AN ANALYSIS OF GEOMETRY LEARNING IN A PROBLEM SOLVING CONTEXT FROM A SOCIAL COGNITIVE PERSPECTIVE

by

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SUMMARY

AN ANALYSIS OF GEOMETRIC LEARNING IN A PROBLEM SOLVING CONTEXT FROM A SOCIAL COGNITIVE PERSPECTIVE

Traditionally, geometry at school starts on a formal level, largely ignoring prerequisite skills needed for formal spatial reasoning. Ignoring that geometry has a sequential and hierarchical nature causes ineffective teaching and learning.

The Van Hiele theory postulates learner progression through levels of geometry thinking, from a Gestalt-like visual level through increasing sophisticated levels of description, analysis, abstraction, and proof. Progression from one level to the next does not depend on biological maturation or development only, but also on appropriate teaching/learning experiences. A higher thinking level is achieved through the application of a series of learning phases, consisting of suitable learning activities. The teacher plays an important facilitating role during this process.

In accordance with the social cognitive learning perspective on self-regulated learning, geometry learners must direct their thoughts and actions while completing activities in order for effective learning to take place. Learners can be described as being self-regulated to the degree that they are metacognitively, motivationally, and behaviorally active in their own learning. The social cognitive theory assumes that students enter learning activities to acquire knowledge, learning how to solve problems and completing learning activities. Self-regulated learners are aware of strategic relations between self-regulatory processes and learning outcomes and feel self-efficacious about using strategies. Self-regulation is similar to metacognitive awareness, which includes task and personal knowledge. Self-regulated learning requires that learners understand task demands, their personal qualities, and strategies for completing a task.

A Van Hiele-based geometry learning and teaching program was designed (with a problem solving context in mind) and implemented in four Grade 7 classes (133 learners) at two schools. The study investigated factors and conditions influencing the effective learning and teaching of spatial concepts, processes and skills in different contexts.

Results suggest that the implementation of a Van Hiele based geometry learning and teaching program in a problem solving context had a positive effect on the learners' concentration, when working on academic tasks, and level of geometric thought. The higher levels of geometric thought included higher categories of thought within these levels. Learners who completed the program reasoned on a higher level, gave more complete answers, demonstrated less confusion, and generally exhibited higher order thinking skills than their counterparts who did not take part in the program. The only prerequisite is that the teacher should consistently teach from a learner-centered approach as the program will deliver little or no advantages if the program is presented in a teacher-centered content-based context.

Words for indexing: learning strategies, self-regulation, self-efficacy, geometry, school mathematics, learning, teaching.

metakognitiewe bewustheid, wat taak- en persoonlike kennis insluit. Self-regulering vereis dat die leerder taakeise, persoonlike kwaliteite, en strategieë vir die voltooiing van 'n taak verstaan.

'n Van Hiele gebaseerde onderrig-en-leer program in 'n probleemoplossingskonteks is ontwikkel en geïmplimenteer in vier Graad 7 klasse (133 leerders) by twee skole. Die studie ondersoek faktore en kondisies wat effektiewe leer en onderrig van ruimtelike konsepte, prosesse en vaardighede in verskillende kontekse beïnvloed.

Resultate toon aan dat die implimentering van 'n Van Hiele gebaseerde onderrig-leer program (in 'n probleemoplossingskonteks) 'n positiewe effek het op die leerders se vlak van konsentrasie, wanneer hulle werk aan akademiese take, en meetkundige denke. Die hoër vlakke van meetkundige denke sluit hoër kategorieë van denke binne hierdie vlakke in. Leerders wat hierdie program voltooi het, redeneer op 'n hoër vlak, gee meer volledige antwoorde, demonstreer minder verwarring, en vertoon oor die algemeen hoër orde denkvaardighede as leerders wat nie aan die program deelgeneem het nie. Die enigste voorwaarde is dat die onderwyser konstant moet onderrig vanuit 'n leerder-gesentreerde benadering, aangesien die program min of geen voordele bied as dit in 'n onderwyser-gesentreerde konteks onderrig word.

Woorde vir indeksering: leerstrategieë, self-doeltreffendheidsoortuiginge, onderrig, self-regulering, skool wiskunde, meetkunde, leer.

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CHAPTER ONE

BACKGROUND AND OVERVIEW OF THE STUDY

1.1 INTRODUCTION AND PROBLEM STATEMENT

Although Great Britain's educational system was mimicked in the founding years of South Africa, the improvements in the teaching of geometry in England's educational system since 1920 had little or no influence in South Africa. Van Niekerk (1997:1) claims that South Africa inherited its Geometry from England at a time when England's teaching was more conservative than that of any other country in the world. The result of this was that any informal approach to geometry teaching in school was looked upon as a waste of time and theorems were introduced as early as possible. As early as 1965, according to Van Niekerk (1997:2), concern was raised over the lack of development in mathematics, especially in geometry.

In primary schools the approach to the teaching of geometry up to 1994 started with the introduction of two- dimensional shapes (squares, rectangles, circles etc.). In the beginning of the senior primary phase formulas were introduced for the calculation of the surface area for these figures (DET, 1991). The new syllabus that was introduced in 1994 (TED, 1994) had a problem-centered approach at its core. Unfortunately, a lack of support in any teacher materials as well as teacher training, accompanying this document, regarding spatial development or presentation of geometry, showed the triviality of geometry in the primary schools in South Africa (Van Niekerk, 1997:2). The little teacher in-service training that did take place, only changed the arrangement of desks with little or no evidence of a change in classroom behaviour (Taylor & Vinjevold, 1999:150).

At present geometry in the school curriculum starts on a formal level, ignoring prerequisite skills needed for formal reasoning. The effect of ignoring the sequential and hierarchical nature of the learning of geometry causes ineffective teaching and learning because learners are expected to perform without the necessary prior knowledge or prerequisite skills (Clements & Battista, 1992:421).

Learners are left with a distorted perception of the difficulty of geometry and a possibly negative self-efficacy belief. It is therefore important that the foundation of formal geometry learning (that starts in grade 7) is laid according to a widely respected and acceptable theory, for example the Van Hiele theory.

The teaching of geometry was a focus point in the Netherlands as far back as the 1950s (Van Hiele-Geldof, 1958) in contrast to South Africa's passive approach to reform in geometry. In 1959 Pierre van Hiele published his first article on thought levels and phases of the learning of geometry.

The Van Hiele theory postulates student progression through levels of thought argumentation in geometry, from a Gestalt-like visual level through increasing sophisticated levels of description, analysis, abstraction, and proof (Van Hiele, 1986:39). The phases of learning of geometry that were developed by Van Hiele identified a way in which a student's level of geometric maturation could be measured, and ways were even suggested to help learners progress through the levels. The five thought levels are Level 1, Visualisation; Level 2, Descriptive / Analytical; Level 3, Abstraction / Relational; Level 4, Formal Deduction and Level 5, Rigor / Metamathematical. Clements and Battista (1992:426) identified the following characteristics of the Van Hiele theory:

- Learning is a discontinuous process, which implies "jumps" in the learning curve. These "jumps" imply the presence of discrete, qualitatively different levels of thinking.

- These levels are sequential and hierarchical. For learners to perform adequately at one level, they must have mastered a large portion of the foregoing level (Mason, 1997:40).

- Concepts implicitly understood at one level become explicitly understood at the next level (Teppo, 1991:213).

- Each level has its own language. This implies that a relation that is "correct" at one level can be "incorrect" at another. Two people who reason at different levels cannot understand each other or follow the thought processes (reasoning) of the other. Language is thus a critical factor in the movement through the levels. New language is introduced in each learning period to make explicit and discuss new objects of study (Teppo, 1991:213).

The progression from one level to the next is more dependent upon educational instruction/experience than on age or maturation (Van Hiele, 1986:50). Certain types of experiences can facilitate (or impede) progress within a level and to a higher level (Mason, 1997:40). The teacher plays a special role in facilitating this progress, especially in providing guidance about expectations. Van Hiele (1986:40) claims that higher levels of geometric thinking are achieved not through direct teacher instruction, but through a suitable choice of learning activities and exercises (Van Hiele, 1982:215; Koehler & Grouws, 1992:123).

It is no longer a question whether these thought levels exist, but how to utilize them so that insight can be gained into the development of learners' spatial abilities (Van Niekerk, 1997:4). When insight is gained, it is possible to design the appropriate materials and instruction for the next teaching episode (Usiskin, 1987:29).

The social cognitive theory assumes that learners enter learning activities to attain or obtain knowledge, learning how to solve problems and completing learning activities.

In accordance with the social cognitive learning perspective on self-regulated learning, geometry learners must be able to direct their thoughts and actions while completing activities in order for effective learning of geometry to take place. Schunk (1989:83) states that self-regulated learning is learning that occurs from learners' self-generated behaviours (thoughts, feelings and actions) that are systematically oriented towards the achievement of their learning goals. Learners can be described as being self-regulated to the degree that they are metacognitively, motivationally, and behaviourally active in their own learning (Zimmerman, 1989a:4; Schunk, 1991:71). Self-regulated learning involves goal-directed cognitive activities (such as attending to instruction, processing and integrating knowledge, and rehearsing information to remember, and beliefs concerning capabilities for learning as well as the anticipated outcomes of learning) that learners generate, modify and sustain. According to Schunk (1996:338; 2000:355), most definitions stress that self-regulation during learning involves the personal activation and sustaining of goal-directed cognition and behaviour.

Schunk (1989:83-88) argues that learners' efforts to regulate themselves during learning are not determined merely by personal elements, such as affect, goals, metacognition and self-efficacy (Schunk, 1996:360; 2000:380). These processes are assumed to be influenced by environmental factors (such as features of the classroom and instruction) and behavioural factors in a reciprocal fashion, which link with the Van Hiele theory (Van Hiele, 1986:39-47) that proposes that progression is dependant upon instruction.

Self-efficacy beliefs influence the learners' choice of tasks and the quantity of effort the learner is willing to give when learning (Bandura, 1986:24). Self-efficacy beliefs influence how much learners are willing to persevere in difficult situations and for how long they are willing to persevere. Learners that perceive themselves to be efficacious will persevere even if the task becomes more demanding, while learners that see themselves as less efficacious will shy away from difficult tasks. Learners may shy away from "threatening" situations such as geometry because they believe that it over estimates their capabilities (Schunk, 1989:89). Learners' differences in how efficacious they feel about being able to attain their goals can be influenced by factors such as learners' abilities, prior experiences, and attitudes towards learning, and by instructional and social factors (Schunk, 1989:88).

Schunk (1996:382) states that from an information processing perspective, self-regulation is similar to metacognitive awareness, which includes task and personal knowledge. Self-regulated learning requires that learners understand task demands, their personal qualities, and strategies for completing a task. This view is in accordance with Van Hiele's view that for learners to perform adequately at one level, they must have mastered a large portion of the foregoing level (and thus also the strategies associated with such a level).

The purpose of this study is not to equate the social cognitive theory to the Van Hiele theory or vice versa, but to allow the researcher to approach the research from different perspectives rather than narrowing it to that of only one paradigm (Shulman, 1986:5).

In accordance with the Van Hiele theory this study will endeavour to identify some aspects of the problems in geometry learning by compiling and testing a series of learning activities to give learners the opportunity to experience geometry at a suitable level in line with their spatial development. These learning activities will form the necessary basis for the development of prerequisite skills to successfully study geometry in secondary school. The learning activities will also help learners to generate behaviours (thoughts and actions) that are directed to systematically achieve their learning goals, thus becoming self-regulated learners of geometry.

The study therefore seeks answers to the following questions:

- How does a Van Hiele geometry program, in a problem solving context, influence learning strategies?
- How does a Van Hiele geometry program, in a problem solving context, influence self-efficacy, intrinsic value of a (mathematical) task as well as self-regulated learning?
- How does a Van Hiele geometry program, in a problem solving context, influence learners' geometric thought levels?

1.2 AIMS OF THE RESEARCH

The primary aim of this research is to investigate geometry learning in a problem solving context from a social cognitive perspective.

The following secondary aims have thus been formulated:

- To define a self-regulated view of learning through a literature study;
- To investigate the learning of geometry by means of a literature study; and
- To develop and implement a Van Hiele based geometry learning and teaching program, in a problem solving context, and to empirically analyze and assess it.

1.3 RESEARCH HYPOTHESES

To achieve the aims stated in paragraph 1.2, the following five hypotheses were set:

1.3.1 HYPOTHESIS 1

The implementation of a Van Hiele based learning and teaching program, in a problem solving context, has an influence on learning strategies (as defined as the ten sub-scales of the LASSI-HS).

1.3.2 HYPOTHESIS 2

The implementation of a Van Hiele based learning and teaching program, in a problem solving context, has an influence on the self-efficacy of the learners.

1.3.3 HYPOTHESIS 3

The implementation of a Van Hiele based learning and teaching program, in a problem solving context, has an influence on the intrinsic value of the learners.

1.3.4 HYPOTHESIS 4

The implementation of a Van Hiele based learning and teaching program, in a problem solving context, has an influence on the self-regulation of the learners.

1.3.5 HYPOTHESIS 5

The implementation of a Van Hiele based learning and teaching program, in a problem solving context, has an influence on the geometric thought levels of the learners.

1.4 METHOD OF RESEARCH

The method of research consisted of a literature study and empirical research.

1.4.1 REVIEW OF THE LITERATURE

A DIALOG search was performed with the following key words: (perceived) difficulty level, prerequisite skills, self-efficacy beliefs, school mathematics, geometry, academic achievement, learning, teaching, spatial ability/development/thinking.

An intensive and comprehensive review of the relevant primary and secondary sources and electronical media was undertaken in order to identify the relationship between the (perceived) task difficulty level, prerequisite skills, and self-efficacy and the learning of geometry in grade 7.

Chapter 2 investigates self-regulated views of learning, while Chapter 3 examines the learning of geometry from a variety of theoretical perspectives.

1.4.2 EMPIRICAL RESEARCH

The research was done in two multi-cultural classrooms for mathematics in Grade 7. The learners' ages were between 12 and 16 years.

A series of Van Hiele based material (see § 4.8.2) was compiled that dealt with the geometry syllabus as prescribed by the Department of Education. Before the beginning of the program the two teachers in the schools that constituted the experimental group were tutored in the Van Hiele theory as well as the developed activities. The activities (see § 4.8.2) were then implemented in the experimental schools and the progression through these activities was continuously videotaped. The researcher planned the activities, but the mathematics teacher presented the classes while being taped for analyses and transcription afterwards.

To collect and interpret the data the following method was adhered to:
All the physical materials (drawings, written calculations and models) made by the learners were collected and categorized. The video material was transcribed in different ways namely:

(i) Verbal transcription

The actual conversations of the group members during the activities were written down or notes were made of the content of the discussions.

(ii) Written or pictorial material transcription

All the activities that involved the process of drawing or writing or sorting during the classroom activities were observed, videotaped and written down. In other words, not only the end product but also the whole process that the learner engaged in, from the moment that the drawing or sorting activity was started until the completion of the drawing or activity was noted.

All the worksheets and other written data like diagrams drawn were interpreted quantitatively or qualitatively depending on the nature of the data.

The method of research is discussed in Chapter 4, which is followed by the results of the statistical analyses of the data, in Chapter 5. In Chapter 6 conclusions are drawn, the review of the literature is summarized and the limitations and recommendations of the study are discussed.

CHAPTER TWO

A SELF-REGULATED VIEW OF LEARNING

2.1 INTRODUCTION

Theories of self-regulated learning seek to explain and describe how a particular learner will learn and achieve despite apparent limitations in mental ability, social environmental background, or in quality of schooling (Zimmerman, 1989a:4). Zimmerman (1989a:4) notes that self-regulated learning theorists assume that learners (a) can personally improve their ability to learn through selective use of metacognitive and motivational strategies; (b) can proactively select, structure and even create advantageous learning environments; and (c) can play a significant role in choosing the form and amount of instruction needed. Self-regulation can furthermore be the best predicator of academic performance (Pintrich & De Groot, 1990:38), while self-efficacy beliefs can be a proactive determinant of academic achievement (Zimmerman & Martinez-Pons, 1992:189).

In this chapter self-regulated learning will be discussed from a social-cognitive point of view. The "why" and "how" learners learn independently and what learners need to know of themselves and of their environment will receive attention. Zimmerman (1989a:11) notes that the social-cognitive theory was initially developed to explain modeling influences on human experience, but now focuses on the relationships between social and cognitive events. Personal processes, such as cognition or affect that are influenced by environmental and behavioural events in a reciprocal fashion determine self-regulated learning. Self-regulation will firstly be described and defined (§ 2.2), after which the assumptions made in self-regulated learning will be discussed (§ 2.4). Lastly, the determinants of self-regulated learning (§ 2.5) will be analyzed to form an overview of self-regulatory learning from a social cognitive perspective.

2.2 A DEFINITION AND DESCRIPTION OF SELF-REGULATED LEARNING

Self-regulated learning can generally be defined as the degree to which learners are metacognitively, motivationally and behaviourally active participants in own learning (Zimmerman, 1989b:329; 1990:4; Schunk, 1991:71). Self-regulated learners can be characterized as active learners who efficiently manage their own learning experiences in many different ways by using a large arsenal of cognitive and metacognitive strategies that they readily deploy to accomplish academic goals (such as to solve a geometry problem) (Wolters, 1998:224). In terms of metacognitive processes, self-regulated learners plan, set goals, organize, self-monitor, and self-evaluate at various points during learning (Zimmerman, 1986:308; Ley & Young, 1998:43).

For a learner to be seen as self-regulated, the learner's learning must involve the use of specified strategies to achieve academic goals on the basis of self-efficacy perceptions (Zimmerman, 1989b:329). The foregoing definition of Zimmerman (1989b:329) assumes the importance of three elements: learners' self-regulated learning strategies, self-efficacy perceptions of performance skill, and commitment to academic goals (Zimmerman, 1989b: 329). Ley and Young (1998:46) note that the individual elements of self-regulation might contribute to achievement but not as much as the combined effect of the elements. These elements (self-regulated learning strategies, self-efficacy perceptions and commitment to academic goals) enable self-regulated learners to be self-aware, knowledgeable, and decisive in their approach to learning (Zimmerman, 1990:5). Motivationally these learners report high self-efficacy, self-attribution, and intrinsic task interest. They seem to be self-starters who display extraordinary effort and persistence during learning (Zimmerman, 1990:5). In their behavioural processes, self-regulated learners select, structure and create optimal environments for learning. Self-regulated learners also seem to be systematic users of metacognitive, motivational and behavioural strategies (Zimmerman, 1990:5).

Zimmerman (1990:4-5) identifies a number of key features common to most definitions of self-regulated learning. The first feature of self-regulatory learning that Zimmerman (1989a:4) identifies is that such learners are assumed to be aware of the potential usefulness of self-regulation processes in enhancing their academic achievement. A further key feature of self-regulated learning is the existence of or role played by a "self-orientated feedback" loop (Zimmerman, 1989a:4). This loop entails a cyclic process in which learners monitor the effectiveness of their own learning methods or strategies and then react to this feedback. Reaction ranges from covert changes in self-perception to overt changes in behaviour such as altering the use of a learning strategy (Zimmerman, 1989a:4; 1990:5). Following Bandura (1986), Zimmerman (1990:5) cautions against viewing this control loop in terms of being only a negative feedback loop (i.e. seeking to reduce differences between one's goals and observed outcomes) because a positive feedback effect (i.e. seeking to raise one's goals based on observed outcomes) is also reported. Zimmerman (1990:5) concludes that regardless of theoretical differences in what is monitored and how outcomes are interpreted, virtually all researchers assume that self-regulation depends on continuing feedback of learning effectiveness.

The indication of how and why learners choose to use a particular strategy or response can be viewed as another feature of self-regulated learning (see § 2.3). Self-regulated learning involves temporal delimiting strategies or responses, which require preparation time, vigilance and effort when learners initiate and regulate strategies proactively. Self-regulated learners may choose not to self-regulate their learning when the opportunity arises – an outcome that requires a comprehensive accounting of their academic motivational processes (Zimmerman, 1989a:4; 1990:6).

A self-regulated learner is assumed to be a strategic learner as the two groups of learners share various characteristics. Weinstein (In press: 3-6) adds to the characteristics of a self-regulated learner by naming attributes normally associated with strategic learners. Firstly, self-regulated learners must be able to set and use learning goals. They need certain types of knowledge, for example self-regulated learners need to know about the nature and characteristics of different academic tasks.

Self-regulated learners, according to Zimmerman (1990:4), approach educational tasks with confidence, diligence and strategic resourcefulness. Such learners are aware when they possess a skill or when not. Furthermore such learners need to know about and how to use a variety of study skills and learning strategies. Finally, learners need to know about the present and future contexts in which they could use what they are trying to learn now (Zimmerman & Martinez-Pons, 1992:186). Ley and Young (1998:47) add another characteristic, namely that self-regulated learners exhibit characteristics that are the antithesis of characteristics associated with low achievers. For example, self-regulated learners persist even if the task becomes more demanding, while less self-regulated learners will disengage if the task becomes demanding (Pintrich & De Groot, 1990:37).

Ley and Young (1998:43) and Zimmerman (1989b:329) state that learners who engage in self-regulation take greater responsibility for their achievement outcomes and initiate efforts to acquire skill and knowledge instead of depending upon external sources. Zimmerman (1989b:330) and Zimmerman and Martinez-Pons (1992:186) theorize that self-regulated learning is a strategic controllable process that occurs to the degree that a learner can use personal processes to strategically regulate own behaviour and the learning environment.

In summary, defining self-regulated learning involves three features: use of self-regulated learning strategies, learners' responsiveness to self-orientated feedback about learning effectiveness, and learners' interdependent motivational processes (Zimmerman, 1990:6).

2.3 DIMENSIONS OF SELF-REGULATION

Another feature of self-regulated learning is the indication of how and why learners choose to use a particular strategy or response. In an effort to analyze and address the question of what constitutes self-regulation, Zimmerman (1994:7) developed a conventional framework which Schunk (2000:357) refined (see table 2.1). Zimmerman (1994:7) notes three purposes for the formulation of this conceptual framework. Firstly, it serves to analyze research on academic self-regulation in terms of its common components in order to show connections with prior forms of learning. The second purpose is to describe the task conditions necessary to self-regulate each component, and the final purpose of this framework is to cross-relate and integrate academic self-regulation findings from different theoretical models.

Table 2.1 Conceptual framework for studying self-regulation

Learning Issues	**Learning Dimensions**	**Learner Conditions**	**Self-regulation Attributes**	**Self-regulation Subprocesses**
Why	Motive	Choose to participate	Self-motivated	Self-efficacy and self-goals
How	Method	Choose method	Planned or automatized	Strategy use or routinized performance
When	Time	Choose time limits	Timely and efficient	Time management
What	Behaviour	Choose outcome behaviour	Self-aware of performance	Self-observation, self-judgement, self-reaction
Where	Physical environment	Choose setting	Environmentally sensitive and resourceful	Environmental structuring
With whom	Social	Choose partner, model, or teacher	Socially sensitive and resourceful	Selective help seeking

The first column, Learning Issues, catalogues important questions in learning: Why should I learn? How should I learn? When should I learn? What should I learn? Where should I learn? With whom should I learn? The question *why* addresses learners' motivation to self-regulate, and the question *how* deals with learners' methods for self-regulating their learning by their use of self-regulated learning strategies. When learners answer the question *what,* their efforts to self-regulate are brought to the foreground. The questions of *where* and *with whom* address learners' efforts to self-regulate their physical and social environment (Zimmerman, 1994:7-8).

The second column, Learning Dimensions, lists the personal or environmental characteristics involved in the relevant aspect of self-regulation. The third column, Learner Conditions, indicates the choices potentially available that are critical for determining the extent of self-regulation. For example, individual differences in performance are greatly reduced if learners are given the opportunity to work at their own pace. The fourth and fifth columns list important attributes and subprocesses involved in each dimension of self-regulation (Schunk, 1996:339; 2000:357-358).

A crucial element of self-regulation is that learners have some choice available (Zimmerman, 1994:9; Schunk, 2000:356) as the middle column of the table indicates. Learners should have a choice in at least one aspect, although learners may not always take advantage of the available choices. This learner choice is necessary because if all task aspects (why, how, when, what, where and with whom) are controlled, the achievement behaviour can be seen as being "externally controlled" or "controlled by others". Self-regulation can vary from low to high depending on how much choice the learners have (Schunk, 1996:339; 2000:357).

To be self-regulated does not require having choices in all six areas (why, how, when, what, where, with whom) but learners should have some choice in elements of the situation.

2.4 SOCIAL COGNITIVE ASSUMPTIONS UNDERLYING SELF-REGULATED LEARNING

Zimmerman (1989b: 330-332) identifies four distinctive assumptions fundamental to self-regulated academic learning. The first assumption is that there is a triadic reciprocality between personal, environmental, and behavioural determinant (see § 2.4.1). Self-efficacy is furthermore viewed as a key feature (see § 2.4.2). The third assumption deals with the subprocesses of self-regulated learning, namely self-observation (§ 2.4.3.1), self-judgement (§ 2.4.3.2) and self-reaction (§ 2.4.3.3). The last assumption is that self-regulation is never an absolute state (see § 2.4.4).

2.4.1 TRIADIC RECIPROCALITY

Self-regulated learning assumes the reciprocal causation between personal, environmental, and behavioural determinants (Zimmerman, 1989b:330). Bandura (1986:454) comments that behaviour is a product of both self-generated and external sources of influence. The response of a learner trying to complete a geometry problem, according to Zimmerman (1989b:330), is assumed to be determined not only by personal (self) perceptions of self-efficacy but also by environmental stimuli such as encouragement from a teacher and by enactive outcomes (i.e. obtaining a correct answer to previous problems).

Zimmerman (1989b:330) provides the following figure to clarify the triadic reciprocality of self-regulated learning:

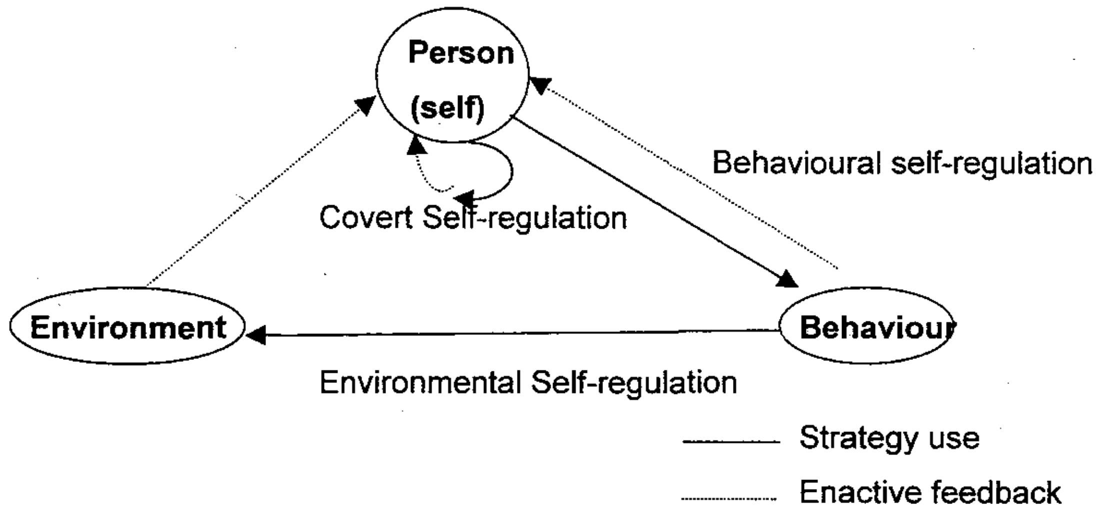


Figure 2.1 Triadic reciprocality of self-regulated learning (Zimmerman, 1989b:330)

Bandura (1986, quoted by Zimmerman, 1989b:330) assumes that the relative strength and the temporal patterning of mutual causation among personal, environmental, and behavioural influences could be altered through (a) personal efforts to self-regulate, (b) outcomes of behavioural performance, and (c) changes in environmental context. Consider, for example, a learner who struggles to memorize the names of the geometric figures. This learner could improve his memory by self-recording the names he could not remember (influences a and b) or if another student arrives seeking help to jointly memorize the list (influence c). Put differently, a learner reorganizes his room to minimize the interference by turning down the TV (an environmental influence), he checks his understanding and knowledge (a personal process) by setting himself a test (a behavioural influence).

2.4.2 SELF-EFFICACY

Schunk (1996:132,448; 2000:108) and Bandura (1997:2) define self-efficacy as conceptually being one's personal beliefs about perceived abilities for learning or performing a task, in other words, beliefs concerning one's capabilities to organize and implement / execute courses of action necessary to attain designated performance levels or to perform behaviours at designated levels or to manage prospective situations.

Efficacy judgements are a learner's beliefs about his/her skills, abilities, and power to achieve the goals in relation to resources and constraints in the task environment (Winne & Butler, 1994:5741). Learner involvement in self-regulated learning is closely tied to learners' efficacy beliefs about their capability to perform classroom tasks (Pintrich & De Groot, 1990:38) (such as solving problems, as seen in the designed learning and teaching program, § 4.8.2).

Lopez and Lent (1992:10) suggest that students largely draw on past math-related performance and emotional arousal information in appraising their course-specific math capabilities. Failure experiences and unfavorable anxiety may significantly diminish students' confidence in their current math ability, and their interest in and motivation to enroll in addition match courses. (See § 2.5.1.4 for a more detailed discussion of self-efficacy.)

2.4.3 THE SUBPROCESSES OF SELF-REGULATED LEARNING

Social cognitive theorists believe that self-regulation involves three classes of subprocesses, namely self-observation, self-judgement, and self-reaction. These subprocesses are not mutually exclusive but rather interact with one another (Schunk, 1989:88).

2.4.3.1 Self-Observation

Self-observation refers to a learner's systematic monitoring of specific aspects of his/her own observable performance/behaviour and situational factors such as pages of homework completed (Zimmerman, 1989b:333; Zimmerman & Martinez-Pons, 1992:187; Graham & Harris, 1994:209). Observing oneself can provide information about how well one is advancing in achieving a set goal such as the number of homework pages completed.

Self-observation provides information necessary for setting realistic performance standards (Schunk, 1989:89; Monteith, 1996:210). Self-observation can also motivate behavioural change by providing information for evaluating ongoing changes in behaviour (Monteith, 1996:210). The two most important criteria for self-observation are regularity and proximity (Schunk, 1989:90). (See § 2.5.2.1 for a more detailed discussion on self-observation.)

2.4.3.2 Self-Judgement

Self-judgement must be distinguished from self-observation because learners may attribute or interpret their performance outcomes according to factors, standards, or goals (Zimmerman & Martinez-Pons, 1992:188).

Self-judgement refers to students' responses that involve systematically comparing their performance with a standard or goal (Zimmerman, 1989b:333). This definition assumes goal setting and knowledge of standards, as well as self-observed responses. Self-judgements can be affected by such factors as the type of standards employed, the properties of the goal, the importance of goal attainment, and the attributions made for one's performance (Schunk, 1989:90). Self-judgement of own behaviour against a goal leads to information needed for self-reaction that will lead to progress in goal attainment. A learner who judges his own performance against a time table for attaining his goal will obtain information of his own performance which will initiate a reaction to work harder or to maintain the speed at which he is going. Whether this student's performance is regarded as favorable or not depends on the evaluation against his personal standard. Self-judgement is necessary, as the information is crucial in determining if actual progress is being made in attaining the set goal. (See § 2.5.2.2 for a more detailed discussion on self-judgement.)

2.4.3.3 Self-Reaction

Self-reaction is initiated after self-judgement in order to arrive at a positive influence in goal attainment; thus self-observation causes self-judgement that will lead to self-reaction.

Self-reaction, according to Zimmerman and Martinez-Pons (1992:188), refers to a wide range of responses ranging from self-praise to self-criticism, from further strategy persistence to strategy change. During self-reaction a geometry learner judges his/her own learning strategies and make changes in the way the strategy is used, or chooses another strategy to ensure that he/she reaches a set goal. Development of judgemental skills and evaluative standards establishes the capacity for self-reactive influence (Bandura, 1986; referred to by Monteith, 1996:210). (See § 2.5.2.3 for a more detailed discussion on self-reaction.)

2.4.4 SELF-REGULATION IS NEVER AN ABSOLUTE STATE

Zimmerman (1989b:332) states that self-regulated learning is never an absolute state of functioning but rather varies in degree, depending on the social and physical context (Zimmerman & Kitsantas, 1999:242; Wolters, 1998:233). The reciprocality between personal, environmental and behavioural processes does not imply equality or symmetry in strength between these processes. Environmental influences may be stronger than behavioural or personal ones in some contexts or at certain points during behavioural interaction sequences. An environment, which allows for freedom in the use of personal processes to strategically regulate their behaviour and environment, is essential for a learner to be truly self-regulated. In schools with a highly structured curriculum or restrictive code for classroom behaviour, learners' self-regulatory processes will be overshadowed by discipline with the result that learners are less self-regulated.

2.5 DETERMINANTS OF SELF-REGULATED LEARNING

Self-regulated learning occurs to the degree that a student can use personal (i.e. self) processes to strategically regulate behaviour and the immediate learning environment (Zimmerman, 1989b:330). This statement implies three determinants namely personal, environmental and behavioural influences or determinants. Zimmerman (1989a:11) argues that mere personal processes, such as cognition or affect, do not determine learners' efforts to self-regulate during learning; these processes are assumed to be influenced by environmental and behavioural events in a reciprocal fashion. Each of the above named determinants consists of various variables that are discussed below.

2.5.1 PERSONAL INFLUENCES

Zimmerman (1989b:331) notes that self-efficacy is the key variable affecting self-regulated learning. Self-efficacy depends, according to Monteith (1996:211) and Schunk (1996:360), in part on each of the four types of personal influences: a learner's knowledge, metacognitive processes, goals and attributions. Among personal influences, strategy awareness is a form of metacognition, and strategy knowledge is a type of knowledge (Zimmerman & Martinez-Pons, 1992:187).

2.5.1.1 Student knowledge

Learners need knowledge from various sources to succeed in their academic objectives. A distinction is drawn between declarative, procedural and conditional knowledge. Declarative knowledge is knowledge about what strategies are and includes knowledge about oneself as a learner and about what factors influence one's performance (Zimmerman, 1989a:21; Schraw, 1998:114). Procedural knowledge is knowledge of how strategies are used, as well as knowledge about doing things. Conditional knowledge refers to knowledge of when and why strategies should be used (Zimmerman, 1989a:21). For self-regulated learning to occur successfully, a learner must possess a variety of learning strategies (declarative knowledge), that he/she knows how to execute (procedural knowledge), as well as when and why to use a specific strategy (conditional knowledge) to ensure success. The latter two forms of knowledge are sometimes referred to as metacognition (Jacobs & Paris, 1987:258-259).

- **Declarative knowledge**

Declarative knowledge is descriptive information or "knowledge that" (Winne & Butler, 1994:5740) or knowing "about" (Schraw, 1998:114). Declarative or propositional knowledge is organized according to its own inherent verbal, sequential, or hierarchical structure (Zimmerman, 1989b:332). Declarative knowledge remains static until changed by learning, such as what happens when a problem is solved (Shuell, 1989:104; 1990:540). Problem solving thus affects or changes declarative knowledge. Terms used to describe organizations in which declarative knowledge is organized or stored in the memory include chunk, concept, frame, image, metaphor and schema (Winne & Butler, 1994:5740).

- **Procedural knowledge**

Procedural knowledge is rules, often called condition-action rules or "if-then" rules, because only if certain conditions arise will this knowledge be invoked (Winne & Butler, 1994:5740). Procedural knowledge is organized around conditions and actions. One of the most common ways of depicting procedural knowledge is in the form of strategies. Zimmerman (1989b: 332) refers to Schneider (1987) who draws a distinction between specific strategies, which are dependent on distinctive task contexts, and general strategies, which can be used more universally. Other theorists such as Paris and Byrnes (1989, as referred to by Zimmerman, 1989b:332) explain procedural knowledge as knowledge of how to use strategies, and conditional knowledge as knowledge of when and why strategies are effective.

- **Conditional knowledge**

Winne and Butler (1994:5740) describe conditional knowledge as one type of procedural knowledge that classifies situations based on their properties. Conditional knowledge defines when, where, and why declarative knowledge or a rule is relevant (Schraw, 1998:114). Conditional knowledge refers to an awareness of the conditions that influence learning, such as when to apply declarative and procedural knowledge and why it is important to do so (Zimmerman, 1989b:332). Conditional knowledge plays a central role in learners' use of cognitive strategies.

A second type of procedural knowledge is simply referred to as a rule, which acts on information to transform knowledge, such as translating information into a visual image. The certainty of the output obtained by applying procedural knowledge can vary, for example: Algorithms which yield reliable outcomes are distinguished from heuristics, which are predictive but do not assure a determinate result (Winne & Butler, 1994:5740).

Self-regulated knowledge, according to Zimmerman (1989b:332), has both procedural and conditional qualities and is thus treated as a single integrated construct.

2.5.1.2 Metacognitive processes

Learners' uses of self-regulated learning strategies depend not only on their knowledge of strategies but also on metacognitive decision-making processes and performance outcomes as well as their explicit knowledge. Metacognition focuses on self-regulated thinking – what learners know and how they apply that knowledge (Jacobs & Paris, 1987:255). Zimmerman and Martinez-Pons (1992:186) view metacognitive monitoring as self-observations of ongoing cognitive actions. Planning and behavioural control are distinguished as parts of metacognitive processes (Zimmerman, 1989b:332).

