

ASPECTS OF PRICE DETERMINATION USING GOAL PROGRAMMING APPROACHES

M.P. TSOGANG Hons. B.Com.

**A mini-dissertation submitted in partial fulfilment of the requirements for the degree
Master of Commerce in Computer Science and Information Systems at the North-West
University**

Supervisor: Prof. H.A. Kruger
Co-supervisor: Prof. J.M. Hattingh

May 2007
Potchefstroom campus

ABSTRACT

The use of goal programming in various real-world areas - including resource allocation, engineering, agriculture and other applications - has increased a lot in the past few years. The aim of this dissertation is to investigate goal programming approaches in determining prices. Various aspects of price determination - such as costs, existing prices, competitors' prices, volume change due to price change and other aspects are incorporated in the model in order to suggest reasonable and realistic prices. Taking just these factors into account will not completely solve the problem, as there are usually certain goals that the decision maker would like to achieve. For example, the decision maker would probably like to attain an acceptable pre-specified minimum profit level without adjusting current prices too much whilst keeping prices competitive to insure that customers are not lost in the process of change. In this study, a goal programming model is developed for the determining of products' prices with consideration of these goals. The model makes provision for the change in demand due to the change in prices.

Keywords

Goal programming technique, price determination, multi-criteria decision making.

OPSOMMING

Die gebruik van doelwitprogrammering in verskillende toepassings gebiede soos brontoekennings, ingenieurswese, landbou en ander velde het geweldig toegeneem gedurende die afgelope aantal jare.

Die doel van hierdie studie is om die gebruik van doelwitprogrammering te ondersoek wanneer pryse van verskillende items bepaal moet word. Verskeie aspekte soos koste, bestaande pryse, mededingers se pryse, volume veranderinge as gevolg van prys veranderinge, ensovoorts, word in ag geneem om realistiese en redelike pryse te bepaal. Daar is gewoonlik sekere doelwitte waarna besluitnemers streef; byvoorbeeld, 'n besluitnemer sal waarskynlik graag 'n voorafbepaalde minimum wins wil realiseer sonder om pryse te veel te verander en dalk sodoende kliënte verloor deur pryse wat nie mededingend is nie. In hierdie studie word 'n doelwitprogrammeringsmodel ontwikkel om die pryse van produkte te bepaal met inagneming van bogenoemde doelwitte. Die model maak ook voorsiening vir verandering in vraag as gevolg van veranderinge in pryse.

ACKNOWLEDGEMENTS

Firstly, I would like to thank God Almighty for strength, patience and the spirit of continuous work that he'd installed in me to initiate and do this research.

There were people who were involved in helping with this research to whom I would like to convey my special thanks. I would like to thank the following persons and institutions:

- ❖ Professor JM Hattingh and Professor HA Kruger for their enthusiastic support and dedication in advising me throughout this research work.
- ❖ The staff of the Department of Computer Science and Information Systems of the North-West University Potchefstroom campus for their various and distinguished support.
- ❖ The staff of the Department of Information Systems of the North-West University Mafikeng campus for their support during my studies.
- ❖ My family, especially my mother and father, for their continued support.
- My friends and fellow-students for their continued input of general ideas regarding research.

Table of contents

ABSTRACT	I
OPSOMMING	II
ACKNOWLEDGEMENTS	III
CHAPTER ONE	1
1.1 INTRODUCTION.....	1
1.2 PROBLEM DEFINITION.....	1
1.3 AIMS AND OBJECTIVES	2
1.4 METHOD OF INVESTIGATION	2
1.5 STRUCTURE OF THE DISSERTATION	3
CHAPTER TWO.....	5
BACKGROUND AND LITERATURE SURVEY.....	5
2.1 INTRODUCTION.....	5
2.2 BACKGROUND.....	5
2.3 DEFINITION AND DESCRIPTION OF CONCEPTS	7
2.3.1 General pricing and economic concepts	7
2.3.2 Multi-criteria decision making	11
2.4 GOAL PROGRAMMING (GP) CONCEPTS	14
2.5 TYPES OF GOAL PROGRAMMING	18
2.5.1 Generic format of goal programming.....	19
2.5.2 Example of Goal programming formulation	21
2.5.3 Goal programming variants	23
2.5.4 Weighted goal programming.....	23
2.5.5 Graphical representation of GP model	24
2.5.6 Normalization techniques	25
2.5.7 Lexicographic goal programming	27
2.5.8 Goal programming solutions	27
2.6 TYPES OF GOAL PROGRAMMING APPLICATIONS	29
2.7 EXISTING PRICING STRATEGIES	32
2.7.1 Pricing based on goal programming.....	32
2.7.2 Pricing for electronic commerce.....	32
2.7.3 Dynamic pricing.....	32
2.8 SUMMARY AND COMMENTS	33
CHAPTER THREE.....	34
PROFIT MODEL FORMULATION	34
3.1 INTRODUCTION.....	34
3.2 PRICING OBJECTIVES	34
3.3 PROFIT DETERMINANTS.....	35
3.4 PROFIT MODELS.....	36
3.5 SUMMARY	44
CHAPTER FOUR	45
GOAL PROGRAMMING APPROACH TO PROFIT MODELS.....	45
4.1 INTRODUCTION.....	45
4.2 GENERAL APPROACH.....	45
4.3 GOAL PROGRAMMING MODELS FOR PRICING DECISION	46
4.4 GOALS FOR PROTECTION OF EXISTING CUSTOMER RELATIONSHIPS	48

4.5	GOALS FOR COMPETITIVENESS	49
4.6	SYSTEM CONSTRAINTS	50
4.7	WEIGHTS FOR GOAL ACHIEVEMENT	51
4.8	OBJECTIVE FUNCTION	51
4.9	NON-NEGATIVITY CONDITIONS	53
4.10	THE COMPLETE GOAL PROGRAMMING MODEL INVESTIGATED IN THIS STUDY.....	54
4.11	SUMMARY	55
CHAPTER FIVE.....		56
ILLUSTRATION OF THE MODEL.....		56
5.1	INTRODUCTION.....	56
5.2	SYSTEM SPECIFICATIONS.....	57
5.2.1	Hardware specifications	57
5.2.2	Software specifications.....	57
5.3	ALGORITHMIC FLOW CHART.....	57
5.4	COMPUTATIONAL EXPERIMENTS.....	58
5.5	SUMMARY	71
CHAPTER SIX.....		72
EMPIRICAL RESULTS.....		72
6.1	INTRODUCTION.....	72
6.2	RESULTS OF COMPUTATIONAL EXPERIMENT A.....	72
6.3	RESULTS OF COMPUTATIONAL EXPERIMENT B	73
6.4	RESULTS OF COMPUTATIONAL EXPERIMENT C	74
6.5	RESULTS OF COMPUTATIONAL EXPERIMENT D.....	74
6.6	RESULTS OF COMPUTATIONAL EXPERIMENT E	75
6.7	RESULTS OF COMPUTATIONAL EXPERIMENT F.....	76
6.8	RESULTS OF COMPUTATIONAL EXPERIMENT G	77
6.9	SUMMARY OF RESULTS OF COMPUTATIONAL EXPERIMENTS	77
6.10	SUMMARY OF DEDUCTIONS	79
6.11	SUMMARY	86
CHAPTER SEVEN		87
CONCLUSIONS AND FURTHER RESEARCH		87
7.1	INTRODUCTION.....	87
7.2	INVESTIGATE SOME OF THE MAIN ASPECTS OF PRICE DETERMINATION AS USED BY SHOP MANAGERS FOR PRODUCTS OR SERVICES.....	87
7.3	CONSIDER MODELS BASED ON WEIGHTED GOAL PROGRAMMING APPROACH FOR PRICE DETERMINATION	88
7.4	DEVELOP AND TEST A DECISION SUPPORT SYSTEM THAT CAN BE USED TO HELP DETERMINE PRICES	88
7.5	FURTHER RESEARCH	89
7.6	CONCLUSIONS	89
APPENDIX A.....		91
ELASTICITY DIAGRAMS.....		91
INTRODUCTION		91
APPENDIX B.....		96
PROGRAM DESCRIPTION.....		96
INTRODUCTION.....		96
REQUIREMENTS FOR RUNNING THE PROGRAM		96
USING THE PROGRAM		96
BIBLIOGRAPHY.....		98

Table of figures

Figure 2-1: Basic pricing process (Rauhala, 2002:4).....	6
Figure 2-2 : Cost-volume relationships (Hanna & Dodge, 1995:44).....	8
Figure 2-3: Pricing strategies matrix (marketingteacher.com, 2006).....	9
Figure 2-4: Concept of price elasticity (Haydam, 1997: 119).....	11
Figure 2-5 : Summary relationship of GP with MS / OR and MCDM (Schniederjans, 1995:13).....	15
Figure 2-6: Graphical solution for a simplified GP model.....	25
Figure 2-7 : Algorithm for solving GP models avoiding inferior solution (Romero, 1990:20)	28
Figure 2-8: Graphical representation of GP topics adopted from Tamiz et al. (1995:42)	31
Figure 3-1: Pricing objective (Crawford, 1997: 3)	34
Figure 3-2: Demand and supply equilibrium point.....	35
Figure 3-3: Price-volume relationship	38
Figure 4-1: Graphical representation of Profit function (DeBrock, 2004:3)	46
Figure 5-1: A view of concert technology for C++ users.....	56
Figure 5-2: Flow chart for optimizing a profit Model	58
Figure 5-3: Determining slope for the first product	64
Figure 6-1: Graphical representation of summary of price and volume changes for shop A	80
Figure 6-2 : Graphical representation of summary of price and volume changes for shop B	81
Figure 6-3: Graphical representation of summary of price and volume changes for shop C	82
Figure 6-4: Graphical representation of summary of price and volume changes for shop D	83
Figure 6-5: Graphical representation of summary of price and volume changes for shop E.....	84
Figure 6-6: Graphical representation of summary of price and volume changes for shop F.....	85
Figure 6-7: Graphical representation of summary of price and volume changes for shop G	86

Table of tables

Table 2-1: Summary of the use of goal constraints.....	21
Table 2-2: Weighting of goals by a decision maker.....	24
Table 2-3 : Results from a weighted goal programming model.....	25
Table 5-1: Data from shop A	59
Table 5-2: Data from shop B	60
Table 5-3: Data from shop C	60
Table 5-4: Data from Shop D.....	61
Table 5-5: Data from Shop E	62
Table 5-6: Data from Shop F	62
Table 5-7: Data from Shop G.....	63
Table 5-8: Summary of products and their related slopes.....	64
Table 5-9: Summary of averaged competition prices for Shop A.....	65
Table 5-10: Demand functions for each product	66
Table 5-11: Computation of constants for model A for illustrative purpose.....	69
Table 5-12 : A summary of old prices.....	69
Table 5-13: A summary of old volumes	70
Table 5-14 : A summary of averages of competition prices	70
Table 5-15 : A summary of costs associated products	71
Table 6-1: Results for experiment A	72
Table 6-2: Results for experiment B	73
Table 6-3: Results for experiment C	74
Table 6-4:Results for Experiment D.....	75
Table 6-5: Results for Experiment E	75
Table 6-6: Results for Experiment F	76
Table 6-7: Results for Experiment G.....	77
Table 6-8: Summary of results from computational experiments.....	78

1.1 Introduction

Product price determination is undoubtedly a decision making task for management that is important because of its direct impact on a firm's performance in the market-place and the overall level of profitability (Hanna & Dodge, 1995:7). According to Marn and Robert (quoted by Hanna & Dodge, 1995:1), the fastest and most effective way for a company to realize its profitability is to get its pricing right. The right price can boost profit faster than increasing volume will; the wrong price can shrink it just as quickly. Determining the price of products or services is referred to as one of the trickiest decisions faced by business managers, especially for businesses such as new home businesses (Maravilla, 2004:1).

Aspects of price determination include amongst other things costs, competitors' prices, volume of sales, profit level to be attained and building good relationships with customers.

1.2 Problem definition

The overall objective of this dissertation is to suggest prices, expected volume changes and optimised profit functions using goal programming approaches. A process of price determination involves taking various aspects into consideration. Setting prices too high or too low can significantly influence a product's profitability. Demand of such products may equivalently deflate as customers are attracted elsewhere by low and reasonable prices (Hanna & Dodge, 1995:2).

In many cases, reduction of prices results in a higher demand. In such a situation, demand is 'price elastic'. In some other cases reduction of prices will not necessarily increase volume of sales. In such situations, demand is 'price inelastic' (Ruby, 2003). These elasticity situations together with other factors such as costs, competitors' prices, customers' behaviour, volume of sales, profit targets and current prices of products - may complicate the pricing decision. Businesses have to capture all these aspects in proper and

realistic price determination with the goal of profit optimisation. The technique chosen for this investigation allows a decision maker to specify goals and constraints to come as close as possible to achieving the goals.

1.3 Aims and objectives

Aims and objectives of this research are to:

- ❖ Investigate some of the main aspects of price determination as used by shop managers for products or services.
- ❖ Consider models based upon the weighted goal programming approach for price determination.
- ❖ Develop and test decision support systems that can be used to help to determine prices.

1.4 Method of investigation

The method of investigation in this study is based on a goal programming methodology, which is a popular management science technique that enables a decision maker to specify a list of goals. The decision maker tries to come as close as possible to satisfying various goals and constraints rather than to seek optimal solutions. The desire of a decision maker is to maximise several objectives simultaneously to satisfactory levels (Moore & Weatherford, 2001: CD12-12). We deemed this technique as appropriate because the pricing problem in this study is a multi-criteria decision problem. Instead of searching for optimal solutions, goal programming strives towards satisfaction of a number of goals and constraints.

The main parts of this work include a literature study and empirical experiments. In this dissertation we have developed the goal programming models that also take possible price elasticity into consideration and created a system that can assist a decision maker regarding pricing decisions. The system was tested and illustrated using real-world data.

1.5 Structure of the dissertation

The remaining chapters of this dissertation are structured as follows:

Chapter two: Background and literature survey

This chapter surveys relevant and existing literature with regard to aspects of price determination. The applications of goal programming techniques are also considered. A survey of the multi-criteria decision making in general and goal programming as a technique of choice are provided in this chapter. The chapter further incorporates a review of critical issues of goal programming and its applications. The description of some of the pricing strategies and development considerations will also be given here.

Chapter three: Profit model formulation

This chapter reviews an existing profit model and relevant factors are considered. An alternative profit model is developed that takes price elasticity into account.

Chapter four: Goal programming approach to profit models

The pricing problem is formulated as a multiple criteria decision making problem. A set of goals and system constraints are formulated as preferred by a decision maker. The process of model formulation takes into consideration factors such as competitors' prices, price restriction and profit goals.

Chapter five: Illustration of the model

The procedure of computational experiments on real-world data are discussed in this chapter. The model formulated in chapter four was tested using data to investigate the behaviour of the model in terms of its multi-criteria capability of coming close to goals and constraints that were specified. This chapter further considers the estimation of values that can be used to incorporate price elasticity concepts.

Chapter six: Empirical results

Results obtained from the experiments described in chapter five (that is, computational experiments results) are discussed in terms of the aims and objectives of this research.

Chapter seven: Conclusion and further research

Some challenges are discussed that could help to make the suggested models more realistic in practical situations.

Appendix A

This appendix contains data used for elasticity estimations.

Appendix B

This appendix contains the user manual of the system that was developed to illustrate the aspects of price determination considered in this dissertation in a C++ programming environment.

CHAPTER TWO

BACKGROUND AND LITERATURE SURVEY

2.1 Introduction

This chapter surveys some of the existing literature associated with multi-criteria decision making (for example, goal programming), pricing models and some software that are available for determining prices for products and / or services. The purpose is to review the existing pricing tactics and consider their relevance in order to incorporate them into multi-criteria decision making, and especially to investigate goal programming solutions for pricing problems in the business sectors.

The development of a price model has always been driven by pricing objectives such as pricing for profit, pricing for volume, pricing for competition, pricing for prestige, and strategic pricing for strengthening existing relationships (Crawford, 1997:3). Due to huge pricing challenges some businesses would like to have computerised pricing strategies which recommend satisfactory pricing solutions. One of the recently emerged business styles is electronic commerce systems whereby buyers and sellers interact via the Internet to make transactions. Irrespective of the type of business, pricing still remains a challenge. Intelligent agents called 'pricebots', are used to determine dynamic prices for products in electronic commerce businesses using dynamic price algorithms (Dasgupta & Hashimoto, 2004:277).

2.2 Background

Price theory concepts are sometimes complicated to implement in practice. The pricing process has always been a challenge as one tries to use various theoretical concepts in a practical application. The end product of the process is a price which has to consider all factors that affects it. A price is often an indicator of quality in the eyes of a customer. Cheap product prices are normally perceived and associated with low quality products

(Crawford, 1997). Many pricing situations exist. Among them there is the pricing of a new product, which generally poses problems such as - but not limited to - markets not existing, no reference price level, extremely uncertain demand and price sensitivity. When a new product is launched, one of the most important things to do is an analysis with regard to cost, competition and demand. Figure 2-1 below depicts the basic pricing process that considers some of the above mentioned factors:

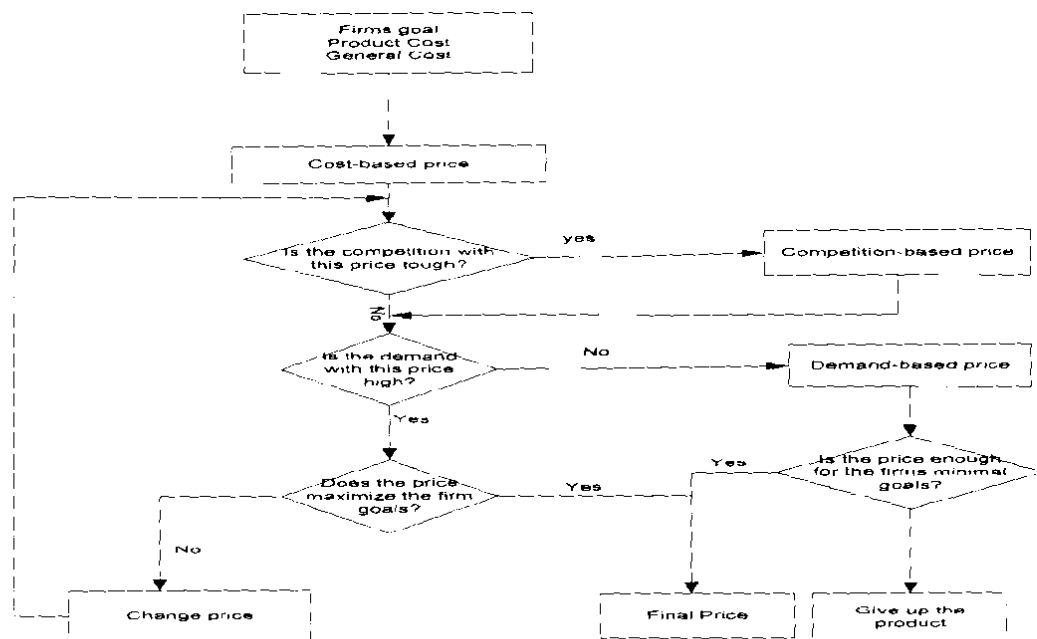


Figure 2-1: Basic pricing process (Rauhala, 2002:4)

When pricing for a new product, challenges such as markets not existing and no reference price level would emerge (Rauhala, 2002:4) and this would necessitate the usage of a pricing strategy such as cost-based pricing and demand pricing, which would stimulate the demand for the product (Hanna & Dodge, 1995). Pricing strategies are normally developed which result in a more generic price model of the business.

2.3 Definition and description of concepts

This section presents definitions and descriptions of some of the concepts related to pricing, multi-criteria decision making and goal programming to be dealt with later in this study.

2.3.1 General pricing and economic concepts

2.3.1.1 Price – definition

Price can be viewed by customers as a monetary expression of the value for dimensions of quality or features for a product or service provided as compared to other products and / or services. In other words, price is a payment in relation to the quality as set and evaluated by the marketplace (Hanna & Dodge, 1995:7). Prices are not confined to monetary value but can be stated in anything of value (Friedman, 1990:100).

2.3.1.2 Cost

Cost is divided into two main categories, namely, fixed and variable costs. Incremental costs are increases in both fixed and variable costs. Fixed costs are the costs of doing business. Fixed costs normally include costs such as space rent, employee salaries, employee benefits, taxes, utilities, security and office equipment. Variable costs are those costs that have a direct relationship to the level of activities such as materials and components required for each unit produced. Formula 2.1 below shows the cost function (also illustrated in figure 2.2):

$$TC = FC + VC \quad (2.1)$$

with

TC = total cost,

FC = fixed cost,

VC = variable cost.

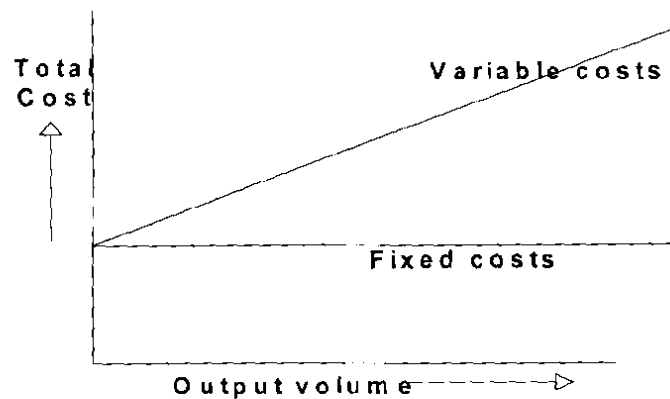


Figure 2-2 : Cost-volume relationships (Hanna & Dodge, 1995:44)

Cost-oriented pricing is the most elementary pricing method. Most pricing methods cover cost associated with production (Maravilla, 2004, Hanna & Dodge, 1995:44).

2.3.1.3 Profit

Profit is generally revenue minus costs. It is important to assume profit to be a function of costs. Equation 2.2 below indicates that profit is actually a difference between total revenue and total costs.

$$\text{Profit} = \text{TR} - \text{TC} \quad (2.2)$$

with

TR = Total revenue for sales of products.

TC = Total costs which is defined in equation (2.1) above.

2.3.1.4 Legal and ethical aspects of pricing

The process of pricing may conflict with social values. Setting prices too high results in the motive of the seller being questioned, and on the other hand the same might prevail if too low prices are set. It has become important that decision makers not only concentrate on profitability but also on legal and ethical aspects associated with pricing. Some

governments impose sanctions upon those who violate pricing law (Nagle & Holden, 1995:360). Listed below are some of the instances in which pricing may be questioned:

- ❖ “The incremental prices in a product line do not seem justified in terms of value increase.”
- ❖ “The price of a ‘new’ product is greater than the value of the change incorporated in the product (false obsolescence).”
- ❖ “There is a disproportionately high price for replacement of a part (foreign cars)” (Hanna & Dodge, 1995:12; Nagle & Holden, 1995:360)

2.3.1.5 Pricing strategies and tactics

There are basically four main pricing strategies, namely; *premium pricing*, *penetration pricing*, *economic pricing* and *skimming pricing*. Figure 2.3 below depicts these pricing strategies.



Figure 2-3: Pricing strategies matrix (marketingteacher.com, 2006)

Other important approaches of pricing strategies are listed below:

Psychological pricing. This pricing approach is used when a marketer wants the customer to respond not only on a rational calculation, but also on an emotional calculation. An example would be price point perspective, for example 99 cents instead of one Rand. This is known as perception of add ending in pricing.

Product line pricing. This pricing approach is used when a marketer want to determine the prices for a range of products or services. This type of price reflects the benefit of part of range.

Promotional pricing is a pricing approach that is common when a product is born. The marketer tries to create a market for a new product. This informs buyers and persuades them to perceive a product as an affordable one.

