The performance of conditional heteroskedastic VAR enhanced Multivariate GARCH models on the time varying integrated data

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DECLARATION

I, Lebotsa Daniel Metsileng, hereby declare that this research report titled: "The performance of conditional heteroskedastic VAR enhanced MGARCH models on the time varying integrated data," is my own original work and that all the sources are have been accurately reported and acknowledged, and that this has not been submitted in part or full for any degree at any other university.

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LD Metsileng **Date**

DEDICATION

I would like to dedicate this work to God who showered me with his grace to complete this study and my parents, my mother Annah Mmaphokwana Metsileng and my late father Lerothodi Jacob Metsileng.

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ABSTRACT

The study investigated the performance of conditional heteroskedastic vector autoregressive (VAR) enhanced Multivariate GARCH models on the time varying integrated data. These models allow the conditional-on-past-history covariance matrix of the dependent variables to follow a flexible dynamic structure. The study evaluated the levels of interdependence and dynamic linkage among the BRICS financial markets (in particular exchange rates) using appropriate univariate and multivariate time-series models.

The study employed the monthly time series data of the BRICS exchange rates ranging from January 2008 to January 2018 and it has 121 observations. The base model used in the study was a VAR model, an ARCH model was fitted with the effects the model presented. Subsequently an extension of ARCH model, which is GARCH, was considered together with its multivariate settings. The focus of the study was to estimate the VAR enhanced Multivariate GARCH using the BEKK and DCC approach on the BRICS exchange rates. The study took a guide from some studies as presented in the literature.

All the statistical properties necessary to test prior to engaging further with the analysis were satisfied. The VAR (1) model was fitted and the parameters were estimated. The results revealed that a linear dependency between the BRICS exchange rates existed. All the linear dependencies took one direction. The squared BRICS exchange rates illustrated the presence of serial correlation and that the ARCH errors were present in the BRICS exchange rates. The LM test for the ARCH model strongly showed the presence of heteroskedasticity of errors for GARCH model for the five countries.

The univariate GARCH (1.1), EGARCH (1.1) and TGARCH (1.1) models for the BRICS exchange rates were fitted to the data and all followed a normal distribution. All the three models were fitted using Student t-distribution (*std*). The GARCH (1.1) model found the unconditional volatility for each of the BRICS exchange rates series. EGARCH (1.1) and TGARCH (1.1) models on the other hand presented the leverage effect. The EGARCH (1.1) model illustrated that the asymmetric effects dominate the symmetric effects except for South Africa as opposed to the TGARCH (1.1) model where the symmetric effects dominates the asymmetric effects. The estimated leverage effect (γ) for all the BRICS exchange rates proved that the bad news has no effect to the volatility as compared to the remaining BRICS exchange rates.

Multivariate GARCH using the BEKK estimates of the diagonal parameters showed that only Russia and South Africa were statistically significant which implied that the conditional variance of Russia and South Africa's exchange rates are affected by their own past conditional volatility and other BRICS exchange rates past conditional volatility. On the other hand, VAR enhanced Multivariate GARCH using the BEKK estimates of the diagonal parameters, showed that only the conditional variance of Brazil, China, India and Russia's exchange rates are affected by their own past conditional volatility and other BRICS exchange rates past conditional volatility. Both methods revealed that there are no spill-over effects in the BRICS exchange rates. The negative impact each of the BRICS exchange rates had did not affect other BRICS exchange rates.

The BEKK-GARCH revealed that only one pair (Russia and South Africa) had a bidirectional volatility transmission whereas on the VAR enhanced BEKK-GARCH did not reveal any bidirectional volatility transmission between the BRICS exchange rates. The BEKK-GARCH model demonstrated the presence of autocorrelation in the residuals while the VAR enhanced BEKK-GARCH model demonstrated the absence of the autocorrelation in the residuals. This implied that the VAR enhanced BEKK-GARCH model was well specified.

The DCC-GARCH model did not follow a normal distribution whereas the VAR enhanced DCC GARCH model follows a normal distribution with some extreme tails. Moreover, the DCC-GARCH revealed that Brazil, China, Russia and South Africa had the highest volatility persistence and India had the least volatility persistence as opposed to the VAR enhanced DCC-GARCH model which revealed that India had the highest volatility persistence followed by Brazil, Russia and South Africa and China with the least volatility persistence. The study contributes to the knowledge base the fresh discussion on the performance of Multivariate GARCH processes and the assessment of the performance of the conditional heteroskedastic VAR enhanced Multivariate GARCH model on the time varying integrated data. Recommendations for further studies were also provided.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF ACRONYMS

ADF	Augmented Dickey Fuller
AIC	Akaike Information Criteria
AR	Autoregressive
ARCH	Autoregressive Conditional Heteroskedasticity
ARMA	Autoregressive Moving Average
BEKK	Baba, Engle, Kraft and Kroner
BHHH	Berndt, Hall, Hall and Hausman
BRICS	Brasil, Russia, India, China and South Africa
CCC	Constant Conditional Correlation
DCC	Dynamic Conditional Correlation
EGARCH	Exponential Generalized Autoregressive Conditional Heteroskedasticity
EWMA	Exponentially Weighted Moving Average
FPE	Final Prediction Error Criterion
GARCH	Generalized Autoregressive Conditional Heteroskedasticity
GDP	Gross Domestic Product
GLS	Generalized-Least-Squares
HQIC	Hannan-Quinn Information Criterion
IGARCH	Integrated Generalized Autoregressive Conditional Heteroskedasticity
IV	Instrumental Variable
LIFFE	London International Financial Futures and Options Exchange
LM	Lagrangian Multiplier
MGARCH	Multivariate Generalized Autoregressive Conditional Heteroskedasticity
MLE	Maximum likelihood Estimation
MOU	Memorandum of Understanding
OECD	Organisation for Economic Cooperation and Development
OLS	Ordinary Least Squares
PP	Phillips and Perron
SC	Schwarz Information Criterion
TGARCH	Threshold Generalized Autoregressive Conditional Heteroskedasticity

VAR	Vector Autoregressive
VCC	Varying Conditional Correlation
VECM	Vector Error Correction Model

CHAPTER 1
OVERVIEW OF THE STUDY

1.1 INTRODUCTION

The study investigated the performance of conditional heteroskedastic vector autoregressive (VAR) enhanced Multivariate Generalized Autoregressive Conditional Heteroskedasticity (Multivariate GARCH) models on the time varying integrated data. Time varying Multivariate GARCH models allow the conditional-on-past-history covariance matrix of the dependent variables to follow a flexible dynamic structure. Multivariate GARCH models implement diagonal VECH and conditional correlation models. Conditional correlation models use nonlinear combinations of univariate GARCH models to represent the conditional covariances. Multivariate GARCH models provide estimators for three popular conditional correlation models, namely: Constant Conditional Correlation (CCC), Dynamic Conditional Correlation (DCC), Varying Conditional Correlation (VCC) which is also known as constant, dynamic, and varying conditional correlation. Mikkonen (2017) indicated that there is an emergence of material to support the existence of time-varying conditional correlation.

The current study explores the suitability of VAR enhanced Multivariate GARCH approach in investigating the dynamic nature of the relationships among the BRICS exchange rates. The study also determines the levels of interdependence and dynamic linkage among the BRICS financial markets using appropriate univariate and Multivariate time-series models. The volatility and interdependence in the BRICS exchange rates play a key role in inter-trade relations. According to Wang and Zivot (2006), interdependence is referred to as “an observed behavioural pattern on a variable due to the influence of another variable”. The behavioural pattern is modelled by a VAR model. VAR models are basically Multivariate extensions of univariate autoregressive (AR) models and are useful in examining the dynamic behaviour and interdependence of financial time series by modeling the conditional mean of time series data.

The VAR model is only effective in modeling the mean or the first order moment of the series (Sims, 1980). It creates a better understanding of the series, modeling and forecasting volatility. VAR models assume a constant one-period forecast variances. In order to generalise the constant one-period forecast variances, Autoregressive Conditional Heteroskedasticity (ARCH) was then introduced by Bachelier (1900) followed by a period of long silence. The concept was however revived by Engle (1982) and Bollerslev (1986), who formally formulated a model to capture all the earlier stylized facts as proposed by Bachelier (1900). ARCH refers to the phenomenon of Conditional Heteroskedasticity in general including all models to capture this phenomenon, and hence does not refer only to Engle's original model. In this research, the focus was to provide an account of recent theoretical advances in VAR-Multivariate GARCH models and their applications in macroeconomic and financial time series. The VAR-Multivariate GARCH model does not only focus on the first moments of the variables, but it also looks at the volatility transmission between the markets (Khalid and Rajaguru, 2006). The VAR model only describes the conditional means, while the Multivariate GARCH model describes the conditional variances. The VAR-Multivariate GARCH framework models conditional variances of the variables in the VAR specification. Application of Multivariate GARCH models is more evident in asset pricing and allocation. Asset pricing is dependent on the covariances of assets in a portfolio, while asset allocation relates to optimal hedging ratios (Anıl, 2008). The model was occasionally applied in financial markets.

1.2 BACKGROUND AND CONTEXT

Lama *et al*. (2015) highlighted that "The Vector Autoregressive (VAR) model is used for modeling the mean or the first order moment of the series".

ARCH model has some drawbacks such as high number of unknown parameters and rapid decay of unconditional autocorrelation function of squared residuals among others. Bollerslev (1986) in countering the above drawback of ARCH, proposed the GARCH model in which conditional variance is also a linear function of its own lags. This model is also a weighted average of past squared residuals, but it has declining weights that never go completely to zero. It gives flexible lag structure and it permits more prudent descriptions in most of the situations. The ability of GARCH model to capture volatility has been widely studied in literature (Lama *et al*., 2015). The Exponential Generalized Autoregressive Conditional Heteroskedasticity (EGARCH), according to Nelson (1991), enables the conditional variance to respond to positive and negative residuals asymmetrically. The issue of proper modeling of the long-run dependencies in the conditional mean of macroeconomic and financial time series led to the formulation of the Integrated Generalized Autoregressive Conditional Heteroskedasticity (IGARCH) by Engle and Bollerslev in 1986. IGARCH models possess many of the features of the unit root processes for the mean.

There are a number of other models where the conditional variance not only depends on the past variance, but is also subject to random noise. For instance, Ait-Sahalia and Kimmel (2006) used a continuous time stochastic asset prices volatility model. Engle and Kroner (1995) introduced a Multivariate structure of GARCH model known as BEKK (Baba, Engle, Kraft and Kroner) model. The model is the direct generalization of univariate GARCH model and is more flexible. A relatively flexible approach known as CCC model which allowed for combination of univariate GARCH models was developed by Bollerslev (1990). The CCC model has an assumption of constant correlation among the series over time. It is also important to note that only the conditional standard deviation is time-varying. Engle (2002) proposed a new class of Multivariate GARCH model known as DCC model which is the extension of the CCC GARCH model. The DCC has the flexibility of the univariate GARCH models coupled with parsimonious parametric model for the correlations. The conditional correlation matrix in DCC GARCH is designed to vary over the time. The use of these models for modeling the degree of interactions among various volatile commodities and markets can be widely seen in literature (Chevallier 2012, Lean and Teng, 2013 and Lin and Li, 2015).

Tse and Tsui (2002) and Bae *et al*. (2003) are of the view that Multivariate GARCH models are suitable for the analysis of volatility and correlation transmission. Conditional variance and covariance adds a bit of flexibility flavor in its dynamic nature and it is expected that the Multivariate GARCH model ought to be more flexible. It is also very essential that the Multivariate GARCH model must make sure that it maintains the positive definiteness of the conditional covariance matrix.

1.3 PROBLEM STATEMENT

The study hoped to build on previous studies conducted to look into the performance of VAR enhanced Multivariate GARCH models on the time-varying integrated data specifically on BRICS exchange rates. The above models were chosen as they are able to deal with data containing heteroskedastic problems as it is a problem contained in exchange rates. The VAR enhanced Multivariate GARCH integration resulted from the aspects brought about by the two methods. VAR model assists in determining the causality relationship and the Multivariate GARCH models take into account the heteroskedastic property of the variance and covariance. Furthermore, the approach allows for the evaluation of the presence of nonlinearity in financial data as well as volatility clustering and heteroskedasticity, which are some of the features in financial time series. Serrano (2009) opined that Multivariate GARCH approach considers volatility spill-overs between markets and assets since conditional covariances and variances are also estimated.

The buying power of the BRICS countries is dependent on the set exchange rates and inter government trade which is also influenced by the exchange rates. BRICS has a set Memorandum of Understanding (MOU) governing their market efficiencies. In the main exchange rates, data are volatile in nature and therefore the variance and covariance ought to be included in modeling any volatility data. The VAR enhanced Multivariate GARCH concept is utilized where series are modeled taking into consideration the changing variances at different time points. The exchange rates change at different time intervals and therefore the VAR enhanced Multivariate GARCH is the most appropriate model to be used.

The exchange rates in foreign economies are regarded as the most liquid of all the asset market. This is because the exchange rates play a major role in all trades involving the cross border trading, more specifically in the BRICS economies. It is therefore important that there is cooperation and Memorandum of understanding (MOUs) among inter trading countries signed to regulate trade. The signed MOUs open for interdependence among financial markets and bring about possible gains of two or more interrelated countries. Losses may be contained by taking into account time-varying variance and covariances. MOUs, therefore, create a clear understanding and linkages between different economies. Exchange rates increase the will for the BRICS countries to work together to formulate and implement relevant policies that helps in governing trade.

1.4 AIM OF THE STUDY

The aim of the study is to determine the levels of interdependence and dynamic linkage among the BRICS financial markets (in particular exchange rates) using both univariate and Multivariate time-series models. The study sought to explore the performance of the conditional heteroskedastic the VAR enhanced Multivariate GARCH models on the time varying integrated data.

1.5 RESEARCH OBJECTIVES

The main objective of this study was to determine the levels of interdependence and dynamic linkage among the BRICS financial markets using appropriate univariate and Multivariate time-series models. The specific objectives were as follows:

- To review and determine the statistical properties of the main time-series models.
- To identify appropriate Multivariate GARCH models for the BRICS exchange rates.
- To estimate VAR-Multivariate GARCH models to the BRICS exchange rates.
- To review and determine the most appropriate VAR-Multivariate GARCH model to the BRICS exchange rates.
- To provide recommendations based on the findings.

1.6 RESEARCH QUESTIONS

The research questions of the study were stated as follows:

- What are the statistical properties of the main time-series models?
- Which appropriate Multivariate GARCH models are suitable for the BRICS exchange rates?
- What are the estimates of the VAR-Multivariate GARCH model to the BRICS market exchange rates?
- What is the most appropriate VAR-Multivariate GARCH model to the BRICS market exchange rates?
- What are recommendations based on the findings?

1.7 IMPORTANCE OF THE STUDY

Given the importance of predicting volatility in macroeconomic and financial time series, many approaches have been proposed in the literature. Notable among them was the class of ARCH processes originally introduced by Engle (1982). In many macroeconomic and financial time series data analyses, the general assumption of constant variance in the disturbance term was violated. The Multivariate GARCH concept was utilized where series are modeled taking into consideration the changing variances at different time points. The exchange rates change at different time intervals, and therefore, Multivariate GARCH was the most appropriate model to be used. As many methods have been proposed in the literature, it is equally important to determine the performance of these methods, focusing only on Multivariate GARCH process. The study seeks to determine the performance of the conditional heteroskedastic VAR enhanced Multivariate GARCH model on the time varying integrated data.

1.8 DELIMITATIONS OF THE STUDY

The study is based on the models originally built by old authors and will have some of the old literature cited as the base for the key theoretical literature. The data used in the study is secondary in nature comprising of the BRICS exchange rates. The data period ranges from January 2008 until January 2018.

1.9 DEFINITION OF TERMS

- BRICS stands for Brazil, Russia, India, China and South Africa
- Vector Autoregressive (VAR)- The VAR model is used for modeling the mean or the first order moment of the series (Lama *et al*., 2015).
- Autoregressive Conditional Heteroskedasticity (ARCH)-As proposed by Nobel laureate Robert Engle in 1982, an ARCH model starts from the premise that we have a static regression model and nonlinear.
- GARCH - allows for asymmetry, or considers nonlinearities in the process generating the Conditional variance
- The Multivariate GARCH models- It is the model that allows the conditional covariance matrix of the dependent variables to follow a flexible dynamic structure and allows the conditional mean to follow a vector autoregressive (VAR) structure.
- Constant Conditional Correlation (CCC)- It is the model that uses nonlinear combinations of univariate GARCH models to illustrate the conditional covariances (Bollerslev, 1986).
- Dynamic Conditional Correlation (DCC)- the diagonal elements of S_t are modeled as

univariate GARCH models.

- Volatility- It is a statistical measure of dispersion around a mean value; or is defined under a theoretical aspect as the changeability or randomness of the underlying asset (Schwert, 1990).

1.10 RESEARCH OUTLINE

Chapter 1 gave the introduction and background of the study, problem statement, aim and objectives of the study. The chapter further looked into the research questions, research methodology, contribution of the study, brief literature review, limitations/delimitation of the study, and definition of terms.

Chapter 2 provides the review of empirical literature. The focus is on the VAR, ARCH, GARCH and the Multivariate GARCH including the enhanced VAR- Multivariate GARCH models.

Chapter 3 discusses the research methodology based on VAR, ARCH, GARCH, VAR Multivariate GARCH and VAR enhanced Multivariate GARCH models. The chapter included the ethical considerations the study undertook and the study philosophy.

Chapter 4 presents the data analysis and interpretation of results in order to achieve the objectives as set in chapter one. The chapter also presents different methods using the BRICS data applied to each method.

Finally, Chapter 5 discusses the conclusions and recommendations in relation to the set research objectives.

1.11 CHAPTER SUMMARY

The chapter gave the overview of the study covering the background and context of the study, the problem statement, and the aims and objective which gave guidance to the study. The chapter further highlighted the importance of the study.

The next chapter gives the literature review related to the study and it gives clear context of each model studied.

CHAPTER 2
LITERATURE REVIEW

2.1 INTRODUCTION

The chapter discusses the empirical literature of the different methods used in the analysis phase of the study. Gaps in the knowledge base are identified. It starts with the univariate approach and describes how it transforms to the Multivariate approach. The VAR enhanced Multivariate GARCH models are also discussed.

The rest of the chapter is organised as follows: In Section 2.2 VAR model is discussed, Section 2.3 looks at the GARCH models, Section 2.4 discusses all the Multivariate GARCH models including the VAR enhanced Multivariate GARCH models and lastly, Section 2.5 presents the chapter summary.

2.2 VECTOR AUTOREGRESSIVE (VAR) MODEL

Vector autoregressive (VAR) processes are well known in economics and other sciences since they are flexible and simple models for Multivariate time series data. Sims (1980) advocated for VAR models as alternatives since he questioned the way classical simultaneous equations models were specified and identified. This model is a generalization or natural expansion of the univariate autoregressive model to dynamic Multivariate time series. VAR has some very attractive features and has provided a valuable tool for analysing dynamics among time series processes (Adenomon *et al.*, 2013). A VAR model posits a set of relationships between lagged values of all variables and the current values of all variable in the system (McMillin, 1991) and (Lu, 2001).

The VAR model has turned out to be particularly helpful for describing the dynamic conduct of economic and financial time series and for forecasting. It regularly gives superior forecasts than those from univariate time series models and elaborate theory-based simultaneous equations models. Forecasts from VAR models are very flexible on the grounds that they can be made conditional on the potential future paths of specified variables in the model.

In addition to data description and forecasting, the VAR model is additionally utilized for structural inference and policy analysis. In structural analysis, certain assumptions about the causal structure of the data under scrutiny are imposed, and the subsequent causal impacts of unexpected shocks or innovations to specified variables on the variables in the model are summarized. These causal impacts are normally compressed with impulse response functions and forecast error variance decomposition.

As of late, because of its flexibility, VAR models are every now and again utilised for modeling economic and financial data. Furthermore, they have been utilised in many empirical studies of different discipline. VAR model was utilised to study Brazilian agricultural prices, industrial prices and money supply (Bessler, 1984); Estenson (1992) used VAR model to explore the dynamics of the Keynesian theory; Backus (1986) also applied the VAR model to elicit the empirical facts concerning the movement of the Canadian-U.S exchange rate; Enders and Sandler (1993) used VAR and Intervention analysis to study various attack modes used by transnational terrorists; Freeman *et al.* (1989) compared VAR model and familiar Structural equation (SEQ) to study politics. Bagliano and Favero (1998) applied the VAR model to measure monetary policy as an evaluation. In fact the empirical literature of VAR process is numerous.

Athanasopoulos *et al.* (2011) conducted a study in which a joint determination of the lag-length, the dimension of the cointegrating space and the rank of the matrix of short run parameters of VAR model using model selection criteria. Monte Carlo simulations were used to measure the improvements in forecasting accuracy. The study applied two empirical of Brazilian inflation and U.S. macroeconomic aggregates growth rates respectively and the results shown the usefulness of the model-selection strategy proposed in the study.

VAR model was utilised to examine the dynamic relationship between rainfall and temperature time series data in Niger State, Nigeria. The data used was of the Meteorological station covering periods January 1981 to December 2010. The impulse response function and the forecast error variance decomposition were further used to interpret the VAR model. The AIC and HQIC selected lag eight for the VAR model. The results showed that modeling rainfall and temperature together in Niger State will improve the forecast of rainfall and temperature respectively (Adenomon *et al*., 2013). VAR model has had numerous successes in the modeling and forecasting of time series data.

Eklund (2007) considered modeling and forecasting Icelandic business cycles. The study used the VAR model to model the general business cycle. The method of selecting monthly variables, coincident and leading, that mimic the cyclical behaviour of the quarterly GDP is described. Using the estimated VAR model bootstrap forecasting procedure is applied, point and interval forecasts of the composite coincident are estimated.

The impact of oil prices on BRIC real returns for the period 1999 to 2009 were examined by Ono (2011) using the VAR model. The findings of the paper revealed that there is a positively significant different response of the oil price indicators to Russia, India and China. The results further revealed a significant asymmetric effect of oil shocks on Indian returns.

Basci and Karaca (2013) examined the relationship between ISE 100 Index and a set of four macroeconomic variables using VAR model. The variables used in the study are Exchange, Gold, Import, Export and ISE 100 Index. In the study 190 observations were used for the sample period from January, 1996 to October, 2011. After determining optimal lag order, it was given one standard deviation shock for each series and their response. However, in variance decomposition carried out subsequently, it has been determined that especially as of the second default of exchange, it was explained 31% by share indices.

The study by Chamalwa and Bakari (2016) investigated the relationship between economic growth (GDP) and some financial deepening indicators (money supply and credit to private sector), using a data obtained from the Central Bank of Nigeria (CBN) statistical bulletin for the period 1981-2012. The study used VAR cointegration and Vector Error Correction Model (VECM) approach. The results indicated that all the three variables are non-stationary at levels, but became stationary after first differencing once. The VAR(1) was selected as the optimum length. The three variables are cointegrated with at most one cointegrating equation; b-bidirectional causality runs among the three variables. The VECM model found a long run relationship amongst the three (Chamalwa and Bakari, 2016).

The study by Enisan and Olufisayo (2009) studied the long run and casual relationship between stock market performance and economic growth from seven sub-Saharan Africa. The study found the presence of a bidirectional relationship between the development of stock markets and economic growth for Cote D'lvoire, Kenya, Morocco and Zimbabwe.

The stock market indexes of South Africa, India and the USA is explored to see if there is any association and existence of short run and long run relationships between them. Monthly data of stock indexes of JALSH (S.A), NIFTY (India) and NASDAQ (USA) is used covering the period of April 2004 to March 2014. The lag length of order one was selected by Final Prediction Error criterion (FPE), the Akaike information criterion (AIC), the Schwarz information criterion (SC) and the Hannan-Quinn information criterion (HQ). According to the VAR model ,the results obtained shows that USA and the South Africa stock markets are predicted by their own past lags (Mohanasundaram and Karthikeyan, 2015).

Ijumba (2013) studied the Multivariate analysis of the BRICS financial markets using the BRICS weekly returns ranging from January 2000 to December 2012. The VAR model was used to determine the linear dependency among the BRICS markets. The study fitted the VAR model with lag length of order one selected by AIC, HQ and SC. The VAR model revealed that there is an evidence of unidirectional dependency of the Indian and Chinese markets on the Brazilian market. However, the study did not forecast the BRICS markets since the VAR(1) model did not pass the any of the diagnostic tests.

2.3 GENERALIZED AUTOREGRESSIVE CONDITIONAL HETEROSKEDASTICITY (GARCH) MODEL

The GARCH models have proven to be able to model conditional volatility and improve the forecasting accuracy of the future volatility of many financial time series (Goodwin, 2012). Engle (1982) introduced the autoregressive conditional heteroskedasticity (ARCH) model to model volatility. Engle (1982) modeled the heteroskedasticity by relating the conditional variance of the disturbance term to the linear combination of the squared disturbances in the recent past. Bollerslev (1986) generalized the ARCH model by modeling the conditional variance to depend on its lagged values as well as squared lagged values of disturbance, which is called generalized autoregressive conditional heteroskedasticity (GARCH). Furthermore, there are other variants of GARCH model.

According to Teräsvirta (2009), modeling volatility is important in asset returns as volatility is considered a measure of risk, and investors wants a premium for investing in risky assets. Goyal (2000) examined various GARCH models for stock market data in terms of their ability of delivering volatility forecasts. Based on the in sample test on actual volatility produces R^2 of less than 8% based on regression. Overall results show that GARCH-M model is outperformed by a simpler ARMA model.

Time series data has a major research problem which is multistep prediction. The challenges encountered are maintaining high prediction accuracy and preserving the data trend across the forecast horizon. The study by Babu and Reddy (2015) proposed a linear hybrid model to address the challenges in TSD. Incorporated in the model are moving average filter based pre-processing, partitioning and interpolation together with quantitative reasoning analysis for justifying the accuracy of the proposed model. The data used in the study was selected from NSE Indian stock market. The proposed linear hybrid model outperforms the other models in terms of prediction accuracy and preserving data trend according to the performance results

Ahmed and Suliman (2011) estimated volatility in the daily returns of the principal stock exchange of Sudan-Khartoum stock exchange (KSE) covering the period of January 2006 to November 2010 using GARCH models. Volatility clustering and leverage effect of index return are captured using both symmetric and asymmetric models. The empirical result showed that the asymmetric models perform better than the symmetric models and they confirm the presence of leverage effect. The overall results revealed that there is a presence of high volatility on index return series in Sudanese stock market over the sample period.

Predescu and Stancu (2011) examined the portfolio risk using ARCH and GARCH models based on the context of current global financial crisis the benefit of choosing an internationally diversified portfolio and the evaluation of the portfolio risk. The three benchmarking indexes from Romania, UK and USA to comprise the portfolio. The results show that international diversification does not reduce risk. Furthermore, ARCH and GARCH models show that evolution of portfolio volatility is as a result of the current global financial crisis. Bala and Asemota (2013) examined exchange-rate volatility with GARCH models for Naira/US dollar return and Naira/British pounds and Naira/Euro returns on a monthly series from 1985:1 to 2011:7 and 2004:1 to 2011:7 respectively. The results reveal the presence of volatility in the three currencies.

Eryılmaz (2015) utilized ARCH, GARCH, EGARCH and TARCH models in modeling and analysing stock return volatility for BIST-100 using a monthly data corresponding to the period of 1997:01 to 2015:03. The results showed that EGARCH (1.1) model is the suitable model in modeling BIST-100 series. Miron and Tudor (2010) used asymmetric GARCH-family models of EGARCH, PGARCH and TGARCH to model volatility estimates using U.S and Romanian daily stock return covering the period of 2002-2010. Based on the results, EGARCH model exhibit the ability to give accurate estimates than the competing models.

In a panel of nineteen of the Arab countries, the study by Abdalla (2012) applied the generalized autoregressive conditional heteroskedastic approach in modeling the exchange rate volatility using daily observations over the period of 1st January 2000 to 19th November 2011. In capturing volatility clustering and leverage effect, the study applied both symmetric and asymmetric models. The results of explosive process of volatility were found based on the estimated GARCH(1.1) model. Furthermore, EGARCH(1.1) shows that there is an evidence of leverage effect for majority of currencies. Overall results show that GARCH model can adequately model exchange rate volatility (Abdalla, 2012).

Wennström (2014) compared the volatility models in terms of the in-sample and out-of-sample fit. In terms of the in sample, the results showed that assuming heavier tail error distribution than normal distribution significantly improves the fit. Furthermore, it was also found that those complex models are better than the parsimonious models in terms of in sample fit. In the out of sample, the results was inconclusive, choosing adequate loss function is important.

Chang *et al*. (2015) compared the ARIMA-GARCH model to the other time series models that have normal innovation. The study aimed to develop the early warning signal models using average value at risk (AVaRs) based on the ARIMA-GARCH model. The empirical results showed that estimating AVaRs for the AVaRs for the ARIMA-GARCH model offers an improvement over prevailing models and provides a suitable warning signal in both extreme events and highly volatile markets using the daily Dow Jones industrial average index, the England financial time stock exchange 100 index and the Japan Nikkei 225. Furthermore, the study by Dhamo *et al*. (2012) evaluated the ARIMA and GARCH models in terms of the prediction of telephone networks and also shows they are advantages and disadvantages of the models in a telephone company in Albania. The data used was a daily data covering a period of 1st January 2009 to 31st May 2011 and all the analysis were done in R environment (Dhamo *et al*., 2012).

Zeitlberger and Brauneis (2016) analysed carbon spot price returns in the European Union Emission Trading scheme for the period of 2008-2012. The study looked to develop an empirical model that can be used to capture the behaviour of carbon price returns. The study applied a broad spectrum of GARCH specifications together with Markov regime switching models for variance equation into consideration. According to the empirical results it is shown that the AGARCH, NARCH and GJR fit the data best.

Goodwin (2012) used the $\text{GARCH}(1.1)$, the GJR $\text{GARCH}(1,1)$ and the $\text{QGARCH}(1.1)$ models in modeling the in-sample and out-of sample volatility of copper spot price returns within the period of 21st July 1993 to 22nd March 2012. The empirical results show that the GARCH models have a satisfactory ability to model both the in-sample and out-of-sample in which case it dominated a random walk model in out-of-sample modeling with a lower mean of absolute errors.

Mwita and Nassiuma (2015) used GARCH model in examining the nature and characteristics of stock market volatility of Kenyan stock markets and its stylized facts. The GARCH (1.1) model explains the volatility of Kenyan stock market and its stylized facts including volatility clustering, fat tails and mean reverting more satisfactory. The overall results depict the evidence of time varying stock return volatility over the sample period. Negative return shocks have a higher volatility than the positive returns shocks.

The paper by Hou and Suardi (2012) modelled and forecasted oil price return volatility using an alternative approach involving nonparametric method using two crude oil markets Brent and West Texas Intermediate (WTI). The results show that the out-of-sample volatility forecast of the nonparametric GARCH models are superior to that of parametric GARCH models (Hou and Suardi, 2012).

E. (2012) estimated both the univariate and Multivariate GARCH models in order to examine which provide the better performance in estimating the Value at Risk of the portfolio using daily returns of the portfolio consisting of the five Colombian financial assets. According to the results, the univariate GARCH model outperforms the Multivariate model in estimating the VaR of the portfolio.

Wang and Wu (2012) used univariate and Multivariate GARCH-class models in forecasting energy market volatility. Based on the results, it is evident that the univariate GARCH models allowing the asymmetric effects display the greater accuracy. Sjöholm (2015) aimed in fitting and comparing the six different classes of heteroskedasticity models in terms of forecasting accuracy of the two different markets: equity and exchange rate. The study forecasted the series to 100 days ahead using MSE as the measurement of error. Based on the results it is evident that the results do not differ much between the chosen models. The study showed that the model provides a very good factors in terms of size, momentum, liquidity and volatility factors (Sohn, 2010).

Obeng (2012) employed ARCH, GARCH and EGARCH models to assess the predictive accuracy of the models in forecasting exchange rate volatility of Canadian dollar, Euro, British pound, Swiss franc and Japanese Yen against a base currency in US dollar. The tests were based on both the in-sample and out-of-sample forecasting. Based on the results the estimated GARCH(1.1) model outperformed all the included models in the in-sample performance, however in terms of the out-of-sample performance the results were inconclusive. At some instance the ARCH model performed. This can be a robust decision that simple models can be given preference in some cases. The study employed GARCH model and estimated the volatility using a historic data and EWMA model to the stock data of PetroChna and TCL on the Shanghai and Shenzhen Stock Exchange Market of China and the results were assessed on the mean square error (Guo, 2012).

Grek and Mantalos (2012) conducted a study to find the best heteroskedasticity model in terms of best forecasting accuracy of the stocks from the Swedish stock market. MSE is employed as a measure for the performance of the models. Based on the results, it is concluded that the stock market with the higher kurtosis were forecasted better by GARCH model and stocks with the lower kurtosis were forecasted better using EGARCH model (Grek and Mantalos, 2014). Wei *et al*. (2010) captured the volatility features of the two crude oil markets: Brent and the WTI using a set of linear and nonlinear GARCH models based on the one, five and twenty day out-of-sample volatility forecast. The performance of the models was evaluated based on the predictive ability test and with more loss function. In general, it is evident that the nonlinear models exhibit a greater forecasting ability than the linear models.

The study by Gabriel (2012) assessed the prediction accuracy of the GARCH-family models in terms of in-sample and out-of-sample forecasting using the daily BET stock index returns series covering the period of 09-03-2001 to 02-29-2012. The empirical results found that TGARCH models is the successful model in terms of forecasting volatility of BET index. Ahmad and Ping (2014) employed symmetric GARCH models (GARCH and GARCH-M) and asymmetric models (TGARCH and EGARCH) in modeling Kijang Emas using model selection criteria of AIC and SIC. Based on the results it is found that TGARCH is the best model to fit the Kijang Emas.

Mokoma and Moroke (2014) used exchange rate, gross domestic product, inflation rate and interest rate in constructing ARCH(1) model, GARCH(1.1) and GARCH(1.2) which were applied in assessing exchange rate volatility in S.A. The study used a time series quarterly data covering the period of 1990: Q1 to 2014: Q2. Based on the results the GARCH(1.1) model was found to best fit the data and it was used for out-of-sample forecasting.

Mathoera (2016) employed GARCH (1.1), TGARCH and EGARCH in predicting option price. The results obtained reveal that GARCH (1.1) model is best in predicting AEX index and TGARCH outperforms the other model in predicting the S&P 500 index. Similarly, Li (2012) examined the performance of in-sample and out-of-sample forecasting for the risk neutral measures using TGARCH, NIG TGARCH with MCMM, NIG TGARCH with Ess and Gaussian-NIG TGARCH with 2nd Ess. The study found that the best model is the Gaussian TGARCH model in terms of in-sample-forecasting of the S&P 500 for both the 2002 and 2004. However, in the out-of-sample forecasting the NIG TGARCH model outperformed the two models.

Paradza and Ericschaling (2015) employed GARCH, EGARCH and TGARCH to model volatility of index return on the Zimbabwe Stock exchange dividing the data into two sets of currency reforms. The first part of the data ranges from January 2004 to April 2008 (pre dollarization period) and the second part ranges from February 2009 to December 2013 (post currency reform). Based on the results, the TGARCH model is able to capture the negative monthly effect.

Atoi (2014) examined the most appropriate error distribution in terms of selecting the best volatility model. A monthly series of all share index of Nigeria was analysed covering the period of January 2nd 2008 to February 11th 2013. The distributions considered are the normal, student's-t and generalized error distributions. The results reveal that the PGARCH (1.1.1) model is the best predictive model in out-of-sample when using the student's-t distribution based on RMSE and Theil inequality coefficient used as error measures.

The study by Li and Begum (2013) compared the GARCH family models of IGARCH, EGARCH, PGARCH, QGARCH and TGARCH with different distributions in forecasting volatility of stock market returns of Japan, U.S.A and Germany. For Japan, the series used is that of closing stock price of Nikkei 225, U.S.A is that of S&P 500 and Germany is DAX index consisting of daily data. The results found that QGARCH with student's t-distribution fit the Nikkei 255 index better, PGARCH with student's t-distribution fits the DAX index for forecasting volatility and overall the GARCH model with student's t-distribution is best in forecasting volatility.

2.4 MULTIVARIATE GARCH MODEL

Bala and Takimoto (2017) used the Multivariate GARCH model and its variants in investigating stock returns volatility spill-overs in emerging and developed markets. Furthermore, the global financial crisis (2007-2009) was analysed on stock market interactions and the BEKK-GARCH-type models is modified by including financial crisis dummies to assess their impact on volatility and spill-overs. The results showed an improved diagnostics with the DCC-with-skewed-t density model as compared to other models because financial returns often present fat tails and skewed features.

The study used data from a sample of selected Asian countries in an attempt to identify and trace the alleged origin and the subsequent path of the currency contagion. In terms of empirical estimation, daily observation (high frequency data) of exchange rate from 1994-2002 was used. The sample was split into four periods (full, pre-crisis, crisis and post-crisis). The empirical evidence shows that there was an increment of currency market links during and after the crisis. However, there is a weak support of the same in the pre-crisis period (Khalid and Rajaguru, 2006).

The study generated the conditional variances of monthly stock exchange prices, exchange rate and interest rates for Turkey using BEKK-GARCH model for the sample period of 2002:M1-2009:M1, before the effects of global economic crisis hit Turkey. According to the empirical results there is volatility among these three financial sectors and an indication of significant transmission of shocks (Türkyılmaz and Balıbey, 2014).

Minović (2017) reviewed both the theoretical and empirical for diagnostic checking of Multivariate volatility processes. The study for empirical analysis used the Ljung-Box statistics (Q-stat) of standardized residuals, those of its squared, as well as of the cross product of standardized residuals to check the model adequacy. The results show for model adequacy the residual-based diagnostics provide a useful check. Furthermore, based on the overall results models performed statistically well.

Harrathi *et al*. (2016) in their study implemented a combination of VAR and Multivariate GARCH models under BEKK specifications (VAR BEKK-GARCH) models with constant correlation (CCC) and dynamic constant correlations (DCC) for daily equity returns of six markets, namely Turkey, Indonesia, Egypt, Mexico, China and Brazil in investigating the volatility spill-over between equity market indexes for Islamic and Non-Islamic emerging countries. According to the results, among the Islamic and Non-Islamic countries there is a strong volatility spill-over in market returns.

Selmi and Hachicha (2014) in their study used VAR DCC-GARCH model regressions in examining the role of oil prices, financial and commercial linkages in the propagation of industrial market crisis during the period 2004-2012. The empirical results show that the European debt crisis has already the same as oil prices to Ireland and Portugal, this also poses risks to other countries: Spain being the probable for financial crisis.

The study by Bunnag (2016) employed four VAR-Multivariate GARCH models: namely VAR (2)-diagonal VECH, the VAR (2)-diagonal BEKK, the VAR (2) CCC and VAR (2) DCC in examining volatility transmission in the crude oil, gold, S and P 500 and USD Index futures. The data used is the daily data covering from 2010 to 2015. The empirical results show that the parameters of all the included models are statistically significant in different series that is used. Heracleous (2003) modified and extended the univariate and Multivariate volatility models viewed as alternative to the GARCH models by using the student's t distribution and follows probabilistic reduction (PR). Since the GARCH models formulations require ad hoc parameter restrictions, the modified and extended models will give rise to internally consistent statistical models.

Chang *et al*. (2012) examined the effectiveness of using the future contracts as hedging instruments using the four estimated Multivariate volatility models (CCC, VARMA-AGARCH, DCC and BEKK). The daily data used was that of major currencies Euro, British and Japanese Yen against the American dollar. The empirical results show that the CCC and AGARCH models show similar hedging effectiveness, there is also a suggestion that dynamic asymmetry may not be crucial empirically, the DCC and BEKK showed some differences.

Mohd *et al*. (2016) employed CCC, DCC and diagonal-BEKK models in identifying the relationship between spot and futures contract exchange rates and spot and forwards contract exchange rates. The daily data used was that of currencies within Asean and Asean+3 of closing prices of spot, futures and 3-month forwards contracts. Based on the empirical results CCC and DCC are the best model for hedging effectiveness. The paper obtained the closed-form expressions for the score of the BEKK model, furthermore the efficient computations are discussed (Lucchetti, 2002).

Optimal portfolio weights and optimal hedge ratios are calculated for the crude oil spot and futures returns of the two major benchmark international crude oil markets, Brent and WTI to suggest a crude oil hedge strategy in examining the performance of Multivariate volatility models CCC, VARMA-GARCH, DCC, BEKK and diagonal BEKK. The results showed that diagonal BEKK (BEKK) is the best (worst) model OHR calculation in terms of the reducing the variance of the portfolio (Chang *et al*., 2011).

Zhao (2010) used VAR and Multivariate GARCH models in analysing the dynamic relationship between real effective exchange rate and stock price. The data used was that of monthly series covering January 1991 to June 2009. The results showed that there exists not a long-term equilibrium relationship between real effective exchange rate and stock price. The study employed four Multivariate GARCH models in investigating volatility spill-over and the dynamics relationship between the stock price and currency markets in the Czech Republic, Poland, Hungary and Russia. According to the results the DCC-S generally yields effective diversification model, this implies that the effectiveness of diversification can improve significantly by using DCC-S (Lee *et al*., 2014).

Malo and Kanto (2006) considers in Multivariate GARCH models the variety of specification tests that are been employed for dynamic hedging in electricity market. Furthermore, hedging performance comparisons in terms of unconditional and conditional ex-post variance portfolio reduction are conducted. The study examined two models the ARMA-GARCH and ARMA-DCC GARCH model for the mean VaR optimization of funds managed by HFC investment limited. The weekly data of the above mentioned funds covering 2009 to 2012 was used. The results reveal that more efficient portfolio is obtained when the VaR is modelled with a Multivariate GARCH (Siaw, 2014).

Ijumba (2013) employed "VAR, univariate GARCH(1.1) and Multivariate GARCH models to investigate the levels of interdependence and dynamic linkages among the five emerging economies well known as BRICS". Based on the univariate GARCH(1.1) models, there is a suggestion of persistence of volatility among all BRICS stock returns as well as it was the case with the Multivariate GARCH model.

Lama *et al*. (2016) employed the VAR-Multivariate GARCH approach in an attempt to mode the volatility pulses prices. The study used employed the student-t distribution as a way of dealing with presence of excess kurtosis and furthermore the variates models of Multivariate GARCH: BEKK, CCC and DCC were also applied. The empirical results showed that the GARCH-DCC is the best model in modeling pulses prices series. Iltuzer and Tas (2012) employed a Multivariate GARCH model in attempt to analyse the bidirectional causal relationships between macroeconomic volatility and stock market volatility for some emerging markets. According to the analysis it is shown that investors followed some macroeconomic variables as a way of showing the riskiness of a particular country.

The study employed VAR model and other tests in order to study the co-movement between Shanghai and New York stock exchange as a way of studying the integration and spill-over effects between the two markets. Furthermore, the Multivariate GARCH model and MSV model were also applied in order to characterize the dynamics of volatility as due to the ARCH effect. The empirical results shows that there exists the spill-over effects (Liu, 2011).

The study by Zhou and Wu (2014) investigated price causal relationship and volatility spill-overs effects between the CSI 300 index futures and spot markets in China by employing the VAR-Multivariate GARCH models for the analysis. The data used was that of 5 min high-frequency data from 4 January 2013 to 31 October 2013. The study compared and contrasted four Multivariate GARCH models: BEKK, diagonal, CCC and DCC. The empirical results showed that the VAR-GARCH-DCC model outperforms the other models and show that there is existence of bidirectional price causal relationship between the CSI 300 index futures and spot markets and also volatility spill-over effects. Ku (2008) conducted a study on comparison on the hedging efficiency of hypothetical portfolios consisting of stock and currency futures in order to justify the DCC- GARCH model based on the student-t distribution.

Chevallier (2012) argued that the interrelations between energy and omissions markets should be modelled by the VAR and Multivariate GARCH model so that the dynamics correlations of CO_2, gas and oil can be reflected. The study employed BEKK, CCC and DCC-GARCH models on a daily data covering the period of April 2005 to December 2008. The study provided strong empirical evidence. Behera (2011) employed recently developed Multivariate GARCH model in examining the onshore-offshore linkages of the Indian rupee. The empirical results show that there are no spill-over effects on the off-shore spot by the off-shore non-deliverable forward market.

Do *et al*. (2016) examined several methodologies including the VAR, GARCH, Copula and DCC, Bayesian approach, Camp and factor models including the VARMA-GARCH asymmetric BEKK models in investigating the integration at industry levels in recommending investment diversification. The study used VAR-Multivariate GARCH model in estimating the dynamic hedge ratios with the aim of examining the hedge effectiveness of futures contracts on a financial asset and commodities in Indian markets. Both the in-sample and out-of-sample performance are compared based on reducing portfolio risk. Based on the results it is shown that the VAR-Multivariate GARCH model provides the highest reduction of the variance as compared to the constant hedge ratio.

