



Development of an effective operational energy envelope for process monitoring

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Abstract

An Effective Operational Energy Envelope (EOEE) is proposed and developed for the purpose of process monitoring, wherein desirable operating conditions form a part of the EOEE. This concept builds on previous work conducted in energy visualisation of systems. Thereafter, the EOEE was evaluated on a FLOWNEX model of the Pebble Bed Micro Model (PBMM). This system was chosen because it is an energy conversion system, which ideally lends itself to energy analysis studies.

The system was simulated in FLOWNEX with a constant Power Turbine (PT) set point of 100 kW. The degradation of compressors and turbines was simulated through the use of the respective characteristic charts. The PBMM system in FLOWNEX compensated for this by increasing the pressure, thereby increasing the mass flow within the system. The results from these simulations were captured using MATLAB and characterised by energy attributed graphs. Following this, the Heterogeneous Euclidean-Overlap Metric (HEOM) function was used. This function quantifies the difference between two matrices of two different operating conditions. The normal (non-degraded simulations/operating conditions) and degraded operating conditions were illustrated through Singular Value Decomposition (SVD).

The Machine Efficiency (ME), or energy conversion efficiency of the system, was used as a performance measure to determine which results would be included in the EOEE and which would not. All operating conditions with an ME below 14% were deemed to fall outside of the EOEE. A second method of creating the EOEE was also proposed. This included a warning front that would act as a buffer between the EOEE or desired operating conditions, as well as the results that were deemed too undesirable. This warning front operated between the machine efficiencies of 13% to 15%. These values formed the basis for the EOEE conceptualisation of the PBMM.

SVD was used to create 3D graphs to visualise the EOEE. The first three SVD values represented the axis and multiple 2D versions. In the 2D versions, the axes were represented by a combination of the first three SVD values. In another version, a single SVD value was plotted against its respective operating point number. This resulted in overlap between points in the EOEE and points not in the EOEE. Consequently, this version is less usable than other methods. These graphs clearly illustrate the difference between the operating points that are included in the EOEE and the points not included in the EOEE or the warning front.

Following this, a second set point of 150 kW was simulated to validate the EOEE as being operating point independent. The same patterns and trend lines were observed for both the

150 kW and the 100 kW points. This could indicate that the EOOE can be used to determine if the system is operating as intended or if an error or degradation has occurred to the level where maintenance is required for multiple operating points.

Keywords: energy graph-based visualisation, degradation, Effective Operational Energy Envelope, Pebble Bed Modular Reactor

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Table of abbreviations

EGBV	Energy Graph-Based Visualisation
EOEE	Effective Operational Energy Envelope
ESF	Efficiency Scaling Factor
FDD	Fault Detection and Diagnosis
FDI	Fault Detection and Isolation
HEOM	Heterogeneous Euclidean-Overlap Metric
HPC	High Pressure Compressor
HPT	High Pressure Turbine
HS	Heat Source
LPC	Low Pressure Compressor
LPT	Low Pressure Turbine
ME	Machine Efficiency
PBMM	Pebble Bed Micro Model
PBMR	Pebble Bed Modular Reactor
PT	Power Turbine
SSC	Steady-State Controller
SDV	Singular Value Decomposition

Chapter 1 Introduction

1.1 Background

In the past, complex industrial systems always required process monitoring to ensure that these industrial systems are running at optimal or acceptable operating conditions. The budget for running these systems mainly consists of maintenance and operating costs. Maintenance is essential as deferring maintenance could increase the potential for more problems, consequently impacting the overall efficiency of a system over time. This could, in some cases, lead to system failure and ultimately result in permanent damage. However, maintenance implemented too soon could waste time; a proper condition monitoring system or method would help keep the proper schedule. For this to work, a method is required that takes into account different industrial processes: a method that regards energy in the system, for example, such as in (Van Schoor and Uren K.R., (2018)). This could subsequently provide more accurate information regarding the operating condition, possible anomalies, and determine if immediate action is required for repairs and/or maintenance.

Many methods and techniques could be used to analyse, monitor, and control different engineering systems; however, each system has unique characteristics and encompasses different physical domains. There are often components in a system that have different functions, which make it difficult to characterise the system as a whole. The majority of industrial plants are complex and encompass multiple physical domains, for instance, fluid, chemical, mechanical, thermal, or electrical.

All these systems use energy, with energy moving from one end to the other. A thermo-fluid system is an example of a system in which energy moves with the use of a fluid as the medium. Energy can therefore be used to characterise the relationship between components in a single system or between larger individual and interlinked systems. This means that the energy characterisation of a single system maps it out in terms of its energy flow or relationship between the individual components.

Using only energy to monitor a plant rather than process variables, reduces the number of variables, leading to a simplified process monitoring approach. This could simplify the process monitoring for companies and organisations that use multiple and different industrial systems in their operation. Considering this, it is a way of characterising large-scale industrial systems and process monitoring, as proposed by (Marais, van Schoor and Uren, 2015). When faults or degradation occur in a system, such as the wearing of components, the energy relationship in a system changes and this change can be measured.

Energy- and exergy-based fault detection has been considered feasible and could be used to isolate faults in previous work. This could include “*Energy-based fault detection and isolation in an industrial steam turbine system*”(Smith, 2020), “*Exergy graph-based fault detection and isolation of a gas-to-liquids process*”(Greyling et al., 2020), and “*Energy-based visualisation of a transcritical CO₂ heat pump cycle*”(De Bruin, no date). There are various methods of energy-based Fault Detection and Diagnosis (FDD) and Fault Detection and Isolation (FDI), including an energy-attributed graph approach, enthalpy-entropy graph

approach, and the area error enthalpy-entropy graph approach (Du Rand and van Schoor, 2012a, 2012b).

To fully utilise energy visualisation for process monitoring, new methods and concepts could be developed to be utilised while the system is running. This can assist in optimising the system. One of these methods could be envisioned by creating a graph with different points, representing various operating conditions of the system. These operating conditions could include, but are not limited to, specific components in a system or entire systems undergoing faults and degradation. Within the graph, some of these operating conditions could illustrate the system working normally, while others may indicate inefficiencies, faults, or degradation of components taking place in a system.

The difference in these operating points could identify an area of optimal or acceptable operating conditions, where the engineering system would still provide its service or purpose with an acceptable performance parameter. The acceptable points could be classified as operating within an acceptable operating area. The points that have degraded beyond acceptable limits or have faults that cause malfunction can be considered beyond the acceptable operating area. This area could be referred to as the Effective Operational Energy Envelope (EOEE), in which all operating points are considered acceptable.

1.2 Research question

The research question can be formulated as follows: How can energy visualisation be used to separate desirable and undesirable operating conditions for process monitoring, and to what extent is it effective?

1.3 Research objectives

1.3.1 Experimental platform

To develop an EOEE for condition monitoring, a simulation of a complex system is required. Different operating conditions are needed to obtain data from which an EOEE could be created. Therefore, a reliable source of experimental data must be identified. Multiple different systems should be considered, with the most suitable used as an experimental platform to simulate the relevant data points for this study.

1.3.2 EOEE conceptualisation

To conceptualise the EOEE, performance measures are required. The EOEE will be characterised in terms of energy; however, the performance measures will determine its boundary. The boundary is the deciding factor of whether an operating point is acceptable (in the EOEE) or unacceptable (outside of the EOEE). The proper performance measures must be explored and selected.

1.3.3 EOEE implementation

The EOEE conceptualisation will have to be implemented on the chosen experimental platform. In order to create the EOEE, the proper method for simulating different operating

points must be explored. There has to be a method by which variance is introduced to the system so that a difference in the energy characterisation of the system can be measured. This can include the degradation of the system as a whole or individual components.

1.3.4 EOOE verification

The results will be illustrated graphically to clearly illustrate the differences between operating conditions within the EOOE and those outside the EOOE.

1.3.5 EOOE validation

The EOOE will be developed and investigated to ensure it can be used universally. Furthermore, the EOOE is required to operate independently at the operating point to ensure that it has merit as a general process monitoring tool.

1.4 Methodology

1.4.1 Experimental platform

This study will build on previous work conducted in energy-based visualisation. Therefore, the study offers a review of previous methods for characterising systems via energy and conduct condition monitoring. The system used for simulating operating conditions will be characterised similarly through an energy-attributed graph, allowing for analysis.

Considering the preceding, a suitable system is required that can be modelled for easy access to the necessary data. This enables different operating conditions and ensures a system with controllable variables for comparing data. It is required that this system has been used previously in order to ensure that the data retrieved from simulations are trustworthy and accurate.

1.4.2 EOOE conceptualisation

To build the EOOE, performance measures are required to determine the boundary operating conditions of the system. This will ensure the separation of operating points into acceptable (within the EOOE) and unacceptable (outside the EOOE). Furthermore, three main performance measures have been identified, namely, the efficiency of a system, the cost of operating the system, and the reliability of the system operating at the current rate.

To determine whether a system is still operating as intended, the difference in normal operating conditions will be tested against non-normal or non-desired conditions. These conditions are expected to represent operating conditions for which components in the system have degraded. The difference in how these operating conditions impact the system will be measured with the Heterogeneous Euclidean-Overlap Metric (HEOM) function.

1.4.3 EOOE implementation

MATLAB will be used to fully control the system and capture data for different operating conditions. MATLAB scripts could control the system running in FLOWNEX to simplify the process of changing the operating conditions and inducing degradation. This will also enable the to capture and manipulation of energy analysis data retrieved from the system into the EOOE. By using both MATLAB and FLOWNEX, the EOOE could be implemented on the chosen system. The results from the system in a steady state can then be verified to ensure the EOOE provides sensible energy analysis data.

1.4.4 EOOE verification

Once the normal and degraded operating points have been collected, they must be processed to create the EOOE. This results in a matrix that characterises the energy flow between the individual components of the system, a node signature matrix. The HEOM function can be used to compare the degraded points to the normal operating point to form a cost matrix.

To reduce the size of these matrices, eigenvalue decomposition can be used, or another similar method. These eigenvalues or similar points can be illustrated on a graph, which will produce the EOOE. The borders of the EOOE can then be tested to determine if all the points inside the borders are present in the EOOE. If crossovers are present, they should be investigated to determine if the EOOE could be resolved.

1.4.5 EOOE validation

Different operating points with different set points will be compared to see if similar trends or assumptions are prevalent. The set point will be a controllable variable of the chosen experimental platform and will be used to determine if the EOOE is operating point independent.

1.5 Dissertation outline

Chapter 2 presents a literature review focusing on previous academic work that involved energy-based graph visualisation and the flow of energy between components.

Chapter 3 discusses the chosen system used in this study, focusing on how it has been used in the past and how it is currently used. This chapter will explain how the system is simulated and how it can be controlled or altered to determine the various operating conditions necessary to create sufficient data for an EOOE.

Chapter 4 conceptualises the EOOE by discussing the performance measures which form the boundary conditions. The most important measures and requirements for the various operating conditions of the system will be discussed. Therefore, this chapter will focus on determining what would cause the system to no longer operate within the EOOE.

Chapter 5 elaborates on the chosen system intended for use and the impact of degradation that will be tested. The operating ranges and mechanisms of degradation will be explained.

Chapter 6 outlines the method of controlling the system through MATLAB, after which the simulation results can be discussed. The results of different operating conditions will be evaluated.

Chapter 7 constructs the EOEE through SVD values obtained in the HEOM equation and the simulation results of different operating conditions in FLOWNEX. The requirements outlined in previous chapters will be used to determine what operating conditions fall inside and outside of the EOEE. The desired warning front is also provided and explained.

Chapter 8 provides a summary of the results, along with recommendations for future studies on how the EOEE can be expanded into a method that could assist in monitoring and controlling different types of engineering systems.

Chapter 2 Literature review

2.1 Introduction

Extensive previous research in the field of Energy Graph-Based Visualisation (EGBV) serves as the foundation for the development of the EOEE. Previous research on the characterisation of systems based on energy could inform the development of the EOEE. This chapter will review and analyse prior research on the field of EGBV, providing justification for the structure and design of the EOEE and focus on methodologies for measuring energy flow within a system and its components. Additionally, relevant concepts and mechanisms that contribute to the construction of the EOEE, especially the envelope or boundary line of the EOEE, will be explored. Finally, a brief overview of degradation mechanisms will be presented. The information provided in this chapter will serve as a foundational basis for the development of the EOEE and guide its implementation.

2.2 Using energy to characterise systems

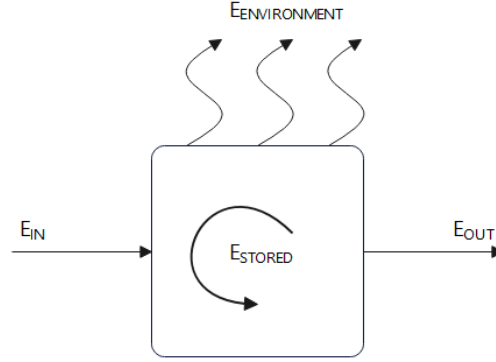
Characterising a system based on energy requires the use of mathematical equations and relevant system information. These may include the characteristics of individual components or the overall system, such as physical attributes like the length, thickness, and diameter of connecting pipes. Additionally, the thermodynamic properties of the fluids and gasses involved, along with the specific characteristics for the gasses and fluids in the system play a crucial role. Furthermore, system properties such as mass flow between components, the temperature of the working fluid, or the pressure could also be considered. To model the system in terms of energy, the most important equations could include the energy balance in the system and the energy balance of components. Therefore, the sum of all energy leaving the component should be equal to the sum of all energy entering it.

As previously mentioned, former research has used energy for fault detection and isolation (Uren and van Schoor, 2016; Uren *et al.*, 2020; Wolmarans, van Schoor and Uren, 2023). There are four categories of energy associated with a system or individual component that can be utilised to establish an energy balance (Marais, van Schoor and Uren, 2016), namely:

1. The energy entering the system. E_{in}
2. The energy leaving the system, as part of the design. E_{out}
3. The energy lost to the environment or energy exiting the system in an unintended manner, such as heat loss from components. $E_{Environment}$
4. The energy stored in the system or individual components in the system. E_{Stored}

This can be seen in Figure 2.1.

Figure 2.1: Energy flow in and out of a component



Source: (Marais, van Schoor and Uren, 2016)

Energy is a multi-domain concept that can be easily understood, although it can be adapted to fit various contexts. In the case of a fluid system, the fluid serves as the medium that carries the energy, regardless of the actual fluid's identity. Energy can be defined as the heat or thermal energy within a system, indicating that the energy flow between components can be interpreted as the heat exchange that occurs between them. The energy balance of a component can be described by:

$$\dot{W} = \dot{Q}_{in} - \dot{Q}_{out} \quad (1)$$

Where $\dot{Q}_{in}$ is the heat flow into the system, $\dot{Q}_{out}$ is the flow out of the system, and $\dot{W}$ is the net energy flow rate. This indicates that when a system's energy transfers are transformed into a heat flow, it can be easily solved, especially in a fluid system. If the heat flow is unknown, it can be calculated from:

$$\dot{Q}_{in} - \dot{Q}_{out} = \dot{m}(h_1) - \dot{m}(h_2) \quad (2)$$

With $\dot{m}$ being the mass flow of the system going through the component, h_2 being the final enthalpy of the component, and h_1 the initial enthalpy. The formula can be shortened to show the change in energy of a component (Marais, van Schoor and Uren, 2016):

$$\dot{Q} = \dot{m}(h_2 - h_1) \quad (3)$$

This facilitates the determination of energy across individual components and may be universally applied to fluid systems, provided that enthalpy and mass flow can be measured. Subsequently, the energy flow between components can be given as:

$$\dot{E}_{12} = \dot{Q}_{c1c2} = \dot{m}_{21}(\Delta h_{c2} - \Delta h_{c1}) \quad (4)$$

With H_{c2} being the enthalpy difference over component 2, or the component downstream, and H_{c1} being the enthalpy of component 1, or the component upstream, in terms of the flow of the fluid in the system (van Graan, van Schoor and Uren, 2017). This could be further shortened to the Δh (difference in enthalpy) between the two components:

$$\dot{E}_{12} = \dot{Q}_{c1c2} = \dot{m}\Delta h_{c1c2} \quad (5)$$

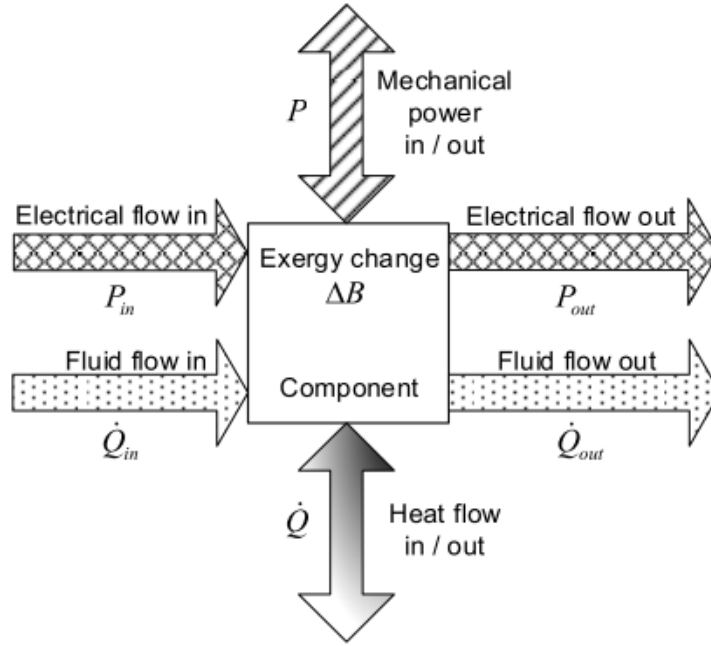
It is important to note that the difference in enthalpy across a component or system can also be expressed as:

$$h_2 - h_1 = C_p(T_2 - T_1) \quad (6)$$

With C_p being the specific heat of the fluid at constant temperature, T_2 being the final or outgoing temperature of the fluid, and T_1 being the initial or in flow, depending on whether a component or system is being measured (du Rand, van Schoor and Nieuwoudt, 2009).

In a multi-domain system, as depicted in Figure 2.2, the same principle applies; however, exergy is also incorporated into the analysis. Figure 2.2 illustrates a multi-domain system depicting various flows, including mechanical power, fluid, heat, and electrical energy, all of which are accompanied by an associated energy flow (van Schoor and Uren K.R., 2018).

Figure 2.2: Energy flow through a multi-domain system



Source: (van Schoor and Uren K.R., 2018)

In Figure 2.2, it is evident that the actions of every domain could change the energy in the component along with the exergy. Both the energy and exergy can be measured, and when exergy is measurable, it is typically possible to measure the entropy and enthalpy of every domain in a system as well. However, if the heat (or energy) exchange of other domains in the system cannot be measured, for example, the mechanical power, exergy should be used to determine the energy flow over a component. The change in exergy can be described as follows (van Schoor and Uren K.R., 2018):

$$\Delta \dot{B} = \Delta \dot{B}_p + \Delta \dot{B}_c = \pm \dot{W} + \dot{B}_{destroyed} \quad (7)$$

ΔB_p represents the physical exergy, and ΔB_c the chemical exergy. $\dot{W}$ signifies the net energy flow rate and is either equal to the mechanical power of the component or the net heat flow rate $\dot{Q}$. Additionally, $\dot{B}_{destroyed}$ refers to the exergy that has been destroyed across the component (Vosloo *et al.*, 2020). The formulas for the physical (ΔB_p) and chemical exergies (ΔB_c) are:

$$\Delta B_p = \dot{m}(h_{in} - h_{out}) - \dot{m}T_0(s_{in} - s_{out}) \quad (8)$$

$$\Delta B_c = \dot{m}(\sum_i x_i b'_{c,i})_{in} - \dot{m}(\sum_i x_i b'_{c,i})_{out} \quad (9)$$

With T_0 either the reference temperature of the system, the initial temperature, or the atmospheric temperature, depending on how the system is set up. The entropy values entering and exiting the component are represented by s_{in} and s_{out} respectively, while the enthalpy values for the entering and exiting states are denoted by h_{in} and h_{out} respectively. The fraction instance of the component given by x_i and $b'_{c,i}$ depicts the chemical exergy of the component. Therefore, the following can be derived from equations (7), (8) and (9):

$$\pm \dot{W} + \dot{B}_{destroyed} = \dot{m}(h_{in} - h_{out}) - \dot{m}T_0(s_{in} - s_{out}) + \dot{m}(\sum_i x_i b'_{c,i})_{in} - \dot{m}(\sum_i x_i b'_{c,i})_{out} \quad (10)$$

In terms of equation (1), it can be derived that:

$$\dot{W} = \dot{m}(h_{in} - h_{out}) \quad (11)$$

However, the preceding is used for simple domain components. Consequently, there are new domains to consider along with the aspect of exergy destruction. From the previous components, it can be derived that:

$$\dot{W} = \dot{m}(h_{in} - h_{out}) + \dot{m}(\sum_i x_i b'_{c,i})_{in} - \dot{m}(\sum_i x_i b'_{c,i})_{out} \quad (12)$$

$$\dot{B}_{destroyed} = - \dot{m}T_0(s_{in} - s_{out}) \quad (13)$$

Although the preceding pertains to a more complex multi-domain system, the equation from (4) remains valid, allowing for the derivation of the following expression:

$$\dot{Q}_{in} = \dot{m}(h_{in}) + \dot{m}(\sum_i x_i b'_{c,i})_{in} \quad (14)$$

$$\dot{Q}_{out} = \dot{m}(h_{out}) + \dot{m}(\sum_i x_i b'_{c,i})_{out} \quad (15)$$

It is worth noting that for most systems and components, especially fluid systems or systems that can be treated as a system or a component with flow, equations (4), (5), and (6) will be sufficient for analysis and modelling. The equation for exergy then becomes:

$$\Delta \dot{B} = \dot{m}(h_{in} - h_{out}) - \dot{m}T_0(s_{in} - s_{out}) \quad (16)$$

If the different energy flows between components are known, the system can be drawn up in terms of these flows (Marais, van Schoor and Uren, 2015; Uren, van Schoor and Auret, 2019). Three previous methods of energy-based visualisation are explained in the following section, in which systems are modelled using energy.

2.3 Previous work in energy graph-based visualisation

Previous methods utilised for analysing and mapping engineering systems based on energy will be discussed, as these methods form the foundation of this study. By building upon these established methods and techniques, significant time and effort can be conserved compared to initiating the analysis from the beginning.

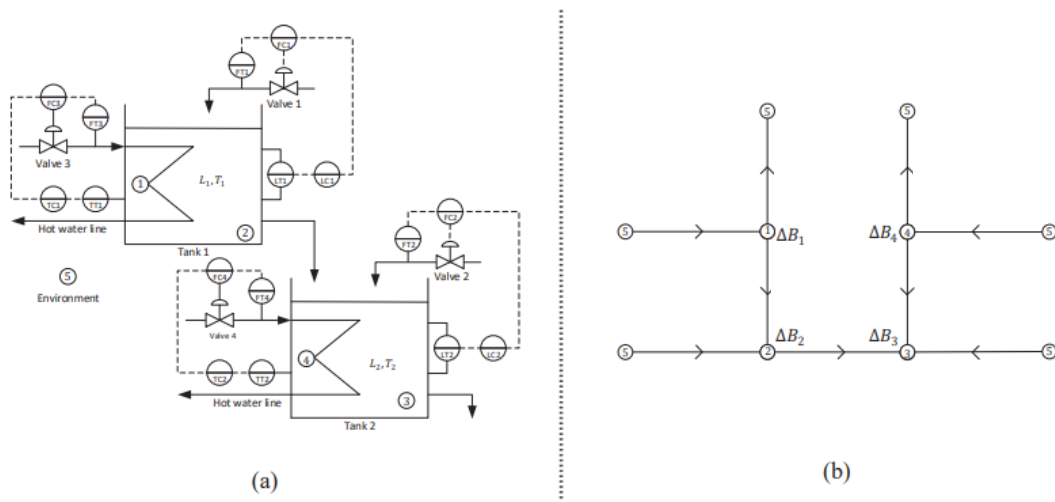
2.3.1 The energy attributed graph

The first step in modelling a system through energy or energy-based visualisation is creating an attributed graph pertaining to the information. A system can be deconstructed into individual nodes, which can be represented in an attributed graph that illustrates the energy flow between and through components. Therefore, the system is described as a series of energy movements between points in space, regardless of what occurs at these points. All points or nodes can represent either the components within a system or the area between the components.

For the purpose of the two-tank system, a diagram of the system is depicted in Figure 2.3 (a) and the energy attributed graph of the system is elucidated in Figure 2.3 (b) (Uren, van Schoor and Auret, 2019). For the two-tank system, there are five primary nodes which are:

1. Water pipe that transfers heat to tank 1
2. Tank 1
3. Water pipe that transfers heat to tank 2
4. Tank 2
5. The environment

Figure 2.3: (a) Diagram of the two-tank system. (b) The energy attributed graph of the two-tank system



Source: (Uren, van Schoor and Auret, 2019)

Where $\dot{Q}_{xy}$ is the energy transfer rate between nodes, x denotes the number of the origin node and y the destination node number. Due to the energy transfers taking place between nodes, there is a change in exergy taking place, which is denoted by ΔB_i , which is defined as:

$$\Delta \dot{B} = \dot{B}_{out} - \dot{B}_{in} \quad (17)$$

By utilising the nodes and the equation, a matrix N_s depicting the energy flow within the system can be established, referred to as a general node signature matrix. This matrix illustrates both the nodes and the links between the nodes, as illustrated in matrix (18):

$$N_s = \left[\begin{array}{c|ccccc} \Delta \dot{B}_1 & \dot{E}_{11} & \dot{E}_{12} & \dot{E}_{13} & \dot{E}_{14} & \dot{E}_{15} \\ \Delta \dot{B}_2 & \dot{E}_{21} & \dot{E}_{22} & \dot{E}_{23} & \dot{E}_{24} & \dot{E}_{25} \\ \Delta \dot{B}_3 & \dot{E}_{31} & \dot{E}_{32} & \dot{E}_{33} & \dot{E}_{34} & \dot{E}_{35} \\ \Delta \dot{B}_4 & \dot{E}_{41} & \dot{E}_{42} & \dot{E}_{43} & \dot{E}_{44} & \dot{E}_{45} \\ \Delta \dot{B}_5 & \dot{E}_{51} & \dot{E}_{52} & \dot{E}_{53} & \dot{E}_{54} & \dot{E}_{55} \end{array} \right] \quad (18)$$

For the purpose of the two-tank system, the specific matrix appears as follows:

$$N_s = \left[\begin{array}{c|ccccc} \Delta \dot{B}_1 & 0 & \dot{E}_{12} & 0 & 0 & \dot{E}_{15} \\ \Delta \dot{B}_2 & \dot{E}_{21} & 0 & \dot{E}_{23} & 0 & \dot{E}_{25} \\ \Delta \dot{B}_3 & 0 & \dot{E}_{32} & 0 & \dot{E}_{34} & \dot{E}_{35} \\ \Delta \dot{B}_4 & 0 & 0 & \dot{E}_{43} & 0 & \dot{E}_{45} \\ \Delta \dot{B}_5 & \dot{E}_{51} & \dot{E}_{52} & \dot{E}_{53} & \dot{E}_{54} & 0 \end{array} \right] \quad (19)$$

The nodes and links that are not used in the system appear as zero in the matrix, with the nodes running along the diagonal from one to five. It can be derived from the preceding matrix that the convention and direction of energy should be considered, changing the matrix to:

$$N_s = \left[\begin{array}{c|ccccc} \Delta \dot{B}_1 & 0 & \dot{E}_{12} & 0 & 0 & \dot{E}_{15} \\ \Delta \dot{B}_2 & -\dot{E}_{21} & 0 & \dot{E}_{23} & 0 & -\dot{E}_{25} \\ \Delta \dot{B}_3 & 0 & -\dot{E}_{32} & 0 & -\dot{E}_{34} & \dot{E}_{35} \\ \Delta \dot{B}_4 & 0 & 0 & \dot{E}_{43} & 0 & \dot{E}_{45} \\ \Delta \dot{B}_5 & \dot{E}_{51} & \dot{E}_{52} & \dot{E}_{53} & \dot{E}_{54} & 0 \end{array} \right] \quad (20)$$

In order to complete the graph matching process, two different N_s matrices are compared, namely, a normal signature matrix $N_{s,n}$ and a fault signature matrix $N_{s,f}$. The Heterogeneous Euclidean-Overlap Metric (HEOM) (van Schoor and Uren K.R., 2018) is used to quantify the difference between the $N_{s,n}$ and the $N_{s,f}$. This method is proposed by (Jouili and Tabbone, (2009) and applied by (Uren, van Schoor and Auret, (2019) on the two-tank heated system described in this study.

The HEOM metric is defined by:

$$HEOM(N_{s,n}, N_{s,f}) = \sqrt{\sum_{a=1}^k \delta(N_{s,n}(i, a), N_{s,f}(j, a))^2} \quad (21)$$

$N_{s,n}(i, a)$ is the (i, a) entry in the normal signature matrix and $N_{s,f}(j, a)$ is the (j, a) entry in the fault signature matrix. Further to this, a is the a – th column of the rows in the matrix and k is the length of the row (van Graan, van Schoor and Uren, 2017). Subsequently, δ is a function defined by:

$$\delta(N_{s,n}(i, a), N_{s,f}(j, a)) = \frac{|N_{s,n}(i, a) - N_{s,f}(j, a)|}{range_a} \quad (22)$$

Furthermore, $range_a$ is defined by:

$$range_a = max_a - min_a \quad (23)$$

This implies that the HEOM compares each row vector of the normal signature matrix with the corresponding row vector in the fault signature node (van Schoor and Uren K.R., 2018). After the HEOM has been calculated, a cost matrix $\mathbf{C}$ can be developed, which will have the same number of dimensions as nodes in the system, signifying that the matrix will be square:

$$\mathcal{C}(N_{s,n}, N_{s,f}) = [HEOM(N_{s,n}, N_{s,f})] \quad (24)$$

If several faults, denoted as k , are to be considered, the cost matrix will possess the same number of eigenvalues (λ_{fk}) with (λ_{f1}) corresponding to the entry $\mathcal{C}(N_{s,n}, N_{s,f1})$ related to fault 1. Normal eigenvalues λ_n for the normal signature matrices, can also be established. Subsequently, the normal and fault eigenvalues can be compared to generate a qualitative fault matrix $\mathbf{Q}_F$ (Uren, van Schoor and Auret, 2019):

$$Q_F(i, j) = \begin{cases} + & \text{if } \lambda_f > \lambda_n \\ - & \text{if } \lambda_f < \lambda_n \\ 0 & \text{if } \lambda_f = \lambda_n \end{cases} \quad (25)$$

Whenever a set of fault eigenvalues differs from the corresponding set of normal eigenvalues, a fault is detectable. If the set of fault eigenvalues is unique from other sets of fault eigenvalues, it is isolable.

2.3.2 Enthalpy-entropy graph approach

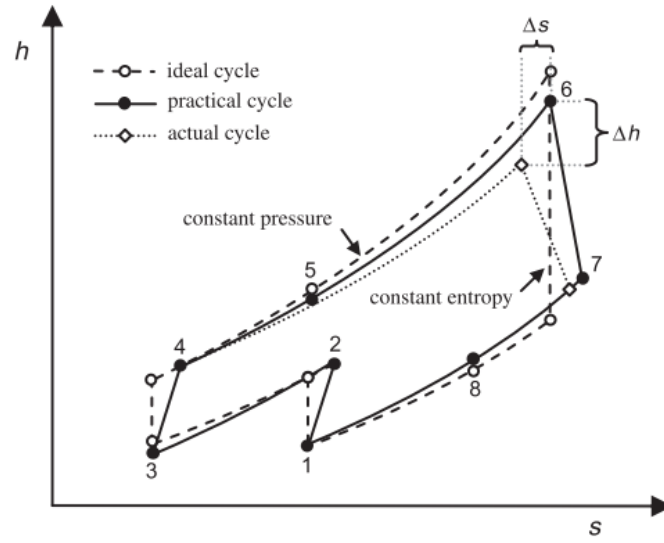
The method used in this section is mainly derived from a research study from (Du Rand and Van Schoor, 2012a). An enthalpy-entropy graph is generated to derive fault patterns and classify faults based on a fault classification index for a generation IV nuclear High Temperature Gas-Cooled Reactor (HTGR). This is depicted in Figure 2.4 (Du Rand and van Schoor, 2012a) illustrating the difference between the ideal, practical, and actual cycles, creating differences in enthalpy (Δh) and entropy (Δs). These residuals are calculated as follows:

$$\Delta h = h_2 - h_1 = c_p(T_2 - T_1) \quad (26)$$

$$\Delta s = s_2 - s_1 = c_p \ln \ln \left(\frac{T_2}{T_1} \right) - R * \ln \left(\frac{P_2}{P_1} \right) \quad (27)$$

Respectively, c_p , T , R , and P represent the constant pressure specific heat, temperature, gas constant, and pressure:

Figure 2.4: Visualisation of the h-s graph approach



Source: (Du Rand and van Schoor, 2012a)

In order to generate the fault signatures for the system, the residuals or errors between the actual system ($S_{actual}(i)$) and the ideal/reference ($S_{ref}(i)$) are used to create fault signature f_s using the following:

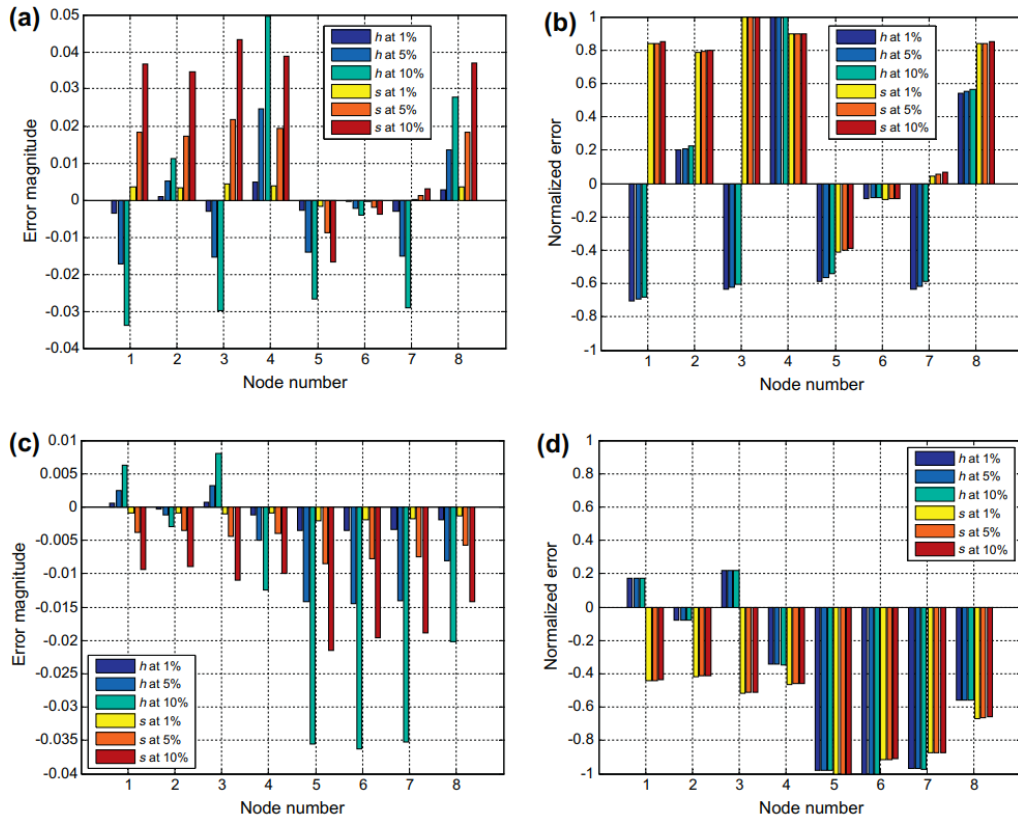
$$f_{s(i)} = \frac{S_{ref}(i) - S_{actual}(i)}{S_{ref}(i)} \quad i = 1, 2, \dots, n \quad (28)$$

For the enthalpy fault values, f_{s_h} , the values are similarly generated using the enthalpy actual and ideal values. The magnitude of the fault increases and decreases according to the manner in which the system functions, as illustrated in Figure 2.5 for faults 11 and 24 (Du Rand and van Schoor, 2012a). Therefore, to generate signatures that are independent of fault magnitude, the signatures should be normalised ($f_{s_{norm}}$):

$$f_{s_{norm}}(i) = \frac{f_s(i)}{(\max_{i=1}^n |f_s(i)|)} \quad i = 1, 2, \dots, n \quad (29)$$

Where $\max |f_s(i)|$ is the maximum value for all the nodes. The results are elucidated in Figure 2.5 and illustrate that there is a strong correlation between the normalised signatures, fault trajectory, and magnitude. It can, therefore, be concluded that one normalised h and s reference signature is required to describe the fault symptoms of the system, which reduces the fault database significantly (Du Rand and van Schoor, 2012a).

