

Astrospheric evolution and cosmic-ray particle transport

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*The most exciting phrase to hear in science, the one that heralds new discoveries, is not
'Eureka!' but 'That's funny...'*

— **Isaac Asimov**

Abstract

The modulation of galactic cosmic rays (GCRs) in astrospheres requires knowledge of large and small scale structures and plasma parameters. These are known for the heliosphere from different spacecraft observations, but observations of all the required parameters are not generally possible or available for astrospheres. Therefore, knowledge of heliospheric conditions needs to be applied. This study presents the use of magneto-hydrodynamic models to calculate some of the plasma parameters that are needed as input for a cosmic ray modulation model. For Luminous Blue Variable (LBV) type stars, periodic variations in their stellar outflow parameters are found to have no influence on their shock structure. However, an eruptive event is able to increase the size of their astrospheres by pushing the astropause or bow shock further out. A one-dimensional stochastic differential equation (SDE) transport model is used to calculate intensities in an astrosphere of a LBV-type star and Proxima Centauri. The calculated intensities in a LBV-type star are higher than observed within the heliosphere, while intensities in Proxima Centauri are higher than reported by other studies; however, the influence of the eruptive event on GCR intensities highlighted the limitations implicit in 1D transport modelling by overestimating diffusive shock acceleration which may be reduced by 3D effects such as drifts. A three-dimensional SDE transport model is therefore used to calculate intensities at exoplanet locations within the selected astrospheres of Proxima Centauri, Trappist-1, LHS 1140 and GJ 436. The investigation of GCR intensities pointed out the importance of the stellar rotation period and winding angle of the astrospheric magnetic field for the modulation of GCRs. For Proxima Centauri this led to counter-intuitive results based on what is known for the heliosphere, and further emphasises the need for three-dimensional models to be used to model the transport of these particles.

Keywords: Astrosphere; magneto-hydrodynamic; Luminous Blue Variables; radiative cooling; cosmic rays; stellar cycle; Proxima Centauri; habitable zone.

Table of abbreviations

A table containing a list of abbreviations that will be used throughout text. The first appearance of these will be written out in full with the abbreviation given in brackets.

1D	One dimensional
2D	Two dimensional
3D	Three dimensional
AU	Astronomical unit
ISM	Interstellar Medium
HP	Heliopause
TS	Termination Shock
AP	Astropause
BS	Bow Shock
SW	Stellar wind
HMF	Heliospheric magnetic field
HCS	Heliospheric current sheet
TTM	Turbulence-Transport Model
HZ	Habitable Zone
eV	Electron volt
GCR	Galactic Cosmic Ray
TPE	Parker Transport Equation
MFP	Mean Free Path
AMF	Astrospheric Magnetic Field
MHD	Magneto-hydrodynamic
LBV	Luminous Blue Variable
IAS	Inner Astrosheath
OAS	Outer Astrosheath
COAS	Cold Outer Astrosheath
HOAS	Hot Outer Astrosheath
SDE	Stochastic Differential Equation
LIS	Local Interstellar Spectrum
ACS	Astrospheric current sheet

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Chapter 1

Introduction

It has long been known that cosmic rays are modulated within the heliosphere with the transport of these particles being governed by the *Parker* (1965) Transport Equation (TPE). The modulation of galactic cosmic rays (GCRs) within other astrospheres has recently attracted the interest of the community with recent studies raising questions regarding the impact of GCRs on the habitability of exoplanets within some of these astrospheres (e.g., *Herbst et al.*, 2020; *Mesquita et al.*, 2022b; *Rodgers-Lee et al.*, 2023). The modulation of GCRs within the heliosphere is well-studied and assisted by spacecraft observations of plasma parameters (e.g., *McComas et al.*, 2012), heliospheric magnetic field (e.g., *Cohen et al.*, 2008), turbulence parameters (e.g., *Pine et al.*, 2020), and the structures present in the heliosphere, such as the heliospheric current sheet (e.g., *Smith*, 2001). Studying the modulation of GCRs within other astrospheres thus requires knowledge or assumptions regarding these very parameters since they cannot be observed directly.

This study aims to provide an outline of the process needed to determine the modulation of GCRs in the heliosphere, and from this apply this knowledge to various astrospheres which host exoplanets. Magneto-hydrodynamic (MHD) models are used to model the astrospheres of some of these stars and used as input for the GCR transport code.

The Heliosphere, Astrospheres and Cosmic Rays chapter introduces the formation and structure of the heliosphere. The various structures and features of the heliosphere, such as cyclic activity, the heliospheric magnetic field, and heliospheric current sheet, are highlighted. The transport of GCRs and the choice of diffusion coefficients used in the rest of this study are discussed and motivated, and a short overview is also given of the previously calculated intensities of GCRs within the astrospheres which are considered in this study.

The Astrospheric Evolution chapter discusses the shock structures of astrospheres and the MHD modelling of astrospheres. The modelling of these astrospheres include the effects of radiative cooling, an interstellar magnetic field and a stellar magnetic field, to be used as input for the transport model in the subsequent chapter. The astrospheres of Luminous Blue Variable (LBV) type stars, particularly HD 99953 and AG Carinae, which

show variations in their outflow parameters, are modelled along with the eruptive-phase of LBV-type stars.

The subject of the Cosmic Ray Modulation chapter is stochastic differential equations (SDEs) and how they are used to model the transport of GCRs. The SDE transport model of *Engelbrecht and Di Felice* (2020) is introduced and the changes that are necessary for astrospheric conditions and to use MHD results as input are discussed. This model is applied to GCR transport within the astrosphere of HD 99953 and an astrosphere influenced by an eruption of an LBV-type star. This chapter closes with the transport of GCRs within the astrosphere of Proxima Centauri, a comparison to results obtained in other studies, and a discussion of the necessity of 3D modelling of GCR transport is highlighted.

The 3D modelling of GCR transport in astrospheres is the topic of the 3D Cosmic Ray Modulation in Selected Astrospheres chapter. The location and choice of boundary spectrum used in this chapter is discussed. The 3D SDE model of *Engelbrecht and Burger* (2015a); *Moloto and Engelbrecht* (2020) and *Engelbrecht and Moloto* (2021), as well as the choice of diffusion coefficients used in this chapter are treated here. The treatment of turbulence quantities within an astrospheric context is considered next, based on what is known from heliospheric observations and numerical modelling.

This model is then applied to the heliosphere to calculate GCR intensities, and compared to observations. This is followed by a parameter study on the modulation of GCRs within the astrosphere of Proxima Centauri. The model is also applied to calculate GCR intensities in the astrospheres of Trappist-1, LHS 1140 and GJ 436, at the locations of their respective exoplanets, and the importance of the stellar rotation period and winding angle of the astrospheric magnetic field is discussed, further emphasising the need for 3D modelling of transport of GCRs.

Chapter 2

Heliosphere, Astrospheres and Cosmic Rays

This chapter will provide an introduction to various aspects pertaining to the heliosphere and expand upon those for other stars and their respective astrospheres. The concept of cosmic rays and their transport within the heliosphere or astrospheres will also be introduced. Further background relevant to specific applications will be covered in subsequent chapters.

2.1 Heliosphere

In order for the Sun to remain in hydrodynamic equilibrium, its outer layer must continually expand into interstellar space (*Parker, 1958*). This continual expansion is achieved through a constantly radially outflowing solar wind carrying heated plasma from the atmosphere of the Sun, creating a cavity in interstellar space. This cavity, the heliosphere, can be described as the region within interstellar space which is directly influenced by the Sun. The Sun is also moving relative to the interstellar medium (ISM) with a velocity of 25 km.s^{-1} (e.g., *Wood et al., 2015*) which gives the heliosphere an asymmetrical shape, resulting in the heliosphere being more compressed in the direction the Sun is travelling relative to the ISM, while the tail region becomes elongated (see *Pogorelov et al., 2017*, and references within). The surface separating the hot solar wind from the cooler ISM plasma is called the heliopause (HP). The shock structure of the heliosphere is composed of the termination shock (TS), the HP and the bow shock (BS). These structures and the processes involved with these will be discussed in more detail in Chapter 3.

The shape of the heliotail, the downwind region of the heliosphere, is still being actively debated. For a recent review, see *Kleimann et al. (2022)*. A brief overview will be given here. The one proposed shape is the comet-like tail, where the solar wind flows around the HP into two jets. The solar wind mixes with the local ISM in the tail direction and the

distinct two-jet structure is eliminated, resulting in a long comet-like tail (*Pogorelov et al.*, 2015). The other structure, as proposed by *Opher et al.* (2015), is that the solar wind is blown into two jets and they are deflected into the tail direction due to the relative motion between the Sun and the ISM. However, these jets do not join into a single comet-like tail; instead, the local ISM flows around the HP and between the two jets and is driven by Rayleigh-Taylor instabilities (*Opher et al.*, 2021), creating a croissant-like shaped heliotail. These contrasting structures, however, are still the subject of active debate.

2.1.1 Cyclic solar activity and solar wind

The Sun goes through an ~ 11 year cycle from solar minimum to solar minimum through solar maximum. During solar minimum there are fewer sunspots, with the number of sunspots increasing up to a maximum during solar maximum. Sunspots are dark areas visible on the surface of the Sun with strong magnetic fields, which lead to lower temperatures (*Hathaway*, 2015). There are other indices for solar activity such as solar-surface magnetic variability, eruption phenomena, tilt angle (discussed below), coronal activity and radiation of the Sun with most of these indices highly correlated with each other (*Balogh et al.*, 2015; *Usoskin*, 2023). For a more in-depth review on solar activity see *Usoskin* (2023).

During solar minimum conditions, the heliospheric magnetic field (HMF, discussed below) can be approximated to be dipole-like with large coronal holes (CH) over the poles which are expected to lead to higher solar wind speeds over the poles with lower solar wind speeds expected at lower latitudes (*Owens and Forsyth*, 2013). The polar CH create open magnetic field lines with the radial solar wind nearly parallel to these magnetic field lines, thus the magnetic field does not inhibit the flow of the solar wind. This allows for the fast solar wind over the polar regions of the Sun. Magnetic field lines near the equatorial region of the Sun form closed loops that start and end on the solar surface creating helmet streamers or the streamer belt. In these regions the radial solar wind and solar magnetic field are almost perpendicular to each other thus inhibiting the flow of the solar wind and leading to the slower solar wind speeds within these regions. It has also been suggested that the edges of CH may generate the slow solar wind (see *Brooks et al.*, 2015, and references within).

The solar wind, thus the heliosphere, is influenced by this cyclic behaviour of the Sun, especially the ~ 11 year solar activity cycle (*Owens and Forsyth*, 2013). The solar wind speed is observed at Earth with speeds of $\sim 400 - 800 \text{ km.s}^{-1}$. These varying solar wind speeds were confirmed by the *Ulysses* spacecraft (see e.g., *McComas et al.*, 2008, 2013). The solar wind thus has a latitude dependence. This can be seen in the left and right panels of Fig. 2.1, which has been adapted from *McComas et al.* (2013), and shows *Ulysses* observations of the solar wind during solar minimum. The fast solar wind of $\sim 800 \text{ km.s}^{-1}$ can be seen over the polar regions of the Sun, while slower solar wind speeds, $\sim 400 \text{ km.s}^{-1}$, are observed around the equatorial regions of the Sun. It should be mentioned that the

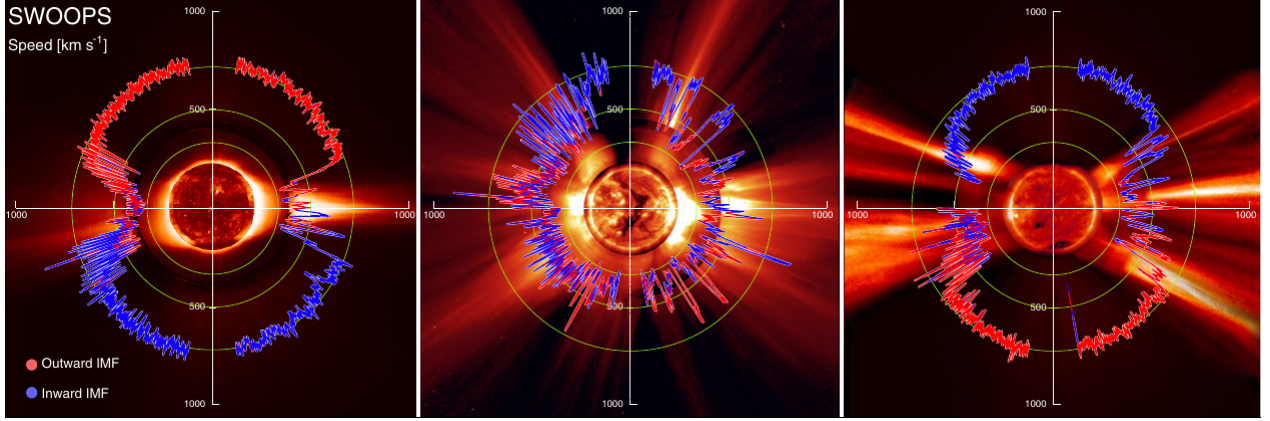


Figure 2.1: Observed solar wind speeds as a function of latitude from *Ulysses* adapted from *McComas et al.* (2013). The outward directed heliospheric magnetic field (red) and inward heliospheric magnetic field (blue) are displayed. The left and right panel are observations for solar minimum, while the middle panel is for solar maximum.

Sun becomes most dynamic during solar maximum, such that the quadrupolar and higher order moments of the solar magnetic field become more prevalent, leading to a decline in polar coronal hole area and thus a decline in the fast solar wind (*Owens and Forsyth, 2013*). This can be seen in the middle panel of Fig. 2.1 showing the observed solar wind speed by *Ulysses* during solar maximum, and the slow solar wind speed is observed at all latitudes, thus filling the heliosphere (see e.g., *McComas et al., 2008, 2013*).

2.1.2 Heliospheric magnetic field

The heliospheric magnetic field (HMF) is created by the solar magnetic field becoming embedded in the solar wind and being dragged out into the heliosphere. This occurs at the Alfvén surface, which is where the Alfvén speed and flow speed of the plasma are equal and within this surface there is a plasma beta of $\beta < 1$ (see e.g., *Cranmer et al., 2023*). Outside the Alfvén surface, the kinetic energy of the wind dominates the magnetic energy stored in the solar wind, causing field lines to be drawn out radially (*Kleimann et al., 2022*). As mentioned above, during solar minimum a dipole-like solar magnetic field is observed with open field lines pointing away from the Sun at the poles and closed loops around the equator. The region separating open magnetic field lines of opposite polarity is the heliospheric current sheet (HCS, discussed below) (*Smith, 2001; Khabarova et al., 2021*).

The first model proposed to describe the HMF was *Parker (1958)* and it is subsequently known as a Parker field. This model assumes a radially directed HMF in the rest frame of the Sun. The rotation rate of the Sun then winds the HMF into Archimedean spirals. The Parker HMF is given by (*Parker, 1958*),

$$B(r, \theta, \phi) = B_0 \left(\frac{r_0}{r} \right)^2 \left(\hat{r} - \frac{\Omega(r - r_s) \sin \theta}{V_{sw}} \hat{\phi} \right), \quad (2.1)$$

where B_0 is the magnitude of the HMF at a heliocentric distance r_0 , $\hat{r}$ and $\hat{\phi}$ are unit vectors in the radial and azimuthal directions, V_{sw} is the solar wind velocity and Ω is the rotation rate at the equator of the Sun. The source surface radius, r_s , is the radius at which the HMF is assumed to be purely radial. The source surface should not be confused with the Alfvén surface as the latter is where the stellar magnetic field is frozen into and dragged away from the star by the stellar while the former is where the stellar magnetic field can be assumed to be purely radial. The polarity of the HMF is then prescribed by the sign of B_0 . If it is positive, $A > 0$, the HMF points away from the Sun in the northern hemisphere and towards it in the southern hemisphere, while a negative value, $A < 0$, indicates the inverse effect.

The Parker field lines form spirals for each colatitude θ , where the winding angle of these spirals can be defined from Eq. 2.1 as

$$\tan \Psi = \frac{\Omega(r - r_s) \sin \theta}{V_{sw}}. \quad (2.2)$$

The winding angle is the angle between the radial direction of the HMF and the direction of the average HMF at a certain position. Using heliospheric conditions, i.e. 400 km.s⁻¹ for the solar wind speed, an equatorial solar rotation rate of $\Omega = 2.67 \times 10^{-6}$ rad.s⁻¹, a polar angle of $\theta = 90^\circ$ at Earth, $r = 1$ AU, a typical value of 45° is obtained (*Smith and Bieber, 1991; Owens and Forsyth, 2013*). The direction of the HMF becomes azimuthal with regard to the radially outflowing solar wind beyond 10 AU. The magnitude of the HMF tends to vary, while some authors believe there is a minimum floor value for the HMF of ~ 2.8 nT at Earth (see *Cliver and Ling, 2011*), with an average solar minimum strength of ~ 5 nT reported by space age observations (see *Cohen et al., 2008*). During solar maximum, the strength of the HMF is observed to be almost double this value (*Caballero-Lopez et al., 2004; Cohen et al., 2008*).

Several modifications have been proposed to the Parker model (see e.g., *Burger, 2005; Hitge and Burger, 2010; Steyn and Burger, 2020*, for a review). These modifications include the ones proposed by *Fisk (1996)* which include the interplay between the differential rotation of footpoints of magnetic field lines (*Snodgrass, 1983*) and the non-radial expansion of these lines which originate from rigidly rotating polar CH, those of *Schwadron and McComas (2003)* which incorporate a latitudinally-dependent solar wind, *Burger et al. (2008)* who introduced a Fisk-Parker hybrid field and *Hitge and Burger (2010)* who introduced a Schwadron-Parker hybrid field. This thesis will mostly be focused on the astrospheres of other stars and an unmodified Parker field will be used.

2.1.3 Heliospheric current sheet

The HCS is a large three-dimensional (3D) corotating structure which expands from the magnetic equator and over which the sign of the HMF changes (*Smith, 2001*). If the rotational and magnetic axes are aligned, a flat HCS would lie in the equatorial plane. However, the rotational and magnetic axis of the Sun are misaligned by the tilt angle of

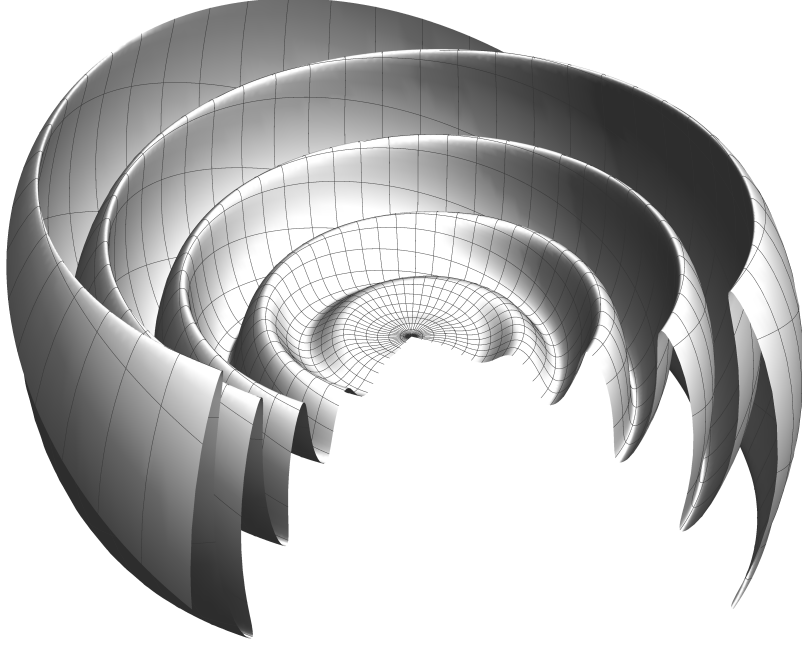


Figure 2.2: The HCS with a tilt angle of 20° to a distance of 30 AU. A section of the HCS has been removed to highlight the wavy structure. Taken from *Moloto* (2020).

the HCS, α . Since the HCS expands from the Sun it also rotates, leading to a wavy or warped structure when the tilt angle is non zero. The tilt angle of the HCS also varies with a solar cycle dependency, resulting in small tilt angles during solar minimum and larger angles during solar maximum. This is also confirmed through observations (see e.g., *Jones et al.*, 2003; *Owens et al.*, 2011; *Khabarova et al.*, 2021; *Richardson et al.*, 2022).

The expression for the neutral HCS as derived by *Jokipii and Thomas* (1981) is given by

$$\theta_{ns} = \frac{\pi}{2} - \tan^{-1} \left[\tan \alpha \sin \left(\phi_0 + \frac{\Omega_r}{V_{sw}} \right) \right]. \quad (2.3)$$

The structure of the HCS with tilt angle of 20° and to a distance of 30 AU is shown in Fig. 2.2. A section of the HCS has been removed to highlight the wavy structure of the HCS. As mentioned above, the tilt angle varies with solar cycle and such the structure of the HCS would also vary. This is discussed and implemented in a cosmic ray transport code in *Moloto and Engelbrecht* (2020). However, in most of this work, a flat current sheet will be assumed.

2.2 Turbulence in the Heliosphere

The large-scale structure of the HMF can be assumed to be uniform over large time scales; however, there are small-scale structures of the HMF which are not, such as turbulence within the solar wind and HMF (see *Matthaeus and Velli*, 2011; *Owens and Forsyth*, 2013; *Bruno and Carbone*, 2013). This study will not attempt to give a comprehensive review of the subject of turbulence in the solar wind and HMF; however, a brief discussion on

the turbulence of the HMF with regard to the transport and modulation of cosmic rays will be given. Reviews on turbulence of the solar wind and HMF can be found in *Bruno and Carbone (2013)*; *Verscharen et al. (2019)*; *Fraternali et al. (2022)*, while *Oughton and Engelbrecht (2021)* also discuss how it affects transport of charged particles.

The HMF can be decomposed into a background HMF $\mathbf{B}$ and a fluctuating component $\mathbf{b}$ such that

$$\mathbf{B} = \mathbf{B}_0 + \delta\mathbf{b}, \quad (2.4)$$

where $\langle \mathbf{B} \rangle = \mathbf{B}_0$ and $\langle \mathbf{b} \rangle = 0$. The brackets denote a suitable ensemble average. The correlation of fluctuations between two points can be described through a correlation function defined as (see e.g., *Matthaeus et al., 2007*),

$$R_{ij}(\mathbf{r}) = \langle \delta\mathbf{b}_i(\mathbf{x})\delta\mathbf{b}_j(\mathbf{x} + \mathbf{r}) \rangle, \quad (2.5)$$

where the fluctuations are separated by a distance r and the angle brackets denote an ensemble average. When this is evaluated at zero lag (separation), the function corresponds to the magnetic variance, $R_{ii}(0) = \delta b_i^2$. A correlation length can then be obtained when the value of the correlation function reaches $1/e$ of the initial value. This is following the definition of the e-folding time as described by *Smith et al. (2001)* and this can be interpreted as the characteristic length for the spatial decorrelation of the fluctuations (*Shalchi, 2009*).

A turbulence spectrum can be obtained from the Fourier transform of the correlation functions, and the spectrum gives an indication of the cascade of energy from large scales to smaller scales (*Oughton and Engelbrecht, 2021*). The power spectrum from *Verscharen et al. (2019)* is shown in Fig. 2.3, which highlights the spectral indices of the energy-containing, inertial and dissipation ranges. The energy-containing range is where energy is added to the spectrum to drive the turbulent eddies and provide a source of energy for the turbulent cascade. In the inertial range, no more energy is added and no driving takes place, energy is transferred to smaller and smaller scales as the eddies decay. The dissipation range is where the fluctuation energy heats the background plasma and dissipates. See *Davidson (2004)* for a complete discussion.

The fluctuations in the solar wind display anisotropic behaviour (see e.g., *Matthaeus et al., 1990*; *Bieber et al., 1996*) such that turbulence properties vary when considered parallel or perpendicular relative to the background HMF (see e.g., *Weygand et al., 2011*). In order to model the anisotropy of the fluctuations, 2D, slab or composite turbulence, which is a combination of the two, is used (*Bieber et al., 1994, 1996*; *Matthaeus et al., 2003*).

The background magnetic field in what follows is assumed to be in the z -direction of a Cartesian coordinate system. The 2D turbulence model assumes that fluctuations are a function of transverse (x, y) coordinates only, such that the decomposed HMF can be written as (see *Engelbrecht, 2008*)

$$\mathbf{B} = \mathbf{B}_0 + \delta\mathbf{b}_{2D}(x, y) = B_0\mathbf{e}_z + \delta b_{2D,x}(x, y)\mathbf{e}_x + \delta b_{2D,y}(x, y)\mathbf{e}_y, \quad (2.6)$$

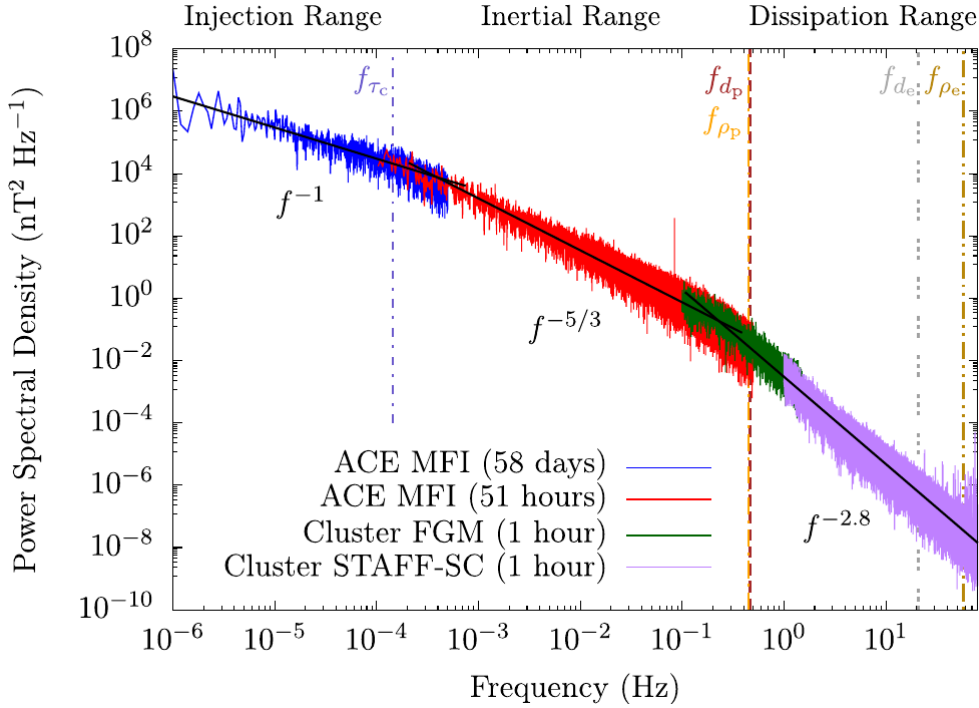


Figure 2.3: Power spectrum of HMF fluctuations, where the black lines show the power laws for the injection (energy-containing) range (-1), inertial range ($-5/3$) and dissipation range (-2.8). Taken from *Verscharen et al.* (2019).

while the 2D variance is then given by,

$$\delta B_{2D}^2 = \delta b_{2D,x}^2 + \delta b_{2D,y}^2 = \delta 2b_{2D,x}^2 = \delta 2b_{2D,y}^2, \quad (2.7)$$

when axisymmetric fluctuations are assumed. The 2D fluctuations are a function of coordinates perpendicular to the background HMF and remain constant for a particular value of z . A set of magnetic field lines at the same z value, but at different positions on the (x, y) plane, would then have different 2D fluctuations added to each of them with increasing z . They would thus create independent flux tubes.

In the case of slab turbulence, the fluctuations are only a function of z , and the HMF can then be given by,

$$\mathbf{B} = \mathbf{B}_0 + \delta \mathbf{b}_{slab}(z) = B_0 \mathbf{e}_z + \delta b_{slab,x}(z) \mathbf{e}_x + \delta b_{slab,y}(z) \mathbf{e}_y, \quad (2.8)$$

with the variances once more assuming axisymmetric turbulence, given by,

$$\delta B_{slab}^2 = \delta b_{slab,x}^2 + \delta b_{slab,y}^2 = \delta 2b_{slab,x}^2 = \delta 2b_{slab,y}^2. \quad (2.9)$$

Field lines that experience purely slab fluctuations would wander in (x, y) with increasing z ; however, field lines that start at the same z coordinate but different (x, y) coordinates would wander exactly the same way. Magnetic flux tubes for pure slab turbulence from *Matthaeus et al.* (2003) are shown in the left panel of Fig. 2.4 where the field lines do not separate.

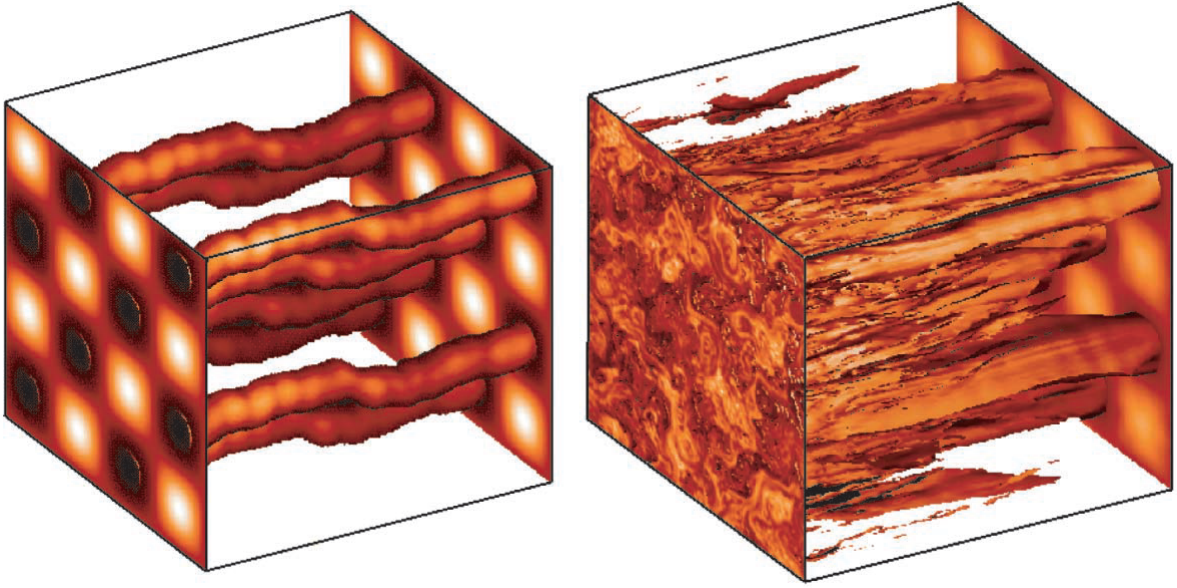


Figure 2.4: Left panel: Magnetic flux tubes for pure slab turbulence. Right panel: Magnetic flux tubes for 80/20 composite 2D/slab turbulence. Note that the flux tubes are followed from right to left. Taken from *Matthaeus et al.* (2003).

The model that is often used to describe turbulence in the solar wind is composite turbulence, which represents a superposition of fluctuations of 2D and slab, such that,

$$\mathbf{b}(x, y, z) = \mathbf{b}_{2D}(x, y) + \mathbf{b}_{slab}(z). \quad (2.10)$$

The total variance of the HMF is then given by a combination of 2D and slab variance by

$$\delta \mathbf{b}^2 = \delta \mathbf{b}_{2D}^2 + \delta \mathbf{b}_{slab}^2. \quad (2.11)$$

Composite turbulence thus assumes that the total turbulent energy is divided between 2D and slab fluctuations. A ratio of 80% of the turbulent energy in 2D turbulence with the remaining 20% in slab turbulence was shown to fit observed mean free path (MFP) values (*Bieber et al.*, 1994) and was also corroborated by observations (*Bieber et al.*, 1996). The magnetic flux tubes for composite turbulence containing a ratio of 80/20 of 2D/slab turbulence is shown in the right panel of Fig. 2.4. For composite turbulence the flux tubes are shredded, compared to the pure slab case in the left panel, due to the (x, y) dependence of the 2D turbulence component.

The variances and correlation scales also change with radial distance, with variances decreasing and correlation scales increasing (see *Zank et al.*, 1996; *Smith et al.*, 2001; *Pine et al.*, 2020). These spatial dependencies can be incorporated into cosmic ray modulation models by using simple power laws (see e.g., *Burger et al.*, 2008; *Strauss et al.*, 2016; *Qin and Shen*, 2017; *Moloto et al.*, 2018). The power laws for the spatial dependence can be obtained from fitting of observational data (*Cuesta et al.*, 2022; *Burger and McKee*, 2023). Fig. 2.5 shows the computed correlation length for various spacecraft measurements of the magnetic field from *Cuesta et al.* (2022). A power law can be fit to the Helios 1 and Voyager 1 data which give the radial dependence of the correlation length of $r^{0.5}$ (*Cuesta*

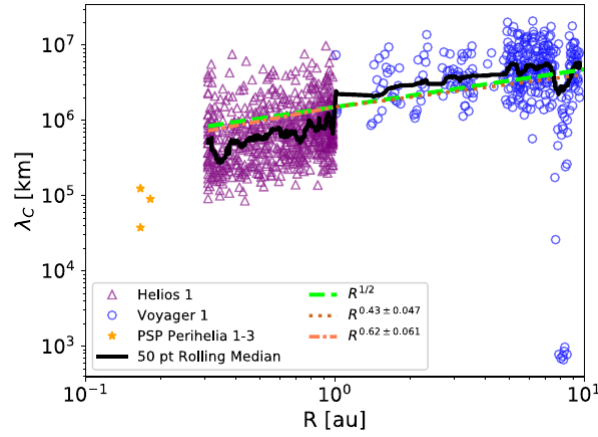


Figure 2.5: Correlation lengths for Parker Solar Probe (PSP), Helios 1, and Voyager 1 magnetic-field measurements as a function of radial distance. Taken from *Cuesta et al.* (2022).

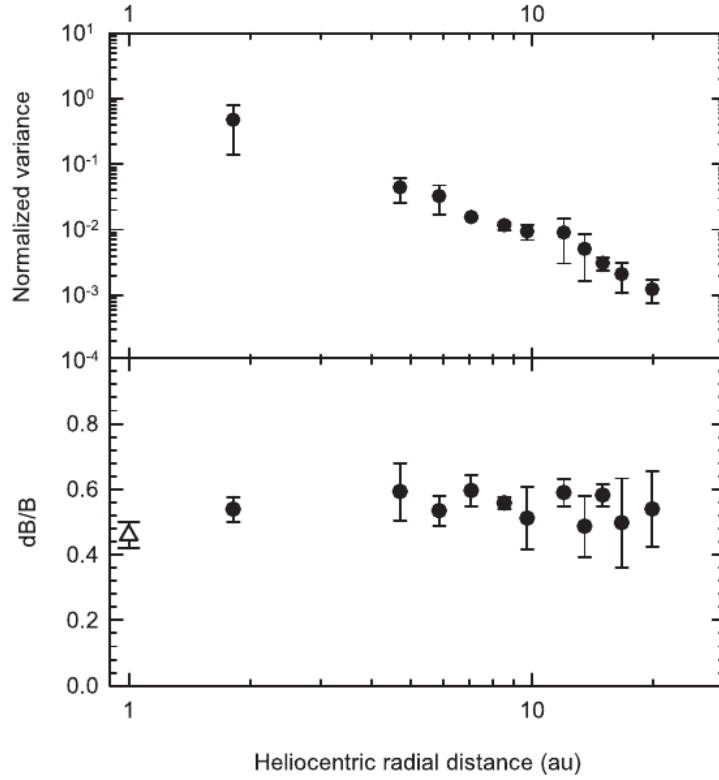


Figure 2.6: Top panel: Normalised magnetic variance as a function of radial distance. Bottom panel: Ratio of $\delta B_N/B_N$ as a function of radial distance. The open triangle shows an ACE data value at 1 AU. Taken from *Burger and McKee* (2023).

et al., 2022). It should also be noted that there is considerable scattering due to variable solar wind conditions (*Cuesta et al.*, 2022), possibly due to a solar cycle dependence (*Engelbrecht and Wolmarans*, 2020).

The magnetic variances as a function of radial distance of Voyager 1 data from *Burger*

and McKee (2023) are shown in the top panel of Fig. 2.6. A similar radial dependence is determined for the magnetic variance with a power law index of -2.4 , while the ratio of $\delta B_N/B_N$, in the bottom panel of Fig. 2.6 has an almost constant value of 0.55 (Burger and McKee, 2023). These power laws for the total magnetic variance and correlation lengths will be used in a later chapter to model the radial dependencies of these values. Turbulence transport models (TTMs) can also be used to model these quantities, such as the models discussed in Breech *et al.* (2008) and Oughton *et al.* (2011), but their implementation is beyond the scope of this work. For more detail see Engelbrecht and Burger (2013).

2.3 Astrospheres

Like the heliosphere of the Sun, other stars also create stellar wind (SW) cavities, called astrospheres (see, e.g., Herbst *et al.*, 2022). Of interest in this work are the stars which have exoplanets orbiting within the Habitable Zone (HZ) of the star. The HZ of a star is defined as the orbital distance where surface liquid water would be sustainable (Kasting *et al.*, 1993) which is a starting point for investigating habitability (Hill *et al.*, 2023). Brief discussions of the astrospheres of several stars considered later in this study, known to host exoplanets in their habitable zone, will be given below.

2.3.1 Proxima Centauri

The closest star to the Sun is Proxima Centauri (from here on refereed to as Proxima), which hosts one confirmed exoplanet within the HZ, namely Proxima Centauri b (from here on referred to as Proxima b) (Garraffo *et al.*, 2022). Proxima is a flaring M5.5 dwarf star, which is 1.3 pc from Earth with a stellar mass of $0.122 M_\odot$ and a radius of $0.154 R_\odot$ (Anglada-Escudé *et al.*, 2016). The rotation period of the star is 82.6 days (Alvarado-Gómez *et al.*, 2020). Flaring activity has been observed from Proxima (Vida *et al.*, 2019), while two candidates for coronal mass ejections have been investigated (Moschou *et al.*, 2019; Zic *et al.*, 2020). A stellar cycle of ~ 7 years was reported by Wargelin *et al.* (2017) and Klein *et al.* (2021).

The exoplanet Proxima b, which is presumed to be an Earth-like rocky exoplanet, has an orbit distance of 0.0485 AU with a period of 11 days, while the mass is similar to Earth with a minimum mass of 1.17 Earth masses (Suárez Mascareño *et al.*, 2020). The equilibrium temperature of Proxima b is 234 K, which is only slightly cooler than that of Earth at 255 K (Alvarado-Gómez *et al.*, 2016). The other two exoplanet candidates are Proxima c (Damasso *et al.*, 2020) and Proxima d (Faria *et al.*, 2022). Proxima c has an orbit of 1.44 AU and a minimum mass of 7 Earth masses (Alvarado-Gómez *et al.*, 2020). Proxima d is orbiting at 0.029 AU with a minimum mass of 0.26 Earth masses (Faria *et al.*, 2022).

The SW of Proxima has been numerically modelled (see e.g., Kavanagh *et al.*, 2021; Gar-

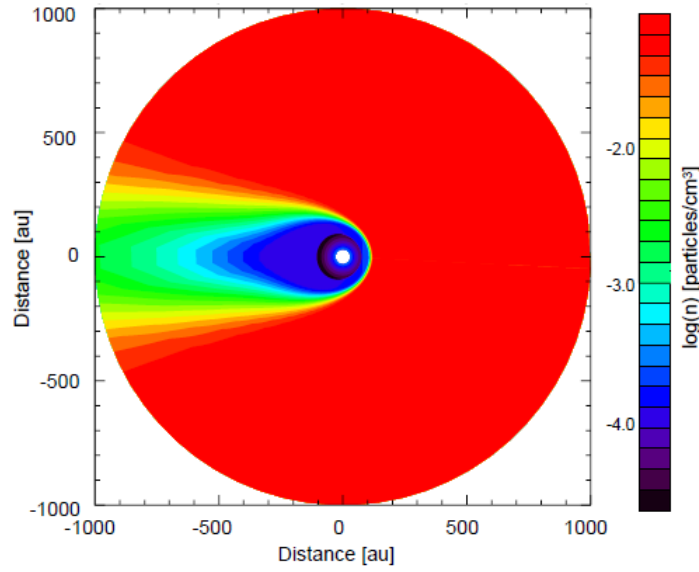


Figure 2.7: Computed density profile of an astrosphere of Proxima with the ISM flow from the right hand side. Taken from *Herbst et al.* (2020).

raffo et al., 2022), while the astrosphere has also been numerically modelled (see e.g., *Herbst et al.*, 2020). The computed astrosphere of Proxima from *Herbst et al.* (2020) is shown in Fig. 2.7. This astrosphere is similar to the heliosphere with a TS at a distance of 76 AU and the AP at 110 AU. It does not show a distinct BS but *Herbst et al.* (2020) do comment on the possibility of a bow wave. From observations a tilt angle for Proxima was calculated at 51° (*Klein et al.*, 2021) while a rotation period of ~ 82.6 days was determined (*Garraffo et al.*, 2022).

2.3.2 Trappist-1

The Trappist-1 system contains seven exoplanets orbiting around the ultra-cool M8 dwarf star which is located 12 pc from the Sun, with three of these exoplanets orbiting within the HZ (*Gillon et al.*, 2017). Trappist-1 has a stellar mass of $0.089 M_\odot$, a stellar radius of $0.121 R_\odot$ and an effective equilibrium temperature of 2516 K (*Van Grootel et al.*, 2018). Trappist-1 also shows high flaring activity with complex and multi-peaked frequent strong flares (*Vida et al.*, 2017) and a short rotation period of 3.3 days (*Luger et al.*, 2017).

The seven exoplanets orbiting Trappist-1 have orbits ranging from 0.011 to 0.062 AU (*Delrez et al.*, 2018). Although they are so close to the host star, the exoplanets Trappist-1 e (0.029 AU), Trappist-1 f (0.038 AU) and Trappist-1 g (0.047 AU) are located within the HZ of Trappist-1 (*Hill et al.*, 2023). These three exoplanets are near-Earth exoplanets with 0.915, 1.05 and 1.15 Earth masses respectively.

While no large scale MHD simulations exist for the astrosphere of Trappist-1, the SW has been modelled close to the star (see e.g., *Garraffo et al.*, 2017). Fig. 2.8 shows the computed

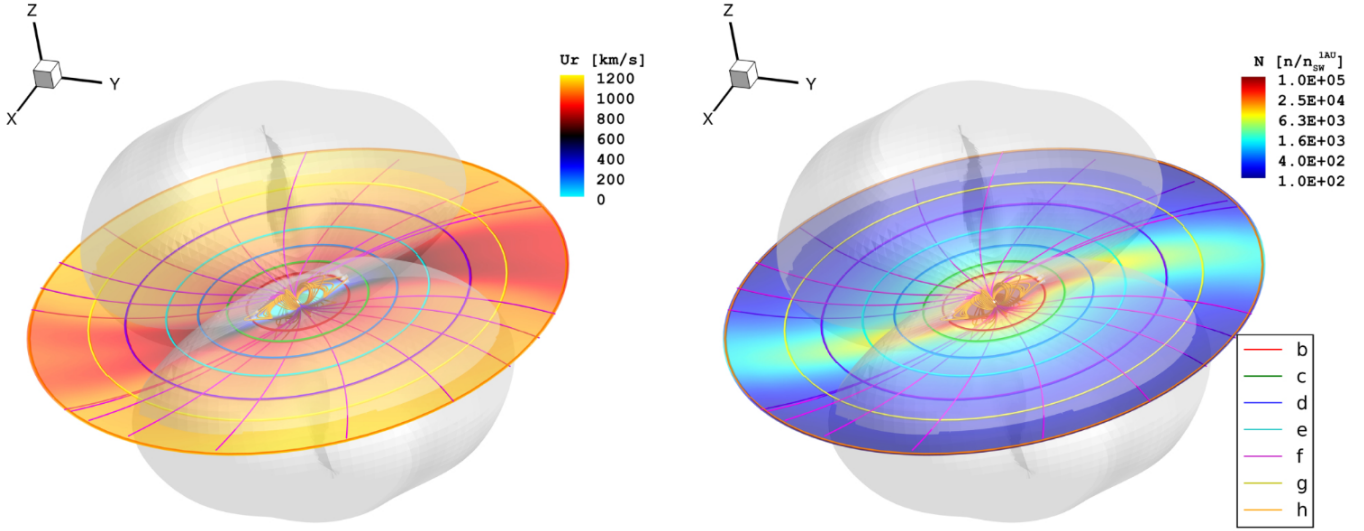


Figure 2.8: Left panel: Contour of the SW speed of Trappist-1. Right panel: Contour of SW density of Trappist-1 normalised to solar wind values at 1 AU. The Alfvén surface (grey shaded area), open magnetic field lines (purple lines), closed magnetic field lines (orange lines) and the orbits of the seven exoplanets (circular lines) are shown. Taken from *Garraffo et al.* (2017).

