Duration as a strategic interest rate risk management tool in financial institutions

Gary Wayne van Vuuren (PhD)

Thesis submitted in the Centre for Business Mathematics and Informatics
of the North-West University (Potchefstroom Campus)
in partial fulfilment of the requirements for the degree of
Philosophiae Doctor (Risk Management)

SUPERVISOR: PROF PAUL STYGER

LONDON, ENGLAND
2005

For my protégé

Marius Botha

I can no other answer make but thanks,
and thanks, and ever thanks;
Too oft good turns
are shuffl'd off with such uncurrent pay.

–William Shakespeare: Twelfth Night
Act III, Scene ii

Preface

Much of the theoretical work described in this thesis was carried out whilst in the employ of Old Mutual Asset Managers (Cape Town, South Africa), Standard Bank (London, UK) and Ernst and Young (London, UK). Some theoretical and practical work was carried out in collaboration with the BMI: University of the North West (Potchefstroom) under the supervision of Professor Paul Styger.

These studies represent the original work of the author and have not been submitted in any form to another University. Where use was made of the work of others, this has been duly acknowledged in the text. Unless otherwise stated, all data were obtained from Bloomberg™ (provider of live and historical financial and economic statistics) and the internal, non-proprietary financial databases of Old Mutual Asset Managers, Standard Bank London and Ernst and Young. Discussions with personnel from all of the above institutions also provided invaluable insight into current investment trends and problems encountered in the fixed-income arena.

The adjustments to the Macaulay duration work (to measure optionality and take account of defaultability) was presented at the annual *South African Finance Association* conference in Cape Town, South Africa in January 2004 (van Vuuren and Styger, 2004a) and – with modifications – at the 57th *International Atlantic Economic Society* conference in Lisbon, Portugal in March 2004 (van Vuuren and Styger, 2004b). This work is in preparation for submission to *The Investment Analyst Journal* under the heading "Towards a Market Value of Equity DGap". Other results, involving new interpretations and the implementation of alternative duration measures has been submitted for publication in *The GARP Risk Review* and *Risk* magazine.

Work based on previous research, but both necessary and important for the analysis discussed in this thesis, has been published in *Risk Magazine* (van Vuuren and Botha, 1998 & 2000), *PJAS* (Botha and van Vuuren, 2001), *The Investment Analysts Journal* (McLeod and van Vuuren, 2004) and *The GARP Risk Review* (van Vuuren, Botha and Styger, 2004).

G.W. VAN VUUREN

Thursday, 11 August 2005

Acknowledgements

I acknowledge an enormous debt of gratitude to all that have contributed in some way or other to the completion of this thesis. In particular, I take special note of the assistance provided – in whatever form – by the folk mentioned below:

- Professor Paul Styger, my supervisor, for endless hours of stimulating conversation, feed-back and ceaseless enthusiasm for this and all my projects. Despite a heavy workload of his own, he has guided me comfortably through two major projects (to date) and I am very grateful indeed for his expert, dependable tutelage,
- Professor Machiel Kruger for his detailed and helpful mathematical insights at a time when it was very difficult to see the wood for the trees,
- my family for their continued – if somewhat bewildered – support,
- my friend, Dorothy Pfister, for her encouragement and wisdom,
- my long-suffering friends, Barry and Luwonda Doo, for the food, conversations, interest and unpaid debts, but above all for being there in the dark hours,
- my companion, Christo Swanepoel, for sharing briefly with me both a planet and an epoch, for fleeting glimpses of freedom and for proof – however ephemeral – that not all those that wander are lost,
- my confidante, moral compass and benchmark of all competence and excellence, Petro Grobler, for her perpetually valuable voice of reason and
- my friends, Marius and Lizl Botha, for their unmerited tolerance, unwarranted companion-ship and wholly undeserved friendship. I am grateful indeed to them for the peaceful sleep on many rainy nights and the silence of several foreboding lambs.

Abstract

Banks and financial institutions record core banking activities (taking deposits and making loans) on the balance sheet of the banking book, but trading book (trading and investment) pursuits are recorded off-balance-sheet. Both books are subject to considerable risk from numerous sources, but regulatory capital reserves – to shield from unexpected market moves – are not required for the banking book as they are for the trading book. Both of these substantial shortcomings will be addressed in the near future. The 2005 initiation of new global accounting standards will ensure that trading book derivative fair values are recorded and reported on the balance sheet while the new Basel accord, due for full implementation in 2007, will regulate the calculation and reservation of capital required for the banking book. These changes are expected to better regulate and manage the transparency of financial institutions' activities and so prevent large scale economic disasters or deliberate corporate fraud. Institutions not compliant with the new rules will face severe financial losses, regulatory fines and possible debilitating legal action.

One of the most commonly-used tools for measuring and managing interest rate risk, the Macaulay duration, has enjoyed almost unchallenged success, but it employs severely restrictive and unrealistic assumptions which constrain its usefulness and reliability in the rapidly-changing world of defaultable securities, those with embedded derivatives and instruments with perpetual maturities. Robust measures which more accurately approximate interest rate risk by relaxing unrealistic assumptions are required.

Applications which significantly improve the accuracy of the Macaulay duration are considered as well as a new look at the duration problem in general. The influence of a more accurate duration measure on duration gap provides a significantly improved *economic* Market Value of Equity. The role of this enhanced measure is crucial for risk management as well as regulatory and accounting compliance.

Uittreksel

Die kernaktiwiteite van banke en finansiële (die neem van deposito's en toestaan van lenings) word gereflekteer in die balansstaat in die bankbedrywighede, terwyl die handelsbedrywighede (handel en investering) nie in die balansstaat opgeteken word nie. Alhoewel beide hierdie bedrywighede blootgestel is aan aansienlike risiko's vanuit verskillende oorde, word geen regulatoriese kapitaalreserwes vereis vir hierdie bedrywighede nie.

Hierdie tekortkoming is egter in proses om die nodige aandag te verkry. Die wereldwye toepassing van nuwe rekenkundige standaarde sal verseker dat waardes van afgeleide instrumente in die handelsboek billik in die balansstaat gereflekteer en verslag oorgedoen sal word. Betreffende die bankbedrywighede, word voorsien dat die nuwe Basel verdrag (implementering 2007) die regulering en berekening van kapitaalvereistes sal orden. Beide hierdie veranderinge sal die deursigtigheid van finansiële instellings se aktiwiteite reguleer en bestuur ten einde grootskaalse ekonomiese rampe of bedrog te voorkom. Instellings wat nie aan die nuwe reels voldoen nie, sal ernstige verliese, boetes en moontlike regsaksie die hoof moet bied.

Een van die mees suksesvolle en gebruikte meetinstrumente vir die kwantifisering en bestuur van rentekoers risiko, is die Macaulay tydsduur. Hierdie maatstaf gebruik egter streng beperkende en onrealistiese aannames wat die bruikbaarheid en betroubaarheid daarvan inperk in 'n vinnig veranderende omgewing van wanbetaalbare sekuriteite sowel as instrumente met ewigdurende looptye. Meer robuuste maatstawwe waar onrealistiese aannames verslap word, word benodig om rentekoers risiko met groter akuraatheid te meet.

In hierdie werkstuk word aanpassings voorgestel wat die akkuraatheid van die bestaande Macaulay tydsduur betekenisvol verbeter. 'n Nuwe beskouing van die tydsduurprobleem in die algemeen, word aan die leser voorgehou. Verder word die invloed van 'n akkurater tydsduurmaatstaf op die tydsduurgaping en gevolglike wesenlike verbeterde ekonomiese markwaarde van ekwiteit in hierdie werkstuk aangetoon. Die rol van hierdie verbeterde maatstaf is krities vir gesonde risiko bestuur sowel as die nakoming van regulatoriese en rekeningkundige vereistes.

Table of Contents

DEDICATION ii
PREFACE iii
ACKNOWLEDGEMENTS iv
ABSTRACT v
UITTREKSEL vi
TABLE OF CONTENTS vii
LIST OF FIGURES xii
LIST OF TABLES xiv
1 INTRODUCTION 1
1.1 THE ECONOMIC ROLE OF BANKS 1
1.2 BANKING RISK 2
1.3 INTEREST RATE RISK 2
1.4 MANAGEMENT OF INTEREST RATE RISK 4
1.5 PROBLEM STATEMENT AND AIMS OF STUDY 6
1.6 THESIS OUTLINE 7
2 BANK RISK 9
2.1 INTRODUCTION 9
2.2 BANKING RISKS 9
2.2.1 CREDIT RISK 10
2.2.2 MARKET RISK 10
2.2.3 OPERATIONAL RISK 11
2.2.4 LIQUIDITY RISK 11
2.2.4.1 Exogenous liquidity risk 11
2.2.4.2 Endogenous liquidity risk 12
2.2.5 COUNTRY RISK 12
2.2.6 FOREIGN EXCHANGE RISK 12
2.2.6.1 Transaction exposure 12
2.2.6.2 Translation exposure 13
2.2.6.3 Economic exposure 13
2.2.7 INTEREST RATE RISK 13
2.2.7.1 Traded interest rate risk 13
2.2.7.2 Non-traded interest rate risk 13
2.3 BANKING VERSUS TRADING BOOKS 14
2.3.1 THE BANKING BOOK 14
2.3.2 THE TRADING BOOK 14
2.4 REGULATIONS GOVERNING INTEREST RATE RISK 15
2.4.1 REGULATORY STANDARDS 16
2.4.1.1 Capital adequacy 17
2.4.1.2 Overview of the Basel accords 18
2.4.2 ACCOUNTING STANDARDS 22

2.4.2.1 History and evolution of IAS 39 23
2.4.2.2 AC 133 requirements - South Africa 26
2.4.3 SUMMARY OF REGULATORY & ACCOUNTING CHANGES IMPACT 28
2.5 CONCLUSION 28
3 OVERVIEW OF MACAULAY & ASSOCIATED DURATION MEASURES 29
3.1 INTRODUCTION 29
3.2 HISTORICAL OVERVIEW 29
3.3 MACAULAY DURATION 31
3.3.1 DEFINITION AND DERIVATION 31
3.3.2 MODIFIED DURATION: DEFINITION AND DERIVATION 35
3.3.3 CONVEXITY 35
3.3.4 DURATION EXAMPLE 38
3.3.5 PROPERTIES OF DURATION 39
3.3.6 KEY RATE DURATIONS 39
3.4 DURATION AS AN ELASTICITY 40
3.5 PORTFOLIO IMMUNISATION USING DURATION 42
3.5.1 PRINCIPLES OF BOND PORTFOLIO IMMUNISATION 43
3.5.2 IMMUNISATION EXAMPLE 44
3.5.3 PROBLEMS WITH IMMUNISATION 45
3.6 CONCLUSION 46
4 INTEREST RATE RISK IN THE BANKING AND TRADING BOOKS 47
4.1 INTRODUCTION 47
4.2 INTEREST RATE RISK IN THE BANKING BOOK 47
4.2.1 EARNINGS AND ECONOOMIC PERSPECTIVES 47
4.2.1.1 Earnings perspective 48
4.2.1.2 Economic perspective 49
4.2.1.3 Numerical example: earnings versus economic perspectives 51
4.2.1.4 Trade-offs in managing earnings and economic exposures 54
4.2.2 THE ROLE OF ALM 55
4.2.3 SOURCES OF INTEREST RATE RISK 56
4.2.3.1 Pricing (maturity mismatch) risk 56
4.2.3.2 Basis risk 57
4.2.3.3 Yield curve risk 58
4.2.3.4 Option risk 59
4.2.4 INTEREST RATE RISK MEASURMENT 60
4.2.4.1 Gap and gap ratio analysis 61
4.2.4.1.1 Benefits of gap reports 62
4.2.4.1.2 Disadvantages of gap reports 62
4.2.4.1.3 Yield curve risk 63
4.2.4.1.4 Option risk 63
4.2.4.1.5 Intra-period gaps 63
4.2.4.1.6 New business 63

4.2.4.2 Duration gap (DGap) analysis 64
4.2.4.2.1 Market value of equity sensitivity analysis 65
4.2.4.2.2 Example of DGap analysis 66
4.2.4.2.3 Embedded options 68
4.2.4.2.4 Advantages of DGap over funding or maturity gap 68
4.2.4.3 Income simulation models 68
4.2.4.3.1 Static net interest income modelling 69
4.2.4.3.2 Dynamic net interest income modelling 69
4.2.4.3.3 Monte Carlo simulation modelling 69
4.2.4.3.4 Construction of Monte Carlo simulation 69
4.2.4.3.5 Advantages of Monte Carlo simulation 71
4.2.4.3.6 Limitations of Monte Carlo simulation 71
4.2.5 REGULATION 71
4.2.6 CONCLUSION 72
4.3 INTEREST RATE RISK IN THE TRADING BOOK 72
4.3.1 INTEREST RATE RISK IN THE TRADING BOOK 74
4.3.2 INTEREST RATE RISK MEASURMENT IN THE TRADING BOOK 74
4.3.2.1 Value at Risk 74
4.3.2.1.1 VaR definition 74
4.3.2.1.2 Calculating Value at Risk 75
4.3.2.1.3 Advantages of the VaR approach 75
4.3.2.1.4 Disadvantages of VaR 76
4.3.2.1.5 VaR alternatives 76
4.3.3 REGULATION 76
4.3.4 CONCLUSION 77
5 MACAULAY DURATION MODIFICATIONS 78
5.1 INTRODUCTION 78
5.1.1 DEFAULTABILITY 79
5.1.2 OPTIONALITY 79
5.1.3 PERPETUAL-MATURITY INSTRUMENTS 80
5.2 DURATION OF DEFAULTABLE BONDS 80
5.2.1 THEORETICAL FRAMEWORK 81
5.2.1.1 Introduction 81
5.2.1.2 Bierwag's contingent cash flows 81
5.2.2 LITERATURE SURVEY: DEFAULTABLE AND OPTION-EMBEDDED SECURITIES 82
5.2.3 PRICING OF DEFAULTABLE SECURITIES 86
5.2.3.1 Introduction 86
5.2.3.2 Literature survey: credit pricing 87
5.2.3.3 General credit pricing model 88
5.2.4 CREDIT CURVE CONSTRUCTION 90
5.2.4.1 Introduction 90
5.2.4.2 Hazard rates 91
5.2.4.3 Li's continuous credit curve model 92

5.2.4.4 Component combination 94
5.2.5 EMPIRICAL RESULTS 94
5.2.6 APPLICATION OF RESULTS 97
5.3 DURATION OF SECURITIES WITH EMBEDDED OPTIONS 99
5.3.1 THEORETICAL FRAMEWORK 99
5.3.1.1 Literature survey 99
5.3.1.2 Bierwag's duration drift 100
5.3.2 EMPIRICAL RESULTS 105
5.3.3 APPLICATION OF RESULTS 107
5.4 DURATION OF NON-MATURITY INSTRUMENTS 109
5.4 CONCLUSION 111
6 ALTERNATIVE DURATION MEASURES 113
6.1 INTRODUCTION 113
6.1.1 OVERVIEW OF MACAUALAY DURATION SHORTCOMINGS 113
6.1.2 DURATION & STOCHASTIC PROCESSES OF THE TERM STRUCTURE 115
6.2 STOCHASTIC PROCESSES 115
6.2.1 ADDITIVE SHIFTS 117
6.2.1.1 Theory 117
6.2.1.2 Empirical testing 120
6.2.2 MULTIPLICATIVE SHIFTS 121
6.2.2.1 Theory 121
6.2.2.2 Empirical testing 122
6.2.3 FISHER-WEIL SHIFTS 123
6.2.3.1 Theory 123
6.2.3.2 Empirical testing 125
6.2.4 LOG-ADDITIVE SHIFTS 126
6.2.4.1 Theory 127
6.2.4.2 Empirical testing 127
6.2.5 LOG-MULTIPLICATIVE SHIFTS 128
6.2.5.1 Theory 128
6.2.5.2 Empirical testing 129
6.2.6 CONCLUSION 130
6.2.7 SOUTH AFRICAN EMPIRICAL DATA 130
6.2.7.1 Data 130
6.2.7.2 Instrument construction 133
6.2.7.3 Applications 134
6.2.7.3.1 Elimination of convexity 134
6.2.7.3.2 Forecasting accuracy 134
6.2.7.3.3 Improvement of DGap measurement 135
6.3 CONCLUSION 138
7 SUMMARY, CONCLUSIONS & SUGGESTIONS FOR FUTURE WORK 140
7.1 SUMMARY 140

7.2 CONCLUSIONS 142
7.2 SUGGESTIONS FOR FUTURE WORK 142
APPENDICES 144
A DERIVATION OF CONTINUOUS COMPOUNDING EQUATION 144
B DEFINITION OF TRACKING ERROR 145
C REGRESSION RESULTS 146
D STATISTICAL DATA TESTS 148
REFERENCES 154

List of Figures

CHAPTER 2

Figure 2.1 Main components of bank risk. 9

Figure 2.2 The new capital adequacy framework of the new Basel accord. 20

Figure 2.3 Schematic representation of bank financial regulations. 27

CHAPTER 3

Figure 3.1 Schematic representation of a bond comprising two cash flows 32

Figure 3.2 The inverse relationship between the price of a bond and its yield to maturity 35

Figure 3.3 First and second order effects used to estimate changes in bond price. 36

Figure 3.4 Actual variation of $\Delta P/P$ with Δy, (ii) $-D_{\text{Mod}} \cdot \Delta y$ only & (iii) $-D_{\text{Mod}} \cdot \Delta y + \frac{1}{2} C \cdot (\Delta y)^2$ .. 37

CHAPTER 5

Figure 5.1 Contingent cash flows used in the pricing of a defaultable corporate bond. 89

Figure 5.2 Inputs and outputs required for defaultable and default-free securities 95

Figure 5.3 (a) through (h) Regression of $-\frac{dP}{dy} \cdot \frac{1}{n \cdot P} \cdot \left(1 + \frac{y}{n}\right)$ on defaultable and default-free Macaulay duration for 1997 through 2004 98

Figure 5.4 Price/yield relationship for an option-free bond, a call option and a callable bond 102

Figure 5.5 Theoretical range of Macaulay duration for a 30-year bond with an embedded option, call able after 10 years 103

Figure 5.6 Time evolution of duration for a callable bond 104

Figure 5.7 Daily, 1-year South African interest rates, measured over 2.5 years 104

Figure 5.8 Summary of South African bond duration data. 106

Figure 5.9 Summary of observed, Macaulay-duration forecast and option-adjusted Macaulay duration forecast price changes, using South African bond data. 107

CHAPTER 6

Figure 6.1 Sequence of T- period rates 116

Figure 6.2 Additive shifts to the yield curve 118

Figure 6.3 (a) Previous period, observed next-period and forecast next-period yield curves using an additive stochastic process. (b) Regression of calculated additive shifts upon observed next-period yield curve.. 120

Figure 6.4 Multiplicative shifts to the yield curve 122

Figure 6.5 (a) Previous period, observed next-period and forecast next-period yield curves using an multiplicative stochastic process. (b) Regression of calculated additive shifts upon observed next-period yield curve.. 123

Figure 6.6 Fisher-Weil shifts to the yield curve 124

Figure 6.7 (a) Previous period, observed next-period and forecast next-period yield curves using a Fisher-Weil stochastic process. (b) Regression of calculated additive shifts upon observed next-period yield curve.. 126

Figure 6.8 Log-additive shifts to the yield curve. 127

Figure 6.9 (a) Previous period, observed next-period and forecast next-period yield curves using a log-additive stochastic process. (b) Regression of calculated additive shifts upon observed next-period yield curve.. 128

Figure 6.10 Log-multiplicative shifts to the yield curve..... 129

Figure 6.11 (a) Previous period, observed next-period and forecast next-period yield curves using a log-multiplicative stochastic process. (b) Regression of calculated additive shifts upon observed next-period yield curve.. 130

Figure 6.12 Evolution in time of the South African yield curve from October 1996 – May 2004... 131

Figure 6.13 Changes in the South African yield curve over the Asian crisis of 1998 and post September 11, 2001 132

Figure 6.14 Example of an inter-week shift in the South African yield curve. This example shows the yield curve for 16 Nov 1997 and 23 Nov 1997. 133

Figure 6.15 Macaulay and log-additive durations over the entire observation period..... 134

Figure 6.16 Actual new bond price after a given yield curve shift, compared with log-additive duration forecast, Macaulay duration forecast and Macaulay duration + convexity adjustment forecast 135

List of Tables

CHAPTER 3

Table 3.1 Sample values for Macaulay and modified duration calculation 38

CHAPTER 4

Table 4.1 Expected cash flows and income (stable interest rates) 52

Table 4.2 Present value of expected cash flows and income (stable interest rates) 52

Table 4.3 Expected cash flows and income – 200bp rise.. 53

Table 4.4 Present value of expected cash flows and income 200bp rise.. 53

Table 4.5 Hypothetical bank balance sheet. All instruments priced at par. 66

Table 4.6 Hypothetical balance sheet: yields to maturity greater than coupon rates (by 1%) 67

CHAPTER 5

Table 5.1 Summary of corporate, option free bond sample results.. 96

Table 5.2 Option-embedded South African bonds grouped by maturity/call period buckets 105

Table 5.3 Recommended durations (in years) for non-maturity deposits 109

Table 5.4 Durations of US money market deposit accounts 111

CHAPTER 6

Table 6.1 Hypothetical balance sheet – base case using log-additive duration 137

Table 6.2 Hypothetical balance sheet – shifted case using log-additive duration and a specific combination of λ and α 137

Table 6.3 Combinations of λ and α that could give rise to yield shifts indicated in Figure 4.1, with corresponding DGaps and ΔMVE's 138

Table 6.4 Comparison of salient features of the Macaulay and log-additive duration measures... 139

CHAPTER 7

Table 7.1 Conclusions and summary 141

Chapter 1

Introduction

1.1 The economic role of banks

Commercial banks play an important role in both the financial system and the economy. As a key component of the financial system, banks allocate funds from savers to borrowers, provide specialized financial services and reduce the cost of obtaining information about both savings and borrowing opportunities. These financial services help to improve the overall efficiency of the economy. Banks also receive, collect, transfer, pay, exchanges, lend, invest and safeguard money for their customers. This broader definition includes many other financial institutions that are not usually thought of as banks, but which nevertheless provide one or more of these broadly-defined banking services. These institutions include finance companies, investment, insurance and mortgage companies, investment banks, pension funds, security brokers, dealers and real estate investment trusts (Cull, 2005:72).

The deposit and loan services provided by banks benefit an economy in several ways. Cheque accounts, because they act like cash, enable the purchase of goods and services and therefore assist both consumers and businesses who would find it inconvenient to transport or transact large cash amounts. Loans enable consumers to improve standards of living by borrowing money to purchase goods and services that would otherwise be unaffordable. Loans also assist businesses finance plant expansion and the production of new goods, and therefore increase employment and economic growth. Finally, since it is in banks' best interests that loans are repaid timeously and in full, borrowers are carefully selected and their subsequent creditworthiness closely monitored. In addition, since the owners (stockholders) of a company receiving a loan desire the company to be profitable and be efficiently managed, bankers act as surrogate observers for stockholders who cannot be present on a regular basis to monitor company managers (Bessis, 2002:6).

Because banks attract large amounts of savings from depositors, many loans – to many different customers in various amounts and for various maturities – are possible. The diversification of loans by banks is therefore promoted and this in turn reduces a bank's overall risk, encouraging even more bank deposits and, therefore, even more loans. This financial intermediation (flow of money from savers through banks to ultimate borrowers) is crucial for the functioning of efficient economies. Understanding, managing and mitigating the risks to which banks are exposed are, therefore, of widespread concern, not only to bank management but to regulatory bodies and governments.

1.2 Banking risk

The acceptance and management of financial risk is inherent to the business of banking and banks' roles as financial intermediaries. To meet customer and community demands and to execute business strategies, banks make loans, purchase securities and take deposits with different maturities and interest rates. These activities leave a bank's earnings and capital exposed to interest rate risk, or movements in interest rates. Changes in banks' competitive environments, products and services have heightened the importance of prudent interest rate risk management. The banking interest rate environment – although relatively stable in the decades following World War II – has now changed as interest rates have become more volatile and banks have become more exposed to such volatility because of the changing character of their liabilities (Santomero, 1997:110). Financial products offered and purchased by banks are becoming more various and complex and many of these pose risk to the bank. Boundaries between financial risks (such as credit, operational, market and interest rate risk) are becoming increasingly blurred and the disparate risks of yesterday are the inextricably intertwined risks of today (Rosenberg, 2004). The structure of banks' balance sheets has also changed. Many commercial banks have increased their holdings of long-term assets and liabilities, whose values are highly sensitive to interest rate changes. Such changes mean that managing interest rate risk is far more important and complex today than it was only a decade ago (de la Torre, 2005:12).

1.3 Interest rate risk

The risk to earnings or capital arising from movement of interest rates (and which take the form of credit risk, prepayment risk, default and market risk) is known as banking interest rate risk (Bessis, 2002:12). These interest rate risks arise from:

- differences between the timing of interest rate changes and timing of the cash flows (repricing risk)
- changing rate relationships among yield curves that affect bank activities (basis risk)
- changing rate relationships across the maturity spectrum (yield curve risk) and
- interest rate-related options embedded in bank products (option risk).

The evaluation of interest rate risk embraces the impact of complex, illiquid hedging strategies or products and also the potential impact on fee income that is sensitive to changes in interest rates. The movement of interest rates affects a bank's reported earnings and book capital by changing the net interest income, the market value of trading accounts (and other instruments

accounted for by market value) and other interest-sensitive income and expenses, such as mortgage servicing fees.

Interest rate changes affect banks' underlying economic value. The value of a bank's assets, liabilities and interest rate related, off-balance sheet contracts are affected by changes in interest rates because the present value of future cash flows (and in some cases, the cash flows themselves) are altered. In banks that manage trading activities separately, the exposure of earnings and capital to those activities – because of changes in market factors – is known as price risk. Price risk (the risk to earnings or capital arising from changes in the value of portfolios of financial instruments) arises from market-making, dealing, and position-taking activities for interest rate, foreign exchange, equity, and commodity markets. The same fundamental principles of risk management apply to both interest rate risk and price risk (Thomas and Wang, 2004:305).

Each financial transaction may affect a bank's interest rate risk profile. Banks differ, however, in the level and degree of interest rate risk they are willing to assume. Some banks seek to minimize their interest rate risk exposure, generally not deliberately taking positions to benefit from a particular movement in interest rates. Rather, they try to match the maturities and repricing dates of their assets and liabilities. Other banks are willing to assume greater levels of interest rate risk and may choose to take interest rate positions or to leave them open. Some banks have attempted to centralise the management of interest rate risk and restrict position-taking to certain 'discretionary portfolios' such as their money market, investment and fixed income portfolios. These banks often use a funds transfer pricing system to isolate the interest rate risk management and positioning in the treasury unit of the bank (Thomas and Wang, 2004:307).

More decentralised approaches are sometimes followed, allowing banks' individual profit centres or business lines to manage and take positions within specified limits (Cebenoyan and Strahan, 2004:32). Some banks choose to confine their interest rate risk positioning to trading activities. Others may choose to take or leave open interest rate positions in non-trading books and activities. A bank may alter its interest rate risk exposure by changing investment, lending, funding, and pricing strategies and by managing the maturities and repricing of these portfolios to achieve a desired risk profile. Many banks also use off-balance-sheet derivatives, such as interest rate swaps, to adjust their interest rate risk profile (Cebenoyan and Strahan, 2004:33).

For banks funded mainly by short-term liabilities, interest rate increases may decrease net interest income at the same time credit quality problems are increasing. When developing a bank's interest rate risk strategy, consideration must be given to liquidity and the ability to access various funding and derivative markets. Banks with ample and stable sources of liquidity may be more able to withstand short-term earnings pressures arising from adverse interest rate move-

ments than those that are heavily dependent on wholesale, short-term funding sources that may cease if earnings deteriorate. Banks that depend solely on wholesale funding may have difficulty replacing existing funds or obtaining additional funds if it has an increasing number of non-performing loans. The ready access of various money and derivatives markets may allow banks to be better able to respond to changing market conditions than those that rely on customer-driven portfolios to alter their interest rate risk positions (Cebenoyan and Strahan, 2004:34).

The nature and complexity of bank's business activities and overall levels of risk determine the sophistication of interest rate risk management. Every well-managed bank, however, will have a process that enables bank management to identify, measure, monitor, and control interest rate risk in a timely and comprehensive manner. The adequacy and effectiveness of a bank's interest rate risk management are important factors in the determination of a bank's level of interest rate risk exposure as the lack thereof poses supervisory concerns or may require additional capital.

1.4 Management of interest rate risk

Interest rates remain key generators of bank profit, and as such the management of interest rate risk cannot be underestimated.

Banks have access to a wide array of financial tools for managing their interest rate risk, such as standard asset-liability management procedures and interest rate derivatives. Banks commonly use one of two approaches when assessing aggregate interest rate risk exposures across their various business lines and portfolios – the traditional earnings approach and the economic value approach. The earnings approach focuses on how interest rate changes affect a bank's overall earnings, which are typically measured as net interest income (the difference between total interest income and total interest expenses). Broader measures that include non-interest income, such as revenue from mortgage servicing activities and expenses have also become more common. This approach examines earnings sensitivity to interest rate fluctuations of different magnitudes (Koch and MacDonald, 2000:186).

The economic value approach offers a longer-term perspective on interest rate risk, taking into account *all* future cash flows generated from a bank's banking book positions. This perspective focuses on how the economic value of all bank assets, liabilities, and interest rate-related, off-balance sheet instruments change with movements in interest rates (Risk Institute, 2005).

Techniques for measuring and managing interest rate risk (as a market risk) have enjoyed decades of relatively unchallenged success in the market place. Credit and operational risk have now begun to attract the attention of quantitative researchers and risk-modellers as corporate accounting scandals and interest rate risk volatility have surged and the deadline for full and final com-

pliance with the new Basel accord regulations draws closer (in 2007). Small incremental interest rate changes over the past year (2004) in the US, UK and Australasia, have done little to cool their over-stretched and over-borrowed economies – fuelled by cheap money and record low interest rates. It is now widely believed that this cannot be sustained and some form of 'correction' is considered inevitable (Miller, 2004). The need for a strategic interest rate management tool that is robust, reliable, accurate and integrates well (on an operational level) with other more 'fashionable' risks has never been greater.

One of the most popular interest rate risk sensitivity measures – the Macaulay duration – is now in its eighth decade of use and has remained remarkably robust despite its application to an ever more complex set of instruments for which it was never originally intended. For example, the Macaulay duration is increasingly unable to cope with financial instruments that suffer the possibility of default. As a result, accurate estimation of price changes for instruments with credit risk is severely obstructed. The burgeoning use of interest rate derivatives embedded in fixed income instruments has also eroded its validity as it is unable to deal with instruments which may be put or called at some time before maturity. It also cannot adequately measure the inherent risk in instruments of perpetual maturity (such as deposit accounts). The numerous assumptions required to justify its use have come under increased criticism recently as the stochastic nature of interest rates, credit (default) events and the exercising of optionality have become better understood (see for example: Fons, 1990; Babbel *et al*, 1997; Duffie and Singleton, 1999; Fisher, 2004 and Rosenberg, 2004).

There are no indications that interest in or use of the Macaulay duration (which remains widely used) shows any signs of abating. As a risk measure for instruments whose value inherently depends on an underlying yield curve, it enjoys relatively unrivalled success. It is a relatively straightforward concept[1] and is easily implemented and applied to both individual instruments and portfolios of instruments. It has survived, virtually unchanged and unchallenged, since its introduction in 1938 (Macaulay, 1938:27). But it is this very immutability that cemented the Macaulay duration firmly into the interest rate risk management arena that has now begun to threaten its survival (Eom *et al*, 2002).

The financial world of 1938 is vastly different to the one ushered in by the new millennium. Tools that were fashionable and reliable in the 1930's must be altered and adapted if they are unable to cope with the highly-evolved, inter-connected and complex contemporary financial environment and discarded completely if these modifications do not yield satisfactory results. The

[1] As with most mathematical concepts in finance, however, both its use and implementation are easily and often misunderstood.

Macaulay duration has undergone only relatively minor adaptations to date; the majority of these have been appended to the existing mathematical infrastructure thereby leaving the original exposition virtually unaltered. Whilst many add-on features have enjoyed varying degrees of success, some underlying requirements of the Macaulay duration – such as the assumption of a flat yield curve – are proving increasingly untenable.

1.5 Problem statement and aims of study

New accounting standards (IAS 39 in January 2005) will soon force financial institutions to account for all derivatives in fair-value terms. New regulatory rules (the new Basel accord in January 2007) will compel banks to determine reserve and maintain regulatory capital for banking book interest rate risk. An increased emphasis on accurate measurement, understanding and interpretation of interest rate risk will inevitably follow. Although still widely used, the Macaulay duration – in its current form – is unable to process interest rate risk associated with embedded optionality (now an almost standard feature of fixed income instruments), credit events (as it plays an ever-increasing role in the fragile and volatile market post Enron (*The Economist*, 2001) and WorldCom (*The Economist*, 2002)), and perpetual maturity (such as a bank's deposit book). A persistent and obdurate problem remains its inadequacy in the accurate forecasting of price changes when large interest rate movements occur (partially rectified by the tedious and complicated convexity correction). It has become clear that the Macaulay duration must either be considerably adapted or discarded completely in favour of a more robust measure.

The major aims of this thesis are to:

1. undertake a literature study of the present academic and practical status of interest rate risk research, and establish the role of the Macaulay duration in this context,
2. provide a background of the historical and mathematical basis of the Macaulay duration measure,
3. review the current **application** of the Macaulay duration to fixed income securities and seek alternative applications in the light of the changing regulatory and accounting standards
4. explore the possibility of **adjustments** to the existing Macaulay duration to take account of defaultable, option-embedded and perpetual maturity instruments' price changes for given interest rate changes,
5. implement and test these adjustments empirically on historical South African data. The results derived from these investigations will be evaluated according to their success in

the accurate estimation of instrument and portfolio price movements (themselves due to underlying yield curve movements which affect probabilities of default and/or the probability of the exercising of optionality),

6. develop **alternative** measures of duration, both mathematical and practical, which allow for, *inter alia*, non-parallel movements in the yield curve. The Macaulay duration alternatives will be evaluated according to their success in the accurate estimation of fixed income price movements due to *actual* changes in the entire yield curve,
7. develop an improved DGap from an *economic* standpoint using the new duration measures and ascertain through testing the efficacy of this economic DGap on the observed Market Value of Equity,
8. indicate the **shortcomings** of the current duration measure and **emphasise** the practical benefits to be derived from new definitions of duration or modifications to the existing Macaulay duration and
9. illuminate the advantages of the new duration measures to financial practitioners in the light of contemporary regulatory requirements.

The above will be placed in the context of alleviating problems associated with the calculation of an economic perspective duration gap. Where possible, practical examples will be used and contextualised in the light of the new accounting (IAS 39) and regulatory (Basel) standards.

1.6 Thesis outline

The Macaulay duration plays an arguably integral part in the many ways to characterize measure and manage interest rate risk. It is used extensively in duration gap analysis, in simulation studies, for hedging and immunisation of portfolios comprising fixed income instruments and has proved to be remarkably robust as an interest rate sensitivity measure. The Macaulay duration suffers severe problems which are now becoming more apparent and difficult to ignore.

This thesis examines various duration measures as instruments for strategic interest rate risk management of a bank, and aims to – if not solve – at least address inherent problems associated with it, contextualize these and point future research in promising directions. The remainder of this thesis is structured as follows:

Chapter 2 sets the scene for the investigation covering aspects of interest rate risk only mentioned in passing in this introductory chapter. The importance of new, soon to be introduced, accounting standards and regulatory guidelines are also examined. The launch of both of these changes will herald renewed concern about the deficiencies of existing interest rate risk measures

and will force institutions to examine both trading and banking book interest rate risk far more closely. The significance of the work conducted is reviewed in the context of the new standards.

The Macaulay duration concept is introduced in Chapter 3. The origins of the measure, its subsequent development and inclusion into the banking and financial world in general are explored here. The measure is examined mathematically and the concept of convexity as a second order correction factor is introduced and evaluated. Whilst these concepts are not new, they provide crucial information to the understanding of the remainder of the thesis. Assumptions employed by the Macaulay duration are examined and subtleties are explored including the measures' limitations and failures. Misunderstandings about duration as an elasticity measure are discussed and portfolio immunisation as a fundamental use is also presented and briefly contextualized here.

Interest rate risk in the banking and trading books is explored in detail in Chapter 4. Risk measurement, monitoring, reporting and control techniques are presented. Distinction between earnings and economic value of a bank – and the role of duration and duration gap in both – is discussed. Interest rate risk models in common use are presented and the regulatory viewpoint is revisited. Shortcomings of the Macaulay duration are examined.

Interest rate sensitivity of financial instruments that are default prone, have embedded optionality or do not mature (i.e., have a perpetual maturity) are not well-described by the Macaulay duration, but recent mathematical advances allow deeper exploration of these now-ubiquitous features. Chapter 5 examines these shortcomings from an application viewpoint and attempts to address them by allowing adjustments to be made to the existing infrastructure.

Many definitions of duration exist, of which the Macaulay duration is arguably the simplest, but not necessarily the best. The yield curve does not shift, for example, in a simple 'parallel' manner, rather, it undergoes stochastic shape changes. If a parameter or parameters can be found that influence the shape of the entire yield curve in a way that is characteristic of observed yield curve changes, duration may be redefined as a rate of change of price with respect to these parameters. Other measures of duration suggested by prominent authors in the 1980's, but never explored further until very recently, are now enjoying some prominence – particularly in the South African market. These descriptions of duration, although not novel, are examined in Chapter 6. The application of the successful models to the measurement and management of interest rate risk – particularly in the potential construction of an improved DGap measure – is also considered from a regulatory viewpoint.

A summary and discussion of the results of this study concludes the thesis in Chapter 7, followed by Appendices A through D and a list of references used in the text.

Chapter 2

Bank risk

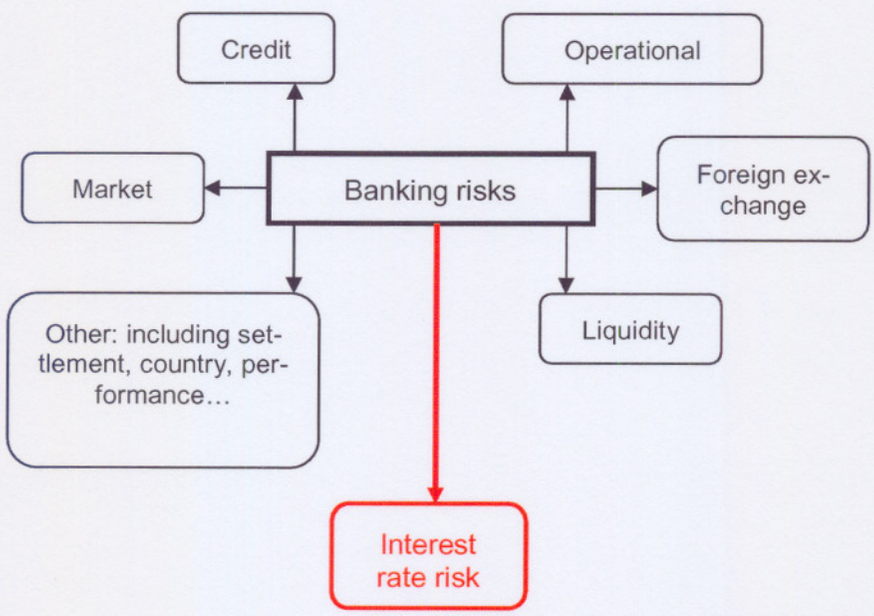
2.1 Introduction

A broad overview of banking risk and interest rate risk in particular was briefly introduced in the previous chapter. The measurement and management of interest rate risk is of paramount importance to any bank wishing to preserve shareholder value and generate profit margins. This chapter discusses bank risk in general and elucidates all of the various risks to which banks are exposed. Interest rates, movements in interest rates and risks associated with these movements are also explored. The different ways in which interest rate risk is made manifest (and hence managed) in the treasury and the trading books are presented in preparation for a more comprehensive overview in Chapter 4.

2.2 Banking risks

Banking risks are defined (Bessis, 2002:7) as those which have the potential to adversely impact the bank's profitability (both accounting and mark-to-market measures). Several distinct sources of uncertainty give rise to these risks. Risk measurement involves the monitoring of the source of this uncertainty and the management of the magnitude of the potential adverse effect on profitability. Since the chief focus of this thesis is interest rate risk, other bank risks will be only cursorily defined. A schematic representation of these risks is summarised in Figure 2.1 below.

Figure 2.1. Main components of bank risk.



(Adapted from Bessis, 2002:8)

The measurement and management of the highlighted risk – interest rate risk – is the principal focus of this thesis.

2.2.1 Credit risk

Credit risk (Cull, 2005:51) refers to the possibility that a borrower will fail to service or repay a debt timeously. The degree of risk is reflected in the borrower's credit rating, which defines the premium over the riskless borrowing rate it pays for funds and ultimately the market price of its debt. Credit risk has two variables: market and firm-specific risk. Credit derivatives allow users to isolate, price and trade firm-specific credit risk by unbundling debt instruments into component parts and transferring each risk to those best suited or most interested in managing it. There are various traditional mechanisms to reduce credit risk including refusal to grant a loan, insurance products, guarantees and letters of credit, but these mechanisms are less effective during periods of economic downturn when risks that normally offset each other simultaneously default and financial institutions suffer substantial loan losses (Cull, 2005:77).

2.2.2 Market risk

Market risk (Bessis, 2002:7) is the risk of loss arising from movements in market variables, including observable variables such as interest and exchange rates and equity prices and others which may be only indirectly observable such as volatilities and correlations. The risk of price movements on securities and other tradable obligations, resulting from general credit and country risk factors and events specific to individual issuers, is also considered market risk.

Market risk is incurred primarily through trading activities and it arises from market-making, facilitation of bank client business and proprietary positions in equities, fixed income and interest rate products, foreign exchange and, to a lesser extent, precious metals and energy (Bessis, 2002:8). Market risk is managed with a short-term focus. Long-term losses are avoided by attempting to prevent losses from one day to the next. On a tactical level, traders and portfolio managers employ a variety of risk metrics, the Greeks and β for example – to assess exposures. These allow for the identification and reduction of excessive exposures.

On a strategic level, banks manage market risk by applying risk limits to traders' or portfolio managers' activities. Increasingly, Value at Risk[2] is used to define and monitor these limits. Scenario generation and stress testing may also be applied to determine market risk (Jorion, 2001:155).

[2] A technique used to estimate the probability of portfolio losses based on the statistical analysis of historical price trends and volatilities (Jorion, 2001). VaR is commonly used by banks, security firms, and companies that are involved in trading energy and other commodities.

2.2.3 Operational risk

Although the definitions of market risk and credit risk are relatively clear, the definition of operational risk has evolved rapidly over the past few years. At first, it was commonly defined (BIS, 2001) as every type of unquantifiable risk faced by a bank. However, further analysis has refined the definition considerably. Operational risk may be defined as the risk of monetary losses resulting from inadequate or failed internal processes, people, and systems or from external events (BIS, 2001).

Losses from external events, such as a natural disaster that damages a firm's physical assets or electrical or telecommunications failures that disrupt business, are relatively easier to define than losses from internal problems, such as employee fraud and product flaws. Because the risks from internal problems will be closely tied to a bank's specific products and business lines, they should be more firm-specific than the risks due to external events.

2.2.4 Liquidity risk

Liquidity risk (Bangia *et al*, 1999:69) is financial risk due to uncertain liquidity. An institution might lose liquidity if its credit rating falls, it experiences sudden unexpected cash outflows, or some other event causes counterparties to avoid trading with or lending to the institution. A firm is also exposed to liquidity risk if markets on which it depends are subject to loss of liquidity (Gibson and Mougeot, 2004:159).

Liquidity risk is an emerging topic in the financial risk world. Adequate liquidity management is crucial as liquidity risk can compound the consequences of other risks. In particular, the effects of credit risk and market risk complications are multiplied when combined with liquidity risk. Conversely, other risks can lead to liquidity risk. For example, credit risk can cause funding problems if counterparties default. The bank in question may be deprived of cash earmarked for current operating expenses and liabilities (Bangia *et al*, 1998).

To manage liquidity risk, banks must be diligent in monitoring potential liquidity. Essential steps include closely noting cash flow, diversifying funding sources and ensuring quick access to liquid assets. Liquidity risk should be estimated under potential stressed market conditions. Stress-testing a portfolio of assets can prepare a firm for possible liquidity problems (Bangia *et al*, 1999:71).

2.2.4.1 Exogenous liquidity risk

Exogenous liquidity risk is the result of market characteristics; it is common to all market players and unaffected by the actions of any one participant (Bangia *et al*, 1999:71). In response to a

market shock (and the resultant loss of predictability), a vicious cycle with a corresponding loss of liquidity is initiated. The perceived need to hold larger prudential reserves in situations of greater uncertainty along with reduced liquidity and leverage may not break the self-reinforcing dynamics of market dislocations. Exogenous liquidity can be affected by the joint action of all or almost all market participants as occurred in several markets in the summer of 1998 (Lowenstein, 2001:110). The market for liquid securities, such as G7 currencies, is typically characterized by heavy trading volumes, stable and small bid-ask spreads, stable and high levels of quote depth. Liquidity costs may be negligible for such positions when marking to market provides a proper liquidation value. In contrast, markets in emerging currencies or thinly traded junk bonds are illiquid and are characterized by high volatilities of spread, quote depth and trading volume.

2.2.4.2 Endogenous liquidity risk

Endogenous liquidity risk in contrast, is specific to the position in the market and varies across market participants (Bangia *et al*, 1998:71). The exposure of any one participant is affected by the actions of that participant. It is mainly driven by the size of the position: the larger the size, the greater the endogenous illiquidity. If the market order to buy/sell is smaller than the volume available in the market at the quote, then the order transacts at the quote. In this case the market impact cost, defined as the cost of immediate execution, will be half of the bid-ask spread. If the size of the order exceeds the quote depth, the cost of market impact will be higher than the half-spread. The difference between the total market impact and half-spread is called the incremental market cost, and constitutes the endogenous liquidity component (Gibson and Mougeot, 2004:164).

2.2.5 Country risk

Country risk refers to the potential volatility of foreign shares, or the potential default of foreign government bonds, due to political and/or financial events in the given country (Moshirian, 2004:272).

2.2.6 Foreign exchange risk

Foreign exchange risk (Bessis, 2002:11) is the risk that the value of an asset or liability will change because of a change in exchange rates. Because these international obligations span different time zones, foreign exchange risk can arise. This risk is made manifest in three important ways:

2.2.6.1 Transaction Exposure

The risk that the domestic cost or proceeds of a transaction may change.

2.2.6.2 Translation Exposure

The risk that the translation of value of foreign-currency-denominated assets is affected by exchange rate changes.

2.2.6.3 Economic Exposure

The risk that exchange rate changes may affect the present value of future income streams (Moshirian, 2004:272).

2.2.7 Interest rate risk

Interest rate risk (Choi and Elyasiani, 1996:268) is identified as the risk that a bank will experience a deterioration in its financial position as interest rates move over time. Banks and regulators typically split interest rate risk into two components: **traded interest rate risk** and **non-traded interest rate risk** (the latter is often referred to as interest rate risk on the balance sheet or in the banking book (Kerkhof and Melenberg, 2004:1853). Both refer to the potential impact of adverse interest rate market movements; the difference lies in terms of where that impact occurs.

2.2.7.1 Traded interest rate risk

Traded interest rate risk is relevant to a bank involved in trading activities and affects, for example, the market value of that a bank's positions in traded interest rate securities, such as government bonds and traded interest rate swaps. Traded interest rate risk is considered a market risk and its measurement and management are therefore governed by the regulatory rules specified in the original Basel accord. These are discussed in Chapter 4.

2.2.7.2 Non-traded interest rate risk

Non-traded, treasury or banking book interest rate risk arises from a bank's core banking activities (deposits and loans). The main source of this type of interest rate risk is repricing risk, which reflects the fact that a bank's assets and liabilities are of different maturities and are priced off different interest rates (Kerkhof and Melenberg, 2004:1851)).

The systems and processes by which a bank identifies and measures risk should be appropriate to the nature and complexity of the bank's operations. Such systems must provide adequate, timely, and accurate information if the bank is to identify and control interest rate risk exposures. Interest rate risk may arise from a variety of sources, and measurement systems vary in how thoroughly they capture each type of interest rate exposure. The nature and mix of a bank's products and activities allow it to establish the measurement system that is most appropriate. A bank's business mix and the risk characteristics of these businesses must be understood before a

bank attempts to identify the major sources of its interest rate risk exposure and the relative contribution of each source to the overall interest rate risk profile. Various risk measurement systems may then be evaluated by how well they identify and quantify the bank's major sources of interest rate risk (Fisher, 2004:30).

The manifestation of interest rate risk, specifically within the two books of the bank, is considered in the next section.

2.3 Banking versus trading books

In a well functioning risk management system, banks broadly position their balance sheet into banking and trading books. The main difference between these two segments is that the 'buy and hold' philosophy prevails for the banking book, contrasting with the trading philosophy of capital markets and it is this (and other) differences between the two books that underlies the rationale for their separation.

2.3.1 The banking book

The banking book is the repository for most conventional banking transactions (loans and deposits). The banking book comprises assets and liabilities, which are contracted basically on account of relationship or for steady income and statutory obligations and are generally held till maturity. Accounting rules for the banking book use accrual accounting of revenues and costs, and rely almost entirely on book values for assets and liabilities.

Asset and liability management applies to the banking portfolio and focuses on both interest and liquidity risks. All assets and liabilities generate accrued revenues and costs – a large fraction of which are interest rate driven. Maturity mismatches between assets and liabilities result in fund excesses or deficits. These mismatches exist in the banking book balance sheet. Financial transactions in capital markets manage such mismatches between commercial assets and liabilities through investment of excess funds or in long-term debt. Whilst the asset side of the banking book also generates credit risk (in addition to interest rate risk), the liability side does not since lenders and depositors are at risk with the bank itself. No market risk is associated with the banking book (Uyemura and van Deventer, 1992).

2.3.2 The trading book

The trading book is a proprietary portfolio for financial instruments held by an institution in its capacity as a dealer. Assets in the trading book are held primarily for generating profit on short-term differences in prices/yields. Market transactions, therefore, are not subject to the same management rules as those in the banking portfolio – since the turnover of tradable positions is vastly

accelerated in the trading portfolio. Earning are profits and losses equal to changes in the mark-to-market values of traded instruments.

The trading portfolio generates market risk – broadly defined (Jorion, 2001:145) as the risk of adverse changes in market values over a given liquidation period – and market liquidity risk (the risk that the transaction volume narrows to such an extent that price movements are triggered by trades themselves). Many market transactions involve non-tradable instruments, often over the counter derivative instruments such as swaps or options, some of which might have long maturities (Jorion, 2001:141).

This section presented the gamut of contemporary risks faced by banks and concentrated on one particular risk – interest rate risk – as a key risk that banks must manage. The manner in which this interest rate risk is managed and measured is different in the banking and trading books (in both manifestation and regulatory treatment) and forms the backbone of this thesis.

The important role of regulatory rules and the speed at which these are changing in the current environment is considered in the following section.

2.4 Regulations governing interest rate risk

The financial world is changing more rapidly today than it has at any other time in the past. This is due to a multitude of factors such as decreasing profit margins from traditional investment vehicles, heightened global awareness of trading possibilities (e.g., over the internet), record-low international interest rates (2004-5), the emergence of China as a global financial participant, the decline of the US dollar as the budget deficit widens, amplified mathematical complexity and understanding of financial instruments and the ever-increasing computerisation of the trading function itself (*The Economist*, 2004). Associated with this vastly augmented participation in the marketplace has been a highly electronic, yet potentially detached and impassive approach to the important role of the profit quest. Computers monitor and alert traders to bargains or arbitrage opportunities on a world-wide scale with disarming accuracy. The global nature of modern computing, however, allows *all* participants to share the same speed of processing data and information so arbitrage opportunities close almost as quickly as they open. Elaborately-constructed financial instruments (designed to squeeze unexploited opportunities for smaller and smaller profits) and clever accounting adaptations (constructed to obscure information – often illegally) have burgeoned (Kerkhof and Melenberg, 2004:1860).

The past decade (1995 – 2005) has witnessed this greatly accelerated and complex market flooded by participants eager for profit, but often unenthusiastic about education, protocol and regulation. Regulatory authorities struggled to keep up with the shifting financial environment

and, although far from idle, had not addressed several glaring gaps in directives aimed at reducing risk. Calamitous consequences were – and remain – inevitable. It became apparent that pricing and risk management of financial instruments were both often little-understood. Existing risk management techniques, which had performed admirably in the undemanding past, were now becoming obsolete and unable to contend with the new financial complexity. Far from delivering never-ending profits, resourceful accounting techniques precipitated first an enormous loss of wealth (Enron (*The Economist*, 2001) and Worldcom (*The Economist*, 2002) in the US and Parmalat in Italy (Accountancy Age, 2005), to name but three such incidents) and then a bout of regulatory activity. Authorities were hurriedly mobilised to seal leaks in what had become an unacceptably dangerous environment. Having endured the consequences of years of regulatory neglect, the financial world will (January 2005 through 2007 and beyond) be forced to comply with directives that significantly enhance investor confidence by reducing mispricing and improving transparency in both risk management and financial reporting (Bauer and Ryser, 2004:344).

There are two key differences in the way interest rate risk is dealt with (one from an accounting viewpoint and one from a regulatory viewpoint) in the trading and banking books:

- Adequate capital is required, at present, only for traded interest rate risk in the trading book[3], but this will change in 2007, when both books will be required to make provision for capital reserves.
- Loans and deposits are reflected on the balance sheet of the banking book, whilst all traded derivative instruments were recorded 'off balance sheet'. This changed in January 2005 with the introduction of new accounting standards[4] which now require traded derivative instruments to be recorded at fair value, 'on balance sheet'.

The next section tracks, *inter alia*, the development of the regulatory milieu both pre- and post the period 2003 – 2007 and discusses the significant and sweeping changes that have transformed (and will attend transformations in) the trading and banking books of financial institutions. The ramifications of the research and results – presented in this thesis – will be argued in the light of these developments in Chapters 5 and 6.

2.4.1 Regulatory standards

Interest rate risk has historically been associated primarily with financial products held in the trading books of banks. However, in the current (2005) environment, it is recognized that the

[3] The quantity of capital is governed by the Basel Accords – see Section 2.4.1.
[4] See Section 2.4.2.

majority of the interest rate risk incurred by banks is in fact based in the banking book. These risks arise from products such as guaranteed deposits and mortgages (OSFI, 2005).

Low interest rate environments, such as those recently experienced, are conducive to banks generating profits by assuming interest rate risk. Value is created when long-dated loans are funded by short-term obligations, but problems arise when short-term interest rates rise, thereby compressing the 'spread' that banks earn between their assets and liabilities. Interest rate risk is a potential source of systemic risk to the financial system unless it is adequately measured and contained by banks.

2.4.1.1 Capital adequacy

Capital adequacy currently refers (BIS, 1988) specifically to the market risk aspect of interest rate risk, or the interest rate risk associated with the trading book and it is one of the most important outcomes of any regulatory review or audit. Reserves lodged with central banks earn no interest: banks therefore strive for the best reviews possible. Current requirements for market Value at Risk are stringent even in a best case scenario, but reserves increase with increasing bank 'riskiness'. These requirements and regulatory concerns are addressed in this section.

Regulators expect all banks to maintain adequate capital for the risks they undertake. The regulator's risk-based and leverage capital standards establish minimum capital thresholds that all banks must meet. Many banks may need capital above these minimum standards to adequately cover their activities and aggregate risk profile. When determining the appropriate level of capital, bank management considers the level of current and potential risks its activities pose and the quality of its risk management processes. Regulators also evaluate whether banks have an earnings and capital base that is sufficient to support their level of short- and long-term interest rate risk exposures and the risk those exposures may pose to their future financial performance (Peura and Jokivuolle, 2004:1819). The following factors are also considered:

- **The strength and stability of a bank's earnings stream and the level of income a bank needs to generate and maintain normal business operations.** A high level of exposure is one that could, under a reasonable range of interest rate scenarios, result in a bank reporting losses or curtailing normal dividend and business operations. In such cases, bank management must ensure that it has the capital and liquidity to withstand the possible adverse impact of such events until it can implement corrective action, such as reducing exposures or increasing capital (Kerkhof and Melenberg, 2004:1853).
- **The level of current and potential depreciation in a bank's underlying economic value due to changes in interest rates.** If a bank has significant unrealized losses in its assets be-

cause of interest rate changes (e.g., depreciation in its investment or loan portfolios), the regulators evaluates the impact such depreciation would have (if recognized) on the bank's capital levels and ratios. In making this evaluation, regulators consider the degree to which the bank's liabilities or off-balance sheet positions may offset the asset depreciation. Such offsets may include non-maturity deposits that bank management can demonstrate represent a stable source of fixed rate funding. Alternatively, a bank may have entered into an interest rate swap contract enabling it to pay a fixed interest rate and receive a floating interest rate. This type of swap contract essentially transforms a bank's floating rate liabilities into a fixed rate source of funds (Hess and Laisathit, 1997:141).

Regulators consider banks to have a high level of exposure if their current or potential change in economic value (based on a reasonable interest rate forecast) would result in capital ratios falling below the 'adequately capitalized' level for prompt, corrective action purposes. This situation often requires additional supervisory attention. At a minimum, bank management should have in place contingency plans for reducing exposures, raising additional capital, or both (Peura and Jokivuolle, 2004:1820).

- **Bank's exposure to other risks that may impair capital.** Consideration must be given to the entire risk profile of a bank relative to its capital.

The regulatory attitude to interest rate risk continues to evolve, with constant input from practitioners 'in the field' and is thus a fluid, highly adaptable and unbounded set of guidelines. The next section traces the evolution of both relevant accounting standards and the Basel accords, both of which are central to the soon to be adopted (2005 through 2007) alterations to the way in which banks conduct business.

2.4.1.2 Overview of the Basel accords

As financial complexity has increased, so the management of interest rate risk has attracted more and more attention of regulatory bodies. The direct and measured result of this regulation is reflected in a bank's capital adequacy, currently governed by the Basel accord of 1988 ('Principles for the Management of Interest Rate Risk') and composed by the Basel Committee on Banking Supervision (BIS, 1988). The original Basel accord, agreed upon in 1988, strengthened the soundness and stability of the international banking system as a result of higher capital ratio requirements. A second accord, the new Basel accord, which has been drafted, discussed, revised and finalised, is essentially a revised version of the original (1988). The new accord, which is due for implementation in January 2007, strongly emphasizes the need for sophisticated risk management methods and tools in the area of asset and liability management. The revision aims

to make the 1988 framework more risk sensitive and representative of modern banks' risk management practices (BIS, 2004). Banks not only need to implement these new Basel requirements, but also require skilled resources and sophisticated tools to cope with an increasing complexity in financial markets.

