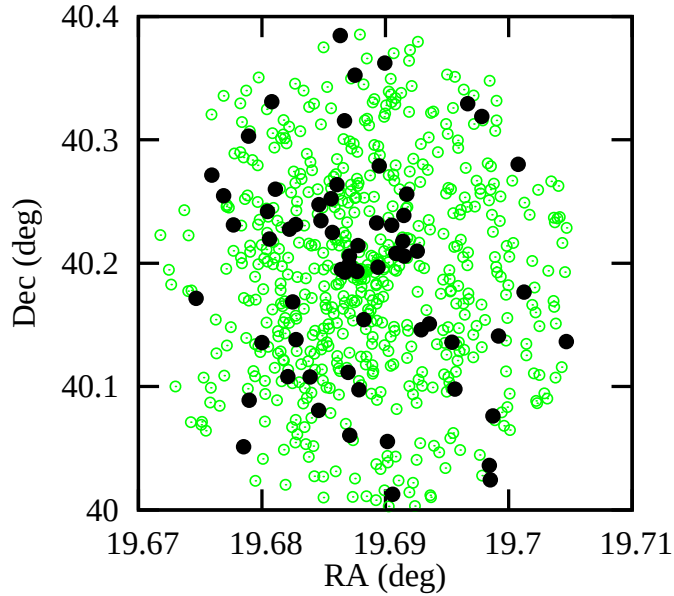


Figure 3.2: The map showing the *Kepler* data for all the stars in the field of NGC 6819 (open circles) while filled circles are the RG stars with solar-like oscillations discovered by visual inspections of the periodogram and light curves (see Balona et al. (2013b) and this work).

The data of RG stars with solar-like oscillations in this chapter were obtained with NASA's *Kepler* space telescope. Figure 3.2 is the map showing the *Kepler* data for all the stars in the field of NGC 6819. Currently, there are more than 13 000 red giants with known solar-like oscillations observed with *Kepler* (Stello et al., 2013). The NASA *Kepler* was successfully launched in March 2009 into earth-trailing orbit (Borucki et al., 2009). The *Kepler* mission has been important in the study of stellar pulsation. This mission is designed to observe the light variations of over 150 000 stars in a large field of 105-square degrees in the direction of Cygnus and Lyra (Figure 3.3). Most of the observations were obtained using an exposure time of about 30 min (long-cadence mode), but a few thousand stars have also been observed with an exposure time of about 1 min (short-cadence mode). The *Kepler* magnitudes are in the range $6.3 < K_p < 18.4$ and are related to the SDSS g and r values by the following⁴:

⁴<http://keplerscience.arc.nasa.gov/CalibrationZeroPoint.shtml>

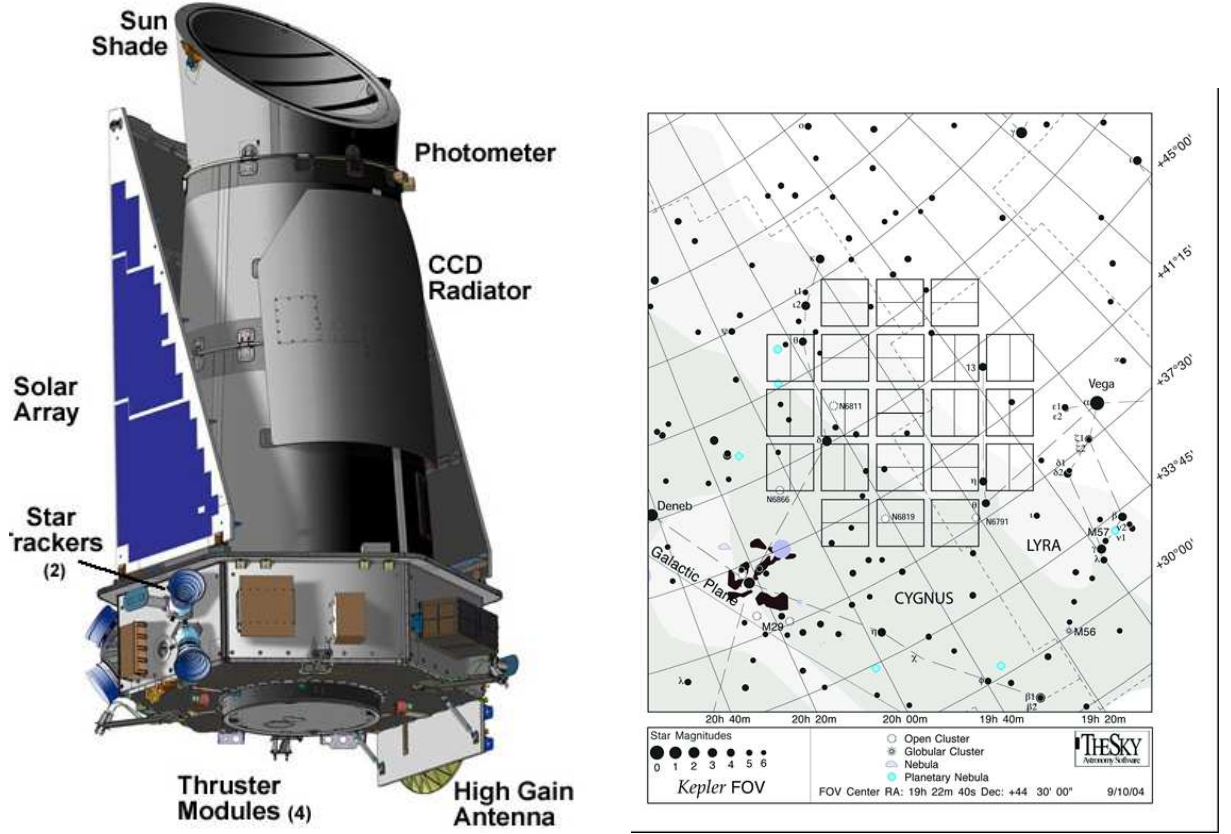


Figure 3.3: The *Kepler* space craft with the photometer and the detailed field of view (FOV) on the right showing the position of all the CCD in the field. Taken from <http://kepler.nasa.gov/>

$$K_p = 0.2g + 0.8r, (g - r) \leq 0.8.$$

$$K_p = 0.1g + 0.9r, (g - r) > 0.8.$$

NGC 6819 is one of the four open clusters in the *Kepler* field of view. *Kepler* studies of pulsating red giants in these clusters allow independent estimates of mass, radius and surface gravity of each star.

The publicly available *Kepler* data (Q0 - Q16) were used in this study, where Q stands for quarters referring to the interval in which the data are downloaded after certain time interval for example, Q0 is a 10-d commissioning run. The stars selected in the KIC within a radius of 12 arcmin of the cluster center were used. The light-curve files of the stars contain simple aperture photometry (SAP) flux and a more processed version of SAP with artefact mitiga-

tion included, also called presearch data conditioning (PDC) flux. Jumps between quarters were calculated and removed using a software written and used by Balona et al. (2013b). The resulting corrected data were used to calculate periodograms. For each star in the field of NGC 6819, Balona et al. (2013b) visually examined the periodogram and attached a variability class, guided by the location of the star in the HR diagram. Solar-like oscillators were classified based on the nature of the periodogram with characteristic Gaussian like envelope, from which red giant stars were identified based on their location in the HR diagram. The search for the RGC stars amongst the RG stars identified by Balona et al. (2013b) were performed using the following procedure:

- (i) applying correction to *Kepler* photometry,
- (ii) calculating the periodogram,
- (iii) identifying solar-like observed frequencies,
- (iv) identifying $\nu_{\max}$ and calculating $\Delta\nu$ using autocorellation,
- (v) constructing echelle diagrams (a plot of ν versus $\nu \bmod \Delta\nu$),
- (vi) measuring median gravity-mode period spacings (ΔP) for each star,
- (vii) calculating the masses, radii and luminosities using equation (1.3), (1.4), and (1.5) respectively.

A total of 84 stars in NGC 6819 were identified by Balona et al. (2013b) as stars with solar-like oscillations, of these, 69 were identified as RG stars based on their location on the HR diagram. The quantities $\nu_{\max}$, $\Delta\nu$, $M/M_{\odot}$, $L/L_{\odot}$ for each of the selected stars were measured. The details of the above procedures are presented in the following subsections.

3.5 Searching for clump stars amongst the red giant stars

According to Miglio et al. (2009), given a population of stars taken from a typical survey such as in the COROT data, the maximum in the $\nu_{\max}$ and $\Delta\nu$ distributions are mostly due to RC stars. However, this was a purely statistical result, meaning that one cannot tell whether an individual star is RC or RG, but given a sufficiently large sample of stars, one can say that a certain percentage of the stars are RC and the rest RG.

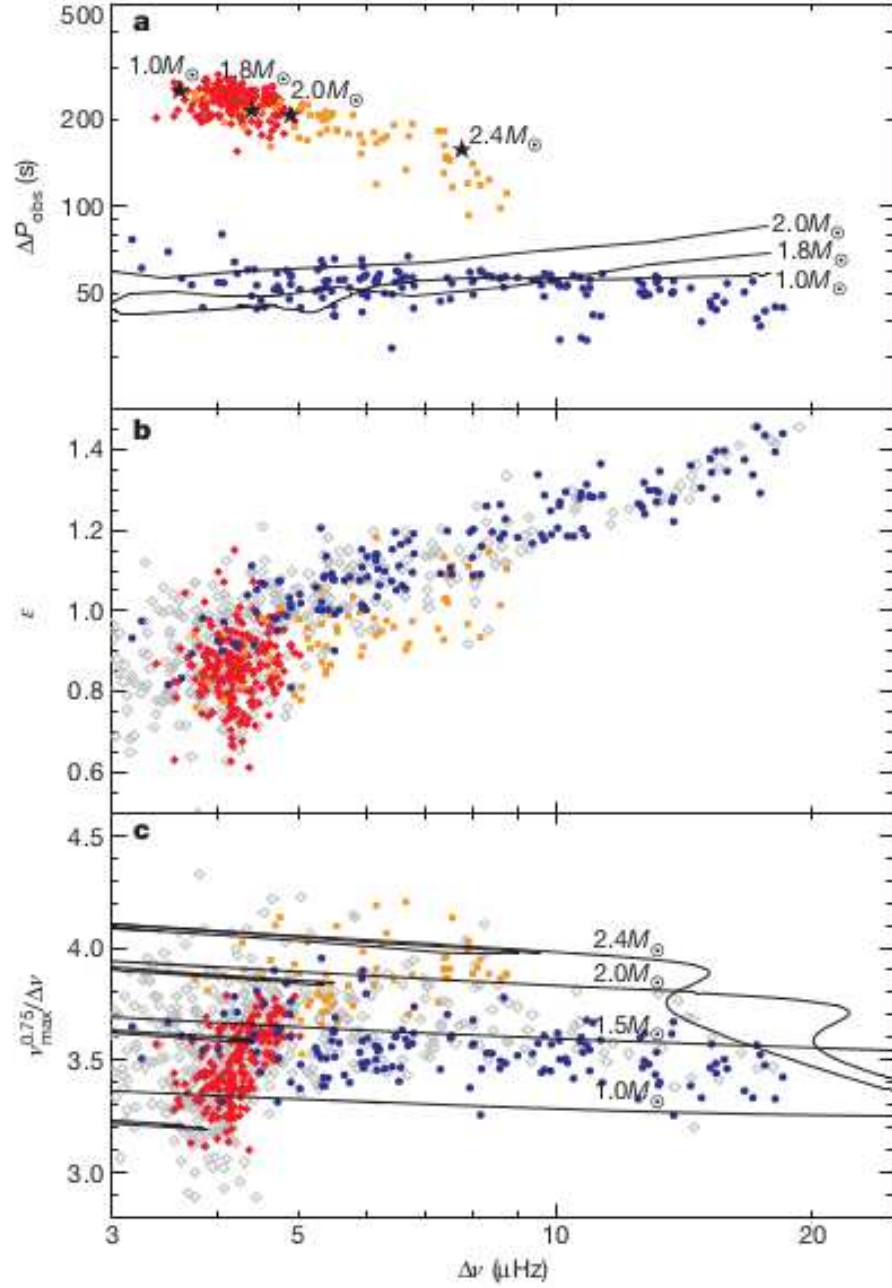


Figure 3.4: The distinction between RGC and RGB stars based on period separation for field stars, ΔP as shown by Bedding et al. (2011). The points with $\Delta P > 100$ s (red and yellow) are the RGC while points with $\Delta P < 100$ s (blue points) are the RGB stars. The solid lines are the theoretical lines calculated with the models indicating various masses.

Bedding et al. (2011) distinguished between H - shell burning RGB stars and He - burning RGC stars using gravity modes as shown in Figure 3.4 (a). They reported observations of g - modes period spacings in red giants and found many stars whose dipole modes show sequences with regular period spacings. The stars considered were categorised into two clear groups, allowing them to distinguish between hydrogen - shell - burning stars (period spacing mostly ~ 50 seconds) and those that were burning helium (period spacing ~ 100 to 300 seconds). The ΔP (s) versus $\Delta \nu$ (μ Hz) plot showed clear separation of H - shell burning (RGB) stars and He - burning RC stars.

Bedding et al. (2011) were able to make the distinction between RGB and RGC stars because the large density gradient outside the helium core in all red giants divides the star into two coupled cavities, with the oscillations behaving like p modes in the envelope and like g modes in the core. In RG stars, the spectrum of g modes in the core is dense. However, these g modes are trapped in the core and not visible at the surface. If a p mode, driven by stochastic convective motions in the envelope, has about the same frequency as a g mode in the core, the energy leaks into the g mode, trapping is reduced and it becomes visible at the surface. The result is a mixed mode with both p and g characteristics. Since the g -mode spectrum is dense, it is likely that more than one g mode is driven for each p mode, as long as its frequency is close to that of the p mode. Therefore for every p mode one expects to see a few g modes of nearly the same frequency to be visible. In the asymptotic approximation, p modes are approximately equally spaced in frequency, but g modes are approximately equally spaced in period. Therefore the period spacing between the g modes will give information about the conditions in the core. Since the conditions in the core of RGB stars differ from that in RGC stars, one expects the period spacing of g modes to be different in the two types of star. The dipole ($l = 1$) mode gives the best chance to see these mixed modes for two main reasons. Firstly, an $l = 1$ g mode in the core is only weakly trapped compared to $l > 1$ mode, therefore it is more easily excited and the mixed $p + g$ mode has a larger amplitude at the surface. Secondly, the $l=1$ modes are well separated from the $l = 0$ and $l = 2$ modes in the echelle diagram, therefore it is easier to identify the $l = 1$ modes.

More recently, Stello et al. (2013) also classified the different population of *Kepler* RG stars based on ΔP and found that the RGB and RGC stars were those with $M/M_{\odot} \lesssim 1.8$, while

the higher - mass SRC stars were those with $M/M_{\odot} \gtrsim 1.8$. It is important to note that Stello et al. (2013) found that there were no RGB stars with $\Delta\nu \lesssim 5 (\mu \text{ Hz})$ for the stars they studied in the *Kepler* field and this was because their technique to measure ΔP eliminated them out (their automated technique omitted those stars because the ΔP could not be reliably measured).

3.6 Testing the software

The software for calculating median gravity-mode period separations were tested on three field stars KIC 6779699, KIC 4902641 and KIC 6928997 (non - members of NGC 6819). Those were stars previously studied by Bedding et al. (2011) and Beck et al. (2011) using *Kepler* data collected for a period of 1 year. The aims were to

- (i) reanalyze the *Kepler* data for the above mentioned three stars collected for a period of four years (Q0 - Q16),
- (ii) check and confirm the previous results calculated and obtained by Bedding et al. (2011) and Beck et al. (2011),
- (iii) check the consistency of our results with that of Bedding et al. (2011) and Beck et al. (2011).

If the results were consistent, that would mean that the software were working perfectly well. In the subsection that follows, detail descriptions and explanations of the steps taken to carry out the analysis in testing the software are presented.

3.6.1 Methods in the reduction of the *Kepler* raw data

The data for the three stars which were collected for a period of 4 years (Q0 - Q16) from the publicly available website: archive.stsci.edu/pub/kepler/lightcurve were downloaded. The original data were in the form of fits files. Conversion of the fits files to readable files using a software called *tablist* available on the NASA's website for public use were made. The readable files consisted of five columns; Barycentric Julian Date, the raw count, error in the raw count, corrected count, error in the corrected count respectively. It should also be noted

that ASCII (readable) files for the same stars are on the KASOC⁵ website. However, the idea was to be able to get experience in extracting data from FITS files rather than using the readily available ASCII files.

Conversion of the raw count to brightness measure (magnitude) in the form of raw signal and corrected signal were done. The problem was that the raw data had drifts and jumps shown in Figure 3.5 for example. There was a need to correct for the drifts and jumps. The corrections were done by:

- (i) removing any linear trend within each quarter,
- (ii) matching the end of one quarter and the start of the following quarter,
- (iii) removing any linear trend in the combined data,
- (iv) fitting a cubic spline curve to medians of the data within a time interval of 0.25 d so that the overall shapes of the light curve were derived,
- (v) removing points (outliers) which deviated much from the curve using an iterative technique, the final output of the results is the corrected light curve (see Figure 3.6).

All of these steps were described and used in Balona (2011). Finally, periodograms for each of the corrected light curves (see Figure 3.7 for example) were obtained. The problem with the periodogram shown in Figure 3.7 is that the peaks are broad. Each peak seems to consist of many peaks all bunched together as a result of oscillations which are stochastic. One needed to smoothen the periodograms so as to be able to extract the observed frequencies. We used the running mean approach to smoothen the periodogram and the result is shown in Figure 3.8. In the running mean, one needs to choose a frequency width, W . In order to get a smooth periodogram, suppose there is j^{th} frequency point in the periodogram, f_j , with amplitude A_j . To get a smooth value of the amplitude at f_j , the amplitudes between $f_j - W$ and $f_j + W$ (let it be B_j) was averaged. Different width of the smoothing box were used since the starting and stopping frequency in the region of interest (smoothing regions) were different. The processes were repeated for all the frequencies in the periodogram and later plotted B_j versus f_j . A sample of a periodogram smoothed according to the method described is shown in Figure 3.8. It should be noted that there was no criterion for setting the noise level limit. It was a matter of judgement on how much smoothing was required and the

⁵Kepler Asteroseismic Science Operations Center

amplitude below which noise were being picked out. If the level is too high (horizontal green lines), you do not pick out ‘real modes’. If it is too low, you start picking out ‘modes’ which are noise. The resulting echelle diagram depends much on how one sets the level as shown in Figure 3.9.

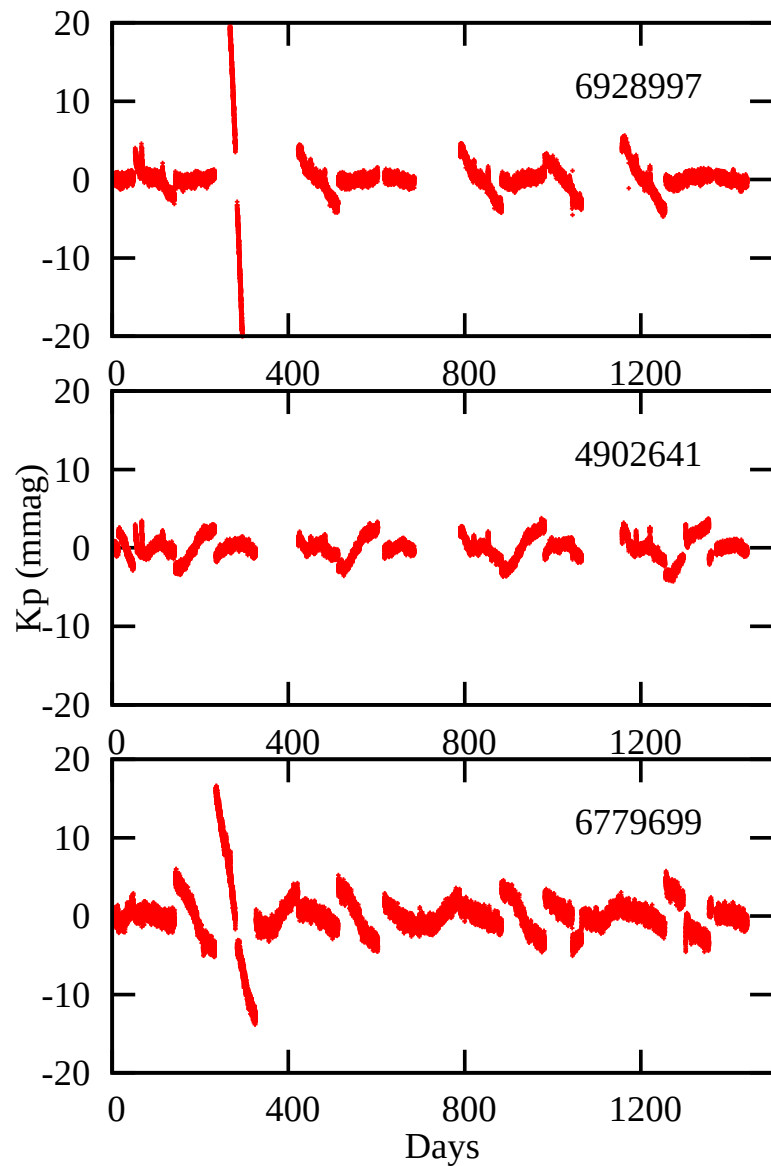


Figure 3.5: *The raw (uncorrected) light curve of stars KIC 6779699, 4902641 and 6928997 observed by Kepler space mission.*