Figure 2.5: Fault signatures of two faults. Fault 11: (a) $F_{s,s}$ and $f_{s,h}$, (b) normalised fault; fault 24: (c) $f_{s,s}$ and $f_{s,h}$, (d) for normalised faults



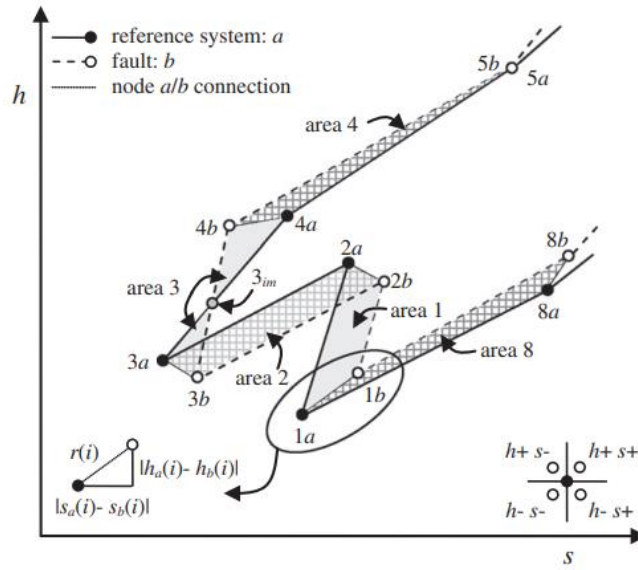
Source: (Du Rand and van Schoor, 2012a)

2.3.3 The area error enthalpy-entropy graph approach

In light of the preceding, the second part of the paper is used as a foundation for this study (Du Rand and van Schoor, 2012b). An area error method is proposed that, as the methods mentioned previously, attempts to generate fault signatures by using enthalpy-entropy graphs or the energy of a system.

In this method, two consecutive nodes (i) and ($i + 1$) are used to create an area between the trajectories and residual shifts of the ideal/reference graph and the actual/fault graph. This is presented in Figure 2.6, where the reference and actual h - s graphs are shown.

Figure 2.6: Illustration of the area error method and definition of the residual areas between ideal and actual h-s graphs



Source: (Du Rand and van Schoor, 2012b)

Considering the preceding, a and b are considered as two nodes for which the distance between the two are given as:

$$r(i) = \sqrt{|h_a(i) - h_b(i)|^2 + |s_a(i) - s_b(i)|^2} \quad (30)$$

Where h_a, h_b, s_a, s_b represent the enthalpy and entropy of the i -th reference and fault nodes, respectively. The residual areas may then be characterised according to the differences as depicted on the graph as follows:

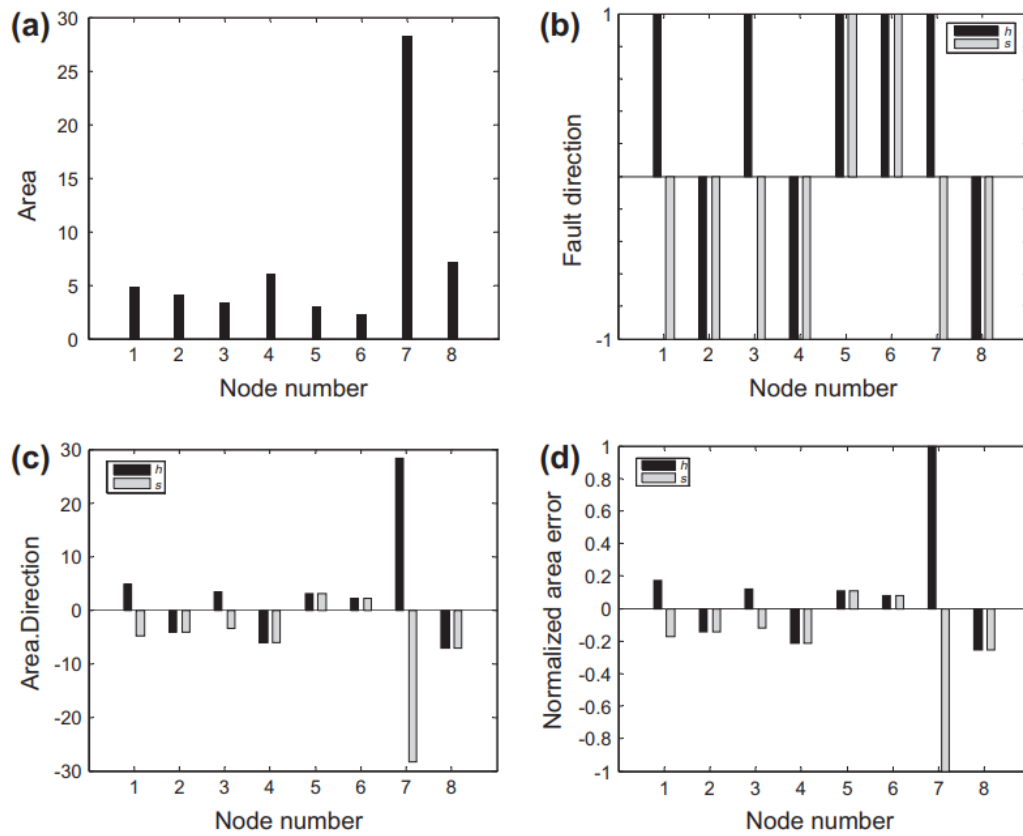
1. Area 1 in Figure 1.4: the area is described by an irregular quadrilateral when the fault nodes (i) and $(i + 1)$ shift in the same direction.
2. Area 4 in Figure 1.4: the area is described as triangular when only one fault node shifts.
3. Area 3 in Figure 1.4: the total area is described as the sum of the two triangular areas that result when two fault nodes (i) and $(i + 1)$ shift in opposite directions from each other.

Following this, the area fault signatures can be calculated using the residual area (i) and the direction of the fault trajectories h and s at node (i) :

$$f_{s_{area}}(i) = \left[\sum_1^n \left(\sqrt{S_p(S_p - x)(S_p - y)(S_p - z)} \right) \right] \cdot dir(i) \quad (31)$$

Where x, y , and z are calculated with equation (14), calculating the side lengths of the triangles while S_p is the semi-perimeter of each triangular area. Furthermore, $dir(i)$ is the direction of the fault trajectory at node (i) for h and s , while n is the number of triangular areas for node (i) . As mentioned earlier in the enthalpy-entropy graph approach, $f_{s_{area}}(i)$ is normalised to obtain $f_{s_{norm}}(i)$. The area error method is elucidated in Figure 2.7, where (c) is the independent h and s fault signatures found by multiplying Figure 2.7 (a) and (b), and (d) is the normalised area error (Du Rand and van Schoor, 2012b).

Figure 2.7: (a) Area of the h-s residuals, (b) individual fault directions, (c) independent fault h and s fault signatures, (d) normalised h and s area error fault signatures



Source: (Du Rand and van Schoor, 2012b)

2.4 Envelope concept

2.4.1 Boundary conditions

To simplify the EOEE concept, the concept will be considered as a set of operating conditions that fall into a certain area due to desirable characteristics. This area may be defined by the characteristics of the system in operation and could differ from the other operating conditions not in the EOEE. Therefore, boundary conditions will separate the operating conditions from those in the EOEE and those not in the EOEE.

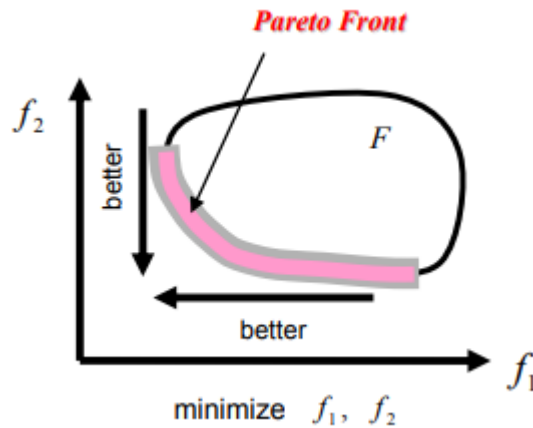
According to the Merriam-Webster dictionary ('Merriam-Webster. (n.d.). Boundary condition', 2024) a boundary condition can be defined as "*a condition which a quantity that varies throughout a given space or enclosure must fulfil at every point on the boundary of that space, especially when the velocity of a fluid at any point on the wall of a rigid conduit is necessarily parallel to the wall*". Furthermore, (Busse, Kach and Wagner, (2017) describes boundary conditions as a theory's generalisability and accuracy of its predictions, given the correct context.

For the purpose of this study, the boundary conditions will make out the borders of the EOEE, or the envelope itself. In a graph, this represents the boundary at which points within the EOEE are distinguished from the points outside the boundary. These boundary conditions that will determine the EOEE will be discussed in Chapter 4.

2.4.2 Pareto Front

According to (Ngatchou, Zarei and El-Sharkawi, (2005), the Pareto Front is a form of multi-objective optimisation that generates a graph where all the optimised solutions appear on a single front. A system with two performance measures (objectives) will be considered for the purpose of this study. For this, f_1 and f_2 will be the two measures that would be ideal to optimise a system. All operating conditions of the system can be expressed in relation to these two measures, allowing for the graphical representation of the impact of varying operating conditions on these measures. On this graph, a line or front will be plotted to represent the optimal or most efficient operating conditions. Considering Figure 2.8, the form of the Front will depend on the direction of what is considered optimal:

Figure 2.8: Pareto Front illustration

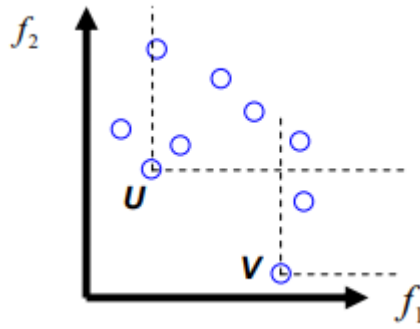


Source: (Ngatchou, Zarei and El-Sharkawi, 2005)

Consider that the preceding f_1 represents cost. Therefore, proximity to the origin indicates a more cost-effective and optimal system operation. Furthermore, let f_2 be defined as $1/\text{efficiency}$, implying that a closer position to the origin also corresponds to high efficiency, representing improved system operations. Therefore, theoretically, the optimal position to operate the system would be as close to the system as possible, as depicted by the pink line in Figure 2.8. Considering this Figure, all operating points are within F ; however, the most optimal operating points are indicated by the pink line, the Pareto front.

A simple method to distinguish what points on the Pareto Front are from those what are not, is by drawing vertical and horizontal lines from a given point away from the origin, as illustrated in Figure 2.9. If any point lies within the region defined by the two lines, it is not on the Pareto Front and can be eliminated. This process can be repeated until the points closest to the origin remain, which do not fall within both lines (Ngatchou, Zarei and El-Sharkawi, 2005).

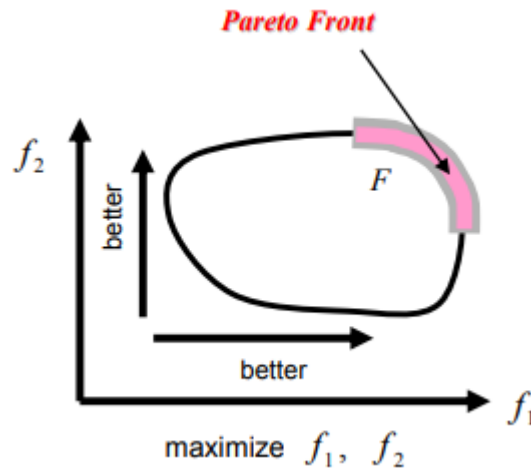
Figure 2.9: Pareto Front elimination process



Source: (Ngatchou, Zarei and El-Sharkawi, 2005)

Considering that the hierarchy of objectives preceding the Front can also be inverted. For instance, if f_1 is defined as $1/\text{cost}$ and f_2 as efficiency, the graph can be represented as depicted in figure 2.10.

Figure 2.10: Reversed Pareto Front



Source: (Ngatchou, Zarei and El-Sharkawi, 2005)

This section discussed the basic concept of the Pareto Front. This approach serves as a potential method for establishing the boundaries of the EOE.

2.5 Measuring energy flow in components

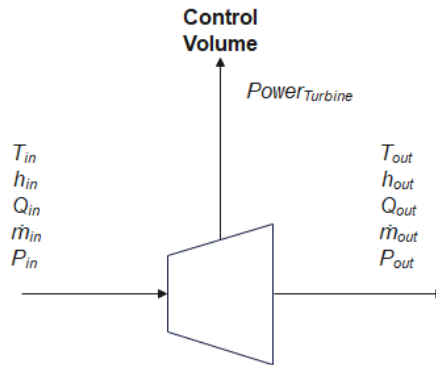
2.5.1 Control volume

In addition to an energy balance, modelling a system through energy requires a clear definition of the system's boundaries, specifying what is inside or outside these boundaries, as well as identifying the system's control volume. The control volume separates the system from the broader environment and can change based on the needs of running the system

(Sonin, 2003). While the entire system can be a single control volume (e.g., a Brayton cycle gas turbine), the system can comprise individual components (e.g., a Compressor), as depicted in Figure 2.11.

By establishing the control volume, it is possible to determine what energy moves into and out of the system or component. This enables the formation of an energy balance, as discussed previously. Following this, the energy graph-based visualisation or any other form of modelling or analysis of the system can be compiled as desired. From the preceding, the following component can be considered:

Figure 2.11: Illustration of a component's control volume



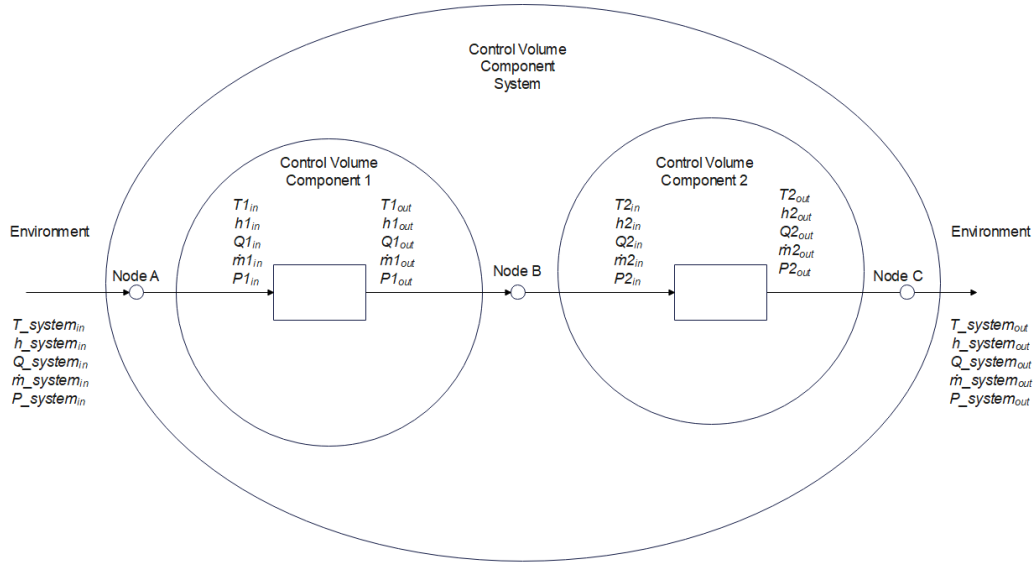
Source: Own composition

All previous equations for energy balance hold true unless there is a change in elevation, as this change would change the mass flow. If no change is prevalent in elevation within the component itself, the following holds true (Uren, van Schoor and Auret, 2019):

$$\dot{m}_{in} = \dot{m}_{out} \quad (32)$$

The heat or energy transfer between components is given in equations (4) and (5). Furthermore, consider Figure 2.12 as illustrating two components. Each component possesses its own control volume while simultaneously sharing a common control volume that represents the total system. (Sonin, 2003) indicates that a control volume should be closed, though it can be connected to other control volumes.

Figure 2.12: Control Volume of a multi-component system



Source: Own Composition

When considering the system, the energy flows from beginning to end, indicating that it changes from $Q_{system_{in}}$ to $Q_{system_{out}}$. This is given by the equation:

$$\dot{W}_{System} = \dot{m}_{system} (h_{system_{in}} - h_{system_{out}}) \quad (33)$$

It is prevalent that this is essentially the same as equation (11), therefore, the system can be perceived as a component if all the required variables are known. A deeper analysis of the system is necessary to identify faults or degradation that may have occurred, particularly if the system exhibits changes in behaviour and/or declines in efficiency.

Previous equations in energy flow provided the change or flow of energy over the components. However, questions still arise regarding the energy flowing between components. While this equation differs, it can be calculated using the same principles. If the enthalpy difference is known, the energy flow for node B, or the energy flow between the two components, will be:

$$\dot{E}_B = Energy_{flow} = \dot{m}_B \Delta H_B \quad (34)$$

As node B is a single point and shares the same mass flow, it is possible that it will not have an enthalpy difference in some cases, but rather a single enthalpy value, which will simplify the equation to:

$$E_B = Energy\ Transfer = \dot{m} H_{node\ B} \quad (35)$$

2.5.2 Mapping individual components

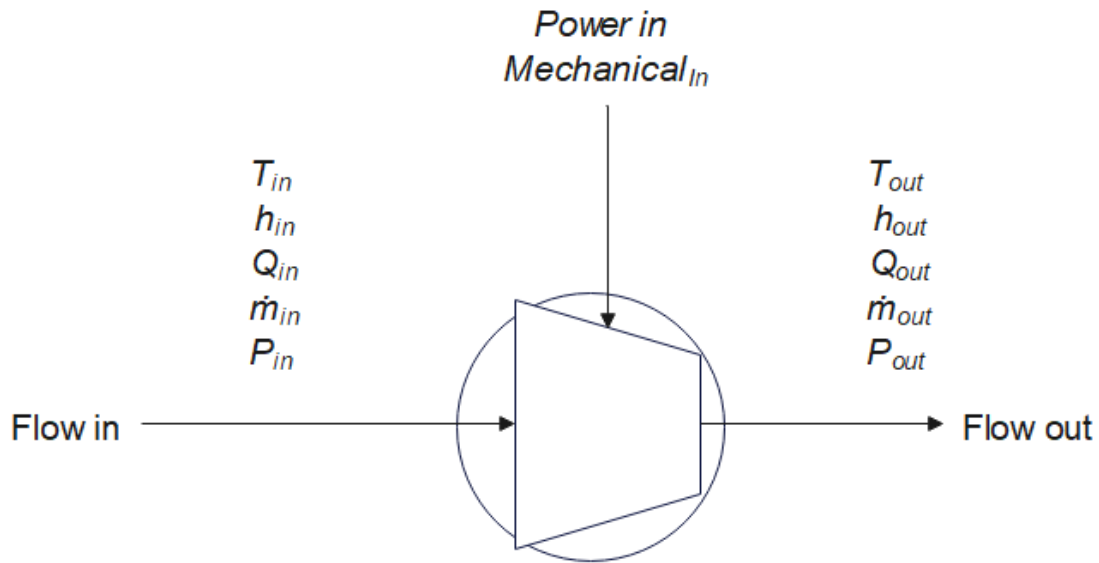
When the enthalpy or change in enthalpy and mass flow over a component is known, it is simple to calculate the energy flow through individual components as well as the energy flow through a system. Different components cause different changes to the enthalpy of a system and, as a result, change the energy flow of a system. This also causes different components to have their own way of measuring the energy flowing through them (Demirel, 2002).

2.5.2.1 Compressors

The function of a compressor is to increase the pressure of a fluid within a system, consequently increasing the enthalpy of the fluid. In terms of energy flow, this implies that the compressor adds energy to the fluid. Consequently, in an energy modelled system, the compressor adds energy to the system. There are two types of compressors, namely, axial and centrifugal. However, for the purpose of this study, the differences between the two are disregarded, as the energy balance will stay consistent.

Compressors, as illustrated in Figure 2.13, utilise characteristic curves to describe the behaviour of the fluid undergoing compression. To model a compressor in terms of energy flow the following equations are required.

Figure 2.13: Flow through a compressor



Source: Own Composition

Considering ideal circumstances, the energy balance is:

$$\dot{m}h_{in} = \dot{m}h_{out} \quad (36)$$

From the preceding Figure, it is evident that the mass flow entering is equal to the mass flow exiting the compressor. This should be true for the majority of compressors when considering an incompressible fluid. The work performed by the compressor, a portion of

which contributes to the energy added to the fluid for compression, is represented as $\dot{Q}$. Therefore, the energy flow of a compressor can be expressed as:

$$\dot{Q} = \dot{m}(h_{in} - h_{out}) \quad (37)$$

As the compressor adds energy to the fluid or system, $\dot{Q}$ will be negative. However, if calculated in terms of the flow, $\dot{Q}$ will be positive as it adds energy. The energy flow $\dot{Q}$ can depict a point in the N_s matrix:

$$\dot{Q} = \dot{E}_{12} = \dot{m}(h_{out} - h_{in}) \quad (38)$$

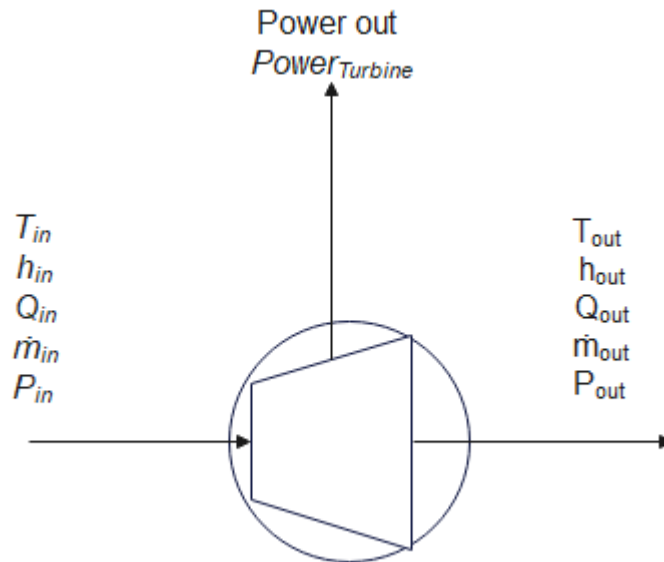
The pressure ratio for a compressor can be calculated as follows:

$$PR = \frac{P_{out}}{P_{in}} \quad (39)$$

2.5.2.2 Turbines

A turbine, as illustrated in Figure 2.14, is a component that uses the flow of the working fluid of a system to generate power for an electrical grid. In terms of energy flow, turbines take the energy flow out of the system and transform it from kinetic flow energy to more usable electrical energy. This indicates that the enthalpy change across a turbine will be negative, resulting in the energy balance as depicted in Figure 2.14.

Figure 2.14: Flow through a turbine



Source: Own Composition

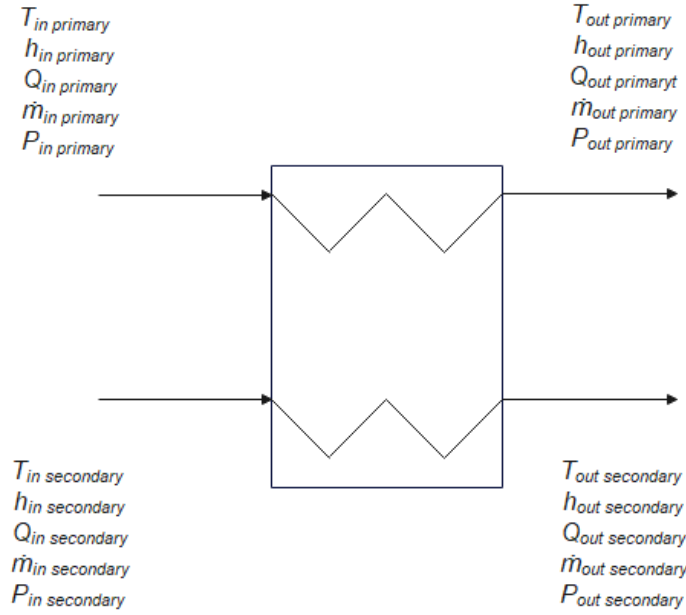
The work performed by the turbine, or the energy extracted from the system, is determined by the turbine's efficiency. The efficiency, η_T , determines the amount of energy remaining after accounting for the sum of the enthalpy difference and mass flow. The power produced by the turbine is:

$$\dot{Q}_{Turbine} = \dot{m} \eta_T (h_{out} - h_{in}) \quad (40)$$

2.5.2.3 Heat exchangers

A Heat Exchanger (HX) can be described as a device that transfers heat between two or more sources. HXs can transfer heat even if different mediums are used, for example, between two fluids or a fluid and another source. A basic HX consists of a primary and a secondary side, where the primary side transfers heat to the secondary side, as depicted in Figure 2.15.

Figure 2.15: Flow through a heat exchanger



Source: (Mohammed Elmardi Suleiman Khayal and Professor Osama Mohammed Elmardi Suleiman Khayal, 2018)

According to (Mohammed Elmardi Suleiman Khayal and Professor Osama Mohammed Elmardi Suleiman Khayal, 2018), HXs can be placed into five categories, namely:

1. Recuperators and regenerators
2. Direct or indirect contact transfer process
3. Construction geometry
4. Single-phase or two-phase heat transfer mechanism
5. Parallel, counter, or cross-flow arrangements

To model an HX in terms of energy, each flow, primary and secondary, can be considered individually. Therefore, equations (4) and (5) remain valid for each individual flow within the system. When considering the location of the HX, the primary and secondary sides each function as distinct nodes in an energy-based visualisation, with each side exhibiting different flows and properties. If one side is outside the boundaries of the system, for example, a cooling tower, that side is considered part of the environment.

Due to inefficiencies and the effectiveness of an HX, a portion of the energy transferred between the two sides will be lost. The maximum amount of heat transferable depends on

the temperature difference between the initial and final temperatures of the side that gives off heat and the side that receives heat, respectively.

Therefore, the side that loses heat, or lowers in temperature, transfers energy to the side that has an increase in temperature. For the purpose of this study, the primary side will give energy to the secondary side of the HX in the following equations:

$$\dot{Q}_{max} = (\dot{m}C_p)_{min}(T_{in,p} - T_{out,s}) = \dot{Q}_{Primary\ to\ Secondary} \quad (41)$$

The amount of energy the secondary side receives can be expressed as:

$$\dot{Q}_{secondary} = \varepsilon \dot{Q}_{primary} \quad (42)$$

With ε being the effectiveness of the HX. Other losses for HXs can also occur; however, this would likely be due to problems arising while using the HX, for instance, fouling or corrosion. As previously mentioned, by considering the two different sides of the HX as different nodes (or components), the same equations can be applied to model each side in terms of energy, with the exception that one side transfers energy to the other.

2.5.2.4 Pipes or fluid transfer

Pipes(s) that transfer fluid from one component to the next are often overlooked. Pipes can lead to energy losses in a system if not properly insulated, as heat can be transferred to the surrounding environment. However, if the pipe is properly insulated and operates within an ideal simulated environment, it can be assumed that the pipe loses no heat or energy to the environment, the mass flow remains constant, and the entropy (and exergy) remain constant.

2.6 Degradation

Degradation affects any mechanical system. Over time, the system may become less efficient as a result. There are many different mechanisms of degradation, which affect turbines, compressors, and HXs. This section discusses a few examples, in accordance with (Kurz and Brun, 2000; Kurz *et al.*, 2014).

2.6.1 Mechanisms of degradation

2.6.1.1 Corrosion

Corrosion occurs when contaminants in either the inlet air or fuel being used react in the component. This results in the removal of material from the turbine, changing the surface roughness, and further removing the anti-oxidation layer of the turbine, resulting in rust. Corrosion reduces the lifespan and performance of components, including turbines and HXs, when impurities in the air are present or fuel corrosion is more pronounced. This is the reason for fuel side corrosion, as the heavy fuel oils used cause more severe damage.

2.6.1.2 Hot corrosion

Hot corrosion is caused by chemical reactions between certain contaminants in the component and the component itself. However, hot corrosion differs from normal corrosion as hot corrosion takes place at higher temperatures. These contaminants include salts, such as sodium, mineral acids, and gases like hydrogen sulphide. Hot corrosion leads to the deterioration of the component through material loss at the flow path. This can be mitigated by filters, although the filters should be regularly monitored to ensure proper moisture control.

2.6.1.3 Fouling

Fouling occurs when contaminants, such as oils or water mists, accumulate within a component, sticking to the surface and altering its roughness, leading to damage. In extreme cases, the shape of the surface can also be altered, leading to the properties of the components being changed at an increased rate. The contaminants that contribute to fouling are typically very small, measuring as small as 10 µm or even smaller. Due to this, an air filtration system could deter fouling. Further to this, if no permanent damage has occurred, for example, the loss of material on the surface, fouling can be reversed by properly cleaning the inside of the component (Kurz *et al.*, 2014).

2.6.1.4 Erosion

Erosion occurs when the fluid in the system contains foreign objects and passes through components at high speed/volume. This causes the removal of surface material, as the particles are usually larger in size than those in fouling (more than 10 µm).

2.6.1.5 Creep

Creep results from the slow change in the microstructure of the component's material caused by constant high stress and temperature. As the microstructure changes, the material may become weaker or more brittle, resulting in cracks in the microstructure that can lead to further degradation. This process is long and gradual; however, the creep in material could eventually be high enough to cause failure, as it will start to cause cracking. This usually occurs in components such as turbine and compressor blades.

2.6.1.6 Thermal stress

When operating in high temperatures, thermal stress occurs, which creates mechanical damage in components such as cracking.

2.6.1.7 Abrasion

Abrasion is the removal of material caused by the rubbing of a moving, usually rotating, surface on a stationary surface. Although most components are designed to prevent two parts from rubbing against each other during operation, prolonged use of parts, for instance, bearings, can increase clearance due to a loss of stiffness. This can cause a part in the component to move more freely, resulting in abrasion.

2.6.1.8 Foreign objects

Although all contaminants can be considered foreign objects, larger objects can fall into a system, for instance, ice into a compressor or turbine inlet, causing the propeller blades to break. Fuel nozzles often experience build-up of carbon, which can cause damage when fragments break off.

2.6.2 Effects of degradation on gas turbines (Brayton cycles)

Degradation causes performance deterioration in components and systems (such as Brayton cycles), with higher loads resulting in a faster rate of degradation (Najjar, Alalul and Abu-Shamleh, 2020). This is determined by three main effects caused by degradation (Kurz and Brun, 2009):

- Increased tip clearance
- Surface (air foil) changes in geometry
- The change in surface quality

Considering the preceding, changes in surface quality may be reversible as the change is usually due to fouling. Therefore, after cleaning or maintenance the component can return to normal. The other two effects are more permanent. Increased tip clearance reduces the stage work, surge margin, and flow capacity for compressors, while changes in the surface geometry and quality result in a lower work capability of the stage. All three, however, cause the system and individual components to have lower efficiency. Since this effect is the easiest to model and measure, it will be the primary focus of degradation examined in this study (Zwebek and Pilidis, 2003a).

Different types of systems will have different behaviours, for example, single-shaft systems vs. three-shaft systems. Although the trend line in terms of degradation vs. effect will be the same, (Zagorowska, Skourup and Thornhill, 2020) demonstrates that a compressor suffering from degradation causes the component to consume more energy to compensate for its loss of efficiency. This is due to the pressure ratio remaining consistent relative to the flow, and consequently more power will be required as it degrades further.

When assessing the impact of this degradation on the overall system, it will be necessary for the system to compensate for the degradation by changing the operating conditions to achieve a consistent result. As previously mentioned, the stage capability can also change as a result of degradation. This results in a knock-on effect where a change in one stage leads to subsequent stages operating under different conditions than what is considered normal (Kurz and Brun, 2009). As a result, the system is unable to function optimally, as each component has to operate within a new range to maintain the desired result.

For the purposes of this study, it is assumed that degradation in a system or individual component reduces the efficiency of a component. Consequently, this affects the overall operating conditions of the system and every individual component due to changes to the stage. If the desired result is kept constant, the system will have to compensate for the degradation by changing the operating conditions and stages through adjustments to the available variables (Zwebek and Pilidis, 2003b). This includes:

- The energy added to the system
- The mass flow of the working fluid in the system
- The pressure of the system
- The working fluid, if possible

While this is not an exhaustive list, there are limitations to the changes that can be made in a gas turbine system without changing the physical geometry or components of the system.

2.7 Critical review

This literature review provides the necessary information to characterise a system through energy. Aspects of each of the methods given can be used to facilitate the development of the EOE.

To develop the EOE, a system should be characterised by energy (Marais, van Schoor and Uren, 2015; van Schoor and Uren K.R., 2018) and incorporated into a general node signature matrix (N_s) (Uren, van Schoor and Auret, 2019). Matrices representing different states of the system can be compared using the HEOM function. This would ensure concrete values in order to illustrate the differences (du Rand, van Schoor and Nieuwoudt, 2009). The HEOM function yields another matrix, the eigenvalues of which can be utilised for the EOE. However, it is essential to investigate the eigenvalues' suitability or consider alternative methods, such as Singular Value Decomposition (SVD).

The enthalpy-entropy graph approach (Du Rand and van Schoor, 2012a) and the area enthalpy-entropy graph approach (Du Rand and van Schoor, 2012b) provide useful concepts, including the utilisation of enthalpy and entropy for energy-based graph visualisation. However, these specific methods will not be utilised in this study, as these methods are not considered adequate for creating the borders of the EOE. The broader concepts established from these methods are valuable for understanding the principles of energy-based graph visualisation.

The boundary of the EOE will have to be explored further. The use of a Pareto Front (Ngatchou, Zarei and El-Sharkawi, 2005) can be explored for establishing boundaries or facilitating the development of the EOE borders. Similar to the EOE, which has a definitive boundary, the system should be defined by its control volume. This control volume should encompass different components, such as compressors and turbines, facilitating the flow of measurable quantities between the components (Demirel, 2002; Sonin, 2003). Through the use of the control volume, the energy flowing to and from components can be mapped in a general node signature matrix for any system.

To create different operating points for the same system without changing the internal mechanisms will require the implementation of standard mechanisms that naturally occur in engineering systems over time. These mechanisms could include degradation as an example (Kurz and Brun, 2000; Kurz *et al.*, 2014). Specific degradation, or the effect that overall degradation has on a system (Zwebek and Pilidis, 2003a, 2003b), can be used to produce different operating conditions for the same controllable variable or set point in a system (Kurz and Brun, 2009; Najjar, Alalul and Abu-Shamleh, 2020; Zagorowska, Skourup and Thornhill, 2020). These different operating conditions will have different general node signature matrices, which can be compared to one another using the HEOM function.

This provides the basis for the development of the EOE.

Chapter 3 Experimental platform

3.1 Introduction

This chapter investigates the requirements for the system that will serve as an experimental platform. Three different systems will be compared, namely the two-tank system, an autothermal reformer, and a Brayton cycle. The most optimal system suitable for developing the EOEE will be selected based on the requirements. Finally, the selected system will be explored and characterised in terms of EGBV. Consequently, this system may provide all the necessary data for the development of the EOEE.

3.2 Requirements

Before discussing a system, it is essential to establish a list of requirements that the system needs to satisfy. This will ensure a clear framework, facilitating the process of finding a system. The requirements are:

- The system must be a closed loop system, with all variables entering or exiting the system, ensuring that all the energy moving into and out of the system can be accounted for.
- It must be a known system, or a system that has been proven to be workable and gives reliable results. A system utilised in previous research would be ideal.
- The system must be replicable, indicating it should be possible to solve the system efficiently multiple times and reconstruct an identical system from its foundational components.
- The system must have different variables that can be changed without physically changing the system, for example, the mass flow of the system. With these variables, the system can be controlled, allowing for different operating conditions to be tested.
- It must be possible to capture and save the results for analysis.
- It must be possible to simulate faults or degradation in the system and nevertheless obtain accurate answers.
- The system must be able to compensate for any faults or degradation introduced while keeping a stable and constant result.
- Considering the preceding, the final requirement is that the system must produce a definitive result or serve a specific purpose. This could include, for example, delivering heat to another system or house, cooling a facility or producing power to an electrical grid.

