A DECISION MAKING PROBLEM IN THE BANKING INDUSTRY

MP. Mulaudzi, Hons. B.Sc

Dissertation submitted in partial fulfilment of the requirements for the degree Magister Scientiae in Applied Mathematics at the Potchefstroom Campus of the North West University (NWU-PC)

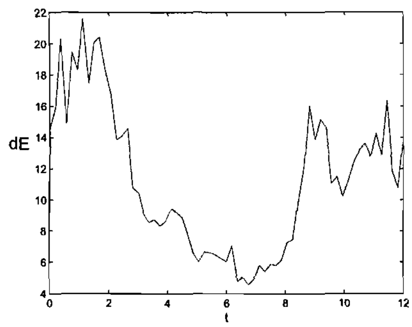

Figure b: Trajectories of Simulated Returns On Equity

Supervisor: Prof. Mark A. Petersen
Co-supervisor: Dr. Ilse M. Schoeman

25 October 2007
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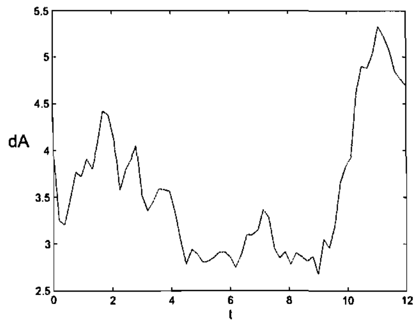

Figure a: Trajectories of Simulated Returns On Assets

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Firstly, I would like to thank the Almighty for His grace in enabling me to complete this dissertation.

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Preface

One of the contributions made by North-West University at Potchefstroom (NWU-PC) to the activities of the financial community in South Africa has been the establishment of an active research group that has an interest in institutional finance. Under the guidance of my supervisor, Prof. Mark A. Petersen, this group has recently made valuable contributions to the existing knowledge about the modeling and optimization of financial institutions.

The work in this dissertation originated from our interest in the connections between concepts that arise in optimization theory and banking. From the outset it became apparent that little work had been done on this topic although it had been identified as an area of potential growth.

Some of the outcomes of this project were collected in 2 research articles that were submitted for possible publication and an accepted proceedings paper for presentation at the IASTED Financial Engineering and Application (FEA2007) Conference at the University of California (UCLA) at Berkeley.

Declaration

I declare that, apart from the assistance acknowledged, the work contained in dissertation is my own work, unaided work. It is being submitted in partial fulfilment of the requirements for the degree of Master of Science in Applied Mathematics at the Potchefstroom Campus of the North West University. It has not been submitted before for any degree or examination to any other University.

Nobody, including Prof. MA. Petersen (Supervisor) and Dr. IM. Schoeman (Co-supervisor), but myself is responsible for the final version of this dissertation.

Signature.................................

Date......................................

Executive Summary

The main categories of assets held by banks are risky assets (loans and intangible assets), Treasuries (bonds issued by the national Treasury) and reserves. Our contribution models how decisions about the allocation of available funds to the former two types are dependent on perceptions about risk and regret. Our discussion is based on utility theory where a regret attribute is considered alongside a risk component as an integral part of the objective function. Preferences among risky assets and Treasuries are described by the maximization of the expected value of a utility function that depends on the funds available to the bank. Moreover, we conjecture that anticipated disutility from regret can dramatically impact the choices of assets types that risk- and regret-averse banks decide to hold. Here we conclude that, compared with the risk-averse case, the bank who takes regret into account will be exposed to higher credit and market risk when the difference between the expected return on risky assets and the Treasuries rate is small but lower risk exposure when this difference is high. We also assess how regret can influence a bank's view of rate of return loan guarantees, as measured by its willingness-to-incur-costs (WTIC) that are related to the screening and monitoring of debtor and guarantor status. We find that regret increases the regret-averse bank's WTIC for a guarantee when the asset portfolio is relatively risky, but decreases when the portfolio is considered to be safe. A feature of our contribution is that the main issues are briefly analyzed and, where possible, the outcomes are represented graphically. In this regard, we comment on the claim that an investment away from risky assets towards Treasuries is responsible for credit crunches in the banking industry.

Contents

Chapter 1

INTRODUCTION TO A BANK DECISIONING PROBLEM

In our contribution, we model how preferences regarding the allocation of available bank funds to risky assets (loans and intangible assets) and Treasuries (bonds issued by the national Treasury) are dependent on perceptions about banking risk and regret. Here regret is the disutility a bank experiences from the gap in value between an actual asset return and the best possible return that the bank could have attained in a particular economic state. More specifically, we evaluate the asset allocation behavior of banks, bearing in mind that the allocation of funds to risky portfolio and the holding of Treasuries may be influenced by the prospect of regret.

A first motivation for wanting to address the above problem is the need for banks to optimize the returns on their asset investments and have no regrets afterwards.

For example, if the return on a specific credit risk type turns out to be very high at the end of a loan period, the bank might regret not having allocated a large enough portion of its funds to that credit type. Conversely, if the credit risk type does poorly, the bank might regret having allocating funds to loans in that risk category. Such anticipated disutility from regret is particularly important in the context of banking, since most banks select an initial risky portfolio at the beginning of a investment period but often do not make much of an effort to manage their portfolio thereafter unless, for instance, a possibility of loan default arises.

A further motivation for discussing asset allocation preferences in a regret theoretic framework is that bank and thrift failures usually spark debates about banking capital, risk and regulatory prescripts to mitigate this risk. The prescriptions are encapsulated in the Basel Accords on capital adequacy requirements (see, for instance, [3] and [4] for the Basel II capital accord), which mandates that all major international banks hold capital in proportion to their perceived credit risks. The 1996 Amendment's Internal Models Approach (IMA) characterizes capital requirements on the basis of the output of the banks' internal risk measurement systems. In many countries, banks are required to report their daily Value-at-Risk (VaR) at the 99 % confidence level over a one-day horizon and over a two-week horizon (ten trading days). The minimum capital requirement on a given day is then equal to the sum of the charge to cover *credit risk* and a charge to cover *market risk*. The credit risk charge is equal to 8 % of risk-weighted and the market risk charge is equal to a multiple of the average reported two-week VaRs in the last 60 trading days. In general, the ratio of capital to assets, also called the *capital adequacy ratio* plays a major role as an index of the sufficiency of capital held by banks. This ratio is the centerpiece of the minimum capital requirement of Basel II and has the form

$$\text{Capital Adequacy Ratio} = \frac{\text{Indicator of Absolute Amount of Capital}}{\text{Indicator of Absolute Level of Risk}}.$$

The Basel II (risk-weighted) and unweighted classes of such capital adequacy ratios play a role in our discussion and may be represented by

$$\begin{aligned} &\text{Basel II Capital Adequacy Ratio (BCAR)} \\ &= \frac{\text{Bank Capital (BC)}}{\text{Risk-Weighted Assets (RWAs)} + \text{VaR Component}} \end{aligned} \tag{1.1}$$

and

$$\text{Leverage Capital Adequacy Ratio (LCAR)} = \frac{\text{Bank Capital (BC)}}{\text{Unweighted Assets (UWAs)}}, \tag{1.2}$$

respectively.

Because capital is more expensive to raise than insured deposits, risk-based capital adequacy requirements (RBCARs) may be viewed as a regulatory tax that is higher on assets in categories that are assigned higher risk weights.

The Basel II Accord on capital adequacy requirements mandates that all major international banks hold capital in proportion to their perceived credit risks. In this regard, all assets and off-balance sheet activities are assigned risk weights between 0 percent and 100 percent according to their perceived credit risks, and banks must hold capital of at least certain percentages against total risk-weighted assets and off-balance sheet items. The more risky assets are assigned a larger weight. Table 1.1 below provides a few illustrative risk categories, their risk-weights and representative items.

Risk Category	Risk-Weight	Banking Items
1	0 %	Cash, Reserves, Treasuries
2	20 %	Shares
3	50 %	Home Loans
4	100 %	Intangible Assets
5	100 %	Loans to Private Agents

Table 1.1: Risk Categories, Risk-Weights and Banking Items

Implementation of RBCARs would encourage allocation to assets in the 0 percent risk category, such as Treasuries, and discourage allocation to assets in the 100 percent risk category, such as commercial loans and intangible assets. Empirical evidence has shown that the allocation of credit away from commercial loans towards Treasuries causes a significant reduction in macroeconomic activity, given that many commercial borrowers cannot easily obtain alternative sources of funding in public markets. This phenomenon can be traced back to the role of RBCARs in modern banking practice and may be responsible for "credit crunches." Such crunches are defined as the significant reduction in the supply of credit available to commercial borrowers.

It is not uncommon in the process of loan issuing to include as part of the financial package a guarantee of the loan by a guarantor. Examples are guarantees by a parent company of loans made to its subsidiaries or government guarantees of loans made to private corporations. At certain junctures in the dissertation, we assume that a guaranteed rate of return on a loan is made available. Loan guarantees may help mitigate the regret experienced by banks, by protecting their funds in economic states where realized loan yields are poor. The benefit of a guarantee is valuable for high levels of investment in the loan. For example, a bank with a substantial portion of its funds invested in loans who finds itself in an economic state with a low realized return on this investment would experience a great deal of regret as a result of such a preference. Of course, a loan guarantee would offer the possibility of hedging the bank's investment against credit risk, which would reduce the banks feeling of regret. In the main, this regret mitigation feature of a guarantee is most beneficial when the proportion of the funds invested in the loan is high. On the other hand, a guarantee also introduces an additional cost to regret-averse banks that could therefore amplify ex-ante regret.

1.1 PRELIMINARIES

In this subsection, we provide preliminaries about bank assets investment behavior under regret aversion.

1.1.1 Bank Assets

After providing liquidity, suppose a bank has initial available funds, f_0, which can be allocated between specific risky assets (loans and intangible assets) and Treasuries (riskless asset). In the sequel, the rate of return on aggregate risky assets, a, is given by a random variable, r^a, while Treasuries, $\mathtt{T}$, yields a deterministic return, $r^{\mathtt{T}}$. In particular, r^a is a function of the loan rate, r^{Λ}, and rate of return on intangible assets, r^I. Also, the aggregate risky assets, a, may be expressed as a weighted sum of the loans, Λ, and intangible assets, I. In making its asset portfolio choice, the bank takes into account the fact that it may regret having preferred a assets investment that proves to be suboptimal after expiry of the investment period. An important assumption throughout our discussion is that banks avoid the deleterious consequences of a result that is worse than the best that could be achieved had knowledge of the loss been known ex-ante. For example, if a bank invests funds of a large value in a risky portfolio and then incurs a large loss to that portfolio, the bank would experience some additional disutility of not having invested less in the risky portfolio.

1.1.2 Utility Theory

Expected utility theory is a major paradigm in decision theory (see, for instance, [23] and [26]). In our contribution, we choose a regret theoretic expected utility of the form

$$\int \left[u(f_\psi) - \rho \cdot g\Big(u(f_\psi^{\max}) - u(f_\psi)\Big)\right] dF(\psi),$$

where $F(\psi)$ is a cumulative distribution function that incorporates institutional views about economic states, ψ, where f_ψ is the result in state, ψ, of action f being taken. With this in mind, we investigate the impact of regret on the banks ex-ante asset allocation by representing the banks preferences as a two-component Bernoulli utility function, $u_\rho : \mathbb{R}^+ \rightarrow \mathbb{R}$, given by

$$u_\rho(f) = u(f) - \rho \cdot g\Big(u(f^{\max}) - u(f)\Big), \tag{1.3}$$

where $u : \mathbb{R}^+ \rightarrow \mathbb{R}$ is the traditional Bernoulli utility (value) function over funding positions.

1.1.3 Risk and Regret

In the above, regret aversion corresponds to the convexity of g, and the bank's preference is assumed to be representable by maximization subject to u. Also, we have that

$$f = f_0\Big(1 + \pi r^a + (1-\pi)r^{\mathrm{T}}\Big)$$

is the actual final fund level and $f^{\max}$ is the value of the ex-post optimal final level of funds, i.e., the fund level that the bank could have attained if it had made the optimal choice with respect to the realized state of the economy. The first term in (1.3) relates to traditional risk aversion and involves the banks utility function $u(\cdot)$ with $u'(\cdot) > 0$ and $u''(\cdot) < 0$. The second term in (1.3) is concerned with the prospect of bank regret. The function $g(\cdot)$ measures the amount of regret that the bank experiences, which depends on the difference between the value it assigns to the ex-post optimal fund level, $f^{\max}$, that it could have achieved, and the value that it assigns to its actual final level of funds, f. The parameter $\rho \geq 0$ measures the weight of the regret attribute with respect to the first attribute expressive of risk aversion. For $\rho = 0$, the bank would be a maximizer of risk-averse expected utility, which means that $u_0(\cdot) = u(\cdot)$. On the other hand, if $\rho > 0$, then the utility function of the bank includes some compensation for regret and we call the bank *regret-averse*. An assumption is that $g(\cdot)$ is increasing and strictly convex, i.e. $g'(\cdot) > 0$ and $g''(\cdot) > 0$, which also implies regret-aversion.

1.2 RELATION TO PREVIOUS LITERATURE

In this section, we briefly review some of the pertinent literature on bank assets, loan guarantees and regret theory.

1.2.1 Assets and Loan Guarantees

In this subsection, we review some of the issues related to loans, Treasuries, intangible assets and loan guarantees. It is important to be able to measure the volume of loans and intangible assets. Banks are interested in establishing the level of Treasuries on demand deposits that the bank must hold. By setting a bank's individual level of assets, roleplayers assist in mitigating the costs of financial distress. For instance, if the minimum level of Treasuries exceeds a bank's optimally determined level of securities, this may lead to deadweight losses. Intangible asset is something of value that cannot be physically touched, such as brand name, franchise, trademark, or patent. Value relevance of intangible investments has been largely recognized by indicating their close relatedness on future operating performance and valuation of banks (see [14]). The financial environment of country (market- or bank-based) is also found to be an important determinant of the economic performance of the bank (also, see [14]). Loan guarantees have been discussed in several interesting contributions. Amongst these are the papers [6], [17] and [24]. The contributions [6] and [17] confirm that riskier debtors will pledge more collateral via loan guarantees which is consistent with the empirical evidence. Also, [24] intimates that where some borrowers are unable to meet the repayment obligations of their debt, guarantors also face material real costs of honoring their guarantee to lenders. The paper highlights the fact that a range of loan guarantee mechanisms might be contemplated for banks, though in practice they tend to take the form of either a *rate of return guarantee* or a *minimum repayment guarantee*. In the present dissertation, we focus on the former structure, in which a guarantor commits to return to the bank the value of the loans issued to the debtor plus some stipulated rate of return. A variation of this theme is a *principal guarantee* that corresponds to guaranteeing a nominal rate of return of zero percent.

1.2.2 Regret Theory

The theory of regret was developed by Bell and Loomes and Sugden in [5] and [20], respectively. Subsequently, the said theory was presented in axiomatized form by Quiggin and Sugden in [23] and [26], respectively. Throughout these contributions, regret is defined as the disutility of failing to choose the ex-post optimal alternative. The exercising of preferences consistent with such a decision theoretic structure has arisen in several contexts. For instance, deviations from expected utility models used in finance and insurance occur in such contributions as [18], [19], [21] and [25]. More recently, regret theory has been used by Gollier and Salanie in [12] to investigate risk-mitigation and the pricing of assets in a complete market setting. Also, in pension fund theory, risk and regret has been discussed, for instance, in [11], [16], [27] and [29]. We note that the above contributions make a distinction between avoiding regret by making regret-avoiding decisions and avoiding regret by suppressing regret-inducing information about the result of the foregone alternative. In the current dissertation, we consider preferences about regret-avoidance for which the bank maximizes its regret theoretic expected utility function. To our knowledge, very little (if any) research has focussed on how behavior compatible with such a decision theoretic structure arises in the banking industry.

