

THE EFFECT OF DRIFT ON THE EVOLUTION OF THE ELECTRON/POSITRON SPECTRA IN AN AXISYMMETRIC PULSAR WIND NEBULA

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ABSTRACT

Charged particles propagating through a structured magnetic field are subject to drift motion. The primary aim of the present paper is therefore to investigate the effects of gradient, curvature, and neutral sheet drift on the evolution of the electron and positron spectra in a pulsar wind nebula, where the drift motion is a direct result of the magnetic field having an Archimedean spiral structure. In order to investigate the evolution of the spectra, the steady-state, axisymmetric Fokker–Planck transport equation is solved numerically using a finite-difference scheme. Apart from drift motion, the transport processes of convection and diffusion, along with the energy loss processes of adiabatic cooling and synchrotron radiation, are also included in the model. It is found that drift, particularly neutral sheet drift, can lead to a quantitative difference in the evolution of the electron and positron spectra. This difference may be of importance when interpreting the positron excess observed by *PAMELA* and AMS-02 near Earth.

Key words: diffusion – ISM: supernova remnants – pulsars: general

1. INTRODUCTION

Pulsars produce highly relativistic magnetized winds that consist of an equal number of electrons and positrons (e.g., Kirk et al. 2009; Arons 2012), and possibly a hadronic component (e.g., Cheng & Ruderman 1980; Horns et al. 2006). Due to the large angular velocity of the pulsar, the magnetic field that is frozen into the out-flowing wind is tightly wound, with the field geometry often described as purely azimuthal (e.g., Bogovalov 1999; Kirk et al. 2009). Moreover, the dipole nature of the pulsar’s magnetic field implies that the polarity of the frozen-in field reverses between the northern and southern hemispheres, leading to the creation of a magnetically neutral sheet in the equatorial region (e.g., Michel 1973; Coroniti 1990). As the rotation and magnetic axes of the pulsar are typically inclined by an angle α with respect to each other, this neutral sheet has a wavy structure, with every wave separating regions of opposing polarity, the so-called striped pulsar wind.

When the ram pressure of the pulsar wind is balanced by the confining pressure of the ambient medium a termination shock is formed (Rees & Gunn 1974) where the leptons (electrons and positrons) are accelerated (e.g., Reynolds & Chevalier 1984). Downstream of the termination shock the leptons interact with the frozen-in magnetic field, emitting synchrotron radiation that is observed from radio to X-ray wavelengths. Additionally, the leptons can also inverse Compton scatter ambient photons to high-energy and very-high-energy gamma rays. This non-thermal emission leads to a luminous nebula commonly known as a pulsar wind nebula (PWN). Polarization measurements show that the azimuthal geometry and dipole character of the magnetic field are largely preserved in the inner parts of the nebula (Zajczyk et al. 2012), with this structure also predicted by recent three-dimensional relativistic magnetohydrodynamic (MHD) simulations (Porth et al. 2014).

The pulsar wind is essentially an extreme form of the solar wind that flows through the heliosphere, where a similar magnetic geometry and wavy neutral sheet is observed (see, e.g., Hoeksema 1995; Smith 2001). Theoretical and experimental studies focusing on the modulation of cosmic rays in the heliosphere have found that an azimuthal magnetic field leads to the gradient and curvature drift of charged particles, while the

presence of a neutral sheet leads to an additional drift motion of particles along this structure (see, e.g., Jokipii & Kopriva 1979; Potgieter & Moraal 1985). It further follows from these studies that drift causes the spectra of particles with opposite electrical charges to evolve differently (see, e.g., Burger & Potgieter 1999; Ferreira & Potgieter 2004). Given the similarities between PWNe and the heliosphere, one thus expects that these drift motions should also be present in PWNe, and that this may lead to the electron and positron spectra evolving differently.

Although PWNe are extended sources, the evolution of the lepton energy spectra is generally calculated using spatially independent particle evolution models, with only a handful of spatially dependent models presented thus far (see, e.g., Schöck et al. 2010; Tang & Chevalier 2012). In a previous paper (Vorster & Moraal 2013) we presented a spherically symmetric model based on the Fokker–Planck transport equation, with this model taking into account the transport processes of convection and diffusion, along with the energy loss processes of adiabatic cooling, synchrotron radiation, and inverse Compton scattering.

In the present paper we extend these simulations by including the additional transport process of gradient, curvature, and neutral sheet drift, with the aim of determining how these processes influence the evolution of the lepton spectra. As these drift processes cannot be included in spherically symmetric simulations, we extend our original model to an axisymmetric model. In order to make the problem more tractable, a steady-state transport model is used for the present simulations. The most prominent limitation of such an approach is that the model cannot take into account the dynamic evolution of the outer boundary of the PWN. However, as we have previously shown (Vorster & Moraal 2013), the evolution of the outer boundary has no qualitative, and only a moderate quantitative effect, on the evolution of the lepton spectra.

Apart from investigating the effect of drift on the evolution of the lepton spectra, the results may also be useful to interpret the positron excess observed at energies above 10 GeV by *PAMELA* (Adriani et al. 2009), *Fermi*-LAT (Ackermann et al. 2012), and more recently, by AMS-02 (Aguilar et al. 2013). To explain this excess, e.g., Büsching et al. (2008), Yüksel et al. (2009), and Linden & Profumo (2013) have argued that leptons escaping from the nearby Geminga PWN and Monogem pulsar could

be responsible for the positron observations. It is therefore necessary to have a clear understanding of how the electron and positron spectra in a PWN evolve.

The outline of the paper is as follows: Section 2 introduces the general Fokker–Planck transport equation, while the axisymmetric model is discussed in Section 3. The results of the simulations are presented and discussed in Section 4, with the most salient results summarized in Section 5.