FLOWNEX is considered the most suitable software for creating a plausible real-life system and accurately simulating its expected behaviour while capturing and calculating the desired results. Additionally, it allows for the implementation of multiple operating conditions for a single system.

To ensure accurate testing, a system previously used in research could be employed to back up and verify the results. This was discussed in the literature review in Chapter 2.

3.3 Possible systems

To simplify the process, only systems that have been previously used in research on energy-based visualisation will be utilised. This is to ensure that the results are easily verifiable.

3.3.1 Two-tank system

The two-tank system consists of two tanks, with the first tank having an outlet to the second tank. Both these tanks exchange hot water to a separate pipe system, and both have a cold-water supply (Uren, van Schoor and Auret, 2019; Wolmarans, van Schoor and Uren, 2023). This can be used to regulate the temperature of the water in both tanks. The system has been previously modelled for energy flow, and there has been a physical system built for it. This system also meets the requirements as follows:

- While the system is not a closed-loop system, it can be manipulated into one if the cold and hot water supply is included into the system. Consequently, external energy will be required to run the pumps and heaters.
- The system has been previously used in research, notably in terms of energy-based visualisation. Considering this, there is only an outdated model in Simulink.
- The system can be replicated, although it has undergone various alterations. Therefore, it is necessary to verify whether the previous simulations and the physical model remain consistent with each other.
- There are different variables that can be controlled in the heat of the hot water and the mass flow between tanks.
- If the simulations remain correct and the model is equipped with sensors, all variables can be captured.
- Faults can be simulated as demonstrated in previous research.
- It is necessary to test if and how the system compensates for faults or degradation.
- There is no predefined objective; an arbitrary temperature for the tanks can be selected. However, calculating how efficiently the system operates would depend on how the data is interpreted by the user.

The two main challenges with this system include the need for a new FLOWNEX model, which has not yet been verified, making it difficult to pinpoint a result.

3.3.2 Autothermal reformer model

An auto thermal reformer has been used in multiple previous studies. Previous research by (Marais, Schoor and Uren, (2016) modelled an auto thermal reformer in terms of energy. Furthermore, the system was modelled while introducing faults. This provided clear results and differences between the standard solved system and the fault-induced system. Therefore, the system adheres to the following requirements:

- It is not completely closed; however, the nodes leading to the environment can be calculated or measured.
- It has been used in previous research, although FLOWNEX or current model software does not comprise a model to simulate different operating conditions.
- It can be replicated.
- Variables can be changed to control the system.

- The results can be captured and analysed.
- Faults can be introduced, as observed in previous research.
- Before modelling the system, there is no guarantee that it will be able to compensate for a loss in efficiency. However, theoretically, it should be possible if modelled correctly.
- An autothermal reformer produces syngas, which is a measurable goal as it can be measured by the mass flow of gas it produces.

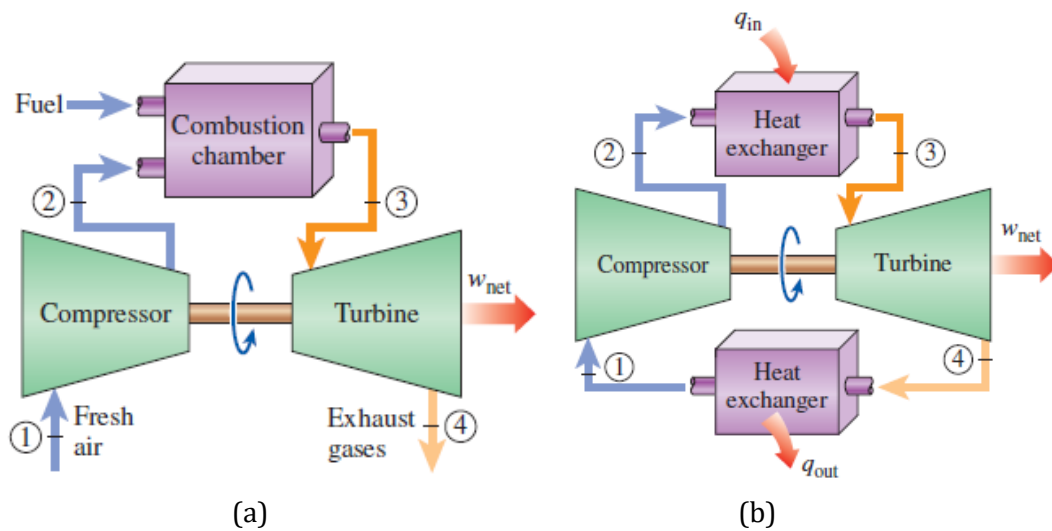
Considering the preceding, this system is a promising candidate; however, a new simulation will have to be set up in FLOWNEX in order for the system to be controlled effectively.

3.3.3 Brayton cycle

There are two types of Brayton cycles, namely open- and closed-loop systems, as illustrated in Figure 3.1 (Çengel, Boles and Kanoğlu Mehmet, 2019). In an open-loop cycle, the working fluid enters the system at one end, typically at a compression system and exits the system at the other end. During the system, heat is added through the combustion of fuel, providing the working fluid with energy to turn a turbine. An example of this includes a gas turbine that uses air as a working fluid, where gas is added to the mixture and then combusted. Subsequently, the combusted mixture is expelled when the turbine is turned.

The working fluid in the closed-loop system is recycled into the compression system, where the heat is then readded. Usually, there is an HX that rejects heat from the working fluid to the environment or recycles it before entering the compressor. By recycling both the working fluid and heat, a more efficient system can be created.

Figure 3.1: Diagram of an (a) open and (b) closed Brayton cycle



Source: (Çengel, Boles and Kanoğlu Mehmet, 2019)

A closed Brayton cycle has previously been modelled in FLOWNEX, and this was used in previous research (Neser, van Schoor and Uren, 2021). This system is known as the Pebble Bed Modular Reactor (PBMR), originally designed by Eskom and its partners (Koster, Matzner and Nicholisi, 2003), comprising a three-shaft Brayton cycle. This system adheres to all the requirements:

- The system has a closed loop, and all energy transfers from outside of the system can be measured and monitored.
- The system has been used in previous research and is an existing system.
- The system can be replicated and, if needed, can be built from scratch.
- The system has multiple variables which can be changed without physically changing the system.
- The results can be saved as it is simulated in FLOWNEX.
- Through the use of features in FLOWNEX, faults and degradation can be simulated.
- Mechanisms exist that make the system compensate for any change in operating conditions to ensure stable and constant results.
- As the system is a Brayton cycle, its primary objective is to create usable electrical energy or give a power output through its Power Turbine (PT). This result is easily measurable as it is already in the form of energy.

The PBMM adheres to all the requirements and has the added benefit of already being modelled in FLOWNEX. Therefore, the system has been tested and no errors can accidentally enter through user error. Therefore, the PBMM will be the system used in this study to build the first EOEE.

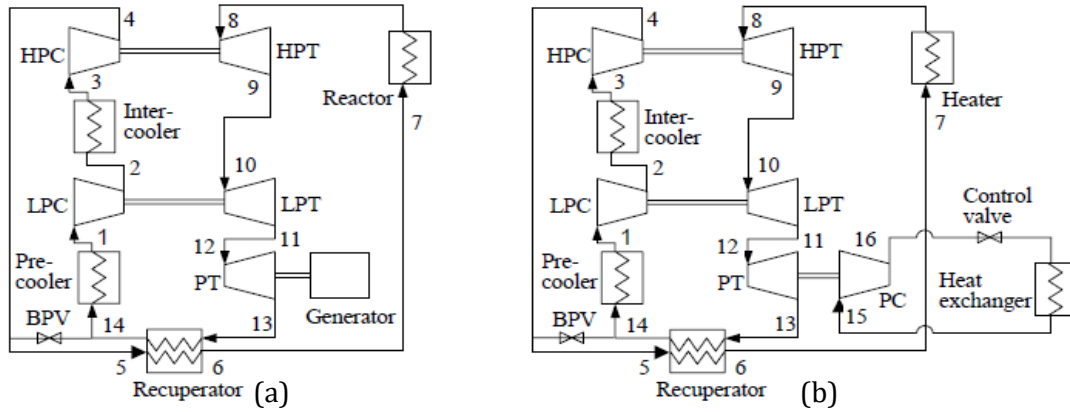
3.3.4 Previous work on modelling the PBMM

Previous attempts to model the PBMM in terms of energy have been conducted by researchers, including (Koster, Matzner and Nicholsi, (2003),; and Naser, van Schoor and Uren, (2021). In these studies, the idea of using energy-based visualisation for FDI in the PBMM has been explored. Furthermore, these studies utilise the eigen decomposition-based method: a residue-based method. Thereafter, the PBMM is modelled using energy and exergy.

3.3.5 The PBMR and PBMM

The PBMM was initially designed in FLOWNET as a model of the Pebble Bed Modular Reactor (PBMR) system. Significant research and development have been conducted on the PBMR, a three-shaft high-temperature gas nuclear reactor. This system utilised a closed recuperative Brayton cycle to generate power and uses Helium as the working fluid (Greyvenstein and Rousseau, 2003; Rousseau, Greyvenstein and Van Niekerk, 2003). The following Figures illustrate the basic design of the PBMR, including Figure 3.2 (a) and the PBMM in Figure 3.2 (b).

Figure 3.2: Diagram of the PBMR (a) and PBMM (b)

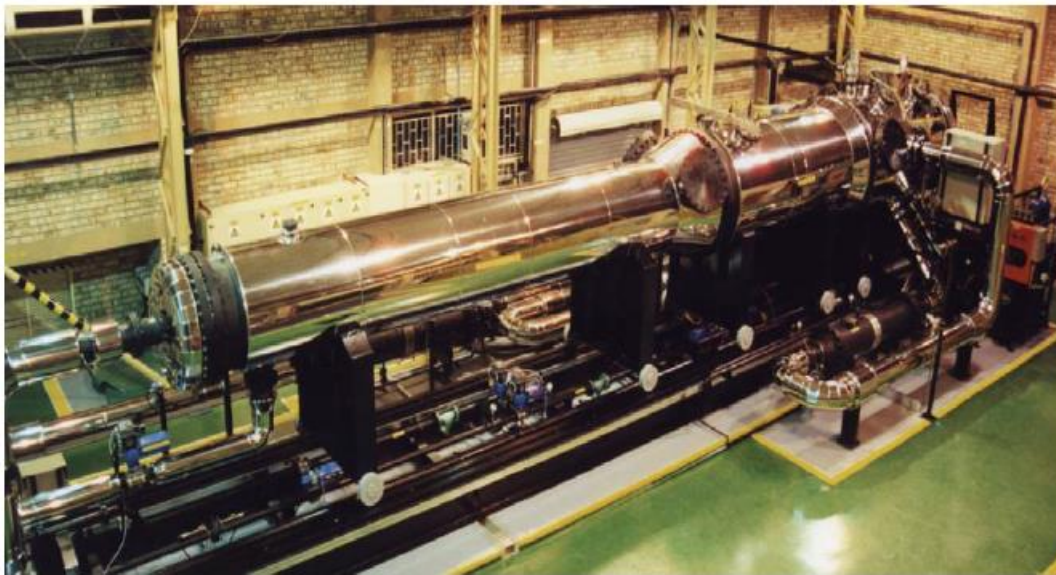


Source: (Greyvenstein and Rousseau, 2003)

The PBMM differs from the PBMR in numerous ways. First, the working fluid, for example, is nitrogen rather than helium. Furthermore, the reactor was switched to an electrical heater (Rousseau and Greyvenstein, 2004), with a maximum temperature of 700°C.

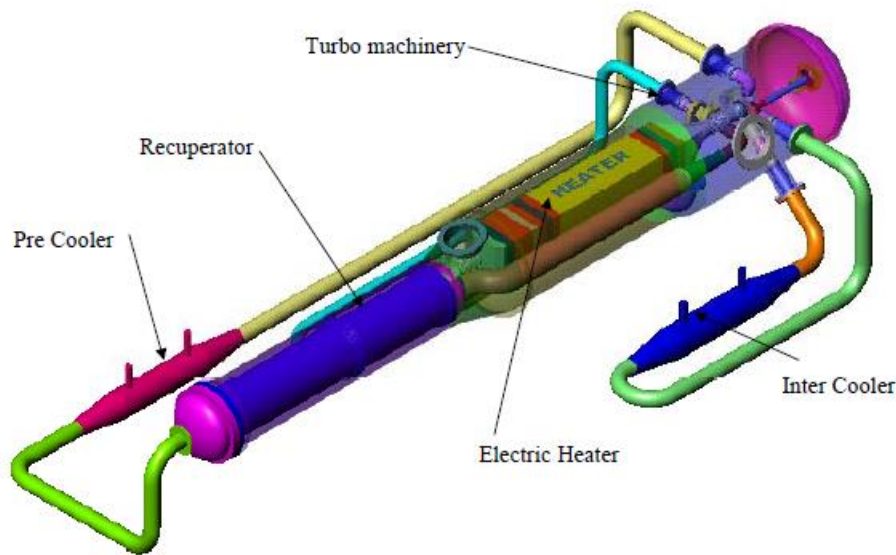
The FLOWNEX model functions as an ideal heat source, and the working fluid has similarly been changed to nitrogen. This model is designed to be a functional model that has similar performance characteristics to gauge how well the PBMR works in practice. It also features the same control mechanics to simulate how it would function in the physical system. FLOWNET has since changed to FLOWNEX. The PBMM model is shown in Figure 3.3, and its layout in Figure 3.4.

Figure 3.3: Photo of the physical PBMM system



Source: (Rousseau and Greyvenstein, 2004)

Figure 3.4: PBMM physical outlay of components



Source: (Van Niekerk *et al.*, 2006)

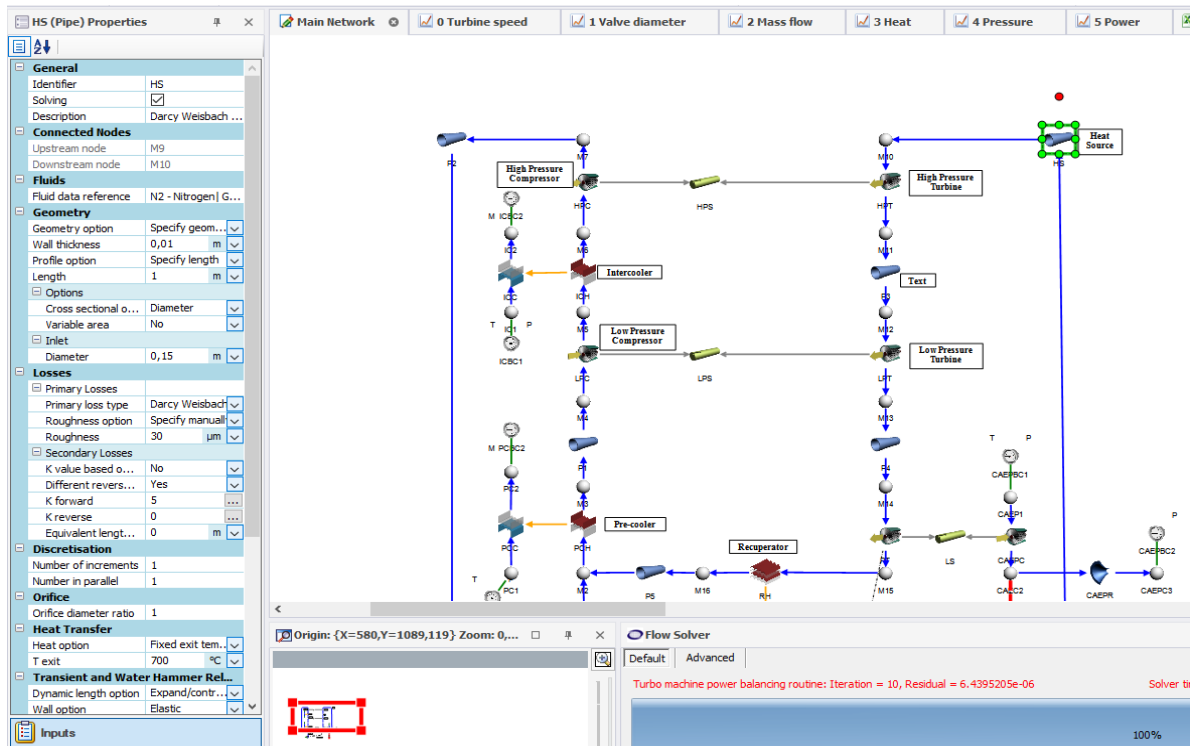
The PBMM simulates the control, operation, and the results of running the system or the steady state answers of the system (Greyvenstein and Rousseau, 2003). The model differs from the original design as the mentioned heat source and the generator are replaced by a compressor in a loop, although this still simulates the generator in the original system. There are nine main components in the system:

- The pre-cooler HX
- The Low-Pressure Compressor (LPC)
- The intercooler HX
- The High-Pressure Compressor (HPC)
- The recuperator
- The heat source, which is modelled as a pipe in FLOWNEX, and therefore functions as an ideal heat source
- The High-Pressure Turbine (HPT)
- The Low-Pressure Turbine (LPT)
- The PT that sends out power to the generator, sending usable electrical power to a grid

The HXs can be further divided into hot and cold sections, which will serve as the basis for modelling the system's energy flow in subsequent analyses. It is prevalent in the FLOWNEX model and the above schematics that a pressure valve is positioned before the pre-cooler, where it injects nitrogen, the working fluid used in the FLOWNEX simulation. As previously mentioned, helium was used in the original system. This valve controls the mass flow in the system and can be used to compensate for the loss of efficiency or degradation.

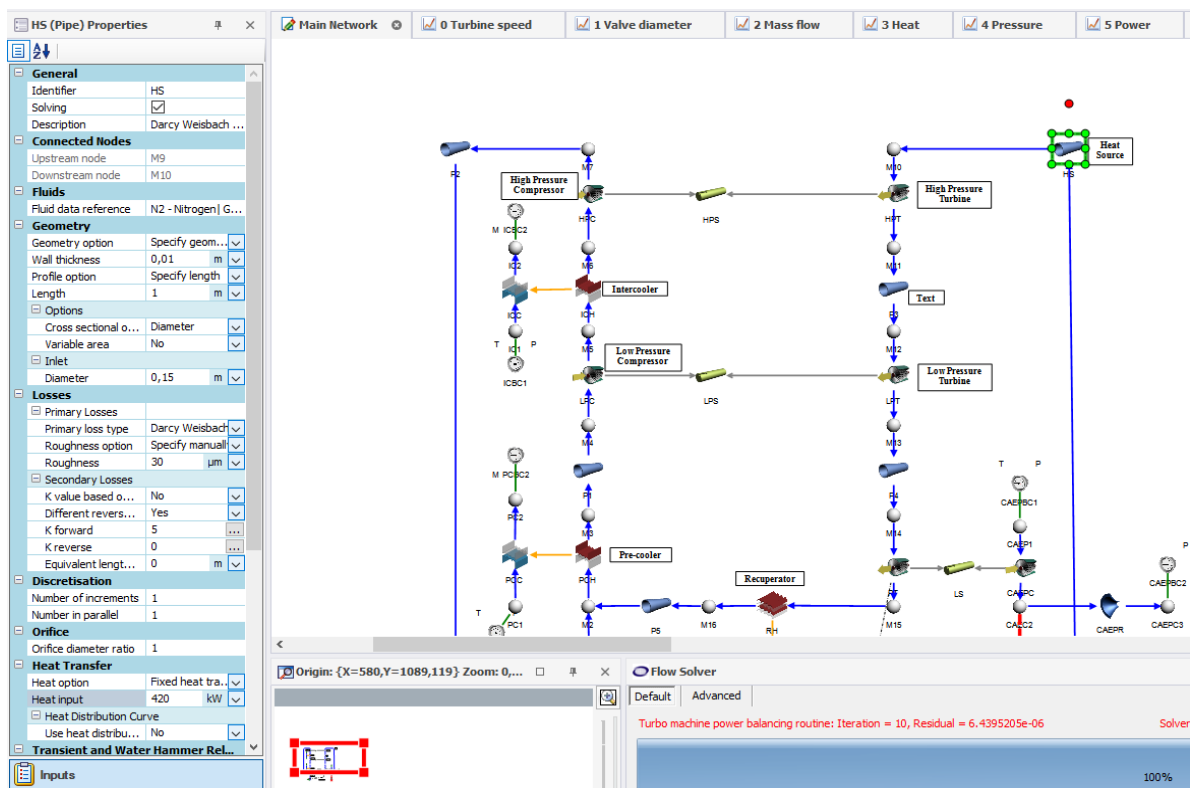
In FLOWNEX, the temperature was specified as 700°C; however, the energy input was omitted. The omission is due to the constraint in FLOWNEX, which only allows a single input. This is illustrated in Figure 3.5, where the exit temperature is used as input and in Figure 3.6, where the added heat is designated as the input. According to (Neser, van Schoor and Uren, (2021), 1078.92 kW enters the system from heat addition. Although this measurement exceeds the 420 kW rating of the physical PBMM for the electrical heater, this is possible when considering the size of the mass flow. According to (Neser, van Schoor and Uren, (2021), the PT provides a 100 kW output and also works at 700°C (or 973.15 K).

Figure 3.5: Heat source showing exit temperature as input



Source: Source: Own Composition using FLOWNEX

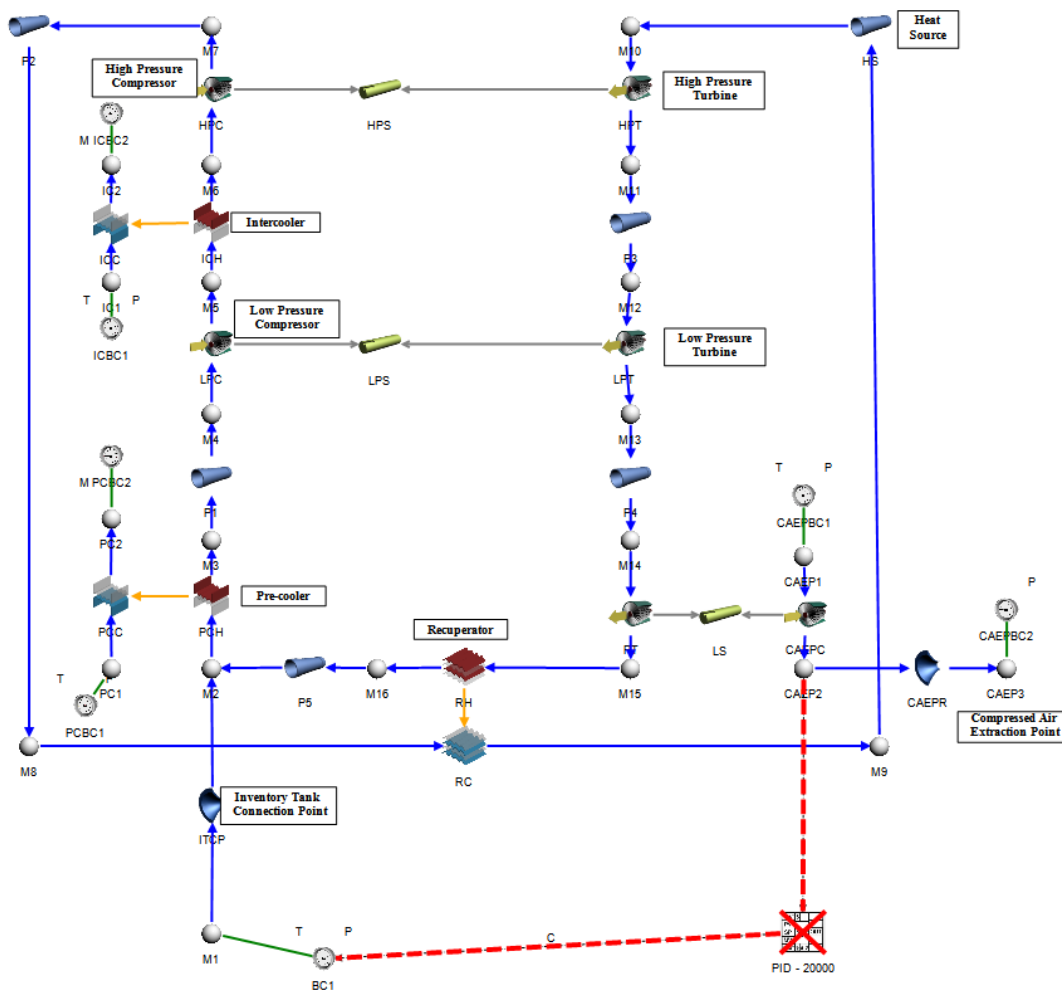
Figure 3.6: Heat source showing heat input



Source: Source: Own Composition using FLOWNEX

Considering the preceding, it can be concluded that it is only necessary to consider the exit temperature from the heat source in this particular study. This is due to the simplification of the process, while still receiving accurate simulated results.

Figure 3.7: The PBMM as modelled in FLOWNEX



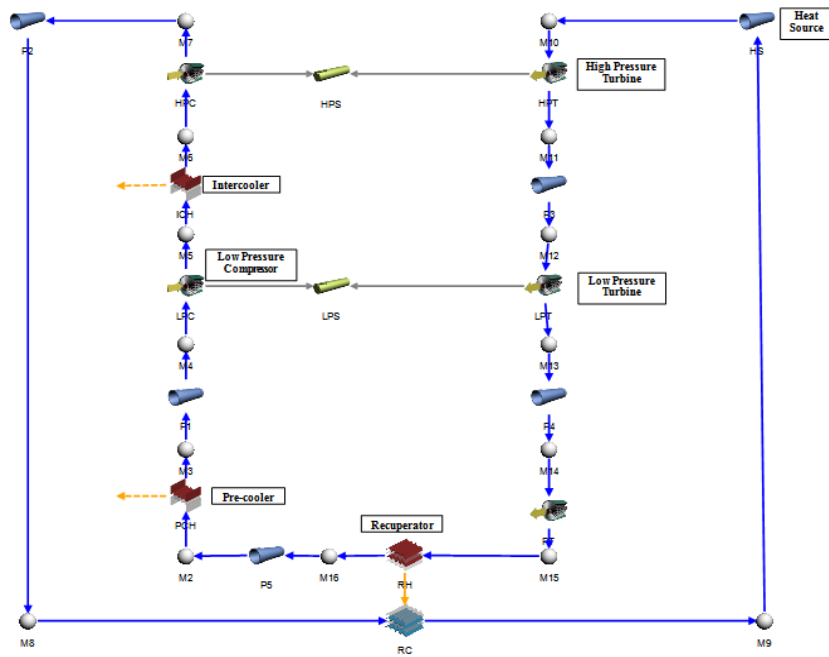
Source: Source: Own Composition using FLOWNEX

It is important to set up the control volume for the PBMM, which can be easily isolated from the surrounding environment. All components that have energy flowing through them and further into the system are part of the work volume. Components that serve as conduits to only get energy out of or into the system, to or from a single component in the system, are part of the environment. This would include parts of the pre-cooler, inter-cooler, and the generator, as these components typically extract energy from the system.

In the case of the PBMM, the PT is in the system; however, the compressor connected by the shaft and the other items in the loop are not in the system. Though these components simulate the generator, the operation of the components is outside of the Brayton cycle.

All the components in the work volume in FLOWNEX are elucidated in Figure 3.8.

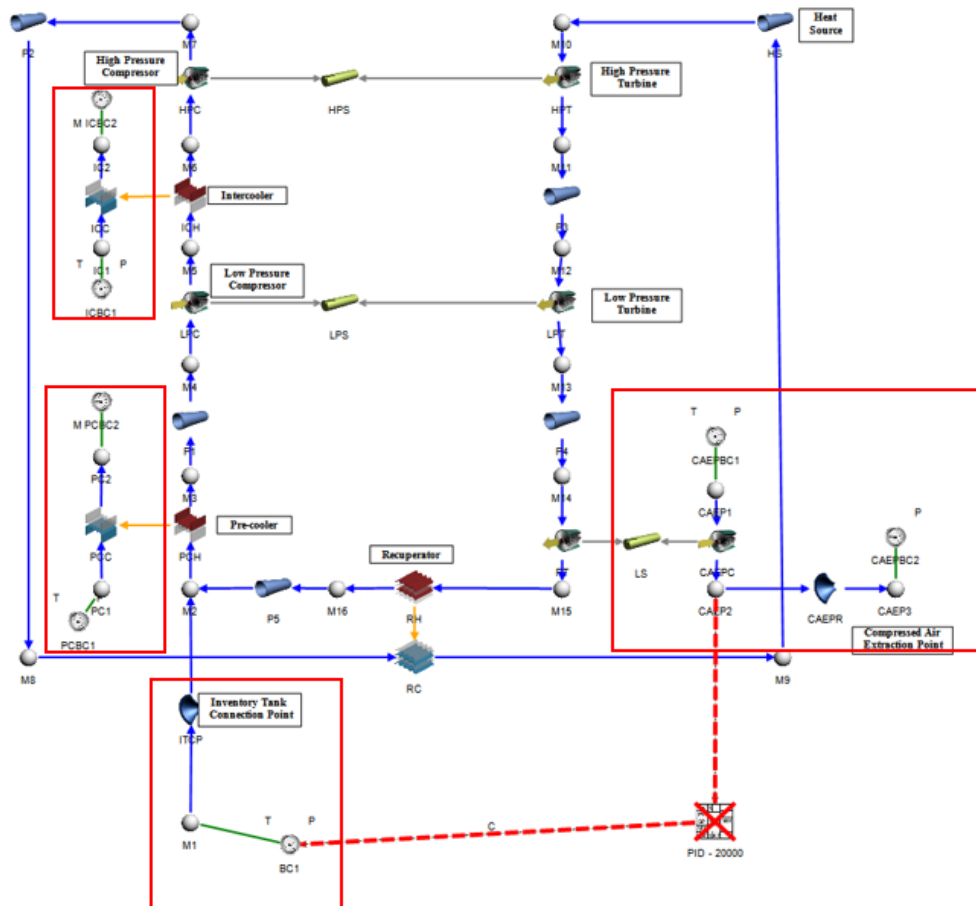
Figure 3.8: The system control volume of the PBMM



Source: Own Composition using FLOWNEX

The other components can be seen as part of the environment and are represented in the red blocks in Figure 3.9.

Figure 3.9: Environment areas (nodes) of the PBMM are highlighted and shown



Source: Own Composition using FLOWNEX

Considering the preceding and Figure 3.9, there is a clear distinction between the environment and the system, as the red blocks indicate the areas that are part of the environment. The variables that can be changed are the pressure (mass flow) of the system and the outlet temperature of the heater. The system can also be controlled and manipulated into operating under different conditions to produce the necessary results to build the EOEE.

3.4 Characterising the PBMM in terms of energy

According to Naser *et al.* (2021), an abridged version of the PBMM was modelled in terms of energy. Therefore, for the purpose of this study, it is important to properly replicate this modelling. As the working volume has been defined from the environment, the system can be modelled in terms of energy. The same equations as discussed in Chapter 2 will be used; however, the system should first be broken down into components. The components are numbered as shown in Table 3.1.

Table 3.1: PBMM Nodes

Node Point	Node Name	Description	Exergy
N1	PCH	Precooler Hot side	$\Delta\dot{B}_1$
N2	LPC	Low-Pressure Compressor	$\Delta\dot{B}_2$
N3	ICH	Intercooler Hot side	$\Delta\dot{B}_3$
N4	HPC	High Pressure Compressor	$\Delta\dot{B}_4$
N5	RC	Recuperator Cold side	$\Delta\dot{B}_5$
N6	HS	Heat source	$\Delta\dot{B}_6$
N7	HPT	High Pressure Turbine	$\Delta\dot{B}_7$
N8	LPT	Low-Pressure Turbine	$\Delta\dot{B}_8$
N9	PT	Power Turbine, the turbine that produces usable energy to a grid or whatever installation it is coupled to.	$\Delta\dot{B}_9$
N10	RH	Recuperator Hot side	$\Delta\dot{B}_{10}$
N11	Environment	Everything else and outside of the control volume	$\Delta\dot{B}_{11}$

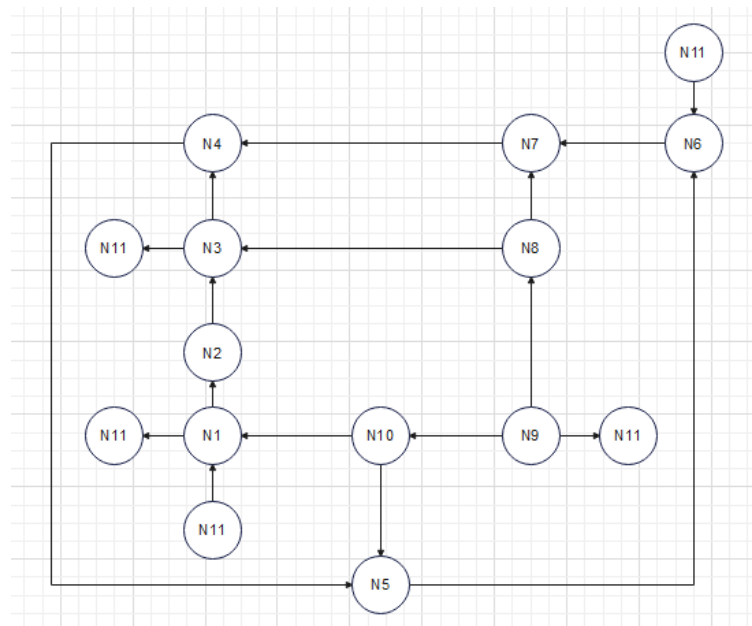
Source: Own composition, 2025

The numbering identified in the previous Table will be used in the energy-based attributed graph with an “N” before it. Following this, the components can be seen as nodes. The environment node includes all other aspects of the system not listed (the red blocks

identified in the previous section). Two controllable variables of the pressure and heat input for the system originate from the environment. The controllable variables in the system include temperature provided by the Heat Source (HS) and the pressure provided by boundary condition 1 (BC1). There is, however, a third, sub-controllable variable that could influence or manipulate the first two. This will be the output power of the PT.

By establishing the nodes, the energy flow through and between components can be calculated, enabling the creation of an energy-based attributed graph of the PBMM. This graph facilitates further visualisation of energy dynamics within the system. The visual representation of the nodes and the energy flow between the nodes are illustrated in Figure 3.10.