Product bundle pricing. This pricing approach is based on selling many different products at once but for a single price (Marketingteacher.com, 2006:1 & Nagle & Holden).

Ramsey pricing. Ramsey pricing is a linear pricing scheme designed for the multi-product natural monopolist named after Ramsey (Ramsey, 1927). Even though this study is not based on monopolistic economy, Ramsey solutions have some significance as it implies that the greatest deviation from marginal cost must be applied to products or services that have least elastic demand (Brown & Norgaard, 1992:674).

2.3.1.6 Concept of price elasticity

Elasticity is considered to be the ratio of the percentage change in quantity demanded to the associated percentage change in price (Cepeda, 2005:1). According to the law of demand, an increase in a price will yield a corresponding decrease in quantity demanded. The question will be how much change is expected as a result of price increase?. The slope of the demand curve will determine the amount of change in the quantity demanded. The diagram below depicts the concept of price elasticity for products A and B.

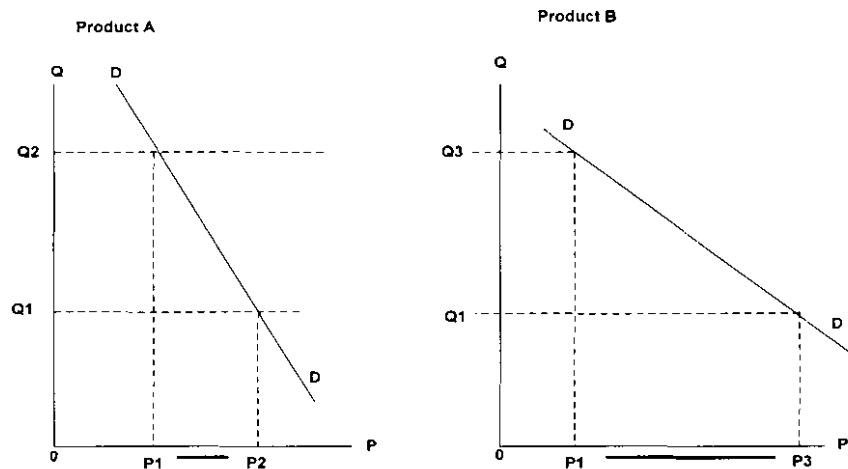


Figure 2-4: Concept of price elasticity (Haydam, 1997: 119)

According to these elasticity diagrams, product B is more price inelastic than product A, because price increase has resulted in the same quantity demanded in product B than in product A. The change in quantity for the product is as follows: Product A $Q2 - Q1$ (that is quantity demanded) and Product B $Q3 - Q1$ with the same amount quantity change as Product A (Haydam, 1997:118 - 120).

There are situations where a reduction in a price may not necessarily result in a large quantity increase. In this case the product is said to be price inelastic. This is shown in the graph for product A (Ruby, 2003). Some price changes will largely change the quantity demanded and others may not.

2.3.2 Multi-criteria decision making

This section presents descriptions, definitions and classifications of concepts of multi-criteria decision making techniques.

Multi-criteria decision making is one of the well-known branches in the field of decision making. This decision making technique is divided into multi-attribute decision making (MADM) and multi-objective decision making (MODM). However, usage of one of the two terms (that is, MADM and MODM) refers to the same category of modeling (Triantaphyllou,

2000:1) Many real-life problems involve multiple objectives. Most linear programming problems are single objective-based (Bonini *et al.*, 1997).

2.3.2.1 Basic concepts of MCDM¹

❖ The “Criterion” tool

Gal *et al* (1999:1-29) define criterion as a tool that is constructed to evaluate and compare potential actions according to a well-defined point of view. The criterion cone is an important concept in multiple objective programming. Each MCDM problem is associated with multiple attributes and these attributes are also referred to as “goals” or “decision criteria”. These goals are often characterised by conflict (Steuer, 1986:170-181; Triantaphyllou, 2000:2).

❖ Optimum

Zenely (1982:61-62) describes the concept of optimum as used when one compares decision alternatives according to a single measure of merit. Gal *et al.* (1999:1-31) illustrate this concept optimum as an action for example: action **a** is optimum in a set **A** as measured by criterion **g** if any other action in A is worse or not better than **a** according to specified criterion.

❖ ‘Satisficing’

Moore & Weatherford (2001: CD12-12) define ‘satisficing’ as a mathematical programming concept that communicates the idea that individuals often do not seek optimal solution, but rather solutions that are good enough to satisfy various goals and constraints.

❖ Alternative

According to Gal *et al.* (1999:1-31), the concept of alternative refers to two or more actions that can in no way be jointly implemented and these actions are basically mutually exclusive to one another. Hence ultimately only one of them will be adopted in a decision making process.

¹ Indicate for Multi-criteria decision making

● Aspirational level

Gal *et al.* (1999:1-27) define 'aspirational level' as a degree on a criterion scale making performance level which, if accomplished, indicates that goals have been satisfactorily attained. According to Zenely (1982:65), improvement on this scale is deemed non-significant. One cannot establish a good and practically achievable aspirational level without firstly exploring associated limits. A decision maker would not like to fall short of an aspirational level.

❖ Conflict of criteria

According to Triantaphyllou (2000:2), different criteria represent different dimensions of the alternatives which are often in conflict with one another. For instance, cost may often in conflict with profit. Zenely (1982:143) indicates that a decision maker attempts to grasp the extent of conflict between means and ends, exploring limits attainable with each important attribute.

2.3.2.2 Classification of MCDM methods

MCDM methods classification is underpinned by various characteristics associated with such methods. However all these methods have got a common goal of making optimal decisions based on given attributes. Another way of classifying the MCDM is by the number of decision makers. That is, a single decision (only one decision maker involved) and a group decision (more than one decision maker involved) (Triantaphyllou, 2000:3). In this study the focus will be on single decision making rather than group decision making.

Although there are many ways to view MCDM methods, we concentrate in this dissertation on the concepts of multiple objectives and goal programming. Below is a description of some of these approaches:

❖ Single objective with others as constraints

A decision maker in this case decides that one objective is of such importance that it overrides the others. The rest of the objectives may be built in as constraints at some

minimal level. The problem then becomes an ordinary linear programming problem of maximizing an objective function subject to a set of constraints (Bonini *et al.*, 1997:159-160).

❖ **Define trade-offs among objectives**

A decision maker in this case specifies the trade-offs among objectives. The objectives would then be traded off with each other. The success of this approach lies in being able to define necessary trade-offs (Bonini *et al.*, 1997:160-161). The trade-off analysis would be the amount of one performance measure that must be sacrificed to achieve a given improvement in another performance measure (Moore & Weatherford, 2001: 61).

❖ **Goal Programming**

A third approach is that of goal programming. The decision maker in this case specifies desirable goals for each objective. Then the problem is formulated as a minimisation of deviational variables (that is, shortfall related to obtaining these goals) of target goals (Bonini *et al.*, 1997:161-162). Unlike in linear programming where optimum solution is sought, goal programming substitutes optimum solution with satisfactory solution (Volpi *et al.*, 2003). The main objective of this study is to apply this technique in the pricing problem.

❖ **Priority Programming**

A decision maker in this case attempts to achieve each objective sequentially rather than simultaneously. This is done by assigning certain priorities to each objective, indicating the order in which each is to be satisfied. A decision maker would list priorities of achieving goals in the order of importance (Bonini *et al.*, 1997:162-163).

2.4 Goal programming (GP) concepts

This section presents definitions, descriptions and classifications of goal programming concepts. It is important to establish how goal programming relates to other classes and super-classes of multi-criteria decision making. The diagram below depicts the relationship

among goal programming, multi-criteria decision making and management science or operations research:

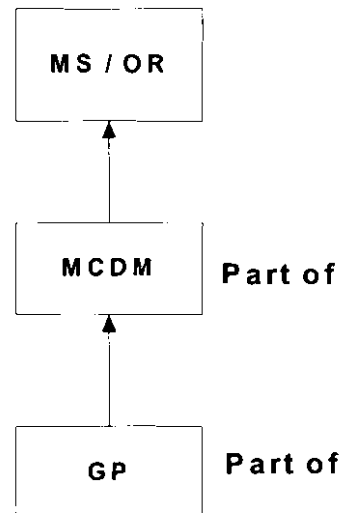


Figure 2-5 : Summary relationship of GP with MS / OR and MCDM (Schniederjans, 1995:13)

The diagram shows that GP is a subject within MCDM and MCDM is a subject within management sciences and / or operations research.

2.4.1 Goal(s)

A goal is a conceptualization of an objective into a target to be attained by minimizing related deviations (Steuer, 1986:282). A target is an acceptable level of achievement for any of the attributes considered by the decision maker, combining attributes with target results in a goal (Romero, 1991:1). Goals are often in conflict with one another and are characterized by competing for resources. Thus in some cases one needs to establish a hierarchy of importance among them (Holzman, 1981:103).

2.4.2 Decision maker

Mateu (2002:7) defines an actor (that is, a decision maker) as an individual for whom the decision-aid tools are developed and implemented. A decision maker from a goal programming point of view, plans to come as close as possible to satisfying various goals and constraints during the optimization process (Moore & Weatherford, 2001:CD12-12).

2.4.3 Variants and goal weighting

Goal programming variants are processes of assigning some weights or priority levels to deviational variables that show order of importance that a decision maker prefers to have in the minimisation of the objective.

Goal weighting is a process whereby a decision maker associates each objective or goal with relative importance in regard to other objectives (MMG², 2005:4). The process of goal weighting is done by assigning coefficients to deviational variables which distinguish the importance among goals (Moore & Weatherford, 2001:CD12-15). A description of how this concept works follows later in this chapter.

2.4.4 Deviational variables

Deviational variables are the amounts by which the plan fails to achieve a target goal. These deviational variables are used to measure deviations such as underachievement and overachievement of a threshold³ (Steuer, 1986:143). Usage of deviational variables follows later in this chapter.

2.4.5 Objective function

Objective function from a goal programming point of view is the sum of weighted or prioritized deviational variables which are to be minimized. This deviational variable may or may not have weighting or priorities (Moore & Weatherford, 2001:CD12-13). Only deviational variables appear in the objective function. Formulation of an objective function will follow later in this chapter.

2.4.6 Hard constraints (System constraints)

Decision makers often may have limits imposed by some physical considerations and company policy within which they are to operate. At any point in modelling, a decision maker has to recognise the presence of constraints (Bonini *et al.*, 1997:10). Hard constraints are also known as system constraints, which are type of constraints that cannot

² In this text MMG stands for management mathematics group

³ Threshold and target will be used interchangeable in this study

be violated, unlike goal constraints which may be violated when needs arise. The general formulation is:

$$f(x) = b \quad (2.3)$$

Where $f(x)$ is a mathematical expression of attributes and b is a restriction to be observed (Romero, 1991:2).

2.4.7 Goal constraints

From a goal programming point of view, goal constraints are soft and do not restrict the original feasible region. Hence measurement of deviational variables to their respective target (Steuer, 1986: 286). A logical structure of a goal is a combination of attributes associated with a target. The general format of a goal constraint is as follows:

$$f(x) + d_i^- - d_i^+ = b_i \quad (2.4)$$

Where $f(x)$ is a mathematical expression of attributes, d_i^- and d_i^+ are underachievement and overachievement of deviational variables respectively and b_i is the target value (Romero, 1991:3; Zanakis & Gupta, 1985:212). Usage of goal constraints will manifest later in this chapter.

2.4.8 Decision variables

Decision variables are those variables entirely under control of the decision maker representing choice of alternative as viewed important by a decision maker (Bonini *et al.*, 1997:9). These variables are usually unknown in terms of what values they will assume, until the solution is attained (Schniederjans, 1995:2). A discussion on usage of decision variables follows later in this chapter. This is in contrast to state variables, in which a decision maker has to accept certain factors. For example, doing business in a situation where a decision maker is far from suppliers, and cannot change this fact.

Goal programming is one of the multi-criteria decision making techniques that allows a decision maker to solve a problem with conflicting goals. The decision maker wants to come as close as possible to satisfying various goals and constraints. This technique has

two major constraints; namely: system constraints (that is, those that cannot be violated) and goal constraints (that is, those that may be violated to come closer to satisfaction of all goals). This method has deviational variables which are measured as overachievement and underachievement of goals. A decision maker wants to minimise the sum of all the deviational variables, and at optimality, at-least one of the deviation variables has to take a zero value. Like any other linear programming model, a goal programming has an objective function which has to be optimised (Moore & Weatherford, 2001). The decision maker may want to achieve goals sequentially rather than simultaneously. In this case goals will be prioritised and achieved one after the other in their order of importance according to the decision the maker (Bonini *et al.*, 1997:163).

2.5 Types of goal programming

In addition to the above mentioned classification of multiple objectives and goal programming, goal programming has several types and related applications. Listed below are the common types of goal programming:

❖ 0-1 Goal programming

A goal programming model in which only two binary values can be assumed by variables is called 0-1 goal programming (Moore & Weatherford, 2001:288). This is applicable to situations when there is a need to represent dichotomous decisions such as true or false decisions. This is sometimes called binary integer programming.

● Fuzzy goal programming

Fuzzy goal programming is based on fuzzy sets which are used to describe imprecise goals. These imprecise goals are associated with objective functions and their achievement is range based. Range in this case is measured by values from zero to one (Schniederjans, 1995:58).

❖ Mixed Integer goal programming

Integer programming is a general term for creating optimisation models with certain conditions imposed. These models in which only some of the variables are restricted to

assume integer values and others to assume any value is referred to as a mixed-Integer linear program (Moore & Weatherford, 2001:288).

❖ Integer goal programming

Integer goal programming approach solves problems that require solution variables to assume only integer values (Lee, 1972:185; Bonini *et al.*, 1997:163). This is applicable when dealing with units that can not be fractional.

❖ Interactive goal programming

Interactive goal programming approach is based on a procedure that employs elicitation of information regarding preferences from the decision maker (Sang & Shim, 1986:571). Demonstration of how interactive goal programming works can be demonstrated by reflecting the decision maker's preferences in the modelling process. Pre-emptive goal programming can also be used interactively (Gal *et al.*, 1999:8-14). The concept of pre-emptive goal programming is described later in this chapter by means of example (Zanakis & Gupta, 1985:212).

2.5.1 Generic format of goal programming

The general format of a goal programming model in which a decision maker imposes weighting on goals can be expressed as follows:

$$\text{Minimize } Z = \sum_{i=1}^m w_i (d_i^+ + d_i^-)$$

Subject to:

$$\sum_{j=1}^n a_{ij} x_j - d_i^+ + d_i^- = b_i, \quad \text{for } i = 1, \dots, m$$

$$d_i^+, d_i^-, x_j \geq 0 \quad \text{for } i=1, \dots, m; \text{ for } j=1, \dots, n$$

where

The assumption is that all variables are continuous.

No systems constraints are indicated in this format.

a_{ij} - coefficient of decision variables.

d_i^- - represent underachievement of the i th goal.

d_i^+ - represent overachievement of the i th goal.

b_i - is a quantitative target value for the i th goal.

X_j - j th decision variable.

w_i - is the relative weight of the goal i .

$w_1 = w_2, \dots, = w_n$ can be used in the case where there is no preference in the goals.

It is good programming practice to include both deviational variables in formulating the goal programming model's constraints. Only in the objective function would one be selective as to which deviational variables to be minimized (Schniederjans, 1995:4).

Because goal programming is an extension of linear programming, it has the same type of components of the model as that of linear programming. The components are decision variables, constraints or goals, and the objective function (Holzman, 1981:103). However goal programming is distinguished from linear programming by the following (Steuer, 1986:282):

- The conceptualisation of objectives to goals;
- ❖ The assignment of priorities and / or weights to the achievement of the specified goals, or application of variants in goal programming which is not applicable to linear programming;
- ❖ The availability of deviational variables d_i^+ and d_i^- to measure underachievement and overachievement for the specified target goals;
- The minimisation of weighted sums of deviational variables to find solutions that best satisfy goals specified.

A decision maker strives to understand and decide which deviational variables to minimise. The following table presents the basic goal programming considerations:

Table 2-1: Summary of the use of goal constraints

Type	Deviations to be minimized	Description
$f_i(x_1, \dots, x_n) + d_i^- - d_i^+ = b_i$	d_i^- and d_i^+	Target
$f_i(x_1, \dots, x_n) + d_i^- \geq b_i$	d_i^-	Minimise underachievement
$f_i(x_1, \dots, x_n) - d_i^+ \leq b_i$	d_i^+	Minimize overachievement
$a_i - d_i^- \leq f_i(x_1, \dots, x_n) \leq b_i + d_i^+$	d_i^- and d_i^+	Goal interval constraints

Goal constraints are written by using non-negative deviation variables d_i^+ and d_i^- . At optimality at least one deviational variable will always be zero. Only deviational variables appear in the objective function (Moore & Weatherford, 2001:CD12-14).

A common procedure in formulating a goal programming model is to: (1) define decision variables, (2) state the constraints, (3) determine the pre-emptive priorities if need be, (4) determine the relative weights if need be, (5) state the objective function, and (6) state non-negativity or given constraints (Schniederjans, 1995:21).

2.5.2 Example of Goal programming formulation

Consider the dilemma faced by the JonesToy corporation - the company is currently producing two types of products, namely: (i) toy elephants and (ii) toy giraffes:

The dynamics of the company are such that:

- ❖ Each elephant requires an eighth of a day of production labour and each giraffe requires a quarter of day of production labour.
- ❖ The profit contribution is R2 per elephant sold and R3 per giraffe sold.
- ❖ JonesToy would like to arrange their business in order to produce at least four elephants and one giraffe a day.

JonesToy corporation's goals

- ❖ Make a sufficient amount of profit, say R20 per day.
- ❖ Workers should work a normal day, no overtime allowed.
- ❖ Produce at least one giraffe per day.
- ❖ Produce at least four elephants per day.

Definition of decision variables

x_1 – number of elephants produced per day.

x_2 – number of giraffes produced per day.

$$\text{Minimize } Z = d_1^- + (d_2^- + d_2^+) + d_3^- + d_4^-$$

Subject to:

$$2x_1 + 3x_2 + d_1^- - d_1^+ = 20,$$

$$x_1/8 + x_2/4 + d_2^- - d_2^+ = 1,$$

$$x_2 + d_3^- - d_3^+ = 1,$$

$$x_1 + d_4^- - d_4^+ = 4,$$

$$\text{and } x_1, d_1^-, d_1^+ \geq 0$$

where

d_1^+ represents overachievement of profit goal.

d_1^- represents underachievement of profit goal.

d_2^+ represents overachievement of the labour working time.

d_2^- represents underachievement of labour working time.

d_3^+ represents overachievement of the giraffe target value.

d_3^- represents underachievement of the giraffe target value.

d_4^+ represents overachievement of the elephant target value.

d_4^- represents underachievement of the elephant target value.

Note that in this case, the decision variables may be constrained to assume integer values, unless some interpretations can be given to fractional or continuous values.

2.5.3 Goal programming variants

Goal programming variants are methods of solving goal programming problems. The two well known programming variants are weighted and lexicographic goal programming methods. Each of these are described in detail in succeeding sections, although the focus of this study is the application of weighted goal programming rather than lexicographic goal programming.

2.5.4 Weighted goal programming

Weighted goal programming places all deviational variables in a single priority level distinguished by different weights to represent their respective importance. The algebraic expression of weighted goal programming has the following generic structure:

$$\begin{aligned} \text{Minimize } a &= \sum_{i=1}^m (\alpha_i d_i^- + \beta_i d_i^+) \\ \text{Subject to} \\ f_i(x) + d_i^- - d_i^+ &= b_i \quad i=1, \dots, m \\ x_i, d_i^-, d_i^+ &\geq 0 \end{aligned}$$

where

a – represents objective function to be minimized,

x – represents a vector of decision variables,

α_i – represents weighting factor for underachievement,

β_i – represents weighting factor for overachievement.

This is defined by Tamiz *et al.* (1995:43-45) and (Romero, 1991:3-4) in the formulation of weighted goal programming model.

2.5.5 Graphical representation of GP model

Suppose JonesToy decides on ranking the objectives as follows:

Table 2-2: Weighting of goals by a decision maker

Goal	Description	Weights
1	Profit	2
2	Labour	5
3	Manufacture (G)	1
4	Manufacture (E)	1

Interpretation of these weights is that this profit (goal 1) is twice as important as manufacturing giraffes (goal 3) and elephants (goal 4). However labour (goal 2) is five times as important as the manufacturing of elephants and giraffes. The objective function in section 2.5.2 will be replaced by the following objective function:

$$\text{Minimize } Z = \frac{2d_1^-}{20} + 5d_2^- + 5d_2^+ + d_3^- + \frac{d_4^-}{4} \quad (2.5)$$

All the weights in the objective function have been divided by their right-hand side value of the goal constraints; this is to express all the deviational variables as percentages from their targets. This concept is known as percentage normalization in the objective function.

Figure 2-6 and table 2-3 show a solution after the objective function deviational variables have been optimised. The optimum point is located at B as seen on the figure 2-6 below:

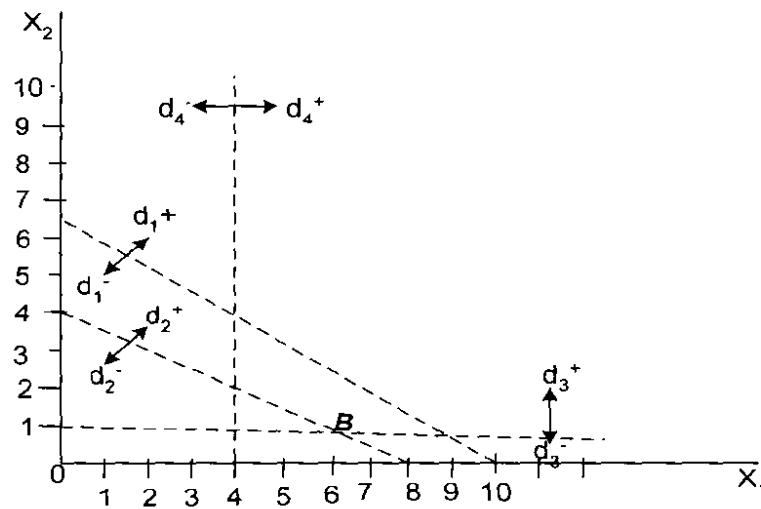


Figure 2-6: Graphical solution for a simplified GP model

The table below reports results analyses of how each of the goals stipulated by JonesToy Corporation performed.

Table 2-3 : Results from a weighted goal programming model

Goals	d_i^-	d_i^+	Results
Goal 1	$d_1^- = 5$	$d_1^+ = 0$	Partially achieved
Goal 2	$d_2^- = 0$	$d_2^+ = 0$	Fully achieved
Goal 3	$d_3^- = 0$	$d_3^+ = 0$	Fully achieved
Goal 4	$d_4^- = 0$	$d_4^+ = 2$	Partial achieved

A decision maker may at one stage have a preference to solve goals of a goal programming model. In this case the decision would have established variants in solving these goals. The two commonly used goal programming variants are weighted goal programming and lexicographic goal programming.

2.5.6 Normalization techniques

Normalization is a process of bringing all deviational variables to a common unit of measurement based on the degree of proximity to the goals. Ossama *et al.* (2004:1837) indicate that un-normalized deviations are undesirable since this implicit weighting could alter the weights which the decision maker effectively allows. There is a number of

normalization techniques listed in the goal programming literature. Listed below are the commonly used normalization techniques:

❖ **Euclidean normalization**

This normalization technique divides each objective by the Euclidean norm of its coefficients. The following example shows how this normalization is done:

$$3x_1 + 4x_2 + n - p = 16 \rightarrow 3/5x_1 + 4/5x_2 + d_j^- - d_j^+ = 16/5 \quad (2.6)$$

Goal programming models with more than one goal to be weighted need to be normalized to avoid incommensurability of goal constraints (Tamiz *et al.*, 1995:44; Schniederjans, 1995:39). This problem is also present in a lexicographic goal programming model when goals making up a certain priority are measured in various units.