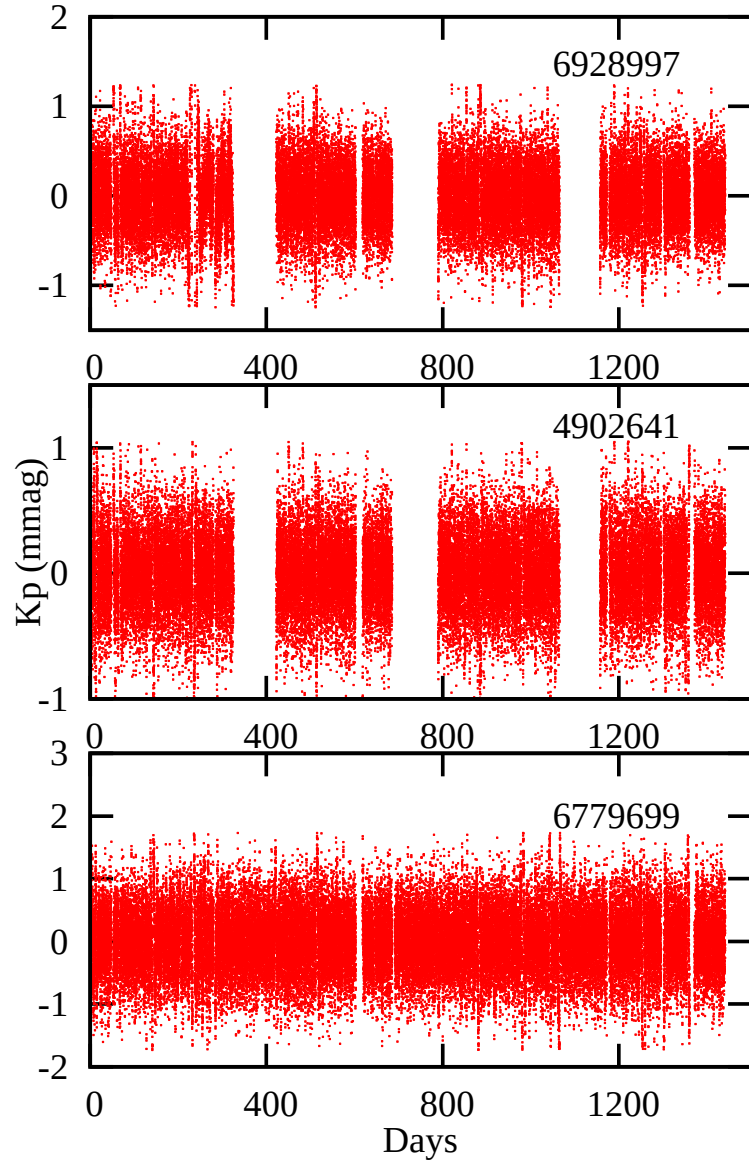


Figure 3.6: *The corrected light curve of stars KIC 6779699, 4902641 and 6928997 observed by Kepler space mission.*

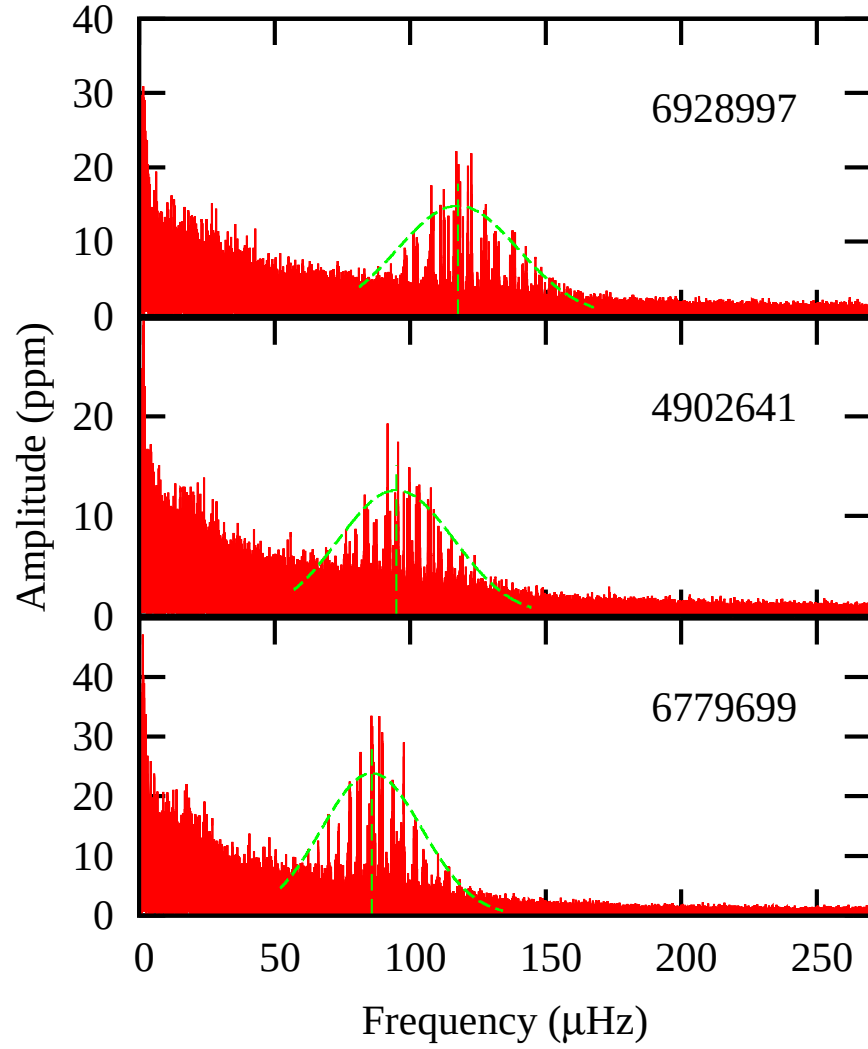


Figure 3.7: The periodogram of KIC 6779699, KIC 4902641 and KIC 6928997 obtained after correcting for the drifts and jumps in the the raw (uncorrected) light curves of the stars. Comb-like structures are clearly seen in all the three periodogram, which are typical characteristics of stars with solar-like oscillations. The location of the frequency of the maximum amplitude is indicated in each plot as a vertical dash line.

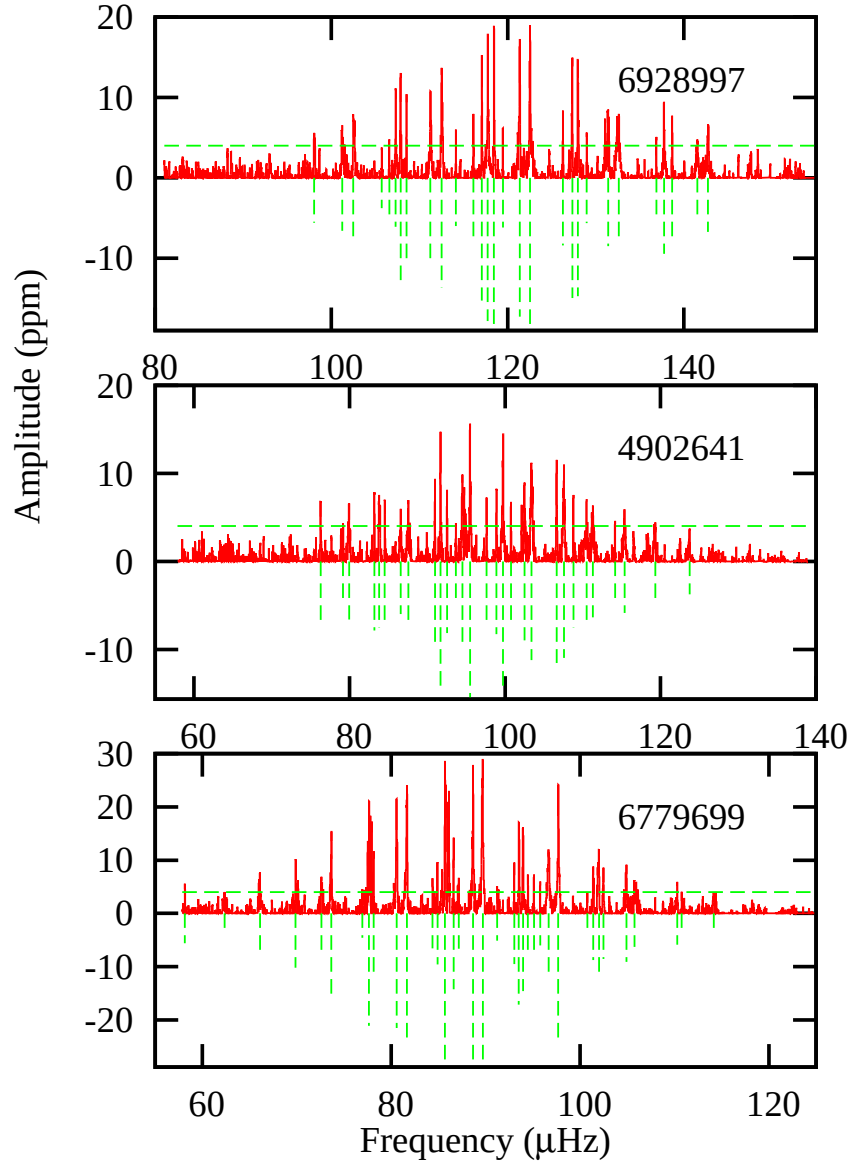


Figure 3.8: *Smoothed periodograms for the three stars we used to test our methods (each star name is indicated in each panel). The Figure shows the observed frequencies extracted from the periodograms in Figure 3.7 (the vertical dashed - green lines indicate the extracted frequencies). We used running mean approach to smooth the periodogram. The horizontal lines are the noise level limit.*

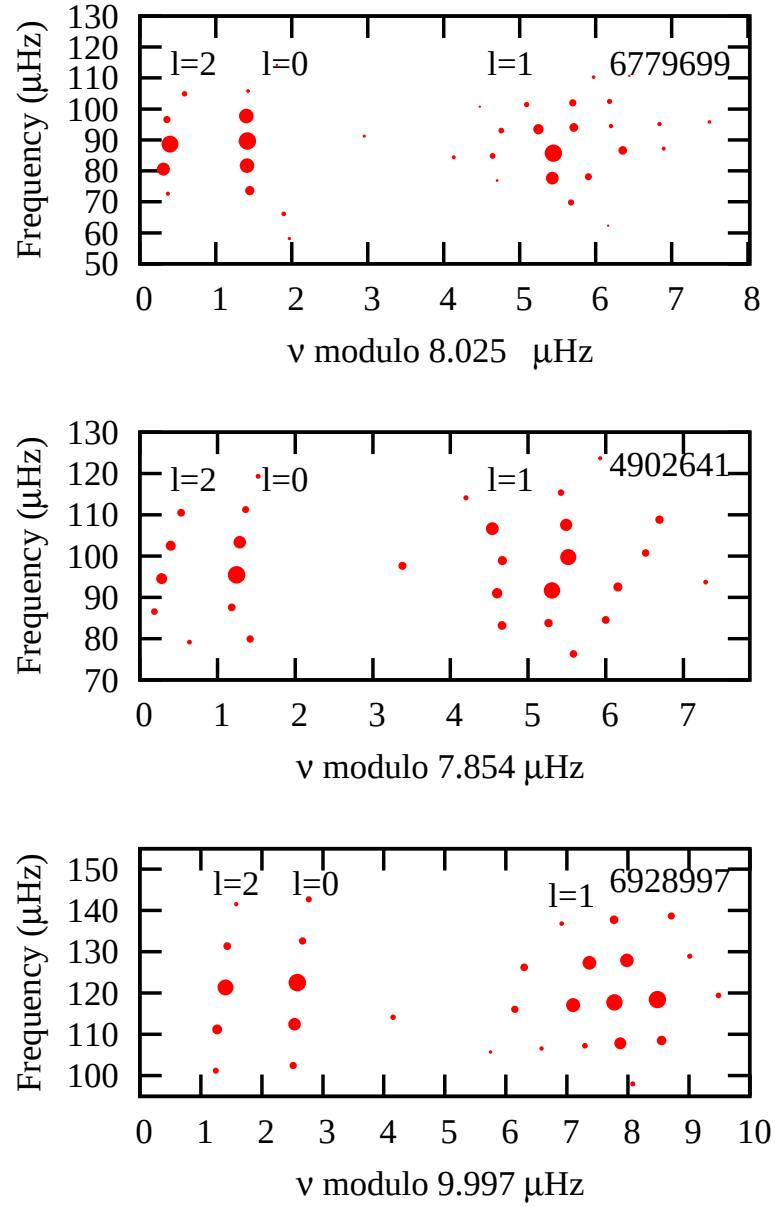


Figure 3.9: Echelle diagram for the three stars constructed using the extracted observed frequencies and the large frequency separations. The modes $l = 0, 1$ and 2 are marked. The star names are also indicated.

3.7 Autocorrelation

The autocorrelation method was used to look for equal frequency separations in the periodogram without measuring individual peaks. To use autocorrelation, one needs to select the frequency region in the periodogram which contains the solar-like oscillations. This is done by making use of the starting and stopping point (f_1 and f_2) of the Gaussian shape in

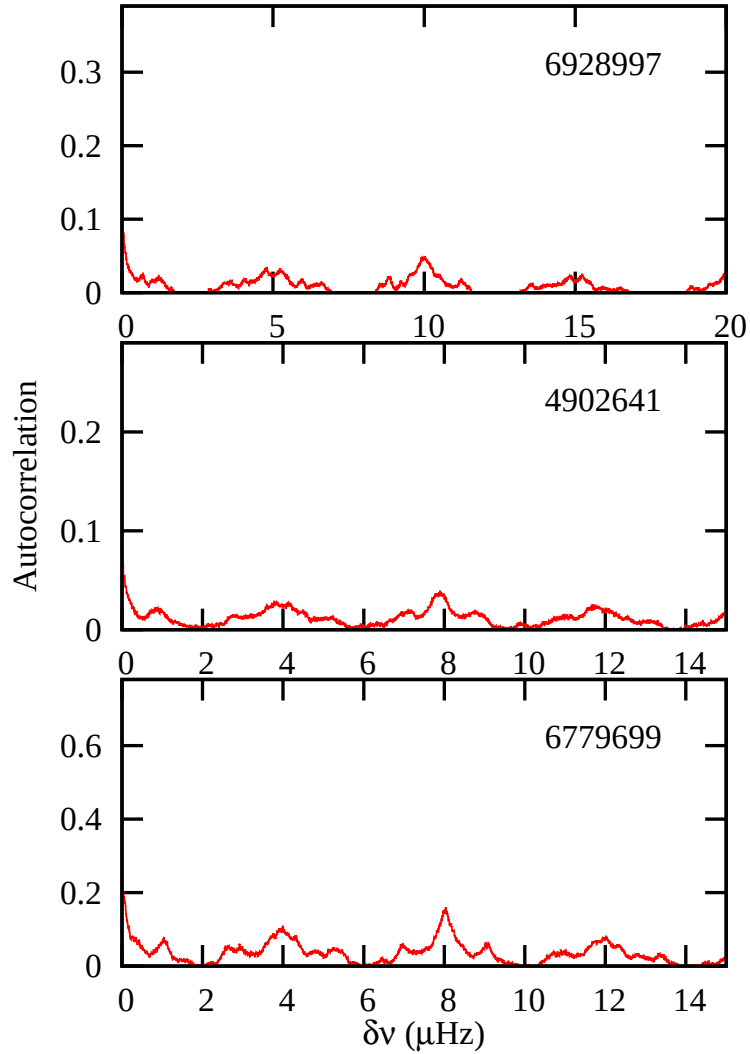


Figure 3.10: Autocorrelation function for KIC 6928997 between $80 < f < 155 \mu\text{Hz}$, KIC 4902641 between $60 < f < 140 \mu\text{Hz}$ and KIC 6779699 between $60 < f < 120 \mu\text{Hz}$. The frequency ranges are the expanded view of relevant regions of interest. The y axis is $A(\delta f)$. Note that the plot is symmetrical around zero Hz.

the periodogram. For example, let the frequency, f , of these oscillations to lie between the lower and upper limits, $f_1 < f < f_2$. Let also the amplitude at any frequency be $y(f)$ and the amplitude at frequency $f + \delta f$ be $y(f + \delta f)$. We can multiply these two amplitudes and sum the result over the frequency limits and call it the sum $A(\delta f)$. Hence,

$$A(\delta f) = \sum_{f=f_1}^{f_2} y(f)y(f + \delta f).$$

In a case where $\delta f = 0$, $A(\delta f)$ will be very large because the peaks match exactly. As δf is increased, the mismatch between peaks becomes larger and larger, therefore $A(\delta f)$ decreases. In case of regular frequency spacings, $\Delta\nu$, as δf approaches $\Delta\nu$, $A(\delta f)$ will increase and reach a local maximum at $\delta f = \Delta\nu$. Further maxima will occur at $\delta f = 2\Delta\nu$ and at $3\Delta\nu$, and so on. If $\Delta\nu$ is the only regular frequency spacing present in the data, a plot of $A(\delta f)$ as a function of δf will show a large peak at $\delta f = 0$ and a series of peaks at integral values of $\Delta\nu$. Alternatively, if one chooses, one can make δf to be negative, in which case the peaks will occur at $-\Delta\nu$, $-2\Delta\nu$ and so on as shown in Figure 3.10.

Stello et al. (2009) first presented an observed relation between the large frequency separation, $\Delta\nu$ and the frequency of maximum amplitude, $\nu_{\max}$ as $\Delta\nu \propto \nu_{\max}^{0.77}$ based on ground-based data. Later, Hekker et al. (2011a) showed that the large frequency separation, $\Delta\nu$, is related to the frequency of maximum amplitude, $\nu_{\max}$ by $\Delta\nu = a\nu_{\max}^b$, with $a = 0.266$, $b = 0.761$. This empirical relationship is based on Kepler data of three open clusters and 662 field stars.

The autocorellation program finds $\nu_{\max}$ by looking for the highest peak in the periodogram and then searching on either side of the peak for peaks which stand out above the rest. It then fits a Gaussian to the high amplitude peaks from which $\nu_{\max}$ is obtained.

In the asymptotic region, the $l = 0$, $l = 2$ modes lie roughly half way between successive $l = 1$ modes. This means that autocorrelation will pick out matches between $l = 1$ and $l = 0$, 2 which occur at about $\Delta\nu/2$. In order to avoid this problem, the software uses Hekker's relationship (Hekker et al., 2011a) between $\nu_{\max}$ and $\Delta\nu$ to resolve this ambiguity. From the $\nu_{\max}$ obtained as described earlier, the software finds an approximate value of $\Delta\nu$ using $\Delta\nu = a\nu_{\max}^b$ and only searches in this region of the autocorrelation for a maximum. This ensures that it is $\Delta\nu$ and not $\Delta\nu/2$.

3.8 Using echelle diagram to separate out different modes

The echelle diagram was first introduced by Grec et al. (1983) in the study of the Sun. It involves dividing the power spectrum into equal segments of length $\Delta\nu$ and stacking them one above the other so that modes with a given degree align vertically in ridges. Any departures from regularity are clearly visible as curvature in the echelle diagram (Bedding, 2014). The echelle diagrams have been widely used in the study of RGC and RGB stars (Bedding et al., 2011; Beck et al., 2011; Corsaro et al., 2012). To construct an echelle diagram, one needs to smooth the periodogram such that the observed frequencies are properly identified.

We used this procedure;

- (i) Smooth the periodogram by applying a running mean method. Smoothing the periodogram eliminates the many close frequencies, giving rise to a single broad peak. Restriction is made to the Gaussian comb-like structure where we are able to approximate the starting and stopping frequencies.
- (ii) specification of the moving window of a certain width of the smoothing filter is done. The larger the width, the smoother the periodogram. Care has to be taken that the periodogram is not too smooth as this results into losing possible peaks. On the other hand, if the periodogram is not smooth enough, one ends up selecting peaks which should belong to the same broad peaks. A choice of width of about ten times the frequency spacing is the best one. This is based on the fact such a choice produces a desired smoother periodogram.
- (iii) The smoothed periodogram will have minimum amplitude and exclusion zone. The minimum amplitude is the amplitude below which there are no significant peaks (the green dashed horizontal lines in the right hand panels of Figure 3.11). The exclusion zone is a zone on either side of the peak where no other peaks will be extracted. A good choice is about ten times the frequency spacing that is printed out by the program. A plot of smoother periodogram will have horizontal line which is the minimum amplitude and the vertical lines indicating the peaks that have been selected as shown in the right hand panels of Figure 3.11.
- (iv) Finally, the program prints out the extracted observed frequencies, ν , ν modulus $\Delta\nu$ and the amplitudes. A plot of the observed frequency, ν versus ν modulus $\Delta\nu$ gives the echelle diagram as shown in Figures 3.9, 3.14, and 3.15. The echelle diagram will clearly show the

modes $l = 0$, $l = 1$ and $l = 2$. The best judgement is obtained by the flow of points belonging to a particular l values and the presence or absence of a particular mode along the flow.

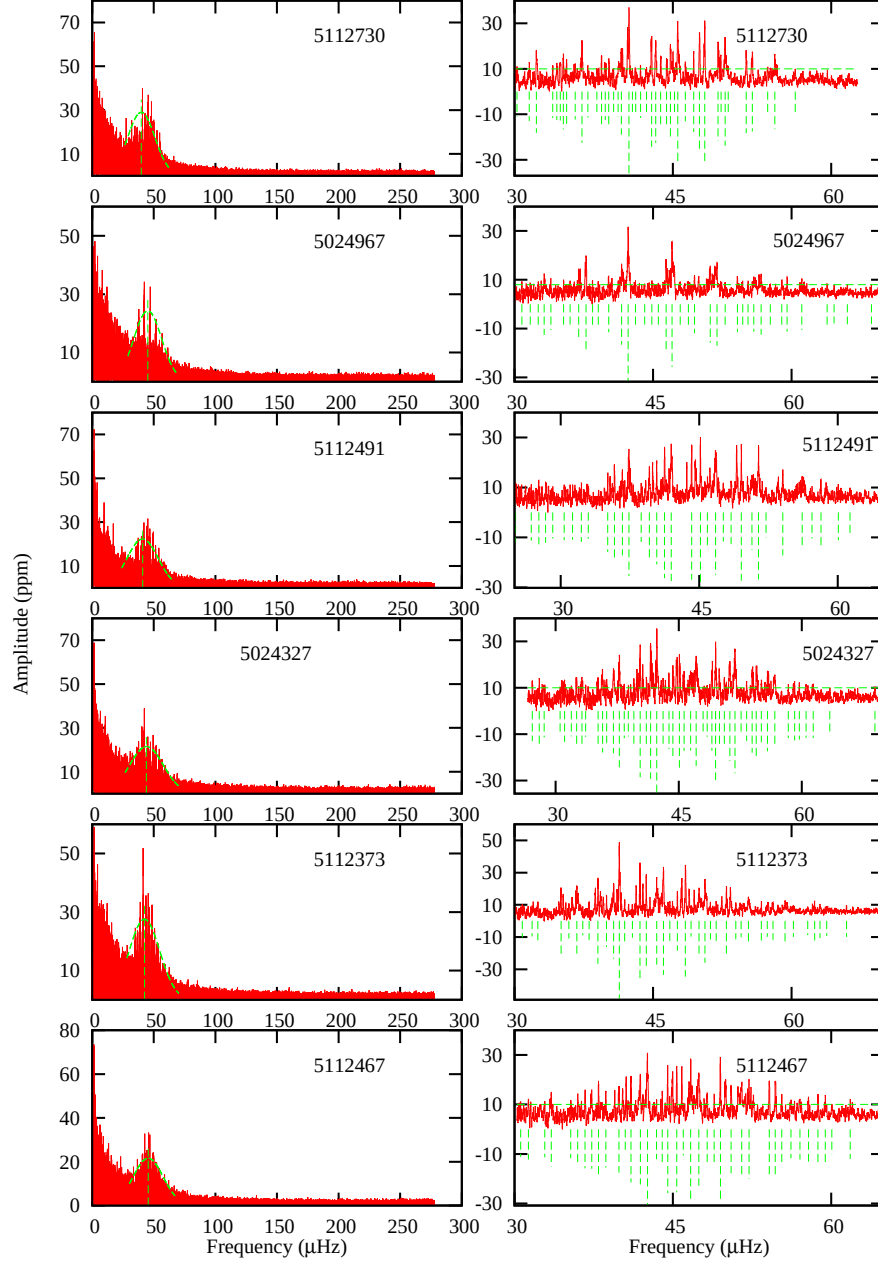


Figure 3.11: Left-hand panel: a periodogram fitted with a Gaussian for the six identified clump stars in NGC 6819 cluster. The peaks in the periodogram are broad and messy as a result of oscillations which are stochastic. Right-hand panel: smoothed periodograms showing observed frequencies extracted from the periodograms from the left-hand panel. We used running mean approach to smoothen the periodogram. The horizontal lines are the noise level limit.

