

Statistical methods for constraining pulsar geometries via multiband light curve fitting

MC Bezuidenhout

 [orcid.org 0000-0001-9541-7439](https://orcid.org/0000-0001-9541-7439)

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Supervisor:	Prof C Venter
Co-supervisor:	Dr AK Harding

Abstract

The Fermi Large Area Telescope (LAT) is a high-energy satellite that operates in the MeV to GeV band. Fermi has revolutionised pulsar physics by its clear detection of more than 200 of these objects in γ -rays. Since its launch in 2008, there have been various attempts to exploit both γ -ray and radio pulsar data to obtain constraints on the geometry and emission sites of pulsars. By concurrently fitting standard geometric model light curves onto Fermi and radio data, researchers have constrained the inclination and observer angles of a number of pulsars. At first this was done by comparing observed and modelled light curves by eye, and later via statistical approaches. Such statistical approaches are, however, complicated by the disparate error margins on the available radio and γ -ray pulsar observations. In this work, I fit modelled light curves of 16 pulsars to radio and γ -ray data both by eye and by optimising a bespoke test statistic developed by Seyffert et al. (in preparation) for combining light curves across the two wavebands, taking their disparate errors into account. I present geometrical constraints found using these approaches, and compare them with results found by Johnson et al. (2014) and Pierbattista et al. (2015) for the same pulsars.

Keywords: astrophysics, pulsars, light curve modelling, goodness of fit, Pearson's χ^2 test, gamma rays, radio.

Nomenclature

CR	Curvature Radiation
FIDO	Force-free Inside, Dissipative Outside
ICS	Inverse Compton Scattering
ISS	International Space Station
LAT	Large Area Telescope
LC	Light Curve
MSP	Millisecond Pulsar
NICER	Neutron Star Interior Composition Explorer
NS	Neutron Star
OG	Outer Gap
PC	Polar Cap
PFF	Pair Formation Front
PPMCC	Pearson Product-Moment Correlation Coefficient
RVD	Rotating Vector Dipole
SFN	Scaled-Flux Normalised
SG	Slot Gap
SR	Synchrotron Radiation
TPC	Two-Pole Caustic

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Chapter 1

Introduction

Pulsar observations have traditionally been dominated by data from the radio wavelength range, with sources in this band totalling 2635 at the time of writing, according to the ATNF Pulsar Catalogue¹ (Manchester et al., 2005). This stands in comparison with the fewer than ten sources which had been detected in the γ -ray domain prior to 2008 (Thompson, 2001). Despite the relative abundance of radio observations, the underlying mechanisms of the emission at these wavelengths have proven most difficult to understand, compounded by the fact that it constitutes a relatively small fraction, in terms of luminosity, of a pulsar's total emission. The γ -ray waveband, on the other hand, often comprises up to 100% of a pulsar's rotational energy loss-powered radiation, making observations in this energy range more powerful tools for probing pulsars' magnetospheric structure and particle acceleration. However, for a long time the lack of available γ -ray data impeded research in these areas.

The LAT's launch in 2008 marked the employment of the first instrument sensitive enough to detect a significant number of γ -ray photons from pulsars. It has discovered² 211 new γ -ray pulsars at high sensitivity and resolution, ushering in what has been called a γ -ray pulsar revolution (see Section 2.1 for a detailed review). Multi-wavelength pulsar studies can now be done that aid our understanding of pulsar emission in both wavelength domains.

A current pertinent question in pulsar astrophysics is reconciling the wealth of multi-band light curve (LC) observations of rotation-powered pulsars supplied by the Large Area Telescope (LAT) aboard the *Fermi* spacecraft, among others, with the predictions made by different available geometric models of these pulsars.

This exciting new avenue for research has not, however, come without complications. Since detected radio photons are much more numerous than detected γ -ray photons, the available radio and γ -ray datasets are characterised by very different errors, impeding their concurrent use. The aim of this thesis is to work towards achieving congruency of these data, so that they can be used simultaneously to constrain the predictions of pulsar models.

One can apply the χ^2 test statistic in each domain to determine how closely the models' predictions fit the observed LCs. By finding the set of model parameters which minimise the value of χ^2 , one can infer the most likely physical configuration in terms of tilt angle (α) and observer angle (ζ). Unfortunately, individual fitting of γ -ray and radio models to the data deliver formally bad fits, with values of χ^2 much larger than the number of degrees of freedom. Moreover, the available γ -ray and radio geometric models often produce very different α and ζ constraints when applied separately. As such, my aim is to find a compromise between the best fits implied by the respective models, i.e. to find the set of model parameters which most satisfactorily reproduce the observations in both domains simultaneously, using a number of

¹ Accessible online at <http://www.atnf.csiro.au/people/pulsar/psrcat/> [date of access: 13 March 2018].

² <https://confluence.slac.stanford.edu/display/GLAMCOG/Public+List+of+LAT-Detected+Gamma-Ray+Pulsars> [date of access: 16 March 2018].

methods for combining χ^2 statistics. These results can then be compared with those found by other studies, namely Johnson et al. (2014) and Pierbattista et al. (2015), for the same pulsars, and I can assess the utility of the methods that we have developed for combining the radio and γ -ray data. In this way we can learn to utilise the admittedly disparate predictions of the available pulsar models to best constrain observed pulsars’ geometries.

1.1 Pulsar geometry

Rotation-powered pulsars are very rapidly rotating, highly magnetised neutron stars (NSs) with beamed radio emission originating from either magnetic pole, and the magnetic axis tilted with respect to the rotation axis. The rotation of a pulsar’s magnetic axis $\boldsymbol{\mu}$ (and therefore emission regions) about the spin axis $\boldsymbol{\Omega}$, along with the beamed nature of this emission, causes the pulsar to be observed as a quickly pulsating light source, raising and dipping in observed flux as the emission beam passes through our line of sight. This is most often represented in the form of LCs, which are plots of a source’s intensity over time. One such LC, for the well-known Vela pulsar (PSR J0835–4510), measured in both radio and γ -rays, is shown in Figure 1.1 for two full rotations. The point in phase labelled $\phi = 0$ is defined as the point where the fiducial plane, chosen to be the plane containing both the pulsar’s rotation and magnetic axes, passes by the observer³. Note that the errors on the γ -ray measurements are much larger than those on the radio data.

Pulsars possess a very strong dipolar-like magnetic field, with a magnitude of around 10^{11-14} G at the surface, tilted with respect to the spin axis, the structure of which is illustrated in Figure 1.2. Important to note is the existence of the so-called *light cylinder*, the distance from the centre of the NS at which the corotation speed equals the speed of light. Further out the magnetic field lines cannot close and are instead swept open by currents. Also indicated in Figure 1.2 is the inclination angle between the pulsar’s spin axis $\boldsymbol{\Omega}$ and magnetic axis $\boldsymbol{\mu}$, or *tilt angle* (α), and the angle to Earth’s line of site with respect to $\boldsymbol{\Omega}$, called the observer angle (ζ). The difference between α and ζ is often labelled $\beta \equiv \zeta - \alpha$, and called the *impact angle*.

1.2 Research context and aims

Johnson et al. (2014) found constraints on α and ζ for 40 millisecond pulsars (MSPs) using multi-wavelength (radio and γ -ray) LC fitting. Pierbattista et al. (2015) did the same for 80 young or middle-aged pulsars. These results were obtained by applying χ^2 tests, which accumulate the square of the difference between the expected and observed LC at each point in phase (see Section 4.2), separately for each waveband. In this way so-called χ^2 *maps* can be constructed that give the value of the test statistic at each point in α - ζ space, the minimum of which indicates the best-fit (α, ζ) pair for a given pulsar.

Normally, the χ^2 maps in radio and γ -rays would simply be added together, with the minimum in the combined map representing the concurrent best-fit parameters. However, the test statistic is inversely proportional to the square of the data’s margins of error, which are much larger for γ -ray than radio LCs. The values of the test statistic for the γ -ray data will therefore be orders of magnitude larger than those for the radio data, meaning that the map produced by their addition would be dominated by the radio dataset, producing relatively poor fits in the γ -ray band.

³In practice, $\phi = 0$ may also be assigned to the phase at which the maximum radio flux occurs.

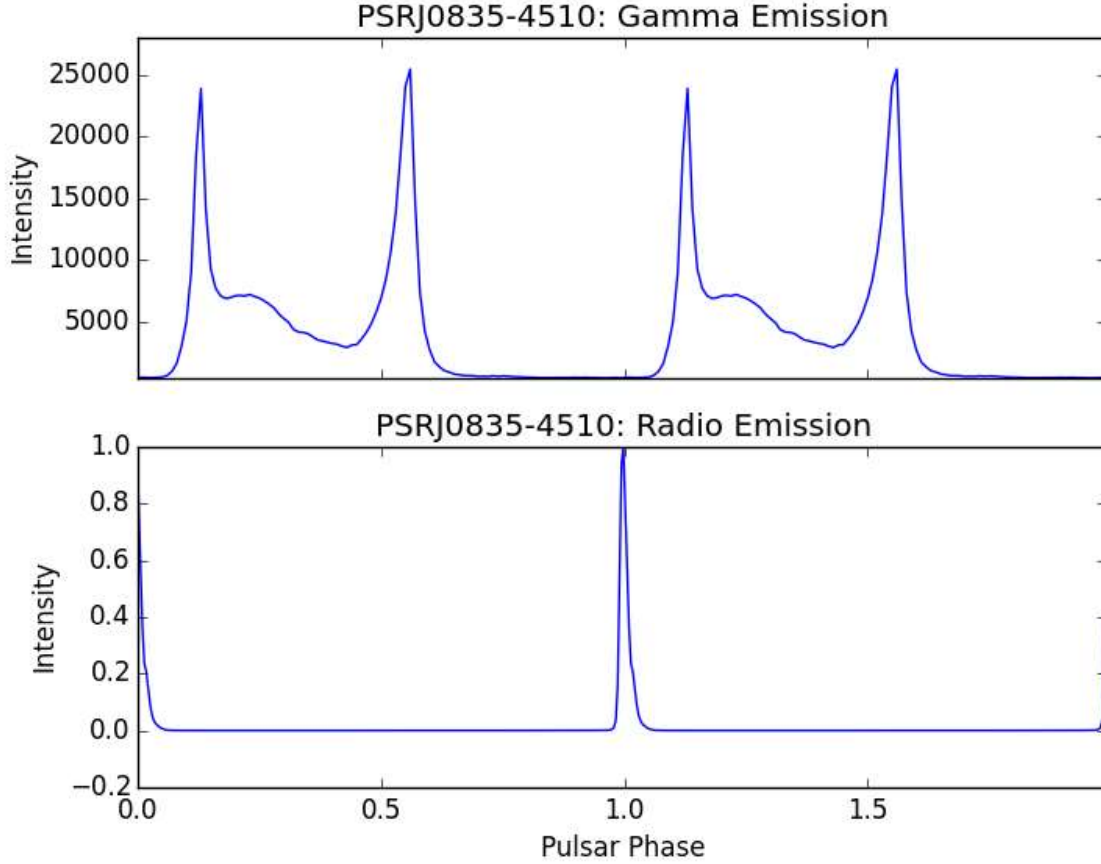


Figure 1.1: Observed light curves of the well-known Vela pulsar (PSR J0835–4510) in the γ -ray (top) and radio (bottom) domains.

Both Johnson et al. (2014) and Pierbattista et al. (2015) circumvented this problem by artificially inflating the errors on the radio data, placing the radio data on equal footing with the γ -ray data, before addition of their χ^2 maps. Although this workaround provided better concurrent fits than would be found by simple addition, the radio LCs were still not well-reproduced. The degree of inflation also had to be tailored to each individual pulsar, a process that is cumbersome to apply in the future. Besides, this approach is unfortunate in that it artificially reduces the quality of the radio data.

In Section 4.3 we introduce a method for combining the radio and γ -ray χ^2 maps suggested by Seyffert et al. (in preparation) that:

- aims to result in a combined map on which neither domain is overrepresented,
- can be applied consistently to any given pulsar without need of arbitrary error adjustment,
- maintains the integrity of the radio dataset.

In Chapter 5 I apply the aforementioned approach to four pulsars chosen from Johnson et al. (2014) and twelve pulsars chosen from Pierbattista et al. (2015). I then compare the constraints found using this new method with what the other studies found for the same pulsars, along with the constraints on α and ζ found by simple by-eye estimation. In Appendix B I present the best-fit LCs produced using each method, overlaid onto the

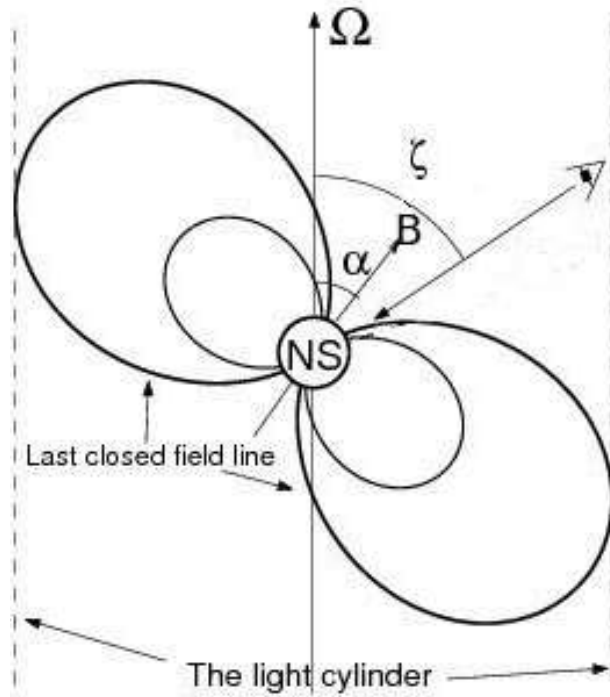


Figure 1.2: A simplified schematic diagram of two geometric pulsar parameters. The spin axis (Ω), magnetic field axis (μ), tilt angle (α), observer angle (ζ) and light cylinder radius are all shown.

observed LC, for all 16 pulsars considered in this study. I draw conclusions as to the utility of this new approach compared to others in Chapter 6.

The main thrust of this work therefore concerns overcoming the disparity between the errors on the radio and γ -ray observational data that complicate the use of traditional goodness-of-fit tests. The secondary aim is to determine which combination of α and ζ best reproduces the observed LCs, in both radio and γ -ray bands, for each of the sample pulsars.

1.3 Thesis structure

The rest of this thesis is structured as follows:

Chapter 2 provides an overview of certain concepts in pulsar astrophysics relevant to this work. I provide a short history of the field, as well as an introduction to the basic properties associated with pulsars. I also give an overview of the emission mechanisms thought to be operating in pulsars, and the emission models based thereupon.

Chapter 3 describes the magnetic field structure assumed by this study as well as the geometric pulsar models that I use to predict LCs, namely the outer gap (OG) and two-pole caustic (TPC) geometric γ -ray models and the empirical radio models.

The purpose of **Chapter 4** is to lay out the methods used for the fitting of modelled to observed LCs. I discuss how LCs can be fit by eye, as well as by using a χ^2 statistical test. I also present the approach of Seyffert et al. (in preparation) towards combining the individual radio and γ -ray χ^2 maps.

In **Chapter 5** I submit the constraints on the α and ζ geometries of the sample pulsars

found using the by-eye and statistical fitting methods. Furthermore, I compare these constraints with those found by Johnson et al. (2014) or Pierbattista et al. (2015) for the same pulsars.

Chapter 6 presents my conclusions as to the propriety of our new fitting method towards obtaining reliable constraints on pulsar geometry, and provides suggestions for further work on this subject.

In **Appendix A** I present for each pulsar four by-eye inclusion contours similar to Figure 5.2, one for each of the four possible model combinations. Overlaid on these is the best-fit (α, ζ) pair (with errors) we found using SFN statistical fitting, and that found by either Johnson et al. (2014) or Pierbattista et al. (2015).

Appendix B collates the best-fit LCs predicted by the geometric models using the constraints found by each fitting method, superimposed onto the observed LC for specific pulsars in both the radio and γ -ray domain.

1.4 Publications

The following publications have resulted from, or are otherwise associated with, this thesis:

- M. C. Bezuidenhout, C. Venter, A. S. Seyffert, and A. K. Harding. *Calibration of statistical methods used to constrain pulsar geometries via multiband light curve modelling*. Proc. 61st Annual Conference of the South African Institute of Physics (SAIP2016), ISBN: 978- 0-620-77094-1, ed. S. Peterson and S. Yacoob, 207–212 (arXiv:1712.08178).
- A. S. Seyffert, C. Venter, A. K. Harding, J. Allison, and WD Schutte. *Implementation of a goodness-of-fit test for finding optimal concurrent radio and gamma-ray pulsar light curves*. Proc. 60th Annual Conference of the South African Institute of Physics (SAIP2015), ISBN:978-0-620-70714-5, ed. Makaiko Chithambo & André Venter, 350–355 (arXiv:1611.01076).
- Seyffert et al. (in preparation)
- Bezuidenhout et al. (in preparation)

Chapter 2

Pulsar astrophysics

This chapter introduces the reader to certain important concepts that are required as background to understand the rest of this study. In particular, I cover the history of pulsar observations, basic pulsar properties, pulsars' formation and classification, the mechanisms by which they convert rotational kinetic energy into the emission we observe, and the available physical models that attempt to explain this emission.

2.1 A history of pulsar observations and interpretation

The existence of NSs, proven by their discovery in 1967, had been anticipated by theorists decades earlier, starting with the calculation of the limit of the mass a white dwarf star can support by electron degeneracy pressure alone (Chandrasekhar, 1931). This so-called *Chandrasekhar limit* indicated that collapsed stellar cores heavier than $\sim 1.4M_{\odot}$ would keep on collapsing, eventually forming a singularity in spacetime. Soon after James Chadwick's discovery of the neutron in 1932 (Chadwick, 1932), Lev Landau speculated about the existence of stars composed entirely of these particles, and supported by neutron degeneracy pressure. Using Fermi-Dirac statistics and basic quantum mechanics, he was able to derive an approximate radius for these stars: $\sim 3 \times 10^5$ cm, impressively close to the modern estimate of $\sim 10^6$ cm (Landau, 1932).

Walter Baade and Fritz Zwicky coined the term *supernova* in 1931, initially believing the phenomenon to be a subclass of *novae*, a sudden brightening of white dwarfs due to cataclysmic nuclear explosions on their surface. Three years later, however, Baade and Zwicky (1934) suggested that supernovae instead accompany the transition of a star into a significantly less massive body through gravitational collapse, and that NSs might be the remnants of these explosions. Oppenheimer and Volkoff (1939) calculated that stars heavier than $\sim 3M_{\odot}$ would collapse into black holes (the so-called Tolman-Oppenheimer-Volkoff limit). This result was obtained using the first detailed model of NS structure, which was based on the equation of state of a cold Fermi gas and which incorporated general relativity. Unfortunately, the idea of NSs remained fairly obscure, offering no observable predictions, until a series of fortuitous discoveries in the 1960s.

On 12 June 1962 an Aerobee rocket launched by a team led by Bruno Rossi and Riccardo Giacconi serendipitously detected X-rays from Sco X-1, now known as the brightest extra-solar X-ray source in the sky. Originally meant to search for fluorescent X-ray photons from the Moon, they instead confirmed the suggestion put forward by Morrison et al. (1954) that certain celestial bodies, such as supernova remnants, produce high-energy cosmic rays which may in turn produce X-rays and γ -rays. Rossi and Giacconi won the 2002 Nobel Prize in physics for this discovery. Tau X-1, a source in the constellation Taurus coinciding with

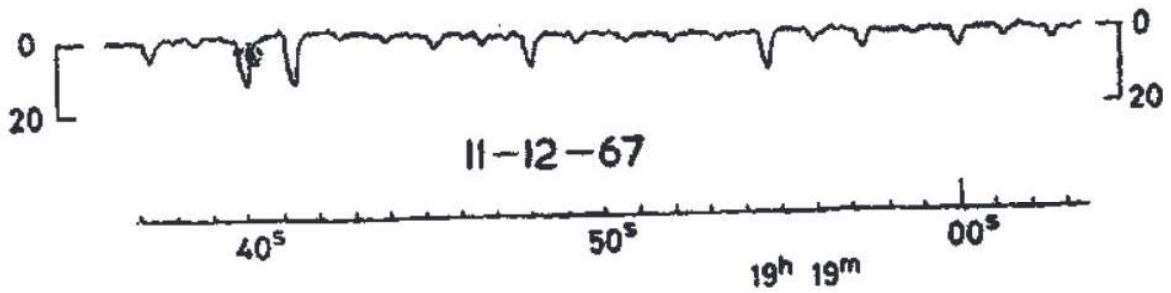


Figure 2.1: “High-speed” chart recording of the first known pulsar, PSR J1921+2153. Adapted from Hewish et al. (1968).

the Crab supernova remnant, was also detected in X-rays during subsequent flights (Bowyer et al., 1964). Chiu and Salpeter (1964) suggested that the detected X-rays were the result of thermal radiation from the surface of a hot NS. However, the early X-ray observations were not considered sensitive enough to confirm the existence of NSs, a task which was left to radio astronomers years later.

Wheeler (1966) and Pacini (1967) independently proposed that the power emitted by the Crab Nebula in the Taurus constellation was provided by the loss of rotational energy of an NS at its centre. At about the same time, Rudolph Minkowski pointed out that one of the stars in a photo of the Nebula emitted peculiar blue light with a spectrum quite different from the surrounding stars, although he could not prove the star to be part of the Nebula (Minkowski, 1964). He could not have known that he had in fact identified what is now known as the Crab pulsar.

Theory had for once, quite unbeknownst to the physicists working on these problems at the time, far outpaced observationalists; 35 years after Landau’s first speculations, the evidence confirming the existence of NSs materialised. In 1967, Jocelyn Bell was working as a graduate student at the University of Cambridge, England, under the supervision of Antony Hewish on a radio telescope designed to look for rapid modulation of radio signals by the solar wind. What Bell noticed amongst the deluge of data available to her was a faint source of very regularly pulsed radio emission, the pulses lasting 0.016 seconds and recurring every 1.34 seconds, with astounding accuracy (Hewish et al., 1968). Figure 2.1 shows the chart in Bell’s survey data in which she identified the manifestation of the first pulsar, later known as CP1919 or PSR J1921+2153. In fact, the extraordinary regularity of the pulse at first led the team to half-jokingly dub the source LGM1 (Little Green Man 1), speculating that its signal might originate from an intelligent extraterrestrial race. They were later relieved to discover three more of these pulsing sources, allowing them to scrap the LGM-hypothesis and rename the phenomenon *pulsars* after ‘pulsating radio sources’. Although Bell had uncovered the signal, it was Hewish, her supervisor, who won the physics Nobel Prize for their discovery in 1974.

Pacini (1967) and Gold (1968) were the first to link pulsars to fast-rotating NSs. The former proposed that the rapid rotation of a highly magnetised NS or pulsar could be an energy source in the Crab Nebula, while the latter recognised that pulsars’ emissions could be at the expense of their rotational kinetic energy, introducing the concept of the *rotation-powered pulsar*. Gold (1968) also speculated that this radiation originates from the plasma in the pulsar’s magnetosphere, where rotational energy is lost via magnetic dipole radiation and relativistic particle ejection. Staelin and Reifenstein (1968) detected radio pulses from the remnant star in the Crab Nebula, finally cementing the link between supernovae, NSs, and pulsars. Goldreich and Julian (1969) demonstrated that pulsars’ magnetospheres must

necessarily be dense with plasma.

The SAS-1 (Small Astronomy Satellite 1), later known as *Uhuru*, was launched by NASA in 1971 to survey the sky in the range 2 – 20 keV (Giacconi et al., 1971). In a little more than two years, the satellite had detected 339 new X-ray sources, the majority of which were accretion-powered pulsars, i.e. NSs in binary systems powered by accretion from their partner. SAS-2 was launched in November 1972, operating in the range 35 – 1 000 MeV, and being the first satellite devoted to γ -ray astronomy. SAS-2 detected γ -rays from the Crab and Vela pulsars (Thompson et al., 1975), as well as from a number of unidentified sources, one of which would turn out to be the Geminga pulsar. Geminga had been identified by SAS-2 as a faint source in Gemini, from which only about ~ 100 γ -ray photons had been detected, and had to wait another 20 years for its final identification. This serves to illustrate the relative scarcity of detectable photons and large positional error boxes that characterised γ -ray astronomy since its infancy, the ramifications of which are partially the subject of this work.

Truemper et al. (1978) were the first to estimate the strength of the pulsar’s magnetic field as $(4\text{--}6) \times 10^{12}$ G, using observations of a strong line feature at ~ 58 keV in the accreting binary Hercules X-1’s X-ray spectrum, which they interpreted as representing resonant electron cyclotron emission or absorption in the hot polar plasma of the rotating NS.

Beginning in 1977, several important pulsar observations were made using a series of large scientific payloads called the *High Energy Astrophysical Observatories* – HEAO 1 (Aug. 1977 – Jan. 1979); HEAO 2 (Nov. 1978 – Apr. 1981); and HEAO 3 (Sep. 1979 – May 1981). The *European X-ray observatory satellite* (EXOSAT) operated from May 1983 to April 1986 in the 0.04 – 2 keV band and made X-ray observations of most classes of astronomical object, including isolated pulsars. ROSAT (short for *Röntgensatellit*) made X-ray observations at a sensitivity $\sim 1\,000$ times greater than *Uhuru* from June 1990 to February 1999, providing valuable information on the fluxes of all known X-ray pulsars. EUVE (*Extreme Ultraviolet Explorer*) observed various pulsars in soft X-rays from June 1992 to January 2001, and RXTE (*Rossi X-ray Timing Explorer*; Dec. 1995 – Jan. 2012) studied X-ray binaries.

Pulsars would go relatively unstudied in γ -rays until 1991, when EGRET (*The Energetic Gamma Ray Experiment Telescope*) was installed aboard the *Compton Gamma Ray Observatory* (CGRO). EGRET operated in the 30 MeV to 30 GeV range, and detected five new pulsars before being decommissioned in 2000. The *Astro-Rivelatore Gamma a Immagini Leggero* (AGILE) satellite, launched in 2007 and operating in both the X-ray and γ -ray domains, also detected these new pulsars. The five newly-discovered pulsars, along with the Crab and Vela pulsars, remained the only to have been found in γ -rays prior to the launch of the LAT aboard *Fermi*.

The LAT is the principal scientific instrument aboard the *Fermi* γ -ray space telescope (formerly known as the *Gamma-ray Large Area Space Telescope* [GLAST]), which was launched into near-earth orbit on 11 June 2008. The LAT covers a range of about 20 MeV to >300 GeV, and scans the sky continuously, covering the full sky once every three hours (Atwood et al., 2009). To date the LAT has found 205 γ -ray pulsars at high sensitivity, representing a dramatic increase in the number of sources in this regime, although, by the nature of γ -ray LC observations, measurements of these sources are still significantly less precise than those of pulsars in the radio band. It is in this context that this study is performed: with so many new γ -ray sources to be studied, we are looking for a way of combining these data with higher accuracy radio data to the benefit of modelling efforts spanning both wavebands.

2.2 Pulsar formation and properties

A supernova occurs when a star has burnt up all its fuel, having converted through fusion all its lighter constituent elements into iron, the thermonuclear burning of which is endothermic. In the absence of radiation pressure from nuclear fusion, the star implodes under its gravitational potential. During the star’s collapse, a “hydrodynamic bounce” may occur when electron degeneracy becomes important, sending a shock through the star and violently ripping away its outer layers in a spectacular supernova explosion. What is left of the core is either a white dwarf, an NS, or a black hole, depending on the progenitor star’s mass, and representing the three major stellar evolutionary endpoints (Heger et al., 2003).

The two fundamental limits that dictate which evolutionary endpoint a star is destined for are the Chandrasekhar limit ($\sim 1.4M_{\odot}$) and the Tolman-Oppenheimer-Volkoff limit ($\sim 3M_{\odot}$). If the remnant of the supernova is less massive than the first limit, it can be supported by electron degenerate pressure. Should the residual stellar core exceed this limit, however, the Fermi energy of the electrons becomes so high that it is energetically advantageous for the stellar matter to neutronise, the electrons combining with the protons through inverse β -decay to form a degenerate neutron gas sometimes called *neutronium*. If the remnant mass exceeds even the second mass limit, the core’s mass can no longer be borne by neutron degenerate pressure, and it collapses to a size smaller than the Schwarzschild radius, at which point the escape velocity from its surface surpasses the speed of light, and a black hole is formed (Schwarzschild, 1916).

It is thought that stars with masses in the approximate range of $8M_{\odot}$ to $30M_{\odot}$ eventually form NSs—lighter stars become white dwarfs, and supernovae from heavier stars result in black holes. Since NSs are supported against gravitational collapse entirely by neutron degeneracy pressure, they are extremely compact, with a mass of $1.4M_{\odot}$ to $3M_{\odot}$ compressed into a sphere with a diameter in the order of tens of kilometres. Retention of the progenitor star’s angular momentum results in extremely rapid rotation – up to hundreds of revolutions per second. If it is assumed that a stellar core’s angular momentum is conserved during its implosion, its angular velocity Ω after collapse is given by the equation

$$\Omega_f = \Omega_i \left(\frac{R_i}{R_f} \right)^2, \quad (2.1)$$

where R represents the core’s radius, and the subscripts i and f denote these values initially (before collapse) and finally (after collapse), respectively. For typical values of $R_i \sim 10^{11}$ cm and $R_f \sim 10^6$ cm, this constitutes a factor $\sim 10^{10}$ increase in angular speed, and yields a spin period in the order of milliseconds.

Assuming the stellar core is fully conductive, the magnetic flux through it ($\Phi_B \equiv \oint \mathbf{B} \cdot d\mathbf{a} \sim BR^2$), is also conserved. The associated magnetic field strength must therefore be similarly amplified:

$$B_f = B_i \left(\frac{R_i}{R_f} \right)^2. \quad (2.2)$$

This yields a typical magnitude of around 10^{12} G at the surface, around a trillion times stronger than that of the sun.

Due to the conversion of its rotational energy into electromagnetic radiation and particle acceleration, an NS will spin down, its period lengthening very slowly over time. A typical pulsar might have a period $\sim 10^{-2}$ seconds decreasing at a rate of $\sim 10^{-14}$ seconds per second. Over the course of a million years ($\sim 10^{13}$ seconds), its period will have changed by

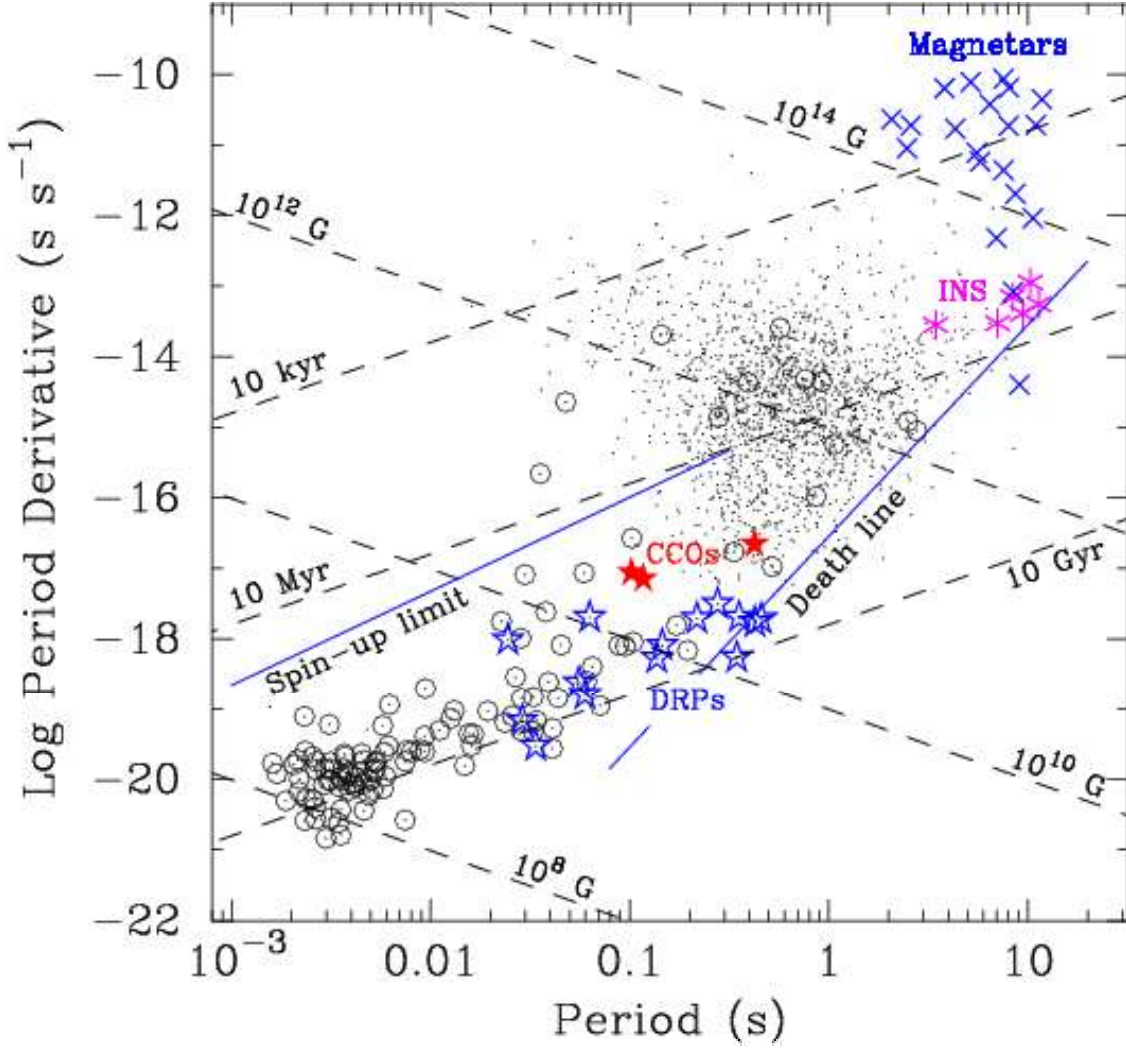


Figure 2.2: $P - \dot{P}$ diagram taken from Gotthelf et al. (2013). The dashed lines with positive slope represent lines of constant rotational age, whereas dashed lines with negative slope represent lines of constant polar magnetic field.

roughly a tenth of a second. It is for this reason that pulsars can be used as reliable clocks to very precisely measure time¹.

By equating a pulsar's rate of rotational energy loss to the vacuum magnetic dipole radiation loss rate, one can find an approximation of the field's strength at the magnetic pole B_0 (Ostriker and Gunn, 1969):

$$\dot{E}_{\text{rot}} = \frac{d}{dt} \left(\frac{1}{2} I \Omega^2 \right) = I \Omega \dot{\Omega} = -\frac{2}{3c^3} \mu^2 \Omega^4 \sin^2(\alpha), \quad (2.3)$$

where $I \sim MR_f^2$ is the moment of inertia, $\mu \equiv \frac{1}{2} B_0 R_f^3$ is the magnetic moment, and α is the angle between the magnetic and rotational axes. Inserting typical values of $I = 10^{45} \text{ g cm}^2$, $R_f = 10^6 \text{ cm}$, and $\alpha = 90^\circ$, and solving for B_0 , we find:

$$B_0 = \frac{\sqrt{6c^3 I P \dot{P}}}{2\pi R^3} \simeq 6.4 \times 10^{19} \sqrt{P \dot{P}} \quad (2.4)$$

in Gaussian units, with P the period in seconds and $\dot{P}$ its derivative in $\text{s} \cdot \text{s}^{-1}$.

¹In practice, timing noise degrades the accuracy of pulsars' rotational period as a measure of time.

We can also find the pulsar’s characteristic *rotational age*, approximately equal to its physical age, by assuming that $\mu_{\perp} \equiv \mu \sin(\alpha)$ stays constant over the pulsar’s lifetime and noting that $\dot{\Omega} \propto -\Omega^3$ (Eq. 2.3), before integrating and substituting $\Omega^2 = -k\frac{\dot{\Omega}}{\Omega}$, with k a constant. This gives:

$$\tau = -\frac{\Omega}{2\dot{\Omega}} \left[1 - \left(\frac{\Omega}{\Omega_0} \right)^2 \right] \approx -\frac{\Omega}{2\dot{\Omega}} \equiv \frac{P}{2\dot{P}}, \quad (2.5)$$

where Ω_0 is the initial rotation rate.

Considering a population of pulsars, it is often useful to plot their period against their period derivative, as in Figure 2.2. From Eq. (2.4) it can be shown that lines on the $P - \dot{P}$ diagram described by the equation

$$\log_{10}(\dot{P}) = -\log_{10}(P) + C_1, \quad (2.6)$$

where $C_1 \equiv 2\log_{10}\left(\frac{B_0}{6.4 \times 10^{19}}\right)$ represent lines of constant B_0 . Similarly, from Eq. (2.5), constant- τ lines are given by

$$\log_{10}(\dot{P}) = \log_{10}(P) + C_2, \quad (2.7)$$

with $C_2 \equiv \log_{10}(2\tau)$.

A pulsar’s evolutionary track is therefore assumed to be along lines of constant B_0 towards the bottom-right corner on the $P - \dot{P}$ diagram, before reaching the so-called death line, where $\dot{E}$ is too low for pair formation to occur (see Section 2.3.3), and the pulsar is thus invisible in the radio waveband. In this regard, $P - \dot{P}$ diagrams are considered the pulsar equivalent of the HR-diagram in astrophysics, showing various properties and evolutionary paths of pulsars.

Figure 2.2 clearly shows two distinct pulsar populations, canonical pulsars being the larger central group, and *millisecond pulsars* (MSPs) clustering around the bottom-left corner. The distinction between these two populations is drawn according to their rotational period: pulsars with periods shorter than ~ 10 ms are typically categorised as MSPs (Carroll and Ostlie, 1996). MSPs are thought to be the result of NSs with weak magnetic fields accreting from a surrounding keplerian disc and being spun up to millisecond periods (Alpar et al., 1982). These pulsars have long apparent ages ($P/\dot{P} \gtrsim 10^8$ years), and pulsed optical, X-ray and γ -ray fluxes significantly lower than those of canonical pulsars. MSPs are therefore considered to be predominately older canonical pulsars which had once spun down into the *graveyard* below the so-called death line indicated on Figure 2.2, before being resurrected by accretion. The accreting material acts to shield out MSPs magnetic fields, leaving these magnetic fields significantly weaker than those of the canonical variety, $\sim 10^8$ G compared to the $\sim 10^{12-13}$ G of canonical pulsars. MSPs’ magnetic fields are therefore less effective at slowing down rotation via magnetic braking, resulting in the low spin-down rate we observe. See Kulkarni (1995) and references therein for a full review of MSPs.

The “Black widow pulsar”, B1957+20, is an example of a binary MSP with a brown dwarf companion being consumed by winds driven by the pulsar’s irradiation. Since this was the first example of such a system to be discovered, the Black widow pulsar was named after the eponymous spider, although the term has subsequently been applied to the class of pulsars with an ablating companion in general.

In addition to rotation-powered MSPs and canonical pulsars, *magnetars* are a class of NS powered by the decay of their immensely powerful magnetic fields. Duncan and Thompson (1992) speculated that these objects are formed when, shortly after gravitational collapse, certain conditions are met, which generate a strong magnetohydrodynamic dynamo effect that briefly accelerates their rotation. Most of these objects have magnetic fields higher than

10^{13} G. They initially spin very fast, $P \sim 1$ ms, but spin down relatively quickly, within about 10^4 years, due to magnetic braking. Anomalous X-ray pulsars (AXPs) and soft γ -ray repeaters (SGRs) are sources thought to be associated with magnetars.

There are also accretion-powered pulsars, a class of NSs in binary systems that accrete matter either from the stellar winds or from an accretion disc that forms if the companion overflows its Roche Lobe. The emission from these pulsars is at least partly powered by the gravitational energy imparted by infalling material. Low-mass X-ray binaries (LMXBs) are binary systems in which a low-mass main sequence star ($M \lesssim M_\odot$) orbits either a NS nor a black hole and emits pulsed X-rays. Binary systems comprised of an intermediate mass star ($M_\odot \lesssim M \lesssim 5M_\odot$) and an NS or black hole are called an intermediate-mass X-ray binary (IMXBs), of which Her X-1 is the most widely known example. If the donor star has a mass greater than about $5M_\odot$, this system is called a high-mass X-ray binary (HMXB), an example being Vela X-1. For a full overview of the variety of sources associated with NSs, see Harding (2013). The sources considered in this study are a mix of canonical pulsars and MSPs, all of which are visible in both radio and γ -rays.

2.3 Radiation mechanisms and pair production

In this section I introduce the radiation mechanisms which are thought to be relevant to pulsars, as well as magnetic $e^\pm$ pair production, and which the physical models attempting to account for pulsars' γ -ray emission incorporate. These physical models are discussed in Section 2.4.

2.3.1 Synchrotron radiation (SR) and curvature radiation (CR)

Charged particles radiate when they are accelerated; for acceleration perpendicular to the particles' velocity along a magnetic field line, this emission is called cyclotron radiation in the case of non-relativistic particles, or SR in the relativistic limit. Specifically, charged particles gyrating about magnetic field lines at relativistic speeds, as is the case with pulsars, emit SR. The equations of motion for a relativistic charged particle in a magnetic field reveals that it will follow a helical path moving along the field, circling about the field lines at a frequency (Rybicki & Lightman, 1979)

$$\omega_B = \frac{qB}{\gamma mc}, \quad (2.8)$$

with q its charge, m its mass, B the field's magnitude, and γ the particle's Lorentz factor. The radius of the particle's oscillation is given by

$$r_B = \frac{v}{\omega_B} = \frac{\gamma m c v}{q B}, \quad (2.9)$$

where v is the particle's speed.

