

**A model for an open-ended task-based approach
in grade 11 mathematics classes**

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Dedicated to my wife Tassie.

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R.K. MAHLOBO

POTCHEFSTROOM

ASOKA ENGLISH LANGUAGE EDITING



DECLARATION

This is to certify that I have English Language edited the dissertation

A model for an open-ended task-based approach in grade 11 mathematics classes

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DISCLAIMER

Whilst the English language editor has used electronic track changes to facilitate corrections, the responsibility for effecting these changes in the final, submitted document remains the responsibility of the candidate in consultation with the supervisor.

ABSTRACT

In this investigation, two schools – a control school and an experimental school – were compared in terms of learner performance in two traditional grade 11 mathematics tests, namely the pre-intervention test and the post-intervention test. Both schools completed the two tests simultaneously. Educators saw both tests before intervention. In the experimental school, four grade 11 mathematics classes were studied. The four classes were given worksheets that complied with an open-ended approach (OEA) to mathematics teaching and learning for learners to work independently on, with the teacher only facilitating. The learner-centredness expressed in the OEA complied with learner-centredness as envisaged by the National Curriculum Statement (NCS), and was predominantly constructivist in character. Throughout the five-month intervention, the author observed proceedings in two of the four classes in the experimental school, ensuring that questions the teacher asked complied with the OEA. The two classes would be referred to as monitored classes. The other two classes at the experimental school worked on the worksheet, with the teacher having been briefed about what was expected of the learners using the worksheet – basically that the learners would have to take own initiatives in solving the mathematics problems with minimal teacher intervention. The two grade 11 mathematics classes were monitored, but not as frequently as the monitored classes. The classes will be referred to as unmonitored classes. At the control school the educators followed their usual (traditional) teaching approach. Both the experimental and control schools followed the same grade 11 mathematics work schedule. The educators in the control school taught without any interference from the author, but the classes at the control school were occasionally observed by the author. In addition to the intervention comparison, the author also gathered qualitative information about participating educators' and learners' experiences and opinions about the OEA at the experimental school by using interviews.

The results of the pre-intervention test showed no statistical difference between the experimental and control school performance, meaning that the learners from both schools were of comparable pre-requisite knowledge. In the post-intervention test, learners from the two monitored classes meaningfully outperformed those from the two unmonitored experimental classes and those from the control school. However,

there was no significant difference in performance between learners from the two unmonitored classes and those from control school. The study concludes that the appropriate OEA intervention was responsible for the good results of the monitored classes, and then uses the gathered qualitative information to design a model for the successful implementation of OEA in mathematics classes.

Keywords for indexing: School mathematics; mathematics teaching; mathematics learning; open-ended approach; problem solving; learner performance; learner achievement; grade 11.

OPSOMMING

'n Model vir 'n oop-einde taakgebaseerde benadering in graad 11-wiskundeklasse

In hierdie ondersoek is twee skole – 'n kontrole en 'n eksperimentele skool – in terme van leerderwerkverrigting vergelyk in twee tradisionele graad 11-wiskundetoetse, naamlik 'n voor-intervensie-toets en 'n na-intervensie-toets. Die skole het die onderskeie twee toetse gelyktydig voltooi. Onderwysers het voor die intervensie insae in beide toetse gehad. In die eksperimentele skool is vier graad 11-wiskundeklasse bestudeer. In die vier klasse het leerders onafhanklik aan werkkaarte gewerk, wat volgens 'n oop-einde benadering (OEB) tot wiskunde-onderrig en –leer saamgestel is, met die onderwyser wat as fasiliteerder teenwoordig was. Die leerdergesentreerdheid in die OEB voldoen aan die eis wat die Nasionale Kurrikulumverklaring (NKV) ten opsigte van leerdergesentreerdheid in die vooruitsig stel, en was hoofsaaklik konstruktivisties van aard. Die outeur het verrigtinge in twee van die vier klasse regdeur die vyf-maande lange intervensie waargeneem en seker gemaak dat die onderwyser vrae gebruik wat aan die OEB voldoen. Na hierdie twee klasse word as die gemoniteerde klasse verwys. Die ander twee klasse in die eksperimentele skool het dieselfde werkkaarte gebruik en die onderwyser is vooraf behoorlik ingelig oor wat van leerders verwag is, naamlik dat hulle met minimale onderwyseringryping eie inisiatief moes gebruik om die wiskunde probleme op te los. Hierdie twee klasse is minder gereeld as die gemoniteerde klasse besoek en waargeneem. Daar word na hierdie twee klasse as ongemoniteerde klasse verwys. In die kontrole skool het die onderwysers op hulle gebruikelike (tradisionele) wyse voortgegaan met onderrig. Sowel die eksperimentele as die kontrole skole het dieselfde graad 11-wiskunde werkskediule gevolg. In die kontrole skool het die onderwysers sonder ingryping van die outeur gewerk, maar die skool is wel by geleentheid deur die outeur besoek. Aanvullend tot die intervensie-vergelyking, het die outeur ook kwalitatiewe inligting oor deelnemende onderwysers en leerders by die eksperimentele skool se ervarings en menings oor die OEB ingesamel.

Die voor-intervensie-resultate toon geen statisties-beduidende prestasieverskille tussen die eksperimentele en kontrole skole nie, wat daarop dui dat die leerders van

die twee skole ten aanvang vergelykbaar ten opsigte van die voorvereiste kennis was. In die na-intervensie toets het leerders in die gemoniteerde klasse betekenisvol beter presteer as sowel die leerders in die ongemoniteerde klasse en in kontrole skool. Daar was egter geen betekenisvolle verskil tussen die leerders in die ongemoniteerde klasse en in die kontrole skool nie. Die studie bevind dat die OEB-intervensie tot die verbeterde werkverrigting in die gemoniteerde klasse gelei het, en gebruik dan die ingesamelde kwalitatiewe inligting om tot 'n model vir die suksesvolle implementering van 'n OEB in wiskunde-klasse te kom.

Sleutelwoorde vir indeksering: Skoolwiskunde; wiskunde-onderrig; wiskundeleer; oop-(einde)-benadering; probleemoplossing; leerderprestasie; leerderwerkverrigting; graad 11.

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CHAPTER 1

STATEMENT OF THE PROBLEM AND MOTIVATION

1.1 STATEMENT OF THE PROBLEM AND MOTIVATION

The education system before democratisation of South Africa was fragmented according to different races and was not equally beneficial to all. Since South African adoption of democracy in 1994, there has been an effort by the government to transform education. The South African government began the process of developing a new curriculum for the school system in 1995 (DoE, 2008:2). The growth and development of knowledge and technology and the demands of the 21st century required learners to be exposed to different and higher-level skills and knowledge than those required by the previous South African General Education curricula. Also, South Africa had changed and the curricula for schools therefore required revision to reflect new values and principles, especially those of the Constitution of South Africa.

The first version of the new curriculum for the General Education Band, known as Curriculum 2005 (C-2005), was introduced into the Foundation Phase in 1997, (DoE, 2008:2). The concerns of educators led to a review of C-2005 in 1999 (DoE, 2008:2). The review of the C-2005 provided the basis for the development of the Revised National Curriculum Statement (RNCS) for General Education and Training (GET) (Grades R-9) in 2002 and the NCS for grades 10-12 in 2005 (DoE, 2005:2). According to the Revised National Curriculum Statement Policy (DoE, 2002:1), outcomes-based education (OBE) forms the foundation of curricula in South Africa. But what exactly is OBE? It is an approach to education that focuses on the pre-defined outcomes. Outcomes are clear learning results that we want learners to demonstrate at the end of significant learning experiences. The outcomes are what learners can actually do with what they know and have learned. OBE requires evidence of changes in attitudes of the learners at the end of learning experiences (Spady, 1998:24). OBE means clearly focusing and organising everything in an educational system around what is essential for all learners to be able to do successfully at the end of their learning experiences. It is an approach to education – an educational philosophy – that governs curriculum design, development and implementation (SAQA *position paper*). C-2005, and subsequently the NCS, is the curriculum that has been developed within an OBE framework.

OBE shifts from the traditional focus on what learners should be taught (content) and how much time they should be allocated to teaching this, to a focus on setting universal standards of what learners are expected to demonstrate they 'know and are able to do'. OBE should be driven by the outcomes displayed by the learner at the end of the educational experience. The kind of teacher envisaged by the National Curriculum Statement (NCS) is one who, amongst other things, can be a mediator of learning, and a developer of learning programmes and material. Learning Programmes specify the scope of learning and assessment activities for each phase. The outcomes specified in the NCS encourage a learner-centred and activity-based approach to education (DoE, 2002). In other words, the NCS envisaged teacher is the developer of learning material in which the teacher will position him/herself as the mediator or facilitator of the learning process in a learner-centred environment.

A common understanding of a mediator is that of a person who intervenes in a solution-seeking process. The mediator needs to have all the facts about the problem whose solution is sought, and how it is proposed to be solved. In most cases, mediation focuses on identifying weaknesses or points of strength in the subjects' proposed solutions, until consensus is reached about a suitable solution. Consequently, the NCS mathematics teacher – a mediator in the mathematics lesson – does not prescribe the learner's mathematics problem solution. It is the learner's problem solution process that guides the teacher's intervention. In this way the teacher is strategically placed to monitor the *learning process*, an important component in the educator – learner interaction. The teacher is well placed even to identify possible misinterpretation of some items that he/she may otherwise have assumed learners would understand. As the designer of the learning activities, it is the teacher's responsibility to ensure the creation of an environment – through the designed activities – in which he/she will facilitate the learning process, rather than act as an absolute source of infallible information.

Does the typical teacher we have in a South African school meet the expectations of NCS? Education and Training in South Africa has seven critical outcomes and five developmental outcomes, which derive from the Constitution (DoE, 2008:10). Each of them describes an essential characteristic of the type of South African citizen the education sector hopes to produce. The document further states that these critical outcomes *should be reflected in the teaching approaches and methodologies that mathematics educators use* [emphasis by the author]. These critical outcomes not only lay a foundation for identification of the 'NCS envisaged citizen' but they also act as a checklist for the 'envisaged NCS teacher's role'. For example, if one considers the critical outcomes, then one can end up with the on 'NCS compliance checklist' (Table 1.1).

TABLE 1.1 "NCS teacher's compliance checklist"

Critical Outcome	The teacher's approach:	Yes	No
1. Identify and solve problems and make decisions using critical and creative thinking.	1. Creates a learning environment in which it is possible for learners to have opportunities to make comprehensive use of <i>their creative thinking</i> .		
2. Work effectively with others as members of a team, group, organisation and community	2. Encourages an active small-group learner participation in lessons and allow the learners to express <i>their</i> ideas frequently.		
3. Organize and manage themselves and their activities responsibly and effectively.	3. a) Provides every learner with an opportunity for reasoning experience b) Positions the teacher as the facilitator, and not the source, of learning.		
4. Communicate effectively using visual, symbolic, and/or language skills in various modes.	4. Makes it possible for every learner to respond to the problem in some significant ways of his/her own.		

To measure how well a student performs, educators have to be able to examine the process of learning, not just the final product (Badger and Thomas, 1992). Such a view of learning and teaching demands an 'open-ended' form of teaching, learning and assessment, based on open-ended tasks and questions (Moschkovich, 2004:51-53; Radford, 2001:251; Elbers, 2003:91; Hershkowitz & Schwarz, 1999:150). In her detailed analysis of two United Kingdom schools, Boaler (1997) argued that the school using an open approach to teaching and learning mathematics produced more sustained outcomes in mathematics learning, than the conventional format used by the other school. In an open-ended approach to teaching and learning, the focus shifts from learning as content knowledge *per se* to learning as the ability to use and interpret knowledge critically and thoughtfully (Badger and Thomas, 1992).

Open-ended questions set by the teacher are those questions that do not explicitly guide the learners in how they should solve the problems. How the learner solves the problem governs how the teacher intervenes. Such tasks, requiring learners to construct their own responses, reportedly open a window to the learners' thinking and understanding (Badger & Thomas, 1992). The teacher, focusing on the learner's activity and response, adapts his/her own schematic representation of the learner's level of understanding, and thus infers the learner's learning needs (Jaworski, 1994:27).

According to French and Nathan (2006:3), it is internationally accepted that open-ended problems form a useful tool for the development of mathematics teaching in schools, in a way that emphasises understanding and creativity. Additional skills needed for teaching may evolve not from a focus on mathematical content but from 'attending to the mathematics in what one's learners are saying and doing, assessing the mathematical validity of their ideas, listening for the sense in children's mathematical thinking even when something is amiss, and identifying the conceptual issues on which they are working' (Schifter, 2001:131).

This study proposes that an open-ended approach, in addition to positioning the teacher as a compatible implementer of NCS, will enhance learning performance. Specifically, the study focused on the impact of an open-ended approach to the learning of school mathematics in grade 11. What constitutes good teaching is consistently controversial and will remain controversial. The goal in this study is not to resolve the controversy but rather to focus on what research can currently tell us about classroom instruction with the intention of making explicit current findings (Franke, Kazemi, & Battery, D. 2007:226). Would an open-ended approach (OEA) enhance learning? If educators listen to children, understand their reasoning, and teach in a manner that reflects this knowledge, the study contends, they will provide children with a mathematics education better than if they did not have this knowledge (Sowder, 2007:163).

The assumption is compatible with what Dossey (1992) said:

'What you have been obligated to discover by yourself leaves a path in your mind which you can use again when the need arises'.

Jaworski (1994:27) refers to Piaget's theory:

'Each time one prematurely teaches a child something he could have discovered himself, the child is kept from inventing it and consequently from understanding it completely (Piaget, 1970).'

In view of the foregoing, this investigation endeavoured to investigate the central question 'What is the influence of open-ended approach on the learning of mathematics in grade 11 classes?' Alternatively, the focus of this study is on whether or not the use of predominantly open-ended mathematical tasks will have any impact on the learner's understanding of mathematical principles, concepts and procedures and if it will help educators to make a shift to creating a better learning environment.

1.2 RESEARCH AIMS

The central aim of the study was to investigate the impact of open-ended questions and / or tasks on the learning and teaching of mathematics in grade 11 classes, and to propose a model for an open-ended approach in those classes. Hence, the research intended to investigate:

- 1.2.1 What distinguishes open-ended from closed questions / tasks;
- 1.2.2 What teaching contexts will be conducive to the use of an open-ended approach in grade 11 mathematics classes;
- 1.2.3 How mathematics educators in those classes will adapt to such an open-ended approach;
- 1.2.4 How mathematics learners in those classes will respond to such an approach;
- 1.2.5 What will be the impact of such an approach on the learning of mathematics in those classes;
- 1.2.6 What model of school mathematics teaching and learning can be proposed in view of the results of the investigation.

1.3 LITERATURE REVIEW

In order to achieve research aims 1.2.1 and 1.2.2, a literature review was conducted to analyse current teaching and learning approaches (CA), as well as open-ended approaches (OEA), with a view to:

1.3.1.1 Distinguishing between, and characterising, them.

1.3.1.2 Identifying adjustments needed to translate from a CA to an OEA in an effort to establish prospects of adaptability of educators, and reaction of learners, to an OEA.

Key words used in Dialog, EBSCOhost and Nexus searches were, among others:

“Open-ended tasks/questions”; “socio-constructivist approach”; “realistic approach”; “collaborative learning”; “appropriation approach to learning”, “problem solving”; “discovery”, “experiential and contextual learning”; “inquiry learning”; “mathematics; grade 11”.

1.4 RESEARCH DESIGN

The approach needed to answer the research questions was a mixed-method approach as depicted in Figure 1.1, adapted from Creswell (2003).

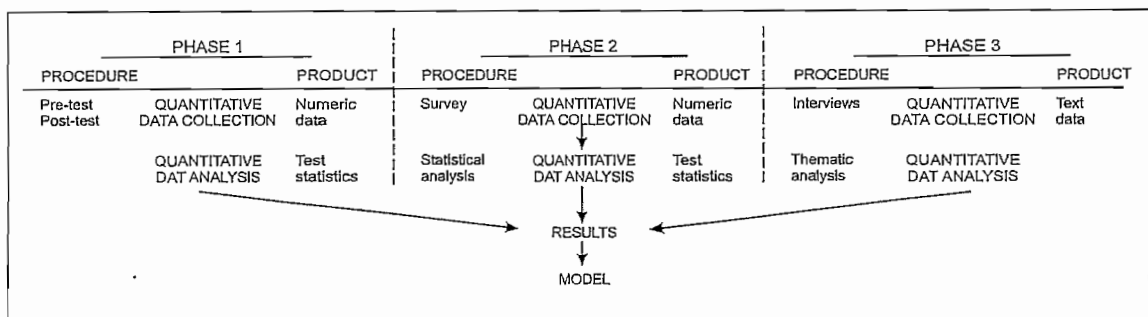


FIGURE 1.1: Research design

The following is a brief summary of the design:

The initial stage of the study was a literature study to obtain information on open-ended tasks and current approaches to teaching and learning mathematics.

Phase 1, the quantitative component of the study, involved a pre-test and post-test to determine the impact of an open-ended approach to teaching and learning on learner performance.

Phase 2 involved a survey of the post-intervention views of a random sample of learners from the monitored class on mathematics teaching and learning.

Phase 3 the qualitative component of the study, involved open-ended interviews with educators and learners from the experimental school.

1.5 METHODOLOGY

1.5.1 Introduction

As reflected in the design, a multi-phased sequential quantitative/qualitative approach was followed. The design progressed through three phases.

Phases 1 and 2 were the quantitative parts of the investigation, with phase 1 dealing with the pre-test and post-test marks, while phase 2 focused on the questionnaire. In phase 3, the qualitative aspect of the investigation, the learner's post-intervention view of mathematics teaching and learning was established through interviews. The main focus of the study – phase 1 - was to investigate the impact of an OEA in grade 11 mathematics classes on learner performance. Phases 2 and 3 were undertaken for triangulation purposes - to establish if their results would corroborate the findings of phase 1.

1.5.2 Phase 1: Quantitative component of the research: Pre-test / Post-test

In the first phase the study focused on establishing the impact of the open-ended approach to teaching and learning in terms of performance in the pre-test and the post-test.

1.5.2.1 Introduction

Figure 1.2 represents the pre-test / post-test control group quasi-experimental design that was used (Leedy & Ormrod, 2003:236):

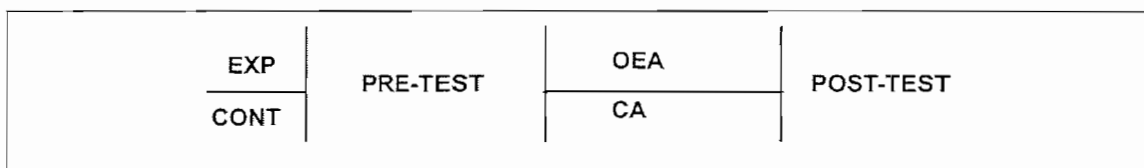


FIGURE 1.2: Pre-test – post-test design

Two high schools in one Gauteng district were used. One acted as the experimental school and the other as a control school. Four grade 11 mathematics classes at the experimental school ($N = 166$) participated in the study, with the main focus of daily monitoring by the author being on two ($N = 93$) of the four experimental classes. The monitored classes were taught by the same teacher. At the control school, the author observed – without any intervention - the dominant approach used in the learning and teaching of mathematics in

two grade 11 mathematics classes ($N = 88$), initially taught by the same teacher, but subsequently divided between two educators.

1.5.2.2 Population and sample

The population and sample is summarised in Table 1.2:

TABLE 1.2. Population and sample

SCHOOL	POPULATION	SAMPLE
Experimental	All maths learners grades 11: $N = 166$	2 monitored Grade 11 maths classes: $N = 93$
Control	Grade 11 maths classes: 2 classes: $N = 88$	Grade 11 maths classes: 2 classes: $N = 88$

A purposeful sub-sample – two grade 11 mathematics classes ($N = 93$) - was selected from the experimental group for monitoring throughout the intervention. The class teacher of these two classes was found to have a greater willingness to try the new approach than the others in the experimental school. The other two classes from the experimental school ($N = 73$), though using the same intervention material, were only occasionally monitored.

1.5.2.3 Variables

Independent variable: Teaching-learning approach characterised by open-ended tasks (e.g. questions).

Dependent variable: Performance in mathematics learning.

1.5.2.4 Measuring Instruments

- The *pre-test* (**Appendix 2**) was developed to test the pre-requisite knowledge of the learners from both the experimental and control schools.
- The *post-test* (**Appendix 3**) was used to compare the two groups in terms of performance in a traditional mathematics test.

Both the pre-test and post-test were discussed with the educators concerned before being written by the learners in order to ensure consistency of expectations from the educators and to accommodate the educators' input.

1.5.2.5 Reliability and validity: post-test

The research methods in this study were designed to minimise plausible alternative explanations for the cause-effect relationships by taking precautionary measures (Trochim, 2006), using relevant statistical tools.

1.5.2.6 Data collection procedures

Efforts were taken to ensure that collection of data was not vulnerable to contamination, for example the possibility of some learners seeing the test before writing it. Synchronisation of test times was a priority for data collection.

1.5.2.7 Data analysis

In the comparison of the pre-tests and post-tests, descriptive, as well as inferential statistical analysis was used. With the quantitative analyses Cronbach Coefficient α was used to establish the reliability of the instruments, while t-tests and effect size (Cohen's Criterion) was used to establish significance of differences between the participating groups' performance. Factor analysis was used to see if factors could be identified by relating the questions.

1.5.3 Phase 2: Questionnaire part of the study

1.5.3.1 Introduction

The main purpose of the questionnaire was to establish the learner's post-intervention view of mathematics teaching and learning. Had their views on mathematics learning changed as a result of their exposure to the OEA? If so, the study assumes, the intervention shall have impacted on the learners as far as post-intervention views on mathematics learning are concerned. Would the learners' post-intervention views on mathematics learning corroborate those of the pre-test / post-test test part of the investigation, in the sense of the intervention impacting positively on both?

1.5.3.2 Population and sample

Population: The study population consisted of grade 11 learners in mathematics classes in the experimental school ($N = 166$).

Samples: Two grade 11 mathematics classes from the experimental school were used ($N = 101$) and these were monitored throughout the intervention. Results of only 93 of these learners were used in the pre-test / post-test analysis. The other 8 learners did not complete all two tests.

1.5.3.3 Questionnaire as a measuring instrument

The questionnaire was based on a survey by Schommer (1990). Only thirty-four of the sixty-three questions from Schommer's survey were used, because they were, in the author's opinion, the most relevant to elicit the learners' post-intervention view of mathematics teaching and learning.

1.5.3.4 Reliability and validity

Validity in the student questionnaire was enhanced by basing it on a validated questionnaire by Schommer (1992) on the mathematical belief scale (Fresen, 2005). It was also read and approved by three research experts.

1.5.3.5 Data analysis

An independent paired t-test and Cronbach α were used to analyse data. A statistical significance test was used to analyse the questionnaire data.

1.5.3.6 Data collection procedure

A pilot study was conducted using 10 of the 101 learners (Hannan, 2007; Zarinpoush & Gumulka, 2006) and then the other 91 were, on the same day, given the questionnaire to fill in and submit.

1.5.4 Phase 3: Qualitative component of the research: Interview

The interview in this study was an *open-ended interview* (Hannan, 2007). The interview was piloted with 5 individual learners from the experimental monitored classes in order to establish their reactions. The learners seemed to be comfortable with the interview.

1.5.4.1 Data collection procedures

Interview statements were transcribed from audio cassette to written text. The interview text was analysed. Statements in the text were categorised according to Flanders's (2004) system.

1.6 PROCEDURE

The study commenced with an investigation of the relevant literature. In order to set up the experimental investigation the researcher asked for permission from the district education authority to use two of their schools for research purposes. The author identified his expectations of those schools, so that the authorities could initiate a meeting between the school principal, involved educators and the researcher. Negotiations around relevant research matters were then entered into. The phased investigation followed. After processing, analysis and interpretation of generated data had been completed, conclusions and recommendations regarding the impact of open-ended tasks on the learning of mathematics in grade 11 classes were drawn up and, finally, a model for the learning and teaching of mathematics in grade 11 classes in South Africa was proposed.

1.7 ETHICAL ASPECTS

The permission of the Gauteng Department of Education District was obtained to undertake the research in the district. Permission was required from and negotiated with the principal, in consultation with the mathematics teacher(s) involved and with the learners. All participants (educators, learners) and parents were informed of the aim and nature of the research, and provided with relevant feedback on the results as requested. Regular monitoring of the experimental programme was done to ensure that no teacher or learner was put at undue risk as a result of the study. All mathematics learners in the grade 11 experimental school voluntarily took part in the intervention, to avoid any potential for unintended discrimination.

1.8 STRUCTURE OF THE THESIS

CHAPTER 1: STATEMENT OF THE PROBLEM

Chapter 1 focused on whether or not educators met the implementation expectations of the NCS. The study then proposed that an OEA would position the educators to reach these

implementation expectations. The investigation also proposed that an OEA would enhance learner performance.

CHAPTER 2: CURRENT APPROACHES TO SCHOOL MATH TEACHING AND LEARNING.

Chapter 2 focused on current approaches to teaching and learning mathematics. A mathematics teacher's approach to teaching is influenced by, among others, the following:

- Her/his view of what it means to understand maths.
- His/her view of what maths is.

It is with these influences in mind that chapter 2 looked at understanding maths from the perspective of this investigation. The chapter also touched on learning theories, as they are based on views of maths. Lastly, the chapter looked at current teaching approaches in South African schools.

CHAPTER 3: AN OPEN-ENDED APPROACH TOWARDS MATH TEACHING AND LEARNING.

Chapter 3 was basically about the open-ended approach (OEA) to teaching and learning. In this chapter consideration is given to different forms of the OEA, the advantages of an OEA, implementation of an OEA, the OEA in an OBE environment, and contexts of an OEA.

CHAPTER 4: RESEARCH DESIGN AND METHODOLOGY

In this chapter the investigation focused on research design and methodology. There are three research items being investigated: a quantitative investigation (Pre-test / post-test), a quantitative investigation (questionnaire) and a qualitative investigation (Interview). For each of these research items the following methodological items were identified (where applicable): Philosophical aspects of the study, motivation for method choice, population and sample, variables, measuring Instruments, reliability and validity, data collection procedures and data analysis.

CHAPTER 5: RESULTS

In this chapter an attempt was made to address research questions by looking at the results of the investigation. Firstly classroom dynamics captured on video recorder were analysed for three scenarios: the monitored class, the unmonitored experimental class, and the control

class. The results of the pre-test / post-test, were then considered, followed by those of the questionnaire and, lastly, those of the interview.

CHAPTER 6: CONCLUSIONS AND RECOMMENDATIONS

The focus of this chapter is on the recommendations that emanate from the interpretation of results in chapter 5.

CHAPTER 2

CURRENT APPROACHES (CA) TO SCHOOL MATHEMATICS TEACHING AND LEARNING

2.1 INTRODUCTION

The view of mathematics held by the teacher, has a strong impact on the way in which mathematics is approached in the classroom (Dossey, 1992). Research confirms that mathematics teaching is deeply rooted in the views of mathematics held by the educators and that they do not discard these views easily (Nieuwoudt 1998). Understanding in mathematics, learning theories, and classroom teaching approaches derive from views about the nature of mathematics. Before focusing on the current approaches to teaching and learning, we explore two dominant views about the nature of mathematics. These views will be discussed and linked to learning theories and teaching approaches with a view to understanding the current approaches prevalent in South African classrooms.

2.2 VIEWS OF SCHOOL MATHEMATICS

The discussion starts by looking at the two different views of mathematics that are prevalent in South African schools.

2.2.1 Formalist-static view of mathematics.

Historically, discussions of the nature of mathematics date back to the fourth century B.C. Among the first major contributions to the dialogue was the contribution of Plato and his student, Aristotle. Plato took the position that the objects of mathematics had an existence of their own, beyond the mind, in the external world. This elevated position for mathematics as an abstract mental activity on externally existing objects that have only representation in the sensual world is also seen in his support for and encouragement of mathematical development in Athens (Dossey, 1992). This Platonian view of mathematics is described as the *formalist-static perspective*. According to this view mathematics is an invariable and static body of knowledge consisting of a logical and meaningful network of inter-related truths [facts, rules and algorithms]. The assumption is that one can gradually and in neat chunks

unfold and discover this body of knowledge, and consequentially, that the mathematics teacher can then transfer these chunks of knowledge to the learner (Nieuwoudt, 1998).

2.2.2 Relativist-dynamic view of mathematics.

In stark contrast to the above rigid view, mathematics can also be viewed from a *relativist-dynamic* perspective. This view is problem-driven and accordingly mathematics is viewed from a 'change and grow' perspective as a continually changing field of human labour, creativity and discovery, aimed at generating patterns through problem solving which is then processed into mathematical knowledge. This view of mathematics bears a strong resemblance to Aristotle's experimental ideas about mathematics (Nieuwoudt, 1998). In Aristotle's view, the construction of a mathematical idea comes through idealisations performed by the mathematician as a result of experience with objects (Dossey, 1992).

Experience and observation of school mathematics educators confirm that many of them hold on to traditional formalistic-static views of mathematics and mathematics education, while only a few reject this view in favour of a dynamic alternative view of mathematics (Nieuwoudt, 1998). Discussion in this investigation concerning current teacher practice in the majority of South African schools will confirm that this is true even today, despite education reform efforts that are compatible with the relativist-dynamic view of mathematics. In terms of education reform efforts, Nieuwoudt (1998) claims that the dynamic view of mathematics and its teaching and learning seems to be winning ground against the static view. The classroom application of the above-mentioned views about mathematics has one common aim – to foster learner understanding of mathematics. What does it mean to understand mathematics? In the following section, understanding in mathematics is described from the perspective of this investigation.

2.3 A MODEL OF UNDERSTANDING IN MATHEMATICS

The goal of many research and implementation efforts in mathematics education has been to promote learning with understanding. Many general theories of learning, including those with different paradigmatic origins, wrestle with the notion of understanding. Drawing from old and new work in the psychology of learning, Hiebert and Carpenter (1992) present a framework for examining issues of understanding. The framework they propose for reconsidering understanding is based on the assumption that knowledge is represented internally, and that these internal representations are structured and linked.

2.3.1 External and internal representation

Communication requires that representations be external, taking the form of spoken language, written symbols, pictures or physical objects. To think about mathematical ideas we need to represent them internally in a way that allows the mind to operate on them. Because mental representations are not observable, discussions of how ideas are represented inside the head are based on a high degree of inference (Hiebert & Carpenter, 1992). For years, the associationist perspective in psychology (Skinner, 1953) ruled out the discussions of mental representations because they cannot be observed. However, work in cognitive science has restored mental representations as a legitimate field of study. Indeed, the notion of mental representations is a central idea that brings together work on cognition from a variety of fields, including psychology, computer science, linguistics, and others (Hiebert & Carpenter, 1992).

The aim of clarifying how ideas are represented in the head is to draw quite heavily on insights provided by work in cognitive science regarding mental or internal representations. Firstly, Hiebert and Carpenter (1992) assume that a relationship exists between external and internal representations. Second, they assume that internal representations can be related or connected to one another in useful ways. They also assume that a relationship between external and internal representations is consistent with much of the work in cognitive science. They admit that it is an assumption not generally held. They mention that there is an ongoing debate, for example, about whether the form of a mental representation mimics in some way the external object or event being represented or whether there is a common form used to represent all information. Although the debate is not resolved, Hiebert and Carpenter (1992) believe it reasonable to assume that the nature of internal representation is influenced and constrained by the external situations being represented. They apply this assumption to mathematical situations by assuming that the nature of external mathematical representations influences the nature of internal mathematical representations. Evidence from a variety of task situations suggests that it is a reasonable assumption. The important point here is that when considering representation in mathematics, one should consider both external and internal representations. That is, the form of an external representation (physical materials, pictures, symbols, etc.) with which a student interacts influences the way the student represents the quantity or relationship internally. Conversely, the way in which a student deals with or generates an external representation reveals something of how the student has represented that information internally.

The second assumption they draw from work in cognitive science is that internal representations can be connected. The connections can be inferred. Hiebert and Carpenter

(1992) propose that when relationships between internal representations are constructed, they produce networks of knowledge. In reaction to the debate about whether or not understanding can fully be described in terms of internal knowledge structures, they argue that this notion of connected representations of knowledge will continue to provide a useful way to think about understanding mathematics. The first reason they give to justify their argument is that the notion provides a level of analysis that makes contact with both theoretical cognitive issues and practical educational issues. Secondly, it generates a coherent framework for connecting a variety of issues in mathematics teaching and learning, both past and present. Thirdly it suggests an interpretation of learners' learning that helps to explain their successes and failures in and out of school.

2.3.2 Learning mathematics with understanding

A mathematical idea, procedure or fact is understood if it is part of an internal network (Hiebert & Carpenter, 1992). More specifically the mathematics is understood if its mental representation is part of a network of representations. The number and strength of the connections determine the degree of understanding. A mathematical idea, procedure, or fact is understood thoroughly if it is linked to existing networks with stronger and more numerous connections. Understanding consists of five interwoven strands of mathematical proficiency: conceptual understanding, procedural fluency, strategic competency, adaptive reasoning and productive disposition. However, one does not develop conceptual understanding first and then the others follow but rather all of the aspects of understanding must be addressed together over time (National Research Council, 2002).

There are four basic mental operations involved in understanding: identification, discrimination, generalisation and synthesis:

- Identification is the main operation involved in acts of understanding – acts that consist in a reorganisation of the field of consciousness so that some objects that, so far, have been in the background, and are now perceived as the 'figure'.
- Discrimination between two objects is an identification of two objects as different objects.
- Generalisation is understood here as that operation of the mind in which a given situation (which is the object of understanding) is thought of as a particular case of another situation.
- Synthesis means the search for a common link (Sierpinska. 1994: 56-60).

2.3.2.1 Building understanding

Hiebert and Carpenter (1992) describe the structuring process that produces understanding by using the above definition of understanding. Networks of mental representations, Hiebert and Carpenter say, are built gradually as new information is connected to existing networks or as new relationships are constructed between previously disconnected information. Understanding grows as the networks become larger and more organised. Understanding can be rather limited if only some of the mental representations of potentially related ideas are disconnected or if the connections are weak. Connections that are weak and fragile may be useless in the face of conflicting or non-supportive situations (Hiebert & Carpenter, 1992). Understanding increases as networks grow and as relationships become strengthened with reinforcing experiences and tighter network structuring. Networks are constantly undergoing realignment and reconfiguration as new relationships are constructed. The processes of reorganising networks and adjoining new representations to existing networks depend, to some degree, on the networks that have already been created.

Cangelosi (2003:173-174) categorises and sub-categorises specifics according to certain commonalities or attributes. The categories provide a mental filing system for storing, retrieving and thinking about information. The process by which a person groups specifics to construct a mental category is referred to as *conceptualising*. The category itself is a *concept*. Constructing concepts in our minds enables us to extend what we *understand* beyond the specific situations we have experienced in the past. Concepts are the building blocks of mathematical knowledge. To construct a concept, learners use inductive reasoning (Cangelosi, 2003: 177). It was earlier mentioned that the view of mathematics held by the teacher, has a strong impact on the way in which mathematics is approached in the classroom (Dossey, 1992). It is reasonable to conclude that a teacher who subscribes to the formalistic-static view of mathematics will have a notion of learner understanding of mathematics that is different from the teacher who subscribes to a relativist-dynamic view of mathematics. Instrumental and relational understanding is used to distinguish between the two views. A brief discussion of the two follows.

2.3.2.2 Instrumental and Relational Understanding in mathematics

Richard R. Skemp (1976) described instrumental understanding as understanding of 'rules without reasons' and relational understanding as 'knowing both what to do and why'. He mentioned that he would until then not have regarded instrumental understanding as understanding at all. He gave examples like 'borrowing' in subtraction, 'turn it upside down

and multiply' for division by a fraction, 'take it over to the other side and change the sign' etc., to illustrate instrumental understanding.

Some of the advantages of instrumental understanding are that it is easier to understand, and that mathematics rewards are more immediate. However, instrumental understanding usually involves a multiplicity of disconnected maths rules. While a case might exist for instrumental mathematics short-term and within a limited context, long-term and in the context of a child's whole education it does not (Skemp, 1976). If pupils are still being taught in a way that promotes instrumental understanding, then a 'traditional' syllabus and evaluation scheme will benefit them more.

One of the advantages of relational understanding is that it is more adaptable to new tasks. Learning relational mathematics consists of building up a conceptual structure (schema) from which its possessor can produce an unlimited number of plans for getting from any starting point within the schema to any finishing point. Relational schemas are organic in quality – if people get satisfaction from relational understanding, they may not only try to understand relationally new material which is put before them, but also actively seek out new material and explore new areas Skemp (1976) also identified the conditions under which it might be more advisable to use instrumental understanding instead of relational understanding. Some of them include conditions like relational understanding taking too long to achieve, relational understanding of a particular topic being too difficult, and the skill being needed for use in another subject.

Associated with these types of understanding are the roles of the teacher. To apply rules without reasoning requires the source of the rules to be from outside the mental structures of the learner. There are reasons why one can conclude that a teaching approach that primarily promotes instrumental understanding is predominantly teacher-centred. If the learners are given an opportunity to discover the rule by themselves, then they should be able to justify their discovery, thus giving reasons for the rule. It will no longer be 'rules without reasons'.

The 'instrumental' teacher may view learners' 'existing networks' as 'pre-requisite mathematical knowledge' learners *are assumed* to have (because there is no effort in trying to prompt the learners to discover the rules), and then prepare a teacher-centred lesson in order to indicate how the rules are to be used. In other words, such a teacher will personally take responsibility for defining what constitutes learner 'existing networks' and build onto it 'new representations'. Transmission teaching and rote learning are characteristics of instrumental understanding and they are premised on a formalist-static view of mathematics.

A 'relational' teacher may develop an approach that is intended to uncover the learner's 'existing networks' schema and then to observe how the learner builds onto the networks to facilitate acquisition of the 'new representations'. Otherwise the learners will not have an opportunity to 'know both what to do and why'. In this case the focus of the classroom dynamics will be on 'getting into the learner's head'. The role of the teacher in this case will be facilitative – using the learner's solution strategies to identify gaps with a view to conscientise the learner with regard to these gaps. Classroom dynamics will be governed by the learner.

Skemp's (1976) sentiments seem to be supported by the above-mentioned model of understanding. He claims that there is no case for instrumental understanding in the child's long-term whole education. One of the reasons for his argument against instrumental understanding is that it usually involves a multiplicity of disconnected maths rules. The model mentions that connections that are weak and fragile may be useless in the face of conflicting or non-supportive situations.

Relational understanding of mathematics consists of building up a conceptual structure (schema) from which its possessor can produce an unlimited number of plans for getting from any starting point within his schema to any finishing point. Hiebert and Carpenter (1992) propose that when relationships between internal representations are constructed, they produce networks of knowledge. It is not only learner understanding of mathematics that will be impacted on by the teacher's view of the nature of mathematics, but also the learning theory.

2.4 LEARNING THEORIES

Associated with a formalist-static view of mathematics is the *behaviourist theory of learning*, and the learning theories associated with relativist-dynamic view of mathematics rely upon the *constructivist* approach to learning and discovery learning.

2.4.1 Behaviourist theory of learning

John B. Watson (as cited by Smith, 1999) is generally credited as being the first proponent of behaviourist theory that relied on laboratory experimentation. What prompted Watson to turn to laboratory experimentation was that inner experiences that were the focus of psychology could not properly be studied as they were not observable. The result was the generation of the *stimulus-response model* (Smith, 1999). In this model the environment is seen as

providing stimuli to which individuals develop responses. In essence three key assumptions underpin this view:

- Observable behaviour rather than internal thought processes are the focus of study. In particular, learning is manifested by a change in behaviour.
- The environment shapes one's behaviour; what one learns is determined by the elements in the environment, not by the individual learner.
- The principles of contiguity (how close in time two events must be for a bond to be formed) and reinforcement (any means of increasing the likelihood that an event will be repeated) are central to explaining the learning process (Smith, 1999).

Other researchers like Edward L. Thorndike built upon these foundations and, in particular, developed an S-R (stimulus-response) theory of learning (1914). He noted that responses (or behaviours) were strengthened or weakened by the consequence of behaviour. This notion was refined by Skinner and is perhaps better known as operant conditioning – reinforcing what you want people to do again; ignoring or punishing what you want people to stop doing (Smith, 1999).

In terms of behaviourist learning, four key principles come to the fore:

- Activity is important: Learning is better when the learner is active rather than passive.
- Repetitions, generalisations and discrimination are important notions: Frequent practice – and practice in varied contexts – is necessary for learning to take place. Skills are not acquired without frequent practice.
- Reinforcement is the cardinal motivator. Positive reinforcements like rewards and successes are preferable to negative events like punishment and failures.
- Learning is helped when objectives are clear. Those who look to behaviourism in teaching will generally frame their activities by behavioural objectives e.g. 'By the end of this session participants will be able to ...' (Hartley, 1998).

2.4.2 Constructivist approach to learning.

Constructivism is a theory of knowledge that says that the world is inherently complex, that there is no objective reality, and that much of what we know is constructed from our beliefs and the social milieu in which we live (Borich & Tombari, 1997:177). There are several

different contemporary interpretations of the concept of constructivism (Killen, 2000: xviii). They all share four common principles: (Snowman & Biehler, 2000)

1. What a learner 'knows' is not just received passively but is actively constructed by the learner – meaningful learning is the active creation of knowledge structures from personal experience.
2. Because knowledge is the result of personal interpretation of experiences, one person's knowledge can never be *totally* transferred to another person.
3. The cultures and societies to which people belong influence their views of the world around them and therefore influence what and how they 'know'.
4. Construction of ideas is aided by systematic, open-minded discussions and debate.

Among the different classifications of constructivism are *cognitive constructivism* and *social constructivism* (Killen, 2000: xviii). *Cognitive constructivism* can be defined as 'an approach to learning in which learners are provided the opportunity to construct their own sense of what is being learned by building internal connections or relationships among the ideas and facts being taught' (Borich & Tombari, 1997:17). This is consistent with Hiebert and Carpenter's (1992) model of understanding. The *social constructivist* approach treats learning as 'a social process whereby learners acquire knowledge through interaction with their environment instead of merely relying on the teacher's lectures (Powers-Collins, 1994:5)

Abbot and Ryan (1999:2), express it this way: 'A person learning something new brings to that experience previous knowledge and present mental patterns. Each new fact or experience is assimilated into a living web of understanding that already exists in that person's mind'. Most efforts will at least share the philosophy that constructivist teaching – a teaching approach that acknowledges constructivism as a theory of knowledge - 'is based on the generalized belief that learners develop understanding when they are active and seek solutions for themselves' (Taylor, 1996:258). Or, as Kamii and Ewing (1996:260) put it, constructivist learning is of 'the view that much learning originates from *inside* the child'. Often, the idea of learning as originating from inside the child is expressed as learners using 'their experience to actively construct understanding in a way that makes sense to them' (Borich & Tombari, 1997: 178). Or, as Dominic and Clark (1996) put it, constructivist teaching involves getting learners to use what they know to figure out what they need to know. The teacher becomes a facilitator of learning rather than a giver of information (Dart, 1994:1). By acknowledging that learning is an internal process, rather than something that a teacher can

impose on learners, 'constructivist learning theory places importance on the learner's point of view' (Garmston & Wellman, 1994:85). A constructivist approach to teaching encourages educators to look for patterns in learners' thinking (Killen, 2000: xxi). Being aware of the thinking patterns that learners typically use helps educators to anticipate and appreciate their learners' understanding. (Dominick & Clark, 1996)

A constructivist approach to teaching and learning does not deny the importance of factual knowledge, but it does emphasise that the best way for learners to retain and apply this knowledge is to 'put it into a larger, more lifelike context that stimulates learners to reflect, organise, analyse, and problem solve' (Borich & Tombari, 1997:180). There are several specific things that educators can do to help learners construct their understanding. One of these is *scaffolding* – providing a learner with enough help to complete a task and then gradually decreasing the help as the learner becomes able to work independently.

2.4.3 Discovery approach to learning

There are various forms of discovery learning methods: pure discovery learning methods, in which the student receives problems to solve with little or no guidance from the teacher and guided discovery learning methods, in which the student receives problems to solve but also the teacher provides hints, direction, coaching, feedback, and/or modelling to keep the learners on track (Mayor, 2004:15).

2.4.3.1 Guided discovery

The author has had an opportunity to interact with educators from different provinces during his tenure as a national trainer for a Gauteng-based Non-governmental organisation (NGO), whose role was to intervene in teaching and learning of mathematics in schools. He was also involved in a research project with educators from North West and Mpumalanga provinces through the North West University (Mahlobo 2000). During the course of discussion on approaches to teaching and learning, it emerged that the context in which discovery learning and teaching was used by the participating educators is different from discovery learning and teaching as expected by the NCS.

In one case, for instance, an educator wanted the learners to learn about addition of fractions with different denominators. He used questions like

"What is the denominator of the first fraction?"

"What is the denominator of the second fraction? "

"What are the common multiples of both denominators? "

"What is the lowest of the common multiples (LCM) of the denominators? "

"How many times does the denominator of the first fraction go into the LCM? "

"What is the number multiplied by the numerator of the first fraction? "

"How many times does the denominator of the second fraction go into the LCM? "

What is the number multiplied by the numerator of the second fraction?

The teacher kept on asking the questions until the learners ultimately were able to determine the sum of the two original fractions. When the educator was asked to analyse his teaching approach, he claimed that the lesson was learner-centred because the learners were actively involved in the discovery of mathematical principles, concepts and procedures.

It appeared from this interaction with the teacher, and subsequently from other educators, that what was actually described as discovery learning involved the teacher asking predominantly recall and recognition types of questions in order to illicit participative response from the learners. The questions asked were mostly of a 'closed' type, in the sense that the answers to these questions were easily predicted by the educator. According to Painter (nd), closed questions do not require complex thought to reach the answer. They are usually used to recall information, assess prior knowledge or knowledge gained after teaching.

2.4.3.2 Pure discovery / inquiry learning strategy

In this mode of discovery, called an inquiry learning strategy, the teacher still asks learners questions, but the difference with the guided discovery is that here the questions asked by the teacher are in response to the learners' process of solving the problem. With inquiry strategies the teacher engages learners in activities in which they interact with information, make observations, and formulate and articulate ideas that lead them toward discovery, conceptual construction, or invention (Cangelosi, 2003:10). In other words, the purpose of the questions asked by the teacher is to get clarity on what goes on in the head of the learner during the problem solution process, instead of influencing directly how the learner should solve the problem. Educators who wish to use constructivist methods of instruction are often

encouraged to focus on pure discovery learning – in which learners are free to work in a learning environment with little or no guidance (Mayer, 2004). This is the type of discovery learning the study assumes is NCS compliant. It is the one to which ‘open-endedness’ applies.

There are contrasting research conclusions regarding the guided and pure discovery learning approaches. There is literature that supports the notion that there is no difference in terms of impact on learning, between guided discovery learning and transmission teaching. According to Jaworski (1994:11), discovery has frequently been used to describe teaching aimed at getting pupils to reason out inductively certain preconceived truths *in the mind of the teacher*.