- Task analysis or planning has been proposed to describe decisional processes for choosing or altering general self-regulatory strategies. Planning is assumed to occur on the basis of task and environment features, one's declarative and self-regulatory knowledge (about strategies), goals, and perceptions of efficacy, affective states, and outcomes of behaviour control (Zimmerman, 1989b:332).

- Behavioural processes guide attentiveness, execution, persistence, and monitoring of strategic and nonstrategic responses in specific contexts (Zimmerman, 1989b:332). Learners' effectiveness in planning and controlling their use of personal, behavioural, and environmental strategies to learn is one of the most visible signs of their degree of self-regulation (Zimmerman, 1989b:332).

Learners' use of self-regulated learning strategies depends not only on their knowledge of strategies but also on their explicit knowledge. Explicit knowledge is knowledge a learner consciously inspects, including knowledge that converts to explicit form by becoming an "object of thought". Explicit knowledge can be divided into conceptual and metacognitive knowledge (Winne & Butler, 1994:5741).

- Conceptual knowledge is information about common forms of and functions of language, as well as other symbol systems used to present conceptual knowledge. Conceptual knowledge is thus information that a learner possesses about his or her physical, social, and mental world. Conceptual knowledge can be used informally (in the context of schooling) or formalized into domains of study, for example spelling, science, music, art and mathematics (Winne & Butler, 1994:5741).

- Learners furthermore need metacognitive knowledge, that is knowledge that provides the means by which learners inquire about knowledge. Metacognition presents reflections on or knowledge about knowledge. Metacognitive knowledge has four facets, namely task knowledge, self-knowledge, strategy knowledge and goals and plans (Winne & Butler, 1994:5741).

2.5.1.3 Goals

Goals are mental constructions that learners create because they are actively inquiring. Goals are also representations of states to be achieved that inherently fuse affects of various kinds of knowledge previously discussed (Winne & Butler, 1994:5741).

The selection or setting of appropriate goals for oneself may be one of the most distinguishing characteristics of self-regulated learners. Learners with a history of academic success set their goals at a realistic level (slightly above their current level), whereas learners who are prone to failure set their goals too high or too low (Zimmerman & Martinez-Pons, 1992:188). Bandura (1986:10) postulates that a perceived negative discrepancy between a goal and present performance creates an incentive for change.

Goals furthermore determine metacognitive decision making. An effective strategy for reaching goals involves setting intermediate goals that are based on their specificity, difficulty level, and proximity in time (Zimmerman, 1989b:333).

- With regard to specificity, learners' motivation do not improve with general goals such as "Do your best" (Zimmerman, 1989b:333), but with more specific goals such as "You can get 80% for geometry." Learners' goals as well as their proactive attempts to reach them are subject to change on the basis of performance feedback (Page-Voth & Graham, 1999:230). Goal accomplishment can lead learners to raise their goals (Zimmerman & Martinez-Pons, 1992:190; Zimmerman & Kitsantas, 1999:230). For example, a learner who has reached his goal of doing 10 geometry problems in class time might try to do 15 geometry problem in the next class.

- Self-regulated learners strategically set goals at a plausible difficulty level. Learners with a low achievement motivation set goals for themselves that are too high or too low to be of much assistance (Zimmerman, 1989b:333). This implies that a strategic geometry learner will set goals that are demanding but reachable for him/her, which will in turn result in an increase in motivation and self-efficacy.

- Goals can also be set on their proximity in time. Learners' goals and use of metacognitive control processes are dependent on perceptions of self-efficacy and affect, as well as self-regulatory knowledge (Zimmerman, 1989b:333). Teaching learners to set goals appropriately can have important academic benefits for learners who have deficits in self-regulation. Teaching learners a goal-setting strategy will help them sustain their motivation and increase their acquisition (Zimmerman & Martinez-Pons, 1992:190; Page-Voth & Graham, 1999:231).

2.5.1.4 Self-efficacy

Social cognitive theorists such as Zimmerman (1989b:331) and Schunk (1996:448) assume that self-efficacy is a key variable affecting self-regulated learning. Self-efficacy expectations, which refer to beliefs about one's ability to perform specific tasks, are presumed to influence a wide range of behavioural outcomes, including one's preferences for particular tasks and one's effort expenditure and persistence at these tasks in the face of obstacles (Lopez & Lent, 1992:2; Schunk, 1996:131; 2000:108).

Schunk (1996:131; 2000:108) indicates that self-efficacy is primarily a domain specific construct. It is therefore meaningful to speak of self-efficacy in relation to drawing conclusions from texts, solving geometry problems and so on. Efficacy is distinguished from the global construct of self-concept, which refers to one's collective self-perceptions formed through experiences with and interactions of the environment and which depends on reinforcement and evaluation by other people that are significant (Schunk, 1996:131-132), for example parents or teachers. Self-efficacy depends in part on the learner's ability. Learners with high ability feel more efficacious about learning compared with low ability learners (Schunk & Ertmer, 1999:251).

Schunk (1989:89) notes that learners differ in how efficacious they feel about being able to obtain goals such as solving problems, finishing workbook pages, and completing experiments due to their level of self-regulation (Schunk & Ertmer, 1999:251). This sense of self-efficacy for learning can be influenced by such factors as learners' abilities, prior experiences and attitude toward learning (Wolters & Pintrich, 1998:44), as well as by instructional and social factors. A learner who evaluates his progress towards a learning goal as being satisfactory will feel more confident about continuing to improve his/her skills.

Schunk (1996:133) remarks that self-efficacy judgements are closely related to performance expectancies. Williams (1994:235) reports that students with greater efficacy expectations generally have higher performance outcome scores. Interestingly, Williams (1994:235) finds that the efficacy/performance relationship is stronger in mathematics than in any other content area for both male and females. Both males and females who perceived themselves to be self-efficacious are also typically more likely to do well on the practice exams. Students may come closer to estimating their efficacy expectations for performance in mathematics as it requires accurate, detailed knowledge of specific rules and precise answers (Williams, 1994:235).

Academic learning becomes self-regulated when learners view acquisition as a strategic process and when they accept responsibility for their achievement outcomes. Self-efficacy is a reliable measure of learners' acceptance of personal responsibility (Zimmerman & Martinez-Pons, 1992:196). Lopez and Lent (1992:2) theorize that young women exhibit significantly higher math course grades and academic self-concepts than do young men. Women's self-efficacy is also higher than that of men. Anxiety and low self-efficacy perceptions can undermine learners' use of metacognitive control processes and can inhibit setting of goals (Zimmerman, 1989b:333).

Bandura (1997:3-5) identifies four main sources of efficacy beliefs. *Mastery experiences* is the most effective way of creating a strong sense of efficacy, as mastery experiences provide the most authentic evidence of whether one can generate whatever it takes to succeed. Successes build a powerful belief in one's personal efficacy. The second way of creating and strengthening efficacy beliefs is through the *vicarious experience* provided by social models. Seeing other learners succeed (in for example solving a geometry problem) by perseverance raises the observer's beliefs that he/she also possesses the capabilities to master comparable activities (Zimmerman & Kitsantas, 1999:241). *Social persuasion* is the third way of strengthening learners' beliefs that they have what it takes to succeed. Learners, who are verbally persuaded that they have the capabilities to succeed, will muster greater effort and perseverance. The fourth source of efficacy is the *physiological and emotional states* of learners. The most effective way of altering efficacy beliefs is to enhance physical status, reduce stress and negative emotional inclination, and correct misinterpretations of bodily states.

Learners' self-efficacy perceptions have been found to be related to two key aspects of the feedback loop, namely learners' use of learning strategies and self-monitoring. A learner with high self-efficacy displays better quality learning strategies and more self-monitoring of his learning outcomes than a learner with low self-efficacy. Learners' perception of self-efficacy is assumed to influence behavioural performance, as well as the reverse (Zimmerman, 1989b:331). Self-efficacy depends in part on the learner's ability. Learners with high ability feel more efficacious about learning compared with low ability learners (Schunk, 1996:132; 2000:108).

Pintrich and De Groot (1990:37) establish that self-efficacy is positively related to learner cognitive engagement and performance. Learners who believe they are capable are more likely to report use of cognitive strategies, to be more self-regulating in terms of reporting more use of metacognitive strategies, and to persist more often at difficult or uninteresting academic tasks.

2.5.1.5 Attributions

Attributions refer to the perceived causes of outcomes (Schunk, 1996:439; Bandura, 1986:10). Attributions explain how learners view the causes of their behaviours and those of others. Attributions identify the reasons, such as a learner's geometry ability or difficulty level of a task, that explain why work on the task yields the predicted outcomes (Winne & Butler, 1994:5741; Sexton, Harris & Graham, 1998:307). Schunk (1991:10) postulates that attribution theorists assume that people desire to explain the causes of significant events. Students who attribute prior successes (failure) to such stable factors as high (low) ability or low (high) task difficulty are apt to hold higher (lower) expectancies for success than those who emphasize the variable factors of high (low) effort or good (bad) luck. Attributions along with judgement of goal progress can affect self-efficacy, motivation, achievement and affective reactions (Schunk, 1996:357).

Attributions, along with judgements on goal progression, can affect self-efficacy, motivation, achievement and affective reactions (Schunk, 1996:357). Learners who are not doing well in geometry may attribute their performance to low ability, which will negatively impact on their expectancies and behaviour. Bandura (1986:349) determines that learners respond self-critically to faulty performances for which they hold themselves responsible but not those they perceive as due to unusual circumstances, to insufficient capabilities, or to unrealistic demands. This feedback, that could be called attributional feedback, can enhance or restrain learners' self-regulated learning. Telling a learner that he/she can do better if he/she works harder could motivate the learner to do so because this feedback indicates that the learner is capable of improvement. An important factor is timing. Early task success makes up a prominent cue for forming ability attributions. Linking early successes with ability should enhance learning efficacy (Schunk, 1996:358; 2000:374).

Lopez and Lent (1992:2) theorize that learners draw mainly on past math-related performance and emotional arousal information in appraising their course-specific math capabilities. Failure experiences may significantly diminish learners' confidence in their current math work, and their interest in and motivation to enroll in additional math courses.

2.5.2 BEHAVIOURAL INFLUENCES

Zimmerman (1989b:333) and Schunk (1996:361; 2000:372) distinguish three classes of student responses: self-observation, self-judgement, and self-reaction. Learners' use of strategies depends on two linked behavioural processes: monitoring the effectiveness of these strategies and attributing academic outcomes to them. The above mentioned strategic processes are subsumed within a more general behavioural category of self-regulation. Strategy monitoring is a form of self-observation, strategy attribution is a type of self-judgement, and strategy use is an important self-reaction (Zimmerman & Martinez-Pons, 1992:187).

2.5.2.1 Self-Observation

Learners can not regulate their own actions if they are not fully aware of them. The function of self-observation is to gain information that is used to determine how well one is progressing towards a set goal. Learners with poor study habits who observed themselves were astonished to learn they waste much study time on nonacademic activities (Schunk, 1989:89). Zimmerman (1989b:333) notices that self-observation is influenced by such personal processes as self-efficacy, goal setting, and metacognitive planning.

Self-recording, regularity and proximity are seen as parts of self-observation. Two behavioural methods of self-recording are (a) verbal or written reporting and (b) quantitative recording of one's actions and reactions, where behaviours are recorded along such features as the time, place and duration of occurrence. In the absence of recording, one's observations may not faithfully reflect one's behaviours due to selective memory (Schunk, 1989:90).

Regularity means that behaviour is observed on a continuous basis – hour by hour, day to day – rather than intermittently. Irregular observation provides misleading results. Proximity means that behaviour is observed close in time to its occurrence rather than long after it (e.g. recall at the end of the day what one did during that day). Trying to recall information long after the occurrence may lead to lack of important information. Proximal observations provide continuous information to use in goal progress (Schunk, 1989:90). (See §2.4.3.1 for more information on self-observation.)

2.5.2.2 Self-Judgement

Social cognitive theorists distinguish self-judgement from self-observation, as learners who self-judge may attribute or interpret their performance outcomes according to various factors, standards, or goals (Zimmerman & Martinez-Pons, 1992:188). Self-judgement, according to Schunk (1989:90), refers to comparing one's performance level with one's goal. Learners form standards for judging their own behaviour partly on the basis of how significant persons in their lives have reacted to it. Standards can be acquired through direct teaching, as well as through the evaluative reactions of others towards one's actions (Bandura, 1986:340). Self-judgement can be affected by such factors as the type of standards employed, the properties of one's goals, and the importance of goal attainment (See § 2.4.3.2 for more information on self-judgement.)

- **Types of standards**

Schunk (1996:355) explains that goals may be cast in terms of absolute or normative standards. Absolute standards are fixed and normative standards are based on the performance of others. Grades or symbols (such as A for 80%) are an example of absolute standards, while normative standards are acquired by observing others. Comparing one's performance socially is important in evaluating the appropriateness of one's behaviour. Comparing one's performance with standards indicates goal progress. Learners who complete more than half their geometry work in less than half the time will believe that they are making progress, which will enhance their self-efficacy that will in turn sustain motivation to complete the task.

- **Goal properties**

Goal properties such as proximity, specificity and attainability enhance motivation and self-efficacy. Regardless of goal properties, goals do not enhance performance (Schunk, 1996:356). Goal setting is influential with long-term tasks as goal setting

Three classes of self-reaction can be distinguished (Zimmerman, 1989b:334-335). Responses that students use to optimize their learning outcomes are behavioural self-reactions. Environmental self-reactions are actions that involve learners' selection, adaptation, or creation of physical settings where learning can readily occur. The last class of self-reaction is personal self-reaction, which seeks to enhance cognitive and effective processes involved in learning (Zimmerman, 1989b:334-335; Zimmerman & Martinez-Pons, 1992:188). For example, learners plagued by anxiety about geometry: (a) behaviourally, might keep a record of problems they can solve; (b) environmentally, might select a quiet area free from intrusions while attempting geometry problems; and (c) personally, might practice relaxation techniques whenever fears of not being able to solve a geometry problem occur.

Two major classes of self-reaction have been identified namely evaluative and tangible motivators. Evaluative motivators refer to personal feelings of satisfaction or dissatisfaction, while tangible motivators refer to self-administered stimuli or consequences, for example food or new clothes (Zimmerman, 1989a:13).

- **Evaluative motivators**

Self-reaction to goal progress motivates behaviour (Bandura, 1986:342). If a learner feels that he/she is making acceptable progress, then self-efficacy is enhanced and motivation is sustained. Negative evaluation does not decrease motivation if the learner believes that he/she is capable of improving. Perceived progress is relative to one's goals; the same level of performance can be evaluated positively, neutrally or negatively (Schunk, 1996:358). Seeing oneself perform successfully can enhance proficiency in at least two ways: It provides clear information on how best to perform skills, and it strengthens beliefs in one's capability (Bandura, 1997:94). Assuming that learners feel capable of improving, higher goals lead to greater effort and persistence that lower goals (Bandura & Cervone, 1983:1020).

- **Tangible motivators**

Learners enter learning activities with certain goals (like completing a worksheet or getting good marks) in mind. With these goals in mind, a learner observers, judges and reacts to his perceived progress. A learner observes his progress and quality of work by judging his progress through comparing his work or progress with an external source and then react by making the necessary adjustments to his learning strategy use or behaviour. For example, a learner spends an hour reading a chapter after which he tests his understanding by answering the questions in the back of the book and makes the necessary reading strategy adjustments (Bandura & Cervone, 1983:1021).

2.5.3 ENVIRONMENTAL INFLUENCES

Schunk (1996:140) remarks that the social cognitive theory argues that learners learn from their social environments. Environmental influences refer to the influence of the social and physical context on learners' behaviour (Zimmerman, 1989b:335). Schunk (1996:361; 2000:380) postulates that environmental factors reside in features of the classroom and instruction, as well as on academic outcomes (e.g. grades).

2.5.3.1 The social context

The social context generally includes people such as teachers, other learners, parents, brothers and sisters (Anderson, Wilson & Fielding, 1988:286). Within the school the following are involved in the social context: modeling; direct assistance from teachers or other learners; verbal persuasion by teachers or by learners themselves; and other forms of information such as diagrams, pictures and formulas (Zimmerman, 1989b:336). Social cognitive theorists have devoted particular attention to the impact of experiences in a social and physical context (Zimmerman, 1989b:335).

- **Modeling**

Schunk (1987:149) views modeling as a means of acquiring skills and beliefs. The modeling of effective self-regulated strategies can improve self-efficacy of learners that are not doing well, as seeing other learners complete a task successfully can lead to the feeling that such a learner could perform on the same level that will equate to success.

Such modeling is theorized to be especially effective if the model is perceived as similar to the observer (Zimmerman, 1989b:335). Schunk (1987:162) defines two types of models, namely mastery and coping models. Mastery models demonstrate faultless performance from the outset, whereas coping models improve their performance and gain self-confidence for a task. Coping models gradually eliminate errors, show high concentration, persistence and increase effort. Learners view the coping model as similar in competence to themselves. Learners also learn mathematics more readily and gain a greater sense of efficacy from the coping model (Zimmerman, 1989b:335).

- **Verbal persuasion**

Verbal persuasion includes talking learners into believing that they are capable or have the qualities that will enable them to achieve the set goal (Bandura, 1986:400). Not only will learners' attention be drawn to important academic tasks (Schunk, 1987:160) but they will also exert greater effort (Bandura, 1986:400) if they are verbally persuaded that they possess the capabilities to master the tasks. Verbal persuasion motivates learners to work harder that in turn will increase self-efficacy if success is achieved, but depends on the learners' level of verbal comprehension (Zimmerman, 1989b:335).

- **Direct assistance**

Zimmerman and Martinez-Pons (1986:615) postulate direct assistance as direct support from teachers, other adults or fellow learners in order to complete a task. Geometry teachers can influence the learners by initiating discussions about a specific answer or allowing fellow learners to participate as tutors in order to help other learners also achieve success. Parents can participate, according to Mathebula (1995:19), by checking learners' homework and by encouraging them to work hard.

Williams (1994:236) suggests that educators should implement instructional methods to enhance efficacy expectations and to reduce efficacy/performance discrepancies.

2.5.3.2 The physical context

Zimmerman (1989b:336) remarks that the physical context determines the structure of the learning environment. An inadequate physical environment, like overcrowding or learners that live in shelters, leads to no private space for such learners. Sharing space may cause learners to try to complete homework in surroundings that are detrimental not only to motivation but also to self-regulated learning. Mathebula (1995:19) advises that to promote self-regulated learning, the child should be advised to study when the other occupants are sleeping or to find a place more suitable for studying.

- **Structure of the learning task**

Zimmerman (1989b:336) refers to the structure of the learning task as the complexity of an academic task and also to the academic setting. When a learner, for example, changes a task to increase its level of difficulty by supplying information that only a very bright learner would be able to solve, e.g. a geometry problem, then he/she is expected to have an effect on self-regulated learning (Mathebula, 1995:20).

Mac Iver (1988:503) argues that task structure may contribute to the development of a pupil's ability perception and may influence pupil-teacher agreement in ability evaluation. Task structures and evaluation practices play an important role in determining whether a student's ability perceptions will reflect this reality by becoming stratified. If the task structure and evaluation practices in the classes encourage stratification of ability perceptions, then low ability students may decide that they can not succeed at mathematics even if they choose to try. This may lead to them "giving up", that will lead to low self-concept learners condemning themselves to failure. These failures confirm their low self-concept, thus creating a failure-prone cycle (Mac Iver, 1988:504).

2.6 CONCLUSION

This chapter focussed on self-regulated learning, with special attention to the social cognitive assumptions underlying self-regulated learning as well as the determinants of self-regulated learning.

Self-regulated learning is determined by personal, behavioural and environmental influences in a reciprocal fashion. Personal influences include student knowledge, metacognitive processes, goals, self-efficacy and attribution. Behavioural influences consist of self-observation, self-judgement and self-reaction, while environmental influences are divided into the social context and the physical context.

CHAPTER THREE

THE LEARNING OF GEOMETRY

3.1 INTRODUCTION

The purpose of the first part of this chapter is to describe (general) factors that influence cognitive development, as discussed by Piaget (see § 3.2). The second part of this chapter deals with three major theoretical perspectives on the development of geometric thinking. The first perspective is the topological primacy theory of Piaget and Inhelder (§ 3.3.1). The second major stream is that of the cognitive sciences, which consists of three models namely Anderson's model of cognition (§ 3.3.2.1), Greeno's model of geometric problem solving (§ 3.3.2.3) and the parallel distributed processing networks (§ 3.3.2.5). The last and influential theoretical perspective in geometry education is Van Hiele's niveau theory of levels of geometric thinking and phases of instruction (see § 3.3.3).

3.2 FACTORS THAT INFLUENCE COGNITIVE DEVELOPMENT

Piaget's theory of cognitive development assumes that learners make sense of the world and create their knowledge through experience with objects, people and ideas. Maturation (see § 3.2.1.1), physical interaction (see § 3.2.1.2.), social experience (see § 3.2.1.3) and the need for equilibrium (see § 3.2.1.4) all influence the way thinking processes and knowledge develop (Piaget, 1968:127; 1970:719-721). Piaget furthermore postulates that thinking processes and knowledge develop through adaptation (that includes the complementary processes of assimilation and accommodation) and changes in the organization of thought (the development of schemes) (Woolfolk, 1995:59).

Piaget believes that learners pass through four stages as they develop cognitively, i.e. the stages of sensorimotor, preoperational, concrete-operational, and formal-operational thought (Woolfolk, 1995:59). Each stage is characterized by the emergence of new abilities, which allow for a major reorganization in the child's thinking (Slavin, 1991:26). In the sensorimotor stage, learners begin to use imitation, memory, and thought. Learners start to recognize that objects do not cease to exist when they are hidden; they thus develop object permanence (Woolfolk, 1995:33). There is also a gradual progression from reflex behaviour to goal-directed behaviour (Slavin, 1991:28). In the preoperational stage, learners develop the use of language, the ability to think in symbolic form (Woolfolk, 1995:33), and the ability to use symbols to represent objects in the world (Slavin, 1991:28). They develop the ability to think operations through but only in one direction, and therefore have difficulty in seeing another person's point of view (egocentricity). Learners in the stage of concrete operations can solve concrete (hands on) problems in logical fashion. Children understand the laws of conservation as well as reversibility. Thinking is decentered and problem solving is less restricted by egocentrism (Slavin, 1991:28). Those who are able to solve abstract problems in a logical fashion, by making use of systematic experimentation, can be classified as belonging to the formal operational stage. This group of learners becomes more scientific in their thinking and develops proportional thinking and hypothetical-deductive reasoning (Woolfolk, 1995:33).

According to Piaget (1968,1970) cognitive development is dependent on four factors namely maturation (§ 3.2.1.1), physical experience (§ 3.2.1.2), social interaction (§ 3.2.1.3) and equilibration (§ 3.2.1.4).

3.2.1 MATURATION

Piaget (1968:118-119) proposes that if one was led to acknowledge certain innate elements, for example the perception of space, the question remains of whether one is dealing with heredity of endogenous origin or heredity stemming from ancestral acquisition as a function of the environment and of experience. In an effort to find clarity, Piaget (1968:119-120) declares that the following two statements should be kept in mind when dealing with maturation. Firstly, maturation is undoubtedly never independent of a certain functional exercise where experience plays a role. Piaget (1968:119) experimented with his own children and found that coordination between hands and eyes differed between his three children due to exercise. His eldest child could only grasp an object outside the visual field and bring it in front of his eyes at the age of six months, while his youngest child could do the same at three months. Piaget admits that the first child had been the object of few experiments, while the youngest child was part of a series of experiments on the imitation of hand movements, from the age of two months. Exercise, therefore, appears to play a role in the acceleration or retardation of certain forms of maturation. The second statement is that the maturation of the nervous system simply opens up a series of possibilities, but without giving rise to an immediate actualization of these possibilities as long as the conditions of material experience or social interaction do not bring about this actualization. Piaget (1968:119-120) declares that the actualization of these possibilities presupposes certain conditions of physical experience (see § 3.2.1.2) and social interaction (see § 3.2.1.3).

Piaget (1970:720) believes that the effects of maturation consist essentially of opening new possibilities for development, giving access to cognitive structures which can not be involved before these possibilities are offered. But between possibility and actualization, there must intervene a set of other factors such as exercise, experience, and social interaction. This actualization presupposes certain conditions of physical experience such as the manipulation of objects, which is also essential for logic, and certain social conditions such as the regulated exchange of information. It is these diverse conditions that determine the completion of what would be impossible through maturation alone (Piaget, 1968:120).

Piaget (1970:719) declares that maturation in itself is not sufficient to explain cognitive development, because the development of cognition does not include hereditary programming factors such as factors underlying instincts. Piaget (1970:719) therefore postulates that experience of the physical environment and the action of the social environment should also be taken into consideration when dealing with cognitive development.

3.2.2 PHYSICAL EXPERIENCE

The second factor is experience acquired through contact with the external physical environment (i.e. objects in the physical environment). Experience is essential to a person's contact with the concrete world, for example experience in the handling of objects (Moll, 1989:719).

Experience, such as the handling of an object, can be grouped in three categories (Piaget, 1970:720). The first is simple exercise, which naturally involves the presence of objects on which action is exerted but does not necessary imply that any knowledge would be extracted from these objects (Piaget, 1970:720). The second category is what Piaget (1970:721) calls physical experience, that Odom and Kelly (1998:33) call physical interaction. Physical interaction consists of extracting information from the objects themselves through a simple process of abstraction. Physical interaction, for example, allows a learner to discover that a round object would fit into a round opening but not into a square opening, while ignoring the color of the object. Knowledge derived from experience is not based on the material properties of the objects in view, but on the cognitive operations exerted on them (Moll, 1989:719).

The learner thus deduces that a round shape would fit into a round opening because he/she has experienced the operation and not because of the material properties such as the color or texture.

The last fundamental category is called logicomathematical experience. It plays an important part at all levels of cognitive development where logical deduction or computations are still impossible. Logicomathematical experience also appears whenever the learner is confronted with problems in which he/she has to discover new deductive instruments. This type of experience also involves acting upon objects (Piaget, 1970:721). For example, a learner could discover that the sum of a number of blocks is independent of the spatial position or order in which they are picked up. The learner does not discover the properties of the blocks but the property of the action (addition).

3.2.3 SOCIAL INTERACTION

Piaget (1970:721) postulates that social influences and physical experience are of equal importance. Social influences and physical experience can have some effect on the learner only if he/she is capable of assimilating (see § 3.2.4) them, and he/she can do this only if he/she already possesses the cognitive instruments or structures.

Social experience consists of experiences with other people. Jacob (1982:266) considers the teacher having a considerable influence on the learner (and the social environment). Pulaski (1971:11) emphasizes verbal instruction by teachers and parents as the primary instrument in the social environment that the learner encounters in his/her social environment.

Moll (1989:719) argues that despite cognitive growth being accelerated or retarded according to a child's socio-cultural environment, the fact that it matures in the same sequence in any social context shows that the social environment can not account for everything. The growth of intelligence is governed, not only by the influence and interaction of factors that lie outside the learner in the external world (like maturation, social transmission), but also by a force from within the child, called equilibration (Jacob, 1982:266 & 269).

3.2.4 EQUILIBRATION

Piaget (1970:722) justifies the forming of this fourth factor by declaring that the first three factors (maturation, physical experience and social interaction) can not explain the sequential development if they are not in some relation to mutual equilibrium. There has, therefore, to exist a fourth organizing factor to coordinate them in a consistent, non-contradictory totality. Lerner (1976:162) proposes that a learner's adaptation to his/her environment involves a balance, an equilibrium, between the activity of the learner on its environment and the activity of the environment on the organism.

Piaget (1970:724) admits, "It is not therefore an exaggeration to say that equilibrium is the fundamental factor of development, and that it is even necessary for the co-ordination of the other three factors." Moll (1989:718) is in agreement with Piaget as he calls equilibrium the fundamental drive that necessitates psychological (and other) development in the learner; in essence he views equilibrium as the "motor" of cognitive growth (Moll, 1989:719).

Ginsburg and Opper (1979:214) define equilibrium as "a state of balance or harmony between at least two elements which have previously been in a state of disequilibrium." Evans (1973:45) views equilibrium as referring to Piaget's self-regulatory model in which new environmental events are assimilated into existing cognitive structures, and existing structures are transformed to fit or accommodate new environmental situations. Moll (1989:718) also conceives equilibrium as a self-regulatory tendency that is not voluntary, but rather a condition of the very existence of any biological system, and which strives constantly for equilibrium at a higher, more complex developmental level.

Assimilation is a form of integration, as new knowledge that is acquired by handling an object is integrated into already existing cognitive structures (Piaget, 1970:706-707; 1972:12; Hiebert & Carpenter, 1992:66, 69). Assimilation is therefore the process by which one alters new incoming information or data to fit one's created reality (Fortosis & Garland, 1990:632; Cooney, Cross & Trunk, 1993:246), or to create new schemata (mental patterns that guide behaviour) (Jacob, 1982:267; Slavin, 1991:26).

Piaget, according to Lerner (1976:161), suggests that the basis of knowledge lies in action. The learner knows objects through the actions performed on them. Cohen (1993:792) suggests that this action should be constructive. Lerner (1976:161) proposes that assimilation involve changing the object, external to the learner, to fit the already existing internal structure of the learner. Hiebert and Carpenter (1992:66), using two metaphors, have explained this internal structure. The first is structured like vertical hierarchies. Some representations subsume other representations, thus representations fit as details underneath or within more general representations (also called umbrella representations). In the second metaphor this internal structure is compared to a spider's web (Hiebert & Carpenter, 1992:67). The nodes can symbolize pieces of represented information and the treads between them as the connections or relationships.

Accommodation is a complementary process of assimilation (Lerner, 1976:161). Accommodation is the process by which one adapts one's created reality or cognitive structures to fit the incoming new information or data (Fortosis & Garland, 1990:632; Cooney *et al.*, 1993:247). Rather than the learner adapting the external object to match the internal cognitive structure of the child (assimilation), accommodation involves the adaptation of already existing cognitive structures in the learner to match new, external stimulus objects (Lerner, 1976:161).

For every assimilation there must be a corresponding accommodation. Just as the learner adapts the object to fit its internal structure, the internal structure of the learner must be adapted to fit the object (Lerner, 1976:162).

Piaget (1968:103) postulates that all behaviours are focussed on reaching a state of equilibrium between accommodation and assimilation. Equilibration is thus the action through which accommodation and assimilation are brought into equivalence (Jacob, 1982:266; Monteith, 1979:44; Flavel, 1963:239).

Piaget (1968:150-151) identifies three characteristics to define equilibrium. The equilibrium is firstly noted for its stability. This stability does not signify immobility, and therefore the equilibrium can be both mobile and stable. Fortosis and Garland (1990:632) note that equilibrium implies an active balance or harmony. Secondly, every (cognitive) system is subjected to external intrusion which tends to modify it, and lastly, equilibrium is not passive, but something essentially active. Therefore, equilibrium is not only a state, but also an actual process (Piaget, 1968:101). Piaget (1968:151) states that the "equilibrium is synonymous with activity". A structure is in equilibrium to the extent that a learner is sufficiently active to be able to counter all intrusion with external compensations.

3.3 THEORETICAL PERSPECTIVES ON THE DEVELOPMENT OF GEOMETRIC THINKING

Piaget gives attention to general factors that influence cognitive development (see § 3.2). Piaget acknowledges logicomathematical experience as being a fundamental category of experience (see § 3.2.1.2). Development of mathematical thinking received much attention by various academics (Piaget and Inhelder, 1963, 1967,1971; Greeno, 1980; Van Hiele, 1982, 1986; Anderson, 1983; Burger & Shaughnessy, 1986; Usiskin, 1987; Teppo, 1991; Pegg & Davey, 1991; Clements & Battista, 1992; Flores, 1993; Battista, 1994; Mason, 1997; Van Niekerk, 1997; Odom & Kelly, 1998).

Geometric thinking has been refined in such detail that today three major perspectives within the development of geometrical thinking exist. The first is Piaget and Inhelder's topological primacy theory (see § 3.3.1). The second theory is the cognitive science theories that consist of Anderson's model of cognition (see § 3.3.2.1); Greeno's model of geometric problem solving (see § 3.3.2.3) and the parallel distributed processing networks (see § 3.3.2.5). The last perspective is the Van Hiele levels of geometric thinking and phases of instruction (see § 3.3.3).

3.3.1 PIAGET AND INHELDER'S TOPOLOGICAL PRIMACY THEORY

Piaget and Inhelder's theory (1971) postulates that depictions of space are constructed through progressive organization of a learner's motor and internalized actions, resulting in operational systems. Holloway (1967:vii) theorizes that Piaget and Inhelder help us to see how long the process of the development of mathematical ideas is and how much it depends on the opportunity to manipulate material. Piaget and Inhelder (1971:10) postulate that representations of space are the building-up from prior active manipulation of that environment and not a mental "reading off" of the spatial environment. A definite order exists in the organization of geometric ideas. Initially topological relations (connectedness or enclosure) are constructed and later projective (rectilinaerity) and Euclidean (angularity or parallelism) relations are constructed (Clements & Battista, 1992:422).

3.3.1.1 Topological Primacy

A learner's building of geometrical representation of space develops very slowly, and for his first perceptions and notions of spatial relationships a branch of mathematics known as "topology" must be investigated. Mathematically speaking this represents a late and advanced level of theory, but it rests on very early modes of perception from which the young learner can most readily form his first elementary spatial representations (Holloway, 1967:3). Piaget and Inhelder (1971:17-52) use haptic, as well as drawing evidence when examining topological primacy.

- **Haptic evidence**

Piaget and Inhelder (1971:17-43) devised experiments where learners were asked to explore hidden objects tactilely (Clements & Battista, 1992:422). The first experiments were concerned with the recognition of shapes (figure 3.1) by means of touch in the absence of visual stimulation (technically known as "haptic perception") (Holloway, 1967:4). Haptic perception thus refers to the ability to acquire information about (the features of) objects by using hands and fingers to touch and thus to discriminate and recognize objects from handling as opposed to looking at them (Catherwood, 1993:702; Bushnell & Boudreau, 1993:1008).

These learners were then asked to name the object or shape and then to identify its duplicate among a visible collection or draw the objects they have felt (Clements & Battista, 1992:422; Holloway, 1967:4).

Shapes on the left (see figure 3.1) were considered Euclidean in haptic perception experiments, while those on the right were considered to be topological forms (Clements & Battista, 1992:424; Holloway, 1967: 5).

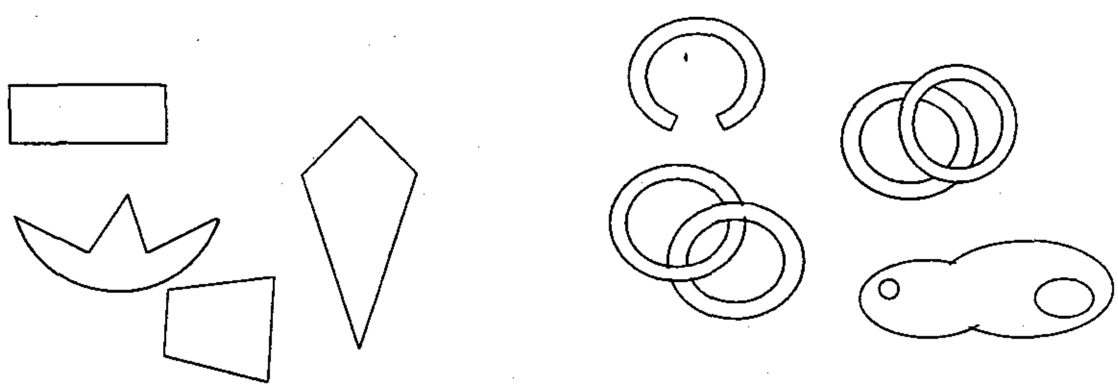


Figure 3.1 Example of shapes used in Piaget and Inhelder's experiment.

Table 3.1 Response stages in haptic evidence stage

Response stage	*Age*	*Characteristic*
1	2years 6months to 4 years	Recognize and draw only closed rounded shapes and those shapes based on simple relations such as openness, closure, and separation. In essence the simplest possible co-ordination of actions is used e.g. following contour step by step, surrounding, separation etc.
2a	4years 1month to approx. 5years	Recognition of simple Euclidean shapes, ideas of equality, straightness and intersection formed from actions as perceived.
2b	Approx. 5years to 7years	
3	Begins at approx. 7 or 8	Connection between shapes and coordinated action becomes apparent in the return to a fixed point of reference.

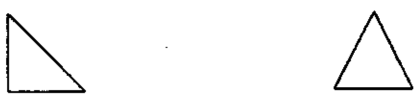
Stage 1 responses (see table 3.1) demonstrated that learners recognized familiar objects but they revealed an inability to recognize unfamiliar shapes due to a lack of sufficient explorations (Holloway, 1967:4). The learners showed no willingness to explore the objects further and this revealed that learners at this stage were passive in their explorations (Clements & Battista, 1992:422-3). For example, a learner would touch one part of a shape and the action would result in a tangible perception; touching another part of the shape involved another action and perception (Clements & Battista, 1992:422-3). For example, the following figure touched at "side 1" could be perceived as being a square, while touching the same figure at "side 2" could be perceived as a circle.