1.3 OUTLINE OF THE DISSERTATION

In the current section, an outline of our contribution is given. Under the conditions highlighted above, the main problems addressed in the rest of our contribution is subsequently identified.

In Chapter 2, we present the banking model with regret. Pertinent facts about loans, Treasuries and intangible assets are presented in Subsections 2.1.1, 2.1.2 and 2.1.3, respectively. In Subsection 2.1.4, our main focus is on characterizing an index of aggregate risky assets for banks. The impact of regret on the bank asset allocation is outlined in more detail in Section 2.2. In this regard, Theorem 2.2.2 in Subsection 2.2.1 proves that a regret-averse bank will always allocate away from $\pi^*_\rho = 0$ and $\pi^*_\rho = 1$, where π^*_ρ denote the optimal risky portfolio. The next important result shows that higher regret amplifies the effect of the bank hedging its bets (see Proposition

2.2.3 from Subsection 2.2.2). Also, Proposition 2.2.4 in Subsection 2.2.3 proposes the existence of a Treasuries rate at which regret has no impact on the bank's optimal proportion invested in risky portfolio.

In Section 3.2 of Chapter 3, we suggest a way of mitigating regret via loan guarantees. Also, we consider the level of costs a bank is willing to incur in order to secure such a guarantee. In particular, Theorem 3.2.1 from Section 3.2 shows that when the fraction of available funds invested in the loan is low, a regret-averse bank values the guarantee less than the risk-averse bank.

In Chapter 4, we analyze the main decision theoretic issues arising from the banking model with regret that we constructed previously. Some of the highlights of this section are mentioned below. A description of the role that bank assets play is presented in Section 4.1. In particular, we consider the contribution to credit crunches of asset allocation away from risky assets (loans and intangible assets) towards Treasuries. Furthermore, we provide more information about the impact of regret on the bank asset allocation in Section 4.2. Loan guarantees and their function of mitigating risk is discussed in Section 4.3.

In addition to the solutions to problems outlined above, Chapter 5 offers a few concluding remarks and possible topics for future research.

The bibliography in Chapter 6 contains all the articles, books and other sources used throughout the dissertation.

Finally, Chapter 7 contains graphical representations of the main conclusions from Theorems 2.2.2 and 3.2.1, Proposition 2.2.4 as well as the credit crunch phenomenon.

Chapter 2

THE BANKING MODEL

In this chapter, we describe some of the components of the banking model with regret like bank assets, regret and loan guarantees.

2.1 ASSETS

In this subsection, the bank assets that we discuss are loans, Treasuries, intangible assets and an aggregate of risky assets.

2.1.1 Loans

We suppose that, after providing liquidity, the bank grants loans at the *interest rate on loans* or *loan rate*, r_t^{Λ}. This rate is assumed to be a random variable which is distributed according to some cumulative distribution function, F. Due to the expenses related to monitoring and screening, we assume that these loans incur a constant *marginal cost*, c^{Λ}. This cost largely depends on the nature of the loan issued. For instance, for a loan with a guarantee the costs of monitoring and screening are generally higher than a loan with no guarantee.

2.1.2 Treasuries

Treasuries are bonds issued by national Treasuries and are the debt financing instruments of the federal government. There are four types of Treasuries: treasury bills, treasury notes, treasury bonds and savings bonds. All of the treasuries besides savings bonds are very liquid and are heavily traded on the secondary market. Furthermore, we denote the *interest rate on Treasuries* or *Treasuries rate* by r_t^{T}. In line with empirical evidence, for all t, it is almost always true that

$$\mathbf{E}[r^{\Lambda}] - c^{\Lambda}(t) > r^{\mathrm{T}}(t). \tag{2.1}$$

2.1.3 Intangible Assets

In the contemporary banking industry, shareholder value is often created by *intangible assets* which consist of patents, trademarks, brand names, franchises and economic goodwill (more specifically, core deposit customer relationships, customer loan relationships as differentiated from the loans themselves, etc.). Economic goodwill consists of the intangible advantages a bank has over its competitors such as an excellent reputation, strategic location, business connections, etc. In addition, such assets can comprise a large part of the bank's total assets and provide a sustainable source of wealth creation. Intangible assets are used to compute Tier 1 bank capital and have a risk weight of 100 % according to Basel II regulation (see Table 1.1 in Chapter 1). In practice, valuing these off-balance sheet items constitutes one of the principal difficulties with the process of bank valuation by a stock analyst. The reason for this is that intangibles may be considered to be "risky" assets for which the future service potential is hard to measure. Despite this, our model assumes that the measurement of these intangibles is possible (see, for instance, [13] and [30]). In the sequel, we denote the value of intangible assets, in the t-th period, by I_t and the return on these assets by $r_t^I I_t$.

2.1.4 Aggregate Risky Assets

After providing liquidity, suppose a bank has initial available funds, f_0, which can be allocated between specific risky assets (loans and intangible assets) and Treasuries (riskless asset). As was mentioned before in Subsection 1.1.1, we suppose that the aggregate risky assets rate is given by a random variable, r^a, whereas Treasuries yields a deterministic return, r^{T}. In particular, r^a is a function of the loan rate, r^{Λ}, and intangible assets rate, r^I, so that

$$r^a = z(r^{\Lambda}, r^I).$$

Also, a_t, the value of the aggregate risky assets may be given by

$$a_t = \alpha \Lambda_t + \beta I_t, \ \alpha + \beta = 1,$$

where α; $0 \leq \alpha \leq 1$ and β; $0 \leq \beta \leq 1$ represent appropriate weights for loans, Λ, and intangible assets, I, respectively. In the sequel, we note that α and β cannot be equal to 1 simultaneously. The main reason for computing aggregate asset prices is to identify the key characteristics of broad swings in asset prices that may be masked by differences in the behavior of individual prices. This may highlight their relationship to macroeconomic performance and monetary policy. Two basic criteria for selecting the assets included in the index should be that they make up a sizeable proportion of bank assets and that they represent both on- and off-balance sheet items.

2.2 REGRET IN BANKING

In order to determine the optimal level of bank funds after the fact, $f^{\max}$, we must make a distinction between the cases where the aggregate risky asset rate, r^a, exceeds the Treasuries rate, $r^a \geq r^{\mathrm{T}}$, and where the opposite is true, i.e., $r^a < r^{\mathrm{T}}$. In the first instance, for optimal returns, the regret-averse bank would have wanted to invest all (having considered the primary mandate of the bank) available funds in the risky portfolio. On the other hand, in the second case, it would have been optimal to invest all funds in the Treasury. Symbolically, this means that

$$f^{\max} = \begin{cases} f_0(1 + r^a), & \text{if } r^a \geq r^{\mathrm{T}}; \\ f_0(1 + r^{\mathrm{T}}), & \text{if } r^a < r^{\mathrm{T}}. \end{cases}$$

A further relationship between r^a and r^{T} that we are interested in is the difference $\mathbf{E}[r^a] - r^{\mathrm{T}}$. In this regard, we will show a particular interest in the situations where

$$\mathbf{E}[r^a] - r^{\mathrm{T}} = 0 \tag{2.2}$$

and

$$\mathbf{E}[r^a] - r^{\mathrm{T}} = \frac{\mathrm{cov}\Big(- r^a, u'(f_0(1 + r^a))\Big)}{\mathbf{E}\Big(u'(f_0(1 + r^a))\Big)}. \tag{2.3}$$

Obviously, (2.2) and (2.3) represent the cases where the expected rate of return from aggregate risky assets and the Treasuries rate coincide and where the difference between the rates is significant, respectively. Note that we will sometimes use the notation $\mathbf{E}[r^a] - r^{\mathrm{T}} >> 0$ when referring to (2.3).

2.2.1 A Decision Theoretic Optimization Problem

In this subsection, we consider how the banks optimal asset allocation is influenced by regret theoretic issues in a stylized framework. Let π_ρ denote the fraction of available bank funds invested in the risky portfolio with regret parameter $\rho \geq 0$. For the case where π_ρ is optimal (denoted by π^*_ρ), we have that $\pi^*_0 = \pi^*$. For the two-attribute Bernoulli utility function (1.3), the objective function is given by

$$J(\pi) = \mathbf{E}[u_\rho(f(\pi))]. \tag{2.4}$$

In order to determine the optimal asset allocation, π^*_ρ, we consider the *set of admissible controls* given by

$$\mathcal{A} = \left\{ \pi_\rho : \ 0 \leq \pi_\rho \leq 1 \text{ and (2.4) has a finite value} \right\}. \qquad (2.5)$$

Also, the *value function* is given by

$$\begin{aligned} V(\pi) &= \max_{\pi \in \mathcal{A}} \mathbf{E}[u_\rho(f(\pi))] \qquad (2.6) \\ &= \max_{\pi \in \mathcal{A}} \mathbf{E}\left[u(f(\pi)) - \rho \cdot g\Big(u(f^{\max}) - u(f(\pi)) \Big) \right]. \end{aligned}$$

The optimal asset allocation problem with regret may be formally stated as follows.

Problem 2.2.1 (Optimal Asset Allocation with Regret): *Suppose that the Bernoulli utility function, u_ρ, objective function, J, and admissible class of control laws, $\mathcal{A} \neq \emptyset$, are described by* (1.3), (2.4) *and* (2.5), *respectively. In this case, characterize $V(\pi)$ in* (2.6) *and the optimal control law, π^*, if it exists.*

The ensuing optimization result demonstrates that a regret-averse bank will always allocate away from $\pi_\rho^* = 0$ and $\pi_\rho^* = 1$. In other words, by comparison with a traditional risk-averse bank, the regret-averse bank will commit to a riskier asset allocation if the difference $\mathbf{E}[r^a] - r^{\mathrm{T}}$ is low, and a less risky allocation if $\mathbf{E}[r^a] - r^{\mathrm{T}}$ is high.

Theorem 2.2.2 (Allocating Away from $\pi_\rho^* = 0$ and $\pi_\rho^* = 1$): *Suppose that u_ρ is the two-attribute Bernoulli utility function defined by* (1.3) *and π is the proportion of available bank funds invested in the risky portfolio that appears in* (2.6). *If* (2.2) *holds then $\pi_\rho^* > 0$ for all $\rho > 0$, with $\pi_0^* = 0$. If* (2.3) *holds then $\pi_\rho^* < 1$ for all $\rho > 0$, with $\pi_0^* = 1$.*

Proof. We use standard maximization arguments to solve the optimization problem stated in Problem 2.2.1. In particular, we must show that the first derivative of (2.6) with respect to π, at $\pi_\rho^* = 0$ and $\pi_\rho^* = 1$ does not vanish. In this regard, we have that

$$f(\pi) = f_0\Big(1 + \pi r^a + (1-\pi) r^{\mathrm{T}}\Big) \text{ and } f^{\max} = f_0\Big(1 + \max(r^a, r^{\mathrm{T}})\Big)$$

denote the banks final fund level and ex-post optimal fund level, respectively. The first- and second-order conditions for (2.6) are

$$\frac{d\mathbf{E}[u_\rho(f(\pi))]}{d\pi} = 0 \tag{2.7}$$

and

$$\frac{d^2\mathbf{E}[u_\rho(f(\pi))]}{d\pi^2} < 0, \tag{2.8}$$

respectively. But

$$\begin{aligned}
\frac{d\mathbf{E}[u_\rho(f(\pi))]}{d\pi} &= \mathbf{E}\Big[\frac{du(f(\pi))}{d\pi} - \rho\frac{dg\Big(u(f^{max}) - u(f(\pi))\Big)}{d\pi}\Big] \\
&= \mathbf{E}\Big[u'(f(\pi))f_0(r^a - r^{\mathrm{T}}) - \rho g'\Big(u(f^{max}) - u(f(\pi))\Big)\Big(- \frac{du(f(\pi))}{d\pi}\Big)\Big] \\
&= \mathbf{E}\Big[f_0(r^a - r^{\mathrm{T}})u'(f(\pi)) + \rho g'\Big(u(f^{max}) - u(f(\pi))\Big)u'(f(\pi))f_0(r^a - r^{\mathrm{T}})\Big] \\
&= \mathbf{E}\Big[f_0(r^a - r^{\mathrm{T}})u'(f(\pi))\Big(1 + \rho g'\Big(u(f^{max}) - u(f(\pi))\Big)\Big)\Big]
\end{aligned}$$

and

$$\begin{aligned}
\frac{d^2\mathbf{E}[u_\rho(f(\pi))]}{d\pi^2} &= \mathbf{E}\Big[f_0(r^a - r^{\mathrm{T}})\frac{du'(f(\pi))}{d\pi}\Big(1 + \rho g'\Big(u(f^{max}) - u(f(\pi))\Big)\Big) \\
&+ f_0(r^a - r^{\mathrm{T}})u'(f(\pi))g''\Big(u(f^{max}) - u(f(\pi))\Big)\Big(- f_0(r^a - r^{\mathrm{T}})u'(f(\pi))\Big)\Big] \\
&= \mathbf{E}\Big[f_0^2(r^a - r^{\mathrm{T}})^2u''(f(\pi))\Big(1 + \rho g'\Big(u(f^{max}) - u(f(\pi))\Big)\Big) \\
&- f_0^2(r^a - r^{\mathrm{T}})^2\rho u'^2(f(\pi))g''\Big(u(f^{max}) - u(f(\pi))\Big)\Big] \\
&= \mathbf{E}\Big[f_0^2(r^a - r^{\mathrm{T}})^2u''(f(\pi))\Big(1 + \rho g'\Big(u(f^{max}) - u(f(\pi))\Big)\Big)\Big] \\
&- \mathbf{E}\Big[f_0^2(r^a - r^{\mathrm{T}})^2\rho u'^2(f(\pi))g''\Big(u(f^{max}) - u(f(\pi))\Big)\Big].
\end{aligned}$$