SW speed and density for an average surface field strength of 600 G taken from *Garraffo et al.* (2017). The fast SW speed can be seen to be accelerated along the open magnetic field lines (purple lines) while the slower SW speed is found around closed magnetic field lines (orange lines). The Alfvén surface and the orbits of the seven exoplanets are also shown. Only the outermost exoplanet orbits outside of the Alfvén surface most of the time, while the inner exoplanets tend to cross the Alfvén surface multiple times during their orbital period (*Garraffo et al.*, 2017).

2.3.3 LHS 1140

LHS 1140 is an M4.5-type main-sequence red dwarf which is located 12.5 pc away (*Dittmann et al.*, 2017). The star has an estimated stellar mass of $0.179 M_{\odot}$ with a stellar radius of $0.214 R_{\odot}$ (*Ment et al.*, 2019) and a stellar rotation rate of 131 days (*Dittmann et al.*, 2017; *Lillo-Box et al.*, 2020).

There are two confirmed exoplanets orbiting around LHS 1140. LHS 1140 b is a rocky super-Earth, with a mass of 6.65 Earth masses and a radius of 1.43 Earth radii (*Dittmann et al.*, 2017) which orbits at 0.096 AU within the HZ of LHS 1140 (*Lillo-Box et al.*, 2020). LHS 1140 c has a mass of 1.81 Earth masses and a radius of 1.28 Earth radii (*Ment et al.*, 2019) and orbits at 0.027 AU (*Lillo-Box et al.*, 2020). A possible third exoplanet, LHS 1140 d, has an estimated mass of 3.9 Earth masses (*Lillo-Box et al.*, 2020).

Large scale MHD simulations for LHS 1140 are shown in Fig. 2.9, indicating that it has a very small astrosphere surrounding it with AP distance of 11.5 AU (*Herbst et al.*, 2020).

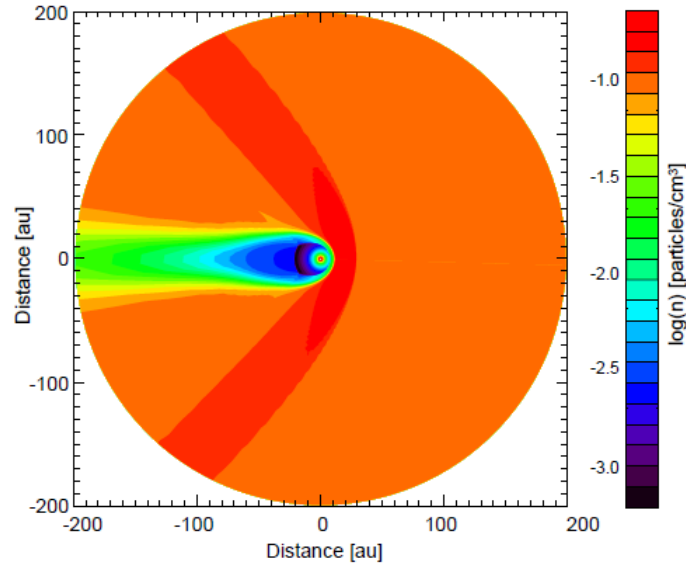


Figure 2.9: Computed density of the astrosphere of LHS 1140 taken from *Herbst et al.* (2020).

The TS distance is only at 8.1 AU. A BS is located at ~ 28.9 AU and is seen to expand to large radial distances by the red bands in Fig. 2.9.

2.3.4 GJ 436

GJ 436 is an M2.5 dwarf at a distance of 9.76 pc (*Bellotti et al.*, 2023), which has an exoplanet, GJ 436 b, orbiting at a distance of 0.028 AU (*Rosenthal et al.*, 2021). This exoplanet was observed to have an extended atmosphere due to the interaction with the SW of GJ 436 (*dos Santos et al.*, 2019). The stellar rotation rate of GJ 436 was determined to be 44 days (*Bourrier et al.*, 2018). GJ 436 b has a mass of 21.72 Earth masses and a radius of 3.85 Earth radii (*Maxted et al.*, 2022).

The SW and large scale magnetic field of GJ 436 was modelled by *Mesquita and Vidotto* (2020); *Mesquita et al.* (2021); *Vidotto et al.* (2023), and the effects these have on the space weather surrounding GJ 436 b are discussed in *Bellotti et al.* (2023) and *Vidotto et al.* (2023). The modelled SW based on two different derived mass-loss rates of GJ 436 is shown in Fig. 2.10 from *Vidotto et al.* (2023). The magnetic field lines (grey lines) can be seen to form closed elongated loops around the rotational equator with open field lines extending from the stellar rotational poles. It is from these open field lines that the faster SW is accelerated, while lower speeds are found at the closed field lines.

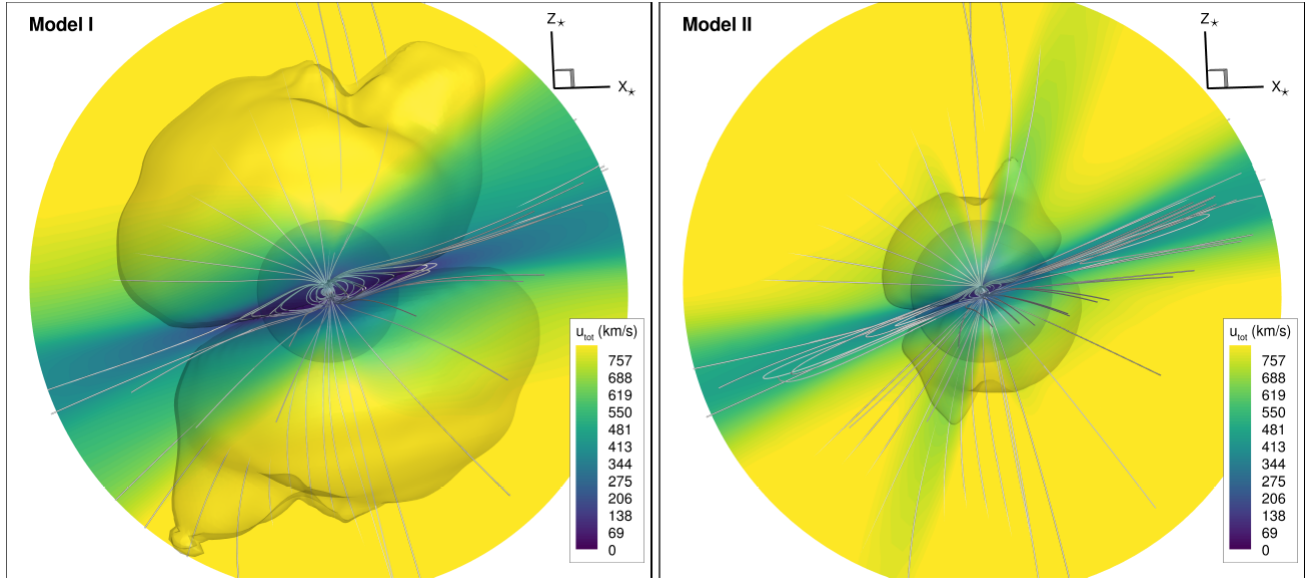


Figure 2.10: The modelled magnetised SW of GJ 436 for two different mass-loss rates from *Vidotto et al. (2023)*. The left panel has a derived mass-loss rate of $1.1 \times 10^{-15} \text{ M}_{\odot} \text{ yr}^{-1}$ and the right panel $2.5 \times 10^{-14} \text{ M}_{\odot} \text{ yr}^{-1}$. The magnetic field lines (grey lines), Alfvén surface (translucent outer surface), orbital distance of GJ 436 b (translucent sphere), and contour of the SW velocity are shown.

2.4 Cosmic rays

Cosmic rays (CRs) are charged particles that have energies ranging from 1 MeV up to 10^{21} eV (e.g., *Potgieter, 2010*). Charged particles that are present in the heliosphere are classified according to their energies and origin into four main populations: Galactic cosmic rays (GCR), anomalous cosmic rays (ACRs), solar energetic particles (SEP) and Jovian electrons. They will be briefly summarised below with the main focus of this thesis being the transport of GCR protons.

Galactic cosmic rays (GCR) are highly energetic fully ionised particles that originate from our galaxy and beyond and enter the heliosphere almost isotropically. They could originate from and potentially be accelerated by supernovae and supernova remnants, pulsars, neutron stars, microquasars, and/or active galactic nuclei (*Aharonian et al., 2012; Peretti et al., 2022; Escobar et al., 2022*). See *Fisk and Gloeckler (2012)* and *Aharonian et al. (2012)* for discussions regarding GCR acceleration and origin.

A composite spectrum of GCRs as a function of energy is shown in Fig. 2.11 from *Fisk and Gloeckler (2012)*. It should be noted that this spectrum is only for GCRs. At energies below a few GeV ($\sim 5 \times 10^9$ eV) the spectrum is flatter due to modulation within the heliosphere. Above these energies, the spectrum displays a spectral index of -2.67 up to the knee at $\sim 8 \times 10^{15}$ eV. The ankle is located at $\sim 5 \times 10^{19}$ with particles above these energies usually assumed to be of extragalactic origin (*Aharonian et al., 2012*). The nuclear cascade of particles above the ankle can also describe the spectrum below the

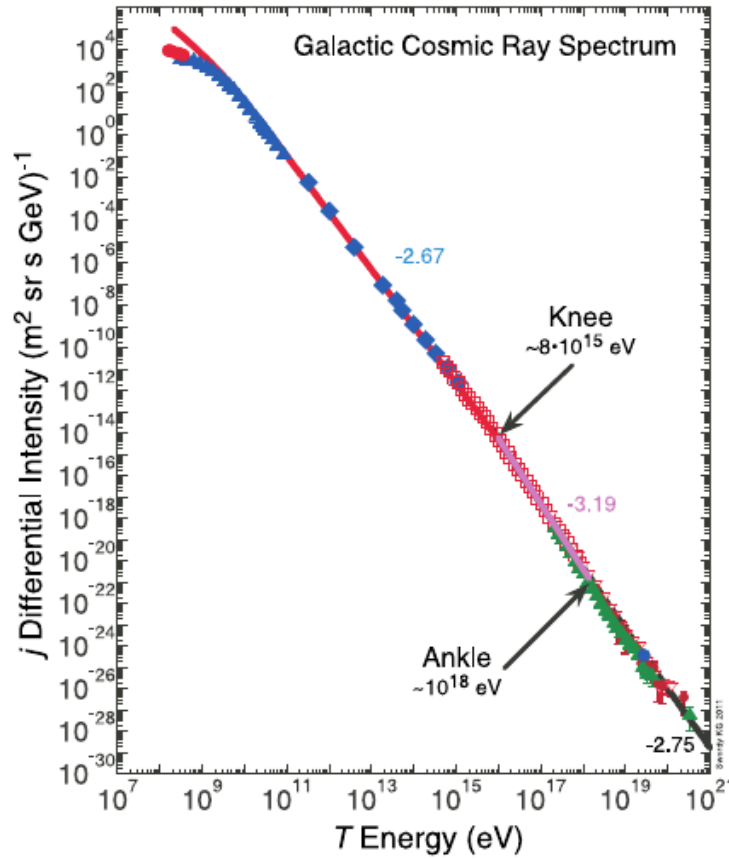


Figure 2.11: Differential intensity of galactic cosmic ray protons as a function of kinetic energy. The spectral index for the three regions is shown. Taken from *Fisk and Gloeckler* (2012).

ankle (*Boncioli et al.*, 2019). The spectral index between the knee and ankle is -3.19 , and -2.75 above the ankle (*Fisk and Gloeckler*, 2012).

Solar energetic particles (SEP) are particles that originate from the Sun and are accelerated by solar flares, radio bursts and coronal mass ejections which have energies ranging from tens of KeV to GeV (*Bruno et al.*, 2018; *Giacalone et al.*, 2020; *Reames*, 2021).

ACRs are accelerated pick-up ions due to interstellar atoms that become ionised due to charge exchange or photo-ionisation and the newly created pick-up ions being transported with the solar wind (*Fisk et al.*, 1974). These particles are believed to be accelerated through diffusive shock acceleration, whereby they continuously move across the TS (*Pesses et al.*, 1981; *Strauss et al.*, 2010). For a recent review of ACRs see *Giacalone et al.* (2022).

Jovian electrons are low energy electrons that dominate the electron population within the heliosphere, which originates from the magnetosphere of Jupiter (*Ferreira*, 2005). These electrons were observed at Earth and by various spacecraft, and typically have energies of a few MeV or more (see *Vogt et al.*, 2018, 2022).

2.5 Transport of Cosmic rays

The transport of GCRs is governed by the *Parker* (1965) transport equation (TPE). The full TPE expression is given by,

$$\frac{\partial f_0}{\partial t} = \nabla \cdot (\mathbf{K} \cdot \nabla f_0) - \mathbf{V}_{sw} \cdot \nabla f_0 + \frac{1}{3} (\nabla \cdot \mathbf{V}_{sw}) \frac{\partial f_0}{\partial \ln p} + Q, \quad (2.12)$$

in terms of the omnidirectional GCR distribution function f_0 , which is a function of position $\mathbf{r}$, momentum p and time t . The term Q describes any sources of CRs within the heliosphere such as Jovian electrons (see, e.g., *Vogt et al.*, 2020, 2022, and references within). When only GCRs are considered this term is set to zero. The terms of Eq. 2.12 describe the physical processes that influence the transport of CRs within the heliosphere. The first term on the right hand side, $\nabla \cdot (\mathbf{K} \cdot \nabla f_0)$ describes cosmic ray diffusion and drift due to the gradient and curvature of the HMF as well as drift along the HCS through the 3D diffusion tensor $\mathbf{K}$. The outward convection of CRs due to the solar wind travelling at a speed of $\mathbf{V}_{sw}$ is described by $\mathbf{V}_{sw} \cdot \nabla f_0$ and adiabatic energy changes are described by $(1/3) (\nabla \cdot \mathbf{V}_{sw}) (\partial f_0 / \partial \ln p)$ (e.g., *Engelbrecht et al.*, 2022a, and references within).

The distribution function can be related to the differential intensity by $j_T = P^2 f_0$ (see, e.g., *Moraal*, 2013), where P is the particle rigidity which can be defined as

$$P = pc/(Ze), \quad (2.13)$$

such that it is dependent on the momentum and charge of a particle and is expressed in units of Volt.

The 3D $\mathbf{K}$ diffusion tensor can be written into HMF aligned coordinates as $\mathbf{K}'$ (see, e.g., *Burger et al.*, 2008), where

$$\mathbf{K}' = \begin{bmatrix} \kappa_{\parallel} & 0 & 0 \\ 0 & \kappa_{\perp,2} & \kappa_A \\ 0 & -\kappa_A & \kappa_{\perp,3} \end{bmatrix}, \quad (2.14)$$

where the accent depicts the tensor differs from $\mathbf{K}$ in Eq. 2.12 which is in spherical coordinates. The perpendicular diffusion coefficients contained within the tensor $\kappa_{\perp,2}$ and $\kappa_{\perp,3}$ indicate that the perpendicular diffusion is in two different directions perpendicular to the HMF. This is necessary when non-axisymmetric turbulence conditions are considered and these coefficients may differ (see *Strauss et al.*, 2016). In this work axisymmetric 2D turbulence is assumed such that $\kappa_{\perp,2} = \kappa_{\perp,3} = \kappa_{\perp}$.

The 3D $\mathbf{K}$ diffusion tensor allows the modelling of diffusion parallel and perpendicular to the HMF through the diffusion ($\kappa_{\parallel}$ and $\kappa_{\perp}$) coefficients. This also includes the modelling of drift effects through the (κ_A) coefficient. These coefficients can be related to the corresponding MFPs by $\kappa_{\perp,\parallel} = v\lambda_{\perp,\parallel}/3$, where v is the particle speed of the GCR.

2.5.1 Particle drifts

Cosmic rays that travel through the heliosphere experience drifts through the gradient and curvature of the large scale HMF, while they also drift along the neutral HCS. The effect of drift within the heliosphere has long been known to have an effect on the amount of modulation of the cosmic ray spectra in the heliosphere (see, e.g., *Jokipii and Levy*, 1977; *Jokipii and Thomas*, 1981). The 22 year cyclic modulation of cosmic rays has been attributed to the effects of drift (see, e.g., *Gil and Mursula*, 2017; *Caballero-Lopez et al.*, 2019; *Mohlolo et al.*, 2022). Drift effects can also account for observed dependency of GCR intensities on the HCS tilt angle and magnetic field polarity (*McDonald et al.*, 1992), while also having a significant influence on the observed latitudinal gradients (*Heber and Potgieter*, 2006). These drift effects may also play a role in the heliosheath (*Kóta*, 2016).

The drift coefficient enters the TPE through the antisymmetric components of the diffusion tensor, $\mathbf{K}$, with a maximum value given by the weak-scattering value (*Forman et al.*, 1974),

$$\kappa_A = \frac{vR_L}{3} \quad (2.15)$$

where v is the particle speed and R_L the Larmor Radius of the particle. However, in the presence of magnetic turbulence, this value is known to be reduced through theory (*le Roux and Webb*, 2007) and numerical simulations (see, e.g., *Minnie et al.*, 2007; *Tautz and Shalchi*, 2012). Reductions to the drift coefficients have been shown to improve the agreement of the results of numerical simulations with spacecraft observations (*Potgieter and Burger*, 1990) but is very sensitive to how these reductions are implemented (*Engelbrecht and Burger*, 2015b). *Engelbrecht et al.* (2017) proposed a turbulence reduced driftscale by revisiting the theory of *Bieber and Matthaeus* (1997) and found that,

$$\lambda_D = R_L \left[1 + \frac{\lambda_{\perp}^2}{R_L^2} \frac{\delta B_T^2}{B_0^2} \right]^{-1}, \quad (2.16)$$

yields results that are congruent with numerical simulations of *Minnie et al.* (2007), while still providing a value of 1, and thus no drift reduction, when pure slab turbulence is present (*Tautz and Shalchi*, 2012).

The direction that particles drift in the heliosphere is affected by the polarity of the HMF. For a positive polarity, $A > 0$, positive particles are expected to drift into the heliosphere over the solar poles and out along the HCS. If negative particles or negative polarity ($A < 0$) would be considered, the drift directions would be reversed (*Jokipii and Thomas*, 1981). This is shown in Fig. 2.12, where the left panel shows how protons are expected to drift in over the poles and outward along the current sheet (grey line) for $A > 0$ and the right panel shows the reversed effect for $A < 0$.

The implementation of drift velocity into numerical simulations can be done using different approaches (see *Burger et al.*, 1985; *Burger*, 2012; *Engelbrecht et al.*, 2019). For a full comparison between the different techniques see *Mohlolo et al.* (2022). The method of *Burger* (2012) will be discussed here, as it will be implemented in this work.

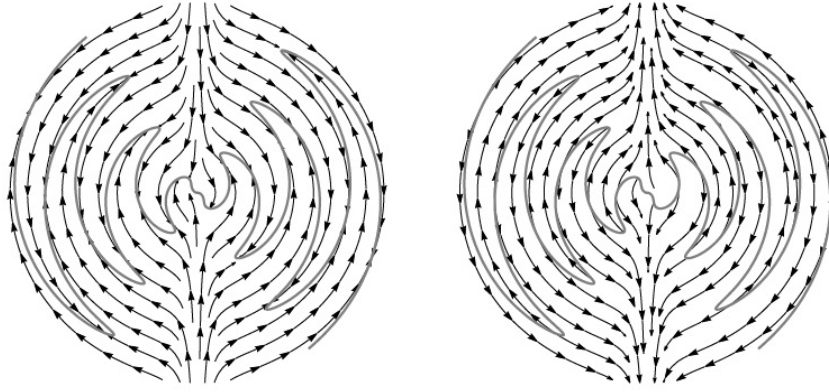


Figure 2.12: Left panel: Proton drift patterns during $A > 0$. Right panel: Proton drift pattern during $A < 0$. The drift along the HCS (grey line) is away from the Sun for $A > 0$ and towards the Sun for $A < 0$. Taken from *Moloto (2020)*.

The drift velocity can be calculated by $\mathbf{V}_d = \nabla \times (\kappa_A \mathbf{e}_B)$ (*Jokipii and Thomas, 1981*). The drift coefficient, κ_A , entering the off diagonal elements of the diffusion tensor and $\mathbf{e}_B$, is a unit vector in the direction of the HMF. The approach outlined in *Burger (2012)* uses hyperbolic tangent functions to model the reversal of the HMF across the HCS and calculates the components of the drift through,

$$\begin{aligned} \mathbf{V}_d = & (\nabla \times \kappa_A \mathbf{e}_B) \tanh[k(\theta_{ns} - \theta) \cos \nu] \\ & - \frac{k \cos \nu}{\cosh^2[k(\theta_{ns} - \theta) \cos \nu]} \kappa_A \mathbf{e}_B \times \nabla(\theta_{ns} - \theta) \\ & - \frac{k(\theta_{ns} - \theta)}{\cosh^2[k(\theta_{ns} - \theta) \cos \nu]} \kappa_A \mathbf{e}_B \times \nabla \cos \nu, \end{aligned} \quad (2.17)$$

The first term describes the gradient and curvature drift away from the HCS, while being zero at the HCS. Current sheet drift is described by the second term and the third term is to ensure that the drift velocity remains divergence free. The angle between the normal vector to the current sheet at the local position and the unit vector directed in the negative θ direction, $-\mathbf{e}_\theta$, is described by ν . The width of the meridional region over which the neutral sheet drift is modelled is adjusted by $\cos \nu$, while the parameter k governs the angular width of the HCS (see *Burger, 2012; Mohlolo et al., 2022*).

2.6 Diffusion Coefficients

There are numerous scattering theories that have been proposed to describe the scatter of charged particles, parallel and perpendicular, to a turbulent magnetic field (see *Shalchi, 2009; Shalchi, 2020*, for a review). Some of these theories are quasilinear theory (QLT) of *Jokipii (1966)*, nonlinear guiding centre (NLGC) theory of *Matthaeus et al. (2003)*, unified nonlinear theory (UNLT) of *Shalchi (2010)*, and the random ballistic decorrelation interpretation of NLGC (RBD/NLGC) of *Ruffolo et al. (2012)*. The various scattering

theories can be used to calculate the MFPs of particles parallel and perpendicular to the magnetic field. The use of different scattering theories and varying assumptions of the behaviour of the turbulence quantities leads to considerable variations in the calculated diffusion and drift coefficients (*Engelbrecht et al.*, 2022a). For a comparison between the various MFPs obtained using these theories, see *Engelbrecht et al.* (2022a).

A key input for the perpendicular MFP is the 2D turbulence power spectrum, and therefore the form of the 2D power spectrum is important, particularly at lower wavenumbers (see *Engelbrecht*, 2019). The derivation of the perpendicular MFP used here assumes a 2D turbulence power spectrum with a Kolmogorov spectral index, $-5/3 = 2\nu$, for the inertial range, which commences at lengthscale λ_{2D} , and a wavenumber-independent energy-containing range. A perpendicular mean free path is then obtained from the NLGC results of *Shalchi et al.* (2004) as modified by *Burger et al.* (2008) to accommodate a general ratio of slab to 2D energies (see, e.g., *Bieber et al.*, 1996) and is given by

$$\lambda_{\perp} = \left[\alpha^2 \sqrt{3\pi} \frac{2\nu - 1}{\nu} \frac{\Gamma(\nu)}{\Gamma(\nu - 1/2)} \lambda_{2D} \frac{\delta B_{2D}^2}{B_0^2} \right]^{2/3} \lambda_{\parallel}^{1/3}, \quad (2.18)$$

where Γ is the standard Gamma function. The value α is constrained based on fits of numerical test particles simulations to $\alpha^2 = 1/3$ based on simulations from *Matthaeus et al.* (2003). The perpendicular MFP described above requires as input an expression for the parallel MFP, as an implicit assumption of some scattering theories is that the pitch angle autocorrelation function of the particle decorrelates after a time $\tau = \lambda_{\parallel}$ (see, e.g., *Matthaeus et al.*, 2003; *Ruffolo et al.*, 2012; *Engelbrecht*, 2019). This parallel MFP dependency has also been confirmed with theory and tests on numerical test particle simulations (*Shalchi*, 2020; *Dundovic et al.*, 2020).

A parallel mean free path expression derived by *Teufel and Schlickeiser* (2003) from the QLT of *Jokipii* (1966) and is given by (*Burger et al.*, 2008),

$$\lambda_{\parallel} = \frac{3s}{(s-1)} \frac{R^2}{k_m} \frac{B_0^2}{\delta B_{sl}^2} \left[\frac{1}{4\pi} + \frac{2R^{-s}}{\pi(2-s)(4-s)} \right], \quad (2.19)$$

where $s = 5/3$ and $R = R_L k_m$. This parallel MFP assumes a slab turbulence spectrum with a spectral index of $-s$ for the inertial range, which commences at $k_m = 1/\lambda_{sl}$, and a wavenumber-independent energy-containing range, similar to the perpendicular MFP.

To motivate the diffusion coefficients chosen for this study, a brief overview of observations within the heliosphere as well as numerical simulations done on test particles will be given here. A more complete review can be found in *Engelbrecht et al.* (2022a).

Diffusion coefficients that are used need to be realistic and as such they need, at least to a degree, to be constrained by observations. However, as MFPs are not quantities that can readily be directly observed, this is achieved through fits of observed differential intensities of various CR species such as solar energetic particles (e.g., *Bieber et al.*, 1994; *Aran et al.*, 2018), galactic cosmic rays (e.g., *Burger et al.*, 2000) and Jovian electrons (e.g., *Vogt et al.*, 2022; *Engelbrecht et al.*, 2022b, and references within). The anisotropy of cosmic rays can

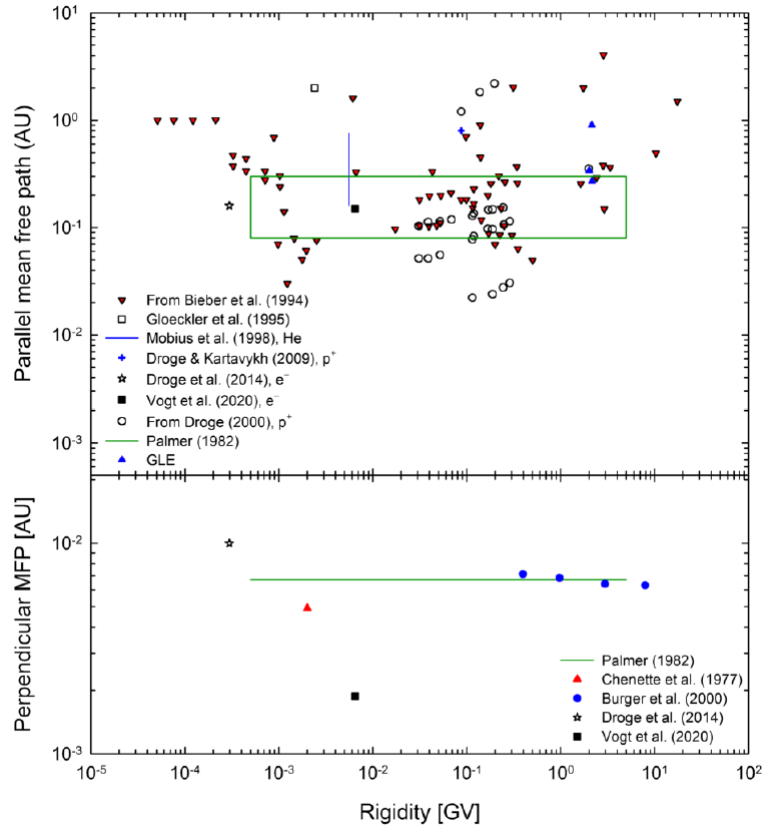


Figure 2.13: Top panel: Parallel mean free paths at 1 AU as function of rigidity. Bottom panel: Perpendicular mean free paths at 1 AU as a function of rigidity. Taken from *Engelbrecht et al. (2022a)*, where a complete list of studies included can be found.

also be used (see *Chen and Bieber, 1993*) as well as pickup ion anisotropy observations (see *Gloeckler et al., 1995*).

The most well known study for parallel and perpendicular MFPs is the *Palmer (1982)* consensus range. This shows the observed MFPs of protons and electrons; however, some of the assumptions made within transport models used to fit observations are severely limiting (see *Engelbrecht et al., 2022a*, and references within). Nevertheless, it still serves as a reasonable constraint on MFPs at 1 AU in the heliosphere, and is used later to guide the eye when values for MFPs are calculated for various astrospheres (See Chapter 5). This consensus was revisited by *Bieber et al. (1994)*, who included more recent studies and separated the MFPs of electron and protons. It was once more revisited by *Engelbrecht et al. (2022b)*.

Fig. 2.13 shows a comparison of various parallel (top) and perpendicular (bottom) MFPs calculated in literature from *Engelbrecht et al. (2022a)*. There is considerable scatter in the reported parallel MFPs from various studies, and MFPs larger than the *Palmer (1982)* consensus range are often reported. *Bieber et al. (1994)* considered results for electron and proton MFPs and separated the parallel MFPs at 25 MV, with protons being above this value. A $\sim P^{1/3}$ rigidity dependence for the parallel MFP of protons can be

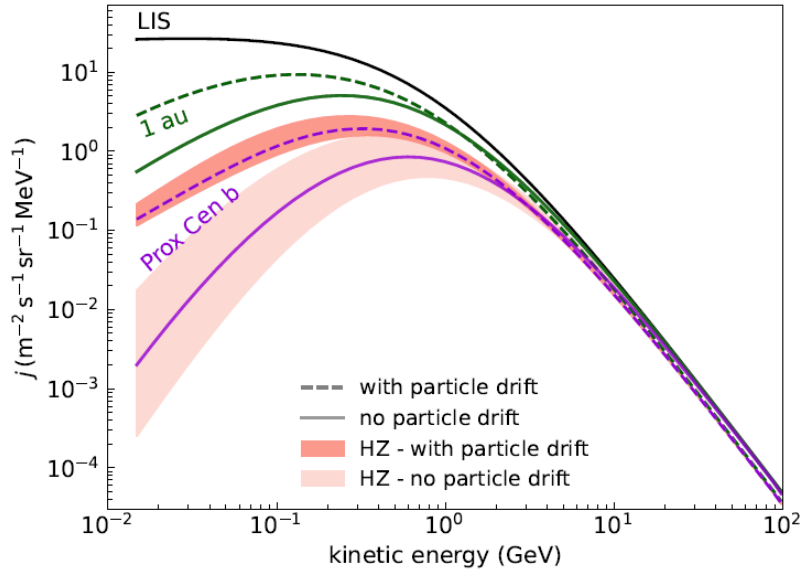


Figure 2.14: Calculated differential intensities at Proxima b with no drift (solid purple line) and with drift (dashed purple line) from *Mesquita et al.* (2022b). The calculated intensities within the HZ are shown by the shaded areas for no drift (light pink) and with drift (dark pink).

seen at higher rigidities. The bottom panel of Fig. 2.13 shows the considerably fewer observational values for the perpendicular MFP values from *Engelbrecht et al.* (2022a). The proton perpendicular MFP shows a relative rigidity independence, with constant values seen in the data of *Burger et al.* (2000) at higher rigidities. The MFPs described above are generally in reasonable agreement with the range of observations discussed above (see *Engelbrecht et al.*, 2022a), thereby motivating their use in what is to follow.

2.7 Cosmic ray transport in astrospheres

The modulation of cosmic rays in astrospheres other than the heliosphere has been done using various diffusion coefficients, turbulence properties and stellar parameters (see e.g., *Scherer et al.*, 2015; *Herbst et al.*, 2020; *Jasinski et al.*, 2020; *Rodgers-Lee et al.*, 2021; *Mesquita et al.*, 2021, 2022b,a; *Light et al.*, 2022). The intensities calculated by these works mostly ignore the effects of drift due to gradient and curvature of the astrospheric magnetic field (AMF) and none assume the presence of an HCS and thus ignore drifts along the HCS. The various authors also utilise different diffusion coefficients with varying degrees of energy dependencies.

The intensities calculated by *Mesquita et al.* (2022b) at Proxima b and within the HZ of Proxima are shown in Fig. 2.14 taken from that work. The authors utilised a 1D transport model and assumed Bohm-type diffusion within the astrosphere of Proxima, while also including drifts due to gradient and curvatures of the AMF. The inclusion of drift at energies below 1 GeV decreased the intensities calculated by ~ 2 orders of

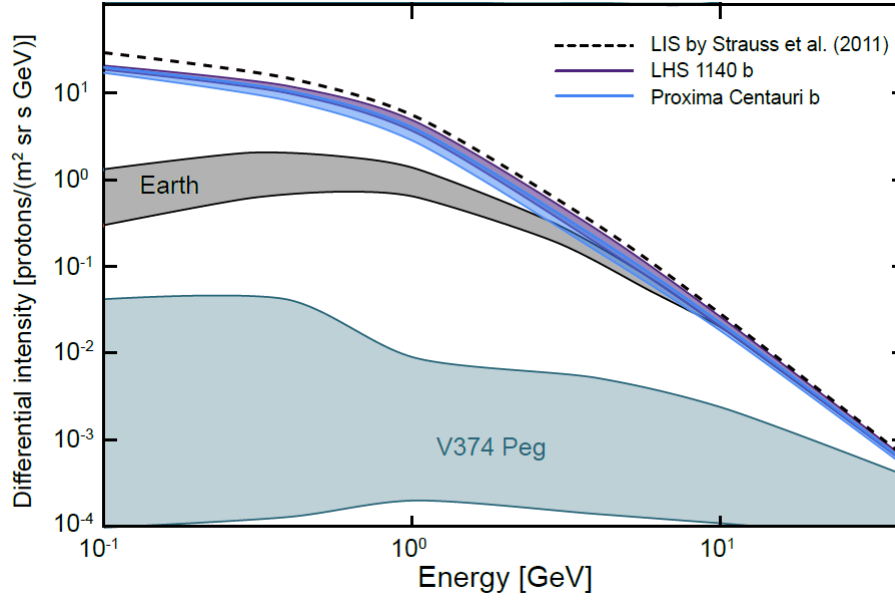


Figure 2.15: Calculated differential intensities at Earth (grey), Proxima b (dark blue), LHS 1140 b (purple) and V374 Peg (light blue) from *Herbst et al.* (2020).

magnitude from the local interstellar spectrum, but even more modulation was calculated for GCRs with intensities ~ 2 orders of magnitude lower than that when no drift is included (*Mesquita et al.*, 2022b). Higher energy GCRs, with energies above 1 GeV, undergo very little modulation within the astrosphere of Proxima, whether drift is included or not. It is however unclear how drift effects can be included in a 1D modulation model.

The differential intensities calculated at Proxima b (dark blue) and LHS 1140 b (purple) are shown in Fig. 2.15 from *Herbst et al.* (2020). These authors also use a 1D transport model, while using a diffusion coefficient that scales based on the ratio between the AMF and HMF strength at 1 AU. They find almost no modulation of GCRs within the astrospheres of Proxima and LHS 1140 with intensities reduced by $\sim 25\%$ from the local interstellar spectrum at energies above 1 GeV and even less modulation at higher energies (*Herbst et al.*, 2020). Even though different local interstellar spectra are utilised by *Herbst et al.* (2020) and *Mesquita et al.* (2022b), the intensities calculated for Proxima b by the latter are considerably lower than the former, due to the different diffusion coefficients used.

Mesquita et al. (2021) calculated the intensities at the location of GJ 436 b, which are shown in Fig. 2.16 for two different plasma parameter cases, which led to different astrosphere sizes. These intensities were used by *Rodgers-Lee et al.* (2023) in top-atmospheric studies of GJ 436 b. The calculated differential intensities at the location of GJ 436 b at energies below 1 GeV are reduced by ~ 7 orders of magnitude compared to the local interstellar spectrum, while the intensities within the HZ are only $\sim 2 - 3$ orders of magnitude lower than the local interstellar spectrum. At energies above 1 GeV these authors find almost no modulation of GCRs. Apart from the study by *Herbst et al.* (2020), the general consensus in the literature seems to be that GCR intensities in various astrospheres appear to be considerably lower than what is observed at Earth.

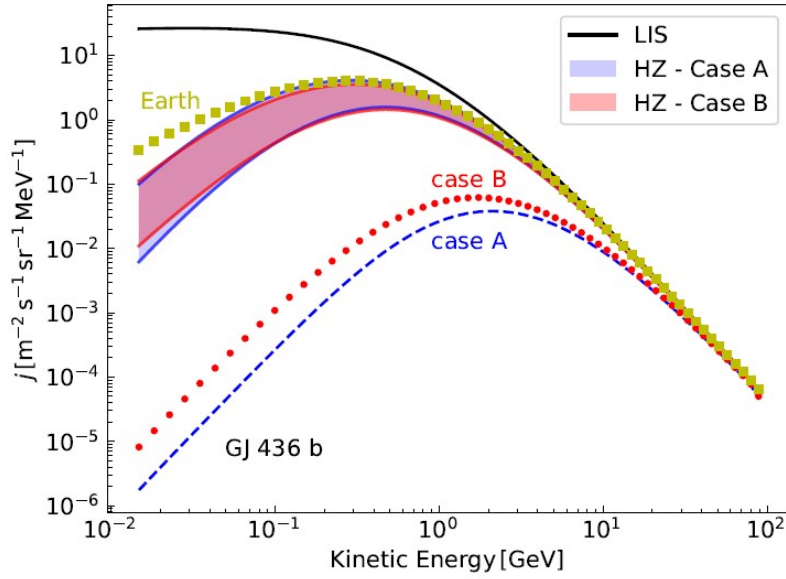


Figure 2.16: Calculated differential intensities at the location of GJ 436 b for two different cases of plasma parameters and astrosphere sizes, Case A (blue dashed) and Case B (red dotted) from *Mesquita et al.* (2021). The calculated differential intensities within the HZ of GJ 436 are shown by the shaded areas for Case A (blue) and Case B (red).

2.8 Summary

This chapter introduced the basic structure and features of the heliosphere which are important to understand the modulation of cosmic rays in astrospheres. Solar activity and variations in the solar wind were discussed. This was followed by introducing the embedded heliospheric magnetic field and the resulting heliospheric current sheet. The turbulence within the heliospheric magnetic field was briefly described, based on the 2D and slab models of turbulence.

The stars, Proxima Centauri, Trappist-1, LHS 1140 and GJ 436, and their respective exoplanets (which will be considered in the following chapters) were introduced. A brief description on the astrospheres and stellar winds of these stars obtained from other studies was given.

The different cosmic ray species were highlighted, while the energy spectrum of galactic cosmic rays was discussed. An introduction to the transport of cosmic rays, particle drift and the diffusion coefficients was given, followed by a brief review of the differential intensities calculated within the astrospheres that will be considered later in this work.

Chapter 3

Astrospheric Evolution

This chapter introduces the magnetohydrodynamic model (MHD) that is used to model the astrospheres surrounding stars. These type of models simulate the MHD flow and astrospheric structures which are required for the study of cosmic-ray modulation within these stellar wind cavities (*Herbst et al.*, 2020; *Rodgers-Lee et al.*, 2021; *Herbst et al.*, 2022; *Mesquita et al.*, 2022a,b). Some aspects of this chapter were already published in *Light* (2020) and *Light et al.* (2022).

A general introduction to the structure of astrospheres and the underlying physics behind their formation will be given. The effect of radiative cooling on model simulations will also be discussed and simulations of Luminous Blue Variable-type (LBV) stars will be shown. Although already published in *Light* (2020), it is important to summarise these results here and to extend them to also include a stellar magnetic field. This is important here because in Chapter 4 cosmic ray transport is calculated in LBV stars, using these results as input into the transport model, which requires as basic input the behaviour of the astrospheric magnetic field.

3.1 Structure of Astrospheres

As discussed for the heliosphere in Sect. 2.1, the expansion of a stellar wind (SW) into the interstellar medium (ISM) surrounding a star creates a stellar wind cavity, also known as an astrosphere. A full review of the different analytical descriptions and modelling efforts for the heliosphere can be found in *Pogorelov et al.* (2017) and *Kleimann et al.* (2022), while a brief description will be given here. Stars may also be moving through the ISM which induces a second, homogeneous wind when observed from the rest frame of the star. The surface separating the hot SW plasma from the cooler ISM plasma is the astropause (AP). The AP is a tangential discontinuity such that no mass is transported across it and the component of the velocity normal to the AP reduces to zero while the thermal pressure of the SW and ISM are matched either side of the AP (*Scherer et al.*, 2016).

The SW released from the star is usually supersonic and often assumed to be isotropically emitted while carrying mass and the magnetic field from the star into the ISM. The SW is accelerated very close to the star and it is assumed for MHD modelling that it has already reached its terminal velocity at an inner boundary in the model. The first analytical description of the solar wind is given by *Parker* (1958) which assumes compressible, isothermal, unmagnetised, purely radial, outflow. At small radial distances coronal heating may justify an isothermal expansion, while observations of propagating waves that are relevant for coronal heating are reported (see the review by *Banerjee et al.*, 2021). However, as supported by measurements (*Richardson et al.*, 2022), it is more realistic to assume the expansion continues adiabatically rather than isothermally (see *Kleimann et al.*, 2022) due to the assumption of constant temperature at larger radial distances not holding. This results in a constant SW and implies that the density has a $\sim r^{-2}$ dependence in order to satisfy the law of mass conservation. The SW is required to decelerate as it approaches the AP and ISM and change in flow direction until it is forced to become tangential. This leads to the creation of the termination shock (TS) where the supersonic SW becomes subsonic and where the SW is redirected into the tail region.

A similar scenario is found on the interstellar side where the ISM wind has to decelerate as it approaches the AP, and flow around it. This happens at the bow shock (BS). The BS can be created by a star moving through the interstellar medium, or for modelling purposes by transforming to the rest frame of the star, and allowing the ambient flow to move past it (see *Baalmann et al.*, 2022, and references therein). The existence of a BS, however, is dependent on the fast magnetosonic speed (e.g., *Scherer and Fichtner*, 2014) with the alternative being a bow-wave which shows enhanced density but no shock (e.g., *McComas et al.*, 2012). For a review on the bow shock of the heliosphere, see e.g., *Mostafavi et al.* (2022) and references within.

The bow shock of the heliosphere, has been modelled extensively (see e.g., *Pogorelov et al.*, 2017; *Opher et al.*, 2021; *Linsky et al.*, 2022), while the bow shocks of O-type stars have also been modelled before (e.g., *Baalmann et al.*, 2022). From these and many other simulations we have a good understanding of the astrospheric structures.

An example of the shock structures of an astrosphere is shown in Fig. 3.1 from *Scherer et al.* (2016). A complete discussion on the shock structures of astrospheres can be found in *Scherer et al.* (2016), while a brief discussion will be given here. The area between the TS (closest red line) and AP (blue line) where the supersonic SW has been slowed to subsonic speeds is called the inner astrosheath (IAS). This area contains extremely hot SW plasma as the supersonic SW is shocked at the TS converting the kinetic energy of the SW to thermal energy creating a region of very hot, incompressible plasma. The shocked ISM plasma is found between the BS (outer red line) and AP in an area called the outer astrosheath (OAS). This area may be further divided into two separate areas (dashed line) when the effect of cooling is included (see Sect. 3.2.2). The structure can also become more complicated when a stellar magnetic field and ISM magnetic field are included (see Sect. 3.2.1).

Dialynas et al., 2022; *Richardson et al.*, 2022), which crossed both the TS and heliopause of the heliosphere.