Figure 3.2 An example of a figure that can result in more than one shape due to different perceptions when touched

As the ability to abstract shape developed, topological shapes were identified, but Euclidean shapes could not be recognized. This illustrated the process of abstraction of a shape as being achieved by virtue of actions (Holloway, 1967:5). Pre-school learners first discriminated objects on the basis of topological features such as being closed e.g. an open ring vs a circle. Later these learners distinguished rectangular from curvilinear forms and finally rectangular closed shapes (squares and diamonds) (Clements & Battista, 1992:422). For example, a learner would be able to make the correct choice for a circle but would be unable to draw it (see figure 3.3B). The same learner would easily recognize the open ring [] and the two inter-twined rings [] (see figure 3.3A), but would not be able to locate the triangle among three models comprising of a circle, a square and a triangle (see figure 3.3C). This would happen even if the learner would touch the apex with both fingers while holding the converging sides between his/her hands.

A. Examples of figures recognized during stage 1

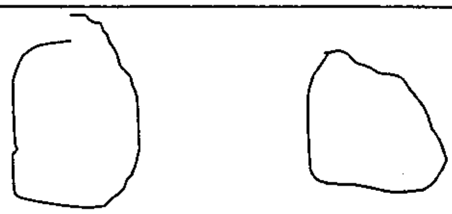
B. An example of a figure recognized during stage 1 but incorrectly drawn



C. Examples of figures not recognized during stage 1

Figure 3.3 Examples of figures used in stage 1

During substage 2a (see table 3.1) progressive recognition of Euclidean shapes were revealed. Exploration became more active but stayed haphazard, which resulted in a number of clues which significance the learner could keep in mind. When learners drew the Euclidean shapes, their drawings did not look like scribbles any more but started to look alike (see figure 3.4A). The drawings showed that the elements of closure were becoming dominant (Holloway, 1967:6). Figures 3.4B – 3.4E illustrate figures recognized (see figure 3.4B); figures recognized and correctly drawn (see figure 3.4C); a figure recognized but incorrectly drawn (see figure 3.4D) and figures not recognized (see figure 3.4E).

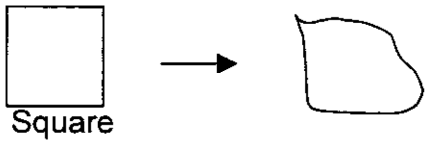


A. Examples of possible drawings of a circle during substage 2a

B. Examples of possible figures recognized during substage 2a

C. Examples of figures recognized and correctly drawn during substage 2a

Figure 3.4 Examples of drawings and figures used during stage 2



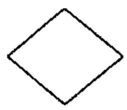
One corner is more or less a rightangle while the other corners are curved

D. Example of a figure recognized but incorrectly drawn during substage 2a

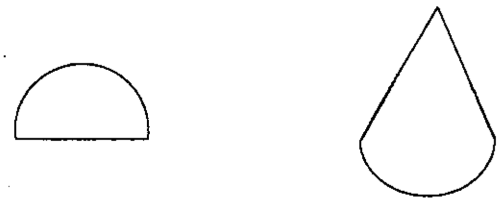
E. Figures not recognized during stage 2a

Figure 3.4(continues) Examples of drawings and figures used during stage 2

Substage 2b (see table 3.1) was characterized by more developed exploration, though not yet complete or systematic. The result is progressive differentiation of angular shapes, but no greater improvement in recognition. Drawing at this stage represented not so much the model visually or tactilely perceived as the tactile activity itself. For example, one child in the experiment was successful with simple shapes and was able to recognize the rhombus (see figure 3.4F) after a series of explorations, but still confused a curved triangle with the semi-circle (see figure 3.4G) (Holloway, 1967:6-7). The learner explored everything but resulting from a of lack of operational guidance, kept moving ahead all the time, never returning systematically to obtain a stable point of reference (Holloway, 1967:7).



F. An example of a figure recognized after a series of explorations in substage 2b (not recognized during substage 2a)



G. Examples of figures still confused in substage 2b

Stage 3 (see table 3.1) was marked by operational co-ordination. An operation can be defined as an ongoing process of arranging information and experience into mental systems or categories (Woolfolk, 1995:30). An operation can further be defined as an action (carried out through logical mental processes) (Slavin, 1991:31), which can return to its starting point, and which can be integrated with other actions also possessing this feature of reversibility (Holloway, 1967:7). An operation is furthermore defined as an ongoing process of arranging information and experience into mental systems or categories (Woolfolk, 1995:30). Achievement of rudimentary reversible co-ordination was also noticed, for example learners now recognized and drew all the simple shapes right away, returning to a central (fixed) point of reference during their exploration (see figure 3.5) (Holloway, 1967:7). The exploration was from then on directed by an operational method, which consisted of grouping the elements perceived in terms of a general plan, and starting from a fixed point of reference to which the learner could always return (Holloway, 1967:7). An accurate representation of a shape is built when a learner can regulate such actions by establishing relations between these actions and perceptions. Abstraction of a shape is therefore not a perceptual abstraction of a physical property but is the result of the coordination of a learner's actions (Piaget & Inhelder, 1971:43; Clements & Battista, 1992:422-3; Holloway, 1967:8).

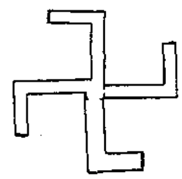


Figure 3.5 An example of a drawing during stage 3 using a fixed point of reference

In conclusion, during stage 1 the only shapes recognized and drawn were closed, rounded shapes and those based on simple relations such as openness, closure and separation that expressed in essence the simplest possible co-ordination of actions. Stage 2 showed the recognition of simple Euclidean shapes, ideas of equality, straightness and intersection being formed from the actions by which we perceive them. At stage 3 the connection between shapes and co-ordination became apparent in the return to a fixed point of reference (Holloway, 1967:8).

- **Drawing as evidence**

Drawing in this section deals with drawings produced either spontaneously with the aid of visual memory or by copying visual models. This type of drawing is distinguished from drawing in the previous section since projective relations played no part in the earlier experiments and the drawings were based on tactile rather than visual perception (Holloway, 1967:9). Piaget and Inhelder advocate that learners' drawn copies of geometric shapes represent topological features (Clements & Battista, 1992:423).

Table 3.2 Stages of drawing of geometrical figures

Drawing stage	*Age*	*Characteristic*
0	Up to age 2years 6 or 11months	Scribbles without variation irrespective of model
1a	Up to around 3years 6months	Different types of scribbles depending on open or closeness of figure
1b	Between 3years 6months to 4years	"Real drawings" with only topological relationships indicated
2	4years to 6years 6months	Euclidean shapes differentiated (first rectangles and squares, then triangles and later a rhombus)
3	6years 6months and onwards	Draw shapes correctly straight away by using mental image drawn up in advance

During stage 0 of drawing (where no experiments in haptic perception were possible) learners only scribbled without any purpose or aim (Piaget & Inhelder, 1971:54). These scribbles showed no variation whatever the model (Clements & Battista, 1992:423; Holloway, 1967:12).

At substage 1a (see table 3.2) different types of scribbles were produced according to whether the learner was looking at open or closed shapes (Piaget & Inhelder, 1971:55; Holloway, 1967:12). For example, a circle was drawn as irregular closed curve (see figure 3.6A), squares and triangles were not distinguished from circles (Clements & Battista, 1992:423). During stage 1b (see table 3.2) "real drawings" emerged, but only topological relationships were indicated with any real degree of accuracy, while Euclidean figures remained undifferentiated (Piaget & Inhelder, 1971:55). For example, a circular shape was represented by means of a single line while squares (see figure 3.6B), triangles, ellipse, rectangles etc. all resulted in the same figure (Holloway, 1967:12).

A. An example of a possible drawing of a square during sub stage 1a

B. An example of a possible drawing of a square during sub stage 1b

Figure 3.6 Examples of possible drawings of a square during stage 1

Topological relationships appeared first because they were inherent in the simplest possible ordering of the actions from which shape was abstracted (Piaget & Inhelder, 1971:55). Topological shapes corresponded to the most elementary forms of such actions, as against the more complex regulatory processes required for the construction of Euclidean figures (Holloway, 1967:14).

Stage 2 (see table 3.2) was marked by progressive differentiation (Piaget & Inhelder, 1971:56). Euclidean shapes were gradually differentiated, starting with rectangles and squares (see figure 3.7), then triangles, but the rhombus was not differentiated till much later (Holloway, 1967:14). The successful reproduction of a square or rectangle can be seen as the criterion for this stage (Clements & Battista, 1992:423).

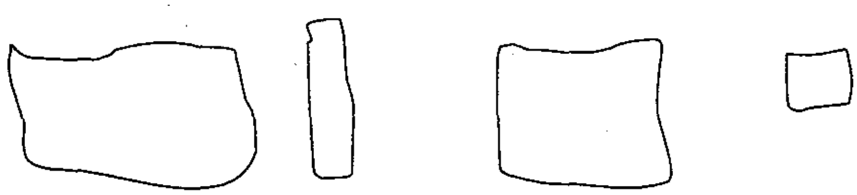


Figure 3.7 Examples of possible drawings of rectangles and squares during stage 2

During stage 3 (see table 3.2) all previous problems were overcome (Clements & Battista, 1992:423), for example, learners drew the shapes correctly straight away (Piaget & Inhelder, 1971:57), and their construction was anticipated by a mental image drawn up in advance (Holloway, 1967:14). Two years of work was needed to pass from copying a square to copying a rhombus because construction of Euclidean shapes requires more than a correct visual impression. The task required a complex interplay of actions (Clements & Battista, 1992:423).

Piaget and Inhelder (1971:68) promulgate that an inaccurate drawing reflects the inadequacy of mental tools for spatial development because the act of making a drawing is an act of representation and not perception (Clements & Battista, 1992:423).

3.3.1.2 Projective Space

The difference between topological and projective or Euclidean relations is the way in which the figures or objects are related to one other (Piaget & Inhelder, 1971:153). Topological relations are internal to particular figures, and have none of the features possessed by a space capable of embracing all possible figures (Clements & Battista, 1992:423; Holloway, 1967:27). Projective and Euclidean relations are more complex in organization (Piaget & Inhelder, 1971:153). Projective relations involve relations between figure and subject, while Euclidean relations involve relations between figures themselves (Clements & Battista, 1992:423).

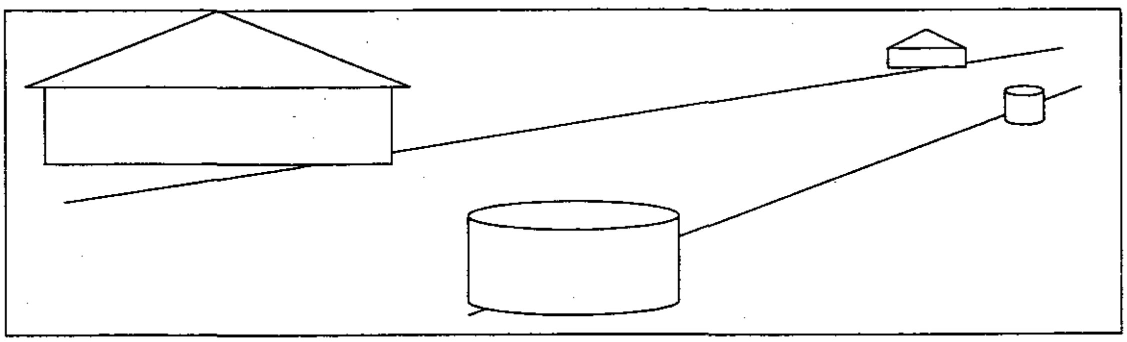


Figure 3.8 An illustration of projective space with projective relations among figures

Projective relations begin psychologically at the point where the figures are no longer viewed in isolation but begin to be considered in relation to a point of view (Piaget & Inhelder, 1971:153-154; Clements & Battista, 1992:423; Holloway, 1967:28) (see figure 3.8). This 'point of view' can either be the viewpoint of the subject (called *ego* by Van Niekerk, 1997:105) or else the viewpoint of other objects on which the first is projected (Holloway, 1967:28). Projective relations are concerned with coordination of objects separated in space rather than the analysis of isolated objects (Holloway, 1967:28). For example, the concept of a straight line is the result of a learner taking aim, or sighting. Very young learners can perceptually recognize a straight line but have an inability to present a straight line mentally. Young learners perceive a straight line, but can not place objects along a straight path that is parallel to the edges of a table even when the learner constantly refers to the edge of the table (Clements & Battista, 1992:423; Holloway, 1967:30). Learners would rather follow the edges of the table or curve the line towards such a path (figure 3.9).

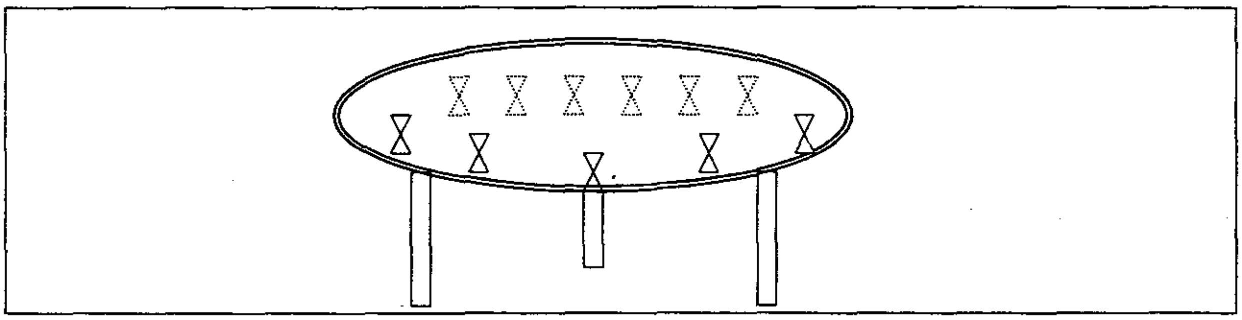


Figure 3.9 An example of a young learner trying to place objects in a straight line

This is not a perceptual problem as they realize that the line is not straight, but they can not construct an adequate representation to make it so. Learners only posses intuitive, spatial representations, an internalized imitation of previous perceptions that can be altered by distracting perceptual configurations (e.g., edges of the tables). Internal representations are based on operations and can limit the influence of perceptual configurations (Clements & Battista, 1992:423).

Table 3.3 Stages of projective space

Stage of projective space	*Age*	*Characteristic*
1	Up to 4 years	Easily recognize and follow straight line but incapable of forming a straight line
2a	4 to 7 years	Form straight line parallel with edge of table but are not able to throw off influence of edge of table
2b		Progressive discrimination of viewpoints and ability to form lines independent of the edge of table
3	Around 7 years and on	Spontaneous "sightings" due to sufficient development of discriminations of viewpoints

Learners in stage 1 (see table 3.3) of projective space can easily recognize a straight line because they can distinguish circles from squares on sight. Learners can also follow a straight line by placing objects on an existing line or the edges of the table, but learners have no clear idea of what constitutes a straight line (Piaget & Inhelder, 1971:160). Learners are incapable of forming a straight line close to and parallel with the edge of the table (Piaget & Inhelder, 1971:163; Holloway, 1967:30).

By stage 2 (see table 3.3.) learners are able to arrange objects parallel with the edge of the table. Learners are even able to arrange objects on neutral ground, but are unable to ignore the influence of the edge of the table when they are asked to construct a line that is no longer parallel to the edge of the table. Learners are satisfied with a line that is approximately straight. They do not think it is worth changing position in order to check because they are still ignorant of the method of "taking aim" (Holloway, 1967:30). The learners can envisage a straight line parallel to some feature of the background but can not imagine the image of a straight line when it has to be independent of lines present in the background. The problem, therefore, is the construction of the intuitive image and not the perception itself (Holloway, 1967:31).

At substage 2b (see table 3.3) a progressive discrimination of viewpoints as well as a gradual liberation from the surrounding shapes develops and eventually the ability to form lines independent of the edge of the table develops (Piaget & Inhelder, 1971:165). As soon as the child goes beyond the perceptual straight line, projective concepts (sighting along a particular line with appreciation of conservation of shape independent of viewpoint) and Euclidean concepts (the shortest path of movement of a constant direction of travel) appear simultaneously and reinforce each other (Holloway, 1967:32).

By stage 3 (see table 3.3) learners' ability to discriminate between points of view is sufficiently developed to carry out spontaneous operations of "sighting" or "taking aim" (Piaget & Inhelder, 1971:169; Holloway, 1967:32). Learners of seven years could construct straight paths by spontaneously aiming or sighting along a trajectory, putting themselves in line with two posts to be linked by a straight line (Clements & Battista, 1992:423). The learners clearly showed their understanding of what a projective straight line is – a topological line with well known features of serial order, with elements ordered relative to a viewpoint in such a way that they succeed one another in terms of the relations 'before; behind' (in essence prepositions), the first concealing the rest (Piaget & Inhelder, 1971:170-171; Holloway, 1967:33). Van Niekerk (1997:110) confirmed that terms which indicate the relational propositions like the two-place predicated "over, under, in front, and behind" are prepositions. These terms should be interpreted in terms of specified sets of vertical and horizontal axes. The terms "before and behind" are vertical predicates that create a conflict between object space and ego space. For vertical predicates "over and under", the conflict is between using object space and environmental space.

The three-mountain experiment (see figure 3.10) confirms these findings, as learners were asked to construct a scene from the perspective of a doll (Clements & Battista, 1992:423). The purpose of the experiment was to trace the learners' developing awareness of their own viewpoint and its relation to others. It is only when the learners were able to co-ordinate a number of viewpoints that the relations involved in elementary perspective were mastered (Holloway, 1967:41). For each new position of the doll, the learners needed to re-create the appropriate viewpoint (Clements & Battista, 1992:423). The learners had the problem of trying to imagine, and reconstruct by a process of interference, the changes in perspective that would accompany the doll's movements, or the different positions which the doll had to occupy, to suit the various perspectives (Holloway, 1967:43). Interestingly it always turned out to be from one perspective – the perspective of the child (Piaget & Inhelder, 1971:212) called ego by Van Niekerk (1997:105). The learners did not get beyond looking for some dominant relation to rectify the results according to the changed viewpoint, whilst ignoring the rest. Thus relations such as left and right, before and in front of, retained a kind of absolute quality, because each remained firmly attached to just the child's own point of view (Piaget & Inhelder, 1971:218; Holloway, 1967:43-44). Piaget and Inhelder (1971:212) deduced that learners constructed systems of reference not from familiarity (due to experience) but from operational thinking and coordination of all possible viewpoints. The conclusion can be made that such global coordination of viewpoints is the basic prerequisite in constructing simple projective relations (Clements & Battista, 1992:423).

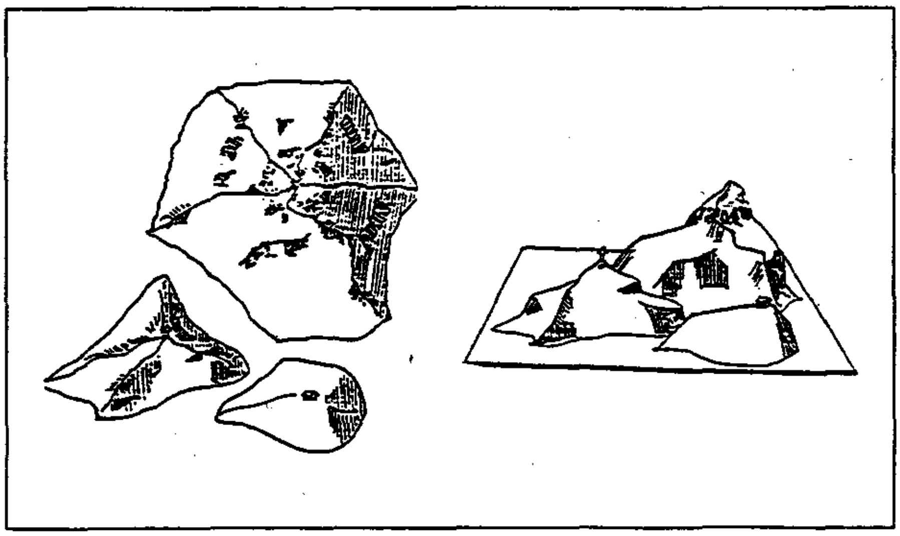
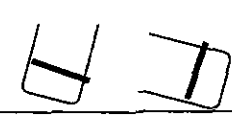
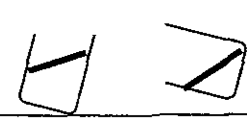
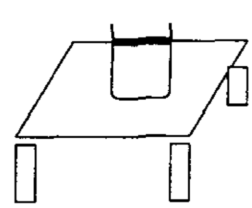


Figure 3.10 The three mountains (Piaget & Inhelder, 1971:211)

3.3.1.3 Euclidean Space

Piaget and Inhelder (1971:303-319) also give attention to the development of notions presumed to be intermediate between projective and Euclidean spaces, for example construction of similar figures. Experiments were conducted that showed the gradual acquisition and conservation of parallelism, the discovery of proportion and conservation of angles, and the development of a simple co-ordinate system of reference required to construct horizontal and vertical axes (Clements & Battista, 1992:4223; Holloway, 1967:55). During the last part of the experiments on Euclidean space, learners "saw" objects as located in a two-dimensional frame of reference. The frame for spatial awareness is a culminating point of development of Euclidean space. Piaget and Inhelder (1971:379-392) use the notion of horizontality to prove their hypothesis. Learners were shown a jar that was half filled with colored water. The learners were asked to predict the spatial orientation of the water level when the jar was tilted (Piaget & Inhelder, 1971:381; Clements & Battista, 1992:422; Holloway, 1967:63).

Table 3.4 Stages of Euclidean space

Stage of euclidean space	*Characteristic*	*Drawings*
1	No understanding of planes	
2a	Horizontal and vertical axes undiscovered	
2b	Water level tilted but not parallel to base of container	
3	Draw on larger frame of reference (table top)	

The experiment had little meaning during stage 1 (see table 3.4), as learners were initially incapable of representing planes that tilted (Piaget & Inhelder, 1971:384). The water plane was represented as a ball (Holloway, 1967:64), or scribbles (Clements & Battista, 1992:422).

During the next stage (stage 2a) of Euclidean space (see table 3.4), horizontal and vertical axes stil remained undiscovered as learners presented the plane as always perpendicular to the sides of the jar, regardless of the tilt (Clements & Battista, 1992:424). Piaget and Inhelder (1971:388) comment that it was striking "how poorly commonly perceived events are recorded in the absence of a schema within which they may be organized". Learners that sensed that the water level moved closer to the edge of the jar, raised the water level or left the lower part of the jar empty (Holloway, 1967:64; Clements & Battista, 1992:424).

During substage 2b (see table 3.4), learners drew the water level as being tilted, but no longer parallel with the base of the jar (Piaget & Inhelder, 1971:392). Sometimes it appears horizontally by accident, but it is not fixed to any fixed frame of reference external to the jar (Holloway, 1967:64).

At the final stage (Stage 3) (see table 3.4) learners became able to apparently draw upon the larger spatial frame of reference (e.g. the tabletop that serves as an external reference system) in order to get the horizon (Piaget & Inhelder, 1971:401; Clements & Battista, 1992:424).

Ultimately, the frame of reference forming Euclidean space was comparable to a container made up of a network of sites or positions (Clements & Battista, 1992:424). The container was relatively independent of the mobile object (water) contained within it (Holloway, 1967:63). The water within this container was mobile but the position was stationary (Clements & Battista, 1992:424), usually perpendicular to the bottom of the container. From the simultaneous organization of all possible positions in the three dimensions emerges the Euclidean coordinate system. This system is rooted in the foregoing construction of the concepts of a straight line, parallels and angle, followed by the coordination of their orientations and inclinations. This leads to a gradual replacement of relations of order and distance between objects with similar relations between the positions themselves. It was as if a space were emptied of objects so as to organize the space itself. Thus the intuition of space was not a "reading" or inherited apprehension of the properties of objects, but a system of relationships born in actions performed on these objects (Clements & Battista, 1992:424).

In summary it can be said that Piaget and Inhelder's theory consists of three topics namely topological primacy, projective space and Euclidean space. Holloway (1967:v) states that all these aspects of space relations which adults find hard to see as problematic, but which for learners are the main stumbling blocks to spatial understanding, are brought to the foreground.

3.3.1.4 Criticism on Piaget and Inhelder's Work

The first criticism in the design is the use of terms such as topological, proximity and Euclidean as well as the application of these and related concepts, which are not mathematically correct (Darke, 1982:126; Martin, 1976:16). The second criticism is the classification of figures as topological or Euclidean. Every figure possesses both Euclidean and topological features / characteristics. This implies that one can not discuss an exclusively Euclidean or topological figure, but Piaget and Inhelder (1971) depend on a mutually exclusive classification of figures into these two categories (Clements & Battista, 1992:424). Many of the figures used (see figure 3.1) were topological equivalent which makes it difficult to judge if learners made choices on the basis of topological characteristics (Martin, 1976:19). A third criticism deals with treatment of irregularities. An example is that the irregularities in choices learners made (mentioned above) were dismissed as a lack of drawing skills, whereas the incapacity to draw a square's straight sides was not dismissed as a lack of drawing skills (Clements & Battista, 1992:424). Bjorklund (1997:147) summarizes the criticism by stating that Piaget (and Inhelder's) theory has been soundly criticized, to the point that much of it is regarded as "flat-out wrong", for example, Somerville and Bryant (1985:612) find that young children master understanding of Euclidean space quite early in life and a great deal earlier than Piaget and Inhelder's analysis of Euclidean understanding suggests.

3.3.2 COGNITIVE SCIENCES

A second major perspective in students' learning of geometry is that of the cognitive sciences. Gardner (1985:6) explains the domain of cognitive science as "a contemporary, empirically based effort to answer long-standing epistemological questions, particularly those concerned with the nature of knowledge, its components, its sources, its development, and its deployment", while Leong (1993:63) adds that it is human knowledge that is emphasized. This perspective endeavours to integrate research and practical work in the fields of psychology, philosophy, linguistics, and artificial intelligence (Clements & Battista, 1992:434). The aim of cognitive science is the understanding of the way in which individuals process information (Leong, 1993:63). Three models are distinguished within this perspective namely Anderson's model of cognition (see § 3.3.2.1), Greeno's model of geometric problem solving (see § 3.3.2.3) and the parallel distributed processing (PDP) networks (see § 3.3.2.5).

3.3.2.1 Anderson's Model of Cognition (ACT)

Anderson (1983:ix) names his theory ACT which stands for Adaptive Control of Thought. Anderson (1983:19) distinguishes between two kinds of knowledge, namely declarative and procedural. Declarative knowledge is "knowing that", while procedural knowledge is "knowing how". Declarative knowledge stores theorems in schemas along with knowledge about their function, form and preconditions while procedural knowledge is stored in the form of production systems or sets of conditional pairs (Clements & Battista, 1992:434).

This model proposes that all knowledge initially comes in declarative form and is interpreted by general procedures (Anderson, 1983:23), for example to bake a cake one would use a general recipe-following procedure. Procedural learning occurs only in the execution of a skill, thus learning by doing. In performing a task, proceduralization gradually replaces the original interpretive application with productions that influenced the behaviour directly. Complementing proceduralization is a composition process combining sequences of productions. Proceduralization together with this composition is called knowledge compilation (Anderson, 1983:34). Knowledge compilation is the creation of task-specific productions through practice. Learning in this theory comprises of the acquisition of declarative knowledge, followed by the application of declarative knowledge to new situations through search and comparison. Thirdly compilations of domain-specific productions was made that are followed by a strengthening of declarative and procedural knowledge (Clements & Battista, 1992:434).

3.3.2.2 Criticism on Anderson's Work

Social cognitive theorists like Schunk (1996:110,360, 2000:82,143), Winne and Butler (1994:5740), and Zimmerman (1989a:21; 1989b:332) describe three types of knowledge namely declarative, procedural and conditional knowledge. These three kinds of knowledge can be classified as student knowledge that is part of personal influences in learning (See § 2.5.1.1 for a detailed discussion).

Winne and Butler (1994:5740) are in agreement with Anderson (1983:19) that declarative knowledge is descriptive knowledge ("knowledge that") but Zimmerman (1989b:332) proposes that declarative knowledge is organized according to its inherent verbal, sequential, or hierarchical structure and that it stays static until changed by learning and is not merely stored as schemas, as Anderson (1983:19) proposes.

Procedural knowledge is called "if then" rules instead of "knowing that", by Winne and Butler (1994:5740), because only if certain conditions arise will this knowledge be invoked. Although Anderson (1983:30) gives attention to the form in which procedural knowledge is stored, no mention is made of what kind of knowledge / situations this procedural knowledge is used in. Schunk (1996:447, 2000:152) argues that procedural knowledge is knowledge of how to do something, for example employ algorithms and rules, identify concepts, and solve problems.

The third kind of knowledge social cognitive theorists distinguish, that is absent from Anderson's theory, is conditional knowledge. Conditional knowledge defines when, where, and why declarative knowledge or a rule is relevant. Conditional knowledge is an awareness of the conditions that influence learning, and when to apply declarative and procedural knowledge and even why it is important to do so (Paris & Byrnes as referred to by Zimmerman, 1989b:332). Zimmerman (1989b:332) furthermore postulates that procedural and conditional knowledge should be treated as a single construct to form self-regulated knowledge.

3.3.2.3 Greeno's model of geometric problem solving

Greeno (1980) invented the second model in the cognitive science perspective of geometry learning in 1980. Greeno (1980:2) designed a computer simulation model that presented the knowledge required for problem solving in geometry. He named this computer simulation model "Perdix". The major source of empirical data used in the development of Perdix was think-aloud protocols obtained from six ninth-grade students. Perdix was designed to solve the same problems that these students were able to solve, and in the same general ways these students solved them by including procedures and structures of knowledge. Perdix was a production system, which meant that each component of its knowledge consisted of a condition and an action that was performed if the condition was tested and found to be true. The productions that make up Perdix's knowledge about geometry were in three groups, and these groups of productions can be considered as three domains of knowledge required for students to solve the problems they are given in their study of geometry (Greeno, 1980:2).

- Firstly, propositions were used in making inferences. The propositions needed in geometric problem solving are the familiar statements about geometric relations for example "If two angles are congruent, they have equal measure" (Greeno, 1980:2). These propositions constituted the main steps in geometry problem solving (Clements & Battista, 1992:435). Each step in solving the problem consisted of an inference in which some new relation was deduced from information that was given or that had previously been inferred. The problem was solved when this chain of inferences reached the relation that was the goal of the problem. Each of the inferential steps was based on one of the "if-then" propositions that the student knew (Greeno, 1980:3).

- Secondly, perceptual concepts were used in recognizing patterns. The perceptual concepts needed for geometric problem solving included patterns that were mentioned in the antecedents of many propositions, for example the proposition "corresponding angles formed by parallel lines and a transversal are congruent" mentions a pattern – corresponding angles (Greeno, 1980:3).

- Thirdly, strategic principles were used in the setting of various kinds of goals and planning. For example, when solutions required showing that two angles were congruent, three alternative approaches were available. One approach was to prove that triangles containing the angles were congruent. The second approach was to use relations between angles that were based on parallel lines, such as corresponding angles or alternative interior angles. The third approach was to use relationships between angles whose vertices were at the same point, such as vertical angles, or angles that were formed by the bisection of another angle (Greeno, 1980:3).

3.3.2.4 Criticism on Greeno's Work

The third domain (strategic knowledge) is not explicitly represented in the instructional materials of geometry (Clements & Battista, 1992:435). References to strategic knowledge in the material are indirect at best, and most teachers do not explicitly identify principles of strategy in their teaching when they teach learners (Greeno, 1980:5). Learners must acquire this knowledge, and the principles of strategic knowledge that must be applied in solving problems, through induction from sequences of steps observed in example solutions. The induced strategic principles are implicit in nature as it is in the form of tacit procedural knowledge, involving processes the learner can perform but can not describe or analyze. Greeno (1980:5) is not sure if this domain-specific strategic knowledge should explicitly be included in the materials of the geometry course and it is not clear if they should be taught directly. Greeno (1980:5) suggests that they should because it is unlikely that unguided discovery is more effective than a more explicit form of instruction, but he still admits "... it is possible that the unguided discovery method now used is more effective than the more explicit form of instruction..." (Greeno, 1980:5). If direct teaching is interpreted as the teacher's imposition of prescribed steps on learners, it contrasts with Van Hiele's characterization of learners finding their own way in the network of relations (Clements & Battista, 1992:435). See § 3.3.3 for a more detailed description of Van Hiele's theory. All of the above confirm Searle's (1990: 26) argument that computational theories in terms of manipulation of formal symbols according to rules in the program do not help in the understanding of mental processes and can not generate mental processes. Another shortcoming in Greeno's model is the lack of inclusion of explicit instruction about problem solving strategies in the instruction given to mathematics teachers. He admits, "I have not studied geometry teachers' understanding of problem solving in a systematic way"

(Greeno, 1980:6), but he assumes that all teachers have a similar impression of the nature of skill in solving problems. Clements and Battista (1992:436) note that the small number of subjects involved makes generalization a concern. Greeno (1980:6) summarizes his own model with these prophetic words: "I close this discussion on geometry by noting that the cognitive analysis of problem solving has not provided strong recommendations about how to teach the subject matter."

3.3.2.5 Parallel Distributed Processing Networks

Clements and Battista (1992:435) postulate that the PDP (parallel distributed processing) network model explains the holistic template representations of the lower levels in the Van Hiele hierarchy (see § 3.3.3.2). This network consists of processing units that represent conceptual objects such as the features, words, or concepts. It furthermore consists of connections with activation weights between these units. The pattern of interconnections among the units forms the processing system's knowledge structure – what it knows and how it responds (McClelland, Rumelhart, & The PDP Research Group, 1986: 26).

Clements and Battista (1992:435) discuss how the PDP networks present learners' knowledge structures at different Van Hiele levels (see § 3.3.3.2). The neural network units that recognize certain frequently occurring visual features are formed, during the pre-recognition level, which results in these features becoming recognizable. Progression to the visual level depends on a sufficient number of visual features becoming recognizable and their detectors interconnecting in patterns that correspond to common shapes. These detector unit networks serve as "shape recognizers" with activation patterns representing maiden schemas for figures. Patterns are activated by figures that closely enough match visual prototypes with the result that the figures are recognized. For example, a nonconvex quadrilateral was "recognized" as a "triangle with a notch", by encoding the basic configuration of a polygon, rather than the sides (see figure 3.11).

With appropriate instruction, property recognition units begin to form, meaning that visual features become observant in isolation and are linked to verbal labels. The capacity to reflect on visual features and the subsequent recognition of shapes' properties leads to level 2 thought (see § 3.3.3.1) (Clements & Battista, 1992:435).

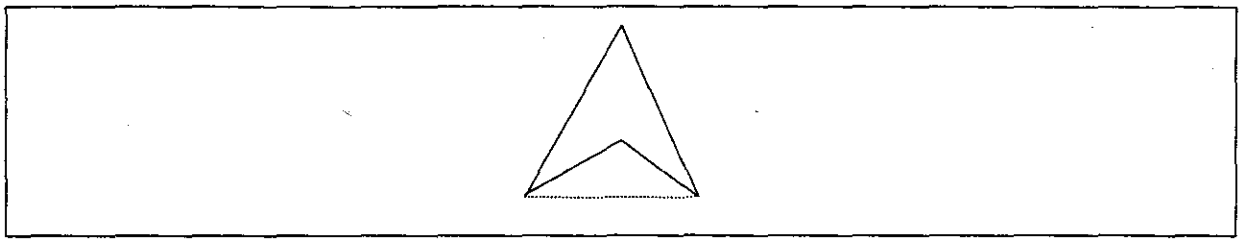


Figure 3.11 A nonconvex quadrilateral perceived as a "triangle with a notch"

3.3.2.6 Criticism on Parallel Distributed Processing Networks

Clements & Battista (1992:435) consider the lack of progress in further clarification of such notions as "network of relations" frustrating. "These ideas are interesting and provocative, but they have not progressed to any greater degree of theoretical specificity than that which they attained at their inception" (Clements & Battista, 1992:435). Learners can furthermore have several different visual subschemas for figures without accepting the "average" case. For example, a child may have a visual subschema (a vertically and horizontally orientated rectangle) instead of the average (an obliquely orientated rectangle). Leong (1993:68) suggests that the notion that the same problem might have different solutions is an important one. Deviations from the "right" solutions in the form of errors and misconceptions in mathematics are important for both diagnoses and instruction.

3.3.3 VAN HIELE'S LEVEL THEORY OF GEOMETRIC THINKING

The levels that came to be known as the "Van Hiele(s) levels" were first published by Pierre van Hiele in *Peadagogische Studiën xxxii* in 1955. Pierre van Hiele notes in his book Structure and insight: A theory of mathematics education (1986:39):"...I discovered the solution, the different levels of thinking. I first introduced my discovery....(Van Hiele, 1955:289)." Pierre does acknowledge the participation of his wife in writing this first article (Van Hiele, 1986:40) but the published article does not accredit Diena as co-author.

Professors Langeveld and Freudenthal note in Diena van Hiele's doctoral dissertation's preface that her work is closely related to Pierre van Hiele's work as seen in his doctoral dissertation also published in 1957 (Van Hiele-Geldof, 1957:vi). It can therefore be deduced that the emergence of the Van Hiele theory is a culmination of this husband and wife team's research, and any reference to this theory implies the contribution of both Pierre van Hiele and Diena van Hiele-Geldof.