2. THE GENERAL TRANSPORT EQUATION

The evolution of the lepton energy spectra in a PWN can be calculated using the Fokker–Planck transport equation (e.g., Ginzburg & Syrovatskii 1964; Parker 1965)

$$\frac{\partial f}{\partial t} + \nabla \cdot \mathbf{S} - \frac{1}{p^2} \frac{\partial}{\partial p} (p^2 \langle \dot{p} \rangle f) = Q(\mathbf{r}, p, t), \quad (1)$$

where $f(\mathbf{r}, p, t)$ is the omnidirectional distribution function and p the momentum. The second term on the left-hand side contains the differential current density

$$\mathbf{S} = 4\pi p^2 (\mathbf{V}f - \mathbf{K} \cdot \nabla f - \mathbf{V}_D f) \quad (2)$$

resulting from convection, diffusion, and gradient and curvature drift, with $\mathbf{V}$ denoting the convection velocity, $\mathbf{K}$ the diffusion tensor that includes diffusion both parallel and perpendicular to the magnetic field, and $\langle \mathbf{V}_f \rangle$ the drift velocity averaged over both the pitch- and phase angles of the particles.

The third term on the left-hand side of Equation (1) describes adiabatic losses, as well as the non-thermal energy loss processes of synchrotron radiation and inverse Compton scattering. In a spherically expanding medium the adiabatic loss rate is given by

$$\frac{\langle \dot{p} \rangle_{\text{ad}}}{p} = -\frac{1}{3} \nabla \cdot \mathbf{V}. \quad (3)$$

Synchrotron radiation and inverse Compton scattering in the Thomson regime have the same momentum dependence, making it possible to describe both non-thermal processes by the single expression

$$\frac{\langle \dot{p} \rangle_{\text{n-t}}}{p} = z(\mathbf{r}, t)p, \quad (4)$$

where

$$z(\mathbf{r}, t) = \frac{4\sigma_T}{3(m_e c)^2} (U_B + U_{\text{IC}}) \quad (5)$$

is a function of the magnetic U_B and photon U_{IC} energy field densities. The other constants in Equation (5) are the Thomson scattering cross-section σ_T and the rest mass m_e of the leptons. As synchrotron radiation and inverse Compton scattering have the same effect on the evolution of the lepton spectra, the latter will be omitted from further discussion. Lastly, the term Q on the right-hand side of Equation (1) describes any sources or sinks of particles.

3. THE MODEL

3.1. The Magnetic Field

The PWN magnetic field is modeled by an Archimedean spiral of the form

$$\mathbf{B} = B_0 \left(\frac{r_0}{r} \right)^2 (\mathbf{e}_r - \tan \psi \mathbf{e}_\phi) [1 - 2H(\theta - \theta_{\text{ns}})], \quad (6)$$

as derived by Jokipii & Levy (1977) for the heliosphere magnetic field. Here r_0 represents the position of the inner boundary of the PWN (defined as the wind termination shock in the current model), B_0 the magnetic field at r_0 , H the Heaviside function, and

$$\theta_{\text{ns}} = \frac{\pi}{2} + \sin^{-1} \left[\sin \alpha \sin \left(\phi - \phi_0 + \frac{\Omega r}{V_r} \right) \right] \quad (7)$$

the angular position of the neutral sheet that separates the outward pointing field in one hemisphere from the inward pointing field in the opposite hemisphere. This sheet has a wavy nature due to the so-called tilt angle α between the rotation Ω and magnetic axes μ of the system, given by

$$\cos \alpha = \frac{\Omega \cdot \mu}{\Omega \mu}, \quad (8)$$

with Ω the angular velocity of the system. The angle ψ between the magnetic field and radial directions is given by

$$\tan \psi = \frac{\Omega r \sin \theta}{V_r}, \quad (9)$$

where V_r is the radial convection velocity with which this field is carried outward. Note that the derivation of Equation (6) assumes that the convection velocity has only a radial component, as is generally assumed for PWN models (e.g., Rees & Gunn 1974; Kennel & Coroniti 1984). Finally, ϕ_0 in Equation (7) is an arbitrary phase constant.

For the Sun and heliospheric magnetic field $\Omega = 2\pi/27$ days and $V_r = 400 \text{ km s}^{-1}$, so that $\tan \psi \approx 1$ at one astronomical unit (at the orbit of Earth). Within this distance the field is quasi radial, while becoming progressively more azimuthal at larger distances. On the other hand, the Crab pulsar has $\Omega \approx 190 \text{ s}^{-1}$ (Nather et al. 1969) and a termination shock radius of $r_0 \approx 10^{17} \text{ cm}$ (Begelman 1998), while the wind immediately upstream of the termination shock is expected to be close to the speed of light c (see, e.g., Gaensler & Slane 2006). Inserting these values into Equation (9) shows that $\tan \psi \gg 1$, implying that the magnetic field in Equation (6) is entirely dominated by B_ϕ (see also, e.g., Kirk et al. 2009). For modeling purposes the value $\Omega = 10^3 \text{ s}^{-1}$ is used. This value, together with the scaled parameter values (see Sections 3.4 and 3.5) ensure that the magnetic field in the PWN has the required azimuthal structure.

As discussed in the Introduction, Porth et al. (2014) found that the azimuthal structure is present in the inner part of the nebula, with the field becoming less structured in the outer parts of the nebula. Nevertheless, for an initial investigation into the effects of drift on the evolution of the lepton spectra it is sufficient to use Equation (6).

3.2. Drift Motion

Particles will experience gradient and curvature drift in the inhomogeneous magnetic field (6). Jokipii & Levy (1977) and Burger (1987) have shown that for an isotropic distribution function, or a distribution function that has only a first-order anisotropy, the pitch- and phase-angle averaged drift velocity that appears in Equation (2) is

$$\mathbf{V}_D = \nabla \times \kappa_T \frac{\mathbf{B}}{B}, \quad (10)$$

where

$$\kappa_T = \frac{\beta p c}{3qB}. \quad (11)$$

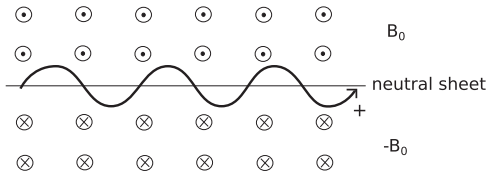


Figure 1. Illustration showing a positively charged particle drifting along the neutral sheet (solid horizontal line). A negatively charged particle will drift in the opposite direction.

Here c denotes the speed of light, $\beta = v/c$ the usual relativistic factor (v is the particle's speed), and q the particle's charge.

Apart from this curvature/gradient and curvature drift, particles will also drift along the neutral sheet, as illustrated in Figure 1.