It then follows that (2.7) and (2.8) take the forms

$$\frac{d\mathbf{E}[u_\rho(f(\pi))]}{d\pi} = \mathbf{E}\Big[f_0(r^a - r^{\mathrm{T}})u'(f(\pi))\Big(1+\rho\cdot g'\Big(u(f^{\max}) - u(f(\pi))\Big)\Big)\Big] = 0 \quad (2.9)$$

and

$$\begin{aligned}\frac{d^2\mathbf{E}[u_\rho(f(\pi))]}{d\pi^2} &= \mathbf{E}\Big[f_0^2(r^a - r^{\mathrm{T}})^2u''(f(\pi))\Big(1+\rho\cdot g'\Big(u(f^{\max}) - u(f(\pi))\Big)\Big)\Big] \quad (2.10)\\ &- \mathbf{E}\Big[f_0^2(r^a - r^{\mathrm{T}})^2\rho u'^2(f(\pi))g''\Big(u(f^{\max}) - u(f(\pi))\Big)\Big] < 0,\end{aligned}$$

respectively. This implies that $\mathbf{E}[u_\rho(f(\pi))]$ is strictly concave in π, so that any solution of the first order condition (2.9) uniquely fixes the global maximum. Furthermore, in this case, a decomposition of (2.9) may be given by

$$\begin{aligned}\frac{d\mathbf{E}[u_\rho(f(\pi))]}{d\pi} &= \mathbf{E}\Big[f_0(r^a - r^{\mathrm{T}})u'(f(\pi))\Big(1+\rho g'\Big(u(f^{max}) - u(f(\pi))\Big)\Big)\Big]\\ &= \mathbf{E}\Big[f_0(r^a - r^{\mathrm{T}})u'(f(\pi)) + \rho f_0(r^a - r^{\mathrm{T}})u'(f(\pi))g'\Big(u(f^{max}) - u(f(\pi))\Big)\Big]\\ &= \mathbf{E}\Big[f_0(r^a - r^{\mathrm{T}})u'(f(\pi))\Big] + \mathbf{E}\Big[\rho f_0(r^a - r^{\mathrm{T}})u'(f(\pi))g'\Big(u(f^{max}) - u(f(\pi))\Big)\Big]\\ &= \frac{d\mathbf{E}[u_0(f(\pi))]}{d\pi} + \int_{-1}^{\infty}\rho f_0(r^a - r^{\mathrm{T}})u'(f(\pi))g'\Big(u(f^{max}) - u(f(\pi))\Big)dF(r^a)\\ &= \frac{d\mathbf{E}[u_0(f(\pi))]}{d\pi} + \int_{-1}^{r^T}\rho f_0(r^a - r^{\mathrm{T}})u'(f(\pi))g'\Big(u(f^{max})) - u(f(\pi))\Big)dF(r^a)\\ &+ \int_{r^T}^{\infty}\rho f_0(r^a - r^{\mathrm{T}})u'(f(\pi))g'\Big(u(f^{max}) - u(f(\pi))\Big)dF(r^a)\\ &= \frac{d\mathbf{E}[u_0(f(\pi))]}{d\pi} + \int_{-1}^{r^T}\rho f_0(r^a - r^{\mathrm{T}})u'(f(\pi))g'\Big(u(f(0)) - u(f(\pi))\Big)dF(r^a)\\ &+ \int_{r^T}^{\infty}\rho f_0(r^a - r^{\mathrm{T}})u'(f(\pi))g'\Big(u(f(1)) - u(f(\pi))\Big)dF(r^a)\end{aligned}$$

If we evaluate this first derivative at $\pi = 0$ and $\pi = 1$, then we obtain

$$\begin{aligned}
\frac{d\mathbf{E}[u_\rho(f(\pi))]}{d\pi}\bigg|_{\pi=0} &= \frac{d\mathbf{E}[u_0(f(\pi))]}{d\pi}\bigg|_{\pi=0} + \rho f_0 u'(f(0))g'(0)\int_{-1}^{r^{\mathrm{T}}}(r^a - r^{\mathrm{T}})dF(r^a) \\
&+ \rho f_0 u'(f(0))\int_{r^{\mathrm{T}}}^{\infty}(r^a - r^{\mathrm{T}})g'\Big(u(f(1)) - u(f(0))\Big)dF(r^a) \\
&> \frac{d\mathbf{E}[u_0(f(\pi))]}{d\pi}\bigg|_{\pi=0} + \rho f_0 u'(f(0))g'(0)\int_{-1}^{r^{\mathrm{T}}}(r^a - r^{\mathrm{T}})dF(r^a) \\
&= f_0 u'(f(0))\Big(\mathbf{E}[r^a] - r^{\mathrm{T}}\Big) + f_0 u'(f(0))\rho g'(0)\Big(\mathbf{E}[r^a] - r^{\mathrm{T}}\Big) \\
&= f_0 u'(f(0))\Big(\mathbf{E}[r^a] - r^{\mathrm{T}}\Big)(1 + \rho g'(0))
\end{aligned}$$

and

$$\begin{aligned}
\frac{d\mathbf{E}[u_\rho(f(\pi))]}{d\pi}\bigg|_{\pi=1} &= \frac{d\mathbf{E}[u_0(f(\pi))]}{d\pi}\bigg|_{\pi=1} \\
&+ \int_{-1}^{r^{\mathrm{T}}}\rho f_0(r^a - r^{\mathrm{T}})u'(f(1))g'\Big(u(f(0)) - u(f(1))\Big)dF(r^a) \\
&+ \int_{r^{\mathrm{T}}}^{\infty}\rho f_0(r^a - r^{\mathrm{T}})u'(f(1))g'(0)dF(r^a) \\
&< \frac{d\mathbf{E}[u_0(f(\pi))]}{d\pi}\bigg|_{\pi=1} + \rho f_0 g'(0)\int_{r^{\mathrm{T}}}^{\infty}(r^a - r^{\mathrm{T}})u'(f(1))dF(r^a) \\
&= \mathbf{E}\Big[(r^a - r^{\mathrm{T}})u'(f(1))\Big] + \rho f_0 g'(0)\mathbf{E}\Big[(r^a - r^{\mathrm{T}})u'(f(1))\Big] \\
&= f_0\mathbf{E}\Big[(r^a - r^{\mathrm{T}})u'(f(1))\Big](1 + \rho g'(0)) \\
&= f_0\mathbf{E}\Big[(r^a - r^{\mathrm{T}})u'(f_0(1 + r^a))\Big](1 + \rho g'(0)),
\end{aligned}$$

respectively. As a result of this, if (2.2) holds, then

$$\frac{d\mathbf{E}[u_\rho(f(\pi))]}{d\pi}\bigg|_{\pi=0} > 0$$

for all $\rho > 0$. On the other hand, if (2.3) holds, i.e.,

$$\mathbf{E}[r^a] - r^{\mathrm{T}} = \frac{\operatorname{cov}\Big(- r^a, u'(f_0(1+r^a))\Big)}{\mathbf{E}\Big(u'(f_0(1+r^a))\Big)}$$

and taking into account that

$$r^{\mathrm{T}} = \mathbf{E}[r^a] - \frac{\operatorname{cov}\Big(- r^a, u'(f_0(1+r^a))\Big)}{\mathbf{E}\Big(u'(f_0(1+r^a))\Big)}$$

then

$$\begin{aligned}
\frac{d\mathbf{E}[u_\rho(f(\pi))]}{d\pi}\bigg|_{\pi=1} &< f_0\mathbf{E}\Big[(r^a - r^{\mathrm{T}})u'(f_0(1+r^a))\Big](1+\rho g'(0)) \\
&= f_0\mathbf{E}\Big[r^a u'(f_0(1+r^a)) - r^{\mathrm{T}}u'(f_0(1+r^a))\Big](1+\rho g'(0)) \\
&= f_0\Big[\mathbf{E}[r^a u'(f_0(1+r^a))] - r^{\mathrm{T}}\mathbf{E}[u'(f_0(1+r^a))]\Big](1+\rho g'(0)) \\
&= f_0\Big[\mathbf{E}[r^a u'(f_0(1+r^a))] - \mathbf{E}[r^a]\mathbf{E}[u'(f_0(1+r^a))] \\
&+ cov\Big(- r^a, u'(f_0(1+r^a))\Big)\Big](1+\rho g'(0)) \\
&= f_0\Big[cov\Big(r^a, u'(f_0(1+r^a))\Big) + cov\Big(- r^a, u'(f_0(1+r^a))\Big)\Big](1+\rho g'(0)) \\
&= f_0\Big[cov\Big(r^a, u'(f_0(1+r^a))\Big) - cov\Big(r^a, u'(f_0(1+r^a))\Big)\Big](1+\rho g'(0)) \\
&= 0.
\end{aligned}$$

Hence

$$\frac{d\mathbf{E}[u_\rho(f(\pi))]}{d\pi}\bigg|_{\pi=1} < 0$$

for all $\rho > 0$. This implies, in the former instance, that $\pi_\rho^* > 0$ for all $\rho > 0$ and $\pi_\rho^* < 1$ for all $\rho > 0$ in the second situation. □

2.2.2 Hedging Against Bank Risk

In the following proposition, we show that higher regret exacerbates the effect of the bank hedging its bets.

Proposition 2.2.3 (Hedging Against Bank Risk): *Suppose that the bank weights regret aversion more strongly than risk aversion (as measured by ρ). Then for* (2.2) *it invests more in the risky portfolio, whereas for* (2.3) *it invests less in the risky portfolio. This means that*

$$\frac{\partial \pi_\rho^*}{\partial \rho} \begin{cases} > 0, & \text{if (2.2) holds;} \\ < 0, & \text{if (2.3) holds.} \end{cases}$$

Proof. Taking the total differential of the first-order condition (2.9) with respect to π and ρ yields

$$d\left[\frac{d\mathbf{E}[u_\rho(f(\pi))]}{d\pi}\right]\Bigg|_{\pi=\pi_\rho^*} = \frac{\partial^2 \mathbf{E}[u_\rho(f(\pi))]}{\partial \pi^2}\Bigg|_{\pi=\pi_\rho^*} \cdot d\pi + \frac{\partial^2 \mathbf{E}[u_\rho(f(\pi))]}{\partial \pi \partial \rho}\Bigg|_{\pi=\pi_\rho^*} \cdot d\rho \;=\; 0.$$

In this case, we therefore have that

$$\frac{\partial \pi_\rho^*}{\partial \rho} = -\frac{\left.\dfrac{\partial^2 \mathbf{E}[u_\rho(f(\pi))]}{\partial \pi \partial \rho}\right|_{\pi=\pi_\rho^*}}{\left.\dfrac{\partial^2 \mathbf{E}[u_\rho(f(\pi))]}{\partial \pi^2}\right|_{\pi=\pi_\rho^*}}.$$

Since it is true that

$$\frac{\partial^2 \mathbf{E}[u_\rho(f(\pi))]}{\partial \pi^2}\Bigg|_{\pi=\pi_\rho^*} < 0$$

we may conclude that

$$\operatorname{sign}\left(\frac{\partial \pi_\rho^*}{\partial \rho}\right) = \operatorname{sign}\left(\frac{\partial^2 \mathbf{E}[u_\rho(f(\pi))]}{\partial \pi \partial \rho}\Bigg|_{\pi=\pi_\rho^*}\right).$$

We observe that the mixed partial derivative yields

$$
\begin{aligned}
\frac{\partial^2 \mathbf{E}[u_\rho(f(\pi))]}{\partial\pi\partial\rho}\bigg|_{\pi=\pi_\rho^*} &= \frac{\partial}{\partial\rho}\left[\frac{\partial \mathbf{E}[u_\rho(f(\pi))]}{\partial\pi}\right]\bigg|_{\pi=\pi_\rho^*} \\
&= \frac{\partial}{\partial\rho}\left[\mathbf{E}\left[f_0(r^a - r^{\mathrm{T}})u'(f(\pi))\left(1+\rho g'\left(u(f^{max}) - u(f(\pi))\right)\right)\right]\right]\bigg|_{\pi=\pi_\rho^*} \\
&= \mathbf{E}\left[\frac{\partial[f_0(r^a - r^{\mathrm{T}})u'(f(\pi))\left(1+\rho g'\left(u(f^{max}) - u(f(\pi))\right)\right)]}{\partial\rho}\right]\bigg|_{\pi=\pi_\rho^*} \\
&= \mathbf{E}\left[f_0(r^a - r^{\mathrm{T}})u'(f(\pi))g'\left(u(f^{max}) - u(f(\pi))\right)\right]\bigg|_{\pi=\pi_\rho^*} \\
&= \mathbf{E}\left[f_0(r^a - r^{\mathrm{T}})u'(f(\pi_\rho^*))g'\left(u(f^{\max}) - u(f(\pi_\rho^*))\right)\right].
\end{aligned}
$$

Furthermore, from the first-order condition (2.9), it follows that

$$
\begin{aligned}
\frac{d\mathbf{E}[u_\rho(f(\pi))]}{d\pi}\bigg|_{\pi=\pi_\rho^*} &= \mathbf{E}\left[f_0(r^a - r^{\mathrm{T}})u'(f(\pi))\left(u(f^{max}) - u(f(\pi))\right)\right]\bigg|_{\pi=\pi_\rho^*} \\
&= \mathbf{E}\left[f_0(r^a - r^{\mathrm{T}})u'(f(\pi)) + \rho f_0(r^a - r^{\mathrm{T}})u'(f(\pi))g'\left(u(f^{max}) - u(f(\pi))\right)\right]\bigg|_{\pi=\pi_\rho^*} \\
&= \mathbf{E}\left[f_0(r^a - r^{\mathrm{T}})u'(f(\pi))\right]\bigg|_{\pi=\pi_\rho^*} \\
&+ \mathbf{E}\left[\rho f_0(r^a - r^{\mathrm{T}})u'(f(\pi))g'\left(u(f^{max}) - u(f(\pi))\right)\right]\bigg|_{\pi=\pi_\rho^*} \\
&= \mathbf{E}\left[f_0(r^a - r^{\mathrm{T}})u'(f(\pi))\right]\bigg|_{\pi=\pi_\rho^*} \\
&+ \rho\mathbf{E}\left[f_0(r^a - r^{\mathrm{T}})u'(f(\pi))g'\left(u(f^{max}) - u(f(\pi))\right)\right]\bigg|_{\pi=\pi_\rho^*} \\
&= \frac{d\mathbf{E}[u_0(f(\pi))]}{d\pi}\bigg|_{\pi=\pi_\rho^*} + \rho\cdot\frac{\partial^2\mathbf{E}[u_\rho(f(\pi))]}{\partial\pi\partial\rho}\bigg|_{\pi=\pi_\rho^*}.
\end{aligned}
$$

Since we have that

$$
\frac{d\mathbf{E}[u_\rho(f(\pi))]}{d\pi}\bigg|_{\pi=\pi_\rho^*} = 0
$$

we can deduce that

$$\operatorname{sign}\left(\frac{\partial\pi^*_\rho}{\partial\rho}\right) = \operatorname{sign}\left(\left.\frac{\partial^2\mathbf{E}[u_\rho(f(\pi))]}{\partial\pi\partial\rho}\right|_{\pi=\pi^*_\rho}\right) = -\operatorname{sign}\left(\left.\frac{d\mathbf{E}[u_0(f(\pi))]}{d\pi}\right|_{\pi=\pi^*_\rho}\right). \qquad (2.11)$$

Our conclusion is that if (2.2) holds then $\pi^*_\rho > 0$ for all $\rho > 0$, and $\pi^*_0 = 0$ according to Theorem 2.2.2. This implies that

$$\left.\frac{d\mathbf{E}[u_0(f(\pi))]}{d\pi}\right|_{\pi=\pi^*_\rho>0} < 0$$

and, as a consequence, we have

$$\frac{\partial\pi^*_\rho}{\partial\rho} > 0$$

as suggested by (2.11).

If, on the other hand, (2.3) holds then $\pi^*_\rho < 1$ for all $\rho > 0$, and $\pi^*_0 = 1$ according to Theorem 2.2.2. By the method used in the above, this implies that

$$\left.\frac{d\mathbf{E}[u_0(f(\pi))]}{d\pi}\right|_{\pi=\pi^*_\rho<1} > 0$$

and thus

$$\frac{\partial\pi^*_\rho}{\partial\rho} < 0$$

by (2.11). □

2.2.3 Risk- and Regret-Averse Banks with Corresponding Asset Allocation

In the main result of this subsection, we show that there exists a Treasuries rate, $\tilde{r}^{\mathrm{T}}$, and therefore a level of $\mathbf{E}[r^a] - r^{\mathrm{T}}$, for which regret does not affect the optimal proportion invested in the aggregate risky assets, π^*. In essence, this means that at the specific $\mathbf{E}[r^a] - r^{\mathrm{T}}$, the asset allocation for a regret-averse bank will correspond to that of a risk-averse bank.