The phases of learning of geometry that were developed by Van Hiele (1955:289-297; 1959:1-31) identified a way in which a student's level of geometric argumentation or thinking could be measured (see § 3.3.3.1), and ways (see § 3.3.3.2) were even suggested to help learners progress through the levels.

3.3.3.1 Levels of geometric thought

The Van Hiele theory postulates that learners progress through levels of thought argumentation in geometry, from a Gestalt-like visual level through increasing sophisticated levels of description, analysis, abstraction, and proof (Van Hiele, 1986:39).

Clements and Battista (1992:426) identify the following characteristics of the Van Hiele theory:

- Learning is a discontinuous process, which implies "jumps" in the learning curve. These "jumps" imply the presence of discrete, qualitatively different levels of argumentation or thinking.

- These levels are sequential and hierarchical. For learners to perform adequately at one level, they must have mastered a large portion of the foregoing level (Mason, 1997:40). The progression from one level to the next is more dependent upon educational instruction / experience than on age or maturation (Van Hiele, 1986:50). Certain types of experiences can facilitate (or impede) progress within a level and progress to a higher level (Mason, 1997:40).

- Concepts implicitly understood at one level become explicitly understood at the next level (Teppo, 1991:213).

- Each level has its own language. This implies that a relation that is "correct" at one level can be "incorrect" at another. Two people who reason at different levels cannot understand each other or follow the thought processes (reasoning) of the other. Language is thus a critical factor in the movement through the levels. New language is introduced in each learning period to make explicit and discuss new objects of study (Teppo, 1991:213).

The Van Hiele theory postulates a learning model that describes the different types of thinking that learners pass through as they move from a global perception of geometric figures to, finally, an understanding of formal geometric proof (Teppo, 1991:210). This model advocates increasing student understanding of geometric properties and relationships. Such understanding requires that learners comprehend the attributes of geometric figures and develop clear concepts of those figures.

Concepts are viewed as building blocks or foundations on which more complex ideas are established. Hence, teaching and learning is concept-based, rather than content-based.

Learners who understand geometric concepts are better equipped to generalize and to transfer their knowledge than learners who merely memorize definitions (Cohen, 1993:793; Shaw, Thomas, Hoffman & Bulgren, 1995:184).

In 1986 Van Hiele (1986:39-47) published his revised thought levels. The first three levels identify thinking within the capacity of elementary school learners. The last two levels involve mathematical thinking typically needed in high school and post-secondary courses (Spear, 1993:393). The levels could be described as follows:

- Level one: Visual

At first learners identify and operate on shapes and other geometric configurations according to their appearance (as visual wholes). Easily put: figures are recognized by appearance alone (Mason, 1997:39; Flores, 1993:152; Spear, 1993:393; Presmeg, 1991:9). Learners, therefore, do not explicitly attend to geometric properties or to traits that are characteristic of the class of figures represented (Battista, 1994:88). Learners recognize figures as visual gestalts, and are thus able to mentally represent these figures as visual images. This is an intuitive observation and at this level the objects are classes of figures, for example a rectangle because "it looks like a door" (see figure 3.12A and figure 3.12B) (Spear, 1993:393). This implies that learners at a visual level think about shapes in terms of images (Battista, 1994:89). Learners do not attend to geometric properties or to characteristic traits of the class of figures represented.

This implies that learners identify, name, compare and operate on geometric figures (triangles, angles, intersecting or parallel lines) according to their appearance (Van Hiele, 1986:39). A learner, at this level, recognizes and names figures and distinguishes a given figure from others that look somewhat the same. Decisions are based on perception, not reasoning (Mason, 1997:39). An example of this visual level of argumentation is the opinion voiced by a grade 7 learner when identifying right-angled triangles: " I can just *see* it is a right angle." When prompted how she could *see* the magnitude so correctly she replied "I just can."

"I can *see* it is a rectangle."
"It is a rectangle because it looks like a door."
"I just know it is a rectangle."

A. Typical responses on a visual level of argumentation to identify a rectangle

"It is a rectangle with skew sides."
"It looks like a window that was cut wrongly."
"I just know it is a parallelogram."

B. Typical responses on a visual level of argumentation to identify a parallelogram

Figure 3.12 Typical responses[1] on a visual level of argumentation

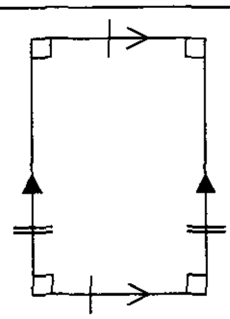
- Level two: Descriptive / Analytic

At this level learners are able to recognize and explicitly characterize shapes by their properties (Van Hiele, 1986:40; Fuys, Geddes & Tischler, 1988:5), but can not recognize relationships between classes of figures (Battista, 1994:89) or even redundancies (repetitions) (Spear, 1993:393). The properties are perceived in

[1] Figures 3.11- 3.13 were "recognized" by learners while sorting quadrilaterals during a group activity (see Chapter 4 for a detailed description of the activities). The comments are noted verbatim

isolation and they are unrelated (Mason, 1997:39; Presmeg, 1991:9). Learners do not see figures as wholes, but now see them as collections of properties rather than as visual gestalts (see figure 3.13A). Properties are established experimentally by observing, measuring (see figure 3.13B), drawing, and modeling. Learners will analyze figures in terms of their components and relationships among components and discover properties/rules of a class of shapes empirically (for example by folding, measuring, using a grid or diagram).

Learners at this level reason on the basis of a network of relations (Van Hiele, 1986:110), and are able to recognize and name properties of geometric figures (Mason, 1997:39). Learners make use of descriptions instead of definitions (Flores, 1993:152).

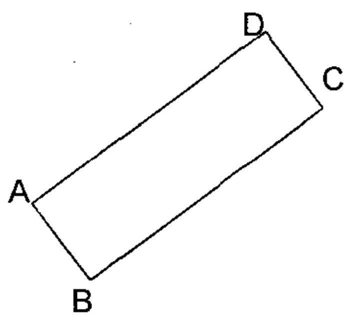


"It is an rectangle because it has two pairs of parallel sides"

"It is a rectangle because it has four right angles"

"It is a rectangle because it has two pairs of sides equal in size."

A. Typical responses on an analytical level of argumentation to identifying a rectangle



" When in doubt – measure."

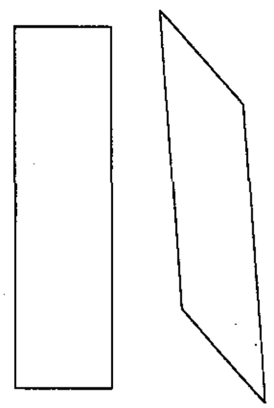
(The learner folded the rectangle so that $\overline{AB} = \overline{DC}$.)

B. A typical respons and action on an analytical level of argumentation

Figure 3.13 Typical responses on an analytical level of argumentation

- Level three: Abstract / Relational

Learners can form abstract meaningful definitions (Mason, 1997:39), distinguish between necessary and sufficient sets of conditions for a concept, classify figures hierarchically (by ordering their properties), give informal arguments to justify their classification (Fuys *et al.*, 1988:5; Battista, 1994:89), and understand and sometimes even provide logical arguments in the geometric domain (Clements & Battista, 1992:427). Learners classify figures hierarchically and give informal arguments to justify their classification / reasoning (Mason, 1997:39). They also discover properties of classes of figures by informal deduction, which leads them to determine relationships of properties both within figures and between figures (Spear, 1993:393). At this level, the objects about which learners reason are properties of classes of figures. The product of this reasoning is the reorganization of ideas achieved by interrelating properties of figures and classes of figures. For learners at this level, definitions provide not only descriptions of shapes but also a logical organization of their properties (Battista, 1994:89), thus logical implications and class inclusions are understood. The role and significance of formal deduction are not understood (Mason, 1997:39; Presmeg, 1991:9).



"A parallelogram is a special kind of rectangle. Both have 4 sides, 4 angles, 2 pairs of equal sides, 2 pairs of parallel sides and opposite angles that are equal. The difference is that a rectangle has four equal angles (all right angles) and a parallelogram has 1 pair of acute angles and 1 pair of obtuse angles."

Figure 3.14 A typical response on an abstract level of argumentation to identifying a parallelogram

- Level four: Formal Deduction

Learners establish theorems within an axiomatic system. They recognize the difference among undefined terms, definitions, axioms, and theorems (Clements & Battista, 1992:428), proving that deduction at this level has become meaningful (Mason, 1997:39; Presmeg, 1991:9). Learners are capable of constructing original proofs (Clements & Battista, 1992:428), as learners understand the meaning of proof in the context of definitions, axioms, and theorems (Flores, 1993:152). They can reason formally by logically interpreting geometric statements such as axioms, definitions, and theorems. The product of their reasoning is the establishment of second-order relationships (Clements & Battista, 1992:428) typically done in a high school geometry class (Mason, 1997:40). (No examples of learner repons on this level were possible, as all the participants in the study were primary school learners.)

- Level five: Rigor / Metamathematical

At the fifth level learners reason formally about mathematical systems. Learners now understand the formal aspects of deduction (Presmeg, 1991:9), establishing and comparing mathematical systems (Fuys *et al.,* 1988:5; Flores, 1993:152; Mason, 1997:40). They can now study geometry in the absence of reference models, and they can reason by formally manipulating geometric statements such as axioms, definitions, and theorems. The product of their reasoning is the establishment, elaboration, and comparison of axiomatic systems of geometry. Learners at this level understand the role and necessity of indirect proof and proof by contra positive, and are able to function in non-Euclidean systems (Mason, 1997:40).

Clements and Batista (1992:429) postulate the following additional level:

- Level zero: Pre-recognition

Learners perceive geometric shapes only by attending to a subset of a shape's visual characteristics. They are unable to identify many common shapes. Learners may distinguish between figures that are curvilinear and those that are rectilinear but not among figures in the same class. Thus, learners at this level may be unable to identify common shapes because they lack the ability to form requisite visual images. The "objects" about which learners reason are specific visual or tactile stimuli, the product of this reasoning is a group of figures recognized visually as "the same shape."

Furthermore it was shown that materials and methodology could be designed in such a way that they match these levels and promote growth through these levels (Burger & Shaughnessy, 1986:31-41).

3.3.3.2 Phases of instruction

Higher levels of geometric thought are reached not through direct telling by the teacher, but through a suitable choice of problem solving activities and exercises (Van Hiele, 1982:215; Fuys *et al.*, 1988:7; Koehler & Grouws, 1992:123). Clements and Battista (1992:431) acknowledge that without the teacher there would be little, if any progress.

Van Hiele (1982:215-218) distinguishes five instructional phases that describe the goal for student learning and the teacher's role in providing instruction that enables this learning:

- Phase one: Information

Learners become familiar with the content domain as the material related to the current level of study is presented to the learners (Fuys *et al.*, 1988:7; Teppo, 1991:212). The teacher discusses material in order to clarifying the content. Through this discussion the teacher learns how learners interpret the language and this discussion provides information about what the learners already know.

- Phase two: Guided orientation

Learners become acquainted with geometric ideas (Clements & Battista, 1992:431), and explore the field of investigation by handling the material, for example by folding, measuring and looking for symmetry (Fuys *et al.*, 1988:7; Presmeg, 1991:9). The teacher's role is to direct learners' activity by guiding them in appropriate and structured explorations (Teppo, 1991:212; Cohen, 1993:795). The teacher also chooses materials and tasks in which the targeted concepts and procedures are salient.

- Phase three: Explicitation

Learners now become conscious of the relations and begin to elaborate on their intuitive knowledge (Fuys *et al.*, 1988:7). Learners describe their geometric conceptualization in their own language. They are also now ready to learn the traditional language for the subject matter. The teacher's role is to bring the objects of study (geometric objects and ideas, relationships, patterns) to an explicit level of awareness by leading learners' discussion of them in their own language (Clements & Battista, 1992:431). It is during this phase that the network of relations is partly formed (Presmeg, 1991:9).

- Phase four: Free orientation

Learners solve more open-ended problems (Teppo, 1991:212) which solutions require the synthesis and utilization of those concepts and relations previously elaborated (Clements & Battista, 1992:431). They are now able to deliberately choose their activities. Learners learn to orient themselves within the network of relations. The teacher's role is to select appropriate materials and geometric problems, to give instructions to permit various performances and to encourage learners to reflect and elaborate on these problems and their solutions, and to introduce terms, concepts, and relevant problem-solving processes as needed.

- Phase five: Integration

Learners build a summary of all they have learned (Fuys *et al.*, 1988:7). They integrate their knowledge into a coherent whole that can easily be described and applied. They can use the appropriate language to describe all the networks. The role of the teacher is to encourage learners to reflect on and to consolidate their geometric knowledge (Clements & Battista, 1992:431).

Progression from one level to the next does not depend on biological maturation or development only; instead, it proceeds under the influence of a teaching/learning process. The teacher plays a special role in facilitating this progress, especially in providing guidance about expectations (Wubbels, Korthagen & Broekman, 1997:1). Van Hiele (1986:40) claims that higher levels of geometric thinking are achieved not through direct teacher telling, but through a suitable choice of (problem solving) learning activities and exercises (Koehler & Grouws, 1992:123).

Research supports the accuracy of the model from assessing learners' understanding of geometry (Hoffer, 1983:205-227). It is no longer a question whether these thought levels exist, but how to utilize them so that insight can be gained into the development of learners' spatial abilities (Van Niekerk, 1997:4). When insight is gained, it is possible to design the appropriate materials and instruction for the next teaching episode (Usiskin, 1987:29).

In accordance with the social cognitive learning perspective on self-regulated learning, geometry learners must be able to direct their thoughts and actions while completing activities in order for effective learning of geometry to take place. Schunk (1989:83) states that self-regulated learning is learning that occurs from learners' self-generated behaviours (thoughts, feelings and actions) that are systematically oriented towards the achievement of their learning goals.

3.3.3.3 Criticism on Van Hiele's work

Clements and Battista (1992:431) state that a clear problem is the ascertainment of when a learner is "at" a level. Burger and Shaughnessy (1986:41) find that individual learners can be assigned a Van Hiele level but learners in transition from one level to the next are difficult to classify reliably. It is further not clear if a learner should be seen as "at" a single level (Clements & Battista, 1992:431). Several studies (Burger & Shaughnessy, 1986:42; Wilson, 1990:232; Pegg & Davey, 1991:12) note the existence of learners that are in transition between levels. Gutiérrez, Jaime and Fortuny (1991:238) theorize that the Van Hiele levels are not discrete and that the transition between levels needs to be studied in more depth. Gutiérrez *et al.* (1991:237) present an alternative method to evaluate and identify those learners who are in transition between levels.

No ac-quisition	Low Acquisition	Intermediate Acquisition	High Acquisition	Complete acquisition

0 15 40 60 85 100

Figure 3.15 Degrees of acquisition of a Van Hiele level (Gutiérrez *et al.*, 1991:238)

Gutiérrez *et al.* (1991:238) quantify the acquisition of a level by representing it with a segment from 0 to 100 (see figure 3.15). A division is also made to divide this continuous process into five stages characterized by the qualitatively different ways in which the learners reason. These periods represent fundamental differences in the degree of acquisition of a given level.

At first learners are not in need or are not conscious of the existence of thinking methods specific to a new level. They have *no acquisition* of this level of reasoning (Gutiérrez *et al.*, 1991:238).

For a learner to be classified as possessing a *low degree of acquisition*, such a learner should be aware of methods of thinking, know their importance and try to use them. These learners make some attempts to work on this level, but have little or no success due to their lack of experience (Gutiérrez *et al.*, 1991:238).

An increase in experience leads to an *intermediate degree of acquisition*. Learners at this level use methods of this level more often and with increasing accuracy. The lack of mastering these methods completely leads learners to fall back on methods of a previous level. Reasoning during this period is marked by frequent jumps between the two levels (Gutiérrez *et al.*, 1991:239).

A *high degree of acquisition* requires progressively strengthened reasoning. Learners still make some mistakes or sometimes go back to the lower level (Gutiérrez *et al.*, 1991:239).

Learners who have attained *complete acquisition* have completely mastered the new level of thinking and use it without difficulties (Gutiérrez *et al.*, 1991:239).

Gutiérrez *et al.* (1991:239) propose an assessment procedure consisting of a series of open-ended items and criteria for evaluating learners' responses to each item. An evaluation of each answer is made that takes into account the thinking level(s) reflected as well as its mathematical accuracy and completeness. The answers are appraised in two specific ways. First the answer is classified according to the Van Hiele levels of thinking it reflected, by using the descriptions of the levels. Secondly, each answer is assigned to one of a number of types of answers, depending on its mathematical accuracy and how complete the solution to the activity is (Gutiérrez *et al.*, 1991:239).

An answer to the open-ended items could be assigned to one of the following types:

Type 0.	No reply, or answers that cannot be categorized.
Type 1.	Answers that indicate that the learner has not reached the given level but has no knowledge of the lower level either.
Type 2.	Answers that contain incorrect and incomplete explanations, reasoning processes, or results.
Type 3.	Correct but insufficient answers that give an indication that the given level of reasoning has been achieved. Answers still contain very few explanations as well as rudimentary reasoning processes, or very incomplete results.
Type 4.	Correct and incorrect answers that clearly show characteristics of two consecutive Van Hiele levels. Answers contain clear reasoning processes and sufficient justifications.
Type 5.	Answers that represent reasoning processes that are complete but incorrect, or answers that reflect correct reasoning that still do not lead to the solution.
Type 6.	Correct answers that reflect the given level of reasoning that are complete or insufficiently justified.
Type 7.	Correct, complete and sufficiently justified answers that clearly reflect a given level of reasoning.

Answers of types 0 and 1 indicate no level. Answers of types 2 and 3 indicate that the learner is beginning to acquire skills of the given level. Type 4 answers indicate that the learner is using two levels of reasoning, but that no level is clearly dominant. Types 5 and 6 answers clearly indicate that a learner is using a higher level of reasoning, thought sometimes a lower level can appear. Type 7 answers are clear indicators that the learner has fully acquired a given level, as such a learner solves problems using only methods of reasoning characteristic of that level (Gutiérrez *et al.*, 1991:240-241).

CHAPTER FOUR

METHOD OF RESEARCH

4.1 INTRODUCTION

In this chapter the empirical investigation is described. The aim of the study is stated in paragraph 4.2, while the population and sample are described in paragraph 4.3, followed by a discussion of the instruments used in paragraph 4.4. The variables used are listed in paragraph 4.5, while the method of research is described in paragraph 4.6. The statistical techniques, which were used to analyze the data are discussed in paragraph 4.7.

4.2 AIM OF THE RESEARCH

The aim of this research (see § 1.2) was to determine whether and how a Van Hiele based program, in a problem solving context, influenced self-regulated and meaningful learning.

4.3 STUDY POPULATION AND SAMPLE

4.3.1 STUDY POPULATION

The experiment was carried out in five primary schools in Potchefstroom with permission of the Department of Education (see Appendix A). An experimental and a control group were used.

One-hundred-thirty-three learners constituted the experimental group while the control group consisted out of eighty-eight learners. School E1 and E2 were part of the experimental group, while school C1 and C2 were part of the control group. School E2 and C2 received first language instruction, while schools E1 and C1 received second language instruction.

School C2 consisted of two farm schools that were treated as one school due to the small number of grade 7s at each school. The ages of the children in school E1 ranged from 12 to 16, while school E2's ages ranged from 12 to 14. School C1's ages ranged from 13 to 22 and school C2's ages ranged from 12 to 14. The relation between girls and boys were respectively: School E1, 27:23; School E2, 40:43; School C1, 10:22; and School C2, 33:23.

4.3.2 SAMPLE

The learners in the experimental group were randomly assigned to groups and the researcher randomly selected a sub-group in each of the classes on which to focus the video taping. These same sub-groups (school E1: 12; school E2: 13) were also used to complete the Van Hiele post-test.

From the study population of the control group, a randomized sample was taken to complete the Van Hiele post-test. From school C1: 11 and school C2: 13 learners were taken as the control group for the Van Hiele post-test.

4.4 INSTRUMENTATIONS

The following tests or questionnaires were used:

4.4.1 THE LEARNING AND STUDY STRATEGIES INVENTORY-HIGH SCHOOL VERSION (LASSI-HS)

The Learning and Study Strategies Inventory-High School Version (LASSI-HS) (see Appendix B) was designed to be used as an assessment tool (consisting of 76 items) to measure learners' use of learning and study strategies and methods at high school level (Weinstein & Palmer, 1990:3). In this study a version of the LASSI was used that was adapted for South African Mathematics Learners by Monteith, Nieuwoudt and Nieuwoudt (see Appendix B). Learners have to respond to the items on a 5-point Likert-type scale. The scale range from 1 = "not at all like me" to 5 = "very much like me". Learners have to answer according to how well the statements describe them, and not how they think they should be, or what others think of them.

The LASSI-HS is meant to be used as:

- a diagnostic measure to help identify areas in which learners could benefit most from educational interventions;
- a counseling tool for learner advising, for academic remediation and enrichment, for learner learning assistance programs;
- a basis for planning individual prescriptions for both remediation and enrichment;
- a pre-post achievement measure for learners participating in programs or courses focusing on learning strategies and study skills; and
- an evaluation tool to assess the degree of success of intervention programs or courses.

In this study, the LASSI-HS was used as a measure to identify learners' use of learning strategies in order to determine the influence of learning strategies on academic achievement in the Van Hiele post-test.

The LASSI-HS consists of ten subscales. A brief description of each subscale is given, the reliability of each subscale (the Alpha levels) as reported in the manual (Weinstein & Palmer, 1990:3), as well as the Alpha coefficients for the LASSI under South African conditions is given in parentheses. Both Alpha coefficients are given, as the LASSI has not yet been standardized under South African conditions for applications to the subjects who participated in this study. These coefficients range from 0,2 to 0,8. The coefficients of motivation and time management are lower than required, indicating that the reliability of these scales may be low. Apart from motivation and time management, whose coefficients were considered to be too low and therefore were not used; the other eight subscales were used. Some sample items of each subscale are also given with item numbers in parentheses.

4.4.1.1 Attitude

The first subscale contains items addressing attitude and interest in education and school. This variable addresses learners' positive or negative interests in education and school. If the relationship between school, their life goals, and their attitudes about themselves and the world is not clear, then it is difficult to maintain a mind-set that promotes good work habits, concentration, and attention to school and its related tasks.

Learners' scores on this scale measure their general attitudes and motivation for succeeding in school and performing the tasks related to school success. Learners who score low on this measure need to work on higher-level goal setting and reassess how school fits into their future plans. If school is not seen as relevant to the learner's life goals and attitudes, then it will be difficult to generate the level of motivation needed to help take responsibility for his/her own learning and for helping to manage one's own study activities (Weinstein & Palmer, 1990:13).

Cronbach Coefficient Alpha = 0,74 (0,67)

"I feel confused and undecided as to what the goals of my maths studies should be." (14)

"I would have preferred not to take maths in school." (18)

4.4.1.2 Motivation

This subscale focuses on learners' diligence, self-discipline, and willingness to work hard. Motivation is the degree in which learners accept responsibility for studying and for their performance. These behaviours include reading textbooks, preparing for class, finishing assignments on time, and being diligent even if the topic is uninteresting.

The learners' scores attained on this scale indicate the degree in which they accept responsibility for performing the specific tasks related to school success. Learners who score low on this measure, need to work on goal setting at the more global levels assessed on the attitude scale, but especially for studying, and achievement outcomes require that learners learn to attribute much of what happens to them in school to their own efforts rather than to outside forces such as luck or teachers (Weinstein & Palmer, 1990:13). As the Alpha Coefficient was low (0,20) this subscale was not used in the analyses.

Cronbach Coefficient Alpha = 0,78 (0,20)

"I work hard to get good marks in maths, even when I don't like the maths being done." (28)

"When doing maths which is difficult for me I either give up or study only the easy parts." (48)

4.4.1.3 Time management

This subscale addresses learners' use of time management for academic work. Managing time is an important support strategy for learning. Most students create schedules and stick to them when studying and this requires knowledge about themselves as learners. This type of knowledge and self-awareness motivates the learners to learn effectively.

The learner scores obtained on this scale indicate the degree in which they create and use schedules. Learners who score low on this measure may need to learn how to create a schedule and how to deal with distractions, competing goals, and procrastination. Accepting more responsibility for studying and achievement outcomes requires that learners set learning goals and create plans that will facilitate goal achievement (Weinstein & Palmer, 1990:14). As the Alpha Coefficient was low (0,44) this subscale was not used in the analyses.

Cronbach Coefficient Alpha = 0,78 (0,44)

"I find it difficult to stick to a study time table for maths." (3)

"When I decide to do my maths homework, I set aside a certain amount of time and stick with it." (57)

4.4.1.4 Anxiety

Anxiety items examine the degree in which learners worry about school and their performance. Cognitive worry is manifested in negative statements that make it difficult for learners to concentrate and learners are easily discouraged about grades. If a learner is worried that he will not have the time to finish a test, he is making matters worse by taking more time away from his performance. This type of self-defeating behaviour often sabotages a learner's efforts.

The learner scores obtained on this scale indicate how tense or fearful they are when approaching academic tasks. Learners who score low on this measure need to learn techniques for coping with anxiety. Many learners are often incapable of demonstrating their true level of knowledge and skills because debilitating anxiety (Weinstein & Palmer, 1990:14) distracts them.

Cronbach Coefficient Alpha = 0,82 (0,70)

"I am worried that I will fail maths at school." (1)

"While I am writing a maths test, worrying about doing poorly gets in the way of keeping my mind on the test." (53)

4.4.1.5 Concentration

This subscale focuses on learners' ability to pay close attention to academic tasks. Concentration helps learners to focus their attention on learning-related tasks such as studying and listening in class. If learners are distracted, there will be a diminished capacity to focus on the task at hand. For the learners, it means that distractions that interfere with concentration divert attention away from school-related activities.

The learner scores obtained on this scale indicate their abilities to concentrate and direct their attention to school and school-related tasks, including study activities. Learners who score high on this measure are effective at focusing their attention and maintaining a high level of concentration. Those who score low on this measure need to learn techniques to enhance concentration and to set priorities so that they can attend to school as well as their other responsibilities (Weinstein & Palmer, 1990:14).

Cronbach Coefficient Alpha = 0,82 (0,60)

"I think of other things during the maths lesson and don't really listen to what is being said in class." (6)

"Problems outside of school such as financial problems, fights with parents, dating (being in love), etc. cause me to not do my maths." (11)

4.4.1.6 Information processing

Information processing items address several sub-areas. These include the use of mental imagery, verbal elaboration, comprehension monitoring and reasoning. Information processing helps to build bridges between what the learners know (prior knowledge) and what they are trying to learn and remember (new knowledge).

The learner scores obtained on this scale indicate how well they can create imaginal and verbal elaborations and organizations to foster understanding and recall. Learners who score low on this measure need to learn methods that they can use to add meaning and organizational schemes and outlining, and the use of inferential, analytical, and synthetic reasoning skills (Weinstein & Palmer, 1990:15).

Cronbach Coefficient Alpha = 0,78 (0,64)

"I try to think through a topic while doing maths and decide what I am supposed to learn from it." (12)

"When I study a topic in maths I try to make the ideas fit together and make sense." (32)

4.4.1.7 Selecting main ideas

This subscale examines learners' ability to pick out important information for further study. Selecting main ideas requires that the learners be able to select the important material for in depth attention. If a learner cannot select the critical information then the learning task becomes complicated by the huge amount of material the individual is trying to acquire. Lacking such a skill means that the learner will not have enough time to study everything that must be covered.

The learner scores obtained on this scale indicate their skills for selecting important information to concentrate on for further study in their classroom lecture. Learners who score low on this measure need to learn more about how to identify important information so that they can focus their attention and information processing strategies on appropriate material (Weinstein & Palmer, 1990:15).

Cronbach Coefficient Alpha = 0,71 (0,53)

"I can tell the difference between more important and less important information in a maths lesson." (2)

"I try to identify the main ideas or most important information in a maths lesson while the lesson is being presented." (8)

notes and the text, thinking of potential questions to guide reading, consolidating new knowledge, integrating related information and identifying (if additional studying must be done), are all important methods for checking understanding.

The learner scores obtained on this scale indicate learners' awareness of the importance of self-testing and reviewing as well as the degree in which they use these methods. Learners who score low on this scale need to learn more about the importance of self-testing and need to learn specific methods to review school material and to monitor their comprehension (Weinstein & Palmer, 1990:16).

Cronbach Coefficient Alpha = 0,74 (0,71)

"I try to think of possible test questions when studying work done in the maths class." (21)

"I check to see if I understand what my teacher is saying during a maths lesson." (36)

4.4.1.10 Test strategies

This final subscale examines learners' approaches to preparing for and taking quizzes and tests. Test-taking strategies include knowing about the characteristics of tests and test items, and how to create an effective test-taking plan.

The leaner scores obtained on this scale indicate their use of the test-taking strategies. Learners who score low on this scale may need to learn more about how to create a plan of attack for taking a test, the characteristics of different types of tests and test items, and how to argue an answer. Knowing about test-taking strategies helps learners to target their study activities, sets up study goals, implements an effective study plan, and demonstrates their knowledge and skill acquisition so it can be accurately evaluated (Weinstein & Palmer, 1990:17).

Cronbach Coefficient Alpha = 0,81 (0,78)

"I have trouble understanding just what a test question in maths is asking." (51)

"When writing maths tests or doing other work in maths, I find I have not understood what is asked of me and lose marks because of it." (74)

4.4.2 THE MOTIVATED STRATEGIES FOR LEARNING QUESTIONNAIRE (MSLQ)

The MSLQ (see Appendix C) includes 44 items on learner motivation, cognitive strategy use, metacognitive strategy, and the management of effort. In this study a version of the MSLQ was used that had been adapted for South African conditions by Monteith and Mathebula. It was necessary to translate this questionnaire into Afrikaans for control group 2. Learners have to respond to the items on a 5-point Likert-type scale. The scale ranges from 1 = "not at all like me" to a 5 = "very much like me". Learners have to answer according to how well the statements describe them, and not how they think they should be, or what others think of them.

Factor analysis of the motivation items revealed three distinct motivational factors: Self-efficacy, intrinsic value, and test anxiety. The Self-Efficacy scale (Alpha = 0,89/0,86)[1] consisted of nine items regarding confidence in class work (e.g. "I expect to do very well in maths." (8), "I know that I will be able to learn the work for maths" (19) (Pintrich and De Groot, 1990:35).

[1] Aplha levels as reported by Pintrich and De Groot (1990). Alpha levels after the forward slash (/) are for South African conditions. Both Alpha levels are given as the MSLQ have not been standardized for the subjects who participated in this study

The Intrinsic Value scale (Alpha = 0,87/0,71) was constructed by taking the mean score of the learners' respons to nine items concerning intrinsic interest in class work, e.g. "It is important for me to learn what is being taught in maths" (4), as well as preference for challenging work and master goals e.g. "I prefer class work that is challenging so that I can learn new things" (1).

The four items in the Test Anxiety scale were included in the questionnaire but not used in the analysis of the data, as text anxiety is already included in the LASSI-HS.

On the basis of the results of the factor analysis, two cognitive scales were constructed, namely cognitive strategy use and self-regulation. The Cognitive Strategy Use scale that consists of 13 items was included in the questionnaire but not used in the analysis of the data as text anxiety is already included in the LASSI-HS.

The Self-Regulation scale (Alpha = 0,74/0,54) was constructed from metacognitive and effort management items. The items on metacognitive strategies, such as planning, skimming, comprehension and persistence at difficult or boring tasks and working diligently were included as part of this scale, e.g. "I ask myself questions to make sure I know the work I have been studying" (25) and "Even when the maths is boring, I keep working until I finish" (33) (Pintrich and De Groot, 1990:35).

4.4.3 A VAN HIELE POST-TEST

The Mayberry Test (Lewin and Pegg Version) includes 58 items that include up to 5 sub-items on a variety of geometric concepts. This test assesses concepts (like congruency, similarity, parallel lines) and shapes (like square, circle and isosceles triangle) over the first four Van Hiele levels (see § 3.3.3.2). In selecting the relevant items it was found that the items from the Mayberry Test were not sufficient and therefore additional items were introduced from a test developed by the unit for Research in Mathematics Education of the University of Stellenbosch (RUMEUS) (1984).

The final product (see Appendix D) includes 21 items (with some sub-items) on concepts (parallel lines) and shapes (square, right angle, isosceles triangle) that deal with the first three Van Hiele levels. Items 1-12 test level 1 responses, items 13-20A deal with level 2 responses and items 20B-21B examine level 3 responses. The answers to the items were quantified according to the acquisition scales of Gutiérrez *et al.* (1991) (see § 3.3.3.3).

4.5 VARIABLES USED

The following independent and dependant variables were used in this study:

4.5.1 INDEPENDENT VARIABLES

A Van Hiele based teaching and learning program in a problem solving context

4.5.2 DEPENDENT VARIABLES

Attitude
Anxiety
Concentration
Information processing
Selecting main ideas
Study aids
Self-testing
Test-taking strategies

Self-efficacy

Intrinsic value

Self-regulation

Geometry performance (Van Hiele levels)

4.6 METHOD OF RESEARCH

Both qualitative and quantitative research were conducted. In doing the quantitative research a pre-test post-test, experimental group-control group design was used as subjects were randomly assigned to groups and different treatments were administered followed by observations and measurements to assess the effects of the treatment.

In executing the qualitative research, a variety of methods were used namely action research (as the researcher investigated whether a Van Hiele based problem-centered teaching and learning program, in a problem solving context, influenced self-regulated and meaningful learning). A type of qualitative research (case and field study research) was used as data were gathered directly from individuals (individual cases) and social groups in their natural environment for the purpose of studying interaction, attitudes and characteristics of individuals and groups (Leedy, 1997:111).

4.7 STATISTICAL PROCEDURES AND TECHNIQUES

The data were processed by using SAS System for Windows Release 6.12 (SAS INSTITUTE, Cary, NC, USA, 1996).

Effect-sizes (Steyn, 1999:3) were used to determine whether the differences between the control group (C) and experimental group (E) were of practical significance.

The following formula was used: $d = \left| \frac{\overline{x_1} - \overline{x_2}}{\text{larger std dev}} \right|$

if d = 0,2 it is a small effect

if d = 0,5 it is a medium effect

if d = 0,8 it is a large effect

Only if d ≥ 0,8 is there a practical significant difference between groups.

The practical significance of the difference between the pre-post tests was determined by using the following effect size (Steyn, 1999:3) formula:

$$d = \left| \frac{\overline{x}_{difference}}{\text{std dev}} \right| \qquad \overline{x}_{difference} = \overline{x}_{post} - \overline{x}_{pre}$$

if d = 0,2 it is a small effect

if d = 0,5 it is a medium effect

if d = 0,8 it is a large effect

Only if d ≥ 0,8 is there a practical significant difference between groups.

As the Van Hiele post-test was only administered to a small random sample of the population, the non-parametric Wilcoxon Rank Sum Test was used. The effect-sizes of only the data that proved to be statistical significant (p value < 0,05) were determined by using the following formula: $d = \left| \frac{\overline{x_1} - \overline{x_2}}{\text{larger std dev}} \right|$

if d = 0,2 it is a small effect

if d = 0,5 it is a medium effect

if d = 0,8 it is a large effect

Only if d ≥ 0,8 is there a practical significant difference between groups.

4.8 PROCEDURE

4.8.1 EXPERIMENTAL BACKGROUND

The researcher visited the two schools who took part in the experiment (hereafter experimental schools) and the learners completed the Motivated Strategies for Learning Questionnaire (MSLQ-HS) (see § 4.4.2) that contained two additional biographical questions dealing with the gender and age (measured in years and months) of each learner. The next day the learners at these schools completed the Learning and Study Strategies Inventory High School (LASSI-HS) (see § 4.4.1). It was not possible to administer these questionnaires at the control schools as these schools worked according to a spiral syllabus system[2], and had thus already started with geometry in the first term, when geometry is usually reserved for the third term. Administering the above mentioned pre-tests at the control schools would not have provided uncontaminated results as these learners had already been exposed to geometry lessons and variables such as attitude, motivation, anxiety, self-efficacy, self-regulation etc. could already be influenced in a negative or positive effect.

A program was developed that was based on a series of Van Hiele material (see § 4.8.2) (following Fuys *et al.*'s 1988 work) that dealt with the geometry syllabus as prescribed by the Department of Education (TED 1994). This program was compiled with a problem solving context in mind, as Schunk (1996:234; 2000:191) postulates that problem solving is relevant to and involve in (self-regulated) learning especially when the learning involves challenges and nonobvious solutions. The learning program was deliberately compiled in such a manner that it offers challenges whose solutions may not be obvious (see § 4.8.2). A problem solving context was central to this program as Shuell (1989:104; 1990:540) states that in mathematics learning is best characterized as problem solving and learning is a form of problem solving. Before the beginning of the experiment the two teachers in the experimental schools

[2] In a spiral syllabus system topics or areas within mathematics are dealt with on a specific day each week, for e.g. Geometry on Monday, Fractions on Tuesday etc.

were well trained in the Van Hiele theory as well as the developed activities (over a period of month). Despite the training one of the teachers (E1) still continued with a teacher-centered teaching approach.