The power radiated by an accelerated charge is given by the relativistic form of the Larmor formula (Jackson, 1998):

$$P = \frac{2q^2}{3c^3} \gamma^4 (a_\perp^2 + \gamma^2 a_\parallel^2). \quad (2.10)$$

If there is no acceleration along the magnetic field ($a_\parallel = 0$), and assuming $a_\perp = \omega v_\perp$, this evaluates to

$$P = \frac{2q^2}{3c^3} \gamma^4 \omega_B^2 v_\perp^2 = \frac{2q^4}{3m^2 c^5} \gamma^2 B^2 v_\perp^2, \quad (2.11)$$

in which $v_{\perp} = v \sin \theta$, with θ the pitch angle between $\mathbf{v}$ and $\mathbf{B}$. By averaging this formula for a given speed $\beta = v/c$ and an isotropic velocity distribution over all angles, one obtains (Padmanabhan, 2000)

$$P_{\text{SR}} = \frac{4}{3} \sigma_{\text{T}} (c\beta^2\gamma^2) U_{\text{B}} \propto E^2 B^2, \quad (2.12)$$

with $\sigma_{\text{T}} \equiv 8\pi r_0^2/3$ the Thompson cross section, $r_0 \equiv e^2/mc^2$ is the classical electron radius, E the particle energy, and $U_{\text{B}} = B^2/8\pi$ the magnetic energy density. SR is strongly beamed in the direction of the emitting particle's motion, with an angular emission cone width $1/\gamma$, where typically $\gamma \sim 10^{6-7}$ for primary electrons. We can therefore approximate SR emission as being tangent to the emitting particles' motion. The power per unit frequency emitted by a single electron can be shown to be (Rybicki and Lightman, 1979)

$$\frac{dP}{d\omega} \equiv P(\omega) = \frac{\sqrt{3}}{2\pi} q^2 \left(\frac{\gamma}{\rho_{\perp}} \right) F \left(\frac{\omega}{\omega_c} \right) = \frac{\sqrt{3} q^2}{2\pi} \frac{qB_{\perp}}{mc^2} F \left(\frac{\omega}{\omega_c} \right), \quad (2.13)$$

where $\rho_{\perp} = \gamma mc^2/(qB_{\perp})$ is the helix's radius of curvature, $\omega_c = 3c\gamma^3/2\rho_{\perp} = \frac{3}{2}\gamma^2 qB_{\perp}/mc$ is the cutoff frequency, and

$$F(x) \equiv x \int_x^{\infty} K_{5/3}(\xi) d\xi, \quad (2.14)$$

where $K_{5/3}$ is the modified Bessel function of order $5/3$, which has the asymptotic forms (Erber, 1966)

$$F(x) \sim \begin{cases} \frac{4\pi}{\sqrt{3}\Gamma(\frac{1}{3})} \left(\frac{x}{2} \right)^{1/3}, & x \ll 1 \\ \left(\frac{\pi}{2} \right)^{1/2} e^{-x} x^{1/2}, & x \gg 1. \end{cases} \quad (2.15)$$

The SR photon spectrum of a single electron therefore extends as a power law of index $2/3$ up to the cutoff frequency, after which it falls exponentially in intensity.

In the case of a population of particles, assuming that the number density $N(E)$ of particles with energies in the range $(E, E + dE)$ may be described by a power law $N(E)dE = CE^{-p}dE$, with C and p constants, the total power radiated per unit volume per unit frequency can be shown to be $P_{\text{tot}}(\omega) \propto \omega^{-s}$, where $s = (p - 1)/2$ is the index of the energy spectrum (Rybicki and Lightman, 1979). A high degree of linear polarisation is expected for this mechanism.

As opposed to SR, which is the result of acceleration (change of direction) transverse to a magnetic field, CR is associated with the movement of charged particles along a curved trajectory, typically following a magnetic field line. In some pulsar models, particles liberated from the NS surface have negligible momentum perpendicular to the magnetic field, meaning that CR is a primary mode of energy loss for particles in pulsars (Sturrock, 1971). Assuming a sufficiently large radius of curvature to justify the use of classical radiation formulae, the instantaneous CR power spectrum is equivalent to that of SR (Eq. 2.19), but inserting the curvature radius $\rho_{\parallel} \simeq 4r/3\theta$ instead of gyration radius $\rho_{\perp}$ (Jackson, 1998). Similar to SR, CR photons can be regarded as originating tangential to the magnetic field lines in the rotating frame of reference, since they are confined to a very narrow emission cone with angular width $1/\gamma$ (Manchester and Taylor, 1977).

The critical frequency of CR is defined as (Jackson, 1998)

$$\omega_{\text{CR}} = \frac{3c}{2\rho_{\parallel}} \gamma^3, \quad (2.16)$$

and the critical energy as

$$E_{\text{CR}} = \hbar\omega_{\text{CR}} = \frac{3\hbar c\gamma^3}{2\rho_{\parallel}}, \quad (2.17)$$

where $\hbar \equiv h/2\pi = 1.055 \times 10^{-27} \text{ erg} \cdot \text{s}$ is the reduced Planck constant. Electrons' CR energy loss rate can be shown to be (Venter, 2008)

$$P_{\text{CR}} = \frac{2e^2 c \gamma^4}{3\rho_{\parallel}^2}, \quad (2.18)$$

where e is the electron charge.

In standard pulsar models, CR is believed to originate from primary electrons in the so-called curvature radiation reaction (CRR) limit. This limit is reached when the energy relativistic electrons gain due to acceleration by an E -field parallel to the B -field is equal to the energy loss via radiation. This can be expressed as (Harding et al., 2005)

$$c |E_{\parallel}| \sim P_{\text{CR}} = \frac{2e^2 c \gamma^4}{3\rho_{\parallel}^2}. \quad (2.19)$$

This yields the Lorentz factor corresponding to CRR as:

$$\gamma = \left(\frac{1.5 E_{\parallel}}{e} \right)^{1/4} \rho_{\parallel}^{1/2}. \quad (2.20)$$

2.3.2 Inverse Compton scattering (ICS)

Compton scattering refers to the lengthening of a photon's wavelength as a result of the transfer of momentum and energy to a low-energy electron during an inelastic collision. ICS is the inverse of this process, in which a low-energy photon is "upscattered" by a high-energy electron, resulting in an increase of the photon's frequency. ICS may equally be considered SR from electrons excited by electromagnetic radiation (Lyne and Graham-Smith, 1990).

When a photon with energy ϵ is upscattered by a relativistic electron with Lorentz factor γ , its final energy E_{γ} may be approximated as (Ramana Murthy and Wolfendale, 1993)

$$E_{\gamma} \sim \begin{cases} \epsilon \gamma^2, & \gamma \epsilon \ll m_e c^2 - \text{Thomson limit} \\ \gamma m_e c^2, & \gamma \epsilon \gg m_e c^2 - \text{Extreme Klein-Nishina limit.} \end{cases} \quad (2.21)$$

In the Thomson limit, power radiated by an electron in an isotropic radiation field due to ICS is the same as in Eq. (2.18) for SR, but with the magnetic field energy density replaced by the photon energy density the electron experiences:

$$P_{\text{Compton}} = \frac{4}{3} \sigma_T (c \beta^2 \gamma^2) U_{\text{rad}}. \quad (2.22)$$

The Thomson limit is valid for photons with $\epsilon \gamma \ll 1$. If both processes occur simultaneously, the ratio of an electron's losses due to SR and ICS is the same as the ratio of U_B to U_{rad} :

$$\frac{P_{\text{SR}}}{P_{\text{Compton}}} = \frac{U_B}{U_{\text{rad}}}. \quad (2.23)$$

In the Klein-Nishina regime, quantum effects become significant and σ_T in Eq. (2.23) must be replaced by the Klein-Nishina cross section $\sigma_{\text{KN}} < \sigma_T$, where (Klein and Nishina, 1929)

$$\sigma_{\text{KN}} = \frac{3}{4} \sigma_T \left[\frac{1+x}{x^3} \left\{ \frac{2x(1+x)}{1+2x} - \ln(1+2x) \right\} + \frac{1}{2x} \ln(1+2x) - \frac{1+3x}{(1+2x)^2} \right], \quad (2.24)$$

in which $x \equiv \gamma hf/m_e c^2$. This signifies a reduction of the classical cross section as the photon energy increases.

The total Compton spectrum is given by (Blumenthal & Gould, 1970):

$$\left(\frac{dN}{dE_\gamma}\right)_{\text{total}} \propto \begin{cases} E_\gamma^{-(p+1)/2} & \text{– Thomson limit} \\ E_\gamma^{-(p+1)} & \text{– Extreme Klein-Nishina limit.} \end{cases} \quad (2.25)$$

The ICS spectrum in the Thomson regime is therefore of similar form to that of SR, whereas the spectrum is much steeper in the Klein-Nishina limit.

2.3.3 One-photon pair production

Sufficiently energetic photons in the pulsar magnetosphere may, through interaction with the pulsar's strong magnetic field, be converted into electron-positron pairs. The number of $e^\pm$ pairs n_p created by n_γ photons travelling a distance d through a magnetic field B is given by the relation (Erber, 1966)

$$n_p = n_\gamma (1 - e^{-\zeta(\chi)d}) \simeq \zeta(\chi)n_\gamma d, \quad (2.26)$$

where the quantity $\chi \equiv \frac{1}{2}(E_\gamma/m_e c^2)(B_\perp/B_{\text{cr}})$, $B_\perp$ is the component of the magnetic field perpendicular to the particles' velocity, $B_{\text{cr}} = m_e^2 c^3 / e \hbar \sim 4 \times 10^{13}$ G is the magnitude of the critical magnetic field for which the gyro-energy of an electron or positron would be equal to its rest mass, and $\zeta(\chi)$ is called the attenuation coefficient (Daugherty and Harding, 1982). The attenuation coefficient effectively represents the chance per unit length of an interaction between a photon and the magnetic field to produce an electron-positron pair, and corresponds to the photon absorption per unit length. This coefficient can be expressed as

$$\zeta(\chi) = \frac{1}{2} \left(\frac{\alpha_f}{\lambda_c}\right) \left(\frac{B_\perp}{B_{\text{crit}}}\right) T(\chi), \quad (2.27)$$

with $\alpha_f \simeq 1/137$ the fine structure constant, $\lambda_c = h/mc \sim 2 \cdot 10^{-10}$ cm is the Compton wavelength, and $T(\chi)$ a dimensionless function with the asymptotic forms

$$T(\chi) = \begin{cases} 0.46e^{-\frac{4}{3\chi}}, & \chi \ll 1 \\ 0.60|\chi|^{-1/3}, & \chi \gg 1. \end{cases} \quad (2.28)$$

In the case where $\chi \ll 1$, which is appropriate for pulsars, Eq. (2.28) becomes (Luo et al., 2000)

$$\zeta(\chi) = 0.46 \left(\frac{\alpha_f}{\lambda_c}\right) \left(\frac{\chi}{\epsilon_\gamma}\right) e^{-\frac{4}{3\chi}} \quad (2.29)$$

where $\epsilon_\gamma \equiv E_{\text{ph}}/m_e c^2$, with E_{ph} the photon energy. Notice that $\zeta \propto \chi e^{-1/\chi}$, which means that $\alpha(\chi)$ scales with the magnetic field strength; stronger magnetic fields are therefore more efficient producers of electron-positron pairs.

2.3.4 Two-photon pair production

An $e^\pm$ pair may also be produced in the absence of a magnetic field, through an interaction between two photons. The threshold photon energy required for this process is $E_\gamma = 2m_e c^2 \sim 1$ MeV. The two-photon pair production cross section is given by the equation (Svensson, 1982)

$$\sigma_{2\gamma} \simeq \frac{3}{8} \sigma_T \begin{cases} (\epsilon_{\text{cm}}^2 - 1)^{1/2} & \epsilon_{\text{cm}} \ll 1 \\ [2 \ln(2\epsilon_{\text{cm}}) - 1] / \epsilon_{\text{cm}}^6 & \epsilon_{\text{cm}} \gg 1, \end{cases} \quad (2.30)$$

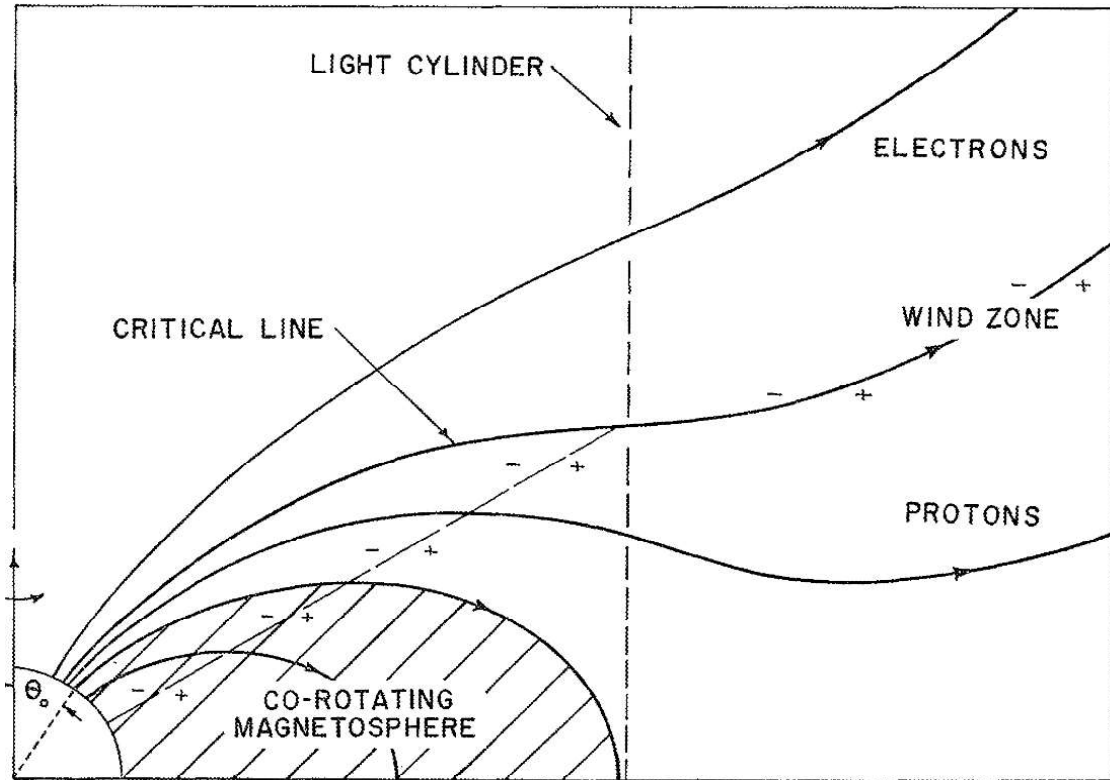


Figure 2.3: A schematic representation of the magnetosphere of an aligned rotator, as envisaged by Goldreich & Julian (1969). The shaded zone indicates the closed B -field region filled with corotating plasma. Field lines extending past the light cylinder (dashed line) sweep open, and particles escaping near the PC rim (colatitude θ_0) travel out along them. The indicated critical line separates regions of positively and negatively charged outflow, as indicated.

where $\epsilon_{\text{cm}} = [\epsilon_1 \epsilon_2 (1 - \cos \theta_{12}) / 2]^{1/2}$ is the photon energy in the centre-of-momentum frame, ϵ_1 and ϵ_2 refer to the photons's respective energies, and θ_{12} is the angle between their directions of propagation.

2.4 Gamma-ray emission models

This study is chiefly concerned with the comparison of observational pulsar data with the predictions of certain geometric models that dictate where in the pulsar magnetosphere emission takes place. However, in order to understand these geometric models, which are presented in the next chapter, one must be familiar with the physical models on which they are based. This section introduces the Goldreich-Julian pulsar magnetosphere model that underlies the geometric models, as well as the γ -ray emission processes associated with this region. The polar cap (PC), outer gap (OG), and slot gap (SG) γ -ray emission models are then introduced.

2.4.1 The Goldreich-Julian magnetospheric model

Goldreich & Julian (1969) showed that despite a pulsar's intense gravitational field, its corotating magnetosphere must necessarily be dense with plasma. In their model they considered the NS to be uniformly magnetised, perfectly conducting and spherical, with an interior magnetic field $\mathbf{B}_{\text{int}} = B_0 \mathbf{e}_z \parallel \boldsymbol{\mu}$, and an external dipole field $\mathbf{B}_{\text{ext}}$ aligned with the rotation axis ($\boldsymbol{\mu} \parallel \boldsymbol{\Omega}$).

Due to the NS's rotation, the charged particles inside it will experience a force which, under the assumption of perfect conductivity, must be counteracted by an electric field $\mathbf{E}_{int}$ established by charge redistribution. The Lorentz force $q\mathbf{E} + \frac{q}{c}\mathbf{v} \times \mathbf{B}$ (with q charge and $\mathbf{v} = \boldsymbol{\Omega} \times \mathbf{r}$ velocity) on particles inside the NS being zero, the interior electric field can be described by the equation:

$$\mathbf{E}_{int} = -\frac{1}{c}(\boldsymbol{\Omega} \times \mathbf{r} \times \mathbf{B}_{int}) = -\frac{\Omega B_0 r \sin \theta}{c}(\sin \theta \mathbf{e}_r + \cos \theta \mathbf{e}_\theta), \quad (2.31)$$

where r and θ are spherical coordinates centred on the NS and B_0 is the magnetic field strength at its surface. Since the magnetic field is assumed to be static, Faraday's law leads to $\boldsymbol{\nabla} \times \mathbf{E}_{int} = 0$, meaning that we can write $\mathbf{E}_{int} = -\boldsymbol{\nabla}\Phi_{int}(r, \theta)$, where Φ is the electric potential. Integration gives us an expression for the potential:

$$\Phi_{int}(r, \theta) = \frac{\Omega B_0 r^2}{2c} \sin^2 \theta + \Phi_0, \quad (2.32)$$

with Φ_0 a constant. The colatitudinal dependence of the potential means that there is a potential difference between the magnetic axis and the rim of the PC (at colatitude $\theta_{PC} \sim \sqrt{\Omega R/c}$) of

$$\Delta\Phi_{int} = \frac{1}{2} \left(\frac{\Omega R}{c} \right)^2 B_0 R. \quad (2.33)$$

Assuming a continuous electric potential at the NS surface and solving the Laplace equation, $\nabla^2\Phi_{out}(r, \theta) = 0$, gives the exterior electric field magnitude:

$$E_{r,ext} = -\frac{9Q}{r^4} \left(\cos^2 \theta - \frac{1}{3} \right), \quad (2.34)$$

$$E_{\theta,ext} = -\frac{6Q}{r^4} \cos \theta \sin \theta, \quad (2.35)$$

with $Q = B_0 \Omega R^5 / 6c$. The force associated with the electric field parallel to the magnetic field, $E_{\parallel}$, is orders of magnitude larger than the gravitational force binding particles to the NS surface, and pulls charged particles from the stellar surface, filling the magnetosphere with plasma.

The charge density of the magnetosphere surrounding the NS is called the Goldreich-Julian charge density, and is given by:

$$\rho_{GJ} = \frac{1}{4\pi} \boldsymbol{\nabla} \cdot \mathbf{E} \approx -\frac{1}{2\pi c} \boldsymbol{\Omega} \cdot \mathbf{B}. \quad (2.36)$$

This equation implies the existence of a null-charge surface where $\boldsymbol{\Omega} \perp \mathbf{B}$, that separates the magnetospheres into positively and negatively charged regions (see Figure 2.3). Furthermore, magnetic field lines extending further than the light cylinder, radius $R_{LC} \equiv c/\Omega$, where the corotation velocity reaches the speed of light, cannot close. Consequently, particles extricated from the stellar surface will travel out along the open field lines, positive and negative charges being separated by the null-charge surface. The magnetic field line that crosses the null-charge surface at the light cylinder is called the critical line.

Important to note is that, due to the uniformity of the conductive plasma without gaps, the Goldreich-Julian model represents a force-free limit. The plasma shields out any accelerating $E_{\parallel}$, and no radiation can be produced; this is of course contrary to observations, which suggests that the model is oversimplified. Subsequent physical models have ameliorated the situation by introducing so-called acceleration gaps, where the plasma density is locally less

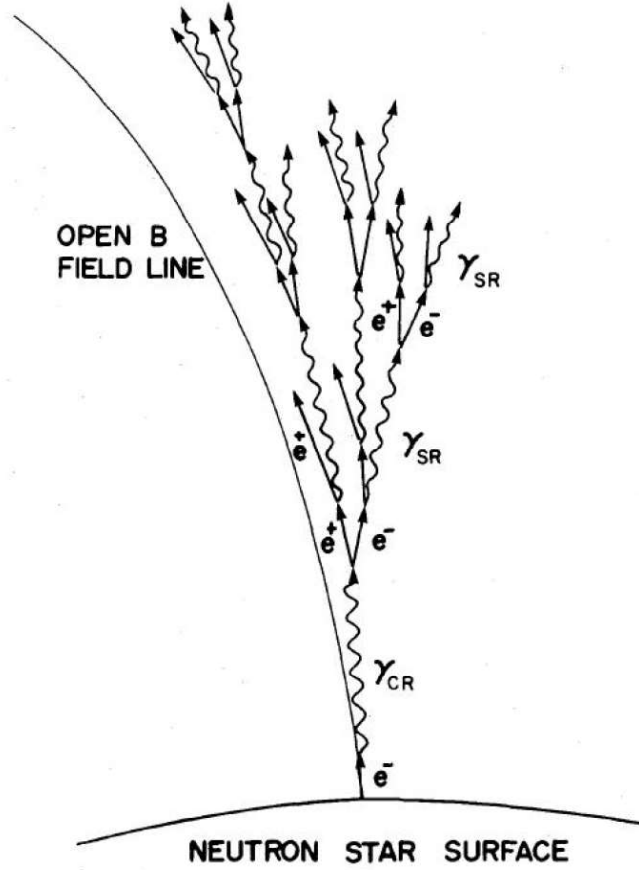


Figure 2.4: Illustration of four generations of a pair cascade resulting from the acceleration of primary electrons along a pulsar’s magnetic field. Photons produced through CR (γ_{CR}), and SR (γ_{SR}) are shown along with secondary $e^\pm$ pairs which result from magnetic pair production.

than ρ_{GJ} , allowing for nonzero $E_\parallel$, and therefore particle acceleration in these regions. The geometric models discussed in Chapter 3 are only concerned with the shape and location of these gaps, not how they are formed. However, it is still necessary to understand the emission mechanisms these models are based on, as will be discussed in the following sections.

2.4.2 The polar cap (PC) model

In the PC emission model, proposed by Daugherty and Harding (1996), emission takes place in the regions immediately above the pulsar’s magnetic poles; the titular PCs refer to the regions surrounding these poles outlined by the footpoints of the last open field lines. According to this model, charged particles ripped from the NS surface by a strong rotation-induced electric field near the PC experience extremely rapid acceleration along the curved B-field lines, producing CR at low altitudes. Some of the highly energetic CR photons will in turn undergo pair production, with the resulting $e^\pm$ pairs generally having components of velocity transverse to the magnetic field which are quickly converted into SR. Whereas electrons will be propelled further out into the magnetosphere by the electric field, positrons will be pulled back towards the NS surface, heating the PC and perpetuating the emission of charged particles in this region. Furthermore, outwardly accelerated electrons will themselves emit CR, which will produce secondary charged pairs, which will emit SR, and so on, creating the self-perpetuating emission cascade shown in Figure 2.4 (Harding and Muslimov, 2003).

The charged particles pulled into the magnetosphere by the process described above leads to a thickening of the plasma moving out from the NS surface. The altitude at which

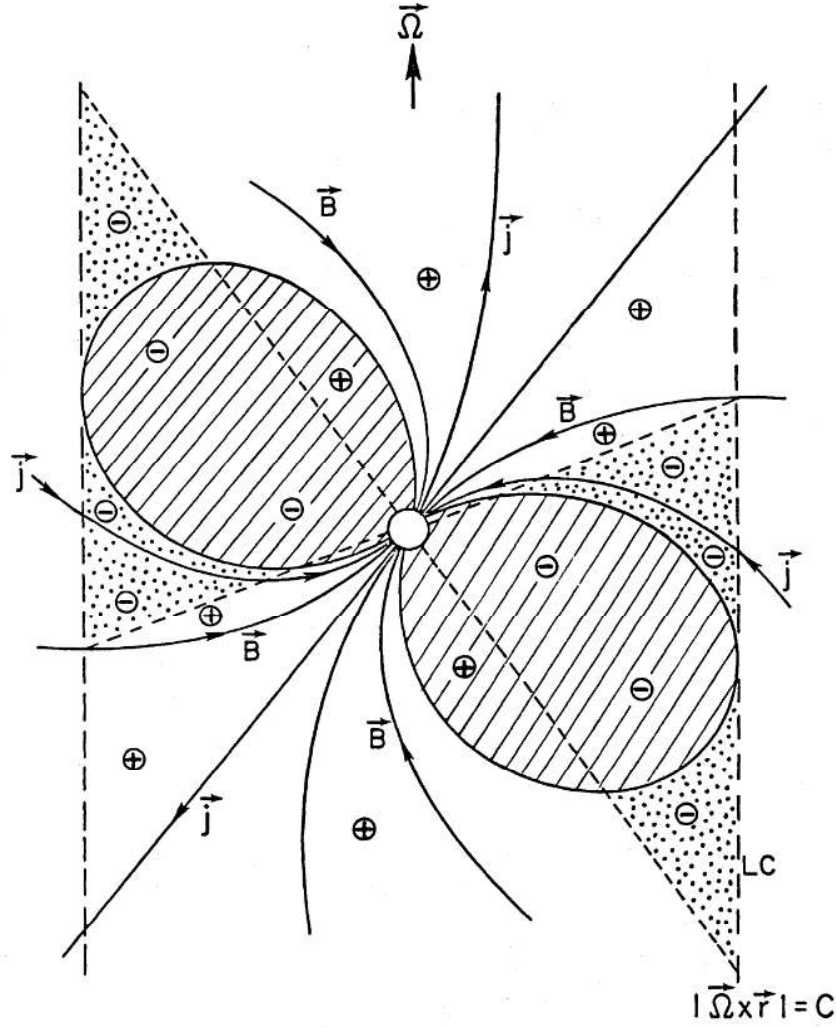


Figure 2.5: A schematic indication of the current flow pattern along open field lines assumed by the OG model, with electrons flowing out in the dotted regions. The shaded area is the closed field line region, where no steady currents flow. An acceleration gap is caused by the net positive charge along the last open field line.

the charge density exceeds the Goldreich-Julian density, thick enough to screen out the accelerating E -field, is called the pair formation front (PFF). In the PC model this PFF is considered to be at a uniform, very low altitude above the PCs, with all emission originating from the region directly below this.

2.4.3 The outer gap (OG) model

Recall from Section 2.3.1 the existence of the so-called null-charge surface where $\boldsymbol{\Omega} \cdot \mathbf{B} = 0$; it is postulated that the return current of positrons flows back through this neutral current sheet, while negative charges continue past the LC on open field lines further from the magnetic axis. Cheng et al. (1986a) speculated that this results in a deficit of negative charge along the last open field line, and consequently the development of an electric potential. Since $\mathbf{E} \cdot \mathbf{B} \neq 0$ in this region, primary particle acceleration will occur; γ -rays from these accelerated particles yield $e^\pm$ pairs which screen out the electric field further out from the field line (toward the magnetic axis), closing the acceleration gap. An illustration of the structure of these gaps is presented in Figure 2.5.

In this model the acceleration gap extends along the last open magnetic field line from

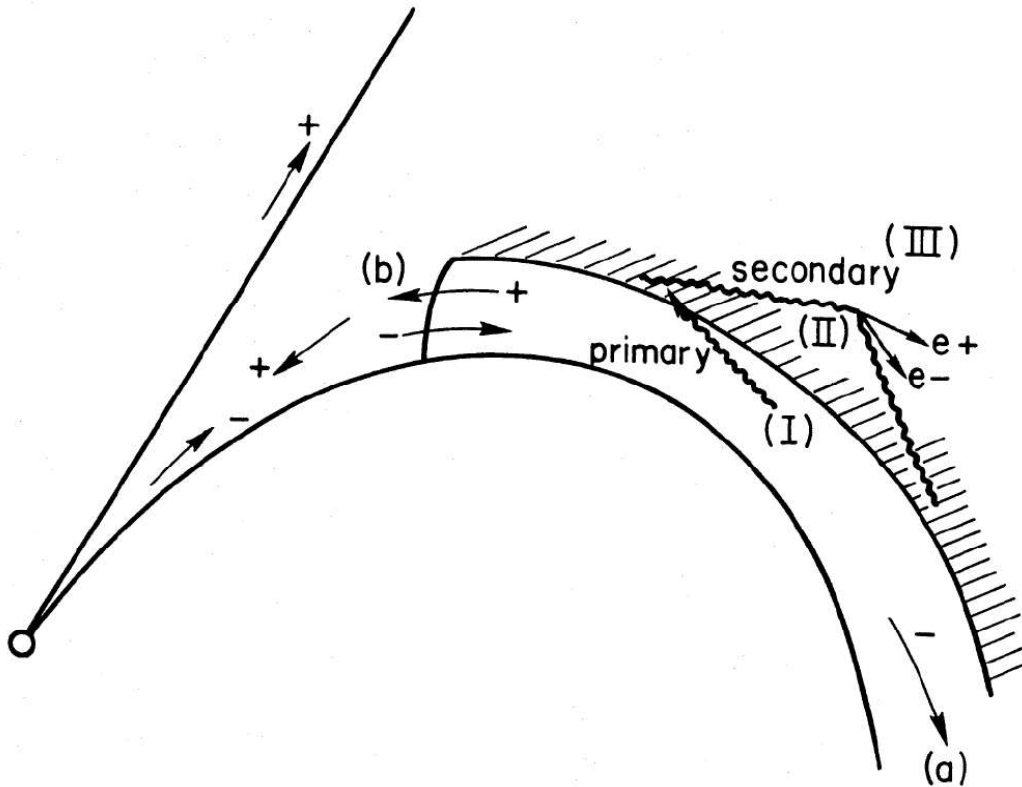


Figure 2.6: Schematic representation of the regions associated with primary (I), secondary (II), and tertiary (III) particle acceleration in the OG model. From Cheng et al. (1986b).

the null-charge surface to beyond the LC (region I in Figure 2.6), bounded by a boundary layer (region II in Figure 2.6). The charged particles accelerated in region I emit primary CR photons, which are converted, in either region I or II, into secondary $e^\pm$ pairs. The primary and secondary pairs in region II eventually build up to quench the nonzero $\mathbf{E}_\parallel$, closing the gap.

In these gaps, X-rays and γ -rays are produced through CR and SR by the secondary charges. However, since the primary electrons and positrons are moving in opposite directions, so are the primary γ -rays and the resultant secondary pairs. The secondary CR and SR radiation is therefore emitted in crossed beams, leading to the creation of tertiary $e^\pm$ pairs by way of photon-photon pair production well beyond the gap (region III in Figure 2.6). The radiation emitted by these less energetic tertiary particles is primarily in the infrared domain, and again stimulate pair production in region I and II through collisions with the primary γ -rays. These infrared photons may also contribute to a pulsar's primary γ -ray flux through *ICS* with primary particles in the gap (Cheng et al., 1986b).

2.4.4 The slot gap (SG) model

The SG model extends the PC model by introducing a colatitude dependence into the PFF height. Since there is a nonzero component of the electric field parallel to the magnetic field ($\mathbf{E}_\parallel$) above the PC, and this component is shielded out entirely in the closed field line region, Arons (1983) reasoned that this quantity must decrease continuously as one moves away from the the magnetic axis. A weaker $\mathbf{E}_\parallel$ closer to the closed field line region results in a more gradual acceleration of the particles at these colatitudes, and a suppression of the CR energy they emit. This effectively increases the mean free path for magnetic pair

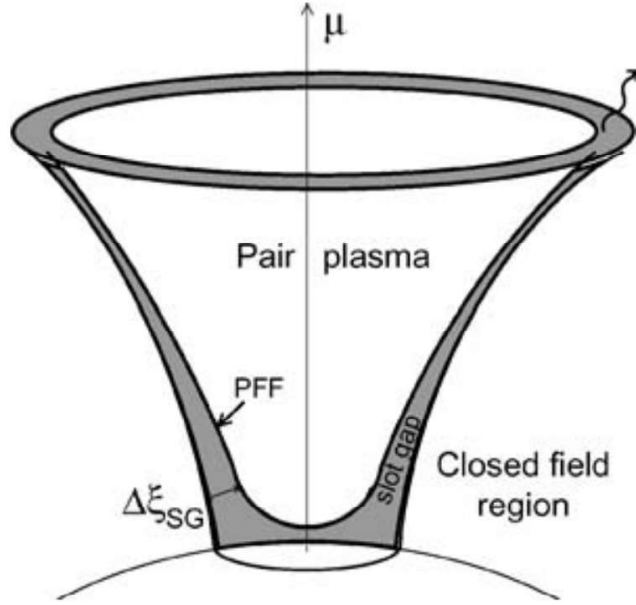


Figure 2.7: Schematic illustration of emission geometry in the SG model. The PFF is formed at low altitude immediately above the PC, but approaches the closed field line region asymptotically. The SG exists between the pair plasma resulting from the pair cascade above the PFF and the boundary to the closed field line region. The $\Delta\xi_{SG}$ represents the thickness of the slot gap. Taken from Harding and Muslimov (2005).

production at greater colatitudes, CR photons moving a greater distance on average before producing $e^\pm$ pairs, and therefore shifting the PFF ever outward. The PFF is thus formed at low altitude directly above the PC, but extends out along the last open field lines, as illustrated in Figure 2.7, forming an SG.

Muslimov and Harding (2004) estimates the width of the SG as the magnetic colatitude at which the variation in height of the PFF becomes comparable to some fraction λ of the stellar radius R , or

$$\Delta\xi_{SG} \approx \begin{cases} 0.2P_{0.1}(\lambda B_{12})^{-4/7}I_{45}^{-3/7} & B_{12} \lesssim 4.4 \\ 0.1P_{0.1}(\lambda B_{12}^{3/4})^{-4/7}I_{45}^{-3/7} & B_{12} \gtrsim 4.4, \end{cases} \quad (2.37)$$

where $I_{45} = I/10^{45} \text{g} \cdot \text{cm}^2$, I is the NS's moment of inertia, $P_{0.1} = P/0.1 \text{ s}$ is its rotation period and $B_{12} = B_0/10^{12} \text{ G}$ is its surface magnetic field.

This model forms the basis of the geometric two-pole caustic (TPC) model discussed in the next chapter.

2.5 Conclusions

In this chapter I introduced pulsars as astrophysical objects, and presented the concepts related to these sources that are most crucial to understanding the rest of this work. I started with a concise history of pulsar observations and theory, before describing these objects' posited origins and most important physical characteristics. Next, I outlined some of the mechanisms by which the observed electromagnetic radiation associated with pulsars is thought to be produced. Finally, I described notable models for how such emission might take place in pulsar magnetospheres.

In the next chapter I will elaborate on pulsar modelling with specific reference to purely *geometric* models, which allow for predicting pulsars' LCs without concern for the complex matter of energetics. It is these models that I later apply to constrain the geometries of a selection of pulsars.

Chapter 3

Geometric pulsar models

As will have become clear from the previous chapter, there is still a great deal of uncertainty with regard to the electrodynamics present in pulsars, making it difficult to make predictions as to the energetics of their emission. One can follow a different approach to this problem, however, by employing strictly geometric models concerned only with the location and extent of the accelerator gap regions in the pulsars' magnetosphere from which γ -rays are thought to originate. As opposed to physical models, which specify the magnetic field, accelerating electric field, radiation mechanisms and particle transport thought to be present in the pulsar system, these geometric models simply assume uniform emissivity originating from acceleration gaps somewhere in the pulsar magnetosphere, along with certain beaming effects. As such, although geometric models cannot provide any information on the characteristics of the emission we ought to observe from pulsars, we *can* use them to infer the LC profile we would have seen for a particular arrangement of acceleration gaps, and, by comparison with observations, determine which configuration most likely matches the physical case. This in turn can be traced back to deduce the physical conditions that would have led to the formation of these gaps, and in turn constrain the properties of physics underlying the very high energy emission from pulsars. This chapter introduces the geometric models whose predictions I compare to *Fermi* data in the next chapter.

Assuming γ -ray emission takes place nearly tangential to the pulsar's magnetic field lines in the corotating frame, it is important to first discuss the structure of this field (Section 3.1). The next section details the outer gap (OG) and two-pole caustic (TPC) geometric models, while Section 3.3 presents the empirical radio geometric model I employ alongside these to predict LCs. Section 3.4 describes how pulsars' rotation and relativistic effects like aberration and time-of-flight delays affect our observations. Finally, Section 3.5 discusses model phase plots and LCs that will be compared to observational data in Chapter 5.

3.1 Magnetic field structure

The magnetic field structure is a fundamental assumption of the geometric models I discuss in the next section. The choice of B -field is not entirely arbitrary, however, as we know it¹ must exist in some intermediate state between the simple retarded vacuum dipole (RVD) of Deutsch (1955), which can accelerate particles but cannot sustain a corotating plasma, and the force-free field (FF; e.g. Contopoulos et al., 1999), which is filled with plasma, but cannot account for particle acceleration (see Section 2.3.1). One must also take into account the sweeping back of field lines close to the light cylinder by displacement currents, which distorts the PC.

¹At least, the dominant dipole structure. Multipolar solutions may exist (see e.g. Pétri, 2016).

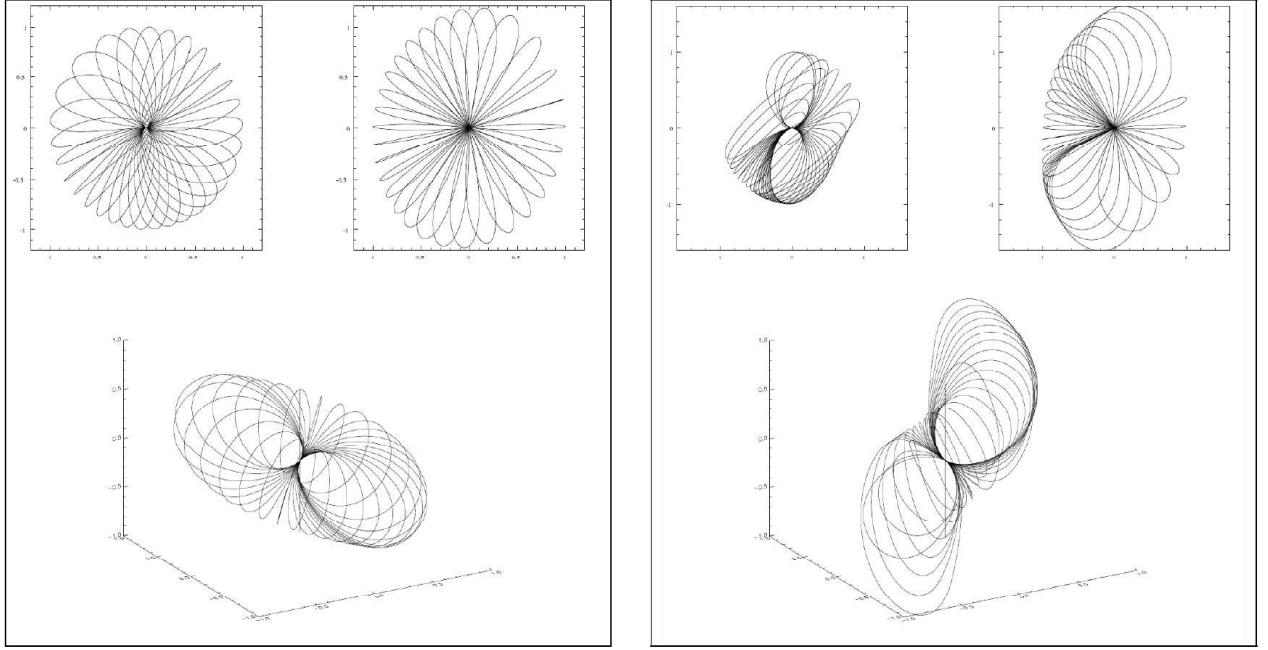


Figure 3.1: The last closed field lines, i.e. those closing on the light cylinder, of an RVD tilted by 30° (left) and 70° (right) with respect to the rotation axis. In each set of panels, the top-left panel shows a projection looking down the rotation axis, with rotation being counter-clockwise from this perspective. The top-right panels are a projection looking down the magnetic axis, and the bottom panels are three-dimensional views with the rotation axis drawn parallel to the $\hat{\mathbf{z}}$ -axis. From Arendt and Eilek (2002).

3.1.1 The RVD magnetic field

Well before the discovery of the first radio pulsar in 1969, Deutsch (1955) had already derived the equations describing the magnetic field of a perfectly conducting sphere rotating rigidly in a vacuum, including the case of the magnetic axis being tilted with respect to the rotational axis. The equation describing the RVD field, taking rotational effects into account, is given in Cheng et al. (2000) as

$$\mathbf{B}_{\text{RVD}} = \hat{\mathbf{r}} \cdot \left[3 \frac{\boldsymbol{\mu}(t)}{r^3} + 3 \frac{\dot{\boldsymbol{\mu}}(t)}{cr^2} + \frac{\ddot{\boldsymbol{\mu}}(t)}{c^2 r} \right] - \left[\frac{\boldsymbol{\mu}(t)}{r^3} + \frac{\dot{\boldsymbol{\mu}}(t)}{cr^2} + \frac{\ddot{\boldsymbol{\mu}}(t)}{c^2 r} \right], \quad (3.1)$$

where $\boldsymbol{\mu}$ represents the time-dependent magnetic moment of the dipole inclined at an angle α with respect to the angular velocity vector ($\boldsymbol{\Omega} \parallel \hat{\mathbf{z}}$). The magnetic dipole moment and its first and second-order derivatives can be written as

$$\begin{aligned} \boldsymbol{\mu}(t) &= \mu(\sin \alpha \cos \Omega t \hat{\mathbf{x}} + \sin \alpha \sin \Omega t \hat{\mathbf{y}} + \cos \alpha \hat{\mathbf{z}}), \\ \dot{\boldsymbol{\mu}}(t) &= \Omega \mu \sin \alpha (-\sin \Omega t \hat{\mathbf{x}} + \cos \Omega t \hat{\mathbf{y}}), \\ \ddot{\boldsymbol{\mu}}(t) &= -\Omega^2 \mu \sin \alpha (\cos \Omega t \hat{\mathbf{x}} + \sin \Omega t \hat{\mathbf{y}}). \end{aligned}$$

The $\mathbf{B}_{\text{RVD}}$ can also be expressed in spherical coordinates as follows (Dyks et al., 2004):

$$\begin{aligned} \mathbf{B}_{\text{RVD},r} &= \frac{2\mu}{r^3} [\cos \alpha \cos \theta + \sin \alpha \sin \theta (r_n \sin \lambda + \cos \lambda)], \\ \mathbf{B}_{\text{RVD},\theta} &= \frac{\mu}{r^3} \{ \cos \alpha \sin \theta + \sin \alpha \cos \theta [-r_n \sin \lambda + (r_n^2 - 1) \cos \lambda] \}, \\ \mathbf{B}_{\text{RVD},\phi} &= -\frac{\mu}{r^3} \sin \alpha [(r_n^2 - 1) \sin \lambda + r_n \cos \lambda], \end{aligned}$$

where $r_n = r/R_{\text{LC}}$, $R_{\text{LC}} = \frac{c}{\Omega}$ is the radius of the light cylinder, and $\lambda = r_n + \phi - \Omega t$. Note that in the case where $\frac{r_n}{R_{\text{LC}}}$ approaches zero, the RVD magnetic field reduces to the static dipole approximation given by the equation (Cheng et al., 2000)

$$\mathbf{B}_{\text{st}} = \frac{1}{r^3} [3(\boldsymbol{\mu} \cdot \hat{\mathbf{r}}) \hat{\mathbf{r}} - \boldsymbol{\mu}]. \quad (3.2)$$

Figure 3.1 shows two examples of RVD magnetic fields, inclined by 30° with respect to the rotational axis in the case of the left panel, and 70° in the case of the right panel (taken from Arendt and Eilek, 2002).