Positively charged particles located above the neutral sheet, where the magnetic field is directed out of the plane, will gyrate in a clockwise direction. Depending on their phase angles, particles with gyrocenters that are less than two gyroradii ($r_g = pc/qB$) from this sheet will cross the neutral sheet into the region where the magnetic field is directed into the plane. As a result, the gyration direction changes to anti-clockwise. This continual changing of gyration direction therefore causes the positively charged particles to propagate toward the right, and negatively charged particles to propagate toward the left.

All aspects of the curvature and neutral sheet drift were originally derived by Jokipii & Levy (1977), Isenberg & Jokipii (1978, 1979), and Jokipii & Kopriva (1979). These authors showed that in the Archimedean spiral magnetic field (6) the radial and latitudinal components of the drift velocity are

$$V_{D,r} = \frac{\kappa_{\top}}{r} \left(-\frac{\sin \psi}{\tan \theta} + \cos \psi (2 \sin^2 \psi - 1) \frac{\partial \tan \psi}{\partial \theta} \right), \quad (12a)$$

and

$$V_{D,\theta} = \kappa_{\top} \left(\cos \psi (1 - 2 \sin^2 \psi) \frac{\partial \tan \psi}{\partial r} + \frac{3 \sin \psi}{r} \right). \quad (12b)$$

When $\tan \psi \gg 1$, as is the case for the toroidal field in a PWN, these expressions reduce to the simple form

$$V_{D,r} = -\frac{2\kappa_{\top}}{r \tan^2 \psi \tan \theta}, \quad (13a)$$

and

$$V_{D,\theta} = \frac{\kappa_{\top}}{r}. \quad (13b)$$

The modeling of neutral sheet drift is more difficult to handle. When the tilt angle α of the sheet is zero, so that it lies at $\theta = 90^\circ$, Jokipii & Levy (1977) have shown that this drift can be modeled as a boundary condition on the drift flux without referring explicitly to the actual drift velocity. However, when the sheet is wavy, this method does not work due to the fact that the sheet does not conveniently lie along a line of grid points in the numerical model.

In order to model the effect of a wavy neutral sheet it is necessary to know the pitch- and phase-angle averaged neutral sheet drift velocity of the particles that partake in neutral sheet drift. This problem was addressed by Burger (1987), who

calculated this velocity to be $v/6$ (exactly), while Caballero-Lopez & Moraal (2003) have shown how to handle this scenario numerically. More specifically, their analysis shows how to calculate the average neutral sheet drift velocity over one full oscillation of the wavy neutral sheet. The radial component is a component of the above-mentioned magnitude $v/6$, while the latitudinal component averages out to zero. There is also a finite azimuthal component due to the explicit ϕ -dependence of the neutral sheet profile in Equation (7), but in the two-dimensional model (radial distance r and latitudinal angle θ) to be introduced in the next section, an azimuthal symmetry is assumed so that this component does not play a role.

The combined effect of gradient/curvature and neutral sheet drift on the motion of a particle is as follows: when $\cos \alpha > 0$ in Equation (8), positively charged particles generally drift radially inward along the (possibly wavy) neutral sheet, while the gradient and curvature drift, described by Equations (12a) and (12b), is radially inward and toward the poles. This drift state was called the $qA > 0$ state by Jokipii & Levy (1977). When $\cos \alpha < 0$, all drift velocities are in the opposite direction, with this drift state referred to as $qA < 0$. For negatively charged particles all directions are reversed, i.e., these particles will drift radially outward and toward the equatorial region when $\cos \alpha > 0$, and vice versa when $\cos \alpha < 0$. For an illustration of this drift motion, the paper by, e.g., Pesses et al. (1981) can be consulted.

3.3. The Axisymmetric Transport Equation

While gradient and curvature drift can be taken into account through Equation (2), Axford (1965) and Parker (1965) have shown that it is possible to incorporate these processes directly into the diffusion tensor $\underline{\mathbf{K}}$; to wit, in a reference frame where the coordinate axes are aligned with the magnetic field, the diffusion tensor should have the form

$$\underline{\mathbf{K}}_B = \begin{bmatrix} \kappa_{\parallel} & 0 & 0 \\ 0 & \kappa_{\perp} & \kappa_{\top} \\ 0 & -\kappa_{\top} & \kappa_{\perp} \end{bmatrix}, \quad (14)$$

where the elements $\kappa_{\parallel}$ and $\kappa_{\perp}$ describe diffusion parallel and perpendicular to the magnetic field, respectively, and $\kappa_{\top}$ gradient and curvature drift. Using the standard rules for tensor transformation it can be shown (e.g., Steenkamp 1995) that $\underline{\mathbf{K}}_B$ transforms to

$$\underline{\mathbf{K}}(r, \theta, \phi) = \begin{bmatrix} \kappa_{rr} & \kappa_{r\theta} & \kappa_{r\phi} \\ \kappa_{\theta r} & \kappa_{\theta\theta} & \kappa_{\theta\phi} \\ \kappa_{\phi r} & \kappa_{\phi\theta} & \kappa_{\phi\phi} \end{bmatrix} = \begin{bmatrix} \kappa_{\parallel} \cos^2 \psi + \kappa_{\perp} \sin^2 \psi & -\kappa_{\top} \sin \psi & (\kappa_{\perp} - \kappa_{\parallel}) \cos \psi \sin \psi \\ \kappa_{\top} \sin \psi & \kappa_{\perp} & \kappa_{\top} \cos \psi \\ (\kappa_{\perp} - \kappa_{\parallel}) \sin \psi \cos \psi & -\kappa_{\top} \cos \psi & \kappa_{\perp} \cos^2 \psi + \kappa_{\parallel} \sin^2 \psi \end{bmatrix} \quad (15)$$

in a spherical coordinate system. The above matrix is valid for any Archimedean spiral magnetic field given Equation (6). Note that, as mentioned previously, Ω is typically large for a PWN. This leads to a magnetic field that is purely azimuthal, i.e., $\psi \rightarrow 90^\circ$. This implies that all diffusion in the model is essentially perpendicular to the magnetic field.