The study by Sherafatmand *et al.* (2014) employed the bivariate BEKK GARCH model in determining time varying hedge ratios. The study show a hedge ratio of dates is 0.7 which is higher than the traditional one from the bivariate BEKK GARCH model. There is also 80% variance reduction of the hedge ratio by the BEKK BGARCH. Gau (2001) examined the temporal dynamics of volatility and correlation across international index futures market by employing the Multivariate GARCH models. The paper studied three index futures S&P500 index, Chicago Merchantile Exchange (CME) and Nikkei 225 index, FT-SE 100 index futures from the London International Financial Futures and Options Exchange (LIFFE). According to the results there is existence of conditional volatility and conditional correlations across index futures.

Guo (2003) fitted the dynamic structure of the conditional volatility and correlations using the Multivariate GARCH models with time varying correlations. The data used was that of the international portfolio of the US, UK and Switzerland stocks for the period of February 1973 to March 2002. The empirical results showed that currency fluctuations can be partially captured by optimal dynamic hedging strategy. Candila (2013) employed both the unconditional and conditional tests in order to assess the different Multivariate models in terms of the ability to forecast volatility with the highest accuracy in both statistical and economic point of view. The Monte Carlo experiment was used as a corner stone for the analysis.

In the study by Mukherjee (2011), a joint VAR-Multivariate GARCH model is estimated in order to assess the relationship between India, United States, the Republic of Korea, Hong Kong and China. It has been found that the return on markets such as the Republic of Korea, China, Singapore and Hong Kong are being affected by the returns of the Indian market. Furthermore, there has been an increase in recent years with other markets with the conditional correlation of the Indian markets. The study reviewed Multivariate GARCH models together with their properties were also discussed (Tas, 2008).

Wei (2016) in the study had two objectives: the first one is to apply both the symmetric and asymmetric models in exploring the return and volatility interaction between electricity and other fuel price markets, the second objective is to investigate both the negative and positive asymmetric effects within and across other energy markets. The models employed are the $\mathrm{VAR}(1)-\mathrm{BEKK}-\mathrm{MGARCH}(1,1), \mathrm{VAR}(1)-\mathrm{CCC}-\mathrm{MGARCH}(1,1), \mathrm{VAR}(1)-\mathrm{DCC}-\mathrm{MGARCH}, \mathrm{VAR}(1)-\mathrm{VARMA}-\mathrm{CCC}-\mathrm{MGARCH}$ and $\mathrm{VAR}(1)-\mathrm{VARMA}-\mathrm{DCC}-\mathrm{MGARCH}$. The empirical results show that volatility spill-over and dynamic structure of the return interactions are captured well by the models and also there are few cross markets effects by the models.

The study estimated the $\mathrm{VAR} - \mathrm{GARCH}$ model in analysing the daily exchange rates in New York, Germany and Japan covering the period 21st June 1996 to 22nd June 1998. The study employed the marginal likelihood criterion in model selection and the model selected is the $\mathrm{VAR} - \mathrm{GARCH} - \mathrm{M}(1,1,2,2)$ model. Based on the results it is shown that the $\mathrm{VAR} - \mathrm{GARCH} - \mathrm{M}$ model has the most accurate forecast with the smaller standard deviation (Polasek and Ren, 1999).

Sheu and Cheng (2011) employed both the VAR and the Multivariate generalised autoregressive conditional heteroskedastic model for two sets of periods: 1996-2005 and 2006-2009 in comparing the effects of volatility for the China and U.S stock market respectively on the Taiwan and Hong Kong. It is found that China's stock market is independent and co-moments with other markets are still insignificant. In their study Kouki *et al*. (2011) examined both the volatility spill-over together with the constant and dynamic of conditional correlation in different sectors. There are five sectors used in the study: banking, financial service, industrial, real estate and oil between international stock markets. Based on the empirical results it is found that the hypothesis of constant conditional correlation is supported and that there exists a cross boarder correlation within sectors.

Minović and Simeunović (2009) gave literature review on the Multivariate GARCH model in the modern finance and economy. Furthermore, it is being documented that Multivariate GARCH model has a variety of applications. The leverage effects of the Multivariate GARCH model are also discussed in the study. Bonga-Bonga and Nleya (2016) compared the performance of the constant conditional correlation (CCC), dynamic conditional correlation (DCC) and asymmetric DCC (ADCC) models in estimating the portfolio at risk in the BRICS countries. The study employed the average deviations, quadratic probability function score and the root mean square error as the performance error measurement. The results showed that portfolio is the way to minimise the losses in BRICS.

The volatility and conditional relationship among inflation rate, exchange rate and interest rates together with constructing a model of Multivariate GARCH DCC and BEKK were investigated using a dataset of Ghana covering the period of January 1990 to December 2013. The results show that both the BEKK model is robust in modeling and forecasting volatility of inflation rate, exchange rate and interest rate whereas the DCC model is robust in modeling the conditional and unconditional correlation of the inflation, exchange and interest rates respectively (Nortey *et al.*, 2015).

Gardebroek *et al.* (2013) employed Multivariate GARCH approach in assessing the interdependence and dynamics of volatility in corn, wheat and soybeans markets in the U.S on a daily, weekly and monthly basis covering 1998 to 2012. Based on the results there was an indication of lack of cross boarder dependence between markets and on weekly basis there exists volatility between these commodities. Caporale *et al*. (2017) estimated the VAR-GARCH (1.1) model in examining the effects of newspapers headlines on the exchange rates on U.S dollar and euro the currencies of the BRICS. The data set used was a daily data covering the period of 03/1/2000 to 12/5/2013. According to the results there is a significant spill-over although it differs across the countries.

Hartman and Sedlak (2013) used a ten years exchange rate data and examined the performance of the two Multivariate GARCH models: BEKK and DCC. The performance is measured based on the OLS regression, MAE and RMSE. Based on the results it is found that the BEKK model performance better than the DCC model. The study employed the Multivariate GARCH model in assessing the interaction between exchange rate and stock market returns. The series used was a daily data of Euro-Dollar exchange rate and the Dow Jones Industrial average and S$P500 index from the US economy (Tastan, 2006).

Efimova and Serletis (2014) compared both the univariate and Multivariate GARCH models in terms of investigating the empirical properties of oil, natural gases and electricity price volatility. The data set used is the daily data of whole sale markets in the US covering the period of 2001 to 2013. The models were compared using the range of performance tests together with assessing the conditional correlation dynamics. Chen and Zapata (2015) employed BEKK-GARCH model in order to model volatility and spill-over effects using the data covering the sample of June 1996 to December 2013. According to the results it is documented that own-price volatility and past unexpected events explains the volatility in China's price hogs, whereas American volatility is explained by its own events.

The study examined both the short run and long run linkages between equity markets in China and the US in terms of exploring and comparing the effects of two financial crises (the 1997 Asian Financial Crisis and the 2007-2010 Subprime Financial Crisis). Furthermore, the BEKK-GARCH model is estimated in order to examine the volatility spill-over effects. According to the results there exists not the cointegration in the stock indexes of the mainland to that of both the US and Hong Kong. However, there exists volatility and spill-over effects in the short run in the different equity markets (Chen and Zapata, 2015).

Santos *et al*. (2012) examined both the Multivariate and univariate GARCH models in terms of comparing them for their forecasting ability of portfolio VaR. Statistical tests were also employed in ranking the models for their predictive accuracy. On the basis of the performance it is being shown that the Multivariate models outperformed their counter parts. Moreover, the asymmetric CCC model with the student-t errors is the most suitable for being employed in modeling portfolios. Allen *et al.* (2015) employed the volatility impulse response analysis to the analysis of Multivariate GARCH model. The data set used is that of New York Stock Exchange Index and the FTSE 100 index from the London stock exchange covering the period of 3rd January 2005 to 31st January 2015. According to the results there is a large impact of the negative shocks due to the effects on both the variance and covariance, but there is a shorter one in difference of duration three and six months.

Wahab (2012) estimated asymmetric conditional returns model which will describe the co-movements of three major European stock markets with the U.S stock market. The $\mathrm{VAR(p)} -$ Multivariate $\mathrm{GARCH(p,q)} - \mathrm{BEKK}$ was used to capture the Multivariate conditional heteroskedasticity. The results show that France offers the best risk-adjusted returns than the U.K or Germany as this is a point of view from the U.S investor. Chen (2015) in the thesis studied the three Multivariate GARCH models: CCC-GARCH, DCC-GARCH and ADCC-GARCH, in which the three univariate GARCH models are used to model the time-varying volatility with the error term assumed to have Gaussian distribution. The thesis adopted Bayesian approach and

Markov chain Monte Carlo is also implemented, Metropolis within Gibbs (MWG) is adopted unlike the maximum likelihood. Finally, value at risk will be computed using the estimated models and their performance will be discussed.

Yi *et al*. (2009) augmented fractionally integrated VECM model with the Multivariate GARCH model to reveal simultaneously the return transmission and volatility spill-over between market return series. The empirical results showed that there is a fractional integration and China's market is strongly tied with Hong Kong market than with the U.S market. Baybogan (2013) estimated the volatility in financial time series econometrics and also investigated the empirical application with respect to estimation applications in the theoretical framework of GARCH models. The two models investigated are both the DCC-GARCH and BEKK-GARCH. Kvasnakova (2009) employed both the copula and Multivariate GARCH model in modeling the returns of the growth pension funds. The study again applied the two models in calculating the VaR and compares them. The results show that copula model produces better VaR estimation.

2.5 CHAPTER SUMMARY

The most recent empirical literature has been reviewed and the difference drawn from various studies. The literature reports on the different models used to model the time varying integrated data that allows the conditional-on-past-history covariance matrix of the dependent variables to follow a flexible dynamic structure. Most of the reviewed literature was based on the international perspective.

A VAR model processes are well known in economics and other sciences since they are flexible and simple models for Multivariate time series data. A VAR model posits a set of relationship between lagged values of all variables and the current values of all variable in the system (McMillin, 1991) and (Lu, 2001). VAR model were utilized in many studies. The study by Chamalwa and Bakari (2016) used VAR(1) to model the relationship between economic growth and some financial deepening indicators. Similarly, Mohanasundaram and Karthikeyan (2015) and Ijumba (2013) fitted the VAR(1) model in their respective studies. On the other hand (Adenomon *et al*. (2013) fitted VAR(8) to model the dynamic relationship between rainfall and temparature in Nigeria. The study by Ahmed and Suliman (2011) and Abdalla (2012 found an evidence of the leverage effect in their variables of interest. Most of the studies such as Predescu and Stancu (2011), Mwita and Nassiuma (2015) revealed the presence of volatility in the stock returns. Goodwin (2012), Obeng (2012) and Mokoma and Moroke (2014) found that GARCH (1.1) models that data better than other models.

The study by Ijumba (2013) on the Multivariate GARCH models suggested that there was a persistence of volatility amongst the BRICS stock market returns and this was also found in the study by Türkyılmaz and Balıbey (2014). Chen and Zapata (2015) employed BEKK-GARCH models in order to model volatility and spill-over effects and the results revealed that own-price volatility and past unexpected events explain the volatility in China price hogs and America's volatility is explained by its own events. Wahab (2012) employed the VAR (p)-BEKK-GARCH (p,q) to capture the Multivariate conditional heteroskedasticity.

In the study by Harrathi *et al*. (2016) a combination of VAR BEKK-GARCH models was used to investigate the volatility spill-over between equity markets indexes for Islamic and Non-Islamic emerging countries and the results showed that there is a strong volatility spill-over effects among the Islamic and the Non-Islamic countries. A similar study was conducted by Zhao (2010) also used the VAR-Multivariate GARCH in analysing the dynamic relationship between the real effective exchange rate and stock price. Lama *et al.* (2016) also employed the VAR-Multivariate GARCH approaches in an attempt to mode the volatility pulse prices. Allen *et al.* (2015) also employed the volatility impulse response analysis to analyse the Multivariate GARCH model and the results revealed that there is a large impact of the negative shocks due to the effects on both the variance and covariance.

Behera (2011) employed the Multivariate GARCH model in examining the onshore-offshore linkages of the Indian Rupee and found that there were no spill-over effects on the offshore spot by the off-shore non-deliverables forward market. In contrary, the study by Yi *et al*. (2009) which augmented fractionally integrated VECM model with Multivariate GARCH model revealed simultaneously the return transmission and volatility spill-over effects between markets return series.

CHAPTER 3
RESEARCH METHODOLOGY

3.1 INTRODUCTION

This chapter discusses the methods used in the study, ranging from univariate to Multivariate techniques. The chapter also discusses different methods used in determining the stationarity of the data used. Under univariate methods VAR, ARCH and GARCH models of the different countries exchange rates are discussed. The Multivariate techniques include Multivariate GARCH and VAR-Multivariate GARCH models. The methodology implemented a series of methods ranging from the simple univariate approach to a more complex Multivariate approaches. The implementation of the methodology was based on the BRICS exchange rates to achieve the following objectives:

- To review and determine the statistical properties of the main time-series models.
- To identify appropriate Multivariate GARCH models for the BRICS exchange rates.
- To estimate VAR-Multivariate GARCH models to the BRICS exchange rates.
- To determine the most appropriate VAR-Multivariate GARCH model to the BRICS exchange rates.
- To provide recommendations based on the findings.

The rest of the chapter is organized as follows: Section 3.2 illustrates the ethical considerations the study undertook. In Section 3.3, the research process picture is painted. Section 3.4 discusses the stationarity tests; Section 3.5 illustrates the VAR model; Section 3.6 looks at the ARCH models. GARCH models are discussed in Section 3.7. This is followed by the discussion of Multivariate GARCH in Section 3.8. VAR-Multivariate GARCH is discussed in Section 3.9 and lastly conclusion is discussed in Section 3.10.

3.2 ETHICAL CONSIDERATION

There is no ethical consideration related to environment, animals or human subject in this study. The study only uses secondary data. However, the researcher always referred to the institutional manual of postgraduate studies for the entire duration of the study. Permission to conduct the study was sought from the university by submitting an application form to the human research Ethics Committee though the supervisor after the proposal idea was approved. Permission to conduct the study was then granted by the human research ethics committee.

3.3 RESEARCH PROCESS

Saunders, Lewis, and Thornhill (2012) describe the research process using the onion figure of speech. The process comprise of six layers namely: research philosophies; Approaches; Strategies; Research choices; Time horizon; and data collection methods.

3.3.1 Research philosophy

Research philosophy paves a distinct direction the study has to follow. Careful consideration has to be placed on choosing the research philosophy. Saunders, Lewis and Thornhill, (2009) alluded to the fact that there are two philosophical dimensions to differentiate the existing research paradigms namely ontology and epistemology. The two philosophical approaches are related to the nature of knowledge (ontology) and the development of that knowledge (epistemology). The one's perception of reality is addressed by ontology approach, with ontology the existence of reality being external and independent of social actors and how they interpret it, which is called objectivist (Saunders *et al*., 2009) or realist (Neuman, 2011). The opposite where reality is deemed as dependent on the social actors is called subjectivist or nominalist. Epistemology on one hand beliefs on the way to generate, understand and usage of knowledge that is valid and acceptable. The two philosophical dimensions guide how to investigate reality which is axiology and methodology. Epistemology is explained by Collis and Hussey (2009) as what constitutes acceptable knowledge.

Epistemology is divided into two elements, which are, positivism and interpretivism. Henning, Van Rensburg and Smit (2004) illustrated that "positivism is concerned with uncovering truth and presenting it by empirical means". The view is also supported by Saunders *et al*., (2012) that positivist believes in observing and describing reality from an objective point of view. Interpretivism is an epistemological position that is opposed to the positivism as the knowledge based on individuals' viewpoint. They believe that it is important to understand the difference between humans in their roles as social actors. It is on the above background that the study chooses the positivism as a philosophical stance to follow.

3.3.2 Research approach

Daniel and Sam (2011) stated that the methodology refers to "the discipline regarding the method and the form of enquiry chosen, as well as referring to the selection of research approaches used to explore the social world". Saunders and Bezzina (2015) highlighted what was introduced by Saunders *et al*. (2012) that there are two approaches to research namely, deductive and inductive approaches. Time series data (it requires quantitative analysis methods) is used in the study and this implies that the deductive approach is used. According to Saunders *et al*. (2009), deductive approach first formulates the hypothesis and tests them while on the other hand in inductive approach theories are derived from data analysis. The study seeks to investigate the performance of conditional VAR enhanced Multivariate GARCH models on the time varying integrated data. This implies that the study follows the deductive approach with the attempt to answer the research objective and questions as set in chapter one.

3.3.3 Research strategy

According to Saunders *et al*. (2012:173), research strategy is "a plan of action to achieve a goal". Saunders further stated that the research strategy links the chosen philosophy with the choice of data and the method of analysing the data. Secondary data is used as the research strategy to reach the intended goals and objectives of the study. Wegner (2016:14) explained secondary data as "data that already exist in a process format". The strategy for the research is a case study from the fundamental that the empirical investigation is undertaken on a specific phenomenon. The research focuses on the BRICS exchange rate. Time horizon is made up of two aspects viz: cross sectional and longitudinal studies. Saunders *et al*. (2009) described cross sectional as the a study concerned with a selected phenomenon at a specific time and period whereas the longitudinal is described as representing events over a long period of time. Therefore, the cross sectional strategy was adopted for the study.

3.3.4 Choice of research

The choice of the study is dependent on the three aspects forming what are the possible choices available. Broadly, qualitative, quantitative and the mixed methods form the base of the research approaches. Creswell (2014:200) indicated that "quantitative research is generally associated with the positivist/post positivist paradigm. It usually involves collecting and converting data into numerical form so that statistical calculations can be made and conclusions drawn". Moule and Goodman (2009:235) stated that the goal of quantitative research is by generating research data that can be analysed using numerical or statistical techniques.

Neuman (2011:165) stated that "qualitative case study research is the approach usually associated with the social constructivist paradigm which emphasises the socially constructed nature of reality". Burns and Grove (2011:73) indicated that "qualitative approaches are mainly appropriate for subjective views on a research problem".

The mixed method was described by Tashakkori and Creswell (2007:207) as "research in which the investigator collects and analyses data, integrates the findings and draws inferences using both qualitative and quantitative approaches/methods in a single study". A mixed approach combines qualitative and quantitative methods in a study. It is used to use the best of both the quantitative and qualitative methods.

The study used the quantitative data and approaches implying that the choice of the study is based on mono-methods. The study employed the monthly time series data. The period of the data covers data from the date South Africa was inducted to be a member of the then BRIC into a new agreement named BRICS. South Africa was officially inducted in April 2010 and this was supposed to give rise to the starting period of the study data, but due to the requirements prescribed in other models extension, the starting point was considered. Therefore, the data covers the scope before the inception of BRICS ranging from January 2008 to January 2018 and it has 121 observations. The reason for including data points before the inception of BRICS is to increase the number of observations since some models require a minimum 100 observations. The study employed the monthly exchange rates of the five BRICS countries. The data involves currency exchange rates monthly average. The BRICS countries are also known as the emerging economies. The data used in this study is a national currency of each of the five countries per US Dollar. It is obtained from the Organisation for Economic Cooperation and Development (OECD) website. Data analyses in this study are carried out using R 3.4.4 programming language.

3.4 STATIONARITY TESTS

Most of the time series used in modeling are non-stationary in nature. By non-stationarity, the mean, variance, and autocovariances may depend on time t. A time series ($X_t: t = 1, 2, 3, \ldots, N$) is said to be stationary if its mean, variance, and autocovariance are independent of time. In Box-Jenkins setting, if the mean of the series is less than its corresponding standard deviation, it is representable as

$$X_t = \sum_{j=1}^{p} \phi_j X_{t-j} + \varepsilon_t + \sum_{k=1}^{q} \theta_k \varepsilon_{t-k} \qquad (3.1.1)$$

where ϕ_j: j=1,2,3,….p are the autoregressive parameters of order p, and $\theta_k: k = 1,2,3, \ldots, q$ are the moving average parameters of order q. If, however, the mean of the series happens to be greater than the standard deviation, an adjustment made to 3.1.1 yields

$$X_t = c + \sum_{j=1}^{p} \phi_j X_{t-j} + \varepsilon_t + \sum_{k=1}^{q} \theta_k \varepsilon_{t-k} \qquad (3.1.2)$$

If the series is driven by a polynomial trend, further adjustments to equation (3.1.2) yields the representation

$$X_t = \sum_{i=0}^{m} \alpha_i t^i + \sum_{j=1}^{p} \phi_j X_{t-j} + \varepsilon_t + \sum_{k=1}^{q} \theta_k \varepsilon_{t-k} \qquad (3.1.3)$$

In equation (3.1.1) to (3.1.3), ε_t is a white noise process with mean zero and variance σ^2, that is

$$\varepsilon_t \backsim i.i.d. N(0, \sigma^2) \qquad (3.1.4)$$

X_t in equation (3.1.3) is non-stationary in levels, but the differenced series given by

$$\Delta X_t = X_t - X_{t-1} \quad (3.1.5)$$

is stationary, thus X_t is said to contain a unit root or simply be a differenced-stationary (DS) series. Consider the case where $p = 1$ and $q = 0$, obtain the autoregressive AR(1), process

$$X_t = \phi_1 X_{t-1} + \varepsilon_t \quad (3.1.6)$$

If $|\phi_1| < 1$, then equation (3.1.6) is said to be stationary so that

$$X_t = \phi_1 X_{t-1} + \varepsilon_t)$$
$$(1 - \phi_1 B)X_t = \varepsilon_t$$
$$X_t = (1 - \phi_1 B)^{-1}\varepsilon_t = (1 + \phi_1 B + \phi_2^2 B^2 + \cdots)\varepsilon_t$$
$$X_t = \varepsilon_t + \phi_1 \varepsilon_{t-1} + \phi_2 \varepsilon_{t-2} \ldots. \quad (3.1.7)$$

It follows that

$$E(X_t) = 0 \quad (3.1.8)$$
$$var(X_t) = \frac{\sigma_\varepsilon^2}{1-\phi_1^2} \quad (3.1.9)$$
$$cov(X_t, X_{t-k}) = \frac{\phi_1^2 \sigma^2}{1-\phi_1^2}, \text{ k =1,2....} \quad (3.1.10)$$

X_t is said to have a unit root if $\phi_1 = 1$.

In this case, equation (3.1.7) becomes

$$X_t = X_{t-1} + \varepsilon_t = X_0 + \varepsilon_t + \varepsilon_{t-1} + \varepsilon_{t-2} + \cdots + \varepsilon_1 \quad (3.1.11)$$

assuming that the process starts at $t = 0$. For the particular case

$$E(X_t) = X_0 \quad (3.1.12)$$
$$var(X_t) = t\sigma^2 \quad (3.1.13)$$

$$cov(X_t, X_{t-k}) = \sqrt{\frac{t-k}{t}}, \qquad k = 1,2,\ldots \qquad (3.1.14)$$

Formal tests for non-stationarity have now become a standard starting point in applied time series analysis. Several test statistics have been proposed to test the need for differencing the series before modeling. Notable among these are due to Dickey and Fuller (1979), Phillips and Perron (1988) and Hall (1989).

The unit root test procedures reviewed in this study are the Augmented Dickey-Fuller ADF test, Phillips-Perron (PP) test, Instrumental Variable (IV) test, multiple unit root tests and joint unit root test. Those unit root test procedures are discussed in the following subsections respectively.

3.4.1 The Augmented Dickey-Fuller (ADF) Test

This section discusses the ADF test. The ADF tests for the existence of the unit root (Dickey and Fuller, 1979). Consider the AR(1) process with

$$X_t = \phi_1 X_{t-1} + \varepsilon_t, \qquad \varepsilon_t \backsim i.i.d.N(0, \sigma_\varepsilon^2) \qquad (3.1.15)$$

Subtracting X_{t-1} from both side of equation (3.1.15) yields

$$\Delta X_t = (\phi_1 - 1)X_{t-1} + \varepsilon_t \qquad (3.1.16)$$

If a constant term is included in the model,

$$\Delta X_t = c + (\phi_1 - 1)X_{t-1} + \varepsilon_t \qquad (3.1.17)$$

Similarly, if X_t is driven by a linear time trend, then the autoregression considered is given by

$$\Delta X_t = (\alpha_0 + \alpha_1 t) + (\phi_1 - 1)X_{t-1} + \varepsilon_t \qquad (3.1.18)$$

It can be shown that if the ε_t are not *i.i.d*, then the autoregressions preferred and the Augmented Dickey-Fuller (ADF) autoregressions for jth differential should be

$$\Delta X_t = (\phi_1 - 1)X_{t-1} + \sum_{j=1}^{p-1} \gamma_j \, \Delta X_{t-j} + \varepsilon_t, \tag{3.1.19}$$

$$\Delta X_t = c + (\phi_1 - 1)X_{t-1} + \sum_{j=1}^{p-1} \gamma_j \, \Delta X_{t-j} + \varepsilon_t, \tag{3.1.20}$$

$$\Delta X_t = (\alpha_0 + \alpha_1 t) + (\phi_1 - 1)X_{t-1} + \sum_{j=1}^{p-1} \gamma_j \, \Delta X_{t-j} + \varepsilon_t, \tag{3.1.21}$$

In what follows, p is selected to ensure that the ε_t are uncorrelated. For the AR(1) process in equation (3.1.15), the maximum likelihood estimator of ϕ_1 the least squares estimator

$$\hat{\phi}_1 = \frac{\sum_{t-1}^{N} X_t X_{t-1}}{\sum_{t-1}^{N} X_{t-1}^2} \tag{3.1.22}$$

Substituting $X_t = \phi_1 X_{t-1} + \varepsilon_t$ in equation (3.1.22) yields

$$\hat{\phi}_1 = \frac{\sum(\phi_1 X_{t-1} + \varepsilon_t) X_{t-1}}{\sum X_{t-1}^2} = \frac{\phi \sum X_{t-1}^2 + \sum X_{t-1}\varepsilon_t}{\sum X_{t-1}^2}$$

$$\hat{\phi}_1 = \phi_1 + \frac{\sum X_{t-1}\varepsilon_t}{\sum X_{t-1}^2}$$

$$\hat{\phi}_1 - \phi_1 = \frac{\sum X_{t-1}\varepsilon_t}{\sum X_{t-1}^2}. \tag{3.1.23}$$

Under the null hypothesis of a unit root, $H_0: \phi_1 = 1$, and hence equation (3.1.23) becomes

$$\hat{\phi}_1 - 1 = \frac{\sum X_{t-1}\varepsilon_t}{\sum X_{t-1}^2}. \tag{3.1.24}$$

The resultant likelihood ratio test is a function of

$$\hat{\tau}_{df} = \frac{\hat{\phi}_1 - 1}{S_e(\hat{\phi}_1 - 1)} \tag{3.1.25}$$

where

$$S_e(\hat{\phi}_1 - 1) = \sqrt{\frac{\sum_{t=2}^{N}\left(X_t - \hat{\phi}_1 X_{t-1}\right)^2}{(N-2)\sum X_{t-1}^2}} \tag{3.1.26}$$

It is obvious that under this null hypothesis, a regression of ΔX_t on X_{t-1} will give a coefficient on X_{t-1} which is an estimate of 0, since $\phi - 1 = 1 - 1 = 0$. However, under the alternative hypothesisH_0: $|\phi_1| < 1$, $\phi - 1 \neq 0$ and hence a regression of ΔX_t on X_{t-1} is appropriate. Similarly, if a constant term is included in the unit root autoregression equation (3.1.15), a regression of ΔX_t on a constant and X_{t-1} is deemed appropriate. Lastly, a linear trend is indicated in equation (3.1.15) suggests regressing ΔX_t on a constant, time and X_{t-1}.

When $\phi_1 = 1$, the process generating X_t is $\Gamma(1)$. This implies that X_{t-1} will not satisfy the standard assumptions needed for asymptotic analysis. Consequently, Dickey and Fuller (1979) employed Monte Carlo methods to compute the non-standard percentiles for the distributions under the null hypothesis of the unit root. The null hypothesis is rejected if the test statistic is less than the corresponding critical values tabulated by Dickey and Fuller. Otherwise, it is accepted.

If the autoregressive model is of higher order, the unit root regressions are augmented by lagged differences and ΔX_{t-j}. For example if the sample partial autocorrelation function (PACF) suggests an AR(2) process, then the appropriate unit root regression to consider is

$$\Delta X_t = (\phi_1 - 1)X_{t-1} + \sum_{j=1}^{2-1} \gamma_j \Delta X_{t-j} + \varepsilon_t \tag{3.1.26}$$

which suggests a regression of ΔX_t on X_{t-1} and ΔX_{t-1}. Where appropriate, a constant term or a linear trend is included in equation (3.1.26). The inclusion of the terms ΔX_{t-j} leaves the asymptotic distribution of the parameters of interest unchanged.

3.4.2 The Phillips-Perron (PP) Test

In this section, the study reviews some theoretical background for a unit root test procedure proposed by Phillips and Perron (1988). The study hereafter refers to this test procedure as the PP test. The PP test corrects for the existence of any serial correlation in the errors by modifying the ADF test statistics (Newey and West, (1987). The unit root test regression is any of the AR(1) processes

$$\Delta X_t = (\phi_1 - 1)X_{t-1} + \varepsilon_t \tag{3.1.27}$$

$$\Delta X_t = c + (\phi_1 - 1)X_{t-1} + \varepsilon_t \quad (3.1.28)$$

$$\Delta X_t = (\alpha_0 + \alpha_1 t) + (\phi_1 - 1)X_{t-1} + \varepsilon_t \quad (3.1.29)$$

The PP test is non-parametric in nature and has the tendency to correct serial correlation that may be present in the error term, ε_t. This test procedure is non-parametric in that the correction in ε_t uses an estimate of the spectrum of ε_t at frequency zero that is robust to heteroskedasticity and autocorrelation of unknown form. The procedure employs the Newey and West (1987) consisting estimate

$$\xi^2 = \Gamma_0 + 2\sum_{k-1}^{\gamma}\left[1 - \frac{k}{q+1}\right]\Gamma_k \quad (3.1.30)$$

where

$$\Gamma_k = \frac{1}{N}\sum_{t=k+1}^{N}\hat{\varepsilon}_t\hat{\varepsilon}_{t-k} \quad (3.1.31)$$

and Γ is the truncation lag determined by the expression

$$\Gamma = floor\left[4\left(\frac{N}{100}\right)^{\frac{2}{9}}\right] \quad (3.1.32)$$

The computed PP test statistic is given by

$$\hat{\tau}_{pp} = \left(\frac{t_{\phi_1 - 1}}{\xi}\right)\Gamma_0^{\frac{1}{2}} - \frac{N}{2}\left(\frac{\xi^2 - \Gamma_0}{\xi\hat{\sigma}}\right)S_e(\phi_1 - 1) \quad (3.1.33)$$

where $t_{\phi_1 - 1}$ is the *t*-statistic of $\phi_1 - 1$, $S_e(\phi_1 - 1)$ is the standard error of $\phi_1 - 1$, and $\hat{\sigma}$ is the standard error of the test regression. The asymptotic distributions of the PP test statistics are the same as those of the ADF test statistics. Here again, the null hypothesis of a unit root $H_0: \phi_1 = 1$ is rejected if $\hat{\tau}_{pp}$ is less than the appropriate critical value at some level of significance.

3.4.3 Instrumental Variable (IV) Unit Root Test

In his Monte Carlo study of the empirical powers of some unit root tests, Schwert (1989) observed that the statistics of an earlier version of unit root test proposed by Phillips (1987a) do not perform well in finite samples in the presence of negative moving average errors. Motivated by the problem, Hall (1989) proposed estimation by instrumental variable (IV) as an alternative to the use of non-parametric corrections. For the AR(1) process

$$X_t = \phi_1 X_{t-1} + \mu_t, \tag{3.1.34}$$

where

$$\mu_t = \varepsilon_t + \sum_{k=1}^{q} \varepsilon_{t-k} \tag{3.1.35}$$

It is shown that under the null hypothesis of a unit root $H_0: \phi_1 = 1$ the instrumental variable $\hat{\phi}_1^{IV}$ of ϕ_1 has the standard Dickey-Fuller distribution. For example, let our date generating process (DGP) be

$$X_t = \phi_1 X_{t-1} + \mu_t, \tag{3.1.36}$$

where $\mu_t = \varepsilon_t - \theta_1 \varepsilon_{t-1}$ and ε_t ~i.i.d, N$(0, \sigma_\varepsilon^2)$. Then the instrumental variable estimator, $\hat{\phi}_1^{IV}$ of ϕ_1 using X_{t-2} as an instrument for X_{t-1} when $\phi_1 = 1$ is given by

$$\hat{\phi}_1^{IV} = \frac{\sum_{t=1}^{N} X_t X_{t-2}}{\sum_{t=1}^{N} X_{t-1} X_{t-2}} \tag{3.1.37}$$

The corresponding test statistic proposed by Hall (1989) is given by

$$\hat{\tau}_N = \left(\hat{\phi}_1^{IV}\right) \sqrt{\frac{\sum_{t=1}^{N} X_{t-1} X_{t-2}}{\hat{\sigma}^2}} \tag{3.1.38}$$

where

$$\hat{\sigma}^2 = \left[\left(1 + \hat{\theta}\right)\hat{\sigma}_\varepsilon\right]^2 \tag{3.1.39}$$

has the ADF *t*-dimensional, and hence the usual ADF critical values are applicable. The null hypothesis of a unit root is rejected if $\hat{\tau}_{IV}$ is less than its corresponding critical value.

3.4.4 The Generalized-Least-Squares (GLS) Unit Root Test

Dickey-Fuller generalized least squares (DF-GLS) test was first proposed by Elliot *et al.* (1996). The test has improved the power against the standard augmented Dickey-Fuller (1979, 1981) ADF test. Let's on a series $(X_t: t = 1, 2, 3, \dots, N)$ assuming the representation

$$X_t = \mu + u_t \tag{3.1.40}$$

$$u_t = \phi_1 u_{t-1} + \varepsilon_t \tag{3.1.41}$$

where $\varepsilon_t \sim i.i.d, N(0, \sigma_\varepsilon^2)$. Concentrating on the t-statistic form of the test for equation (3.1.40), the t-statistic for $\phi_1 = 1$ is obtained by estimating by ordinary least squares (OLS), the autoregression

$$\Delta X_t = \mu + (\phi_1 - 1)X_{t-1} + \varepsilon_t \tag{3.1.42}$$

Then to order N^{-1}, this is equivalent to computing the ADF test statistic $\hat{\tau}_{df}$ from the reparameterized autoregression

$$\Delta\tilde{X}_t = \mu + (\phi_1 - 1)\tilde{X}_{t-1} + \tilde{\varepsilon}_t \tag{3.1.43}$$

where $\tilde{X}_t = X_t - \mu$, and $\hat{\mu} = \sum_{t=0}^{N} {X_t}/{(N+1)}$ is the OLS estimator of μ. Next, denote the generalised-least-squares (GLS) test statistic by $\hat{\tau}_{gls}$. Then $\hat{\tau}_{gls}$ is obtained simply by calculating the ADF test statistic using the autoregression in equation (3.1.43), replacing $\tilde{X}_t$ by a demeaned series using a psuedo-GLS estimator of the mean ($\hat{\mu}_{gls}$), rather than the OLS estimator, $\hat{\mu}$. Based on the testing the hypotheses

H_0: $\phi_1 = 1$
vs.
H_1: $|\phi_1| < 1$

the $\hat{\tau}_{gls}$ statistic is defined as the regression t-statistic on the coefficient of X^*_{t-1} in the OLS autoregression

$$\Delta X^*_t = (\phi^*_1 - 1)X^*_{t-1} + \varepsilon^*_t \qquad (3.1.44)$$

where

$$X^*_t = X_t - \hat{\mu}_{gls} \qquad (3.1.45)$$

The corresponding test statistic becomes

$$\hat{\tau}_{gls} = \frac{\phi^*_1 - 1}{Se(\phi^*_1 - 1)} \qquad (3.1.46)$$

where $Se(\phi^*_1 - 1)$ is the standard deviation of $(\phi^*_1 - 1)$. The same critical values used in the case of the ADF and PP tests apply. H_0 is rejected if the test statistic is less than the corresponding critical value.

3.4.5 Multiple Unit Roots Test

Much as the study considered testing for the presence of a unit root in a given time series, it must be admitted that not all time series processes can well be represented by any of the autoregressions

$$\Delta X_t = (\phi_1 - 1)X_{t-1} + \varepsilon_t \tag{3.1.47}$$

$$\Delta X_t = c + (\phi_1 - 1)X_{t-1} + \varepsilon_t \tag{3.1.48}$$

$$X_t = \sum_{i=1}^{m} \alpha_i t^i + \varepsilon_t + (\phi_1 - 1)X_{t-1} + \varepsilon_t \tag{3.1.49}$$

and their respective higher-order autoregressions

$$\Delta X_t = (\phi_1 - 1)X_{t-1} + \sum_{j=1}^{p-1} \gamma_j \Delta X_{t-j} + \varepsilon_t \tag{3.1.50}$$

$$\Delta X_t = c + (\phi_1 - 1)X_{t-1} + \sum_{j=1}^{p-1} \gamma_j \Delta X_{t-j} + \varepsilon_t \tag{3.1.51}$$

$$X_t = \sum_{i=1}^{m} \alpha_i t^i + (\phi_1 - 1)X_{t-1} + \sum_{j=1}^{p-1} \gamma_j \Delta X_{t-j} + \varepsilon_t \tag{3.1.52}$$

In rare instances, one might suspect more than one unit root. For such cases, Dickey and Pantula (1987) have proposed a simple extension of the ADF methodology capable of handling multiple unit roots. This is essentially nothing but more than performing the ADF tests on successive differences of the series, X_t. For instance, if two unit roots are suspected, the appropriate autoregression to consider is any of the following:

$$\Delta^2 X_t = + (\phi_1 - 1)X_{t-1} + \sum_{j=1}^{p-1} \gamma_j^* \Delta X_{t-j} + \varepsilon_t \tag{3.1.53}$$

$$\Delta^2 X_t = C_{1,2} + \left(\phi_{1,2} - 1\right)X_{t-1} + \sum_{j=1}^{p-1} \gamma_j^* \Delta^2 X_{t-j} + \varepsilon_t^* , \tag{3.1.54}$$

$$\Delta X_t = \sum_{i=1}^{m} \alpha_i^* t^i + \left(\phi_{1,2} - 1\right)X_{t-1} + \sum_{j=1}^{m} \gamma_j^* \Delta^2 X_{t-j} + \varepsilon_t^2 , \tag{3.1.55}$$

where $\sum_{i=1}^{m} \alpha_i^* t^i$ is a polynomial time trend of order *m* employing the test statistic.

$$\hat{\tau}_{df} = \frac{\hat{\phi}_{1,2} - 1}{Se(\hat{\phi}_{1,2} - 1)} \tag{3.1.56}$$

and the same critical values used in the case of the ADF and PP tests, the null hypothesis $H_0 : \phi_{1,2} = 1$ is rejected if the test statistic is less than the corresponding critical value.

3.4.6 Joint Unit Root Test: A Multivariate Setting

Here, the study outlines a simple joint unit root test developed in the Multivariate setting and due to Fountis and Dickey (1989). This methodology requires the examination of the eigenvalue and eigenvector. Steps involved are as follows:

Step 1: Fit the linear Multivariate time series. That is

$$X_{1,t} = \phi_{1,1}X_{1,t-1} + \phi_{1,2}X_{1,t-2} + \cdots + \phi_{1,p}X_{1,t-p} + \varepsilon_{1,t}$$
$$X_{2,t} = \phi_{2,1}X_{2,t-1} + \phi_{2,2}X_{2,t-2} + \cdots + \phi_{2,p}X_{2,t-p} + \varepsilon_{2,t}$$
$$X_{n,t} = \phi_{n,1}X_{n,t-1} + \phi_{n,2}X_{n,t-2} + \cdots + \phi_{n,p}X_{n,t-p} + \varepsilon_{n,t}$$
$$\Rightarrow \qquad X_t = \Phi_1 X_{t-1} + \Phi_2 X_{t-2} + \cdots . + \Phi_p X_{t-p} \tag{3.1.57}$$

Step 2: Obtain the largest eigenvalue, λ_{max}, based on the characteristic equation

$$\left|\lambda^p \mathrm{I} - \Phi_1 \lambda^{p-1} - \Phi_2 \lambda^{p-2} - \cdots - \Phi_p\right| = 0 \tag{3.1.58}$$

where I is the p × p identity matrix.

Step 3: Test the following hypotheses

$H_0 = X_t$ has a unit root,
vs.
$H_1 = X_t$ does not have a unit root,

based on the following test statistic

$$\hat{\tau}_{mfd} = N(\lambda_{max} - 1), \tag{3.1.59}$$

where λ_{max} is the largest eigenvalue based on Step 2.

Step 4: For some nominal level, α, obtain the critical value from the usual Dickey-Fuller table.

H_0 is rejected if $|\hat{\tau}_{mfd}| > Critical\ Value.$

The current study only focuses on the ADF and the PP tests for unit root since the two methods have the same distribution. The asymptotic distributions of the PP test statistics are the same as those of the ADF test statistics. ADF and PP unit root tests use the same critical values.

3.5 THE VECTOR AUTOREGRESSION MODEL

This section discusses both the univariate vector autoregression model and multivariate vector autoregression model. The VAR model is one of the flexible and easy to use models for the analysis of Multivariate time series. VAR provides for a build up towards the VAR-Multivariate GARCH model and it forms the basis of the study.

The VAR model is an extension of the autoregressive (AR) model to dynamic Multivariate time series. The VAR model has proven to be especially useful for describing the dynamic behaviour of economic and financial time series and for forecasting. It gives better forecasts to those from univariate time series models. The forecasts derived from VAR models are flexible and can be made conditional on the potential future paths of specified variables in the model (Zhang, Zhou, Zhang and Li, 2016). Therefore the VAR model was used to determine the dynamic behaviour between the BRICS exchange rates.

VAR model was introduced by Sims (1980) and is used to capture the dynamics and the interdependency of Multivariate time series. It is considered as a generalization of univariate AR models or a combination between the two or more equations models and the univariate time series models. Each variable in a VAR is explained by its own lagged values and the lagged values of all the other variables in the equation.

Let $Y_{t=}y_{1t}, y_{2t}, y_{3t}, \dots, y_{nt}$ denote an (n×1) vector of time series variables. The basic p-lag vector autoregressive VAR(p) model has the form:

$$Y_{t=} A + B_1 Y_{t-1} + B_2 Y_{t-2} + \cdots + B_p Y_{k-p} + \varepsilon_t \qquad (3.2.1)$$

where, A is n × 1 vector of intercepts , B_i is k × k matrices of parameters where i =1, 2, …, p and $\varepsilon_t \sim iid, N(0, \sum_{\mu})$.

The number of parameters to be estimated in the VAR model is k(1+kp) which increases with the number of variables (k) and number of lags (p). The inclusion criterion of the lags (p) in the equations is done using a test of system reduction and the AIC (Akaike Information Criterion is used to determine the lag length of VAR model. The following Criterions are also used: HQ (Hannan Quinn Information Criterion), SIC (Schwarz Information Criterion) and FPE (Final Prediction Error) information criteria. The commonly used information criteria are AIC, HQ and SIC and they are represented using the following equation respectively:

$$AIC\ (p) = \ ln\left|\overline{\sum}(p)\right| + \frac{2}{T} pk^2 \qquad (3.2.2)$$

$$HQ\ (p) = \ ln\left|\overline{\sum}(p)\right| + \frac{2lnlnT}{T} pk^2 \qquad (3.2.3)$$

$$SIC\ (p) = \ ln\left|\overline{\sum}(p)\right| + \frac{lnT}{T} pk^2 \qquad (3.2.4)$$

where T is the sample size and $\overline{\sum}(p) = T^{-1} \sum_{t-1}^{T} \hat{\mu}_t \hat{\mu}_t^{'}$. According to Lutkepohl (1991), the AIC criterion asymptotically is said to be overestimating the lag order with positive probability, whereas the BIC and HQ criteria does not overestimate. Therefore the selection is based on the lowest value of the minimum value of the three criterion.

3.5.1 Model Parameter Estimation

The VAR (p) coefficients can be estimated efficiently using either the Ordinary Least Squares (OLS) or the Maximum likelihood Estimation (MLE) methods. Tsay (2005) confirms that the Ordinary Least Squares (OLS) or the Maximum likelihood methods are asymptotically similar.