3.2 Modelling of Astrospheres

The astrospheric structures mentioned above were modelled from 1D to full 3D geometries (e.g., *van Marle et al.*, 2015; *Alvarado-Gómez et al.*, 2016; *Pogorelov et al.*, 2017; *Scherer et al.*, 2020; *Meyer et al.*, 2021; *Opher et al.*, 2021; *Baalmann et al.*, 2022; *Herbst et al.*, 2022). The model mostly used in this chapter is that used by *Fahr et al.* (2000); *Scherer and Ferreira* (2005); *Ferreira and de Jager* (2008), while the 3D model used later in the chapter is discussed in Sect. 3.5. These models solve the well-known HD and MHD equations given by

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \quad (3.1)$$

$$\frac{\partial(\rho \mathbf{v})}{\partial t} + \nabla \cdot (\rho \mathbf{v} \otimes \mathbf{v} + P^* \mathbf{I} - \frac{1}{4\pi} \mathbf{B} \otimes \mathbf{B}) = 0 \quad (3.2)$$

$$\frac{\partial e}{\partial t} + \nabla \cdot [(e + P^*) \mathbf{v} - \frac{1}{4\pi} \mathbf{B}(\mathbf{B} \cdot \mathbf{v})] = -n_e n_H \Lambda(T) \quad (3.3)$$

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \times (\mathbf{B} \times \mathbf{v}) = 0 \quad (3.4)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (3.5)$$

which describe the conservation of mass, momentum and energy, and also include the induction equation and a closure equation, which is a condition that is imposed which requires the magnetic field to remain divergence free. The density is given by ρ , the velocity by $\mathbf{v}$ and the magnetic field by $\mathbf{B}$. The total pressure is described through $P^* = P + \mathbf{B}^2/8\pi$, a combination of the thermal pressure P and the magnetic pressure $\mathbf{B}^2/8\pi$. Lastly, $\mathbf{I}$ is the unit matrix and $\otimes$ is the dyadic product. It can be seen from these equations that radiative cooling is included on the right-hand side of the energy equation through $-n_e n_H \Lambda(T)$. Radiative cooling is discussed further in Sect. 3.2.2. The total energy is calculated using

$$e = \frac{\rho |\mathbf{v}|^2}{2} + \frac{P}{\gamma - 1} + \frac{\mathbf{B}^2}{8\pi}, \quad (3.6)$$

with the adiabatic index given by γ , and in this work a single fluid approach is taken with $\gamma = 5/3$.

3.2.1 ISM and stellar magnetic field

The structure and size of the astrosphere can be influenced by the density, temperature, and direction of the ISM flow as well as the magnitude of stellar and interstellar magnetic fields. The first part of this work covering the modelling of astrospheres does not include a stellar magnetic field. The stellar magnetic field can sometimes be neglected as it is negligible compared to the ram pressure or thermal pressure of the SW. Even for large

	Unit	Sect. 3.2.2	Sect. 3.2.2.1	HD 99953	AG Carinae
Mass-loss rate	$M_{\odot}/\text{year}$	2.88×10^{-6}	1.0×10^{-6}	$8.0 \times 10^{-8(a)}$ $2.2 \times 10^{-7(b)}$	$3.16 \times 10^{-5(c)}$ $1.58 \times 10^{-4(d)}$
Stellar wind velocity	km.s^{-1}	1000	1000	250 ^(a) 700 ^(b)	150 ^(c) 300 ^(d)
Inner boundary	pc	0.03	0.03	0.03	0.03
Stellar wind density	particles.cm^{-3}	1.0	3.48	1.115 ^(a) 1.095 ^(b)	$7.338 \times 10^{2(c)}$ $1.835 \times 10^{3(d)}$
Stellar wind temperature	K	1×10^5	1×10^5	1×10^5	1×10^5
ISM density	particles.cm^{-3}	1.0	10.0	1.0	1.0
ISM $\mathbf{B}^e$	μG	3.0	3.0	3.0	3.0
ISM temperature	K	9×10^3	10^2	9×10^3	9×10^3

Table 3.1: Simulation parameters used in sections below. ^aMinimum stellar outflow parameters for HD 99953. ^bMaximum stellar outflow parameters for HD 99953. See *Fraser et al.* (2010) and *Haucke et al.* (2018) for more details regarding the outflow parameters of HD 99953. The ^cminimum and ^dmaximum stellar outflow parameters for AG Carinae can be found in *Stahl et al.* (2001). ^eThe assumed ISM magnetic field strength based on the galactic magnetic field strength found in *Kleimann et al.* (2019), where the turbulent part has been neglected.

stellar magnetic fields (see *Schöller et al.*, 2017) it can be mostly neglected, as shown in *van Marle et al.* (2015). The stellar magnetic field can be neglected due to the $\sim r^{-2}$ radial dependence and in turn the $\sim r^{-4}$ radial dependence of the magnetic pressure, which becomes very small in large astrospheres.

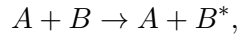
However, the stellar magnetic field or more importantly the magnitude and direction of the magnetic field frozen into the SW, which can be seen as the astrosphere magnetic field (AMF), is an important parameter for the transport of particles throughout these astrospheres and will be included in later chapters when it is required for the GCR modulation.

The ISM magnetic field is an important parameter influencing stellar wind cavity evolution where it increases the total pressure of the ISM that the SW experiences as it expands into the ISM. The expansion of the astrosphere into the ISM causes the ISM matter to be shocked by the BS, creating a dense shell surrounding the astrosphere. The pressure in this dense shell is then increased by a larger compressed magnetic field. In the 2D model which is first used, an azimuthal ISM magnetic field is assumed to determine the maximum effect that it may have on the evolution of the astrosphere (*Light*, 2020). Due to this assumption the results could be seen as 1D. The strength of the ISM magnetic field is assumed to be $3 \mu\text{G}$, which is an approximation of the strength of the galactic magnetic field found in *Kleimann et al.* (2019), while the turbulent part of this field has been ignored.

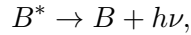
3.2.2 Radiative Cooling

The outer structure of an astrosphere may be influenced through the process of radiative cooling. This process is described in *Dalgarno and McCray (1972)* and *Dyson and Williams (1997)* with a summary of this process following (see also *Van der Schyff, 2016*).

In a medium that is optically thin, a partially ionised gas is able to cool by emitting radiation. The thermal energy is transformed into radiation through collisions. Following some collisions, an atom, ion, or molecule in the gas may become excited. The additional energy acquired through excitation can be emitted as a photon, provided the photon can escape through an optically thin medium. The escaping photon carries away the energy from the surrounding environment, reducing the total thermal kinetic energy of the environment. *Dyson and Williams (1997)* describe this process with



where the collision of A and B leads to B being in an excited state. Given sufficient time



with $h\nu$ being the photon which may escape, given an optically thin medium.

Radiative cooling is included in the numerical model through the additional term, $-n_e n_H \Lambda(T)$ on the right hand side of the energy conservation equation. Here n_e is the electron density, n_H the proton density and $\Lambda(T)$ is a cooling function, the subject of the next section.

3.2.2.1 Cooling functions

The term $\Lambda(T)$ in Eq. 3.3 contains the cooling function, which is dependent on the temperature. There are a multitude of cooling functions covering various ranges of temperatures (see, e.g., *Sutherland and Dopita, 1993*; *Mellema and Lundqvist, 2002*; *Schure et al., 2009*; *Lykins et al., 2013*), which could have different effects on the evolution of astrospheres. A full discussion on the effect that cooling has can be found in *Van der Schyff (2016)* and *Light (2020)*, with details being discussed here.

In this work a comparison between the cooling functions of *Mellema and Lundqvist (2002)* (CFA) and *Schure et al. (2009)* (CFB) is shown in Fig. 3.3. It should be noted that CFB only extends down to 10^4 K while CFA covers a temperature range of $10^2 \sim 10^8$ K. This is motivation for the choice of using CFA in this work. In the range $10^4 \sim 10^6$ K there is also a factor ~ 2 difference between CFA and CFB, leading to significantly different cooling rates.

The evolution of an astrosphere with CFA and CFB included is shown in Fig. 3.4 at different simulation times. The parameters used for the modelling of these astrospheres are given in Table 3.2.1. The cooling function CFB is able to cool more effectively due to the factor ~ 2 difference. This leads to a slightly smaller sized astrosphere at all stages

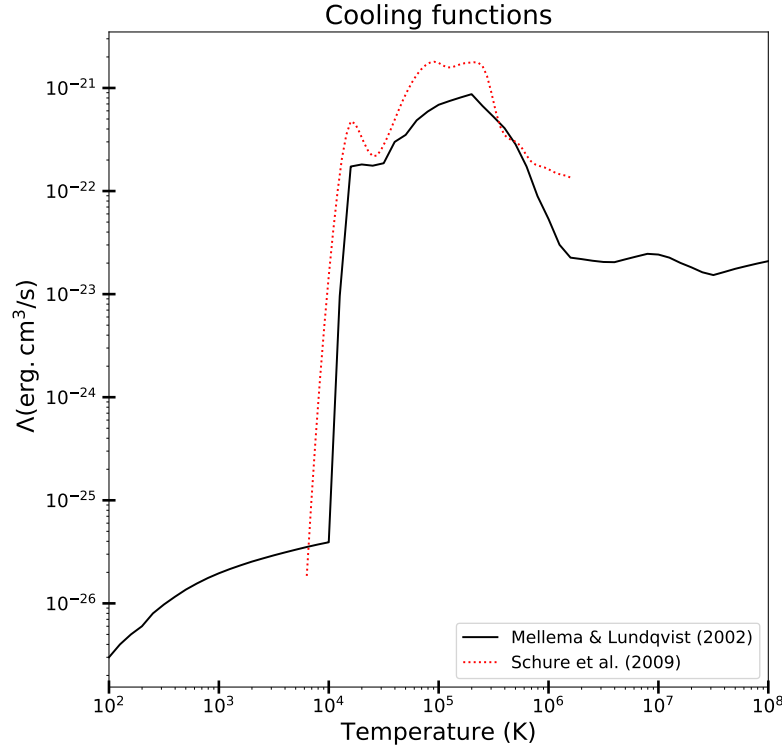


Figure 3.3: Cooling functions of *Mellema and Lundqvist* (2002) (CFA, solid black line) and *Schure et al.* (2009) (CFB, dotted red line) as a function of temperature. Adapted from *Light et al.* (2022).

of evolution seen in Fig. 3.4. However, the effects of the respective cooling functions on the structures of the astrospheres are similar. The difference in cooling of CFB compared to CFA is more apparent when comparing the compression ratios of the BS of the astrospheres. The compression ratios are taken by comparing the density immediately before and after a shock front.

A comparison between the compression ratios for the BS using CFA and CFB for a pure hydrodynamic and magnetohydrodynamic case is shown in Fig. 3.5. The compression ratio of the BS using CFB increases early compared to CFA, showing the increased effectiveness, in the cooling function shown in Fig. 3.3. In the pure hydrodynamic case CFA cools more than CFB, leading to a higher compression ratio; however, for the magnetohydrodynamic case both cooling functions lead to similar profiles for the compression ratio of the BS due to the added magnetic pressure. This is further motivation for the use of CFA for this work.

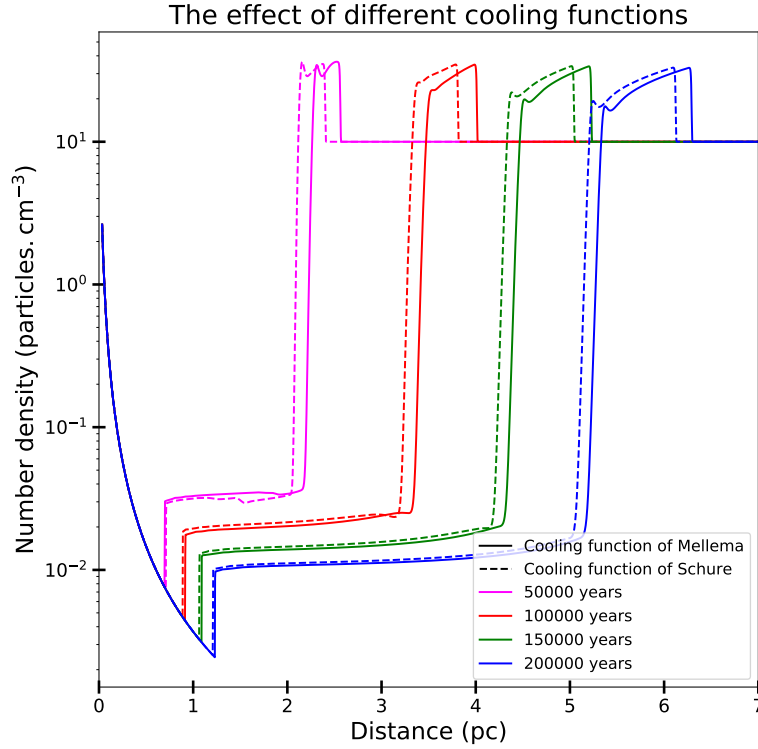


Figure 3.4: Computed radial profiles showing the evolution of an astrosphere for 4 different simulation times. The solid and dashed lines correspond to the inclusion of the different cooling functions CFA (solid lines) and CFB (dashed lines) in the model. Taken from *Light et al. (2022)*.

3.2.2.2 Effect on evolution of astrospheres

Radiative cooling reduces the thermal kinetic energy in the outer shell during the evolution of the astrosphere. There is almost no radiative cooling within the supersonic outflow as radiative cooling is dependent on the density squared, and hence with a relatively high density at inner boundary it is not possible to cool. A decrease in thermal pressure reduces the total pressure of the ISM in the OAS leading to higher compression. However, the increased compression of the OAS leads to a compression of the ISM magnetic field lines within the OAS. The compressed magnetic field lines thus increase the magnetic pressure in the OAS and the increased magnetic pressure decompresses the OAS into a more distinctive two-part structure in the OAS, namely a hot part (HOAS) and a cool part (COAS). This can be seen by the dashed line in Fig. 3.1. This occurs once the medium within the OAS has cooled and the increased magnetic pressure is able to dominate over the ram pressures of the SW and ISM. Due to the azimuthal nature of the magnetic field and of the results presented here, the addition of radiative cooling can be seen as including the maximum cooling effect in the outer shell.

The evolution of two astrospheres, one with the inclusion of cooling and one without, is

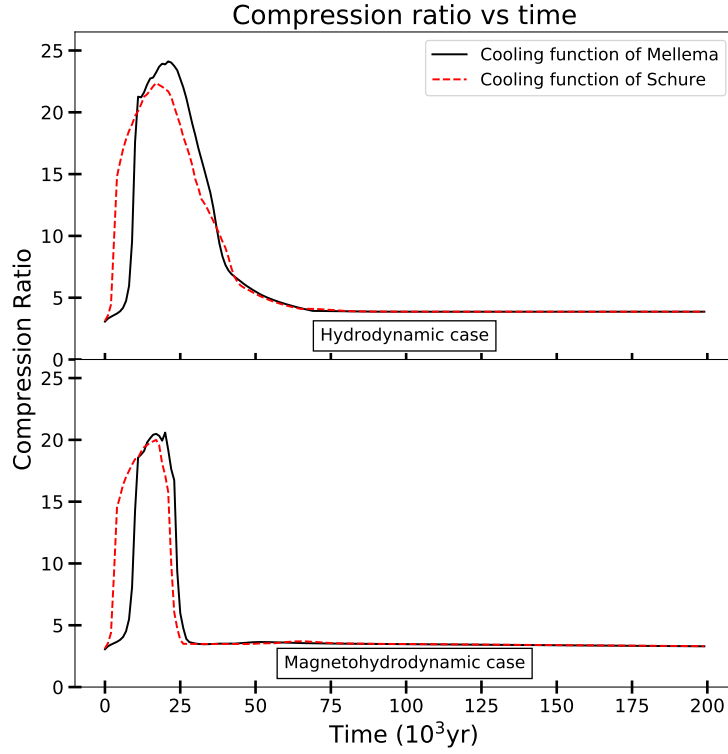


Figure 3.5: Compression ratio of the BS for a hydrodynamic case with no magnetic field (top panel) and a magnetohydrodynamic case with a ISM magnetic field of $3 \mu\text{G}$. CFA shown by the solid black lines and CFB by the dashed red lines. Taken from *Light et al.* (2022).

shown in Fig. 3.6 at two different stages of evolution. The solid orange line and dashed blue line are density as a function of radius after 25 000 years simulation time, where the astrosphere is still relatively young. At this time, the OAS is experiencing the maximum effect of radiative cooling, where all the shocked ISM matter is cooled, reducing the thermal pressure of the OAS to such a degree that it is compressed into a thin, very dense shell compared to the astrosphere without cooling which has a much less dense and thicker shell. The continual evolution of the astrosphere allows the ISM magnetic field and increasing magnetic pressure in the OAS to decompress the thin dense shell into a COAS at the AP and a HOAS inside the BS.

The outer structures of the two astrospheres have similar structures, given enough time once the astrosphere has cooled as shown by the solid red and dotted green line in Fig. 3.6. These radial profiles are for 300 000 years simulation time. At this point the effect of cooling is only visible by the slightly higher density of the COAS compared to astrosphere without cooling. The thin dense shell surrounding the astrosphere during the early stages is decompressed by the increased magnetic pressure inside the shell. The magnetic pressure decompresses the OAS radially outward. This radial decompression reduces the size of the AS as the AP is pushed closer to the inner boundary while the OAS decompresses.

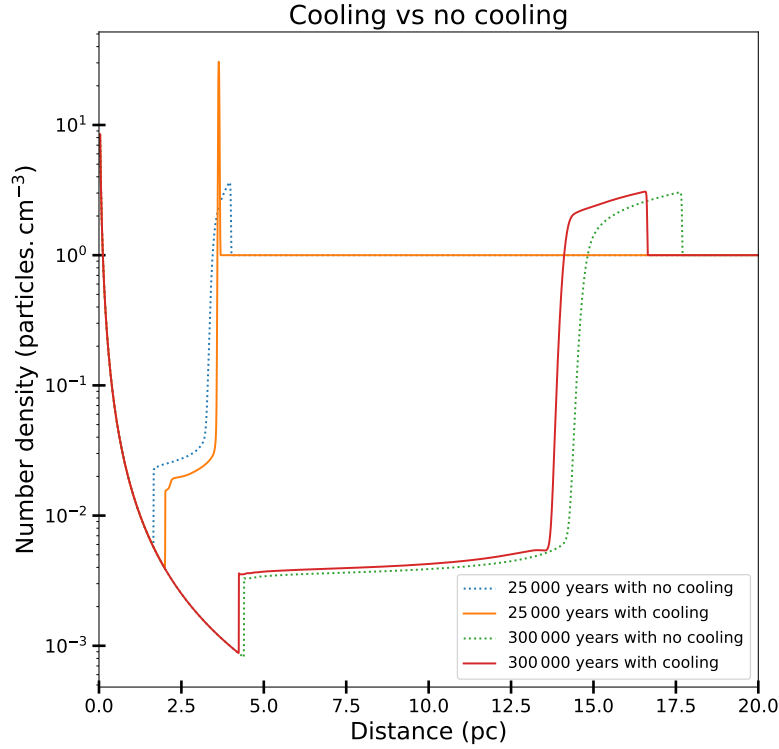


Figure 3.6: Computed density profiles of an astrosphere with and without cooling at different stages of evolution. The mass-loss rate for both astrospheres is $2.88 \times 10^{-6} M_{\odot}/\text{year}$ and the outflow velocity is 1000 km.s^{-1} . The cooling function used here is that of *Mellema and Lundqvist* (2002). Taken from *Light et al.* (2022).

The effect of radiative cooling on the outer structure of an astrosphere as it evolves through time is more visible from the compression ratio of the BS as a function of time. The BS being compressed due to the cooling effect of radiative cooling, increases the compression ratio of the BS. As shown in Fig. 3.7 for a pure hydrodynamic case without cooling effects (blue dashed line) the compression ratio remains roughly constant at a value of 4. As stated in the previous paragraph, for simulations of an astrosphere, the addition of radiative cooling causes the outer structure to become thinly compressed during the early stages and then decompressed as it continues to evolve.

The compression ratio of the BS, when cooling is included, increases from the expected compression ratio of ~ 4 for a hydrodynamic case to ~ 34 . As shown in Fig. 3.7 the compression of the outer structure has a gradual start, followed by a rapid compression as the plasma within the OAS is completely cooled. Once the plasma within the OAS is cooled it remains as such and the effect of cooling is reduced as the new plasma shocked by the BS forms the HOAS. This is followed by the decompression of the OAS due to the ram pressure dominating as the astrosphere evolves. At this stage, the astrosphere is cooled and a constant compression ratio of four continues, similar to the case without cooling.

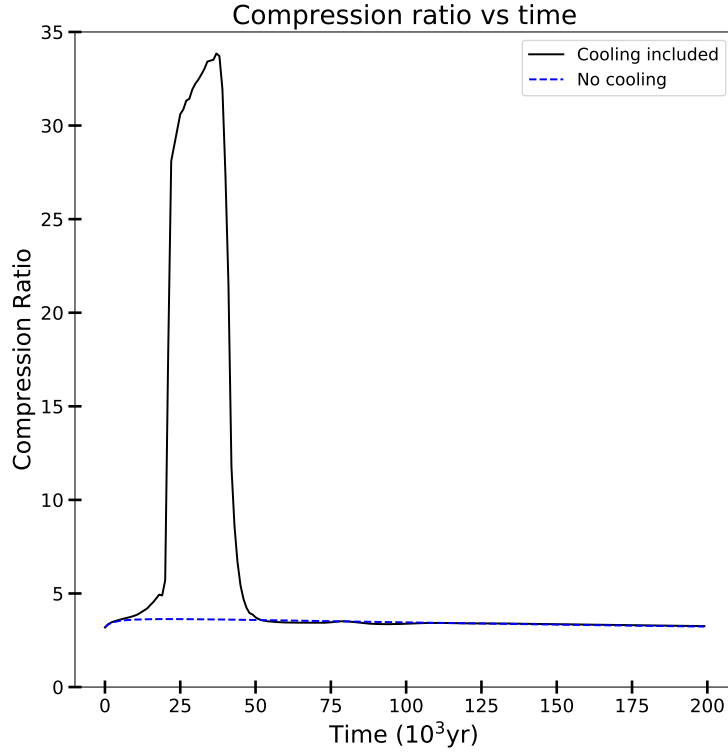


Figure 3.7: Compression ratio of the BS of a pure hydrodynamic astrosphere without cooling (dashed blue) and with cooling (solid black) as a function of time as the astrosphere evolves. Taken from *Light et al.* (2022).

Magnetic pressure within the OAS will reduce the compression of the OAS even when radiative cooling is included. When an ISM magnetic field is included, as is done for Fig. 3.8, there is additional magnetic pressure along with the ram pressure of the shocked ISM decompressing the OAS once it is cooled. A larger ISM magnetic field will increase the magnetic pressure within the OAS reducing the compression more.

A comparison of the compression ratios of the BS for different ISM magnetic field strengths and a hydrodynamic case is shown in Fig. 3.8. A very large compression ratio for the hydrodynamic case of ~ 34 is again visible until the astrosphere has cooled and then reduces to a constant compression ratio of 4 which remains as the astrosphere evolves. Including the ISM magnetic field reduces the maximum compression ratio of the OAS. An ISM magnetic field strength of $1.5 \mu\text{G}$ reduces the maximum compression ratio to ~ 27 while a magnetic field strength of $3 \mu\text{G}$ reduces this further to ~ 18 . The added magnetic pressure within the OAS reduces the amount of time it takes the OAS to decompress once it has been cooled. A very large ISM magnetic field, $10 \mu\text{G}$, adds such a large amount of magnetic pressure into the OAS that it dominates over the ram pressure of the SW to such a degree that the compression ratio of the BS never reaches the expected value of a hydrodynamic shock, starting off at ~ 4 but reducing quickly to ~ 2 . There are no visible signs of the effect that radiative cooling has on the OAS when such a large ISM magnetic

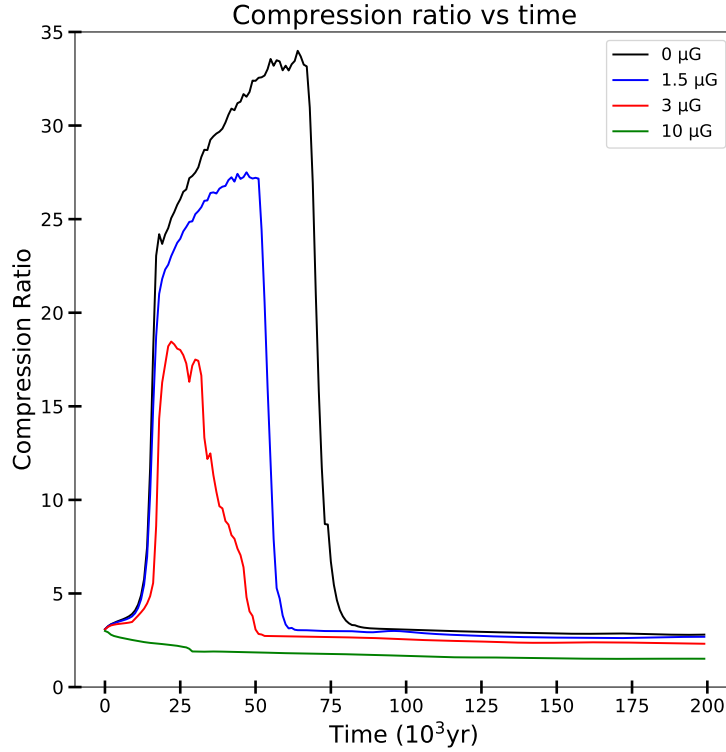


Figure 3.8: Compression ratio of the BS for a pure hydrodynamic astrosphere (black) and the compression ratios of the BS for different ISM magnetic field strengths, $1.5 \mu\text{G}$ (blue), $3 \mu\text{G}$ (red) and a large field, $10 \mu\text{G}$ (green). Taken from *Light et al.* (2022).

field is included.

3.3 Astrospheres of Luminous Blue Variable Stars

Luminous blue variable stars (LBV) are massive O-type stars that do not complete their evolution to red supergiants. These stars instead enter a phase of very high mass-loss with an increase in temperature (*Weis and Duschl*, 2002). Stars that go through the LBV phase can lose up to three orders of magnitude of solar mass and the outflow parameters of the star, such as SW density, SW velocity and temperature vary throughout the phase (*Burggraf et al.*, 2015; *Solov'yeva et al.*, 2019). These types of stars may also undergo eruptions, during which they lose an extreme amount of solar masses during a very short period of time, which can be mistaken for supernova remnants. See *Humphreys and Davidson* (1994) and *Humphreys et al.* (2016) for a review on LBV-type stars.

LBV stars can undergo different types of variability with four types of major photometric variability, namely microvariations, S Dor phases, stochastic variability and eruptions (*Sterken*, 2003). These types of variations are not unique to LBVs and it is common for supergiants to undergo microvariations (*Vink*, 2012). Of interest in this work is the S Dor

phases, which can be divided into short S Dor phases and long S Dor phases. Short S Dor phases are on the timescales of years, while long S Dor phases are on timescales of decades.

The stars chosen for this section are HD 99953 and AG Carinae, and the eruptive phase will be done with similar conditions as those of η Carinae and P Cygni, which are the two best-documented cases of eruptions for LBV-type stars (see *Sterken*, 2003).

3.3.1 Astrosphere of HD 99953

HD 99953 is an early B-supergiant which shows variation in the stellar outflow parameters (see *Fraser et al.*, 2010; *Haucke et al.*, 2018). Even though HD 99953 is not an LBV star, the variations in the stellar outflow parameters of this star still make it an ideal candidate to study the effect that these types of variations have on the evolution of the astrospheres of LBV stars. During the minimum phase, where the outflow parameters are lower, the mass-loss rate of this star is $8 \times 10^{-8} M_{\odot}/\text{year}$ with a SW velocity of 250 km.s^{-1} while for the maximum phase the mass-loss rate increases to $2.2 \times 10^{-7} M_{\odot}/\text{year}$ with an increase in the SW velocity to 700 km.s^{-1} .

Using the mass-loss rate, the SW velocity and the inner boundary of the model, 0.03 pc, a SW density is calculated at the inner boundary. The inner boundary is constrained due to the grid size of the model but is also large enough so that an uniform stellar wind may be assumed. For the minimum phase the SW density is found to be $1.115 \text{ particles.cm}^{-3}$ while for the maximum phase this decreases to $1.095 \text{ particles.cm}^{-3}$ at the inner boundary. A constant SW density is assumed of $1.095 \text{ particles.cm}^{-3}$ for both the minimum and the maximum phase due to the densities being very similar in both phases. A sinusoidal variation in SW outflow velocity is implemented at the inner boundary with a period of 17 years to simulate the variation of the outflow parameters. An ISM magnetic field strength of $3 \mu\text{G}$ is assumed to be azimuthal to the astrosphere for the maximum effect on the evolution of the astrosphere.

Radial profiles for the minimum, maximum and periodic variation astrospheres are shown for HD 99953 in Fig. 3.9. There is a distinct difference in size of the astrospheres when the minimum and maximum outflow parameters of HD 99953 are used to model these astrospheres. Both the minimum and maximum outflow parameters show similar structures and evidence of radiative cooling within the OAS, showing a COAS and a HOAS. A sinusoidal variation between these two phases within a period of 17 years produces the astrosphere shown by the solid green line in Fig. 3.9. During the maximum phases the higher outflow velocity creates clumps of high density that flow radially outwards. This is due to the higher velocity catching up to the slower flowing plasma from the minimum phase. These high and low density clumps can be seen to travel from the inner boundary out to the TS. Halfway towards the TS, these clumps become less defined as the stellar wind slows down and the velocity of the wind becomes more uniform. At the TS, the stel-

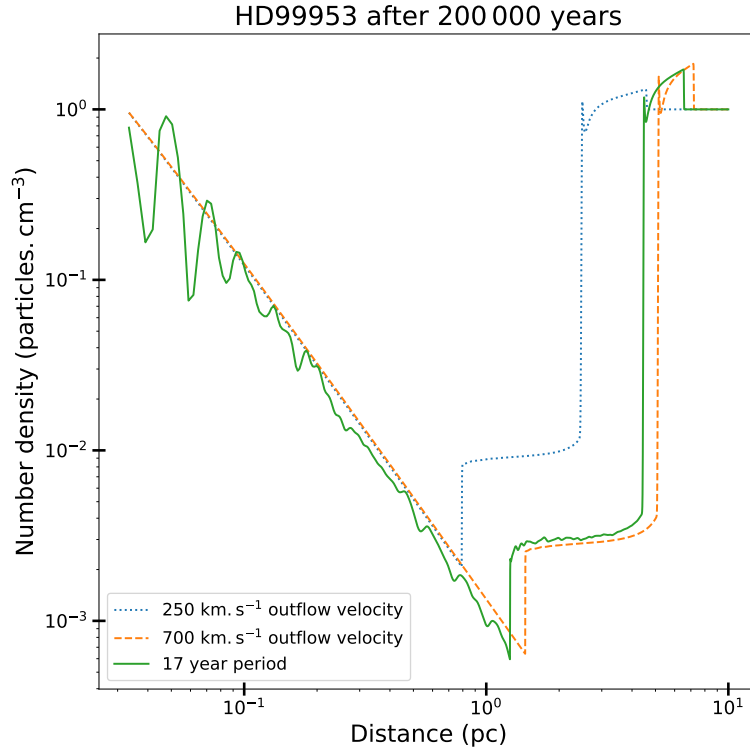


Figure 3.9: Radial profiles of the minimum (dotted blue) and maximum (dashed orange) phases of HD 99953 compared with the radial profile of HD 99953 with the inclusion of a sinusoidal variation between minimum and maximum phase within a period of 17 years (solid green). Taken from *Light et al.* (2022).

lar wind becomes shocked and the kinetic energy of the wind is transformed into thermal energy; the high and low density clumps are smoothed by this shock as there is negligible density variation after the TS.

3.3.1.1 Stellar magnetic field of HD 99953

The effect of the ISM magnetic field on the structure and evolution of an astrosphere was discussed in Sect. 3.2.1, where it was also mentioned that the stellar magnetic field which gets frozen into the stellar wind creating the AMF as it is dragged away from the star, can be neglected. However, GCR modulation in the following chapter requires as an input the magnitude of the stellar magnetic field as a function of radial distance within the astrosphere, as such a comparison is made for HD 99953 between an astrosphere only containing an ISM magnetic field and one containing an ISM and stellar magnetic field.

This can be seen in Fig. 3.10. Shown here by the the black dotted line is the astrosphere of HD 99953 with only an ISM magnetic field of $3 \mu\text{G}$ and the solid blue line shows an astrosphere containing an ISM magnetic field of $3 \mu\text{G}$ and a stellar magnetic field of $1.5 \mu\text{G}$

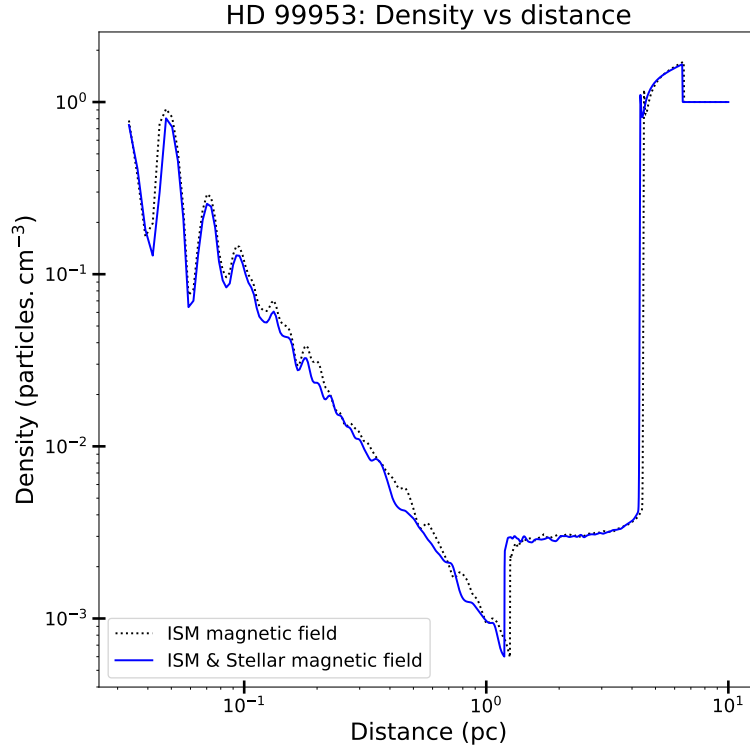


Figure 3.10: Astrosphere of HD 99953 similar to Fig. 3.9 with a 17 year sinusoidal variation in the outflow parameters. An astrosphere with no stellar magnetic field and only an ISM magnetic field is shown in the black dotted line, while the solid blue line shows the density profile of an astrosphere with a stellar and ISM magnetic field included.

at the inner boundary of the simulation. There is very little to no difference in the size and structure of the two astrospheres with a small variation in the density oscillations due to the varying outflow parameters from minimum to maximum phase within a period of 17 years. The magnetic pressure that is added by the addition of the stellar magnetic field is small compared to the ram pressure of SW since the magnetic pressure of the AMF has a $\sim r^{-2}$ dependency from the inner boundary out to the TS.

A radial profile of the magnetic strength of both the AMF and ISM magnetic field can be seen in Fig. 3.11. The solid lines show the AMF of HD 99953 decrease with distance squared out to the TS after 50 000 years (black) and after 200 000 years (green) simulation time. The ISM magnetic field (dotted lines) can be seen to remain constant within the ISM and increases sharply once the ISM becomes shocked by the BS. The ISM magnetic field keeps increasing within the OAS up to the AP where it drops off to zero similarly for the AMF, where the AMF drops off to zero within the AP. The higher magnetic field strength within the OAS after 50 000 years compared to the strength within the OAS after 200 000 years displays the higher magnetic pressure due to the compressed magnetic field lines from the effect of radiative cooling compressing the OAS. These magnetic profiles will be used in the following chapter as input to the cosmic ray transport model for the

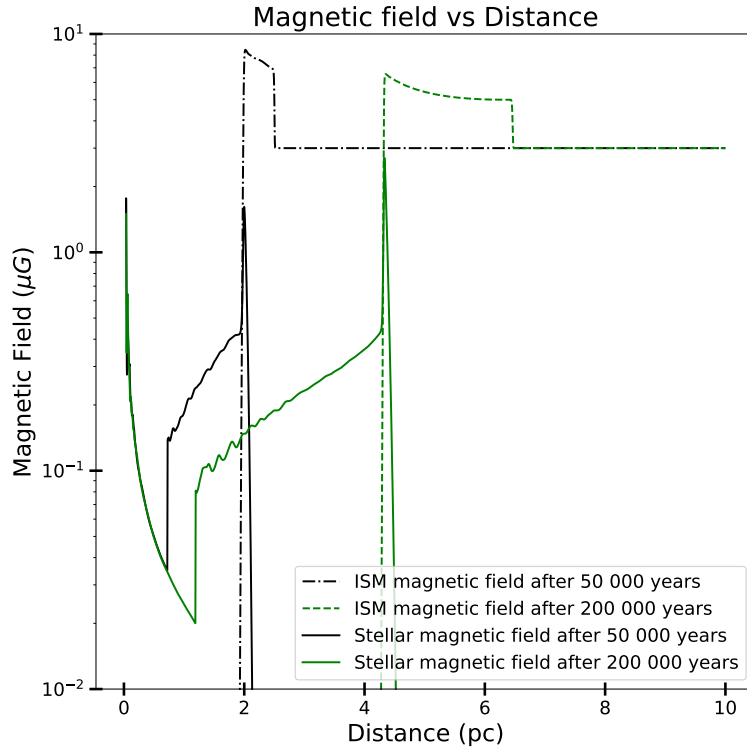


Figure 3.11: Computed Stellar and Interstellar Magnetic profiles of HD 99953 as a function of radial distance from the inner boundary. The stellar (solid lines) and ISM (dotted lines) magnetic field profiles are shown. The black lines correspond to an astrosphere 50 000 years into its evolution while the green lines show the same astrosphere 200 000 years after it started.

modulation of GCRs.

3.3.2 Astrosphere of AG Carinae

AG Carinae is a well-studied typical LBV-type star (see *Stahl et al.*, 2001). This star undergoes variability on timescales of days to decades, with photometric, spectroscopic, and polarimetric variability (*Groh et al.*, 2009). Quantitative spectroscopic analysis has been done for different epochs of AG Carinae and different mass-loss rates have been reported during different S Dor phases (see *Vink*, 2012). AG Carinae transitions from visual minimum to the following visual minimum within a period of ~ 15 years (*Groh et al.*, 2009). The stellar parameters used in this chapter for AG Carinae are taken from *Stahl et al.* (2001). For the minimum phase, the outflow velocity is taken as 150 km.s^{-1} with a mass-loss rate of $3.16 \times 10^{-5} M_{\odot}/\text{year}$, while for the maximum phase the outflow velocity increases to 300 km.s^{-1} and the mass-loss rate increases up to $1.58 \times 10^{-4} M_{\odot}/\text{year}$.

The increase in mass-loss rate is approximately a factor of ~ 5 from minimum to maximum

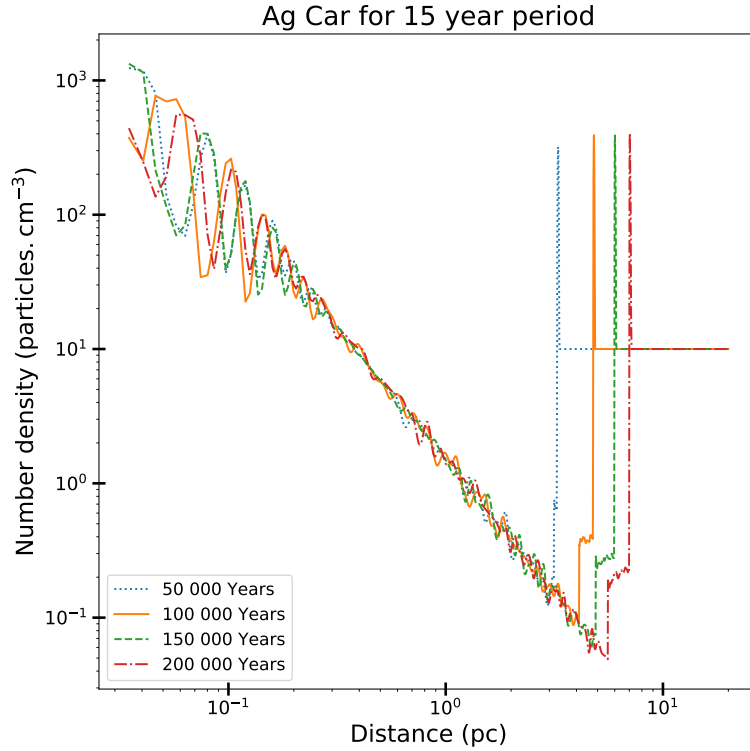


Figure 3.12: Radial profiles for AG Carinae at different stages of evolution. Dotted blue line corresponds to 50 000 years, solid orange to 100 000 years, dashed green to 150 000 years and finally dashed dotted red to 200 000 years simulation time. Taken from *Light et al.* (2022).

phase, similar to the increase for HD 99953. Different from HD 99953 is that the mass-loss rates are much higher and the outflow velocities are considerably lower. Assuming the mass-loss rates and stellar wind velocities as stated above as inner boundary conditions, and with an assumed inner boundary at 0.03 pc, the minimum phase outflow density at the inner boundary is found to be 645.0 particles.cm⁻³ and the outflow density for the maximum phase is 1612.0 particles.cm⁻³. An ISM magnetic field strength of 3 μ G is also assumed, while the ISM density is assumed to be 10.0 particles.cm⁻³ surrounding AG Carinae.

The transition time from minimum to following minimum for AG Carinae is between 10 and 18 years (see e.g., *Groh et al.*, 2009). However, in this work to simulate a period between a short S Dor and a long S Dor phase, the stellar parameters vary with a period of 15 years. Even though a higher ISM density is assumed around AG Carinae, the larger assumed mass-loss rate, stellar wind density and consequently larger ram pressure of the stellar wind of AG Carinae, is able to create an astrosphere of similar size compared to HD 99953.

Even though the astrosphere has a similar size, the TS distance for AG Carinae is larger

than the TS distance for HD 99953, due to the higher ram pressure. The TS for AG Carinae is located at ~ 5.5 pc after 200 000 years while the TS for HD 99953 is located at ~ 1.2 pc after the same amount of simulation time. The periodic variations of AG Carinae creates similar density waves, as seen for HD 99953, travelling from the inner boundary up to the TS. The evolution of these density waves can be seen in Fig. 3.12. As the astrosphere evolves from 50 000 years to 200 000 years simulation time the astrosphere increases in size and expands into the ISM. The OAS of AG Carinae is thinner and denser compared to HD 99953 due to more effective cooling.

3.4 Luminous Blue Variables Eruptive Events

As mentioned in Sect. 3.3, LBV-type stars can undergo eruptions during which they lose a significant amount of mass during a very short time frame. The mass lost during this phase can also be accompanied by an increase in luminosity of many magnitudes (*Humphreys and Davidson*, 1994; *Humphreys et al.*, 2016). These eruptions can be so large that some of them have been classified as supernovas, only to be later reclassified as LBV eruptions as the star still remained intact (*Humphreys et al.*, 2016). Two well documented cases of LBV-stars that erupted are η Carinae and P Cygni (see *Sterken*, 2003; *Humphreys et al.*, 2016, and references within).

The first case considered here is η Carinae, which erupted in 1840. During this eruption, the star lost $\sim 10M_{\odot}$ which corresponds to a mass-loss rate of $0.1 - 0.5 M_{\odot}/\text{year}$ (see *Puls et al.*, 2008, and references within), an increase of $\sim 10^2$ from the current mass-loss rate of $1.6 \pm 0.3 \times 10^{-3} M_{\odot}/\text{year}$ (*van Boekel et al.*, 2003). The mass-loss rates from these eruptions can be calculated by obtaining the mass and kinetic energy of the nebulae surrounding these stars (see *Smith and Hartigan*, 2006, and references within for a discussion on the procedure). The eruption of P Cygni was smaller than the one for η Carinae as the mass lost during that eruption was $\sim 0.1M_{\odot}$ (*Smith*, 2007). The eruptive event of η Carinae is assumed to have lasted for ~ 20 years (*Smith et al.*, 2003), while the eruptive event of P Cygni lasted for ~ 55 years (*Smith and Hartigan*, 2006). The parameters of LBV-type stars that may have undergone eruptions, including η Carinae and P Cygni, can be found in *Clark et al.* (2009) and references within. The two candidates used in this work can be seen to illustrate the likely range of mass that is lost during an eruption of an LBV-type star (*Smith*, 2007).