Table 3.1: *Results analysis and comparison with Bedding et al. (2011) work. The 2nd and 3rd columns are the results for $\Delta\nu$ and ΔP obtained in this thesis while the 4th and 5th columns are the results obtained by Bedding et al. (2011).*

KIC	$\Delta\nu(\mu\text{Hz})$	ΔP (s)	$\Delta\nu(\mu\text{Hz})$	ΔP (s)
This work			Bedding et al. (2011)	
6779699	8.0	59.6 ± 3.3	8.0	53.0
4902641	7.9	93.1 ± 2.6	7.9	96.0
6928997	10.0	58.0 ± 3.0	10.0	54.0

3.9 Measuring period spacing (ΔP)

In this section, descriptions of how the observed frequencies of $l = 1$ are converted to the period (P) and period spacings (ΔP) are presented.

The average period spacing for mixed modes can be obtained by first of all removing the data associated with the $l = 0$ and $l = 2$ modes. We identify them in the echelle diagram. What is left are just the $l = 1$ modes. Conversion of the $l = 1$ frequencies into periods (in seconds) and sorting the periods into ascending or descending order are made. This is obtained by subtracting the periods of adjacent modes to obtain a list of period spacings. The number of modes detected versus the period spacings are plotted. The period spacing that corresponds to the largest number of modes is considered as being the main period spacing for a star in question as shown in Figure 3.12. The ΔP chosen are indicated by the arrows in Figure 3.12.

3.10 Using ΔP to distinguish between RGB and RGC stars

A graph of ΔP as a function of large separation, $\Delta\nu$ will clearly show the separation of RG stars into distinct groups, this is because the RGC have distinctly longer values of ΔP than those on the red giant branch. As mentioned in section 3.5, if a star has $\Delta P \sim 50$ seconds it is on the red giant branch, while stars with $\Delta P \sim 100$ to 300 seconds are in the RGC state of evolution. The results of such analysis are shown in Table 3.1.

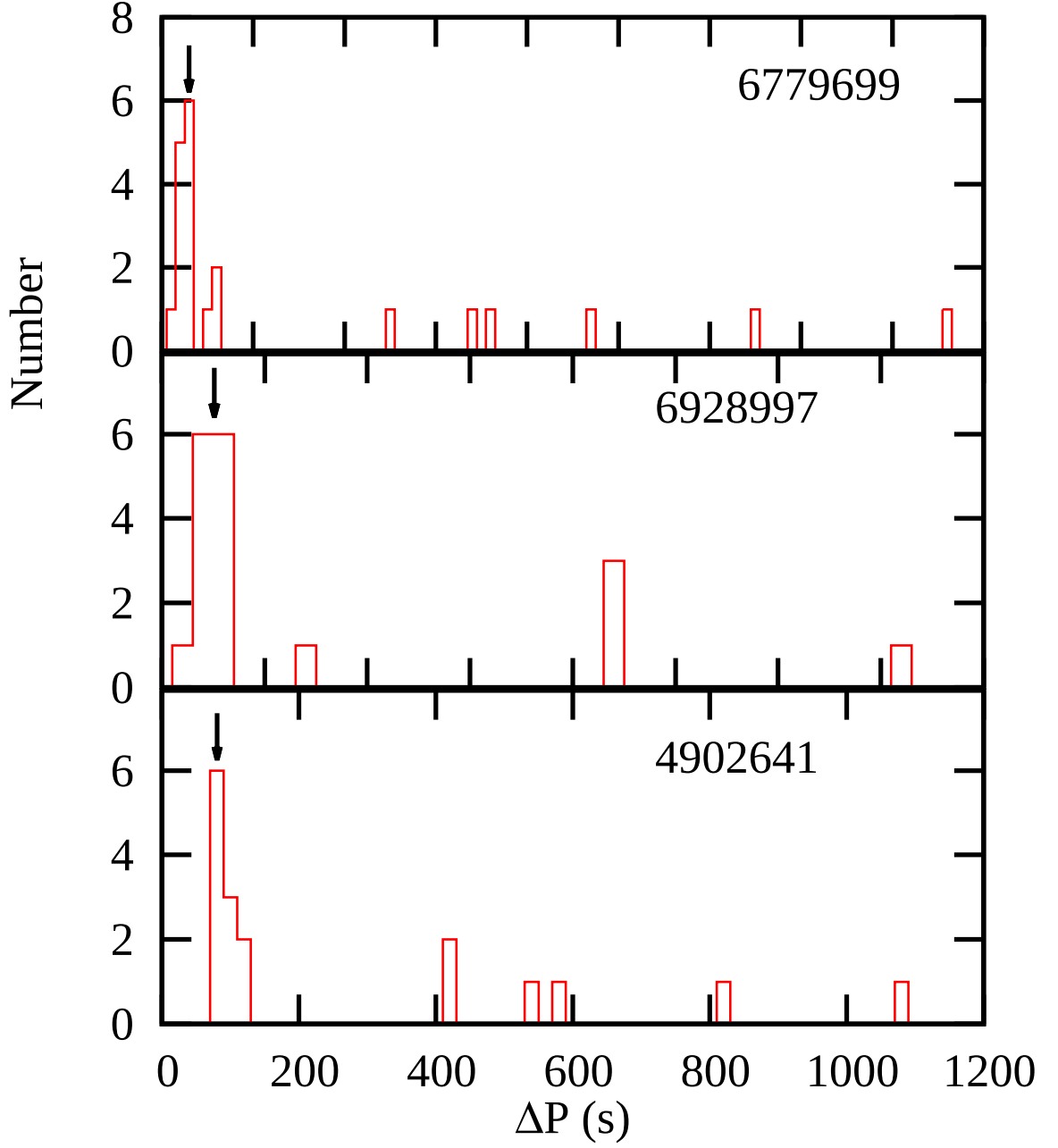


Figure 3.12: The distribution of gravity-mode period spacings, ΔP for the stars in Table 3.1. The vertical arrows indicate the ΔP chosen.

Table 3.1 clearly shows that the conclusions pertaining to the three stars were in line with the previous studies such as Bedding et al. (2011) although the median values differ slightly from the previous one. We attributed the differences in the values median ΔP to the improved data. Our data spans 4 years, whereas Bedding et al. (2011) data was for 13

months. Most importantly the final deductions were the same. We therefore, concur with Bedding et al. (2011) based on the ΔP values that (i) KIC 6779699 and KIC 6928997 are undergoing hydrogen-shell-burning on the red giant branch (RGB stars), while KIC 4902641 is burning helium in its core (Secondary Red Clump) since its median ΔP value is far much different from the other two (see Bedding et al. (2011)). This is a clear indication that our software is reliable for the analysis of stars in the *Kepler* clusters NGC 6819. The errors quoted on ΔP are internal errors obtained by taking the standard deviation of the mean. The errors in $\Delta\nu$ were estimated by (i) fitting a parabola of the form $y = a_0 + a_1x + a_2x^2$ to the points at the peak in the correlation function using least squares, (ii) getting $x_{\max}$ by setting $dy/dx = 0$ which resulted in $x_{\max} = -a_1/(2a_2)$, (iii) from the least squares solution, values of a_0 , a_1 , a_2 and corresponding errors σ_{a_0} , σ_{a_1} , σ_{a_2} were obtained (iv) using the errors and applying propagation of errors, $\sigma_{x_{\max}}$ were obtained. The same technique was applied to obtain the error in $\nu_{\max}$ by (i) determining the position of $\nu_{\max}$ by fitting a Gaussian to the peaks and measuring the centre point of the Gaussian, (ii) fitting a parabola to a few points surrounding the peaks (iii) following the remaining procedures as described before for $\Delta\nu$ as shown in Table 3.1. However, it turned out that there were little values (not significant) in the error in $\Delta\nu$ and $\nu_{\max}$. Thus, for this reason, $\Delta\nu$ and $\nu_{\max}$ were recorded without errors in this work. In addition, Hekker et al. (2012) have investigated the issue of uncertainties in $\Delta\nu$ and $\nu_{\max}$ by means of simulations using different methods and found that for a time series duration of about 500 d, the uncertainty in $\nu_{\max}$ is about 2 per cent and in $\Delta\nu$ is about 1 per cent.

In the section that follows, the methods described in sections 3.8 - 3.10 are applied to stars in the field of *Kepler* open cluster NGC 6819. First, the red giant stars that were identified by Balona et al. (2013b) in NGC 6819 were searched for solar-like oscillations. Solar-like oscillations in these stars were identified by visual inspection of their periodograms. The results of data reduction for stars in which period separation could be measured are shown in Table 3.2. Figure 3.11 (left panel) is the periodogram for some of the stars. Echelle diagrams are shown in Figures 3.14 and 3.15. In order to see in detail the mode pattern for different stars, we also show some of the echelle diagrams plotted with opposite aspect ratio in Figures 3.16 and 3.17. Period spacing distribution for KIC 5024327, KIC 5024476 and KIC 5024967 are shown in Figure 3.18.

Using echelle diagrams in Figures 3.14 - 3.17, we calculated period spacing ΔP presented in Table 3.2. Given the difficulty in identifying the modes we requested Dr E Corsaro to send us period spacing for the same stars in NGC 6819 (listed in Table 3.2). His ΔP measurements are listed in column 9 in Table 3.2. It is not easy to compare our calculated ΔP s with those of Dr Corsaro directly since we do not have error estimates. However, a plot of ΔP s from Corsaro vs our ΔP s, presented in Figure 3.13 shows a linear relationship with a slope of nearly 1.00. This shows that our determination of ΔP is comparable to that of Dr Corsaro. This gives confidence that our identifications are the same.

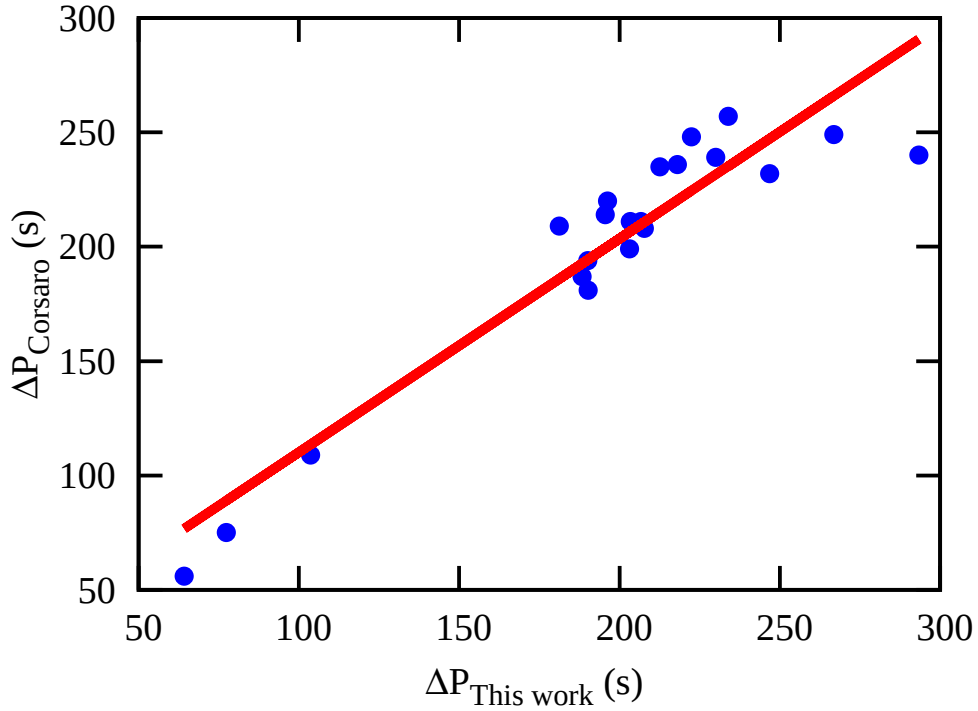


Figure 3.13: Comparison of ΔP obtained by Corsaro (2012) and this work with a slope of 0.934 ± 0.076 . The data plotted are in Table 3.2.

Table 3.2: A list of RG stars in NGC 6819 in which we were able to construct echelle diagrams and there after calculate the median gravity-mode period spacings. Comparison is made with the values of the frequency of maximum amplitude, $\nu_{\max}$, the large frequency separation, $\Delta\nu$, median gravity-mode period spacings with the work of Corsaro (2012). The columns are: the Kepler Input Catalogue number - KIC WIYN OPEN CLUSTER STUDY - WOCS, Membership - Mem (Single Member - SM, Binary Likely Member - BLM, Binary Member - BM): 4th - 6th columns are the results in this work while 7th - 9th columns are from Corsaro (2012). The radial velocity memberships were obtained from Hole et al. (2009).

KIC	WOCS	Mem	$\nu_{\max}(\mu\text{Hz})$	$\Delta\nu(\mu\text{Hz})$	$\Delta P(s)$	$\nu_{\max}(\mu\text{Hz})$	$\Delta\nu(\mu\text{Hz})$	$\Delta P(s)$
			This work			Corsaro		
4937056	002012	BM	46.867	4.748	203.4	43.4	4.76	211.
4937770	009024	SM	95.169	7.842	103.6	93.8	7.82	109.
5023953	003011	BLM	46.043	4.741	195.5	50.0	4.76	214.
5024327	011002	SM	42.484	4.709	206.7	43.9	4.73	211.
5024404	003004	SM	43.699	4.777	190.2	48.9	4.86	181.
5024476	001006	BLM	64.347	5.735	203.1	67.0	5.69	199.
5024582	009002	BLM	43.024	4.766	222.4	46.3	4.78	248.
5024967	006009	SM	43.754	4.711	190.1	46.9	4.72	194.
5111949	004011	SM	45.571	4.799	212.6	48.7	4.96	235.
5112361	004008	BM	69.588	6.140	77.4	67.4	6.18	75.
5112373	005005	SM	42.401	4.591	188.3	43.7	4.67	187.
5112387	003007	SM	43.123	4.681	207.8	45.7	4.75	208.
5112401	003009	SM	33.030	4.042	181.2	38.2	4.05	209.
5112467	006003	SM	45.367	4.730	196.3	46.3	4.71	220.
5112491	010002	SM	40.883	4.654	293.4	45.0	4.68	240.
5112730	004005	SM	39.829	4.537	246.8	45.7	4.55	232.
5112938	002006	SM	42.613	4.752	233.9	44.1	4.75	257.
5112950	003005	SM	41.009	4.334	266.8	42.8	4.30	249.
5112974	004009	SM	38.453	4.289	230.0	41.3	4.36	239.
5113441	012016	SM	150.899	11.671	64.2	154.8	11.71	56.
5200152	003021	SM	43.546	4.679	218.0	46.3	4.79	236.

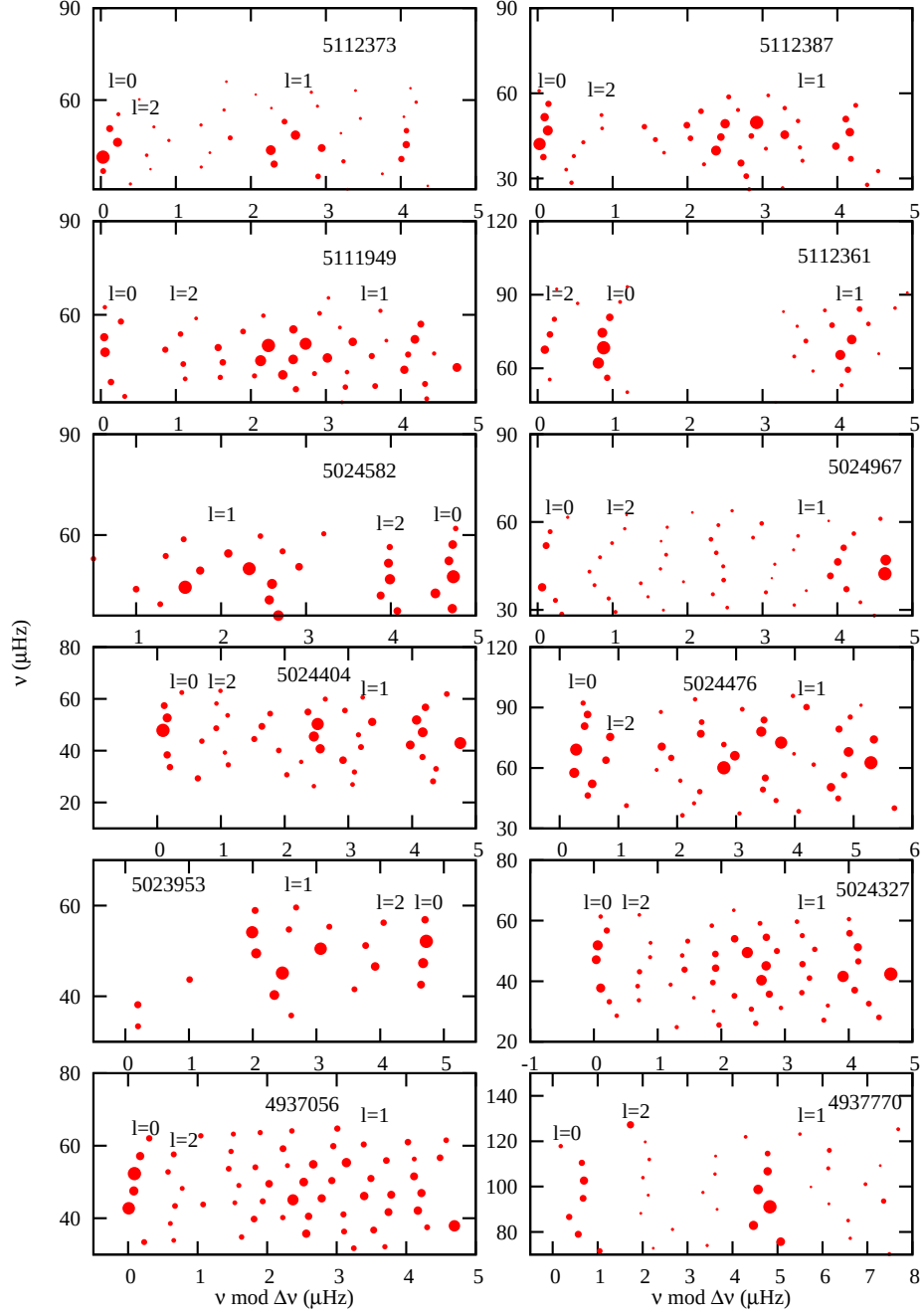


Figure 3.14: Echelle diagram for some of the red giant stars in NGC 6819 constructed using the extracted observed frequencies and the large frequency separations taken from Table 3.2. The modes $l = 0$, $l = 1$ and $l = 2$ are clearly marked. In each plot the name of the star is indicated. In calculating the median gravity-mode period spacings, we used only points for $l = 1$ for which gravity modes dominate. To distinguish between $l = 0$ and $l = 2$, one needs to rely on the fact that $l = 0$ modes generally have higher amplitudes. Symbol sizes are proportional to the amplitude.

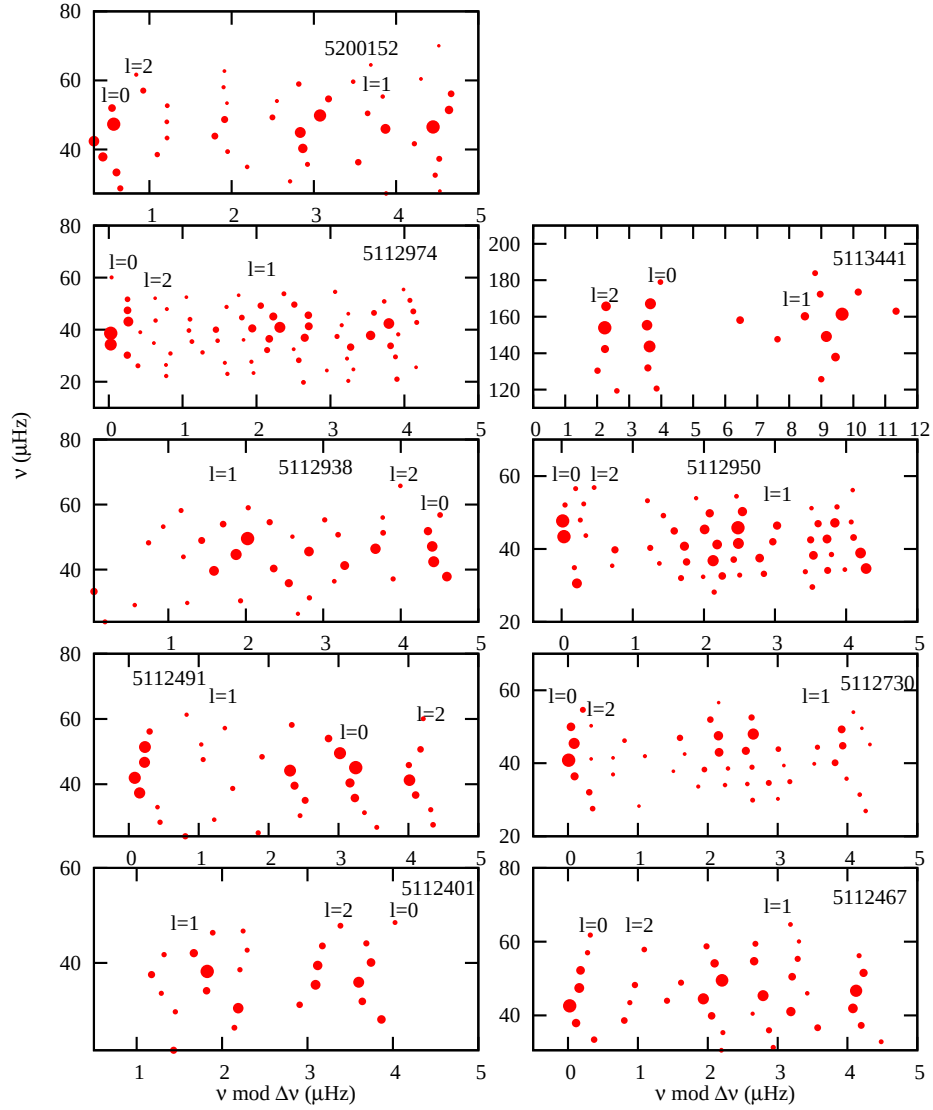


Figure 3.15: Echelle diagram for some of the red giant stars in NGC 6819 constructed using the extracted observed frequencies and the large frequency separations taken from Table 3.2. The modes $l = 0$, $l = 1$ and $l = 2$ are clearly marked. In each plot the name of the star is indicated. In calculating the median gravity-mode period spacings, we used only points for $l = 1$ for which gravity modes dominate. To distinguish between $l = 0$ and $l = 2$, one needs to rely on the fact that $l = 0$ modes generally have higher amplitudes. Symbol sizes are proportional to the amplitude.