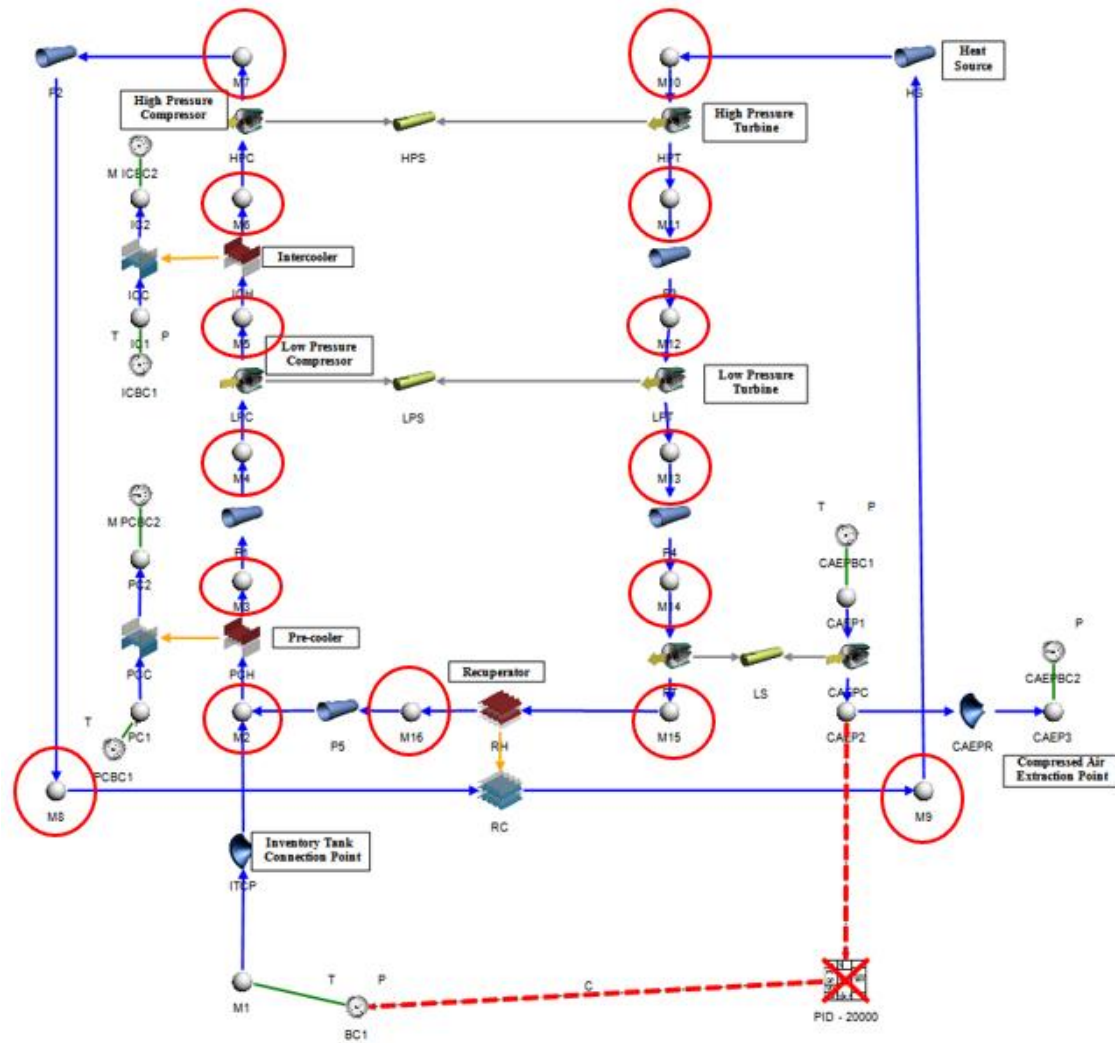
Figure 3.10: The Component nodes of the PBMM



Source: Own Composition

The preceding identified the component nodes and equation (3), as discussed in Chapter 2, and can be used to determine the energy flow or change through the component. However, calculating the energy flow between the components is required, which is calculated using equation (5). The FLOWNEX system includes other marked nodes, which represent the energy flow between components and are denoted with an M before it. These nodes will serve as a reference point for the enthalpy values used to calculate the energy flow between components. The nodes are circled in red in Figure 3.11.

Figure 3.11: The nodes between components of the PBMM



Source: Own Composition Using FLOWNEX

Many nodes occur over pipes in the FLOWNEX model. The pipes do not change the conditions of the system in any way, and the nodes before and after the pipes provide the same information:

- $M3 = M4$
- $M5 = M6$
- $M7 = M8$
- $M11 = M12$
- $M13 = M14$
- $M16 = M2$

This is due to the pipe component being considered ideal in this context. Consequently, no energy losses occur during the system operation, resulting in identical energy flow and other flow characteristics at the two nodes situated before and after the pipe component. Therefore, for the purposes of energy characterisation, the nodes before and after a pipe can be treated as a single node.

It is possible to model the pipe component with faults and losses such as fouling or friction. This could impact the flow characteristics as well as the energy flow, most likely lowering

it. For the purposes of this study, this is not necessary; however, it will be necessary for future research on the EOOE. From the preceding, a general node signature matrix can be derived for the system, as can be seen in matrix (43).

$$N_S = \begin{bmatrix} \Delta \dot{B}_1 | & \dot{E}_{11} & \dot{E}_{12} & \dot{E}_{13} & \dot{E}_{14} & \dot{E}_{15} & \dot{E}_{16} & \dot{E}_{17} & \dot{E}_{18} & \dot{E}_{19} & \dot{E}_{1\ 10} & \dot{E}_{1\ 11} \\ \Delta \dot{B}_2 | & \dot{E}_{21} & \dot{E}_{22} & \dot{E}_{23} & \dot{E}_{24} & \dot{E}_{25} & \dot{E}_{26} & \dot{E}_{27} & \dot{E}_{28} & \dot{E}_{29} & \dot{E}_{2\ 10} & \dot{E}_{2\ 11} \\ \Delta \dot{B}_3 | & \dot{E}_{31} & \dot{E}_{32} & \dot{E}_{33} & \dot{E}_{34} & \dot{E}_{35} & \dot{E}_{36} & \dot{E}_{37} & \dot{E}_{38} & \dot{E}_{39} & \dot{E}_{3\ 10} & \dot{E}_{3\ 11} \\ \Delta \dot{B}_4 | & \dot{E}_{41} & \dot{E}_{42} & \dot{E}_{43} & \dot{E}_{44} & \dot{E}_{45} & \dot{E}_{46} & \dot{E}_{47} & \dot{E}_{48} & \dot{E}_{49} & \dot{E}_{4\ 10} & \dot{E}_{4\ 11} \\ \Delta \dot{B}_5 | & \dot{E}_{51} & \dot{E}_{52} & \dot{E}_{53} & \dot{E}_{54} & \dot{E}_{55} & \dot{E}_{56} & \dot{E}_{57} & \dot{E}_{58} & \dot{E}_{59} & \dot{E}_{5\ 10} & \dot{E}_{5\ 11} \\ \Delta \dot{B}_6 | & \dot{E}_{61} & \dot{E}_{62} & \dot{E}_{63} & \dot{E}_{64} & \dot{E}_{65} & \dot{E}_{66} & \dot{E}_{67} & \dot{E}_{68} & \dot{E}_{69} & \dot{E}_{6\ 10} & \dot{E}_{6\ 11} \\ \Delta \dot{B}_7 | & \dot{E}_{71} & \dot{E}_{72} & \dot{E}_{73} & \dot{E}_{74} & \dot{E}_{75} & \dot{E}_{76} & \dot{E}_{77} & \dot{E}_{78} & \dot{E}_{79} & \dot{E}_{7\ 10} & \dot{E}_{7\ 11} \\ \Delta \dot{B}_8 | & \dot{E}_{81} & \dot{E}_{82} & \dot{E}_{83} & \dot{E}_{84} & \dot{E}_{85} & \dot{E}_{86} & \dot{E}_{87} & \dot{E}_{88} & \dot{E}_{89} & \dot{E}_{8\ 10} & \dot{E}_{8\ 11} \\ \Delta \dot{B}_9 | & \dot{E}_{91} & \dot{E}_{92} & \dot{E}_{93} & \dot{E}_{94} & \dot{E}_{95} & \dot{E}_{96} & \dot{E}_{97} & \dot{E}_{98} & \dot{E}_{99} & \dot{E}_{9\ 10} & \dot{E}_{9\ 11} \\ \Delta \dot{B}_{10} | & \dot{E}_{10\ 1} & \dot{E}_{10\ 2} & \dot{E}_{10\ 3} & \dot{E}_{10\ 4} & \dot{E}_{10\ 5} & \dot{E}_{10\ 6} & \dot{E}_{10\ 7} & \dot{E}_{10\ 8} & \dot{E}_{10\ 9} & \dot{E}_{10\ 10} & \dot{E}_{10\ 11} \\ \Delta \dot{B}_{11} | & \dot{E}_{11\ 1} & \dot{E}_{11\ 2} & \dot{E}_{11\ 3} & \dot{E}_{11\ 4} & \dot{E}_{11\ 5} & \dot{E}_{11\ 6} & \dot{E}_{11\ 7} & \dot{E}_{11\ 8} & \dot{E}_{11\ 9} & \dot{E}_{11\ 10} & \dot{E}_{11\ 11} \end{bmatrix} \quad (43)$$

From (43) and Figure 3.6, the true energy flow of the PBMM from node to node can be deduced, as shown in matrix (44). Note that the 0 indicates that there is no energy flow between the two components at that point.

$$N_S = \begin{bmatrix} \Delta \dot{B}_1 | & 0 & \dot{E}_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dot{E}_{1\ 10} & \dot{E}_{1\ 11} \\ \Delta \dot{B}_2 | & \dot{E}_{21} & 0 & \dot{E}_{23} & 0 & 0 & 0 & 0 & \dot{E}_{28} & 0 & 0 & 0 \\ \Delta \dot{B}_3 | & 0 & \dot{E}_{32} & 0 & \dot{E}_{34} & 0 & 0 & 0 & 0 & 0 & 0 & \dot{E}_{3\ 11} \\ \Delta \dot{B}_4 | & 0 & 0 & \dot{E}_{43} & 0 & \dot{E}_{45} & 0 & \dot{E}_{47} & 0 & 0 & 0 & 0 \\ \Delta \dot{B}_5 | & 0 & 0 & 0 & \dot{E}_{54} & 0 & \dot{E}_{56} & 0 & 0 & 0 & \dot{E}_{5\ 10} & 0 \\ \Delta \dot{B}_6 | & 0 & 0 & 0 & \dot{E}_{64} & \dot{E}_{65} & 0 & \dot{E}_{67} & 0 & 0 & 0 & \dot{E}_{6\ 11} \\ \Delta \dot{B}_7 | & 0 & 0 & 0 & \dot{E}_{74} & 0 & \dot{E}_{76} & 0 & \dot{E}_{78} & 0 & 0 & 0 \\ \Delta \dot{B}_8 | & 0 & \dot{E}_{82} & 0 & 0 & 0 & 0 & \dot{E}_{87} & 0 & \dot{E}_{89} & 0 & 0 \\ \Delta \dot{B}_9 | & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dot{E}_{98} & 0 & \dot{E}_{9\ 10} & \dot{E}_{9\ 11} \\ \Delta \dot{B}_{10} | & \dot{E}_{10\ 1} & 0 & 0 & 0 & \dot{E}_{10\ 5} & 0 & 0 & 0 & \dot{E}_{10\ 9} & 0 & 0 \\ 0 | & \dot{E}_{11\ 1} & 0 & \dot{E}_{11\ 3} & 0 & 0 & \dot{E}_{11\ 6} & 0 & 0 & \dot{E}_{11\ 9} & 0 & 0 \end{bmatrix} \quad (44)$$

From this matrix, the PBMM can be used for energy-based visualisation and can be used to create the EOOE.

3.5 Conclusion

When considering this section, the PBMM is the ideal system to develop the EOOE, as it has been used in previous research and is therefore verifiable. It is also easy to characterise the system by energy, making it possible to describe the system through a general node signature matrix. Furthermore, the system has also been built in FLOWNEX by FLOWNEX, meaning that the system can be used without user-induced errors. Therefore, different operating conditions can be tested and compared to each other using FLOWNEX's features.

Chapter 4 EOEE conceptualisation

4.1 Introduction

This chapter aims to identify various factors. This includes the performance measures to create the borders of the EOEE as well as the factors that determine if the operating condition of a system falls inside or outside of the EOEE. The performance measures of efficiency, reliability, and cost will be explored as possible measures or determining factors that shape the boundaries of the EOEE. Following this, functions will be identified or created to visually illustrate these measures with clear variables. The method of how these performance measures' operating conditions will be displayed on a graph using the HEOM function will also be discussed. Together, these factors will determine the boundary of the EOEE.

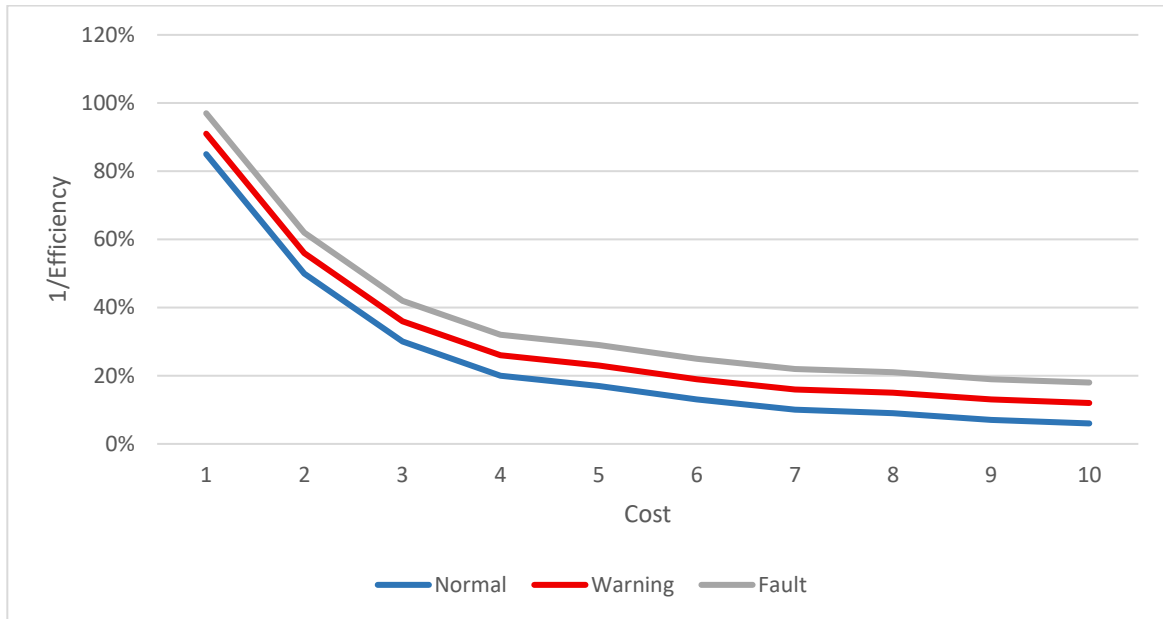
4.2 Concept of the EOEE

In creating the EOEE, a general node signature matrix should first be set up for the system. Following this, different matrices representing the normal and degraded states may be compared using the HEOM function. From this, a new matrix could be constructed. The subsequent question pertains to how this matrix can be graphically represented or how the data can be visualised. This is further influenced by the nature of the resulting values, which, in turn, will inform the most appropriate configuration for the graph.

One method to accomplish this is by using the HEOM matrix to identify eigenvalues that can then be graphed individually or against each other. In theory, the HEOM will find a greater distance between operating points as more degradation occurs. However, this would have to be tested in order to determine if it is true.

Another method is to create a Pareto front in which efficiency, reliability and cost comprising the axes as illustrated in Figure 4.1. In order to illustrate this method, 1/efficiency and cost would respectively be the y- and x-axis.

Figure 4.1: EOOE Pareto Front graph



Source: Own Composition

The operating points that symbolise degradation should, in theory, move further away from the optimal position. Following this, a normal frontline could be created for acceptable operating conditions. To create an EOOE, an additional front that aligns with the Pareto front should also be generated. This secondary front, referred to as the warning front, defines the extent to which the system's operating conditions can acceptably change before corrective action becomes necessary. This warning front, alongside the Pareto front, will create an area between the two graphs that will make up the EOOE. A third fault line will then represent the point at which system degradation is considered too severe to continue running. This boundary may also be defined using eigenvalue analysis.

The energy values that correspond to the Pareto front and the warning front will form the EOOE and should be investigated if these values are unique. The enthalpy and entropy can be used in the enthalpy-entropy graph approach. However, other forms of energy visualisation should be investigated in order to find the optimal and simplest option. Furthermore, tests should determine if the energy values are sufficient to construct a Pareto front. If it is possible to construct a Pareto front with the energy values, it will be graphed against the nodes of the system (or the enthalpy vs. entropy).

To measure if a working parameter is suitable for the EOOE, performance measures will be critical for comparison to act as a gauge. In other words, if the results of a certain operating point are favourable, it is prevalent that the point can be put onto the EOOE. However, it is important to first define and formulate how these performance measures can work.

4.3 Finding the correct measures for the EOEE

The initial performance measures considered for defining the EOEE were efficiency, cost, and reliability. Efficiency refers to the performance of a system or individual component, while cost pertains to the operational expenses associated with running the system. Reliability encompasses the system's ability to perform consistently over its intended lifespan. A system that operates optimally but for a fraction of the intended time cannot be considered optimal, as frequent repairs or replacements would compromise both its efficiency and cost-effectiveness.

Some problems arise when considering how to model these measures. Efficiency is the easiest to model as it can either be defined by the efficiency of a component or a system. A Brayton cycle is an example of this, where the efficiency is defined by the energy provided by the system divided by the energy input to the system.

However, cost is more complicated to model as it can either be the cost to run a system (i.e., the gas burned during the Brayton cycle to run it) or it can include any modifications designed into the system. Cost can also be defined by the initial purchasing of the system and/or any maintenance costs required for the system over time. Therefore, cost as a parameter will be difficult to determine without a comprehensive study of a physical system.

Reliability is the most challenging parameter to quantify, as there is no universal or specific formula for specific systems. Although literature could facilitate this through generalisation, however, there will be no concrete numbers to utilise for the model. On a physical system, the number of breakdowns can be counted and the frequency thereof can be calculated. However, this will not facilitate the development of a general condition monitoring tool such as the EOEE.

Considering the preceding, the measures of the EOEE will be changed. The measures will also differ for every different type of system, although the general principles will remain the same. The efficiency of a system could still be measured, while the cost of a system could be considered as the cost of the energy that the system uses while operating. Other measures could also be used. For example, as an alternative for reliability, degradation could be considered, as a degraded system will function worse and produce suboptimal results.

It is important to consider the function of the system being analysed. For example, the purpose of a Brayton cycle is to generate electric power. Therefore, if operating conditions or other factors, including degradation, hinder this function, it can be assumed that the particular set of operating conditions is not part of the EOEE. Conditions that generate the most power or the required quantity with the least amount of input cost while pushing the system to a breaking point can be considered optimal and would be included on the EOEE.

4.4 Efficiency

Efficiency serves as a fundamental metric for evaluating the performance of any mechanical system. Every system uses energy or converts energy from one form to another. The efficiency of a component refers to how well it utilises that energy to perform its function. A more efficient component will use less energy to produce the same result. A turbine, which extracts energy from the flow within the system and converts it into mechanical energy, inevitably loses some of the energy to the environment during the conversion process. The more efficient the turbine, the fewer the losses will be.

In a complex system, such as a three-shaft Brayton cycle, the efficiencies of every individual component should be considered. Moreover, the overall efficiency of the system should also be considered, which can be defined as the workable energy received from the PT. In other words, efficiency is the ratio of usable output power, such as for lighting, to the total energy input:

$$Efficiency = E_{out}/E_{in} \quad (44)$$

The energy that enters the system is usually obtained through a reactor or the burning of gas, as in a gas turbine.

After efficiency has been calculated, another problem could arise pertaining to the optimal time to consider if the system is no longer efficient. FLOWNEX can provide the individual efficiencies of every component while also calculating the overall efficiency of the PBMM system. However, either the efficiency of the system or the efficiency of the component will reach a point where the efficiency is too low due to degradation and can no longer be considered part of the EOEE. The PBMM is inherently an inefficient system, as it is modelled using a small-scale electric heater that provides energy. When solving the system for normal operating conditions for a power output of 100 kW, the system has an efficiency of 19.44%. According to (Neser, van Schoor and Uren, 2021), the set point for the PT in a modified PBMM is set as 100 kW. Therefore, 100 kW will be used as the set point for the PT used for this study. This efficiency will be referred to as the machine efficiency (ME) going forward, and is calculated with the following formula:

$$ME = \frac{Heat\ Transfer\ In\ Heat\ Source}{Power\ Turbine\ Output} \quad (45)$$

Table 4.1 provides the full specifications of the system being solved under normal, non-degraded operating conditions.

Table 4.1: Normal Operating Conditions

Specification	Value
Inlet Pressure	432.86 kPa
Mass flow	2.05 kg/s
Heat source outlet Temperature	700°C
Power Turbine set point	100 kW
Machine Efficiency	19.44%

Source: Own Composition

Going forward, the PT output energy will be recognised as the set point of the system or the set point of the current operating point. Although the efficiency of the system is extremely low, it is a simulation of a system using an electric heater at what can be considered a low temperature for a Brayton cycle. A maximum allowable level of system degradation should be defined before it is considered to fall outside the EOEE, which will be visible in the ME. For the purposes of this study, when the ME falls below 13%, the operating conditions are considered to be too far from normal, and the system is outside of the EOEE boundaries. As previously stated, if possible, a warning front should be created. Therefore, before 13% ME, the EOEE should indicate that the system is close to performing below what is desired. Each system, especially a practical system, would require comprehensive research and tests to identify in which region this warning front should be. This region would be the area in which maintenance and check-ups of the system should be completed to keep the system from leaving the EOEE and potentially failing or suffering irreparable damage.

For the purposes of this study, however, the range of the warning front will be 13% to 15%, as it provides an adequate range to test the theory. Furthermore, at 15% the system is almost three-quarters as efficient as it is under normal operating conditions, suggesting a considerable decrease in efficiency for any system. When no warning front is utilised, indicating that the conditions are either within or outside of the EOEE, the cut-off point will be 15% ME. Table 4.2 provides the data for ME in relation to the EOEE.

Table 4.2: Corresponding values for the EOEE and machine efficiency (ME)

Place on the EOEE	ME Value
Within the EOEE (Operating as intended)	>14%
Warning area (damage and degradation sustained, acceptable but action is required to restore the system to previous operating condition.)	13%-15%
Damage or fault line (All operating conditions beyond this point are deemed unacceptable and the system must be stopped.)	<13%

Source: Own Composition

4.5 Cost

When considering cost as a variable, it would determine if a system can be created or not, regardless of the efficiency of the system or the input costs. In many instances, the cheapest solution is most likely inefficient or unreliable; however, it is mostly used due to monetary issues such as burning coal. Considering the preceding, more efficient, reliable, and environmentally friendly methods to produce energy are possible. However, coal is more affordable than, for example, nuclear power.

When considering a three-shaft Brayton cycle in FLOWNEX, increased system efficiency may correlate with higher associated costs. For example, replacing a heater with a nuclear reactor (as is intended for the PBMR) could immensely increase the cost. Therefore, this metric will attempt to measure the actual operating cost of the system at its intended operating condition. This metric of the EOEE serves to ground the analysis in practical reality by evaluating the system's feasibility for real-world application. However, a

significant challenge arises from the fact that the study is based on simulation, raising the question of how to accurately determine cost within a virtual modelling environment.

The simplest method would be to multiply the power consumed within the heater by the R/kWh. However, the system produces power from an electric heater, resulting in higher input costs to continue running the system. Although this could be considered a problem, it could be overlooked as it is a simulation, and the purpose of the PBMM is to act as a guide for the PBMR. Considering the preceding, the efficacy of cost as a parameter in this study could be reconsidered. Therefore, it is essential to visually graph cost compared to efficiency in order to determine the impact on the creation of the EOOE for this study. Consequently, this can be further analysed by considering the power supplied by the heat source, which may also be interpreted as a proxy for cost per energy unit supplied.

It is possible to calculate the cost of constructing an entire system, as well as calculating the individual components and running costs. The profit, or projected profit from the system, could then be used as a powerful performance parameter. This parameter would be important as most businesses would want to compare it with the efficiency of the system to ensure that a proper balance is found. As a system degrades, prolonged and more expensive maintenance would be required for remediation, or the running cost would have to increase as the system becomes less efficient. For example, if leakages occur in the PBMR, the system would have to run at higher temperatures to maintain the same power output. This would consequently impact the profit and increase the cost.

However, this falls beyond the confines of the study due to its scope. The power provided by the increase in temperature for the heat source is all-encompassing for this study. An increase in energy inputs would consequently lead to higher costs. If the pressure is required to increase, it would increase the capacity at which the pumps are required to operate. This would consequently increase costs. However, this would have an impact on the energy provided by the heat source, as it will increase the mass flow receiving thermal energy from the heat source.

Therefore, for the purpose of this study, the increase in energy provided to the PBMM by the heat source can be considered as an increase in cost.

4.6 Reliability/degradation

If a system demonstrates a high degree of efficiency and/or cost effectiveness, it is likely to be considered highly effective or of superior quality. However, if such a system is prone to frequent breakdowns, it cannot be deemed reliable and is likely to result in substantial long-term costs due to the need for repeated replacements. Higher loads contributing to higher degradation suggest that a preferred operating point is prevalent to maximise efficiency and the rate of degradation (Najjar *et al.*, 2020). While this would likely be difficult to model, it is possible to model the extent of degradation a system has undergone, especially in FLOWNEX. The degradation could be simulated until the system fails, or FLOWNEX can no longer simulate it. This will indicate that, when implemented in practice, the system would either cease to function entirely or experience component failure. This could easily be tested for all compressors and turbines, even if a combination of components experiences degradation simultaneously.

The effect of this would have to be tested by incrementally increasing degradation until the system becomes unsolvable. Degradation could be tested in terms of the overall system or

individual components. However, in FLOWNEX, the tests would comprise individual components. Further to this, the rate of degradation remains beyond the scope of current simulation capabilities. This refers to the extent to which the system's operating conditions contribute to the wear and deterioration of its components or the system's reliability in maintaining its current level of performance over time. There are studies on degradation in gas turbine cycles where the rate of degradation is determined by each individual system. In these studies, the method in which the system operates is analysed and could be determined after obtaining data while the system is running and after the components have already degraded. This could be a study in itself, as data are required to plot the rate of degradation as the system is running, and therefore, this will not be included in this study. This would, however, be a relevant topic to investigate in further studies for the EOEE, as it would determine how reliability is impacted by a system's operating conditions.

Therefore, for the purpose of this study, reliability will not be included, although it would be interesting to be investigated in future studies.

4.7 Sensitivity analysis

For the system utilised in this study, data will be generated for all the different operating conditions. In order to utilise this data, a sensitivity analysis is required, and therefore, a Monte Carlo analysis is considered the best option.

A Monte Carlo study (Kroese *et al.*, 2014) is a statistical model where numerous quantities of data are gathered through simulating the behaviour of a system by having different inputs and/or measures. By simulating the system with an array of different inputs, the expected performance and variations of the system's performance could be tested. It is especially useful for systems that have uncertainties.

For this study, the Monte Carlo analysis will be used as follows:

- Multiple operating conditions will be tested in which the system degrades, and the energy flows entered into to a matrix.
- These matrices will be compared to one another, and the differences will be converted into eigenvalues to facilitate graphing.
- These graphs will determine the first iteration of the EOEE and this will be tested to determine if the EOEE can be used for condition monitoring of a complex system.

4.8 Graphing the EOEE

As previously discussed, the EOEE will be created by graphing eigenvalues derived from the differences in energy flow values between the normal operating conditions and degraded operating conditions. These differences will be derived through the use of the HEOM function as discussed in Chapter 2. The energy flow between components will be calculated through equation (4) and will appear in the matrix derived from the general node signature matrix (44) in Chapter 3.

$$N_S = \begin{bmatrix} \Delta \dot{B}_1 & | & 0 & \dot{E}_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dot{E}_{110} & \dot{E}_{111} \\ \Delta \dot{B}_2 & | & \dot{E}_{21} & 0 & \dot{E}_{23} & 0 & 0 & 0 & \dot{E}_{28} & 0 & 0 & 0 & 0 \\ \Delta \dot{B}_3 & | & 0 & \dot{E}_{32} & 0 & \dot{E}_{34} & 0 & 0 & 0 & 0 & 0 & 0 & \dot{E}_{311} \\ \Delta \dot{B}_4 & | & 0 & 0 & \dot{E}_{43} & 0 & \dot{E}_{45} & 0 & \dot{E}_{47} & 0 & 0 & 0 & 0 \\ \Delta \dot{B}_5 & | & 0 & 0 & 0 & \dot{E}_{54} & 0 & \dot{E}_{56} & 0 & 0 & 0 & \dot{E}_{510} & 0 \\ \Delta \dot{B}_6 & | & 0 & 0 & 0 & \dot{E}_{64} & \dot{E}_{65} & 0 & \dot{E}_{67} & 0 & 0 & 0 & \dot{E}_{611} \\ \Delta \dot{B}_7 & | & 0 & 0 & 0 & \dot{E}_{74} & 0 & \dot{E}_{76} & 0 & \dot{E}_{78} & 0 & 0 & 0 \\ \Delta \dot{B}_8 & | & 0 & \dot{E}_{82} & 0 & 0 & 0 & 0 & \dot{E}_{87} & 0 & \dot{E}_{89} & 0 & 0 \\ \Delta \dot{B}_9 & | & 0 & 0 & 0 & 0 & 0 & 0 & \dot{E}_{98} & 0 & \dot{E}_{910} & \dot{E}_{911} & 0 \\ \Delta \dot{B}_{10} & | & \dot{E}_{101} & 0 & 0 & 0 & \dot{E}_{105} & 0 & 0 & 0 & \dot{E}_{109} & 0 & 0 \\ 0 & | & \dot{E}_{111} & 0 & \dot{E}_{113} & 0 & 0 & \dot{E}_{116} & 0 & 0 & \dot{E}_{119} & 0 & 0 \end{bmatrix} \quad (44)$$

This cost matrix fully describes the energy flow between components. When the energy flow changes due to degradation, the general node signature matrix will consequently change. Following this, these matrices (the normal and the degraded matrix) will be compared through the use of the HEOM function as discussed in Chapter 2. The resulting matrix will appear as an 11x11 matrix due to the number of nodes in the matrix. Eigenvalues can be derived from the cost matrix to ensure that the EOEE can be graphed. Eigenvalues are derived from eigen decomposition, which is a method from linear algebra for matrix factorisation. The eigenvalues of a matrix are the characteristic scalar values of a matrix. Therefore, different matrices generally do not have the same eigenvalues, making eigenvalues particularly useful for simplifying a matrix while preserving its characteristics, thereby enabling meaningful comparisons between matrices. The eigenvalues of a matrix are determined by the equation:

$$Ax = \lambda x \quad (48)$$

In terms of the prior equation, A is a $n \times n$ matrix and x is a nonzero vector, also referred to as the eigenvector of A. If both of these characteristics are true, then λ is the eigenvalue or eigenvalues of A, depending on the matrix size. It is also possible that multiple eigenvectors could exist for a matrix; however, it will be linearly dependent on a single set of eigenvalues. For the 11×11 matrix calculated using the HEOM function through determining the differences of the normal and degraded general node signature matrix, the eigenvalues will appear as follows:

$$\lambda = \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \lambda_5 \\ \lambda_6 \\ \lambda_7 \\ \lambda_8 \\ \lambda_9 \\ \lambda_{10} \\ \lambda_{11} \end{bmatrix} \quad (49)$$

These values produced by different operating conditions should provide points on a graph that illustrate the difference between the normal and degraded values. Chapters 6 and 7 will discuss the results and these graphs.

4.9 Conclusion

When considering the different performance measures discussed in this chapter, the most suitable measure for the development of the first EOEE is the efficiency of the overall system (ME). Both cost and reliability should be investigated in further studies but are not suitable for the current system and current understanding of the EOEE. With the EOEE having been first developed, these measures and others can be added to an expanded understanding of the EOEE.

Operating conditions can be easily categorised by the ME into different regions, including those within and outside the EOEE. A Monte Carlo analysis, which involves numerous operating conditions, can be reworked into eigenvalues that are easily converted into a graph in order to visually illustrate the first EOEE.

Chapter 5 EOEE implementation

5.1 Introduction

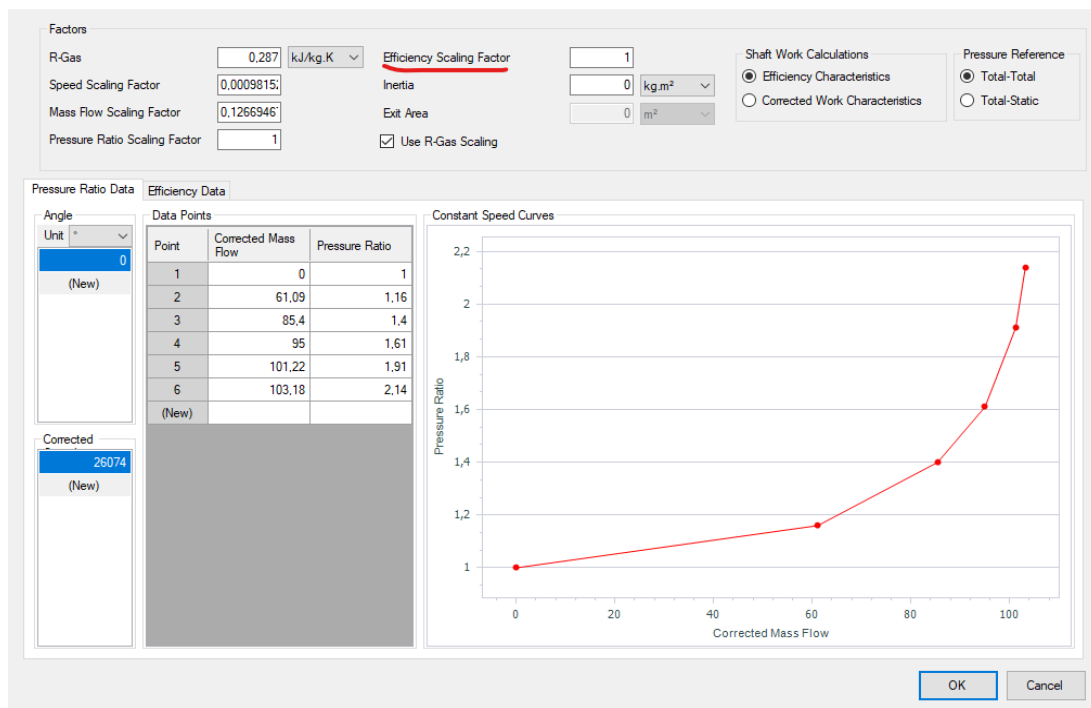
The purpose of this chapter is to illustrate how the chosen system, the PBMM, will be controlled by MATLAB to produce the necessary results for developing the EOEE. The chapter will also illustrate how degradation will be utilised in FLOWNEX to provide the different operating conditions necessary for utilisation by the HEOM function for analysis. A test will be conducted to determine how the system reacts to different operating conditions. The setup for the final simulation which will provide the required data, will also be discussed. The preceding will provide all the necessary information to utilise the PBMM, enabling implementation of the EOEE.

5.2 FLOWNEX charts

In FLOWNEX, the operating points of the compressors and turbines are modelled via characteristic charts. These charts represent the design range of the devices, or the possible range in which a compressor or turbine can be solved and produce results. If at any point, the steady state that is being solved is not accounted for by the chart or cannot be interpolated from the graph on the chart, it will produce an error instead of solving the system. Figure 5.1 illustrates the chart used for the PT.

It is important to note that mass flow scaling and speed scaling factors are illustrated in Figure 5.1. These factors originate from the original equipment manufacturer. The charts, likewise, were also previously set up. The PT's characteristic curve chart has an efficiency curve, as evident in Figure 5.2, that determines the efficiency of the component for the corrected mass flow provided.

Figure 5.1: Characteristic curve chart used to simulate degradation for PT

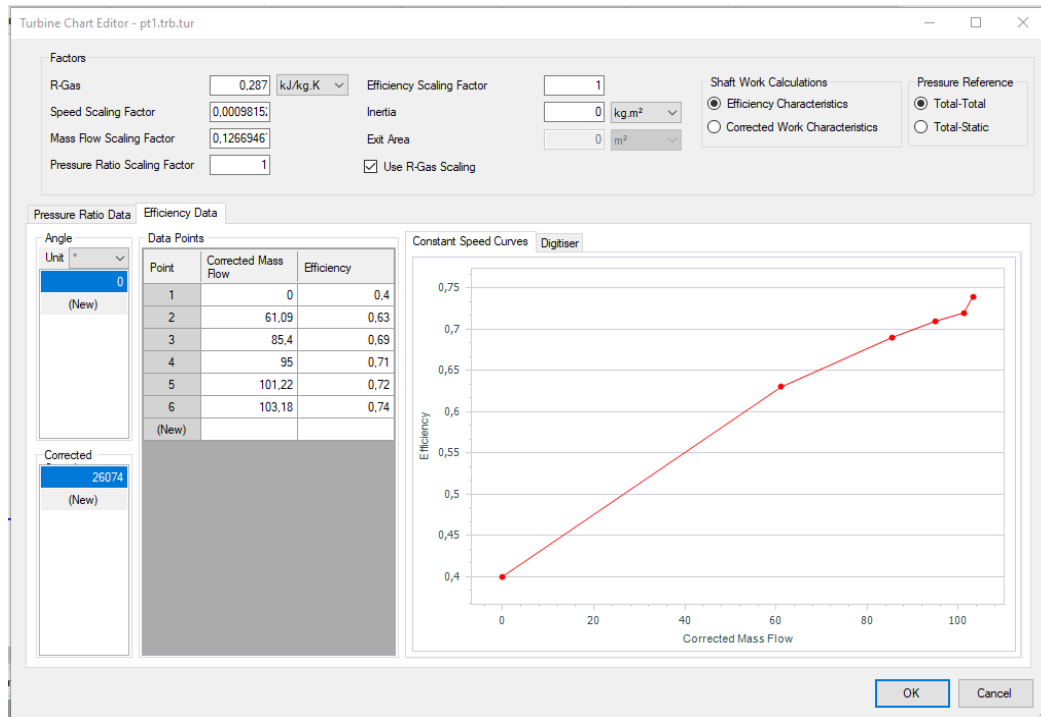


Source: Own Composition Using MATLAB

The speed and mass flow scaling factors determine the corrected speed and mass flow, respectively, used in the charts. Due to these factors originating from the OEM, they can only be indirectly controlled by a user. However, these factors are not important to the study, as they only determine how the system operates. Whether the corrected speeds, pressure ratios, and mass flow rates of the compressors and turbines fall within the permissible operational limits determines if the system can converge to a steady-state solution. Any other restrictions placed on the system, such as a 700°C maximum temperature from the heat source, is determined by the user's discretion.