Using Equation (15), along with the adiabatic and synchrotron energy loss rates given by Equations (3) and (4), respectively,

the transport Equation (1) in an axisymmetric system becomes

$$\begin{aligned} & \kappa_{rr} \frac{\partial^2 f}{\partial r^2} + \frac{\kappa_{\theta\theta}}{r^2} \frac{\partial^2 f}{\partial \theta^2} \\ & + \left[\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \kappa_{rr}) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta \kappa_{\theta r}) - V_r \right] \frac{\partial f}{\partial r} \\ & + \left[\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \kappa_{r\theta}) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta \kappa_{\theta\theta}) \right] \frac{\partial f}{\partial \theta} \\ & + \left[\frac{1}{3r^2} \frac{\partial}{\partial r} (r^2 V_r) + \frac{1}{3r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta V_\theta) + zp \right] \frac{\partial f}{\partial \ln p} \\ & + 4zp f + Q(r, \theta, p, t) \\ & = 0. \end{aligned} \quad (16)$$

Note that the terms involving $\kappa_{\theta r}$ and $\kappa_{r\theta}$ are, respectively, the radial $V_{D,r}$ and polar $V_{D,\theta}$ components of the combined gradient and curvature drift velocities. These expressions are augmented with neutral sheet drift according to the prescription described in the previous section.

3.4. Scaling of the Model

To keep the model as general as possible, it is useful to scale the variables present in Equation (16) to dimensionless ones. The scaling used in Vorster & Moraal (2013) is again used. The radial distance is scaled as

$$r = \frac{\tilde{r}}{\tilde{r}_S}, \quad (17)$$

where $\tilde{r}$ is the unscaled variable and $\tilde{r}_S$ a characteristic length, chosen as the size of the system. This leads to a system with a scaled radial dimension of $r_0 < r \leq 1$. A similar scaling is used for momentum

$$p = \frac{\tilde{p}}{\tilde{p}_S}, \quad (18)$$

with the difference that $\tilde{p}_S$ is not chosen as the maximum particle momentum, but rather an intermediate value. For the present paper, the scaled momentum range is chosen as $10^{-3} \leq p \leq 10^4$. It is also possible to scale the particle energy in a similar fashion, $E = \tilde{E}/\tilde{E}_S$. Since $\tilde{E} = \tilde{p}c$ for relativistic particles, it follows that $E = p$. For the numerical calculations the scale energy is chosen as $\tilde{E}_S = 1$ TeV, with the scaled momentum range translating to the energy range $1 \text{ GeV} \leq \tilde{E} \leq 10 \text{ PeV}$. This energy range includes the electron energy range needed to produce the non-thermal emission observed from PWNe.

The convection velocity in the system is expressed as a fraction of the speed of light

$$V_r = \frac{\tilde{V}_r}{c}, \quad (19)$$

while the diffusion and drift coefficients are scaled as

$$\kappa_{i,j} = \frac{\tilde{\kappa}_{i,j}}{\tilde{r}_S c}. \quad (20)$$

3.5. Parameter Values Used

The well-known spherically symmetric, steady-state MHD simulations of Kennel & Coroniti (1984) show that the spatial dependence of the radial convection velocity V_r depends on the ratio of electromagnetic to particle energy σ . When $\sigma = 1$,

the convection velocity remains almost independent of r . When $\sigma = 0.01$, the velocity falls off as $V_r \propto 1/r^2$ close to the termination shock, but falls off slower after one termination shock radius, approaching a constant value toward the edge of the nebula. For the current model the profile is chosen to be $V_r \propto 1/r$, while it is assumed that V_r has no θ dependence. The convective flow downstream of the pulsar wind termination shock is subsonic, with the sound speed in a magnetized plasma being of the order $c/\sqrt{3}$ (e.g., Reynolds & Chevalier 1984). Based on these considerations, the velocity at the termination shock is chosen to be $\tilde{V}_0 = 0.3c$.

From Equation (6) it follows that the radial profile of the magnetic field depends on the convection velocity, with the chosen profile $V_r \propto 1/r$ leading to a magnetic field that is constant with radius. The Crab Nebula has an average magnetic field strength of $B \sim 260 \mu\text{G}$ (e.g., de Jager et al. 1996), whereas it is estimated that the Vela PWN has a lower value of $B \sim 3 \mu\text{G}$ (e.g., De Jager & Djannati-Ataï 2009). It is therefore difficult to specify a characteristic value for the magnetic fields in PWNe, and the value $\tilde{B}_0 = 35 \mu\text{G}$ is chosen for the simulations.

As discussed in Section 3.3, the purely azimuthal magnetic field geometry used in the model implies that all diffusion is essentially perpendicular to the magnetic field. Diffusion occurs as a result of particles interacting with irregularities in the magnetic field, and experience with the propagation of cosmic rays in the heliosphere shows that the perpendicular diffusion coefficient can be modeled as (e.g., Caballero-Lopez et al. 2004, and references therein)

$$\kappa_{\perp} = \kappa_{\perp,0} \frac{B_0}{B} p. \quad (21)$$

To account for the loss of particles with $\tilde{E} > 100 \text{ GeV}$ from the gamma-ray emission region in the Vela PWN, a spatially independent diffusion coefficient of $\tilde{\kappa}_0 \sim 10^{27} \text{ cm}^2 \text{ s}^{-1}$ has been estimated by Hinton et al. (2011). A similar $\tilde{\kappa}_0$ has also been derived by Van Etten & Romani (2011) to explain the TeV gamma-ray emission observed from the source HESS J1825-137. There is however one caveat that should be kept in mind. It is believed that the elongated, cigar-like shape of both these PWNe, with the pulsar located toward the edge, is the result of the interaction between the PWN and supernova reverse shock (e.g., Vorster et al. 2013). It is thus possible that the interaction has transformed the initial azimuthal magnetic field geometry to one with a significant radial component, thereby increasing the role of diffusion parallel to the magnetic field. The values derived by Hinton et al. (2011) and Van Etten & Romani (2011) may therefore represent a combination of both radial and perpendicular diffusion. For the model the smaller value $\kappa_{0,\perp} = 10^{26} \text{ cm}^2 \text{ s}^{-1}$ is therefore used.

It follows from Equation (11) that the drift coefficient $\kappa_{\top} \propto 1/B$, similar to the spatial dependence chosen for $\kappa_{\perp}$. In contrast to the diffusion coefficient, the value of $\kappa_{\top,0}$ is not chosen arbitrarily, but is directly determined by the magnitude of the magnetic field.