Examples of how E1's teacher persisted in the teacher-centered approach in spite of the training and specially developed program can clearly be seen on the video-taped procedures in her class. Instead of allowing the learners to sort for themselves (for example sorting triangles in groups, see § 4.8.2 – activity 8.1-9.3) and discovering properties, this teacher sorted the triangles herself on the board (with little to no learner participation) and later even provided the properties in the way she wanted them "back" in the test/exam. Re-training also deemed little improvement as this teacher believed that a program such as this one would take up too much time, which could result in her not completing the syllabus.

The activities (see § 4.8.2) were implemented in the experimental schools after completion of the training and the progression through these activities was continuously videotaped.

The normal periods, on two non-consecutive days, that were allocated to mathematics were used to complete the series of activities. This constituted two hours per week allocated to Geometry from May to September. This period also included the June exams as well as the July holidays of 4 weeks. The researcher planned the activities, but the mathematics teacher presented the classes while they were being taped for analyses and transcription afterwards.

Before the experiment started, a desensitizing period was undertaken where the researcher with the video camera videotaped randomly selected algebra lessons. After this period the learners were randomly arranged into sub-groups and a sub-group was randomly selected to be the group on which the videotaping would focus.

The other groups in each class were still included in the videotaping for comparison when analyzing the progress. The learners remained in the same groups for the duration of the study.

In the same time that the experimental groups were progressing through the designed program, the control schools continued with their spiral syllabus system[2] where the teacher and the textbook formed the main sources of information with little or no learner involvement in the classroom activities. The control schools' spiral syllabus system allowed for post-testing to take place in September, as by that time they had already completed the geometry section within the syllabus.

After conclusion of the program in September, a set of post-tests was administered at both the experimental and control schools. All the learners completed the Motivated Strategies for Learning Questionnaire (MSLQ-HS) (see § 4.4.2 and Appendix C) that contained two additional biographical questions that dealt with the gender and age (measured in years and months) of each learner. The next day the learners at these schools completed the Learning and Study Strategies Inventory High School (LASSI-HS) (see § 4.4.1 and Appendix B) as well as the Van Hiele post-test (see § 4.4.3 and Appendix D).

After completion of the field work, the data were computerized for analysis (see chapter 5).

4.8.1.1 Data collection and interpretation

The following method was adhered to:
All the physical materials (drawings, written calculations and models) made by the learners were collected and categorized. The video material was transcribed in different ways namely:

(i) Verbal transcription

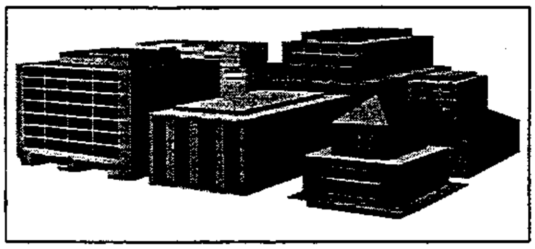
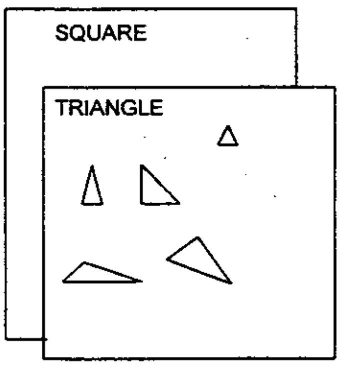
The actual conversations of the group members during the activities were written down or notes were made of the content of the discussions.

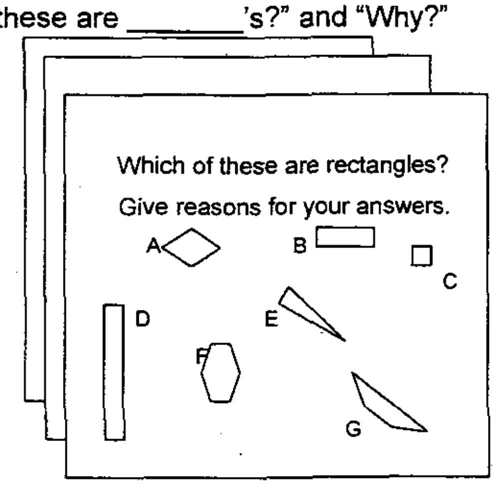
(ii) Written or pictorial material transcription

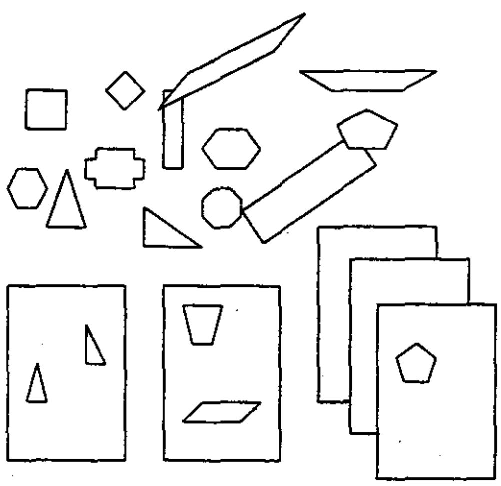
All the activities that involved the process of drawing or writing or sorting during the classroom activities were observed, videotaped and written down. In other words, not only the end product but the whole process that the learner engaged in, from the moment that the drawing or sorting activity was started until the completion of the drawing or activity was noted.
All the worksheets and other written data like diagrams drawn were interpreted quantitatively or qualitatively depending on the nature of the data.

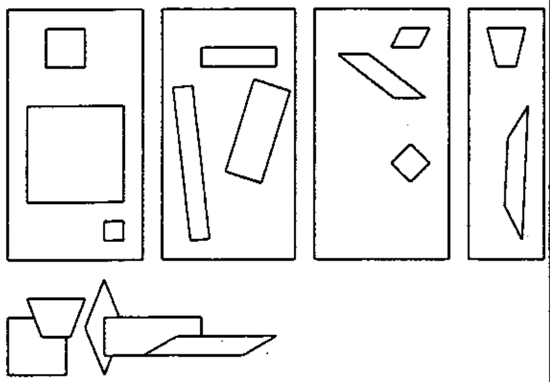
An enormous amount of data was therefore gathered, but only the most prominent differences between the two experimental schools and their respective control schools could be reported in Chapter 5.

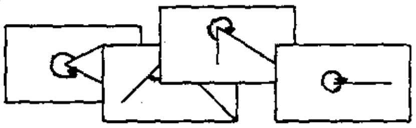
4.8.2 ACTIVITIES

ACTIVITY		OBJECTIVE
1.1	<u>Introductory activity</u> First the learners are shown pictures of a city environment (skyline of buildings). They are then asked to find "geometrical ideas" in the pictures. For the shapes identified, they are asked to find other examples of the shapes in the picture and to decide on some examples and non-examples selected by the teacher. The concepts (named adjacently) are required in later work, and are pursued in this way. (All other responses are simply praised.) For any concepts that are not mentioned, learners are shown examples and are asked if they recognize the configuration and can name it – if not, they are shown "word-and-picture-cards."   	This activity assesses learners' familiarity with some basic geometric concepts, namely: 1. Concepts of shapes: a. Triangle b. Square c. Rectangle d. Parallelogram 2. Concepts of components of shapes: a. Angles, right angles b. Parallel sides c. Opposite sides and angles

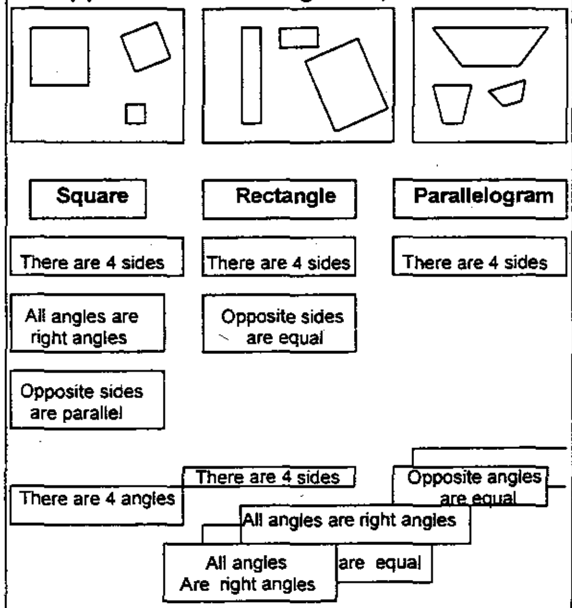
1.2	<u>Drawing</u> Three concepts are explored in greater depth as they arise namely: rectangle, right angle and parallelism. Learners are asked to draw the figures. Geometry concepts like point and line segment are also dealt with. Learners then describe their idea (figure) to someone who didn't know what the figure looked like "over the phone."	Only three concepts are pursued in depth because it was found to be time-consuming and a bit boring for the learners to discuss more at this point. The questioning about rectangles, right angles and parallelism in this activity does not allow for responses at either 0 (looking at individual shapes, descriptions in terms of "looking like") or at level 1 (invoking properties of a class of figures).
1.3	<u>Selection sheets</u> Selection sheets that allow for detailed assessment of learner's grasp of these concepts. Learners are asked: "Which of these are ________'s?" and "Why?" 	This activity was prolonged when correct responses are unlikely, but learners were given every opportunity to produce their own non-standard vocabulary before standard terms were introduced. If a non-standard term was produced, the interviewer used it together with the standard term thereafter, until the learners seemed comfortable with the standard term.

2.	<u>Sorting of properties of polygons</u> Learners are shown a collection of cardboard cutout polygons and some mats. The teacher says: "These shapes came from several different boxes but they got all mixed up. This is how someone tried to put them back into groups which belong together." The teacher then places a couple of pieces on each mat, sorting by number of sides, and says: "Can you guess where this will go? Why? And this? Why? Can you arrange the rest of the pieces using this idea?" Finally learners are asked to describe the way the pieces were sorted, and if they know names for triangle and quadrilateral piles.  Learners thinking at level 0 tend to explain placement of pieces by phrases such as "they look like", while level 1 thinkers try to find a common characteristic such as the number of sides.	This activity was designed to assess the learner's ability to think about shapes in terms of properties rather that merely a shape's appearance. The first sorting activity was presented in a "Guess my rule" format because it was found that when the challenge to arrange pieces was presented in a less structured manner, learners often tried to put pieces together in a puzzle format and it was awkward to establish what they meant by sorting. The open sort proved to be too time-consuming, although many interesting ideas arose.

3.1	<u>Sorting (Quadrilaterals)</u> Learners are shown a collection of quadrilaterals. Again learners are asked: "How can you place these into groups of things that belong together?" They are asked to explain their thinking as they sort. Eventually a sort by square, rectangle, parallelogram etc is expected. If learners sort in any other way, their thinking is discussed and their work praised, and they are then asked to try it another way. If the standard sort is not produced, a "Guess My Rule" format is followed. 	Again level of thinking was assessed by the extent to which learners describe placement of pieces in terms of properties. This activity provided a richer context for sorting than the previous one. Learners thinking at level 0 may place pieces together in roughly similar pairs, making judgements by eye, while learners thinking at level 1 might spontaneously invoke properties (I'm putting all the ones with four angles here.")
3.2	<u>Sorting (angles)</u> Learners are shown a collection of angles. Learners are then asked to sort these angles into groups (maybe from big to small). They are asked to explain their thinking as they sort. "Angle meters" are provided for initial explanations. Learners are given some more examples of the same angles and	This activity was designed to activate learners' prior knowledge concerning angles. Learners thinking at level 0 may place pieces together in roughly similar pairs, making judgements by eye, while learners thinking at level 1 might spontaneously invoke properties.

	are asked to correctly sort them. Learners are encouraged to give each angle a name. The teacher uses name cards to indicate the name of each angle. "Angle meter"   	If learners sorted in any other way, their thinking was discussed and their work praised, and they were then asked to try it another way. If the standard sort was not produced, a "Guess My Rule" format was followed.
3.3	Listing properties (angles) The teacher points to a group of angles and says: "If you were talking with your friend over the phone and you wanted to describe these angles, what could you say about them?" For each group of angles there is a name card and a set of color-coded property cards. As learners mention a name or a property, the teacher places the corresponding card on the group. Learners are encouraged to say as much as they can about one angle type, then go on to other angles.	This activity was a continuation of the previous activity.

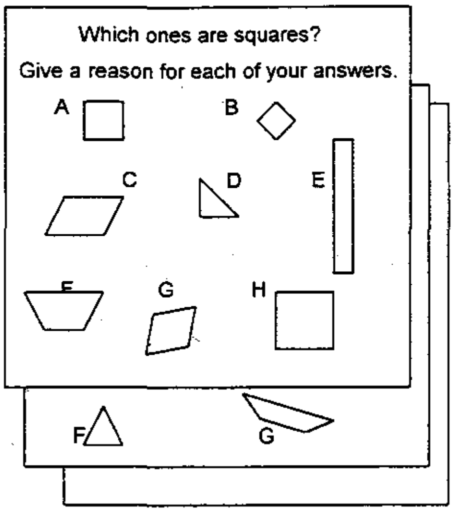
of opposite sides are green.)



As learners mention a name or a property, the teacher places the corresponding card on the group. Learners are encouraged to say as much as they can about the squares, then go on to other shapes. Prompts are given as needed, for example "Is there anything you can say about the size of the angles?" If necessary, sticks are placed on shapes to remind learners of parallelism or angle sizes. This continues until all properties are listed.

Then learners are asked to look at selection sheets for each angle, on which they identify examples and non-examples, and to explain their thinking.

The selection sheets allowed for detailed assessment of learners' grasp of the concepts. In the course of the session the teacher kept track of the learners' understanding of the concepts.

		
5.	<u>Quadrilateral subclass relation</u> The teacher says: "When you sorted the first set of shapes, do you remember that you had a group of triangles, and one of quadrilaterals or four-sided figures, and five-sided ones, and six-sided ones? Where would all these shapes on the table have gone?" When learners have responded correctly, the teacher picks up a square, saying "So I could move this square to the quadrilateral or four-sided group - a square is a special kind of quadrilateral. What makes it special?" The teacher then asks: "Can we move this square to the rectangle group? Why? (or Why not?)"	This activity assessed if a learner could identify and explain subclass relations – for example, all rectangles are parallelograms.

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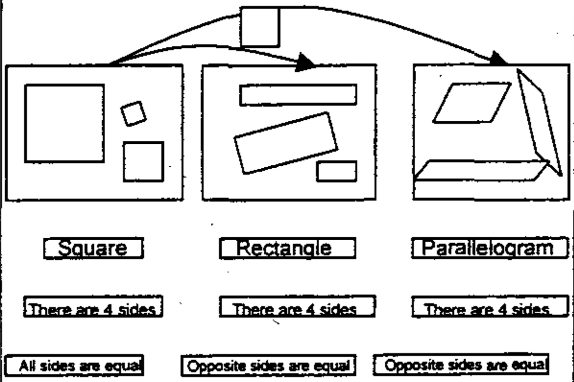

The procedure is repeated for a rectangle, parallelogram and a rhombus.
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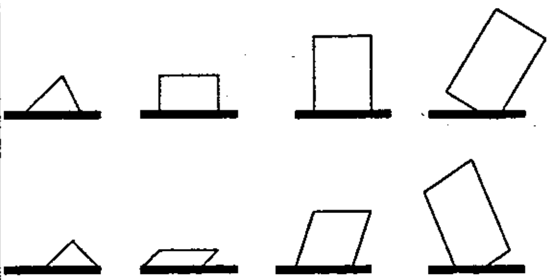
<td>6.1</td>
<td>
<u>Uncovering shapes (Quadrilaterals)</u>
The teacher uncovers a cardboard cut-out in four stages, asking at each stage "What could this be? Could it be anything else? Why? What couldn't it be? Why?" The process is repeated for a second shape if the learners seemed to be confused about the directions the first time. This process is repeated until a square, a rectangle, a parallelogram, a rhombus and a trapezium have been dealt with.


</td>
<td>These activities (6.1-6.2) were similar in format, but one was presented visually and the other more abstractly. This activity was designed to assess how the learner used partial information about the figure (a partial view or a few properties) to make judgements about what the figure could be or could not be.</td>
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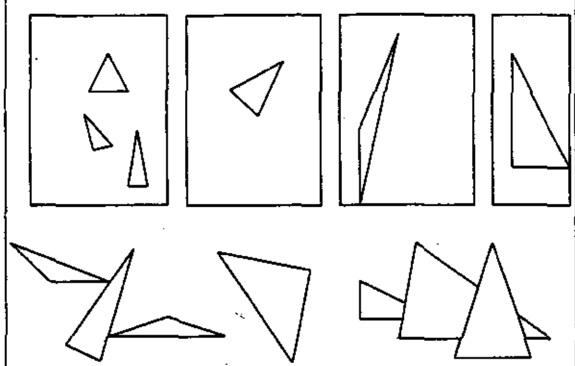
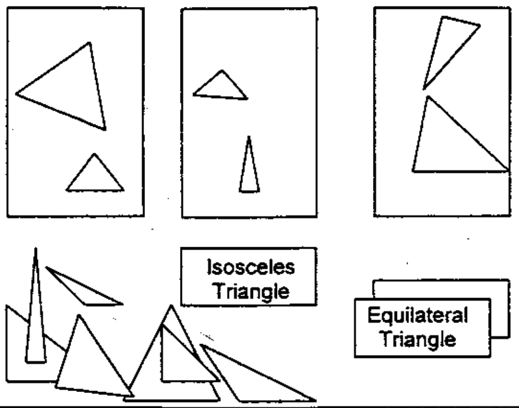
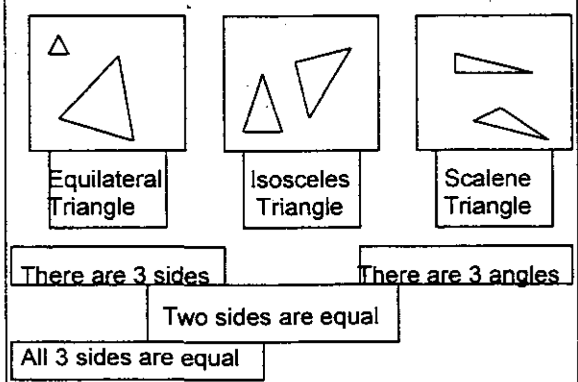
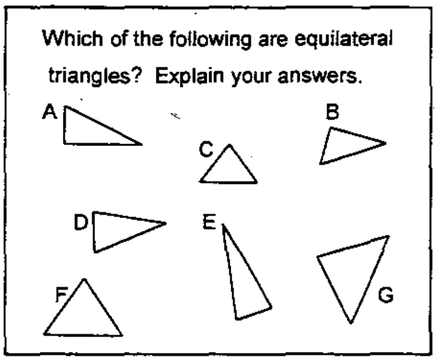
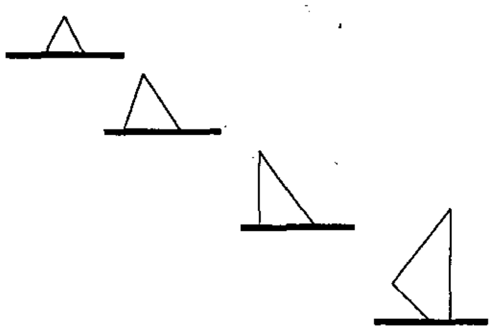
6.2	Clues (Quadrilaterals) The teacher says: "Let's try a guessing game. You will try to figure out what shape I'm thinking of, only now you won't see any of the shape, instead I'll just give you some clues about it. I'll show you the clues one by one – after each tell me all you can. What COULD it be? Could it be anything else? What COULDN'T it be?" The teacher slides a piece of paper down the sheet to uncover clues one by one. At the end the learner is asked: "Can you think of other clues I could put down? Are you sure? Why?" This process is repeated for two sets of clues for each figure. The shape has 4 sides Opposite sides are equal It has at least one right angle One side is longer than another side There are 4 angles Opposite sides are equal Opposite sides are parallel One side is longer than another side	
7.	Minimum properties (Quadrilaterals) Learners are shown a collection of "clues cards" for a square. The teacher says: "Here is a lot of properties of a square that we have talked about. Suppose that you wanted to make up some clues for a	This activity assessed if learners could deduce some property from another, within the context of their knowledge. They were encouraged to use a drawing to check their thinking. This activity

	square for your friend. Do you think that your friend would need to see ALL of these properties to know that you were talking of a square? Which cards could you take away? Why? Any others?" When the learners are finished, the teacher takes away first a card which is necessary, then one which is not necessary (if any remain) and asks for each: "Could I take this away? Why?" If the explanation is incomplete, the teacher asks: "Can you show me why in a drawing?" This procedure is repeated for a rectangle, a parallelogram and a trapezium. There are 4 angles There are 4 sides All sides are equal All angles are right angles Opposite angles are equal Opposite sides are parallel Opposite sides are equal After successful completion of the above-mentioned activity, the teacher continues: "Now suppose that you wanted to give your friend clues for a parallelogram. What is the smallest number of clues you could give her so that she would know you were thinking of a parallelogram? Why?"	prompted a learners to give the fewest properties necessary to describe a shape. It was not expected that students would be able to give complete arguments about why some properties implied others. This activity did allow the teacher to assess the learners' recognition of the need for such arguments, and the students' ability to search for counter-examples through drawing. Level 1 thinkers might respond by saying that "yes, a parallelogram has all those properties" without seeing duplication or logical relations. Level 2 thinkers would either reason ("You don't need *opposite angles are equal* because *all four angles are right angles* and so they are equal.") or try to test a hypothesis ("if the opposite sides are parallel, then the opposite sides will always be equal").

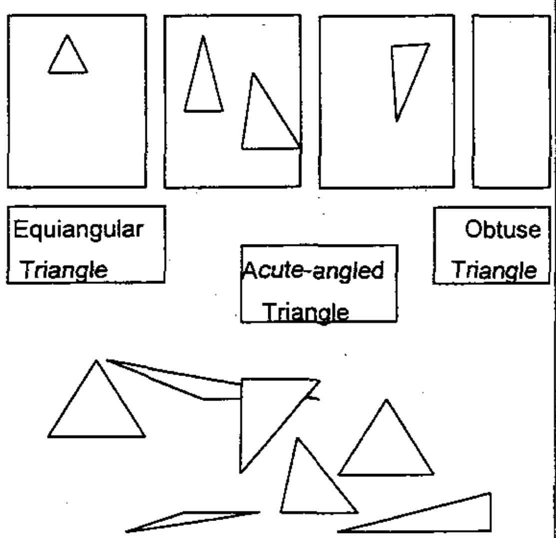
	The teacher then shows the sheet at the bottom. She/he then asks: "Three people were arguing about their description of a parallelogram. These are the only clues that they have – they were talking about four-sided figures. Who was correct? Why?" Clues for a parallelogram *"A parallelogram is a four-sided figure where: "* A says: Opposite sides are parallel (both sides) B says: Opposite sides are equal (both sides) C says: The shape has four angles	
8.1	<u>General sorting (Triangles)</u> Learners are shown a collection of cutout triangles. The teacher says: "These shapes came from several different boxes but they got all mixed up. Try to put them back into groups which belongs together." The teacher gives no indication of the number of groups in which the learners must sort. They are asked to explain their thinking as they sort. Their thinking is discussed and their work praised.	Activities 8.1 to 11.3 were designed to assess the learners' ability to think about shapes in terms of properties rather than merely by the triangle's appearance. Activities 8.1 to 11.3 used the same cutout triangles and it was decided to number the triangles randomly to make further discussion easier. This activity was designed to assess whether the learners sort on a "look like" basis, or by thinking about the properties.

		
8.2	<u>Sorting triangles according to sides</u> Learners are shown a collection of cutout triangles. The teacher says: "These shapes came from several different boxes but they got all mixed up. This is how someone tried to put them back in groups which belong together." The teacher then places a couple of pieces on each page, sorting according to sides, and says: "Can you guess where this will go? Why? And this? Why? Can you arrange the rest of the pieces using this idea?" Finally learners are asked to describe the way the pieces were sorted, and if they know names for each pile. They are expected to sort the triangles as equilateral, isosceles and scalene triangles. For each group of triangles there is a name card and after sorting learners are allowed to use these	This activity was designed to assess the learner's ability to think about shapes in terms of properties rather that merely a shape's appearance. This sorting activity was again presented in a "Guess my rule" format because it was found that when the challenge to arrange pieces was presented in a less structured manner, learners often tried to put pieces together in a puzzle format and it was awkward to establish what they meant by sorting.

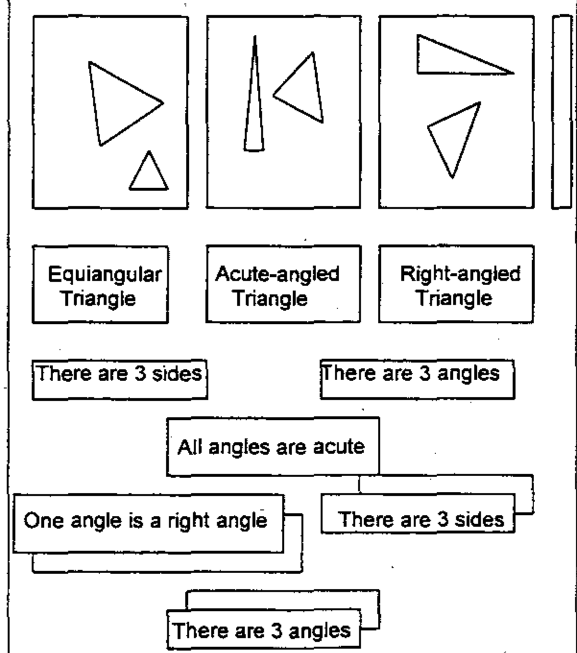
		
9.1	<u>Listing properties (Triangles)</u> The teacher points to a group of triangles and says: "If you were talking with your friend over the phone and you wanted to describe these triangles, what could you say about them?" For each group of triangles there is a name card and a set of color-coded property cards. As learners mention a name or a property, the teacher places the corresponding card with the group. Learners are encouraged to say as much as they can about one type of triangle, then go on to other triangles. Prompts are given as needed. This continues until all properties are listed. 	This activity assessed a learner's ability to characterize the groups of triangles in terms of properties, and also through guided questioning, to instruct the learners in this area.

	Learners then are asked to look at selection sheets for each triangle, on which they identify examples and non-examples, and to explain their thinking. 	The selection sheets allowed for detailed assessment of learners' grasp of the concepts. In the course of the session the teacher kept track of the learners' understanding of the concepts.
9.2	<u>Uncovering shapes (Triangles)</u> The teacher uncovers a cardboard triangle cutout in four stages, asking at each stage "What could this be? Could it be anything else? Why? What couldn't it be? Why?" The process is repeated for a second triangle if the learners seemed to be confused about the directions the first time. This process is repeated until equilateral-, isosceles- and scalene triangles have been dealt with. 	This activity was designed to assess how the learner uses partial information about the figure (a partial view or a few properties) to make judgements about what the figure could be or could not be.

9.3	<u>Minimum properties (Triangles)</u> Learners are shown a collection of "clues cards" for an equilateral triangle. The teacher says: "Here is a lot of properties of an equilateral triangle that we have talked about. Suppose that you wanted to make up some clues for a equilateral triangle for your friend. Do you think that your friend would need to see ALL of these properties to know that you were talking of an equilateral triangle? Which cards could you take away? Why? Any others?" When the learners have finished, the teacher takes away first a card which is necessary, then one which is not necessary (if any remains) and asks for each: "Could I take this away? Why?" If the explanation is incomplete, the teacher asks: "Can you show me why in a drawing?" This procedure is repeated for isosceles and scalene triangles. There are 3 sides There are 3 angles None of the sides are equal	This activity assessed if learners could deduce some property from another, within the context of their knowledge. They were encouraged to use a drawing to check their thinking. This activity asked a learner the fewest properties necessary to describe a shape. It was not expected that students would be able to give complete arguments about why some properties implied others. This activity did allow the teacher to assess the learners' recognition of the need for such arguments, and the students' ability to search for counter-examples through drawing. Level 1 thinkers might respond by saying that "yes, a equilateral triangle has all those properties" without seeing duplication or logical relations. Level 2 thinkers would reason ("You don't need *There are 3 angles* because *There are 3 sides* implies the forming of 3 angles") or try to test a hypothesis ("If there are 3 angles, then there will be 3 sides").

10.1	<u>Sorting triangles according to angles</u> Learners are shown a collection of cutout triangles. The teacher says: "These shapes came from several different boxes but they got all mixed up. This is how someone tried to put them back in groups which belong together." The teacher then places a couple of pieces on each page, sorting according to angles, and says: "Can you guess where this will go? Why? And this? Why? Can you arrange the rest of the pieces using this idea?" Finally learners are asked to describe the way the pieces were sorted, and if they know names for each pile. They are expected to sort the triangles as equiangular, acute-angled-, right-angled and obtuse angles triangles. For each group of triangles there is a name card and after sorting learners are allowed to use these. 	This activity was designed to assess the learner's ability to think about shapes in terms of properties rather that merely a shape's appearance. This sorting activity was again presented in a "Guess my rule" format because it was found that when the challenge to arrange pieces was presented in a less structured manner, learners often tried to put pieces together in a puzzle format and it was awkward to establish what the meant by sorting.

10.2	<u>Listing properties (Triangles)</u> The teacher points to a group of triangles and says: "If you were talking with your friend over the phone and you wanted to describe these triangles, what could you say about them?" For each group of triangles there is a name card and a set of color-coded property cards. As learners mention a name or a property, the teacher places the corresponding card with the group. Learners are encouraged to say as much as they can about one type of triangle, then go on to other triangles. Prompts are given as needed. This continues until all properties are listed.	This activity assessed a learner's ability to characterize the groups of triangles in terms of properties, and also through guided questioning, to instruct the learners in this area.



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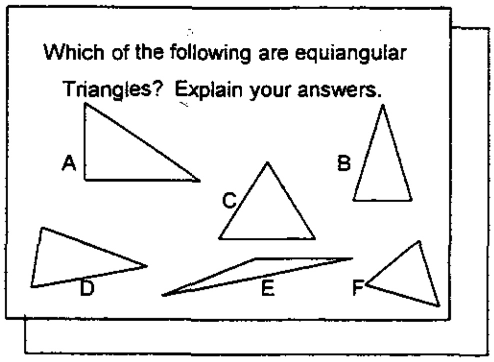
<td>Learners then are asked to look at selection sheets for each triangle, on which they identify examples and non-examples, and to explain their thinking.

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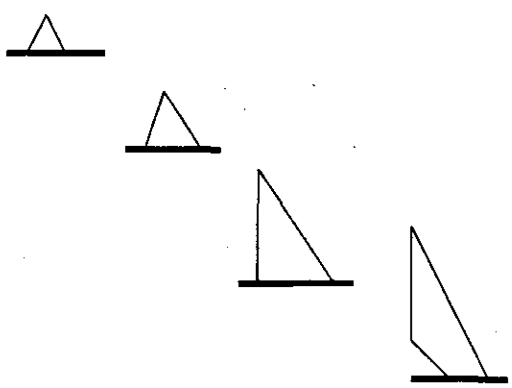
<tr>
<td>11.1</td>
<td><u>Uncovering shapes (Triangles)</u>
The teacher uncovers a cardboard triangle cutout in four stages, asking at each stage "What could this be? Could it be anything else? Why? What couldn't it be? Why?" The process is repeated for a second triangle if the learners seemed to be confused about the directions the first time. This process is repeated until equilateral, isosceles and scalene triangles have been dealt with.
</td>
<td>This activity was designed to assess how the learner uses partial information about the figure (a partial view or a few properties) to make judgements about what the figure could be or could not be.</td>
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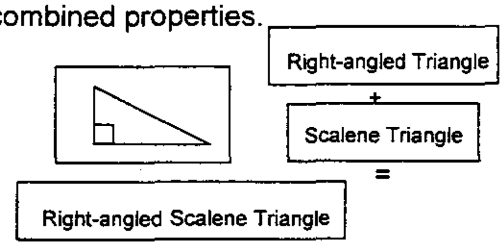
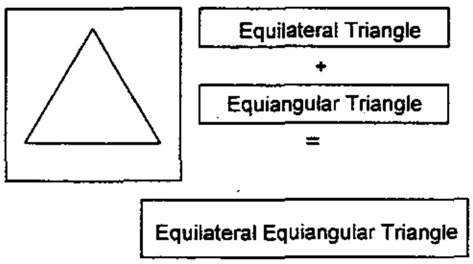
11.2	<u>Minimum properties (Triangles)</u> Learners are shown a collection of "clue cards" for an equiangular triangle. The teacher says: "Here is a lot of properties of an equiangular triangle that we have talked about. Suppose that you wanted to make up some clues for an equiangular triangle for your friend. Do you think that your friend would need to see ALL of these properties to know that you were talking of an equiangular triangle? Which cards could you take away? Why? Any others?" When the learners have finished, the teacher takes away first a card which is necessary, then one which is not necessary (if any remain) and asks for each: "Could I take this away? Why?" If the explanation is incomplete, the teacher asks: "Can you show me why in a drawing?" This procedure is repeated for acute-angled-, right-angled- and obtuse-angled triangles. There are 3 sides There are 3 angles All angles are the same	This activity assessed if learners could deduce some property from another, within the context of their knowledge. They were encouraged to use a drawing to check their thinking. This activity asked a learner the fewest properties necessary to describe a shape. It was not expected that students would be able to give complete arguments about why some properties implied others. This activity did allow the teacher to assess the learners' recognition of the need for such arguments, and the students' ability to search for counter-examples through drawing. Level 1 thinkers might respond by saying that "yes, a equiangular triangle has all those properties" without seeing duplication or logical relations. Level 2 thinkers would reason ("You don't need *There are 3 angles* because *There are 3 sides* implies the forming of 3 angles") or try to test a hypothesis ("If there are 3 angles, then there will be 3 sides").

<table>
<tr>
<td>11.3</td>
<td><u>Classification of triangles according to angles and sides</u>
Learners are shown a collection of cutout triangles. They are asked to sort these triangles by using the knowledge they already possess. If the standard sort is not produced, a "Guess My Rule" format is followed. Eventually a sort according to sides AND angles are expected. Learners are encouraged to give names to their new groups based on their combined properties.



</td>
<td>This activity was designed to assess the learner's ability to think about shapes in terms of properties rather that merely a shape's appearance. This concluding activity is designed to summarize both classifications of triangles according to sides and angles.

(This activity was received with great enthusiasm as learners enjoyed "making up new names" for triangles.)</td>
</tr>
</table>

4.9 SUMMARY

In this chapter the study population and sample were discussed in paragraph 4.3, the instrumentation in paragraph 4.4, the variables used in paragraph 4.5, and the method of research in paragraph 4.6. The statistical procedures and techniques were outlined in paragraph 4.7 and the procedure used in paragraph 4.8. In Chapter 5 statistics will be analyzed and the results interpreted.

CHAPTER FIVE

ANALYSES AND INTERPRETATION OF RESULTS

5.1 INTRODUCTION

The aim of the study was to investigate geometry learning in a problem solving context from a social cognitive perspective. In this chapter, the hypotheses are stated (see § 5.2) followed by summary statistics (see § 5.4) and the testing of hypotheses (see § 5.5 – 5.9).

5.2 HYPOTHESES

Five main hypotheses were set of which one differentiated into ten sub-hypotheses.

5.2.1 MAIN HYPOTHESIS 1

The implementation of a Van Hiele based learning and teaching program, in a problem solving context, has an influence on learning strategies (as defined as the ten sub-scales of the LASSI-HS).

5.2.1.1 Sub-hypothesis 1.1

The implementation of a Van Hiele based learning and teaching program, in a problem solving context, has an influence on the attitude of the learners.

5.2.1.2 Sub-hypothesis 1.2

The implementation of a Van Hiele based learning and teaching program, in a problem solving context, has an influence on the anxiety of the learners.

5.2.1.3 Sub-hypothesis 1.3

The implementation of a Van Hiele based learning and teaching program, in a problem solving context, has an influence on the concentration of the learners.

5.2.1.4 Sub-hypothesis 1.4

The implementation of a Van Hiele based learning and teaching program, in a problem solving context, has an influence on the way information is processed by learners.

5.2.1.5 Sub-hypothesis 1.5

The implementation of a Van Hiele based learning and teaching program, in a problem solving context, has an influence on the selection of main ideas by the learners.

5.2.1.6 Sub-hypothesis 1.6

The implementation of a Van Hiele based learning and teaching program, in a problem solving context, has an influence on the use of study aids by the learners.

5.2.1.7 Sub-hypothesis 1.7

The implementation of a Van Hiele based learning and teaching program, in a problem solving context, has an influence on self-testing by the learners.

5.2.1.8 Sub-hypothesis 1.8

The implementation of a Van Hiele based learning and teaching program, in a problem solving context, has an influence on the test-taking strategies of the learners.

5.2.2 MAIN HYPOTHESIS 2

The implementation of a Van Hiele based learning and teaching program, in a problem solving context, has an influence on the self-efficacy of the learners.

5.2.3 MAIN HYPOTHESIS 3

The implementation of a Van Hiele based learning and teaching program, in a problem solving context, has an influence on the intrinsic value for a (mathematical) task.

5.2.4 MAIN HYPOTHESIS 4

The implementation of a Van Hiele based learning and teaching program, in a problem solving context, has an influence on the self-regulation of the learners.

5.2.5 MAIN HYPOTHESIS 5

The implementation of a Van Hiele based learning and teaching program, in a problem solving context, has an influence on the geometric thought levels of the learners.