❖ **Percentage normalization**

In this normalization method, each coefficient is divided by its (target) right-hand side and then multiplied by 100. The new deviations will result in percentage deviations from target goals. The following example illustrates percentage normalization:

$$\begin{aligned} 120x_1 + 180x_2 + d_i^- - d_i^+ &= 4000 \\ 3x_1 + 4.5x_2 + d_j^- - d_j^+ &= 100 \end{aligned} \quad (2.7)$$

The critical factor in this method is the goal target value b_i . This method works well except in the situation where the target value is very small.

These normalization techniques were considered originally by Romero (1991:36) as a critical issue of goal programming and later by Tamiz *et al.* (1995:44) in a review of goal programming and its applications.

2.5.7 Lexicographic goal programming

The second well known variant is called lexicographic, pre-emptive or priority goal programming. In this case a decision maker arranges objectives in order of importance and assigns levels of priority to these objectives. Suppose that JonesToy decides to prioritise the objectives as follows:

- ❖ Priority level 1: Satisfying manufacturing levels (objective 3 and 4).
- ❖ Priority level 2: Achieving profit level (objective 1).
- Priority level 3: Maintain work force level (objective 2).

$$\text{Minimize } Z = P_1(d_3^- + d_4^-) + P_2d_1^- + P_3(d_2^- + d_2^+) \quad (2.8)$$

Where

$P_1 \gg P_2 \gg P_3$, this means that the first goal is much more important than the second which in turn is much more important than the third.

The rationale of this is to minimise the first term $(d_3^- + d_4^-)$. After minimisation of this term, the second term will be minimised (d_1^-) and the last term $(d_2^- + d_2^+)$ one after the other (MMG, 2005). The prioritisation of goals in goal programming must accurately reflect the decision environment, failure to establish priority among goals in a goal programming model brings with it a criticism of naïve prioritisation (Schniederjans, 1995:29).

2.5.8 Goal programming solutions

A decision maker would like to obtain solutions that suffice from the optimisation process. Because some solutions are unacceptable, there exists a general framework for solving goal programming models to avoid such solutions. A general framework (or algorithm) with four steps has been suggested by Romero (1990:20). The first step suggests that a

decision maker solves the initial WGP⁴ or LGP⁵. If there is no other optimal solution, a decision maker adopts the existing one. If there are other optimal solutions, the algorithm goes to step 2, if a decision maker is interested in exploring the set of optimal and GP-efficient solutions (Romero, 1990:19-20), the algorithm will flow through other relevant steps as indicated in figure 2-7 below.

Some solutions are sometimes inferior to other feasible GP solutions (which are alternative GP optima) with regard to true meaning of the goals and priorities as stipulated by a decision maker. An illustration of a procedure relating to Nondominance⁶ in goal programming is found in the literature (Hannan, 1980:304-308) through exploring alternative (if any) optima solutions. The following is a general framework to search for 'satisficing' solutions:

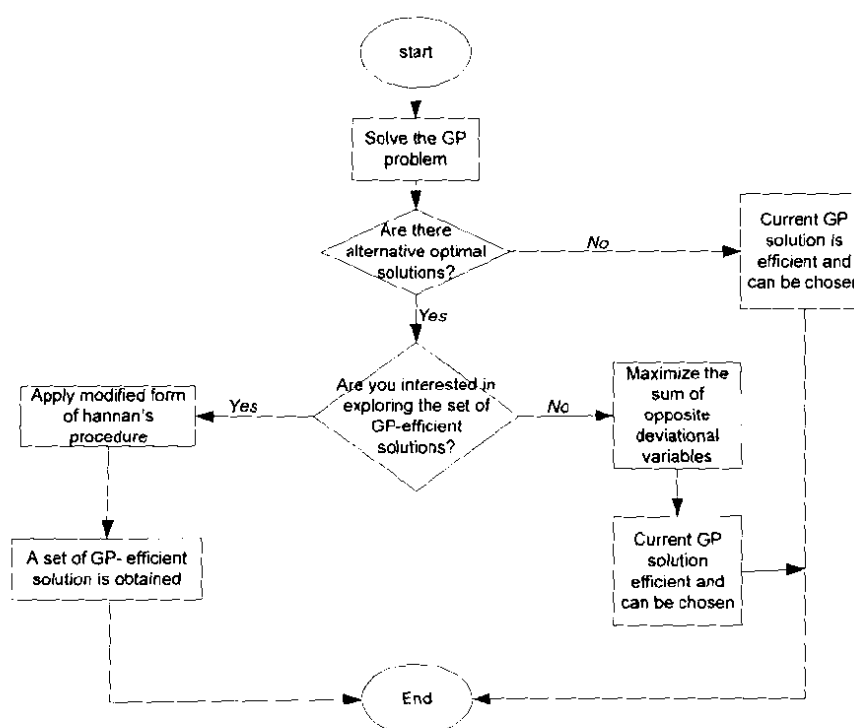


Figure 2-7 : Algorithm for solving GP models avoiding inferior solution (Romero, 1990:20)

⁴ Weighted goal programming

⁵ Lexicographic goal programming

⁶ Achievement with respect to two-sided goals and is not dominated by any other solution

The algorithm above suggests that it may be important to test many different weighting structures in relation to a decision maker's preferences, in order to obtain the most efficient and optimal solution for a GP model. The implementation of the profit model optimisation in this study will embrace the application of the general framework to avoid inferior solutions. This is to ensure that the decision maker's preference will motivate usage of the algorithm to explore more optimal solutions.

2.6 Types of goal programming applications

Goal programming is one of the widely utilized techniques in multiple objective decision making. The common applications of goal programming are listed but not restricted to the following:

2.6.1 Resource allocation

The application of goal programming technique in resource allocation related decisions has shown success. Volpi *et al.* (2003:165-178) developed a project of land allocation in which goal programming techniques were used in order to achieve goals such as maximisation of tourism and wood harvests in a resource allocation problem.

2.6.2 Agriculture

The application of goal programming techniques in agricultural problems are described in Adejobi *et al.* (s.a) where farmers solve conflicting objectives such as providing food for families throughout the year, accumulating more monetary income and minimising labour expenses.

2.6.3 Finance

The application of goal programming techniques to finance related decisions has shown success. Booth and Blessler (1989:81-82) have developed an asset and liability management model.

2.6.4 Engineering

The application of goal programming techniques in the field of engineering has solved many engineering problems. In chemical waste reduction problems, Chang and Hwang (1996: 3951) have shown that goal programming can be used in an effective manner in a waste reduction problem.

2.6.5 Academic planning and research development

The use of the goal programming technique in academic planning has shown great success. Time-table scheduling using goal programming developed by Croucher (1984:146-151) has solved the problem of allocating courses to various time slots. Optimisation of university tuition and fee structure via goal programming (Greenwood & Moore, 1987: 601) has shown the capability of the goal programming technique in this regard. Besides academic planning, the development of goal programming through research has proliferated widely as indicated in the literature by Zanakis and Gupta (1985: 211), Tamiz *et al.* (1995:44), Saul (1986:779), Sherali and Soyster (1983:173) and Ossama *et al.* (2004:1833). These publications have played a significant role in the increase of literature on goal programming applications.

2.6.6 Portfolio analysis

Multi-criteria decision making as an umbrella to the goal programming technique has also shown success in solving problems in portfolio comparisons compared to the usage of probabilistic estimates as indicated in Martel *et al.* (1988:617). The selection of a portfolio for research and development through multi-criteria decision support systems by Stewart (1991: 17) in a large electricity supply corporation has added to the arsenal of multi-criteria decision making methods in portfolio analysis.

2.6.7 Business

The application of the goal programming technique to the business related decisions has also proliferated. Small businesses have begun to apply decision support systems to decision making. Sang and Shim (1986: 571) have developed an interactive GP system on the microcomputer for small businesses, and the system determines the best priority

structure for the goals established by small business owners. Universities considered as businesses also use goal programming to optimise their processes. Greenwood and Moore (1987:601) have considered fee structures related to students' tuition obligations. Modelling the Telecommunication pricing decision using a weighted sum linear goal programming approach by Brown and Norgaard (1992:675-676) has shown applicability of the approach. The model simulates prices based on the profit maximisation goal, cost-covering goal and Ramsey regulatory prices.

The diagram below depicts a summary of applications and some general concepts about goal programming:

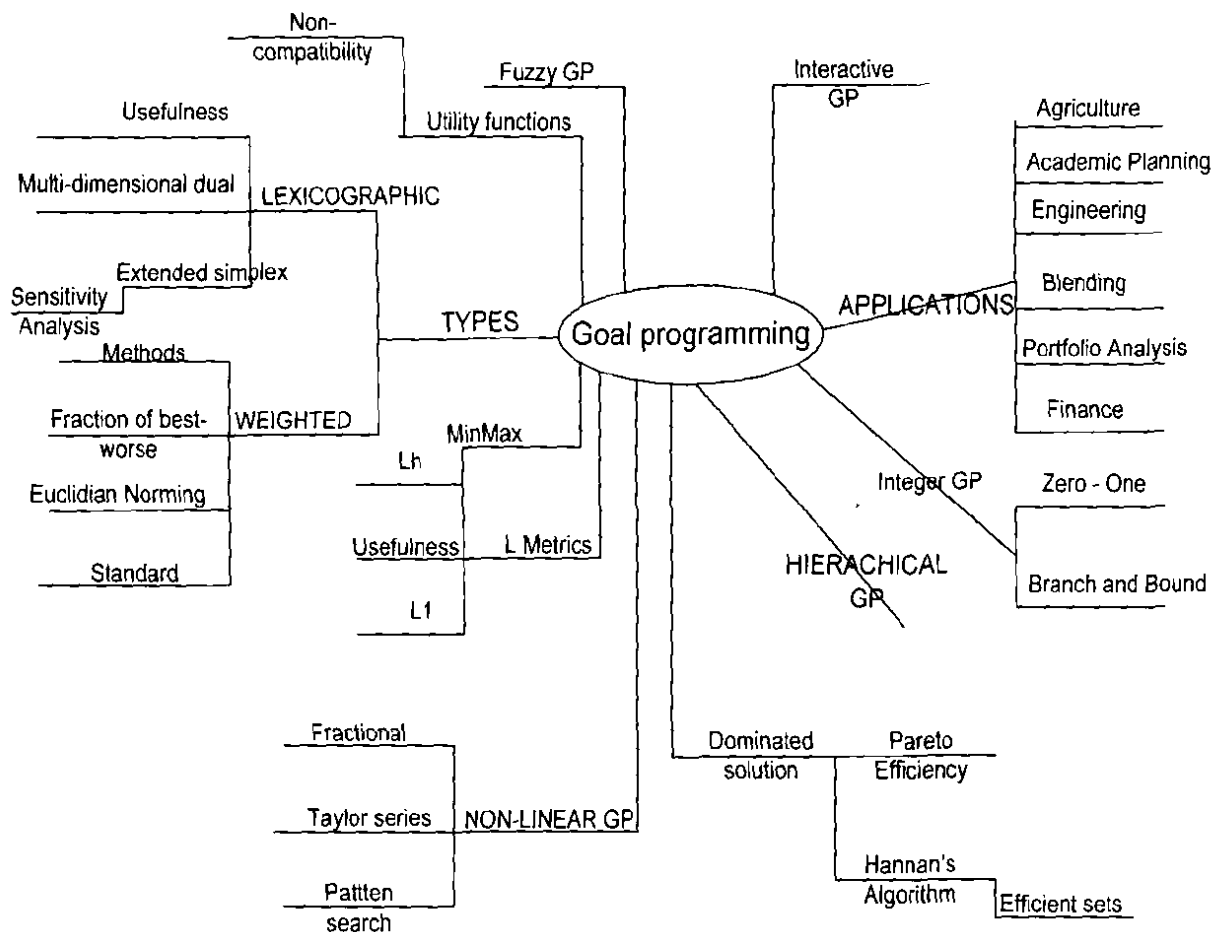


Figure 2-8: Graphical representation of GP topics adopted from Tamiz et al. (1995:42)

The figure above shows, as some experts have indicated, that goal programming is one of the foremost management science techniques to consider when dealing with problems related to multiple objectives (Gal *et al.*, 1999:8-2). It is also a great goal-setting tactical device which complements the pursuit of objectives in different fields (Zenely, 1982:280-281).

2.7 Existing pricing strategies

2.7.1 Pricing based on goal programming

A number of price models exist that suggest price determination. As many factors or price aspects exist, goal programming models have been developed to take into account some of these factors to suggest reasonable prices having considered all the goals and restrictions of being close to competitors' prices and not too far from existing prices (Bell, 2003:63-70).

2.7.2 Pricing for electronic commerce

As a result of the growing way of doing business such as electronic commerce, some price models are suitable for e-commerce systems. In this investigation, the price model consists of a description of product price components such as product transaction and contract. All components of this price model, which result in a reduction or increase of a basic price, can be reconciled in a price model (Kelkar *et al.*, 2002:366-375).

2.7.3 Dynamic pricing

"Pricebots" is an intelligent agent that does the pricing for online markets. Pricebots dynamically calculates a profit maximizing price of a product for sellers. This agent determines a price in response to parameters such as prices and profit of competitors and prices that buyers are willing to pay (Dasgupta & Hashimoto, 2004:277- 284). SiteSell.com has recently developed a system by the name of MYPS⁷ which is a web application price model and does the pricing work on-line. This system mathematically determines a price for a product (SiteSell.com, 2005)⁸.

⁷ MYPS stands for Make Your Price Sell from www.sitesell.com

The dynamic algorithm of Pricebot works as follows; the algorithm assumes that a product has a reservation price distribution that follows standard distributions, whose parameters are unknown and must be determined by the pricebot. There are a number of sellers and buyers, sellers are profit maximizers and buyers would like to pay a reserved price. The calculations are done by the seller for each product attribute. Mean and standard deviation have to be estimated with reasonable accuracy so that sellers can determine their profit maximizing price, bearing in mind that every buyer has reservation price. The algorithm further uses parameters such as market prices, price update interval and cost to calculate an optimum price for a product.

A hypothetical market shows that buyers have got preferred attributes of products sold by different sellers. A buyer would request a quotation from different sellers in respect to these attributes and selects the seller who offers the best price (Dasgupta & Hashimoto, 2004:277- 284).

2.8 Summary and comments

This chapter has surveyed general pricing concepts, multi-criteria decision making concepts, general application and formulation of goal programming models. Other aspects that were addressed are:

- ❖ Price elasticity showing the ratio of the percentage change in quantity demanded to the associated percentage change in price.
- ❖ Goal programming variants describing the goal programming solution approach. The examples in this case are weighted and lexicographic goal programming models.
- ❖ Pricebots show a situation whereby buyers and sellers are price determiners. These also allow customers to negotiate prices.

⁸ More information may be obtained from 450-458-1064 about MYPS product and www.sitesell.com

CHAPTER THREE

PROFIT MODEL FORMULATION

3.1 Introduction

This chapter is aimed at reviewing one of the existing profit models (Bell, 2003:66 – 68) and formulating a more detailed profit model which a decision maker can utilize through proper and realistic pricing of products / or services. Profit maximization through good pricing strategies is a challenge especially to most small and emerging businesses.

Profit is defined to be the difference between total revenue and total cost. It is linked with price, such that the higher the price the more the profit and the lower the price the lower the profit. The only problem to be tackled is the relationship between unit price and the number of units sold. Businesses survive because they make profit (Rogers, 1990:25-26 & DeBrock, 2004:3-6). If they are greedy however, their volume of sales may decrease.

3.2 Pricing objectives

A decision maker determines prices for products and / or services with some objectives in mind. The commonly known pricing objectives are pricing for profit maximisation, pricing for growth, pricing for maintaining good image or prestige and pricing for maximised volumes. The following diagram portrays common pricing objectives:

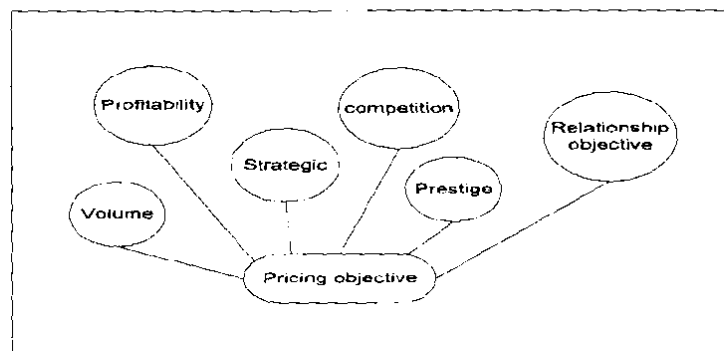


Figure 3-1: Pricing objective (Crawford, 1997: 3)

Some practical problems of price theory and profit maximisation are that it may be difficult to estimate the demand because some products are price elastic and others are not (Crawford, 1997:13). Again some pricing objectives may be in conflict with one another as some decision makers would like to price for more than one objective.

3.3 Profit determinants

A good profit is normally determined by a related demand. The demand for a product is mainly determined by price (a major determinant). Other factors also exist such as income and wealth of potential buyers (Rogers, 1990:42). In every demand situation there should be supply, therefore supply is also directly or indirectly influential on the profit realised.

Another important concept in profit formulation and maximization is that of a break-even point. A break-even point is a point at which after the process of pricing, selling and incurring cost, total profit equals zero. That means total revenue was exactly equal to total cost. Equilibrium is also vital especially when intending to maximise profit in a perfectly competitive business environment. The following graph depicts equilibrium as a result of demand and supply:

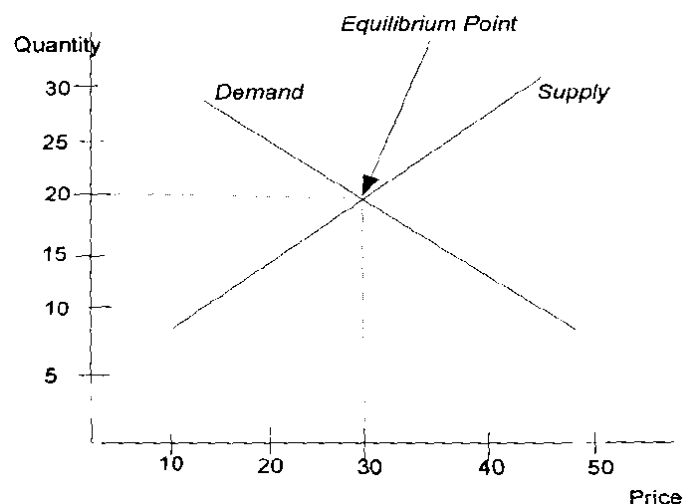


Figure 3-2: Demand and supply equilibrium point

Equilibrium is desirable because it implies that pricing was good and that the price at which quantity was supplied by sellers equals to the quantity demanded by buyers (Crawford, 1997:7). Good pricing should result in equilibrium or excess demand to signal that a decision maker still have willing buyers.

3.4 Profit models

This section presents a generalized formulation of profit models. These profit models are designed for the small and emerging home-based businesses to overcome profit optimization challenges through proper and realistic pricing.

3.4.1 Profit model with assumption of Independence between volume and price

Some profit models (Bell, 2003:66 – 68) assume that even if a price changes, it does not influence volume much, that is, volume is relatively independent of price change. This work may raise criticism because price and demand usually have a relationship of some sort. Some studies by Cepeda (2005:1-4), Haydam (1997:118 – 120) and Rogers (1990:44-53) have shown that volume and price often have a linear relationship. This profit model like most others, computes profit using two components, namely revenue and cost. The expected revenue is computed as a product of volume (v) and price (x) for k items. The formula below describes revenue:

$$\sum_{i=1}^k v_i \cdot x_i \quad (3.1)$$

The cost (c) of acquiring items and / or products is adjusted per item. This is calculated as a product of volume, cost adjustment (if any) and current cost of item i . The following formula was used to describe cost component of the profit model:

$$\sum_{i=1}^k v_i \cdot ((1 + \sigma) \cdot c_i) \quad (3.2)$$

The factor $(1 + \sigma)$ indicates the expected increase in cost at some future date and depends on the planning horizon.

Two components constitute the profit function namely, revenue from equation (3.1) and cost from equation (3.2). The following formula describes the profit based on revenue and cost which a decision-maker wishes to maximize:

$$\sum_{i=1}^k v_i \cdot x_i - \sum_{i=1}^k v_i \cdot ((1 + \sigma) \cdot c_i) \quad (3.3)$$

3.4.2 Profit model with assumption of dependence between volume and price

An assumption made in this study is that volume is dependent on price changes unlike in the profit model by Bell (2003:66 – 68). This means that any change in a price is highly likely to influence volume of sales either positively or negatively. Depending on what relationship volume does have with price (for example, a linear relationship), such a relationship will determine what change in the volume will be expected to occur.

Thus we consider models where volume of an item v_i is a function of the price x_i denoted by $v_i(x_i)$.

Assuming a linear relationship, the diagram below depicts the expected relationship between price and volume indicating that a lower price (x_i) will generally result in increased volumes and vice versa.

Assume (change in price of the i th product) $x_{ch_i} \geq 0$, for explanation but later can assume negative values.

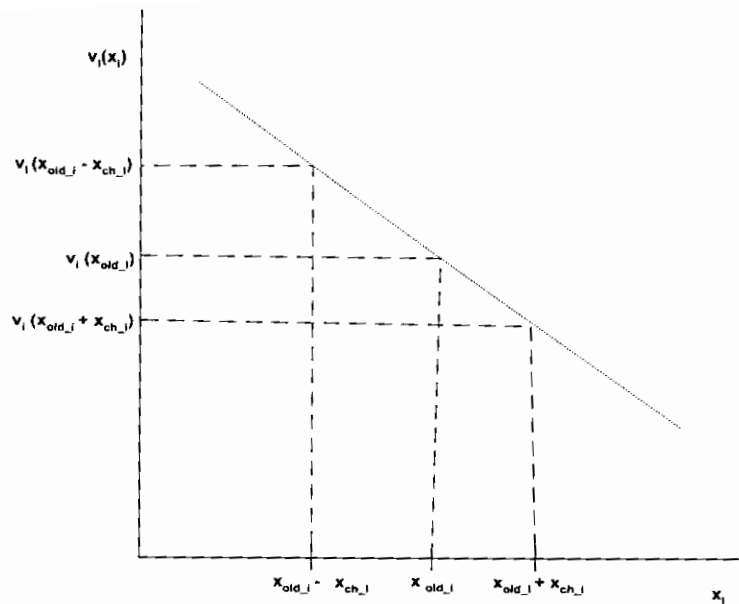


Figure 3-3: Price-volume relationship

Based on the price-volume relationship graph above, increasing a price by x_{ch_i} would result in a probable drop of volume as denoted by $v_i(x_{old_i}) - v_i(x_{old_i} + x_{ch_i})$ etc.

3.4.2.1 Elasticity and its slope

The model to be implemented in this study assumes a linear relationship between price and volume, the slope being the price elasticity of demand.

Based on the price-volume relationship above, one can deduce that there is an old price and old volume, new price and new volume. The profit function has to differentiate between the old volume and the new volume as a result of price change in terms of formulation.

