

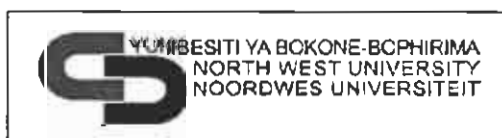


INVESTIGATION INTO THE VIABILITY OF AN AXIAL COMPRESSOR FOR A SINGLE SHAFT PBMM APPLICATION

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ABSTRACT

Title: Investigation into the viability of an axial compressor for a single shaft PBMM application
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The purpose of this study was to investigate the possibility of implementing axial compressors in the proposed single shaft Pebble Bed Micro Model (PBMM) cycle. Software for the development of preliminary axial compressor design was written with the aid of Engineering Equation Solver (EES). The preliminary design parameters obtained from the EES program was then used as input for the compressor analysis in Concepts NREC. An extensive literature review was conducted to obtain the required technical information concerning cycle analysis and compressor design.

Cycle simulations were carried out with both Helium and Nitrogen as the working fluid to rationalise the most suitable cycle for this application.

EES and Concepts NREC (Axial) were used to validate the software programs written for the cycle analysis and the preliminary compressor design. The EES output parameters of both programs closely resemble that of the verification software.

Axial and radial compressors were considered during the selection of the most appropriate compressor to be utilized in the cycle. As a result of this study it could be concluded that radial compressors with Nitrogen as the working fluid will be the best option in terms of complexity and expenses. A detailed cost analysis is recommended as it was not part of this study's scope. A detailed cost investigation could advance the conclusion of this study. The implementation of a high speed generator was also considered in the study. During the study it was concluded that a high speed generator would be most suitable for a micro model application.

UITTREKSEL

Titel: Ondersoek tot die lewensvatbaarheid van 'n aksiale kompressor vir 'n enkel-as PBMM toepassing
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Studieleier: Dr. B.W Botha
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Hierdie studie is gedoen om te bepaal of dit moontlik is om aksiale kompressors in die voorgestelde enkel-as Gepakte Bed Mikromodel (PBMM) te implimenteer. Sagteware vir die ontwikkeling van voorlopige ontwerpe van aksiale kompressors is geskryf met behulp van "*Engineering Equation Solver*" (EES). Die voorlopige ontwerp parameters wat met behulp van "*EES*" bepaal is, is vervolgens gebruik as inset parameters vir die "*Concepts NREC*" analise. 'n Uitgebreide literatuurstudie is uitgevoer om die nodige inligting aangaande siklus analises en die ontwerp van aksiale kompressors te bestudeer.

Siklus simulaties is uitgevoer met Helium sowel as Stikstof as die vloeier. Die rede hiervoor was om te bepaal watter een van die betrokke alternatiewe die beste sal wees vir die toepassing.

"*EES*" en "*Concepts NREC (Axial)*" is gebruik om die siklus analise en voorlopige kompressor-ontwerp programme wat in "*EES*" geskryf is, te valideer. Die "*EES*" uitset parameters van albei programme is baie na aan dié van die valideringsagteware.

Aksiale en radiale kompressors is oorweeg tydens die proses van selektering vir die mees geskikte kompressor wat in die siklus gebruik kan word. In aggenome kompleksiteit en koste, is die radiale kompressors met Stikstof as vloeier op hierdie stadium die mees geskikte opsie. Aangesien 'n koste-analise 'n belangrike bepaler vir die finale keuse van die kompressor is, maar nie deel van hierdie studie uitgemaak het nie, word aanbeveel dat so 'n analise as 'n toekomstige studie uitgevoer word. Die implimentering van 'n hoë-spoed generator is ook tydens die studie ondersoek. Na aanleiding van die studie is gevind dat 'n hoë-spoed generator geskik is vir 'n mikromodel toepassing.

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NOMENCLATURE

C	Absolute velocity	m/s
Ca	Axial velocity	m/s
CFD	Computational fluid dynamics	-
Cp	Specific heat	kJ/(kg.K)
D	Diameter	m
DC	Direct current	-
Dhr	De Haller number at rotor	-
Dhs	De Haller number at stator	-
EES	Engineering Equation Solver	-
f	Friction factor	-
h	Blade height	m
HP	High Pressure	-
htr	Hub-tip-ratio	-
KB	Blockage factor	-
LP	Low Pressure	-
M	Mach number	-
<i>m</i>	Mass flow rate	kg/s
N	Rotational speed	rpm
P	Pressure	kPa
PBMM	Pebble Bed Micro Model	-
PBMR	Pebble Bed Module Reactor	-
r	Radius	m
S	Entropy	J/kg.K
SF	Safety factor	-
T	Temperature	K
U	Tangential velocity	m/s
UA	Heat transfer coefficient	W/K
V	Relative velocity	m/s
W	Relative velocity	m/s
WDF	Work done factor	-
Z	Length	m



SUBSCRIPTS

b	Blade
c	Compressor
HPC	High pressure compressor
HS	Heat source
IC	Intercooler
LPC	Low pressure compressor
m	Mean
PC	Pre-cooler
r	Root
RXHP	Recuperator HP side
RXLP	Recuperator LP side
t	Tip
T	Turbine
w	Whirl



GREEK SYMBOLS

α	Air angle	°
β	Air angle	°
ϕ	Flow coefficient	-
ρ	Density	kg/m ³
ψ	Temperature coefficient	-
γ	Ratio of specific heats	-
σ	Stress	MPa
ε	Heat exchanger effectiveness	-
ω	Rotational speed	rad/s
η	Turbomachine efficiency	-
Λ	Stage reaction	-



1. **INTRODUCTION**

1.1 Background

Due to the world energy crisis a three-shaft Brayton cycle experimental facility was constructed at the North-West University, Potchefstroom. The world tendency to rather look at a single-shaft configuration resulted in the investigation of a single-shaft configuration as an alternative option for a PBMM application.

In the three-shaft PBMM, standard off-the-shelf components were used in the turbo-units. These components were not reliable due to high pressure levels during operation. Thus the need for an alternative option, namely to develop components for the PBMM, was identified. This resulted in the question if axial compressors would be suitable for this application. Using axial compressors in the PBMM will therefore closely resemble the commercial units.

1.2 Aim of study

The purpose of this project was to investigate the viability of an axial compressor for a single-shaft PBMM application. The compressor had to be designed to be as efficient as possible to enhance cycle efficiency.

It was decided to use similar design parameters to that of the existing three-shaft micro model. By using similar design parameters, it might be possible to use certain components from the existing three-shaft cycle to construct the single-shaft system in order to reduce overall expenses.

1.3 Problem statement

The three-shaft PBMM components were used in the turbo-units. These components were not reliable due to high pressure levels during operation. The need to develop components for the PBMM was identified. Development of axial compressors for a PBMM application became a viable option.



1.4 *Study objectives*

The three most important objectives of the study is the cycle analysis, conceptual compressor design and the selection of the most appropriate generator for the cycle. These objectives are discussed in more detail in the following paragraphs.

1.4.1 Cycle analysis

Extensive research on cycle analysis should be done to obtain sufficient knowledge on how to optimize the cycle. Different types of cycles must be considered to obtain the most suitable configuration for this specific application.

1.4.2 Compressor design

Research on axial machines should be performed to enable the design of the most appropriate compressors for the cycle. The compressor design is a conceptual design solely to determine the compressors' aptness for the application.

1.4.3 Generator selection

The selection and implementation of a generator in the cycle improves the viability of constructing a single-shaft test facility. It was therefore decided to include the selection of a generator in this project. Different types of generators should be studied also investigating the availability of commercial generators.

1.5 *Method of investigation*

In order to have successfully performed this study, a wide variety of information on cycle analysis were gathered and examined. This ensured the required technical background in order to successfully complete the project.



At first the different shaft arrangements were considered and a cycle analysis was carried out. The single-shaft cycle had to be programmed in accessible software to perform the cycle analysis. Hereafter the software code had to be validated with a similar software code. The compressor design parameters were then obtained from the verified software.

With the inlet and outlet conditions of the compressors known, a conceptual design of the compressor could be programmed in the provided software. This design then had to be validated by way of the compressor analysis software.

Power conversion formed part of the study and the most appropriate generator to be used in the cycle had to be selected. Different types of generators were considered as well as the availability of commercial generators.

2. LITERATURE STUDY

2.1 *Introduction*

In order to proficiently complete this study, a sufficient background on shaft arrangements, types of cycles, the optimization of cycles, compressor design and the types of generators on the market for this application was obtained. As a result, single shaft as well as multi-shaft layouts is discussed to give a wider perspective concerning shaft arrangements. Furthermore a discussion also follows on the different types of cycles to explain the layout theory of different types of cycles.

Information on the subject of cycle analysis and cycle optimization is also reported in this chapter. The knowledge was then fittingly used to compile the software for the cycle analysis.

Section two of this chapter considers a short history, basic operation and the design of axial compressors. The explanation of compressor designs continues with a background on generators, with specific thought on types of generators and their application.

2.2 *Cycle configurations*

The single shaft arrangement is most suitable when a gas turbine is required to operate at a fixed speed and fixed load conditions such as power generation schemes (*Rogers, Cohen & Saravanamuttoo, 2001:6*). However, flexibility of operation is not important in this application. High inertia due to the drag of the compressor is an advantage because it reduces the danger of over speeding in the event of a loss of electrical load.

In spite of this, the twin shaft arrangement has a significant advantage in ease of starting compared to a single-shaft arrangement. The starter needs only to be sized to turn over the gas generator. A disadvantage of a multi-shaft arrangement is that a shedding of electrical load can lead to rapid over speeding of the turbine, requiring a control system to prevent the over speeding (*Rogers, Cohen & Saravanamuttoo, 2001:6*)



2.2.1 Open cycles

Basic Brayton cycle

The single-shaft micro model is based on the basic Brayton cycle. The Brayton cycle in its ideal form consists of two isobaric and two isentropic processes as shown in Figure 2-1. By looking at the basic T-S diagram it is made easier for the reader to understand more complex layouts of the Brayton cycle.

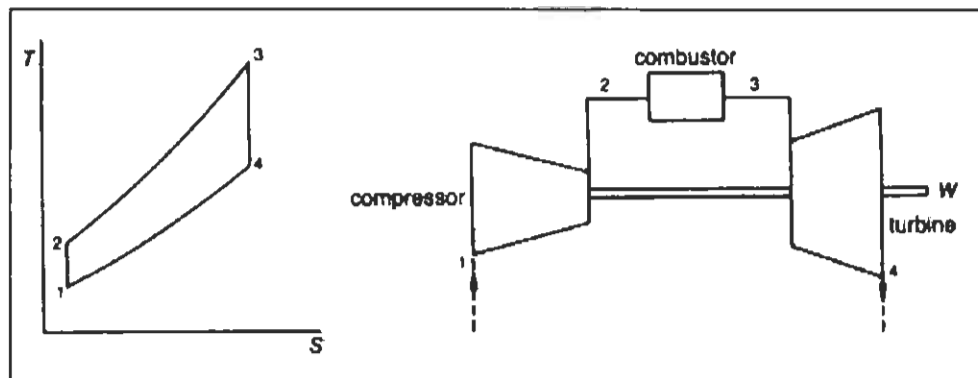


Figure 2-1: *Ideal Brayton cycle*

It was mentioned in Chapter 1 that the single-shaft cycle will be designed to be as efficient as possible. One way of increasing the cycle efficiency is to increase the turbine inlet temperature (T_3 shown in Figure 2-1)

The maximum temperature of the cycle has a extensive affect on the thermal efficiency of the basic Brayton cycle. Thermal efficiency increases with the ratio of the maximum and minimum temperature of the cycle and so does the optimum pressure ratio for maximum efficiency.

Recuperative cycle

The thermal efficiency of the power plant can further be increased by incorporating a recuperator, exchanging heat between the hot exhaust gas and the colder compressor gas. To build in a heat exchanger can improve the thermal efficiency of the power plant. In a simple gas turbine cycle the turbine exit temperature is almost always considerably higher than the temperature of the air leaving the compressor. The required input energy can be lowered by utilizing a recuperator or a regenerative heat exchanger. The hot turbine

exhaust gas then pre-heats the gas between the compressor and the heat source (Refer to Figure 2-2).

A larger or more efficient recuperator will increase the cycle efficiency significantly. Increasing the effectiveness of a recuperator necessitates more heat transfer surface area, which increases the cost, the pressure drop and the space required for the unit.

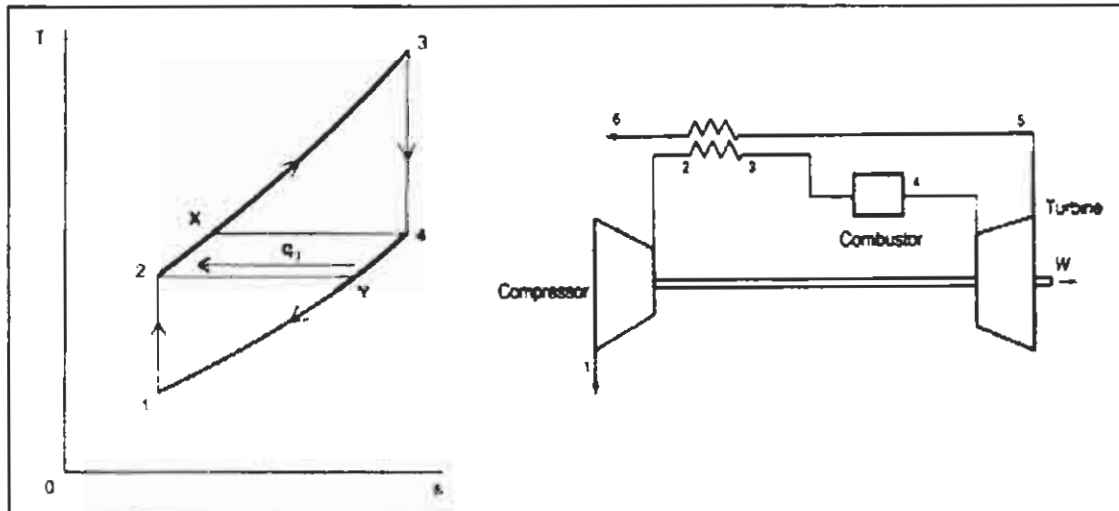


Figure 2-2: Recuperative cycle

Inter-cooled and Reheated cycles

Figure 2-3 illustrates that the incorporation of inter-cooling and reheating can enhance the power output of a gas turbine. In addition, decreasing the compressor work or increasing the turbine work, can also increase the generator output of a gas turbine cycle. Lowering the compressor inlet temperature reduces the effort required to compress the gas. In the case of reheating, the gas is expanded at a higher temperature and more work is delivered.

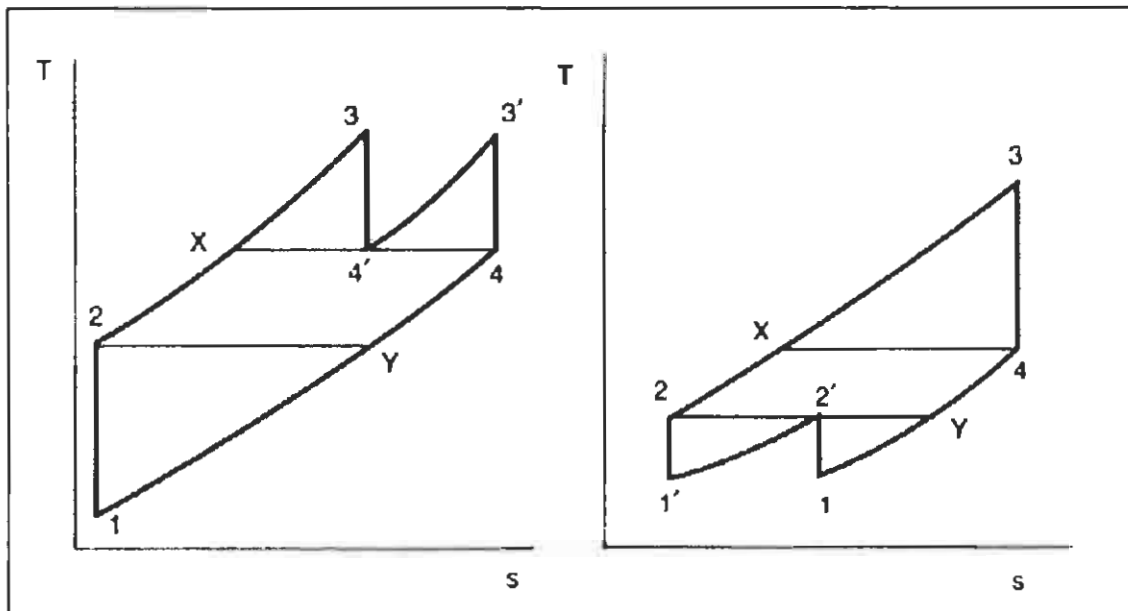


Figure 2-3: Reheated and intercooled cycles

Multiple compressor configurations

Using multiple compressors makes it possible to cool the gas between compressors to reduce the total work input. The following conditions and assumption result in a maximum work output when the pressure ratios of the LP (low pressure) and HP (high pressure) compressors are equal:

- When splitting the compression
- When inter-cooling the gas between the LP and HP compressors; and
- When assuming that the air is cooled to ambient temperature.

This concept is illustrated in Figure 2-4. Both the compression and expansion are divided into a large number of small processes. The efficiency approaches that of a Carnot cycle, since all the heat is supplied at the maximum temperature and all the heat is rejected at the minimum temperature (*Horlock, 2003:31-33*). By applying these concepts the cycle can be made even more efficient. This concept was only discussed to illustrate the effect of using more than one stage of compression and expansion. Although the cycle efficiency can be enhanced by applying this concept, it is not a viable to use multiple compressors and turbines, due to excessive expenses.

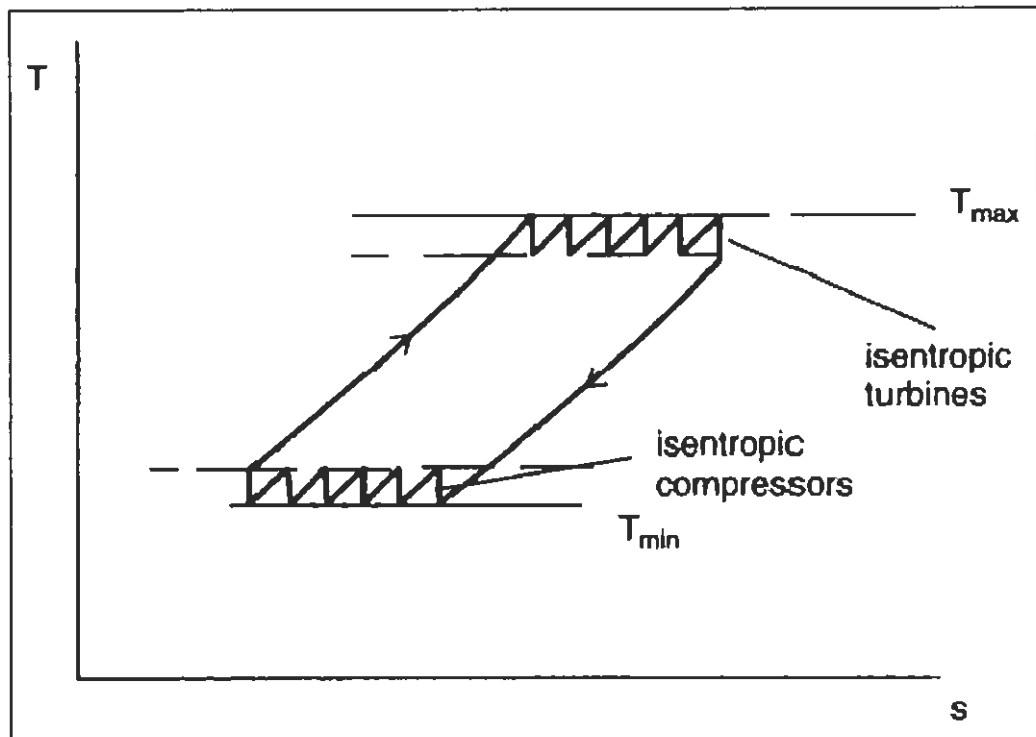


Figure 2-4: Multi-stage configuration

2.2.2 Closed cycles

Amongst the many advantages claimed for closed cycles (Figure 2-5), is the possibility of using a higher pressure throughout the cycle. This would result in a reduction in size of the turbo machinery for a given output. This makes it possible to change the power output by changing the pressure level in the circuit. Still, the pre-requisite of an external heating source exists as the main disadvantage of the closed cycle. This is, however, only a problem in the case of indirect cycles and not in the case of cycles like that of the PBMR (PBMR, 2005). The use of an auxiliary cycle is required and introduces a temperature difference between the combustion gasses and the working fluid. An upper limit is imposed due to the allowable working temperature of the surfaces in the heater and will therefore limit the maximum temperature of the main cycle (Rogers, Cohen & Saravanamuttoo, 2001:10)

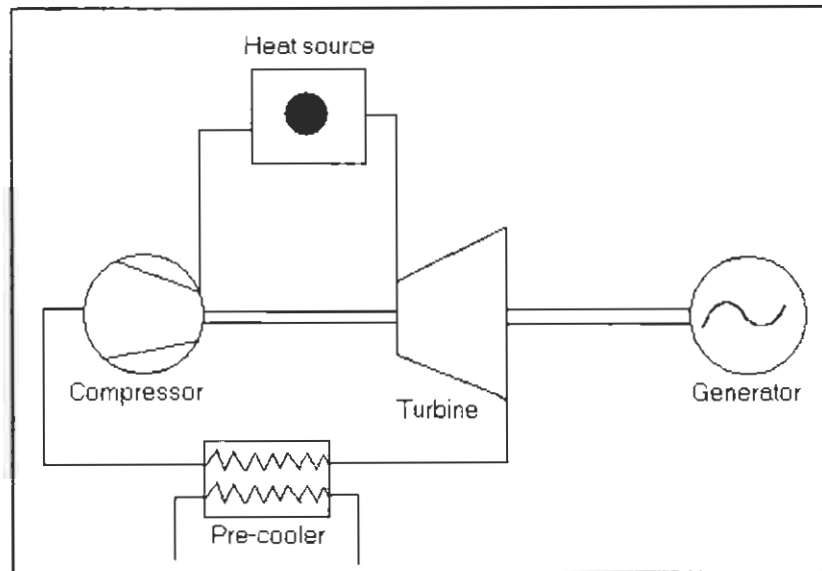


Figure 2-5: Closed Brayton cycle

Apart from the advantage of a smaller compressor and turbine and efficient control, the closed cycle also avoids erosion of the turbine blades and other detrimental effects due to the products of combustion. The need for filtration of the incoming air is eliminated, which is a severe problem in the use of open-cycle units operating in contaminated atmospheres. The closed cycle opens up the field for the use of gasses other than air having more advantageous thermal properties (*Rogers, Cohen & Saravanamuttoo, 2001:11*).

The noticeable difference in the values of specific heats for air and a monatomic gas such as Helium does not have an effect on the efficiency as much as might be expected. Higher fluid velocities can be used with Helium and optimum cycle pressure ratios are lower. This implies that regardless of lower density, the turbo machinery may not be larger. Nevertheless, the better heat transfer characteristics of helium enables smaller physical size (about half of units designed for use with air) of the heat exchangers used in the layout (*Rogers, Cohen & Saravanamuttoo, 2001:96*).

2.3 Cycle analysis

The overall pressure ratio of the cycle is an important parameter when it comes to cycle analysis. The effect of altering the pressure ratio and the consequences it has on the cycle efficiency will be discussed regarding cycles without heat exchanging.