This study uses the MLE method to draw approximation of the coefficients of VAR (p). The VAR (p) matrix process can be written as follows:

$$Y = DW + \xi \quad (3.2.5)$$

where

$$Y = (y_1 \dots \dots \dots y_T)' \quad (3.2.6)$$

$$D = (c. A_1 \dots \dots \dots A_p)' \quad (3.2.7)$$

$$W_t = (1. y_t \dots \dots \dots y_{t-p+1}) \quad (3.2.8)$$

$$W = (W_0 \dots \dots \dots W_{T-1})' \quad (3.2.9)$$

$$\xi = (\mu_1 \dots \dots \dots \mu_T)' \quad (3.2.10)$$

Y, D, W and ξ are $(N \times T), (N \times (Np + 1)), ((Np + 1) \times T)$, and $(N \times T)$ matrices respectively. The MLE of the VAR (p) model is as follows:

$$y = vec\ (Y) \quad (3.2.11)$$

$$d = vec\ (D') \quad (3.2.12)$$

$$\mu = vec\ (\xi) \quad (3.2.13)$$

$$Y^* = (y_{1-\mu} \dots \dots \dots y_{T-\mu}) \quad (3.2.14)$$

$$X = (Y_0^*, \dots \dots \dots, Y_{T-1}^*) \quad (3.2.15)$$

$$\alpha = (A_1 \dots \dots \dots A_p) \quad (3.2.16)$$

where $y, d, \mu\ and\ \alpha$ are $(NT \times 1), ((N^2p + N) \times 1), (NT \times 1)$, and $(N^2p \times 1)$ vectors respectively. Y^* and X are $(N \times T)$ and $(Np \times T)$ matrices. The probability density function of μ is presented as follows:

$$f_\mu(\mu) = \frac{1}{(2\pi)^{\frac{NT}{2}}} \left|\Sigma_\mu\right|^{-\frac{1}{2}} \exp\left(-\frac{1}{2}\mu' \Sigma_\mu \mu\right) \quad (3.2.17)$$

where

$$\mu = y - \mu^* - (X' \otimes I_N)\alpha \tag{3.2.18}$$

such that

$$\mu^* = (\mu', \dots, \mu') \tag{3.2.19}$$

Using equation (3.2.18)

$$f_y(y) = \left|\frac{\partial \mu}{\partial y'}\right| f_\mu(\mu)$$
$$= \frac{1}{(2\pi)^{\frac{NT}{2}}} |I_T \otimes \textstyle\sum_u|^{-\frac{1}{2}} exp\left(-\frac{1}{2}(y - \mu^* - (X' \otimes I_N)\alpha)'(I_T \otimes \textstyle\sum_u^{-1}\right)(y - \mu^* - (I_N \otimes$$
$$X')\alpha)' \tag{3.2.20}$$

Therefore, the log-likelihood function

$$logL(\mu, \alpha, \textstyle\sum_u) = -\frac{NT}{2}\log(2\pi) - \frac{T}{2}(\sum_u) - \frac{1}{2}(y - \mu^* - (X' \otimes I_N)\alpha)'((I_T \otimes \sum_u^{-1}) \times$$
$$((I_T \otimes \textstyle\sum_u^{-1}))$$
$$= -\frac{NT}{2}\log(2\pi) - \frac{T}{2}log|\textstyle\sum_u| - \frac{1}{2}\sum_{t=1}^{T}((y_t - \mu) - \sum_{i=1}^{P} A_i\,(y_{t-i} - \mu))'\sum_\mu$$
$$\times ((y_t - \mu) - \sum_{i=1}^{P} A_i\,(y_{t-i} - \mu))$$
$$= -\frac{NT}{2}\log(2\pi) - \frac{T}{2}log|\textstyle\sum_u| - \frac{1}{2}\sum_t(y_t - \sum_i A_i y_{t-i})' \sum_u^{-1}(y_t - \sum_i A_i y_{t-i}) +$$
$$\mu'(I_N - \textstyle\sum_i A_i)'\sum_u^{-1}\sum_t(y_t - \sum_i A_i y_{t-i}) - \frac{T}{2}\mu'(I_N - \sum_i A_i)'\sum_u^{-1}(I_N - \sum_i A_i)\mu$$
$$= -\frac{NT}{2}\log(2\pi) - \frac{T}{2}log|\textstyle\sum_u| - \frac{1}{2}tr[(Y^* - AX)'\sum_u^{-1}(Y^* - AX)] \tag{3.2.21}$$

To find the MLE of $\mu, \alpha, \sum_\mu$, first order of the partial derivative of the log-likelihood function is considered:

$$\frac{\partial logL}{\partial \mu} = \left(I_N - \sum_i A_i\right)\Sigma_u^{-1}\sum_t\left(y_t - \sum_i A_i\, y_{t-i}\right) - \left(I_N - \sum_i A_i\right)' \Sigma_u^{-1}\left(I_N - \sum_i A_i\right)\mu$$

$$= (I_N - A[k \otimes I_N])'\Sigma_u^{-1}\left(\sum_t (y_t - \mu - AY_{t-1}^*)\right)$$

$$\frac{\partial logL}{\partial \mu} = (X \otimes I_N)\left(I_T \otimes \Sigma_\mu^{-1}\right)(y - \mu^* - (X' \otimes I_N)\alpha)$$

$$= (X \otimes \Sigma_u^{-1})(y - \mu^*) - (XX' \otimes \Sigma_u^{-1})\alpha$$

$$\frac{\partial logL}{\partial \Sigma_u} = -\frac{T}{2}\Sigma_u^{-1} + \frac{1}{2}\Sigma_u^{-1}(Y^* - AX)(Y^* - AX)'\Sigma_u^{-1} \tag{3.2.22}$$

where K is a $p \times 1$ vector of ones. The following MLE will result from equating the system of derivatives to zero:

$$\hat{\mu} = \frac{1}{T}\left(I_N - \sum_i \hat{A}_i\right)' \sum_t\left(y_i - \sum_i \hat{A}_i\, y_{t-i}\right) \tag{3.2.23}$$

$$\hat{\alpha} = \left(\left(\hat{X}\hat{X}'\right)^{-1}\hat{X} \oplus I_N\right)(y - \hat{\mu}^*) \tag{3.2.24}$$

$$\hat{\Sigma}_u = \frac{1}{T}\left(\hat{Y}^* - \hat{A}\hat{X}\right)\left(\hat{Y}^* - \hat{A}\hat{X}\right)' \tag{3.2.25}$$

3.5.2 Diagnostic tests

Diagnostic tests are meant to test the adequacy of the model. After fitting VAR (p) model, it is important to check whether the fitted residuals satisfy the model's assumptions. The following are the three main assumptions of a VAR (p) model:

- "The absence of the serial correlation of errors, tested using a Portmanteau test;
- The absence of heteroskedasticity in the errors, tested using an ARCH test; and
- Normal distribution of the residuals, tested using a Jarque-Bera test".

3.5.2.1 Portmanteau test

Edgerton and Shukur (1999) introduced Portmanteau test to test for the absence of serial correlation. The following are the hypotheses tested:

H_0: the residual are not serially correlated

H_1: the residual are serially correlated

The test statistics is described as follows:

$$Q_h = T^2 \sum_{i=1}^{h} \frac{1}{T-i} tr\left(\hat{C}_i' \hat{C}_0^{-1} \hat{C}_j' \hat{C}_0^{-1}\right) \tag{3.2.26}$$

where $\hat{C}_i = \frac{1}{T} \sum_{t=i+1}^{T} \hat{u}_t \, \hat{u}_{t-i}$. The test statistics is asymptotically distributed as a $\chi^2(N^2 h - n)$ where n denotes determistics term of a VAR (p) model. The limiting distribution is valid for h tending to infinity at an approximate growing sample size rate. Therefore, the trade-off is between a descent approximation of the χ^2 distribution and a loss in power of the test when the selected h is too large.

3.5.2.2 Jarque-Bera test

Lutkepohl (2007) introduced Multivariate Jarque-Bera test (JB) a test method which was initially introduced by Jarque and Bera (1980). According to Pfaff (2008), "the test can be computed using the residuals standardized by a Choleski decomposition of the variance-covariance matrix of a VAR (p) model". It is also based on the third and fourth $(E(y^3 = 0)$ and $E(y^4 = 3))$ moments (skewness and kurtosis) of a Gaussian distribution. The following are the hypothesis tested for the JB test:

H_0: the residual are normally distributed

H_1: the residual are not normally distributed

The Multivariate JB test statistics is described as follows:

$$JB_{mv} = \tau_s + \tau_k \tag{3.2.27}$$

Represented as a $\chi^2(2N)$.

where τ_s and τ_k are calculated as

$$\tau_s = \frac{T b_1' b_1}{6} \tag{3.2.28}$$

$$\tau_k = \frac{T(b_2 - 3_N)'(b_2 - 3_N)}{24} \quad (3.2.29)$$

where b_1 and b_2 are third and fourth non–central moment vector of the standardized residuals $\hat{\mu}_t^s = \hat{P} - (\hat{\mu}_t - \bar{\hat{\mu}}_t)$ and $\hat{P}$ denotes a lower triangular matrix. It comprises of diagonal positive values such that $\hat{P}\hat{P}' = \widehat{\sum}_u$ representing the Choleski decomposition of the residual covariance matrix.

3.5.2.3 Multivariate ARCH-LM test

Breusch (1978) introduced Multivariate ARCH-LM test and it is used to test for heteroskedasticity in the fitted residuals. Supposing the error vector, $u_t = B_1 u_{t-1} + \cdots + B_h u_{t-h} + \eta_t$, where $\eta_t$is a white noise. The Multivariate ARCH-LM test is based on the following equation:

$$\hat{u}_t = c + A_1 y_t + \cdots + A_p y_{t-p} + \cdots + B_1 \hat{u}_{t-1} + \cdots + B_h \hat{u}_{t-h} + \epsilon_t \quad (3.2.30)$$

where A_i and B_i are coefficients matrices and $\epsilon_t$is the regression error term. The following are the hypothesis tested for Multivariate ARCH-LM:

H_0: $B_1 = B_2 = \cdots = B_h = 0$ (absence of ARCH errors) alternatively

H_1: $B_i \neq 0$

The Multivariate ARCH-LM test statistic is denoted as:

$$LM_h = T\hat{c}_h' \widehat{\sum}_c^{-1} \hat{c}_h \quad (3.2.31)$$

where $c_h = (C_1 \ldots \ldots C_h)'$ such that $C_h = \frac{1}{T}\sum_{t=h+1}^{T} u_t u_{t-h}'$,$\widehat{\sum}_c$ is the covariance matrix of the residuals.

3.5.3 Forecasting with the VAR model

One of the objectives of Multivariate time series analysis is to predict future values based on the past observed values of a time series. After a VAR model was found adequate from the relevant diagnostic tests, it may be used for predicting future values. For a given VAR (p), h-step ahead forecast is computed using the chain-rule of forecasting as

$$y_{T+h|T} = c + A_1 y_{T+h-1} + \cdots + A_p y_{T+h-p|T} \tag{3.2.32}$$

where $y_{T+j|T} = t_{T+j}$ for $j \leq 0$. The h-step prediction errors are expressed as follows

$$y_{T+h} - y_{T+h|T} = \sum_{s=0}^{h-1} \Psi_s u_{t+h-s} \tag{3.2.33}$$

The matrices Ψ_s are determined by a recursive substitution

$$\Psi_s = \sum_{j=1}^{p-1} \Psi_{s-j} A_j \tag{3.2.34}$$

where $\Psi_0 = I_N$ and $A_j = 0$ for $j > p$. Because all the forecast errors a zero expectation value then the forecasts are unbiased and MSE matrix of $y_{t+h|T}$ is

$$\begin{aligned} \sum(h) &= MSE\left(y_{T+h} - y_{T+h|T}\right) \\ &= \sum_{j=0}^{h-1} \Psi_j \sum \Psi_j' \end{aligned} \tag{3.2.35}$$

The confidence interval of the forecasts was represented as follows:

$$\left[y_{k,T+h|T} - c_{1-\frac{\gamma}{2}} \sigma_k(h), y_{k,T+h|T} + c_{1-\frac{\gamma}{2}} \sigma_k(h)\right] \tag{3.2.36}$$

where $c_{1-\frac{\gamma}{2}}$ implies the $\left(1 - \frac{\gamma}{2}\right)$ percentage point of the normal distribution and $\sigma_k(h)$ is the standard deviation of the k^{th} variable h-step ahead.

3.6 AUTOREGRESSIVE CONDITIONAL HETEROSKEDASTICITY (ARCH) PROCESSES

As proposed by Nobel laureate Robert Engle in 1982, an ARCH model starts from the premise that there is a static regression model. The underlying property of the ARCH process is its ability to capture the tendency for volatility in macroeconomic and financial time series. The ARCH models take account of time-varying variance of a single variable time series. The ARCH models exclude the interaction of the variances. In a dynamic linear regression model, the series $\{X_t: t = 1,2, \dots, N\}$ takes the form:

$$X_t = Y_t'\beta + \varepsilon_t \tag{3.3.1}$$

where $\varepsilon_t = \sigma_t w_t$, $\mathrm{w_t} \sim iid, (0,1)$. Y_t' is an $m \times 1$ vector of independent variables, which may be lagged values of the dependent variable, X_t, and β is an $m \times 1$ vector of regression parameters. In the basic ARCH process, the square of the disturbance term, ε_t, is described as itself following an AR(q) process:

$$\varepsilon_t^2 = \lambda_0 + \sum_{h=1}^{q} \lambda_h \varepsilon_{t-h}^2 + v_t \tag{3.3.2}$$

$$\varepsilon_t^2 = \lambda_0 + \lambda_1 \varepsilon_{t-1}^2 + \lambda_2 \varepsilon_{t-2}^2 + \cdots + \lambda_r \varepsilon_{t-q}^2 + v_t \tag{3.3.3}$$

where $\mathrm{v_t} \sim iid, (0, \delta^2)$. The conditions $\lambda_0 > 0$ and $\lambda_1 \geq 0$ for $i = 1.2,3, \dots.q$ ensure that the conditional variance is always positive. In equation (3.3.3), the distribution of ε_t conditional ξ_{t-1} is

$$\varepsilon_t | \xi_{t-1} \sim N(0, \sigma_t^2), \tag{3.3.4}$$

where

$$\xi_{t-1} = X_{t-1}, Y_{t-1}, X_{t-2}, Y_{t-2}, \dots \tag{3.3.5}$$

3.6.1 Estimation of the ARCH Processes

In a more convenient way, the ARCH process is represented as

$$\sigma_t^2 = \lambda_0 + \sum_{k=1}^{q} \lambda_k \varepsilon_{t-k}^2 \qquad (3.3.6)$$

$$\sigma_t^2 = \lambda_0 + \lambda_1 \varepsilon_{t-1}^2 + \lambda_2 \varepsilon_{t-2}^2 + \cdots + \lambda_q \varepsilon_{t-q}^2 \qquad (3.3.7)$$

where

$$\varepsilon_t = \sigma_t w_t, \qquad \mathrm{w_t} \sim iid, (0,1). \qquad (3.3.8)$$

If σ_t^2 evolves according to equation (3.3.7), then

$$\mathrm{E}(\varepsilon_t^2 | \varepsilon_{t-1}, \varepsilon_{t-2}, \dots) = \lambda_0 + \lambda_1 \varepsilon_{t-1}^2 + \lambda_2 \varepsilon_{t-2}^2 + \cdots + \lambda_q \varepsilon_{t-q}^2 \qquad (3.3.9)$$

and hence

$$\varepsilon_t | \varepsilon_{t-1}, \varepsilon_{t-2}, \dots, \sim N(0, \sigma_\varepsilon^2). \qquad (3.3.10)$$

Now, squaring (3.3.8) yields

$$\varepsilon_t^2 = \sigma_t^2 w_t^2 \qquad (3.3.11)$$

Then, by substituting equation (3.3.11) and (3.3.7) in (3.3.8) and simplifying yields

$$\sigma_t^2 . w_t^2 = \sigma_t^2 v_t \qquad (3.3.12)$$

$$v_t = \sigma_t^2 (w_t^2 - 1) \qquad (3.3.13)$$

$$v_t^2 = \sigma_t^2 (w_t^2 - 1)^2 \qquad (3.3.14)$$

The expectation of equation (3.3.14) is

$$E(v_t^2) = E(\sigma_t^2) \times E[(w_t^2 - 1)^2] \qquad (3.3.15)$$

Equation (3.3.15) implies that the second moment (or the variance) of v_t does not exist for all stationary ARCH processes. For the simple case where the series X_t assumes the AR(1) representation

$$X_t = C + \phi_1 X_{t-1} + \varepsilon_t \qquad\qquad \varepsilon_t \sim iid, (0, \sigma_\varepsilon^2). \tag{3.3.16}$$

Then,

$$\varepsilon_t = \lambda_0 + \lambda_1 \varepsilon_{t-1}^2 + v_t \tag{3.3.17}$$

and

$$\sigma_t^2 = \lambda_0 + \lambda_1 \varepsilon_{t-1}^2 \tag{3.3.18}$$

Squaring both sides of equation (3.3.18) yields

$$[\sigma_t^2]^2 = [\lambda_0 + \lambda_1 \varepsilon_{t-1}^2]^2 = \lambda_0^2 + 2\lambda_0\lambda_1\varepsilon_{t-1}^2 + \lambda_1^2\varepsilon_{t-1}^4 \tag{3.3.19}$$

Hence

$$E[(\sigma_t^2)^2] = \lambda_0^2 + 2\lambda_0\lambda_1 E[\varepsilon_{t-1}^2] + \lambda_1^2 E[\varepsilon_{t-1}^4] \tag{3.3.20}$$

Now,

$$\text{var}[\varepsilon_{t-1}^2] = \text{E}[\varepsilon_{t-1}^4] - \lfloor \text{E}[\varepsilon_{t-1}^2]^2 \rfloor \tag{3.3.21}$$
$$\text{E}[\varepsilon_{t-1}^4] = \lfloor \text{E}[\varepsilon_{t-1}^2]^2 \rfloor + \text{var}[\varepsilon_{t-1}^2] \tag{3.3.22}$$

Thus, equation (3.3.20) becomes

$$E[(\sigma_t^2)^2] = \lambda_0^2 + 2\lambda_0\lambda_1 E[\varepsilon_{t-1}^2] + \lambda_1^2\{\text{var}[\varepsilon_{t-1}^2] + \lfloor \text{E}[\varepsilon_{t-1}^2]^2 \rfloor\} \tag{3.3.23}$$

By (3.3.17), since $E[\varepsilon_t] = E[\varepsilon_{t-1}]$ then

$$E[\varepsilon_t^2] = \lambda_0 + \lambda_1 E[\varepsilon_{t-1}^2] + E[v_t] \tag{3.3.24}$$

$$E[\varepsilon_t^2] = \lambda_0 + \lambda_1 E[\varepsilon_{t-1}^2] + 0 \tag{3.3.25}$$

$$E(\varepsilon_t^2) = \frac{\lambda_0}{1-\lambda_1} \tag{3.3.26}$$

Similarly,

$$\text{var}[\varepsilon_t^2] = 0 + \lambda_1^2 \text{var}[\varepsilon_{t-1}^2] + \text{var}[v_t] \tag{3.3.27}$$

$$(1-\lambda_1^2).\text{var}[\varepsilon_t^2] = \text{var}[v_t] \tag{3.3.28}$$

since $\text{var}(\varepsilon_t^2) = \text{var}(\varepsilon_{t-1}^2)$ and $\text{v}_\text{t} \sim iid, (0, \delta^2)$, equation (3.3.28) simplifies to give

$$\text{var}[\varepsilon_t^2] = \frac{\text{var}[v_t]}{1-\lambda_1^2} = \frac{\delta^2}{1-\lambda_1^2} \tag{3.3.29}$$

Substituting equation (3.3.26) and equation (3.3.29) in equation (3.3.23) and simplifying further yields

$$E[(\sigma_t^2)^2] = \lambda_0^2 + 2\lambda_0\lambda_1 \left[\frac{\lambda_0}{1-\lambda_1}\right] + \lambda_1^2 \left\{\frac{\delta^2}{1-\lambda_1^2} + \left(\frac{\lambda_0}{1-\lambda_1}\right)^2\right\} \tag{3.3.30}$$

$$E[(\sigma_t^2)^2] = \frac{\lambda_1^2\delta^2}{1-\lambda_1^2} + \frac{\lambda_0^2}{(1-\lambda_1)^2} \tag{3.3.31}$$

Also, by equation (3.3.15) then

$$\sigma^2 = \left\{\frac{\lambda_1^2\delta^2}{1-\lambda_1^2} + \frac{\lambda_0^2}{(1-\lambda_1)^2}\right\}.E(w_t - 1)^2 \tag{3.3.32}$$

Now, since $\text{w}_\text{t} \sim iid, (0,1)$, implies

$$\begin{aligned} E[(w_t-1)^2] &= E(w_t^2 - 2w_t + 1) \\ &= E(w_t^2) - 2Ew_t + 1 \\ &= 1 - 2(0) + 1 \end{aligned}$$

$$= 2 \tag{3.3.33}$$

Hence, equation (3.3.32) becomes

$$\sigma^2 = 2\left\{\frac{\lambda_1^2\delta^2}{1-\lambda_1^2} + \frac{\lambda_0^2}{(1-\lambda_1)^2}\right\} = \frac{2\lambda_0^2(1-\lambda_1^2)}{(1-3\lambda_1^2)(1-\lambda_1)^2} \tag{3.3.34}$$

Equation (3.3.34) shows that if $3\lambda_1^2 < 1$, then the 4th moment of ε_t (or the kurtosis) is greater than 3 for positive λ_1, and so the ARCH process yields observations with heavier tails than those of a normal distribution. If $\lambda_1 < 1$, ε_t follows a white noise process while ε_t^2 follows an AR(q) process, yielding volatility clustering (Shepard, 1996).

3.6.2 Testing for ARCH

The study stated that the series X_t follows an ARCH(q) process if it satisfies the mean equation specification:

$$X_t = Y_t'\beta + \varepsilon_t \tag{3.3.35}$$

where $\varepsilon_t = \sigma_t w_t$, $w_t \sim iid, (0,1)$. Y_t' is an $m \times 1$ vector of independent variables, which may be lagged values of the dependent variable, X_t, and β is an $m \times 1$ vector of regression parameters. Then

$$X_t \sim N(Y_t'\beta, \sigma_t^2) \tag{3.3.36}$$

If $\Re_t$ in equation 3.3.37 below is a vector of observations obtained through date *t*, then the conditional distribution of X_t is normal with mean $Y_t'\beta$ and variance σ_t^2 (i.e. by equation (3.3.36)):

$$f(X_t|Y, \Re_t) = \frac{1}{\sigma_t\sqrt{2\pi}} \exp\left[\frac{\varepsilon_t^2}{2\sigma_t^2}\right] \tag{3.3.37}$$

$$f(X_t|Y, \Re_t) = \frac{1}{\sigma_t\sqrt{2\pi}} \exp\left[-\frac{(X_t - Y_t'\beta)^2}{2\sigma_t^2}\right] \tag{3.3.38}$$

since $\varepsilon_t = X_t - Y_t'\beta$. Denoting the parameters which index the model by Θ, the conditional likelihood and the log conditional likelihood are, respectively, given by

$$L = \sum_{t=1}^{N} f(X_t|Y, \Re_t; \Theta) = \left(\frac{1}{\sigma_t\sqrt{2\pi}}\right)^N \exp\left[\sum_{t=1}^{N} -\frac{(X_t - Y_t'\beta)^2}{2\sigma_t^2}\right] \quad (3.3.39)$$

$$\ln L = -\frac{N}{2}\ln(2\pi) - \frac{1}{2}\ln\sum_{t=1}^{N}\ln(\sigma_t^2) - \frac{1}{2\sigma_t^2}\sum_{t=1}^{N}(X_t - Y_t'\beta)^2 \quad (3.3.40)$$

The log likelihood function equation (3.3.40) can then be maximised with respect to the unknown parameters $\lambda = (\lambda_0, \lambda_1, \lambda_2, \dots, \lambda_r)'$ and β. Consider the simplest ARCH(1) process

$$\sigma_t^2 = \lambda_0 + \lambda_1\varepsilon_{t-1}^2 + v_t \quad (3.3.41)$$

where

$$\varepsilon_t = \sigma_t w_t,\ \ w_t \sim i.i.d(0,1) \quad (3.3.42)$$

The log conditional likelihood is

$$\ln(\mathrm{L}) = f(\mathrm{X_t}|\mathrm{X_0}; \Theta) = -\frac{1}{2}\sum_{\mathrm{i}=1}^{\mathrm{N}}\ln(\sigma_\mathrm{t}^2) - \frac{1}{2\sigma_\mathrm{t}^2}\sum_{\mathrm{t}=1}^{\mathrm{N}}\mathrm{X_t^2} \quad (3.3.43)$$

where $\Theta = (\lambda_0, \lambda_1)'$. The null hypothesis, $H_0{:}\,\lambda_1 = 1$, that there is no volatility clustering in the series, turns out to be the usual analogue of the Box-Pierce Portmanteau test for the AR(1) process or the MA(1) process, but in squares. With no specific alternative to the test, Engle (1982) recommends a Lagrangian Multiplier (LM) test of the alternative hypothesis of ARCH(q) disturbances since such a test can be computed from running the auxiliary regression

$$\hat{\varepsilon}_t^2 = \hat{\lambda}_0 + \hat{\lambda}_1\hat{\varepsilon}_{t-1}^2 + \hat{\lambda}_2\hat{\varepsilon}_{t-2}^2 + \cdots + \hat{\lambda}_r\hat{\varepsilon}_{t-q}^2 \quad (3.3.44)$$

Under the null hypothesis of no volatility

$H_0 : \lambda_1 = \lambda_2 = \cdots = \lambda_q = 0$

The appropriate test statistic is given by the following equation:

$$TS = NR^2 \qquad (3.3.45)$$

where R^2 is the coefficient of determination from the auxiliary regression (3.3.44), is tested as $\chi^2(q)$. The hypothesis of no serial correlation (no volatility) is rejected if test statistic is greater than the corresponding chi-square value. Alternatively, reject the null if the probability of obtaining such a chi-square value is much less than a certain nominal value, say 0.05.

3.6.3 Forecasting with an ARCH Process

In time series analysis, one important aim is to be able to model the series and also to be able to forecast. The relation Section 3.3.3

$$\varepsilon_t^2 = \lambda_0 + \lambda_1 \varepsilon_{t-1}^2 + \lambda_2 \varepsilon_{t-2}^2 + \cdots + \lambda_r \varepsilon_{t-q}^2 + v_t \qquad (3.3.46)$$

where $v_t \sim i.i.d(0, \delta^2)$ implies that ε_t^2 follows an AR(q) process. Thus, the unconditional variance of ε_t is

$$\begin{aligned} \text{var}(\varepsilon_t) = E(\varepsilon_t^2) &= E\left(\lambda_0 + \lambda_1 \varepsilon_{t-1}^2 + \lambda_2 \varepsilon_{t-2}^2 + \cdots + \lambda_r \varepsilon_{t-q}^2 + v_t\right) \\ &= \lambda_0 + \lambda_1 E(\varepsilon_{t-1}^2) + \lambda_2 E(\varepsilon_{t-2}^2) + \cdots + \lambda_r E\left(\varepsilon_{t-q}^2\right) + E(v_t) \end{aligned}$$

$$E(\varepsilon_t^2) = \lambda_0 + \lambda_1 E(\varepsilon_t^2) + \lambda_2 E(\varepsilon_t^2) + \cdots + \lambda_q E(\varepsilon_t^2) + 0 \qquad (3.3.47)$$

since $E(\varepsilon_t^2) = E(\varepsilon_{t-1}^2) = E(\varepsilon_{t-2}^2) = \cdots = E\left(\varepsilon_{t-q}^2\right)$ Simplify (3.3.47) further yields

$$\text{var}(\varepsilon_t) = E(\varepsilon_t^2) = \frac{\lambda_0}{1 - \lambda_1 - \lambda_2 - \cdots - \lambda_q} \qquad (3.3.48)$$

or

$$\sigma^2 = \frac{\lambda_0}{1-\lambda_1-\lambda_2-\cdots-\lambda_q} \tag{3.3.49}$$

The *s*-period-ahead linear forecast is

$$\hat{\varepsilon}^2_{t+s|t} = \hat{E}(\varepsilon^2_{t+s}|\varepsilon^2_t, \varepsilon^2_{t-1}, \dots) \tag{3.3.50}$$

From equation (3.3.49), then

$$\lambda_0 = \sigma^2 - \lambda_1\sigma^2 - \lambda_2\sigma^2 - \cdots \lambda_q\sigma^2 \tag{3.3.51}$$

Substituting equation (3.3.51) in equation (3.3.46) and simplifying the results gives

$$(\varepsilon^2_t - \sigma^2) = \lambda_1(\varepsilon^2_{t-1} - \sigma^2) + \lambda_2(\varepsilon^2_{t-2} - \sigma^2) + \cdots + \lambda_q\left(\varepsilon^2_{t-q} - \sigma^2\right) + v_t \tag{3.3.52}$$

and hence

$$(\hat{\varepsilon}^2_t - \sigma^2) = \lambda_1(\hat{\varepsilon}^2_{t-1} - \sigma^2) + \lambda_2(\hat{\varepsilon}^2_{t-2} - \sigma^2) + \cdots + \lambda_q\left(\hat{\varepsilon}^2_{t-q} - \sigma^2\right) + v_t \tag{3.3.53}$$

The s-period-ahead forecast can be calculated from

$$\left(\hat{\varepsilon}^2_{t+k|t} - \sigma^2\right) = \lambda_1\left(\hat{\varepsilon}^2_{t+k-1|t} - \sigma^2\right) + \lambda_2\left(\hat{\varepsilon}^2_{t+k-2|t} - \sigma^2\right) + \cdots + \lambda_q\left(\hat{\varepsilon}^2_{t+k-q|t} - \sigma^2\right) \tag{3.3.54}$$

for $k = 1,2,3 \dots, s$, with $\hat{\varepsilon}^2_{u/t} = \varepsilon^2_u$ for $u \le t$.

3.6.4 Extensions of the ARCH Process: A Review

The ARCH concept has been extended in several ways since its introduction. The most important of these extensions is the Generalised ARCH (GARCH) process due to Bollerslev (1986). In this section, the brief discussion of some of these extensions are done. GARCH models are used in modeling volatility. The GARCH model emphasise on the conditional variance which is the variance conditional of the past.

3.6.4.1 The ARCH-in-Mean (ARCH-M) Process

The ARCH-Mean process due to Engle, Lilien and Robins (1987) is an extension of the basic ARCH concept to allow the mean of a series to depend on its own conditional variance. The motivation has been derived from the fact that the mean and the variance of a return are expected to move in the same direction. The process is therefore suitable to the study of the relationship between risky asset and level of volatility. Denote the mean by μ_t, where

$$\mu_t = \beta_0 + bf(\sigma_t^2) \tag{3.3.55}$$

A time series $\{X_t: t = 1,2, \dots, N\}$ follows an ARCH-in-Mean process if it satisfies the mean equation

$$X_t = Y_t'\beta + bf(\sigma_t^2) + \varepsilon_t \tag{3.3.56}$$

where

$$\varepsilon_t | \xi_{t-1} \sim N(0, \sigma_t^2) \tag{3.3.57}$$

and

$$\sigma_t^2 = \lambda_0 + \lambda_1 \varepsilon_{t-1}^2 + \lambda_2 \varepsilon_{t-2}^2 + \cdots + \lambda_r \varepsilon_{t-q}^2 \tag{3.3.58}$$

where $f(\sigma_t^2)$ is a function of σ_t^2, with $f(\lambda_0) = 0$. In finance, $bf(\sigma_t^2)$ represents the expected rate of return due to an increase in the variance of the return (i.e. the risk premium). For the simple ARCH-M process where $\varepsilon_t \sim ARCH(1)$

$$X_t = bf(\sigma_t^2) + \varepsilon_t \tag{3.3.59}$$

Then

$$f(\sigma_t^2) = \lambda_0 + \lambda_1 \varepsilon_t^2 \tag{3.3.60}$$

$$\mu_t = \beta_0 + b[\lambda_0 + \lambda_1 \varepsilon_t^2] \quad (3.3.61)$$

and

$$X_t = b[\lambda_0 + \lambda_1 \varepsilon_t^2] + \varepsilon_t \quad (3.3.62)$$

or

$$X_t = b\lambda_0 + b\lambda_1 \varepsilon_{t-1}^2 + \varepsilon_t \quad (3.3.63)$$

Then using the fact that

$$E(\varepsilon_t^2) = E(\varepsilon_{t-1}^2) = \frac{\lambda_0}{1-\lambda_1} \quad (3.3.64)$$

it follows immediately that

$$E(X_t) = b\lambda_0 \left[1 + \frac{\lambda_0}{1-\lambda_1}\right] \quad (3.3.65)$$

Equation (3.3.65) is viewed as the unconditional expected return of holding a risky asset. In a similar fashion, it can be shown that

$$\mathrm{var}(X_t) = \frac{\lambda_0}{1-\lambda_1} + \frac{2\lambda_0^2(b\lambda_1)^2}{(1-\lambda_1)^2(1-3\lambda_1^2)} \quad (3.3.66)$$

In the absence of a risk premium, $bf(\sigma_t^2) = b\lambda_1 = 0$, and so (3.3.66) becomes

$$\mathrm{var}(X_t) = \frac{\lambda_0}{1-\lambda_1} \quad (3.3.67)$$

Other statistical properties of the ARCH-M process have been considered in Hong (1991). In most applications, using

$$f(\sigma_t^2) = \ln(\sigma_t^2) \tag{3.3.68}$$

has been found to work better in the estimation of time-varying risk premiums (Engle *et al.*, 1987). The use of the ARCH-M process for measuring risk has been criticised in the literature, for instance Backus, Gregory and Zin (1989) and Backus and Gregory (1993). It is argued that there does not necessary exist any relationship between risk premium and conditional variances.

3.7 THE GENERALISED ARCH (GARCH) PROCESS

The GARCH model was introduced by Bollerslev (1986) as an extension of the ARCH model. The GARCH model has the ability to capture volatility in the simplest form. A time series $\{X_t : t = 1,2,\ldots,N\}$ follows the Generalised ARCH or GARCH(*p.q*) process if it satisfies the mean equation specification

$$X_t = Y_t'\beta + \varepsilon_t \tag{3.4.1}$$

where $\varepsilon_t = \sigma_t w_t$, $w_t \sim i.i.d(0,1)$. Y_t' is an $m \times 1$ vector of independent variables, which may be lagged values of the dependent variable, X_t, and β is an $m \times 1$ vector of regression parameters. The specified conditional variance equation is representable as

$$\sigma_t^2 = \lambda_0 + \sum_{i=1}^{q} \lambda_1 \varepsilon_{t-1}^2 + \sum_{i=1}^{p} \alpha_i \sigma_{i-1}^2 \tag{3.4.2}$$

where

$\lambda_0 > 0$,

$\lambda_1 \geq 0$ for $i = 1,2,\ldots,q$,

$\alpha_i \geq 0$ for $i = 1,2,\ldots,p$,

and

$\varepsilon_t = \sigma_t w_t$ with $w_t \sim i.i.d(0,1)$

The disturbance term is weakly stationary if

$$\left(\sum_{i=1}^{q} \lambda_i + \sum_{i=1}^{p} \alpha_i\right) < 1 \tag{3.4.3}$$

Writing equation (3.4.2) as

$$\sigma_t^2 = \lambda_0 + \lambda_1(B)\varepsilon_{t-1}^2 + \alpha(B)\sigma_t^2 \tag{3.4.4}$$

where $\lambda(B) = \lambda_1 B + \lambda_2 B^2 + \cdots + \lambda_q B^q$, $\alpha(B) = \alpha_1 B + \alpha_2 B^2 + \cdots + \alpha_p B^p$ and B, the backshift operator, equation (3.4.4) becomes

$$\sigma_t^2 - \alpha(B)\sigma_t^2 = \lambda_0 + \lambda(B)\varepsilon_t^2 \tag{3.4.5}$$

$$[1 - \alpha(B)]\sigma_t^2 = \lambda_0 + \lambda(B)\varepsilon_t^2 \tag{3.4.6}$$

$$\sigma_t^2 = \frac{\lambda_0}{1-\alpha(B)} + \frac{\lambda(B)}{1-\alpha(B)}\varepsilon_t^2 \tag{3.4.7}$$

If the roots z=$z_1, z_2, \dots z_p$ of $1 - \alpha(B)$ lie outside the unit circle, equation (3.4.7) becomes

$$\sigma_t^2 = \frac{\lambda_0}{1-\alpha(1)} + \frac{\lambda(B)}{1-\alpha(B)}\varepsilon_t^2 \tag{3.4.8}$$

$$\sigma_t^2 = \lambda_0^* + \sum_{i=1}^{\infty} h_i \varepsilon_{t-i}^2 \tag{3.4.9}$$

where $\lambda_0^* = \frac{\lambda_0}{1-\alpha(1)}$ and h_i is the coefficient of B^i in the expansion of $\frac{\alpha(B)}{1-\alpha(B)}$. Equation (3.4.9) is simply a GARCH(p,q) process with an infinite order ARCH process. Nelson and Cao (1992) have shown that even though the conditions under Section 3.4.4.1 are sufficient to ensure a strictly positive conditional variance, setting

$$\lambda_0^* > 0 \quad \text{and } h_i \geq 0 \tag{3.4.10}$$

where $i = 1,2,3, \dots, \infty$ will equally ensure a strictly positive conditional variance. Consider, for instance, the GARCH(1,1) process

$$\sigma_t^2 = \lambda_0 + \lambda_1 \varepsilon_{t-1}^2 + \alpha_1 \sigma_t^2 \qquad (3.4.11)$$

Nelson and Cao (1992) were able to show that the conditional variance is strictly positive if based on the following conditions:

$$\lambda_0 > 0, \lambda_1 \geq 0, \alpha_1 \geq 0, \text{ and } \alpha_1\lambda_1 + \lambda_2 \geq 0, \qquad (3.4.12)$$

As in the case of ARCH(1) process, in the most commonly used GARCH(1,1) process,

$$\sigma_t^2 = \lambda_0 + \lambda_1 \varepsilon_{t-1}^2 + \alpha_1 \sigma_t^2 \qquad (3.4.13)$$

Hwang and Satchell (1998) have shown in Knight and Satchell (1998) that the logarithmic likelihood function is

$$\ln \mathrm{L}\,(\lambda_0, \lambda_1, \alpha_1) = -\frac{1}{2}\ln(2\pi) - \frac{1}{2}\sum_{i=0}^{n}\left[\ln(\sigma_t^2) + \frac{X_t^2}{\sigma_t^2}\right] \qquad (3.4.14)$$

Hwang and Satchell (1998) further showed that the s-step-ahead forecast from the GARCH(1.1) process is given by

$$E(X_{t+s}^2) = \lambda_0 \sum_{i=0}^{s-1}(\lambda_1 + \alpha_1)^i + (\lambda_1 + \alpha_1)^{s-1}\lambda_1\alpha_t^2 + (\lambda_1 + \alpha_1)^{s-1}\alpha_1 X_t^2 \quad \text{for } s > 1 \qquad (3.4.15)$$

and

$$E(X_{t+s}^2) = \lambda_0 \sum_{i=0}^{s-1}(\lambda_1 + \alpha_1)^i + (\lambda_1 + \alpha_1)^{s-1}\sigma_{t+1}^2 \quad \text{for} \quad s > 2. \qquad (3.4.16)$$

Thus, for large s and $\lambda_1 + \alpha_1 < 1$, then

$$E(X_{t+s}^2) = \lambda_0 \sum_{i=0}^{s-1}(\lambda_1 + \alpha_1)^i = \frac{\lambda_0}{1-\lambda_1-\alpha_1} \quad \text{as } s \longrightarrow \infty \qquad (3.4.17)$$

Lastly, from the GARCH(1.1) process, the condition

$$3\lambda_1^2 + 2\lambda_1\alpha_1 + \alpha_1^2 < 1 \tag{3.4.18}$$

means the 4th moment (or the kurtosis) of ε_t is greater than that of a normal random variable. Consequently, the GARCH process is capable of producing outliers. One important feature of $GARCH(q.p)$ processes is that the conditional variance of the disturbances of the series X_t follows an $ARMA(r.q)$ process. That is if

$$\varepsilon_t^2 = \sigma_t^2 + u_t \tag{3.4.19}$$

then

$$\varepsilon_t^2 = \lambda_0 + \sum_{i=1}^{r}(\lambda_1 + \alpha_i)\,\varepsilon_{t-1}^2 + u_t - \sum_{i=1}^{p}\alpha_i(\varepsilon_{t-1}^2 - \sigma_{i-1}^2) \tag{3.4.20}$$

equation (3.4.20) can be written as

$$\varepsilon_t^2 = \lambda_0 + \sum_{i=1}^{r}(\lambda_1 + \alpha_i)\,\varepsilon_{t-1}^2 + u_t - \sum_{i=1}^{q}\alpha_i u_{t-1} \tag{3.4.21}$$

where $r = max(q,p)$, $\lambda_i = 0$ for $i > p$, $\alpha_i = 0$ for $i > q$. It comes from equation (3.4.21) that ε_t^2 has an $ARMA(r.q)$ representation. Therefore, it is expected that the residuals from the fitted ARMA process follow a white noise process. The autocorrelation function of the squared residuals, $\hat{\varepsilon}_t^2$, aid in determining the order of the GARCH process. In fact, McLeod and Li (1983) suggest estimating the best-fitting ARIMA model (or regression model) and calculating the sample autocorrelation (ACF) of the squared residuals, ε_t^2:

$$\hat{\rho}_k(\varepsilon) = \frac{\sum_{t=k+1}^{N}(\hat{\varepsilon}_t^2 - \hat{\sigma}^2)(\hat{\varepsilon}_{t-k}^2 - \hat{\sigma}^2)}{\sum_{t=1}^{N}(\hat{\varepsilon}_t^2 - \hat{\sigma}^2)} \tag{3.4.22}$$

where

$$\hat{\sigma}^2 = \sum_{t=1}^{N}\frac{\hat{\varepsilon}_t^2}{N} \tag{3.4.23}$$

The Box-Pierce Portmanteau statistic

$$\varrho(\varepsilon) = N(N+2)\sum_{k=1}^{m} \frac{\hat{\gamma}_{k(\varepsilon)}}{(N-K)} \tag{3.4.24}$$

which is asymptotically distributed as $\chi^2(m)$, where m is the number of autocorrelations used in the test, can then be used to test for groups of significant coefficients. Rejecting the null hypothesis,

$H_0: \hat{\varepsilon}_t^2$ are uncorrelated,

is equivalent to rejecting null hypothesis of no ARCH or GARCH errors. Equivalently, the LM test proposed by Engle (1982) and discussed in Section 3.4.3 can be used. Researchers have revealed that a process greater than GARCH(1.2) or GARCH(2.1) are very uncommon.