3.4.2.2 Current revenue ("old" revenue)

Profit in this regard relates to previous sales of products / or services and their related costs. $v_i(x_{old_i})$ is a volume of sales and (x_{old_i}) is the price at which volume was sold, the two components determine current revenue. The formula below describes current revenue:

$$\sum_{i=1}^k v_i (x_{old_i}) \cdot x_{old_i} \quad (3.4)$$

3.4.2.3 Current cost ("old" cost)

A decision maker also incorporates cost as an important component in profit realization. In this case, it is the cost of acquiring product and / or service in certain volumes. $v_i (x_{old_i})$ is a volume and c_i is a related cost. The following formula describes the current cost part of the old profit:

$$\sum_{i=1}^k v_i (x_{old_i}) \cdot c_i \quad (3.5)$$

3.4.2.4 Current profit

Current profit is the difference between equations (3.4) and (3.5) above. A break-even point would be realized when current profit equals zero, this means that total revenue was equal to total cost / expenses. The formula below gives the current profit:

$$\text{Current Profit} = \sum_{i=1}^k v_i (x_{old_i}) \cdot x_{old_i} - \sum_{i=1}^k v_i (x_{old_i}) \cdot c_i \quad (3.6)$$

where

$v_i (x_{old_i})$ = represents volume of product and / or service i at price x_{old_i} .

x_{old_i} = represents old price of product and / or service i.

c_i = represents cost associated with product and / or service i.

A decision maker usually changes prices with the intention of maximizing profit. As prices change the volume is normally affected and a corresponding change in profit will result. In formulating the new profit, it is important to determine changes in price and volume.

3.4.2.5 New price ("Suggested" price)

A new price which a decision maker wishes to establish (that is, after price change) is described as:

$$x_i = x_{old_i} + x_{ch_i} \quad (3.7)$$

3.4.2.6 Change in volume of sales

Assuming a linear relationship, the following formula describes the new expected change in volume:

$$v_{ch_i} = v_i (x_{old_i} + x_{ch_i}) - v_i (x_{old_i}) \quad (3.8)$$

where

v_{ch_i} - represents the change in expected volume as a result of the price change.

3.4.2.7 Demand / volume function

Volume which is also known as quantity demanded is described by the following generic formula in the case of a linear model assumption:

$$\text{Volume} = t_i + a_i \cdot x_i \quad (3.9)$$

where

t_i – represents an intercept on the volume axis.

a_i - represents a (typically non-positive) factor that describes the slope.

x_i - represents a price of product / or service.

The above generic volume function is used in this study. The new expected volume is described by the following formula:

$$v_{\text{new}_i} = v_i(x_{\text{old}_i}) + a_i \cdot (x_{\text{ch}_i}) \quad (3.10)$$

We thus have:

$$a_i \cdot x_{\text{ch}_i} = v_{\text{new}_i} - v_i(x_{\text{old}_i}) \quad (3.11)$$

This means that equation (3.8) can be rewritten as:

$$v_{\text{ch}_i} = a_i \cdot x_{\text{ch}_i} \quad (3.12)$$

3.4.2.8 New revenue ("Expected" revenue)

The expected revenue after the price change is now:

$$\sum_{i=1}^k (v_{\text{ch}_i} + v_i(x_{\text{old}_i})) \cdot (x_{\text{old}_i} + x_{\text{ch}_i}) \quad (3.13)$$

Note that we relax the assumption that x_{ch_i} is non-negative for modelling purposes.

$$= \sum_{i=1}^k [(v_{ch_i} \cdot x_{old_i}) + (v_{ch_i} \cdot x_{ch_i}) + (v_i(x_{old_i}) \cdot x_{old_i}) + (v_i(x_{old_i}) \cdot x_{ch_i})] \quad (3.14)$$

If new notations for the constants in the formula are introduced, equation (3.14) simplifies to:

$$\sum_{i=1}^k [v_{ch_i} \cdot K_{1i} + v_{ch_i} \cdot x_{ch_i} + K_{3i} + K_{2i} \cdot x_{ch_i}] \quad (3.15)$$

where

K_{1i} – represents the old or existing price of item i , which is already known.

K_{2i} - represents the old or existing volume of item i , which is already known.

K_{3i} - represents the old / previous revenue of item i , which is already known.

Substituting $a_i \cdot x_{ch_i}$ for v_{ch_i} the model changes from a linear to a quadratic function as follows:

$$\sum_{i=1}^K [a_i \cdot (x_{ch_i})^2 + (K_{1i} \cdot a_i + K_{2i}) \cdot x_{ch_i} + K_{3i}] \quad (3.16)$$

The formula above describes expected revenue which is one of the components of total profit which we ultimately strive to maximise. It is thus also important to establish new cost as a result of price and volume changes. We address this below.

3.4.2.9 New cost

For the purposes of this dissertation, we have considered the possibility that cost may have change for this planning horizon. We have assumed that a general increase by a factor σ is

applicable and have not considered the case where different (σ_i) may be applicable to the items i .

We have also not considered fixed costs. It is accepted that fixed cost will have an impact on total costs and profit. However the breakdown of fixed costs and the allocation thereof to individual product items are beyond the scope of the model illustrated in this study.

The total cost after price changes and cost change will then be:

$$\sum_{i=1}^k (v_{ch_i} \cdot c_i(1 + \sigma)) + (v_i(x_{old_i}) \cdot c_i(1 + \sigma)) \quad (3.17)$$

Substituting $a_i \cdot x_{ch_i}$ for v_{ch_i} from equation (3.12) and introducing notation for constants, we have:

$$\sum_{i=1}^k [a_i \cdot x_{ch_i} \cdot c_i(1 + \sigma) + K_{4i}] \quad (3.18)$$

where

K_{4i} = old / previous volume times the current cost.

3.4.2.10 A revised profit function

A general profit function will be:

$$\text{New Profit} = \sum_{i=1}^k [(v_{ch_i} + v_i(x_{old_i})) \cdot (x_{old_i} + x_{ch_i}) - (v_{ch_i} + v_i(x_{old_i})) \cdot (c_i(1 + \sigma))] \quad (3.19)$$

In the light of equations (3.16) and (3.18) we thus find the profit function to be:

$$\sum_{i=1}^k (a_i \cdot (x_{ch_i})^2 + (K_{1i} \cdot a_i + K_{2i}) \cdot x_{ch_i} + K_{3i} - (a_i \cdot x_{ch_i} \cdot c_i \cdot (1 + \sigma) + K_{4i})) \quad (3.20)$$

The model above is a quadratic function. The convexity and concavity properties of such functions have to be considered in the optimisation process.

3.5 Summary

This chapter has reviewed one of the existing profit models and introduced a more detailed profit model formulation that accounts for a linear relationship between price and quantity demanded. A number of concepts that are important to profit formulations have been discussed as follows:

- ❖ Pricing objectives are decision makers' pricing motives looking at circumstance in which business is been made. Such commonly used objectives are pricing for competition, image, volume, profitability. Some decision makers combine a number of these objectives.
- Revenue is gross sales of products and / or services before subtracting associated costs. A decision maker intends to have high revenue through proper and realistic prices to keep customers buying.
- Cost is a component of profit that should be subtracted from revenue in order to realize profit
- ❖ Profit is total difference between total revenue and total costs. A decision maker strives to maximize profit in order to survive in business.
- ❖ Price / volume elasticity concepts were introduced in this chapter and some model assumptions were discussed.

CHAPTER FOUR

GOAL PROGRAMMING APPROACH TO PROFIT MODELS

4.1 Introduction

This chapter is aimed at formulating a goal programming model for profit maximisation using a multiple goals approach. All necessary constraints emanating from general pricing and profit optimisation problems are modelled to try to approach realistic pricing strategies. The technique used in this study is capable of considering multiple and often conflicting objectives. The conflicting objective, in this case is that, a decision maker would like to maximise profit without setting high prices. A decision maker may also observe competition prices and move closer to such prices. In order not to shock customers by prices that are far away from previous prices of products and / or services, it could also be a goal to make more incremental changes. At the same time cost must obviously be reckoned with.

Model formulation is a process of transforming a real-world decision-oriented problem into a management decision model. In the process of formulating a goal programming model it is often the custom to minimize a summation of deviational variables with certain priorities and / or weights (Lee, 1972:39 - 41). According to Schniederjans (1995:21), Lee (1972:61) and Romero (1991:2 - 4), the process of model formulation starts with determining decision variables, followed by determining goals and constraints, weights and / or priorities, objective function and non-negativity conditions. The same approach will be adopted in this study regarding model formulation.

4.2 General approach

The demand and price of a single product are some of the determinants of profit. Demand cannot be estimated in absence of a price. The demand is considered to be the total quantity that will be purchased by customers at a given price level from which profit will be realizable after total costs are removed (Rogers, 1990:37). It is a goal of every firm to

maximise the profit function and realise a good profit. The following figure portrays the curve of a typical profit function:

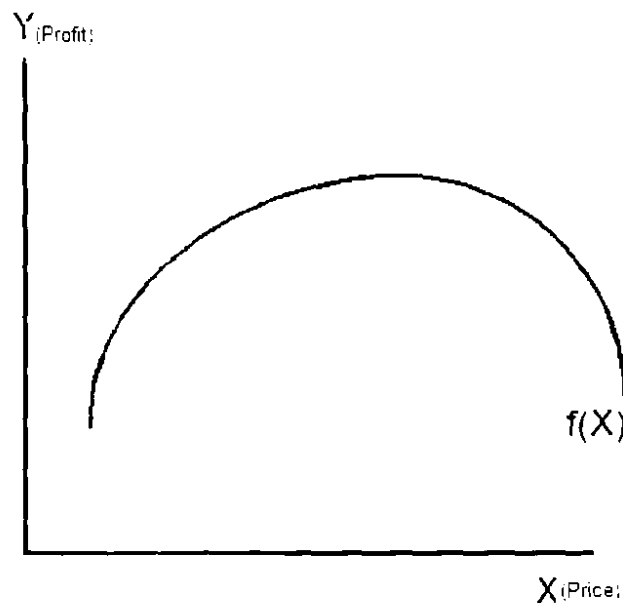


Figure 4-1: Graphical representation of Profit function (DeBrock, 2004:3)

The figure above has a curve which will be at its optimum point when the slope is equal to zero. The slope of this function is “rise over run”, or the proportion of the change in the vertical measure to the horizontal measure (DeBrock, 2004:3-6). A decision maker would like to simulate prices of products and / or services around a point where it is the maximum point $y = f(x)$ (that is, at the point where slope of the graph is zero).

4.3 Goal programming models for pricing decision

4.3.1 Introduction

Profit goals formulations differ in most models. Some models formulate a firm’s profit goal as only deviation from average marginal profit with a certain desirable target (Lee & Shim, 1986:572) whereby a decision maker wishes to minimize the under achievement of the profit goal. Some profit goals are the difference between revenue and cost (Brown & Norgaard, 1992:676) which is the profit function that a decision maker would like to maximise.

4.3.2 Choice of goal programming approach

We will concentrate in this dissertation on a weighted goal programming model and not a lexicographic goal programming model. The reason for a choice of weighted goal programming is that goals in this study are assigned weights and optimised in a weighted order. A decision maker often decides on the weights for the pre-specified goals to indicate the relative importance of the goals.

4.3.3 Profit goals considered in this study

Profit goal in this model incorporates the total change in price which influences demand / volume. A decision maker would like to come closer to old profit with certain adjustment. For example, a decision maker may wish to increase previous profit by 10% or any preferable value that would have fairly maximised profit.

A decision maker may wish to achieve a target level of profit. The profit goal restriction will be to stay as close as possible to a desirable profit goal. This goal can be achieved by minimising negative and positive deviational variables P^r and P^{r+} from a profit goal. Sometimes a decision maker will be more interested in avoiding under-achievement of profit and thus would ignore P^{r+} in the objective function of the weighted goal programming model. The profit goal is formulated in terms of (3.20) of chapter 3:

$$\sum_{i=1}^K (a_i \cdot (x_{ch_i})^2 + (K_{1i} \cdot a_i + K_{2i}) \cdot x_{ch_i} + K_{3i} - (a_i \cdot x_{ch_i} \cdot c_i \cdot (1 + \sigma) + K_{4i})) + P^r - P^{r+} = P(1 + \gamma) \quad (4.1)$$

where

P = Represents previous total profit associated with sales of products.

$(1 + \gamma)$ = represents the target of adjusting the profit from the previous.

P^r = represents underachievement the profit goal.

P^{r+} = represents overachievement the profit goal.

x_{ch_i} = represents change in a price of product i .

a_i = slope of a linear function of volume as a function of price for product i .

The profit function above accounts for costs associated with doing business. Even though some costs might not easily be apportioned, we consider a simple case where costs are independent, additive and linear. Some pricing profit models assign a price to a product and / or service and calculate its profit after cost and assessing its contribution to the overall profit (Rogers, 1990:269-270).

4.4 Goals for protection of existing customer relationships

Goals for price restrictions in this study are used to represent a set of goals or soft constraints which can be violated during the optimisation process if necessary. Deviation variables make it possible for these goals to be violated in a structured way.

A decision maker may also feel that it is important to increase the price of a product and / or service only slightly (that is, not sudden high prices which may shock customers). The old price restriction will be to stay as close as possible to the product's old price. These goals can be sought by minimizing both negative and positive deviation variables d_i^- and d_i^+ for all price restriction goals. Goals for price restrictions are formulated as follows:

$$\begin{aligned}
 x_1 + d_1^- - d_1^+ &= x_{old_1} \\
 x_2 + d_2^- - d_2^+ &= x_{old_2} \\
 &\vdots \\
 x_k + d_k^- - d_k^+ &= x_{old_k} \\
 i &= 1, \dots, k
 \end{aligned} \tag{4.2}$$

where

x_i = represent new price of product i after goal i is sought.

x_{old_i} = Old price of product i, which is target of goal i.

d_i^- = represent underachievement of goal i.

d_i^+ = represent overachievement of goal i.

(The concepts of underachievement and overachievement here should be interpreted with care and a case could be made for reversing the definitions).

4.5 Goals for competitiveness

Goals for competitiveness in this study are used to represent a set of goals or soft constraints which can be violated during the optimization process if necessary. Deviation variables make it possible for these goals to be violated.

A decision maker may also feel that it is important to observe the price level of competitors for a product and try to close gaps that might exist. The competitors' price goal restriction will be to stay as close as possible to average competition prices. These goals can be sought by minimizing both negative and positive deviation variables (n_i and p_i) for all competitiveness goals. The competitiveness goals are formulated as follows:

$$\begin{aligned}
 x_1 + n_1 - p_1 &= \left(\sum_{i=1}^n x_{\text{comp}_1} \right) / n \\
 x_2 + n_2 - p_2 &= \left(\sum_{i=1}^n x_{\text{comp}_2} \right) / n \\
 &\vdots \\
 x_k + n_k - p_k &= \left(\sum_{i=1}^n x_{\text{comp}_k} \right) / n \\
 i &= 1, \dots, k
 \end{aligned} \tag{4.3}$$

where

x_i = represents new price of product i after goal i is sought.

x_{comp_i} = averaged competitors' price of product and /or service i , which is a target of goal i .

n_i = represents underachievement of goal i , (price is lower than those of competitors).

p_i = represents overachievement of goal i , (price is higher than those of competitors).

4.6 System constraints

System constraints are constraints that have to be considered during the optimization process. This is also known as hard or rigid constraints. The following is an example of a system constraint:

$$x_1 + x_2 \leq 100 \quad (4.4)$$

Sum of x_1 and x_2 cannot exceed **100** (Moore & Weatherford, 2001:CD12-12). These constraints do not have associated deviational variables.

4.6.1 System constraints in this study

The first set of hard constraints (that is, system constraint) is that of employing bounds to ensure that each price to be suggested falls within certain parameters. For example, a decision maker may wish to adjust a price by not more than the value of theta (θ) which is a percentage adjustment, thus staying within certain bounds in price determination. The system constraint is formulated as follows:

$$(1 - \theta) x_{old_i} \leq x_i \leq (1 + \theta) x_{old_i} \quad (4.5)$$

$i=1, \dots, k$

where

x_{old_i} = represents old price of product i .

x_i = represent new price product i .

θ = represent percentage of bounds adjustment as preferred by decision maker.

4.6.2 Example of system constraints

If the decision maker, for example, decides to set prices within $\theta = 10\%$ adjustment, then bounds imposed will be as follows:

$$0.9 \cdot x_{old_i} \leq x_i \leq 1.10 \cdot x_{old_i} \quad (4.6)$$

The second set of hard constraints is a limit on volume change. A decision maker may want to impose limits on a volume change to restrict high volume changes. Thus we consider

$$\begin{aligned} v_{new_i} - v_{old_i} &= a_i \cdot (x + x_{ch_i}) + b_i - a_i \cdot (x_i) - b_i \\ &= a_i \cdot x_{ch_i} \end{aligned}$$

If we limit the changes as a percentage of v_{old_i} we have the following bounds:

$$- \frac{1}{2} \cdot \beta \cdot v_{old_i} \leq a_i x_{ch_i} \leq \beta \cdot v_{old_i} \quad (4.7)$$

$i = 1, \dots, k$

where

β = a factor by which a decision maker may wish to restrict volume change as a percentage of existing volume.

For example in this study it has been chosen to have a lower limit volume change of half the current volume and upper limit of one.

4.7 Weights for goal achievement

In general, a decision maker will be faced with multiple goals to achieve. It is practically impossible to achieve all goals simultaneously, as indicated by Volpi *et al.* (2003:174), for example, considering the goal to maximise profit without overcharging customers and to come closer to competition prices. Distinguishing importance of goals through assigning weights will yield several solutions which will not be obtained if goals were of equal importance. Different weighting structures will be used to seek the most satisfactory solution. See also paragraph 4.8.2.

4.8 Objective function

The objective function of a linear programming model is to maximize or minimize the objective by complying with some constraints or goals (Lee, 1972:20). It is often important

to choose the objective and weights in a goal programming model realistically as it has a strong influence on the outcomes.

4.8.1 Objective function in this study

The objective function of the goal programming model in this study seeks to minimize the sum of the weighted deviations of the goals. Weighting structures can also be applied repetitively until the 'satisficing' solution is found. The objective function of the pricing model is given below:

$$\text{Minimize } Z = \beta_{01} \cdot p^r + \beta_{02} \cdot p^{r+} + \sum_{i=1}^k \beta_{1i} (d_i^+ + d_i^-) + \sum_{i=1}^k \beta_{2i} (n_i + p_i) \quad (4.8)$$

where

Z = represents the objective function to be minimized.

β_{0i} = represents a weighting factor as desired by decision-maker regarding total profit goal deviations.

It is possible to use $\beta_{01}p^r + \beta_{02}p^{r+}$ for the situation where for example overachievement of profit may be weighted less than under underachievement ($\beta_{02}p^{r+} < \beta_{01}p^r$).

β_{1i} = represents a weighting factor as desired by decision-maker regarding old price goal deviations for product i .

β_{2i} = represent a weighting factor as desired by decision-maker regarding competitors' price goal deviations for product i .

4.8.2 Choosing the weighting factors

Since prices differ according to the item referred to, it is usually better to either scale the goal constraints or (equivalently) - as we have decided here - to rather work with the goal weights on the deviational variables that amount to penalizing the percentage deviation from the goal. As illustration we can thus use $1/x_{old_i}$ as the weight (β_{1i}) for d_i^+ (or d_i^-) in the goal programming model's objective function.

4.9 Non-negativity conditions

Variables are constrained to non-negativity as is the custom in Linear programming models. Simple transformations can be used to achieve this in case of unconstrained variables.

4.10 The complete goal programming model investigated in this study

This section presents the integration of different components of the goal programming model into one complete program. All the descriptions of the decision variables are given in the preceding sections:

$$\text{Minimize } Z = \beta_{01} \cdot P^r + \beta_{02} \cdot P^+ + \sum_{i=1}^k \beta_{1i} (d_i^+ + d_i^-) + \sum_{i=1}^k \beta_{2i} (n_i + p_i)$$

Subject to

$$x_1 + d_1^- - d_1^+ = x_{old_1}$$

$$x_2 + d_2^- - d_2^+ = x_{old_2}$$

.

$$x_k + d_k^- - d_k^+ = x_{old_k}$$

$$x_1 + n_1 - p_1 = \left(\sum_{i=1}^n x_{comp_1} \right) / n$$

$$x_2 + n_2 - p_2 = \left(\sum_{i=1}^n x_{comp_2} \right) / n$$

.

.

$$x_k + n_k - p_k = \left(\sum_{i=1}^n x_{comp_k} \right) / n$$

$$(1 - \theta) x_{old_i} \leq x_i \leq (1 + \theta) x_{old_i}$$

$$x_{ch_i} = x_i - x_{old_i}$$

$$-1/2 \cdot \beta \cdot v_{old_i} \leq a_i \cdot x_{ch_i} \leq \beta \cdot v_{old_i}$$

K

$$\sum_{i=1}^K (a_i \cdot (x_{ch_i})^2 + (K_{1i} \cdot a_i + K_{2i}) \cdot x_{ch_i} + K_{3i} - (a_i \cdot x_{ch_i} \cdot c_i \cdot (1 + \sigma) + K_{4i})) + P^r - P^+ = P (1 + \gamma)$$

$$d_i^+, d_i^-, n_i, p_i, P^r, P^+ \geq 0 \text{ for } i=1, \dots, k$$

4.11 Summary

This chapter has formulated a goal programming model with different (perhaps conflicting) goals. Each set of goals have deviational variables with regard to overachievements and underachievements. The formulation allows for the following goals:

- ❖ A profit goal of a desired level;
- ❖ Goals for competitiveness are desired levels that a decision maker wants to reach or come closer to, in order to comply with competition prices and to maintain good customer relationships;
- ❖ Goals for old price restriction are desired levels that a decision maker wants to reach or come closer to, in order not to shock customers by drastic price changes;
- ❖ Bounds are margins within which a decision maker wants to determine prices. This restriction was implemented as hard constraints.

5.1 Introduction

This chapter gives the implementation of the price model and profit optimisation, languages to be used and set of experiments to showcase the selection of different parameters for satisfactory solutions. One of the popular programming languages used in this case is C++. This language was designed and originally implemented for the first time by Dr. B. Stroustrup. This language is used in this dissertation in conjunction with one of the market leading optimisation software tool called ILOG Cplex. ILOG Cplex is a tool for solving linear optimisation problems, commonly referred to as linear programming (LP) problems. Several extensions of linear programming problems that can also be solved using ILOG Cplex are network flow problems, quadratic programming problems and mixed integer programming problems.

The model's data is represented in a Cplex file format called MPS-files (i.e., Mathematical programming systems files). A C++ program was designed to create an MPS-file on the UNIX platform. The Cplex software is used to do the optimisation. Below is a figure that illustrates a view of the concert technology of Cplex for C++ programmers:

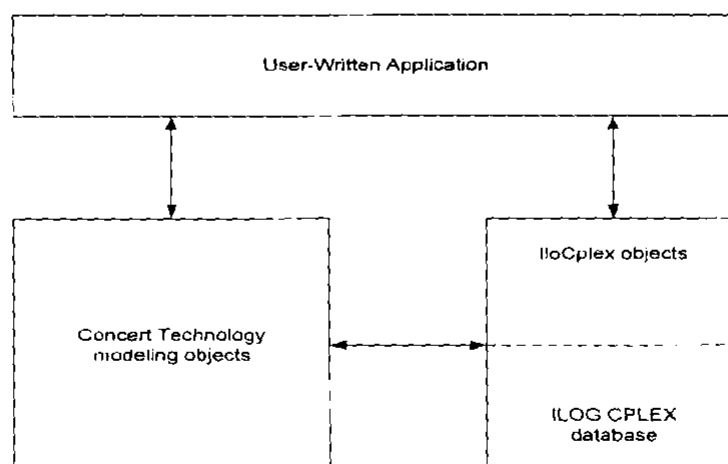


Figure 5-1: A view of concert technology for C++ users

The concert technology framework has being included in this study because the empirical testing of the model is implemented in C++ programming language in conjunction with ILOG CPLEX optimisation software which optimises the model. Figure 5-1 above illustrates how an application written in C++ would integrate with ILOG CPLEX database objects for optimisation purposes.