3.4.1 Eruptions within LBV astrospheres

In *Light* (2020) and *Light et al.* (2022), the eruptive phases of LBV-type stars are modelled by increasing the mass-loss rate by 10^2 or 10^3 and also doubling the outflow velocity of the SW for the duration of the eruption. First, an astrospheric cavity is modelled for 100 000 years simulation time before the eruptive phase is simulated. The 100 000 years simulation time is chosen to have a well established astrospheric cavity before the eruptive

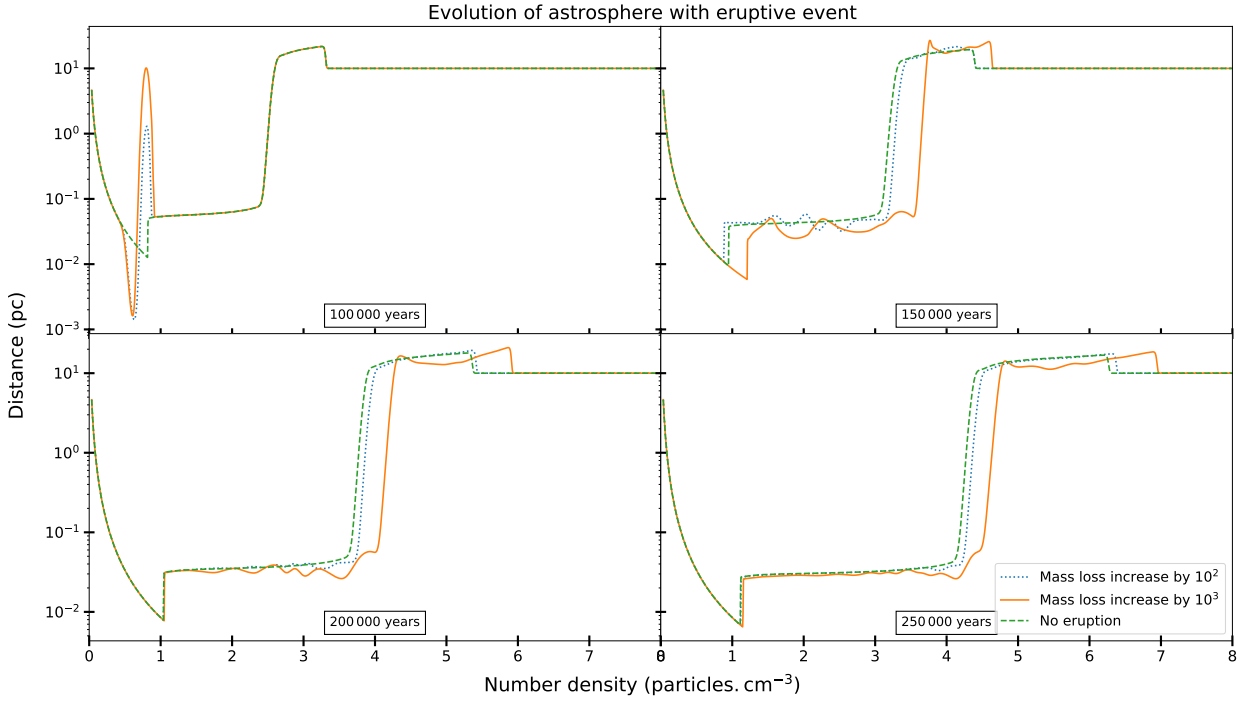


Figure 3.13: Evolution of an astrosphere at 4 different times after an eruption takes place. Solid orange lines shows an eruption where velocity doubles and the mass-loss rate increases by 10^3 , while dotted blue line is an eruption where velocity doubles and mass-loss rate increases by 10^2 . Top panel is 100 000 years after the astrosphere has started to evolve and when the eruption takes place. The second panel is after 150 000 years, third panel is after 200 000 years and fourth panel is after 250 000 years simulation time.

event takes place. The astrospheric cavity is modelled by assuming a mass-loss rate of $1 \times 10^{-6} M_{\odot}/\text{year}$ and an outflow velocity of 500 km.s^{-1} at an assumed inner boundary of 0.03 pc . Different from the eruptive phases modelled in those works, a stellar magnetic field of $1.5 \mu\text{G}$ is included in this work.

For the eruptive phase, the mass-loss rate increases to $1 \times 10^{-4} M_{\odot}/\text{year}$ and $1 \times 10^{-3} M_{\odot}/\text{year}$ to simulate two eruptive events of different amounts of mass being lost during the eruptive phase. The outflow parameters for the eruptive phase are maintained for 100 years, thereafter the outflow parameters before the eruptive phase is assumed once more. A 100 year simulation time eruptive phase is chosen to simulate an amount of mass lost during this period, similar to the amount of mass η Carinae lost during its ~ 20 year eruption.

The eruption phase of LBV-type stars causes a large increase in the SW density over a short period of time causing a high density wave travelling radially outwards from the inner boundary. The top left panel of Fig. 3.13 shows an already established astrosphere after 100 000 years simulation time. An eruption which increases the mass-loss rate for 100 years by 10^2 (blue dotted) and 10^3 (solid orange) creates a high density wave followed by a decrease in density behind the wave as the fast very dense wind of the eruption phase travels radially outward. Such a high density wave compared to the lower density

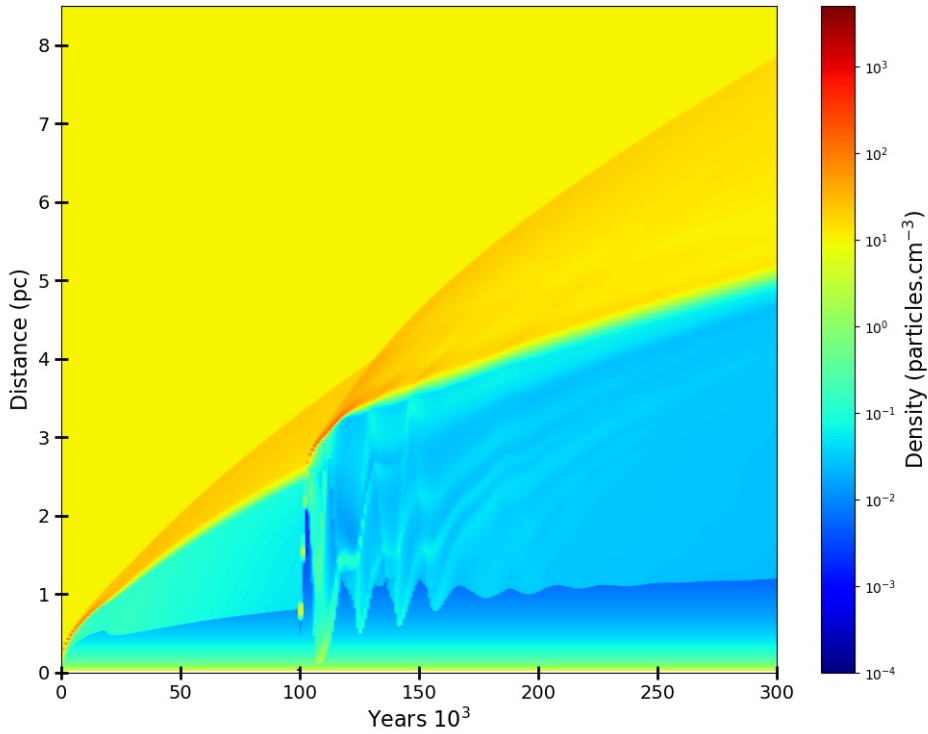


Figure 3.14: A contour showing the density of an astrosphere as a function of distance and time. At 100 000 years an eruption takes place increasing mass-loss rate by 10^3 and velocity doubles.

oscillation discussed for the other LBV phases is able to traverse past the TS and affect the outer structure of the astrosphere. Density oscillations are seen within the IAS in the top right panel of Fig. 3.13, 50 000 years after the eruption has taken place. The large increase in mass-loss rate over a short time period, relative to the time the astrosphere takes to evolve, is able to increase the size of the astrosphere and affect the OAS. The bottom left panel of Fig. 3.13 shows how the wave caused by the eruption travels up to the AP. The density waves caused by the eruption dissipate, given enough time, seen in the bottom right panel of Fig. 3.13. The addition of the stellar magnetic field shows no effect on the structure of the astrosphere modelled in *Light* (2020).

The high density waves can be seen more clearly in Fig. 3.14 which shows the density of the astrosphere as a function of time and distance and in Fig. 3.15 which shows the distance from inner boundary to the TS, AP and BS of the astrosphere as a function of time. Both these figures show how the eruption creates a high density wave that travels radially outward and pushes the TS up the AP. The wave is reflected at the AP and pushes the TS back closer to the inner boundary. The density wave along with the TS oscillates between the inner boundary and the AP for $\sim 100\,000$ years after the eruption has taken place. After this period of time the astrosphere returns to a similar state as it was in before the eruption took place.

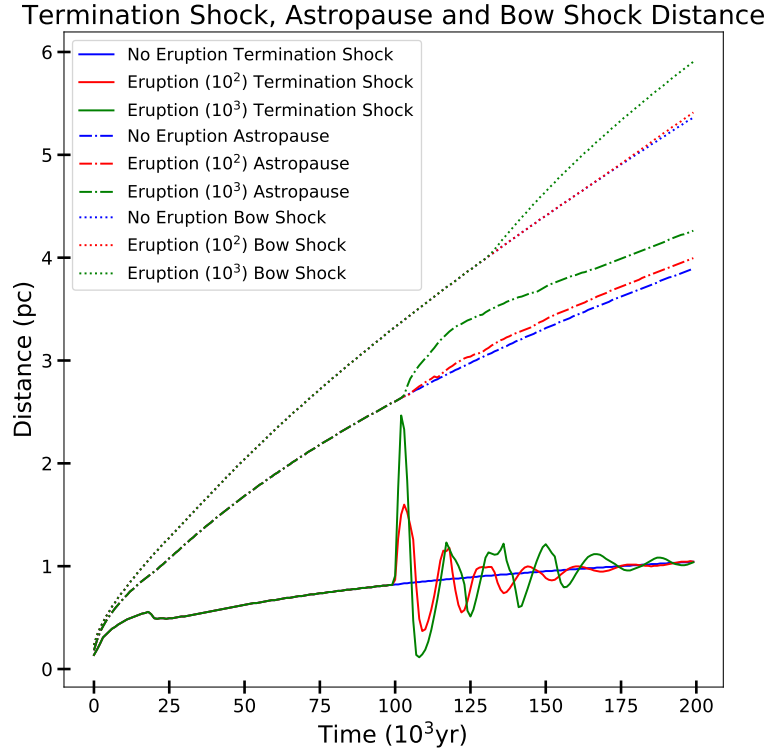


Figure 3.15: Distance from the inner boundary to the shock structures of an evolving astrosphere with the inclusion of an eruptive event after 100 000 years.

The eruptive phase also affects the outer structure. The large amount of energy that is carried through the high density wave is able to increase the distance from the inner boundary to the AP. For both eruptive cases, namely the 10^2 and 10^3 mass-loss rate increases, a density oscillation between the inner boundary and the AP is created and both cases increase the distance from the inner boundary to the AP. Although the 10^3 case does increase the distance by ~ 0.1 pc more compared to the 10^2 case. Both eruptions are able to push the BS further from the inner boundary; however, the smaller eruption creates an extremely small increase in the distance while the larger eruption increases the distance from the inner boundary to the BS by ~ 0.5 pc compared to the non-eruptive case.

3.5 3D Modelling of astrospheres

The results up to this point have shown the modelling of astrospheres in 2D but due to the assumption of an azimuthal ISM magnetic field, results could be seen as 1D. This section expands on the previous sections by extending the modelling of astrospheres to 3D.

3.5.1 3D MHD model

In this section, the MHD model of *Pen et al.* (2003), as described in *Strauss* (2013), is adapted to be used for LBV-type stars, specifically the eruptions of LBV-type stars as discussed in Section 3.4. Similar to the 2D case, a single-fluid MHD approach is taken, where neutrals are neglected. In this section, Eqs. 3.1-3.5 are solved on a 3D Cartesian grid. Similar to the approach discussed in *Strauss* (2013), the magnetic field is saved in the cell faces while the fluid variables are stored in the cells. Since this model is explicit in time, it has to satisfy the Courant-Friedrichs-Lewy (CFL) stability criterion (*Courant et al.*, 1928), which constrains the numerical time step Δt through,

$$\Delta t = a \cdot \frac{\Delta x}{\max\{|\mathbf{u}|, V_S, V_A, \sqrt{V_S^2 + V_A^2}\}}, \quad (3.7)$$

where Δx is the size of the numerical cells, $V_S = \sqrt{\Gamma p / \rho}$ is the local sound speed, $V_A = B / \sqrt{4\pi\rho}$ is the local Alfvén speed, and a is constant with $a \in (0, 1)$. This constant can be chosen to provide a faster computation speed by choosing a value closer to 1 or to provide greater numerical accuracy by choosing a smaller value. A typical value of $a = 0.5$ is used. The CFL condition forces the numerical scheme to update faster than information can travel through the system.

3.5.2 Eruptions of LBVs

The eruption event of an LBV-type star modelled in Sect. 3.4 is modelled here using the 3D model described in Sect. 3.5.1. As discussed in the previous section, the 3D model employed here is solved on a Cartesian grid. The grid size used for these simulations corresponds to a simulation volume of 10 pc^3 with 350 grid points in each Cartesian direction, namely $N_x = N_y = N_z = 350$. The stellar parameters assumed here are similar to those in Sect. 3.4, being a stellar wind speed of 500 km.s^{-1} , a mass-loss rate of $1 \times 10^{-6} \text{ M}_\odot/\text{year}$ and a stellar magnetic field strength of $1.5 \mu\text{G}$. Surrounding the astrosphere the ISM is assumed, in the model, to have a density of $10 \text{ particles.cm}^{-3}$ and a magnetic field strength of $3 \mu\text{G}$.

The eruption occurs after 50 000 years, when the mass-loss rate is increased by a factor of 10^2 for a period of 100 years. After 100 years, the mass-loss rate is reduced to the value assumed before the eruption takes place, namely $1 \times 10^{-6} \text{ M}_\odot/\text{year}$.

The density contours, taken in the ecliptic plane, for four different times during the evolution of the astrosphere are shown in Fig. 3.16. The astrosphere can be seen after 50 000 years in the top left panel. With no relative velocity between the star and the ISM the astrosphere forms a relatively spherical shape, due to the spherically symmetric outflow conditions. However, the ISM magnetic field is aligned from the top right of the panels, pointing towards the bottom left corner of the panels. The ISM magnetic field causes the slight deviation from a spherical astrosphere due to the higher pressure from

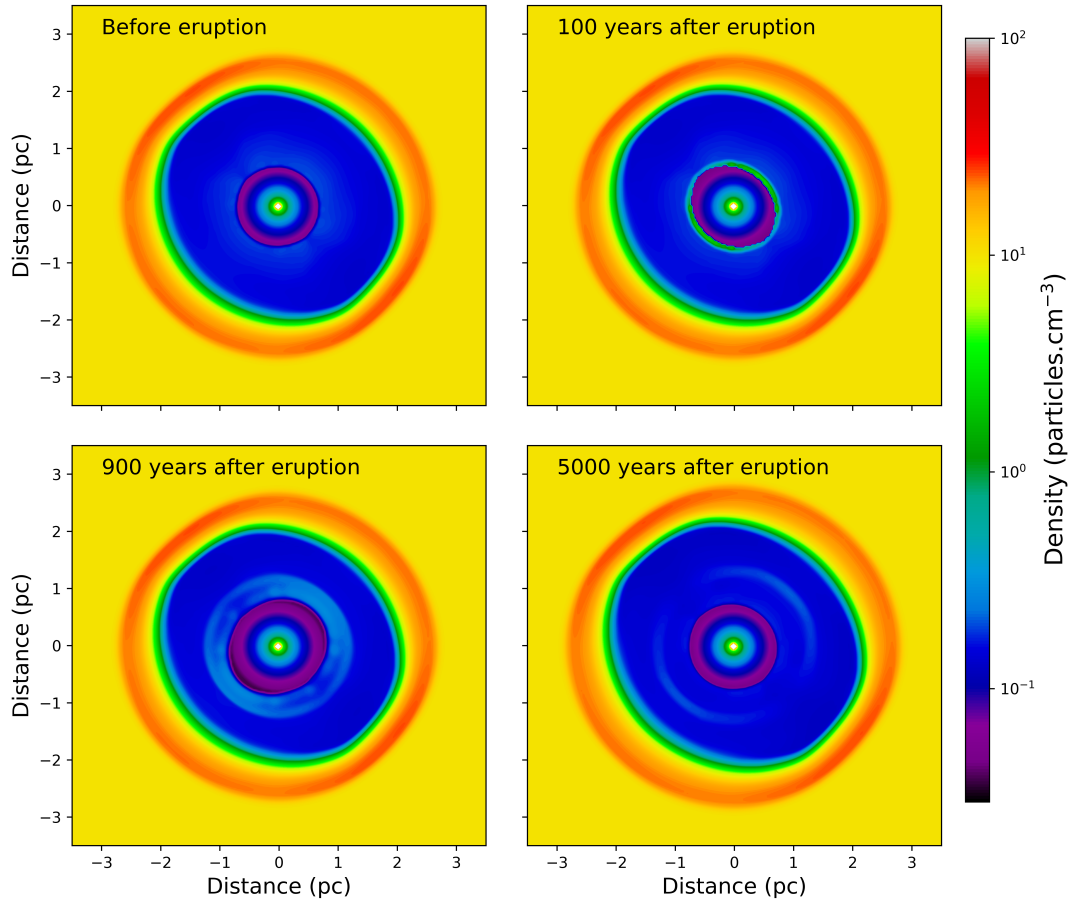


Figure 3.16: Density contours as a function of distance for the computed astrosphere of an LBV-type star undergoing an eruption. Top left panel: 50 000 years after evolution before the eruption occurs. Top right panel: 100 years after the eruption has taken place. Bottom left panel: 900 years after the eruption has taken place. Bottom right panel: 5 000 years after the eruption has taken place.

the ISM. The BS or bow wave can be seen from the red circle around the astrosphere, while the AP is shown by the green line, showing the highest density surrounding the astrosphere. The purple circle in the centre of the panel shows the TS.

Following the eruption, the higher density wave that is ejected from the star can be seen crossing over the TS by the green circle surrounding the purple ring, in the top right panel, 100 years after the eruption has taken place. The rest of the astrosphere is unaffected at this point. The density wave continues travelling past the TS towards the AP, which can be seen as the higher density (light blue) waves in the bottom left panel of Fig. 3.16. Similar to the 2D case, the higher density clumps travel towards the AP and are eventually dissipated within the IAS as can be seen from the bottom right panel, showing the astrosphere another 5 000 years after the eruption has taken place. The eruption that increases the mass-loss rate by a factor of 10^2 has no influence on the size of the astrosphere or the distance from the inner boundary to the TS, AP and BS, which is similar to what was found for the 2D case with similar outflow parameters and parameters chosen for the

eruption.

3.6 Summary and Conclusion

In this chapter the different shock structures of astrospheres were discussed and these were shown by presenting model results of an astrosphere. It was shown that the addition of radiative cooling separates the outer structure of the astrosphere between a cold outer astrosheath (COAS) and a hot outer astrosheath (HOAS), while creating a thin dense shell surrounding the astrosphere, while the astrosphere is cooling. The effect of cooling is especially visible when looking at the compression ratio of the bow shock (BS) as a function of time, Fig. 3.7. Shown here is that with cooling, the maximum compression ratio of the BS increases from 4 to ~ 34 .

Luminous blue variable (LBV) type stars undergo variations in their outflow parameters. These variations have been found to have no effect on the outer structures of the astrospheres of HD 99953 and AG Carinae. The variations are dissipated at the termination shock (TS) in the case of HD 99953 and within the inner astrosheath (IAS) in the case of AG Carinae. The addition of a stellar magnetic field also shows no effect on the structure of the astrosphere, due to the size of the astrospheres in question.

LBV-type stars can also undergo eruptions, during which a large amount of mass is carried away by the stellar wind over a short period of time. For results where ISM and stellar magnetic fields are included, these eruptions were able to increase the size of the astrosphere, as was shown in Fig. 3.15, depending on the strength of the eruption. The inclusion of a stellar magnetic field again showed no effect on the structure of the astrosphere.

In 3D, only the smaller eruption was considered. Here, the mass-loss rate was increased by two orders of magnitude for a period of 100 years. Fig. 3.16 shows that this eruption dissipates within the OAS without affecting the structure and size of the astrosphere.

Chapter 4

Cosmic Ray Modulation

This chapter introduces the stochastic differential equation (SDE) transport model that is utilised to calculate the modulation of galactic cosmic rays (GCRs) in the astrosphere of HD 99953, during an eruptive event of an LBV-type star and Proxima Centauri. Parts of this work have been published in *Light et al.* (2022).

The use of MHD model output as input for a GCR transport code is also discussed. In particular the numerical interpolation of the large scale plasma parameters which are necessary as input for the transport model of GCRs is addressed.

4.1 Galactic Cosmic Ray Modulation Modelling

As discussed in Sect. 2.4, the transport of GCRs is governed by the TPE given in Eq. 2.12, given here again for reference,

$$\frac{\partial f_0}{\partial t} = \nabla \cdot (\mathbf{K} \cdot \nabla f_0) - \mathbf{v}_{sw} \cdot \nabla f_0 + \frac{1}{3} (\nabla \cdot \mathbf{v}_{sw}) \frac{\partial f_0}{\partial \ln p}. \quad (4.1)$$

The ideal would be to solve the TPE in 3D and time-dependently with the various transport mechanisms also modelled time-dependently and in 3D. However, 3D time-dependent modelling of the modulation of GCRs presents unique difficulties (see, e.g., *Engelbrecht*, 2019; *Engelbrecht and Moloto*, 2021). These difficulties include the uncertainties as to the parameters used for the GCR modelling, especially parameters pertaining to the turbulence quantities within astrospheres other than the heliosphere. These turbulence quantities directly influence the cosmic ray diffusion and drift coefficients (*Matthaeus and Velli*, 2011; *Oughton and Engelbrecht*, 2021; *Engelbrecht et al.*, 2022a).

As a first approach, the approach taken in this work will solve the 1D TPE, which is given by,

$$\frac{\partial f}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \kappa_{rr} \frac{\partial f}{\partial r}) + \frac{1}{3r^2} \frac{\partial}{\partial r} (r^2 V_{sw}) \frac{1}{p^2} \frac{\partial}{\partial p} (p^3 f) - \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 V_{sw} f), \quad (4.2)$$

which incorporates the adiabatic energy changes in a more physical manner (*Caballero-Lopez and Moraal, 2004*) compared to the more analytic Force-Field (FF) approach (see *Gleeson and Axford, 1968*). The FF approach uses a force-field potential, which is a single measure that contains information regarding the effective diffusion coefficient, stellar wind speed, and modulation volume which can not be retrieved, while multiple combinations of these parameters are able to give the same potential making the results not unique (*Engelbrecht and Di Felice, 2020*). The advantage of solving the TPE over the more analytic methods is that it is able to incorporate more theoretically defensible expressions for the diffusion coefficient, while being able to model the complicated interplay between diffusion coefficients, stellar wind and modulation volume (*Engelbrecht and Di Felice, 2020*). For a more detailed discussion see *Caballero-Lopez et al. (2019)* and *Engelbrecht and Di Felice (2020)*.

4.2 Stochastic Differential Equation

The transport of non-thermal particles through turbulent plasmas, such as the modulation of cosmic rays within an astrosphere, can be studied by solving a Fokker-Planck type diffusion equation. These equations can be solved using Stochastic Differential Equations (SDEs). See *Strauss and Effenberger (2017)* for a review on SDEs. A brief discussion will also be given here. SDEs can be defined as equations that can be rewritten into a general form

$$\frac{dx(t)}{dt} = a(x, t) + b(x, t)\zeta(t), \quad (4.3)$$

where $a(x, t)$ is the deterministic part of the SDE while $b(x, t)\zeta(t)$ is the diffusive or random part. The diffusive part is composed of a vector field $b(x, t)$ coupled with Gaussian white noise or randomness $\zeta(t)$. In this work, only SDEs of the Itô type are considered. An Itô type of SDE is an SDE which is solvable using Itô calculus. Itô calculus extends the methods of calculus to be able to solve stochastic or random processes. For a mathematical description applicable to these type of SDEs, see e.g., *Zhang (1999)*. Considering only Itô type of SDEs, Eq. 4.3 can be rewritten as

$$dx(t) = a(x, t)dt + b(x, t)dW(t), \quad (4.4)$$

with $W(t)$ being a Wiener process. A Wiener process is a process where the displacements are random but Markovian. It has a Gaussian distribution with a mean value of 0. See again e.g., *Zhang (1999)*. This equation is usually integrated numerically using the Euler-Maruyama numerical scheme (*Maruyama, 1955*); with a finite time step, Δt . Eq. 4.4 can then be solved iteratively,

$$x(t + \Delta t) = x(t) + a(x, t)\Delta t + b(x, t)\Delta W(\Delta t), \quad (4.5)$$

with an initial position defined by $x = x_0$ at $t = 0$ up to a boundary defined by $x = x_b$ at $t = t_b$. This boundary can also be a temporal integration limit at $t = T$. The Wiener

process can also be discretized as

$$\Delta W(\Delta t) = \sqrt{\Delta t} \cdot \Lambda(t), \quad (4.6)$$

where $\Lambda(t)$ is a Gaussian-distributed pseudo-random number between 0 and 1. Each Wiener process is then composed of a varying amount of Rayleigh flights. The trajectory formed by the temporal evolution of the spatial component through phase space is generally known as the trajectory of a pseudo-particle. In this case, where charged particle propagation is considered, *Strauss and Effenberger* (2017) showed that the pseudo-particle actually represents the temporal evolution of an ensemble of real particles. An ensemble of CR particles is constructed in such a way that it is a gyro- and isotropic collection of particles. This can rather be seen as a phase space density element (*Strauss and Effenberger*, 2017).

Eq. 4.5 needs to be integrated over a large number of possible trajectories to have any significance. A generalised description of Eq. 4.4 can be given by an n -dimensional set of SDEs,

$$dx_i = a_i(x_i, s)ds + \sum_{j=1}^n b_{ij}(x_i, s)dW_j(s), \quad (4.7)$$

where $\mathbf{a}$ is an n -dimensional vector and $\mathbf{b}$ is a $n \times n$ matrix. This system of SDEs can be integrated forward or backward in time. Eq. 4.7 can be equivalent to two different Fokker-Planck equations depending on whether time forward or time backwards integration is used. In the case of charged particle transportation, the time forward integration approach can be described as tracing the pseudo-particle trajectories from the source towards the direction of some observer. There is no guarantee, however, that all or even any of the pseudo particles may reach the observer point (*Strauss and Effenberger*, 2017).

A time-backward approach is therefore mostly used in astrophysical applications. This approach starts from an observational point and pseudo-particles are traced backwards in time until a model boundary is reached. The time backward approach guarantees that the pseudo-particles reach the observational point, since they start there. The time backward Fokker-Planck equivalent to Eq. 4.4 is given by

$$-\frac{\partial \rho(x_i, s)}{\partial s} = \sum_{i=1}^n a_i(x_i, s) \frac{\partial \rho(x_i, s)}{\partial x_i} + \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n C_{ij}(x_i, s) \frac{\partial^2 \rho(x_i, s)}{\partial x_i \partial x_j}, \quad (4.8)$$

where $\rho(x_i, s)$ is a conditional probability density. A new time-marching coordinate is also introduced, s , which for the time backward integration is defined by $s = T - t$. The diffusion tensor C_{ij} is given by

$$C_{ij}(x_i, s) \equiv (b_{ij}(x_i, s) \cdot b_{ji}(x_i, s)). \quad (4.9)$$

The 1D TPE, given by Eq. 4.2, is solved in this study using the 1D solver employed by *Engelbrecht and Di Felice* (2020) as applied in *Light et al.* (2022). The TPE can be written

as a set of Itô stochastic differential equations, as mentioned above (see *Zhang, 1999; Pei et al., 2010; Strauss and Effenberger, 2017; Moloto et al., 2019*, for more detail) given by,

$$dr = \left(\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \kappa_{rr}) - V_{sw} \right) dt + \sqrt{2\kappa_{rr}} dW_r, \quad (4.10)$$

$$dp = \frac{p}{3r^2} \frac{\partial}{\partial r} (r^2 V_{sw}) dt, \quad (4.11)$$

where $dW_r = \eta(t)\sqrt{dt}$ is a Wiener process, with η being a Gaussian-distributed pseudo-random number between 0 and 1. This pseudo-random number is generated, in this work, using the Mersenne-Twister algorithm (see *Matsumoto and Nishimura, 1998*). The 1D TPE, Eq. 4.2, is solved as described by *Engelbrecht and Burger (2015a)* and *Moloto et al. (2019)*, in an iterative, time-backward manner, using the Euler-Maruyama scheme, as discussed in Sect. 4.2. It should be noted that the signs of stellar wind velocity, V_{sw} in Eq. 4.10, and the energy change, dp in Eq. 4.11, are chosen as such to explicitly show the time-backwards approach taken to solve Eq. 4.2.

In this work, $N = 10\,000$ pseudo-particles per considered energy bin is assumed. These particles represent trajectories in phase space and are released at the specified inner boundary and traced backwards in time towards the outer boundary. For each Rayleigh flight, the transport coefficients that enter into Eq. 4.2 need to be calculated. The inner boundary of the SDE model is set to be similar to the inner boundary of the MHD model, at 0.032 pc, while the outer boundary is set to be located past the AP of the astrosphere, obtained from the MHD model output. Once the pseudo-particles reach the outer boundary, the trajectories are weighted using a source spectrum, the local interstellar spectrum (LIS), according to their energies and also the number of pseudo-particle trajectories. The weighting is done using (e.g., *Engelbrecht and Burger, 2015a*)

$$j_T = \frac{1}{N} \sum_{i=1}^N j_{lis}^T(E_i^{exit}), \quad (4.12)$$

with summation over i pseudo-particle trajectories. It should be noted that if a pseudo-particle trajectory reaches the inner boundary they are simply discarded and rereleased at the inner boundary to maintain the same statistics. The propagation or residence time, which is the time it takes for a pseudo-particle to travel from the inner boundary to the LIS, for the time-backward approach, can be directly calculated from the numerical model. The average residence time per pseudo-particle for each energy bin is calculated by adding the time of each Rayleigh flight for each pseudo-particle and averaging over all the pseudo-particles in each respective energy bin.

4.2.1 3D SDE Transport equation

The TPE given in Eq. 4.1 can be written for a fully 3D case where drift and a full 3D $\mathbf{K}$ diffusion tensor including cross terms as (see *Pei et al.*, 2010),

$$\begin{aligned} \frac{\partial f}{\partial t} = & \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \kappa_{rr} \frac{\partial f}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\kappa_{\theta\theta} \sin \theta \frac{\partial f}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial}{\partial \phi} \left(\kappa_{\phi\phi} \frac{\partial f}{\partial \phi} \right) \\ & + \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi} \left(\kappa_{r\phi} \frac{\partial f}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial r} \left(r \kappa_{r\phi} \frac{\partial f}{\partial \phi} \right) + \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 V_{sw}) \frac{\partial}{\partial p^3} (p^3 f) \\ & - \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 V_{sw} f) - \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 V_{dr} f) - \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta V_{d\theta} f) - \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi} (V_{d\phi} f), \end{aligned} \quad (4.13)$$

where, κ_{ij} refers to an element of the diffusion tensor with $i, j \in [r, \theta, \phi]$, V_{sw} the stellar wind velocity and V_{di} the drift velocity with $i \in [r, \theta, \phi]$. The diffusion tensor can be transformed from HMF aligned coordinates into spherical coordinates, as is shown in *Burger et al.* (2008).

Using the same method as described above the TPE can be written as a set of Itô type SDE equations. Different to the Itô SDE equations considered for the 1D case, here these equations are dependent on each spherical coordinate, r, θ, ϕ , and energy, E . Similar to the 1D case though, dW_i still satisfies a Weiner process. Assuming a generalised 3D HMF, *Pei et al.* (2010) and *Engelbrecht and Burger* (2015a) showed that the components of $\mathbf{B}_{i,j}$ are given by,

$$\begin{aligned} B_{1,1} &= \frac{\sqrt{2 \left(\kappa_{\phi\phi} \kappa_{r\theta}^2 - 2 \kappa_{r\phi} \kappa_{r\theta} \kappa_{\theta\phi} + \kappa_{rr} \kappa_{\theta\phi}^2 + \kappa_{\theta\theta} \kappa_{r\phi}^2 - \kappa_{rr} \kappa_{\theta\theta} \kappa_{\phi\phi} \right)}}{\sqrt{\kappa_{\theta\phi}^2 - \kappa_{\theta\theta} \kappa_{\phi\phi}}}, \\ B_{1,2} &= \frac{\kappa_{r\phi} \kappa_{\theta\phi} - \kappa_{r\theta} \kappa_{\phi\phi}}{\kappa_{\theta\phi}^2 - \kappa_{\theta\theta} \kappa_{\phi\phi}} \sqrt{2 \left(\kappa_{\theta\theta} - \frac{\kappa_{\theta\phi}^2}{\kappa_{\phi\phi}} \right)}, \\ B_{1,3} &= \frac{\sqrt{2} \kappa_{r\phi}}{\sqrt{\kappa_{\phi\phi}}}, \\ B_{2,2} &= \frac{\sqrt{2 \left(\kappa_{\theta\theta} - \kappa_{\theta\phi}^2 / \kappa_{\phi\phi} \right)}}{r}, \\ B_{2,3} &= \frac{\kappa_{\theta\phi}}{r} \sqrt{\frac{2}{\kappa_{\phi\phi}}}, \\ B_{3,3} &= \frac{\sqrt{2 \kappa_{\phi\phi}}}{r \sin \theta}, \\ B_{2,1} &= B_{3,1} = B_{3,2} = 0, \end{aligned} \quad (4.14)$$

where κ refers to the elements of the diffusion tensor $\mathbf{K}$ in the TPE, Eq. 4.13. The

components of the vector $\mathbf{A}$ are given as

$$\begin{aligned}
A_r &= \frac{1}{r^2} \frac{\partial}{\partial r} r^2 \kappa_{rr} + \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi} \kappa_{r\phi} + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} \kappa_{r\theta} \sin \theta - V_{sw} - V_{d,r}, \\
A_\theta &= \frac{1}{r^2} \frac{\partial}{\partial r} r^2 \kappa_{r\theta} + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \kappa_{\theta\theta} \sin \theta + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \phi} \kappa_{\theta\phi} - \frac{V_{d,\theta}}{r}, \\
A_\phi &= \frac{1}{r^2 \sin^2 \theta} \frac{\partial}{\partial \phi} \kappa_{\phi\phi} + \frac{1}{r^2 \sin^2 \theta} \frac{\partial}{\partial r} r \kappa_{\phi\phi} + \frac{1}{r^2 \sin^2 \theta} \frac{\partial}{\partial \theta} \kappa_{\theta\phi} - \frac{V_{d,\phi}}{r \sin \theta}, \\
A_E &= \frac{1}{3r^2} \frac{\partial}{\partial r} (r^2 V_{sw}) \frac{(E + 2E_0)}{E + E_0} E.
\end{aligned} \tag{4.15}$$

Here E_0 represents the rest-mass energy of the GCRs. It should be noted that similar to the 1D case, the signs of the stellar wind velocity, V_{sw} , the drift velocity, $V_{d,i}$, and of the energy term, A_E , are the opposite to those which are reported in *Pei et al.* (2010). This is to show the explicit nature of the time-backward approach that is taken in this work to solve the SDE equations from the point of observation to the boundary, where the LIS is assumed. A time-backward approach is utilised as it is computationally more efficient (*Strauss et al.*, 2011). The 3D SDE model is employed in the next chapter.

4.3 Galactic Cosmic Ray Modulation within Astrospheres

Using an SDE method to solve the 1D TPE, the modulation of cosmic rays within an astrosphere can be modelled. The modulation of cosmic rays within the heliosphere has been studied and modelled before using a model as described above (see, e.g., *Engelbrecht and Burger*, 2015a; *Moloto and Engelbrecht*, 2020). This work is different from the work done on the heliosphere in that it will focus on the modulation of GCRs within astrospheres of other stars, here in particular the astrospheres of LBV-type stars. The difficulty in studying the modulation of GCRs within these astrospheres is that very little of what is known for heliospheric modulation of GCRs is available for other astrospheres.

An approach to solve this problem is to use the output from an MHD model as input for the SDE modulation model. The astrospheres of stars can be modelled using an MHD model as is discussed in the previous Chapter. The MHD model can provide details of the distances to the TS, AP and BS while also providing a spatial profile of the density, velocity and stellar magnetic field of the astrosphere. These properties can be used as base input in the SDE GCR model while the parameters that are still unknown can be scaled based on heliospheric observations.

4.3.1 Model

As a preliminary study to calculate GCR modulation in astrospheres, for example the astrospheres of LBV-type stars, a 1D TPE is used as discussed above. Following *Herbst*

et al. (2020), the mean free path is assumed to be

$$\lambda = \lambda_0 \left(\frac{B_0}{B} \right) \left(\frac{P}{P_0} \right)^{2/3}, \quad (4.16)$$

with P the particle rigidity, and $P_0 = 1$ GV. The astrospheric magnetic field (AMF) magnitude, B_0 , is assumed at the inner boundary at 0.032 pc (the inner boundary of the model), while B is the astrospheric magnetic field at the particle position. The mean free path is normalised to a value $\lambda_0 = ar_1/r(1au)$, as is done in *Light et al.* (2022), where $r_1 = 6892.2AU$ and a has values of 0.01 or 0.02 based on typical heliospherical values for the mean free path (e.g., *Ferreira and Potgieter*, 2003). These values are also well within the range for heliospheric values for this quantity as reported in other studies (see, e.g., *Engelbrecht et al.*, 2022a, and references therein). The mean free path shown in Eq. 4.16 is only assumed within the astrosphere. Once the particle leaves the astrosphere, a radial mean free path of $\lambda_r = 1$ pc is chosen and also assumed to be spatially constant. This allows no modulation to occur outside of the astrosphere.

The rigidity dependence of the mean free path in Eq. 4.16, is chosen to be similar to the rigidity dependence of the perpendicular mean free path at high energies using the nonlinear guiding center (NLGC) theory (see *Matthaeus et al.*, 2003) and assuming a quasilinear theory (QLT) parallel mean free path (*Jokipii*, 1966). Heliospheric modulation using NLGC-derived expressions for the perpendicular MFP have yielded GCR intensities in good agreement with observations (e.g., *Moloto et al.*, 2018; *Moloto and Engelbrecht*, 2020). For a more detailed discussion, see, e.g., *Shalchi et al.* (2004); *Burger et al.* (2008) and *Moloto and Engelbrecht* (2020). Compared to what is assumed, there exist more theoretically and physically motivated expressions to model the cosmic ray diffusion coefficients (see, e.g., *Shalchi et al.*, 2010; *Strauss et al.*, 2016; *Gammon and Shalchi*, 2017; *Dempers and Engelbrecht*, 2020). These have been applied to various degrees to heliospheric cosmic ray modulation studies (see *Oughton and Engelbrecht*, 2021, and references within). The use of Eq 4.16 in this work is motivated by the uncertainties in the turbulence quantities and their behaviour associated with the astrospheres in question (see, e.g., *Gallegos-Garcia et al.*, 2020).

The LIS used in this work is the one employed by *Strauss et al.* (2011), following the approach used by *Herbst et al.* (2020). This LIS, which describes the source spectrum for galactic protons is given by

$$j_{LIS} = \begin{cases} e^{(4.64 - 0.08 \log(T)^2 - 2.91 \sqrt{T})}, & \text{if } T < 1 \\ e^{(3.22 - 2.86 \log(T) - 1.5/T)}, & \text{if } T \geq 1 \end{cases} \quad (4.17)$$

in units of $\text{particles.m}^2.\text{s}^{-1}.\text{sr}^{-1}.\text{MeV}^{-1}$, with T being the particle kinetic energy. This LIS is used as a first estimate of the boundary GCR intensities, although the GCR intensities may change throughout the galaxy (see *Amato and Blasi*, 2018), and this spectrum is only applicable to the region of interstellar space surrounding the heliosphere, as is noted by *Herbst et al.* (2020).

Although the radial component of Eq. 4.10 is calculated using a pseudo-random number between 0 and 1 and thus inherently stochastic, the transport parameters, given by Eq. 4.2, may need to be calculated at locations that do not correspond to grid points from the MHD simulations. Numerical interpolation is then required between MHD simulation grid points. This is discussed next.

4.3.2 Numerical Interpolation

As mentioned in the previous section the SDE method of solving for the modulation of GCRs does not use a grid but rather a diffusive process. The data required from the output of an MHD-model are the magnitude of the magnetic field, $|\mathbf{B}|$, and the magnitude of the stellar wind speed, $|\mathbf{V}_{sw}|$. These two values are used as input for the SDE model and numerical interpolation are required between spatial points to calculate the values of these variables as the pseudo-particles complete their diffusion from the starting boundary to the outer boundary. To test the interpolation scheme to be used in the SDE model, values for the magnitude of $\mathbf{B}$ and $\mathbf{V}_{sw}$ for the astrosphere of Proxima Centauri (Proxima), modelled using the CRONOS code (*Herbst et al.*, 2020), are interpolated in this section.

Figure 4.1 shows the magnitude of the magnetic field in nanoTesla, (nT), in the top panel, and the stellar wind speed in units of km.s^{-1} in the bottom panel. Both are plotted as a function of radial distance in AU. The difference between radial points for the modelled astrosphere of Proxima is $\Delta r = 0.47$ AU. The magnitudes of $|\mathbf{B}|$ and $|\mathbf{V}|$ as obtained from the output of the MHD model are shown by the blue crosses. A linear interpolation was done by calculating the slope between each radial point using $m = (y_2 - y_1)/(x_2 - x_1)$. A random point was then found between each set of radial distance points, which would correspond to the position of the pseudo-particle, for each Rayleigh flight, and the slope corresponding to the two radial points on either side of the pseudo-particle position was used to calculate the value for $|\mathbf{B}|$ and $|\mathbf{V}|$ respectively, using

$$|\mathbf{B}| = (r - r_1)m + |\mathbf{B}_1|,$$

where r_1 is the radial point on the MHD grid before the radial point of the pseudo-particle, which is located at r , $|\mathbf{B}_1|$ is the magnitude of the magnetic field at r_1 and $|\mathbf{B}|$ the magnitude of the magnetic field at the pseudo-particle position. The same is done for the stellar wind velocity.

A similar method was used to test a power law interpolation for the magnetic field. The power law interpolation is done by simply taking the logarithm of the data before calculating the slopes and values at the radial position of the pseudo-particle. The linear (solid red) and power law (dashed green) interpolations are shown in Fig. 4.1. Both the linear and power law interpolation with regard to the magnetic field fit the data well when the radial difference between points is $\Delta r = 0.47$ AU. The linear interpolation also shows a good fit to the speed in the bottom panel of Fig. 4.1.