This revised capital adequacy framework will further reduce the probability of consumer loss or market disruption as a result of prudential failure. It will do so by seeking to ensure that the financial resources held by a bank are commensurate with the risks associated with the business profile and the control environment within the bank. There are four main components to the new framework:

- it is more sensitive to the risks that face financial institutions,
- it includes an explicit measure for operational risk and includes more risk sensitive risk weightings against credit risk,
- it reflects improvements in financial institutions' risk management practices by the introduction of the internal ratings based approach that allows institutions to rely on their own risk estimates and
- it provides incentives to improve risk management practices, with more risk sensitive risk weights as firms adopt more sophisticated approaches to risk management.

The new Basel accord, which will directly affect banks and building societies and certain types of investment firms, rests on three mutually reinforcing pillars: (1) minimum capital requirements, (2) supervisory review, and (3) market discipline – see Figure 2.2.

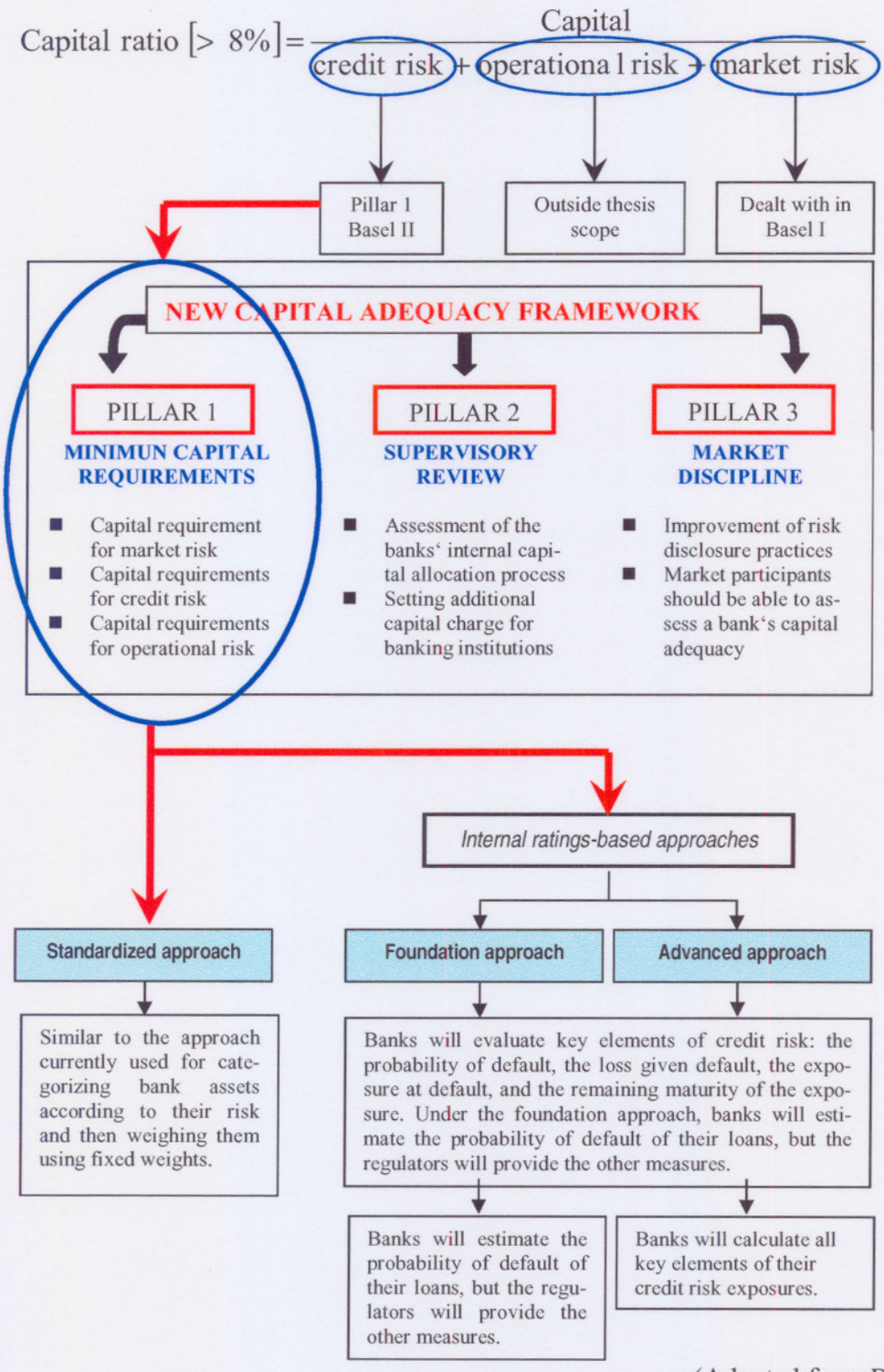
Pillar 1 outlines the calculation procedures of the capital requirements for banking organizations. Under Basel I, the minimum required capital ratio (set at 8%) is calculated as the regulatory capital divided by the risk exposure (measured by the risk-weighted assets). The difference under the new Basel accord will be that the risk exposure will be evaluated as the total of the credit risk, market risk, and operational risk exposure of the bank, where more refined measures will be incorporated to calculate credit and operational risk.

Pillar 2 addresses the supervisory review process in ensuring sound capital management and comprehensive assessment of the risks and the capital adequacy of the banking institutions. This pillar seeks to increase the transparency and accountability of the banking system and to a large extent has already been incorporated in the United States.

Pillar 3 aims at improving market discipline by requiring banks to publicly disclose key information regarding their risk exposures and capital positions. Because the new Basel accord gives

banking institutions greater discretion in calculating their own capital requirements, it is anticipated that the disclosure statements will allow market participants to better assess the safety and soundness of the banking environment and thus exert stronger market discipline.

Figure 2.2. The new capital adequacy framework of the new Basel accord.



(Adapted from BIS, 2004)

The new Basel accord will include three options for measuring credit risk and another three options for measuring operational risk. The options for calculating credit risk are the standardized approach and two internal ratings-based approaches – the foundation approach and the advanced approach (BIS, 2004). The standardized approach is similar to the approach currently used for categorizing bank assets according to their risk and then weighing them using fixed weights. Under the internal ratings-based approaches, banks will evaluate key elements of credit risk: the probability of default, the loss given default, the exposure at default, and the remaining maturity of the exposure. Under the foundation approach, banks will estimate the probability of default of their loans, but the regulators will provide the other measures. Finally, under the advanced approach, banks will calculate all key elements of their credit risk exposures (Tikam, 2005).

At present, South Africa's supervisory regime (the Bank Supervision Department of the South African Reserve Bank – SARB) requires that banks hold capital against the interest rate risk arising from taking trading positions in interest rate markets (i.e., interest rate risk inherent in banks' trading books). Banks are not, at present, required to hold capital against interest rate risk in the banking book. This will change going forward, however, as the new accord provides, *inter alia*, the SARB and other national banking supervisors with the discretion to require capital to be held against interest rate risk in the banking book. The Basel Committee is of the opinion that interest rate risk in a bank's balance sheet is a potentially significant risk and must be incorporated into the capital framework. Interest rate risk in the banking book will be dealt with under Pillar 2 of the new accord (SARB, 2005).

The proposed new accord recognises banks' internal systems as the principal tool for monitoring risk. Where bank's own internal systems are not appropriate, a prescribed method for calculating this capital charge is proposed. Each method, own or prescribed, will require a bank to determine the potential decline in the net present value of future cash flows (i.e., the bank's economic value or market value of equity) if its balance sheet were subjected to an interest rate shock – taking the form of a yield curve shift based on either a specified parallel shift or a statistical approach using a specified confidence interval, holding period and observation period. From these results, the regulator can determine whether the bank should reduce its risk profile and/or hold additional capital against the interest rate risk which it is running on the balance sheet (SARB, 2005 and IAS Board, 2004:32).

Traditionally, regulatory attention to interest rate risk has focused on the trading book. As a result, there has been insufficient guidance regarding policies and procedures for dealing with interest rate risk in the banking book (OSFI, 2005). The new Basel accord's ALM rules focus on 14

principles of which 13 are applicable independent of whether the positions are part of the trading book or reflect the banks' non-trading activities (BIS, 2004).

Accounting standards – which historically only affected the banking book since traded instruments were recorded 'off balance sheet' – have also undergone significant revision in the light of the recent accounting scandals at Enron (*The Economist*, 2001) and Parmalat (Accountancy Age, 2005). These new changes – which require that all instruments move from 'off' to 'on' the balance sheet and all derivative instruments[5] are recorded at fair value – will have a significant impact on the interest rate risk milieu and are introduced in the next section.

2.4.2 Accounting standards

Accounting rules for banks in the European Union (IAS Board, 2004) distinguished between financial instruments held for trading purposes (in the trading book) and those intended to be held to maturity (in the banking book) up until December 2004. Instruments held in the trading book were valued at market (fair value) prices. A profit or loss arising from the revaluation of trading book instruments was recognised in the profit and loss account. The accounting rules for the trading book thereby takes all market risks (i.e., price risk, interest rate risk, foreign exchange risk and liquidity risk) into account. Banking book instruments, by contrast, were recorded on the balance sheet at the lower of historical cost and market value. Whereas a loss on a banking book instrument was transferred to the profit and loss account, unrealised gains were not recognised and could therefore become hidden reserves in the balance sheet. Therefore, the accounting rules for the banking book did not take market risks into account (except for the foreign exchange risk, where the end-period value is usually applied to almost all balance sheet items).

New accounting standards, introduced in January 2005 have been initiated mainly by the US, but have been widely embraced by the international financial community in anticipation of ever increasing accounting audit scrutiny. These new standards go by different names around the world, depending on the local accounting standards board. The US accounting standards are grouped under FAS 133 (FAS, 2004), Europe and the UK collectively incorporate these in 'IAS 39' (for International Accounting Standards: Article 39 (IAS Board, 2004:22)) whilst in South Africa these are grouped by the SARB under 'AC 133' (AC 133, 2004). Whilst known by different monikers, the initials refer to essentially the same new standards. The differences that do exist between AC 133 and IAS 39 are expected to narrow over the coming months (KPMG, 2004).

[5] It is a requirement of IAS 39 accounting standards that instruments used specifically for *hedging* demonstrate – within a range of acceptable values – the hedge effectiveness of the hedging portfolio.

The standards will impact AC 133 models, processes and data requirements and must be addressed as South African banks move towards reporting under International Accounting Standards. The International Accounting Standards Board also proposed a series of changes to IAS 39 during 2004 which will impact South African banks in 2005 and beyond.

A brief history of the evolution of IAS 39 (and all its various prior guises), covering the reasons for its existence and the scope of its influence, is presented in the next section.

2.4.2.1 History and evolution of IAS 39

Note: All Exposure Drafts, discussion papers, SIC papers, IAS drafts and amendments thereto, referred to in this section (Section 2.4.2.1), are available in condensed format from the report issued by the Office of the Superintendent of Financial Institutions (OSFI, 2005).

The US Exposure Draft E26: 'Accounting for Investments' document became available to deal, in part, with the burgeoning use of derivatives in financial institutions as investment instruments in October 1984. Amendments to E26: 'IAS 25 Accounting for Investments', were released in March 1986 with an effective date of 1 January 1987. September 1991 saw the announcement of Exposure Draft E40: 'Financial Instruments', later modified in January 1994 and re-exposed as Exposure Draft E48: 'Financial Instruments'. The disclosure and presentation portion of E48 was adopted as IAS 32 and published in June 1995. Work on recognition and measurement continued and, in March 1997, the discussion paper 'Accounting for Financial Assets and Financial Liabilities' was circulated amongst affected parties for approval. Exposure draft E62: 'Financial Instruments: Recognition and Measurement', announced in June 1998 saw further refinements to the standards.

SIC (Standards Interpretations Committee) 33 was issued on 12 September 2001 and approved by the IASB (International Accounting Standards Board) in December 2001. SIC 33 was superseded by IAS 27 and by IAS 28 (both revised in 2003) and became effective for annual financial reporting periods beginning 1 January 2005. SIC 33 addresses whether the proportion allocated to the parent and minority interests in preparing consolidated financial statements, and the proportion allocated to an investor that accounts for its investment in an associate using the equity method, should be determined based on present ownership interests or ownership interests that would be held if the potential voting rights were exercised or converted. Under SIC 33, the proportion allocated should be determined based solely on present ownership interests. An enterprise may, in substance, have a present ownership interest when for example, it sells and simultaneously agrees to repurchase, but does not lose control of, access to economic benefits associated with an ownership interest. In this circumstance, under SIC 33 the proportion allocated

should be determined taking into account the eventual exercise of potential voting rights and other derivatives that, in substance, presently give access to the economic benefits associated with an ownership interest.

The first appearance of 'IAS 39 Financial Instruments – Recognition and Measurement' occurred in December 1998 and in April 2000 IAS 25 was withdrawn following the approval of 'IAS 40 Investment Property'. Limited revisions to IAS 39 (effective 1 January 2001) were released in October 2000.

As discontentment about the failure of accounting standards to reflect derivative instruments' fair value grew and as the magnitude of corporate scandals became apparent, the necessity for further, far-reaching amendments to be made, arose. On 21 August 2003 a further Exposure Draft entitled 'Fair Value Hedge Accounting for a Portfolio Hedge of Interest Rate Risk' was issued for public comment. On 17 December 2003, the revised version of IAS 39 was issued by the IASB and the revised version released on 31 March 2004 IAS 39 to reflect Macro Hedging. The effective date for the implementation of IAS 39 was 1 January 2005[6].

IAS 39 requires recognition of a financial asset or a financial liability when, and only when, the entity becomes a party to the contractual provisions of the instrument, subject to the following provisions in respect of regular way purchases (IAS Board, 2004:14):

- a regular way purchase or sale of financial assets is recognised and derecognised using either trade date or settlement date accounting,
- the method used is to be applied consistently for all purchases and sales of financial assets that belong to the same category of financial asset as defined in IAS 39 (note that for this purpose, assets held for trading form a different category from assets designated at fair value through profit or loss) and
- the choice of method is an accounting policy (IAS Board, 2004:38).

In addition, IAS 39 requires that all financial assets and all financial liabilities be recognised on the balance sheet – this includes all derivatives. Historically, in many parts of the world, derivatives have not been recognised on company balance sheets. The argument has been that at the time the derivative contract was entered into, there was no amount of cash or other assets paid. Zero-cost justified non-recognition, notwithstanding that as time passes and the value of the un-

[6] It is important to note that 'IAS 27 Consolidated Financial Statements and Accounting for Investments in Subsidiaries', 'IAS 28 Investments in Associates', and 'IAS 39 Financial Instruments: Recognition and Measurement' should all be considered together (OSFI, 2005).

derlying variable (rate, price, or index) changes, the derivative has a positive (asset) or negative (liability) value.

Initially, financial assets and liabilities should be measured at fair value (including transaction costs, for assets and liabilities not measured at fair value through profit or loss). Subsequently, financial assets and liabilities (including derivatives) should be measured at fair value, with the following exceptions (IAS Board, 2004:46):

- loans and receivables, held-to-maturity investments, and non-derivative financial liabilities should be measured at amortised cost using the effective interest method,
- investments in equity instruments with no reliable fair value measurement (and derivatives indexed to such equity instruments) should be measured at cost. Financial assets and liabilities that are designated as a hedged item or hedging instrument are subject to measurement under the hedge accounting requirements of the IAS 39. Financial liabilities that arise when a transfer of a financial asset does not qualify for de-recognition, or that are accounted for using the continuing-involvement method, are subject to particular measurement requirements,
- fair value is the amount for which an asset could be exchanged, or a liability settled, between knowledgeable, willing parties in an arm's length transaction. IAS 39 provides a hierarchy to be used in determining the fair value for a financial instrument (IAS Board, 2004:30),
- quoted market prices in an active market are the best evidence of fair value and should be used, where they exist, to measure the value of the financial instrument. If a market for a financial instrument is not active, an entity establishes fair value by using a valuation technique that makes maximum use of market inputs and includes recent arm's length market transactions, reference to the current fair value of another instrument that is substantially the same, discounted cash flow analysis, and option pricing models. An acceptable valuation technique incorporates all factors that market participants would consider in setting a price and is consistent with accepted economic methodologies for pricing financial instruments,
- if there is no active market for an equity instrument and the range of reasonable fair values is significant and these estimates cannot be made reliably, then an entity must measure the equity instrument at cost less impairment,
- amortised cost is calculated using the effective interest method,

- the effective interest rate is the rate that exactly discounts estimated future cash payments or receipts through the expected life of the financial instrument to the net carrying amount of the financial asset or liability and
- financial assets not carried at fair value though profit and loss are subject to an impairment test and if the expected life cannot be determined reliably, the contractual life is used instead.

This concludes the historical developments of the US Exposure Draft E26 through its inception in 1984 to its current manifestation as IAS 39 internationally and AC 133 in South Africa. The impact of AC 133 in the South African context is discussed in the next section.

2.4.2.2 AC 133 requirements – South Africa

The introduction of AC 133[7] precipitated significant volatility as a greater number of financial instruments are now recorded at fair value with the changes in fair value reflected either in the income statement or directly within reserves. Although the impacts on the most recent year's results are sometimes difficult to glean from the financial statements, they are significant. First-Rand, for example, recognised R136 million additional profits in the 2003 year due to AC 133, together with an increase in reserves of R643 million (SAICA, 2005).

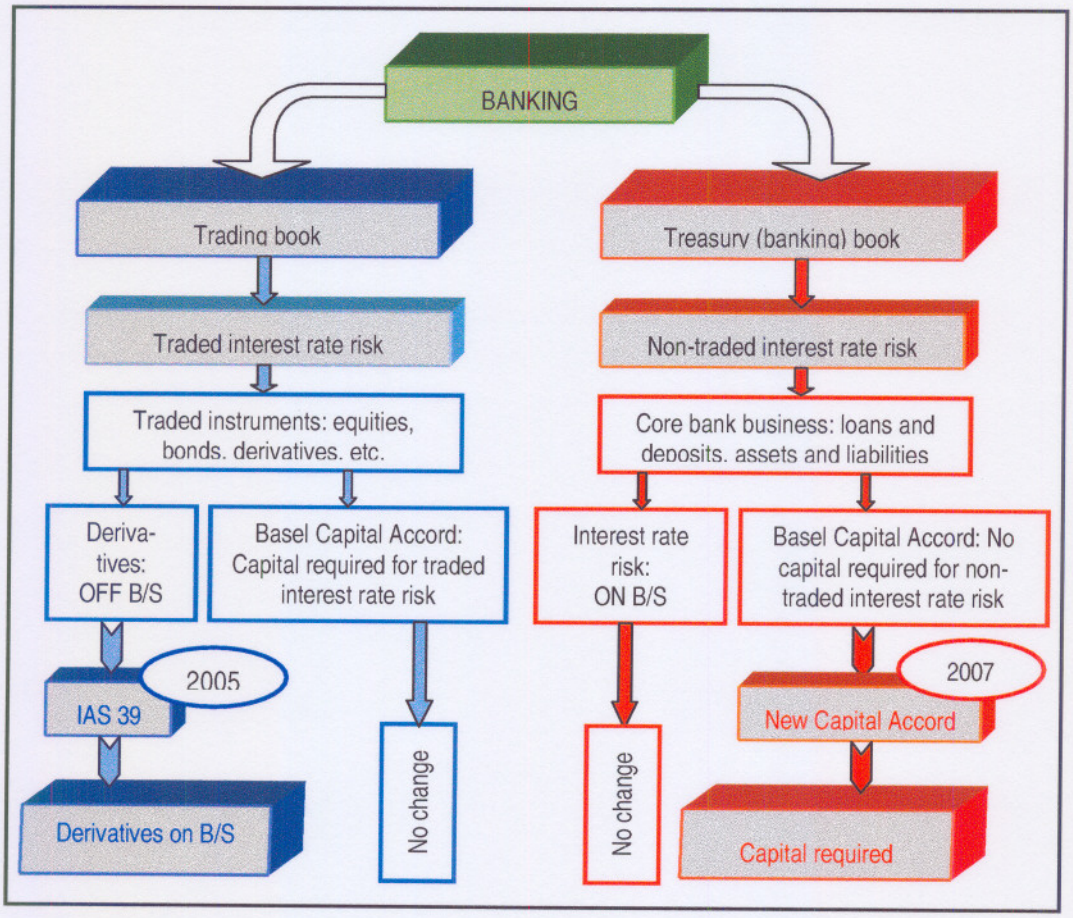
AC 133 has posed a number of operational challenges to banks because of its increased focus on financial modelling. Other than the increased need for fair valuation models for the accounting purposes, AC 133 also imposes strict rules around hedge accounting and loan-loss provisioning. These rules have necessitated the development of complex financial models for accounting purposes, with material data requirements.

The approach of the local banks to implementing the requirements of AC 133 has tended to be tactical in nature, with AC 133 models and processes remaining outside of the existing reporting infrastructure (AC 133, 2004). This approach is consistent with the trend currently emerging from Europe and the UK which stems largely from the short timeframe within which the banks have had to implement the international equivalent of AC 133 (IAS 39), and the fact that the final rules remain a moving target. The trend will have to become more strategic, however, since over the next two years (from 2005) local banks will have to increase the sophistication of their AC 133 models and increasingly embed the models, processes and data requirements into their businesses (KPMG, 2004).

[7] AC 133 requires that all financial assets and all financial liabilities be recognised on the balance sheet – this includes all derivatives.

The onerous requirements of AC 133 come at a time of wide sweeping change in the local financial services sector. Implementation of AC 133 is running in parallel with other major change programmes, including the implementation of the new Basel accord. Figure 2.3 below provides a summary of the major changes facing banks' trading and banking books in the near term.

Figure 2.3. Schematic representation of bank financial regulations both pre- and post the inauguration of new rules.



(Adapted from BIS, 2004)

This landscape has imposed strain on banks' internal resources and resulted in significant costs associated with system and process conversions. It is thus clear that banks must acquire a deeper understanding of their off-balance sheet interest rate risk. Hedge accounting and the effectiveness of hedging strategies in particular, must adapt to measure interest rate risk more accurately than ever before if banks and financial institutions are to avoid unexplained earnings volatility, regulatory/legal penalties and even possible credit rating downgrades. The necessity of timely and accurate pricing of interest rate-sensitive instruments in general (but particularly those with embedded optionality or in which the probability of default is non-negligible) is becoming increasingly important (SAICA, 2005). Some required amendments to the existing Macaulay duration measure, which take account of precisely these characteristics, are explored in Chapter 5.

2.4.3 Summary of regulatory and accounting changes impact

Banks and other financial institutions have been under the regulatory spotlight for several decades. The Basel accord of 1988 (and versions thereof) established rules for regulatory capital for banks' trading books. As a large component of market risk, trading interest rate risk has thus been effectively dealt with by the 1988 Basel accord (BIS, 1988). The new Basel accord (BIS, 2004) adds little to the measurement, management or understanding of interest rate risk in the *trading book* from a *market risk* perspective. The interest rate risk associated with the banking book, however, has hitherto been largely ignored by the regulator and the new Basel accord now addresses this breach. By January 2007, all banks must comply with these regulations or face penalties and fines imposed by local regulatory authorities.

Regulations governing trading interest rate risk are mandatory and capital requirements are regularly reported and audited. However, risks associated with derivative instruments in the trading book have not been historically recognised in that these risks have been 'off balance sheet'. The new accounting standards (FAS 133, IAS 39, AC 133) are now set to change this, with on-balance sheet reporting becoming an obligatory requirement and the requirement that derivative instruments be recorded at fair value.

2.5 Conclusion

This chapter provided a broad overview of banking risk in general and interest rate risk in particular. The current situation regarding interest rate risk measurement and management was explored from the viewpoint of the banking and trading books. Regulatory and accounting standards have already undergone and continue to undergo severe alterations to the way in which interest rate risk is measured, reported and managed, and thus these perspectives were also examined.

Although still extensively used after some eight decades since its conception, the Macaulay duration – an important interest rate risk measure – continues to demonstrate its inability to cope with a significantly advanced financial market place. Chapter 3 introduces the Macaulay duration and later chapters then suggest both potential amendments and renewed definitions. The introduction of these might promote renewed interest in the measure as early results are encouraging.

Chapter 3

Overview of the Macaulay and associated duration measures

3.1 Introduction

The field of interest rate risk measurement and management has many tools at its disposal, developed and tested over many years of academic research and practical application. This chapter focuses on one particular aspect of the wide and deep field of interest rate risk, namely duration in its various guises. The fundamental concepts underlying the measurement of duration are rooted in financial mathematics. The duration measure itself has survived for many decades more or less unaltered – in spite (or because) – of its relative simplicity. The acceleration of increasing financial complexity has imposed a burgeoning of corrective adaptations. Many financial measures are inadaptable to the new environment, whilst the kudos of others – such as the Sharpe ratio and Macaulay duration – remain securely intact, if constantly tailored to suit the complex, volatile new millennium. For all its limitations and limiting assumptions, the Macaulay duration has survived relatively unchanged from its original definition. That it has allowed constant adaptations and modifications to accommodate financial instruments of increasing complexity is an indication of its robust flexibility.

The important role of interest rate risk in the wider arena of banking risk was established in the previous chapter as well as the increasing regulatory and accounting pressure to accurately measure interest rate risk in the banking book. An exploration of the historical development of the duration concept, including its successes and failures, limitations and new adaptations as well as the reasons behind these, will be explored in this chapter. This provides an important and necessary background to a deeper, more in-depth investigation into other interpretations of 'duration' as well as substantial amendments (with the aim of improving interest rate risk measurement) which follow in proceeding chapters. It is important to note that the literature on this subject and its variants is vast. This chapter, therefore, does not provide an exhaustive overview.

3.2 Historical overview

Macaulay (1938:31) was the first to use the word 'duration' as the measure of the average life of a security. After rejecting a future value weighting model as inappropriate, Macaulay found that weighting each time period by the proportion of each payment, in *present value* terms, to the present value of the total payment stream resulted in a measure of duration exhibiting the desired

properties[8]. In particular, while the maturity of a loan only considers when the *final* payment occurs, duration assigns weights to the time periods when each cash payment occurs thereby considering the timing *and* size of the payments. This property of duration is important because it measures the effective life of a payment stream rather than the maturity of the stream. In this way, Macaulay's duration can be considered as expressing the 'time element of a loan' more precisely than does maturity. It is not, however, the only definition of duration.

Hicks (1946:44) formulated an independent definition of duration, calling it instead the 'average period'. Hicks' definition of duration was as an elasticity of the bond price with respect to the discount factor $(1+r)^{-t}$ and he showed that changes in the interest rate r at time t would not affect the relative prices of two securities if both had the same duration. Samuelson (1945:22) was concerned with the ability of institutions to meet their long-term obligations in the face of continuing low interest rates to be received on their investments. He pointed out that if the Macaulay duration of the assets were equal to the Macaulay duration of the liabilities, the institution's portfolio would be 'immune' to the effects of interest-rate changes.

Redington (1952:307) analysed portfolio immunisation independently and in far greater detail. He suggested that 'first-order immunisation' was achieved when Samuelson's condition was met[9]. Perks (1952:324), in discussing Redington's paper, suggested that it was equally appropriate to immunise a portfolio at a zero interest rate. This was the same as an alternative measure of duration that Macaulay (1938:33) had rejected earlier (Altman, 1968:600).

Fisher and Weil (1971:418) extended Redington's work and in doing so noted the usefulness of the duration measure in the development of immunisation strategies for bond investments. Fisher and Weil examined the overall tracking error[10] of immunizing a par bond with a zero coupon bond (or a pair of bonds) that had the same Macaulay duration versus immunizing it with a zero-coupon bond due at maturity. They found that the absolute tracking error was almost always less with the duration-matched portfolios than for the maturity-matching for 5, 10, and 20 year overlapping time horizons during the years 1926-1968. Macaulay had used the yield of a security to estimate the present value of each of its payments while Fisher and Weil used a variant, namely the spot interest rate on each payment period. Fisher and Weil's work led to the development of

---

[8] A change in underlying interest rate results in a corresponding change in the bond price. The factor which, when multiplied by the change in the interest rate, gives the change in the bond price was called the Macaulay duration and exhibited Macaulay's requisite properties.

[9] Redington (1952:292) suggested further that immunisation would be more accurate if a second condition were met: namely, that the variance of the time to payment of the assets be set equal to or greater than the variance of the time to payment of the liabilities.

[10] Tracking error is an important supplementary factor in the consideration of interest rate (and other) risk measures. An exposition at this point detracts from the main argument of the Chapter and is thus presented instead in Appendix B. See also McLeod and van Vuuren (2004).

the first bond immunisation strategies which explicitly took non-flat term structures into account. This necessitated a new definition of duration.

Bierwag (1977:726) and Bierwag and Kaufman (1977:365) suggested using variants of the Macaulay duration by first showing that many definitions of duration were possible. Macaulay and Hicks's duration research was extended by Hopewell and Kaufmann (1973:752), particularly in the explanation of debt security price behaviour. Despite this pioneering work, little has been done subsequently to supplant the measure for a trendier alternative.

The Cox, Ingersoll, Ross (henceforth CIR) (1979:12, 1985:388) duration measure assumes that the square root of the short-term spot interest rate and the departure of that rate from its long-run expected value are important parameters. However, empirical evaluations of the CIR measure have included several additional, *ad hoc* variables (and other variants of the Macaulay duration have also included additional parameters), but these have limited, rather than broadened its use.

Recent work (e.g., Fons, 1990, Kim *et al*, 1993, Fooladi *et al*, 1997, Duffie, and Singleton, 1999, Nawalkha, 1999, Rzadkowski and Zaremba, 2000, Guntay, 2002, Jacoby and Roberts, 2003 and Jacoby, 2003) has produced better variants and adaptations of the Macaulay duration. Alternative measures and modifications are explored in the next chapter. The next section provides an overview of the mathematical definition and derivation of the original Macaulay duration. In addition, the modified duration – an adjusted Macaulay duration which takes into account the value of the yield to maturity at the point of measurement – is also discussed.

3.3 Macaulay duration

3.3.1 Definition and derivation

The Macaulay duration is an average of the dates on which cash flows are promised weighted by the present values of the cash flows to be received on those dates. Whilst the maturity of a zero coupon bond is an appropriate measure of its life, it is an ambiguous measure of the life of a security that promises cash flows at many future times. It is also true that the price behaviour of a zero coupon bond is different from that of a non-zero coupon bond with the same maturity.

Consider an option-free, one year bond (nominal $=N$) with semi-annual coupon payments C and annual yield to maturity y – shown schematically in Figure 3.1. The cash flows associated with such an option-free bond are C after 6 months and $C+N$ at maturity (i.e., after one year). The discount factor associated with the first coupon payment is $DF_1 = 1/(1+y/2)$ and the discount factor associated with the final payment is $DF_2 = 1/(1+y/2)^2$. The present, fair value of

this bond, P, then, is given by the sum of present values of the cash flows (Fabozzi, 1996, 1997):

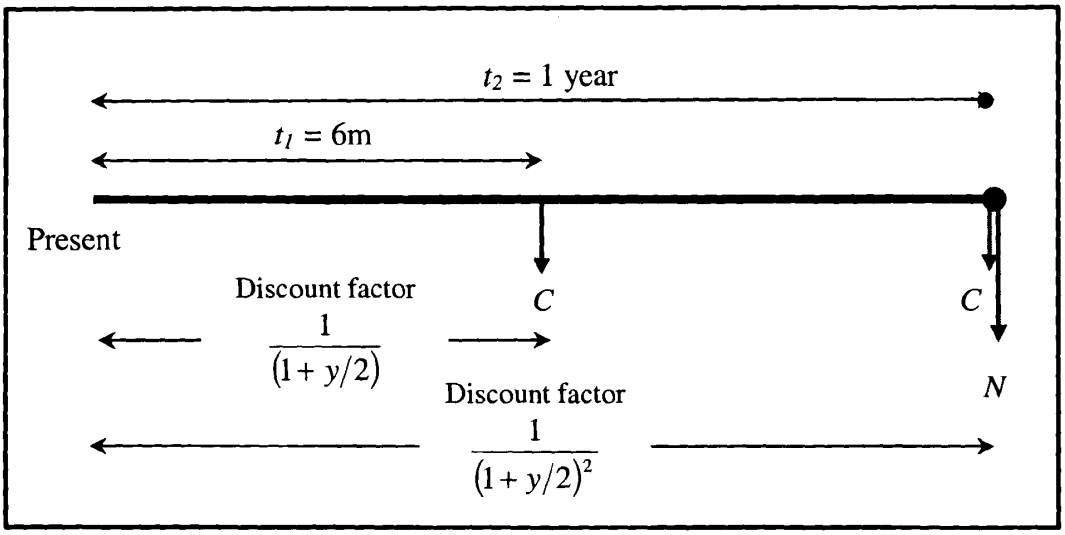
$$P = \frac{C}{\left(1+{}^{y}/_{2}\right)} + \frac{C+N}{\left(1+{}^{y}/_{2}\right)^{2}},$$

or in general:

$$P = \sum_{t=1}^{M} \frac{CF_t}{\left(1+{}^{y}/_{n}\right)^{t}}, \tag{3.1}$$

where CF_t is the cash flow at time t, M is the number of periods to maturity, n is the number of periods in one year and y is the annual yield to maturity.

Figure 3.1: Schematic pricing of a bond comprising two cash flows.



(Source: Fabozzi, 1997)

The price of this security is a function of two different cash flows. The value of the first cash flow (as a proportion of the overall price of the security) is:

$$w_1 = \frac{C \cdot \left(1+{}^{y}/_{2}\right)^{-1}}{P} = \frac{C \cdot DF_1}{P},$$

and the value of the second as a proportion of the price is

$$w_2 = \frac{(C+N) \cdot \left(1+{}^{y}/_{2}\right)^{-2}}{P} = \frac{(C+N) \cdot DF_2}{P},$$

where $DF_1 = \left(1+{}^{y}/_{2}\right)^{-1}$ and $DF_2 = \left(1+{}^{y}/_{2}\right)^{-2}$ are the 1- and 2-period discount factors, respectively. More generally

$$w_t = \frac{CF_t \cdot \left(1+{}^{y}/_{2}\right)^{-t}}{P} == \frac{CF_t \cdot DF_t}{P}, \tag{3.2}$$

where w_t is the weight assigned to the t^{th} cash flow, CF_t.

Both w_1 and w_2 are positive fractions which necessarily sum to unity, i.e., $w_1 + w_2 = 1$. These weights reflect the relative importance of the promised cash flows and the values of those weights change as the coupon rate and the yield to maturity change. The Macaulay duration of this bond is given by the weighted sum the times of promised cash flows:

$$D_{\text{Mac}} = w_1 \cdot t_1 + w_2 \cdot t_2,$$

or, more generally:

$$D_{\text{Mac}} = \sum_{n=1}^{M} w_t \cdot t_n. \tag{3.3}$$

As an example, consider a one year bond in which $N = \text{R}100$, the annual coupon rate is 4% (i.e., $C = \text{R}100 \times 4\%/2 = \text{R}2$) and the annual yield to maturity is 5%. In this case $P = \text{R}99.0363$ using Equation 3.1, $w_1 = 0.0197$ and $w_2 = 0.9803$ using Equation 3.2, assuming semi-annual compounding. Using Equation 3.3,

$$\begin{aligned} D_{\text{Mac}} &= 0.0197 \cdot \tfrac{1}{2} + 0.9803 \cdot 1 \\ &= 0.9901 \text{ years.} \end{aligned}$$

This value assigns a single number to the timing and magnitude of the cash flows. If the value were closer to 0.5 years, this might be due to an earlier payment than the first one at $t = 0.5$ years or a large payment at the earlier time or a combination of both.

The mathematical reasoning behind the Macaulay duration becomes clearer if the measure is thought of as a scaled gradient. Taking the first derivative with respect to the yield to maturity y of a bond with M periods to maturity and comprising a set of cash flows CF_t occurring at times t, where $t \in 1, M$, (i.e., differentiating Equation 3.1) gives:

$$\frac{dP}{dy} = (-1) \cdot \frac{1}{n} \cdot CF_1 \cdot \left(1+\frac{y}{n}\right)^{-2} + (-2) \cdot \frac{1}{n} \cdot CF_2 \cdot \left(1+\frac{y}{n}\right)^{-3} + \cdots + (-M) \cdot \frac{1}{n} \cdot CF_M \cdot \left(1+\frac{y}{n}\right)^{-(M+1)}.$$

The term $-\left(1+\frac{y}{n}\right)^{-1} = \frac{-1}{\left(1+\frac{y}{n}\right)}$ may be factored out giving:

$$\frac{dP}{dy} = \frac{-1}{\left(1+\frac{y}{n}\right)}\left[(1)\cdot CF_1\cdot\left(1+\frac{y}{n}\right)^{-1} + (2)\cdot CF_2\cdot\left(1+\frac{y}{n}\right)^{-2} + \cdots + (M)\cdot CF_M\cdot\left(1+\frac{y}{n}\right)^{-M}\right]\cdot\frac{1}{n}$$

$$\frac{dP}{dy} = \frac{-1}{\left(1+\frac{y}{n}\right)}\left[(1)\cdot \mathrm{PV}\cdot CF_1 + (2)\cdot \mathrm{PV}\cdot CF_2 + \cdots + (M)\cdot \mathrm{PV}\cdot CF_M\right]\cdot\frac{1}{n}$$

$$\frac{dP}{dy}\frac{1}{P} = \frac{-1}{\left(1+\frac{y}{n}\right)}\left((1)\frac{\mathrm{PV}\cdot CF_1}{P} + (2)\frac{\mathrm{PV}\cdot CF_2}{P} + \cdots + (M)\frac{\mathrm{PV}\cdot CF_M}{P}\right)\cdot\frac{1}{n}$$

$$\frac{dP}{dy}\frac{1}{P} = \frac{-1}{\left(1+\frac{y}{n}\right)}\left(\sum_{t=1}^{M} t\cdot\frac{\mathrm{PV}\cdot CF_t}{P}\right)\cdot\frac{1}{n}$$

$$\frac{dP}{P} = \frac{-dy}{\left(1+\frac{y}{n}\right)}\cdot\frac{D_{\mathrm{Mac}}}{n},$$

where D_{Mac} is the Macaulay duration. The Macaulay duration may thus be thought of as a *scaled* gradient of the line tangential to the price/yield relationship curve measured at a given yield to maturity. This is obvious from the derivation above, namely:

$$\frac{dP}{dy}\cdot\frac{1}{P} = \frac{-1}{\left(1+\frac{y}{n}\right)}\cdot\left(\sum_{t=1}^{M} t\cdot\frac{\mathrm{PV}\cdot CF_t}{P}\right)\cdot\frac{1}{n}$$

thus

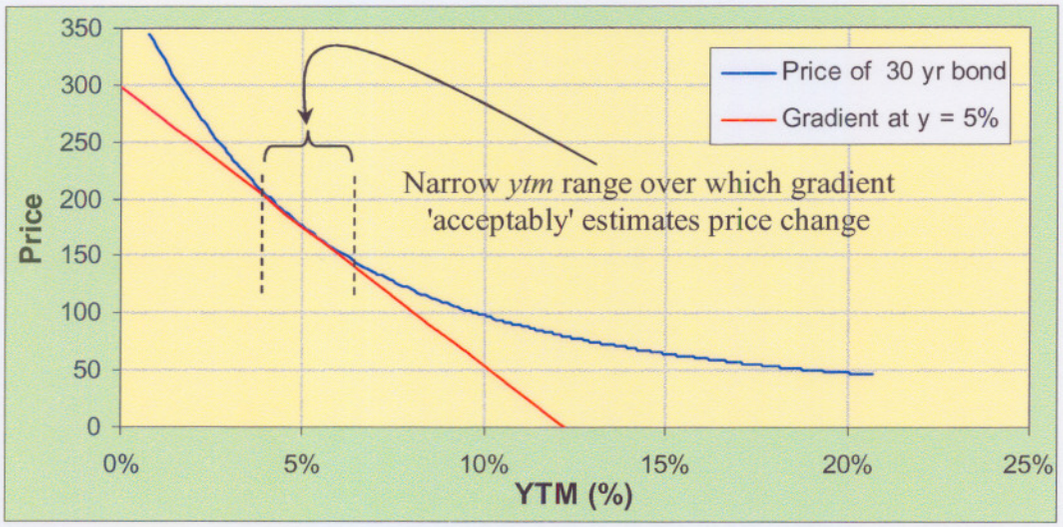
$$D_{\mathrm{Mac}} = -\frac{dP}{dy}\cdot\frac{1}{n\cdot P}\cdot\left(1+\frac{y}{n}\right). \tag{3.4}$$

Multiplying the gradient $\left(\frac{dP}{dy}\right)$ by $-\left(\frac{1}{n\cdot P}\right)\cdot\left(1+{}^{y}/_{n}\right)$ (scale factors) then, gives the Macaulay duration. The price/yield relationship of a 30-year, 10% semi-annual coupon, option-free bond is given in Figure 3.2 below.

The line tangential to this curve at an annual yield to maturity of 5% is also shown in Figure 3.2 with gradient $(dP/dy)\big|_{y=5\%}$. Scaling this value by $-\left({}^{1}/_{2\cdot P\big|_{y=5\%}}\right)\cdot\left(1+{}^{y\big|_{y=5\%}}/_{2}\right)$ since $n=2$ in

this case, gives D_{Mac} – the Macaulay duration. This linear measure of the change in the price of a bond given a change in the yield to maturity, whilst mathematically appealing, is clearly only effective for a small range of changes in yield to maturity.

Figure 3.2: The inverse price/yield relationship for an option-free bond. The tangential line to this curve (at *ytm* = 5%) is also shown.



3.3.2 Modified duration: definition and derivation

The modified duration simply combines two of the quantities in Equation 3.4 to produce a scaled duration measure, i.e., $D_{\text{Mod}} = D_{\text{Mac}} \Big/ \left(1 + {}^{y}\!/_{n}\right)$ (Fabozzi, 1997:233). Using Equation 3.4 gives

$$\frac{D_{\text{Mac}}}{\left(1+\frac{y}{n}\right)} = -\frac{dP}{dy}\cdot\frac{1}{P}\cdot\frac{1}{n}$$

$$\frac{dP}{P} = \frac{-D_{\text{Mac}}\cdot dy\cdot n}{\left(1+\frac{y}{n}\right)} \qquad (3.5)$$

resulting in a relationship between the percentage change in the price of a security dP/P for any given change in the yield to maturity (dy) to the first order.

Second order changes, which take into account the convex shape of the relationship between prices and yield of fixed income securities, are discussed in the next section.

3.3.3 Convexity

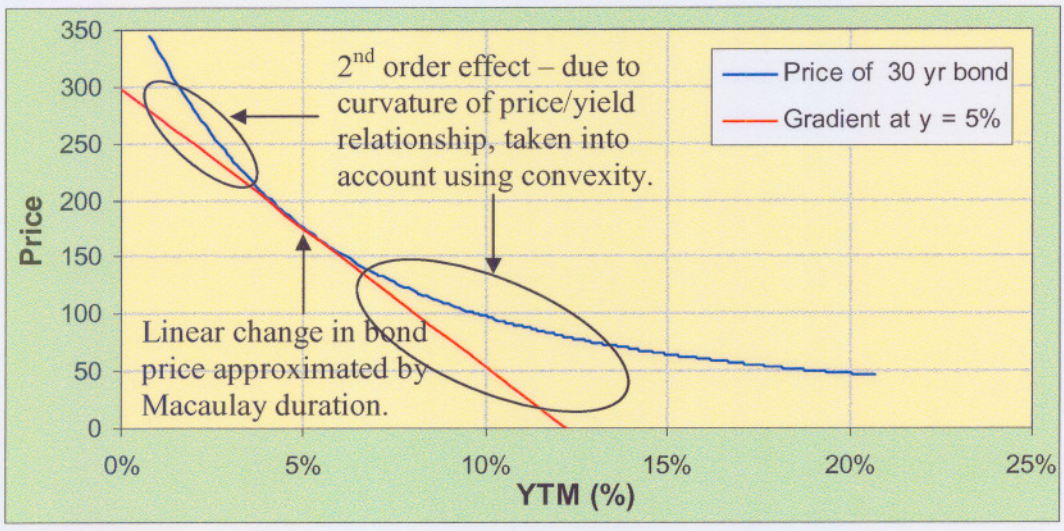
The non-linear relationship between the change in the bond price and the change in the yield may be approximated using the Taylor expansion around its initial value (Fabozzi, 1997:267):

$$P_1 = P_0 + \Delta y \cdot \left.\frac{\partial P}{\partial y}\right|_{y=y_0} + \frac{1}{2} \cdot (\Delta y)^2 \cdot \left.\frac{\partial^2 P}{\partial y^2}\right|_{y=y_0} + \cdots$$

where y_0 is the original yield, Δy is the small change in yield, P_1 is the price approximated at $y = y_0 + \Delta y$, P_0 is the original price measured at $y = y_0$, $\left.\frac{\partial^n P}{\partial y^n}\right|_{y=y_0}$ is the n^{th} order partial derivative of the price with respect to yield measured at $y = y_0$.

For all practical purposes, only the first two terms (linear and quadratic) are ever used in finance – since they provide a good approximation to changes in prices relative to other assumptions that must be made about pricing assets. If the increment in y is extremely small, even the quadratic term is negligible (Jorion, 2001:87). For large changes in yield, the linear (first order) duration measure does not adequately describe the change in the bond price as shown below in Figure 3.3 using the same parameters for the bond as used in Figure 3.2.

Figure 3.3: First (Macaulay duration) and second order (convexity) effects used to estimate changes in bond price.



Using Equation 3.1 and taking the second derivative with respect to the yield gives:

$$\frac{d^2 P}{dy^2} = (-1)\cdot(-2)\cdot\frac{1}{n^2}\cdot CF_1 \cdot \left(1+\frac{y}{n}\right)^{-3} + (-2)\cdot(-3)\cdot\frac{1}{n^2}\cdot CF_2 \cdot \left(1+\frac{y}{n}\right)^{-4} +$$
$$\cdots + (-M)\cdot(-M-1)\cdot\frac{1}{n^2}\cdot CF_M \cdot \left(1+\frac{y}{n}\right)^{-(M+2)}.$$

The term $\left(1+\frac{y}{n}\right)^{-2} = \frac{1}{\left(1+\frac{y}{n}\right)^2}$ may be factored out giving:

$$\frac{d^2P}{dy^2}=\frac{1}{\left(1+\frac{y}{n}\right)^2}\left[\begin{array}{l}(1)\cdot(2)\cdot CF_1\cdot\left(1+\frac{y}{n}\right)^{-1}+(2)\cdot(3)\cdot CF_2\cdot\left(1+\frac{y}{n}\right)^{-2}+\\ \cdots+(M)\cdot(M+1)\cdot CF_M\cdot\left(1+\frac{y}{n}\right)^{-M}\end{array}\right]\cdot\frac{1}{n^2}$$

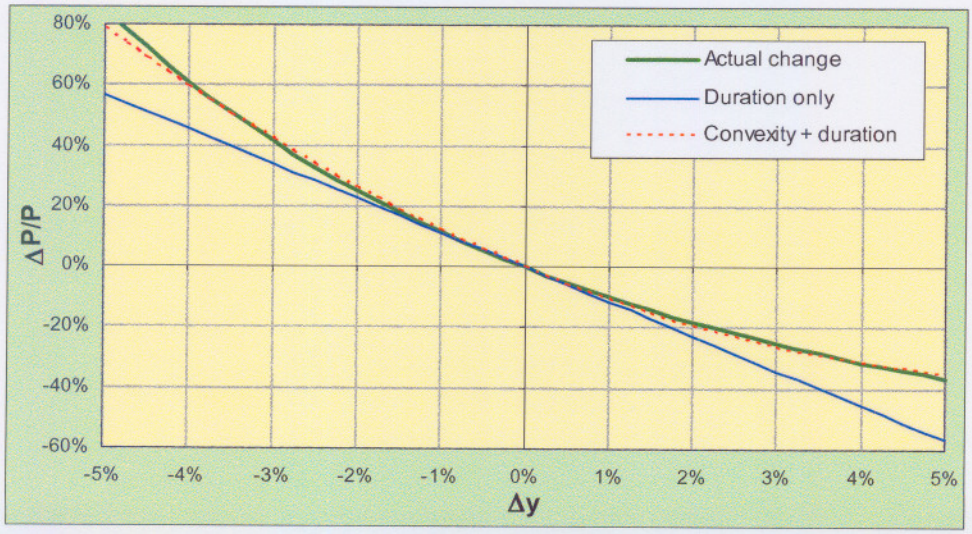
$$\frac{d^2P}{dy^2}=\frac{1}{\left(1+\frac{y}{n}\right)^2}\left[(1)\cdot(2)\cdot\mathrm{PV}(CF_1)+(2)\cdot(3)\cdot\mathrm{PV}(CF_2)+\cdots+(M)\cdot(M+1)\cdot\mathrm{PV}(CF_M)\right]\cdot\frac{1}{n^2}$$

$$\frac{d^2P}{dy^2}=\frac{1}{\left(1+\frac{y}{n}\right)^2}\left((1)\cdot(2)\cdot\mathrm{PV}(CF_1)+(2)\cdot(3)\cdot\mathrm{PV}(CF_2)+\cdots+(M)\cdot(M+1)\cdot\mathrm{PV}(CF_M)\right)\cdot\frac{1}{n^2}$$

$$\frac{d^2P}{dy^2}=\frac{1}{\left(1+\frac{y}{n}\right)^2}\left(\sum_{t=1}^{M}t\cdot(t+1)\cdot\mathrm{PV}(CF_t)\right)\cdot\frac{1}{n^2}$$

In addition: $\frac{P_1-P_0}{P_0}=\frac{\Delta P}{P_0}=-D_{\text{Mod}}\cdot\Delta y+\frac{1}{2}C\cdot(\Delta y)^2$ where C is known as the convexity adjustment and is calculated using $C=\frac{1}{\left(1+\frac{y}{n}\right)^2}\left(\sum_{t=1}^{M}t\cdot(t+1)\cdot\mathrm{PV}(CF_t)\right)\cdot\frac{1}{n^2}$. If the convexity adjustment is added to Equation 3.4 – the change in price due to a change in rates becomes: $\frac{\Delta P}{P}\cong-D_{\text{Mod}}\cdot\Delta y+\frac{1}{2}C\cdot(\Delta y)^2$. Figure 3.4 below depicts the problem of estimating the change in bond price when the yield to maturity changes using duration alone and the solution provided by the addition of convexity.

Figure 3.4: (i) Actual variation of $\Delta P/P$ with Δy, (ii) $-D_{\text{Mod}}\cdot\Delta y$ only and (iii) $-D_{\text{Mod}}\cdot\Delta y+\frac{1}{2}C\cdot(\Delta y)^2$.



3.3.4 Duration example

Before advances in computing technology made simulations of net present values under multiple interest rate scenarios feasible, some bankers used duration as a proxy for estimating the net economic value of their institution. Duration is still used by many bank managers as a basis for evaluating the relative risks of different financial instruments, portfolios, or investment strategies. Duration incorporates an instrument's remaining time to maturity, the level of interest rates, and intermediate cash flows. If a fixed income instrument has only one cash flow, as a zero coupon bond does, duration will equal the maturity of the instrument: a zero coupon bond with five years remaining to maturity has a duration of 5 years. If coupon payments are received before maturity, the duration of the bond declines, reflecting the fact that some cash is received before final maturity (Jorion, 2001:91).

Duration is calculated by weighting the present value of an instrument's cash flows by the time to receipt of those cash flows. Table 3.1 illustrates the calculation of the duration of a R100 000 two-year bond that pays interest semi-annually, has a 7.5% coupon, and was purchased at par to yield 7.5%.

Table 3.1: Sample values for Macaulay and modified duration calculation.

Period	Years	Cash flows	Discount factors	Discounted CF's	Time weighted, discounted CF's
1	0.5	3.75	0.9638	3.6145	3.6145
2	1	3.75	0.9290	3.4838	6.9676
3	1.5	3.75	0.8954	3.3579	10.0737
4	2	103.75	0.8630	89.5438	358.1753
				SUM/Coupon freq	**189.41**
			Macaulay	**Divide by price**	**1.894 years**
			Modified	**Divide by** $1/(1+\text{ytm}/2)$	**1.83 years**

This bond has a modified duration of 1.83 years. If rates were to increase 100 basis points (1%), the value of this bond would be expected to decline by approximately 1.83%. The calculations set out in Table 3.1 do *not* adjust the expected cash flows of the bond to changes in interest rates. Hence, this calculation is not valid for instruments, such as callable bonds and mortgage-backed securities, whose options will change their cash flows as interest rates move. To partially adjust for this shortcoming, banks make use of the 'effective duration'. Effective duration is derived by using simulation techniques to calculate the change in price of an instrument for a given change in interest rates.

3.3.5 Properties of duration

Duration exhibits the following characteristics (Fabozzi, 1997:223):

- The higher the duration, the greater the price sensitivity of the instrument to changes in market interest rates.
- For two instruments with the same maturity, a high-coupon instrument will have a lower duration than a low-coupon instrument and will also be less price-sensitive. A larger proportion of a high coupon's cash flows will be received sooner and thus the average time to receipt of the cash flows will be less.
- A given fixed income instrument will have a higher duration in a low interest rate environment than in a high interest rate environment.
- Duration may be positive or negative. A fixed rate instrument would have a positive duration, and an increase in interest rates would generally decrease the market value of the instrument. Mortgage servicing rights and interest-only mortgage-backed securities generally have a negative duration, since an increase in interest rates would decrease the prepayment speed of the underlying mortgages, increasing the market value of the instruments.
- Durations are *additive* when weighted by the amount of the contract – that is, they combine linearly. For example, if a portfolio consists of 2 bonds of equal market value, one with a duration of 6 and the other with a duration of 2, the portfolio duration is four. Mathematically, $w_1 \cdot D_1 + w_2 \cdot D_2 = D_3$. Thus, for example, if $w_1 = w_2 = 50\%$ then $D_3 = \frac{D_1 + D_2}{2}$.

3.3.6 Key-rate durations

Key-rate duration calculates the spot durations of each of the 11 'key' maturities along a spot rate curve[11]. These eleven key maturities are at the 3-month and 1, 2, 3, 5, 7, 10, 15, 30, 25, and 30-year portions of the curve (Ho, 1992:38 and Knez *et al*, 1994:1869).

In essence, key-rate duration allows the duration of a portfolio to be calculated for a one-basis-point change in interest rates while holding the yield for all other maturities constant. The key-rate method is most often used for portfolios such as the bond-ladder, which consists of fixed-income securities with differing maturities. The equation for key-rate duration is given by (Ho, 1992:33):

[11] These specific eleven key rates are used as most countries employ this level of detail when defining local yield curves. Developed economies generally employ far greater granularity when describing the local yield curve (Ho, 1992:31 and Knez *et al*, 1994:1868).

$$\frac{\text{Security price after 1\% decrease in yield - Security price after 1\% increase in yield}}{2\times\text{Initial price of the security}\times 1\%}.$$

The sum of the key-rate durations along the curve is equal to the 'effective duration'.

An important interpretation of the duration measure is as an elasticity measure. Historically, duration has been mistakenly assigned time as the unit of its measurement. This leads to a paradox since elasticities are, by definition, dimensionless. Recent work has shown this to be due to fallacious assignment – to the constituent elements – of incorrect units. The next section considers the Macaulay duration as an elasticity measure.

3.4 Duration as an elasticity

Hicks (1939:186) asserted that the Macaulay duration (not then called by that name), was equal to the elasticity of the price of the bond with respect to the 'present value factor' $(1+y)$ of the bond. Hicks noted a difficulty with his proof: the Macaulay duration has a dimension: that of time, but elasticities should be pure numbers. According to Hicks (1939:187) (generalizing slightly and changing notation) if the yield to maturity is y, then the discount ratio is:

$$\frac{1}{\left(1+\frac{y}{n}\right)},$$

where y is the annual yield compounded n times a year.

Hicks (1939:187) asserted that the elasticity of the present value of a payment due after t years with respect to the discount ratio was t. For a stream of payments, Hicks averaged the 'elasticities' in the same manner that Macaulay did, but he called the result the average period of the income stream. However, Hicks's elasticities cannot be elasticities as the term is normally defined as Hicks himself acknowledged in a footnote:

> 'The reader may...find it rather surprising that an elasticity, usually supposed to be a pure number, independent of units, turns out to be equal to a length of time' (Hicks 1939:187).

But elasticities *are* pure numbers and are independent of the units used to measure the inputs. Hicks's explanation is equally unsatisfactory; his footnote continues:

> 'This is a consequence of compound interest. The interest rate for two years is not double that for one; so that time cannot be eliminated by considering proportional changes.' (ibid.)

The problem has little to do with compound interest. Hicks observed that the duration is an average time to payment, but the compound interest rate has nothing to do with the investment pe-

riod. The problem appears to arise from confusing an amount of interest with an interest rate (Fisher, 2004).

Rewriting Equation 3.4 as an elasticity measure:

$$D_{\text{Mac}} = -\frac{\left(dP/P\right)\cdot\frac{1}{n}}{dy\Big/\left(1+\frac{y}{n}\right)}, \tag{3.6}$$

or, in terms of the modified duration:

$$D_{\text{Mod}} = \frac{D_{\text{Mac}}}{\left(1+\frac{y}{n}\right)} = -\frac{\left(dP/P\right)\cdot\frac{1}{n}}{dy} = \frac{1}{n\cdot P}\frac{dP}{dy}. \tag{3.7}$$

Any elasticity must be independent of the units used to measure the variables. But in Equation 3.4, the Macaulay duration depends on the units used to measure time. That is, if the units of time are changed from years to months, the number describing duration becomes 12 times as big.

The problem has been neatly solved recently (Hallerbach, 2001). The bond price valuation equation, namely

$$P = \sum_{t=1}^{M} \frac{CF_t}{\left(1+y'\right)^t}, \tag{3.8}$$

(where $y' = y/n$ and other symbols have their usual definitions) presents a 'sloppy definition of the present value' (Hallerbach, 2001) and it is this which provides the source of the interpretation error of duration. In order to show that duration is indeed dimensionless – as expected from its definition as an elasticity – the correct general expression for the value of an option-free bond on a coupon date is (Hallerbach, 2001):

$$P = \sum_{i=1}^{M} \frac{CF_i}{\left(1+\frac{y}{n}\right)^{i\cdot\Delta t\cdot n}}, \tag{3.9}$$

where: i is the index number for the cash flows, M is the number of remaining cash flows until maturity, n is the compounding frequency within one year and Δt reflects the time interval (in years) between the cash flow dates.

Hence, the remaining time to maturity in years is $M \cdot \Delta t$, the compounding interval for the yield is n^{-1} (in years) and the number of cash flows per year (the cash flow frequency) is given by $(\Delta t)^{-1}$. In this representation the cash flows are:

$CF_i = \Delta t \cdot c \cdot N$ for $i = 1, \cdots, M-1$:

$$CF_M = (1 + \Delta t \cdot c) \cdot N,$$

where c is again the coupon rate per annum and N is the face value of the bond. For notational simplicity but without loss of generality, it has been assumed that the cash flow dates are equally spaced; the necessary adjustment for accommodating unevenly spaced cash flows is easily accommodated. Equation 3.8 clearly shows the dimensions of the relevant parameters. There are two crucial differences with Equation 3.7. First note that the summation index i now is indeed a pure number and has no longer the dimension time attached. In addition, the compounding frequency n appears explicitly in the expression of the discount factor thus allowing the familiar transformation of discrete compounding to continuous compounding:

$$\lim_{n \to \infty} \left(1 + \frac{y}{n}\right)^{-i \cdot \Delta t \cdot n} = e^{-y \cdot i \cdot \Delta t}.$$

In Equation 3.8 above it is incorrectly assumed that $n = 1$. Defining the dimension operator $\dim(\cdot)$, gives: $\dim(y) = \text{time}^{-1}, \dim(n) = \text{time}^{-1}, \dim(\Delta t) = \text{time}, \dim(\text{c}) = \text{time}^{-1}$. Hence, n has the dimension time^{-1}, so y/n and $\Delta t \cdot n$ are just numbers without any time dimension attached, thereby resolving the 'paradox'. Nevertheless, the idea that duration has the dimensions of time is still largely prevalent in the literature, even in authoritative texts such as Fabozzi (1996), Fabozzi (1997), Haugen (1997) and Sharpe *et al,* (1999).