3.8 Integrated GARCH (IGARCH) Process

A time series X_t following a standard GARCH(1.1) process takes the following mean equation specification and conditional variance equation:

$$X_t = Y_t'\beta + \varepsilon_t \tag{3.4.25}$$

and

$$\sigma_t^2 = \lambda_0 + \lambda_1\varepsilon_{t-1}^2 + \alpha_1\sigma_t^2 \quad \text{where} \quad \varepsilon_t = \sigma_t w_t. \tag{3.4.26}$$

Now,

$$\varepsilon_t^2 = \sigma_t^2 + \alpha_1\varepsilon_{t-1}^2 + \varepsilon_t^2 - \sigma_t^2 - \alpha_1\varepsilon_t^2 \tag{3.4.27}$$

From equation (3.4.27)

$$\varepsilon_t^2 = (\sigma_t^2 + \alpha_1\varepsilon_{t-1}^2) + (\varepsilon_t^2 - \sigma_t^2) - \alpha_1\varepsilon_t^2 \tag{3.4.28}$$

Equation (3.4.28) can be written as

$$\varepsilon_t^2 = (\sigma_t^2 + \alpha_1\varepsilon_{t-1}^2) + \left(\frac{\varepsilon_t^2}{\sigma_t^2} - 1\right) - \alpha_1\varepsilon_t^2 \tag{3.4.29}$$

Substituting the relation $\sigma_t^2 = \lambda_0 + \lambda_1\varepsilon_{t-1}^2 + \alpha_1\sigma_{t-1}^2$ in equation (3.4.29) yields

$$\varepsilon_t^2 = (\lambda_0 + \lambda_1\varepsilon_{t-1}^2 + \alpha_1\sigma_{t-1}^2 + \alpha_1\varepsilon_{t-1}^2) + \alpha_1^2\left(\frac{\varepsilon_t^2}{\sigma_t^2} - 1\right) - \alpha_1\varepsilon_t^2 \ ,$$

$$\varepsilon_t^2 = \lambda_0 + (\lambda_1 + \alpha_1)\varepsilon_{t-1}^2 + \alpha_1^2\left(\frac{\varepsilon_t^2}{\sigma_t^2} - 1\right) - \alpha_1(\varepsilon_{t-1}^2 - \sigma_{t-1}^2),$$

$$\varepsilon_t^2 = \lambda_0 + (\lambda_1 + \alpha_1)\varepsilon_{t-1}^2 + \alpha_1^2\left(\frac{\varepsilon_t^2}{\sigma_t^2} - 1\right) - \alpha_1\sigma_{t-1}^2\left(\frac{\varepsilon_t^2}{\sigma_t^2} - 1\right) \tag{3.4.30}$$

Using the relation $\varepsilon_t = \sigma_t w_t$, the study have

$$e_t = \sigma_t^2(w_t - 1) \ \text{ and } e_{t-1} = \sigma_{-1t}^2(w_{t-1} - 1) \tag{3.4.31}$$

Hence, equation (3.4.30) becomes

$$\varepsilon_t^2 = \lambda_0 + (\lambda_1 + \alpha_1)\varepsilon_{t-1}^2 + e_t - \alpha_1 e_{t-1} \tag{3.4.32}$$

Equation (3.4.32) implies that the GARCH process can be written as an ARMA process. If $\lambda_1 + \alpha_1 < 1$, then the original series $\{X_t: t = 1,2, \dots, N\}$ is covariance stationary. If $\lambda_1 + \alpha_1 = 1$, equation (3.4.32) becomes

$$\varepsilon_t^2 = \lambda_0 + \varepsilon_{-t-1}^2 + e_t - \alpha_1 e_{t-1} \tag{3.4.33}$$

Equation (3.4.33) is then rewritten as

$$\varepsilon_t^2 - \varepsilon_{-t-1}^2 = \lambda_0 + e_t - \alpha_1 e_{t-1} \quad \text{or} \quad \varepsilon_t^2 - B\varepsilon_t^2 = \lambda_0 + e_t - \alpha_1 e_{t-1}, \tag{3.4.34}$$

where B is the backshift operator. Equation (3.4.34) can compactly be written as

$$(1 - B)\varepsilon_t^2 = \lambda_0 + e_t - \alpha_1 e_{t-1}, \tag{3.4.35}$$

Equation (3.4.35) leads to an analogy with an ARIMA(0,1,1) process with an intercept in terms of defining an autocorrelation function of squared observations. Equation (3.4.35) is called Integrated GARCH or IGARCH since the squared observation are stationary in first differences, but does not follow that ε_t^2 will behave like an integrated process. For many empirical studies using high-frequency data, $\lambda_1 + \alpha_1$ is estimated to be close to 1, suggesting that volatility has quite persistent shocks. That is, the null hypothesis of a unit root in variance

$$H_0: \lambda_1 + \alpha_1 = 1$$

is mostly accepted using high-frequency data. For example, French, Schwert and Stambaugh (1987), Chou (1988), Pagan and Schwert (1990) do not reject the null hypothesis of unit root in variance $\left(\lambda_1 + \alpha_1 + \lambda_2 + \alpha_2 + \cdots + \lambda_q + \alpha_p = 1\right)$ when the IGARCH process was applied to different stock market data.

3.9 Exponential GARCH (EGARCH) Process

A possible limitation of the GARCH process is that the conditional variance σ_t^2 responds to positive and negative residuals ε_{t-i} in the same manner, i.e. σ_t^2 may be symmetric in ε_{t-i}. Nelson (1991) argued that a symmetric conditional variance function may be inappropriate for modeling volatility of returns on stocks since it cannot represent the leverage effect which is negative correlation between volatility and past returns. Nelson (1991) therefore proposed the concept of Exponential GARCH or EGARCH. The EGARCH process enables the conditional variance to respond to positive and negative residuals asymmetrically. A time series $\{X_t: t = 1,2, \ldots, N\}$ follows an EGARCH(*p*,*q*) process if it satisfies the following specifications:

$$\mathrm{X_t = Y_t'\beta + \varepsilon_t \ \text{ with } \varepsilon_t = \sigma_t w_t \text{ where } \ w_t \sim N(0,1)} \tag{3.4.36}$$

$$\ln(\sigma_t^2) = \omega + \beta \ln(\sigma_{t-1}^2) + \gamma \frac{\varepsilon_{t-1}}{\sqrt{\sigma_{t-1}^2}} + \alpha \left[\frac{|\varepsilon_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - \sqrt{\frac{2}{\pi}} \right] \tag{3.4.37}$$

where ω is a constant parameter. $\beta \ln(\sigma_{t-1}^2)$ denotes the fitted variance from the previous period, γ is the value of the leverage term, α is the symmetric effect and β denotes the past volatility coefficient. If the value of $\gamma > 0$ then it is concluded that there is a larger impact for negative shocks on the conditional variance.

3.10 Threshold GARCH (TGARCH) Process

The application of the EGARCH process to represent asymmetric responses in the conditional variance to positive and negative errors has motivated to the proposal of the Threshold GARCH or the TGARCH(*p*,*q*) process. Proposed independently by Zakoian (1994) and Glosten, Jaganathan, and Runkle (1993), the specification for the conditional variance is

$$\sigma_t^2 = \lambda_0 + \sum_{i=q}^{q} \lambda_i \varepsilon_{i-1}^2 + c_1 \varepsilon_{t-1}^2 d_{t-1} + \sum_{i=1}^{p} \alpha_i \sigma_{t-1}^2, \tag{3.4.38a}$$

where

$$d_t = \begin{cases} 1 & , \quad \varepsilon_t > 0 \\ 0 & , \quad \varepsilon_t \leq 0. \end{cases} \tag{3.4.38b}$$

In this specification, news has differential impacts on the conditional variance, σ_t^2. Consider the simple TGARCH(1.1) process

$$\sigma_t^2 = \lambda_0 + \lambda_1 \varepsilon_{t-1}^2 + c_1 \varepsilon_{t-1}^2 d_{t-1} + \alpha_1 \sigma_{t-1}^2, \tag{3.4.39}$$

For good news, $\varepsilon_t \leq 0$ and $d_t = 0$. Hence, (3.4.39) becomes

$$\sigma_t^2 = \lambda_0 + \lambda_1 \varepsilon_{t-1}^2 + \alpha_1 \sigma_{t-1}^2 \tag{3.4.40}$$

Similarly, for bad news, $\varepsilon_t > 0$ and $d_t = 1$. The specification equation (3.4.39) is

$$\sigma_t^2 = \lambda_0 + (\lambda_1 + c)\varepsilon_{t-1}^2 + \alpha_1\sigma_{t-1}^2 \tag{3.4.41}$$

Equation (3.4.40) and (3.4.41) show that the impact of good news is λ_1, while bad news has an impact of $\lambda_1 + c$. Leverage effects exist if $c_1 > 0$. News impact is asymmetric if $c_1 \neq 0$.

The focus of the study is on the standard GARCH, TGARCH and the EGARCH models. The EGARCH and TGARCH processes were included as an extended form of GARCH as they enable the conditional variance to respond to positive and negative residuals asymmetrical effects. Nelson (1991) suggested the EGARCH model as an extension of the GARCH to deal with overcoming the weakness encountered in using the standard GARCH. TGARCH allows for asymmetric effects of good and bad news. Lim and Sek (2013) proposed that the EGARCH model uses its exponential nature to capture the effect of the external unexpected shocks on the predicted volatility. The models (EGARCH and TGARCH) were selected due to their asymmetrical nature common characteristic as opposed to other GARCH family models.

3.11 MULTIVARIATE GARCH MODEL

The Multivariate GARCH model is basically the extension of the univariate GARCH models that it is significant to predict the dependence in the co-movement of the BRICS countries. There are several Multivariate GARCH model formulations which have been proposed in the literature, and the most popular of these are the diagonal VECH, the diagonal BEKK, CCC and DCC models. For a Multivariate time series $\mathrm{Y_{t=}y_{1t}, y_{2t}, y_{3t}, \ldots, y_{kn}}$ the Multivariate GARCH model is given by

$$\mathrm{y_t = P_t^{1/2}\varepsilon_t} \tag{3.5.1}$$

where, P is k × k positive-definite matrix and of the conditional variance of C_t, k is the number of series and t = 1,2,…,n (number of observations). It is with the specification of conditional variance that the Multivariate GARCH model changes. Bollerslev (1986) describes a general GARCH (p. q) as follows

$$h_t = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \cdots + \alpha_p\varepsilon_{t-p}^2 + \beta_1 h_{t-1} + \cdots + \beta_q h_{t-q,} \quad (3.5.2)$$

$$\alpha_1 > 0, \beta_1 > 0, \alpha_1 + \beta_1 < 1$$

where h_t is conditional variance dependent on the previous error term as well as the previous conditional variance of the process. The main issue in Multivariate GARCH is to develop the conditional variance-covariance matrix (S) from equation (3.5.2). It is transferred into Multivariate GARCH model with a generalization of the resulting variance matrix S_t below

$$S_t=\begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix} \quad (3.5.3)$$

Every element of S_t depends on the p delayed values of the squared ε_t, the cross product of ε_t and on the q delayed values of elements from S_t.

3.11.1 The diagonal VECH

The diagonal VECH is the first general model introduced by Bollerslev et al. in 1988. In the VECH model, every conditional variance and covariance is a function of all lagged conditional variances and covariances, as well as lagged squared returns and cross-products of returns. The model can be expressed below

$$\text{VECH}\ (H_t) = c + \sum_{j=1}^{q} D_j\ VECH\ (\varepsilon_{t-j}\varepsilon'_{t-j}) + \sum_{j-1}^{p} E_j\ VECH(H_{t-j}) \quad (3.5.4)$$

where, VECH (H_t) is an operator that stacks the columns of the lower triangular part of its argument square matrix, H_t is the covariance matrix of the residuals, N presents the number of variables, t is the index of the t^{th}observation, c is an $\frac{N(N+1)}{2} * 1$ vector, D_j and E_j are $\frac{N(N+1)}{2} * \frac{N(N+1)}{2}$ parameter matrices and ε is an $N \times 1$ vector. The condition for H_t is to be positive definite for all t is not restrictive.

To ensure that positive definiteness is enforced, a new parameterization of the conditional variance matrix H_t was defined by Baba et al. (1990) and became known as the BEKK model.

The model is viewed as another restricted version of the VECH model. The positive definiteness of the conditional variance is achieved by formulating the model to suite the model structure.

3.11.2 The diagonal BEKK

Engle and Kroner (1995) introduced the BEKK model which is the direct generalization of the univariate GARCH model. The outcome variance is dependent on the state of the information present. The form of the BEKK model is as

$$H_t = CC' + \sum_{j=1}^{q}\sum_{j=1}^{k} D'_{kj}\varepsilon_{t-j}\varepsilon'_{t-j}D_{kj} + \sum_{j=1}^{q}\sum_{j=1}^{k} E'_{kj}H_{t-j}E_{kj} \tag{3.5.5}$$

where D_{kj}, E_{kj} and C are $N \times N$ parameter matrices. and C is a lower triangular matrix. The reason for decomposing the constant term into a product of two triangle matrices is to guarantee the positive semi-definiteness of H_t. Whenever K > 1, an identification problem would be generated for the reason that there are not only single parameterizations that can obtain the same representation of the model. The first order BEKK model is given as

$$H_t = CC' + D'\varepsilon_{t-1}\varepsilon'_{t-1}D + E'H_{t-1}E \tag{3.5.6}$$

The BEKK model also has its diagonal form by assuming D_{kj}, E_{kj}matrices are diagonal. This model is a restricted version of the diagonal VECH model.

3.11.3 The CCC models

Bollerslev in 1990 introduced the CCC model which was primarily intended to model the condition covariance matrix indirectly by estimating the conditional correlation matrix. It follows that the conditional correlation is assumed to be constant and in the conditional variances are varying nature. Consider the CCC model of Bollerslev (1990)

$$y_t = F\langle y_t|G_{t-1}\rangle + \varepsilon_t, \qquad \varepsilon_t = D\eta_t \tag{3.5.7}$$

$$Var\,\langle \varepsilon_t|G_{t-1}\rangle = D_t\Gamma D_t \tag{3.5.8}$$

where, $y_t = (y_{1t}, \dots \dots y_{mt})'$ $\eta_t = \eta_{1t}, \dots \dots \eta_{mt}$ is a sequence of independently and identically distributed random vector. G_t is the past information available at time t. $D_t = Diag\left(h_t^{\frac{1}{2}}, \dots h_m^{\frac{1}{2}}\right)$

The CCC model assumes that the conditional variance for each exchange rates h_{it}, i=1........,m and it follows a univariate GARCH process, which follows

$$\sigma_t^2 = \lambda_0 + \sum_{i=1}^{q} \lambda_1 \varepsilon_{t-1}^2 + \sum_{i=1}^{p} \alpha_i \sigma_{i-1}^2, \qquad (3.5.9)$$

where λ_0, λ_1 and α_i , are nonnegative and $\sum_{i=1}^{q} \lambda_i + \sum_{i=1}^{p} \alpha_i$ for i = 1 ...k.

3.11.4 The DCC models

The CCC model was deemed to be inconsistent with reality in accordance with (Longin and Solink, 1995, 2001). Therefore, Engle (2002) developed a new Multivariate dynamic conditional correlation GARCH model to address the inconsistencies raised in relation with the CCC model. Due to the dynamic nature of the model, it was termed DCC-GARCH model, and it has the dynamic presumption of conditional correlation coefficients among different variables. Engle (2002) introduced the DCC model and was illustrated below

$$H_t = D_t R_t D_t \qquad (3.5.10)$$

where R_t is the conditional correlation matrix of the exchange rates vector $r_t = r_{1t}, \dots \dots r_{nt}$, $D_t = diag\{\sqrt{hit}\}$ is a 5*5 diagonal matrix and R_t matrix is given by

$$R_t = diag(Q_t)^{-1} Q_t diag(Q_t)^{-1} \qquad (3.5.11)$$

$$Q_t = (1 - \upsilon_1 - \upsilon_2)\bar{Q} + \upsilon_1(\eta_{t-1}\eta_{t-1}') + \upsilon_2 Q_{t-1} \qquad (3.5.12)$$

where $Q_t = \{\rho_{ij}\}$ is a (time-invariant) K*K positive definite parameter matrix with unit diagonal elements. The DCC-GARCH model is process is estimated by the MLE method and the log-likelihood is expressed as follows

$$L = \frac{-1}{2}\sum_{t=1}^{T}(nlog(2\pi) + 2log|D_t| + log|R_t| + \varepsilon_t' R_t' \varepsilon_t) \qquad (3.5.13)$$

From the above four models, diagonal VECH, the diagonal BEKK, CCC and DCC models, the study will focus on the BEKK model and the DCC. An advantage of the BEKK model is that E' is positive definite if the diagonal elements of C is positive and DCC has a K*K positive definite parameter matrix with unit diagonal elements. The main reason is to make sure that there is the condition of a positive-definite conditional-variance matrix in the process of optimization. The other advantage is that the number of parameters will reduce/decrease, but the positive definiteness will not be lost in the process.

3.11.5 Model Estimation for Multivariate GARCH

Following the conditional normality assumption, the parameters of BEKK-GARCH models can be estimated using the maximisation of the log likelihood function

$$L(\upsilon) = -\frac{TN}{2} \text{Log}2\pi - \frac{1}{2}\sum_{t=1}^{T}\langle \log|H| + \varepsilon_1' H_t^{-1} \varepsilon_t \rangle \qquad (3.5.14)$$

where υ represents all the unknown parameters to be estimated. N is the number of the series in the system and T is the number of observations. BHHH (Berndt, Hall, Hall and Hausman) algorithm is used to maximize the above log likelihood function.

3.12 VAR MULTIVARIATE GARCH MODEL

Elder and Serletis (2010) introduced a VAR-Multivariate GARCH two-step approach that can account for market independencies. The first step in VAR-Multivariate GARCH framework is to fit the VAR model to the data series for the conditional mean equations then the standard VAR technique is extended by admitting time coefficient specified by a Multivariate GARCH model. According to Bollerslev *et al*. (1992), time series data of return generally possesses time varying heteroskedastic volatility structure or ARCH-effect. Therefore, the VAR-Multivariate GARCH model considers the ARCH effect of the time series and calculate time varying hedge ratio. Consider the following equation for the combination of VAR and Multivariate GARCH model

$$Y_t = \alpha + \sum_{i=1}^{p} \Phi_i Y_{t-1} + \varepsilon_t \tag{3.6.1}$$

where Y_t is an $n \times 1$ vector of changes in monthly exchange rate at time t, $\varepsilon_t \sim N(0, \sum_t)$ and

$$\Phi_i = \begin{pmatrix} \varphi_{11}^i \varphi_{12}^i & \cdots & \varphi_{1n}^i \\ \vdots & \ddots & \vdots \\ \varphi_{n1}^i \varphi_{n2}^i & \cdots & \varphi_{nn}^i \end{pmatrix}, i = 1, 2, .., p. \tag{3.6.2}$$

The $n \times 1$ vector α represents the long term drift coefficients, ε_t denotes the $n \times 1$ vector of innovative at each market at time t with its corresponding $n \times n$ conditional variance covariance matrix $\sum_t$. The elements of the matrix Φ_i 's are the degree of mean spill-over effect across markets and measures the transmission in mean from one market to another. The current study adopts the BEKK model and the DCC model. In the BEKK model, the variance-covariance matrix of the system of equations at time t depends on the squares and cross products of innovation ε_{t-1} and volatility $\sum_t$ for each market (Engle and Kroner, 1995 and Bauwens and Giot, 2003).

3.12.1 VAR-BEKK-GARCH Parameter estimation

The BEKK parameterisation of Multivariate GARCH model is computed using the following equation

$$\sum_t = B'B + C'\varepsilon_{t-1}\varepsilon_{t-1}C + G'\sum_{t-1}G \tag{3.6.3}$$

where B is a 5×5 lower triangular matrix with intercept parameters, C and G are 5×5 square matrices of parameters. The $\sum_t$, B, C, G and $\varepsilon_t' \varepsilon_t$ are given by the following equations respectively

$$\sum_t = \begin{pmatrix} \sigma_{11,t}\sigma_{12,t} & \cdots & \sigma_{1n,t} \\ \vdots & \ddots & \vdots \\ \sigma_{n1,t}\sigma_{n2,t} & \cdots & \sigma_{nn,t} \end{pmatrix} \tag{3.6.4}$$

$$B_t = \begin{pmatrix} b_{11}b_{12} & \cdots & b_{1n} \\ \vdots & \ddots & \vdots \\ b_{n1}b_{n2} & \cdots & b_{nn} \end{pmatrix} \quad (3.6.5)$$

$$C = \begin{pmatrix} c_{11}c_{12} & \cdots & c_{1n} \\ \vdots & \ddots & \vdots \\ c_{n1}c_{n2} & \cdots & c_{nn} \end{pmatrix} \quad (3.6.6)$$

$$G = \begin{pmatrix} g_{11}g_{12} & \cdots & g_{1n} \\ \vdots & \ddots & \vdots \\ g_{n1}g_{n2} & \cdots & g_{nn} \end{pmatrix} \quad (3.6.7)$$

$$\varepsilon_t'\varepsilon_t = \begin{pmatrix} \varepsilon_{1t}^2 \ \varepsilon_{1t}\varepsilon_{2t} & \cdots & \varepsilon_{1t}\varepsilon_{nt} \\ \vdots & \ddots & \vdots \\ \varepsilon_{nt}\varepsilon_{1t} \ \varepsilon_{nt}\varepsilon_{2t} & \cdots & \varepsilon_{nt}^2 \end{pmatrix} \quad (3.6.8)$$

The c_{ij} of the $n \times n$ symmetric matrix C measures the degree measures the degree innovation from market i to j. The g_{ij} of the $n \times n$ symmetric matrix G measures the persistence in conditional volatility between market i and market j. The equation (3.6.1) and (3.6.2) are estimated by the use of estimated through maximum likelihood estimation procedures. The log-likelihood for Multivariate GARCH model under Gaussian errors is computed by the following equation

$$L(\theta) = -\frac{Tn}{2} + \ln(2p) - \frac{1}{2}\sum(ln|\sum_t| + \varepsilon_t'|\sum_t^{-1}|\,\varepsilon_t) \quad (3.6.9)$$

where T denotes the effective sample size, n represent the number of markets and θ is the vector of parameters defined in equation (3.6.1) and (3.6.2). The traditional Berndt, Hall, Hall and Hausman (BHHH) algorithm is used to produce the maximum likelihood parameters and the corresponding standard errors.

3.12.2 VAR-DCC-GARCH Parameter estimation

According to Savva, Osborn and Gill (2005), the VAR-DCC-GARCH model is represented using the following

$$R_{it} = \beta_{i0} + \sum_{j=1}^{n} \beta_{ij} R_{j,t-1} + U_{it} \tag{3.6.10}$$

$$\sigma_{i.t}^2 = exp\left[\alpha_{i0} + \sum_{j=1}^{n} \alpha_{ij}\, f_j\left(Z_{j,t-1}\right) + \delta_i ln\left(\sigma_{i,t-1}^2\right)\right] \tag{3.6.11}$$

$$f_j\left(Z_{j,t-1}\right) = \left(\left|Z_{j,t-1}\right| - E\left|Z_{j,t-1}\right|\right) + r_j Z_{j,t-1} \tag{3.6.12}$$

where R_{it} is a function of own past exchange and other BRICS exchange rate, $R_{j,t-1}$. The parameter coefficient of β_{ij} captures the spill-over relationship in different BRICS exchange rates, for $i \neq j$. The conditional variance in every BRICS country is an exponential function of the past standardized innovations$(Z_{j,t-1} = \varepsilon_{j,-1t-1}|b_{j,t})$. If the $\delta_i = 1$ and the unconditional variance does not exist and the conditional variance follows a I(1) process. The coefficient of α_{ij} measure the spill-over effects, $r_j < 0$ implies asymmetry. The conditional variance is captured by $\left(\sum_{j=1}^{n} \alpha_{ij}\, f_j\left(Z_{j,t-1}\right)\right)$. A positive $i_j$together with a negative (positive) r_j indicates that there is a negative shock, j impact significantly on the volatility market i than positive (negative) shock. $\left(\left|Z_{j,t-1}\right| - E\left|Z_{j,t-1}\right|\right)$ measures the size effects which shows a positive α_{ij}. The disturbance error term of the mean equation is assumed to be conditionally Multivariate normal with means equal to zero and the conditional covariance matrix H_t is presented as

$$\varepsilon_t|\psi_{t-1} \sim N(0, H_t), H_t = D_t S_t D_{t,}\sigma_{ij,t} = q_{ij,t}\sigma_{i,t}\sigma_{j,t} \tag{3.6.13}$$

where D_t is a 5×5 diagonal matrix with time-varying standard deviations of equation on the diagonal and S_t is a time-varying symmetrical correlation matrix. D_t and S_t are given in the following equations respectively

$$D_t = \begin{pmatrix} \sigma_{1,t} & 0 \cdots & 0 \\ 0 & b_{2,t} & 0 \\ \vdots & \ddots & \vdots \\ 0 & 0 & b_{5,t} \end{pmatrix} \tag{3.6.14}$$

$$S_t = \begin{pmatrix} S_{1,1,t} & S_{1,2,t} \cdots & S_{1,5,t} \\ S_{2,1,t} & S_{2,2,t} & S_{2,5,t} \\ \vdots & \ddots & \vdots \\ S_{5,1,t} & S_{5,2,t} & S_{5,5,t} \end{pmatrix} \tag{3.6.15}$$

The DCC model is a specification of the dynamic correlation matrix S_t. The dynamic correlations are captured by the asymmetric general diagonal DCC equation given as

$$Q_t = \left(\bar{Q} - A'^{\bar{Q}}A - B'^{\bar{Q}}B - C'^{\bar{N}}C\right) + A'^{Z_{t-1}} + B'^{Q_{t-1}} + A'^{\eta_{t-1}\eta_{t'-1}}C \quad (3.6.16)$$

where $\bar{Q}$ and $\bar{N}$ are the unconditional correlation matrix of Z_t and η_t with $\eta_{i,t} = I\left(Z_{i,t<0}\right)Z_{i,t}$, where $I\left(Z_{i,t<0}\right)$ is the indicator function that takes the value unity when $Z_{i,t<0}$ (Engle, 2002; Cappiello, McAleer and Tansuchat, 2003). The A, B and C are scalars. The DCC model can be estimated by maximum likelihood in which the log-likelihood function can be expressed as

$$\mathrm{L}(Q) = -\frac{1}{2}\sum_{t=1}^{T}(\mathrm{klog}(2\pi) + \log(|H_t|) + \varepsilon_t' H_t^{-1}\varepsilon_t \quad (3.6.17)$$

$$= -\frac{1}{2}\sum_{t=1}^{T}(\mathrm{klog}(2\pi) + \log(|D_t S_t D_t|) + \varepsilon_t' D_t^{-1} S_t^{-1} D_t^{-1}\varepsilon_t \quad (3.6.18)$$

where the number of equations is denoted by k, T is the number of observations, Q is the parameter vector to be estimated, the vector of innovation at time t is denoted by ε_t and H_t is the time-varying conditional variance-covariance matrix with the diagonal elements and cross diagonal elements.

3.12.3 Model diagnostics

To determine the model adequacy of the two models (VAR-BEKK-GARCH and VAR-DCC-GARCH), the following tests were employed: Ljung-Box test for serial correlation, the ARCH-LM test for constant correlation and the normality test.

3.12.3.1 Ljung-Box test

Ljung-Box test was first introduced by Ljung and Box (1978) to test for the presence of serial correlation. The presence of serial correlation is tested using the squared standardised residual. The Ljung-Box test is computed using the following equation

$$Q_p = N(N+2)\sum_{k=1}^{p}\frac{\hat{\rho}_k^2}{N-k} \quad (3.6.19)$$

where N is the sample size and $\hat{\rho}_k^2$ represents the k-lag sample autocorrelation of the absolute or squared residuals.

3.12.3.2 ARCH-LM test

The Multivariate ARCH-LM test is discussed in detail in section 3.5.2.3 above.

3.12.3.3 Normality test

The goodness-of-fit test is the test used under the normality testing to determine the model fit. It compares the observed standardised residuals with the expected if the selected distribution is correct. Palm (1996) suggested a test to alter for the observation that is not i.i.d by categorising the standardised residuals by magnitude and not by value. The Adjusted Pearson goodness-of-fit statistics is computed as

$$P(g) = \sum_{i=1}^{g} \frac{(n_i - En_i)^2}{En_i} \tag{3.6.20}$$

where n_i is the number of observations in cell i and En_i is the predicted number of observations using the MLE. The null hypothesis to be tested is H_0: the data follows a given distribution (Normally distributed) and the alternative hypothesis is H_A: the data does not follows a given distribution. If the p-value is < 0.05 then reject the H_0.

3.12.4 The Q-Q Plot

The Q-Q plot is used to confirm the distribution the data follows (Mad'ar, 2014). The plot approximates the data around the straight line near the centre. If the data values deviates from the straight line, the null hypothesis of the assumed distribution for the data set is rejected.

3.13 CONCLUSION

This chapter presented the methodology applied to the five BRICS countries exchange rates. It started by presenting the process the methodology followed in the study process. The chapter presented the ethical considerations the study undertook. The research process picture that the study followed was also painted for ease of reference. This provided the roadmap the study followed in its entirety. Traditionally, time series data is non stationary in nature and before such data is used the stationarity must prevail hence test for stationarity are conducted to ensure that the data is indeed stationary. The VAR model procedures are also presented including model parameter estimation, diagnostic tests, and forecasting future values. The ARCH models and GARCH models were discussed followed by the extension of GARCH models then Multivariate GARCH and lastly VAR-Multivariate GARCH model procedures were illustrated. Data analysis and interpretation of the results are presented in the next chapter 4.

CHAPTER 4

DATA ANALYSIS AND INTERPRETATION OF THE RESULTS

4.1 INTRODUCTION

The chapter presents the data analysis and interpretation of results in order to achieve the objectives as set in chapter one. Graphical presentation of results is provided to illustrate the nature of the series. The chapter also presents different methods using the BRICS data applied to each method. The chapter presents the following methods: the stationarity testing methods (ADF and PP); univariate and the Multivariate methods. The tests administered under univariate are VAR, ARCH and GARCH models of the different countries exchange rates. The Multivariate techniques include Multivariate GARCH and VAR-Multivariate GARCH models.

The rest of the chapter is organised as follows: Section 4.2 presents the preliminary data analysis. In section 4.3, the Vector Autoregressive (VAR) model is presented, and Section 4.4 presents the Autoregressive Conditional Heteroskedasticity (ARCH). Section 4.5 presents the Generealised ARCH (GARCH). Section 4.6 presents the Exponential GARCH. Section 4.7 presents the BEKK-GARCH. In Section 4.8, the DCC-GARCH-DCC is presented. Section 4.9 presents the VAR enhanced BEKK-GARCH. In Section 4.10, the VAR enhanced DCC-GARCH is presented, and lastly Section 4.11, presents Chapter Summary.

4.2 PRELIMINARY DATA ANALYSIS

The following presents the preliminary results using both graphical and tables including descriptive statistics. Figure 4.1 presents the original plot of BRICS countries.

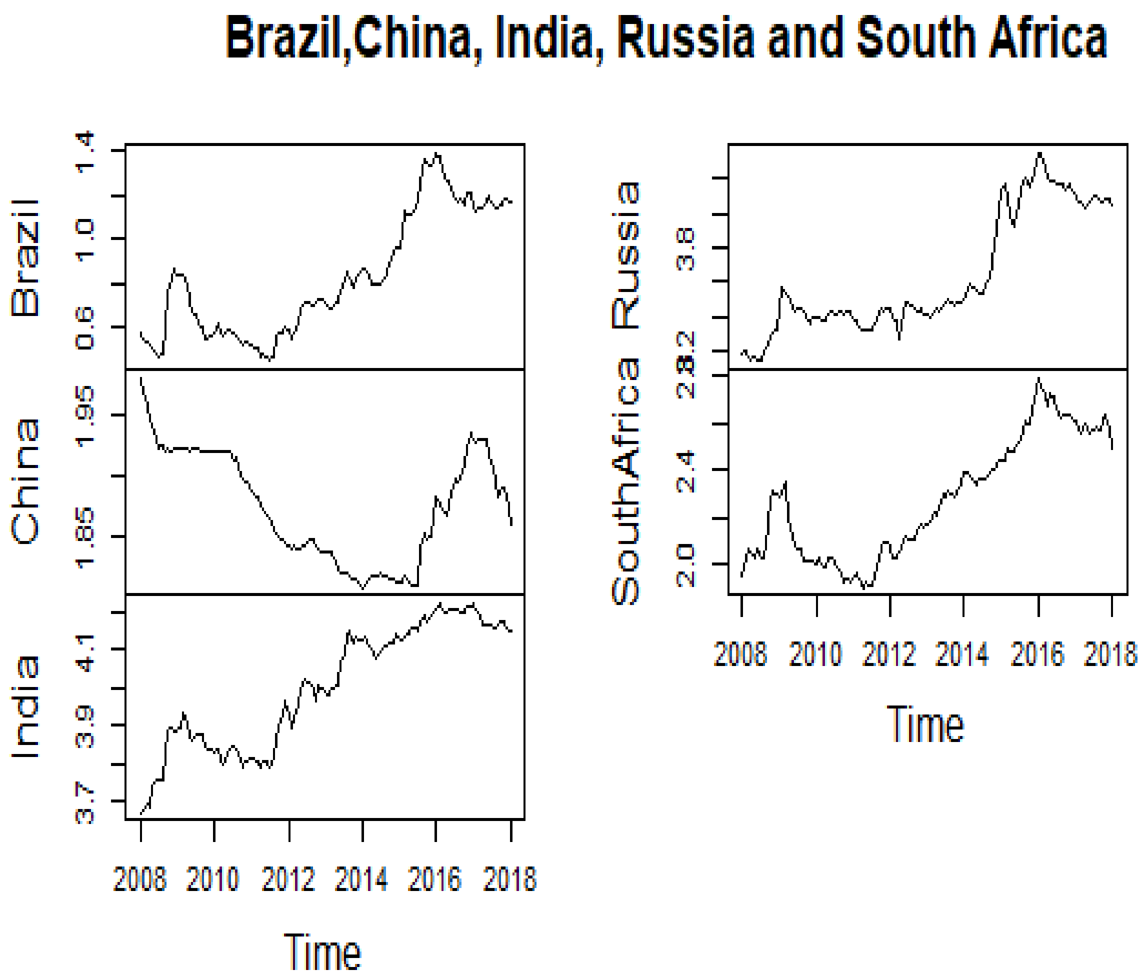


Figure 4.1Original plots of BRICS countries

Figure 4.1 above shows the different countries' original plots. Brazil illustrates a steady increase for a sampled period and declines in the period between 2016 and 2018. Russia is on a plateau mode for the substantial period between 2008 through to 2015 and a sudden increase of a period between 2015 and 2016. It then shows a decline in the last period of 2017 and 2018. China on one hand starts on the high and declines for a period between 2011 to mid 2015. It then takes on a sudden increase in the last period between mid 2015 to 2017 and drops in the first quarter of 2018. South Africa shows a constant increase between the period 2011 through to 2016 and decline in the period after 2016. India on the other hand shows a constant increase throughout the sampled period with some minor fluctuations. The picture above shows that the BRICS data is non stationary. There is no sign of mean reversion.

The next Figure 4.2 presents the overlay plots of BRICS countries.

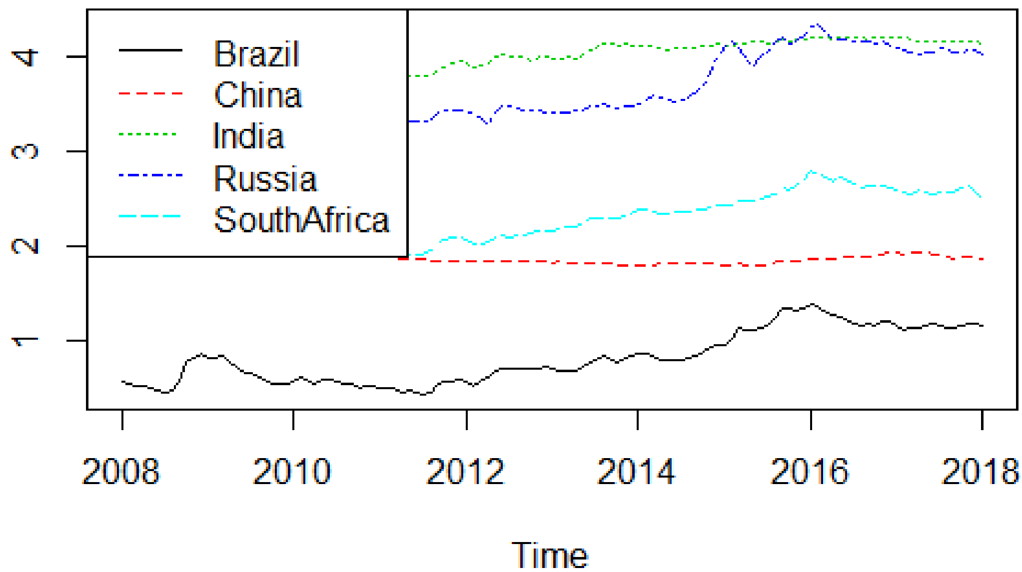


Figure 4.2 Overlay plots of BRICS countries

Figure 4.2 illustrates the overlay plots showing that India and Russia are somehow moving together and China is constantly on a plateau with some minor dips around the period between 2015 and 2016. This shows that India has a high exchange rate as compared to all the four countries with Brazil taking the least position. South Africa has a constant upward movement until the period 2016 and dips in the last period of the sample.

The data presented below shows the differenced data.

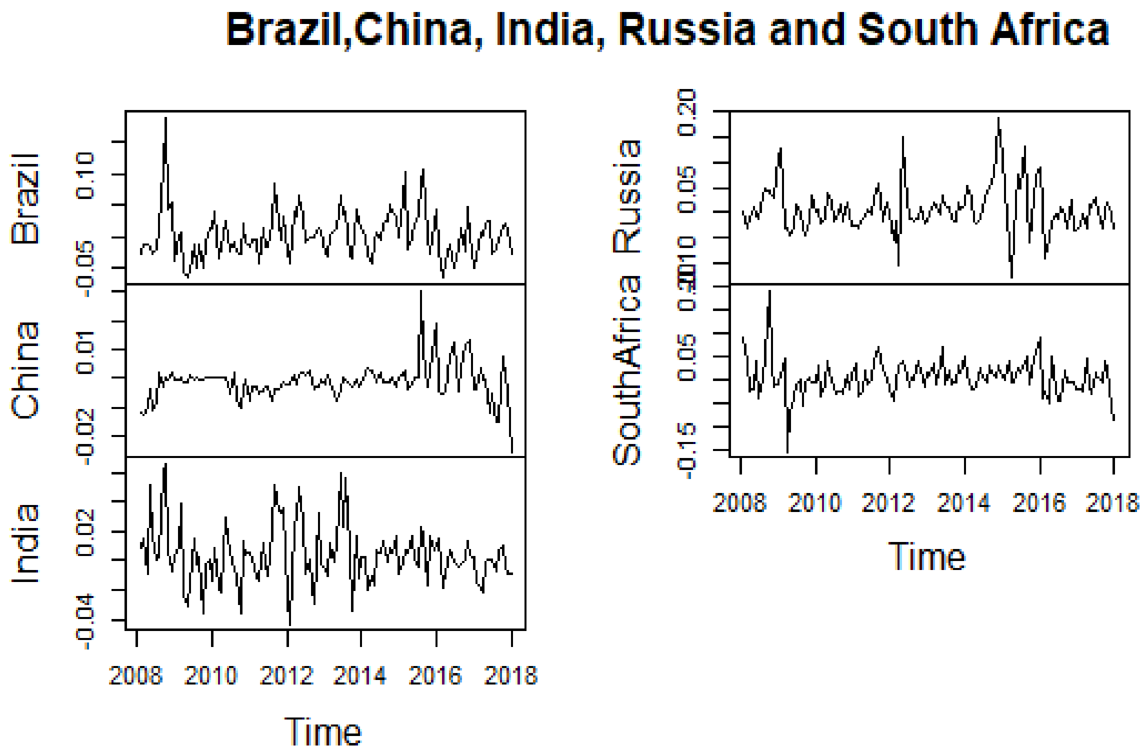


Figure 4.3 Differenced data of BRICS countries

Figure 4.3 above shows a stationary set at first logged difference by eye inspection. The formal test of stationarity will be conducted to confirm the assertion. The following section presents the summary, correlation and unit root tests of the data, followed by the VAR modeling of the data.

Table 4.1 Descriptive statistics of BRICS countries

Variable	Brazil	China	India	Russia	South Africa
Mean	0.828	1.875	4.003	3.639	2.280
Median	0.786	1.876	4.003	3.468	2.292
Maximum	1.398	1.981	4.223	4.347	2.795
Minimum	0.447	1.809	3.673	3.151	1.908
Std. Dev.	0.278	0.045	0.162	0.345	0.256
Skewness	0.465	0.059	-0.228	0.610	0.267
Kurtosis	1.873	1.724	1.645	1.837	1.727
Jarque-Bera	10.772	8.283	10.310	14.325	9.604
Probability	0.005	0.016	0.006	0.001	0.008
Observations	121	121	121	121	121

Table 4.1 presents the summary of the BRICS exchange rate. The country with the highest mean value of 4.003 as per the Table 4.1 is India with the standard deviation of 0.345 and the country with the lowest mean value (0.828) is Brazil with the standard deviation of 0.278. None of the BRICS countries appears to be normally distributed since all the p-values of the JB test are less than 0.05. India is the only country illustrating a negative skewness and the rest of the BRICS countries are positively skewed. Since the kurtosis values are close to 2, they are said to be mesokurtic.

Table 4.2 presents the correlation analysis of the BRICS countries' exchange rates.

Table 4.2 Correlation analysis of BRICS countries

Variable	BRAZIL	CHINA	INDIA	RUSSIA	SOUTH AFRICA
BRAZIL	1				
CHINA	-0.154	1			
INDIA	0.897	-0.463	1		
RUSSIA	0.949	-0.143	0.861	1	
SOUTH AFRICA	0.976	-0.185	0.916	0.913	1

Table 4.2 illustrates how each country is correlated to the other. Brazil shows a weak negative correlation with China and a strong positive correlation with India, Russia and South Africa. China shows a weak negative correlation with India, Russia and South Africa. India illustrates a strong positive correlation with Russia and South Africa. Russia is highly positively correlated to South Africa. The weakness of China's correlation is resulting from China's economy which surpasses the rest of the BRICS countries.

Table 4.3 presents the unit root tests of BRICS countries exchange rates.

Table 4.3 Unit root test of BRICS countries

	ADF test		PP test	
	Level	**First difference**	**Level**	**First difference**
BRAZIL	0.554	<0.010 ***	0.564	0.010 **
CHINA	0.663	**0.549**	0.651	0.010 **
INDIA	0.697	<0.010 ***	0.579	0.010 **
RUSSIA	0.739	<0.010 ***	0.625	<0.010 ***
SOUTH AFRICA	0.584	<0.010 ***	0.703	<0.010 ***

*Note: '***', '**', '*' and '·' indicates significant codes at 0.001, 0.01, 0.05 and 0.1 respectively*

Table 4.3 shows that both ADF and PP tests at level show no statistically significant difference. This illustrate that the data is non stationary at level. The ADF p-values for Brazil, India, Russia and South Africa show a statistically significant difference at 10%. China shows an insignificant difference. The PP test revealed that all the countries are stationary at first difference.

Section 4.3 presents the VAR model to the different BRICS series of data.

4.3 VECTOR AUTOREGRESSIVE (VAR) MODEL

This section presents the results of the procedure carried out for fitting a VAR model. The lag length selection is presented in the Table 4.4 below.

Table 4.4 Lag length selection

Fit	Model	AIC	HQ	SC
1	VAR(1)	-38.653	-38.362	**-37.937**
2	VAR(2)	**-39.083**	**-38.550**	-37.770
3	VAR(3)	-38.922	-38.147	-37.012
4	VAR(4)	-39.749	-37.732	-36.243
5	VAR(5)	-38.751	-37.492	-35.645
6	VAR(6)	-38.828	-37.326	-35.128

Table 4.4 above shows that AIC and HQ selected lag length 2, while SC selected lag length 1 as an optimal length. Therefore VAR (1) was fitted and the parameters were estimated and presented in the following Table 4.5.