Begle (1979: 11) compared the traditional methods of exposition and guided discovery:

"...About as many studies show mixed results or else no significant differences as show significant differences. Of the latter, more show expository teaching as more effective than discovery methods, but not enough to allow us to come to a definite conclusion. "

This sentiment was earlier described by Bittinger (1968):

"Many research studies into the value of discovery methods in teaching were not convincing of its value over methods of traditional instruction. "

Mayer (2004:15) claimed that in one pioneering study, the pure discovery group performed the worst and the guided discovery group performed the best on tests of immediate retention, delayed retention, and transfer to solving new problems.

2.5 TEACHING APPROACHES

The teaching approaches that this study will focus on are transmission teaching, discovery teaching, and then problem-solving and modelling.

2.5.1 Transmission teaching

Transmission teaching is the teaching approach in which knowledge is ‘transmitted’ to the learner by the teacher. It is influenced by a behaviourist view of learning. In this mode of teaching, the teacher is the main source of the knowledge acquired by the learner (Van de Walle, 2004:12). It is sometimes known as conventional instruction or traditional teaching. By transmission teaching the author refers to an approach in classroom in which the teacher

does most of the talking, with learners passively listening, and then working on the problems given by the teacher. This teacher-centred instruction is consistent with what Silver, Smith and Nelson (1995:18) describe as *conventional mathematics instruction*. It emphasises whole-group instruction, with learners mainly listening to a teacher's explanation and watching the teacher work sample problems, followed by learners working alone on similar problems presented in their textbooks or on worksheets (Silver *et al.*, 1995:18). Educators tend to view the mathematics classroom as consisting of alternate moments of exposition (for which they are responsible) and practice (to be carried out by the learners). Practice is constituted by exercises and takes up most of the teaching time; the learning situations tend to be highly structured and the most important interaction is that between teacher and student (Ponte *et al.*, 1994:348).

Cangelosi (2003:133) made this comment when demonstrating perception about mathematics: 'You are well aware of how mystifying many people find mathematics. For example, did you ever attend a social gathering where it became known that you are or plan to be a mathematics teacher? If so, you probably heard comments such as 'Math?! So you are some kind of a genius!' Cangelosi (1996:12) highlighted some of the problems associated with such a teacher-dominated approach to teaching and learning of school mathematics:

'The type of teaching that dominates mathematics classrooms in today's school tends to produce undesirable side effects, including leaving learners with the following impressions:

- Mathematics is a boring sequence of technical vocabulary, rules and algorithms to be memorised for the purpose of passing tests in schools.
- Mathematics is a complex, mystifying subject that was handed down to us by some ancient mystics (from Greek mythology).
- Only people with exceptional aptitude for mathematics can creatively do mathematics.'

2.5.2 Problem solving and modelling as teaching and learning strategies

One of the primary aims of mathematics education for secondary schools is to enable pupils to acquire the necessary mathematical knowledge and skills and to develop thinking processes so as to apply them in mathematical situations in real life. In the process of delivering the curriculum, it is hoped that mathematics learners will not only appreciate the beauty of the subject but also the usefulness and power of mathematics. In practice, however, the emphasis has been on solving routine mathematical problems in a context-free environment. Even on the odd occasion when a "real life" problem or example is discussed in

the classroom, it is typically a rather artificial problem created for the purpose of fitting it into the topic in question. Such practice makes it difficult to convince the learner that real life applications of mathematics do indeed exist (Cheng, 2001).

Among mathematics educators, there is growing recognition that a serious mismatch exists between the low-level skills emphasised in test-driven curriculum materials and the kind of understanding and abilities that are needed for success beyond school (Lester & Kehle, 2003). In mathematics education, Polya-style problem solving strategies such as 'draw a picture, work backwards, look for a similar problem, or identify the given and goals', have long histories of being advocated as important abilities for learners to develop. Research has not linked these strategies to improved problem solving performance. Interpretation of Polya's heuristics is that the strategies are intended to help problem solvers think about, reflect on, and interpret problem situations, more than they are intended to help them decide what to do when 'stuck' during a solution attempt (Lesh & Zawojewski, 2007). Little progress has been made in problem-solving research and literature on problem-solving has little to offer to school practice. There is evidence that the amount of research on problem-solving appears to be on the decline (Lester & Kehle, 2003, Stein, Boaler, & Silver, 2003). However, an indicator of a pendulum swing back towards problem-solving is apparent from recent research that emphasises how mathematics is used in fields such as engineering, medicine and business management. Experts in such fields report that the nature of problem-solving has changed dramatically during the past 20 years (Lesh & Zawojewski, 2007:764). The new definition of problem-solving embraces the notion that people learn problem-solving through creating mathematical models. Problem solving is defined as the process of interpreting a situation mathematically (mathematical modelling), which usually involves several iterative cycles of expressing, testing and revisiting mathematics interpretations, and of sorting out, integrating, modifying, revising, or refining clusters of mathematics concepts from various topics within and beyond mathematics. (Lesh & Zawojewski, 2007:782).

As figure 2.1. shows, from the traditional perspective of problem-solving, the assumption tends to be made that 'real life' applied problems are the most difficult types of problems to solve. Therefore, they are commonly addressed only after computational procedures have been learned, the procedures have been practised on sets of story problems, and problem-solving strategies have been taught. Thus, only in the final stages of instruction are learners solving realistic and complex applied problems. In this traditional perspective, the applied problem (if time permits) is a small subset of the problem-solving experiences in which learners engage. From a model-and-modelling perspective, the assumption is that the learning of mathematics takes place through modelling. In other words, learners begin their

learning experience by developing conceptual systems (i.e. models) for making sense of real-life situations where it is necessary to create, revise, or adapt a mathematical way of thinking. Given model-eliciting activities, learners are expected to bring their own personal meaning to bear on a problem, and to test and revise their interpretation over a series of modelling cycles. Learners are assumed simultaneously to gain an increasing understanding of both the problem situation and their own mathematization of the problem. Therefore, learners' applied problem-solving experiences (mathematics modelling), drive the learning in the conventional curriculum, and traditional story problems become a subset of the applied problems through which learners learn maths (Lesh & Zawojewski, 2007:783).

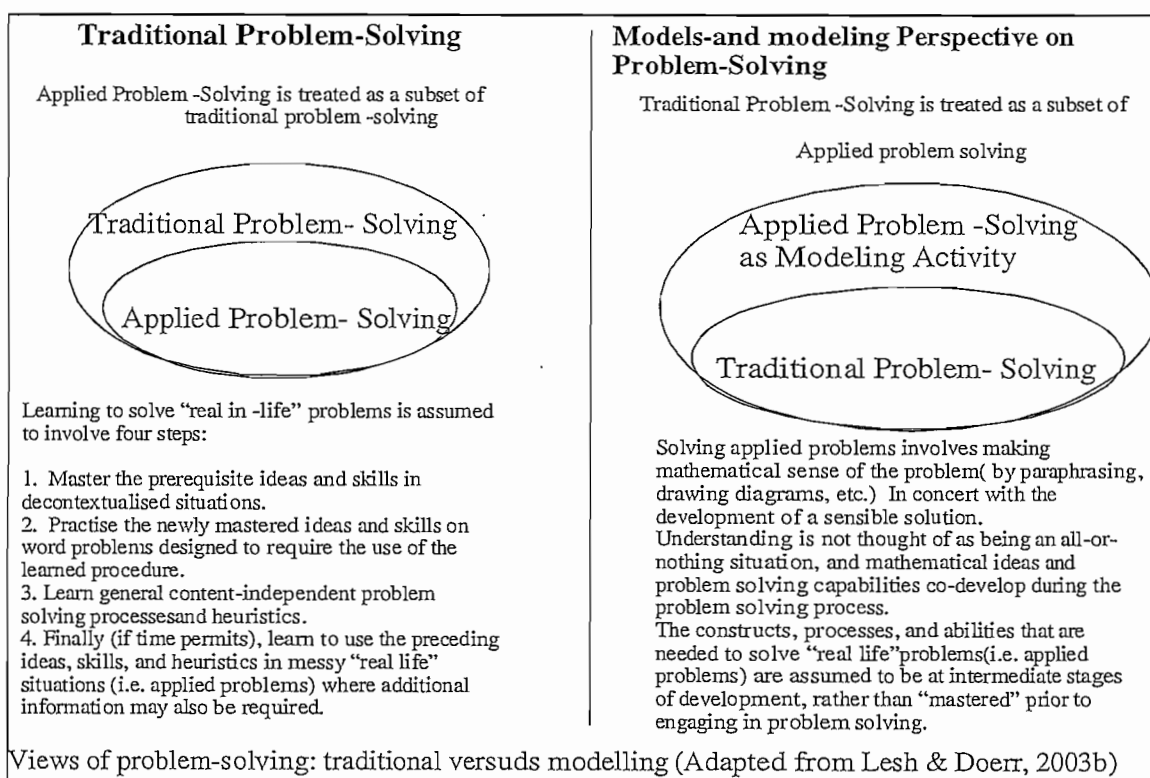


FIGURE 2.1 Traditional versus Modelling perspectives of Problem-Solving.

Mathematical modelling is a process of representing real world problems in mathematical terms in an attempt to find a solution to the problem (Cheng, 2001). A mathematical model can be considered as a simplification or abstraction of a complex real world problem or situation into a mathematical form, thereby converting the real world problem into a mathematical problem. The mathematical problem can then be solved using whatever known techniques to obtain a mathematical solution. This solution is then interpreted and translated

into real terms as the simplified view of the process of mathematical modelling in Figure 2.2. shows.

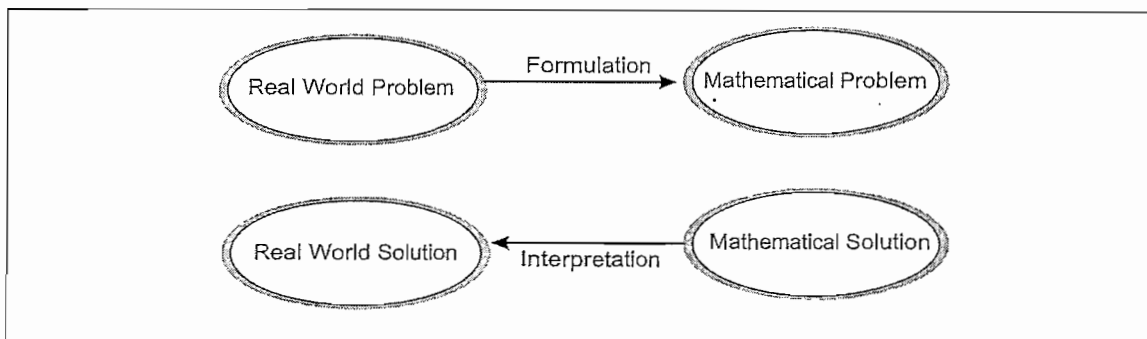


FIGURE 2.2 A simple view of mathematical modelling process

Example (Cheng, 2001): The car park problem

A typical school precinct would normally have some car park space. The parking lots are usually already painted, lines drawn and so on. Suppose we wish to check if the existing plan has made maximum use of the car park space. If not, we wish to re-design the space to increase the number of lots.

Such a problem would involve questions like the following:

- How many cars can be parked along one curb using parallel parking?
- How much space should there be for traffic within the car park?

For angle parking, we could consider the relationship between x the curb space and angle θ the lines make with the curb as depicted by Figure 2.3.

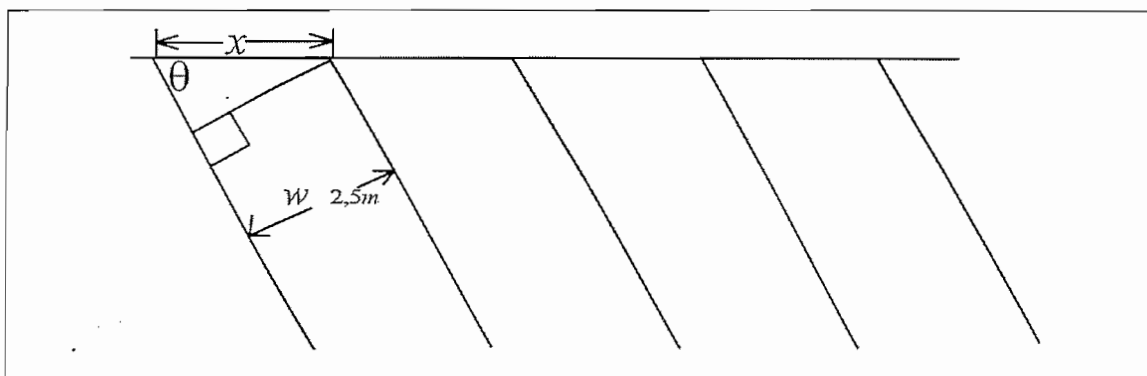


FIGURE 2.3 Car Park Problem

We can first assume that a typical width, w , for a lot is 2,5m.

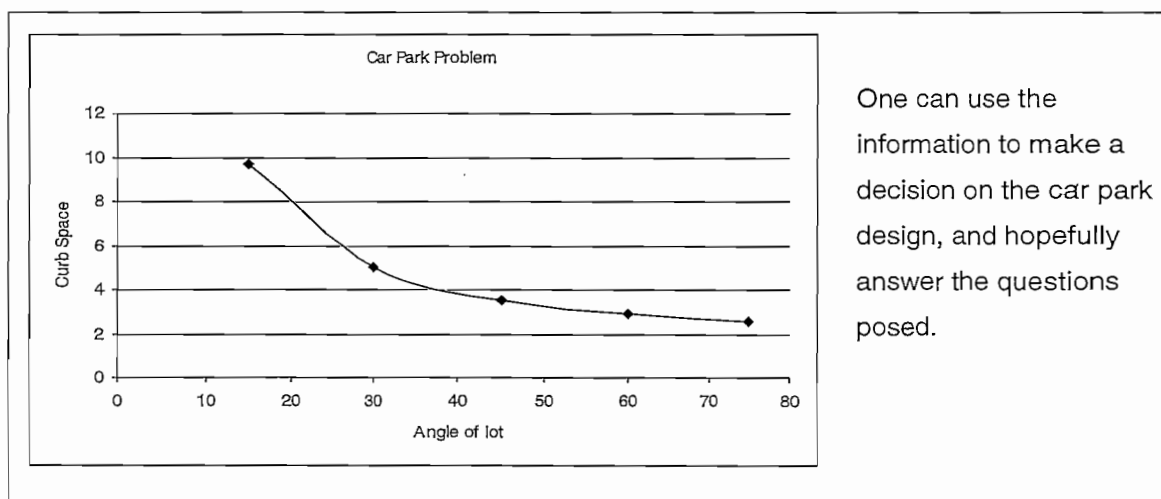
It is not hard to see that the relationship between the three variables, x , θ and w is

$$\sin \theta = \frac{w}{x}$$

Suppose we fix the lot width w and wish to see how the curb space varies with the angle of the lot. We can generate the table 2.1. and its graph:

TABLE 2.1 Values of w and θ

θ	x
15	9,7
30	5
45	3,54
60	2,9
75	2,6



Mathematical modelling as an aspect of problem-solving is compatible with a constructivist approach to teaching and learning. It is because given model-eliciting activities, learners are expected to bring their own personal meaning to bear on a problem, and to test and revise their interpretation over a series of modelling cycles. Learners are assumed simultaneously to gain an increasing understanding of both the problem situation and their own mathematisation of the problem (Lesh & Doerr, 2003b:686). It therefore requires the use of an OEA.

In all of the situations where I managed to obtain information about the educators approach to teaching and learning, there was neither an explicit nor implicit reference to any of the two above-mentioned notions of problem-solving.

2.6 THE CURRENT SITUATION IN SOUTH AFRICAN SCHOOLS

In 1999, teaching and learning in the majority of South African schools were described in terms of teacher-centredness and rote learning (Taylor & Vinjevold, 1999:131). Unfortunately, the description is relevant in most schools even today. This is despite the fact that the National Curriculum Statement is in the process of being implemented. The author's interaction with educators in schools from eight provinces, as well as a research project he conducted among educators from Northwest and Mpumalanga provinces (Mahlobo 2000), and mathematics workshops he conducted on behalf of the Mathematics, Science and Technology (MST) project with educators in the Gauteng province in 2003, confirmed Taylor and Vinjevold's (1999:131) observation about teaching and learning in the majority of South African high schools as being characterised by teacher-centredness.

In May 2007, a provincial science week was held in Mpumalanga province, in line with the National Science Week that was launched by the Department of Science and Technology. One component of the weekly activities was a workshop for educators that the author conducted. The topic under discussion was the *introduction of the sine function* in high school. In introducing this workshop, the author asked 69 educators from different schools in that region, who were present at the workshop, to indicate how they would introduce the topic. The educators were split into small groups with a view to choosing one representative to give a feedback to the whole audience. The author's idea – not communicated to the participants - was to establish if the educators' approach would be teacher-centred or learner-centred. The summary on the educators' responses, which were written on flipcharts, and which have not been published elsewhere, is written and included in **Appendix 1**.

For instance, one group (Group 2) mentioned that they would talk of voice pitches of men and women as expressible in sine wave terms. They said they would inform the learners that the equation governing the human voice pitch is $y = A \sin \theta$, with A being small for men's low pitch and large for women's high pitch. They would then plot the graph of $y = A \sin \theta$ by using chosen values of θ . In all of the cases, the lessons were clearly teacher-centred. The new curriculum, as has already been mentioned, is clearly meant to be learner-centred.

A frequent criticism of school mathematics taught in such a teacher-dominated manner is that procedures for solving problems in such classes often cannot be used flexibly to solve problems other than those which have been demonstrated to learners and which have been practised. As a result problem-solving skills do not develop and transfer well in learners being taught in traditional classes according to (Hiebert & Carpenter,1992). The formalist-static view of mathematics 'is harmful to persons holding that belief because it can result in false impression about how mathematics is done' (Nieuwoudt, 1998).

When the author introduced the research project early in 2007, all grade 10-12 mathematics educators at the school attended the author's briefing. The first priority was baseline assessment. Amongst other things, the author wanted to find out about the current approach to teaching and learning at the school. The author asked the team how they would introduce factorising quadratic expressions - the first topic in the common work schedule the grade 11 educators were given by their Gauteng District office. The work schedule was to be used by all grade 11 mathematics classes, implementing the new curriculum for the first time, in the District schools. It was also hoped that grade 10 educators, already a year into implementing the NCS requirements, would somehow reflect the new approach in their responses. Those educators responding indicated that:

- They would start by explaining what a quadratic expression is and what factorisation means.
- They would then briefly introduce the procedure to be followed in identifying the factors of a quadratic expression.
- The learners will then be given a number of quadratic expressions to factorise.

The role of learners was predominantly that of being listeners and careful executioners of educators' instructions. There was no evidence of preparedness of the educators to embrace a learner-centred approach – an approach with focus on the active learner involvement - at all. The author then asked whether the educators, especially grades 10 to 11 educators, thought there was any difference between how they used to teach and how they were

expected to teach according to the National Curriculum Statement (NCS). There seemed to be a feeling that only terminology changed, but most things remained the same.

Carnoy, Chisholm & Baloyi (Sowetan July 10 2008:13), conducted a small- scale pilot study that focused on the role that educators' skills and practice play in South African learners' learning within the socio-economic and administrative conditions in selected Gauteng schools. The study was conducted on a sample of grade 6 mathematics lessons in 40 primary schools in Gauteng. Learners, educators and principals filled in questionnaires, learners took tests at two points in the year to measure gains, and educators' grade 6 mathematics classes were video-taped and analysed. They report that classes generally revolved around a considerable amount of teacher-led presentation, with the teacher asking the learners to reply individually or in chorus to questions while making his or her presentation.

The author also captured one grade 11 mathematics lesson in the control school on video tape. The teacher talked most of the time, with learners passively listening. The only engagement of the learners was through their working on problems after the teacher gave examples. Again, there clearly was no sign of learner-centredness in this lesson. Furthermore, the author was invited by the provincial Association of Mathematics Educators of South Africa (AMESA), Mpumalanga, to co-conduct a plenary session on mathematics, as well as to present a workshop to the educators on the NCS approach to teaching a mathematics topic. This provincial AMESA conference was held on 10 – 11 May 2008. Upon enquiry by the author about how the educators would introduce a mathematics topic selected by the researcher, teacher domination was picked up in the educators' responses. This, according to the researcher, shows how deeply entrenched teacher-dominated so-called conventional or transmission teaching is in the mind of educators at these schools. All these observations corroborate what Taylor and Vinjevoid, (1999:131) claim in their statement that teaching and learning in the majority of South African schools involves teacher-centredness and rote learning.

For a good insight into transmission teaching and rote learning, the author found a book *Getting It Right*, based on the Report of President Education Initiative (PEI) by Nick Taylor and Penny Vinjevoid, to be quite authoritative and informative. Taylor and Vinjevoid (1999:132) attribute transmission teaching and rote learning, which as explained above, are predominant in many schools to the doctrine of what they call *fundamental pedagogy*. According to them, Hartshorne (1992), the NEPI Teacher Education Report (NEPI, 1992a)

they all suggest that the doctrine of fundamental pedagogy has had profoundly detrimental effects on educators' thinking and practice.

Fundamental pedagogy is an indigenous product which, drawing on Dutch phenomenological philosophy, claims to have developed a science of education. During the apartheid years it was prominently associated with the Department of Education at the University of South Africa, by far the largest provider of both pre- and in-service education for educators, and supported by a number of Afrikaans and homeland campuses. It is based on premises, which can be interpreted as authoritarian (for example, the teacher as knowing adult, leads the child to maturity). It was the dominant theoretical discourse in education departments at South Africa's black universities and colleges (Taylor and Vinjevoold, 1999:132).

The NEPI (1992a:17) report claims that fundamental pedagogy has 'debilitating effects' and prevents educators from 'developing an understanding of the relationship between education and the context in which knowledge and understanding are created and shared'. Fundamental pedagogy 'heads off the possibility of critical reflection on that system by making reflection illegitimate' (Enslin,1990: 83). She describes fundamental pedagogy as 'an ontology, which produces useful and docile educators' (p.100). Chisholm (1993:3) asserts that the values and approaches of fundamental pedagogic 'block and hinder the development of critical and innovative teaching strategies'. Policy makers in opposition to apartheid saw the uprooting and replacement of this philosophy of education as one of the primary tasks to be undertaken when restructuring teacher education. Laridon (1992), in talking about transmission teaching and rote learning, claimed that what needed to be broken in mathematics education in the country was the vicious circle generated by the tradition of transmission teaching and rote learning at all levels in the education system. Learners come into our methodology (didactic) courses from high school or university courses in mathematics, which have largely been taught in this mode. When they go out on teaching experience or during their early years in the schools, the social pressures brought to bear by colleagues practically negate all efforts for the change that some are already advocating in their pre-service courses. Novice educators aspiring to implement new ideas are thus inevitably once again drawn into the vortex of the vicious circle. Studies of children's conceptual development in various domains of knowledge have demonstrated that children's thinking is qualitatively different from, and not just an imperfect copy of, adult thought. So an adult educator trying to dominate the 'transmission of knowledge' to the learner may easily miss this difference, in the process limiting knowledge acquisition by the learner (Black & Ammon, 1992).

2.7 CONCLUSION

The argument above seems to justify the conclusion that the approaches to teaching and learning adopted by educators in most South African schools do not comply with the National Curriculum Statement. An approach that is teacher-centred, which promotes rote learning, does not comply with the following notions of an 'envisaged NCS learner' characteristics:

- a) The learner takes control of his/her learning with limited intervention by the teacher (Taylor and Vinjevd, 1999:108).
- b) The learner is given an opportunity to develop him-/her-self mathematically (DoE, 2005:8).
- c) The learner is involved in mathematics which is accepted at times as being based *on observing patterns* (DoE,2005:7).
- d) Classroom approach to teaching and learning prepares the learner to be a citizen described by the critical outcomes (See table 1.1).

The type of questions asked in a teacher-centred environment, according to the author's observation, are *closed questions*. Closed questions and tasks are, those questions or tasks that typically have one correct response or expected outcome. The teacher gives direction as to how the learners should solve the problems or execute the task through questions, which are intended to lead to the predetermined outcomes (Zevenberg, Sullivan and Mousley, 2001:5). If a learner appears to deviate from the teacher's intended solution path, the teacher will redirect the learner to this path through facilitating/guiding questions.

How can the educators be helped? What will be the effect of an open-ended approach to teaching and learning on the NCS compliance? In the next chapter, the focus will be placed on an open-ended approach.

CHAPTER 3

AN OPEN ENDED APPROACH TOWARDS MATHEMATICS TEACHING AND LEARNING

3.1 INTRODUCTION

This investigation is premised on the notion that an open-ended approach (OEA) to teaching and learning will satisfy the compliance requirements of the NCS, as well as enhance learner performance in grade 11 traditional mathematics tests. The contention is that educators need assistance in deciding on the activities that will produce the learners envisaged by NCS, the learners who can *develop themselves mathematically* (DoE, 2005:8). It is the claim of this study that an OEA is a relevant approach to facilitate NCS compliance.

In world-wide attempts to find a new teaching method that might meet the challenges set by constructivism; the so-called open-ended approach was developed in the 1970s in Japan (Pehkonen, 2007:3). In other words, the OEA is a vehicle through which constructivist ideals can be achieved. It was earlier argued that constructivist lessons are characterised by the focus of teacher-learner interaction being on 'looking inside the head of the learner', and using the information to facilitate the learning process (See page 5). This study will argue that the types of questions used by the teacher should not prescribe the learner's solution process, but rather, the learner's solution process should determine the kinds of questions the teacher will ask in order to facilitate the learner's solution (See chapter 2). The latter can only be ensured by the use of open-ended questions and/or tasks.

3.2 VARIOUS DESCRIPTIONS OF THE OEA

Two types of open-ended problems can be identified, namely, those that have several correct answers, and/or those that have several ways to find a correct answer for (Takahashi, nd; Zevenberg, 2001:5, Mewborn, *et al.*, 2005:413, Harms, 2007). The two types are illustrated below in figure 3.1.

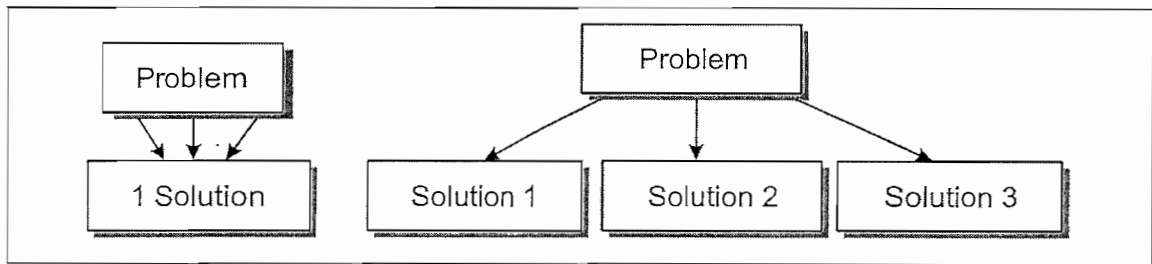


FIGURE 3.1 Two types of Open-ended Problem-Solving

The first problem in the figure refers to problems with one solution but diverse approaches and the second problem in the figure refers to a problem with multiple correct answers. Sometimes the multiple solutions can give rise to a targeted mathematical idea, concept, principle or rule as shown (Takahashi, nd):

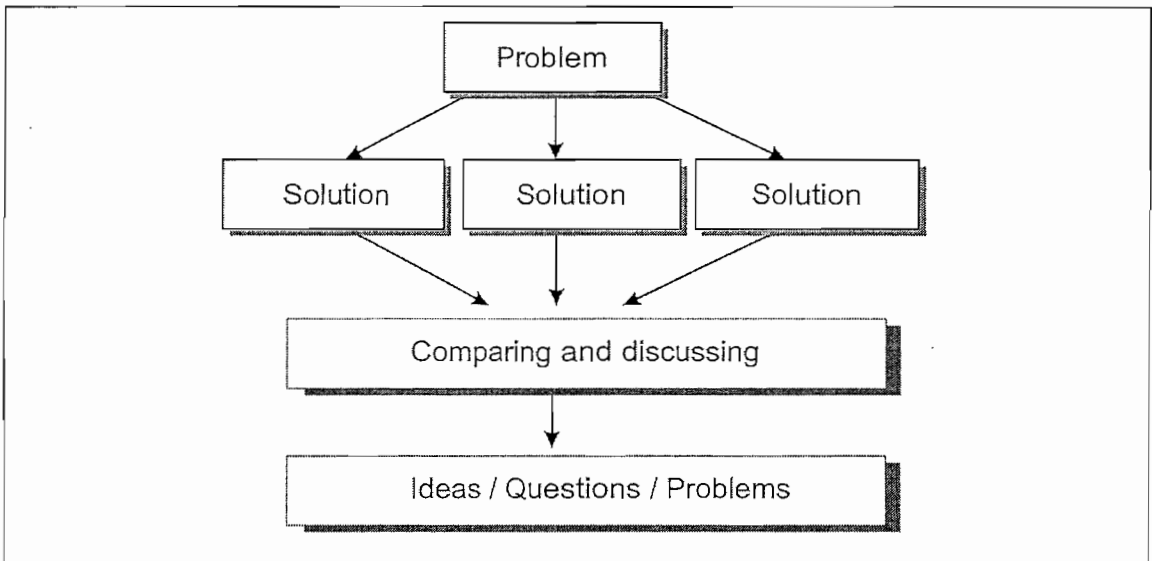


FIGURE 3.2 Targeted math idea

An OEA is an approach in which the questions asked by the educator are determined by the solution process of the learner. It is characterised by the educator building on *what is in the head of the learner*. The role of the educator here is to facilitate the learner’s attempt to solve the problem. The use of *open-ended questions* is intended to help the educator facilitate the learner’s construal of knowledge, not to guide the learner in deducing what is in the educator’s head. The educator, focusing on the learner’s activity and response, adapts his or her own schematic representation to that of the learner’s level of understanding, and thus

infers the learner's needs' (Jaworski, 1994:27). A constructivist approach to teaching and learning is characterised by the use of open-ended tasks and/or questions, and it is thus OEA compliant.

An open-ended task refers to a task structure which allows learners to determine their own approach when solving problems (Moskal, 1997). Since the requirements of open-ended tasks may resemble realistic problems that are external to the school setting, activities that appear in open-ended form are considered to be appropriate to the preparation of learners to be contributing members of society (NCTM, 1995). Problems that emerge outside the school setting are likely to be open-ended (Moskal, 1997). In open tasks, pupils are given freedom in the solving of the task (Pehkonen, 2007). Open-ended problems can even be described as 'ill-structured' because they comprise missing data or assumptions with no fixed procedures that guarantee a solution (Foong, 2002). An open-ended approach can also be defined in terms of varied problem-solving strategies. A problem occurs when learners are confronted with a task, which is usually given by the teacher, and there is no prescribed way of solving the problem. It is generally not a problem that the learners can immediately solve it (Nohda, 2000).

Our discussion thus far has justified the conclusion that learner-centredness in an open-ended approach encompasses the following:

The learners:

- (a) Take the initiative in solving mathematical problems and do not depend on the teacher;
- (b) Determine their own approach when solving problems (Moskal, 1997);
- (c) Express their own ideas more frequently when solving mathematical problems.
- (d) Modify other learners' ideas;
- (e) Can stimulate the exploration of concepts and ideas and facilitate creative and critical thinking processes.

Open-ended questions involve complex thinking and require a great deal of explanation and detail in their answers and probably time to think and reflect. They are used to build up information, to allow for more personal responses and to generate further discussion and questions. In an open-ended classroom environment, participants build on, shape and modify one another's ideas. We would hear the kinds of questions, answers, hypotheses,

ponderings and explanations which reflect the nature of inquiry as open-ended, yet shaped by logic which is both general and specific to each discipline or subject (Painter, nd).

In summary, there are three senses – not mutually exclusive - in which an open-ended approach can be used in the teaching and learning of mathematics:

- (a) The sense in which learners respond to multiple-approach single-answer mathematics questions.
- (b) The sense in which learners respond to a mathematics question with many possible answers.
- (c) The senses of learner-centredness in a mathematics class in which the teacher's role is to respond to learner mathematics solutions through prompting questions. Prompting questions from the teacher like 'What do you think?', 'Explain...', 'How do you...?' . characterise the open-ended approach. However, it is not necessarily words that define the open-ended approach, but rather the classroom environment created.

In all of the above contexts, the role of the teacher was facilitative, rather than prescriptive.

3.3 MATHEMATICAL EXAMPLES OF OPEN-ENDED QUESTIONS.

In order to illustrate the above-mentioned three senses of an OEA, various examples of open-ended questions are given.

3.3.1 Sum of numbers (Zevenberg, 2001:5)

One can illustrate an open-ended question with multiple correct answers using a mathematics example. Where a closed question typically has one correct response – for example, 'What is the sum of 3,5 and 10?' – an open-ended question is one where there are multiple correct answers and learners can answer at a level that is appropriate to, and represents, their current level of understanding. An open-ended task that is similar in content to the previous example could be 'What three numbers add up to 18?'

3.3.2 Sequences

This example is given with a view to stress Painter's (nd) point that open-endedness is not necessarily described in terms of words, but rather the classroom environment created.

Consider the following question: 'What terms follow in the sequence 2, 4...?'

INCOMPLETE SEQUENCE	COMPLETED SEQUENCE
2; 4;.....	

This apparently closed question may be open-ended, because in the process of addressing the question, learners may come up with different answers.

INCOMPLETE SQUARE	COMPLETED SQUARE
2; 4;.....	2, 4, 7, 11, 16, ... (Add successively 2; 3; 4; ...) 2, 4, 6, 8, 10, ... (Add common difference of 2) 2, 4, 8, 16, ... (Multiply by common ratio of 2)

Some learners may view the sequence as a sequence with difference between the first two terms being 2, the difference between the second and third term being 3, so that they can continue the sequence as 2, 4, 7, 11, etc. Others may view the sequence as an arithmetic sequence with 2 as the common difference, while others may view the sequence as a geometric sequence with 2 as the common ratio. What makes the question open-ended is the fact that there are multiple answers to the question. In other words, a suitable environment in which sequences – arithmetic and geometric - can be introduced would be to ask an 'open-ended' question like 'Add more terms to the following sequences':

The following could be the possible responses:

Learners can be required to explain their observations. What would have to be expected is that in the course of their explanation, categorisation of the sequences would follow. This would lay the basis for distinguishing between arithmetic and geometric sequences. However, requiring learners to add three terms to a sequence 2, 4, 6, 8is engaging in a closed task, because there is only one answer to the question, namely 2, 4, 6, 8, 10, 12, 14. In contrast, requiring extension of the sequence 2, 4, by three terms is, as shown above, open-ended.

3.3.3 Broken Calculator Problem (Mewborn *et al.*, 2005)

The broken calculator problem is an open-ended problem by virtue of having multiple solutions. Mewborn, *et al.*, (2005:413) introduced the calculator problem by posing the question: ‘How would you make a calculator display the number 75 if the 5 key was broken?’ They refer to such a problem as open-ended because there are multiple solution strategies and multiple correct answers. They also talk about the other questions that this problem can be modified to, like:

- How would you get your calculator to display 75 if all odd-numbered keys were broken?
- How would you get your calculator to display 75 if the 5 key and the + key were broken?

They claim that the problems bring into question the learners’ understanding of place value and the operation of additions. Open-ended problems elicit reasoning, problem-solving, and communication.

3.3.4 Area (Adapted from Mewborn *et al.*, 2005)

Consider the following question: ‘Draw a rectangle whose area is twice the area of the rectangle below’ (inches have been changed to centimetres).

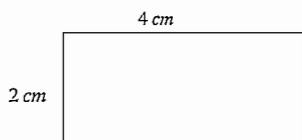


FIGURE 3.3 Area problem

The problem requires that learners understand area as well as the relationship between the two linear measures (length and width). If the directions had been more explicit by asking learners to first find the area of the new rectangle and then provide possible dimensions, the problem would have been less open, less challenging, and less revealing of learners’ thinking. If the problem had been more open by merely asking learners to produce rectangles with an area greater than that of the given rectangle, it would have been less challenging and less revealing of learners’ thinking about the relationship between area and the linear measures that comprise the area measure (Mewborn, *et al.*, 2005:414).

3.3.5 First 'Terrible Tommy' problem (Mewborn, *et al.*, 2005:414)

It appears that the context of openness characterising this problem is the one in which learners express themselves, and it seems to be consistent with the third sense of open-endedness (*See last part of 3.2.*) especially if the teacher allows the self-expression through asking relevant non-restricting questions. Giving reasons for or against a given solution creates a platform for self-expression by the learner.

The question posed was: 'What is wrong with Terrible Tommy's reasoning?'

$$\begin{array}{r} 70 \\ - 53 \\ \hline 23 \end{array}$$

Many learners did not understand why they had to explain if they 'just knew' the answer. Getting learners to explain what they were thinking was initially difficult, but has since become easier as a result of using open-ended assessment items.

3.3.6 The car park problem (Cheng, 2001)

The car park problem that was discussed *as a model-eliciting problem (Section 2.5.2.)*, is such that if a learner is able to solve it, then the learner:

- Brought own personal meaning to bear on a problem.
- Developed conceptual systems (i.e. models) for making sense of real-life situations where it is necessary to create, revise, or adapt a mathematical way of thinking (Lesh, & Zawojewski, 2007:783).

Such a learning situation would have satisfied the majority of the criteria on the table above. In other words, the solution process by the learner would have been compatible with an OEA. It is again the author's contention that the second 'mathematical modelling' problem, described below, would also use the OEA criteria.

3.3.7 Biggest box problem (Cheng, 2001)

Suppose we intend to make an open-top box using a square piece of card of side s by cutting a square (of side, say, x). The resulting piece is then folded to form a box.

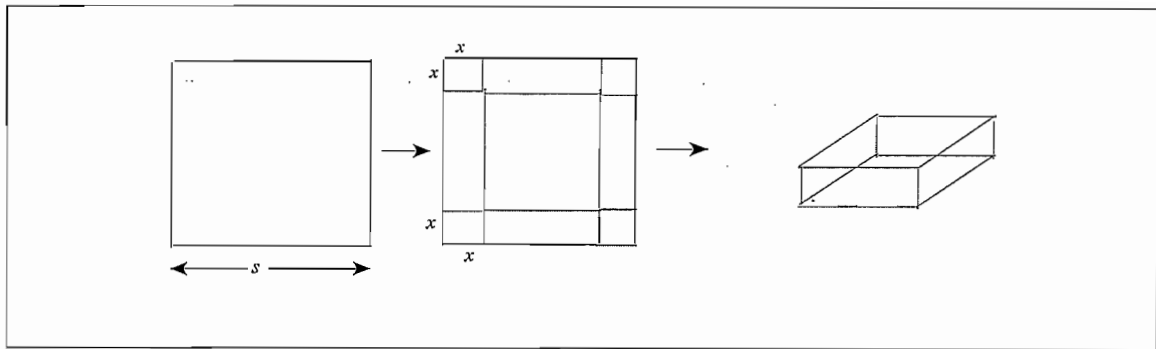


FIGURE 3.4: Biggest box problem

The question is: What should x be if we wish to make the biggest box in terms of volume? (Cheng, 2001)

There are several approaches to this problem. Here, two are described (Cheng, 2001). Note that Cheng's comment that there are several approaches to this problem qualifies this problem as open-ended. Cheng describes the two approaches below:

3.3.7.1 Empirical approach

The empirical model involves actually constructing the boxes and taking measurements. This has to be done systematically just like in performing a scientific experiment. Since we are particularly interested in the relationship between the size of the smaller square (i.e. x) and the volume of the box, we systematically make boxes using different values of x . The sides of the box can then be measured and volume calculated for each case. Alternatively, the volume may be estimated by first pouring sand to completely fill the box. The amount of sand used can be measured using a measuring cylinder. Still another variant could be to weigh the sand instead. Whichever approach is used, the result can be presented in the form of a graph (Cheng, 2001) in Figure 3.5.

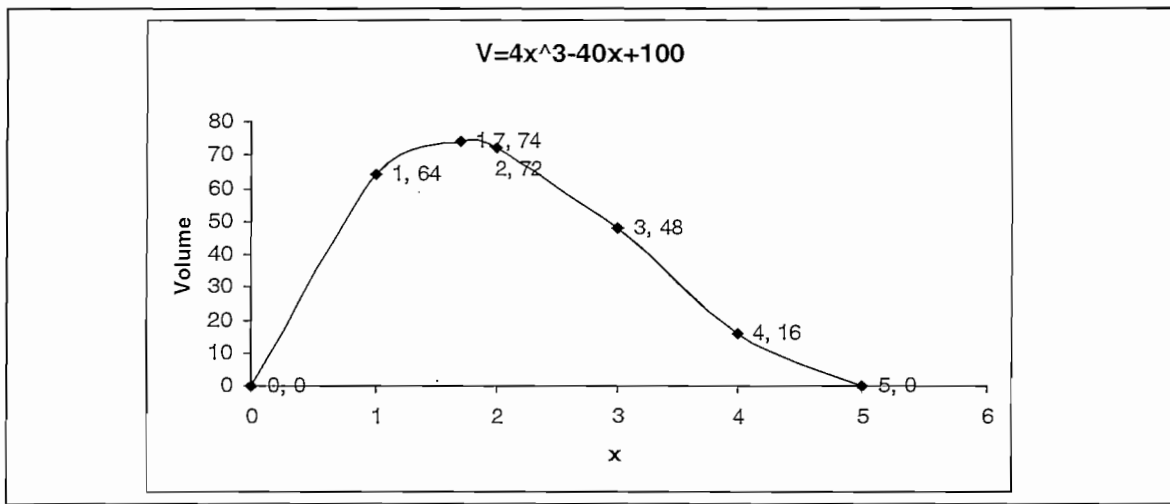


FIGURE 3.5: Graph of box problem

3.3.7.2 Analytical approach

An analytical or theoretical model may also be constructed to solve the problem. This approach is more abstract and involves the use of algebra and geometry. We model the box by a geometric diagram as indicated in figure 3.4 above. We then find the volume of the box in terms of the dimensions s and x : $V(s, x) = x(s - 2x)^2 = 4x^3 - 4sx^2 + s^2x$

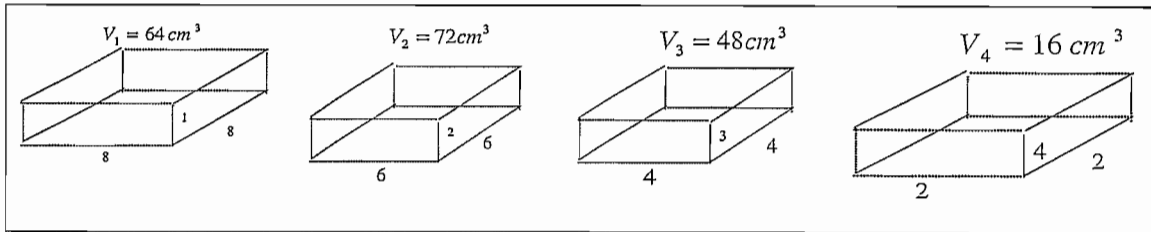
Suppose the original square cards have sides of dimension, say, $s = 10\text{cm}$. Then we have $V(x) = 4x^3 - 40x^2 + 100x$.

This is perhaps a good point at which to introduce the cubic function. In this particular case, the function models the relationship between the volume of the box and the size of the cut-off square. It now remains for us to find the value of x that makes V maximum. How this is done depends on the mathematical ability and maturity of the learner. For instance, a student familiar with calculus may choose to find the derivative and the turning point of the function to obtain the maximum. Another may use a graphical tool to plot a graph of V against x (See figure 3.5.) to estimate the maximum (Cheng, 2001).

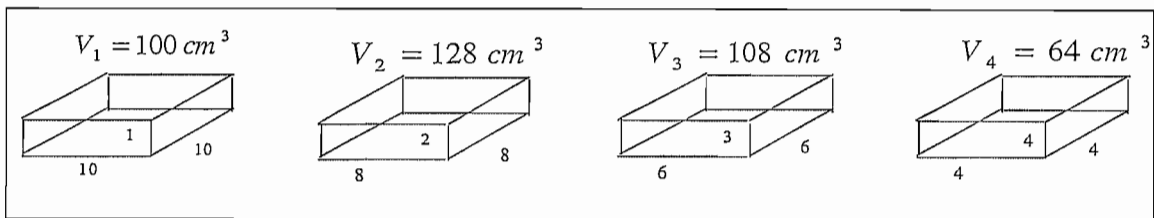
3.3.7.3 Some remarks about the Big Box Problem

One cannot simply dismiss a mathematical problem or task as being ‘not open-ended’ without looking at the expectation on the learner as far as attempting to solve the problem is concerned. For instance, a reader can argue that the “Biggest box” problem can be viewed

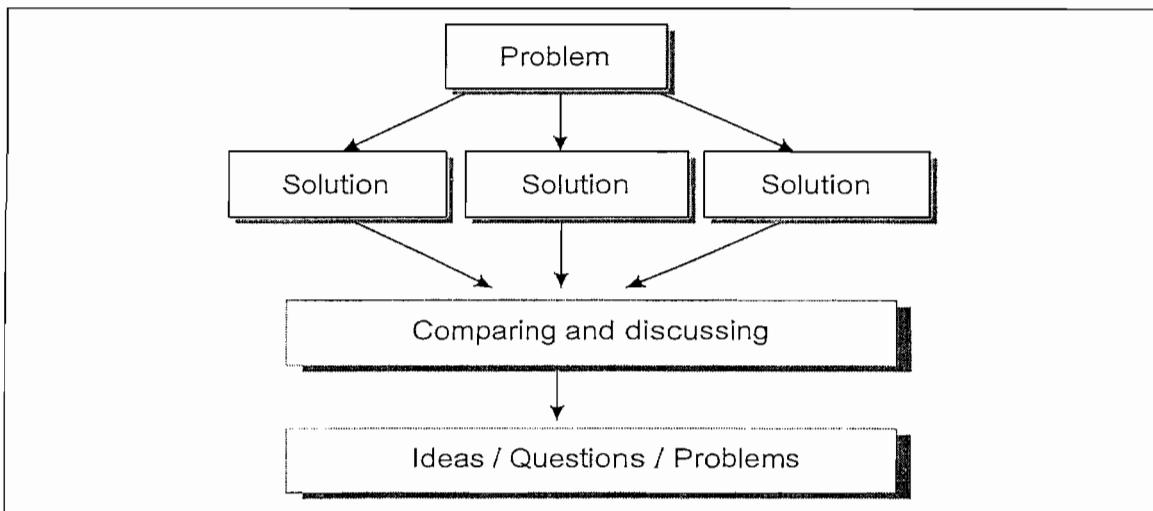
as a closed question on the basis that all learners could come up with the same solution, indicated below, to conclude that the required volume is $72cm^3$:



A counter-argument could be that one learner may not even think of going beyond the first arrangement $V_1 = 64cm^3$, and then conclude that the required volume is $V_1 = 64cm^3$. The other argument could be that another learner may have assumed that $s = 12cm$, thus giving a different sets of answers:



The third argument could be that even if, say, all learners agreed to use $s = 10cm$, it would be possible for one learner to have, instead of all four solutions, only V_1, V_3 and V_4 while the other could have V_2, V_3 and V_4 . In that case the model below could be used to resolve the impasse:



Lastly, even if all four learners obtained the same four solutions and came to the same conclusion, whether the problem is really open-ended or closed does not depend on these same solutions, but rather on whether the learners came up with the solutions *on their own* or through the guidance from the teacher. If the learners came up with the same solutions, independently of each other, on their *own accord*, then the questions they were answering could not necessarily be assumed to be closed. The point here is that closed questions will always elicit same answers, but same answers do not necessarily imply closed questions.