3.1.2 Shape of the PC

The PC is the region on the NS’s surface outlined by the footpoints of the closed magnetic field lines originating closest to the magnetic poles (tangential to the light cylinder). Since the pulsar’s rotation results in sweep back of the field lines, the shape of the PC is different than what it would have been in a static case. The first panel of Figure 3.2 shows the shape of the PC under the assumption of a static dipole magnetic field (dashed line), an RVD aligned with the rotational axis (thin solid line), and an RVD inclined by 45° with respect to the rotational axis (thick line). Note that, while in both the static dipole and aligned RVD cases the PC is circular with a radius of $r_{\text{PC}} \simeq (\Omega R_{\text{NS}}^3 c^{-1})^{1/2}$, prominent indentations or *notches* appear in the PC of an inclined RVD field, which may play an important role in explaining certain features on pulsar phase plots (see Section 3.5.) In Figure 3.2 the distorted PC rim is divided into two parts, R1 and R2, by the critical points C1 (the tip of the notch) and C2, where a second notch appears at greater tilt angles α .

In our computer simulations the PC is typically described using open volume coordinates (OVCs; r_{OVC} and l_{OVC}). In the second panel of Figure 3.2, the PC is divided into rings of constant parameter $r_{\text{OVC}} \equiv 1 \pm d_{\text{OVC}}$, where d_{OVC} is the minimum distance of a point to the rim normalised to the PC radius r_{PC} , such that $r_{\text{OVC}} = 1$ specifies the location of the last open field lines. The l_{OVC} coordinate denotes the physical distance along constant- r_{OVC} “rings” on the stellar surface starting from the fiducial plane containing both the magnetic and spin axes where $l_{\text{OVC}} = 0$.

3.2 The geometric γ -ray models

In this study, observational pulsar data are compared with the predictions of two model pulsar geometries, namely the geometric OG model of Cheng et al. (1986a) and the geometric TPC model of Dyks and Rudak (2003). In both of these models, γ -ray photon emissivity is assumed to be uniform in the corotating frame within an acceleration gap somewhere along the last open magnetic field lines, up to the light cylinder. Caustic effects are employed to explain the characteristic double-peak profile of pulsars (see Section 3.4). These models differ only in the exact position and extent of the respective gaps. Figure 3.3 illustrates the geometries of the acceleration gaps assumed in the OG and TPC models, the former bound by the thick dashed lines and the latter represented by the shaded area.

Based on the OG emission model described in Section 2.3.5, the geometric OG model supposes a γ -ray photon emission region extending from the null-charge surface, where the Goldreich-Julian charge density ρ_{GJ} is zero, out to the light cylinder along, but slightly interior to, the last open field lines. The OG geometry only allows the emission from a single pole to reach the observer at any given moment, and as such may be referred to as a “single-pole caustic model”. Furthermore, due to the lack of radiation emanating from close

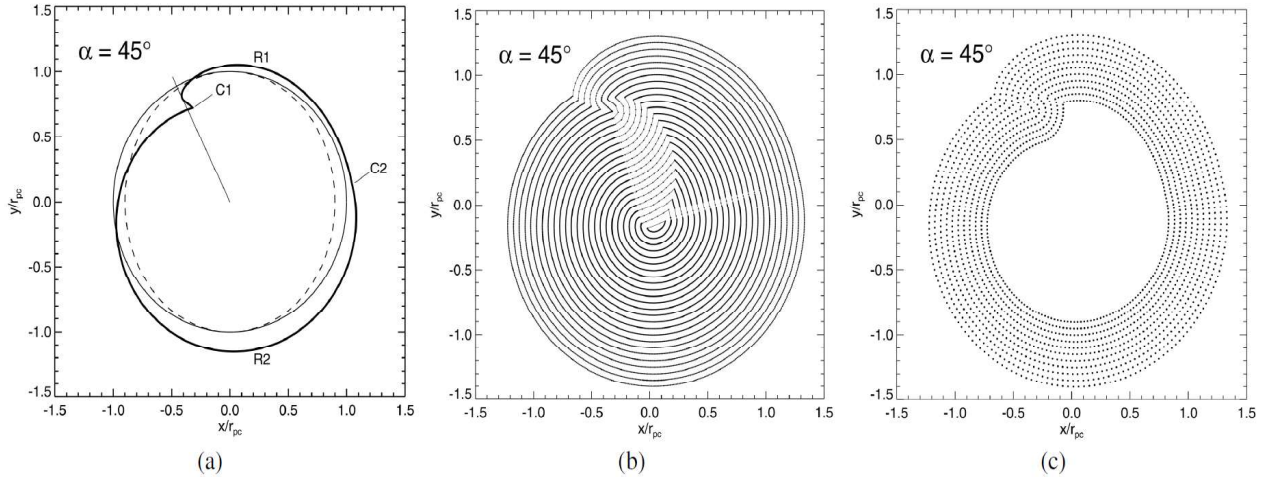


Figure 3.2: (a) The shape of the PC under the assumption of a static dipolar magnetic field (dashed line), an RVD field aligned with the rotation axis (thin solid line), and an RVD field inclined by 45° with respect to the rotation axis (thick line). Note that whereas in the case of both a static dipole and an aligned rotator the PC is circular with a radius $r_{\text{PC}} \simeq (\Omega R_{\text{NS}}^3 c^{-1})^{1/2}$, the sweep back of field lines close to the light cylinder deforms the misaligned RVD PC, creating characteristic *notches* which can be used to explain certain features on pulsar phase plots (see Section 3.5.) (b) A set of concentric rings of constant r_{OVC} (from 0.05 to 1.25 in steps of 0.05) in the case of an inclined RVD magnetic field (see text for details.) (c) Sample of the footpoints of magnetic field lines along which photon emission is modelled in our simulations. In this case there are 180 footpoints per “ring” ranging from $r_{\text{OVC}} = 0.75$ to $r_{\text{OVC}} = 1.25$ in steps of 0.05. All panels taken from Dyks et al. (2004).

to the NS surface, emission from this model is invisible in γ -rays for low values of α and ζ . The OG model also prefers more oblique rotators.

The TPC model was developed in 2003 by J. Dyks and B. Rudak in response to the OG model’s preference for highly inclined rotators, and a perceived inability of the OG model to account for off-peak emission in the Crab pulsar. In this model γ -rays originate along the last open field lines all the way from the NS surface to the light cylinder. The TPC model has the advantage of producing double-peak emission profiles for a larger region of $\alpha - \zeta$ phase space, and allows more aligned rotators ($\alpha \sim 0^\circ$). The TPC model also produces off-peak emission necessary to fit certain pulsar LCs.

3.3 The empirical radio models

This study employs two geometric radio models, separating the “cone” and “core” components of the radio model described in Gonthier et al. (2004). The first of these models, referred to as the *core* model throughout the rest of this work, assumes that the radio emission takes on the form of a simple Gaussian beam of fixed width and height.

The second, more involved, radio model is called the *cone* model, and is identical to the one used in Seyffert (2014), which is itself a modification of the “cone” component of Gonthier et al. (2004)’s empirical model. In this model, radio emission is assumed to originate from simple annular regions above the PCs whose altitudes in units of stellar radius R_{NS} are given by the equation

$$r_{\text{KG}} = 40 \nu_{\text{GHz}}^{-0.26} \dot{P}_{-15}^{0.07} P^{0.3}, \quad (3.3)$$

where $\nu_{\text{GHz}} = \frac{\nu}{1\text{GHz}}$ is the frequency of the radiation emitted at that altitude in GHz, and

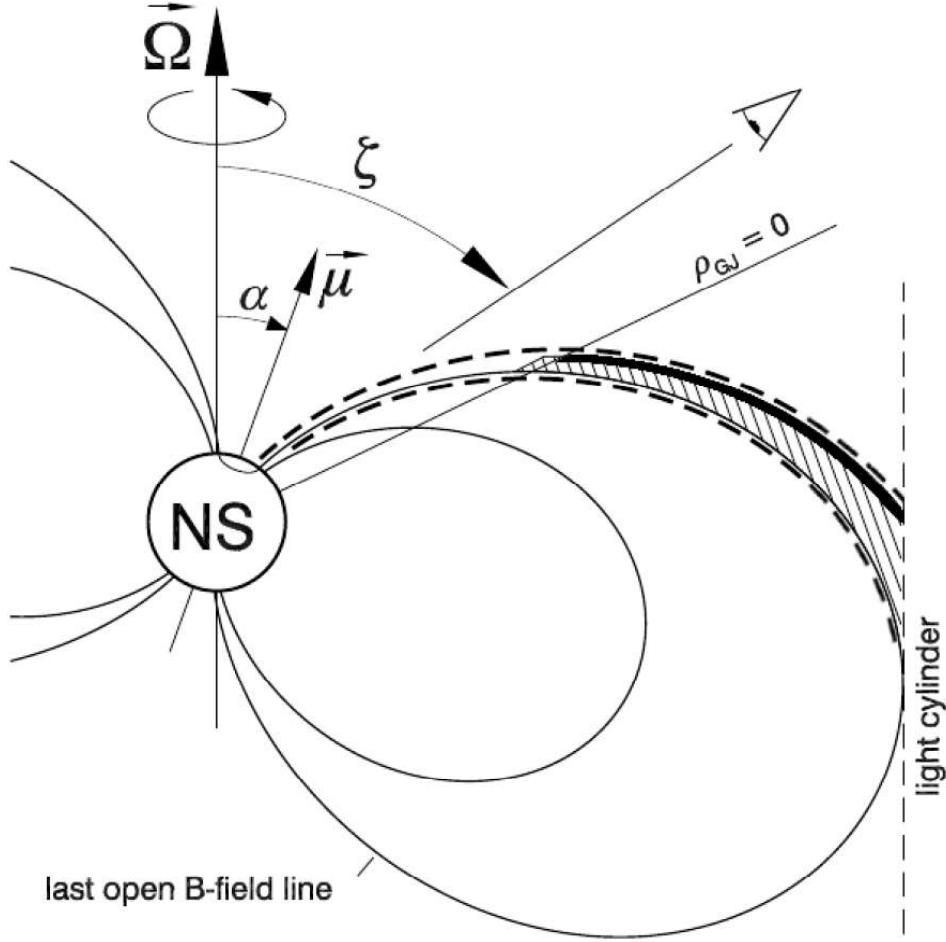


Figure 3.3: A schematic representation of the acceleration gaps from which γ -ray photons are uniformly emitted according to the geometric OG and TPC models. In the TPC model the emission zone is assumed to extend along the last open magnetic field lines, all the way from the NS surface out to the light cylinder, between the dashed lines. Conversely, the OG acceleration gap, represented by the shaded area, only extends from the null-charge surface denoted by ρ_{GJ} to the light cylinder. Taken from Dyks & Rudak (2003).

$\dot{P}_{-15} = \frac{\dot{P}}{10^{-15}}$ is the derivative of the pulsar's period in units of $10^{-15} \text{ s} \cdot \text{s}^{-1}$ (Kijak and Gil, 1998).

An important feature of this model is the frequency dependence of the emission height, which implies that higher-energy radio emission occurs closer to the NS surface. This is likely caused by changes in the local plasma frequency with height. Another implication of this model is that the opening angle of the emission cone is smaller for higher radio frequencies, as reflected in the expression for the opening angle of the beam:

$$\rho_{\text{cone}} = 1.24^\circ \sqrt{\frac{r_{\text{KG}}}{P}}. \quad (3.4)$$

3.4 Rotation and caustics

An important factor for consideration relevant to the geometric γ -ray models is the implications of the emission being produced at varying altitudes above the NS. As opposed to the radio model, where all photons in this band are assumed to have originated from a single altitude, two γ -ray photons observed at the same point in phase may have been emitted

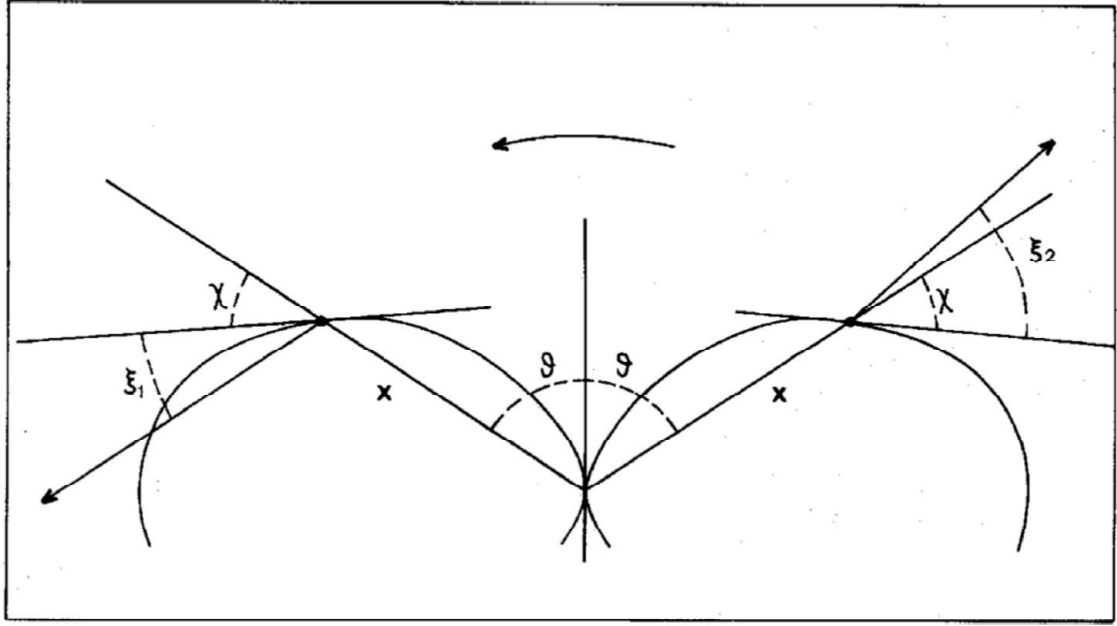


Figure 3.4: Magnetic field line geometry of an orthogonal rotator. The arrow indicates the direction of rotation. From Morini (1983).

at very different heights, i.e. one photon may have been emitted physically closer to the observer, but at an earlier time than the other.

The shift in photons' observed phase due to differing travel distances is called the *time-of-flight delay*, and is given, to first order in r/R_{LC} , by

$$\Delta\phi_{\text{tof}} \simeq -\frac{r}{R_{LC}}, \quad (3.5)$$

where the negative indicates that a photon emitted closer to the observer will *always* be observed earlier because of the finite speed of light (Dyks and Harding, 2004).

A second effect is caused by the shape of the magnetic field lines tangent to which the observed radiation is emitted: neglecting time-of-flight delays, photons emitted at a lower altitudes may reach the observer either before or after those emitted higher up, depending on whether they are emitted from leading or trailing field lines. Figure 3.4 demonstrates the geometry causing this, showing a simple orthogonal rotator turning in a counter-clockwise direction; lines tangential to the magnetic field are also drawn at two points on either side of the magnetic axis, along with the angles χ they make with the local radial direction. Consider an observer located directly to the top of Figure 3.4, parallel to the symmetry axis. On the trailing field lines, i.e. those on the right of Figure 3.4, the tangent line pointing towards the observer at any given moment starts off very close to the NS surface, then moves higher and higher as the rotator turns anti-clockwise. Radiation emitted closer to the NS surface therefore reach the observer earlier than radiation emitted higher up. On the trailing edge of the magnetic field, this effect therefore directly *counteracts* the time-of-flight delay, and photons emitted there tend to be bunched together in phase. On the leading field lines, on the left of Figure 3.4, the opposite is true, i.e. the altitude where the observer-directed emission originates decreases as the pulsar rotates. This effect therefore serves to *amplify* the difference in phase that photons emitted at different altitudes on leading field lines arrive at the observer. Emission from this edge is thus smeared out in phase. In short, the pulsar's rotation shapes emission from its trailing edge into a caustic surface, bunching large portions of the emission from this region in phase, and creating the peaks we observe

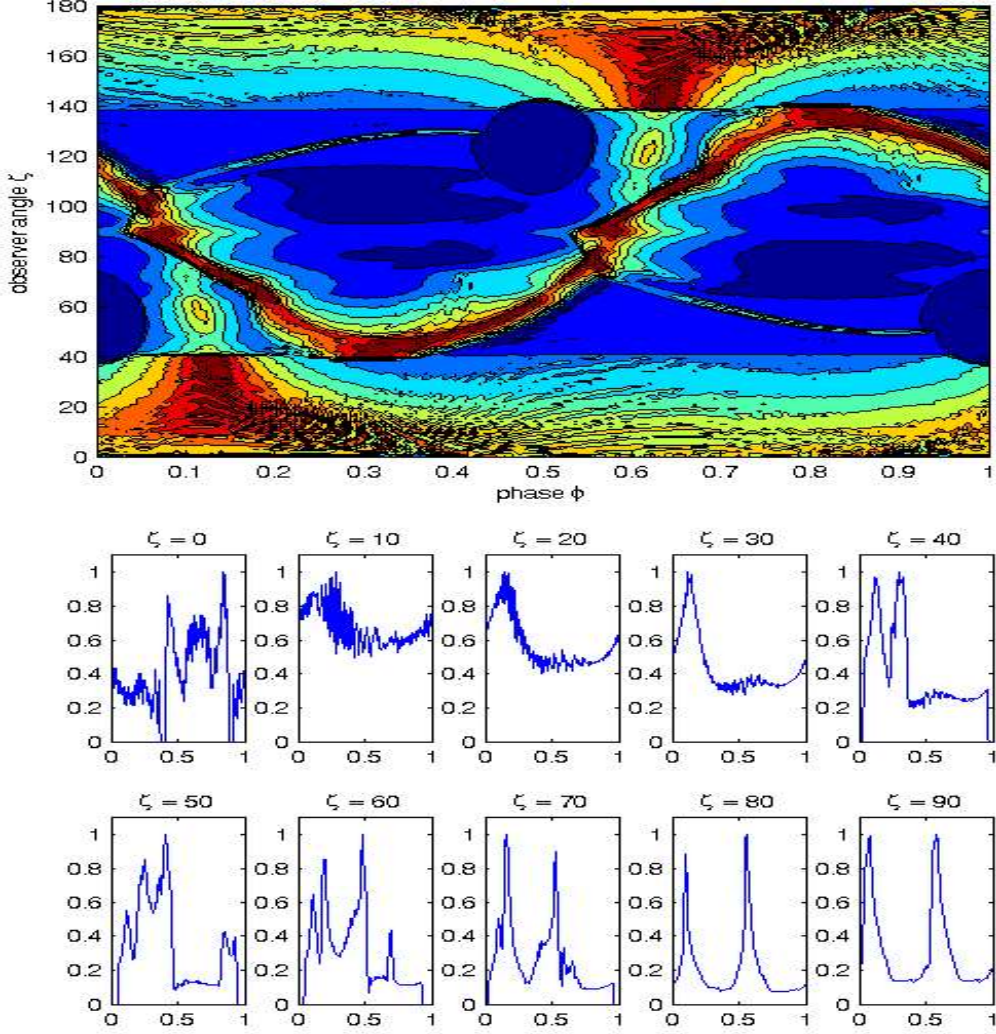


Figure 3.5: Phaseplot (top) and LCs (bottom) for a 55° tilt-angle pulsar modelled using the TPC geometric model.

in pulsar profiles. A twin-peak profile is formed when our line of sight intersects the caustic twice in one rotation.

The third factor relevant to the formation of caustics is the emitting particles' rotational velocity relative to the observer frame. While in the corotating frame emission occurs nearly tangential to the magnetic field lines, from the observer's perspective this emission is aberrated, increasingly so at higher altitudes. The perceived shift in phase as a result of this aberration is

$$\Delta\phi_{\text{ab}} \simeq -\frac{r}{R_{\text{LC}}} \quad (3.6)$$

to first order in r/R_{LC} (Dyks and Harding, 2004). This effect is very small for radio photons due to their low emission height, but can be important to the γ -ray phase lag.

3.5 Phaseplots and light curves

A method commonly used for representing modelled pulsar emissions is the phaseplot, as shown in the top panel of Figure 3.5. These are plots of a pulsar's intensity per solid angle on axes of phase and observer angle, with tilt angle fixed. A second method for depicting pulsar emission, of which examples are given in the bottom panel of Figure 3.5, are LCs.

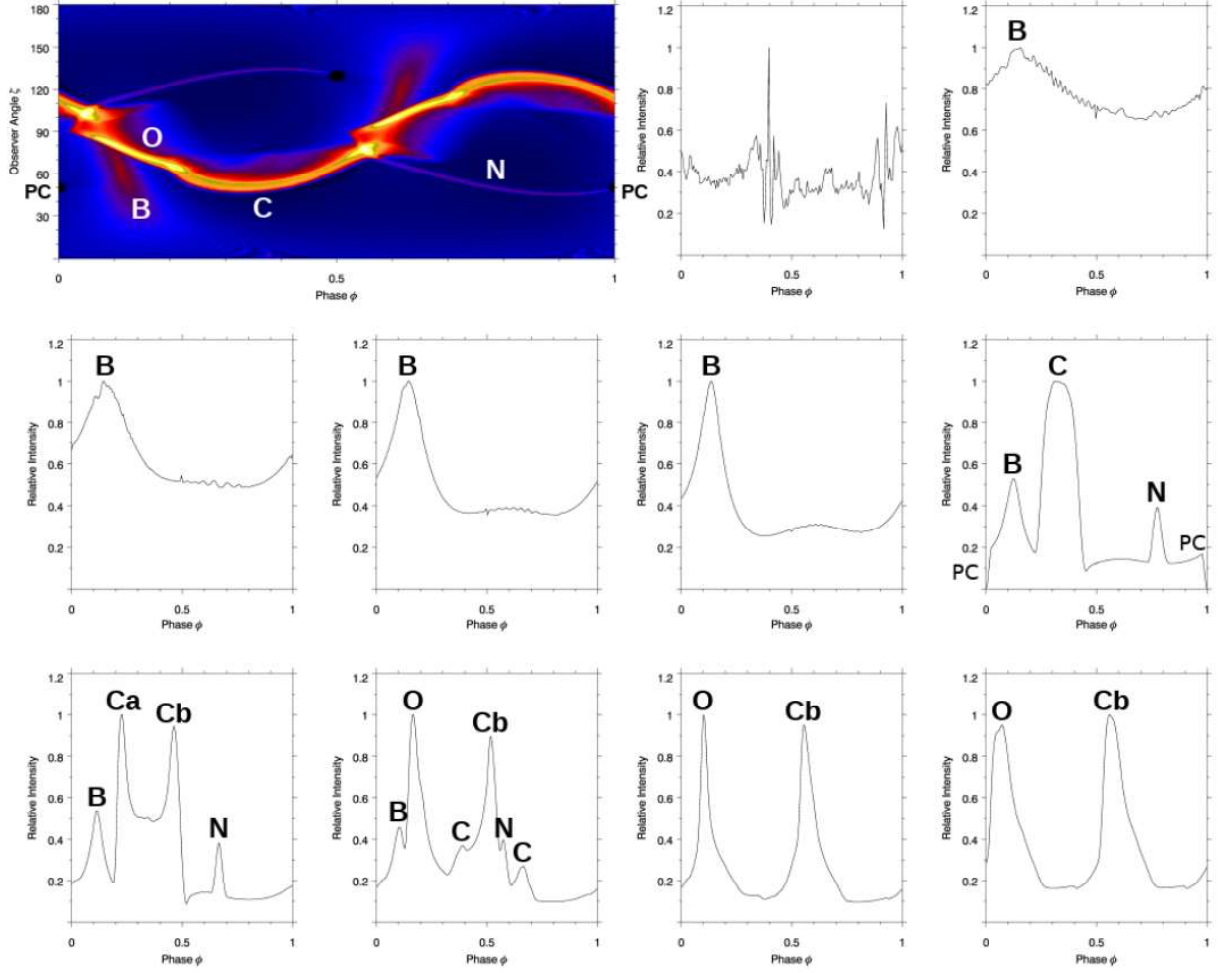


Figure 3.6: An example pulsar phase plot and common features exemplified with the corresponding LCs. Each LC is for a different ζ value, going from 0° to 90° in steps of 10° , increasing from left to right and top to bottom. See the text for details. Taken from Seyffert (2014).

LCs are normalised plots of a pulsar's observed intensity per solid angle $I(\alpha, \zeta, \phi_L)/\Delta\Omega$ as a function of observed phase ϕ_L , i.e. constant- ζ cuts of the phase plot. As such, phase plots contain the LCs at all possible observer angles for a single pulsar, and might therefore be considered a projection of its radiation beam. Notice in Figure 3.5 that ζ -cuts of the phase plot which intersect the caustic twice, e.g. $\zeta = 70^\circ$, produce double-peak LCs: physically this represents the emission cone crossing the observer's line of sight twice in one rotation. Single-peak profiles therefore characterise cases where, in the pulsar's reference frame, the observer crosses its cone tangentially, as opposed to secantially. LCs are the primary method used in this work for depicting pulsars' radiation. In the context of this work, I determine statistically which combination of α and ζ best reproduce the LCs, both in radio and γ -rays, of a number of sources published in *Fermi's* Second Pulsar Catalog (Abdo et al. 2013).

Figure 3.6 shows certain features commonly found on the phase plot, and the LCs associated with these features. As ζ increases, the first feature to appear, labelled *B*, is called the *bump*, and appears earlier in phase for greater ζ values due to its negative slope. Note that the black dots at $\alpha \sim 50^\circ$ and $\alpha \sim 130^\circ$ represent the PCs, where no emission takes place; the PCs manifest as dips to zero intensity on LCs, as can be seen on the sections labelled *PC*. At $\alpha \sim 50^\circ$, the observer intersects both the caustic (*C*), and the emission from the *notch* (*N*, see Section 3.1.2), which are represented by the second and third peaks on the LC for this observer angle. At $\zeta \sim 60^\circ$, the observer cuts the caustic twice, represented

by the peaks Ca and Cb on the LC; this implies a secantial intersection by the observer of the emission cone. The last common phase plot feature, O , is called the *overlap*, where a transition from the N feature to another occurs. Here, the small feature just after the second caustic peak is associated with N , while the last peak is produced by cutting the small tail of the caustic.

3.6 Conclusions

This chapter laid out the geometric models employed further on to generate the predicted pulsar LCs that I compare with observations. First I introduced the RVD magnetic field which represents the basic assumption for the pulsar magnetospheric structure, and then the various γ -ray and radio models utilised in this work. I also described certain effects that contribute to the characteristic beaming of pulsars' emission, as well as how this emission is commonly depicted graphically.

In the next chapter I describe the methods through which observed pulsar LCs can be compared with the predicted LCs obtained using the geometric models described in this chapter.

Chapter 4

Light curve fitting methods

This chapter presents the various methods employed to compare modelled pulsar LCs with observed LCs from the Second *Fermi* Pulsar Catalog (Abdo et al., 2013), and how we used these comparisons to obtain estimates for the α and ζ geometric parameters of 16 pulsars. First, I detail the process by which we constrain these geometries by comparing modelled and observed LCs by eye. I then describe Pearson’s χ^2 test statistic, a test statistic which is often used for LC fitting, and discuss its shortcomings with respect to our aims of concurrently fitting LCs from two wavebands with disparate errors. Next, I introduce the bespoke test statistic that we have developed to facilitate improved cross-waveband LC fitting. These fitting methods are applied to a sample of pulsars in the next chapter.

4.1 By-eye fitting

The first approach used in this work towards finding best-fit (α, ζ) pairs is simple by-eye fitting of modelled LCs onto observed data. For a given pulsar, γ -ray LCs are modelled using a *C* code (see e.g., Dyks et al. (2004)) taking as most important parameters the pulsar’s spin period P , α , ζ , the radial extent of the acceleration gap, its position and width, and the phase lag between radio and γ -ray peaks $\Delta\phi$. We model radio LCs using another code with P , $\dot{P}$, radio emission height, α , ζ , $\Delta\phi$, and the radio frequency as notable parameters (see e.g., Story et al. (2007)).

In both of these codes we fix all parameters besides α , ζ , and $\Delta\phi$ to generate both a radio and γ -ray LC assuming a particular model combination. We then superimpose these LCs on the pulsar’s observed LCs in both wavebands, and make a qualitative decision as to whether or not the experimental data are satisfactorily reproduced by these model realisations. In making this judgement, particular attention is paid to whether or not the number of γ -ray and radio peaks in the modelled LC matches that of the observed LC, whether the peaks have similar shapes, and whether the peaks occur at similar phases. This process is iterated to cover all possible combinations of α , ζ , and $\Delta\phi$, keeping all other model parameters fixed. Finally, we draw an inclusion contour in (α, ζ) parameter space, defining the border between (α, ζ) pairs adjudged to adequately reproduce the observed LCs for that particular pulsar and those which do not. This is repeated four times per pulsar, using every combination of the OG/TPC geometric γ -ray models and cone/core radio models discussed in the previous chapter.

The best-fit (α, ζ) pair is taken to be in the centre of the inclusion contour, with the errors on this estimate comprising a square encompassing all of the contour. Note that in cases where two or more disconnected closed contours appear on the map, we study the LCs at the centre of each contour and choose the best fit among these, again purely qualitatively.

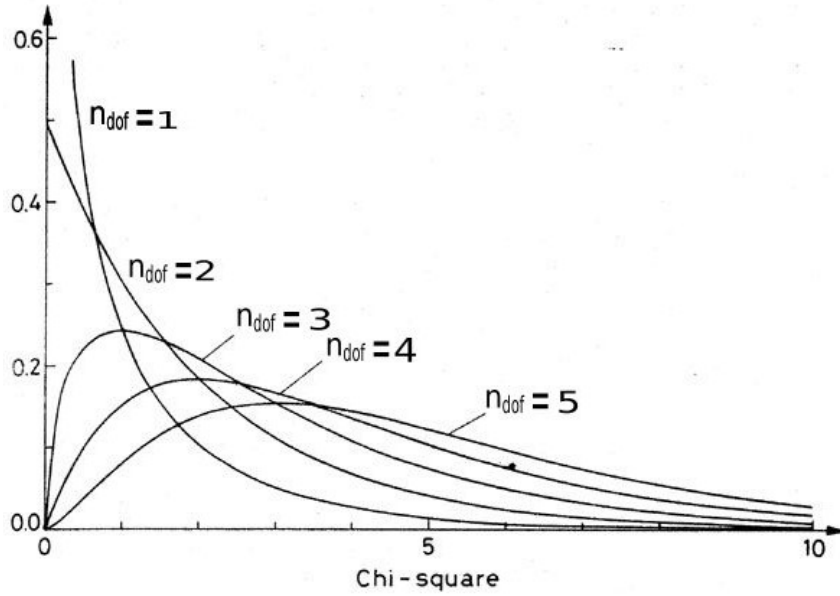


Figure 4.1: The χ^2 distribution for a number of different degrees of freedom, n_{dof} . Important to note is that $0 < \chi^2 < \infty$. Adapted from Leo and Haase (1990).

This fitting method is, of course, rather subjective: two people applying the same method for the same pulsar using the same geometric model will find two slightly differing answers. This is especially true when there are multiple (α, ζ) pairs plausibly replicating the observed data for a single pulsar. As such, this fitting method is not seen as an attempt at constraining pulsar geometries in and of itself, but rather as a sanity check, or a basis for (qualitatively) judging the accuracy of more rigorous methods: if a statistical approach produces an answer far out of line with what by-eye fitting delivers, the former result may be cast into doubt. Conversely, a strong correspondence between the results of by-eye fitting and a given statistical fit only serves to strengthen the case for the suitability of the statistical fitting method in question.

4.2 The χ^2 test statistic

4.2.1 Background

Considering the limitations of the by-eye fitting method, a more rigorous alternative is desired, such as the pervasive Pearson's χ^2 goodness-of-fit test (Pearson, 1900).

For a set of n_{dof} independent variables x_i , normally distributed about a mean of μ_i and each having a variance of σ_i^2 , one can define a test statistic

$$\chi^2 \equiv \sum_{i=1}^{n_{\text{dof}}} \frac{(x_i - \mu_i)^2}{\sigma_i^2} \quad (4.1)$$

commonly referred to as Pearson's χ^2 test statistic.

The χ^2 statistic can be used to assess the goodness-of-fit between a set of experimentally measured quantities x_i and a corresponding set of predicted values μ_i , assuming a set of σ_i values representing uncertainties on the former.

Assuming a Gaussian distribution of errors, the χ^2 probability distribution¹, can be

¹Formally, this should be called the $\chi^2_{n_{\text{dof}}}$ distribution, since each value of n_{dof} corresponds to a different

written as

$$f(\chi^2) = \frac{1}{2^{n_{\text{dof}}/2} \Gamma(n_{\text{dof}}/2)} e^{-\chi^2/2} (\chi^2)^{(n_{\text{dof}}/2)-1} \quad (4.2)$$

with n_{dof} the number of degrees of freedom, a positive integer, and $\Gamma(p)$ the “Gamma function” defined by $\Gamma(q+1) \equiv \int_0^\infty x^q e^{-x} dx$. This probability distribution is normalised (i.e. $\int_0^\infty f(\chi^2) d\chi^2 = 1$), has a mean value of n_{dof} , a variance of $2n_{\text{dof}}$, and reduces to the Gaussian distribution for $n_{\text{dof}} \gg 1$. The χ^2 distribution is shown for a number of different degrees of freedom in Figure 4.1.

If the experiment in question is carried out N times, for example, we can assign to each data sample a value of x_i , and compare these to model predictions, assigned μ_i . If the observed data were well matched by our model hypothesis, our sample of χ^2 values should take on the distribution described by (4.2). Note that this distribution will be one of $n_{\text{dof}} = N - r$ degrees of freedom, where r is some number of relations or constraints connecting the x_i .

If we pick a single value of χ^2 from our N collected samples, it will have a certain probability, say p , of being greater than χ_p^2 , where χ_p^2 is defined as

$$\int_{\chi_p^2}^\infty f(\chi^2) d\chi^2 = p. \quad (4.3)$$

Thus we can choose a requisite significance level p (e.g. 5% or 10%) and, knowing the relevant number of degrees of freedom, derive χ_p^2 . Normally this value can be looked up in a reference table such as Table 4.1. If the χ^2 we calculated from comparing observed and modelled data has a value approximately equal to the value of χ_p^2 , it is considered an indication that the respective datasets match to the extent of satisfying the desired level of significance. If $\chi^2 > \chi_p^2$ we might conclude that our model is poorly chosen, and if $\chi^2 < \chi_p^2$ we may suspect either over-estimation of the values of σ_i or untrustworthy data, i.e. data that are “too good to be true” (smoothed).

A handy rule of thumb is that a value of χ^2 approximately equal to n_{dof} represents a good model fit. To see this, note that if each model prediction μ_i is within one standard deviation of the corresponding observation x_i , then $(x_i - \mu_i)^2 \sim \sigma_i^2$, and thus from (4.1) $\chi_{n_{\text{dof}}}^2 \sim n_{\text{dof}}$.

4.2.2 Application to pulsar LCs

We can apply Eq. (4.1) to our purposes of pulsar LC fitting by replacing x_i with the modelled (expected) intensity in the i^{th} bin (E_i), μ_i with the corresponding observed intensity (O_i), and σ_i with the error on the observed intensity (U_i). In this context Pearson’s χ^2 test statistic can thus be written as

$$\chi^2 = \sum_{i=1}^{n_{\text{bins}}} \left(\frac{E_i - O_i}{U_i} \right)^2, \quad (4.4)$$

where n_{bins} designates the number of phase bins the LC is divided into.

If our LC models have $n_{\text{parameters}}$ free parameters, the minimum value of χ^2 would ideally be approximately equal to the number of degrees of freedom, $N_{\text{dof}} = n_{\text{bins}} - n_{\text{parameters}}$, specifying a good match between modelled and observed data. However, this rarely occurs for the models used in this study, with the minimum test statistic often being far larger than N_{dof} . This indicates that the models are still somewhat rough approximations of the real phenomena.

distribution, but for the sake of brevity and established convention I drop the subscript from here on. Also note that χ^2 here refers to a single statistical variable, and not the square of some variable χ . The notation is strictly evocative of its construction as a sum of squares.

n_{dof}	0.1%	0.5%	1.0%	2.5%	5.0%	10.0%	12.5%	20.0%	25.0%	33.3%	50.0%
1	0.000	0.000	0.000	0.001	0.004	0.016	0.025	0.064	0.102	0.186	0.455
2	0.002	0.010	0.020	0.051	0.103	0.211	0.267	0.446	0.575	0.811	1.386
3	0.024	0.072	0.115	0.216	0.352	0.584	0.692	1.005	1.213	1.568	2.366
4	0.091	0.207	0.297	0.484	0.711	1.064	1.219	1.649	1.923	2.378	3.357
5	0.210	0.412	0.554	0.831	1.145	1.610	1.808	2.343	2.675	3.216	4.351
6	0.381	0.676	0.872	1.237	1.635	2.204	2.441	3.070	3.455	4.074	5.348
7	0.598	0.989	1.239	1.690	2.167	2.833	3.106	3.822	4.255	4.945	6.346
8	0.857	1.344	1.646	2.180	2.733	3.490	3.797	4.594	5.071	5.826	7.344
9	1.152	1.735	2.088	2.700	3.325	4.168	4.507	5.380	5.899	6.716	8.343
10	1.479	2.156	2.558	3.247	3.940	4.865	5.234	6.179	6.737	7.612	9.342

Table 4.1: Reference table of χ^2 values given a number of degrees of freedom n_{dof} and a desired significance p . Adapted from Haynam et al. (1982).

Typically, finding the (α, ζ) pair that leads to the best fit of a pulsar’s modelled and observed LCs entails the following: first, the χ^2 test is applied to all possible (α, ζ) pairs in radio and γ -rays separately, yielding two “maps” of test statistic values on axes of α and ζ . These maps can then be minimised to find the best-fit combination of α and ζ for each waveband. This minimum may or may not be co-located in the radio and γ -ray bands.

The purpose of our work is to find a *combined* γ -ray and radio fit, i.e. an (α, ζ) pair corresponding to an adequate simultaneous radio and γ -ray fit. The standard approach to finding such a combined fit is to add the separate radio and γ -ray χ^2 maps, and then minimise the resultant. However, the dependence of Pearson’s test statistic on observational uncertainties U_i creates problems. In our case, the relative errors on the observed γ -ray data are much larger than those of the radio data, so that the typical values of χ^2 are much smaller for the LC in the γ -ray domain than for the radio domain. Simply adding the test statistics in the γ -ray and radio domains therefore creates a combined χ^2 map that is very much radio-dominated. When using this combined fitting method it is therefore common to end up with a model fit that reproduces one waveband’s observed LC qualitatively well (although not formally), but the other waveband’s observed LC not at all.

Other studies have circumvented the problem of vastly differing χ^2 values associated with the γ -ray and radio datasets by artificially inflating the errors on the radio observations to match those of the γ -ray observations. Johnson et al. (2014) applied this approach to MSPs, while Pierbattista et al. (2015) applied it to young and middle-aged pulsars with longer periods of rotation. While this method invariably performs better than simple addition of the χ^2 maps, different pulsars often require different degrees of radio error inflation, and in some cases these errors even need to be decreased to match the wide range of errors for different γ -ray pulsar LCs (in other words, arbitrary tuning is necessarily involved in this approach). Crucially, radio error inflation defeats the purpose of concurrent radio and γ -ray LC fitting, i.e. taking advantage of good radio observations and relatively well-developed γ -ray geometrical models of pulsars, and offsetting the problems presented by cruder radio models and large-error γ -ray observations.

Seyffert et al. (2016) expounds an alternative approach to multiband pulsar LC fitting: scaling the radio and γ -ray χ^2 maps before combining them. In this method the dynamic ranges of the χ^2 maps for each waveband are equalised before adding them, forcing each domain to carry equal weight in determining the best concurrent fit. Bezuidenhout et al. (2017) applied this approach to a sample of eleven pulsars, but obtained mixed results.