The Macaulay duration – interpreted as an elasticity – is an important application of this measure. Another chief role of the Macaulay duration is in managing the immunisation of interest rate-sensitive portfolios, considered in the next section.

3.5 Portfolio immunisation using duration

The presence of interest rate uncertainty presents serious problems to financial institutions whose profits rely on exposing their cash flow to as little interest rate risk as possible. Portfolio immunisation refers to the process devised to minimise this interest rate risk.

The Macaulay duration lies at the heart of portfolio immunisation and constitutes a crucial component of both its use and implementation. As a result, the literature dedicated to this field of research is extensive and ongoing. Fong and Vasicek (1984) introduced a risk minimising strategy

for portfolio immunisation whilst Gultekin and Rogalski's (1984) work used empirical tests on basis risk and provided illuminating insights into portfolio immunisation using alternative duration specifications. Garbade (1985, 1986) provided a generalised approach to immunisation and Barrett *et al,* (1995) presented a selection of different immunisation strategies using duration. Barber and Cooper (1996) was first to introduce principal component analysis to immunise portfolios against interest rate risk, Chambers *et al,* (1988) used a duration vector for this purpose and Balbas *et al,* (1998, 2002) used dispersion measures as an immunisation risk measure. This brief résumé does not do justice to the burgeoning portfolio immunisation field, but does provide a starting point for the interested reader.

Whilst the significance of the duration measure on portfolio immunisation cannot be underestimated, research pertaining to this field lies outside the scope of this thesis. The focus here is instead on the *application* of new duration descriptions to the field of risk management from regulatory and compliance points of view. Nevertheless, the general description and example of portfolio immunisation using duration which follows, provides the reader with an introduction to this important field.

Assume a bank has a customer deposit due in 2 years. The bank's main function is to transform the deposit into a profitable investment that grows to a certain (matured) amount to meet the obligation (say, R1m). One way to ensure that the targeted cash flow is free of value fluctuation is to invest the fund in a zero coupon bond that has the same maturity time as the obligation time. If a 3 year zero bond is available, the bank would rigidly prefer this investment instrument over any other maturity instruments. Market segmentation (or preferred habitat) refers to this type of rigid investment behaviour. Risk taking or speculating the future interest rate movement is not the objective of these types of management; even the compensation of bearing the interest rate risk in the form of liquidity risk premium is not sufficient to vary the management's planned investment horizon. The inability to meet the future obligation is a stigma for banks. The planned investment horizon, however, can be arbitrary and even if it is a reasonable wholesome time (assume 2 years in this example) there may not be a zero coupon bond with maturity time that matches with the planned time. The concept of duration may now be applied to solve situations such as this in which the bond market is incomplete (and thus insufficient) to cater for the need of the investor to ensure a certain cash flow in the presence of interest rate uncertainty.

3.5.1 Principles of bond portfolio immunisation

A portfolio of investments in bonds is said to be immunised if its value at the end of the holding period must be as large as it would have been had interest rates remained unchanged i.e., regard-

less of the course of interest rates during the period (Morgan, 1986:91). A simple example of immunisation would be that if a pure zero coupon bond exists maturing at the end of the holding period (the planned horizon or the target date), then a trivial case of immunised investment (the 2 year bond in the above discussion) arises. More generally, it will be necessary to hold a bond portfolio with payments received both before and after the target date. This subjects the portfolio to both reinvestment risk and price risk that arises because of fluctuations in future interest rates (Soto, 2004:1001). If the term structure of interest rates shifts in an uncertain, but sufficiently harmonious, way ('parallel' or 'horizontal' shifts) then it is possible to design a dynamic structure involving offsetting fluctuation between long- and short-term (coupon) bonds, in order to ensure that the effects of price and reinvestment risk will effectively cancel each other out.

3.5.2 Immunisation example

Consider a simple situation in which one and only one cash outflow (to make an amount equal to R1m) is to be paid in 2 years. Since there is only one cash outflow, its duration is simply 2 years. The ideal would be to have a 2 year zero coupon bond traded so that the amount can be fully immunised in the presence of future interest rates uncertainty, but suppose it is not available. Assume investment in two different bond issues is considered with the following details: the first bond issue is a 3 year coupon bond with 8% annual coupon, the second bond is a 1 year coupon bond with 7% annual coupon. The future value of these two bonds are R1,000 and their PV are R950.25 and R972.73 respectively given that the yield curve is flat and that the current yield to maturity is 10%.

All the funds could be invested in the 1 year bonds, with the presumption that proceeds from the maturing bonds are reinvested one year from now in another 1 year issue. But note that if 1 year interest rates were to decline over the next year, then the funds from the maturing 1 year bonds will have to be reinvested at a lower rate than the currently available 10%. Thus, reinvestment risk arises due to the possibility that the funds one year from now may only be reinvested at a lower rate.

A second alternative would be to invest the funds in the 3 year bond but this also entails risk. In particular, the 3 year bond will have to be sold after two years in order to yield the R1m. The risk is that interest rates will have risen before then, meaning that bond prices, in general, will have fallen and the bonds will not have a selling price that is equal to or greater than R1m. Thus this strategy faces price risk. However, although the first and second alternatives involve risks due to interest rate fluctuations, the two kinds of risk described above offset each other.

Immunisation takes advantage of the offsetting reinvestment risk and the price risk in a bond portfolio. Continuing the example, immunisation implies partially investing the initial fund in the 1 year bond and in the 3 year bond in order to attain the 2 year targeted horizon. That is, if w_1 and w_3 denote the weights of the funds that are to be invested in Bond 1 and Bond 3, solving for the solution w_1 and w_3 to the following simultaneous set of simultaneous linear equations:

$$w_1 + w_3 = 1$$
$$w_1 \cdot D_{\text{Mod}1} + w_3 \cdot D_{\text{Mod}3} = 2$$

gives $D_{\text{Mod}1} = 1$ and $D_{\text{Mod}3} = 2.78$ (the durations of Bond 1 and Bond 3 respectively). Note that the first equation is merely the budget constraint whereas the second equation indicates that the weighted average of the durations of the bonds in the portfolio must equal the duration of the cash outflows: 2 years. The solution to these two equations is $w_1 = 0.4382$ and $w_3 = 0.5618$. Given the desired R1m, at $t = 2$ the present value of the needed initial investment is $PV = \frac{1\,000\,000}{1.10^2} = \text{R}826\,446$. Then 43.82% of this amount is used to purchase bond 1 and 56.18% of this amount is used to buy Bond 3.

$\frac{\text{R}362\,149}{\text{R}972.73}$ = quantity of Bond 1 bought and $\frac{\text{R}464\,297}{\text{R}950.25}$ = quantity of Bond 3 bought.

If interest rates were to rise the loss due to liquidating the 3 year bonds at a discount after two years will be exactly offset by the gains from reinvesting the maturing 1 year bonds at the higher rate. Alternatively, if interest rates were to fall, loss from being able to reinvest the maturing 1 year bonds at a lower rate will be exactly offset by being able to liquidate the 3 year bonds after two years at a premium. In this sense, the portfolio is 'immunised' from the consequence of any movements in interest rates in the future (Soto, 2004:1002).

3.5.3 Problems with immunisation

It has been assumed that the yield curve is flat in order to conduct the parallel (vertical) shift of the yield curve due to an interest rate shock. The assumption that the yield curve undergoes parallel (random) shifts is a strong one and the effect of, *inter alia*, hedging interest risk in the duration model depends crucially on this assumption. The rationale behind the assumption of a flat yield curve was justified on the following grounds.

The construction of the Macaulay duration is based on using the yield to maturity as the factor for discounting coupons to find their present values. In the absence of a complete set of zero bonds, the yield to maturity of a coupon bond may be used as a legitimate proxy by the market

participants to obtain 'unobserved' future interest rates. Prior to the innovation of modern financial instruments such as interest rate options, futures and swaps, this was the preferred method of obtaining these obscured future rates. The introduction of these new interest rate derivative products has allowed bank managers to hedge interest rate uncertainty without relying on the strong assumption employed by the duration model (Babbel *et al*, 1997:36). Convexity is often introduced to minimize the error of hedging due to the exclusive use of the Macaulay duration. As financial instruments grow ever more complex, however, the justification for flat yield curves – in which all movements are considered to be parallel – diminishes. These are considered in later chapters.

Wu (2000:922) considered the use of a stochastic duration (based on the Vasicek (1977) and CIR (1985) models). Within the one-factor framework, Wu proposed the use of a longer zero-curve yield instead of the original instantaneous interest rate as a proxy for the relevant risk source(s). Bond immunization[12] showed that the stochastic duration in some cases outperformed Macaulay's duration.

3.6 Conclusion

This chapter examined the origins, subsequent development, interpretation, limitations and adjustments of the Macaulay duration. The measure's computational simplicity and durability have allowed it to survive relatively intact despite decades of ongoing development. Recent changes in interest rate markets and financial instrument complexity have, however, necessitated more severe adaptations and alterations in order to deal with the limitations imposed by the Macaulay duration. The next chapter deals with the measurement and management of interest rate risk in the banking and trading books as well as the important role played by the Macaulay duration in both the measurement and management of this risk.

[12] Using Belgian data.

Chapter 4

Interest rate risk in the banking and trading books

4.1 Introduction

The previous chapter introduced the Macaulay duration as an important tool for the measurement and management of interest rate risk. It was shown in Chapter 2 that interest rate risk affects both books of the bank since both comprise instruments affected directly by movements in interest rates. The measurement, management and regulation of these, however, differ significantly between the two books. It was also shown in Chapter 2 that interest rate risk measurement in the trading book has advanced significantly in the last decade. With the new requirements set out in the new Basel accord, the same development should take place for the banking book. This chapter focuses on aspects of interest rate risk in both the banking and trading books.

4.2 Interest rate risk in the banking book

To evaluate the potential impact of interest rate risk on a bank's operations, a well-managed bank will consider the effect on both its earnings (the **earnings** or **accounting perspective**) and underlying economic value (the **economic** or **capital perspective**). Both viewpoints must be assessed to determine the full scope of a bank's interest rate risk exposure, especially if the bank has significant long-term or complex interest rate risk positions. These perspectives are considered and discussed in the next section.

4.2.1 Earnings and economic perspectives

From an earnings perspective, banks consider the effect of interest rate risk on net income and net interest income. It also assists in the assessment of the contribution of non-interest income and operating expenses to the interest rate risk exposure of the bank (ICC, 2004).

From a capital, or economic value, viewpoint, banks consider how intermediate (two years to five years) and long-term (more than five years) positions affect future financial performance. Since the value of instruments with intermediate and long maturities can be particularly sensitive to interest rate changes, it is important for a bank to monitor and control the level of these exposures. Banks also consider how interest rate risk acts jointly with other risks they face. In a rising rate environment, for example, loan customers may be unable to meet interest payments because of the increase in the size of the payment or a reduction in earnings. The result

will be a higher level of non-performing loans. An increase in interest rates exposes a bank with a significant concentration of adjustable rate loans to credit risk (ICC, 2004).

Bank management decisions concerning a bank's interest rate risk exposure need to take into account the risk/reward trade-off of interest rate risk positions. Management should compare the potential risk (impact of adverse-rate movements) of an interest rate risk position or strategy against the potential reward (impact of favourable-rate movements).

4.2.1.1 Earnings perspective

The earnings perspective considers how interest rate changes will affect a bank's reported earnings, for example, a decrease in earnings (caused by changes in interest rates) can reduce earnings, liquidity and capital (O'Brien, 2000). Earnings-based approaches tend to focus on short run variability in net interest income (typically one to two years), thus linking directly to reported profitability. Fluctuations in interest rates generally affect reported earnings through changes in a bank's net interest income. Net interest income will vary because of differences in the timing of accrual changes (repricing risk), changing rate and yield curve relationships (basis and yield curve risks) and options positions (Bessis, 2002:59).

Changes in the general level of market interest rates also may cause changes in the volume and mix of a bank's balance sheet products. For example, when economic activity continues to expand while interest rates are rising, commercial loan demand may increase while residential mortgage loan growth and prepayments slow. Changes in the general level of interest rates also may affect the volume of certain types of banking activities that generate fee-related income. The volume of residential mortgage loan originations, for example, declines as interest rates rise, resulting in lower origination fees. In contrast, mortgage servicing pools often face slower prepayments when interest rates rise since borrowers are less likely to refinance. Fee income and associated economic value arising from mortgage servicing related businesses thus may increase or remain stable in periods of moderately rising interest rates (Beck *et al*, 2000:68).

Declines in the market values of certain instruments may diminish near-term earnings when accounting rules require a bank to charge such declines directly to current income (known as price risk). Banks with large trading account activities generally employ separate measurement and limit systems to manage this risk. Evaluating interest rate risk solely from an earnings perspective may not be sufficient if a bank has significant positions that are intermediate-term (between two years and five years) or long-term (more than five years). This is because most earnings-at-risk measures consider only a one-year to two-year time frame. As a result, the

potential impact of interest rate changes on long-term positions is often not fully captured (ICC, 2004).

4.2.1.2 Economic perspective

Economic value approaches offer a longer-term perspective on interest rate risk, taking into account all future cash flows generated from a bank's banking book positions (Risk Institute, 2005). The economic perspective provides a measure of the underlying value of the bank's current position and seeks to evaluate the sensitivity of that value to changes in interest rates. This perspective focuses on how the economic value of all bank assets, liabilities, and interest-rate-related, off-balance-sheet instruments change with movements in interest rates. The economic value of these instruments equals the present value of their future cash flows. By evaluating changes in the present value of the contracts that result from a given change in interest rates, the change to a bank's economic value (or economic value of equity) may be estimated (Risk Institute, 2005).

In contrast to the earnings perspective, the economic perspective identifies risk arising from long-term repricing or maturity gaps. By capturing the impact of interest rate changes on the value of all future cash flows, the economic perspective can provide a more comprehensive measurement of interest rate risk than the earnings perspective. The future cash flow projections used to estimate a bank's economic exposure provides a *pro forma* estimate of the bank's future income generated by its current position. Because changes in economic value indicate the anticipated change in the value of the bank's future cash flows, the economic perspective can provide a leading indicator of the bank's future earnings and capital values[13]. The longer the view, however, the more difficult it becomes to accurately model these earnings streams (ICC, 2004).

Changes in economic value can also affect the liquidity of bank assets because the cost of selling depreciated assets to meet liquidity needs may be prohibitive. The growing complexity of many bank products and investments heightens the need to consider the economic perspective of interest rate risk. The financial performance of bank instruments is increasingly linked to pricing and cash flow options embedded within those instruments. The impact of some of these options, such as interest rate caps on adjustable rate mortgages and the prepayment option on fixed-rate mortgages, may not be discernable if the impact of interest rate changes is evaluated only over a short-term (earnings perspective) time horizon. For newly originated products, a short-term hori-

[13] In the interest of reporting stable earnings streams, those banks that have adopted both earnings and economic approaches tend to place stronger emphasis on measuring and managing net interest income, typically focusing on future earnings one year ahead but sometimes considering earnings out over the next two or three years.

zon may underestimate the impact of caps[14] and prepayment options because loan rates are unlikely to exceed caps during the early life of a loan. In addition, borrowers may be unlikely to refinance until the transaction costs associated with originating a loan have been absorbed. As time passes, however, interest rate caps may become binding or borrowers may be more likely to refinance if market opportunities become favourable. Similarly, some structured notes offer relatively high initial coupon rates to the investor at the expense of potentially lower-than-market interest rates of return at future dates (Risk Institute, 2005, Choi and Elyasiani, 1996:271).

Failure to consider the value of future cash flows under a range of interest rate scenarios may leave the bank with an instrument that underperforms the market or provides a rate of return below the bank's funding costs. Although it is a powerful tool in the management of interest rate risk exposure, the economic perspective often is more difficult to quantify than the earnings perspective. Measuring risk from the economic perspective requires a bank to estimate the future cash flows of all of its financial instruments. Since many retail bank products (such as savings and demand deposits) have uncertain cash flows and indefinite maturities, measuring the risk of these accounts can be difficult and requires numerous assumptions by the bank (FDIC, 2002).

Because of the difficulty of precisely estimating market values for every product, many economic measurement systems track the relative change or sensitivity of values rather than the absolute change in value (Choi, 1996:275). Economic value analysis facilitates risk/reward analysis because it provides a common benchmark (present value) for evaluating instruments with different maturities and cash flow characteristics. Many banks have found this type of analysis to be useful in decision making and risk monitoring (FDIC, 2002).

In line with international regulators, the SARB plans to accept only the economic value approach to measuring interest rate risk in the banking book for the purposes of calculating regulatory capital. This acceptance of only the economic – longer term – approach to measuring interest rate risk, will force banks to develop accurate measures of this interest rate risk (SARB, 2005).

It is clear that the current Macaulay duration measure is woefully ill-prepared to deal with the vagaries of contemporary financial instruments. The yield curve is *not* flat and even if it were, shifts in the yield curve are almost never parallel. Rather, yield curve movements twist and contort in a stochastic manner – captured far more effectively by duration interpretations and analysis that allow for these stochastic yield curve changes. These both negate the necessity for con-

[14] An interest-rate cap is an over the counter derivative that protects the holder from rises in short-term interest rates by making a payment to the holder when an underlying interest rate (the 'index' or 'reference' interest rate) exceed a specified strike rate (the 'cap rate'). Caps are purchased for a premium and typically have expirations between 1 and 7 years. They may make payments to the holder on a monthly, quarterly or semi-annual basis, with the period generally set equal to the maturity of the index interest rate (Hull and White, 2001:14).

vexity and are more accurate in general in the description of interest rate movements and hence interest rate risk, than the traditional Macaulay duration. These measures require a complete re-thinking of the duration measure as an interest rate risk tool, but the mathematics is not onerous and the implementation and interpretation of results are both relatively straightforward with regard to the benefits realised. Results obtained from the research undertaken for this thesis are presented in Chapter 6.

The two perspectives (earnings and economic) have been introduced and discussed. A numerical example follows which illustrates the principles discussed in this section, is given in the next section.

4.2.1.3 Numerical example: earnings versus economic perspectives

A bank's interest rate risk should not be viewed solely in terms of its effects on either economic value or earnings. These two perspectives are complementary and both are necessary to capture interest rate risk comprehensively. The economic perspective focuses on the value of the bank in the current (2005) interest rate environment and the sensitivity of that value to changes in interest rates. It also captures future exposure by evaluating the impact of potential interest rate changes on market values of all assets, liabilities, and off-balance-sheet contracts. The earnings perspective, which captures the *timing* of income effects, helps risk managers determine what action to take to offset or hedge the exposure. In the example given below, the earnings perspective indicates that earnings problems will not develop until the second year in which the assets and liabilities are on the bank's balance sheet. The bank in the example is exposed to interest rate risk arising from the repricing gap between a four-year asset and a one-year liability. Both instruments are accounted for on an historic cost basis. Table 4.1 illustrates the expected annual income and cash flows for this bank, assuming that interest rates remain at their current levels. The example uses the following simplifying assumptions:

- the bank has equity capital of R200,000,
- the bank purchases a four-year bond carrying an annual 8% coupon,
- the notional amount is R1.2 million and the current market value is par,
- the bank funds the bond with a one-year certificate of deposit (CD) with a face amount of R1 million,
- the current rate on the CD is 6% and interest is paid annually,
- the bank pays all of its income to shareholders as dividends and pays no taxes. It has no other income or operating expenses and

- at the end of the 4th year, the bank plans to liquidate and distribute any residual equity to shareholders.

Under this scenario, the bank expects to earn R36,000 each year on its asset/liability spread. Shareholders would receive R36,000 in dividends in each year. At the end of the fourth year, the bank receives approximately R1.2 million in cash from the bond but must pay out approximately R1 million in cash to the CD customer. The entire bank's net income was distributed as dividends, therefore the equity available to shareholders equals the original equity (R200,000).

Table 4.1: Earnings perspective: Expected cash flows and income (stable interest rates).

Values in R 000's	Initial outlay and book values	Cash flows			
		Year 1	Year 2	Year 3	Year 4
Bond	-R1,200	R96	R96	R96	R1 296
CD	R1,000	-R60	-R60	-R60	-R1 060
Net Income		R36	R36	R36	R36
Dividends		-R36	-R36	-R36	-R36
Equity	R200				-R200

Table 4.2 shows the present values of the asset, liability, dividend, and equity cash flows, assuming that interest rates do not change. The bond's cash flows are discounted at 8% and the CD cash flows are discounted at 6%. The present values for dividends (net income) and equity reflect the differences in the present value of the bond and CD cash flows. Note that if interest rates remain at their current level, the present value of the expected cash flows equals the par value of the instruments.

Table 4.2: Economic perspective: Present value of expected cash flows and income (stable interest rates).

Values in R 000's	Initial outlay and book values	PV of cash flows			
		Year 1	Year 2	Year 3	Year 4
Bond	R1,200	R88.9	R82.3	R76.2	R952.6
CD	-R1,000	-R56.6	-R53.4	-R50.4	-R839.6
Dividends and equity	-R200	-R32.3	-R28.9	-R25.8	-R113.0

Because this bank is funding a four-year asset with a one-year liability, it is exposed to rising interest rates. Table 4.3 shows cash flows and net income if interest rates were to rise immedi-

ately by 200 basis points (bps). The bank's reported earnings in Year 1 remain unchanged because funding rates for the first year are locked. After Year 1, however, the CD reprices by 200 bps to a new rate of 8%. As a result, net income for the remaining three years will decline by R20,000 per year. The bank's cumulative net income and corresponding dividends paid to shareholders over the four-year period declines from R144,000 to R84,000.

Table 4.3: Earnings perspective: expected cash flows and income – 200bp rise.

Values in R 000's	Cash flows			
	Year 1	Year 2	Year 3	Year 4
Bond	R96	R96	R96	R1 296
CD	-R60	-R80	-R80	-R1 080
Net Income	R36	R16	R16	R16
Dividends/Equity	-R36	-R16	-R16	-R216
Change in net income versus stable rate scenario	R0	-R20	-R20	-R20

Table 4.4 illustrates the present value of the bank's expected cash flows under the new rate scenario. Note that the present value of both the bond and the CD decline. The decline in the present value of the bond reflects the fact that, although the cash flows from the bond remain constant, those cash flows are now discounted at a higher (10%) rate. In essence, the bank has forgone more profitable investment opportunities and now holds a bond that offers below-market returns. As in Table 4.3, the PV of net income, dividends, and equity represents the difference between the present values of the bond and CD cash flows. Table 4.4 shows that the net economic value of the bank declines by R57.6 million in comparison with net economic value in the stable rate scenario. This decline in net economic value represents the decline in the present value of the bank's future cash flows.

Table 4.4: Economic perspective: present value of expected cash flows and income – 200bp rise.

Values in R 000's	Net present values	Present value of cash flows			
		Year 1	Year 2	Year 3	Year 4
Bond	R1,123.9	R87.3	R79.3	R72.1	R885.2
CD	-R981.5	-R55.6	-R68.6	-R63.5	-R793.8
Dividends/equity	-R142.4	-R31.7	-R10.7	-R8.6	-R91.4
Change in equity vs stable rate scenario	-R57.6				

These examples illustrate that if a bank evaluates its earnings exposure over only a short time horizon, it may incorrectly assume that it has little or no exposure. This bank shows no earnings exposure for the first year. Yet, as the example illustrates, the bank's earnings in future periods may decline significantly if interest rates increase. The change in the economic value of equity (as measured by the change in the present value of the bank's assets less the present value of its liabilities) can be a leading indicator of the expected decline in future earnings and capital.

Having discussed the earnings and economics perspectives in some detail and illustrated their use by way of a numerical example in this section, the trade-offs between the two perspectives are presented in the next section.

4.2.1.4 Trade-offs in managing earnings and economic exposures

When immunizing earnings and economic value from interest rate risk, bank management must make certain trade-offs. When earnings are immunised, economic value becomes more vulnerable and vice versa. The economic value of equity, like that of other financial instruments, is a function of the discounted net cash flows (profits) it is expected to earn in the future. If a bank has immunised earnings, such that expected earnings remain constant for any change in interest rates, the discounted value of those earnings will be lower if interest rates rise. Hence, although the bank's earnings have been immunised, its economic value will fluctuate with rate changes. Conversely, if a bank fully immunises its economic value, its periodic earnings must increase when interest rates rise and decline when interest rates fall. A simple example illustrates this point (ICC, 2004).

Consider a bank that has R100 million in earning assets and R90 million in liabilities. If the assets are earning 10%, the liabilities are earning 8%, the cost of equity is 8%, and the bank's net non-interest expense (including taxes) totals R2 million, the economic value of the bank is R10 million. This value is obtained by discounting the net earnings of R0.8 million — R10 million in interest income less R7.2 million in interest expense and R2 million in non-interest expense — as a perpetuity at 8%[15]. If net non-interest expenses are not affected by interest rates, the bank can immunise its net income and net interest income by placing R10 million of its assets in perpetuities and the remainder of assets and all liabilities in overnight funds. If this is undertaken, a general 200bp increase in interest rates leaves the bank's net income at R0.8 million. The bank earns R11.8 million on its assets (R10 million perpetuity at 10% and R90 million overnight assets at 12%) and incurs interest expenses of R9 million (R90 million at 10%) and non-interest expenses of R2 million. The economic value of its equity, however, declines to R8 million. (The

[15] A perpetuity is an annuity that pays interest forever. Its present value equals the periodic payment received divided by the discount rate.

net earnings of R0.8 million are discounted as a perpetuity at 10%). As a result of this trade-off, many banks that limit the sensitivity of their economic value will not set a zero risk tolerance (i.e., attempt to maintain current economic value at all costs), but rather will set limits around a range of possible outcomes. In addition, because banks generally have some fixed operating expenses that are insensitive to interest rate changes (as in the above example), some banks have determined that their risk-neutral position is a slightly long, net asset, position. The bank's fixed operating expenses from a cash flow perspective behave in a similar way to a long-term fixed rate liability that must be offset or hedged by a long-term, fixed-rate asset (Risk Institute, 2005).

This concludes the comparison between the earnings and economic perspectives.

Over half of a bank's interest rate risk resides in the traditional domain of asset and liability management (ALM), non-trading activities. The importance of asset and liability management – as a strategic component of bank interest rate risk management – is explored in the following section.

4.2.2 The role of ALM

ALM is the coordinated management of the entire portfolio of a financial institution. For commercial banks in particular, an adequate ALM practice is today considered an inevitable and indispensable instrument for controlling risk and optimizing the risk-return-relationship (BIS, 2004). It considers both the acquisition of funds from various sources and the allocation of funds to profitable investments. These activities leave a bank's earnings and capital exposed to interest rate risk, or unexpected movements in interest rates[16]. Interest rate risk also arises from options embedded in banking products, such as prepayment options of individual borrowers who are able to renegotiate the interest rate payable on loans when interest rates decline. The traditional focus of ALM has been on these two types of interest rate risk (Uyemura and van Deventer, 1992:121).

Strategic interest rate risk management is crucial to any financial institution since the interest rate is still the single most important generator of income and hence profit contributor. Over the past decade, the adoption of trading risk management tools has strengthened immensely asset-liability managers' ability to measure and manage interest rate risk exposure. In the 1990s, ALM's increasing complexity coupled with rising concern over interest rate risk, forced risk managers to go beyond their traditional toolbox of metrics and methodologies. Although ALM units continue to use accrual accounting, and although traditional metrics serve a useful purpose, market value and statistical metrics have been embraced by large banks. Senior executives are becoming so convinced of the importance of managing economic rather than pure accounting

[16] It is important to note, however, that expected changes in interest rates are not true risks as these may be hedged or priced.

results that they are implementing compensation plans based on shareholder value-added, rather than revenues or earnings (Austin, 2005).

In addition to adding an economic perspective, the intermingling of ALM and trading risk management has brought the application option-adjusted valuation[17] (OAV) methodology to mortgages and non-maturity deposits (Austin, 2005). Before the use of OAV, the challenges of modelling prepayment risk spawned a variety of disparate methodologies that were more art than science. OAV now provides a relatively uniform way to gauge embedded optionality (Guntay, 2002:12) but to date has been unable to address the issue of duration measures for instruments with embedded optionality.

Another critical contribution has been the introduction of stochastic (i.e., probabilistic) simulations which yield EaR (earnings-at-risk) and VaR (value-at-risk). Although these have some shortcomings, the consensus among industry professionals (and regulators) as to their robustness and applicability has firmly established both as 'market standard' metrics. Stress testing, which captures extreme risk not covered by VaR, has also become an essential component in the risk management toolkit. Most large banks use all three to allocate market risk capital (Austin, 2005).

The merging of ALM and trading risk management, however, is not straightforward. While the advantages of economic and statistical analyses are clear, market value and VaR conflict with ALM due to fundamental differences in instrument characteristics and time horizons. However, the most daunting obstacle to bridging the gap between an earnings perspective and an economic one is understanding and managing the trade-offs. Actions that reduce earnings risk and dampen earnings volatility do not translate into similar impacts on economic value. In fact, the opposite can occur. Consequently, asset-liability managers must optimise hedging strategies and in this regard, a combination of an increase in bank balance sheet liquidity and the focus on contract and market/fair value (enabled by technological advancements) has met with some success (Uyemura and van Deventer, 1992:124).

4.2.3 Sources of interest rate risk

The interest rate risk that arises from a bank's core loans and deposits business may be broken down into four broad categories, discussed in each of the next four subsections.

4.2.3.1 Pricing (maturity mismatch) risk

Repricing risk results from differences in the timing of rate changes and the timing of cash flows that occur in the pricing and maturity of a bank's assets, liabilities, and off-balance-sheet instru-

[17] Option-adjusted valuation of financial instruments takes the effect of embedded optionality into account and adjusts interest rate spreads, credit spreads and so on, accordingly (Austin, 2005).

ments. Repricing risk is often the most apparent source of interest rate risk for a bank and is often gauged by comparing the monetary value (book value) of a bank's assets that mature or reprice within a given time period with the monetary value (book value) of liabilities that do so. Some banks intentionally take repricing risk in their balance sheet structure in an attempt to improve earnings (Kwan and Eisenbeis, 1995:122).

Because the yield curve is generally upward-sloping (long-term interest rates are higher than short-term interest rates), banks can often earn a positive spread by funding long-term assets with short-term liabilities (Choi, 1996:282). The earnings of such banks, however, are vulnerable to an increase in interest rates that raises cost of funds. Banks whose repricing asset maturities are longer than their repricing liability maturities are liability-sensitive because their liabilities will reprice more quickly. The earnings of a liability-sensitive bank generally increase when interest rates fall and decrease when they rise. Conversely, an asset-sensitive bank (asset repricings shorter than liability repricings) will generally benefit from a rise in interest rates and hurt by a fall in interest rates (Kwan and Eisenbeis, 1995:123) – see Section 4.2.4.2 on gap analysis.

Repricing risk is often, but not always, reflected in a bank's current earnings performance. Repricing imbalances may be created by a bank that will not be manifested in earnings until sometime in the future. A bank that focuses only on short-term repricing imbalances may be induced to take on increased interest rate risk by extending maturities to improve yield. When evaluating repricing risk, therefore, it is essential that the bank considers not only near term imbalances, but also long-term ones. Failure to measure and manage material, long-term repricing imbalances can leave a bank's future earnings significantly exposed to interest rate movements (Uyemura and van Deventer, 1992:133).

4.2.3.2 Basis risk

Basis risk arises from a shift in the relationship of the interest rates in different financial markets or on different financial instruments (Wall and Shrikhande, 2005). Basis risk occurs when market interest rates for different financial instruments – or the indices used to price assets and liabilities – change at different times or by different amounts. This change affects a bank's current net interest margin through changes in the earned/paid spreads of instruments that are being repriced. It also affects the anticipated future cash flows from such instruments, which in turn affects the underlying net economic value of the bank. Basis risk also includes changes in the relationship between managed interest rates, or interest rates established by the bank, and external interest rates. For example, basis risk may arise because of differences in the prime rate and a bank's offering interest rates on various liability products, such as money market deposits and savings accounts (Wall and Shrikhande, 2005). Because consumer deposit interest rates tend

to lag behind increases in market interest rates, many retail banks may observe an initial improvement in their net interest margins when interest rates are rising. As interest rates stabilise, however, this benefit may be offset by repricing imbalances and unfavourable spreads in other key market interest rate relationships; and deposit rates gradually catch up to the market. Certain pricing indices have a built-in 'lag' feature such that the index will respond more slowly to changes in market interest rates. Such lags may either accentuate or moderate the bank's short-term interest rate exposure (Wall and Shrikhande, 2005).

Some banks use interest rate derivatives as an alternative to other investments; others use them to manage their earnings or capital exposures. Banks can use interest rate derivatives to limit downside earnings exposures, preserve upside earnings potential, increase yield, and minimize income or capital volatility. Although interest rate derivatives can be used to hedge interest rate risk, they expose a bank to basis risk because the spread relationship between cash and derivative instruments may change. A bank using off-balance-sheet interest rate instruments such as futures, swaps, and options to hedge or alter the interest rate risk characteristics of on-balance-sheet positions needs to consider how the off-balance-sheet contract's cash flows may change with changes in interest rates and in relation to the positions being hedged or altered (IAS Board, 2004:31).

Derivative strategies designed to hedge or offset the risk in a balance sheet position will typically use derivative contracts whose cash flow characteristics have a strong correlation with the instrument or position being hedged. Banks must also consider the relative liquidity and cost of various contracts, selecting the product that offers the best mix of correlation, liquidity, and relative cost. Even if there is a high degree of correlation between the derivative contract and the position being hedged, the bank may be left with residual basis risk because cash and derivative prices do not always move together. Banks holding large derivative portfolios or actively trading derivative contracts should determine whether the potential exposure presents material risk to the bank's earnings or capital.

4.2.3.3 Yield curve risk

Yield-curve risk arises from variations in the movement of interest rates across the maturity spectrum. It involves changes in the relationship between interest rates of different maturities of the same index or market. The relationships change when the shape of the yield curve for a given market flattens, steepens, or becomes negatively sloped (inverted) during an interest rate cycle. Yield curve variation can accentuate the risk of a bank's position by amplifying the effect of maturity mismatches (FDIC, 2002).

As an example, consider a coupon whose rate is based on the relationship:

$$\text{Coupon} = 10\text{yr swap rate} + (300\text{bp} - 3\text{month government rate (or LIBOR)}).$$

Since the coupon on this bond adjusts as interest rates change, a bank may incorrectly assume that it will always benefit if interest rates increase. If, however, the increase in 3-month LIBOR exceeds the increase in the 10-year swap rate, the coupon on this instrument will fall, even if both LIBOR and swap rates are increasing. Banks holding these types of instruments should evaluate how their performance may vary under different yield curve shapes.

4.2.3.4 Option risk

Option risk arises when a bank or a bank's customer has the right (but not the obligation) to alter the level and timing of the cash flows of an asset, liability, or off-balance-sheet instrument (Jorion, 2001:187). Options give option holder's rights to buy (calls) or sell (puts) a financial instrument at a specified price (strike price) over a specified period of time. For the seller (or writer) of an option, there is an obligation to perform if the option holder exercises the option. The option holder's ability to choose whether to exercise the option creates an asymmetry in an option's performance. Generally, option holders exercise their right only when it is to their benefit. As a result, option holders face limited downside risk (the premium or amount paid for the option) and unlimited upside reward (Jorion, 2001:182). The option seller faces unlimited downside risk (an option is usually exercised at a disadvantageous time for the option seller) and limited upside reward (if the holder does not exercise the option and the seller retains the premium).

Options often result in an asymmetrical risk/reward profile for the bank. If the bank has written (sold) options to its customers, the amount of earnings or capital value that a bank may lose from an unfavourable movement in interest rates may exceed the amount that the bank may gain if interest rates move in a favourable direction. As a result, the bank may have more downside exposure than upside reward. For many banks, their written options positions leave them exposed to losses from both rising and falling interest rates. Some banks buy and sell options on a 'stand-alone' basis. The option has an explicit price at which it is bought or sold and may or may not be linked with another bank product. A bank does not have to buy and sell explicitly priced options to incur option risk, however. Almost all banks incur option risk from options that are embedded or incorporated into retail bank products. These options are found on both sides of the balance sheet (Uyemura and van Deventer, 1992:126).

On the asset side, prepayment options are the most prevalent embedded option. Most residential mortgage and consumer loans give the consumer an option to prepay with little or no prepayment

penalty. Banks may also permit the prepayment of commercial loans by not enforcing prepayment penalties (usually to remain competitive in certain markets). A prepayment option is equivalent to having written a call option to the customer. When interest rates decline, as they have done worldwide since late 2001 (although this trend is beginning to reverse (Miller, 2004)), customers will exercise call options by prepaying loans, and the bank's asset maturities will shorten at a time when the bank would prefer to be extending them. When interest rates rise, customers maintain their mortgages, making it difficult for banks to shorten asset maturities at the time they most desire to do so (Uyemura and van Deventer, 1992:116).

On the deposit side of the balance sheet, the most prevalent option given to customers is the right of early withdrawal (which behave as put options on deposits). When interest rates increase, the market value of customer deposits declines, and customers have the right to 'put' the deposit back to the bank – to the depositor's advantage. Thus, management's discretion in pricing such retail products as non-maturity deposits may also be viewed as a type of option – usually working in the bank's favour. For example, deposits may be pegged at interest rates that lag market rates when interest rates are increasing and that lead when they are decreasing (FDIC, 2002).

Products that contain interest 'caps' or 'floors' are other sources of option risk. Such products are often loans and may have a significant effect on interest rate exposure. For the bank, a loan cap is equivalent to selling a put option on a fixed income security, and a floor is equivalent to owning a call. The cap or floor interest rate is the strike price. When market interest rates exceed the cap rate, the borrower's option moves 'in the money' as the borrower is paying interest at a rate lower than market. When market interest rates decline below the floor, the bank's option moves 'in the money' because the rate paid on the loan is higher than the market rate. Floating rate loans that do not have an explicit cap may have an implicit one at the highest rate that the borrower can afford to pay. In high rate environments, the loan rate may have to be capped, renegotiated to a lower rate or face a default. Non-maturity deposits, such as money market demand accounts and savings accounts also may have implicit caps and floors on the interest rates that the bank is willing to pay (Madan and Unal, 1998:126).

The measurement of interest rate risk – from both an earnings and economic perspective – is considered in the next section.

4.2.4 Interest rate risk measurement

Measurement techniques used to determine the potential impact of non-traded interest rate risk on a bank's financial position can take a number of forms. The technique used depends, in part, on whether the bank focuses on earnings or economic value as an indicator of its financial posi-

tion. Many banks choose to analyse the impact of interest rate movements on earnings (a significant component of which is net interest income) whilst others measure the effect of changing interest rates on the bank's economic value or market value, (cash flows expected to be generated from the bank's on and off-balance sheet positions, all discounted at that interest rate which adequately reflects the riskiness of those cash flows (Austin, 2005)).

Principal methods to measure interest rate risk in the banking book are gap analysis (from an earnings perspective) and income simulation (using static, dynamic and Monte Carlo methods) and DGap analysis (from an economic perspective). These are considered in turn in the following sections.

4.2.4.1 Gap and gap ratio analysis

Gap reports stratify all of a bank's assets, liabilities, and off-balance-sheet instruments into maturity segments (time bands) based on the instrument's next repricing or maturity date and are commonly used to assess and manage interest rate risk exposure – specifically a bank's repricing and maturity imbalances (Austin, 2005).

Gap is the Rand difference between rate-sensitive assets and rate-sensitive liabilities[18] with respect to a specific time frame. Gap management involves the management of all three of its constituents: assets, liabilities, and time (Austin 2005). Assets and liabilities are assigned to time periods (0-1 month, 1-2 months, etc) based on the repricing of their maturities. This is relatively simple for components such as investments and certificates of deposit in which the total notional amount matures on a specific date, but more complex if the rate-sensitive asset or rate-sensitive liability does not have a stated maturity such as a savings account. If the total outstanding balance of the rate-sensitive asset or rate-sensitive liability does not mature simultaneously, but rather over a specified time period such as member loans, problems are also encountered (WCCU, 2002).

Gap reports can be particularly useful in identifying the repricing risk of a bank's current balance sheet structure before assumptions are made about new business or how to effectively reinvest maturing balances. Within a given time band, a bank may have a positive, negative, or neutral gap. A bank will have a positive gap when more assets (in monetary terms) reprice or mature than liabilities. Because this bank has more assets than liabilities subject to repricing, the bank is said to be 'asset sensitive' for that time band. An asset sensitive bank is generally expected to benefit from rising interest rates because assets of a larger value are expected to reprice more quickly than its liabilities.

[18] Rate-sensitive assets and liabilities are any interest bearing instrument that can be repriced to a market rate in a given time frame (WCCU, 2002).

A bank has a negative gap and is 'liability sensitive' when a larger value of liabilities reprice within a given time band than assets. A bank that is liability-sensitive usually benefits from falling interest rates. A bank whose assets equal liabilities within a time band is said to have a 'neutral' gap position. A bank in a neutral gap position is not free of exposure to changes in interest rates, however. Although the bank's repricing risk may be small, it can still be exposed to basis risk or changes in interest rate relationships. Traditionally, most bankers have used gap report information to evaluate how a bank's repricing imbalances will affect the sensitivity of its net interest income for a given change in interest rates (Koch and MacDonald, 2000:81).

The gap ratio is also sometimes used in gap reports. This quantity divides the gap by total assets in the balance sheet and places the gap in perspective to the size of the bank. A gap of R500,000 would not be material for a bank with R10m in assets, but it would be for a bank with R2m in assets. The gap ratio assumes that all rate-sensitive accounts reprice equally.

4.2.4.1.1 Benefit of gap reports

Gap reports are conceptually simple and are relatively straightforward to explain to board members. Data accumulation is uncomplicated and no sophisticated or expensive software models are required. Gap reports provide an indication of the direction and degree of interest rate risk.

4.2.4.1.2 Disadvantages of gap reports

Gap reports are useful as a primary interest rate risk measurement tool only for those banks with very simple balance sheets. If a bank has any material balances in mortgage-related loans or investments or any investments with uncertain maturities (for example, option embedded fixed income instruments), they should move beyond gap analysis. Even if gap is acceptable, regulatory entities will still expect the bank to perform some degree of income simulation analysis (see Section 4.2.4.3).

A gap report is similar to a balance sheet in that it is only a snapshot in time – it does not measure the effect of multiple interest rate changes over time (WCCU, 2002). Gap reports do not quantify risk and make the assumptions that asset and liability characteristics are symmetrical and cash flows are static cash flows – regardless of interest rates or interest rate movements. Most contemporary interest rate scenario shocks assume parallel shifts in interest rates across all maturities and gap reports are no different in this respect. Misleading results are common (Austin, 2005 and Koch and MacDonald, 2000:162) since simple gap reports do not identify and quantify yield curve risk, option risk, intra-period gaps and dynamic gaps. Modified gap reports, however, have been produced which are able to take account of these shortcomings. These are discussed in the sections below.

4.2.4.1.3 Yield curve risk

To measure a bank's cumulative repricing risk over several periods or time bands, most users of gap reports simply sum the gaps across each time band to produce a net cumulative gap position. Implicit in this act is an assumption that movements in interest rates will be perfectly correlated across the time bands and will move in a parallel fashion. This assumption can be amended by applying different weights to each time band (Bessis, 2002:80). For example, gaps in the shorter time bands could be weighted more heavily than those in the longer time bands because short-term interest rates are usually more volatile and usually move by larger amounts than long-term interest rates. The pattern of a bank's repricing gaps across the various time bands can provide an indication of the bank's exposure to changes in yield curve shapes.

A bank that is liability sensitive (has negative gaps) in the short- and long-term time bands and asset sensitive in the intermediate time bands is exposed to a flattening of the yield curve when short-term interest rates go up and long-term interest rates remain stable. The bank's net interest margin deteriorates as the interest rates on its short-term liabilities increase (FDIC, 2002).

4.2.4.1.4 Option risk

As noted earlier, it is difficult to capture option risks with gap reports. Options introduce an asymmetrical and nonlinear element to a bank's risk profile. Although techniques such as preparing multiple gap reports and reporting options by their delta-equivalent values attempt to overcome some of these weaknesses, they are unable to fully capture all of the dimensions of option risk. To do so, a bank that has significant option risk must supplement its gap reports with simulation or option pricing models.

4.2.4.1.5 Intra-period gaps

Although gap reports rely on stratifying balances into broad time bands, they do not detect imbalances within those bands. Some bankers have partly overcome this weakness by reporting the weighted average repricing maturity within each time band. Another method is to reduce the width of the bands (ICC, 2004).

4.2.4.1.6 New business

Many gap reports used by banks consider only the bank's current financial positions. These reports are called 'static' reports because they capture only the risk that arises from the bank's existing balance sheet structure and do not incorporate any assumptions about new business. Some banks may also prepare 'dynamic' gap reports. Typically, these reports are generated from the

bank's earnings simulation models and show how the bank's gap would appear at some point in the future, after new business assumptions are incorporated into the risk measure (ICC, 2004).

Duration gap – discussed in the next section – takes the gap concept further and considers the duration gap rather than the simple maturity gap considered in this section.

4.2.4.2 Duration gap (DGap) analysis

Duration gap is defined as the difference between a balance sheet's weighted average duration[19] of assets and a scaled, weighted average duration of liabilities (Koch and MacDonald, 2000:92) and is, therefore, an economic rather than an earnings measure. The *scaling* applied to the weighted duration of the liabilities is the ratio of the market value of the balance sheet assets (MVA) to the market value of the balance sheet liabilities (MVL),

$$\text{DGap} = DA - \left(\frac{MVA}{MVL}\right) \cdot DL, \tag{4.1}$$

where

DA is the weighted average duration of assets and

DL is the weighted average duration of liabilities. The *weighting averaging* applied here to both assets and liabilities employs the useful additive property of the Macaulay duration[20], namely that if a portfolio comprises R100 of instrument A (Macaulay duration: 6 years) and R200 notional of instrument B (Macaulay duration: 12 years) – the portfolio duration may be calculated using:

$$\frac{\text{R100}}{\text{R300}} \times 6 + \frac{\text{R200}}{\text{R300}} \times 12 = 10 \text{ years}.$$

Four steps are involved determining the duration gap (Koch and MacDonald, 2000):

1. bank management develops an interest rate forecast,
2. the market value of the bank's assets, liabilities and stockholders' equity is estimated,
3. the weighted duration of assets and weighted duration of liabilities are calculated. The effects of both on- and off-balance sheet items are incorporated and these estimates are used to calculate duration gap and finally
4. management forecasts changes in the market value of stockholders' equity across different interest rate environments.

[19] Here and elsewhere, unless otherwise specified, 'duration' refers specifically to the Macaulay duration.
[20] See Section 3.3.5.

A positive DGap indicates that assets are more price sensitive than liabilities, on average. Thus, when interest rates rise, assets will fall proportionately more in value than liabilities and the market value of equity (MVE) will fall accordingly. Similarly, when interest rates fall, assets will fall proportionately less in value than liabilities and the MVE will rise accordingly. A negative DGap indicates that weighted liabilities are more price sensitive than assets. The reverse of the situation for positive DGap is now true: when interest rates rise/fall, assets will fall proportionately less/more in value than liabilities and the MVE will rise/fall (Koch and MacDonald, 2000:94).

4.2.4.2.1 Market value of equity sensitivity analysis

An MVE sensitivity analysis effectively involves the same steps as earnings sensitivity analysis. In MVE sensitivity analysis, however, the bank focuses on the relative durations of assets and liabilities, the extent to which durations change in different interest rate environments, and the effect of different rate environments on the market value of equity. Duration gap analysis focuses on managing the market value of equity whilst taking the timing of cash flows into account. Either the market value of equity or the book value of net interest income may be protected using duration Gap, but not both (Beck *et al*, 2000:66).

The change in the market value of equity stems from the well-known accounting definition of equity, namely:

$$MVE = MVA - MVL \tag{4.2}$$

where the terms refer to the market value of equity, assets and liabilities, respectively. Changes in these quantities are given by:

$$\Delta MVE = \Delta MVA - \Delta MVL\,. \tag{4.3}$$

It has been shown (Equation 3.5) that

$$\frac{\Delta MVA}{MVA} = \frac{-DA \cdot \Delta y}{1+y}$$

Substituting into Equation 4.2 gives:

$$\Delta MVE = -\left[DA - \left(\frac{MVA}{MVL}\right) \cdot DL\right] \cdot \left(\frac{\Delta y}{1+y_{\text{Total assets}}}\right) \cdot MVA$$

And using Equation 4.1 gives

$$\Delta MVE = -\text{DGap} \cdot \left(\frac{\Delta y}{1+y_{\text{Total assets}}}\right) \cdot MVA\,. \tag{4.4}$$

To protect the bank from interest rate changes causing changes in the market value of equity, it suffices for the bank to reduce DGap to zero in Equation 4.4 above.

4.2.4.2.2 Example of DGap analysis

As a demonstration of the use of DGap, consider the balance sheet shown in Table 4.5 below. The bank to whom this balance sheet belongs owns assets comprising cash, a 3-year commercial loan and a 6-year government bond receiving coupons as shown. It has two interest bearing liabilities – a 1-year time deposit and a 3-year certificate of deposit, paying the coupons as indicated. Assume all assets and liabilities are currently priced at par – therefore yields to maturity equal coupons received or paid. The yield to maturity of the total earning assets is 11.11%, calculated as a weighted sum of the individual asset yields to maturity. For the total liabilities, this is 5.65%. The Macaulay durations, in the rightmost column are calculated using the yields to maturity in the YTM column.

Table 4.5. Hypothetical bank balance sheet. All instruments priced at par.

	Par R 1,000	Coupon	Maturity	YTM	Market Value	Duration
Assets						
Cash	100				100	
Earning assets						
3-yr Commercial loan	700	12%	3	12%	700	2.69
6-yr Treasury bond	200	8%	6	8%	200	4.99
Total Earning Assets	900			11.11%	900	
Non-cash earning assets	0				0	
Total assets	1000			10.00%	1000	2.88
Liabilities						
Interest bearing liabs.						
1-yr Time deposit	620	5%	1	5%	620	1.00
3-yr Certificate of deposit	300	7%	3	7%	300	2.81
Tot. Int Bearing Liabs.	920			5.65%	920	
Tot. non-int. bearing	0				0	
Total liabilities	920			5.65%	920	1.59
Total equity	80				80	
Total liabs & equity	1000				1000	

(Adapted from Koch and MacDonald, 2000).

The duration gap is calculated using Equation 4.1.

$$DA = \frac{700}{1000} \times 2.69 + \frac{200}{1000} \times 4.99 = 2.88$$

$$DL = \frac{620}{920} \times 1.00 + \frac{300}{920} \times 2.81 = 1.59$$

$$\text{DGap} = 2.88 - \frac{920}{1000} \times 1.59 = 1.42 \text{ years}$$

The duration gap of 1.42 years implies that the average duration of assets is greater than for the liabilities, hence asset values change by more than liability values for any given yield movement. The minimum risk position, according to Equation 4.4 occurs when DGap = 0 as this eliminates the risk of changes in the MVE. To achieve this, a change of –1.42 in the weighted assets duration is required ($\Delta DA = -1.42$) or a change in the scaled, weighted liabilities duration of +1.54 from $\Delta DL = +1.42 \cdot (920/1000) = 1.54$. Either of these changes will reduce the DGap to zero and thus immunise the portfolio from yield changes.

As a further example, consider the balance sheet as before (Table 4.5), but assume a 1% increase in all yields as shown below in Table 4.6.

Table 4.6. Hypothetical balance sheet: yields to maturity greater than coupon rates (by 1%).

	Par R 1,000	Coupon	Maturity	YTM	Market Value	Duration
Assets						
Cash	100				100	
Earning assets						
3-yr Commercial loan	700	12%	3	13%	683.47	2.69
6-yr Treasury bond	200	8%	6	9%	191.03	4.97
Total Earning Assets	900			12.13%	874.5	
Non-cash earning assets	0				0	
Total assets	1000			10.88%	974.5	2.86
Liabilities						
Interest bearing liabs.						
1-yr Time deposit	620	5%	1	6%	614.15	1.00
3-yr Certificate of deposit	300	7%	3	8%	292.27	2.81
Tot. Int Bearing Liabs.	920			6.64%	906.42	
Tot. non-int. bearing	0				0	
Total liabilities	920			6.64%	906.42	1.58
Total equity	80				68.08	
Total liabs & equity	1000				974.5	

(Adapted from Koch and MacDonald, 2000).

The duration gap is again calculated using Equation 4.1.

$$DA = \frac{683}{974} \times 2.68 + \frac{191}{974} \times 4.97 = 2.86$$

$$DL = \frac{614}{906} \times 1.00 + \frac{292}{906} \times 2.80 = 1.58$$

$$\text{DGap} = 2.86 - \frac{906}{974} \times 1.58 = 1.36 \text{ years.}$$

The duration gap of 1.36 years again implies that the average duration of assets is greater than for the liabilities, hence asset values change by more than liability values for any given yield movement. The change in the MVE may now be determined using Equation 4.4.

$$\begin{aligned}\Delta MVE &= -\text{DGap} \cdot \left(\frac{\Delta y}{1 + y_{\text{Total assets}}} \right) \cdot MVA \\ &= -1.42 \times \left(\frac{0.01}{1.10} \right) \times 1000 \\ &= -\text{R}12.90.\end{aligned}$$

This example will be revisited in Chapter 6 (Section 6.2). Other duration definitions will be explored and tested empirically on South African yield curve data. Differences between the DGap and ΔMVE forecast by the standard Macaulay duration measure and those forecast by alternative duration definitions will be discussed and compared in Chapter 6.

4.2.4.2.3 Embedded options

Embedded options strongly influence the estimated volatility in MVE. A bond being called will shorten duration, a deposit that is withdrawn early will shorten duration, a deposit that is not withdrawn as expected will lengthen duration, a deposit that is withdrawn early will shorten duration, a deposit that is not withdrawn as expected will lengthen duration (Koch and MacDonald, 2000:96).

4.2.4.2.4 Advantages of DGap over funding or maturity Gap

DGap analysis has the advantage of focusing on all cash flows from the underlying assets and liabilities and not just cash flows that are expected to arise over short time intervals. Interest rate risk can be summarized in one measure for the entire portfolio.

4.2.4.3 Income simulation models

Income simulation models allow a method to enable banks to forecast the way in which net interest income reacts to changes in interest rates. The South African regulator encourages banks to perform at least rudimentary net interest income simulation testing even if gap analysis is used. Income simulation quantifies risk in terms of income and, by extension, future capital accumulation. Although regulatory bodies encourage up to five years of forecast testing, one to two years is usually the optimal modelling period (NCUA, 2005) since the longer the period, the less reliable the results of the forecast. The complexity of the simulation modelling should be governed by the complexity of the banks' balance sheet. If the balance sheet contains mortgage-related products or complex, option-embedded investments, the model should be able to incorporate these characteristics (FDIC, 2002).

4.2.4.3.1 Static net interest income modelling

The simplest form of income simulation - static income simulation modelling – determines changes in net interest income for immediate, sustained, parallel shifts in the yield curve of both up and down 100, 200 and 300 bps and are measured from the base model (rate shocks). Static modelling requires little, if any, subjective assumptions and is relatively inexpensive and uncomplicated to implement. The shocks to the yield curve, however, are prescribed and are not representative of economic reality. Using this method, the full risks presented by longer-term instruments could remain hidden (ICC, 2004).

4.2.4.3.2 Dynamic net interest income modelling

Dynamic net interest income simulation modelling is used to manage the bank's balance sheet and earnings and, rather than consider only prescribed parallel shifts in the yield curve, examines 'what-if' scenarios. In this way, bank management may explore best, worst and most likely case scenarios and may investigate alternative budgeting scenarios. Dynamic modelling, however, requires increased subjective assumptions (ICC, 2004).

4.2.4.3.3 Monte Carlo simulation modelling

Monte Carlo simulation measures may be used to determine the probable outcomes of events, such as a movement in interest rates that have a random or stochastic element. Simulation models discussed in previous sections measure the value of the bank under a limited number of interest rate scenarios (ICC, 2004). Such approaches are 'deterministic' because the possible interest rate paths are predetermined and controlled by the model user. Although deterministic models are valuable, their outcomes depend on the interest rate scenarios. If actual interest rates differ from assumptions, the risk to the bank may be substantially different from the measured risk (Bessis, 2002:613). The outcomes of Monte Carlo simulations are less preordained than those of deterministic simulations because the statistical modelling technique involved generates thousands of randomly determined interest rate paths. These interest rate paths result in a distribution of possible interest rate scenarios. The value of the bank or the bank's portfolios is then evaluated for each of the possible interest rate paths, yielding a range of possible values or outcomes (Koch and MacDonald, 2000:100).

4.2.4.3.4 Construction of a Monte Carlo simulation

Formulating a Monte Carlo model is complex and involves the following principles:

- Develop the underlying probability distribution for interest rates that will generate the random interest rate paths. Typically, the current forward yield curve is used to anchor the probability distribution (ICC, 2004).

- A model generates a multitude of random interest rate paths (typically several thousand). However, certain properties are usually built into this process to ensure that the average interest rate generated is consistent with the current structure of interest rates and that the dispersion (distribution) of possible interest rates is consistent with observed volatility. These properties are important to ensure that the model does not introduce the possibility of 'risk-free' arbitrage. Essentially, the properties assume that markets efficiently and fairly price securities, such that one cannot construct instruments with equivalent risk and higher returns than what the market commands (Dimson and Marsh, 1996:1532).

- The cash flows corresponding to each of the randomly developed interest rate paths are calculated. That is, the bank specifies the relationships between the interest rates and the cash flows of the bank's portfolios. For example, the bank would develop a prepayment function that relates mortgage prepayments with each interest rate path. Once adjusted for prepayments and other interest-rate effects, the cash flows are said to be 'option-adjusted.'

- The option-adjusted cash flows for each rate path are discounted by the risk-free rate to obtain their net present value. All of these outcomes are summed, and the total is divided by the total number of rate paths evaluated to produce an expected net present value for the distribution. If the cash flows have been adjusted correctly and the interest rate paths correctly reflect market expectations about the distribution of possible future interest rates outcomes, this expected net present value represents the base-case market price. If the model's assumptions are accurate, the cash flows have been adjusted for all risks, and the market for the instrument under consideration operates according to the underlying theory (which assumes risk neutral valuation), this base-case price should be within a few basis points of observable market prices. If the net present value does not match the market price, common practice is to add a fixed spread known as the option-adjusted spread to the risk-free rate (James and Webber, 2000:137).