To test the accuracy of the interpolation, the distance between the radial points is in-

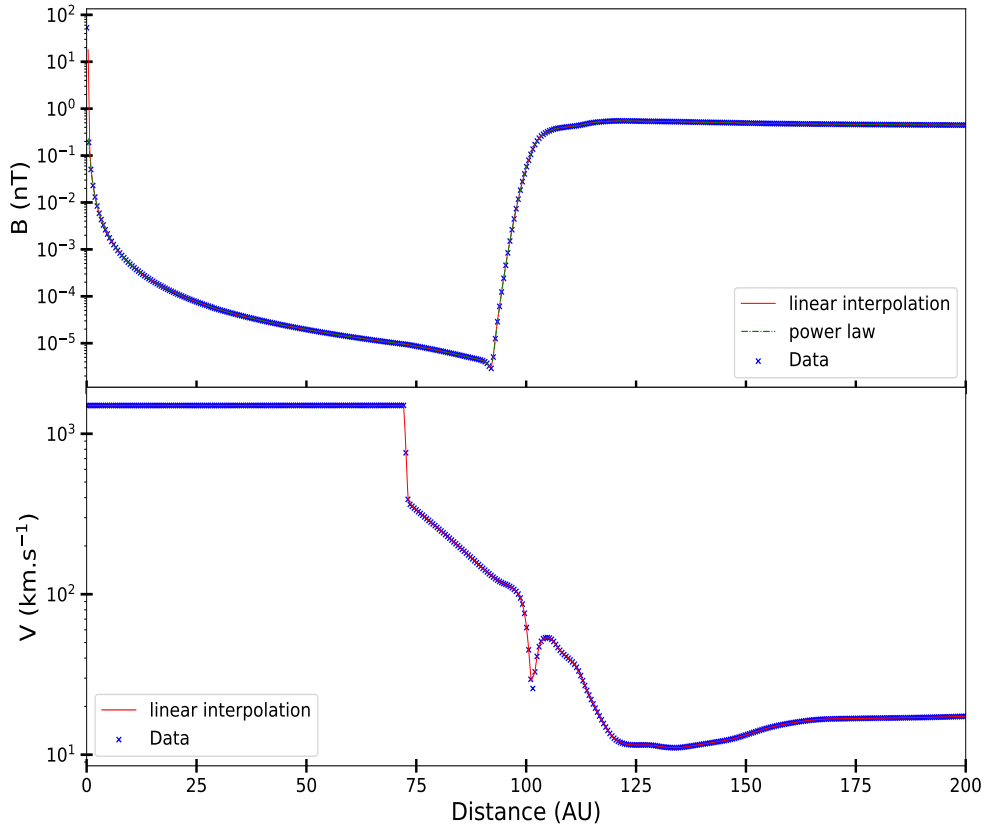


Figure 4.1: Radial profiles of the magnetic field (blue crosses). Linear (red) and power law (green dashed) interpolation of the magnetic field, and linear interpolation (red) of the magnitude of the stellar wind speed. The difference between radial points is $\Delta r = 0.47$ AU.

creased. For Figure 4.2 the same interpolation methods were used to the same data set but now every second data point has been removed. This increases the radial distance between the values in the data set to $\Delta r = 0.94$ AU, thus increasing the distance over which the interpolation needs to be performed. Fig. 4.2 shows that the linear interpolation and power law interpolation still fit the data well for the magnitude of the magnetic field, while the linear interpolation still fits the data well for the stellar wind speed. To further test the limits of the interpolation schemes and their accuracy, every second point is removed from the data set once more. This reduces the data set for Proxima to a quarter of the original size and increases the radial distance between points from $\Delta r = 0.47$ AU to $\Delta r = 1.88$ AU. This is shown in Fig. 4.3. When the radial distance between the points gets too large the interpolation becomes less accurate, especially in regions where there are large increases/decreases and sharp changes in values, as those that occur around ~ 100 AU. Both interpolation methods show deviations from the data when there are such large radial differences, especially where the distance between steep changes in the data occur, at ~ 90 AU. The linear interpolation for the magnitude of the stellar wind speed, bottom panel Fig. 4.3, is in good agreement with the data, except now where there

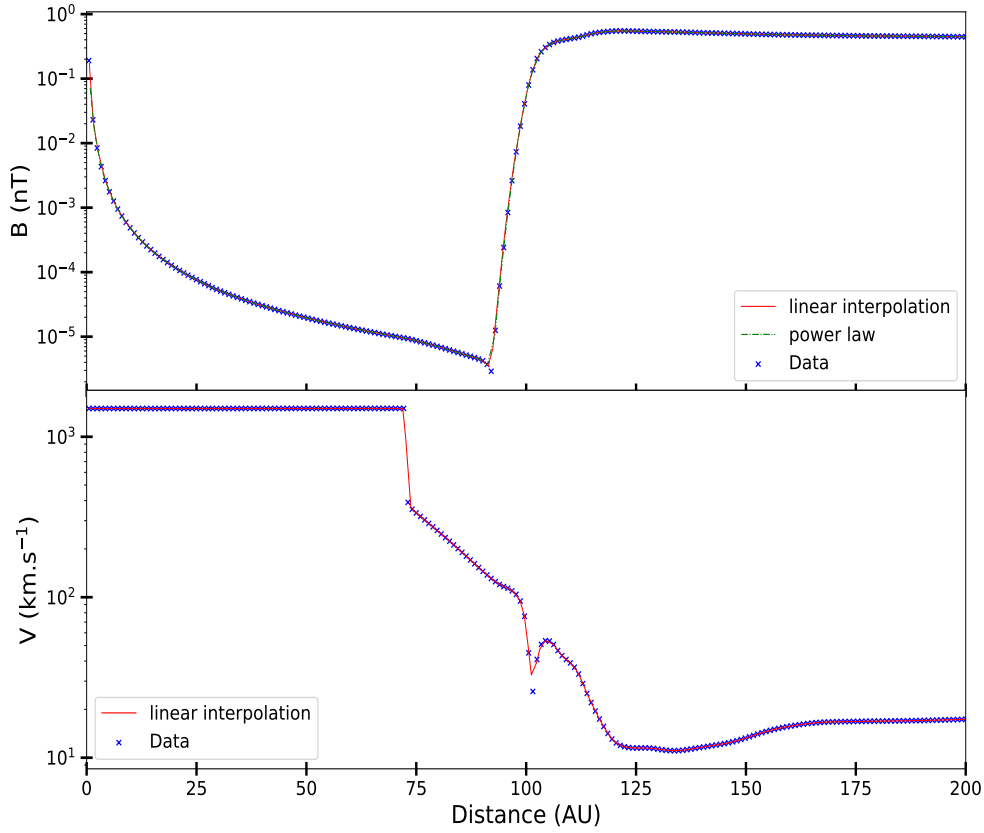


Figure 4.2: Similar to fig. 4.1; however, every second point has been removed, increasing the radial distance between points to $\Delta r = 0.94$ AU.

are sharp changes in the values. A sharp change occurs at ~ 100 AU, where it is shown that the interpolation scheme does not reach the low values seen in the data. Therefore, a very coarse grid being used in the MHD model will lead to a less accurate interpolation resulting in a less accurate result from the SDE model.

4.3.2.1 3D MHD interpolation

The 3D MHD model discussed in the previous chapter simulates the astrospheres on a Cartesian grid. In order to use the output from the MHD as input for the SDE, which is written in spherical coordinates, it is a lot simpler if the Cartesian MHD data are transformed into spherical coordinates as the SDE solves along a radial spoke outwards from an inner boundary towards an outer boundary. The transformation from Cartesian to Spherical data is done by using a trilinear interpolation method. Simply put, trilinear interpolation is a linear interpolation method applied in three steps along each of the Cartesian coordinates in order to determine the value at a point located at the center of a cube with vertices located at $x(i)$, $x(i+1)$, $y(i)$, $y(i+1)$, $z(i)$ and $z(i+1)$, with i the nearest

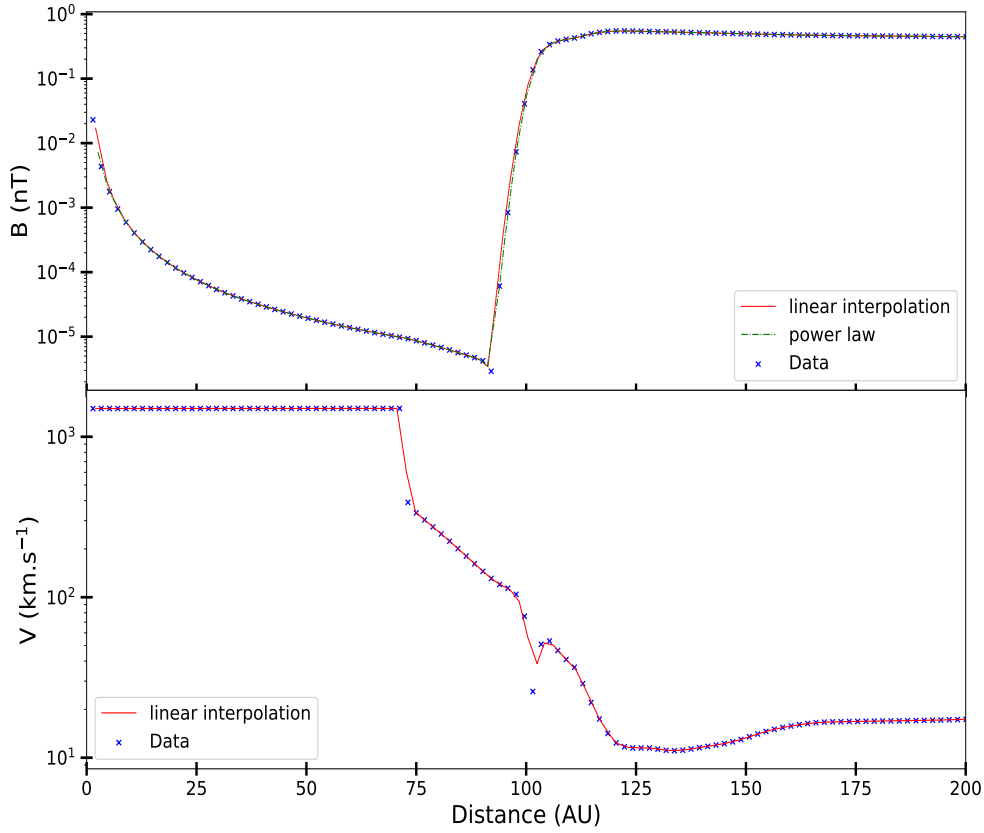


Figure 4.3: Similar to fig. 4.2 but every second point is removed again. This increases the radial distance between the data points to $\Delta r = 1.88$ AU.

radial point from the where the transportation parameters need to be calculated. Note that the radial point is between the inner boundary and the current position. To obtain the MHD output necessary for the 1D SDE model, the 3D MHD output is trilinearly interpolated with a constant meridional and equatorial angle.

It is also possible to transform the 3D Cartesian results into 3D Spherical results by specifying the amount of radial, meridional and equatorial points. The top-left panel of Fig. 4.4 shows the density of a simulated astrosphere of Proxima using a 3D MHD model (see Sect. 3.5.1) that solves on a Cartesian grid with $350 \times 350 \times 350$ cells contained within the grid. The other three contours are the results of trilinear interpolations from the Cartesian to a Spherical coordinate system with varying amounts of grid points used for the interpolation. The top-right panel has 99 radial points, 32 meridian and 44 equatorial points that were used for the trilinear interpolation. The use of too few points creates a loss in some of the quality and features of the astrosphere. It can be seen here that the TS (black ring) in the Cartesian result is mostly spherical while the lowest resolution of spherical coordinates has an ellipsoid shape for the TS. The AP also does not line up at the transition region from 360° to 0° . Using a finer grid as those shown in the bottom

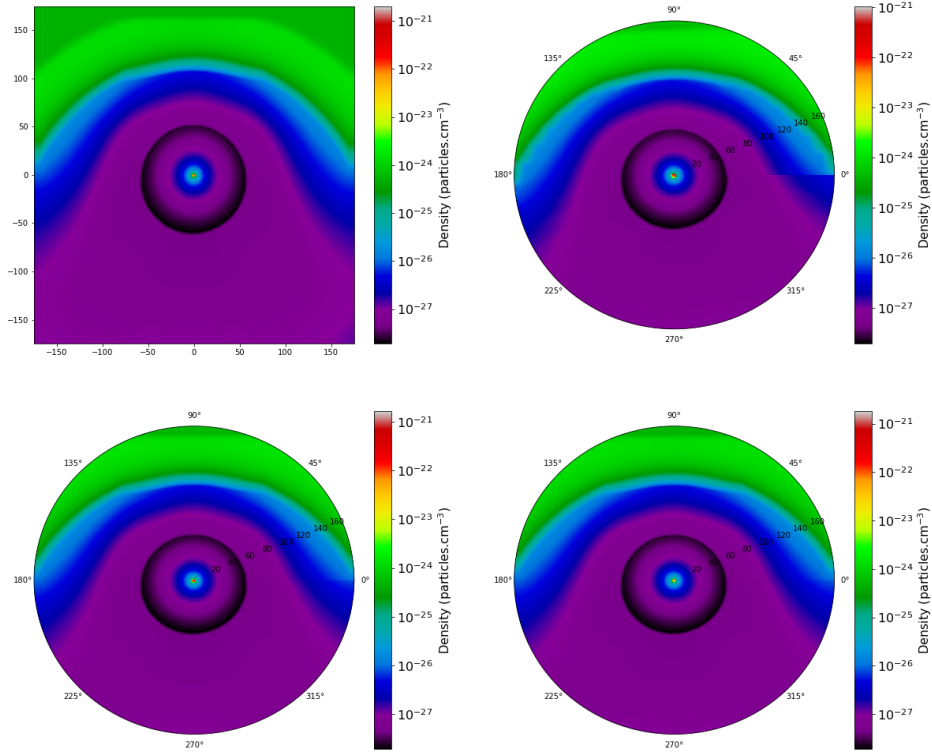


Figure 4.4: Density contour plots in the equatorial plane. Top left panel: Cartesian MHD data with grid size of $350 \times 350 \times 350$. The other three panels show contour plots of spherical data obtained from trilinear interpolation of Cartesian coordinates. Top right panel grid size: $99 \times 32 \times 44$, bottom left panel: $174 \times 90 \times 180$ and bottom right panel: $349 \times 180 \times 360$.

panels, where the resolution of the bottom-left is given by $174 \, r \times 90 \, \theta \times 180 \, \phi$ and the bottom-right by $349 \, r \times 180 \, \theta \times 360 \, \phi$, the amount of deformation that occurs when transitioning from Cartesian to Spherical coordinates has been reduced. The bottom-left panel still shows somewhat of an ellipsoid-shaped TS, but it is more spherical compared to the lowest spherical resolution contour, while the bottom-right panel shows the most spherical TS and with the highest resolution, it also shows the smoothest structures with regards to the TS, AP and BS. These interpolations are shown to highlight the possibility of using Cartesian MHD results as input even though the modulation codes that are discussed in the following chapters use Spherical coordinates.

4.4 Modulation within HD99953

Modulation of GCRs by astrospheres has been done before in 1D (see *Van der Schyff*, 2016; *Herbst et al.*, 2020; *Rodgers-Lee et al.*, 2021; *Mesquita et al.*, 2021, 2022a,b). These prior studies focused on modulation within astrospheres of O-type stars and astrospheres hosting Earth-like exoplanets. This study differs from those works in that for the first time, the astrospheres of Luminous Blue Variable (LBV) type stars will be considered and the modulation of GCRs within their astrospheres calculated. These stars are considered due to the variation of their stellar outflow parameters. The astrospheres considered in this work are those of HD 99953, and of an LBV type star which is undergoing an eruptive event (see Sect. 4.5). HD 99953 was chosen as it features a well-evolved astrospheric structure, featuring an inner astrosphere, astrosheath, cold outer astrosheath and hot outer astrosheath. This work was published in *Light et al.* (2022). The astrosphere of HD 99953 was modelled using the MHD model discussed in the previous chapter of this work. The MHD model used to model the astrosphere in this case is a 2D model; however, results are only shown in 1D (see Sect. 3.3.1), and the SDE modulation model used here is also 1D as described in Sect. 4.3.1. As a first approach to the modulation of GCRs within the astrosphere of HD 99953, a model used to calculate the modulation of GCRs within the heliosphere is used. The approach taken to solve the diffusion coefficients is similar to the approach taken by *Herbst et al.* (2020) as described in Sect. 4.3.1.

4.4.1 Changes from Heliospheric model

The transport model used here is the same as discussed in *Engelbrecht and Di Felice* (2020) for heliospheric conditions. Adapting this model from calculating modulation within the heliosphere to astrospheres, and using the MHD model results as input for the modulation code, requires some changes that need to be made.

4.4.1.1 Stellar wind

The transport model as discussed in *Engelbrecht and Di Felice* (2020) and used to calculate cosmic ray intensities in the heliosphere assumes a constant radial solar wind. The assumption of a radially constant solar wind reduces the derivative of the energy term in Eq. 4.11 to $dp = (2p/3r)V_{sw}dt$, where there is no dependence on the change of the solar wind with radial distance. For the case of the astrosphere, an MHD model was used, which produces a radial stellar wind profile which is used as an input for the transport code. The MHD model produces a calculated stellar wind which is changing as function of radial distance and this changes its divergence. This changes Eq. 4.11 to

$$dp = \frac{2p}{3r}V_{sw}dt + \frac{p}{3}\frac{\partial V_{sw}}{\partial r}dt, \quad (4.18)$$

which includes an additional second term.

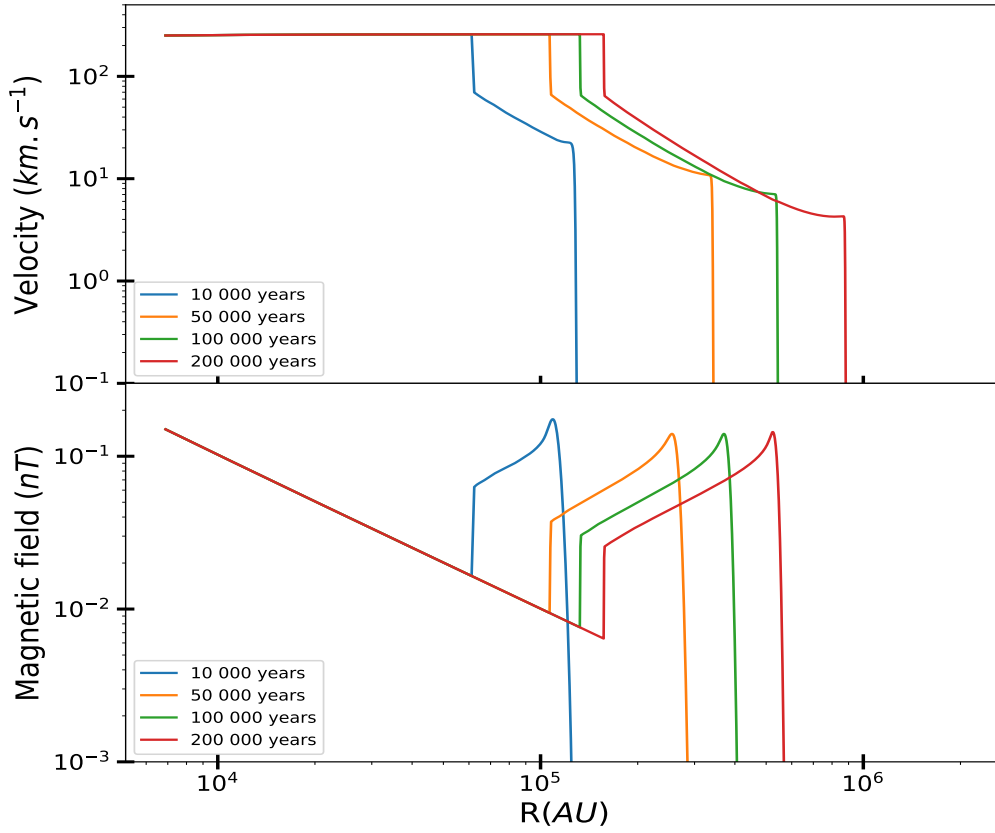


Figure 4.5: Computed radial profiles for HD 99953. The radial velocity profile is shown in the top panel while the magnitude of the magnetic field is shown in the bottom panel. The radial profiles are shown on a log-log scale.

The computed stellar wind velocity profiles as a function of radial distance from the MHD simulations are shown in the top panel of Fig. 4.5. The stellar wind velocity profiles are shown for four different simulation times during the evolution and expansion of the astrosphere, namely 10 000 years (blue), 50 000 years (orange), 100 000 years (green) and 200 000 years (red). Each case shows there is no change in the stellar wind velocity from the inner boundary to the TS; at the TS however there is a sharp decrease in the stellar wind velocity. Within the IAS, past the TS, the stellar wind has been shocked to subsonic velocities and within this region the change in the stellar wind velocity can be brought into effect with the modulation of GCRs.

4.4.1.2 Stellar magnetic field

Following a similar approach as for the stellar wind, the magnitude of the magnetic field as a function of radial distance from the MHD output can be used instead of assuming a constant or a simple Parker magnetic field (*Parker, 1958*). The stellar magnetic field

radial profile is shown in the bottom panel of Fig. 4.5, for four different times in the evolution of the astrosphere. Similar to the stellar wind velocity, the stellar magnetic field follows a simple $1/r$ dependence within the TS, but at the TS there is a sharp increase in the magnitude of the magnetic field and an increase in the magnetic field strength within the IAS towards the AP.

4.4.1.3 Mean Free Paths

The MFP described by Eq. 4.16 is slightly different to the MFPs employed by *Herbst et al.* (2020), even though it follows a similar approach. The radial MFP for the astrosphere of

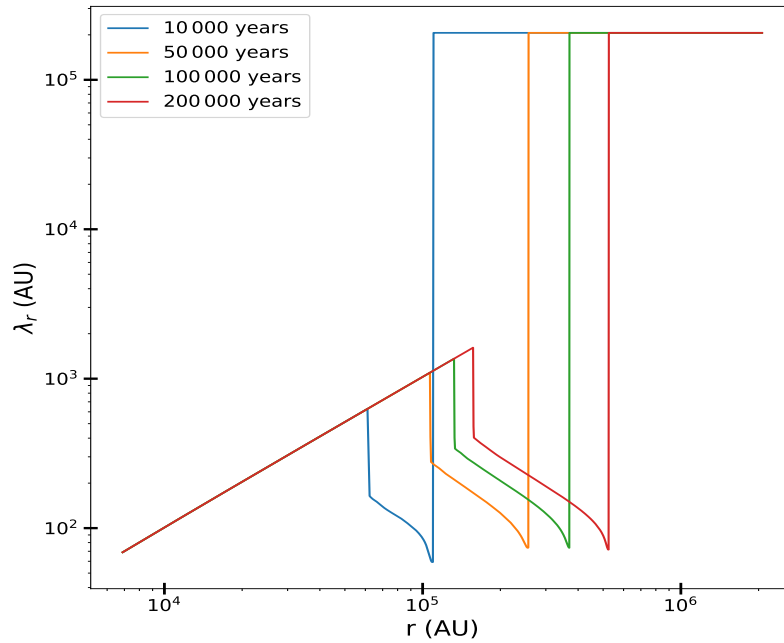


Figure 4.6: Radial mean free path for 1 GV GCR proton in the astrosphere of HD 99953 at various stages during the astrospheric evolution as a function of radial distance. The MFP for 10 000 years (blue), 50 000 years (orange), 100 000 years (green) and 200 000 years (red) are shown.

HD 99953 as a function of radial distance is shown in Fig. 4.6. The $\sim 1/B$ dependence is visible as the MFP has a steadily increasing value from the inner boundary to the TS, where the MFP drastically decreases. Once the particle reaches the AP, it is no longer considered to be within the astrosphere, and a MFP value of ~ 1 pc is assumed. This can be seen by the sudden increase in the size of the MFP. Fig. 4.6 also shows the MFP for various evolutionary times of the astrosphere. This shows a time dependence for the MFP, based on the size of the astrosphere. After 10 000 years (blue), the astrosphere is still expanding and the MFP shows a steady increase but a much shorter radial distance where it assumes lower values in the astrosheath compared to later times, such as 50 000 years

(orange), 100 000 years (green) and 200 000 years (red).

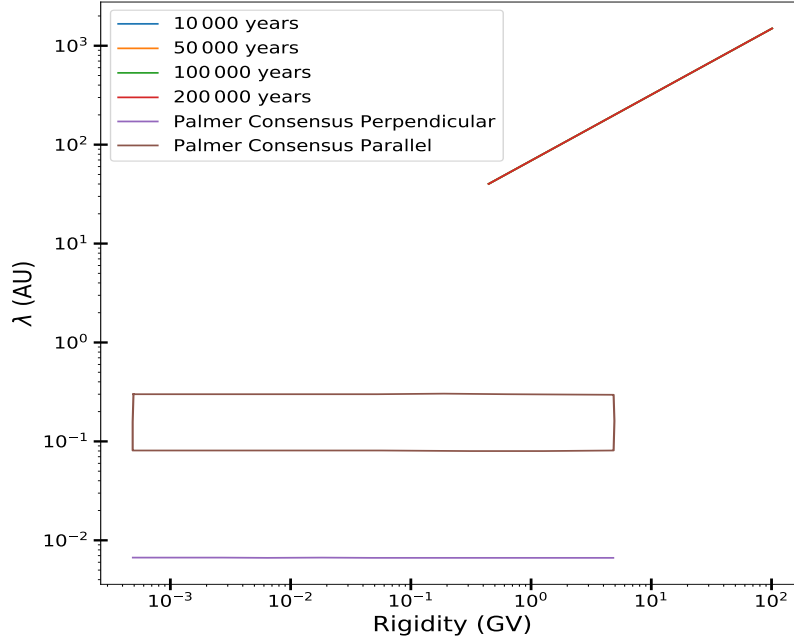


Figure 4.7: The radial mean free path of GCR protons in the astrosphere of HD 99953 as a function of rigidity. The mean free paths are taken at 0.032 pc which is equivalent to 7×10^3 AU. The MFP for 10 000 years (blue), 50 000 years (orange), 100 000 years (green) and 200 000 years (red) are shown. The *Palmer* (1982) consensus ranges for the parallel (brown box) and perpendicular (purple line) at 1 AU are shown to guide the eye.

The rigidity dependence of these MFPs is shown in Fig. 4.7. Even though the MFPs are shown for different times during the evolution of the astrosphere of HD 99953, they all have the same values and show the same steady increase due to the $P^{2/3}$ dependence in Eq. 4.16. The rigidity dependence of the MFP is thus not time dependent or dependent on the size of the astrosphere, while the radial dependence is. The *Palmer* (1982) consensus ranges for the parallel (brown box) and perpendicular (purple line) MFP at 1 AU in the heliosphere are also shown to guide the eye. Even though the astrosphere of HD 99953 is much larger than the heliosphere, the MFPs of GCRs in the astrosphere of HD 99953 are ~ 3 orders of magnitude larger than the MFPs for the *Palmer* (1982) consensus.

4.4.1.4 Timestep change

The heliospheric version of the transport model uses a constant timestep, $dt = 0.004$ days, in Eqs. 4.10 and 4.11 (see *Strauss et al.*, 2011; *Moloto et al.*, 2019; *Engelbrecht and Di Felice*, 2020). However, for the astrospheric version, the changes in stellar wind velocity and magnetic field with radial distance creates regions where there are large gradients in the stellar wind velocity and magnetic field magnitude. These reduce the computational

speed of the transport code. The regions where there are large gradients in magnetic field magnitude and stellar wind velocity may also not be properly sampled by pseudo-particles if a too large timestep was chosen. For a smaller timestep the pseudo-particles should sample all the changes in the large plasma quantities. However, choosing a smaller timestep increases the computational time of the model. Implementing a variable timestep increases the computational speed of the model without sacrificing physical accuracy. The timestep implemented in this section follows the same approach as taken in *Strauss and Effenberger (2017)*,

$$\begin{aligned}
 L &= \Delta r/2, \\
 t1 &= \frac{L}{\max\left(\frac{2}{r\kappa_{rr}}, \frac{\partial\kappa_{rr}}{\partial r}, V_{sw}\right)}, \\
 t2 &= \frac{L^2}{\kappa_{rr}}, \\
 dt &= 0.5 \min(t1, t2),
 \end{aligned} \tag{4.19}$$

where L is half the distance between radial grid points from the MHD output. This is chosen as the transport parameters are changing over each grid point of the MHD output. Therefore, the timestep needs to be chosen such that it is able to sample each transition over the grid points, which is accomplished by limiting the timestep based on a lengthscale that is smaller than the distance between grid points of the MHD model. Applying a variable timestep allows for smaller timesteps where the diffusion coefficients change rapidly and a larger one where they do not, making the model more computationally efficient (*Strauss and Effenberger, 2017*).

4.4.2 Results

With the astrospheric changes now implemented in the heliospheric model as described in Sect. 4.4.1, the GCR intensity within the astrosphere of HD 99953 is modelled within the range of 0.09 - 100 GeV for different evolutionary stages of the astrosphere. The computed differential intensities for GCRs within the astrosphere of HD 99953 are shown in Fig. 4.8. The unmodulated LIS from *Strauss et al. (2011)* shown by the black line is placed outside of the AP of the astrosphere such that only modulation within the astrosphere is considered.

Shown here is that the amount of modulation at higher energies, $E > 10$ GeV, is small due to the large mean free path of these particles. The amount of modulation that these higher energy particles undergo, namely $\sim 2.9\%$, is less than that reported by *Scherer et al. (2015)*. This is not unexpected as the astrosphere used in that work was λ Cephei while HD 99953 was considered here. These astrospheres differ in stellar wind speed, where HD 99953 has a stellar wind speed of $250 - 700 \text{ km.s}^{-1}$ while λ Cephei has a stellar wind speed of 2500 km.s^{-1} . They also differ in mass-loss rate where the mass-loss rate of HD 99953 ranges between $8 \times 10^{-8} \text{ M}_{\odot}/\text{year}$ and $2.2 \times 10^{-7} \text{ M}_{\odot}/\text{year}$, whereas the mass-loss rate of λ Cephei is $1.5 \times 10^{-6} \text{ M}_{\odot}/\text{year}$. Furthermore, while the MFPs have

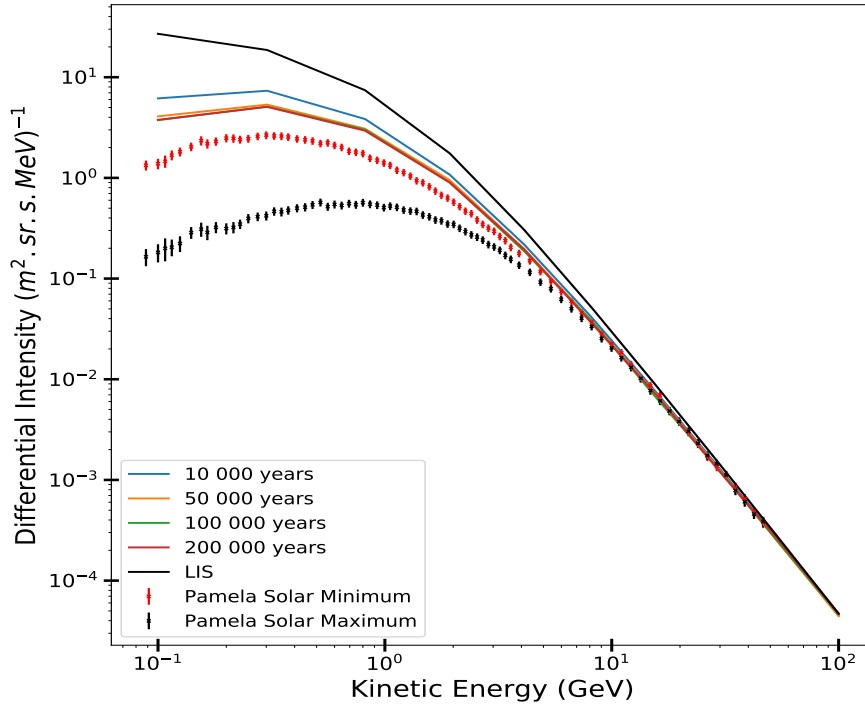


Figure 4.8: Differential intensities calculated for HD 99953 at four different stages of evolution for the astrosphere. The black line shows the LIS, the blue line corresponds to an astrosphere after 10 000 years, orange line is after 50 000 years, green line after 100 000 years and the red line is for the astrosphere after 200 000 years of evolution. Heliospheric observations from PAMELA taken from *Martucci et al.* (2018) are shown in red (solar minimum conditions) and black (solar maximum conditions) crosses, to guide the eye. Taken from *Light et al.* (2022).

similar rigidity dependence at higher energies, *Scherer et al.* (2015) use a value of $P_0 = 4$, while in this work, $P_0 = 1$ GV was used for the MFP (Eq. 4.16).

The lower amount of modulation at higher energies is due to the steeper rigidity dependence assumed in the mean free paths in this work. A larger degree of modulation can be seen at intermediate energies, $1 - 10$ GeV, relative to the LIS. The amount of modulation that occurs at these energy levels is similar to what has been reported by *Scherer et al.* (2015). The amount of modulation increases for lower energies where the differential intensity is seen to be flattened. Observations from PAMELA for the observed modulation within the heliosphere are shown in Fig. 4.8 as black crosses for solar maximum (during early 2014), and red crosses for solar minimum (beginning of 2010) taken from *Martucci et al.* (2018). The heliospheric observations are included to show the relative amount of modulation within the astrosphere of HD 99953 compared to what is observed within the heliosphere. Even though the heliospheric observations are obtained at 1 AU, compared to the GCR intensities modelled for HD 99953 near the inner boundary of the MHD model,

located at 7×10^3 AU, the amount of modulation within the heliosphere is considerably larger than that found in the astrosphere. This is regardless of solar cycle dependence of the heliospheric observations. This is due to the large MFPs used here (see Figs. 4.6 and 4.7), relative to the heliosphere. There is an increase in modulation with increasing astrosphere size as the astrosphere evolves, as would be expected from the increase in modulation volume; however, intensities still remain above what is observed within the heliosphere. It should also be noted that the differential intensity spectra computed after 50 000 years are virtually identical, even though the modulation volume is still increasing. This implies an upper limit to the amount of modulation possible for the scenarios that are considered here, possibly due to the large mean free paths that are employed here.

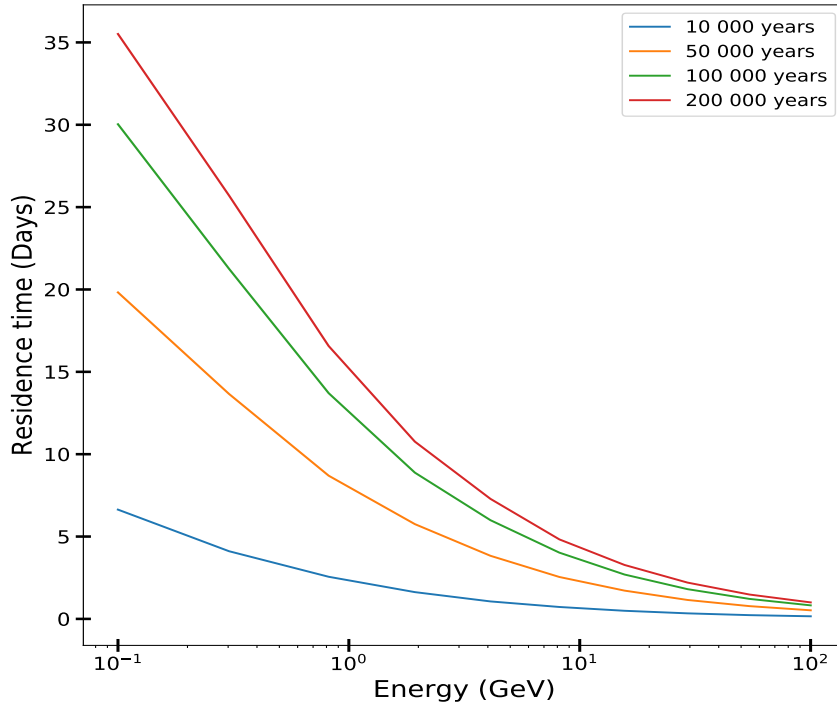


Figure 4.9: The residence time as a function of energy for the pseudo-particles shown in Fig. 4.8 for the same stages of astrospheric evolution. Taken from *Light et al.* (2022).

The residence times of the GCRs travelling through the astrosphere of HD 99953 as a function of energy are shown in Fig. 4.9. The residence time is calculated by summation over all the Rayleigh flights for each of the 10 000 pseudo-particles per energy bin considered with subsequent averaging over each energy bin. The particles at lower energies have a short residence time, ~ 6 days, when they enter the astrosphere at 10 000 years, due to the smaller size of the astrosphere. As the astrosphere evolves, it increases in size, which can also be seen from the increase in the residence time as evident from the residence time at 50 000 years, where at lower energies the residence time is ~ 20 days. This trend is continued with further evolution of the astrosphere, however the increased residence times at 100 000 and 200 000 years for the lower energies do not reflect in an increased amount

of modulation for these energies. The residence times at the higher energies are relatively similar for all stages of evolution of the astrosphere, similar to what would be expected from the amount of modulation at these energies in Fig. 4.8.

The residence times calculated for the GCRs within the astrosphere of HD 99953 show a similar energy dependence as is reported by *Strauss et al.* (2011) for GCRs within the heliosphere; however, the residence times for HD 99953 are considerably lower than those of the heliosphere. *Strauss et al.* (2011) report a max residence time of ~ 90 days for $A > 0$ and ~ 190 days for $A < 0$, while for the largest astrosphere of HD 99953 the max residence time is ~ 35 days. This is related to the difference in diffusion coefficients used in that study compared to those used in this one. Even though the astrosphere of HD 99953 is much larger compared to the heliosphere, there are significant differences in the sizes of the mean free paths. The mean free path employed in this work for HD 99953 is of the order 10^2 AU at the inner boundary of the MHD model, located at $r_1 = 6892.2$ AU, while the mean free path used by *Strauss et al.* (2011) for the heliosphere is of the order 10^{-3} at 1 AU. It should also be noted that the mean free path in this work is proportional to $P^{2/3}$, while that employed by *Strauss et al.* (2011) is proportional to P . The larger mean free path used for HD 99953 allows the particles to traverse the larger astrosphere in a much shorter time, also lessening the particles' exposure time to the modulation mechanisms modelled by Eq. 4.2, leading to lower amounts of modulation overall, even though the astrosphere of HD 99953 is considerably larger than the heliosphere.

4.5 Eruptive Events of LBV type stars

The LBV-type stars can also undergo eruptive stages where large amounts of mass are ejected over a short period of time, as discussed in Sect. 3.4. The eruptions increase the mass-loss rate of the star over a short period of time, while this would also cause fluctuations in the magnitude of the stellar magnetic field which is frozen into the stellar wind. The same approach used in Sect. 4.4, to model the modulation of GCRs within the astrosphere of HD 99953, is used here to model the GCR modulation within the astrosphere of an LBV-type star undergoing an eruptive phase as discussed in Sect. 3.4. In what follows, the *Strauss et al.* (2011) LIS is again employed.

A comparison of differential intensities of GCRs as a function of kinetic energy before and at small intervals following an eruption within the astrosphere of an LBV-type star is shown in Fig. 4.10. The dark blue line, which shows the intensities just before an eruption shows a large amount of modulation at lower energies, similar to what was seen for HD 99953. Different from HD 99953, there is a significant amount of modulation at the higher energies with the average intensity for the whole time interval considered being $\sim 39.6\%$ lower than the LIS (black line). Immediately following the eruption, at 100 000 years (orange line), the differential intensity of the GCRs is above the LIS, pointing towards significant diffusive shock acceleration as particles get trapped at the shock, where the local stellar wind has a high divergence, and gain significant amounts of energy.

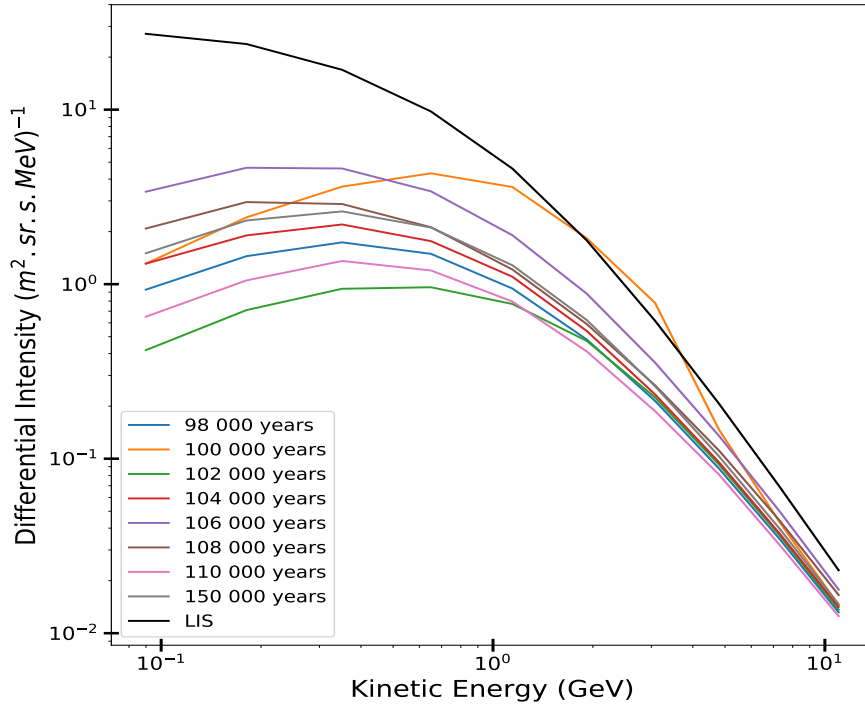


Figure 4.10: The differential intensities before (dark blue line, 98 000 years) an eruption occurs and at various stages during and as the eruption travels through the astrosphere as discussed in Sect. 3.4. Taken from *Light et al.* (2022).

This effect is more clearly seen in Fig. 4.11, where the differential intensity of 1 GeV GCRs is shown as a function of time, from immediately before, during and after the eruptive event has taken place. This shows how the oscillating termination shock and density wave that travels between the inner boundary and AP, as shown in Figs. 3.13 and 3.15, increases and then decreases the amount of modulation that takes place. This effect could be mostly due to the 1D nature of the GCR modulation code employed here, due to the particles getting trapped in the vicinity of the shock, without any alternative transport mechanisms such as drift and diffusion parallel to the perpendicular shock, that are present in a 3D model, to transport the particles away from the accelerating region. The large amount of diffusive shock acceleration could thus be interpreted as an upper limit for this effect in this situation.

The MFPs for different stages during an eruptive event in an astrosphere of an LBV-type star is shown in Fig. 4.12. Just before the eruption takes place at 98 000 years (blue) the MFP shows the simple $\sim 1/B$ dependence with a steady increase in the inner astrosphere, a sharp decline at the TS and assuming the ISM value past the AP. Once the eruption takes place at 100 000 years (orange), the stellar magnetic field is dragged with the dense wave of plasma travelling through the astrosphere. Thus the MFP shows a sharper increase at the TS and dropping to lower values just past the TS, where it might be possible

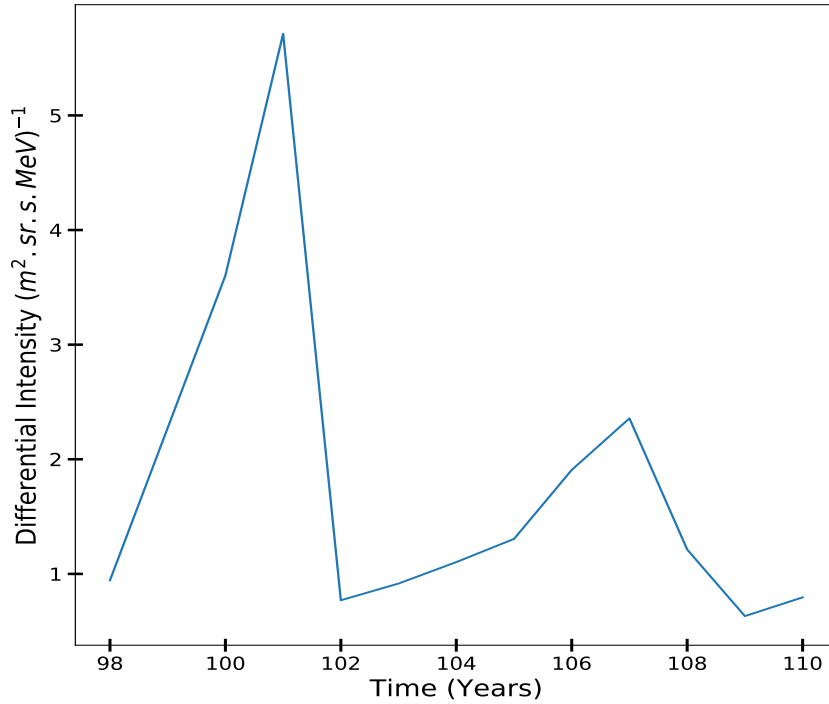


Figure 4.11: The differential intensity of a 1 GeV GCR as function of time as an eruption takes place within an astrosphere as discussed in Sect. 3.4 similar to the differential intensities shown in Fig. 4.10. Taken from *Light et al.* (2022).

for particles to experience shock acceleration. At 101 000 years (green), the eruption has travelled through the astrosphere and pushed the TS closer to the AP. The MFP at this stage shows a much steeper decrease at the TS leading to diffusive shock acceleration.

4.5.1 Effect of Timestep

The timestep discussed in Sect. 4.4.1.4 was used to calculate the transport coefficients of GCRs during the eruptive phase of an LBV-type star. However, this resulted in a significant increase in computation time. The timestep that was utilised in this section is given by,

$$dt = \lambda_r / v_{part}, \quad (4.20)$$

where λ_r is the radial mean free path of the particle and v_{part} is the speed of the particle at every given timestep. This timestep takes into consideration the changes in the particles' diffusion coefficient as they move through the astrosphere and also how many of these changes the particles will experience due to the speed that the particles will have.