As an example, the dynamic range scaling approach of Seyffert et al. (2016) is applied

to PSR J0205+6449 using the TPC γ -ray model and the cone radio model in Figure 4.2. The colour scales represent the value of the χ^2 test statistic for γ -rays, radio, and a scaled χ^2 combination of the two (from left to right). The white space is where the models predict the pulsar to be invisible to us; these regions of parameter space are ignored. It is important to note that the χ^2 values vary greatly in each of these plots, and that the minimum of

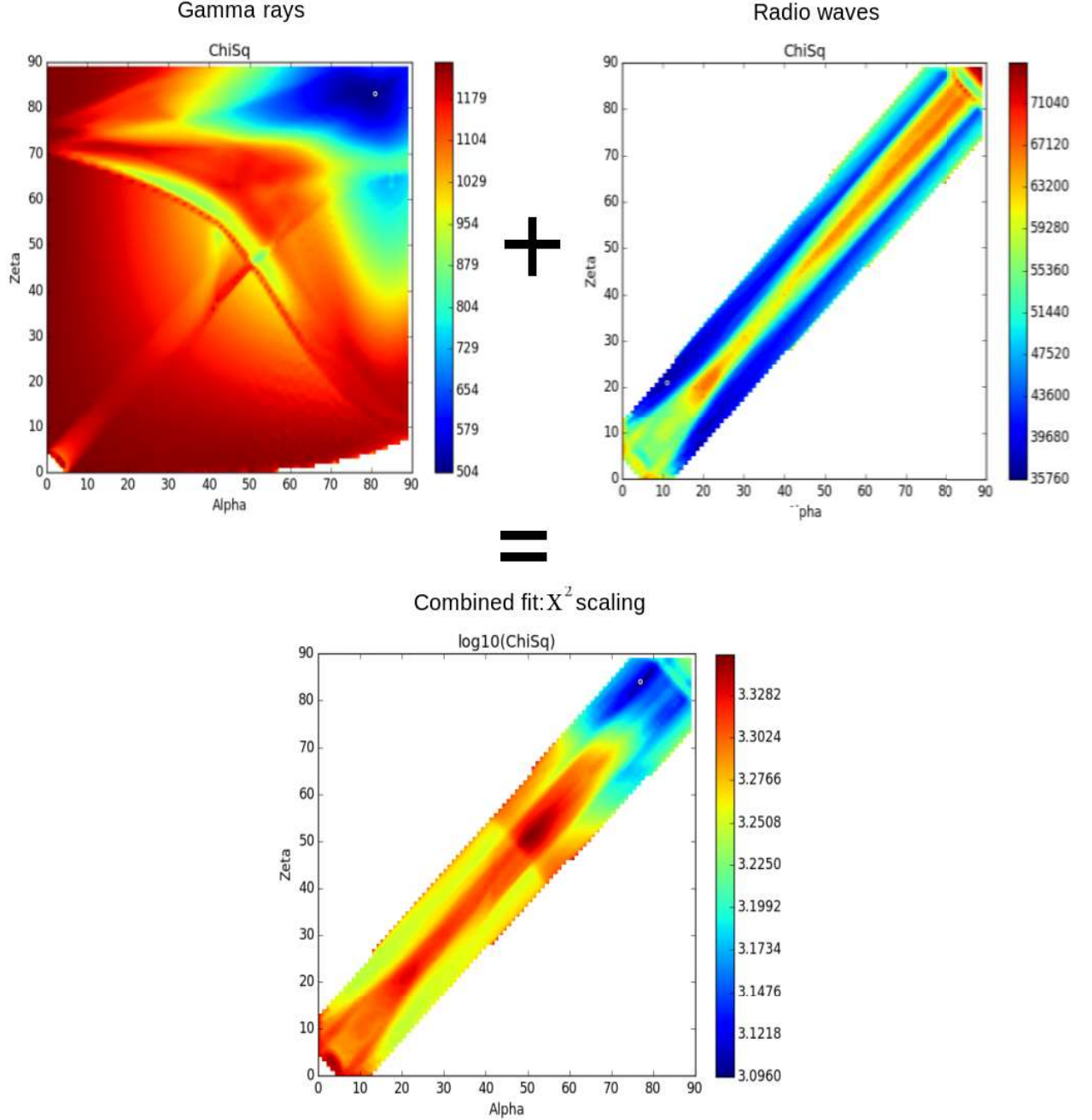


Figure 4.2: Maps of Pearson’s χ^2 test statistic on axes of α and ζ for PSR J0205+6449 using the TPC γ -ray model and the cone radio model. The first two maps are for the γ -ray and radio regimes separately (on a linear scale), while the third is a combination of the two obtained using the dynamic range scaling approach laid out in Seyffert et al. (2016), on a logarithmic scale. The minimum χ^2 value in each map is marked by a white ring. The white regions are where the model predicts the pulsar to be invisible. It is important to note the disparity in the values of the test statistic in each map prior to combination, as represented by the colour scale.

the combined fit lies much closer to that of the γ -ray band than of the radio band. This illustrates the difficulty obtaining a combined radio and γ -ray fit that does not over-represent one band in favour of the other.

4.3 The Scaled-Flux Normalised χ^2 test statistic

It is clear that the Pearson χ^2 test statistic’s dependence on observational data uncertainties makes it ill suited for combining fits across datasets with disparate errors. For this reason Seyffert et al. (in preparation) developed an alternate test statistic which minimises the impact of such data error disparities, called the Scaled-Flux Normalised (SFN; χ_ϕ^2) test statistic. The SFN test statistic is defined as

$$\chi_\phi^2 = \frac{\phi^2 - \chi^2}{\phi^2 - n_{\text{dof}}} \quad (4.5)$$

where

$$\phi^2 = \sum_i \left(\frac{B_i - D_i}{\epsilon_i} \right)^2 \quad (4.6)$$

is the usual χ^2 value from Eq. 4.4 for the background flux.² The B_i , D_i , and ϵ_i are the background flux, observed flux, and error on the observed flux in the i^{th} bin, respectively. The test statistic’s name refers to the fact that it represents a normalised measure of the goodness-of-fit of the “excess”, non-background, flux.

It is instructive to consider the case of a very good match between modelled and observed LCs: under these conditions $\chi^2 \sim n_{\text{dof}}$, so that the value of the test statistic will be approximately unity. On the other end of the scale, e.g. in the case where the model predicts nothing but background noise, $\chi^2 \sim \phi^2$, and thus the SFN test statistic approaches zero. A negative value (indicating a worse fit than a flat model LC) is unlikely, and a value larger than one is analogous to the case of $\chi^2 < \chi_p^2$ in representing a suspicious over-fit. Therefore the value of the SFN test statistic can be thought of as a normalised measure of goodness-of-fit, scaling from zero (no fit) to one (good fit). Maximising this test statistic across all possible combinations of α and ζ results in the best possible estimation of a given pulsar’s geometry.

The most crucial facet of the SFN test statistic is that it partially negates the problem presented by comparisons between datasets with dissimilar errors. As opposed to Pearson’s test statistic, which returns values that can vary greatly depending on the data errors, the SFN test statistic nearly always returns values around the range of zero to one. This means that SFN test statistic values are much more consistently comparable between datasets, regardless of uncertainties, than is the case with Pearson’s test statistic, rendering any sort of error inflation or scaling unnecessary. Maps of χ_ϕ^2 can be simply added together, and the resultant will weigh each constituent approximately equal, leading to a combined LC fit that favours neither the radio nor the γ -ray fit.

4.3.1 A geometrical heuristic

Here follows a simple geometrical framework for thinking about test statistics that we hope is illustrative of the differences between Pearson’s χ^2 test statistic and our SFN test statistic,

²For our purposes, we used the background flux reported in the second *Fermi* pulsar catalog, which is derived through a complicated probability analysis described in Abdo et al. (2013). One should not make the mistake of conflating background flux levels with off-peak flux levels, since some of the latter is of magnetospheric origin.

and why the latter is better suited to multiband LC fitting.

One can envision an n_{bins} -dimensional space where each dimension corresponds to one of n_{bins} phase bins into which a pulsar’s LC is divided. The diagram in Figure 4.3 is a two-dimensional analogue of this space. Any particular LC is represented by a single point in this space, e.g. a modelled LC $\vec{M}$, an observed LC $\vec{D}$, and a flat off-peak level background LC $\vec{B}$ as in the figure. Furthermore, we can construct vectors $\vec{M}$, $\vec{D}$, and $\vec{B}$ from the origin to these points. Goodness-of-fit tests can be thought of as being concerned with measuring and comparing the length of and separation between such vectors in this space.

Pearson’s χ^2 test statistic measures the square of the distance between $\vec{M}$ and $\vec{D}$ in this space in units of $\vec{\epsilon}$, the error on observations. While it is valid to compare these “distances” from different model predictions to a single set of observations (say, a radio LC), these distances are not necessarily comparable to distances from a second set of observations (say, a γ -ray LC). The difference in error values $\vec{\epsilon}$ of various observations mean that χ^2 distances in this space are measured in different units, and thus cannot be simply added.

The SFN test statistic scales the χ^2 distance between $\vec{M}$ and $\vec{D}$ to units of Φ , the square of the distance between $\vec{B}$ and $\vec{D}$, thus *normalising this distance*. We can draw two concentric circles centred on $\vec{D}$, as in green on the figure, the inner circle being defined by $\vec{\epsilon}$, and the outer circle by $\vec{B}$. The closer $\vec{M}$ is to the outer circle, the closer χ_ϕ^2 ’s value is to zero; the closer $\vec{M}$ is to the inner circle, the closer χ_ϕ^2 ’s value is to one. This makes it clear that the SFN test statistic is significantly less dependent on observational uncertainties, and that distances measured in this way are more comparable between wavebands.

The SFN test statistic is therefore well suited to our purposes of multi-wavelength LC fitting because it uses the signal-to-noise ratio as a ruler for measuring goodness of fit, as opposed to the χ^2 test statistic, which uses errors on observations as a ruler. Because this excess-to-background ratio is nearly ϵ -independent, the SFN test statistic to a large extent negates the problem posed by disparate radio and γ -ray errors on pulsar LC observations.

We can intuitively understand this using the following argument: if the errors in each phase bin are approximately equal to some average error $\langle\epsilon\rangle$, then from Eq. (4.6) $\phi^2 \sim 1/\langle\epsilon\rangle^2 \sum_i (D_i - B_i)^2$, and from Eq. (4.5)

$$\begin{aligned} \chi_\phi^2 &\simeq \frac{1/\langle\epsilon\rangle^2 [\sum_i (D_i - B_i)^2 - \sum_i (D_i - M_i)^2]}{1/\langle\epsilon\rangle^2 \sum_i (D_i - B_i)^2 - n_{\text{dof}}} \\ &= \frac{\sum_i (D_i - B_i)^2 - \sum_i (D_i - M_i)^2}{\sum_i (D_i - B_i)^2 - \langle\epsilon\rangle^2 n_{\text{dof}}} \end{aligned} \quad (4.7)$$

which is approximately independent of $\langle\epsilon\rangle$ when the product $\langle\epsilon\rangle n_{\text{dof}}$ is small.

4.3.2 Best-fit errors

Once the SFN test has been applied to a given pulsar, combined, and mapped onto (α, ζ) axes, the best-fit (α, ζ) parameters are chosen as those that minimise the value of the combined test statistic. In order to obtain error estimates on these best-fit values, we use a Monte Carlo simulation to draw confidence contours around the minimum in (α, ζ) space. This entails successively simulating random variations on the observed data within the bounds of the errors ϵ_i , and submitting these synthetic datasets to the SFN test procedure so as to determine the probability distribution of the χ_ϕ^2 test statistic. The errors on our best-fit estimates are then chosen to encompass all (α, ζ) combinations which produce a χ_ϕ^2 value within one standard deviation of the maximum. For more information on error estimation using Monte Carlo simulation, see e.g. Hastings (1970).

The fitting process is repeated four times per pulsar, once using each possible combination of the OG/TPC γ -ray models and the cone/core radio models. In the next chapter error estimates are found on each of these fits via Monte Carlo simulation, and compared to what was found by-eye.

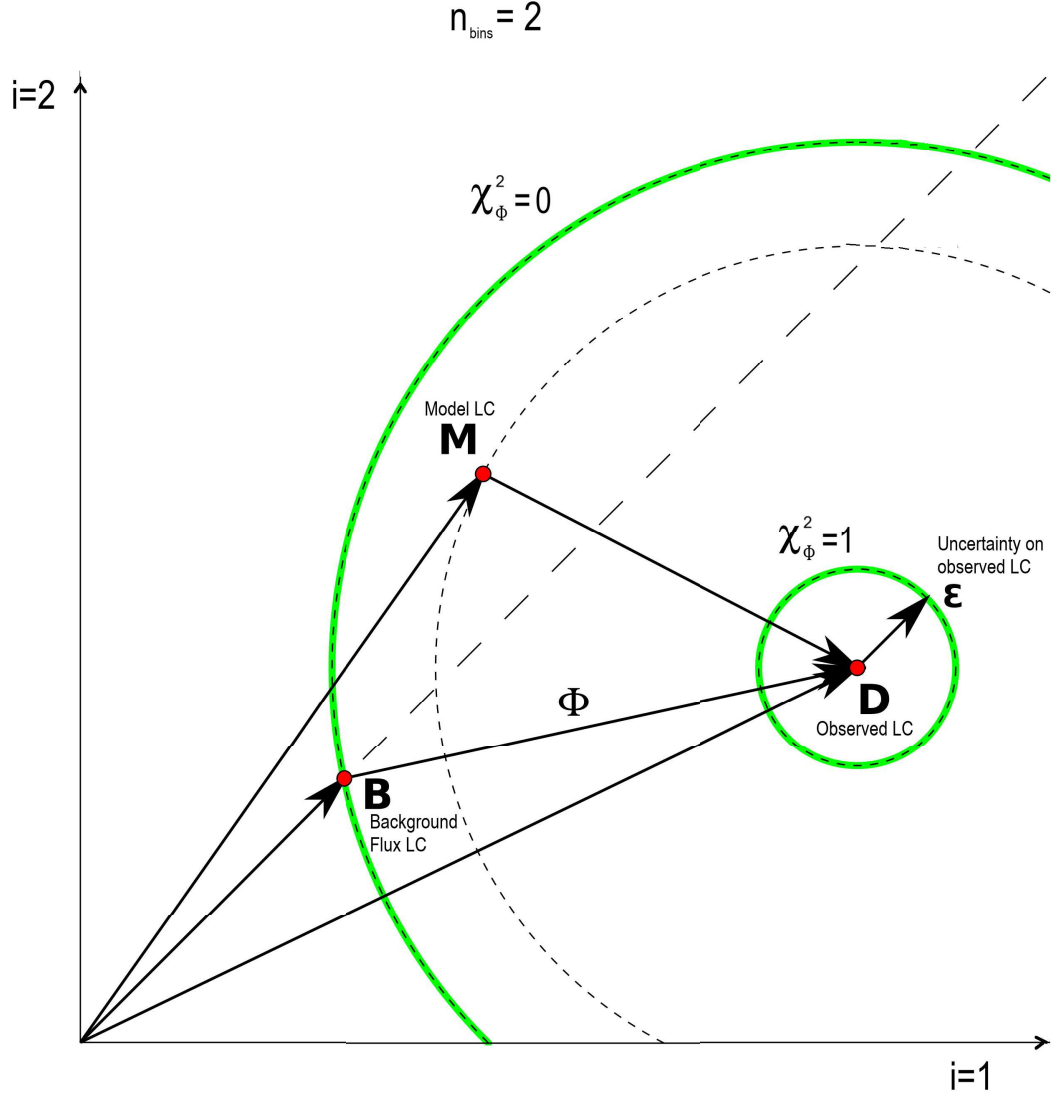


Figure 4.3: A two-dimensional representation of an n_{bins} -dimensional space corresponding to n_{bins} phase bins of a pulsar LC. A representative model LC, observed LC, and background flux LC are represented by the points M , D , and B in this space. Pearson's χ^2 test statistic measures the distance between M and D in this space in units of observational errors, $\vec{\epsilon}$, and the SFN test statistic scales this distance to units of Φ , the square of the distance between $\vec{B}$ and $\vec{D}$. The outer green circle centred on D represents the zero-value of the χ_{ϕ}^2 test statistic, and the inner circle represents its unity value. Note that for this figure we assumed equal errors in both phase bins of the LC—in general each bin will have a slightly different relative error, and thus the inner boundary will be irregularly shaped. See the text for details.

4.4 Conclusions

In this chapter I outlined various methods through which modelled and observed pulsar LCs can be compared simultaneously in the radio and γ -ray wavebands. Although one can get an idea of goodness-of-fit via by-eye comparison, a more rigorous, statistical technique is desired that removes the subjective human element. Moreover, any such statistical approach would have to take into account the disparate errors on the available radio and γ -ray pulsar observations.

Johnson et al. (2014) and Pierbattista et al. (2015) attempted concurrent radio and γ -ray LC fitting by artificially inflating the radio errors before applying Pearson's χ^2 test statistic in each band separately, and then adding the two. I do not consider this method optimal because it relies on reducing the quality of the radio data. Seyffert et al. (2016) suggested applying Pearson's χ^2 test statistic separately in each band without radio error inflation, scaling the results to have equal dynamic range, and then combining across bands. When this method was applied by Bezuidenhout et al. (2017), the results were not clearly better than those of Johnson et al. (2014) or Pierbattista et al. (2015).

Seyffert et al. (in preparation) introduced a test statistic called the SFN test statistic, which was developed specifically to minimise the impact of disparate observational errors, and thus allow for direct cross-band comparison. The principal objective of this thesis is to apply this test statistic to a sample of pulsars and assess whether it leads to improved dual-band LC fits.

Chapter 5

Results

In this chapter I relay the results we found applying the LC fitting methods described in the previous chapter to a selection of 16 pulsars using four combinations of radio and γ -ray geometric models. First, I recount our source-selection process. Second, I report the (α, ζ) constraints we obtained for our sample of pulsars and compare them with those found by Johnson et al. (2014) and Pierbattista et al. (2015). Third, I present an example best-fit LC we found for PSR J0030+0451, superimposed on its observed LCs, and compared with the other authors' best-fit LCs (our best-fit LCs for the rest of the sample pulsars can be found in Appendix B). Lastly, I show plots comparing the constraints on α and ζ found using the various fitting methods. In Appendix A I show the inclusion contours (as discussed in the previous chapter) we found by eye and through statistical fitting.

5.1 Source Selection

Our primary consideration with regard to selecting pulsars for LC fitting was to obtain a sample with a wide spread of observed spin-down rates ($\dot{E}$, see Eq. [2.3]). This was done with the work of Brambilla et al. (2015) in mind, who compared *Fermi* LAT pulsar spectra with those predicted by the FIDO (Force-free Inside, Dissipative Outside) pulsar magnetosphere model of Kalapotharakos et al. (2014). The former found a positive correlation between pulsars' $\dot{E}$ values and the optimal conductivity (σ) of their magnetospheres outside the light

MSPs	$\dot{E}$ (erg/s)	Canonical pulsars	$\dot{E}$ (erg/s)
PSR J0030+0451	3.5×10^{33}	PSR J0205+6449	3.0×10^{37}
PSR J0437−4715	1.2×10^{34}	PSR J0631+1036	1.7×10^{35}
PSR J1231−1411	1.8×10^{34}	PSR J0659+1414	3.8×10^{34}
PSR J1614−2230	1.3×10^{34}	PSR J0742−2822	1.4×10^{35}
		PSR J1124−5916	1.2×10^{37}
		PSR J1410−6132	1.0×10^{37}
		PSR J1420−6048	1.0×10^{37}
		PSR J1509−5850	5.2×10^{35}
		PSR J1513−5908	1.8×10^{37}
		PSR J1718−3825	1.3×10^{36}
		PSR J1833−1034	3.4×10^{37}
		PSR J2229+6114	2.2×10^{37}

Table 5.1: The sample of 16 pulsars used in this work grouped into MSPs and canonical pulsars, and listing the spin-down rate $\dot{E}$ of each.

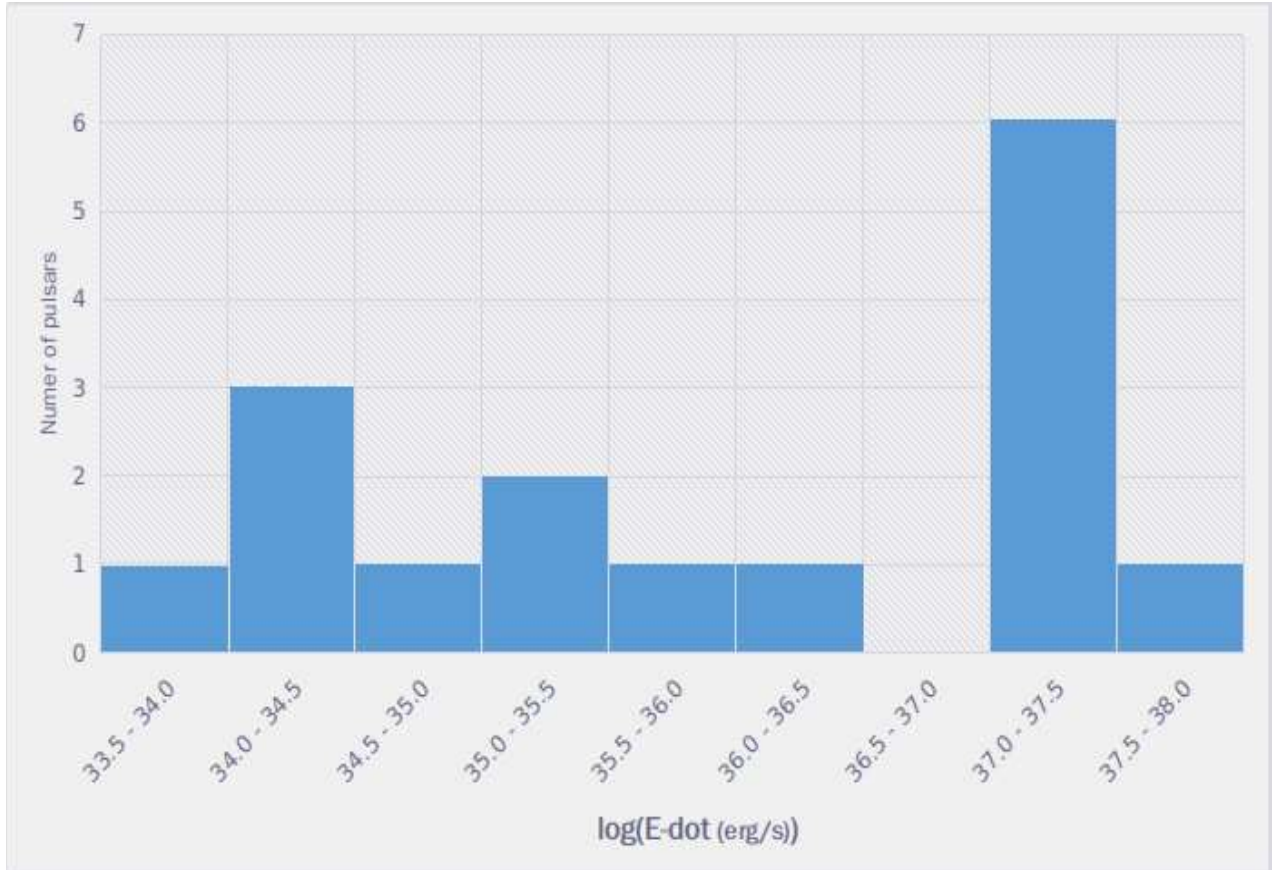


Figure 5.1: Histogram of the sample pulsars’ observed $\log_{10} \dot{E}$ values. Our aim was to select pulsars with as even a spread of these values as possible.

cylinder as derived from said spectral fitting. We want to revisit this trend in the future, but using LC fitting to constrain σ instead of spectral fitting (Seyffert et al., in preparation), and the fits obtained in this work might provide a useful point of reference. For this reason it was important that we choose pulsars with a wide range of values of $\dot{E}$. A histogram of the selected pulsars’ $\log_{10} \dot{E}$ values is shown in Figure 5.1.

Furthermore, we opted for a mix of four MSPs previously fit by Johnson et al. (2014) and twelve younger pulsars fit by Pierbattista et al. (2015) in order to have two points of comparison for our obtained constraints. We also chose pulsars which have good statistics available (i.e. small relative errors) for both their radio and γ -ray LCs found in the second *Fermi* pulsar catalog (Abdo et al., 2013).

Lastly, we made sure to pick some of the top targets for observation of the Neutron Star Interior Composition Explorer (NICER) mission aboard the International Space Station (ISS), which aims to make highly accurate measurements of these pulsars’ masses and radii (Gendreau et al., 2012). To do this they require constraints on α to use as priors, which we might be able to provide.

The sources we selected are reported in Table 5.1, grouped into MSPs and canonical pulsars, with the spin-down rate of each reported adjacently.

5.2 Obtained parameter constraints

In this section I compile the constraints on the sample pulsars’ inclination angle α and observer angle ζ (see Figure 1.2) that we obtained through by-eye fitting and statistical

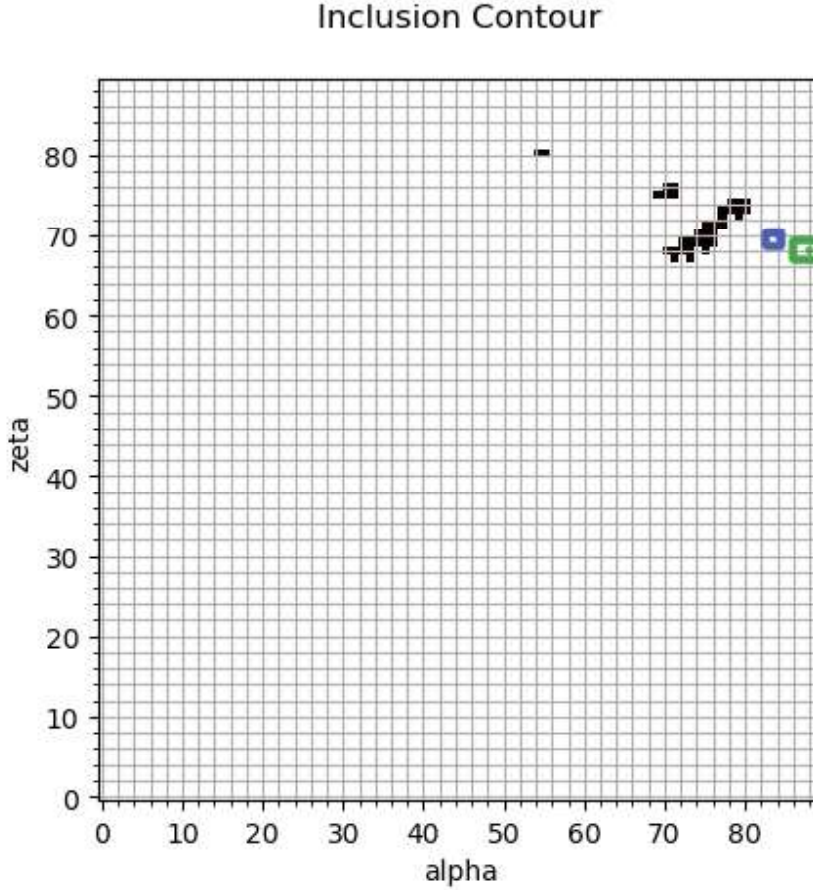


Figure 5.2: By-eye fitting inclusion contour for PSR J0030+0451 using the OG geometrical γ -ray model and core radio model. Black regions represent combinations of α and ζ for which the modelled and observed LCs are adjudged to match in both wavebands. In green is the best-fit (α, ζ) constraints found by Johnson et al. (2014) for PSR J0030+0451 using the OG γ -ray model, and in blue is what we obtained for this pulsar using SFN LC fitting.

fitting using the SFN test statistic. Table 5.2 lists the best-fit (α, ζ) pairs that we found using these methods along with those found by either Johnson et al. (2014) or Pierbattista et al. (2015) for each of the sample pulsars.

5.2.1 By-eye fitting

We applied the by-eye fitting method as detailed in Section 4.1 to each of the sample pulsars listed in Table 5.1 four times: once for each possible combination of the TPC/OG γ -ray models and cone/core radio models. This entails fixing all model parameters besides LC phase difference $\Delta\phi$, and the pulsar's α and ζ angles, and selecting the combinations of α and ζ for which we adjudge the models to adequately replicate the pulsar's observed LCs (simultaneously in both the radio and γ -ray bands). We optimised $\Delta\phi$ by manually shifting the modelled LCs forward and backward in phase until they best match the observed LCs. The obtained inclusion contour encloses all of these selected (α, ζ) ¹ pairs. The best-fit parameter pair is specified as the centre of a box drawn to encompass the entire contour,

¹Note that throughout this work, pairs of the α and ζ angles are denoted by (α, ζ) , always in that order, and always as measured in degrees.

Pulsar	BY-EYE FITS				INDEPENDENT FITS				FITS VIA THE SCALED-FLUX NORMALISED TEST STATISTIC			
	OG + Cone	OG + Core	TPC + Cone	TPC + Core	Pierbattista OG	Pierbattista TPC	Johnson OG	Johnson TPC	OG + Cone	OG + Core	TPC + Cone	TPC + Core
J0030+0451	(60±8, 78±4)	(75±5, 70±4)	(75±7, 59±6)	(75±5, 57±8)			(88 ⁺¹ ₋₂ , 68±1)	(74±2, 55 ⁺³ ₋₁)	(85±1, 67±1)	(84±1, 70±1)	(73±1, 60±1)	(73±1, 60±1)
J0205+6449	(65±5, 88±2)	(86±3, 80±3)	(86±4, 60±7)	(85±5, 79±4)	(73±2, 90±2)	(75±2, 86±2)			(74±1, 85±1)	(84±1, 86±1)	(74±1, 85±1)	(84 ⁺² ₋₃ , 86 ⁺¹ ₋₃)
J0437-4715	(31±10, 62±4)	(58±6, 40±7)	(57±9, 20±10)	(52±7, 32±10)			(76±1, 46±1)	(35±1, 64±1)	(22±1, 65±1)	(35 ⁺³ ₋₁ , 61±1)	(54±1, 47 ⁺¹ ₋₂)	(34 ⁺¹⁸ ₋₁ , 62 ⁺¹ ₋₁₃)
J0631+1036	(54±10, 53±9)	(58±8, 57±8)	(49±7, 39±9)	(52±2, 51±2)	(87±2, 72±2)	(48±2, 67±2)			(69 ⁺⁸ ₋₄ , 61 ⁺⁷ ₋₄)	(66 ⁺² ₋₃ , 65±2)	(62 ⁺⁸ ₋₄ , 54 ⁺⁶ ₋₄)	(60 ⁺³ ₋₂ , 59 ⁺⁴ ₋₁)
J0659+1414	(57±3, 49±2)	(52±5, 52±3)	(42±9, 34±8)	(34±6, 34±9)	(78±2, 73±2)	(30±2, 32±2)			(54 ⁺⁹ ₋₈ , 48 ⁺¹⁴ ₋₃)	(56 ⁺¹ ₋₃ , 52 ⁺⁴ ₋₁)	(53±1, 46±1)	(51±2, 51 ⁺² ₋₃)
J0742-2822	(77±4, 64±4)	(82±7, 81±7)	(48±5, 38±5)	(40±13, 39±11)	(76 ⁺² ₋₃ , 86 ⁺³ ₋₂)	(62±2, 77±2)			(77±1, 88 ⁺¹ ₋₁)	(85±3, 85 ⁺² ₋₃)	(66 ⁺⁸ ₋₇ , 79 ⁺⁶ ₋₂)	(83 ⁺² ₋₃ , 83 ⁺² ₋₃)
J1124-5916	(56±8, 86±3)	(83±6, 84±5)	(84±6, 62±6)	(82±8, 82±8)	(83±2, 88±2)	(84±2, 89±2)			(71±1, 88±2)	(89±3, 83±1)	(83±1, 80±1)	(83±1, 82±2)
J1231-1411	(79±5, 50±4)	(81±8, 53±10)	(75±5, 46±7)	(73±5, 48±9)			(88±1, 67±1)	(26 ⁺³ ₋₄ , 69±1)	(82±1, 65±1)	(80±1, 66±1)	(71±1, 59±1)	(71±1, 59±1)
J1410-6132	(63±4, 40±5)	(59±3, 52±4)	(53±3, 35±9)	(37±8, 44±9)	(87±2, 76±2)	(19 ⁺² ₋₄ , 6±2)			(52 ⁺³⁷ ₋₁₇ , 84 ⁺⁵ ₋₅₁)	(83 ⁺⁶ ₋₃₄ , 89 ⁺¹ ₋₄₀)	(6 ⁺⁹ ₋₅ , 17 ⁺¹² ₋₃)	(5 ⁺¹⁰ ₋₃ , 10 ⁺⁵ ₋₁)
J1420-6048	(60±6, 61±6)	(61±2, 60±2)	(54±3, 46±2)	(50±1, 50±1)	(55±2, 57±2)	(52±2, 53±2)			(65±1, 50±1)	(60±1, 58±1)	(60±1, 46±1)	(54 ⁺² ₋₁ , 53 ⁺² ₋₁)
J1509-5850	(58±5, 59±6)	(54±4, 55±5)	(54±2, 52±2)	(54±3, 54±5)	(85±2, 76±2)	(46±2, 66±2)			(76 ⁺⁷ ₋₁₄ , 65 ⁺⁶ ₋₁₄)	(67 ⁺³ ₋₁₁ , 66 ⁺² ₋₁₀)	(48 ⁺³ ₋₁ , 60±2)	(51 ⁺² ₋₁ , 53 ⁺¹ ₋₃)
J1513-5908	(58±4, 42±4)	(52±4, 52±4)	(48±8, 33±7)	(33±8, 38±8)	(60±2, 59±2)	(50±2, 54±2)			(56 ⁺¹ ₋₂ , 47±1)	(52 ⁺¹ ₋₂ , 51 ⁺¹ ₋₁)	(55±1, 46 ⁺² ₋₁)	(52±1, 51±1)
J1614-2230	(54±6, 84±4)	(52±8, 83±6)	(85±5, 54±7)	(84±5, 51±6)			(64 ⁺⁸ ₋₂₀ , 88 ⁺² ₋₅)	(80 ⁺⁸ ₋₂₀ , 80 ⁺⁶ ₋₄)	(89 ⁺¹ ₋₅ , 84 ⁺² ₋₃)	(88 ⁺¹ ₋₃ , 84 ⁺² ₋₄)	(84 ⁺² ₋₁ , 82±1)	(85 ⁺¹ ₋₆ , 83 ⁺³ ₋₁)
J1718-3825	(51±4, 51±4)	(54±4, 53±4)	(46±7, 34±8)	(34±7, 40±7)	(80±2, 55±2)	(45±2, 65±2)			(67 ⁺⁵ ₋₂ , 56 ⁺² ₋₅)	(67±3, 63±3)	(47±1, 54±1)	(57±1, 58±1)
J1833-1034	(55±10, 78±3)	(78±5, 72±5)	(75±5, 52±8)	(53±8, 78±2)	(65±2, 87±2)	(55±2, 75±2)			(87 ⁺² ₋₆ , 65 ⁺² ₋₆)	(85 ⁺³ ₋₁ , 81±2)	(67±1, 80±1)	(80 ⁺⁶ ₋₁ , 83 ⁺³ ₋₂)
J2229+6114	(60±5, 38±6)	(50±5, 50±5)	(51±10, 33±8)	(34±6, 41±7)	(75±2, 55±2)	(42±2, 55±2)			(64±1, 50±1)	(61 ⁺¹ ₋₂ , 59±2)	(51 ⁺¹ ₋₃ , 65 ⁺¹ ₋₃)	(54±1, 45±1)

Table 5.2: Best-fit (α, ζ) pairs (in degrees) for each sample pulsar using by-eye fitting, those found by other authors, and those we found using the SFN test statistic. We used four different combinations of the OG and TPC γ -ray models and the core and cone radio models for fitting. Note that we assumed a minimum error of ± 1 on each of these values, i.e. equal to the resolution in α and ζ .

also defining the errors on the best-fit parameters. In Figure 5.2, such an inclusion contour is shown for PSR J0030+0451 using the OG γ -ray model and core radio model, for which we found a best-fit (α, ζ) combination of $(75 \pm 5, 70 \pm 4)$. Also shown is the best-fit (α, ζ) constraints found by Johnson et al. (2014) for this source using the OG γ -ray model in green [best fit $(88^{+1}_{-2}, 68 \pm 1)$], and what we found for this source and model combination using SFN fitting as described in Section 4.3 in blue (best fit $(84 \pm 1, 70 \pm 1)$).

Note that among the by-eye fits in Table 5.2 there are cases where multiple best-fit (α, ζ) pairs are listed. In these situations, each pair represents a separate, closed inclusion contour.

In Appendix A I present for each pulsar four by-eye inclusion contours similar to Figure 5.2, one for each of the four possible model combinations. Overlaid on these is the best-fit (α, ζ) pair (with errors) we found using SFN statistical fitting, and that found by either Johnson et al. (2014) or Pierbattista et al. (2015).

5.2.2 Statistical fitting using the SFN test statistic

Figure 5.3 shows a map of the SFN test statistic (see Section 4.3) in parameter space which we found when applying the method to PSR J1509–5850 using the OG γ -ray model and the cone radio model. The red cross designates the location of the maximum value of the test statistic on this map, in this case $(76^{+7}_{-14}, 65^{+6}_{-14})$, and therefore the best-fit (α, ζ) . The 1σ errors on the best fit are defined by the white contours, which are obtained using Monte Carlo simulation (see Section 4.3.2).

Note that the colour scale representing the value of the combined (radio + γ -ray) SFN test statistic in Figure 5.3 ranges from zero to one—compare this with the wide variety of ranges seen in Figure 4.1, for example. This illustrates the fact that different SFN test statistic maps are in general much more comparable with one another than is the case with maps of Pearson’s χ^2 test statistic.

5.3 Best-fit light curves

Figure 5.4 shows the combined best-fit LCs we found for PSR J0030+0451 through by-eye fitting (blue), and using the SFN test statistic (green), as well as best-fit LCs found by Johnson et al. (2014) (orange), all normalised and covering two rotations of the pulsar. These are superimposed on the pulsar’s observed flux in black. Note that the observed γ -ray LCs are represented using only error bars, instead of the customary histograms, for the sake of clarity. Each subfigure shows the fitting results found using one of the four possible combinations of the radio and γ -ray geometric models we used in this work. The legend in each subfigure also states the best-fit (α, ζ) pairs found using each method. Note that in each subfigure the γ -ray LCs are shown in the top plot, and the radio band LCs are shown below.

I present a similar figure for each of the 16 sample pulsars in Appendix B. The reader is encouraged to scrutinise each figure and evaluate the quality of these fits themselves.

5.4 Comparison of results

In Figures 5.5–5.10 I compare directly the best-fit geometries found using different LC fitting methods. Figure 5.5 plots the best-fit estimates for the tilt angle α for each sample pulsar found through statistical fitting using the SFN test statistic against those found through by-eye fitting. Each colour represents a different combination of radio and γ -ray used to

obtain the best-fit α . The closer a point is to the diagonal on this plot, the greater the agreement between the fit we found by eye and that found using the SFN test statistic.

Note that in the legend of each figure a value of Pearson’s product-moment correlation coefficient (PPMCC), r , is assigned to every model combination. This is a measure of the linear correlation between the two variables compared in the plot. The PPMCC may assume values between -1 and $+1$, where $+1$ indicates a total positive correlation, -1 a total negative correlation, and 0 no correlation. Table 5.3 compiles values of the PPMCC found when comparing the constraints for each fitting method and each possible model combination.

It is evident from Figure 5.5 that although there is general agreement between the α constraints we found using SFN fitting and those we found by eye, this correlation is fairly sensitive to the combination of radio and γ -ray models employed. In particular, the combinations of the OG + Cone models (blue) and TPC + Core models (red) produce a large number of outliers and significantly lower PPMCC values than is the case for the OG +

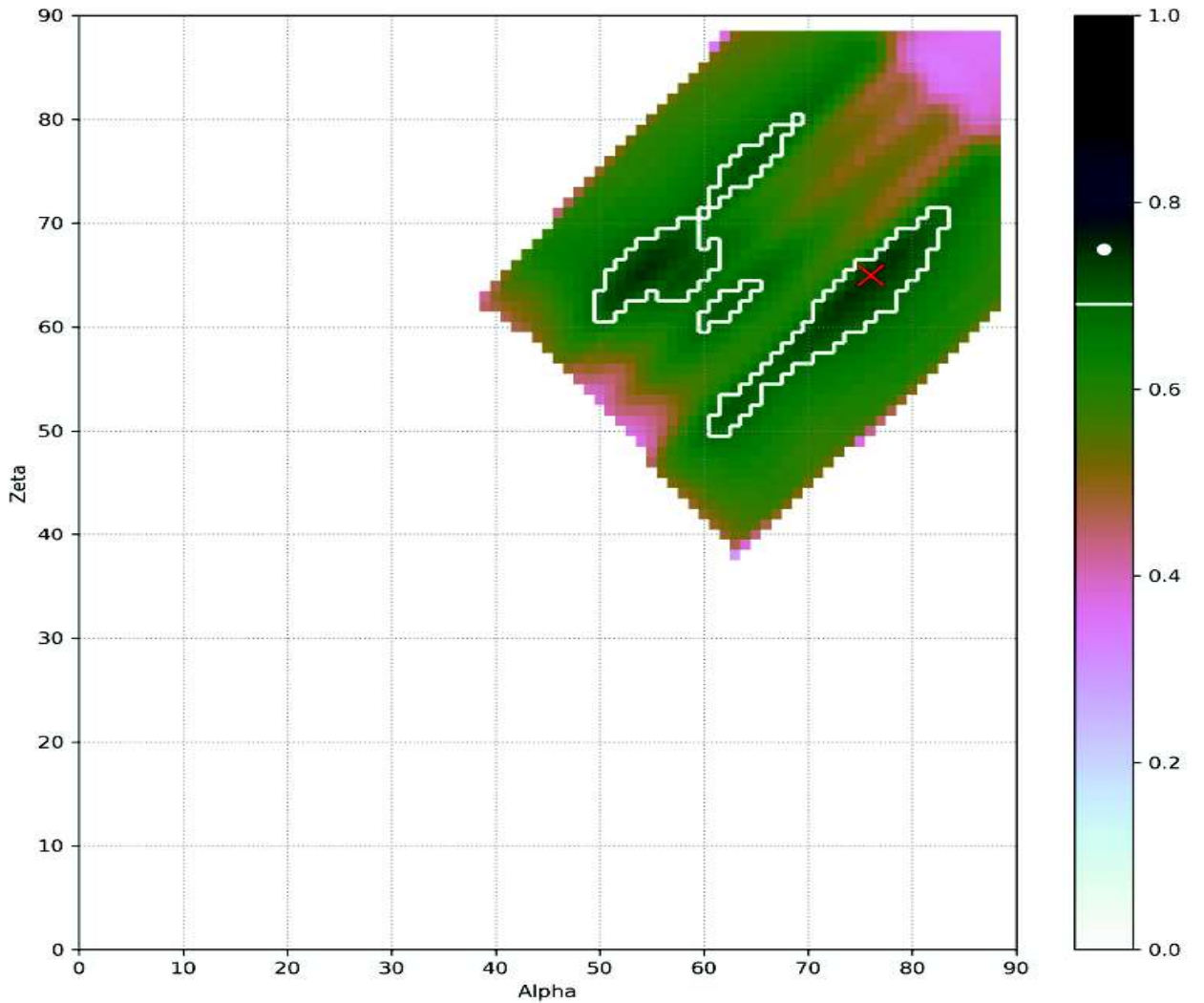


Figure 5.3: Combined SFN test statistic map for PSR J1509–5850 fit using the OG γ -ray model and the cone radio model. The red cross denotes the location of the maximum of the test statistic, and therefore the best-fit parameter combination. The white dot on the colour scale shows the maximum value of the test statistic on the map. The white contours are found using Monte Carlo simulation, and represent 1σ errors on the combined best fit. The white regions indicate geometries for which the models predict the pulsar should not be visible to us.