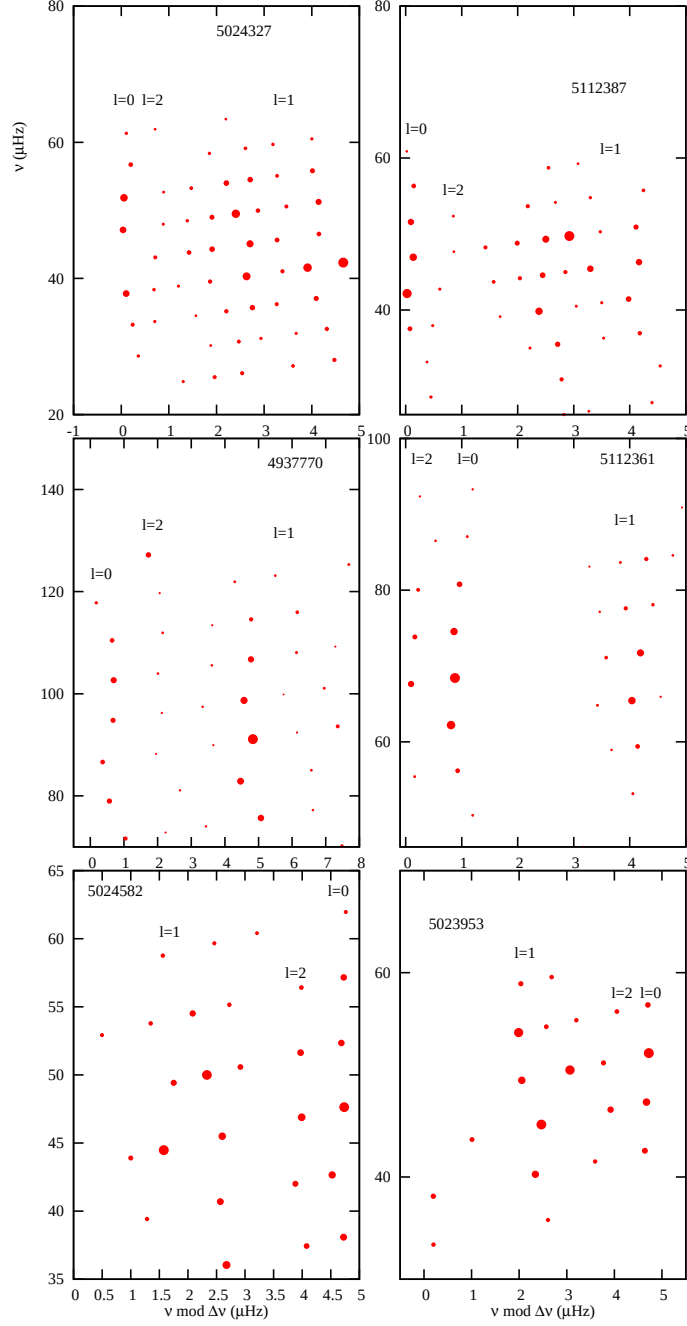


Figure 3.16: Echelle diagram for some of the red giant stars in NGC 6819 constructed using the extracted observed frequencies and the large frequency separations taken from Table 3.2. The modes $l = 0$, $l = 1$ and $l = 2$ are clearly marked. In each plot the name of the star is indicated. In calculating the median gravity-mode period spacings, we used only points for $l = 1$ for which gravity modes dominate. To distinguish between $l = 0$ and $l = 2$, one needs to rely on the fact that $l = 0$ modes generally have higher amplitudes. Symbol sizes are proportional to the amplitude.

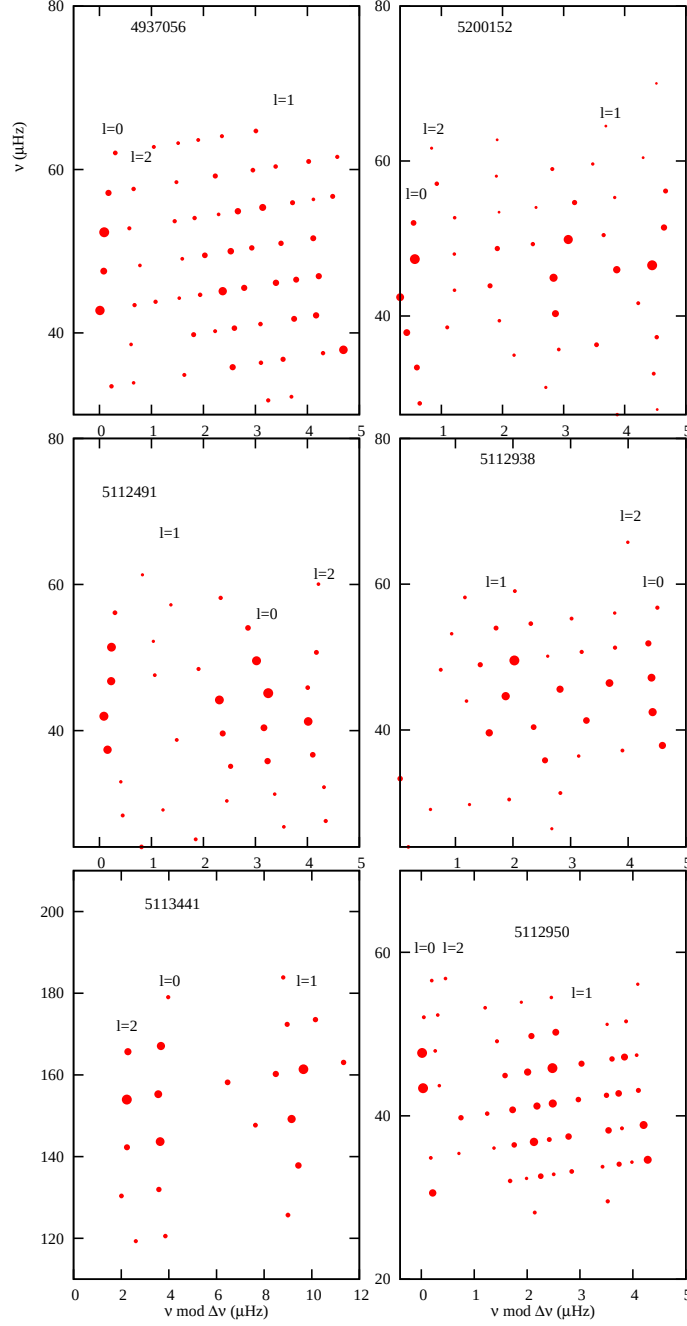


Figure 3.17: Echelle diagram for some of the red giant stars in NGC 6819 constructed using the extracted observed frequencies and the large frequency separations taken from Table 3.2 . The modes $l = 0$, $l = 1$ and $l = 2$ are clearly marked. In each plot the name of the star is indicated. In calculating the median gravity-mode period spacings, we used only points for $l = 1$ for which gravity modes dominate. To distinguish between $l = 0$ and $l = 2$, one needs to rely on the fact that $l = 0$ modes generally have higher amplitudes. Symbol sizes are proportional to the amplitude.

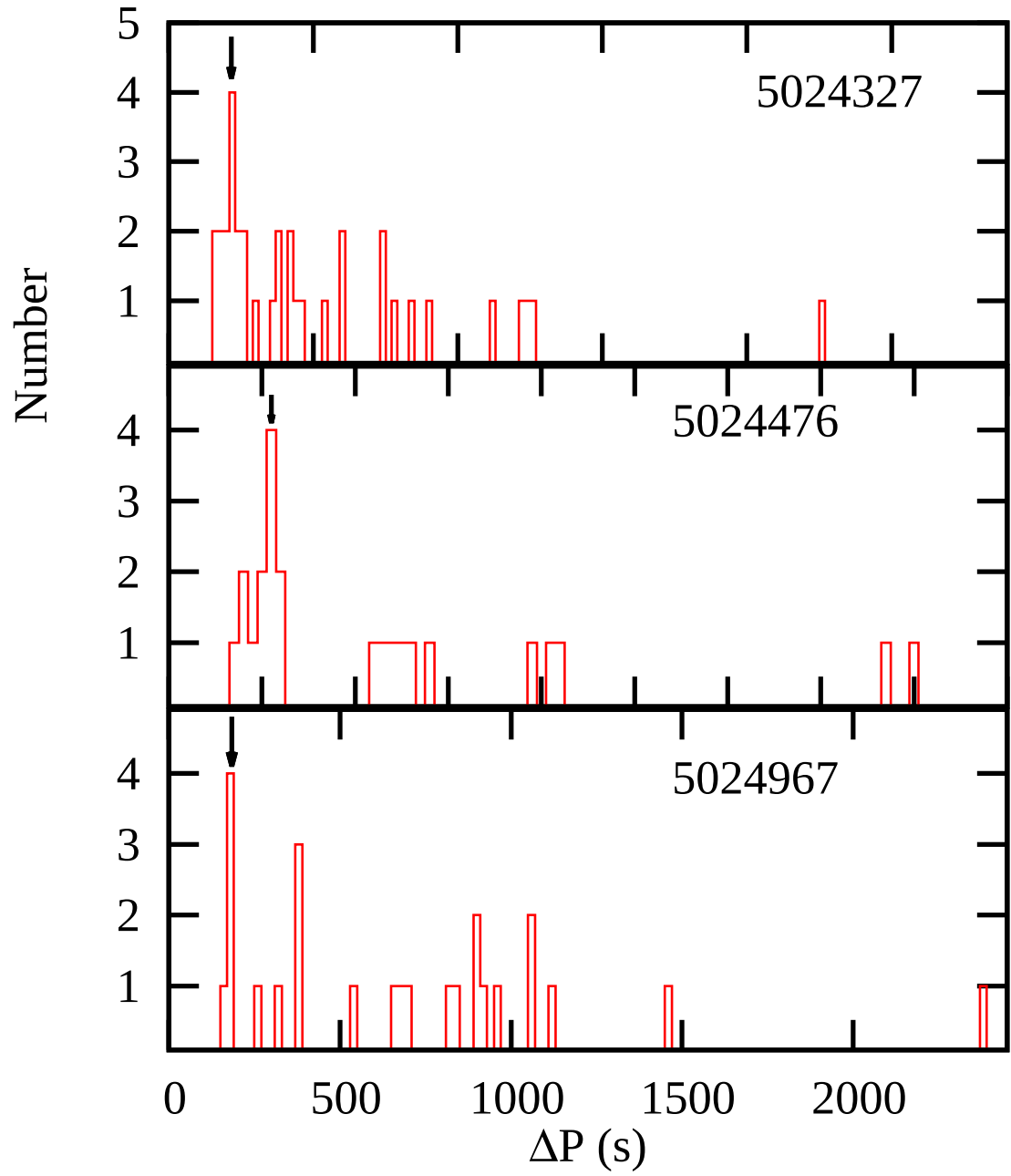


Figure 3.18: The distribution of gravity-mode period spacings, ΔP for some of the stars identified in Table 3.2. The vertical arrows indicate the ΔP chosen.

3.11 Special case - The distribution of gravity-mode period spacings

The period distribution diagram is presented in Figure 3.19 for the two stars (KIC 5112948 and 5024297). We select the period spacing from the value with the highest number in the period distribution graph. Clearly in the case of stars KIC 5112948 and 5024297 there are two bins with the same number value. It is difficult to exactly specify the ΔP for these stars because their period distributions peak at two different values indicated by the arrows in Figure 3.19. However, according to the literature (Stello et al., 2011a; Basu et al., 2011), KIC 5112948 & 5024297 are classified as RGB stars.

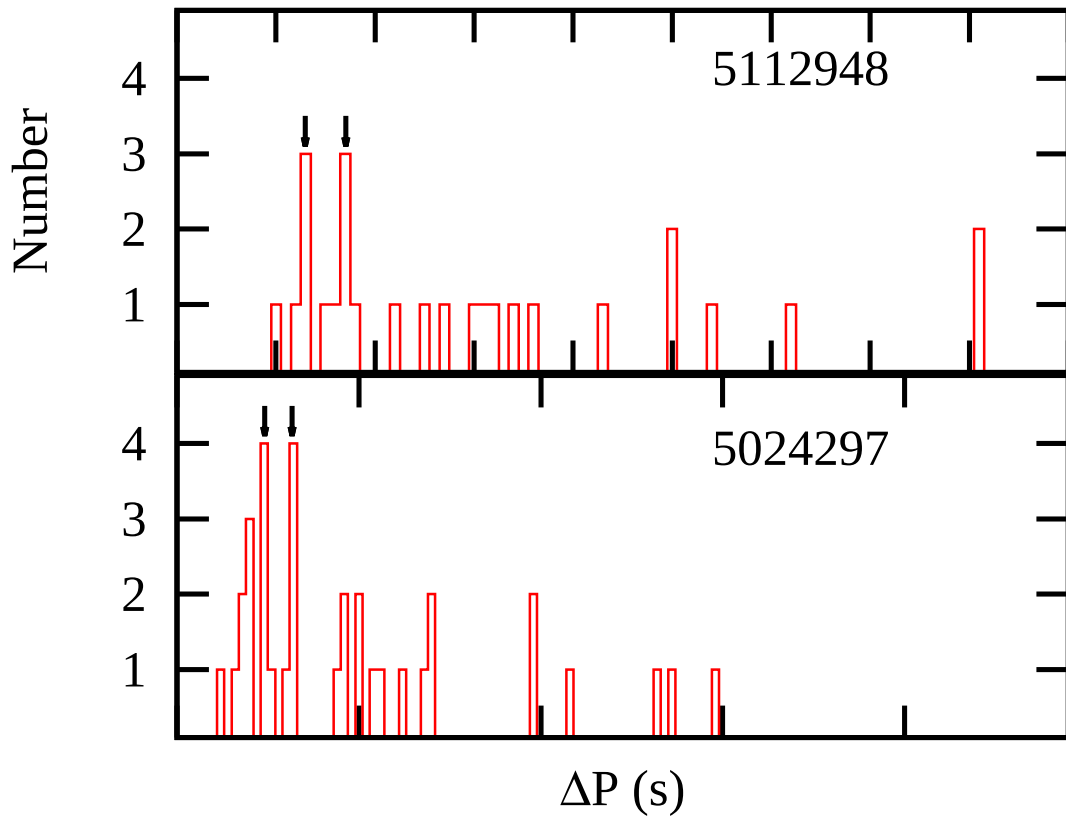


Figure 3.19: The distribution of gravity-mode period spacings, ΔP for some of the stars identified in Table 3.2. The two vertical arrows indicate the ΔP with the same number. In this case, it is difficult to specify the ΔP for such stars.

3.12 Selection of RGB and RGC stars

In determining distance moduli, reddening and mass loss, we selected stars that are single members (SM) of the cluster. Preference was also given to stars that were not marked by Corsaro et al. (2012) as outliers, misclassified CMD or evolved. We avoided stars listed as discrepant by Balona et al. (2013b). In addition, the classification of the stars into RGC/RGB were also based on the measurement of ΔP . In the case where ΔP could not be reliably measured, the stars previously identified by Basu et al. (2011) as those on the RGB and by Stello et al. (2011a) as single member & not clump were used as shown in Table 3.3. All stars which are marked as RGC in this work were previously classified by Corsaro et al. (2012) as RGC based on the measurement of ΔP . For each of the selected stars in Table 3.3, $\nu_{\max}$ and $\Delta\nu$ have been used together with the estimate for T_{eff} to obtain $\log L/L_{\odot}$, $\log R/R_{\odot}$, and $\log M/M_{\odot}$ using equations (1.3) - (1.5).

Table 3.3: A list of RG stars in the field of NGC 6819. The columns are: the *Kepler* Input Catalogue number - KIC, WIYN OPEN CLUSTER STUDY - WOCS, Membership - Mem, the frequency of maximum amplitude, $\nu_{\max}$, the large frequency separation, $\Delta\nu$, estimated effective temperature, T_{eff} , the mass, radius and luminosity calculated from the solar-like oscillations, $M/M_{\odot}$, $R/R_{\odot}$, and $(\log L/L_{\odot})$. The majority of the RGB stars were those previously identified by Basu et al. (2011) as those on the RGB and by Stello et al. (2011a) as single member & not clump stars.

KIC	WOCS	Mem	$\nu_{\max}(\mu\text{Hz})$	$\Delta\nu(\mu\text{Hz})$	T_{eff}/K	$M/M_{\odot}$	$R/R_{\odot}$	$\log L/L_{\odot}$
RGB			STARS					
4937576	005016	SM	30.881	3.515	4462	1.43	12.81	1.7661
5023732	005014	SM	25.855	3.089	4493	1.42	13.93	1.8511
5111718	008018	SM	124.841	10.534	4811	1.31	5.99	1.2365
5112072	009010	SM	119.537	10.013	4809	1.41	6.34	1.2861
5112744	005011	SM	42.470	4.406	4529	1.54	11.30	1.6830
5112880	002004	SM	23.964	2.789	4421	1.66	15.72	1.9279
5112948	005007	SM	40.517	4.297	4538	1.48	11.34	1.6890
5113041	004007	SM	35.456	3.945	4503	1.38	11.73	1.7060
5113441	012016	SM	150.899	11.671	4782	1.52	5.88	1.2100
RGC			STARS					
5024327	011002	SM	44.022	4.704	4675	1.39	10.44	1.6692
5024967	006009	SM	45.066	4.716	4657	1.46	10.61	1.6768
5111949	004011	SM	46.234	4.791	4723	1.51	10.62	1.7022
5112373	005005	SM	44.885	4.588	4673	1.62	11.19	1.7287
5112491	010002	SM	40.981	4.649	4680	1.17	9.96	1.6302
5112730	004005	SM	39.768	4.534	4646	1.17	10.12	1.6310
5112938	002006	SM	43.724	4.736	4667	1.32	10.22	1.6480
5200152	003021	SM	43.195	4.671	4751	1.38	10.47	1.7002
5112387	003007	SM	43.123	4.681	4682	1.33	10.33	1.6631
5112467	006003	SM	45.367	4.730	4664	1.48	10.62	1.6807

3.12.1 Distance determination and reddening

In this section I outline how the distance moduli of the cluster were calculated from the distance modulus of the RGC stars. Only clump stars are used in this case because they are the ones that are useful in the determination of cluster distances, reddening, and to investigate mass loss & internal mixing in evolving stars as first pointed out by Cannon (1970). This is done in several steps. The first step was to convert the KIC *griz* to standard Sloan Digital Sky Survey (SDSS) *griz* using the following equation given by Pinsonneault et al. (2012)

$$g_{SDSS} = g_{KIC} + 0.0921(g - r)_{KIC} - 0.0985, \quad (3.1)$$

$$r_{SDSS} = r_{KIC} + 0.0548(r - i)_{KIC} - 0.0383, \quad (3.2)$$

$$i_{SDSS} = i_{KIC} + 0.0696(r - i)_{KIC} - 0.0583, \quad (3.3)$$

$$z_{SDSS} = z_{KIC} + 0.1587(i - z)_{KIC} - 0.0597. \quad (3.4)$$

Where subscript KIC implies magnitude in the KIC system and SDSS implies magnitude in the SDSS system.

$$A_g = 1.196A_V, \quad (3.5)$$

$$A_r = 0.874A_V, \quad (3.6)$$

$$A_i = 0.672A_V, \quad (3.7)$$

$$A_z = 0.488A_V. \quad (3.8)$$

The total extinction values are those from An et al. (2009) and are given by equations (3.5) - (3.8).

$$g_0 = g - 1.196A_V, \quad (3.9)$$

$$r_0 = r - 0.874A_V, \quad (3.10)$$

$$i_0 = i - 0.672A_V, \quad (3.11)$$

and

$$z_0 = z - 0.488A_V. \quad (3.12)$$

Using A_V , the unreddened magnitude in g and r were determined using equations (3.9) - (3.12). In order to determine the distance modulus for each of the individual stars in the *Kepler* open cluster NGC 6819 and the mean distance modulus of the cluster, we needed to calculate absolute magnitude, apparent magnitude, absolute bolometric magnitude and bolometric correction. Given the distance, d , in parsecs, the absolute magnitude, M is given by

$$M = m - 5(\log_{10}d - 1), \quad (3.13)$$

where m is the apparent magnitude in the particular wave-band. The absolute bolometric magnitude, M_{bol} , is the magnitude corresponding to luminosity, L . The absolute bolometric magnitude of the Sun is $M_{bol\odot} = +4.75$ (see Sandage (2004)), and the relationship between the absolute magnitude of the Sun and of any star is given by

$$M_{bol} = 4.75 - 2.5 \log_{10}L/L_{\odot}. \quad (3.14)$$

The unreddened distance modulus is given by

$$\mu_0 = m_{bol} - M_{bol} = m_{bol} - 4.75 + 2.5 \log_{10}L/L_{\odot}, \quad (3.15)$$

where $m_{bol} = r_0 + BC_r$, and BC_r is the bolometric correction. The BC_r were all obtained from⁶. The values of $\log_{10}L/L_{\odot}$ were obtained in the way explained in section 3.12. The results for the distance modulus calculation of the RGC stars are shown in Table 3.4. It should be noted that the A_V in the *Kepler* catalogue is a crude value which is derived from a model of the galactic disk which assumes a certain smooth distribution of dust. If one wishes, instead of using *Kepler* A_V , one could use the A_V values by employing the fact that $A_V = 3.1E(B - V)$ with fixed $E(B - V) = 0.15$ (Hekker et al., 2011a; Basu et al., 2011) and finally deriving the mean A_V for the cluster as a whole. Using the mean A_V , one could use the value for each of the Red Giant stars in the determination of distance modulus of the cluster. This is presented in Table 3.5. Such a method was used by Balona et al. (2013b) in the determination of distance modulus of *Kepler* Open cluster NGC 6819 and they obtained $\mu_0 = 12.20 \pm 0.06$ mag.