The effect of changing the timestep from a constant timestep, $dt = 0.004$ days (orange line), to the timestep given in Eq. 4.20 (dark blue line) can be seen in Fig. 4.13. The

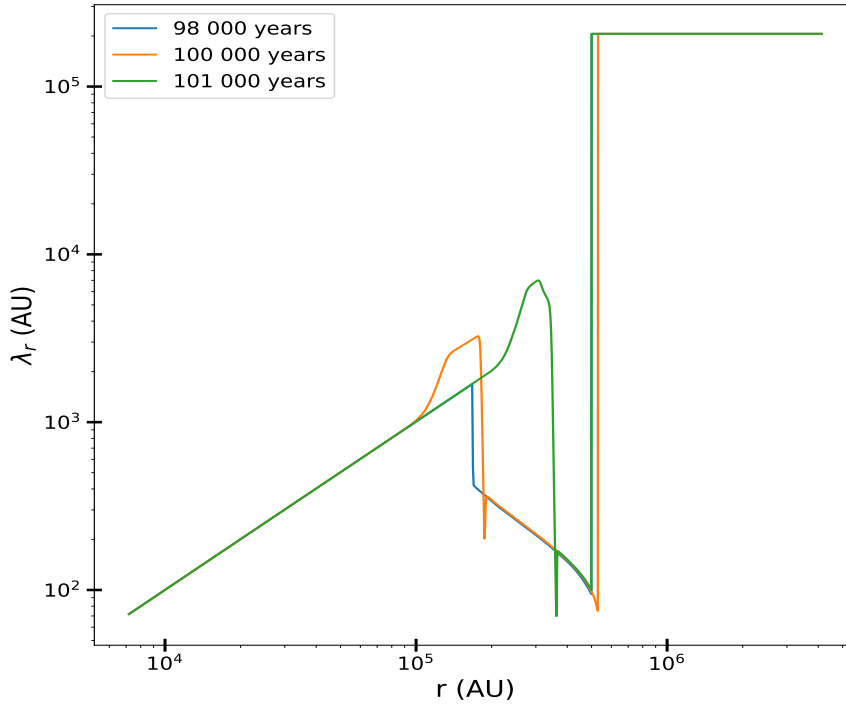


Figure 4.12: The MFPs of 1 GeV GCR protons in the astrosphere of an LBV-type star which is undergoing an eruptive event as function of radial distance. The MFPs just before the eruptive event takes place at 98 000 years (blue), as the eruption takes place at 100 000 years (orange) and immediately following the eruption at 101 000 years (green) are shown.

differential intensities of the GCRs during this time in the eruptive phase of an LBV-type star shows a very large gain in energy as the particles get stuck at the shock. Fig. 4.13 shows a larger energy gain than shown in Fig. 4.10; however of note in this figure is how the differential intensity spectra change as a larger timestep is considered to solve Eq. 4.2. As shown, there is no significant difference between the differential intensity when using a constant time step of $dt = 0.004$ days (orange line) compared to using the timestep of Eq. 4.20 (dark blue line). There is, however, a reduction in computational time when using Eq. 4.20. Increasing the size of the timestep by an order of magnitude, $10 \times \lambda_r / v_{part}$ (green line), shows a decrease in the diffusive shock acceleration effect and a reduction in the amount of energy that the particles gain. A larger timestep reduces the probability and time that the particles get trapped in an accelerating region of the astrosphere when an eruption takes place. Particles with larger timesteps may sample less of the large changes in density, stellar wind velocity and magnetic field as the TS and density wave traveled between the inner boundary and the AP. The diffusive shock acceleration effect may be completely negated (brown line) when a large enough timestep is considered; however this may not be physical.

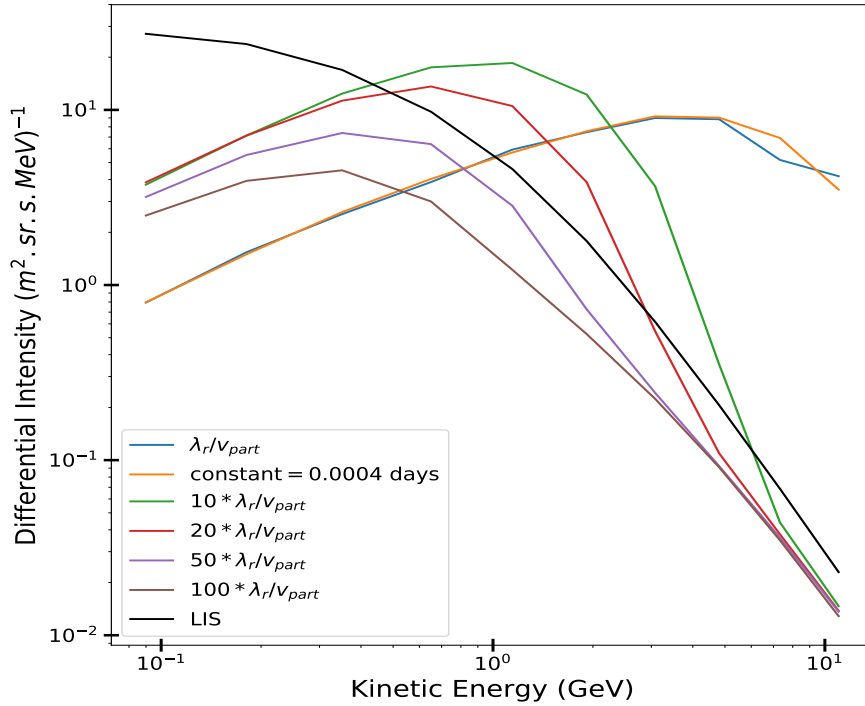


Figure 4.13: The differential intensities of GCRs in an astrosphere where an eruption takes place with a constant timestep (orange line) and different sizes of timesteps.

4.6 Proxima in 1D

The GCR intensities at Proxima Centauri b (Proxima b) have been calculated before using different 1D models (see *Herbst et al.*, 2020; *Mesquita et al.*, 2022b). The model described here and used for the modulation of GCRs in the astrosphere of HD 99953, described above, is used to calculate the GCR modulation at the location of Proxima b, 0.05 AU, using the MHD results of *Herbst et al.* (2020) as inputs. It should also be noted that the LIS used here is the LIS of *Strauss et al.* (2011), same as used above for HD 99953 and also the LIS used by *Herbst et al.* (2020). *Mesquita et al.* (2022b) used a different LIS from *Vos and Potgieter* (2015).

The top panels of Fig. 4.14 show the MHD results for the magnetic field in the top left and stellar wind speed in the top right for Proxima from *Herbst et al.* (2020). These results are taken along the stagnation line of the astrosphere. The MHD results (black) start at a distance of ~ 30 AU, although the exoplanet, Proxima b, is located at a distance of 0.05 AU. In order to obtain plasma parameters at the location of the exoplanet, an analytical extrapolation (red) is done. The bottom panels of Fig. 4.14 show the MFP used in this work for Proxima. The bottom left panel shows the rigidity dependence of the radial MFP, while the bottom right panel shows the radial MFP as a function of radial distance.

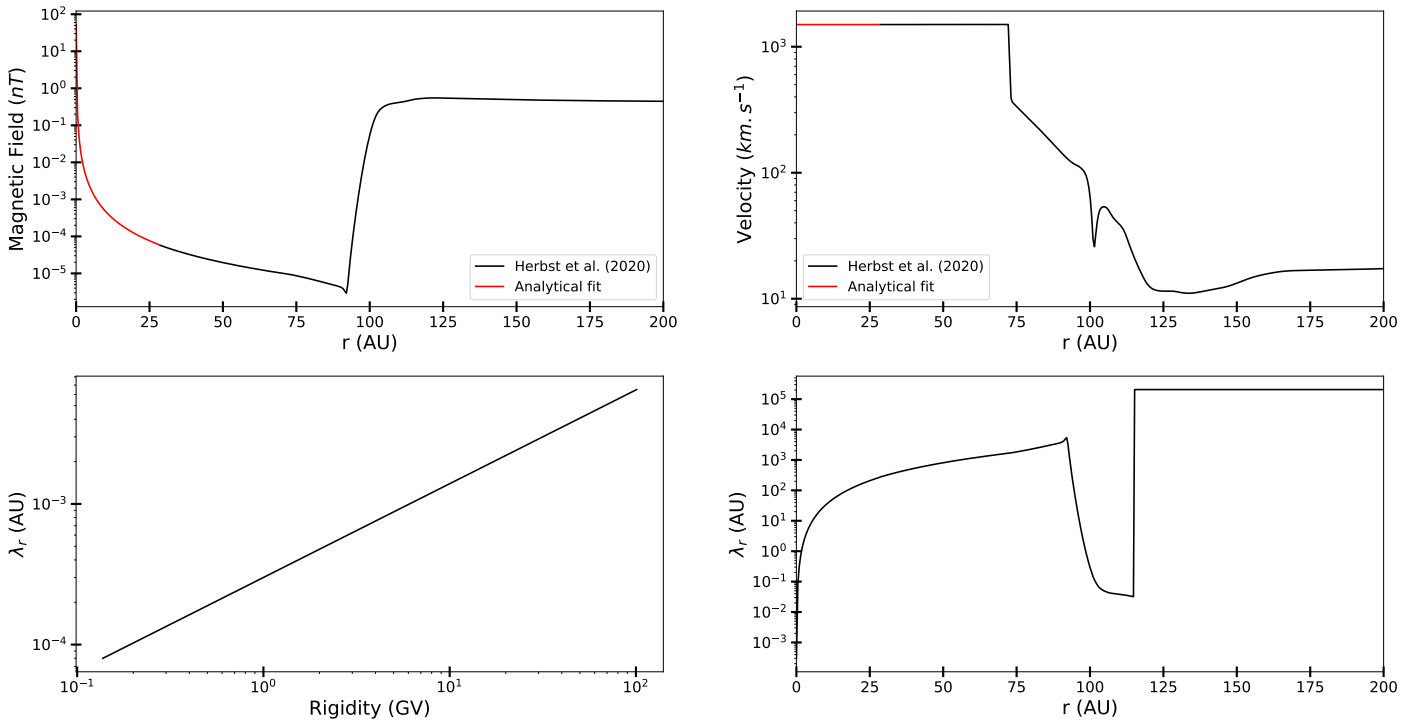


Figure 4.14: Top left panel: Computed radial profile of the AMF of Proxima. Top right panel: Computed stellar wind speed of Proxima as function of radial distance. These radial profiles are taken from the MHD results of *Herbst et al. (2020)* for Proxima. Bottom left panel: The radial MFP (Eq. 4.16) as a function of rigidity at 0.05 AU. Bottom right panel: The radial MFP of 1 GeV GCRs as function of radial distance.

The differential intensities calculated at Proxima b are shown in Fig. 4.15. The differential intensities calculated in this work (blue) are similar to the intensities calculated for stellar minimum (red dots) and stellar maximum (grey dots) by *Herbst et al. (2020)*. At energies below $E < 1$ GeV, the difference between this work and *Herbst et al. (2020)* is due to the different rigidity dependence, as a higher rigidity dependence is assumed in this work. Differential intensities calculated by *Mesquita et al. (2022b)* using a 1D code (green) where they have also included drift (orange) is also shown. The intensities calculated here are ~ 2 orders of magnitude higher than the intensities obtained by *Mesquita et al. (2022b)* for their no-drift solution, while intensities are ~ 3.5 orders of magnitude higher than the drift inclusive solution of *Mesquita et al. (2022b)*.

4.7 Summary and Conclusion

This chapter introduced the stochastic differential equation (SDE) method of solving the 1D transport equation, Eq. 2.12, to calculate the modulation of galactic cosmic rays (GCRs) through an astrosphere, such as those discussed in the previous chapter. The 3D SDE TPE, which will be used in the following chapter, was also discussed. The model used to calculate the modulation of GCRs through an astrosphere, in 1D, was described,

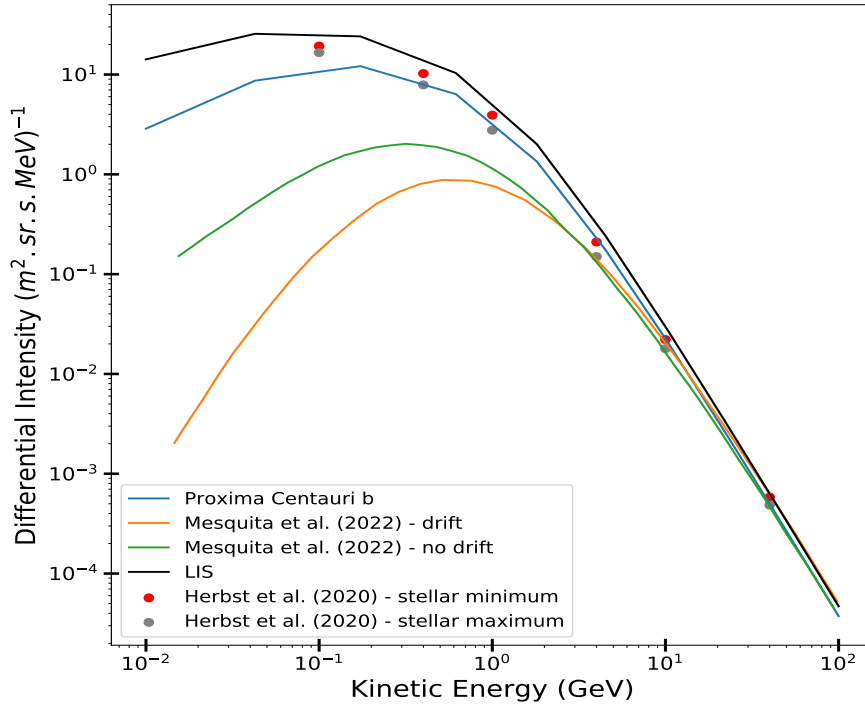


Figure 4.15: Computed differential intensities for GCRs in the astrosphere of Proxima at Proxima Centauri b, 0.05 AU. The results of this work (blue) are compared to the results of *Herbst et al.* (2020) for stellar minimum (red dots) and stellar maximum (grey dots). The results of *Mesquita et al.* (2022b) for their no-drift (green) and drift (orange) 1D results are also included.

which is similar to the approach discussed in *Herbst et al.* (2020). The necessity and method of numerical interpolation of MHD output was discussed in order to use these as input for the transport model. The possibility of numerically interpolating 3D MHD outputs was discussed and it was shown that it is possible to interpolate 3D MHD output from a Cartesian to a spherical coordinate system, given an adequate grid size for the spherical coordinate system.

The changes made to the model to move from heliospheric conditions to those of the astrosphere of HD 99953 were discussed (see *Light et al.*, 2022). It was necessary to include the radial dependence of the stellar wind obtained via the MHD-model in the transport model. The modulation of GCRs within the astrosphere of HD 99953 was calculated. As expected, the amount of modulation that occurs within HD 99953 is dependent on the energy. At high energies, very little modulation occurs, similar to what is found in the heliosphere. A larger amount of modulation was found for lower energies, namely $E < 10$, but intensities were still larger than those found within the heliosphere. Even though larger residence times were found for larger astrospheres as they evolve, a limit was reached at and beyond 50 000 years. This is due to the large diffusion coefficients that

are modelled in this chapter.

The eruption events of LBV-type stars caused an oscillation of the TS which in turn caused a significant variation in the amount of modulation at lower energies. Diffusive shock acceleration also took place around 1 GeV, where particles were accelerated during the early stages of the eruption. However, this effect was very large, and due to the limitations implicit of the 1D model, may represent an upper limit as it was modelled here. This latter point, however, indicated the limitations inherent to 1D approaches to solving the Parker TPE, as it was unclear whether the large amounts of diffusive shock acceleration would occur if pseudo-particles could escape the shock region via 3D transport mechanisms like drift.

The differential intensities of GCR protons were calculated at Proxima Centauri b using the 1D SDE transport model and MHD outputs from *Herbst et al.* (2020). The MHD outputs had to be analytically extrapolated to the location of Proxima Centauri b. The intensities calculated in this work are similar to intensities calculated by *Herbst et al.* (2020). However, they are several orders of magnitude higher than intensities calculated by *Mesquita et al.* (2022b), where these authors have included drift effects. In order to include drift effects, a 3D approach is required. The 3D study of GCR transport in various astrospheres, including Proxima, is therefore the subject of the next chapter.

Chapter 5

3D Cosmic ray modulation in selected astrospheres

5.1 Introduction

The previous chapter demonstrated the use of a SDE transport model to solve the 1D TPE equation, given in Eq. 4.2, for the transport of GCRs within selected astrospheres. This is similar to the results presented in *Van der Schyff* (2016); *Scherer et al.* (2020); *Herbst et al.* (2020) and *Light et al.* (2022). The results obtained in that chapter could in some cases, as for the eruption phase of LBV-type stars, be seen as a first order estimate for the amount of modulation that these particles may undergo due to the limiting 1D nature of the model used. In order to obtain a more accurate and physical measure of the amount of modulation that these particles undergo as they travel through astrospheres, a 3D model is employed in this chapter to allow for 3D physical processes.

Modulation studies done on GCRs within the heliosphere have shown that the physical processes that play a key role in the transport of GCRs through the heliosphere are diffusion, perpendicular and parallel to the heliospheric magnetic field (HMF), as well as drifts (see, e.g., *Ruffolo et al.*, 2012; *Shalchi*, 2020; *Engelbrecht et al.*, 2022a, and references therein). When considering the transport of GCRs through the heliosphere, the turbulent nature of the plasma should also be taken into account, which is incorporated into the diffusion coefficients that are used to describe the transport of GCRs.

5.2 3D SDE Model

The transport of GCRs are described by the TPE such as given in Eq. 2.12, and in absence of local sources is given by *Parker* (1965),

$$\frac{\partial f_0}{\partial t} = \nabla \cdot (\mathbf{K} \cdot \nabla f_0) - \mathbf{v}_{sw} \cdot \nabla f_0 + \frac{1}{3} (\nabla \cdot \mathbf{v}_{sw}) \frac{\partial f_0}{\partial \ln p}, \quad (5.1)$$

where drift and diffusion are described through the 3D diffusion tensor $\mathbf{K}$. As mentioned in Sect. 2.5, this also describes the outward convection due to the stellar wind and the adiabatic energy changes of the omnidirectional GCR distribution function f_0 . This distribution function is a function of position $\mathbf{r}$, momentum p , and time. The distribution function can be once more related to the differential intensity by $j_T = P^2 f_0$ (see, e.g., *Moraal*, 2013) where P is the particle rigidity.

The 3D $\mathbf{K}$ diffusion tensor can be written into HMF aligned coordinates as $\mathbf{K}'$ (see, e.g., *Burger et al.*, 2008), where

$$\mathbf{K}' = \begin{bmatrix} \kappa_{\parallel} & 0 & 0 \\ 0 & \kappa_{\perp,2} & \kappa_A \\ 0 & -\kappa_A & \kappa_{\perp,3} \end{bmatrix}. \quad (5.2)$$

The accent indicates that the tensor differs from $\mathbf{K}$ in Eq. 5.1 which is in spherical coordinates. The perpendicular diffusion coefficients contained within the tensor $\kappa_{\perp,2}$ and $\kappa_{\perp,3}$ indicate that the perpendicular diffusion is in two different directions perpendicular to the HMF. In this work, however, axisymmetric 2D turbulence is assumed such that $\kappa_{\perp,2} = \kappa_{\perp,3} = \kappa_{\perp}$ (see, however, *Strauss et al.* (2016) for a study on diffusion coefficients in non-axisymmetric turbulence).

The 3D diffusion tensor allows the modelling of diffusion parallel and perpendicular to the HMF through the diffusion ($\kappa_{\parallel}$ and $\kappa_{\perp}$) coefficients. This also includes the modelling of drift effects through the (κ_A) coefficient. Similar to the 1D case the coefficients can be related to the corresponding mean free paths by $\kappa_{\perp,\parallel} = v\lambda_{\perp,\parallel}/3$ (*Shalchi*, 2009), where v is the GCR particle speed.

The 3D SDE model discussed in Sect. 4.2.1 is solved here using a time-backward approach as was done for the 1D case. As such, the GCR intensities are computed at the position of the exoplanets within each unique astrosphere. A time-backward approach allocates an observational point (exoplanet position) from which the pseudo-particles are traced backward to the outer boundary. This gives a measure of the amount of modulation that GCRs will experience as they travel through the astrospheres and the intensities that reach these exoplanets.

5.3 Boundary Spectrum

Different to the previous chapter where an LIS was used, in this chapter a boundary spectrum is placed at the TS location. Even though there are MHD simulations for many astrospheres (see e.g., *Herbst et al.*, 2020; *Baalmann et al.*, 2022; *Herbst et al.*, 2022), there are astrospheres for which detailed MHD simulations do not exist. A common factor among MHD simulations for astrospheres is that within the TS the SW remains relatively constant and the astrospheric magnetic field (AMF) can be described as a Parker field as is done for the HMF. This allows for a relatively simple analytical description for

the SW and AMF that have long been used in the heliosphere. Furthermore, using an analytical description allows for extensive parameter studies to be done. A parameter study requiring MHD runs as input for each parameter change would be prohibitively computationally expensive.

When only considering the modulation of GCRs within the TS of astrospheres, the diffusion coefficients that have been derived for the heliosphere can be used (discussed below). Within the supersonic SW, the turbulence is expected to be mostly transverse as is the case for the heliosphere (*Matthaeus et al.*, 1990; *Bieber et al.*, 1996), while within the heliosheath or astrosheath compressive turbulence is expected (*Burlaga et al.*, 2018; *Fraternale et al.*, 2020). Following a similar approach to *Herbst et al.* (2022), possible compressive turbulence (which has been observed within the heliosheath see e.g., *Gutynska et al.* (2010); *Fichtner et al.* (2020); *Zhao et al.* (2020)), is not taken into account in this study as we have a limited understanding of the transport and evolution of compressive turbulence in the heliosphere and of how this influences the transport coefficients of charged particles (see *Engelbrecht et al.*, 2022a). Although it is unclear if compressive turbulence will be present within other astrospheres, if compressive turbulence is assumed, the diffusion coefficients derived for transverse turbulence can not necessarily be used. Therefore, this study confines itself to studying GCR modulation within the TS of various astrospheres, which allows for the use of existing expressions for the diffusion coefficients of GCRs that have been previously employed successfully in heliospheric modulation studies. However, it should be noted that significant modulation within the heliosheath has been reported (see *Stone et al.*, 2013), which may also be possible within the astrosheath. A subject of future study would be to include the modulation that occurs within the astrosheath.

The GCR proton boundary spectrum employed here, is a boundary spectrum that was constructed by *Moloto et al.* (2018) to agree with *Voyager* observations of intensities at 85 AU as reported by *Webber et al.* (2008) within the heliosphere. This boundary spectrum is used as a first approach to the intensities at the TS of other astrospheres and is given by (*Moloto et al.*, 2018; *Moloto and Engelbrecht*, 2020)

$$j_B = \frac{17.0(P/P_0)^{-2.4}}{2.2 + 2.1(P/P_0)^{-3}}, \quad (5.3)$$

which is given in units of $m^2 s^{-1} sr^{-1} MeV^{-1}$, P in GV and $P_0 = 1$ GV. A time dependent boundary spectrum is not assumed here, although it may be possible (*Moloto and Engelbrecht*, 2020; *Engelbrecht and Moloto*, 2021). This boundary spectrum may be an overestimate, as the heliosheath is not the same as the astrosheath, but a full LIS will, potentially be an even greater overestimate, given it is known that modulation could occur in such astrosheaths.

5.3.1 Diffusion coefficients

The model used here in this work is the one discussed in *Engelbrecht and Burger* (2015a); *Moloto and Engelbrecht* (2020); *Engelbrecht and Moloto* (2021) while the coefficients used

here are those employed by *Moloto et al.* (2018); *Moloto and Engelbrecht* (2020); *Engelbrecht and Wolmarans* (2020) and *Engelbrecht and Moloto* (2021). These coefficients have been shown to yield modulation results that are in good agreement with GCR intensities within the heliosphere (see e.g., *Moloto et al.*, 2018; *Moloto and Engelbrecht*, 2020). These coefficients are also discussed and compared to other expressions derived from different scattering theories in *Engelbrecht et al.* (2022a).

A parallel mean free path expression derived by *Teufel and Schlickeiser* (2003) from the *Jokipii* (1966) quasilinear theory (QLT) is used, and is given by (*Burger et al.*, 2008),

$$\lambda_{\parallel} = \frac{3s}{(s-1)} \frac{R^2}{k_m} \frac{B_0^2}{\delta B_{sl}^2} \left[\frac{1}{4\pi} + \frac{2R^{-s}}{\pi(2-s)(4-s)} \right], \quad (5.4)$$

where $s = 5/3$ and $R = R_L k_m$. The parallel mean free path expression is dependent on the maximal proton Larmor radius, R_L , which is defined as $R_L = P/Bc$ (see e.g., *Moraal*, 2013), with P the rigidity, B the magnetic field and c the speed of light. The parallel mean free path is also dependent on the wavenumber at which the slab spectrum inertial range commences, $k_m = 1/\lambda_{sl}$. The background magnetic field is given by B_0 , while δB_{sl}^2 denotes the slab variance, which will be discussed further in Sect. 5.3.3.

The perpendicular mean free path is obtained from the Nonlinear Guiding Center (NLGC) results of *Shalchi et al.* (2004) as modified by *Burger et al.* (2008) to accommodate a general ratio of slab to 2D energies (see, e.g., *Bieber et al.*, 1996) and is given by

$$\lambda_{\perp} = \left[\alpha^2 \sqrt{3\pi} \frac{2\nu - 1}{\nu} \frac{\Gamma(\nu)}{\Gamma(\nu - 1/2)} \lambda_{2D} \frac{\delta B_{2D}^2}{B_0^2} \right]^{2/3} \lambda_{\parallel}^{1/3}, \quad (5.5)$$

where $\nu = s/2 = 5/6$ denotes half the Kolmogorov inertial range spectral index and it is assumed that $\alpha^2 = 1/3$ (from *Matthaeus et al.*, 2003). The perpendicular mean free path is dependent on the 2D variance, δB_{2D}^2 , which was discussed in Sect. 2.2, and the turnover scale where the inertial range begins on the assumed 2D turbulence power spectrum, λ_{2D} . The dependency of the perpendicular mean free path on the parallel mean free path, through $\lambda_{\parallel}^{1/3}$, has been confirmed with theory and tests on numerical test particle simulations (*Shalchi*, 2020; *Dundovic et al.*, 2020).

5.3.2 Heliospheric vs astrospheric large scale plasma quantities

When shifting from heliospheric conditions to the various conditions associated with each unique astrosphere, some assumptions need to be made regarding the large scale plasma parameters that are required to describe the transport coefficients. MHD simulations of various astrospheres (see *Opher et al.*, 2021; *Meyer et al.*, 2021; *Herbst et al.*, 2022) show that the resulting spatial behaviour of the computed AMF within the TS closely resembles that of a *Parker* (1958) HMF model. The first assumption that can be made is that the AMF can be described using a Parker field, as described in Sect. 2.1.2, and given by,

$$B(r, \theta) = AB_0 \left(\frac{r_0}{r} \right)^2 \left(\hat{r} - \tan \Psi \hat{\phi} \right), \quad (5.6)$$

where the value of B_0 is a normalisation value, which is set as the magnitude of the AMF at $r_0 = 1$ AU for the various astrospheres. The AMF magnitude of Proxima at 1 AU is set to 1.8 nT (*Herbst et al.*, 2020). The next assumption is that the polarity of the AMF behaves similar to the polarity of the HMF, in that it has a cyclic behaviour and is given by the sign of A . This would imply that when $A > 0$ the magnetic field in the northern hemisphere points away from star, while the field in the southern hemisphere points towards the star. This effect is flipped when $A < 0$. This equation is used to determine the magnetic field values of the AMF.

As discussed in Sect. 2.1.2, the magnetic field is directed radially outwards with regard to the star and the rotation of the star cause the Parker field lines to form Archimedean spirals. The angle between the AMF and the radial direction is defined as Ψ . For each colatitude θ , the winding angle of these spirals are then given by

$$\tan \Psi = \frac{\Omega(r - r_s) \sin \theta}{V_{sw}}, \quad (5.7)$$

which is dependent on the stellar wind speed of the star, V_{sw} , and the rotation rate of the star, Ω . It is also assumed that the field source surface of stars are located at $r_s = 0.005$ AU, as for the HMF. This assumes that the exoplanets (or point of observation for GCR modulation) lies outside of the Alfvén surface of the star, which is the case for Proxima Centauri and the exoplanet Proxima Centauri b (see *Klein et al.*, 2021). Following the MHD simulations presented by *Herbst et al.* (2020), a constant V_{sw} can also be assumed.

There are however more sophisticated stellar wind models which produce a truly 3D distribution of densities, stellar wind velocities and magnetic fields, with the magnetic fields based on reconstructions made from observations (see e.g., *Kavanagh et al.*, 2021; *Vidotto et al.*, 2023). However, as a first approach to solving the transport of GCRs within other astrospheres in 3D a constant stellar wind and *Parker* (1958) magnetic field are assumed.

5.3.3 Turbulence quantities within astrospheres

The modelling of the GCR diffusion coefficients, from Sect. 5.3.1, requires knowing the turbulence quantities associated with the AMF. The complication regarding the turbulence quantities is that even though within the heliosphere there are extensive observations (*Bruno and Carbone*, 2013), it may be difficult to obtain these quantities when another astrosphere is considered (see, e.g., *Herbst et al.*, 2022).

A first approach is to follow the route of *Herbst et al.* (2022). This involves estimating the behaviour of astrospheric turbulence parameters by using our existing knowledge of heliospheric turbulence, for which extensive observations exist. The turbulence parameters can be related to the behaviour of observable parameters, such as stellar magnetic field and other stellar wind parameters. A first approach to being able to achieve this would

be to scale the turbulence quantities within the astrospheres relative to their behaviour within the heliosphere. This can be done by scaling the astrospheric variances depending on how the AMF scales relative to the HMF at a specific point, say 1 AU. Heliospheric observations have indicated that this is possible (*Matthaeus and Goldstein, 1986; Burger et al., 2022*). This would allow an estimate for the turbulence parameters required for the MFPs described in Sect. 5.3.1. The spatial dependencies of these parameters can be modelled based on results from advanced turbulence transport models for heliospheric conditions and heliospheric observations (see *Herbst et al., 2022*, and references therein), or by using simple power laws motivated by heliospheric observations, which is what is done in this study.

The magnetic variances and correlation scales are scaled according to the observed values in the heliosphere at 1 AU. The values are scaled by taking the ratio of the AMF and HMF magnitude at this distance. This scaling is applied to the observed quantities for the slab and 2D magnetic correlation scales reported by *Weygand et al. (2011)*. For the magnetic variance, the scaling will be applied to the observed variance of 12 nT² reported by *Smith et al. (2006)*. Similar to the approach discussed by *Herbst et al. (2022)*, as a first approach, a 80 : 20 ratio between 2D and slab variances according to the slab/2D variance anisotropy is assumed (*Bieber et al., 1994*). The influence of different ratios of slab and 2D variances will be investigated in Sect. 5.5.2.4. The radial dependencies for the total transverse magnetic variance and correlation scales are assumed to be simple power laws, such that $\delta B^2 \sim r^{-2.4}$, while it is assumed that the MFPs, λ_{sl} and λ_{2D} have $r^{0.5}$ dependencies, following the observations in the heliosphere reported by *Zank et al. (1996); Smith et al. (2001); Cuesta et al. (2022)* and *Burger and McKee (2023)*.

5.3.4 Drift within astrospheres

Along with diffusion, GCRs are also effected by the gradients and curvatures of the HMF causing the particles to drift. In addition, they also drift along the heliospheric current sheet (HCS) as discussed in Sect. 2.5.1. It is not yet observationally clear whether other astrospheres have a structure equivalent to the HCS. Such a structure is, however, to be expected if a dipole-like astrospheric magnetic field exists. An astrospheric current sheet (ACS) is still included in this work in the same manner as it is included for the heliosphere, where the ACS angle is given by (*Kota and Jokipii, 1983*),

$$\theta_{ns} = \frac{\pi}{2} - \tan^{-1} \left[\tan \alpha \sin \left(\frac{\Omega r}{V_{sw}} \right) \right]. \quad (5.8)$$

It should be noted that $\alpha = 0^\circ$ is assumed for most of this study, as a simple first approach.

The drift velocity in this work is calculated following the approach outlined in *Burger (2012)*, using $V_d = \nabla \times (\kappa_A \mathbf{e}_B)$, where $\mathbf{e}_B$ is a unit vector in the direction of the Parkerian AMF. The components of the drift velocity are then calculated as described in *Burger*

(2012) by using

$$\begin{aligned} \mathbf{V}_d = & (\nabla \times \kappa_A \mathbf{e}_B) \tanh[k(\theta_{ns} - \theta) \cos \nu] \\ & - \frac{k \cos \nu}{\cosh^2[k(\theta_{ns} - \theta) \cos \nu]} \kappa_A \mathbf{e}_B \times \nabla(\theta_{ns} - \theta) \\ & - \frac{k(\theta_{ns} - \theta)}{\cosh^2[k(\theta_{ns} - \theta) \cos \nu]} \kappa_A \mathbf{e}_B \times \nabla \cos \nu, \end{aligned} \quad (5.9)$$

where hyperbolic tangent functions are utilised to model the change in the sign of the AMF over the ACS.

The drift coefficient enters the TPE through the antisymmetric components of the diffusion tensor, K_A , with a maximum value given by the weak-scattering value (*Forman et al.*, 1974),

$$\kappa_A = \frac{v R_L}{3}, \quad (5.10)$$

where v is the particle speed and R_L the Larmor Radius of the particle. As discussed in Sect. 2.5.1, in the presence of turbulence, a reduction factor should be included (see *Engelbrecht et al.*, 2017, and references within). However, due to the unknown nature of the turbulence that may be present within other astrospheres, the weak-scattering limit value is assumed as the drift coefficient for the diffusion tensor here.

5.4 Heliospheric modulation

The model discussed in Sect. 5.2 is first applied to the heliosphere as a benchmark. Heliospheric conditions are used, with a solar wind speed of 400 km.s⁻¹ in the equatorial plane which increases to 800 km.s⁻¹ over the poles (*McComas et al.*, 2008). The HMF strength of 5 nT at 1 AU is assumed (see *Cohen et al.*, 2008; *Cliver and Ling*, 2011) and the differential intensity is calculated at 1 AU. The boundary spectrum is placed at 85 AU (*Webber et al.*, 2008).

The resulting GCR proton diffusion MFPs and drift lengthscales are shown in the panels of Fig. 5.1, with the left panel displaying MFPs and lengthscales for 1 GV GCR protons as a function of radial distance, and the right panel the MFPs and lengthscales as a function of rigidity at 1 AU. The parallel mean free path (blue) increases gradually with radial distance, which reflects the steady decrease in slab variance. The perpendicular mean free path (orange) shows a smaller steady increase over radial distance, due to the dependence on $\lambda^{1/3}$. The drift lengthscale (green), which for the weak-scattering limit is simply the Larmor radius of the particle, also shows a steady increase similar to the parallel mean free path with radial distance. This increase is due to the inverse dependency on the AMF, in the Larmor radius.

The radial component (black) of the diffusion tensor is given by, $\kappa_{rr} = (\kappa_{\parallel} \cos^2 \psi + \kappa_{\perp,3} \sin^2 \psi)$, and thus the radial mean free path, since $\kappa_{rr} = v \lambda_{rr} / 3$, is a combination of the parallel and perpendicular mean free path. There is a stronger dependence on the parallel mean

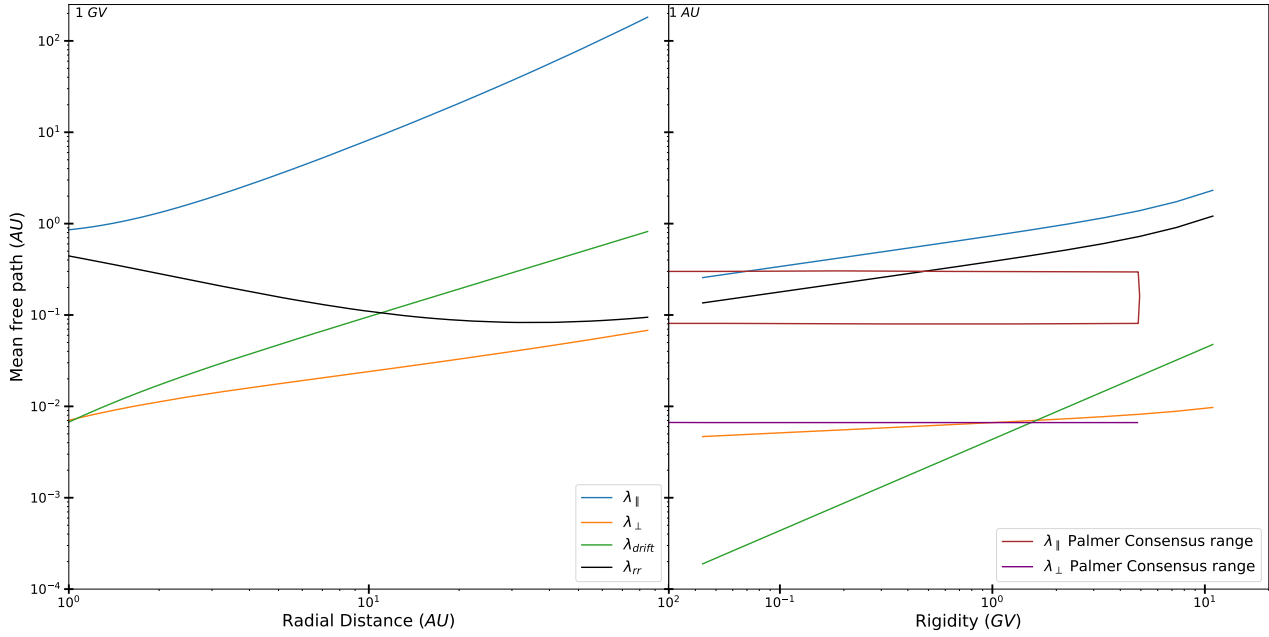


Figure 5.1: Left panel: Mean free paths of 0.01 GV GCR protons in the heliosphere as function of radial distance. Right panel: Mean free paths of GCR protons within the heliosphere at 1 AU as function of rigidity. The different mean free paths are shown, parallel mean free path (blue, given by Eq. 5.4), perpendicular mean free path (orange, given by Eq. 5.5) and the driftscale (green, given by Eq. 5.10), while the combined radial component of the mean free paths is also shown (black). The *Palmer* (1982) consensus range for $\lambda_{\parallel}$ (brown) and $\lambda_{\perp}$ (purple) is also shown.

free path closer to 1 AU but with increasing radial distance the perpendicular mean free path starts to dominate and the radial mean free path approaches the perpendicular mean free path value. This decrease is due to the winding angle of the HMF, $\tan \psi = B_{\phi}/B_r$, which approaches 90° as radial distance increases. As the magnetic field lines travel outward, carried by the solar wind, the Parker spirals become more azimuthal which leads to a more perpendicular diffusion dominated scenario. This is due to the inward diffusion of particles occurring mostly transverse to the HMF.

The MFPs and lengthscales are shown as a function of rigidity in the right panel of Fig. 5.1. The parallel MFP increases gradually with rigidity due to the dependence on rigidity through the Larmor radius, while the perpendicular MFP shows a similar gradual increase due to the dependence on $\lambda_{\parallel}$. The *Palmer* (1982) consensus range for $\lambda_{\parallel}$ (brown box) and $\lambda_{\perp}$ (purple line) within the heliosphere is shown. The perpendicular MFP shows a good agreement with the *Palmer* (1982) consensus range, while at low rigidity $P < 0.1$ GV the parallel MFP is also within the range. The drift lengthscales shows the dependence on rigidity of the Larmor radius with a steady increase with rigidity. The radial MFP shows the dominance of $\lambda_{\parallel}$ at 1 AU with values being closer to $\lambda_{\parallel}$ than $\lambda_{\perp}$.

Fig. 5.2 shows the computed differential intensities of GCRs at 1 AU as function of kinetic energy. The boundary spectrum that is discussed in Sect. 5.3 can be seen as the black

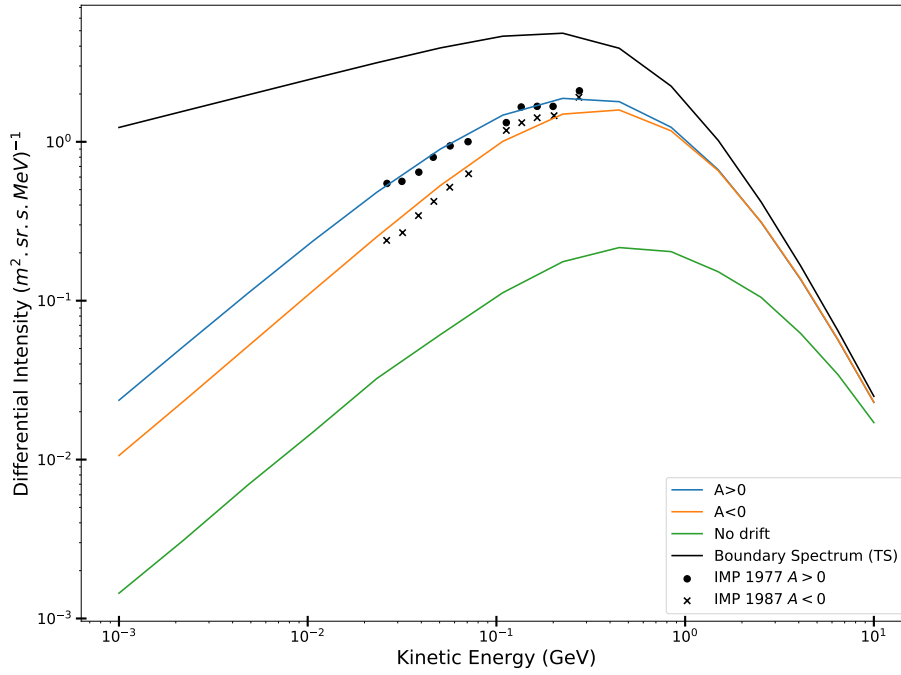


Figure 5.2: Computed differential intensities at 1 AU in the Heliosphere for $A > 0$ (blue) and $A < 0$ (orange) polarities and a no drift (green) solution. The boundary spectrum used here is a boundary spectrum at 85 AU as employed by *Moloto and Engelbrecht (2020)*. Observational data are taken from *McDonald et al. (1992)* for $A > 0$ (black dots) and $A < 0$ (black crosses), to guide the eye.

line in the figure. Observations for GCR proton intensities measured at Earth taken from *McDonald et al. (1992)* for $A > 0$ (black circles) and $A < 0$ (black crosses) are included to guide the eye. Both $A > 0$ and $A < 0$ cases show reasonably good agreement with observational data. At higher energies the intensities are reduced slightly from the boundary spectrum, for all cases, due to the relatively large mean free paths for these particles as can be seen from Fig. 5.1. The no drift case (green) shows a larger reduction in intensities at all energies as the GCRs are not drifting inward from the boundary due to the gradients and curvature of the HMF or along the HCS. This shows the drastic effect that drift has within the heliosphere and the necessity for 3D modelling in order to include these effects. The inclusion of drift reduces the differential intensities by approximately, one order of magnitude compared to the $A < 0$ (orange) case and by more than an order of magnitude compared to the $A > 0$ (blue) case at low energies. The computed differential intensities during $A < 0$ can also be seen here to be less than during $A > 0$ for the lower energy GCRs (< 1 GeV).

5.5 Proxima Centauri

Expanding on what has been done in the previous section, the modulation of GCRs within astrospheres will now be computed using a 3D model with the inclusion of drifts. This

approach expands on the approach followed by *Herbst et al.* (2022) for the transport coefficients of LHS 1140, and applies it to other astrospheres as well. As a first approach the GCR intensities within the astrosphere of Proxima Centauri will be considered.

Proxima Centauri (from here on referred to as Proxima) is a star which hosts an exoplanet, Proxima Centauri b (from here on referred to as Proxima b), which is located at 0.05 AU within the habitable zone (HZ) (see, e.g., *Alvarado-Gómez et al.*, 2020; *Garraffo et al.*, 2022, and references within). There are also two other exoplanet candidates Proxima Centauri c (*Damasso et al.*, 2020) and Proxima Centauri d (*Faria et al.*, 2022), but these are yet to be confirmed. The GCR intensities at the location of Proxima b can be modelled using the model discussed in Sect. 5.2, using the heliosphere as an example as was done in Sect. 5.4, and scaling the values for the turbulence quantities based on heliospheric equivalents as discussed in Sect. 5.3.3.

5.5.1 Stellar parameters of Proxima Centauri

The large scale plasma quantities and stellar parameters that are required to model the GCR intensities at the location of Proxima b are the stellar wind speed of Proxima, the AMF magnitude, specifically at 1 AU, for scaling purposes of the turbulence quantities as discussed in Sect. 5.3.3, the location of the exoplanet, the location of the TS (which is where the boundary spectrum will be placed), and the rotation rate of the star.