Table 4.5 Parameter estimation

Stock returns	Parameter	Variable	Estimate	Std. Error	t-value	p-value
Brazil	$AR(1)_{11}$	$Brazil_{t-1}$	0.906	0.075	12.017	< 2e-16 ***
	$AR(1)_{12}$	$China_{t-1}$	-0.375	0.119	-3.154	0.002 **
	$AR(1)_{13}$	$India_{t-1}$	-0.154	0.082	-1.873	0.064 ·
	$AR(1)_{14}$	$Russia_{t-1}$	0.030	0.033	0.909	0.365
	$AR(1)_{15}$	$SouthAfrica_{t-1}$	0.137	0.073	1.879	0.063 ·
China	$AR(1)_{21}$	$Brazil_{t-1}$	0.009	0.013	0.740	0.461
	$AR(1)_{22}$	$China_{t-1}$	0.968	0.020	48.782	<2e-16 ***
	$AR(1)_{23}$	$India_{t-1}$	-0.004	0.014	-0.306	0.760
	$AR(1)_{24}$	$Russia_{t-1}$	0.001	0.005	0.213	0.832
	$AR(1)_{25}$	$SouthAfrica_{t-1}$	-0.002	0.012	-0.188	0.851

Stock returns	Parameter	Variable	Estimate	Std. Error	t-value	p-value
India	$AR(1)_{31}$	$Brazil_{t-1}$	-0.020	0.039	-0.524	0.601
	$AR(1)_{32}$	$China_{t-1}$	-0.214	0.061	-3.504	0.001 ***
	$AR(1)_{33}$	$India_{t-1}$	0.843	0.042	19.942	< 2e-16 ***
	$AR(1)_{34}$	$Russia_{t-1}$	-0.002	0.017	-0.114	0.909
	$AR(1)_{35}$	$SouthAfrica_{t-1}$	0.098	0.037	2.618	0.010 *
Russia	$AR(1)_{41}$	$Brazil_{t-1}$	0.052	0.092	0.565	0.573
	$AR(1)_{42}$	$China_{t-1}$	-0.364	0.145	-2.517	0.013 *
	$AR(1)_{43}$	$India_{t-1}$	-0.162	0.100	-1.620	0.108
	$AR(1)_{44}$	$Russia_{t-1}$	0.923	0.040	23.123	<2e-16 ***
	$AR(1)_{45}$	$SouthAfrica_{t-1}$	0.127	0.089	1.426	0.157
South Africa	$AR(1)_{51}$	$Brazil_{t-1}$	0.146	0.076	1.916	0.058 ·
	$AR(1)_{52}$	$China_{t-1}$	-0.297	0.120	-2.469	0.015 *
	$AR(1)_{53}$	$India_{t-1}$	-0.117	0.083	-1.400	0.164
	$AR(1)_{54}$	$Russia_{t-1}$	-0.035	0.033	-1.047	0.298
	$AR(1)_{55}$	$SouthAfrica_{t-1}$	0.930	0.074	12.604	< 2e-16 ***

*Note: '***', '**', '*' and '·' indicates significant codes at 0.001, 0.01, 0.05 and 0.1 respectively*

Table 4.5 presents the VAR (1) model parameter estimates, Std. Errors, t-values and significance based on the p-values. All the parameter estimates with the p-values less the 0.1 are considered significant. From the above Table 4.5 the following are the significant autoregressive matrix coefficients: $AR(1)_{11}$, $AR(1)_{12}$, $AR(1)_{13}$, $AR(1)_{15}$, $AR(1)_{22}$, $AR(1)_{32}$, $AR(1)_{33}$, $AR(1)_{35}$, $AR(1)_{42}$, $AR(1)_{44}$, $AR(1)_{51}$, $AR(1)_{52}$, and $AR(1)_{55}$ implying that there exist a linear dependency between Brazil and its own past values, Brazil and past values of China, Brazil and past values of India, Brazil and past values of South Africa, China and its own past values, India and past values of China, India and its own past values, India and past values of South Africa, Russia and past values of China, Russia and its own past values, South Africa and past values of Brazil, South Africa and past values of China and lastly South Africa and its own past values. All the linear dependencies take one direction. The equations of the VAR (1) model for every variable which possesses the significant parameters are written as follows

$$Brazil = 0.906(\pm 0.075)Brazil_{1,t-1} - 0.375(\pm 0.119)China_{2,t-1} - 0.154(\pm 0.082)India_{4,t-1} + 0.137(\pm 0.073)SouthAfrica_{5,t-1} + \mu_{1,t} \quad (4.1)$$

$$China = 0.968(\pm 0.020)China_{2,t-1} + \mu_{2,t} \quad (4.2)$$

$India =$

$-0.214(\pm 0.061)China_{2,t-1} + 0.843(\pm 0.042)India_{3,t-1} +$

$0.098(\pm 0.037)SouthAfrica_{5,t-1} + \mu_{3,t}$ (4.3)

$Russia = -0.364(\pm 0.145)China_{2,t-1} + 0.923(\pm 0.040)Russia_{4,t-1} + \mu_{4,t}$ (4.4)

$SouthAfrica =$

$0.146(\pm 0.076)Brazil_{1,t-1} - 0.297(\pm 0.120)China_{2,t-1} +$

$0.930(\pm 0.074)SouthAfrica_{5,t-1} + \mu_{1,t}$ (4.5)

Table 4.6 below presents the covariance matrix of the BRICS exchange rates.

Table 4.6 Covariance matrix

Variable	**BRAZIL**	**CHINA**	**INDIA**	**RUSSIA**	**SOUTH AFRICA**
BRAZIL	1.393e-03	5.877e-05	0.0004356	6.990e-04	9.318e-04
CHINA	5.877e-05	3.882e-05	0.0000138	7.618e-05	4.497e-05
INDIA	4.356e-04	1.380e-05	0.0003683	2.545e-04	4.135e-04
RUSSIA	6.990e-04	7.618e-05	0.0002545	2.067e-03	5.456e-04
SOUTH AFRICA	9.318e-04	4.497e-05	0.0004135	5.456e-04	1.431e-03

The results presented above Table 4.6 illustrate that there is a presence of concurrent relationship amongst all the BRICS exchange rates.

Table 4.7 below presents the model diagnostic tests of the VAR (1).

Table 4.7 Diagnostic tests

Test	**Statistic**	**DF**	**p-value**
Portmanteau Test	243.75	125	<0.001 ***
JB-Test	381.00	10	<0.001 ***
Skewness	52.783	5	<0.001 ***
Kurtosis	328.220	5	<0.001 ***
ARCH	1468.100	1350	0.0131 **

*Note: '***', '**', '*' and '·' indicates significant codes at 0.001, 0.01, 0.05 and 0.1 respectively*

Table 4.7 above gives the summary of the diagnostic tests results for the fitted VAR (1) model. All the p-values including the ARCH p-value are significant at 5%. This implies that the residuals of the fitted VAR (1) model are serially correlated, do contain ARCH errors and are not normally distributed. The model VAR (1) does not pass all the diagnostic tests and cannot be used to forecast future values of the BRICS exchange rates.

4.4 AUTOREGRESSIVE CONDITIONAL HETEROSKEDASTICITY (ARCH)

This section presents the results for plotting ACF plots and their squares of the BRICS exchange rates. It further provides for the parameter estimations and tests for ARCH disturbances using residuals.

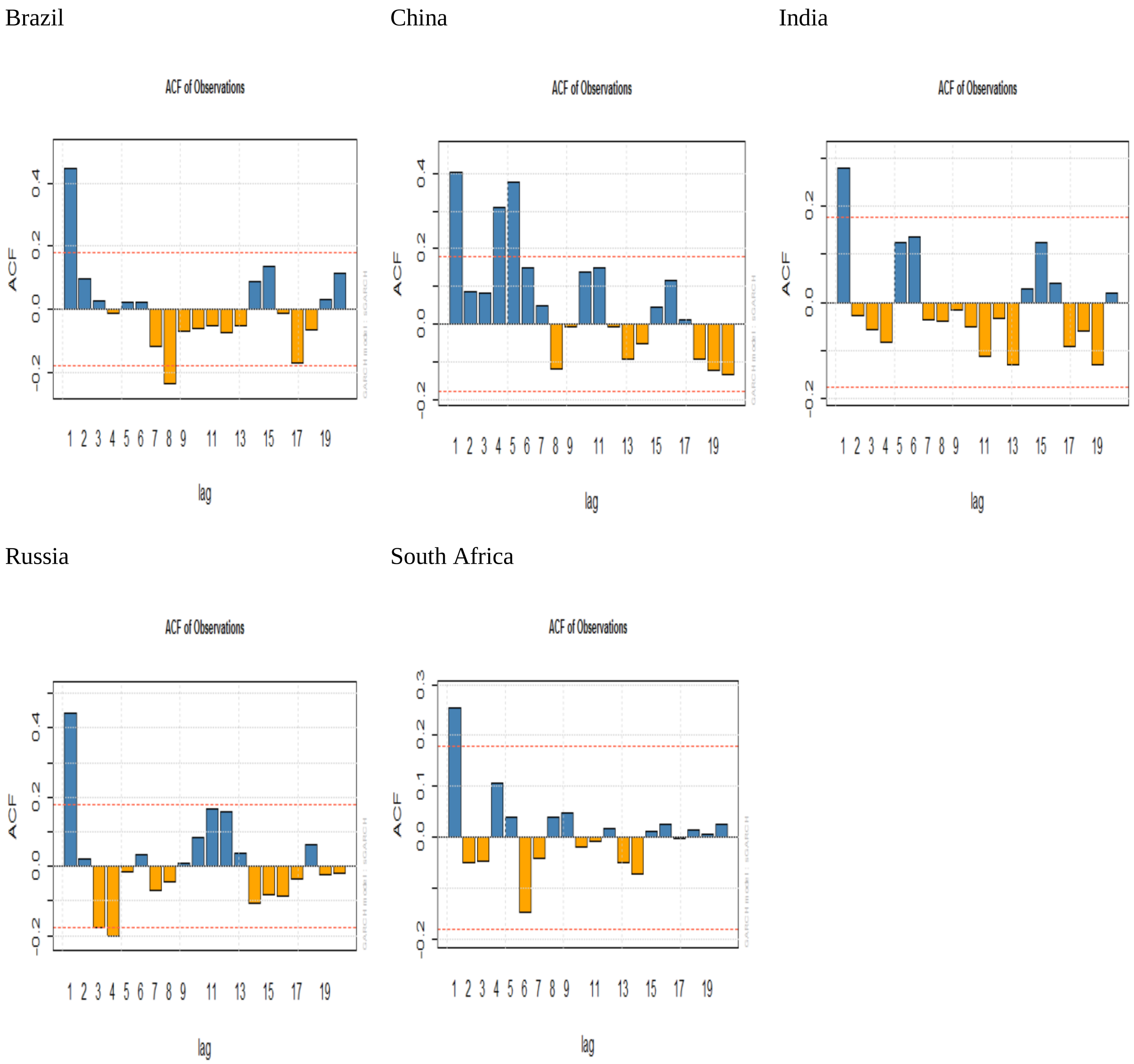


Figure 4.4 ACF plots of BRICS exchange rates

Figure 4.4 above shows that all the BRICS exchange rates appear to be serially correlated. The following Figure 4.5 presents the ACF plot of BRICS squared exchange rates.

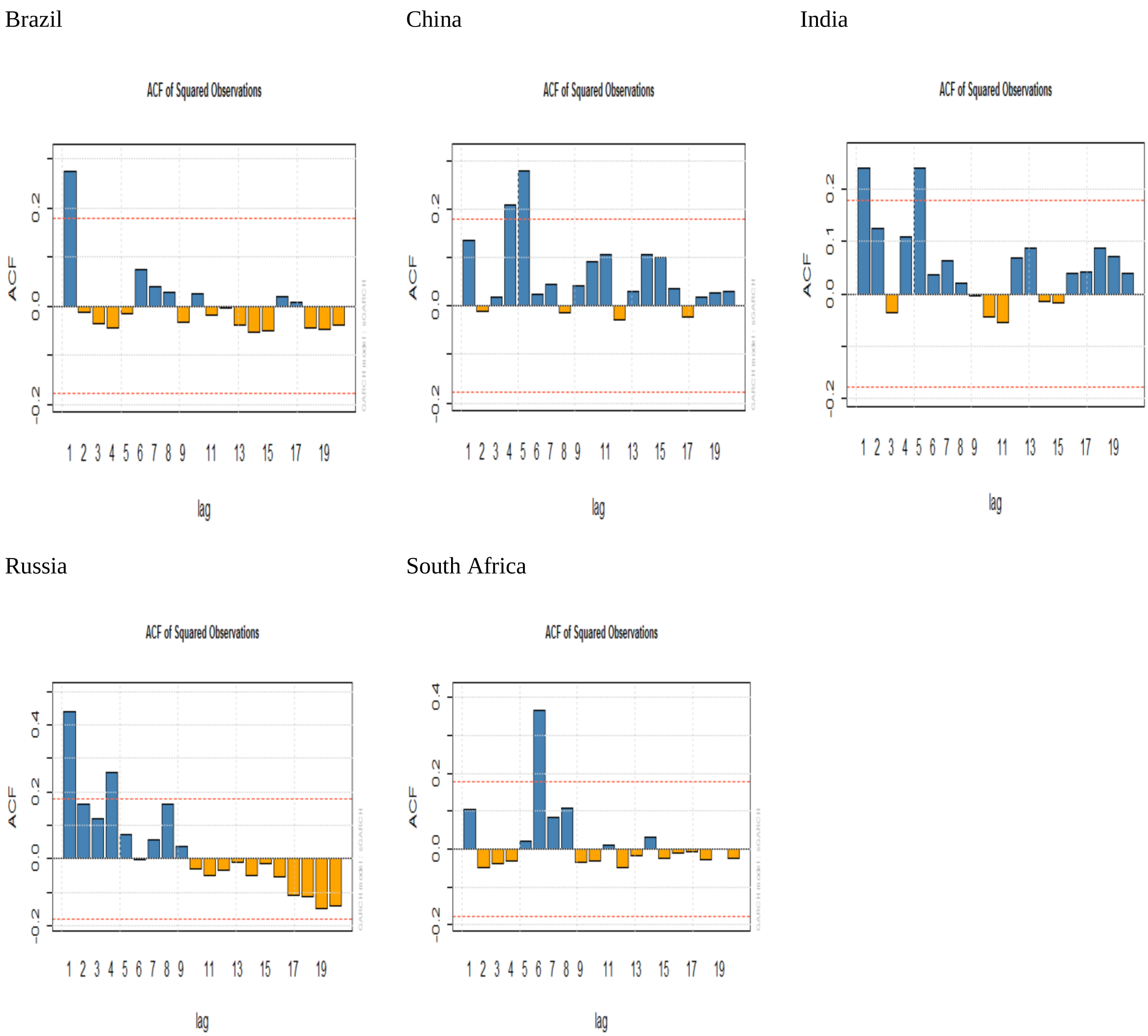


Figure 4.5 ACF plots of BRICS squared exchange rates

Figure 4.5 shows that the squared BRICS exchange rates illustrate the presence of serial correlation and also showing that the ARCH errors are present in the BRICS exchange rates. The visual presentations above in Figure 4.4 and 4.5 will be confirmed with the relevant test in the next two Tables 4.8 and 4.9.

Table 4.8 presents the selected ARCH models fitted.

Table 4.8 Parameter estimation

Exchange Rates	**Parameter**	**Estimate**	**Std. Error**	**t-value**	**p-value**
Brazil	Mu	0.001	0.0001	7.098	0.000 ***
	ar1	0.411	0.147	2.806	0.005 **
China	Mu	0.00003	0.000002	17.362	<.00002 ***
	ar1	0.504	0.181	2.784	0.005 **
India	Mu	0.0003	0.00003	8.128	<0.0001 ***
	ar1	0.435	1.978	2.201	0.278
Russia	Mu	0.001	0.0001	5.434	<0.0001 ***
	ar1	0.852	0.247	3.447	0.001 ***
South Africa	Mu	0.001	0.0002	3.907	0.0001 ***
	ar1	0.765	0.143	5.360	<0.0001 ***

*Note: '***', '**', '*' and '·' indicates significant codes at 0.001, 0.01, 0.05 and 0.1 respectively*

The ARCH (1) effect is significant with probability values below all the levels of significance except for India. Since the ARCH (1) model is significant according to the results, this is an indication that this mean equation could be fit to the GARCH variance equation.

Table 4.9 presents the tests for ARCH disturbances based on residuals.

Table 4.9 Tests for ARCH disturbances based on residuals

Exchange Rates	**Box-Ljung**	**p-value**	**ARCH-LM**	**p-value**
Brazil	2.879	0.090	35.581	<0.0001 ***
China	0.011	0.915	207.800	0.000 ***
India	0.113	0.737	37.140	<0.0001 ***
Russia	0.086	0.769	101.470	0.000 ***
South Africa	0.656	0.418	187.311	0.000 ***

*Note: '***', '**', '*' and '·' indicates significant codes at 0.001, 0.01, 0.05 and 0.1 respectively*

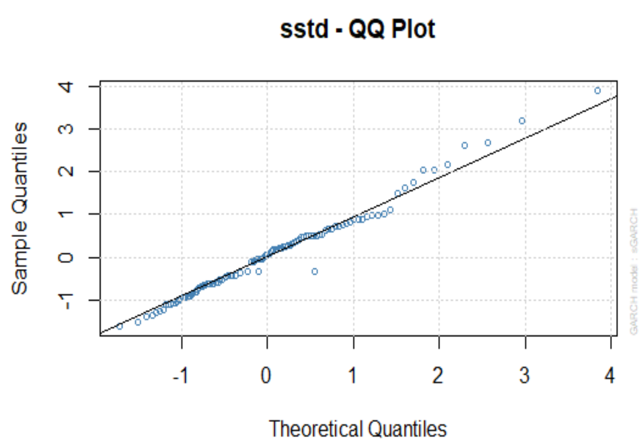
In Table 4.9 above, the testing for heteroskedasticity was done using the Engle's Lagrange Multiplier (LM), squared residual test. In essence, the ARCH effects were tested by scrutinising whether or not the BRICS exchange rates are heteroskedastic. The Box-Ljung squares of the BRICS exchange rates show that the BRICS exchange rates are serially correlated, with probability values greater than 0.05. The LM test strongly shows that there is heteroskedasticity, with p-values less than 0.05. The LM test further suggests a strong heteroskedasticity of errors for GARCH model for the five countries. Section 4.5 presents the GARCH model.

4.5 GENEREALISED ARCH (GARCH)

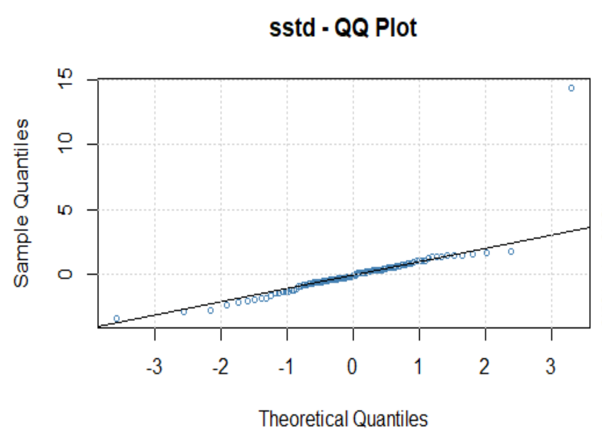
This section presents the univariate GARCH (1.1) model for the BRICS exchange rates. As seen in the previous section on ARCH, all the BRICS exchange rates showed some presence of ARCH errors. Depending of the nature of the time series, the model (GARCH (1.1)) may present different assumptions of conditional distribution. The Gaussian normal distribution appeared to be the most common conditional distribution. The first step in fitting the GARCH (1.1) is to plot the Q-Q plots. Figure 4.6 below presents the Q-Q plots of the BRICS exchange rates and in theory the plot takes the shape of $y = x$.

4.5.1 Q-Q plots

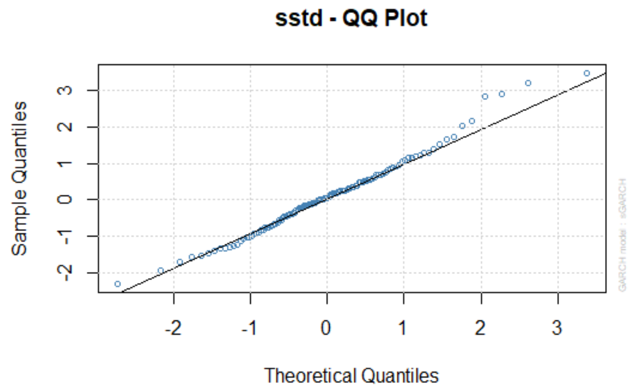
Brazil



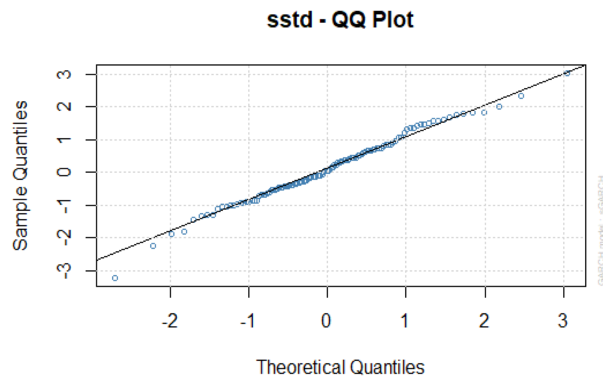
China



India



Russia



South Africa

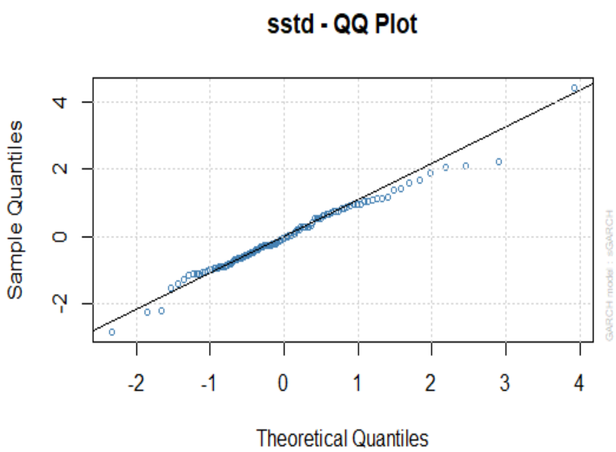


Figure 4.6 Q-Q plots for BRICS exchange rates

The above Figure 4.6 depicts that most of the BRICS exchange rates points lie on the normal line. All the BRICS exchange rates Q-Q plots follow a normal distribution with some extreme tails. Both the left and the right tail distribution of the exchange rate illustrate some differences and therefore advisable to keep the distribution as skewed. There are two conditional distributions namely: *std* and *sstd*. The Table 4.10 below shows the fitted *std* and *sstd* on GARCH (1.1).

Table 4.10 AIC values of the GARCH (1.1) model under *std* and *sstd* conditional distributions for each of the BRICS exchange rates

Exchange Rates	**AIC**	
	Std	**sstd**
BRAZIL	-3.786	-3.831
CHINA	-7.947	-7.931
INDIA	-5.035	-5.028
RUSSIA	-3.682	-3.668
SOUTH AFRICA	-3.775	-3.789

Table 4.10 indicated that *std* has the most lowest AIC values of all the BRICS exchange rates. Therefore GARCH (1.1) model was fitted using the *std*.

The results are presented in the Table 4.11 below.

4.5.2 Parameter estimation

Table 4.11 Summary table of GARCH (1.1) model parameter estimates for each of the BRICS exchange rates

Exchange Rates	**Parameter**	**Estimate**	**Std. Error**	**t-value**	**p-value**	**Longrun variance**
Brazil	μ	-0.001	0.003	-0.322	0.748	0.0019
	ω	0.001	0.0002	3.550	0.000 ***	
	α_1	0.455	0.226	2.013	0.044 *	
	β_1	0.014	0.103	0.133	0.894	
China	μ	-0.001	0.0003	-2.053	0.040 *	0.0010
	ω	0.000001	0.000003	0.304	0.761	
	α_1	0.408	0.134	3.043	0.002 **	
	β_1	0.591	0.095	6.228	0.000 ***	
India	μ	0.001	0.001	0.890	0.373	0.0007
	ω	0.00003	0.00003	0.836	0.403	
	α_1	0.201	0.133	1.506	0.132	
	β_1	0.753	0.143	5.280	0.000 ***	
Russia	μ	-0.002	0.003	-0.638	0.523	0.0064
	ω	0.0003	0.0001	2.553	0.011 **	
	α_1	0.713	0.225	3.168	0.002 **	
	β_1	0.240	0.125	1.922	0.055 *	
South Africa	μ	0.005	0.003	1.547	0.122	0.0015
	ω	0.0003	0.0004	0.714	0.475	
	α_1	0.150	0.178	0.842	0.400	
	β_1	0.653	0.382	1.711	0.087 ·	

*Note: '***', '**', '*' and '·' indicates significant codes at 0.001, 0.01, 0.05 and 0.1 respectively*

The following models are deduced from the above Table 4.11, the GARCH (1.1) model equations for each BRICS exchange rates are written as follows

$x_r(Brazil) = -0.001(\pm 0.003) + \varepsilon_t,$

Brazil: $\sigma_t^2 = 0.001(\pm 0.0002) + 0.455(\pm 0.226)\sigma_{t-1}^2 + 0.014(\pm 0.103)\sigma_{t-1}^2$ (4.6)

$x_r(China) = -0.001(\pm 0.0003) + \varepsilon_t,$

China: $\sigma_t^2 = 0.000001(\pm 0.000003) + 0.408(\pm 0.134)\sigma_{t-1}^2 + 0.591(\pm 0.095)\sigma_{t-1}^2$ (4.7)

$x_r(India) = 0.001(\pm 0.001) + \varepsilon_t,$

India: $\sigma_t^2 = 0.00003(\pm 0.00003) + 0.201(\pm 0.133)\sigma_{t-1}^2 + 0.753(\pm 0.143)\sigma_{t-1}^2$ (4.8)

$x_r(Russia) = -0.002(\pm 0.003) + \varepsilon_t,$

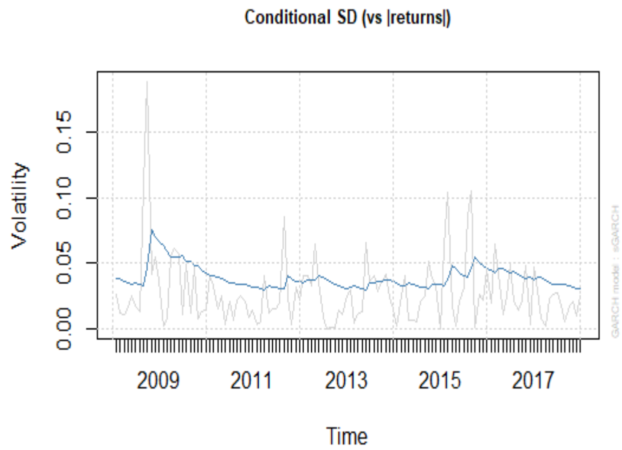
Russian: $\sigma_t^2 = 0.0003(\pm 0.0001) + 0.713(\pm 0.225)\sigma_{t-1}^2 + 0.240(\pm 0.125)\sigma_{t-1}^2$ (4.9)

$x_r(SouthAfrica) = 0.005(\pm 0.003) + \varepsilon_t,$

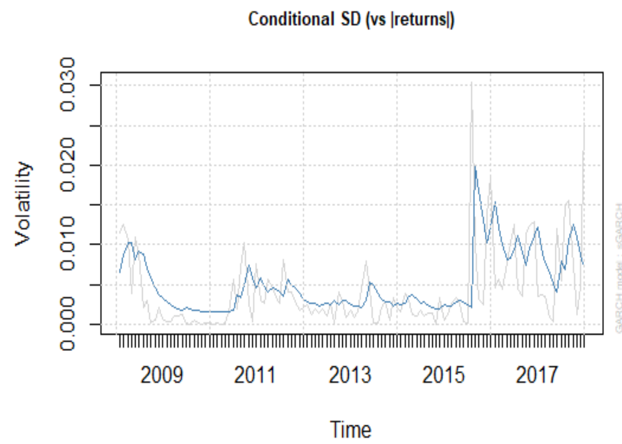
South Africa: $\sigma_t^2 = 0.0003(\pm 0.0004) + 0.150(\pm 0.178)\sigma_{t-1}^2 + 0.653(\pm 0.382)\sigma_{t-1}^2$ (4.10)

x_r represents the exchange rates for each of the BRICS countries whereas σ_t^2 symbolises the volatility part of the GARCH (1.1) model equation for each BRICS exchange rates. The sum of the estimates $\hat{\alpha}_1$ and $\hat{\beta}_1$ of all the BRICS exchange rates series are less than one meaning that the unconditional volatility for each of the BRICS exchange rates series is finite. The results further revealed that China has the highest volatility persistence value of $\hat{\alpha}_1 + \hat{\beta}_1 = 0.999$, followed by India with the value of $\hat{\alpha}_1 + \hat{\beta}_1 = 0.954$, followed by Russia with the value of $\hat{\alpha}_1 + \hat{\beta}_1 = 0.943$, followed by South Africa with the value of $\hat{\alpha}_1 + \hat{\beta}_1 = 0.803$ and the least is Brazil with volatility persistence value of $\hat{\alpha}_1 + \hat{\beta}_1 = 0.469$. The Figure 4.7 below shows the BRICS conditional volatility.

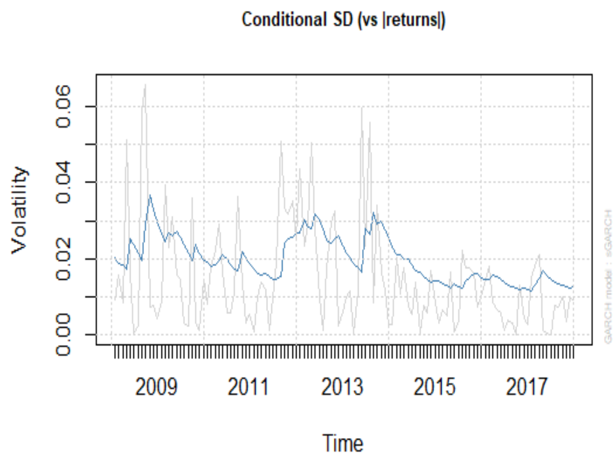
Brazil



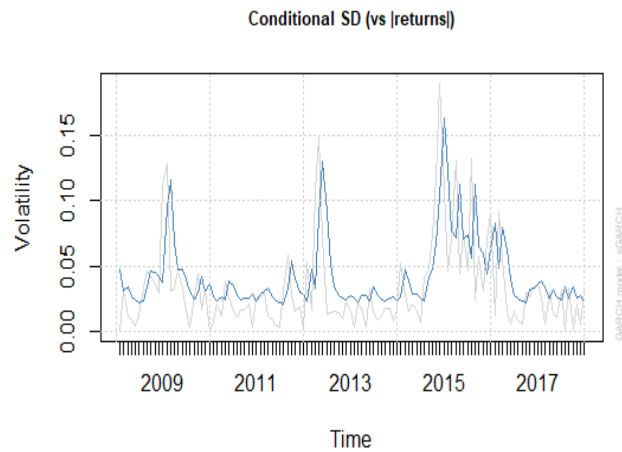
China



India



Russia



South Africa

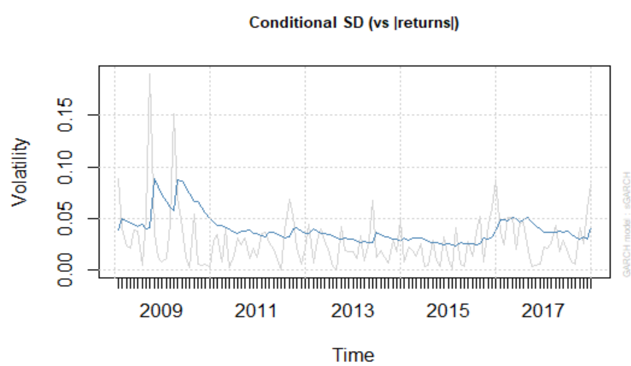


Figure 4.7 BRICS conditional volatility

The volatility scales in Figure 4.7 above shows that Brazil, Russia and South Africa have the highest volatility followed by India and the least volatile is China. The next subsection 4.5.3 illustrates the diagnostic tests.

4.5.3 Diagnostic tests

Model adequacy testing is done using the following diagnostic tests: goodness of fit test; Ljung-Box (R), Ljung-Box (R^2), and ARCH-LM.

Table 4.12 Diagnostic test of the GARCH (1.1) model

Exchange Rates	Diagnostic test	Statistic	p-value
Brazil	Goodness of fit test	15.670	0.679
	Ljung-Box (R)	17.090	3.567e-05 ***
	Ljung-Box (R^2)	2.377	0.1232
	ARCH-LM	0.3834	0.5358
China	Goodness of fit test	19.330	0.456
	Ljung-Box (R)	3.365	0.067 ·
	Ljung-Box (R^2)	0.030	0.863
	ARCH-LM	0.027	0.870
India	Goodness of fit test	8.667	0.979
	Ljung-Box (R)	7.111	0.008 **
	Ljung-Box (R^2)	0.142	0.706
	ARCH-LM	1.884	0.170
Russia	Goodness of fit test	20.670	0.356
	Ljung-Box (R)	11.260	0.001 ***
	Ljung-Box (R^2)	0.044	0.834
	ARCH-LM	0.928	0.335
South Africa	Goodness of fit test	23.670	0.209
	Ljung-Box (R)	5.650	0.017 *
	Ljung-Box (R^2)	0.108	0.743
	ARCH-LM	0.150	0.698

*Note: '***', '**', '*' and '·' indicates significant codes at 0.001, 0.01, 0.05 and 0.1 respectively.*

The data in the above Table 4.12 shows that all the BRICS exchange rates have no ARCH errors, since all the p-values of the ARCH-LM test are greater than 0.05 level of significance. The Ljung-Box (R^2) revealed that the residuals of the squared BRICS exchange rates do not have serial correlation. All the BRICS exchange rates show that the fitted residuals are normally distributed except for Russia which has a p-value less than 0.05. The Q-Q plots in Figure 4.6 for BRICS exchange rates are in support of the above assertion that the fitted residual are normally distributed. Therefore, GARCH (1.1) under the *std* conditional distribution appears to be adequate and can be used for further analysis. Forecasting is demonstrated in the following subsection.

4.5.4 Forecasting

The forecasts of the GARCH (1.1) are presented in the following Table 4.13. The volatility of each BRICS exchange rate was forecasted for five periods ahead.

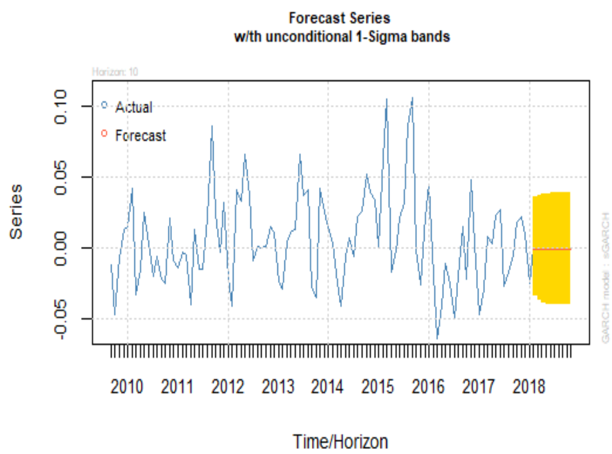
Table 4.13 Forecasting

Exchange Rates	Time (months)	Mean forecast	Mean error	95% Lower CI	95% Upper CI
Brazil	1	-0.001	0.033	-0.055	0.113
	2	-0.001	0.037	-0.055	0.113
	3	-0.001	0.038	-0.056	0.114
	4	-0.001	0.039	-0.056	0.114
	5	-0.001	0.039	-0.056	0.115
China	1	-0.001	0.017	-0.012	0.016
	2	-0.001	0.017	-0.012	0.016
	3	-0.001	0.017	-0.012	0.015
	4	-0.001	0.017	-0.012	0.015
	5	-0.001	0.017	-0.012	0.015
India	1	0.001	0.013	-0.049	0.047
	2	0.001	0.014	-0.053	0.050
	3	0.001	0.015	-0.056	0.053
	4	0.001	0.015	-0.058	0.055
	5	0.001	0.016	-0.061	0.058
Russia	1	-0.002	0.033	-0.067	0.060
	2	-0.002	0.037	-0.077	0.070
	3	-0.002	0.040	-0.087	0.079
	4	-0.002	0.043	-0.095	0.088
	5	-0.002	0.048	-0.103	0.096
South Africa	1	0.005	0.049	-0.041	0.104
	2	0.005	0.047	-0.043	0.108
	3	0.005	0.045	-0.046	0.112
	4	0.005	0.044	-0.048	0.116
	5	0.005	0.042	-0.050	0.120

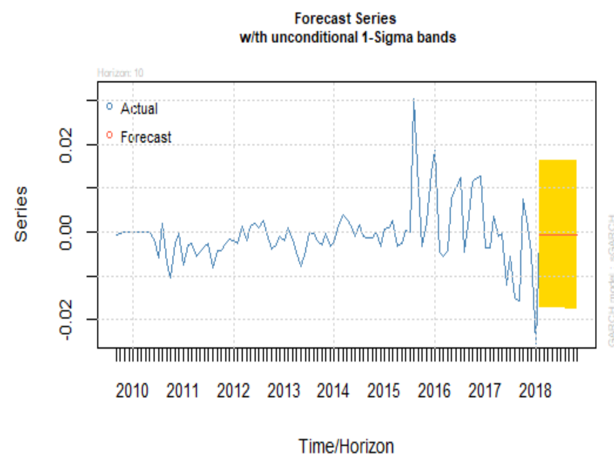
Table 4.13 above presents the mean and volatility forecasts of the BRICS exchange rates. The mean forecasts falls within the 95% confidence interval.

Figure 4.8 presents the volatility forecast plots with 95% CI.

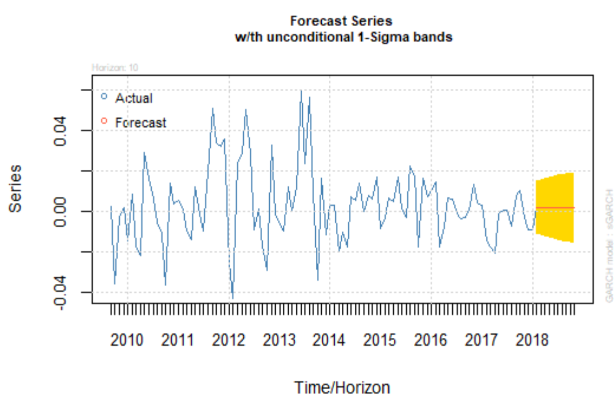
Brazil



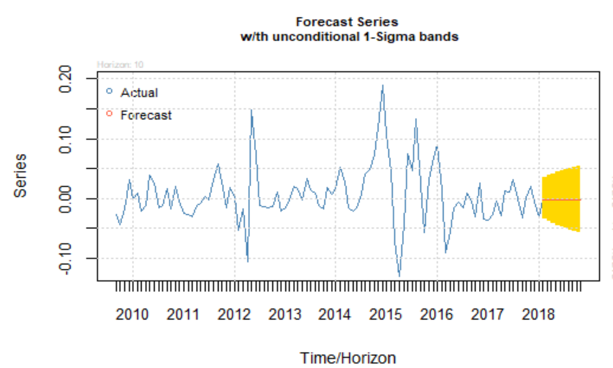
China



India



Russia



South Africa

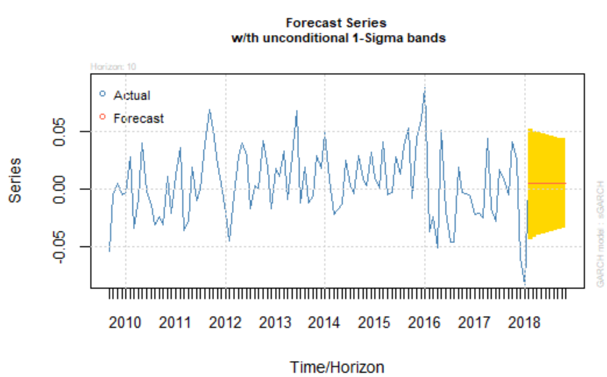
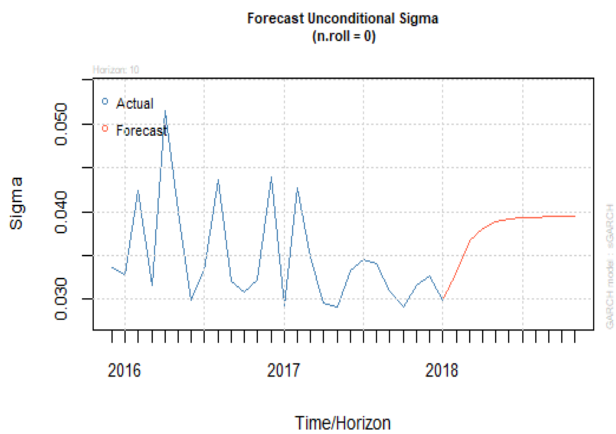


Figure 4.8 Volatility Forecast plots with 95% CI

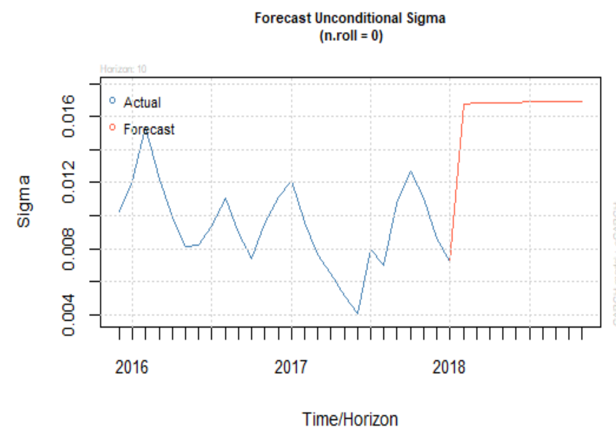
All the volatility forecasts in Figure 4.8 above confirm that the volatility forecasts are within the 95% confidence limit.

Figure 4.9 illustrates the volatility forecasts plots with a five period ahead.

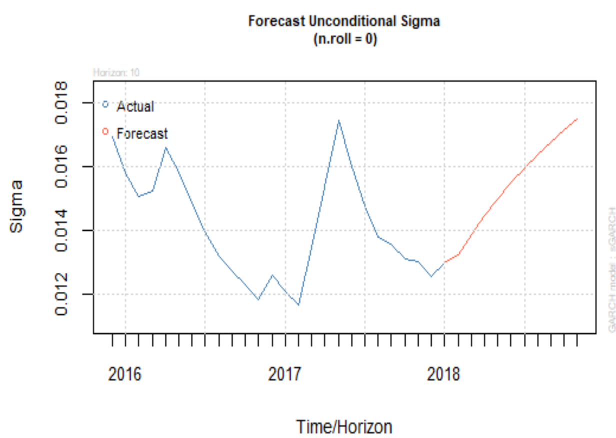
Brazil



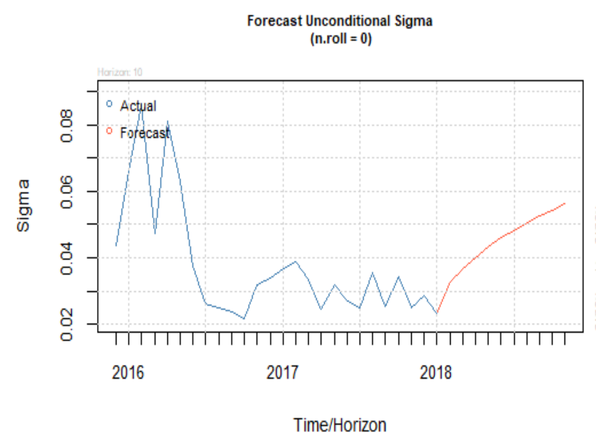
China



India



Russia



South Africa

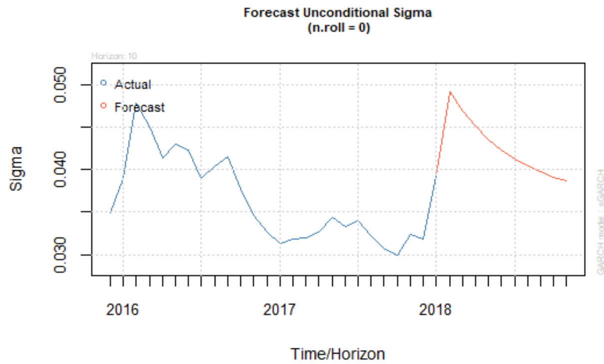


Figure 4.9 Volatility Forecast plots

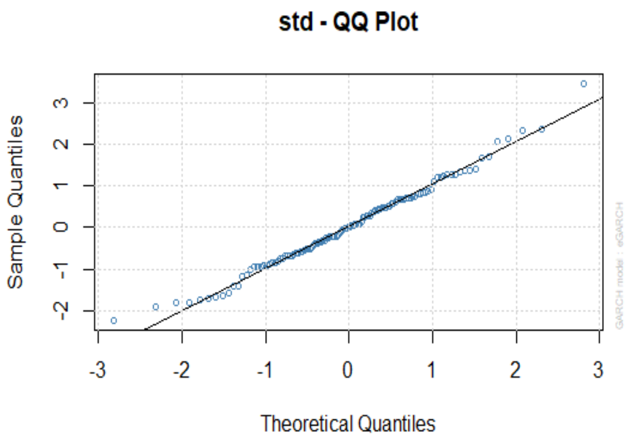
The Figure 4.9 above shows that all the BRICS exchange rates volatility forecasts plots are on the rise except for China and South Africa.

The next Section 4.6 presents the EGARCH results.

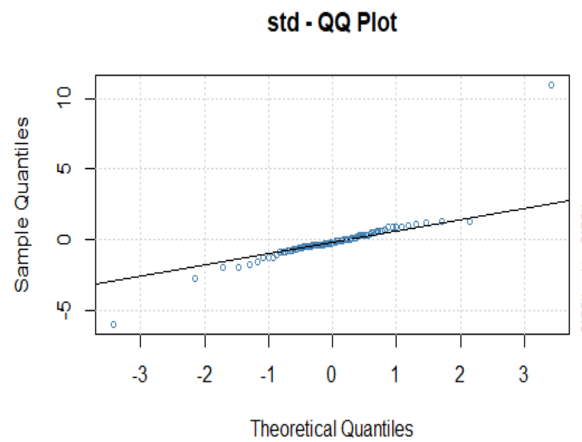
4.6 EXPONENTIAL GARCH

This section presents the univariate exponential GARCH (1.1) model for the BRICS exchange rates. As seen in the previous Section on ARCH, all the BRICS exchange rates showed some presence of ARCH errors. Depending on the nature of the time series, the model (EGARCH (1.1)) may present different assumptions of conditional distribution. The Gaussian normal distribution appeared to be the most common conditional distribution. The first step in fitting the EGARCH (1.1) is to plot the Q-Q plots. Figure 4.10 below presents the Q-Q plots of the BRICS exchange rates and in theory the plot takes the shape of $y = x$.