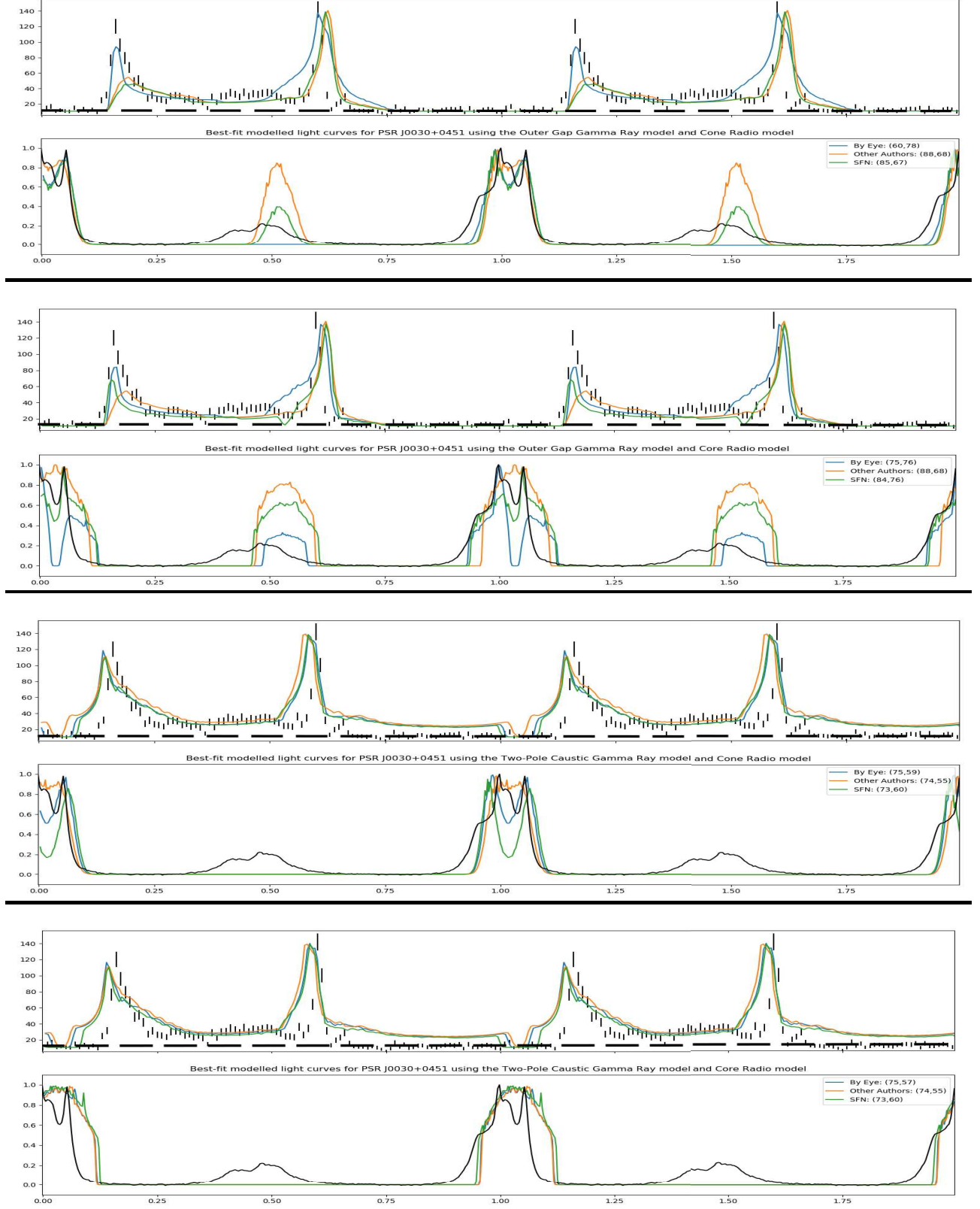


Figure 5.4: Best-fit LCs for PSR J0030+0451. Each subfigure (separated by the thick black lines) represent a different combination of the OG/TPC γ -ray models and cone/core radio models. In each case the radio LCs are shown on top and the γ -ray LCs on the bottom. The observed LCs are in black, the best-fit LC found by eye is in blue, that found by Johnson et al. (2014) is in orange, and what we found through SFN fitting is in green. The γ -ray background flux levels, as reported in *Fermi*'s Second Pulsar Catalog (Abdo et al., 2013), are indicated with dashed lines.

	OG + Cone		OG + Core		TPC + Cone		TPC + Core	
	α	ζ	α	ζ	α	ζ	α	ζ
SFN vs by eye	0.6640	0.7167	0.7756	0.7398	0.8918	0.8454	0.6213	0.6562
Other authors vs by-eye	-0.0102	0.6371	0.1777	0.7380	0.6639	0.7623	0.5895	0.6299
SFN vs other authors	0.0108	0.7308	0.2143	0.7843	0.7629	0.8913	0.7803	0.9156

Table 5.3: Values of the PPMCC for the α and ζ best-fit estimations found using all different fitting methods and model combinations.

Core (green) and TPC + Cone (yellow) combinations of models. The former pair of model combinations also seem to produce fits with generally larger error bars – probably indicative of a failure of these models to produce any one particularly good candidate LC, but rather a range of different LCs that all somewhat match the observed LC. It is important to bear in mind that the purpose of by-eye fitting is not to serve as a rigorous fitting method in and of itself, but purely as a “sanity check” to ensure that statistical fitting methods are not producing superficially incredible fits. I would suggest that the fact of broad accord between our SFN and by-eye fits supports the case of the SFN fitting method as useful, at least in the sense that it does not demonstrate the opposite.

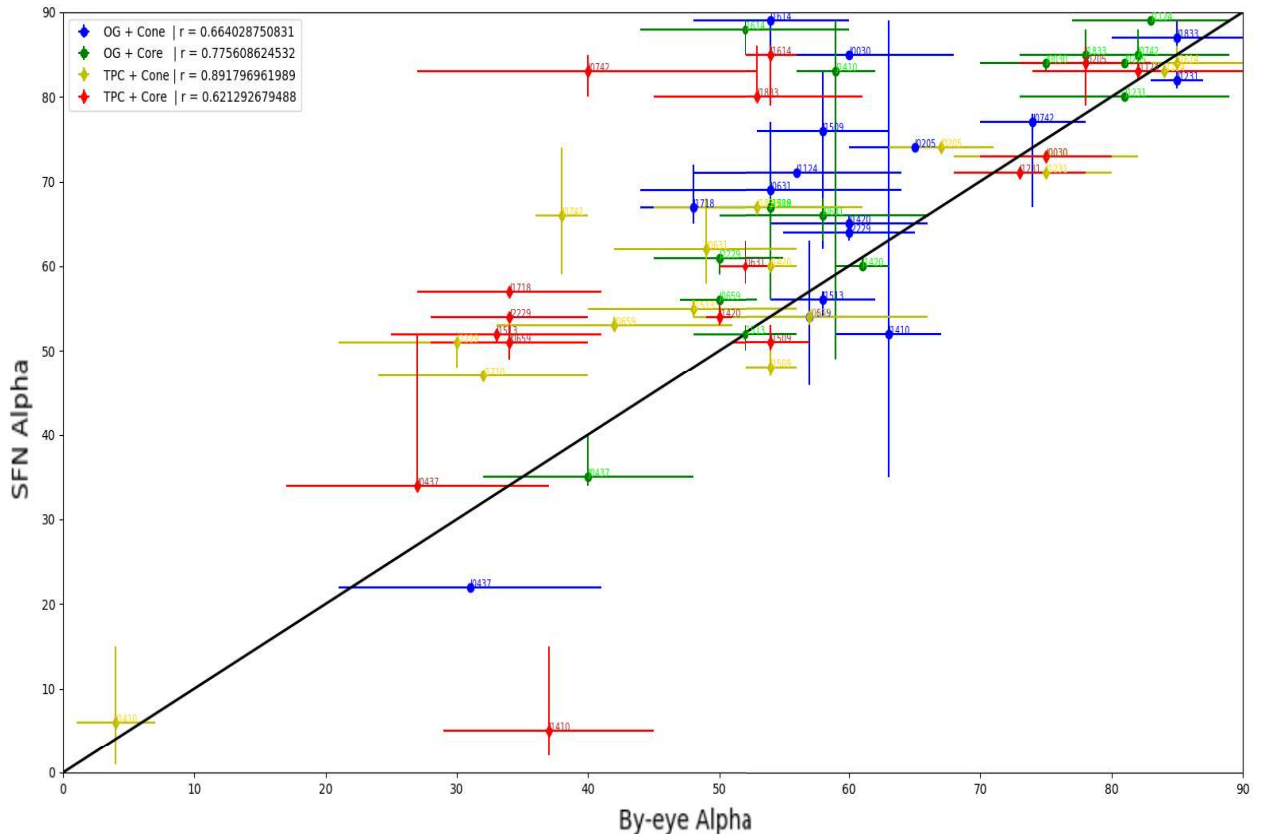


Figure 5.5: Comparison of best-fit tilt angles (α) found using the SFN test statistic vs. those found by eye. Each colour represents constraints found by employing a different combination of radio and γ -ray models. The r value shown in the legend for each model combination is the value of the PPMCC, a measure of the correlation between the plotted variables.

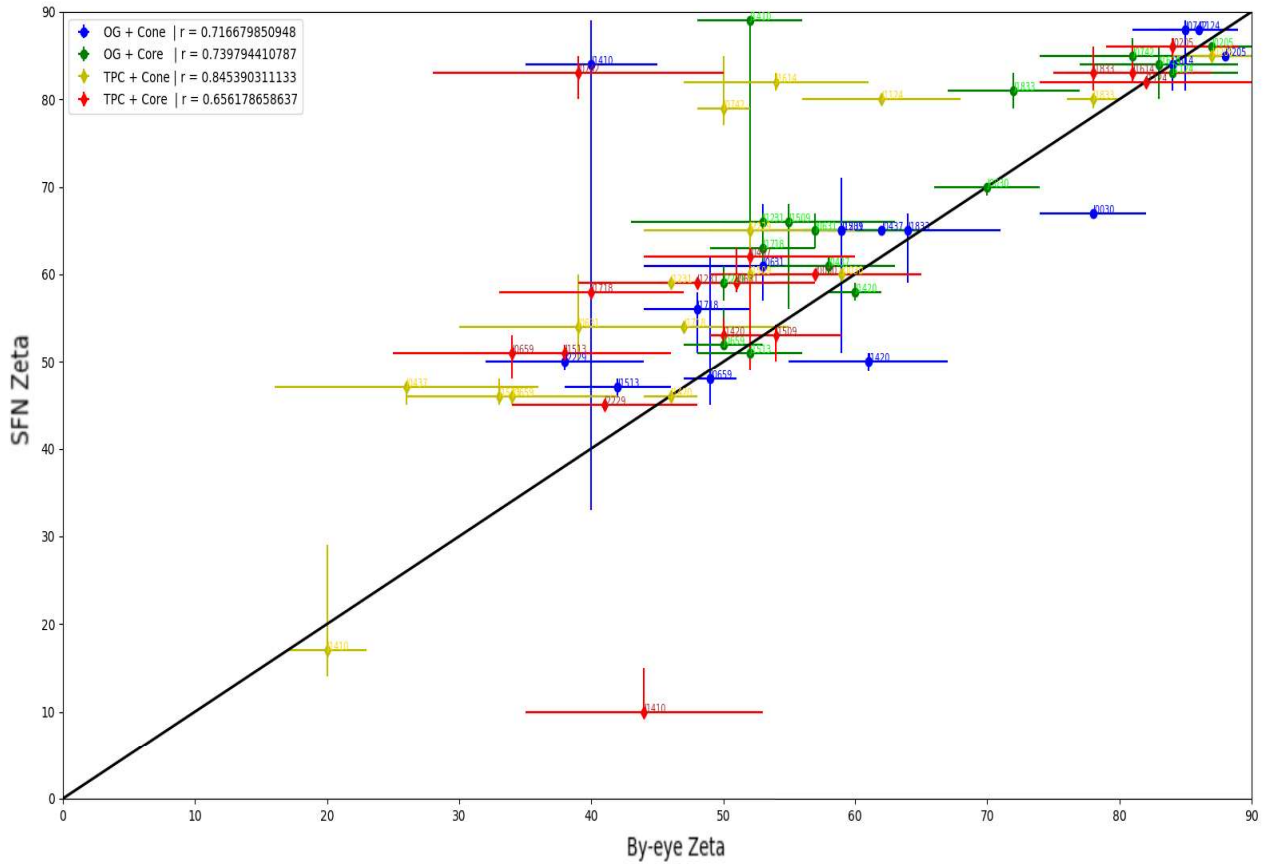


Figure 5.6: Comparison of best-fit observer angles (ζ) found using the SFN test statistic vs. those found by eye.

Figure 5.6 is equivalent to Figure 5.5, but for the observer angle ζ . Judging by PPMCC values, it is again the case here that the TPC + Cone model combination produces the best correlation between SFN fits and by-eye fits, and that the TPC + Core combination results in the worst correlation. It seems to be the case that, besides a couple of outliers, the constraints on ζ we found by eye are not too far away from those produced by SFN fitting, and on this basis SFN fitting again passes the sanity check.

Figures 5.7 and 5.8 compares the fits reported in Johnson et al. (2014) and Pierbattista et al. (2015) with those we obtained by eye. Notably, these authors' fits do not seem to correlate with the by-eye fits as well as the SFN fits do. In fact, as demonstrated by the respective PPMCC values in Table 5.3, they found constraints which are across the board less in line with the by-eye fits than the SFN fits are. While the lower correlation between the by-eye fits and Johnson et al. (2014) and Pierbattista et al. (2015)'s fits does not necessarily establish the inadequacy of the fitting methods used by these authors, it does suggest that SFN fitting produces LC fits that are more pleasing to the eye. Interestingly, the TPC + Cone model combination again produces the greatest agreement between by-eye fits and statistical fit for both α and ζ , while the OG + Cone and OG + Core combinations produce particularly poor α fits. It is curious that the α fits in Figure 5.7 are so much farther from the by-eye constraints than is the case for the ζ fits in Figure 5.8, a fact for which I can offer no explanation.

Lastly, in Figures 5.9 and 5.10 I plot the constraints found on α and ζ (respectively) that we obtained with SFN fitting against those published by Johnson et al. (2014) and Pierbattista et al. (2015). These figures demonstrate a general agreement between these sets of constraints for both parameters, besides two outliers, namely the α constraints found

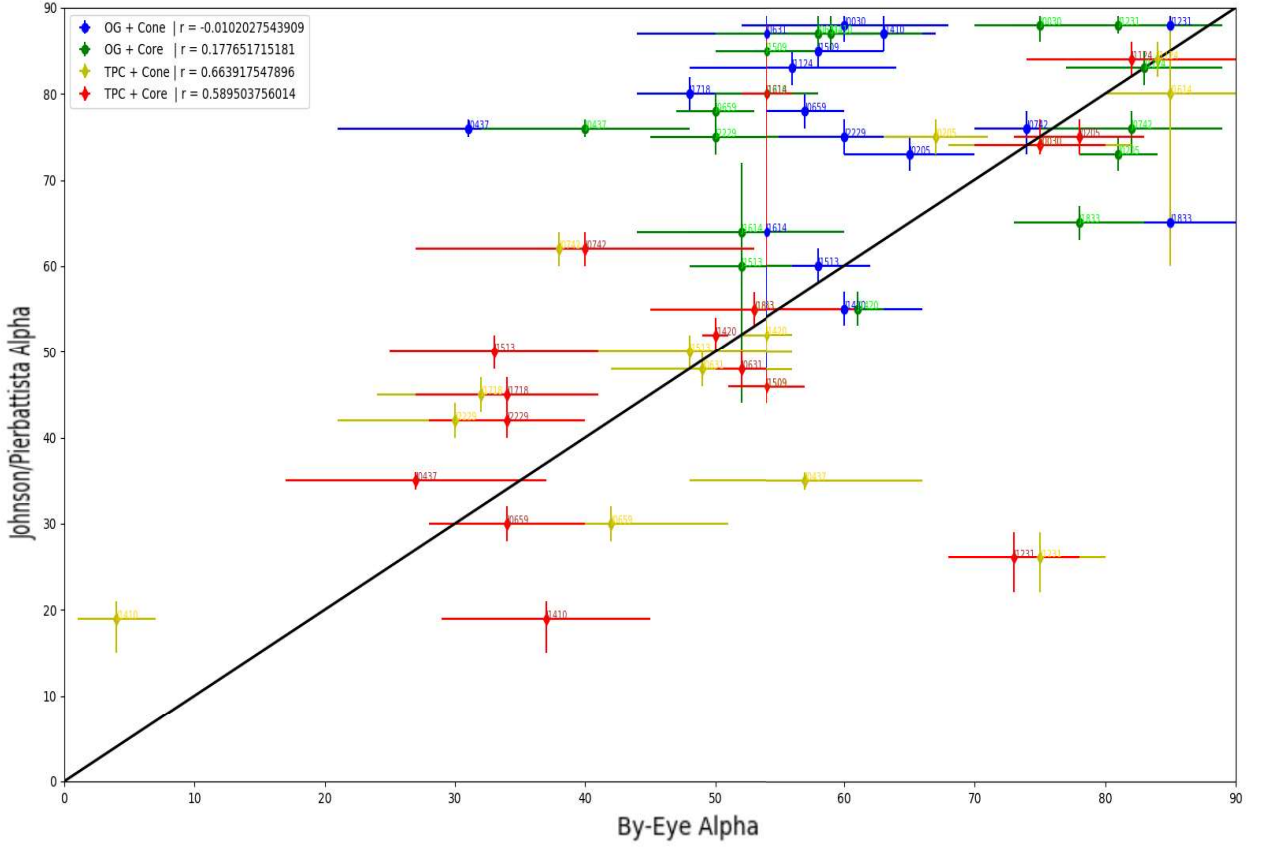


Figure 5.7: Comparison of best-fit tilt angles (α) found by either Johnson et al. (2014) or Pierbattista et al. (2015) using Pearson’s χ^2 test statistic and radio error inflation vs. those found by eye.

using the OG + Cone and the OG + Core combinations of models. It is telling to view these poor correlations in the light of the similar lack of agreement between these α fits found by the other authors and those we found by eye. This fact leads me to conclude that Johnson et al. (2014) and Pierbattista et al. (2015)’s α constraints found using the OG γ -ray model might not be as trustworthy, and that their disaccord with what we found through SFN fitting is not necessarily evidence of a problem with this fitting method. Apart from these two outliers, however, there seems to be a strong correlation overall between the parameter constraints we found using the SFN test statistic and those found by Johnson et al. (2014) and Pierbattista et al. (2015) using Pearson’s χ^2 test statistic.

5.5 Conclusions

This chapter relays the constraints on α and ζ found for a number of pulsars using various LC fitting methods, as compiled in Table 5.2, and aims to assess whether the SFN test statistic in particular is useful as an alternative to Pearson’s χ^2 goodness-of-fit test for multiband pulsar LC fitting.

Taken together, Figures 5.5–5.10 seem to paint a picture of SFN pulsar LC fitting as delivering best-fit LCs that are more convincing to the eye—in the radio and γ -ray bands simultaneously—than those produced by the manipulation of Pearson’s χ^2 test statistic as put into practice by Johnson et al. (2014) and Pierbattista et al. (2015). The reader is encouraged to look through the LCs in Appendix B to decide for themselves whether or not this assessment is borne out. If this is the case then it is reasonable to conclude that SFN

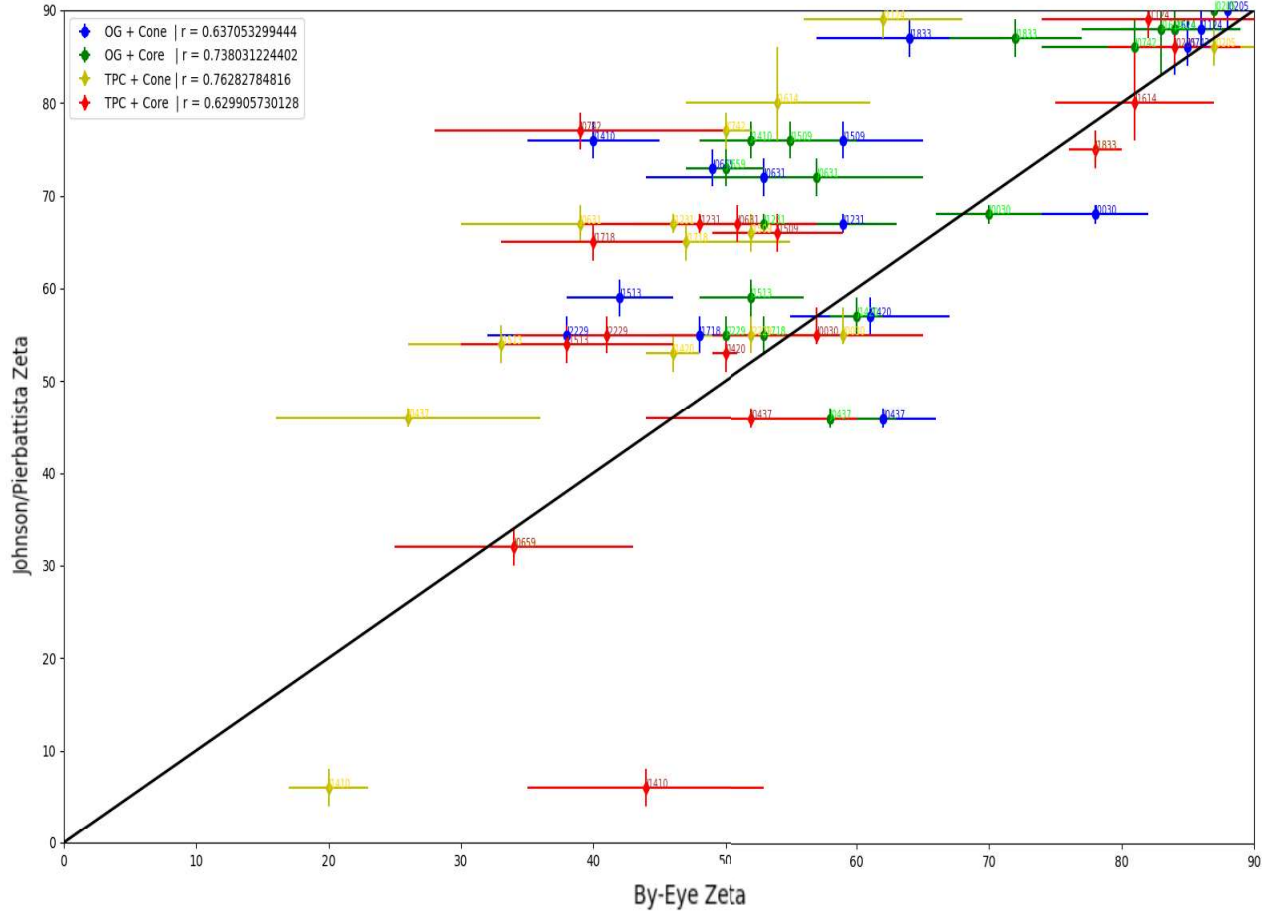


Figure 5.8: Comparison of best-fit observer angles (ζ) found by either Johnson et al. (2014) or Pierbattista et al. (2015) using Pearson's χ^2 test statistic and radio error inflation vs. those found by eye.

fitting is suitable for dual-band pulsar LC fitting, and at least partially negates the problems presented in this regard by disparate uncertainties on the radio and γ -ray observations of pulsars.

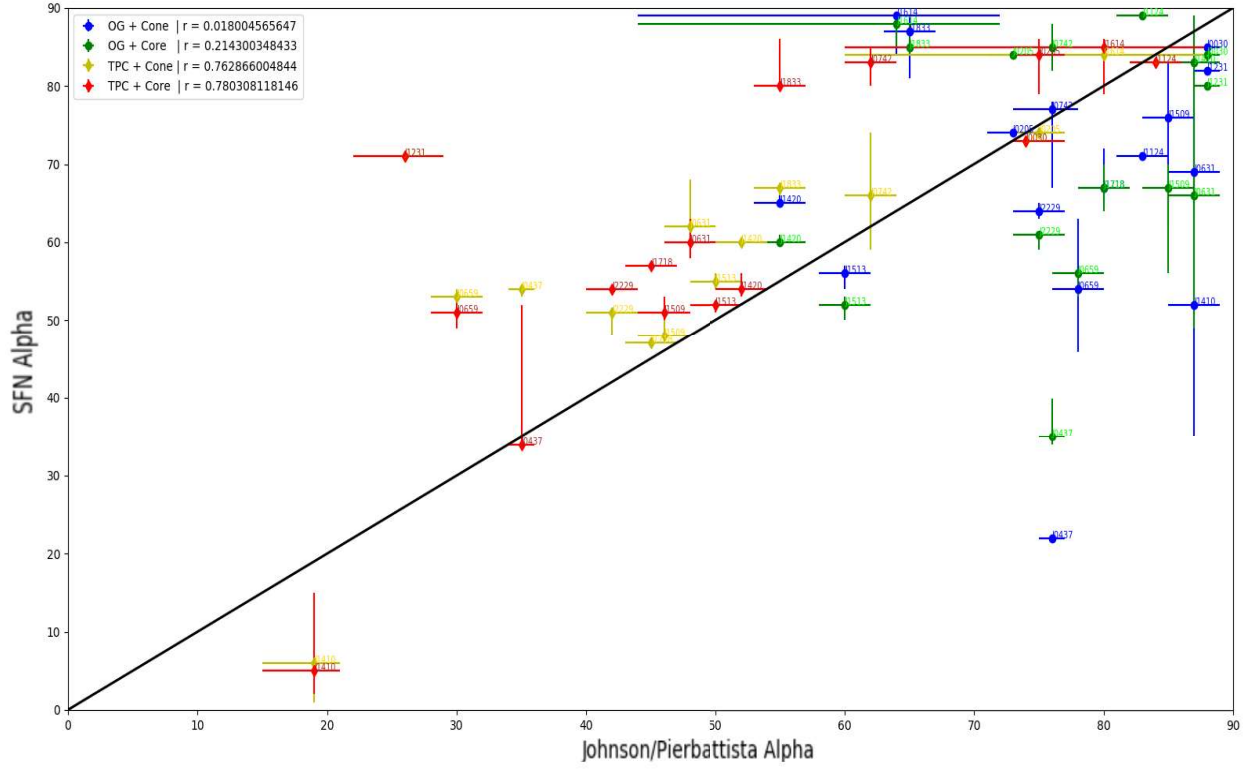


Figure 5.9: Comparison of best-fit tilt angles (α) found using the SFN test statistic vs. those found by either Johnson et al. (2014) or Pierbattista et al. (2015) using Pearson's χ^2 test statistic and radio error inflation.

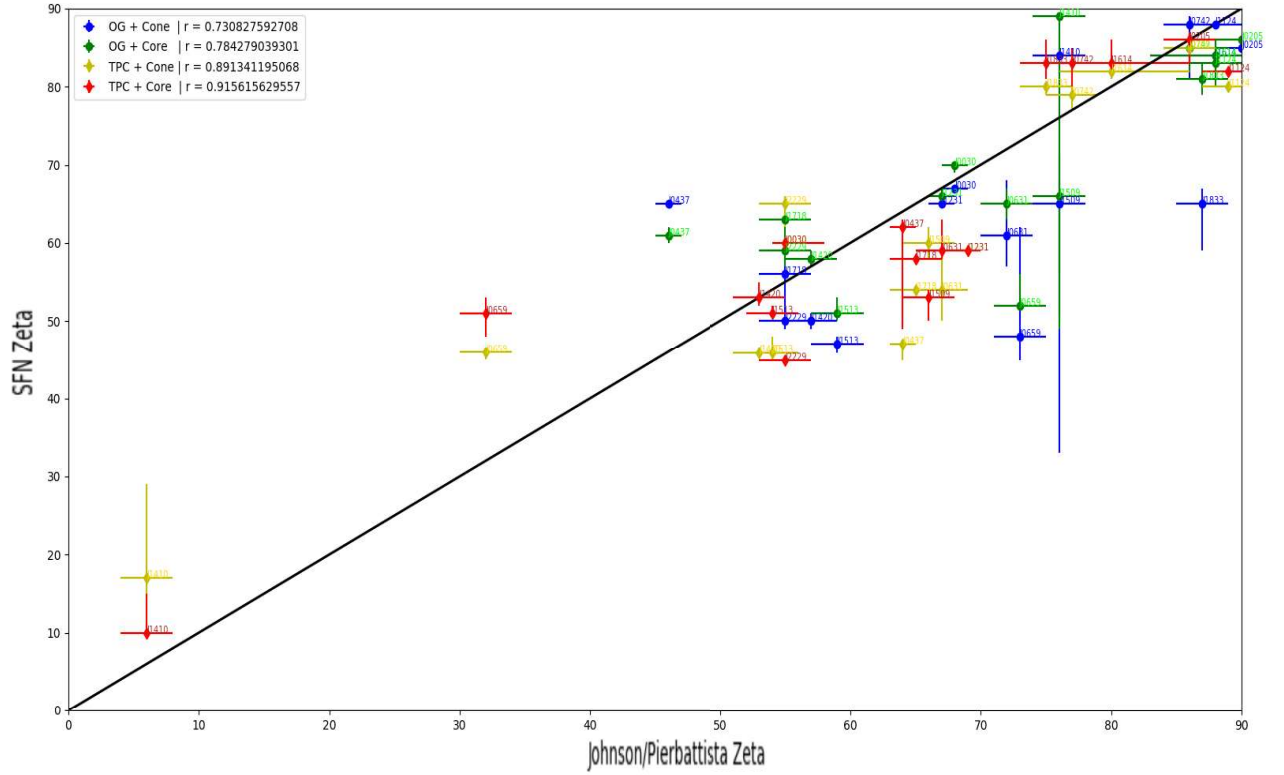


Figure 5.10: Comparison of best-fit observer angles (ζ) found using the SFN test statistic vs. those found by either Johnson et al. (2014) or Pierbattista et al. (2015) using Pearson's χ^2 test statistic and radio error inflation.

Chapter 6

Conclusions

Pulsars have been observed in the radio band since the 1960s, but only since the deployment of the *Fermi* LAT in 2008 has there been enough γ -ray data to perform multi-wavelength studies of these sources. Various studies have been done that look at, for example, the spectrum and polarisation of their emission, but multiband study of their LCs directly is complicated by the limitations of the available observational data. Specifically, disparities in the errors on γ -ray and radio wavelength observations render traditional joint fitting approaches ineffective. This thesis is presented as a contribution towards improved multiband LC fitting, a subject that has proven more problematic than anticipated.

First, let me recapitulate what has been said so far: Chapter 1 introduced pulsars as astrophysical sources, as well as the research context within which this thesis is placed.

Chapter 2 gave an overview of the history of pulsar observations and theory. I presented the current conception of pulsars' formation as the remnants of supernovae, and the physical mechanisms thought to produce the emission we observe from pulsars. I also introduced the concept of acceleration gaps in pulsars' magnetospheres on which many geometric pulsar models are based.

In Chapter 3 I described the geometric models we employed to obtain pulsar LCs in this study. We assumed that pulsars have RVD magnetic field structures and magnetospheres dense with plasma, except for regions of low density where charged particles are accelerated and produce electromagnetic radiation. The OG and TPC γ -ray models used in this work both assume constant emissivity from these gaps, and differ only in where in the magnetosphere they are located. I also described the considerably more crude geometrical radio models we employed—like the geometrical γ -ray models, these radio models make no reference to emission mechanisms, instead simply assuming conical or Gaussian beam-shaped radio emission from near the pulsar's magnetic poles.

Chapter 4 dealt with fitting LCs modelled using the models described in Chapter 3 to observed pulsar LCs, in both the radio and γ -ray bands *simultaneously*. I delineated the fitting methods previously used for joint pulsar LC fitting, and the advantages and drawbacks associated with each. In particular, the disparity in margins of error associated with the available radio and γ -ray observational data render joint fitting approaches that use the traditional Pearson's χ^2 goodness-of-fit test ineffective without modifying or manipulating the data in some way. Various LC fitting procedures based on such manipulations have been applied to pulsar LCs [e.g. Johnson et al. (2014), Pierbattista et al. (2015), Seyffert et al. (2016)], but with mixed results. I also described an LC fitting method newly proposed by Seyffert et al. (in preparation) involving a bespoke test-statistic that minimises the impact of the data error disparity.

Finally, in Chapter 5 I relayed the results we obtained applying the above approaches to LC fitting of a sample of 16 pulsars. I then compared the constraints that we found on

α and ζ , with errors, with those reported by Johnson et al. (2014) and Pierbattista et al. (2015) for the same pulsars, and drew conclusions as to the merit of the respective fitting methods.

6.1 Evaluating the utility of SFN LC fitting

Appendix B, along with Tables 5.2 and 5.3, contain the thrust of this work with regard to assessment of the various LC fitting methods we applied. In making a judgement as to the efficacy of the SFN fitting method proposed by Seyffert et al. (in preparation), I am foremost guided by the apparent (albeit subjective) goodness of fit of the best-fit LCs assembled in Appendix B, viz. whether this approach produces best-fit model LCs that might reasonably be said to match the observed counterparts. I would encourage the reader to scrutinise these fits for themselves, but it is clear to me that the SFN-derived best-fit LCs presented in Appendix B uniformly represent acceptable facsimiles of the observed LCs in both the radio and γ -ray wavebands.

Table 5.3 substantiates this impression. The correlation coefficient r between the constraints found through SFN fitting and through by-eye fitting ranges from ~ 0.62 to ~ 0.89 , depending on the combination of models used. I propose that this agreement is more impressive than it seems at first, considering the limitations of by-eye fitting. For any given pulsar and model combination, there is often a number of different α - ζ combinations that might lead to an adequate match with the observed LC, and deciding which of these comprises *the* best-fit parameter pair can be a matter of the fitter’s personal preference¹.

Having established that SFN fitting does not produce entirely improper best-fit LCs, the next question is how these fits compare to those found by Johnson et al. (2014) and Pierbattista et al. (2015) through modified χ^2 fitting. Figures 5.9 and 5.10, which plot the respective α and ζ constraints found by these two methods against one another, seem to suggest that they deliver broadly similar fits, although less so for α . This agreement is sensitive to the combination of radio and γ -ray models used, however, with α fits found using the OG γ -ray model frequently being at odds. The correlation coefficients reported in Table 5.3 support this assessment

It is elucidating to highlight the case of PSR J0437–4715 modelled using the OG γ -ray and core radio models, the best-fit LCs of which are shown in Figure 6.1. The best-fit (α, ζ) constraints on this pulsar reported by Johnson et al. (2014) are $(76 \pm 1, 46 \pm 1)$, while using SFN fitting we found best-fit parameters of $(35^{+5}_{-1}, 61 \pm 1)$ ². As is apparent from Figure 6.1, the constraints found by Johnson et al. (2014) lead to a (superficially) closer fit to the observed LC in the radio band, but a considerably *worse* fit in the γ -ray band. This is an encouraging sign that SFN fitting has accomplished what we set out to achieve: a joint fitting approach that produces adequate fits in *both* bands without favouring one or the other.

Moreover, the correlation between the SFN fits and by-eye fits is uniformly greater than between the previous studies’ fits and the by-eye fits, which ranges between $r \simeq -0.1$ and $r \simeq 0.76$. It is therefore reasonable to say that SFN fitting produces fits that agree with what the eye suggests—in both the radio and γ -ray bands, crucially—more than those Johnson et al. (2014) and Pierbattista et al. (2015) found.

In conclusion, I put forward that this thesis presents compelling corroboration for SFN

¹This is compounded by the fact that quite often an (α, ζ) combination leads to an LC that is indistinguishable from that produced by the inverted pair (ζ, α) . Also indicative of this α - ζ ambiguity is the large degree of symmetry along the diagonal evident in test-statistic maps like Figure 4.2.

²However, it is worth mentioning that the Johnson et al. (2014) best-fit ζ is closer to that reported by Hotan et al. (2006), namely $\zeta = 46$.

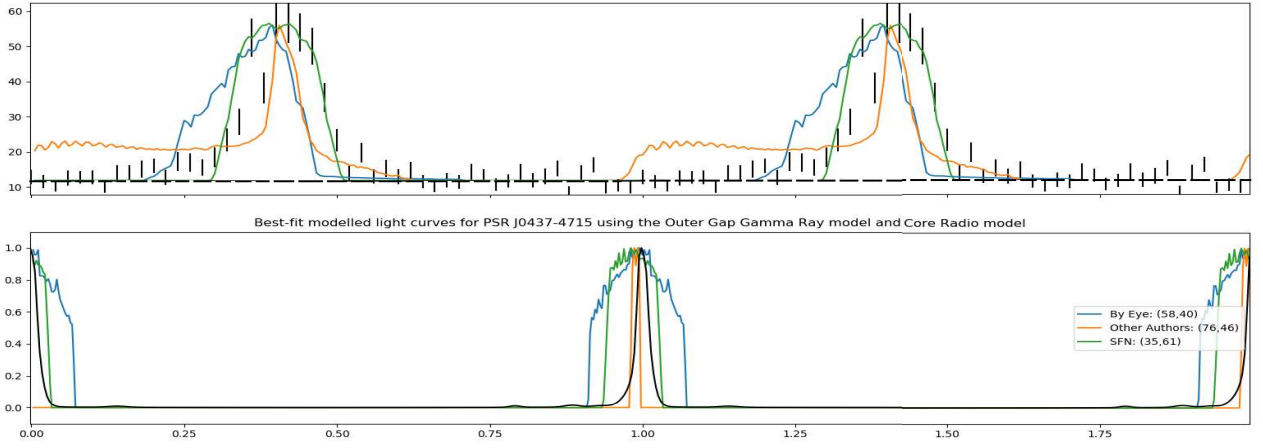


Figure 6.1: Best-fit LCs for PSR J0437–4715 found using the OG γ -ray and core radio models. Each coloured line represents a best-fit LC found using a different fitting method, and the observed LCs are in black. The top plot contains the γ -ray LCs and the bottom contains the radio LCs. The γ -ray background flux level, as reported in *Fermi*’s Second Pulsar Catalog, is indicated with the dashed line.

fitting to be regarded as a valid and useful approach to fitting modelled and observed pulsar LCs in multiple bands concurrently.

6.2 Suggestions and future work

Multi-wavelength LC studies of pulsars is an exciting new area of research that this work aims to facilitate. Having attested to the value of SFN fitting as a tool for comparing pulsar observations and modelling, it can now be used to test further modelling attempts. In particular, Seyffert et al. (in preparation) will apply the approach to the FIDO γ -ray model of Kalapotharakos et al. (2014) to investigate the relationship between pulsars’ spin-down rates ($\dot{E}$) and the conductivity of their magnetospheres (σ). SFN fitting can be applied to any number of future pulsar LC models, and can be extended to incorporate other wavebands, e.g. X-rays, as data and models become available.

SFN fitting might even be useful for other fitting endeavours, e.g. spectral fitting. I would suggest that this approach might prove serviceable for any joint fitting across datasets with disparate associated errors and a clear background level.

Furthermore, the constraints on α and ζ I obtained in this study for each of the sample pulsars, using both by-eye fitting and SFN fitting, might be used as priors in other studies of these pulsars. The NICER Mission aboard the ISS aims to make high-precision measurements of a number of pulsars’ masses and radii using rotation-resolved spectroscopy in the soft (0.2–12 keV) X-ray band (Gendreau et al., 2012). To do this they require constraints on α (and ζ in the case of isolated MSPs) to use as priors, which we might be able to provide. We have already submitted a *Fermi* Cycle-11 GI proposal in this regard.

Finally, as pulsar observations improve, the process followed in this thesis can be re-applied to obtain tighter constraints on pulsar geometry. In particular, this analysis can be redone using the new and improved Pass 8 LAT data, representing a reprocessing of the entire mission dataset, including improved event reconstruction, a wider energy range, better energy measurements, and significantly increased effective area. This should result in significantly better constraints on α and ζ .