3.3.8 Closed versus Open-ended items (Unknown (nd))

The examples from this unknown author are taken from a website that is specified in the reference section of this thesis. On the website is the article titled *Open-ended assessment in math, a searchable collection of 450+ questions* (<http://books.heinemann.com/math/construct.cfm>) . There are more than 450 open-ended questions. Table 3.1. gives some of the mathematical examples :

TABLE 3.1 Closed versus open questions

ORIGINAL CLOSE-ENDED ITEM	REVISED OPEN-ENDED ITEM
Which of the following numbers are prime? 7, 57, 67, 117	Fred thinks that 57 and 67 are prime because they both end in 7, which is a prime number. Dick says he is wrong. Who is correct and why?
What are the next three numbers in the following sequence? 1,4,7,10,13,...	Consider the following sequence: 1,4,7,10,13,...Is 100 a member of this sequence? Explain your reasoning.
Round 37,67 to the nearest 10 th .	Generate three different numbers that when rounded to the nearest 10 th give 37,7.
Find the LCM of 18 and 24	Why can't 48 be the LCM of 18 and 24?

In conclusion what determines whether a question is open-ended or not depends on what is expected of the learner in terms of solving it, and not on the words used to formulate the question.

3.4 THE USE/ROLE OF QUESTIONS IN AN OEA

Open-ended questions are those that promote open-mindedness and invite many answers or possibilities. They can stimulate the exploration of concepts and ideas and facilitate creative and critical thinking processes. These are the kinds of questions that challenge learners and

their thinking. A good question should breed more questions and the desire to find answers. Good questions need to take the learner beyond the recall of basic information and they should be challenging. However, it is important to ensure that the questions are appropriate to the learning situations and that they allow learners to build on their prior knowledge and experience so they can make connections (Jaworski, 1994).

An educator who uses an OEA tries to elicit explanations from the learner by asking questions such as ‘What do you mean?’, ‘How does that relate to...?’, ‘How did you come to that conclusion?’ etc. (Jaworski, 1994). The following are examples of six types of open-ended questions, as articulated by Painter (nd). The questions in italics are from Unknown-1 (2003).

TABLE 3.2 Painter’s open-ended questions

Category of questions	Examples
1. Questions of clarification.	<ul style="list-style-type: none"> • What do you mean by that? • Can you give an example?
2. Questions that probe assumptions	<ul style="list-style-type: none"> • What is being assumed? • Why would somebody say that? • <i>Why do you suppose....?</i>
3. Questions that probe reason and evidence.	<ul style="list-style-type: none"> • What are your reasons for saying that? • What criteria do you base your argument on? • <i>What patterns do you see?</i> • <i>Why do you think that.....?</i> • <i>What evidence do you have?</i> • <i>What reasons did you have...?</i>
4. Questions that probe implications and consequences.	<ul style="list-style-type: none"> • What might be the consequences of behaving like that? • <i>What else might have caused....?</i>
5. Questions about viewpoints or perspectives.	<ul style="list-style-type: none"> • How do Maria’s ideas differ from Peter’s?
6. Questions about the question.	<ul style="list-style-type: none"> • How is the question going to help us? • Can you think of any other question that might be useful?

However, one should be cautious against using the questions as descriptors of open-ended and closed questions. What really produces closure [or open-endedness] is neither the question nor the answer but the environment in which questions are considered (See some of the mathematical examples above). If the environment encourages the formation of questions as an important activity in its own right, and if it encourages learners to use a variety of strategies regarding questions and activities as a step to further inquiry, then even questions that sound closed may be open (Painter,nd). Unknown-1 (2003) suggests the following strategies to ignite classroom discussions and support active thinking and reasoning:

- Ask open questions that encourage observation, reflection, evaluation and new questions.
- Minimise factual questions that have just one right answer or those that require yes or no.

3.5 CHECKLIST FOR AN OEA

Following these paragraphs one can compile an OEA checklist to see if an approach is open-ended:

TABLE 3.3 An OEA checklist

DOES THE CLASSROOM APPROACH:	YES	NO
1. Produce the learners who can <i>develop themselves mathematically</i> ?		
2. Create an environment in which the teacher adapts – through asking open-ended questions - his or her own schematic representation to that of the learner's level of understanding, and thus infer the learner's needs?		
3. Encourage posing of problems with one solution but diverse approaches or single-answer problems with multiple correct answers?		
4. Stimulate the exploration of concepts and ideas and facilitate creative and critical thinking processes?		
5. Encourage questions appropriate to the learning situations and allow learners to build on their prior knowledge and experience so they can make connections?		

DOES THE CLASSROOM APPROACH:	YES	NO
6. Allow learners to determine their own approach when solving problems?		
7. Encourage questions that are within the learners' abilities?		
8. Encourages learners to express <i>their</i> ideas frequently?		
9. Makes it possible for every learner to respond to the problem in some significant ways of his/her own?		

3.6 OPEN-ENDEDNESS IN A NCS ENVIRONMENT

OBE and the OEA learners have a lot in common. The following table summarises some of their characteristics:

Table 3.4 OBE versus the OEA learner

OBE LEARNER:	OEA LEARNER
<ul style="list-style-type: none"> ○ Develops him-/her-self mathematically (DoE, 2005:8) 	<ul style="list-style-type: none"> ○ Determines own approach when solving problems (Moskal, 1997)
<ul style="list-style-type: none"> ○ Takes control of his/her learning with limited intervention by the teacher (Taylor and Vinjevoid, 1999:108) 	<ul style="list-style-type: none"> ○ Takes the initiative and does not depend on the teacher to solve mathematical problems.
<ul style="list-style-type: none"> ○ Is a critical and creative thinker and works effectively with others (see table 1.1) 	<ul style="list-style-type: none"> ○ Expresses own ideas more frequently when solving mathematical problems. ○ Modifies other learners' ideas. ○ Can stimulate the exploration of concepts and ideas and facilitate creative and critical thinking processes.

It should be obvious to the reader that the OEA is, to a large extent, compatible with the NCS approach.

3.7 OTHER TEACHING AND LEARNING ENVIRONMENTS THAT ARE OEA-COMPLIANT

It has already been indicated that a socio-constructivist approach to teaching and learning requires the use of open-ended questions and/or tasks. The following are other teaching and learning environments in which an open-ended approach can be used.

3.7.1 Realistic Mathematics Education environment

Following Freudenthal (1983), the realistic approach stresses the idea of mathematics as a human activity. The most important part of mathematics as the human activity is mathematising. This mathematising has two components: horizontal and vertical components. Horizontal mathematising, concentrates on 'reality', finding examples from real-life with similar mathematical structures. Vertical mathematising, on the other hand, focuses on developing mathematical structures (axioms, definitions, theorems, rules *etc.*). The realistic approach stresses both the horizontal and vertical components of mathematising (Oldham *et al.*,1999).

The horizontal and vertical mathematising in realistic mathematics arise from adapting to the learner's point of view - What should learners do to 'reinvent' mathematics? For this, contexts, in particular contexts from real life, are essential. Contexts have three roles. The first role is context as an area of application: applying mathematics to real-life situations. By applying their mathematics, learners are prepared to use mathematics in out-of-school situations. The second role is that of context as a source of mathematics – exploring mathematics in real life. The third is context as a tool or a support for vertical mathematisation – helping learners to develop mathematical structures (Oldham *et al.*,1999).

3.7.2 Environment of appropriation of mathematical practices

Appropriation of mathematical practices is compatible with a view of *learners as actively constructing knowledge*. During appropriation an expert interprets a student's cognitive product within the student's knowledge framework and subsequently engages the student in an activity reflecting this expert understanding of the situation (Moschkovich, 2004:51). An expert uses a learner product (an action or a statement), and engages the learner in that new task. Evidence for the success of the appropriation process lies in relating subsequent actions carried out independently by the novice to the expert framework originally revealed through joint productive activity. Radford (2001:251) has used the notion of appropriation to

focus on the individual appropriation of 'technical mathematical expressions' that include recreation.

Since all of the above contexts require the use of an open-ended approach to teaching and learning, we shall, for the discussion below, use an open-ended approach to teaching and learning to describe each context. For instance, each time constructivist approach, competence approach, inquiry approach, appropriation approach, for instance, are referred to, an open-ended approach to teaching and learning should be assumed to be implied.

3.8 PERSPECTIVES ON AN OPEN-ENDED APPROACH.

Here we focus on the mathematical communities who are employing an open-ended approach, with interest being on whether those communities consider open-ended approach to be successful or not.

- In Finland, the idea of an open-ended approach has received wide-spread acceptance in teacher in-service courses, in educators' journals, and in teacher pre-service education for more than twenty years. The leading idea has been to increase openness and creativity in mathematics teaching (Harms, 2007).
- Harms (2007:1.), described a conference that was to be held in 2007:

'Professor Becker shows educators how to use an open-ended approach to teaching, in which educators encourage learners to find a variety of ways to solve the same problem'.

Subsequent to the conference Harms (2007) reported that Marie Schilling, a retired fourth-grade teacher...said she felt more confident as a teacher and began using problems that Becker had suggested. The problems, which are more like puzzles than conventional story problems, prompted her learners to come up with multiple solutions, each tailored to their individual understanding of how to go about solving a mathematics problem.

Among the many educators who found the programme rewarding was Marvin Neely:

'Because of our participation in the program, fear of teaching math subsided and the results were that in the past five years our school went from having below 20 percent of

our learners at or above grade level on the ISAT mathematics test, to over 50 percent' (Harms, 2007:1)

- 'During 1970s and 1980s, open-ended approach had emerged as a method to reform mathematical teaching of Japanese classrooms and has been spreading around the world. Open-ended approach... has become an innovative mathematics teaching to improve teacher education in Thailand. Using open-ended approach in order to create **a rich mathematical activity** [emphasis by Inprasitha] is the most important part of making the first study lesson' (Inprasitha, 2006:1). However, the OEA does not suit all classroom cultures:

'... Thai educators are familiar with introducing new contents to learners through examples and exercises. It is very difficult for them to organise many mathematical concepts into a problem situation, which is an important part of open-ended problems. This kind of problem situation has to be formulated so that mathematical activity can be naturally generated from it' (Inprasitha, 2006:101)

- The 1990s saw growing support for open or process-based forms of learning of mathematics where learners were engaged in open-ended, practical tasks (Chan *et al.*, 2001).

3.9 ADVANTAGES OF AN OEA

There are 5 advantages of an open-ended approach to teaching and learning (Takahashi,nd):

Firstly, learners participate more actively in lessons and express their ideas more frequently. Usually learners work in small groups, exchanging ideas, assessing each others' opinions, and reaching consensus. Using open-ended questions could lead to benefits like that of learners solving realistic problems when incomplete information was given, in that they were required to make their assumptions on the missing information (Van den Heuvel-Panhuizen, 1996).

Secondly, learners have more opportunities to make comprehensive use of their mathematical knowledge and skills. When learners learn mathematics through such a problem-based approach, struggling with the difficulties facing them instead of relying on memorisation or any pre-determined rule to search for solutions, it promotes 'deep understanding' of the mathematics that is valued (Hiebert, *et al.*, 1996).

Thirdly, every student can respond to the problem in some significant way of his/her own. 'The only important reality is in the learner's mind, and the goal of learning is to construct in the learner's mind its own, unique conception of events.' In a constructivist approach, 'the learners actively participate in their learning process by discovery, with the instructor as the mediator of the process' (Carswell, 2001:3). The collaborative effort of finding solutions creates a platform for each learner to be heard.

*Fourthly, the open-ended problems provide every student with a reasoning experience.'*Using constructivist principles, a teacher may develop discussion topics that are open-ended enough to allow the individual learner to incorporate individual experiences, interpretations, reactions, and opinions into discussion responses' (Carwile, 2007:1). Consensus is usually preceded by sometimes robust debates that are characterised by a need for reasoned justifications.

Lastly, there are rich experiences for learners to have the pleasure of discovery and to receive approval from fellow learners. Hodgson and Watland (2004:1), in talking about an OEA, said: 'Through groups and other learning interactions with their online peers, learners acquire deeper understanding because of the opportunities for exposure to multiple perspectives and interpretations'.

'I have noted significant improvement in my learners' self-confidence and their willingness to share their thinking with others. In fact, they begin to take pride in their explanations and find satisfaction in being able to explain what they are doing and why. They begin to see that there is a point to explaining their thinking. This leads to learners feeling more ownership of their mathematical learning' (Mewborn, *et al.*, 2005:416).

3.10 POTENTIAL BARRIERS TO SUCCESSFUL IMPLEMENTATION OF AN OPEN-ENDED APPROACH TO TEACHING AND LEARNING

There are possible factors that can impact negatively on the implementation of an OEA to teaching and learning. The following are the main ones.

3.10.1 Learner background

3.10.1.1 Expected learner initiative

The open-ended approach to teaching and learning relies on the learners being able to think for themselves and to solve problems – both academic problems and interpersonal problems (De Bono, 1996). It is important for learners to have appropriate thinking skills, that is, ways of moving from one arrangement of knowledge to a better one. Inability to think for themselves can prove to be a potential barrier for successful implementation of an open-ended approach to teaching and learning.

Another difficulty lies within group dynamics. Quieter learners lessen the value of group work. It is not possible for the teacher to monitor the involvement of each learner in the class. The perception of the teacher as the source of truth can also hinder progress. The teacher's presence is often sought to clarify thoughts to determine if the learners were 'on the right track' in getting 'the right answers'.

3.10.1.2 School culture

Learners who are not from English speaking, middle-class backgrounds have less synergy with the culture of schools and classrooms than their peers from such backgrounds (Lamb, 1997). There is less chance of them being seen as effective learners since they cannot crack the code of classroom life. When learners are given directives, they may respond in ways different from the expectation of the educator because of cultural misinterpretation (Zevenberg *et al.*, 2001). The fundamental distinction between the formal knowledge of schooling and everyday knowledge, according to Taylor and Vinjefold (1999), is well illustrated by research undertaken by Bernstein (1996). Two groups of seven-year old children from the same school, one from middle-class homes and the other of working class origin, were given a series of cards showing pictures of the food on offer for school lunch. After making sure that the children recognized the pictures, they were asked to group those pictures which they thought belonged together: they could use all or only some of the cards, and they could use any reason for grouping which they saw fit.

Working-class children predominantly used criteria drawn from their own life contexts as a principle for classification ('I have this for breakfast', 'I cook this for my mum'). For these children the reason for grouping is embedded in the local context and personal experience of the learner. Middle-class children, on the other, were far more likely to use as their principle of classification something the pictures have in common ('They come from the sea'; 'They

are vegetables'). The children were then asked to put the cards together in another way. This time a significant number of the middle-class children switched their classificatory principle to one based on local context and experience, while the working-class children merely used another reason based on their personal experience. In short, middle-class children have access to two principles of classification: one formal and specialized (school knowledge) and the other personal and localized (everyday knowledge). Working-class children have access only to non-specialised principles of classification, based on their personal experience.

Middle-class children, because of factors such as the kinds of conversations which occur in their homes and social circles, and access to books, computers, travel and other sources of information and experience, have ready entry into the principles which underlie school knowledge. Language of middle-classes tends to be rich and embellished, according to Zevenberg *et al.* (2001), whereas the language of the working class tends to be more functional. This is important because open-ended tasks involve the considerable use of language. Middle-class parents are more likely to interact with their children in ways that resemble the school context when undertaking pseudo-school work. In contrast, working-class families are less likely to engage in such patterns of interaction.

In the majority of South African schools, English is the medium of instruction. Most of the learners do their subjects, including mathematics, in English, and not in their own languages. The type of mathematical problems posed may not fall within the learner's cultural experiences.

3.10.2 Educator factor

There are also some barriers that Takahashi (nd), mentioned, such as difficulty of posing problems successfully and difficulty of developing meaningful problem situations. While the open-ended approach to teaching and learning requires that educators play a more covert role than they would in teacher-centred models, they nevertheless make far greater demands on the teacher (Taylor & Vinjefold, 1999:116). A competence curriculum is likely to require high teacher-training costs because of the sophisticated theoretical base of competence (Bernstein,1996).

To be effective learners of mathematics, learners must construct knowledge that resembles that which is seen as legitimate in the field. In this setting, learners are not rewarded for the construction of their own meaning, but rather, for the construction and reproduction of

'legitimate' [or conventional] knowledge (Zevenbergen, 1996). Sometimes a teacher, despite moving towards a constructivist [open-ended] position, is nevertheless thinking from within an absolutist [closed] perspective. This results in a tension coming from a desire for learners to discover particular mathematical 'facts', and the reluctance of the teacher to 'tell' these facts when it seemed that the learners were not going to discover them in quite the form the teacher desired.

'The issue of what and when to tell had been important for me and for the two educators' (Jaworski, 1994:85-86).

The educators referred to were those whose constructivist lessons were observed by Jaworski. It seemed that 'telling' indicated a lapse back into 'transmission' teaching, and was therefore something to be avoided. There is the realisation that expecting learners to discover everything for themselves was equally not making sense. It is becoming clearer that learners' construal of mathematics included making sense of what educators told them just as it included making sense of their own discoveries.

'This reinvention of the wheel cannot be a model for education. A responsibility of education is to enable learners to process the beliefs of others around them. The beliefs of educators are not an exception. A mistake is for learners to infer that the beliefs of educators are in some sense absolute and not open to processing and interpretation' (Underhill & Jaworski, 1991:32).

'This seemed to me to be part of the whole dilemma of trying to teach from a constructivist philosophy, that despite trying to behave like constructivists, we nevertheless find ourselves trapped within our own expectations regarding the outcomes of our teaching. The distinction is between existing comfortably within a limited knowledge base, compared to the insecurity of coping with uncertainty. The two poles seemed to correspond to the end of a transmission-constructivism continuum' (Jaworski, 1994:86).

In the South African context, there is the question of capacity. An OEA places a great deal of responsibility on the teacher. To be able to fill in the gaps in the mind of the learner, the teacher needs to be ready for unanticipated problems. This requires a great deal of subject expertise, and full command of the OEA. Furthermore, it requires a great deal from the teacher to actually prepare the material that will position him/her as the facilitator.

3.11 DEVELOPMENT OF A PROGRAMME FOR PRESENT STUDY

3.11.1 Introduction

Attempting to determine the impact of the OEA, the main focus of this study requires that the OEA be practised at the time of the investigation. For the educators from the experimental school to be implementers of the OEA would require a lengthy period of pre-investigation teacher in-service training in an OEA. Instead of expecting the educators to know how to use the OEA in class, the author produced worksheets that learners would use throughout the intervention period. A brief description of the worksheet follows.

3.11.2 The worksheet

The worksheet was used by the learners from the experimental school.

3.11.2.1 Introduction

Section 3.7 of this chapter identified some of the open-ended mathematics examples. These are the appropriate examples as far as clarifying the OEA is concerned.

Most of the mentioned open-ended examples are about items that are not addressed by grade 11 mathematics topics. As a result, the author was forced to think of open-endedness within the restriction of the mathematics topics which all the grade 11 mathematics learners in the district were expected to address. For instance, the first three topics that the learners were expected to engage with were:

- Factorisation of quadratic expressions;
- Completing the square;
- Drawing the parabola of the form $y = ax^2 + bx + c$.

3.11.2.2 Development of the worksheet.

A number of considerations went into the development of the worksheet. The main focus was consideration of OEA-compliance. The mathematics topics that grade 11 learners had to engage with during the investigation were of a conceptual nature, not ideal for illustration of an OEA. The study recognises that mathematical conceptual understanding is also

important. Conceptual understanding is mental connections among mathematical facts, procedures and ideas (Hiebert & Grouws, 2007:383).

Hiebert and Grouws, (2007:382) claim that a clear pattern across a range of empirical studies is that learners can acquire conceptual understanding of mathematics if teaching attends explicitly to concepts – to connections among mathematical facts. The author, in preparing the worksheet, focused on facilitating conceptual understanding, as far as possible, within the framework of the OEA. To ignite classroom discussions and support active thinking and reasoning, open questions that encouraged observation and reflection were asked. Factual questions that have just one right answer or those that require yes or no were not entertained (Unknown – 1, 2003).

Most of the exercises in the worksheet followed the pattern of Mewborn *et al.* (2005:414) First ‘Terrible Tommy’ problem. They were mainly based on explaining given solutions, in most cases the explanation was preceded by identification of a pattern in the table.

The following OEA learner-role checklist was uppermost in the author’s mind when developing the worksheet:

TABLE 3.5 OEA learner-role checklist

DO THE LEARNERS:	YES	NO
• Determine their own approach when solving problems?		
• Take the initiative in solving mathematical problems without depending on the teacher ?		
• Express their own ideas more frequently when solving mathematical problems?		
• Modify other learners' ideas?		
• Stimulate the exploration of concepts and ideas and facilitate creative and critical thinking processes?		

3.11.2.3 Main features of the worksheet.

The bulk of the worksheet attended to the relationships among mathematical ideas. The learners would investigate the given table with a view to identifying the pattern, and then investigate the pattern in order to generate conclusions. This is consistent with the NCS view of mathematics as the study of patterns. They would then apply their conclusions to the standard exercises. The following example highlights this point:

Example: Factorisation of quadratic expression

This is how the material on factorisation of quadratic expressions was presented:

If $a = 1$.

John uses the Table 3.6 to fill in the missing information and then concludes that

$x^2 + 2x - 15 = (x + 5)(x - 3)$. Phindile disagrees, concluding that $x^2 + 2x - 15 = (x - 5)(x + 3)$.

What do you think about John and Phindile's solutions?

TABLE 3.6. Factorising

EXPRESSION					FACTORS OF THE QUADRATIC EXPRESSION
i) $x^2 + 5x + 6$	5	6	3 and 2, -3 and -2, 6 and 1, -6 and -1	2 and 3	$(x + 2)(x + 3)$
ii) $x^2 - 7x + 12$	-7	12	6 and 2, -6 and -2, 4 and 3, -4 and -3, 12 and 1, -12 and -1.	-4 and -3	$(x - 4)(x - 3)$
iii) $x^2 + 3x - 18$	3	-18	9 and -2, -9 and 2, -6 and 3, 6 and -3, 18 and -1, -18 and 1.	6 and -3	$(x + 6)(x - 3)$
iv) $x^2 + 6x - 7$					
v) $x^2 + 3x - 10$					

b) The table summarises what is called *factorising quadratic expressions of the form $x^2 + bx + c$* . How do you explain factorisation of the general quadratic expressions $x^2 + bx + c$?

c) Factorise the following expressions:

i) $x^2 - x - 6$

ii) $x^2 - 2x - 8$

iii) $x^2 + x - 12$

iv) $x^2 + 5x - 6$

The exercises were the normal traditional exercises. For further exercises, see appendix 5.

Expected response

The educator must facilitate the process of successful identification of the patterns arising from the table. Through facilitation, the learners must be able to see that the second column represents coefficient of the middle term, and the third the constant term. They must further see that the fourth column represents factors of the constant term, and that the fifth column represents factors of the constant whose sum is the coefficient of the middle term. The last column represents factors of the quadratic expression

The main reason the author felt that these exercises – questions (a) and (b) - complied with an OEA to teaching and learning was the assumption that the facilitative role of the teacher would ensure compliance with the checklist items on table 3.4. The author and the teacher from the monitored experimental classes spent some time brainstorming the possible learner responses in order to establish how the teacher would respond to the learners' attempts to solve the problems. It was also agreed that the initial lessons would be led by the author, so that the teacher could get the relevant exposure to the approach. Would the teacher solve the problem for the learners or use prompting questions to interrogate learners' solutions?

In other cases learners would be asked to explain a particular learner's solution. To check if this problem is open-ended, one could contrast it with the 'Terrible Tommy' problem (Mewborn, *et al.*, 2005:414) (See section 3.3.5.). It might have surprised the reader that the 'Terrible Tommy' problem is open-ended. The instruction 'Identify the wrong in Terrible Tommy's reasoning' sounds 'closed'. What qualifies it as open-ended is the part of encouraging learners to express their ideas freely from teacher influence by giving their reasons for Terrible Tommy's 'wrongness'. In the same way, in explaining a learner's solution, learners could express their ideas freely.

3.11.3 Conclusion

In general, the pattern followed in the compilation of the worksheet can be summarised as follows:

- A table is presented to the learners with a view to them identifying the pattern.
- Open-ended questions are asked with a view to expecting the learners to discover the mathematical procedure, rule, concept or principle by themselves, using the pattern.

CHAPTER 4

RESEARCH DESIGN AND METHODOLOGY

4.1 INTRODUCTION

In chapters 2 and 3 current approaches to mathematics teaching and learning on one hand, and an open-ended approach to mathematics teaching and learning on the other, were looked at. It was established that current approaches still entail teacher domination. In contrast the OBE or NCS-envisaged approach is learner-centred. In this chapter the impact of an open-ended approach is investigated, both empirically and qualitatively. Four experimental classes in one school were selected to follow an open-ended approach and two control classes from another school were selected as a control group in order to compare with those from the experimental school. Data was collected from these classes to answer the following research questions:

- (a) *What will be the impact of an open-ended approach on the learning of mathematics in grade 11 mathematics classes at the selected experimental school?*
- (b) *How will mathematics learners in the 'open-ended approach' experimental classes respond to such an approach?*
- (c) *How will mathematics educators in the 'open-ended approach' experimental classes adapt to an open-ended approach?*

As far as the first question (a) is concerned, there were two contexts in which 'impact' was established. The first one was in terms of *learner performance* in the tests, while the second was in terms of the learners' *post-intervention view of mathematics learning* as a result of exposure to the intervention. The latter was established through a questionnaire, and it was intended to find out if it corroborated the former. In the second question (b), the response of the learners was established through verbal expression of their attitude to the open-ended approach. The verbal communication was done during an interview. In the third question (c), the educators' interview responses were used to establish their adaptation to the open-ended approach to teaching and learning. These ideas are summarised in table 4.1., based on example by Summer and Tribe (2004:11).

TABLE 4.1 Design of the research

RESEARCH DESIGN	
RESEARCH QUESTION	DATA COLLECTED TO ADDRESS THE QUESTION
<i>“What will be the impact of such an approach on the learning of mathematics in experimental classes?”</i>	<i>Quantitative data:</i> <ul style="list-style-type: none"> • <i>Test marks (Phase 1).</i> • <i>Questionnaire (Phase 2)</i>
<i>“How will mathematics educators in experimental schools adapt to such an open-ended approach?”</i>	<i>Qualitative data:</i> <ul style="list-style-type: none"> • <i>Interview of educators (N = 2) from the experimental school (Phase 3)</i>
<i>“How will mathematics learners in experimental classes respond to such an approach?”</i>	<i>Qualitative data:</i> <ul style="list-style-type: none"> • <i>Interview of learners (N = 17) from the monitored experimental classes (Phase 3)</i>

4.2 APPROACHES IN THE CLASSROOMS

In general, the study focused on the impact of ‘the open-ended approach’ on teaching and learning, either in terms of how educators and learners would respond to it or how learners would perform when using it. The fact that all classes in the experimental school were using open-ended approach learner material was not enough to guarantee that the classroom proceedings were compliant with the approach. There was still a question of the teacher’s intervention role. If the teacher was using an open-ended approach material *inappropriately* - dominating the classroom proceedings in contravention to the expectations of the approach - then the character of the class would be that of teacher-centredness (as described by teacher role) and *not compliant to the* open-ended approach (as necessitated by the learner material). The impact of the intervention can only be attributed to the open-ended approach if it can be established that classroom dynamics (as determined by teacher role, learner role and learning material) of the better performing classes were reasonably more compliant with the approach itself, while those of the low performing groups were not complying with the approach.

In this study, four grade 11 mathematics classes from the experimental school were divided into two groups, namely two monitored classes and two unmonitored classes. This came about as a result of the author's consideration of the probability that the educators in the experimental school may find it difficult, within the short intervention period, to be ready to appropriately implement the open-ended approach. Consequently the author decided to follow one willing teacher in the experimental school, giving him support throughout the intervention period, ensuring that an open-ended approach to learning was, as far as possible, adhered to. This the author did by initially demonstrating to the teacher how to implement the approach in the classroom. The demonstration was followed by brainstorming with the teacher on the possible learner responses to the open-ended questions or tasks, with a view to identifying possible teacher interventions in reaction to the learners' responses. The teacher that the researcher followed had two classes. The choice of one teacher at this experimental school was motivated by two factors. The first was the need for an intensive interaction with one teacher, hoping that the teacher would ultimately perfect his skills of implementing the approach. The second reason was to avoid possible time-table clashes associated with following different educators. The other two classes were conducted by a teacher who had been briefed about the approach, and monitored occasionally, but who was not followed up regularly enough for the author to be confident of his appropriate implementation of the open-ended approach to learning. This was the unmonitored class. In the end, the three different classroom situations – control, monitored experimental and unmonitored experimental - were separated mainly by the role the educators played from the open-ended approach perspective. The pre-test / post-test impact of the open-ended approach to teaching and learning was therefore measured in terms of the learner performance versus teacher's degree of implementation of open-ended approach to teaching and learning mathematics.

In order to establish the approaches to teaching and learning in the three groups of learners – monitored experimental, unmonitored experimental and control groups – a video recording of proceedings in each of the classrooms was made sometime during the period of intervention. The video recording was made without prior arrangement with the educators in those classes. The recordings are described below. The descriptions were confirmed by two independent researchers as being a true reflection of the video proceedings. Unfortunately, because of a confidentiality agreement with the educators and learners from the experimental and control schools, the video clip cannot be made available to the reader. The classroom approaches were looked at from the point of view of an open-ended approach as depicted in Table 3.4. on learner-role checklist in the previous chapter. The following are some of the extracts from the video clips of the classroom dynamics.

Unmonitored experimental classroom (See Appendix 6)

The following extract gives evidence of the classroom dynamics in the unmonitored experimental classroom.

Learners wanted to use 'completing the square' to solve $x^2 + \frac{5x}{3} = \frac{119}{36}$. The teacher called a volunteer to come and solve the problem. While the volunteer was busy solving the problem on the chalkboard, the other learners were listening and looking at the chalkboard. The teacher was moving around as if monitoring them. The first volunteer got the problem wrong. The teacher asked the class if the volunteer was correct. A second volunteer came with the idea to solve the problem. Again the teacher asked the class if the second volunteer's solution was correct. The learners did not respond. A third volunteer came. At that stage a group of late-comers arrived in the class. It was later established that a bus had been late. The teacher asked the third volunteer to explain to the class what he was doing. The learner had written $x^2 + \frac{5x}{3} = \frac{\sqrt{-119}}{6}$. After a bit of a struggle by the learner, the teacher asked the class if the solution was correct. It appeared from the video clip that the learners did not know. The teacher then required the learners to compare what was happening with what was in the worksheet. The teacher then briefly explained to the learners what 'solving for x ' meant. The teacher then advised the learners to follow what was in the worksheet. This instruction was not very clear to the author. Subsequently the teacher gave the learners 5 minutes to check if the solution on the chalkboard was correct. The teacher then moved around to find out what the learners were doing. If one compares the approach of the teacher with the envisaged open-ended approach, one realises that the teacher interpreted learner-centredness in terms of the learners focusing on the solution of one volunteering learner. This approach certainly did not comply with table 3.4. checklist (See chapter 3) of the open-ended approach to teaching and learning, despite the checklist having been given to all the educators in the experimental school during the briefing at the beginning of the study.

Monitored experimental classroom (Appendix 6)

In this classroom small groups of learners were busy solving the problems in the worksheet. There was an attempt by the teacher to comply with Table 3.4 (See chapter 3) of the checklist on the open-ended approach to teaching and learning. The teacher moved around, asking the groups to explain their solutions. Where the learners got stuck, the teacher either asked the group to review their solution or asked prompting open-ended questions like 'What do you mean by that?' 'Can you support your argument?' etc., to facilitate learner solutions.

When a group got the right solution, they would clap hands. The following is an example of what transpired between a group of learners - group 1 - and the teacher. The author was seated next to group 1, and was able to monitor the teacher's interaction with the group.

1. Factorising $ax^2 + bx + c$.

1.1 If $a = 1$.

a) Discuss what you observe, and then completely fill in the table 4.2., and then answer the questions below:

TABLE 4.2 Classroom approach: Quadratic factorisation

EXPRESSION					FACTORS OF THE QUADRATIC EXPRESSION
i) $x^2 + 5x + 6$	5	6	3 and 2, -3 and -2, 6 and 1, -6 and -1	2 and 3	$(x+2)(x+3)$
ii) $x^2 - 7x + 12$	-7	12	6 and 2, -6 and -2, 4 and 3, -4 and -3, 12 and 1, -12 and -1.	-4 and -3	$(x-4)(x-3)$
iii) $x^2 + 3x - 18$	3	-18	9 and -2, -9 and 2, -6 and 3, 6 and -3, 18 and -1, -18 and 1.	6 and -3	$(x+6)(x-3)$
iv) $x^2 + 6x - 7$					
v) $x^2 + 3x - 10$					

b) Explain the numbers in the second row. The process summarised in that row is called *factorising quadratic expressions $x^2 + 5x + 6$* . How do you explain factorising the general quadratic expressions $x^2 + bx + c$?

The group initially seemed not to make sense of the above exercise in terms of what was actually expected of them. The teacher asked the group what they thought of the numbers in the columns. One learner in the group immediately said they could not make sense of the numbers in the column. The teacher then mentioned that the group needed a bit more time to think about the columns and then left this group to attend to another group. After some time the Group 1 learners raised their hands to attract the teacher's attention. They had

successfully identified where numbers in columns two and three came from. They filled in the titles for the two columns, middle term and constant term of the quadratic expression. However, they could not explain the third column. The teacher left the group after advising them to think more about what column 3 was about. The group seemed rather weak, because they had not made any progress by the time the teacher came round to see them again. They could still not reply to the teacher's question about what column 3 was about.

The teacher then gave the group an example that $8 = 4 \times 2$ and then asked them how the equation could be used to help explain column 3. Initially the group started focusing on the two numbers 3 and 2. After some deliberations some group members looked at these numbers 3 and 2 as giving 5 as the sum or 6 as the product. This was very helpful in the group, because they could suddenly understand other numbers in columns 3 and 4, giving them relevant titles (respectively possible products of constant term and products of constant term whose sum is the middle term). Other exercises were also done without many problems.

Control classroom

In this classroom the teacher was showing the learners how a given problem was solved. For the bulk of the time the learners were just listening to the teacher's explanation. After a thorough explanation, the learners were given exercises to work on.

Summary: Classroom environment.

We can summarise the classroom dynamics as per table 4.3.

TABLE 4.3 Classroom approaches of the three different groups.

GROUP	LEARNER ROLE	TEACHER ROLE
Control	Listen to teacher explanation and then do exercises.	To give and explain examples and then to give exercises.
Unmonitored	Look at one volunteering learner solving the problem on the chalkboard, and then discuss the volunteer's solution.	To encourage learners to volunteer to solve problems, and to require others to discuss the volunteer's solution.
Monitored	To solve problems on their own.	To interrogate learner solutions using open-ended questions.

These are the classroom environments that we shall compare the data against.

4.3 RESEARCH DESIGN

The approach needed to answer the research questions was a mixed-method approach as depicted by Figure 4.1 below, adapted from Creswell (2003). It was a pragmatic evaluation study.

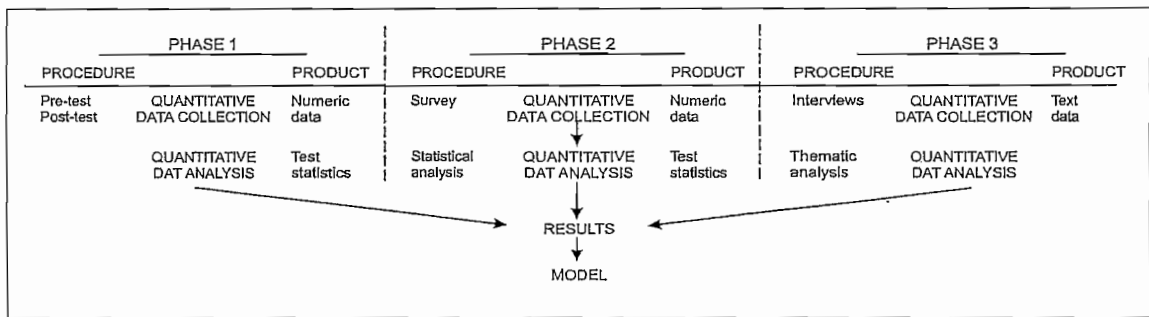


FIGURE 4.1 Diagram of the research design

Phase 1 of the investigation was the quantitative part of the investigation where results from experimental and control schools were compared to explore the impact of an open-ended approach on the learning of mathematics. Learners from the experimental school and control school wrote a pre-test at the beginning of the study to establish and compare their pre-requisite knowledge. They wrote a post-test at the end of the intervention period, compiled by the author in consultation with the educators from both schools. The purpose of the post-test was to see if there was any post-intervention difference in performance between the two groups. All the classes that formed part of study covered the same work schedule and wrote the same tests. Care was taken by the research design to limit other possible factors accounting for different behaviours of the groups as far as performance in the tests is concerned.

Regular monitoring of the implementation of the open-ended approach to teaching and learning was done in two of the four experimental classes. The other two were only supplied with the worksheet to use throughout the period of intervention. The design of the investigation was such that the monitored class was the only one in which the *teacher's compliance* to the open-ended approach was adequately monitored by the author. Because of lack of consistent monitoring of the other classes in the experimental school, there was not enough evidence of compliance or otherwise to the open-ended approach to teaching and

learning in those classes. However, a video of the unmonitored class, recorded during the intervention period, pointed to passiveness among other learners while a volunteer learner attempted to solve the problem on the chalkboard. The video of the control school, also taken during the intervention period, showed a teacher approach that definitely did not comply with the open-ended approach to teaching and learning.

The main focus of monitoring in the two classes was to observe the classroom dynamics and to establish if the type of questions the teacher was asking the learners during the solution process complied with the expectations of the open-ended approach to teaching and learning. The initial briefing of the experimental school educators by the author at the beginning of the study was intended to explain to the educators exactly what constituted compliance to the open-ended approach to teaching. However, it was subsequently revealed that some of the proceedings in the unmonitored classes were not fully compliant with the approach.

Phase 2 of the investigation was about learner responses to a questionnaire, which was analysed quantitatively. The questionnaire statements were intended to elicit learner responses in terms of post-intervention attitude towards mathematics learning. Only learners from the monitored experimental classrooms responded to a questionnaire whose statements were either in favour of an open-ended-approach that was learner-centred or in favour of a teacher-centred approach. The study assumed that the pre-intervention approach to teaching and learning was predominantly teacher-centred. This assumption was based on the experimental school teacher responses to the author's questions during the initial briefing about how they would teach grade 11 'factorisation of quadratic expressions'. Their responses definitely pointed to teacher-centredness. It was the contention of the study that if the learners' post-intervention responses to the questionnaire statements were still teacher-centred, then the intervention would not have impacted on their post-intervention view of mathematics teaching and learning. If their views favoured learner-centredness, the study assumed, then the intervention was considered to have impacted positively on their post-intervention view of mathematics teaching and learning.

Phase 3 of the study was a qualitative investigation. Interviews were conducted with 17 learners from the monitored classes to establish the learners' attitudes towards the open-ended approach to teaching and learning. Interviews were also conducted with two educators, one from the monitored class and other from the unmonitored class, in order to establish the educators' sense of readiness or adaptability to the open-ended approach to teaching and learning.

4.4 RESEARCH PROCEDURES AND METHODS

4.4.1 Introduction

In order to set up the experimental investigation the researcher asked for permission from the district education authority to use two of their schools for research purposes. The author identified his expectations of those schools, so that the authorities could initiate a meeting between the school principal, involved educators and the author. Negotiations around relevant research matters were then conducted. Then the phased investigation followed. Unfortunately, the study was interrupted by a general teacher strike between late April 2007, when meetings culminating in a strike, in May 2007. Interviews resumed after the strike, and data collection was completed by the end of the first semester.

4.4.2 Quantitative component of the research

4.4.2.1 *Phase 1: Pre-test / post-test component of the study*

a) *Philosophical aspects of the study*

The philosophical foundation of the quantitative pre-test / post-test component of the study is premised on a positivist epistemology. Epistemology is the meaning ascribed to knowledge and its creation (Darlaston-Jones, 2007:25; Summer and Tribe, 2004:3). Knowledge that develops through a postpositivist lens is based on careful observation and measurement of the objective reality that exists 'out there' in the world (Creswell 2003:6, Summer & Tribe 2004:4). The 'goal of academic enquiry' is, from the positivist perspective, 'acquisition of the truth' (Summer & Tribe 2004:4) This quantitative aspect of the study is consistent with the *positivist* epistemology. In attempting to establish the impact of an open-ended approach (OEA) intervention in grade 11 mathematics classes at the chosen schools, there is an underlying *assumption* that the intervention would somehow impact on the teaching and learning at these schools.

b) *Motivation for method choice*

The *pre-test / post-test* quantitative component of the study fits into what is called a quasi-experimental design (Creswell 2003:169). Two schools, one experimental and the other control, were used in the study. A common pre-test was written by both schools. Intervention was made in the experimental school. A common post-test was then completed by both

schools with a view to comparing the impact of the intervention. In this study, the focus was on the impact of the *new* approach, namely the open-ended approach to teaching and learning mathematics. The approach was new in the sense, as it emerged during the intervention, of being (contrary to new curriculum implementation expectation) unfamiliar to the teacher.

c) Population and sample

The experimental and control schools were assigned to the author by the Education District Office in Gauteng province. The following is the background information about the two schools:

i) Background of the two schools

The learners of both the experimental and control schools used predominantly buses as their mode of transport. In both schools, the first period suffered because buses sometimes arrived 10-15 minutes late. Each of the schools is built in a farm area, and the schools are about 20 kilometres apart. The majority of the learners from the experimental school were from a township which was about 6 kilometres from the school. These learners were not allowed admission at the control school and vice versa. The reason given centred around transport logistics. It was also stated that the learners coming from the areas targeted by each school would be disadvantaged if criss-cross movement was allowed.

Both schools had libraries that were used as staff rooms. Both had photocopiers that needed repair from time to time. The educators who were involved in the teaching of mathematics all had at least an M+3 qualification in mathematics teaching. Those involved in the study had diplomas in teaching and had majored in mathematics as one of their courses. Supply of textbooks by the government was done at both schools. There were regular visits from subject advisors at both schools.

ii) Population and sample

The population and the sample are summarised in table 4.4.

TABLE 4.4 Population and sample

SCHOOL	POPULATION	SAMPLE
Experimental	All grade 11 maths learners $N = 166$.	Monitored Grade 11 maths learners: 2 classes $N = 93$
Control	All grade 11 maths classes $N = 88$.	Grade 11 maths classes: 2 classes $N = 88$

As the table shows, the sample comprised two monitored grade 11 mathematics classes from the experimental school and two from the control school. All grade 11 mathematics classes in the experimental schools were issued with the intervention worksheet.

The study sampling fits what Trochim (2006) describes as purposive non-probability sampling. Non-probability in the sense that the sample was not randomly chosen, and purposive in the sense that the two grade 11 mathematics classes – the sample in the experimental group – and the two grade 11 classes in the control school, were chosen with a purpose in mind. The author had targeted grade 11 mathematics classes as the focus of this study.

The purposeful sample – two grade 11 mathematics classes ($N = 93$) - was selected from the experimental group for monitoring throughout the intervention. The class teacher of these two classes was found to have a greater openness to trying the new approach than the others in the experimental school. The other two classes from the experimental school ($N = 73$), though using the same intervention material, were only occasionally monitored. The idea was also to compare the two sets from the experimental school with a view to establishing the impact of the intervention from the perspective of the teacher's role. The main focus of monitoring was to establish if the reaction of the teacher to learner solutions and /or questions would comply with an open-ended approach to teaching and learning of mathematics.

d) *Variables*

Independent variable: Teaching-learning approach characterised by open-ended tasks (e.g. questions).

Dependent variable: Performance in mathematics learning.

e) *Measuring Instruments*

The *pre-test* (**Appendix 2**) was developed to test the pre-requisite knowledge of the learners from both the experimental and control schools. Since the intervention included extensive introduction and application of quadratic equations, the items tested by the pre-test included:

- o Substitution
- o Simplifying quadratic expressions
- o Understanding terms like 'coefficients, square of half coefficients etc.'
- o Graphical representation of the parabola of the form $y = ax^2 + q$.

The *post-test* (**Appendix 3**) tested the following items:

- o Factorisation of quadratic expressions.
- o Solving quadratic equations by completing the square.
- o Drawing the parabola of the form $y = ax^2 + bx + c$
- o Applying quadratic equations to real-life situations.
- o Identifying equations of the parabola of the form $y = ax^2 + bx + c$
- o Solving quadratic inequalities.

These are the only topics that the post-test was based on, because they were the topics that were covered at the time the post-test was written. The timing of the writing of the post-test was influenced by the impending teacher strike.

f) *Reliability and validity: post-test.*

The research design and methods in this study were focused on minimising the plausible alternative explanations for the cause-effect relationships by taking precautionary measures (Trochim, 2006). The discussion below outlines three of Trochim's (2006) five ways to minimise threats to reliability and validity. The three – minimising the threats by preventative measures, by observation and by analysis - were chosen because of their relevance to this study.

i) By preventative measures

The reader might be interested to know whether or not the questions in the post-test were not more directly linked to those appearing in the worksheet when compared to those questions that might have been asked by the teacher in the control school. Obviously if it were the case, then any change in performance of the experimental group as compared to the control group could be attributed not only to the intervention itself but also to the kind of questions asked. As a preventative measure, the control school teacher was advised (by the author) that the questions in the post-test would be modelled according to those in the prescribed NCS text book the teacher used. The same textbook was used by the two schools. The worksheet also asked the questions that were similar to those in the same text book.

Furthermore, the author gave both the pre-test and post-test to the educators from both schools for scrutiny and input before finalisation. This was done at the beginning of the study. The control school teacher was aware of the type of questions the post-test would contain.

ii) By Observation

The focus here was on establishing whether or not any of the two groups had an unfair advantage over the other in terms of mathematics study opportunity during the intervention. In both the experimental and control schools, the same work schedule was followed, and mathematics periods were of the same duration. There were cases where some learners would continue with the exercises on the worksheet at home. Learners from the control group would also do homework given to them by their teacher. There was definitely no discernible difference between the two groups in terms of one group having any prolonged assistance in mathematics learning as compared to the other.

iii) By analysis

There are a number of ways to rule out alternative explanations using statistical analysis. The plausibility of alternative explanations might be minimised using *co-variance analysis*.

g) Data collection procedures

In this phase a common pre-requisite test (pre-test) on selected mathematical topics, relevant to the research mathematics topics on which the intervention was done, was administered in the experimental (EXP) and control (CONT) schools.