5.3 PROCEDURE

A series of effect-sizes was used to determine the influence of the independent variable (see § 4.5.1) on the dependant variables (see § 4.5.2)[1]. (Also see §4.8 for a more detailed description on the procedure used in this study.)

5.4 SUMMARY STATISTICS

The summary statistics for each variable in the pre- and post-tests of the experimental and control groups were calculated (see table 5.1 – 5.3).

Table 5.1 Summary statistics for pre-test of experimental groups 1 and 2

Variables	Mean		Standard deviation		Smallest Value		Largest Value	
	E1	E2	E1	E2	E1	E2	E1	E2
Attitude	27,20	29,16	5,86	5,30	13,00	16,00	39,00	39,00
Anxiety	25,86	25,64	5,65	6,36	12,00	13,00	36,00	38,00
Concentration	23,18	21,94	5,47	5,33	12,00	12,00	35,00	34,00
Information processing	27,27	26,06	5,15	4,94	9,00	15,00	36,00	40,00
Selecting main ideas	16,27	15,04	3,56	3,31	6,00	7,00	23,00	24,00
Study aids	27,73	24,94	5,08	5,52	15,00	12,00	40,00	35,00
Self-testing	29,18	26,95	5,60	5,68	12,00	13,00	38,00	40,00
Test-taking strategies	24,78	22,39	5,78	6,11	9,00	9,00	37,00	26,00
Self-efficacy	3,89	3,75	0,73	0,65	1,78	1,33	5,00	5,00
Intrinsic value	4,34	4,12	0,48	0,50	3,00	3,00	5,00	5,00
Self-regulation	4,46	4,21	0,61	0,52	3,00	2,56	5,67	5,44

[1] Intervals are small, medium, and large

Table 5.2 Summary statistics for post-test of experimental groups 1 and 2

Variables	Mean		Standard deviation		Smallest Value		Largest Value	
	E1	E2	E1	E2	E1	E2	E1	E2
Attitude	30,52	28,86	4,50	5,75	19,00	17,00	40,00	40,00
Anxiety	24,60	25,65	7,53	6,41	9,00	11,00	40,00	40,00
Concentration	27,69	27,45	5,58	5,89	14,00	13,00	40,00	40,00
Information processing	24,90	23,66	5,60	5,28	8,00	12,00	37,00	37,00
Selecting main ideas	14,48	16,42	3,13	3,38	7,00	8,00	21,00	25,00
Study aids	24,35	22,55	4,85	5,73	8,00	10,00	34,00	37,00
Self-testing	25,65	25,42	5,70	5,40	11,00	15,00	35,00	39,00
Test-taking strategies	23,71	27,43	5,61	5,84	12,00	14,00	37,00	40,00
Self-efficacy	3,72	3,49	0,69	0,70	2,22	1,89	4,78	5,00
Intrinsic value	4,14	3,73	0,60	0,61	2,22	2,33	5,00	5,00
Self-regulation	4,15	3,76	0,95	1,32	0,44	0,33	5,22	5,33

Table 5.3 Summary statistics for post-test of control groups 1 and 2

Variables	Mean		Standard deviation		Smallest Value		Largest Value	
	C1	C2	C1	C2	C1	C2	C1	C2
Attitude	29,81	27,77	7,77	7,24	11,00	12,00	40,00	40,00
Anxiety	24,00	24,89	4,38	8,02	18,00	10,00	35,00	36,00
Concentration	29,47	26,32	6,61	7,17	18,00	13,00	40,00	40,00
Information processing	27,06	25,23	5,63	6,14	18,00	12,00	39,00	38,00
Selecting main ideas	16,19	16,25	4,51	4,32	10,00	6,00	25,00	25,00
Study aids	27,88	22,57	5,79	5,26	16,00	12,00	40,00	38,00
Self-testing	29,56	24,68	6,30	5,78	16,00	12,00	39,00	36,00
Test-taking strategies	26,69	26,52	5,61	7,51	16,00	11,00	38,00	39,00
Self-efficacy	3,44	3,53	0,92	0,73	2,11	1,89	5,00	5,00
Intrinsic value	4,06	3,62	0,79	0,76	2,11	2.11	5,00	5,00
Self-regulation	3,94	3,62	0,62	0,62	2,67	2,56	5,11	4,89

5.5 THE EFFECT THE IMPLEMENTATION OF A VAN HIELE BASED LEARNING AND TEACHING PROGRAM, IN PROBLEM SOLVING CONTEXT, HAS ON LEARNING STRATEGIES

To test sub-hypotheses 1.1 to 1.8, that *the implementation of a Van Hiele based learning and teaching program, in a problem solving context, had an influence on the various learning strategies* as defined by the subscales of the LASSI, effect-sizes were calculated to determine whether the program had any effect on the learners' learning strategies.

An analysis of the effect-sizes revealed only one effect size of practical significance, and that was concentration, d = 0,86 (see table 5.4). Therefore, only sub-hypothesis 1.3 can be accepted. The acceptance of this sub-hypothesis implies that learners who partook in this experiment are better in directing their attention to school and school-related tasks, including study activities (also see § 4.4.1.5).

As no effect-sizes of practical significance for the other sub-hypotheses could be found, none of these hypotheses could be accepted.

Table 5.4 Effect-sizes (d-values) for the effect of the Van Hiele based treatment on concentration

	E1 post-test (d)	E2 post-test (d)	C1 post – test (d)	C2 post-test (d)	Total E pre-test (d)	Total C post-test (d)
E1 pre-test	0,5*	———	———	———	———	———
E2 pre-test	———	0,5*	———	———	———	———
E1 post-test	———	0,1	0,3	———	———	———
E2 post-test	———	———	———	0,2	———	———
Whole E post-test	———	———	———	———	0,9**	0,0
Small effect d = 0,2 Medium effect d = 0,5* Large effect d = 0,8**						

5.6 THE EFFECT THE IMPLEMENTATION OF A VAN HIELE BASED LEARNING AND TEACHING PROGRAM, IN PROBLEM SOLVING CONTEXT, HAS ON SELF-EFFICACY

To test the hypothesis, *that a Van Hiele based learning and teaching program, in a problem solving context, has an effect on self-efficacy*, effect-sizes were used to determine the practical significance of the program (see table 5.5).

Table 5.5 Effect-sizes (d-values) for the effect of the Van Hiele based treatment on self-efficacy

	E1 post-test (d)	E2 post-test (d)	C1 post – test (d)	C2 post-test (d)	Total E pre-test (d)	Total C post-test (d)
E1 pre-test	0,5*	———	———	———	———	———
E2 pre-test	———	0,3	———	———	———	———
E1 post-test	———	0,4	0,3	———	———	———
E2 post-test	———	———	———	0,1	———	———
Whole E post-test	———	———	———	———	0,3	0,1
Small effect d = 0,2 Medium effect d = 0,5* Large effect d = 0,8**						

The hypothesis can not be accepted because of the low d-values (effect sizes).

5.7 THE EFFECT THE IMPLEMENTATION OF A VAN HIELE BASED LEARNING AND TEACHING PROGRAM, IN PROBLEM SOLVING CONTEXT, HAS ON INTRINSIC VALUE

To test the hypothesis, *that a Van Hiele based learning and teaching program, in a problem solving context, has an effect on intrinsic value*, effect-sizes were used to determine the practical significance of the program (see table 5.6).

Table 5.6 Effect-sizes (d-values) for the effect of the Van Hiele based treatment on intrinsic value

	E1 post-test (d)	E2 post-test (d)	C1 post – test (d)	C2 post-test (d)	Total E pre-test (d)	Total C post-test (d)
E1 pre-test	0,4	——	——	——	——	——
E2 pre-test	——	0,4	——	——	——	——
E1 post-test	——	0,7*	0,1	——	——	——
E2 post-test	——	——	——	0,1	——	——
Whole E post-test	——	——	——	——	0,5*	0,1
Small effect d = 0,2 Medium effect d = 0,5 * Large effect d = 0,8 **						

The hypothesis can not be accepted due to the low d-values (effect sizes).

In comparing E1 and E2's Van Hiele post-test a medium effect-size (d=0.7) for learners' intrinsic value (or intrinsic interest in class work well as preference for challenging work and master goals, see § 4.4.2) could be reported (see table 5.6). An effect size of 0.7 approaches the practical or educational significant value of 0.8.

5.8 THE EFFECT THE IMPLEMENTATION OF A VAN HIELE BASED LEARNING AND TEACHING PROGRAM, IN A PROBLEM SOLVING CONTEXT, HAS ON SELF-REGULATION

To test the hypothesis, *that a Van Hiele based learning and teaching program, in problem solving context, has an effect on self-regulation*, effect-sizes were used to determine the practical significance of the program (see table 5.7).

Table 5.7 Effect-sizes (d-values) for the effect of the Van Hiele based treatment on self-regulation

	E1 post-test (d)	E2 post-test (d)	C1 post – test (d)	C2 post-test (d)	Total E pre-test (d)	Total C post-test (d)
E1 pre-test	0,3	———	———	———	———	———
E2 pre-test	———	0,3	———	———	———	———
E1 post-test	———	0,2	0,2	———	———	———
E2 post-test	———	———	———	0,3	———	———
Whole E post-test	———	———	———	———	0,3	0,3
			Small effect Medium effect Large effect	d = 0,2 d = 0,5 d = 0,8		

The hypothesis can not be accepted in view of the low d-values (effect sizes).

5.9 THE EFFECT THE IMPLEMENTATION OF A VAN HIELE BASED LEARNING AND TEACHING PROGRAM, IN A PROBLEM SOLVING CONTEXT, HAS ON GEOMETRIC THOUGHT LEVELS

To test the hypothesis, *that a Van Hiele based learning and teaching program, in a problem solving context, has an effect on geometric thought levels*, effect-sizes were used to determine the practical significance of the program (see table 5.8).

Table 5.8 Effect-sizes (d-values) and statistical significant value (p-values) for the effect of the Van Hiele based treatment on geometric thought levels

		C1 post-test (p)	C2 post-test (p)	E2 post-test (p)		C1 post-test (d)	C2 post-test (d)	E2 post-test (d)	Total C post-test (d)
E1 post	Level 1	0,02*	—	0,01*		1,2**	—	1,3**	—
	Level 2	0,00*	—	0,01*		1,4**	—	1,1**	—
	Level 3	0,80	—	0,08		0,2	—	1,1**	—
E2 post	Level 1	—	0,00*	—		—	2,1**	—	—
	Level 2	—	0,00*	—		—	1,9**	—	—
	Level 3	—	0,03*	—		—	0,9**	—	—
Total E past	Level 1	—	—	—		—	—	—	1,3**
	Level 2	—	—	—		—	—	—	1,5**
	Level 3	—	—	—		—	—	—	0,3
p < 0,05* = statistically significant						Small effect d = 0,2 Medium effect d = 0,5* Large effect d = 0,8**			

The hypothesis, *that a Van Hiele based learning and teaching program, in a problem solving context, has an effect on geometric thought levels,* can therefore be accepted due to the large effect-sizes.

A change in language and behavior are critical factors in the movement through the Van Hiele levels (see § 3.3.3.1) as changes in these factors make progression visible. In determining the effect of the Van Hiele based program on learners' use of language and behaviour, video recordings were used to make comprehensive transcriptions (as taped during the duration of the program). These transcriptions were used to make comparisons between the two classes in experimental group 2 (see § 5.9.1.1) as well as the classes in experimental group 1 (see § 5.9.1.2) regarding learner behaviour and the use of language. A comparison between experimental groups 1 and 2 was made using transcriptions of classroom activities and interviews with the respective teachers (see § 5.9.1.3).

To compare the experimental groups and control groups the Van Hiele post-test was administered (see § 4.4.3 and Appendix D). The Van Hiele post-test consists of questions in which learners can give a variety of answers. A more in depth analysis (see § 5.9.2 - § 5.9.6) was made to determine the (possible) differences in thought level. A few concepts or areas within geometry (general acquisition {§ 5.9.2}, spatial orientation {§ 5.9.3}, general identification of triangles {§ 5.9.4}, confusion between right angles and right-angled triangles {§ 5.9.5}, and the identification and characterization of isosceles triangles {§ 5.9.6}) were focussed on, to explain the difference between the experimental and control groups as well as between experimental groups 1 and 2.

5.9.1 COMPARISONS WITHIN THE EXPERIMENTAL GROUPS

5.9.1.1 Comparing experimental group 2 classes

Learners in this school were placed in classes according to various characteristics such as academic performance or behaviour. Two distinct classes were formed that were homogeneous with reference to academic achievement.

5.9.1.1.1 Evaluating the differences in language and behaviour of class number 1

Both the teacher and principal described this class as the more "better behaved and more intelligent". Teachers expected good academic performance and behaviour of this class due to previous behaviour and achievement.

In the beginning of the program (see § 4.8.2, activity 1.3), learners were asked to complete selection sheets that allowed for detailed assessment of the learners' grasp of concepts, such as squares, rectangles, parallelograms, triangles etc. From the observations it was deduced that learners used vocabulary of a "high level of knowledge" concerning the relevant figures as the following verbal transcriptions[2], of learners' characterization of a rectangle, indicate:

"...the opposites are the same..."

"...you have to have 2 pairs of opposite sides..."

"...2 opposite sides are equal..."

In identifying triangles (activity 1.3) learners demonstrated a misconception about the qualifying property of a triangle. In looking at a right-angled scalene triangle, learners remarked: "...it has one straight side... it is not a triangle" and "...opposite sides (are) not equal...not a triangle". It later became evident (when analyzing the written Van Hiele post-test, see § 5.9.3) that learners in both the experimental and control groups conceived a triangle to be a figure where all three the sides had to be equal (equilateral triangle).

When dealing with lesser known or less frequently dealt with figures such as the rhombus (activity 1.2), these learners used vocabulary reminiscent of their counterparts in class number 2 as the following transcriptions indicate:

"...not parallelogram, it is more like a square..."

"...all sides are equal but it is only slanted..."

[2] These answers are written down in verbatim as transcribed from video recording of class activities

The interaction in the beginning of the program among the learners was slow and teacher encouragement became necessary. The use of measuring instruments to verify "assumed" characteristics of figures was non-existent. Learners decided on properties by visual judgements and recalled knowledge of previous years' work.

The rate at which these (the "better behaved and more intelligent") learners completed the activities steadily increased. By the middle of the program these learners were one activity or more ahead of their counterparts in class number 2. The use of a ruler or a protractor came naturally as the learners realized that the only way to prove an answer was to have concrete proof of a specific fact, that could be done only by a ruler or a protractor depending on the task, typical level 2 behaviour (see § 3.3.3.1 for a detailed description of the Van Hiele levels). Learners by this time were evaluating their own as well as their friends' answers or comments by intense debating followed by measurement, clear signs of self-regulation (see § 2.4.3 and 2.5.2 for a detailed discussion on the subprocesses and behavioural influences in self-regulation). Mistakes in the use of the protractor were brought to light in the differences in answers. Learners tried to convince one another of the correctness of their answers by showing their method of measuring. This interaction led to a form of "peer-teaching" where learners corrected their mistakes through the help and instruction of fellow learners. It can be theorized that the above mentioned behaviour is a result of the impact of problem solving.

In the final activity (activity 12.3) of the program, learners were requested to sort triangles by using all the knowledge they had gathered in this program. Learners sorted first by either attending to sides or angles but through debate realized that sorting according to both sides and angles, were possible. Spontaneous measuring took place of both sides and angles if any doubt existed. This activity led to learners changing the activity into a type of game in which one group of learners displayed a triangle while the other group had to give a "comprehensive name". Some learners made their "own triangles" in an effort to "out smart" other learners, an indication of level 2 behaviour (see § 3.3.3.1 for a detailed discussion on the Van Hiele levels).

The teacher was amazed that learners could spontaneously change such a dull repetitious exercise into a really fun activity. It can be theorized that learners did benefit from taking part in at least the increase in social skills.

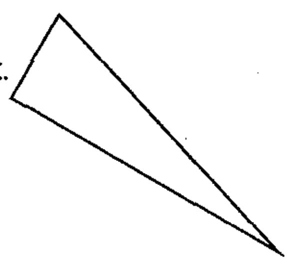
5.9.1.1.2 Evaluating the differences in language and behaviour of class number 2

At the beginning of the program this class was clearly characterized as the "difficult" group (even the principal wished the researcher luck when starting the program in this class). This class was seen as the less intelligent class, with mostly the "trouble makers and friends" making life a bit of a challenge for all the teachers in the school.

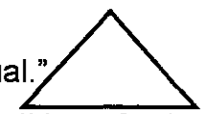
In the beginning of the program (see § 4.8.2, activity 1.3), learners (as was the case with class 1) were asked to complete selection sheets that allowed for detailed assessment of the learners' grasp of concepts, such as squares, rectangles, parallelograms, triangles etc.

In completing this activity, no specifications on the use of measuring instruments were given. Learners were left to interact with these selection sheets in the manner that they would naturally do with no teacher intervention. The learners could therefore use any instrument that they deemed necessary, such as rulers, but none of the learners even attempted to measure rectangular figures to determine whether a figure was a rectangle or a square when confusion arose.

In this early stage of the program somewhat diverse and even erroneous answers emanated from class 2 learners as the following conversation, dealing with the identification of triangles (such as the one below) shows:



Learner 1: "It is not a triangle because the bottom line is squint.
Learner 2: No, it is too squashed up.
Learner 3: Not a triangle because it doesn't look like a triangle.
Learner 4: No because the sides are askew."

Confirming a triangle's status as a triangle seemed to be determined by visual judgement of the sides. The following figure was deemed a triangle:
"It is a triangle because everything is perfect and they (sides) are all equal." 
The sides had to be equal (as in a equilateral triangle) with strict conditions for its orientation on the paper.

The above mentioned language indicates that learners used language that was not subject specific, but "general terms" that were not very good descriptions of figures (e.g. "squint, squashed up", and "askew"). Their judgements were obviously based on what they could visually perceive without giving much attention to more accurate "findings" by means of measurement – true level 1 behaviour (see § 3.3.3.1 for a more detailed description of the Van Hiele levels).

Not only was the learners' language on a low level but their participation was sluggish and needed a lot of prompting from the teacher. The following conversation between the teacher and a learner demonstrates the level of argumentation and the constant prompting of the teacher to get the learner to elaborate at this early stage :

Teacher: "Is this a triangle?"

Learner: "Yes, two sides are equal."

Teacher: "How do you know it's equal?"

Learner: " You can see them, like...look at them, they look equal."

Teacher: "If I ask you to give a definition of a triangle, what would you say?"

Learner: "A triangle is a...yo...aaa...3 corners and all sides must be equal."

Learners perceived a triangle to always have 3 equal sides in a "conventional straight up" orientation (). This visual orientation was also adapted for a square, with a square becoming a rhombus as soon as the orientation "turned" () (see § 5.9.3 for a more detailed discussion on learners' concept of spatial orientation).

Later in the program, the learners took over the role of the teacher, asking questions and actively questioning one another's answers, notions and thought directions. The following transcription is an excellent example of this increase in more specific language but also the break through to the next level of argumentation (level 2, see § 3.3.3.1 for a detailed description of the Van Hiele levels).

Learner 1: "The sides are much more longer than the long one are, but it has got 3 corners...has 3 sides...mmm. *It is a triangle* because it doesn't matter if its got equal sides...because it still got 3 corners."

Learner 2: "The 2 long ones (sides) are supposed to be equal."

Learner 1: "It doesn't matter, it's still got 3 corners."

Learner 2: "This line, if you take a ruler, is totally squint, it's suppose to be straight.

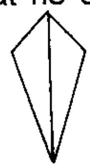
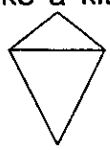
Learner 3: "I think it's a triangle, no matter how the shape looks like or how it's formed. Its gotto have 3 sides to be a triangle."

Question (by researcher): "As long as it's got 3 sides it's a triangle? Any shape?"

Learner 3: "Yes....NO! "(Drawing is made: ) "It has 3 sides but it is not closed up."

(Make drawing:  "This is now a triangle").

Learners here demonstrated that they were unconsciously constructing properties of a triangle that are usually not formally taught in primary school (by implying that a triangle needed to be a closed figure).

As the program progressed, learners felt more comfortable to experiment with the paper figures and their participation increased to such an extent that the teacher even became worried about the noise level. The worry about the noise level was, overshadowed by the amazement of the teacher about the amount of "extra non-syllabus" information that started to flow through the class. A surprising and spontaneous activity was when learners started to disseminate a kite into "elements". One learner started by noticing that he could make a kite by placing two triangles together (in two different ways)  and  .

This discovery caused an eruption on ideas of different ways in which a kite could be formed, using different figures in different combinations. Figures such as squares also were discussed with amazing learner involvement. This lesson was an excellent staring point for other topics in the syllabus such as tessellations and struts with in figures.

Slowly words such as "opposite", "equal to" and "parallel to" started appearing in learner vocabulary (class 2). By the time learners started dealing and sorting equilateral, isosceles and scalene triangles (see § 4.8.2, activity 9.1-10.1) their thoughts and vocabulary were structured and well defined as the following answers prove:

Learner 1: "An isosceles has 3 sides."

Learner 2: "And 2 sides are equal and 1 side is longer."

Learner 3: "Only 2 sides are equal."

Learner 4: "With at least 2 acute angles."

Learner 5: "2 acute angles or even three."

The information concerning the angles of an isosceles triangle is of a more complicated nature than what the syllabus prescribes. Learners started to "invent" information by constantly measuring with rulers *and protractors.* An order by the teacher was never given to use protractors; learners started to use them as a tool to verify their own hypotheses concerning the figures that they were working with. The use of a protractor to "invent" more specific information concerning a figure ensured that almost all learners not only bought but also brought their own protractors to school. Measuring, by using a ruler (or protractor), had increased by such measures that it became an object that was refused when asked to be borrowed by someone else. Measuring became a verifying meganism for answers. Learners became more vocal in their thoughts: "Only 2 sides are equal and one is not ..I am not sure...so measure (learner uses ruler to measure sides)....2 sides *are equal* ...I measured it...just check!" This is a comment from a learner who was usually quiet and reserved. Her excitement on successfully completing this activity and knowing that she was correct, prompted her to challenge the rest of the group, an action never before witnessed by the researcher. This quiet girl was smiling broadly and confidence was emanating from her. Success bred success in this mathematics class (class 2) as the learners' speed of completion of activities increased to such an extent that they completed the program in less time (one session of 1 hour) than what was scheduled for it.

The above mentioned details are by no means a full account of what happened in this class, but should serve as highlights of how learners who were reluctant to participate became active participants in their own learning. Learners that started with vague terms completed this program with a strong subject specific vocabulary to describe quadrilaterals and various triangles.

There were bad days too, with numerous interruptions, learners that became rowdy and a teacher that became exhausted trying to keep up with this "new" method and a class that seemed to have an inexhaustible source of energy and inquisitiveness. Learners did not always reach the level of argumentation that was hoped for by the researcher, as learners in the other "smarter" class did. But at the end of the day it can be stated that the progress in this class surprised the teacher the most as he noted that this class (class 2) usually worked so slowly that he sometime in the past had to leave out ("less important") work in order to cover more important work. And those "troublemakers" became problem solvers.

5.9.1.1.3 Comparing conceptual changes in classes 1 and 2 by means of teacher comments

The teacher (and principal) held different expectations for each of these classes based on previous academic achievement and behaviour. Class 2 was not expected to complete this program, and in the end surpassed all expectations.

Both classes made undeniable progress. Steady progress was made by class 1 while class 2 started off not only on a lower level of acquisition but also at a slower speed. In evaluating the overall increase in conceptualization it should be noted that class 1 ended on a higher level (although they started on a higher level). Based on the grouping system in the school which was done on the grounds of academic achievement, class 2 learners performed at a lower level than class 1 learners. Class 2 learners were thus also at a lower end of conceptualization as was proved/suggested by their longer reaction time compared to class 1 learners. Class 2 may have started on a lower level of conceptualization but the increase in knowledge in relation to time spend was higher and more dramatic. Class 2 may still not be on the level of conceptualization of class 1, but class 2's "learning profit" seems to be more evident.

The comments of the mathematics teacher in a discussion about the program and its effects brought the following facts to light. The teacher noted that he at first

- could not see the need for changing his teaching style or method of instruction as it worked in the past – according to him; and
- was worried that this program would take too long and that the syllabus would not be covered.

The teacher also noted that during the running of the program that he realized that

- there was a more positive attitude toward mathematics;
- he felt more exhausted because the learners' inquisitiveness kept him running ("Ek kon voel ek *het skool gegee*, en tog het ek minder klas gegee."); and
- he felt comfortable with his role as facilitator and liked the guiding role more than the role of "policeman".

The teacher came to the conclusion that

- this program was noisier but the increased level of interaction between teacher and learners, and learners and learners compensated for this;

- the spontaneous reaction of learners was surprising and learners reacted well to discussion compared to just an indication of the correctness of an answer by the teacher;
- next years' progress will be quicker (and maybe better), as *he* now knows how to use this program; and
- he was contemplating trying to use the general principles (of this program) in other topics such as volume.

5.9.1.2 Comparing experimental group 1 classes

Learners in this school (experimental group 1) were randomly placed in classes. Little difference existed in academic performance or behaviour. From the start of the program the teacher was skeptical about the need for change and of the need for group work.

She did divide the learners into groups but only after a long discussion with the researcher on the use of group work. The researcher even then needed to move the tables in suitable group formations before the learners arrived. Even when learners were moved into these groups they were given little chance of working on their own before the teacher intervened to give her input. Sorting activities were sabotaged as the teacher dominated these activities by either giving too little time for independent group work or allowing no time for group work. She gave distinct "clues" of how she expected the information in tests and even directly told learners what to do, for example: In activity 9.2 learners sorted triangles, by focussing on the length of the sides (see § 4.8.2). Here the teacher simply took three triangles and directly told the learners to sort the triangles according to the length of the sides. She then pasted these triangles on the board as examples. Learners were instructed to use their rulers.

Numerous discussions and "re-training" sessions failed to completely change this teacher's perspective on teaching and learning of mathematics.

During the small amount of group work that took place it became evident that these learners demonstrated a low level of argumentation and conceptualization. A learner that described a trapezium said: "...look like a bowl...a pudding dish..."
After five activities learners still described figures according to visual judgements. In one sorting activity (activity 3.1) learners sorted kites with all the rhombuses with the comment: "This one (kite) come here because the sides are the same...it's skew..." A rhombus was described as: "...looks like a diamond". The teacher gave little time for learners to explain their sorting, but rather demonstrated her sorting by asking a rhetorical question if the learners agreed. This behaviour of the teacher resulted in little or no difference in the rate of learner involvement in the class room activities when comparing the beginning, middle and end of the program.

During some sessions (of an hour) the only learner participation was the occasional "YES" to acknowledge something the teacher said (for example activities 4 and 7.2). Some activities were simply left out by the teacher as she could either not see the use for the activity, or blamed her fear of not completing the syllabus (for example activities 3.1, 6.2, 10.3 and 12.1). The completion of the home work sheets was deemed more important than the designed classroom activities.

By the end of the program, the learner participation was almost non-existent with the teacher dominating every component of the classroom activities. This small amount of learner participation let to insignificant information on the increase of learner conceptualization and it is unclear if any change did take place. The increase in the level of conceptualization or argumentation (if any) is a bonus in a program that was nullified by teacher action (based on her beliefs, perceptions and expectations) (See Fennema & Franke, 1992:147-164, for a detailed discussion on teachers' knowledge and its impact on learning and teaching) .

5.9.1.3 Comparing experimental groups 1 and 2

Differences between these groups existed from the start of the program. The first difference between E1 and E2 was the language of instruction, which in it self caused difficulties in comparing these two groups. Non the less, some comparisons are possible by giving attention to the attitude of the teacher towards various aspects within the learning environment, the change in learner activity and the general benefit for learners involved in this program.

Although both teachers entered the program voluntarily, their attitude towards teaching mathematics, their views about their learners' abilities to learn the prescribed work and their attitude towards the program differed dramatically.

The teacher in E1 viewed mathematics as a fixed construct of knowledge (as prescribed by the syllabus) that she had to teach directly to her learners. Mathematics was seen as a staircase, with concepts making up the steps. Each concept should be dealt with (taught) thoroughly (by using a "recipe" that the teacher gave) before she could move on to the next "step". She saw no need for the change in her teaching style or learners' involvement as she felt her success rate in previous years was good enough. Her teaching was directed at completing the syllabus, a characteristic synonymous with "content-based teaching". She did not believe that her learners could successfully learn within this program (See Fennema & Franke, 1992:147-164 and Thompson, 1992:127-146, for a detailed discussion on teachers' knowledge and beliefs and its impact on teaching and learning). She turned out to be hesitant to give control of classroom activities over to the learners.

E2's teacher's main aim was to get learners to understand the concepts that were being dealt with (a characteristic of "concept-based teaching"), even if it meant that fewer topics would be covered. He was hesitant in changing his teaching style, as it worked in the past, but he was willing to try a new approach if it meant that his learners would benefit. His general teaching style resembled a form of learner centered teaching but he still taught a considerable quota of concepts in a direct teaching style (as observed in the desensitizing period). He held different expectations for each of his classes based on their previous academic performances.

E1's learners did (hopefully) benefit from the program but it is possible that the benefits could have been bigger or more significant if the teacher had co-operated in her approach during the program. The possibility that the learners did not benefit at all from the program was also not determinable as no data were available on the knowledge these learners possessed at the beginning of the program. It is also possible that the program was not on the level of these learners' language use, but this seems unlikely as the vocabulary used was on a very rudimentary level. Learners in this group did have a disadvantage in trying to study in their second language.

The learners in E2 not only learned in their first language but also had a teacher that placed a greater value on their understanding than on the completion of the syllabus. It is possible that E2 learners started on a higher level of conceptualization than E1. But the progress made was to the satisfaction (and sometimes "amazement") of the teacher. One group's (number 2) progress was more astounding since this group started on a (perceived) lower level of conceptualization and made such dramatic progress that it was possible that they may have reached the level (or almost the level) of conceptualization of their "more intelligent" counterparts in the other class.

5.9.2 GENERAL ACQUISITION

The Van Hiele levels can be characterized as very complex structures that involve the development of both concepts and reasoning processes (Burger & Shaughnessy, 1986:42). This dual nature of geometric understanding became evident in analyzing the Van Hiele post-test. Both levels of acquisition (see § 5.9.2.1) and categories of acquisition (see § 5.9.2.2) were investigated.

5.9.2.1 Levels of acquisition

Figure 5.1 indicates the average level of acquisition (in percentage) achieved in the Van Hiele post-test.

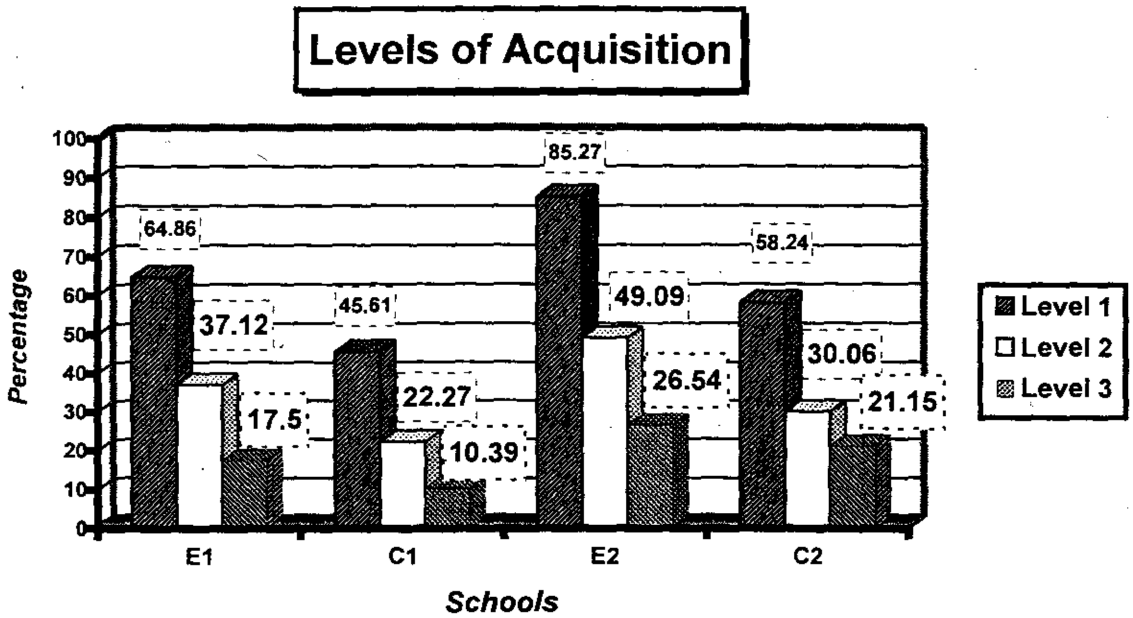


Figure 5.1 Average level of acquisition

An analysis of the data (average level of acquisition for level 1) indicated that the difference between experimental group 1 (E1) and control group 1 (C1) was 19,25 percentage points, but E1 was 42,2% higher than C1. The difference between experimental group 2 (E2) and control group 2 (C2) was 27,03 percentage points, and comparing E2 and C2 indicated that E2 was 46,4% higher than C2. The difference between experimental groups 1 and 2 was 20,41 percentage points, but E2 was 31,5% higher than E1.

An analysis of level 2 acquisition (figure 5.1) revealed that the difference between experimental group 1 (E1) and control group 1 (C1) was 14,85 percentage points and E1 was 66,7% higher than C1. The difference between experimental group 2 (E2) and control group 2 (C2) was 19,03 percentage points and E2 was 63,3% higher than C2. The difference between experimental groups 1 and 2 was 11,97 percentage points but E2 was 32,2% higher than E1.

In acquiring level 3 (figure 5.1), the difference between experimental group 1 and control group 1 was 7,11 percentage points while E1 was 68,4% higher than C1. The difference between experimental group 2 and control group 2 was 5,39 percentage points but E2 achieved 25,5% higher than E2. Experimental groups 1 and 2 differed by 9,04 percentage points and E2 was 51,2% higher than E1.

The experimental groups consistently achieved higher levels of acquisition than their control groups, leading to the conclusion that the program did have a positive effect on the acquisition of high(er) levels of geometric thought. In conclusion, it is evident that experimental group 2 achieved the highest levels of acquisition in all 3 levels. In comparing experimental groups 1 and 2 it is clear that a difference exists. Possible reasons can include the difference in mother tongue and medium of instruction, as E1's mother tongue is mainly African languages and the medium of instruction is English, while E2's mother tongue and language of instruction are both English. A second and maybe the most important reason for the difference between the two groups is the teaching approach the teachers followed. E1's teacher followed a content-based teacher-centered approach while E2's teacher followed a problem orientated learner-centered teaching approach.

5.9.2.2 Categories of acquisition

Figure 5.2 indicates the number of learners in each of the five categories of acquisition (no, low, intermediate, high and complete) for level 1 in the Van Hiele post-test. The number in parenthesis is the number of learners in each group.

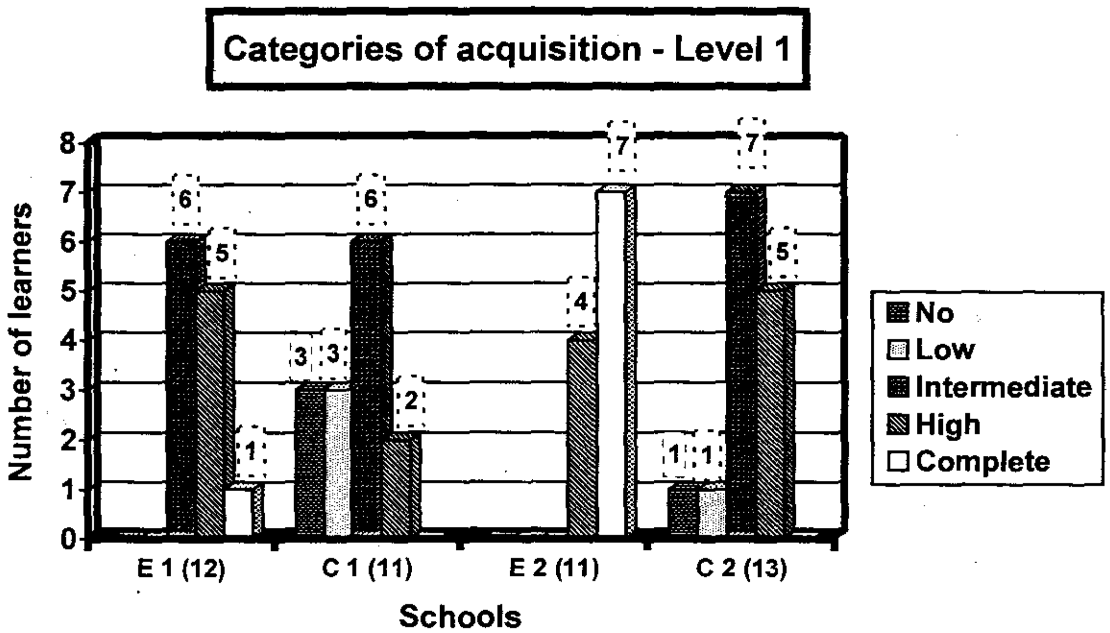


Figure 5.2 Number of learners in the categories of acquisition for level 1

In examining figure 5.2 it became clear that the experimental groups' categories of acquisition started at intermediate and rose to complete acquisition, while the control groups both started at the lowest category namely "no acquisition" and did not go higher than "high acquisition". None of the learners in the control groups reached complete acquisition. The average category of acquisition for E1 was intermediate (50% of the learners), and intermediate (54,5% of the learners) for C1. 63,3% of the learners reached complete acquisition in E2 while 53,8% of C2's learners reached the average level of intermediate acquisition.