The results of the component, either a turbine or a compressor, are obtained from these charts. As prevalent in Figure 5.1 and Figure 5.2, the pressure ratio and efficiency of the component are all reliant on the corrected mass flow.

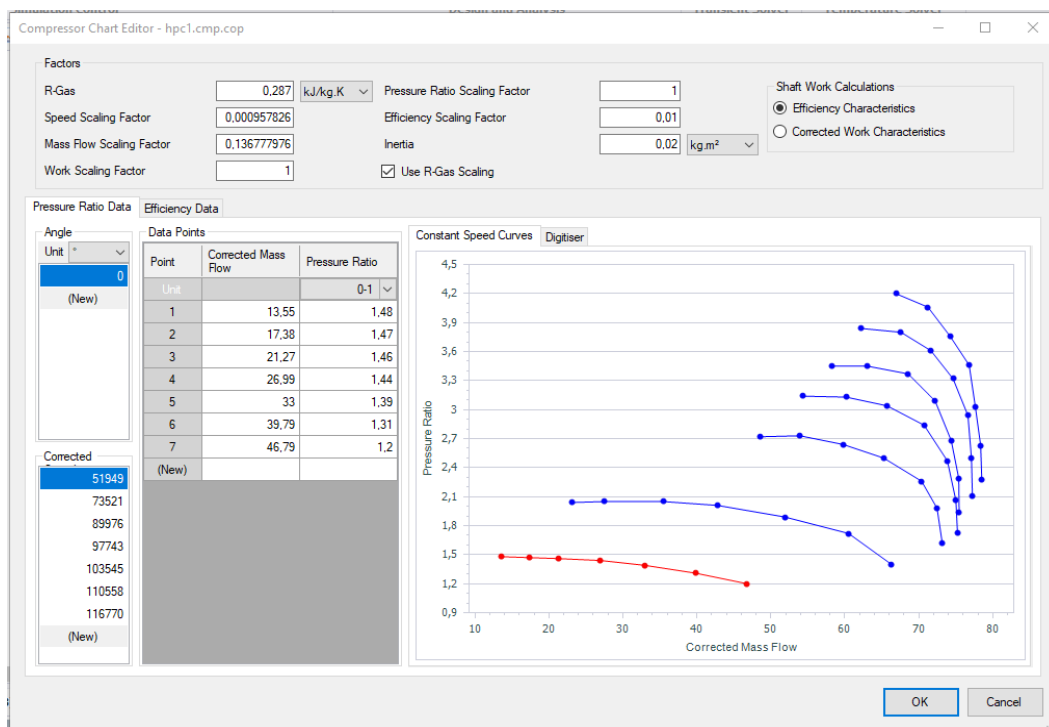
Figure 5.2: Characteristic efficiency curve for PT



Source: Own Composition Using MATLAB

In Figure 5.3, it can be observed that there are multiple lines, each representing a corrected speed (prevalent in the lower left). The different corrected speeds illustrated in Figure 5.3 and Figure 5.4 indicate that there is an extra factor to consider when FLOWNEX solves the efficiency and pressure ratio of the compressors in the PBMM.

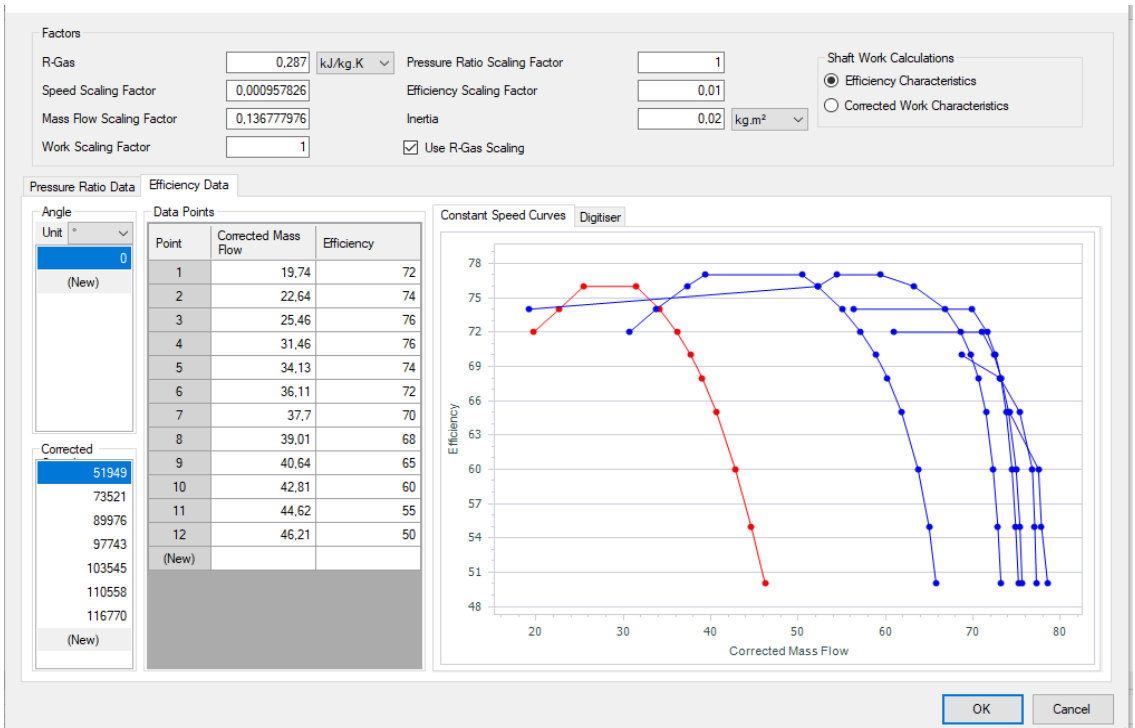
Figure 5.3: Characteristic curve chart for the HPC



Source: Own Composition Using MATLAB

FLOWNEX performs interpolation between the corrected speeds to determine the location of the operating points on the graph, as well as to estimate the corresponding corrected mass flow rate and efficiency. Only the compressors have charts with multiple corrected speed lines, whereas the turbines are represented by a single corrected speed curve. The red line serves solely as a visual aid to highlight the currently selected corrected speed by the cursor and does not correspond to any specific configuration or operational setting. Figure 5.4 illustrates the corresponding efficiency data graph for the HPC.

Figure 5.4: Characteristic efficiency curve chart for the HPC



Source: Own Composition Using MATLAB

In the initial FLOWNEX model, the shafts connecting the HPC and the HPT, as well as the LPC and the LPT, are provided with a constant speed. This leads to inaccurate results with the compressors consuming more energy than provided by the turbines. When the compressor is switched to automatically calculate the speed, more accurate results are obtained. This method will therefore be used in this study.

5.3 Simulating degradation in the system

To successfully utilise the PBMM, it is essential that the model is applied in accordance with best practices. As mentioned previously, it is necessary to simulate degradation in order to create the EOEE. Furthermore, the PBMM FLOWNEX model has two variables that can be used to control the system and adjust its operating conditions.

During closer analysis of the system, a third variable was identified that can control or influence the system. This includes the Efficiency Scaling Factor (ESF) of components, as depicted in the characteristic curve illustrated in Figure 5.1. Only compressors and turbines have an ESF as part of the characteristic curves in FLOWNEX. This factor can be used to simulate degradation in the system as it controls the overall efficiency of the component. This does not depict the efficiency at which the component operates (i.e., when the system

is solved in steady state), and each component seemingly runs at its own efficiency. Instead, the ESF gauges the value of the compressor charts efficiency vs the corrected mass flow.

The ESF multiplies with the efficiency outcomes of the charts to provide the efficiency with which the component operates. Therefore, an ESF of 1 indicates that the component operates as normal, and a factor of, for example, 0.9 indicates that an event or condition occurred within the system resulting in operational performance that is less efficient than originally intended in the design specifications. In practical terms, when a system operates differently despite no intentional changes to its configuration or input, the deviation is typically attributable to degradation or the presence of faults in the component.

If a system's ESF is 0.8, it indicates that, for a specific corrected mass flow, the turbine will operate at 80% of its nominal efficiency. For instance, instead of achieving an efficiency of 70%, the turbine would operate at only 56% efficiency. Therefore, the ESF can be used to represent faults or degradation within a system. However, quantifying the impact of a specific fault is challenging, as the extent to which it may reduce the ESF is often unknown. Degradation levels can vary, and a component may continue to function adequately despite some degree of degradation. Considering this, the ESF provides a means to model and assess varying levels or percentages of degradation within components.

Considering the preceding, by lowering the ESF, degradation is simulated in the system, indicating the progressive decline in the system's power output as degradation increases. When adjusting the value to 0.95 or 0.9, a 5% and 10% degradation is simulated, respectively, and further reductions can be used to represent increasing levels of degradation. Other values may be employed to simulate various degrees of degradation. Currently, there is no precise method to quantify the extent to which components such as turbines or compressors have degraded. Therefore, it is assumed that an ESF of 0% represents a complete failure of a component, indicating that it is no longer functional and requires replacement. When the PT is set to zero, some system components can still function; however, the PT itself no longer provides any output. If, however, a constant power output is set to be maintained, the system will crash at some point, as it is no longer possible to maintain the constant power output. As will be demonstrated subsequently, the system predictably shuts down well before a single component reaches a 0 ESF, as this would correspond to the component achieving 0% efficiency, effectively rendering the system incapable of fulfilling its intended purpose.

The downside of this method is that only the percentage of degradation can be simulated. In practice, when degradation occurs, it is not possible to precisely determine the extent of degradation that has occurred. However, the method still captures the general trend that the actual system would follow. When the simulated system becomes non-operational and/or when the level of degradation is deemed excessive, it reflects the same outcome in the actual system, prompting the need for maintenance or repairs.

The same degradation method cannot be applied to simulating degradation in HXs or pipes. For the purpose of this study, pipes have been excluded from the degradation study to avoid unnecessary complexity. However, these components can be included in future research.

There are two types of HXs in this system, namely the Intercooler and Pre-cooler HX (which are standard effectiveness HXs that transfer heat from the system to the environment), and the Recuperator HX (which retains heat within the system as a counterflow-type HX). Including HXs in the degradation study would introduce significant complexities. While this would strengthen the argument for the validity of the EOEE across a broader range of

components and systems, it would also considerably extend the duration of the study. Simulating degradation in HXs present greater challenges, as there is no ESF that can be directly applied to represent such degradation. In the simpler Pre-cooler and Intercooler HXs, effectiveness values are used; however, these primarily reflect the initial design specifications rather than operational deterioration. The more complex Recuperator HX includes controlling measures such as fouling factors and other mechanisms for inducing performance losses. However, accurately implementing these functions would require further research and a dedicated preliminary study.

Therefore, to create the first EOEE, sufficient data can be obtained by stimulating degradation in the turbines and compressors alone. Degradation in HXs and pipes will be excluded from this phase of the study to maintain simplicity. However, it is expected that energy transfer values within the HXs will still exhibit variations from the baseline as a consequence of performance changes in the degraded compressors and turbines.

5.4 MATLAB as platform for control and analysis

5.4.1 Background

To ensure the system is adequately controlled, MATLAB scripts are used for the input of different operating conditions and for saving data. This data will then be inserted into a workable form, namely the EOEE.

The MATLAB code will employ “for” loops to simulate the system under various conditions of temperatures, pressures, and degradation levels. The resulting data can subsequently be displayed in tabular formats, and an energy transfer matrix may be constructed to facilitate further analysis. The resulting energy cost matrix is a valuable analytic tool, as it enables the application of EGBV methods. By facilitating comparisons across various working/operating conditions, the matrix supports the identification of scenarios that merit inclusion on the EOEE.

Although the coding process can be time-consuming, its development ultimately streamlines the overall workflow by facilitating system control, enabling efficient modification of input variables, and ensuring the automatic storage of all necessary data.

For the purpose of this study, a MATLAB script was created that controls and changes the variables in FLOWNEX, as well as records these changes. MATLAB stores the results in a table format and can illustrate the data in graphs. This data can also be displayed as cost matrices that can be converted into eigenvalues using the HEOM function.

Given the complexity of the scripts used, separate scripts are utilised to model the degradation of each individual component, with an additional script dedicated to the construction of the cost matrix. This modular approach enhances clarity and manageability, as consolidating all processes into a single script would result in excessive data output and complicate the development of accurate MATLAB code.

To fully utilise the system and to ensure the received data is correct, specific conditions and results may be compared. To maintain consistency within the system, a single variable must remain constant, regardless of the degradation occurring in individual components. This

allows for the analysis of how the system adjusts or compensates in response to various degrees of component degradation.

The most suitable candidate for the constant variable is the output from the PT to the generator/environment. This can be effectively implemented using the designer function in MATLAB. To evaluate the system's response to degradation, either the heat input or pressure must remain constant while the order is varied to compensate for the effect of degradation. However, the non-tested variable may still be altered independently to assess the system behaviour under multiple operating conditions.

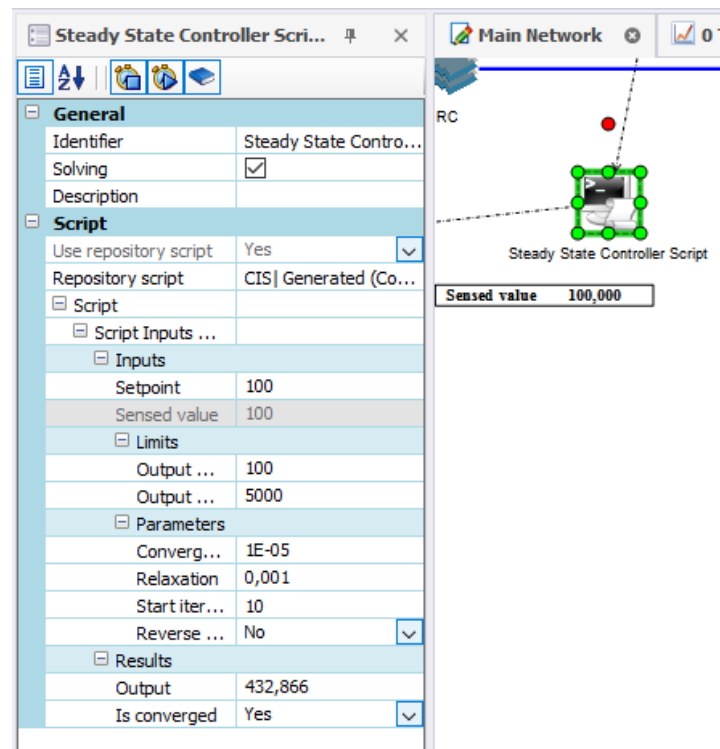
In practical terms, temperature should remain constant while pressure is adjusted to compensate for component degradation. This approach is justified by the relative ease of varying mass flow rates compared to altering the temperature or energy output of a nuclear reactor, which remains the foundational basis of the system. Variations in mass flow are expected to yield sufficient insight into the system's response to degradation, and this methodology can be applied to assess the impact of degradation across different system components.

5.4.2 Set up of simulated system

A first test is necessary to verify that the system can effectively compensate for degrading components. In this preliminary analysis, a minor degree of degradation will be introduced to the PT, and the resulting effects on the overall system performance will be recorded. As previously discussed, the system has two primary controllable variables, excluding graphical outputs, namely the input heat of the heat source and the system pressure. For the purpose of this test, the temperature of the heat source will remain at 700°C. In this configuration, system pressure effectively governs the mass flow of the system, with increases or decreases in pressure corresponding to proportional changes in mass flow.

Following consultations with FLOWNEX personnel, a Steady-State Controller (SSC) script was implemented to maintain a constant power output of 100 kW from the PT, with system pressure used as the variable input. A comparison of these results with those obtained from the "designer" function revealed consistent outcomes for the degradation test of the PT. This consistency confirms the validity of the SSC-based results within the simulation framework. The configuration is illustrated in Figure 5.5, where the input setpoint is fixed at 100, thereby regulating the PT's power output to remain at 100 kW. The "Results Output" is allowed to vary accordingly to ensure that the PT power output remains stable at 100 kW.

Figure 5.5: FLOWNEX Steady State Controller



Source: Own Composition Using FLOWNEX

For the test, the system was solved under steady-state conditions for five iterations: initially at nominal conditions, followed by successive increments of 5% degradation applied to the PT, up to a maximum of 20% degradation.

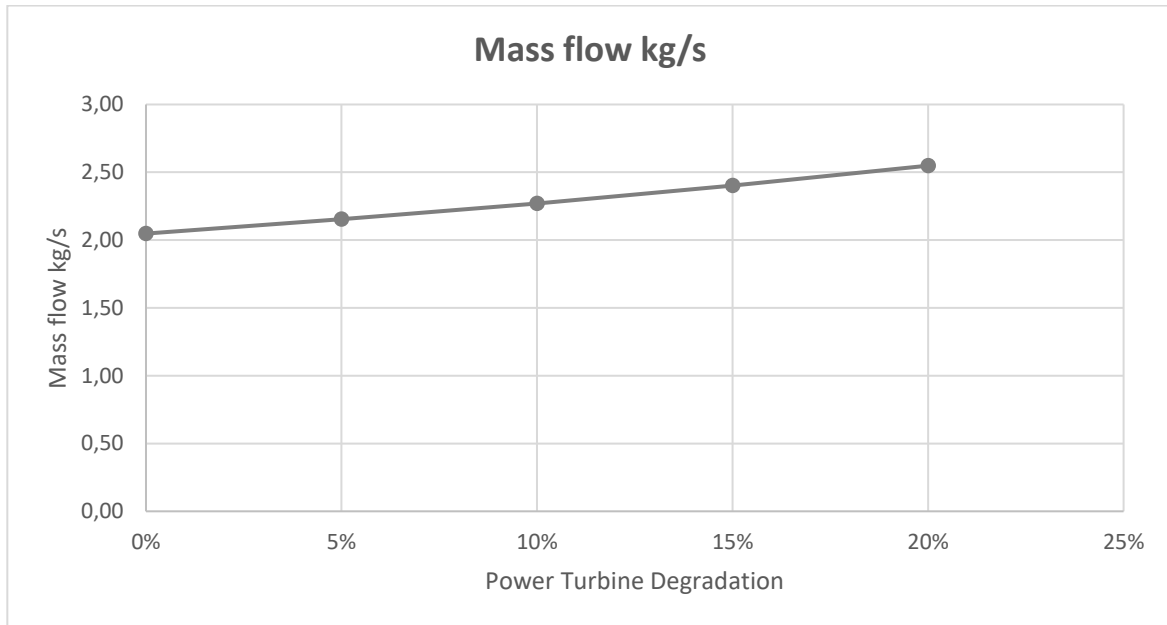
Table 5.1: First Degradation Test Results

PT Degradation	Pressure kPa	Difference kPa	Mass flow kg/s	Power Output kW	Heater kW	Machine Efficiency
0%	432,87	0,0	2,05	100	514,381	19,44%
5%	454,81	21,94	2,15	100	541,337	18,47%
10%	479,22	24,41	2,27	100	571,831	17,49%
15%	506,55	27,33	2,40	100	606,488	16,49%
20%	537,33	30,78	2,55	100	645,928	15,48%

Source: Own Composition Using FLOWNEX

As illustrated in Table 5.1, the system compensates for degradation by increasing the mass flow. This increase results in a greater quantity of working fluid passing through the heater, thereby absorbing more heat and introducing additional energy into the system, as depicted in Figure 5.6. Concurrently, the ME decreases as a result of the degradation.

Figure 5.6: First Test Mass Flow Results



Source: Own Composition using FLOWNEX

The variations in mass flow and energy input from the heater demonstrate that the system's energy flow dynamics are altered as degradation progresses. Consequently, the general node signature matrix characterising the system will differ for each distinct operating condition.

5.5 Setting up the final simulation

To develop the EOOE, numerous operating conditions are required to create sufficient data points to adequately capture system behaviour variations. To achieve this, a final simulation must be conducted encompassing multiple components subjected to varying degrees of degradation under different operating scenarios. Additionally, in some cases, more than one component will be degraded simultaneously to test if the trend will remain consistent.

Considering the preceding, the system will be simulated as normal, while certain components will be set to degrade at a rate of 2%. This will continue until the component has reached 22% degradation, equivalent to 78% operational capability, or until the system becomes unsolvable under the given conditions. The output of the PT is set to remain 100 kW, and the pressure is set vary between 100 and 5000 kPa throughout the simulation. The temperature is pre-set at 700°C being the maximum temperature of the electrical heater.

In certain simulations, multiple components will be degraded simultaneously to evaluate the system's response to compound degradation scenarios. Degradation will be applied in 2% increments per iteration and will continue until the system either fails or can no longer be solved, indicating operational breakdown or automatic shutdown. Within the MATLAB framework, one component will essentially act as a "master" variable, maintained at 100% functionality, while the second component degrades at 2% increments until failure. As the second component incrementally degrades, the master will be reduced by 2% (therefore being 98%), and the process for the second component will continue to degrade at 2%

incements until failure. This cycle will continue until the master can no longer sustain the amount of degradation, resulting in system failure.

5.6 Conclusion

Using MATLAB, the PBMM system can be simulated for multiple operating conditions to facilitate a Monte Carlo analysis. The data produced by these operating conditions can be captured and processed through the HEOM function, enabling the extraction of eigenvalues necessary to create the EOEE.

By integrating appropriate features within FLOWNEX, component degradation can be effectively simulated, adding an additional dimension to the EOEE framework. This approach aligns with ongoing research in energy-based visualisation methods focused on fault detection. Incorporating degradation into the EOEE not only enhances the diagnostic capability of the system but also allows for the identification of potential faults before their occurrence by revealing the characteristics of a degraded operating condition.

Chapter 6 EOOE verification

6.1 Introduction

The purpose of this Chapter is to illustrate and discuss the results of the simulation set up in Chapter 5. Several logical tests are conducted to verify the results and the concept of the EOOE. Finally, eigenvalues and SVD are compared in order to validate the use of SVD values in the development of the EOOE. This would provide more adequate information for the EOOE, as well as provide consistency in the visual graphs.

6.2 Final simulation results

The use of MATLAB to control FLOWNEX through a steady-state solver was demonstrated in Chapter 5 to produce the same results for steady-state conditions as when FLOWNEX solved it independently. A substantial number of results were obtained; therefore, only the most significant findings will be discussed in this chapter.

When simulating with degradation, most components were unable to reach the 22% degradation threshold, especially when two or more components experienced degradation simultaneously. Table 6.1 presents the final level of degradation by each individual component and combination of components prior to failure.

Table 6.1: Component Degradation Values and Combinations

Case of degradation	Component	Degradation
Case 1	HPC	18%
Case 2	HPC	12%
	LPC	12%
Case 3	HPT	12%
Case 4	HPT	12%
	HPC	10%
Case 5	HPT	12%
	LPC	12%
Case 6	HPT	12%
	LPT	12%
Case 7	LPC	22%
Case 8	LPT	22%
Case 9	LPT	12%
	HPC	12%
Case 10	LPT	12%
	LPC	12%
Case 11	PT	22%
Case 12	PT	22%
	LPC	22%
Case 13	PT	22%
	LPT	22%

Source: Own Composition

Simulating these various operating conditions generated a substantial volume of data—far too much to present in full. Therefore, only a selection of results will be presented, specifically those obtained from degrading the PT in isolation, as outlined in Table 6.2, and subsequently in combination with the LPC. These selected cases also serve as a partial validation of the results, as certain values should be consistent across both datasets. In the simulation where both the PT and LPC undergo degradation, any instance in which the LPC remains at 0% degradation should yield results identical to the results produced when only the PT is degraded. If discrepancies were observed, it would suggest a potential error in either the MATLAB code or the FLOWNEX PBMM. As demonstrated in the results below, the values match, indicating that consistent outcomes are achieved when simulated independently. This supports the validity of the results within the context of the PBMM system.

Table 6.2: Power Turbine Degradation Results

PT degradation	Pressure kPa	Mass flow	Energy out KW	Energy In KW	Energy difference KW	Machine efficiency
100%	432,8655	2,0479	507,982	514,381	6,399	19,44%
98%	441,3689	2,0888	518,245	524,773	6,528	19,06%
96%	450,2305	2,1313	529,018	535,681	6,663	18,67%
94%	459,4760	2,1756	540,333	547,136	6,803	18,28%
92%	469,1314	2,2219	552,221	559,171	6,949	17,88%
90%	479,2208	2,2703	564,729	571,831	7,102	17,49%
88%	489,7744	2,3209	577,894	585,156	7,262	17,09%
86%	500,8255	2,3738	591,762	599,192	7,429	16,69%
84%	512,4121	2,4293	606,386	613,991	7,605	16,29%
82%	524,5651	2,4875	621,763	629,552	7,789	15,88%
80%	537,3268	2,5486	637,946	645,928	7,982	15,48%
78%	550,7520	2,6129	654,967	663,151	8,185	15,08%

Source: Own Composition

The results of the PT and LPC degradation simulations are presented in Table 6.3, beginning with the PT at 0% degradation. This allows for observation of the system's response when only the LPC experiences degradation. It is evident that when both components start to degrade simultaneously, the overall ME declines at a significantly faster rate. To clearly distinguish between acceptable and non-acceptable MEs, as discussed in Chapter 4, warning values of ME (i.e., warning fronts) are depicted in beige, while the non-acceptable values are depicted in red. Given the large number of operating conditions, only a subset of the results is depicted—the initial degradation range of the PT from 0% to 4%.

Table 6.3: Power Turbine and Lower Pressure Compressor Degradation Results

PT degradation	LPC degradation	Pressure kPa	Mass flow kg/s	PT to environment KW	Heat transfer kW	Machine efficiency
0%	0%	432,87	2,05	100,0	514,38	19,44%
0%	2%	475,34	2,15	100,0	533,66	18,74%
0%	4%	522,62	2,27	100,0	555,20	18,01%
0%	6%	574,48	2,40	100,0	578,92	17,27%
0%	8%	631,14	2,53	100,0	604,78	16,53%
0%	10%	695,01	2,68	100,0	633,96	15,77%
0%	12%	777,83	2,86	100,0	672,92	14,86%
0%	14%	886,18	3,10	100,0	724,74	13,80%
0%	16%	1007,61	3,37	100,0	784,25	12,75%
0%	18%	1144,03	3,66	100,0	852,52	11,73%

PT degradation	LPC degradation	Pressure kPa	Mass flow kg/s	PT to environment KW	Heat transfer kW	Machine efficiency
0%	20%	1298,14	3,98	100,0	930,32	10,75%
0%	22%	1711,03	4,63	100,0	1070,99	9,34%
2%	0%	441,37	2,09	100,0	524,77	19,06%
2%	2%	484,72	2,20	100,0	544,67	18,36%
2%	4%	532,98	2,32	100,0	566,88	17,64%
2%	6%	585,91	2,44	100,0	591,31	16,91%
2%	8%	643,74	2,58	100,0	617,83	16,19%
2%	10%	708,88	2,73	100,0	647,66	15,44%
2%	12%	793,42	2,92	100,0	687,75	14,54%
2%	14%	904,00	3,17	100,0	741,28	13,49%
2%	16%	1027,93	3,44	100,0	802,65	12,46%
2%	18%	1167,18	3,73	100,0	872,92	11,46%
2%	20%	1324,44	4,06	100,0	952,61	10,50%
2%	22%	1745,25	4,72	100,0	1097,50	9,11%
4%	0%	450,23	2,13	100,0	535,68	18,67%
4%	2%	494,49	2,24	100,0	556,22	17,98%
4%	4%	543,77	2,36	100,0	579,14	17,27%
4%	6%	597,83	2,49	100,0	604,26	16,55%
4%	8%	656,86	2,63	100,0	631,43	15,84%
4%	10%	723,35	2,79	100,0	661,97	15,11%
4%	12%	809,65	2,98	100,0	703,36	14,22%
4%	14%	922,57	3,23	100,0	758,67	13,18%
4%	16%	1049,11	3,51	100,0	821,97	12,17%
4%	18%	1191,28	3,81	100,0	894,21	11,18%
4%	20%	1351,86	4,15	100,0	975,91	10,25%
4%	22%	1780,93	4,82	100,0	1125,31	8,89%

Source: Own Composition

Table 6.4 depicts the final higher stages of degradation for the PT. When comparing Table 6.3 and Table 6.4, it becomes evident that significant differences in the pressure, mass flow, and ME of the system occur as degradation increases.

Table 6.4: Second Part of Degradation Results for PT & LPC

PT Degradation	LPC Degradation	Pressure kPa	Mas s flow kg/s	PT to Env KW	Heat Transfer kW	Machine Efficiency
20%	0%	537,33	2,55	100,0	645,93	15,48%
20%	2%	590,51	2,68	100,0	672,19	14,88%
20%	4%	649,82	2,83	100,0	701,16	14,26%
20%	6%	714,86	2,98	100,0	733,09	13,64%
20%	8%	785,90	3,15	100,0	768,31	13,02%
20%	10%	865,44	3,34	100,0	808,51	12,37%
20%	12%	968,96	3,57	100,0	863,26	11,58%
20%	14%	1104,75	3,87	100,0	935,45	10,69%
20%	16%	1256,78	4,20	100,0	1015,85	9,84%
20%	18%	1427,71	4,57	100,0	1107,07	9,03%
20%	20%	1620,58	4,97	100,0	1212,37	8,25%
20%	22%	2131,70	5,77	100,0	1408,07	7,10%
22%	0%	550,75	2,61	100,0	663,15	15,08%
22%	2%	605,34	2,75	100,0	690,25	14,49%
22%	4%	666,19	2,90	100,0	720,35	13,88%
22%	6%	732,91	3,06	100,0	753,59	13,27%
22%	8%	805,77	3,23	100,0	790,27	12,65%
22%	10%	887,32	3,42	100,0	832,00	12,02%
22%	12%	993,50	3,66	100,0	888,79	11,25%
22%	14%	1132,77	3,97	100,0	963,19	10,38%
22%	16%	1288,78	4,31	100,0	1046,21	9,56%
22%	18%	1464,09	4,68	100,0	1140,94	8,76%
22%	20%	1661,93	5,10	100,0	1250,07	8,00%
22%	22%	2185,63	5,92	100,0	1453,13	6,88%

Source: Own Composition

One notable difference is the change in energy through the heat exchange, which represents one of the primary nodes of energy flow. Such a significant change has a cascading effect, resulting in a distinct node signature matrix for each operating condition. Consequently, the impact on the HEOM function becomes observable in the results.

6.3 General node signature results and verification

Each general node signature matrix contains a diagonal of zeros, corresponding to the position of Q_{nn} which serves as a means of verifying that the data is, in fact, correct. If the data were not captured correctly, this expected pattern will not be present. Another validation test involves confirming that the inverse of an energy flow is represented by multiplication by -1. For example:

$$\dot{Q}_{ni} = -\dot{Q}_{in} \quad (50)$$

Or:

$$\dot{Q}_{12} = -\dot{Q}_{21} \quad (51)$$

As shown in Table 6.5, the presence of the expected diagonal confirms that the general node signature matrix was constructed correctly.

Table 6.5: Normal General Node Signature Matrix

Node	Δ	$\dot{Q}_{n1}$	$\dot{Q}_{n2}$	$\dot{Q}_{n3}$	$\dot{Q}_{n4}$	$\dot{Q}_{n5}$	$\dot{Q}_{n6}$	$\dot{Q}_{n7}$	$\dot{Q}_{n8}$	$\dot{Q}_{n9}$	$\dot{Q}_{n10}$	$\dot{Q}_{n11}$
$\dot{Q}_{n1}$	-0,0190	0	0,6263	0	0	0	0	0	0	0	-0,8746	0,2483
$\dot{Q}_{n2}$	-0,0598	-0,6263	0	0,7876	0	0	0	0	-0,1646	0	0	0
$\dot{Q}_{n3}$	-0,0084	0	-0,7876	0	0,6279	0	0	0	0	0	0	0,1597
$\dot{Q}_{n4}$	0,0595	0	0	-0,6279	0	0,7802	0	-0,1554	0	0	0	0
$\dot{Q}_{n5}$	0,1848	0	0	0	-0,7802	0	1,6264	0	0	0	0,8462	0
$\dot{Q}_{n6}$	0,1636	0	0	0	0	-1,6264	0	2,1408	0	0	0	-0,5144
$\dot{Q}_{n7}$	-0,0842	0	0	0	0,1554	0	-2,1408	0	1,9854	0	0	0
$\dot{Q}_{n8}$	-0,0958	0	0,1646	0	0	0	0	-1,9845	0	1,8208	0	0
$\dot{Q}_{n9}$	-0,0564	0	0	0	0	0	0	0	-1,8208	0	1,7208	0,1000
$\dot{Q}_{n10}$	-0,2028	0,8746	0	0	0	-0,8462	0	0	0	-1,7208	0	0
$\dot{Q}_{n11}$	0	-0,2483	0	-0,1597	0	0	0,5144	0	0	-0,1000	0	0

Source: Own Composition

Table 6.6 depicts the operating conditions when both the PT and the LPC were degraded at 22%.

Table 6.6: Example of a Degraded General Node Signature Matrix

Node	Δ	$\dot{Q}_{n1}$	$\dot{Q}_{n2}$	$\dot{Q}_{n3}$	$\dot{Q}_{n4}$	$\dot{Q}_{n5}$	$\dot{Q}_{n6}$	$\dot{Q}_{n7}$	$\dot{Q}_{n8}$	$\dot{Q}_{n9}$	$\dot{Q}_{n10}$	$\dot{Q}_{n11}$
$\dot{Q}_{n1}$	-0,0355	0	1,7957	0	0	0	0	0	0	0	-2,8353	1,0395
$\dot{Q}_{n2}$	0,0299	-1,7957	0	20797	0	0	0	0	-0,2898	0	0	0
$\dot{Q}_{n3}$	-0,0034	0	-2,0797	0	1,7785	0	0	0	0	0	0	0,3013
$\dot{Q}_{n4}$	0,0411	0	0	-1,7785	0	2,0995	0	-0,3275	0	0	0	0
$\dot{Q}_{n5}$	0,1946	0	0	0	-2,0995	0	4,7478	0	0	0	2,6483	0
$\dot{Q}_{n6}$	0,1602	0	0	0	0	-4,7478	0	6,2009	0	0	0	-1,4531
$\dot{Q}_{n7}$	-0,0642	0	0	0	0,3275	0	-6,2009	0	5,8734	0	0	0
$\dot{Q}_{n8}$	-0,0582	0	0,2898	0	0	0	0	-5,8734	0	5,5836	0	0
$\dot{Q}_{n9}$	-0,0228	0	0	0	0	0	0	0	-5,5836	0	5,4836	0,1000
$\dot{Q}_{n10}$	-0,2412	2,8353	0	0	0	-2,6483	0	0	0	-5,4836	0	0
$\dot{Q}_{n11}$	0	-1,0395	0	-0,3013	0	0	1,4531	0	0	-0,1000	0	0

Source: Own Composition

Table 6.7 below presents the results of the HEOM function when the normal operating conditions are compared to itself, followed by a comparison with a degraded case, as shown in Table 6.8.