Proposition 2.2.4 (Asset Allocation of Risk- and Regret-Averse Banks): *There exists a Treasuries rate, $\widetilde{r}^{\mathrm{T}}$, such that*

$$0 < \mathbf{E}[r^a] - \widetilde{r}^{\mathrm{T}} < \frac{cov\Big(- r^a, u'(f_0(1+r^a))\Big)}{\mathbf{E}\Big[u'(f_0(1+r^a))\Big]}$$

and, for all $\rho > 0$, we have that $\pi^*_\rho = \pi^*_0$.

Proof. We have proved in Theorem 2.2.2, for any fixed $\rho > 0$, that

$$\begin{cases} \pi^*_\rho > 0 \text{ and } \pi^*_0 = 0, & \text{if (2.2) holds;} \\ \pi^*_\rho < 1 \text{ and } \pi^*_0 = 1, & \text{if (2.3) holds.} \end{cases}$$

Furthermore, the Intermediate Value Theorem suggests the existence of a Treasuries rate, $\widetilde{r}^{\mathrm{T}}$, with the property that

$$\mathbf{E}[r^a] > \widetilde{r}^{\mathrm{T}} > \mathbf{E}[r^a] - \frac{\mathrm{cov}\Big(- r^a, u'(f_0(1+r^a))\Big)}{\mathbf{E}\Big[u'(f_0(1+r^a))\Big]}$$

and $\pi^*_\rho = \pi^*_0$. The first-order derivative conditions

$$\frac{d\mathbf{E}[u(f(\pi))]}{d\pi}\bigg|_{\pi=\pi^*_0} = \mathbf{E}\Big[f_0(r^a - \widetilde{r}^{\mathrm{T}}(\rho))u'(f(\pi^*_0))\Big] = 0$$

and (2.9) at $\pi = \pi^*_\rho$, i.e.,

$$\frac{d\mathbf{E}[u_\rho(f(\pi))]}{d\pi}\bigg|_{\pi=\pi^*_\rho}$$
$$= \mathbf{E}\Big[f_0(r^a - \widetilde{r}^{\mathrm{T}}(\rho))u'(f(\pi^*_0))\Big(1 + \rho g'\Big(u(f^{\max}) - u(f(\pi^*_0))\Big)\Big)\Big] = 0.$$

It then follows that

$$\mathbf{E}\Big[f_0(r^a - \widetilde{r}^{\mathrm{T}}(\rho))u'(f(\pi^*_0))\Big] \;=\; \mathbf{E}\Big[f_0(r^a - \widetilde{r}^{\mathrm{T}}(\rho))u'(f(\pi^*_0))\Big(1 + g'\Big(u(f^{\max}) - u(f(\pi^*_0))\Big)\Big)\Big].$$

Assuming a continuous time environment, we write

$$\int_{-1}^{\infty} f_0(r^a - \widetilde{r}^{\mathrm{T}}(\rho))u'(f(\pi_0^*))dF(r^a)$$

$$= \int_{-1}^{\infty} f_0(r^a - \widetilde{r}^{\mathrm{T}}(\rho))u'(f(\pi_0^*))\Big(1 + \rho g'\Big(u(f^{max}) - u(f(\pi_0^*))\Big)\Big)dF(r^a).$$

The above expression holds if and only if,

$$f_0(r^a - \widetilde{r}^{\mathrm{T}}(\rho))u'(f(\pi_0^*)) \;=\; f_0(r^a - \widetilde{r}^{\mathrm{T}}(\rho))u'(f(\pi_0^*))\Big(1 + \rho g'\Big(u(f^{max}) - u(f(\pi_0^*))\Big)\Big),$$

which simply mean

$$f_0(r^a - \widetilde{r}^{\mathrm{T}}(\rho))u'(f(\pi_0^*))\rho g'\Big(u(f^{max}) - u(f(\pi_0^*))\Big) \;=\; 0.$$

Therefore,

$$r^a - \widetilde{r}^{\mathrm{T}}(\rho) \;=\; 0,$$

since $f_0 > 0$, $\rho > 0$, $u'(.) > 0$ and $g'(.) > 0$. Thus, we conclude that for all $\rho > 0$, we have $\widetilde{r}^{\mathrm{T}}(\rho) = r^a$. □

Chapter 3

SPECIAL CASE OF LOAN GUARANTEES

We look at the special case of loan guarantees, where

$$\alpha = 1; \ \beta = 0; \ r^{\Lambda} = r^{a}.$$

In this chapter, we examine how banks may value a loan guarantee by comparing the willingness-to-incur-costs (WTIC) for such a guarantee between a risk- and a regret-averse bank. As is the case in many situations, the WTIC is derived from an indifference relation between a bank portfolio with and without the guarantee, so that a measure of how much the bank values the guarantee can be established. As was noted before, for a loan with a guarantee the costs of monitoring and screening are generally higher than a loan with no guarantee.

3.1 RATES OF RETURN ON LOAN GUARANTEES

In the sequel, we suppose that $r^{\Lambda g} \geq -1$ is the guaranteed return on the loan with $r^{\Lambda g} = -1$ describing the situation where no guarantee is provided. As a consequence, the return on the loan contract, is given by

$$R^{\Lambda g} = \max(r^{\Lambda}, r^{\Lambda g}). \tag{3.1}$$

As was mentioned before, the guarantee does not alter the ex-post optimal level of funds, $f^{\max}$. In this regard, the ex-post optimal preference is for the bank to invest all its available funds in the loan, in the event that its realized return, r^{Λ}, is above the Treasuries rate, r^{T}, and all of it in the Treasury otherwise. Symbolically, this may be expressed as

$$f^{\max} = f_0\Big(1 + \max(r^{\Lambda}, r^{\Lambda g})\Big).$$

Let $c_\rho(r^{\Lambda g}, \overline{\pi})$ denote the maximum cost that the bank, with regret parameter $\rho \geq 0$, is willing to incur for the guaranteed return, $r^{\Lambda g}$, if its loan allocation were fixed at $\overline{\pi}$. In this case, the bank's WTIC is governed by the indifference equation

$$\begin{aligned} &\mathbf{E}\Big[u_\rho\Big(f_0\Big(1 + \overline{\pi} r^{\Lambda} + (1-\overline{\pi}) r^{\mathrm{T}}\Big)\Big)\Big] \\ = &\mathbf{E}\Big[u_\rho\Big(\Big(f_0 - c_\rho(r^{\Lambda g}, \overline{\pi})\Big)\Big(1 + \overline{\pi} R^{\Lambda g} + (1-\overline{\pi}) r^{\mathrm{T}}\Big)\Big)\Big]. \end{aligned} \tag{3.2}$$

In the case where no guarantee is provided, i.e., where $r^{\Lambda g} = -1$, the banks WTIC is zero. This means that

$$c_\rho(-1, \overline{\pi}) = 0, \quad \text{for all } 0 \leq \overline{\pi} \leq 1.$$

For $R^{\Lambda g}$ given by (3.1), if we put $R(r^{\Lambda g}, \overline{\pi}) = 1 + \overline{\pi} R^{\Lambda g} + (1-\overline{\pi}) r^{\mathrm{T}}$, then (3.2) can be rewritten as

$$\mathbf{E}\Big[u_\rho\Big(f_0 R(-1, \overline{\pi})\Big)\Big] = \mathbf{E}\Big[u_\rho\Big(\Big(f_0 - c_\rho(r^{\Lambda g}, \overline{\pi})\Big) R(r^{\Lambda g}, \overline{\pi})\Big)\Big]. \tag{3.3}$$

In addition, if all the banks funds were allocated to Treasuries, its WTIC for the loan guarantee is zero, so that

$$c_\rho(r^{\Lambda g}, 0) = 0, \quad \text{for all } -1 \leq r^{\Lambda g} \leq r^{\mathrm{T}}.$$

3.2 THE MAIN LOAN GUARANTEE RESULT

In the following theorem, if the proportion of available funds invested in the loan is low, then we have that a risk-averse bank values the guarantee more than the regret-averse bank. On the other hand, a risk-averse bank will find the guarantee less valuable than the regret-averse bank when the proportion of available funds invested in the loan is high and the level of guaranteed return is low.

Theorem 3.2.1 (Loan Guarantee Value for Risk- and Regret-Averse Banks): *We have that*

$$c_\rho(r^{\Lambda g}, \overline{\pi}) < c_0(r^{\Lambda g}, \overline{\pi}) \tag{3.4}$$

for low levels of $\overline{\pi}$ *and all* $r^{\Lambda g}$*. On the other hand, it is true that*

$$c_\rho(r^{\Lambda g}, \overline{\pi}) > c_0(r^{\Lambda g}, \overline{\pi}) \tag{3.5}$$

for high levels of $\overline{\pi}$ *and low levels of* $r^{\Lambda g}$*.*

Proof. The bank's willingness to incur costs (WTIC) is implicitly defined through the conditional indifference equation (3.3). The regret-averse bank is willing to incur less costs for the guarantee than the risk-averse bank, i.e., (3.4) holds for all $r^{\Lambda g}$, if and only if

$$\begin{aligned}\mathbf{E}\left[u\left(\left(f_0 - c_\rho(r^{\Lambda g}, \overline{\pi})\right)R(r^{\Lambda g}, \overline{\pi})\right)\right] &> \mathbf{E}\left[u\left(\left(f_0 - c_0(r^{\Lambda g}, \overline{\pi})\right)R(r^{\Lambda g}, \overline{\pi})\right)\right] \\ &= \mathbf{E}\left[u\left(f_0 R(-1, \overline{\pi})\right)\right]\end{aligned}$$

for all $r^{\Lambda g}$. Define the function $h : [0, 1] \to \mathbb{R}$ as

$$h(\overline{\pi}) = \mathbf{E}\Big[u\Big(\Big(f_0 - c_\rho(r^{\Lambda g}, \overline{\pi})\Big)R(r^{\Lambda g}, \overline{\pi})\Big)\Big] - \mathbf{E}\Big[u\Big(f_0 R(-1, \overline{\pi})\Big)\Big]. \tag{3.6}$$

A first observation is that for $\overline{\pi} = 0$, we have $h(0) = 0$. In order to prove that (3.4) holds for small $\overline{\pi}$ and all $r^{\Lambda g}$, we thus have to show that $h'(0) > 0$. Finding the derivative of h with respect to $\overline{\pi}$ yields

$$h'(\overline{\pi}) = \frac{d\mathbf{E}[u(f_0 - c_\rho(r^{\Lambda g}, \overline{\pi}))R(r^{\Lambda g}, \overline{\pi})]}{d\overline{\pi}} - \frac{d\mathbf{E}[u(f_0 R(-1, \overline{\pi})]}{d\overline{\pi}}$$

Recall that $R(r^{\Lambda g}, \overline{\pi}) = 1 + \overline{\pi} R^{\Lambda g} + (1 - \overline{\pi}) r^{\mathrm{T}}$ and $R^{\Lambda g} = \max(r^{\Lambda}, r^{\Lambda g})$. Then $R(-1, \overline{\pi}) = 1 + \overline{\pi} r^{\Lambda} + (1 - \overline{\pi}) r^{\mathrm{T}}$, since $R^{\Lambda g} = \max(r^{\Lambda}, -1) = r^{\Lambda}$. Furthermore, we have

$$\begin{aligned}
h'(\overline{\pi}) &= \mathbf{E}\Big[\frac{du((f_0 - c_\rho(r^{\Lambda g}, \overline{\pi}))R(r^{\Lambda g}, \overline{\pi}))}{d\overline{\pi}}\Big] - \mathbf{E}\Big[\frac{du(f_0 R(-1, \overline{\pi}))}{d\overline{\pi}}\Big] \\
&= \mathbf{E}\Big[u'((f_0 - c_\rho(r^{\Lambda g}, \overline{\pi}))R(r^{\Lambda g}, \overline{\pi}))\Big[- R(r^{\Lambda g}, \overline{\pi})\frac{\partial c_\rho(r^{\Lambda g}, \overline{\pi})}{\partial \overline{\pi}} + (f_0 - c_\rho(r^{\Lambda g}, \overline{\pi}))\frac{\partial R(r^{\Lambda g}, \overline{\pi})}{\partial \overline{\pi}}\Big]\Big] \\
&- \mathbf{E}\Big[f_0 u'(f_0 R(-1, \overline{\pi}))\frac{\partial R(-1, \overline{\pi})}{\partial \overline{\pi}}\Big] \\
&= \mathbf{E}\Big[u'((f_0 - c_\rho(r^{\Lambda g}, \overline{\pi}))R(r^{\Lambda g}, \overline{\pi}))\Big[- R(r^{\Lambda g}, \overline{\pi})\frac{\partial c_\rho(r^{\Lambda g}, \overline{\pi})}{\partial \overline{\pi}} + (f_0 - c_\rho(r^{\Lambda g}, \overline{\pi}))(R^{\Lambda g} - r^{\mathrm{T}})\Big]\Big] \\
&- \mathbf{E}\Big[f_0 u'(f_0 R(-1, \overline{\pi}))(r^{\Lambda} - r^{\mathrm{T}})\Big] \\
&= \mathbf{E}\Big[- u'((f_0 - c_\rho(r^{\Lambda g}, \overline{\pi}))R(r^{\Lambda g}, \overline{\pi}))R(r^{\Lambda g}, \overline{\pi})\frac{\partial c_\rho(r^{\Lambda g}, \overline{\pi})}{\partial \overline{\pi}} \\
&+ (f_0 - c_\rho(r^{\Lambda g}, \overline{\pi}))(R^{\Lambda g} - r^{\mathrm{T}})u'((f_0 - c_\rho(r^{\Lambda g}, \overline{\pi}))R(r^{\Lambda g}, \overline{\pi}))\Big] \\
&- \mathbf{E}\Big[f_0(r^{\Lambda} - r^{\mathrm{T}})u'(f_0 R(-1, \overline{\pi}))\Big]
\end{aligned}$$

$$
\begin{aligned}
&= \mathbf{E}\Big[-u^{'}((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi})\frac{\partial c_\rho(r^{\Lambda g},\overline{\pi})}{\partial \overline{\pi}}\Big] \\
&+ \mathbf{E}\Big[(f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))(R^{\Lambda g} - r^{\mathrm{T}})u^{'}((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi}))\Big] \\
&- \mathbf{E}\Big[f_0(r^{\Lambda} - r^{\mathrm{T}})u^{'}(f_0 R(-1,\overline{\pi}))\Big] \\
&= -\frac{\partial c_\rho(r^{\Lambda g},\overline{\pi})}{\partial \overline{\pi}}\mathbf{E}\Big[u^{'}((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi})\Big] \\
&+ \mathbf{E}\Big[(f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))(R^{\Lambda g} - r^{\mathrm{T}})u^{'}((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi}))\Big] \\
&- \mathbf{E}\Big[f_0(r^{\Lambda} - r^{\mathrm{T}})u^{'}(f_0 R(-1,\overline{\pi}))\Big].
\end{aligned}
$$