The two schools, control and experimental, that the study is based on, followed a common work schedule. In other words, the topics that the educators were expected to teach were the same. Both schools also used the same prescribed grade 11 mathematics textbook. All four grade 11 mathematics classes in the experimental school and two in the control school wrote the same pre-test. Permission was sought and obtained from relevant class educators to allow the mathematics test to be written at the same time in each of the two schools. This was intended to prevent possible contamination in each school – stopping learners in subsequent mathematics classes from discussing test items with their predecessors, before writing it. Furthermore, the learners wrote down their answers on the pre-test paper, which was then collected for marking. Class educators invigilated the writing of the test. The pre-test consisted of ten multiple-choice questions. The memorandum was prepared by the researcher. Class educators marked the scripts, which were then moderated by the researcher. Marks were then recorded.

Similar procedures were followed for the post-test. However, here learners were expected to use loose sheets of paper to write their answers on. Contamination in each school was also prevented by synchronising test time for each of the two schools.

h) Data analysis

In both pre-test / post-tests, descriptive, as well as inferential statistical analysis was used. With the quantitative analyses Cronbach Coefficient α was used to establish the reliability of the instruments, while t-tests and effect size (Cohen's Criterion) was used to establish significance of differences between the participating groups' performance. The following were the data analysed:

i) Pre-test

In all the comparisons a t-test was used to detect if differences between the compared groups were significant. This was done by comparing pre-test performances in mathematics classes of both

- unmonitored experimental E ($N = 73$) and control C ($N = 88$) schools.
- monitored experimental ME ($N = 93$) and control C ($N = 88$) schools
- unmonitored experimental N ($N = 73$) and monitored experimental Y ($N = 93$)

j) Post-test

Again a t-test was used to detect if differences between the compared groups were significant. This was done by comparing post-test performances in mathematics classes of both

- unmonitored experimental E ($N = 73$) and control C ($N = 88$) schools.
- monitored experimental ME ($N = 93$) and control C ($N = 88$) schools
- unmonitored experimental N ($N = 73$) and monitored experimental Y ($N = 93$)

The post-test results were also analysed on a question-by-question basis. The questions were categorised by factor analysis, and the questions comprising each factor were analysed to identify commonality.

4.3.2.2 Phase 2: The questionnaire component of the study

a) *Philosophical aspects of the study*

The questionnaire aspect of the research fits into what could be termed empiricism. Empiricism is based on the belief that all concepts are derived from sense experience (O'Leary, 2004:10, Carroll, 2005). *Empiricism*, the foundation of positivism, views reality as universal, objective, and quantifiable (Darlaston-Jones 2007:19). The term also refers to the method of observation and experiment used in the natural sciences Carroll (2005). In philosophy, empiricism is a theory of knowledge which asserts that knowledge arises from experience. In the philosophy of science, empiricism emphasises those aspects of scientific knowledge that are closely related to evidence, especially as discovered in experiments. Empiricism is based on the view that experience, especially of the senses, is the only source of knowledge (Dictionary.com, MLA). Hence, science is considered to be methodologically empirical in nature.

Empirical evidence is observable by the senses. The term *semi-empirical* is sometimes used to describe theoretical methods which make use of basic axioms, established scientific laws, and previous experimental results in order to engage in reasoned model-building and theoretical inquiry.

b) *Motivation for method choice*

The questionnaire was used to measure post-intervention attitudes (Zarinpoush & Gumulka, 2006) of the learners towards mathematics teaching and learning. Despite attitudes being subjective, the questionnaire itself provides data amenable to quantification (Hannan, 2007, McCarthy, 2007). The survey questionnaire is a traditional empirical quantitative technique applicable to market research (Davies, 2000).

The respondents of the questionnaire in the study had been exposed to a teacher-centred approach. This is the impression the author got from the experimental school educators' responses at the beginning of the study, to the question about how they would teach selected mathematical topics. The purpose of the questionnaire was to determine learners' post-intervention impression of teaching and learning. Would they still view learning in teacher-domination terms or in OEA-compliant learner-initiation terms? In other words, would the intervention impact on the learners in terms of their view towards teaching and learning?

c) *Population and sample*

Population: The study population consisted of grade 11 learners in mathematics classes in the experimental school.

Samples: Two grade 11 mathematics classes from the experimental schools ($N=101$) that were monitored throughout the intervention. (Only 93 of these wrote both the pre-test and post-test, with 8 writing only one of the tests, and thus not included in the pre-test / post-test analysis).

The reason for the choice of the sample was that the question addressed by the questionnaire related to the impact of the intervention on the learning and teaching of grade 11 mathematics based on learners' post-intervention view of mathematics teaching and learning. Depending on the teacher's role in the teaching-learning situation using the worksheet, learner exposure would either be teacher-centred or learner-centred. Intense monitoring in the two classes from the experimental school was aimed at ensuring an open-ended approach compliance. Consequently the two classes that were monitored were chosen as a sample for a relevant response to the questionnaire. The author argues that response to the intended intervention (an open-ended approach) is more representative of all participating learners from the experimental school than the response to intervention clouded by an inadequate teacher's anticipated role.

d) *Questionnaire as a measuring instrument.*

The questionnaire was based on a model by Schommer (1990) on the mathematical belief scale. Epistemological beliefs affect academic performance by playing a crucial role in the planning and assessment of learners' comprehension (Schommer *et al.*, 1992:435). They could affect ways in which learners plan to study, which could involve the teacher selecting specific study strategies. The motivation for studies on beliefs about the nature of knowledge on comprehension is the assumption that epistemological beliefs affect comprehension in important ways. For instance, Schommer *et al.* (1992:441) found that belief in simple knowledge (belief that knowledge is isolated facts) is negatively associated with comprehension.

It is within this line of argument that questionnaire statements were chosen from Schommer's model of teacher mathematical belief scale. In this study, however, the questionnaire was administered to the learners. Learners' pre-intervention view of mathematics teaching and learning was probably that of teacher domination, the apparent mode of classroom teacher-learner interaction as established by the author at the beginning of the study.

The learners were then exposed to the intervention. What are their post-intervention views of mathematics teaching and learning? If intervention has influenced their views, then the influence should manifest itself through learners' pro-learner-centred responses to the questionnaire. Learners were to cross out the degree to which they agreed or disagreed with the questions in the questionnaire. There was a scale from 1 to 5, with 1 representing strongly disagree and 5 strongly agree. Only thirty-four of the sixty-three questions, perceived by the researcher to be suitable to indicate views towards mathematics teaching and learning, were used.

e) *Validity: questionnaire*

Validity in the student questionnaire was enhanced by basing it on a validated survey by Schommer (1992) on the mathematical belief scale (Fresen, 2005). It was also read and approved by three professors involved in different math education fields of research.

f) *Data collection procedures.*

The following are some of the important points that were considered when the questionnaire was used as a research tool (Hannan, 2007; Zarinpoush & Gumulka, 2006).

- A pilot trial run was made with 10 of the 101 learners. The idea was to determine if their interpretation of the questionnaire statements would be as anticipated.

- The 10 learners gave feedback on their views about the questionnaire itself – things that they could not understand. Their responses seemed to indicate that they understood the statements.
- The scales on the questionnaire were used for coding responses.
- The learners were requested not to indicate their names on the questionnaire in order to ensure confidentiality.

After the trial run, ninety-one learners were given the outstanding questionnaires to fill in. This was intended to give the learners enough time to reflect on the questionnaire statements.

g) Data analysis.

The following were the steps to be taken in analysing the questionnaire (Nolan, 2008):

- The questionnaire was split according to teacher-centred (TC) and learner-centred (LC) statements.
- The teacher-centred statements were reversed so that analysis could be undertaken from one perspective, a learner-centred perspective, by replacing, for instance, 5 (strongly agree with a teacher-centred statement) with 1 (strongly disagree as viewed from a learner-centred perspective).
- The averages of each of the questions from reversed TC and then from LC statements were determined.
- The average for each category (reversed TC or LC) was determined.

4.3.3 Phase 3: Interview: Qualitative component of the research

The reason for the interview was to obtain information about the educators' and learners' feeling about the open-ended approach to teaching and learning. Would there, for instance, be consistency between how the approach impacted on learner performance and how the learners felt about the approach?

The interview in this study was open-ended (Hannan, 2007). The same open-ended question – 'What do you think about the approaches to teaching and learning that you have so far been exposed to?' - was asked to all interviewees.

a) *Philosophical aspects of the qualitative research*

The philosophical foundation of the qualitative component of the study – interviews - is premised on a socio-constructivist epistemology. Darlaston-Jones (2007:25) claims that it can be argued that the use of qualitative methodologies is predicated upon social constructionism and the adherence to a social constructionist philosophy requires the use of qualitative research methods. She concludes: ‘...in this manner we see a natural relationship between interview techniques as a data collection method and a social constructionist epistemology’. Darlaston-Jones (2007:19) mentions that ‘the basic contention of the social constructionist argument is that reality is socially constructed by and between the persons who experience it (Gergen, 1999)’. Social constructionism, argues Darlaston-Jones (2007:20), provides a different perspective with which to view the world that allows the unique differences of individuals to come into focus, while at the same time permitting the essential sameness that unites human beings to be identified (Ashworth, 2003). Research conducted within a social constructionist epistemology is more likely to involve a heavy reliance on the spoken word through conversation, interviews, narrative, and similar (Gergen, 2001b; Padgett, 2004). The ‘goal of academic enquiry’ is, and, from the constructivist perspective, ‘a more informed construction of the world’.

In the qualitative aspect of the study, the interview was conducted with seventeen learners and two educators in order to address some of the research questions.

b) *Population and sample*

i) *Learners*

While it cannot be claimed that the post-intervention interview response of the seventeen learners was representative of the feeling of all mathematics learners in the experimental school (Hannan, 2007), focus of the learner interview was intended to determine if the interview results would corroborate quantitative post-intervention findings. On that basis, corroboration of other findings by the seventeen learners would justify the conclusion that their views were fairly representative of other learners in the experimental school.

The interview was piloted with 5 learners from the experimental monitored classes. The pilot interview was not captured on tape and consequently was not transcribed. Their responses were consistent with those given by the majority of the seventeen learners whose responses were captured on the audiotape and subsequently transcribed. The seventeen learners were basically expected verbally to express their opinions about their experiences of the two

teaching approaches they had so far been exposed to. The focus of the study was basically on which of the two approaches they, the learners, liked.

Sample: Seventeen grade 11 mathematics learners from the two monitored classes were used. The seventeen learners were randomly chosen – every fifth learner in the class, ten from one class and seven from the other.

ii) *Educators*

Two educators were interviewed after the intervention. One was from the monitored class while other was from the unmonitored class.

Sample: Two educators.

c) *Interview as a measuring instrument*

To ensure the good quality of coding response to the interview text, Flanders's (2004:pp 111-116) ten-category system for classroom interaction analysis was used. Classroom interaction analysis refers not to one system, but to many systems for coding spontaneous verbal communication, arranging the data into a useful display, and then analysing the results in order to study patterns of teaching and learning (Flanders, 2004:111).

The Flanders's 10-category system was developed by Flanders and others at the University of Minnesota between 1955 and 1960. In the following table, ten categories – seven when the teacher is talking, two when the pupil is talking, and the last category indicating silence or confusion – are used. So far as communication is concerned, these three categories – teacher talk, pupil talk and silence or confusion – are said to exhaust all possibilities. Category systems which exhaust all possibilities are totally inclusive of all possible events, and since any event can be classified, a totally inclusive system permits coding at a constant rate throughout the observation (Flanders 2004:113).

Even if the Flander's system was quantitatively analysed, the categories are, in the author's opinion, relevant for qualitative analysis. Flanders's system is relevant to this interview study, mainly because:

- Preference of teacher-centredness or learner-centredness of the interview subjects was expected to be informed by their actual experience of learner-teacher classroom interaction. Flanders's ten-category system could be implied in the subjects' final decision of pro-teacher-centredness or pro-learner-centredness, even if those categories were not explicitly mentioned.

- Instead of *coding* spontaneous pupil-teacher classroom communication processes as 'teacher talk' or 'pupil talk' as per Flanders's system, subjects in the interview study responded in a way that facilitated categorisation of the statements in the questionnaire as 'teacher dominated' or 'learner dominated'.
- *Coding* a pupil's response as 'silence' or 'confusion' in the Flanders's system would be suited to a respondent in the questionnaire indicating the response scale 3 – pointing neither to teacher domination nor pupil domination.

The following table depicts Flanders's 10 category system. For the reader's convenience, the table has been split into two sections, Tables 4.5 and 4.6, respectively dealing with teacher talk and the other with pupil talk (rather than presenting this as a table dealing with both, which would be too big to fit into one page).

TABLE 4.5 Flanders 10 – category system: Teacher talk aspect.

Teacher talk	Response	<ol style="list-style-type: none"> 1. <i>Accepts feeling.</i> Accepts and clarifies an attitude or the feeling tone of a pupil in a non-threatening manner. Feelings may be positive or negative. Predicting and recalling feelings are included. 2. <i>Praises or encourages.</i> Praises or encourages pupil action or behaviour. Jokes that release tension, but not at the expense of another individual; nodding head or saying 'Um hm?' or 'go on' are included. 3. <i>Accepts or uses ideas of pupils.</i> Clarifying, building, or developing ideas suggested by a pupil. Teacher extensions of pupil ideas are included but as the teacher brings more of his own ideas into play, shift to Category 5.
		<ol style="list-style-type: none"> 4. <i>Asks questions.</i> Asking a question about content or procedure, based on teacher's ideas, with the intention that a pupil should answer accordingly.

Teacher talk	Initiation	<p>5. <i>Lecturing</i>. Giving facts or opinions about content or procedures, expressing <i>the teacher's own</i> ideas, giving <i>his/her own</i> explanation, or citing an authority other than a pupil.</p> <p>6. <i>Giving directions</i>. Directions , commands, or orders to which a pupil is expected to comply.</p> <p>7. <i>Criticising or justifying authority</i>. Statements intended to change pupil behaviour from non-acceptable to an acceptable pattern; bawling someone out; stating why the teacher is doing what he/she is doing; extreme self-reference.</p>
Pupil talk	Response	8. <i>Pupil-talk – response</i> . Talk by pupils in response to the teacher. Teacher initiates the contact or solicits pupil statements or structures the situation. Freedom to express own ideas is limited.
	Initiation	9. <i>Pupil-talk - initiation</i> . Talk by pupils which they initiate. Expressing own ideas; initiating a new topic; freedom to develop opinions and a line of thought, like asking thoughtful questions; going beyond the existing structure.
Silence		10. <i>Silence or confusion</i> . Pauses , short periods of silence and periods of confusion in which communication cannot be understood by the observer

d) *Validity: The interview analysis instrument*

Validity in the student interview was enhanced by basing it on a validated model by Flanders (2007).

e) *Data collection procedures*

i) Introduction

A date was set for the interview to be conducted. An audio system was set up on that day, and the learner responses to the open-ended questions were captured on the three audiotapes. A classroom was chosen for the interview, since it was recognised as familiar to the interviewees. The interview was conducted during break, the time chosen by the interviewees. Interview statements were later transcribed. The interview text was analysed.

Statements in the text were categorised according to Flanders's categories. Not all of Flanders's categories were covered by the interviewees.

Some of the interview candidates were reluctant to be individually interviewed. It emerged later that their concerns were that the interview would be conducted in the presence of their educators. After the procedure was explained to them, and it had been explained that they did not need to indicate their names, some still felt comfortable to be interviewed in groups. Consequently the learners were split according to four groups of three and five who were prepared to be individually interviewed. The learners seemed no longer to have a problem in identifying themselves during the interview.

Learners would come in for interviews, either in groups or as individuals. In the case of groups, the interviewer, the research author, would urge the learners to be honest about their opinions, not to influence each other in terms of their responses. For further details, see appendix on the interview transcripts.

ii) Analysis of tapes

Items in the interview text were fitted into the relevant Flanders categories. For instance, the text on Tape 2, page 1, paragraph 12, lines 4 to 5 (2:1:12:4-5) says: 'When the teacher is in the class I can understand better because he explains the basics...' fits well into Flanders's category 5, which described lecturing in terms of giving facts or opinions about content or procedures, expressing his own ideas, giving his own explanation, or citing an authority other than a pupil. Finally, the general concentration of the interview texts within the Flanders's system was used to determine the general feeling of the learners.

4.5 CONCLUSION

As described in this chapter, data was collected in three phases, namely, pre-test / post-test data in phase 1, questionnaire data in phase 2 and interview data in phase 3. The quantitative data in phases 1 and 2 was organised and presented to the Statistical Services for processing. The Statistical Services assisted in statistical calculations of comparison, as well as in the calculation of questionnaire statistics. The interview data in phase 3 was transcribed from audio tapes by the author, who then analysed it. After the quantitative data collection was completed and processed, it was interpreted by the author with the help of the statistician. The qualitative data was also interpreted and reflected upon. In the next chapter representation, interpretation and reflection on the data will be undertaken.

CHAPTER 5

RESULTS

5.1 INTRODUCTION

Chapter 4 dealt mainly with the research procedures and methodologies that culminated in the production of the data needed to facilitate the answering of the research questions. In this chapter, the data is represented, interpreted – first quantitatively, then qualitatively - and reflected upon. The t-test was used to compare the groups. The difference, if any, of the means of the compared groups was considered to be significant if the *significance level* p was at most 0.05 ($p \leq 0.05$). In cases where significant differences were obtained, the effect size d was calculated to establish the practical significance of the result (see Steyn, 2009, Chapter 4).

The focus will now be placed on representation and interpretation of and reflection on the data according to the three research phases as identified in chapter 4. We shall deal with the data as it was collected according to the model under section 1.3. of chapter 1.

5.2 PHASE 1: RESULTS: PRE-TEST / POST-TEST

Research question: *'What will be the impact of the open-ended approach on the learning of mathematics in those classes?'*

There are issues that can influence the impact of an intervention. If two groups are found to perform differently in the post-intervention test, then the reason for one group outperforming the other in the test needs not necessarily be the intervention itself. The reason could, for instance, be that the better performing group consisted of above-average learners as compared to the other group. To counter this possibility, all three groups wrote a common pre-test.

5.2.1 Pre-test results

The aim of the pre-test was to establish the prerequisite knowledge of the learners. The prerequisite knowledge mentioned here was the mathematical knowledge required to

facilitate the learners' understanding of the mathematical topics covered during the period of intervention.

5.2.1.1 Pre-test comparison results: Unmonitored experimental E (N = 73) versus control C (N = 88) groups.

The pre-test scores of the unmonitored classes (N = 73) from the experimental school and the classes from the control school (N = 88) were compared by means of a t-test. The results are displayed in Table 5.1 and Figure 5.1

TABLE 5.1: Two-sample t-test: Unmonitored versus control

T-tests; Grouping: Group (RMahlobo_data.sta)										
Group 1: E										
Group 2: C										
Include condition: Monitor='N'										
Variable	Mean E	Mean C	t-value	df	p	t separ. var.est.	df	p 2-sided	Valid N E	Valid N C
Pretest	21.64384	21.93182	-0.134293	159	0.893341	-0.132258	141.8137	0.894968	73	88

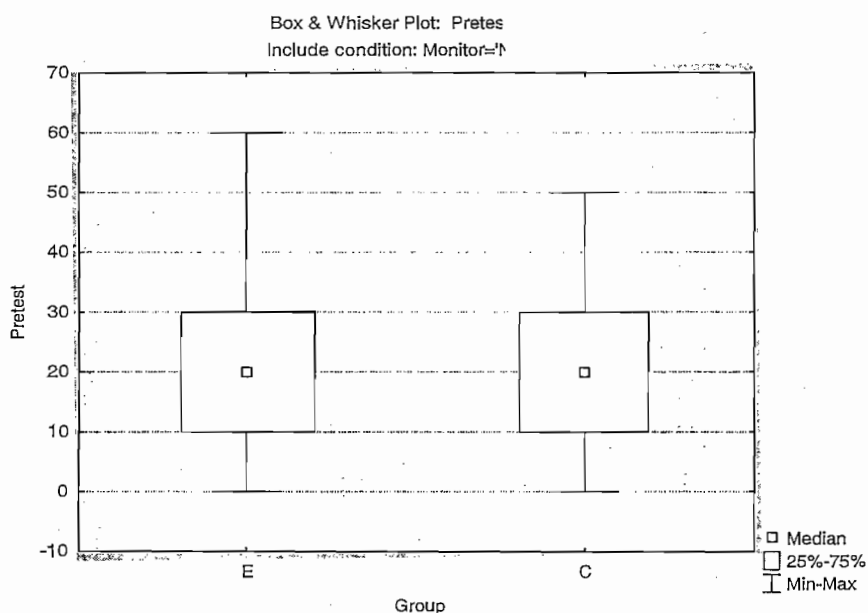


FIGURE 5.1: Box & Whisker Plot Unmonitored versus control

The t-test in Table 5.1 does not suggest any statistical difference between the two groups. The fact that $p = 0,895 > 0,05$ implies that the p-value is well above the significant level of 0,05. The Box and Whisker Plot in Figure 5.1 also corroborates this point.

One can conclude from the data interpretation that there was no significant difference in pre-requisite knowledge between the groups – from the unmonitored experimental school and from the control school.

5.2.1.2 Pre-test performance of grade 11 mathematics classes of both monitored experimental E(N = 93) and control C(N = 88) schools

Here the pre-test scores of the monitored classes (N = 93) from the experimental school and the classes from the control school (N = 88) were compared, again by means of a t-test. The data was represented using Table 5.2. and Figure 5.2.

TABLE 5.2: Monitored versus control

T-tests; Grouping: Group (RMahlobo_data.sta)										
Group 1: E										
Group 2: C										
Variable	Mean E	Mean C	t-value	df	p	t separ. var.est.	df	p 2-sided	Valid N E	Valid N C
Pretest	22.04301	21.93182	0.054997	179	0.956202	0.055230	177.3181	0.956017	93	88

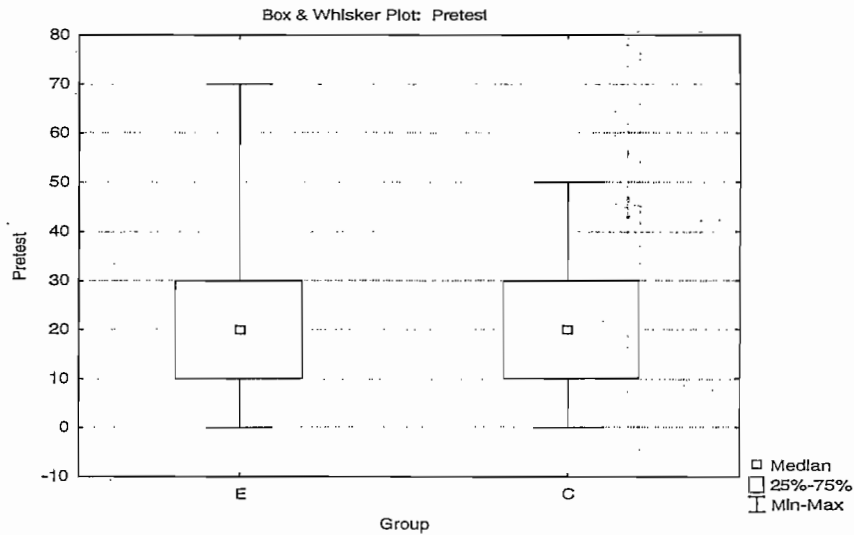


FIGURE 5.2: Monitored versus control

Again, there was no reason to believe that the monitored group from the experimental school performed differently in the pre-test from the group from the control school.

One can conclude from the data interpretation that there was no significant difference in pre-requisite knowledge between the monitored group and the group from the control school.

5.2.1.3 Pre-test performance of grade 11 mathematics classes of both unmonitored experimental N (N = 73) and monitored experimental Y (N = 93)

The two groups from the experimental school were compared, again using a t-test, in order to establish if there was any significant difference between them in terms of pre-requisite knowledge.

The data was represented using Table 5.3 and Figure 5.3.

TABLE 5.3: Monitored versus Unmonitored

T-tests; Grouping: Monitor (RMahlobo_data.sta)										
Group 1: Y										
Group 2: N										
Include condition: group='E'										
Variable	Mean Y	Mean N	t-value	df	p	t separ. var.est.	df	p 2-sided	Valid N Y	Valid N N
Pretest	22.04301	21.64384	0.174458	164	0.861721	0.174230	153.9915	0.861913	93	73

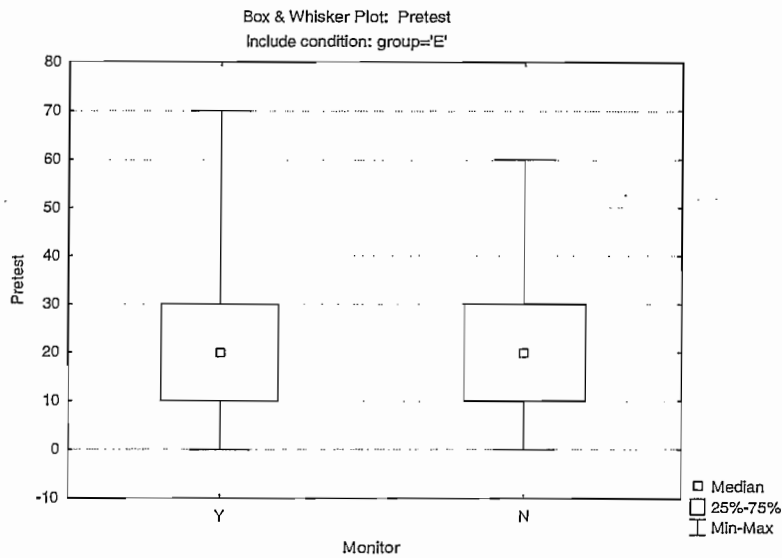


FIGURE 5.3: Monitored versus Unmonitored

Again, there was no reason to believe that the monitored group performed differently in the pre-test from the unmonitored group.

One can conclude from the data interpretation that there was no significant difference in pre-requisite knowledge between the monitored group and the group from the control school.

5.3.1.4 Conclusion: Pre-test

There was no significant difference in performance between each pair of the three groups: unmonitored experimental ($N = 73$), monitored experimental ($N = 93$) and control ($N = 88$), as far as their pre-requisite knowledge is concerned. This seems to validate the conclusion that the groups the study investigated were of comparable pre-requisite knowledge.

5.2.2 Post-test results

There were two contexts within which to examine the post-test results. The first one was in terms of comparing averages of the post-test marks themselves, while the second was in terms of looking at the post-test marks on a question-by-question basis. We start with the post-test averages.

5.2.2.1 Average post-test performance of grade 11 mathematics classes of both monitored experimental ($N = 93$) and control ($N = 88$) schools.

Here the post-test scores of the monitored classes ($N = 93$) and the classes from the control school ($N = 88$) were compared, again by means of a t-test.

The data is reflected in Table 5.4. and Figure 5.4.

TABLE 5.4: Average post-test performance: monitored versus control

T-tests; Grouping: Group (RMahlobo_data.sta)										
Group 1: E										
Group 2: C										
Include condition: Monitor='Y' or group='C'										
Variable	Mean	Mean	t-value	df	p	t separ.	df	p	Valid N	Valid N
	E	C				var.est.		2-sided	E	C
Post-test	41.89247	16.68182	9.641450	179	0.000000	9.756880	156.6394	0.000000	93	88

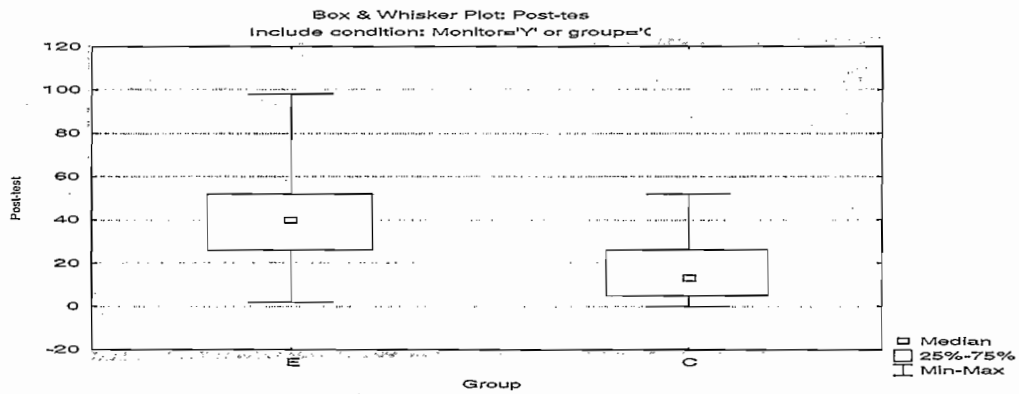


FIGURE 5.4 Average post-test performance: monitored versus control

The monitored experimental group E performed significantly better than the control group in the post-test. This is confirmed by the Box and Whisker Plot.

One can conclude from the data interpretation that the monitored group out-performed the control group on the post-test. The difference in means is also practically significant, since the effect size $d=1.43$, which is a significant effect.

5.2.2.2 Average post-test performance of grade 11 mathematics classes of both unmonitored experimental N (N = 73) and monitored experimental Y (N = 93) groups.

Here the post-test scores of the unmonitored classes (N = 73) and monitored classes (N = 93) from the experimental school were compared by means of a t-test. The data was represented using Table 5.5 and Figure 5.5.

TABLE 5.5 Average post-test performance: Monitored versus unmonitored

T-tests; Grouping: Monitor (RMahlobo_data.sta)										
Group 1: Y										
Group 2: N										
Include condition: group='E'										
Variable	Mean Y	Mean N	t-value	df	p	t separ. var.est.	df	p 2-sided	Valid N Y	Valid N N
Post-test	41.89247	20.19178	7.552087	164	0.000000	7.876580	161.8501	0.000000	93	73

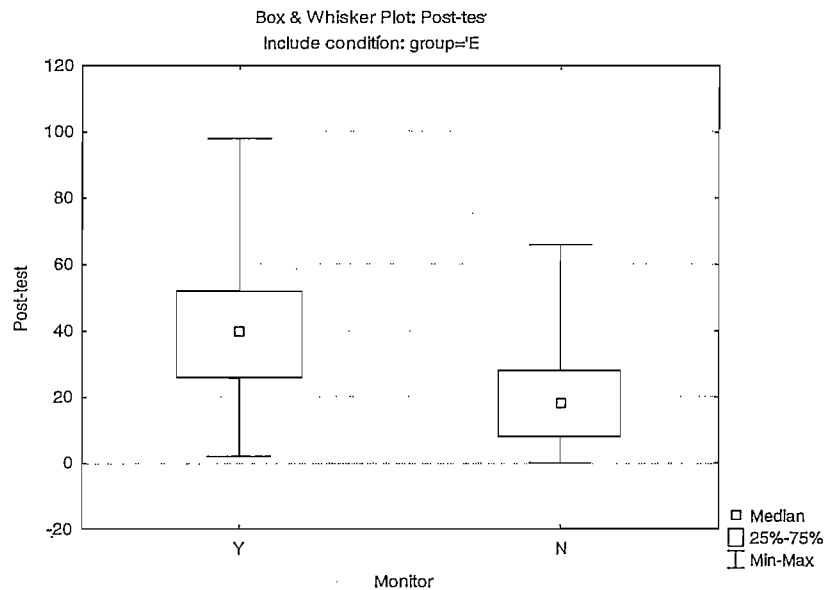


FIGURE 5.5 Average Post-test performance: Monitored versus unmonitored

The monitored group Y performed significantly better than the unmonitored group N in the post-test ($p = 0.00 < 0.05$). This is confirmed by the Box Whisker Plot. The difference in means is also practically significant, since the effect size $d=1.18$, which is a significant effect.

One can conclude from the data interpretation that there was a significant difference in post-intervention knowledge between the monitored and unmonitored groups, with the former showing superiority.

5.2.2.3 Post-test performance of grade 11 mathematics classes of unmonitored experimental ($N = 73$) and control ($N = 88$) schools.

The post-test scores of the unmonitored classes ($N = 73$) from the experimental school and control classes ($N = 88$) were compared by means of a t-test.

The data was represented using the table as shown in Table 5.6 and Figure 5.6.

TABLE 5.6 Average post-test performance: Control versus unmonitored

T-tests; Grouping: Group: =1*(group='E')+0*(group='C') (RMahlobo_post.sta)										
Group 1: C										
Group 2: N										
Variable	Mean C	Mean N	t-value	df	p	t separ. var.est.	df	p 2-sided	Valid N C	Valid N N
perc_Post	16.68182	20.19178	-1.60280	159	0.110963	-1.58848	147.2244	0.114323	88	73

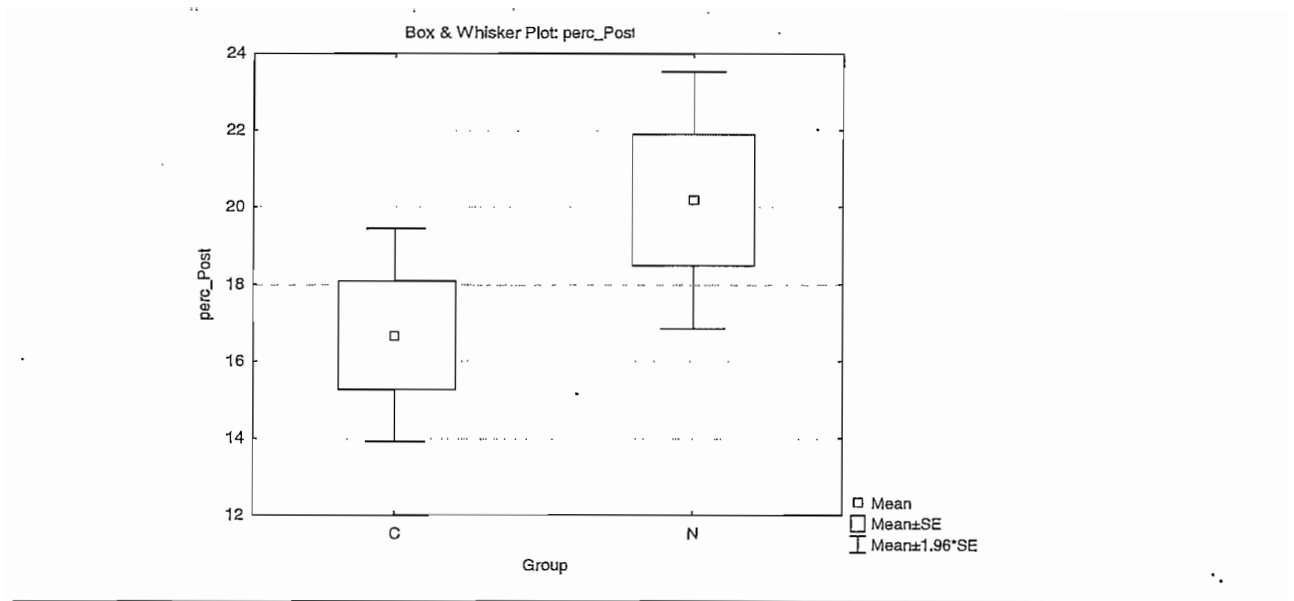


FIGURE 5.6 Average post-test performance: Control versus unmonitored

The difference between the means of the unmonitored experimental class and the control class was not statistically significant.

The statistical information seems to suggest that there is no difference between the two groups in terms of post-test performance.

5.2.2.4 Average post-test performance: Conclusion

The monitored group outperformed both the control group and the unmonitored experimental group as far as average performance in the post-test was concerned.

5.2.3 Post-test question-by-question

5.2.3.1 Question-by-question comparison of monitored group E ($N = 93$) and control group C ($N = 88$)

Here comparison in terms of post-test performance is made between the monitored group and the control group on a question-by-question basis as was described in chapter 4. The data was represented using Table 5.7

TABLE 5.7: Question-by-question: Monitored versus control

	Mean E	Mean C	t separ.	df	p	Valid N(E)	Valid N(C)	Std. Dev.(E)	Std. Dev.(C)	Effect size (d)
Q1	1.57	1.41	0.89	179.00	0.374	93	88	1.25	1.18	0.13
Q2	2.75	2.31	1.48	176.15	0.140	93	88	2.21	1.83	0.22
Q3	3.45	1.91	4.65	177.72	0.000	93	88	2.38	2.07	0.69
Q4	3.94	0.77	12.31	141.45	0.000	93	88	2.18	1.15	1.83
Q5	1.98	0.69	6.99	137.51	0.000	93	88	1.57	0.79	1.04
Q6	3.08	0.48	12.21	137.97	0.000	93	88	1.82	0.92	1.82
Q7	2.80	0.36	9.85	105.61	0.000	93	88	2.30	0.61	1.47
Q8	1.39	0.48	4.21	119.67	0.000	93	88	1.94	0.74	0.63

For questions 1 and 2 there was no statistically significant difference in performance between the monitored experimental group and the control group.

However, for questions 3 to 8, there were significant improvements (which were also practically significant due to the large effect sizes), to a varying degree per question, in performance of the monitored group as compared to the control group. In these questions learners from the monitored class performed better than those of the control class.

The data indicates that in general, the monitored group outperformed the control group in terms the marks they obtained in the majority of the questions.

5.2.3.2 Question-by-question comparison of unmonitored group N ($N = 73$) and control group C ($N = 88$)

Here comparison in terms of post-test performance is made between the unmonitored group N and the control group C. The data was represented using Table 5.8.

TABLE 5.8: Question-by-question: Control versus unmonitored

	Mean C	Mean N	t separ.	df	p	Valid N(C)	Valid N(N)	Std.Dev.(C)	Std.Dev.(N)	Effect size (d)
Q1	1.41	1.74	-1.86	157.79	0.064	88	73	1.18	1.07	-0.30
Q2	2.31	2.40	-0.32	154.97	0.752	88	73	1.83	1.79	-0.05
Q3	1.91	2.37	-1.30	144.57	0.195	88	73	2.07	2.36	-0.21
Q4	0.77	1.11	-1.69	142.95	0.093	88	73	1.15	1.34	-0.27
Q5	0.69	0.89	-1.50	148.41	0.135	88	73	0.79	0.86	-0.24
Q6	0.48	0.56	-0.63	158.92	0.531	88	73	0.92	0.78	-0.10
Q7	0.36	0.41	-0.46	145.82	0.647	88	73	0.61	0.68	-0.07
Q8	0.48	0.59	-1.01	158.10	0.315	88	73	0.742	0.663	-0.16

There was no significant difference in performance between the two groups ($p > 0.05$) in each of the eight questions.

The data does not suggest that the two groups performed differently on a question-by-question basis.

5.2.3.3 Question-by-question comparison of monitored group ($N = 93$) and unmonitored group ($N = 73$).

Here comparison in terms of question-by-question post-test performance is made between the monitored group and the unmonitored group.

The data was represented using table 5.9.:

TABLE 5.9 Question-by-question: unmonitored versus monitored

	Mean E	Mean N	t separ.	df	P	Valid N(E)	Valid N(N)	Std. Dev.(E)	Std. Dev.(N)	Effect size (d)
Q1	1.57	1.74	-0.94	162.70	0.346	93	73	1.25	1.07	-0.15
Q2	2.75	2.40	1.15	163.83	0.253	93	73	2.21	1.79	0.18
Q3	3.45	2.37	2.92	155.48	0.004	93	73	2.38	2.36	0.46
Q4	3.94	1.11	10.29	155.84	0.000	93	73	2.18	1.34	1.61
Q5	1.98	0.89	5.68	147.85	0.000	93	73	1.57	0.86	0.89
Q6	3.08	0.56	11.99	131.09	0.000	93	73	1.82	0.78	1.87
Q7	2.80	0.41	9.49	112.13	0.000	93	73	2.30	0.68	1.48
Q8	1.39	0.59	3.70	118.10	0.000	93	73	1.94	0.66	0.58

For questions 1 and 2 there was no statistically significant difference in performance between the monitored experimental group and the unmonitored group.

However, for questions 3 to 8, there was a significant improvement, to a varying degree per question, in performance of the monitored group as compared to the unmonitored group. Differences in means for questions 4 to 7 can be regarded as practically significant. In these questions learners from the monitored class performed better than those of the unmonitored class. However, questions 3 and 8 had only medium effect sizes, indicating not necessarily practically important differences.

In general, the monitored group outperformed the unmonitored group in the post-test on a question-by-question basis.

5.2.3.4 Question-by-question comparison: Factor Analysis

The questions were analysed to establish if it was possible for some questions to be grouped together under some underlying factor, resulting in the reduction of the questions. All questions which have high loadings under a particular factor would constitute that factor. The principal components method was used to extract two factors, followed by an oblique rotation in order to see the factor structures more clearly (see Bartholomew *et. al*, 2002,

Chapter 6). Table 5.10. summarises the questions that were found to be classifiable under two factors, Factors 1 and 2, which explained 67% of the total variance.

TABLE 5.10. Question-by-question: Factor Analysis

Variable	Factor Loadings (Quartimax raw) (RMahlobo_post.sta) Extraction: Principal components (Marked loadings are >.700000)	
	Factor 1	Factor 2
Q1	0.018334	0.803176
Q2	0.362868	0.771864
Q3	0.459519	0.689468
Q4	0.854138	0.143159
Q5	0.614190	0.400706
Q6	0.852461	0.096284
Q7	0.866859	-0.079504
Q8	0.690622	0.271628
Expl.Var	3.405041	1.986665
Prp.Totl	0.425630	0.248333

Table 5.10 shows that post-test questions 4, 6 and 7 constitute Factor 1, while post-test questions 1, and 2 comprise Factor 2. A look at the Factor 2 questions – factorising a quadratic expression and solving by completing the square – suggest that the questions depended on specific predetermined procedures to solve. Questions 4 and 7 from Factor 2 – word problems – required more than specific procedures to solve. Question 6 from Factor 1 – solving for x – does not specify the procedure needed to solve it. It thus requires more than using predetermined procedures to solve it. In other words, one can classify Factor 1 questions as being more cognitively demanding to solve than Factor 2 questions. We now look at how the learners fared in these questions.

a) Monitored ($N = 93$) versus control ($N = 73$) learners: Factors 1 and 2 questions

For Factor 2 questions 1 and 2 there was no statistically significant difference in performance between the monitored experimental group ($N = 93$) and the control group ($N = 73$) (See table 5.7). However, in Factor 1 questions 4, 6 and 7, there was a significant improvement, to a varying degree per question, in performance of the monitored group as compared to the control group. In these questions learners from the monitored class performed better than those of the control class. This tempts one to conclude that the performance of the monitored group in more demanding questions was better than that of the control group.

b) Unmonitored group N ($N = 73$) and control group C ($N = 88$): Factors 1 and 2 questions

There was no significant difference in performance in Factors 1 and 2 questions ($p > 0.05$) between the two groups (See table 5.8.).

c) Monitored group ($N = 93$) and unmonitored group ($N = 73$): Factors 1 and 2 questions

For Factor 2 questions 1 and 2 there was no statistically significant difference in performance between the monitored experimental group and the unmonitored group (See Table 5.9.). However, for Factor 1 questions 4,6, and 7, there was a significant improvement, to a varying degree per question, in performance of the monitored group as compared to the unmonitored group. In these questions learners from the monitored class performed better than those of the unmonitored class.

A similar pattern is found in those questions – questions 3, 5 and 8 – that do not belong to a factor. There was significant difference, to a varying degree per question, between the monitored group and the other groups. The monitored group performed better than the other two groups. However, there was no significant difference in performance between the unmonitored experimental group and the control group.

5.2.4. PHASE 1 Conclusions: Pre-test and Post-test phase

In general, the pre-test / post-test data showed that the monitored group outperformed both the unmonitored group and the control group in the post-intervention test. There was, however, no significant difference between the unmonitored experimental group and control group. This is despite the fact that in the unmonitored experimental school the open-ended approach compliant worksheet was used. With the prerequisite knowledge, similar school resource environment, etc. all being the same, one can attribute the difference in results to the role the teacher played during the intervention. If one compares the monitored and unmonitored classes, one realises that both were of similar pre-requisite knowledge, both were of the same school, and both were using the same intervention material. The main difference was in terms of the approaches adopted by the educators in both classes. It was earlier mentioned that in the unmonitored class, most learners were left passive while the volunteering one was doing the problem on the chalkboard. This level of passiveness was not very different from the one exercised in the control school. In both classes the majority of the learners were deprived of the opportunity to initiate the solution processes, to interrogate

and question each others solutions and to reach consensus. The fact that there was not much to separate the control and unmonitored classes in terms of post-test performance meant that the teacher's role, rather than the material used in the class, was crucial for learner understanding. The relatively better performance by the learners in the monitored classes as opposed to the others could consequently be attributed to the teacher's adherence to the open-ended approach. In other words, the learner improvement was as a result of the approach itself. Improved performance in the post-test results for the monitored experimental group prompts one to agree with the statement of Hiebert *et al.* (1996) that when learners learn mathematics through such a problem-based approach, struggling with the difficulties facing them instead of relying on memorisation or any pre-determined rule to search for solutions, it promotes 'deep understanding' of the mathematics that is valued. Hodgson and Watland (2004:1), in talking about an OEA, said: 'Through groups and other learning interactions with their online peers, learners acquire deeper understanding because of the opportunities for exposure to multiple perspectives and interpretations'.

Mewborn *et al.* (2005:416) also had a positive comment to make about the open-ended approach to teaching and learning:

'I have noted significant improvement in my learners' self-confidence and their willingness to share their thinking with others. In fact, they begin to take pride in their explanations and find satisfaction in being able to explain what they are doing and why. They begin to see that there is a point to explaining their thinking. This leads to learners feeling more ownership of their mathematical learning.'

5.3 PHASE 2: QUESTIONNAIRE

A questionnaire was used as a means of determining the monitored experimental learners' post-intervention view of mathematics learning. It was intended to address the question about the impact of the open-ended approach to teaching and learning from the point of view of post-intervention mathematics learning perception.

Research question: 'What will be the impact of the open-ended approach on the learning of mathematics in those classes?'

The context in which 'impact' is used here is 'influencing one's view'. In other words, what this research question wants to establish is whether or not the learners' view of mathematics learning was in any way influenced by their exposure to the intervention. Before the

intervention, the only view of learning that the learners had, according to this study, was probably the one informed by the classroom teaching approach of the teacher. As was previously mentioned, a request by the author for the educators at the experimental school to indicate how they would teach 'factorisation of quadratic expressions' pointed to a teacher-centred tendency of teaching.

Teacher-centredness inevitably leads to the role of the learners being that of 'paying attention' to the teacher. Paying attention in the sense of listening to the teacher's explanations, paying attention in the sense of the learner using the teacher's examples as a model to solve mathematical exercises similar to the examples, or paying attention in the sense of looking at the teacher as the 'source of truth'. The author's impression of the learners' pre-intervention view of mathematics learning, obtained from experimental educators responses to approaches they used in teaching, was that of teacher-centredness.