Observations of PWNe indicate that the leptons injected at the termination shock should be described by a broken power law (e.g., Gaensler & Slane 2006; De Jager & Djannati-Ataï 2009, and references therein), with various spectral evolution models predicting a break-energy of $p_{bc} = 0.1 \text{ TeV}$ (e.g., Fang & Zhang 2010; Tanaka & Takahara 2011). In order to not unduly complicate the results and their discussion, a single power law $f(p, r_0) \propto p^{-4}$ is used. This is comparable to the spectral

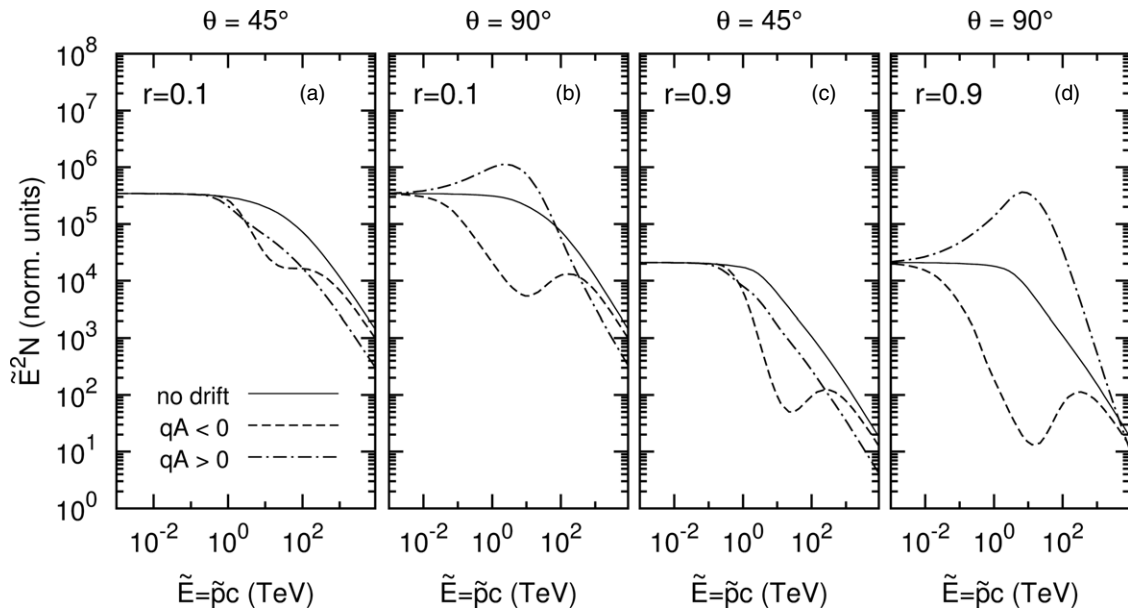


Figure 2. Particle spectra in the inner ($r = 0.1$) and outer ($r = 0.9$) parts of the pulsar wind nebula at the latitudes $\theta = 45^\circ$ and $\theta = 90^\circ$. For a definition of qA , see the last paragraph of Section 3.2. Note that the flat ($\alpha = 0^\circ$) neutral sheet is located at $\theta = 90^\circ$.

index derived from X-ray observations. For example, Mangano et al. (2005) observed $f(p, r_0) \propto p^{-4.00 \pm 0.04}$ for Vela X, Schöck et al. (2010) observed $f(p, r_0) \propto p^{-4.32 \pm 0.04}$ for MSH 15-52, and Slane et al. (2004) observed $f(p, r_0) \propto p^{-2.2 \pm 0.2}$ for 3C 58. Due to the fact that both diffusion and drift scale with momentum in the present model, the use of a single power-law injection spectrum is not particularly restrictive as the effects of these processes will largely be present at higher energies, i.e., the part of the lepton spectrum responsible for the X-ray emission.

3.6. Numerical Setup

The axisymmetric transport Equation (16) is solved numerically using the Douglas (1962) *Alternating Direction Implicit* finite-difference method. The model used for the simulations is adapted from one that was initially developed to calculate the modulation of cosmic rays in the heliosphere, with this latter model used extensively by, e.g., Moraal & Gleeson (1975), Potgieter & Moraal (1985), and Reinecke et al. (1993).

As the simulations are time-independent, momentum plays the numerical role usually assigned to time, and is thus used as the stepping parameter of Equation (16). Because particles migrate downward in momentum (or energy) as a result of energy losses, the numerical scheme steps from high to low momentum values in logarithmic steps. The momentum interval under consideration ranges between $10^{-3} \leq p \leq 10^4$, with the step size chosen to be $\Delta \ln p = -0.2$. Particles propagating away from the termination shock immediately lose energy, and particles with momentum $p_{\max}$ can therefore only exist at the location of the shock ($r_0 = 0.01$). This furnishes the “initial” condition $f(p_{\max}, r) = 0$ for all $r > r_0$.

The simulations are performed over a radial grid that ranges between $r_0 \leq r \leq 1$, with the numerical scheme having a resolution of $\Delta r = 10^{-3}$. The inner boundary condition is similar to the one that has also been derived by Ng & Gleeson (1975) to describe the evolution of the energy spectrum of cosmic rays produced by solar flares: the number of particles per momentum interval that flows through the inner boundary must be equal to the total number of particles produced per time and momentum interval, $Q = Q^* p^{-4}$, with Q^* a normalization

constant. This implies that

$$\oint \mathbf{S} \cdot d\mathbf{A} = Q, \quad (22)$$

where $d\mathbf{A} = r_0^2 \sin \theta d\theta d\phi \mathbf{e}_r$ is the surface element, and $\mathbf{S}$ is given by Equation (2). Integrating Equation (22) leads to the inner boundary condition

$$CV_0 f - (\kappa_{rr,0} + \kappa_{\theta r,0}) \frac{\partial f}{\partial r} = Q^* \frac{1}{4\pi r_0^2} p^{-4} \quad (23)$$

that is solved simultaneously with Equation (16). To simulate particles escaping from the system, the free-escape (Dirichlet) condition $f(r = 1, p) = 0$ is imposed at the outer boundary.