Table 6.7: The HEOM Function Results for Normal Operating Conditions

0	0,9709	1,1518	0,8718	1,2295	1,5025	1,0253	0,9281	1,2133	1,1576	0,6912
0,9709	0	1,0342	1,1325	1,0905	1,3183	1,1296	1,0752	1,1044	1,4595	0,7459
1,1518	1,0342	0	0,9418	1,3767	1,4876	1,0488	1,1148	1,0950	1,2263	0,7829
0,8718	1,1325	0,9418	0	1,0037	1,4260	1,0237	0,9680	1,0384	1,2995	0,5351
1,2295	1,0905	1,3767	1,0037	0	1,3355	1,5171	1,2780	1,1136	1,5162	0,8767
1,5025	1,3183	1,4876	1,4260	1,3355	0	1,4821	1,6239	1,5379	1,5159	1,1909
1,0253	1,1296	1,0488	1,0237	1,5171	1,4821	0	1,0571	1,3405	1,1832	0,9323
0,9281	1,0752	1,1148	0,9680	1,2789	1,6239	1,0571	0	1,0979	1,3358	0,8118
1,2133	1,1044	1,0950	1,0384	1,1136	1,5379	1,3405	1,0979	0	1,2360	0,8754
1,1576	1,4595	1,2263	1,2995	1,5162	1,5159	1,1832	1,3358	1,22360	0	1,0945
0,6912	0,7459	0,7829	0,5351	0,8767	1,1909	0,09323	0,8118	0,8754	1,0945	0

Source: Own Composition

Table 6.8: Example of a Degraded HEOM Function Result

0,5885	1,3655	1,6250	1,1721	1,7551	2,2312	1,5454	1,4151	1,8982	1,8039	0,7862
1,4299	0,5033	1,4452	1,5116	1,5924	2,0449	1,6238	1,5173	1,6973	2,1226	0,7619
1,6287	1,4124	0,5157	1,2255	1,8986	2,1982	1,5462	1,5646	1,7085	1,8499	0,8680
1,3646	1,6017	1,3807	0,4122	1,5342	2,2071	1,5401	1,4331	1,6659	1,9619	0,6276
1,6788	1,4755	1,8134	1,2824	0,0146	2,0605	2,0117	1,6553	1,5813	2,0725	0,9119
1,9671	1,6489	1,8220	1,7308	1,7633	0,8239	1,8717	2,0389	2,0067	1,9592	1,2485
1,4632	1,4798	1,4321	1,2862	2,0420	2,1492	0,6202	1,5196	1,9932	1,8133	1,0543
1,3673	1,4329	1,5356	1,2351	1,7350	2,3349	1,5707	0,5956	1,7190	2,0306	0,8941
1,6915	1,4552	1,4853	1,2991	1,4840	2,2154	1,8833	1,5454	0,7348	1,8511	0,9522
1,5554	1,8363	1,5926	1,5628	1,9228	2,0758	1,6636	1,8558	1,8202	0,7595	1,2483
1,2548	1,2091	1,2788	0,9084	1,4244	1,9630	1,5249	1,3598	1,5759	1,8069	0,2123

Source: Own Composition

As shown in Table 6.8, the HEOM function quantifies the distance between the normal and the degraded operating conditions. The normal condition is consistently identifiable by the diagonal of zeros. This also serves as a basic verification test—if the diagonal of zeros is not present when the normal condition is compared to itself, it could be indicative of a potential error.

6.4 Singular value decomposition

As discussed previously, eigenvalues can be utilised to construct the EOOE. However, conventional eigenvectors provide only the invariant direction, while SVD vectors yield both the direction and the magnitude of maximum action. Additionally, the eigenvalues, which represent the magnitude associated with the corresponding vectors, are not inherently ordered. For example, λ_1 may represent the largest eigenvalue under certain operating conditions, while in others, λ_2 or another eigenvalue may assume that role.

In practical terms, for the EOOE, this implies that plotting all values of λ_1 against each other, they can result in a confusing graph, as there is no discernible pattern or clearly defined region that can be interpreted as an envelope. The same applies to other eigenvalues. Therefore, a matrix of values that is consistently ordered in terms of importance is required, allowing for meaningful comparison and interpretation.

To simplify the process of constructing the EOOE, SVD is used rather than conventional eigenvalue analysis. SVD is a matrix factorisation technique that determines the rank of a matrix rank (Chengwang, 2010), producing an orthonormal matrix as a basis that effectively provides an eigen-decomposition. Unlike standard eigenvalue methods, SVD can be applied to any $m \times n$ matrix and yield singular values in descending order (Chengwang, 2010). This ordering ensures that, across matrices which may differ significantly under different operating conditions, the corresponding singular values are more consistently aligned in terms of relative importance. As a result, this facilitates easier comparison and graphical representation. The formula for SVD is:

$$M = U\Sigma V^T \quad (52)$$

In this formulation, M represents the original matrix. The matrix U is an $m \times m'$ matrix containing the eigenvectors and is the left singular matrix. Furthermore, V contains the eigenvectors and is the right singular matrix and is given by $m' \times m$. Σ is the $m \times n$ matrix containing the descending, diagonal eigenvalues or in the case of an $n \times n$ matrix, Σ is also an $n \times n$ matrix (Ford, 2015).

When using the SVD function in MATLAB, the results return the eigenvalues of the matrix in simple descending order, as seen in equation (53):

$$\lambda_S = \begin{bmatrix} \lambda_{S1} \\ \lambda_{S2} \\ \lambda_{S3} \\ \lambda_{S4} \\ \lambda_{S5} \\ \lambda_{S6} \\ \lambda_{S7} \\ \lambda_{S8} \\ \lambda_{S9} \\ \lambda_{S10} \\ \lambda_{S11} \end{bmatrix} \quad (53)$$

An example of the SVD values is presented in Table 6.9. This Table displays the SVD values for the normal operating conditions at a set point of 100 kW, alongside the SVD values for a

degraded operating condition at the same set point. This comparison illustrates how the singular values differ between the normal and degraded states.

Table 6.9: Normal SVD Values vs Degraded SVD Values

Normal SVD	Degraded SVD
11,6414	16,9476
2,0289	1,9544
1,8012	1,7465
1,4589	1,3370
1,2573	1,1829
1.1397	1,0466
1,0348	0,9067
0,9425	0,7671
0,8612	0,6643
0,6541	0,4878
0,4628	0,3360

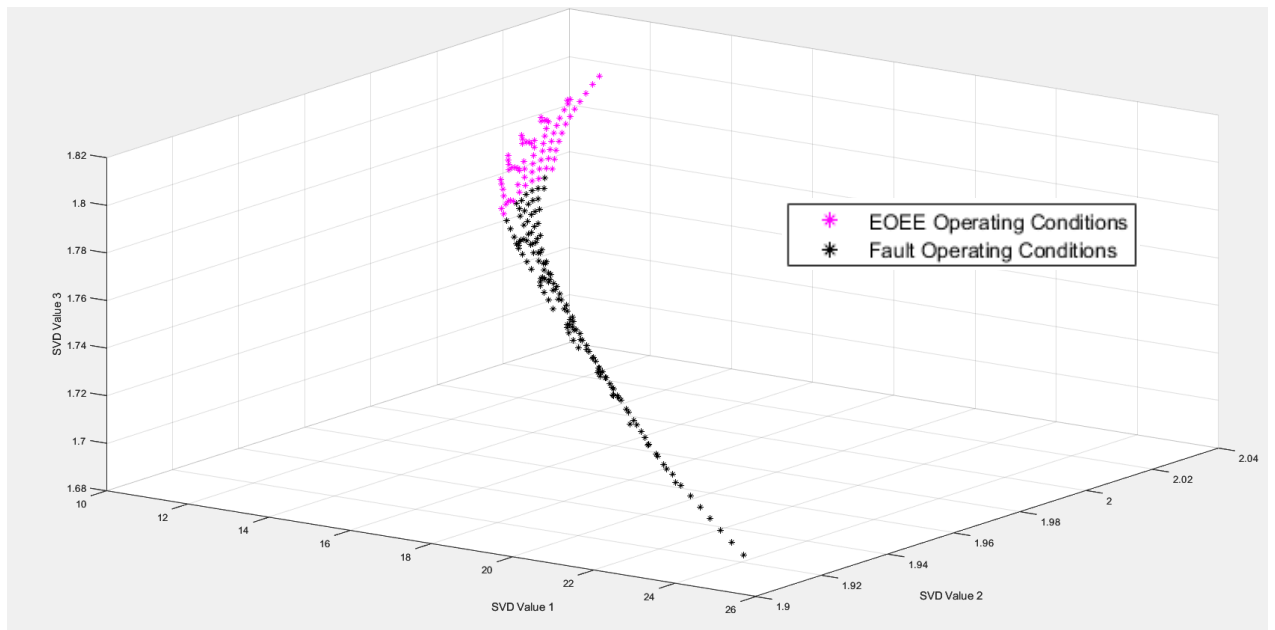
Source: Own Composition

The SVD values essentially represent the EOOE; the larger the singular value, the greater the deviation from the normal operating condition, and thus the higher the likelihood that the system lies outside the EOOE. The most important SVD values are those with the highest values, or in other words, the values depicting the most significant difference. These values can be plotted to form the EOOE.

6.5 3D SVD value map

To visually illustrate the EOOE, the three largest SVD values can be plotted on a 3D map. As previously noted, an ME below 14%, is considered unacceptably low and thus lies outside the EOOE. A clear difference can be observed between the SVD values corresponding to operating conditions within the EOOE and the values outside it. For improved visual clarity, all SVD values outside the EOOE are marked in black. The highest point in the corner, shown in purple, represents the normal operating condition.

Figure 6.1: First EOOE Results 3D Map



Source: Own Composition using MATLAB

It is prevalent in the preceding Figure that, beyond a certain distance from the normal operating conditions, all SVD values correspond to points that fall outside the EOOE. However, within the middle region, it becomes difficult to clearly distinguish between points inside and outside the EOOE. This suggests that the boundary of the EOOE is governed by a complex function that is not immediately evident from the current visualisation.

6.6 Conclusion

This chapter presented a sample of the numerical results obtained from the simulations conducted on the PBMM. The various result matrices were introduced and explained, highlighting their role in the construction of the EOE. SVD was selected over traditional eigenvalue analysis for visualising the EOE, as the results of the singular values are arranged in descending order, making the overall trends easier to interpret and compare.

The first visualisation of the EOE was presented in Figure 6.1. It was illustrated how the operating conditions originate from the normal point and progressively degrade, moving downward to represent increasingly deteriorated states of ME. A clear distinction emerged between the operating points; this lies within the EOE, and the points that fall outside of the EOE.

Chapter 7 EOOE validation

7.1 Introduction

The final objective of the EOOE is to facilitate the simplification of condition monitoring in complex industrial systems. Analysing large matrices and extensive datasets can be challenging, as it is often difficult to interpret the meaning of individual values. Similarly, 3D visualisations containing multiple points and colour variations may be visually overwhelming and offer limited practical insight. By simplifying the data representation, the EOOE enhances interpretability, making it easier to assess system performance and compare different operating conditions across various set points (in this case, the output of the PT).

In this chapter, the EOOE concept is further developed and finalised by simplifying the results previously presented through the normalisation of SVD points. Additionally, different operating points are compared to assess the consistency of the EOOE across operating regimes. This comparison serves to validate the EOOE and determine whether it is independent of specific input or output variables. If such independence is confirmed, the EOOE can be generalised and applied beyond a single dataset or system, enhancing its practical utility in a broader condition monitoring context.

7.2 Normalised SVD value graph

To simplify the EOOE and reduce visual complexity, it is necessary to scale down the values depicted in the graph without diminishing their interpretive value. This can be achieved through normalisation.

Specifically, each λ_s value is divided by the corresponding normal condition value λ_{nS} , thereby minimising the magnitude of differences while retaining meaningful distinctions. When comparing the normal to itself, this process should yield an 11×1 matrix composed entirely of ones. Any deviation from a value of one indicates that the SVD values represent a degraded operating condition. This is illustrated in Table 7.1, where the first column displays the normalised SVD values for the normal operating condition, and the second column shows the normalised values for a degraded operating condition. The resulting normalised SVD values can be graphed to clearly visualise and assess the extent of deviation from the normal state.

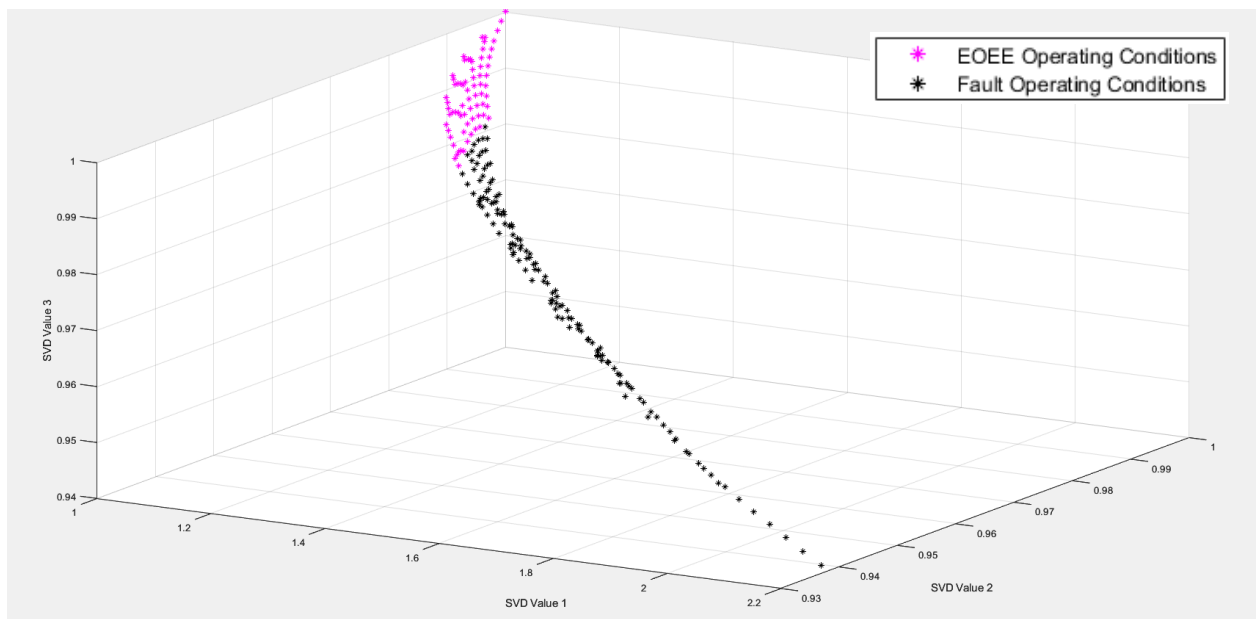
Table 7.1: Normalised SVD Values

Normal	Normalised SVD
1	1,4558
1	0,9633
1	0,9696
1	0,9165
1	0,9409
1	0,9183
1	0,8762
1	0,8138
1	0,7714
1	0,7458
1	0,7260

Source: Own Composition

The process of normalising all degraded values is facilitated by the continued use of MATLAB. Figure 7.1 depicts the resulting 3D map of the normalised SVD values.

Figure 7.1: Normalised 3D Map



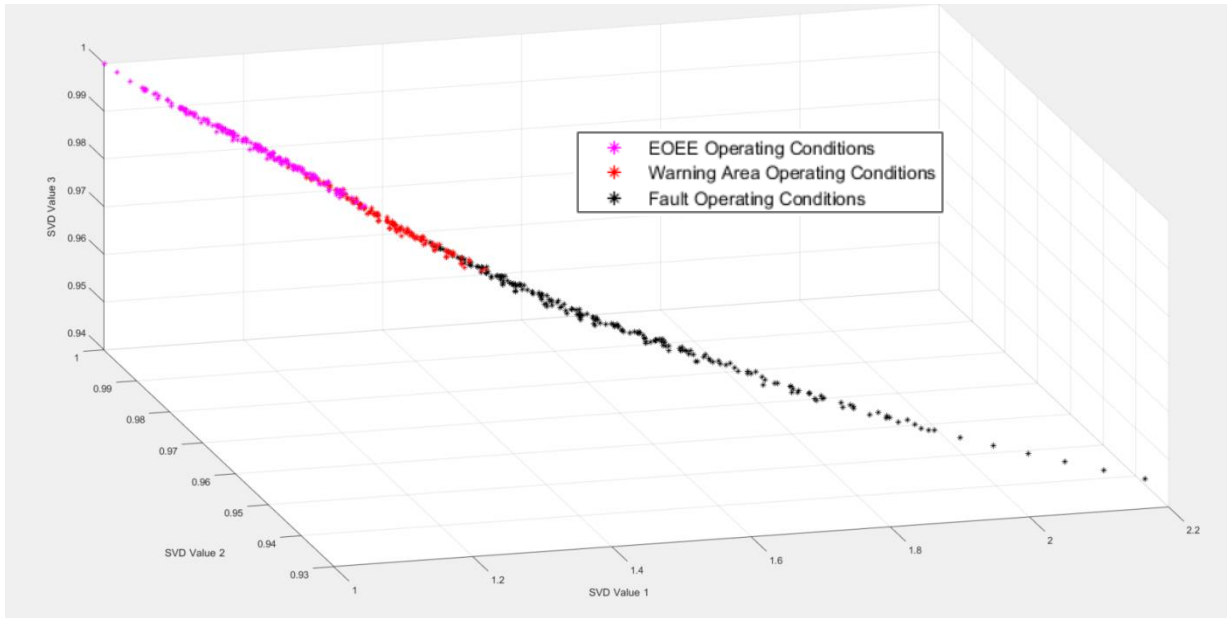
Source: Own Composition using MATLAB

As a result of the normalisation, the axes in the 3D map represent significantly lower and more closely aligned values, which enhances the ability to directly compare different operating points.

7.3 Warning front

As previously discussed, the aim is to establish a warning front, or, in the case of a 3D graph, a defined area. When applied to the ME range of 13% to 15%, the resulting visualisation is shown in Figure 7.2.

Figure 7.2: 3D Map with Added Warning Front



Source: Own Composition using MATLAB

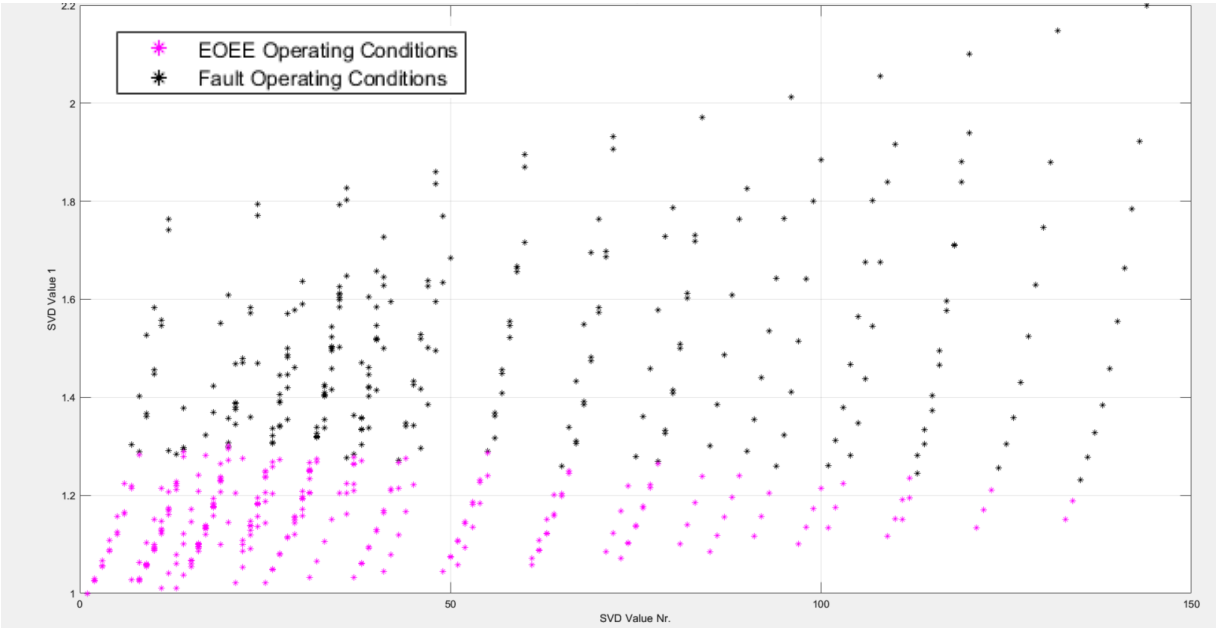
In Figure 7.2, the purple points represent the EOOE, the red points indicate the warning front, and the black points correspond to operating conditions deemed excessively degraded. While some overlap exists, especially between the warning front and degraded area, this does not discredit the EOOE. Notably, all overlapping operating conditions are already significantly degraded and would have previously passed through the warning front. This implies that, under proper monitoring, system operators should have received sufficient warning prior to reaching these critical states.

Additionally, alternative methods of visualising the EOOE may offer clearer or more intuitive representations than a 3D map and warrant further exploration.

7.4 Single SVD value graphs

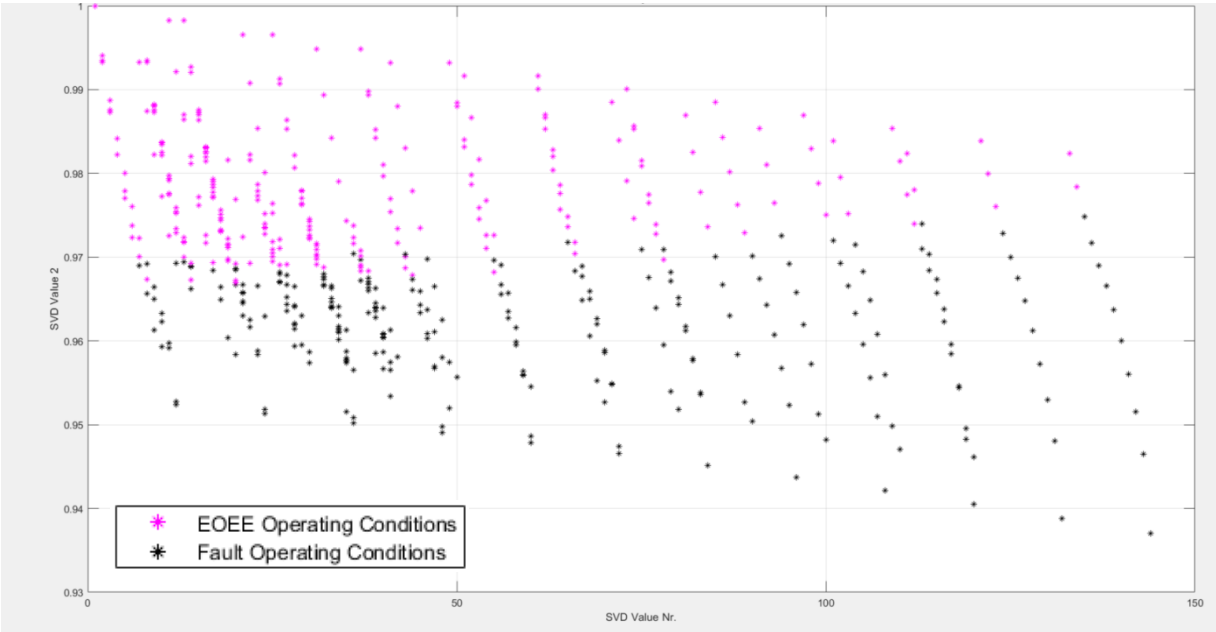
To further simplify the EOOE for practical application in condition monitoring, a reduced number of SVD values can be graphed. Plotting the individual normalised SVD values by index allows for a clearer assessment of how each singular value responds to varying operating conditions. The first five normalised SVD values are presented individually in Figures 7.3, 7.4, 7.5, 7.6, and 7.7. For these figures, no warning front is included, thus all data points corresponding to ME levels below 14% are depicted in black.

Figure 7.3: Normalised Degraded Graph SVD Value 1



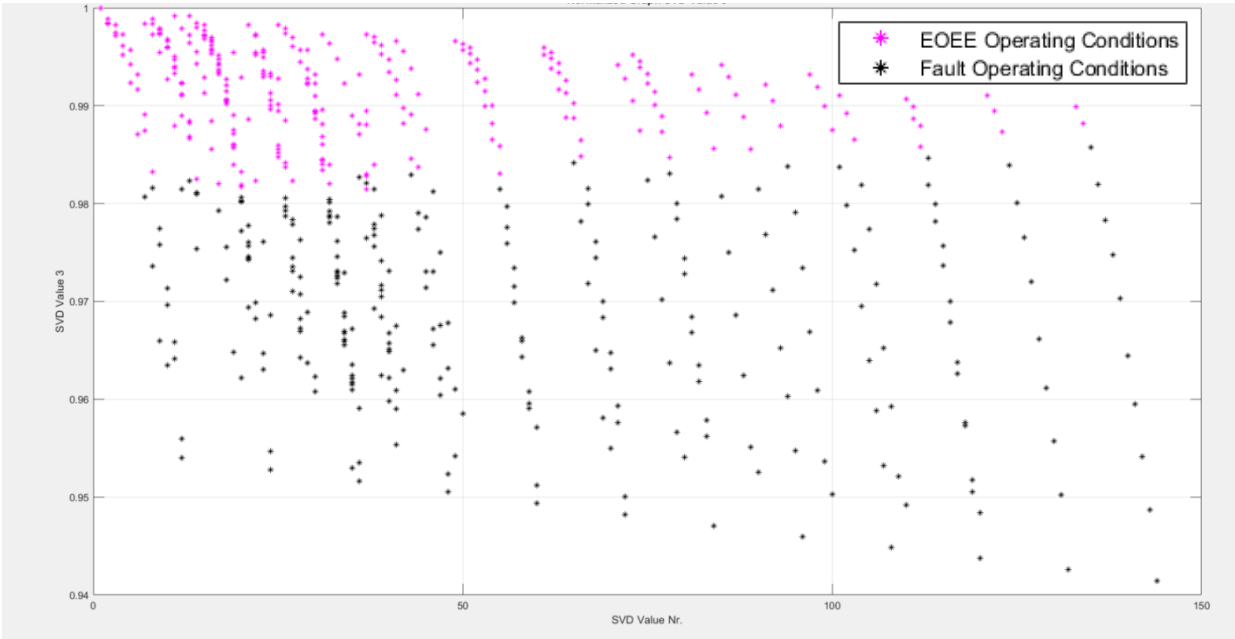
Source: Own Composition using MATLAB

Figure 7.4: Normalised Degraded Graph SVD Value 2



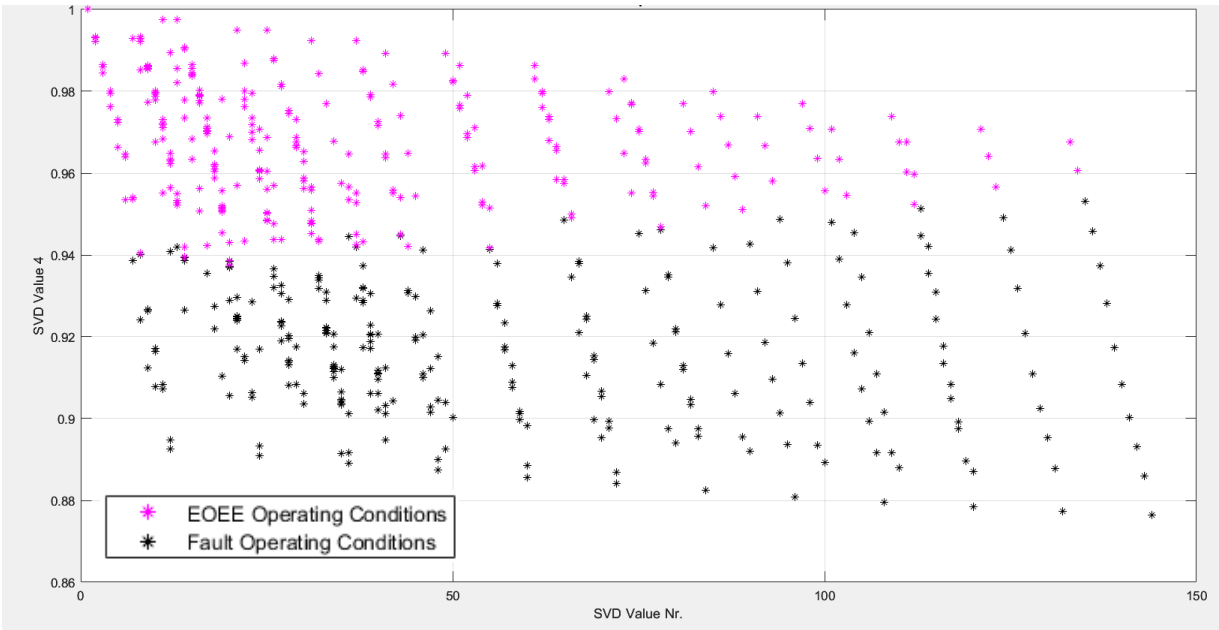
Source: Own Composition using MATLAB

Figure 7.5: Normalised Degraded Graph SVD Value 3



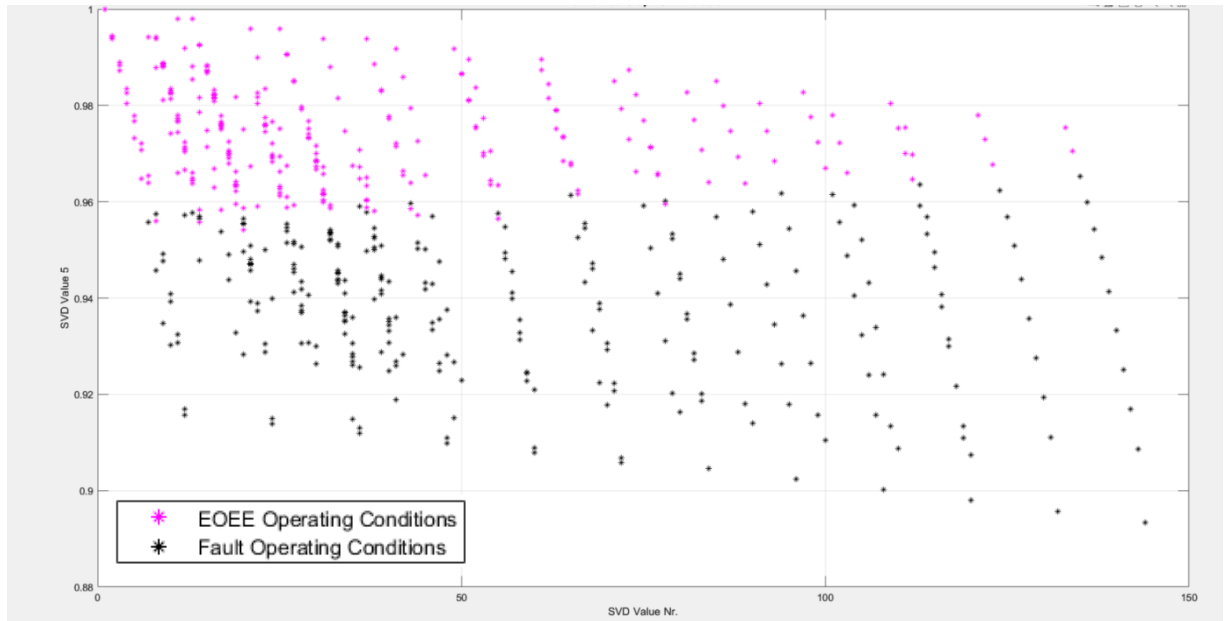
Source: Own Composition using MATLAB

Figure 7.6: Normalised Degraded Graph SVD Value 4



Source: Own Composition using MATLAB

Figure 7.7: Normalised Degraded Graph SVD Value 5

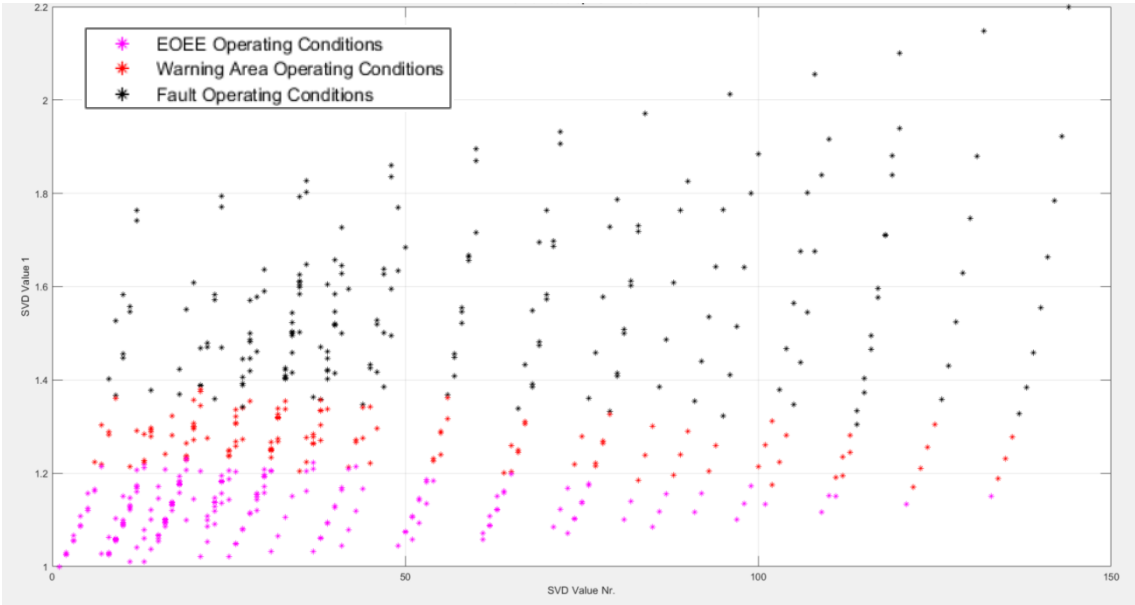


Source: Own Composition using MATLAB

From these graphs, it is evident that the more degraded operating points differ significantly from the values within the EOOE. However, some overlap remains, especially in later simulations. These specific simulations involve dual-component degradation, and when both components are moderately degraded, the resulting machine efficiencies tend to cluster near the 14% threshold. This observation suggests that, in some cases, operating conditions may fall outside the EOOE despite energy flow remaining within acceptable limits.

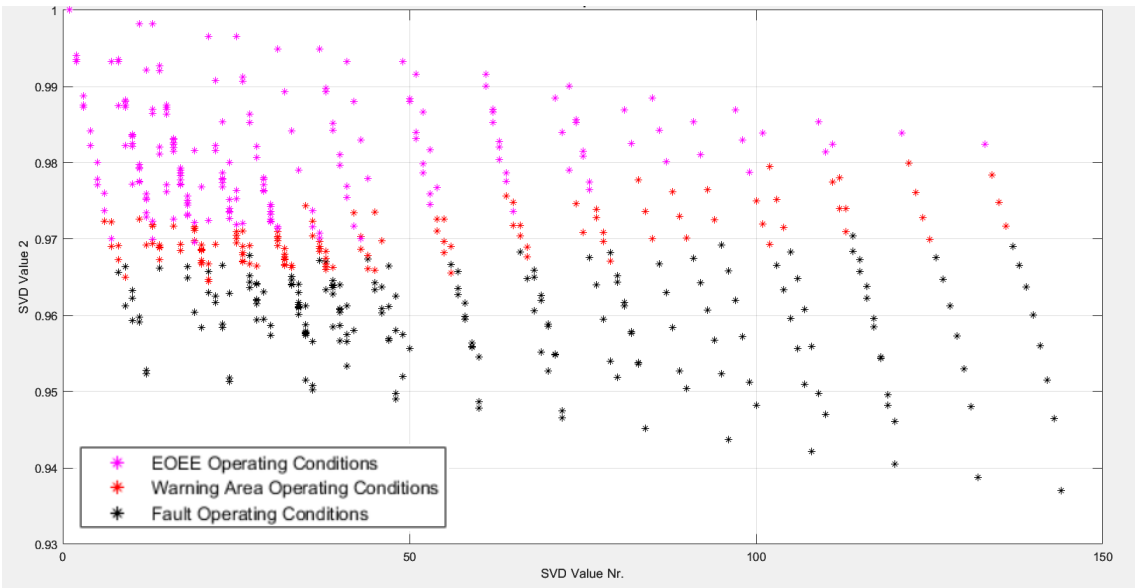
To address the remaining overlap, a warning front is introduced, as illustrated in Figures 7.8, 7.9, and 7.10 for the first three normalised SVD values, respectively. Although some overlap persists, the implementation of a warning front provides an added safety buffer, offering an early indication when the system approaches unacceptable levels of degradation.