The stellar wind speed of Proxima is assumed to be 1500 km.s^{-1} (*Alvarado-Gómez et al.*, 2020; *Herbst et al.*, 2020). The surface magnetic field strength of Proxima is assumed to be 600 G (*Reiners and Basri*, 2008; *Klein et al.*, 2021). Under the assumption that the AMF of Proxima behaves similar to the HMF, the magnetic field strength of Proxima is assumed to be 1.8 nT at 1 AU. This is used with the heliospheric value of the HMF at 1 AU, 5 nT, to scale the turbulence quantities. The rotation rate for Proxima is taken as ~ 82.6 days (*Alvarado-Gómez et al.*, 2020; *Garraffo et al.*, 2022). The differential intensities that are calculated are taken at the location of Proxima b, at 0.05 AU. The boundary spectrum for Proxima is placed at 76 AU, which is the TS distance as determined through MHD modelling (see *Herbst et al.*, 2020).

5.5.2 Cosmic ray modulation of Proxima Centauri b

Utilising the 3D SDE model the differential intensities of GCRs at Proxima b can be calculated.

The parallel MFP (blue), perpendicular MFP (orange), drift lengthscale (green) and radial MFP (black) for Proxima are given as a function of radial distance for a 1 GV GCR in the left panel of Fig. 5.3. Comparing the MFPs for Proxima in Fig. 5.3 with those obtained for the heliosphere, Fig. 5.1, there are significant differences from the heliospheric values. The values obtained for all MFPs and lengthscales within the astrosphere of Proxima are

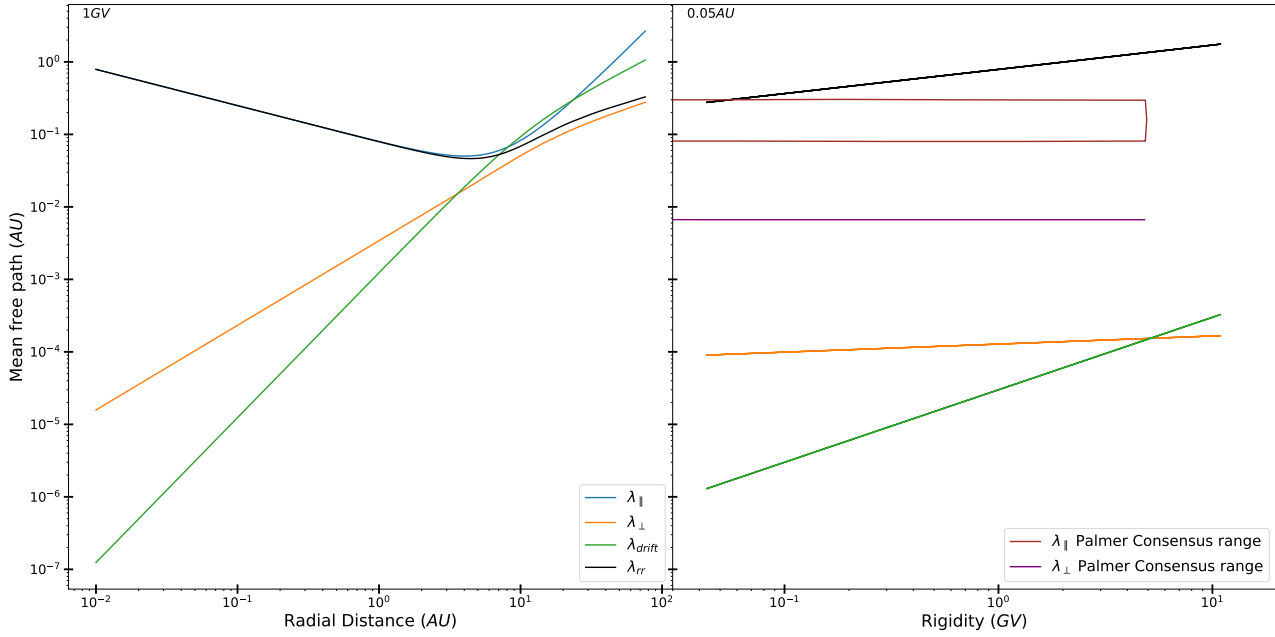


Figure 5.3: Left panel: The parallel (blue) and perpendicular (orange) mean free path and the driftscale (green) of 1 GV GCR protons as a function of radial distance from the boundary at 76 AU. These are compared to the radial mean free path (black). Right panel: The mean free paths as function of rigidity, measured at the position of the exoplanet, at 0.05 AU. The *Palmer* (1982) consensus range for $\lambda_{\parallel}$ (brown) and $\lambda_{\perp}$ (purple) is shown to guide the eye.

considerably smaller than those found in the heliosphere. This is due to the lower AMF strength of Proxima compared to the HMF and the weaker turbulence. The parallel MFP (blue) steadily decreases from the location of Proxima b till ~ 6 AU, from where it starts steadily increasing. The steady decrease of $\lambda_{\parallel}$ is due to the dependence on the background magnetic field, B_0 , which has a $1/r^2$ dependence. The value assumed for the AMF at 1 AU is $B_0 = 1.8$ nT. The AMF therefore increases from 1 AU towards 0.05 AU, the position of Proxima b. The steady increase of $\lambda_{\parallel}$ from ~ 6 AU outwards towards the boundary at 76 AU is due to radial dependence of slab variance, δB_{sl}^2 . The perpendicular MFP shows similar behaviour as is seen and expected for the heliosphere; from ~ 6 AU onwards, a steady increase with radial distance owing to the dependence on the parallel MFP. The drift lengthscale, which in the weak-scattering limit is dependent on the particle speed and rigidity, shows extremely small values near the position of Proxima b and relatively large values from ~ 6 AU onwards. Close to Proxima b, the radial MFP (black) is dominated by the parallel MFP (blue). At radial distances past ~ 6 AU the diffusion is a combination of parallel and perpendicular components similar to the heliosphere.

The rigidity dependence of the parallel MFP (blue), perpendicular MFP (orange), drift lengthscale (green) and radial MFP (black) at Proxima b, 0.05 AU, can be seen in the right panel of Fig. 5.3. The *Palmer* (1982) consensus range for $\lambda_{\parallel}$ (brown box) and $\lambda_{\perp}$ (purple line) within the heliosphere is shown to guide the eye. The parallel MFP, $\lambda_{\parallel}$,

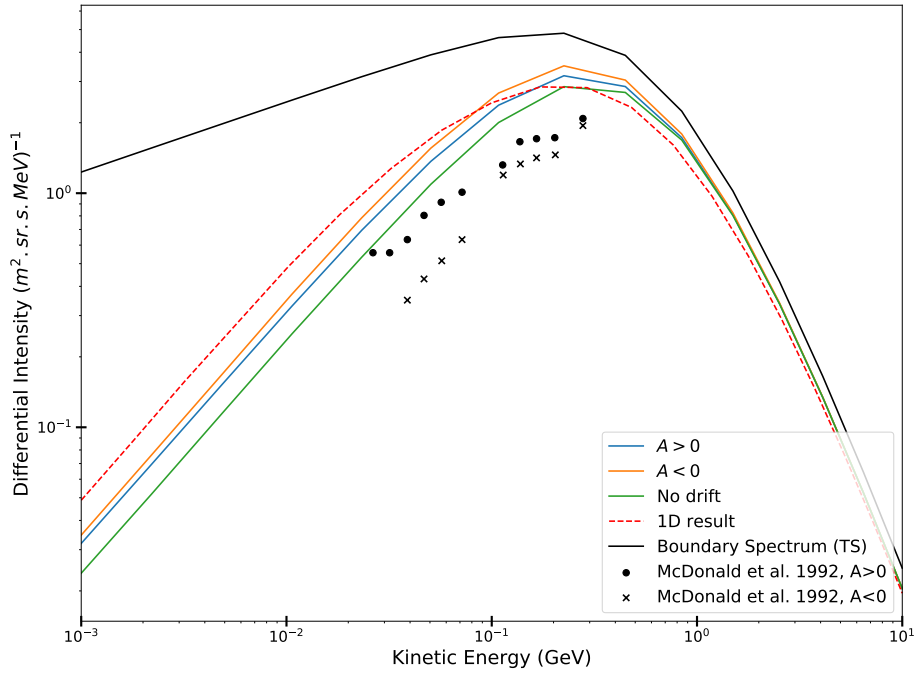


Figure 5.4: Computed differential intensities at 0.05 AU, the position of Proxima b. The same boundary spectrum is used as done for Fig. 5.2. The three different cases correspond to $A > 0$ (blue), $A < 0$ (orange) and a no drift case (green). The 1D results (red dashed line) from the previous chapter are also shown.

shows a $\sim P^{1/3}$ dependence. There is very little rigidity dependence for $\lambda_{\perp}$ since the only rigidity dependence is in the form of a $\lambda_{\parallel}^{1/3}$ dependence. The drift lengthscale shows a steady increase with rigidity as is expected in the weak-scattering limit. The parallel diffusion dominated modulation at Proxima b can clearly be seen in Fig. 5.3 as the radial MFP (black) matches $\lambda_{\parallel}$ at all rigidities.

The amount of modulation that takes place within the TS of Proxima is less than what is found for the heliosphere, with higher intensities for both $A > 0$ and $A < 0$. This can be seen from Fig. 5.4, showing computed differential intensities at Proxima b for $A > 0$ (blue), $A < 0$ (orange) and no drift (green) scenarios with the boundary spectrum (black) given by Eq. 5.3 at the TS distance of 76 AU. The effect of drift (green) is considerably less for Proxima than it was for the heliosphere, shown in Fig. 5.2, but still plays a crucial role for Proxima decreasing the differential intensity, by a factor ~ 2 compared to the $A > 0$ and $A < 0$ cases.

The 1D result (red dashed) from the previous chapter is also shown. This highlights the need for 3D modelling, as even though the intensities are similar at energies above 1 GeV, at lower energies the 1D results have higher intensities than all the 3D results. When 3D modelling is considered, lower intensities are calculated for these energies, while the inclusion of drift increases the intensities but the intensities are still a factor of ~ 1.5 smaller than the 1D results.

An interesting result is found when comparing the heliosphere to Proxima for $A > 0$ and $A < 0$ AMF polarities. For the heliosphere the intensities calculated for a polarity of $A > 0$ is higher, while for Proxima the opposite seems to be the case. The differential intensity for $A < 0$ polarity of the AMF is higher than the intensities that are found for $A > 0$. This may be due to an underwound AMF resulting from a longer stellar rotation period, reducing the effects of gradient and curvature drifts. The differential intensities found for Proxima are also higher than those found for the heliosphere by a factor of ~ 2 for $A > 0$ and $A < 0$ AMF polarities. At higher energies, $E > 1$ GeV, there is no difference between the different polarities of the AMF as very little modulation is found for GCRs at Proxima b. The higher energy GCRs also experience no drift effects.

The intensities calculated for Proxima are several orders of magnitude higher than the intensities reported by *Mesquita et al.* (2022b), while they are slightly lower than the intensities reported by *Herbst et al.* (2020). It should be noted that these authors made use of 1D models, although *Mesquita et al.* (2022b) reported the inclusion of drift effects. The boundary spectra employed by these authors also differ to the one used here.

Based on stellar parameters, such as stellar wind speed, AMF magnitude and the scaling of turbulence quantities based on heliospheric quantities, Proxima and the astrosphere created by Proxima along with the parameters needed for GCR transportation could be described as being similar to the heliosphere. Therefore differential intensities for GCRs are expected to be somewhat similar or at least behave similarly to what is known from heliospheric modulation. This is found to not be the case, as the differential intensities for Proxima are a factor ~ 2 higher for 1 GeV GCRs at Proxima b than what is found for the heliosphere at Earth. The higher intensities for Proxima are due to the parallel MFP dominating most of the modulation within the TS while for the heliosphere this is not the case. The parallel dominant diffusion is due to the underwound AMF of Proxima, as will be demonstrated below.

The winding angles, Ψ , given by Eq. 5.7, as a function of radial distance for the heliosphere (orange) and Proxima (blue) are shown in the left panel of Fig. 5.5. The expected value for the HMF winding angle at 1 AU of 45° is obtained for the heliosphere while the slower rotation rate of Proxima, 82.6 days, has a more underwound field having a value of $\sim 5^\circ$ at 1 AU. The faster stellar wind speed also creates an AMF which is more underwound compared to the heliosphere. This leads to the AMF of Proxima being basically purely radial at the location of Proxima b, which in turn allows parallel diffusion to dominate. The parallel mean free path dominates the radial component of the diffusion tensor when the winding angle is below $\sim 45^\circ$ and due to the slow rotation rate the winding angle of the AMF of Proxima remains below 45° up to ~ 12 AU.

The more underwound field of Proxima can also be seen in the right panel of Fig. 5.5, showing the Parker spiral, Eq. 5.7, for the HMF (orange) and the AMF of Proxima (blue) from the source surface of the magnetic field, which for both cases is set to 0.005 AU, to 5 AU. The slower rotation rate and faster stellar wind speed for Proxima compared to the

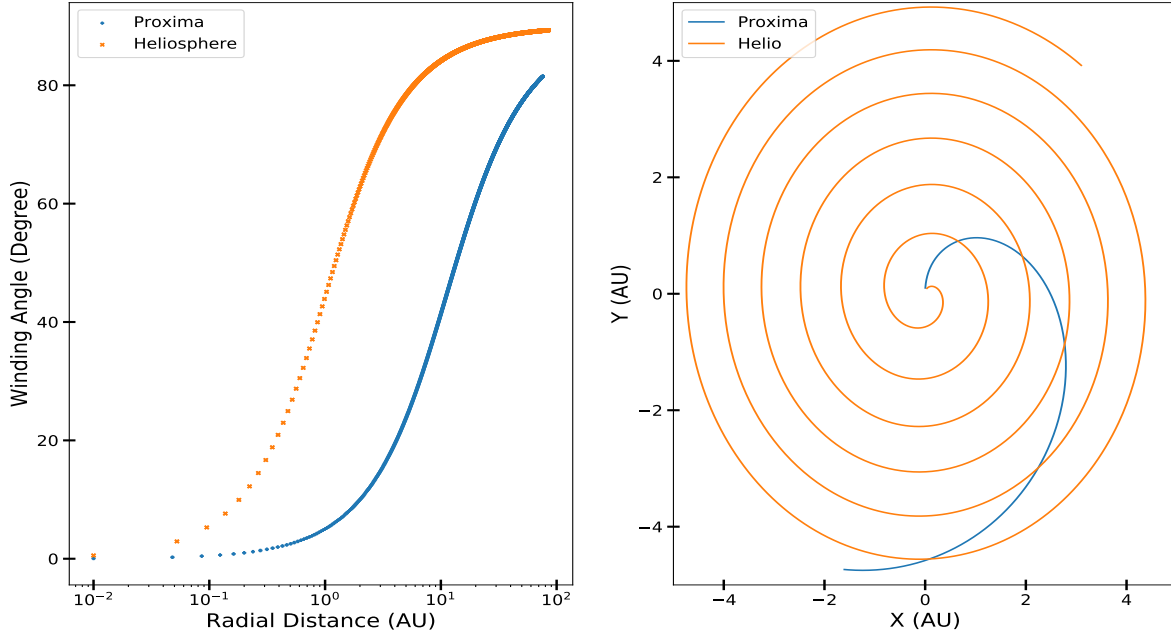


Figure 5.5: Left panel: The winding angle, Eq. 5.7, of the HMF in the Heliosphere (orange) as a function of radial distance compared to the winding angle of the AMF within the astrosphere of Proxima (blue). Right panel: A single Parker spiral of the HMF (orange) and AMF of Proxima (blue) at a constant latitude.

heliosphere creates a Parker spiral which is extremely underwound.

The underwound Parker Spirals of Proxima due to the slower rotation rate leads to an AMF which is almost purely radial at the location of Proxima b, allowing particles to rely more on parallel diffusion compared to perpendicular diffusion. Due to the parallel MFP being considerably larger than perpendicular MFP, the resulting intensities are higher. While drift effects are still necessary and have an impact on the amount of modulation that takes place, it has less of an effect than compared to the heliosphere as the underwound AMF reduces the effects of gradient and curvature drifts. HCS drift, however, remains unaffected, resulting in higher $A < 0$ intensities.

5.5.2.1 Effect of rotation period

The difference in differential intensity at Proxima b within the astrosphere of Proxima and at Earth within the heliosphere can be attributed to the difference in magnetic field magnitude, stellar wind speed and rotation rate of the star. The slower rotation period of Proxima creates an underwound Parker field, creating a diffusion scenario that is more dominated by parallel diffusion compared to what is known for the heliosphere. The change of rotation period is thus a variable that could be influential when GCR modulation is

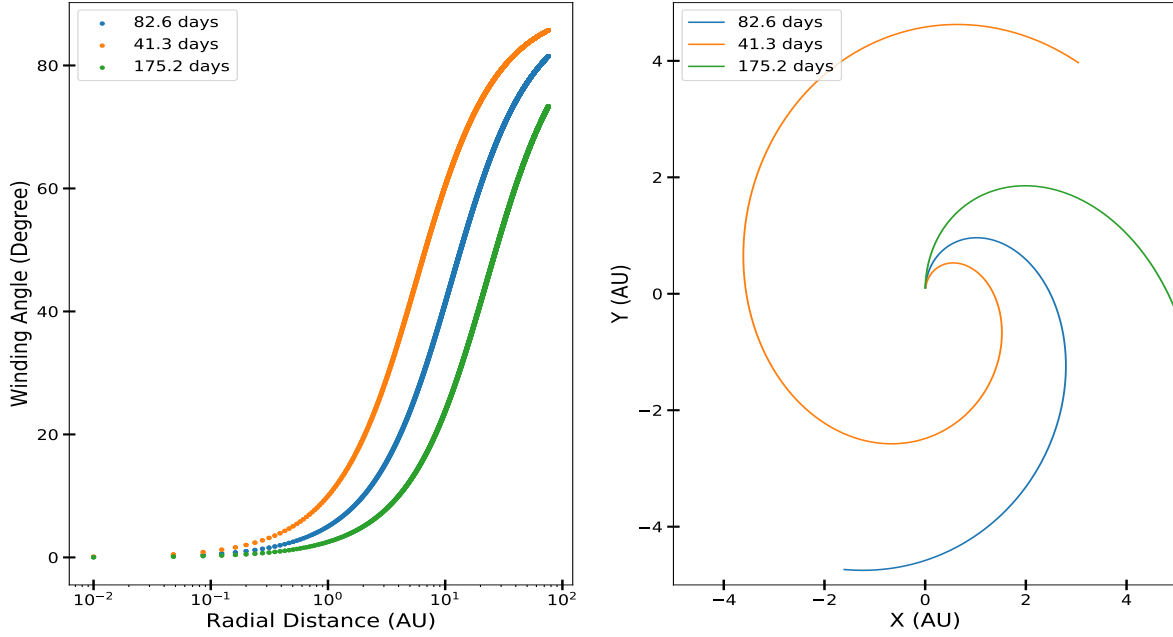


Figure 5.6: Left panel: The winding angle of the AMF within the astrosphere of Proxima (blue), where Proxima has a rotation period of 82.6 days. Double the rotation period, 165.2 days (orange) and half the rotation period, 41.3 days (green) are also shown as a function of radial distance. Right panel: Parker spirals for rotation period of Proxima (blue), double the rotation period, 175.2 days (green), and half the rotation period, 41.3 days (orange).

concerned.

The effect that different rotation periods have on differential intensities of GCRs is tested here for Proxima. The winding angle of the AMF as a function of radial distance for the rotation period of Proxima, 82.6 days, is shown in left panel of Fig. 5.6 in blue. The winding angle for half that rotation period, 41.3 days (orange) is more comparable to what is observed within the heliosphere. However, it is still underwound due to the faster stellar wind speed dragging the AMF. The faster rotation period has a winding angle of 45° at a radial distance of ~ 6 AU compared to the 12 AU for normal rotation period of Proxima. Doubling the rotation period to 175.2 days (green) creates an even more underwound Parker spiral for the AMF with a winding angle of 45° at a radial distance of ~ 23.5 AU.

The effect of changing the rotation period of Proxima on the winding angle of the AMF can also be seen in the right panel of Fig. 5.6 showing the Parker spiral of the AMF. This highlights the underwound field for the slower rotation period while the faster rotation period creates a more wound field.

The differential intensities for $A > 0$, computed for different rotation periods can be seen

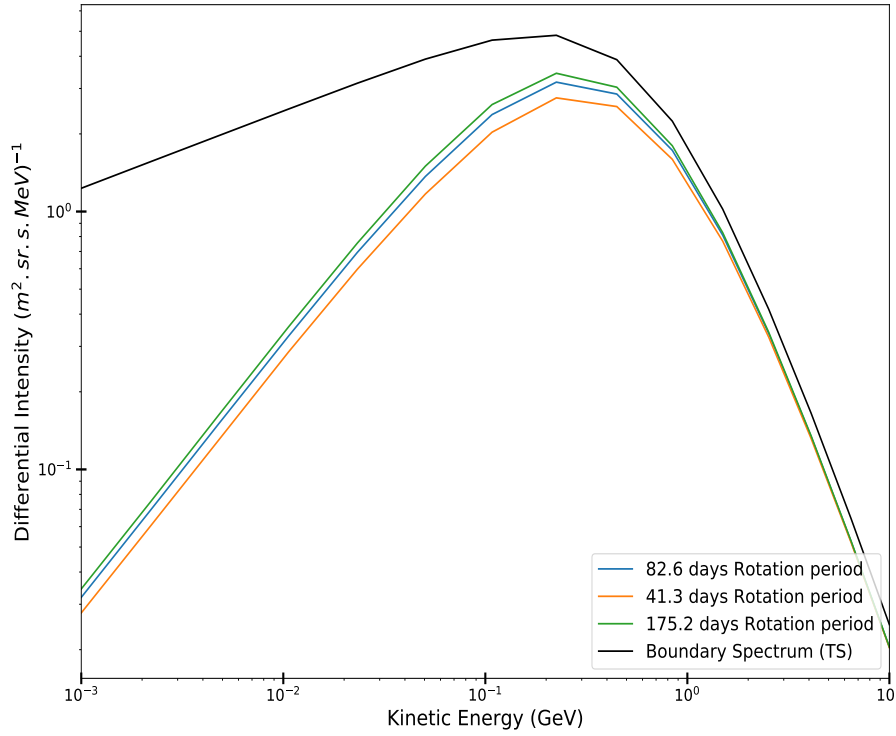


Figure 5.7: Computed differential intensities at Proxima b, 0.05 AU, with a stellar rotation period of 82.6 days (blue). Intensities for twice the rotation period, 165.2 days (green), and half the rotation period, 41.3 days, are also shown. The intensities shown here are for a $A > 0$ AMF polarity.

in Fig. 5.7. The faster rotation period (orange) shows a decrease in intensity at lower energies, where $E < 1$ GeV, as would be expected with a more tightly wound Parker spiral. The more wound AMF reduces the dominance of parallel diffusion. If parallel diffusion does not dominate as much, there is less of a streamline for particles travelling along magnetic fieldlines towards Proxima b, thus reducing the intensity and effectively shielding Proxima b more effectively from GCRs. The slower rotation period (green) shows the opposite effect at the same energies, where the field is more unwound compared to the actual rotation period of Proxima (blue). The more unwound field even further exacerbates the parallel diffusion dominance and creates more of a streamline for particles to flow from the outer boundary, the TS in this case, to the location of Proxima b. At higher energies however, $E > 1$ GeV, there is very little modulation of GCRs for differing rotation periods similar to what was found in Fig. 5.4.

The comparison of the rotation period of the Sun with that of Proxima and the increase and decrease of the rotation period of Proxima has shown that the rotation rate, thus the winding angle of the AMF within which the exoplanet is located, plays a crucial role in the modulation of GCRs, at least within the TS.

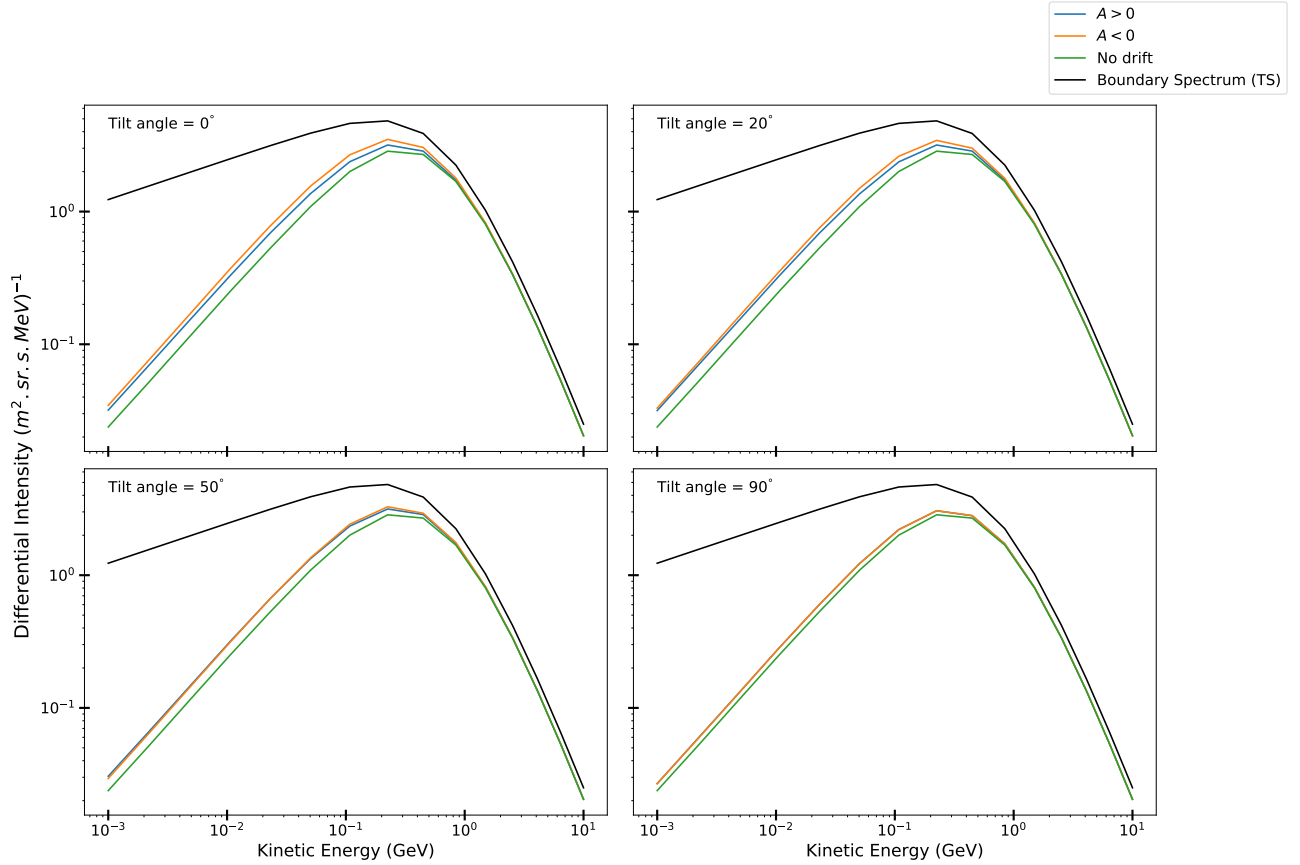


Figure 5.8: The differential intensities calculated at Proxima b for $A > 0$ (blue), $A < 0$ (orange), no drift (green) compared to the boundary spectrum (black) located at 76 AU. Top-left panel shows intensities for a tilt angle of 0° , top-right panel shows a tilt angle of 20° , bottom-left panel shows a tilt angle of 50° and bottom-right panel shows a theoretical maximum at a tilt angle of 90° .

5.5.2.2 Tilt angles

The tilt angle, α , is the angle between the rotational and magnetic axis of the Sun which determines the extent of the HCS. The effect it has on the modulation of GCRs has been shown before (see, e.g., *Burger et al.*, 2008; *Burger*, 2012; *Moloto and Engelbrecht*, 2020). Similar to the heliosphere, it can be assumed that the tilt angle of any assumed ACS would presumably have an effect on the modulation of GCRs within that astrosphere. Although the existence of the ACS has not been confirmed through observations, a tilted dipole field has been observed for Proxima (see, e.g., *Klein et al.*, 2021; *Garraffo et al.*, 2022). The tilt angle of Proxima has been found to reach 51° (*Klein et al.*, 2021). The tilt angle of the ACS will change how particles drift along the ACS as they travel from the TS to the location of Proxima b.

The differential intensities obtained during both polarities of the AMF, $A > 0$ (blue) and $A < 0$ (orange), for varying tilt angles of the ACS are compared to the scenario where

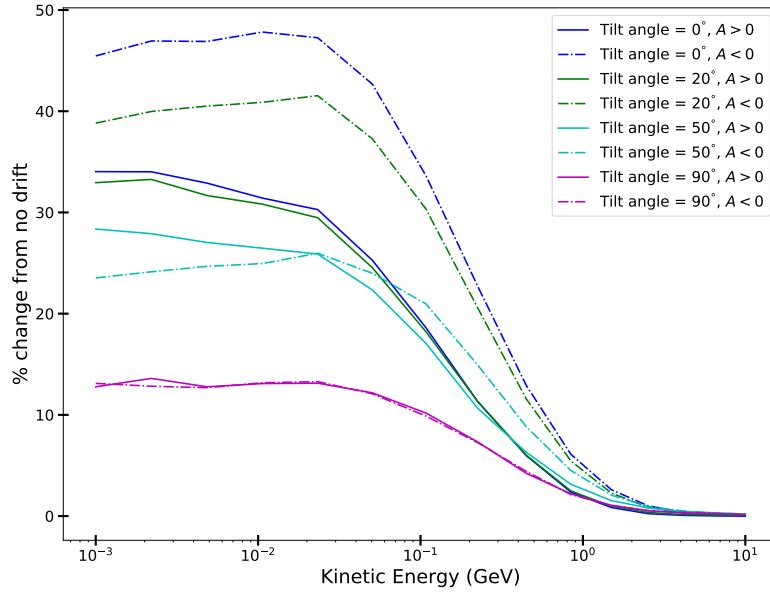


Figure 5.9: The percentage change in differential intensities between $A > 0$ (solid lines), $A < 0$ (dashed dotted line) and the no drift cases in Fig. 5.8. The differential intensities for tilt angles of 0° (blue), 20° (green), 50° (cyan) and 90° (magenta) are shown.

there is no drift (green) in Fig. 5.8. The tilt angles that are considered here, to show the effect varying tilt angles would have on the differential intensities of GCRs, are 0° (top left), 20° (top right), 50° (bottom left) and 90° (bottom right).

The top-left panel of Fig. 5.8 shows that the 0° tilt angle results in the same differential intensities that are shown and described in Fig. 5.4. Increasing the tilt angle to 20° , top-right panel, $A > 0$ and $A < 0$ intensities at the lower energies tend to be closer to each other compared to the 0° case. The difference between the intensities where drift is included to where there is no drift (green) also decreases. This trend continues for higher tilt angles, e.g., 50° (bottom-left), where the intensities for $A > 0$ and $A < 0$ tend to be nearer the intensities obtained when no drift was assumed. For different polarities of the AMF the intensities are more similar for larger tilt angles. The bottom-right panel shows the intensities for a theoretical maximum tilt angle of 90° . For the maximum tilt angle there is very little difference between the intensities found for $A > 0$, $A < 0$ and no drift. For all tilt angles the differential intensities remain the same at higher energies, such that very little modulation is visible at energies higher than 1 GeV.

In order to better visualise the effect that varying tilt angles have on the modulation of GCRs within the astrosphere of Proxima, the percentage changes relative to the no drift solution between both polarities of the AMF for all the tilt angles in Fig. 5.8 are shown in Fig. 5.9 as a function of kinetic energy. Fig. 5.9 shows that for higher energies, where $E > 1$ GeV, there is very little to no change in the differential intensities when no drift is assumed compared to when drift is included. The higher energy GCRs undergo very

little modulation within the TS of Proxima and thus the inclusion of drift has very little effect. At lower energies, $E < 1$ GeV, a 0° tilt angle with an AMF polarity of $A < 0$ (dashed blue) shows the largest change in differential intensity with $\sim 50\%$ increase in intensity, while the opposite polarity $A > 0$ at this tilt angle (solid blue) shows an increase of $\sim 35\%$ compared to no drift. An increase in the tilt angle from 0° to 20° lowers the percentage change from the no drift case to $\sim 40\%$ for $A < 0$ (dashed green) but a similar value for $A > 0$ (solid green). When the tilt angle is increased to 50° there is a change in how the particles behave with regard to polarity. At this tilt angle and at the lower energies, there is a $\sim 29\%$ increase in intensity for $A > 0$ (solid cyan) while there is only a $\sim 24\%$ increase for $A < 0$ (dashed cyan) compared to when no drift is assumed. This change over where the increase is larger for $A > 0$ than for $A < 0$ occurs at energies lower than $E < 0.03$ GeV, while at higher energies the differential intensities for $A < 0$ remain higher and have a larger percentage increase than those of $A > 0$. Fig. 5.9 shows that the maximum tilt angle would reduce the effect that drift has on the modulation of GCRs by only increasing the differential intensities for $A > 0$ (solid magenta) and $A < 0$ (dashed magenta) by $\sim 12\%$.

5.5.2.3 Colatitudinal stellar wind speed effects

When GCR modulation of the heliosphere is calculated, a colatitudinal dependence for the solar wind is often assumed (see, e.g., *Moloto et al.*, 2018; *Moloto and Engelbrecht*, 2020) based on observations. The stellar wind of Proxima is here assumed to behave in a similar manner to the solar wind, with a slow stellar wind component, 750 km.s^{-1} , at the equator and a fast stellar wind component, 1500 km.s^{-1} , at the poles (*Alvarado-Gómez et al.*, 2020). This colatitudinal dependence is modelled following a similar approach as for the heliosphere, following the approach of *Moloto et al.* (2018), where the latitudinal dependence of the stellar wind is modelled through a hyperbolic tangent function using,

$$V_{sw}(\theta) = 750 \begin{cases} \frac{3}{2} - \frac{1}{2} \tanh \left[8 \left(\theta - \frac{\pi}{2} + \alpha + \delta_t \right) \right], & \theta \leq \frac{\pi}{2} \\ \frac{3}{2} + \frac{1}{2} \tanh \left[8 \left(\theta - \frac{\pi}{2} - \alpha - \delta_t \right) \right], & \theta > \frac{\pi}{2} \end{cases} \quad (5.11)$$

with $\delta_t = \pi/9$ radians and α is the tilt angle of the AMF.

The stellar wind speed as a function of colatitude is shown in Fig. 5.10. Here it can be seen that the stellar wind speed remains at the maximum value of 1500 km.s^{-1} at the poles of Proxima, while the minimum stellar wind speed of 750 km.s^{-1} is obtained at colatitudes of $75 - 105^\circ$. Note that a tilt angle of $\alpha = 0^\circ$ is assumed for this section.

The differential intensities for a colatitudinal dependent stellar wind speed is shown in Fig. 5.11. The lower stellar wind speed assumed at the equatorial plane of Proxima, increases the differential intensities for both polarities of the AMF, $A > 0$ (solid blue) and $A < 0$ (solid orange). At higher energies, very little modulation occurs, and although not very visible in Fig. 5.11, there is a $\sim 10\%$ increase in the differential intensities when a colatitudinal stellar wind speed is assumed compared to a constant stellar wind speed. At

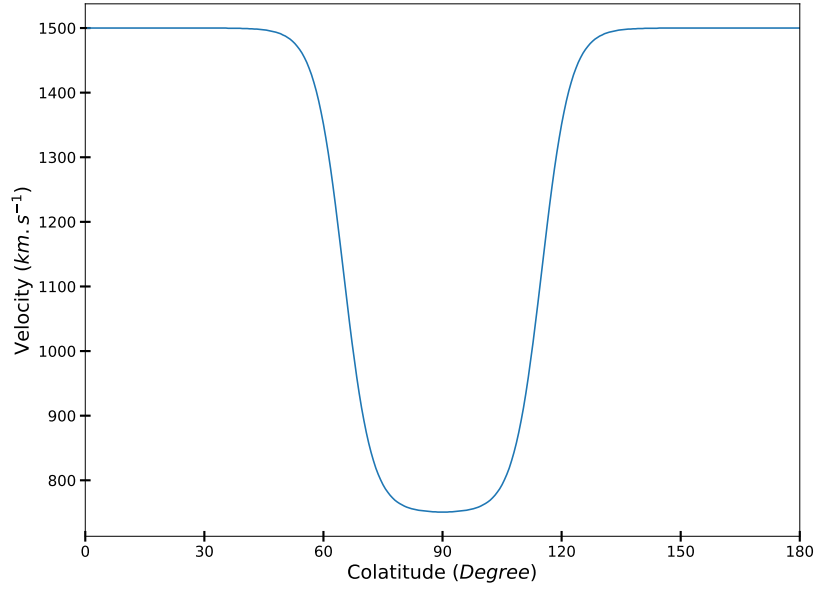


Figure 5.10: Stellar wind speeds of Proxima, taken at 0.05 AU, the position of Proxima b, as a function of colatitude.

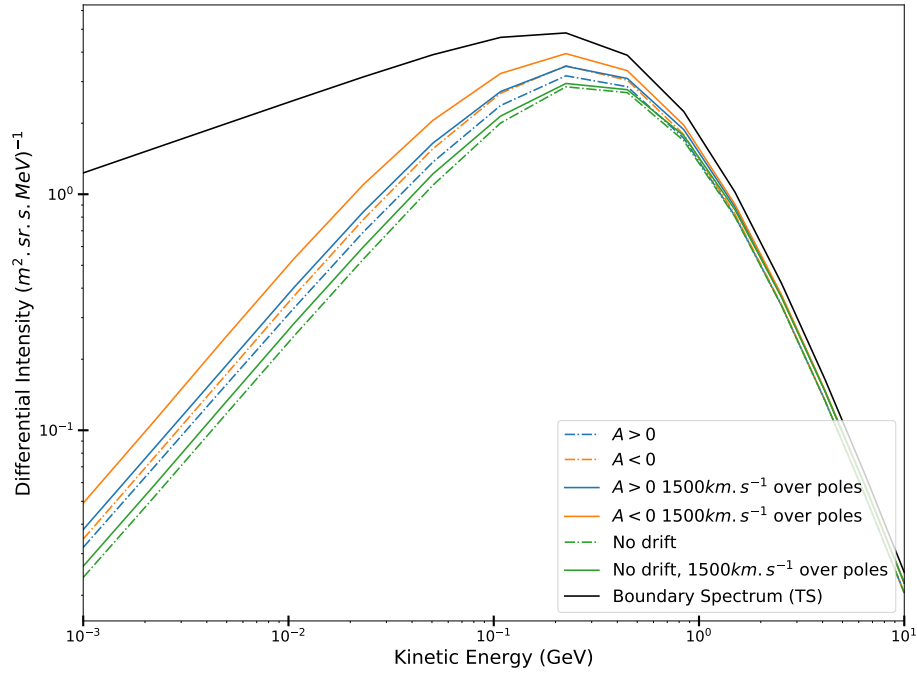


Figure 5.11: The differential intensities for $A > 0$ (blue dashed), $A < 0$ (orange dashed) and no drift (green dashed) for a constant colatitudinal speed of 1500km.s^{-1} similar to Fig. 5.4. Stellar wind speeds varying from 750km.s^{-1} to 1500km.s^{-1} with colatitude as shown in Fig. 5.10 are shown for $A > 0$ (solid blue), $A < 0$ (solid orange) and no drift (solid green).

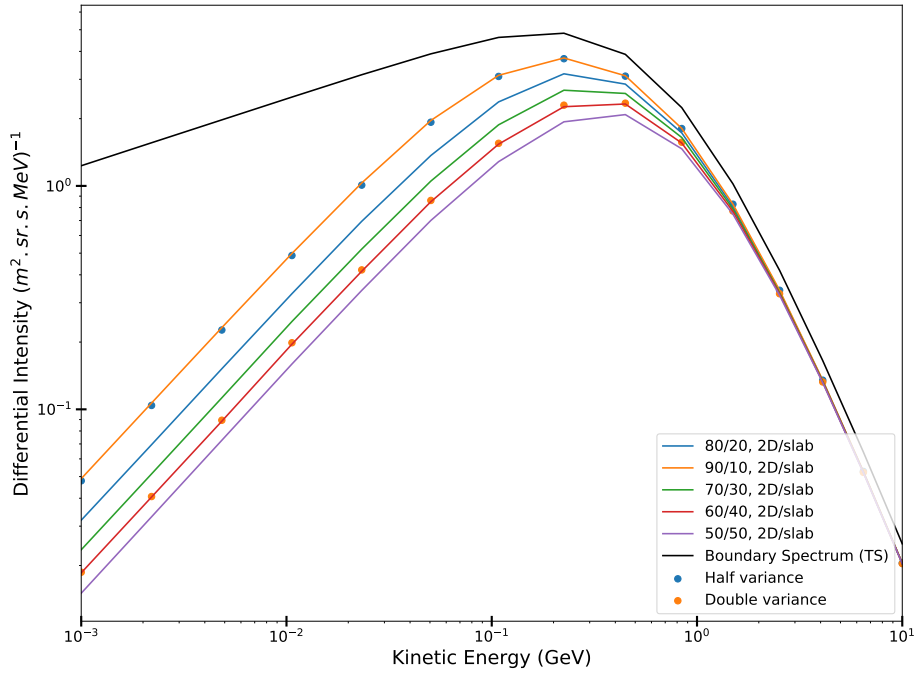


Figure 5.12: Computed differential intensities of GCR protons at Proxima b with the same composition of 2D and slab turbulence as for heliosphere, namely 80/20 (blue). Variations on these compositions are also shown, namely, 90/10 (orange), 70/30 (green), 60/40 (red) and 50/50 (purple). The turbulent variance used here is scaled, as discussed in Sect. 5.3.3, from the 12 nT^2 variance of the heliosphere. The effect of halving (blue dots) and doubling (orange dots) this variance on the differential intensities is displayed.

the lower energies, when no drift is assumed, there is an increase in differential intensity of $\sim 12\%$. At lower energies, for $A > 0$, there is a $\sim 20\%$ increase in differential intensity as the particle drifts in from polar regions, therefore experiencing the larger stellar wind speed for a longer period of time. There is still an increase in the differential intensity as the GCRs are exposed to the lower stellar wind speed once they have drifted from polar regions to the ACS. For $A < 0$ this increase in differential intensity at lower energies increases to $\sim 42\%$ as the particles drift in from the ACS, where they are convected by a stellar wind speed of 750 km.s^{-1} and only experience the faster stellar wind regions once they have drifted towards the polar regions of Proxima.

5.5.2.4 Proportion of Slab and 2D turbulence

Using the scaling discussed in Sect. 5.3.3, different ratios of slab and 2D turbulence can be tested within the astrosphere of Proxima. Due to unknown nature of the turbulence levels within these astrospheres, different values for the total variance will also be considered.

All other results thus far have considered a 80/20 assumed ratio of 2D and slab turbulence anisotropy. A comparison between different ratios of turbulence anisotropy within the astrosphere of Proxima is shown in Fig. 5.12, for an $A > 0$ polarity, with the 80/20 ratio

shown by the blue line. It should be noted that due to the nature of modulation within the astrosphere of Proxima, at energies above 1 GeV, the amount of modulation that GCRs undergo remains constant with regard to different 2D/slab turbulence ratios. Assuming a 90/10 2D/slab ratio (orange line) increases the differential intensity at energies lower than 1 GeV as this ratio assumes an increase in 2D turbulence and a decrease in slab turbulence. A decrease in slab turbulence increases the size of the parallel MFP due to the $\sim 1/\delta B_{slab}^2$ dependence in Eq. 5.4, while the increase in 2D turbulence also increases the size of the perpendicular MFP due to Eq. 5.5 dependence on $\sim (\delta B_{2D}^2)^{2/3}$ and $\sim (\lambda_{\parallel})^{1/3}$. Larger MFPs allow the particles to travel further along the magnetic field lines before being diffused. Due to the parallel diffusion dominated nature of Proxima's astrosphere, the particles are able to reach the exoplanet easier.

Changing the ratio of 2D/slab composite turbulence to 70/30 (green), 60/40 (red) and 50/50 (purple) shows lower computed differential intensities for GCR protons in Fig. 5.12. Increasing the amount of slab turbulence leads to a smaller parallel MFP while decreasing the 2D turbulence also results in smaller perpendicular MFP. Larger ratios of slab to 2D composite turbulence leads to lower intensities, which is not unexpected due to smaller parallel and perpendicular MFPs.