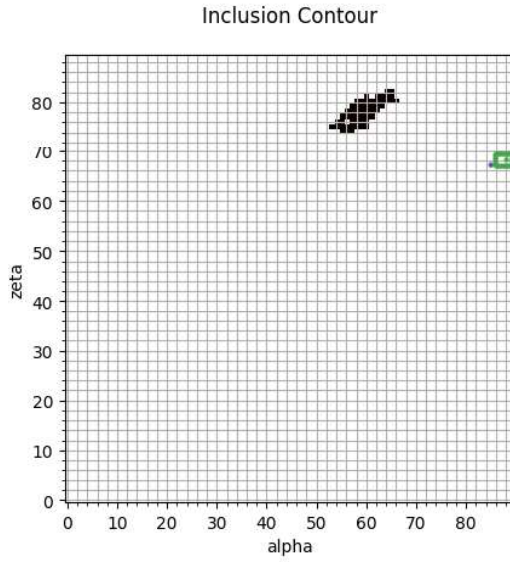
Appendices

Appendix A

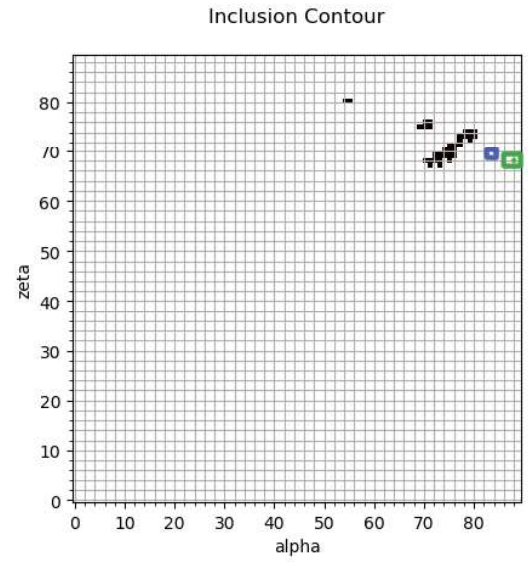
Inclusion contours

This Appendix compiles the inclusion contours associated with each pulsar, as described in Section 5.2.1. The black regions in each figure represents combinations of α and ζ for which I adjudged, purely by eye, that the modelled pulsar LC satisfactorily matches its observational counterpart. The green box overlaid on each figure represents the corresponding constraints reported in either Johnson et al. (2014) or Pierbattista et al. (2015), with errors. The blue box shows the (α, ζ) constraints, with errors, that I obtained in this work using SFN fitting. All angles are given in degrees.

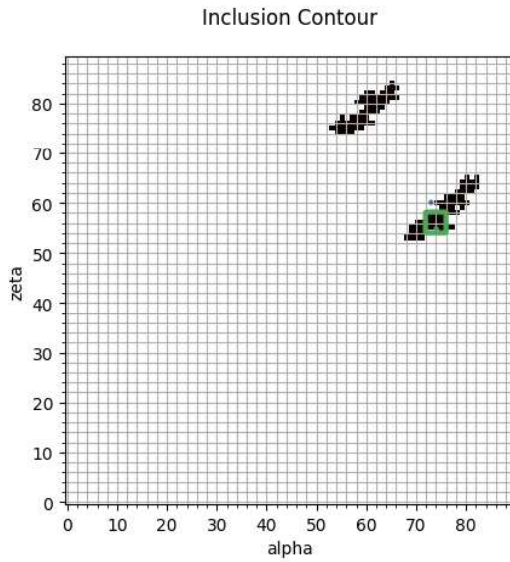
Note for each pulsar there are four figures, each corresponding to one of the four radio and γ -ray model combinations used in this study.



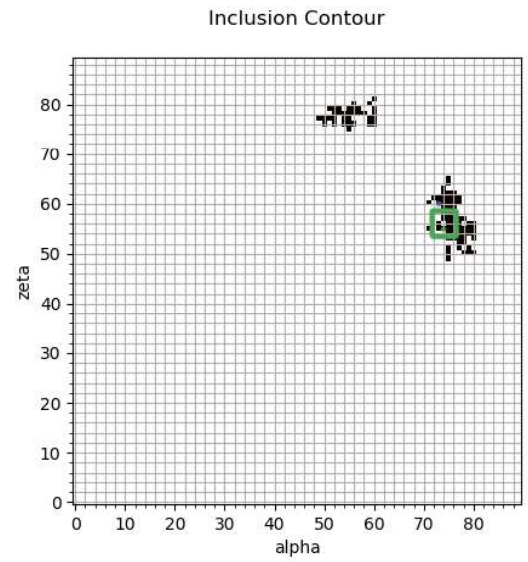
(a) Inclusion contours of PSR J0030+0451
OG + Cone models.



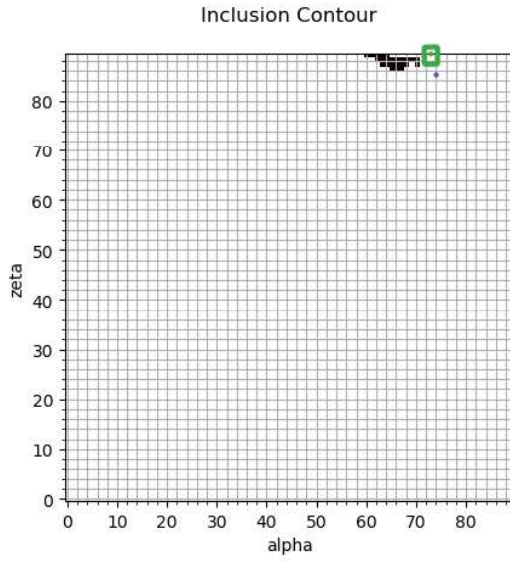
(b) Inclusion contours of PSR J0030+0451
OG + Core models.



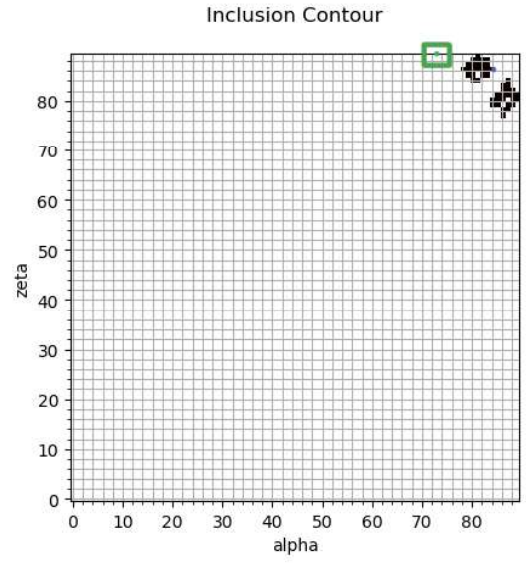
(a) Inclusion contours of PSR J0030+0451
TPC + Cone models.



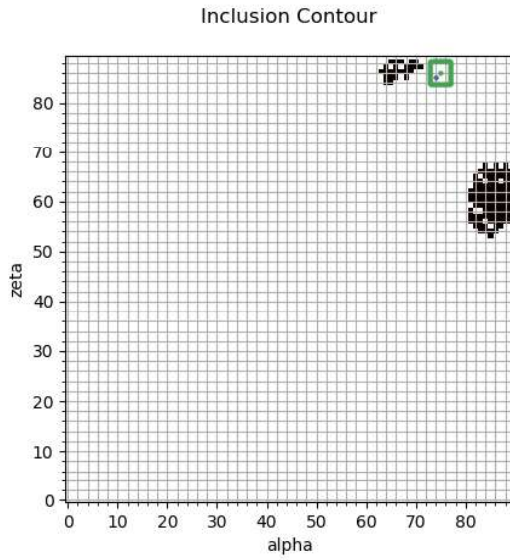
(b) Inclusion contours of PSR J0030+0451
TPC + Core models.



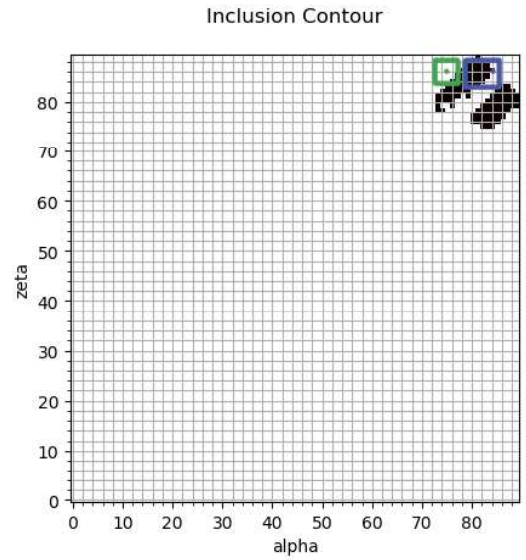
(a) Inclusion contours of PSR J0205+6449
OG + Cone models.



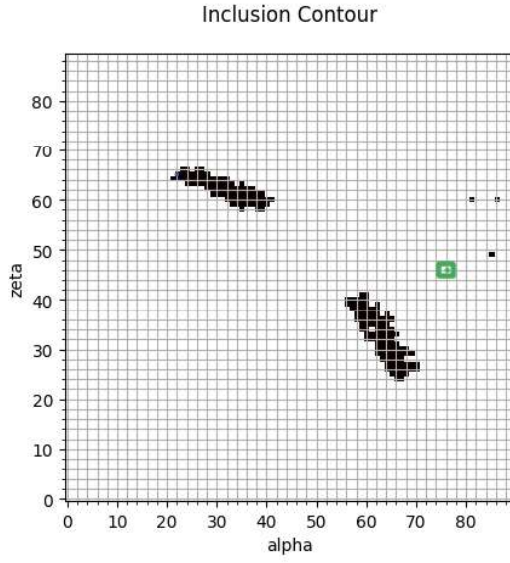
(b) Inclusion contours of PSR J0205+6449
OG + Core models.



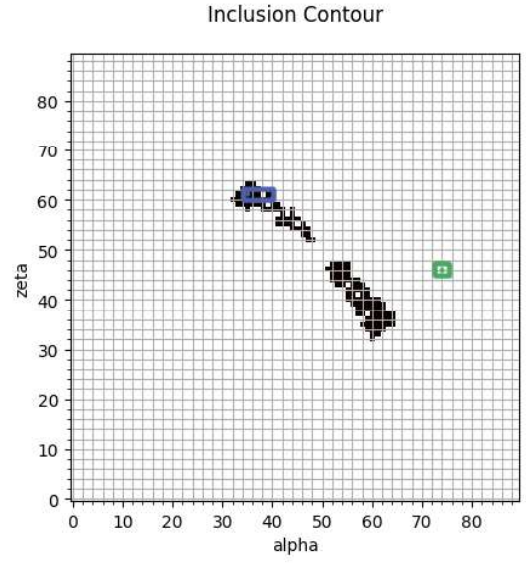
(a) Inclusion contours of PSR J0205+6449
TPC + Cone models.



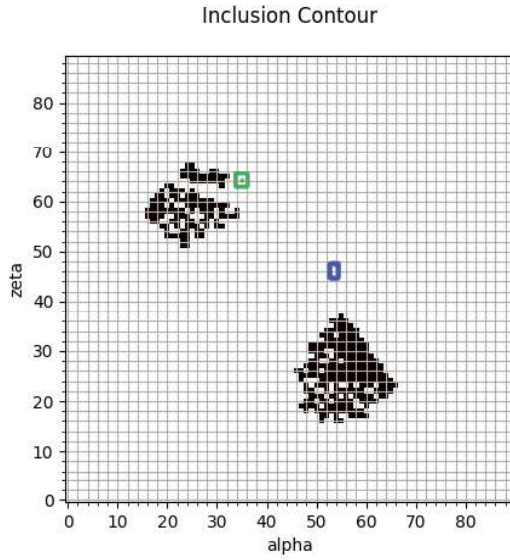
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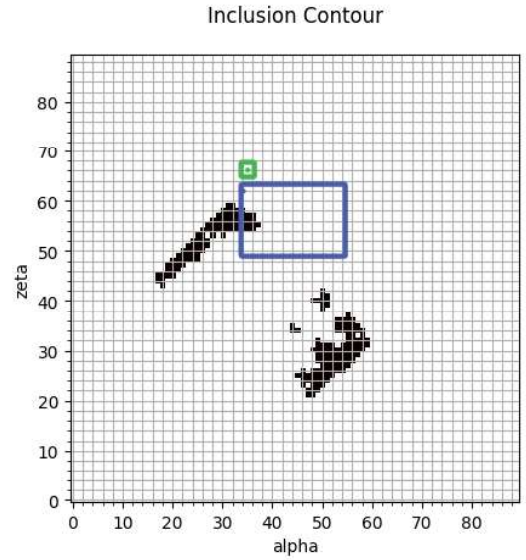
(a) Inclusion contours of PSR J0473-4715
OG + Cone models.



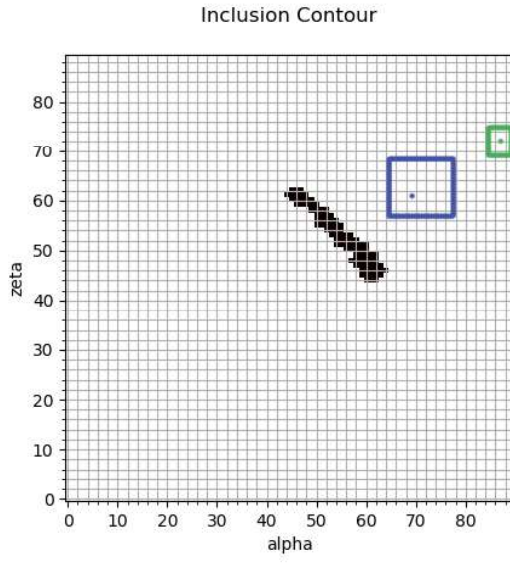
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OG + Core models.



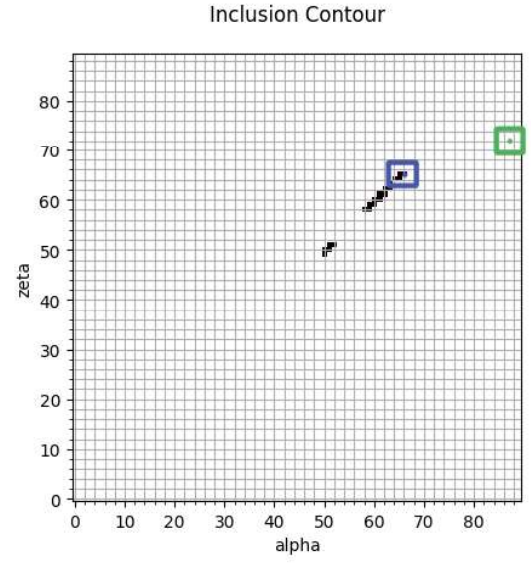
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TPC + Cone models.



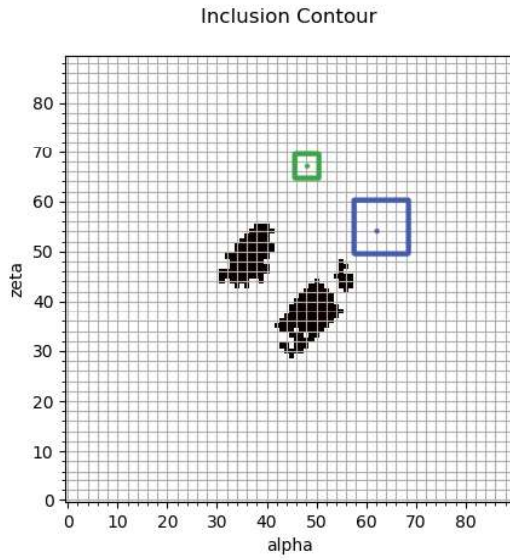
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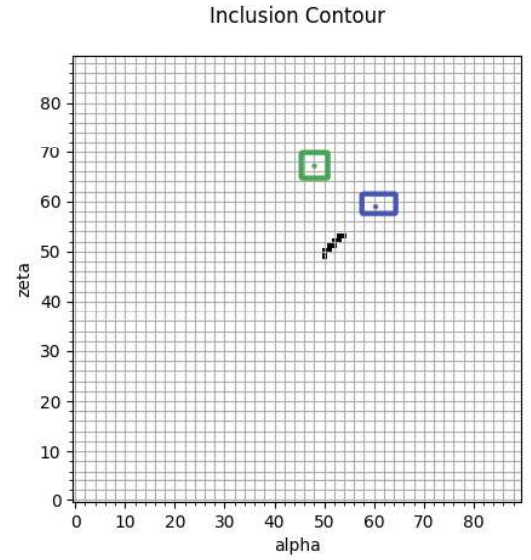
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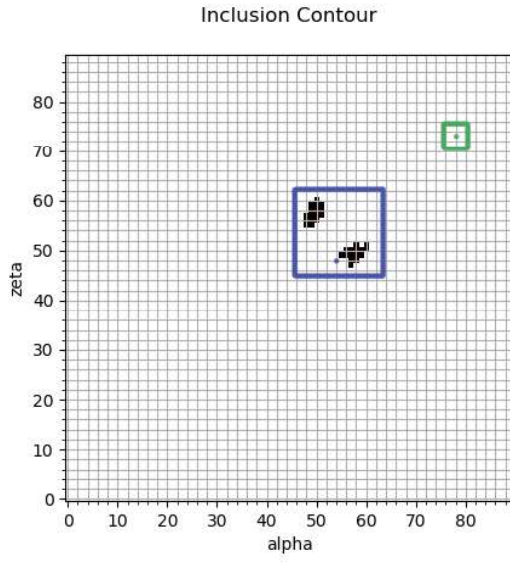
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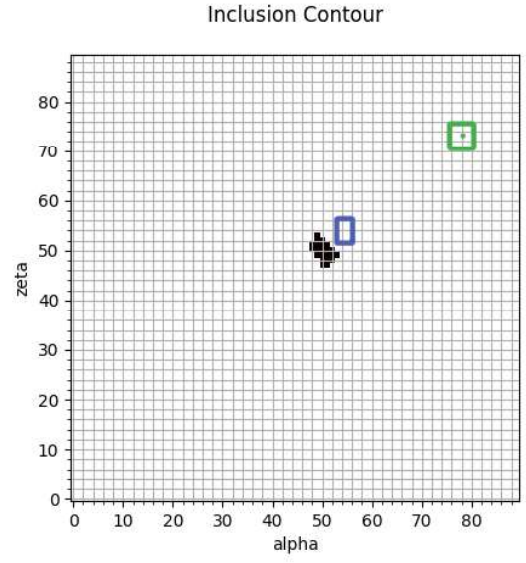
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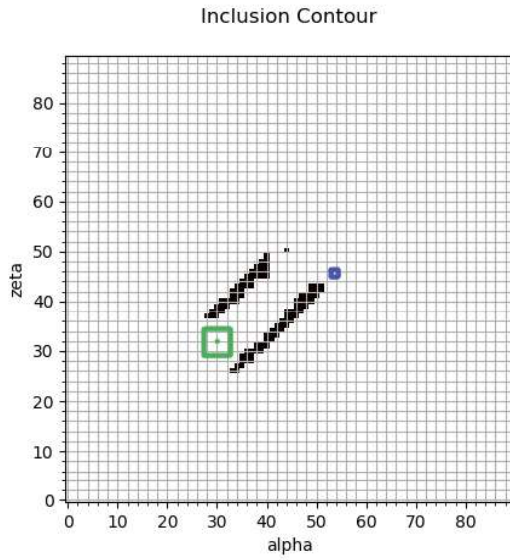
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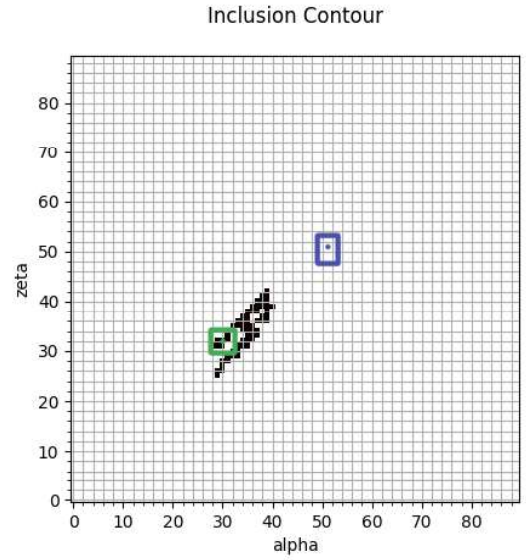
(a) Inclusion contours of PSR J0659+1414 OG + Cone models.



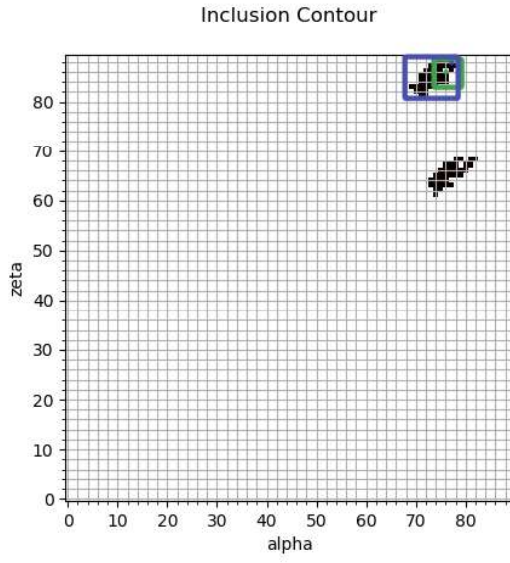
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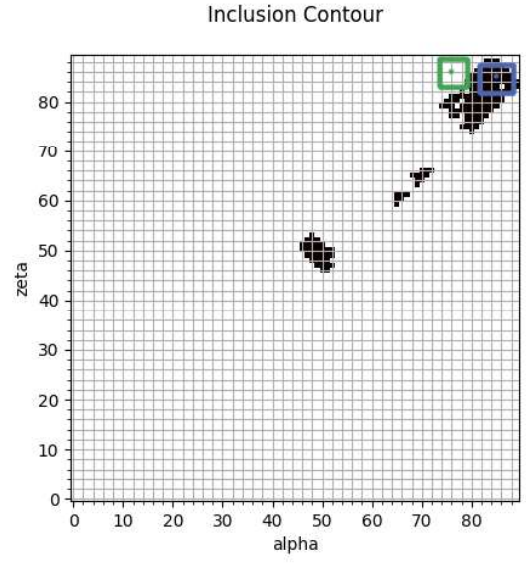
(a) Inclusion contours of PSR J0659+1414 TPC + Cone models.



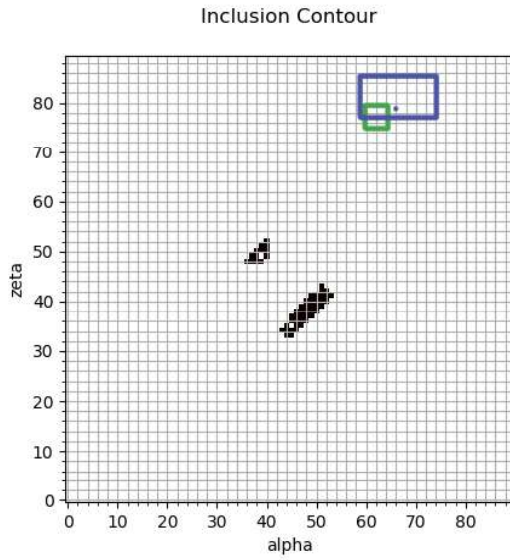
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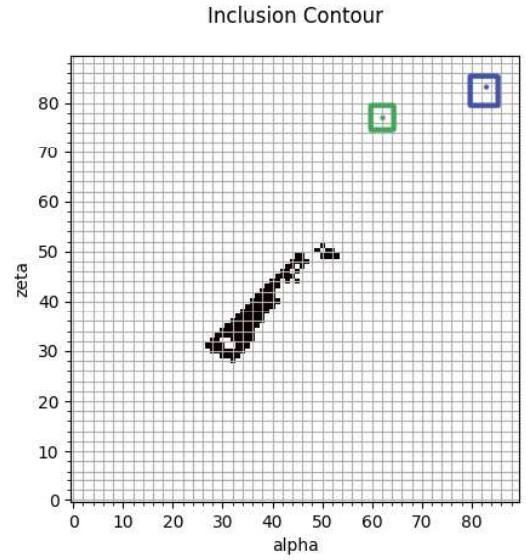
(a) Inclusion contours of PSR J0742-2822
OG + Cone models.



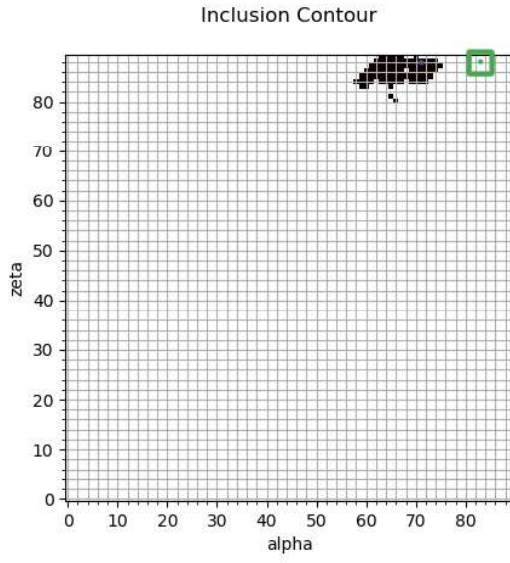
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OG + Core models.



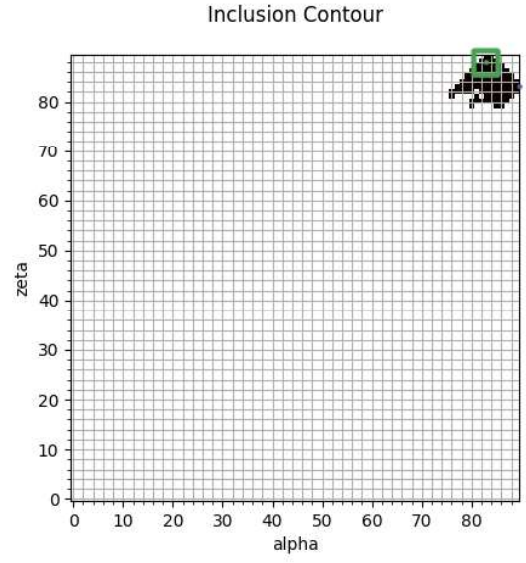
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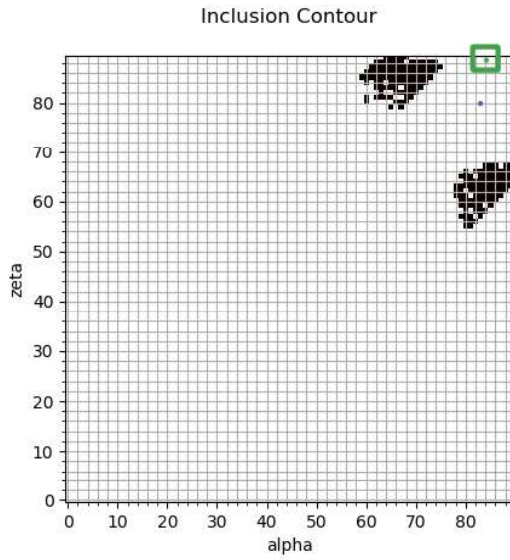
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TPC + Core models.



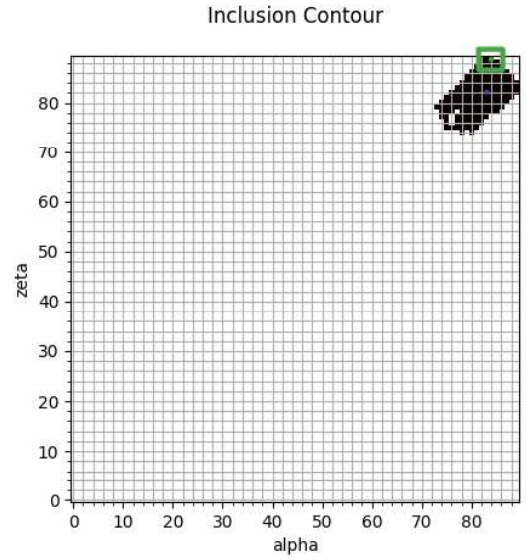
(a) Inclusion contours of PSR J1124-5916
OG + Cone models.



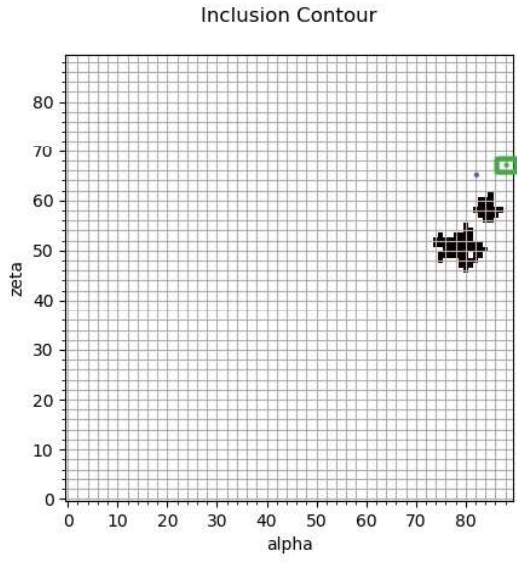
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OG + Core models.



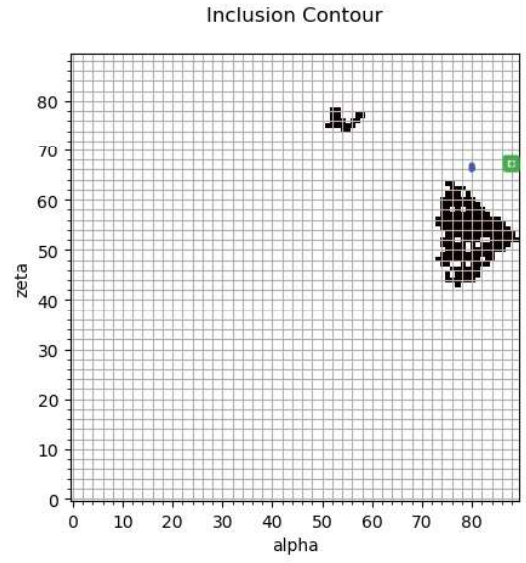
(a) Inclusion contours of PSR J1124-5916
TPC + Cone models.



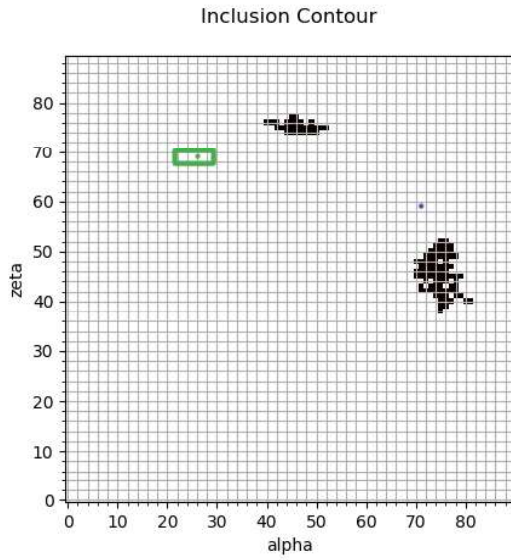
(b) Inclusion contours of PSR J1124-5916
TPC + Core models.



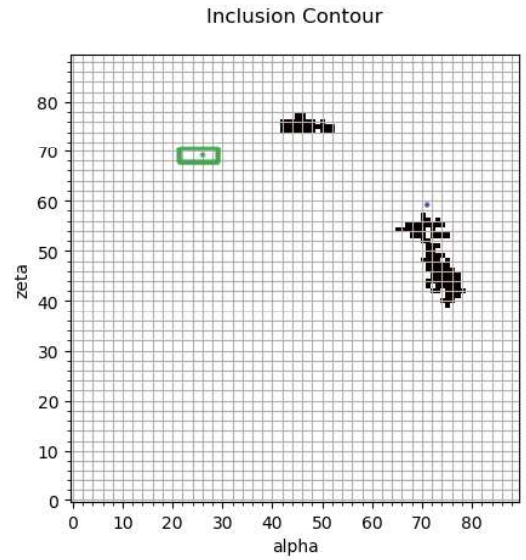
(a) Inclusion contours of PSR J1231-1411
OG + Cone models.



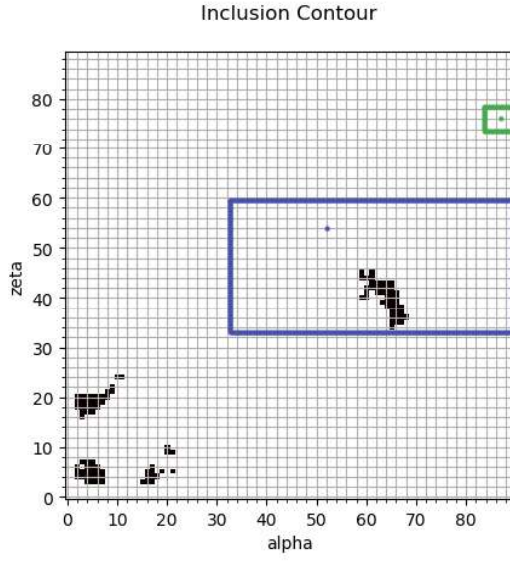
(b) Inclusion contours of PSR J1231-1411
OG + Core models.



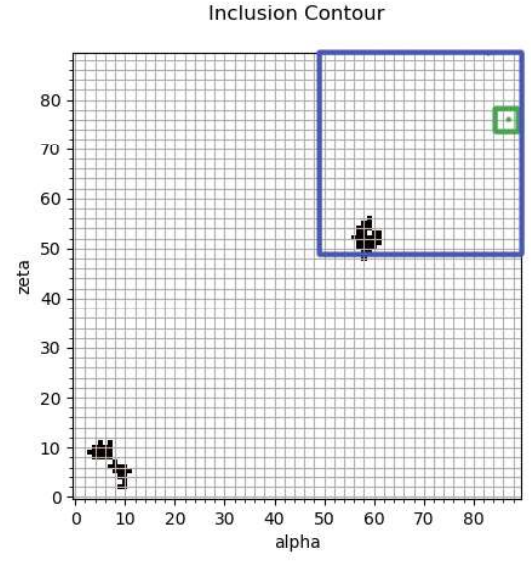
(a) Inclusion contours of PSR J1231-1411
TPC + Cone models.



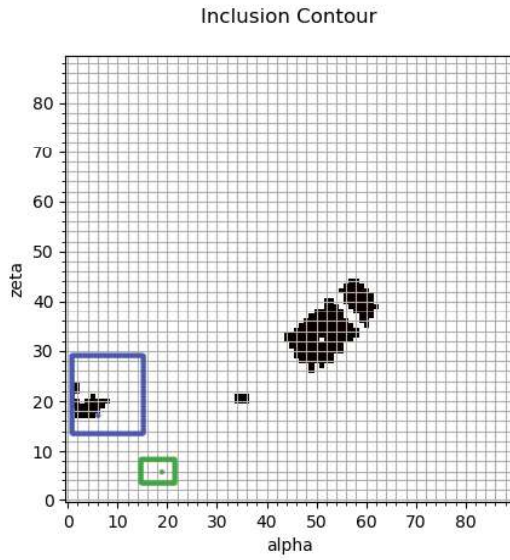
(b) Inclusion contours of PSR J1231-1411
TPC + Core models.



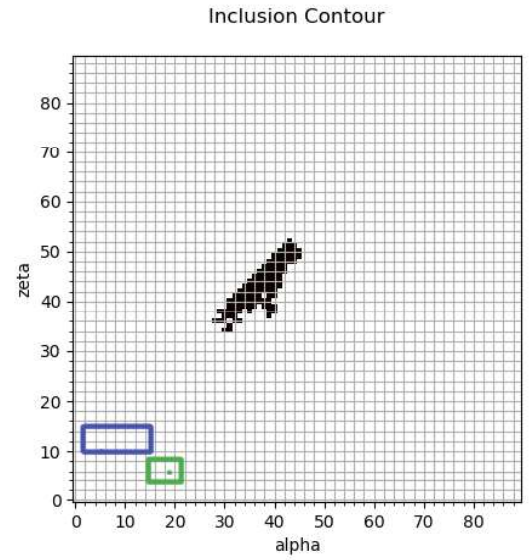
(a) Inclusion contours of PSR J1410-6132
OG + Cone models.



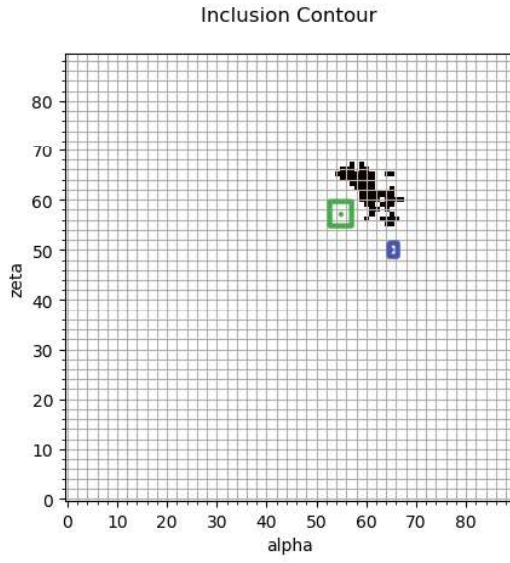
(b) Inclusion contours of PSR J1410-6132
OG + Core models.



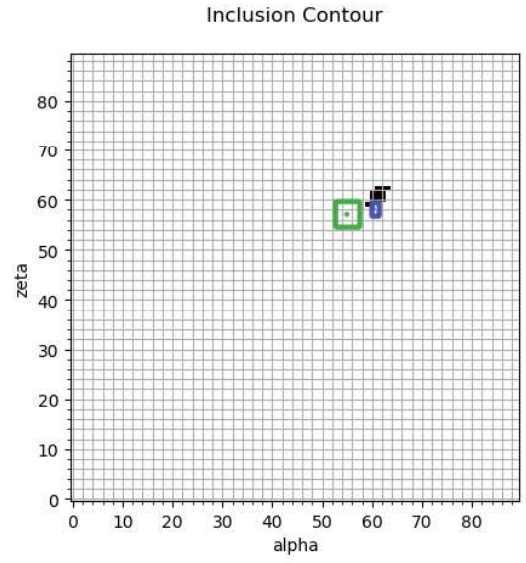
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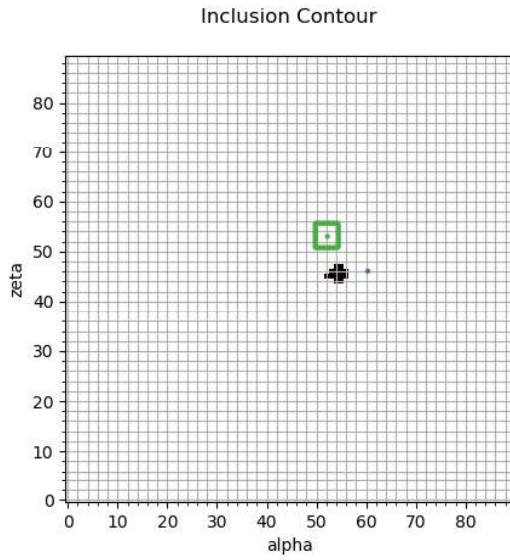
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TPC + Core models.



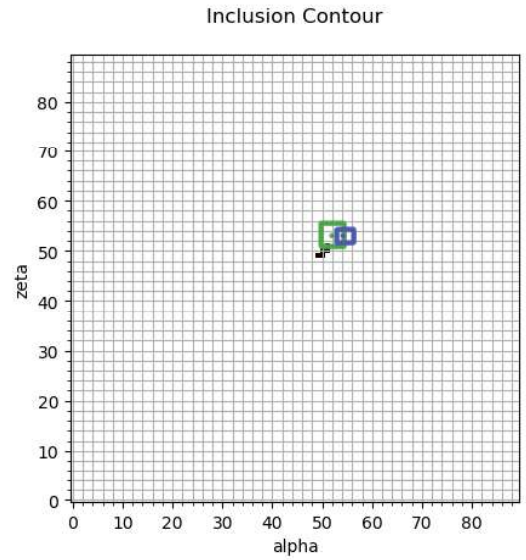
(a) Inclusion contours of PSR J1420-6048
OG + Cone models.



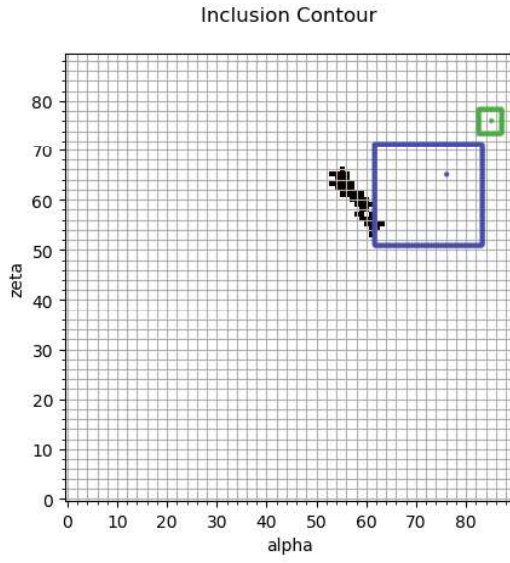
(b) Inclusion contours of PSR J1420-6048
OG + Core models.



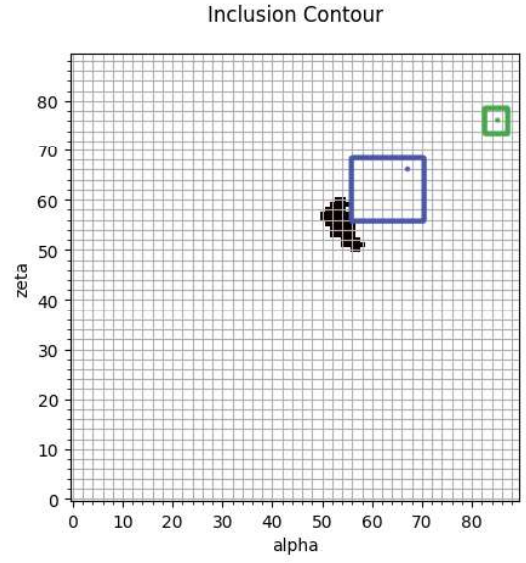
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TPC + Cone models.