Figure 7.8: Normalised SVD Value 1 with Warning Front



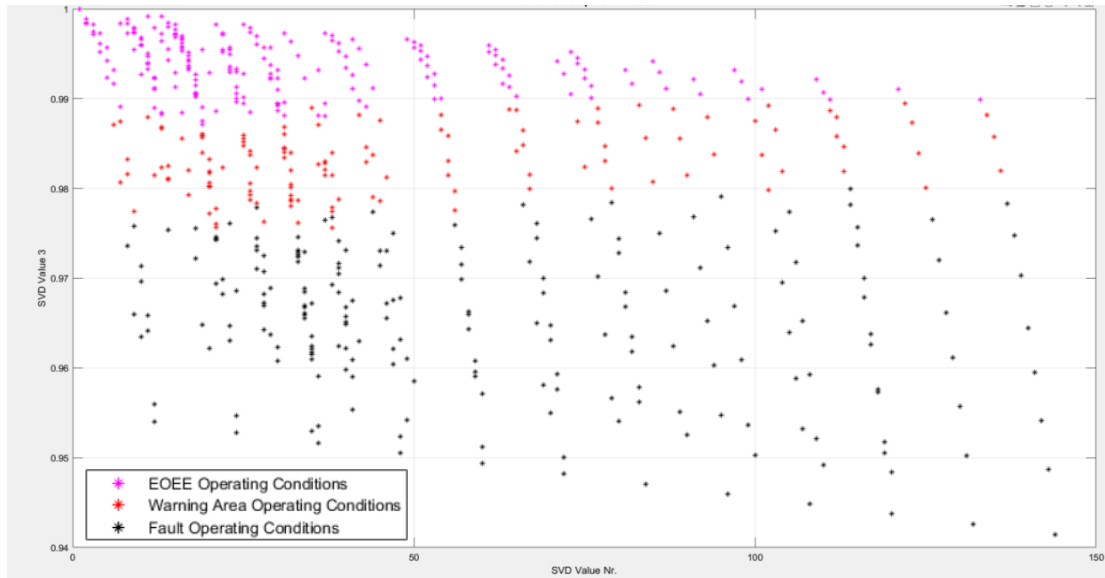
Source: Own Composition using MATLAB

Figure 7.9: Normalised SVD Value 2 with Warning Front



Source: Own Composition using MATLAB

Figure 7.10: Normalised SVD Value 3 with Warning Front



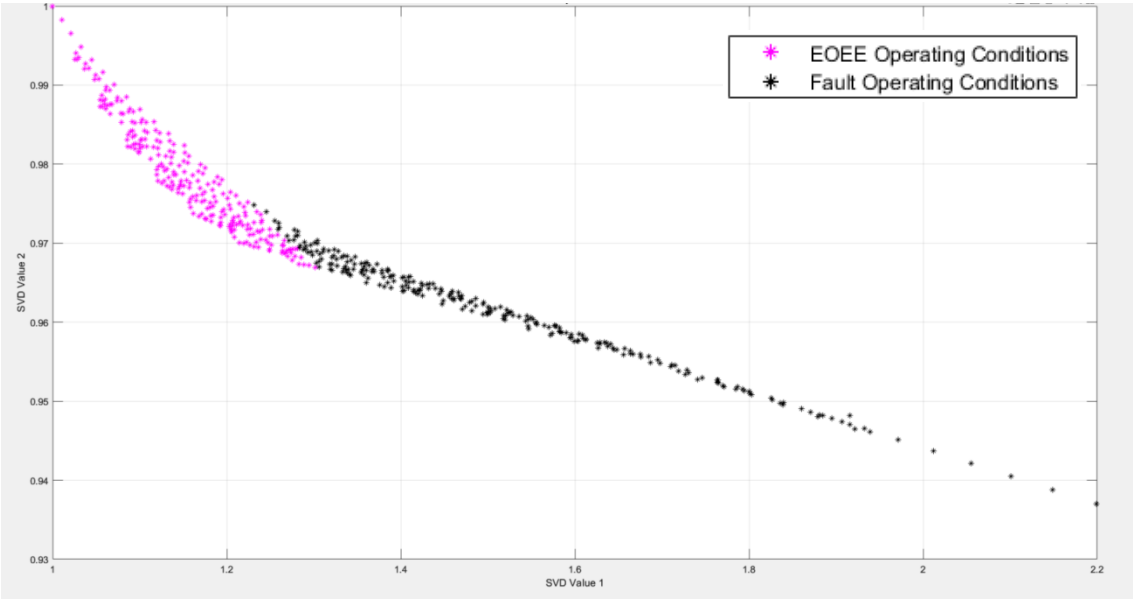
Source: Own Composition using MATLAB

In the visualisations, the red points represent the warning area, the black points indicate the fault area, and the purple points correspond to operating conditions within the EOOE. It is important to note that, due to the presence of overlap, these graphs do not allow for definitive classification of all operating points. However, these graphs remain useful in identifying general trends and boundaries between acceptable and unacceptable system behaviour. For instance, a tentative threshold may be suggested, such as SVD values exceeding 1.2, indicating that the system is likely operating outside the EOOE.

7.5 2D two SVD value EOOE

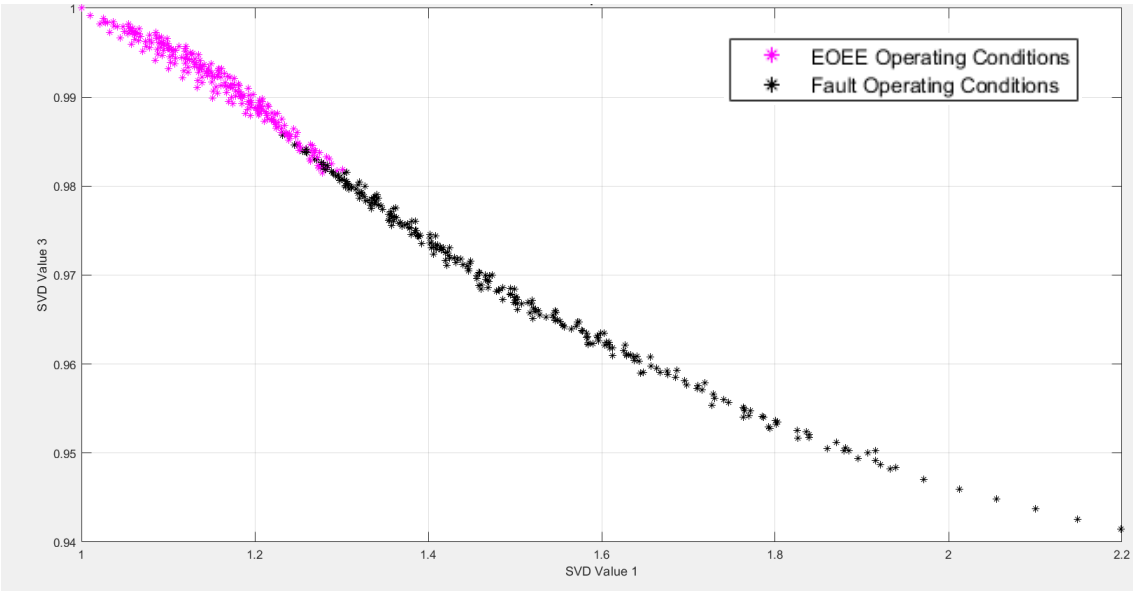
The SVD values have been analysed in both 3D and 1D spaces, each revealing a degree of overlap. To further investigate the effectiveness of the EOOE, a two-dimensional approach is now considered. Specifically, two SVD values are plotted against each other to evaluate if this approach offers improved distinction. Figure 7.11 presents a plot of SVD values 1 and 2, Figure 7.12 illustrates the relationship between SVD values 1 and 3, and Figure 7.13 displays the interaction between SVD values 2 and 3.

Figure 7.11: Normalised SVD Values 1 & 2 Graph



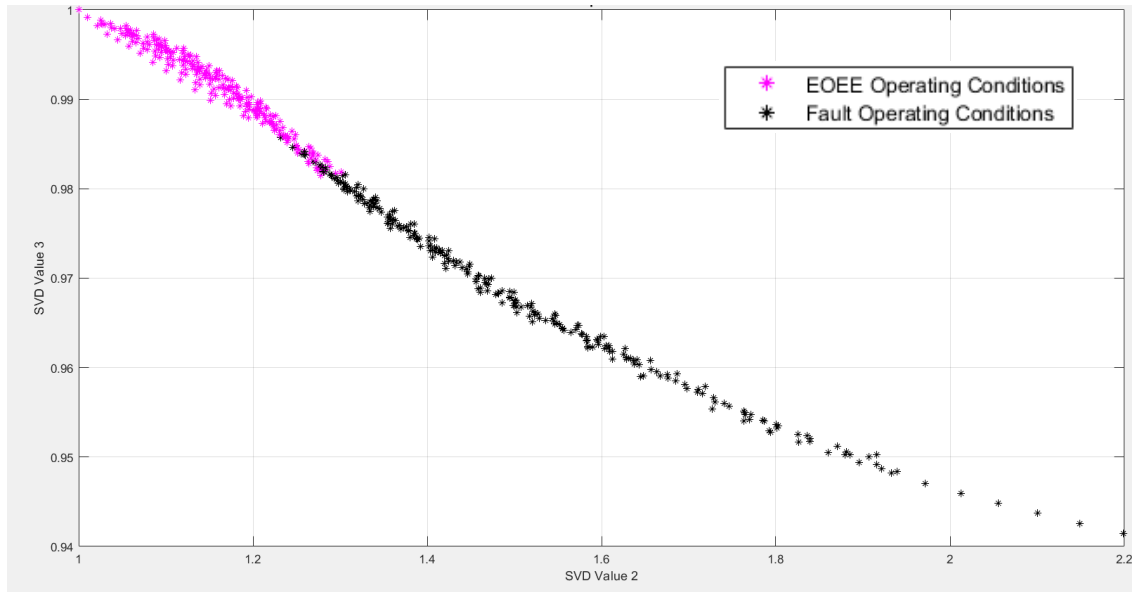
Source: Own Composition using MATLAB

Figure 7.12: Normalised SVD Values 1 & 3 Graph



Source: Own Composition using MATLAB

Figure 7.13: Normalised SVD Values 2 & 3 Graph

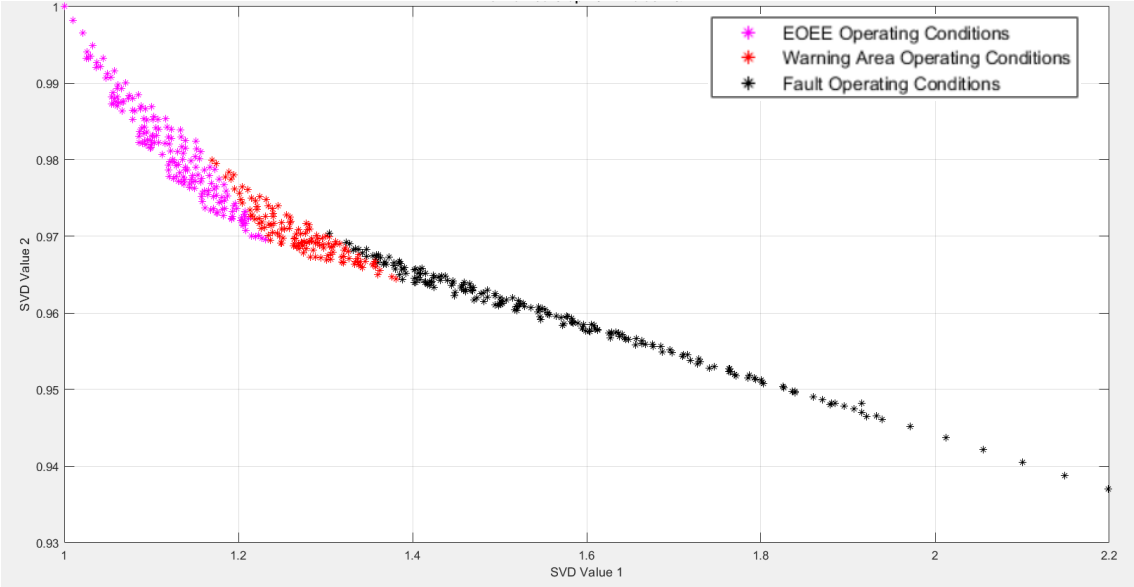


Source: Own Composition using MATLAB

A distinct separation is evident between the operating conditions within the EOEE and the conditions outside the EOEE. Although some overlap remains in some extreme cases, these instances typically correspond to machine efficiencies that are extremely close to the 14% threshold. From a system monitoring perspective, these points can be interpreted as early warnings rather than failures and may be regarded as acceptable exceptions. This interpretation extends to all points that appear in the “incorrect” region. While a formal Pareto front does not emerge, a clear boundary is observable, particularly when comparing the first and second normalised SVD values, effectively delineating the EOEE from non-conforming operating states.

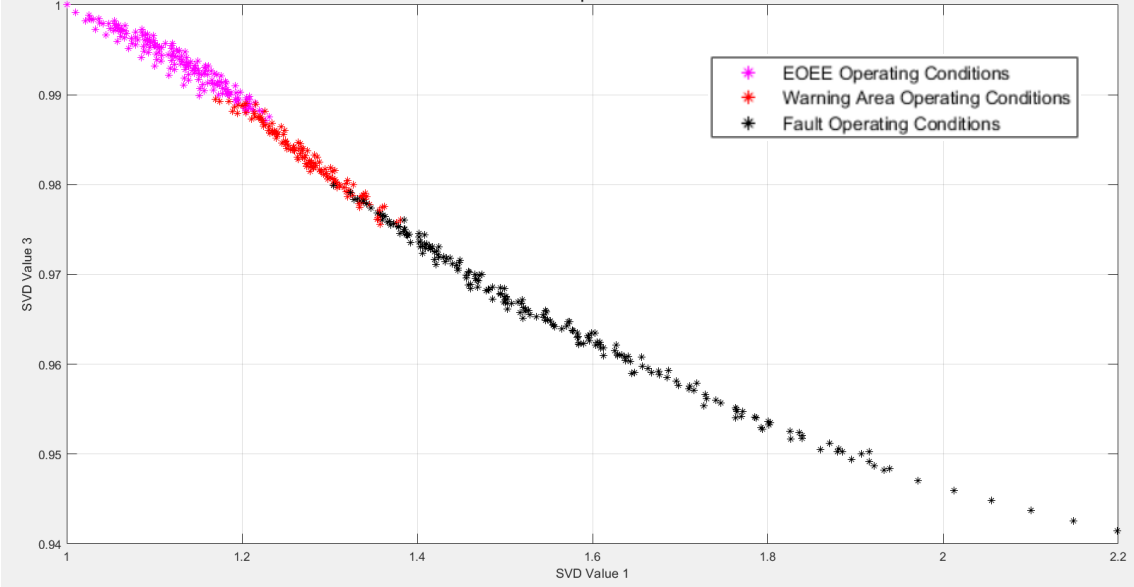
When analysing the normalised SVD value graphs for SVD values 1 and 2, it becomes apparent that a boundary can be established to separate the EOEE area from the degraded region, with only minimal misclassification (typically one or two points). As in previous analyses, a warning front can be introduced to enhance the detection of degradation before significant performance loss occurs. This allows for early intervention before critical thresholds are reached. The effectiveness of this approach is illustrated in Figures 7.14, 7.15, and 7.16.

Figure 7.14: Normalised SVD Values 1 & 2 Graph with Warning Area



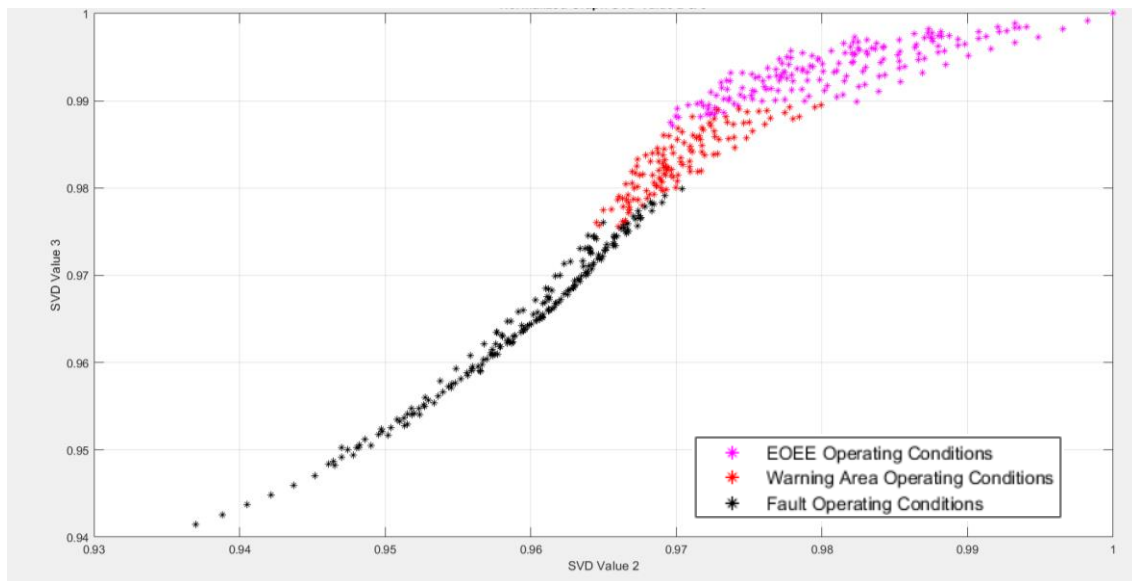
Source: Own Composition using MATLAB

Figure 7.15: Normalised SVD Values 1 & 3 Graph with Warning Area



Source: Own Composition using MATLAB

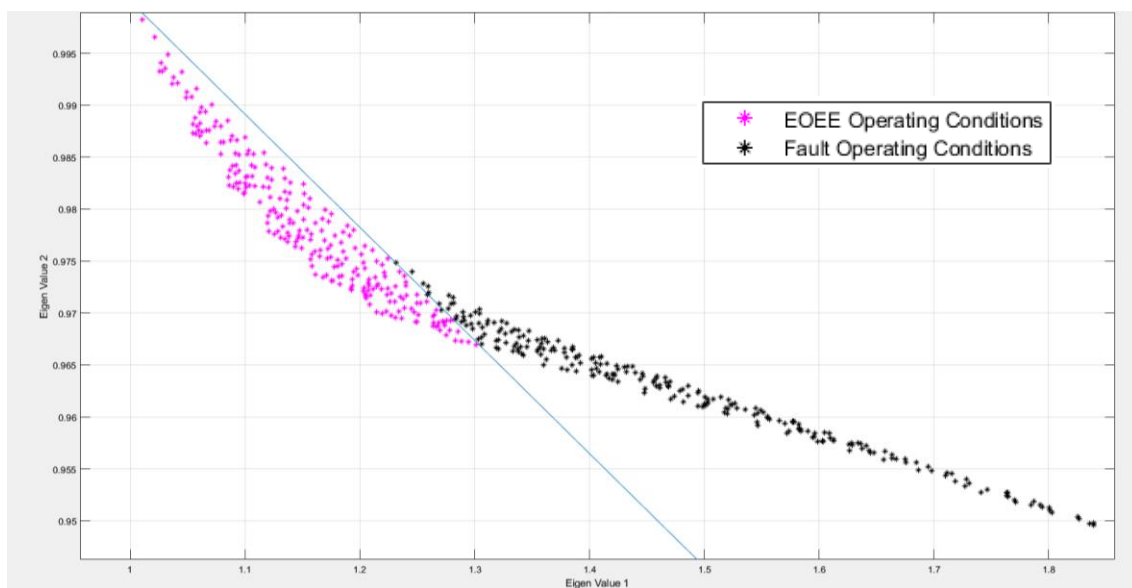
Figure 7.16: Normalised SVD Values 2 & 3 Graph with Warning Area



Source: Own Composition using MATLAB

In the graphs comparing SVD values 1 and 3, as well as 2 and 3, considerable overlap is observed along the boundaries between the EOOE and non-EOOE regions. Despite this, these visualisations still offer useful insight into the general classification of operating points. In contrast, the graph plotting SVD values 1 and 2 provides a markedly clearer distinction between the two regions, with minimal or no overlap at all. In fact, this particular graph allows for the formulation of two definitive conclusions regarding the separation between normal and degraded operating conditions.

Figure 7.17: Normalised SVD Values 1 & 2 Graph with Separating Line Function



Source: Own Composition using MATLAB

When defining the EOOE as encompassing all operating points with a ME greater than 14%, the area located beneath the normal point (1.1) and below the demarcating line $f(x)$, as illustrated in Figure 7.17, can be classified as the EOOE:

$$f(x) = -0,10884x + 1,10884$$

All remaining points are considered to represent excessively degraded or damaged operating conditions. While some points may fall on the incorrect side of the demarcating line, these typically lie near the boundary and thus reflect marginal cases. In a properly monitored system, any operating condition approaching this boundary should prompt increased scrutiny. Notably, the demarcating line intersects the normal point, indicating that the area between the origin and the line represents the EOOE. Although this boundary does not constitute a true Pareto curve, it exhibits characteristics that closely resemble one.

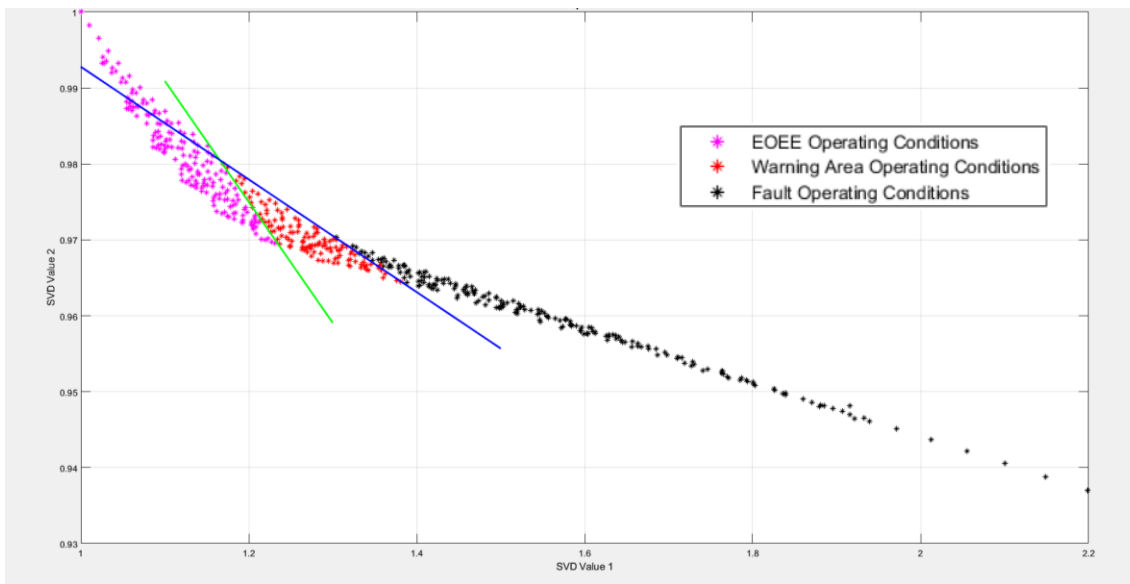
When considering a warning front for ME in the range of 13% to 14%, the EOOE can be defined as the area encompassing all points between the normal point (1.1) and the boundary:

$$g(x) = -0,159339x + 1,1662$$

The warning front can be defined as the area between the function $g(x)$ and the boundary:

$$u(x) = -0,074197x + 1,06699$$

Figure 7.18: Normalised SVD Values 1 & 2 Graph with Warning Area and Separating Functions



Source: Own Composition using MATLAB

In the graph, $g(x)$ is depicted in green and $u(x)$ in blue. As shown in Figure 7.18, the best-fit line distinguishing the EOOE and the warning area intersects the boundary of the EOOE. This intersection does not pose an issue; rather, it facilitates clearer visual differentiation. The area between $g(x)$ and $u(x)$ defines the warning front. All remaining points outside this area are classified as either too far from the EOOE or as degraded conditions. A minor overlap can be observed between some points within the warning front and points in the degraded area. These overlapping points can reasonably be categorised as part of the fault or

degraded area, as the corresponding operating conditions are already approaching the critical threshold. Consequently, system operation within this area warrants immediate attention and assigning these points to the fault category aligns with the intended function of the EOEE framework.

7.6 Validation of the EOEE

To ensure the validity and reliability of the results, it is essential to demonstrate repeatability under varying operating conditions of the system. This verification step is necessary to ensure that the EOEE is independent of specific operating points. If the EOEE, developed earlier in the chapter, is validated, similar trends should emerge when applying a different set point for the PT.

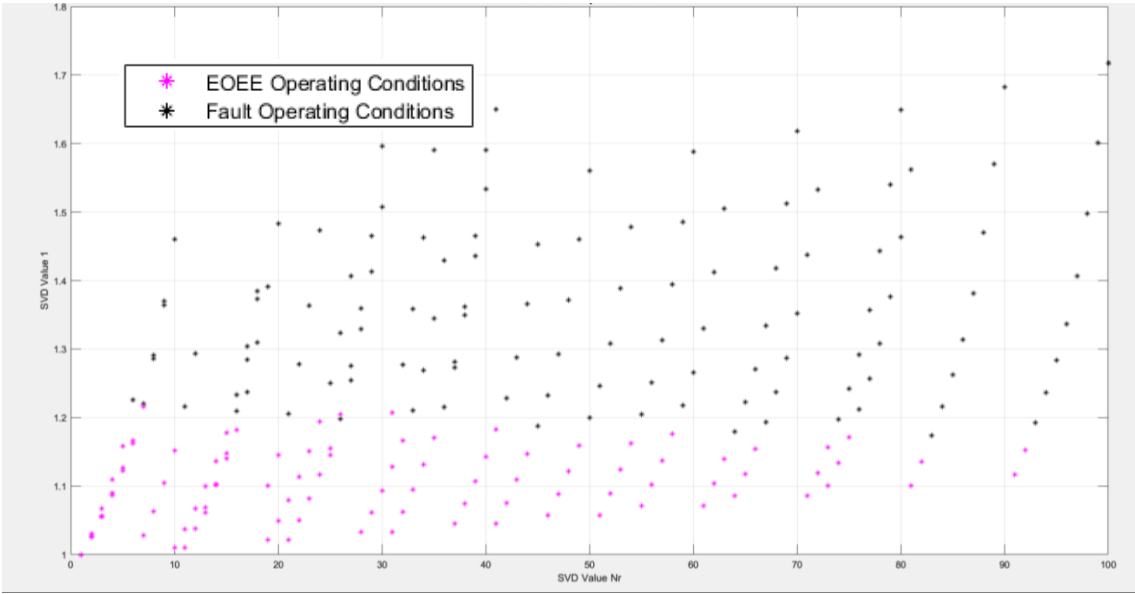
For this study, a set point of 150 kW will be used, which is 50% higher than the initial set point, to ensure a sufficiently large gap between the two conditions. Choosing a lower set point than the initial point risks producing an artificially high ME, and since the study is not focused on optimising the PBMM, such data would not be as informative. The same boundary conditions for the EOEE will be maintained, and the resulting SVD values will be graphed below.

As noted in (Neser, van Schoor and Uren, (2021), an elevated set point of 120 kW has been used previously, demonstrating that operating above 100 kW is feasible. Regarding the PBMM setup in FLOWNEX, the validity of using a 150 kW set point is well supported. As discussed in Chapter 5, the component charts indicated the feasible operating range for the system, with any infeasible set points triggering errors. For example, a 400 kW set point for the PT leads to errors when trying to solve and an unsolvable steady state, confirming the upper limits of system operations.

A set point of 150 kW represents a sufficiently high operating condition that pushes the system beyond the baseline of 100 kW, while still remaining within a solvable range that produces accurate results. It is not only a feasible set point, but also one that is far enough removed from 100 kW to ensure that the results are meaningfully distinct. Therefore, if the resulting SVD values exhibit similarities with the results of the 100 kW set point, it would provide strong evidence that the EOEE is operating point independent and applicable across a wide range of operating conditions.

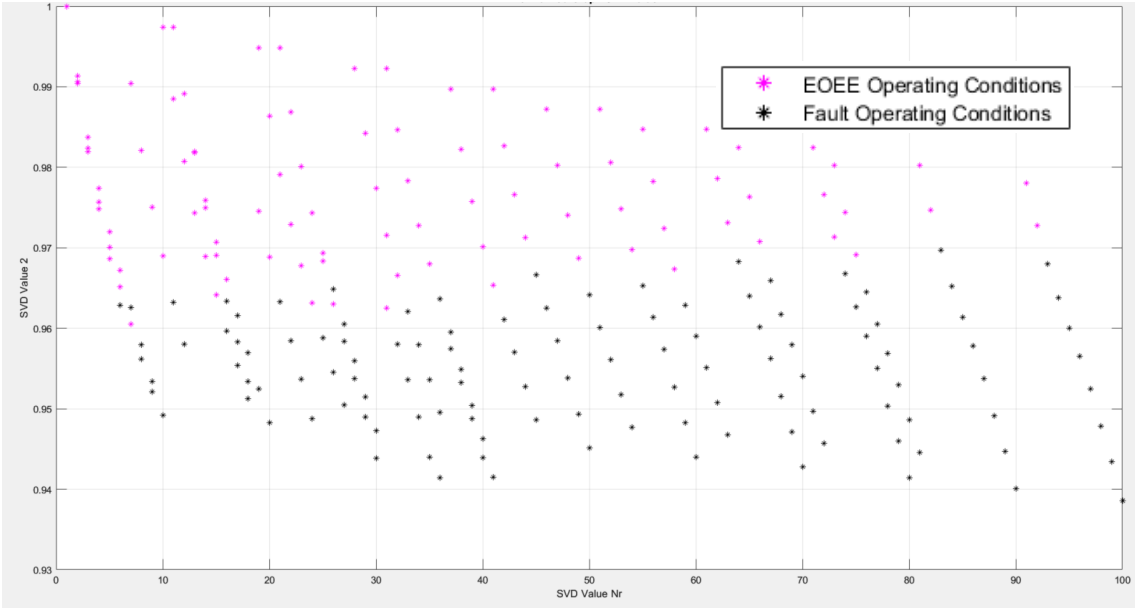
The first three SVD values will be graphed individually, in the same manner as before, and are illustrated in Figures 7.19, 7.20, and 7.21.

Figure 7.19: 150 kW SVD 1



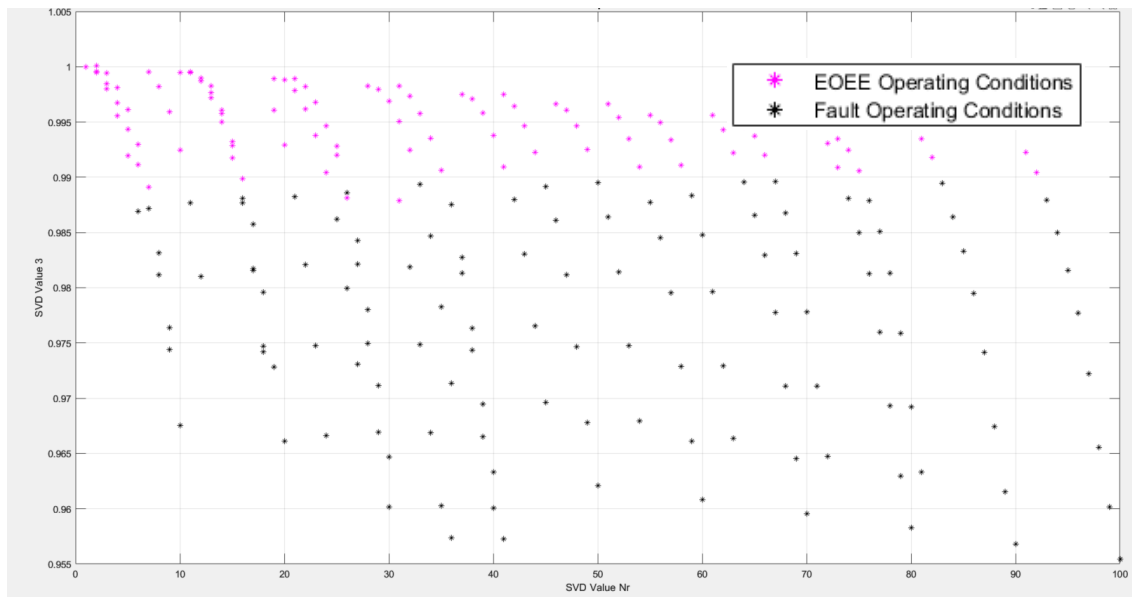
Source: Own Composition using MATLAB

Figure 7.20: 150 kW SVD 2



Source: Own Composition using MATLAB

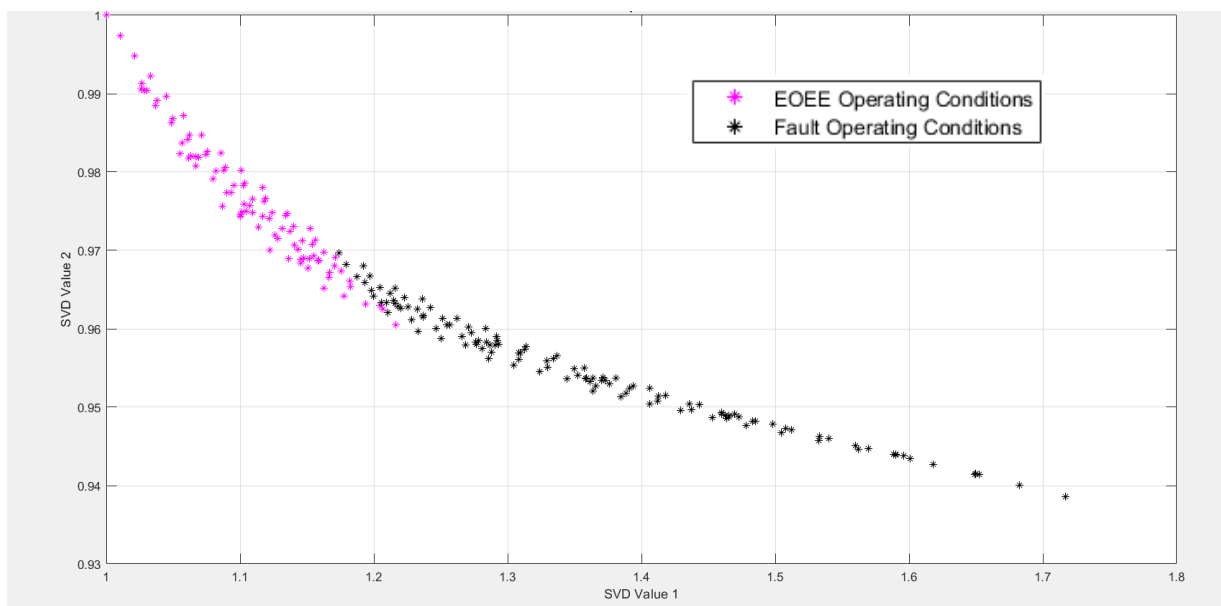
Figure 7.21: 150 kW SVD 3



Source: Own Composition using MATLAB

As observed in the preceding Figures, the scale of the SVD values changes; however, the overall trend remains consistent. Beyond a certain point, the normalised SVD values clearly indicate that degradation has exceeded acceptable limits and the system is no longer operating within the EOEE. Furthermore, the SVD values can be plotted against each other once again. Figure 7.22 illustrates the same trend observed previously in Figure 7.11, where SVD values 1 and 2 are graphed against one another.