Table 3.4: Unreddened distance moduli, μ_0 , for RGC single members of NGC 6819 (Hole et al., 2009). Values obtained with A_V from KIC. Mean distance moduli of $\mu_0 = 11.520 \pm 0.105$ mag is obtained.

KIC	m_{bol} (mag)	M_{bol} (mag)	μ_0 (mag)
5024327	12.078	0.577	11.501
5024967	12.162	0.558	11.604
5111949	12.172	0.494	11.677
5112373	12.088	0.428	11.660
5112491	12.029	0.674	11.355
5112730	12.079	0.672	11.407
5112938	12.126	0.630	11.496
5200152	11.913	0.500	11.413
5112387	12.184	0.592	11.592
5112467	12.045	0.548	11.497

⁶http://pleiadi.pd.astro.it/isoc_photsys.01/isoc_sloan/index.html

Table 3.5: Unreddened distance moduli, μ_0 , for RGC single members of NGC 6819 (Hole et al., 2009). Values obtained with uniform reddening for all stars using the mean value $E(B - V) = 0.15$. Mean distance moduli of $\mu_0 = 11.747 \pm 0.086$ mag is obtained.

KIC	m_{bol} (mag)	M_{bol} (mag)	μ_0 (mag)
5024327	12.342	0.577	11.765
5024967	12.368	0.558	11.810
5111949	12.320	0.494	11.825
5112373	12.330	0.428	11.902
5112491	12.277	0.674	11.603
5112730	12.297	0.672	11.625
5112938	12.316	0.630	11.686
5200152	12.247	0.500	11.747
5112387	12.359	0.592	11.767
5112467	12.291	0.548	11.743

3.13 Mass loss estimation

3.13.1 Introduction

As was mentioned in section 1.5.2.2, Red giants are expected to lose mass on the RGB, nearly all of it at the tip of the RGB, but the amount of mass loss is a major unsolved problem. The mass obtained from asteroseismology, equation (1.3) can in principle answer this question. However, there are several obstacles. Firstly, there are likely to be systematic differences in the way $\nu_{\max}$ and $\Delta\nu$ are measured. In particular, there is no precise definition of how $\nu_{\max}$ should be measured. However, only the relative mass difference between stars in the RGB and RC branches is required. In this section, an attempt to try to answer the question of mass loss in *Kepler* open cluster NGC 6819 is presented. To address mass loss problem, we first present summary of age estimate of NGC 6819 which are available in the literature. This is followed by details of our approach use to estimate mass loss.

3.13.2 Summary of age estimate of NGC 6819 in the literature

In order to study mass loss of RGC, one needs a good estimate for the age of the cluster. In Table 3.6, a summary of the age estimates of NGC 6819 in the literature is presented. The age that seems the best is chosen and later an isochrone of the best age using models which assume no mass loss was constructed. The criterion used in choosing the best age estimate was based on the modal value of the ages listed in Table 3.6. The age of NGC 6819 was taken to be 2.5 Gyr as this seems to be the best age estimate of this cluster. Of course, if one chooses to trust more the value derived from binaries, then its still consistent with the value of 2.5 Gyr. An isochrone of the age using models which assume no mass loss was constructed as shown in Figure 3.20. The observational data of RGB and RGC stars plotted in Figure 3.20 are those in Table 3.3. As seen in Figure 3.20, the isochrone fits temperature and luminosities of the RGB stars which are cluster members but does not fit the cluster member RGC stars. The fit was made using Padova Evolution Code to obtain isochrones of known age and metallicity. The advantage of the Padova code is that it allows mass loss free parameter, η , metallicity, Z , Helium content, Y and age to be varied. The Marigo et al.

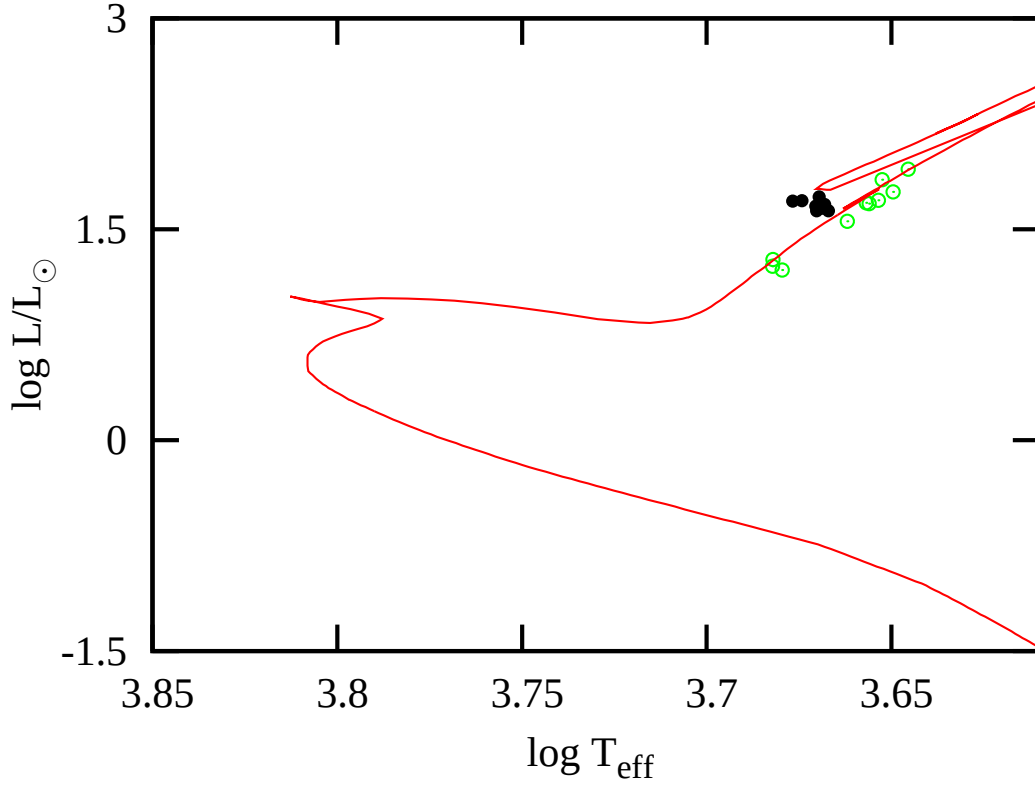


Figure 3.20: *Theoretical isochrone for $\log t = 9.4$ and $Z = 0.017$, $Y = 0.30$ with mass loss rate free parameter η set to 0.0 calculated with Padova Evolution code (Marigo et al., 2008). The filled circles are the RGC stars while the open circles are the RGB stars.*

(2008) evolutionary track consists of models with the Padova stellar evolution. Padova uses the OPAL high-temperature opacities complemented in the low temperature regime with the tables of Marigo & Aringer (2009). Thus suggesting that mass loss has occurred or some other physics is missing in the models. Going by the former, if there has been no mass loss, the isochrone should fit both the RGB and RGC stars.

3.13.3 Statistical techniques to study mass loss

To study mass loss one needs to provide values of $\log L/L_\odot$, $\log R/R_\odot$, and $\log M/M_\odot$. In our case the selected stars with the associated parameters are in Table 3.3. The estimates of effective temperature, T_{eff} and radius are also available in the KIC. These estimates are based on multicolour photometry and are subject to large uncertainties. The error in T_{eff}

Table 3.6: A list of NGC 6819 cluster age estimate by different authors. The columns are: the age (Gyr), methods used for determining the age, and the last column: authors.

Age (Gyr)	Method	Author
2.0	main sequence stars	Lindoff (1972)
2.0	isochrone	Rosvick & Vandenberg (1998)
2.5	isochrone fitting	Bragaglia et al. (2001)
2.5	isochrone fitting	Kalirai et al. (2001)
1.5	Open cluster data (WEBDA)	Mermilliod & Paunzen (2003)
2.5	K magnitude MS turn-off	Beletsky et al. (2009)
2.4	isochrone	Hole et al. (2009)
2.0 - 2.4	asteroseismology	Basu et al. (2011)
2.1	isochrone fitting	Miglio et al. (2012)
2.5	isochrone fitting	Balona et al. (2013b)
2.65 ± 0.25	binaries	Sandquist et al. (2013)
2.5	isochrone fitting	Jeffries et al. (2013)
3.1	mass - radius of binary star	Jeffries et al. (2013)
1.6	Taken from WEBDA	Mészáros et al. (2013)
1.9 ± 0.1	isochrone fitting	Wu et al. (2014)

is approximately 100-200 K (Brown et al., 2011), which means the standard deviation in $\log(T_{\text{eff}})$ is about 0.038 for $T_{\text{eff}} \sim 4000$ K. The radius is derived from photometric estimates of $\log g$ which have typical errors of about 0.5 dex (Brown et al., 2011), together with the mass estimated from evolutionary models. A standard deviation of 0.1 in mass, which is about 10 percent of the mass of a typical star on the RGB, is assumed as a reasonable value. The resulting error in $\log L/L_{\odot}$ is about 0.2 (taking into consideration that the standard deviation in luminosity assumes a standard deviation in absolute magnitude of about 0.5 mag which is the typical standard deviation in photometric estimate). We have also assumed that ν_{max} can be measured to about 2 percent, $\Delta\nu$ to about 1.5 percent (Chaplin et al., 2008; Huber et al., 2009). Thus, the approximate standard deviations can be written as $\sigma(M)/M = 0.100$, $\sigma(L)/L = 0.200$, $\sigma(T)/T = 0.038$. The exact values of these standard deviations is not important because we try to minimize the square of the distance from a particular observed point (M , L , T_{eff}) to the locus of the evolutionary track which describes a line in 3-dimensional (M , L , T_{eff}) space.

The goodness-of-fit criterion is then:

$$\chi^2 = \sum_{i=1}^n \left(\frac{(t_i - T)^2}{2\sigma_t^2} + \frac{(l_i - L)^2}{2\sigma_l^2} + \frac{(m_i - M)^2}{2\sigma_m^2} \right), \quad (3.16)$$

where $\log T_{\text{eff}} = t$, $\log L/L_{\odot} = l$, $\log M/M_{\odot} = m$, (T , L , M) are true values on the isochrone and σ_t^2 , σ_l^2 and σ_m^2 are the variance (as already explained in the first paragraph of section 3.13.3). The χ^2 depends on the relative values of the three standard deviations and not on the actual values. Therefore one can multiply σ_M , σ_L and σ_T by the same factor (no matter how big or small) and end up with the same answer.

As a starting point to the fit, an age of 2.5 Gyr is used as a best age estimate and metallicity $Z = 0.017$. The assumption that there is no mass loss at all for the RGB stars is taken. In this case the observed $\log L/L_{\odot}$, $\log R/R_{\odot}$, and $\log M/M_{\odot}$ for each RGB should fall on the isochrone line with the same $\log L/L_{\odot}$, $\log R/R_{\odot}$, and $\log M/M_{\odot}$. However, because of the observational errors and possible systematic differences, the observed points do not all lie on the isochrone line as shown in Figure 3.20. We assume that there are errors in quantities t , l and m such that the true values are (T , L , M) in our discussions.

Suppose that the errors in t , l and m are normally distributed in a way that the probability densities are given by the following equation:

$$\begin{aligned} P(t) &= \frac{1}{\sqrt{2\pi\sigma_t^2}} \exp\left(-\frac{(t-T)^2}{2\sigma_t^2}\right), \\ P(l) &= \frac{1}{\sqrt{2\pi\sigma_l^2}} \exp\left(-\frac{(l-L)^2}{2\sigma_l^2}\right), \\ P(m) &= \frac{1}{\sqrt{2\pi\sigma_m^2}} \exp\left(-\frac{(m-M)^2}{2\sigma_m^2}\right), \end{aligned} \quad (3.17)$$

The probability that the point (t, l, m) comes from a distribution with the true values (T, L, M) is given as

$$P(t, l, m) = P(t) \times P(l) \times P(m), \quad (3.18)$$

Equation (3.16) also gives goodness-of-fit criterion. The closer the observed values are to the true value, the smaller the value of χ^2 . In other words, χ^2 gives the scaled distance between an observed point (t, l, m) and the true value on the isochrone (T, L, M) .

Making use of equation (3.16) by assuming that the true values are given by the isochrone and knowing the observed values (t, l, m) , we find (T, L, M) by minimizing χ^2 . For each RGB star we calculate χ^2 according to equation (3.16). These points give the most likely true values for the stars.

Taking into consideration that there are systematic errors in t, l, m and given the fact that the most likely true values (T, L, M) corresponding to (t, l, m) are known, we find the average differences given by the following equation,

$$\begin{aligned} \delta t &= \frac{1}{n} \sum_{i=1}^n (T_i - t_i), \\ \delta l &= \frac{1}{n} \sum_{i=1}^n (L_i - l_i), \\ \delta m &= \frac{1}{n} \sum_{i=1}^n (M_i - m_i), \end{aligned} \quad (3.19)$$

for the n stars on the RGB. The values $(t + \delta t, l + \delta l, m + \delta m)$ are better estimates of the true values. If the systematic corrections are applicable to RGB stars, then they must also apply to

Table 3.7: Mass loss results by varying age with constant metallicity $Z = 0.017$, $Y = 0.30$. No correction to the $\Delta\nu$ scaling was applied to clump stars.

Age (Gyr)	Ratio of observed mass to mass from the isochrone
1.9	0.926 ± 0.031
2.0	0.926 ± 0.031
2.1	0.927 ± 0.031
2.4	0.928 ± 0.031
2.5	0.928 ± 0.031

RGC stars. This means that even if one has made some systematic error in measuring $\nu_{\max}$, $\Delta\nu$ or $\log T_{\text{eff}}$, such systematic error can be compensated provided the number, n , of the RGB cluster members is such that one can estimate δt , δl and δm with reasonable accuracy.

Following the same approach for RGB stars in studying RGC stars. Applying the corrections determined from the the RGB stars to (t, l, m) to get corrected values (t', l', m') . As before, we determine (T, L, M) by finding the smallest χ^2 . This gives the true values for each RGC star. The mass loss is calculated by obtaining the average value $\Delta M = \sum_{i=1}^p (m'_i - M_i)$. The mass loss (ΔM) is expressed in the form of the ratio between observed mass and mass from the isochrone. If no mass loss has occurred, the ratio between observed mass and mass from the isochrone must be equal to 1.00, while if mass loss of the RGC stars occurred, then the the ratio between observed mass and mass from the isochrone must be less than 1.00. We use isochrones of slightly different ages in order to determine how the result is affected by uncertainty in age. Further more, we also vary metallicities in order to determine how the result depends on metallicity. The summary of the analysis of the results are presented in Tables 3.7 and 3.8 (no correction applied to $\Delta\nu$ of RGC stars).

Miglio et al. (2012) studied mass loss in NGC 6819 and found out that a correction of 1.9 % needs to be applied to $\Delta\nu$ of RGC stars in order to fit the stellar evolution model. We applied a correction of 1.9 % to $\Delta\nu$ of our RGC stars in order to determine the effect on the determination of mass loss (Tables 3.9 and 3.10).

Table 3.8: Mass loss results by varying Z, Y with constant age of 2.5 Gyr. No correction to the $\Delta\nu$ scaling was applied to clump stars.

Z	Y	Ratio of observed mass to mass from the isochrone
0.020	0.30	0.932 ± 0.030
0.019	0.30	0.929 ± 0.031
0.017	0.28	0.934 ± 0.030

Table 3.9: Mass loss results by varying age with constant metallicity Z = 0.017, Y = 0.30. Correction to the $\Delta\nu$ scaling was applied to clump stars.

Age (Gyr)	Ratio of observed mass to mass from the isochrone
1.9	1.000 ± 0.033
2.0	1.000 ± 0.033
2.1	1.001 ± 0.033
2.4	1.002 ± 0.033
2.5	1.002 ± 0.033

Table 3.10: Mass loss results by varying Z, Y with constant age of 2.5 Gyr. Correction to the $\Delta\nu$ scaling was applied to clump stars

Z	Y	Ratio of observed mass to mass from the isochrone
0.020	0.30	1.005 ± 0.032
0.019	0.30	1.005 ± 0.032
0.017	0.28	1.006 ± 0.032

Table 3.11: Summary of the results of mass-loss in red giants from asteroseismology of RGB and RGC stars using different ages and metallicities for NGC 6819. The fourth column shows mass loss derived directly from equation (1.3) and (1.5) while the last column shows the mass loss when a correction of 1.9 % is applied to $\Delta\nu$ for RGC stars.

Age (Gyr)	Z	Y	Mass loss (%) without correction	Mass loss (%) with correction
2.5	0.017	0.30	7.0 ± 3.1	-0.2 ± 3.3
2.5	0.020	0.30	6.8 ± 3.1	-0.5 ± 3.2
1.9	0.017	0.30	7.4 ± 3.1	0.0 ± 3.3

3.14 Results and discussions

Echelle diagrams for 21 stars in the field of *Kepler* open cluster NGC 6819 of which 17 are confirmed single members of the cluster based on the photometric and radial velocity measurements as shown in Table 3.2 were constructed.

The results for distance modulus estimations are shown in Tables 3.4 and 3.5 with mean distance moduli of $\mu_0 = 11.520 \pm 0.105$ mag and $\mu_0 = 11.747 \pm 0.086$ mag for varying A_V from KIC and fixed $E(B - V) = 0.15$ for the cluster respectively. These values are lower than the values obtained by Balona et al. (2013b) of $\mu_0 = 12.20 \pm 0.06$ mag using RG stars with solar-like oscillations, $\mu_0 = 11.88 \pm 0.08$ using luminosities derived from effective temperatures and radii, and $\mu_0 = 11.94 \pm 0.04$ for stars with well-defined lower main sequence of the cluster. We attribute the slight variation in our derived value of μ_0 to the different way we estimated T_{eff} hence different luminosities. The differences in distance modulus can also be as a result of different assumptions concerning the cluster reddening.

Shown in Table 3.2 are the stars for which we could construct echelle diagrams. Most of the stars are RGC stars given the fact that $\Delta P > 100$ s. We were unable to construct echelle diagram and measure ΔP for most of the RGB stars because most RGB stars do not show mixed modes, so one cannot measure ΔP for them. The reason why they do not show mixed modes (or at least fewer mixed modes than RGC stars) is because the resonance between the solar-like oscillations and the core eigenfrequencies is much narrower (Miglio et al., 2012).

In Figure 3.21, we show a plot comparing ν_{max} obtained by Corsaro (2012) and this work. We have found that Corsaro et al. (2012) ν_{max} differ from our work by 5.4 %, so $\nu_{\text{max}}(\text{Corsaro})/\nu_{\text{max}}(\text{this work}) = 1.054$.

Figure 3.22 shows a comparison of $\Delta\nu$ obtained by Corsaro (2012) and this work. We found that Corsaro (2012) values of $\Delta\nu$ are more than the values in this work by 0.4 % ($\Delta\nu(\text{Corsaro})/\Delta\nu(\text{this work}) = 1.004$).

In Figure 3.13, we made comparison of ΔP obtained by Corsaro et al. (2012) and this work. We have found that the ΔP from Corsaro (2012) were more than the ΔP in this work by 1.9 % ($\Delta P(\text{Corsaro})/\Delta P(\text{this work}) = 1.019$). The agreement between $\Delta\nu$ in this work and that of Corsaro et al. (2012) is good, $\Delta\nu$ is a well defined quantity. In comparison for

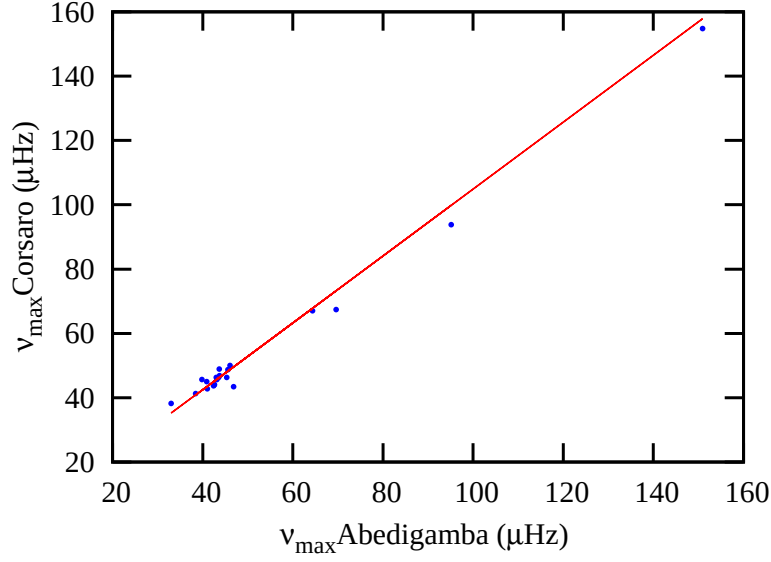


Figure 3.21: Comparison of $\nu_{\max}$ obtained by Corsaro (2012) and this work. The data plotted are in Table 3.2.