Learners who reached the intermediate to complete category were able to identify, name, compare and operate with geometric figures (triangles, angles or parallel lines) according to their appearance (Van Hiele, 1986:39). A learner could also recognize, name figures and distinguish a given figure from others that look somewhat the same with decisions that were based on perception and not reasoning (Mason, 1997:39).

Figure 5.3 indicates the number of learners in each of the five categories of acquisition (no, low, intermediate, high and complete) for level 2 in the Van Hiele post-test. The number in parenthesis is the number of learners in each group.

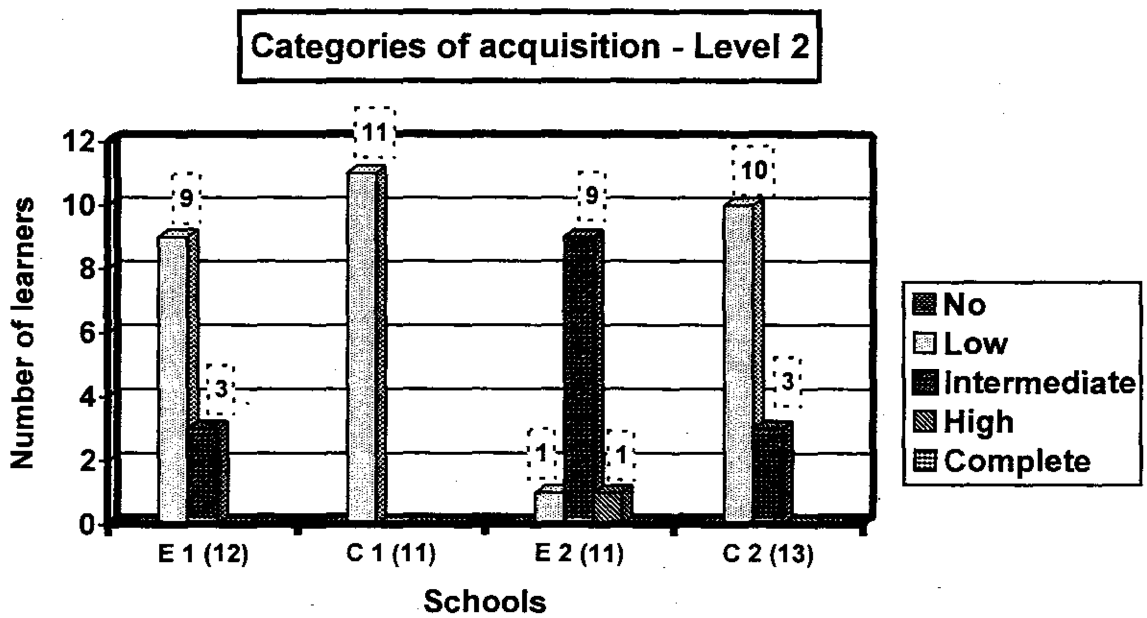


Figure 5.3 Number of learners in the categories of acquisition for level 2

In level 2 acquisition, the difference between the experimental and control groups became more evident. E2 was the only group that could manage to reach the "high acquisition" category, with 81,8% of the learners reaching the "intermediate" category. The bulk of the control group's learners could only manage a low level of acquisition for level 2 compared with the experimental group's learners who managed to reach an average of intermediate level of acquisition. Learners reaching the intermediate category at level 2 were able to recognize and explicitly characterize shapes by their properties (Van Hiele, 1986:40; Fuys *et al.*, 1988:5), but were not able to recognize relationships between classes of figures (Battista, 1994:89).

Figure 5.4 indicates the number of learners in each of the five categories of acquisition (no, low, intermediate, high and complete) for level 3 of the Van Hiele post-test. The number in parenthesis is the number of learners in each group.

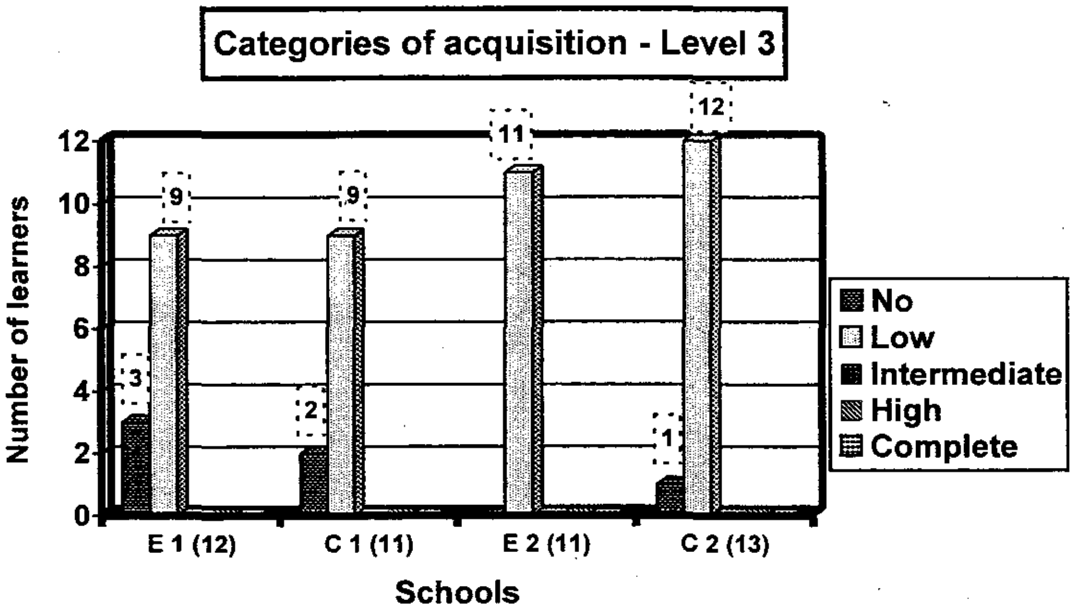


Figure 5.4 Number of learners in the categories of acquisition for level 3

None of the learners (in the experimental or control group) could reach the intermediate category of acquisition, indicating that none of the learners could form abstract meaningful definitions (Mason, 1997:39) or provide logical arguments in the geometric domain (Clements & Battista, 1992:427).

In summary it became evident that the experimental groups clearly achieved higher levels of acquisition as well as higher categories of acquisition which led to the conclusion that the program did have a positive effect on the experimental groups' geometric thought levels.

5.9.3 SPATIAL ORIENTATION

Some questions (6, 10 & 11) in the Van Hiele post-test require learners to identify specific concepts in a variety of orientations. These concepts (like a square, right-angled triangle and isosceles triangle) appear with other concepts in the "classical or conventional textbook form" () as well as in a "unconventional or turned form" (). When analyzing the answers (see table 5.9) to these three questions it seems that recognizing one figure (for example a square) in a "rotated form" does not guarantee recognition of a different figure (for example an isosceles triangle) in a "rotated form". This belief is confirmed, as recognition of the one kind of triangle (right-angled triangle) in a "rotated form" does not guarantee recognition of a different kind of triangle (isosceles triangle) as illustrated in the data shown in table 5.9.

Table 5.9 Identification of a variety of shapes in different spatial orientations

	E1 (12)	C1 (11)	E2 (11)	C2 (13)
Identify □ only in "traditional form"	6 / 2[a]	0 / 2[a]	4 / 1[a]	5
Do not identify □ in "traditional form"	0	1	0	0
Identify □ only in "rotated form"	0	0 / 1[a]	0	0
Identify □ in both forms	4	6 / 1[a]	6	5 / 4[a]
Identify right-angled ∇ only in "traditional form"	7 / 1[a]	1 / 5[a]	0	3
Do not identify right-angled ∇ in "traditional form"	1	3	1	3
Identify right-angled ∇ only in "rotated form"	0	0 / 2[a]	1	1 / 1[a]
Identify right-angled ∇ in both forms	3	0	9	1 / 3[a]
Identify isosceles ∇ only in "traditional form"	1 / 2[b]	0 / 4[a]	0 / 1[a]	0 / 1[b]
Do not identify isosceles ∇ in "traditional form"	2	1	2	5
Identify isosceles ∇ only in "rotated form"	0	0	0	0
Identify isosceles ∇ in both forms	7	3 / 2[a] / 1[b]	6 / 1[a] / 1[b]	3 / 3[a] / 1[b]

[a] Learners identified figure in specified manner but also identified incorrect shapes with the correct answer. [b] Learners correctly identified figure in specified manner, but the answer is incomplete.

Of the four categories for each figure, the last category that entailed identification of figures in both the "traditional" and "rotated" form, is considered the highest category of identification. E2 consistently achieved the highest number of answers (when comparing the number of learners in this category with the number of learners in the group) in all three these categories. It is interesting to note that only 54,5% of E2's learners could recognize the square in both forms, but their recognition of the two kinds of triangles were respectively 81,8% and 72,7%. Figures such as these lead to the hypothesis that if a learner could recognize one kind of triangle (right-angled) in both forms, he/she could most probably recognize any other triangle (isosceles) in both forms. This hypothesis can not be accepted in the light of C2's figures. Only 30,8% of the learners could recognize a right-angled triangle in the "traditional" and "turned" form, but managed to reach 53,8% in recognizing an isosceles triangle in both forms. It is noticeable that E1, C1 and C2 found recognizing the right-angled triangle problematic, as it is here where they scored the lowest marks of all three figures. It becomes evident that E1 fared as well or as poorly as the control groups in many aspects, leading to the conclusion that programs in a problem solving context are not efficient in raising understanding if it is not implemented by using a learner-centered teaching approach.

5.9.4 GENERAL IDENTIFICATION OF TRIANGLES

In testing the general acquisition of triangles, learners were asked to decide if certain figures were triangles and to explain their answers (see question 2, Appendix D). The answers are depicted in table 5.10.

Table 5.10 General identification of triangles

		Schools (with number of learners in each group)			
Question no	Answers	E 1 (12)	C 1 (11)	E 2 (11)	C 2 (13)
2	Correct	6	3 1[a]	10	8 4[a]
	Incorrect	6	7	1	1
(Reasons:)	3 angles	4	1	7	7
	3 sides	3	1	9	2
	More specific answer	3	1	3	

[a] These answers were classified as being correct, but unconvincing or incorrect motivation was given.

Experimental group 1

Learners in this group who decided that the shapes were not triangles rejected them because the sides were not equal (as for an equilateral triangle). Learners pertinently stated" no, because triangle all side are equal" and "no, triangles it has two equal size at the sides and the bottom one is shorter than two equal size in the sides[3]". Most of the learners who decided that the figures were triangles gave more than one answer that usually consisted of a reference to the angles and sides.

Control group 1

Learners who decided that the figures were not triangles all motivated their answers by referring to the visual gestalt of the shapes and therefore rejected the figures as the following answers[3] demonstrate: "no it is not triangls because is not the same" and " no Because they don't look like the triangles". Responses such as these demonstrate classical level 1 responses. Only one learner tried to give a more specific answer, but even this learner's thoughts seem scattered: "Yes Because they are all the same but the shape is very different but is all the same way but the shape are not the same[3]".

[3] These answers are written down verbatim from the (written) Van Hiele post-test

Experimental group 2

Most learners gave a combination of answers[3] that included references to the number of sides and angles needed to be classified as a triangle, for example: "Yes, it has three sides, three angles" and "Yes, Because they all have three sides and they are have three angles they just have different names and sizes." Three learners gave some additional information even when not required to for example: "Yes, There are 3 corners, They don't have to be equal as long Their are 3 sides and 2 acute angles[3]".

It must be noted that this group was at the same level of geometric thought as the level that the relevant control is at at present as the answers[4] that the experimental group learners gave in the first selection sheet (see activity 1.3) concerning the identification of triangles demonstrate[3].

Q: Which of these are triangles? Explain your answers.
A: No (it is not a triangle) ... has one straight side
A: No (it is not a triangle) ... opposite sides not equal

Learners gave answers such as these although they could indicate that a triangle is a triangle because "it has 3 points" and "it has 3 sides" (see activity 1, § 4.8.2). It therefore stands to reason that these learners could only "reproduce" the information they were taught in previous years, but the reproduction was without insight as the answers to activity 1.3 demonstrate.

Control group 2

Learners judged shapes by appearance as this learner's answer[3] demonstrates: " ... all 4 looks like triangles..."- typical level 1 reasoning as defined by various authors (Mason, 1997:39; Flores, 1993:152; Spear, 1993:393; Presmeg, 1991:9). Motivation for defining a triangle was varied, ranging from answers[3] referring to angle shapes ("it

[4] These answers are written down verbatim as transcribed from the videos recorded during the class activities

has 3 sharp points") to answers referring to sides ("their sides al have angles....they are sharp"). A learner that indicated that the shapes are not triangles motivated his answer by writing: "the sides are not the same length".

It seems to be a common phenomenon that learners perceive a triangle to have equal sides, as learners in all the groups rejected isosceles and scalene triangles on the grounds that their sides were not all equal.

In conclusion it can be theorized that E2's answers again proved to be more correct and complete with a total of 19 reasons (form 11 learners) explaining why the figures were triangles (see table 5.11), compared with C2's 9 reasons (from 13 learners), E1's 10 answers (from 12 learners) and C2's 3 answers (from 11 learners). .

5.9.5 CONFUSION BETWEEN RIGHT ANGLE AND RIGHT-ANGLED TRIANGLE

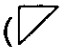
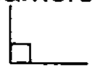
While doing qualitative analysis of the relevant data confusion was noticed between right angles and a right-angled triangle in some groups (see table 5.11). In the Van Hiele post-test (see Appendix D) the concepts of right angles and right–angled triangles are tested.

Table 5.11 Identification of right angle and drawings of right-angled triangle

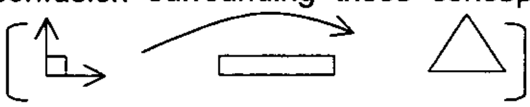
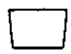
	E1 (12)	C1 (11)	E2 (11)	C2 (13)
Correct identification of right ∠ and correct drawing of right-angled ∇	3	3	9	8
Correct identification of right ∠ but incorrect drawing of right-angled ∇	4	4	1	3
Incorrect identification of a right ∠ and correct drawing of right-angled ∇	1	1	1	1
Incorrect identification of a right ∠ and incorrect drawing of right-angled ∇	4	3	0	1

It is not clear whether the incorrect answers concerning a right angle and/or a right-angled triangle were given because of confusion between the concepts or just carelessness on the part of the learners. To shed light on this uncertainty more in depth investigation would be necessary.

Experimental group 1

In total four learners correctly drew a right-angled triangle. Three of the four drawings are similar to "typical text-book" right-angled triangles () while one learner drew the right-angled triangle in an unusual position (). The eight learners who drew the right angled-triangle incorrectly al drew it as a right angle (). Seven learners in total could correctly identify the right-angle but only three could also correctly draw a right-angled triangle.

Control group 1

Of the four learners who incorrectly drew a right angled-triangle, one learner tried to draw the triangle but only managed to draw an acute-angled triangle (). Two learners' confusion surrounding these concepts was clear when looking at their drawings  and  . It is noticeable that the learners who incorrectly identified right angles even drew more "strange" figures () that resembled a rectangle or a trapezium, instead of a right-angled triangle.

Experimental group 2

An overwhelming number (82%) of learners correctly identified a right angle and correctly drew a right-angled triangle. Most of these learners' drawings also appeared to be "classical text-book" right-angled triangles () while one learner drew the right-angled triangle in an unusual position (). Some of the learners also exhibited more detailed drawings by placing a right angle sign in the relevant angle ().

Control group 2

All four learners who drew the right-angled triangle incorrectly, drew it as an acute angled triangle (△). It stands to reason that these learners may have focussed on the "triangle" part of the question without giving attention to the specification (right-angled). Confusion surrounding right angles in this group seems to be deeper than first discovered as the following drawing shows (see figure 5.5) when learners were asked to draw a rectangle in question 7 in the Van Hiele post-test.

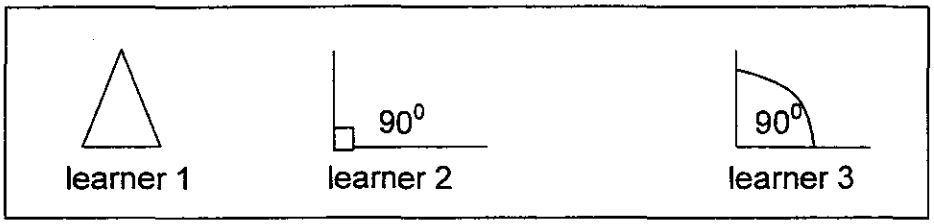


Figure 5.5 Drawings of rectangles

A synopsis of the above mentioned data reveals that E2 is emerging to be the top achiever once again with 81,8% of the learners being able to correctly identify a right angle and drawing a right-angled triangle (see table 5.12). It is disappointing to notice that E1 faired very poorly in correctly drawing a right-angled triangle. Their statistics are as low as C1's and even worse in correctly identifying a right angle and drawing a right-angled triangle (see table 5.11). This poor performance, together with the previous results (see § 5.9.2 to 5.9.4), underlines that a program in a problem solving context without the appropriate teaching method delivers little if any advantages to learners.

5.9.6 IDENTIFICATION AND CHARACTERIZATION OF ISOSCELES TRIANGLES

The data in table 5.12 indicate the number of answers that can be categorized into the indicated groups, and not the number of learners giving such answers, as some learners gave more complex answers that could be divided into more than one "single" answer when identifying and characterizing an isosceles triangle.

Table 5.12 Number of answers providing specific definitions of Isosceles Triangle

Question no	Answers	E1 (12)	C1 (11)	E2 (11)	C2 (13)
4	Identify figure correctly	5	1	6 3[a]	6
4	Identify figure incorrectly	3 3[b]	8 2[c]	2	4 3[b]
16	3 sides	2	1	1	
	sides =	1			3
	2 sides =	5	3	4	3
	only 2 sides =			1	2
	2 sides = with 3rd different	4	1	5	3
	incorrect answer	1	6 1[c]	1	2
17	2 angles =	1	1	1	
	only 2 angles =				1
	2 angles = with 3rd different	6	1	4	2
	3 angles	2	1	1	3
	3 acute angles	1		3	1
	2 acute angle			2	
	1 acute angle	3	1	2	
	incorrect or unclassifiable	1	6 1[c]		8

Note: [a] Learners identified the triangles not as an isosceles but as an obtuse and two acute angled triangles. [b] These learners identified the triangles by indicating angle sizes eg. "An (one) acute angle" or "Two angles are acute" or "Acute-angles triangle"
[c] Not answered.

Experimental group 1

From the 12 learners in the sample, 5 correctly identified the triangles in question 4 (level 1 question, see table 5.12) as isosceles triangles. The answers of the three learners that referred to angle sizes can be grouped in two categories. The first category (that consisted of one learner) answered: "Two enger[5] (angles) are equal" This answer could have been taken as being correct but this learner demonstrated a

[5] Answers are given verbatim with assumed corrections (or meanings) in parenthesis

misconception and gave unclassifiable answers in later questions dealing with the isosceles triangles. An example of this learner's unclassifiable answer[5] relates to information given on the angles of an isosceles triangle: "the engel (angle) of isosceles is two eyes of two person". In the second category three learners only answered "an acute angle". The three learners' whose answers[3] were classified as incorrect ranged from an unsure answer of what to call this type of triangles ("equaletaral...isosceles...scallen[3]"), to referring to size ("it small one[3]"), and an answer indicating an absolute inaccurate concept of isosceles triangles, parallelograms and quadrilaterals ("B (probably the second triangle) is a parallelogram and A and C (probably the first and last triangle) is a Quadrilateral").

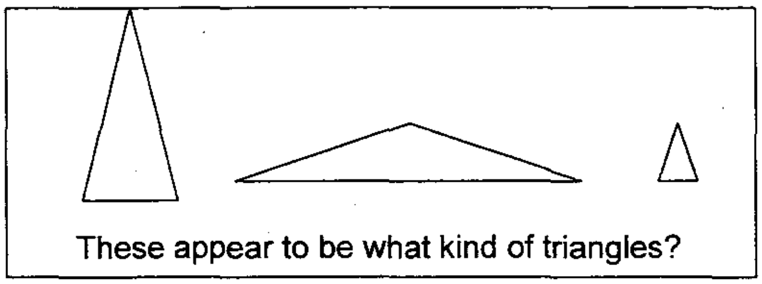


Figure 5.6 Question 4 in Van Hiele post-test

Questions 16 and 17 (level 2 reasoning) required the learners to respond by describing respectively the sides and angles of an isosceles triangle. Eight of the 12 learners gave answers that consisted of only one answer while 4 learners gave more complex answers[3] that could be placed in more than one category for example: "A isoscels triangl have tree angles and two of them are ecual and one is not the same size as the others." It is noticeable that even of these 4 learners only 2 could indicate that all isosceles triangles are not right-angled triangles (question 21 a in the Van Hiele post test, see Appendix D). Although most learners demonstrated level 1 reasoning, a few learners reasoned in ways consisted with level 2 reasoning.

Control group 1

Control group 1's learners struggled even with the level one question (question 4 see table 5.12) in identifying the kind of triangle displayed, as only 1 learner out of 11 could correctly identify the triangles. The ten wrong answers ranged from simple answers such as "triangle", "parallelogram", "straight angle(s)" to "equilacteral" or "scalene" and two non-respondents.

It became evident when interpreting questions 16 and 17 that the learners in this control group (11 learners) had little or no concept of what an isosceles triangle was. A few of these faulty answers are given.

Answers[3] to question 16
"The isosceles triangle line wish to go up..."
"the isosceles triangle is 3 square two are width and I on the bottom is Birdth ..."
"the side of isosceles triangle are not the same because they are not straight"

Answers[3] to question 17
"the hok[5] (hook meaning angle?) is look like the house of the triangle"
"the angles of an isoceles triang is width and straight"
"the triangles look like triangles angles"
"the angles are all the same way but are not the same way and the same part ...
... Because are not the same way"

It was interesting to note that some of the learners that could not recognize the kind of triangle in question 4 were able to define an isosceles triangle, for example one "non-respondent" answered[3]: "Two of the sides are the same length". The inability to answer question 4, therefore, may be contributed to a lack of proficiency in English. It was noted that a learner with a wrong perception consistently reasoned from the same perspective. For example, a learner that called the triangles equilateral triangles in question 1 now writes[5]: "they (being the triangles?) are three angle and they are no all equal".

The geometrical reasoning ability of these learners, as demonstrated by the above mentioned facts and examples, is far below the anticipated and expected level. Most of the learners demonstrated level 1 reasoning, with some learners that could be described as being in transition between level 0 (as defined by Clements and Battista, 1992:429) and level 1.

Experimental group 2

The majority of learners (9) identified the triangles in question 1 correctly, although 3 of these 9 learners identified the triangles as 1 obtuse and 2 acute-angled triangles. One of the two learners who identified the triangles incorrectly, as equilateral and scalene respectively, still gave an acceptable answer in question 16 when describing properties of the sides of an isosceles triangle.

In evaluating answers to question 16, it became clear that this group was reasoning in more clear terms. An example of this reasoning is illustrated in the answer a boy from this school gave in describing the sides of an isosceles triangle[3]:

"Two sides are equal
3 sides
Opposite sides are not parellel"

The only incorrect answer was that of the girl that identified the triangles in question 1 as scalene. Her reasoning stayed consistent as she here describes the properties of an isosceles triangle's sides as: "None of the sides are equal[3]", and the properties of the angles as: "There must be at least 1 Acute angle[3]". Her answer, concerning the properties of the angles, was not classified as incorrect as it was valid even for both isosceles and scalene triangles. It is noticeable that all learners in this group named valid properties of the angles (question 17) of an isosceles triangle, while the control group had 8 incorrect or unclassifiable answers to the same question – a truly remarkable difference.

The comprehensiveness of the answers given by this group (compared with any other groups) persists as the following example in which learners had to give information about the angles of an isosceles triangle reveals:
"they have two equal angles...they are one obtuse or right angle and two acutes[3]"

This group demonstrated higher, more complex and complete geometrical reasoning than any other group in this study. The majority of the learners demonstrated level 2 thinking if all answers are taken into consideration.

Their level 2 thinking is also clearly demonstrated in their class activities as seen on the videos of the classroom activities. In activity 10.1 learners name properties of equilateral, isosceles and scalene triangles. The following are transcriptions[4] of their words in describing an isosceles triangle:

Response 1: " There are 3 sides...There are 3 angles...Not all angles are the same...At least 2 angles are acute"
Response 2: "Only 2 sides are equal and they are all acute angles"
Response 3: "...they have obtuse angles too!"

As the learners engaged in activity 10.3, spontaneous measuring of sides occurred to determine if the triangle was an equilateral, isosceles or scalene. The learners' reasoning and discussion were structured and specific answers[4] were required and given, for example:
"It's a isosceles triangle...only two sides are equal."
Q: Did you measure it?
A: YES! (Learner shows ruler to friend)

Control group 2[6]

Less than 50% (6 out of 13) of the learners in this group could correctly identify an isosceles triangle. Seven learners in total identified the triangles incorrectly, with 2 learners naming the triangles acute angled triangles and one learner naming the triangles obtuse-angled triangles. These answers were classified as incorrect as there were both acute-angled- and obtuse-angled triangles (see figure 5.6). For such an answer to be classified as correct both names were expected (see experimental group 2).

In answering question 16, only 2 learners gave incorrect answers but giving properties on the angles of an isosceles triangle proved to be a too demanding task as 8 learners gave incorrect or unclassifiable answers as the following examples prove.

Answers to question 16
"Their sides are equal in length . . . their sides are sharp"

Answers to question 17
"It consists of triangles... their breadth ("breete")[6] is all the same length and all is the same . . . all their thickness ("dikte")[6] is the same"
"that his legs are the same"
In evaluating the answers of this group, most learners' reasoning abilities fall within the parameters of level 1. More testing would be required to determine whether the remainder of this group are in transition between level 0 and level 1 and these learners demonstrated lower than expected reasoning levels.

[6] All the quotations of control group 2 have been translated from Afrikaans. In some instances the Afrikaans version is given in parenthesis when translation proved difficult

In conclusion it can be theorized that reaching higher order thinking in geometry relies not merely on a suitable choice of activities, but also on the active participation of both the teacher and the learners. The program in a teacher-centered environment (as in experimental group 1) can produce results that are higher than the relevant control group, but it stands to reason that the results could have been more dramatic if the program was taught in a learner-centered problem based environment (as in experimental group 2). The results that were achieved in experimental group 1 (second language education) are comparable with control group 2 (mother tongue education) which may indicate that experimental group 1 has shown some progress, and to such an extent that their results are equal or higher than the learners who received first language instruction like control group 2 (and whose school is considered to be an advantaged school).

Many learners made considerable progress, and further investigation is needed to determine whether these results are permanent over a longer period of time, and to what extent the learners retain what they have learned.

In spite of the progress of most learners, there were a few learners that made little progress. Fuys *et al.* (1988:139) give possible reasons that may explain this lack of progress:

- lack of prerequisite knowledge;
- poor vocabulary / lack of precision of language;
- lack of realization of what was expected of them;
- lack of experience in reasoning / explaining; and
- inadequate time to assimilate new concepts and experiences.

The hypothesis, *that a Van Hiele based learning and teaching program, in a problem solving context, has an effect on geometric thought*, can be accepted in light of the all the data (quantitative, see table 5.8, and qualitative, see § 5.9.1 to 5.9.6) with the provision that the program is presented in a learner-centered and not a teacher-centered teaching approach.

5.10 CONCLUSION

The aim of this chapter was to investigate whether and how a Van Hiele based learning and teaching program, in a problem solving context, influenced learning strategies, self-efficacy, intrinsic value, self-regulation and the geometric thought levels of learners.

The program rendered small and medium effect-sizes in learning strategies, self-efficacy, intrinsic value and self-regulation, which led to the conclusion that these hypotheses could not be accepted. The short period of time in which the program was implemented and the interval (holidays) in the middle of the program can be two of the major factors for the small and medium effect-sizes (see § 6.8 for a more detailed description of the limitations of the study). Concentration (see table 5.4) did provide a large effect-size which could be an indication that implementing the program over a longer period of time without an interruption could possibly produce better effect-sizes.

The exceptionally large effect-sizes (see table 5.8) as well as the quantitative results (see § 5.9.1 to 5.9.6) resulted in the acceptance of the hypothesis that this Van Hiele based program, in a problem solving context, did have a positive effect on geometric thought levels, with the provision that the program is presented in a learner-centered and not teacher-centered teaching approach.

Not only did the experimental groups consistently achieve higher levels of acquisition than their control groups, but they also achieved higher categories of acquisition (see § 5.9.2), which substantiate that the experimental groups' general acquisition was higher than that of the control groups.

In analyzing the recognition of concepts (square, right-angled triangle and isosceles triangle) in different spatial orientation (see table 5.9), it became evident that recognizing one figure (square / right-angled triangle) in the conventional orientation does not guarantee recognition of a different figure (isosceles triangle) in an unconventional orientation.

In the identification of triangles it can be hypothesized that the learners who followed a Van Hiele based program, in a problem solving context, combined with a learner-centered teaching approach will be able to correctly identify triangles and will give more complete answers to substantiate their answers. Such learners will furthermore be less likely to confuse right angles and right-angled triangles.

Combining a Van Hiele based teaching and learning program with a teacher-centered teaching approach can deliver results that are better (sometimes just slightly) than the conventional content-based teacher-centered teaching approach. The results achieved in the teacher-centered classroom compare dismally with the learner-centered classroom's results.

CHAPTER SIX

SUMMARY, RECOMMENDATIONS AND CONCLUSIONS

6.1 INTRODUCTION

This chapter consists of a summary of the research. A statement of the problem is given in paragraph 6.2. The review of the literature is summarized in paragraph 6.3, followed by the method of research in paragraph 6.4, the procedure of the research in paragraph 6.5 and the results and conclusion in paragraphs 6.6 and 6.7 respectively. The limitations of the study are discussed in paragraph 6.8, and finally, some recommendations are made in paragraph 6.9.

6.2 STATEMENT OF THE PROBLEM

South Africa has a history of a lack of adequate geometry instruction in primary schools (Van Niekerk, 1997:270). Taylor and Vinjevold (1999:143) state that lessons are generally characterized by a lack of structure, and the absence of activities which promote higher order skills such as investigation and understanding relationships.

Van Hiele's niveau theory (Van Hiele, 1986:39-47) postulates a learning model that describes the different types of thinking that learners pass through as they move from a global perception of geometric figures to, finally, an understanding of formal geometric proof (Teppo, 1991:210). Higher levels of geometric thought are reached not through direct instruction by the teacher, but through a suitable choice of problem solving activities and exercises (Van Hiele, 1982:215; Fuys *et al.*, 1988:7; Koehler & Grouws, 1992:123).

The aim of the research was, therefore, to determine how the implementation of a Van Hiele based learning and teaching program, in problem solving context, influences:

- learning strategies;
- self-efficacy;
- intrinsic value of a task; and
- geometric thought levels of learners.

6.3 REVIEW OF THE LITERATURE

6.3.1 A SELF-REGULATED VIEW OF LEARNING

Self-regulated learning can generally be defined as the degree to which learners are metacognitively, motivationally and behaviourally active participants in own learning (Zimmerman, 1989b:329; 1990:4; Schunk, 1991:71).

Zimmerman (1989b:330-332) identifies four distinctive assumptions fundamental to self-regulated academic learning. The first assumption is that there is a triadic reciprocality (see § 2.4.1) between personal, environmental, and behavioural determinants. The second, self-efficacy (see § 2.4.2), is viewed as a key feature. The third assumption deals with the subprocesses of self-regulated learning (see § 2.4.3) namely self-observation (see § 2.4.3.1), self-judgement (see § 2.4.3.2) and self-reaction (see § 2.4.3.3). The last assumption is that self-regulation is never an absolute state (see § 2.4.4).

Self-regulated learning occurs to the degree that a student can use personal (i.e. self) processes to strategically regulate behaviour and the immediate learning environment (Zimmerman, 1989b:330). This statement implies three determinants (see § 2.5) namely personal, environmental and behavioural influences or determinants.

Zimmerman (1989a:11) argues that mere personal processes, such as a learner's knowledge (see § 2.5.1.1), metacognitive processes (see § 2.5.1.2), goals (see § 2.5.1.3) and attributions (see § 2.5.1.4) do not determine learners' efforts to self-regulate during learning. These processes are assumed to be influenced by environmental and behavioural events in a reciprocal fashion. Zimmerman (1989b: 333) and Schunk (1996:361, 2000:372) distinguish three classes of student responses (behavioural influences / variables) (see § 2.5.2): self-observation (see § 2.5.2.1), self-judgement (see § 2.5.2.2), and self-reaction (see § 2.5.2.3). Environmental influences (see § 2.5.3) refer to the influence of the social (see § 2.5.3.1) and physical (see § 2.5.3.2) context on learners' behaviour (Zimmerman, 1989b:335).

6.3.2 THE LEARNING OF GEOMETRY

Piaget's theory of cognitive development assumes that learners make sense of the world and create their knowledge through experience with objects, people and ideas. Maturation (see § 3.2.1), physical experience (see § 3.2.2.), social interaction (see § 3.2.3) and the need for equilibrium (see § 3.2.4) all influence the way thinking processes and knowledge develop (Piaget, 1968:127; 1970:719-721).

Geometric thinking has been refined in such detail that today three major perspectives within the development of geometrical thinking exist. The first is Piaget and Inhelder's topological primacy theory (1963,1967,1971) (see § 3.3.1) which claims that depictions of space are constructed through progressive organization of a learner's motor and internalized actions, resulting in operational systems. A definite order exists in the organization of geometric ideas. Initially topological relations (see § 3.3.1.1 - connectedness or enclosure) are constructed and later projective (rectilinaerity - see § 3.3.1.2) and Euclidean (angularity or parallelism-see § 3.3.1.3) relations are constructed (Clements & Battista, 1992:422).

The second theory is the cognitive science theory (see § 3.3.2). This perspective endeavours to integrate research and practical work in the fields of psychology, philosophy, linguistics, and artificial intelligence (Clements & Battista, 1992:434). The aim of cognitive science is the understanding of the way in which individuals process information (Leong, 1993:63). The cognitive science theory consists of:

- Anderson's model of cognition (ACT - see § 3.3.2.1) that distinguishes between two kinds of knowledge, namely declarative (knowing that) and procedural (knowing how);

- Greeno's model of geometric problem solving (see § 3.3.2.3) that is a computer simulation model that presents the knowledge required for problem solving in geometry; and

- The parallel distributed processing network model (see § 3.3.2.5) that explains the holistic template representations of the lower levels in the Van Hiele hierarchy (see § 3.3.3.1 - Clements and Battista, 1992:435).

The last major perspective is the Van Hiele theory of geometric thinking (see § 3.3.3). The Van Hiele theory postulates that, in geometry, learners progress through levels of thought argumentation (see § 3.3.3.1), from a Gestalt-like visual level through increasing sophisticated levels of description, analysis, abstraction, and proof (Van Hiele, 1986:39). Clements and Battista (1992:426) identify the following characteristics of the Van Hiele theory: Learning is a discontinuous process, which implies "jumps" in the learning curve. These levels are sequential and hierarchical. Concepts implicitly understood at one level become explicitly understood at the next level (Teppo, 1991:213). Each level has its own language.

At the first level (Visual) learners identify and operate on shapes and other geometric configurations according to their appearance (as visual wholes). Easily put: figures are recognized by appearance alone (Mason, 1997:39; Flores, 1993:152; Spear, 1993:393; Presmeg, 1991:9).

At the second level (Descriptive / Analytic) learners are able to recognize and explicitly characterize shapes by their properties (Van Hiele, 1986:40), but can not recognize relationships between classes of figures (Battista, 1994:89) or even redundancies (repetitions) (Spear, 1993:393).

Learners at level three (Abstract / Relational) can form abstract meaningful definitions (Mason, 1997:39), distinguish between necessary and sufficient sets of conditions for a concept, classify figures hierarchically (by ordering their properties), give informal arguments to justify their classification (Battista, 1994:89), and understand and sometimes even provide logical arguments in the geometric domain (Clements & Battista, 1992:427).

Level four (Formal Deduction) requires learners to establish theorems within an axiomatic system. They recognize the difference among undefined terms, definitions, axioms, and theorems and are capable of constructing original proofs (Clements & Battista, 1992:428).

At the fifth level (Rigor / Metamathematical) learners reason formally about mathematical systems. Learners now understand the formal aspects of deduction (Presmeg, 1991:9), establishing and comparing mathematical systems (Mason, 1997:40; Flores, 1993:152). They can now study geometry in the absence of reference models, and they can reason by formally manipulating geometric statements such as axioms, definitions, and theorems.

Clements and Batista (1992:429) postulate an additional level namely level zero (Pre-recognition). At this level learners perceive geometric shapes by only attending to a subset of a shape's visual characteristics. Learners may distinguish between figures that are curvilinear and those that are rectilinear but not among figures in the same class.