The choice of cycle pressure ratio will depend on whether the cycle is optimized for high efficiency or high specific work output. In the case of the simple cycle without heat exchanging the efficiency depends only on the pressure ratio. This can be illustrated by referring to Equation (2.1), with η the cycle efficiency and PR the cycle pressure ratio.

$$\eta = 1 - \left(\frac{1}{PR} \right)^{\left(\frac{\gamma-1}{\gamma} \right)} \quad (2.1)$$

This equation shows that an increase in pressure ratio will enhance the cycle efficiency. The need to cool down the turbines blades when the cycle is operating at a high efficiency limits the efficiency of the simple cycle. This also means that the material used for the turbine blades can have a great effect on the cost of a turbine operating at high temperature.

The introduction of a heat exchanger leads to higher efficiency at a lower pressure ratio. Heat exchange increases the efficiency significantly and noticeably reduces the optimum pressure ratio for maximum efficiency. Major benefits of the addition of reheating and inter-cooling to the un-recuperated plants are to increase the specific work. Then again, coupling these features with heat exchanging one obtains the full benefits on efficiency (*Horlock, 2003:30*).

Equation (2.2) illustrates that an increase in pressure ratio (PR), in the case where a heat exchanger is included, will have a negative result on the cycle efficiency. The “t” in Equation (2.2) is a ratio of the compressor and turbine inlet and outlet temperatures.

$$\eta = 1 - \frac{PR^{\left(\frac{\gamma-1}{\gamma} \right)}}{t} \quad (2.2)$$

(*Rogers, Cohen & Saravanamuttoo, 2001:49*)

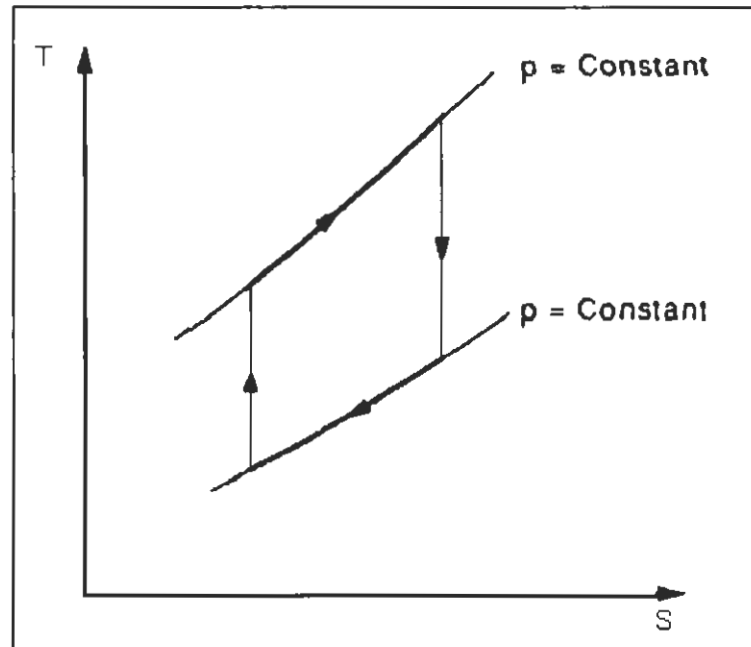
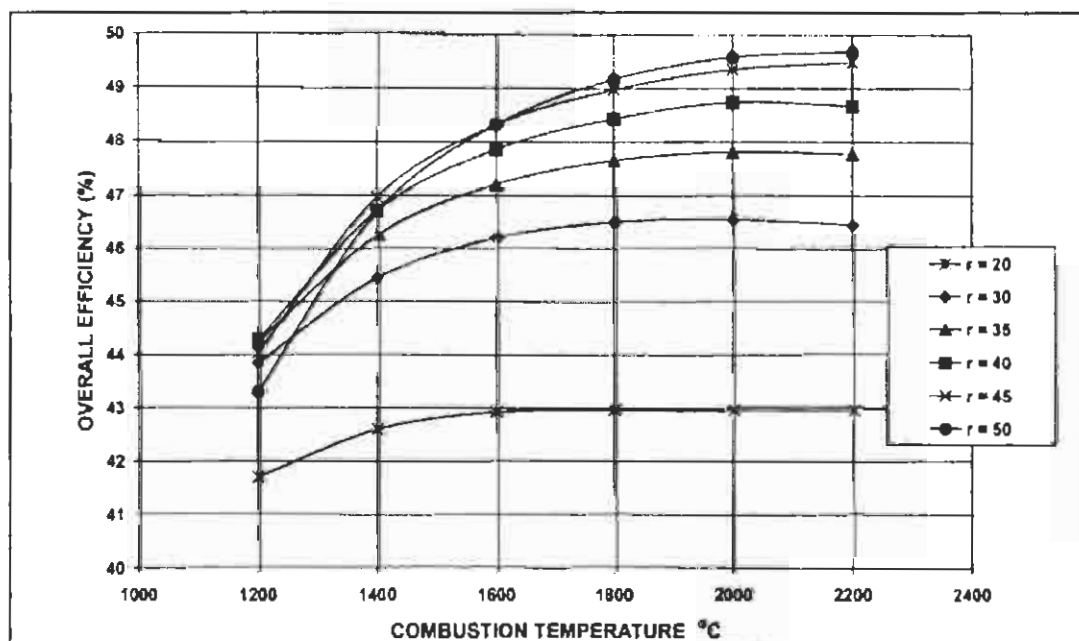


Figure 2-6: Thermal efficiency

Increasing the maximum temperature of the cycle and decreasing the minimum temperature bring about high thermal efficiency (illustrated in Figure 2-6). This is due to the diverging constant pressure lines, less work is needed for compression at low temperatures. At high temperatures, the turbine work output increases as shown in Figure 2-7 (Horlock, 2003:44).

Figure 2-7: Effect of T_{max} on cycle efficiency

2.4 *Axial compressors*

The following two paragraphs briefly discuss the history and origin of axial flow compressors. This will put the subject into perspective and highlight the development of axial compressors over the past years.

The idea of using a form of reversed turbine as an axial compressor is as long-standing as the reaction turbine itself. Sir Charles Parsons obtained a patent for such an arrangement as early as 1884. By simply reversing, a turbine for use as a compressor gives efficiencies that are less than 40% for machines of high-pressure ratio.

It was not until 1926 that any further development on axial compressors was undertaken when A. A. Griffith outlined the basic principles of his aerofoil theory of compressor and turbine design. There is a close link between the subsequent history of the axial compressor and that of the aircraft gas turbine. The work of the team under Griffith at the Royal Aircraft Establishment, led to the conclusion that small stages with low-pressure ratios per stage achieve efficiencies of at least 90% (*Dixon, 1998:137*).

2.4.1 *Basic operation*

For a designer to be able to design an axial compressor, the basic operation must first be understood. Basic flow through the compressor and the main components will be discussed.

The axial-flow compressor compresses its working fluid by first accelerating the fluid and then diffusing it to obtain the rise in pressure increase. The fluid is accelerated by a row of rotating airfoils (blades) called the rotor, and then diffused in a row of stationary blades (the stator). The diffusion in the stator converts the velocity increase gained in the rotor to a pressure increase. A compressor consists of several stages. One rotor and stator make up a stage in a compressor. One additional row of fixed blades (inlet guide vanes) operates at the compressor inlet to ensure that the air enters the first stage rotors at the desired angle. As indicated in Figure 2-8 the length of the blades, and the annulus area, which is the area between the hub and the shroud, decreases through the length of the compressor. This

reduction in flow area compensates for the increase in fluid density as it is compressed, permitting a constant axial velocity (Boyce, 2002:275).

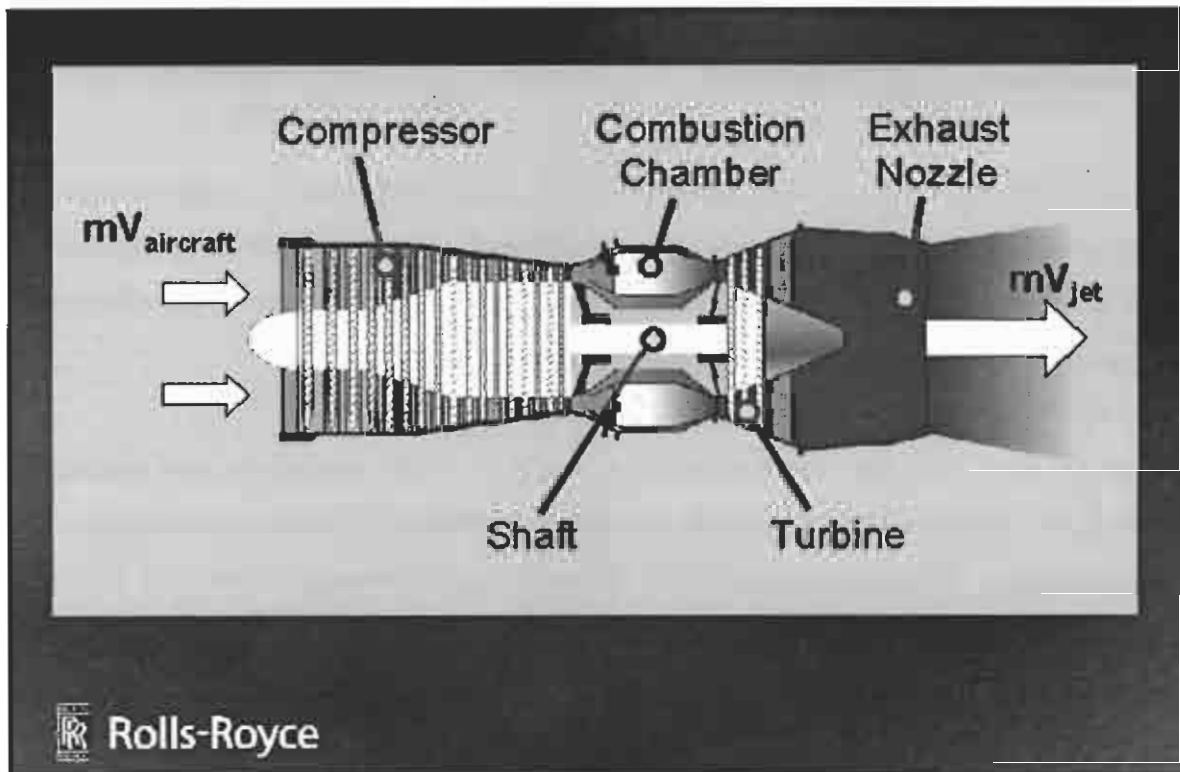


Figure 2-8: Compressor annulus

2.4.2 Velocity triangles

Velocity triangles are one of the building blocks of any turbomachine design. It is important for the designer to understand and be able to determine the angles and velocities of the gas entering and leaving the compressor stage.

The gas entering and leaving a compressor stage can be calculated by utilizing velocity triangles. Figure 2-9 illustrates the velocity diagrams for an axial stage. As for axial turbine stages, a normal compressor stage is one where the absolute velocities and flow directions at stage outlet are the same as at stage inlet. The flow from a previous stage has a velocity c_1 and direction α_1 . By subtracting, the blade speed U , as a vector gives the inlet relative velocity w_1 at angle β_1 . Relative to the blades of the rotor, the flow turned to the direction β_2 at outlet with a relative velocity w_2 . By adding the vectors, the blade speed U

on to w_2 gives the absolute velocity from the rotor, c_2 at angle α_2 . The stator blades deflect the flow towards the axis and the exit velocity is c_3 at angle α_3 (Dixon, 1998:140).

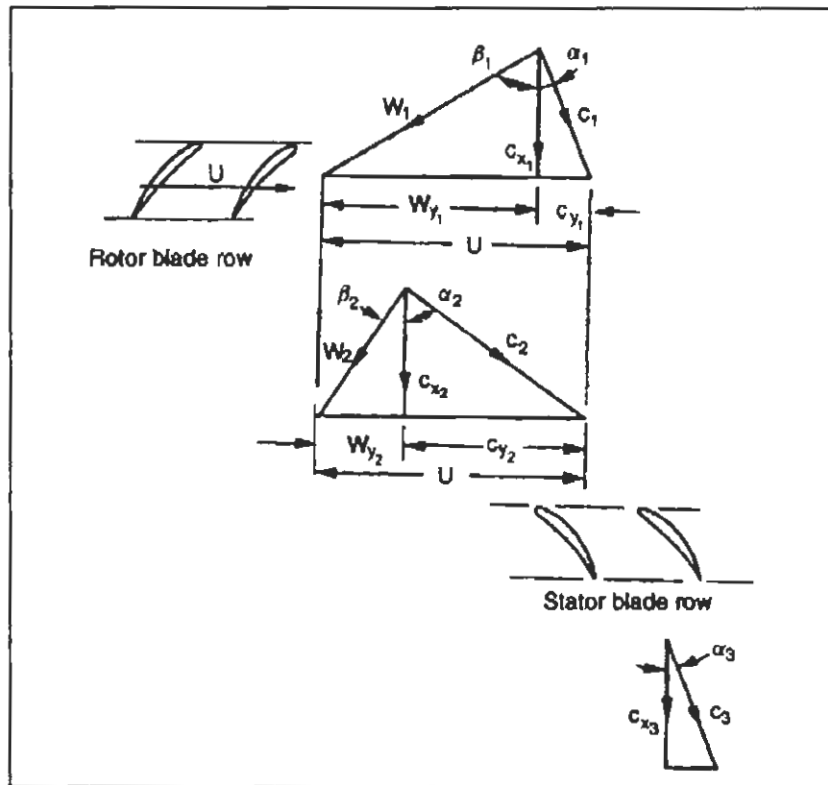


Figure 2-9: Axial compressor velocity triangles

2.4.3 Compressor design considerations

Axial velocity

The axial velocity through a compressor stage is a critical parameter, which should be considered during the compressor design process. This value must be estimated to simplify the initial stages of the design procedure.

Axial velocities for industrial gas turbines are usually in the order of 150 m/s and axial velocities in advanced turbo engines can go up to 200 m/s. Early compressors had to be designed that the Mach number at the rotor tip was subsonic. In the early 1950's, it became possible to use transonic Mach numbers up to about 1.1 without introducing excessive losses (Rogers, Cohen & Saravanamuttoo, 2001:189).

Blade root stress

The compressor design conducted in this study does not include a detailed stress analysis of the compressor blades. Instead, a short formula was used to make sure that the blade stresses is kept within reasonable limits.

The acoustic velocity increases in successive stages because of the progressive increase in static temperature. Mach numbers become less of a problem in the last stages. From a mechanical point of view, the later stages will not be problematic, because of the shorter blades that imply low stresses as shown in Equation (2.3).

$$\sigma_{\max} = \frac{\rho_b}{2} U_t \left[1 - \left(\frac{r_r}{r_t} \right)^2 \right] \quad (2.3)$$

When air is the working fluid, compressibility effects become critical before stress considerations. With a monatomic gas such as Helium, the gas constant is much higher than that of the air and the acoustic velocity is correspondingly higher. This makes it feasible to consider using helium as the working fluid in the compressor because the flow will not tend to choke as easy as air or nitrogen. In the case of helium, the Mach numbers are low but blade stresses become more of a problem (*Rogers, Cohen & Saravanamuttoo, 2001:190*).

Blockage in compressor annulus

Because of the pressure gradient in compressors the boundary layers along the annulus thickens towards the back of the compressor. The boundary layer causes a reduction in effective annulus area as the flow progresses (refer to Figure 2-10). The design process should compensate for this, as it will have a considerable effect on the axial velocity through the compressor.

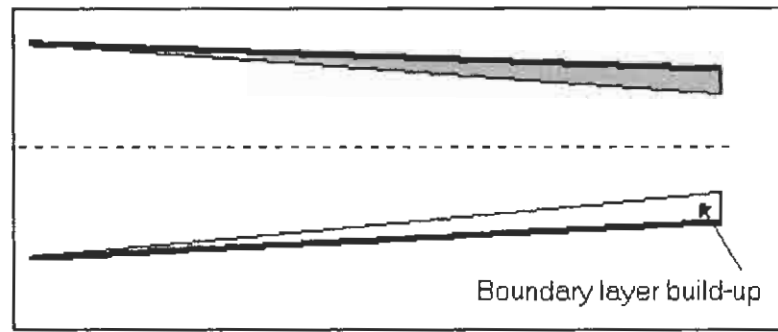


Figure 2-10: *Boundary layer along annulus*

The use of Blockage factors account for the effect of the boundary layer build-up. Table 2-1 shows typical values that can be used in compressor designs with air as the working fluid (*Botha, 2005:103*).

Table 2-1: *Blockage factor*

# Stage	1	2	3	4	5 and following
Blockage factor	0.99	0.95	0.92	0.9	0.88

Temperature coefficient

The stage loading factor ψ is another important design parameter of a compressor stage and is one that strongly affects the off-design performance characteristics.

$$\psi = \frac{\Delta H}{U^2} \quad (2.4)$$

The stage loading factor is an indication of the energy exchange that occurs per unit mass for a given blade speed. A high stage-loading factor results in a high rise in pressure (*Dixon, 1998:146*).

2.2.4 Axial compressor geometry

The analysis of the EES design in Concepts NREC, required more detail concerning the compressor geometry. Parameters including the stagger angle, blade pitch, aspect ratio and the solidity were determined or assumed in order to analyze the design. These parameters are briefly discussed in the following paragraphs.

Different definitions of the blade geometry exist and expand the understanding thereof (refer to Figure 2-11 for a schematic illustration of the parameters).

As seen from Figure 2-11, b is the chord (width of the blade) and s the pitch (distance between consecutive blades). The ratio of the chord / pitch is the solidity and is commonly taken as one.

Camber angle (θ)

According to *Rogers, Cohen & Saravanamuttoo (2001:235)* the camber-line of the blade profile is a circular arc and the solidity is one. The following equation derives from these assumptions:

$$\delta = 0.273\theta, \text{ with } \delta \text{ the incidence angle and } \theta \text{ the camber angle.} \quad (2.5)$$

Refer to Figure 2-11 for an illustration of the camber angle. Assuming zero incidence it can be shown that the camber angle can be determined as follows:

$$0.727\theta = \alpha_1 - \alpha_2, \text{ with } \alpha_1 \text{ as the rotor inlet angle and } \alpha_2 \text{ the rotor outlet angle.} \quad (2.6)$$

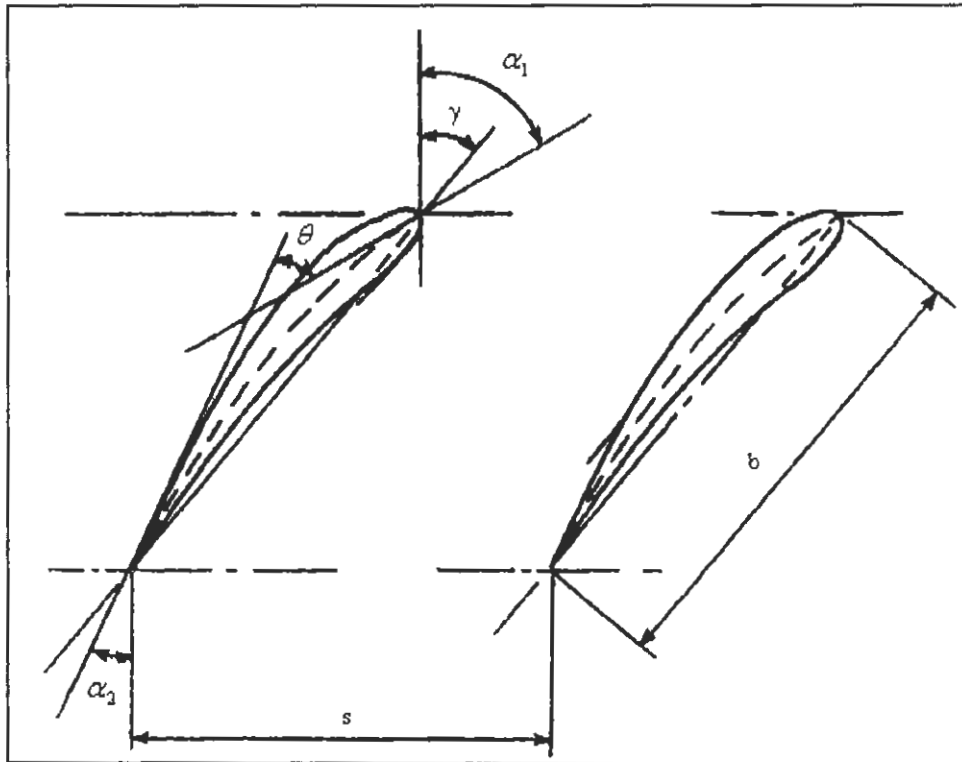


Figure 2-11: Compressor blade nomenclature

Stagger angle (γ)

The stagger angle is the inclination to the axial direction of the cord, the line joining the leading and the trailing edges. This angle (shown in Figure 2-11) can be determined with the following equation:

$$\lambda = \alpha_1 - \frac{\theta}{2} \quad (2.7)$$

(Cumpsty, 1989:128)

2.2 Generators

This section discusses the different types of generators and some applications, followed by a comparison between high- and low-speed generators in terms of size, efficiency and overall cost. The section also rationalizes the advantages of high-speed generators over lower-speed generators.

Generators are most widely used in the production of electricity. Since the 1990's, micro gas turbines, with a power output of 300kW or lower, have attracted attention in the United States, Europe and Japan as high-efficiency, low-cost and small-distributed power-generation equipment. The development of micro gas turbines is now in progress. Since the early 1960's, Toyota Motor Corporation has devoted attention to gas turbines because of their lightweight, small size, and low emissions, and has developed gas turbines mainly for various automobile application. The result of this development is the production of high-speed generators (*Ryo Sakai, Koji Ishibashi, Akio Mori, 2002:1-12*).

Operating speed of micro turbines tend to be quite high and can often exceed 100 000 rpm. The speed generally vary over a wide range (50 000 rpm to 120 000 rpm) to accommodate diverge loads while maintaining high operating efficiency (*Staunton & Ozpimeci, 2003:1-29*).

2.2.6 Types of generators

The turbo-unit usually drives a generator that may be either synchronous or asynchronous. According to *Staunton and Ozpimeci (2003:1-29)* cage rotor design in asynchronous generators tends to make it a less expensive alternative to synchronous generators. Synchronous generators contain a magnetic rotor, designed to use either rare earth permanent magnets or coils. Although rare in the industry, asynchronous generators are the generator of choice in wind and hydro generator applications.

2.2.6.1 Synchronous

Synchronous generators can operate at a constant speed, and with the proper regulation of speed, enable direct coupling to the grid. This is not the case in high-speed microturbine applications because the turbo-unit is not rotating at 50Hz, but at much higher frequencies. Synchronous generators require an external field coil excitation. They therefore contain brushes that require regular maintenance. Permanent magnet machines do not need brushes, but are expensive. (*Staunton & Ozpimeci, 2003:1-29*).

Permanent magnet motors/generators offer very high power density for minimum size simplifying integration. The high efficiency of these motors/generators also maximizes the overall system efficiency. Offered in air-cooled or liquid-cooled versions, these machines offer the latest technology in high strength rare earth materials for very high power density (minimum size) and very high efficiency (*Calnetix, 2003*).

Calnetix applies this configuration (refer to Figure 2-14) from low- to high-speed applications, making it well suited for a wide range of applications. Designs that operate over 450,000 rpm are achievable. The typical operating range for Calnetix machines range from 30,000 to 100,000 rpm.

Calnetix not only offers a wide range of operating speeds, but also a robust rotor construction maximizes rotor stiffness for higher bending mode frequencies. A low number of rotor poles reduce stator losses by lower drive frequency requirements. The latest technology in stator lamination materials minimizes stator iron losses (*Calnetix, 2003*).

2.2.6.2 Asynchronous

Induction generators are low cost and they have a robust construction. They do not require external excitation and therefore have a simpler control system. Thus, in this case there is no need for brushes. Without brushes, they are virtually maintenance free. Asynchronous generators are not used often in the industry since their speed depends on the load. They can therefore not be connected to the grid without a power converter. This power conversion increases the overall system cost. (*Staunton & Ozpimeci., 2003:1-29*).