5.2 System specifications

This section specifies the minimal hardware and software requirements, especially those that were used during the implementation of this goal programming application.

5.2.1 Hardware specifications

Implementation of the investigated model in this dissertation was done on a computer with a hard-drive of approximately 18 GB of disk space and a processor with 256MB of RAM.

5.2.2 Software specifications

The specific software required to implement the model are: a UNIX operating system; CPLEX optimisation software (version 8.0 or any later version) containing all the executable and callable library files such as Makefile: datafile for encapsulating a model in an MPS or LP file. The CPLEX software is used as an LP solver and optimiser. A C++ compiler is also needed to compile the code.

5.3 Algorithmic flow chart

The following flow-chart shows logical steps that are followed in solving and / or optimising the model in this study.

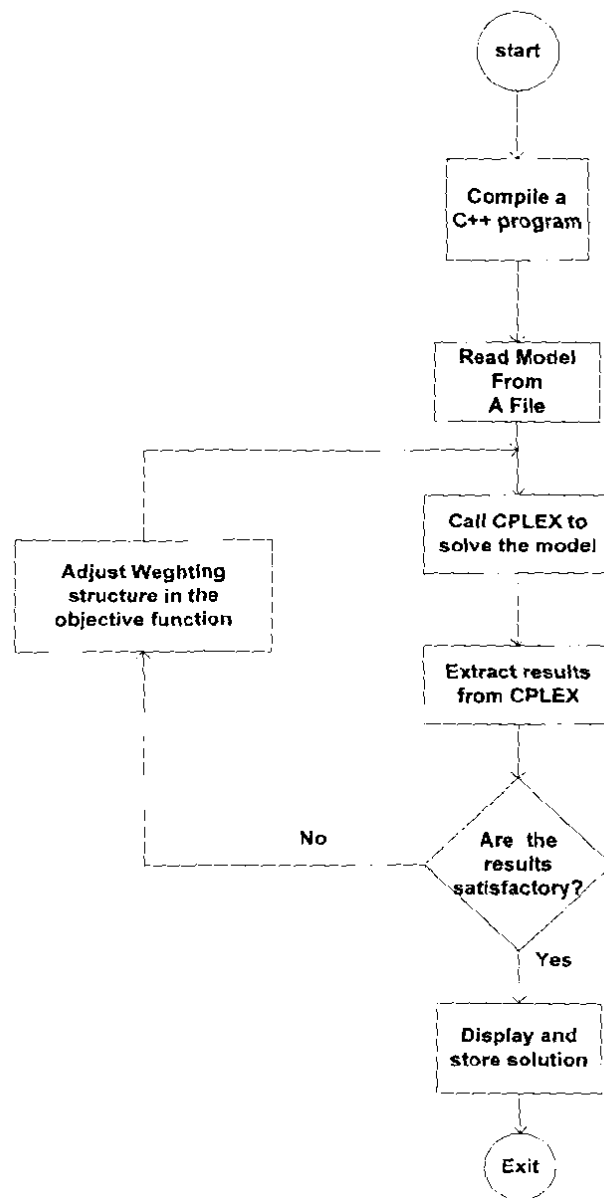


Figure 5-2: Flow chart for optimizing a profit Model

5.4 Computational Experiments

The empirical testing of the model was based on real data collected from small and medium shops in the rural areas around Mafikeng in the North West Province of South Africa. Seven shops were selected. From each shop ten products were selected and attributes of interest were selling price, cost price and volume of sales per month for each product.

This experimental data contains fields that relate to all the goals previously stated for example, old price restrictions, goals for competitiveness, cost factor adjustment etc.

The following tables contain data from seven shops regarding the above mentioned parameters:

Table 5-1: Data from shop A

Item	Actual Monthly volume of sales	Price	Cost Price	Range of sales
1) Colgate Toothpaste (100 ml)	90	R 7.95	R 6.99	0 – 100
2) Black Kiwi Shoe Polish (50 ml)	120	R 5.95	R 3.86	100 – 200
3) Huletts White Sugar (2,5 kg)	50	R 16.50	R 12.77	0 – 100
4) Sunlight Washing Powder (1kg)	200	R 17.65	R 11.60	100 – 200
5) Frisco coffee (250g)	110	R 13.99	R 10.55	100 – 200
6) Coca-cola (340 ml)	250	R 4.95	R 4.00	200 – 300
7) White star maize meal (2,5 kg)	900	R 8.95	R 6.70	900 – 1000
8) Mayonnaise (750 g)	20	R 15.95	R 11.77	0 – 100
9) Snowflake flower (10 kg)	120	R 39.95	R 37.00	100 – 200
10) Kellogg's cornflakes (500g)	10	R 19.50	R 17.99	0 – 100

In chapter four, profit models were developed. Among those models, one goal with respect to old profit (that is, current profit) was named. For data in **table 5 -1**, we can calculate:

$$\begin{aligned}
 \text{Current Profit} &= \sum_{i=1}^k v_i (x_{\text{old}_i}) \cdot x_{\text{old}_i} - \sum_{i=1}^k v_i (x_{\text{old}_i}) \cdot c_i \\
 &= 21\,923.90 - 17\,096.60 \\
 &= 4827.30
 \end{aligned}$$

The profit that a decision maker will realize from the above sales and costs is **4827.30** before any price change and or volume change.

Table 5-2: Data from shop B

Item	Actual Monthly volume of sales	Price	Cost Price	Range of sales
1) Colgate Toothpaste (100 ml)	345	R 5.79	R 4.47	300 – 400
2) Black Kiwi Shoe Polish (50 ml)	410	R 4.69	R 4.06	400 – 500
3) Hulett's White Sugar (2,5 kg)	110	R 15.99	R 12.08	100 – 200
4) Sunlight Washing Powder (1kg)	445	R 17.49	R 13.37	400 – 500
5) Frisco coffee (250g)	380	R 11.99	R 9.00	300 – 400
6) Coca-cola (340 ml)	890	R 3.69	R 2.90	800 – 900
7) White star maize meal (2,5 kg)	965	R 7.99	R 7.00	900 – 1000
8) Mayonnaise (750 g)	480	R13.79	R10.90	400 – 500
9) Snowflake flower (10 kg)	300	R 35.99	R 27.98	200 – 300
10) Kellogg's cornflakes (500g)	300	R 15.79	R 12.05	300 – 400

For data in **table 5-2**, we have:

$$\begin{aligned}
 \text{Current Profit} &= \sum_{i=1}^k v_i (x_{\text{old}_i}) \cdot x_{\text{old}_i} - \sum_{i=1}^k v_i (x_{\text{old}_i}) \cdot c_i \\
 &= 51166.20 - 40482.20 \\
 &= 10\ 684
 \end{aligned}$$

Table 5-3: Data from shop C

Item	Actual Monthly volume of sales	Price	Cost Price	Range of sales
1) Colgate Toothpaste (100 ml)	400	R 5.95	R 5.63	4000 – 4100
2) Black Kiwi Shoe Polish (50 ml)	1500	R 4.95	R 4.91	1500 – 1600
3) Hulett's White Sugar (2,5 kg)	600	R 14.95	R 13.01	600 – 700
4) Sunlight Washing Powder (1kg)	600	R 16.95	R 15.23	600 – 700
5) Frisco coffee (250g)	1200	R 11.95	R 9.92	1200 – 1300
6) Coca-cola (340 ml)	300	R 3.99	R 3.31	300 – 400
7) White star maize meal (2,5 kg)	1000	R 7.99	R 7.00	1000 – 1100
8) Mayonnaise (750 g)	1200	R12.95	R12.49	1200 – 1300
9) Snowflake flower (10 kg)	400	R 38.99	R 35.90	300 – 400
10) Kellogg's cornflakes (500g)	500	R 19.95	R 14.30	500 – 600

For data in **table 5-3**, we have:

$$\begin{aligned}\text{Current Profit} &= \sum_{i=1}^k v_i (x_{\text{old}_i}) \cdot x_{\text{old}_i} - \sum_{i=1}^k v_i (x_{\text{old}_i}) \cdot c_i \\ &= 93\,583 - 82\,956 \\ &= 10\,627\end{aligned}$$

Table 5-4: Data from Shop D

Item	Actual Monthly volume of sales	Price	Cost Price	Range of sales
1) Colgate Toothpaste (100 ml)	150	R 4.85	R 3.96	100 – 200
2) Black Kiwi Shoe Polish (50 ml)	90	R 5.99	R 4.89	0 – 100
3) Hulett's White Sugar (2,5 kg)	280	R 13.29	R 10.28	200 – 300
4) Sunlight Washing Powder (1kg)	90	R 17.29	R 12.63	100 – 200
5) Frisco coffee (250g)	210	R 11.95	R 9.92	200 – 300
6) Coca-cola (340 ml)	360	R 4.30	R 2.95	300 – 400
7) White star maize meal (2,5 kg)	80	R 7.89	R 7.49	0 – 100
8) Mayonnaise (750 g)	90	R13.49	R9.28	0 – 100
9) Snowflake flower (10 kg)	120	R 41.79	R 39.70	100 – 200
10) Kellogg's cornflakes (500g)	150	R 21.99	R 15.13	100 – 200

For data in **table 5-4**, we have:

$$\begin{aligned}\text{Current Profit} &= \sum_{i=1}^k v_i (x_{\text{old}_i}) \cdot x_{\text{old}_i} - \sum_{i=1}^k v_i (x_{\text{old}_i}) \cdot c_i \\ &= 20\,471.40 - 16\,662.30 \\ &= 3\,809.10\end{aligned}$$

Table 5-5: Data from Shop E

Item	Actual Monthly volume of sales	Price	Cost Price	Range of sales
1) Colgate Toothpaste (100 ml)	220	R 4.95	R 4.50	200 – 300
2) Black Kiwi Shoe Polish (50 ml)	240	R 5.50	R 4.00	200 – 300
3) Hulett's White Sugar (2,5 kg)	250	R 13.69	R 12.19	200 – 300
4) Sunlight Washing Powder (1kg)	360	R 16.95	R 14.44	300 – 400
5) Frisco coffee (250g)	240	R 13.95	R 11.85	200 – 300
6) Coca-cola (340 ml)	280	R 4.30	R 3.44	200 – 300
7) White star maize meal (2,5 kg)	180	R 8.60	R 6.88	100 – 200
8) Mayonnaise (750 g)	240	R13.00	R11.00	200 – 300
9) Snowflake flower (10 kg)	150	R 38.99	R 35.00	200 – 300
10) Kellogg's cornflakes (500g)	30	R 19.95	R 14.30	0 – 100

For data in **table 5-5**, we have:

$$\begin{aligned}
 \text{Current Profit} &= \sum_{i=1}^k v_i (x_{\text{old}_i}) \cdot x_{\text{old}_i} - \sum_{i=1}^k v_i (x_{\text{old}_i}) \cdot c_i \\
 &= 27\,600.50 - 23\,560.50 \\
 &= 4\,040
 \end{aligned}$$

Table 5-6: Data from Shop F

Item	Actual Monthly volume of sales	Price	Cost Price	Range of sales
1) Colgate Toothpaste (100 ml)	170	R 5.79	R 5.20	1700 – 17100
2) Black Kiwi Shoe Polish (50 ml)	230	R 5.59	R 4.90	500 – 600
3) Hulett's White Sugar (2,5 kg)	200	R 13.99	R 12.90	2000 – 2100
4) Sunlight Washing Powder (1kg)	340	R 17.69	R 17.69	300 – 400
5) Frisco coffee (250g)	290	R 11.69	R 9.93	200 – 300
6) Coca-cola (340 ml)	800	R 3.99	R 3.50	1500 – 15100
7) White star maize meal (2,5 kg)	640	R 9.19	R 8.18	600 – 700
8) Mayonnaise (750 g)	280	R13.49	R12.43	2800 – 2900
9) Snowflake flower (10 kg)	220	R 42.49	R 35.90	200 – 300
10) Kellogg's cornflakes (500g)	230	R 13.49	R 12.07	200 – 300

For data in **table 5-6**, we have:

$$\begin{aligned}\text{Current Profit} &= \sum_{i=1}^k v_i (x_{\text{old}_i}) \cdot x_{\text{old}_i} - \sum_{i=1}^k v_i (x_{\text{old}_i}) \cdot c_i \\ &= 49\,067 - 43\,610.50 \\ &= 5\,456.50\end{aligned}$$

Table 5-7: Data from Shop G

Item	Actual Monthly volume of sales	Price	Cost Price	Range of sales
1) Colgate Toothpaste (100 ml)	300	R 5.39	R 4.89	300 - 400
2) Black Kiwi Shoe Polish (50 ml)	800	R 5.49	R 4.80	800 - 900
3) Hulets White Sugar (2,5 kg)	250	R 13.69	R 13.01	200 - 300
4) Sunlight Washing Powder (1kg)	100	R 17.79	R 17.02	100 - 200
5) Frisco coffee (250g)	150	R 12.15	R 11.80	200 - 300
6) Coca-cola (340 ml)	4000	R 2.99	R 2.50	4000 - 4100
7) White star maize meal (2,5 kg)	800	R 8.39	R 7.20	800 - 900
8) Mayonnaise (750 g)	500	R13.59	R12.90	500 - 600
9) Snowflake flower (10 kg)	200	R 44.59	R 39.90	200 - 300
10) Kellogg's cornflakes (500g)	100	R 16.49	R 13.89	100 - 200

For data in **table 5 – 7**, we have:

$$\begin{aligned}\text{Current Profit} &= \sum_{i=1}^k v_i (x_{\text{old}_i}) \cdot x_{\text{old}_i} - \sum_{i=1}^k v_i (x_{\text{old}_i}) \cdot c_i \\ &= 21\,923.90 - 17\,096.60 \\ &= 4827.30\end{aligned}$$

5.4.1 Determining the slope values

Determining slope values in this study was done by fitting a straight-line to the data points for each item. The following figure shows slope value regarding the product *black kiwi shoe polish 50ml* from the seven different shops:

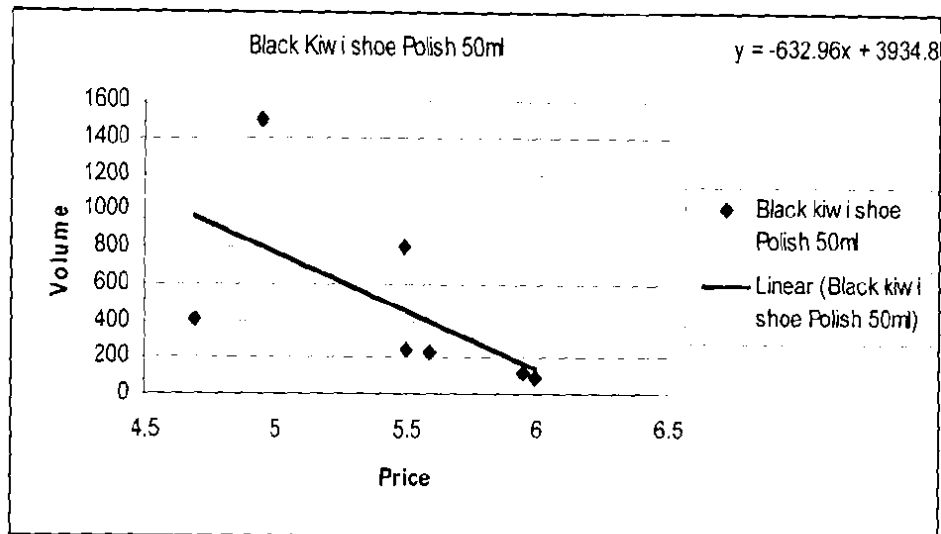


Figure 5-3: Determining slope for the first product

The sample is rather small and may not display a clear picture of the elasticity concept. However negative estimates of slopes were obtained in this case. The interpretation is that we estimate that an increase of R1.00 in the price of kiwi black shoe polish will result in the loss of 633 sales opportunities.

The same procedure is followed in determining fitted lines for the other products. Table 5-8 shows a summary of obtained slopes:

Table 5-8: Summary of products and their related slopes

	Item	Slope	a_i
1	Colgate Toothpaste (100 ml)	-32.15	a_1
2	Black Kiwi Shoe Polish (50 ml)	-632.96	a_2
3	Hulett's White Sugar (2,5 kg)	-44.25	a_3
4	Sunlight Washing Powder (1kg)	-300.19	a_4
5	Frisco coffee (250g)	-135.43	a_5
6	Coca-cola (340 ml)	-164.10	a_6
7	White star maize meal (2,5 kg)	-81.93	a_7
8	Mayonnaise (750 g)	-199.45	a_8
9	Snowflake flower (10 kg)	-14.44	a_9
10	Kellogg's cornflakes (500g)	-7.97	a_{10}

5.4.2 Determining average of competition prices

All the competition prices for each product have been averaged to be used in goals for competitiveness in the model. The following table summarizes the averages:

Table 5-9: Summary of averaged competition prices for Shop A

	Item	Price
1	Colgate Toothpaste (100 ml)	05.49
2	Black Kiwi Shoe Polish (50 ml)	05.37
3	Hulets White Sugar (2,5 kg)	14.27
4	Sunlight Washing Powder (1kg)	17.36
5	Frisco coffee (250g)	12.28
6	Coca-cola (340 ml)	03.88
7	White star maize meal (2,5 kg)	08.34
8	Mayonnaise (750 g)	13.39
9	Snowflake flower (10 kg)	40.47
10	Kellogg's cornflakes (500g)	17.94

The averages of competitors of shops B, C, D, E, F and G (not shown here) were computed in a similar way.

5.4.3 Demand function

The demand functions associated with each of the products are shown in the table below. Demand functions below are interpreted as follows: For example, $-32.15x + 413.02$ means that if x is increased by one, there will be a drop of -32.15 in quantity demanded for the Colgate tooth paste product. If x is increased by two, there will be a drop of -64.30. A decrease of x by two will increase quantity demanded by 64.30. The same interpretation holds for each item's demand function.

Table 5-10: Demand functions for each product

	Item	Demand functions
1	Colgate Toothpaste (100 ml)	- 32.15x + 413.02
2	Black Kiwi Shoe Polish (50 ml)	- 632.96x + 3934.80
3	Hulets White Sugar (2,5 kg)	- 44.25x + 886.80
4	Sunlight Washing Powder (1kg)	-300.19x + 5528.70
5	Frisco coffee (250g)	-135.43x + 2064.70
6	Coca-cola (340 ml)	-164.10x + 1129.90
7	White star maize meal (2,5 kg)	- 81.93x + 1311.30
8	Mayonnaise (750 g)	-199.45x + 3145.30
9	Snowflake flower (10 kg)	-14.44x + 799.14
10	Kellogg's cornflakes (500g)	-7.97x + 333.33

The following section presents a goal programming model that is formulated in respect to shop A. The data used in the model is obtained from **tables 5-1** and **5-14**.

5.4.4 Goal programming model A

The following is a goal programming model formulated for shop A. The objective function indicates that minimization of the profit deviations is the most important, followed by minimization of the competitors' goal deviation and existing price deviations.

We refer to overachievement and underachievement of deviational variables in the example as U_i and V_i (for old price goals) and N_i and P_i (for goals of competitiveness) for purpose of distinguishing between the two set of goals' deviational variables.

$$\text{Minimize } Z = (1/7.95 V_1 + 1/ 5.95 V_2 + \dots + 1/19.50 V_{10} + 1/5.49 P_1 + 1/5.37 P_2 + \dots + 1/17.94 P_{10} + 20 P^r$$

Subject to:

$$X_1 + U_1 - V_1 = 7.95$$

$$X_2 + U_2 - V_2 = 5.95$$

$$X_3 + U_3 - V_3 = 16.50$$

$$X_4 + U_4 - V_4 = 17.65$$

$$X_5 + U_5 - V_5 = 13.99$$

$$X_6 + U_6 - V_6 = 4.95$$

$$X_7 + U_7 - V_7 = 8.95$$

$$X_8 + U_8 - V_8 = 15.95$$

$$X_9 + U_9 - V_9 = 39.95$$

$$X_{10} + U_{10} - V_{10} = 19.50$$

The following goal constraints relate to the average of all competitors' prices

$$X_1 + N_1 - P_1 = 5.49$$

$$X_2 + N_2 - P_2 = 5.37$$

$$X_3 + N_3 - P_3 = 14.27$$

$$X_4 + N_4 - P_4 = 17.36$$

$$X_5 + N_5 - P_5 = 12.28$$

$$X_6 + N_6 - P_6 = 3.88$$

$$X_7 + N_7 - P_7 = 8.34$$

$$X_8 + N_8 - P_8 = 13.39$$

$$X_9 + N_9 - P_9 = 40.47$$

$$X_{10} + N_{10} - P_{10} = 17.94$$

The following system constraints relate to the boarder adjustment on existing-prices for both upper and lower limits where θ is set to be twenty percent.

$$6.36 \leq X_1 \leq 9.54$$

$$4.76 \leq X_2 \leq 7.14$$

$$13.20 \leq X_3 \leq 19.80$$

$$14.12 \leq X_4 \leq 21.18$$

$$11.19 \leq X_5 \leq 16.79$$

$$3.96 \leq X_6 \leq 5.94$$

$$7.16 \leq X_7 \leq 10.74$$

$$12.76 \leq X_8 \leq 19.14$$

$$31.96 \leq X_9 \leq 47.94$$

$$15.60 \leq X_{10} \leq 23.40$$

The following system constraints relates to limit on volume change due to price change where β is set to half (0.5) for the lower limit and one (1) for the upper limit.

$$-45 \leq -32.15 (X_1 - 7.95) \leq 90$$

$$-60 \leq -632.96 (X_2 - 5.95) \leq 120$$

$$\begin{aligned}
-25 &\leq -44.25 (X_3 - 16.60) \leq 50 \\
-100 &\leq -300.19 (X_4 - 17.50) \leq 200 \\
-55 &\leq -135.43 (X_5 - 13.99) \leq 110 \\
-125 &\leq -164.10 (X_6 - 4.95) \leq 250 \\
-450 &\leq -81.93 (X_7 - 8.95) \leq 900 \\
-10 &\leq -199.45 (X_8 - 15.95) \leq 20 \\
-60 &\leq -14.44 (X_9 - 39.95) \leq 120 \\
-5 &\leq -632.96 (X_{10} - 19.5) \leq 10
\end{aligned}$$

$$\sum_{i=1}^k (a_i \cdot (x_{ch_i})^2 + (K_{1i} \cdot a_i + K_{2i}) \cdot x_{ch_i} + K_{3i} - (a_i \cdot x_{ch_i} \cdot c_i \cdot (1 + \sigma) + K_{4i})) + P^r \cdot P^{r*} = 5792.76$$

For illustration purposes γ is assigned the value of **0.2** and σ is assigned to **0**. The right-hand side of the equation is computed from current profit in section 5.4.

If we substitute the expression $x_{chi} = x_i - K_{1i}$ the profit goal changes to:

$$\sum_{i=1}^k [a_i \cdot (x_i)^2 - K_{1i} \cdot x_i \cdot a_i + K_{2i} \cdot (x_i - K_{1i}) \cdot a_i \cdot x_i \cdot c_i (1 + \sigma) + a_i \cdot K_{1i} \cdot c_i (1 + \sigma)] + P^r \cdot P^{r*} = P(1 + \gamma) + \sum_{i=1}^k (K_{4i} \cdot K_{3i})$$

5.4.5 Computation of constants in the model

The following table contains values that are associated with constants found in the goal programming model A. Computation of K_{4i} is based on the fact that σ is assigned the value of 0.