Figure 7.22: 150 kW SVD 1 & 2

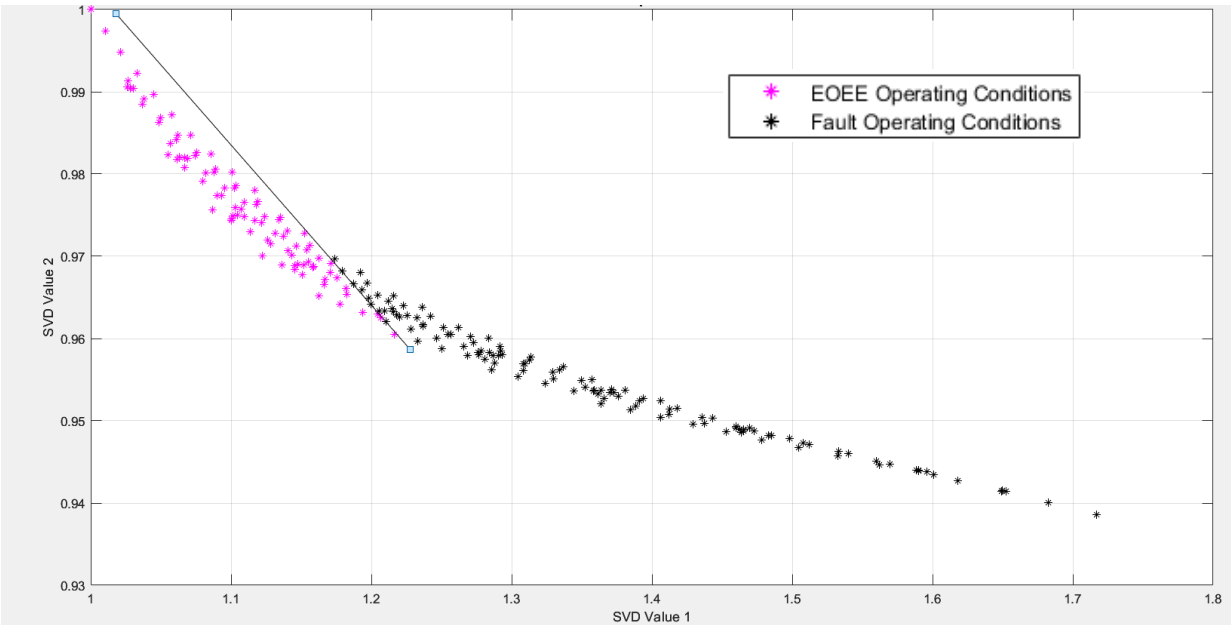


Source: Own Composition using MATLAB

When considering the preceding, it is observed that a line can be drawn to distinguish the points inside and outside of the EOEE, as depicted in Figure 7.23. This demonstrates that

the EOOE can be applied across different operating settings or output goals for the same system and that the information it provides offers an accurate representation of the system's current condition.

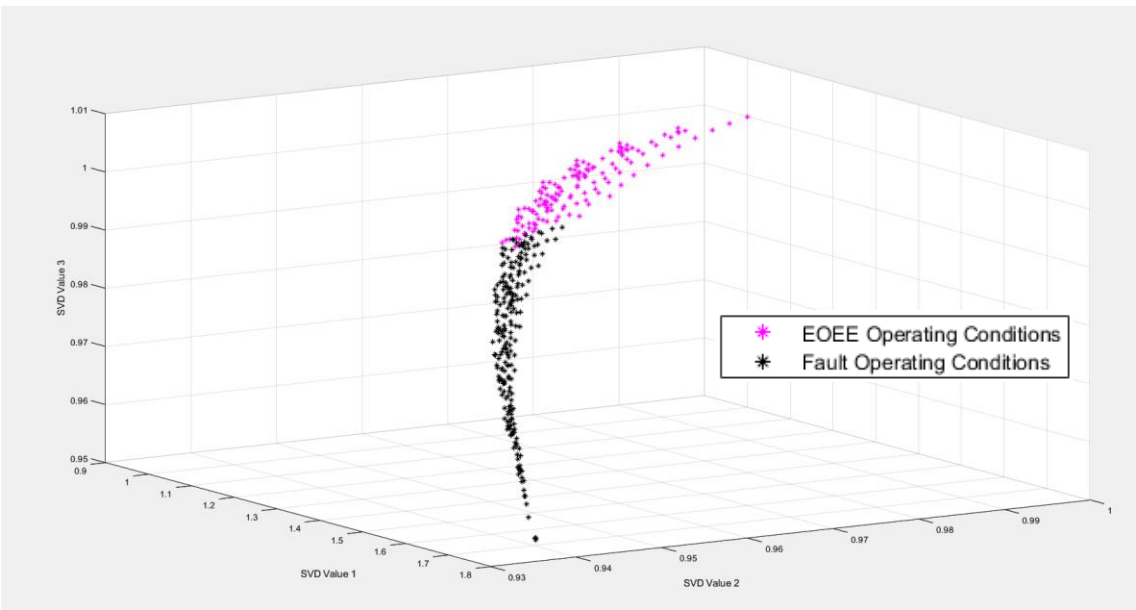
Figure 7.23: 150 kW SVD 1 & 2 with a Separation Line



Source: Own Composition using MATLAB

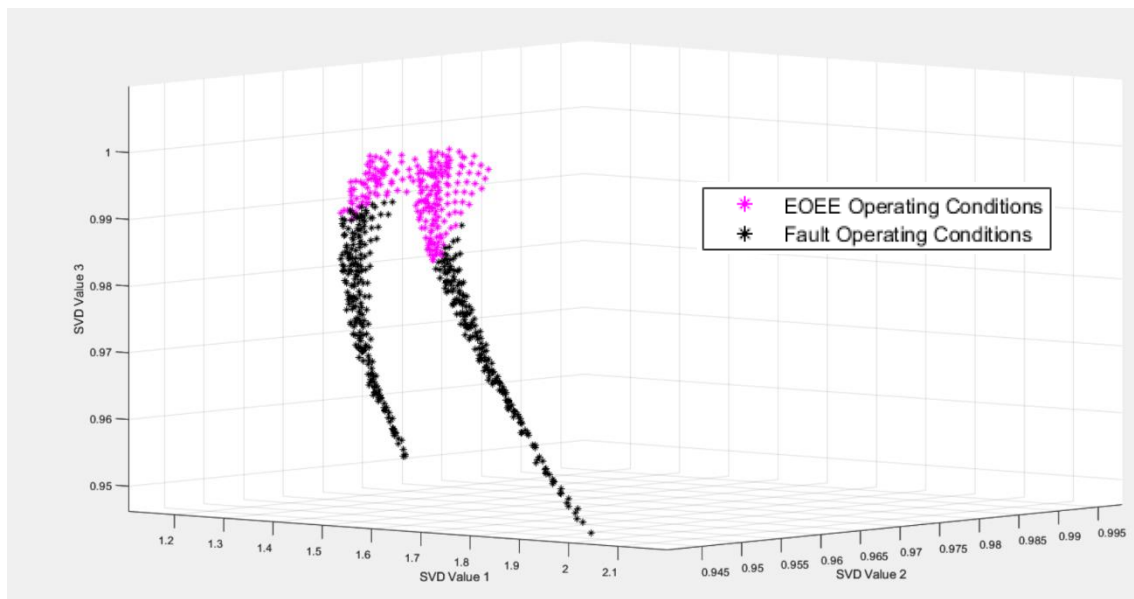
To properly compare the 100 kW and 150 kW set points, the corresponding 3D graphs should also be examined. In Figure 7.24, the SVD points 1, 2, and 3 for the 150 kW set point are plotted in 3D. Figure 7.25 overlays the 3D graphs for both the 100 kW and 150 kW set points, allowing for a direct visual comparison.

Figure 7.24: 150 kW 3D SVD Graph



Source: Own Composition using MATLAB

Figure 7.25: 100 kW vs 150 kW 3D Graph



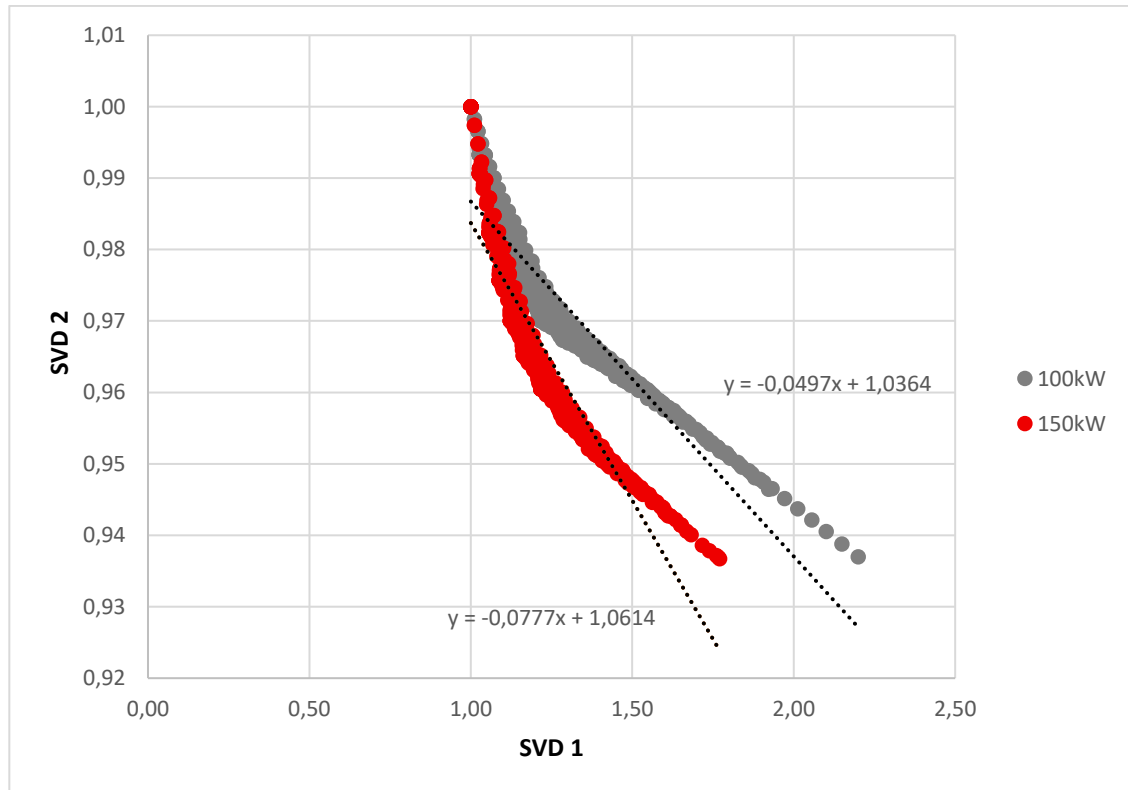
Source: Own Composition using MATLAB

The cluster of points on the right in Figure 7.25 represents the 100 kW set point, while the cluster on the left corresponds to the 150 kW set point. Although the EOOE points (shown in purple) are not identical across the two set points, this discrepancy is likely due to the differences in how degradation effects scale with increased power output. Despite this, the same general pattern is evident. Notably, there appears to be some overlap in the “border” region of the 150 kW set point, whereas the 100 kW data do not exhibit this overlap. This overlap is likely a result of the greater variety of operating conditions introduced by the higher power demand.

7.7 EOOE feature extraction

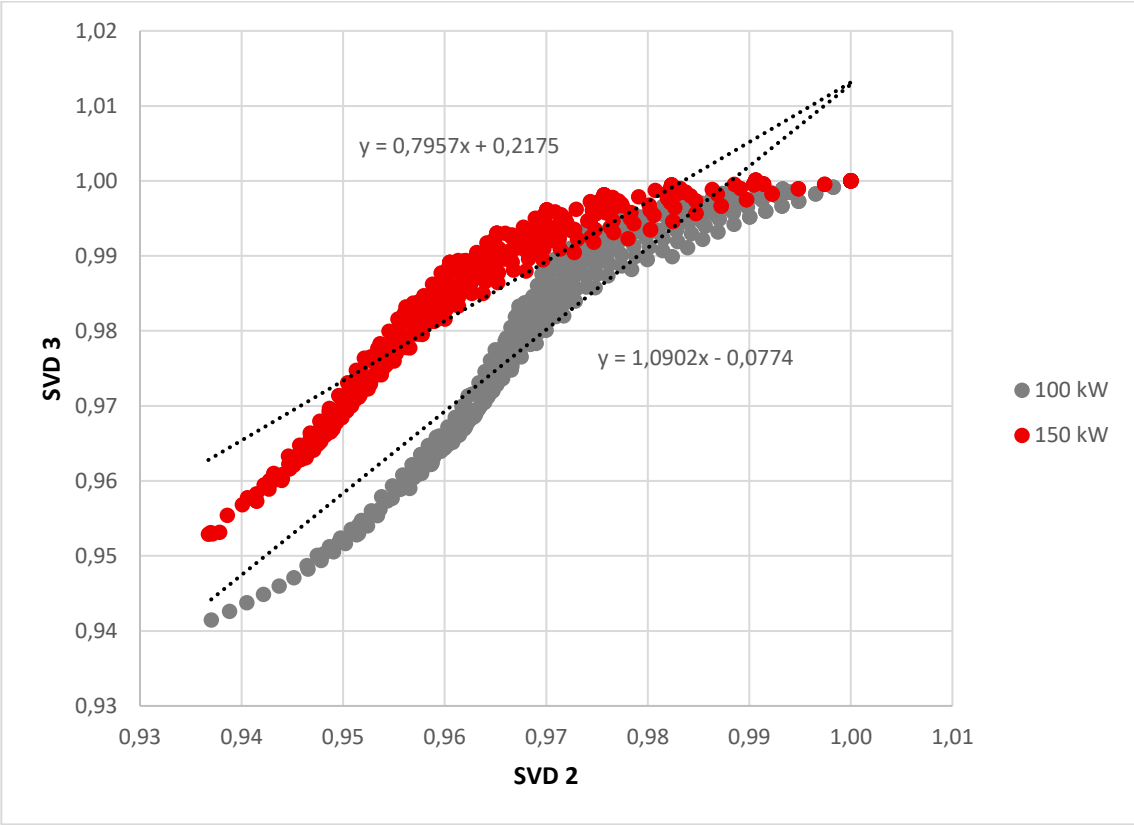
For the EOOE to be effective across different operating conditions, operating point-independent features must be identified. This section compares trends from the 100 kW and 150 kW set points to determine shared characteristics. By plotting results from both operating conditions, with varying levels of degradation, on a two-dimensional scale, a trend line can be derived. The resulting trend demonstrates consistency of the EOOE across operating points, thereby supporting the independence of operating points. This relationship is illustrated in Figures 7.26, 7.27, and 7.28.

Figure 7.26: SVD 1 & 2 Comparison of 100 kW vs 150 kW Operating Point



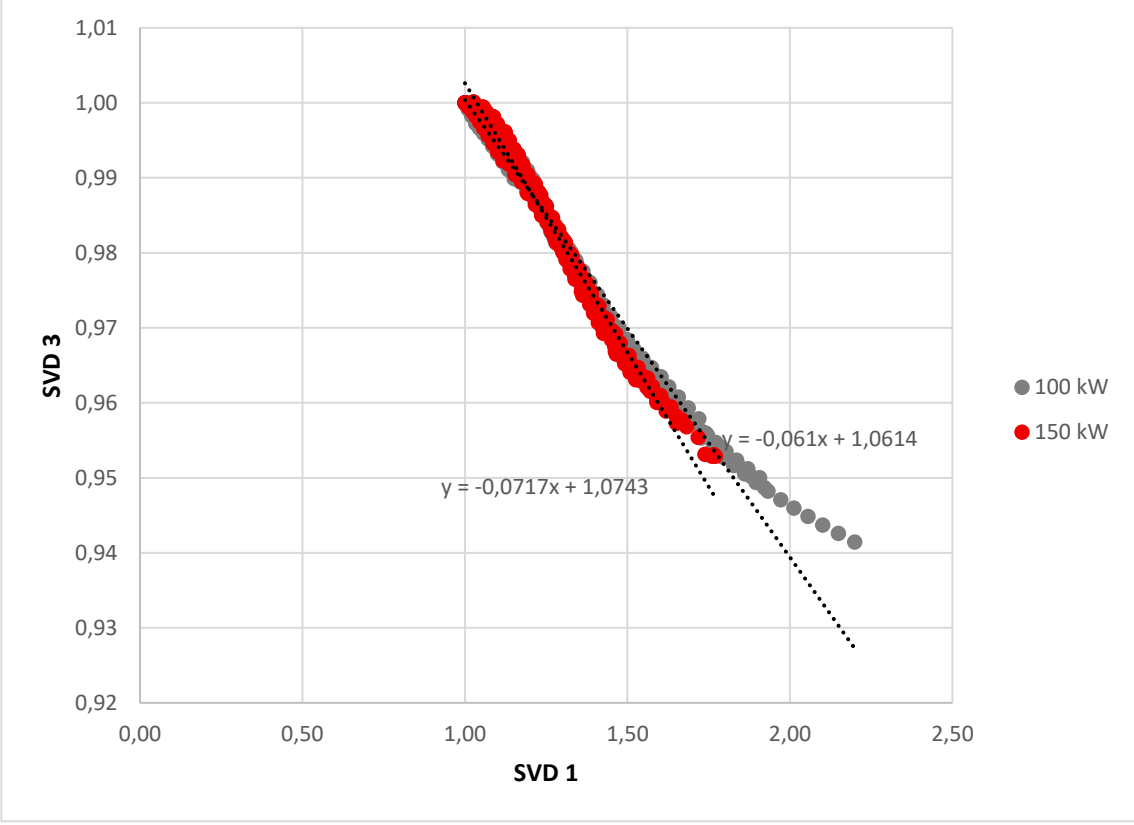
Source: Own Composition using MATLAB

Figure 7.27: SVD 2 & 3 Comparison of 100 kW vs 150 kW Operating Point



Source: Own Composition using MATLAB

Figure 7.28: SVD 1 & 3 Comparison of 100 kW vs 150 kW Operating Point



Source: Own Composition using MATLAB

From Figures 7.26, 7.27, and 7.28, it is evident that trendlines intersect but display a consistent pattern. Data points from different operating conditions are clustered and form similar shapes as degradation increases. This indicates that variation in operating conditions primarily reflects a difference in scale. As a result, the EOEE demonstrates applicability independent of specific operating points. Trendlines for all graphs are shown in Table 7.2 and provide support for the initial analysis, with the 150 kW operating point aligning with the 100 kW results.

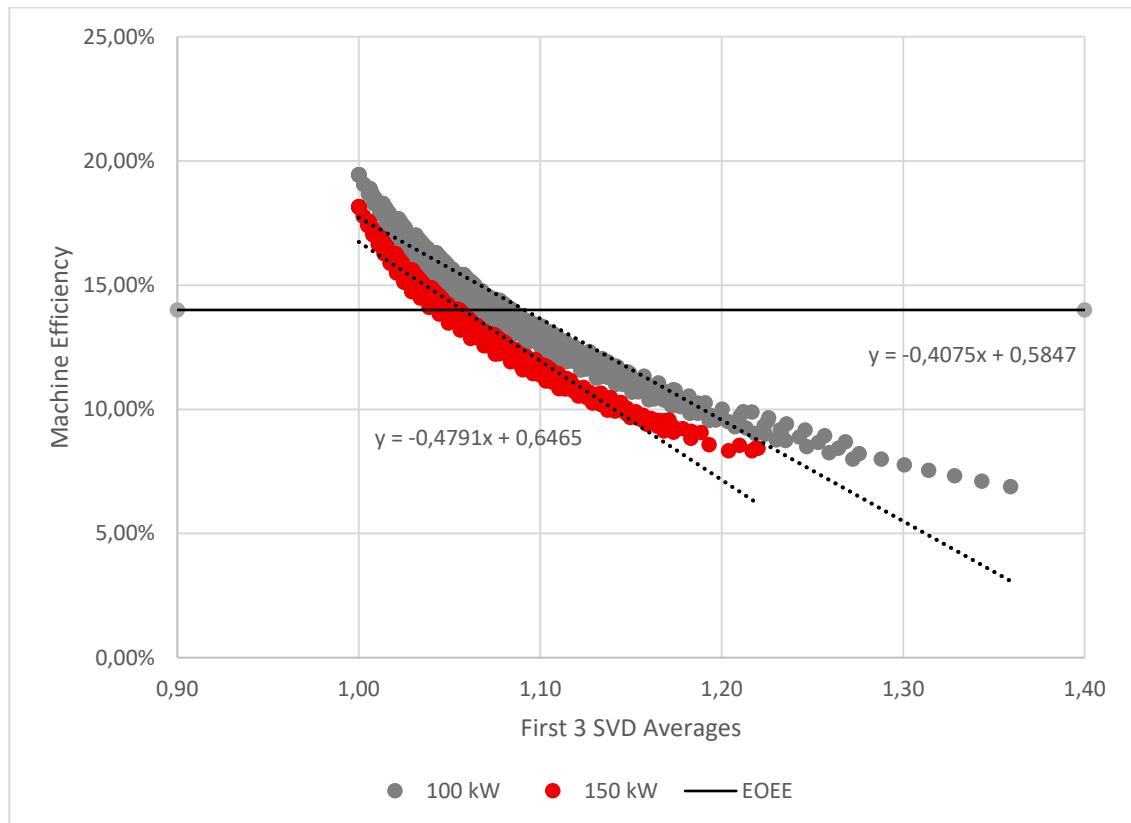
Table 7.2: Trendlines of SVD Graphs

SVD Valus	100 kW trendline	150 kW trendline
1&2	$y = -0,0497x + 1,0364$	$y = -0,0777x + 1,0614$
2&3	$y = 1,0902x - 0,0774$	$y = 0,7957x + 0,2175$
1&3	$y = -0,061x + 1,0614$	$y = -0,0717x + 1,0743$

Source: Own Composition

When averaging the first three SVD values for each operating point, the results can be graphed against ME, as illustrated in Figure 7.29. Both set points, 100 kW and 150 kW, respectively, produce the same pattern when plotting operating points as a result of degradation. All points above the grey EOEE line are within the EOEE. Some points overlap, indicating that average SVD values coincide, while one point lies inside the EOEE and another outside.

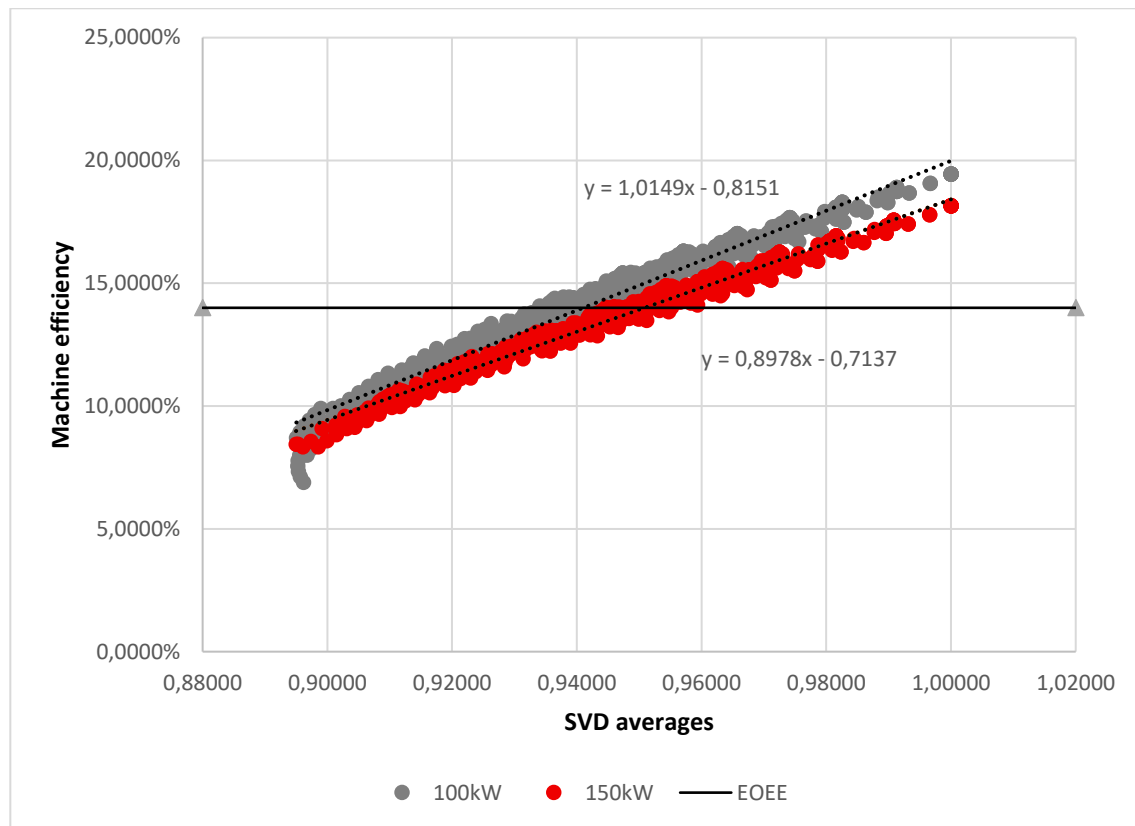
Figure 7.29: First 3 SVD Values Average



Source: Own Composition using MATLAB

It is necessary to consider all the SVD points, as there are more than just the first three; a total of 11 points exist for each operating condition and each level of degradation. To address this, the average of all SVD points is calculated and plotted against ME, the metric that determines whether a point lies inside or outside the EOEE. In Figure 7.30, the trendlines shown in Table 7.3, although not parallel, are closely aligned, and the points form similar shapes and patterns. The trendlines are closer when only the average of the first three SVD values is considered. This further suggests that only the first three SVD values are required to define the EOEE, as these values contain the most important information.

Figure 7.30: SVD Averages vs Machine Efficiency



Source: Own Composition using MATLAB

In Figure 7.29, all points above the line are within the EOEE, and all points below the line are outside of it. It is also evident that no single vertical line can separate all points inside and outside of the EOEE into distinct regions. These points represent the overlap, meaning the average SVD values could correspond to either being inside or outside the EOEE. Table 7.2 compares the trendlines for both set points. The gradient of the average of the first three SVD values is negative, while the gradient of the average of all SVD values is positive. The difference in gradients between using the first three SVD values and all SVD values is not critical; it indicates that including the additional eight SVD values slightly alters the gradient, but the change is minimal due to the small values involved. What is, however, important is that the gradient remains consistently negative or positive for the two set points, regardless of whether the first three or all SVD points are used.

Table 7.3: SVD Average Trendline

	100 kW	150 kW
First 3 SVD average trendlines	$y = -0,4075x + 0,5847$	$y = -0,4791x + 0,6465$
All SVD points average trendline	$y = 1,0149x - 0,8151$	$y = 0,8978x - 0,7137$

Source: Own Composition

In Table 7.4, all the simulations run for an operating set point are shown as the total number of points. The number of overlapping points represents the number of simulations that overlap in Figure 7.29—points with average SVD values that fall both inside and outside of the EOOE. The low number indicates the lowest average SVD value that overlaps, while the high number indicates the highest average SVD value that overlaps.

Referring to Figure 7.30, the low number corresponds to the lowest average SVD value that is out of place; in other words, the lowest value for which the operating point is still within the EOOE, but the average SVD value suggests it should be outside the EOOE. Conversely, the high number represents the highest average SVD value that, based on Figure 7.30, should be inside the EOOE, but the ME places it outside the EOOE.

When calculating the number of operating points between the high and low values as a proportion of the total number of points, the error for the 100 kW set point is 14.61% and for the 150 kW set point, it is 15.07%. The mean value refers to the midpoint between the highest and lowest SVD average values—in other words, the centre between the two most extreme outliers for each set point. The error improves when calculated as the difference between the mean and both extremes, divided by the mean. The difference in mean values between the two set points is expected, considering that the 150 kW values are consistently larger than the 100 kW values. This indicates that the results scale proportionally based on the operating point and set points of the PT.

Table 7.4: EOOE Set Point Comparison

	100 kW	150 kW
Number of overlapping points/simulations	70	66
Total number of points/simulations	479	438
Error	14,61%	15,07%
Low number	0,9340 2	0,9447 0
High number	0,9465 4	0,9547 4
Mean	0,9402 8	0,9497 2
Difference	0,0062 6	0,0050 2
Error	0,67%	0,53%

Source: Own Composition

7.8 Conclusion

Based on the various graphs presented in this Chapter, the EOEE can be clearly delineated. A boundary line can be established to distinguish between operating points that fall within the EOEE and the points that fall outside of the EOEE. This implies that future operating conditions can be classified in accordance with the position relative to this boundary. Furthermore, the EOEE has been shown to be independent of the operating point, indicating that the EOEE application remains valid across different operating conditions. As such, the EOEE provides a reliable indication of whether a system is functioning within acceptable performance measures.

Furthermore, a warning area, as outlined in Chapter 4, can be defined to further distinguish the EOEE from unacceptable operating conditions. This area also holds practical significance, as it serves as an early indicator that maintenance or corrective action may be required to prevent system failure.

After comparing the two different set points of 100 kW and 150 kW, the validity of the EOEE concept is reinforced, as the results exhibit consistent patterns, shapes, and trends, despite variations in size or magnitude. This consistency is evident in the trendlines of the graphs and the comparable distribution of the plotted operating points. The mean value for each case can serve as the threshold for determining whether an operating condition falls within or outside the EOEE. By applying a 1% margin of error to the mean SVD average value, the EOEE can be defined with a high degree of accuracy.

Chapter 8 Conclusion

8.1 Introduction

From previous work and the work done in this study, it is proven that the PBMM can be characterised through energy. From this characterisation, the condition of the system can be monitored to see the effect of faults, or in the case of this study, degradation, which results in different operating conditions. These conditions can be converted into SVD values and plotted in an EOEE, although the process can be refined and improved. This chapter provides a summary of the findings obtained through characterising the PBMM through energy, introducing degradation and using the different operating conditions to create the first iteration of the EOEE.

8.2 Conclusion on simulation results of the EOEE

The degradation of a component affects the overall efficiency of the larger system and impacts the energy flow between components. This energy flow can be used to find the EOEE that determines whether an operating point or condition falls within or outside of it. There are multiple ways of looking at the EOEE:

- 3D Map
- Singular value SVD graph
- 2D SVD value graph
- Machine efficiency over the average of SVD values

All the EOEE graphs (except for the machine efficiency vs. average SVD) can be given a warning front or error front to help separate the EOEE from degraded or damaged operating conditions. Different regions can be created and clearly differentiated from each other. Lines can be drawn to completely separate the different regions.

The regions of the EOEE, the points outside of the EOEE, and a third warning region can also be implemented to create a more gradual shift between acceptable and non-acceptable operating conditions. The 3D map shows all the necessary information, though it can be a bit confusing. These graphs clearly indicate the operating conditions and desirable operating ranges.

Further, the data and graphs obtained from the first three, or dominant SVD values, provide enough information to create an EOEE. The SVD graph gives a good overview; however, it gives too little information and, as a result, causes a great deal of overlap on the different operating conditions. Distinct lines can be drawn on the 2D graphs to separate the operating points in the EOEE from points outside of it; the SVD values 1 and 2 are especially useful for this purpose. Further, a line can be drawn on the 2D graph to separate the EOEE and other regions. The lines drawn, however, are only relevant to a specific set point.

When creating the EOEE, a simple approach could be to calculate the SVD average of a few simulations and determine a mean that represents the EOEE dividing line. When applying

the correct margin of error, the mean should be accurate every time; however, it is important to note that each mean will only apply to a specific operating output.

8.3 Reflection on Objectives

8.3.1 Experimental platform

The PBMM was the platform used in this study. Through the use of FLOWNEX, different operating points with different levels of degradation were simulated for the required data. These different points provided enough diversity for the development of the EOEE. The use of different set points with different levels of degradation made it possible for verification of the data and methods used for the EOEE. This would suggest that the PBMM fulfilled its intended function.

8.3.2 EOEE conceptualisation

Machine efficiency was used as the performance parameter that provided the boundary condition for what operating points were acceptable and non-acceptable, or the points that fell into the EOEE and those that did not. It also showed how a warning front or area could be created to further develop the EOEE into a functional process monitoring system. The clearly visible, distinct areas which formed the EOEE proves that conceptually the EOEE can be used to determine whether a system is operating as intended. Though there is still some overlap at the border regions, it is very small and can be improved upon in the future.

The EOEE is therefore a viable method to determine the state of a system or its individual components. In the future, other performance measures can be looked at, such as cost and reliability.

8.3.3 EOEE implementation

MATLAB was used to control the PBMM in FLOWNEX and store the operating point data from the simulations. The different characteristic charts of each component were used to simulate degradation to introduce a variance in the operating point of the system. By routing the data to MATLAB, it became possible to quickly introduce a new set point and utilise the HEOM function on a large dataset, yielding quick and accurate results.

These results could further be controlled by calculating the SVD of each matrix.

8.3.4 EOEE verification

After the HEOM function calculated the distances between the normal and degraded matrices, it became necessary to take the result and display it visually. The best way to do this was determined to be through the use of SVD and then normalising these points. This made it easier to read the values of the SVD points. From these SVD points, the singular value SVD, 2D, and 3D graphs were drawn to visualise the EOEE.

These graphs showed clear differences in regions both within and outside the EOEE, allowing a distinct line to be drawn to separate the regions from each other. In some cases,

there was an overlap of operating points, though these were few and far between, with little impact on the EOEE still making it a very accurate measure of acceptable operating conditions and non-acceptable conditions.

8.3.5 EOEE validation

To validate the EOEE, a second set point of 150 kW was simulated for the PBMM. The same degradation levels were also simulated; every parameter was kept the same except for the PT output. The result was that the same trends were observed for the set point of 100 kW. The trendlines and shape of the operating conditions for both set points were very similar, though they differed in scale.

This would suggest that the EOEE is indeed operating point independent; however, the EOEE created by operating conditions would only be useful for a specific set point. Meaning the EOEE of one set point is not usable as a process monitoring tool for the operating points of another set point.

The EOEE is therefore universally useful for multiple operating conditions; however, each change in controllable variable or set point will require the development of its own EOEE.

8.4 Recommendations

To better develop and utilise the EOEE, further studies and more than simulations are required. A physical system has to be tested while it is running. The EOEE must be set up exactly as in this study, with all three variations of the EOEE, namely the 3D maps, the single SVD value graphs, and the 2D SVD value 1 and 2 graphs. For the 2D graph, whichever combination of SVD values gives the clearest answer should be used, and both the warning front and non-warning front variants should be used.

While the system is running, these graphs should continuously be created and updated so that the system's condition is constantly modelled. To ensure that the EOEE works as intended, its faults or degraded components should be implemented to test that the operating point will indeed fall outside of the EOEE and into the warning zone. The EOEE can also be refined to show exactly which component is suffering from degradation and how severe it is, to gauge whether a replacement component or simple maintenance is needed. If possible, different multi-domained systems should be compared against each other.

Currently, the EOEE was created for a specific set point, which will have different SVD values, meaning that the EOEE will change for every system and subsequent desired output. A programme or method should be created in which the desired output can be changed, and automatically generates graphs to show the EOEE. Other performance characteristics, such as cost, should also be explored and integrated into the EOEE.

A future study could look as follows:

- Set up the two-tank system in FLOWNEX.
- Run the same type of simulations for both degradation and faults and work the resulting data into an EOEE.
- Run the physical two-tank system for the same operating conditions as the simulation and implement faults and degradation as much as possible.

- It will be important that the faults and degradation for both the simulation and practical systems be the same so that the results can be compared with each other.
- An EOEE for the physical system has to be created and compared to the simulated one.
- If the simulated EOEE is accurate to the physical system, it must be run until the simulated system fails. The EOEE data must be noted, and the physical system must similarly be run until failure.
- It can then be compared to see if the simulated system accurately predicted the failure of the physical system.
- If the EOEE proves accurate this far, a larger study is required, for example, on an actual gas turbine or even a real energy plant.
- The plant must be modelled and simulated for at least one to two years of running. The physical plant data must be gathered and compared to the simulated one.
- If both the physical and simulated plant data are correct, the EOEE can be taken as an accurate tool for energy-based visualisation.

The EOEE can then be used to operate similar plants or engineering systems at an optimal level and can determine what components are suffering from degradation and require maintenance.

8.5 Closure

This study developed the concept of the EOEE and demonstrated its applicability in the condition monitoring of large and complex systems. All the objectives set out in Chapter 1 were satisfactorily met. To fully realise the functionality of the EOEE, it must be tested with a physical running system. Functioning as it did in this study, it will prove the EOEE as a practical tool useful to see the overall condition of a system at a glance and whether actions such as maintenance are required.

If properly developed further, the EOEE can help take the study of energy graph-based visualisation and all other forms of condition monitoring through energy flow to the next level. It is the stepping stone for moving past fault detection and isolation and going towards system optimisation.

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Appendix A

FLOWNEX System

The FLOWNEX system included in the link, <https://dayta.nwu.ac.za/account/home#/projects/228678>, is the complete system used for this study. It is set up with the steady-state controller script ready to go. Refer to Chapter 5 for more information.

MATLAB Code

The code included in the link, <https://dayta.nwu.ac.za/account/home#/projects/228678>, has all the code used to manipulate the FLOWNEX system and the relevant formulas. All the code MATLAB uses to communicate commands to FLOWNEX, as well as the individual scripts used to simulate all the results, are included. It is worth noting that there are individual scripts for each degradation instance.

MATLAB Results

All the results used to create the EOOE are included, named according to the operating point, and the component that underwent degradation can be found at the link, <https://dayta.nwu.ac.za/account/home#/projects/228678>, and is labelled according to the component undergoing degradation.