2.2.7 Advantages of high speed generators

An advantage of a high-speed generator is that the size of the machine decreases almost in direct proportion to the increase in speed. This leads to a very small unit that can be integrated with the gas turbine (*Aglen, 2000:1-5*). According to a study done at PBMR by *Greyvenstein Renee*, the overall cost of axial turbomachines is a function of the tip radius and the number of stages. This simply means that, the higher the operating speed of the turbo-unit, the less expensive the turbomachinery will be. Consequently, the total cost of the layout will be reduced in the case of a high speed application.

High-speed alternators can act as the starter and as the generator. This eliminates the need for a starter motor and for a large, heavy, expensive speed reduction gearbox. High-speed generators are reliable and rugged in construction and deliver low maintenance costs. (*Bowman 2004:1*).

2.2.8 High-speed generators available on the market

Both Calnetix and S2M require detailed information concerning the turbo-unit. This includes the layout, exact geometry of the turbomachine and the power delivered by the turbo-unit. It is impossible to include all this detail in the study. This study does not include the design of the turbine for this specific cycle. At this stage it would be difficult to decide on a generator from either Calnetix or S2M. It should be considered that both alternatives are a viable solution and available from on the market.

2.2.8.1 S2M

Permanent magnet motor/generator development began ten years ago at S2M. Responding to the needs of the machine tool industry, permanent magnet motor technology replaced less efficient, lower-speed asynchronous induction motors/generators. Figure 2-12 shows the available power range and speed at which the available machines operate (*S2M, 2004*).

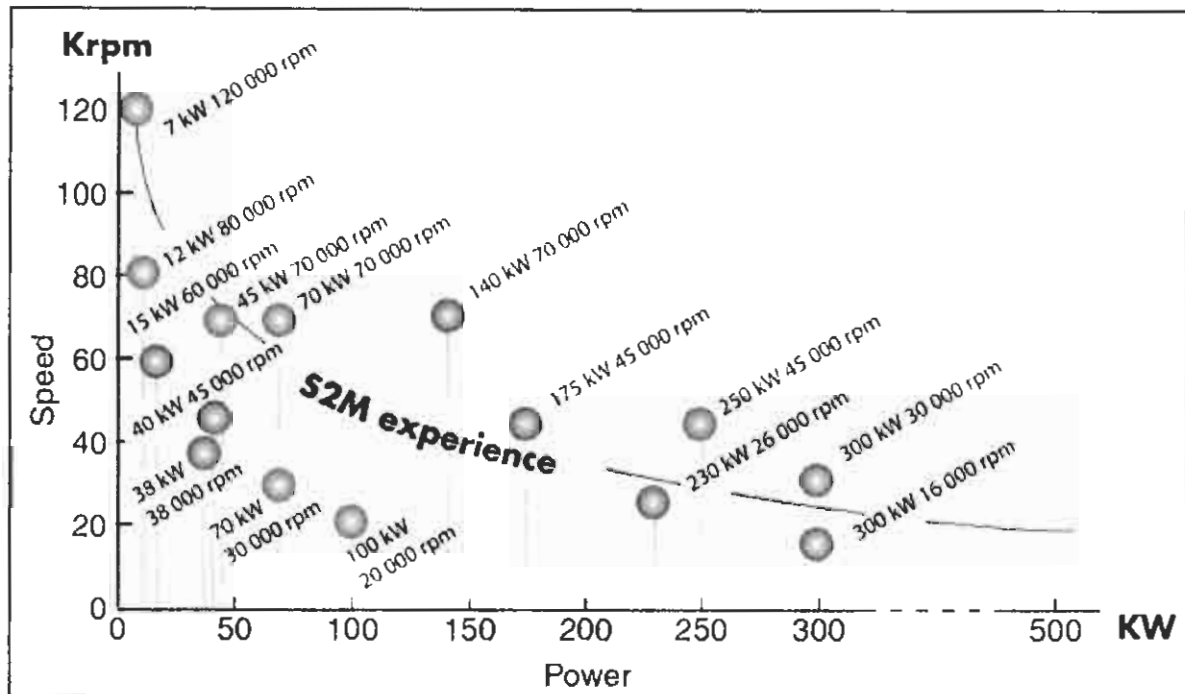


Figure 2-12: S2M Motors and generators range

2.2.8.2 Calnetix

Calnetix designs, develops and manufactures a wide range permanent magnet generators for a variety of applications and industries. According to their website, Calnetix has the largest installation of high speed permanent magnet motors and generators in the field.

They state:

“This expertise in integration from the system perspective optimizes the benefits of this high speed technology to meet performance and cost goals for the system. Whether the machine is coupled to the drive, or integrated onto a single shaft, Calnetix offers the expertise to integrate this technology. Integrated magnetic bearings and high speed motors for turbocompressor applications provide a true oil-free system with higher efficiency and reduced maintenance.” (Calnetix, 2003)

Figure 2-13 is an example of a conventional system without the use of a high-speed generator. Figure 2-14 however, shows the less complex system with the implementation of a high-speed generator. The use of a high-speed system eliminates the necessity for a reduction gearbox along with the lubricating system.

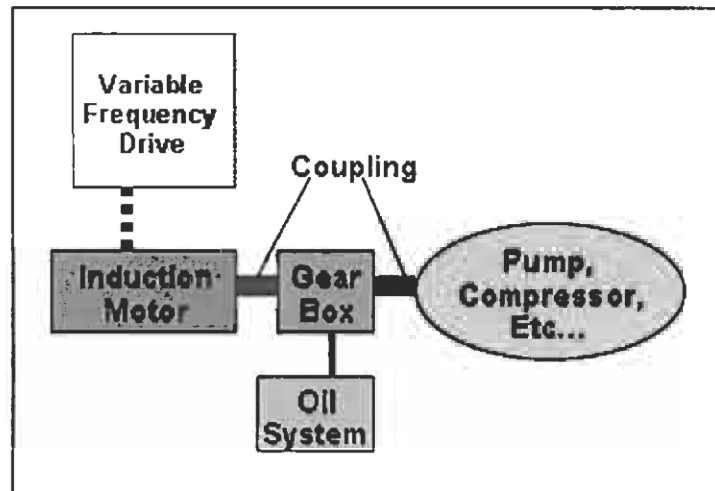


Figure 2-13: *Conventional systems*

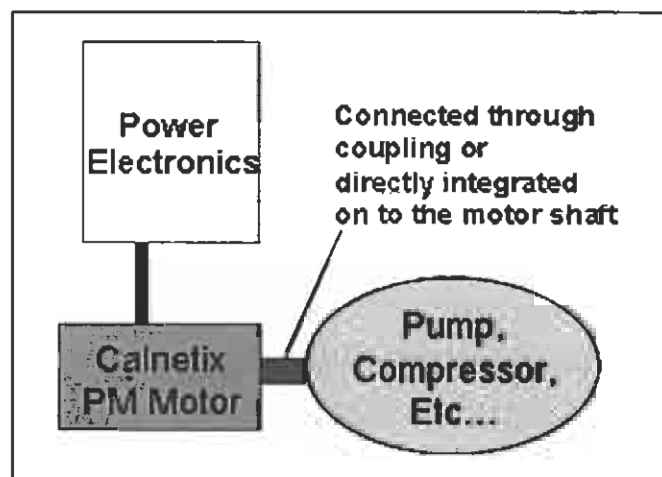


Figure 2-14: *Calnetix system*

Table 2-2 summarizes the specifications of the MPD 100 high-speed generator available from Calnetix that can operate as a typical generator in the micro model.

Table 2-2: Calnetix MPD 100 High speed generator

MPD 100 specifications (Calnetix)	
Operating Speed	60,000 rpm
Overspeed	66,000 rpm
Efficiency	96.60 %
Cooling	Air-cooled
Weight	100 kg
Dimensions	50 cm, 28 cm in diameter
Rated Output Power	100 kW DC
Machine Type	Permanent Magnet, Synchronous

2.2.9 Summary

From the literature obtained regarding high-speed generators the following conclusions was made:

- High-speed machinery is less complex
- High-speed machinery is less expensive
- It takes up less space, which is another cost benefit
- A modern high-speed generator is more efficient
- and requires less maintenance

This concludes that for smaller scale power generation, the high-speed application is a viable option.

2.3 Conclusion

Different shaft layouts as well as the effect of several components on the cycle efficiency and power output were examined by way of an extensive literature study. One of the aims of this study is to simulate a single-shaft cycle with the same design parameters as that of the previous three-shaft cycle. Other cycles were also considered, but not studied in detail.

A heat exchanger is essential for high efficiency when the cycle pressure ratio is low. However, it becomes less advantageous when the cycle pressure ratio becomes higher.

In the case of the multi-stage configuration, including an intercooler between the two compressors can reduce the work needed for compression. Intercooling between



compressors will help to increase the cycle efficiency, but on the other hand increase the complexity and costs of the power plant.

An increase in heat exchanger effectiveness raises the cycle efficiency appreciably and also reduces the value of the optimum pressure ratio of the cycle. Since the optimum pressure ratio for maximum efficiency is below that for maximum specific output it is inevitable that a plant designed for high efficiency will suffer a weight and space penalty, when a large recuperator is included in the cycle.

Multi-stage configurations and the effect of more turbo-machines in the cycle were briefly discussed. As seen from the literature review, a great number of compressors in conjunction with intercoolers will result in a very efficient cycle. Yet, total cost limits the implementation of many multi-stage compressors.

The history of axial compressors was discussed in short to serve as an introduction to the section. Basic operation and the design of axial compressors were debated.

Different types of generators were considered for use in the cycle. As previously mentioned the micro model cycle must be as close as possible to that of large commercial units. Small scale, high speed generators were studied to be implemented in the cycle. At this stage there are no standard “off the shelf” high-speed generators available in the industry. S2M and Calnetix supply high-speed generators, but these are manufactured according to the specifications of the turbomachines.

The next chapter explores cycle analysis in order to accomplish the objective to optimize cycle analysis.

3. CYCLE ANALYSIS

3.1 Introduction

A cycle analysis was done to determine the overall cycle efficiency, power available to drive a high-speed generator and thirdly to specify the inlet and outlet conditions of the compressors in the cycle. The cycle was simulated with the aid of EES (Engineering Equation Solver). This chapter discusses the layout of the cycle as well as the methodology to simulate the cycle in EES. Results obtained from the analysis are discussed and verified with other software programs.

3.2 Cycle analysis of single shaft configuration

3.2.1 Cycle layout

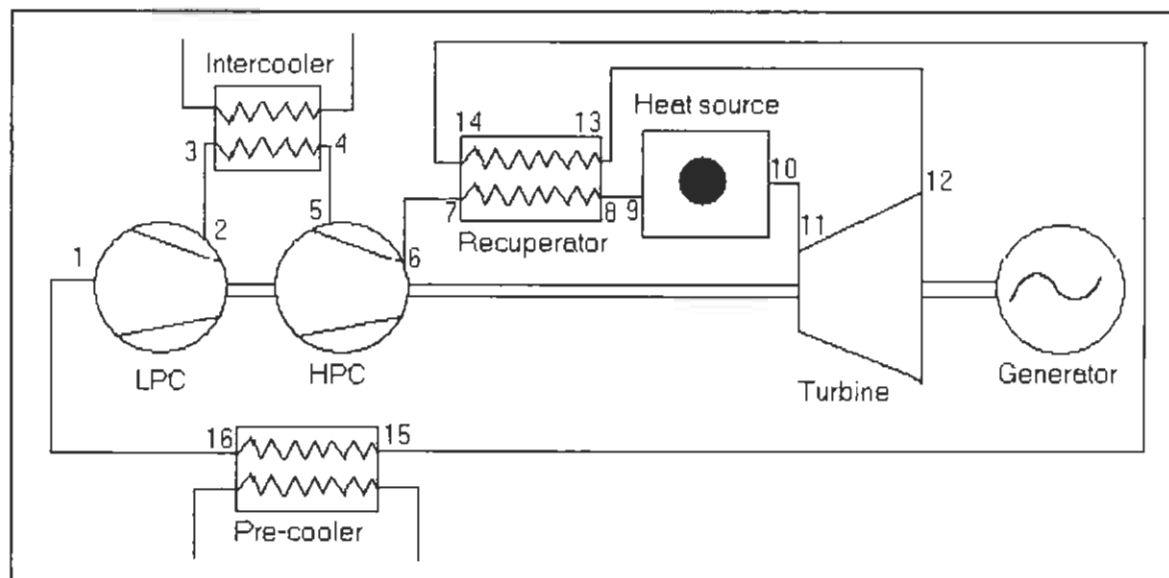


Figure 3-1: Cycle layout

Figure 3-1 is a schematic illustration of the proposed single shaft layout. The cycle consists of two axial compressors and one axial turbine. The incorporation of a recuperator and intercooler enhances the cycle efficiency. A high-speed generator can be directly coupled to the turbo-unit, which means that no gearbox is utilized in the layout. Figure 3-2 is a schematic of the cycle T-S diagram.

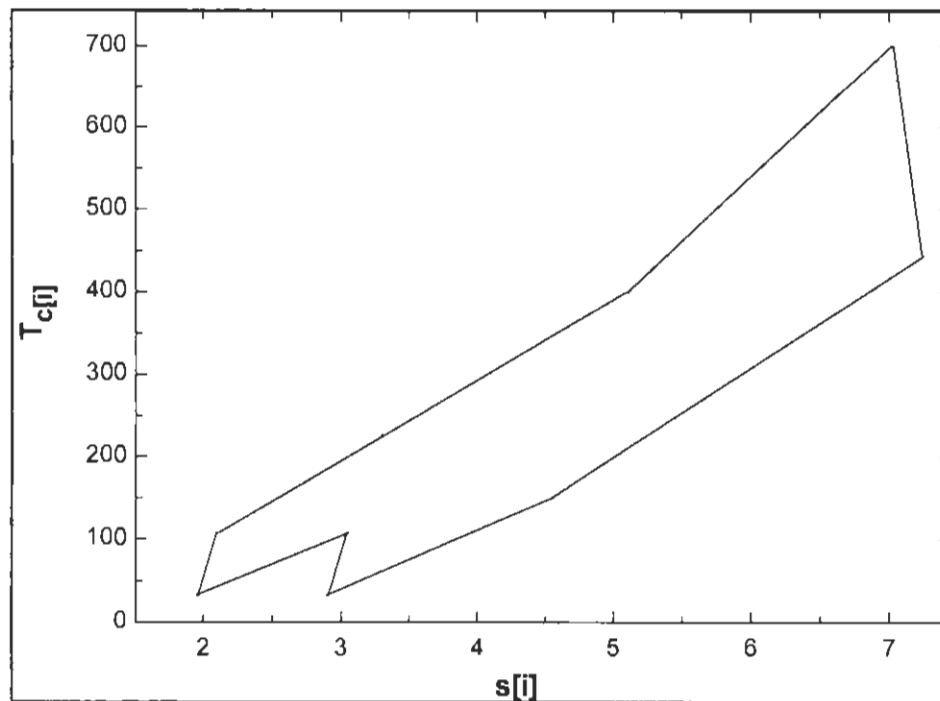


Figure 3-2: Cycle T-S Diagram

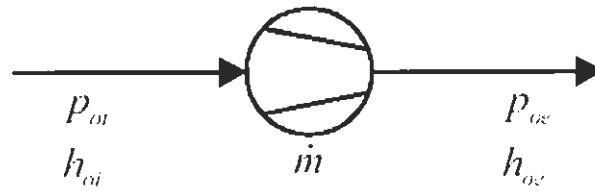
Table 3-1 is a summary of the inlet conditions and the gas properties. These values act as constant input values in the program. These values will act as constant input values in the cycle analysis software discussed in this chapter.

Table 3-1: Input values

Boundary values		
	Value	Unit
γ	1.666	kJ/kg K
R	2.078	kJ/kg K
P_{01}	250	kPa
T_{water}	20	°C
T_{max}	700	°C

3.2.2 Program methodology

This section explains the methodology of the software written to simulate the cycle and to obtain the compressor design parameters to carry out the compressor design in Chapter 4. The reader can refer to Figure 3-1 to obtain better understanding of the design procedure. Each step in the program was numbered according to the numbering in Figure 3-1.

LP Compressor (1-2)**Figure 3-3: Compressor schematic**

The compressor efficiency was assumed as:

$$\eta_{LPC} = 0.85$$

A value of 0.85 for the compressor efficiency was chosen to contribute to high overall cycle efficiency. The reason for keeping the cycle efficiency high is to keep it similar to larger cycles.

The compressor work was calculated at a constant γ of 1.667.

$$Q_c = \frac{T_{01}}{\eta_c} \left[\left(\frac{P_{02}}{P_{01}} \right)^{\frac{\lambda-1}{\lambda}} - 1 \right] \quad (3.1)$$

$$Q_c = \dot{m} c_p (T_{02} - T_{01}) \quad (3.2)$$

Initially a guess value for the pressure ratio was used in order to solve the equation. The guess value was later replaced with an optimum value obtained from a lookup table, which optimized the pressure ratio versus cycle efficiency.

$$PR_c = \frac{P_{02}}{P_{01}} \quad (3.3)$$

Intercooler (3-4)

Efficiency of the intercooler was assumed as:

$$\varepsilon_{IC} = 0.96$$

Pressure loss factor due to friction was assumed as:

$$f_{IC} = 0.02$$

It was assumed that a 2-5 % pressure drop would occur in the heat exchanger. According to Rousseau (2006:23) a 2-5 % pressure drop through a heat exchanger is a good, conservative approximation.

The pressure drop through the intercooler was calculated as follows:

$$\Delta p_{IC} = f_{IC} \cdot p_3 \quad (3.4)$$

Outlet pressure of the intercooler:

$$p_4 = p_3 - \Delta p_{IC} \quad (3.5)$$

Heat exchanged in the intercooler can be calculated using Equations (3.6) and (3.7). The intercooler outlet temperature was calculated as well.

$$Q_{IC} = \dot{m} c_p (T_4 - T_3) \quad (3.6)$$

$$Q_{IC} = \varepsilon_{IC} \dot{m} c_p (T_{water} - T_3) \quad (3.7)$$

HP Compressor (5-6)

The compressor efficiency was assumed as:

$$\eta_{HPC} = 0.89$$

The compressor work was calculated at a constant γ of 1.667.

$$Q_c = \frac{T_{05}}{\eta_c} \left[\left(\frac{P_{06}}{P_{05}} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \quad (3.8)$$

$$Q_c = \dot{m} c_p (T_{06} - T_{05}) \quad (3.9)$$

Pressure ratio

$$PR_c = \frac{P_{06}}{P_{05}} \quad (3.10)$$

Recuperator

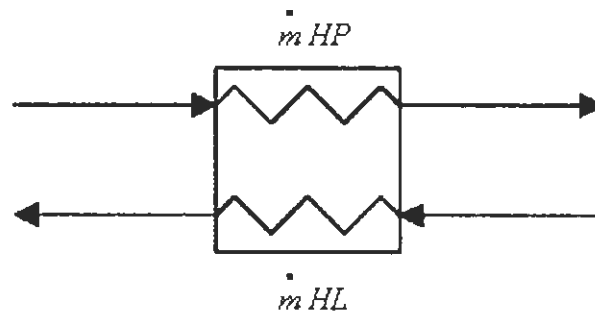


Figure 3-4: Recuperator schematic

LP Side (7-8)

Pressure loss factor due to friction was assumed as:

$$f_{RXHP} = 0.02$$

Effectiveness of the intercooler was assumed as:

$$\varepsilon_{RX} = 0.87$$

Applying the NTU method, the total heat exchanged in the recuperator can be calculated using Equations (3.11) and (3.14).

$$Q_{RXHP} = C_{\min} \cdot \varepsilon_{RX} \cdot (T_{13} - T_7) \quad (3.11)$$

The pressure drop through the recuperator was calculated as follows:

$$\Delta p_{RXHP} = f_{RXHP} \cdot p_7 \quad (3.12)$$

Outlet pressure of the recuperator:

$$p_8 = p_7 - \Delta p_{RXHP} \quad (3.13)$$

$$Q_{HXL.P} = \dot{m} c_p (T_8 - T_7) \quad (3.14)$$

Heat source (9-10)

Pressure loss factor due to friction was assumed as:

$$f_{HS} = 0.001$$

The pressure drop through the heat source was calculated as follows:

$$\Delta p_{HS} = f_{HS} \cdot p_9 \quad (3.15)$$

Size of the heat source

$$Q_{HS} = \dot{m} c_p (T_{10} - T_9) \quad (3.16)$$

Turbine (11-12)

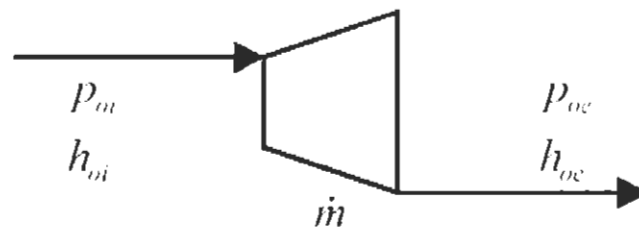


Figure 3-5: Turbine Schematic

The turbine efficiency was assumed as:

$$\eta_T = 0.91$$

The turbine efficiency was chosen high to contribute to a high cycle efficiency.

The turbine work was calculated as follows:

$$Q_{turb} = \dot{m} c_p \eta_t T_{\max} \left[1 - \left(\frac{1}{P_{011} / P_{012}} \right)^{\frac{\lambda-1}{\lambda}} \right] \quad (3.16)$$

Turbine pressure ratio.

$$PR_T = \frac{p_{11}}{p_{12}} \quad (3.17)$$

$$Q_t = \dot{m} c_p (T_{12} - T_{11}) \quad (3.18)$$

Recuperator

HP Side (13-14)

Pressure loss factor due to friction was assumed as:

$$f_{RXLP} = 0.011$$

The pressure drop through the recuperator was calculated as follows:

$$\Delta p_{1314} = f_{RXLP} \cdot p_{13} \quad (3.19)$$

$$Q_{RXLP} = -Q_{RXHP} \quad (3.20)$$

Outlet pressure of the recuperator:

$$p_{14} = p_{13} - \Delta p_{1314} \quad (3.21)$$

$$Q_{HXHP} = \dot{m} c_p (T_{14} - T_{13}) \quad (3.22)$$

Heat exchanger

Precooler (15-16)

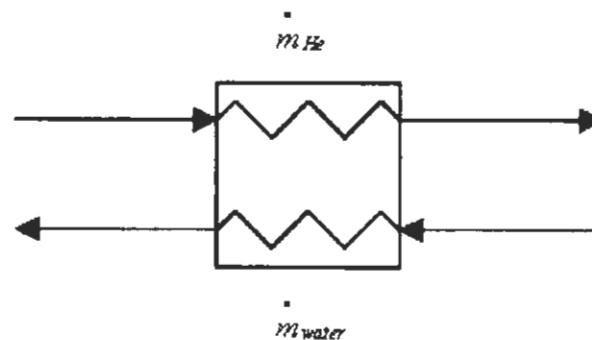


Figure 3-6: Precooler schematic

Effectiveness of the pre-cooler was assumed as:

$$\varepsilon_{PC} = 0.9$$

Pressure loss factor due to friction was assumed as:

$$f_{PC} = 0.02$$

The pressure drop through the pre-cooler was calculated as follows:

$$\Delta p_{PC} = f_{PC} \cdot p_{15} \quad (3.23)$$

Outlet pressure of the pre-cooler:

$$p_{16} = p_{15} - \Delta p_{PC} \quad (3.24)$$

Heat exchanged in the pre-cooler.

$$Q_{PC} = \dot{m} c_p (T_{16} - T_{15}) \quad (3.25)$$

$$Q_{PC} = \varepsilon_{PC} \dot{m} c_p (T_{water} - T_{15}) \quad (3.26)$$

Generator

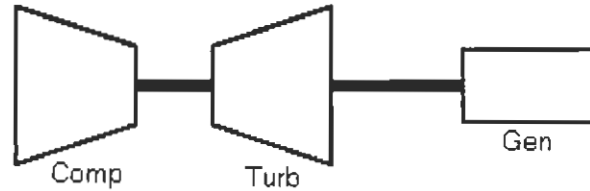


Figure 3-7: Shaft energy balance

Shaft energy balance.

$$Q_{gen} = Q_T \cdot \eta_m - Q_{LPC} - Q_{HPC} \quad (3.27)$$

Mechanical efficiency.

$$\eta_m = 1$$

Pipes

The pressure drop through the pipes due to friction was calculated as follows:

$$\Delta P = P_{out} \cdot f_{pipe} \quad (3.28)$$

Pressure loss factor due to friction was assumed as:

$$f_{pipe} = 0.001 \quad (3.29)$$

Energy lost in the pipe was assumed as zero..

$$\dot{Q}_{pipe} = 0$$

The calculation of the pressure drop through the pipes was done for the following steps:

(2-3), (4-5), (6-7), (8-9), (10-11), (12-13), (14-15) and (16-1).