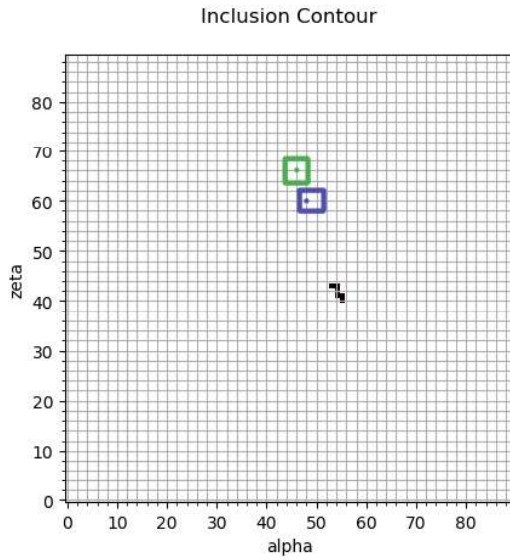
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TPC + Core models.



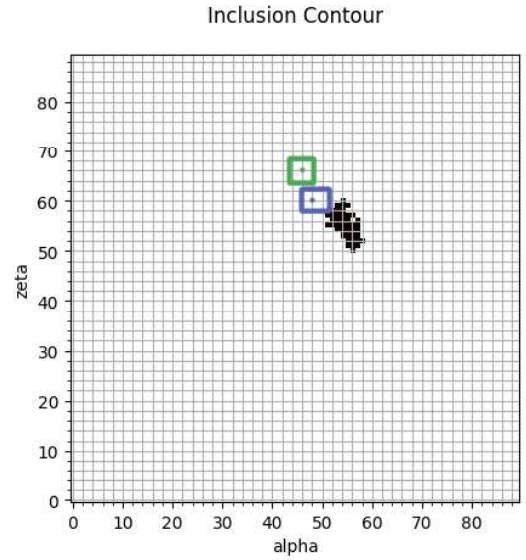
(a) Inclusion contours of PSR J1509-5850
OG + Cone models.



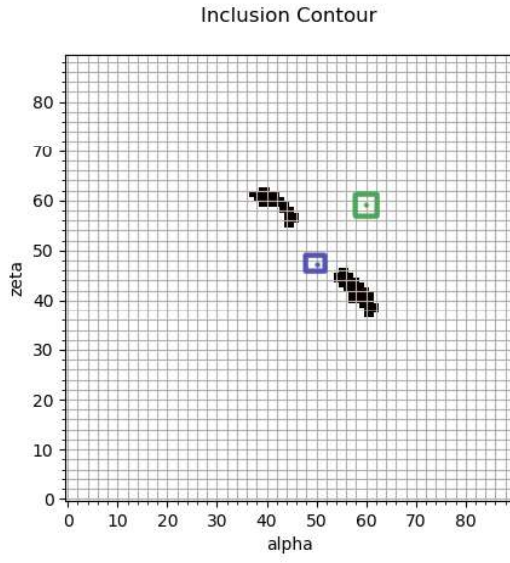
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OG + Core models.



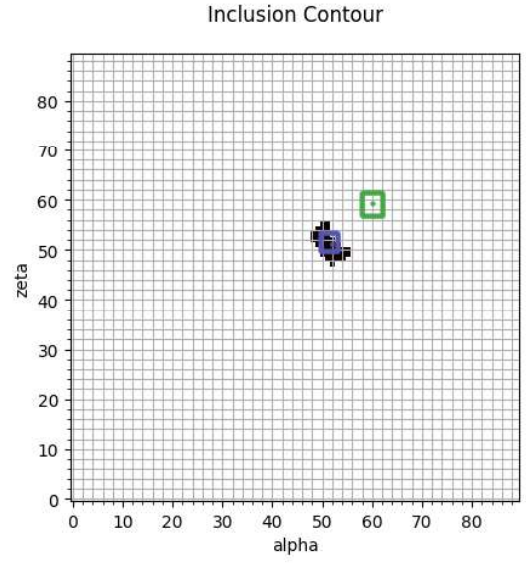
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TPC + Cone models.



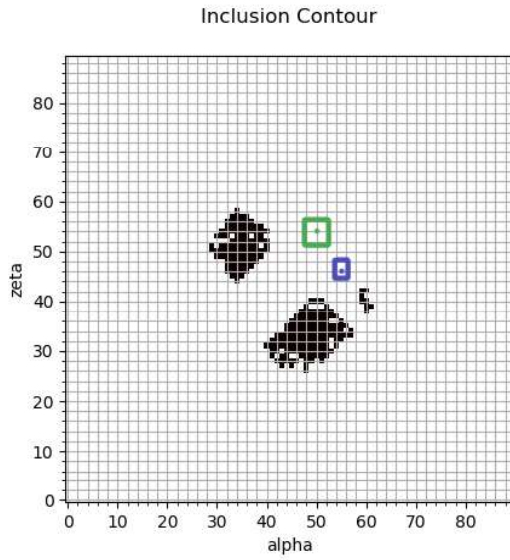
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TPC + Core models.



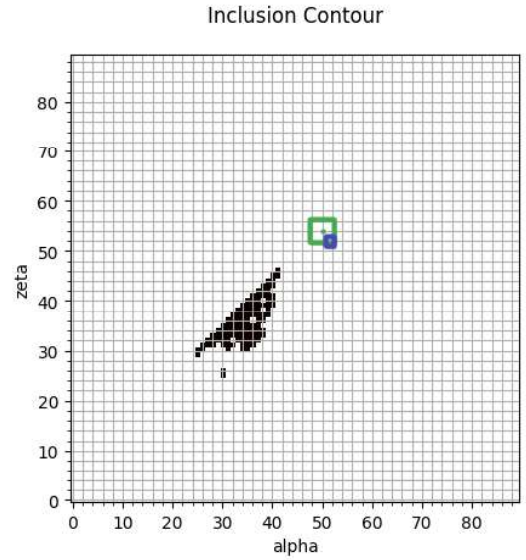
(a) Inclusion contours of PSR J1513-5908
OG + Cone models.



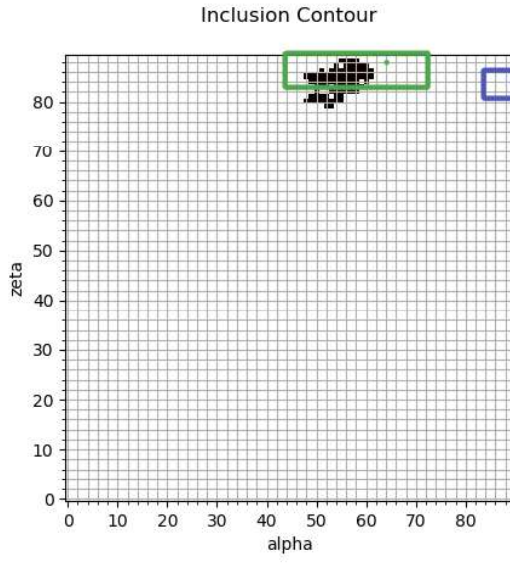
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OG + Core models.



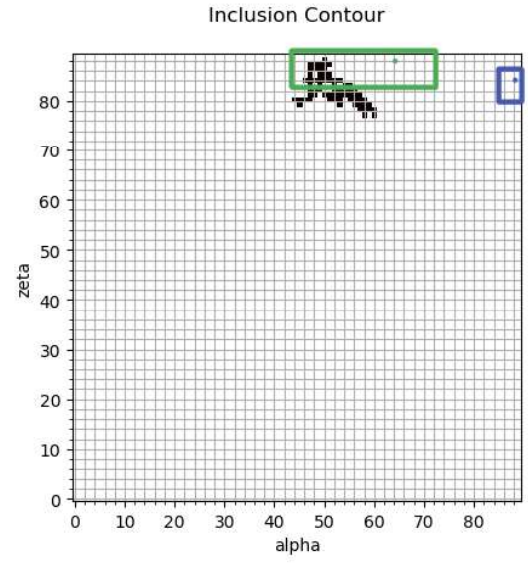
(a) Inclusion contours of PSR J1513-5908
TPC + Cone models.



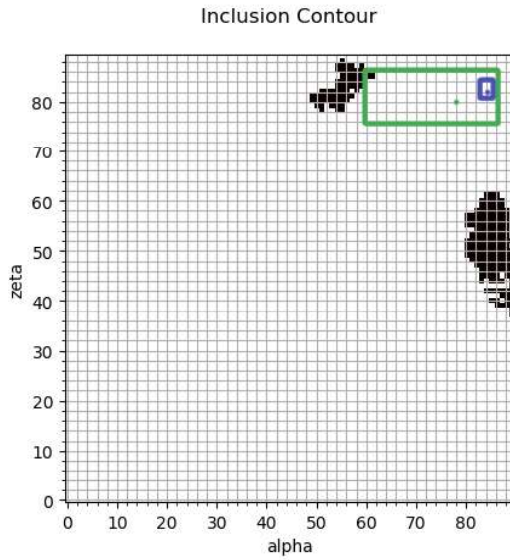
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TPC + Core models.



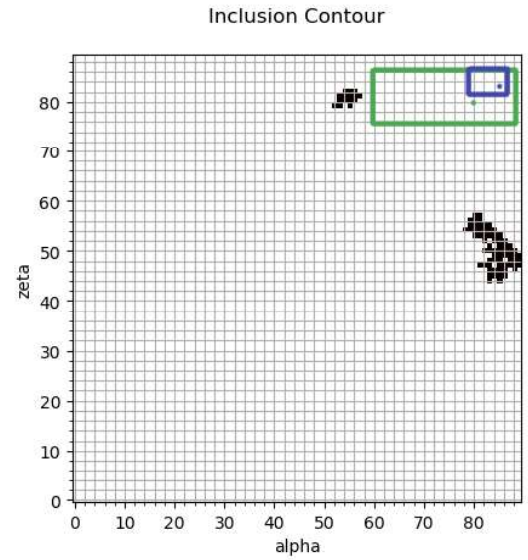
(a) Inclusion contours of PSR J1614-2230
OG + Cone models.



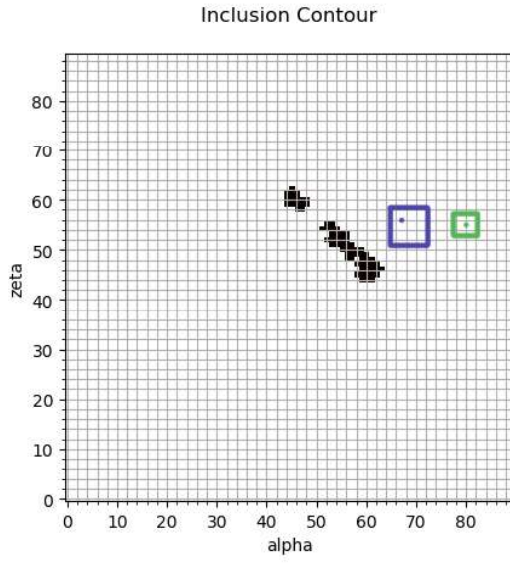
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OG + Core models.



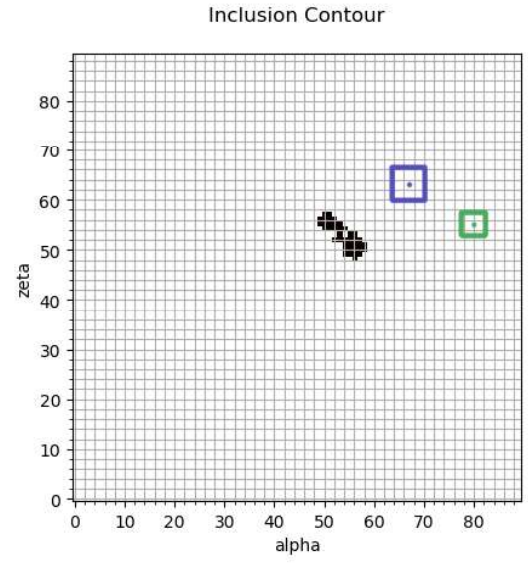
(a) Inclusion contours of PSR J1614-2230
TPC + Cone models.



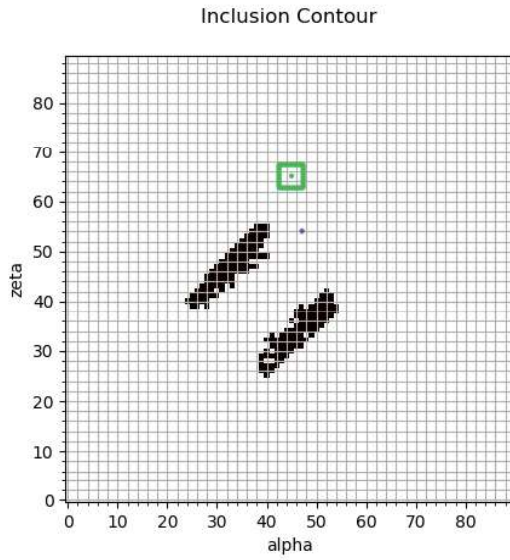
(b) Inclusion contours of PSR J1614-2230
TPC + Core models.



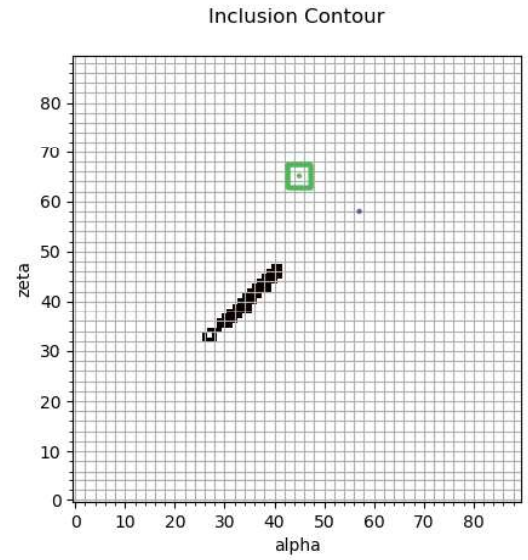
(a) Inclusion contours of PSR J1718-3825
OG + Cone models.



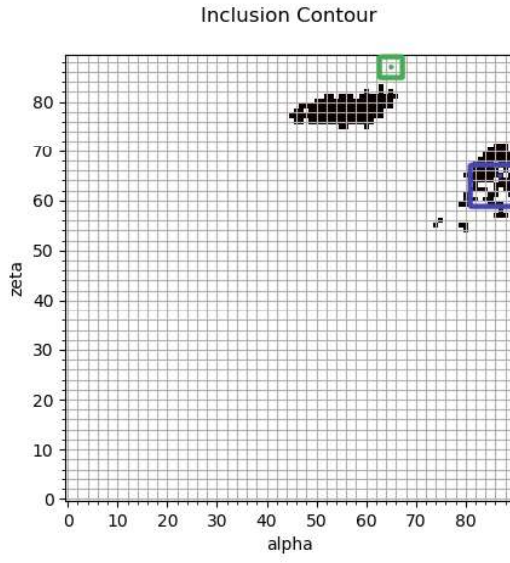
(b) Inclusion contours of PSR J1718-3825
OG + Core models.



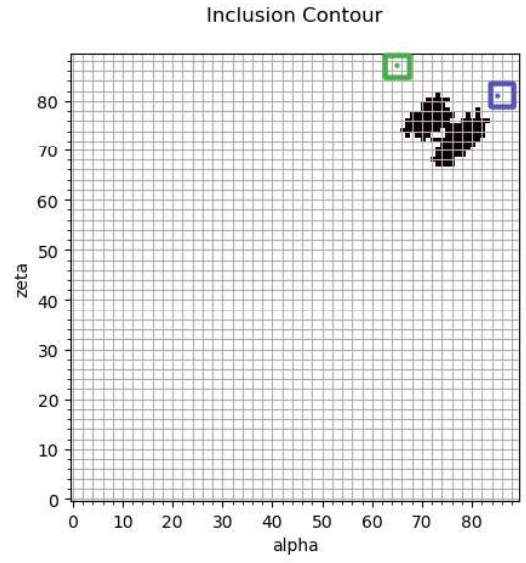
(a) Inclusion contours of PSR J1718-3825
TPC + Cone models.



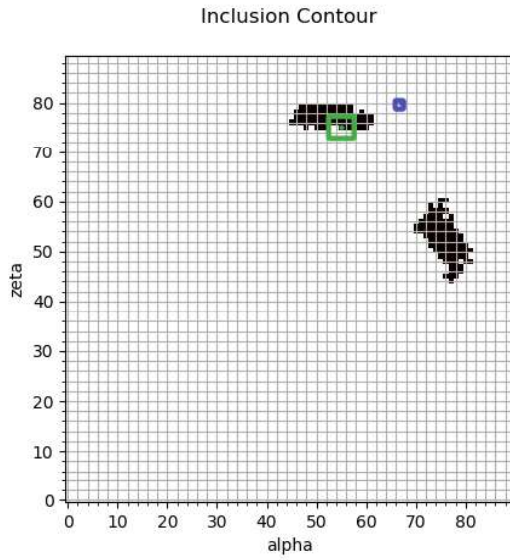
(b) Inclusion contours of PSR J1718-3825
TPC + Core models.



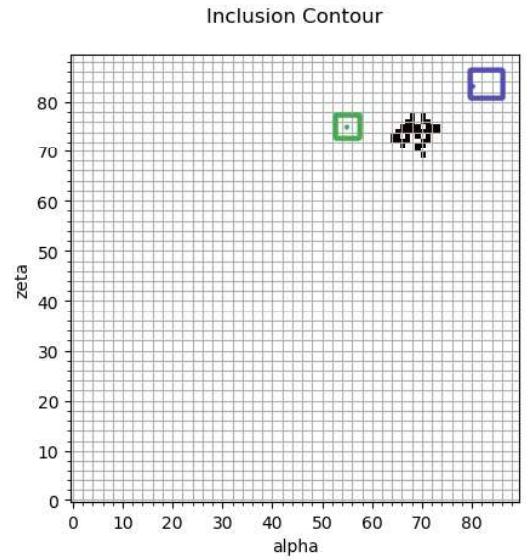
(a) Inclusion contours of PSR J1833-1034
OG + Cone models.



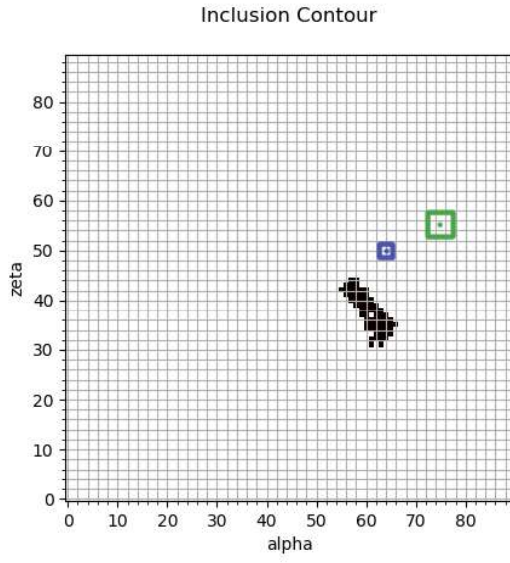
(b) Inclusion contours of PSR J1833-1034
OG + Core models.



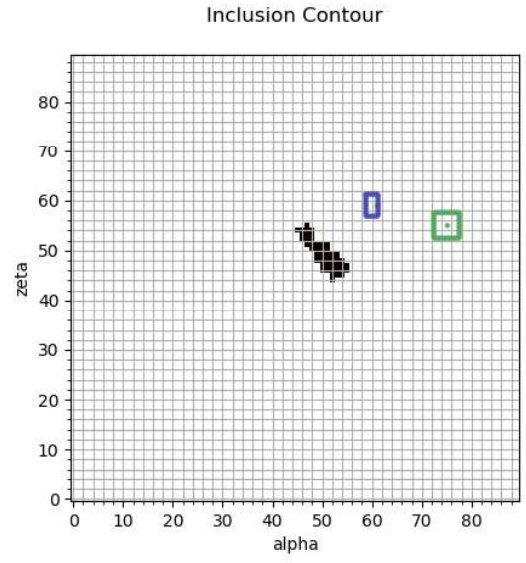
(a) Inclusion contours of PSR J1833-1034
TPC + Cone models.



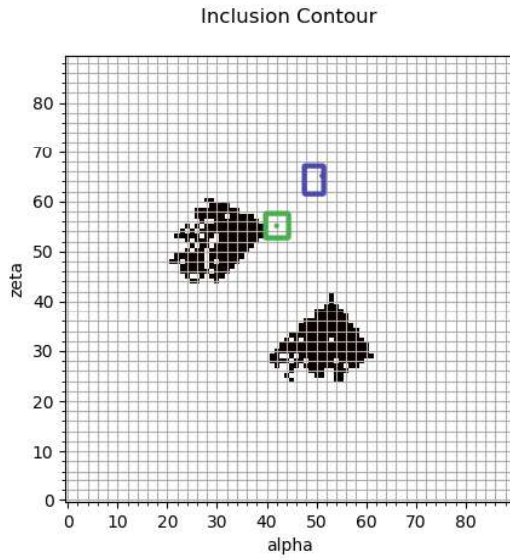
(b) Inclusion contours of PSR J1833-1034
TPC + Core models.



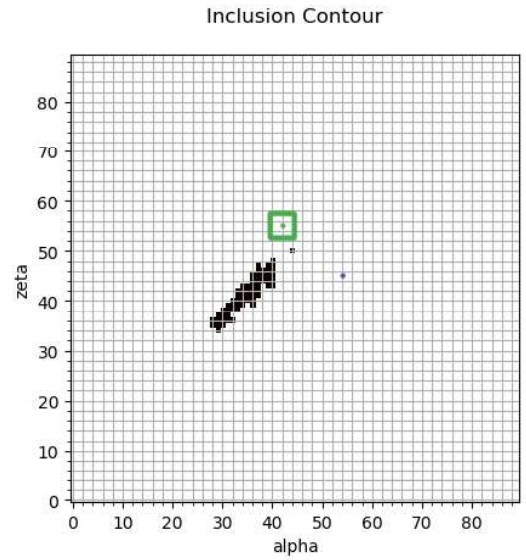
(a) Inclusion contours of PSR J2229+6114
OG + Cone models.



(b) Inclusion contours of PSR J2229+6114
OG + Core models.



(a) Inclusion contours of PSR J2229+6114
TPC + Cone models.



(b) Inclusion contours of PSR J2229+6114
TPC + Core models.

Appendix B

Best-fit light curves

Here I present the best-fit LCs we found using by-eye fitting (blue curve), using the SFN test statistic (green), and those found by either Johnson et al. (2014) or Pierbattista et al. (2015) (orange), superimposed on the observed LC taken from the second *Fermi* pulsar catalog (black) for each of our 16 sample pulsars. Each figure contains four subfigures—one for each of the four possible combinations of the OG and TPC γ -ray models and the cone and core radio models. In each subfigure the γ -ray LCs are shown in the top plot, and the radio LCs below. The best-fit (α, ζ) pairs found using the respective fittings method are reported in the legends.

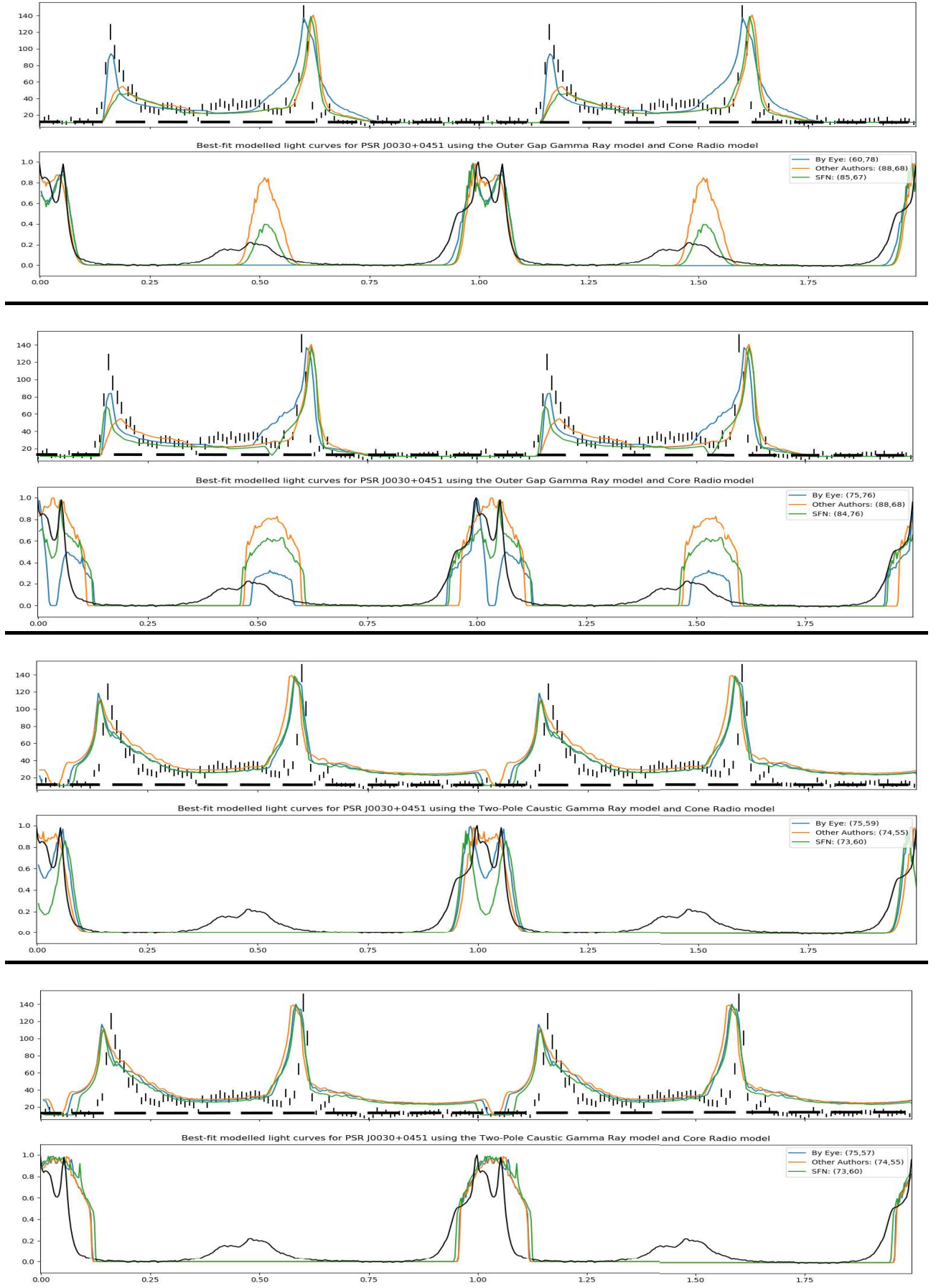


Figure B.1: Best-fit LCs for PSR J003+0451.

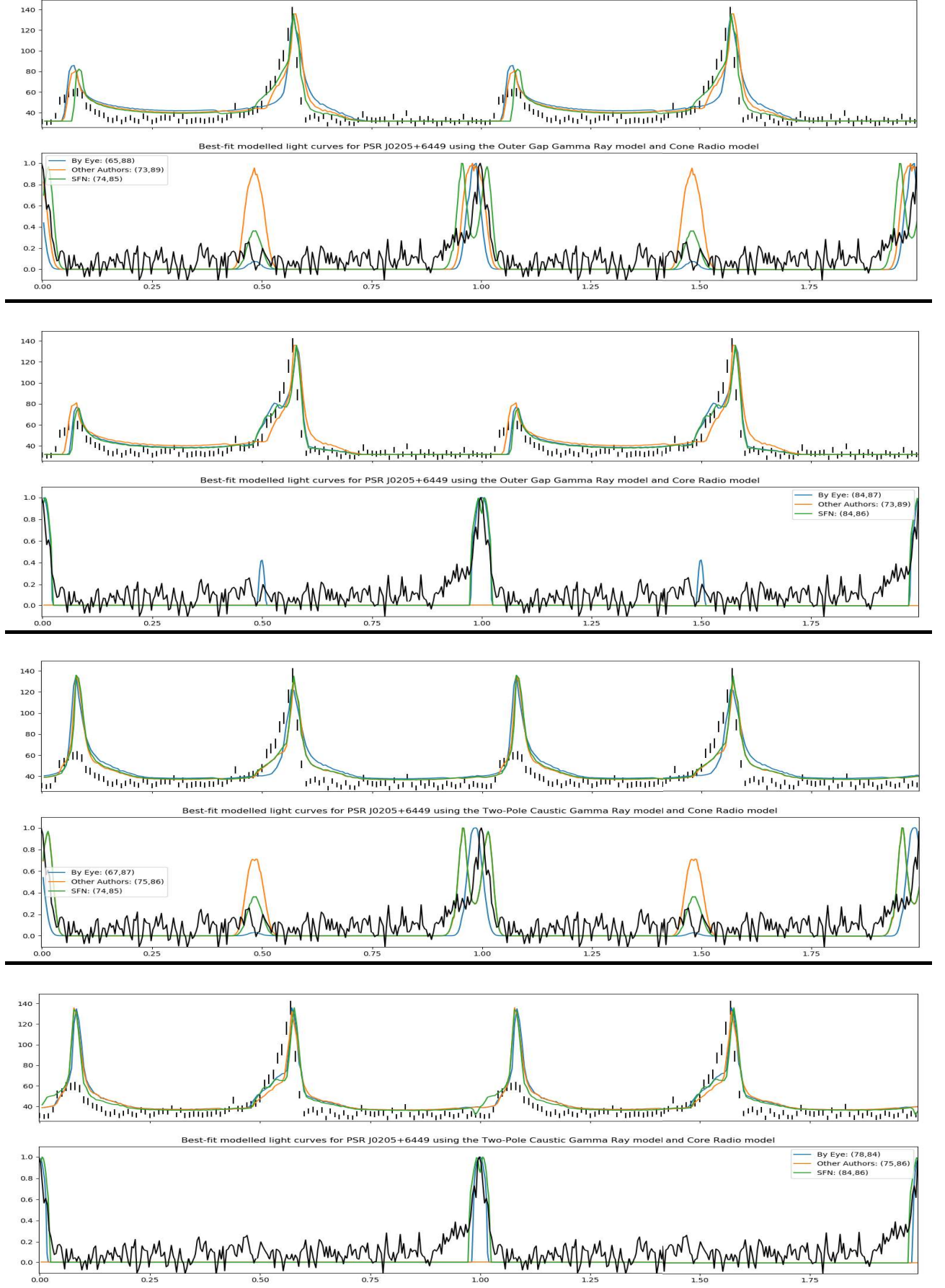


Figure B.2: Best-fit LCs for PSR J0205+6449.

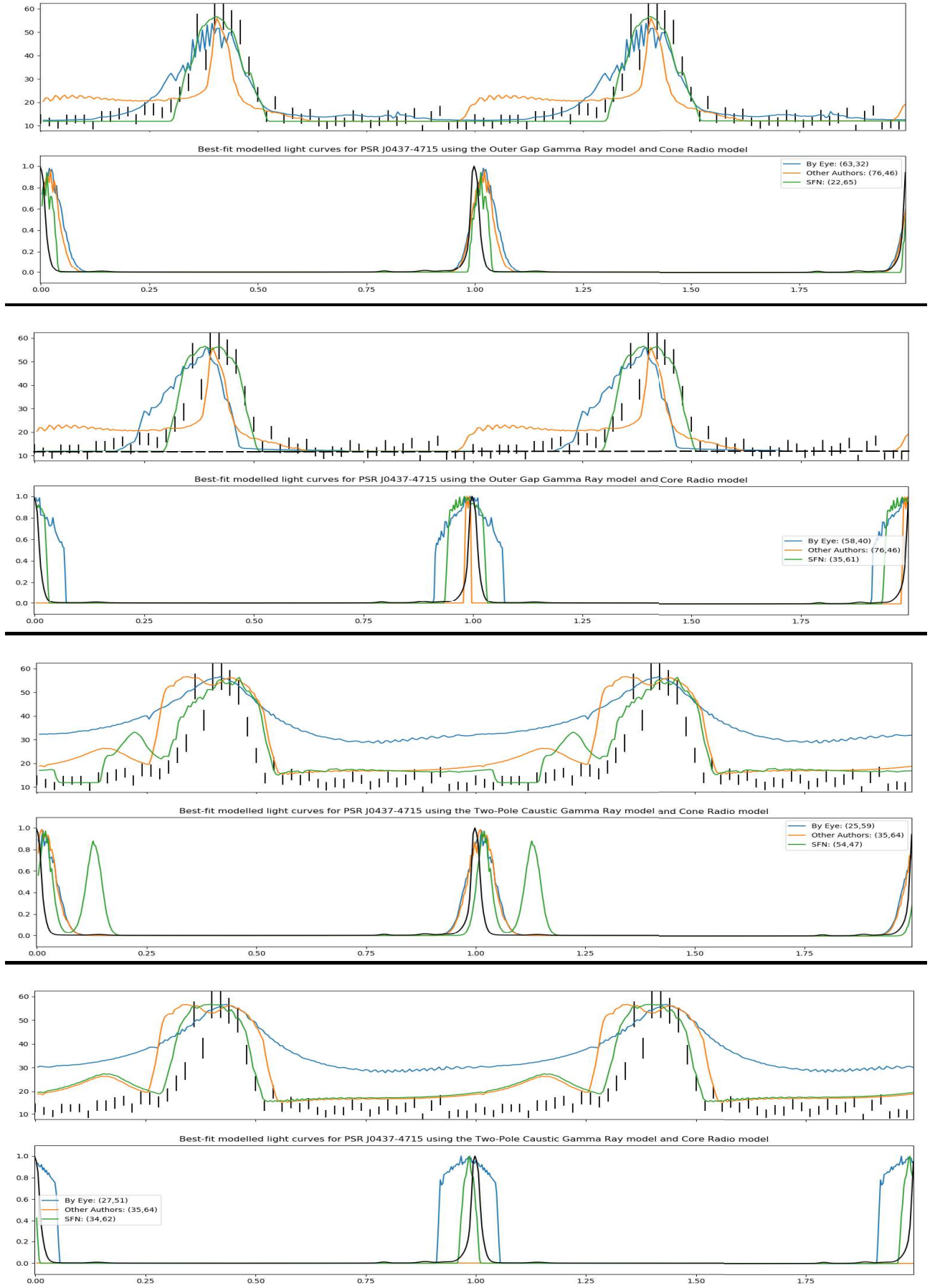


Figure B.3: Best-fit LCs for PSR J0437–4715.

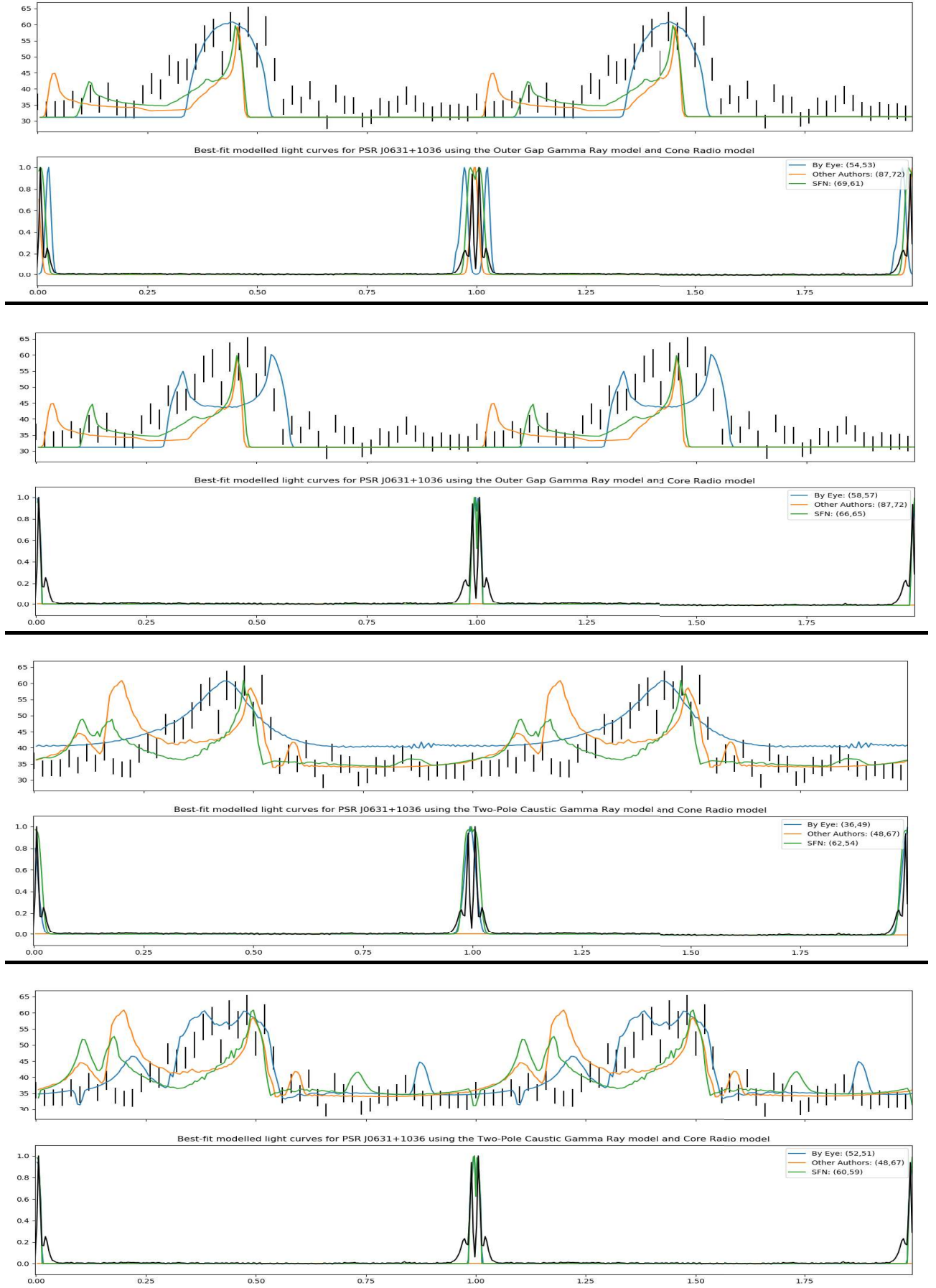


Figure B.4: Best-fit LCs for PSR J0631+1036.

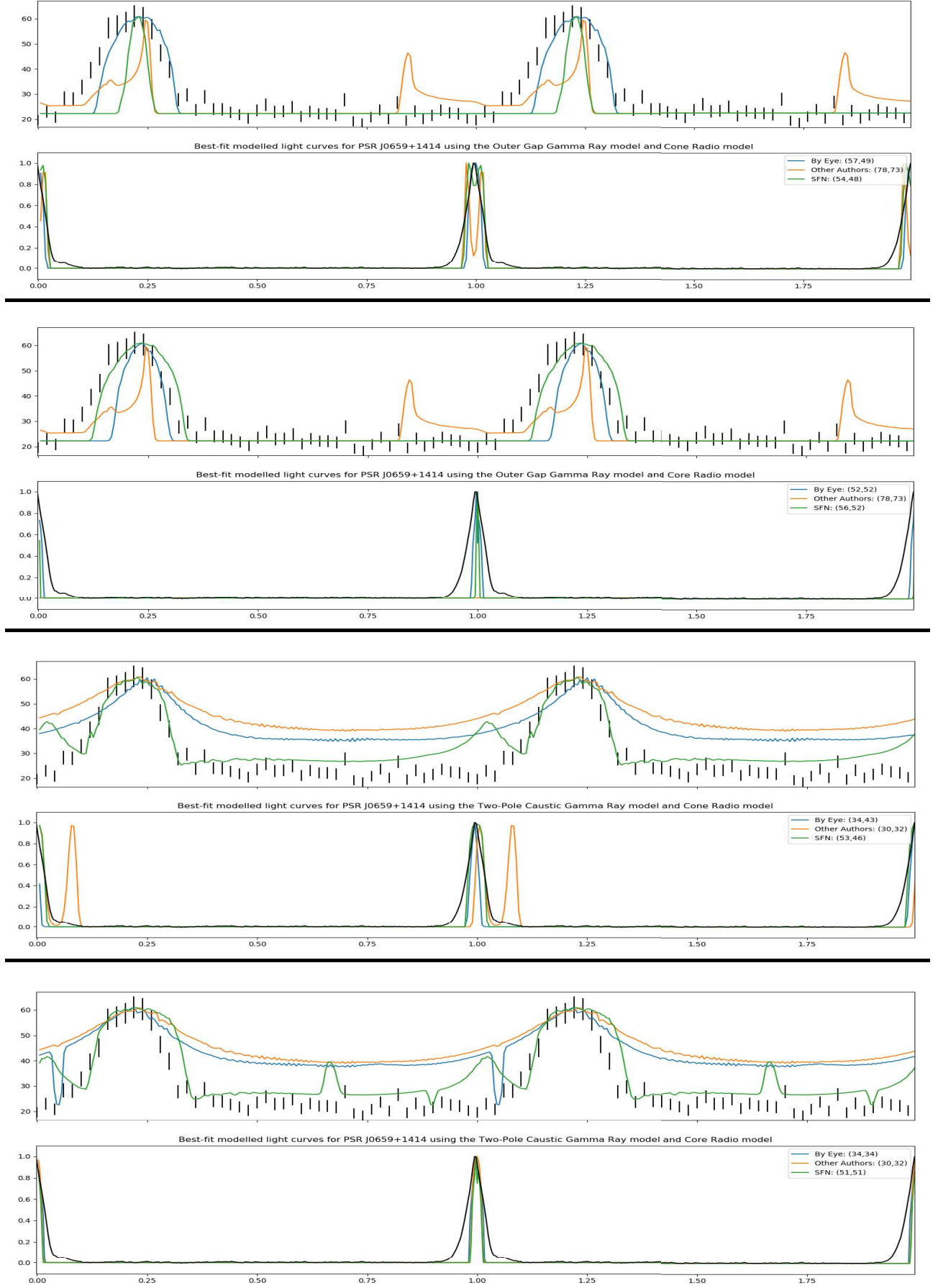


Figure B.5: Best-fit LCs for PSR J0659+1414.