- After obtaining the base-case price in the previous step, the current forward yield curve is 'shocked' for each of the interest rate scenarios that banks consider in their risk analysis. For example, if the bank is evaluating its risk for a parallel 200-basis-point increase in interest rates, it would shift the underlying distribution of interest rates (developed in the

first step) by 200 bps such that the expected mean (average) is 200 bps higher across the maturity spectrum. These steps are then repeated, except that the 'market price' that is generated as a result represents the price that would result if interest rates were to change as assumed for that rate scenario (Dimson and Marsh, 1996:1533).

4.2.4.3.5 Advantages of Monte Carlo simulation

Monte Carlo simulation is a powerful risk analysis tool because it can accurately and clearly adjust risk estimates for optionality and convexity. Capital markets employ Monte Carlo techniques to price interest rate derivative products and residential mortgage products using option adjusted spread analysis. Monte Carlo techniques are employed to understand and evaluate current market pricing as well as their economic Value at Risk. This technique provides banks with a valuable tool for measuring and managing interest rate risk (ICC, 2004).

4.2.4.3.6 Limitations of Monte Carlo simulation

Monte Carlo simulations, like all interest rate risk measurement systems, are only as good as the data and assumptions underlying the analysis. Two critical assumptions in Monte Carlo analysis are the process used to derive the interest rate paths and the cash flow relationships developed for each interest rate path. If these assumptions are faulty, the results of the simulation will be potentially inaccurate. Monte Carlo simulations are complicated to develop and require substantial computing technology. To derive and apply this modelling process, a bank must utilise personnel with considerable expertise in financial and statistical theory (ICC, 2004).

Inadequate risk management can result in heavy fines at best and – at worst – complete closure of a bank, so it is in a bank's best interests to constantly review and keep current, its risk practices and processes. The next section presents a summary of regulatory and accounting rules and procedures relevant for the banking book.

4.2.5 Regulation

At present, South Africa's supervisory regime for banks requires that they hold capital against the interest rate risk arising from taking trading positions in interest rate markets (i.e., the interest rate risk inherent in a banks' trading book). Banks are not required to hold capital against interest rate risk in the banking book. This will change going forward, however, as the new accord provides the SARB and other national banking supervisors with the discretion to require capital to be held against interest rate risk in the banking book. The Basel Committee (BIS, 2004) is of the view that interest rate risk in a bank's balance sheet is a potentially significant risk and should be incorporated into the capital framework. However, given the heterogeneity among international banks in terms of the nature of the underlying risk and the processes for monitoring and measur-

ing that risk, the Committee has determined that interest rate risk in the banking book should be dealt with in Pillar Two of the new accord. This means that each national supervisor will have discretion as to whether there is a need to require all, or some, banks to hold capital against this type of risk.

4.2.6 Conclusion

This section established an overview of the sources, exposure and measurement of interest rate risk in the banking book. In addition, current regulatory and accounting rules and future changes to these (which have or will have a direct impact on this book) were outlined. Interest rate risk in the bank's trading book – or trading portfolio – is considered in the next section.

4.3 Interest rate risk in the trading book

Although credit risk continues to be a cause of great concern, the threat posed by market risk – interest rate risk in particular – is rising. The dictum that *no bank can fail due to market risk* no longer holds: interest rate risk is now the dominant market risk (Miller, 2004). Banks have increased their awareness of risk management practices as a result of high profile failures, advances in technology and developments in the regulatory process. There has been an increased emphasis on the quality of internal management systems as a key defence against large trading losses such as those experienced at Barings, Daiwa, Sumitomo and National Westminster[21]. Advances in technology have enabled institutions to develop more sophisticated systems for monitoring and controlling risk. Regulatory developments have contributed by recognising the more rigorous risk management methodology contained in banks' internal models for the purpose of setting market risk capital standards.

Risk measurement methodologies are becoming increasingly sophisticated and some form of VaR model is generally used to analyse and monitor market risks. VaR models aim to measure the potential loss on a portfolio that would result if relatively large adverse price movements were to occur. VaR is used predominantly as a high level management tool with structural limits, such as basis point values and net open positions, used to influence trader behaviour. VaR is, however, starting to be driven down to the dealer level as traders become more sophisticated and

[21] Barings made losses totalling $1.89 billion due to the unauthorised trading activities of Nick Leeson. These activities went undetected as a consequence of a failure of management and other internal controls of the most basic kind (Rawnsley, 1995). At Sumitomo Corporation, the company's chief copper trader lost an estimated $1.8 billion on futures contracts. Sumitomo did not separate its front office trading activities from its back office processing and control unit. The trader also reportedly declined to take a holiday in his ten years at Sumitomo, thus making it more difficult for auditors to discover his wrongdoing (Krugman, 2002). At Daiwa bank, Toshihide Iguchi accumulated $1.1 billion of losses over 11 years of bond trading in New York. There were a number of internal control weaknesses at the bank including the fact that Iguchi was responsible for both securities trading and custody operations and some related back office functions (FDIC, 1995). The NatWest Group recently announced a £77 million charge against pre tax profits as a result of losses incurred in the London interest rate options business (Wolfe, 2001).

measures such as risk adjusted performance (based on VaR) shape behaviour and move capital to its most efficient use (Matz, 2005:16).

A comprehensive stress testing program is an essential supplement to a VaR model. Stress testing involves subjecting trading portfolios to unexpected but possible shocks in market or political conditions. This enables an institution to evaluate its capacity to absorb potentially large losses and to identify steps that it can take to reduce its risk and conserve capital. The move moving towards more regular stress testing is in part being driven by the market risk capital requirements whereby banks using internal models will be required to submit the results of their stress testing scenarios to the Reserve Bank on a quarterly basis (SARB, 2005).

For a number of institutions with large balance sheets the interest rate risk lying within the banking book is substantially greater than the market risk sitting within the trading book. Therefore, it is essential that a comprehensive risk management framework is developed, that effectively identifies, measures, monitors and controls interest rate risk exposures. In general, increasing resources and attention is being devoted to balance sheet risk management. Systems used are becoming more sophisticated as institutions move away from traditional gap analysis to simulation of net interest income and market value of equity. State of the art techniques including simulation and option type analysis are being used to analyse the risks underlying the balance sheet in greater detail. The aim is not only to manage these risks but to add value to the entire process (SARB, 2005).

The integrity and timeliness of data on current positions is a key component of the risk measurement process. One of the biggest hurdles is obtaining accurate, timely data across the entire operation from retail banking to Treasury. The problem arises because of the large number of disparate systems that are used. This data aggregation problem implies that detailed analysis of interest rate risk is usually only conducted on a monthly basis. In the interim, however, major movements in the balance sheet are monitored. At smaller institutions, the problem of aggregating data over a number of systems is substantially reduced and this enables the management of interest rate risk on a more frequent basis, often using simpler techniques.

The complexity involved in modelling interest rate risk has meant that even the Basel Committee has deferred the development of a capital standard. The complexity arises from the fact that as well as technical assumptions and economic forecasts one needs to take into account customer behavioural patterns, such as break-outs and prepayment behaviour, which is hard to model accurately. In addition, the difficulties associated with determining objective mark to market values for assets and liabilities (e.g. loans) introduces another level of subjectivity to the process. De-

spite these complexities, the magnitude of potential interest rate risk on the balance sheet implies that banks as well as supervisors will devote more attention to this area over the next few years.

4.3.1 Interest rate risk in the trading book

Market risk is defined as the potential loss arising from changes in market prices, namely, foreign exchange, interest rates and equity prices. Market risks are calculated using Value-at-Risk methodologies (variance/covariance and simulation models) and are managed using a framework of market risk management policies and risk control procedures, as well as risk and loss limits. Transactions undertaken for trading purposes are marked-to-market and the resulting gains and losses are taken up in the profit and loss accounts. Transactions designated as hedges are accounted for in a manner consistent with the accounting treatment of hedged items. Stress and scenario tests are performed on the trading and investment portfolios to provide early warning of potential worst case losses so as to facilitate proactive management of these risks in the rapidly changing financial markets. A risk-reward measure of Earnings-at-Risk is used as a standard measurement of the risks against related rewards across different products and business types.

4.3.2 Interest rate risk measurement in the trading book

4.3.2.1 Value at Risk

Value at Risk (VaR) is a relatively new – but important – player in the arena of interest rate risk management. Essentially, VaR is the amount that a bank, for a given probability, can lose at most when the market (i.e., interest rates) changes adversely. In the VaR approach, probability distributions of interest changes, levels and term structure, are estimated (Pritsker, 1996:213).

There are many ways to develop VaR models. Much research is devoted to the question of how best can the interest rate sensitivity of the many individual financial instruments on and off the balance sheet of a bank be aggregated.

4.3.2.1.1 VaR definition

Market risk measurement is a field that has seen some breakthrough advances in the past few years, but still offers plenty of room for improvement. The tool that revolutionized market risk measurement is value at risk (VaR), a scenario analysis technique that expresses market risk with relation to a confidence level. To calculate VaR, the following preliminary steps must be completed:

- A risk horizon must be specified. A risk horizon is the time period during which losses may occur.

- A confidence level must be defined. A confidence level may range from 0% to 100%. A 95% confidence level, for example, indicates that there is a 95% chance that the worst case scenario will not exceed the computed VaR.
- A calculation frequency must be chosen. The period of time between calculations should not exceed the risk horizon.

Once the above mentioned variables have been fixed, a VaR can be calculated. The result is a monetary value, R x, that represents total losses due to market risk at a specific level of certainty, for example 95%. This means that a risk manager calculating this VaR can be 95% sure that his firm will not lose more than R x. Generally, 100% is not used as a confidence level because the intuitive solution is that the entire portfolio of assets can be lost. Decreasing the confidence level, even by a few percentage points, provides a far more accurate and realistic assessment of possible market risk losses (Marshall and Siegel, 1996:107).

4.3.2.1.2 Calculating Value at Risk

There are three main techniques that VaR may be calculated, namely the parametric (or variance-covariance 'VCV') approach, historical simulation, and Monte Carlo analysis. First, the VCV approach calculates VaR as a multiple of the standard deviation of a portfolio return. This method is convenient because it computes VaR quickly, but it is unreliable if the portfolio contains nonlinear instruments, such as options. In addition, it assumes that portfolio returns are normally distributed – which is often not true in practice (Matz, 2005:19).

Historical simulation works on the principle that the future will behave exactly like the past. The distribution of returns is re-valued using only historical data. As a result, no assumptions need to be made regarding the distribution. The downside of this method is that its dependency on past data reduces its ability to provide a full range of possible outcomes in the futures (Matz, 2005:18).

Monte Carlo analysis derives VaR from random market (or risk factor) movements. This flexible approach offers a wide range of possible outcomes and can be used for any distribution of return. However, it is also the most costly and time intensive approach. In particular, Monte Carlo is not recommended for especially large or complex portfolios.

4.3.2.1.3 Advantages of the VaR approach

VaR approaches provide general tools to manage interest rate risk. VaR is a single risk measure which employs market value accounting and uses an explicit empirical probability distribution for interest rate changes rather than a hypothetical x %-change scenario.

4.3.2.1.4 Disadvantages of VaR

VaR employs a number of assumptions – most of which are invalid. For example, VaR aims at value gains/losses for specific time horizons, usually very short time horizons (days or months). Standard VaR considers interest rates to be randomly drawn from a probability distribution of interest rates. This is adequate for short horizons (reflecting the assumption of random walks in financial market prices), but is inadequate for long horizons. Market turbulence badly affects VaR precision, market liquidity is not well-monitored and when VaR is exceeded, the potential loss cannot be calculated (Botha *et al*, 2001:26).

All of these are concerns of risk managers that are addressed through alternate market risk measurement techniques. Stress tests, one such approach, can measure the potential loss when VaR is exceeded. Over long periods, interest rates are affected by systematic components in monetary policy, inflation, business cycle, etc. These are not random changes but persistent, cyclical changes. The true risks to banks are longer-term, persistent changes in interest rates, not short-term, temporary changes. Popular interest rate models such as Cox, Ingersol, Ross (1985), Ho and Lee (1986), Hull and White (1990) are complex and require a detailed understanding of the modelling procedures involved and must be incorporated into the VaR model in order to measure long-term interest rate risk.

4.3.2.1.5 VaR alternatives

Through scenario analysis, stress tests reveal portfolio losses under worst case scenarios. They are very useful tools for determining if a portfolio requires diversification, however, unlike VaR, they do not compute the likelihood of these scenarios. Extreme value theory (EVT) is another alternate technique and is used to model unusual and rare circumstances. Like stress tests, EVT can help measure losses when VaR fails (Coles, 2001:132).

4.3.3 Regulation

At present, AC 133 governs the accounting standards for South African banks. Banks in this country are moving towards reporting under International Accounting Standards (IAS 39) which themselves changed in January 2005 to embrace fair value accounting for *all* off-balance sheet derivatives – not only those used directly purely for hedging purposes. Improved accuracy in the pricing of interest rate derivatives and the measurement of the risk these instruments present is increasing and shows no signs of abating. Defaultability and embedded optionality are two important features of contemporary interest rate instruments that must be taken into account when determining their interest rate risk. This will involve a better estimation of duration: the current

Macaulay duration framework is incapable of accurately estimating the duration of such instruments.

4.4 Conclusion

This chapter established an overview of the sources, exposure and measurement of interest rate risk in the banking and trading books. In addition, current regulatory and accounting rules and future changes to these (which have or will have a direct impact on these books) were outlined. Improvements to the existing Macaulay duration – aimed at enhancing the performance of this measure in a modern context – are considered in the next chapter. A later chapter will consider the possibility of entirely new definitions of duration as interest rate risk measures.

Chapter 5

Macaulay duration modifications

5.1 Introduction

The historical development of the Macaulay duration was presented in Chapter 3. Mathematical subtleties which define and place the measure in context were also discussed. As financial instruments develop and evolve into more and more complex conglomerates and the field of structured finance becomes more elaborate via the use of derivatives and the ever-increasing demands of an exacting investment community, the tools required to measure risk associated with these instruments must evolve too[22]. The Macaulay duration is based on several restrictive assumptions which are untenable in the modern, rapidly-developing financial world of complex securities, stochastic probabilities of default and embedded optionality. This is not due to careless mathematics of formulation, but rather a failure to foresee the explosive growth of defaultable and perpetual-maturity instruments as well as those with embedded option features. Financial markets have evolved substantially, but the Macaulay duration has struggled to keep pace.

Improving the accuracy of the measurement of interest rate risk in the banking and trading books is the key focus of this thesis. Fair value measurement techniques, such as DGap, employ duration as a chief component to quantify interest rate risk. If the duration measurement employs limiting assumptions or requires highly restrictive conditions in which to operate, estimates of the fair value will be subject to these same limitations. If the Macaulay duration is to survive at all as a key measure of interest rate sensitivity (and this is by no means certain), structural modifications are urgently required.

Revisions and amendments which address significant shortcomings of the Macaulay duration are developed and explored in detail in this chapter. Models are first constructed using contemporary mathematics of finance and these are then tested empirically using South African market data. If found to be reliable and robust these may enable market practitioners to incorporate complex instrument properties into the current Macaulay duration formulation.

---

[22] Interest rate risk has also been explored in the context of yield curve term structure models and a number of alternative ways of estimating these has been considered. Some are purely used as interpolation functions, while others are fully specified, dynamic term structure models. Vasicek (1977) and CIR (1985) made use of multi-factor term structure models whilst Nelson and Siegel (1987) proposed an approximating function using principal component analysis or PCA. PCA is widely used, but the technique employs limiting assumptions. The effects of taxes and transaction costs, embedded options and both option and convertibility features, for example, are all ignored. A comprehensive review of both the theory and application of PCA is given by Hansen (1982), Litterman and Scheinkman (1991), Knez *et al* (1994), Basilevsky (1995), Shao and Tu (1995), Rebonato (1996), Rebonato and Cooper (1996), Singh (1997), Golub and Tilman (1997), Thomas and Wilner (1997) and Buhler *et al* (1999).

Within the last 30 years, duration-based investment strategies have been intensively developed and implemented with professional fixed income portfolio managers employing duration concepts to manage, hedge and mitigate interest rate risk (Fisher, 2004). More recently, bond and mortgage portfolios that include instruments with embedded optionality have been explored in a variety of ways using a duration framework (e.g., Sarkar, 2000, Guntay, 2002 and Jacoby, 2003). As credit risk becomes increasingly established as a new realm of investigation and scrutiny amongst regulators and investors alike, defaultable securities and the influence of defaultability upon duration (and hence ultimate payoff) have also begun to be studied (Nawalkha, 1996 and 1999). The major shortfalls of the Macaulay duration are discussed in the following sections.

5.1.1 Defaultability

A criticism levelled at Macaulay's duration – that of its inability to correctly handle defaultable bonds or other defaultable fixed income instruments – could, in principle, have been dealt with in the 1940's when the Macaulay duration began to establish itself as the preferred fixed income volatility measure. An important factor that hampered early improvement was the then immaturity of the fixed income market. Default – though always present – was not then considered to be of catastrophic proportions and it was not until the debt market explosion of the 1990's, that disasters such as those of Barings (Leeson and Whiteley, 1996) or Long Term Capital Management (Lowenstein, 2001) were even considered possible – let alone the corporate meltdowns and Chapter 11 bankruptcies recently experienced by Enron (*The Economist*, 2001) and Worldcom (*The Economist*, 2002). These experiences led to the swift development of more flexible duration models.

Section 5.2 makes use of new, but existing, credit default models and then applies these results to the existing Macaulay duration measure. This approach has not been used previously and early research shows promising theoretical and empirical results.

5.1.2 Optionality

Optionality of fixed income instruments required Black and Scholes to solve their eponymous equation – achieved only in 1973 – before analysis on this problem could proceed with any earnestness. Optionality was in any case a rare bond feature and not worthy of impeding the rolling-out of the Macaulay duration measure in 1938.

Embedded optionality has become a feature of many retail products[23] as it provides investors with the option to alter the cash flow of products, often in response to changing interest rates,

[23] The US-based Fixed Investment Securities Database recorded some 18,500 US-dollar fixed-rate corporate bonds in April 2000, of which 65% (both in number and in offering amount) were callable (FISD, 2000).

and thus impacting on the amount of interest rate risk to which the bank is exposed. Some fixed rate mortgage products, for example, give borrowers the option to prepay the loan early without penalty. For these mortgages, prepayments tend to increase as interest rates fall and decrease as interest rates rise. The result is a divergence between scheduled and actual cash flows attached to the mortgage. Another example of embedded optionality is an at-call deposit, where investors have the option of withdrawing the deposit funds at any time. The treatment of such deposits is made all the more difficult by the fact that the interest rates received by customers are administered by banks, being driven largely by competition and internal cost factors.

Section 5.3 applies the well-known Black formula to the determination of the price of a bond with embedded optionality, but then introduces a probability measurement technique to estimate the duration of bonds with option-embedded features. This approach is new, but empirical results are encouraging.

5.1.3 Perpetual-maturity instruments

The Macaulay duration does not consider open-ended investments – such as bank deposits – despite the obvious need to be able to handle such instruments. These often constitute a large part of the banking book.

The above three problems will be considered empirically, using South African data where possible, to determine whether the Macaulay duration may be adapted to deal with these objections.

5.2 Duration of defaultable bonds

Institutions are increasingly facing credit (or counterparty) risk arising from many financial instruments, but exposure to corporate bond credit risk continues to be one of the leading sources of risk in financial institutions world-wide. The effective management of this credit risk is a critical component of a comprehensive approach to risk management and essential to the long-term success of any financial institution. The identification, measurement, monitoring and controlling of credit risk as well as the determination of adequate capital against these risks has called for increased attention to such bond features as optionality and defaultability, both of which alter the timing and quantity of promised cash flows for callable, corporate bonds. Prior research (e.g., Sarkar, 2000:509) has indicated the theoretical importance of adjusting Macaulay duration for the impacts of default and call.

This thesis, however, focuses on interest rate risk and thus considers defaultability from this perspective. Default-prone financial instruments will have an impact upon the earnings (net interest income) as well as the fair value of the book (economic impact).

5.2.1 Theoretical framework

5.2.1.1 Introduction

Financial institutions have faced difficulties over the years for many reasons, the major cause of serious problems continues to be directly related to lax credit standards for borrowers and counterparties, poor portfolio risk management, or a lack of attention to changes in economic or other circumstances that can lead to a deterioration in the credit standing of an institution's counterparties (BIS, 2001). Credit risk is defined as the potential that a borrower or counterparty will fail to meet its obligations in accordance with agreed terms and the aim of credit risk management is to maximise an institution's risk-adjusted rate of return by maintaining credit risk exposure within acceptable parameters (Schönbucher, 2003:56). The effective management of credit risk, then, is a critical component of a comprehensive approach to risk management and essential to the long-term success of any financial organisation. For most organisations, loans are the largest and most obvious source of credit risk, although credit risk (or counterparty risk) from instruments other than loans (including acceptances, inter-bank transactions, trade financing, foreign exchange transactions, financial futures, swaps, bonds, equities, options, and in the extension of commitments and guarantees, and the settlement of transactions) also exists. Institutions are increasingly realising the need to identify, measure, monitor and control credit risk as well as to determine that they hold adequate capital against these risks and that they are adequately compensated for risks incurred.

Among these measures, the Macaulay duration still features prominently (e.g., Nawalkha, 1999:13, Sarkar, 2000:522, and Jacoby, 2003:2308), but standard Macaulay duration – which determines the price sensitivity of loan instruments to interest rates – ignores two important loan features: the possibility of exercising embedded optionality and the probability of default. The former is considered in Section 5.3, the latter is discussed in the next section.

5.2.1.2 Bierwag's contingent cash flows

Bierwag (1987) recognised that calculating the Macaulay duration for the defaultable bond above may be severely inaccurate. Clearly, the promised cash flows may not materialise because of default losses and the yield to maturity may therefore not be realised either. Bierwag observed that, if F_t is the promised cash flow at time t, it may be replaced by an expected cash flow, F_t^* which adjusts for default losses. Assuming no other risks, the expected default losses may be accounted for using the risk free rate of return, r_F. The actual yield to maturity, $r(> r_F)$, includes the risk premium. Discounting the adjusted flows at the adjusted rate should be equivalent to discounting the promised flows at the risk free rate. If P is the price of such a security, then

$$P = \sum_t F_t^* \cdot (1 + r_F)^{-t} = \sum_t F_t \cdot (1 + r)^{-t}. \quad (5.1)$$

Two durations may thus be calculated: one using F_t^* and r_F and the other using F_t and r. There is, of course, no reason to expect the two durations to be equivalent. Using data from the previous sections and combing them with bond and bond-option pricing theory, the relevant Macaulay durations may be measured.

In order to calculate F_t^* and r, constituents of Equation 5.1 are first required. Measurements of both absolute and marginal probabilities of default are, however, non-trivial. Bierwag did not have at his disposal the tools and techniques which are in abundant use today. The remainder of Section 5.2 concerns the history of the efforts to determine and calculate the components embedded in Equation 5.1.

5.2.2 Literature Survey: defaultable and option-embedded securities

The duration research since the late 80's has focused on either defaultability or optionality and sometimes on both. Some work has attempted to isolate the effects of default and optionality and almost all of it has approached the problems from the point of view of claims models.

Fons (1990:21) presented a theoretical framework for the adjustment of duration for the risk of default and concluded that risk-adjusted duration should be *lower* than its Macaulay counterpart. His conclusion, however, depends heavily on his assertion that a bond's credit quality is negatively correlated with the level of the riskless term structure. Fons' empirical evidence demonstrates that, during the 1980–1988 period, the risk-adjusted duration of corporate bond indices of all ratings was significantly lower than their Macaulay duration. It is also important to note that Fons ignored optionality as a factor in measuring the bond's price sensitivity to changes in the riskless rate. Many of the corporate bonds included in the indices he used were, in fact, callable and subsequent studies have shown that optionality is perhaps an even more important adjustment factor for highly rated corporate bonds relative to the adjustment for default risk (e.g., Ho and Lee, 1986:1017, Jacoby, 2003:2301 and Acharya and Caprenter, 2002:1359).

Babbel, Merrill and Panning (BMP) (1997) compared interest sensitivities of risk-prone and riskless bonds and isolated the effects of default risk. BMP utilized option-pricing technology to calculate risk-adjusted duration and compared it to the duration of matched, synthetic Treasury bonds. They found that risk-adjusted duration was lower than riskless duration for all levels of yield spreads, with the difference monotonically increasing with yield spreads. The statistical robustness of BMP's results, however, has been questioned (see, for example, (Jacoby and Roberts, 2003:2300) and it has been pointed out that the implications of their results are less impor-

tant to bond portfolio managers who use Macaulay duration with the risky yield. This is because the underlying reference to estimate the sensitivity of the value of a risky bond changes in the riskless term structure.

Fooladi, Roberts and Skinner (FRS) (1997:2) derived a risk-adjusted duration measure for corporate bonds while considering both risk aversion of investors and a delay period in recovery in the default process (a feature introduced in a different form by Bierwag and Kaufman, 1988:41). FRS showed that incorporating the delay period, especially for zero-coupon bonds with a material risk of default, may result in a risk-adjusted duration greater than the bond's time to maturity. This result was attributed to the positive probability that the issuer of a zero-coupon bond would default at maturity and bondholders would receive any recovered amount only after a certain waiting period (delay period). Jacoby (2003:138) also included risk aversion and delay in recovery, and found a similar result. Jacoby's risk-adjusted duration may also be greater than the bond's maturity. However, it will always be greater than the Fisher-Weil duration (or Macaulay duration) when flat term structures are assumed, due to the delay period. Like FRS, Jacoby assumed independence between the probability of default and the riskless term structure.

Chance (1990:266) provided the first contingent claims model in an effort to understand the effects of default risk on corporate bond duration. His model was based on fairly restrictive assumptions (namely that the duration of a firm's underlying stock could only be greater than zero since the duration of the firm's assets were assumed to be zero). Merton's (1974:452) option-pricing model for the valuation of risky pure discount bonds was used (by Chance (1990)) as well as results obtained by Garman (1995:311) for the sensitivity of an option's value to changes in interest rates. Chance expressed the duration of the discount bond as the weighted average of the duration of a riskless discount bond and that of a put option written on the firm's assets. Like Merton, Chance assumed that the firm's assets followed a diffusion process independent of the short riskless interest rate process and, therefore, the duration of the firm's assets must be zero. Although Chance's model is intuitively appealing, it is difficult to construct in practice because the duration of a firm's assets cannot be directly observed and is thus assumed to be zero. Zero asset duration is also implicitly assumed in the work of Merton (1974:453) and Galai and Masulis (1976:119) since they are both based on the Black and Scholes option pricing framework.

Nawalkha (1996:230) extended Chance's model by allowing dependence between the two processes, implying a nonzero duration of the firm's assets. Unlike Chance's duration, Nawalkha's duration was not restricted to be lower than the maturity of the risky discount bonds. Since the Macaulay duration of a pure discount bond is its time to maturity, Nawalkha's model implied that the risk-adjusted duration of a pure discount corporate bond could be either lower than, equal to,

or higher than its Macaulay duration. Nawalkha (1999:7) provided a contingent claims analysis of the interest rate characteristics of corporate liabilities by using Vasicek's (1977:179) mean-reverting term structure model in Merton's (1974) option pricing model. He found that the duration of defaultable securities, issued by firms having assets with low interest rate-sensitivity, was an increasing function of the bond's default risk: the reverse is true for bonds having assets with a high interest rate-sensitivity. An important consequence of Nawalkha's model is that the duration of a defaultable zero coupon bond will be greater than the corresponding duration measure proposed by Chance.

Acharya and Carpenter (2002:1362) developed a model for the valuation of callable defaultable bonds. In their paper, both interest rates and firm value were stochastic, and the call and default decisions were endogonised. With respect to interest-rate sensitivity, as in other models applying option-pricing technology, their model implied that default risk alone reduced the bond's duration. Acharya and Carpenter also showed that call risk, when isolated from the risk of default, shortened bond duration. When considering both default and call risks together, they demonstrated that as the probability of default increased, the impact of the call option on the bond duration diminished. A higher default risk provided an incentive to the issuer to wait longer before calling the bond, and resulted in a higher effective duration for the bond (Ho and Lee, 1986:1018).

Kihn (1994:33) explored default and call risks for corporate bonds. He paid special attention to the case where both credit quality and interest rates decline. For low grade bonds the credit deterioration depresses bond prices to a level low enough to discourage issuers from exercising the call provision. Kihn provided empirical support for his hypothesis by analyzing return volatility of high-grade and low-grade bonds. His results supported the theoretical implications of Acharya and Carpenter's (2002) bond pricing model.

Bierwag *et al*, (1983a:17, 1983b:118, 1983c:109), Morgan (1986:88) and Ott (1986:927) all extended work on the Macaulay duration measure used for corporate securities, but all ignored the effects of default risk. Bierwag and Kaufmann (1988:41) did further work in which the assumption that a simple linear relationship exists between the risk adjusted return and a defaultable duration was made. The error in estimating the Macaulay duration (by not taking stochastic processes of default into account) was found to be larger the greater the default risk premium.

Xie *et al,* (2002) used the Cox, Ingersol, Ross (1985:386) term structure model to evaluate duration of defaultable securities. Xie *et al,* found that the duration for defaultable bonds could be longer or shorter than default-free bonds, depending on the relationship between interest rates and the default intensity of defaultable bonds.

Several approaches have been developed regarding optimal exercise strategies for embedded call options. In particular, one can distinguish between models using partial differential equation (PDE) techniques and the Martingale approach. Regarding the PDE-based literature, models with default were advocated by Merton (1974), who argued that prices of a callable, defaultable bond solve a PDE subject to boundary conditions that describe default and call events. Closed-form solutions do not exist therefore finite-difference methods are applied. Kim *et al,* (1993:120) extend this work by allowing for stochastic interest rates, but found no difference between these results and those using finite-difference methods.

In Sarkar (2000:511), imperfections in the capital structure (refunding costs, taxes, bankruptcy costs) alter the optimal call policy. Martingale methods simplify the calculation of prices of bonds with embedded options. Duffie and Singleton (1999:692) relied on the Martingale approach to price callable, defaultable bonds assuming that the issuer calls the bond to minimize its market value, which is optimal assuming perfect capital markets and the absence of other motives to exercise bond calls or to postpone exercise. Acharya and Carpenter's (2002:1360) model used call and default options as American options written on a non-callable, default-free bond with fixed continuous coupons. In a related paper, Guntay (2002) proposed a double-hazard (see Section 4.2.4) framework to price callable, defaultable coupon bonds. He modelled call risk and default risk as two correlated hazard processes, and allowed taxes and refunding costs to affect the arrival rate of the call, and firm characteristics to influence the arrival of default. Peterson and Stapleton (2003:182), in a recent contribution to the pricing of options on credit sensitive bonds, built a three-factor model for the term structures of default-free yields and for correlated credit spreads. They priced Bermudan-style options on defaultable bonds using a recombining log-binomial tree methodology. These models are complex in both formulation and implementation and are thus of limited use in banking environments which require the accurate, but prompt, measurement of duration.

All of the work discussed in this section was considered for the provision of a robust yet straightforward duration measure in the South African market. Whilst results gathered from – and techniques employed by – these authors tempered the direction of study, their work was ultimately not used. Regulatory and accounting standards demand that an interest rate risk measure be not only sufficiently accurate for everyday use, but also easily implemented, maintained and updated. In the light of these requirements, the work suggested by Bierwag (1987:324) was used as a starting point. These ideas are then expanded in this chapter to embrace financial instruments which face the probability of default.

Bierwag (1987:324) suggested – but did not develop further – ideas on embedded optionality and defaultability, now Bierwag considered the problems of defaultability and optionality from the point of view of simple discounted cash flow pricing models. Using the definition of the Macaulay duration as a price elasticity, he considered the evolution of the Macaulay duration in terms of its constituent factors. He argued that different cash flows arise from different contingencies and thus give rise to different Macaulay durations. For defaultable securities, the Macaulay duration changes because of different cash flow outcomes (see Section 5.2.1.2). Bierwag (1987) did not explore these ideas much beyond this point, possibly because the then fledgling credit market was woefully unsophisticated. Only a rudimentary body of knowledge governing the probability of default, recovery rates and time to default of financial instruments existed.

The recent upsurge of interest in the field of credit risk has provided quantitative analysts with an array of tools for the estimation of default probabilities, recovery rates and times to default (e.g. see Sarkar (2002), Xie and Lui (2002), Schönbuchler (2003), Rutkowski (2003) and Peterson and Stapleton (2003)). These tools range from fairly straightforward models to highly complex, stochastic descriptions of the credit environment, but almost none were available nor in frequent use in 1987. These tools have now been adapted, implemented and tested in this thesis to extend and embellish Bierwag's suggestions regarding different cash flows for different contingencies.

The next section presents a background to the pricing of defaultable securities with the aim of measuring the different cash flows suggested by Bierwag (1987).

5.2.3 Pricing of defaultable securities

5.2.3.1 Introduction

A duration measure capable of accounting for default-prone securities first requires a pricing model to establish the fair value of contingent cash flows. Recall that the Macaulay duration is the present value of the time-weighted sum of relevant cash flows (Equation 3.3). The measurement of the Macaulay duration for a default-prone security, therefore, requires knowledge of all cash flow amounts – both contingent and known – as well as probabilities associated with each event. More complex models require information about two more stochastic parameters, namely times to default and recovery rates. The market is a rich source of such information. Real-time prices and yields of most publicly traded instruments are easily obtained from data vendors such as Reuters™ and Bloomberg™, but encoded in these data is the market's assessment of obligors' default risk, as well as price structures of various other prevailing risk premia. This embedded information may be recovered and utilised to price new securities (about which little else may be

known), provide an understanding of default probabilities and the timing thereof during the life of instruments or simply supply a risk profile assessment to expose arbitrage opportunities.

Extraction of these encoded data, whilst not necessarily deeply complex, does require some effort and comprehension of the mechanics of probability theory (structural models) or the Black and Scholes equation (closed form models). The effort is warranted only if certain assumptions about the market and its participants are made. Firstly, some degree of market efficiency must be assumed. If this is true, the market as a whole will embrace more information than any single participant, and the amassed information will be aggregated into market prices (Audet and Doshi, 2003). Secondly, models which are based upon and calibrated to prices of traded assets will be immune to simple arbitrage strategies using these traded assets (though this does not exclude using other traded instruments to create arbitrage opportunities). Finally, implied default probabilities that have been recovered from market prices of traded assets also supplies risk premia associated with this type of risk (Caouette *et al*, 1998:89).

A brief literature survey regarding the pricing of default-prone corporate securities is presented in the next section. This survey provides a general background to work whose aim is the linking default probability and frequency to market-observable credit spreads.

5.2.3.2 Literature survey: credit pricing

The theory of pricing credit risk was originally developed for valuing risky corporate debt. Credit pricing models are usually split into two distinct groups: *asset-based* (or structural) models and *intensity-based* (or reduced form) models.

Asset-based models assume a value process for the firm's underlying assets. The firm defaults when the value of the firm's underlying assets drops below its financial commitments or below an exogenously specified boundary. An aspect of the structural models may also involve debtor's incentive patterns. This approach was proposed by Merton (1974) and developed further in work performed by Black and Cox (1976), Fischer (1989), Hull and White (1995) and others (e.g., Longstaff and Schwartz, 1995).

Reduced form models assume an exogenously specified bankruptcy process. These intensity based or reduced form approaches were developed by Jarrow and Turnbull (1995) and refined by Jarrow *et al,* (1997), Lando (1998), Duffie and Singleton (1999), and others (e.g., Frey and McNeil, 2003). Recent research has focused on imposing specific dependence structures on observed market data. Research in this direction includes Li (1998), Frey *et al*, (2001), Schönbucher and Schubert (2001), Hull and White (2001), and Jouanin *et al,* (2001). All of the above

may be mapped into the so-called hybrid approach, which integrates the important advantages of both the structural and reduced form approaches.

Berndt *et al*, (2004) estimated default risk premia for U.S. corporate debt, based on a close relationship between default probabilities, as estimated by Moody's KMV expected default frequencies and credit default swap market rates. The default-swap data, obtained from 22 US banks and specialty dealers, allowed the authors to establish a strong link between actual and risk-neutral default probabilities for the 69 companies analysed. Risk premia were also found to vary dramatically over time. A simple linear model of the relationship between a firm's 5-year credit default spread and the 5-year expected default frequency (measured in bps on the same day) was used. Bohn (2000:60), Delianedis and Geske (1998:27), Delianedis *et al*, (1998), and Huang and Huang (2003) used structural approaches to estimating the relationship between actual and risk-neutral default probabilities, assuming that the asset value process is described by the Black-Scholes-Merton and volatility is constant.

South African corporate default frequencies are extremely difficult to source and those that do exist have relatively short histories (ICP, 2005). The corporate defaults, therefore, provide insufficient data for determining risk premia using Berndt's methods. Instead, a generalised reduced form model – adapted from Schönbucher and Schubert (2001) – was used. This model was chosen as it is both widely used and simple to understand. In addition, Bierwag (1987:324) alluded to a similar idea[24] and indeed, it is the underlying model used by Li (1998:41) in the construction of the credit curve. Li's credit curve construction is discussed in Section 5.2.4.

5.2.3.3 General credit pricing model

A defaultable corporate bond, with parameters as defined below, may be priced using:

$$P = \underbrace{\sum_{i=1}^{n} c_i \cdot \left(1 - P_{\mathrm{A}}(i)\right) \cdot e^{-rt_i}}_{\text{PVs of coupon cash flows (no default)}} + \underbrace{\sum_{j=1}^{n} N \cdot R \cdot \left(P_{\mathrm{M}}(j)\right) \cdot e^{-rt_j}}_{\text{PVs of nominal (conditional on default)}} + \overbrace{N \cdot e^{-rt_n} \cdot \left(1 - P_{\mathrm{A}}(n)\right)}^{\text{PV of nominal (no default)}}, \qquad (5.2)$$

where it is implicitly assumed that default can only occur on coupon payment dates and:

c_i is the i$^{\text{th}}$ coupon cash flow,

r is the yield to maturity,

e^{-rt_i} is the continuously compounded discount factor for the i$^{\text{th}}$ period,

t_i is the i$^{\text{th}}$ coupon period,

[24] As stated previously, these ideas were never developed further until now.

n is the number of payment periods until maturity,

$P_A(i)$ is the cumulative, absolute probability of default,

N is the face value of the bond,

R is the recovery rate, expressed as a percentage and

$P_M(i)$ is the marginal probability of default in the period $i-1$ to i (Schönbucher, 2003:55).

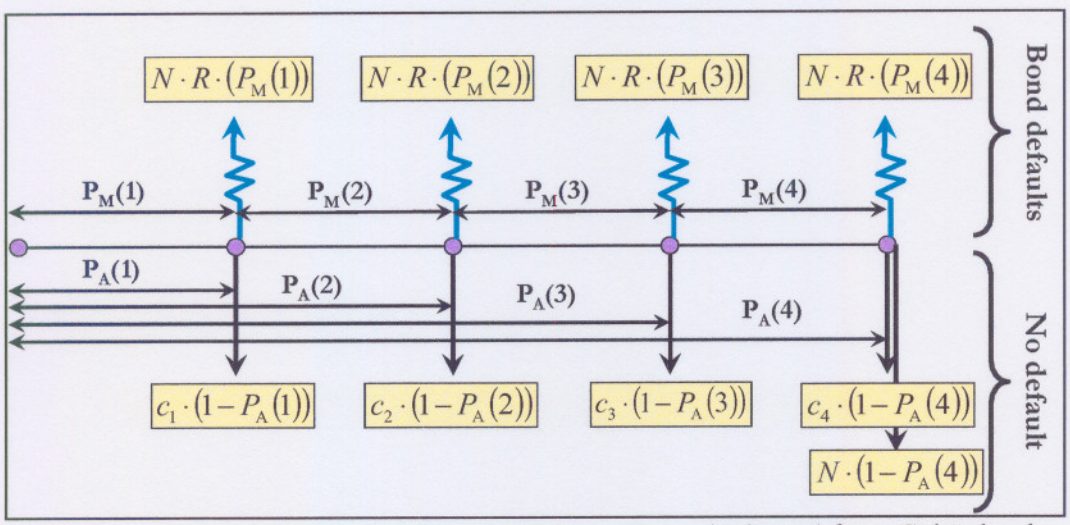
Thus, for a two-year, defaultable, semi-annual, coupon bond (four payment dates) may be valued by discounting the cash flows indicated below in Figure 5.1, by the continuously compounded yield to maturity, r. Blue lines (upwards) indicate payments payable in the event of default. Black solid lines (downwards) indicate payments made if no default occurs. Thus, at Node 1 (end of first coupon period):

- **Default**: (probability of defaulting between $t=0$ to $t=1$ is $P_M(1)=P_A(1)$), coupon paid; only a fraction (R) of the face value (N) is paid out – given by $N \cdot R \cdot (P_M(1))$.
- **No default**: (probability $1-P_A(1)$), coupon (c_1) paid.

At Node 2 (end of second coupon period):

- **Default**: (probability of defaulting between $t=1$ to $t=2$ is $P_M(2)$), coupon not paid; only a fraction (R) of the face value (N) is paid out – given by $N \cdot R \cdot (P_M(2))$.
- **No default**: (probability $1-P_A(2)$ where $P_A(2)=\int_{t=0}^{2} P_A(t)\cdot dt$), coupon ($c_2$), and so on.

Figure 5.1 Contingent cash flows used to price a defaultable corporate bond.



(Adapted from Schönbucher, 2003)

Equation 5.2 provides a robust means of establishing the price of a default-prone security. If the assumption is made that recovery rates and times to default are deterministic (requirements of this model, see (Schönbucher, 2003:57)), the problem of determining default probabilities still remains. These may be measured using Li's (1998) work and their extraction is discussed in the next section.

5.2.4 Credit curve construction

5.2.4.1 Introduction

The work described in this section was based upon Li (1998:42), who used a hybrid approach of the credit pricing models described in the previous section. Hazard rates (discussed in the next section) were obtained using observed credit spreads and default probabilities were derived directly from these.

The credit curve is as central to the pricing of credit derivatives and defaultable securities as the yield curve is to fixed income instruments. A credit curve gives the instantaneous probability of default of a party at any time in the future. Li (1998:42) proposed a hybrid of the reduced-form and intensity-based models described above for the construction of a credit curve, based entirely on market-observable data. Yields to maturity of corporate securities may be obtained from market prices and these may be compared with the yields to maturity of treasury securities. A spread curve is thus obtained. An exogenous assumption about the recovery rates for these securities is made using the seniority and rating of these securities coupled with knowledge of the industrial sector to which the issuing corporation belongs (Houweling and Vorst, 2001).

Li's approach differs from rating agencies such as Moody's Investor Service (Dwyer *et al*, 2004) or Standard & Poors in that market information – rather than historical information – is used at all times. This approach was justified on the basis that:

- Rating agencies react more slowly than the market to changes in future credit quality of corporate securities.
- Rating agencies are primarily used to estimate default *frequency* rather than default *severity*. Credit derivative values depend on both these parameters.
- Rating agencies usually provide the 1-year default probability and the rating migration index for each rating group. Neither of these is necessarily stable over long periods. Many corporate securities and credit derivatives have maturities beyond one year, necessitating the use of long term marginal default probabilities (Caouette *et al*, 1998:88, Houweling and Vorst, 2001).

- Calculation of profit and loss for a trading desk can only be based on current market information since it is that information that reflects the market-agreed perception about the evolution of movements in the market in the future (Rutkowski, 2003:18). Current default rates may differ considerably from historical default rates (Audet and Doshi, 2003).

Using only market information also has associated setbacks. Only a few large corporates have a series of outstanding debts with the same seniority and multiple maturities (both required as inputs for Li's model). However, in the absence of this information, asset swap spreads from similar firms may be used as a proxy.

In the next section, the rich field of hazard rate theory is discussed and contextualised. The probabilities of occurrence of cash flows are determined using hazard rates which may in turn be derived from corporate bond spreads. Spreads of highly liquid securities are market-priced and those of illiquid securities are derived from the spreads of comparable, liquid securities. Knowledge of hazard rates leads directly to knowledge of default probabilities and these allow the calculation of all cash flows (and hence the price) associated with default-prone securities.

5.2.4.2 Hazard rates

Assume that the instantaneous rate of default of a given corporate security is given by h (the 'hazard rate') such that the probability of default, p, in a short period of time δt is given by $h \cdot \delta t$. The quantity δt may be defined as $\delta t = t/n$ where t is the full period length and n is the number of small divisions in the full period. The probability of surviving, $(1-p)$, a period of length t is the probability of surviving each period of length δt raised to the power n. Assuming the probability of default follows a geometric probability, the probability of surviving is given by:

$$P(\text{surviving period } t) = \left(1 - h \cdot \frac{t}{n}\right)^n.$$

Borrowing from proofs derived for continuous compounding, it can be shown that[25]:

$$\lim_{n\to\infty}\left(1 - h \cdot \frac{t}{n}\right)^n = e^{-ht},$$

and thus

$$P(\text{surviving period } t) = e^{-ht}$$
$$P(\text{defaulting in period } t) = 1 - e^{-ht}.$$

The probability of surviving a period of length mt, assuming a geometric distribution, is simply:

[25] See Appendix A for the proof of this limit statement.

$$P(\text{surviving period } mt) = P(\text{surviving period } t)^m$$
$$= \left(e^{-ht}\right)^m$$

or

$$P(\text{surviving period } mt) = e^{-mht}. \tag{5.3}$$

Thus:

$$P(\text{defaulting in period } mt) = 1 - e^{-mht}.$$

Setting $t = 1$ gives:

$$P(\text{default in 1st year}) = 1 - e^{-h}$$
$$p_1 = 1 - e^{-h}$$
$$h = -\ln(1 - p_1).$$

Thus an annual probability of default of 5% is equivalent to a hazard rate of 5.13%.

The advantages of using hazard rates to drive the regular geometric distribution approach are that they are simpler to apply in practice, they combines well with fixed income mathematics and they may be consistently applied to each period. Hazard rates translate directly into default probabilities and are extensively used in Li's (1998) model. A new methodology introduced by Li[26] (1998:41) makes direct use of observed market credit spreads to determine hazard rates and hence default probabilities. It was this model that was employed in the testing of South African data and is presented in the following section.

5.2.4.3 Li's continuous credit curve model

The assumptions made by Li's (1998) model are:

- when default occurs, the bond holder receives a fixed percentage of the bond price (i.e., the recovery rate) immediately prior to default (see Duffie and Singleton (1999:693) for a similar argument),
- the credit curve consists of a series of default probabilities over each period, conditional on the survival to the beginning of the period (conditional marginal Martingale default probabilities),
- the current time is used as the time origin to allow use of current market information to build credit curves and
- the meaning of 'default' has been defined by a number of rating agencies, such as Moody's (Moody's, 2003 and Dwyer *et al*, 2004).

Li's starting point was the Duffie and Singleton (1999) model for pricing defaultable fixed income securities, given by:

[26] This work was later refined by Schönbucher (2003:65).

$$V(t_0) = \sum_{i-1}^{n} P(t_0, t_i) \cdot \left(\prod_{j=0}^{i-1} \left[p_j + (1 - p_j) \cdot R(t_{i+1}) \right] \right) \cdot C_i . \tag{5.4}$$

where $V(t_0)$ is the defaultable bond price measured at t_0,

$P(t_0, t_i)$, the discount factor, is measured at t_0 for a time t_i,

p_j, the conditional survival probability is measured over the period t_i to t_{i+1},

$R(t_{i+1})$, the recovery rate measured over the period t_i to t_{i+1} and

C_i is the i^{th} cash flow, occurring at t_i.

Equation 5.4 above is also referred to as the discrete model in which default is assumed to occur only at the end of the default period. The continuous model of default, which allows for default at any stage in the period under consideration, is given by:

$$p_n = P(\text{surviving period } nt) = e^{-\int_{t_n}^{t_{n+1}} h(s)ds} . \tag{5.5}$$

Expanding the square brackets in Equation 5.4 above and making use of Equation 5.5 gives (if $\Delta t = t_{i+1} - t_i$ is small):

$$p_n + (1 - p_n) \cdot R(t_{i+1}) = e^{-\int_{t_n}^{t_{n+1}} h(s)ds} + \left(1 - e^{-\int_{t_n}^{t_{n+1}} h(s)ds} \right) \cdot R(t_{n+1})$$

$$\approx e^{-(1-R(t_{n+1}))\int_{t_n}^{t_{n+1}} h(s)ds} .$$

The credit discount factor (large round brackets in Equation 5.4 above) is:

$$\prod_{n=0}^{i-1} \left[p_n + (1 - p_n) \cdot R(t_{i+1}) \right] \approx \prod_{n=0}^{i-1} e^{-(1-R(t_{n+1}))\int_{t_n}^{t_{n+1}} h(s)ds}$$

$$\approx e^{\sum_{n=0}^{i-1} \int_{t_n}^{t_{n+1}} (1-R(t_{i+1}))h(s)ds} .$$

Dividing the time interval $(0, t_i]$ into many smaller time intervals of length $\Delta t = t_i / n$, and allowing $n \to \infty$ allows Equation 5.4, the discrete model, to be reformulated into the continuous model, namely:

$$V(t_0) = \sum_{i-1}^{n} C_i \cdot e^{-\int_0^{t_i} [r(s) + (1-R(s))h(s)]ds} , \tag{5.6}$$

where the symbols have the same definitions as for Equation 5.4 and it has been assumed that

$$P(t_0, t_i) = e^{\int_0^{t_i} r(s)ds} ,$$

where $r(s)$ is the short term rate.

The equations governing the components required for Bierwag's Macaulay duration comparison (Equation 5.1) are complete. The description of their combination is discussed next.

5.2.4.4 Component combination

Using Equation 5.6, coupled with a known spread-over-Treasury curve for a given defaultable security, generates a term structure of hazard rates peculiar to that security (Houweling *et al*, 2001:312). From these hazard rates, the marginal (and hence cumulative) default probabilities for the security are generated using Equation 5.5. These probabilities are used in Equation 5.2 to generate the price of the defaultable security and finally, using Equation 5.1, the defaultable cash flows are now used to estimate the Macaulay duration instead of the default free cash flows. In this manner the Macaulay duration – adjusted for defaultable cash flows – is produced.

The techniques and methodologies which combine to determine the constituents of Equation 5.1 (as well as embedded components) are not new, but the combination of these disparate procedures – and the empirical testing of these on relevant data – has not been attempted until now. This combination and testing is discussed in the next section.

5.2.5 Empirical results

South African market data were used for all inputs[27]. A sample of 120 option-free, corporate bonds was selected spanning the 8-year period from 1 January 1997 to 31 December 2004. Bond maturities ranged from $5 \leq t \leq 30$ years and were of varying credit quality[28] – Aaa (investment grade) to Caa (highly speculative[29] grade). Credit spreads over the South African yield curve were obtained from Bloomberg™, stated[30] either as a fixed, parallel spread (single value over all maturities) or as a term structure (different for all maturities). Option-free bonds with long dated maturities ($t > 25$ years) are virtually non-existent. As the period selected approaches the present (2005), option-free corporate bonds with shorter maturities become increasingly difficult to source, reflecting both issuer and purchaser reluctance to lock in interest rates for even short periods. Coupons ranged from 8% to 18% and were paid quarterly, semi-annually or annually. Recovery rates used were those associated directly with the issuer (as reported by corporate news services, such as Reuters™ and Bloomberg™), those appropriate for the specified credit rating or, where data were particularly limited – 30% being the average recovery rate for South African corporate bonds over the period under examination (Moody's, 2005).

Figure 5.2 shows the results for a typical defaultable corporate bond (maturity 30 years, semi-annual 8% coupon, risk-free rate 7.5%, recovery rate 30%).

---

[27] The South African corporate bond market is small compared with those of developed economies (Tikkam, 2005).
[28] Moody's™ ratings – [Internet] Bloomberg™.
[29] Speculative grade corporate bonds, although plentiful in the marketplace, have notoriously high and unreliable credit spreads coupled with low recovery rates. These parameters, used in Equation 5.6, give rise to highly variable hazard rates and both absolute and cumulative default probabilities. Corporate bonds with speculative grade credit ratings were therefore used sparingly, comprising about 10% of the total sample.
[30] In bond contract confirmations (where available) from corporates themselves or their relevant auditors.

Figure 5.2. Inputs and outputs to produce the duration/yield relationship for defaultable and default-free securities.

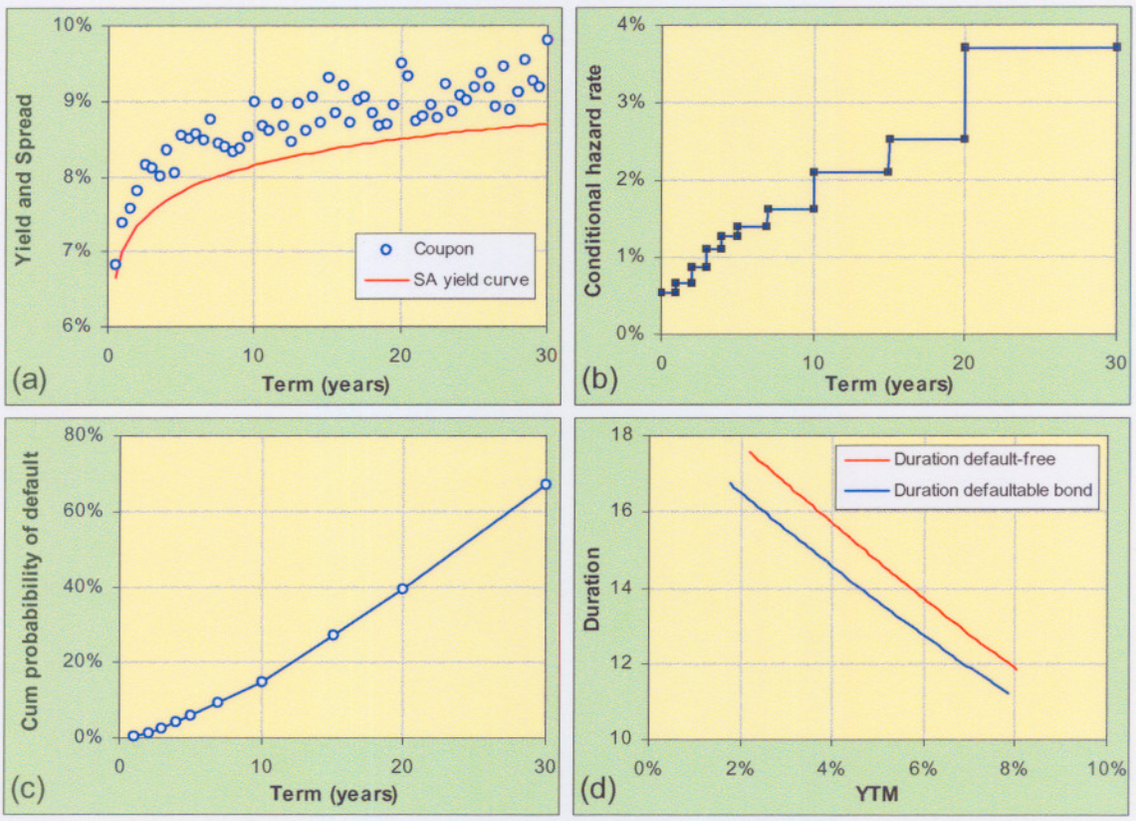


Figure 5.2(a) shows that the spreads over the South African treasury curve increase by 50bp (1y) to 100bp (30y) with high variability. The yield curve is upward-sloping from 6% (1y) to 9% (30y). Hazard rates range from 0.5% (1y) to 3.5% (30y) (Figure 5.2(b)). Cumulative default probabilities are shown in Figure 5.2(c) and are generated using Equation 5.6 and known spreads from Figure 5.2(a). The aim of this investigation is shown in Figure 5.2(d), namely, the effect of the possibility of default (Equation 5.2) on the Macaulay[31] duration.

Corporate securities with higher and lower recovery rates, shorter and longer maturity periods and higher and lower credit spreads at various periods during the economic cycle were tested and the results remain broadly the same, i.e., default free Macaulay duration is higher than the defaultable Macaulay duration by between 6% and 8% for all yields, with the difference more pronounced at higher yields. A summary of results is presented in Table 5.1.

Measurements were taken at the mid-yield to maturity for the corporate bond in question. The rightmost column represents the percentage difference between the defaultable Macaulay duration and the default-free Macaulay duration.

[31] The effect of defaultability on the bond convexity was also examined in this analysis. The convexity was large for low yields, small for high yields, and of considerably smaller magnitude ($<10^{-2}$) than the duration effect. These results add nothing new to the understanding of the effect of defaultability on duration since all of these effects are well known from analysis performed on default-free bonds.

Table 5.1. Summary of corporate, option free bond sample results.

	Sample size	Range	Average difference $D_{\text{Mac}} - D_{\text{Defaultable}}$ (%)
Coupon	31	8% - 10%	6.00
	49	10% - 12%	6.75
	28	12% - 14%	7.25
	12	14% - 18%	8.00
Spreads	23	25 – 50 bps	6.00
	41	51 – 100 bps	7.00
	56	101 – 200 bps	8.00
Maturity	47	5 - 10y	8.00
	38	10 – 15y	7.25
	29	15 – 20y	6.50
	6	20 – 30y	6.00
Credit rating	16	Aa	6.00
	32	A	6.50
	27	Baa	6.75
	19	Ba	7.00
	14	B	7.25
	12	Caa	8.00
Recovery rate	12	< 15%	8.00
	12	15 - 25%	7.50
	76	25 - 35%	7.00
	17	35 - 50%	6.50
	3	> 50%	6.00

In order to test these results against market prices empirically, a sample of 80 option-free corporate bonds was selected. The securities had maturities ranging from 10 to 30 years and paid annual or semi-annual coupons of between 8% and 18%. The market data were sampled over the 8-year period 1 January 1997 $\leq t \leq$ 31 December 2004 thereby encompassing periods of both market stability and expansion (1997 to mid-1998, 2003 to 2005) and market turbulence and contraction (mid 1998 – 2003). The price, yield to maturity and Macaulay durations (both default-free (Equation 3.4) and defaultable (Equation 5.2)) of each security were calculated at the end of each month during the period under examination. The actual change in price between one month end and the next was then measured as well as the known change in yield over this period. The two sides of Equation 3.4 were regressed upon each other, ie, $-(dP/dy)\cdot 1/(n\cdot P)\cdot(1+y/n)$ versus D_{Mac} (defaultable and default-free). If Equation 3.4 is indeed an accurate description of the way in which interest sensitive securities react to changes in yield, the gradient of this regression line should be 1. If the Macaulay duration, D_{Mac}, *over*estimates the change in price of an

interest rate-sensitive security, the gradient will be less than 1 and if D_{Mac}, *under*estimates the change in price, the gradient will be greater than 1. The results obtained in this study – summarised in Figure 5.2 – indicate that the default free Macaulay duration measure consistently *over*-estimates the change in price of defaultable securities. This overestimation leads to the result that

$$\frac{\left[-\frac{dP}{dy}\cdot\frac{1}{n\cdot P}\cdot\left(1+\frac{y}{n}\right)\right]}{D_{\text{Mac}}}<1,$$

for the default free case. The same quantity for the defaultable Macaulay duration gives gradients statistically closer to unity (see R^2 values in Figure 5.3 below). The results for each of the years 1997 through 2004 are presented in Figures 5.3(a) through (h) below.