Lastly, the polar grid extends over the interval $0^\circ \leq \theta \leq 180^\circ$ with a resolution of $\Delta \theta = 0.5^\circ$, while both the inner and outer polar boundaries have a Neumann condition

$$\frac{\partial f}{\partial \theta} = 0 \Big|_{\theta=0^\circ, 180^\circ} \quad (24)$$

imposed on them. Note that the boundaries in the θ direction coincide with the positions of the poles of the PWN, and that the equator is located at $\theta = 90^\circ$.

The discussions presented in this section were taken from Vorster (2013), where a more detailed description of the model can be found.

4. RESULTS

4.1. Synchrotron Losses Neglected

Figure 2 shows the evolution of the spectra for the $qA < 0$ and $qA > 0$ scenarios when convection, diffusion, drift, and adiabatic losses are taken into account. For these simulations a flat neutral sheet is included in the equatorial plane, i.e., at $\theta = 90^\circ$. Also shown is the evolution of the spectra is also shown for a scenario where drift is neglected. To better illustrate the effect of drift on the evolution of the spectra, synchrotron

losses are neglected in this section, but will be discussed in the next. Although the results are primarily presented for the purposes of illustration, they may be of importance if there is a hadronic (proton) component in PWNe, as these particles are not subjected to significant synchrotron losses.

As discussed in Vorster & Moraal (2013), the no-drift particle spectra can be divided into two energy regimes: at lower energies convection and adiabatic losses are important, leading to a reduction in the intensity of the spectra, while the spectral shape remains similar to that of the injection spectrum. At higher energies radial diffusion is the dominant process, leading to a softening of the spectrum. This softening is directly related to the energy dependence of the diffusion coefficient. For the present simulations with $\kappa \propto p$ the spectra become softer by one power of momentum (or energy).

$qA < 0$. Compared to the no-drift scenario the spectrum at $r = 0.1$, $\theta = 45^\circ$ (Figure 2(a)) develops a “depression”-like feature in the energy range $1 \text{ TeV} \lesssim \tilde{E} \lesssim 300 \text{ TeV}$, while the spectrum at higher energies is similar to that of the no-drift spectrum. This behavior becomes more pronounced toward lower latitudes and larger radial distances. When interpreting the results it should be kept in mind that the spectra in Figure 2, and subsequent figures, have been multiplied by E^2 in order to enhance spectral features. This depression thus represents a spectrum that initially becomes softer, before becoming harder at higher energies. Note that the intensity of the spectrum nevertheless decreases monotonically.

In order to understand the behavior of the spectra in Figure 2, consider a spherically symmetric system where drift is unimportant. Particles injected into the system at the inner boundary will propagate toward the outer boundary where they can freely escape. This leads to the presence of a radial gradient in the particle intensity. Furthermore, the spherically symmetric nature of the system implies an absence of gradients in the polar direction and all diffusion will therefore be in a radial direction. Next, consider a system where a neutral sheet is present in the equatorial region. If the neutral sheet drift velocity is larger than the convection velocity, as is the case for the spectra in Figure 2, particles in a $qA < 0$ scenario particles will drift inward along the neutral sheet. As a result, particles injected at the equatorial region of the inner boundary will effectively be trapped in the very inner regions of the system. Note that drift, like diffusion, requires the presence of a gradient. However, unlike diffusion, particles can drift either against or along the direction of the gradient.

The absence of particles in the equatorial region at larger radial distances leads to the presence of a gradient in the polar direction. In the $qA < 0$ scenario gradient and curvature drift will transport the particles to higher latitudes, whereas particles will simultaneously diffuse from higher latitudes to the equatorial region. Drift and diffusion therefore transport particles in opposite directions, both in the radial and polar directions.

A particle with an energy $1 \text{ TeV} \lesssim \tilde{E} \lesssim 300 \text{ TeV}$ located at, e.g., an intermediate radial and polar position can therefore diffuse to the equatorial region, where it will be transported inward along the neutral sheet to a smaller radial position. On the other hand, gradient and curvature drift can also transport the particle toward higher latitudes, where it will again be transported outward by convection. Note, however, that the present radial position of the particle will be smaller than its initial radial position before it was transported inward along the neutral sheet. Consequently, this particle will suffer

more adiabatic losses during its propagation toward the outer boundary of the system than a particle in the no-drift scenario, leading to the formation of the depression feature visible in Figure 2. This increase in adiabatic losses occurs up to an energy of $\tilde{E} \sim 10 \text{ TeV}$, with diffusion becoming more important at larger energies.

As both diffusion and drift scale with momentum, one would expect these processes to be of comparable importance at higher energies. However, Figure 2 shows that for a given energy either diffusion or drift is the dominant transport process. To understand this behavior, consider Equation (16). Convection and drift are described by terms that only contain first-order spatial derivatives of f , and it is therefore possible to view drift as a form of energy dependent convection. However, diffusion is not only determined by the gradient, but also by the second-order spatial derivative of f . It is this difference that leads to diffusion becoming the dominant transport process at higher energies. This is not completely unexpected as a similar result has also been found from the study of the modulation of cosmic rays in the heliosphere (see, e.g., Potgieter & Moraal 1985).

$qA > 0$. Figures 2(a) and (c) show that the spectra have a similar spectral shape as those of the no-drift scenario, albeit with reduced intensities in the energy range where diffusion becomes the dominant transport process. However, Figures 2(b) and (d) show that a feature appearing to be the inverse of the one seen in the $qA < 0$ scenario develops in the equatorial region. Again the development of this feature is related to neutral sheet drift. At energies $\tilde{E} \lesssim 10 \text{ TeV}$ particles are effectively transported outward by neutral sheet drift, and as a result these particles suffer less adiabatic losses than particles only transported by convection. Due to the momentum dependence of drift, the reduction in the adiabatic losses, and consequently the reduction in particle intensity, is more pronounced at larger energies, leading to the formation of the “peak”-like structure visible in Figures 2(b) and (d). In contrast to the $qA < 0$ scenario, the spectral feature associated with drift is only visible in the equatorial region. This can be ascribed to the fact that in the $qA < 0$ scenario convection and neutral sheet drift are opposing transport processes, whereas these processes both transport particles outward in the $qA > 0$ scenario.