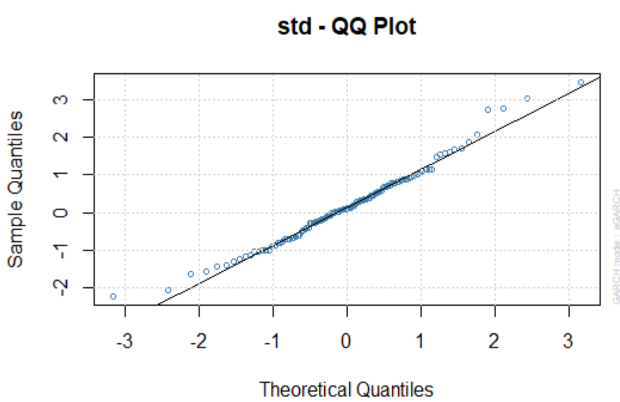
Brazil



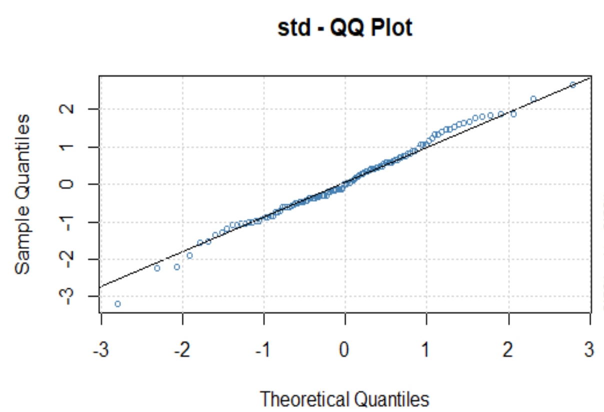
China



India



Russia



South Africa

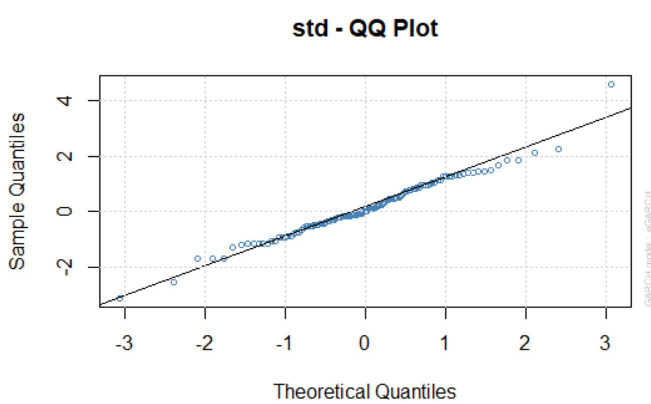


Figure 4.10 Q-Q plots for BRICS exchange rates

The above Figure 4.10 depicts that the most of the BRICS exchange rates points lie on the normal line. All the BRICS exchange rates Q-Q plots follow a normal distribution with some extreme tails. Both the left and the right tail distribution of the exchange rate illustrate some differences and therefore advisable to keep the distribution as skewed. There are two conditional distributions namely: *std* and *sstd*.

The Table 4.14 below shows the fitted *std* and *sstd* on EGARCH (1.1).

Table 4.14 AIC values of the EGARCH (1.1) model under *std* and *sstd* for each of the BRICS exchange rates

Exchange Rates	AIC	
	Std	**sstd**
BRAZIL	-3.857	-3.820
CHINA	-7.998	-8.072
INDIA	-5.046	-5.030
RUSSIA	-3.682	-3.667
SOUTH AFRICA	-3.777	-3.762

Table 4.14 indicates that *std* has the most lowest AIC values of all the BRICS exchange rates. Therefore EGARCH (1.1) model was fitted using the *std*.

The results are presented in Table 4.15 below.

4.6.1 Parameter estimation

Table 4.15 Summary table of EGARCH (1.1) model parameter estimates for each of the BRICS exchange rates

Exchange Rates	Parameter	Estimate	Std. Error	t-value	p-value
Brazil	ω	-5.871	1.709	-3.435	0.001 ***
	α_1	0.346	0.161	2.148	0.032 *
	β_1	0.136	0.251	0.540	0.589
	γ_1	0.673	0.223	3.022	0.003 **
China	ω	-0.249	0.228	-1.091	0.275
	α_1	-0.373	0.154	-2.424	0.015 *
	β_1	0.985	0.021	46.958	0.000 ***
	γ_1	1.001	0.351	2.849	0.004 **
India	ω	-0.889	0.682	-1.305	0.192
	α_1	0.168	0.101	1.657	0.097 ·
	β_1	0.892	0.086	10.364	0.000 ***
	γ_1	0.279	0.180	1.555	0.120
Russia	ω	-1.880	0.789	-2.381	0.017 *

	α_1	0.158	0.126	1.254	0.210
	β_1	0.717	0.119	6.041	0.000 ***
	γ_1	0.957	0.216	4.429	0.000 ***
South Africa	ω	-0.838	0.633	-1.323	0.186
	α_1	0.148	0.095	1.548	0.122
	β_1	0.879	0.094	9.321	0.000 ***
	γ_1	0.126	0.152	0.829	0.407

*Note: '***','**', '*' and '·' indicates significant codes at 0.001, 0.01, 0.05 and 0.1 respectively.*

The above table 4.15 shows the leverage effects, γ_1, of all the BRICS countries exchange rates is greater than zero or positive coefficients implying that an increase in the BRICS exchange rate have greater impact on the conditional volatility as compare to the decrease in the BRICS exchange rate. The impact for South Africa is very weak γ (0.126) and smaller than the symmetric effect α (0.148). The impact for the rest of the BRICS countries (Brazil γ (0.673); China γ (1.001); India γ (0.279); Russia γ (0.957)) appears to be very strong and larger than the symmetric effect of those BRICS countries (Brazil α (0.346); China α (-0.373); India α (0.168); Russia α (0.158)). The relative size of the two groups of coefficients (γ and α) suggests that the asymmetric effects dominates the symmetric effects except for South Africa which illustrated the opposite. All the BRICS countries stationarity is also assured by the past volatility coefficient β less than one. It must be noted however that β for China, India, Russia and South Africa implies that there is the presence of high shock persistence in the exchange rates. Brazil on one hand has low shock persistence in their exchange rates. The following models are deduced from the above Table 4.15, the EGARCH (1.1) conditional variance equations for each BRICS exchange rates are written as follows:

Brazil

$$\ln(\sigma_t^2) = -5.871 + 0.136 ln(\sigma_{t-1}^2) + 0.673 z_{t-1} + 0.346\left(|z_{t-1}| + \sqrt{2/\pi}\right) \quad (4.11)$$

China

$$\ln(\sigma_t^2) = -0.249 + 0.985 ln(\sigma_{t-1}^2) + 1.001 z_{t-1} - 0.373\left(|z_{t-1}| + \sqrt{2/\pi}\right) \quad (4.12)$$

India

$$\ln(\sigma_t^2) = -0.889 + 0.892 ln(\sigma_{t-1}^2) + 0.279 z_{t-1} + 0.168\left(|z_{t-1}| + \sqrt{2/\pi}\right) \quad (4.13)$$

Russia

$$\ln(\sigma_t^2) = -1.880 + 0.717ln(\sigma_{t-1}^2) + 0.957z_{t-1} + 0.158\left(|z_{t-1}| + \sqrt{2/\pi}\right) \quad (4.14)$$

South Africa

$$\ln(\sigma_t^2) = -0.838 + 0.879ln(\sigma_{t-1}^2) + 0.126z_{t-1} + 0.148\left(|z_{t-1}| + \sqrt{2/\pi}\right) \quad (4.15)$$

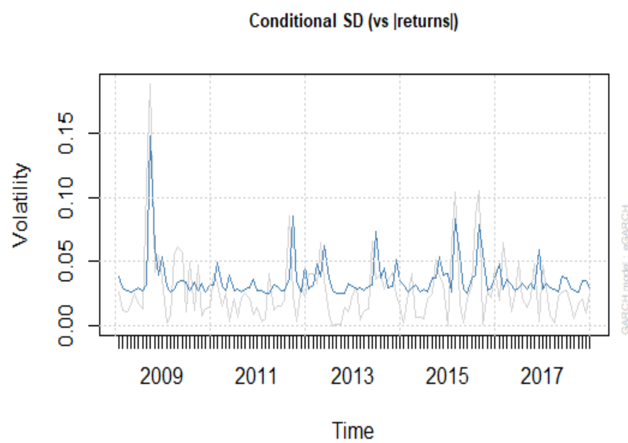
where

$$z_t = \frac{\varepsilon_t}{\sqrt{\sigma_t^2}}$$

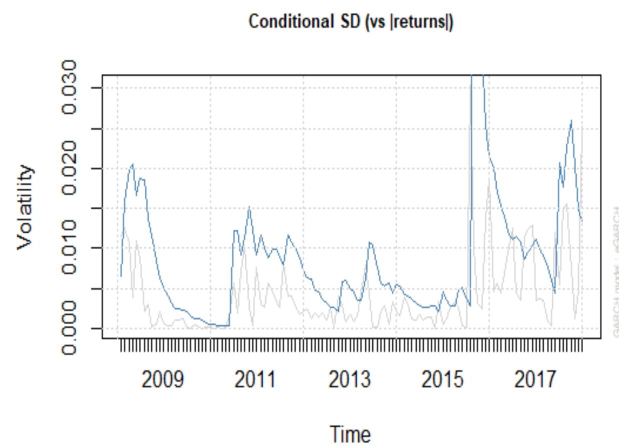
The sum of the estimates $\hat{\alpha}_1$ and $\hat{\beta}_1$ of all the BRICS exchange rates series are less than one except for India and South Africa which are slightly greater than one. This means that the unconditional volatility for the three BRICS exchange rates series is finite. The results further revealed that India has the highest volatility persistence value of $\hat{\alpha}_1$ + $\hat{\beta}_1$= 1.060, followed by South Africa with the value of $\hat{\alpha}_1$ + $\hat{\beta}_1$= 1.027, followed by Russia with the value of $\hat{\alpha}_1$ + $\hat{\beta}_1$= 0.875, followed by China with the value of $\hat{\alpha}_1$ + $\hat{\beta}_1$= 0.612 and the least is Brazil with volatility persistence value of $\hat{\alpha}_1$ + $\hat{\beta}_1$= 0.482.

The Figure 4.11 below shows the BRICS conditional volatility.

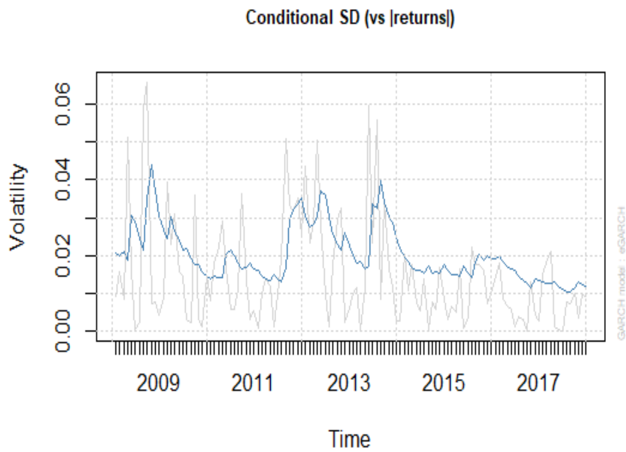
Brazil



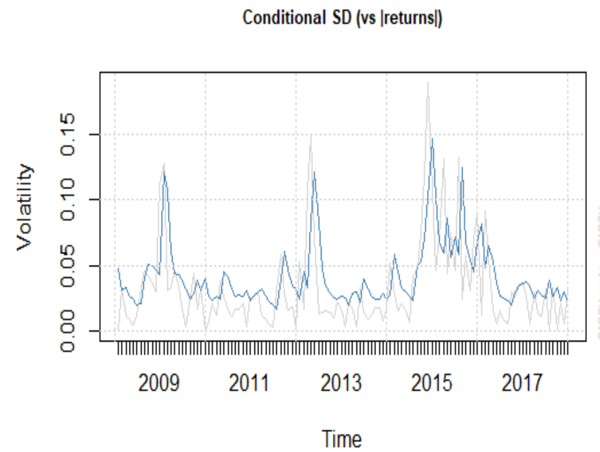
China



India



Russia



South Africa

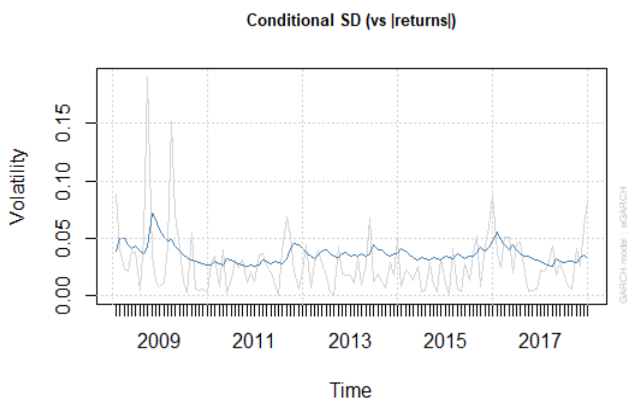


Figure 4.11 BRICS conditional volatility

The volatility scales in Figure 4.11 above shows that Russia has the highest volatility followed by Brazil, India and South Africa and the least volatile is China.

The next subsection 4.6.2 illustrates the diagnostic tests.

4.6.2 Diagnostic tests

Model adequacy testing is done using the following diagnostic tests: goodness of fit test; Ljung-Box (R), Ljung-Box (R^2), and ARCH-LM.

Table 4.16 Diagnostic test of the EGARCH (1.1) model

Exchange Rates	**Diagnostic test**	**Statistic**	**p-value**
Brazil	Goodness of fit test	17.670	0.545
	Ljung-Box (R)	15.060	1.042e-04 ***
	Ljung-Box (R^2)	0.070	0.791
	ARCH-LM	0.1115	0.739
China	Goodness of fit test	33.670	0.201
	Ljung-Box (R)	0.786	0.375
	Ljung-Box (R^2)	0.048	0.827
	ARCH-LM	0.039	0.843
India	Goodness of fit test	17.330	0.567
	Ljung-Box (R)	5.572	0.018 **
	Ljung-Box (R^2)	0.250	0.617
	ARCH-LM	2.286	0.131
Russia	Goodness of fit test	16.000	0.657
	Ljung-Box (R)	10.770	0.001
	Ljung-Box (R^2)	0.005	0.943
	ARCH-LM	0.974	0.324
South Africa	Goodness of fit test	28.000	0.083
	Ljung-Box (R)	6.721	0.010 **
	Ljung-Box (R^2)	0.549	0.459
	ARCH-LM	0.410	0.522

*Note: '***', '**', '*' and '·' indicates significant codes at 0.001, 0.01, 0.05 and 0.1 respectively.*

The data in the above Table 4.16 shows that all the BRICS exchange rates have no ARCH errors, since all the p-values of the ARCH-LM test are greater than 0.05 level of significance. The Ljung-Box (R^2) revealed that the residuals of the squared BRICS exchange rates do not have serial correlation. All the BRICS exchange rates show that the fitted residuals are normally distributed. The Q-Q plots in Figure 4.10 for BRICS exchange rates are in support of the above assertion that the fitted residual are normally distributed. Therefore, EGARCH (1.1) under the *std* conditional distribution appears to be adequate and can be used for further analysis.

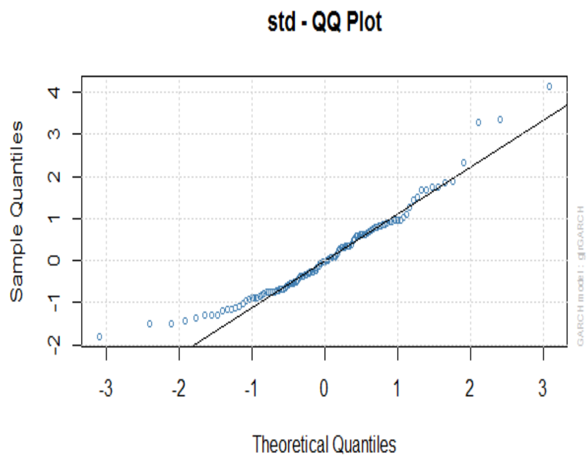
The next Section 4.7 presents the TGARCH model.

4.7 Threshold GARCH (TGARCH)

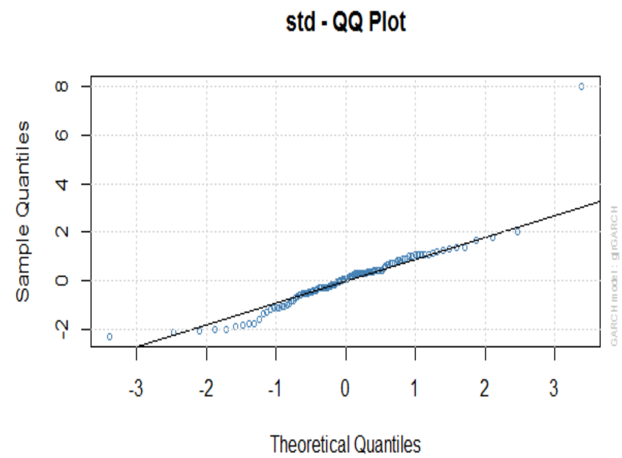
This section presents the univariate Threshold GARCH (1.1) model for the BRICS exchange rates. As seen in the previous section on ARCH, all the BRICS exchange rates showed some presence of ARCH errors. Depending of the nature of the time series the model (TGARCH (1.1)) may present different assumptions of conditional distribution. The Gaussian normal distribution appeared to be the most common conditional distribution. The first step in fitting the TGARCH (1.1) is to plot the Q-Q plots. Figure 4.12 below presents the Q-Q plots of the BRICS exchange rates and in theory the plot takes the shape of $y = x$.

4.7.1 Q-Q plots

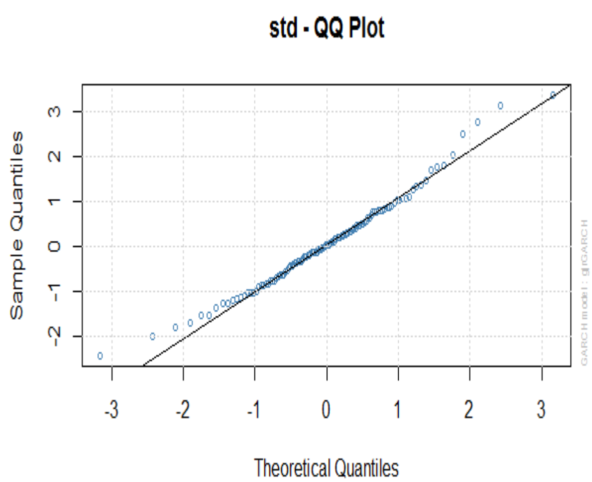
Brazil



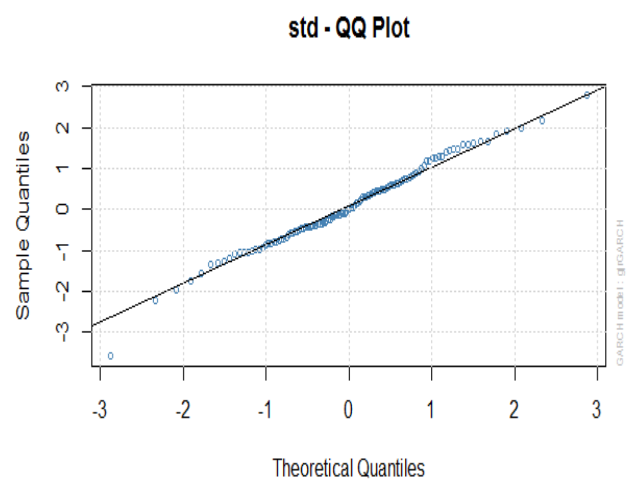
China



India



Russia



South Africa

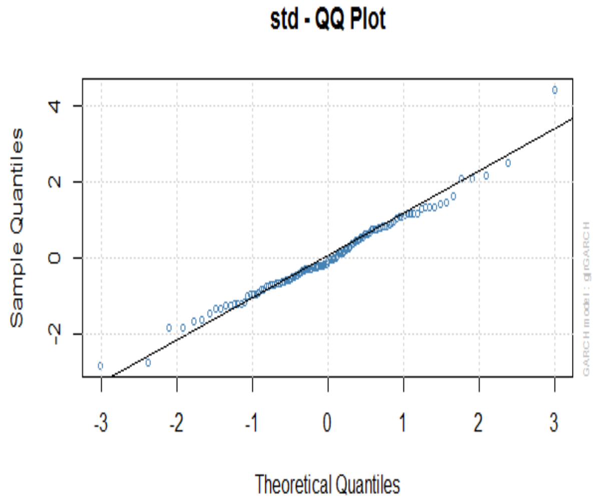


Figure 4.12 Q-Q plots for BRICS exchange rates

The above Figure 4.12 depicts that the most of the BRICS exchange rates points lie on the normal line. All the BRICS exchange rates Q-Q plots follow a normal distribution with some extreme tails. Both the left and the right tail distribution of the exchange rate illustrate some differences and therefore advisable to keep the distribution as skewed. There are two conditional distributions namely: *std* and *sstd*.

The Table 4.17 below shows the fitted *std* and *sstd* on TGARCH (1.1).

Table 4.17 AIC values of the TGARCH (1.1) model for each of the BRICS exchange rates

Exchange Rates	**AIC**	
	Std	**Sstd**
BRAZIL	-3.785	-3.901
CHINA	-7.872	-------
INDIA	-5.045	-5.036
RUSSIA	-3.674	-3.536
SOUTH AFRICA	-3.788	-3.784

Table 4.17 indicates that *std* has the most lowest AIC values of all the BRICS exchange rates. Therefore TGARCH (1.1) model was fitted using the *std*.

The parameter estimation results are presented in Table 4.18 below.

4.7.2 Parameter estimation

Table 4.18 Summary table of TGARCH (1.1) model parameter estimates for each of the BRICS exchange rates

Exchange Rates	Parameter	Estimate	Std. Error	t-value	p-value
Brazil	μ	0.001	0.00005	29.157	0.000 ***
	ω	0.0001	0.00001	12.383	0.000 ***
	α_1	0.078	0.015	5.309	0.000 ***
	β_1	0.935	0.0001	13928.338	0.000 ***
	γ_1	-0.297	0.026	-11.593	0.000 ***
China	μ	-0.001	0.0003	-4.195	0.000 ***
	ω	0.000	0.000001	0.114	0.909
	α_1	0.060	0.005	11.982	0.000 ***
	β_1	0.997	0.0002	4886.005	0.000 ***
	γ_1	-0.124	0.006	-20.885	0.000 ***
India	μ	0.002	0.001	1.197	0.231
	ω	0.00003	0.00003	1.120	0.263
	α_1	0.248	0.149	1.673	0.094
	β_1	0.777	0.117	6.671	0.000 ***
	γ_1	-0.255	0.162	-1.577	0.115
Russia	μ	-0.001	0.002	-0.222	0.825
	ω	0.0003	0.0001	2.618	0.009 **
	α_1	0.853	0.312	2.737	0.006 **
	β_1	0.254	0.127	2.011	0.044 *
	γ_1	-0.394	0.371	-1.063	0.288
South Africa	μ	0.004	0.002	1.802	0.072
	ω	0.000003	0.00001	0.465	0.642
	α_1	0.039	0.003	13.003	0.000 ***
	β_1	1.000	0.0001	16044.983	0.000 ***
	γ_1	-0.102	0.007	-13.664	0.000 ***

*Note: '***', '**', '*' and '·' indicates significant codes at 0.001, 0.01, 0.05 and 0.1 respectively.*

The above table 4.18 shows the leverage effects, γ_1, of all the BRICS countries exchange rates is less than zero or negative coefficients implying that a decrease in the BRICS exchange rate has lesser impact on the conditional volatility as compare to the increase in the BRICS exchange rate except for Brazil. The impact of all the BRICS countries (Brazil γ (-0.297); China γ (-0.124); India γ (-0.255); Russia γ (-0.394); South Africa γ (-0.102)) appears to be very weak γ (0.126) and smaller than the symmetric effect of those BRICS countries (Brazil α (0.078); (China α (0.060); India α (0.248); Russia α (0.853); South Africa α (0.039)). The estimated γ for all the BRICS exchange rates proves that the bad news has no effect to the volatility. The relative size of the two groups of coefficients (γ and α) suggests that the symmetric effects dominates the asymmetric effects. All the BRICS countries stationarity is also assured by the past volatility coefficient β less than one except for South Africa. It must be noted however that β for Brazil, China, India and South Africa implies that there is the presence of high shock persistence in the exchange rates. Russia on one hand has low shock persistence in their exchange rates.

The following models are deduced from the above Table 4.18, the TGARCH (1.1) model equations for each BRICS exchange rates are written as follows

$x_r(Brazil) = 0.001(\pm 0.000045) + \varepsilon_t,$

Brazil: $\sigma_t^2 = 0.0001 + 0.078\epsilon_{t-1}^2 + 0.935\epsilon_{t-1}^2 + 0.297\epsilon_{t-1}^2(\epsilon_{t-1}^2 > 0)$ (4.16)

$x_r(China) = -0.001(\pm 0.0003) + \varepsilon_t,$

China: $\sigma_t^2 = 0.000 + 0.060\epsilon_{t-1}^2 + 0.997\epsilon_{t-1}^2 - 0.124\epsilon_{t-1}^2(\epsilon_{t-1}^2 > 0)$ (4.17)

$x_r(India) = 0.002(\pm 0.001) + \varepsilon_t,$

India: $\sigma_t^2 = 0.00003 + 0.248\epsilon_{t-1}^2 + 0.777\epsilon_{t-1}^2 - 0.255\epsilon_{t-1}^2(\epsilon_{t-1}^2 > 0)$ (4.18)

$x_r(Russia) = -0.0006(\pm 0.003) + \varepsilon_t,$

Russia: $\sigma_t^2 = 0.0003 + 0.853\epsilon_{t-1}^2 + 0.254\epsilon_{t-1}^2 - 0.394\epsilon_{t-1}^2(\epsilon_{t-1}^2 > 0)$ (4.19)

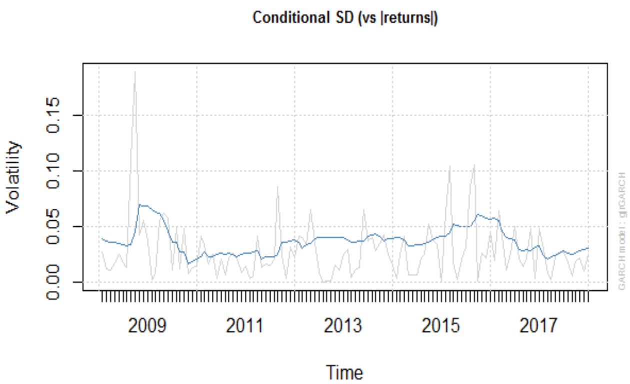
$x_r(SouthAfrica) = 0.005(\pm 0.002) + \varepsilon_t,$

South Africa: $\sigma_t^2 = 0.000003 + 0.039\epsilon_{t-1}^2 + 1.000\epsilon_{t-1}^2 - 0.102\epsilon_{t-1}^2(\epsilon_{t-1}^2 > 0)$ (4.20)

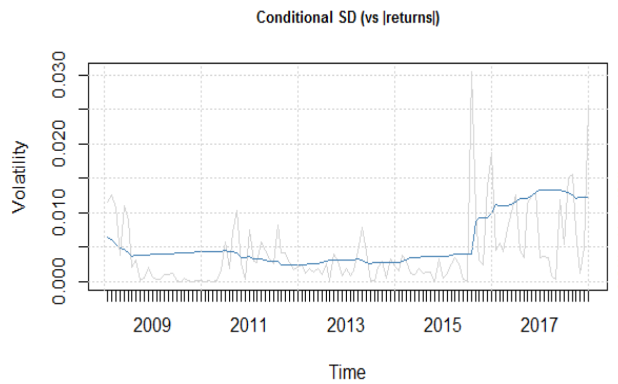
The sum of the estimates $\hat{\alpha}_1$ and $\hat{\beta}_1$ of all the BRICS exchange rates series are all slightly greater than one. This means that the unconditional volatility for all the BRICS exchange rates is finite. The results further revealed that Russia has the highest volatility persistence value of $\hat{\alpha}_1 + \hat{\beta}_1 = 1.107$, followed by China with the value of $\hat{\alpha}_1 + \hat{\beta}_1 = 1.057$, followed by South Africa with the value of $\hat{\alpha}_1 + \hat{\beta}_1 = 1.039$, followed by India with the value of $\hat{\alpha}_1 + \hat{\beta}_1 = 1.025$ and the least is Brazil with volatility persistence value of $\hat{\alpha}_1 + \hat{\beta}_1 = 1.013$.

The Figure 4.13 below shows the BRICS conditional volatility.

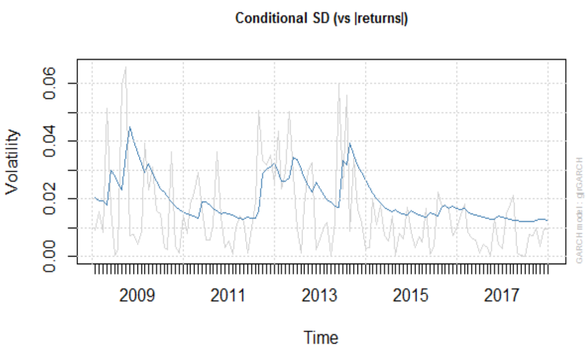
Brazil



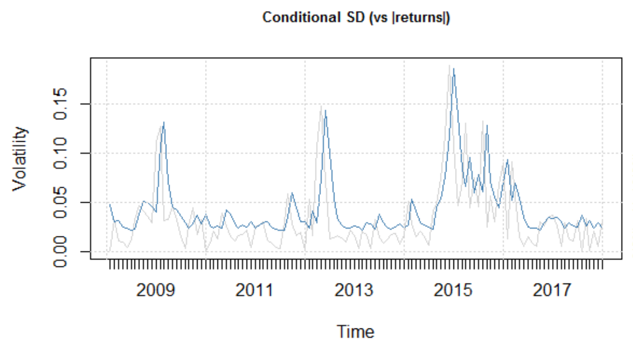
China



India



Russia



South Africa

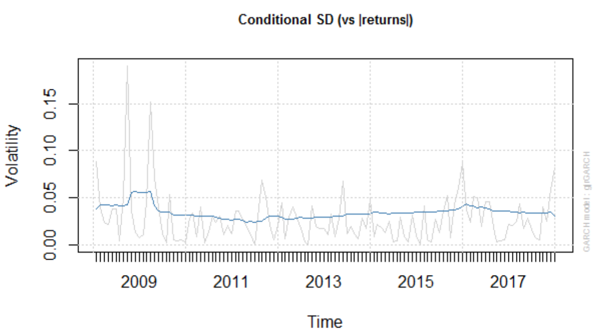


Figure 4.13 BRICS conditional volatility

The volatility scales in Figure 4.13 above shows that Brazil, Russia and South Africa have the highest volatility followed by India and the least volatile is China.

The next subsection 4.7.3 illustrates the diagnostic tests.

4.7.3 Diagnostic tests

Model adequacy testing is done using the following diagnostic tests: goodness of fit test; Ljung-Box (R), Ljung-Box (R^2), and ARCH-LM.

Table 4.19 Diagnostic test of the TGARCH (1.1) model

Exchange Rates	Diagnostic test	Statistic	p-value
Brazil	Goodness of fit test	23.67	0.2093
	Ljung-Box (R)	18.69	1.538e-05 ***
	Ljung-Box (R^2)	8.068	0.004506 **
	ARCH-LM	0.7918	0.3735
China	Goodness of fit test	34.00	0.01838 **
	Ljung-Box (R)	15.69	7.470e-05 ***
	Ljung-Box (R^2)	0.003951	0.9499
	ARCH-LM	0.06435	0.7997
India	Goodness of fit test	13.00	0.8386
	Ljung-Box (R)	6.144	0.013 **
	Ljung-Box (R^2)	0.2175	0.6410
	ARCH-LM	2.607	0.1064
Russia	Goodness of fit test	13.00	0.8386
	Ljung-Box (R)	9.356	0.002 **
	Ljung-Box (R^2)	0.2649	0.6068
	ARCH-LM	0.9126	0.3394
South Africa	Goodness of fit test	13.67	0.8028
	Ljung-Box (R)	9.105	0.003 **
	Ljung-Box (R^2)	0.9062	0.3411
	ARCH-LM	0.3894	0.5326

*Note: '***', '**', '*' and '·' indicates significant codes at 0.001, 0.01, 0.05 and 0.1 respectively.*

The data in the above Table 4.19 shows that all the BRICS exchange rates have no ARCH errors, since all the p-values of the ARCH-LM test are greater than 0.05 level of significance. The Ljung-Box (R^2) revealed that the residuals of the squared BRICS exchange rates do not have serial correlation. All the BRICS exchange rates show that the fitted residuals are normally distributed except for Russia which has a p-value less than 0.05. The Q-Q plots in Figure 4.12 for BRICS exchange rates are in support of the above assertion that the fitted residual are normally distributed. Therefore, TGARCH (1.1) under the *std* conditional distribution appears to be adequate and can be used for further analysis. Forecasting is demonstrated in the following subsection.

4.7.4 Forecasting

The forecasts of the TGARCH (1.1) are presented in the following Table 4.20. The volatility of each BRICS exchange rate was forecasted for five periods ahead.

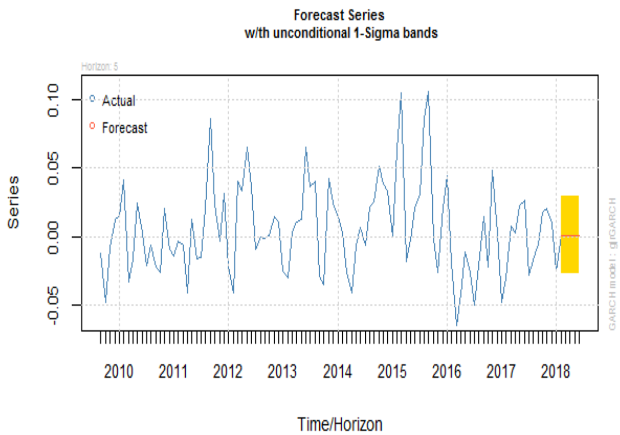
Table 4.20 Forecasting

Exchange Rates	Time (months)	Mean forecast	Mean error	95% Lower CI	95% Upper CI
Brazil	1	0.001	0.028	-0.494	0.494
	2	0.001	0.028	-0.494	0.494
	3	0.001	0.028	-0.494	0.494
	4	0.001	0.028	-0.494	0.494
	5	0.001	0.028	-0.494	0.494
China	1	0.001	0.011	-0.100	0.100
	2	0.001	0.011	-0.100	0.100
	3	0.001	0.011	-0.100	0.100
	4	0.001	0.011	-0.100	0.100
	5	0.001	0.011	-0.100	0.100
India	1	0.002	0.012	-0.400	0.400
	2	0.002	0.013	-0.400	0.400
	3	0.002	0.014	-0.400	0.400
	4	0.002	0.014	-0.400	0.400
	5	0.002	0.015	-0.400	0.400
Russia	1	-0.001	0.030	-0.700	0.700
	2	-0.001	0.034	-0.700	0.700
	3	-0.001	0.037	-0.700	0.700
	4	-0.001	0.040	-0.700	0.700
	5	-0.001	0.042	-0.700	0.700
South Africa	1	0.004	0.022	-0.461	0.461
	2	0.004	0.022	-0.461	0.461
	3	0.004	0.022	-0.461	0.461
	4	0.004	0.021	-0.461	0.461
	5	0.004	0.021	-0.461	0.461

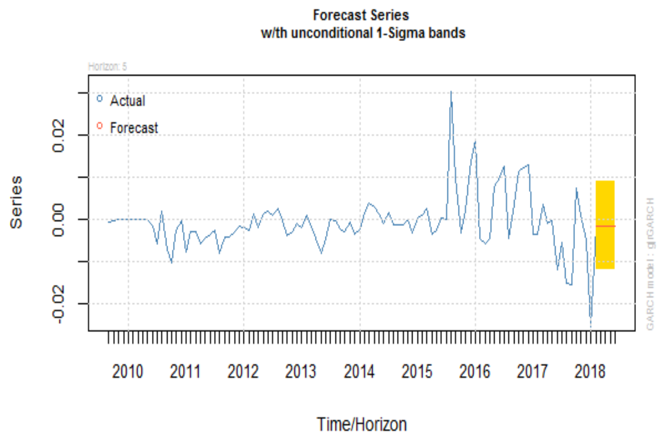
Table 4.20 above presents the mean and volatility forecasts of the BRICS exchange rates. The mean forecasts falls within the 95% confidence interval.

Figure 4.14 presents the volatility forecast plots with 95% CI.

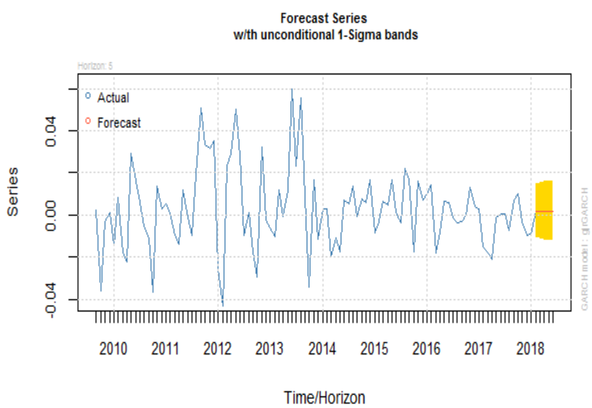
Brazil



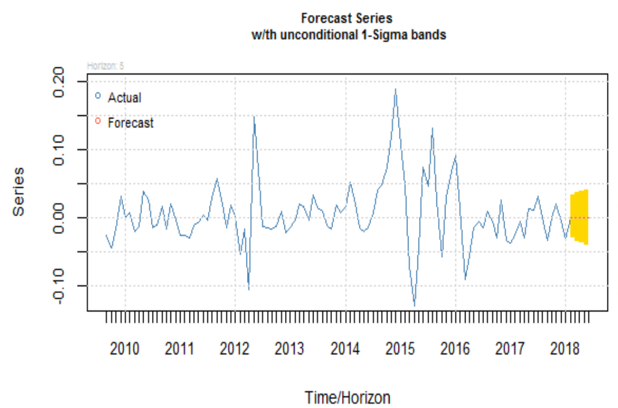
China



India



Russia



South Africa

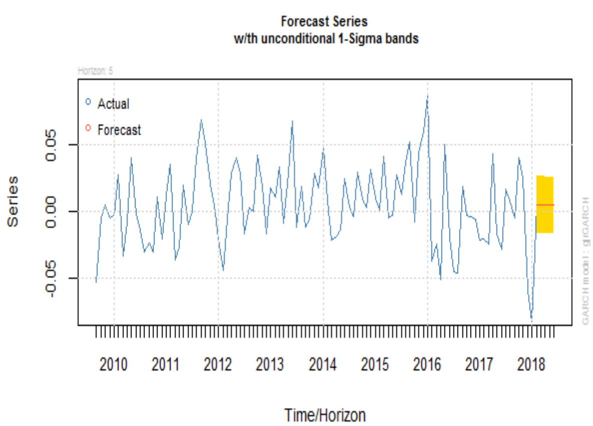
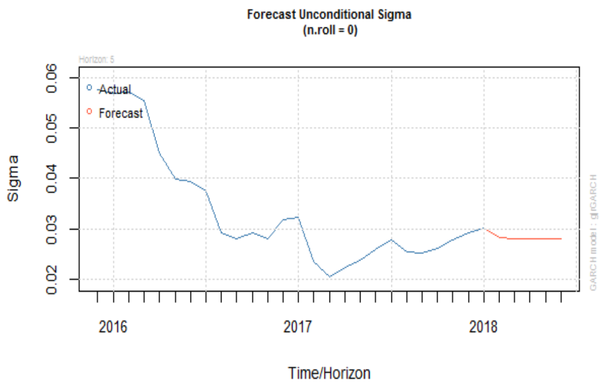


Figure 4.14 Volatility Forecast plots with 95% CI

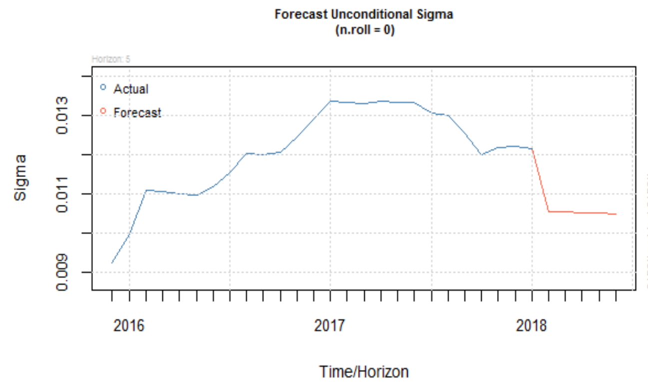
All the volatility forecasts in Figure 4.14 above confirm that the volatility forecasts are within the 95% confidence limit.

Figure 4.15 illustrates the volatility forecasts plots with a five period ahead.

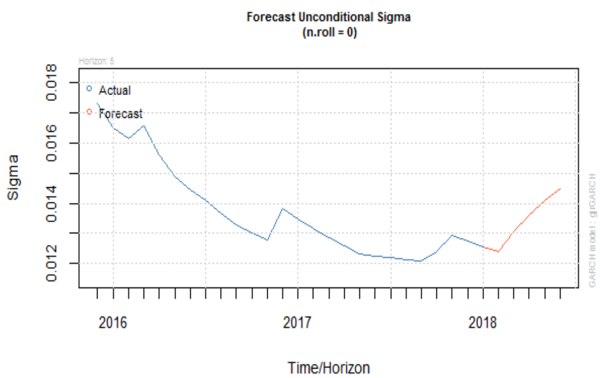
Brazil



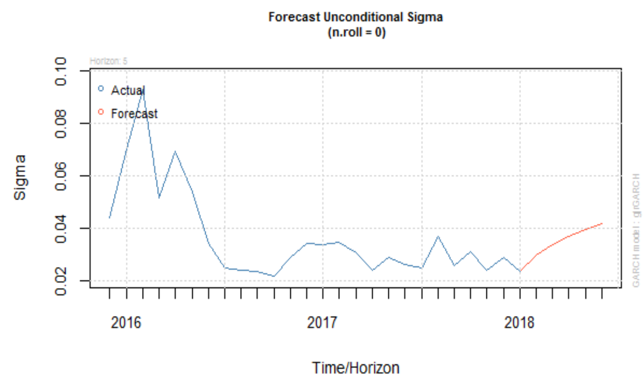
China



India



Russia



South Africa

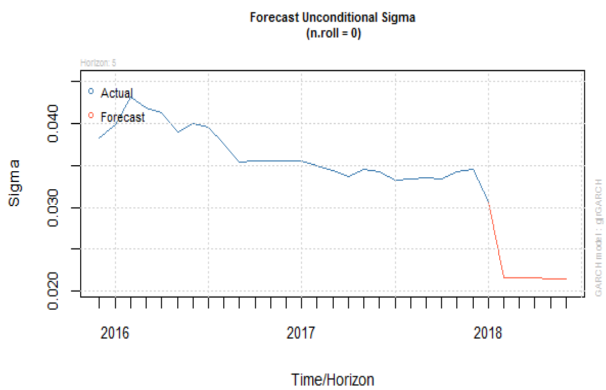


Figure 4.15 Volatility Forecast plots

The Figure 4.15 above shows that all the BRICS exchange rates volatility forecasts plots are on the rise except for China and South Africa.

The next Section 4.8 presents the BEKK-GARCH.

4.8 MULTIVARIATE GARCH USING BEKK APPROACH

The section presents the extension of the univariate GARCH model using a Multivariate BEKK approach. It provides the dynamic relations amongst the BRICS exchange rates. Table 4.21 – 4.23 presents the volatility spill-overs which are mainly results from the BEKK-GARCH.