The worksheet itself was constructed to be learner-centred, an important component of an open-ended approach to teaching and learning. Since the monitored classes were the only classes in which the open-ended approach was monitored, the hundred-and-one learners chosen to respond to the questionnaire came from these classes. This was done to eliminate the possibility of other clouding factors in the learners' responses to the questionnaire that could emanate from, for instance, their teacher's perpetuation of teacher-centredness even when using the worksheet.

In simpler terms, the research question posed attempted to establish the following from the learner: 'Your initial view of mathematics learning was that of teacher domination. You were then exposed to a learner-dominated approach. What is your post-intervention view of mathematics learning?' If the learners' post-intervention view of mathematics learning is that of teacher domination, then the intervention shall not have impacted on the learners in terms of views towards mathematics learning. If, on the other hand, the learners' post-intervention view of mathematics learning is that of learner initiative, then the intervention shall have impacted on the learners. The questionnaire itself consisted of teacher-centred statements as well as learner-centred statements. The learners had to agree or disagree with the statements. The Likert scale used in this question ranged from 1 to 5, with 1 representing strongly disagree and 5 representing strongly agree (See **Appendix 7**).

The questionnaire was split according to learner-centred (LC) and teacher centred (TC) statements. In order to facilitate the comparison of the two groups from a learner-centred perspective, the teacher-centred statements were reversed. If, for instance, a student strongly agreed with a teacher-centred statement (indicating 5), this was viewed from a

learner-centred perspective to mean strongly disagree (1). The reversal exercise was meant to analyse the data from the learner-centred perspective only. The following is the statistical information about the two groups.

Comparison of the two groups was represented using Table 5.11 and Figure 5.7.

TABLE 5.11 Response to learner centred (LC) statements versus teacher centred (TC) statements

T-test for Dependent Samples (LC_TC.sta)								
Marked differences are significant at $p < .05000$								
Variable	Mean	Std.Dv.	N	Diff.	Std.Dv. Diff.	t	df	p
LC	3.850339	0.465980						
TC	3.700678	0.476957	101	0.149661	0.543712	2.766309	100	0.006754

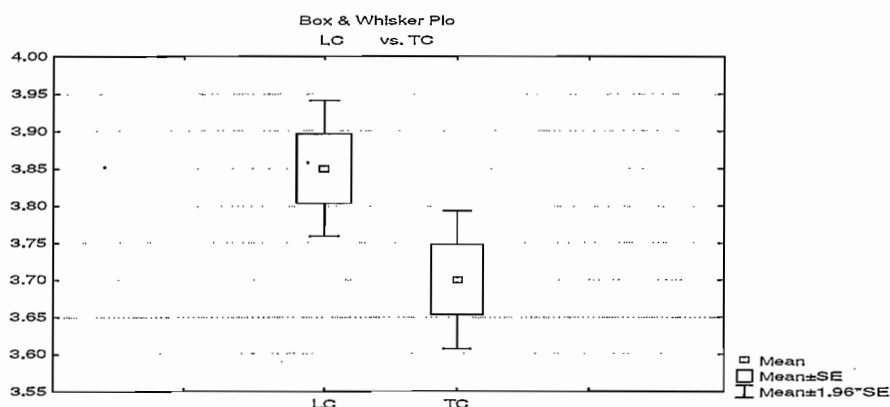


FIGURE 5.7 Response to learner centred (LC) statements versus teacher centred (TC) statements

The difference between the means of the TC and LC statements was statistically significant but had a medium effect ($d = 0.32$), indicating not necessarily a practically significant difference. However, the fact that both means leaned more towards 4 means that in general, the learners were inclined to favour learner-centredness over teacher-centredness.

One can deduce from the data that the learners who participated in the questionnaire were slightly swayed by the intervention in terms of views towards a learner-centred position.

5.4 PHASE 3: INTERVIEWS

The interviews were intended to establish the reaction of the learners and educators in the monitored experimental classes, to the open-ended approach to teaching and learning.

5.4.1 Introduction

There were two interview targets. One target was the learners. The interview purpose was to get the learners' post-intervention attitude towards the open-ended approach to teaching and learning. The choice of the seventeen learners from the monitored classes was motivated by the fact that the author was satisfied that in those classes, a reasonable amount of care had been taken to ensure compliance with the open-ended approach to teaching and learning. This would, the author contends, ensure that the learners' responses were genuinely in relation to an open-ended approach to teaching and learning.

The other target of the interview was two educators, one from the monitored classes and the other one from the unmonitored classes. The purpose was to establish if the educators would be able to adapt to the open-ended approach to learning and teaching. The two interviews are now discussed, starting with the interview in which learners were the target.

5.4.2 Learner interview

The interview was conducted at the end of the intervention period, after the public teacher strike. The seventeen learners were chosen randomly from the monitored classes. Twelve learners were reluctant to be individually interviewed. They were interviewed in groups of three. Five were individually interviewed. Their interviews were captured on audiotape. Transcripts of the interview were independently confirmed by two research colleagues as truly representing the interview proceedings. During the interview, learners were informed that they could decide whether or not to disclose their names. Where the learners decided not to disclose their names, they were labelled Learner 1, 2 etc. Otherwise their names were used. It appears that the learners were suspicious that their interview responses would be disclosed to their educators, and they feared repercussions should their educators find out that they criticised their teaching approaches. The interviews were conducted in the absence of the educators. The following was the research question addressed at the interview.

Research question: How will mathematics learners in the 'open-ended approach' classes respond to such an approach?

To address the research question, one concern immediately came to mind. Will the learners be responding to that particular open-ended approach? In other words, did those learners who responded have sufficient experience of the open-ended approach to make an informed choice? This question has already partly been addressed by ensuring that the respondents all came from the monitored classes. The other part of addressing the concern was in terms of classifying the interview text of their responses according to themes or categories that would illuminate the respondents' experiences. It needs to be highlighted here that the analysis of the interview data was not done using grounded theory. Firstly, the interview was conducted at the end of the intervention, which is not conducive for the implementation of grounded theory. Grounded theory should have no preconceived ideas or hypothesis. To ensure the good quality of coding responses to the interview text, Flanders's (2004: 111-116), ten-category system for classroom interaction analysis was used. Classroom interaction analysis refers not to one system, but to many systems for coding spontaneous verbal communication, arranging the data into a useful display, and then analysing the results in order to study patterns of teaching and learning (Flanders, 2004:111). See chapter 4 for a discussion on Flanders' category system. What is significant about the system is that the categories contained support teacher-centredness or open-ended compliant learner-centredness. Analysis of the interview text would hopefully reflect the respondents experience or otherwise of the open-ended approach to teaching and learning. The focus of analysis was on extracting information from the text and seeing where the text would fit on the Flanders's ten-category system – whether it would fit on the pupil-talk side of the system or teacher-talk side. This would give an indication of whether or not the learner's interpretation of their classroom experience would be compatible with the open-ended approach or teacher-centred approach. A decision would then be taken on the basis of where the fitted text would be concentrated in the Flanders's system.

5.4.2.1 Representation of the coded data in the Flanders's category system

Tables 5.12 and 5.13 summarise the interview text categories as they fit into the Flanders's category system. Text code 1:3:7:4-6, for instance, refers to the interview text found on Tape 1, page 3, paragraph 7 and lines 4 to 6.

TABLE 5.12 Flanders 10 – category system: Teacher talk

Teacher talk	Response	<p>3. <i>Accepts or uses ideas of pupils.</i> Clarifying, building, or developing ideas suggested by a pupil. Teacher extensions of pupil ideas are included.</p> <table border="1"> <tr> <th colspan="2">TAPE 1 GROUP 1</th> </tr> <tr> <td>Learner 1</td> <td>1:3:7:4-6, 1:1:5:4-8, 1:4:3:6</td> </tr> <tr> <td>Learner 2</td> <td>1:2:9:7-8, 1:4:3:5</td> </tr> <tr> <td>Learner 3</td> <td>1:2:11:2, 1:2:3:4-5</td> </tr> <tr> <th colspan="2">TAPE 1 GROUP 2</th> </tr> <tr> <td>Learner 1</td> <td>1:9:8:4-6</td> </tr> <tr> <td>Learner 2</td> <td>1:6:6:6-7</td> </tr> <tr> <td>Learner 3</td> <td></td> </tr> <tr> <th colspan="2">TAPE 2 GROUP 1</th> </tr> <tr> <td>Evelyn</td> <td>2:3:9:4, 2:3:11:3</td> </tr> <tr> <td>Catherine</td> <td>2:1:16:2-3, 2:3:17:1</td> </tr> <tr> <th colspan="2">TAPE 2 GROUP 2</th> </tr> <tr> <td>Dieketseng</td> <td></td> </tr> <tr> <td>Brandeline</td> <td>2:4:14:6-7</td> </tr> <tr> <td>Sydney</td> <td>2:5:3:3-4, 2:5:3:2</td> </tr> <tr> <td></td> <td></td> </tr> <tr> <td></td> <td></td> </tr> </table>	TAPE 1 GROUP 1		Learner 1	1:3:7:4-6, 1:1:5:4-8, 1:4:3:6	Learner 2	1:2:9:7-8, 1:4:3:5	Learner 3	1:2:11:2, 1:2:3:4-5	TAPE 1 GROUP 2		Learner 1	1:9:8:4-6	Learner 2	1:6:6:6-7	Learner 3		TAPE 2 GROUP 1		Evelyn	2:3:9:4, 2:3:11:3	Catherine	2:1:16:2-3, 2:3:17:1	TAPE 2 GROUP 2		Dieketseng		Brandeline	2:4:14:6-7	Sydney	2:5:3:3-4, 2:5:3:2				
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Brandeline	2:4:14:6-7																																			
Sydney	2:5:3:3-4, 2:5:3:2																																			
	Initiation	<p>10. <i>Lecturing.</i> Giving facts or opinions about content or procedures, expressing <i>his own</i> ideas, giving <i>his own</i> explanation, or citing an authority other than a pupil.</p> <table border="1"> <tr> <th colspan="2">TAPE 2 GROUP 1</th> </tr> <tr> <td>Calvin</td> <td>2:1:12:4, 2:2:11:1-2, 2:3:1:2</td> </tr> <tr> <th colspan="2">TAPE 2 GROUP 2</th> </tr> <tr> <td>Brandeline</td> <td>2:4:14:1-2</td> </tr> </table>	TAPE 2 GROUP 1		Calvin	2:1:12:4, 2:2:11:1-2, 2:3:1:2	TAPE 2 GROUP 2		Brandeline	2:4:14:1-2																										
TAPE 2 GROUP 1																																				
Calvin	2:1:12:4, 2:2:11:1-2, 2:3:1:2																																			
TAPE 2 GROUP 2																																				
Brandeline	2:4:14:1-2																																			

TABLE 5.13 Flanders 10 – category system: Pupil talk

Pupil talk	Response	<p>8. <i>Pupil-talk – response.</i> Talk by pupils in response to teacher. Teacher initiates the contact or solicits pupil statement or structures the situation. Freedom to express own ideas is limited.</p> <table border="1"> <tr> <th colspan="2">TAPE 1 GROUP 2</th> </tr> <tr> <td>Learner 1</td> <td>1:8:13:2-4</td> </tr> <tr> <td>Learner 2</td> <td>1:8:5:4</td> </tr> <tr> <th colspan="2">TAPE 2 GROUP 2</th> </tr> <tr> <td>Brandeline</td> <td>2:4:8:4</td> </tr> </table>	TAPE 1 GROUP 2		Learner 1	1:8:13:2-4	Learner 2	1:8:5:4	TAPE 2 GROUP 2		Brandeline	2:4:8:4																														
	TAPE 1 GROUP 2																																									
Learner 1	1:8:13:2-4																																									
Learner 2	1:8:5:4																																									
TAPE 2 GROUP 2																																										
Brandeline	2:4:8:4																																									
	Initiation	<p>9. <i>Pupil-talk - initiation.</i> Talk by pupils which they initiate. Expressing own ideas; initiating a new topic; freedom to develop opinions and a line of thought, like asking thoughtful questions; going beyond the existing structure.</p> <table border="1"> <tr> <th colspan="2">TAPE 1: GROUP 1</th> </tr> <tr> <td>Learner 1</td> <td>1:2:5:3-4, 1:5:7:1, 1:3:7:4</td> </tr> <tr> <td>Learner 2</td> <td>1:2:9:6, 1:5:3:3</td> </tr> <tr> <td>Learner 3</td> <td>1:2:3:2-3, 1:2:3:2-3</td> </tr> <tr> <th colspan="2">TAPE 1: GROUP 2</th> </tr> <tr> <td>Learner 1</td> <td>1:7:7:1, 1:9:13:3-4, 1:7:5:3-6</td> </tr> <tr> <td>Learner 2</td> <td>1:7:9:1-3, 1:4:3:2</td> </tr> <tr> <td>Learner 3</td> <td>1:8:3:1-2</td> </tr> <tr> <th colspan="2">TAPE 2 : GROUP 1</th> </tr> <tr> <td>Evelyn</td> <td>2:1:8:3</td> </tr> <tr> <td>Catherine</td> <td>2:2:9:1, 2:1:16:4, 2:2:5:1, 2:2:5:2, 2:1:16:4</td> </tr> <tr> <th colspan="2">TAPE 2:GROUP2</th> </tr> <tr> <td>Dieketseng</td> <td>2:5:9:1, 2:5:11:1</td> </tr> <tr> <td>Brandeline</td> <td>2:4:14:5, 2:4:14:6-7</td> </tr> <tr> <td>Sydney</td> <td>2:5:1:2, 2:5:3:3, 2:5:3:3-4, 2:5:3:2</td> </tr> <tr> <th colspan="2">TAPE 3: INDIVIDUAL RESPONDENTS</th> </tr> <tr> <td>Thandeka</td> <td>3:2:5:1-4, 3:2:9:4-5, 3:3:4:2-4</td> </tr> <tr> <td>Sheila</td> <td>3:3:2:5</td> </tr> <tr> <td>Brenda</td> <td>3:4:2:3</td> </tr> <tr> <td>Phumzile</td> <td>3:4:18:3</td> </tr> </table>	TAPE 1: GROUP 1		Learner 1	1:2:5:3-4, 1:5:7:1, 1:3:7:4	Learner 2	1:2:9:6, 1:5:3:3	Learner 3	1:2:3:2-3, 1:2:3:2-3	TAPE 1: GROUP 2		Learner 1	1:7:7:1, 1:9:13:3-4, 1:7:5:3-6	Learner 2	1:7:9:1-3, 1:4:3:2	Learner 3	1:8:3:1-2	TAPE 2 : GROUP 1		Evelyn	2:1:8:3	Catherine	2:2:9:1, 2:1:16:4, 2:2:5:1, 2:2:5:2, 2:1:16:4	TAPE 2:GROUP2		Dieketseng	2:5:9:1, 2:5:11:1	Brandeline	2:4:14:5, 2:4:14:6-7	Sydney	2:5:1:2, 2:5:3:3, 2:5:3:3-4, 2:5:3:2	TAPE 3: INDIVIDUAL RESPONDENTS		Thandeka	3:2:5:1-4, 3:2:9:4-5, 3:3:4:2-4	Sheila	3:3:2:5	Brenda	3:4:2:3	Phumzile	3:4:18:3
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Brenda	3:4:2:3																																									
Phumzile	3:4:18:3																																									

5.4.2.2 Interpretation of the data

The majority of the coded interview texts - all but seven - are allocated mainly in the section of the Flanders's system that are pro-learner-centred, and hence compatible with an open-ended approach to teaching and learning. The interpretation of this could be that the learner experience of the intervention was, to a reasonable extent, compatible with the open-ended approach to teaching and learning, as intended to be established by the study.

The analysis of the interview text was intended only to establish if the experience of the interview respondents was compatible with an open-ended approach to teaching and learning. To address the research question, we need to analyse the interview text for statements that reflect the learners' attitude towards the open-ended approach to teaching and learning. The researcher, in analysing the interview text, 'scavenged' for any information that reflected the learners' attitude to the open-ended approach by looking at any response to the implicit question: *Do you prefer the 'worksheet approach'?* Implicit in the sense that the question was not directly asked, but rather used to pick out those responses reflecting the learner attitude to the approach. These responses will, in the author's opinion, illuminate learners' reaction the question: *How will mathematics learners in the 'open-ended approach' classes respond to such an approach?* In other words, will they respond positively or negatively? Table 5.14 reflects the respondents' attitudes towards the open-ended approach:

TABLE 5.14 Interview responses that reflect attitude to the approach. *Do you prefer the 'worksheet approach'?*

TAPE 1					
LEARNER1	LEARNER2	LEARNER3	LEARNER4	LEARNER5	LEARNER6
YES	N/A	YES	YES	N/A	N/A
1:1:5:3		1:2:3:1	1:7:5:1		
TAPE 2: RESPONSES IN GROUPS					
EVELYN	CALVIN	CATHERINE	DIEKETSING	BRANDELINÉ	SYDNEY
YES	NO	YES	YES	YES	YES
2:1:8:1	2:1:12:1	2:1:16:1	2:5:9:1	2:4:14:1	2:5:3:2

TAPE 3: INDIVIDUALS				
THANDEKA	SHEILA	BRENDA	PHUMZILE	NQUBILE
YES	YES	YES	YES	YES
3:1:2:3	3:3:2:2	3:4:2:2	3:4:18:2	3:5:7:1

Alternatively, the researcher looked for themes in the texts that related to *teacher role*, *learner role*, *learner attitude*, *group role* that emerged from the interview data, in order to establish the learners' interpretation of the approach. This resulted in the Tables 5.15 and 5.16. below:

TABLE 5.15 Responses from learners in groups

Learner	Takes initiative		Views teacher as an <i>initiating</i> source of information		Expresses own ideas		Acts as a member of a group	
	Yes	No	Yes	No	Yes	No	Yes	No
TAPE 1 : GROUP 1								
Learner 1	1:2:5:3-4, 1:5:7:1			1:3:7:4-6, 1:1:5:4-8, 1:4:3:6	1:3:7:4		1:5:1:3, 1:1:7:1, 1:4:1:2, 1:3:13:2	
Learner 2	1:2:9:6, 1:5:3:3			1:2:9:7-8, 1:4:3:5	1:4:3:2		1:3:9:3, 1:4:5:2-3	
Learner 3	1:2:3:2-3			1:2:11:2, 1:2:3:4-5	1:2:3:2-3		1:3:2:2, 1:4:7:2-5	
TAPE 1 : GROUP 2								
Learner 1	1:7:7:1, 1:9:13:3-4, 1:7:5:3-6	1:8:13: 2-4	1:9:2:1	1:9:8:4-6				
Learner 2	1:7:9:1-3	1:8:5:4		1:6:6:6-7			1:7:11:2, 1:8:9:4-5	
Learner 3	1:8:3:1-2							
TAPE 2 : GROUP 1								
Evelyn	2:1:8:3			2:3:9:4, 2:3:11:3			2:1:8:6	
Calvin			2:1:12:4, 2:2:11:1- 2, 2:3:1:2				2:3:13:1	
Catherine	2:2:9:1, 2:1:16:4, 2:2:5:1, 2:2:5:2			2:1:16: 2-3, 2:3:17:1	2:1:16:4			

TAPE 2: GROUP2								
Dieketseng	2:5:9:1, 2:5:11:1							
Brandeline	2:4:14:5	2:4:8:4	2:4:14:1-2	2:4:14:6-7				
Sydney	2:5:1:2, 2:5:3:3			2:5:3:3-4, 2:5:3:2				

TABLE 5.16 Responses from individual learners

Learner	Takes initiative		Views teacher as an initiating source of information		Expresses own ideas		Acts as a member of a group	
	Yes	No	Yes	No	Yes	No	Yes	No
TAPE 3								
Thandeka	3:2:5:1-4, 3:2:9:4-5				3:2:7:1, 3:1:2:5, 3:1:4:1-3, 3:1:6:1-3, 3:2:1:1, 3:2:6:2-4, 3:2:3:4	3:3:4:2-4		3:2:1:2
Shella	3:3:2:5				3:3:2:3			3:3:4:3, 3:3:8:2, 3:3:2:3
Brenda	3:4:2:3				3:4:2:6, 3:4:4:1, 3:4:6:1			
Phumzile	3:4:18:3				3:4:20:1, 3:4:22:3			
Nqobile					3:5:7:2-5			3:5:7:3

A scrutiny of the table suggests a high concentration of responses in the sections of the table that are compatible with learner-centredness. In the main, the learners' experiences are compatible with the open-ended approach to learning mathematics. Table 5.8 could be used to conclude that the learners' response to the open-ended approach was positive.

5.4.2.3 Reflection

It appears from the argument above that the interviewed learners' response to the question *How will mathematics learners in the 'open-ended approach' classes respond to such an approach?* was generally positive. Only Calvin responded negatively to the approach, while learners 2, 5 and 6 did not give a response that directly reflected their attitude to the approach. This conclusion seems to corroborate the findings of pre-test / post-test results, in the sense that liking something results in commitment to it, shown by the relatively good performance in it as opposed to if you did not like it.

5.4.3 Teacher interview response

Two educators, one from the monitored classes and the other from the unmonitored experimental classes were interviewed. The following is the research question the interview was intended to answer:

Research question: How will mathematics educators in those 'open-ended approach' classes adapt to the approach?

Again there is a concern: Adapt to which approach? Was the educators' experience of the approach as intended? In order to establish this, Table 5.3. summarises the educators' responses. This table is based on the assumption that a teacher will appropriately adapt to a 'new' situation if he/she is comfortable with it and if the teacher's understanding of the new situation is appropriate. Consequently the educators' interview analysis focused on text items that related to attitude, the teacher's perceived teacher role and the teacher's perceived learner role. The educators were not directly asked questions related to attitude, the teacher's perceived teacher role and the teacher's perceived learner role. The only question each of the educators was asked was: *What do you think about the approach adopted in the worksheet?* The interview text was scrutinised for responses relevant to attitude, the perceived teacher role and the perceived learner role.

5.4.3.1 Data representation and interpretation

TABLE 5.17 Educators' interview response

TEXT ITEM	TEACHER 1		TEACHER 2	
Teacher attitude to 'worksheet' approach	Positive <ul style="list-style-type: none"> • 4:1:3:3; • 4:2:3:5; • 4:2:5:4-5 	Negative	Positive <ul style="list-style-type: none"> • 4:2:9:1; • 4:2:9:4-5; • 4:3:4:2-3 	Negative
Perceived teacher role in the 'worksheet' approach	Facilitator <ul style="list-style-type: none"> • 4:1:5:3; • 4:1:7:2 	Source of info	Facilitator	Source of info
Perceived learner role in the 'worksheet' approach	Learner working on his/her own <ul style="list-style-type: none"> • 4:1:3:1; • 4:1:3:5; 4:1:5:2 	Learner depends on the teacher	Learner working on his/her own <ul style="list-style-type: none"> • practise at home on their own 4:3:2:5 	Learner depends on the teacher
Learner attitude towards the 'worksheet' approach as observed by the teacher.	Positive <ul style="list-style-type: none"> • 4:1:5:1; • 4:1:7:3; • 4:1:3:4 	Negative	Positive <ul style="list-style-type: none"> • 4:2:9:2 	Negative

TEXT ITEM	TEACHER 1		TEACHER 2	
	Resources for implementation	Available	Needed • 4:1:8:1	Available
Teacher support for implementation	Available	Needed • 4:1:10:1-5	Available	Needed • 4:3:6:1 • 4:2:9:3

According to the table, both educators were positive about the open-ended approach to teaching and learning as a viable alternative to the teacher-centred approach. None of the educators supported the notion of the teacher as the sole source of information. Furthermore, the teacher from the monitored class viewed the teacher role in the 'worksheet' approach as being that of a facilitator, while the teacher from the unmonitored class did not comment on the role expected of the teacher. Again, Teacher 1 saw the role of learners as solving problems on their own, while Teacher 2 saw the role of the learner as solving the problem on their own at home. Both educators saw learner attitude towards the approach as being positive. Lastly, both educators thought they needed support and resources in order to be to implement the approach.

5.4.4 Conclusions

A number of observations arose from the data. The first one was that if an open-ended approach to teaching and learning was appropriately implemented, there was enhancement of learning. The second observation was that learners who were engaged in the open-ended approach to teaching and learning responded positively to the approach, and their post-intervention views towards mathematics learning was influenced by the approach. This was corroborated by the learner interview responses, as well as those of the two educators interviewed, which showed that the learners and educators found the approach to be favourable as opposed to their pre-intervention approach. There are other studies that also vindicated some of the results of this investigation. In the 2002 academic year, the Faculty of Education at Khon Kaen University, Thailand, in an attempt to improve the teacher education programme, conducted a project to investigate how student educators using an open-ended approach developed their worldview on teaching practice and to investigate how school learners in the classrooms using the open-ended approach responded to their learning experience (Inprasita, 2006). In order to have a chance to share their experiences of teaching using the open-ended approach, the 15 student educators involved in the project attended a special seminar organised by the researcher every Friday. One of the results was that participating in Friday seminars made most of the student educators in the project gradually change their views on the educators' role. Some of the changes involved the

educators realising that emphasising learners' learning processes and original ideas was more important than they originally assumed. The other results showed a very high consistency that most of the school learners liked doing activity in the classroom using an open-ended approach, and that the classroom activity enhanced student thinking to a greater extent than before (Inprasita, 2006:105). Consistent with the result of Thailand learners' liking the activity was how the learners in this investigation found fulfilment in getting answers by themselves.

The third observation was that the use of learning material that complied with the open-ended approach to learning was not enough to guarantee success of learning. The teacher's role in using the material was crucial.

CHAPTER 6

CONCLUSIONS, RECOMMENDATIONS AND THE MODEL

6.1 INTRODUCTION

In addressing the research question *'What will be the impact of the open-ended approach on the learning of mathematics in those classes?'* the empirical part of the investigation has shown that an open-ended approach to teaching and learning had a positive impact on learner performance. It also prompted learners to view mathematics learning from a learner-centred perspective. In the qualitative part of the investigation, which addressed the research question *'How mathematics learners in those classes will respond to such an approach'* learners expressed their positive attitude to the approach. Furthermore, an open-ended approach to teaching and learning has been found to be conducive to the successful implementation of the National Curriculum Statement. This was confirmed by the fact that towards the end of 2008, some time after the intervention, the teacher from the monitored experimental class came to the author for assistance in preparing material on the grade 11 mathematics topic 'Remainder Theorem'. His school was to be visited by education officials from the district. The teacher's verbal feedback to the author suggested that the officials were very pleased with his approach and use of the material. They requested that a copy of the learner material be given to them for use in other schools. His explanation of how he intervened during the lesson seemed to point to appropriate intervention - complying with an open-ended approach to teaching and learning. This the author found pleasing. The downside, however, was that the monitored teacher was still unable to prepare learner material that complied with an open-ended approach. Before recommending what should be done to deal with the inability of educators to implement the open-ended approach to teaching and learning, we need to interrogate the approach in relation to other findings.

6.2 THE OEA VERSUS OTHER FINDINGS

The discussion that arises out of the study results involves a consideration of whether or not the research finding justifies implementation of the open-ended approach to teaching and learning in schools, and if so, how it should be implemented. Hiebert and Grouws's

(2007:383-387) study on features that promote conceptual understanding seemed to touch on approaches that are compatible with an open-ended approach to teaching and learning. For instance, the design of this study on an open-ended approach complied with what Hiebert and Grouws's (2007:383-387) referred to as *a second feature that promotes conceptual development*, namely that of *engaging learners in struggling or wrestling with mathematical ideas*. They use the word *struggle* to mean that learners expend effort to make sense of mathematics, to figure something out that is not immediately apparent. The struggle Hiebert and Grouws's (2007:383-387) had in mind comes from solving problems that are within reach and grappling with key mathematical ideas that are comprehensible but not yet well formed.

However, the conclusion that the open-ended approach to teaching and learning enhanced learning did not necessarily imply that at all times there would be one approach to teaching and learning that was better than others. There was no reason to believe, based on empirical findings or theoretical arguments, that a single method of teaching was the most effective for achieving all types of learning goals. One consequence of recognising that different teaching methods might be effective for different learning goals was the fact that empirical studies that compare one teaching method to another using blunt learning measures were difficult to interpret (Hiebert & Grouws,2007:374).

The ways in which concepts are developed in the classroom can vary from educators actively directing classroom activity to educators taking less active roles (Hiebert & Grouws, 2007:383-387). The evidence did not justify a single or 'best' method of instruction to facilitate conceptual understanding. Concepts could be developed through teacher-centred and highly structured formats or through student-centred and less structured formats. Any aspect of classroom practice could evolve differently depending on the classroom, the teacher, the student, and the broader social, cultural, and political context. However, we do not want these concerns so to override the interpretations of the research of this study that we become unable to make progress in our understanding of the investigation (Franke *et al.*, 2007:226).

In the South African context, a predominantly learner-centred approach has been endorsed through the adoption of outcomes based education and the National Curriculum Statement recommendations. The open-ended approach to teaching and learning is the vehicle relevant to ensuring the learner-centredness conducive to the meeting of the requirements of the NCS. One of the possible reasons why the open-ended approach to teaching and learning cannot be ignored is that it is compatible with the creation of an individual envisaged by the

Critical Outcomes, which is the foundation stone of Outcomes-based education and the National Curriculum Statement (See table 1.1.). In short, successfully to implement the NCS, we need, among others, an open-ended approach to teaching and learning.

The results of the study revealed that there are educators' needs that require to be addressed in order for them successfully to implement the open-ended approach to teaching and learning. Even the monitored experimental teacher who was expected to be conversant with the use of the open-ended approach did not develop the confidence needed successfully to execute the expectations of the approach. Probably the teacher's lack of confidence stemmed from his inability to prepare the OEA-compliant learner material, and this was highlighted by his post-intervention request for learner material on grade 11 'Remainder Theorem'.

In order ultimately to meet the teacher's needs for successful implementation of the approach, a three-tier model is recommended. The model deals with the instructional approach and then workshops for subject advisors and educators. This model will now be discussed.

6.3 RECOMMENDED IMPLEMENTATION MODEL

The instructional aspect of the model deals with a description of what is intended to happen in the classroom – what role is expected of a learner, what role is expected of a teacher etc.- in order to facilitate successful implementation of the open-ended approach to teaching and learning of mathematics. The workshop part of the model is intended to prepare the subject (mathematics) advisers and educators for meaningful open-ended approach facilitation in their respective interaction with the educators and learners. The instructional part of the model will be considered first.

6.3.1 Instructional part of open-ended approach to teaching and learning

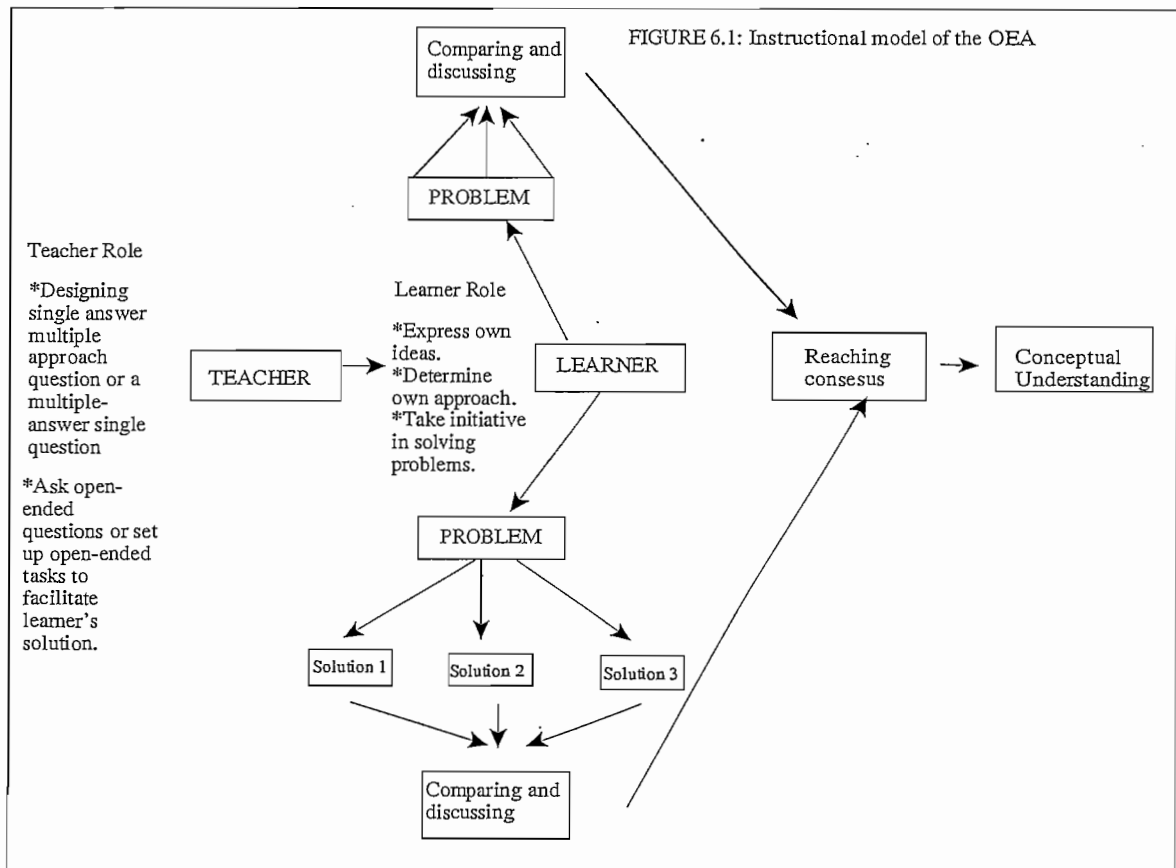


FIGURE 6.1 Summarises the instructional part of the OEA.

Figure 6.1. is a summary of previously discussed items of the open-ended approach to teaching and learning (Section 3.2.). Those items relate to the role of the teacher and the role of the learner. The two components are explained below.

6.3.1.1 Teacher role

The teacher will ask an open-ended question as described in Chapter 3 (See section 3.2.) to facilitate learner solutions. The learners would then be given an opportunity to come up with the answers, discuss their answers among themselves with a view to reaching consensus. The teacher's intervention will be informed by the proposed learner role. In other words, the facilitative questions that the teacher should ask should be posed in such a way that they make provision for the learners to express their own ideas, determine their own approach, as well as take initiatives in solving the problems.

6.3.1.2 *Learner role*

The learners basically take initiative in solving mathematics problems. They should express their own ideas, determine their own approach as well as take initiatives in solving the problems. In other words, the learner's role should satisfy Table 3.4. (See Chapter 3) on the OEA checklist.

The second aspect of the three-tier model deals with training of subject advisers in successful implementation of the approach. The role of a subject adviser is, among others, to advise educators on matters relating to the subject. They have the authority to run workshops for educators.

6.3.2 Recommended intervention at subject advisory level

The mathematics adviser's appreciation of the author's OEA compliant learner material on 'Remainder Theorem' and decision to use it in other schools in the education district, prompted the author to think of addressing the OEA at subject advisory level in a holistic way. One of the findings of the study was that use of the OEA material by the teacher was not enough to guarantee the teacher's successful implementation of an OEA. In other words, the study findings did not justify a situation where OEA-compliant material was just handed over to the teacher to use without monitoring if the teacher's facilitative intervention complied with the OEA requirements. The choice of a subject adviser as a stakeholder was motivated by access of the subject adviser to the educators. If the subject adviser had the authority to distribute learner materials to educators, the advisor must be empowered to facilitate appropriate use of the material. The study recommends an intensive three-week training of the subject advisers on the implementation of the NCS / OEA. The fact that schools are using a common work schedule will be helpful in the facilitating the training. Table 6.1. summarises how the training can be done:

TABLE 6.1: Workshops on training of subject advisers

Week 1	Week 2	Week 3
Describing and distinguishing between open-ended and closed questions/tasks	<ul style="list-style-type: none">• Using open-ended questions in selected mathematical topics• Brainstorming possible responses of learners to open-ended questions	Development of OEA compliant lesson plans.

The following is a brief explanation of Table 6.1.

Week 1

During the week, a facilitator of the open-ended approach to teaching and learning meets Further Education and Training (FET) mathematics advisers. In the first week, the facilitator splits the mathematics advisers into small groups, and requests each group of advisers to choose an FET level mathematics topic, and to indicate to the facilitator how they would teach the topic in class. They should indicate the questions they would ask their imagined learners. Each group should appoint a spokesperson to give feedback. The aim of the workshop would be to conduct a needs analysis. The facilitator's interest will be on the type of questions the advisers would be asking the learners, and whether or not the approach the subject advisers would use would be teacher-centred or learner-centred. The facilitator would record the questions asked. The facilitator would then briefly explain to the advisers what the open-ended approach to teaching and learning entails. If the questions asked by the advisers were mostly of a closed nature, the facilitator would then take a few of those questions, and then indicate how they could be rephrased in order for the questions to be open-ended (See examples of Section 3.3.8. in Chapter 3). The advisers would then be required to rephrase their closed questions in order for the questions to be open-ended. The result of this exercise would be an indication from the mathematics advisers of their ability to ask learners the anticipated open-ended questions required for facilitating the process of problem solution by the learners.

Week 2

The advisers would split into groups according to grades 10, 11 and 12. Each group would look at the topics that would be covered in the common work schedules schools would follow

for the year. The groups would then choose two topics – from geometry, algebra, calculus, analytical geometry etc. - and then indicate the open-ended questions they would ask the learners in class when teaching such topics. They would also brainstorm the potential learner answers to their questions, and their follow-up questions to the learners' anticipated answers. The result would be open-ended questions, anticipated answers, and the follow-up questions.

Week 3

The advisers would use the information of Week 2 work to generate lesson plans. In addition to that, the advisers would develop lesson plans for the first month of teacher work in the work schedule. If, in the facilitator's view, the advisers' lines of questioning complied with the requirements for an open-ended approach to teaching and learning, then they would be considered to be ready to train the educators on this approach. Otherwise more examples would need to be given in order to qualify them to be ready. A grade 5 example of an OEA compliant lesson plan by Takahashi (2002) is attached as **Appendix 9**. The grade 5 example has been chosen firstly because there was no grade 11 example, and secondly because the lesson plan had all the relevant components needed to illuminate the properties of an OEA-compliant lesson plan.

The third component of the model deals with teacher training.

6.3.3 Recommended intervention at teacher level

According to the National Curriculum Statement, as articulated in the policy document on Learning Programmes, designing lesson plans is the individual educators' responsibility (See Chapter 1). The results of the investigations of the study seemed to justify the contention that educators need help in designing their NCS / OEA-compliant lessons. The observed tendency towards teacher-centredness in current approaches used by educators clearly indicates that educators have to be empowered to enable them to prepare appropriate NCS / OEA compliant lesson plans. Constructing a good open-ended problem is not an easy task for a teacher whose approach has been predominantly teacher-centred. One related overseas study in Thailand found that it was difficult even for Thai educators to organise many mathematical concepts into a problem situation (Inprasita, 2006:101). For successful intervention at the teacher level, two major areas needed attention, namely for the teacher to develop material to be used by the learners and for the teacher to intervene appropriately in learner solutions when using the developed material. The reader should recall that this study

has indicated that the teacher's use of OEA compliant material did not guarantee success, unless the teacher knew how to use the material. Pre-service and in-service training will now be considered.:

6.3.3.1 *Pre-service teacher level*

The South African schools are currently expected to implement the NCS recommendations, whose compatibility with the open-ended approach to teaching and learning has already been described. An implementation of an OEA could be done on a sustained basis if the approach could be taught at pre-service level. The production of a course or module on the OEA should be implemented at tertiary institutions providing the study. Table 6.2. summarises some of the steps needed for successful preparation of pre-in-service teacher training for implementation of the NCS recommendations through the use of an open-ended approach to teaching:

TABLE 6.2 Items and their time frames: Pre-service educators

ITEM	TIME FRAME
Supervision of the student educators	3 times per week in the first semester.
Student educators attend seminar or meet their research advisers, where they express their common concerns, interesting points etc. about the OEA.	Once a week in the first semester.
Prior to the teaching practice, prospective educators construct lesson plans to be used during the teaching practice as part of course work.	<ul style="list-style-type: none"> • 1st semester • 3 periods per week prior to the teaching practice.
Prospective educators conduct practical teaching at selected schools	2 nd semester

What the table implies is that in one year of study, the student studying to be a teacher should attend a course specifically designed to address (as one component of their courses) all aspects of the open-ended approach to teaching and learning conveying what it is, examples, construction of lesson plans etc. This should happen within the first semester in three periods per week. This should culminate into the construction of the OEA-compliant lesson plans by each student by the end of the first semester that would cover the topics to

be addressed during the teaching practice. In the second semester, the student teacher should go for a six-month teaching practice in local schools, where a lecturer from the tertiary institution offering the teacher qualification would come and observe his/her lessons three times a week. There should be a once-a-week meeting between the institution's representative and student educators with a view to brainstorming items in the lesson plan that will need revisiting.

6.3.3.2 Recommendations for in-service educators

After the educators in a school have met to develop learning programmes as per requirements of the NCS policy document, the subject advisers should then facilitate the educators' development of the OEA-compliant lesson plans for the topics they would be teaching in the first few weeks of classroom lessons. 4 weeks should be reserved for the facilitation, between the completion of the previous final examination and the time of starting with lessons in the current year. Table 6.3. identifies the items needed to be addressed by the advisers and their time frames:

TABLE 6.3 Items and their time frames: in-service educators

Item	Time frame
Workshops on open-ended mathematical tasks.	First week
Preparation of OEA-compliant lesson plans covering 10 mathematical topics.	Two weeks
Monitoring of classroom implementation of the NCS / OEA	One week

What the table suggests is that a subject adviser, in collaboration with a facilitator, should conduct 4 workshops for in-service educators. In the first week, the focus should be on the open-ended approach to teaching and learning. All definitive aspect of the approach should be dealt with during the workshops. During the second and third weeks, educators should identify 10 topics that they would be teaching, and prepare lesson plans on these topics that will ensure compliance to the open-ended approach. It is hoped that all educators in the same education district will be doing a common work schedule. If not, the subject adviser should recommend strongly that they do. They would then discuss each of the topics, in order possibly to exhaust all anticipated learner responses to open-ended tasks for each

topic. In the fourth week, the adviser should monitor each teacher for a week to see if there is compliance with the open-ended approach during the class proceedings. It is further recommended that the educators successfully preparing lessons should be given some incentives. Possibly an achievement certificate linked to an increase in remuneration.

6.4 LIMITATIONS OF THE STUDY

There are two levels at which the limitations of the study were considered. The first limitation related the difficulties inherent in the study design. The second limitation related to the ability to generalise the study results to the entire South African school population.

6.4.1 Difficulties inherent in the study design

The main difficulty was to monitor the activity of every learner at any given time, especially in a group discussion where there was domination of some learners by others. This was compounded by some learners' reluctance to engage in group discussions.

6.4.2 Generalising of the study results

There is not enough evidence to suggest that the post-test results in this study could be reproduced by a similar study done elsewhere. Neither is there a reason to believe that the results could not be reproduced.

6.5 CONTRIBUTION OF THE STUDY

The author's pre-investigation observation was that teacher domination was widespread in many South African schools. The author's pre-investigation interaction with many schools either through conducting workshops for educators or observing teacher lessons confirmed this point. In that sense the pre-intervention classroom practice at the experimental and control schools mirrored what happens in many schools in South Africa. What necessitated the curriculum change in South Africa was precisely the need to address the teacher-dominated classroom practices. Unfortunately, the classroom practices do not seem to have changed up to the time of the investigation of this study. Nieuwoudt's (1998) observation that many of school mathematics educators hold on to traditional formalistic-static views of mathematics and mathematics education, while only a few reject this view in favour of a dynamic alternative view of mathematics, seems to have been confirmed by the researcher.

What this study has highlighted is that educators have not yet changed their teaching and learning approaches, and are consequently not ready to implement the NCS requirements.

The post-test results of this investigation have shown that the open-ended approach to grade 11 mathematics teaching and learning, if properly practised, did have a positive impact on the learner performance. In that sense the study has identified one area that needs to be explored further. Would the same results be reproduced in other schools? The National Curriculum Statement, which is currently practised in South Africa, requires the type of teacher practice that could adequately be served by the teacher's competence in applying the open-ended approach to teaching. In other words, the study is a relevant vehicle for preparing the educators for the successful implementation of the NCS requirements.

6.6 A FINAL WORD

There have been claims that Outcomes Based Education has failed. These sentiments have been expressed in the South African print media. The findings of this study have indicated that OBE is currently not appropriately being practised in many schools in South Africa. This could either be attributed to resistance by the educators to change, or by the inadequate training of the educators to implement it. Under the circumstances, it would be difficult to have a proper assessment of whether OBE has failed or not. This study has given a glimpse of the possibility of its success if properly implemented and managed. The study has also identified the needs that require to be addressed in order to rectify the situation.

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APPENDIX 1

NATIONAL SCIENCE WEEK: 12-19 MAY 2007

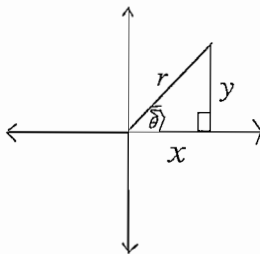
VENUE: NORTH WEST PROVINCES

17 MAY 2007: EDUCATORS' WORKSHOP

Educators were given an opportunity briefly to outline how they would introduce – at any grade level - the sine function in a classroom situation. The main idea of the author was to establish whether schools in that region encourage learner-centredness or not. The educators were split into groups of six. They were required to discuss in the small groups and give feedback to the audience using flipcharts. The following were the group responses:

Group 1

One group referred to a figure drawn on the flipchart that looked like the one below:



They then defined the trigonometric functions as follows:

$$\sin \theta = \frac{Opp}{Hyp} = \frac{y}{r}$$

$$\cos \theta = \frac{Adj}{Hyp} = \frac{x}{r}$$

$$\tan \theta = \frac{Opp}{Adj} = \frac{y}{x}$$

$$\operatorname{cosec} \theta = \frac{Hyp}{Opp} = \frac{r}{y}$$

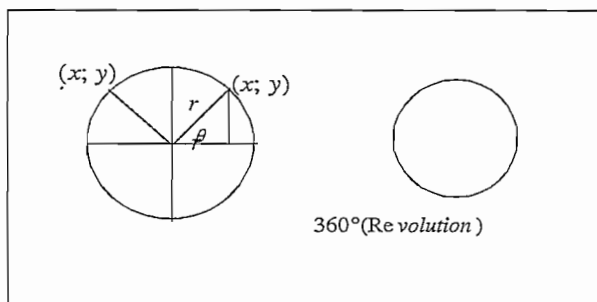
$$\sec \theta = \frac{Hyp}{adj} = \frac{r}{x}$$

$$\cot \theta = \frac{Adj}{Opp} = \frac{x}{y}$$

Group 2

This group mentioned that they would talk of voice pitches of men and women as expressible in sine wave terms. They said they would inform the learners that the equation governing the human voice pitch is $y = A \sin \theta$, with A being small for men's low pitch and large for women's high pitch. They would then plot the graph of $y = A \sin \theta$ by using chosen values of θ .