In a more realistic scenario one would have $\alpha > 0^\circ$, and simulations were thus also preformed for the case $\alpha = 30^\circ$. It was found that the effect of neutral sheet drift is strongly reduced, and the spectra of both the $qA > 0$ and $qA < 0$ scenarios closely resemble those of the no-drift scenario. For $\alpha > 0^\circ$ the neutral sheet has a wavy appearance, and the radial component associated with the neutral sheet drift velocity therefore decreases as α increases. Consequently the role of diffusion in transporting the particles (relative to neutral sheet drift) becomes more important. Additionally, the average θ component will approach zero due to the fact that a particle will, in an alternating fashion, drift to higher and lower latitudes along the sheet.

4.2. Synchrotron Losses Included

The next step is to investigate the effect of drift on the evolution of the particle spectra when synchrotron losses become important. In contrast to the simulations of Section 4.1, the more realistic value $\alpha = 30^\circ$ is used for the simulations in this section. The result of these simulations are shown in Figure 3. The $qA < 0$ and $qA > 0$ spectra are again shown together with the spectra for the no-drift scenario.

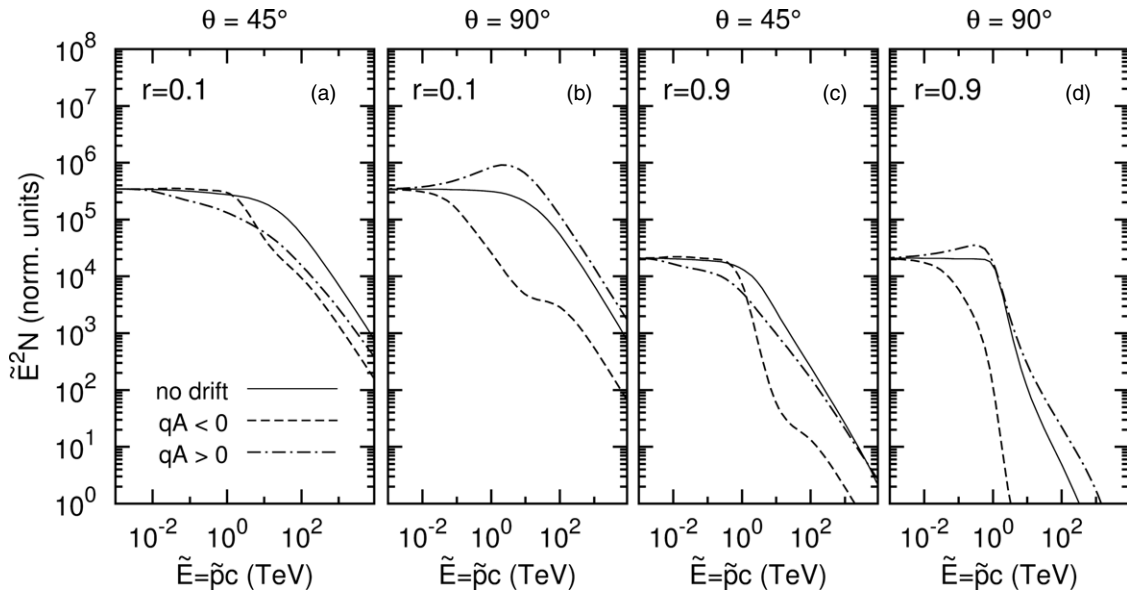


Figure 3. Same as Figure 2, but for a scenario where synchrotron losses is included and $\alpha = 30^\circ$.

Figures 3(a) and (b) show that in the absence of drift the leptons in the inner part ($r = 0.1$) of the system have not suffered significant synchrotron losses, and the spectral evolution at higher energies ($\tilde{E} \gtrsim 10$ TeV) is therefore determined by diffusion, and is comparable to the results shown in Figures 2(a) and (b). In the absence of diffusion, synchrotron losses lead to a sharp cut-off in the spectrum. However, when diffusion is an important transport mechanism this synchrotron cut-off is transformed into a significantly harder spectrum (e.g., Vorster & Moraal 2013). This behavior can be clearly seen from the spectra in the outer parts ($r = 0.9$) of the system, as shown in Figures 3(c) and (d). As the magnetic field increases toward the equatorial region (see Section 3.1), the particles at $\theta = 45^\circ$, Figure 3(c), have been subjected to less synchrotron losses, leading to a spectrum that is harder than the one in the equatorial plane, Figure 3(d).

$qA < 0$. Figure 3(a) shows that the high-energy ($\tilde{E} \gtrsim 100$ TeV) leptons at $r = 0.1$, $\theta = 45^\circ$ have been subjected to more synchrotron losses than those of the no-drift scenario, leading to a decrease in the intensity of the spectrum. This is also true for the spectra at $r = 0.1$, $\theta = 90^\circ$, as shown in Figure 3(b). As a result, the depression-like feature seen in Figure 2 is significantly less prominent in Figure 3.

In the outer part of the system ($r = 0.9$), Figures 3(c) and (d) show that, compared to the no-drift scenario, the synchrotron cut-off occurs at lower energies. As discussed in Section 4.1, the inward drift of particles along the neutral sheet significantly increases the residence time in the system, consequently leading to larger synchrotron losses and a smaller cut-off energy.

$qA > 0$. Compared to the no-drift scenario, Figure 3 shows that drift significantly reduces the synchrotron losses suffered by particles in the equatorial plane. This is primarily due to the fact that, apart from convection and diffusion, particles are additionally transported outward by neutral sheet drift, thereby reducing the residence time of the particles in the system. This effective transport of particles in the equatorial plane causes a gradient to development, leading to the effective transport of particles from higher latitudes to the equatorial plane. The intensity of the spectra at higher latitudes are therefore generally lower than those of the no-drift scenario, as shown in Figures 3(a) and (c).

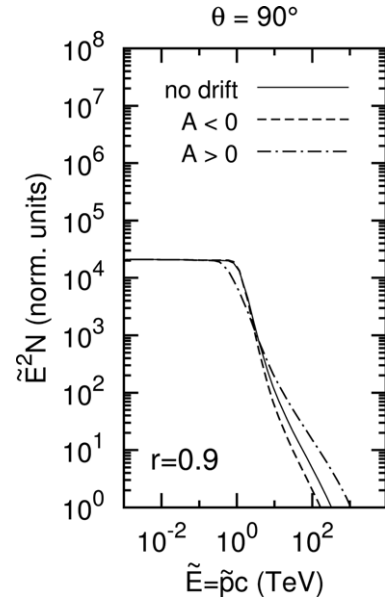


Figure 4. Same as Figure 3(d), but with the effect of neutral sheet drift neglected.