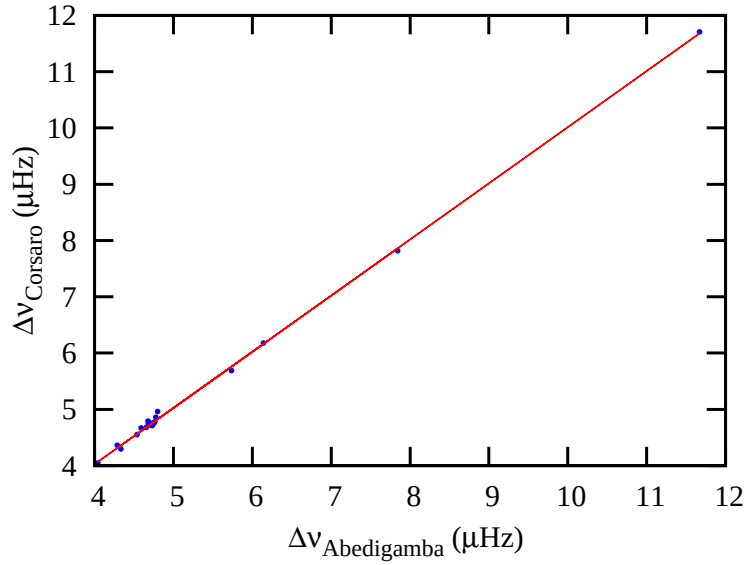


Figure 3.22: Comparison of $\Delta\nu$ obtained by Corsaro (2012) and this work. The data plotted are in Table 3.2.

$\nu_{\max}$, we see that the agreement is 5.4 % since $\nu_{\max}$ is not a well defined quantity. There is no proper theory that describes why $\nu_{\max}$ should be closely related to the critical frequency and the equation that is used is an empirical relationship that is assumed to be true. We attribute this to the substantial difference in our $\nu_{\max}$ and those obtained by Corsaro et al. (2012). In

addition the reason for the differences in the measured quantities could be in the quality of the data (Q0 - Q16) while previous work had data with less observing quaters or the value of the adopted metallicity and reddening in estimating temperature.

It should be noted that Hekker et al. (2011b) discussed and summarized the different method of measuring $\nu_{\max}$. For example, the autocorellation method described by Mosser & Appourchaux (2009) where $\nu_{\max}$ is obtained by the centroid of a Gaussian fit to the smoothed power spectrum. Other methods are: $\nu_{\max}$ is measured by the centroid of a Gaussian fitted to the unsmoothed power spectrum, while other authors measure $\nu_{\max}$ by fitting a Gaussian to the smoothed power spectrum (Mathur et al., 2010). Hekker et al. (2011b) showed how the different values of $\nu_{\max}$ changes when measured using different methods and how $\Delta\nu$ does not change much as compared to $\nu_{\max}$. This clearly explains why our measured $\nu_{\max}$ is slightly different as compared to that of other authors found in the literatures.

For example, Hekker et al. (2011b) showed how the different values of $\nu_{\max}$ changes when measured using different methods and how $\Delta\nu$ does not change much as compared to $\nu_{\max}$.

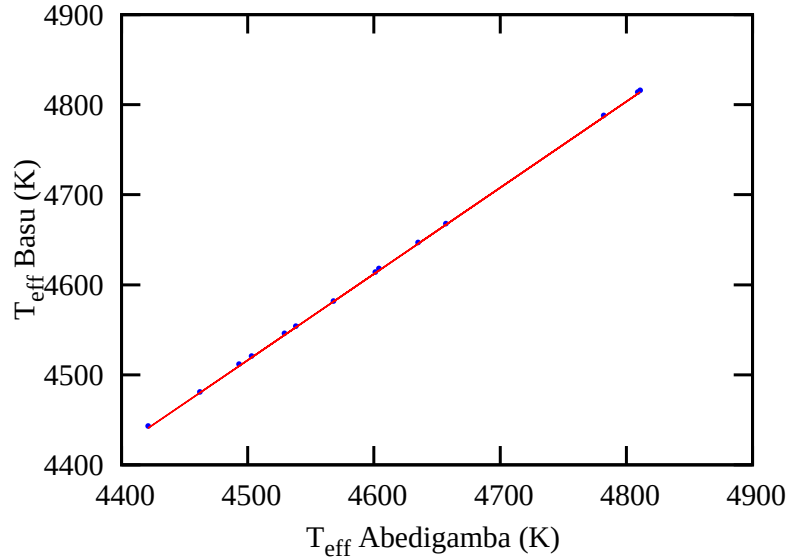


Figure 3.23: Comparison of T_{eff} obtained by Basu et al. (2011) and this work. The data plotted are found in Table 3.12.

Figure 3.23 is a comparison of T_{eff} obtained by Basu et al. (2011) & this work (data is from Table 3.12). We found a very small systematic trend between our best T_{eff} estimates

and those estimated by Basu et al. (2011) with difference being 0.3 %. We attribute the small difference to the value of $[\text{Fe}/\text{H}]$ used. We adopted a value of +0.1 while Basu et al. (2011) used a value of $[\text{Fe}/\text{H}] = +0.09$.

As shown in Table 3.7 for the age estimate of $\log \text{Age} = 9.4$ (2.5 Gyr) and $Z = 0.017$, $Y = 0.30$, we obtained the average ratio between the observed mass and the mass from the isochrone for the RGC branch to be 0.93 ± 0.03 , which means that the mass loss at the tip of the RGB is about 7 %. We investigated the effect of using isochrone of slightly different ages of NGC 6819 to determine how the result is affected by uncertainties in age. As shown in Table 3.7, no significant differences are seen. In Table 3.8, we varied metallicity in order to find out how the result depends on metallicity. As seen in Table 3.8, the results are not affected by varying metallicity slightly. Table 3.11 is the summary of the interpretation of the results with and without correction of ΔP of the RC stars.

We further used isochrones with mass loss and varied the mass loss rate free parameter η in order to determine the exact value for which η was able to give a ratio between observed mass and mass from the isochrone of 1.00. We used isochrone of $\log \text{Age} = 9.4$ (2.5 Gyr) and $Z = 0.017$, $Y = 0.30$. We found that the value of η which gives a ratio between observed mass and mass from the isochrone of 1.00 is 0.318.

Looking at the results without correction applied to $\Delta\nu$ and with correction applied to $\Delta\nu$ (Table 3.11), we see that the mass loss are approximately + 0.07 and 0.00 $M_{\odot}$ respectively. The fact that in some cases we get a negative value of mass loss when we apply a correction to $\Delta\nu$ means that there is no mass loss when the $\Delta\nu$ correction is applied. Such similar result was obtained by Miglio et al. (2012) who studied mass loss of RGB by comparing the average mass of stars in the red clump (RC) with that of stars in the low-luminosity portion of the RGB (stars with $L \lesssim L(\text{RC})$). Miglio et al. (2012) found a mass loss for NGC 6819 RGB of + 0.09 $M_{\odot}$ without correction applied to $\Delta\nu$ and - 0.03 $M_{\odot}$ with correction applied to $\Delta\nu$. They concluded that the mass loss results of RGB stars in NGC 6819 was weaker than those in NGC 6791 due to lack of an independent and accurate distance measurement. From the analysis, it appears that the mass loss on the tip of the RGB all depends on how one applies correction to $\Delta\nu$.

Table 3.12: Stars that were studied in this thesis as well as in previous studies by Basu et al. (2011) and Corsaro (2012). Column 4th - 5th (this work), 6th - 7th (Basu et al., 2011), 8th - 9th (Corsaro, 2012).

KIC	WOCS	Mem	$\nu_{\max}$ (μHz) ^a	$\Delta\nu$ (μHz)	$\nu_{\max}$ (μHz) ^b	$\Delta\nu$ (μHz)	$\nu_{\max}$ (μHz) ^c	$\Delta\nu$ (μHz)
4937576	005016	SM	30.881	3.515	32.290	3.560	31.1	3.56
5023732	005014	SM	25.855	3.089	27.450	3.110	26.9	3.11
5023931	007009	BM	45.490	4.899	51.270	4.890	50.1	4.93
5024297	008003	SM	43.223	4.544	46.030	4.550	46.1	4.57
5024405	004001	SM	91.965	8.259	97.840	8.230	96.4	8.28
5111718	008018	SM	124.841	10.534	132.960	10.450	135.2	10.52
5111940	005012	SM	53.428	5.150	52.890	5.140	51.9	5.18
5112072	009010	SM	119.537	10.013	126.510	9.980	125.0	10.01
5112744	005011	SM	42.470	4.406	44.740	4.440	45.4	4.42
5112880	002004	SM	23.964	2.789	26.650	2.800	26.3	2.81
5112948	005007	SM	40.517	4.297	43.960	4.280	43.3	4.28
5113041	004007	SM	35.456	3.945	37.570	3.940	37.7	3.97
5024404	003004	SM	43.698	4.777	49.260	4.780	48.9	4.86
5113441	012016	SM	150.899	11.671	152.580	11.570	154.8	11.71

^aThis work

^bBasu et al. (2011)

^cCorsaro (2012)

From the discussion, it is evident that the exact amount of mass-loss in RGB stars in NGC 6819 is not known. We notice that the result we get here using our new method is interesting and slightly lower than that obtained by Miglio et al. (2012) but still showing evidence of mass loss at the tip of the RGB for the case of NGC 6819 with no correction applied to $\Delta\nu$. It is important to mention that our mass loss estimate also included the use of correction to the $\Delta\nu$ of all RGC as was done by Miglio et al. (2012) when they compared their model results and observations.

Chapter 4

Search for RGB and RGC stars amongst solar-like stars in the open cluster

NGC 6866

4.1 Introduction

The *Kepler* mission has been helpful in the study of solar-like oscillations in Red Giant (RG) stars. *Kepler* field has four open clusters, NGC 6866 is one of them and the youngest (age 0.65 Gyr). It is centered at (RA = 20:03:55, Dec. = +44:09:30; $l = 79^\circ.560$, $b = +6^\circ.839$) and with a main-sequence turn-off spectral type of about A3. NGC 6866 is roughly located in the direction of solar apex (the point towards which the Sun is moving) which means that all stars (members and non-members) have similar proper motion. Therefore, one cannot use proper motion to discriminate between members and non-members. No radial velocity measurements for stars located in the field of NGC 6866 are available to help in discriminating between members and non-members. The only method left in discriminating between members and non-members in the field of NGC 6866 is the photometric distance method. However, with only a single method one cannot be quite sure of the membership.

Molenda-Żakowicz et al. (2009) searched the cluster for variables and discovered three δ Scuti and two γ Doradus stars. They also found that most of the stars were placed on the main sequence and that the red giant clump were not visible. Loktin et al. (1994) derived $[\text{Fe}/\text{H}] = +0.10$ from photometry and obtained an age = 0.66 Gyr. Gunes et al. (2012) found the age value of 0.8 ± 0.1 Gyr, while Joshi et al. (2012) estimated the age from the color-

magnitude diagram using the theoretical isochrones of solar metallicity and found an age ~ 0.63 Gyr. Balona et al. (2013a) estimated the age of *Kepler* open cluster NGC 6866 to be 0.65 ± 0.1 Gyr with isochrones of solar composition. Kharchenko et al. (2005) obtained an age of 0.5 Gyr using isochrone-based procedure. Joshi et al. (2012) obtained UBVRI photometry and time series photometry of several stars in NGC 6866. In fact, they discovered 19 additional new variables that included δ Scuti and γ Doradus stars. Joshi et al. (2012) also found a distance modulus of ~ 11.84 mag. Another distance modulus calculation of NGC 6866 was performed by Gunes et al. (2012) who found a value of 11.08 using decontamination techniques. According to Hekker et al. (2011b), no red giants have been observed in NGC 6866, however the study carried out by Balona et al. (2013a) on the field of *Kepler* open cluster NGC 6866 reveals that there are 23 Red Giants (RG) with solar-like oscillations as shown in Figure 4.1. Balona et al. (2013a) used photometric distance method to claim that 5 RG stars out of the 23 RG with solar-like oscillations are members of the *Kepler* open cluster NGC 6866. They made comparison between the photometric distance of each of the star with the one of the cluster. Thus, if the distance of the star is the same with that of the cluster then the star is a member.

The aim of this chapter is to determine if the RG stars in which solar-like oscillations have been detected in the field of *Kepler* open cluster NGC 6866 are either RGB or RC stars. This will improve our knowledge of the stages of stellar evolution of RG stars in NGC 6866.

4.2 The data

The data of RG with solar-like oscillations in this chapter were obtained with NASA's *Kepler* space telescope. Most of the observations are obtained using an exposure time of about 30 min (long-cadence mode), but a few thousand stars have also been observed with an exposure time of about 1 min (short-cadence mode). *Kepler* studies of pulsating red giants in these clusters allow independent estimates of mass, radius and surface gravity of each star.

The publicly available *Kepler* data (Q0 - Q16) were used in this study, where Q stands for quarters referring to the interval in which the data were downloaded after certain time interval, where Q0 is a 10-d commissioning run. Stars selected in the KIC within a radius of

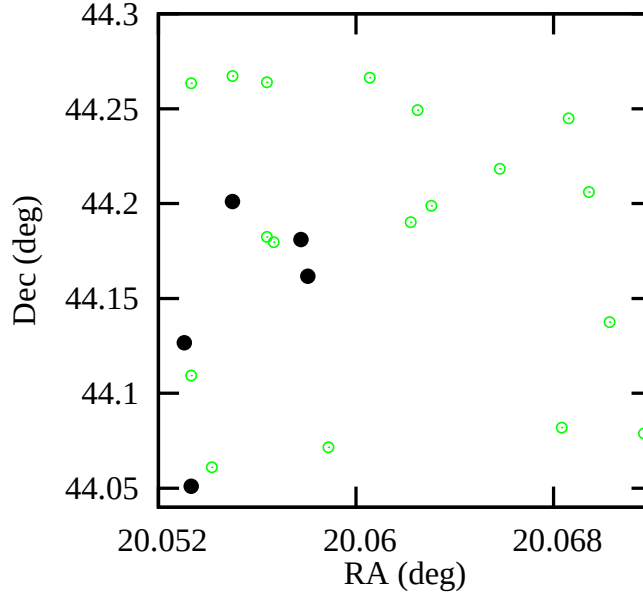


Figure 4.1: The map of the field showing the location occupied by the RG stars with solar-like oscillations in NGC 6866. Open circles are the 23 RG stars discovered by visual inspections of the periodogram and light curves Balona et al. (2013a). Filled circles are the clump and RGB stars identified in this work.

10 arcmin of the cluster center were used. The light-curve files of the stars contain simple aperture photometry (SAP) flux and a more processed version of SAP with artefact mitigation included. Thus we used the presearch data conditioning (PDC) flux. Jumps between quarters were calculated. The resulting corrected data were used to calculate periodograms as described in chapter 3. Periodograms of five RG stars with solar-like oscillations are shown in Figures 4.2 and 4.3. Part of the resulting smoothed periodograms are shown in Figure 4.4 and the corresponding echelle diagrams in Figure 4.5. The data was reduced in the same way as in chapter 3 and similar procedure described in section 3.4. A total of 23 stars in NGC 6866 were identified by Balona et al. (2013a) using visual inspections of the light curves and periodograms as stars with solar-like oscillations as shown in Figure 4.1. All the 23 RG stars in this study were used to determine the population of the RGC and RGB stars. $\nu_{\max}$, $\Delta\nu$, $M/M_{\odot}$, and $L/L_{\odot}$ for each of the stars were measured.

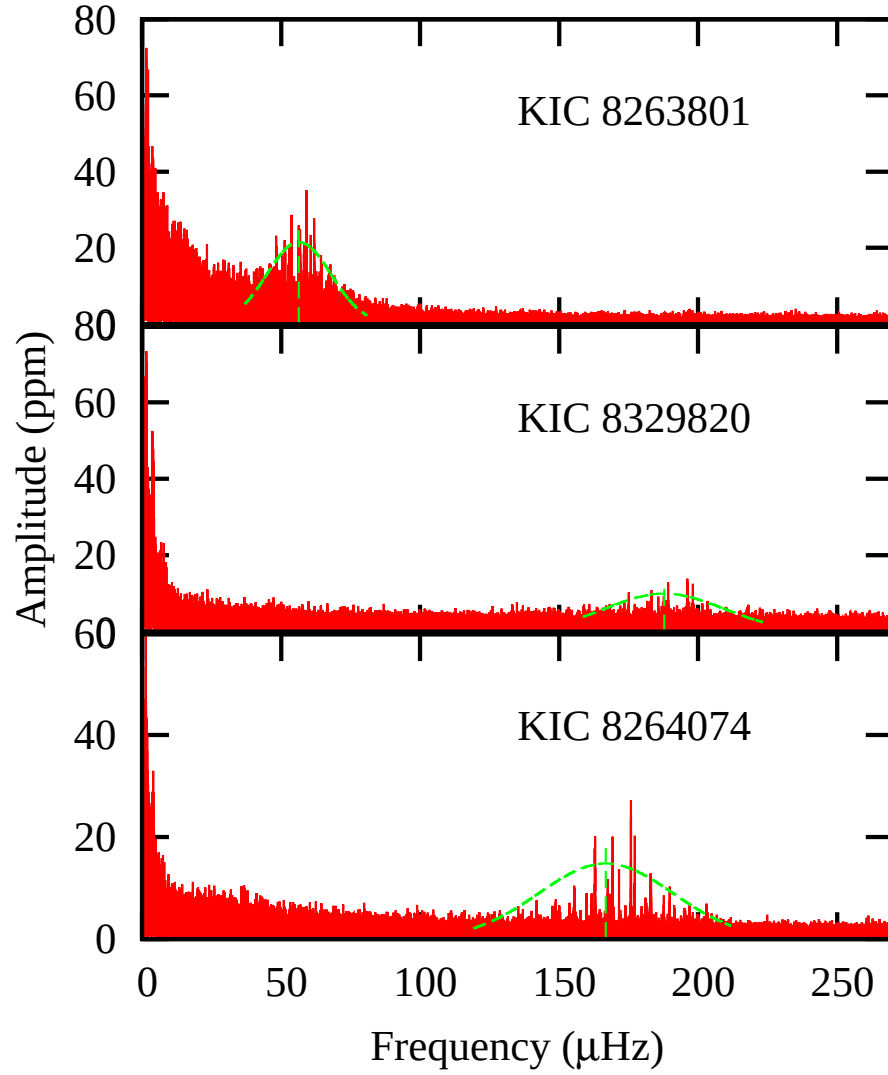


Figure 4.2: The periodograms of KIC 8263801, KIC 8329820 and KIC 8264074 obtained after correcting for the drifts and jumps in the the raw (uncorrected) light curves of the stars. Comb-like structures which are typical characteristics of stars with solar-like oscillations are clearly seen in all the three periodograms. The location of the frequency of the maximum amplitude gives what is known as the $\nu_{\max}$.

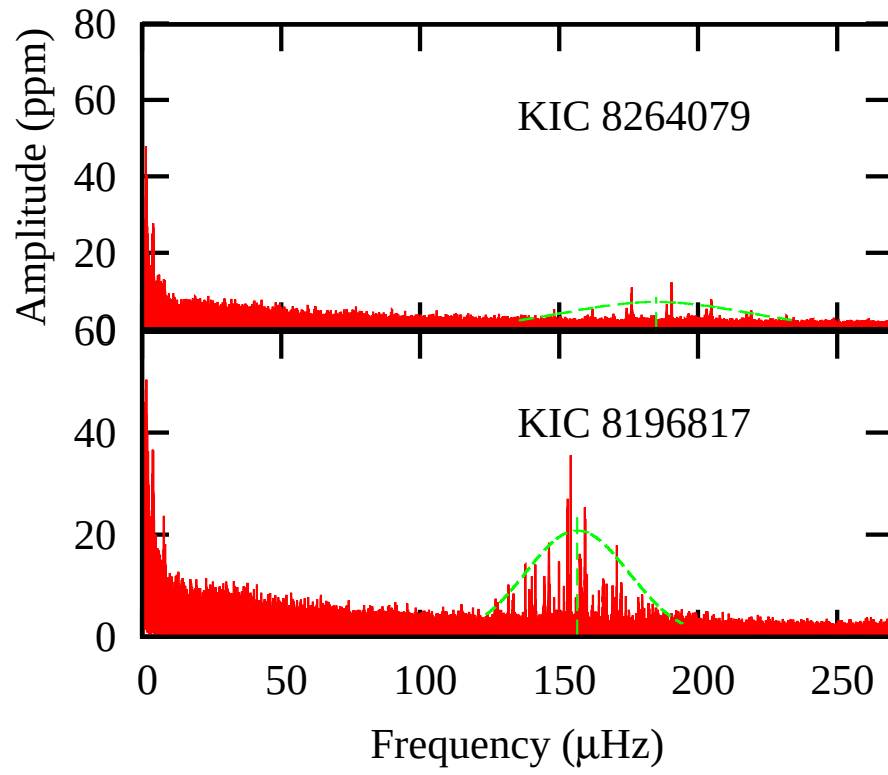


Figure 4.3: The periodograms of KIC 8196817 and KIC 8264079 obtained after correcting for the drifts and jumps in the the raw (uncorrected) light curves of the stars. Comb-like structures which are typical characteristics of stars with solar-like oscillations are clearly seen in the two periodograms. The location of the frequency of the maximum amplitude gives what is known as the ν_{max} .