For a similar pressure-loss, the flow velocity of helium can be double that of air, while the heat transfer coefficient of helium is almost twice that of air (*Rogers, Cohen & Saravanamuttoo, 2001:96*).

3.3 Cycle optimization

This section reviews the methods to optimize the cycle along with the results obtained during the optimization process.

3.3.1 Optimization of pressure ratio

Figure 3-8 shows the plot of determining the optimum cycle efficiency and power output as a function of the overall pressure ratio. The cycle efficiency and power output data were obtained by way of a parametric table in EES. The cycle was optimized for efficiency and not for power output. This results in an optimum pressure ratio of about 2.6 for maximum cycle efficiency and about 4.2 for maximum power output.

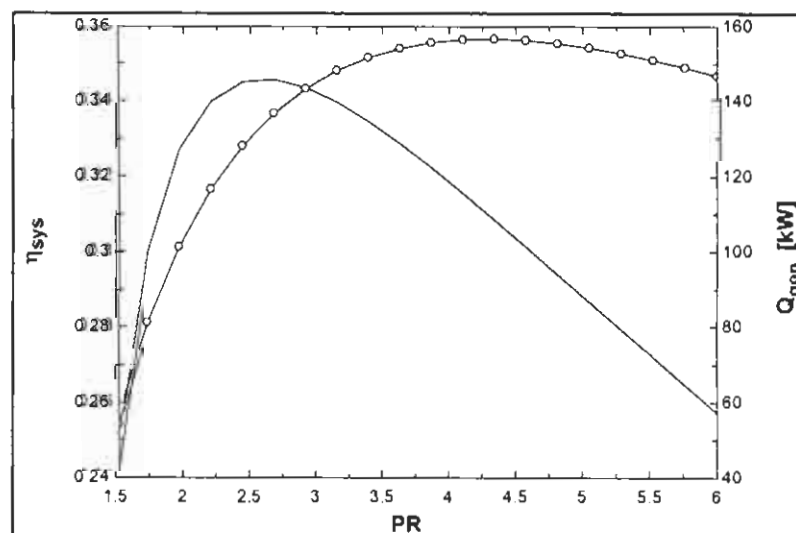


Figure 3-8: Optimum pressure ratio



3.3.2 Heatexchanger effectiveness

The efficiency of the recuperator has a great effect on the cycle efficiency. A more expensive recuperator will make the cycle more efficient. Figure 3-9 illustrates that a 15% (from 75% to 90%) increase of the recuperator efficiency will result in a 6% increase in cycle efficiency.

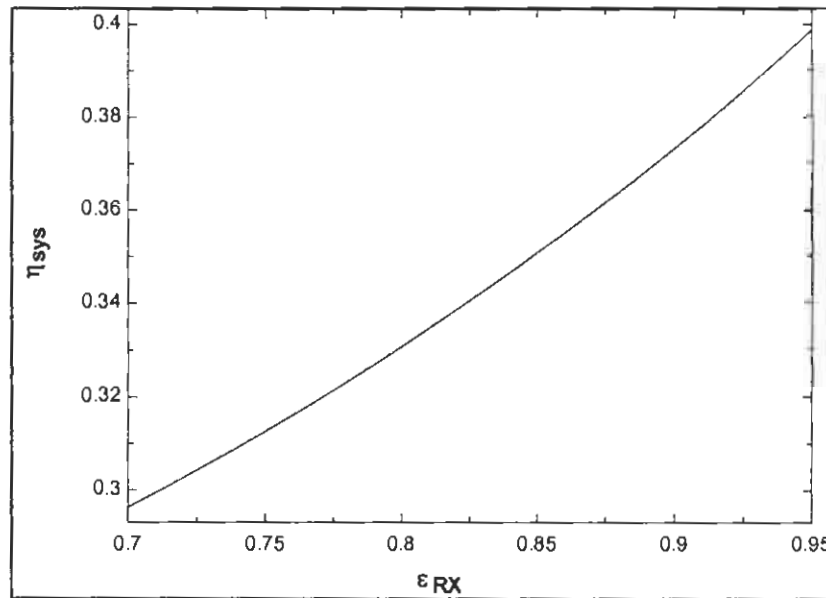


Figure 3-9: *Recuperator efficiency vs. cycle efficiency*

The efficiency of the heat exchangers used in the cycle analysis was between 85% and 90%. The application of the NTU-method constructed a plot of heat exchanger efficiency versus UA (heat transfer coefficient multiplied by the total area). As seen from Figure 3-10, UA becomes increasingly larger as the efficiency of the heat exchanger increases. This means that the overall cost of a heat exchanger with an effectiveness of greater than 90% will increase and become more expensive.

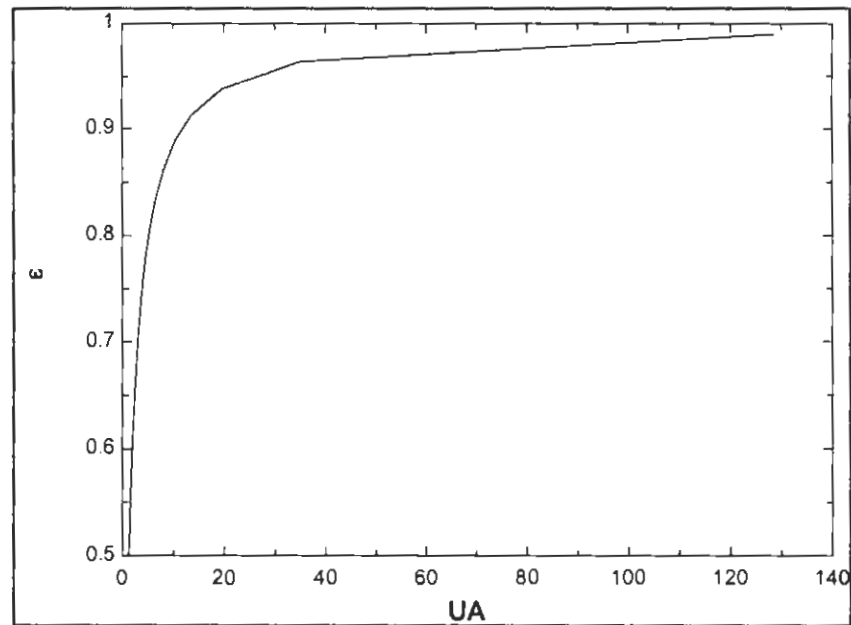


Figure 3-10: Heat exchanger efficiency versus heat transfer coefficient

3.3.3 Maximum temperature

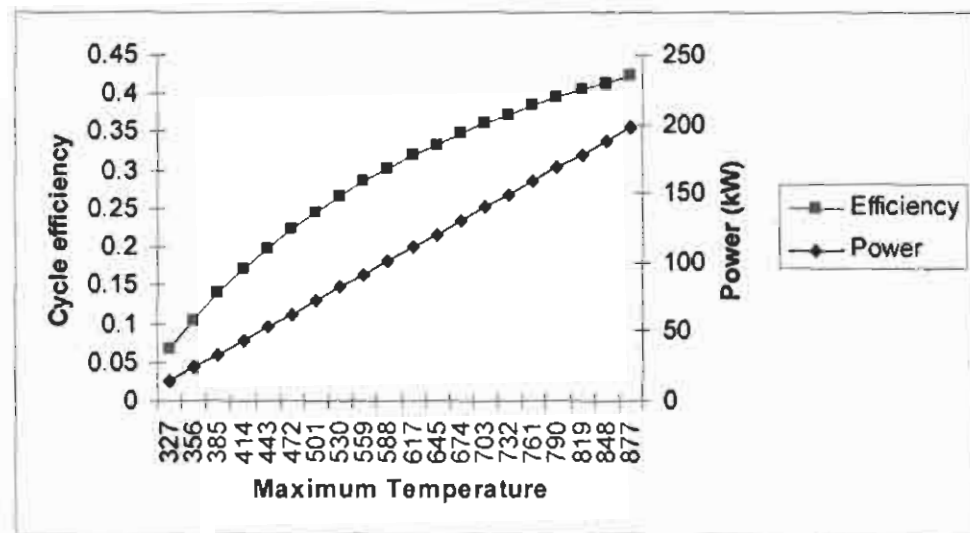


Figure 3-11: Turbine inlet temperature

Figure 3-11 shows the effect of higher turbine inlet temperatures on the cycle efficiency and the power output. One of the methods to optimize cycle efficiency and increase the power output is by increasing the turbine inlet temperature. To put this into perspective the cycle efficiency can be increased by 5% if the turbine inlet temperature is increased by 100°C.

3.3.4 EES output variables

Table 3-2 shows the output variables of the simulation. The Brayton cycle was simulated with nitrogen and helium to be able to design compressors with nitrogen and helium as the working fluid. The mass flow of the helium cycle is five (5) times less than the nitrogen cycle (for the same power output). This is due to the fact that the specific heat capacity of helium (5.193 kJ/kgK) is five (5) times greater than that of nitrogen (1.046 kJ/kgK).

According to *Rogers, Cohen and Saravanamuttoo (2001:96)* the flow velocity of helium can be double that of air for similar pressure losses. This means that the diameters of pipes used in the helium cycle can be much smaller than those of the nitrogen cycle. It is assumed that this will be the case when nitrogen is used in the cycle, seeing that the properties of nitrogen are close to that of air. The pipe diameters can be smaller for a more cost effective system.

As seen from Table 3-2 the size of the heat exchangers of both the cycles, are more or less the same in terms of heat exchanging. The heat transfer coefficient of helium is almost twice that of air, which means it will be almost twice that of nitrogen. This implies that the physical size of heat exchangers used in a helium system will be half the size of heat exchangers used in the nitrogen cycle.

The compressor work required in both cycles is very close. Table 3-2 reveals that the compressors in the nitrogen cycle consume about 5 kW less than those of the helium cycle.

Table 3-2: Cycle output values

Cycle output values and comparison		
	1-Shaft (Helium)	1-Shaft (Nitrogen)
Cycle efficiency	0.3509	0.3516
Cycle mass flow	0.255 [kg/s]	1.291 [kg/s]
Cycle pressure ratio	2.6	3.5
Generator power	140.4 [kW]	140.6 [kW]
Compressor work (LP)	98.99 [kW]	93.31 [kW]
Compressor work (HP)	98.99 [kW]	93.47 [kW]
Turbine work	341.8 [kW]	330.7 [kW]
Heat source size	400.0 [kW]	400.0 [kW]
Intercooler size	98.99 [kW]	94.57 [kW]
Pre-cooler size	157.2 [kW]	161.0 [kW]
Recuperator size	389.7 [kW]	446.0 [kW]

The maximum efficiencies of the cycles are almost identical as seen from Figure 3-12. Figure 3-13 explains that for the same power output, the cycle pressure ratio of the Nitrogen cycle is slightly higher. As seen from Figure 3-12 and Figure 3-13, the cycles are very close in terms of performance. The cost of components used in the different cycles will most likely determine the outcome of the final decision, of which one of the two cycles will be the most appropriate.

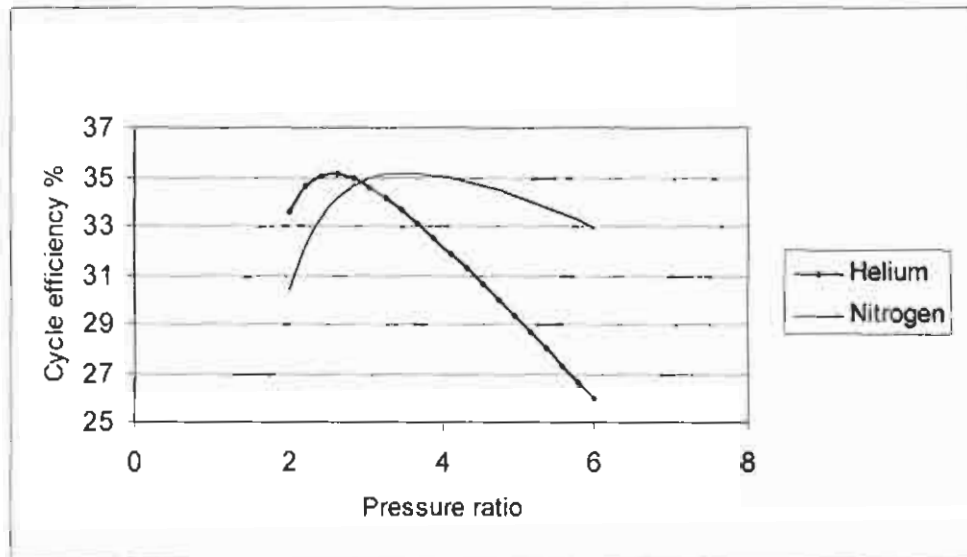


Figure 3-12: Comparison of cycle efficiency

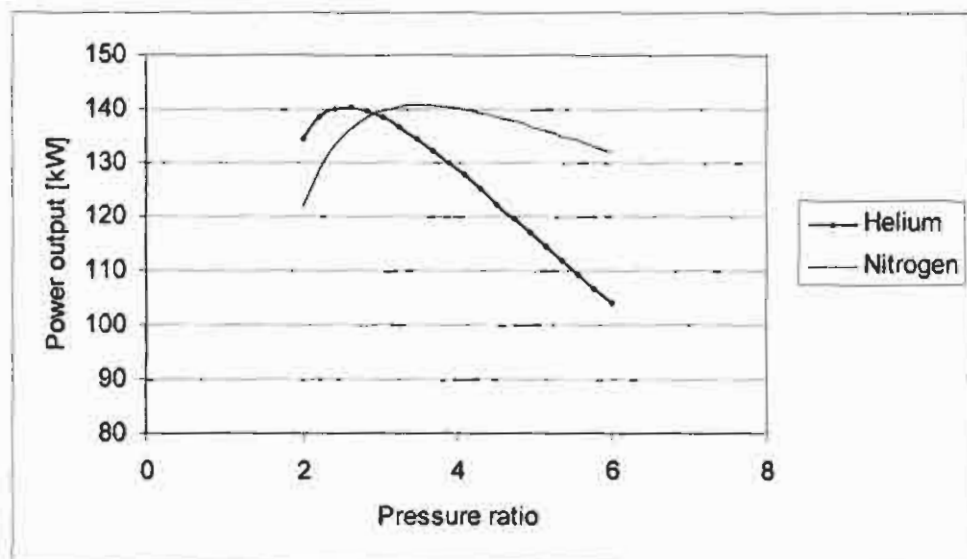


Figure 3-13: Comparison of cycle power output

3.4 Compressor design parameters

Table 3-3 is a summary of the compressor inlet and outlet conditions obtained from the cycle analysis with the aid of EES. These values are used in the preliminary compressor design in Chapter 5.

Table 3-3: Compressor design parameters

Compressor design parameters				
	LP Compressor		HP Compressor	
	Inlet conditions	Outlet conditions	Inlet conditions	Outlet conditions
Temperature [°C]	32.89	103.8	32.57	103.4
Pressure [kPa]	250	395.3	386	611.3

3.5 Cycle verification

Earlier in this chapter the cycle analysis was carried out to determine the compressor design parameters. After the software was written in EES, a similar code was obtained from PBMR. The program from PBMR was used to validate the EES program which was written for the cycle analysis carried out during the project. Inlet conditions of the EES program were added to the PBMR program to validate the results and the software. Helium was the working fluid throughout the validation.



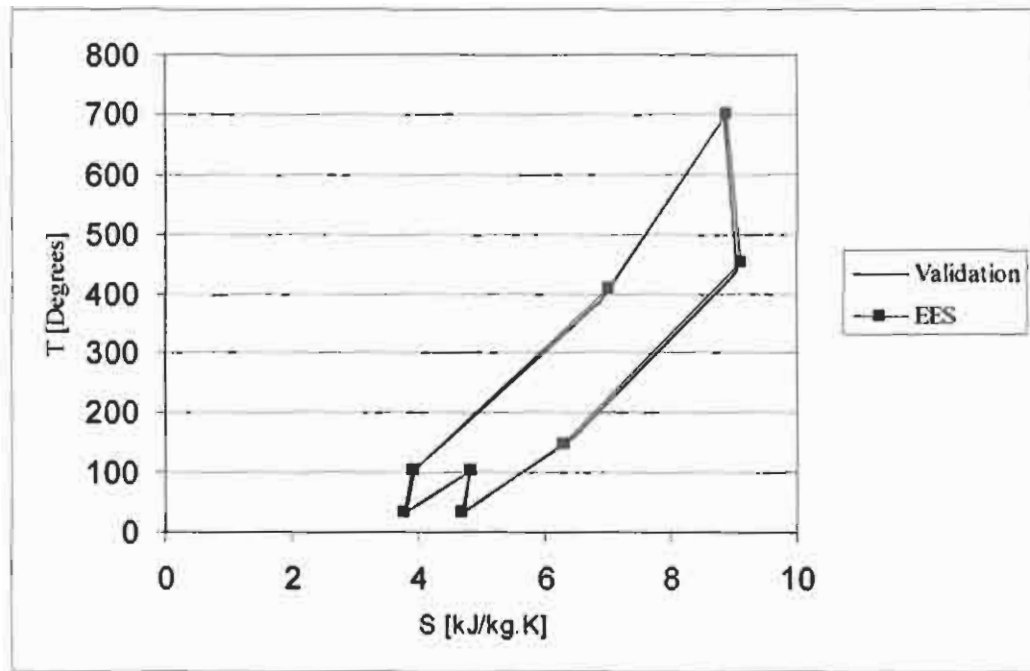


Figure 3-14: *T-s Diagram validation*

Figure 3-14 shows the overlay plot of the T-s diagram obtained from the cycle analysis and the T-s diagram attained from the validation software. The output temperatures and entropy of both cycles are very close, which means that the EES program is quite accurate when compared to the validation software.

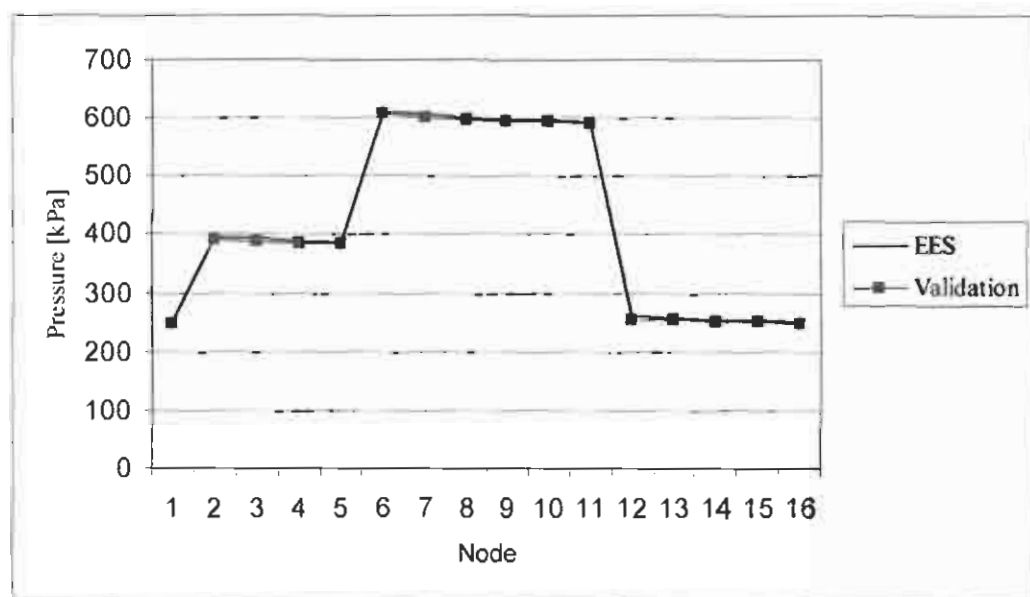


Figure 3-15: *Cycle pressure validation*

The pressure level of the working fluid was plotted at every node in the system. This allowed for a comparison of the pressure levels in both cases. Figure 3-15 confirms that the pressure levels of both alternatives are very close.

3.5.1 Comparison of results

As illustrated in Table 3-4 there are some differences in the output values of the software programs. The main reason for this display is because the validation software has more detail incorporated into the program. Leak flows as well as more complex pressure drop factors were incorporated.

The reason for not incorporating complex loss factors in the EES program is to simplify the software. It is preliminary design software to solely determine the in- and outlet conditions of the compressors.

Table 3-4: Cycle comparison

Cycle output values and comparison				
		EES	Validation	% Err
Cycle efficiency		0.3509	0.348	0.826
Cycle mass flow	[kg/s]	0.255	0.2429	4.745
Generator power	[kW]	140.4	139.2	0.854
Compressor work (LP)	[kW]	98.99	94.52	4.515
Compressor work (HP)	[kW]	98.99	90.55	8.526
Turbine work (PT)	[kW]	341.8	327.6	4.154
Heat source size	[kW]	400	400	0
Intercooler size	[kW]	98.99	90.55	8.526
Pre-cooler size	[kW]	157.2	167	5.868
Recuperator size	[kW]	389.7	354.6	9.006

3.6 Conclusion

The final decision of the best alternative (helium or nitrogen) depends on the outcome of the compressor design. Both Figure 3-12 and Figure 3-13, demonstrate that the performances of both cycles are very close. The nitrogen cycle requires larger heat exchangers, but the gas is cheaper than helium. Currently, both cycles offer viable solutions.

After the cycle analysis was done, the cycle was verified with software obtained from PBMR. As discussed in the previous section, most of the EES output parameters closely resemble that of the verification software. The mass flow rate deviates by 4.7 % due to leak flows incorporated in the validation software. Different types of loss factors were used in the validation software, which explains the deviation of the output variables. The T-s diagram and the cycle pressure of the EES program and the validation software are, however, very similar.

4. AXIAL COMPRESSOR DESIGN

4.1 *Introduction*

In the previous chapter the cycle analysis was conducted to obtain the design parameters for the compressor design in this chapter. This chapter firstly discusses the methodology of the preliminary design software written with the aid of EES. One of the study objectives of this project is to carry out a conceptual design of an axial compressor for a single shaft PBMM application. The compressor must be designed to be as small as possible to reduce expenses. Furthermore, the design has to be as efficient as possible to more closely resemble larger commercial units.

This chapter mainly reflect on the methodology of the preliminary design software as well as the verification of the software. The output values of the EES code is then validated by using turbomachine design software. A comparison between the EES and Concepts NREC results is drawn to determine the accuracy of the EES program.

The EES code was written according to a software program developed at Cranfield University. This software was developed in Excel for preliminary design of turbojet and turbofan engines. Other important resources utilized in the construction of the program are: *Gas Turbine Theory* by Saravanamutto, Rogers and Cohen, and *Gas Turbine Theory and Performance* by B.W. Botha.

4.2 *Preliminary compressor design procedure (EES Program)*

The starting point of the design procedure is the inlet conditions of the compressors. These values were obtained from the cycle analysis conducted in Chapter 3. Six of the most important inlet parameters were identified to act as the starting point of the conceptual compressor design.