Table 4.21 Volatility spill-overs: Results from BEKK-GARCH model

Triangular matrix of constant				
Variable	**Coefficient**	**Std. Error**	**t-statistic**	**p-value**
B_{11}	1.259	0.506	2.488	1.259
B_{21}	0.605	0.384	1.574	0.605
B_{22}	1.069	0.201	5.308	1.069
B_{31}	0.152	0.411	0.370	0.152
B_{32}	0.076	0.336	0.225	0.076
B_{33}	1.067	0.023	46.573	1.067
B_{41}	-18.341	19.349	-0.948	-18.341
B_{42}	-12.543	3.516	-3.567	-12.543
B_{43}	-0.499	1.535	-0.325	-0.499
B_{44}	6.142	3.140	1.956	6.142
B_{51}	17.249	20.126	0.857	17.249
B_{52}	11.878	6.584	1.804	11.878
B_{53}	0.705	5.042	0.140	0.705
B_{54}	-5.526	5.806	-0.952	-5.526
B_{55}	71.665	5.786	12.385	71.665

*Note: '***', '**', '*' and '·' indicates significant codes at 0.001, 0.01, 0.05 and 0.1 respectively.*

Table 4.22 Volatility spill-overs: Results from BEKK-GARCH model

ARCH effect				
Variable	**Coefficient**	**Std. Error**	**t-statistic**	**p-value**
C_{11}	-0.221	2.885	-0.076	0.939
C_{12}	-0.117	2.943	-0.040	0.968
C_{13}	0.008	9.406	0.001	0.999
C_{14}	11.447	136.477	0.084	0.933
C_{15}	-10.424	169.354	-0.062	0.951
C_{21}	-0.608	2.862	-0.213	0.832
C_{22}	-0.354	3.683	-0.096	0.924
C_{23}	-0.087	6.896	-0.013	0.990
C_{24}	25.217	68.084	0.370	0.712
C_{25}	-23.212	91.522	-0.254	0.800

ARCH effect				
Variable	**Coefficient**	**Std. Error**	**t-statistic**	**p-value**
C_{31}	-1.417	3.845	-0.368	0.713
C_{32}	-0.878	1.329	-0.661	0.510
C_{33}	-0.304	5.334	-0.057	0.955
C_{34}	53.811	167.695	0.321	0.749
C_{35}	-49.752	66.333	-0.750	0.455
C_{41}	-1.287	3.147	-0.409	0.683
C_{42}	-0.803	1.393	-0.577	0.565
C_{43}	-0.276	4.492	-0.061	0.951
C_{44}	49.276	134.230	0.367	0.714
C_{45}	-45.538	136.807	-0.333	0.740
C_{51}	-0.768	3.706	-0.207	0.836
C_{52}	-0.467	5.440	-0.086	0.932
C_{53}	-0.135	10.354	-0.013	0.990
C_{54}	30.834	58.633	0.526	0.600
C_{55}	-28.417	72.308	-0.393	0.695

*Note: '***', '**', '*' and '·' indicates significant codes at 0.001, 0.01, 0.05 and 0.1 respectively.*

Table 4.23 Volatility spill-overs: Results from BEKK-GARCH model

GARCH effect				
Variable	**Coefficient**	**Std. Error**	**t-statistic**	**p-value**
G_{11}	-0.542	1.757	-0.309	0.758
G_{12}	-0.236	1.039	-0.227	0.821
G_{13}	0.210	0.302	0.697	0.487
G_{14}	12.848	71.470	0.180	0.858
G_{15}	-11.738	67.671	-0.173	0.863
G_{21}	-0.352	2.991	-0.118	0.906
G_{22}	-0.114	1.822	-0.062	0.950
G_{23}	0.104	0.697	0.149	0.882
G_{24}	12.850	114.642	0.112	0.911
G_{25}	-11.742	107.884	-0.109	0.914
G_{31}	-0.187	0.725	-0.259	0.796
G_{32}	-0.108	0.615	-0.176	0.861
G_{33}	0.088	0.716	0.123	0.903
G_{34}	12.885	21.567	0.597	0.551
G_{35}	-11.708	19.939	-0.587	0.558
G_{41}	-0.320	0.002	-209.431	0.000 ***
G_{42}	-0.207	0.002	-97.155	0.000 ***
G_{43}	-0.101	0.003	-38.715	0.000 ***
G_{44}	11.771	0.097	120.884	0.000 ***
G_{45}	-10.988	0.123	-89.070	0.000 ***
G_{51}	-0.321	0.002	-153.158	0.000 ***
G_{52}	-0.214	0.003	-80.943	0.000 ***
G_{53}	-0.121	0.008	-15.947	0.000 ***
G_{54}	11.907	0.226	52.616	0.000 ***
G_{55}	-11.126	0.281	-39.664	0.000 ***

*Note: '***', '**', '*' and '·' indicates significant codes at 0.001, 0.01, 0.05 and 0.1 respectively.*

The results presented in Table 4.21 to Table 4.23 depict that most of the variables are statistically significant. The estimated BEKK-GARCH model can be found by substituting the following matrices into equation (3.5.5)

$$B = \begin{pmatrix} 1.259 & 0 & 0 & 0 & 0 \\ 0.605 & 1.069 & 0 & 0 & 0 \\ 0.152 & 0.076 & 1.067 & 0 & 0 \\ -18.341 & -12.543 & -0.499 & 6.142 & 0 \\ 17.249 & 11.878 & 0.705 & -5.526 & 71.665 \end{pmatrix} \tag{4.21}$$

$$\mathrm{C} = \begin{pmatrix} -0.221 & -0.117 & 0.008 & 11.447 & -10.424 \\ -0.608 & -0.354 & -0.087 & 25.217 & -23.212 \\ -1.417 & -0.878 & -0.304 & 53.811 & -49.752 \\ -1.287 & -0.803 & -0.276 & 49.276 & -45.538 \\ -0.768 & -0.467 & -0.135 & 30.834 & -28.417 \end{pmatrix} \tag{4.22}$$

$$G = \begin{pmatrix} -0.542 & -0.236 & 0.210 & 12.848 & -11.738 \\ -0.352 & -0.114 & 0.104 & 12.850 & -11.742 \\ -0.187 & -0.108 & 0.088 & 12.885 & -11.708 \\ -0.320 & -0.207 & -0.101 & 11.771 & -10.988 \\ -0.321 & -0.214 & -0.121 & 11.907 & -11.126 \end{pmatrix} \tag{4.23}$$

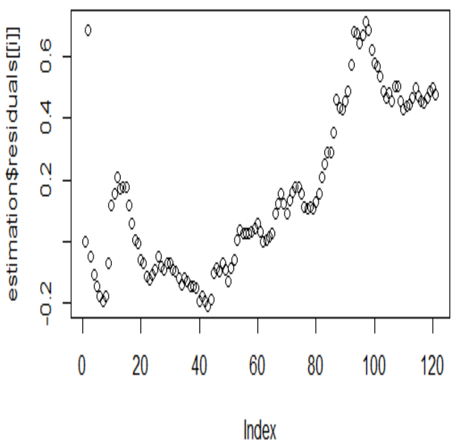
Table 4.22 and Table 4.23 above presenting the estimates of the diagonal parameters show that only G_{44} and G_{55} are statistically significant at 5% level of significance. This implies that the conditional variance of Russia and South Africa's exchange rates are affected by their own past conditional volatility and other BRICS exchange rates past conditional volatility. However, C_{11}, C_{22}, C_{33}, C_{44}, C_{55}, G_{11}, G_{22} and G_{33} are not significant implying that the past conditional volatility does not influence volatility in the BRICS exchange rates.

The off diagonal elements of the matrix C captures the cross BRICS exchange rate shock. All the off diagonal elements of the matrix C are statistically insignificant, meaning that there is no spill-over effect between Brazil, India, China, Russia and South Africa's exchange rates. The negative impact each of the BRICS exchange rates have does not affect other BRICS exchange rates.

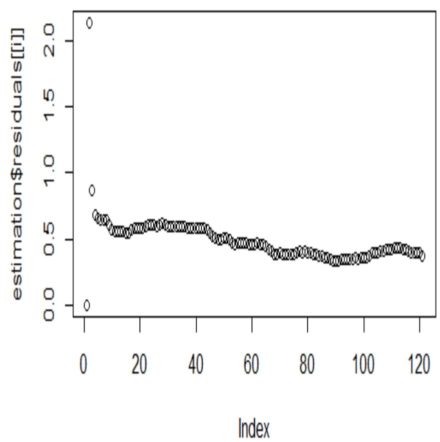
The off diagonal element of the matrix G captures the BRICS exchange rate volatility transmission. Only one pair (G_{45} and G_{54}) of the off diagonal parameter is statistically significant at 5% level of significance illustrating a bidirectional volatility transmission between Russia and South Africa. The coefficients of G_{41}, G_{42}, G_{43}, G_{51}, G_{52}, G_{53} and G_{54} are statistically significant at 5% level of significant whereas their counterparts (G_{14}, G_{24}, G_{34}, G_{15}, G_{25}, G_{35} and G_{45}) are not statistically significant. This means that there is a unidirectional volatility transmission between Russia and Brazil; Russia and China; Russia and India, South Africa and Brazil; South Africa and China; South Africa and India; and South Africa and Russia.

The next Figure 4.16 illustrates the Residual Series for BEKK-GARCH model.

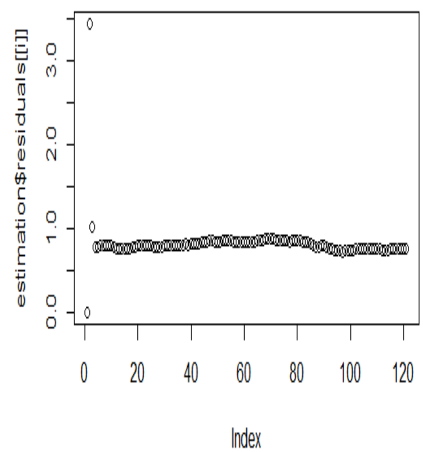
Brazil



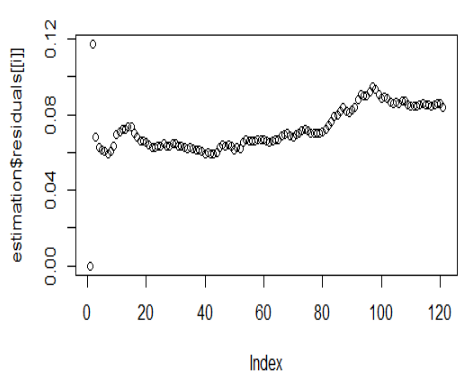
China



India



Russia



South Africa

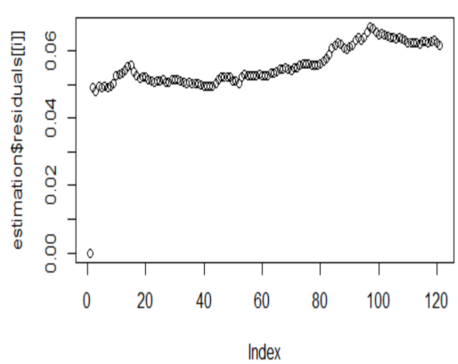


Figure 4.16 Residual Series for BEKK-GARCH model

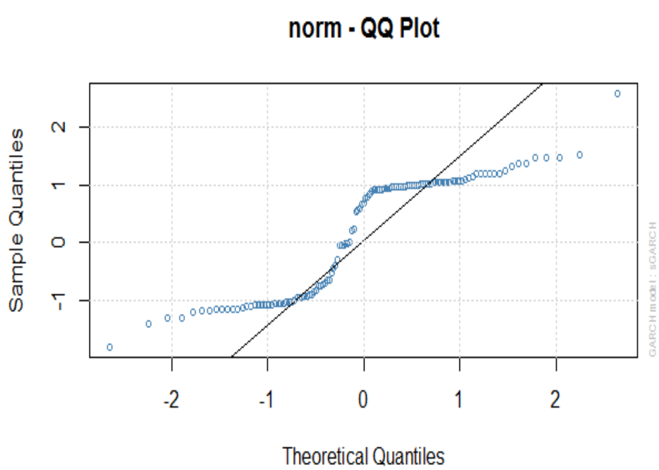
Figure 4.16 above shows that the residual series depicts a particular pattern for each of the BRICS exchange rates. Therefore, BEKK-GARCH model demonstrates the presence of autocorrelation in the residuals.

Section 4.9 presents the DCC-GARCH model.

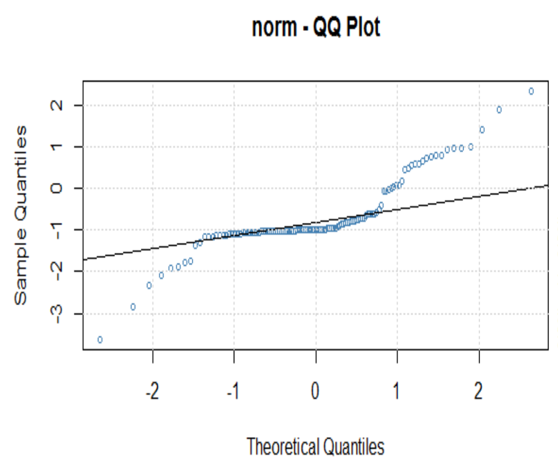
4.9 MULTIVARIATE GARCH USING DCC APPROACH

The section presents the Multivariate GARCH model using a DCC approach. It provides the dynamic relations amongst the BRICS exchange rates. Figure 4.17 presents the Q-Q plots for BRICS exchange rates

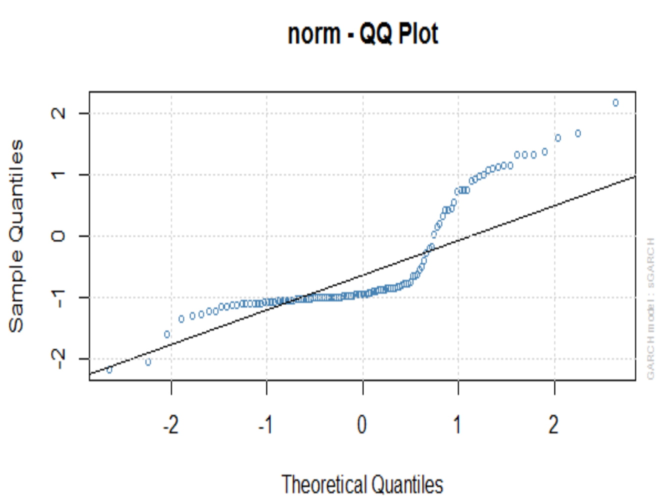
Brazil



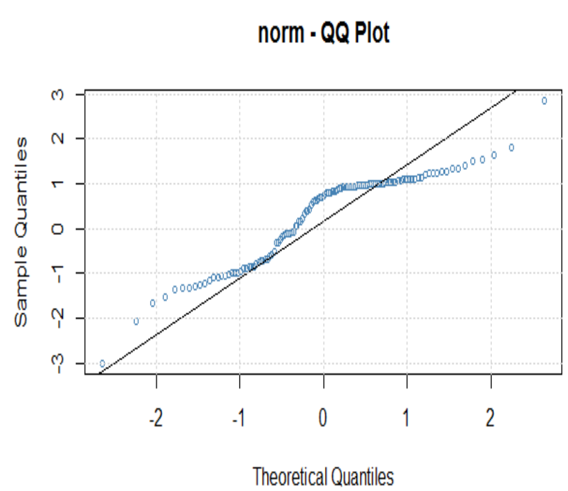
China



India



Russia



South Africa

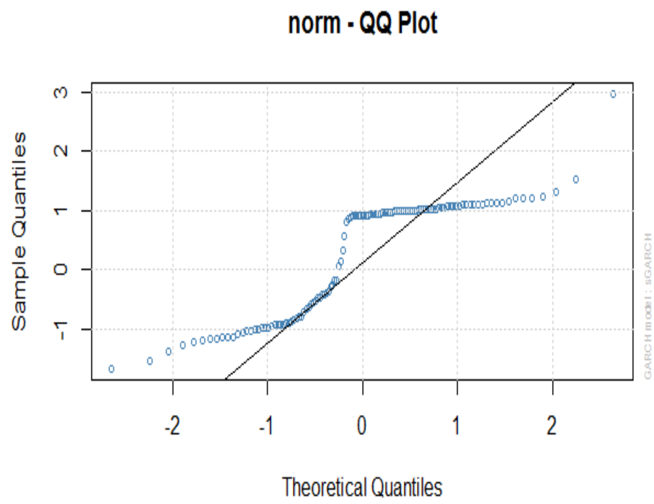


Figure 4.17 Q-Q plots for BRICS exchange rates

The above Figure 4.17 depicts that all the BRICS exchange rates points lie outside of the normal line. Therefore it is concluded that all the BRICS exchange rates Q-Q plots does not follow a normal distribution.

The Table 4.24 below shows the summary table of DCC-GARCH (1.1) model parameter estimates for each of the BRICS exchange rates.

Table 4.24 Summary table of DCC-GARCH (1.1) model parameter estimates for each of the BRICS exchange rates

Exchange Rates	Parameter	Estimate	Std. Error	t-value	p-value
Brazil	μ	0.709	0.025	28.637	0.000 ***
	Ω	0.001	0.001	1.031	0.302
	α_1	0.999	0.413	2.419	0.016 *
	β_1	0.000	0.427	0.000	1.000
China	μ	1.922	0.026	73.831	0.000 ***
	Ω	0.000	0.0001	0.006	0.995
	α_1	0.852	0.327	2.604	0.009 **
	β_1	0.147	1.753	0.084	0.933
India	μ	4.162	0.022	186.973	0.000 ***
	Ω	0.00004	0.00003	1.592	0.111
	α_1	0.774	0.088	8.773	0.000 ***
	β_1	0.208	0.081	2.558	0.011 *
Russia	μ	3.432	0.012	290.482	0.000 ***

Exchange Rates	Parameter	Estimate	Std. Error	t-value	p-value
	Ω	0.0004	0.0002	2.124	0.034 *
	α_1	0.820	0.094	8.721	0.000 ***
	β_1	0.179	0.109	1.651	0.099 ·
South Africa	μ	2.120	0.014	150.892	0.000 ***
	Ω	0.001	0.002	0.607	0.544
	α_1	0.962	0.159	6.040	0.000 ***
	β_1	0.037	0.179	0.209	0.834

*Note: '***', '**', '*' and '·' indicates significant codes at 0.001, 0.01, 0.05 and 0.1 respectively.*

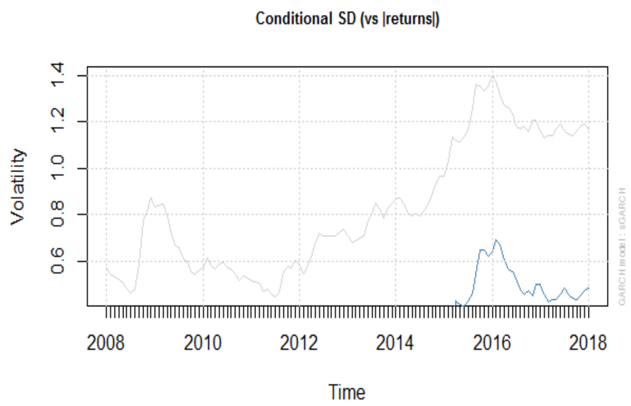
The following models are deduced from the above Table 4.24, the DCC-GARCH (1.1) model equations for each BRICS exchange rates are written as follows

$$x_r(Brazil) = 0.709(\pm 0.025) + \varepsilon_t,$$
$$\sigma_t^2 = 0.001(\pm 0.001) + 0.999(\pm 0.413)\sigma_{t-1}^2 + 0.000(\pm 0.427)\sigma_{t-1}^2 \quad (4.24)$$
$$x_r(China) = -0.000(\pm 0.0001) + \varepsilon_t,$$
$$\sigma_t^2 = 0.000(\pm 0.0001) + 0.852(\pm 0.327)\sigma_{t-1}^2 + 0.147(\pm 1.753)\sigma_{t-1}^2 \quad (4.25)$$
$$x_r(India) = 4.162(\pm 0.022) + \varepsilon_t,$$
$$\sigma_t^2 = 0.00004(\pm 0.00003) + 0.774(\pm 0.088)\sigma_{t-1}^2 + 0.208(\pm 0.081)\sigma_{t-1}^2 \quad (4.26)$$
$$x_r(Russia) = 3.423(\pm 0.012) + \varepsilon_t,$$
$$\sigma_t^2 = 0.0004(\pm 0.0002) + 0.820(\pm 0.094)\sigma_{t-1}^2 + 0.179(\pm 0.109)\sigma_{t-1}^2 \quad (4.27)$$
$$x_r(SouthAfrica) = 2.120(\pm 0.014) + \varepsilon_t,$$
$$\sigma_t^2 = 0.001(\pm 0.002) + 0.962(\pm 0.159)\sigma_{t-1}^2 + 0.037(\pm 0.179)\sigma_{t-1}^2 \quad (4.28)$$

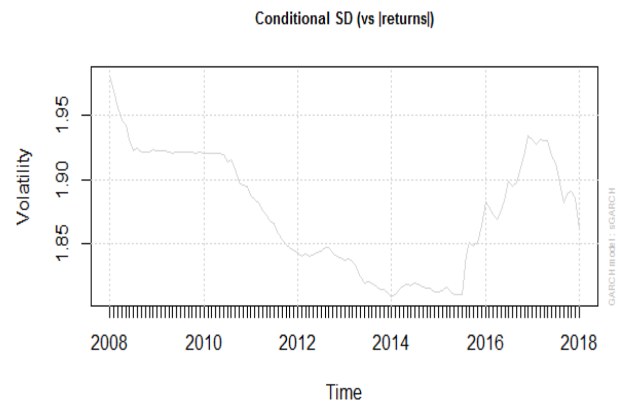
x_r represents the exchange rates for each of the BRICS countries whereas σ_t^2 illustrates the volatility part of the DCC-GARCH (1.1) model equation for each BRICS exchange rates. The sum of the estimates $\hat{\alpha}_1$ and $\hat{\beta}_1$ of all the BRICS exchange rates series are less than one meaning that the unconditional volatility for each of the BRICS exchange rates series is finite. The results further revealed that Brazil, China, Russia and South Africa has the highest volatility persistence value of $\hat{\alpha}_1$ + $\hat{\beta}_1$= 0.999, and India has the least volatility persistence value of $\hat{\alpha}_1$ + $\hat{\beta}_1$= 0.982.

The Figure 4.18 below shows the BRICS conditional volatility.

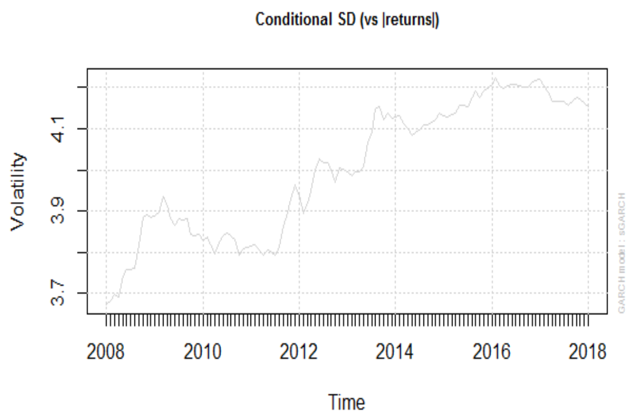
Brazil



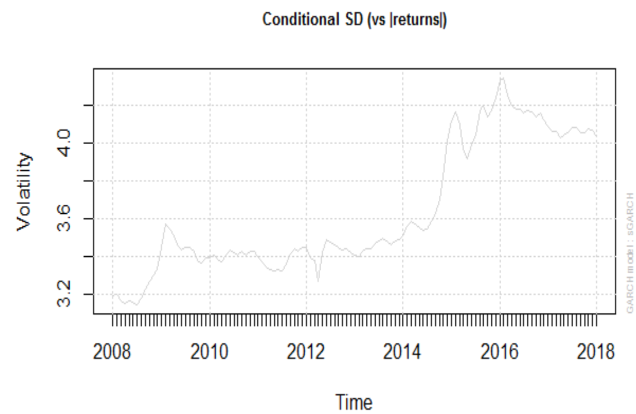
China



India



Russia



South Africa

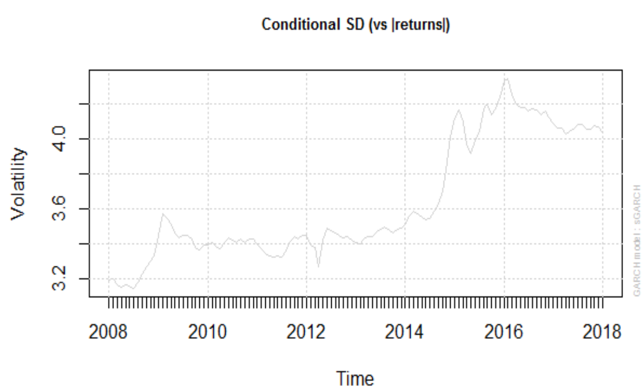
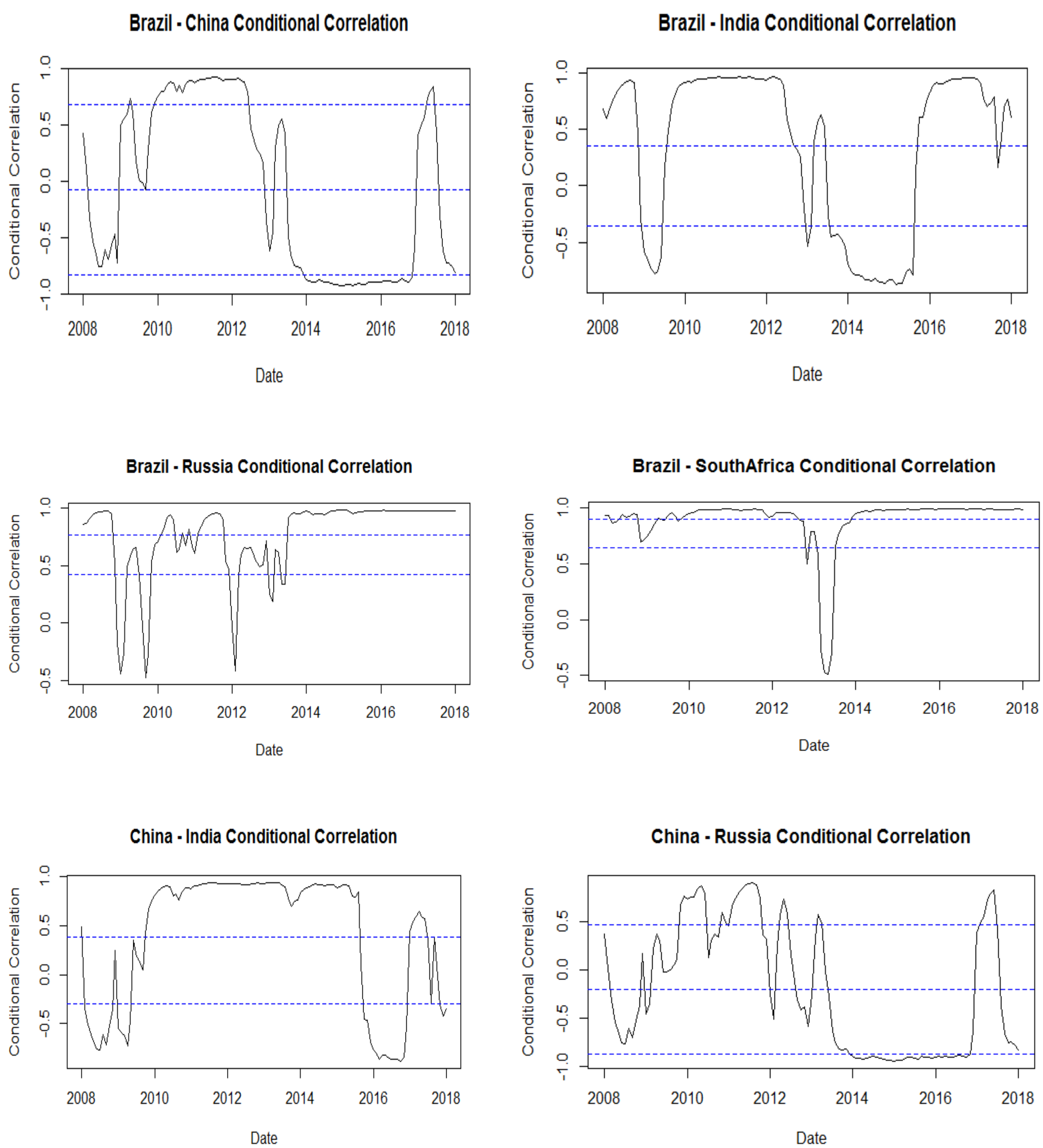


Figure 4.18 BRICS conditional volatility

The volatility scales in Figure 4.18 above shows that India has the highest volatility followed by Russia, South Africa, China and Brazil is least volatile.

The next Figure 4.19 illustrates Time-varying conditional correlations from the DCC model.



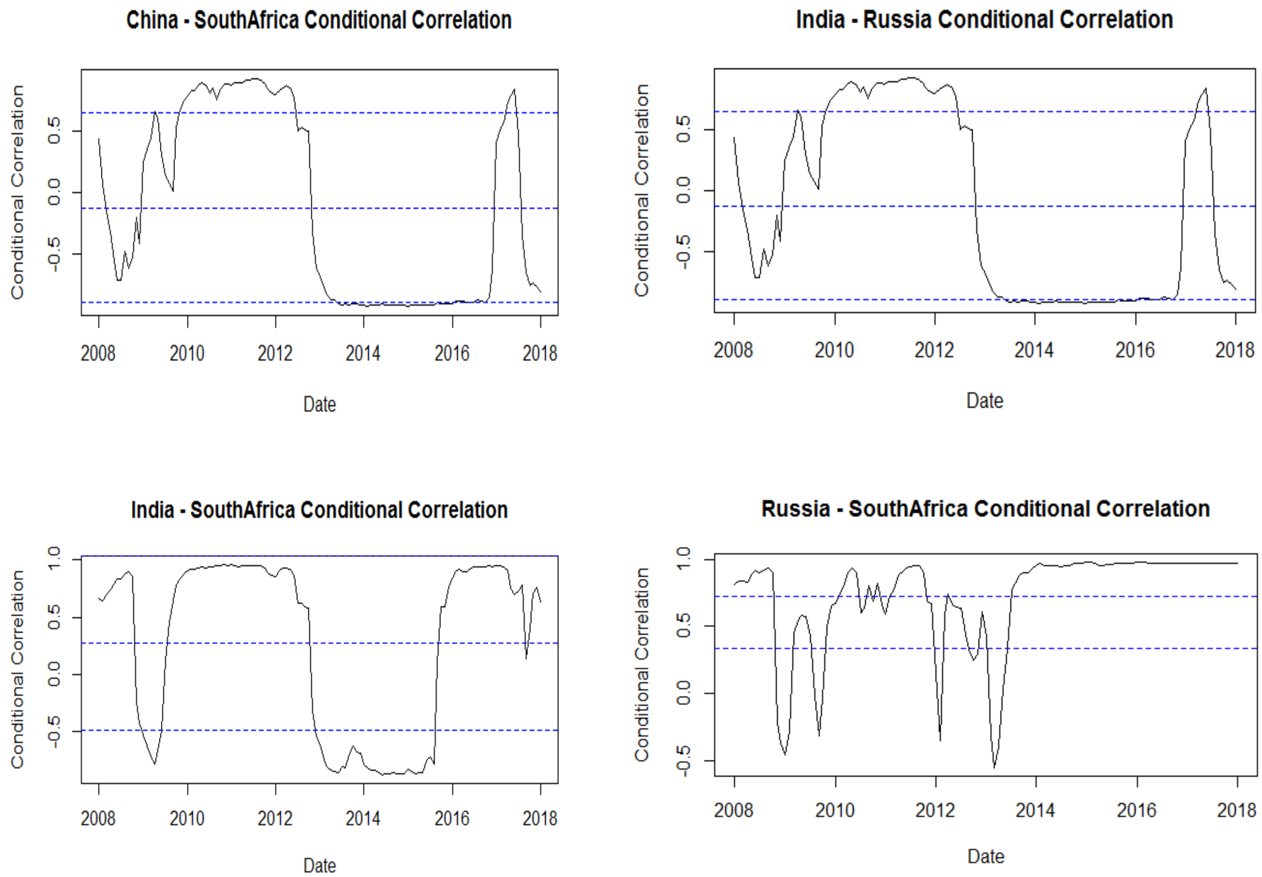


Figure 4.19 Time-varying conditional correlations from the DCC model

The above Figure 4.19 illustrates the time-varying conditional correlation between two countries at a time. DCC model was used in the construction of such conditional correlations. Brazil-China, Brazil-India, China-South Africa, and India-South Africa presented a similar pattern. The conditional correlation dynamic ranges between -1.0 and 1.0 except for China-South Africa are within the ranges of between -0.5 and 1.0.

Brazil-Russia conditional correlation is on the main in the range between 0.0 and 1.0 indicating that the majority of the data values are on the positive side of the correlation. Brazil-South Africa conditional correlation is also on the main positively correlated. The China-South Africa and India-Russia presents a similar pattern with the conditional correlation dynamic ranging between -0.5 and 1.0. China-Russia and Russia-South Africa also presents the similar pattern with the conditional correlation dynamic ranging between -0.5 and 1.0.

4.9.1 Diagnostic tests

Model adequacy testing is done using the following diagnostic tests: goodness of fit test; Ljung-Box (R), Ljung-Box (R^2), and ARCH-LM.

Table 4.25 Diagnostic test of the DCC-GARCH (1.1) model

Exchange Rates	**Diagnostic test**	**Statistic**	**p-value**
Brazil	Goodness of fit test	209.700	3.891e-34 ***
	Ljung-Box (R)	102.100	0.000 ***
	Ljung-Box (R^2)	4.593	0.032 *
	ARCH-LM	0.409	0.523
China	Goodness of fit test	361.800	3.685e-65 ***
	Ljung-Box (R)	60.860	6.106e-15 ***
	Ljung-Box (R^2)	9.869	0.002 **
	ARCH-LM	2.510	0.113
India	Goodness of fit test	373.000	1.730e-67 ***
	Ljung-Box (R)	87.170	0.000 ***
	Ljung-Box (R^2)	0.290	0.590
	ARCH-LM	0.057	0.812
Russia	Goodness of fit test	0.383	0.702
	Ljung-Box (R)	77.440	0.000 ***
	Ljung-Box (R^2)	0.769	0.381
	ARCH-LM	1.404	0.236
South Africa	Goodness of fit test	332.400	4.406e-59 ***
	Ljung-Box (R)	97.210	0.000 ***
	Ljung-Box (R^2)	0.281	0.596
	ARCH-LM	1.273	0.259

*Note: '***', '**', '*' and '·' indicates significant codes at 0.001, 0.01, 0.05 and 0.1 respectively.*

The data in the above Table 4.25 shows that all the BRICS exchange rates have no ARCH errors, since all the p-values of the ARCH-LM test are greater than 0.05 level of significance. The Ljung-Box (R^2) revealed that the residuals of the squared BRICS exchange rates do not have serial correlation. All the BRICS exchange rates show that the fitted residuals are not normally distributed except for Russia which has a p-value more than 0.05. The Q-Q plots in Figure 4.17 for BRICS exchange rates are in support of the above assertion that the fitted residual are not normally distributed except for Russia.

The next Section 4.10 presents the VAR enhanced Multivariate GARCH model using the BEKK approach.

4.10 VAR-BEKK GARCH

The section presents the VAR enhanced Multivariate GARCH model using the BEKK approach. The VAR-Multivariate GARCH model considers the ARCH effect of the time series and calculate time varying hedge ratio.

Table 4.26 Volatility spill-overs: Results from VAR (1) BEKK-GARCH model

Triangular matrix of constant				
Variable	**Coefficient**	**Std. Error**	**t-statistic**	**p-value**
B_{11}	-0.019	0.003	-5.766	6.442e-08 ***
B_{21}	0.000	0.000	-0.326	0.745
B_{22}	0.003	0.001	3.251	1.492e-03 ***
B_{31}	-0.013	0.003	-4.969	2.258e-06 ***
B_{32}	0.001	0.005	0.217	0.828
B_{33}	0.008	0.002	3.599	4.661e-04 ***
B_{41}	-0.020	0.007	-3.000	3.283e-03 *
B_{42}	0.013	0.006	2.313	0.022 *
B_{43}	-0.003	0.007	-0.356	0.722
B_{44}	-0.020	0.007	-3.009	0.003 **
B_{51}	-0.004	0.005	-0.745	0.457
B_{52}	-0.005	0.012	-0.435	0.665
B_{53}	0.008	0.005	1.508	0.134
B_{54}	-0.013	0.006	-2.222	0.028 *
B_{55}	-0.008	0.009	-0.903	0.368

*Note: '***', '**', '*' and '·' indicates significant codes at 0.001, 0.01, 0.05 and 0.1 respectively.*

Table 4.27 Volatility spill-overs: Results from VAR (1) BEKK-GARCH model

ARCH effect				
Variable	**Coefficient**	**Std. Error**	**t-statistic**	**p-value**
C_{11}	-0.004	0.197	-0.021	0.983
C_{12}	-0.093	0.037	-2.486	0.014 *
C_{13}	0.020	0.080	0.249	0.804
C_{14}	-0.502	0.233	-2.156	0.033 *
C_{15}	-0.112	0.214	-0.526	0.600
C_{21}	1.385	1.514	0.915	0.362
C_{22}	1.100	0.190	5.786	0.000 ***
C_{23}	0.471	0.556	0.846	0.399
C_{24}	0.154	1.584	0.097	0.923
C_{25}	1.577	1.286	1.226	0.223
C_{31}	0.421	0.348	1.209	0.229
C_{32}	0.029	0.027	1.091	0.278
C_{33}	0.303	0.153	1.981	0.050 *
C_{34}	-0.026	0.336	-0.077	0.939
C_{35}	0.146	0.319	0.459	0.647
C_{41}	0.058	0.127	0.454	0.651

ARCH effect				
Variable	**Coefficient**	**Std. Error**	**t-statistic**	**p-value**
C_{42}	-0.004	0.011	-0.411	0.682
C_{43}	-0.010	0.059	-0.161	0.872
C_{44}	0.798	0.234	3.415	0.001 ***
C_{45}	0.129	0.114	1.129	0.261
C_{51}	0.060	0.157	0.380	0.705
C_{52}	0.064	0.065	0.995	0.322
C_{53}	0.012	0.080	0.151	0.880
C_{54}	0.190	0.396	0.480	0.632
C_{55}	0.309	0.264	1.171	0.244

*Note: '***', '**', '*' and '·' indicates significant codes at 0.001, 0.01, 0.05 and 0.1 respectively.*

Table 4.28 Volatility spill-overs: Results from VAR (1) BEKK-GARCH model

GARCH effect				
Variable	**Coefficient**	**Std. Error**	**t-statistic**	**p-value**
G_{11}	0.732	0.094	7.761	0.000 ***
G_{12}	0.011	0.032	0.350	0.727
G_{13}	0.346	0.141	2.453	0.016 *
G_{14}	0.066	0.177	0.372	0.711
G_{15}	0.936	0.468	1.999	0.048 *
G_{21}	-0.720	0.840	-0.857	0.393
G_{22}	-0.136	0.272	-0.499	0.619
G_{23}	-0.806	0.508	-1.588	0.115
G_{24}	-0.224	0.338	-0.663	0.509
G_{25}	-1.124	0.445	-2.523	0.013 *
G_{31}	-1.629	0.266	-6.114	0.000 ***
G_{32}	-0.077	0.102	-0.753	0.453
G_{33}	-0.248	0.205	-1.211	0.228
G_{34}	-0.431	0.342	-1.261	0.210
G_{35}	-1.249	0.397	-3.148	0.002 **
G_{41}	0.225	0.126	1.792	0.076 ·
G_{42}	-0.012	0.025	-0.483	0.630
G_{43}	-0.075	0.060	-1.246	0.215
G_{44}	0.114	0.101	1.129	0.261
G_{45}	-0.085	0.254	-0.334	0.739
G_{51}	-0.016	0.169	-0.092	0.927
G_{52}	-0.009	0.012	-0.700	0.485
G_{53}	0.005	0.192	0.024	0.981
G_{54}	0.003	0.050	0.067	0.947
G_{55}	-0.033	0.424	-0.079	0.938

*Note: '***', '**', '*' and '·' indicates significant codes at 0.001, 0.01, 0.05 and 0.1 respectively.*

The results presented in Table 4.26 to Table 4.28 depicts that most of the variables are statistically significant. The estimated VAR BEKK-GARCH model can be found by substituting the following matrices into equation (3.6.3)

$$B = \begin{pmatrix} -0.019 & 0 & 0 & 0 & 0 \\ 0.000 & 0.003 & 0 & 0 & 0 \\ -0.013 & 0.001 & 0.008 & 0 & 0 \\ -0.020 & 0.013 & -0.003 & -0.020 & 0 \\ -0.004 & -0.005 & 0.008 & -0.013 & -0.008 \end{pmatrix} \tag{4.29}$$

$$\mathrm{C} = \begin{pmatrix} -0.004 & -0.093 & 0.020 & -0.502 & -0.112 \\ 1.385 & 1.100 & 0.471 & 0.154 & 1.577 \\ 0.421 & 0.029 & 0.303 & 0.026 & 0.146 \\ 0.058 & -0.004 & -0.010 & 0.798 & 0.129 \\ 0.060 & 0.064 & 0.012 & 0.190 & 0.309 \end{pmatrix} \tag{4.30}$$

$$G = \begin{pmatrix} 0.732 & 0.011 & 0.346 & 0.066 & 0.936 \\ -0.720 & -0136 & -0.806 & -0.224 & -1.124 \\ -1.629 & -0.077 & -0.248 & -0.431 & -1.249 \\ 0.225 & -0.012 & -0.075 & 0.114 & -0.085 \\ -0.016 & -0.009 & 0.005 & 0.003 & -0.033 \end{pmatrix} \tag{4.31}$$

Table 4.27 and Table 4.28 above presenting the estimates of the diagonal parameters show that only C_{22}, C_{33}, C_{44} and G_{11} are statistically significant at 5% level of significance. This implies that the conditional variance of Brazil, China, India and Russia's exchange rates are affected by their own past conditional volatility and other BRICS exchange rates past conditional volatility. However, C_{11}, C_{55}, G_{22}, G_{33}, G_{44} and G_{55} are not significant implying that the past conditional volatility does not influence volatility in the BRICS exchange rates.

The off diagonal elements of the matrix C captures the cross BRICS exchange rate shock. The coefficient of C_{12}, and C_{14}, are statistically significant. There are no pairs which are both significant implying a unidirectional influence in the exchange rates. There is a unidirectional influence in the exchange rates of China and Brazil and Russia and Brazil. The remaining off diagonal elements (C_{13}, C_{15}, C_{23}, C_{31}, C_{51} and C_{32}) of the matrix C are statistically insignificant meaning that there is no spill-over effect.

The off diagonal element of the matrix G captures the BRICS exchange rate volatility transmission. The coefficient of G_{13}, G_{15}, G_{25}, G_{31}, and G_{35} are statistically significant. The following pair (G_{13} and G_{31}) of the off diagonal parameter is statistically significant at 5% level of significance illustrating that there is a bidirectional volatility transmission between the BRICS exchange rates of Brazil and India. The rest of the counterparts are not statistically significant. This means that there is a unidirectional volatility transmission between Brazil and South Africa; China and South Africa; India and South Africa; and Russia and Brazil.

The next Figure 4.20 illustrates the residual Series for VAR BEKK-GARCH model.