Group 3



The group indicated that they would first introduce division. For example, they would demonstrate that $\frac{2}{2} = \frac{1}{1}$. They would then talk of a point (x;y) that moves in a circle as indicated by the figure as the one below. Once the point

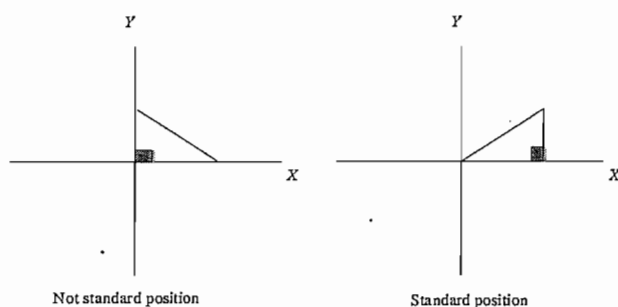
has completed a revolution, they would inform the learners, it would have made 360°

The movement of the point (x;y), they would tell the learners, would result in the variations of the y-component. They would then introduce the sine function.

Group 4

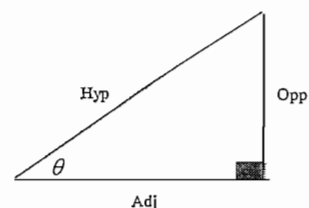
The following are the steps the group said it would take to introduce the sine function:

1. Introduction of Cartesian plane and coordinates (x & y).
2. Introduce right-angled triangle on the Cartesian plane and the position



of the right-angled triangle. The group drew the following diagrams:

3. Emphasising the naming of the sides of the right-angled triangle. The group drew the following diagram to illustrate their point.



The group mentioned that it is important to indicate that the sine is a function or ratio. The group would then

introduce the function $\sin \theta = \frac{opp}{hyp}$.

Group 5

This is what the group indicated on their flip chart:

Pre-knowledge: 1. Able to plot a linear function.

2. The shapes of a function.

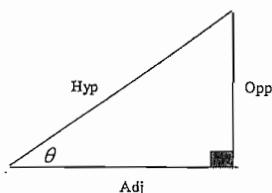
3. Real-life example. The group did not specify the example.

Introduction:

1. We compare the linear function with the sine function.
2. We use a table with x values and ask them [learners] to fill in the table.
3. Discuss the features of functions from function $y = \sin x$ drawn.

Group 6

This group said it would draw the right-angled triangle as shown below:

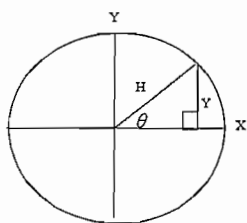


They would then familiarise learners with SOH, CAH and TOA rules. They would then focus on SOH, namely

$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

Group 7

This group indicated that it would draw the following diagram, and then define the sine function.



They would tell the learners that $\sin \theta = \frac{\text{opp}}{\text{hyp}}$

The educators were subsequently given worksheets on the open-ended approach to the introduction of the sine function. The worksheets would be used by the learners the following day.

The reason the author did not specify exactly in what way the sine function would have to be introduced to the learners was mainly, as has already been mentioned, because the author just wanted to check if learner-centredness is the focus of such educators. The content was not the author's main concern. It is clear from the above responses that learner-centredness is not the educators' priority in this particular exercise.

18 MAY 2007: LEARNERS' WORKSHOP

The high school learners were given worksheets to immediately start working on, without any further explanation. The worksheets were about an open-ended approach towards introducing the sine function and its graphical representation (See section on mathematical examples below). After about three hours of working on the worksheets, the learners – in small groups - were asked to answer the question: *What do you think about today's lesson? What they said – captured on flipcharts – were as follows:*

Group 1

- It's a more practical way of understanding sine function;
- The piston is something that is used in everyday life scenarios (cars, water, pumps). In such ways it will be easier to remember;
- It is beneficial to work in groups. In that way everyone obtains more information and it encourages others to enjoy doing maths and science. All should participate. More pleasure will come out of it.

Group 2

- It is much easier to understand when things are done practically by relating it to real-life situations;
- The method of teaching this way makes it easier to understand, and we are able to answer questions;
- Learners will enjoy maths and science, and the academic results will improve;
- By working together we can gain more knowledge.

Group 3

- In this lesson we learned and gained more knowledge;
- It was very uplifting and must be done more often;
- We shouldthis way of teaching, in guidance of a teacher, so that he/she can help;
- If only we had more time to finish the activities.

Group 4

- We did activities in a way that we don't usually do, like doing activities without the teacher's guides;
- We managed to go through them except for a few questions which some of us didn't know how to approach;
- Some of the activities were challenging but we managed to complete them due to team-work;
- It was fun;
- We learned the sine function which we haven't done before.

Group 5

We liked the method that was used to teach us, because it is challenging and it helps us to be able to develop our learning skills and we can be able to tackle any problem that we come across.

Group 7

- It's easy to work as a team because we can help each other;
- We should be given more time;
- I think today's lesson was very interesting, because we had met several tricky questions and activities. We think this science sessions should be done more often.

Group 8

- This lesson was very difficult for us, but we would like to do it more often at school;
- It challenged us, because we were not prepared, but we would like to try and put more effort on it;
- It was difficult because there was no one to explain it to us.

Another Group 8 (Same group number by mistake)

- They [educators?] should follow this method of teaching;
- Learners are being able to assist each other;
- It will be a privilege if this kind of teaching method is followed;
- This will assist in producing better results;
- One gets to see his/her weaknesses especially in problem solving and will assist in improving one's standard of problem solving.

Group 9

- The activities were challenging and lack of enough time;
- We have learnt mathematical techniques and how to solve problems involving mathematical equations or system;
- We wish the lesson could continue at our school;
- They made us learn from each other's ideas;
- The lesson was interesting as we were discussing to make it seem simple.

Unspecified group (a)

- I think we should do it sometimes, because it motivates us and others;
- We are seeing this for the first time;
- I like it because it gave me the opportunity to express/share my knowledge with them;
- I felt good to share my thoughts with other learners.

Unspecified group (b)

- We really enjoyed this lesson;
- I was very interested about what we learned today;
- It is what we need most when we come in mathematics;
- It was very important;
- It is the biggest challenge in our future.

Unspecified group (c)

- So far the activity which we are still doing is quite flexible & mind challenging, but fun at the same time;
- We should do this more often.

Group 10

- We think it is a good method of teaching, because it gives us a chance to express our feelings and opinions;
- This activity needs a lot of concentration;
- We think we should be taught the basics before we start the activity;
- It needs more time;
- This activity has taught us how to work as a group/team.

APPENDIX 2 : PRE-TEST

Pre-test

Grade 11

22 January 2007

Answer all the questions. The questions test the prerequisite knowledge to completing the square.

1. If $a = 1, b = 3$ and $c = -2$, then $b^2 - 4ac =$
 - a. -7
 - b. 9
 - c. 6
 - d. 10
 - e. None of the above.
2. $\frac{4ac - b^2}{4a} =$
 - a. $c - b^2$
 - b. $-\frac{b^2 + 4ac}{4a}$
 - c. $\frac{-b^2 - 4ac}{4a}$
 - d. $-\frac{b^2 - 4ac}{4a}$
 - e. None of the above.
3. In $x^2 + 6x - 3$, the coefficient of x is
 - a. $6x$
 - b. 3
 - c. 6
 - d. -3
 - e. None of the above.
4. In $x^2 + 10x$, the square of half the coefficient of x is
 - a. 25
 - b. 50
 - c. $\frac{1}{4}$
 - d. All of the above
 - e. None of the above.

5. If $a(x + \frac{b}{2a})^2 = \frac{b^2 - 4ac}{4a}$, then
- $(x + \frac{b}{2a}) = \frac{\pm\sqrt{b^2 - 4ac}}{2}$
 - $(x + \frac{b}{2a}) = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$
 - $(x + \frac{b}{2a}) = (\frac{b^2 - 4ac}{4a})^2$
 - All of the above.
 - None of the above.
6. In $x^2 + 10x$, of half the square of the coefficient of x is
- 25
 - 50
 - $\frac{1}{4}$
 - All of the above
 - None of the above.
7. If $a = -1, b = -1$ and $c = 6$, then $\frac{4ac - b^2}{4a} =$
- 5
 - $\frac{23}{4}$
 - $\frac{25}{4}$
 - All of the above
 - None of the above.
8. $3(x^2 - \frac{x}{3} + \frac{1}{15}) - \frac{1}{5} =$
- $x^2 - x - \frac{2}{15}$
 - $x^2 - x$
 - All of the above
 - $3x^2 - x$
 - None of the above.
9. The graph of $y = ax^2 + c$, where $a \leq 0$, has :
- A minimum value
 - Both a minimum and a maximum value.
 - None of the above
 - All of the above
 - A maximum value.
10. The graph of $y = ax^2 + c$, where $a \geq 0$, is
- Facing upward.
 - Facing downward
 - A line.
 - All of the above

e. None of the above

APPENDIX 3 : POST-TEST

Post - test

18 April 2007

Subject: Maths

Gade: 11

L.O covered: 2

A.Ss.11.2.3, 11.2.4, 11.2.5, 11.2.6.

Marks: 50

Marks

Examiner: Mahlobo RK

1. Factorise $2x^2 + x - 15$ [3]

2. Solve by completing the square: $-3x^2 - 5x + 12 = 0$ [7]

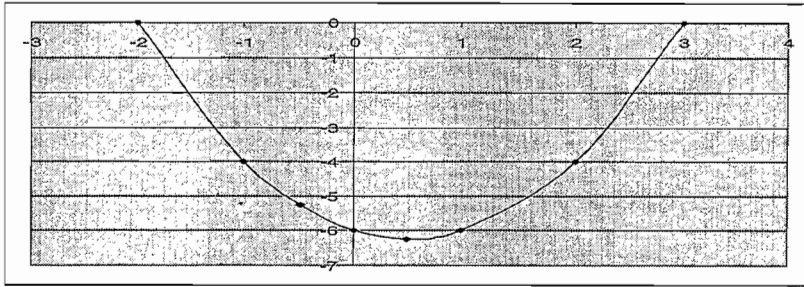
3. Draw the parabola $y = 3x^2 - 22x + 24$ [9]

4. A spaza shop buys a certain number of sweets for R100. When left with 5 sweets, the spaza shop has already made a profit of R504. If the shop sold each sweet for R1 more than it originally paid for it, how many sweets did the shop buy? Let x be the number of sweets bought. Fill in the following table in order to answer the question.

	No of sweets	Total Price	Price per sweet
Cost Price	x		
Selling Price	$x - 5$		

[8]

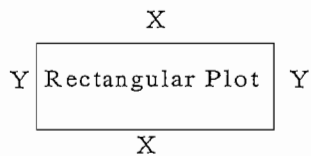
5. Identify the equations of the following graph and identify the coordinates of the turning point. [4]



6. Solve for x : $x - 2 = \sqrt{3x - 8}$ [5]

7. In a self-help scheme, community members interested in farming are given rectangular plots to grow vegetables in. One member has 24 metres of fence to cover his rectangular plot with. What will be the dimensions of his garden for *maximum* size?

[7]



8. Solve for x : $x^2 + 5x - 6 < 0$ [7]

TOTAL: 50

APPENDIX 4 : INTERVIEWS

TRANSCRIPT OF INTERVIEWS

TAPE 1

Group 1

Researcher: Eh ..God morning lady and...and.. gentlemen..

Eh.. The purpose of this interview is basically to get the feeling of the learners in relation to whatever methods they have been exposed to.

You have been grouped into...eh... small groups of three.

A question will be asked. Let each one feel free to answer.

You can support or disagree with the response of your fellow learner, and ...anything that you need clarified...clarification on before we start?... No.

1st learner: Ah...I think everything is fine. You can start.

Researcher (R): Ok. Right and then ...eh... feel free to say... eh... anything

if you need clarification. Right, ok...let us...eh...let me...start by the question. What do you think about the approaches to teaching mathematics that you have been exposed to?

2nd learner: Well our first approach ...eh.. on the first term was we were given some papers which has questions on it so and we had to work them out on our own as to we can get more knowledge in to how do we work on problems on equations and some of the learners were understanding that method better than we are doing now because now the teacher comes in front and gets in the board and writes and after he writes whatever he writes he wants you to understand and know and after that he will give you a class work or a home activity and he

expects that you must do it and do it perfectly because he or she has shown you the way to do it and so you have to do it.

R: Ok. And...any other response?

1st learner: Ah... and I think to what he have already said is that at the beginning of the year we were introduced..eh.. the educators introduced a handout to us so that we can solve the problems of mathematics.. ah.. according to my side that is very cool because.. like some...some other learners ...they rely on educators and sometimes you find out they don't understand what's going on and but when the teacher said "Do you understand what's going on?" they will just all say "Yes, sir, we did understand" but deep inside they know that they don't understand, they end up failing the test.

R: Ok. And then ...eh...eh.. just on that point, so what do you think that method....what type of role do you think that method has played in trying to solve that particular problem?

1st learner: Ah...I think that the role that it has played is that....ok...when you are given that handout...all of us maybe in the group of six we are...like we have to be able to express ourselves about the answers that we'll be getting and then...thereby each and everyone of us will be getting an answer, and will agree in one answer.

R: Oh ...so what you mean is that in a way you are able to help each other up to the point of identifying each other's problem?

1st learner: Yes

R: Ok. And then ...what does the other one say?

3rd learner: Ok...about the teacher giving us the pamphlet I think it is a very good idea because once the teacher has given us the pamphlet for me to go and try it on myself.. on my own...I'm sorry... then I think it is very easy because when the teacher comes the

following day the teacher will be able to see where exactly my problem is, than the teacher standing in the board and teaching.

R: Ok. So it appears to me that you are favourable to this particular method. Eh...what about...eh.. the little problems here and there? What is it that you feel we shall need to sort of improve on in order to maximise the potential of this approach?

1st learner: Ah...to me I think the little problems that we come and get across I think ...ah.. to a teacher it is not like a big deal for him or her to solve with us those problems I mean like...this is the New South Africa .. we have to do it by ourselves and we have to investigate. There are better people and good skilled people in mathematics that they can be able to help us and teach us how to solve those problems than our educators.

R: Oh...so you feel that if you help each other out you will understand each other better?

1st learner: Yes, we will understand each other better.

R: Ok...and then ..eh...now what do you think needs to be done in order to make this approach really successful?

2nd learner: I think that when educators are going to really give us those pamphlets they need to also invigilate, go around groups to see as to who is participating and who is not participating so that they can see who will be working very well and who will not be working very well and at home when you are given some equations on those pamphlets to do at home, we must all participate on that, and when we come back tomorrow, then each and every one must show that I did this and that and that and that's where I could not go further so can you please help me here and there so that we can help each other and proceed.

R: Ok. And then anyone... any other contribution?

3rd learner: Ok. The other thing that I think the teacher must do is, when the teacher asks the question, the teacher must not only point those that their hands are up. They must also try

those that don't even know the answer because if the teacher asks the person whose hand is up it is like the teacher is moving on with those who understand and leave us behind ...those ones that our hands are not up.

R: Can I get clarification. Are you talking about the situation where now the teacher is the one who is in control?

3rd learner: Yes.

R: Ok. So how do you think that this thing of having to be given worksheets to work on is helping in that respectHow do you think it is avoiding the problem of the teacher when the teacher is dominating?

3rd learner: Yeah I think it is a very good idea teacher giving us the pamphlet because you are all...I mean like ...in a group we can all try and help each other than the teacher standing there in front and teaching us.

R: Do you think you understand each other better than when an expert is there in front of you?

3rd learner: Yes, sir.

R: Why do you think so?

3rd learner: Because...

1st learner: Ah...because...ah...to us we are getting...ah...we are getting... more invigilate to get like...to solve those equations by...like...by ourselves because as I have already said previously at the end we come with different answers then maybe teacher...is whereby the

teacher can come and say: No here you were supposed to do this guys and here you were not supposed to do this you are supposed follow these steps, such things.

R: Hm. But how do you think that this thing of having to get different answers can ultimately lead you to one answer. Is it not going to result into you arguing and everybody saying my one is the correct one? How do you solve a problem like that?

2nd learner: Eh...to solve the problem like that I think it is when you have someone in the group who understands better than the others, not that I mean that he is the cleverest and then he can come in and help and say guys even though you have done one...two...three and then here our answer will be like this because they say that there are many ways of killing a cat but there is only one way to eat it. And then even though you have different steps, but at the end you will end up having one answer which is the correct one and then teacher will then say it is the correct or not the correct.

R: Am I right to say what you are saying is that even if you have different answers...eh ...you are each going to justify your answers up to the point where the right one is always going to come on top?

2nd learner: Yes.

R: Ok. And then any other comment relating to your experiences?

1st learner: I think...like...when you are given those pamphlets to solve by our own...I mean like...it's all about working together and understanding each other because...like...if we can ...maybe...argue with the answers...even though....I know that we are going to argue with the answers ...but that is going...is not going to help us with nothing because we'll end up like having a wrong answer and agreeing with that wrong answer. So like as Sifiso [1st learner] has already said ...ah ...the one who understands better than those among that group then she will...he will be able to help us: 'no guys, you were supposed to do this here...' 'you were not supposed to do this there...'

R: Ok. And then...eh...what do you think if this method were to be applied...what do you think....what type of results would you think it would have on your performance in general?

2nd learner: Well, I think our performances as well will have been great because each and everyone would have had a chance to express him or herself and then get to understand as to what we are working on and what we are not working on because some people when teacher comes in and say 'do you understand' then they can say 'yes we understand' even if they know they don't understand, because they are afraid that if they say they don't understand some people will laugh at them and say 'which part don't you understand, because this is like this and that'. And you don't blame them because that's the way they are and ...eh...one day one day when the teacher teaches you won't understand some things and you won't expect people to laugh at you because it isn't right.

R: So what about now if you are working in a group? Does it make it easier to understand or whator are you less afraid now to express yourself as compared to when it is the teacher because you say when the teacher is available you are afraid maybe to raise your hand to say 'I don't understand. How do you think that this thing of working on a pamphlet will help you?

2nd learner: Working on a pamphlet is helpful because when you don't understand you ask us as a group because you have group mates which you understand which you stay with each and every day and when you ask them questions they will be able to answer you and then without any..[inaudible]...because you are in a group and then you are brainstorming and then after that you in a group you are asking a question no one will see you if you are asking a question and laughing at you because they will be doing their thing in their group and you will be your thing in a group.

R: Ok. Is there any addition to this?

3rd learner: Ok. About us having a pamphlet ..yes it's great because sometimes you can find out that...I mean... those kids...the learners that I'm staying with in a group I understand better than the teacher because at lunch I am with them....anytime I am with them then I cannot be afraid to ask questions to them than asking a question to a teacher.

R: Ok. So ...eh...so what you are saying is that even after school you can still continue...maybe...discussing this together.

3rd learner: Yes.

R: Ok. Any other comment?

1st learner: Yeah...I think...ah...it's better if you are given the pamphlet to solve with a group because...like...we as youth...then you find sometimes you are maybe same age group and are not afraid of each other...No that one is old. If I say this he or she will get cross with me... like we are always expressing ourselves how do we feel about what we are given by the educators. In everything that we are doing will be like expressing ourselves to those people how do we feel about ourselves. I think that is very much better.

R: Now I don't know if there is anything to add.

2nd learner: There is anything to add because as the last time we said on class that those pamphlets will be great because when we get to tertiary institutions educators will only be lecturing us and this thing will groom us as to when we get to tertiary institution when it comes to lecturing so we can understand as to what to do because those pamphlets will be helping us as now when the teacher comes is the same as the teacher comes and say 'where have you ended'? 'Can you come and explain to me how did you solve?'

R: Anything else?

1st learner:

Ah...I think like...ah...as I said before....because we have meet last time...I think it is better because here in South Africa it has been like a culture when the teacher is coming in front of you and teaching. Things are ... technology is changing and things are changing. We are getting to Eurocentric life ...we as Africans, so then I think it is better when we are given those pamphlets because when we knock off here at school and go to FET level that's where

we are going to see the differences. Then ...ah... I think if we follow that culture of a teacher coming in front of you, when you go to tertiary you are going to get more problems because a lecturer will come in and give you a topic, explain a little about it and leave you. You understand?

R: Hm. So, do I understand you well to mean that you say that the method is promoting independence?

1st learner: Yeah...I think...like...yeah it is promoting independence because....like...this is new South Africa. We have to do it by our own. We have to stand for our country.

R: Oh....alright. Anything else?

3rd learner: Ok...the other thing I can say is us as learners we should always ask ourselves a question after we have finished with the chapter a question like "Am I on track?" and ask yourself a question like "Can I do this and this and this?" If you can't you can go straight to your teacher and ask the teacher whether you can do it and at the end you'll be able to understand everything.

R: Ok, lady and gentlemen. Thank you very much for this opportunity.

Group 2

R: Thank you ladies and gentleman for coming here. Remember that the main purpose of this meeting is to try and get your feelings about the type of things that have been happening sometimes in the past. Eh...basically [the purpose of] this interview is to obtain your attitude or opinion about your experiences of the approaches you have been exposed to.

Now you are....you have been split into small groups of three and a question is going to be asked and then feel free to answer. You do not necessarily have to agree to what your fellow learner is going to say. So if maybe there is something that you would like to say that opposes what your previous learner has said, feel free to do that because we really want honest answers. Rightand then...eh... if I ask a question and you need clarification, just make me aware of that, so that we can always be moving on the same wavelength.

The question that I want to ask is: 'What do you think about the approaches to learning mathematics that you have so far been exposed to?' Anybody?

1st learner: Ah....nna I think like...what I experienced from January and up to now....it has been really challenging and difficult but the fact is that I'm here to learn. Ah...if I was not here to learn I wouldn't be here this time. So I'm here to learn but it is very challenging. Yeah, I will learn and if I don't understand I'll ask somebody who does so that he can expreciate [explain] to me....whoever he is to expreciate [explain] to myself.

R: Yah. The main purpose of the question I'm going to ask is to find out if there is any particular method of teaching mathematics that you would say you find easier to understand than the other. What's your comment.

2nd learner: Nna I think the easier method to understand mathematics is for that teacher to be free when he comes to class because there are these other educators who come to class...you see a teacher being revolted [?] coming to class with this....I don't know....face. Then it happens that we don't understand what he is trying to say and when he asks "You all understand?" ,we say "Yes" and when he goes out and gives us work to do we don't do the

work because really don't understand exactly what he meant. I know it is a bit boring and everything but as a teacher you have to come, approach us very goodly [?], I mean, be fine when you approach us and everything.

1st learner: Ok. That is very good of you to say that. And one more thing about that is that when the teacher is in front of us and teaching a lesson, then the main thing that we are supposed to do as learners...we should all concentrate as we know that everything is ...(inaudible) to do but not everything is beneficial so we are all hear to learn and for us to be able to learn it's for you as a teacher to be polite towards ourselves. Don't be rude or something like that because being rude towards us learners it leads us to the point that we say we understand what you are teaching while we don't. Why I'm saying that is because of your appearance and your attitude to yourself. So be polite so that even us can be free to talk.

R: Yeah...it appears from your answers that you are talking about the situation where the teacher is always going to be in control of the class. Are you saying that from the beginning of the year up to now you have always been exposed to one method where the teacher is always at the front and doing everything for you, or can you say there is another method that you have been exposed to and if so, can you describe the method?

2nd learner: Ok, the teacher would be in front of us and teach us and ask us if we understand. Then if we do understand he will give us a method to solve then you go in front to solve, then you expatiate (?) to the class how did you find the answer in that way. Yeah that's how we sometimes understand.

1st learner: I think the teacher have to teach us, then give us work to do so that we can see if we managed to do that work individually.

R: Yeah...let me put it this way: There was a time when the teacher always came into the class to teach you and then there was a time when another method was used other than for the teacher to always to be at the front. Can you describe that particular method and what do you think about it?

1st learner: I think that method was about those copies that they gave to us. Nna I don't think it is bad for them to give those copies because as they are giving those copies they are like lecturing us, because if we can pass matric and go to higher level studies like university and everything, there are not going to educators standing in front of you, telling you what to do and how to do things on your own and not depend on anyone.

R: So what you are suggesting is that when you get those worksheets - let's use the word worksheets – you found it easier to learn than when the teacher was...is that what you are saying?

1st learner: Yes, that is what I am saying because sometimes it's good to learn on your own than for someone to come and tell you. In English they say that no one has to teach you how to eat fish. It's better for him to teach you how to get it so that one day you can get it yourself.

R: Ok. Is there any other comment relating to this method we are talking about...where you are given those worksheets to work on? What do you think about it...what do others think?

2nd learner: Ah...it is a very interesting thing to do and it's very challenging because it also teaches others in some way and somehow those people that teach us...they won't be there. Then there would be ourselves and we should understand and it's fine like that.

R: So are you suggesting that if you are given those problems to work on yourself you end up learning better than when the teacher is standing at the front?

2nd learner: Yeah, we understand it very well and one thing that I like about it is that when we work on these worksheets as a group, there will be one person who understands better than us, then he or she will expatiate to us very well. Then if you don't get exactly to the conclusion he is the first person we should call to confirm if we are wrong or right but it's fine.

R: What about the other one [author referring to 3rd learner]? You have been quite. Let's hear.

3rd learner: I think the worksheets are right because we have to work on our own so that we can develop our skills, so they are right.

R: Ok. Any other comment?

2nd learner: Again as you know there are disadvantages and advantages. On the other hand they [worksheets] are not right because other people when they [educators] give us the worksheets and when they [educators] stay upfront trying to teach us something, they [the other people] don't concentrate. They say 'I have a question paper ...I have the worksheet and I'll work it on my own' only to find that when the person goes home maybe she or he does not have the time to do those things and then just deceiving himself that 'I will do it on my own' while he knows that he won't understand exactly what is happening on that worksheet.

R: So what is it - just point of clarification – What is it that you say will make it difficult for you to understand what you are going to do if you are in a small group?

2nd learner: May you please repeat the question?

R: Eh... you mentioned that sometimes people can think they understand the problem. Does it mean that if you are given a worksheet to work on in small groups, you might end up not understanding everything? Is that my understanding of what you are saying?

2nd learner: Yes, because we came here to school to learn. We don't understand everything – some will understand, others we don't – and that is the reason why we came here in school because if we understood everything we wouldn't have been here. So what we understand we have to share with the other people as a small group and what we don't understand we have to ask so that we may learn.

R: So in other words you are also for working in small groups on the worksheets?

2nd learner: Yes.

R: Ok. Any other comment?

1st learner: Ah...one more thing to say is that when we work in groups...let's say a minimum of about five learners, you find that maybe two or three learners are participating while others don't, and when you tell a person to listen you go like

'Why should I listen because I've got a question paper here and answers on top of it. What's the main plan of participating here? I'll do it on my own. That thing I don't know how to deal with it.

R: So, is it something that will make you feel better if the teacher is at the front or are you still feeling that it would be better if you are still working in small groups on the worksheets?

1st learner: It is very better to work in small groups and also for the teacher to teach us, because a learner when he says 'I do understand even when he does not understand.' At the end who's going to fail? Him- or her-self and you find that for sure sir [teacher] is blaming himself like 'I'm not teaching those learners in a decent way' – stuff like that – but while a learner... I don't know.

2nd learner: *Nna* [Setswana for I] I think I can answer her.

R: Ok.

2nd learner: If a learner he or she doesn't concentrate on anything that you are doing in a group I think it is better for us to tell the teacher and the teacher should deal with the learner and if the learner repeats we should move on with our own things. At the end of the day who's going to fail? He is going to fail so there is no need to worry about someone who does not worry about her or himself.

1st learner: one more thing...

R: Ok.

1st learner: When we are in a class and sir is teaching us in front, there are some learners that they don't understand and there are some learners that they do understand very better than others. Let's say it's 40 learners in the class and you find that 18 learners understand better than others. Then the other learners don't and sir sometimes goes with people who say they understand and leave us. The other learners that don't understand are left behind.

R: So in that way which of the two methods do you think will help to address that problem? Is it the one where you are addressed by the teacher or the one in which you are working in small groups?

2nd : It's the one where you are working together as a group.

R: Why?

2nd learner: Because....

1st learner [interjecting] I think the best way for solving that problem in each and every school each and every learner has to have her or his own book [set of worksheets] so that even if I'm alone, even if the maths teacher didn't come in the class to teach, I may go on with those things so that when he comes I have something. Where I don't understand I'll ask on top of something that I've learned.

R: So you mean that this worksheet will help you identify what you don't understand?

1st learner: That's what I think because copying – the teacher is going to copy something from the book, not everything - just maybe something that is important they'll [the educators]

ask , only to find that in the final exam or international exams and everything, they [spotted questions] don't come, only those things [questions] that he left behind will come out.

R: I want us to feel free that sometimes that when a person is thinking about something, we should give her a chance to say whatever she wants to say before we move on to the next person. Is there any other thing that you'd like to say?

2nd learner: Ok. My question goes like this about what Sharon says....yeah it's something that is obviously ...it's happening and that's the point. At the end it makes us to fail. So I don't know how to deal with the thing. Because sir gives us....

R: What is it that makes you fail?

2nd learner: Like you find that we are going through this pamphlet that they give us. There's also the answer on top of those pamphlets. So it's a work that we have to learn and let the other.....the other[learner quiet for some time].

R: You can talk in vernacular.

2nd learner: And o kereye gore o sia tiro enngwe ka morago and then o re ruta ka e e leng gore o bona e kare at the end e tlo tswa mo exameng and you find that at the end that work he gave us ga e tlhagelle mo exameng and you find out that re a faila [You find that he omits some part of work, teaching only the part he expected to be asked in an examination and you find that at the end that work he gave us does not appear in the examination and you find out that we ultimately fail]. I don't know how to deal with that.

R: So what are you talking about now? Are you talking about the situation where you might fail because the teacher is at the front or are you sayingWhich of the methods are you talking about now?

2nd learner: Ok, I'm talking about the method e e leng gore o kereya a re fa dipampiri tse eleng gore is work e e leng gore re tshwanetse re bereke ka yona.. My point is that mmereko o a re fang gore re tshwanetse go bereka ka ona ke o e leng gore at the end mo exameng o tshwanetse go tthagella mare o se tthagelle. Rona as learners re tshwanetse go bereka ka mmereko oo or not. [I'm talking about the method where you get pamphlets with questions that you have to work on in preparation for examinations, which may not actually be covered by the examination itself. Must we as learners work on such pamphlets or not?]

Premature end of tape 1.

Tape 2: Interviews

	Group 1
R	Thank you ladies and gentlemen for coming here Remember that this is one session in which we want to know about the experiences of our own learners in relations to...you know... the different teaching styles that they've been exposed to. You have been... eh... split into small groups. Then we are going to ask questions and everyone is free to answer and if you disagree with the answer please feel free to disagree. Now... eh... can you briefly tell us who you are and how old you are.
Catherine	Ok. Hi my name is Catherine. I'm 16 years old. I'm in grade11.
R	Okay... right.
Calvin	Eh... my name is Calvin. I'm 16 years old and I'm also in grade11.
R	Okay
Evelyn	Hi... my name Evelyn. I'm 18 years old and I'm in grade 11.
R	Ok. So what do you think about the approaches to teaching mathematics that you have so far been exposed to?

Evelyn	Well, about the methods especially the sheets that the educators gave us, I think it's a very good thing to do because when they give us the pamphlets... they are giving us more chance to learn... especially when they give us the pamphlets to go and do some research at home because many people are going to.... like.... going to give us ideas on how we should do this and should do that ...so we going to come up with different ideas to the teacher
R	Oh ...So what you mean is that it is easier to take home to continue with the work once you have been given the worksheets.
Evelyn	Yes
R	What do others feel?
Calvin	Eh... I think it's not right to give us these pamphlets to do at home because some learners don't understand what... eh... the pamphlets... eh... is written within there. Ok, and then me... I'm talking for myself and then when... eh... the educators is in the class I can understand better because he explains the basics and other things and what she is teaching or he's teaching.
R	Ok. So what you feel is that the best of the two methods is the one in which the teacher just comes into the class and teaches you.
Calvin	Yes, yes.
R	Ok. And what about the others?
Catherine	The method that I think is right is the one of giving us the pamphlet because ...when the teacher is in the class ...on the front of us teaching us, she teaches what she or he knows and he only got the information for us, but when he give us pamphlets we look for research and come with different opinions, whereby we all have different answers and I will do more... that is not on the pamphlet because I will know that by next time they might bring things like this.
R	So in other words what you are saying is that with this pamphlet it gives you an opportunity to....
Catherine	...to do more.
R	Ok. And what do others feel? Any other comment?
Evelyn	Yah...about that again... you know like now... if the teacher comes into the class and teaches us like Catherine has said ...he teaches what he knows but

	then... if he gives the pamphlets then we are taking the pamphlets and we... going out. While we are going out we are going to find more ideas, then meanings. When we find more ideas ...then we going to tell the teacher ...then giving him those ideas. He is going to differentiate which one is right and which one is wrong so ...you see... in that way that we kind of learn better, because we ...asking other people... I mean... we getting different opinions from different people.
R	Ok. So you mean there is always an advantage in hearing different opinions?
Evelyn	Yes there is... amongst the group members.
R	Ok. Any other comment?
Catherine	I think that by this pamphlet... neh... the subject gets more fun because everyone will be participating... what she had got from asking the people from outside... from other schools.
R	Ok.
Catherine	And the teacher will also...[inaudible]... us on "Why do you say the answer it's like this?" "What goes with this?" and "Why do you think so?" until we find the relative correct answer.
R	So in other words the teacher does not give you the answers directly?
Catherine	Yah ...he does not give the answer directly like when he is on the front teaching us.
R	Ok and then.... any other suggestion?
Calvin	Eh... when the teacher is in the class... Ok... when she teaches or he teaches and we can do things in groups or in pairs to understand more... and ask him or her: "What is this?" and you know a thing like that... I prefer when the teacher comes to the class.
R	So ...in other words... you still feel that working in small groups is a good idea?
Calvin	Yes
R	And then... eh... but then you feel that you still need to...what did you say about the teacher?

Calvin	Asking him or her ...eh... the question asking him or her what I don't understand.
R	What you don't understand? So what you feel is...it's easier... eh... to know the answer if you asked the teacher directly as compare to when you ask the other member in the group? Which of the two do you think is something that will benefit you more?
Calvin	Eh... I think in the class... there are some learners who understand things and others don't. So when you asked them: "What is this?" they can explain to you what is this and then get other information from the teacher.
R	Okay so basically if you were to suggest... eh... one of the two methods to be the one to be continued which one would it be? Would be just a teacher standing in front of you and teaching you or would you say there is value in always being given problems to work on your own. What do you think?
Calvin	Eh ...I think eh... I prefer the teacher standing in the front and teaching us.
R	Okay. Eh... is there any particular reason why you feel that it would be the best if the teacher is in front as compared to when you are working amongst yourself
Calvin	As I've said ...eh... when the teacher is in the class, she teaches or he teaches and then if I don't understand something I ask him... I raise up my hand and ask him: "I don't understand there and there and there and there."
R	And then what about if... lets say... eh... you were to ask your members within a small group. Do you think you would not be able to be helped?
Calvin	In the small group? You mean the learners?
R	Yes
Calvin	I will get help but ah...
R	But you still feel the best help will be from...
Calvin	From the teacher.
R	Okay. What do others say?
Evelyn	Well, according to my side I prefer the one that you know the teacher will give us pamphlets ... because if I just prefer the teacher.... like ...standing in front of us and teaching, then he will teach and teach and he asks... eh... those who

	understand. It will be the one who'll be talking to him, then if I don't understand.... you know us learners... we become so afraid ...you know... to ask questions or to tell the teacher that sir I don't understand this and that.
R	Yes
Evelyn	You see... so the teacher will give the pamphlets... I prefer that 'cause we are going to work in groups and working in groups means different answers. Different answers the teacher will tell us which one is right and we are going to follow that.
R (looking at Calvin)	Okay then what do you think about her point when she says some people will be afraid to ask the teacher directly? You were mentioning that you feel that the teacher is best, if he's at the front, now she's mentioning that sometime it is difficult...you know... for some learners to raise their hands if they don't understand. Do you think the problem could be solved?
Calvin	I think... eh... they should ask other learners in the class. You know in the class there are some learners who are very smart, you know this sir
R	So in that way you perceive that it is a good idea to work in a small group rather than to depend on the teacher. Is that your point? I just want to understand.
Calvin	What I am saying is... you can work in groups and then you should include the teacher there. If ...something you don't understand, you go to the teacher as a group or one of them goes there and asks her [the teacher].
R	Okay and then what do you say? Is there anything you want to add?
Catherine	Well by the way as Evelyn said that ... there are some learners who are afraid to ask the educators when they do not understand. The cause of the problem... it might be the teacher ...maybe.... he is too cheeky ... so they are afraid to ask the teacher or he or she goes with those who understand the method better than the others.
R	Okay. Thank you very much, unless there's something you'd like to add.
No response	
R	Okay thank you very much ladies and gentlemen.
	Group 2

R	Er...Ladies and Gentlemen. It is our honour for you to be here. Remember that the main purpose here is just to seek you feelings about things that affect you in your learning. Right, can you briefly introduce yourself? Can somebody get the mic [microphone] there. You may not necessarily indicate the school that you come from. Just tell as your name and how old you are.
Dieketseng	Hi my name is Dieketseng and I'm 18 years old.
R	Okay thank you
Brandeline	My name is Brandeline and I am in Grade 11 and am 16
R	Okay.
Sydney	My name is Sydney and I'm 18 years old.
R	Right... eh... now there are three of you. Eh... I'm actually going to ask a question and then please feel free to answer the question, and then you do not necessarily have to agree. Sometimes if you feel that, no, I don't agree with what the previous respondent said you can just express that. The main thing is for us to get what you feel as individuals. So... eh... I'm going to start by asking this question: "What do you think about the approaches to learning mathematics that you have been exposed to? Anyone?"
Brandeline	Hm... nowadays educators like standing in front of you guys and.... like... teaches them... which is much simpler because you can ask the question if you feel like. It's that bond between you learners and educators. There is that interaction between you and the teacher so I think it is better if the teacher standing in front you delivers. It feels free to be in classes.
R	Yah... eh... okay let me put it this way... eh.... do you know any other method other than when the teacher stands in front of you?
Dieketseng	Hm... I think the method of when the teacher is standing in front of you and teaching, it's... it's ...I don't know how to put it. But I prefer when the teacher wrote something on the...on the chalkboard.
R	You remember at the beginning there was a time when you were given some pamphlets to work on.
Dieketseng	Yes

R	Right. Eh... how would you compare that type of situation with the situation where the teacher would come into the class and teach?
Brandeline	Hm... the pamphlets... like... motivates... like... the learners to go out there and fight for information... like... it is obvious that you... like... once you answer that question paper the teacher is giving you... If the teacher come standing in front of you, some of the learners just take it as like "Aai we are used to the teacher standing in front of us" so if the teacher gives the learners the pamphlets... eh... the learner will go out there to find the information so that the teacher will understand how... eh...how...how much information the learners has concerning the work the learners she or he has given to them.
R	Okay. Anybody else?
Sydney	I think is very challenging because if you ...eh... the teacher hands a pamphlet, you are going to find the solution by yourself. I... I... thought think it's very challenging because you are going to do find a solution for yourself.
R	So is it an advantage to find the solution yourself or not, as compare to being...
Sydney (interjecting)	I think it's an advantage because when the teacher stands in front of teaching, some of the learners are not listening...they are doing things. So it's better if you... a teacher hands you pamphlets and you'll find a solution for yourself in order to understand. If you don't understand you go to the teacher and the teacher will explain everything to you
R	Okay. Anything further?
Dieketseng	Ah... I also think is challenging 'cause you get...
R	Hm... I'm listening.
Dieketseng	Yes you get a chance to...
R	Yes...just continue please.
Dieketseng	Okay I also think it's challenging because you get a chance to find information by yourself ... yah...
R	So the main thing is to find information by yourself?

Dieketseng	Eh ...yeah and you get used to the idea that you can do things by yourself.
R	So it provides this sense of being independent when it comes to studying. Any other contributions or suggestions?
No response	Okay , if you don't have any further contribution, thank you very much.

	Tape 3 Individual interviews
R	Eh ... hello... We are going to talk about... eh... your experiences in terms of different methods that you have been exposed to... for mathematics. Can ...eh... you... eh...first introduce yourself and tell us about the different approaches towards teaching mathematics that you have so far been exposed to. What do you think about those approaches towards teaching mathematics that you have exposed to?
Thandeka	Hi... My name is Thandeka. Eh... we have been exposed to two different methods which is ...a teacher standing in front of us and teaching us and other one are the pamphlets. Eh... myself I will say that I prefer the pamphlets, 'cause when you having pamphlets in front of you it's like you reading it and you're trying to understand it as far as you can. If you don't understand it, it's still much better 'cause the teacher will make you understand and I think it improves English skills doing maths or whatever. So I think it is the best.
R	If I understand you well, you mean... eh... sometimes something can be written and you are in a better position to try and ask the teacher to explain it to... to you. Is that what you mean?
Thandeka	Yes. That's exactly what I mean, 'cause sometimes there are some words that you find difficult and sometimes the teacher will say them in front of you and he's like just talking. He's not... like... eh... emphasizing them that you can ask him: "Sir what does that word mean?" but when it's written down it's like you see it everyday and if you don't understand you have a better chance to ask him as to what it

	means.
R	But it appears that learners generally have a problem in term of asking educators. What is wrong if a teacher mentions the word and you don't understand it, as compared to when you read a word on ...eh... a pamphlet and ... and you don't understand it? Why is it that you appear to find it easier to ask the teacher when it is something on the pamphlet as compared to just raising your hand and ask the teacher to explain that word?
Thandeka	Some other words are... are pronounced in the same way but you find that they are not written in the same way. So when it's written down you can ask a teacher if it's not what you thought it was, but when the teacher talks you can think he means something else whereas he means the other thing, so it's easy to say it.
R	Ok... eh... I understand that. Now what do you think is the role of members in your group? When you are given...you know... a worksheet or a pamphlet - as you call it – to work on, what do you think will be the role of the learners in the group?
Thandeka	Some of the learners... they find it difficult to face the teacher and ask him as to say: "Sir, I didn't understand you here or there" but they find it much easy to ask their colleagues... like their friend in a group or group mates. They find it more easier than going straight to the teacher
R	Ok ...eh... then what do you think are the other...eh... advantages of...eh... using the pamphlets? Let's suppose now that you are given the pamphlets and you are expected to do work based on it at home. What do you think will happen?
Thandeka	Well...if you have a pamphlet and you have to refer to it... doing something...it is much easier 'cause you can even take pamphlets home and at home there are many people who can assist you as to what's going on here and what's going on there, unlike having a teacher in front of you and it's like you just have to get everything from him. Sometimes you cannot understand a teacher... he cannot explain in the best way as the other person can do.
R	Ok ...eh... from your experience would you say there were many situations where you felt comfortable mainly because you were able to solve many problems more often when they were in the pamphlets as opposed to when the teacher gave you

	the problems? What do you think about those situations?
Thandeka	Eh... it's like in the pamphlet it's very nice 'cause sometimes it gives you pride when you... like... solve something in the paper and then all of the sudden you find that you're right and it's like "Yes I did it". So I like this, unlike having the teacher explain. It's like he's feeding you ... telling you each and everything.
R	Ok. So in other words you feel that, if you use the pamphlets you have control over your lesson as compared to when the teacher is in front taking control?
Thandeka	Yes it's like I'm having all the control. If I don't understand well I can still go to him [teacher] and ask him: "Sir, here I don't understand. Can you explain further" or whatever that I need.
R	Ok. Thank you very much. I'm not sure if there is any that you would like to add
Thandeka	No. There is not much I like to add, except for ...well... I would like them [educators] to give us [learners] a paper [pamphlet] as a person ...not as a group 'cause if we share them it's like one person will have to take it and the others would like to take it but if each any every person have her or his own paper you can finish the whole paper in... like... a week, 'cause we can even practise at home with others.
R	Thank you very much. If I understand the last statement you made, every learner can go according his pace using these pamphlets because it depends on how fast the learner participating can understand. Is that what you mean?
Thandeka	Yes. It's exactly what I mean 'cause even if you see that you are a bit slower or you don't understand exactly what is needed of you, you can still refer to the others or to the teacher.
R	Thank you very much.
	2 nd learner

R	Eh... I just what you to remind us of different methods that you have been exposed to but before that... just introduce yourself. Tell us who you are and then tell us what... eh... different approaches of teaching mathematics you have been exposed to, and which of the two you like and why.
Sheila	Hi. My name is Sheila. We've been introduced to two different methods ...which is the pamphlets and the teacher teaching. Eh... I prefer the pamphlets because when we use the pamphlets we get the chance to interact as learners and come up with the solution or sampler solution [?] that suits us than listening to the teacher coming up with his solution that he prefers, but with us learners... we come up with our own solution that comfort our need and how to do mathematics.
R	Ok. So it appears that you value learner contribution. Somebody can argue that sometimes you are going to have different answers as learners. How do you think that can be resolved if you are given the pamphlets and you find that you are arguing because somebody is having her own solution and so on?
Sheila	From my understanding I think that when we are using pamphlets... the thing is that we have to come up with our own solution. But the solution that we have...that we come up with, has to give us the same answer but doing it in the different methods. So the teacher is going to be there to assist us with the correct answer but we have to come with our solution to the correct answer.
R	From your personal experience how does this thing of having to come up with your own solution make you feel? Does it make you feel good that you are in control? How will you describe your experiences?
Sheila	Eh...for me ...I feel great because I come up with the solution that I can use when maybe I write a test. It is simpler and more understandable for me and straight than when I listen to the teacher coming up with his own method which... maybe.... I don't understand.
R	Oh ...so you are raising a very important point... that if you know something and you discover it on your own it becomes easier to use it at the later stage. Is it what you mean?
Sheila	Yes, that is exactly what I mean because when I come up with the solution as a group we sit down and I teach them how did I come up with solution and maybe that will help somebody else and somebody else benefit from what I have done.
R	Oh... so in other words, if I understand you well, what you mean is that, in the

	situation of group work, you are able to explain things in such a way that even the ones who are right or wrong are able to identify the wrong or right of their solutions. Is that what you mean?
Sheila	Yes that's what I mean because if one of the group members understands better than the other one, we sit down as a group. We come up with our own solution and we choose which the best is and then the person that did not understand chooses which one he or she is comfortable with.
R	Ok. So... eh... is there any other thing that you would like to add? I understand all the point that you have raised so far.
Sheila	I like to say that I think next year we should use pamphlets because when we use pamphlets we get a chance to do things on our own and we can even practise at home. So next year each an everyone has to get his or her own pamphlets so that we can go home with that because this year we experience that in a groupthere a group of four people... we get one pamphlets and then after the session the teacher takes back the pamphlets so we can't practise at home.
R	Okay. Thank you very much.
	3 rd learner
R	I want you to introduce yourself first and then... eh... tell us about the different approaches toward teaching mathematics that you have so far been exposed to and tell us which of the two you are more comfortable with and give us the reason on for you answer.
Brenda	Hi. I'm Brenda and the method that we have been exposed to is pamphlets and the teacher standing in front of us...hm... teaching us, so I would go with pamphlets. You know why? Because we go home and do things on our own. And if we don't understand we come back and ask our educators rather than the teacher standing in front of you and writing things on the chalk board that at the end of the day you don't understand and the second one I will go for the group mates. Some of us understand better when our classmates explain things to us. Thank you.
R	So in other words ... you are comfortable in terms of the group discussions... if I understand you well. Let's suppose that the teacher is at the front teaching. Why is it that you feel that you can understand better when you are in a group than listening to the teacher? What do you thing will be advantage when you are in a group?