4.2.1 Compressor inlet conditions

The following parameters were specified at the inlet of the compressor:

- Inlet temperature
- Inlet pressure
- Compressor pressure ratio
- Mass flow rate
- Inlet hub-tip-ratio
- Compressor polytropic efficiency

4.2.2 Gas properties

The following gas properties were taken as constants:

- $\gamma = 1.666$
- $R = 2.078 \text{ kJ/kg K}$
- $C_p = f(\text{Temperature, Pressure}) \text{ kJ/kg K}$

4.2.3 Program interface

INPUT VARIABLES		DE HALLER NUMBERS																																	
Compressor inlet conditions $T_{in} = 308 \text{ [K]}$ $P_{in} = 250 \text{ [kPa]}$ $\dot{m}_a = 0.2588$ $PR_{overall} = 1.8$ Compressor work $\dot{Q}_c = 99 \text{ [kW]}$ Polytropic efficiency $\eta_p = 0.88$ Rotational speed $N = 80000 \text{ [rpm]}$ Maximum allowable blade stress $\sigma_{max} = 7.440 \times 10^8 \text{ [Pa]}$		Loss factor $f_{loss} = 0.75$ Inlet hub-tip-ratio $h_{in} = 0.77$ Number of stages $n_{stage} = 8$ Blade material $P_{blade} = 7770 \text{ [kg/m}^2\text{]}$ Maximum tip speed $U_{tip} = 420 \text{ [m/s]}$ Calculate																																	
OUTPUT VARIABLES		<table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="text-align: center;">Rotor</th> <th style="text-align: center;">Stator</th> </tr> </thead> <tbody> <tr><td>$Dhs_1 = 0.7164$</td><td>$Dhs_1 = 0.764$</td></tr> <tr><td>$Dhs_2 = 0.7156$</td><td>$Dhs_2 = 0.7655$</td></tr> <tr><td>$Dhs_3 = 0.7176$</td><td>$Dhs_3 = 0.767$</td></tr> <tr><td>$Dhs_4 = 0.7198$</td><td>$Dhs_4 = 0.7683$</td></tr> <tr><td>$Dhs_5 = 0.7216$</td><td>$Dhs_5 = 0.7696$</td></tr> <tr><td>$Dhs_6 = 0.7234$</td><td>$Dhs_6 = 0.7708$</td></tr> <tr><td>$Dhs_7 = 0.725$</td><td>$Dhs_7 = 0.7719$</td></tr> <tr><td>$Dhs_8 = 0.7265$</td><td>$Dhs_8 = 0.7729$</td></tr> <tr><td>????</td><td>????</td></tr> <tr><td>????</td><td>????</td></tr> <tr><td>????</td><td>????</td></tr> <tr><td>????</td><td>????</td></tr> <tr><td>????</td><td>????</td></tr> <tr><td>????</td><td>????</td></tr> <tr><td>????</td><td>????</td></tr> </tbody> </table>		Rotor	Stator	$Dhs_1 = 0.7164$	$Dhs_1 = 0.764$	$Dhs_2 = 0.7156$	$Dhs_2 = 0.7655$	$Dhs_3 = 0.7176$	$Dhs_3 = 0.767$	$Dhs_4 = 0.7198$	$Dhs_4 = 0.7683$	$Dhs_5 = 0.7216$	$Dhs_5 = 0.7696$	$Dhs_6 = 0.7234$	$Dhs_6 = 0.7708$	$Dhs_7 = 0.725$	$Dhs_7 = 0.7719$	$Dhs_8 = 0.7265$	$Dhs_8 = 0.7729$	????	????	????	????	????	????	????	????	????	????	????	????	????	????
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$C_a = 152.8 \text{ [m/s]}$ $\eta_c = 0.8884$ $A_{in,stage} = 9.287$ $D_{hub,1} = 0.1045$ $D_{tip,1} = 0.1343$ $U_{m,1} = 373.4 \text{ [m/s]}$ $M_{a,1} = 0.1461$ $\psi = 0.9186$ $\phi = 0.3632$ $SF = 2.752$ $r_{hub,m} = 0.05171 \text{ [m]}$ $r_{tip,m} = 0.06715$ $\Delta T_{st} = 74.3$ $A_1 = 0.8286$		DIAGRAMS Annulus diagram Temperature rise Pressure rise Stage loading																																	

Figure 4-1: EES Diagram window

Figure 4-1 is a screenshot of the EES program interface and shortcuts to the annulus diagram as well as the pressure and temperature distribution throughout the compressor.

4.2.4 Determining temperature rise per stage

With the inlet conditions specified and the gas constants known, the overall compressor efficiency can be determined. The overall compressor efficiency was first determined in order to determine the overall temperature rise (refer to Equation (4.1)). A poly-tropic efficiency of 88 % was assumed to determine the overall compressor efficiency (Botha, 2005:59-103).

$$\eta_c = \frac{PR_{overall} \left[\frac{\gamma - 1}{\gamma} \right] - 1}{PR_{overall} \left[\frac{\gamma - 1}{\gamma \cdot \eta_p} \right] - 1} \quad (4.1)$$

The second step was to calculate the overall temperature rise during compression (refer to Equation (4.2)).

$$\Delta T_{c} = \frac{T_{in}}{\eta_c} \cdot \left(PR_{overall} \left[\frac{\gamma - 1}{\gamma} \right] - 1 \right) \quad (4.2)$$

Assuming equal temperature rise for all stages (to simplify the software) the temperature rise per stage was calculated by dividing the overall temperature rise by the number of stages as shown in Equation (4.3).

$$\Delta T_{stage} = \frac{\Delta T_c}{n_{stage}} \quad (4.3)$$

4.2.5 Calculation of the mean blade speed

The inlet hub-tip-ratio is specified as a constant value. A value of 0.8 was first adopted and then optimized during the design process. This high value was chosen because of the high rotational speed of the machine.

$$\frac{r_{hub,in}}{r_{tip,in}} = htr_{in} \quad (4.4)$$

The tip diameter was kept constant in order to simplify the design procedure. The tip diameter was then calculated as a function of the rotational speed and the maximum tip speed. The compressor's rotational speed was chosen as 60 000 rpm in order to determine the tip speed.

4.2.6 *Calculation of the annulus outlet area*

By calculating the outlet annulus area of each stage, a basic profile of the compressor annulus was obtained. With the annulus area of each stage known, the blade length of each stage was determined.

Initially, the axial velocity of the helium through the compressor was assumed at a constant value of 150 m/s. With this value known, the outlet area of each stage was determined as a function of the hub and tip diameters. Refer to Appendix B for more detail on the calculation of the annulus areas

The outlet area at each stage was calculated with Equation (4.5).

$$A_{out,0} = \frac{\pi}{4} \cdot D_{tip,0}^2 \cdot \left[1 - \left(\frac{D_{hub,0}}{D_{tip,0}} \right)^2 \right] \quad (4.5)$$

4.2.7 *Calculation of air angles*

The next step was to calculate the air angles at the outlets and inlets of the rotors and stators for each stage. By making use of velocity triangles, the velocities of the gas entering each stage can be calculated. Other important values obtained from the velocity triangles were the angles of the gas entering and leaving the compressor stages. With these values known the blade angles was calculated. These calculations were carried out with the aid of Figure 4-2.

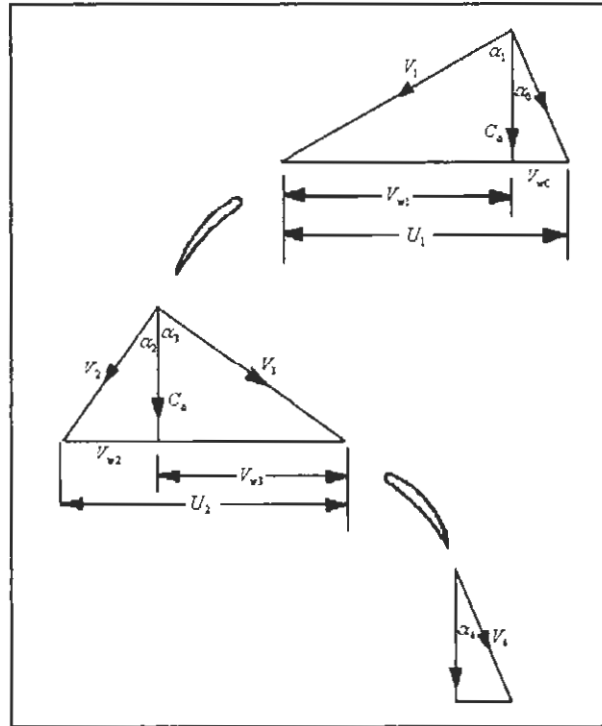


Figure 4-2: Velocity triangles

The velocity diagrams for one stage are shown in Figure 4-2 and the convention is adopted throughout this chapter.

4.2.8 De Haller numbers

The designer must have some method for assessing the allowable diffusion. One of the earliest criteria is the De Haller number and is defined as V_2/V_1 (Ratio of relative velocities) (Rogers, Cohen & Saravanamuttoo, 2001:191).

The De Haller numbers were calculated to ensure the allowable diffusion throughout the compressor. This was done by determining the relative velocities at the inlet and outlet of each stage (refer to Equation (4.6)). The ratio of these relative velocities was determined to obtain an indication of the amount of diffusion in the compressor as well as the manufacturability of the blades. Excessive diffusion will cause pressure losses in the compressor and a limit of 0.72 was set.

$$Dhr_1 = \frac{\cos(\alpha_{1,1})}{\cos(\alpha_{2,1})} \quad (4.6)$$

4.2.9 Stage reaction

The reaction ratio (R) is a design parameter that has an important influence on stage efficiency. The degree of reaction provides a measure of the extent to which the rotor contributes to the overall static pressure rise in the stage. Stages having 50% (0.5) reaction are widely used, as the adverse pressure gradient through the rotor and stator rows are equally shared. This choice of reaction minimizes the tendency of the blade boundary layers to separate from the solid surfaces, thus avoiding large stagnation pressure losses.

If $R = 0.5$, then $\alpha_1 = \beta_2$ from Equation (4.7), and the velocity diagram is symmetrical. The stage enthalpy rise is equally distributed between the rotor and stator rows.

$$R = \frac{\frac{1}{2} + (\tan \beta_2 - \tan \alpha_1) * Ca}{2U} \quad (4.7)$$

If $R > 0.5$ then $\beta_2 > \alpha_1$ and the velocity diagram is skewed to the right as shown in Figure 4-3. The static enthalpy rise in the rotor exceeds that in the stator (this is also true for the static pressure rise).

If $R < 0.5$ then $\beta_2 < \alpha_1$ and the velocity diagram is skewed to the left as indicated in Figure 4-3. In this case the stator enthalpy (and pressure) rise exceeds that in the rotor (*Dixon, 1998:143*).

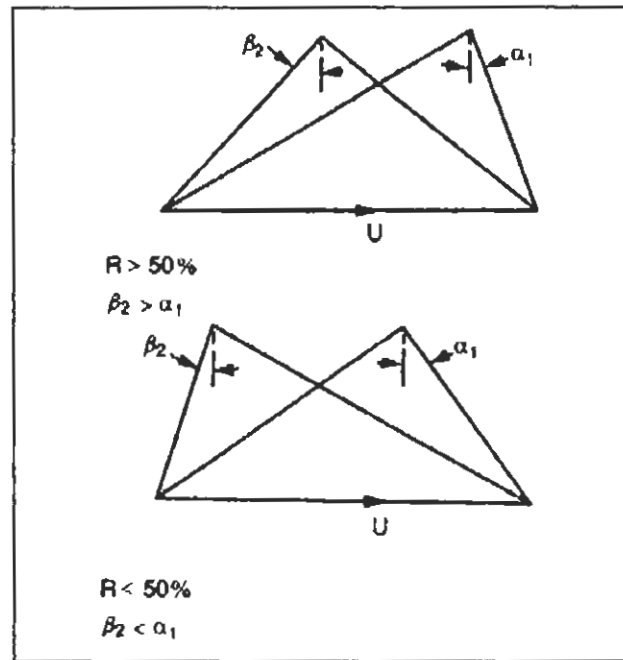


Figure 4-3: Stage reaction

The stage reaction was calculated from the rotor inlet and outlet angles as shown in Equation (4.8)

$$\Lambda_1 = \frac{C_a}{2 \cdot U_{m,1}} \cdot (\tan(\alpha_{1,1}) + \tan(\alpha_{2,1})) \quad (4.8)$$

4.2.10 Calculation of the safety factor for maximum blade stress at the root.

The compressor design conducted in this study does not include a detailed stress analysis of the compressor blades. Instead, a short formula was used to make sure that the blade stresses is kept within reasonable limits.

The centrifugal stress in the rotor blades depends on the rotational speed, the blade material and the length of the blade. The maximum centrifugal tensile stress occurs at the root of the blade and is given by:

The density of the blade is given by ρ_{blade} , U_t is the tip speed and $r_{\text{hub}} / r_{\text{tip}}$ is the hub-tip ratio. From Equation (4.9), it can be seen that low hub-tip-ratios will result in lower root stresses due to shorter blades.

$$\sigma_{\text{blade}} = \frac{\rho_{\text{blade}}}{2} \cdot U_{t,\text{in}}^2 \cdot \left[1 - \left(\frac{r_{\text{hub},1}}{r_{\text{tip},1}} \right)^2 \right] \quad (4.9)$$

(Rogers, Cohen & Saravanamuttoo, 2001:188)

The root stress of the longest rotor blade was calculated to obtain an indication of the safety factor. By applying Equation (4.9), the root stress of the longest blade was calculated. If the maximum blade stress is known, the safety factor can be calculated. Maximum blade stress was assumed as 700 MPa.

$$\text{SF} = \frac{\sigma_{\text{max}}}{\sigma_{\text{blade}}} \quad (4.10)$$

4.2.11 Blade geometry

Finally, the blade geometry had to be determined to serve as the input values in the validation software. The solidity was assumed as one and the blade profile as a circular-arc. From Equations (4.11) and (4.12) the chamber and stagger angles were calculated.

$$0.727 \cdot \text{theta}_1 = \alpha_{1,1} - \alpha_{2,1} \quad (4.11)$$

From Equation (4.12), the stagger angle can be calculated as:

$$\phi_1 = \alpha_{1,1} - \frac{\text{theta}_1}{2} \quad (4.12)$$

The same procedure was followed to determine the stagger of the stator blades.

4.3 EES Results

This section discusses the results obtained from the EES program. Furthermore, it examines and summarizes only the most important output parameters. Appendix A displays the rest of the results in detail.

4.3.1 LP Compressor

At first, the compressor design was done with helium as the working fluid. The results displayed in Table 4-1 is a summary of the compressor design parameters. According to *Rogers, Cohen and Saravanamuttoo (2001:189)* the axial velocity should be in the order of 150 m/s to minimize pressure losses in the annulus. As seen from Table 4-1 the axial velocity is 148.6 m/s, which is acceptable in this case.

Due to the high rotational speed of the compressor, the blade tip speed is quite high. This is, however, not a critical problem, because of a high hub-tip-ratio at the inlet of the compressor. The blades are very short, which ensures lower root stresses. A safety factor of 2.589 was calculated for the root stress of the longest blade (Refer to Equation (4.9)).

Table 4-1: EES Results (LP)

EES Results (LP)			
Compressor inlet and outlet conditions		Compressor characteristics	
Temperature	306 [K]	Axial velocity through compressor	148.6 [m/s]
Inlet pressure	250 [kPa]	Compressor efficiency	0.8684
Outlet pressure	400 [kPa]	Temperature rise per stage	7.43 [K]
Pressure ratio	1.6	Safety factor (for root stress)	2.836
Mass flow	0.2566 [kg/s]	Rotational speed	60 000 [rpm]
		Inlet hub-tip-ratio	0.77
		Maximum tip speed at inlet	400 [m/s]
		Compressor work	99 [kW]
		Number of stages	10

Figure 4-4 is an illustration of the LP compressor annulus diagram. The overall pressure ratio of the compressor is low (1.6) with the result of a low pressure rise per stage and a small change in density per stage. This is the reason for a small change in compressor annulus area as seen from Figure 4-4.

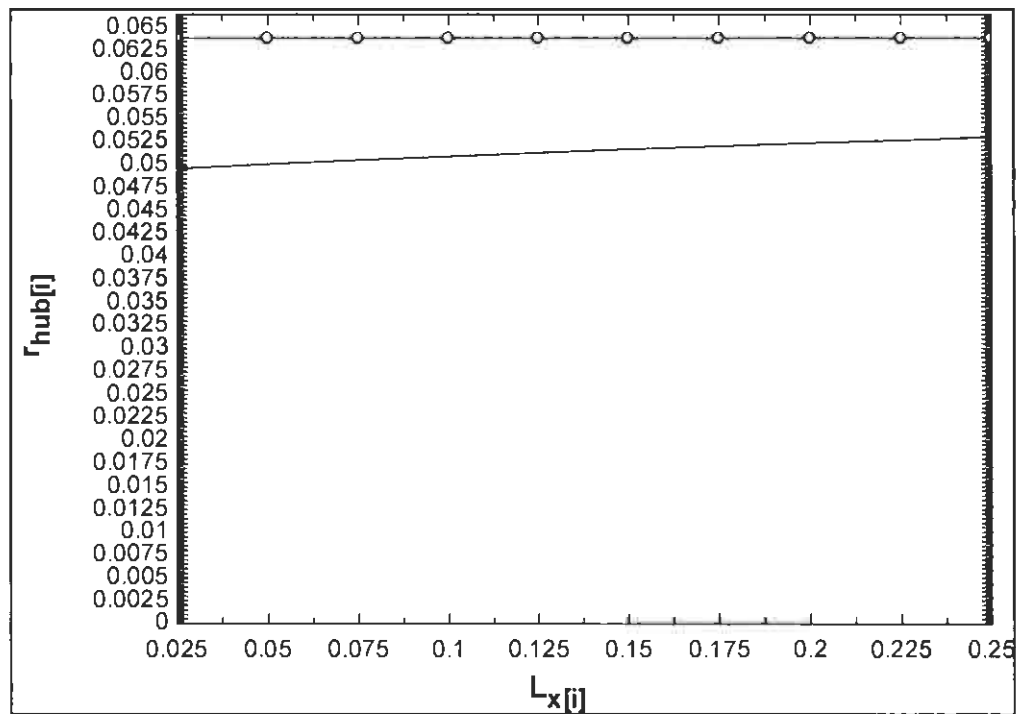


Figure 4-4: LP Compressor annulus diagram (EES)

The pressure rise through the LP compressor is shown in Figure 4-5. This is the average pressure rise over the rotor and the stator. A more detailed schematic of the pressure rise over the rotor and the stator will be shown in the Concepts analysis in order to verify the results obtained from the EES program.

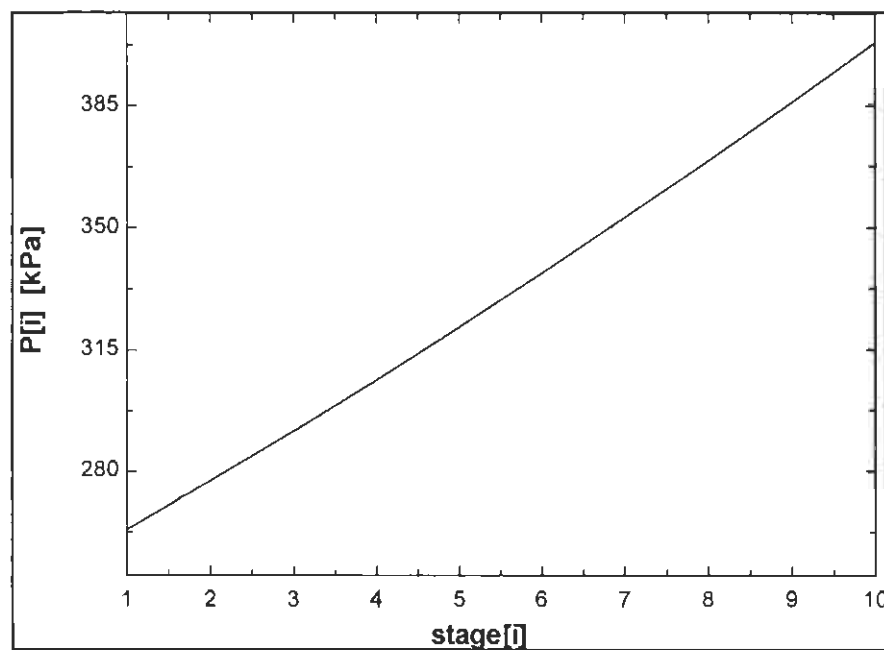


Figure 4-5: LP Compressor pressure rise

Figure 4-6 shows the average temperature rise of the helium over the rotor and the stator. The temperature rise per stage is low due to a low overall compressor pressure ratio and a large number of stages. A more detailed illustration is shown in the Concepts NREC analysis.

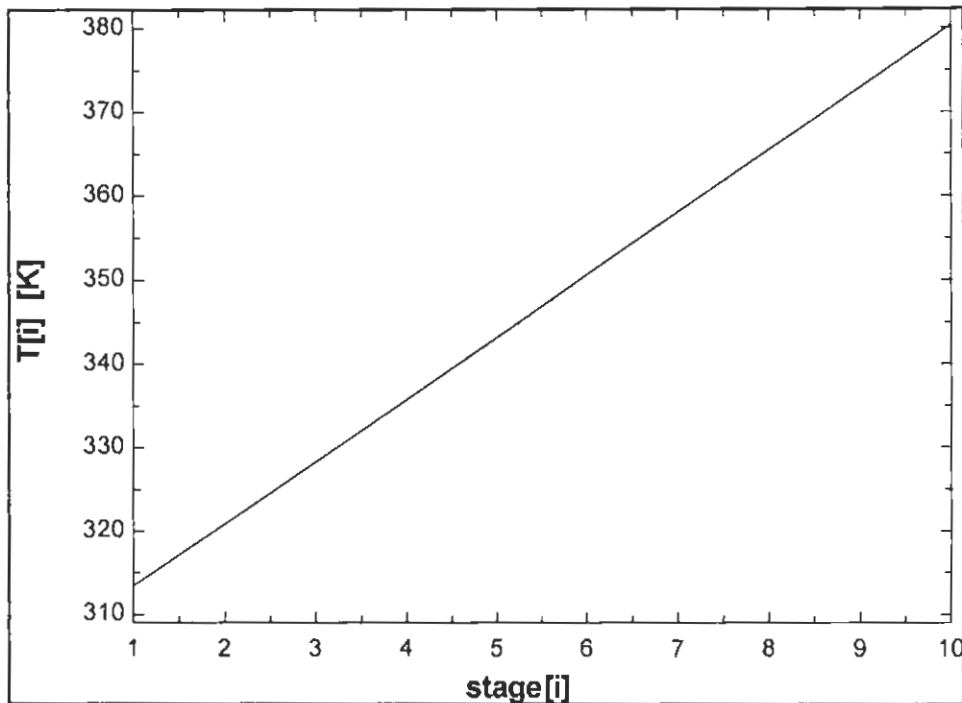


Figure 4-6: LP Compressor temperature rise

4.3.2 HP Compressor

A preliminary design of the HP compressor was done in EES as well as the validation of the design in Concepts. The design and analysis procedure of the HP compressor is the same as that of the LP compressor and is therefore discussed in less detail. Detailed results of the EES design and the Concepts NREC analysis are displayed in Appendix A. Table 4-2 summarises the design parameters obtained from the EES program.