Evaluating $h^{'}(\overline{\pi})$ at $\overline{\pi} = 0$, we have

$$
\begin{aligned}
h^{'}(0) &= -\frac{\partial c_\rho(r^{\Lambda g},\overline{\pi})}{\partial \overline{\pi}}\Big|_{\overline{\pi}=0}\mathbf{E}\Big[u^{'}((f_0 - c_\rho(r^{\Lambda g},0))R(r^{\Lambda g},0))R(r^{\Lambda g},0)\Big] \\
&+ \mathbf{E}\Big[(f_0 - c_\rho(r^{\Lambda g},0))(R^{\Lambda g} - r^{\mathrm{T}})u^{'}((f_0 - c_\rho(r^{\Lambda g},0))R(r^{\Lambda g},0))\Big] \\
&- \mathbf{E}\Big[f_0(r^{\Lambda} - r^{\mathrm{T}})u^{'}(f_0 R(-1,0))\Big] \\
&= -\frac{\partial c_\rho(r^{\Lambda g},\overline{\pi})}{\partial \overline{\pi}}\Big|_{\overline{\pi}=0}\mathbf{E}\Big[(1 + r^{\mathrm{T}})u^{'}(f_0(1 + r^{\mathrm{T}}))\Big] \\
&+ \mathbf{E}\Big[f_0(R^{\Lambda g} - r^{\mathrm{T}})u^{'}(f_0(1 + r^{\mathrm{T}}))\Big] - \mathbf{E}\Big[f_0(r^{\Lambda} - r^{\mathrm{T}})u^{'}(f_0(1 + r^{\mathrm{T}}))\Big]
\end{aligned}
$$

$$= -\frac{\partial c_\rho(r^{\Lambda g},\overline{\pi})}{\partial \overline{\pi}}\bigg|_{\overline{\pi}=0}(1+r^{\mathrm{T}})u'(f_0(1+r^{\mathrm{T}}))$$

$$+ \; f_0u'(f_0(1+r^{\mathrm{T}}))\mathbf{E}[R^{\Lambda g}-r\mathrm{T}] - f_0u'(f_0(1+r^{\mathrm{T}}))\mathbf{E}[r^{\Lambda}-r^{\mathrm{T}}]$$

$$= -\frac{\partial c_\rho(r^{\Lambda g},\overline{\pi})}{\partial \overline{\pi}}\bigg|_{\overline{\pi}=0}(1+r^{\mathrm{T}})u'(f_0(1+r^{\mathrm{T}})) + f_0u'(f_0(1+r^{\mathrm{T}}))\mathbf{E}[(R^{\Lambda g}-r^{\mathrm{T}})-(r^{\Lambda}-r^{\mathrm{T}})]$$

$$= -\frac{\partial c_\rho(r^{\Lambda g},\overline{\pi})}{\partial \overline{\pi}}\bigg|_{\overline{\pi}=0}(1+r^{\mathrm{T}})u'(f_0(1+r^{\mathrm{T}})) + f_0u'(f_0(1+r^{\mathrm{T}}))\mathbf{E}[(R^{\Lambda g}-r^{\Lambda})],$$

that is,

$$h'(0) \;=\; u'\Big(f_0(1+r^{\mathrm{T}})\Big)\Big[-\frac{\partial c_\rho(r^{\Lambda g},\overline{\pi})}{\partial \overline{\pi}}\bigg|_{\overline{\pi}=0}(1+r^{\mathrm{T}}) + f_0\mathbf{E}[R^{\Lambda g}-r^{\Lambda}]\Big]. \tag{3.7}$$

If we differentiate (3.3) with respect to $\overline{\pi}$, we obtain

$$\frac{d\mathbf{E}\Big[u_\rho(f_0R(-1,\overline{\pi}))\Big]}{d\overline{\pi}} = \frac{d\mathbf{E}\Big[u_\rho((f_0-c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi}))\Big]}{d\overline{\pi}}.$$

But

$$\frac{d\mathbf{E}\Big[u_\rho(f_0R(-1,\overline{\pi}))\Big]}{d\overline{\pi}} = \frac{d\mathbf{E}\Big[u(f_0R(-1,\overline{\pi})) - \rho g'\Big(u(f^{max}-u(f_0R(-1,\overline{\pi}))\Big)\Big]}{d\overline{\pi}}$$

$$= \mathbf{E}\Big[\frac{du(f_0R(-1,\overline{\pi}))}{d\overline{\pi}} - \rho\frac{dg\Big(u(f^{max})-u(f_0R(-1,\overline{\pi}))\Big)}{d\overline{\pi}}\Big]$$

$$= \mathbf{E}\Big[f_0u'(f_0R(-1,\overline{\pi}))(r^{\Lambda}-r^{\mathrm{T}})$$

$$- \; \rho g'\Big(u(f^{max})-u(f_0R(-1,\overline{\pi}))\Big)\Big(-\frac{du(f_0R(-1,\overline{\pi}))}{d\overline{\pi}}\Big)\Big]$$

$$= \mathbf{E}\Big[f_0(r^{\Lambda}-r^{\mathrm{T}})u'(f_0R(-1,\overline{\pi}))$$

$$+ \; \rho g'\Big(u(f^{max})-u(f_0R(-1,\overline{\pi}))\Big)f_0(r^{\Lambda}-r^{\mathrm{T}})u'(f_0R(-1,\overline{\pi}))\Big]$$

$$= \mathbf{E}\Big[f_0(r^{\Lambda}-r^{\mathrm{T}})u'(f_0R(-1,\overline{\pi}))\Big[1+\rho g'\Big(u(f^{max})-u(f_0R(-1,\overline{\pi}))\Big)\Big]\Big]$$

and

$$\frac{d\mathbf{E}\Big[u_\rho((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi}))\Big]}{d\overline{\pi}} = \frac{d\mathbf{E}\Big[u((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi}))\Big]}{d\overline{\pi}} - \rho\frac{d\mathbf{E}\Big[g\Big(u(f^{max}) - u((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi}))\Big)\Big]}{d\overline{\pi}}. \tag{3.8}$$

Now, we have

$$\begin{aligned}
\frac{d\mathbf{E}\Big[u((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi}))\Big]}{d\overline{\pi}} &= \mathbf{E}\Big[\frac{du((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi}))}{d\overline{\pi}}\Big] \\
&= \mathbf{E}\Big[u^{'}((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi}))\Big(- R(r^{\Lambda g},\overline{\pi})\frac{\partial c_\rho(r^{\Lambda g},\overline{\pi})}{\partial\overline{\pi}} \\
&+ (f_0 - c_\rho(r^{\Lambda},\overline{\pi}))\frac{\partial R(r^{\Lambda g},\overline{\pi})}{\partial\overline{\pi}}\Big)\Big]
\end{aligned}$$

$$\begin{aligned}
&= \mathbf{E}\Big[u^{'}((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi}))\Big(- R(r^{\Lambda g},\overline{\pi})\frac{\partial c_\rho(r^{\Lambda g},\overline{\pi})}{\partial\overline{\pi}} \\
&+ (f_0 - c_\rho(r^{\Lambda},\overline{\pi}))(R^{\Lambda g} - r^{\mathrm{T}})\Big)\Big] \\
&= \mathbf{E}\Big[- u^{'}((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi})\frac{\partial c_\rho(r^{\Lambda g},\overline{\pi})}{\partial\overline{\pi}} \\
&+ (f_0 - c_\rho(r^{\Lambda},\overline{\pi}))(R^{\Lambda g} - r^{\mathrm{T}})u^{'}((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi}))\Big)\Big] \\
&= -\frac{\partial c_\rho(r^{\Lambda g},\overline{\pi})}{\partial\overline{\pi}}\mathbf{E}\Big[R(r^{\Lambda g},\overline{\pi})u^{'}((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi}))\Big] \\
&+ \mathbf{E}\Big[(f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))(R^{\Lambda g} - r^{\mathrm{T}})u^{'}((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi}))\Big]
\end{aligned}$$

and

$$\frac{d\mathbf{E}\Big[g\Big(u(f^{max}) - u((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi}))\Big)\Big]}{d\overline{\pi}}$$

$$= \mathbf{E}\Big[\frac{dg\Big(u(f^{max}) - u((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi}))\Big)}{d\overline{\pi}}\Big]$$

$$= \mathbf{E}\Big[g'\Big(u(f^{max}) - u((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi}))\Big)\Big(-\frac{du((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi}))}{d\overline{\pi}}\Big)\Big]$$

$$= \mathbf{E}\Big[-g'\Big(u(f^{max}) - u((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi}))\Big)u'((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi}))$$

$$\times\Big(-R(r^{\Lambda g},\overline{\pi})\frac{\partial c_\rho(r^{\Lambda g},\overline{\pi})}{\partial\overline{\pi}} + (f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))(R^{\Lambda g} - r^{\mathrm{T}})\Big)\Big]$$

$$= \mathbf{E}\Big[g'\Big(u(f^{max}) - u((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi}))\Big)u'((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi})$$

$$\times\frac{\partial c_\rho(r^{\Lambda g},\overline{\pi})}{\partial\overline{\pi}} - g'\Big(u(f^{max}) - u((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi}))\Big)$$

$$\times u'((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi}))(f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))(R^{\Lambda} - r^{\mathrm{T}})\Big]$$

$$= \frac{\partial c_\rho(r^{\Lambda g},\overline{\pi})}{\partial\overline{\pi}}\mathbf{E}\Big[g'\Big(u(f^{max}) - u((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi}))\Big)$$

$$\times u'((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi})\Big]$$

$$- \mathbf{E}\Big[g'\Big(u(f^{max}) - u((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi}))\Big)u'((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi}))$$

$$\times(f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))(R^{\Lambda g} - r^{\mathrm{T}})\Big].$$

From (3.8), we see that

$$\frac{d\mathbf{E}\Big[u_\rho((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi}))\Big]}{d\overline{\pi}}$$

$$= -\frac{\partial c_\rho(r^{\Lambda g},\overline{\pi})}{\partial \overline{\pi}}\mathbf{E}\Big[R(r^{\Lambda g},\overline{\pi})u'((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi}))\Big]$$

$$+ \quad \mathbf{E}\Big[(f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))(R^{\Lambda g} - r^{\mathrm{T}})u'((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi}))\Big]$$

$$- \quad \rho\frac{\partial c_\rho(r^{\Lambda g},\overline{\pi})}{\partial \overline{\pi}}\mathbf{E}\Big[g'\Big(u(f^{max}) - u((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi}))\Big)u'((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi}))\Big]$$

$$+ \quad \rho\mathbf{E}\Big[g'\Big(u(f^{max}) - u((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi}))\Big)u'((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi}))$$

$$\times (f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))(R^{\Lambda g} - r^{\mathrm{T}})\Big]$$

$$= \quad -\frac{\partial c_\rho(r^{\Lambda g},\overline{\pi})}{\partial \overline{\pi}}\Big[\mathbf{E}\Big[R(r^{\Lambda g},\overline{\pi})u'((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi}))\Big]$$

$$+ \quad \rho\mathbf{E}\Big[g'\Big(u(f^{max}) - u((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi}))\Big)u'((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi}))\Big]\Big]$$

$$+ \quad \mathbf{E}\Big[(f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))(R^{\Lambda g} - r^{\mathrm{T}})u'((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi}))\Big]$$

$$+ \quad \mathbf{E}\Big[\rho g'\Big(u(f^{max}) - u((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi}))\Big)u'((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi}))$$

$$\times (f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))(R^{\Lambda g} - r^{\mathrm{T}})\Big]$$

$$\begin{aligned}
= &\quad -\tfrac{\partial c_\rho(r^{\Lambda g},\overline{\pi})}{\partial \overline{\pi}}\mathbf{E}\Big[R(r^{\Lambda g},\overline{\pi})u^{'}((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi}))\Big(1 \\
+ &\quad \rho g^{'}\Big(u(f^{max}) - u((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi}))\Big)\Big)\Big] \\
+ &\quad \mathbf{E}\Big[(f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))(R^{\Lambda g} - r^{\mathrm{T}})u^{'}((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi}))\Big(1 \\
+ &\quad \rho g^{'}\Big(u(f^{max}) - u((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi}))\Big)\Big)\Big].
\end{aligned}$$

It then follows that

$$\begin{aligned}
&\mathbf{E}\Big[f_0(r^{\Lambda} - r^{\mathrm{T}})u^{'}\Big(f_0R(-1,\overline{\pi})\Big) \\
&\quad\times\Big(1 + \rho g^{'}\Big(u(f^{\max}) - u\Big(f_0R(-1,\overline{\pi})\Big)\Big)\Big)\Big] \\
&= -\frac{\partial c_\rho(r^{\Lambda g},\overline{\pi})}{\partial \overline{\pi}}\mathbf{E}\Big[R(r^{\Lambda g},\overline{\pi})u^{'}\Big(\Big(f_0 - c_\rho(r^{\Lambda g},\overline{\pi})\Big)R(r^{\Lambda g},\overline{\pi})\Big) \\
&\quad\times\Big(1 + \rho g^{'}\Big(u(f^{\max}) - u\Big(\Big(f_0 - c_\rho(r^{\Lambda g},\overline{\pi})\Big)R(r^{\Lambda g},\overline{\pi})\Big)\Big)\Big)\Big] \\
&+\mathbf{E}\Big[\Big(f_0 - c_\rho(r^{\Lambda g},\overline{\pi})\Big(R^{\Lambda g} - r^{\mathrm{T}}\Big)u^{'}\Big(\Big(f_0 - c_\rho(r^{\Lambda g},\overline{\pi})\Big)R(r^{\Lambda g},\overline{\pi})\Big)\Big) \\
&\quad\times\Big(1 + \rho g^{'}\Big(u(f^{\max}) - u\Big(\Big(f_0 - c_\rho(r^{\Lambda g},\overline{\pi})\Big)R(r^{\Lambda g},\overline{\pi})\Big)\Big)\Big)\Big].
\end{aligned}$$

If we set $\overline{\pi} = 0$, it follows that

$$\begin{aligned}
&\mathbf{E}\Big[f_0(r^{\Lambda} - r^{\mathrm{T}})u^{'}(R(-1,0))\Big(1 + \rho g^{'}\Big(u(f^{max}) - u(f_0R(-1,0))\Big)\Big)\Big] \\
&\quad = -\frac{\partial c_\rho(r^{\Lambda g},\overline{\pi})}{\partial \overline{\pi}}\bigg|_{\overline{\pi}=0}\mathbf{E}\Big[R(r^{\Lambda g},0)u^{'}((f_0 - c_\rho(r^{\Lambda g},0))R(r^{\Lambda g},0)) \\
&\qquad\times\Big(1 + \rho g^{'}\Big(u(f^{max}) - u((f_0 - c_\rho(r^{\Lambda g},0))R(r^{\Lambda g},0))\Big)\Big)\Big] \\
&\quad +\mathbf{E}\Big[(f_0 - c_\rho(r^{\Lambda g},0))(R^{\Lambda g} - r^{\mathrm{T}})u^{'}((f_0 - c_\rho(r^{\Lambda g},0))R(r^{\Lambda g},0)) \\
&\qquad\times\Big(1 + \rho g^{'}\Big(u(f^{max}) - u((f_0 - c_\rho(r^{\Lambda g},0))R(r^{\Lambda g},0))\Big)\Big)\Big]
\end{aligned}$$

$$
\begin{aligned}
&\mathbf{E}\Big[f_0(r^{\Lambda}-r^{\mathrm{T}})u'(f_0(1+r^{\mathrm{T}}))\Big(1+\rho g'\Big(u(f^{max})-u(f_0(1+r^{\mathrm{T}}))\Big)\Big)\Big] \\
&= -\frac{\partial c_\rho(r^{\Lambda g},\overline{\pi})}{\partial \overline{\pi}}\bigg|_{\overline{\pi}=0}\mathbf{E}\Big[(1+r^{\mathrm{T}})u'(f_0(1+r^{\mathrm{T}}))\Big(1+\rho g'\Big(u(f^{max})-u(f_0(1+r^{\mathrm{T}}))\Big)\Big)\Big] \\
&+ \quad \mathbf{E}\Big[f_0(R^{\Lambda g}-r^{\mathrm{T}})u'(f_0(1+r^{\mathrm{T}}))\Big(1+\rho g'\Big(u(f^{max})-u(f_0(1+r^{\mathrm{T}}))\Big)\Big)\Big],
\end{aligned}
$$

that is,

$$
\begin{aligned}
&f_0u'\Big(f_0(1+r^{\mathrm{T}})\Big)\mathbf{E}\Big[(r^{\Lambda}-r^{\mathrm{T}})\Big(1+\rho g'\Big(u(f^{\max})-u\Big(f_0(1+r^{\mathrm{T}})\Big)\Big)\Big)\Big] \\
&= -\frac{\partial c_\rho(r^{\Lambda g},\overline{\pi})}{\partial \overline{\pi}}\bigg|_{\overline{\pi}=0}(1+r^{\mathrm{T}})u'\Big(f_0(1+r^{\mathrm{T}})\Big)\mathbf{E}\Big[\Big(1+\rho g'\Big(u(f^{\max})-u\Big(f_0(1+r^{\mathrm{T}})\Big)\Big)\Big)\Big] \\
&\quad +f_0u'\Big(f_0(1+r^{\mathrm{T}})\Big)\mathbf{E}\Big[(R^{\Lambda g}-r^{\mathrm{T}})\Big(1+\rho g'\Big(u(f^{\max})-u\Big(f_0(1+r^{\mathrm{T}})\Big)\Big)\Big)\Big]
\end{aligned}
$$

which, in turn, implies that

$$
\begin{aligned}
&\frac{\partial c_\rho(r^{\Lambda g},\overline{\pi})}{\partial \overline{\pi}}\bigg|_{\overline{\pi}=0} \qquad (3.9)\\
&= \frac{f_0\mathbf{E}\Big[(R^{\Lambda g}-r^{\Lambda})\Big(1+\rho g'\Big(u(f^{\max})-u\Big(f_0(1+r^{\mathrm{T}})\Big)\Big)\Big)\Big]}{(1+r^{\mathrm{T}})\mathbf{E}\Big[1+\rho g'\Big(u(f^{\max})-u\Big(f_0(1+r^{\mathrm{T}})\Big)\Big)\Big]}.
\end{aligned}
$$