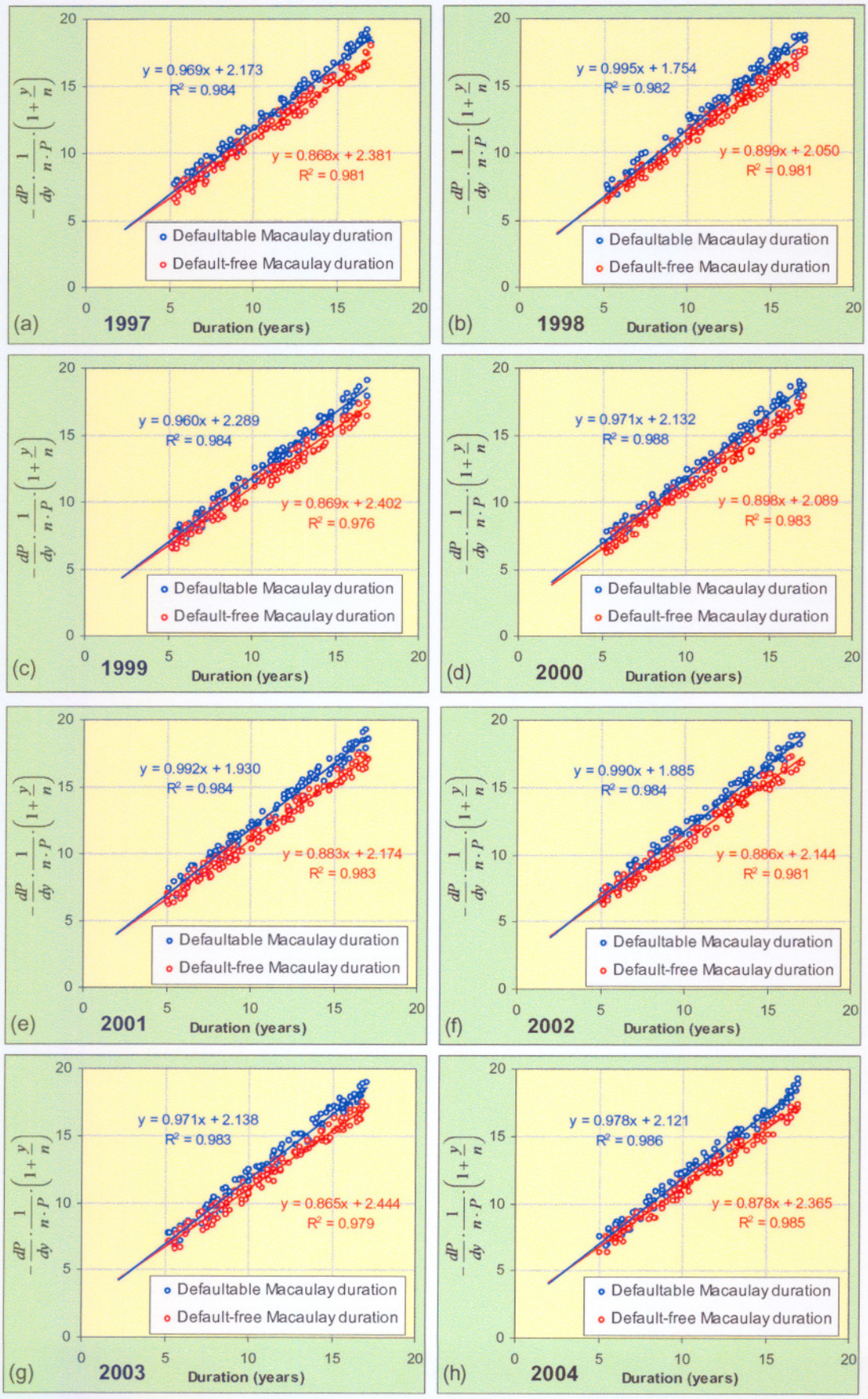
For short periods remaining to maturity ($t \approx 5$ years) – there is no statistical difference between the default-free and defaultable Macaulay duration regression lines. However, for longer maturities, a pattern emerges which *is* statistically significant. The defaultable Macaulay duration regression lines have higher gradients for each year and all are closer to unity than the default-free Macaulay duration regression lines. High R^2 values are observed for all years (see Appendix C.)

These results are consistent with those of Fons (1990:37), Babbel, Merril and Panning (1997:41) and Acharya and Carpenter (2002:1380), but contradict those of Fooladi, Roberts and Skinner (1997:4), Nawalkha (1999:241) and Xie *et al,* (2002). The consistency of results between empirical with theoretical tests is, however, encouraging. Potential applications of these results to bank portfolios are discussed in the next section.

5.2.6 Application of results

The credit market is enormous and growing and the appetite for credit derivatives shows no signs of slowing. Any financial institution's portfolio which comprises even few defaultable securities can potentially make use of these new results. The defaultable Macaulay duration calculation is simple to implement in itself, but requires some preliminary work to measure corporate credit spreads, estimate hazard and recovery rates and calculate default probabilities. These measurements and calculations, however onerous and time consuming, are nevertheless becoming a more commonplace feature of banks' trading books, especially since the mandatory (January 2005) implementation of FAS 133 (IAS 39) on a worldwide basis. Traded interest rate risk is now required to be 'on balance sheet' in contrast to prior treatment of traded interest rate-sensitive securities. The accurate measurement of the interest rate risk of these securities is thus of significant concern to financial institutions in general and banks in particular.

Figure 5.3(a) – (e). Regression of $-\frac{dP}{dy}\cdot\frac{1}{n\cdot P}\cdot\left(1+\frac{y}{n}\right)$ on defaultable and default-free Macaulay duration, D_{Mac} for 1997 through 2004.



Measuring and reporting default probabilities is expected to become widespread in the near future as banks endeavour to comply with strict new regulatory rules regarding the accurate measurement of these parameters (Caouette *et al*, 1998:91). With these in place, implementing the default-adjusted Macaulay duration requires relatively little manipulation of existing data used in any case to calculate the default-free Macaulay duration.

Overestimating portfolio duration (and hence portfolio value sensitivity to interest rate changes) by even a few percent, can have adverse effects upon banks' balance sheets – especially if the prevailing interest rate environment is volatile. Larger institutions and banks are supported by separate technology within specialist systems, but the majority – 85% – still rely on spreadsheets (Mosher, 2005). Adjustments to the existing Macaulay duration measure, suggested in this section, may be easily incorporated into existing spreadsheet models and show promising early results toward the prevention of inaccurate forecasts made by the default-free Macaulay duration.

Having examined the consequences of **defaultability** upon the Macaulay duration, the next section reviews and extends the mathematics of the Macaulay duration measure when the probability of exercising an option embedded in a fixed income security becomes non-negligible.

5.3 Duration of securities with embedded options

Embedded optionality in retail products is in many ways analogous to (and thus may be modelled by) the call (or put) option provisions present in the majority of corporate bonds. For such bonds, the issuer may redeem the bond only a pre-specified price and usually only after it has been outstanding for a pre-specified number of years.

The duration of corporate securities with embedded optionality is, however, not yet well understood and the mathematics governing the probability of exercising embedded options is non-trivial. Section 5.2.2 presented an overview of work performed in the field of embedded optionality (coupled with work regarding corporate security defaultability). The next section briefly introduces further recent work which attempted to separate bond optionality from bond defaultability (without success).

5.3.1 Theoretical framework

5.3.1.1 Literature survey

Berndt (2003) extracted credit pricing information from the prices of callable corporate debt, by disentangling the components of callable corporate bond prices associated with discounting at market interest rates, discounting for default risk, and optionality. The value of the call option was found to be sensitive to both interest rate risk and to the risk of changes in credit quality, but

two unresolved issues were identified. First, data limitations did not allow Berndt to break down short spreads into the risk-neutral mean fractional loss rate due to credit risk and the risk-neutral mean fractional cost rate due to illiquidity. Second, research showed that investors 'do not necessarily price callable debt as if the issuer would redeem it rationally' (Berndt, 2003:1). Modelling this phenomenon via an intensity based model for the call arrival was suggested, relying on a related theoretical framework by Guntay (2002) and Jarrow *et al,* (2003). These are intractable and as yet unresolved problems. No attempt was therefore made here to disentangle the two aspects of corporate debt – optionality and defaultability: these were dealt with separately.

Brooks and Gup (1997a) maintain that ignoring embedded options can result in serious reduction in equity value for various duration gaps. This includes (Brooks and Gup, 1997b) cases in which the yield curve experiences non-parallel shifts. Gilkeson and Ruff (1996:351) analyse the withdrawal option in retail portfolios of certificates of deposit (CDs). A method of valuing a portfolio of CDs was developed which explicitly considers early withdrawals in retail CDs (under $100,000) and include a case study for a small US bank. Gilkeson and Smith (1992:22) note the large increase in securitisation of residential mortgages and address interest rate risk of portfolios of mortgage-backed securities financed by fixed rate liabilities. They describe a convexity trap where losses to equity occur in both high and low interest rate environments. Gilkeson and Smith (1992:17) provide no computational procedure for their results while Gilkeson and Ruff (1996:341) have a model for adjusted cash flows, but ignore volatility of interest rates and consider only one future path for interest rates. Gilkeson and Ruff (1996:334) note these as obvious criticisms of their procedures.

The brief literature survey presented here serves only to augment the more extensive survey presented in Section 5.2.2. The extension of Bierwag's (1987) work, which was chosen for its robustness and ease of use, explanation and implementation, is presented in the next Section.

5.3.1.2 Bierwag's duration drift

Bierwag (1987:324) first suggested the possibility of 'duration drift' for bonds with defaultable and option features, but never explored the ideas further. The work described in this section is loosely based on Bierwag's (1987) proposals. A new theoretical framework was established to explore the effects of optionality on the Macaulay duration and to test its empirical implications for South African bond index data. While the approach is as yet unable to isolate the optionality and default duration components, early results show consistency with the available research and promising avenues for further work.

Given the standard inverse law governing bond prices and yields (Fabozzi, 1997:188), the call price will have an associated call yield, r^*. Bierwag (1987:129) approached the problem of callable bond duration from the perspective of a 'shift' in the Macaulay duration near r^*. Assuming a callable bond with underlying maturity m and earliest call date n, Bierwag noted that such a bond would behave like a non-callable bond of maturity m if the current yield to maturity, $r_t > r^*$ (and it is obvious the bond will not be called). In addition, the bond will behave like a bond of maturity n if the current yield to maturity $r_t < r^*$ and it is obvious the bond will, with certainty, be called. Bierwag did not expand his analysis beyond these intuitive observations.

In this thesis, Bierwag's suggestions are used as a starting point. Bond option pricing and probability theory are then combined to explore and extend Bierwag's ideas – the results of this research are reported in the next section.

Black's (1995:1372) equation for the price of a call option on a bond was used, namely

$$c = P(0,T)\cdot[F_0 \cdot N(d_1) - K \cdot N(d_2)], \tag{5.7}$$

where c is the call option price,

$P(t,T)$ is the price of a zero coupon bond paying 1 at time T

F_0 is the forward price of the bond measured at $t = 0$,

K is the call strike price,

σ is the bond forward price volatility,

$d_1 = \dfrac{\ln(F_0/K) - \frac{1}{2}\sigma^2 \cdot T}{\sigma \cdot \sqrt{T}}$ and $d_2 = d_1 - \sigma \cdot \sqrt{T}$.

Note that $N(d_2)$ is the probability of exercising the call option, where $N(\cdot)$ represents the cumulative probability distribution function for a standardised normal distribution.

The price of the callable bond is a combination of the prices of the call-free bond and the call option, given by

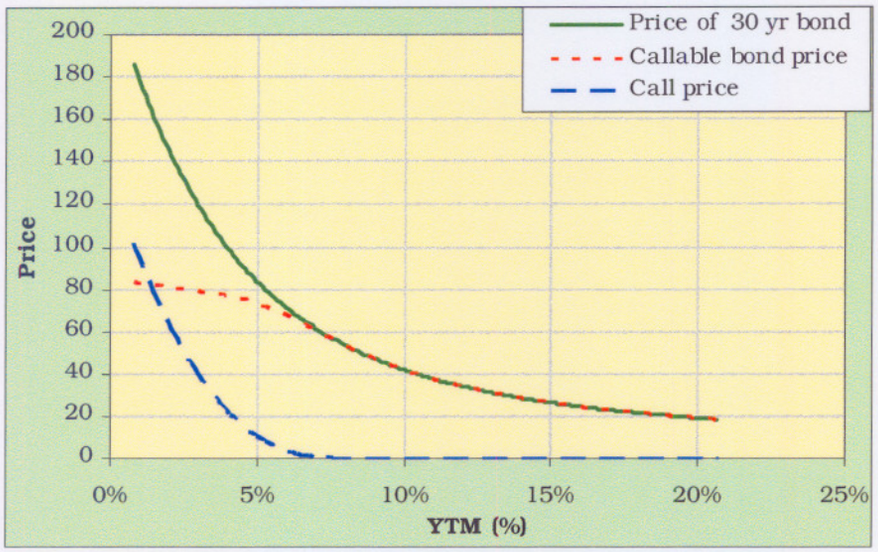
$$P_{\text{callable bond}} = P_{\text{call-free}} - c \tag{5.8}$$

where c is the price of the call option.

At high yields, the price of the call is negligible and the callable bond price is the price of the option-free bond. At low yields, the call value is high and reduces the price of the call-free

bond[32] significantly as shown in Figure 5.4 below. For lower yields where $r < r^*$, the callable bond exhibits negative convexity as the likelihood of being called increases and the price does not stray far from the callable value, i.e., par. For high yields ($r > r^*$) the probability of call and corresponding call value are diminished, and the bond price is that of an option free bond.

Figure 5.4. Price/yield relationship for an option-free bond of maturity 30 years, a call option and a 10-year callable bond.



The bond will either be called (with probability $N(d_2)$) or not (with probability $1-N(d_2)$). The conjecture is thus made that the duration of the callable bond is:

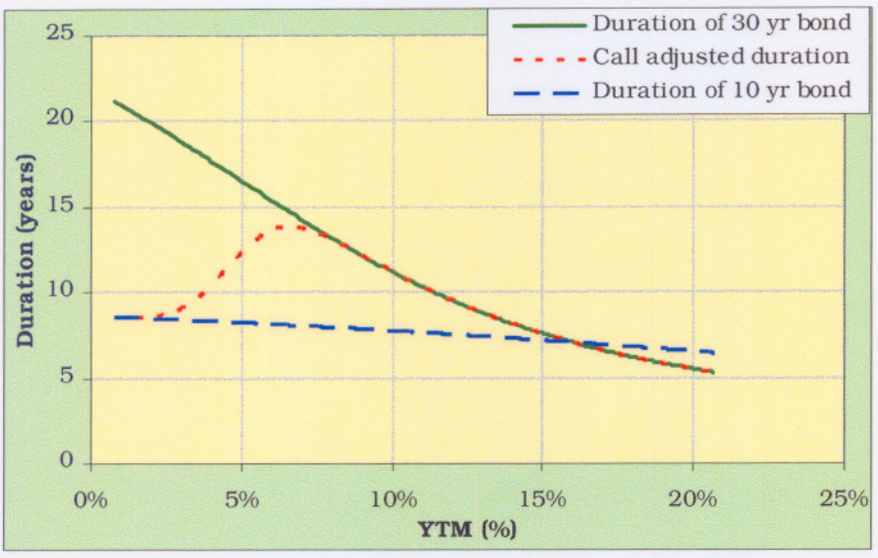
$$D_{\text{callable}} = D_{\text{call-free bond}} \cdot [1-N(d_2)] + D_{\text{call option}} \cdot [N(d_2)] \qquad (5.9)$$

It is clear that when interest rates are much lower than the strike rate, r^*, i.e., $r_t < r^*$, the probability of calling the option is high ($N(d_2) \approx 1$) and the callable bond duration is $D_{\text{callable}} \approx D_{\text{call option}}$. On the other hand, if interest rates are much higher than the strike rate, i.e., $r_t > r^*$, then the probability of calling the option is low ($N(d_2) \approx 0$) and the callable bond duration shifts to that of the call-free bond, i.e., $D_{\text{callable}} \approx D_{\text{call-free bond}}$. In between these values, i.e., where $r_t \approx r^*$, Equation 5.8 applies. Figure 5.5 below displays these results for a bond with the same valuation parameters as those given for Figure 5.4. Note that this graph is produced for a single instant in time – i.e., $t = 0$ – and represents the possible values of duration for any given yield. At low yields ($r < r^*$) the duration is that of a bond with the same maturity as the time to

[32] Parameters are: maturity: 30 years, coupon 4% paid semi-annually, first callable date 10 years from present, volatility of forward price 10%, risk free rate in 10 years 8%. Current calculated bond forward price = R5.41.

first call. At high yields ($r > r^*$) the duration is that of the option-free bond. In between, the callable bond's duration is bounded by these two possible durations.

Figure 5.5. Theoretical range of Macaulay duration for a 30-year bond with an embedded option, callable after 10 years.



To determine the evolution in time of the changing duration curve given in Figure 5.5 the duration of the call option and the underlying bond were evaluated at 6 month intervals from $t = 0$ until the option expiry date ($t = 10$ years) using Equation 5.9 above.

After 6 months, the callable bond comprises an option free bond with 29.5 years to maturity and a call option with 9.5 years until first callable date. After 5 years, the option free bond has 25 years to maturity and the call 5 years until the first callable date, and so on. This time-changing profile is shown in Figure 5.6, below. The duration surface becomes steeper (for $r < r^*$) as first call date approaches. If yields are low ($r < r^*$) the probability that the bond will be called increases and the duration of the bond will tend to that of a bond with the same maturity as the call option. If $t \to 10$, the time to first call date $\Delta t \to 0$, so the Macaulay duration, D, will tend to zero. At and beyond $t = 10$ the duration is 0 for low ($r < r^*$) yields (since in this case the bond will be called immediately) and equal to the duration for the remaining life of the bond if $r > r^*$.

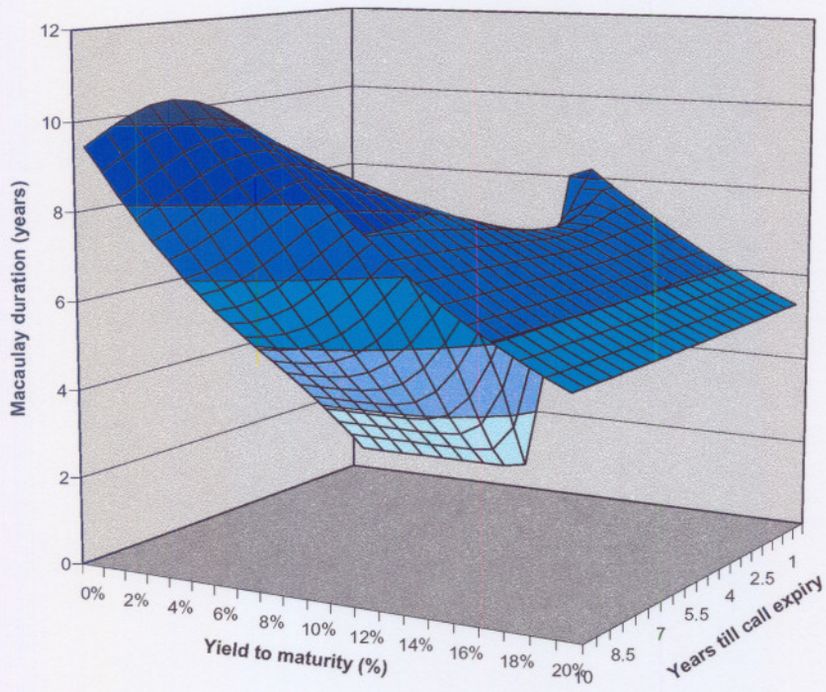
In the period from January to May 1986, US interest rates dropped precipitously. Bierwag (1987:144) pointed out that, using the standard elasticity definition of duration, namely

$$\frac{\Delta P}{P} \cong -D \cdot \Delta r,$$

large price increases should correspond to large decreases in interest rates. However, if the duration also decreases as interest rates fall, the accompanying increase in price will not be as large.

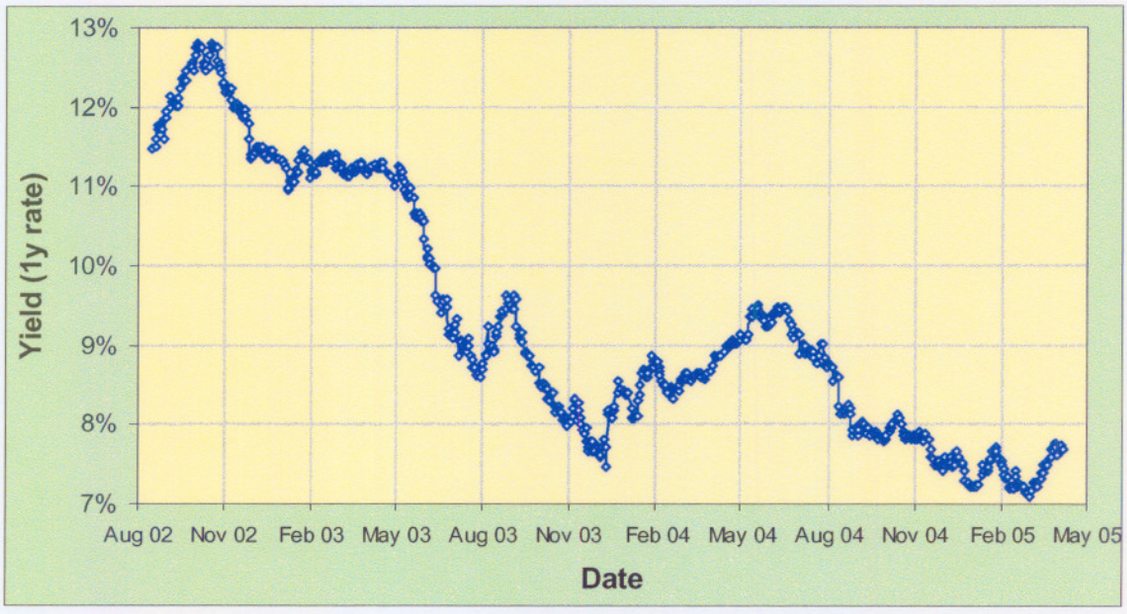
Many callable bonds experienced such a lacklustre increase in prices in 1986 despite the large drops in interest rates.

Figure 5.6. Time evolution of duration for a callable bond.



Similar, significant, rapid interest rate decreases have also been experienced in the South African market[33] in recent years (namely Oct 2002, Jun 2003 and Sept 2003 – see Figure 5.7).

Figure 5.7. Daily, 1-year South African interest rates, measured over 2.5 years.



(Source: Reuters™,2005)

[33] The South African 1-year through 3-year interest rates. Source: Reuters™.

Similar effects on bond portfolio values have been observed amongst many asset managers e.g., MacDonald (2004) and Botha (2004). As interest rates drop to levels near to but still above r^*, the duration also decreases. The percentage increase in bond prices represented insufficient increments to bond portfolio values to offset the reduced future earnings from reinvesting at lower rates. Previously experienced high yields could not be locked in over long planning periods and durations of bond portfolios became too short to immunise over such planning periods. The realised rates of return over such planning periods were thus likely to be significantly less than many investment managers projected for clients.

5.3.2 Empirical results

To test these results against market prices empirically, a sample of 108 corporate bonds – all with embedded call options – was selected. The bonds had maturities ranging from 8 to 15 years and paid annual or semi-annual coupons of between 10% and 18%. The call options all had exercise dates of 5 years ± 3 months. All market data were sampled over the 4-year period 1 January 2001 $\leq t \leq$ 31 December 2004. The price, yield to maturity and Macaulay durations (both option-free (Equation 3.4) and option-embedded (Equation 5.9)) of each bond were calculated at the end of each month during the period under examination. The actual change in price between one month end and the next was then measured as well as the known change in yield over this period. The bonds were grouped into 4 maturity/call period buckets as shown in Table 5.2 below.

Table 5.2. Option-embedded South African bonds grouped by maturity/call period buckets.

Sample size	Maturity range	Call period	Graph in Figure 5.8
27	7y 10m – 8y 2m	5y±3m	a
30	9y 10m – 10y 2m		b
28	11y 10m – 12y 2m		c
23	14y 10m – 15y 2m		d

To limit controls, each of the bonds in the relevant buckets was of the same credit quality and had similar coupons. Each bond in the buckets indicated was examined during a period of interest rate 'volatility' – e.g., the period between July 03 and July 04, when large (> 100 bps) interest rate movements occurred over relatively short periods of time (< 3 months). Using the Macaulay duration, the forecast price movement for given (observed) changes in yield to maturities was calculated. Note that the convexity adjustment was also added to both the option-free and the option-adjusted Macaulay durations. For the sake of clarity, for the remainder of this section, the 'duration' refers to the duration measure plus the convexity adjustment. These data were then compared with actual (observed) market changes in the bond prices to determine which of Equa-

tions 5.4 and Equation 5.9 were more accurate in the determination of these price changes. Figure 5.8 shows the underlying bond duration, the embedded call option duration and the option-adjusted duration as suggested by Equation 5.9 for the maturity ranges indicated in Table 5.2.

Figure 5.8. Summary of South African bond duration data.

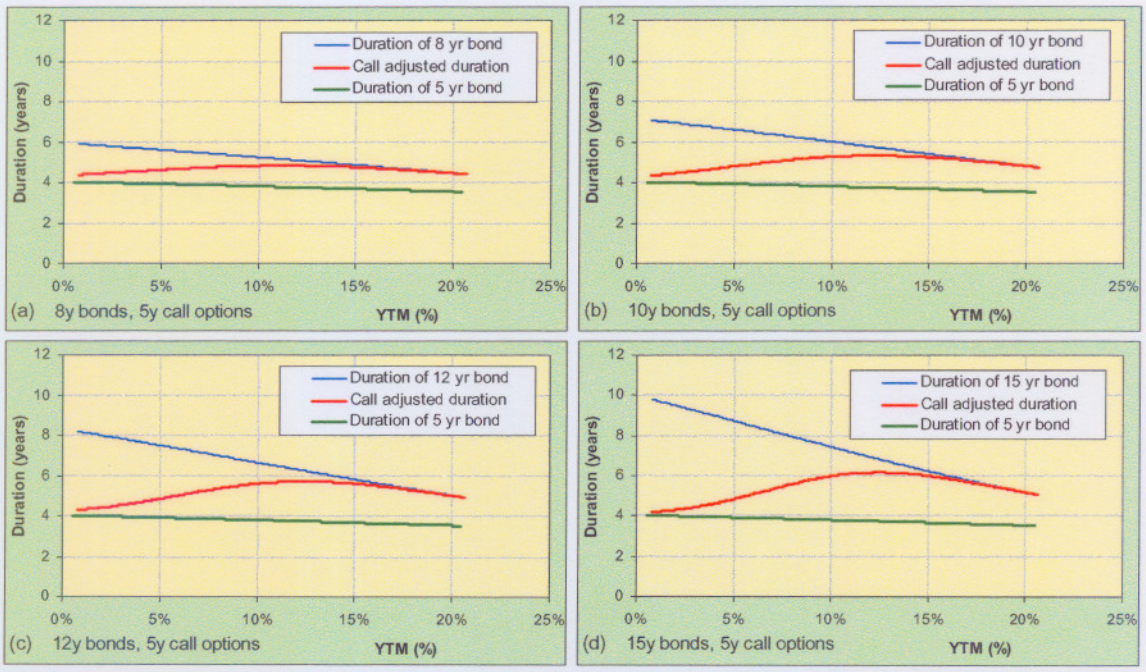


Figure 5.9 below presents a summary of results. The four graphs (a) through (d) represent bond data from the samples indicated in Table 5.2. In each graph, the *y*-axis is absolute percentage differences. The red points represent the difference between the actual change in the market price of the option-embedded bond and the price change forecast by the Macaulay duration of the underlying bond for a given change in yield, i.e.,

$$\Delta\left(\Delta P_{\text{Observed}} - \Delta P_{\text{Macualay}}\right).$$

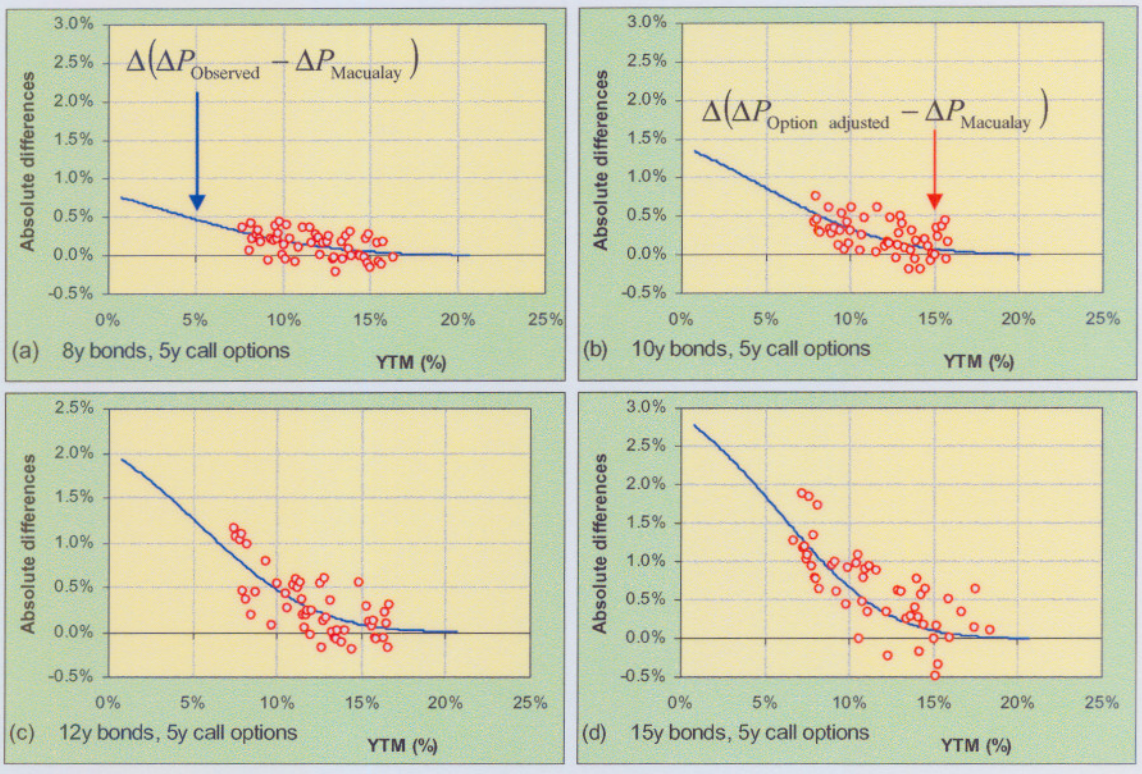
The blue line represents the theoretical change in the price of a bond forecast by an option-adjusted Macaulay duration and the change forecast by an unadjusted Macaulay duration, i.e.,

$$\Delta\left(\Delta P_{\text{Option adjusted}} - \Delta P_{\text{Macualay}}\right).$$

The wide scatter in observed bond price changes prevents an unambiguous decision as to the accuracy and durability of the option-adjusted Macaulay duration, especially for option-embedded bonds with relatively short maturities (< 10y). For longer maturity bonds, the results are encouraging. The overestimation of option-embedded bond price changes by the Macaulay duration have been observed empirically in the market during periods in which interest rates were volatile prior to this study, e.g., Bierwag and Kaufmann (1988:42), Kihn (1994:37),

Acharya and Carpenter (2002:1379) and Guntay (2002) as well as South African asset managers e.g., MacDonald (2004) and Botha (2004).

Figure 5.9. Summary of observed, Macaulay-duration forecast and option-adjusted Macaulay duration forecast price changes, using South African bond data.



5.3.3 Application of results

Global investment in fixed income has been increasing since early 2002 and is now approximately double that of global equity investments (ABN Amro, 2005). Despite signs of an increasing interest rate environment (*The Economist*, 2005), worldwide investments continue to flow into fixed income portfolios. The 'dotcom crash' in April 2000, precipitated by the non-materialisation of the dreaded and widely proclaimed 'Millennium Bug', led to further disenchantment in equity returns. This wariness had been observed worldwide since the 1998 equity crash – itself caused by the default of Russian government fixed income securities, but blamed on the collapse of the massively swollen US hedge fund, Long Term Capital Management (Lowenstein, 2001). The shift in investment emphasis from risky equity to (perceived) less-risky fixed income securities has led directly to an increase in option features of fixed income instruments. Today (2005), more corporate and government bonds are issued *with* embedded optionality than without[34] as the investment community endeavours to protect itself from potentially volatile interest rate markets. Most fixed income portfolios thus comprise optionality of some

[34] Roughly 80% of corporate and government bonds are issued with embedded option features (Mosher, 2005).

form, an attribute that alters the sensitivity of the portfolio to interest rate movements and hence, to a large extent, the risk profile. The Macaulay duration measure is not able to (and its original formulation did not require it to) deal with this burgeoning optionality characteristic, yet its use remains widespread, particularly in spreadsheet models (Mosher, 2005). In addition, corporates and banks must cope with changes in accounting standards in the form of AC 133 and IAS39. Companies must have the flexibility to cope with these changes and ensure that they are not left at a dead-end. Proactive companies may actually benefit from these changes, as more investment may be encouraged by full disclosure resulting from the new international standards.

The true health of a company will no longer be confused by inconsistent treatment of derivatives. Historically, derivatives could be recorded at either cost, fair value or even completely omitted from the balance sheet, but no longer. On the other hand, without proper management of the issues raised by the new regulations, large fluctuations may now appear on a company's financial statements and actual financial health may be difficult to determine. As a result, there has been speculation that some companies may even stop their use of derivatives, a 'knee-jerk' response to the implications of change to their existing risk management strategy. For the first time, treasurers and risk managers must become aware of the accounting implications of their hedging activity (SARB, 2005). In order to cope with the evolving regulations companies are hurriedly reviewing their internal processes, re-educating personnel, and searching for new systems with true compliance capabilities. The potential for IAS39 to disrupt prudent risk management and impact performance statements is huge. Becoming as informed as possible and sharing that information within the company, remaining flexible as regulations and their interpretations evolve, and making careful preparations are the best means to ensure a smooth transition (KPMG, 2004).

New research results, presented in this section, are encouraging and have been shown to be reliable for a broad range of interest rate environments and widely disparate fixed income instruments. The model is easily incorporated into spreadsheet models and, given the stochastic nature of option exercising (which, though modelled on the 'rational investor', must cope with irrational human behaviour), shows great promise.

Most prior research has explored the theoretical importance of adjusting Macaulay duration for the dual impacts of defaultability and optionality. Simple, robust duration measures have been developed within a new theoretical framework to account for the differences observed between durations of defaultable and default-free, and callable and option-free, bonds. Observed differences between portfolios with and without defaultable or callable bonds are well explained by the new methodology and the results showed consistency with much of the prior research. The duration of defaultable securities is consistently higher than that of corresponding default-free

bonds and the duration of callable bonds, although subject to many variables, is bounded by the duration of the underlying option-free bond and that of a bond with a maturity equal to the first callable date.

Instruments with open-ended or unknown maturities are considered next.

5.4 Duration of non-maturity instruments

It has long been acknowledged that the treatment of non-maturity deposits is, for many banks, the single most important assumption in measuring their exposure to interest rate movement (Janosi *et al*, 1999:22). This has also recently come to the attention of regulators (NCUA, 2004). Regulators continue to rely on the bank's internal modelling systems to determine the value and interest rate sensitivity of such accounts. This presents banks with the difficult task of accurately calculating the market value of deposits and credit card loans, as well as developing effective hedging strategies to protect this value against market rate movements. Similarly, with the continued consolidation trend in the banking industry, accurate valuation of demand deposits, in particular, becomes critical in determining the value of an institution or a single branch (van Deventer and Imai, 1997:301). Acquiring or banks being targeted for acquisition need to measure the deposit franchise value based on the unique characteristics of an institutions deposit portfolio.

In the valuation of any security, total cash flow is the basic building block of valuation. For most securities, the maturity date is fixed and the cash flow from the principal stems from a predetermined payment schedule associated with that security. In the case of non-maturity deposits and credit card loans, however, the security is perpetual and the aggregate principal is never returned in totality. Instead, changes in balances are simply another source or use of the cash flow. Hence, a crucial component in valuing non-maturity deposits and credit card loans is to isolate the total cash flows from these franchises. The total cash flow in a given time period for such accounts includes: interest paid, non-interest expense of servicing, non-interest revenue, net change in balances, losses (credit cards only) (Janosi *et al*, 1999:28).

Using a market segmentation argument, Jarrow and van Deventer (1998:266) used the interest rate derivative's arbitrage-free methodology to value both demand deposit liabilities and credit card loan balances in markets where deposits and loan rates may be determined under imperfect competition. In this context, these financial instruments were shown to be equivalent to a particular interest rate swap, where the principal depends on the past history of market rates. Solutions were obtained which were independent of any particular model for the evolution of the

term structure of interest rates. Jarrow and van Deventer's (1998) work built on previous study by van Deventer and Imai (1997:297).

The duration of non-maturity instruments such as demand deposits "is a much-debated topic for which there is no right answer" (Holton, 2003:2). Holton proposed that the duration for these maturity types should be zero since depositors may withdraw funds at any time, but he acknowledged that it was unlikely all depositors would simultaneously withdraw all funds. Holton further proposed that although a bank's total demand deposits fluctuate over time, in principle it is possible to identify a 'core level' of deposits below which the total never drops. This portion might then be treated as 'permanent'. A simplistic argument, then, may ascribe to demand deposits any duration from zero to infinity. This argument may be invalid as duration here has been treated as average time to maturity, a statement that is only valid if cash flows are fixed. The more general definition of duration as a metric of sensitivity to parallel shifts in the yield curve may also be used. With this approach, some means of ascribing a market value to the demand deposits as a function of the level of interest rates would be required. Holton permits that not only is this difficult to achieve, but it would entail a host of assumptions. Duration is thus "poorly equipped" to explain interest rate sensitivities of non-maturity instruments (Farin, 2004).

Research is, however, ongoing in the field. Ellis and Jordan (2001:49) found that the estimated durations of non-maturity deposits in the US vary according to the degree of interest rate sensitivity of the deposit type. Deposits that are interest rate-sensitive tend to have shorter durations, such as money market deposit accounts durations, which are approximately one year or less. They reprice quickly as rates change in a manner similar to floating rate notes, which have low durations. Overnight deposit accounts tend to have the highest durations, followed by transactions accounts and money market deposit accounts. A summary of Ellis and Jordan's results in tabular form are shown in Table 5.3 below which summarises their research. These durations are recommended as the final ('effective') maturities of non-maturity deposits for banks that do not explicitly model non maturity deposit cash flows. They are acceptable for assigning non maturity deposits to maturity buckets and for the final maturity when a non maturity deposit is treated as a zero-coupon bond.

Table 5.3: Recommended durations (in years) for non-maturity deposits.

	Min	Mid	Max
Share Drafts (Transaction Accounts)	2.0	2.5	2.8
Regular Shares (deposits)	2.5	3.0	3.5
Money market deposit accounts	0.5	1.0	1.5

(Source: Ellis and Jordan, 2001)

O'Brien (2000) reported that durations for median US bank deposits are mostly less than one year, but exceed one year for market rate increases. Under variable deposits, the durations are not only smaller, but are negative except for the positive rate shocks under asymmetric deposit rate adjustments (a positive interest rate elasticity is represented by a negative duration). For variable deposits, the present values of the longer-term interest rate change reversals get relatively more weight due to growing deposits. For example, O'Brien found the duration for an annuity with fixed monthly payments[35] is approximately 4.3 years (O'Brien, 2000). In Table 5.4 below, the durations of US money market deposit accounts are shown. 'Asymmetric adjustments' refers to non-parallel movements of the yield curve (with an average shift of the amount indicated); 'symmetric adjustments' refers to parallel yield moves. 'Inc' implies an upwards movement in yields and 'Dec' refers to a downwards movement.

Table 5.4. Durations of US money market deposit accounts.

	Fixed deposits				**Variable deposits**			
	Asymmetric adjustment		**Symmetric adjustment**		**Asymmetric adjustment**		**Symmetric adjustment**	
Market rate changes (bps)	Inc	Dec	Inc	Dec	Inc	Dec	Inc	Dec
50	0.46	0.32	0.17	0.14	0.41	0.34	0.19	0.19
100	0.50	0.25	0.16	0.14	0.42	0.32	0.17	0.20
150	0.54	0.17	0.16	0.14	0.42	0.25	0.16	0.21
200	0.58	0.14	0.16	0.14	0.40	0.21	0.15	0.19
250	0.61	-0.02	0.16	0.13	0.39	0.13	0.14	0.21
300	0.64	-0.17	0.16	0.13	0.36	-0.07	0.13	0.24

(Source: O'Brien, 2000)

The duration of non-maturity deposit accounts in banks present unique problems for interest rate risk managers. The problems are intractable and difficult to solve not only because of the inherent assumptions involved, but also because results depend critically on inputs. Institutions which use different inputs will therefore obtain different results – and these can often conflict even if similar inputs were used. Future research will focus on the use of the Fisher-Weil duration to non-maturity deposit accounts in the South African context.

5.5 Conclusion

Modifications to the existing Macaulay duration measure to account for default-prone, option embedded and perpetual-maturity securities have yielded interesting and largely positive results.

[35] Scant data and little research exist for equivalent work in the South African market. US data were used here for illustrative purposes, instead.

The bulk of these data agree with previously published research. Whilst individual components of the description of defaultability and optionality are not new, the way in which they have been applied (i.e., combined, tested and implemented) is entirely new. These applications were explored in this chapter.

The requirement for an entirely new approach to the problem of interest rate risk measurement in which the assumption of a flat yield curve is relaxed is, however, becoming more and more apparent. An approach in which the definition of duration itself is re-examined is presented in the next chapter.

Chapter 6

Alternative duration measures

6.1 Introduction

Previous chapters have focused on the nature of banking risk in general, the management, mitigation, and regulatory controls of these risks and the Macaulay duration, as the chief measure of a bank's interest rate risk. The Macaulay duration makes limiting assumptions and imposes restrictions on its interpretation and use. Some of these inadequacies – addressed in Chapter 5 – may be partially mitigated by innovative adaptations to the Macaulay duration, modifications to input data or amendments to well-known fixed income pricing models.

Adapting the existing Macaulay duration measure is but one solution. A *new* definition of estimating duration is, however, possible. Recall that the Macaulay duration is a scaled measure of the gradient, dP/dy, of the price/yield to maturity relationship measured at a given yield to maturity (Equation 3.4). A limiting assumption has, however, *already* been made: namely that the yield to maturity is flat and the change in the yield to maturity occurs at all points on the (flat) yield curve, hence the shift is, literally, 'parallel'[36]. Establishing a duration definition (and its accompanying mathematical construct) that does *not* make these initial limiting assumptions yet is still capable of estimating interest rate risk to a greater degree of accuracy than the Macaulay duration is the goal of this chapter. While these *definitions* have been described previously (e.g., Fisher and Weil (1971) and Bierwag (1987)), the *application* of these measures to South African data – particularly in the context of new accounting rules and regulatory pressures – is novel.

The next section considers problems with the existing definition of the Macaulay duration. These have been partially dealt with by the use of convexity and other short cuts, but the complexity of modern financial instruments is forcing ever newer, better measures of interest rate risk.

6.1.2 Overview of Macaulay duration shortcomings

The Macaulay duration has long been criticised for its failings as a robust measure of interest rate risk. Macaulay used many limiting assumptions and requirements in his definition of duration, but these were not necessarily all oversights or examples of shoddy model building. Much of the hardware technology available in the early part of the 20th Century was simply incapable of executing the calculations taken for granted today. Monte Carlo simulations and scenario gen-

[36] As opposed to a 'parallel shift' commonly used in the discipline in which an n basis-point shift (either up or down) at all points on the non-linear yield curve is experienced. In the strict definition of the term, this shift is of course, highly non-parallel.

eration – two core tools of modern risk management – require substantial computing resource reserves and were unthinkable 70 years ago. The markets of the 1940's through 80's were considerably less developed, in terms of both volume and complexity, than the modern, almost frictionless, trading brought about by modern communications and trading systems. Yield curve modelling was in its infancy and several more decades of intense research was required before the complex interest rate models in use today could be developed. Indeed, the development of entirely novel analysis techniques (such as stochastic calculus and Martingale measures combined with the vastly accelerated computing power only available recently), none of which were available in the 1940's, was instrumental in solving many duration problems.

The Macaulay duration assumes parallel shifts in the term structure of interest rates, i.e., it uses an extremely rare occurrence as the basis for risk estimates, Macaulay (1938) used parallel shifts in the definition and derivation of his duration measure, despite the very low frequency (and low probability) of such events. It was acknowledged that hedging, for example, using the Macaulay duration was only guaranteed when term structure changes were parallel. As a result, extensive research to extend the Macaulay duration to *general* patterns of interest rate changes was suggested and many other duration concepts were explored. For example, Khang (1979:1036) incorporated the empirical result that short term interest rates are more volatile than long-term interest rates[37] into his work on log-stochastic process durations, Ho (1992:38) explored key-rate durations in cases where changes in several key interest rates' determine changes of other interest rates and Rzadkowski and Zaremba (2000:29) explored multidimensional duration when several factors affect changes of the term structure.

The feature common to these approaches was that interest rate changes were specified exogenously by a parametric model, implying that the resulting duration measures work well if term structures change according to specified models, but perform poorly if they do not. Some empirical tests found that there was little difference in the hedging effectiveness among different duration strategies (including Principal Component Analysis) and concluded that the simplest Macaulay duration is the most cost effective immunisation method (Jorion and Khoury, 1996:33). Empirical evidence using South African interest rate data (van Vuuren and Styger, 2004a, 2004b) suggests otherwise.

6.1.3 Duration and stochastic processes of the term structure

The term structure of the yield curve is critical in the analysis and formulation of investment strategies of fixed-income securities. This approach – which examined the underlying yield

[37] Considerable arbitrage opportunities arise if this empirical fact is ignored (Khang 1979:1037).

curve structure to plumb the depths of duration – began with Fisher and Weil (1971:426) who developed bond immunisation strategies which explicitly took non-flat term structures into account. Bierwag (1977:731) and Bierwag and Kaufmann (1977:366) later showed that many definitions of duration were possible. In particular, Bierwag (1977:732) illustrated that appropriate definitions of duration depended upon the manner in which the term structure was assumed to shift from period to period. The manner in which the term structure shifts is known as the *stochastic process* which – unlike the flat yield curve assumption - is assumed to involve changes to only a few (or one) parameters which define it. These stochastic processes are described in the next section.

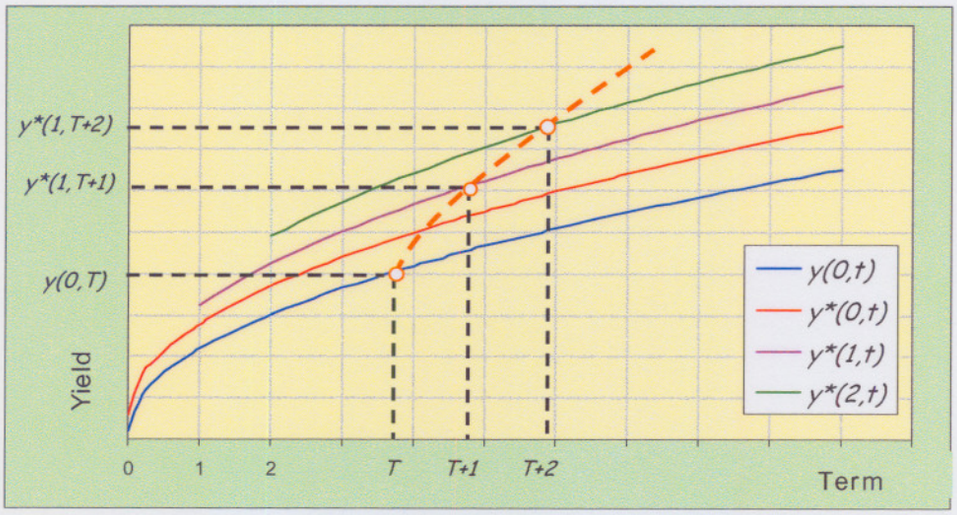
6.2 Stochastic processes

Assume the initial term structure is defined by the vector $y(0,t)$, $t = 1,2,3,\cdots$ where the first term in brackets establishes the *time* of measurement (in most cases considered here, $t = 0$ refers to the present moment) and the second term, the *term* over which the rate, y, is defined. For simplicity and uniformity, annual rates are assumed throughout this thesis, unless otherwise stated. Thus, $y(3,5)$ refers to the 5 year annual rate measured 3 years from the present time.

Assume that instantly after the observation of the term structure ($y(0,t)$) given above, the term structure changes to, $y^*(0,t)$, $t = 1,2,3,\ldots$ and that the forward rates generated from $y^*(0,t)$ using the principal of no-arbitrage (Fabozzi, 1997:201) become the actual future rates. This statement of the shift in the term structure, albeit fairly simple, forms the basis of a class of stochastic processes each of which is made explicit when the difference $y^*(0,t) - y(0,t)$, $t = 1,2,3,\ldots$ is explicitly specified as a function of one or many random variables. This approach treats shifts in the yield curve as functions of a few (or one) variables – a quite different approach to the idea that yield curve node points may shift independently of one another. The Macaulay duration concept (as defined in previous chapters) requires the reduction of the yield curve to a *single* value – the yield to maturity – and treats all subsequent yield curve changes as being parallel shifts in the true sense of 'parallel': the flat yield curve moves either up or down all along its length by a single value. This is, of course, far from reality, but the concept was introduced for its simplicity and its ease of use in measuring yield curve risk. Another assumption implicit in the Macaulay duration definition is that, after the shift in the yield curve, these rates remain at this level thereafter (until a later change). When considering the stochastic process approach, the entire term structure shifts from $y(0,t)$ to $y^*(0,t)$, $t = 1,2,3,\cdots$ and that these forward rates become the actual rates hereafter. This is illustrated in Figure 6.1 below.

Here, $y(0,t)$ is the term structure as it is at present. Instantly after the observation is made, the structure shifts to $y^*(0,t)$ and, at the start of the next period, the forward structure, $y^*(1,t)$ becomes the actual set of rates. If there are no further shifts in the term structure, the future rates, $y^*(2,t)$, at the start of the next period become the new term structure and so on through time. The evolution of the $T-$period rate through time may thus be traced. At time 0, this rate is $y(0,T)$ as indicated. At the start of the next period the term structure is $y^*(1,T)$ and at the start of the next period it is $y^*(2,T)$ and so on. The evolution of the $T-$period rate is given by the dashed line in Figure 6.1, beginning at $y(0,T)$ then instantly moving on to $y^*(0,T)$ and thereafter in successive periods it becomes $y^*(1,T+1)$, $y^*(2,T+2)$, $y^*(3,T+3)\ldots$. If there had been no initial random shift, the successive $T-$period interest rates would have been $y(0,T)$, $y(1,T+1)$, $y(2,T+2)\ldots$, where the rates starting with the period after time 0 would be taken from the forward rates calculated from the initial term structure. Since the initial shift is random, it is possible that a great variety of future interest rates can evolve. Thus, there exists a future scenario – for all interest rates – that corresponds to every possible initial random shift. If the initial shift is related to a single random variable, the entire sequence of future interest rates depends on the value of that single, random variable. In this way, an association may be made between an entire sequence of rates over time and a single random variable or, in more advanced versions of this idea, an entire sequence of rates over time may be associated with a single vector of random variables.

Figure 6.1: Sequence of $T-$period rates in which actual current rates undergo a shock and then become future rates at future times.



Contrast this evolution of rates, above, with the special case in which the yield curve is completely flat. If the flat structure shifts additively by some random amount, say λ, then the interest rates $y^*(1,T+1)$, $y^*(2,T+2)$, …, are read off flat curves because the forward structure of

a flat curve is always the same structure, by definition. In such a case, the initial term structure may be represented by y $(= y(0,t)$ for all $t)$ and the ensuing interest rates by y^* $(= y^*(0,t)$ for all $t)$ which are precisely the definitions used for the Macaulay duration.

Portfolio strategies that are appropriate or optimal for ever-changing non-flat term structures depend on the specification of the stochastic process of the term structure. This specification explicitly stipulates how $y^*(0,t)$ is formed as a function of one or more variables. This is sufficient to establish the difference, $y^*(\tau,\tau+T) - y(\tau,\tau+T)$, for all T and measured at time τ as a function of relevant random variables. Some of these explicit specifications and their implications for duration-based models are developed in this section. A major point is that there are *many* different measures of duration. Recall that the Macaulay duration has been defined in previous chapters as a weighted average at which future cash flows occur, namely (Equation 4.3):

$$D_{\text{Mac}} = \sum_{t=1}^{M} w_t \cdot t \,. \tag{6.1}$$

Using a flat term structure, the appropriate weights at time t comprised the value of the cash flow occurring at time t relative to the total value of the income stream. A different measure of duration results if the weights are differently defined. Each stochastic process of the term structure implies a duration measure. Different stochastic processes imply different duration measures. In the next sections, five simple stochastic processes will be considered. Each is a one-factor process, i.e., each instantaneous random shift is a function of a single random variable[38]. In each process, it is this shift that plays the same role as the yield to maturity did in the exposition of the Macaulay duration. The random factor is an index of an entire term structure as the yield to maturity is an index of the discount rate applicable to future periods. In the analysis that follows, all yield curves generated from stochastic shifts are arbitrage-free.

6.2.1 Additive shifts

6.2.1.1 Theory

The simplest way to specify the difference $y^*(0,t) - y(0,t)$, $t = 1,2,3,\cdots$ is to contend that this difference is both random and independent of the term to maturity. Thus:

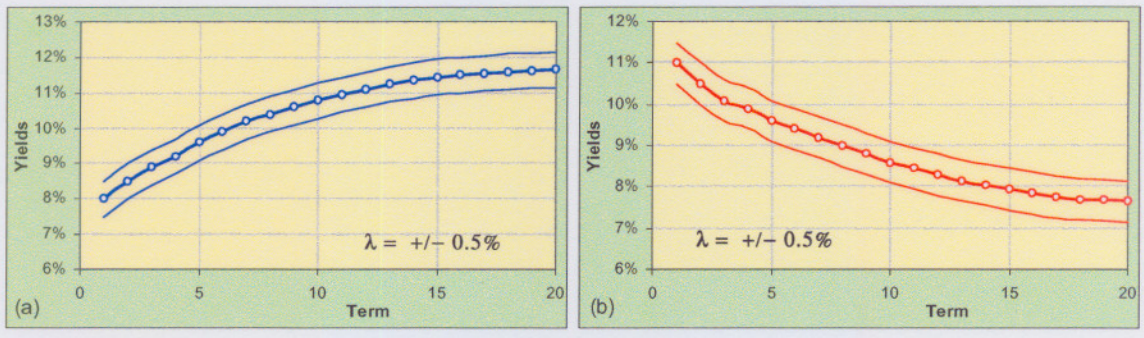
$$y^*(0,t) = y(0,t) + \lambda \,, \tag{6.2}$$

where λ is a random variable. Figure 6.2a below shows an original upward-sloping yield curve (with tenors up to 20 years) and a possible upward additive shift $(\lambda = +0.5\%)$ and possible

[38] Yield curve research has consistently shown that single sources of uncertainty do not provide a sufficiently accurate description of yield curve movements (e.g., Rzadkowski and Zaremba (2000)).

downward shift ($\lambda = -0.5\%$). Figure 6.2b shows an original downward-sloping yield curve with the same potential additive shifts.

Figure 6.2: Additive shifts to the yield curve.



This is a natural extension of the yield to maturity concept in which $y^* = y + \lambda$ for flat term structures. Having obtained $y^*(0,t)$ in this manner, future rates evolve as forward rates derived from $y^*(0,t)$. The difference $y^*(\tau,\tau+T) - y(\tau,\tau+T)$ is an explicit function of λ. This is called an additive process because λ is simply added to the existing term structure in order to compute the new instantaneous term structure. Although this is sometimes called a parallel shift, it does not – of course – produce a straight line unless the existing term structure is flat. The value of the random variable, λ, is also known as the instantaneous shock. For reasonable ranges of interest rates (e.g., $0 < y(0,t) < 20\%$) the difference $y^*(\tau,\tau+T) - y(\tau,\tau+T)$ will be almost constant for all T. As a result, if the stochastic process is additive, the future rate $y^*(\tau,\tau+T)$ is found by adding a random variable to $y(\tau,\tau+T)$. This added variable will be a function of the instantaneous shock, λ.

The price of a fixed-income security, P, is a function of both the cash flows it will experience during its life and the underlying yield curve. Previously, a flat yield curve, y, has been used and the price of the security was given by (Equation 4.7):

$$P = \sum_{t=1}^{M} CF_t (1+y)^{-t} . \tag{6.3}$$

With the additive process in effect, the price of a security instantly after the shock to the term structure may be written:

$$P(\lambda) = \sum_{t=1}^{M} CF_t \left(1+\lambda+y(0,t)\right)^{-t} .$$

The derivative with respect to λ is then:

$$\frac{dP}{d\lambda} = P'(\lambda) = -\sum_{t=1}^{M} t \cdot CF_t \cdot (1+\lambda+y(0,t))^{-t-1} . \tag{6.4}$$

Evaluating this function at the value of $\lambda = 0$ (the value which gives the original price of the security) gives:

$$\left.\frac{dP}{d\lambda}\right|_{\lambda=0} = P'(0) = -\sum_{t=1}^{M} CF_t \cdot \left[t \cdot (1+y(0,t))^{-1}\right] \cdot \left[(1+y(0,t))^{-t}\right].$$

Dividing through by the original price, $P(0)$, gives:

$$\frac{1}{P(0)} \left.\frac{dP}{d\lambda}\right|_{\lambda=0} = \frac{-\sum_{t=1}^{M} CF_t \cdot \left[\frac{t}{(1+y(0,t))^{-1}}\right] \cdot \left[(1+y(0,t))^{-t}\right]}{P(0)} . \tag{6.5}$$

Recall from Equation 4.6 that

$$D_{\text{Mod}} = \frac{1}{P}\frac{dP}{dy},$$

or

$$\frac{D_{\text{Mac}}}{(1+y)} = \frac{1}{P}\frac{dP}{dy}, \tag{6.6}$$

and in the context of a shifting term structure as opposed to a shifting yield to maturity, an equation such as Equation 6.5 above may be compared to Equation 6.6. The analogous relationship between these equations gives:

$$\frac{1}{P(0)} \left.\frac{dP}{d\lambda}\right|_{\lambda=0} = \frac{D_{\text{Add}}}{(1+y(0,D_{\text{Add}}))} \tag{6.7}$$

$$\frac{D_{\text{Add}}}{(1+y(0,D_{\text{Add}}))} = \frac{\sum_{t=1}^{M} CF_t \cdot \left[\frac{t}{(1+y(0,t))^{-1}}\right] \cdot \left[(1+y(0,t))^{-t}\right]}{P(0)}, \tag{6.8}$$

where D_{Add} is the duration measured using the **additive** stochastic process applied to the entire term structure[39] and $y(0,D_{\text{Add}})$ is the yield measured at the present time with term to maturity D_{Add}. D_{Add} may not be made the subject of the formula and must be solved iteratively using a Newton-Raphson technique (Abramowitz and Stegun, 1972:18). The percentage price change of the security for a change in the random variable λ is (using Equation 6.7):

[39] This stochastic additive duration term may not be made the subject of Equation 6.8 as it is an implicit argument of the additively-shifted yield curve, $y(0,D_{\text{Add}})$. An iterative solution, such as Newton-Raphson is easily implemented.