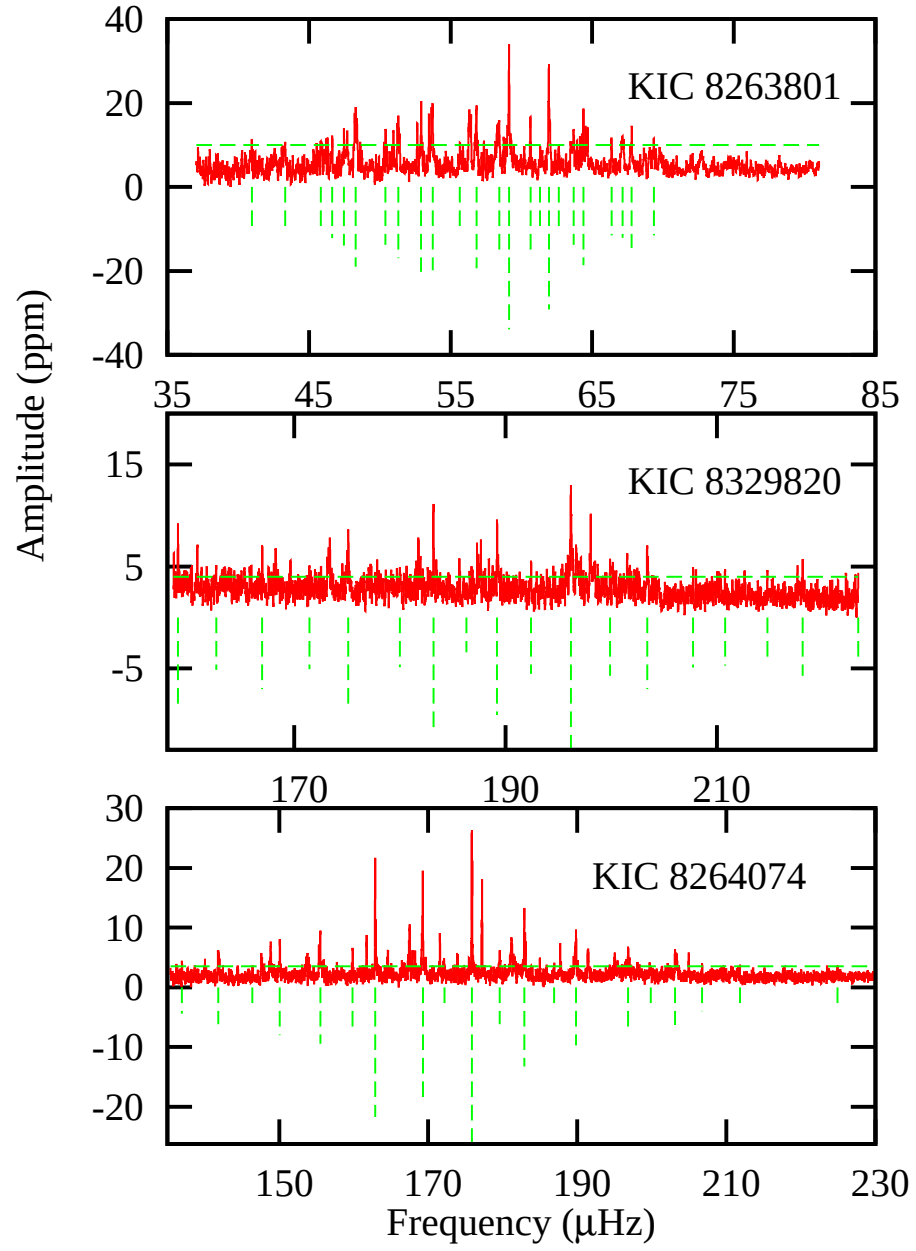


Figure 4.4: *Smoothed periodograms showing observed frequencies extracted from the periodograms in Fig. 4.2. We used running mean approach to smoothen the periodogram. The horizontal lines are the noise level limit.*

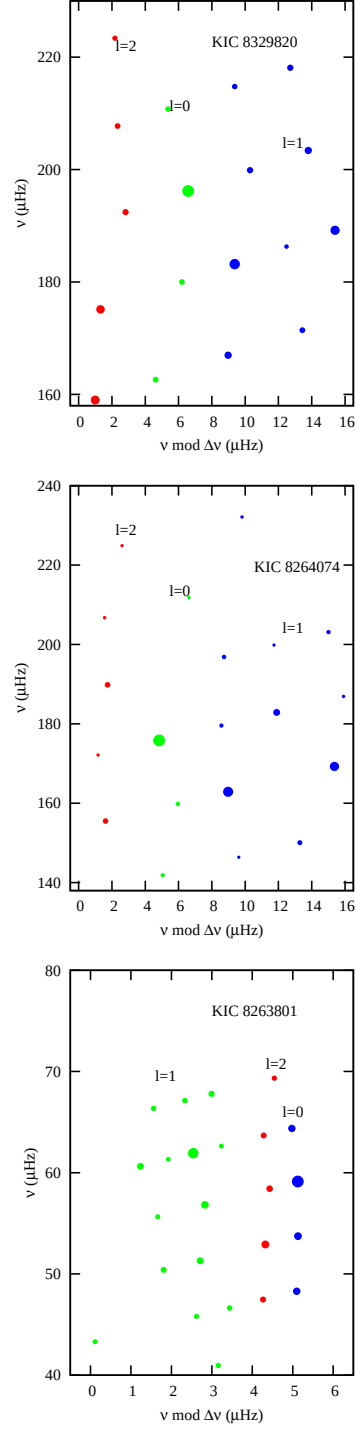


Figure 4.5: Echelle diagram for the three stars constructed using the extracted observed frequencies and the large frequency separations. Vertical points running parallel and closer together are the $l = 0$ and $l = 2$ modes while points scattered and far away from the two parallel lines are the $l = 1$ mixed modes. In calculating the median gravity-mode period spacings, we used only points for $l = 1$ for which gravity modes dominate. Symbol sizes are proportional to the amplitude.

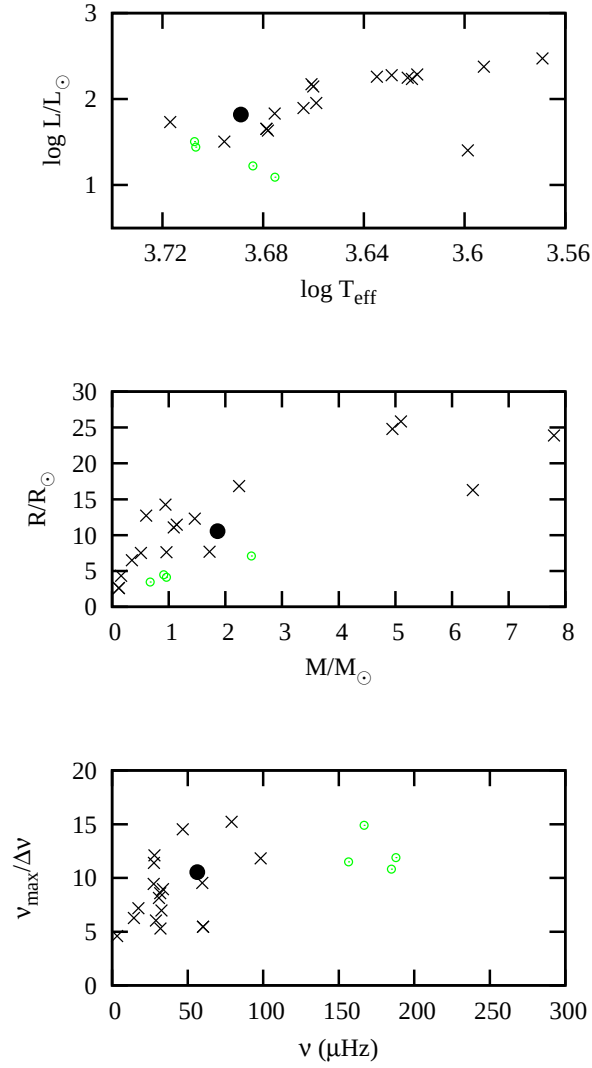


Figure 4.6: *Top panel: HR diagram for the RG stars identified by Balona et al. (2013a), their Table 7. The open circles are the RGB stars and filled circle is the RC star identified in this thesis while the crosses are the stars which we could not construct echelle diagram for and therefore could not assign them as RGB or RGC using median gravity mode period separations, ΔP . Middle panel: Calculated radius versus mass. Bottom panel: $\nu_{\text{max}}/\Delta\nu$ versus ν_{max} (symbols as in the top panel).*

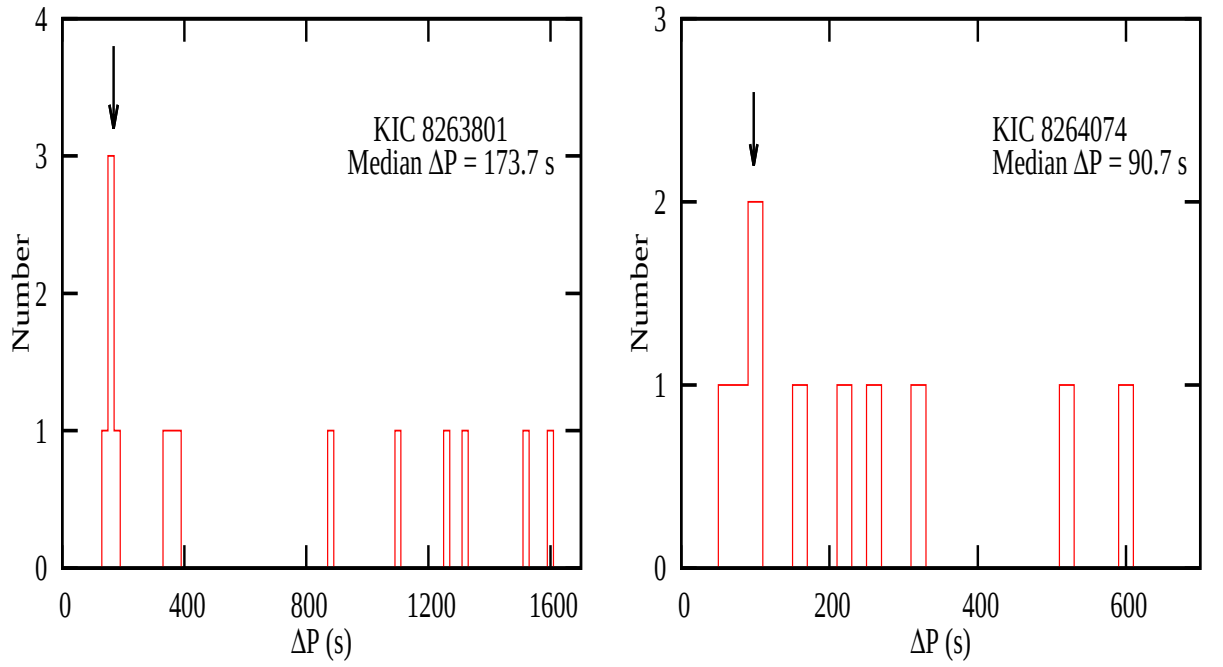


Figure 4.7: The distribution of gravity-mode period spacings, ΔP . Left panel: Histogram for KIC 8263801 which we have classified as SRC star. Right panel: Histogram for KIC 8264074 which we have classified as RGB star. The vertical arrows indicate the ΔP chosen.

Echelle diagrams for some of the stars are constructed, three of which have been identified as cluster members by Balona et al. (2013a) in their Table 7. The results are summarized in Table 4.1. Figure 4.6 is a plot of different calculated parameters of the stars in the field of NGC 6866 (more elaborate explanations are in the caption of the plot). The bottom panel of Figure 4.6 shows a clear distinction of RGB stars from the rest of the stars in the field of NGC 6866. Figure 4.7 shows the distribution of gravity-mode period spacings, ΔP with the left panel showing the histogram for KIC 8263801 which we have classified as SRC star while the right panel is the histogram for KIC 8264074 which has been classified as RGB star (the vertical arrows indicate the ΔP chosen).

4.3 Results and Discussions

Based on the value of the median gravity period separation, ΔP , KIC 8264074, KIC 8196817, KIC 8329820, and KIC 8264079 are found to be RGB stars while KIC 8263801 is a SRC star (Table 4.1). This is the first time that stars located in the field of NGC 6866 are classified in this manner. However, much as the echelle diagrams for the stars have been constructed, the only one in which the method seems to have worked perfectly well is for KIC 8263801 since the modes are clearly seen with well defined patterns.

It is also found that a star KIC 8263801 located in the field of NGC 6866 is a RGC star based on the value of median gravity period separation, $\Delta P = 173.7$ s (Table 4.1). Unfortunately, based on the photometric distance membership determination, KIC 8263801 was already classified as non-member of NGC 6866.

Table 4.1: A list of stars in NGC 6866 for which we were able to construct echelle diagrams and there after calculated the median gravity-mode period spacings. The columns are: the Kepler Input Catalogue number - KIC, Membership, the logarithm of the corrected effective temperature, the corrected effective temperature, the KIC luminosity, the calculated luminosity $(\log L/L_{\odot})_{\nu}$, the frequency of maximum amplitude, $\nu_{\max}$, the large frequency separation, $\Delta\nu$, the mass, radius and the period spacing calculated from the solar-like oscillations, $M/M_{\odot}$, $R/R_{\odot}$. *M* means the star which is a member while *N* is for non member.

KIC	Mem	$\log T_{\text{eff}}$	T_{eff}	$\log L/L_{\odot}$	$(\log L/L_{\odot})_{\nu}$	$\nu_{\max}(\mu\text{Hz})$	$\Delta\nu(\mu\text{Hz})$	$M/M_{\odot}$	$R/R_{\odot}$	$\Delta P(s)$
8264074	M ^a	3.6841	4832	1.2238	1.3905	166.842	11.205	2.46	7.09	90.7
8196817	M ^a	3.6753	4735	1.0912	0.9531	156.557	13.613	0.91	4.46	56.7
8329820	N ^a	3.7073	5097	1.5046	1.0116	187.872	15.809	0.96	4.12	84.4
8264079	M ^a	3.7068	5091	1.4401	0.8581	184.891	17.108	0.67	3.46	82.0
8263801	N ^a	3.6889	4885	1.8190	1.7555	56.422	5.354	1.86	10.56	173.7

^aAccording to Balona et al. (2013a)

Chapter 5

Main conclusions

5.1 The amplitude ratios and phase shifts

I performed calculations of amplitude ratios and phase shifts in the atmosphere of the Sun by modifying Phorah (2007) code (new code) so as to obtain amplitude ratios and phase shifts at a height of 200 km above the photosphere. This was done by solving the non - adiabatic radial pulsation equations with non - local mixing length theory of convection and consistent treatment of radiation. I also used Houdek et al. (1995) and Houdek (1996) code (old code) to calculate amplitude ratios and phase shifts in the atmosphere of the Sun by solving the non - adiabatic radial pulsation equations with non - local mixing length theory of convection and Eddington's approximation to radiation. The results of amplitude ratios and phase shifts using modified Phorah (2007) and Houdek et al. (1995) codes were fitted to the observational data. The summary of the main results are as follows;

- By including full treatment of radiation in modeling the atmosphere of the Sun, we are able to see features which are not noticeable when we treat radiation by Eddington's approximation which is an approximation to radiation.
- The new code produced results for the amplitude ratios and phase shifts that differ by about 32 % for frequency between 0 and 6 mHz when compared to the old code.
- The new code differs by about 18 % for frequency between 2.5 and 4.5 mHz (5 minutes range) when compared to the old code.
- The estimate of amplitude ratios and phase shifts using both pulsation modeling codes significantly depends on the equilibrium models that are being used.

- A mean value of amplitude ratios of $0.207 \text{ ppm s cm}^{-1}$ was obtained with the new code between 2.5 and 4.0 mHz compared to a mean value of amplitude ratios of $0.235 \text{ ppm s cm}^{-1}$ obtained from the observational data of Schrijver et al. (1991) in the same frequency range were obtained.
- Mean value of amplitude ratios of $0.180 \text{ ppm s cm}^{-1}$ was obtained with the old code for frequency between 2.5 and 4.0 mHz compared to a mean value of amplitude ratios of $0.235 \text{ ppm s cm}^{-1}$ obtained from the observational data of Schrijver et al. (1991) in the same frequency range.
- I obtained a mean value of phase shifts of -114° with the new code for frequency between 2.4 and 4.3 mHz compared to a mean value of phase shifts of -119° obtained from the observational data of Schrijver et al. (1991) in the same frequency range.
- A mean value of phase shifts of -99° was obtained with the old code for frequency between 2.4 and 4.3 mHz compared to a mean value of phase shifts of -119° between 2.4 and 4.3 mHz obtained from the observational data of Schrijver et al. (1991) in the same frequency range.

5.2 *Kepler* open cluster NGC 6819 study

From the median gravity-mode period spacings calculated from the echelle diagrams, we searched for RC stars amongst the RG stars in the *Kepler* open cluster NGC 6819. We compared the observations with theoretical isochrones and determined the mass loss on the tip of the RGB of $7 \pm 3\%$ with no correction applied to $\Delta\nu$ for RGC stars as suggested by Miglio et al. (2012). From the study, we conclude the following;

- We found distance moduli $\mu_0 = 11.520 \pm 0.105 \text{ mag}$ (A_V obtained from KIC) and $\mu_0 = 11.747 \pm 0.086 \text{ mag}$ with fixed $E(B - V) = 0.15$ for the cluster when we use only single member RGC stars.
- The mass loss seems not to be affected by small changes in metallicity.

- Using isochrones of slightly different ages in the range of NGC 6819 age in the literature do not affect the mass loss results significantly.
- The value of $\eta = 0.318$ for a ratio between observed mass and mass from the isochrone of 1.00 (for no mass loss to occur at the tip of RGB for the case with no correction to the $\Delta\nu$ values of RGC stars).
- There is no mass loss when the $\Delta\nu$ correction of 1.9% is applied as shown by Miglio et al. (2012).

5.3 *Kepler* open cluster NGC 6866 study

- We found that KIC 8263801 located in the field of NGC 6866 is a RGC star based on the value of median gravity period separation.

References

- Aerts C., Christensen-Dalsgaard J., Cunha M., Kurtz D. W., 2008, *SoPh*, 251, 3
- Aerts C., Christensen-Dalsgaard J., Kurtz D. W., 2010, *Asteroseismology*. Springer Science+Business Media B.V
- Allen C. W., 1973, *Astrophysical quantities*. London: University of London, Athlone Press, —c1973, 3rd ed
- An D., Pinsonneault M. H., Masseron T., Delahaye F., Johnson J. A., Terndrup D. M., Beers T. C., Ivans I. I., Ivezić Ž., 2009, *ApJ*, 700, 523
- Auer L. H., Mihalas D., 1970, *MNRAS*, 149, 65
- Bahcall J. N., 1964, *Physics Letters*, 13, 332
- Bahcall J. N., Peña-Garay C., 2004, *New Journal of Physics*, 6, 63
- Baker N., Kippenhahn R., 1965, *ApJ*, 142, 868
- Baker N. H., Gough D. O., 1979, *ApJ*, 234, 232
- Balmforth N. J., 1992, *MNRAS*, 255, 603
- Balona L. A., 2011, *MNRAS*, 415, 1691
- Balona L. A., Joshi S., Joshi Y. C., Sagar R., 2013a, *MNRAS*, 429, 1466
- Balona L. A., Medupe T., Abedigamba O. P., Ayane G., Keeley L., Matsididi M., Mekonnen G., Nhlapo M. D., Sithole N., 2013b, *MNRAS*, 430, 3472
- Basu D., 1998, *SolPhys*, 183, 291

- Basu S., Grundahl F., Stello D., Kallinger T., Hekker S., Mosser B., García R. A., Mathur S., Brogaard K., Bruntt H., Chaplin W. J., Gai N., Elsworth Y., Esch L., Ballot J., Bedding T. R., Gruberbauer M., Huber D., Miglio A., Yildiz M., Kjeldsen H., Christensen-Dalsgaard J., Gilliland R. L., Fanelli M. M., Ibrahim K. A., Smith J. C., 2011, *ApJ*, 729, L10
- Beck P. G., Bedding T. R., Mosser B., Stello D., Garcia R. A., Kallinger T., Hekker S., Elsworth Y., Frandsen S., Carrier F., De Ridder J., Aerts C., White T. R., Huber D., Dupret M.-A., Montalbán J., Miglio A., Noels A., Chaplin W. J., Kjeldsen H., Christensen-Dalsgaard J., Gilliland R. L., Brown T. M., Kawaler S. D., Mathur S., Jenkins J. M., 2011, *Science*, 332, 205
- Bedding T. R., 2014, *Solar-like oscillations: An observational perspective*, Pallé P. L., Esteban C., eds., p. 60
- Bedding T. R., Mosser B., Huber D., Montalbán J., Beck P., Christensen-Dalsgaard J., Elsworth Y. P., García R. A., Miglio A., Stello D., White T. R., De Ridder J., Hekker S., Aerts C., Barban C., Belkacem K., Broomhall A.-M., Brown T. M., Buzasi D. L., Carrier F., Chaplin W. J., di Mauro M. P., Dupret M.-A., Frandsen S., Gilliland R. L., Goupil M.-J., Jenkins J. M., Kallinger T., Kawaler S., Kjeldsen H., Mathur S., Noels A., Aguirre S. V., Ventura P., 2011, *Nature*, 471, 608
- Beletsky Y., Carraro G., Ivanov V. D., 2009, *A&A*, 508, 1279
- Bessell M. S., 2005, *ARA&A*, 43, 293
- Borucki W., Koch D., Batalha N., Caldwell D., Christensen-Dalsgaard J., Cochran W. D., Dunham E., Gautier T. N., Geary J., Gilliland R., Jenkins J., Kjeldsen H., Lissauer J. J., Rowe J., 2009, in *IAU Symposium*, Vol. 253, *IAU Symposium*, Pont F., Sasselov D., Holman M. J., eds., pp. 289–299
- Bowen G. H., 1988, *ApJ*, 329, 299
- Bragaglia A., Carretta E., Gratton R. G., Tosi M., Bonanno G., Bruno P., Calì A., Claudi R., Cosentino R., Desidera S., Farisato G., Rebescini M., Scuderi S., 2001, *AJ*, 121, 327

Brickhill A. J., 1991, MNRAS, 251, 673

Brown T. M., Latham D. W., Everett M. E., Esquerdo G. A., 2011, AJ, 142, 112

Cannon R. D., 1970, MNRAS, 150, 111

Carroll B. W., Ostlie D. A., 2006, An introduction to modern astrophysics and cosmology.
Pearson Education, Inc., Addison - Wesley

Chaplin W. J., Appourchaux T., Arentoft T., Ballot J., Christensen-Dalsgaard J., Creevey
O. L., Elsworth Y., Fletcher S. T., García R. A., Houdek G., Jiménez-Reyes S. J., Kjeldsen
H., New R., Régulo C., Salabert D., Sekii T., Sousa S. G., Toutain T., rest of asteroFLAG
Group, 2008, Astronomische Nachrichten, 329, 549

Chaplin W. J., Miglio A., 2013, ARA&A, 51, 353

Chevalier C., 1971, A&A, 14, 24

Christensen-Dalsgaard J., 2002a, Reviews of Modern Physics, 74, 1073

—, 2002b, in ESA Special Publication, Vol. 485, Stellar Structure and Habitable Planet Find-
ing, Battrick B., Favata F., Roxburgh I. W., Galadi D., eds., pp. 25–34

Christensen-Dalsgaard J., Frandsen S., 1983a, , 82, 165

—, 1983b, SoPh, 82, 165

—, 1983c, SoPh, 82, 469

Christensen-Dalsgaard J., Gough D. O., Thompson M. J., 1991, ApJ, 378, 413

Corsaro E., Stello D., Huber D., Bedding T. R., Bonanno A., Brogaard K., Kallinger T.,
Benomar O., White T. R., Mosser B., Basu S., Chaplin W. J., Christensen-Dalsgaard J.,
Elsworth Y. P., García R. A., Hekker S., Kjeldsen H., Mathur S., Meibom S., Hall J. R.,
Ibrahim K. A., Klaus T. C., 2012, ApJ, 757, 190

Corsaro E. M. N., 2012, PhD thesis, “Ph.D. Thesis, UNIVERSITÀ DEGLI STUDI DI CATA-
NIA, (2012)”

Cox J. P., 1957, AJ, 62, 90

—, 1958, ApJ, 127, 194

Cugier H., Dziembowski W. A., Pamyatnykh A. A., 1994, A&A, 291, 143

Cunha M. S., Aerts C., Christensen-Dalsgaard J., Baglin A., Bigot L., Brown T. M., Catala C., Creevey O. L., Domiciano de Souza A., Eggenberger P., Garcia P. J. V., Grundahl F., Kervella P., Kurtz D. W., Mathias P., Miglio A., Monteiro M. J. P. F. G., Perrin G., Pijpers F. P., Pourbaix D., Quirrenbach A., Rousselet-Perraut K., Teixeira T. C., Thévenin F., Thompson M. J., 2007, , 14, 217

Deheuvels S., Michel E., 2010, Astrophysics and Space Science, 328, 259

Domogatsky G. V., Zatsepin G. T., Eramjan R. A., 1965, International Cosmic Ray Conference, 2, 1028

Eddington A. S., 1926, The Internal Constitution of the Stars. Cambridge: Cambridge University Press, 1926. ISBN 9780521337083.