If we substitute (3.9) into (3.7), we may conclude that

$$h'(0) = f_0 u'\Big(f_0(1+r^{\mathrm{T}})\Big)$$

$$\times\Bigg(- \frac{(1+r^{\mathrm{T}})\mathbf{E}\Big[(R^{\Lambda g} - r^{\Lambda})\Big(1+\rho g'\Big(u(f^{\max}) - u\Big(f_0(1+r^{\mathrm{T}})\Big)\Big)\Big)\Big]}{(1+r^{\mathrm{T}})\mathbf{E}\Big[1+\rho g'\Big(u(f^{\max}) - u\Big(f_0(1+r^{\mathrm{T}})\Big)\Big)\Big]} + \mathbf{E}[R^{\Lambda g} - r^{\Lambda}]\Bigg)$$

$$= f_0 u'\Big(f_0(1+r^{\mathrm{T}})\Big)$$

$$\times\Bigg(- \frac{\mathbf{E}\Big[(R^{\Lambda g} - r^{\Lambda})\Big(1+\rho g'(u(f^m ax) - u(f_0(1+r^{\mathrm{T}})))\Big)\Big]}{\mathbf{E}\Big[1+\rho g'\Big(u(f^{max}) - u(f_0(1+r^{\mathrm{T}}))\Big)\Big]} + \mathbf{E}\Big[R^{\Lambda g} - r^{\Lambda}\Big]\Bigg)$$

$$= - \frac{f_0 u'(f_0(1+r^{\mathrm{T}}))\rho}{\mathbf{E}\Big[1+\rho g'\Big(u(f^{\max}) - u\Big(f_0(1+r^{\mathrm{T}})\Big)\Big)\Big]} \cdot \operatorname{cov}\Big(R^{\Lambda g} - r^{\Lambda}, g'\Big(u(f^{\max}) - u\Big(f_0(1+r^{\mathrm{T}})\Big)\Big)\Big).$$

Also, we observe that

$$\operatorname{cov}\Big(R^{\Lambda g} - r^{\Lambda}, g'\Big(u(f^{\max}) - u\Big(f_0(1+r^{\mathrm{T}})\Big)\Big)\Big)$$

$$= \operatorname{cov}\Big(R^{\Lambda g} - r^{\Lambda}, g'\Big(u(f_0(1+\max(r^{\Lambda}, r^{\mathrm{T}}))) - u\Big(f_0(1+r^{\mathrm{T}})\Big)\Big)\Big) < 0.$$

In this case, we may conclude that $h'(0) > 0$, which implies $h(\overline{\pi}) > 0$ for small $\overline{\pi}$ since $h(0) = 0$. From this we can deduce that (3.4) holds for low levels of $\overline{\pi}$ and all $r^{\Lambda g}$.

Next, we would like to show that (3.5) holds for high levels of $\overline{\pi}$ and small $r^{\Lambda g}$. This inequality holds if and only if $h(\overline{\pi}) < 0$ for $\overline{\pi}$ and small $r^{\Lambda g}$ (see (3.6) for the definition of $h(\cdot)$). A first observation is that

$$h(1) = \mathbf{E}\Big[u\Big(\Big(f_0 - c_\rho(r^{\Lambda g}, 1)\Big)(1+R^{\Lambda g})\Big)\Big] - \mathbf{E}\Big[u(f_0(1+r^{\Lambda}))\Big].$$

At $r^{\Lambda g} = -1$, we have that

$$\begin{aligned} h(1)|_{r^{\Lambda g}=-1} &= \mathbf{E}\Big[u((f_0 - c_\rho(-1,1))(1+r^{\Lambda}))\Big] - \mathbf{E}\Big[u(f_0(1+r^{\mathrm{T}}))\Big] \\ &= \mathbf{E}\Big[u(f_0(1+r^{\Lambda}))\Big] - \mathbf{E}\Big[u(f_0(1+r^{\Lambda}))\Big] \\ &= 0. \end{aligned}$$

If we differentiate $h(1)$ with respect to $r^{\Lambda g}$, we obtain

$$\begin{aligned} \frac{\partial h(1)}{\partial r^{\Lambda g}} &= \mathbf{E}\left[\frac{\partial u((f_0 - c_\rho(r^{\Lambda g},1))R(r^{\Lambda g},1))}{\partial r^{\Lambda g}}\right] - \mathbf{E}\left[\frac{\partial u(f_0 R(-1,1))}{\partial r^{\Lambda g}}\right] \\ &= \mathbf{E}\Big[u'((f_0 - c_\rho(r^{\Lambda g},1))R(r^{\Lambda g},1))\Big(-R(r^{\Lambda g},1)\frac{\partial c_\rho(r^{\Lambda g},\overline{\pi})}{\partial r^{\Lambda g}}\Big|_{\overline{\pi}=1} \\ &+ (f_0 - c_\rho(r^{\Lambda g},1))\frac{\partial R(r^{\Lambda g},1)}{\partial r^{\Lambda g}}\Big)\Big] \\ &= \mathbf{E}\Big[-u'((f_0 - c_\rho(r^{\Lambda g},1))(1+R^{\Lambda g}))(1+R^{\Lambda g})\frac{\partial c_\rho(r^{\Lambda g},\overline{\pi})}{\partial r^{\Lambda g}}\Big|_{\overline{\pi}=1} \\ &+ (f_0 - c_\rho(r^{\Lambda g},1))u'((f_0 - c_\rho(r^{\Lambda g},1))(1+R^{\Lambda g}))\frac{\partial(1+R^{\Lambda g})}{\partial r^{\Lambda g}}\Big] \\ &= -\frac{\partial c_\rho(r^{\Lambda g},\overline{\pi})}{\partial r^{\Lambda g}}\Big|_{\overline{\pi}=1}\mathbf{E}\Big[(1+R^{\Lambda g})u'\Big((f_0 - c_\rho(r^{\Lambda g},1))(1+R^{\Lambda g})\Big)\Big] \\ &+ \mathbf{E}\Big[\frac{\partial R^{\Lambda g}}{\partial r^{\Lambda g}}(f_0 - c_\rho(r^{\Lambda g},1))u'\Big((f_0 - c_\rho(r^{\Lambda g},1))(1+R^{\Lambda g})\Big)\Big] \\ &= -\frac{\partial c_\rho(r^{\Lambda g},\overline{\pi})}{\partial r^{\Lambda g}}\Big|_{\overline{\pi}=1}\mathbf{E}\Big[(1+R^{\Lambda g})u'\Big((f_0 - c_\rho(r^{\Lambda g},1))(1+R^{\Lambda g})\Big)\Big] \\ &+ (1+r^{\Lambda g})(f_0 - c_\rho(r^{\Lambda g},1))u'\Big((f_0 - c_\rho(r^{\Lambda g},1))(1+r^{\Lambda g})\Big). \end{aligned}$$

Determining the value at $r^{\Lambda g} = -1$ yields

$$\frac{\partial h(1)}{\partial r^{\Lambda g}} = -\frac{\partial c_\rho(r^{\Lambda g},\overline{\pi})}{\partial r^{\Lambda g}}\Big|_{\overline{\pi}=1,\ r^{\Lambda g}=-1}\mathbf{E}\Big[(1+r^{\Lambda})u'(f_0(1+r^{\Lambda}))\Big].$$

Furthermore, if we differentiate (3.3) with respect to $r^{\Lambda g}$, it follows that

$$\frac{\partial \mathbf{E}\Big[u_\rho((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi}))\Big]}{\partial r^{\Lambda g}} = 0$$

$$\frac{\partial \mathbf{E}\Big[u((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi})) - \rho g\Big(u(f^{max}) - u((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi}))\Big)\Big]}{\partial r^{\Lambda g}} = 0$$

$$\mathbf{E}\Big[u^{'}((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi}))\Big(- R(r^{\Lambda g},\overline{\pi})\frac{\partial c_\rho(r^{\Lambda g},\overline{\pi})}{\partial r^{\Lambda g}} + (f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))\overline{\pi}\frac{\partial R^{\Lambda g}}{\partial r^{\Lambda g}}\Big)$$

$$+\rho g^{'}\Big(u(f^{max}) - u((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi}))\Big)$$

$$\times u^{'}((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi}))\Big(- R(r^{\Lambda g},\overline{\pi})\frac{\partial c_\rho(r^{\Lambda g},\overline{\pi})}{\partial r^{\Lambda g}} + (f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))\overline{\pi}\frac{\partial R^{\Lambda g}}{\partial r^{\Lambda g}}\Big)\Big] = 0$$

$$\mathbf{E}\Big[- R(r^{\Lambda g},\overline{\pi})u^{'}((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi}))\frac{\partial c_\rho(r^{\Lambda g},\overline{\pi})}{\partial r^{\Lambda g}}$$

$$\times\Big(1 + \rho g^{'}\Big(u(f^{max}) - u((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi}))\Big)\Big)$$

$$+(f_0 - c_\rho(r^{\Lambda g},\overline{)})\overline{\pi}u^{'}((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi}))\frac{\partial R^{\Lambda g}}{\partial r^{\Lambda g}}$$

$$\times\Big(1 + \rho g^{'}\Big(u(f^{max}) - u((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi}))\Big)\Big)\Big] = 0,$$

therefore,

$$-\frac{\partial c_\rho(r^{\Lambda g},\overline{\pi})}{\partial r^{\Lambda g}}\mathbf{E}\Big[R(r^{\Lambda g},\overline{\pi})u^{'}\Big((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi})\Big)$$
$$\times\Big(1 + \rho g^{'}\Big(u(f^{\max}) - u\Big((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi})\Big)\Big)\Big)\Big] \tag{3.10}$$
$$+\mathbf{E}\Big[(f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))\overline{\pi}\frac{\partial R^{\Lambda g}}{\partial r^{\Lambda g}}u^{'}(f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi})\Big)$$
$$\times\Big(1 + \rho g^{'}\Big(u(f^{\max}) - u\Big((f_0 - c_\rho(r^{\Lambda g},\overline{\pi}))R(r^{\Lambda g},\overline{\pi})\Big)\Big)\Big)\Big] = 0.$$

For $\overline{\pi} = 1$, we obtain

$$-\frac{\partial c_\rho(r^{\Lambda g}, \overline{\pi})}{\partial r^{\Lambda g}}\bigg|_{\overline{\pi}=1} \mathbf{E}\bigg[R(r^{\Lambda g}, 1)u^{'}((f_0 - c_\rho(r^{\Lambda g}, 1))R(r^{\Lambda g}, 1))$$

$$\times\bigg(1 + \rho g^{'}\bigg(u(f^{max}) - u((f_0 - c_\rho(r^{\Lambda g}, 1))R(r^{\Lambda g}, 1))\bigg)\bigg)\bigg]$$

$$+\mathbf{E}\bigg[(f_0 - c_\rho(r^{\Lambda g}, 1))u^{'}((f_0 - c_\rho(r^{\Lambda g}, 1))R(r^{\Lambda g}, 1))\frac{\partial R^{\Lambda g}}{\partial r^{\Lambda g}}$$

$$\times\bigg(1 + \rho g^{'}\bigg(u(f^{max}) - u((f_0 - c_\rho(r^{\Lambda g}, 1))R(r^{\Lambda g}, 1))\bigg)\bigg)\bigg] = 0$$

$$-\frac{\partial c_\rho(r^{\Lambda g}, \overline{\pi})}{\partial r^{\Lambda g}}\bigg|_{\overline{\pi}=1} \mathbf{E}\bigg[(1 + R^{\Lambda g})u^{'}((f_0 - c_\rho(r^{\Lambda g}, 1))(1 + R^{\Lambda g}))$$

$$\times\bigg(1 + \rho g^{'}\bigg(u(f^{max}) - u((f_0 - c_\rho(r^{\Lambda g}, 1))(1 + R^{\Lambda g}))\bigg)\bigg)\bigg]$$

$$\mathbf{E}\bigg[(f_0 - c_\rho(r^{\Lambda g}, 1))u^{'}((f_0 - c_\rho(r^{\Lambda g}, 1))(1 + R^{\Lambda g}))\frac{\partial R^{\Lambda g}}{\partial r^{\Lambda g}}$$

$$\times\bigg(1 + \rho g^{'}\bigg(u(f^{max}) - u((f_0 - c_\rho(r^{\Lambda g}, 1))(1 + R^{\Lambda g}))\bigg)\bigg)\bigg] = 0.$$

Now, if we simplify the above expression further, we get

$$-\frac{\partial c_\rho(r^{\Lambda g}, \overline{\pi})}{\partial r^{\Lambda g}}\bigg|_{\overline{\pi}=1} \mathbf{E}\bigg[(1 + R^{\Lambda g})u^{'}\bigg((f_0 - c_\rho(r^{\Lambda g}, 1))(1 + R^{\Lambda g})\bigg) \tag{3.11}$$

$$\times\bigg(1 + \rho g^{'}\bigg(u(f^{\max}) - u\bigg((f_0 - c_\rho(r^{\Lambda g}, 1))(1 + R^{\Lambda g})\bigg)\bigg)\bigg)\bigg]$$

$$+(1 + r^{\Lambda g})(f_0 - c_\rho(r^{\Lambda g}, 1))u^{'}\bigg((f_0 - c_\rho(r^{\Lambda g}, 1))(1 + r^{\Lambda g})\bigg)$$

$$\times\bigg(1 + \rho g^{'}\bigg(u(f_0(1 + r^{\mathrm{T}})) - u\bigg((f_0 - c_\rho(r^{\Lambda g}, 1))(1 + r^{\Lambda g})\bigg)\bigg)\bigg) = 0.$$