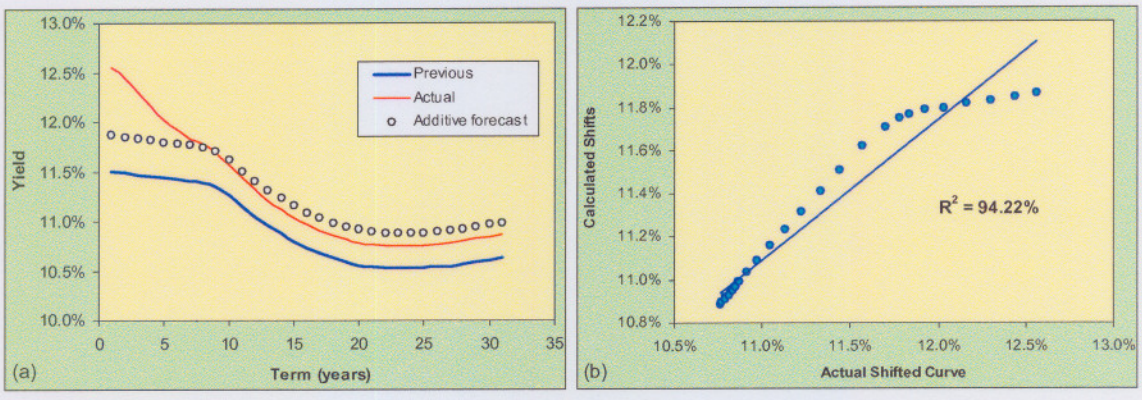
$$\frac{\Delta P}{P} \cong \frac{-D_{\text{Add}}}{(1+y(0,D_{\text{Add}}))} \cdot \lambda . \tag{6.9}$$

The value of λ in Equation 6.9 is also $\Delta\lambda$ since the initial value of λ is 0. The derivation of D_{Add} is achieved using Equation 6.9, which requires the calculation of the right hand side of Equation 6.8.

6.2.1.2 Empirical testing

Given the multitude of potential yield curve shapes and changes from period to period, it seems intuitively unlikely that a simple 'parallel' shift added to a current yield curve in order to forecast a next-period yield curve will ever (or very frequently) adequately describe reality. In order to empirically test the effectiveness of the additive stochastic process in the accurate forecasting of interest rate movements, some 7½ years of weekly interest rate data were used. These data are described in detail in Section 6.2.6, for the moment it suffices to indicate the salient result from the analysis, shown in Figure 6.3a below for two observed South African yield curves, separated in time by a single week (September 2002). The period under observation was volatile and the yield curve was steepening proportionately more in the short end than in the long end of the curve. This behaviour is not usual, but neither is it entirely uncommon.

Figure 6.3: (a) Previous period, observed next-period and forecast next-period yield curves using an additive stochastic process. (b) Regression of calculated additive shifts upon observed next-period yield curve.



The blue line indicates a yield curve at some arbitrary time $t = n$, whilst the red line is the observed yield curve one period later, i.e., at $t = n+1$. The black open circles represent the best-fit-possible forecast using the stochastic additive shift process. This 'best fit' was calculated using the ordinary least squares technique for the differences between observed and forecast yield curve node points. The calculated yield curve was regressed upon the observed yield curve and the result shown in Figure 6.3b. The R^2 value is also displayed on the graph. Whilst the R^2

value seems to indicate a high degree of correlation ($R^2 = 0.9422$), comparisons with other suggested stochastic processes (see later sections in this chapter) will clearly show that this is far from the superior model. Also, in the interests of brevity and clarity, only one interest rate shift (out of an infinite number of potential shifts) has been illustrated (see Figure 6.3). Several years of data were, however, tested (see Figure 5.7 and Appendix D) and in all cases the conclusion reached regarding the best-fit interest rate model was the same.

6.2.2 Multiplicative shifts

6.2.2.1 Theory

The process here may be specified by:

$$y^*(0,t) = \lambda \cdot y(0,t) \tag{6.10}$$

where λ is a positive random variable. Thus, if $\lambda = 1$ the term structure does not shift; if $\lambda > 1$ the term structure shifts upwards and if $\lambda < 1$ the term structure shifts downwards. If the term structure is upward sloping and $\lambda > 1$ the term structure shifts upwards and long term rates instantaneously shift more than short term rates. In a similar way, if the term structure is upward sloping and $\lambda < 1$ long term rates decrease instantaneously more than short term rates. Note that the above entails an instantaneous interest rate change; it does not follow that over time the same results prevail. Figure 6.4 shows the possible effect of an upward ($\lambda = 1.05$) and downward ($\lambda = 0.95$) multiplicative shift applied to an original (a) upward-sloping and (b) downward-sloping yield curve. Note that changes at the short end of the curve are less pronounced than those at the long end.

With the multiplicative process in effect, the price of a security instantly after the shock to the term structure may be written:

$$P(\lambda) = \sum_{t=1}^{M} CF_t \left(1 + \lambda \cdot y(0,t)\right)^{-t}. \tag{6.11}$$

The derivative with respect to λ is then:

$$\frac{dP}{d\lambda} = -\sum_{t=1}^{M} CF_t \cdot \left(\frac{t \cdot y(0,t)}{1 + \lambda \cdot y(0,t)}\right) \cdot \left(1 + \lambda \cdot y(0,t)\right)^{-t}. \tag{6.12}$$

The corresponding duration is

$$\frac{D_{\text{Mult}} \cdot y(0, D_{\text{Mult}})}{\left(1 + y(0, D_{\text{Mult}})\right)} = \frac{\sum_{t=1}^{M} CF_t \cdot \left(\frac{t \cdot y(0,t)}{1 + y(0,t)}\right) \cdot \left(1 + y(0,t)\right)^{-t}}{P}, \tag{6.13}$$

where D_{Mult} is the duration for the **multiplicative** stochastic process and $y(0, D_{\text{Mult}})$ is the yield measured at the present time with term to maturity D_{Mult}. D_{Mult} may not be made the subject of the formula and must be solved iteratively using a Newton-Raphson technique (Abramowitz and Stegun, 1972:18). Evaluating $dP(\lambda)/d\lambda$ at $\lambda = 1$ (the value which gives the original price of the security) and dividing by $P(1)$ gives (using Equation 6.13):

$$\frac{1}{P(1)} \cdot \left. \frac{dP}{d\lambda} \right|_{\lambda=1} = \frac{D_{\text{Mult}} \cdot y(0, D_{\text{Mult}})}{(1 + y(0, D_{\text{Mult}}))}.$$

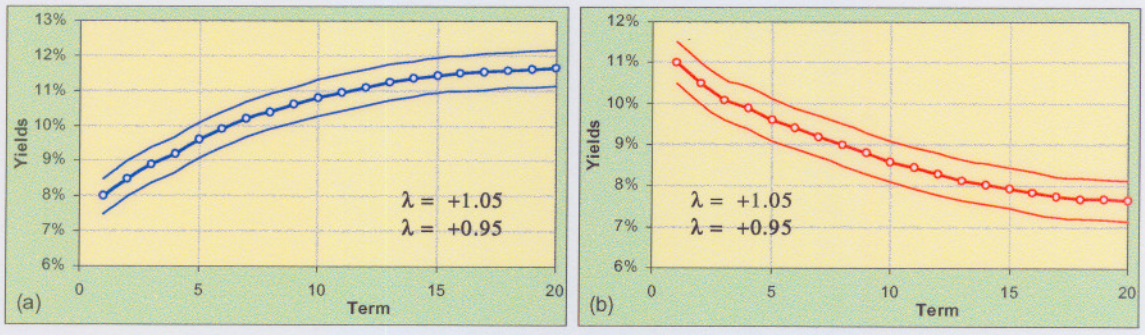
The approximation equation is $\dfrac{\Delta P}{P} \cong \dfrac{-D_{\text{Mult}} \cdot y(0, D_{\text{Mult}})}{(1 + y(0, D_{\text{Mult}}))} \cdot \Delta\lambda$

or

$$\frac{\Delta P}{P} \cong \frac{-D_{\text{Mult}} \cdot y(0, D_{\text{Mult}})}{(1 + y(0, D_{\text{Mult}}))} \cdot (\lambda - 1) \qquad (6.14)$$

Since $\lambda = 1$ at the start of the analysis, so any change in λ, $\Delta\lambda$ is given by $\lambda - 1$.

Figure 6.4: Multiplicative shifts to the yield curve.

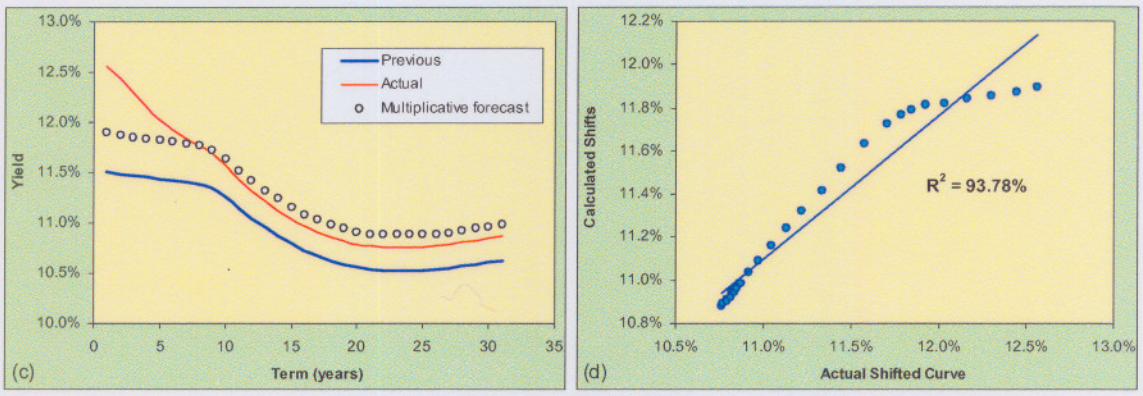


6.2.2.2 Empirical testing

The multitude of potential yield curve shapes and changes from period to period, again intuitively preclude the possibility that a simple multiplicative shift to a current yield curve will accurately forecast a next-period yield curve. To empirically test the effectiveness of the multiplicative stochastic process in the accurate forecasting of interest rate movements, the same data as described in Section 6.2.1.2 were used. The blue line in Figure 6.5a again indicates a yield curve at some arbitrary time $t = n$, whilst the red line is the observed yield curve one period later, i.e., at $t = n + 1$. The black open circles represent the best-fit-possible forecast using the stochastic multiplicative shift process. The calculated yield curve was regressed upon the observed yield curve and the result shown in Figure 6.5b. The R^2 value is also displayed on the graph. Whilst the R^2 value again seems to indicate a high degree of correlation (R^2 = 0.9378), comparisons

with other suggested stochastic processes (see later sections in this chapter) will clearly show that this is far from the superior model.

Figure 6.5: (a) Previous period, observed next-period and forecast next-period yield curves using a multiplicative stochastic process. (b) Regression of calculated multiplicative shifts upon observed next-period yield curve.



6.2.3 Fisher-Weil shifts

6.2.3.1 Theory

Fisher and Weil (1971) were the first to devise immunisation strategies based upon shifts and changes in the term structure rather than merely upon shifts in the yield to maturity. Whilst their framework employed continuous compounding, this will not be used here in line with the non-continuous derivation of all other yield curve shifts. An analogous, non-continuous yield curve shift for the Fisher-Weil process is:

$$1+y^*(0,t)=\lambda\cdot\left(1+y(0,t)\right) \tag{6.15}$$

where λ is again a positive random variable with a range that implies that $y^*(0,t)>0$. Equation 6.15 may be rewritten as:

$$y^*(0,t)=\left(\lambda-1\right)+\lambda\cdot y(0,t) \text{ for } t=1,2,3,\ldots$$

thus the process is a combination of both an additive shift ($\lambda-1$) and a multiplicative shift (λ). Clearly, both shifts are related to the random shift, λ.

With the Fisher-Weil process in effect, the price of a security instantly after the shock to the term structure may be written:

$$P(\lambda)=\sum_{t=1}^{M} CF_t\cdot\lambda^{-t}\cdot\left(1+y(0,t)\right)^{-t}.$$

The derivative with respect to λ is then:

$$\frac{dP}{d\lambda} = -\sum_{t=1}^{M} t \cdot CF_t \cdot \lambda^{-t-1} \cdot (1 + y(0,t))^{-t}. \qquad (6.16)$$

Evaluating $dP(\lambda)/d\lambda$ at $\lambda = 1$ (the value which gives the original price of the security) and dividing by $P(1)$ gives:

$$\frac{1}{P(1)} \cdot \left.\frac{dP}{d\lambda}\right|_{\lambda=1} = \frac{-\sum_{t=1}^{M} t \cdot CF_t \cdot (1 + y(0,t))^{-t}}{P} = -D_{\text{FW}} \cdot$$

where D_{FW} is the duration for the **Fisher-Weil** stochastic process. The approximation equation is:

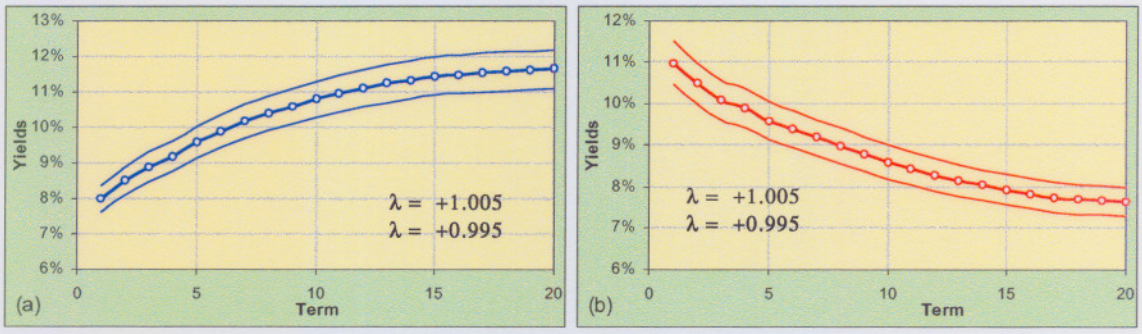
$$\frac{\Delta P}{P} \cong -D_{\text{FW}} \cdot \Delta\lambda$$

or

$$\frac{\Delta P}{P} \cong -D_{\text{FW}} \cdot (\lambda - 1)$$

Since $\lambda = 1$ at the start of the analysis, so any change in λ, $\Delta\lambda$, is given by $\lambda - 1$. Figure 6.6 below shows the possible effects of an upward ($\lambda = 1.005$) and downward ($\lambda = 0.995$) Fisher-Weil shift on an original (a) upward-sloping and (b) downward-sloping yield curve. In practice, least squares minimisation techniques are used to determine the value of λ in which the value of λ which best forecasts the next day's yield curve shape is chosen. The value of λ for the Fisher-Weil duration can never be negative.

Figure 6.6: Fisher-Weil shifts to the yield curve.



An alternative interpretation of the Fisher-Weil duration measure is that it takes advantage of the forward markets to trade interest rate risk directly. The spot rate is defined as the geometric average of the current spot and forward rates, and thus a change in any forward rate can impact upon a wide range of spot rates. A measure of duration that is linked to forward/spot rate changes can

be defined as follows. Let ${}_0r_t$ denote the spot yield to maturity of a t-period discount bond. The current price of cash flows, $CF_1,\cdots,CF_T$, is given by

$$P=\frac{CF_1}{(1+{}_0r_1)}+\frac{CF_2}{(1+{}_0r_2)^2}+\cdots+\frac{CF_T}{(1+{}_0r_T)^T}.$$

A parallel shift in the yield curve then consists of perturbing every yield by a constant amount, say δ. Let ${}_0i_t={}_0r_t+\delta$ and let $P(\delta)$ be the price of the bond at the perturbed yield curve. Then, the duration may be measured by taking the derivative of $P(\delta)$ with respect to δ evaluated at $\delta=0$. This yields, from Equation 3.3:

$$D_{\text{Mac}}=\sum_{t=1}^{T}t\cdot w_t=\sum_{t=1}^{T}t\cdot\frac{CF_t}{P(1+{}_0r_t)^t}.$$

Finally, a measure of duration may be defined directly in terms of perturbations in the forward rates. Let f_t denote the forward rate between periods $t-1$ and t. Then $(1+{}_0r_t)^t=\prod_{j=1}^{t}(1+f_j)$. If all forward rates are perturbed by δ, then

$$(1+{}_0r_t(\delta))^t=\prod_{j=1}^{t}(1+f_j+\delta),\text{ and }P(\delta)=\frac{CF_1}{(1+{}_0r_1(\delta))}+\frac{CF_2}{(1+{}_0r_2(\delta))^2}+\cdots\frac{CF_T}{(1+{}_0r_T(\delta))^T},$$

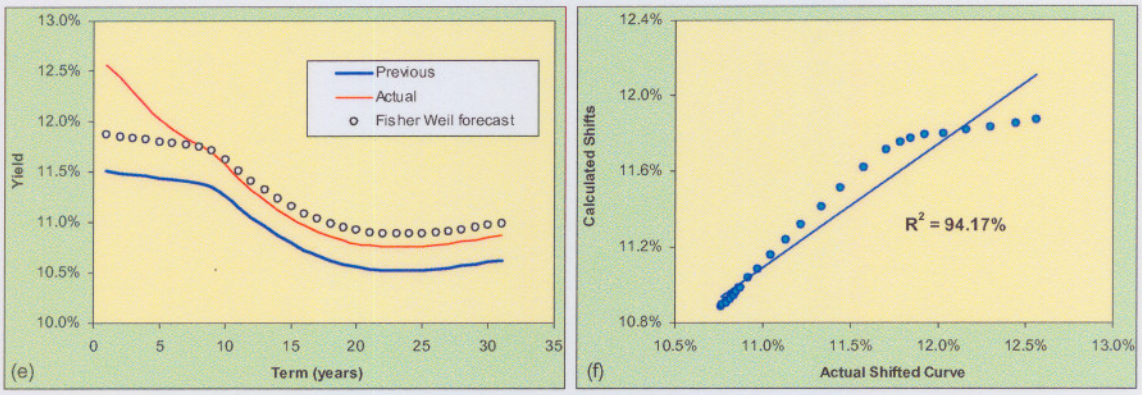
which leads to the Fisher-Weil duration measure:

$$D_{\text{FW}}=\sum_{t=1}^{T}t\cdot w_t=\sum_{t=1}^{T}t\cdot\frac{CF_t}{P(1+{}_0r_t(0))^t}. \qquad (6.17)$$

6.2.3.2 Empirical testing

Whilst the Fisher-Weil duration enjoys some success in the market (Au and Thurston 1995:372; Munk, 2000:171 and Jacoby, 2003:148), the Fisher-Weil stochastic process does not sufficiently accurately describe yield curve shifts in a volatile interest rate environment as shown below in Figure 6.7a. To empirically test the effectiveness of the Fisher-Weil stochastic process in the accurate forecasting of interest rate movements, the same data as described in previous sections were again used. The calculated yield curve was regressed upon the observed yield curve and the result shown in Figure 6.7b. The R^2 value is also displayed on the graph. Whilst the R^2 value again seems to indicate a high degree of correlation (R^2 = 0.9417), comparisons with other suggested stochastic processes (see later sections in this chapter) will clearly show that this is far from the superior model.

Figure 6.7: (a) Previous period, observed next-period and forecast next-period yield curves using a Fisher-Weil stochastic process. (b) Regression of calculated Fisher-Weil shifts upon observed next-period yield curve.



6.2.4 Log-additive shifts

6.2.4.1 Theory

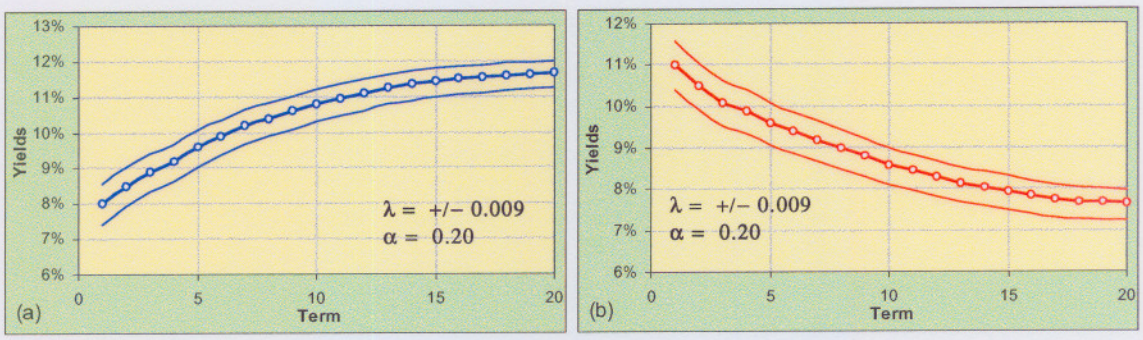
None of the above stochastic processes allow for short term rates to fluctuate instantaneously more than long term rates for both downward and upward sloping term structures. Khang (1979:1036) devised two processes that alleviate this problem (although Khang also worked in a continuous compounding framework, the analogous non-continuous time description is presented here). Khang's log-additive process was specified as:

$$y^*(0,t) = \frac{\lambda \cdot \ln(1+\alpha t)}{\alpha t} + y(0,t) \text{ for } t = 1,2,3,\ldots \qquad (6.18)$$

where λ is a random variable and α is a positive parameter. Since $\ln(1+\alpha t)$ (> 0) increases with t at a slower rate than αt so that the ratio $\ln(1+\alpha t)/\alpha t$ decreases at t increases. Thus, for any given value of α, short-term rates are instantaneously shocked by larger increments than long-term rates. The given positive value of α regulates the degree to which short-term rates fluctuate more than long-term rates. As $\alpha \to 0$, $\ln(1+\alpha t)/\alpha t \to 1$ and so, if α is made small enough, the log-additive process behaves like an additive process. Figure 6.8 below shows the possible effects of an upward ($\lambda = +0.009, \alpha = 0.2$) and downward ($\lambda = -0.009, \alpha = 0.2$) log-additive shift applied to an original (a) upward-sloping and (b) downward-sloping yield curve.

In practice, a numerical solver technique such as Newton-Raphson (Abramowitz and Stegun, 1972:18), was used to determine the optimal value of both α and λ. The simultaneous values of α and λ that forecast the most accurate yield curve values for the next day, are the ones chosen. A least squares method determines this degree of accuracy.

Figure 6.8: Log-additive shifts applied to the yield curve.



The derivative with respect to λ and evaluating it at $\lambda = 0$ is:

$$\left.\frac{dP}{d\lambda}\right|_{\lambda=0} = -\sum_{t=1}^{M} CF_t \left(\frac{1}{\alpha}\cdot\frac{\ln(1+\alpha t)}{(1+y(0,t))}\right)\cdot(1+y(0,t))^{-t}. \tag{6.19}$$

This allows an implicit definition of duration:

$$\frac{\ln(1+\alpha D_{\mathrm{LA}}(\alpha))}{(1+y(0,D_{\mathrm{LA}}(\alpha)))} = \frac{\sum_{t=1}^{M} CF_t \cdot \left(\frac{\ln(1+\alpha t)}{(1+y(0,t))}\right)\cdot(1+y(0,t))^{-t}}{P} \tag{6.20}$$

where $D_{\mathrm{LA}}(\alpha)$ is the duration for the **log-additive** stochastic process as a function of α and $y(0, D_{\mathrm{LA}}(\alpha))$ is the yield measured at the present time with term to maturity $D_{\mathrm{LA}}(\alpha)$. $D_{\mathrm{LA}}(\alpha)$ may not be made the subject of the formula and must be solved iteratively using a Newton-Raphson technique (Abramowitz and Stegun, 1972:18). The approximation equation is

$$\frac{\Delta P}{P} \cong \frac{-\ln(1+D_{\mathrm{LA}}(\alpha))}{\alpha\cdot(1+y(0,D_{\mathrm{LA}}(\alpha)))}\cdot\Delta\lambda,$$

or

$$\frac{\Delta P}{P} \cong \frac{-\ln(1+D_{\mathrm{LA}}(\alpha))}{\alpha\cdot(1+y(0,D_{\mathrm{LA}}(\alpha)))}\cdot\lambda \tag{6.21}$$

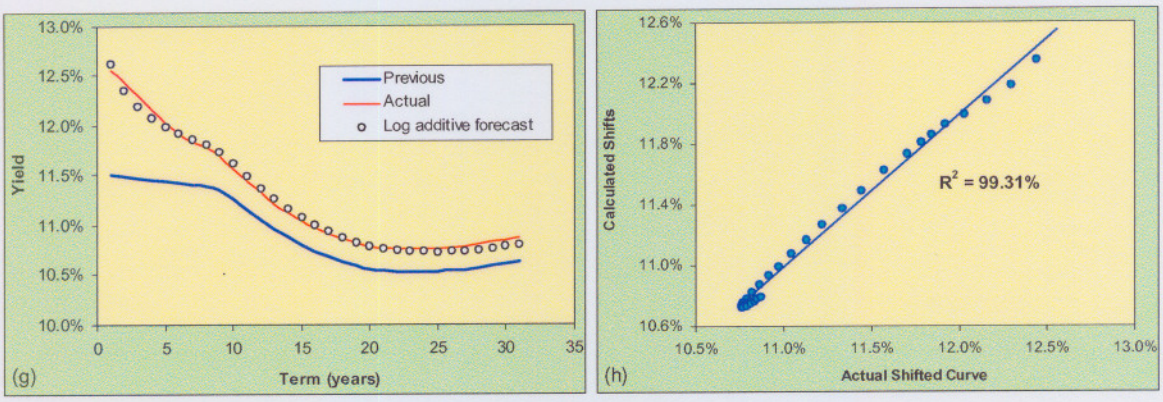
Since $\lambda = 0$ at the start of the analysis, therefore any change in λ, $\Delta\lambda$ is given by λ.

6.2.4.2 Empirical testing

To empirically test the effectiveness of the log-additive stochastic process in forecasting interest rate movements, the same data as described in previous sections were again used. It is clear from Figure 6.9a that the log-additive process best describes the observed shifts in the yield curve. The black open circles represent the best-fit-possible forecast using the stochastic log-additive shift process. The Newton-Raphson method was used to determine the optimal value of both λ and

α. The regression of the calculated yield curve upon the observed yield curve (Figure 6.9b) also consistently yields the highest R^2 value from all techniques examined, no matter the interest rate volatility of the period observed.

Figure 6.9: (a) Previous period, observed next-period and forecast next-period yield curves using a log-additive stochastic process. (b) Regression of calculated log-additive shifts upon observed next-period yield curve.



6.2.5 Log-multiplicative shifts

6.2.5.1 Theory

Khang's log-multiplicative process (Khang, 1979:1037) was specified as:

$$y^*(0,t) = \frac{\lambda \cdot \ln(1+\alpha t)}{\alpha t} + \left[1 + \frac{\lambda \cdot \ln(1+\alpha t)}{\alpha t}\right] \cdot y(0,t) \text{ for } t = 1,2,3,\ldots$$

where λ is a random variable and α is a positive parameter. It is a combination of both an additive and a multiplicative shock and it preserves the feature of short rates fluctuating instantaneously more than long-term rates. Both the log-additive and the log-multiplicative processes involve term-dependent shocks. The additive and multiplicative components are themselves functions of the term to maturity. This, then, is a manner in which the shock may vary with term to maturity, t. As $\alpha \to 0$, the log-multiplicative shift tends to a Fisher-Weil shock and the larger the value of α, the larger the ratio of short term rate changes to long term rate changes. Figure 6.10 below shows the possible effects of an upward ($\lambda = +0.009, \alpha = 0.2$) and downward ($\lambda = -0.009, \alpha = 0.2$) log-multiplicative shift applied to an original (a) upward-sloping and (b) downward-sloping yield curve. Note that changes in the long end of the curve are less pronounced than those in the short end.

With the log-multiplicative process in effect, the price of a security instantly after the shock to the term structure may be written:

$$P(\lambda) = \sum_{t=1}^{M} CF_t \cdot \left(1 + \frac{\lambda \cdot \ln(1+\alpha t)}{\alpha t}\right)^{-t} \cdot (1 + y(0,t))^{-t}.$$

The derivative with respect to λ and evaluating it at $\lambda = 0$ is:

$$\left.\frac{dP}{d\lambda}\right|_{\lambda=0} = -\sum_{t=1}^{M} CF_t \cdot \left(\frac{\ln(1+\alpha t)}{\alpha}\right) \cdot (1 + y(0,t))^{-t}. \tag{6.22}$$

This allows an implicit definition of duration:

$$\ln(1 + \alpha D_{\mathrm{LM}}(\alpha)) = \frac{\sum_{t=1}^{M} CF_t \cdot \ln(1+\alpha t) \cdot (1 + y(0,t))^{-t}}{P}$$

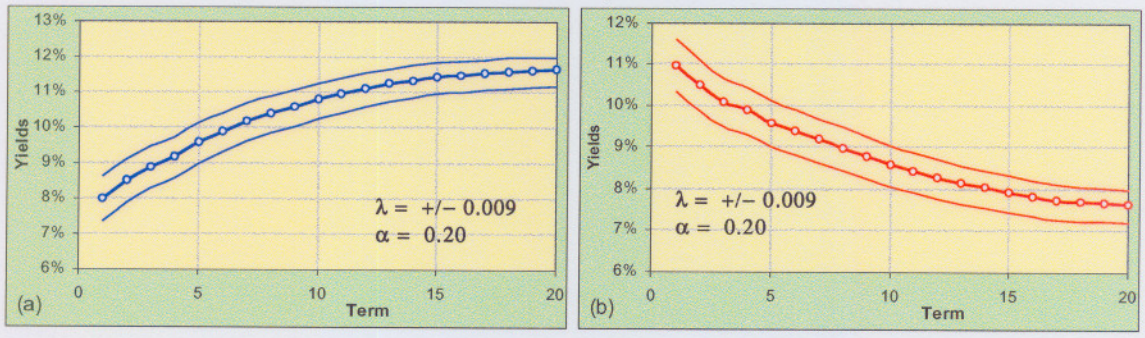
where D_{LM} is the duration for the **log-multiplicative** stochastic process as a function of α. The approximation equation is $\frac{\Delta P}{P} \cong \frac{-\ln(1+\alpha \cdot D_{\mathrm{LM}}(\alpha))}{\alpha} \cdot \Delta\lambda$.

or

$$\frac{\Delta P}{P} \cong \frac{-\ln(1+\alpha \cdot D_{\mathrm{LM}}(\alpha))}{\alpha} \cdot \lambda \tag{6.23}$$

Since $\lambda = 0$ at the start of the analysis, therefore any change in λ, $\Delta\lambda$ is given by λ.

Figure 6.10: Log-multiplicative shifts to the yield curve.

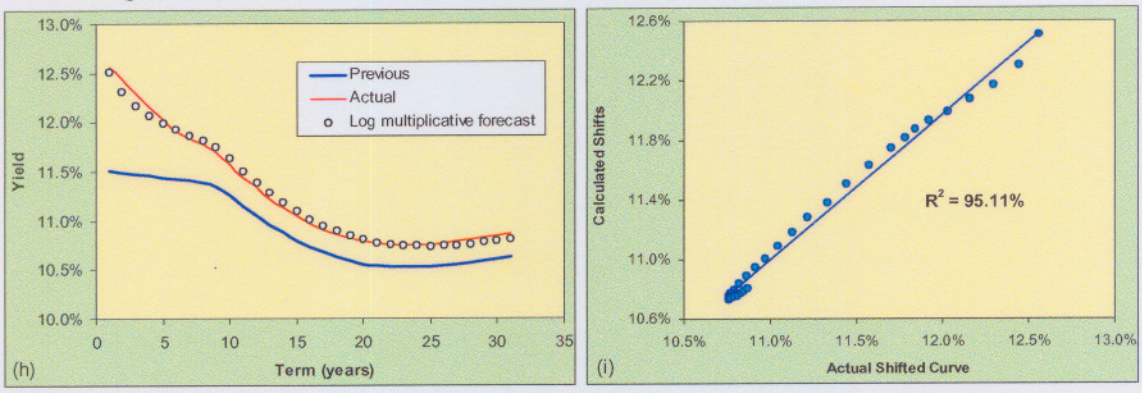


6.2.5.2 Empirical testing

To empirically test the effectiveness of the log-multiplicative stochastic process in the accurate forecasting of interest rate movements, the same data as described in previous sections were again used. Figure 6.11a shows that the log-multiplicative process describes the observed shifts in the yield curve reasonably accurately – certainly more accurately than the additive, multiplicative or Fisher-Weil stochastic processes, but not as statistically accurate as the log-additive process as indicated by a comparison of the respective R^2 values. The black open circles represent the best-fit-possible forecast using the stochastic log-multiplicative shift process. The Newton-Raphson method was again used to determine the optimal value of both λ and α. The regres-

sion of the calculated yield curve upon the observed yield curve (Figure 6.11b) consistently yields high R^2 values for all periods observed, no matter the interest rate volatility.

Figure 6.11: (a) Previous period, observed next-period and forecast next-period yield curves using a log-multiplicative stochastic process. (b) Regression of calculated log-multiplicative shifts upon observed next-period yield curve.



6.2.6 Conclusion

Five potential stochastic processes were employed in an attempt to describe the observed changes in the South African yield curve over the past 7½ years. Of these, only two (log-additive and log-multiplicative) showed statistically significant results when data – separated by one period – were regressed upon each other. A simple computer programme may be written to determine the values of variable α and the stochastic parameter λ on a period to period basis. This need not be onerous nor time-consuming from a computational point of view. The associated duration[40], once installed (for example) in a spreadsheet, requires little adjustments over and above those required for the Macaulay duration. This new duration, however, takes into account twists and bends of the underlying yield curve by relaxing the 'flat yield curve' requirement of the Macaulay duration. The implementation and testing of this new duration measure on empirical South African yield curve data is discussed in the next section.

6.2.7 South African empirical data

6.2.7.1 Data

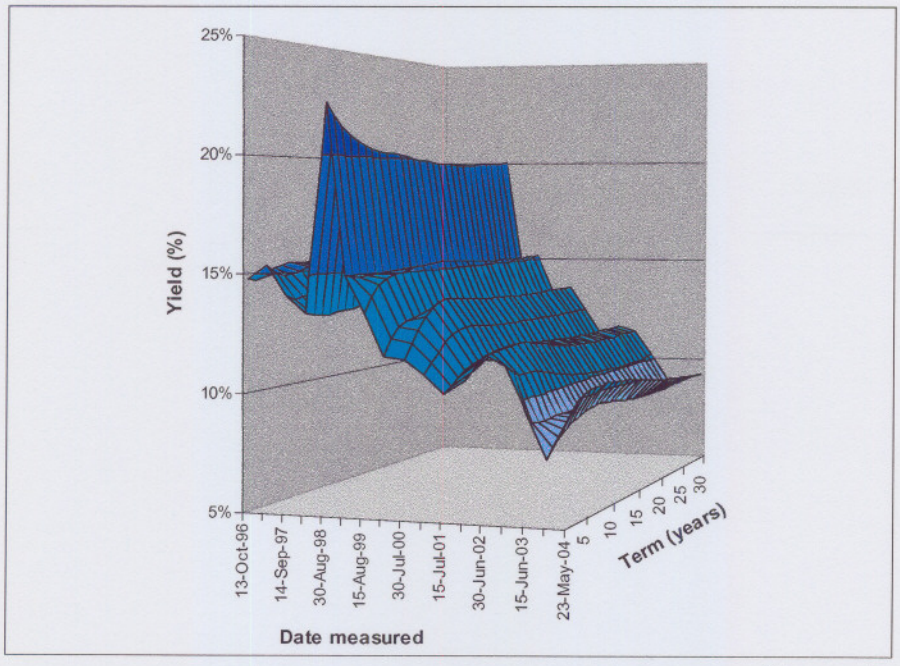
Empirical yield curve data used in the testing of the respective stochastic processes considered in Sections 6.2.1 to 6.2.5 are shown in Figure 6.12. This figure shows the yield curve (from 1 to 30-years, in annual steps) for the South African market over the 7½ years from October 1996 to May 2004.

It is clear that, during that time, the yield curve underwent severe twists, significant steepening and pronounced interest rate decreases. This turbulent period is, therefore, a good one in which

[40] This new duration is only marginally more complicated to implement than the standard Macaulay duration.

to test the applicability of the various stochastic processes. If a particular stochastic process performs well under the conditions exhibited by the South African yield curve in the period considered, it is likely to work well under *all* conditions[41], given the gamut of yield curve shapes and changes in this time. It is important to note that this era embraces the Asian (or Long Term Capital Management) crisis of 1998, the 9/11 terrorist attacks in the US and subsequent interest rate recoveries of both events.

Figure 6.12: Evolution in time of the South African yield curve from October 1996 – May 2004.



(Source Reuters™,2004)

Close-ups of the yield curve when rates were highly-variable are shown below in Figures 6.13(a) – 9/98 and (b) – 9/01

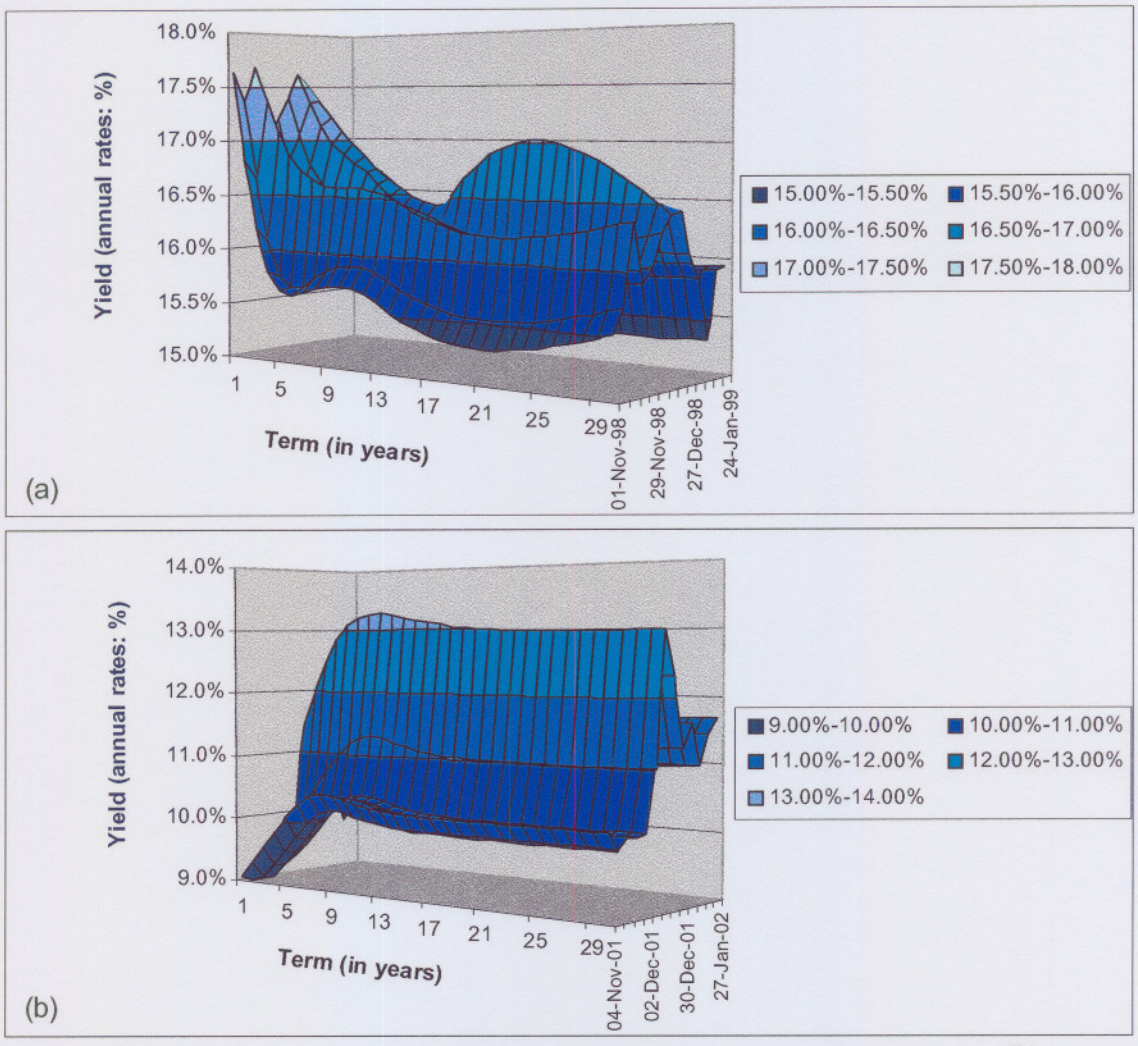
The data used were the 1-year through 30-year interest rates measured on a weekly basis using the Reuters™ 'Actuaries Index'[42]. Let $w = 0$ be the index given to the first data point, e.g., 1 October 1996. The subsequent week's index is $w = 1$, and the following $w = 2$, and so on. In any given week, w_i, the yield curve vector, measured from $t = 0$ up to term t, is given by $y(0,t)\big|_{w_i}$, $t = 1,2,3\ldots30$. The yield curve vector in the subsequent week is $y(0,t)\big|_{w_{i+1}}$, $t = 1,2,3\ldots30$. The

[41] Further rigorous testing may be required, but these early results are promising.

[42] These data take the form of quoted annual interest rates, compounded annually. The JSE Actuaries Index series, which measures the domestic South African market, converted to an enhanced design methodology on 25 June 2002 and became part of the FTSE/JSE Africa Index Series. FTSE indices are used extensively by investors world-wide for investment analysis, performance measurement, asset allocation and for creating a wide range of index tracking funds. Committees of senior fund managers, derivatives experts, actuaries and experienced practitioners independently review all changes to the indices ensuring that they are accurate, timely and without bias. Real time FTSE indices are calculated on systems managed by Reuters™.

stochastic processes detailed in the previous section were applied in turn to yield curve vectors $y(0,t)|_{w_i}$, $t = 1,2,3\ldots30$, and a 'best fit' curve (to $y(0,t)|_{w_{i+1}}$, $t = 1,2,3\ldots30$) established using the ordinary least squares technique. The calculated values (according to the relevant stochastic process, $y'(0,t)|_{w_{i+1}}$, $t = 1,2,3\ldots30$) were regressed on actual values $y(0,t)|_{w_i}$, $t = 1,2,3\ldots30$ and the R^2 measured. This was performed on each of the data points for the 7½ years under scrutiny.

Figure 6.13: Changes in the South African yield curve over (a) the Asian crisis of 1998 and post September 11, 2001.

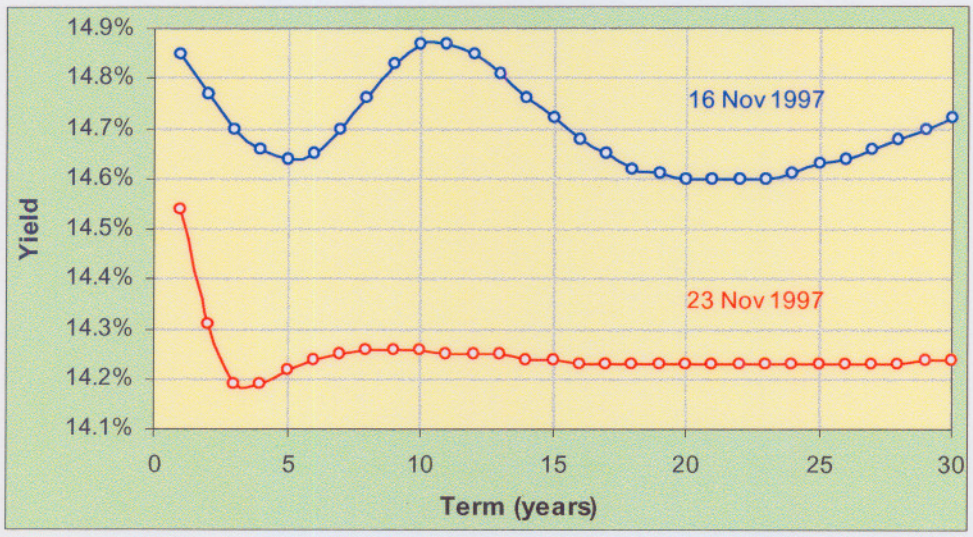


(Source Reuters™,2004)

The results showed that simple additive and multiplicative stochastic processes fare the worst, although this is not surprising given that inter-week shifts can frequently be far from 'parallel' upward or downward shifts, as shown in Figure 6.14 in which a flattening (in the long end of the curve), twisting and steepening (in the short end) were all experienced over one week. In cases such as these (and others), Fisher-Weil shifts are also highly inaccurate, returning relatively low R^2 values. Additive, multiplicative and Fisher-Weil shift processes are unable to cope with nei-

ther the rapid steepening (or flattening) of the yield curve nor some of the more severe twists that sometimes occur. For a slowly changing yield curve, these methods do, however, work best[43].

Figure 6.14: Example of an inter-week shift in the South African yield curve. This example shows the yield curve for 16 Nov 1997 and 23 Nov 1997.



(Source Reuters™:2004)

The log stochastic shift processes – both additive and multiplicative – returned consistently high R^2 values. The log-additive process was finally selected as the candidate process on which to test duration forecast accuracy.

Having established the most accurate stochastic process generator for South African yield data, the associated log-additive duration values were now determined. The log-additive duration was calculated using Equation 6.20. The real test of value for the stochastic processes above is not in the accuracy of their forecast of yield curve movements, but rather in the accuracy of their measure of bond price change with a given yield curve change (here using Equation 6.21).

6.2.7.2 Instrument construction

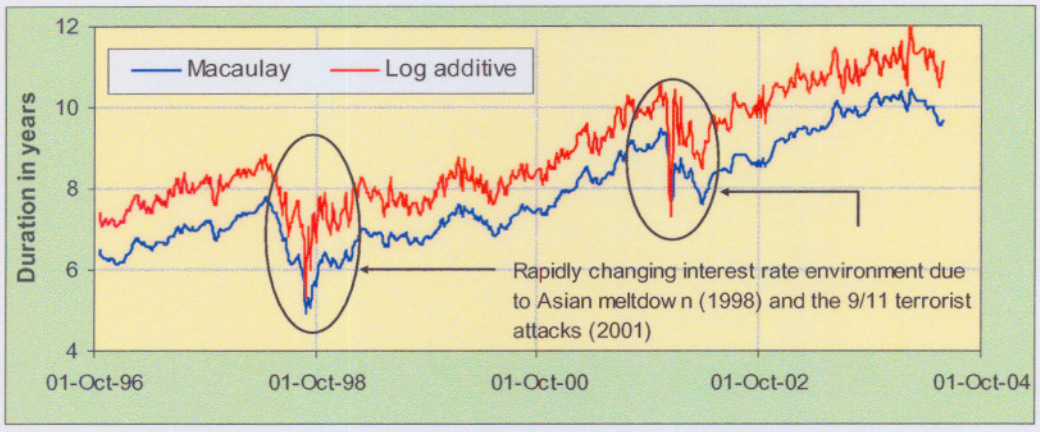
A hypothetical 'rolling' 30-year bond was constructed and its Macaulay (and corresponding log-additive) duration calculated at weekly points in time over the period in which the yield curve was measured (i.e., October 1996 to June 2004). The change in the bond price was then determined using these duration measures and (i) the change in the yield to maturity (for the Macaulay duration) and (ii) the change in the stochastic parameter, λ, for the log-additive duration. The percentage change in the bond price as forecast by the two measures could then be compared (see Appendix C for statistical p- and t- tests for yield curve forecasting accuracy as well as determination of actual change in bond prices, ΔP). In all cases, the log-additive duration

[43] Tabulated results, including t- and p- tests are presented in Appendix C.

bond price change forecasts were *considerably* more accurate. While the percentage bond price forecast improves with the combination of the convexity term in the case of the Macaulay duration, no such improvement was required for the log-additive measure.

Figure 6.15, shows the evolution of the Macaulay and log-additive durations measured over the entire observation period. Times when the yield curve was changing rapidly are clearly shown e.g., the period immediately following the Asian crisis (beginning in September 1998) and the months following September 2001. Both the Macaulay and the log-additive durations fall here as interest rates rise and then slowly recover afterwards as rates fall again and stability returns.

Figure 6.15: Macaulay + log-additive durations over the entire observation period.



6.2.7.3 Applications

6.2.7.3.1 Elimination of convexity

Estimating changes in the yield curve using the log-additive method may be easily included in existing spreadsheet models. Whilst the calculation of the stochastic variable, λ, and parameter α is marginally more complex to implement, once set-up, require minimal adjustments. The technique used to determine these is the standard statistical methodology of ordinary least squares which is itself simple to both understand and implement. The complexity involved with log-additive implementation is arguably no more time consuming than calculating and incorporating the convexity adjustment, a requirement of the Macaulay duration interest rate sensitivity calculation. The log-additive duration technique makes no assumptions about the manner in which yield curves twist and move in general – hence no limiting suppositions are required.

6.2.7.3.2 Forecasting accuracy

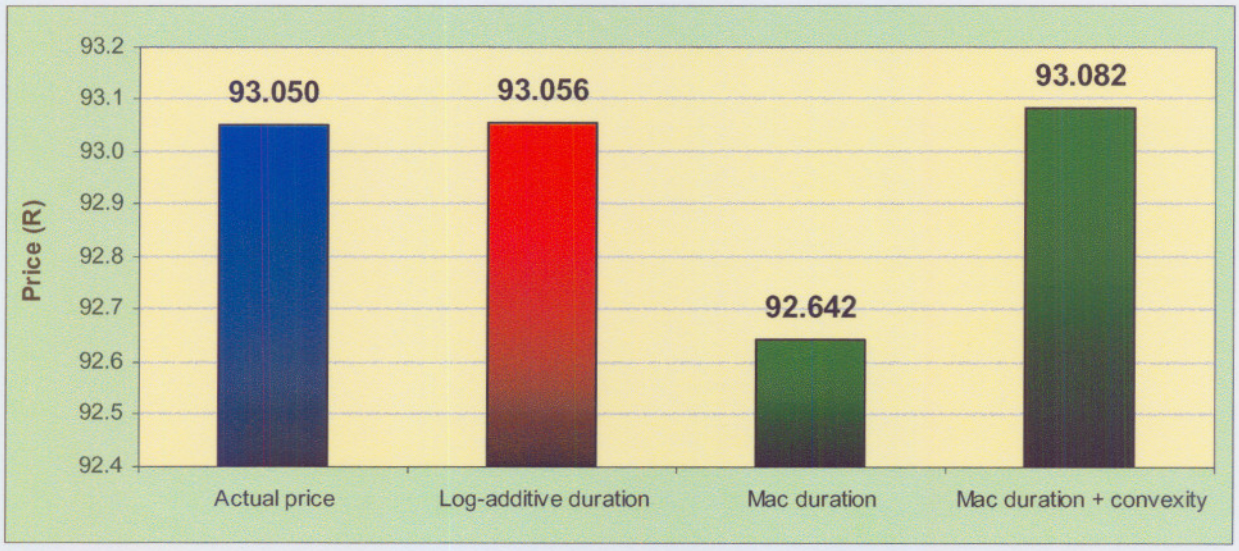
Appendix C gives the statistical *p*- and *t*- tests for both yield curve forecasting accuracy as well as accuracy in estimating changes in bond price. The log-additive technique produces signifi-

cantly better results without the need to adjust for convexity. Indeed, it is even more accurate than the first and second order terms of Equation 3.5 combined, i.e.,

$$P_1 = P_0 + \Delta y \cdot \left.\frac{\partial P}{\partial y}\right|_{y=y_0} + \frac{1}{2} \cdot (\Delta y)^2 \cdot \left.\frac{\partial^2 P}{\partial y^2}\right|_{y=y_0} + \cdots$$

The values given above the columns in Figure 6.16 indicate the new bond price after a given, observed yield curve shift. This value is compared with the log-additive duration forecast in red, the Macaulay duration forecast and the sum of the Macaulay duration plus convexity adjustment forecast in green. Whilst the sum of the Macaulay duration and convexity estimate the new bond price with admirable accuracy (< 0.034%), the log-additive duration process provides an even more accurate estimate (< 0.006%).

Figure 6.16. Actual new bond price after a given yield curve shift, compared with log-additive duration forecast, Macaulay duration forecast and Macaulay duration + convexity adjustment forecast.



6.2.7.3.3 Improvement of DGap measurement

Recall from Equation 6.2, that

$$\frac{\Delta P}{P} \cong \frac{-\ln(1 + D_{\mathrm{LA}}(\alpha))}{\alpha \cdot (1 + y(0, D_{\mathrm{LA}}(\alpha)))} \cdot \lambda$$

where $D_{\mathrm{LA}}(\alpha)$ is the log-additive duration. This duration was established as being the most accurate measure of interest rate risk from a suite of stochastic yield curve shape change possibilities. This may be further tested on other applications where the Macaulay duration characteristically enjoys some success, for example DGap (see Section 4.2.4.2). Consider the identical balance sheet as that presented in Section 4.2.4.2. (Tables 4.5 and 4.6), but assume instead that the

log-additive duration has been employed and calculated using a full yield curve – not one in which the yields at all maturities as assumed to be the same (i.e., the flat yield curve requirement for the Macaulay duration has been relaxed). Yield curves were chosen that were both upward and downward sloping, but chosen such that, in each case, all instruments were priced at par in the initial period or base case (Table 6.1) – as was achieved using the flat yield curve described in Section 4.2.4.2. As discussed in Section 6.2.4, $\lambda = 0$ for the initial (or base) case.

To minimise controls, a stochastic yield curve shift was applied such that it gave essentially the same 1% upward shift in all yields as in the example in Section 4.2.4.2. Many stochastic shifts are possible – each having a different combination of λ and α and each equally probable. Several combinations were tested and the results are tabulated in Table 6.3 below. Table 6.2 shows a possible balance sheet for one such combination and corresponding log-additive durations.

The DGap in this example is now 1.38 years – using the same technique again as that used in Section 4.2.4.2. The change in the market value of equity is

$$\begin{aligned}\Delta MVE &= -\text{DGap} \cdot \left(\frac{\lambda}{\alpha \cdot (1 + y(0, D_{\text{LA}}(\alpha)))} \right) \cdot MVA \\ &= -\text{R}12.22\end{aligned},$$

where the factor

$$\left(\frac{\lambda}{\alpha \cdot (1 + y(0, D_{\text{LA}}(\alpha)))} \right),$$

is the scale factor that replaces

$$\left(\frac{\Delta y}{1 + y_{\text{Total assets}}} \right),$$

in Equation 4.4 as a direct consequence of the definition of log-additive duration (Equation 6.21).

Table 6.1. Hypothetical balance sheet – base case using log-additive duration.

	Par R 1,000	Coupon	Maturity	Parameters	Market Value	Duration
Assets				$\alpha = 0.282$		
Cash	100			$\lambda = 0.000$	100	
Earning assets						
3-yr Commercial loan	700	12%	3		700	2.69
6-yr Treasury bond	200	8%	6		200	4.99
Total Earning Assets	900				900	
Non-cash earning assets	0				0	
Total assets	1000				1000	2.88
Liabilities						
Interest bearing liabs.						
1-yr Time deposit	620	5%	1		620	1.00
3-yr Certificate of deposit	300	7%	3		300	2.81
Tot. Int Bearing Liabs.	920				920	
Tot. non-int. bearing	0				0	
Total liabilities	920				920	1.59
Total equity	80				80	
Total liabs & equity	1000				1000	

(Adapted from Koch and MacDonald, 2000).

Figure 6.2. Hypothetical balance sheet – shifted case using log-additive duration and a specific combination of λ and α.

	Par R 1,000	Coupon	Maturity	Parameters	Market Value	log additive duration
Assets				$\alpha = 0.241$		
Cash	100			$\lambda = 0.0051$	100	
Earning assets						
3-yr Commercial loan	700	12%	3		683.41	2.65
6-yr Treasury bond	200	8%	6		191.08	4.93
Total Earning Assets	900				874.49	
Non-cash earning assets	0				0	
Total assets	1000				974.49	2.83
Liabilities						
Interest bearing liabs.						
1-yr Time deposit	620	5%	1		613.09	1.00
3-yr Certificate of deposit	300	7%	3		291.87	2.73
Tot. Int Bearing Liabs.	920				904.96	
Tot. non-int. bearing	0				0	
Total liabilities	920				904.96	1.56
Total equity	80				69.53	
Total liabs & equity	1000				974.49	

(Adapted from Koch and MacDonald, 2000).

Table 6.3. Combinations of λ and α that could give rise to yield shifts indicated in Figure 4.1, with corresponding DGaps and ΔMVE's.

		D_{Mac}Gap = 1.42		**D_{Mac} ΔMVE = -R12.40**
λ	α	D_{LA}Gap	ΔMVE's from D_{LA}	% diff from D_{Mac} ΔMVE
0.0051	0.241	1.377	-R12.20	-5.4%
0.0047	0.288	1.346	-R11.93	-7.6%
0.0041	0.312	1.333	-R11.81	-8.4%
0.0037	0.403	1.306	-R11.57	-10.3%
0.0033	0.551	1.291	-R11.44	-11.4%

The log-additive duration measure has been shown to be a superior forecaster of changes in both the yield curve and bond prices and thus is an improved measure of interest rate risk. The log-additive duration does not entail vast computational complexity and indeed, to forecast the next period yield curve, requires an estimate of the change in only one stochastic parameter – λ – from period to period and one variable α. The Macaulay duration achieves similar ends and also requires only an estimate of the change in only one parameter: the yield to maturity, in order to forecast bond price changes. But it is there that the similarities between the two durations end.

A comparison of the pros, cons, limitations and successes of the two duration measures is given below in Table 6.4.

The log-additive duration does not suffer from the same drawbacks or limiting assumptions that shackle the Macaulay duration, yet it consistently produces better forecasts of yield curve changes and does not require a cumbersome corrective term in the form of convexity. For the South African fixed income market at least, the log-additive duration seems to be the measure of choice. Other markets may benefit from other stochastic processes detailed in this section. A combination of refinements to volatility measurements (Botha and van Vuuren, 1999; van Vuuren *et al*, 2000, van Vuuren *et al*, 2003), and this more accurate log-additive duration measure could lead to overall superior risk measurement of fixed income securities.

6.3 Conclusion

The improvement to fixed income pricing estimation due to stochastic yield curve parameter movements, at least for the South African fixed-income market, has been established empirically. Early results (see Appendix C) indicate that the log-additive duration measure is signifi-

cantly better at fixed income pricing estimation than the Macaulay duration and, in addition, requires no convexity adjustment.

Table 6.4: Comparison of salient features of the Macaulay and log-additive duration measures.