Higher levels of geometric thought are reached not through direct telling by the teacher, but through a suitable choice of activities and exercises (Van Hiele, 1982:215; Koehler & Grouws, 1992:123). Van Hiele (1982:215-218) distinguishes five instructional phases (see § 3.3.3.2) that describe the goal for student learning and the teacher's role in providing instruction that enables this learning. During phase one (Information) learners become familiar with the content domain as the material relates to the current level of study as presented to the learners (Teppo, 1991:212). Learners in phase two (Guided orientation) become acquainted with geometric ideas (Clements & Battista, 1992:431), and explore the field of investigation by handling the material (Presmeg, 1991:9). Learners become conscious of the relations and begin to elaborate on their intuitive knowledge during phase three (Explicitation). Learners now describe their geometric conceptualization in their own language. Phase four (Free orientation) requires learners to solve more open-ended problems (Teppo, 1991:212) which solutions require the synthesis and utilization of those concepts and relations previously elaborated (Clements & Battista, 1992:431). They are now able to deliberately choose their activities. During the last phase (Integration), learners build a summary of all they have learned. They integrate their knowledge into a coherent whole that can easily be described and applied.

6.4 METHOD OF RESEARCH

6.4.1 SUBJECTS

Grade 7 learners (n=221) from five primary schools in Potchefstroom constituted the study population, with 133 learners in the experimental group and 88 learners in the control group (see § 4.3).

6.4.2 INSTRUMENTS

The following instruments were used:

6.4.2.1 THE LEARNING AND STUDY STRATEGIES INVENTORY-HIGH SCHOOL VERSION (LASSI-HS)

The Learning and Study Strategies Inventory-High school Version (LASSI-HS) (see § 4.4.1) is designed to be used as an assessment tool (consisting of 76 items) to measure learners' use of learning and study strategies and methods at high school level (Weinstein and Palmer, 1990:3). In this study a version of the LASSI was used that had been adapted for South African Mathematics Learners by Monteith, Nieuwoudt and Nieuwoudt. Learners have to respond to the items on a 5-point Likert-type scale. The scale ranges from 1 = "not at all like me" to a 5 = "very much like me". The LASSI-HS consists of ten subscales namely: attitude, motivation, time management, anxiety, concentration, information processing, selecting main ideas, study aids, self-testing and test strategies.

6.4.2.2 THE MOTIVATED STRATEGIES FOR LEARNING QUESTIONNAIRE (MSLQ)

The MSLQ (see § 4.4.2) includes 44 items on learner motivation, cognitive strategy use, metacognitive strategy use, and the management of effort. In this study a version of the MSLQ as adapted for South African conditions by Monteith and Mathebula was used. Learners have to respond to the items on a 5-point Likert-type scale. The scale ranges from 1 = "not at all like me" to a 5 = "very much like me".

The four items in the Test Anxiety scale and the 13 items in the Cognitive Strategy Use scale were included in the questionnaire but not used in the analysis of the data as both are already included in the LASSI-HS.

6.4.2.3 A VAN HIELE POST-TEST

The Mayberry Test (Lewin and Pegg Version) includes 58 items that include up to 5 sub-items on a variety of geometric concepts. In selecting the relevant items (12 items) it was found that the selected items were not sufficient and therefore 9 additional items were introduced from a test developed by the unit for Research in Mathematics Education of the University of Stellenbosch (RUMEUS) (1984).

The final product (see § 4.4.3) includes 21 items (with some sub-items) on the concept of parallel lines and shapes such as the square, right angle and isosceles triangle. The answers to the items were quantified according to the acquisition scales of Gutiérrez *et al.* (1991) (see § 3.3.3.3).

6.5 PROCEDURE

Both qualitative and quantitative research methods were used. In doing the quantitative research a pre-test post-test, experimental group-control group design was used.

In executing the qualitative research, a variety of methods were used namely action research (as the researcher investigated whether a Van Hiele based teaching and learning program, in a problem solving context, influenced self-regulated and meaningful learning). A type of qualitative research (case and field study research) was used as data was gathered directly from individuals (individual cases) and social groups in their natural environment for the purpose of studying interaction, attitudes and characteristics of individuals and groups (Leedy, 1997:111).

After the questionnaires had been completed, they were processed by using the SAS System for Windows Release 6.12 (SAS INSTITUTE, Cary, NC, USA, 1996). Effect-sizes (Steyn, 1999:3) were used to determine whether differences between the control group (C) and experimental group (E) were of practical significance. As the Van Hiele post-test was only administered to a small random sample of the population, the non-parametric Wilcoxon Rank Sum Test was used.

6.6 RESULTS

6.6.1 HYPOTHESIS 1

With reference to learning strategies only concentration (table 5.4), a sub-scale of learning strategies, was positively influenced (with a practical significance d-value > 0,8) by the treatment.

6.6.2 HYPOTHESIS 2

With relation to self-efficacy (see table 5.5), no effect-sizes of practical significance could be reported.

6.6.3 HYPOTHESIS 3

With regard to intrinsic value (see table 5.6), no effect-sizes of practical significance could be reported.

6.6.4 HYPOTHESIS 4

With relation to self-regulation (see table 5.7), no effect-sizes of practical significance could be reported.

6.6.5 HYPOTHESIS 5

With regard to the change in geometric thought levels, it was found that the program produced large effect-sizes (see table 5.8), and quantitative results (see § 5.9.1 to 5.9.6) indicated more refined advantages of implementing such a program.

The experimental groups consistently achieved higher levels and categories of acquisition (see § 5.9.2) than the relevant control group. Learners that followed the program in a problem solving context and a learner-centered teaching environment could recognize figures (square, right-angled triangle and isosceles triangle) in different spatial orientation (see table 5.9), although recognition of one figure (square / right-angled triangle) in the conventional orientation did not guarantee recognition of a different figure (isosceles triangle) in an unconventional orientation.

These learners correctly identified triangles and gave more complete answers to substantiate their answers. Such learners will furthermore be less likely to confuse right angles and right-angled triangles.

6.7 CONCLUSION

Hypothesis 1 (see § 5.5), that *the implementation of a Van Hiele based learning and teaching program, in a problem solving context, has an influence on learning strategies*, could therefore be accepted only in sub-hypothesis 1.5 (see § 5.5). Thus *the implementation of a Van Hiele based learning and teaching program, in a problem solving context, has an influence on the concentration abilities of learners.* This implies that a learning and teaching program in a problem solving context has a positive influence on the learners' concentration when working on academic tasks.

Hypothesis 5 (see § 5.9), *that the implementation of a Van Hiele based learning and teaching program, in a problem solving context, has an influence on the geometric thought levels*, could therefore be accepted. This implies that a learning and teaching program, in a problem solving context, has a very strong and positive effect on the development of learners' geometric thought levels.

6.8 LIMITATIONS OF THE STUDY

The study might have suffered because of the following limitations:

6.8.1 MISSING DATA

Due to either a lack of clear understanding of the questionnaires, or a negative attitude towards the questionnaires, some learners failed to complete them fully. This resulted in missing data and an inconsistency in the numbers of sample sizes for the various analyses.

6.8.2 INSTRUMENTATION

Questionnaires such as the Learning and Study Strategies Inventory-High School Version (LASSI-HS) and the Motivated Strategies for Learning Questionnaire (MSLQ) are questionnaires developed for use in the United States of America. These questionnaires were adapted for South African learners but had not been standardized for use in South Africa. Questionnaires developed and standardized for South African conditions were not available, and therefore no alternative existed other than to use the above mentioned questionnaires.

6.8.3 LANGUAGE MEDIUM

Some of the subjects were African language speakers, while the questionnaires were in English, for some their second and for others their third language. The assumption can be made that learners did not understand the questionnaires, and therefore failed to answer the questions correctly.

6.8.4 THE DISTANCE PROBLEM

Some of the subjects live far from school. Such learners need to walk a distance up to 7 km to school each day. They become physically and mentally tired from walking such a long distance and may therefore not be able to perform as well as expected in the questionnaires.

6.8.5 HUNGER

The majority of the subjects appears to come from low socio-economic status families, where there is little money for food. The only food for some of the subjects is the government feeding scheme, which consists of a biscuit and a glass of cooldrink. This is sometimes the only food for the whole day and hungry learners cannot perform well in the questionnaires.

6.8.6 AVAILABLE LITERATURE

Not much research has been done about the influence of variables such as self-regulated learning, self-efficacy and the learning of geometry on South African learners in the African situation. Literature and results based on the Western world thus had to be used for generalization and application in the predominantly rural situation in South Africa.

6.8.7 INTERRUPTION IN PROGRAM

The interruption in the middle of the program because of the school holidays could have negatively influenced the results. The learners' progress could have been higher as the culminating effect of the program was thus shortened that may have lead to a decreased effect.

6.9 RECOMMENDATIONS

It is recommended that:

- More research be done on South African learners, to determine the variables that affect mathematics performance, with special regard to geometry.
- Teachers should be made sensitive to the value of Van Hiele based learner-centered teaching methods as well as to the variables that influence and affect mathematics performance.
- Comparative studies in a bigger study population should be undertaken. Longitudinal studies are also needed to determine whether results, of a program such as this one, are permanent over a longer period of time as well as the extent of retention.

6.10 CONCLUDING REMARKS

In this research the effect of a Van Hiele based learning and teaching program, in a problem solving context, was analyzed. The implementation of a Van Hiele based learning and teaching program had a positive effect not only on the concentration of the learners but also on their level of geometric thought. These learners showed not only higher levels of geometric thought but they also proved to be on a higher category of thought within these levels. Such learners reasoned on a higher level, gave more complete answers, demonstrated less confusion of figures in different spatial orientations, and generally exhibited higher order thinking skills than their counterparts who did not take part in the program. These noteworthy advantages can only be achieved if the teacher consistently teaches from a learner-centered approach. This program will deliver little or no advantages if the program is presented in a teacher-centered content-based environment.

It is hoped that the conclusions / results will help to convince educators of the benefits of the active participation of both teacher and learner in the learning process. Geometry (and mathematics as a whole) should be enjoyed, understood and made part of everyday life and not feared, learned by rote and only be seen as a classroom activity reserved for school.

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APPENDIX A

PERMISSION FROM THE DEPARTMENT OF EDUCATION TO CONDUCT RESEARCH IN SCHOOLS

5 Vanadium Street
CARLETONVILLE
2499

26 November 1998

Mr. S. H. N. Komane
Private Bag X 919
POTCHEFSTROOM
2520

Dear Sir

REQUEST : PERMISSION FOR RESEARCH IN PRIMARY SCHOOLS

I hereby request permission to do research in the following primary schools as part of the M.Ed-degree at PU for CHE:

Keagile
Laerskool Mooirivier
Potchefstroom Central
Potchefstroom Primary
President Pretorius.

DEPT. OF EDUCATION ART CULTURE
SPORT AND RECREATION
CIRCUIT MANAGER / KRINGBESTUURDER
PRIVATE BAG / PRIVAATSAK X 1266
2520 POTCHEFSTROOM
1998-11-30
POTCHEFSTROOM KRINGKANTOOR
POTCHEFSTROOM OOS
DEPT ONDERWYS KUNS KULTUUR
SPORT EN REKREASIE

...e research will be conducted during 1999. The aim of the research is to determine the influence of perceived task difficulty, prerequisite skills and self-efficacy beliefs on the learning of Geometry in Grade 7. The research will contribute to more effective maths instruction and effective learning of maths.

Principals of the above mentioned schools will be contacted personally for further arrangements.

Yours sincerely

[signature] 98

Suriza van der Sandt

Permission is hereby granted to Suriza van der Sandt to conduct Research at the identified schools. Please afford her the opportunity.

[signature] Komane
C.M. 26/11/98

APPENDIX B

LEARNING AND STUDY STRATEGIES INVENTORY - HIGH SCHOOL VERSION (LASSI-HS)

LASSI-HS LEARNING AND STUDY STRATEGIES INVENTORY - HIGH SCHOOL VERSION

by **Claire E. Weinstein & David Palmer**,
Department of Educational Psychology, University of Texas at Austin.

Adapted for South African Mathematics Learners by JL de K. Monteith, HD Nieuwoudt (PU for CHE), and S.M. Nieuwoudt (PCE)

DIRECTIONS

The Learning and Study Strategies Inventory - High School Version (LASSI-HS) is designed to find out how you learn, how you study, and how you feel about learning and studying. On the following pages you will find 76 statements about learning and studying of mathematics. Read each statement and then mark one of these choices on the answer sheet:

1. **Not at all like me**
2. **Not very much like me**
3. **Somewhat like me**
4. **Fairly much like me**
5. **Very much like me**

To help you decide which choice to mark, we will explain what is meant by each one.

By **Not at all like me,** we do not necessarily mean that the statement would never describe you, but that it would be true of you only rarely. Cross out number 1 for this choice.

By **Not very much like me,** we mean that the statement generally would not be true of you. Cross out number 2 for this choice.

By **Somewhat like me,** we mean that the statement would be true of you about half of the time. Cross out number 3 for this choice.

By **Fairly much like me,** we mean that the statement would generally be true of you. Cross out number 4 for this choice.

By **Very much like me,** we do not necessarily mean that the statement would always describe you, but that it would be true of you almost all the time. Cross out number 5 for this choice.

Cross out the number that describes you best.

Example	1	2	3	4	5

Try to answer according to how well the statement describes you, not how you think you should be or what others do. There is no right or wrong answers to these statements. Please work as quickly as you can without being careless and please answer all the items.

STATEMENTS

1. I am worried that I will fail maths at school.
2. I can tell the difference between more important and less important information in a maths lesson.
3. I find it difficult to stick to a study time table for maths.
4. After a maths class, I look over the work we did in class to help me better understand it.
5. I don't care if I finish maths in high school as long as I can get a job.
6. I think of other things during the maths lesson and don't really listen to what is being said in class.
7. I use special study aids, such as main headings and words printed in italics or bold face, that are in my maths textbook.
8. I try to identify the main ideas or most important information in a maths lesson while the lesson is being presented.
9. I get discouraged because of low marks for a maths test or examination.
10. I am up-to-date in my maths assignments.
11. Problems outside of school such as financial problems, fights with parents, dating (being in love), etc. cause me to not do my maths.
12. I try to think through a topic while doing maths and decide what I am supposed to learn from it.
13. Even when some parts of the maths are dull and not interesting, I keep on working until I finish.
14. I feel confused and uncertain as to what my goals of my maths studies should be.
15. I learn new maths terms and concepts by visualizing a situation in which they occur.
16. I come to a maths class unprepared.
17. When studying for a maths test or exam, I think of questions that I think might be asked.
18. I would have preferred not to take maths in school.
19. The notes I take as I read my maths text book are helpful when I review the work.
20. I do poorly on maths tests because I find it hard to plan my work within a short period of time.

21. I try to think of possible test questions when studying work done in the maths class.

22. I only study maths when I have to write a test.

23. I change the maths I am studying into my own words.

24. I compare my maths class work / homework with other students to make sure it is correct.

25. I am very tense when I study maths.

26. I look at the maths of the previous lesson before the next lesson.

27. I have trouble summarizing the maths that I have just heard or done in class.

28. I work hard to get good marks in maths, even when I don't like the maths being done.

29. I often feel like I have little control over what happens to me in the maths class.

30. I stop often while doing maths and think over or review what I have been doing.

31. Even when I am well prepared for a maths test, I feel very upset when writing it.

32. When I study a topic in maths I try to make the ideas fit together and make sense.

33. I talk myself into believing some excuse for not doing a homework assignment in maths.

34. When I study maths, I have trouble figuring out just what to do to learn the work.

35. When I begin a maths test, I feel pretty sure that I will do well.

36. I check to see if I understand what my teacher is saying during a maths lesson.

37. I do not want to learn a lot of maths in school. I just want to learn what I need to get a good job.

38. I am sometimes unable to keep my mind on my maths work because I am restless or moody.

39. I try to find links between the maths I am learning and what I already know.

40. I set high standards or goals for myself in maths.

41. I end up "cramming" (learning a lot of maths in a very short period) for almost every test.

42. I find it hard to pay attention during a maths lesson.

43. I pay special attention to the first and/or last parts of most paragraphs when reading my maths text book.

44. I only study the parts of maths I like.

45. I am very easily distracted from the maths I'm doing.

46. I try to find connections between the maths I am studying and my own experiences.

47. I make good use of study hours after school to also study maths.

48. When doing maths that is difficult for me I either give up or study only the easy parts.

49. I make drawings or sketches to help me understand the maths I am studying.

50. I dislike most of the maths done in class.

51. I have trouble understanding just what a test question in maths is asking.

52. I use symbols, key words, diagrams, or tables in summarizing my maths.

53. While I am writing a maths test, worrying about doing poorly gets in the way of keeping my mind on the test.

54. I don't understand some sections of maths because I do not listen carefully.

55. I use my maths text book to prepare worksheets.

56. I feel very anxious when writing an important maths test.

57. When I decide to do my maths homework, I set aside a certain amount of time and stick with it.

58. When I write a maths test I realize I have studied the wrong material.

59. It is hard for me to know which parts of my maths text book are important to remember.

60. I pay attention fully when studying maths.

61. I use the headings of paragraphs and parts put in blocks as guidelines for important ideas in my maths text book.

62. I get so tense and confused when writing a maths test that I don't answer questions to the best of my ability.

63. I learn mathematical rules, formulas, techniques, etc., without understanding them.

64. I test myself to be sure I know the maths I have been studying.

65. I put off the maths I'm suppose to do more than I should.

66. I try to see how the maths I am studying would apply to my everyday living.

67. My mind wanders a lot when I do maths.

68. In my opinion, the maths I learn at school is not worth learning.

69. I go over homework tasks when looking over the maths done in class.

70. I have a hard time knowing how to study for different parts of maths.

71. Often when doing maths I seem to get lost in details and can't remember the main ideas.

72. When they are available, I go to review sessions or extra classes in maths.

73. I spend so much time with my friends that my maths suffers.

74. When writing maths tests or doing other work in maths, I find I have not understood what is asked of me and lose marks because of it.

75. I try to make connections between the many ideas in what maths I am studying.

76. I have a hard time finding important aspects of the work done in the maths class.

a:/las_e_g7-lassiehsvirgr7

NAME: ______________________________ SCHOOL : ____________________

LASSI

TEST NUMBER	2	1

Key				
1	2	3	4	5
Not at all like me	Not very much like me	Fairly much like me	Much like me	Very much like me

1	1	2	3	4	5	(2)	39	1	2	3	4	5	(40)
2	1	2	3	4	5	(3)	40	1	2	3	4	5	(41)
3	1	2	3	4	5	(4)	41	1	2	3	4	5	(42)
4	1	2	3	4	5	(5)	42	1	2	3	4	5	(43)
5	1	2	3	4	5	(6)	43	1	2	3	4	5	(44)
6	1	2	3	4	5	(7)	44	1	2	3	4	5	(45)
7	1	2	3	4	5	(8)	45	1	2	3	4	5	(46)
8	1	2	3	4	5	(9)	46	1	2	3	4	5	(47)
9	1	2	3	4	5	(10)	47	1	2	3	4	5	(48)
10	1	2	3	4	5	(11)	48	1	2	3	4	5	(49)
11	1	2	3	4	5	(12)	49	1	2	3	4	5	(50)
12	1	2	3	4	5	(13)	50	1	2	3	4	5	(51)
13	1	2	3	4	5	(14)	51	1	2	3	4	5	(52)
14	1	2	3	4	5	(15)	52	1	2	3	4	5	(53)
15	1	2	3	4	5	(16)	53	1	2	3	4	5	(54)
16	1	2	3	4	5	(17)	54	1	2	3	4	5	(55)
17	1	2	3	4	5	(18)	55	1	2	3	4	5	(56)
18	1	2	3	4	5	(19)	56	1	2	3	4	5	(57)
19	1	2	3	4	5	(20)	57	1	2	3	4	5	(58)
20	1	2	3	4	5	(21)	58	1	2	3	4	5	(59)
21	1	2	3	4	5	(22)	59	1	2	3	4	5	(60)
22	1	2	3	4	5	(23)	60	1	2	3	4	5	(61)
23	1	2	3	4	5	(24)	61	1	2	3	4	5	(62)
24	1	2	3	4	5	(25)	62	1	2	3	4	5	(63)
25	1	2	3	4	5	(26)	63	1	2	3	4	5	(64)
26	1	2	3	4	5	(27)	64	1	2	3	4	5	(65)
27	1	2	3	4	5	(28)	65	1	2	3	4	5	(66)
28	1	2	3	4	5	(29)	66	1	2	3	4	5	(67)
29	1	2	3	4	5	(30)	67	1	2	3	4	5	(68)
30	1	2	3	4	5	(31)	68	1	2	3	4	5	(69)
31	1	2	3	4	5	(32)	69	1	2	3	4	5	(70)
32	1	2	3	4	5	(33)	70	1	2	3	4	5	(71)
33	1	2	3	4	5	(34)	71	1	2	3	4	5	(72)
34	1	2	3	4	5	(35)	72	1	2	3	4	5	(73)
35	1	2	3	4	5	(36)	73	1	2	3	4	5	(74)
36	1	2	3	4	5	(37)	74	1	2	3	4	5	(75)
37	1	2	3	4	5	(38)	75	1	2	3	4	5	(76)
38	1	2	3	4	5	(39)	76	1	2	3	4	5	(77)

APPENDIX C

MOTIVATED STRATEGIES FOR LEARNING QUESTIONNAIRE (MSLQ)

MOTIVATED STRATEGIES FOR LEARNING QUESTIONNAIRE

National Center for Research to Improve Postsecondary Teaching and Learning (NCRIPTAL)
School of Education, The University of Michigan, Ann Arbor, Michigan

Adapted by
J.L. de K. Monteith (Potchefstroom University for CHE)
and
M.J. Mathebula

The questionnaire asks you about your study habits, your learning skills, and your motivation for learning or studying.

THERE ARE NO RIGHT OR WRONG ANSWERS TO THE QUESTIONNAIRE. THIS IS NOT A TEST.

We want you to respond to the questionnaire as accurately as possible, reflecting your attitudes and behaviors in this course.

Directions: Read each statement and then mark one of these choices on the answer sheet

1. **Not at all like me**
2. **Not very much like me**
3. **Somewhat like me**
4. **Fairly much like me**
5. **Very much like me**

To help you decide which choice to mark, we will explain what is meant by each one.

By **Not at all like me,** we do not necessarily mean that the statement would never describe you, but that it would be true of you only rarely. Cross out number 1 for this choice.

By **Not very much like me,** we mean that the statement generally would not be true of you. Cross out number 2 for this choice.

By **Somewhat like me,** we mean that the statement would be true of you about half of the time. Cross out number 3 for this choice.

By **Fairly much like me,** we mean that the statement would generally be true of you. Cross out number 4 for this choice.

By **Very much like me,** we do not necessarily mean that the statement would always describe you, but that it would be true of you almost all the time. Cross out number 5 for this choice.

MOTIVATIONAL BELIEFS

The following questions ask about your motivation for and attitudes about this class.
Remember there are no right or wrong answers, just answer as accurately as possible.
Use the scale below to answer the questions.

Not at all like me	1	2	3	4	5	Very much like me

Read each statement and then mark one of these choices on the answer sheet:

1. **Not at all like me**
2. **Not very much like me**
3. **Somewhat like me**
4. **Fairly much like me**
5. **Very much like me**

1. I prefer class work that is challenging so that I can learn new things.
2. Compared with other students in maths I expect to do well.
3. I am so tense during a test that I cannot remember facts I have learned.
4. It is important for me to learn what is being taught in maths.
5. I like what I am learning in maths.
6. I'm sure I can understand the ideas taught in maths.
7. I think I will be able to use what I learn in maths in other classes.
8. I expect to do very well in maths.
9. Compared with others in maths, I think I'm a good student.
10. I often choose topics I will learn something from even if they require more work.
11. I am sure I can do a very good job on the problems and tasks given in maths.
12. I have a nervous, upset feeling when I take a test.
13. I think I will receive a good grade in maths.
14. Even when I do badly in a test I try to learn from my mistakes.
15. I think that what I am learning in maths is useful for me to know.
16. My study skills are very good compared with others in mathematics.
17. I think that what we are learning in maths is interesting.
18. Compared with other students in this class I think I know a great deal about maths.
19. I know that I will be able to learn the work for maths.
20. I worry a great deal about tests.
21. Understanding maths is important to me.
22. When I take a test I think about how poorly I am doing.

PART B. SELF-REGULATED LEARNING STRATEGIES

The following questions ask about your learning strategies and study skills for this class. **There are no right or wrong answers. Answer the questions about how you study in this class as accurately as possible.**

Use the scale below to answer the questions.

Not at all like me	1	2	3	4	5	Very much like me

Read each statement and then mark one of these choices on the answer sheet:

1. **Not at all like me**
2. **Not very much like me**
3. **Somewhat like me**
4. **Fairly much like me**
5. **Very much like me**

Use the same scale to answer the remaining answers.

23. When I study for a test, I try to put together the information from class and from the book.
24. When I do homework, I try to remember what the teacher said in class so I can answer the questions correctly.
25. I ask myself questions to make sure I know the work I have been studying.
26. It is hard for me to judge what the main ideas are in what I read. (*R)
27. When work is hard I either give up or study only the easy parts. (*R)
28. When I study I put important ideas into my own words.
29. I always try to understand what the teacher is saying even if it doesn't make sense.
30. When I study for a test I try to remember as many facts as I can.
31. When studying, I copy my notes over to help me remember the work.
32. I work on practice exercises and answer end of chapter questions even when I don't have to.
33. Even when the maths are boring, I keep working until I finish.
34. When I study for a test I practice saying the important facts over and over to myself.
35. Before I begin studying I think about the things I will need to do to learn.
36. I use what I have learned from old homework and the textbook to do new work.
37. I often find that I have been reading for class but don't know what it is all about. (*R)
38. I find that when the teacher is talking I think of other things and don't really listen to what is being said. (*R)
39. When I am studying a topic, I try to make everything fit together.
40. When I'm reading I stop once in a while and go over what I have read.
41. When I read work for maths, I say the words over and over to myself to help me remember.
42. I outline the chapters in my book to help me study.
43. I work hard to get a good grade even when I don't like a class.
44. When reading I try to link the things I am reading about with what I already know.

MSLQ - HS

If you think the statement is very much like you, cross out 5; if a statement is not at all like you, cross out 1. If the statement is more or less like you, find the number between 1 and 5 that best describes you. Cross out this number.

Test number [1] (1)

Surname: .... Name: ....

Not at all like me	1	2	3	4	5	Very much like me

PART A: MOTIVATION

1	1	2	3	4	5	(2)
2	1	2	3	4	5	(3)
3	1	2	3	4	5	(4)
4	1	2	3	4	5	(5)
5	1	2	3	4	5	(6)
6	1	2	3	4	5	(7)
7	1	2	3	4	5	(8)
8	1	2	3	4	5	(9)
9	1	2	3	4	5	(10)
10	1	2	3	4	5	(11)
11	1	2	3	4	5	(12)
12	1	2	3	4	5	(13)
13	1	2	3	4	5	(14)
14	1	2	3	4	5	(15)
15	1	2	3	4	5	(16)
16	1	2	3	4	5	(17)
17	1	2	3	4	5	(18)
18	1	2	3	4	5	(19)
19	1	2	3	4	5	(20)
20	1	2	3	4	5	(21)
21	1	2	3	4	5	(22)
22	1	2	3	4	5	(23)

PART B: LEARNING STRATEGIES

23	1	2	3	4	5	(24)
24	1	2	3	4	5	(25)
25	1	2	3	4	5	(26)
26	1	2	3	4	5	(27)
27	1	2	3	4	5	(28)
28	1	2	3	4	5	(29)
29	1	2	3	4	5	(30)
30	1	2	3	4	5	(31)
31	1	2	3	4	5	(32)
32	1	2	3	4	5	(33)
33	1	2	3	4	5	(34)
34	1	2	3	4	5	(35)
35	1	2	3	4	5	(36)
36	1	2	3	4	5	(37)
37	1	2	3	4	5	(38)
38	1	2	3	4	5	(39)
39	1	2	3	4	5	(40)
40	1	2	3	4	5	(41)
41	1	2	3	4	5	(42)
42	1	2	3	4	5	(43)
43	1	2	3	4	5	(44)
44	1	2	3	4	5	(45)

School		(46)
Girl	1	(47)
Boy	2	(48)

Age: Years [ ][ ] Months [ ][ ] (49-52)

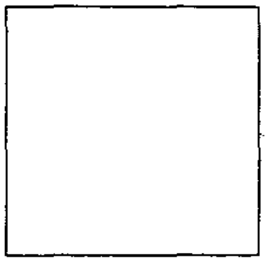
APPENDIX D

A VAN HIELE POST-TEST

Surname: ______________________ Name: __________________________

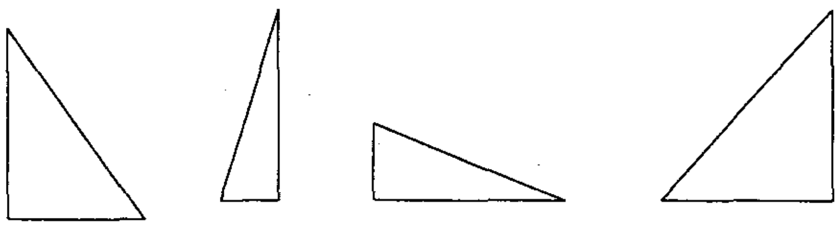
School: ______________________ Student id number: ____________ Gender: B or G

1. This figure is which of the following?



a) Triangle
b) Quadrilateral
c) Square
d) Parallelogram
e) Rectangle

2.



a) Are all of these triangles? YES NO

Explain:

____

____

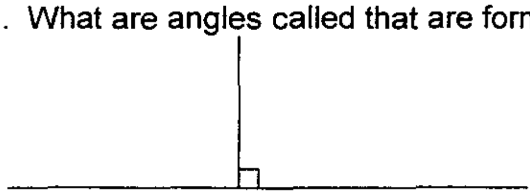
____

____

b) Do they appear to be a special kind of triangle? YES NO

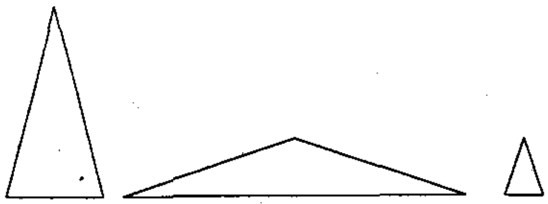
If so what kind? ______________________________

3. What are angles called that are formed like this?



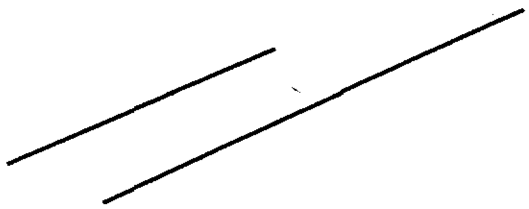
Answer: ______________________

4.



These appear to be what kind of triangles? ____________________

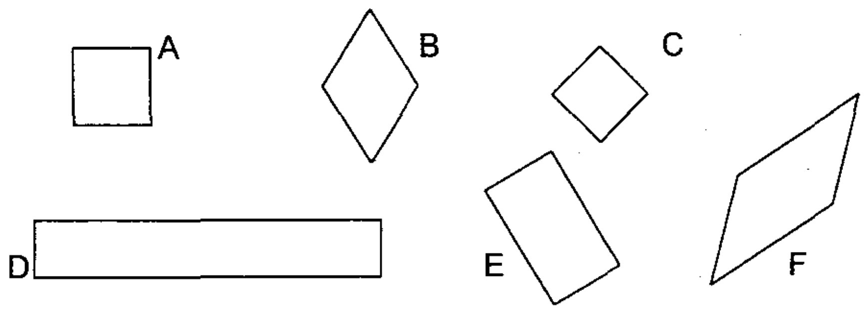
5.



Suppose these two lines will never meet no matter how far we draw them.

What word describes this? ____________________

6.



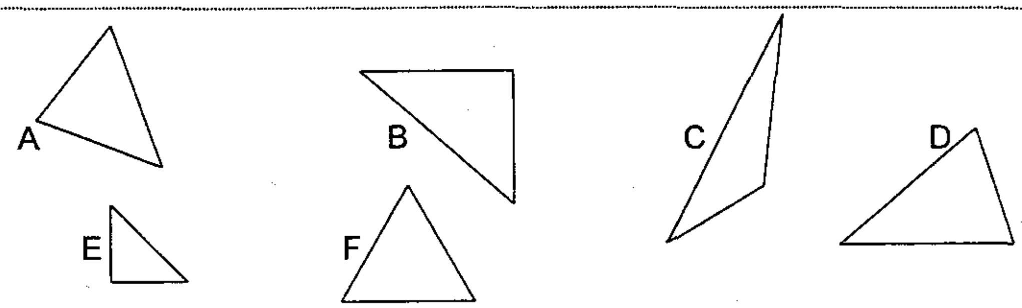
Which of these figures are squares? ____________________

7. Draw a rectangle

8. Draw a right-angled triangle

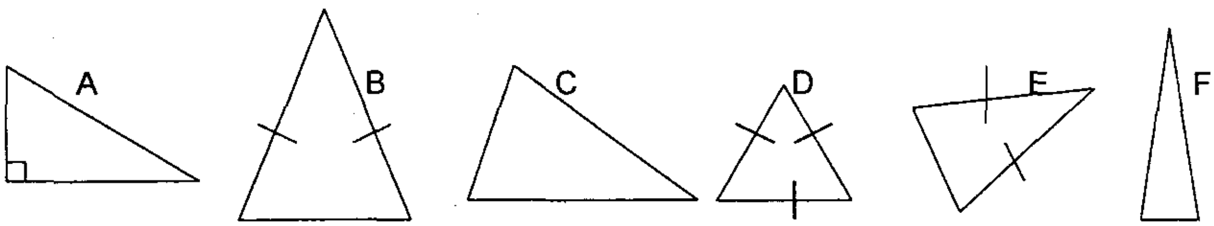
9. Draw a parallelogram

10.



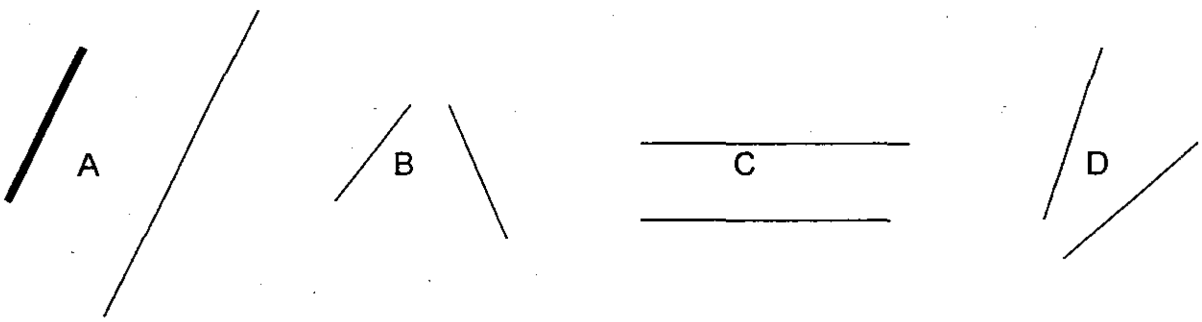
Which of these appear to be a right-angled triangle? ________________________________

11.



Which of the following are isosceles triangles? ________________________________

12.



Which pair(s) of lines appear to be parallel? ________________________________

13.a. Draw a square.

b. What must be true about the sides? ________________________________

c. What must be true about the angles? ________________________________

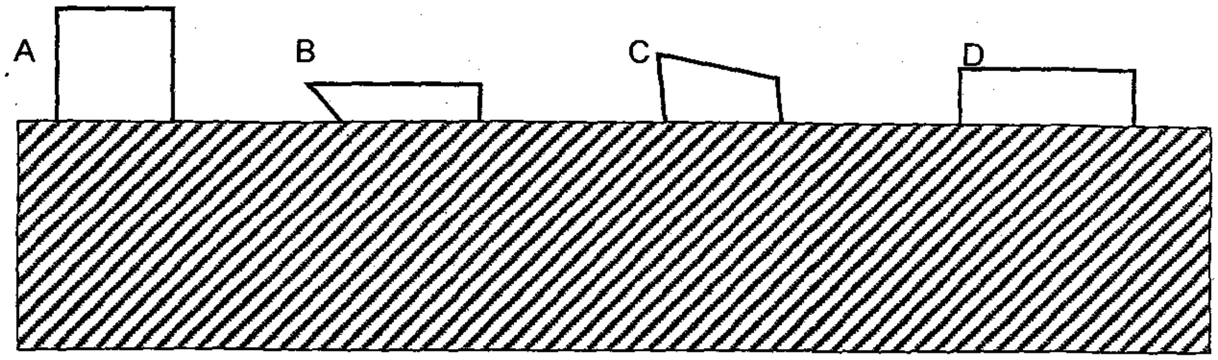
14. Does a right-angled triangle always have a longest side? YES NO
If so, which one?

15. Does a right-angled triangle always have a largest angle? YES NO
If so, which one?

16. What can you tell me about the sides of an isosceles triangle?

17. What can you tell me about the angles of an isosceles triangle?

18. Four figures are partially hidden in the drawing below. They are a square, a triangle, a rectangle and a parallelogram. Write down the correct name above each figure.



Give a reason for each of your answers:

A Why? ____

____

B Why? ____

____

C Why? ____

____

D Why? ____

____

19. How do you recognize parallel lines?

____

____

____

____