Evaluating at $r^{\Lambda g} = -1$ implies

$$\frac{\partial c_\rho(r^{\Lambda g}, \overline{\pi})}{\partial r^{\Lambda g}}\bigg|_{\overline{\pi}=1,\ r^{\Lambda g}=-1} = 0$$

and thus

$$\frac{\partial h(1)}{\partial r^{\Lambda g}}\bigg|_{r^{\Lambda g}=-1} = 0.$$

If we differentiate again we obtain

$$\begin{aligned}
\frac{\partial^2 h(1)}{\partial (r^{\Lambda g})^2} = & -\frac{\partial^2 c_\rho(r^{\Lambda g},\overline{\pi})}{\partial (r^{\Lambda g})^2}\bigg|_{\overline{\pi}=1} \mathbf{E}\Big[(1+R^{\Lambda g})u^{'}\Big((f_0 - c_\rho(r^{\Lambda g},1))(1+R^{\Lambda g})\Big)\Big] \\
& -\frac{\partial c_\rho(r^{\Lambda g},\overline{\pi})}{\partial r^{\Lambda g}}\bigg|_{\overline{\pi}=1} \mathbf{E}\Big[\frac{\partial R^{\Lambda g}}{\partial r^{\Lambda g}}u^{'}\Big((f_0 - c_\rho(r^{\Lambda g},1))(1+R^{\Lambda g})\Big)\Big] \\
& +\Big(\frac{\partial c_\rho(r^{\Lambda g},\overline{\pi})}{\partial r^{\Lambda g}}\bigg|_{\overline{\pi}=1}\Big)^2 \mathbf{E}\Big[(1+R^{\Lambda g})^2 u^{''}\Big((f_0 - c_\rho(r^{\Lambda g},1))(1+R^{\Lambda g})\Big)\Big] \\
& -\frac{\partial c_\rho(r^{\Lambda g},\overline{\pi})}{\partial r^{\Lambda g}}\bigg|_{\overline{\pi}=1}(f_0 - c_\rho(r^{\Lambda g},1)) \\
& \times\mathbf{E}\Big[(1+R^{\Lambda g})\frac{\partial R^{\Lambda g}}{\partial r^{\Lambda g}}u^{''}\Big((f_0 - c_\rho(r^{\Lambda g},1))(1+R^{\Lambda g})\Big)\Big] \\
& +(f_0 - c_\rho(r^{\Lambda g},1))u^{'}\Big((f_0 - c_\rho(r^{\Lambda g},1))(1+r^{\Lambda g})\Big) \\
& -(1+r^{\Lambda g})\frac{\partial c_\rho(r^{\Lambda g},\overline{\pi})}{\partial r^{\Lambda g}}\bigg|_{\overline{\pi}=1}u^{'}\Big((f_0 - c_\rho(r^{\Lambda g},1))(1+r^{\Lambda g})\Big) \\
& -(1+r^{\Lambda g})^2(f_0 - c_\rho(r^{\Lambda g},1))\frac{\partial c_\rho(r^{\Lambda g},\overline{\pi})}{\partial r^{\Lambda g}}\bigg|_{\overline{\pi}=1} \\
& \times u^{''}\Big((f_0 - c_\rho(r^{\Lambda g},1))(1+r^{\Lambda g})\Big) \\
& +(1+r^{\Lambda g})(f_0 - c_\rho(r^{\Lambda g},1))^2 u^{''}\Big((f_0 - c_\rho(r^{\Lambda g},1))(1+r^{\Lambda g})\Big).
\end{aligned}$$

At $r^{\Lambda g} = -1$, we have

$$\frac{\partial c_\rho(r^{\Lambda g},\overline{\pi})}{\partial r^{\Lambda g}}\bigg|_{\overline{\pi}=1} = 0$$

and thus

$$\frac{\partial^2 h(1)}{\partial (r^{\Lambda g})^2}\bigg|_{r^{\Lambda g}=-1} = -\frac{\partial^2 c_\rho(r^{\Lambda g},\overline{\pi})}{\partial (r^{\Lambda g})^2}\bigg|_{\overline{\pi}=1,\ r^{\Lambda g}=-1} \mathbf{E}\Big[(1+r^{\Lambda})u^{'}\Big(f_0(1+r^{\Lambda})\Big)\Big] + f_0 u^{'}(0).$$

If we differentiate (3.11) with respect to $r^{\Lambda g}$ and determine a value at $r^{\Lambda g} = -1$,then it follows that

$$-\frac{\partial^2 c_\rho(r^{\Lambda g},\overline{\pi})}{\partial(r^{\Lambda g})^2}\Big|_{\overline{\pi}=1,\ r^{\Lambda g}=-1}$$

$$\times\mathbf{E}\Big[(1+r^{\Lambda})u'\Big(f_0(1+r^{\Lambda})\Big)\Big(1+\rho g'\Big(u(f^{\max})-u\Big(f_0(1+r^{\Lambda})\Big)\Big)\Big)\Big]$$

$$+f_0u'(0)\Big(1+\rho g'\Big(u(f_0(1+r^{\mathrm{T}}))-u(0)\Big)\Big)=0.$$

From this it follows that

$$\frac{\partial^2 c_\rho(r^{\Lambda g},\overline{\pi})}{\partial(r^{\Lambda g})^2}\Big|_{\overline{\pi}=1,\ r^{\Lambda g}=-1}$$

$$=\frac{f_0u'(0)\Big(1+\rho g'\Big(u(f_0(1+r^{\mathrm{T}}))-u(0)\Big)\Big)}{\mathbf{E}\Big[(1+r^{\Lambda})u'(f_0(1+r^{\Lambda}))\Big(1+\rho g'\Big(u(f^{\max})-u\Big(f_0(1+r^{\Lambda})\Big)\Big)\Big)\Big]}.$$

We may also conclude that

$$\frac{\partial^2 h(1)}{\partial(r^{\Lambda g})^2}\Big|_{r^{\Lambda g}=-1}$$

$$=-\frac{f_0u'(0)\Big(1+\rho g'\Big(u(f_0(1+r^{\mathrm{T}}))-u(0)\Big)\Big)}{\mathbf{E}\Big[(1+r^{\Lambda})u'(f_0(1+r^{\Lambda}))\Big(1+\rho g'\Big(u(f^{\max})-u\Big(f_0(1+r^{\Lambda})\Big)\Big)\Big)\Big]}$$

$$\times\mathbf{E}\Big[(1+r^{\Lambda})u'(f_0(1+r^{\Lambda}))\Big]+f_0u'(0)$$

$$=\frac{\rho f_0u'(0)\mathbf{E}\Big[(1+r^{\Lambda})u'(f_0(1+r^{\Lambda}))\Big(g'\Big(u(f^{\max})-u(f_0(1+r^{\Lambda}))\Big)-g'\Big(u(f_0(1+r^{\mathrm{T}}))-u(0)\Big)\Big)\Big]}{\mathbf{E}\Big[(1+r^{\Lambda})u'(f_0(1+r^{\Lambda}))\Big(1+\rho g'\Big(u(f^{\max})-u\Big(f_0(1+r^{\Lambda})\Big)\Big)\Big)\Big]}.$$

Following on from this expression, for $r^{\Lambda} < r^{\mathrm{T}}$, we have

$$g'\Big(u(f^{\max})-u(f_0(1+r^{\Lambda}))\Big)-g'\Big(u(f_0(1+r^{\mathrm{T}}))-u(0)\Big)$$

$$=g'\Big(u(f_0(1+r^{\mathrm{T}})\Big)-u\Big(f_0(1+r^{\Lambda}))\Big)-g'\Big(u(f_0(1+r^{\mathrm{T}}))-u(0)\Big)<0.$$

On the other hand, for $r^{\Lambda} > r^{\mathrm{T}}$, we have

$$g^{'}\Big(u(f^{\max}) - u(f_0(1+r^{\Lambda}))\Big) - g^{'}\Big(u(f_0(1+r^{\mathrm{T}})) - u(0)\Big)$$

$$= g^{'}\Big(u(f_0(1+r^{\Lambda})) - u(f_0(1+r^{\Lambda}))\Big) - g^{'}\Big(u(f_0(1+r^{\mathrm{T}})) - u(0)\Big)$$

$$= g^{'}(0) - g^{'}\Big(u(f_0(1+r^{\mathrm{T}})) - u(0)\Big) < 0.$$

As a result of the above deductions, we have that

$$\left.\frac{\partial^2 h(1)}{\partial (r^{\Lambda g})^2}\right|_{r^{\Lambda g}=-1} < 0.$$

Since we have

$$\left.\frac{\partial^2 h(1)}{\partial (r^{\Lambda g})^2}\right|_{r^{\Lambda g}=-1} < 0 \text{ and } \left.\frac{\partial h(1)}{\partial r^{\Lambda g}}\right|_{r^{\Lambda g}=-1} = 0,$$

it follows that $h(1) < 0$ for small guarantee levels, i.e. close to $r^{\Lambda g} = -1$. This, in turn, confirms that (3.5) holds for large $\overline{\pi}$ and small $r^{\Lambda g}$. □

Chapter 4

ANALYSIS OF THE MAIN ISSUES

In this chapter, we provide an analysis of the main decision theoretic issues emanating from the banking model with regret presented in the above.

4.1 ASSETS

In this subsection, we discuss decision theoretic matters arising from the discussion about loans, Treasuries, intangible assets and aggregate risky assets presented earlier. Of particular importance in this regard, is the so-called "credit crunch" phenomenon that arises from the allocation of available funds into Treasuries rather than risky assets (e.g., loans and intangible assets).

4.1.1 Loans

Profit maximizing banks set their loan rates, r^{Λ}, as a sum of the risk-free Treasuries rate, r^{T}, the expected loss ratio, $\mathbf{E}(d)$, and of the risk premium, k. Furthermore, expressing the expected losses, $\mathbf{E}(d)$, as a rate of return per unit time, we obtain the expression

$$r^{\Lambda} = r^{\mathrm{T}} + k + \mathbf{E}(d).$$

Here, the risk premium, k, under the CAPM model could be quantified by the relation

$$k = \beta(r^{m} - r^{\mathrm{T}}),$$

where r^{m} is the rate of return of the market portfolio. The sum $r^{\mathrm{T}} + k$ provides the remuneration for the cost of monitoring and screening of loans and of capital, c^{Λ}. The $\mathbf{E}(d)$ component is the amount of provisioning that is needed to match the average losses faced by the loans. The representation of the banks' interest setting shows that banks will experience excess returns in good times when the actual rate of default, r^{d}, is lower than the provisioning for expected losses, $\mathbf{E}(d)$, and will not be able to cover their expected losses when $r^{d} > \mathbf{E}(d)$. In this case, bank capital will be needed to cover these excess losses. If this capital is not enough then the bank will face insolvency.

An important aspect of the loan issuing process is related to credit risk and its association with regret or disappointment. The dissertation demonstrates that the results from classical risk theory can be affected when regret is taken into account. In particular, if the return on a specific credit risk type turns out to be very high at the end of a contract period, the bank might regret not having allocated a large enough portion of its funds to that risky portfolio type which is constituted by loans and intangible assets. Conversely, if the credit risk type does poorly, the bank might regret having allocating funds to risky portfolio in that risk category.

The aggregate credit reallocation or credit crunch effect is expected to be accentuated in the following situations.

1. the greater the number of banks that were below the capital adequacy standards prior to RBCAR implementation (RBCAR aggregate effect);
2. the greater the proportion of aggregate assets held by these capital-deficient banks (aggregate effect).

We briefly examine the effect that these situations had on the U.S. banking industry in the 80's and 90's. In both 1. and 2. above, the evidence would predict a relatively strong credit allocation effect from risk-based capital adequacy requirements (RBCARs). The replacement of the constant-rate capital adequacy standards of the 1980s with RBCARs increased by more than 20 % the numbers of banks below the regulatory capital minima (see, for instance, (1.1)). More important for the aggregate effect, capital adequacy requirements were more often binding for the very largest banks, so that banks representing more than one-fourth of total U.S. assets did not meet the capital adequacy standards as of December 1989 (see [7] and the references contained therein). These banks were faced with the prospects of raising their ratios of capital to risk-weighted assets to meet the capital adequacy standards either by raising expensive capital or by reducing risk-weighted assets through substituting out of assets with high risk weights, such as commercial loans (compare with (1.1)). Consistent with these expectations, U.S. banks did reduce their commercial loans and increase their holdings of Treasuries in the early 1990s. According to some observers, RBCARs played a major role in this aggregate asset reallocation, and was responsible for a credit crunch. That is, they assert that RBCARs caused a leftward shift in the supply function for bank credit in which significant numbers of borrowers who otherwise would have been funded were denied credit or priced out of the market. We refer to this as the *CAR credit crunch hypothesis* (see [7] and the references contained therein). However, a number of alternative explanations for this change in bank behavior have been offered. According to some observers, implementation of the leverage requirement by U.S. regulators (see, for instance, (1.2)) concomitantly with the RBCARs caused much of the reduction in commercial lending. This requirement, which mandates that banks hold capital of at least a certain percentage of unweighted bank assets, may have forced banks to shrink the size of their asset portfolios. In addition, because the minimum leverage capital percentage depends upon the bank's examination rating and the discretion of the regulator, banks may also have switched out of assets with high perceived credit risks, such as commercial loans, and into safer assets, such as Treasuries, to reduce the required leverage capital adequacy ratio as given by (1.2). Some have also argued that the leverage requirement may have been difficult to meet because the capital of many banks was depleted by loan losses. We refer to this as the *leverage credit crunch hypothesis* (see [7] and the references contained therein). A third regulatory explanation of the observed shifts in bank portfolio behavior of the early 1990s is that regulators may have scrutinized bank loan portfolios more severely in response to heightened concerns about bank risk.

4.1.2 Treasuries

There is a possibility to attain equality in (2.1), by augmenting the right hand-side with further expenses incurred due to the holding of loans. For example, we can consider the rate of loan consumption by dividend payments and provisions for loan losses. In the case of equality in (2.1), we call these cumulative loan expenses together with c^{Λ}, the *threshold loan cost* which we denote by c.

Treasuries are bonds issued by national Treasuries and is modeled as a risk-free asset (bond) in this contribution. It is important to be able to measure the volume of Treasuries. In particular, banks are interested in establishing the level of Treasuries on demand deposits that the bank must hold. By setting a bank's individual Treasuries level, roleplayers assist in mitigating the costs of financial distress. For instance, if the minimum level of Treasuries exceeds a bank's optimally determined Treasuries level, this may lead to deadweight losses.

4.1.3 Intangible Assets

As is evidenced by Section 1.1, we consider intangible assets to be part of this dissertation. In reality, valuing this off-balance sheet item constitutes one of the principal difficulties with the process of bank valuation (see, for instance, [13] and [30]). However, analysts should continually update their valuation procedures for measuring intangible assets for the following reasons. Firstly, the nature and structure of intangibles are not static. Secondly, accounting and other disciplines are developing new methodologies to value such assets. Finally, the valuation models use a causal framework that links the nature and structure of intangible assets to opportunities for future wealth generation.

4.1.4 Aggregate Risky Assets

The discussion in Subsubsection 2.1.4 is related to the principles governing the weighting of credit risk exposure (loan) types as prescribed by new banking regulation in the form of Basel II (see, for instance, [3] and [4]). Banks categorize banking-book exposures into broad classes of loans with different underlying risk characteristics. For interest sake, 15 credit risk exposure types are identified by Basel II and may be listed as follows.