As the evolution of the spectra is to a large degree determined by the presence of a neutral sheet, and consequently neutral sheet drift, it is useful to investigate the evolution of the spectra when neutral sheet drift is neglected. For these simulations neutral sheet drift is artificially set to zero, while gradient and curvature drift are still taken into account. For this scenario the value of α plays no role as this parameter influences the shape of the neutral sheet, and consequently the drift velocity along the neutral sheet.

As an example, Figure 4 shows the spectra at $r = 0.9$ and $\theta = 90^\circ$. Although greatly reduced, gradient and curvature drift still lead to a quantitatively different evolution of the $qA < 0$ and $qA > 0$ spectra for energies $\tilde{E} \gtrsim 5$ TeV. A similar quantitative difference is also found at higher latitudes in the outer part of the system.

5. SUMMARY AND CONCLUSIONS

Charged particles in a PWN will be subjected to gradient and curvature drift due to the Archimedean spiral structure of the nebular magnetic field. Furthermore, the dipolar nature of this magnetic field implies the presence of a neutral sheet separating regions of opposing magnetic polarity. Consequently, particles will also be subjected to neutral sheet drift. The aim of this paper was therefore to investigate the effects of these drift processes on the evolution of the electron and positron spectra in a PWN.

For this investigation a model based on the steady-state, axisymmetric Fokker–Planck transport equation was used. Apart from drift, this model also includes convection, diffusion, adiabatic cooling, and synchrotron radiation. For studies of this kind it is important to keep in mind that in systems with a dipolar magnetic field, e.g., the heliosphere and PWNe, it is the product qA that determines the direction of the drift motion, where q is the charge of the particle and A is the orientation of the magnetic and rotation axes of, e.g., the Sun or pulsar, relative to each other (the definition of qA is described at the end of Section 3.2).

In order to obtain a better understanding of the effect of drift, initial simulations focused on a system with a flat neutral sheet while synchrotron losses were neglected. For the case $qA < 0$ it was found that in the specified magnetic field configuration convection and drift transport low and intermediate energy particles in opposite directions, thereby leading to longer residence times. As a result, these particles suffer more adiabatic losses compared to a scenario where drift is absent. By contrast, the high-energy particles are primarily transported by diffusion. This transition from a drift/convection dominated system to a diffusion dominated system with increasing energy occurs gradually, leading to the formation of depression-like feature in the spectra. This behavior of the spectra is present at all latitudes and radial distances, but is particularly pronounced at large radial distances in the equatorial regions (where the neutral sheet is located).

For the case $qA > 0$ it was found that particles drifting outward along the neutral sheet will suffer fewer adiabatic losses, leading to the formation of a peak-like structure in the equatorial spectrum at lower and intermediate energies. At higher energies diffusion again becomes the dominant transport mechanism, leading to a softening of the spectrum. In contrast to $qA < 0$, the effect of drift on the spectra is only important in the equatorial region. At higher latitudes the spatial evolution of the spectra was found to be similar to that of a system where drift is absent.

A more realistic scenario with $\alpha = 30^\circ$, where α is the angle between the rotation and magnetic axes of the pulsar, was also investigated. When $\alpha > 0^\circ$ the neutral sheet is no longer flat, but has a wavy structure. It was found that this modification greatly reduces the effect of drift on the spatial evolution of the spectra, indicating that neutral sheet drift is considerably more important than gradient and curvature drift.

These results may be of importance when protons and anti-protons are present in the PWN as these particles do not suffer significant synchrotron and inverse Compton losses. However, as mentioned above, the effect of drift becomes less important as the value of α increases.

For the second set of simulations synchrotron losses were included, while the value $\alpha = 30^\circ$ was again used. Although the drift-related spectral features that develop in the absence of synchrotron losses were present in the spectra, these features are largely masked by the effect of synchrotron losses. The reason is that the diffusion and drift currents are proportional to

momentum p , whereas the synchrotron loss rate is proportional to p^2 . However, when $qA < 0$ it was found that the longer residence time causes the particles to suffer more synchrotron losses when compared to a scenario where drift is not present. On the other hand, when $qA > 0$ the energy loss suffered by the particles due to synchrotron radiation is reduced as a result of a shorter residence time. Drift therefore leads to $qA < 0$ and $qA > 0$ spectra that are clearly distinguishable from each other.

As mentioned previously, of the two drift processes neutral sheet drift plays the largest role. A scenario with neutral sheet drift (artificially) neglected was therefore investigated, and it was found that the effects of drift are strongly reduced. Nevertheless, a quantitative difference still remains between the $qA < 0$ and $qA > 0$ spectra when synchrotron losses are important. The main result of the simulations is thus that the spectra of the particles and anti-particles evolve differently in space, regardless of whether synchrotron losses are neglected or included.

This asymmetric development of the particle and anti-particle spectra has some implication for the positron excess observed in the heliosphere (Adriani et al. 2009; Ackermann et al. 2012; Aguilar et al. 2013). Büsching et al. (2008), Yüksel et al. (2009), and Linden & Profumo (2013), among others, have suggested that this excess may result from leptons escaping from the nearby Geminga PWN and Monogem pulsar. Although the Monogem pulsar has no PWN associated with it, Büsching et al. (2008) have suggested that it is possible for the Monogem PWN to have already broken up. Although the above-mentioned models do not require a different evolution of the electron and positron spectra in the PWN, the present results suggest that a positron excess may already be present when the leptons escape from the PWN (this is specifically for the case where the angle α between the rotation and magnetic axis is $\alpha < 90^\circ$, i.e., $A > 0$, so that the $qA > 0$ spectra in Figure 3 represent the positron spectra).

There is, however, a caveat that should be kept in mind: drift will only be effective as long as the magnetic field has a clearly defined structure. The MHD simulations of Porth et al. (2014) predict that this should indeed be the case in the inner parts of the nebula, where an Archimedean structure is to be expected. By contrast, the magnetic field becomes more randomized in the bulk of the nebula, and the effect of drift would therefore be reduced.

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