Brenda	Okay for an example, if my class mate... hm... is explaining something to me I think for me it will be easy to me rather the teacher standing in front and teaching me.
R	Why is it easier, when you are with the fellow learner than when the teacher is the one to get the explanation to you?
Brenda	Some educators are not good on explaining. Yah... I prefer learners, not educators.
R	Am I right to assume that you prefer learners than educators because it is easier to ask the learner if you don't understand than to ask the teacher? Is that what you mean?
Brenda	Yes, sir
R	Eh...But why is it that you think is easier to ask your fellow learner than to ask the teacher?
Brenda	Sometimes if you ask teacher and you tell them you don't understand, they will tell you that they don't understand. They also don't understand.
R	So what you mean is sometimes the educators are not in a position to exactly know what the problem is while the other learners will do.
Brenda	Yes sir
R	Eh... any other thing you can think about.
Brenda	No sir
R	Okay ...eh... thank you very much.
	4 th learner
R	Right... eh... I want you to introduce yourself and then if you recall the different approaches of teaching or learning of mathematics you were exposed to, please remind us, as to what they are and tell us which... er...of those methods you are more comfortable with and give us the reasons.
Phumzile	Hi I am Phumzile and at the beginning of the year, we did the pamphlets method of teaching and then later on we use the one of teacher teaching us. And then I prefer the pamphlets because you can take the pamphlets home and you practise on your own and you can ask your peer to help you.

R	Yah... eh... I understand what you are saying but somebody can say "I can as well ask the teacher." Is there any special advantage in asking the peers as compared to asking the teacher?
Phumzile	Eh... their level of understanding is the same as yours, so they will explain better than the teacher.
R	Okay. You also talk about taking the pamphlets home. What do you think will be an advantage if you have to use the pamphlets as compared to when the teacher gives you the work to do at home?
Phumzile	Eh... the pamphlets you can take it to your friend and practise... ah... with your friend and then you can also understand on your own. When the teacher gives you work he will put it on his own understanding but when the pamphlets come the way you practise on your own then you understand it.
R	Okay from... eh... from your personal experience would you say there were times when there were some words on the pamphlets that you didn't understand and how did you deal with those?
Phumzile	Eh... I would take my dictionary or ask from other educators.
R	Okay. So there might be the number of reasons why you've been so comfortable with pamphlets. I am not sure if there is anything you would like to add.
Phumzile	Eh... I would like to add this. Eh ...I would advise educators to photocopy more paper for us help an individual to have his or her own paper not as a group because when you have as a group you have to give it to somebody else day tomorrow is somebody else. I think everybody should have his or her own pamphlets
R	Thank you very much.
	5 th learner
MH	Hello. I would like you to start by introducing yourself and then tell us about the different methods of learning mathematic that you were exposed to and which one you like better and the reasons
Nqubile	Hi. I'm Nompilo. I prefer pamphlets because you can be able to participate rather than the teacher teaching on the chalkboard. Hmm ...some of us... we are slow learners... we can't understand while the teacher is teaching. So if we practise with the pamphlets we can be able to help each other to understand.

R	Okay. That one is quite a good point. Is there any other thing that you would like to add?
Nqubile	Eh... what I can add is eh... next year if they can provide us with many pamphlets so then we can be able to work on our own at homes... eh.... even at schools cause... hm... they use to give us those pamphlets to use one per group so I think it's something to improve.
R	Eh... what is the difference between doing the homework at home on pamphlets and doing the work given by the teacher?
Nqubile	I think eh... doing eh... homework at home...hm... is much better cause you understand ...eh.. you can be with your friend that can help you like... yeah... classmates. They can help you, and when you are at home you can meet somebody at the library then help each other. Yah.
R	Oh. If I understand, you are raising a very important point... that with pamphlets you are motivated enough to concentrate. It can continue to happen in the classroom till the late hours. Is that what you mean
Nqubile	Yes is that I mean
R	Okay, thank very much okay o right

	Tape 4: Teacher from experimental school whose class was consistently monitored
R	Eh...Good..... (Tape not audible at the beginning.)
R	Eh...for the purpose of testing this thing of having to use worksheets...(Tape inaudible)
Skhumbuso	...things for them, and then they work on those worksheet on their own and sometimes we group them so that they work with those worksheets. So it was a very ...very useful tool and I think in future, if we can ...you know....use that, we're going to manage because learners - it seems - understand what they are doing and they can even take those worksheets' home to work...you know... on their own ...maybe work in groups and to have more... more knowledge.
R	Okay... eh... from... eh... your experience. How do you think the learners have receive this?
Skhumbuso	I think some enjoyed it most becausebecause... ah...you know they will come back with the feedback to say that... "Hm...we've done 123... eh... this and that... is what we did". So as an educator you just facilitate the whole process because - that's where I come in - of the feedback that "okay it is expected this way" and then maybe give them more examples of their own doing in the particular... eh...example of work.
R	Okay ...eh... from your experience... eh... there might be a lot of expectance from the teacher in relation to this... eh... method of learning with the worksheet. What do you think?
Skhumbuso	The worksheet expectation from me... myself....as the educator is to see these kids managing it ...eh... and able to do well in the investigations, the test and assignment. So looking at that I could see that some of them... they did manage and enjoy these exercises because it was...[inaudible, break in communication]
Skhumbuso	That we needed the equipment. As the school in Diepsloot we know we lack some of the equipment like enough computers...eh... you can name them. This particular thing of worksheet - it needs more time- it needs equipment. So that is why I think we resorted to ... eh... the old system so that we quickly pushed the programme and tried to...you know...to make sure that when they sit for the final exam at least we have covered certain topics. I think that was the reason why we went back to that old system of method.

R	<p>But... eh... other than the strikes and so on, and the fact that you indicate that you feel that that could have been the best... eh... method, what do you think the authorities would need to do in order to help our educators? Do you feel that somehow you might need more workshops? What do you think is to be done in order for the teacher to come to the point of confidence when it comes to implementing to method?</p>
Skhumbuso	<p>I think educators need support. What kind of support?</p> <ol style="list-style-type: none"> 1. Workshops... they need to be equipped... you know... they need to attend workshops because this is new curriculum. Educators need to be conversant with it. 2. Also support on the materials which use the equipment. They need to be trained on how to prepare worksheets using computers and lot of material. I think it is very very important so that department comes in and assist educators.
R	<p>Okay. I understand what you mean when you talk about computers because in the pamphlets that I gave you sometimes there were these drawing that require this specialised software [Corel Draw]. Sometimes you needed some... eh... the mathematical symbols [Equation Editor] and so on. But if you were to suggest and... you were to talk to the authorities, what would be your advice... eh... so that we can really ensure successful implementation of the new curriculum.</p>
Skhumbuso	<p>I think the main thing is workshops ...you know... If they can arrange workshops for educators... you know... especial on the software of this curriculum so that they became aware of what is happening around the world ...you know. Because we need to adapt... definitely. If the educators can be equipped ...from my point of view..., I think we can go far with this new curriculum and it's very very nice and so educators need more... more workshops and I think we can go far with in this country.</p>
R	<p>Eh... thank you very much ... but... eh... just one last question. Eh... as a person who has been exposed to both methods against the background of all this requirement and as a head of the department, what do you think you are going to do from here? What little contribution do you think you are going to make towards ensuring that educators are conversant with the new approach?</p>
Skhumbuso	<p>I'll make recommendations to the EMT - the entire Management- about the whole thing and ...you know... try to conscientise them thatthat.... "Guys this is the way we have to take our route, because you know that nated 550 [?] is over now. So we need to focus ...eh... in the future because if we still going to hang on back on the</p>

	previous... eh... method of teaching, we're not going to win." So, I think I'll recommend to the management that "Guys, this is the route that we need to take...you know ...we need to take it up maybe to the department, the facilitators of different learning areas or subjects so that...you know... we start working this thing together because if we are divided is going to be difficult to us."
R	Thank you very much. The commitment that I personally am prepared to make is if maybe at the beginning of the year we can start communicating and that there would be a need to help in terms of preparation of the material, I think I'll be available for that. Thank you.
Skhumbuso	Thank you Mr. Mahlobo. There's also one another thingthat I'll recommend to the management that... you know.... even NGOs, people outside there who will feel that they can come and assist, I'll make also that recommendation to them. Thank you.
	2nd teacher in the experimental group whose class was not constantly monitored
R	Eh... Good afternoon madam. Eh... I just want you to introduce yourself, I know that you are one of the Grade 11 mathematics educators... eh... educators. And I would like you just to give a brief overview of what you think about this method of having to use worksheets as compared to the method that you might have used earlier... eh...and then give your views about the whole thing and then to indicate to us, where you are at the moment and what will you recommend.
Educator	Eh... the methodit was very good for us and also for the learners. Eh... our learners were enjoying the method where we used worksheets. Eh... the problem is that we don't have... hm... enough material and then... eh... it is because of time that we stopped that the method, but I think we can continue with that method next time. I think is better.
R	Yeah... One of the things that learners kept on mentioning was that... eh... sometimes they had a problem because you may find that four learners were given one material to work on, with the result that they cannot take it home. What do you think can be done in order to try and rectify that?
Educator	Eh... I think... if we can have... eh... may be the problem is that... eh... we don't have the material. Like eh., we...we don't have..... Our photocopy machines is sometimes broken and we fail to make more copies for the learners. I think our learners can have more books, more... eh...eh... more papers to work on. I think it is better for the learners to take the material home and practise at home on their own, and also to practise at school because if we have the material only in class and they

	don't take material home... It's where we've got a problem. They cannot cope with ...eh... one pamphlet although they are five. I think we can have more materials.
R	Ok. Thank you madam. Eh... so now there've been a lot of things... there was this public strike, there was this thing... you know... that... and then I understand that as the result of all those things, you were so much pressurised with time. You ended up, having to do...eh... you know... things that - after the introduction to that method - would not be appealing. What do you think about them....would you say now that you have been pressurised and as a result you did not have enough time to spread this method over a lengthy period of time, what would you suggest should happen next year, especially if we are not anticipating any strikes. What do you feel... what role do you think you can play to make sure that things are going to be done differently and how are you going to do to ensure that?
Educator	I... I think if we can take or start with the method next year it will be better, to start early January then we continue with the method. As long as we got all materials then, I think the method is right. We can continue with the method...eh...as long as we start early... having all materials to use. It is right. Then we can go on with the method.
R	Okay... eh...so ...what about this demand on educators? Eh.. do you think... eh... with a bit of time you have been exposed to the worksheet method and ... eh...few weeks of [teacher] strikes sometime in May, would you say... eh... you had acquired enough knowledge tosort of.... prepare material on your own? Or you'd say you just need a little bit more of help until you are confident? Where would you say you are at the moment?
Educator	I think if we can have somebody to help us in preparing them. From there we can go on.... yes we know but we still need somebody to help us, in preparation of that pamphlets and so on
R	Okay, as we should know by now the pamphlets were encouraging open-ended types of questions. Eh... nowhow would you suggest we go? Do you think it will be a good idea for us to organise workshops for educators? Or do you think that when you say you need somebody, you need somebody just to prepare the material and from there on...on the basis of little experience that you have acquired.... you can be able to help other educators on how to use those particular worksheets?
Educator	I think workshops are going to work. If we can get a few educators to attend the workshops so that when they come back [to their schools] they can workshop other ...eh... other educators and another thing... at the workshop if we can have more

	pamphlets... eh to use. If we got less then we can make copies at our schools then go on with the method of teaching.
R	Thank you very much, thank you
Educator	Thank you for the day, thank you

APPENDIX 5 : WORKSHEET

Solving quadratic equations by factorization

Please Note:

- *The worksheet has to be used in conjunction with the prescribed textbook, so that further relevant exercises can be obtained therein.*
- *In the following worksheet the learners are not provided with shaded information written 'Expected response'. It is meant for the teacher.*

1. Factorising $ax^2 + bx + c$.

1.1 If $a=1$.

- a) John uses the table to fill in the missing information and then concludes that $x^2 + 2x - 15 = (x+5)(x-3)$. Phindile concludes that $x^2 + 2x - 15 = (x-5)(x+3)$. What do you think about John and Phindile's solutions?

Expression					Factors of the quadratic expression
i) $x^2 + 5x + 6$	5	6	3 and 2, -3 and -2, 6 and 1, -6 and -1	2 and 3	$(x+2)(x+3)$
ii) $x^2 - 7x + 12$	-7	12	6 and 2, -6 and -2, 4 and 3, -4 and -3, 12 and 1, -12 and -1.	-4 and -3	$(x-4)(x-3)$
iii) $x^2 + 3x - 18$	3	-18	9 and -2, -9 and 2, -6	6 and -3	$(x+6)(x-3)$

			and 3, 6 and -3, 18 and -1,		
			-18 and 1.		
iv) $x^2 + 6x - 7$					
v) $x^2 + 3x - 10$					

b) The table summarises what is called *factorising quadratic expressions of the form $x^2 + bx + c$* . How do you explain factorisation of the general quadratic expressions $x^2 + bx + c$?

c) Factorise of the following expressions:

a) $x^2 - x - 6$

b) $x^2 - 2x - 8$

c) $x^2 + x - 12$

d) $x^2 + 5x - 6$

Expected response

The educator must facilitate the process of successful identification of the patterns arising from the table. Through facilitation, the learners must be able to see that the second column represents the coefficient of the middle term, and the third the constant term. They must further see that the fourth column represents factors of the constant term, and that the fifth column represents factors of the constant whose sum is the coefficient of the middle term. The last column represents factors of the quadratic expression

1.2 We want to factorise $ax^2 + bx + c, a \neq 1$

a) Study the table below and then indicate what you think about Oregomoditse's conclusion that $3x^2 - 2x - 5 = (3x - 5)(x + 1)$.

Expression	Solution	Conclusion	
$2x^2 + x - 15$	<p>Cross sum = $-2x + 15x = 13x$</p>	$2x^2 + x - 15 \neq (2x + 15)(x + 1)$	
$2x^2 + x - 15$	<p>Cross sum = $-6x + 5x = -x$</p>	$2x^2 + x - 15 \neq (2x + 5)(x - 3)$	
$2x^2 + x - 15$	<p>Cross sum = $6x - 5x = x$</p>	$2x^2 + x - 15 = (2x - 5)(x + 3)$	
$3x^2 - 22x + 24$	<p>Cross sum = $9x + 8x = 17x$</p>	$3x^2 - 22x + 24 \neq (3x + 8)(x + 3)$	
$3x^2 - 22x + 24$	<p>Cross sum = $-6x - 12x = -18x$</p>	$3x^2 - 22x + 24 \neq (3x - 12)(x - 2)$	
$3x^2 - 22x + 24$	<p>Cross sum = $-18x - 4x = -22x$</p>	$3x^2 - 22x + 24 = (3x - 4)(x - 6)$	

Expected response

Through facilitation, the learners should observe that:

- The first 'cross column' represents factors of the first term;
- The second 'cross column' represents factors of the constant term;
- 'Cross-sum' is the sum of 'cross-multiplied' terms;
- In all of the cases where there is an inequality (\neq) sign between factors and the quadratic expression, the cross sum is not the middle term;
- The factors of the first and last terms of the quadratic expression-, whose cross-sum is the middle term of the quadratic expression, are accepted as correct factors of the expression.

b) What are the factors of the following quadratic expressions?

i. $2x^2 + 5x - 3$

ii. $5x^2 - 13x - 6$

iii. $6x^2 - 24x + 18$

2 Roots of a quadratic equation

2.1 Fill in the following table:

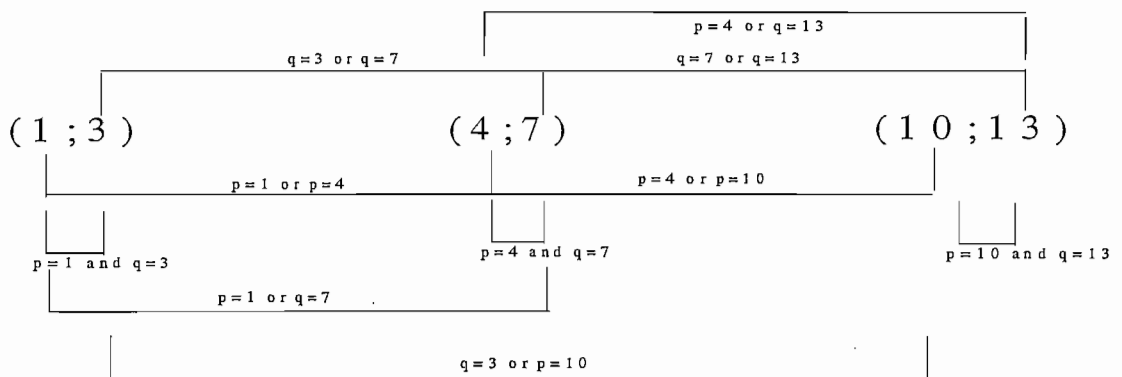
Statement	Possible values of p and q , Write them as (p, q)
i) $p \times q = 0$	
ii) $p \times q = 1$	
iii) $p \times q = 4$	

Expected results

The learners should be allowed to make as many choices for p and q as possible. For instance, the learner could have as pairs p and q - denoted below as $(p;q)$ - the following:

Statement	Possible values of p and q
i) $p \times q = 0$	$(0;3), (5;0), (12;0), (0;15), (0;0)$ etc
ii) $p \times q = 1$	$(1;1); (\frac{1}{3};3), (5;\frac{1}{5}),$ etc
iii) $p \times q = 4$	$(2;2), (4;1), (1;4), (\frac{1}{2};8), (16;\frac{1}{4}),$ etc

2.2 In the diagram below, coordinates are used. What would you advise a person who thinks that 'and' and 'or' mean the same thing?



Expected responses

'and' describes two numbers of the same pair – those that occur simultaneously - while 'or' (generally speaking) describes two numbers from different pairs.

NB: *There are circumstance under which 'or' is inclusive of 'and', but this relationship is not stressed here.*

2.3 Give your impressions of the following statements by 8 learners. Fully explain your answers.

Name of learner	Statement	Statement generally correct or incorrect
i) Lerato	If $p \times q = 0$, then $p = 0$ or $q = 0$	
ii) Jabulile	If $p \times q = 1$, then $p = 1$ or $q = 1$	
iii) Simphiwe	If $p \times q = 4$, then $p = 2$ or $q = 2$	
iv) Rethabile	If $p \times q = 0$, then $p = 0$ and $q = 0$	
v) Mooketsi	If $p \times q = 1$, then $p = 1$ and $q = 1$	
vi) David	If $p \times q = 4$, then $p = 2$ and $q = 2$	
vii) Palesa	If $p = 2$ or $q = 2$, then $p \times q = 4$.	

viii) Oregomoditse	If $p=1$ and $q=1$, then $p \times q = 1$	
--------------------	-----------------------------------------------	--

Expected response

The idea is to stress the significance of “or” as opposed to “and”.

Learner	Statement	Statement correct/not correct
i) Lerato	If $p \times q = 0$, then $p = 0$ or $q = 0$	$p = 0$ or $q = 0$ means $p = 0$ while q is another number or $q = 0$ while p is another number. Statement correct.
ii) Jabulile	If $p \times q = 1$, then $p = 1$ or $q = 1$	$p = 1$ or $q = 1$ means $p = 1$ while q is another number (including 1) or $q = 1$ while p is another number (including 1). Statement is generally incorrect.
iii) Simphiwe	If $p \times q = 4$, then $p = 2$ or $q = 2$	$p = 2$ or $q = 2$ means $p = 2$ while q is another number or $q = 2$ while p is another number. Statement generally incorrect.
iv) Rethabile	If $p \times q = 0$, then $p = 0$ and $q = 0$	$p \times q = 0$ has other solutions than $p = 0$ and $q = 0$ e.g., if $p = 0$ then q can be another number. In general, statement is incorrect.
v) Mooketsi	If $p \times q = 1$, then $p = 1$ and $q = 1$	$p \times q = 1$ has other solutions than $p = 1$ and $q = 1$ e.g., $p = a$ and $q = \frac{1}{a}, a \in \mathbb{R}, a \neq 0$. In

		general, statement is incorrect.
vi) Palesa	If $p=2$ or $q=2$, then $p \times q = 4$.	$p=2$ or $q=2$ means $p=2$ while q is any number or $q=2$ while p is any number. Statement generally incorrect.
vii) Oregomoditse	If $p=1$ and $q=1$, then $p \times q = 1$	Statement is correct. This is the converse of statement (e) by Mooketsi.

If the learners seem to be lost, draw their attention to their responses in 2(a) above. They should then be facilitated to discover, for instance, that (1;1) means $p=1$ and $q=1$, while (0;3), (5;0), mean respectively that $p=0$ and $q=3$ or $p=5$ and $q=0$. In short, if we focus on the first components only, it means $p=0$ or $p=5$. In other words, $p=0$ or $p=5$ means $p=0$ while q is an unspecified number or $p=5$ while q is not specified.

2.4 Peter uses the following table to conclude that $3x^2 - 8x + 5 = 0 \Rightarrow x = -\frac{6}{7}$ or $x = 2$, and that there is no difference between a root of an equation and the factor of a quadratic expression. What is wrong with Peter's conclusions?

Quadratic Equation	Factors in the quadratic equation	Roots of the quadratic equation
$x^2 + 5x + 6 = 0$	$(x+2)(x+3) = 0$	$x = -2$ or $x = -3$
$x^2 - 7x + 12 = 0$		$x = 4$ or $x = 3$
$x^2 + 3x - 18 = 0$		$x = -6$ or $x = 3$
$3x^2 - 8x + 5 = 0$	$(3x-5)(x-1) = 0$	
$5x^2 + 13x - 6 = 0$		

Expected response

- Learners should be referred to the previous exercise on factorisation to explain factors.
- They should also be prompted towards realising that $(x+2)(x+3)=0$ should be treated as $pq=0$, in which case it means $p=0$ or $q=0$
- They should be prompted towards realizing that a *root is the value* of the unknown – in this case x – which satisfies the equation, for which the *factor is zero*.

d) What are the roots of the following quadratic equations?

i) $2x^2 + 5x - 3 = 0$

ii) $5x^2 - 13x - 6 = 0$

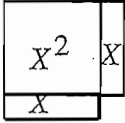
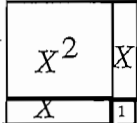
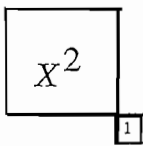
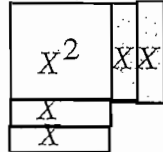
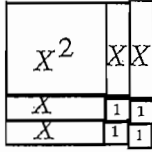
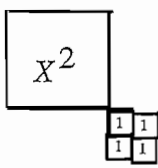
iii) $6x^2 - 24x + 18 = 0$

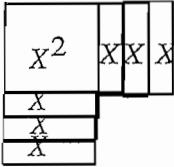
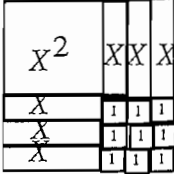
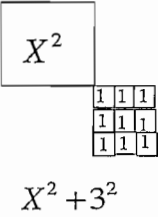
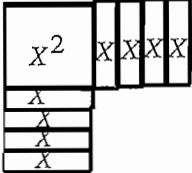
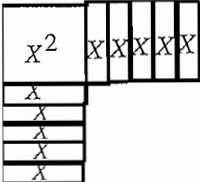
3. Completing the square

For this exercise we need the following components:

$$\begin{array}{c} 1 \\ \boxed{1} \\ \text{One} \end{array} \quad \begin{array}{c} X \\ \boxed{} \\ X \end{array} \quad X \begin{array}{c} \boxed{X^2} \\ X \end{array}$$

3.1 What conclusions can you draw from the following table?

Incomplete Square Components	Complete square	What you added to the incomplete square to make it complete.	Describe components	Half the coefficient of X	
 $X^2 + 2X$	 $(X+1)^2$	1	 $X^2 + 1$	$\frac{1}{2} \times 2 = 1$	$(\frac{1}{2} \times 2)^2 = 1$
 $X^2 + 4X$	 $(X+2)^2$	4	 $X^2 + 2^2$	$\frac{1}{2} \times 4 = 2$	$(\frac{1}{2} \times 4)^2 = 4$

Incomplete Square	Complete square	What you added to the incomplete square.	Describe components	Half the coefficient of X	
 $X^2 + 6X$	 $(X + 3)^2$	9	 $X^2 + 3^2$	$\frac{1}{2} \times 6 = 3$	$(\frac{1}{2} \times 6)^2 = 9$
 $X^2 + 8X$					
 $X^2 + 10X$					


An observant learner claims that the table is mainly about **completing the square**. What do you think the learner means?

Expected response:

- Learners should be prompted towards observing that the last column is the square of half the coefficient of x .
- Learners are expected to observe, through facilitation, that 'completing the square' is the process of *adding the square of half the coefficient of x to components of an incomplete square* in order to end up with the complete square i.e. $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$.

The other thing that the learners are expected to observe through facilitation is that $x^2 + a^2 \neq (x + a)^2$

3.2 Another learner concludes that the following table is still about completing the square. Justify the conclusion.

Incomplete square: quadratic expression		
a) $X^2 + 6X$	9	$(X + 3)^2$
b) $X^2 + 8X$	16	
c) $X^2 + 10X$		
d) $X^2 + 12X$		
e) $X^2 + 14X$		
f) $X^2 + 16X$		
g) $X^2 + 7X$		
h) $X^2 + 9X$		
i) $X^2 + BX$		

3.3 Siphso, Lerato, and Vusi solve quadratic equations using the method of completing the square. Fully explain what they are doing.

Siphso

$$x^2 - x = 20$$

$$x^2 - x + \left(-\frac{1}{2}\right)^2 = 20 + \left(-\frac{1}{2}\right)^2$$

$$\left(x - \frac{1}{2}\right)^2 = \frac{81}{4}$$

$$\left(x - \frac{1}{2}\right) = \frac{9}{2} \text{ or } -\frac{9}{2}$$

$$x = 5 \text{ or } x = -4.$$

Lerato

$$2x + 3 - \frac{5}{x} = 0$$

$$2x^2 + 3x - 5 = 0$$

$$x^2 + \frac{3x}{2} - \frac{5}{2} = 0$$

$$x^2 + \frac{3}{2}x = \frac{5}{2}$$

$$x^2 + \frac{3}{2}x + \left(\frac{3}{4}\right)^2 = \frac{5}{2} + \frac{9}{16}$$

$$\left(x + \frac{3}{4}\right)^2 = \frac{49}{16}$$

$$x + \frac{3}{4} = \pm \frac{7}{4}$$

$$x = 1 \text{ or } -\frac{5}{2}$$

Vusi

$$-2x^2 - x + 15 = 0$$

$$-2\left(x^2 + \frac{x}{2} - \frac{15}{2}\right) = 0$$

$$x^2 + \frac{x}{2} - \frac{15}{2} = 0$$

$$x^2 + \frac{x}{2} = \frac{15}{2}$$

$$x^2 + \frac{x}{2} + \left(\frac{1}{4}\right)^2 = \frac{15}{2} + \frac{1}{16} = \frac{121}{16}$$

$$\left(x + \frac{1}{4}\right)^2 = \frac{121}{16}$$

$$x + \frac{1}{4} = \pm \frac{11}{4}$$

$$x = \frac{5}{2} \text{ or } x = -3$$

3.4 Solve the following by using the method of completing the square.

a) $2x^2 + x - 15 = 0$

b) $-3x^2 - 5x + 12 = 0$

c) $x^2 + x - 1 = 0$

d) $ax^2 + bx + c = 0$

Expected response

Learners should consistently be made to look at their previous solutions. Their deviation from intended solutions should be contrasted with their previous solutions, so that they can, through facilitation - *correct themselves*. The last problem (d) – dealing with variable coefficients - is the one that the learners may not have done before. They should, through facilitation, be encouraged to follow the same procedure as when they were dealing with integer coefficients. The following is what the learners are expected to do:

$$ax^2 + bx + c = 0$$

$$ax^2 + bx = -c$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a} = \frac{b^2 - 4ac}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

3.5. Hazel says completing the square in the equation $ax^2 + bx + c = 0$ is the same as expressing $ax^2 + bx + c = 0$ in the form $a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a} = 0$. Use the solution on the left to explain Hazel's statement.

$$ax^2 + bx + c = 0$$

$$a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right) = 0$$

$$a\left[x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + \frac{c}{a}\right] = 0$$

$$a\left[\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + \frac{4ac}{4a^2}\right] = 0$$

$$a\left[\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b^2}{4a^2} - \frac{4ac}{4a^2}\right)\right] = 0$$

$$a\left[\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b^2 - 4ac}{4a^2}\right)\right] = 0$$

$$a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a} = 0$$

3.6 Surprise claims that $ax^2 + bx + c = a(x + \frac{b}{2a})^2 - \frac{b^2 - 4ac}{4a}$. What do you think?

Expected response

Since $ax^2 + bx + c = 0 \Rightarrow a(x + \frac{b}{2a})^2 - \frac{b^2 - 4ac}{4a} = 0$, $ax^2 + bx + c = a(x + \frac{b}{2a})^2 - \frac{b^2 - 4ac}{4a}$

3.7. Simplify the following by using the equation $ax^2 + bx + c = a(x + \frac{b}{2a})^2 - \frac{b^2 - 4ac}{4a}$:

a) $-x^2 - x + 6$.

b) $2x^2 - x - 15$

c) $x^2 + 4x + 1$

Expected response:

a) $-x^2 - x + 6$, $a = -1$, $b = -1$, $c = 6$

$$-x^2 - x + 6 = -1\left(x + \frac{-1}{2(-1)}\right)^2 - \frac{(-1)^2 - 4(-1)(6)}{-4}$$
$$= -1\left(x + \frac{1}{2}\right)^2 + \frac{25}{4}$$

Check!

$$-1\left(x + \frac{1}{2}\right)^2 + \frac{25}{4} = -1\left(x^2 + x + \frac{1}{4}\right) + \frac{25}{4}$$
$$= -x^2 - x - \frac{1}{4} + \frac{25}{4}$$
$$= -x^2 - x + 6$$

b) $2x^2 - x - 15$, $a = 2$, $b = -1$, $c = -15$

$$2x^2 - x - 15 = 2\left(x + \frac{(-1)}{2(2)}\right)^2 - \frac{(-1)^2 - 4(2)(-15)}{4(2)}$$
$$= 2\left(x - \frac{1}{4}\right)^2 - \frac{121}{8}$$

Check!

$$2\left(x - \frac{1}{4}\right)^2 - \frac{121}{8} = 2\left(x^2 - \frac{x}{2} + \frac{1}{16}\right) - \frac{121}{8}$$
$$= 2x^2 - x - \frac{120}{8}$$
$$= 2x^2 - x - 15$$

$$c)x^2 + 4x + 1, a = 1, b = 4, c = 1$$

$$\begin{aligned} x^2 + 4x + 1 &= 1\left(x + \frac{4}{2}\right)^2 - \frac{4^2 - 4(1)(1)}{4(1)} \\ &= (x + 2)^2 - \frac{12}{4} \\ &= (x + 2)^2 - 3. \end{aligned}$$

4 Quadratic formula

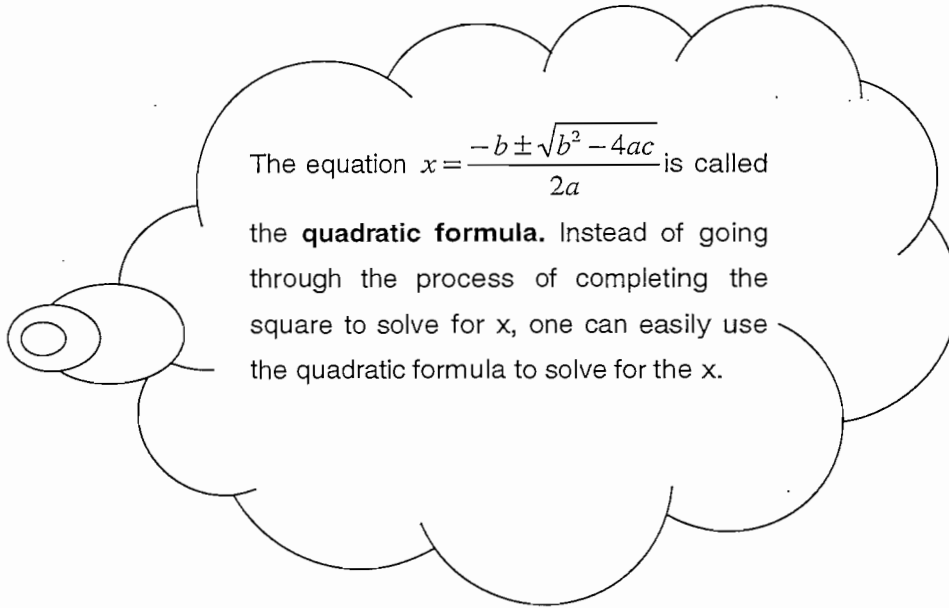
4.1 Serame, one of very good mathematics learners, makes the following statement:

"If $ax^2 + bx + c = 0$, then one can use completing the square to show that $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$." Fully explain Serame's statement.

Expected response:

The main focus of facilitation by the educator should be geared (without explicitly saying it) towards the realisation that $ax^2 + bx + c = 0$ implies that $\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$. Taking the square root on both sides will give $\left(x + \frac{b}{2a}\right) = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$, so that, as Serame has suggested, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. The facilitator should, as far as possible, avoid temptation to

give the learners the solution. Their familiarity with the procedure of completing the square should make it possible for them to be led towards the solution.



4.2. Choose any quadratic equation $ax^2 + bx + c = 0$ such that $b^2 - 4ac$ satisfies the conditions under second column. Use the quadratic formula to determine the roots of your quadratic equation, and then fill in the table.

Equation $ax^2 + bx + c = 0$	$b^2 - 4ac$	Roots: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Nature of roots
	0		
	Perfect non-zero square		
	Non-perfect square		
	negative		

Expected response

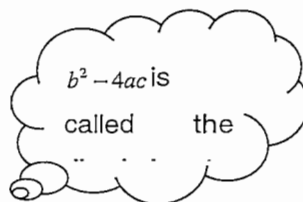
This is a good opportunity for the teacher to establish if the learners understand the terms used in the table. Instead of starting by telling them what the terms – perfect square, non-perfect squares etc.- mean, the teacher should, when asked, give examples of such numbers with the expectation that the learners should describe them. Terms like rational numbers, irrational numbers should be similarly introduced.

4.3 What do you think about Modiri's statements below?

- a) If $b^2 - 4ac = 0$, then the equation $ax^2 + bx + c = 0$ will have two *rational*, equal roots (or alternatively, one repeated rational root).
- b) If $b^2 - 4ac \geq 0$, then the equation $ax^2 + bx + c = 0$ will have two unequal roots.
- c) If $b^2 - 4ac \geq 0$ and $b^2 - 4ac$ is a non-zero perfect square, then the equation $ax^2 + bx + c = 0$ will have two *rational*, unequal roots.
- d) If $b^2 - 4ac \geq 0$ and $b^2 - 4ac$ is a not a perfect square, then the equation $ax^2 + bx + c = 0$ will have two *rational*, unequal roots.
- e) If $b^2 - 4ac$ is a not a perfect square, then the equation $ax^2 + bx + c = 0$ will have two *irrational*, unequal roots.
- f) If $b^2 - 4ac < 0$, then the equation $ax^2 + bx + c = 0$ will have no real roots.

Expected response

Learners are expected to consolidate their observations in 4.2 above.



4.4 Use the quadratic formula to find the roots of the following quadratic equations:

a) $x^2 + 6x + 9 = 0$

b) $x^2 - 7x + 12 = 0$

c) $2x^2 + 13x - 15 = 0$

d) $x^2 + 6x + 13 = 0$

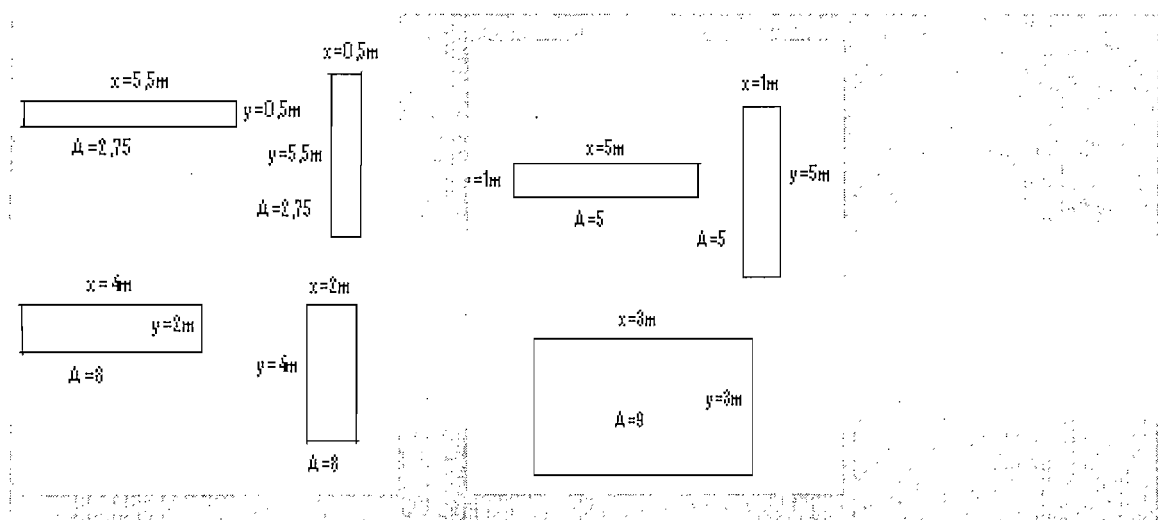
5. **Drawing a parabola:**

5.1 Using context

a) You are given a fence of length 12m with which to enclose a rectangular vegetable garden with. In small groups of four, identify the dimensions of your

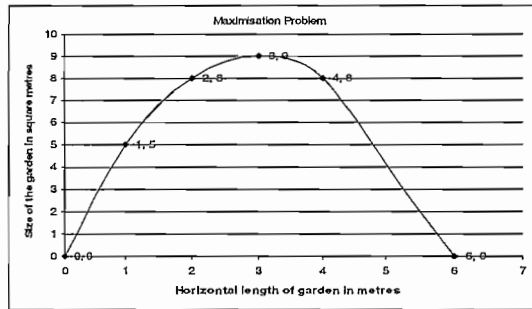
Expected response

The following were some of the responses given (not according to scale). In the figures below A stands for area.



The example with fractional lengths showed that there are an infinite number of possible dimensions e.g. $x = 5,4m$, $y = 0,6m$, $x = 4,4m$, $y = 1,6m$ etc. Initially the learners were not in agreement that $x = 3m$ and $y = 3m$ is one of the possible answers — thinking that a square is not a rectangle. This was a reflection of how parallelograms were taught in the learners' earlier studies — in isolation from each other.

b) An observant learner used the above information to come up with the following graph:



Fully explain how the graph was arrived at.

Expected responses

Basically, the graph is obtained by plotting the horizontal dimension x of each garden and its resulting area. The learners are also expected to see the connection between components of the turning point and the horizontal dimension giving rise to the maximum area of the garden.

c) Another learner uses the information to generate the following table:

Perimeter	Area	Context:	Equation		$(x; A(x))$
		Length of fence needed to cover a rectangular vegetable garden			
i) $2x+2y=12$	$A = xy$	12m	$A(x) = 6x - x^2$	$A(x) = -(x-3)^2 + 9$	(3;9)
ii) $2x+2y=16$	$A = xy$	16m	$A(x) = 8x - x^2$	$A(x) = -(x-4)^2 + 16$	(4;16)

iii) $2x + 2y =$	$A = xy$	20m	$A(x) =$	$A(x) =$	
iv) $2x + 2y =$	$A = xy$	24m	$A(x) =$	$A(x) =$	
v) $2x + 2y =$	$A = xy$	28m	$A(x) =$	$A(x) =$	

Fully explain and then complete the table.

Expected response

The exercise requires:

- Application of one variable from one equation to the other equation.
- Completing the square by using $ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a}$
- They must also be able to see that $(x; A(x))$ is a point that identifies the horizontal dimension x as well as the corresponding maximum area $A(x)$ that they obtained in the arrangement of the garden. This is when terms like critical number – the x -value giving rise to the maximum - can be introduced.
- Finally, the learners are expected to compare $(x; A(x))$ with the form of an equation in a completed square format e.g. between $A(x) = -(x - 3)^2 + 9$ and $(3; 9)$.

Perimeter	Area	Context:	Equation	Completing the square	x-value and its corresponding maximum area $(x; A(x))$
		Length of fence needed to enclose a rectangular vegetable garden			

i) $2x + 2y =$	$A = xy$	12m	$A(x) = 6x - x^2$	$A(x) = -(x-3)^2 + 9$	(3;9)
i) $2x + 2y =$	$A = xy$	16m	$A(x) = 8x - x^2$	$A(x) = -(x-4)^2 + 16$	(4;16)
i) $2x + 2y =$	$A = xy$	20m	$A(x) = 10x - x^2$	$A(x) = -(x-5)^2 + 25$	(5;25)
i) $2x + 2y =$	$A = xy$	24m	$A(x) = 12x - x^2$	$A(x) = -(x-6)^2 + 36$	(6;36)
i) $2x + 2y =$	$A = xy$	28m	$A(x) = 14x - x^2$	$A(x) = -(x-7)^2 + 49$	(7;49)

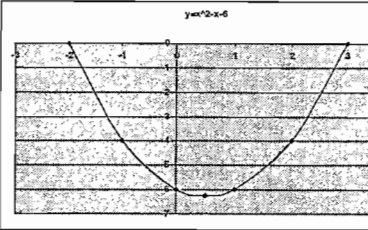
At this stage the learners are expected to use contextual information (using the resulting dimensions and area of the vegetable gardens, as they did above), to draw the graph. One major observation the learners are expected to make is the following:

- o If the equation in completing the square is expressed as $A(x) = -(x-p)^2 + q$, then q represents the *maximum value*, while the vertical line through p represents the *axis of symmetry* of the graph. $(p;q)$ represents the turning point.

- o Of importance will be the linking of, for instance, (3,9) as an aspect of the rectangle with horizontal length 3 and area 9 and (3,9) as the turning point of the graph.

5.2. Using turning point and intercepts

a) Tumelo uses the argument in 5.1. to generate the following table and graph.

Function	X-intercepts	Y intercept	Turning point	Graph
$f(x) = x^2 - x - 6$	(3;0) or (-2;0)	(0;-6)	$(\frac{1}{2}; -6\frac{1}{4})$	
$f(x) = 2x^2 - x - 6$				
$f(x) = 3x^2 - x - 1$				

Fully explain Tumelo's graph and table, and then complete the table.

Expected response

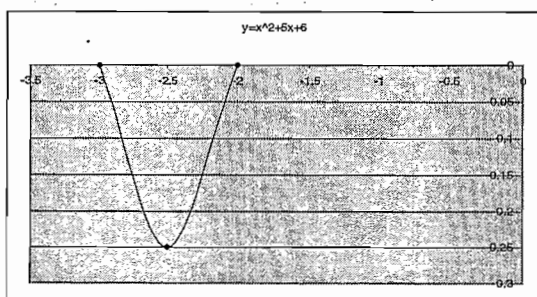
Learners should be prompted towards realising that equating $f(x) = ax^2 + bx + c$ to zero and solving for x is tantamount to identifying the x values for which the corresponding y is zero.

Since $y = 0$ are all the points along the x axis, equating $f(x) = ax^2 + bx + c$ to zero is a way of identifying the x intercepts. Learners could be asked to identify any points on the x -axis, so that they can see for themselves that any point along the x -axis has zero as the corresponding y value.

b) Fully explain and then complete the following table. Draw the resulting graph.

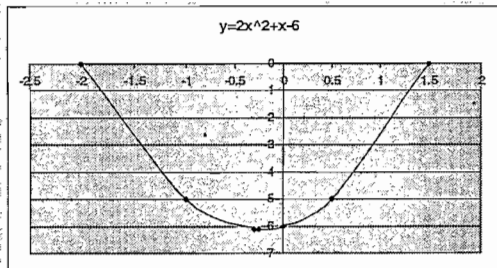
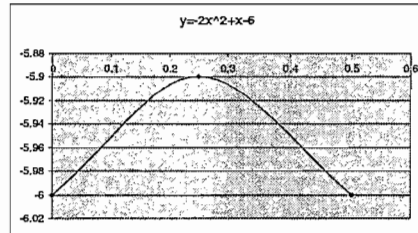
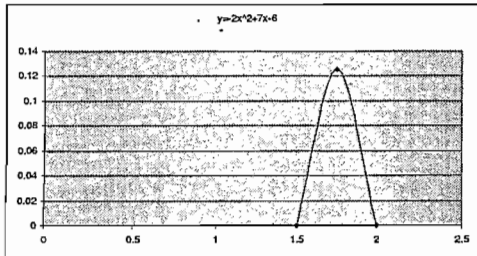
$y = f(x) = ax^2 + bx + c$	Complete the square	Axis of symmetry	Min ($a > 0$) or Max ($a < 0$) value	x intercepts
$y = f(x) = x^2 + 5x + 6$	$f(x) = (x + \frac{5}{2})^2 - \frac{1}{4}$	$x = -\frac{5}{2}$	Min $y = -\frac{1}{4}$	$(-3;0)$ or $(-2;0)$
$y = f(x) = -2x^2 + 7x - 6$				
$y = f(x) = -2x^2 + x - 6$				
$y = f(x) = 2x^2 + x - 6$				

Expected response



$y = f(x) = ax^2 + bx + c$	Complete the square	Axis of symmetry	Min ($a > 0$) or Max ($a < 0$) value	x intercepts
$y = f(x) = -2x^2 + 7x - 6$	$f(x) = -2(x - \frac{7}{4})^2 + \frac{1}{8}$	$x = \frac{7}{4}$	Max $y = \frac{1}{8}$	$(1,5;0), (2;0)$
$y = f(x) = -2x^2 + x - 6$	$f(x) = -2(x - \frac{1}{4})^2 - \frac{47}{8}$	$x = \frac{1}{4}$	Max $y = -5,875$	No roots

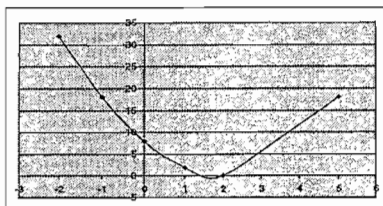
$y = f(x) = 2x^2 + x - 6$	$f(x) = 2\left(x + \frac{1}{4}\right)^2 - \frac{49}{8}$	$x = -\frac{1}{4}$	$y = -6,125$	$(1,5;0), (-2;0)$
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- c) Lucky, in trying to draw the graph of $y = 2(x-2)^2$, decided to simplify to the equation $y = 2x^2 - 8x + 8$ and then used the following coordinates to draw the graph:

x	-2	-1	0	1	2	5
y	32	18	8	2	0	18

He then got the following graph:



What is your comment about Lucky's solution?