Table 4-2: EES results (HP)

EES Results (HP)				
Compressor inlet and outlet conditions			Compressor characteristics	
Temperature	306	[K]	Axial velocity through compressor	141.2 [m/s]
Inlet pressure	250	[kPa]	Compressor efficiency	0.8684
Outlet pressure	640	[kPa]	Temperature rise per stage	6.192 [K]
Pressure ratio	1.6		Safety factor	4.172
Mass flow	0.2566	[kg/s]	Rotational speed	60 000 [rpm]
			Inlet hub-tip-ratio	0.8
			Maximum tip speed at inlet	350 [m/s]
			Compressor work	99 [kW]
			Number of stages	12

The results obtained from the EES model (refer to Table 4-2) can now be used as input for the Concepts NREC analysis. The Concepts NREC analysis will serve as verification of the EES model and to obtain more detailed results concerning the compressor design.

4.4 *Concepts NREC analysis and EES validation*

The axial compressor design was analyzed and verified with the aid of Concepts NREC. As explained in the previous section a program was written in EES to obtain the preliminary design parameters. These output values were used as input values in Concepts NREC. The output values consist of the blade geometry and inlet conditions of the compressor.

4.4.1 *Concepts output*

Figure 4-7 and Figure 4-8 were obtained after the compressor analysis was carried out in Concepts. The graphs displayed in this section are only that of the LP compressor. Appendix A summarizes the results concerning the HP compressor.

Figure 4-7 shows the pressure rise through the LP compressor. This graph was obtained by using the inlet conditions and the compressor geometry of the EES design in the Concepts NREC design. This is a more detailed illustration of the pressure rise through the compressor because the pressure rise over the stator as well as the rotor is shown.

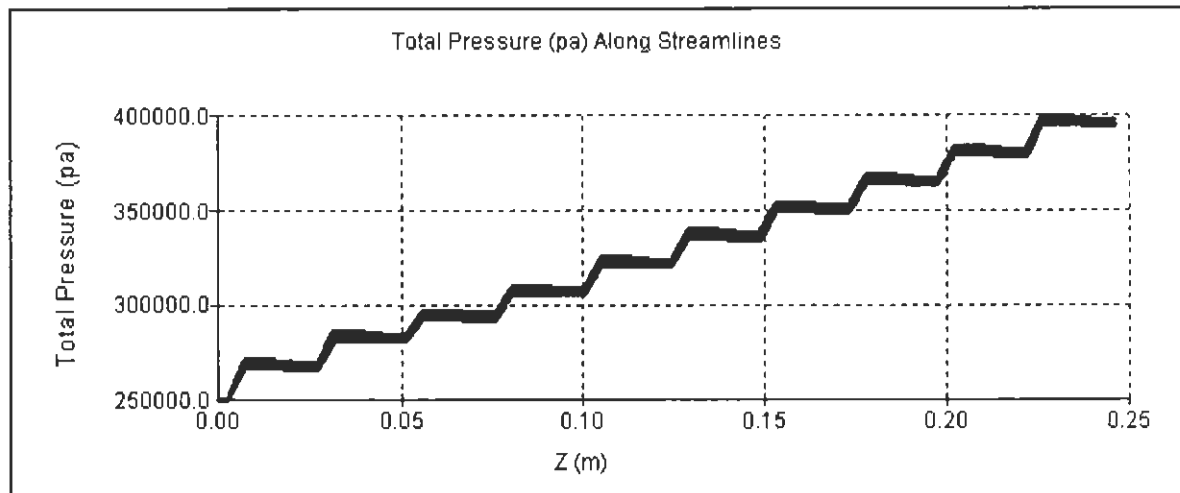


Figure 4-7: Pressure rise

Figure 4-8 illustrates the temperature rise through the LP compressor. The variation of the temperature rise at the hub and the tip is due to a constant profile of the blade geometry. Figure 4-8 also shows the temperature rise over the stator and the rotor.

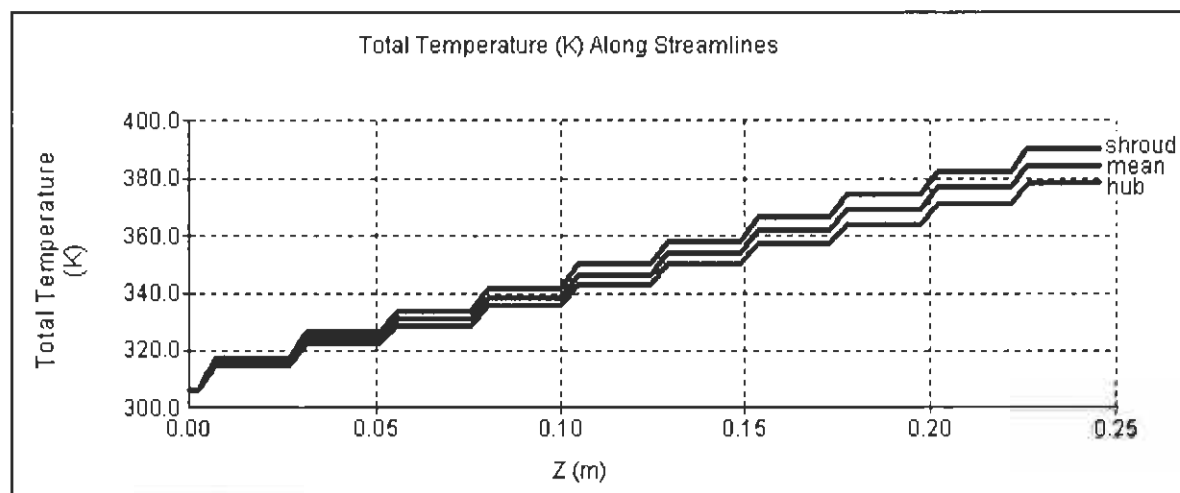


Figure 4-8: Temperature rise

Figure 4-9 eliminates the annulus diagram and the dimensions of the compressor simulated in Concepts. This gives a good indication of the size of the compressor relative to the size of the shaft and the annulus.

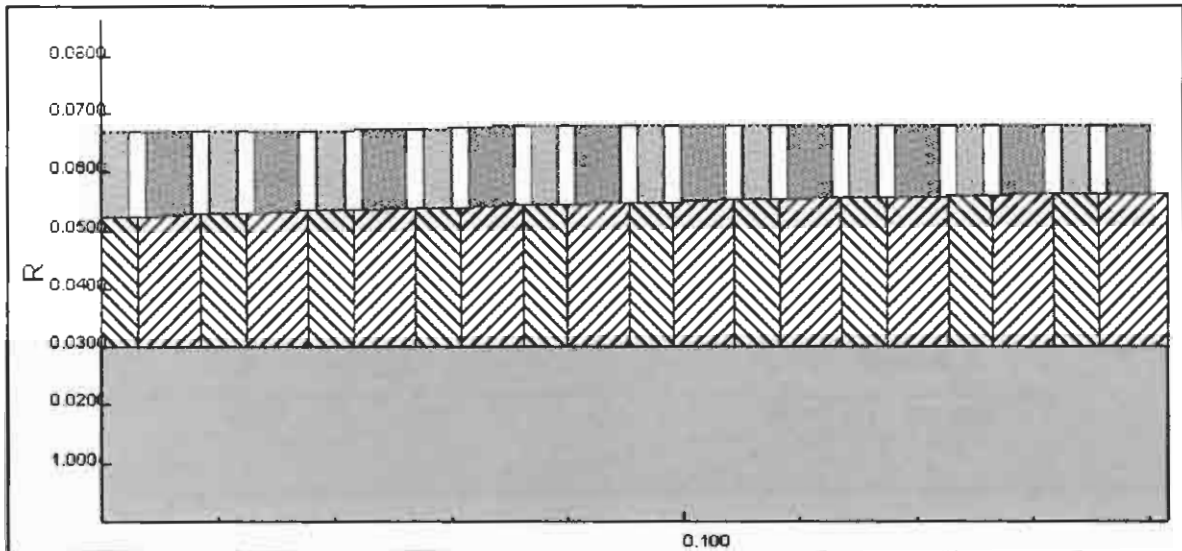


Figure 4-9: LP compressor annulus diagram (Concepts)

4.4.2 Temperature and pressure comparison

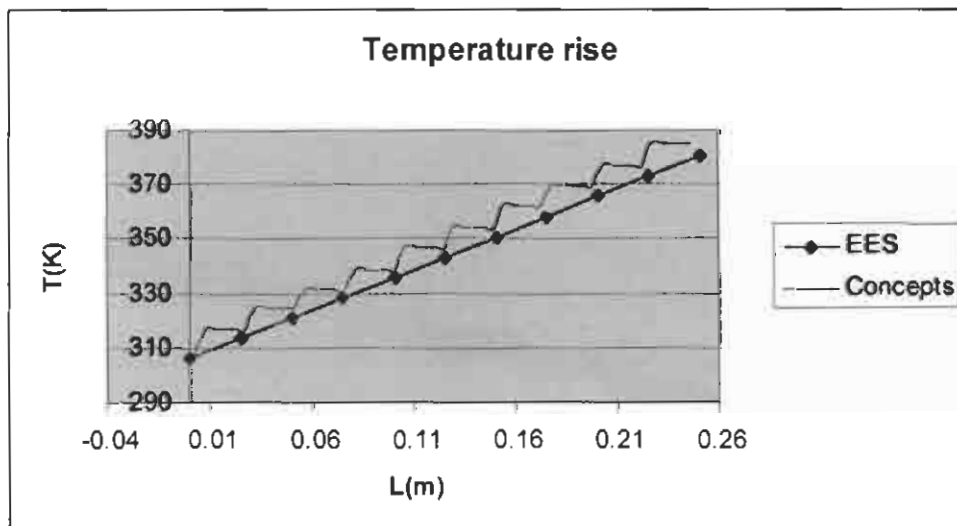


Figure 4-10: Temperature validation

As previously mentioned, no loss models were implemented in the EES program to compensate for pressure and other losses. The result was that the pressure and temperature rise of the compressors analysed in Concepts deviated by about 15 % from those designed in EES. Blockage factors were then incorporated into the EES program to compensate for pressure losses due to boundary layer build-up. The annulus area was effectively increased to reduce the axial velocity of the helium through the compressor annulus. As seen from Figure 4-10 and Figure 4-11 much better results were obtained after the blockage factors were incorporated into the EES program.

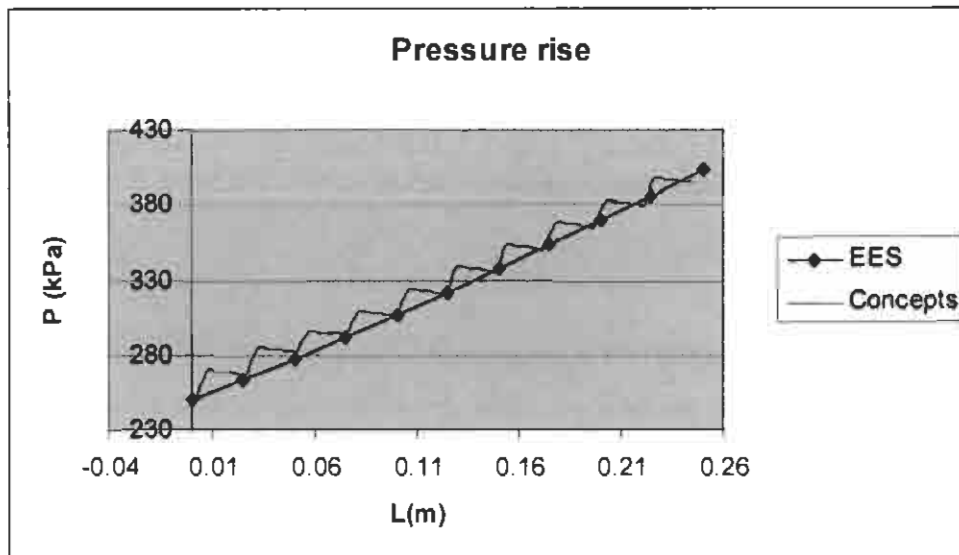


Figure 4-11: Pressure validation

4.4.3 Compressor map generation

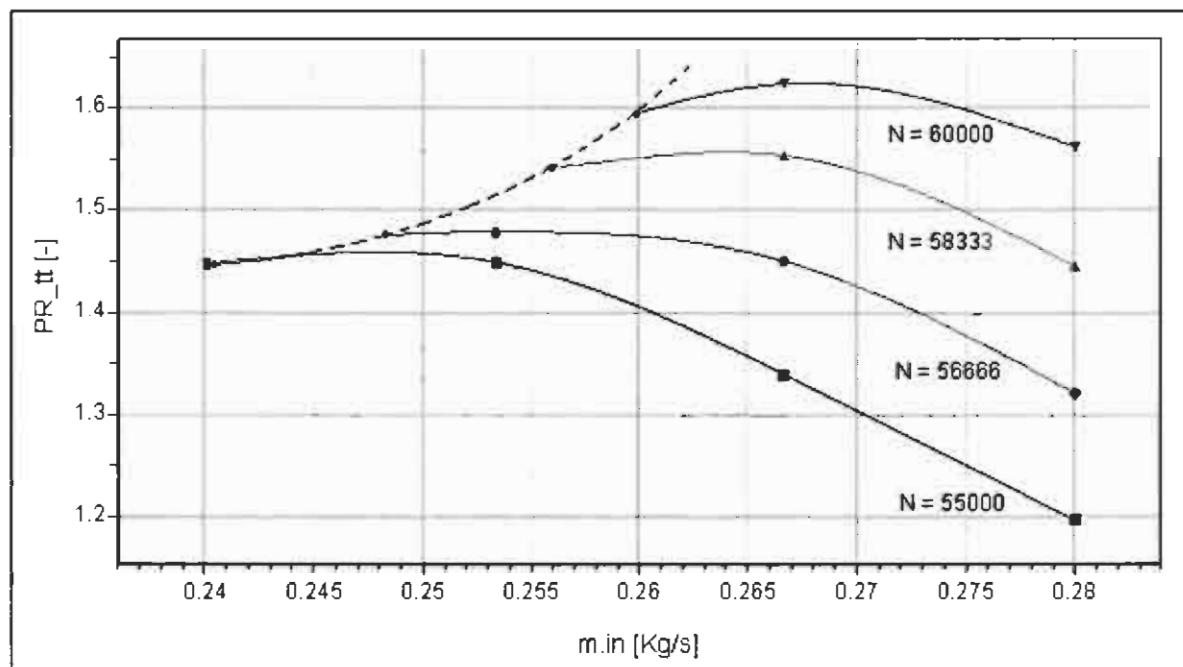


Figure 4-12: LP Compressor map (Helium)

A compressor map was generated with the aid of Concepts to determine the operating range of the compressor at off-design conditions. At a rotational speed of 55 000 rpm and a mass flow rate of 0.25 kg/s the compressor will produce a pressure ratio of 1.46. At these conditions the power available for the generator would be about 93 kW. For a maximum power output of 133 kW, the compressor will be rotating at a speed of 60 000 rpm along

with a mass flow rate of 0.265 kg/s. As shown in Figure 4-12, the mass flow rate can be varied between 0.24 kg/s (surge margin) and 0.28 kg/s (choke margin), which allows for a 0.04 kg/s change in mass flow rate from 55 000 rpm to 60 000 rpm. This means that the mass flow rate can be positively and negatively varied by 9%, from the average mass flow rate of 0.26 kg/s.

During the compressor map generation it was found that Concepts NREC does not indicate the positions of the surge or choke lines. The mass flow rate had to be reduced until the software indicated that stall occurred at the first stage of the compressor. It was assumed that surge will occur soon after compressor stall. The choke line was obtained by increasing the mass flow rate until the software indicated that the compressor is choked.

4.4.4 Comparison of axial compressors with different working fluids

After the design was completed with helium as the working fluid, the design was repeated with nitrogen. A compressor design using nitrogen as the working fluid was first completed in the EES program. It was found that the blades are much longer than in the case of the helium compressor. The reason for this is that, for the same generator power output of the nitrogen cycle, the mass flow had to be nearly five times more than that of the helium cycle. A much larger annulus area is required in the case of the nitrogen design. The problem is that the compressor blades cannot be extended, because the tip diameter was already determined at the maximum tip speed. The only option is to reduce the hub-tip-ratio, which will cause longer blades. By applying this, the safety factor of the root stress for the longest blade dropped to 1.47 compared to 2.836 of the helium compressor. According to Krige P, root stresses of industrial compressors are usually determined at 50% of the material's yield strength. This results in a safety factor of about two. The tip speed is 450 m/s which is quite high and higher strength material would be required to cope with the blade stresses. For this reason the Nitrogen design at 60 000 rpm is not a viable solution.

However, the nitrogen compressor requires fewer stages to obtain the specified pressure ratio. This would reduce the overall cost of the compressor. One of the major problems



concerning the axial nitrogen compressor is the narrow operating range, which makes it a less practical design.

Another compressor simulation was done at a rotation speed of 30 000 rpm in order to decrease inlet hub-tip-ratio and reduce the tip speed. A safety factor of 2.518 was obtained, which is much better than the previous 1.47. The only drawback is that the compressor will be more expensive due to the increase in size. A reduced rotating speed will also have an effect on the size of the generator. As previously mentioned, the size of the generator is almost inversely proportional to the speed of rotation.

4.4.5 HP Compressor

The validation procedure of the HP compressor is the same as that of the LP compressor and will not be discussed in this Chapter. Appendix A lists more detail on the validation of the HP compressor design.

4.5 Discussion

The Concepts NREC analysis indicates that the operating range of the compressors in both cases is very narrow. The helium compressor has a wider operating range than the nitrogen compressor. During the analysis it was concluded that an axial compressor using nitrogen as the working fluid is less suitable for this application.

A short simulation was carried out, of a compressor with nitrogen as the working fluid and a rotational speed of 30000 rpm. This is not a viable solution due to excessive expenses of a larger turbo-unit and generator.

The first problem with the axial nitrogen compressor is that a larger annulus area is required, because of the higher mass flow rate through the system. As previously mentioned in the cycle analysis, the mass flow in the nitrogen cycle will be five times that of the helium cycle for the same power output. For the same rotational speed the tip radius will be kept constant, but the hub radius will be reduced significantly. This results in longer blades, which results in much higher root stresses. Higher root stresses will influence the material cost of the compressor blades.

In both cases the compressor blades are very small due to the high rotational speed of the compressor and the relative low mass flow. The outcome is that a very small change in incidence angle causes the flow to separate as illustrated in Figure 4-14 (*Lakshminarayana, 1996:100*).

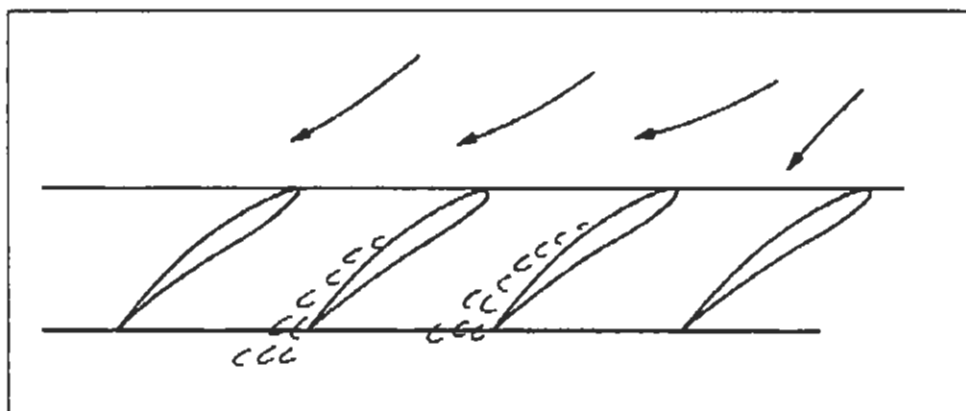


Figure 4-13: Flow separation due reduced incidence

A thin profile with a sharp leading edge is less sensitive to the inlet Mach number, but more sensitive to a variation of the incidence angle. According to *Kühnel, Gyarmathy and Ortman P (2005:1-8)*, the losses of an axial compressor cascade depend mainly on the inlet incidence angle and the inlet Mach number. Figure 4-14 exemplifies that the losses increase as the Mach numbers and incidence angles increases.

The loss distribution shows only small losses for low Mach numbers. For higher Mach numbers the losses increase considerably (Refer to Figure 4-14), especially for higher incidences. In the case of the nitrogen compressor, the losses will increase due to a higher Mach number. *Shames (1992:480-482)* defines the Mach number as:

$$M = \frac{V}{\sqrt{\gamma RT}} \quad (4.13)$$

For the same axial velocity (V) and temperature (T), the Mach number in the case of nitrogen will be much higher (nearly 3 times) due to a higher gas constant (R) and the ratio of specific heats (γ).

Furthermore, the profile is less sensitive to negative incidences than to positives incidences. This means that the losses rise more drastically for higher mass flow rates than

for reduced mass flow rates. The reason for this is that the flow will tend to separate at high axial velocities as shown in Figure 4-13. This will also have a negative effect on the nitrogen compressor.

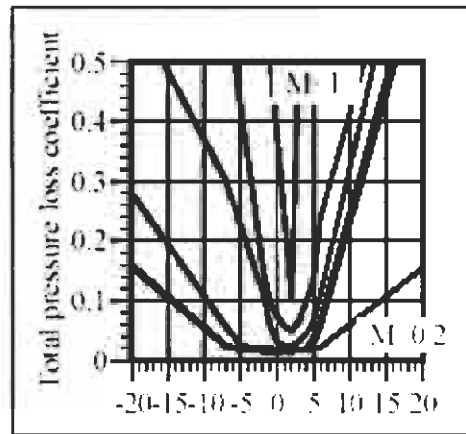


Figure 4-14: Compressor losses as a function of Mach number and incidence

Another problem in small scale turbomachines is the annulus boundary layer build-up. The reason for this is that the effective annulus area is affected more by boundary layer build-up. The thickness of the boundary layer is a function of the velocity. This insinuates that a small annulus with a gas flowing at the same velocity than in a large annulus will suffer a larger percentage reduction of effective flow area due to the boundary layer (Figure 4-15).

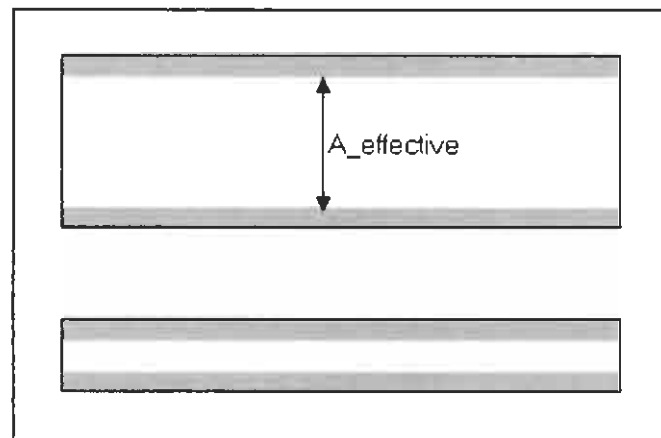


Figure 4-15: Effective flow area

The boundary layer thickness is a function of the Reynolds number. For a higher Reynolds number a thicker boundary layer will accumulate (*Shames, 1992:358*). The Reynolds number is formulated as follows:

$$\text{Re} = \frac{\rho V D}{\mu} \quad (4.14)$$

Equation (4.14) demonstrates that a compressor that uses nitrogen as the working fluid will have a higher Reynolds number for the same velocity and diameter, due to a much higher density. It can be shown with EES that the viscosity of helium and nitrogen does not differ much under the same conditions. The helium compressor will therefore be less affected by the problem of boundary layer build-up.

As previously mentioned, the axial compressors designed for this application were designed with high efficiencies in order to more closely resemble commercial units (Refer to Figure 4-1). It, however, turns out that small axial compressors are much more sensitive to small changes in mass flow rate than initially anticipated. Axial machines with lower efficiencies can be used in the system. However, the question arise why not use radial machines instead.

A design procedure and the testing of a 6.5 pressure ratio centrifugal compressor incorporating 30 deg back swept vanes, giving an isentropic total-to-total efficiency for the impeller of over 85 % was carried out. The overall total-to-total efficiency for the stage was 76.5 %. This proves that radial compressors can be designed with relative high efficiencies (*Wilson & Korakianitis, 1998*).