Edmonds P. D., Gilliland R. L., 1996, ApJ, 464, L157

Frandsen S., Carrier F., Aerts C., Stello D., Maas T., Burnet M., Bruntt H., Teixeira T. C., de Medeiros J. R., Bouchy F., Kjeldsen H., Pijpers F., Christensen-Dalsgaard J., 2002, A&A, 394, L5

Goldberg L., 1979, QJRAS, 20, 361

Goldreich P., Keeley D. A., 1977, ApJ, 212, 243

Gough D. O., 1977, in IAU Colloq. 36: The Energy Balance and Hydrodynamics of the Solar Chromosphere and Corona, Bonnet B., Delache P., eds., pp. 3–36

—, 1986, in NATO Advanced Science Institutes (ASI) Series C, Vol. 169, NATO Advanced Science Institutes (ASI) Series C, Gough D. O., ed., pp. 125–140

—, 1993, in Astrophysical Fluid Dynamics - Les Houches 1987, Zahn J.-P., Zinn-Justin J., eds., pp. 399–560

Grec G., Fossat E., Pomerantz M. A., 1983, *SoPh*, 82, 55

Gunes O., Karatas Y., Bonatto C., 2012, *NewA*, 17, 720

Guzik J. A., Kaye A. B., Bradley P. A., Cox A. N., Neuforge C., 2000, *ApJ*, 542, L57

Hart A. B., 1954, *MNRAS*, 114, 17

Hatzes A. P., Cochran W. D., 1998, in *Astronomical Society of the Pacific Conference Series*, Vol. 154, *Cool Stars, Stellar Systems, and the Sun*, Donahue R. A., Bookbinder J. A., eds., p. 311

Hekker S., Basu S., Stello D., Kallinger T., Grundahl F., Mathur S., García R. A., Mosser B., Huber D., Bedding T. R., Szabó R., De Ridder J., Chaplin W. J., Elsworth Y., Hale S. J., Christensen-Dalsgaard J., Gilliland R. L., Still M., McCauliff S., Quintana E. V., 2011a, *A&A*, 530, A100

Hekker S., Elsworth Y., De Ridder J., Mosser B., García R. A., Kallinger T., Mathur S., Huber D., Buzasi D. L., Preston H. L., Hale S. J., Ballot J., Chaplin W. J., Régulo C., Bedding T. R., Stello D., Borucki W. J., Koch D. G., Jenkins J., Allen C., Gilliland R. L., Kjeldsen H., Christensen-Dalsgaard J., 2011b, *A&A*, 525, A131

Hekker S., Elsworth Y., Mosser B., Kallinger T., Chaplin W. J., De Ridder J., García R. A., Stello D., Clarke B. D., Hall J. R., Ibrahim K. A., 2012, *A&A*, 544, A90

Höfner S., 2009, in *Astronomical Society of the Pacific Conference Series*, Vol. 414, *Cosmic Dust - Near and Far*, Henning T., Grün E., Steinacker J., eds., p. 3

Hole K. T., Geller A. M., Mathieu R. D., Platais I., Meibom S., Latham D. W., 2009, *AJ*, 138, 159

Homer Lane J., 1869, *AJS*, 50, 57

Houdek G., 1996, PhD thesis, “Ph.D. Thesis, Formal- und Naturwissenschaftliche Fakultät” at der Universität Wien, (1996)”

- , 2006, in ESA Special Publication, Vol. 624, Proceedings of SOHO 18/GONG 2006/HELAS I, Beyond the spherical Sun
- , 2010, *Ap&SS*, 328, 237
- Houdek G., Balmforth N. J., Christensen-Dalsgaard J., 1995, in ESA Special Publication, Vol. 376, *Helioseismology*, p. 447
- Houdek G., Balmforth N. J., Christensen-Dalsgaard J., Gough D. O., 1999, *A&A*, 351, 582
- Huber D., Stello D., Bedding T. R., Chaplin W. J., Arentoft T., Quirion P.-O., Kjeldsen H., 2009, *Communications in Asteroseismology*, 160, 74
- Iglesias C. A., Rogers F. J., Wilson B. G., 1992, *ApJ*, 397, 717
- Innis J. L., Isaak G. R., Brazier R. I., Belmonte J. A., Palte P. L., Roca Cortés T., Jones A. R., 1988, in ESA Special Publication, Vol. 286, *Seismology of the Sun and Sun-Like Stars*, Rolfe E. J., ed., pp. 569–573
- Jeffries Jr. M. W., Sandquist E. L., Mathieu R. D., Geller A. M., Orosz J. A., Milliman K. E., Brewer L. N., Platais I., Brogaard K., Grundahl F., Frandsen S., Dotter A., Stello D., 2013, *AJ*, 146, 58
- Jiménez A., Roca Cortés T., Severino G., Marmolino C., 1999, *ApJ*, 525, 1042
- Joshi Y. C., Joshi S., Kumar B., Mondal S., Balona L. A., 2012, *MNRAS*, 419, 2379
- Judge P. G., Stencel R. E., 1991, *ApJ*, 371, 357
- Kalirai J. S., Richer H. B., Fahlman G. G., Cuillandre J.-C., Ventura P., D’Antona F., Bertin E., Marconi G., Durrell P. R., 2001, *AJ*, 122, 266
- Kallinger T., Mosser B., Hekker S., Huber D., Stello D., Mathur S., Basu S., Bedding T. R., Chaplin W. J., De Ridder J., Elsworth Y. P., Frandsen S., García R. A., Gruberbauer M., Matthews J. M., Borucki W. J., Bruntt H., Christensen-Dalsgaard J., Gilliland R. L., Kjeldsen H., Koch D. G., 2010, *A&A*, 522, A1

- Kharchenko N. V., Piskunov A. E., Röser S., Schilbach E., Scholz R.-D., 2005, *A&A*, 438, 1163
- Kivelson M. G., Russell C. T., 1995, *Science*, 269, 862
- Kjeldsen H., Bedding T. R., 1995, *A&A*, 293, 87
- Kosovichev A. G., Schou J., Scherrer P. H., Bogart R. S., Bush R. I., Hoeksema J. T., Aloise J., Bacon L., Burnette A., De Forest C., Giles P. M., Leibbrand K., Nigam R., Rubin M., Scott K., Williams S. D., Basu S., Christensen-Dalsgaard J., Däppen W., Rhodes Jr. E. J., Duvall Jr. T. L., Howe R., Thompson M. J., Gough D. O., Sekii T., Toomre J., Tarbell T. D., Title A. M., Mathur D., Morrison M., Saba J. L. R., Wolfson C. J., Zayer I., Milford P. N., 1997, in *IAU Symposium*, Vol. 181, *Sounding Solar and Stellar Interiors*, Provost J., Schmider F.-X., eds., p. 203
- Kurucz R. L., 1991, in *NATO ASIC Proc. 341: Stellar Atmospheres - Beyond Classical Models*, Crivellari L., Hubeny I., Hummer D. G., eds., p. 441
- Lal P., Bhatnagar P. L., 1956, *ZAP*, 41, 21
- Leighton R. B., Noyes R. W., Simon G. W., 1962, *ApJ*, 135, 474
- Libbrecht K. G., 1988, *ApJ*, 334, 510
- Lindoff U., 1972, *A&AS*, 7, 497
- Loktin A. V., Matkin N. V., Gerasimenko T. P., 1994, *Astronomical and Astrophysical Transactions*, 4, 153
- Marigo P., Aringer B., 2009, *A&A*, 508, 1539
- Marigo P., Girardi L., Bressan A., Groenewegen M. A. T., Silva L., Granato G. L., 2008, *A&A*, 482, 883
- Mathur S., García R. A., Régulo C., Creevey O. L., Ballot J., Salabert D., Arentoft T., Quirion P.-O., Chaplin W. J., Kjeldsen H., 2010, *A&*, 511, A46

- Medupe R., Christensen-Dalsgaard J., Phorah M., 2009, in American Institute of Physics Conference Series, Vol. 1170, American Institute of Physics Conference Series, Guzik J. A., Bradley P. A., eds., pp. 506–511
- Medupe T. R., 2002, PhD thesis, “Ph.D. Thesis, University of Cape Town, (2002)”
- Merline W. J., 1999, in Astronomical Society of the Pacific Conference Series, Vol. 185, IAU Colloq. 170: Precise Stellar Radial Velocities, Hearnshaw J. B., Scarfe C. D., eds., p. 187
- Mermilliod J.-C., Paunzen E., 2003, *A&A*, 410, 511
- Mészáros S., Holtzman J., García Pérez A. E., Allende Prieto C., Schiavon R. P., Basu S., Bizyaev D., Chaplin W. J., Chojnowski S. D., Cunha K., Elsworth Y., Epstein C., Frinchaboy P. M., García R. A., Hearty F. R., Hekker S., Johnson J. A., Kallinger T., Koesterke L., Majewski S. R., Martell S. L., Nidever D., Pinsonneault M. H., O’Connell J., Shetrone M., Smith V. V., Wilson J. C., Zasowski G., 2013, *AJ*, 146, 133
- Miglio A., Brogaard K., Stello D., Chaplin W. J., D’Antona F., Montalbán J., Basu S., Bressan A., Grundahl F., Pinsonneault M., Serenelli A. M., Elsworth Y., Hekker S., Kallinger T., Mosser B., Ventura P., Bonanno A., Noels A., Silva Aguirre V., Szabo R., Li J., McCaulliff S., Middour C. K., Kjeldsen H., 2012, *MNRAS*, 419, 2077
- Miglio A., Montalbán J., Baudin F., Eggenberger P., Noels A., Hekker S., De Ridder J., Weiss W., Baglin A., 2009, *A&A*, 503, L21
- Mihalas D., 1978, *Stellar atmospheres* /2nd edition/
- Molenda-Żakowicz J., Kopacki G., Steślicki M., Narwid A., 2009, *Acta Astro*, 59, 193
- Mosser B., Appourchaux T., 2009, *A&A*, 508, 877
- Mullan D. J., 1978, *ApJ*, 226, 151
- Origlia L., Ferraro F. R., Fusi Pecci F., Rood R. T., 2002, *ApJ*, 571, 458
- Pasquini L., de Medeiros J. R., Girardi L., 2000, *A&A*, 361, 1011
- Phorah M. W., 2007, PhD thesis, “Ph.D. Thesis, University of Cape Town, (2007)”

- Pinsonneault M. H., An D., Molenda-Żakowicz J., Chaplin W. J., Metcalfe T. S., Bruntt H., 2012, ApJS, 199, 30
- Plaskett H. H., 1916, ApJ, 43, 145
- Poon A. W. P., 2002, ArXiv High Energy Physics - Experiment e-prints
- Ramírez I., Meléndez J., 2005, ApJ, 626, 465
- Reimers D., 1975, MSRSL, 8, 369
- Reines F., 1964, in Interaction Between Cosmic Rays High Energy Physics, p. 1
- Rosvick J. M., Vandenberg D. A., 1998, AJ, 115, 1516
- Roxburgh I. W., 1996, Bulletin of the Astronomical Society of India, 24, 89
- Samadi R., Belkacem K., Goupil M.-J., Ludwig H.-G., Dupret M.-A., 2008, Communications in Asteroseismology, 157, 130
- Samadi R., Goupil M., 2001, A&A, 370, 136
- Sandage A., 2004, ArXiv Astrophysics e-prints
- Sandquist E. L., Mathieu R. D., Brogaard K., Meibom S., Geller A. M., Orosz J. A., Milliman K. E., Jeffries Jr. M. W., Brewer L. N., Platais I., Grundahl F., Bruntt H., Frandsen S., Stello D., 2013, ApJ, 762, 58
- Schou J., Antia H. M., Basu S., Bogart R. S., Bush R. I., Chitre S. M., Christensen-Dalsgaard J., Di Mauro M. P., Dziembowski W. A., Eff-Darwich A., Gough D. O., Haber D. A., Hoeksema J. T., Howe R., Korzennik S. G., Kosovichev A. G., Larsen R. M., Pijpers F. P., Scherrer P. H., Sekii T., Tarbell T. D., Title A. M., Thompson M. J., Toomre J., 1998, ApJ, 505, 390
- Schrijver C. J., Dappen W., Jimenez A., 1991, A&A, 251, 655
- Scuflaire R., 1974, A&A, 36, 107

- Skrutskie M. F., Cutri R. M., Stiening R., Weinberg M. D., Schneider S., Carpenter J. M., Beichman C., Capps R., Chester T., Elias J., Huchra J., Liebert J., Lonsdale C., Monet D. G., Price S., Seitzer P., Jarrett T., Kirkpatrick J. D., Gizis J. E., Howard E., Evans T., Fowler J., Fullmer L., Hurt R., Light R., Kopan E. L., Marsh K. A., McCallon H. L., Tam R., Van Dyk S., Wheelock S., 2006, *AJ*, 131, 1163
- Smith P. H., McMillan R. S., Merline W. J., 1987, *ApJ*, 317, L79
- Spiegel E. A., 1963, *ApJ*, 138, 216
- Stello D., Chaplin W. J., Basu S., Elsworth Y., Bedding T. R., 2009, *MNRAS*, 400, L80
- Stello D., Huber D., Bedding T. R., Benomar O., Bildsten L., Elsworth Y. P., Gilliland R. L., Mosser B., Paxton B., White T. R., 2013, *ApJ*, 765, L41
- Stello D., Huber D., Kallinger T., Basu S., Mosser B., Hekker S., Mathur S., García R. A., Bedding T. R., Kjeldsen H., Gilliland R. L., Verner G. A., Chaplin W. J., Benomar O., Meibom S., Grundahl F., Elsworth Y. P., Molenda-Żakowicz J., Szabó R., Christensen-Dalsgaard J., Tenenbaum P., Twicken J. D., Uddin K., 2011a, *ApJL*, 737, L10
- Stello D., Meibom S., Gilliland R. L., Grundahl F., Hekker S., Mosser B., Kallinger T., Mathur S., García R. A., Huber D., Basu S., Bedding T. R., Brogaard K., Chaplin W. J., Elsworth Y. P., Molenda-Żakowicz J., Szabó R., Still M., Jenkins J. M., Christensen-Dalsgaard J., Kjeldsen H., Serenelli A. M., Wohler B., 2011b, *ApJ*, 739, 13
- Tassoul M., 1980, *ApJS*, 43, 469
- , 1990, *ApJ*, 358, 313
- Thompson M. J., Toomre J., Anderson E. R., Antia H. M., Berthomieu G., Burtonclay D., Chitre S. M., Christensen-Dalsgaard J., Corbard T., De Rosa M., Genovese C. R., Gough D. O., Haber D. A., Harvey J. W., Hill F., Howe R., Korzennik S. G., Kosovichev A. G., Leibacher J. W., Pijpers F. P., Provost J., Rhodes Jr. E. J., Schou J., Sekii T., Stark P. B., Wilson P. R., 1996, *Science*, 272, 1300
- Ulrich R. K., 1986, *ApJL*, 306, L37

Vernazza J. E., Avrett E. H., Loeser R., 1981, *ApJS*, 45, 635

Vitense E., 1953, *ZfAp*, 32, 135

Woltjer Jr. J., 1936, *BAIN*, 8, 17

Wood P. R., 2007, in *Astronomical Society of the Pacific Conference Series*, Vol. 374, *From Stars to Galaxies: Building the Pieces to Build Up the Universe*, Vallenari A., Tantaló R., Portinari L., Moretti A., eds., p. 47

Wu T., Li Y., Hekker S., 2014, *ApJ*, 781, 44

Zombeck M., 2007, *Handbook of Space Astronomy and Astrophysics: Third Edition*. Cambridge University Press

Appendices

Appendix A1: The Global Parameters of the Sun

In order to determine the physical processes which occur in the Sun, we need to know certain properties of the Sun. In this section, summary of the relevant information are presented. Taken from Zombeck (2007), and Kivelson & Russell (1995) as shown in Table A.1.

Table A.1: *Global list of solar parameters. Taken from Zombeck (2007), and Kivelson & Russell (1995).*

Solar Parameter	Value
Mass, $M_{\odot}$	1.9891×10^{30} kg
Radius, $R_{\odot}$	6.96×10^8 m
Effective temperature, T_{eff}	5777 K
Solar luminosity, $L_{\odot}$	2.839×10^{26} W
Surface Gravity, g_s	$2.74 \times 10^2 \text{ m s}^{-2}$
Mean Density, $\bar{\rho}$	$1.41 \times 10^3 \text{ kg m}^{-3}$
Escape speed from the solar surface	617.7 km s^{-1}
Semi-diameter at mean distance	$15' 59'' .63$
Motion relative to nearby stars	$1.94 \times 10^4 \text{ m s}^{-1}$
Period of sidereal rotation	25.38 days
Period of synodic rotation (ϕ = latitude)	$26.90 + 5.2 \sin^2 \phi$ days
Age	4.5×10^9 years
Volume	$1.422 \times 10^{27} \text{ m}^3$
Chemical constituents	H (92.1%, He (7.8%))
Mean distance from Earth	1AU = 1.5×10^8 km
Pressure (Center)	2.334×10^{16} Pa
Magnetic field (sunspots)	0.1 - 0.4 T

Appendix A2: Propagation of errors in Mass, Radius and Luminosity

In order to fit the model to the data correctly, we needed to take into account the standard deviation (uncertainties) in $M/M_\odot$ and $L/L_\odot$. Equation (1.3), (1.4) and (1.5) can also be written as

$$\frac{M}{M_\odot} \simeq \left(\frac{\nu_{\max}}{\nu_{\max\odot}} \right)^3 \left(\frac{\Delta\nu}{\Delta\nu_\odot} \right)^{-4} \left(\frac{T_{\text{eff}}}{T_{\text{eff}\odot}} \right)^{1.5}, \quad (1)$$

$$\frac{R}{R_\odot} \simeq \left(\frac{\nu_{\max}}{\nu_{\max\odot}} \right) \left(\frac{\Delta\nu}{\Delta\nu_\odot} \right)^{-2} \left(\frac{T_{\text{eff}}}{T_{\text{eff}\odot}} \right)^{0.5}, \quad (2)$$

$$\frac{L}{L_\odot} \simeq \left(\frac{\nu_{\max}}{\nu_{\max\odot}} \right)^2 \left(\frac{\Delta\nu}{\Delta\nu_\odot} \right)^{-4} \left(\frac{T_{\text{eff}}}{T_{\text{eff}\odot}} \right)^5, \quad (3)$$

The standard deviation in $M/M_\odot$ and $L/L_\odot$ can only be determined once the standard deviations in T_{eff} , $\nu_{\max}$ and $\Delta\nu$ are known. Brown et al. (2011) showed that the standard deviations in T_{eff} is ~ 200 K while Chaplin et al. (2008) and later Huber et al. (2009) showed that the uncertainties in $\nu_{\max}$ and $\Delta\nu$ can be estimated as $\sigma\nu_{\max} = 0.01 \times 2\nu_{\max}$ and $\sigma\Delta\nu = 0.01 \times (3/2)\Delta\nu$ (on the assumption that such errors allow the radius to be calculated with the precision of 1%) respectively. Once we know the standard deviations in T_{eff} , $\nu_{\max}$ and $\Delta\nu$, we apply error propagation law to equation (1), (2). The error propagation law says that given any arbitrary function $w = f(x, y, z)$, the variance of w (σ_w) is found as follows;

$$\sigma_w^2 = \left(\frac{\partial f}{\partial x} \right)^2 \sigma_x^2 + \left(\frac{\partial f}{\partial y} \right)^2 \sigma_y^2 + \left(\frac{\partial f}{\partial z} \right)^2 \sigma_z^2. \quad (4)$$

Applying the form of equation (4) to equation (1) - (3), we get equation (5), (6) and (7) respectively

$$\sigma M = M \sqrt{9 \left(\frac{\sigma\nu_{\max}}{\nu_{\max}} \right)^2 + 16 \left(\frac{\sigma\Delta\nu}{\Delta\nu} \right)^2 + 2.25 \left(\frac{\sigma T_{\text{eff}}}{T_{\text{eff}}} \right)^2} \quad (5)$$

$$\sigma R = R \sqrt{\left(\frac{\sigma \nu_{\max}}{\nu_{\max}}\right)^2 + 4 \left(\frac{\sigma \Delta \nu}{\Delta \nu}\right)^2 + 0.25 \left(\frac{\sigma T_{\text{eff}}}{T_{\text{eff}}}\right)^2} \quad (6)$$

$$\sigma L = L \sqrt{4 \left(\frac{\sigma \nu_{\max}}{\nu_{\max}}\right)^2 + 16 \left(\frac{\sigma \Delta \nu}{\Delta \nu}\right)^2 + 25 \left(\frac{\sigma T_{\text{eff}}}{T_{\text{eff}}}\right)^2} \quad (7)$$

We used the standard deviations in T_{eff} , $\nu_{\max}$ and $\Delta \nu$ to calculate the standard deviation in $M/M_{\odot}$ and $L/L_{\odot}$ in equation (5) and (7) respectively.