Table 5-11: Computation of constants for model A for illustrative purpose

Item	K_{1i}	K_{2i}	K_{3i}	C_i	K_{4i}
1) Colgate Toothpaste (100 ml)	7.95	90	715.50	6.99	629.10
2) Black Kiwi Shoe Polish (50 ml)	5.95	120	714	3.86	463.20
3) Hulets White Sugar (2,5 kg)	16.50	50	825	12.77	638.50
4) Sunlight Washing Powder (1kg)	17.65	200	3530	11.60	2320
5) Frisco coffee (250g)	13.99	110	1538.90	10.55	1160.50
6) Coca-cola (340 ml)	4.95	250	1237.50	4.00	1000
7) White star maize meal (2,5 kg)	8.95	900	8055	6.70	6030
8) Mayonnaise (750 g)	15.95	20	319	11.77	235.40
9) Snowflake flower (10 kg)	39.95	120	4794	37.00	4440
10) Kellogg's cornflakes (500g)	19.50	10	195	17.99	179.90

The values of the constants in table 5-11 are computed based on the descriptions in chapter 3 under formulas in (3.15) and (3.18). Computation of the constants for models for shops B, C, D, E, F and G (not shown here) was done in a similar manner.

5.4.6 Computation of parameters for other models

This section presents computation of the parameters (for example, old price, old volume etc.) needed for the formulation of the models for the remaining shops. The following table shows a summary of old prices for shops under consideration:

Table 5-12 : A summary of old prices

Shop	A	B	C	D	E	F	G
x_{old_i}							
x_{old_1}	R 7.95	R 5.79	R 5.95	R 4.85	R 4.95	R 5.79	R 5.39
x_{old_2}	R 5.95	R 4.69	R 4.95	R 5.99	R 5.50	R 5.59	R 5.49
x_{old_3}	R 16.50	R 15.99	R 14.95	R 13.29	R 13.69	R 13.99	R 13.69
x_{old_4}	R 17.65	R 17.49	R 16.95	R 17.29	R 16.95	R 17.69	R 17.79
x_{old_5}	R 13.99	R 11.99	R 11.95	R 11.95	R 13.95	R 11.69	R 12.15
x_{old_6}	R 4.95	R 3.69	R 3.99	R 4.30	R 4.30	R 3.99	R 2.99
x_{old_7}	R 8.95	R 7.99	R 7.99	R 7.89	R 8.60	R 9.19	R 8.39

x_{old_8}	R 15.95	R13.79	R12.95	R13.49	R13.00	R13.49	R13.59
x_{old_9}	R 39.95	R 35.99	R 38.99	R 41.79	R 38.99	R 42.49	R 44.59
x_{old_10}	R 19.50	R 15.79	R 19.95	R 21.99	R 19.95	R 13.49	R 16.49

Table 5-13 below presents a summary of old volumes for products of all the shops:

Table 5-13: A summary of old volumes

Shop V_{old_i}	A	B	C	D	E	F	G
V_{old_1}	90	345	400	150	220	170	300
V_{old_2}	120	410	1500	90	240	230	800
V_{old_3}	50	110	600	280	250	200	250
V_{old_4}	200	445	600	90	360	340	100
V_{old_5}	110	380	1200	210	240	290	150
V_{old_6}	250	890	300	360	280	800	4000
V_{old_7}	900	965	1000	80	180	640	800
V_{old_8}	20	480	1200	90	240	280	500
V_{old_9}	120	300	400	120	150	220	200
V_{old_10}	10	300	500	150	30	230	100

The following table shows a summary of averages of the competition prices. These competitors' prices are computed in terms of formula shown in (4.3):

Table 5-14 : A summary of averages of competition prices

Shop ΣX_{comp_i}	A	B	C	D	E	F	G
A_{vrg_1}	5.49	5.81	5.79	5.96	5.87	5.95	5.81
A_{vrg_2}	5.37	5.58	5.54	5.45	5.36	5.44	5.43
A_{vrg_3}	14.27	14.35	14.53	14.74	14.42	14.74	14.69
A_{vrg_4}	17.36	17.36	17.39	17.48	17.42	17.48	17.35
A_{vrg_5}	12.28	12.28	12.61	12.62	12.59	12.29	12.66
A_{vrg_6}	3.88	4.09	4.04	4.20	3.99	3.99	4.04
A_{vrg_7}	8.34	8.50	8.50	8.44	8.52	8.40	8.30

A_{vrg_8}	13.39	13.75	13.89	13.79	13.80	13.88	13.80
A_{vrg_9}	40.47	41.13	40.63	39.70	40.17	40.63	40.05
A_{vrg_10}	17.94	18.56	17.87	18.45	17.53	17.87	18.95

The table below shows a summary of costs for all the products for each shop:

Table 5-15 : A summary of costs associated products

Shop	A	B	C	D	E	F	G
C_j							
C₁	R 6.99	R 4.47	R 5.63	R 3.96	R 4.50	R 5.20	R 4.89
C₂	R 3.86	R 4.06	R 4.91	R 4.89	R 4.00	R 4.90	R 4.80
C₃	R 12.77	R 12.08	R 13.01	R 10.28	R 12.19	R 12.90	R 13.01
C₄	R 11.60	R 13.37	R 15.23	R 12.63	R 14.44	R 17.69	R 17.02
C₅	R 10.55	R 9.00	R 9.92	R 9.92	R 11.85	R 9.93	R 11.80
C₆	R 4.00	R 2.90	R 3.31	R 2.95	R 3.44	R 3.50	R 2.50
C₇	R 6.70	R 7.00	R 7.00	R 7.49	R 6.88	R 8.18	R 7.20
C₈	R11.77	R10.90	R12.49	R9.28	R11.00	R12.43	R12.90
C₉	R 37.00	R 27.98	R 35.90	R 39.70	R 35.00	R 35.90	R 39.90
C₁₀	R 17.99	R 12.05	R 14.30	R 15.13	R 14.30	R 12.07	R 13.89

The formulation of the goal programming models for the other shops (that is, B, C, D, E, G not shown here) were computed in the same manner as the one in section 5.4.4. All the parameters used in those models were obtained from preceding tables (that is, tables in section 5.4.6). The profit constraints values were obtained from computation of current profits of individual shops in section 5.4.

5.5 Summary

In this chapter, the various goal programming models were given for each shop. Each model conforms to the generic goal programming model investigated in this study. Computation of constants contained in the model was illustrated. Weighting structures are flexible and a specific decision maker can adapt them for his / her own circumstances.

CHAPTER SIX

EMPIRICAL RESULTS

6.1 Introduction

This chapter is aimed at the empirical testing and discussion of the results achieved by applying the models indicated in chapter five to the data collected of the seven shops.

6.2 Results of computational experiment A

Table 6-1 below gives results for the empirical testing of the goal programming model applied to shop A. In these results, variables such as k_{1i} (old price) x_i (New price), x_{ch_i} (change in price), U_i (underachievement) and V_i (overachievement) are displayed with respective values after model optimization. The weights chosen for the model were as follows:

20 for P^r and those indicated in section 5.4.4 for other variables.

Table 6-1: Results for experiment A

	K_{1i}	x_i	x_{ch_i}	v_{ch_i}	U_i	V_i	a_i	v_{old_i}	v_{new_i}
Coalgate tooth paste 100ml	7.95	6.36	-1.59	51	1.59	0	-32.15	90	141
Kiwi black shoe polish 50ml	5.95	5.76	-0.19	120	0.19	0	-632.96	120	240
Hulets White sugar 2.5kg	16.50	15.37	-1.13	50	1.13	0	-44.25	50	100
Sunlight Washing Powder 1kg	17.65	17.36	-0.29	87	0.29	0	-300.19	200	287
Frisco coffee 250g	13.99	13.17	-0.81	110	0.81	0	-135.43	110	220
Coca-cola 340ml	4.95	3.96	-0.99	162	0.99	0	-164.1	250	412
White star maize meal	5.95	8.34	-0.61	49	0.61	0	-81.93	900	949
Mayonnaise 750g	15.95	15.84	-0.10	20	0.10	0	-199.45	20	40
Snowflake flower 10kg	39.95	40.47	0.52	-7	0	0.52	-14.44	120	113
Kellogg's cornflakes 500g	19.50	18.24	-1.26	10	1.26	0	-7.97	10	20

Data collected for shop A indicates that shop A is a high price shop with low volumes and thus a small shop. The averaged competition prices would in this case tend to bring prices

down. This is denoted by predominantly negative entries in the column x_{ch_i} . Because of the linear relationship that exists between price and volume, any decrease in price will result in a rise in expected volume and vice versa. Note also that v_{new_j} is generally higher than v_{old_j} as seen from the last two columns.

The estimated profit for this shop is 5792.76 reflecting an estimated increase of 965.46 from the original profit level of 4827.30.

6.3 Results of computational experiment B

Table 6-2 gives a summary of results obtained from the goal programming model applied for shop B.

Table 6-2: Results for experiment B

	K_{11}	x_i	x_{ch_i}	v_{ch_i}	U_i	V_i	a_i	v_{old_i}	v_{new_i}
Coalgate tooth paste 100ml	5.79	5.81	0.02	1	0	0.02	-32.15	345	344
Kiwi black shoe polish 50ml	4.69	5.01	0.32	-205	0	0.32	-632.96	410	205
Hulett's White sugar 2.5kg	15.99	14.35	-1.64	72	1.64	0	-44.25	110	182
Sunlight Washing Powder 1kg	17.49	17.36	-0.13	39	0.13	0	-300.19	445	484
Frisco coffee 250g	11.99	12.28	0.29	-39	0	0.29	-135.43	380	341
Coca-cola 340ml	3.69	4.09	0.40	-65	0	0.04	-164.1	890	825
White star maize meal	7.99	8.5	0.51	-41	0	0.51	-81.93	965	924
Mayonnaise 750g	13.79	13.75	-0.04	7	0.04	0	-199.45	480	487
Snowflake flower 10kg	35.99	41.13	5.14	-74	0	5.14	-14.44	300	226
Kellogg's cornflakes 500g	18.56	18.56	2.77	-22	0	2.77	-7.97	300	278

According to the data, shop B is an average price shop with medium volume and thus a middle sized shop. Most of the suggested prices are above the current prices and these result in a decrease in the expected volume. The averaged competition prices are slightly above the current prices - hence a suggestion of price increase.

The estimated profit for this shop is 12820.85 which reflects an estimated increase of 2395.80 from the original profit level of 10425.80.

6.4 Results of computational experiment C

Table 6-3 below gives results obtained from the experiment with the goal programming model applied to shop C.

Table 6-3: Results for experiment C

	K_{1i}	x_i	x_{chi}	v_{chi}	U_i	V_i	a_i	v_{old_i}	v_{new_i}
Coalgate tooth paste 100ml	5.95	5.79	-0.16	5	0.16	0	-32.15	400	405
Kiwi black shoe polish 50ml	4.95	5.54	0.59	-373	0	0.59	-632.96	1500	1127
Hulett's White sugar 2.5kg	14.95	14.53	-0.42	18	0.42	0	-44.25	600	618
Sunlight Washing Powder 1kg	16.95	17.39	0.44	-132	0	0.44	-300.19	600	468
Frisco coffee 250g	11.95	12.61	0.66	-89	0	0.66	-135.43	1200	1111
Coca-cola 340ml	3.99	4.04	0.05	-8	0	0.05	-164.1	300	292
White star maize meal	7.99	8.50	0.51	-41	0	0.51	-81.93	1000	959
Mayonnaise 750g	12.95	13.89	0.94	-187	0	0.94	-199.45	1200	1013
Snowflake flower 10kg	38.99	40.63	1.64	-23	0	1.64	-14.44	400	377
Kellogg's cornflakes 500g	19.95	17.87	-2.08	16	2.08	0	-7.97	500	516

According to the data, shop C is a high volume shop with average prices and thus a big shop. Suggested prices are both increased and decreased in some cases. This shop has an estimated profit of 12752.40 which reflects an estimated increase of profit by 2125.40 from the previous profit of 10627. Note that the v_{new_i} is generally lower than the v_{old_i} due to the suggestion of price increases in the x_{ch_i} column.

6.5 Results of computational experiment D

Table 6-4 provides results for the experiment with the goal programming model applied to shop D.

Table 6-4: Results for Experiment D

	K_{ti}	x_i	x_{chi}	v_{chi}	U_i	V_i	a_i	v_{old_i}	v_{new_i}
Coalgate tooth paste 100ml	4.85	5.82	0.97	-31	0	0.97	-32.15	150	119
Kiwi black shoe polish 50ml	5.99	5.84	-0.15	90	0.15	0	-632.96	90	180
Hulelts White sugar 2.5kg	13.29	14.74	1.45	-64	0	1.45	-44.25	280	216
Sunlight Washing Powder 1kg	17.29	17.43	0.14	-45	0	0.14	-300.19	90	45
Frisco coffee 250g	11.95	12.62	0.67	-90	0	0.67	-135.43	210	120
Coca-cola 340ml	4.30	4.20	-0.1	16	0.1	0	-164.1	360	376
White star maize meal	7.89	8.37	0.48	-40	0	0.48	-81.93	80	40
Mayonnaise 750g	13.49	13.71	0.22	-45	0	0.22	-199.45	90	45
Snowflake flower 10kg	41.79	39.70	-2.09	30	2.09	0	-14.44	120	150
Kellogg's cornflakes 500g	21.99	18.45	-3.54	28	3.54	0	-7.97	150	178

Data collected for Experiment D shows that shop D is an averaged price shop with medium volumes. Suggested prices are mostly higher than the existing prices because the existing prices have been below the averaged competitors' prices and the importance of increasing profit from the existing one. The estimated profit for this shop is 4917.24 showing an increase target of 819.80 from the original profit of 4097.70.

6.6 Results of computational experiment E

Table 6-5 below gives the results for the experiment with the goal programming model applied to shop E.

Table 6-5: Results for Experiment E

	K_{ti}	x_i	x_{chi}	v_{chi}	U_i	V_i	a_i	v_{old_i}	v_{new_i}
Coalgate tooth paste 100ml	4.95	5.87	0.92	-29	0	0.92	-32.15	220	191
Kiwi black shoe polish 50ml	5.50	5.36	-0.14	88	0.14	0	-632.96	240	328
Hulelts White sugar 2.5kg	13.69	14.8	1.11	-49	0	1.11	-44.25	250	201
Sunlight Washing Powder 1kg	16.95	17.42	0.47	-141	0	0.47	-300.19	360	219
Frisco coffee 250g	13.95	12.59	-1.36	184	1.36	0	-135.43	240	424
Coca-cola 340ml	4.30	3.99	-0.31	50	0.31	0	-164.1	280	330
White star maize meal	8.60	8.52	-0.08	6	0.08	0	-81.93	180	186
Mayonnaise 750g	13.00	13.60	0.60	-120	0	0.60	-199.45	240	120
Snowflake flower 10kg	38.99	40.17	1.18	-17	0	1.18	-14.44	150	233
Kellogg's cornflakes 500g	19.95	17.53	-2.42	19	2.42	0	-7.97	30	49

Data collected for E shows that shop E is an average price shop with medium sized volumes. Suggested prices are mostly higher than the existing prices because the existing prices have been below the average competitors' prices and the importance of reaching the target profit from the previous one. This shop has a target profit of 4848 which reflects an estimated increase of 808 from the previous profit of 4040. Note that these price increases have resulted in a v_{new_i} to be lower than v_{old_i} as seen in the last two columns.

6.7 Results of computational experiment F

Table 6-6 below gives the results for the experiment with the goal programming model applied to Shop F.

Table 6-6: Results for Experiment F

	K_{1i}	x_i	x_{chi}	v_{chi}	U_i	V_i	a_i	v_{old_i}	v_{new_i}
Coalgate tooth paste 100ml	5.79	5.95	0.16	-5	0	0.16	-32.15	170	165
Kiwi black shoe polish 50ml	5.59	5.44	-0.15	94	0.15	0	-632.96	230	324
Hulets White sugar 2.5kg	13.99	14.74	0.75	-33	0	0.75	-44.25	200	167
Sunlight Washing Powder 1kg	17.69	17.48	-0.21	63	0.21	0	-300.19	340	404
Frisco coffee 250g	11.69	12.29	0.60	-81	0	0.60	-135.43	290	209
Coca-cola 340ml	3.99	3.99	0	0	0	0	-164.1	800	800
White star maize meal	9.19	8.40	-0.79	64	0.79	0	-81.93	640	704
Mayonnaise 750g	13.49	13.88	0.39	-77	0	0.39	-199.45	280	203
Snowflake flower 10kg	42.49	40.63	-1.86	26	1.86	0	-14.44	220	246
Kellogg's cornflakes 500g	13.49	16.19	2.70	-21	0	2.70	-7.97	230	209

Data collected for F shows that shop F is an average price shop with medium sized volumes. Suggested prices are almost equally increased and decreased, because some of existing prices have been below and others have been above the averaged competitors' prices and the importance of increasing new profit from the previous one. This shop has a target profit of 4918.80, showing the profit increase of 819.80 from the existing profit of 4099. Note that, because some suggested prices are above and other below the existing prices, this has resulted in the related variation in the both v_{new_i} and v_{old_i} .

6.8 Results of computational experiment G

Table 6-7 below gives the results for the experiment with the goal programming model applied shop G.

Table 6-7: Results for Experiment G

	K_{1i}	x_i	x_{chi}	v_{chi}	U_i	V_i	a_i	v_{old_i}	v_{new_i}
Coalgate tooth paste 100ml	5.39	5.81	0.42	-13	0	0.42	-32.15	300	287
Kiwi black shoe polish 50ml	5.49	5.43	-0.06	37	0.06	0	-632.96	800	837
Hulett's White sugar 2.5kg	13.69	14.69	1	-44	0	1	-44.25	250	206
Sunlight Washing Powder 1kg	17.79	17.45	-0.33	100	0.33	0	-300.19	100	200
Frisco coffee 250g	12.15	12.66	0.51	-69	0	0.51	-135.43	150	81
Coca-cola 340ml	2.99	3.59	0.60	-98	0	0.60	-164.1	4000	3902
White star maize meal	8.39	8.30	-0.09	7	0.09	0	-81.93	800	807
Mayonnaise 750g	13.59	13.80	0.21	-41	0	0.21	-199.45	500	459
Snowflake flower 10kg	44.59	40.05	-4.54	65	4.54	0	-14.44	200	265
Kellogg's cornflakes 500g	16.49	18.95	2.46	-19	0	2.46	-7.97	100	81

Data collected for G shows that shop G is an average price shop with medium sized volumes. Suggested prices are almost equally increased and decreased because some of existing prices have been below and others have been above the averaged competitors' prices and the importance of increasing new profit from the previous one. This shop has a target profit of 6547.92 reflecting an estimated increase of 1091.42 from the previous profit of 5456.50. The overachievement of the profit goal has resulted in 745.94 being realised.

6.9 Summary of results of computational experiments

Table 6-8 below summarizes the results. The results of the first shop's goal programming model are referred to in the table as "A", the second as "B" etc. In each case the changes suggested by the goal programming model are given.

Table 6-8: Summary of results from computational experiments

Product	A	B	C	D	E	F	G
Colgate tooth paste 100ml							
• Current price	7.95	5.79	5.95	4.85	4.95	5.79	5.39
• Suggested price	6.36	5.81	5.79	5.82	5.87	5.95	5.81
• Change in price	-1.59	0.02	-0.16	0.97	0.92	0.16	0.42
• Change in Volume	51	0	5	-31	-29	-5	-13
• New volume	141	345	405	119	191	165	287
Kiwi Black shoe polish 50ml							
• Current price	5.95	4.69	4.95	5.99	5.50	5.59	5.49
• Suggested price	5.76	5.01	5.54	5.84	5.36	5.44	5.43
• Change in price	-0.10	0.32	0.59	-0.15	-0.14	-0.15	-0.06
• Change in Volume	120	-205	-373	90	88	94	37
• New volume	240	205	1127	180	328	324	837
Hulett's White Sugar 2.5kg							
• Current price	16.50	15.99	14.95	13.29	13.69	13.99	13.69
• Suggested price	15.37	14.35	14.53	14.74	14.80	14.74	14.69
• Change in price	-1.13	-1.64	-0.42	1.45	1.11	0.75	1
• Change in Volume	50	72	18	-64	-49	-33	-44
• New volume	100	182	618	216	201	167	206
Sunlight Washing powder 1kg							
• Current Price	17.65	17.49	16.95	17.29	16.95	17.69	17.79
• Suggested price	17.36	17.36	17.39	17.43	17.42	17.48	17.45
• Change in price	-0.29	-0.13	0.44	0.14	0.47	-0.21	-0.33
• Change in Volume	87	39	-132	-45	-141	63	100
• New volume	287	484	468	45	219	403	200
Frisco coffee 250g							
• Current price	13.99	11.99	11.95	11.95	13.95	11.69	12.15
• Suggested price	13.18	12.28	12.61	12.62	12.59	12.29	12.66
• Change in price	-0.82	0.29	0.66	0.67	-1.36	0.60	0.51
• Change in Volume	110	-39	-89	-90	184	-81	-69
• New volume	220	341	1111	120	424	209	81
Coca-cola 340ml							
• Current Price	4.95	3.69	3.99	4.30	4.30	3.99	2.99
• Suggested price	3.96	4.09	4.04	4.20	3.99	3.99	3.59
• Change in price	-0.99	0.40	0.05	-0.10	-0.31	0	0.60
• Change in Volume	162	-65	-8	16	50	0	-98
• New volume	412	825	292	376	330	800	3902
White star maize meal 2.5kg							
• Current price	8.95	7.99	7.99	7.89	8.60	9.19	8.39
• Suggested price	8.34	8.50	8.50	8.37	8.52	8.40	8.30
• Change in price	-0.61	0.51	0.51	0.48	-0.08	-0.79	-0.09
• Change in Volume	49	-41	-41	-40	6	64	7
• New volume	949	924	959	40	186	704	807
Mayonnaise 750g							
• Current price	15.95	13.79	12.95	13.49	13.00	13.49	13.59
• Suggested price	15.84	13.75	13.89	13.71	13.60	13.88	13.80
• Change in price	-0.11	-0.04	0.94	0.22	0.60	0.39	0.21
• Change in Volume	20	7	-187	-45	-120	-77	-41

• New volume	40	487	1013	45	120	203	459
Snowflake flower 10kg							
• Current price	39.95	35.99	38.99	41.79	38.99	42.49	44.59
• Suggested price	40.47	41.13	40.63	39.70	40.17	40.63	40.05
• Change in price	0.52	5.14	1.64	-2.09	1.18	1.86	-4.54
• Change in Volume	-7	-74	-23	30	-17	26	65
• New volume	113	226	377	150	133	246	265
Kellogg's cornflakes 500g							
• Current price	19.50	15.79	19.95	15.13	19.95	13.49	16.49
• Suggested price	18.85	18.56	17.87	18.45	17.53	16.19	18.95
• Change in price	-1.25	2.77	-2.08	-3.54	-2.42	2.70	2.46
• Change in Volume	10	-22	16	28	19	-21	-19
• New volume	20	278	516	178	49	209	81
Profit							
• Existing profit	4827.30	10425.80	10627	4097.70	4040	4099	5456.50
• Target Profit	5792.76	12820.85	12752.40	4917.24	4848	4918.80	6547.92
• Actual Profit	5792.76	12820.85	12752.40	4917.24	4848	4918.80	7293.86
• Profit Shift	965.46	2395.80	2125.40	819.54	808	819.80	1837.36

6.10 Summary of deductions

In this section graphical representations are summarised to show the price and volume changes for each shop.

Figure 6-1 below displays a graphical representation of price and volume changes for shop A. The numbers on the X-axis represent different product categories. For example in figure 6-1 below, the number 1 represents the first product which is Colgate toothpaste. See table 5-11 in which these products have been stated in sequence.