$i = 1:$ Project Finance (PF);
$i = 2:$ Object Finance (OF);
$i = 3:$ Commodities Finance (CF);
$i = 4:$ Income Producing Real Estate (IPRE);
$i = 5:$ Specialized Lending High Volatility Commercial Real Estate (SLHVCRE);
$i = 6:$ Specialized Lending Not Including High Volatility Commercial Real Estate (SLNIHVCRE);
$i = 7:$ Bank Exposure (BE);
$i = 8:$ Sovereign Exposure (SE);
$i = 9:$ Retail Residential Mortgage (RRM);
$i = 10:$ Home Equity Line of Credit (HELOC);
$i = 11:$ Other Retail Exposure (ORE);
$i = 12:$ Qualifying Revolving Retail Exposure (QRRE);
$i = 13:$ Small to Medium Size Enterprises with Corporate Treatment (SMECT);
$i = 14:$ Small to Medium Size Enterprises with Retail Treatment (SMERT);
$i = 15:$ Equity Exposure Not Held in the Trading Book (EENHTB)

with i = 1-6 and i = 9-12 constituting corporate and retail exposures, respectively. The precise definitions of the aforegoing credit risk categories are provided in [4] (see also [3]). With certain minimum conditions and disclosure requirements in place, banks that have received supervisory approval to use the Internal Ratings Based (IRB) approach may use their own internal estimates of risk components in determining the capital requirement for a given exposure. The derivation of RWAs for the aforementioned credit risk categories is dependent on estimates of risk components such as the probability of default (PD), loss given default (LGD), exposure at default (EAD) and, in some cases, effective maturity (EM). In the sequel, the actual values of PD, LGD, EAD and EM are denoted by p_d, l_d, e_d and m, respectively. Throughout we have that

$$0 \leq p_d \leq 1, \quad 0 \leq l_d \leq 1$$

and that e_d is measured in a monetary unit. Also, the unit of measurement of the effective maturity, m, is years. In some cases, banks may be required to use a supervisory value as opposed to an internal estimate for one or more of the risk components.

With regard to the above, we can identify a special risk-weight denoted by $\omega(M_t)$ that is a decreasing function of current macroeconomic conditions, i.e.,

$$\frac{\partial \omega(M_t)}{\partial M_t} < 0.$$

This is in line with the procyclical notion that during booms, when macroeconomic activity increases, the risk-weights will decrease. On the other hand, during recessions, risk-weights may increase because of an elevated probability of default (PD) and/or loss given default (LGD) on loans (see, for instance, [8] and [9]).

4.2 REGRET IN BANKING

We have seen that the introduction of regret in the banking model can alter some of the classical risk theory results. In this section, we consider some of the amendments that are made when regret is introduced. Our interest is specifically in a strategy that allocates away from $\pi_\rho^* = 0$ and $\pi_\rho^* = 1$ and that involves the bank hedging its bets. Also, the behavior of risk-averse banks is compared with that of regret-averse banks.

4.2.1 A Decision Theoretic Optimization Problem

The conclusions from Theorem 2.2.2 are presented in Figure 1 in Appendix (see Chapter 7). In addition, Theorem 2.2.2 has some interesting ramifications for risk and regret in the banking industry. The result intimates that regret-averse banks invest a positive proportion of their available funds in the risky portfolio, even if the $\mathbf{E}[r^a] - r^{\mathrm{T}} = 0$. By contrast, a risk-averse bank would hold all funds in Treasuries in that case. This means that the credit crunch effect would be greater for risk- than regret-averse banks when $\mathbf{E}[r^a] = r^{\mathrm{T}}$. On the other hand, for a sufficiently large $\mathbf{E}[r^a] - r^{\mathrm{T}}$, the regret-averse bank always invests a positive amount in the Treasury, whereas the risk-averse bank holds all available funds in risky portfolio. The effect of a credit crunch is less marked in this case. This may be explained intuitively, by noting that $\pi_\rho^* = 0$ exposes the bank to the possibility of facing extreme regret if risky portfolio do well. By contrast, where $\pi_\rho^* = 1$, the bank will feel less regret if risky portfolio do well but, in return, the bank will feel some regret if it does badly. By way of summary, we can say that regret aversion results in the suboptimality of preferences in favour of $\pi_\rho^* = 0$ and $\pi_\rho^* = 1$. In Figure 2 (see the Appendix in Chapter 7), we demonstrate the behavior of credit crunches for risk- and regret-averse banks.

4.2.2 Hedging Against Bank Risk

In Proposition 2.2.3, banks that weigh regret-aversion heavily, i.e., have large values for ρ, are more likely to hold risky assets in its portfolio under the assumption that $\mathbf{E}[r^a] - r^{\mathrm{T}}$ is low. A credit crunch is less likely to occur under these conditions. Conversely, the bank will hold less risky assets if $\mathbf{E}[r^a] - r^{\mathrm{T}}$ is high. Under these circumstances, the probability of a credit crunch is higher.

4.2.3 Risk- and Regret-Averse Banks with Corresponding Asset Allocation

Proposition 2.2.4 claims that for some intermediate level of $\mathbf{E}[r^a] - r^{\mathrm{T}}$, a regret-averse bank chooses a asset portfolio allocation as if regret was not considered. In the situation where $\mathbf{E}[r^a] - r^{\mathrm{T}} = 0$, the risk-averse bank would invest all of its available funds in the Treasury, T. By contrast, the regret-averse bank would place some of its funds in the risky portfolio. This means that, under the above conditions, a regret-averse disposition would mitigate against credit crunches. As the level of regret aversion rises, i.e., the value of ρ increases, the amount of available funds invested in the risky portfolio increases. With a relatively large $\mathbf{E}[r^a] - r^{\mathrm{T}}$, the risk-averse bank allocates all of its available funds to risky portfolio, while the regret-averse bank invests some money in the Treasury, T. As the level of regret aversion increases, with a high $\mathbf{E}[r^a] - r^{\mathrm{T}}$, the amount of available funds invested in risky portfolio decreases. The effect of these scenarios on credit crunches is obvious. Hence, the certainty equivalent is the point $\mathbf{E}[r^a] = \tilde{r}^{\mathrm{T}}$ where a regret-averse bank chooses an optimal asset portfolio allocation as if regret was not considered. We illustrate this result in the Figure 3 (refer to Appendix in Chapter 7).

4.3 SPECIAL CASE OF LOAN GUARANTEES

In this subsection, we discuss some of the issues related to loan guarantees. Our interest is specifically in the rates of return of such guarantees and the comparison between the behavior of risk- and regret-averse banks.

4.3.1 Rates of Return on Loan Guarantees

It is generally accepted that if the collateral pledged in a loan contract exceeds a critical value, the debtors project may be inefficiently liquidated once he becomes financially distressed. Our conjecture is that a fairly priced loan guarantee provided by a guarantor can partially alleviate this inefficient liquidation problem.

4.3.2 The Main Loan Guarantee Result

The costs of providing loan guarantees depends, of course, on how the guarantees are designed. Firstly, it depends on how often the guarantor must honor its promise. For example, it might be sufficient to structure the program so that the minimum return is evaluated only at the expiry of the loan period, rather than annually or more frequently. Second, the cost of the loan guarantee depends on how much investment risk is borne by the bank. Banks could make the guarantee more valuable, and hence more costly, if they have an opportunity to choose riskier loans after receiving the guarantee. This moral hazard problem has been recognized and has prompted some countries to impose regulations on the banks asset allocations. For instance, some countries required that banks hold most of its available funds in Treasuries. Alternatively, regulators could offer banks some protection from market fluctuations without making a portfolio with Treasuries compulsory. This can be accomplished by providing a guaranteed return on the loan.

Theorem 3.2.1 demonstrates that if the portfolio allocation to the loan is low, the regret-averse bank would be willing to incur less costs for the guarantee than would a risk-averse bank. In this case, the benefits from the loan guarantee in mitigating regret are small and the additional regret cost through the price weighs more. On the other hand, when investment in the loan is large and the guaranteed rate of return is small, the benefits of regret mitigation would be large and would outweigh its cost. Under these conditions, a regret-averse bank would value the loan guarantee more than a risk-averse bank.Furthermore, we present the results of Theorem 3.2.1 in the form of graphically representation as shown in the Figure 4 (see the Appendix in Chapter 7).

Chapter 5

CONCLUSION AND FUTURE INVESTIGATIONS

5.1 CONCLUSIONS

This dissertation shows how regret can influence asset allocations for individual banks. Our research imply that banks with regret-averse attributes will select asset allocations that are less extreme than those predicted by conventional expected utility. If a very risky portfolio were selected by a purely risk-averse bank, its regret-averse counterpart will elect a less risky portfolio. Conversely, when the purely risk-averse bank is predicted to choose a non-risky portfolio, the regret-averse bank would prefer a riskier portfolio. In essence, banks who are regret-averse will tend to hedge their bets, taking into account the possibility that their preferences may turn out to be suboptimal after the expiry of the risky portfolio period. From Proposition 2.2.3, we conclude that banks that weight regret-aversion more strongly than risk-aversion (as measured by ρ), are more likely to hold risky assets in its portfolio when $\mathbf{E}[r^a] - r^{\mathrm{T}}$ is low. Conversely, the bank will hold less risky assets if $\mathbf{E}[r^a] - r^{\mathrm{T}}$ is high. Proposition 2.2.4 claims that at the event where $\mathbf{E}[r^a] = \widetilde{r}^{\mathrm{T}}$, a regret-averse bank chooses a asset portfolio allocation as if regret was not considered.

We also commented on how much a regret-averse bank is willing to pay for a rate of return guarantee on the loan, given a fixed asset portfolio allocation. Theorem 2.2.2 shows that regret allows bank decisions to move away from $\pi_\rho^* = 0$ and $\pi_\rho^* = 1$, if no guarantee is present. This means that banks who take regret into account will hold more risky assets (including loans) when $\mathbf{E}[r^a] - r^T$ is low, but less risky assets when $\mathbf{E}[r^a] - r^T$ is high. However, in the presence of a guarantee, Theorem 3.2.1 shows that regret-averse banks value guarantees less than purely risk-averse banks, when the investment in the loan is small. On the other hand, regret-averse banks value guarantees more than risk-averse banks when the investment in the loan is large and the guarantee is small. As an added feature, the dissertation relates the aforementioned conclusions to the credit crunch phenomenon.

Table 5.1 below provides a summary of the main results obtained in this dissertation.

Cases	**Behavior with Respect to:**	**Risk Averse Bank**	**Regret Averse Bank**
Case 1: $\mathbf{E}[r^a] = r^T$	Probability of Investing in risky portfolio	$= 0$ $\therefore\ \pi_0^* = 0$	> 0 $\therefore\ \pi_\rho^* > 0$
Case 2: $\mathbf{E}[r^a] >> r^T$	Probability of Investing in Treasuries	$= 0$ $\therefore\ \pi_0^* = 1$	> 0 $\therefore\ \pi_\rho^* < 1$
Case 1: $\mathbf{E}[r^a] = r^T$	Credit Crunch Effect	High	Low
Case 2: $\mathbf{E}[r^a] > r^T$ $\mathbf{E}[r^a] >> r^T$	Credit Crunch Effect	Low Lower	Low if $\rho >> 0$. Higher if $\rho >> 0$.

Table 5.1: Effect of Regret on Asset Allocation and Credit Crunches

5.2 FUTURE INVESTIGATIONS

Extensions of our research appear to potentially include some interesting topics. An open problem is to extend the risky assets from one loan type to multiple loan types.

This would enable us to analyze, for instance, the relationship between banking risk and regret for commercial and home loans. Another possible research topic is related to [28] where a risk measure that has a concave distortion function above a given reference point but a convex distortion function below that point, is considered. It would be interesting to translate their work to the regret context, using the ex-post optimal fund level as the relevant reference point. Additionally, we could investigate what happens if the fraction of the bank asset portfolio invested in loans cannot be fixed ex-ante. In this case, it would be of interest to ask whether there is an incentive-compatible contract that would still permit an attractive guarantee, without being prohibitively expensive.

Chapter 6

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Chapter 7

APPENDIX

In this section, we include appendix containing the graphical representations of the results for Theorems 2.2.2 and 3.2.1, Proposition 2.2.4, and also the credit crunch phenomenon.

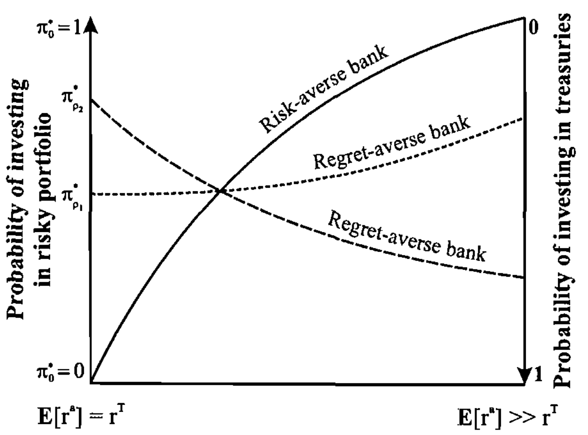

Figure 1: Optimal Risk and Regret Management in Banking

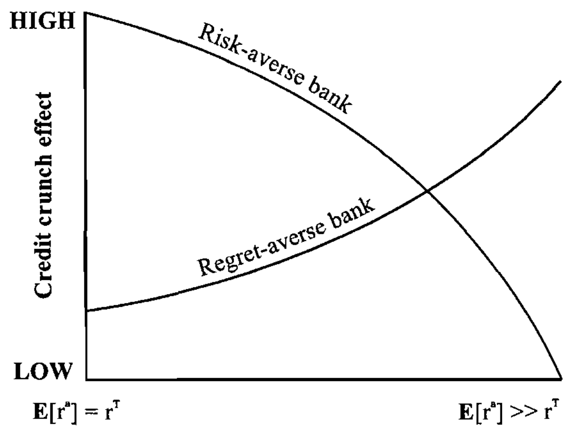

Figure 2: The Credit Crunch Effect

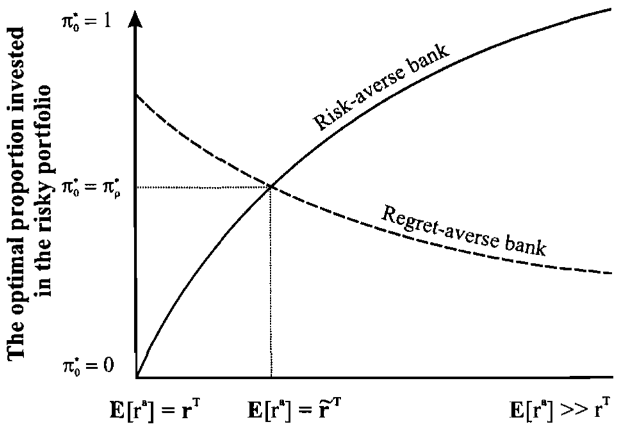

Figure 3: The Certainty Equivalent

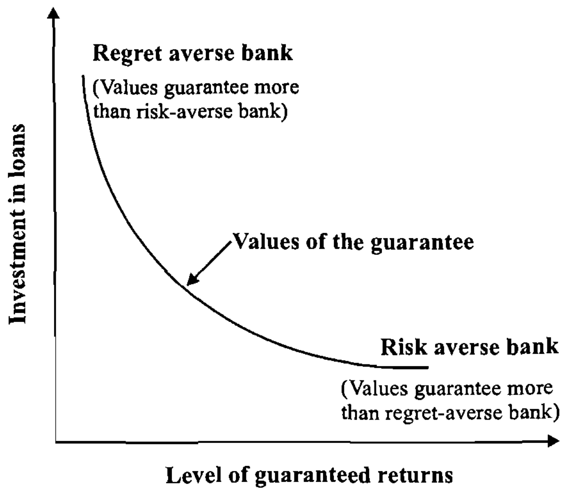

Figure 4: How Banks Value a Guarantee