At very low air mass-flow rates, the efficiency of axial compressors drops rapidly. Blading of small axial turbines are difficult to manufacture accurately. It is better to make use of a centrifugal compressor in the case of low mass flow rate.

Radial-flow machines, for the same flow and pressure rise, are usually less expensive to produce than axial machines. With radial machines fewer stages are required and one-piece rotors and possible one piece stators can be produced. Accurate manufacturing of radial machines can be done with relative inexpensive investment casting (*Wilson & Korakianitis, 1998*).



The axial compressor's capital cost is higher than that of a centrifugal, but may well be justified by energy cost in an overall evaluation. The pressure ratio per stage is less than that of the centrifugal compressor. It takes approximately twice as many stages to perform the same pressure ratio as would be required by a centrifugal compressor (*Brown, 1997:226*).

It can be concluded that radial compressors will be the better option for the use in small power conversion plants (low mass flow rate), for example in the three-shaft nitrogen micro model. Nitrogen as the working fluid along with radial turbomachines will be the best option in the case of the micro model. This confirms the theory stating that centrifugal compressors should be used for low to medium mass flow applications and axial machines for medium to high flow rates. When the current three-shaft micro model is considered, radial machines seem to be the better option for this application. It is already proven by the three-shaft system that radial machines offer a good solution for this kind of system.

In order to decide whether to use helium or nitrogen in the cycle, a detailed cost analysis should first be conducted. This, however, does not fall within the scope of this project and a detailed cost analysis will not be included in the study. Factors and components affecting the overall cost of the cycles are briefly discussed.

Firstly, the implications of constructing a nitrogen system are discussed. It is possible to modify standard off-the-shelf turbochargers to fit the cycle layout. A larger turbine would, however, be required to drive both compressors. The expenses would still be less if a radial turbo-unit is manufactured for a cycle with nitrogen as the working fluid in stead of an axial turbo-unit with helium as the working fluid.

If a helium system is used, the gas will be more expensive. One great advantage of the helium cycle is that the size of the heat exchangers will be half the size of those used in the nitrogen system. Another advantage is the physical size of the pipe work, as the diameter of the pipes used in the helium cycle can be reduced by 15 % for the same pressure losses (*Rogers, Cohen & Saravanmuttoo, 2001:97*). Leakages will be a larger problem in the helium cycle and extra care should be taken to minimize leakages. All these factors should be taken into account to select the most appropriate system. It is recommended that a

further study on the expenses of each cycle should be done, in order to select the most appropriate system according to cycle performance and overall cost.

4.6 Conclusion

It can be concluded that radial compressors with nitrogen as the working fluid will be the best option. This decision is based on the successful construction of the existing three-shaft PBMM as well as the results obtained from the axial compressor design and analysis. During the compressor design it was found that the operating range of an axial compressor is severely compromised by low mass flow rate through the compressor annulus. The result is that a very precise control system will be required. Literature has shown that modern radial compressors can be designed to be very efficient and at a lower cost than that of axial machines.

5. POWER CONVERSION

5.1 *Introduction*

This chapter will discuss reasons for implementing a high-speed generator in the single shaft micro model. According to the information gathered in the literature study, the advantages for using a high-speed generator are summarized. An assessment of possible suppliers and their products also follows.

5.2 *Generator selection*

- High-speed generators are smaller than conventional generators. This is due to the fact that machine size being inversely proportional the speed. The unit is smaller and uses less material, which makes it less expensive.
- High-speed alternators can act as the starter to get the turbo-unit up to speed, and as the generator to produce the required power. This eliminates the need for a dedicated starter motor or an auxiliary unit, as used in conventional generator applications. This makes the power plant less complex and more compact.
- A reduction gearbox is not required if a high-speed generator is used in the power plant. This requires no lubrication system for the reduction gearbox and makes the configuration even less expensive and more compact. As previously mentioned a high-speed generator has very low maintenance and is quite reliable.

5.3 *Cost impact of a high-speed generator*

The overall cost of the turbo-unit is reduced when a high-speed generator is utilized in the cycle. According to a correlation derived at PBMR the overall cost of axial turbomachines is a function of the tip radius and the number of stages. It is clear that the higher the rotational speed the lower the compressor cost will be. A compressor with a rotational

speed of 60 000 rpm was chosen simply because of mechanical limitations. The blade tip speed will be in excess of 450 m/s and the root stresses will be unacceptably high if the rotational speed is further increased. It is proven that a high-speed turbo-unit will be less expensive if the cost of the turbo-machines and the generator is taken into account.

5.4 *Generator manufacturers*

Either S2M or Calnetix can supply a high-speed generator for the micro model. The only problem is that both companies require detailed information regarding the application and the size of the turbo-machines utilized in the configuration in order to manufacture and supply a generator. No standard off-the-shelf generators are available from these companies. One advantage is that both the companies offer integrated high-speed magnetic bearings along with the high-speed generator.

Currently, it is difficult to estimate the cost of such a generator due to a lack of structural and rotor dynamic analysis of the turbo-unit. However in future, consideration should be given to Calnetix who offers complete generator design, bearing system design, rotor dynamic analysis, structural analysis, thermal analysis, and electronics (produced by either Calnetix or its partners).

5.5 *Conclusion*

It can be concluded that a high-speed generator will be most suitable for this specific application. By implementing a high-speed generator, the system will be less expensive and simpler. High-speed generators are smaller than conventional generators, which makes it less expensive. Extra components like a reduction gearbox or an auxiliary power unit, required for start-up, is eliminated when a high-speed generator is used.

6. **CONCLUSIONS**

6.1 *Introduction*

This chapter summarizes the most important results obtained from the study. Also is described the outcome of each chapter as well as a conclusion and the identification of further studies.

6.2 *Summary*

The first chapter of the thesis focussed on the identification of the study and background concerning the need for the study. The study objectives and the method of investigation were set to ensure successful completion of the project.

In Chapter Two a literature study was conducted to gather the required technical information to be utilized in the study. Information concerning open and closed cycles were gathered to form a background against which a detailed cycle analysis was done in chapter three. Methods of cycle optimization and different cycle layouts were discussed. Axial compressors were studied, with the focus on design and selection of the most appropriate compressor to be implemented in the cycle. Information concerning the detail design of compressors and their applications were also studied. The last section of Chapter two discussed the implementation of a high-speed generator in the proposed cycle.

In Chapters Three, a cycle analysis was done in order to determine the overall cycle efficiency, power available to drive a high speed generator and thirdly to specify the inlet and outlet conditions of the compressors used in the cycle. After the cycle analysis was done, the cycle was verified with software obtained from PBMR.

Chapter Four consists of the compressor design and the final selection of the compressor to be used in the cycle. It is concluded that radial compressors will be the better option for the use in small power conversion plants, as in the three-shaft nitrogen micro model. Thus,



nitrogen as the working fluid along with radial turbomachines will be the best option in the case of the micro model.

Chapter Five concluded that a high-speed generator will be most suitable for this specific application. By implementing a high-speed generator, the system will be less expensive and simpler.

6.3 *Recommendation of further studies*

In order to make a definite final decision on the type of compressor and the working fluid for the cycle, the following aspects must first be considered:

- Expenses concerning the manufacturing of axial and radial compressors.
- The cost and size of the heat exchangers used in the helium and nitrogen cycles should be compared.

Other aspects to be considered:

- Dry gas seals should be developed for a PBMM application including all turbo-machine inlets and outlets (CFD). This is to prevent or minimize any gas leakage in the system
- The selection or design of an appropriate coupling between the generator and the turbo-unit should be done.
- A physical layout of the cycle (including pipe work and couplings) must be put together to determine the geometry of the pipework and to determine the space required to construct the micro model.

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APPENDIX A: ILLUSTRATIONS

This appendix displays the detailed results of the different compressor designs obtained from the EES simulation and the Concepts NREC analysis. Table A.1 gives a short description for each of the Figures displayed in the appendix.

Table A.1: Figure description

Figure	Description
A.1	Annulus dimensions of the HP (Helium) compressor obtained from the EES simulation.
A.2	The pressure rise per stage of the HP (Helium) compressor, determined in EES
A.3	The temperature rise per stage of the HP (Helium) compressor obtained from the EES program.
A.4	Annulus dimensions of the HP (Helium) compressor simulated in Concepts NREC.
A.5	The pressure rise per stage of the HP (Helium) compressor, obtained from Concepts NREC.
A.6	The temperature rise per stage of the HP (Helium) compressor, obtained from Concepts NREC.
A.7	The HP (Helium) compressor map, generated with the aid of Concepts NREC
A.8	Annulus dimensions of the LP (Nitrogen) compressor obtained from the EES simulation.
A.9	The pressure rise per stage of the LP (Nitrogen) compressor, determined in EES.
A.10	The temperature rise per stage of the LP (Nitrogen) compressor, determined in EES
A.11	The pressure rise per stage of the LP (Nitrogen) compressor, obtained from the Concepts NREC analysis.
A.12	The temperature rise per stage of the LP (Nitrogen) compressor, obtained from Concepts NREC.

Helium HP Compressor results

EES results

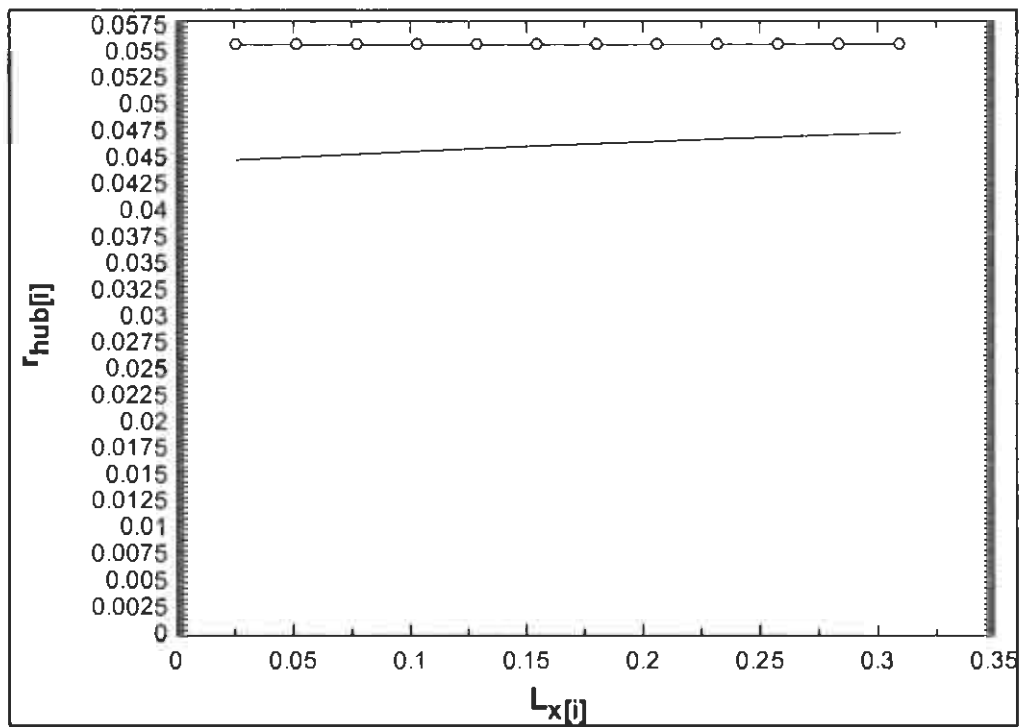


Figure A.1: HP compressor annulus diagram (He)

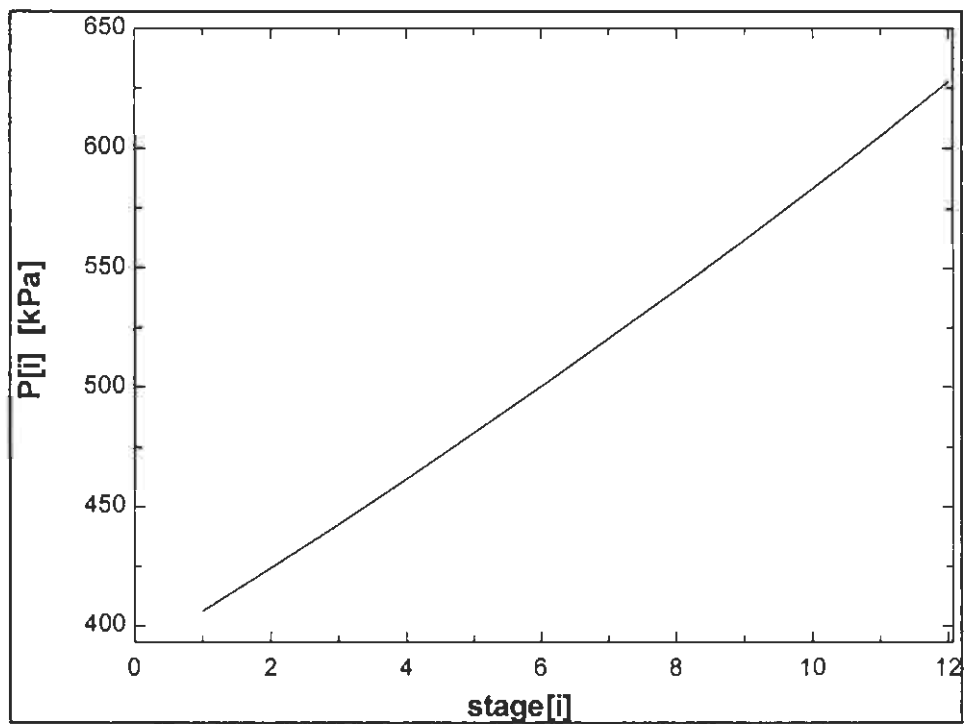


Figure A.2: HP compressor pressure rise (He)

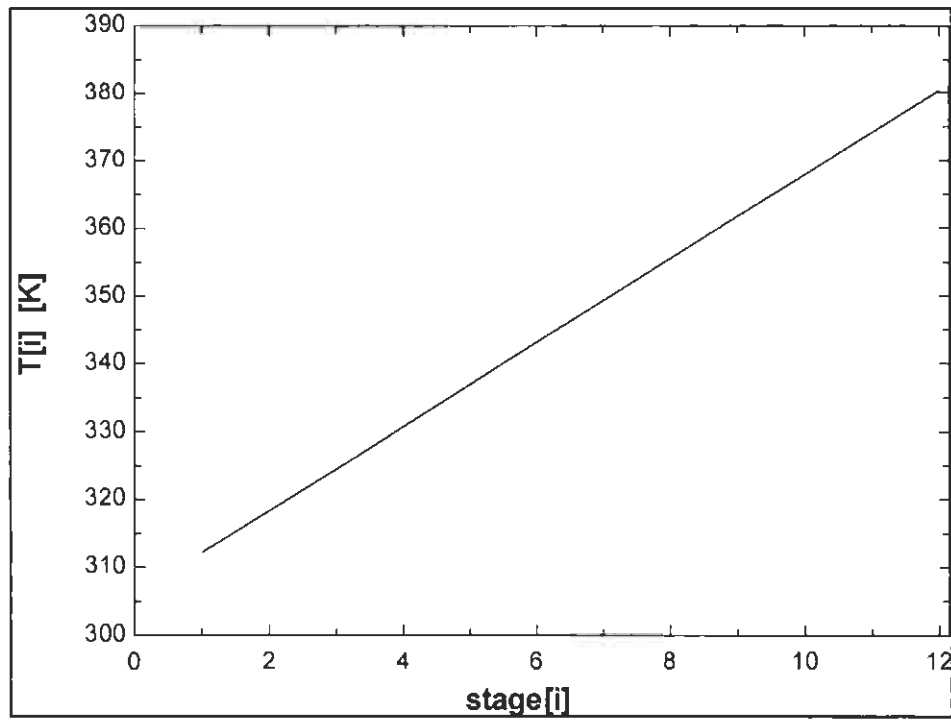


Figure A.3: HP compressor temperature rise (He)

Concepts Results

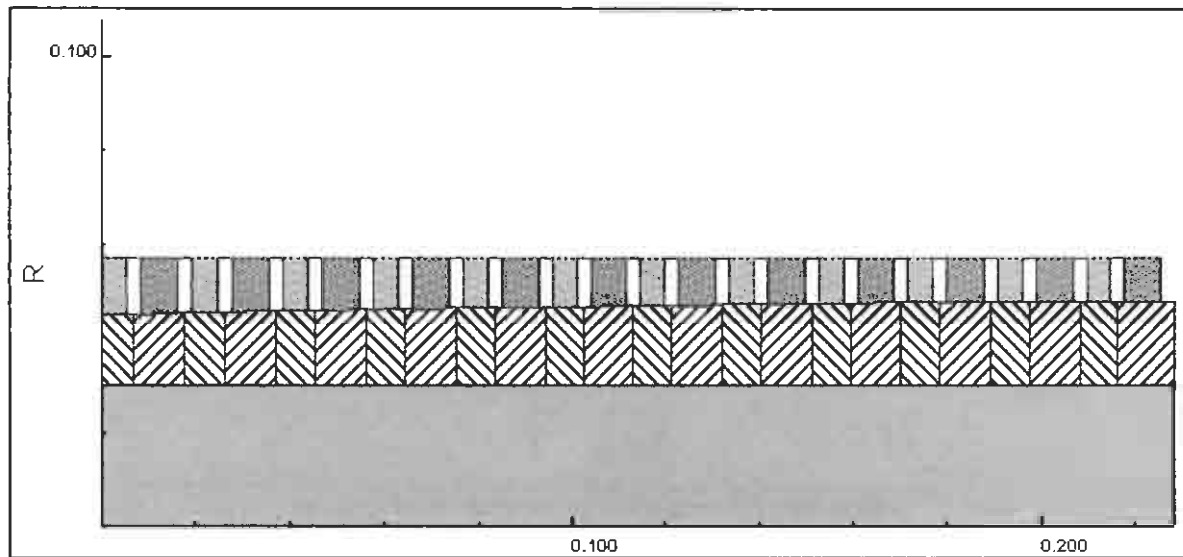


Figure A.4: HP compressor annulus diagram (Concepts)

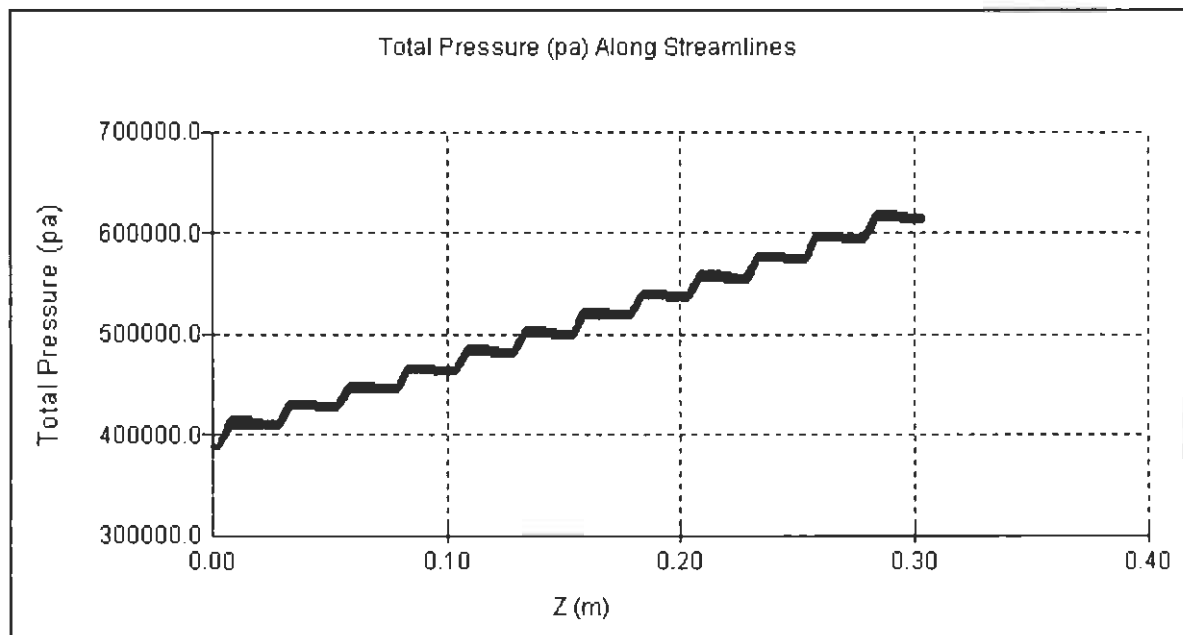


Figure A.5: HP compressor pressure rise (Concepts)

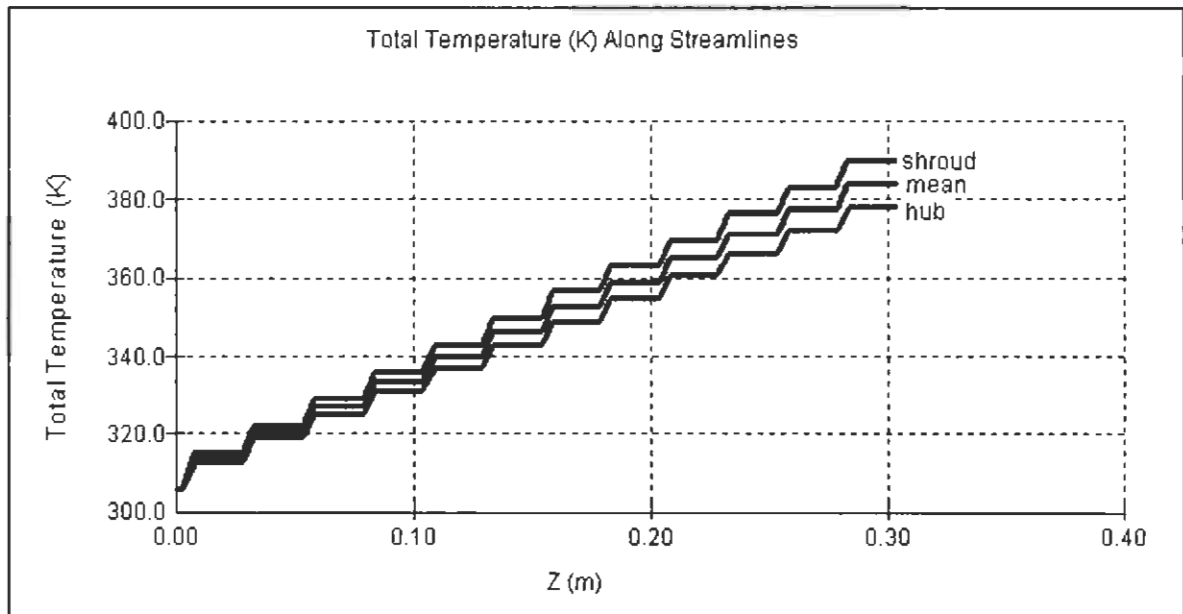


Figure A.6: HP compressor temperature rise (Concepts)