The total variance for the results thus far has been kept constant by taking the 12 nT² solar minimum value observed at Earth and scaling it using the magnitude of the AMF and HMF at 1 AU, as discussed in Sect. 5.3.3. Fig. 5.12 shows the differential intensities of GCR protons as a function of kinetic energy when the total variance is halved (blue dots) and doubled (orange dots). When composite turbulence is assumed the total variance is given by Eq. 2.10. While the total variances are changed, a constant 80/20 ratio of 2D/slab composite turbulence is assumed. A reduction in the total variance by halving it, reduces the amount of turbulence within the inner astrosphere of Proxima. Reducing the total variance, when composite turbulence is assumed, reduces the total amount of 2D and slab variance. Higher intensities are found for half the total variance (blue dots) due to the larger parallel MFP, while the perpendicular MFP becomes smaller due to the dependence on 2D variance. Doubling the variance reduces the size of the parallel MFP, while the perpendicular MFP becomes larger. The combination of smaller $\lambda_{\parallel}$ and larger $\lambda_{\perp}$ when the variance is doubled (orange dots) leads to lower intensities as shown in Fig. 5.12. This again points to parallel diffusion being the dominant transport mechanism within the TS.

5.6 Selection of other stars containing habitable exoplanets

Like Proxima, there are other astrospheres that contain exoplanets within their HZ, such as Trappist-1 (*Gillon et al.*, 2017) and LHS 1140 (*Dittmann et al.*, 2017). There are also exoplanets around stars such as GJ 436 (*Rosenthal et al.*, 2021) for which GCR modulation has been calculated (see, e.g., *Mesquita et al.*, 2021; *Rodgers-Lee et al.*, 2023). The approach used to calculate the modulation of GCR protons, in Sect. 5.5.2, for the TS

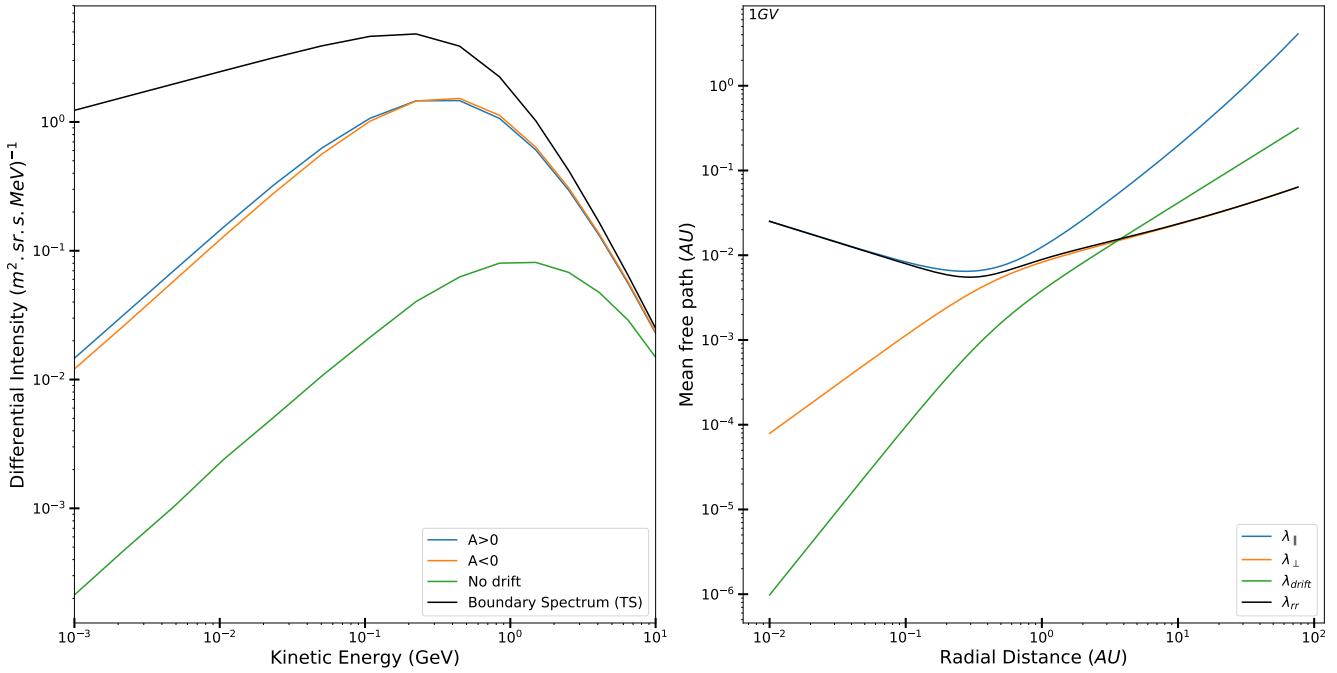


Figure 5.13: Left panel: Computed differential intensities of GCR protons calculated at Trappist-1e. The boundary spectrum (black) is located at the TS. Positive, $A > 0$ (blue) and negative, $A < 0$ (orange), polarities are shown, also a no drift case (green). Right panel: Parallel (blue), perpendicular (orange), drift (green) and radial (black) lengthscales as a function of radial distance for 1 GV GCR protons.

of Proxima, is now applied to these astrospheres.

5.6.1 Trappist-1

Similar to Proxima, the large scale plasma quantities are required as input for the transport model. The stellar wind speed of Trappist-1 has been found to reach 1400 km.s^{-1} (see *Garraffo et al.*, 2017; *Harbach et al.*, 2021), while the magnetic field is estimated to be similar to that of Proxima (see *Reiners and Basri*, 2008; *Herbst et al.*, 2020). The rotation period of Trappist-1 is much faster than Proxima and the heliosphere, having been measured at 3.3 days (*Luger et al.*, 2017). Currently there are no MHD simulations to give the TS distance for Trappist-1; however, as it is very similar to Proxima as a first order estimate it is assumed to be located at 76 AU.

There are seven exoplanets orbiting Trappist-1, however only three of these are located within the HZ of Trappist-1, namely Trappist-1e, Trappist-1f and Trappist-1g (*Gillon et al.*, 2017). Even though they are located within the HZ, these are very close to the star with orbits of 0.028 AU for Trappist-1e, 0.037 AU for Trappist-1f and 0.045 AU for Trappist-1g (see *Gillon et al.*, 2017). The GCR proton differential intensities in this section will be calculated at Trappist-1e, at a distance of 0.03 AU.

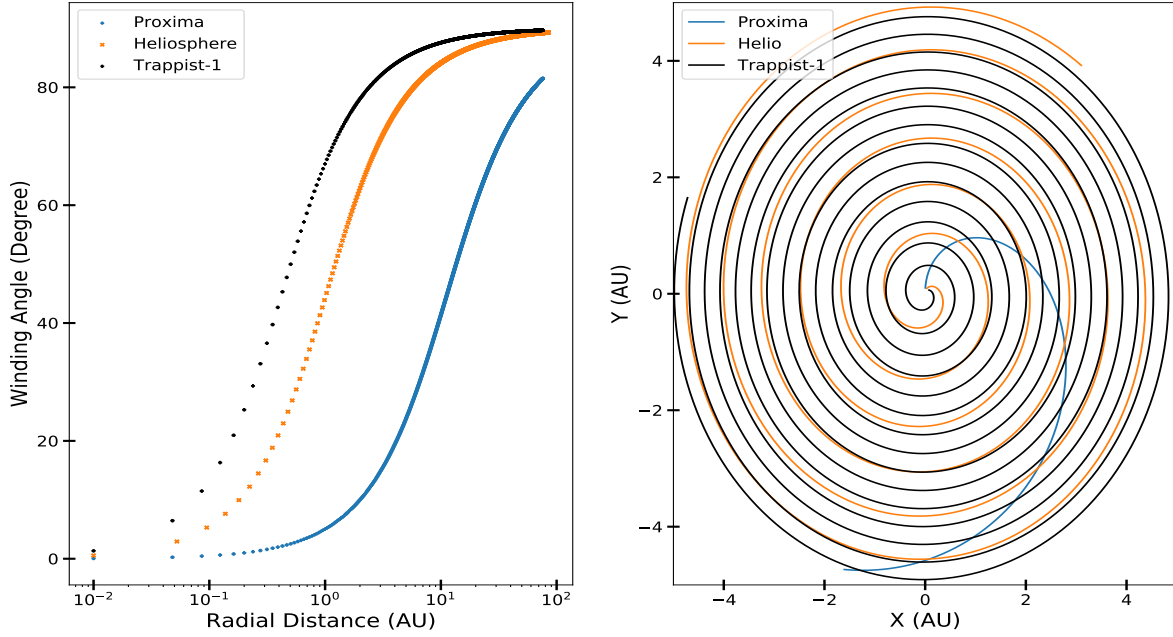


Figure 5.14: Left panel: Comparisons of the winding angle of Trappist-1 (black), Proxima (blue) and the heliosphere (orange) as a function of radial distance. Right panel: The trace of a magnetic field line Parker spiral for Trappist-1 (black), Proxima (blue) and the heliosphere (orange) within 5 AU of the star.

The computed differential intensities at Trappist-1e for $A > 0$ (blue), $A < 0$ (orange) and no-drift (green) as a function of kinetic energy is shown in the left panel of Fig. 5.13. Drift plays a large role as at low energies, $E < 1$ GeV, the addition of drift increases the differential intensities by ~ 2 orders of magnitude. At higher energies the effect is significantly less, however, still significant as including drift increases the intensities by a factor of ~ 2 . At lower energies, the intensities are similar to what is calculated for the heliosphere, and the behaviour with regard to polarity is similar to the heliosphere, although not as pronounced, with intensities for $A > 0$ being higher than for $A < 0$. This is different to what was found for Proxima.

The similarity with the heliosphere and difference from Proxima can be explained through the mean free paths calculated for Trappist-1, shown in the right panel of Fig. 5.13. Close to Trappist-1 and Trappist-1e, the radial lengthscale (black), which corresponds to the radial diffusion coefficient is shown to be dominated by the parallel lengthscale (blue). At radial distances past ~ 0.4 AU the perpendicular lengthscale (orange) dominates. This suggests that the diffusion of particles from the outer boundary inward is perpendicular diffusion dominated for the case of Trappist-1, similar to what is found for the heliosphere, and is related to the higher winding angle.

The similarities in the differential intensities and mean free paths of Trappist-1 and the

heliosphere are not unexpected, as the winding angle and consequently the Parker spirals of the magnetic field lines are very similar. The winding angle of Trappist-1 (black), the heliosphere (orange, during solar minimum) and Proxima (blue) are shown in the left panel of Fig. 5.14. At a distance of 1 AU, the heliosphere has a winding angle of 45° , while at the same distance Proxima has a winding angle of $\sim 5^\circ$, as mentioned in Sect. 5.5.2. Trappist-1, with a much faster rotation rate than both the heliosphere and Proxima, has a winding angle of $\sim 65^\circ$ at a distance of 1 AU. This is considerably larger than the winding angle of even the heliosphere at this distance.

The Parker spirals created by tracing the magnetic field lines are shown in the right panel of Fig. 5.14 for Trappist-1 (black), the heliosphere (orange) and Proxima (blue). The addition of the fast stellar wind of Trappist-1 creates Parker spirals that closely resemble those of the heliosphere. This helps to illustrate why the intensities for Trappist-1 more closely resemble those found for the heliosphere than those found for Proxima. Due to the fast rotation rate and fast stellar wind speed, GCR transport within the TS of Trappist-1 is perpendicular diffusion dominated, similar to what is found in the heliosphere.

5.6.2 LHS 1140

The astrosphere of LHS 1140 is very different to other astrosphere considered thus far, as this astrosphere is extremely small in size with very low stellar outflow parameters. The stellar wind speed is assumed to be 250km.s^{-1} and a TS distance of 8.1 AU was calculated by *Herbst et al.* (2020). The stellar magnetic field magnitude assumed for LHS 1140 is 0.3 nT at 1 AU (*Herbst et al.*, 2020). The rotation period of LHS 1140 is taken to be 131 days (see *Dittmann et al.*, 2017; *Lillo-Box et al.*, 2020). The exoplanet within the HZ around LHS 1140, is LHS 1140b, which orbits at a distance of 0.096 AU (see *Lillo-Box et al.*, 2020). This is the distance at which the differential intensities are calculated.

The differential intensities calculated for GCR protons at LHS 1140b are shown in the left panel of Fig. 5.15. GCR protons with energies higher than $E > 0.1$ GeV undergo no modulation within the TS of LHS 1140, due to the small size of the astrosphere. LHS 1140 has a very small astrosphere, with the boundary spectrum in this case placed at 8.1 AU, while the boundary spectrum for the heliosphere is placed at 85 AU. Drift also has no effect on particles with energies higher than 0.1 GeV. Drift does have a small effect on particles with energies less than 0.1 GeV when a negative polarity, $A < 0$ (orange), is assumed for the AMF. When a positive polarity ($A > 0$, blue) is assumed, drift has a negligible effect within the TS of LHS 1140. The right panel of Fig. 5.15 shows the parallel (blue), perpendicular (orange), drift (green) and radial (black) lengthscales as a function of radial distance. The diffusion of particles within the TS of LHS 1140 is parallel diffusion dominated since the radial lengthscale is mainly following the behaviour of the parallel MFP. The drift lengthscale (green) is small at all radial distances except at the TS of LHS 1140, hence the relatively small effect that drift has in this case.

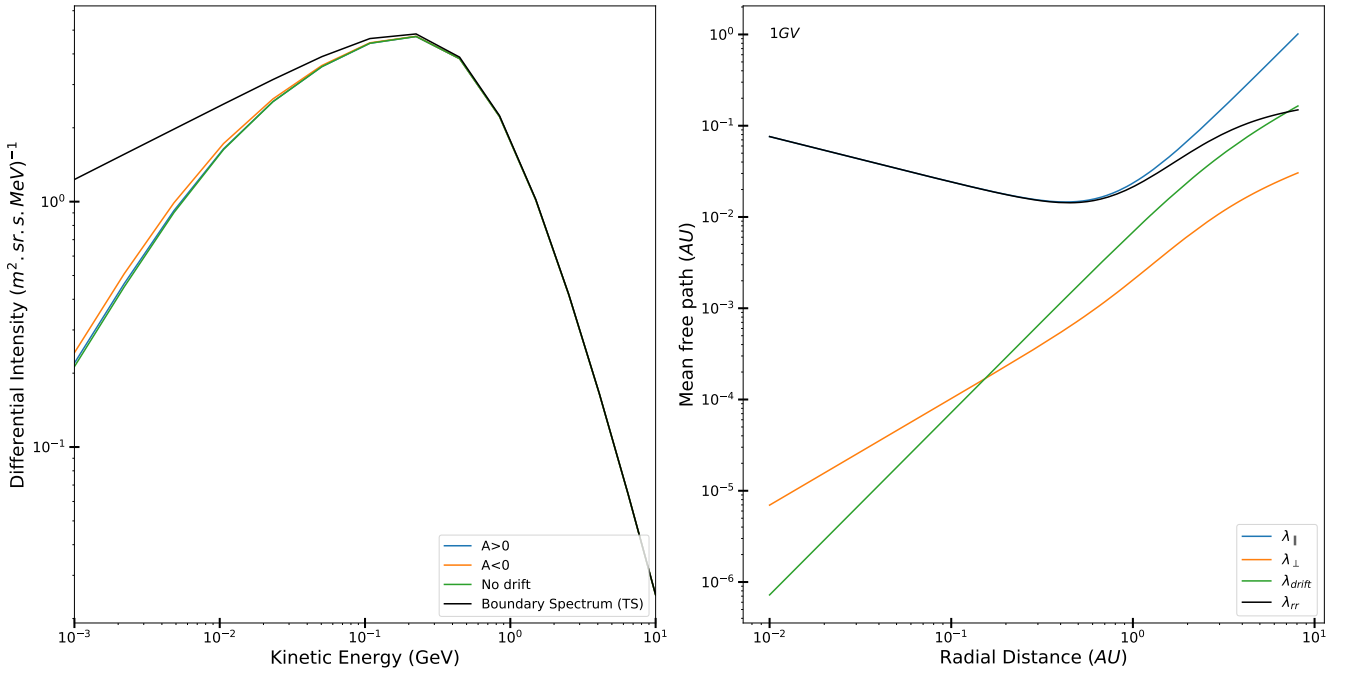


Figure 5.15: Left panel: Computed differential intensities of GCR protons at LHS 1140b for $A > 0$ (blue), $A < 0$ (orange) and no drift (green). Right panel: Computed mean free paths for 1 GV GCR protons as a function of radial distance. The parallel (blue), perpendicular (orange), drift (green) and radial lengthscales are shown.

The winding angle as a function of radial distance for LHS 1140 (black) is shown in the left panel of Fig. 5.16. The winding angle of the heliosphere (orange) and Proxima (blue) are also shown for comparison. It should be noted that the winding angles are shown up to the boundary assumed in the model for each astrosphere, 8.1 AU in the case of LHS 1140. The AMF of LHS 1140 has a winding angle of $\sim 19^\circ$ at 1 AU with a rotation rate of 131 days. Even though the rotation rate is the slowest of the stars considered in this work, the winding angle is a factor of ~ 4 larger than that of Proxima at a similar distance. The larger winding angle of LHS 1140 is due to the considerably lower stellar wind speed of 250 km.s^{-1} . This also reflects in the Parker spiral created by the AMF of LHS 1140, shown in right panel of Fig. 5.16. Even though the AMF of LHS 1140 (black) is more wound than that of Proxima (blue), it is still considerably less wound than the HMF explaining the parallel diffusion dominated transport of GCRs within the TS of LHS 1140.

5.6.3 GJ 436

The star GJ 436, discussed in Sect. 2.3.4, has an exoplanet GJ 436b orbiting at a distance of 0.028 AU (*Rosenthal et al.*, 2021). The GCR intensities within this astrosphere were calculated previously at the position of the exoplanet, see e.g., *Mesquita et al.* (2021); *Rodgers-Lee et al.* (2023). These authors used a stellar wind model to determine the stellar wind speed and magnetic field profiles. They obtained a stellar wind speed of 1250 km.s^{-1}

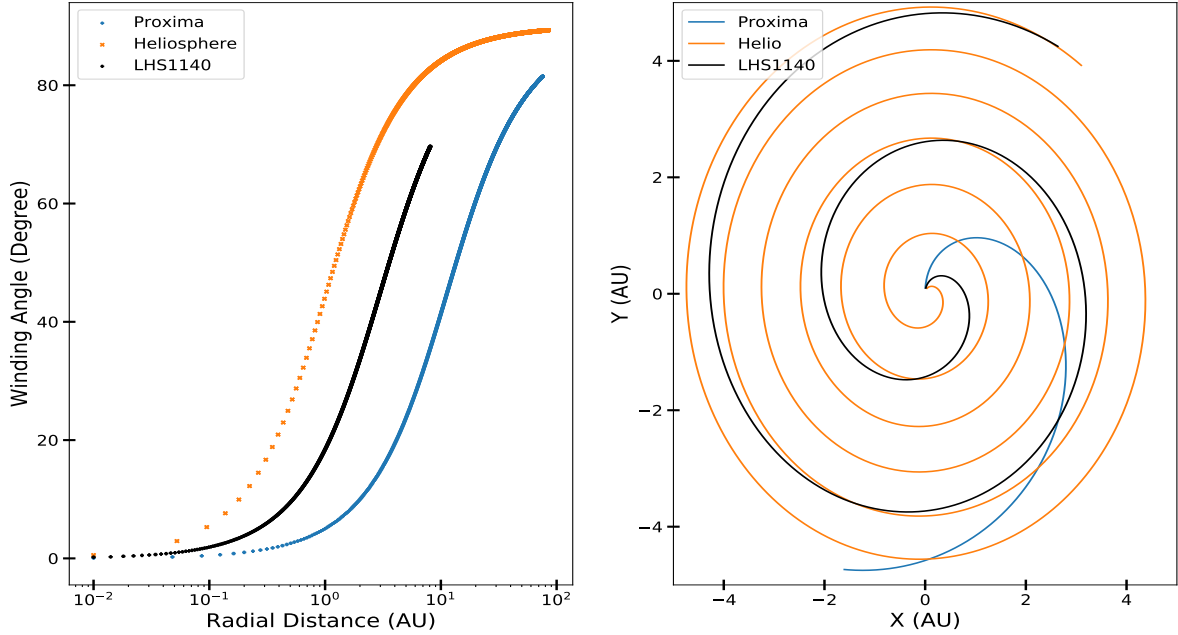


Figure 5.16: Left panel: Winding angle of LHS 1140 (black), heliosphere (orange) and Proxima (blue) as a function of radial distance. Right panel: Parker spiral created by tracing an AMF field line for LHS 1140 (black), heliosphere (orange) and Proxima (blue).

and an AMF value of $B = 3.85$ nT at 0.6 AU (see *Mesquita et al.*, 2021). The stellar wind speed of GJ 436 were also determined by fitting Lyman α observations and found to range from $250 - 460$ km.s⁻¹ (see *Villarreal D'Angelo et al.*, 2021). The magnitude of B is extrapolated to a distance of 1 AU assuming a $B \sim r^{-2}$ dependence, and a value of $B = 1.386$ nT at 1 AU was obtained, as is necessary for input to the transport model. The rotation period of GJ 436 was determined to be 44 days (see *Bourrier et al.*, 2018). Currently, no large scale MHD results exist for GJ 436 which shows TS locations. As such, the TS location of 33 AU as reported by *Mesquita et al.* (2021) is used. The stellar outflow parameters used here will be those used by *Mesquita et al.* (2021) in order to compare results.

The calculated differential intensities of GCRs at GJ 436b are shown for $A > 0$ (blue), $A < 0$ (orange) and no drift (green) in the left panel of Fig. 5.17. For energies above 1 GeV, almost no modulation is present, with drift having no effect. At lower energies, $0.01 < E < 1$ GeV, drift increases the intensities by $\sim 50\%$. A positive polarity, $A > 0$ (blue), of the AMF yields intensities that are slightly lower than intensities for a negative polarity, $A < 0$ (orange). The intensities obtained for GJ 436 are ~ 4 orders of magnitude larger than those obtained by *Mesquita et al.* (2021) at the position of GJ 436b. It should be noted that these authors made use of a 1D transport model and a different LIS, namely the LIS from *Vos and Potgieter* (2015), while the boundary spectrum from *Moloto et al.* (2018) is used in this work in a full 3D transport model. The LIS of *Vos*

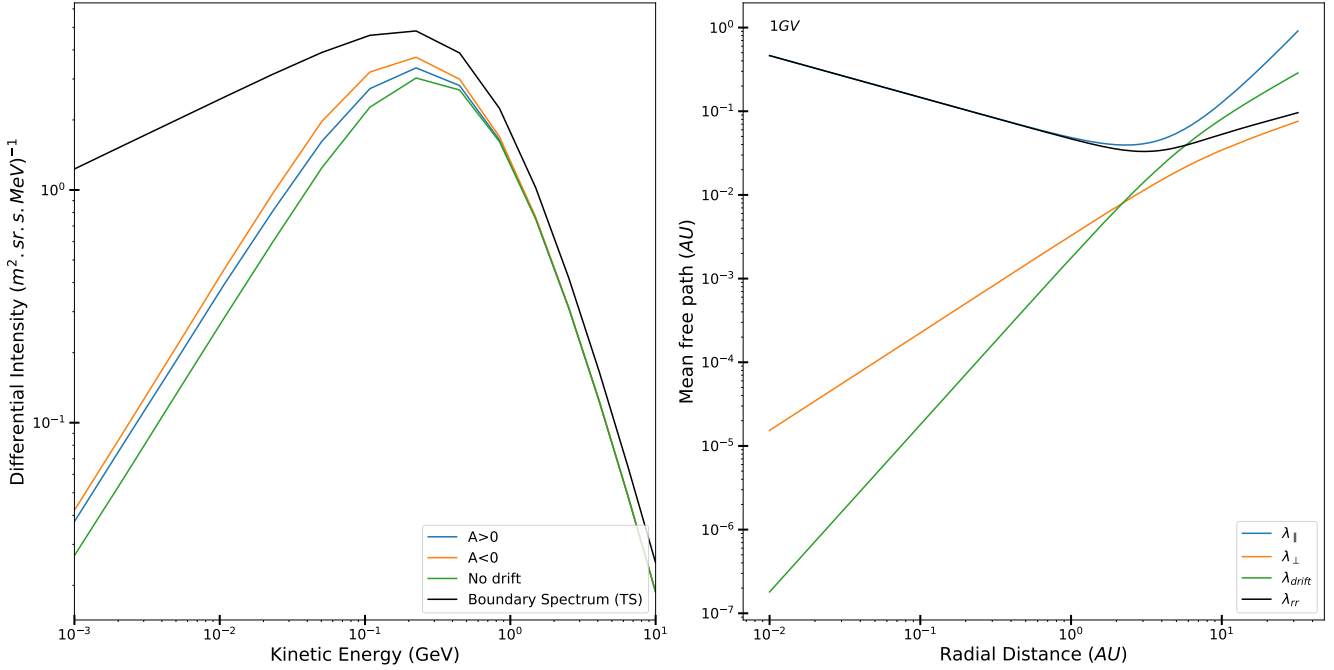


Figure 5.17: Left panel: Computed differential intensities of GCR protons at GJ 436b for $A > 0$ (blue), $A < 0$ (orange) and no drift (green). Right panel: Computed mean free paths for 1 GV GCR protons as a function of radial distance. The parallel (blue), perpendicular (orange), drift (green) and radial (black) lengthscales are shown as a function of radial distance.

and Potgieter (2015) has a value of $3.93 \text{ m}^2 \text{s}^{-1} \text{sr}^{-1} \text{MeV}^{-1}$ at 1 GeV while the boundary spectrum used here from Moloto *et al.* (2018) is a factor ~ 2 smaller with a value of $1.76 \text{ m}^2 \text{s}^{-1} \text{sr}^{-1} \text{MeV}^{-1}$. These differences may account for a factor ~ 2 difference between calculated intensities, but cannot account for the ~ 4 orders of magnitude differences between the results presented here and those of Mesquita *et al.* (2021).

As is the case for Proxima, the higher intensities calculated for a negative polarity of the AMF is due to the parallel diffusion dominating λ_{rr} close to the orbital distance of the exoplanet. This is shown in the right panel of Fig. 5.17, where the parallel (blue), perpendicular (orange), drift (green) and radial (black) lengthscales are shown as a function of radial distance. The radial (black) lengthscale is dominated by the parallel (blue) from the exoplanet location, 0.028 AU, to a distance of 1 AU. At distance greater than 1 AU, the radial MFP approaches the perpendicular (orange) MFP, but is still strongly influenced by $\lambda_{\parallel}$.

The dominance of parallel diffusion close to the exoplanet location and perpendicular diffusion closer to the TS is due to the behaviour of the winding angle of GJ 436. The winding angle of GJ 436 (black) is shown in the left panel of Fig. 5.18 as a function of radial distance. The winding angle of the heliosphere (orange) and Proxima (blue) are also shown for comparison. The winding angle of GJ 436 is similar to that of Proxima, with a rotation rate of 44 days and a stellar wind speed of 1250 km.s^{-1} compared to Proxima stellar

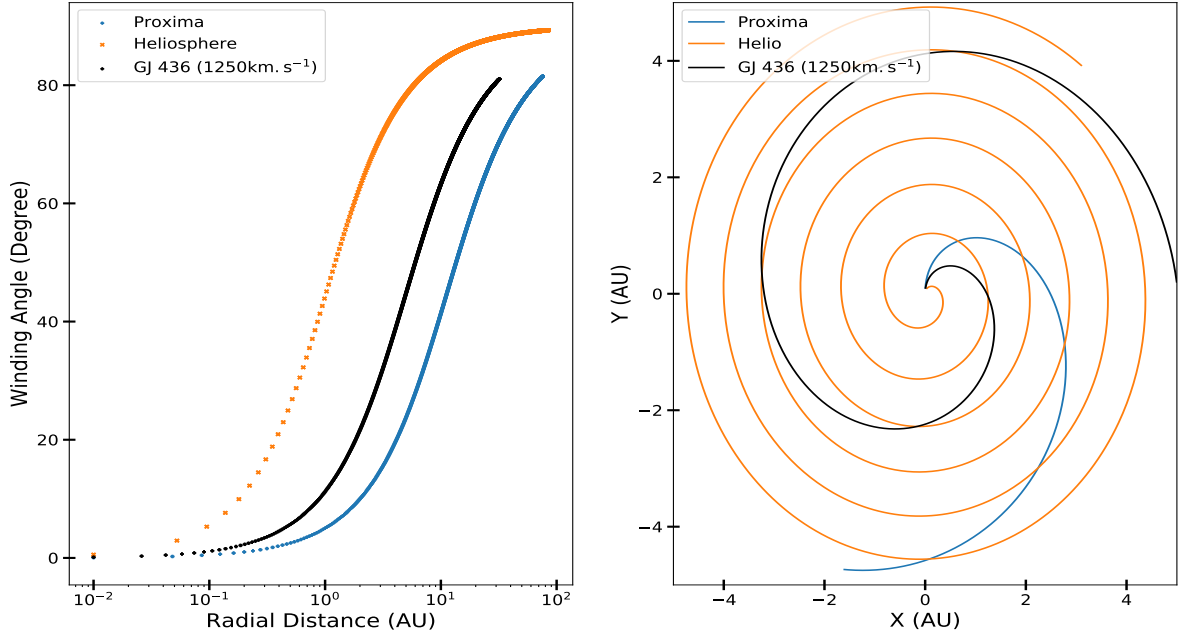


Figure 5.18: Left panel: Winding angle of GJ 436 (black), heliosphere (orange) and Proxima (blue) as a function of radial distance. Right panel: Parker spiral created by tracing an AMF field line for GJ 436 (black), heliosphere (orange) and Proxima (blue).

conditions (a rotation rate of 83 days and a stellar wind speed of 1500 km.s⁻¹). GJ 436 has a winding angle of $\sim 12^\circ$ at 1 AU compared to Proxima with 5° and the heliosphere with $\sim 45^\circ$ at the same distance. The Parker spirals of the AMF for a single magnetic field line are shown in the right panel of Fig. 5.18 for GJ 436 (black), the heliosphere (orange) and Proxima (blue). The AMF of GJ 436 is very underwound compared to the heliosphere which lends itself to parallel diffusion dominated transport. The AMF at the location of the exoplanet is almost purely radial, similar to Proxima.

5.7 Effect of winding angle

The effect of the rotation period and stellar wind speed on the modulation of GCR protons was shown in Sect. 5.5.2 and Sect. 5.6. To highlight the impact of these parameters heliospheric conditions are assumed while varying the rotation period and the stellar wind speed of the star. The boundary spectrum is placed at the TS of the heliosphere, 86 AU, while the intensities are calculated at Earth, 1 AU.

The intensities of 0.1 GeV GCR protons are shown as a function of rotation period, for $A > 0$ (blue) and $A < 0$ (orange) polarities of the AMF, in the left panel of Fig. 5.19. The stellar wind speed in this case has been kept at a constant 400 km.s⁻¹. The intensity computed for the rotation rate of the Sun, 27 days, is shown by the green circle, to guide the

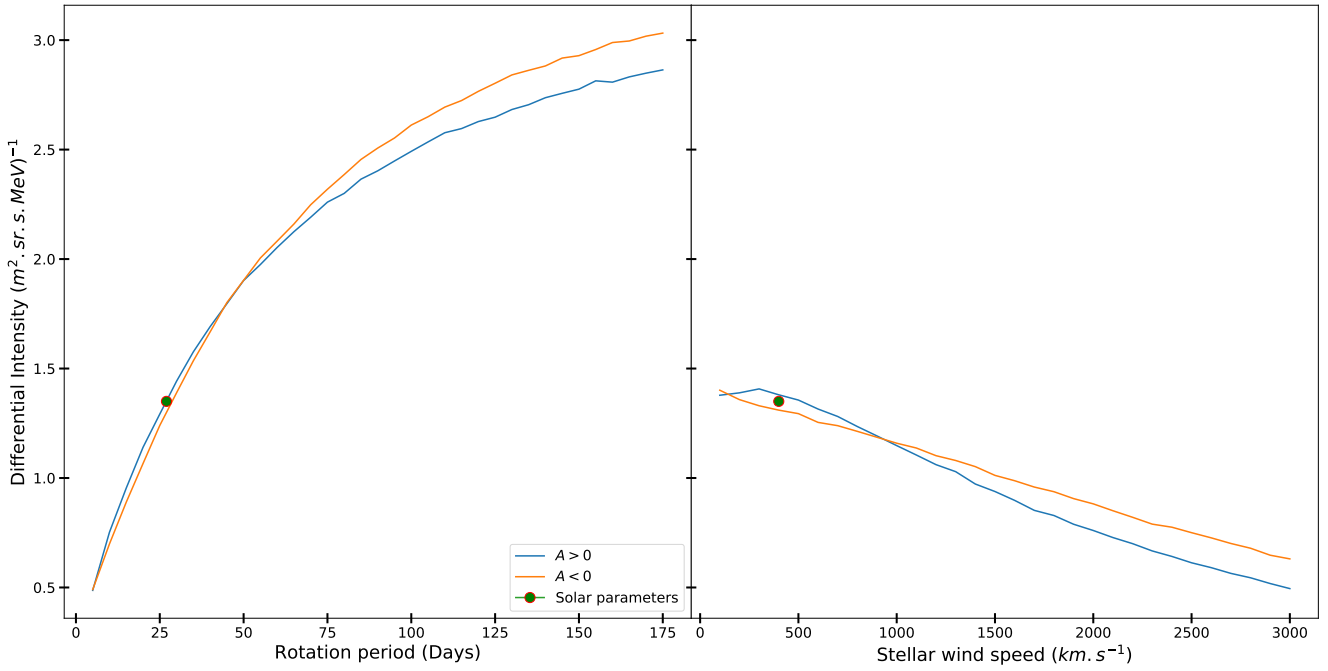


Figure 5.19: Left panel: Computed differential intensities of 0.1 GeV GCR protons as a function of different rotation rates, for $A > 0$ (blue) and $A < 0$ (orange), calculated at Earth for heliospheric conditions. Right panel: Computed differential intensities of 0.1 GeV GCR protons as a function of different stellar wind speeds, for $A > 0$ (blue) and $A < 0$ (orange), calculated at Earth for heliospheric conditions. The intensities for heliospheric conditions are marked with the green circle.

eye. Decreasing the rotation period from heliospheric conditions leads to a sharp decrease in differential intensity of GCR protons. A faster rotation rate creates a AMF that is more wound which leads to perpendicular dominated diffusion. A slower rotation rate leads to Parker spirals that are less wound and more radial. In a more radial AMF, parallel diffusion dominates. Particles travelling through an underwound AMF where parallel diffusion dominates can follow magnetic field lines from the boundary spectrum towards the exoplanet or observational point, increasing the calculated or observed intensities at the exoplanet. When heliospheric conditions are assumed there is a factor of ~ 6 increase from intensities calculated for a rotation period of 5 days to intensities calculated for a rotation period of 175 days. At slower rotation period, $P_{rot} > 50$ days, the intensities for negative AMF polarity, $A < 0$ (orange), are larger than for a positive AMF polarity, $A > 0$ (blue).

The right panel of Fig. 5.19 shows the differential intensities of 0.1 GeV GCR protons for a rotation period of 27 days as a function of stellar wind speed. Once more the intensity for the solar wind speed, $400 km \cdot s^{-1}$ (green circle) is shown to guide the eye. The differential intensities of GCR protons decrease with increasing stellar wind speed. This is again related to the winding angle of the HMF, as a larger stellar wind speed leads to a less tightly wound field.

It is not, however, the change in rotation period between different astrospheres or the change in stellar wind speed that has the largest influence on the modulation of GCRs, but rather the interplay of rotation rate and stellar wind speed in the winding angle, Eq. 5.7. A fast rotation rate creates an AMF that is more wound leading to the magnetic field lines being almost purely azimuthal. Due to the frozen in nature of the magnetic field in the stellar wind, the addition of a slow stellar wind speed that is unable to drag the AMF quickly further away from the star, yields a more wound AMF. When the field is almost azimuthal, as it is with a very wound Parker spiral, perpendicular diffusion dominates. When a fast rotation rate is coupled with a fast stellar wind speed, the AMF becomes less wound. The faster stellar wind speed is able to drag the AMF out unwinding the Parker spirals of the magnetic field lines.

A parallel diffusion dominated scenario can occur when a slow rotation rate, which creates an underwound AMF, is assisted by a fast stellar wind that is able to drag out magnetic field lines. This is similar to what was found for the case of Proxima. This leads to a massively underwound field, to such an extent that the AMF is almost purely radial at the location of the exoplanet. In an almost purely radial AMF, where parallel diffusion dominates with large parallel MFPs, the particles are able to move extremely effectively along magnetic field lines to the exoplanet. This increases the differential intensities by factor ~ 6 that was found when comparing fast and slow rotation rates.

5.8 Summary and Conclusions

This chapter introduces the 3D SDE model used by *Engelbrecht and Burger (2015a)*; *Moloto and Engelbrecht (2020)* and *Engelbrecht and Moloto (2021)* to calculate GCR modulation within the heliosphere and discussed the assumptions and changes that were made for the calculation of GCR modulation within other astrospheres. The diffusion coefficients used in this chapter were also discussed followed by how turbulence quantities were estimated within these astrospheres. As a first approach the modulation of GCRs is only considered within the inner astrosphere, and the boundary spectrum is placed at the TS. For comparison the calculation of GCR modulation within the heliosphere is compared to observational data and found to produce a good fit for both HMF polarities, ($A > 0$) and ($A < 0$).

The GCR modulation within the inner astrosphere of Proxima Centauri, at the location of Proxima Centauri b, was calculated. The behaviour of GCRs within the astrosphere of Proxima was different to what is expected from the heliosphere, as higher intensities were found for $A < 0$ and lower intensities for $A > 0$, where for the heliosphere the opposite is the case. Higher intensities are also found compared to other studies. Due to the slower rotation period and faster stellar wind speed of Proxima, it produced an underwound AMF, which leads to parallel diffusion dominating. This was followed by investigating the effect of varying the rotation period of Proxima, where faster rotation led to lower intensities while the opposite was true for slower rotations. A parameter study

was performed on how different tilt angles, colatitudinal dependent speed and different ratios of composite turbulence affect the differential intensities of GCRs at Proxima b.

The same approach that was used to calculate the modulation of GCRs in the astrosphere of Proxima, was then applied to other astrospheres containing exoplanets, namely Trappist-1 and the exoplanet Trappist-1e (*Gillon et al.*, 2017), LHS 1140 with exoplanet LHS 1140b (*Lillo-Box et al.*, 2020) and GJ 436 and the exoplanet GJ 436b (*Rosenthal et al.*, 2021). These astrospheres all have different stellar parameters, such as stellar wind speed, rotation period, AMF magnitude and astrosphere size, which influence the modulation of GCRs. Finally, in this chapter the effect that different stellar rotation periods and stellar wind speeds have on the modulation of GCRs for heliospheric conditions was shown. It is not simply the stellar rotation period or the stellar wind speed that greatly influences the intensities that are calculated for GCRs, but rather the combination of the two that lead to more underwound or overwound AMF Parker spirals which have a large influence. The influence of this inherently 3D phenomenon highlights the need to study GCR transport in astrospheres using fully 3D models. This is further emphasised by the difference in GCR intensities, calculated here vs prior 1D models.

Chapter 6

Summary and Conclusion

This thesis studied the transport and modulation of galactic cosmic rays within different astrospheres. Using a transport model, the modulation of galactic cosmic rays was computed for various astrospheres and the necessity for 3D modelling was highlighted.

Chapter 2 (Heliosphere, Astrospheres and Cosmic Rays) introduced the formation and structure of the heliosphere. The transport of galactic cosmic rays based on the *Parker* (1958) Transport Equation (TPE) was discussed. The various structures and features of the heliosphere that influence the modulation of GCRs were highlighted.

Chapter 3 (Astrospheric Evolution) discussed the different shock structures of astrospheres. It was shown that the addition of radiative cooling separates the outer structure of the astrosphere between a cold outer astrosheath (COAS) and a hot outer astrosheath (HOAS), while creating a thin dense shell surrounding the astrosphere, while the astrosphere is cooling. The effect of cooling results in the increasing of the compression ratio of the BS. The variations in the stellar outflow parameters from Luminous Blue Variable (LBV) type stars were shown to have no effect on the outer structures of the astrospheres of HD 99953 and AG Carinae. The variations are dissipated at the termination shock (TS) in the case of HD 99953 and within the inner astrosheath (IAS) in the case of AG Carinae. The addition of a stellar magnetic field also showed little effect on the structure of the astrosphere compared to when no stellar magnetic field was assumed. Using a 2D model, the eruptive phases of LBV-type stars were shown to be able to increase the size of the astrosphere, depending on the strength of the eruption. In a 3D model, it was shown that a smaller eruption dissipated within the OAS without affecting the structure and size of the astrosphere.

Chapter 4 (Cosmic Ray Modulation) introduced the stochastic differential equation (SDE) method of solving the 1D transport equation to calculate the modulation of galactic cosmic rays (GCRs) in an astrosphere. The model used to calculate the modulation of GCRs through an astrosphere, in 1D, was described. It was shown to be necessary to include the radial dependence of the stellar wind obtained via the MHD-model in the

transport model, as a constant solar wind was assumed when calculating intensities in the heliosphere. The modulation of GCRs within the astrosphere of HD 99953 was calculated and intensities at the inner boundary were higher than observed at 1 AU within the heliosphere, even though the astrosphere is considerably larger. At high energies, very little modulation occurred, while a larger amount of modulation was found for lower energies. The residence times calculated for GCRs in the astrosphere of HD 99953 were also considerably lower than those found in the heliosphere (e.g. *Strauss et al.*, 2011).

The modulation of GCRs within an astrosphere of an LBV-type star undergoing an eruption was calculated. Diffusive shock acceleration was found to take place. This effect is very large, and due to the limitations implicit to the 1D model, may represent an upper limit for this effect. It was unclear whether this effect would occur if 3D modelling was done, and points to the necessity of 3D modelling and possible limitations of 1D approaches.

The differential intensities of GCR protons were also calculated at Proxima Centauri b using the 1D SDE transport model, and compared to intensities from other studies (*Herbst et al.*, 2020; *Mesquita et al.*, 2022b). Intensities calculated were several orders of magnitude higher than some other studies. The necessity for 3D modelling was pointed out, as processes such as drift effects need to be included.

Chapter 5 (3D Cosmic Ray Modulation in Selected Astrospheres) introduced the 3D SDE model and calculated GCR modulation within the heliosphere. The changes that were necessary for the calculation of GCR modulation within other astrospheres (compared to the heliosphere) were discussed and it was shown how turbulence quantities that are used in the model needed to be estimated. In order to perform the parameter study, only modulation within the TS of the astrospheres was considered.

The GCR modulation within the TS of Proxima Centauri, at the location of Proxima Centauri b, was calculated. The behaviour of the differential intensities calculated for the astrosphere of Proxima Centauri was found to be opposite to what is expected from the heliosphere with higher intensities found for $A < 0$ and lower intensities for $A > 0$. Higher intensities were also found compared to other studies. The slower rotation period and faster stellar wind speed of Proxima produced an underwound astrospheric magnetic field (AMF), which led to parallel diffusion dominating. It was also found that faster rotation led to lower intensities.

The GCR differential intensities in other exoplanet containing astrospheres, namely Trappist-1, LHS 1140 and GJ 436, were calculated. The different stellar parameters of these stars highlighted the effect of rotation period and led to different intensities. This effect was further illustrated by varying the rotation period and stellar wind speed of the heliosphere. It was shown that the rotation period and stellar wind speed greatly influenced the intensities that were calculated for GCRs, but the combination of the two that led to more underwound or overwound AMF Parker spirals had a larger influence. It was subsequently shown that this inherently 3D phenomenon highlights the need to study GCR transport in astrospheres using fully 3D transport models. This is further emphasised by the difference

in GCR intensities calculated here compared to previous attempts.

Avenues for future work could be to use 3D MHD model results for the astrospheres as input for the 3D transport model, and also include turbulence transport models such as those of *Oughton et al.* (2011) and *Wiengarten et al.* (2016) to model the turbulence quantities within these astrospheres instead of scaling heliospheric observations. The sophisticated 3D stellar wind models of *Vidotto et al.* (2023) can also be used as input for a potentially more accurate representation of the stellar magnetic field. The model could also be made time-dependent, as was done for the heliosphere (see, e.g., *Moloto and Engelbrecht*, 2020), and include the observed ~ 7 year stellar cycle of Proxima Centauri. A larger survey of astrospheres with varying stellar parameters could also be considered. A 3D model could investigate the diffusive shock acceleration for LBV stars undergoing eruptions.

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