Expected response

- The graph does not look like a parabola.
- The learners should be prompted towards realising that it is easier to draw a parabola if its equation is such that the axis of symmetry (or turning point) can be located i.e. that it is expressed in 'completing the square' format.
- They should also be prompted towards the realisation that obtaining points on one side of the axis of symmetry would be sufficient to enable them to draw the graph, as the points on the other side of the axis can be located by using the equal distance from the axis. Some learners may argue that Luckyboy should have chosen the values around the axis of symmetry. This will reinforce the need to identify the axis of symmetry. If the equation of the parabola is written in a way that one cannot locate the axis of symmetry, then it would not be advisable to use coordinates to plot graphs of the parabolas of the form $y = f(x) = ax^2 + bx + c$.

6 Quadratic Inequalities

6.1 Fully explain the solutions to the following quadratic inequalities

i) $x^2 - 4x + 3 < 0$

ii) $x^2 - 4x + 3 \leq 0$

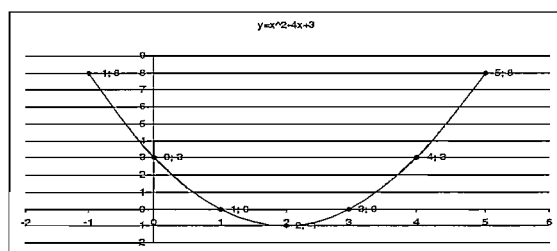
iii) $x^2 - 4x + 3 > 0$

iv) $x^2 - 4x + 3 \geq 0$

a) Ntumi's solution:

Inequality	Arithmetic signs of $(x-1)(x-3)$ at different intervals					Solution
$i) (x-1)(x-3) < 0$	$x < 1$	$x = 1$	$1 < x < 3$	$x = 3$	$x > 3$	$1 < x < 3$
	+	0	-	0	+	
$ii) (x-1)(x-3) \leq 0$	$x < 1$	$x = 1$	$1 \leq x \leq 3$	$x = 3$	$x > 3$	$1 \leq x \leq 3$
	+	0	-	0	+	
$iii) (x-1)(x-3) > 0$	$x < 1$	$x = 1$	$1 < x < 3$	$x = 3$	$x > 3$	$x < 1$ or $x > 3$
	+	0	-	0	+	
$iv) (x-1)(x-3) \geq 0$	$x < 1$	$x = 1$	$1 < x < 3$	$x = 3$	$x > 3$	$x \leq 1$ or $x \geq 3$
	+	0	-	0	+	

b) Jake's solution:



From the graph he comes up with the values on the following table:

Inequality	Solution
$i)(x-1)(x-3) < 0$	$1 < x < 3$
$ii)(x-1)(x-3) \leq 0$	$1 \leq x \leq 3$
$iii)(x-1)(x-3) > 0$	$x < 1 \text{ or } x > 3$
$iv)(x-1)(x-3) \geq 0$	$x \leq 1 \text{ or } x \geq 3$

Expected response: Ntumi's solution

If the learners seem not to make sense of the table, prompting questions like: "What do the numbers in shaded part represent?", "How were +, 0 and - arrived at?" may be appropriate. The learners are expected to realise that:

1. Roots are identified.
2. x values on or around roots are considered.
3. Arithmetic signs of product of factors are identified (at the x values on or around roots).
4. Intervals satisfying the inequalities are identified as solutions.

It is important to avoid giving the learners the answers. Use questions like: "What do you think about..." , "Why do you say that" etc.

Note that $(x-1)(x-3) = x^2 - 4x + 3$, so that the original inequality could have been written $x^2 - 4x + 3 < 0$, instead of $(x-1)(x-3) < 0$. The solution, according to the table, is $1 < x < 3$.

At this stage 'table method', 'graph method' can be mentioned explicitly

Jake's solution

i) $1 < x < 3$ is the interval over which the graph – the set of y -values – is *below* the x -axis, meaning that the y -values are negative, in other words $1 < x < 3$ is the interval satisfying $(x-1)(x-3) < 0$.

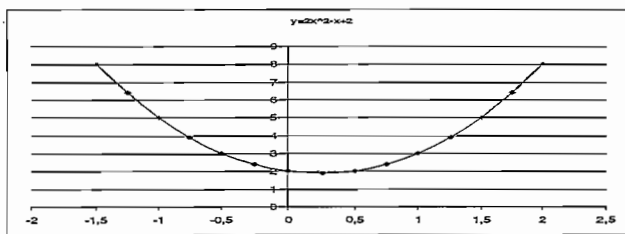
ii) $1 \leq x \leq 3$ is the interval over which the graph – the set of y-values – is *on or below* the x-axis, meaning that the y-values are zero or negative, in other words $1 \leq x \leq 3$ is the interval satisfying $(x-1)(x-3) \leq 0$.

iii) $x < 1$ or $x > 3$ is the interval over which the graph – the set of y-values – is *above* the x-axis, meaning that the y-values are positive, in other words $x < 1$ or $x > 3$ is the interval satisfying $(x-1)(x-3) > 0$.

iv) $x \leq 1$ or $x \geq 3$ is the interval over which the graph – the set of y-values – is *on or above* the x-axis, meaning that the y-values are zero or positive, in other words $x \leq 1$ or $x \geq 3$ is the interval satisfying $(x-1)(x-3) \geq 0$.

b) Mpho is impressed with the table method of solving the inequality, while Lebogang likes the graphical method. They want to apply their preferred methods to the inequalities i) $2x^2 - x + 2 < 0$, ii) $2x^2 - x + 2 > 0$.

Lebogang's method:



Inequality	Solution
i) $2x^2 - x + 2 < 0$	No solution
ii) $2x^2 - x + 2 > 0$	All real numbers $x \geq 1\frac{7}{8}$.

Fully explain Lebogang's conclusions.

Expected response

- There are no negative y -values ($y < 0$) covered by the graph.
- $y = 2x^2 - x + 2 = 2(x - \frac{1}{4})^2 + 1\frac{7}{8} \geq 1\frac{7}{8} \forall x \in \mathfrak{R}$, so that the solution is $x \geq 1\frac{7}{8}$, where $x \in \mathfrak{R}$.

b) Mpho's attempt

Mpho claims to be frustrated in trying to solve this problem. Why?

Expected results:

In the case of **irreducible** quadratic inequalities – quadratic inequalities that cannot be broken into linear factors – only the graphical method would be possible. Sometimes learners may waste time trying to find factors of an irreducible quadratic expression by trial and error. They should through facilitation be prompted towards determining $b^2 - 4ac$ first in order to establish if the roots exist or not. In this case $b^2 - 4ac = (-1)^2 - 4(2)(2) = -15$, so that there are no real roots.

c) Determine the solutions to the following inequalities using any method of your preference:

a) $x^2 - 3x - 10 \geq 0$

b) $-2x^2 + x + 1 < 0$

c) $5x^2 - 3x - 2 \leq 0$

d) $x^2 + 1 \geq 0$

Expected response:

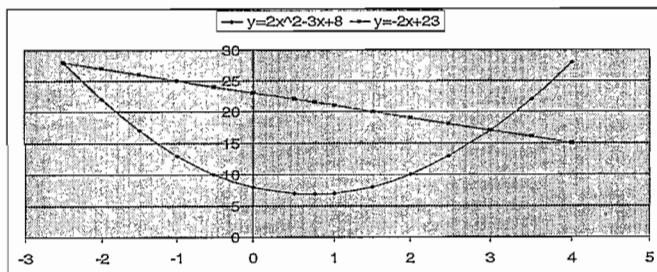
The learners should be encouraged to use $b^2 - 4ac$ in order to predict the nature of roots.

7 Simultaneous equations: One linear and one quadratic.

7.1 Fully explain the solutions to the simultaneously equation

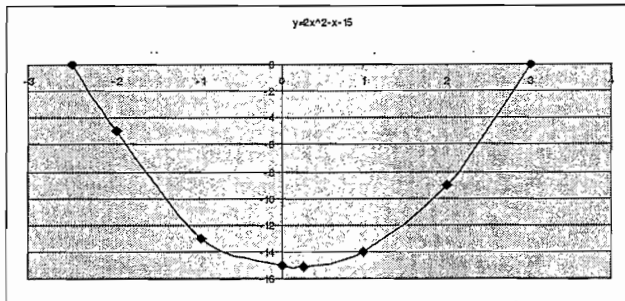
$$2x^2 - 3x + 8 = -2x + 23.$$

a) Leonard's solution:



Leonard's answer is $x = -2,5$ or $x = 3$.

b) **Maria's solution:**



Maria's answer is $x = -2,5$ or $x = 3$.

c) **Mpho's solution:**

$$2x^2 - 3x + 8 = -2x + 23$$

$$2x^2 - x - 15 = 0$$

$$(2x + 5)(x - 3) = 0$$

$$x = -\frac{5}{2} \text{ or } x = 3$$

Expected response

- Leonard has drawn the graphs $y = 2x^2 - 3x + 8$ and $y = -2x + 23$, and then read off the x components of the points of intersection.
- Maria has drawn the graph of $y = 2x^2 - x - 15$ and read off the x intercepts.
- Leonard and Maria's solutions are equivalent because $y = 2x^2 - 3x + 8$ and $y = -2x + 23 \Rightarrow 2x^2 - 3x + 8 = -2x + 23 \Rightarrow 2x^2 - x - 15 = 0$.
- Mpho decided to make the right-hand side of the equation zero, and then factorised to solve the quadratic equation.

7.2 Use any method to solve the following simultaneous equations:

a) $3x^2 - x + 5 = 6x + 1$

b) $5x^2 - x + 1 = -4x + 3$

c) $-7x^2 + x - 5 = 2x - 11$

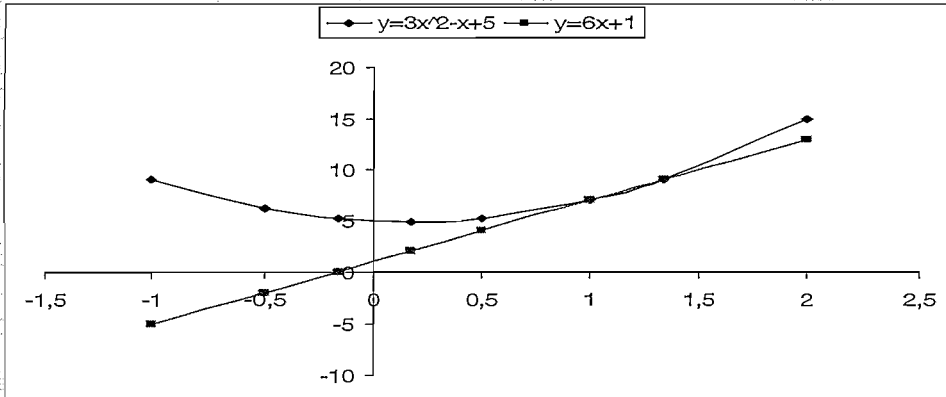
Expected response

a) $3x^2 - x + 5 = 6x + 1$

i) Leonard's solution:

- $y = 3x^2 - x + 5 = 3\left(x - \frac{1}{6}\right)^2 + \frac{59}{12}$; Axis of symmetry = 0,17 Min value = 4,92
- y-intercept: (0;5) x-intercepts: $b^2 - 4ac = -59$, no x-intercepts.
- $y = 6x + 1$: y-intercept: (0;1) x-intercept: (0,17;0).

Graph reading: $x = 1$ or $x \approx 1,33$



ii) Mpho's solution:

$$3x^2 - x + 5 = 6x + 1$$

$$3x^2 - 7x + 4 = 0$$

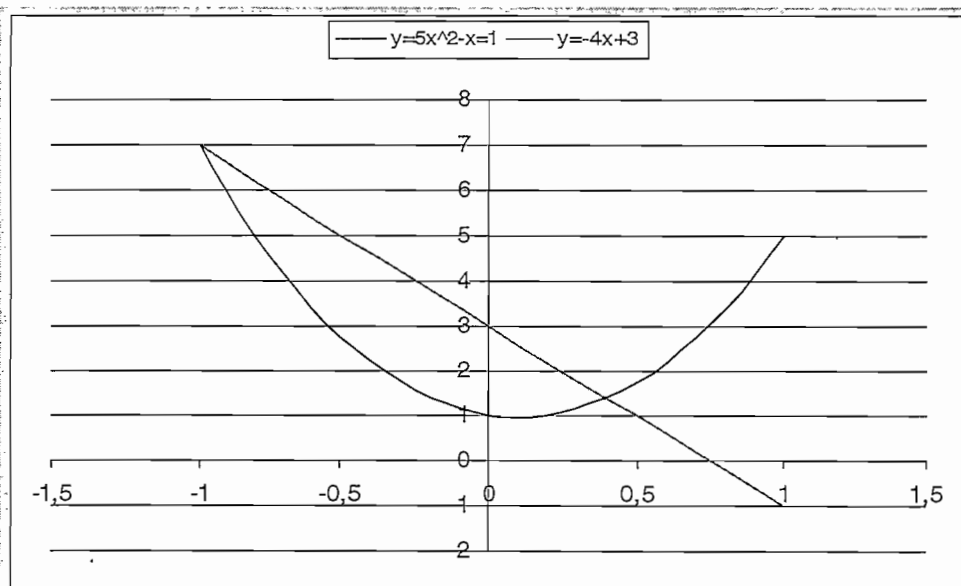
$$(3x - 4)(x - 1) = 0$$

$$x \approx 1,33 \text{ or } x = 1$$

b) $5x^2 - x + 1 = -4x + 3$

i) Leonard's solution:

- $y = 5x^2 - x + 1 = 5\left(x - \frac{1}{10}\right)^2 + \frac{19}{20}$ Axis of symmetry = 0,1. Min value = 0,95. $b^2 - 4ac = -19$, no x-intercepts. y-intercept = (0;1).
- $y = -4x + 3$. y-intercept (0;3). x-intercept (0,75;0)



Answer: $x = -1$ or $x = 0,4$

ii) Mpho's solution:

$$5x^2 - x + 1 = -4x + 3$$

$$5x^2 + 3x - 2 = 0$$

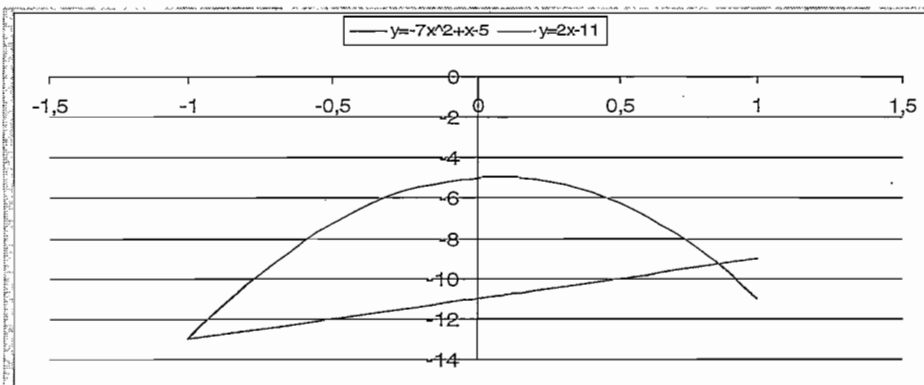
$$(5x - 2)(x + 1) = 0$$

$$x = \frac{2}{5} \text{ or } x = -1$$

c) $-7x^2 + x - 5 = 2x - 11$

i) Leonard's method:

- $y = -7x^2 + x - 5 = -7\left(x - \frac{1}{14}\right)^2 - \frac{139}{28}$ Axis of symmetry = 0,07. Min value = -
- $5. b^2 - 4ac = -139$, no x-intercepts. y-intercept = (0,-5).
- $y = 2x - 11$. y-intercept (0,-11). x-intercept (5,5;0)



Answer: $x = -1$ or $x \approx 0,86$

ii) Mpho's method:

$$-7x^2 + x - 5 = 2x - 11$$

$$-7x^2 - x + 6 = 0$$

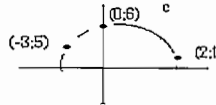
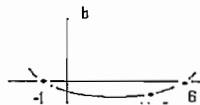
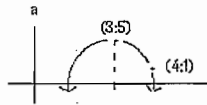
$$(-7x+6)(x+1) = 0$$

$$x = \frac{6}{7} \text{ or } x = -1$$

8. Manipulating Algebraic Expressions.

8.1 Determining quadratic equations

- a) Below are the graphs (a), (b) and (c) and attempts by three learners to identify their equations.



Peter:

$$y = a(x+1)(x-6)$$

$$-1 = a(5)(-2)$$

$$a = \frac{1}{10}$$

$$\therefore y = \frac{1}{10}x^2 - \frac{1}{2}x - \frac{3}{5}$$

Dimakatso

$$y = ax^2 + bx + 6$$

$$-5 = 4a + 2b$$

$$-1 = 9a - 3b$$

$$a = -\frac{17}{30}, b = -\frac{41}{30}$$

Pontsho

$$y = a(x-3)^2 + 5$$

$$1 = a + 5$$

$$a = -4$$

$$\begin{aligned} \therefore y &= -4(x-3)^2 + 5 \\ &= -4x^2 + 24x - 31 \end{aligned}$$

Determine the approximate **x** and **y** intercepts of each of the graphs.

Expected responses

- Pontsho's solution is for graph (a).

$$y = -4x^2 + 24x - 31$$

$$x = \frac{-24 \pm \sqrt{576 - 496}}{-8} \approx 1,9 \text{ or } 4,1$$

y-intercept = (0;-31), x-intercepts (1,9;0) or (4,1;0)

- Peter's solution is for graph (b).

$$y = \frac{1}{10}x^2 - \frac{1}{2}x - \frac{3}{5}$$

$$y - \text{int intercept} = (0; -\frac{3}{5})$$

- Dimakatso's solution is for graph (c)

$$y = -\frac{17}{30}x^2 - \frac{41}{30}x + 6$$

$$x = \frac{\frac{41}{30} \pm \sqrt{(\frac{41}{30})^2 + \frac{68}{5}}}{\frac{34}{30}} \approx -2,26 \text{ or } 4,68$$

x-intercepts (-2,26;0) or (4,68,0)

b) Use the above solutions to describe how we could identify the equations of the graphs with the following details:

- Given: One point and roots α and β .
- Given: One point and turning point $(p; q)$
- Coordinates and three points – one the y-intercept $(0; c)$ - on the parabola.

Expected response

- i) Use $y = a(x - \alpha)(x - \beta)$ and substitute the point in the equation to obtain a . If only roots α and β given, assume that $a = 1$.
- ii) Use $y = a(x - p)^2 + q$ and substitute the point in the equation to obtain a .
- iii) Use $y = ax^2 + bx + c$. Substitute the coordinates of the two points into $y = ax^2 + bx + c$, then solve the resulting equations simultaneously to obtain a .

c) For each of the following, find a quadratic equation with the given roots:

- i) 3;7 ii) -1;4 iii) $\frac{1}{4}; 5$ iv) $-1\frac{1}{4}; \frac{3}{4}$ $-1\frac{1}{4}; \frac{3}{4}$:

d) Donald is given the following problems to solve:

- i) 3 is one root of the equation $x^2 - px + 3 = 0$. Determine the value of p and the other root.
- ii) $\frac{1}{2}$ is one root of the equation $ax^2 - 5x + \frac{3}{4} = 0$. Determine the value of a and the other root.
- iii) $\frac{1}{2}$ and 2 are roots of the equation $2x^2 + ax + b = 0$. Determine the values of a and b .
- iv) 1 and 2 are roots of the equation $x^2 - px + c = 0$. Determine the values of a and c .

Donald solved two of the four problems as follows:

$$3^2 - 3p + 3 = 0$$

$$3p = 12$$

$$p = 4$$

$$\therefore x^2 - 4x + 3 = 0$$

$$(x-3)(x-1) = 0$$

$$x = 3 \text{ or } x = 1$$

\therefore The other root is $x = 1$.

$$(x-1)(x-2) = 0$$

$$x^2 - 3x + 2 = 0$$

$$p = -3 \text{ and } c = 2.$$

Explain Donald's solutions and then solve the other two problems.

e) If $x + \frac{1}{x} = 5$, determine i) $x^2 + \frac{1}{x^2}$ ii) $x^4 + \frac{1}{x^4}$ and iii) $x^8 + \frac{1}{x^8}$

Lomao solved i) as follows:

$$i) x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2 = 5^2 - 2 = 23$$

Explain Lomao's solution, and then solve the other two

Expected response

$$ii) x^4 + \frac{1}{x^4} = \left(x^2 + \frac{1}{x^2}\right)^2 - 2 = 23^2 - 2 = 527$$

$$iii) x^8 + \frac{1}{x^8} = \left(x^4 + \frac{1}{x^4}\right)^2 - 2 = 527^2 - 2 = 277727$$

- f) Find the equations of the following quadratic equations:
- Turning point $(-1;2)$ passing through $(2;11)$
 - Turning point $(2;-5)$ and y-intercept 15.
 - y-intercept 27, x-intercept 3, axis of symmetry 3.

Expected response

$$\begin{aligned}
 i) y &= a(x+1)^2 + 2 \\
 (2;11) : 11 &= 9a + 2 \\
 a &= 1 \\
 \therefore y &= x^2 + 2x + 3
 \end{aligned}$$

$$\begin{aligned}
 ii) y &= a(x-2)^2 - 5 \\
 (0;15) : 15 &= 4a - 5 \\
 a &= 5 \\
 \therefore y &= 5x^2 - 20x + 15
 \end{aligned}$$

$$\begin{aligned}
 iii) y &= ax^2 + bx + 27 \\
 -\frac{b}{2a} &= 3 \Rightarrow b = -6a \\
 \therefore y &= ax^2 - 6ax + 27 \\
 (3;0) : 0 &= -9a + 27 \\
 \therefore a &= 3, b = -18 \\
 \therefore y &= 3x^2 - 18x + 27
 \end{aligned}$$

2. Rational quadratic expressions

- a) Mahlatse wants to solve for x , given that $i) \frac{x}{x-1} - \frac{2x+4}{3x+1} = -4$ and

$$ii) \frac{2x}{x-1} - \frac{x+3}{x^2-1} = 3$$

This is how he proceeds:

$$i) \frac{x}{x-1} - \frac{2x+4}{3x+1} = -4$$

The denominator will be zero if

$x=1$ or $x=-\frac{1}{3}$ These must be avoided in the solution.

Multiply across by $(x-1)(3x+1)$:

$$3x^2 + x - (2x^2 + 2x - 4) = -12x^2 + 8x + 4$$

$$x^2 - x + 4 = -12x^2 + 8x + 4$$

$$13x^2 - 9x = 0$$

$$x(13x-9) = 0$$

$$x=0 \text{ or } x = \frac{9}{13}$$

$$ii) \frac{2x}{x-1} - \frac{x+3}{x^2-1} = 3$$

Reject $x = \pm 1$ in the final solution

because for these x - values the denominator is zero.

$$2x(x+1) - (x+3) = 3(x^2-1)$$

$$2x^2 + x - 3 = 3x^2 - 3$$

$$-x^2 + x = 0$$

$$-x(x-1) = 0$$

$$x=0. \text{ Reject } x=1.$$

Fully explain Mahlatsé's solution.

Expected response

In *ii)* $x=1$ is rejected because if $x=1$, then the original equation will be zero in the denominator, which invalidates the equation.

b) Solve for x to 2 decimal places if necessary:

$$i) \frac{6}{x+1} - \frac{3x}{1-x^2} = \frac{2x}{x-1}$$

$$ii) 2x^2 - x - 1 + \frac{15}{2x^2 - x + 2} = 5$$

$$iii) x - 1 + \frac{1}{x-3} = 0$$

$$iv) \frac{3}{x-2} + \frac{3}{x+2} = 2$$

Expected solutions

$$\begin{aligned}
 \text{i)} \quad & \frac{6}{x+1} - \frac{3x}{1-x^2} = \frac{2x}{x-1} \\
 & 6x - 6 - 3x = -2x(1+x) \\
 & 2x^2 + 5x - 6 = 0 \\
 & x = \frac{-5 \pm \sqrt{25 + 48}}{4} \\
 & = \frac{-5 \pm \sqrt{73}}{4} \\
 & = 0,89 \text{ or } -3,39
 \end{aligned}$$

$$\text{ii)} \quad 2x^2 - x - 1 + \frac{3}{2x^2 - x + 2} = 5$$

Let $k = 2x^2 - x - 1$. Then

$$2x^2 - x - 2 = (2x^2 - x - 1) - 1 = k - 1.$$

$$k + \frac{3}{k-1} = 5$$

$$k^2 - k + 3 = 5(k-1)$$

$$k^2 - 6k + 8 = 0$$

$$(k-4)(k-2) = 0$$

$$k = 4 \text{ or } k = 2$$

This means $2x^2 - x - 1 = 4$ or $2x^2 - x - 1 = 2$

$$2x^2 - x - 5 = 0 \text{ or } 2x^2 - x - 3 = 0$$

$$x = \frac{1 \pm \sqrt{41}}{4} \text{ or } x = \frac{1 \pm \sqrt{25}}{4}$$

$$x \in \{-1,35; -1; 1,85; \frac{3}{2}\}$$

$$\begin{aligned}
 \text{iii)} \quad & x - 1 + \frac{1}{x-3} = 0 \\
 & (x^2 - 4x + 3) + 1 = 0, x \neq 3 \\
 & x^2 - 4x + 4 = 0 \\
 & (x-2)^2 = 0 \\
 & x = 2
 \end{aligned}$$

$$\text{iv)} \quad \frac{3}{x-2} + \frac{3}{x+2} = 2$$

$$3(x+2) + 3(x-2) = 2(x^2 - 4)$$

$$x \neq \pm 2$$

$$2x^2 - 6x - 4 = 0$$

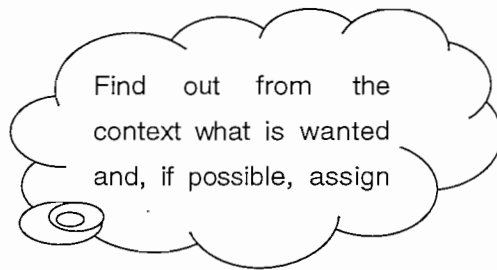
$$x^2 - 3x - 1 = 0$$

$$x = \frac{3 \pm \sqrt{13}}{2}$$

$$x \approx 3,3 \text{ or } x \approx -0,30$$

3 Mathematical modeling

Mathematical modelling is the process by which mathematical equations are assigned to contextual – usually word or verbal – problems, in order to solve the problems.



Mmemme is given the following problem to solve:

- a) A calculator shop buys a certain number of calculators for R2000. When left with 4 calculators, the shop has already made a profit of R300. If the shop sold each calculator for R10 more than it originally paid for it, how many calculators did the shop buy?

This is how Mmemme tried to solve the problem.

Let x be the number of calculators bought.

	No of calculators	Total Price	Price per calculator
Cost Price	x	R2000	$\frac{2000}{x}$
Selling Price	$x-4$	R2300	$\frac{2300}{x-4}$

Selling price – cost price (per calculator) = 10

$$\frac{2300}{x-4} - \frac{2000}{x} = 10$$

$$2300x - 2000(x-4) = 10(x-4)x$$

$$10x^2 - 340x - 8000 = 0$$

$$x^2 - 34x - 800 = 0$$

$$(x-50)(x+16) = 0$$

$$x = 50$$

The shop bought 50 calculators at R40 per calculator and sold 46 at R50 per calculator by the time it made a profit of R300.

Fully explain Mmemme's method of solving the problem.

b) Use Mmemme's method of solving the above problem to solve the following:

- i) A motorist paid R400 for petrol at the garage. Two days later he went to the garage and found that petrol had increased by R1,00 per litre. He paid R400 for petrol, but left the garage with 5 litres less than on previous occasion. Determine the approximate number - to the nearest unit - of litres of petrol the motorist bought on the first occasion, and the cost per litre of petrol before the increase.

Expected response

Let x be the number of litres of petrol bought on the first occasion.

	No of litres	Total Price	Price per litre
First occasion	x	R400	$\frac{400}{x}$
Second occasion	$x-5$	R400	$\frac{400}{x-5}$

2nd Selling price - 1st Selling price (per litre) = 1

$$\frac{400}{x-5} - \frac{400}{x} = 1$$

$$400x - 400(x-5) = x^2 - 5x$$

$$x^2 - 5x - 2000 = 0$$

$$x = \frac{5 \pm \sqrt{8025}}{2}$$

$$x \approx -42,29 \text{ or } x \approx 47,29$$

The motorist bought approximately 47 litres of petrol on the first occasion. The cost of 1 litre of petrol then was approximately R8,46.

- ii) A group of soccer fans hired a taxi to go to a soccer game. The taxi owner charged the fans R600. Four of the fans did not turn up at the collection point, with the result that to make up the R600, each fan had to pay an extra R8. How many fans went to the stadium? What was the increased fare?

Expected response

Let x be the number of fans.

	No. of fans	Total Price	Price per fan
Original agreement	x	R600	$\frac{600}{x}$
Emergency agreement	$x-4$	R600	$\frac{600}{x-4}$

$$\frac{600}{x-4} - \frac{600}{x} = 8$$

$$600x - 600(x-4) = 8x^2 - 32x$$

$$8x^2 - 32x - 2400 = 0$$

$$x^2 - 4x - 300 = 0$$

$$x = \frac{4 \pm \sqrt{1216}}{2}$$

$$x \approx -15,44 \text{ or } x \approx 19,44$$

About 15 (19-4) fans went to the stadium, each paying about R40.

iii) A painter bought paint for R800. A week later he returned to the paint shop and found that the price has been increased by R20 per litre. After again paying R800, the painter realised that he got 4 litres less paint than on earlier occasion. How many litres did the painter buy originally? What was the original price of paint per litre?

Expected response

Let x be the number of litres of petrol bought on the first occasion.

	No of litres	Total Price	Price per litre
First occasion	x	R800	$\frac{800}{x}$
Second occasion	$x - 4$	R800	$\frac{800}{x - 4}$

2nd Selling price $-$ 1st Selling price (per litre) = 20

$$\frac{800}{x - 4} - \frac{800}{x} = 20$$

$$800x - 800(x - 4) = 20x^2 - 80x$$

$$20x^2 - 80x - 3200 = 0$$

$$x^2 - 4x - 160 = 0$$

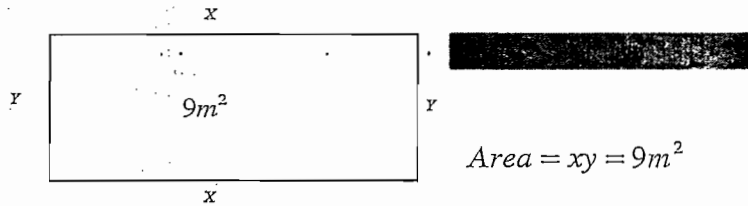
$$x = \frac{4 \pm \sqrt{656}}{2}$$

$$x \approx -10,81 \text{ or } x \approx 14,81$$

The painter bought approximately 15 litres of paint on the first occasion. The cost of 1 litre of paint then was approximately R53.

c) The following is the problem that Sello has got to solve:

The area of a rectangle is $9m^2$ and its perimeter is 12m. What are the dimensions of the rectangle?



The following is how Sello solve the problem:

$$2x + 2y = 12 \Rightarrow y = 6 - x$$

$$9 = xy \Rightarrow 9 = x(6 - x) = 6x - x^2$$

This means

$$x^2 - 6x + 9 = 0$$

$$(x - 3)^2 = 0$$

$$x = 3.$$

Substituting y in $y = 6 - x$, we get $y = 3$

The dimensions are $x = 3m$ and $y = 3m$

Fully explain Sello's method.

d) Solve the following problems:

i) The perimeter of a rectangle is 14m and its area is $12m^2$. What are the dimensions of the rectangle?

Expected response

We solve simultaneously for x and y .

$$2x + 2y = 14 \Rightarrow y = 7 - x$$

$$12 = xy \Rightarrow 12 = x(7 - x) = 7x - x^2$$

This means

$$x^2 - 7x + 12 = 0$$

$$(x - 3)(x - 4) = 0$$

$$x = 3 \text{ or } x = 4$$

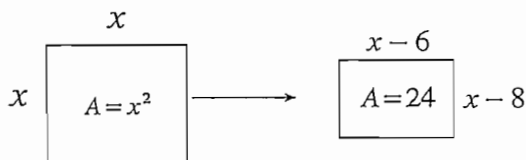
Substitute y in $y = 7 - x$:

$$\text{If } x = 3, y = 4$$

$$\text{If } x = 4, y = 3$$

The dimensions are $x = 3\text{m}$ and $y = 4\text{m}$ or $x = 4\text{m}$ and $y = 3\text{m}$

ii) A square plot of ground is divided by decreasing one side by 6m and decreasing the other side by 8m. The rectangular area thus formed is 24 square metres. Find the original dimensions of the plot.



Expected response

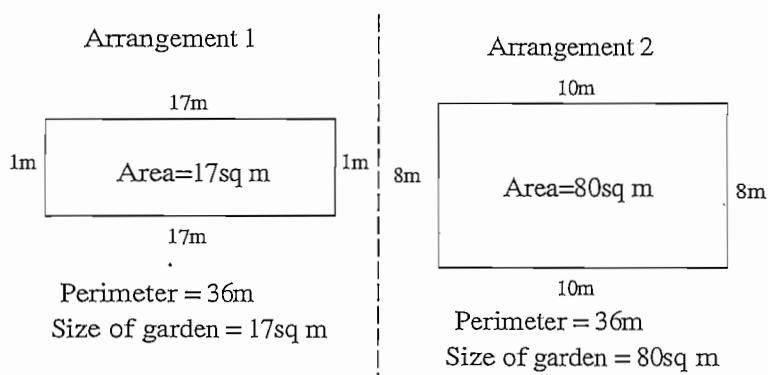
We want x - the original side - such that $(x - 8)(x - 6) = 24$

$$(x - 8)(x - 6) = 24 \Rightarrow x^2 - 14x + 24 = 0 \Rightarrow (x - 12)(x - 2) = 0 \Rightarrow x = 12 \text{ or } x = 2$$

Reject $x = 2$ to avoid negative sides. The original dimensions of the plot were 12×12 .

c) In a self-help scheme, community members interested in farming are given rectangular plots to grow vegetables in. One member has 36 metres of fence to enclose his rectangular plot with. What will be the dimensions of his garden for *maximum size*?

In trying to solve this problem, Boitumelo thinks that arrangement 1 below is the solution. However, she realises that arrangement 2 is also a possibility.



Ultimately she realises that these are not the only possibilities – there are actually many more possible arrangements of the 36m fence. She also realises that each arrangement gives rise to a different garden size. She then proceeds in the following way to solve the problem.

$$\text{Perimeter: } 2x + 2y = 36 \Rightarrow x + y = 18 \Rightarrow y = 18 - x$$

$$\text{Size: } A = xy \Rightarrow A(x) = x(18 - x) = -x^2 + 18x$$

This is an ordinary parabola whose maximum ($a < 0$) value can be obtained

by completing the square or formula as follows:

$$\text{Max value} = -\frac{b^2 - 4ac}{4a} = -\frac{(18)^2 - 4(-1)(0)}{4(-1)} = 81$$

$$\text{or } A(x) = -x^2 + 18x = -1\left(x - \frac{18}{2}\right)^2 + 81, \text{ meaning that maximum} = 81.$$

So the maximum possible size the garden can have is $81m^2$

But then $81 = 18x - x^2$ or $x^2 - 18x + 81 = 0 \Rightarrow (x - 9)^2 = 0 \Rightarrow x = 9$.

Substituting $y = 18 - x$ yields $y = 9$

So the dimensions for maximum size of the rectangular garden are, Boitumelo concludes, 9m by 9m.

Explain in details Boitumelo's solution.

Use Boitumelo's method to answer the following question:

- i) In a self-help scheme, community members interested in farming are given rectangular plots to grow vegetables in. One member has 12 metres of fence to enclose his rectangular plot with. What will be the dimensions of his garden for *maximum size*?

Expected solution

Perimeter: $2x + 2y = 12 \Rightarrow x + y = 6 \Rightarrow y = 6 - x$

Size: $A = xy \Rightarrow A(x) = x(6 - x) = -x^2 + 6x$

This is an ordinary parabola whose maximum ($a < 0$) value can be obtained

by completing the square or formula as follows:

$$\text{Max value} = -\frac{b^2 - 4ac}{4a} = -\frac{(6)^2 - 4(-1)(0)}{4(-1)} = 9$$

or $A(x) = -x^2 + 6x = -1\left(x - \frac{6}{2}\right)^2 + 9$, meaning that maximum = 9.

So the maximum possible size the garden can have is $9m^2$

But then $9 = 6x - x^2$ or $x^2 - 6x + 9 = 0 \Rightarrow (x - 3)^2 = 0 \Rightarrow x = 3$.

Substituting $y = 6 - x$ yields $y = 3$

So the dimensions for maximum size of the rectangular garden are 3m by 3m.

- ii) In a self-help scheme, community members interested in farming are given rectangular plots to grow vegetables in. One member has 24 metres of fence to his rectangular plot with. What will be the dimensions of his garden for *maximum size*?

Expected solution

$$\text{Perimeter: } 2x + 2y = 24 \Rightarrow x + y = 12 \Rightarrow y = 12 - x$$

Size:

This is an ordinary parabola whose maximum ($a < 0$) value can be obtained

by completing the square or formula as follows:

$$\text{Max value} = \frac{b^2 - 4ac}{4a} = \frac{(12)^2 - 4(-1)(0)}{4(-1)} = 36$$

$$\text{or } A(x) = -x^2 + 12x = -1\left(x - \frac{12}{2}\right)^2 + 36, \text{ meaning that maximum} = 36.$$

So the maximum possible size the garden can have is $36m^2$

$$\text{But then } 36 = 12x - x^2 \text{ or } x^2 - 12x + 36 = 0 \Rightarrow (x - 6)^2 = 0 \Rightarrow x = 6.$$

Substituting $y = 12 - x$ yields $y = 6$

So the dimensions for maximum size of the rectangular garden are, Boitumelo concludes, 6m by 6m

APPENDIX 7 : QUESTIONNAIRE

Learner survey

Directions: There are no right or wrong answers for the following questions. We want to know what *you* really believe. For each statement cross out the degree to which you agree or disagree.

Strongly disagree	1	2	3	4	5	Strongly agree
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If you strongly agree with a statement, cross out 5; if you strongly disagree with the statement, cross out 1. If you more or less agree or disagree with a statement, find the number between 1 and 5 that best describes your level of agreement and cross out the number.

You have been exposed to two approaches towards teaching and learning. Indicate your feelings about each of the statements below:

1. Finding your own solutions is better than depending on the educator to give the solutions.
2. Sometimes learners explain better to each other than educators would explain to them.
3. A good educator's job is to keep his learners from wandering from the right track.
4. Wisdom is not knowing the answers, but knowing how to find the answers.
5. I like an educator who directs the lesson by giving answers.
6. Educators should encourage learners to find their own ways of working out mathematics computations.
7. Educators should teach exact procedures for solving problems.
8. Mathematics should be taught in such a way that learners can discover relationships for themselves.
9. It is important for a learner to be a good listener to the teacher's explanations in order to learn how to do mathematics well.

10. The majority of the learners can work out a way to solve mathematical problems without the help of the teacher.
11. An effective mathematics educator always demonstrates the right method for solving mathematical problems.
12. Learners should be told to solve the problems the way the teacher has taught them.
13. The majority of learners have to be shown how to do mathematical computations.
14. The best way of teaching problem-solving is to show learners how to solve problems.
15. Learners learn mathematics best by working out their own ways (methods) of doing computations.
16. The majority of learners cannot solve their problems without the help of the teacher.
17. Learners can usually work out for themselves how to do mathematical investigations.
18. Allowing learners to discuss their thinking with other learners helps them to understand mathematics.
19. Learners can work out ways to solve word problems without teacher or adult help.
20. The aims of teaching school mathematics are best achieved when learners find their own methods for working out computations.
21. Educators should allow learners who are having difficulty solving a non-routine problem to try to find a solution for themselves.
22. Learners' explanations of their solutions to computations are good indicators of their mathematical learning.
23. To be a good problem-solver, it is important for a learner to know how to follow a teacher's directions.
24. Educators should allow learners who are having difficulty solving a word problem to continue to try to work it out.
25. To be successful in mathematics, a learner must listen carefully to the teacher's explanations.
26. The most important issue in mathematics is not whether the answer to a problem is correct, but whether the learners can explain their answers.
27. Having learners work in small groups interferes with learning mathematics.
28. If learners disagree over the right way to solve a problem it can interfere with their learning of mathematics.
29. Because learners' mathematics abilities differ so much it is better to explain a new concept to the whole class group and then to observe them drilling and practising it afterwards.
30. Discussing their work with each other in class interferes with learners' learning of school mathematics.
31. It is motivating to see the learners solving problems on their own.

32. Learners who solve problems on their own develop self-confidence.
33. Learners who solve problems on their own tend to become good problem-solvers.
34. Each time an educator teaches a learner something the learner could have discovered himself, the learner is kept from understanding it completely.

APPENDIX 8 : LESSON PLAN

6.3.3. Open-ended approach lesson plan: Takahashi (2002), Fifth Grade

6.3.3.1. Title of the lesson: How do you find the number of the dots?

6.3.3.2. Goals of the lesson:

- a) Learners will begin to recognise equations as a way to represent quantitative relations mathematically;
- b) Learners will begin to be able to communicate with other learners by:
 - i) Expressing their thinking processes using equations.
 - ii) Inferring possible thinking processes of other learners from the equations they present.

Note that the above stated communication goals described the anticipated learner role in the lesson. In a way it also restricts teacher domination of the lesson.

6.3.3.3. Instructions of the lesson

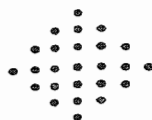
The NCTM expects learners to be able to:

- a) Understand patterns, relations and functions;
- b) Represent and analyse mathematical situations and structures using algebraic symbols;
- c) Use mathematical models to represent and understand quantitative relationships;
- d) Analyse change in various contexts.

This lesson is designed to provide learners with an opportunity to experience representational activities. By employing an open-ended approach, this lesson is designed to provide learners with an opportunity to develop their competence to use mathematical expressions and equations. The lesson presents the learners with the open-ended problem

to compare and discuss a variety of ways to describe the arrangement of 25 dots by using equations. This type of open-ended problem is known as a problem with multiple solutions.



Find the number of dots and describe your way to find the number of dots by using a

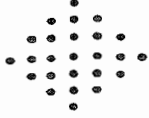
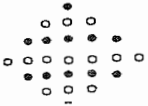
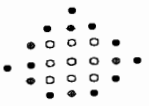
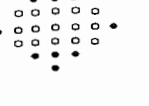


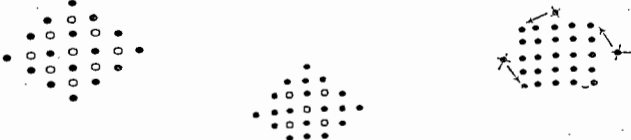
Because of the nature of the open-ended approach, the main concern for the teacher during this lesson is to facilitate discussions meaningfully by including all the learners in the class.

6.3.3.4. Flow of the lesson

The introductory part of the equation starts with the question dealing with 5 dots. Table 6.2. gives a summary of the lesson flow:

Steps, learning activities, questions and expected reactions	Teacher's support	Points of evaluation
<p>1. Introduction</p> <p>Describe the number of dots.</p>  <p><i>a) Anticipated learners' responses:</i></p> <p>i) one in centre and four outside</p> <p>ii) two, one, two</p> <p>iii) three and two</p> <p><i>b) Anticipated learners' responses:</i></p>  <p>i) one in centre and four outside</p> <p>ii) two, one, two</p> <p>iii) three and two</p>	<p>Help learners see the major focus of today's activities.</p> <p>Help learners realise that they can use equations to describe the arrangement of dots by translating learners' verbal expression into an equation.</p> <p>a) i) $1+4=5$</p> <p>ii) $2+1+2=5, 2 \times 2+1=5$</p> <p>iii) $3+2=5$</p> <p>b) i) $3+3=6, 2 \times 3=6$</p> <p>ii) $2+2+2=6, 3 \times 2=6$</p>	<ul style="list-style-type: none"> • Do learners realise that there are multiple ways to describe the arrangement of dots? • Do learners understand that they can describe the arrangement of dots by using equations? • Are learners motivated to find different ways to describe the arrangement of dots by using different equations?

2. Posing problem		
<p>Find the number of dots and describe your way to find the number of dots by using a </p>		
<p>Learners should use worksheets to write down their equations so that they can use them during the whole class discussion to share and describe their equations.</p>	<p>Provide learners worksheets with a large picture of the dots for the problem.</p> <p>Let learners write an equation that describes their counting method on each worksheet. Learners will be able to have as many worksheets as they need.</p> <p>Learners might work with their partners.</p>	<p>Do learners understand how to use the worksheet?</p> <p>Do learners works comfortably with their partners?</p>
3. Solving problem		
<div style="display: flex; justify-content: space-around; align-items: flex-start;"> <div style="text-align: center;">  <p>$1+3+5+7+3+1=25$</p> </div> <div style="text-align: center;">  <p>$3 \times 3 + 4 \times 4 = 25$</p> </div> <div style="text-align: center;">  <p>$3 \times 5 + 2 \times 4 + 2 \times 1 = 25$</p> </div> </div> <p>$2 \times 1 + 2 \times 3 + 2 \times 5 + 7 = 25$ $4 \times 4 + 9 = 25$ $3 \times 5 + 2 \times 3 + 4 \times 1 = 25$</p> <p>$7 + 2(1 + 3 + 5) = 25$</p>		<p>Does each student find the number of dots, and is he/she able to describe his/her way by using the diagram on the worksheet?</p> <p>Does each student write an equation to describe his / her counting methods?</p>

 <p> $4 \times 4 + 3 \times 3 = 25$ $5 \times 5 = 25$ $4 + 4 + 4 + 4 + 3 \times 3 = 25$ $4 \times 5 + 5 = 25$ </p>	
<p>4. Share their equations</p> <p>Share your equations to the class but do not tell others where the equations come from.</p> <p>Other learners: Let's figure out where the equation comes from.</p>	<p>Facilitate opportunities for learners to interpret an equation. Encourage all the learners to be engaged in the representing and interpreting activity because it is possible that some equations might come from different ways of representing the arrangement of the dots. The unique solutions will be discussed later.</p> <p>Do learners see relationships between a diagram and an equation?</p>

<p>5. Summing up</p> <p>Reflecting on what we learned by looking at the board writing and recognising mathematical expression / equation can be a way to express mathematical situations.</p>	<p>Let learners write their learning experience as a journal reflection.</p>	
<p>6. Evaluation</p> <p>a) Did the learners represent quantitative relations by using equations?</p> <p>b) Did the learners communicate with other learners by expressing their thinking processes using equations, and inferring possible thinking processes of other learners from the equations they represent?</p>		