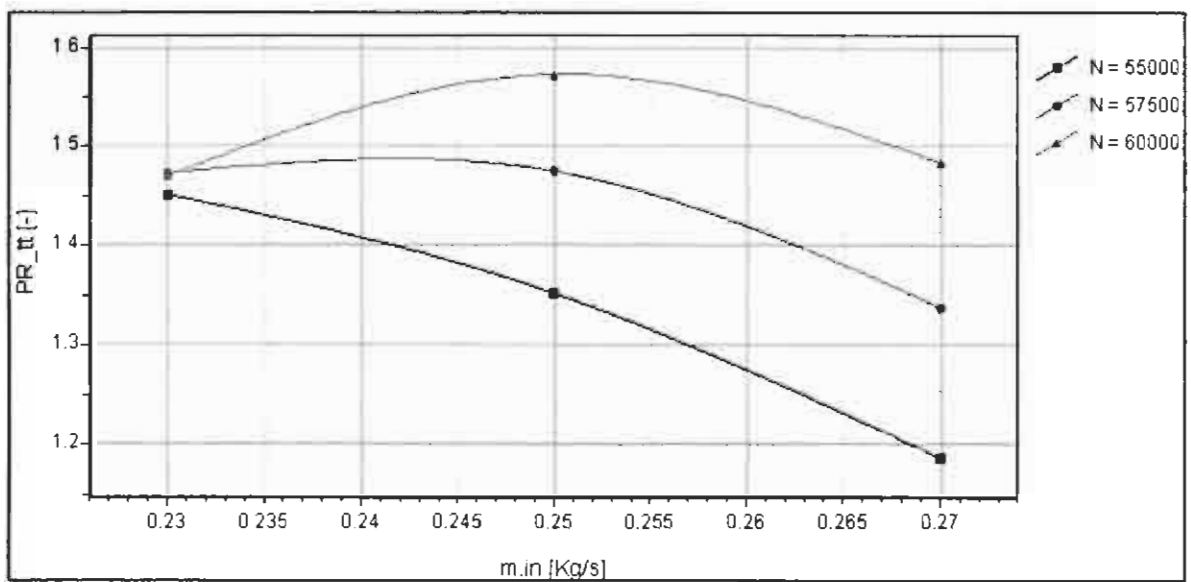


Figure A.7: HP compressor map

Nitrogen LP Compressor results

EES Results

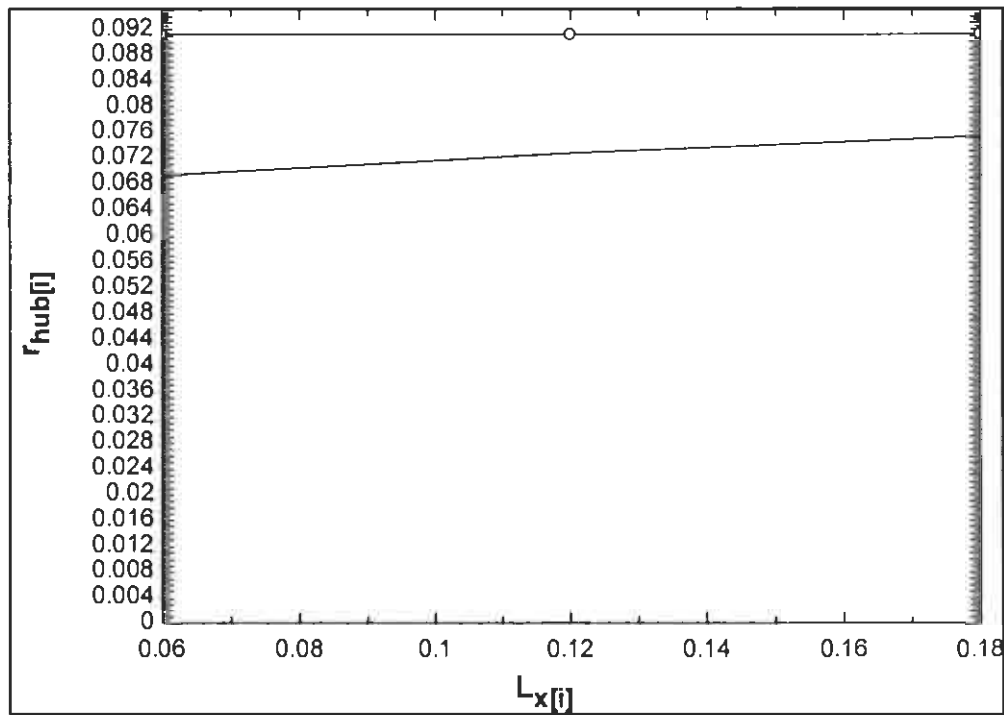


Figure A.8: LP compressor annulus diagram

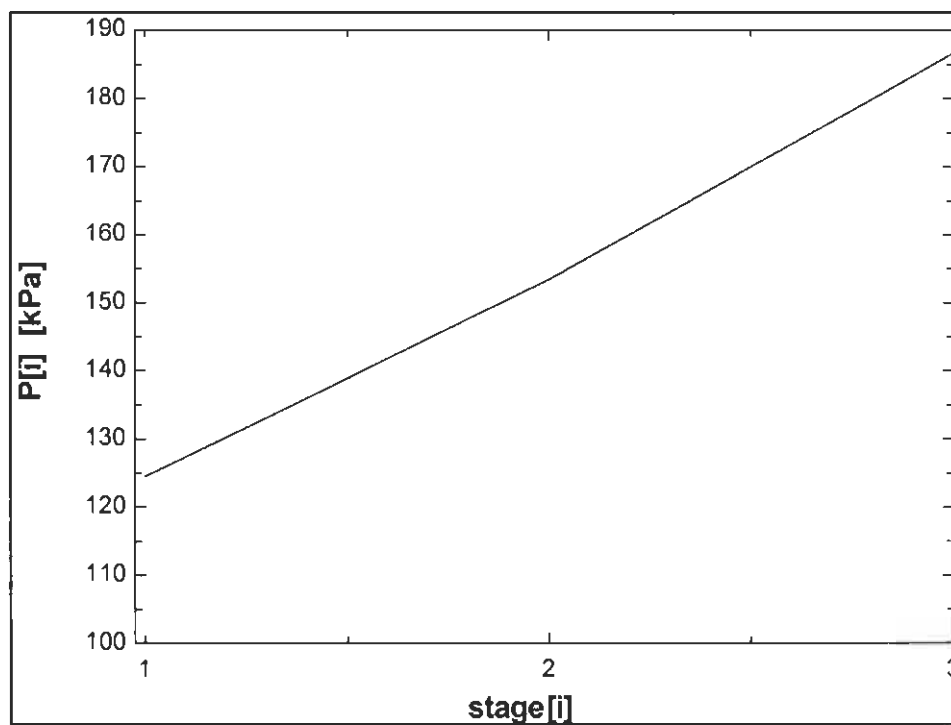


Figure A.9: HP compressor pressure rise

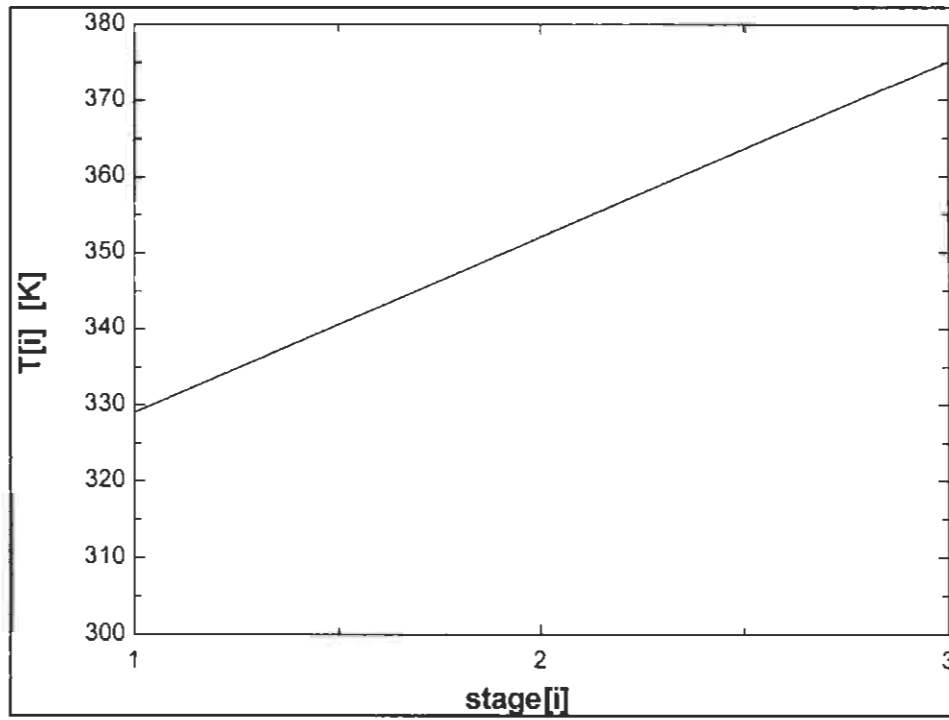


Figure A.10: HP compressor temperature rise

Concepts Results

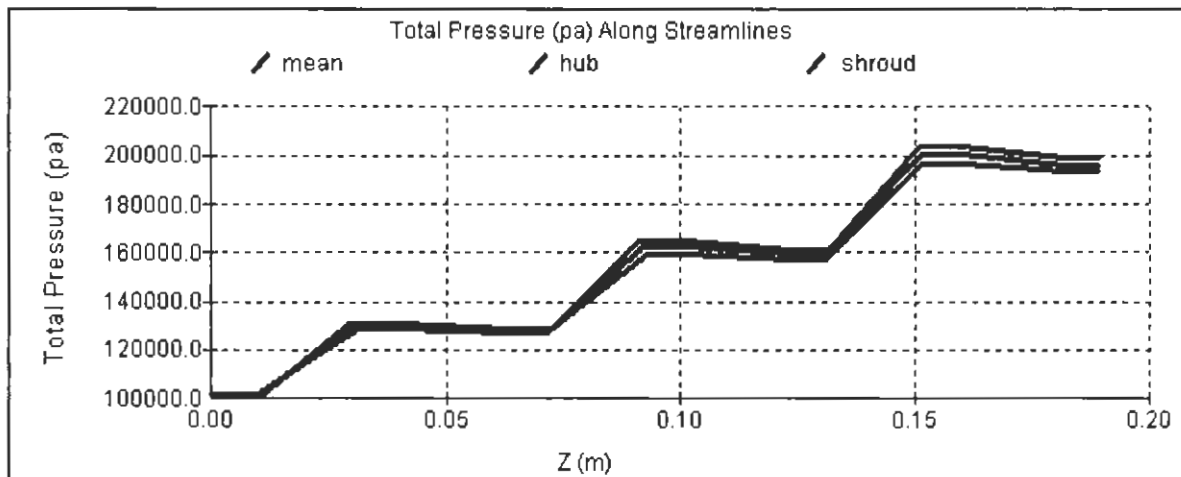


Figure A.11: HP compressor pressure rise

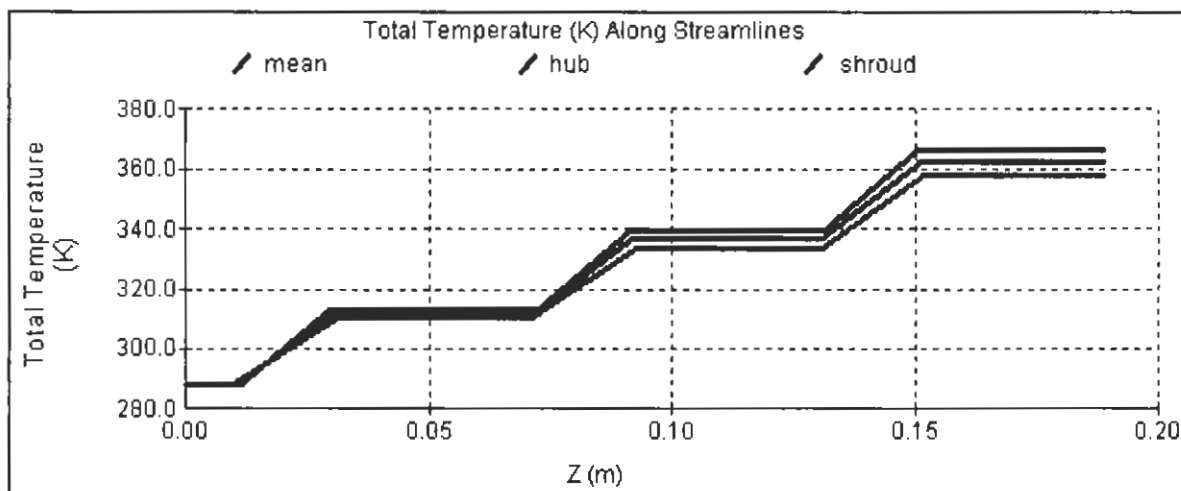


Figure A.12: HP compressor pressure rise

APPENDIX B: COMPRESSOR DESIGN PROCEDURE

The EES model was written according to a software program developed at Cranfield University. This software was developed in Excel, for preliminary designs of turbojet and turbofan engines. Other important resources utilized in the construction of the program are: *Gas Turbine Theory* by Saravanamutto, Rogers and Cohen, and *Gas Turbine Theory and Performance* by B.W. Botha.

At first the inlet conditions and the compressor design parameters had to be specified to design the compressor according to the output values of the cycle analysis.

Compressor inlet conditions

The following parameters were specified at the inlet of the compressor:

- Inlet temperature
- Inlet pressure
- Compressor pressure ratio
- Mass flow
- Inlet hub-tip-ratio
- Compressor polytropic efficiency

Gas properties

- $\gamma = 1.666$
- $R = 2.078 \text{ kJ/kg K}$
- $C_p = f(\text{Temperature, Pressure}) \text{ kJ/kg K}$

Determining temperature rise per stage

The overall compressor efficiency was first determined in order to determine the overall temperature rise. A poly-tropic efficiency of 88 % was assumed to determine the overall compressor efficiency.

$$\eta_c = \frac{PR_{overall} \left[\frac{\gamma - 1}{\gamma} \right] - 1}{PR_{overall} \left[\frac{\gamma - 1}{\gamma \cdot \eta_p} \right] - 1} \quad (B.1)$$

The overall temperature rise during compression was determined as follows:

$$\Delta T_{c,c} = \frac{T_{in}}{\eta_c} \cdot \left(PR_{overall} \left[\frac{\gamma - 1}{\gamma} \right] - 1 \right) \quad (B.2)$$

Assuming equal temperature rise for all stages (to simplify the software) the temperature rise per stage was determined by using the following equation:

$$\Delta T_{stage} = \frac{\Delta T_{c,c}}{n_{stage}} \quad (B.3)$$

Calculation of the mean blade speed

The inlet hub-tip-ratio is specified as a constant value. This value can be optimized after all the inlet parameters have been specified.

$$\frac{r_{hub,in}}{r_{tip,in}} = htr_{in} \quad (B.4)$$

Tip diameter is kept constant throughout all the compressor stages. The tip diameter is determined as a function of the rotational speed and the maximum tip speed.

$$D_{tip,in} = r_{tip,in} \cdot 2 \quad (B.5)$$

A value for the hub diameter is initially guessed to get the program solving. The hub diameter will later be solved using equation (B.18)

$$D_{hub,in} = r_{hub,in} \cdot 2 \quad (B.6)$$

Mean blade diameter can then be calculated:



$$D_{\text{mean},0} = \frac{D_{\text{hub},0} + D_{\text{tip},1}}{2} \quad (\text{B.7})$$

The user specifies rotational speed of the compressor.

$$N = \frac{U_{m,1} \cdot 60}{\pi \cdot D_{\text{mean},0}} \quad (\text{B.8})$$

Equation (B.8) was derived from:

$$\omega = N \cdot 2 \cdot \frac{\pi}{60} \quad (\text{B.9})$$

and

$$U = \omega \cdot r \quad (\text{B.10})$$

$U_{t,m}$ is the maximum tip speed specified.

$$U_{t,in} = \frac{D_{\text{tip,in}} \cdot U_{m,1}}{D_{\text{mean},1}} \quad (\text{B.11})$$

Calculation of the annulus outlet area

Axial velocity of the helium was first assumed at a constant value of 150 m/s. The static temperature is calculated from the following equation:

$$T_0 = T_{\text{in}} - \frac{C_a^2}{2 \cdot C_p \cdot 1000} \quad (\text{B.12})$$

With the static temperature known, the Mach number at the outlet of each stage can be calculated.

$$M_{e,0} = \frac{C_a}{\sqrt{\gamma \cdot R_{\text{He}} \cdot T_0}} \quad (\text{B.13})$$

The following procedure was written according to a software program (written in Excel) developed at Cranfield University. This equation is used to set up tables for compressible

flow of dry air. In this case, the properties of helium are used to set up similar tables to be used in the EES program. This equation programmed in a loop “duplicate” to determine the value of Q at various mach numbers.

$$Q_0 = K1 \cdot \frac{\left[\frac{\gamma}{R_{He}} \right]^{0.5}}{\left[1 + \left(\frac{\gamma - 1}{2} \right) \cdot K1^2 \right]^{K2}} \quad (B.14)$$

$K1$ is determined as a function of the Mach number and the gas inlet angle.

$$K1 = \frac{M_{e,0}}{\cos \left[\alpha_0 \cdot \frac{\pi}{180} \right]} \quad (B.15)$$

$K2$ is determined as a function of the ratio of specific heats.

$$K2 = \frac{\gamma + 1}{2 \cdot (\gamma - 1)} \quad (B.16)$$

From equation (B.17), the outlet area of the stage can be determined. As previously mentioned, the user specifies the mass flow and inlet temperature and pressure. KB is a blockage factor incorporated into the equation.

$$Q_0 = \frac{\dot{m} \cdot \sqrt{T_0}}{KB_1 \cdot A_{out,0} \cdot P_0 \cdot 1000} \quad (B.17)$$

The outlet area at each stage is calculated with equation (B.18).

$$A_{out,0} = \frac{\pi}{4} \cdot D_{tip,0}^2 \cdot \left[1 - \left(\frac{D_{hub,0}}{D_{tip,0}} \right)^2 \right] \quad (B.18)$$

Calculation of air angles

From the velocity triangles it can be seen that:

$$\tan \alpha_1 = \frac{U_m}{C_a} \quad (\text{B.19})$$

and

$$V_{w1} = U_m - V_{w0} \quad (\text{B.20})$$

thus

$$\alpha_1 = \arctan \left(\frac{U_m - V_{w0}}{C_a} \right) \quad (\text{B.21})$$

and

$$\tan \alpha_0 = \frac{V_{w0}}{C_a} \quad (\text{B.22})$$

Equation (B.23) can then be derived from equations (B.19) to (B.22). The rotor inlet angle is calculated by solving this equation.

$$\alpha_{1,1} = \arctan \left[\frac{U_{m,1}}{C_a} - \tan \left(\alpha_0 \cdot \frac{\pi}{180} \right) \right] \quad (\text{B.23})$$

Stator blade height is calculated.

$$h_{\text{out,stator},1} = \frac{D_{\text{tip},1} - D_{\text{hub},1}}{2} \quad (\text{B.24})$$

Rotor blade height is calculated.

$$h_{\text{in,rotor},1} = \frac{D_{\text{tip},1} - D_{\text{hub},0}}{2} \quad (\text{B.25})$$

Stator mean diameter can be calculated using the output values of equations (B.24) and (B.25).

$$D_{\text{mean,stator},1} = D_{\text{tip},1} - \left[\frac{h_{\text{in,rotor},1} + h_{\text{out,stator},1}}{2} \right] \quad (\text{B.26})$$

Mean blade speed can be calculated when the mean blade diameter is known.

$$U_{3\text{mean},1} = \frac{\pi \cdot N \cdot D_{\text{mean,stator},1}}{60} \quad (\text{B.27})$$

Stator inlet angle can be calculated. The influence of boundary layers on the annulus walls and the blade tip clearance has an unfavorable effect on the total work capacity. This phenomenon becomes more pronounced as the total number of stages is increased. At this stage, the work done factor (WDF) is assumed as one (*Rogers, Cohen and Saravanamuttoo, 2001:194*).

$$\alpha_{3,1} = \arctan \left[C_p \cdot 1000 \cdot \frac{\Delta T_{\text{stage}}}{\text{WDF} \cdot C_a \cdot U_{3\text{mean},1}} + \tan \left(\alpha_0 \cdot \frac{\pi}{180} \right) \right] \quad (\text{B.28})$$

The rotor outlet angle can be obtained.

$$\alpha_{2,1} = \arctan \left[\frac{U_{3\text{mean},1}}{C_a} - \tan(\alpha_{3,1}) \right] \quad (\text{B.29})$$

De Haller numbers

Calculating De Haller number for the rotor. (*Rogers, Cohen and Saravanamuttoo, 2001:213*)

$$Dhr_1 = \frac{\cos(\alpha_{1,1})}{\cos(\alpha_{2,1})} \quad (\text{B.30})$$

Calculating De Haller number for the stator. The stator outlet angle is assumed as zero.

$$Dhs_1 = \frac{\cos(\alpha_{3,1})}{\cos(\alpha_{4,1})} \quad (\text{B.31})$$

Stage reaction

The stage reaction can be determined from the rotor inlet and outlet angles.

$$\Lambda_1 = \frac{C_a}{2 \cdot U_{m,1}} \cdot (\tan(\alpha_{1,1}) + \tan(\alpha_{2,1})) \quad (\text{B.32})$$

Flow coefficient

The flow coefficient is determined at the mean axial velocity and tip speed at the inlet.

$$\phi = \frac{C_a}{U_{t,in}} \quad (B.33)$$

Temperature coefficient

$$\psi = C_p \cdot \frac{\Delta T_c}{U_{t,in}} \quad (B.34)$$

Calculation of the safety factor for maximum blade stress at the root.

Root stress of rotor blade at inlet of compressor (longest blade) is calculated.

$$\sigma_{blade} = \frac{\rho_{blade}}{2} \cdot U_{t,in}^2 \cdot \left[1 - \left(\frac{r_{hub,1}}{r_{tip,1}} \right)^2 \right] \quad (B.35)$$

The safety factor can then be calculated if the maximum blade stress is known. Maximum blade stress was assumed as 700 MPa.

$$SF = \frac{\sigma_{max}}{\sigma_{blade}} \quad (B.36)$$

Variation of air angles from root to tip

Inlet angle at the tip of the rotor is calculated for each stage.

$$\alpha_{1,tip,1} = \arctan \left[\frac{U_{tip,1}}{C_a} \right] \quad (B.37)$$

Inlet angle at the hub of the rotor calculated for each stage.

$$\alpha_{1,hub,1} = \arctan \left[\frac{U_{hub,1}}{C_a} \right] \quad (B.38)$$

The mean radius is determined in order to calculate the stator whirl velocity.

$$r_{mean,1} = \frac{D_{mean,1}}{2} \quad (B.39)$$

Whirl velocity at the hub is calculated as follows:



$$V_{w3,hub,1} = V_{w3,1} \cdot \frac{r_{mean,1}}{r_{hub,1}} \quad (B.40)$$

Whirl velocity at the tip is calculated as follows:

$$V_{w3,tip,1} = V_{w3,1} \cdot \frac{r_{mean,1}}{r_{tip,1}} \quad (B.41)$$

Stator inlet angle at the root is calculated for each stage.

$$\alpha_{3,hub,1} = \arctan \left[\frac{V_{w3,hub,1}}{C_a} \right] \quad (B.42)$$

Stator inlet angle at the tip is calculated for each stage.

$$\alpha_{3,tip,1} = \arctan \left[\frac{V_{w3,tip,1}}{C_a} \right] \quad (B.43)$$

Rotor inlet angle at the root is calculated for each stage.

$$\alpha_{2,hub,1} = \arctan \left[\frac{U_{hub,1} - V_{w3,hub,1}}{C_a} \right] \quad (B.44)$$

Rotor inlet angle at the tip is calculated for each stage.

$$\alpha_{2,tip,1} = \arctan \left[\frac{U_{tip,1} - V_{w3,tip,1}}{C_a} \right] \quad (B.45)$$

Blade geometry

With the solidity assumed as one and a circular-arc blade profile, the following equation could be used to determine the camber angle. The symbols $\alpha_{1,1}$ and $\alpha_{2,1}$ represents the rotor inlet and outlet angles.

$$0.727 \cdot \theta_{1,1} = \alpha_{1,1} - \alpha_{2,1} \quad (B.46)$$

From equation (B.46), the stagger can be calculated.

$$\phi_1 = \alpha_{1,1} - \frac{\theta_{1,1}}{2} \quad (B.47)$$

The same procedure was followed to determine the stagger of the stator blades.