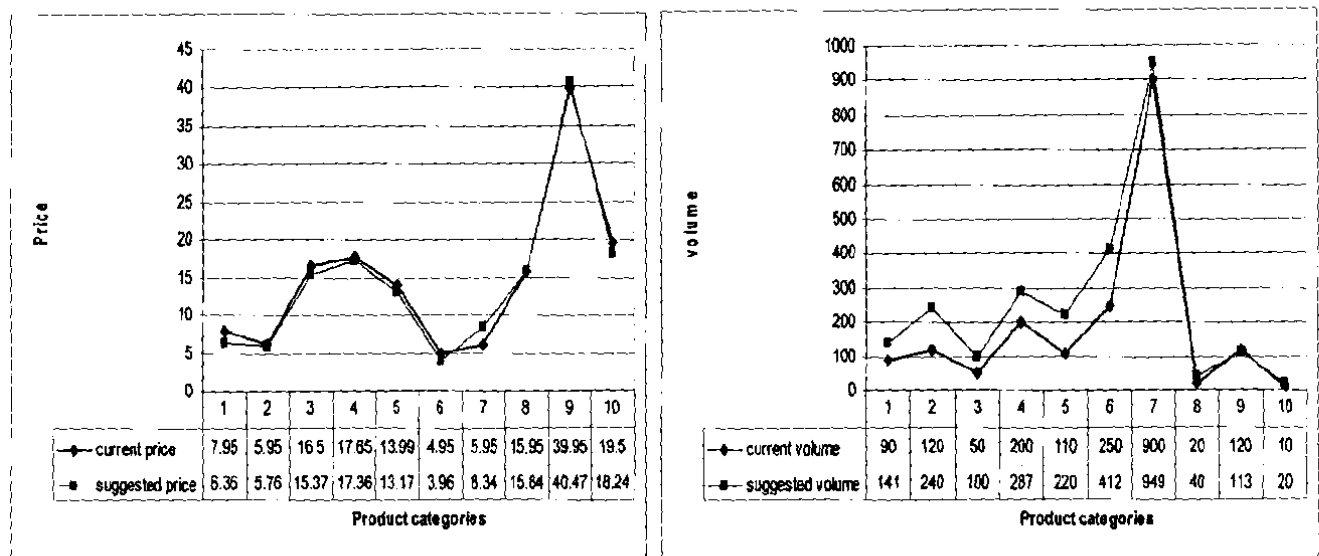


Figure 6-1: Graphical representation of summary of price and volume changes for shop A

It can be seen from figure 6-1 above that the suggested prices for shop A are generally lower for all products (except for product seven). The new volumes are however considerably higher than the existing volumes. This is an indication that current prices for shop A might be too high. A strategy to lower prices and try to increase volumes would probably be more successful when trying to increase profit instead of just increasing prices.

Figure 6-2 below displays a graphical representation of price and volume changes for shop B.

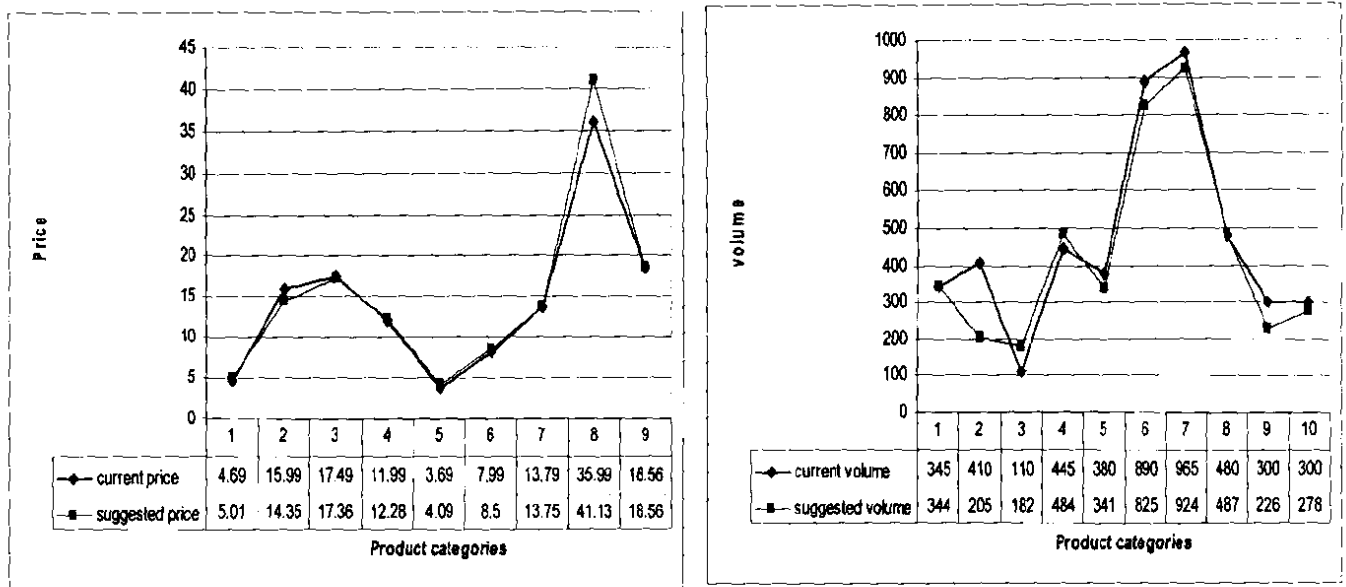


Figure 6-2 : Graphical representation of summary of price and volume changes for shop B

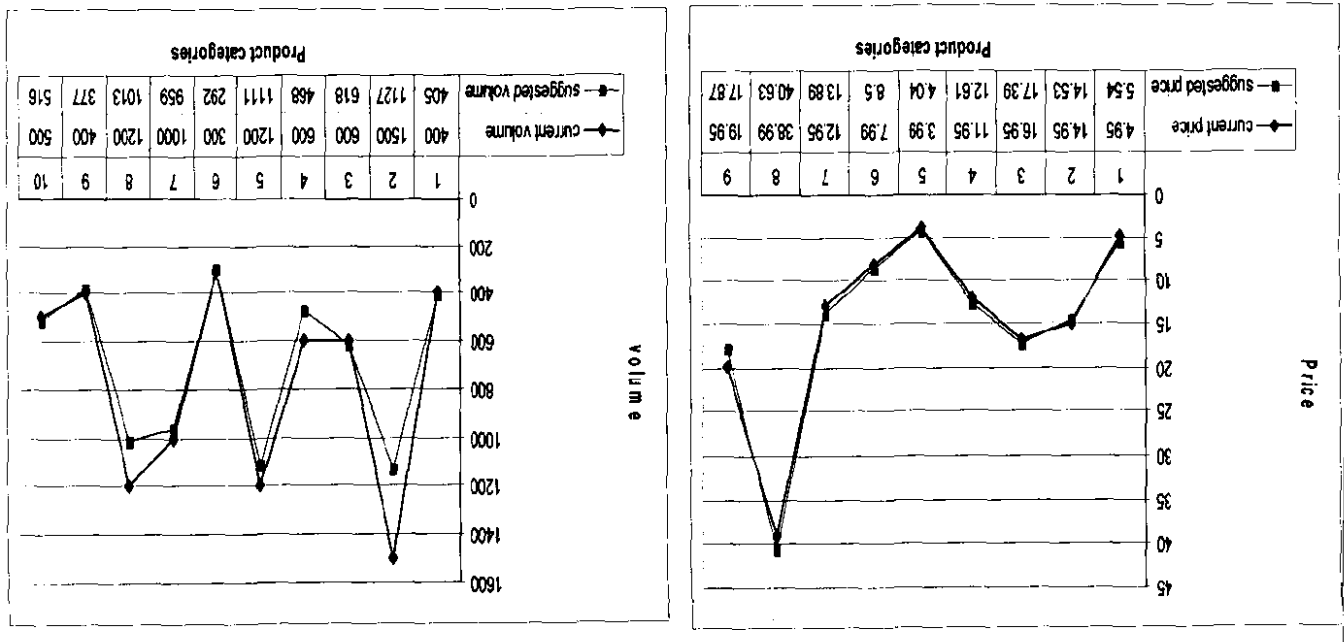
It can be deduced from figure 6-2 above that some products are sensitive to price change. Product 2 has had a minor price change but volume has largely changed. Most of the suggested prices are increases. This has resulted in the new volume (that is, the suggested prices) have been reduced.

Figure 6-3 below displays a graphical representation of price and volume change for the computational experiment C.

Figure 6-3: Graphical representation of summary of price and volume changes for shop C

It can be seen from figure 6-3 above that most of the suggested prices were increased above the current prices. This has resulted in suggested volumes being generally reduced across all products.

Figure 6-4 below depicts a graphical representation of price and volume changes for the computational experiment D.



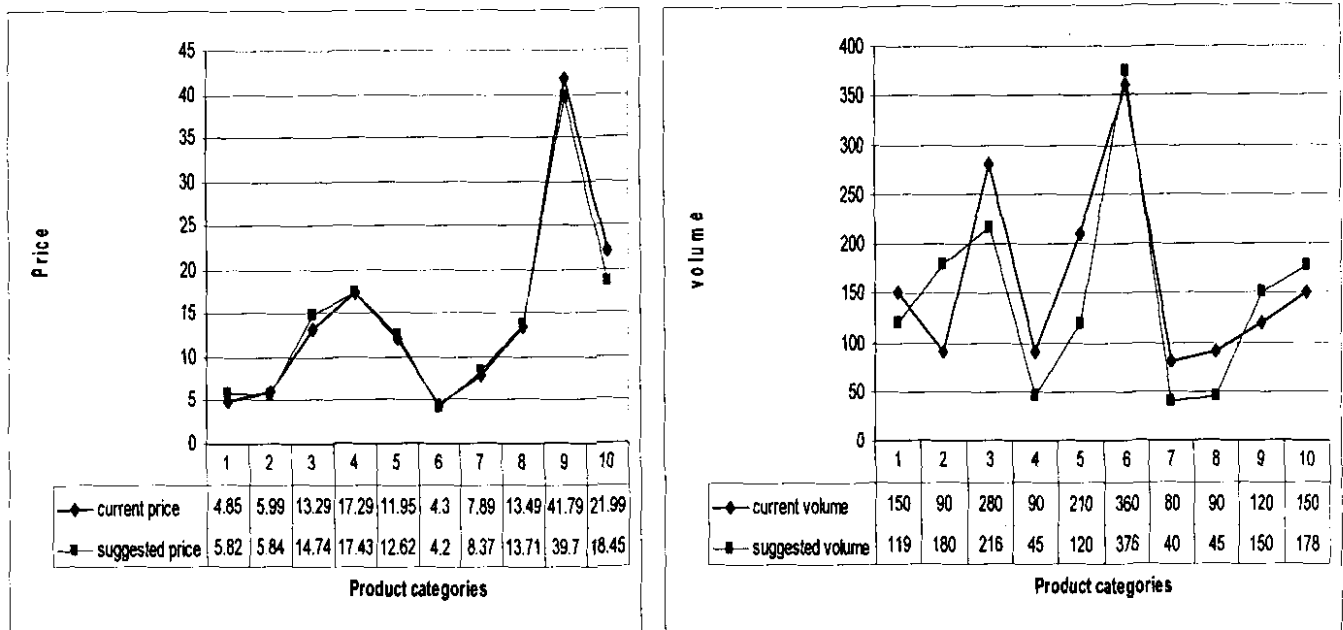


Figure 6-4: Graphical representation of summary of price and volume changes for shop D

From figure 6-4 above, it is deducible that most of the suggested prices have gone above the current prices. This has caused new volumes to drop in most cases except for product 2, product 6, product 9 and product 10.

Figure 6-5 below displays a graphical representation of price and volume changes for shop E.

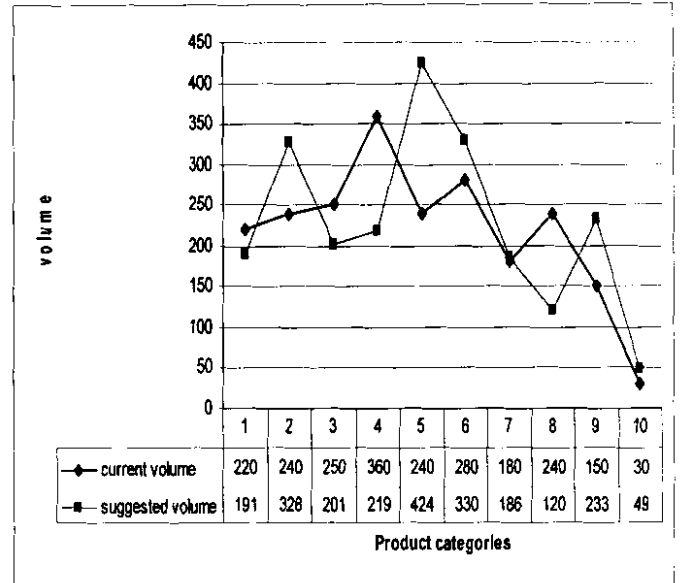
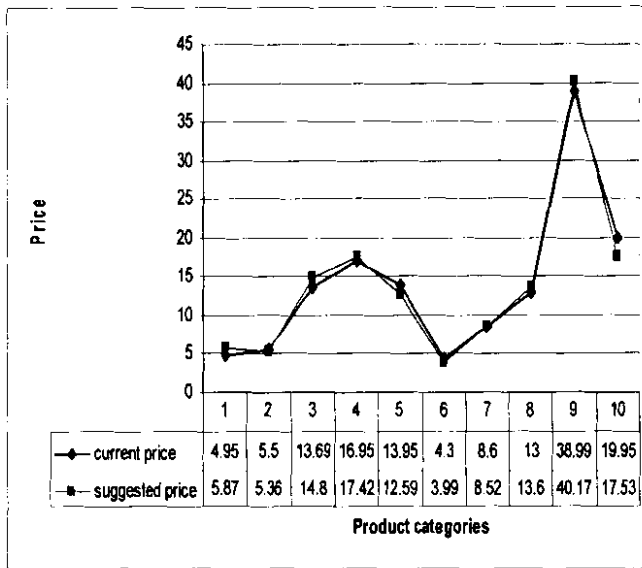


Figure 6-5: Graphical representation of summary of price and volume changes for shop E

It can be deduced from figure 6-5 above that most of the products in this experiment are price elastic because every little price change has had huge volume change and resulted new volumes being highly affected. The graph's current volume and suggested volume show the relationship between current or existing volume and the suggested or expected volume.

Figure 6-6 below displays a graphical representation of price and volume changes for the computational experiment F.

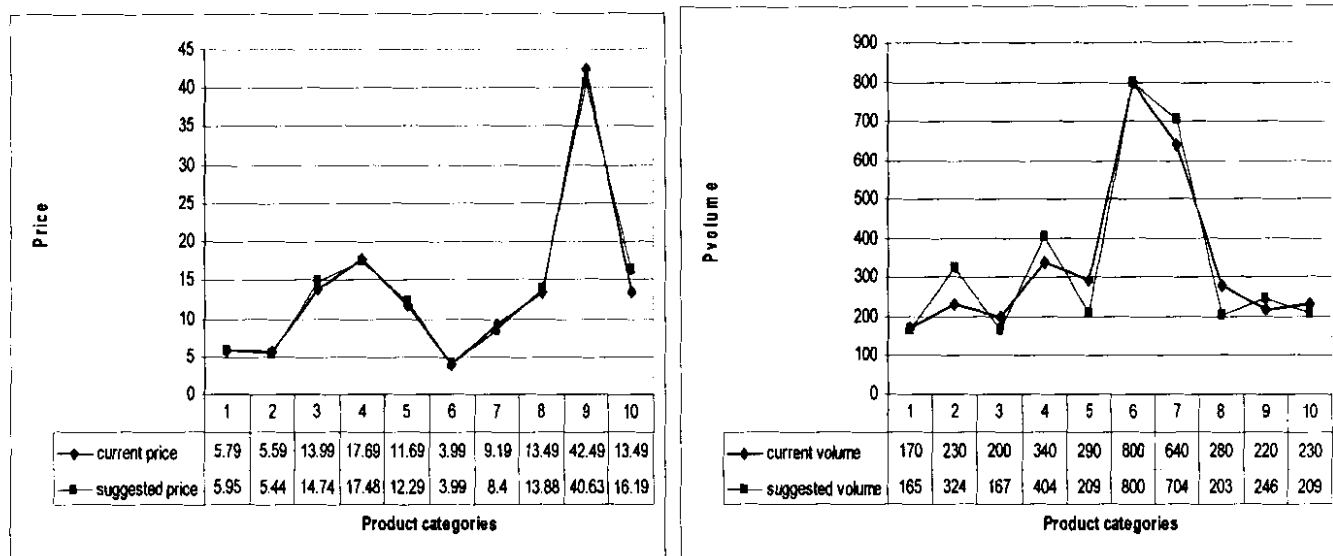


Figure 6-6: Graphical representation of summary of price and volume changes for shop F

It can be deduced from figure 6-6 that current prices and suggested prices have not differed a lot. This has resulted in current volumes and suggested volumes being smoothly related, unlike in the results of shop E.

Figure 6-7 below displays a graphical representation of price and volume changes for shop G.

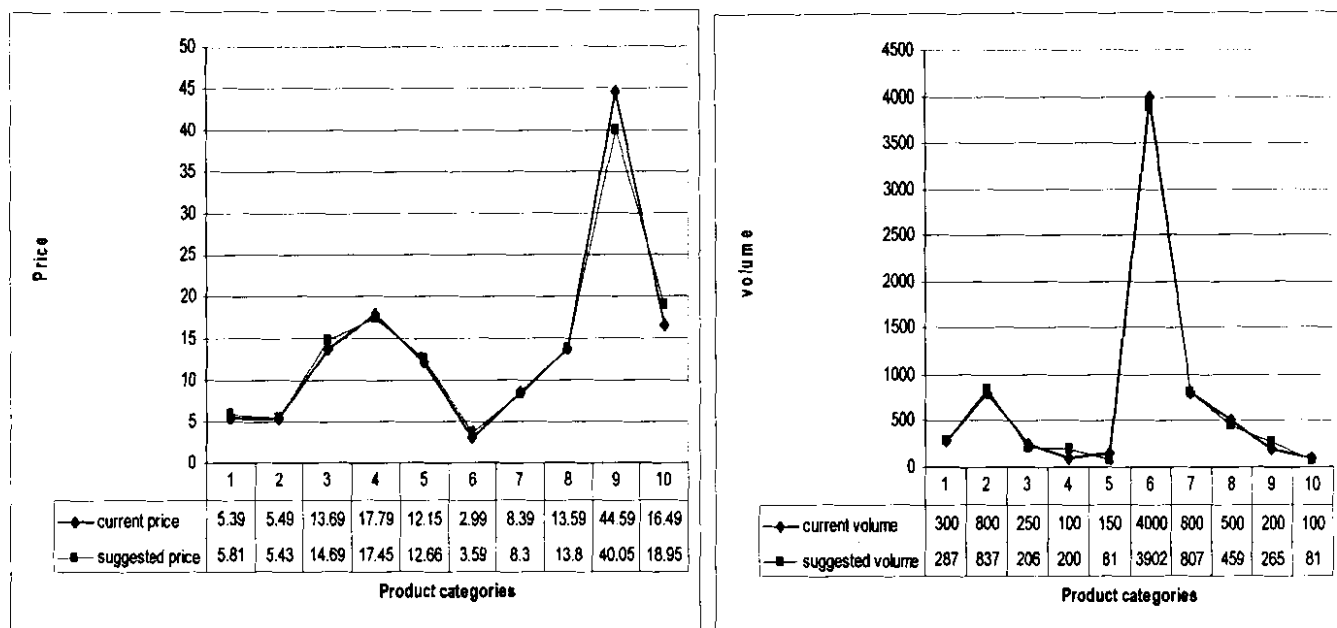


Figure 6-7: Graphical representation of summary of price and volume changes for shop G

It can be seen from figure 6-7 that the suggested prices have also not differed a lot to current prices. This has resulted in the existing volume and suggested volume being almost the same due to the amount of price changes.

6.11 Summary

The results obtained by applying the model to some real-world data were discussed in this chapter. The main results of this application have been that it seems feasible to create a decision support system for price determination using the goal programming approach. In the next chapter, the conclusions and limitation are identified. Further possible research work and enhancement are also indicated.

7.1 Introduction

In chapter one the goals for this research were stated. In summary the aims and objectives of this study have been to:

- ❖ Investigate some of the main aspects of price determination as used by shop managers for products or services.
- Consider models based upon the weighted goal programming approach for price determination.
- Develop and test decision support systems that can be used to help to determine prices.

The extend to which these aims and objectives have been realised is shown in section 7.2, 7.3 and 7.4 below.

7.2 Investigate some of the main aspects of price determination as used by shop managers for products or services

The first part of the study (that is, chapters two and three) was devoted to a survey of relevant and existing literature. Definitions and descriptions of the concepts pertaining to general pricing and economic aspects as well as multi-criteria decision aspects were given. The goal programming concepts for example the types of goal programming and applications were also discussed comprehensively.

7.3 Consider models based on weighted goal programming approach for price determination

This part of the work dealt with a review of an existing price model (Bell, 2003) and extending the model to include aspects such as price elasticity. A weighted goal programming model was then formulated in order to consider multiple and often conflicting objectives. The goals that are considered in the model were discussed in chapter four and can be summarized as follows:

- ❖ A profit goal is to stay as close as possible to a pre-determined target profit. The profit goal also takes into account price and elasticity.
- ❖ Existing or old price goal. The objective of this goal is to keep new prices as close as possible to the existing prices. Decision makers need to protect the existing customer relationships and should try to prevent sudden high price increases that may be unacceptable to customers.
- ❖ Competitors' prices were also taken into account by implementing a goal constraint to stay as close as possible to competitors' prices. This goal will also assist with customer relationships as customers may turn to those businesses who offer considerably lower prices for their products.
- System constraints, for example to stay within certain limits of prices, were also incorporated into the final suggested model
- ❖ Finally, weighting factors were derived as discussed in section 4.8.2.

7.4 Develop and test a decision support system that can be used to help determine prices

A decision support system to implement and illustrate the price model and profit optimization was developed using the C++ programming language and a tool for solving linear optimisation problems called ILOG CPLEX. Chapter five provides detailed systems specifications and an algorithmic process.

To illustrate and test the model, real-world data from seven different shops in the Mafikeng area were collected and used in the model. Prices and volumes for ten different items were

obtained from each shop and used in the model. The data and how it was used in the model was described in chapter five. Results were presented and discussed in chapter six. The main result of the application was that it seems feasible to create a decision support system for price determination using a weighted goal programming approach. The technique used in this study was capable of considering multiple and often conflicting objectives and will certainly assist shop managers in making price determination decisions.

7.5 Further research

The development of the model in this study has highlighted various opportunities for enhancement and possible further research projects.

- ❖ Expanding the area of experiments may assist in validating results. Seven shops were used in the illustration and obtaining more data may for example improve the accuracy of the elasticity coefficients.
- ❖ The number of products used in the study can also be increased to show model's capability for large volumes.
- ❖ The model can easily be expanded to cater for other business policies or framework, for example to include the Ramsey price constraints to insure that the suggested prices are in line with Ramsey pricing index.
- ❖ One approach for assigning weights was described in this study. Other methods for determining weights may be investigated. Different weights for different products and different businesses may also provide additional insight into the price determination process.

7.6 Conclusions

The work presented in this study described a weighted goal programming model that may be used to assist decision makers when new prices for products are to be determined. Real world data were used to illustrate the model and a decision support system was developed to apply the model. Satisfactory results were obtained and it proves to be feasible to use a weighted goal programming approach to assist decision makers in the process of price

determination. Possible enhancements and future research following from the study were highlighted.