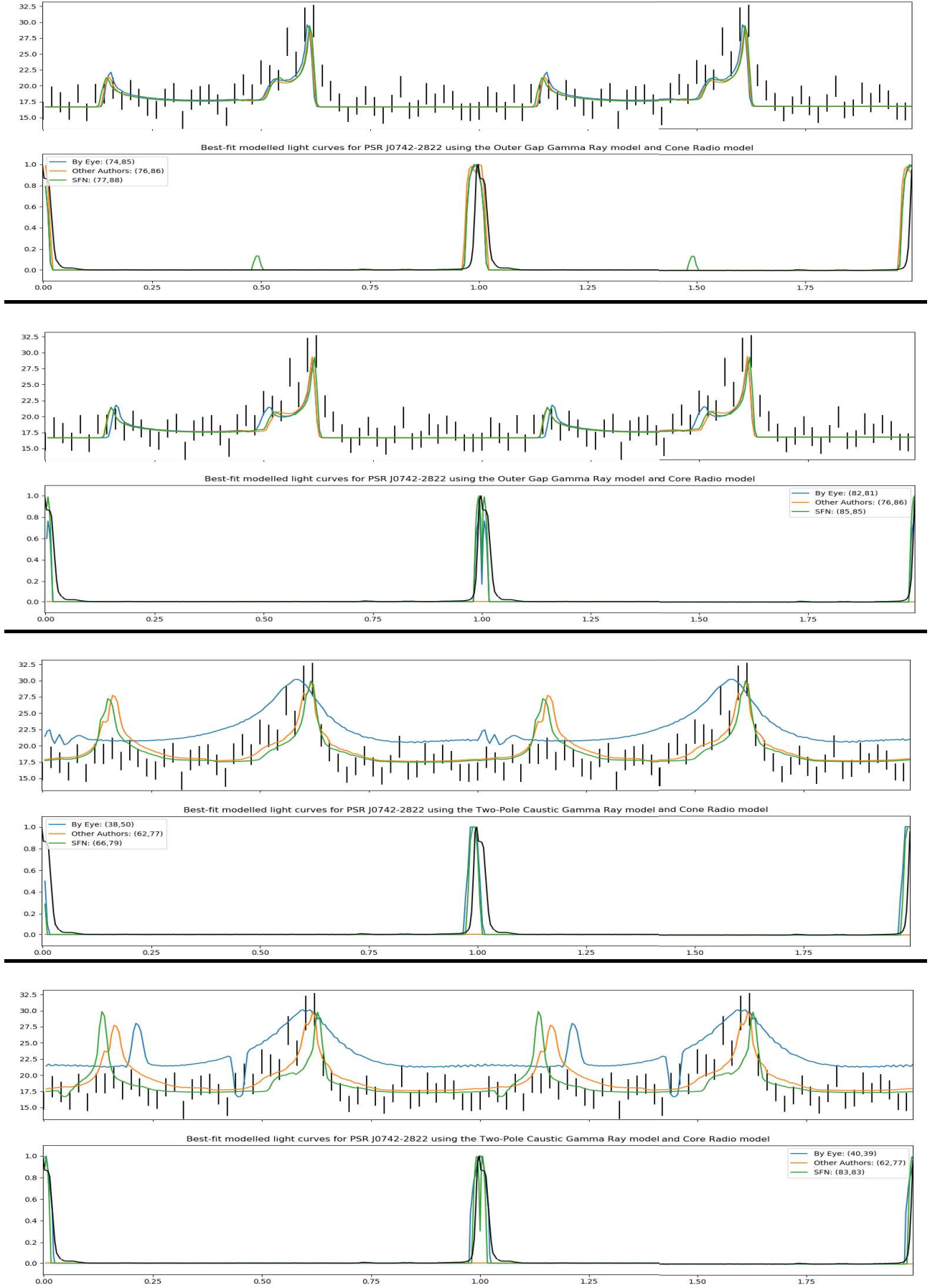


Figure B.6: Best-fit LCs for PSR J0742–2822.

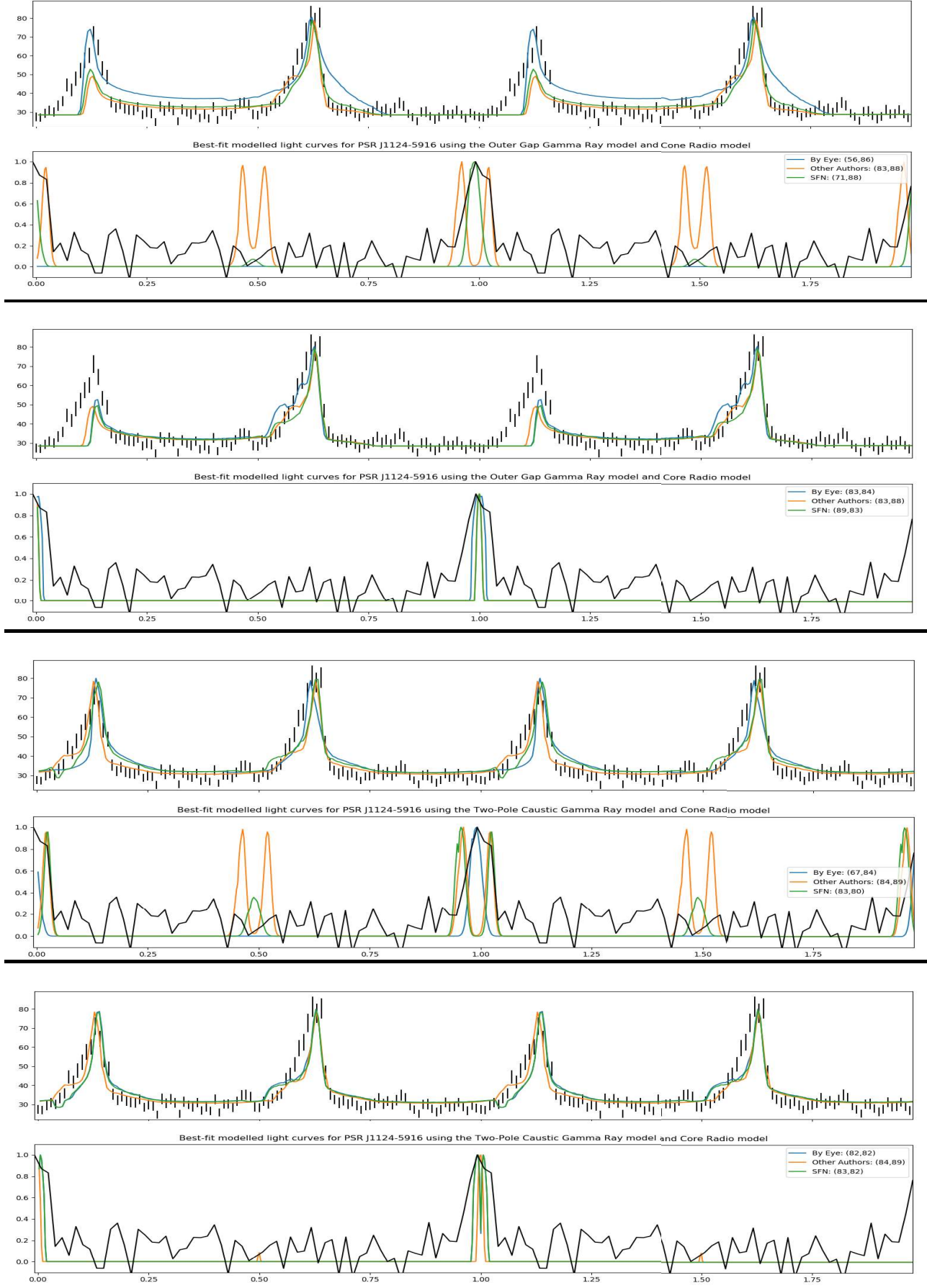


Figure B.7: Best-fit LCs for PSR J1124–5916.

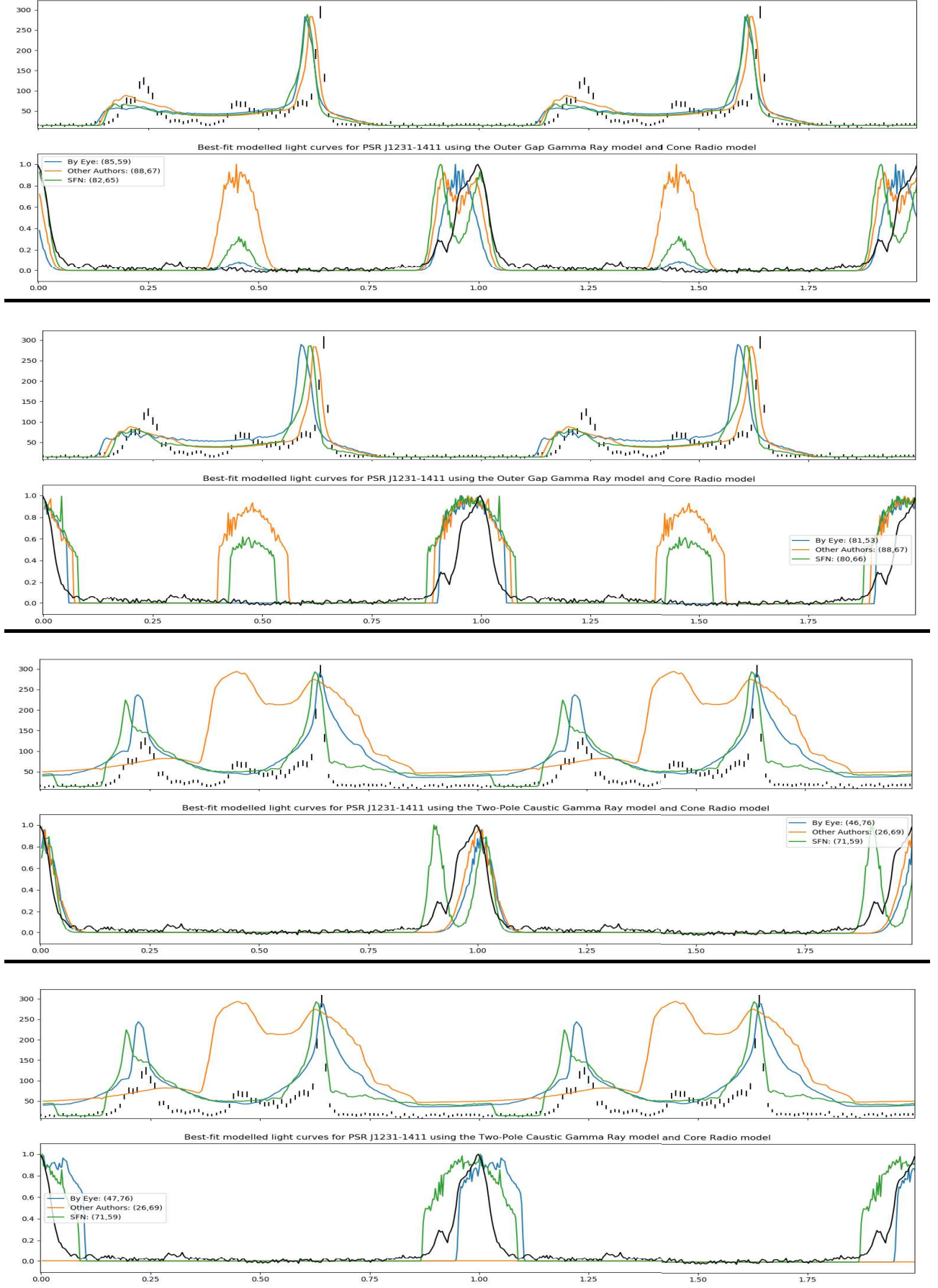


Figure B.8: Best-fit LCs for PSR J1231–1411.

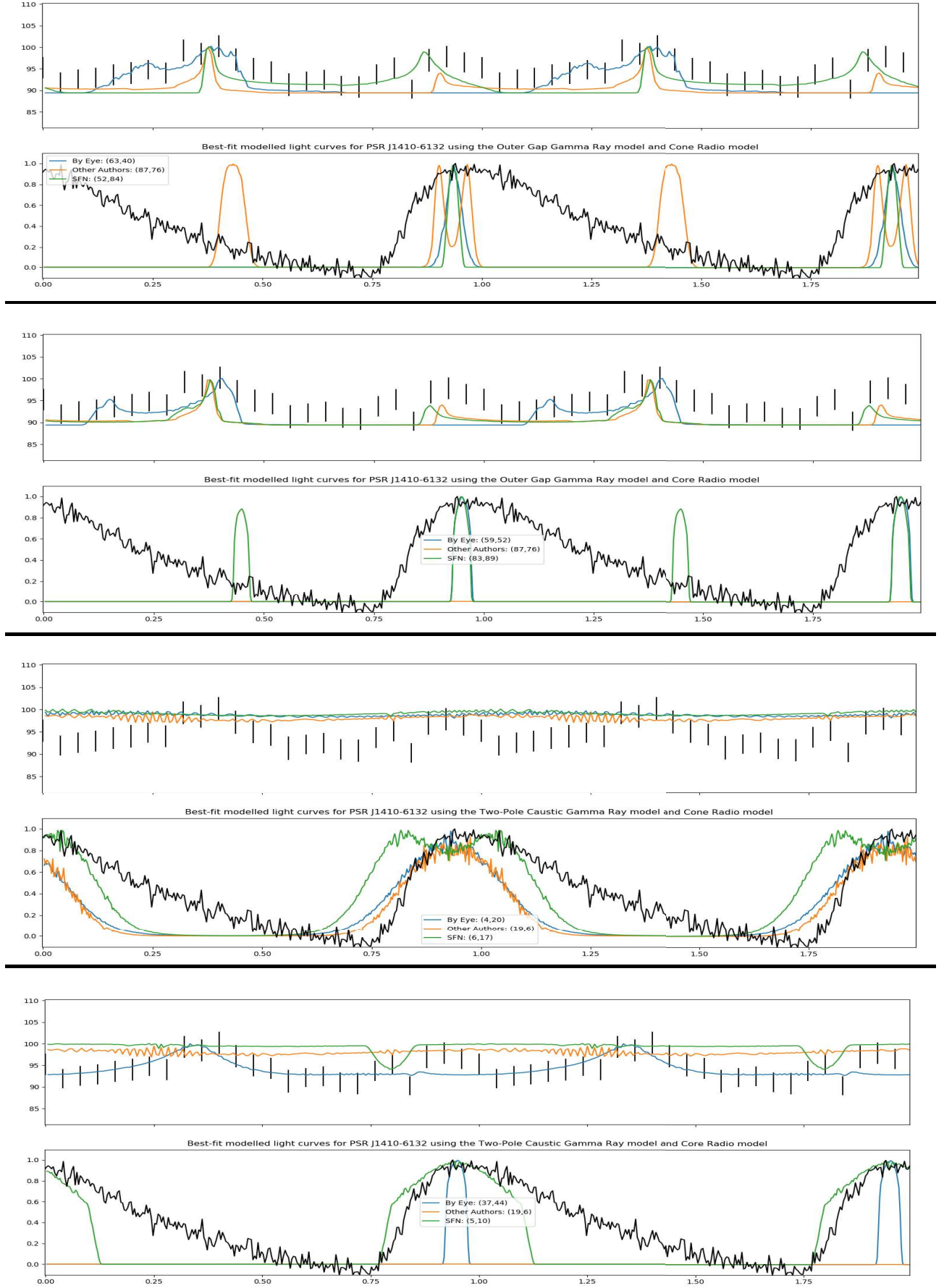


Figure B.9: Best-fit LCs for PSR J1410–6132.

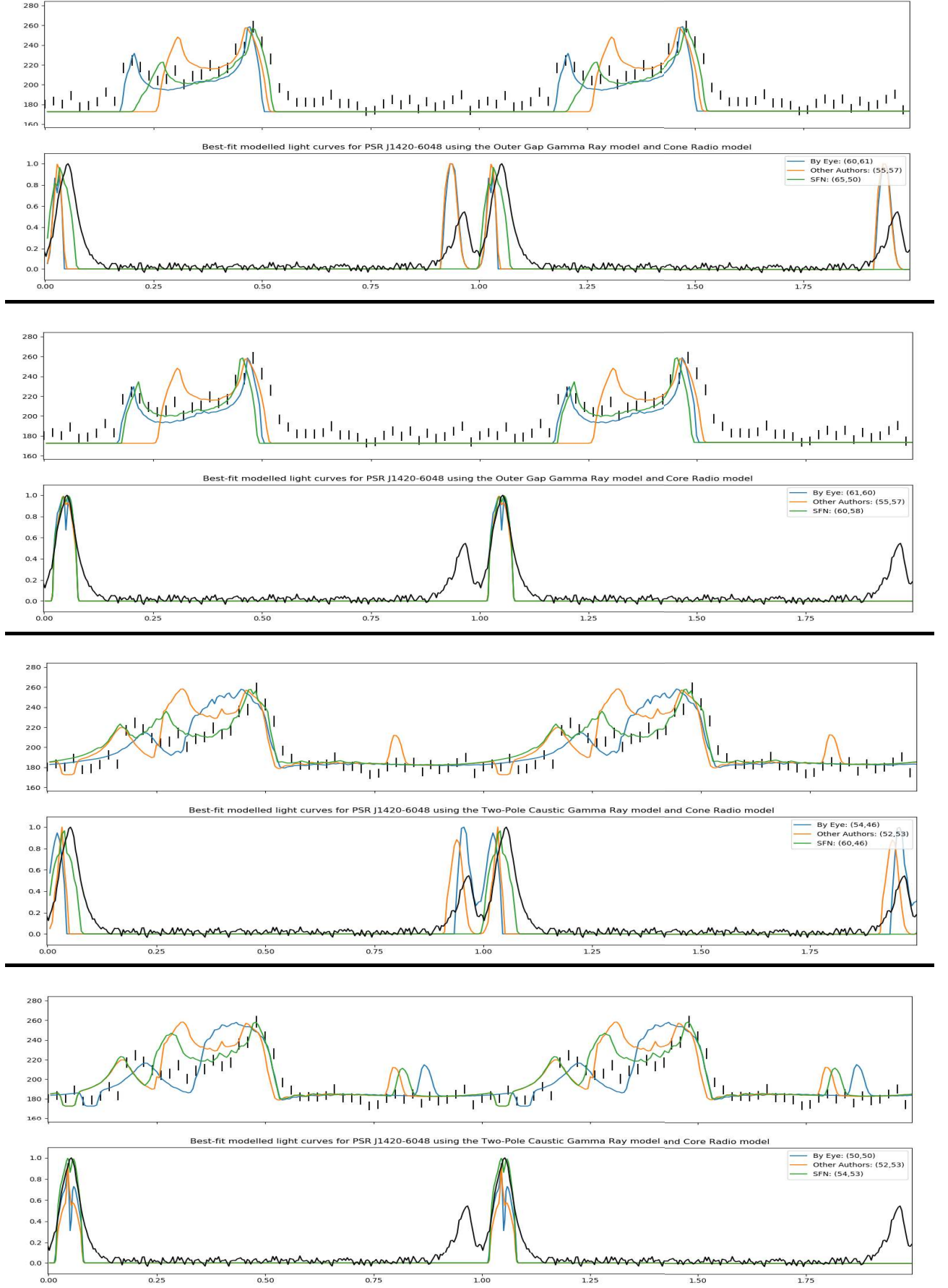


Figure B.10: Best-fit LCs for PSR J1420–6048.

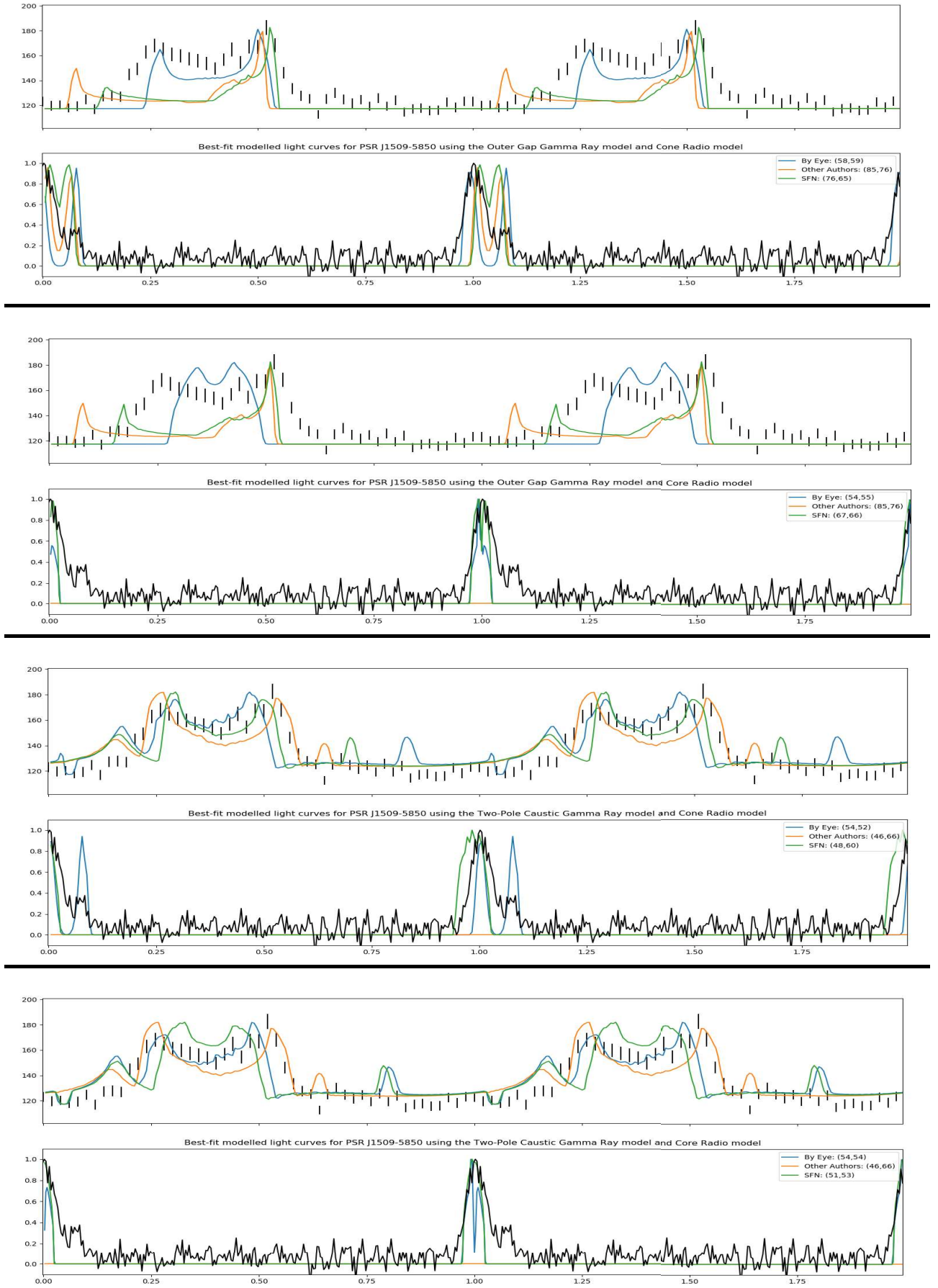


Figure B.11: Best-fit LCs for PSR J1509-5850.

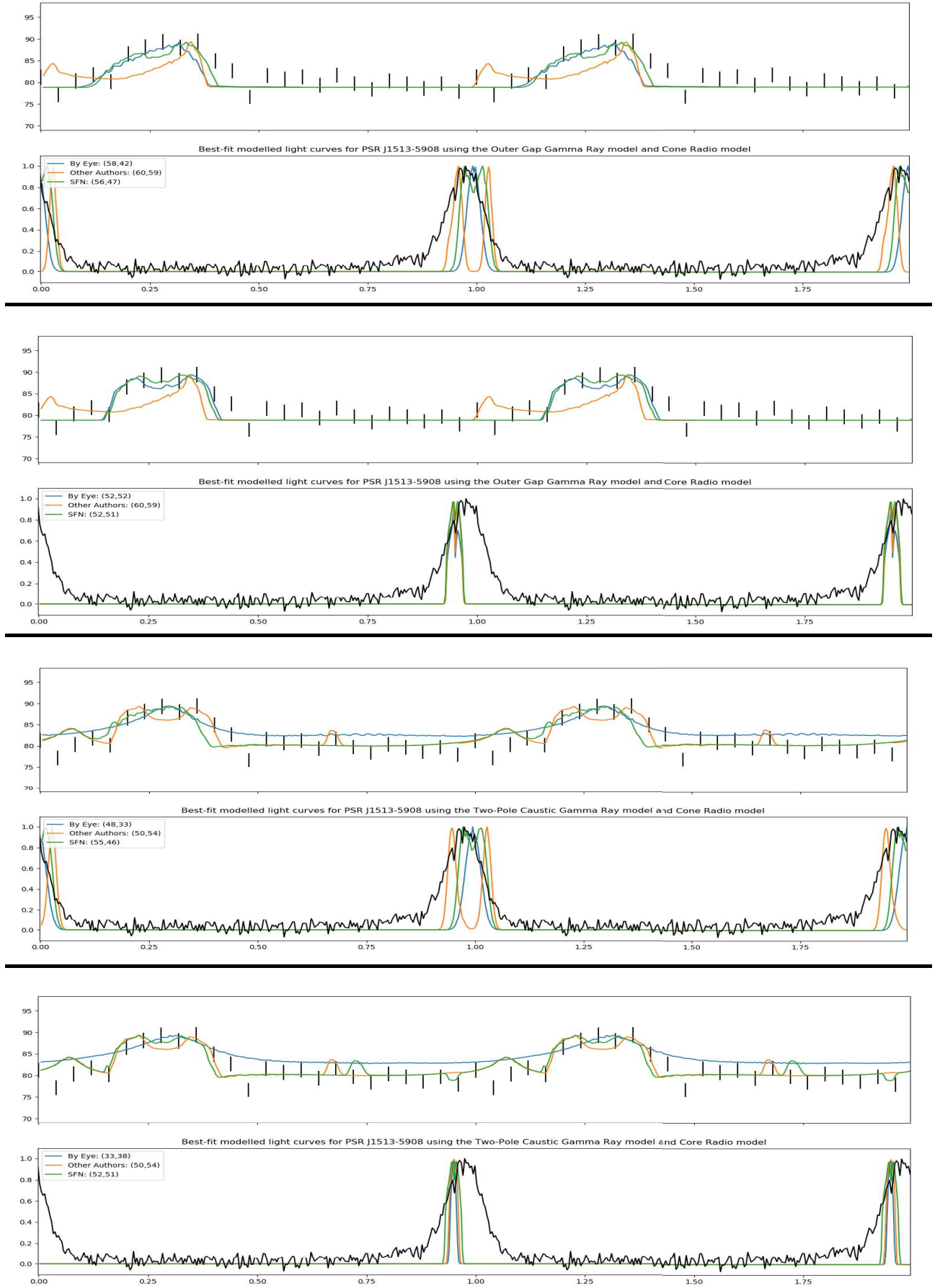


Figure B.12: Best-fit LCs for PSR J1513–5908.

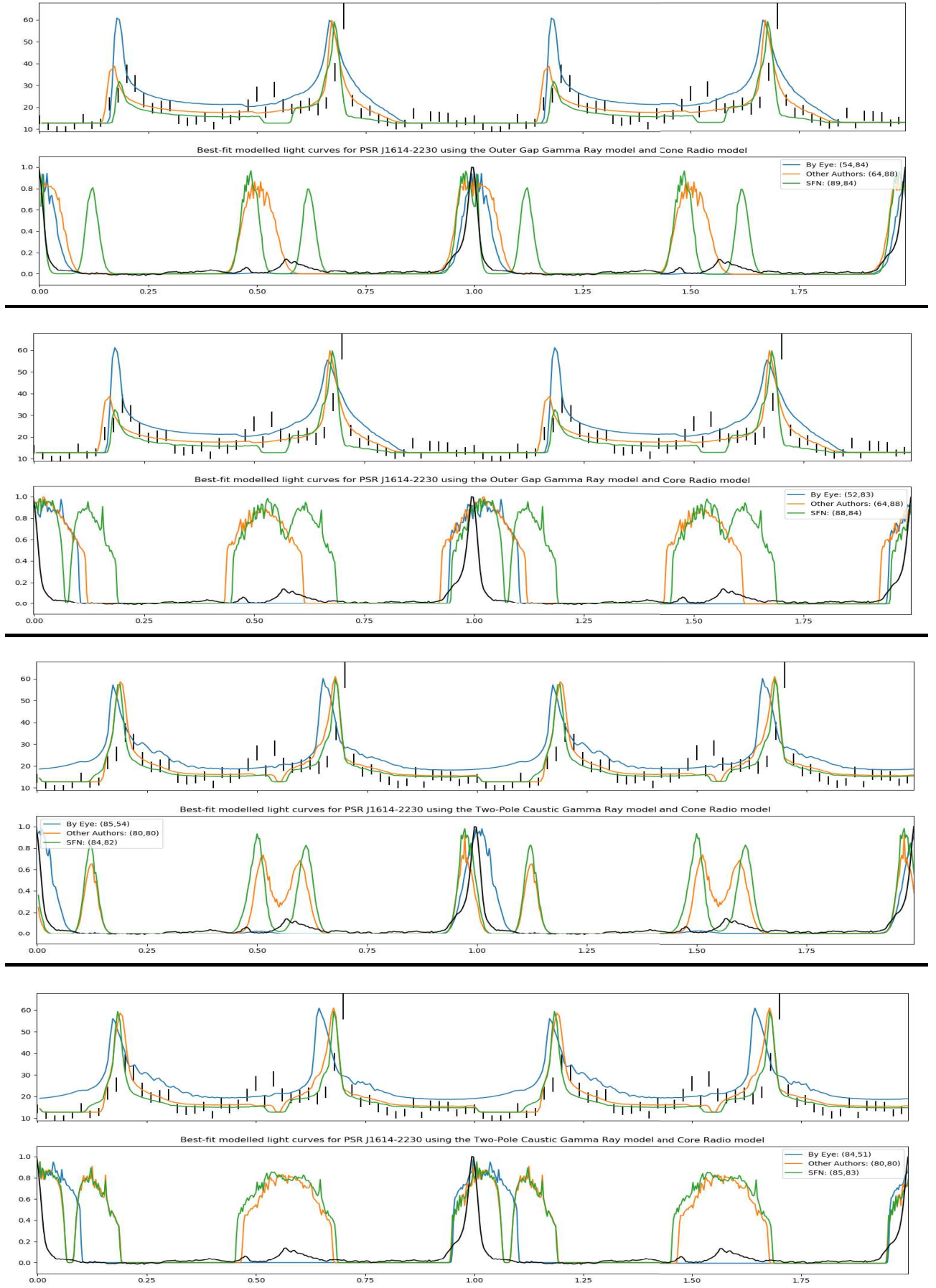


Figure B.13: Best-fit LCs for PSR J1614–2230.

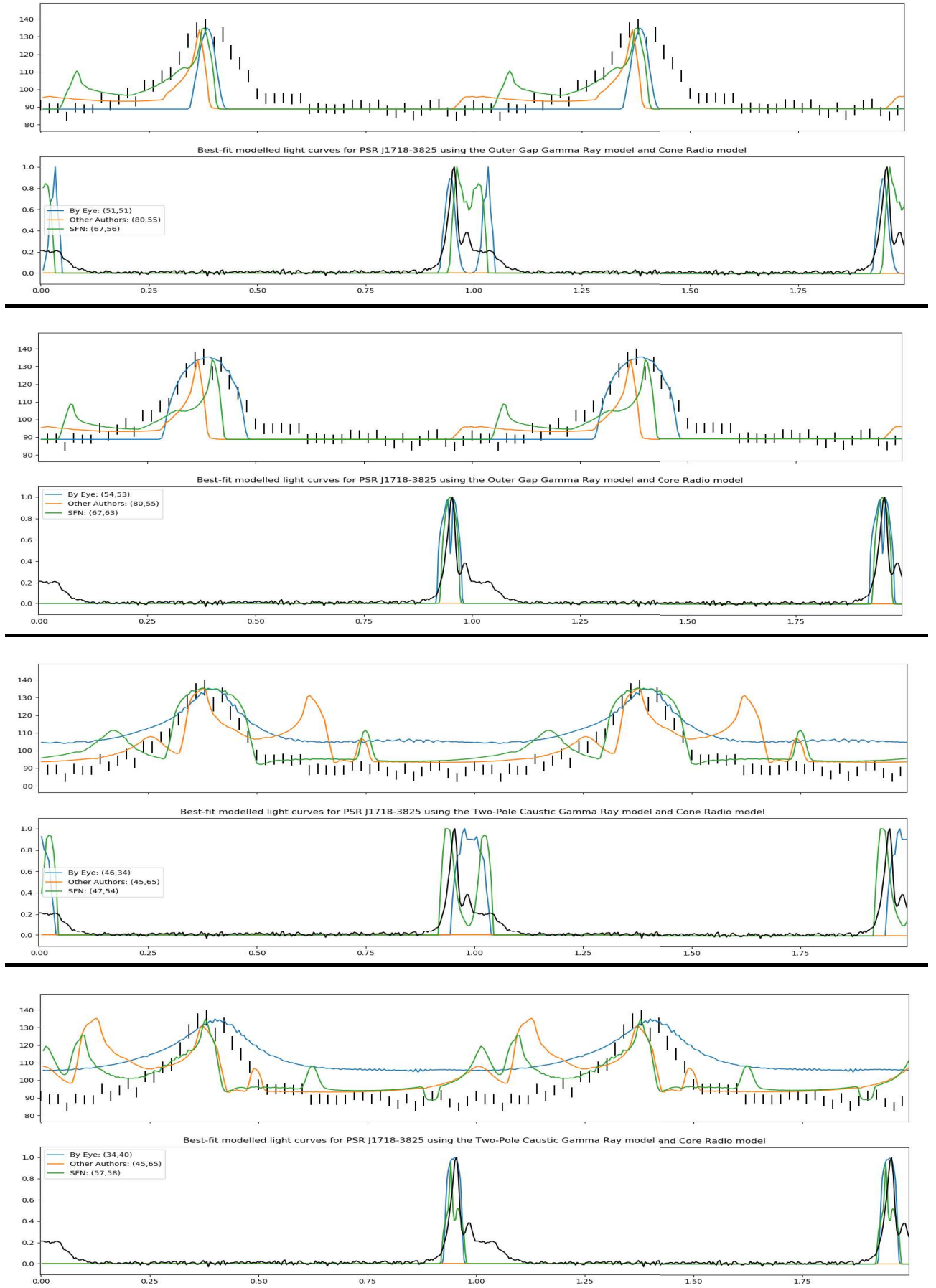


Figure B.14: Best-fit LCs for PSR J1718–3825.

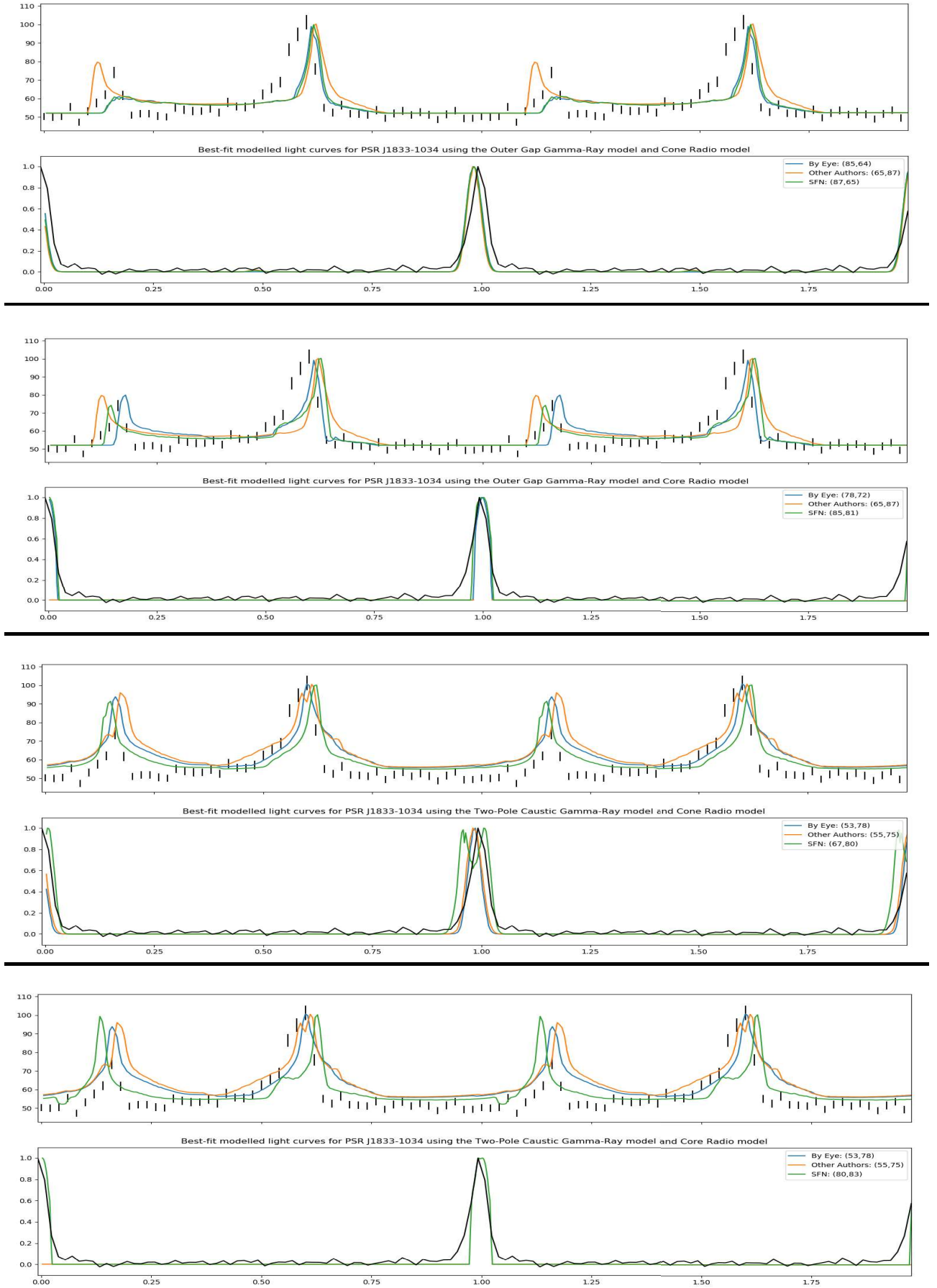


Figure B.15: Best-fit LCs for PSR J1833–1034.

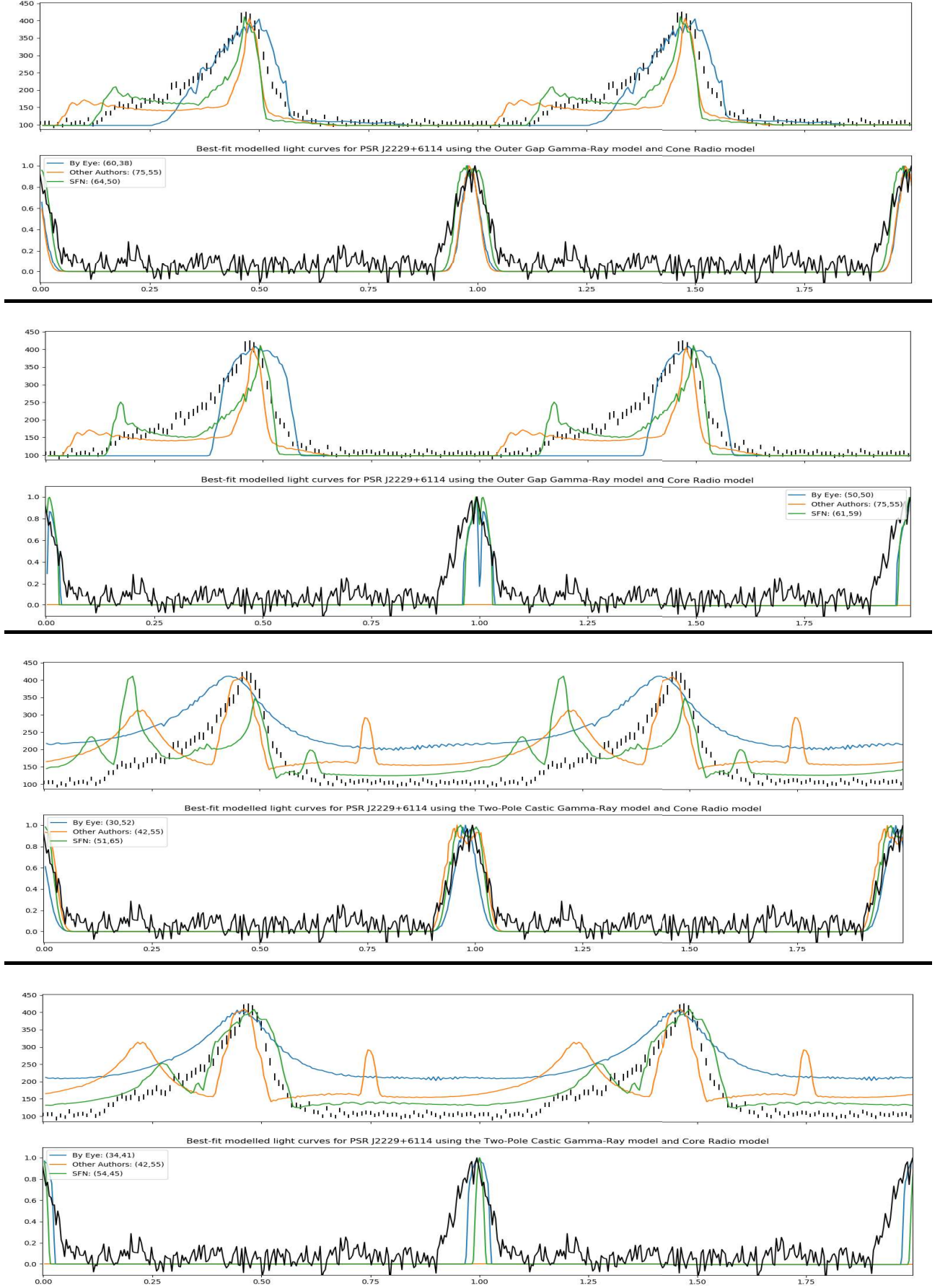


Figure B.16: Best-fit LCs for PSR J2229+6114.

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