	MACAULAY	LOG-ADDITIVE
Input required	Cash flow table and current yield curve	
Output	Macaulay duration	Log-additive duration
Required to estimate bond price change	Change in yield to maturity	Change in stochastic yield curve shape variable λ and parameter α
Duration calculation	Time weighted cash flows divided by original price using Equations 5.2 and 5.3: $w_t = \frac{CF_t \cdot \left(1+{}^{y}/_{2}\right)^{-t}}{P} == \frac{CF_t \cdot DF_t}{P}$ $D_{\text{Mac}} = \sum_{n=1}^{M} w_t \cdot t_n$	Duration is implicit – can be calculated using an iterative process using Equation 6.20: $\frac{\ln(1+\alpha D_{\text{LA}}(\alpha))}{(1+y(0,D_{\text{LA}}(\alpha)))}$ $= \frac{\sum_{t=1}^{M} CF_t \cdot \left(\frac{\ln(1+\alpha t)}{(1+y(0,t))}\right) \cdot (1+y(0,t))^{-t}}{P}$
Assumptions	• Flat yield to maturity curve at all tenor points • Yield to maturity curve moves only in a parallel fashion • Yield to maturity movements change in a linear manner	• No assumption about yield curve shape – any change is possible • Any change in yield curve possible • Any movement is possible
Forecast statistics	See Appendix C	
Calculation of $\frac{\Delta P}{P}$	Equation 3.4: $\frac{\Delta P}{P} = \frac{-n \cdot D_{\text{Mac}} \cdot \Delta y}{\left(1+{}^{y}/_{n}\right)}$	Equation 6.21 $\frac{\Delta P}{P} \cong \frac{-\ln(1+D_{\text{LA}}(\alpha))}{\alpha \cdot (1+y(0,D_{\text{LA}}(\alpha)))} \cdot \lambda$
Amendments	Convexity partially corrects for curvature of price/yield relationship	No convexity correction required for curvature of price/yield relationship

An overall summary of the thesis, main conclusions drawn from the research and suggestions for possible future exploratory directions are presented in the next, final chapter.

Chapter 7

Summary, conclusions and suggestions for future work

7.1 Summary

The rapidly-changing interest rate risk environment of banks and other financial institutions is under increasing pressure from investors, regulators and governments to provide accurate, timely and transparent reports of risks taken and profits/losses recorded. These were more easily concealed in the past due to loopholes in the complex nature of derivative profit and loss reporting, lack of capital required for the banking book's interest rate risk and a generally loose approach to some aspects of banks' interest rate risk.

Present-day banks are being forced to adapt to far more stringent regulatory and accounting environments. Whilst trading book, derivative profit and loss went largely unreported in previous years, this moved from being off-balance sheet to on-balance sheet only a few months ago (January 2005), due largely to the convergence of internationally acceptable accounting rules embodied in the IAS 39 document and local variations thereof (e.g., AC 133). Financial instruments which comprise embedded optionality will now be reported on balance sheet and not – as before – only at maturity. In addition, all instruments which face the possibility of default must now be priced accordingly if the correct fair value of the derivative in question is to be obtained. Interest rate risk of a portfolio cannot be measured accurately if the instruments that comprise the portfolio are themselves inaccurately priced or if the way in which they react to underlying shifts in risk factors (i.e., shifts in the yield curve) is unknown or erroneously modelled. The existing Macaulay duration – as a tool for the measurement of interest rate risk – is clearly inadequate in the current financial environment. This thesis examined possible amendments to the Macaulay duration and tested the application of these amendments to empirical data. The alterations that best embraced both embedded optionality and implied defaultability were explored further and considered with regards to the imminent introduction of new, stringent regulatory pressures (in the form of the new Basel accord) and accounting standards (in the from of AC 133).

The new Basel accord is now less than two years away from full implementation (time of writing: June 2005). Banks will, at that stage, be forced – *inter alia* – to calculate and then reserve regulatory capital for interest rate risk that arises from banking books. Whilst there are several methods in which to do this, the current Macaulay duration methodology is incapable of accu-

rately measuring potential capital requirements for movements in the yield curve. Among its many shortfalls is its implicit assumption that the yield curve is both flat and that all subsequent movements are parallel to this flat yield curve. New interpretations of the Macaulay duration as a measure of interest rate risk are required in which such limiting assumptions are not made. This thesis introduced and tested some potential new measures which did not rely on restrictive conjectures and considered the application of these measures to interest rate-sensitive financial instruments. The successful application of these results to a potentially improved *economic* measure of the market value of equity (via an improved duration measurement) was undertaken and confirmed empirically.

The relevance of all of these new measures and interpretations were placed in the context of the previous, current and forthcoming regulatory and legal environments and considered new impending accounting standards.

Table 7.1 Summary of thesis conclusions from the banking and trading book perspectives.

Table 7.1. Conclusions and summary.

	TRADING BOOK	**BANKING BOOK**
Activity	Instruments traded for hedging and speculation	Core banking activity
Instruments involved	Bonds, interest rate swaps, options, futures, forwards, exotic derivatives	Deposits and loans (assets and liabilities)
New legislation	**IAS 39**	The new Basel accord
Basel I: capital reserve	Required	Not currently required
The new Basel accord: capital reserve		Will be required (2007)
Effect of IAS39/AC 133	Derivatives move from 'off' to 'on' to balance sheet (January 2005)	Transactions remain on balance sheet (unaltered)
Measurement of IRR	VaR, scenario testing, stress testing	Gap, duration gap, Monte Carlo interest rate scenario generation
Will benefit from:	Improved valuation accuracy of instruments with embedded optionality and which have credit risk	**Improved measurement of duration gap**
Therefore important that:	Defaultable and option-embedded instruments are *correctly* priced and interest rate risk for derivative instruments accurately dealt with	Reserve capital is both adequate (not too small) and constrained (not too large) – accurate measurement of gap – hence duration of assets and liabilities
Thesis chapters:	Chapter 5	Chapter 6
Perspective	**Market**	**Economic**
	Short term (trading) horizons	Longer term perspective
		On and off balance sheet

7.2 Conclusions

The existing Macaulay duration measure has enjoyed almost unparalleled success in the financial arena. It has undergone several modifications over its lifetime to adapt to new and innovative financial instruments which were unavailable in the 1930's. The Macaulay duration functions tolerably well if these amendments are correctly implemented. Financial instruments which have embedded options and the probability of default were both dealt with in this thesis and the Macaulay duration was altered in a way which allowed it to take account of both these important features. Initial results are good, reproducible and explicable within the context of changing interest rate environments. For all the effort dedicated to resuscitating the Macaulay duration, the feeling remains that an entirely new measure must ultimately be found – not a patchwork of recalibrations.

Returning to original definitions of duration and far more realistic, stochastic descriptions of yield curve movements allows precisely such a new measure or measures to be defined. Whilst the mathematics underpinning the yield curve shifts is not new, it has not previously been applied and tested in the current, rapidly-evolving financial environment. These shifts were explored in detail and found to provide accurate measures of the interest rate risk associated with many shape shifts of the South African yield curve. These are easily implemented and seem to negate the necessity for the convexity adjustment.

Accounting standards and the regulatory milieu will continue to demand more stringent requirements as the process of financial engineering becomes ever-more complex. Adopting sensible, robust measures which approximate a bank's interest rate risk accurately – whether trading or non-trading – can only protect a bank's bottom line and provide greater assurance to a nervous investment community.

7.3 Suggestions for future work

Further research could include stochastic descriptions of yet other possible yield curve shifts, including the rigorous back-testing of these measures. Within the mathematical framework presented in this thesis, the development of a several 'standard' stochastic measures could in principal be developed and implemented according to the relevant economic environment.

The contradictory results between theory and observation, reported in Section 4.2, necessitate further theoretical examination.

The log-additive duration seems to be the measure of choice for the South African fixed income market (in terms of accuracy of forecasting the effect of interest rate movements on fixed in-

come prices). International fixed income markets may benefit from different stochastic processes detailed in this thesis or, indeed, entirely new definitions of stochastic movements in interest rates. This important question deserves further investigation.

A combination of refinements to volatility measurements (Botha and van Vuuren, 1999; van Vuuren, 2000 and 2001), and this more accurate duration measure could lead to overall superior risk measurement of fixed income securities.

A further project could include synthesising the embedded optionality and defaultability mathematics of this thesis with the stochastic yield curve shift duration mathematics. This could result in yet a more accurate measure of duration and hence interest rate risk in general.

APPENDICES

Appendix A

Derivation of continuous compounding equation

The limit of the pay-off function of interest rate r as the length of the compounding period tends to zero is e^r, i.e.,

$$\lim_{n\to\infty}\left\{\left(1+\frac{r}{n}\right)^n\right\}=e^r.$$

where n is the number of compounding periods. This may be demonstrated by performing a binomial expansion on the left hand side of the above equation:

$$\begin{aligned}\left(1+\frac{r}{n}\right)^n &= \sum_{i=0}^{n}\frac{n!}{i!(n-i)!}\cdot\left(\frac{r}{n}\right)^i\\ &=\sum_{i=0}^{n}\frac{1}{i!}\cdot\frac{n\cdot(n-1)\ldots(n-i+1)}{n^i}\cdot r^i\\ &=\sum_{i=0}^{n}\frac{r^i}{i!}\cdot\left\{\frac{n}{n}\cdot\frac{n-1}{n}\cdots\frac{n-(i-1)}{n}\right\}\\ &=\sum_{i=0}^{n}\frac{r^i}{i!}\cdot\left\{1\cdot\left(1-\frac{1}{n}\right)\ldots\left(1-\frac{i-1}{n}\right)\right\}.\end{aligned}$$

As

$$\lim_{n\to\infty}\left\{1.\left(1-\frac{1}{n}\right)\ldots\left(1-\frac{i-1}{n}\right)\right\}=1\ ,$$

then

$$\lim_{n\to\infty}\left[\sum_{i=0}^{n}\frac{r^i}{i!}\left\{1.\left(1-\frac{1}{n}\right)\ldots\left(1-\frac{i-1}{n}\right)\right\}\right]=\sum_{i=0}^{\infty}\frac{r^i}{i!}=e^r\ .$$

Appendix B

Tracking error

Used in the context of general equities, tracking error is the standard deviation of the difference between the portfolio return and the desired investment benchmark return measured over a given time period.

$$TE = \sqrt{\frac{1}{T-1} \cdot \sum_{t=1}^{T} \left[\left(r^p - r^b \right)_t - \overline{\left(r^p - r^b \right)} \right]^2} ,$$

where T is the number of time periods under investigation, r^p is the portfolio return over a single period, r^b is the benchmark return over a single period and the overbar represents the average of the quantities concerned. Tracking error thus provides a measure of *relative* risk and is popular in asset management portfolio risk analysis.

Note the similarity to the standard and most commonly applied measure of risk: standard deviation, σ:

$$\sigma = \sqrt{\frac{1}{T-1} \cdot \sum_{t=1}^{T} [r_t - \bar{r}]^2} .$$

Appendix C

Regression results: Defaultable and default free Macaulay duration

Regression data used to analyse the results presented in Chapter 6, are given below in Tables C.1 and C.2, for the years 1997 through 2004.

Table C.1. Summary of regression statistics for $D_{\text{Mac}} = -\frac{dP}{dy} \cdot \frac{1}{n \cdot P} \cdot \left(1 + \frac{y}{n}\right)$ against the *defaultable* Macaulay duration.

		1997	1998	1999	2000	2001	2002	2003	2004
	Multiple R	0.993	0.992	0.993	0.992	0.993	0.994	0.992	0.992
	R^2	0.985	0.985	0.986	0.984	0.986	0.988	0.984	0.985
	Adjusted R^2	0.985	0.985	0.985	0.984	0.986	0.987	0.983	0.985
	Standard Error	0.426	0.455	0.431	0.435	0.411	0.393	0.452	0.410
	Observations	118	120	111	120	110	112	108	120
ANOVA	df	SS							
Regression	1	1426	1569	1497	1382	1412	1447	1441	1306
Residual	118	21.43	24.43	21.88	22.31	19.93	18.22	24.08	19.82
Total	119	1447	1593	1519	1405	1432	1466	1465	1326
MS	Regression	1426	1569	1497	1382	1412	1447	1441	1306
	Residual	0.182	0.207	0.185	0.189	0.169	0.154	0.204	0.168
Regression	F	7850	7579	8074	7312	8362	9372	7061	7777
	Significance F	8.5E-110	6.5E-109	1.7E-110	5.3E-108	2.2E-111	2.8E-114	4.0E-107	1.5E-109
Coefficients	Intercept	1.691	1.801	2.030	1.856	2.062	1.919	2.140	1.913
	Yield	1.010	0.995	0.980	0.991	0.977	0.992	0.965	0.993
Standard Error	Intercept	0.138	0.133	0.124	0.127	0.127	0.119	0.131	0.134
	Yield	0.011	0.011	0.011	0.012	0.011	0.010	0.011	0.011
t Stat	Intercept	12.224	13.506	16.318	14.629	16.195	16.179	16.396	14.302
	Yield	88.599	87.059	89.856	85.512	91.445	96.811	84.030	88.188
P-value	Intercept	1.1E-22	1.1E-25	4.9E-32	2.8E-28	9.1E-32	9.9E-32	3.3E-32	1.6E-27
	Yield	8.5E-110	6.5E-109	1.7E-110	5.3E-108	2.2E-111	2.8E-114	4.0E-107	1.5E-109
Lower 95%	Intercept	1.417	1.537	1.784	1.605	1.810	1.684	1.881	1.648
	Yield	0.987	0.972	0.959	0.968	0.955	0.971	0.942	0.971
Upper 95%	Intercept	1.965	2.065	2.276	2.107	2.315	2.154	2.398	2.178
	Yield	1.032	1.018	1.002	1.014	0.998	1.012	0.988	1.016

Table C.2. Summary of regression statistics for $D_{\text{Mac}} = -\frac{dP}{dy} \cdot \frac{1}{n \cdot P} \cdot \left(1 + \frac{y}{n}\right)$ against the *default free* Macaulay duration.

		1997	1998	1999	2000	2001	2002	2003	2004
	Multiple R	0.990	0.991	0.991	0.991	0.991	0.989	0.990	0.991
	R^2	0.981	0.983	0.983	0.982	0.981	0.978	0.980	0.981
	Adjusted R^2	0.980	0.982	0.983	0.982	0.981	0.978	0.980	0.981
	Standard Error	0.435	0.420	0.432	0.443	0.428	0.441	0.455	0.413
	Observations	117	111	112	116	114	115	115	117
ANOVA	df	SS							
Regression	1	1129	1173	1255	1279	1136	1016	1186	1061
Residual	118	22.30	20.85	22.01	23.12	21.59	22.95	24.43	20.10
Total	119	1152	1194	1277	1302	1157	1039	1210	1081
MS	Regression	1129	1173	1255	1279	1136	1016	1186	1061
	Residual	0.189	0.177	0.187	0.196	0.183	0.194	0.207	0.170
Regression	F	5975	6642	6728	6527	6206	5222	5728	6226
	Significance F	6.4E-103	1.4E-105	6.6E-106	3.8E-105	7.1E-104	1.5E-99	7.3E-102	5.9E-104
Coefficients	Intercept	2.150	2.343	2.556	2.186	1.966	2.294	2.056	2.156
	Yield	0.892	0.873	0.860	0.888	0.908	0.876	0.894	0.877
Standard Error	Intercept	0.137	0.127	0.122	0.135	0.134	0.141	0.136	0.125
	Yield	0.012	0.011	0.010	0.011	0.012	0.012	0.012	0.011
t Stat	Intercept	15.713	18.499	20.914	16.211	14.716	16.216	15.133	17.199
	Yield	77.297	81.497	82.025	80.788	78.778	72.265	75.684	78.905
P-value	Intercept	1.0E-30	1.1E-36	1.7E-41	8.4E-32	1.8E-28	8.2E-32	2.1E-29	6.1E-34
	Yield	6.4E-103	1.4E-105	6.6E-106	3.8E-105	7.1E-104	1.5E-99	7.3E-102	5.9E-104
Lower 95%	Intercept	1.879	2.093	2.314	1.919	1.702	2.014	1.787	1.908
	Yield	0.869	0.852	0.839	0.866	0.886	0.852	0.871	0.855
Upper 95%	Intercept	2.421	2.594	2.798	2.454	2.231	2.574	2.325	2.405
	Yield	0.915	0.894	0.881	0.910	0.931	0.900	0.918	0.899

Appendix D

Statistical tests

Using data from Chapters 6 of this thesis, statistical *t*- and *p*- tests were conducted as follows. Two consecutive days of *known* yield curve data, d_t and d_{t+1} were used. The log-additive methodology was then employed to generate the value of λ_t and α_t that best approximated d_{t+1}'s yield curve from d_t 's yield curve data, using the ordinary least squares method. Values of λ_t and α_t were applied to the previous day's data using $y^*(0,t) = \frac{\lambda \cdot \ln(1+\alpha t)}{\alpha t} + y(0,t)$ (Equation 6.18) in which y^*_{LA} is the log-additive forecast yield curve vector and y is today's actual yield curve vector. Regressing y^*_{LA} on y gives R^2_{LA} (first column) and an estimate of both the gradient (3rd column) and associated $t-$ and $p-$statistics (5th and 7th columns respectively). Yields to maturity were found for d_t and d_{t+1} (note that the implicit assumption has been made that the yield curve is flat), as well as the difference between these, Δytm. This value was then added to y: d_t's actual yield curve vector to give d_{t+1}'s forecast yield curve vector, y^*_{Mac}. Regressing y^*_{Mac} on y gives R^2_{Mac} (second column) and an estimate of both the gradient (column 4) and the associated $t-$ and $p-$statistics (columns 6 and 8 respectively). The change in λ, $\Delta\lambda = \lambda_{t+1} - \lambda_t$ and recorded in bps gives the 9th column (note: $t = 1$, $\lambda_1 = 1$ by definition). The 10th column gives $\Delta ytm = ytm_{t+1} - ytm_t$ in bps. Forecast values of the change in the bond price under investigation using the log-additive duration (discussed in Chapter 7), ΔP_{LA}, is given in the 11th column, whilst those estimated by the Macaulay duration, ΔP_{Mac}, are shown in the 12th column. Adding the convexity adjustment to the change in bond price as estimated by the Macaulay duration is shown in the 13th column and given by $\Delta P_{\text{Mac+C}}$. The final (14th) column, gives the actual change in the bond price, ΔP_{Actual} as obtained using actual yield curves and simulated bonds with known coupon and maturity. Whilst it is clear that the combination of Macaulay duration and convexity are sufficient to give accurate values of ΔP, the log-additive duration requires no such adjustment and enjoys far better overall statistics than the Macaulay duration in all instances. This technique was used on multiple-security portfolios and the same results were obtained.

LA	Mac	LA	Mac	LA	Mac	LA	Mac	LA	Macaulay	LA	Macaulay		Actual
R^2		Gradient		*t-stat*		*p-stat*		$\Delta\lambda$ (bps)	Δytm (bps)	ΔP_{LA}	ΔP_{Mac}	ΔP_{Mac+C}	ΔP_{Actual}
0.983	0.985	1.081	1.037	30.96	31.78	2.49E-24	1.16E-24	4.70	5.17	-0.2208	-0.2478	-0.2200	-0.2199
0.965	0.886	1.018	0.702	20.63	8.04	2.82E-19	5.62E-09	33.75	39.36	-1.6310	-1.6405	-1.6310	-1.6288
0.994	0.993	1.168	1.036	56.25	46.23	5.67E-32	1.91E-29	11.35	13.37	-0.5377	-0.5702	-0.5369	-0.5353
0.978	0.977	1.031	1.061	26.24	26.16	3.02E-22	3.28E-22	-3.10	-3.16	0.1260	0.1224	0.1260	0.1256
0.984	0.974	0.994	0.863	29.45	20.05	1.07E-23	6.28E-19	11.75	13.03	-0.5176	-0.5211	-0.5176	-0.5151
0.992	0.993	0.953	0.919	40.61	41.77	8.80E-28	3.83E-28	3.21	3.38	-0.1342	-0.1391	-0.1325	-0.1324
0.985	0.979	1.222	1.324	37.80	34.44	7.26E-27	1.12E-25	-9.36	-9.71	0.3829	0.3638	0.3829	0.3818
0.988	0.970	0.858	0.812	29.60	17.77	9.28E-24	1.80E-17	12.92	15.61	-1.8106	-1.8111	-1.8106	-1.8062
0.981	0.994	1.011	0.952	27.77	46.87	5.87E-23	1.27E-29	19.97	20.81	-2.3410	-2.3447	-2.3410	-2.3314
0.976	0.991	0.753	0.799	18.46	31.13	6.31E-18	2.14E-24	-12.42	-10.90	1.2139	1.1657	1.2139	1.2105
0.997	0.999	0.929	0.979	63.04	100.52	1.91E-33	1.70E-39	-14.53	-15.21	1.7358	1.6487	1.7358	1.7282
0.989	0.987	0.944	0.926	34.39	30.87	1.16E-25	2.73E-24	5.83	7.10	-0.8148	-0.8529	-0.8148	-0.8121
0.998	0.998	1.029	1.017	93.56	97.25	1.45E-36	4.57E-39	3.30	3.14	-0.3580	-0.3807	-0.3580	-0.3567
0.852	0.838	1.363	1.373	13.79	13.42	1.60E-14	3.22E-14	-11.40	-17.58	2.0262	1.9715	2.0262	2.0200
0.999	0.997	0.971	0.963	133.80	62.56	3.25E-43	2.40E-33	5.64	6.32	-0.7359	-0.7634	-0.7359	-0.7329
0.983	0.991	1.094	1.088	31.16	42.98	2.07E-24	1.65E-28	10.11	9.84	-1.1280	-1.1446	-1.1280	-1.1249
0.996	0.997	0.996	0.999	56.84	72.37	4.15E-32	3.12E-35	-2.08	-1.92	0.2191	0.2118	0.2187	0.2179
0.988	0.987	1.021	1.069	35.72	35.02	3.82E-26	6.85E-26	-21.83	-23.52	2.7390	2.5934	2.7390	2.7327
0.856	0.939	0.927	0.992	9.50	15.33	1.50E-10	9.77E-16	25.34	25.40	-2.9598	-2.9889	-2.9598	-2.9462
0.979	0.968	0.980	0.983	25.45	20.89	7.28E-22	2.00E-19	6.30	7.94	-0.8970	-0.9258	-0.8970	-0.8936
0.988	0.929	0.835	0.821	28.77	11.81	2.11E-23	8.37E-13	24.78	28.64	-3.1380	-3.1534	-3.1380	-3.1241
0.814	0.728	0.492	0.454	4.48	3.55	9.98E-05	1.28E-03	-25.34	-38.92	4.2978	4.1984	4.2978	4.2841
0.988	0.960	1.000	0.939	33.81	17.86	1.92E-25	1.57E-17	-26.59	-32.10	3.7625	3.7084	3.7625	3.7611
0.995	0.987	0.969	0.995	52.04	32.55	5.70E-31	5.80E-25	14.57	17.34	-2.0609	-2.0891	-2.0609	-2.0580
0.997	0.994	1.015	0.981	67.89	49.40	2.10E-34	2.68E-30	-7.60	-8.72	0.4036	0.3735	0.4036	0.4017
0.767	0.871	0.601	1.143	4.95	12.32	2.67E-05	2.88E-13	104.04	122.87	-5.2783	-5.3043	-5.2783	-5.2651
0.998	0.990	0.951	1.068	88.31	39.48	8.17E-38	2.03E-27	24.77	28.77	-1.1241	-1.1301	-1.1241	-1.1201
1.000	0.999	0.876	0.926	175.51	124.61	9.59E-47	2.74E-42	11.08	13.32	-0.5065	-0.5464	-0.5065	-0.5057
0.993	0.960	0.713	0.588	31.25	11.31	1.91E-24	2.43E-12	-41.35	-47.94	1.8666	1.8009	1.8666	1.8586
0.999	0.998	0.840	0.804	116.48	63.81	2.06E-41	1.33E-33	-14.79	-17.17	0.6943	0.6663	0.6927	0.6926
1.000	0.994	0.894	1.032	151.52	51.08	7.84E-45	9.93E-31	55.55	65.97	-2.5878	-2.6020	-2.5878	-2.5836
1.000	0.999	1.007	1.070	332.15	162.16	4.73E-55	1.03E-45	25.59	31.02	-1.1465	-1.1745	-1.1465	-1.1454
0.984	0.994	1.101	1.338	32.94	63.86	4.11E-25	1.30E-33	79.25	99.03	-3.4054	-3.4054	-3.4054	-3.3897
0.999	0.979	0.979	1.641	136.03	42.49	1.98E-43	2.31E-28	225.08	275.21	-7.7154	-7.7154	-7.7154	-7.6951
0.993	0.978	0.814	0.548	36.06	14.08	2.89E-26	9.38E-15	-114.48	-143.86	3.7478	3.6868	3.7478	3.7429
0.994	0.994	1.178	1.376	58.43	64.82	1.83E-32	8.32E-34	73.21	89.73	-2.4094	-2.4209	-2.4094	-2.3987
0.999	0.972	0.883	0.634	131.03	14.51	6.08E-43	4.25E-15	-101.13	-123.81	3.3840	3.2793	3.3840	3.3703
0.989	0.982	1.016	0.803	36.87	22.26	1.51E-26	3.32E-20	-86.18	-108.68	3.3669	3.1403	3.3669	3.3514
0.999	0.999	1.039	1.119	104.71	110.23	5.01E-40	1.08E-40	31.17	38.28	-1.2280	-1.2694	-1.2280	-1.2267
0.991	0.972	1.138	0.879	45.72	20.00	2.66E-29	6.79E-19	-86.10	-105.41	3.5184	3.2834	3.5184	3.5117
0.998	0.988	1.117	0.900	97.13	30.69	4.74E-39	3.24E-24	-65.58	-79.30	2.9527	2.8067	2.9527	2.9421
0.986	0.958	1.181	1.225	37.04	22.73	1.32E-26	1.83E-20	10.87	12.42	-0.6346	-0.6387	-0.6346	-0.6324
0.990	0.924	0.970	1.002	36.38	13.92	2.23E-26	1.25E-14	16.62	18.62	-0.9272	-0.9349	-0.9293	-0.9286
0.985	0.987	1.035	1.037	31.31	34.62	1.80E-24	9.55E-26	-12.01	-13.92	0.6931	0.6518	0.6931	0.6919
0.998	0.966	1.056	1.044	99.14	21.53	2.57E-39	8.48E-20	11.79	13.50	-0.6742	-0.6878	-0.6742	-0.6713
0.988	0.987	0.963	0.963	33.04	31.72	3.74E-25	1.23E-24	-0.70	-0.80	0.0375	0.0373	0.0396	0.0395
0.989	0.996	1.028	1.032	36.67	61.32	1.77E-26	4.36E-33	-4.93	-5.73	0.2849	0.2780	0.2842	0.2837
0.997	0.981	0.976	0.973	66.16	26.35	4.53E-34	2.66E-22	-6.45	-7.32	0.3676	0.3510	0.3676	0.3660
0.990	0.957	1.244	1.229	45.70	22.49	2.69E-29	2.46E-20	-12.23	-13.64	0.6931	0.6575	0.6931	0.6914
0.980	0.967	0.891	0.878	23.57	18.13	6.53E-21	1.04E-17	-3.10	-3.54	0.1829	0.1786	0.1819	0.1817
0.992	0.897	0.840	0.749	36.17	9.02	2.65E-26	4.75E-10	-16.66	-19.25	1.0075	0.9747	1.0075	1.0028
0.967	0.952	1.167	1.202	24.21	20.82	3.02E-21	2.19E-19	7.23	8.51	-0.4438	-0.4486	-0.4484	-0.4463
0.990	0.950	1.103	1.033	41.72	17.58	3.97E-28	2.40E-17	-9.89	-10.97	0.5779	0.5694	0.5777	0.5763
0.944	0.674	1.691	1.710	27.15	12.41	1.12E-22	2.42E-13	30.63	35.31	-1.8326	-1.8471	-1.8326	-1.8254
0.799	0.866	0.832	1.766	7.32	18.85	3.75E-08	3.49E-18	36.53	42.35	-2.0806	-2.1180	-2.0806	-2.0773
0.949	0.695	1.377	2.033	23.05	15.19	1.23E-20	1.25E-15	27.84	31.76	-1.4885	-1.5101	-1.4885	-1.4832
0.732	0.195	0.294	0.093	2.28	0.53	2.99E-02	5.98E-01	-16.51	-17.23	0.7951	0.7485	0.7982	0.7971
0.483	-0.004	0.721	-0.007	4.35	-0.04	1.43E-04	9.71E-01	-39.58	-47.51	2.2911	2.2316	2.2938	2.2935
0.986	0.964	0.791	0.934	24.69	18.63	1.73E-21	4.88E-18	8.78	9.74	-0.4838	-0.5319	-0.4838	-0.4823
0.983	0.937	0.955	0.805	27.29	12.24	9.68E-23	3.41E-13	-10.38	-12.09	0.6021	0.5771	0.6021	0.5993
0.967	0.965	1.457	1.498	30.43	30.32	4.15E-24	4.59E-24	2.25	2.62	-0.1285	-0.1537	-0.1309	-0.1308
0.982	0.718	0.905	0.617	25.58	4.76	6.28E-22	4.58E-05	-13.57	-15.36	0.7745	0.7375	0.7745	0.7731
0.990	0.749	0.764	0.512	28.66	4.17	2.35E-23	2.41E-04	-14.05	-16.18	0.8341	0.8070	0.8328	0.8320
0.993	0.918	1.032	0.824	46.49	11.04	1.62E-29	4.39E-12	-10.42	-11.83	0.6224	0.5878	0.6224	0.6201
0.970	0.895	1.074	0.856	23.27	10.18	9.34E-21	3.01E-11	-8.73	-9.52	0.5076	0.4936	0.5076	0.5062
0.988	0.700	0.906	0.496	31.63	3.76	1.35E-24	7.27E-04	-16.85	-19.32	1.0492	1.0244	1.0492	1.0485
0.971	0.941	1.061	1.266	23.32	19.80	8.88E-21	8.94E-19	8.78	10.09	-0.5493	-0.5793	-0.5520	-0.5511
0.988	0.987	0.810	0.720	27.35	23.52	9.11E-23	6.90E-21	-5.31	-6.35	0.3436	0.3424	0.3464	0.3459
0.999	0.944	0.869	0.710	85.74	11.48	1.98E-37	1.67E-12	-11.36	-13.06	0.7236	0.6952	0.7215	0.7211
0.978	0.929	1.207	1.309	30.51	18.81	3.83E-24	3.73E-18	9.08	10.27	-0.5685	-0.6001	-0.5685	-0.5683
0.818	0.953	0.713	0.780	6.56	13.70	2.93E-07	1.90E-14	-9.04	-10.85	0.5988	0.5747	0.6018	0.6005
0.980	0.989	0.948	0.976	25.00	35.13	1.21E-21	6.25E-26	-2.20	-2.37	0.1334	0.1245	0.1326	0.1322
0.960	0.935	0.890	0.840	16.71	12.57	9.57E-17	1.74E-13	3.38	3.80	-0.2099	-0.2375	-0.2126	-0.2119
0.885	0.890	0.535	0.542	6.07	6.39	1.15E-06	4.76E-07	0.88	0.50	-0.0255	-0.0454	-0.0281	-0.0280
0.996	0.995	0.874	1.025	52.87	53.62	3.57E-31	2.36E-31	-11.46	-12.85	0.7232	0.6719	0.7232	0.7210
0.995	0.984	1.308	1.742	67.95	52.14	2.04E-34	5.40E-31	-26.71	-29.47	1.7123	1.6854	1.7123	1.7042
0.994	0.992	0.778	0.728	38.70	30.85	3.64E-27	2.79E-24	4.43	4.29	-0.2547	-0.2794	-0.2531	-0.2528
0.994	0.980	1.021	1.328	47.90	35.34	6.67E-30	5.24E-26	-23.44	-26.09	1.5627	1.4852	1.5627	1.5600
0.993	0.982	0.851	0.985	38.34	27.84	4.79E-27	5.47E-23	-12.44	-14.06	0.8696	0.8162	0.8696	0.8655
0.996	0.958	1.005	0.758	56.52	14.11	4.91E-32	8.85E-15	22.28	24.81	-1.5160	-1.5467	-1.5160	-1.5153
0.998	0.998	1.042	1.019	98.14	90.17	3.48E-39	4.39E-38	1.83	1.97	-0.1162	-0.1597	-0.1184	-0.1182
0.996	0.977	0.949	0.766	59.00	18.96	1.37E-32	2.99E-18	16.14	18.11	-1.0734	-1.0820	-1.0734	-1.0692
0.979	0.927	1.038	0.711	26.98	10.09	1.35E-22	3.68E-11	25.51	28.65	-1.6417	-1.6460	-1.6417	-1.6366
0.974	0.976	1.397	1.341	32.71	32.56	5.05E-25	5.75E-25	2.20	2.88	-0.1613	-0.2005	-0.1613	-0.1607
0.723	0.732	0.768	0.404	5.88	3.19	1.94E-06	3.36E-03	46.77	52.12	-2.8103	-2.8220	-2.8103	-2.8024
0.918	0.946	0.849	0.825	11.34	13.52	2.27E-12	2.68E-14	10.09	11.49	-0.5899	-0.6077	-0.5920	-0.5911
0.727	0.777	0.579	0.978	4.46	8.26	1.06E-04	3.19E-09	68.25	78.07	-3.7983	-3.7983	-3.7983	-3.7843
0.987	0.994	0.560	0.584	18.46	29.04	6.26E-18	1.61E-23	3.96	3.69	-0.1693	-0.1841	-0.1696	-0.1695

LA	Mac	LA	Mac	LA	Mac	LA	Mac	LA	Macaulay	LA	Macaulay		Actual
R^2		Gradient		*t-stat*		*p-stat*		$\Delta\lambda$ (bps)	Δytm (bps)	ΔP_{LA}	ΔP_{Mac}	ΔP_{Mac+C}	ΔP_{Actual}
0.999	0.997	0.895	0.855	116.18	63.11	2.23E-41	1.85E-33	14.72	16.49	-0.7526	-0.7891	-0.7526	-0.7506
0.998	0.997	0.813	0.789	72.67	51.75	2.76E-35	6.76E-31	12.12	12.74	-0.5712	-0.5876	-0.5712	-0.5690
0.999	0.999	0.967	0.975	147.57	153.15	1.73E-44	5.68E-45	-2.82	-3.43	0.1528	0.1464	0.1528	0.1522
0.999	0.997	1.127	1.205	145.89	83.78	2.44E-44	3.94E-37	-29.91	-33.14	1.5127	1.4462	1.5127	1.5064
0.995	0.995	1.106	1.113	60.15	59.07	7.73E-33	1.32E-32	-3.87	-3.64	0.1701	0.1582	0.1698	0.1694
0.999	1.000	0.977	1.003	162.86	174.81	9.02E-46	1.08E-46	-8.89	-10.14	0.4776	0.4457	0.4767	0.4762
0.999	0.999	1.005	0.956	109.52	95.89	1.31E-40	6.95E-39	17.04	19.46	-0.9095	-0.9413	-0.9089	-0.9089
1.000	0.998	1.038	1.124	207.51	88.99	6.32E-49	6.49E-38	-29.05	-32.89	1.5505	1.4470	1.5504	1.5493
0.999	0.999	0.985	0.989	109.30	107.41	1.39E-40	2.34E-40	-1.39	-1.34	0.0662	0.0614	0.0648	0.0647
0.997	0.993	1.072	1.142	69.64	49.55	9.83E-35	2.44E-30	-24.37	-27.16	1.3361	1.2698	1.3361	1.3338
0.994	0.994	1.116	1.183	52.99	56.48	3.34E-31	5.02E-32	-20.23	-23.35	1.1895	1.1258	1.1895	1.1863
0.998	0.995	1.134	1.225	109.38	65.01	1.36E-40	7.62E-34	-24.92	-28.34	1.4967	1.4662	1.4967	1.4915
0.986	0.980	0.903	0.792	29.06	21.33	1.57E-23	1.10E-19	33.83	38.63	-2.0228	-2.0579	-2.0228	-2.0193
0.999	0.997	1.030	1.081	113.12	80.77	4.97E-41	1.18E-36	-14.94	-16.81	0.8659	0.8189	0.8659	0.8656
0.999	1.000	1.003	1.047	103.28	279.00	7.55E-40	8.83E-53	-11.89	-14.05	0.7411	0.7093	0.7397	0.7389
1.000	0.996	1.047	1.125	204.89	63.78	9.25E-49	1.35E-33	-21.52	-24.10	1.3015	1.2672	1.3015	1.3010
0.999	0.999	0.957	0.990	130.19	133.28	7.38E-43	3.65E-43	-8.83	-9.81	0.5449	0.5273	0.5449	0.5425
1.000	1.000	1.005	0.992	367.34	345.48	2.31E-56	1.45E-55	3.73	4.24	-0.2363	-0.2612	-0.2362	-0.2354
0.991	0.992	0.912	0.929	36.37	38.22	2.26E-26	5.24E-27	-4.22	-4.62	0.2573	0.2396	0.2572	0.2565
0.995	0.985	0.976	1.100	53.93	33.84	1.98E-31	1.86E-25	-38.48	-42.54	2.4491	2.4003	2.4491	2.4418
0.996	0.996	0.924	0.902	53.76	52.80	2.17E-31	3.71E-31	8.04	9.56	-0.5638	-0.6018	-0.5638	-0.5617
0.999	0.987	0.925	0.811	120.29	27.17	7.88E-42	1.10E-22	42.78	48.09	-2.7204	-2.7581	-2.7204	-2.7128
1.000	0.996	1.089	1.206	231.74	73.44	2.31E-50	2.01E-35	-39.11	-44.52	2.5078	2.4198	2.5078	2.5050
0.999	0.997	0.964	0.922	126.28	63.16	1.84E-42	1.80E-33	14.54	16.19	-0.9336	-0.9712	-0.9336	-0.9290
0.997	0.998	0.993	0.957	69.88	76.67	8.87E-35	5.56E-36	12.66	14.53	-0.8182	-0.8437	-0.8182	-0.8162
1.000	0.998	1.072	1.124	210.87	91.51	3.91E-49	2.82E-38	-17.83	-20.34	1.1495	1.1082	1.1481	1.1472
0.929	0.832	0.945	0.651	13.51	6.25	2.74E-14	6.90E-07	-36.17	-39.95	1.5019	1.4255	1.5019	1.4974
1.000	0.997	0.985	0.838	220.17	56.14	1.07E-49	6.02E-32	-18.29	-21.62	0.8438	0.7871	0.8438	0.8409
0.997	0.952	1.079	0.655	75.09	11.44	1.04E-35	1.82E-12	-47.49	-55.82	2.2879	2.2368	2.2879	2.2778
0.993	0.996	0.871	1.308	39.08	80.34	2.71E-27	1.38E-36	54.30	65.09	-2.6541	-2.6541	-2.6541	-2.6410
0.999	0.999	1.033	1.042	101.50	102.98	1.27E-39	8.25E-40	1.15	1.48	-0.0597	-0.0788	-0.0578	-0.0575
0.995	0.983	1.192	0.853	61.62	24.95	3.76E-33	1.28E-21	-33.16	-39.85	1.5894	1.5313	1.5894	1.5884
0.930	0.899	1.013	0.423	14.62	5.24	3.48E-15	1.18E-05	-60.00	-72.11	3.0992	3.0318	3.0992	3.0842
0.955	0.944	1.572	1.310	27.93	20.99	4.98E-23	1.74E-19	-15.54	-17.57	0.7973	0.7581	0.7973	0.7955
0.955	0.955	1.430	1.428	25.47	25.49	7.05E-22	6.92E-22	0.63	0.97	-0.0465	-0.0540	-0.0445	-0.0444
0.791	0.528	0.568	0.280	4.91	1.81	2.98E-05	8.11E-02	-42.60	-49.90	2.3664	2.2930	2.3664	2.3590
0.950	0.983	0.631	0.621	10.66	17.88	1.02E-11	1.50E-17	-10.95	-13.67	0.6765	0.6611	0.6765	0.6739
0.849	0.813	1.502	1.471	15.04	13.38	1.64E-15	3.49E-14	12.36	15.68	-0.7737	-0.7808	-0.7737	-0.7719
0.909	0.857	0.962	0.902	12.24	9.28	3.38E-13	2.52E-10	6.93	8.12	-0.3922	-0.4093	-0.3948	-0.3935
0.998	0.871	0.913	0.746	82.82	8.09	5.57E-37	5.02E-09	19.31	22.11	-1.0532	-1.0890	-1.0532	-1.0502
0.811	0.820	0.645	0.664	5.84	6.15	2.21E-06	9.05E-07	-1.31	-0.66	0.0328	0.0295	0.0312	0.0311
0.994	0.960	1.106	1.349	55.50	25.64	8.43E-32	5.86E-22	-21.89	-25.27	1.2094	1.1833	1.2094	1.2037
0.997	0.981	0.889	0.813	59.37	22.09	1.14E-32	4.12E-20	7.25	8.21	-0.3964	-0.4453	-0.3964	-0.3954
0.992	0.997	0.873	0.944	35.66	65.95	4.00E-26	4.98E-34	-6.14	-7.27	0.3493	0.3353	0.3501	0.3500
0.993	0.990	0.916	0.886	41.38	33.37	5.03E-28	2.79E-25	3.06	3.38	-0.1614	-0.1999	-0.1637	-0.1633
0.985	0.965	0.695	0.577	21.25	11.73	1.23E-19	9.81E-13	16.52	19.32	-0.9230	-0.9410	-0.9230	-0.9188
0.933	0.927	0.969	0.696	14.26	9.88	6.73E-15	6.02E-11	47.52	55.86	-2.5349	-2.5349	-2.5349	-2.5298
0.999	0.997	0.942	0.980	125.06	63.81	2.46E-42	1.33E-33	-7.26	-8.02	0.3526	0.3358	0.3526	0.3520
0.998	1.000	0.882	0.851	83.15	189.94	4.94E-37	8.97E-48	7.39	9.35	-0.4104	-0.4371	-0.4104	-0.4101
0.940	0.956	1.183	1.654	18.41	29.95	6.79E-18	6.59E-24	-63.98	-75.50	3.4695	3.2609	3.4695	3.4580
0.994	0.978	0.684	0.622	33.10	15.87	3.54E-25	3.91E-16	12.56	14.32	-0.6842	-0.7024	-0.6842	-0.6822
0.994	0.992	0.942	0.914	46.85	38.61	1.28E-29	3.88E-27	5.47	5.54	-0.2593	-0.3101	-0.2617	-0.2607
0.999	0.994	0.803	0.740	78.73	36.71	2.53E-36	1.71E-26	15.05	16.90	-0.7859	-0.7859	-0.7859	-0.7832
1.000	0.997	0.910	1.005	170.54	65.06	2.27E-46	7.46E-34	-22.01	-24.87	1.1570	1.1134	1.1594	1.1586
0.997	0.998	0.769	0.798	50.53	61.69	1.37E-30	3.63E-33	-6.38	-7.45	0.3534	0.3388	0.3551	0.3544
0.998	0.998	0.862	0.824	73.47	62.31	1.98E-35	2.70E-33	14.45	16.46	-0.7819	-0.7882	-0.7819	-0.7786
0.999	0.997	1.016	1.078	112.62	78.81	5.67E-41	2.45E-36	-21.34	-23.53	1.1234	1.1029	1.1234	1.1185
0.993	0.993	1.075	0.961	48.44	43.12	4.78E-30	1.50E-28	36.35	41.69	-1.9616	-1.9804	-1.9616	-1.9579
0.995	0.997	1.049	0.981	57.01	69.92	3.80E-32	8.69E-35	21.52	25.41	-1.1465	-1.1796	-1.1465	-1.1421
0.997	0.998	0.992	1.036	71.24	84.63	4.98E-35	2.92E-37	-13.90	-16.21	0.7255	0.7141	0.7255	0.7243
0.999	0.999	0.979	1.009	139.52	134.87	9.28E-44	2.56E-43	-9.51	-10.87	0.4967	0.4698	0.4967	0.4945
1.000	1.000	1.066	1.074	235.25	190.53	1.47E-50	8.18E-48	-3.21	-3.42	0.1565	0.1489	0.1576	0.1571
0.928	0.923	1.174	1.156	16.70	15.86	9.78E-17	3.96E-16	4.94	5.69	-0.2448	-0.2956	-0.2477	-0.2468
0.994	0.955	1.106	1.037	52.06	18.59	5.64E-31	5.15E-18	12.05	14.06	-0.6004	-0.6270	-0.6045	-0.6026
0.966	0.768	0.808	0.847	16.61	7.07	1.13E-16	7.35E-08	-33.42	-38.54	1.6810	1.6443	1.6810	1.6779
0.983	0.978	1.012	1.010	29.22	25.72	1.34E-23	5.36E-22	-2.55	-2.53	0.1149	0.1075	0.1133	0.1129
0.996	0.988	0.982	0.974	55.46	33.18	8.63E-32	3.31E-25	-6.66	-7.47	0.3346	0.3248	0.3371	0.3359
0.998	0.991	1.120	1.129	99.12	43.51	2.59E-39	1.14E-28	7.50	8.93	-0.4023	-0.4122	-0.4023	-0.4012
0.996	0.993	0.967	0.962	60.07	41.86	8.02E-33	3.59E-28	-2.52	-2.75	0.1255	0.1218	0.1235	0.1232
0.991	0.958	1.180	1.186	45.73	21.99	2.64E-29	4.70E-20	10.69	12.44	-0.5510	-0.5947	-0.5541	-0.5532
0.981	0.881	0.894	0.835	24.41	9.40	2.40E-21	1.90E-10	-16.35	-18.50	0.8238	0.7707	0.8278	0.8261
0.985	0.835	0.915	0.779	28.08	7.56	4.26E-23	1.99E-08	-23.74	-26.80	1.2311	1.1561	1.2326	1.2325
0.992	0.990	1.026	1.032	43.82	38.52	9.30E-29	4.18E-27	1.60	1.59	-0.0749	-0.1197	-0.0749	-0.0745
0.993	0.992	0.966	0.963	42.93	40.54	1.71E-28	9.23E-28	-0.78	-0.58	0.0276	0.0255	0.0272	0.0271
0.992	0.823	0.943	0.755	40.09	7.10	1.29E-27	6.69E-08	-29.75	-33.96	1.6274	1.5779	1.6274	1.6234
0.992	0.903	0.955	0.800	39.47	9.91	2.04E-27	5.58E-11	-22.12	-24.85	1.2329	1.2156	1.2377	1.2353
0.990	0.972	1.129	1.243	42.69	28.18	2.02E-28	3.83E-23	14.83	17.40	-0.8685	-0.9012	-0.8698	-0.8696
0.993	0.971	0.976	1.031	43.79	22.85	9.50E-29	1.57E-20	9.30	10.35	-0.5099	-0.5410	-0.5086	-0.5075
0.994	0.800	0.853	0.664	40.57	5.91	9.04E-28	1.77E-06	-34.20	-39.47	1.9804	1.9112	1.9804	1.9744
0.999	1.000	1.098	1.139	151.30	246.22	8.19E-45	3.75E-51	11.86	14.55	-0.5596	-0.6095	-0.5612	-0.5609
0.995	0.988	1.109	0.891	58.55	30.33	1.72E-32	4.56E-24	-54.75	-65.49	2.6125	2.4808	2.6125	2.6059
0.990	0.987	1.289	1.334	48.90	44.11	3.62E-30	7.66E-29	11.75	12.91	-0.5332	-0.5332	-0.5332	-0.5307
0.921	0.971	0.990	1.383	13.45	30.91	3.06E-14	2.63E-24	62.69	76.69	-2.9955	-3.0313	-2.9955	-2.9836
0.990	0.980	1.258	0.894	47.14	23.81	1.07E-29	4.89E-21	-46.11	-55.68	2.1435	2.0485	2.1435	2.1383
0.995	0.992	0.940	1.012	50.73	42.01	1.22E-30	3.22E-28	10.32	11.35	-0.4457	-0.4621	-0.4477	-0.4475
0.996	0.998	1.012	1.183	60.19	94.48	7.55E-33	1.09E-38	23.20	27.69	-1.0687	-1.0772	-1.0687	-1.0667
0.451	0.442	0.425	0.523	2.52	3.11	1.74E-02	4.04E-03	36.38	32.54	-1.2106	-1.2309	-1.2106	-1.2088
0.793	0.760	0.869	0.817	7.55	6.68	2.05E-08	2.13E-07	-9.49	-3.68	0.1335	0.1306	0.1350	0.1344

LA	Mac	LA	Mac	LA	Mac	LA	Mac	LA	Macaulay	LA	Macaulay		Actual
R^2		Gradient		*t-stat*		*p-stat*		$\Delta\lambda$ (bps)	Δytm (bps)	ΔP_{LA}	ΔP_{Mac}	ΔP_{Mac+C}	ΔP_{Actual}
0.999	0.992	0.932	0.827	113.85	33.73	4.10E-41	2.05E-25	38.48	43.65	-2.4247	-2.4308	-2.4247	-2.4219
0.997	0.998	1.003	1.013	73.67	76.19	1.83E-35	6.71E-36	-3.29	-3.73	0.2005	0.1926	0.2018	0.2014
0.999	0.999	0.998	0.964	115.50	132.16	2.66E-41	4.71E-43	12.21	14.03	-0.7536	-0.7908	-0.7536	-0.7518
0.999	0.999	1.000	1.053	114.05	104.11	3.88E-41	5.95E-40	-18.47	-20.88	1.1285	1.0475	1.1285	1.1239
1.000	0.995	0.955	0.889	298.10	49.43	1.21E-53	2.63E-30	24.71	27.66	-1.4801	-1.5176	-1.4831	-1.4817
0.999	0.998	0.907	0.877	129.52	72.05	8.61E-43	3.56E-35	13.01	14.83	-0.7738	-0.7769	-0.7722	-0.7717
1.000	0.998	1.031	1.069	228.59	94.78	3.48E-50	9.87E-39	-15.25	-16.97	0.8873	0.8729	0.8873	0.8844
1.000	0.994	0.911	0.830	271.59	40.98	1.98E-52	6.73E-28	35.60	40.36	-2.0785	-2.1185	-2.0785	-2.0700
1.000	1.000	1.026	1.036	439.53	371.83	1.06E-58	1.60E-56	-4.13	-4.75	0.2393	0.2226	0.2380	0.2376
0.999	0.995	0.965	0.905	152.74	50.43	6.16E-45	1.46E-30	25.57	28.38	-1.3998	-1.4092	-1.3986	-1.3985
0.994	0.996	1.033	0.999	52.01	58.56	5.82E-31	1.71E-32	13.71	16.36	-0.7851	-0.8115	-0.7851	-0.7825
1.000	0.999	1.138	1.141	201.81	188.12	1.46E-48	1.20E-47	-2.40	-2.96	0.1409	0.1353	0.1402	0.1401
1.000	0.999	1.068	1.109	353.24	120.85	7.46E-56	6.85E-42	-15.56	-18.00	0.8669	0.8096	0.8669	0.8649
0.996	0.995	0.959	1.061	55.86	56.37	6.96E-32	5.32E-32	-36.43	-41.44	2.0747	2.0066	2.0747	2.0729
0.999	0.999	1.022	0.976	141.18	152.11	6.51E-44	6.98E-45	16.20	18.52	-0.9444	-0.9751	-0.9444	-0.9407
1.000	1.000	1.021	1.015	284.44	404.22	4.95E-53	1.31E-57	1.95	2.30	-0.1133	-0.1277	-0.1157	-0.1153
0.999	0.999	0.984	1.022	116.55	120.75	2.03E-41	7.03E-42	-13.63	-15.58	0.7882	0.7423	0.7882	0.7872
0.998	0.998	0.958	0.998	72.30	79.42	3.21E-35	1.95E-36	-13.97	-15.81	0.8190	0.8031	0.8190	0.8162
0.986	0.983	1.082	1.161	34.89	33.04	7.59E-26	3.74E-25	-27.87	-32.01	1.7105	1.6751	1.7105	1.7076
0.999	0.999	0.899	0.908	122.71	144.57	4.34E-42	3.20E-44	-2.41	-2.31	0.1240	0.1230	0.1270	0.1264
0.999	1.000	0.933	0.943	127.17	161.08	1.49E-42	1.25E-45	-2.95	-3.28	0.1807	0.1707	0.1802	0.1801
1.000	0.999	0.978	0.997	248.10	147.32	2.98E-51	1.82E-44	-7.72	-8.18	0.4545	0.4480	0.4545	0.4525
1.000	0.999	1.004	1.046	201.83	122.51	1.45E-48	4.56E-42	-16.58	-18.57	1.0474	1.0148	1.0474	1.0462
1.000	0.999	1.076	1.118	472.67	122.22	1.20E-59	4.89E-42	-16.19	-18.46	1.0730	1.0197	1.0730	1.0677
1.000	0.998	0.961	0.908	1020.56	78.12	1.13E-69	3.18E-36	20.72	23.34	-1.3511	-1.3832	-1.3511	-1.3451
1.000	0.999	1.066	1.112	557.27	123.00	8.59E-62	4.05E-42	-17.09	-19.53	1.1262	1.0661	1.1262	1.1223
1.000	0.999	0.980	0.943	400.16	132.53	1.77E-57	4.33E-43	13.60	15.44	-0.8942	-0.9180	-0.8942	-0.8903
1.000	1.000	0.986	0.971	625.36	309.67	2.70E-63	3.87E-54	6.04	6.87	-0.3916	-0.4383	-0.3909	-0.3896
1.000	0.999	0.988	0.965	235.06	114.09	1.51E-50	3.84E-41	8.95	9.87	-0.5548	-0.5911	-0.5546	-0.5535
1.000	1.000	1.006	1.016	357.43	239.36	5.24E-56	8.75E-51	-3.80	-4.21	0.2363	0.2324	0.2353	0.2350
1.000	0.999	1.000	1.037	320.02	139.36	1.44E-54	9.60E-44	-14.09	-15.76	0.8956	0.8614	0.8956	0.8927
1.000	1.000	1.010	1.017	284.35	309.34	4.99E-53	3.99E-54	-2.39	-2.73	0.1543	0.1511	0.1574	0.1566
1.000	0.996	0.976	0.913	298.81	57.29	1.13E-53	3.29E-32	23.73	26.40	-1.4915	-1.5227	-1.4915	-1.4901
1.000	1.000	1.114	1.120	306.54	243.51	5.25E-54	5.22E-51	-3.21	-3.98	0.2219	0.2060	0.2215	0.2211
1.000	0.999	1.015	0.986	394.58	126.85	2.70E-57	1.61E-42	9.52	10.44	-0.5777	-0.6108	-0.5777	-0.5771
1.000	0.994	1.067	1.170	267.40	56.85	3.16E-52	4.13E-32	-33.27	-37.55	2.1206	2.0660	2.1206	2.1168
1.000	1.000	1.004	1.014	215.96	268.16	1.91E-49	2.90E-52	-3.13	-3.73	0.2141	0.2061	0.2168	0.2165
1.000	1.000	1.016	1.029	617.18	359.23	4.01E-63	4.51E-56	-3.95	-4.54	0.2640	0.2530	0.2649	0.2649
0.999	0.999	1.036	1.034	133.92	137.25	3.17E-43	1.52E-43	0.35	0.30	-0.0164	-0.0200	-0.0178	-0.0178
1.000	0.995	1.054	1.152	403.90	62.04	1.34E-57	3.07E-33	-28.47	-32.09	1.9267	1.8592	1.9267	1.9227
1.000	1.000	0.969	0.967	337.13	337.00	3.03E-55	3.06E-55	0.60	0.89	-0.0564	-0.0836	-0.0550	-0.0548
1.000	0.999	0.999	1.057	309.18	112.30	4.06E-54	6.18E-41	-16.80	-18.79	1.1695	1.1364	1.1695	1.1667
1.000	0.998	1.028	1.082	356.03	89.48	5.90E-56	5.51E-38	-15.74	-17.72	1.1352	1.0714	1.1352	1.1302
1.000	0.999	0.987	0.960	374.46	144.42	1.30E-56	3.30E-44	7.90	8.88	-0.5679	-0.6063	-0.5702	-0.5699
0.999	1.000	0.927	0.969	106.01	373.11	3.46E-40	1.45E-56	-11.70	-12.84	0.8292	0.8168	0.8284	0.8268
1.000	0.999	1.032	1.000	419.89	159.48	4.18E-58	1.69E-45	9.37	10.20	-0.6595	-0.6936	-0.6595	-0.6581
1.000	0.999	1.038	1.065	368.77	165.02	2.05E-56	6.07E-46	-7.72	-8.81	0.5697	0.5472	0.5697	0.5677
1.000	1.000	1.018	1.045	416.37	250.65	5.38E-58	2.20E-51	-7.75	-8.69	0.5675	0.5516	0.5673	0.5669
1.000	0.998	1.078	1.163	221.09	94.25	9.45E-50	1.16E-38	-21.96	-25.02	1.6724	1.6241	1.6752	1.6743
1.000	0.997	1.150	1.235	258.19	80.42	9.02E-52	1.34E-36	-19.67	-22.54	1.5701	1.4857	1.5701	1.5639
1.000	0.999	1.033	1.065	248.83	121.49	2.73E-51	5.85E-42	-7.31	-8.24	0.5879	0.5722	0.5879	0.5851
1.000	0.997	1.092	1.153	239.03	82.04	9.11E-51	7.38E-37	-12.90	-14.67	1.0628	1.0378	1.0628	1.0600
1.000	0.999	1.034	1.065	309.53	122.06	3.92E-54	5.08E-42	-6.43	-7.14	0.5263	0.4929	0.5258	0.5243
0.999	0.996	1.169	1.243	142.80	75.13	4.62E-44	1.02E-35	-13.35	-15.64	1.1707	1.0999	1.1707	1.1701
0.999	0.987	0.875	0.778	115.89	25.22	2.41E-41	9.38E-22	19.26	21.88	-1.6317	-1.6375	-1.6317	-1.6285
0.999	0.986	0.904	0.794	141.45	25.35	8.16E-44	8.14E-22	23.56	26.53	-1.9119	-1.9502	-1.9119	-1.9024
1.000	0.994	0.993	0.901	174.31	45.19	1.18E-46	3.75E-29	19.38	21.53	-1.4907	-1.5004	-1.4907	-1.4884
0.998	0.993	1.168	1.275	108.25	57.24	1.85E-40	3.37E-32	-20.28	-23.34	1.6170	1.5062	1.6170	1.6161
1.000	1.000	0.987	0.970	186.67	189.32	1.51E-47	9.89E-48	-0.36	-0.22	0.0145	0.0144	0.0153	0.0152
1.000	0.989	0.953	0.835	168.44	30.20	3.28E-46	5.15E-24	23.77	26.71	-1.8485	-1.8613	-1.8465	-1.8449
0.999	1.000	1.033	1.065	169.18	194.47	2.88E-46	4.43E-48	-6.07	-7.09	0.4820	0.4733	0.4828	0.4825
0.999	0.998	1.046	1.105	111.57	88.30	7.50E-41	8.19E-38	-10.64	-12.50	0.8649	0.8391	0.8643	0.8633
1.000	0.999	1.022	1.035	186.29	144.01	1.61E-47	3.60E-44	-2.42	-2.65	0.1848	0.1747	0.1854	0.1853
1.000	0.993	1.037	1.170	188.24	52.26	1.17E-47	5.04E-31	-22.84	-25.60	1.8298	1.8061	1.8298	1.8280
1.000	1.000	1.036	1.046	246.86	224.52	3.47E-51	5.96E-50	-1.65	-1.94	0.1394	0.1352	0.1424	0.1417
0.999	0.999	1.050	1.015	161.79	142.25	1.10E-45	5.19E-44	5.47	5.94	-0.4299	-0.4809	-0.4327	-0.4315
1.000	0.996	1.070	1.169	251.46	69.56	1.99E-51	1.01E-34	-15.09	-17.12	1.2599	1.2111	1.2599	1.2550
0.998	0.979	0.801	1.002	74.72	26.17	1.20E-35	3.25E-22	-36.06	-38.97	2.9943	2.8123	2.9943	2.9855
0.998	0.988	1.229	1.350	92.70	46.03	1.91E-38	2.18E-29	-19.95	-22.97	1.8517	1.7257	1.8517	1.8476
1.000	0.997	1.134	1.192	226.11	88.43	3.70E-50	7.85E-38	-8.55	-9.80	0.8100	0.7660	0.8098	0.8089
0.997	0.965	0.844	0.703	60.27	14.23	7.27E-33	7.10E-15	23.37	26.57	-2.1697	-2.2053	-2.1697	-2.1647
0.999	0.998	0.851	0.830	111.57	73.58	7.50E-41	1.90E-35	4.51	5.75	-0.4575	-0.4888	-0.4575	-0.4563
0.999	0.978	1.056	1.273	155.87	32.65	3.36E-45	5.33E-25	-38.91	-43.34	3.5576	3.4257	3.5576	3.5463
0.998	0.992	1.086	1.177	91.84	48.95	2.53E-38	3.51E-30	-15.74	-17.74	1.5294	1.5016	1.5294	1.5244
0.999	0.999	0.980	0.994	123.77	142.57	3.35E-42	4.85E-44	-2.26	-2.33	0.2062	0.1985	0.2045	0.2038
0.999	0.997	0.979	1.029	116.70	75.35	1.95E-41	9.36E-36	-8.57	-9.41	0.8278	0.7963	0.8307	0.8295
0.999	0.983	0.911	0.795	126.02	22.67	1.96E-42	1.97E-20	21.88	24.61	-2.1462	-2.1849	-2.1462	-2.1432
0.999	0.997	0.973	0.926	153.17	63.13	5.67E-45	1.83E-33	8.84	9.75	-0.8282	-0.8439	-0.8282	-0.8260
1.000	1.000	0.976	0.997	237.87	212.48	1.05E-50	3.11E-49	-3.76	-4.05	0.3422	0.3357	0.3422	0.3412
0.999	0.999	1.027	1.039	120.08	103.38	8.31E-42	7.35E-40	-2.35	-2.59	0.2199	0.2165	0.2199	0.2196
1.000	0.999	0.957	0.925	225.95	146.45	4.93E-50	2.18E-44	6.48	7.37	-0.6246	-0.6714	-0.6246	-0.6221
0.996	0.997	0.849	0.899	51.91	62.28	6.15E-31	2.74E-33	-10.29	-10.72	0.9119	0.8587	0.9119	0.9077
1.000	0.999	1.058	1.083	216.44	174.99	1.79E-49	1.05E-46	-5.52	-6.50	0.5600	0.5473	0.5600	0.5579
0.998	0.997	1.038	0.968	99.88	63.37	2.06E-39	1.63E-33	14.21	15.28	-1.3088	-1.3256	-1.3088	-1.3028
1.000	0.999	1.024	1.088	199.06	135.10	2.20E-48	2.44E-43	-12.57	-14.03	1.1957	1.1552	1.1953	1.1946
1.000	0.998	1.020	1.084	177.69	89.92	6.62E-47	4.76E-38	-12.43	-13.58	1.1840	1.0996	1.1840	1.1823
1.000	0.999	1.054	1.084	231.15	120.05	2.49E-50	8.38E-42	-5.89	-6.85	0.6093	0.5651	0.6093	0.6063
0.982	0.976	0.948	1.002	26.64	24.52	1.95E-22	2.11E-21	-11.63	-12.15	1.0964	1.0783	1.0964	1.0925