APPENDIX A

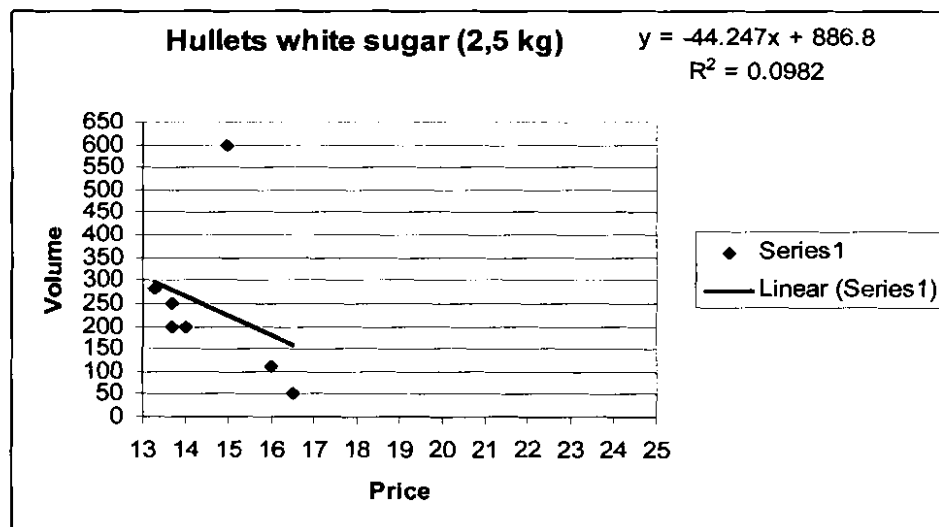
ELASTICITY DIAGRAMS

Introduction

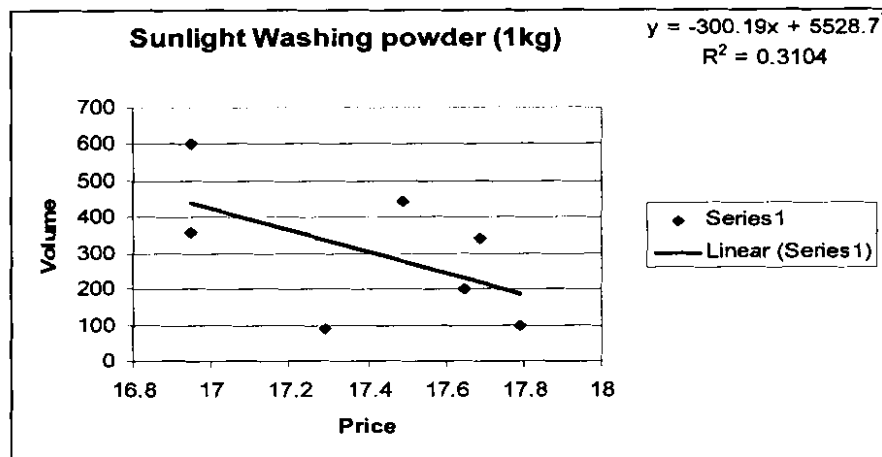
This appendix contains the diagrams relating to elasticity determinations used in this study. Each of the ten selected products will have its own diagram with seven data points as collected from businesses.

The first two products' elasticity determinations have been shown in chapter five. The elasticity determination relates to the remaining product that have not been shown anywhere in this text.

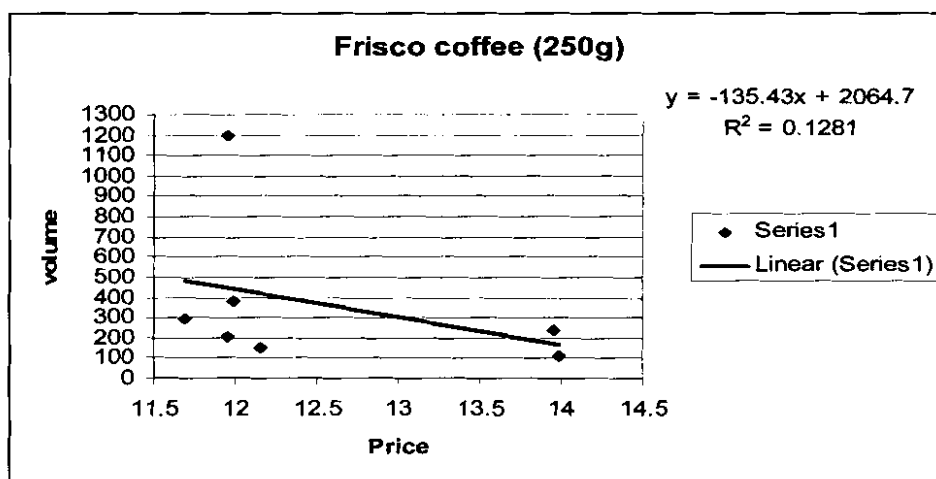
The following diagram shows elasticity determination of the Huletts White sugar (2,5 kg):



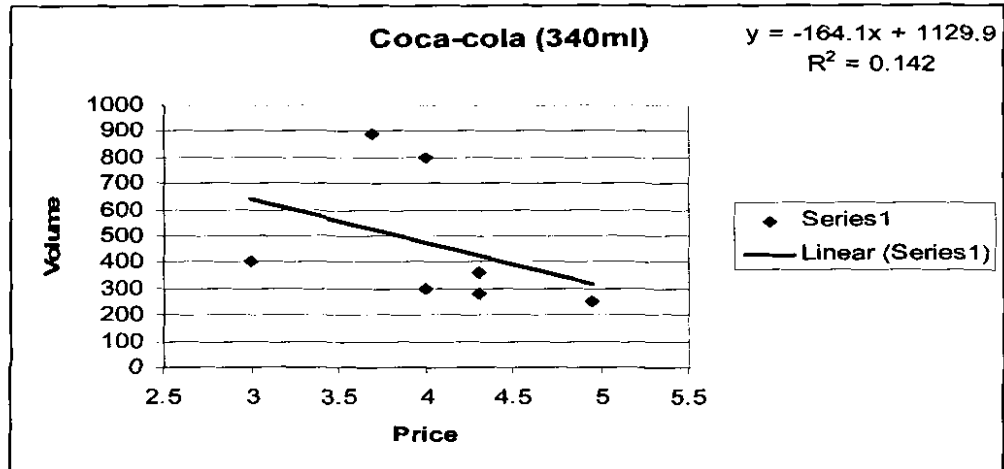
The following diagram shows elasticity determination of the Sunlight washing powder (1kg):



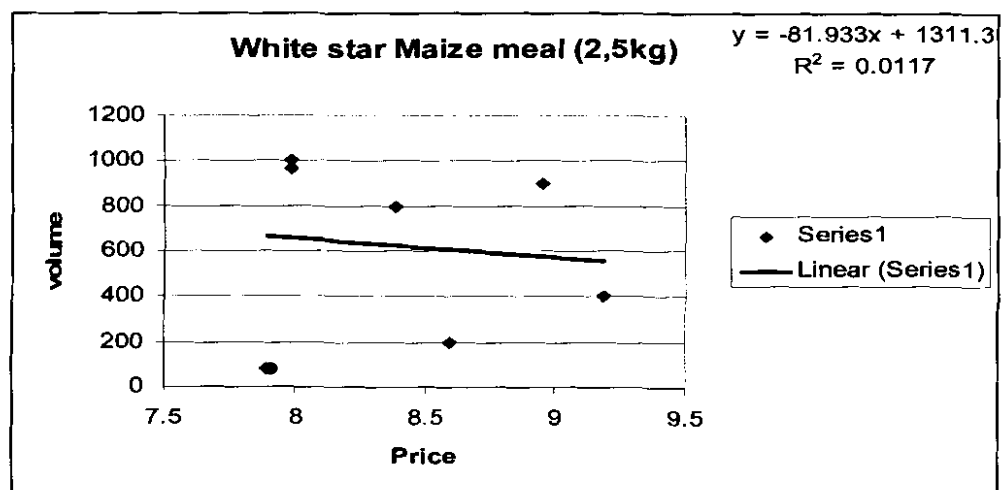
The following diagram shows elasticity determination of the Frisco coffee (250g):



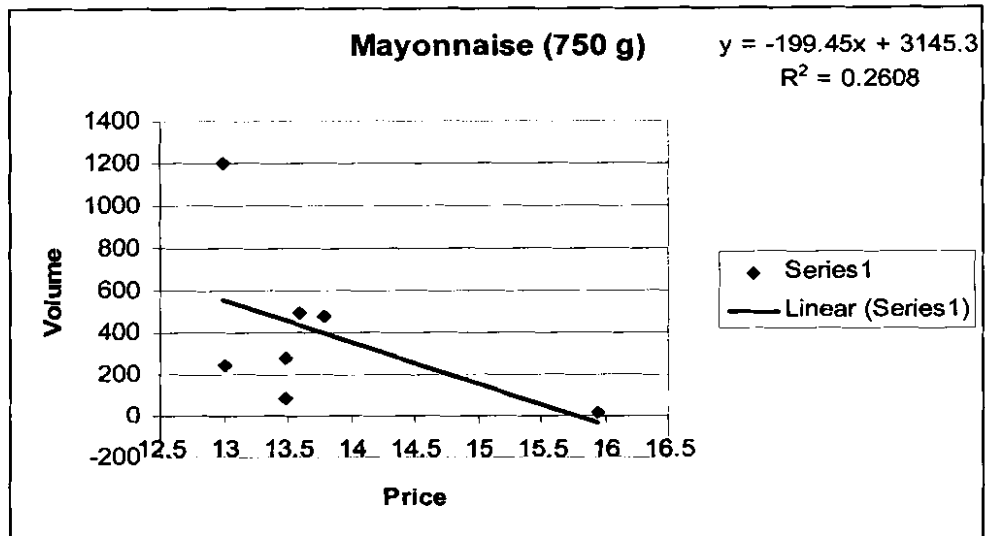
The following diagram shows elasticity determination of the Coca-cola (340ml):



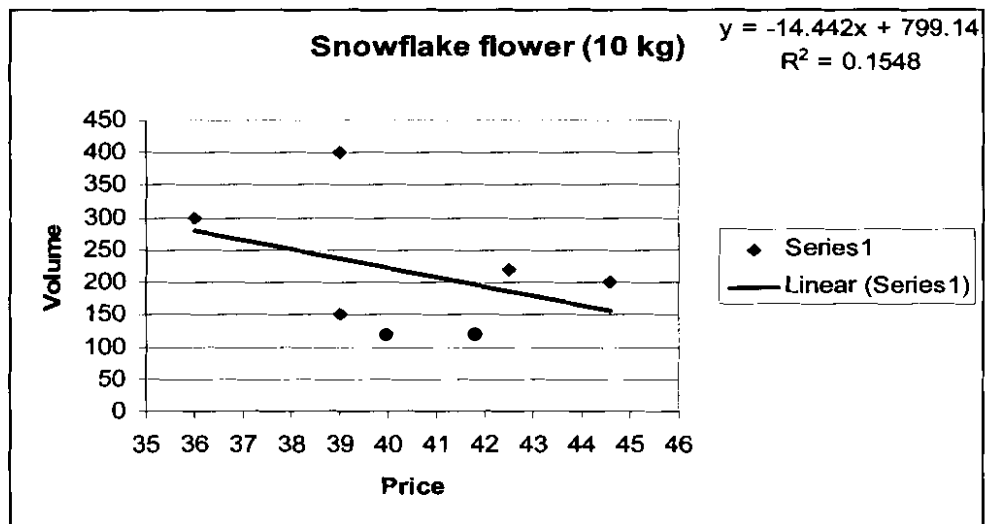
The following diagram shows elasticity determination of the White star maize meal (2, 5 kg):



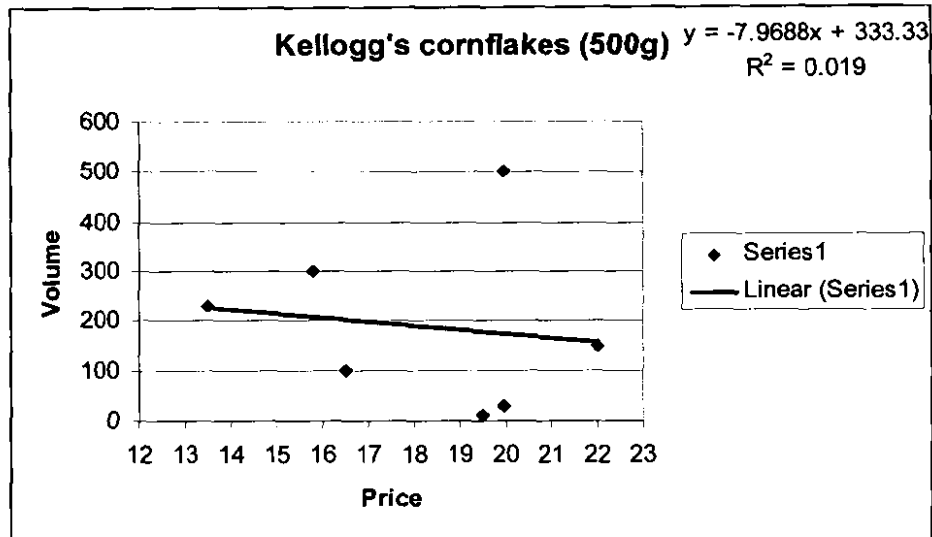
The following diagram shows elasticity determination of the Mayonnaise (750g):



The following diagram shows elasticity determination of the Snowflake flower (10kg):



The following diagram shows elasticity determination of the Kellogg's cornflakes (500g):



The determination of these elasticity values is based on the assumption that price and volume are of linear relationship. These values will always be negative because of the inverse relationship between price and volume. This means that any increase in price will result in corresponding decrease in volume.

APPENDIX B

PROGRAM DESCRIPTION

Introduction

This appendix contains the description of the source code (that source code included in the attached CD) underpinning the goal programming model developed in this study to suggest products' prices. The description covers the system requirements for the environment in which this program was developed.

Requirements for running the program

The program requires an installation of ILOG CPLEX 9.0 or any of the late versions. The computer must be running Windows or UNIX operating systems - preferable late versions of these operating systems. Other specifications are indicated in chapter five regarding hardware requirements.

Using the program

The following steps are meant to guide a user of the program encoded in the CD accompanying this research report. It is important to execute these steps in the sequence listed below:

❖ Step 1

Log on to the terminal and type in command "*make lloqcpexg.cpp*" which will create a make file for the program above. 'Click enter' and the following message will be displayed "*lloqcpexg updated*". The command for running the program is described in step 2.

❖ Step 2

To run the program and obtain the results, type in the following command "*lloqcpexg*" and click enter to view the results. These results are based on data for experiment A, a command for inserting another data file is described in step 3.

❖ Step 3

In order to run the program for the next computational experiments, the source code reads various data files. The data file for the second computational experiment is named "price1.dat". To select this data file remove "/" in front of it and place it front of the previous data file. This procedure will remain the same for the remaining data files. Interpretation of the results is indicated in the following step:

● Step 4

For all computational experiments the format of the results is the same. Firstly a series of current prices are displayed, followed by suggested prices, underachievements, overachievements, current or existing profit, change in prices, expected change in volume, slopes associated with products, expected new volume and the profit after price changes. The following step describes how to exit the results window:

❖ Step 5

To exit the window in which the results have been displayed type in the following command "*exit*". The results window will then close. The following window describes how to exit the CPLEX Environment:

❖ Step 6

To exit the working environment type in the following command "*exit*". Then the working environment will close down.

BIBLIOGRAPHY

ADEJOBI, A.O., KORMAWA, P.M., MANYONG, V.M. & OLAYEMI, J.K. Optimal crop combinations: application of linear goal programming (LPG) model to smallholder farmers in the Drier Savannah Zone of Nigeria [Available on line]. <http://www.tropentag.de/2003/abstracts/full/347.pdf> Date of access: 19 Oct 2006.

BELL, A. 2003. 'n Ondersoek na die gebruik van multikriteriametodes vir strategiese prysbeleid. Potchefstroom: PU vir CHO (Dissertation – M. Sc.) 147 p.

BONINI, C.P., HUISMAN, W.H. & BIERMAN H jr. 1997. Quantity analysis for management 9th Edition Boston: The McGraw-hill companies. 540 p.

BOOTH, G., G. & BESSLER, W. 1989. Goal programming models for managing interest-rate risk. *OMEGA International journal management sciences* 17(1) 81-89.

BROWN, K. & NORGAARD, R. 1992. Modelling the telecommunication pricing decision. *Decision sciences* 23(3) 673-686.

CEPEDA, R.,H.,C., 2005. Slope, Elasticity, Demand, and inverse demand [Available online] <http://are.berkeley.edu/~cavazos/teaching/eep1/slopeelasticity.pdf> Date of access: 04 May 2006.

CHANG, C.T. & HWANG, J.R., 1996. A multi-objective programming approach to waste minimization in the utility systems of chemical process. *Chemical Engineering sciences* 51(16) 3951-3965.

CRAWFORD, I.M.1997. Chapter 8: Pricing Decision. Agricultural and food marketing management. [Available online]

<http://www.fao.org/DOCREP/004/W3240E/W320E08.htm>

Date of access: 20 Nov. 2005.

CROUCHER J.S., 1984. A goal programming model for time-table scheduling, *Journal of operations research* 21(3): 145- 152.

DASGUPTA, P. & HASHIMOTO, Y. 2004. Multi-attribute dynamic pricing for online markets using intelligent agents. (*In IEEE computer society – vol 1, Proceedings of the 3rd international joint conference on autonomous agents and multiagents systems. New York: ACM. P 277-284*) Available: ACM

DeBROCK, L. 2004. [Available online]

Profits.<http://www.business.uiuc.edu/ldebrock/econ102s04/oll/Chap07/> Date of access: 05 Apr. 2005.

FRIEDMAN, D.D, 1990. Price theory, an intermediate text 2nd ed. Cincinnati Ohio: South-Western Publishing CO.

GAL, T., STEWART, T.J. & HANNE, T., editors. 1999. Multi-criteria Decision making: advances in MCDM Models, Algorithms, Theory, and applications. Boston : Kluwer Academic Publishers.

GREENWOOD, A.G. & MOORE L.J. 1987. An Inter-temporal Linear Programming model for optimizing University Tuition and fee structure. *Journal of operational research society*, 38(7): 599 – 613.

HANNA, N. & DODGE, H.R. 1995. Pricing: Policies and procedures, Antony Rowe Ltd, Great Britain: chippenhan, Wiltshire.

HANNAN, E.,L. 1980. Nondominance in goal programming, *INFOR*, 18(4) 300 - 309

HAYDAM, N. 1997. The principle of micro-economics. Pretoria: Van schaik publishers, p 118 – 151.

HOLZMAN, A.G. 1981. Mathematical Programming for operations research and computer scientists. New York and Basel: Marcel Dekker, inc. 373 p.

KELKAR, O., LEUKEL, J. & SCHMITZ, V. 2002. Price modeling in standards for electronic product catalogs based on XML, (in *Proc. of the 11th inter. Conference on World Wide Web*, ACM press)

LEE, S.M. 1972. Goal programming for decision analysis. Philadelphia: Auerbach publishers inc. 397 p.

LEE, S.M. & SHIM, J.P. 1986. Interactive Goal Programming on the Microcomputer to establish Priorities for Small Business. *Journal of operational research society*. 37(6) 571 – 577.

MANAGEMENT MATHEMATICS GROUP. 2005. Goal programming information, Goal PROGRAMMING TUTORIAL, UNIVERSITY OF PORTSMOUTH [Available online] <http://www.tech.port.ac.uk/research/mmg/GPInfoSiteMain.html#GPTutorials> Date of access: 16 Jan. 2006.

MARAVILLA, N. 2004. Pricing: The key to your survival [Available online] <http://www.powerhomebiz.com/vol35/pricing.htm> Date of access: 06 Jun 2005.

MARKETINGTEACHER.COM. 2000-2006. Pricing strategies. [Available online] http://www.marketingteacher.com/Lessons/lesson_pricing.htm Date of access: 20 Feb. 2006.

MARTEL, J.M., KHOURY, N.T., & BERGERON, M. 1988. An application of a Multicriteria approach to portfolio comparisons, *Journal of operational research society*, 39(7): 617 – 628.

MATEU, A.V. 2002. ClusDM a multiple criteria Decision making method for heterogeneous data set. Dept. de la Llenguatges i Sistemes Informatics, Programa de Doctorat en Intel.ligencia artificial, Universitat Politecnica De Catalunya. (Thesis – Ph.D.)

MOORE, J.H. & WEATHERFORD, L.R. 2001. Decision modeling with Microsoft excel 6th edition. Upper Saddle River. New Jersey: prentice hall. [CD]. 693 p.

NAGLE, T.T. & HOLDEN, R.K. 1995. The Strategy and Tactics of Pricing: A guide to profitable decision Making 2nd edition. Englewood-Cliffs: Prentice-hall. 409 p.

OSSAMA, K., BELALID, A. & JEAN-MARC, M. 2004. The double role of the weight factor in goal programming model. *Computer and operations Research* 31: 1833-1845. Available: ScienceDirect.

RAMSEY, E.P.A. 1927. A contribution to the theory of taxation. *Economic journal*, 10, 47- 61

RAUHALA, E. 2002. Problems in pricing digital content products. [Available online] <http://www.hiit.fi/de/core/DigitalContentPricing.pdf> Date of access: 11 Apr. 2006

ROGERS, L. 1990. Pricing for profit. Oxford: Basil Blackwell

ROMERO, C. 1991. Handbook of critical issues in Goal Programming. Oxford: Pergamon Press.

RUBY, D. A. 2003. Price elasticity of demand. [Available online] <http://www.digitaleconomist.com/elasticity.html> Date of access: 07 Jan. 2006.

SAUL, I.G. 1986. A process for determining Priorities and Weights for Large-Scale Linear goal Programmes. *Journal of operational research society*, 37(8): 779 – 785.

SCHNIEDERJANS, M.J. 1995. Goal programming methodology and applications. Boston: kluwer academic Publisher.

SHERALI, H.D. & SOYSTER, A.L. 1983. Preemptive and Nonpreemptive Multi-objective programming: Relationships and Counterexamples. *Journal of optimization theory and application*, 39(2): 173 – 186, February.

SITESELL.COM. 2005. Price with complete confidence...

[Available online] <http://mysps.sitesell.com/> Date of access: 22 Nov. 2005.

STEUER, R.E. 1986. Multiple criteria optimization: theory, computation and application. New York: John Wiley & sons, Inc. 546 p.

STEWART, T.J. 1991. A multi-criteria decision support systems for R & D project selection. *Journal of the operational Research Society*, 42(1) 17 – 26.

STROUSTRUP, B. 2000. The C++ programming language, Special edition. Florham Park, New Jersey: Addison-Wesley 1019 p.

TAMIZ, M., JONES, D.F. & EL-DARZI, E. 1995. A Review of Goal Programming and its Applications. *Annals of Operations Research*, 58: 39 – 53.

TRANTAPHYLLOU, E. 2000. Multi-criteria decision making methods: A comparative study, applied optimization. Boston: Kluwer Academic Publishers. 288 p.

TRANTAPHYLLOU, E., SHU, B., SANCHEZ, S.N. & RAY, T., 1998. Multi-criteria decision making: An operations research approach. (*In Encyclopedia of Electrical and Electronic engineering* vol. 15, 175-186)

VOLPI, N.M.P. DE OLIVEIRA, F. & SANQUETTA, C.R. 2003. Goal programming in a planning problem. *Applied mathematics and computation*, 140:165-178, Available: ScienceDirect.

ZANAKIS, S.H. & GUPTA, S.K. 1985. A categorized bibliographic survey of goal programming. *OMEGA International journal of management science*, 13(3) 211 - 222

ZENELY M. 1982. Multiple criteria decision making. New York: McGraw-hill. 563 p.