LA	Mac	LA	Mac	LA	Mac	LA	Mac	LA	Macaulay	LA	Macaulay		Actual
R^2		Gradient		*t-stat*		*p-stat*		$\Delta\lambda$ (bps)	Δytm (bps)	ΔP_{LA}	ΔP_{Mac}	ΔP_{Mac+C}	ΔP_{Actual}
0.999	0.992	1.015	1.136	112.46	47.05	5.91E-41	1.13E-29	-24.90	-27.32	2.5475	2.4677	2.5475	2.5388
1.000	0.994	0.979	0.900	210.03	42.79	4.40E-49	1.88E-28	16.62	18.39	-1.7234	-1.7499	-1.7234	-1.7214
0.999	0.998	1.034	1.055	161.83	99.92	1.09E-45	2.03E-39	-4.61	-5.26	0.4885	0.4533	0.4876	0.4869
0.999	0.999	1.108	1.080	121.86	136.80	5.35E-42	1.67E-43	5.05	4.86	-0.4520	-0.4901	-0.4517	-0.4505
0.993	0.767	0.878	0.546	38.87	4.59	3.20E-27	7.46E-05	80.56	89.12	-7.6683	-7.6828	-7.6683	-7.6515
0.983	0.267	0.848	0.261	24.32	1.48	2.66E-21	1.49E-01	177.63	197.59	-13.6443	-13.6443	-13.6443	-13.6070
0.999	0.957	1.008	1.199	131.42	21.83	5.57E-43	5.78E-20	-58.59	-65.92	4.1093	3.8898	4.1093	4.0989
0.976	0.954	1.048	1.468	25.48	26.08	7.01E-22	3.57E-22	-96.47	-108.43	7.7107	7.1867	7.7107	7.6863
0.993	0.988	1.105	0.961	50.81	32.78	1.16E-30	4.74E-25	33.60	36.87	-2.7664	-2.8063	-2.7664	-2.7579
0.996	0.963	0.951	1.188	58.30	23.49	1.95E-32	7.16E-21	-55.78	-61.79	4.7272	4.4733	4.7272	4.7137
0.997	0.962	1.322	1.097	97.05	21.39	4.87E-39	1.02E-19	42.96	46.08	-3.5614	-3.5614	-3.5614	-3.5584
0.994	0.975	0.861	0.781	42.38	18.78	2.50E-28	3.91E-18	17.75	20.42	-1.5025	-1.5025	-1.5025	-1.4971
1.000	0.986	0.985	0.895	255.87	28.89	1.18E-51	1.87E-23	19.32	21.55	-1.5284	-1.5362	-1.5299	-1.5297
0.990	0.980	1.087	1.142	40.74	30.07	8.00E-28	5.86E-24	-12.97	-14.58	1.0322	1.0044	1.0322	1.0291
0.993	0.960	1.120	1.207	49.18	23.01	3.06E-30	1.29E-20	-22.96	-26.02	1.8995	1.7648	1.8995	1.8956
0.998	0.986	0.986	0.916	74.25	29.33	1.45E-35	1.20E-23	21.51	24.09	-1.7587	-1.7983	-1.7579	-1.7573
0.999	0.919	0.973	0.855	116.49	11.50	2.06E-41	1.60E-12	40.27	45.23	-3.1341	-3.1341	-3.1341	-3.1299
0.996	0.998	1.018	0.989	60.95	85.78	5.21E-33	1.95E-37	13.20	14.79	-0.9770	-0.9951	-0.9781	-0.9780
0.997	0.997	1.051	1.029	67.95	70.49	2.04E-34	6.84E-35	14.37	16.04	-1.0409	-1.0570	-1.0409	-1.0367
0.994	0.940	0.912	0.859	43.44	13.37	1.20E-28	3.60E-14	22.59	25.73	-1.6208	-1.6254	-1.6208	-1.6131
0.998	0.987	0.994	0.955	79.88	31.51	1.64E-36	1.50E-24	21.74	24.79	-1.5022	-1.5417	-1.5022	-1.4977
0.972	0.858	1.072	1.054	24.16	10.92	3.22E-21	5.66E-12	-35.43	-40.57	2.4887	2.3631	2.4887	2.4796
0.979	0.942	1.002	0.967	26.29	15.33	2.85E-22	9.89E-16	-13.69	-15.74	1.0034	0.9704	1.0034	1.0025
0.961	0.937	0.911	0.875	17.36	13.32	3.38E-17	3.95E-14	-8.10	-9.36	0.6102	0.5893	0.6102	0.6075
0.906	0.650	0.825	0.517	10.29	3.67	2.32E-11	9.28E-04	-41.56	-48.76	3.3116	3.0992	3.3116	3.3040
0.981	0.943	0.905	0.732	24.67	11.70	1.78E-21	1.04E-12	-22.53	-26.54	1.9134	1.8850	1.9134	1.9040
0.996	0.995	1.032	1.099	63.72	55.70	1.38E-33	7.61E-32	8.69	10.29	-0.7474	-0.7877	-0.7474	-0.7470
0.995	0.988	0.983	0.884	53.53	30.93	2.47E-31	2.56E-24	-12.27	-14.41	1.0524	0.9837	1.0524	1.0498
0.995	0.995	0.857	0.860	46.25	47.02	1.88E-29	1.16E-29	-0.47	-0.99	0.0722	0.0688	0.0729	0.0727
0.987	0.988	1.243	1.221	41.23	41.94	5.60E-28	3.40E-28	-1.21	-0.58	0.0402	0.0423	0.0429	0.0427
0.999	0.998	0.864	0.818	114.32	65.24	3.62E-41	6.86E-34	-6.87	-7.88	0.5875	0.5654	0.5867	0.5853
0.991	0.991	1.034	1.058	41.81	41.25	3.73E-28	5.54E-28	3.50	4.11	-0.3076	-0.3458	-0.3076	-0.3060
0.999	0.999	0.962	1.059	144.26	120.50	3.41E-44	7.48E-42	12.86	14.60	-1.0770	-1.0826	-1.0770	-1.0726
0.998	0.998	1.010	1.010	95.21	95.52	8.62E-39	7.81E-39	0.12	0.29	-0.0223	-0.0660	-0.0215	-0.0214
0.975	0.920	0.775	0.568	18.59	7.78	5.17E-18	1.12E-08	-34.03	-40.05	3.0039	2.9693	3.0039	2.9993
0.993	0.981	0.901	0.730	39.45	19.87	2.06E-27	8.06E-19	-29.66	-34.70	2.7575	2.6312	2.7575	2.7552
0.999	0.999	1.033	1.021	162.40	157.91	9.81E-46	2.27E-45	-1.72	-1.98	0.1632	0.1524	0.1624	0.1619
0.997	0.997	1.151	1.132	79.11	79.67	2.18E-36	1.77E-36	-1.73	-1.56	0.1288	0.1244	0.1284	0.1278
0.989	0.989	0.857	0.887	30.33	31.83	4.55E-24	1.11E-24	3.98	4.49	-0.3672	-0.4149	-0.3672	-0.3669
1.000	1.000	0.967	0.958	306.55	308.03	5.24E-54	4.54E-54	-1.71	-2.00	0.1654	0.1594	0.1638	0.1634
1.000	1.000	0.990	0.992	360.26	373.24	4.14E-56	1.43E-56	0.33	0.25	-0.0187	-0.0446	-0.0207	-0.0207
0.999	0.999	0.935	1.037	114.09	155.51	3.84E-41	3.60E-45	18.10	20.20	-1.6323	-1.6598	-1.6323	-1.6246
1.000	1.000	0.911	0.963	201.97	264.26	1.42E-48	4.50E-52	9.76	10.66	-0.8356	-0.8723	-0.8381	-0.8369
1.000	1.000	0.945	0.973	356.09	289.77	5.87E-56	2.84E-53	5.58	6.18	-0.4795	-0.5135	-0.4791	-0.4786
0.997	0.996	0.825	0.811	52.48	49.70	4.44E-31	2.24E-30	-5.54	-7.86	0.6108	0.5779	0.6108	0.6096
0.999	0.999	0.985	1.044	133.36	148.72	3.59E-43	1.37E-44	15.11	17.02	-1.3156	-1.3440	-1.3156	-1.3103
1.000	1.000	0.837	0.786	169.67	159.70	2.64E-46	1.62E-45	-17.94	-21.97	1.7020	1.6080	1.7020	1.6980
1.000	1.000	0.944	1.010	264.18	216.11	4.54E-52	1.87E-49	20.89	23.82	-1.8421	-1.8421	-1.8421	-1.8384
0.999	0.999	0.956	0.870	139.27	94.44	9.80E-44	1.10E-38	-29.96	-35.70	2.7903	2.7400	2.7903	2.7817
0.999	0.999	1.140	1.233	116.66	125.30	1.97E-41	2.32E-42	29.89	35.24	-2.7482	-2.7870	-2.7482	-2.7467
1.000	1.000	1.076	1.023	212.74	186.79	3.00E-49	1.48E-47	-14.00	-15.36	1.1806	1.1535	1.1799	1.1782
1.000	1.000	1.182	1.135	358.96	308.16	4.61E-56	4.48E-54	-9.22	-9.64	0.7553	0.7104	0.7553	0.7541
0.995	0.994	1.068	0.991	56.29	49.23	5.54E-32	2.96E-30	-16.06	-17.89	1.4286	1.3707	1.4317	1.4308
0.996	0.996	0.890	0.828	51.30	46.67	8.75E-31	1.45E-29	-16.20	-19.80	1.6368	1.5616	1.6368	1.6318
0.998	0.998	0.920	0.860	78.27	69.56	3.01E-36	1.02E-34	-16.68	-19.84	1.6893	1.5836	1.6893	1.6883
0.990	0.990	1.235	1.147	47.32	42.78	9.61E-30	1.89E-28	-16.66	-16.77	1.4715	1.4493	1.4715	1.4699
0.995	0.995	0.913	0.909	50.56	50.11	1.35E-30	1.76E-30	-1.77	-2.91	0.2578	0.2527	0.2600	0.2594
0.998	0.998	1.103	1.078	88.83	88.82	6.85E-38	6.88E-38	-4.19	-4.13	0.3710	0.3550	0.3710	0.3698
1.000	1.000	1.002	1.048	223.62	198.32	6.73E-50	2.46E-48	9.85	11.22	-0.9974	-1.0181	-1.0009	-0.9997
0.972	0.972	0.939	0.933	21.08	21.03	1.55E-19	1.64E-19	-1.09	-1.86	0.1653	0.1596	0.1649	0.1648
1.000	1.000	0.988	0.945	341.86	253.96	1.99E-55	1.48E-51	-10.38	-11.76	1.0555	1.0256	1.0555	1.0508
1.000	0.999	0.965	0.902	303.88	143.86	6.81E-54	3.71E-44	-15.67	-18.06	1.6564	1.6045	1.6558	1.6542
0.994	0.995	0.885	0.908	42.88	47.54	1.77E-28	8.34E-30	4.59	3.95	-0.3675	-0.4087	-0.3675	-0.3661
0.999	0.999	1.009	1.030	162.18	159.05	1.02E-45	1.83E-45	5.50	6.45	-0.5936	-0.6163	-0.5936	-0.5928
1.000	0.998	0.920	0.837	180.75	79.37	3.96E-47	1.98E-36	-24.92	-29.27	2.7435	2.6066	2.7435	2.7409
0.973	0.973	1.013	1.009	23.38	23.18	8.21E-21	1.04E-20	-1.05	-0.58	0.0578	0.0547	0.0559	0.0557
0.999	0.999	1.029	1.044	158.65	164.02	1.98E-45	7.29E-46	3.96	5.11	-0.4903	-0.5279	-0.4903	-0.4887
0.999	0.999	0.946	0.937	155.38	155.39	3.69E-45	3.68E-45	-2.95	-4.30	0.4085	0.3988	0.4108	0.4108
0.998	0.998	1.061	1.044	98.42	99.91	3.20E-39	2.04E-39	-3.56	-3.37	0.3218	0.3184	0.3243	0.3239
1.000	1.000	1.114	1.121	193.23	194.81	5.36E-48	4.20E-48	2.79	4.20	-0.4055	-0.4153	-0.4041	-0.4033
1.000	0.999	1.095	1.169	193.00	157.65	5.56E-48	2.39E-45	16.62	19.57	-1.8518	-1.8757	-1.8518	-1.8445
1.000	1.000	0.942	0.933	262.61	273.45	5.43E-52	1.61E-52	-2.53	-3.58	0.3340	0.3273	0.3330	0.3326
0.999	1.000	0.960	0.902	125.25	154.99	2.35E-42	3.98E-45	-14.13	-16.44	1.5591	1.5276	1.5591	1.5540
0.999	0.998	1.012	1.030	100.38	96.10	1.77E-39	6.53E-39	4.60	5.32	-0.5065	-0.5150	-0.5078	-0.5075
0.999	0.999	1.041	1.054	130.85	126.74	6.35E-43	1.65E-42	3.35	3.90	-0.3698	-0.4009	-0.3693	-0.3693
0.999	0.998	0.905	0.856	88.81	77.37	6.89E-38	4.25E-36	-13.31	-15.74	1.5110	1.4854	1.5110	1.5050
0.999	0.999	1.080	1.070	125.88	133.17	2.02E-42	3.75E-43	-1.56	-1.06	0.1018	0.0998	0.1030	0.1028
0.997	0.996	0.955	0.992	61.53	56.47	3.92E-33	5.04E-32	9.38	10.09	-0.9741	-0.9866	-0.9741	-0.9711
0.995	0.997	1.140	1.089	61.09	69.58	4.86E-33	1.01E-34	-10.54	-10.45	1.0105	0.9539	1.0105	1.0062
1.000	1.000	0.985	0.949	307.48	175.78	4.79E-54	9.15E-47	-8.40	-9.44	0.9284	0.9080	0.9284	0.9246
0.995	0.997	1.122	1.078	60.07	73.23	8.03E-33	2.19E-35	-8.55	-8.65	0.8615	0.8340	0.8615	0.8598
0.998	0.995	1.024	0.927	95.07	49.29	9.01E-39	2.86E-30	-20.66	-22.72	2.3270	2.2275	2.3270	2.3203
0.990	0.995	1.043	0.909	39.68	46.09	1.74E-27	2.09E-29	-29.14	-31.70	3.3954	3.2261	3.3954	3.3923
0.991	0.994	1.504	1.409	58.83	67.57	1.49E-32	2.41E-34	-13.02	-11.25	1.2559	1.1888	1.2559	1.2500
0.976	0.953	1.285	1.387	31.40	24.26	1.65E-24	2.85E-21	17.26	20.54	-2.2691	-2.2793	-2.2691	-2.2631
0.613	0.628	0.659	0.667	4.41	4.60	1.21E-04	7.22E-05	0.84	3.70	-0.4009	-0.4151	-0.3996	-0.3989
0.942	0.813	1.088	0.899	17.22	8.23	4.25E-17	3.50E-09	26.00	28.54	-3.0097	-3.0219	-3.0097	-2.9949
0.896	0.898	0.558	0.552	6.64	6.64	2.36E-07	2.38E-07	1.99	2.31	-0.2352	-0.2739	-0.2359	-0.2358

LA	Mac	LA	Mac	LA	Mac	LA	Mac	LA	Macaulay	LA	Macaulay		Actual
R^2		Gradient		*t-stat*		*p-stat*		$\Delta\lambda$ (bps)	Δytm (bps)	ΔP_{LA}	ΔP_{Mac}	ΔP_{Mac+C}	ΔP_{Actual}
0.952	0.878	1.055	1.069	18.24	11.87	8.70E-18	7.30E-13	-24.84	-28.03	2.9305	2.8527	2.9305	**2.9289**
0.926	0.938	0.837	0.850	11.76	13.06	9.32E-13	6.57E-14	-1.68	-0.19	0.0193	0.0197	0.0206	**0.0206**
0.972	0.974	1.058	1.011	23.79	23.57	4.99E-21	6.51E-21	13.45	14.52	-1.5368	-1.5793	-1.5361	**-1.5348**
0.414	0.382	0.551	0.547	3.20	3.15	3.24E-03	3.68E-03	-19.47	-19.95	2.1254	2.0980	2.1254	**2.1192**
0.952	0.981	1.028	1.071	17.82	29.25	1.67E-17	1.31E-23	9.10	9.08	-0.9751	-1.0009	-0.9751	**-0.9738**
0.983	0.984	0.964	1.095	27.72	32.88	6.20E-23	4.32E-25	20.93	22.73	-2.3748	-2.4093	-2.3748	**-2.3712**
0.943	0.915	0.904	0.848	14.36	11.15	5.54E-15	3.42E-12	-6.73	-8.38	0.8658	0.8035	0.8658	**0.8630**
0.825	0.842	0.804	0.797	7.54	7.86	2.10E-08	9.00E-09	-1.43	0.36	-0.0375	-0.0484	-0.0375	**-0.0375**
0.930	0.945	0.819	0.826	11.75	13.34	9.33E-13	3.80E-14	-2.80	-1.64	0.1728	0.1588	0.1711	**0.1703**
0.999	0.995	0.945	0.970	107.65	52.53	2.18E-40	4.32E-31	-11.52	-12.34	1.3005	1.2806	1.3005	**1.2975**
0.994	0.999	0.884	0.915	43.95	95.89	8.55E-29	6.95E-39	-10.13	-10.45	1.1242	1.0485	1.1242	**1.1202**
0.994	0.959	1.030	1.070	49.18	19.97	3.06E-30	7.09E-19	-20.31	-22.58	2.5036	2.4194	2.5036	**2.4917**
0.999	0.993	0.927	0.959	101.72	44.23	1.19E-39	7.07E-29	-14.29	-15.22	1.7425	1.6746	1.7425	**1.7354**
0.991	0.990	0.888	0.884	35.30	33.46	5.41E-26	2.59E-25	1.81	2.76	-0.3160	-0.3497	-0.3195	**-0.3179**
0.991	0.998	0.954	0.992	38.15	85.27	5.55E-27	2.33E-37	-10.60	-11.00	1.2814	1.2088	1.2814	**1.2778**
0.939	0.956	1.374	1.288	21.12	23.28	1.47E-19	9.22E-21	5.65	8.10	-0.3175	-0.3332	-0.3192	**-0.3186**
0.996	0.888	1.077	0.803	60.15	9.31	7.73E-33	2.36E-10	19.08	22.24	-0.8628	-0.8877	-0.8628	**-0.8590**
0.994	0.992	0.929	0.989	45.86	41.46	2.42E-29	4.77E-28	-4.35	-5.37	0.2035	0.2016	0.2056	**0.2051**
0.985	0.984	0.904	0.894	27.64	26.68	6.75E-23	1.86E-22	1.00	1.00	-0.0394	-0.0423	-0.0385	**-0.0384**
0.848	0.849	0.756	0.785	7.54	7.85	2.07E-08	9.31E-09	-2.75	-3.36	0.1266	0.1231	0.1293	**0.1288**
0.987	0.889	0.827	1.020	27.30	11.82	9.57E-23	8.20E-13	-28.89	-33.96	1.3383	1.2416	1.3383	**1.3337**
0.980	0.915	0.860	0.925	23.00	12.15	1.32E-20	4.10E-13	-18.85	-22.00	0.8970	0.8657	0.8970	**0.8943**
0.999	0.992	0.914	0.935	111.20	39.76	8.29E-41	1.63E-27	-7.43	-8.81	0.3665	0.3502	0.3665	**0.3651**
0.991	0.975	0.989	1.084	38.75	25.90	3.51E-27	4.39E-22	-24.73	-28.08	1.1932	1.1791	1.1932	**1.1910**
0.986	0.924	0.934	0.973	29.72	13.51	8.18E-24	2.74E-14	-22.30	-25.51	1.1226	1.1073	1.1226	**1.1196**
0.989	0.946	1.014	1.018	36.31	16.71	2.37E-26	9.63E-17	-17.66	-19.70	0.8921	0.8350	0.8903	**0.8902**
0.998	0.987	0.975	0.972	83.32	32.22	4.64E-37	7.80E-25	-8.96	-10.24	0.4730	0.4444	0.4718	**0.4717**
0.986	0.908	1.026	0.991	32.99	12.54	3.92E-25	1.86E-13	25.40	28.87	-1.3173	-1.3495	-1.3173	**-1.3140**
0.990	0.979	0.986	0.970	36.74	25.24	1.67E-26	9.20E-22	6.23	6.97	-0.3077	-0.3504	-0.3104	**-0.3100**
0.996	0.983	0.957	0.976	58.97	27.86	1.39E-32	5.33E-23	-9.87	-11.24	0.5033	0.4820	0.5033	**0.5010**
0.994	0.932	1.153	1.070	57.53	15.61	2.90E-32	6.03E-16	23.49	27.16	-1.2028	-1.2350	-1.2028	**-1.1989**
0.995	0.992	0.957	0.963	53.15	39.42	3.05E-31	2.11E-27	-3.09	-3.33	0.1458	0.1383	0.1451	**0.1447**
0.997	0.995	1.031	1.051	72.03	58.63	3.59E-35	1.65E-32	10.84	13.40	-1.5519	-1.5837	-1.5519	**-1.5472**
0.997	0.994	1.017	1.025	67.46	51.25	2.53E-34	8.98E-31	6.15	7.88	-0.8914	-0.9200	-0.8941	**-0.8934**
0.999	0.999	0.976	0.981	97.89	123.87	3.75E-39	3.27E-42	2.45	2.11	-0.2397	-0.2737	-0.2382	**-0.2373**
1.000	1.000	1.025	1.019	266.11	244.42	3.65E-52	4.67E-51	-3.49	-3.83	0.4320	0.4110	0.4320	**0.4308**
1.000	1.000	0.969	0.977	315.02	241.15	2.31E-54	7.00E-51	4.65	4.90	-0.5530	-0.5828	-0.5509	**-0.5507**
0.999	0.988	1.028	1.060	101.93	35.83	1.12E-39	3.50E-26	22.16	25.80	-2.8286	-2.8393	-2.8286	**-2.8243**
0.994	0.989	0.961	0.947	46.31	34.11	1.81E-29	1.47E-25	-7.99	-10.65	1.1522	1.1007	1.1503	**1.1502**
0.999	1.000	0.955	0.968	95.10	163.78	8.90E-39	7.62E-46	5.35	5.39	-0.5872	-0.6026	-0.5872	**-0.5853**
0.998	0.998	0.966	0.964	87.35	79.75	1.13E-37	1.72E-36	-1.79	-2.87	0.3121	0.2945	0.3110	**0.3107**
0.996	0.986	1.004	1.040	61.33	33.63	4.32E-33	2.23E-25	19.77	23.56	-2.5093	-2.5103	-2.5093	**-2.5055**
0.998	0.995	1.019	1.030	79.35	56.94	1.99E-36	3.94E-32	7.09	9.07	-0.9411	-0.9705	-0.9411	**-0.9375**
0.994	0.980	1.072	1.105	53.92	29.75	1.99E-31	8.00E-24	24.48	29.19	-2.9315	-2.9322	-2.9315	**-2.9221**
0.999	1.000	1.022	1.020	169.45	209.67	2.75E-46	4.63E-49	-1.49	-1.23	0.1201	0.1179	0.1200	**0.1200**
0.996	0.991	0.995	0.975	62.97	39.01	1.97E-33	2.87E-27	-13.69	-16.42	1.6368	1.5172	1.6368	**1.6297**

References

ABN Amro, 2005. Global Investment Returns Yearbook 2005. [Internet] http://www.london.edu/assets/documents/PDF/GIRY2005abstract.pdf [Date of use: January 24 2005]

Abramowitz, M. and Stegun, I. A., (Editors) 1972. Handbook of mathematical functions with formulas, graphs, and mathematical tables, 9th edition, New York: Dover, p. 18.

AC 133, 2004. IFAD GAAP Convergence studies, country update: South Africa, *South African Accounting Standards Board.* [Internet] http://www.iasplus.com/country/safrica.htm [Date of use: June 16 2004]

ACCOUNTANCY AGE, 2005. Parmalat: the aftermath. [Internet] http://www.accountancyage.com/specials/1136019 [Date of use: February 22 2005]

ACHARYA, V. V. and Carpenter, J. N. 2002. Corporate bonds: valuation, hedging, and optimal call and default policies, *Review of Financial Studies*, **15**, pp. 1355-1383.

ALTMAN, E. I. 1968. Financial ratios, discriminant analysis and the prediction of corporate bankruptcy, *Journal of Finance*, Sep-1968, pp. 589-609.

AU, K. T. and Thurston, D. C. 1995. A new class of duration measures, *Economics Letters*, 47, pp. 371–375.

AUDET, N. and Doshi, N. 2003. Pricing of defaultable bonds by modelling the term structure, *Michigan engineering.* [Internet] http://www.engin.umich.edu/class/ioe553/finalschedule.htm [Date of use: July 12 2004]

AUSTIN, S, 2005. Asset Liability Management 101, CorpAnalytics: *Southeast Corporate Consulting Services.* [Internet] http://www.secorp.org [Date of use: May 1 2005]

BABBEL, D. F., Merrill, C. and Panning, W. 1997. Default risk and the effective duration of bonds, *Financial Analysts Journal,* January/February, pp. 35-44.

BALBAS, A. and Ibanez, A. 1998. When can you immunize a bond portfolio?, *Journal of Banking and Finance*, 22, pp. 1571-1595.

BALBAS, A., Ibanez, A. and Lopez, S. 2002. Dispersion measures as immunization risk measures, *Journal of Banking and Finance*, 26, pp. 1229-1244.

BANGIA, A., Diebold, F. X., Schuermann, T. and Stroughair, J. 1998. Modelling liquidity risk. [Internet] http://www.ssc.upenn.edu/~fdiebold/papers/paper25/bds.pdf [Date of use: May 11 2005]

BANGIA, A., Diebold, F. X., Schuermann, T. and Stroughair, J. 1999. Liquidity on the outside, *Risk*, **12**, pp. 68-73.

BARBER, J. R. and Cooper, M. L. 1996. Immunizing using principal component analysis, *The Journal of Portfolio Management*, Fall, pp. 99-105.

BARRETT, W. B., Gosnell, T. F. and Heuson, A. J. 1995. Yield curve shifts and the selection of immunization strategies, *The Journal of Fixed Income*, September, pp. 53-64.

BASILEVSKY, A. 1995. Statistical Factor Analysis and Related Methods, *John Wiley & Sons*, New York. 288p.

BAUER, W. and Ryser, M. 2004. Risk management strategies for banks, *Journal of Banking & Finance*, **28**, 2, pp. 331-352.

BECK, K. L., Goldreyer, E. F. and D'Antonio, L. J. 2000. Duration gap in the context of a bank's strategic planning process, *Journal of Financial and Strategic Decisions*, **13**, 2, pp. 57-71.

BERNDT, A. 2003. Estimating the term structure of credit spreads from callable corporate bond price data, Working paper, *School of Operations Research and Industrial Engineering*, Cornell University, November, 2003. [Internet] http://www.orie.cornell.edu/~aberndt/papers.html [Date of use: April 17 2005]

BERNDT, A., Douglas, R., Duffie, D., Ferguson, M. and Schranzk, D. 2004. Measuring default risk premia from default swap rates and EDFs, Working paper, March 2004. [Internet] http://www.orie.cornell.edu/~aberndt/papers.html [Date of use: November 25 2005]

BESSIS, J. 2002. Risk management in banking, *John Wiley & Sons Ltd.*, 2nd edition, London.

BIERWAG, G. O. 1977. Immunisation, duration and the term structure of interest rates, *Journal of Financial and Quantitative Analysts,* December, pp. 725-742.

BIERWAG, G. O. 1987. Duration analysis, *Ballinger Publishing Company*, USA, pp. 324-329.

BIERWAG, G. O. and Kaufman, G. G. 1977. Coping with the risk of interest rate fluctuations, *Journal of Business,* July, pp. 364-370.

BIERWAG, G. O. and Kaufman, G. G. 1988. Durations of non-default-free securities, *Financial Analysts Journal*, July/August, pp. 39-46.

BIERWAG, G. O., Kaufman, G. G. and Toevs, A. L. 1983a. Duration: its development and uses in bond portfolio management, *Financial Analysts Journal.* **39**, 4, pp. 15-35.

BIERWAG, G. O., Kaufman, G. G. and Toevs, A. L. 1983b. Immunisation strategies for funding multiple liabilities, *Journal of Financial and Quantitative Analysis,* **18**, pp. 113-123.

BIERWAG, G. O., Kaufman, G. G. and Toevs, A. L. 1983c. Recent developments in bond portfolio immunisation strategies, *Contemporary Studies in Economic and Financial Analysis*, **41**, pp. 105-157.

BIS Consultative Document, 1988. International convergence of capital measurement and capital standards, July 1988. [Internet] http://www.bis.org/publ/bcbsc111.htm#pgtop [Date of use: March 17 2005]

BIS Consultative Document, 2001. The standardised approach to credit risk: supporting document to the new Basel capital accord, *Issued for comment by 31 May 2001.* [Internet] http://www.bis.org/publ/bcbsca04.pdf [Date of use: June 1 2005]

BIS Consultative Document, 2004. International convergence of capital measurement and capital standards: a Revised Framework. [Internet] http://www.bis.org/publ/bcbs107.htm [Date of use: June 16 2004]

BLACK, F. 1995. Interest rates as options, *Journal of Finance*, **50**, 2, pp. 1371-1376.

BLACK, F. and Cox, J. C. 1976. Valuing corporate securities: some effects of bond indenture provisions, *Journal of Finance*, **31**, 2, pp. 351-367.

BOHN, J. 2000. An empirical assessment of a simple contingent-claims model for the valuation of risky debt, *Journal of Risk Finance*, Summer, pp. 55-77.

BOTHA, M. 2004. Private communication.

BOTHA, M. and van Vuuren, G. W. 1999. The silence of the lambdas, *South Africa: A Risk Special Report*, Risk Supplement, June, pp. 22-24.

BOTHA, M., van Vuuren, G. W. and Styger, P. 2001. The effect of a dynamic exponential decay factor on volatility and VaR, *PJAS*, **1**, 1, January, pp. 24-32.

BROOKS, R. and Gup, B. 1997a. Embedded derivatives impact on interest rate risk and capital adequacy. In G. Kaufman (Ed.), *Research in Financial Services,* Stamford, CT: JAI Press, pp. 232 – 290.

BROOKS, R. and Gup, B. 1997b. The effects of nonparallel shifts in the yield curve on bank capital adequacy: a study of options and structured liabilities. In J. Lederman, T. Cornyn, & R. Klein (Eds.), *Controlling and Management Interest Rate Risk.* Englewood Cliffs, NJ: Prentice Hall. 319p.

BUHLER, W., Uhrig, M., Walter, U. and Weber, T. 1999. An empirical comparison of forward- and spot-rate models for valuing interest-rate options, *Journal of Finance*, 54, pp. 269-305.

CAOUETTE, J. B., Altman, E. I., and Narayanan, P. 1998. Managing credit risk: The next great financial challenge, *John Wiley and Sons*, New York, 1998. 265p.

CEBENOYAN, A. S. and Strahan, P. E. 2004. Risk management, capital structure and lending at banks, *Journal of Banking & Finance*, **28**, 1, pp. 19-43.

CHAMBERS, D. R., Carleton, W. T. and McEnally, R. M. 1988. Immunizing default-free bond portfolios with a duration vector, *Journal of Financial and Quantitative Analysis*, 23, pp. 89-104.

CHANCE, D. M. 1990. Default risk and the duration of zero coupon bonds, *Journal of Finance,* **35**, pp. 265-274.

CHOI, J. J. and Elyasiani, E. 1996. Derivative exposure and the interest rate and exchange rate risks of U.S. banks, *Journal of Financial Services Research*, **12**, 2-3, pp. 267-286.

COLES, S. 2001. An introduction to statistical modelling of extreme values
Springer Verlag, *University of Bristol*, 208p.

COX, J. C., Ingersoll, J. E. Jr. and Ross, S. A. 1979. Duration and the measurement of basis risk, *Journal of Business*, **52**, 1, January, pp.1-61.

COX, J. C., Ingersoll, J. E. Jr. and Ross, S. A. 1985. A theory of the term structure of interest rates, *Econometrica*, **53**, pp. 385-408.

CULL, R. J. 2005. Deposit insurance and financial development, *Journal of money credit and banking,* 37, pp. 43-82.

DE LA TORRE, A. 2005. Coping with risks through mismatches: domestic and international financial contracts for emerging economies, *International Finance,* **7**, pp. 1-42.

DELIANEDIS, G. and Geske, R. 1998. Credit risk and risk neutral default probabilities: information about rating migrations and defaults, *UCLA Working Paper*, pp. 19-98.

DELIANEDIS, G., Geske, R. and Corzo, T. 1998. Credit risk analysis with option models: estimation and comparison of actual and risk-neutral default probabilities, Working Paper, *Anderson Graduate School of Business*, University of California, Los Angeles.

DIMSON, E. and Marsh, P. 1996. Stress tests of capital requirements, *Journal of Banking & Finance*, 21, 11-12, pp. 1515-1546.

DUFFIE, D. and Singleton, K. J. 1999. Modelling term structures of defaultable bonds, *Review of Financial Studies*, **12**, pp. 687-720.

DWYER, D. W., Kocagil, A. E. and Stein, R. M. 2004. The Moody's KMV EDF™ RiskCalc™ v3.1 Model Next-Generation Technology for Predicting Private Firm Credit Risk, White paper. [Internet] http://www.defaultrisk.com/pp_score_33.htm [Date of use: August 30 2004]

ELLIS, D. M. and Jordan, J. V. 2001. The evaluation of credit union non-maturity deposits. *Prepared for National Credit Union Administration*, National Economic Research Associates, September 10, 2001, pp. 1-76.

EOM, Y. H., Helwege, J. and Huang, J. J. 2002. Structural models of corporate bond pricing: an empirical analysis. *EFA,* February 2002, Berlin Meetings.

FABOZZI, F. J. (ed.) 1997. The Handbook of Fixed Income Securities, *Irwin.* Chicago, Ill. 491p.

FABOZZI, F. J. 1996. Bond markets: analysis and strategies, *Prentice Hall*, Upper Saddle River, NJ. 532p.

FARIN, T. A. 2004. Managing non-maturity deposits: Part 2, *Farin & Associates, Inc*, October 2004. [Internet] http://www.farin.com/media/documents/pdf/Non-Maturity%20Deposits2.pdf [Date of use: January 5 2005]

FAS. 2004. Financial Accounting Standards Board: Derivatives Implementation Group. [Internet] http://www.fasb.org/ [Date of use: September 22 2004].

FDIC (Federal Deposit Insurance Corporation), 1995. Regulators terminate the US operations of Daiwa Bank, Ltd, Japan, *PR-67-95*, November 11, 1995.

FDIC (Federal Deposit Insurance Corporation), 2002. The rise of risk management: Basel and beyond, symposium organised by *Credit Suisse First Boston*. [Internet] http://www.fdic.gov/bank/analytical/fyi/2002/073002fyi.html [Date of use: December 13 2004]

FISD, 2000. The Fixed Investment Securities Database, *LJS Global Investment Services*, Inc, New York. 284p.

FISHER, E., Heinkel R. and Zechner J. 1989. Dynamic capital structure choice: theory and tests, *Journal of Finance*, **44**, pp. 19-40.

FISHER, L. 2004. Forecasting interest-rate risk over time. [Internet] http://business.rutgers.edu/departments/fin-papers/04Spring/lfisher.pdf [Date of use: June 16 2004]

FISHER, L. and Weil, R. 1971. Coping with the risk of interest rate fluctuations and returns to bondholders from naïve and optimal strategies, *Journal of Business*, **44**, October, pp. 408-431.

FONG, H. and Vasicek, O. 1984. A risk minimizing strategy for portfolio immunization, *The Journal of Finance*, December, pp. 1541-1546.

FONS, J. S. 1990. Default risk and duration analysis. In: Altman, E. I. (Ed.), The High Yield Debt Market, *Dow Jones Irwin*, New York, pp. 18-40.

FOOLADI, I. J., Roberts, G. S. and Skinner, F. 1997. Duration for bonds with default risk, *Journal of Banking and Finance*, 21, pp. 1-6.

FREY, R. and McNeil, A. 2003. Dependent defaults in models of portfolio credit risk, *Journal of Risk*, **6**, 1, pp. 59-92.

FREY, R., McNeil, A. and Nyfeler, N. 2001. Copulas and credit models, *Risk*, pp. 111-114.

GALAI, D. and Masulis, R. W. 1976. The option pricing model and the risk factor of stock, *Journal of Financial Economics*, November, pp.115-146.

GARBADE, K. D. 1985. Managing yield curve risk: a generalized approach to immunization, *Topics in Money and Securities Markets*, Bankers Trust Company, New York, August.

GARBADE, K. D. 1986. Modes of fluctuation in bond yields: an analysis of principal components, *Topics in Money and Securities Markets*, Bankers Trust Company, New York, January.

GARMAN, M. 1995. The duration of option portfolios, *Journal of Financial Economics*, **14**, 2, pp. 309-315.

GIBSON, R. and Mougeot, N. 2004. The pricing of systematic liquidity risk: Empirical evidence from the US stock market, *Journal of Banking & Finance*, **28**, 1, pp. 157-178.

GILKESON, J. and Ruff, C. 1996. Valuing the withdrawal option in retail CD portfolios. *Journal of Financial Services Research,* 10, pp. 333-358.

GILKESON, J. and Smith, S. 1992. The convexity trap: pitfalls in financing mortgage portfolios and related securities, *Economic Review,* 77, pp. 14-27.

GOLUB, B.W. and Tilman, L. M. 1997. Measuring yield curve risk using principal components analysis, Value at Risk, and key rate durations, *Journal of Portfolio Management*, Summer 1997, pp. 72-84.

GULTEKIN, N. B. and Rogalski, R. J. 1984. Alternative duration specifications and the measurement of basis risk: empirical tests, *Journal of Business*, 57, pp. 241-264.

GUNTAY, L. 2002. Pricing the risks of callable defaultable coupon bonds, *Working Paper*, University of Maryland.

HALLERBACH, W. M. 2001. Duration and dimension, *Tinbergen Institute Working Paper No. TI 99-047/2*. [Internet] http://ssrn.com/abstract=488623 [Date of use: March 8 2005]

HANSEN, L. 1982. Large sample properties of generalized methods of moments estimators, *Econometrica*, 50, pp. 1029-1054.

HAUGEN, R. A. 1997. Modern Investment Theory, *Prentice Hall.* Upper Saddle River, NJ. 542p.

HESS, A. C. and Laisathit, K. 1997. A market-based risk classification of financial institutions, *Journal of Financial Services Research*, **12**, 2-3, pp. 133-158.

HICKS, J. R. 1939. Value and Capital, *London*: Oxford University Press, pp. 186-188.

HICKS, J. R. 1946. Value and Capital, 2nd Edition. *Oxford, Clarendon Press.* 177p.

HO, T. S. Y. 1992. Key rate durations: measures of interest rate risks, *Journal of Fixed Income,* **2**, 2, pp. 29-44.

HO, T. S. Y. and Lee, S. B. 1986. Term structure movements and the pricing interest rate contingent claims, *Journal of Finance*, 41, December, pp. 1011-1029.

HOLTON, G. 2003. discussion on the duration of demand deposits and non-maturity instruments. [Internet] http://www.contingencyanalysis.com/archive/archive03-1/000046ce.htm [Date of use: October 19 2004]

HOPEWELL, M. and Kaufmann, G. G. 1973. Bond price volatility and years to maturity, *American Economic Review,* September, pp. 749-753.

HOUWELING P. and Vorst, T. 2001. An empirical comparison of default swap pricing models, *Economics Working Paper*, Erasmus University, Rotterdam, December.

HOUWELING, P., Hoek, J. and Kleibergen, F. 2001. The joint estimation of term structures and credit spreads, *Journal of Empirical Finance*, **8**, pp. 297-323.

HUANG, J. and Huang, M. 2003. How much of the corporate-treasury yield spread is due to credit risk? A new calibration approach, Working Paper, *Department of Finance*, Pennsylvania State University.

HULL, J. and White, A. 1990. Pricing interest rate derivative securities, *Review of Financial Studies*, **3**, 4, pp. 573- 592.

HULL, J. and White, A. 1995. The impact of default risk on the prices of options and other derivative securities, *Journal of Banking and Finance*, **19**, pp. 299-322.

HULL, J. and White, A. 2001. Valuing credit default swaps II: modelling default correlations, *Journal of Derivatives*, **8**, 3, pp. 12-22.

IAS Board, 2004. Amendments to IAS 39 Financial Instruments: Recognition and measurement, the fair value option. [Internet] http://www.iasb.org/uploaded_files/documents/ias39fv.pdf [Date of use: February 1 2005]

ICC, 2004. Interest rate risk: comptroller's handbook, Narrative - June 1997, Procedures - March 1998 2004. Comptroller of the Currency Administrator of National Banks. [Internet] www.occ.treas.gov/handbook/irr.pdf [Date of use: May 8 2005]

ICP – International Company Profile, 2005. [Internet] http://www.icpcredit.com/companies/Ed.asp [Date of use: April 12 2005]

JACOBY, G. 2003. A duration model for defaultable bonds, *Journal of Financial Research*, **26**, 1, pp. 129-146.

JACOBY, G. and Roberts, G. S. 2003. Default- and call-adjusted duration for corporate bonds, *Journal of Banking & Finance*, **27**, pp. 2297-2321.

JAMES, J. and Webber, N. 2000. Interest rate modelling, *John Wiley and Sons*, New York, 672p.

JANOSI, T., Jarrow, R. and Zullo, F. 1999. An empirical analysis of the Jarrow van Deventer model for valuing non-maturity demand deposits, *The Journal of Derivatives*, Fall, pp. 8-31.

JARROW, R. A. and Turnbull, S. M. 1995. Pricing derivatives on financial securities subject to credit risk, *Journal of Finance*, **50**, 1, pp. 53-85.

JARROW, R. A., Lando, D. and Turnbull, S. 1997. A Markov model for the term structure of credit risk spreads, *Review of Financial Studies*, **10**, 2, pp. 481-523.

JARROW, R. A., Li, H, Liu, S. and Wu, C. 2003. Valuing callable corporate bonds in a reduced form model using a call intensity process, Working Paper, Cornell University.

JARROW, R. and van Deventer, D. R. 1998. The arbitrage-free valuation and hedging of demand deposits and credit card loans, *Journal of Banking and Finance*, **22**, 3, March 1998, pp. 249-272.

JORION, P. 2001. Financial risk manager handbook: 2001 – 2002, GARP, *Wiley Finance*, John Wiley and Sons.

JORION, P. and Khoury, S. J. 1996. Financial risk management, *Blackwell*, New York, 1996.

JOUANIN J. F., Rapuch G., Riboulet, G. and Roncalli, T. 2001. Modelling dependence for credit derivatives with copulas, *Groupe de Recherche Op´erationnelle, Cr´edit Lyonnais.* [Internet] http://gro.creditlyonnais.fr [Date of use: June 22 2005]

KERKHOF, J. and Melenberg, B. 2004. Backtesting for risk-based regulatory capital, *Journal of Banking & Finance*, **28**, 8, pp. 1845-1865.

KHANG, C. 1979. Bond immunisation when short-term rates fluctuate more than long-term rates, *Journal of Financial and Quantitative Analysis*, **14**, pp. 1035-1040.

KIHN, J. 1994. Unravelling the low-grade bond risk/reward puzzle, *Financial Analysts Journal*, July/August, pp. 32-42.

KIM, I. J., Ramaswamy, K. and Sundareasan, S. 1993. Does the default risk in coupons affect the valuation of corporate bonds?, *Financial Management,* **22**, pp. 117-131.

KNEZ, P. J., Litterman, R. and Scheinkman, J. 1994. Explorations into factors explaining money market returns, *Journal of Finance*, 49, pp. 1861-1882.

KOCH, T. W. and MacDonald, S. S. 2000. Bank management, 4th edition, *Harcourt Inc*, New York. 381p.

KPMG, 2004. Banking Survey South Africa: 2004, 14th International Financial Services Group survey. [Internet] http://www.kpmg.co.za/download/sa%20banking%20survey_2004.pdf [Date of use: March 7 2005]

KRUGMAN, P. 2002. How copper came a cropper: Sumitomo's robber-baron tactics make the case for regulation. [Internet] http://web.mit.edu/krugman/www/copper.html [Date of use: July 28 2004]

KWAN, S. and Eisenbeis, R. A. 1995. Bank risk, capitalization and operating efficiency, *Journal of Financial Services Research*, 2, 2/3, pp. 117-131.

LANDO, D. 1998. On Cox processes and credit risky securities, *Review of Derivatives Research*, **2**, pp. 99-120.

LEESON, N. W. and Whiteley, E. 1996. Rogue trader: How I brought down Barings Bank and shook the financial world, *Little Brown and Company*, New York. 249p.

LI, D. 1998. Constructing a credit curve, Credit Risk Special Report, *Risk Magazine*, November, pp. 40-44.

LITTERMAN, R. and Scheinkman, J. 1991. Common factors affecting bond returns, *Journal of Fixed Income*, 1, pp. 62-74.

LONGSTAFF, F. A. and Schwartz, E. S. 1995. A simple approach to valuing risky fixed and floating rate debt, *Journal of Finance*, **50**, 3, pp. 789-819.

LOWENSTEIN, R. 2001. When genius failed: The rise and fall of Long-Term Capital Management, *Random House Trade*, New York. 323p.

MACAULAY, F. R. 1938. Some theoretical problems suggested by the movements of interest rates, bond yields and stock prices in the US since 1856, *New York: Columbia University Press.*

MACDONALD, S. 2004. Private communication.

MADAN, D. and Unal, H. 1998. Pricing the risks of default, *Review of Derivatives Research,* **2**, 3, pp. 121-160.

MARSHALL, C. and Siegel, M. 1996. Value at risk: implementing a risk measurement standard, *Journal of Derivatives*, **4**, pp. 91-111.

MATZ, L. 2005. Use and misuse of Value-at-Risk analysis for bank balance sheet risk analysis, *Bank Accounting & Finance*, December–January, 2005, pp. 15-20.

MCLEOD, W. and van Vuuren, G. W. 2004. More risk for less return?, *Investment Analysts Journal*, 59, pp. 15-20.

MERTON, R. C. 1974. On the pricing of corporate debt: The risk structure of interest rates, *Journal of Finance*, 29, pp. 449-470.

MILLER, K. 2004. Bubble capitalism: Corporate Crisis, US deficits and fall of the dollar. [Internet] http://www.globalpolicy.org/socecon/crisis/ [Date of use: December 3 2004]

MOODY'S, 2003. Moody's approach to rating SME CLOs, International Structured Finance Special Report. [Internet] http://www.moodyskmv.com/research/whitepaper/moodysjapan_eng.pdf [Date of use: January 30 2005]

MOODY'S, 2005. Moody's credit ratings: South Africa – the decade from 1995 to 2005. [Internet] http://www.moodys.com/cust/default.asp

MORGAN, G. E. 1986. Floating rate securities and immunisation: some further results, *Journal of Financial and Quantitative Analysis*, 21(**1**), pp. 87-94.

MOSHER, M. 2005. Broadening your horizons. [Internet] http://www.ftmandate.com/news/fullstory.php/aid/5/Broadening_your_horizons.html [Date of use: August 8 2004]

MOSHIRIAN, F. 2004. Financial services: global perspectives, *Journal of Banking & Finance*, **28**, 2, pp. 269-276.

MUNK, C. 2000. Stochastic duration and fast coupon bond option pricing in multi-factor models, *Review of Derivatives Research*, **3**, pp. 157–181.

NAWALKHA, S. K. 1996. A contingent claims analysis of the interest rate risk characteristics of corporate liabilities, *Journal of Banking & Finance*, **20**, pp. 227-245.

NAWALKHA, S. K. 1999. Duration 60 years later. In: Nawalkha, S., Chambers, D. (Eds.), Interest rate risk measurement and management, *Institutional Investor*, New York, pp. 3-17.

NCUA (National Credit Union Administration), 2004. NCUA Combined annual performance report 2003 and initial annual performance plan 2005. [Internet] http://www.ncua.gov/ReportsAndPlans/plans-and-reports/NCUACombinedReport2003_2005.pdf [Date of use: March 2 2005]

NCUA (National Credit Union Administration), 2005. *NCUA Press Release*. [Internet] http://www.ncua.gov/news/press_releases/2004/NR04-0427-2.htm [Date of use: November 28 2004]

NELSON, C. R and Siegel, A. F. 1987. Parsimonious modelling of yield curves, *Journal of Business*, **60**, 4, pp. 473-89.

O'BRIEN, J. M. 2000. Estimating the value and interest rate risk of interest-bearing transactions deposits, White paper, *Division of Research and Statistics*, Board of Governors, *Federal Reserve System*, November 2000.

OSFI (Office of the Superintendent of Financial Institutions), 2005. Guideline impact analysis statement, guideline B-12 - Interest Rate Risk Management. [Internet] http://www.osfi-bsif.gc.ca/app/DocRepository/1/eng/ [Date of use: April 29 2005]

OTT, R. A. Jr. 1986. The duration of an adjustable-rate mortgage and the impact of the index, *Journal of Finance*, **41**, 4, pp. 923-934.

PERKS, W. 1952. Abstract of the discussion of Redington (1952). pp. 324-326.

PETERSON, S. and Stapleton, R. C. Pricing of options on credit-sensitive bonds, *Schmalenbach Business Review*, **55**, July 2003, pp. 178-193.

PEURA, S. and Jokivuolle, E. 2004. Simulation based stress tests of banks' regulatory capital adequacy, *Journal of Banking & Finance*, **28**, 2, pp. 1801-1824.

PRITSKER, M. 1996. Evaluating Value at Risk methodologies: accuracy versus computational time, *Journal of Financial Services Research*, **12**, 2, pp. 201-242.

RAWNSLEY, J. 1995. Going for broke: Nick Leeson and the collapse of Barings Bank, Harper Collins, *London*, 1995.

REBONATO, R. 1996, Interest-rate option models, *John Wiley & Sons*, New York. 166p.

REBONATO, R., and Cooper, I. 1996. The limitations of two-factor interest rate models, *Journal of Financial Engineering*, 5, pp. 1-16.

REDINGTON, F. M. 1952. Review of the principle of life office valuations, *Journal of the Institute of Actuaries*, 18, pp. 286-430.

RISK Institute, 2005. Sources and effects of interest rate risk. [Internet] http://riskinstitute.ch/138860.htm [Date of use: December 18 2004]

ROSENBERG, J. 2004. A general approach to integrated risk management with skewed, fat-tailed risks. *Federal Reserve Bank of New York*, Rutgers University Finance Department Seminar.

[Internet] http://business.rutgers.edu/departments/fin-papers/04Spring/josh.pdf [Date of use: April 13 2005]

RUTKOWSKI, M. 2003. Modelling of default times and hedging of credit risk, PRMIA Meeting, *Leszno*, June 2003, pp. 15-22.

RZADKOWSKI, G. and Zaremba, L. S. 2000. New formulas for immunizing durations, *Journal of Derivatives*, **8**, 2, pp. 28-36.

SAICA (South African Institute of Chartered Accountants) 2005. Guide on the application of AC133 financial instruments: recognition and measurement, to liabilities arising from long-term insurance contracts. [Internet] https://www.saica.co.za/documents/guideAC133.PDF [Date of use: July 9 2004]

SAMUELSON, P. A. 1945. The effect of interest rate increases on the banking system, *American Economic Review,* March, pp. 16-27.

SANTOMERO, A. M. 1997. Commercial bank risk management: an analysis of the process, *Journal of Financial Services Research*, **12**, 2/3, pp. 83-115.

SARB – South African Reserve Bank, 2005. New directives. [Internet] http://www.sarb.co.za/ [Date of use: May 22 2005]

SARKAR, S. 2000. Probability of call and likelihood of the call feature in a corporate bond, *Journal of Banking and Finance*, **25**, pp. 505-533.

SCHÖNBUCHER, P. and Schubert, D. 2001. Copula-dependent default risk in intensity models, *Working paper*, Universität Bonn.

SCHÖNBUCHER, P. J. 2003. Credit derivatives pricing models, *John Wiley and Sons.* England.

SHAO, J. and Tu, D. 1995, The jackknife and the bootstrap, *Springer-Verlag,* New York.

SHARPE, W. F., Alexander, G. J. and Bailey, J. V. 1999. Investments, *Prentice Hall.* Upper Saddle River, NJ. 418p.

SHUMWAY, T. 2001. Forecasting bankruptcy more accurately: a simple hazard model, *Journal of Business*, **74**, 1, pp. 101–124.

SINGH, M. 1997. Value at Risk using principal components analysis, *Journal of Portfolio Management*, Fall 1997, pp. 101-110.

SOTO, G. M. 2004. Duration models and IRR management: A question of dimensions?, *Journal of Banking & Finance*, **28**, 5, pp. 1089-1110.

The Economist, 2001. Enron's fall: Upended, *The Economist print edition, Global Agenda,* November 29th 2001.

The Economist, 2002. WorldCom's bankruptcy mess, *The Economist print edition, Global Agenda,* July 23rd 2002.

The Economist, 2004. Private jets in China, *The Economist print edition, Global Agenda,* December 2nd 2004.

The Economist, 2005. Not feeling so good, *The Economist print edition, Global Agenda,* April 8th 2005.

THOMAS, H. and Wang, Z. 2004. The integration of bank syndicated loan and junk bond markets, *Journal of Banking & Finance*, **28**, 2, pp. 299-329.

THOMAS, L. and Willner, R. 1997. Measuring the duration of an internationally diversified bond portfolio, *Journal of Portfolio Management*, Fall 1997, pp. 93-99.

TIKAM, J. 2005. Private communication.

UYEMURA, D. and van Deventer, D. 1992. Financial risk management in banking: the theory and application of asset/liability management, *Dearborn Financial Publishing*, 1st ed, New York. 431p.

VAN DEVENTER, D. R. and Imai, K. 1997. Financial risk analytics: A term structure model approach for banking, insurance and investment management, *Irwin Professional*, 1997, Chapter 15, pp. 283-305.

VAN VUUREN, G. W. and Styger, P. 2004a. Adjustments to the Macaulay duration measure, 23rd *South African Financial Association* conference, Cape Town, South Africa, Jan 2004.

VAN VUUREN, G. W. and Styger, P. 2004b. Immunisation using alternative duration methods, 57th *International Atlantic Economic Society* conference in Lisbon, Portugal, March 2004.

VAN VUUREN, G. W., Botha, M. and Styger, P. 2000. Effects of a changing lambda, *South Africa: A Risk Special Report*, Risk Supplement, June, pp. 22-25.

VAN VUUREN, G. W., Botha, M. and Styger, P. 2003. Risky business, *GARP Risk Review*, November. pp. 16-19.

VASICEK, O. 1977. An equilibrium characterisation of the term structure, *Journal of Financial Economics*, **5**, pp. 177-188.

WALL, L. D. and Shrikhande, M. M. 2005. Managing the risk of loans with basis risk sell, hedge or do nothing, *Departmental Working Papers* from Tor Vergata University, CEIS.

WCCU – World Council of Credit Unions, 2002. Development best practices in credit union supervision. [Internet] http://www.woccu.org/best_practices/asset_liability.pdf [Date of use: October 22 2004]

WOLFE, E. 2001. NatWest markets. ERisk – October 2001. [Internet] http://www.erisk.com/Learning/CaseStudies/NatWestCaseStudy.pdf [Date of use: October 1 2004]

WU, X. 2000. A new stochastic duration based on the Vasicek and CIR term structure theories, *Journal of Business Finance & Accounting*, **27**, September: 7-8, pp. 911-932.

XIE, A. Y., Liu, S. and Chunchu, W. 2002. Estimating duration for bonds with default risk. [Internet] http://www.cab.latech.edu/public/DEPTS/econ_fin/papers/xie.htm [Date of use: June 16 2004]