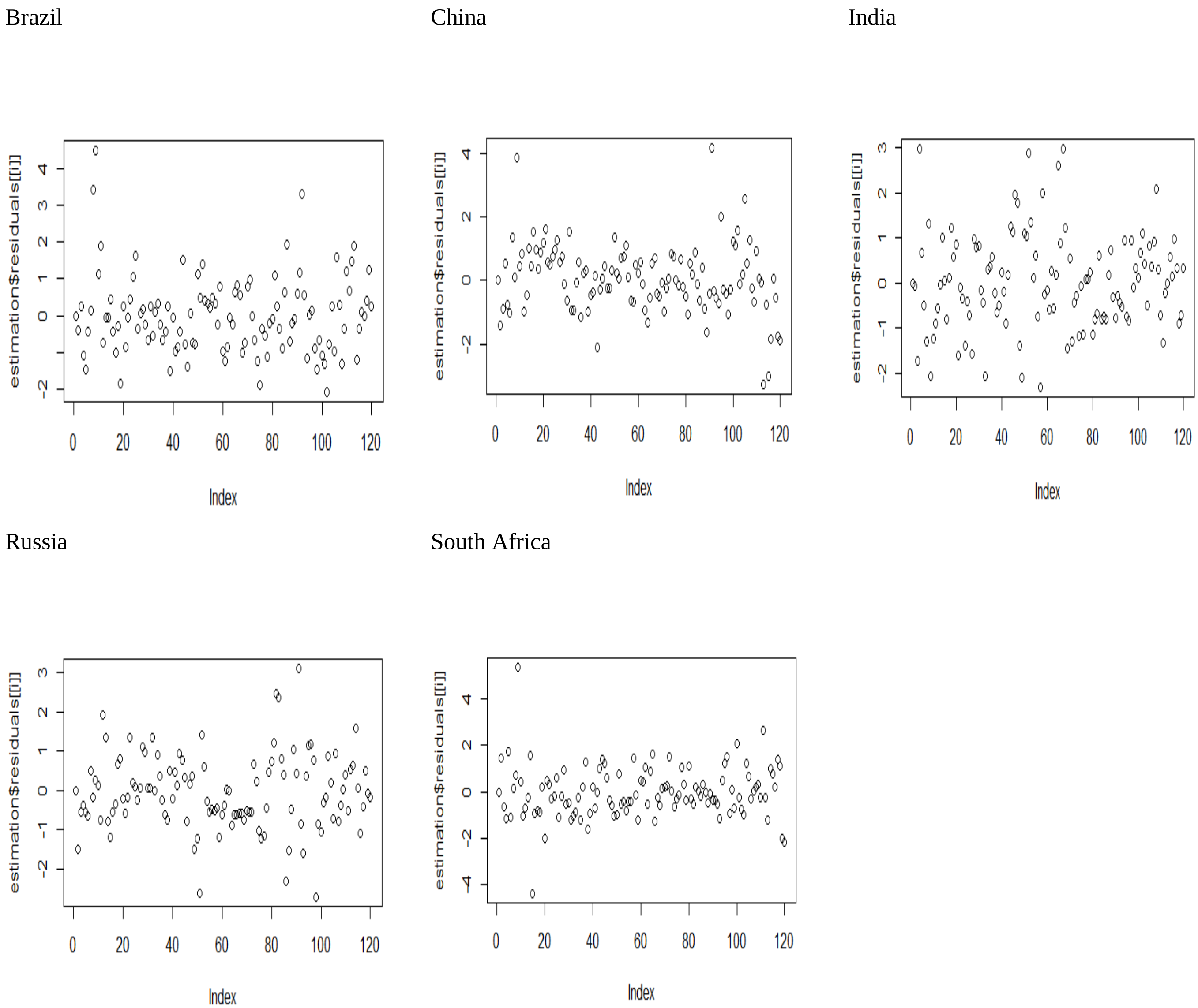


Figure 4.20 Residual Series for VAR BEKK-GARCH model

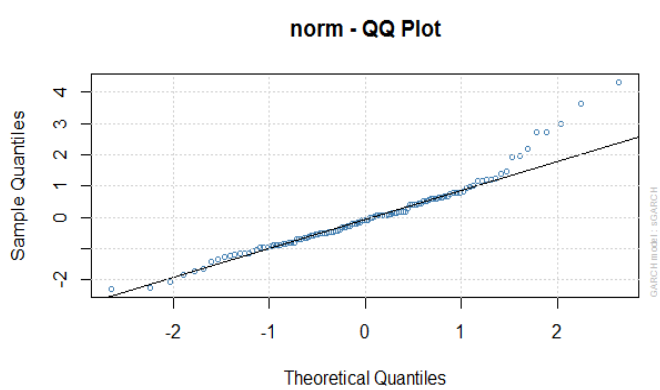
In Figure 4.20 above the residual series depicts no particular pattern for each of the BRICS exchange rates. Therefore, VAR BEKK-GARCH model demonstrates the absence of the autocorrelation in the residuals. This implies that the model is well specified. The following section presents the enhanced VAR Multivariate GARCH model using the DCC approach.

4.11 VAR-DCC GARCH

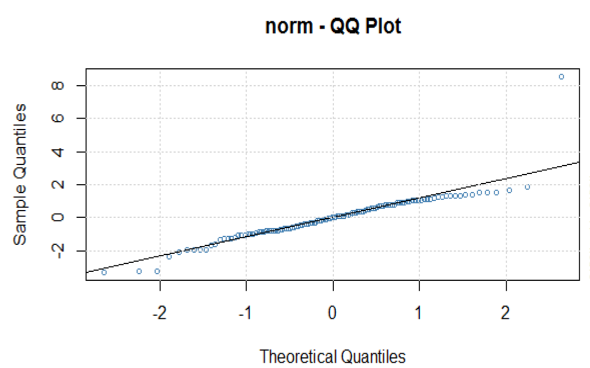
The section presents the enhanced VAR Multivariate GARCH model using the DCC approach. The VAR Multivariate GARCH model considers the ARCH effect of the time series and calculate time varying hedge ratio.

The next Figure 4.21 Q-Q plots for BRICS exchange rates

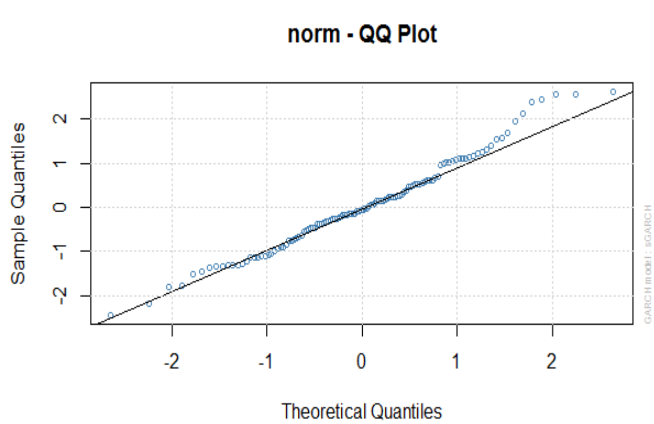
Brazil



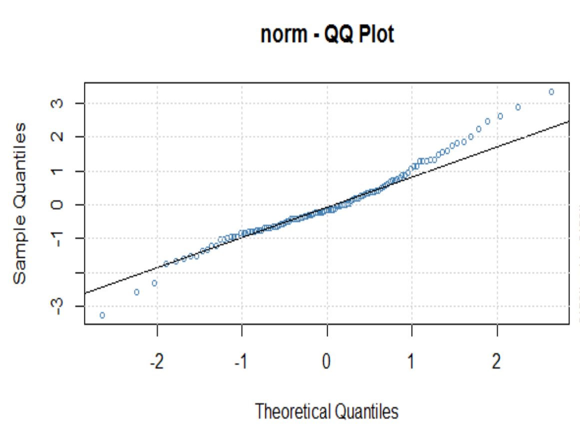
China



India



Russia



South Africa

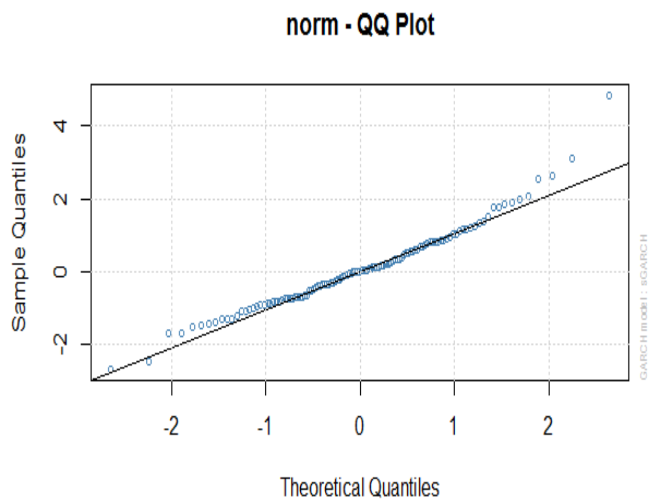


Figure 4.21 Q-Q plots for BRICS exchange rates

The above Figure 4.21 depicts that most of the BRICS exchange rates points lie on the normal line. All the BRICS exchange rates Q-Q plots follow a normal distribution with some extreme tails. Both the left and the right tail distribution of the exchange rate illustrate some differences and therefore advisable to keep the distribution as skewed.

Table 4.29 Summary table of VAR DCC-GARCH (1.1) model parameter estimates for each of the BRICS exchange rates.

Table 4.29 Summary table of VAR DCC-GARCH (1.1) model parameter estimates for each of the BRICS exchange rates

Exchange Rates	Parameter	Estimate	Std. Error	t-value	p-value
Brazil	μ	0.000	0.008	0.000	1.000
	ω	0.000001	0.000002	0.544	0.587
	α_1	0.051	0.045	1.125	0.260
	β_1	0.900	0.095	9.433	0.000 ***
China	μ	0.000000	0.003	0.000	1.000
	ω	0.000000	0.000024	0.002	0.999
	α_1	0.050	0.074	0.673	0.501
	β_1	0.900	0.177	5.083	0.000 ***
India	μ	0.000000	0.002	0.000	1.000
	Ω	0.000001	0.000006	0.085	0.932
	α_1	0.102	0.113	0.907	0.365

Exchange Rates	Parameter	Estimate	Std. Error	t-value	p-value
	β_1	0.895	0.088	10.178	0.000 ***
Russia	μ	0.000000	0.010	0.000	1.000
	ω	0.000002	0.000004	0.542	0.588
	α_1	0.051	0.036	1.409	0.159
	β_1	0.900	0.083	10.815	0.000 ***
South Africa	μ	0.000000	0.007	0.000	1.000
	ω	0.000001	0.000007	0.200	0.842
	α_1	0.051	0.136	0.375	0.708
	β_1	0.900	0.291	3.098	0.002 **

*Note: '***', '**', '*' and '·' indicates significant codes at 0.001, 0.01, 0.05 and 0.1 respectively.*

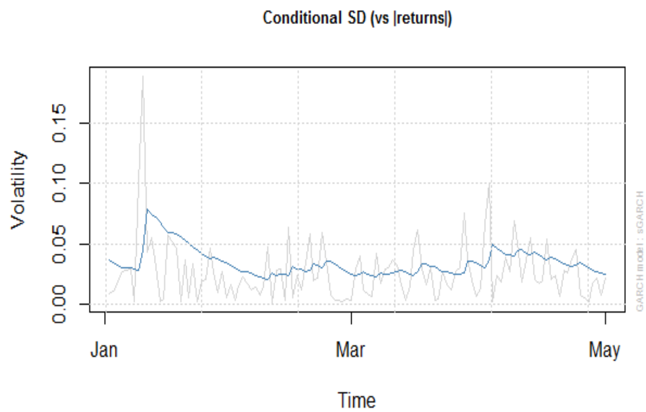
The following models are deduced from the above Table 4.29, the VAR DCC-GARCH (1.1) model equations for each BRICS exchange rates are written as follows

$$x_r(Brazil) = 0.000(\pm 0.008) + \varepsilon_t,$$
$$\sigma_t^2 = 0.000001(\pm 0.000002) + 0.051(\pm 0.045)\sigma_{t-1}^2 + 0.900(\pm 0.095)\sigma_{t-1}^2 \quad (4.32)$$
$$x_r(China) = -0.000(\pm 0.003) + \varepsilon_t,$$
$$\sigma_t^2 = 0.000(\pm 0.000024) + 0.050(\pm 0.074)\sigma_{t-1}^2 + 0.900(\pm 0177)\sigma_{t-1}^2 \quad (4.33)$$
$$x_r(India) = 0.000000(\pm 0.002) + \varepsilon_t,$$
$$\sigma_t^2 = 0.000001(\pm 0.000006) + 0.102(\pm 0.113)\sigma_{t-1}^2 + 0.895(\pm 0.088)\sigma_{t-1}^2 \quad (4.34)$$
$$x_r(Russia) = 0.000000(\pm 0.010) + \varepsilon_t,$$
$$\sigma_t^2 = 0.000002(\pm 0.000004) + 0.051(\pm 0.036)\sigma_{t-1}^2 + 0.900(\pm 0.083)\sigma_{t-1}^2 \quad (4.35)$$
$$x_r(SouthAfrica) = 0.00000(\pm 0.007) + \varepsilon_t,$$
$$\sigma_t^2 = 0.000001(\pm 0.000007) + 0.051(\pm 0.136)\sigma_{t-1}^2 + 0.900(\pm 0.291)\sigma_{t-1}^2 \quad (4.36)$$

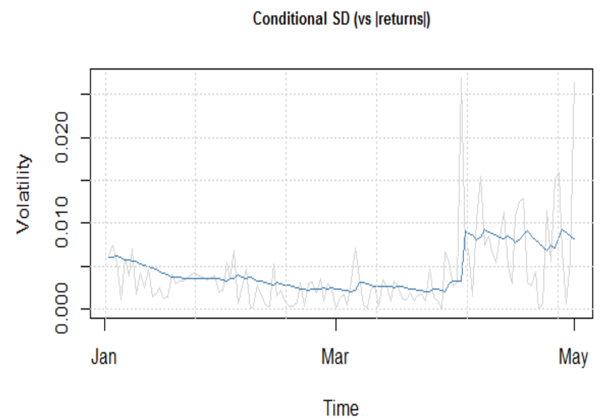
x_r represents the exchange rates for each of the BRICS countries whereas σ_t^2 denotes the volatility part of the VAR DCC-GARCH (1.1) model equation for each BRICS exchange rates. The sum of the estimates $\hat{\alpha}_1$ and $\hat{\beta}_1$ of all the BRICS exchange rates series are less than one meaning that the unconditional volatility for each of the BRICS exchange rates series is finite. The results further revealed that India has the highest volatility persistence value of $\hat{\alpha}_1$ + $\hat{\beta}_1$= 0.999, Brazil, Russia and South Africa has the second highest volatility persistence value of $\hat{\alpha}_1$ + $\hat{\beta}_1$= 0.951, and China has the least volatility persistence value of $\hat{\alpha}_1$ + $\hat{\beta}_1$= 0.950.

The Figure 4.22 below shows the BRICS conditional volatility.

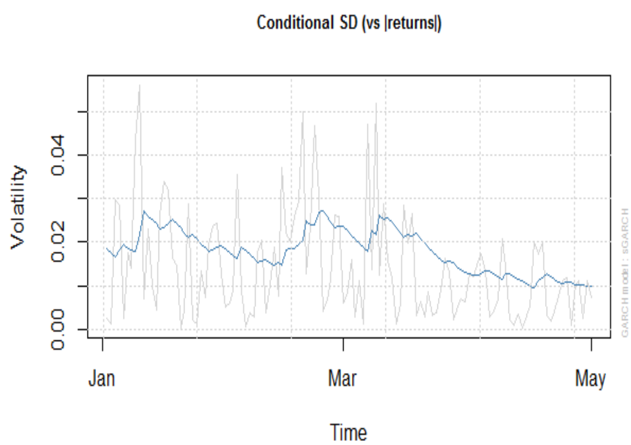
Brazil



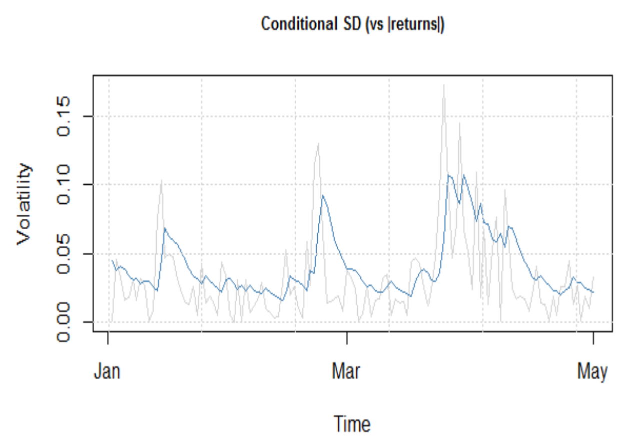
China



India



Russia



South Africa

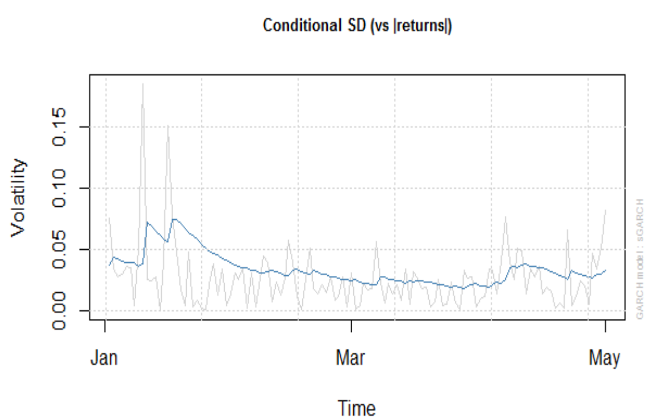
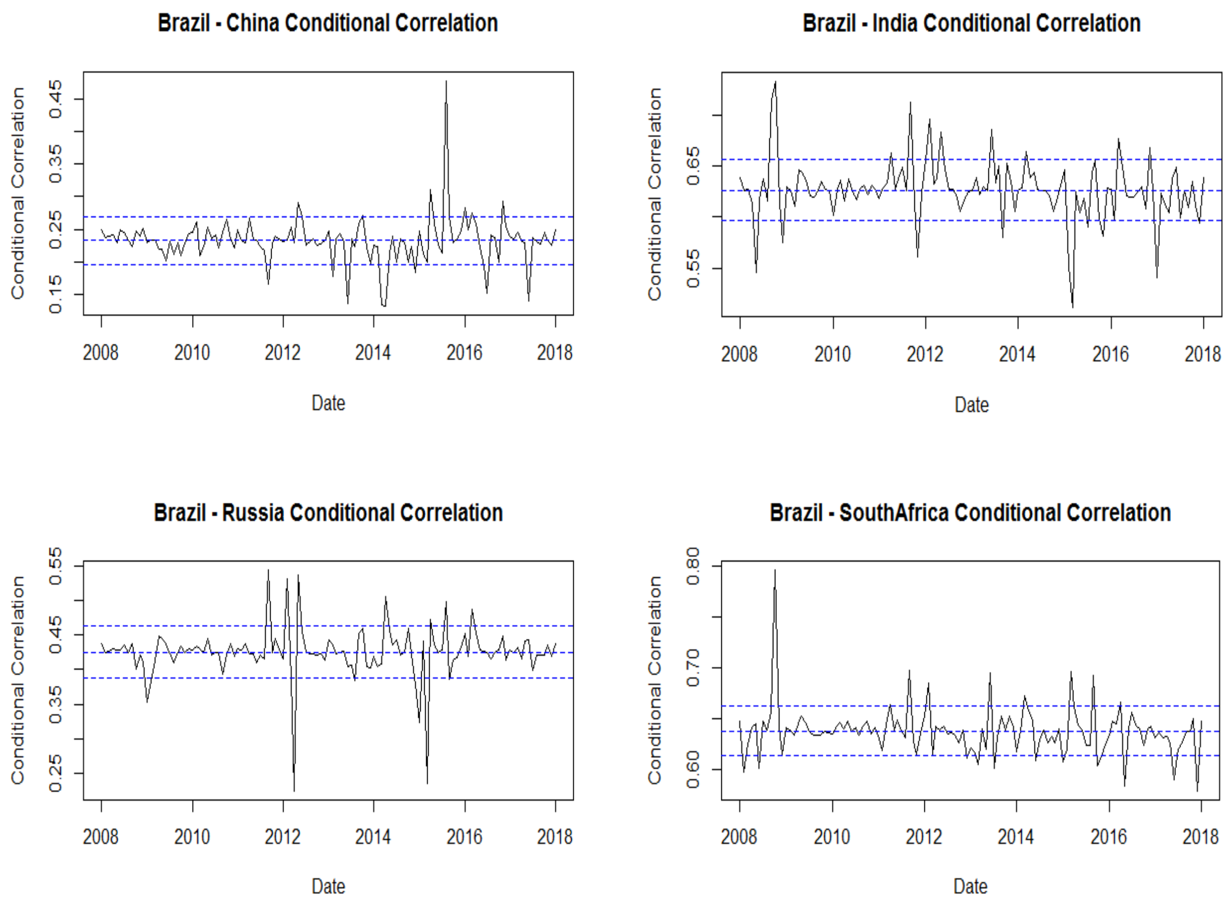


Figure 4.22 BRICS conditional volatility

The volatility scales in Figure 4.22 above shows that Brazil, Russia and South Africa has the highest volatility followed by India and the least volatile is China.

The next Figure 4.23 Time-varying conditional correlations from the VAR DCC-GARCH model



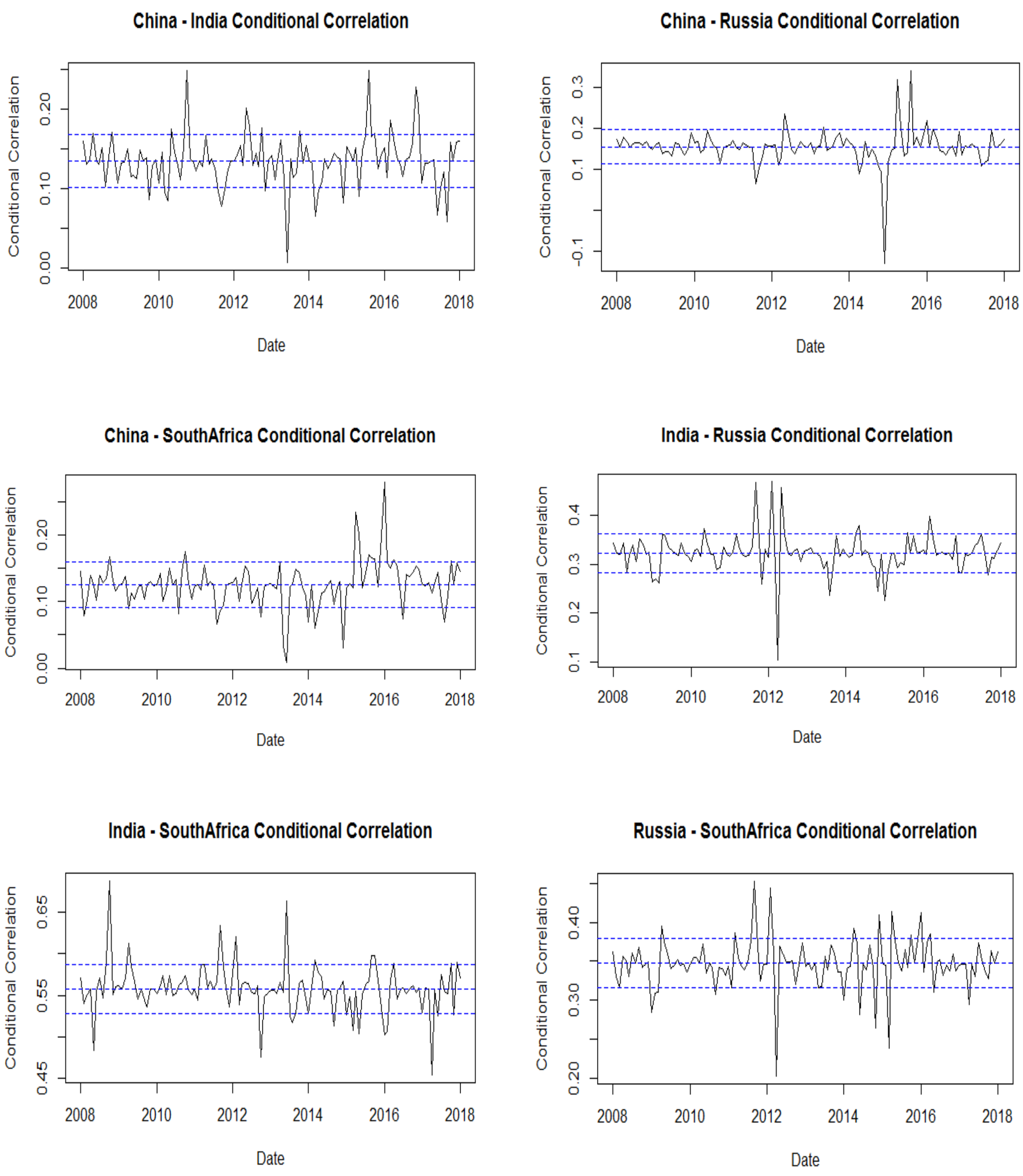


Figure 4.23 Time-varying conditional correlations from the VAR DCC-GARCH model

The above Figure 4.23 illustrates the time-varying conditional correlation between two countries at a time. DCC model was used in the construction of such conditional correlations. All the conditional correlations presented a similar pattern and the only difference is the ranges within which they fall.

4.11.1 Diagnostic tests

Model adequacy testing is done using the following diagnostic tests: goodness of fit test; Ljung-Box (R), Ljung-Box (R^2), and ARCH-LM.

Table 4.30 Diagnostic test of the VAR DCC-GARCH (1.1) model

Exchange Rates	Diagnostic test	Statistic	p-value
Brazil	Goodness of fit test	18.000	0.522
	Ljung-Box (R)	19.190	1.185e-05 ***
	Ljung-Box (R^2)	10.360	0.001 ***
	ARCH-LM	0.669	0.413
China	Goodness of fit test	18.000	0.522
	Ljung-Box (R)	9.149	0.004 **
	Ljung-Box (R^2)	0.016	0.899
	ARCH-LM	0.004	0.952
India	Goodness of fit test	19.000	0.457
	Ljung-Box (R)	7.724	0.005 **
	Ljung-Box (R^2)	1.506	0.2198
	ARCH-LM	1.490	0.222
Russia	Goodness of fit test	23.670	0.209
	Ljung-Box (R)	19.100	1.239e-05 ***
	Ljung-Box (R^2)	6.054	0.014 *
	ARCH-LM	2.069	0.150
South Africa	Goodness of fit test	17.670	0.545
	Ljung-Box (R)	3.183	0.074 ·
	Ljung-Box (R^2)	0.145	0.703
	ARCH-LM	0.004	0.950

*Note: '***', '**', '*' and '·' indicates significant codes at 0.001, 0.01, 0.05 and 0.1 respectively.*

The data in the above Table 4.30 shows that all the BRICS exchange rates have no ARCH errors, since all the p-values of the ARCH-LM test are greater than 0.05 level of significance. The Ljung-Box (R^2) revealed that the residuals of the squared BRICS exchange rates do not have serial correlation. All the BRICS exchange rates show that the fitted residuals are normally distributed. The Q-Q plots in Figure 4.21 for BRICS exchange rates are in support of the above assertion that the fitted residual are normally distributed.

The next Section 4.12 presents the chapter summary

4.12 CHAPTER SUMMARY

The study investigated the performance of conditional heteroskedastic VAR enhanced Multivariate GARCH models on the time varying integrated data. The stationarity testing methods (ADF and PP); univariate and the multivariate methods. The tests administered under univariate are VAR, ARCH, GARCH, TGARCH and EGARCH models of the different countries exchange rates. The Multivariate techniques include Multivariate GARCH for BEKK and DCC; and VAR-Multivariate GARCH models for BEKK and DCC. The next chapter presents conclusions and recommendations.

CHAPTER 5
CONCLUSION AND RECOMMENDATIONS

5.1 INTRODUCTION

The chapter follows on the data analysis and interpretation of results presented in chapter 4 to provide the account in relation to the summary of the findings of the research, the envisaged contribution of the research, limitations, and draws conclusions in relation to the objectives and suggested recommendations, including areas for future research.

The rest of the chapter is presented as follows: In Section 5.2 the results of the research in respect of the objectives is discussed and works on individual objective achievements. Section 5.3 brings the contribution to the body of knowledge. Section 5.4 discusses the limitations of the study, and in Section 5.5 conclusions are drawn based on the discussions of individual objective. In Section 5.6, the recommendations are discussed and areas for future study, and finally, Section 5.7 gives the summary of the research.

5.2 DISCUSSION OF RESULTS

The study investigated the performance of conditional heteroskedastic VAR enhanced Multivariate GARCH models on the time varying integrated data. The BRICS exchange rates were used as the base for analysis. The base model used in the research was VAR model, an ARCH model were fitted with the effects the model presents. Subsequently an extension of ARCH, which is GARCH, was considered together with its Multivariate settings. The Multivariate techniques include Multivariate GARCH and VAR-Multivariate GARCH models.

Results from the BRICS exchange rates based on the statistical properties

The preliminary results using both graphical and tables including descriptive statistics were presented. The original plots of BRICS countries were presented. Different countries data fluctuates at different points in time. The original data was non stationary. The original pictorial representation of the BRICS data showed non stationary picture. There was no sign of mean reversion. The data was then differenced to achieve stationarity. The data showed a stationary set at first logged difference by eye inspection. The formal tests of stationarity (ADF and PP) were conducted to confirm the assertion.

The country with the highest mean value of 4.0.3, as per the Table 4.1, was India with the standard deviation of 0.162, while the country with the lowest mean value (0.828) is Brazil with the standard deviation of 0.278. None of the BRICS countries appears to be normally distributed. India was the only country that illustrated a negative skewness and the rest of the BRICS countries were positively skewed. Since the kurtosis values were close to 2, they were said to be mesokurtic. The correlation analysis of the BRICS countries' exchange rates was presented. Brazil shows a weak negative correlation with China and a strong positive correlation with India, Russia and South Africa. China shows a weak negative correlation with India and weak negative correlation with Russia and South Africa. India, on one hand, illustrated a strong positive correlation with Russia and South Africa. Russia was strongly positively correlated to South Africa. The weakness of China's correlation is resulting from the scale of China's economy which far surpasses the rest of the other four BRICS countries.

The unit root tests of BRICS countries exchange rates were presented and both ADF and PP tests at level show no significant difference. This illustrated that the data was non stationary at level. The ADF p-values for Brazil, India, Russia and South Africa show a statistically significant difference at 10%. China shows an insignificant difference. The data was ready and stationery at first difference to continue with further analysis. All the statistical properties necessary to test prior to engaging further with the analysis were satisfied.

Results from the BRICS exchange rates based on VAR model

The VAR (1) model was fitted and the parameters were estimated. All the parameter estimates with the p-values less the 0.1 were considered significant. The results revealed that there is an existence of a linear dependency between Brazil and its own past values, Brazil and past values of China, Brazil and past values of South Africa, China and its own past values, India and past values of China, India and its own past values, India and past values of South Africa, Russia and past values of China, Russia and its own past values, South Africa and past values of Brazil, South Africa and past values of China and lastly South Africa and its own past values. All the linear dependencies take one direction. The study by Mohanasundaram and Karthikeyan (2015) revealed similar results of the VAR model.

Results from the BRICS exchange rates based on ARCH and GARCH models

The ACF plots and their squares of the BRICS exchange rates were presented and the parameter estimations and tests for ARCH disturbances using residuals were also presented. The squared BRICS exchange rates illustrated that there was the presence of serial correlation and that the ARCH errors were present in the BRICS exchange rates. The ARCH (1) model is statistically significant according to the results. This was an indication that this mean equation could be fit to the GARCH variance equation. The ARCH (1) effect was found to be significant with probability values below all the levels of significance except for India. The LM test strongly shows that there is heteroskedasticity, with p-values less than 0.05. The LM test further suggests a strong heteroskedasticity of errors for GARCH model for the five countries.

Univariate GARCH (1.1) model for the BRICS exchange rates was fitted to the data. All the BRICS exchange rates Q-Q plots followed a normal distribution with some extreme tails. Both the left and the right tail distribution of the exchange rate illustrated some differences and therefore it was advisable to keep the distribution as skewed. The results found that *std* had the most lowest AIC values of all the BRICS exchange rates and therefore GARCH (1.1) model was fitted using the *std*. The results are in line with the views by Mokoma and Moroke (2014). Model adequacy testing was done using the following diagnostic tests: goodness of fit test; Ljung-Box (R), Ljung-Box (R^2), and ARCH-LM. Diagnostic results revealed that GARCH (1.1) under the *std* conditional distribution appeared to be adequate and was used for further analysis. The mean and volatility forecasts of the BRICS exchange rates were also presented and it was found that the mean forecasts falls within the 95% confidence interval. The views were supported by Teräsvirta (2009) and Goyal (2000). The results were also supported by Minovic (2017).

Results from the BRICS exchange rates based on EGARCH and TGARCH models

The univariate EGARCH (1.1) model for the BRICS exchange rates was also presented. The Q-Q plots of the BRICS exchange rates were presented and most of the BRICS exchange rates points lie on the normal line. The leverage effects, γ_1, of all the BRICS exchange rates was greater than zero or positive coefficients, thus implying that an increase in the BRICS exchange rate have greater impact on the conditional volatility as compare to the decrease in the BRICS exchange rate. The assertion was supported by Mwita and Nassiuma (2015). The relative size of the two groups of coefficients (γ and α) suggests that the asymmetric effects dominates the symmetric effects except for South Africa which illustrated the opposite. Wang and Wu (2012) presented similar results. All the BRICS countries stationarity is also assured by the past volatility coefficient β less than one. It must be noted, however, that β for China, India, Russia and South Africa implies that there is the presence of high shock persistence in the exchange rates. Brazil on one hand has low shock persistence in their exchange rates. The results are in line with Grek and Mantalos (2014) and supported by Abdalla (2012).

The univariate TGARCH (1.1) model for the BRICS exchange rates was also presented. All the BRICS exchange rates Q-Q plots follow a normal distribution with some extreme tails. The leverage effects, γ_1, of all the BRICS countries exchange rates is less than zero or negative coefficients implying that an decrease in the BRICS exchange rate have lesser impact on the conditional volatility as compare to the increase in the BRICS exchange rate. The estimated γ for all the BRICS exchange rates proves that the bad news has no effect to the volatility. The relative size of the two groups of coefficients (γ and α) suggests that the symmetric effects dominates the asymmetric effects except for Brazil which illustrated the opposite. All the BRICS countries stationarity is also assured by the past volatility coefficient β less than one except for South Africa. It must be noted, however, that β for Brazil, China, India and South Africa implies that there is the presence of high shock persistence in the exchange rates. Russia on one hand has low shock persistence in their exchange rates. The results are in line with Grek and Mantalos (2014) and supported by Ahmed and Suliman (2011).

Results from the BRICS exchange rates based on Multivariate GARCH models

The extension of the univariate GARCH model using a Multivariate approach was investigated (BEKK-GARCH) and presented. The results showed that most of the variables were statistically significant. The estimates of the diagonal parameters shows that only Russia and South Africa were statistically significant which implied that the conditional variance of Russia and South Africa's exchange rates are affected by their own past conditional volatility and other BRICS exchange rates past conditional volatility. This is supported by the study by Bala and Takimoto (2017)

There was only one pair (G_{45} and G_{54}) of the off diagonal parameter which was found to be statistically significant thus illustrating a bidirectional volatility transmission between Russia and South Africa. There was a unidirectional volatility transmission found between Brazil and India; Russia and Brazil; Russia and China; South Africa and Brazil; South Africa and China; and South Africa and India. This results are supported by the study by Ijumba (2013).

The Multivariate GARCH model using a DCC approach was also presented and it provided for the dynamic relations amongst the BRICS exchange rates. All the BRICS exchange rates Q-Q plots did not follow a normal distribution. The results further revealed that Brazil, China, Russia and South Africa had the highest volatility persistence and India has the least volatility persistence. The time-varying conditional correlation between two countries at a time were presented using a DCC model. All the BRICS exchange rates show that the fitted residuals are not normally distributed except for Russia. The results are supported by the study by Bala and Takimoto (2017) and Ijumba (2013).

Results from the BRICS exchange rates based on VAR Multivariate GARCH models

The enhanced VAR Multivariate GARCH model using the BEKK approach was presented. The results showed that most of the variables were statistically significant. The estimates of the diagonal parameters shows that only Brazil, China, India and Russia are statistically significant which implied that the conditional variance of Brazil, China, India and Russia's exchange rates are affected by their own past conditional volatility and other BRICS exchange rates past conditional volatility. There are no spill-over effects in the BRICS exchange rates. The results are supported by Behera (2011) and Zhou and Wu (2014). The results were also supported by Mukherjee (2011).

None of the pairs of the off diagonal parameter was statistically significant, thus implying that there was no bidirectional volatility transmission between the BRICS exchange rates. There is a unidirectional volatility transmission between Brazil and China; China and India; Russia and Brazil; and Russia and China. The residual series for VAR BEKK-GARCH model depicts no particular pattern for each of the BRICS exchange rates. Therefore, VAR BEKK-GARCH model demonstrates the absence of the autocorrelation in the residuals. This implies that the model is well specified. The study is in line with Türkyılmaz and Balıbey (2014)

The enhanced VAR Multivariate GARCH model using the DCC approach was presented. All the BRICS exchange rates Q-Q plots follow a normal distribution with some extreme tails. The sum of the estimates $\hat{\alpha}_1$ and $\hat{\beta}_1$ of all the BRICS exchange rates series are less than one meaning that the unconditional volatility for each of the BRICS exchange rates series is finite. The results further revealed that India has the highest volatility persistence followed by Brazil, Russia and South Africa and China had the least volatility persistence. The time-varying conditional correlations between two countries at a time were constructed using the DCC model and all the conditional correlations presented a similar pattern. All the BRICS exchange rates showed that the fitted residuals are normally distributed. All the conditional correlations presented a similar pattern and the only difference is the ranges within which they fall. The results are supported Nortey et al., (2015) and Bonga-Bonga and Nleya (2016).

5.3 CONTRIBUTION TO THE STUDY

The study contributes to the knowledge base the fresh discussion on the performance of Multivariate GARCH processes and the assessment of the performance of the conditional heteroskedastic VAR enhance Multivariate GARCH model on the time varying integrated data. The study also contributes to the current set of literature on how Multivariate GARCH develops the conditional variance-covariance matrix. The findings highlighted that there was a bidirectional volatility transmission between Russia and South Africa. There was also a unidirectional volatility transmission found between Brazil and India; Russia and Brazil; Russia and China; South Africa and Brazil; South Africa and China; and South Africa and India. The data used in the study gives a fresh view on the methods used in the study and it also allows BRICS countries to measure themselves against other economies on issues of exchange rates.

Different studies conducted on the VAR Multivariate GARCH do not cover the exchange rates in the BRICS countries. Therefore, the study contributes to the knowledge base the application of the VAR enhanced Multivariate GARCH models. The study looked at the univariate aspect of each of the BRICS countries and how they individually perform and then move to the more complex Multivariate aspect.

The study is also novel as it showed a linkage and how the models have been transforming over time from VAR, ARCH, GARCH, EGARCH**,** TGARCH, Multivariate GARCH to VAR enhanced Multivariate GARCH models (BEKK and DCC) with data applied at each stage of the study. The study further contributes to the limited empirical evidence on the application of the VAR enhanced Multivariate GARCH models on the BRICS exchange rates. Studies conducted on the BRICS exchange rates do not cover the "The performance of conditional heteroskedastic VAR enhanced Multivariate GARCH models on the time varying integrated data." Most of the studies conducted on the BRICS exchange rates covered the Multivariate GARCH models. The application in the current study was done differently to encompass the VAR aspect as compared to the standard Multivariate GARCH models. The application of the VAR enhanced Multivariate models (BEKK and DCC) will attracts researchers and scholars to draw comparison with other Multivariate models on the VAR enhanced.

5.4 LIMITATIONS

The study employed the monthly time series data. The period of the data covers data the date South Africa was inducted to be a member of the then BRIC into a new agreement named BRICS. South Africa was officially inducted in April 2010 and this was supposed to give rise to the starting period of the study data, but due to the requirements prescribed in other models, extension to the starting point was considered. Therefore, the data covered the scope before the inception of BRICS ranging from January 2008 to January 2018 and has 121 observations. The findings of this study may not be generalised as they only apply to the data used in the study. The findings may not be transferable to other time frames. The study followed the Ijumba (2013) study that investigated the levels of interdependence and dynamic linkages among the five emerging economies well known as the BRICS. Literature on the subject matter is not adequate and as a results the study looked into how the VAR enhanced Multivariate GARCH models will perform using the exchange rates of the BRICS countries. The study only focused of the application of VAR BEKK-GARCH on BRICS exchange rate.

5.5 CONCLUSION

The study investigated the performance of conditional heteroskedastic VAR enhanced Multivariate GARCH models on the time varying integrated data. BRICS countries exchange rates were used as the base for analysis. The base model used in the research was VAR model, an ARCH model was fitted with the effects the model presents. Subsequently, an extension of ARCH which is GARCH was considered together with its Multivariate settings. The focus of the study was to estimate the VAR enhanced Multivariate GARCH using the BEKK and DCC approach on the BRICS exchange rates. All the statistical properties necessary to test prior to engaging further with the analysis were satisfied.

Using the Multivariate BEKK-GARCH estimates of the diagonal parameters showed that only Russia and South Africa are statistically significant. On the other hand using VAR enhanced BEKK-GARCH estimates of the diagonal parameters showed that only Brazil, China, India and Russia’s exchange rates are statistically significant. Both the methods revealed no spill-over effects in the BRICS exchange rates. The Multivariate BEKK-GARCH further revealed a bidirectional volatility transmission between Russia and South Africa whereas on the VAR enhanced BEKK-GARCH showed no bidirectional volatility transmissions. The Multivariate BEKK-GARCH model demonstrated the presence of autocorrelation in the residuals, while the VAR enhanced BEKK-GARCH model demonstrated the absence of the autocorrelation in the residuals. Therefore, the VAR enhanced BEKK-GARCH model was well specified and is the best of the two models. The Multivariate DCC-GARCH revealed that Brazil, China, Russia and South Africa had the highest volatility persistence while India has the least volatility persistence as opposed to the VAR enhanced DCC-GARCH which revealed that India has the highest volatility persistence followed by Brazil, Russia and South Africa and China had the least volatility persistence.

The study reported that conditional covariances modelled through conditional variances and correlations paves way to several novel models used currently. Also reported by this study is the feasibility of conditional correlation models in estimation and interpretation of parameters as also confirmed by Mustafa (2008). It is further evident judging from the findings the accurateness of the VAR BEKK-GARCH which gives the time-varying conditional correlations as opposed to the VAR DCC-GARCH model. It is clear according to the study findings that modelling the conditional correlation matrix with VAR BEKK GARCH provided flexible and parsimonious parameters.

The results are supported by the studies amongst others by Bala and Takimoto (2017), Ijumba (2013), Behera (2011), Zhou and Wu (2014), Mukherjee (2011), Türkyılmaz and Balıbey (2014), Nortey et al., (2015) and Bonga-Bonga and Nleya (2016). The study successfully investigated the performance of conditional heteroskedastic VAR enhanced Multivariate GARCH models on the time varying integrated data.

5.6 RECOMMENDATIONS FOR FURTHER STUDIES

The study recommends the following areas for future studies and policy implementation:

- The same study could be undertaken to determine the strength of different models in VAR enhanced BEKK-GARCH, VAR enhanced CCC-GARCH, and VAR enhanced DCC-GARCH models on the time varying integrated data. The strength of each model will be tested against each other and the best model recommended.
- One can consider, on a small scale (article), to draw a comparison on the different types of GARCH model on the time varying integrated data. This may enlighten the discussion on the up and coming scholars on which GARCH model is more reliable for the time varying integrated data.
- The same study could be undertaken to determine the strength of different models in VAR diagonal BEKK-GARCH and VAR Scalar BEKK-GARCH models on the time varying integrated data. Most literature assumes the two methods do almost everything the same. Therefore, it will be advisable to test that assumption and see what the difference is if any. The study will also highlight which model performs the better of the two.
- One can consider, on a small scale (article), to draw a comparison on the different types of Multivariate GARCH model on the time varying integrated data. This will determine which of the GARCH model perform the best of all the models.
- A similar study to investigate the performance of conditional heteroskedastic VEC enhanced GARCH models on the time varying integrated data may be conducted. This

will assist in checking the results of the current study with other views relating to VEC enhanced GARCH model.

- The study found a highly positive correlation between South Africa and all the BRICS countries except for China. Therefore, the study recommends that South Africa should continue maintaining collaborative relations with those BRICS countries and work towards improving policies and memorandums of understanding with China. Apart from the BRICS agreement South Africa must continue maintaining its bilateral collaborations with these BRICS countries.
- The study found that the leverage effects of all the BRICS exchange rates was greater than zero or positive coefficients, thus implying that an increase in the BRICS exchange rate have greater impact on the conditional volatility as compare to the decrease in the BRICS exchange rate. The study recommends that BRICS countries should develop policies that allow for the very slow increase of the exchange rates to encourage trade amongst the BRICS countries. The weak exchange rate makes currency more attractive and volatile exchange rates negatively affect trade and reduce investor confidence.
- Government should sponsor the researchers to assist develop the research on relevant policies relating to the current study. The sponsorship will assist in ensuring that all the above mentioned recommendations are put into effect.

5.7 SUMMARY OF THE THESIS

The research was arranged into five chapters to respond to the research objectives as proposed in chapter one. Chapter 1 introduced the research and provided some background of the study. It further established the problem statement, rationale of the study, aim and objectives of the study and the research questions. Furthermore, the significance of the study, scope limitations or delimitations of the study, and definition of terms were outlined. Literature was thoroughly reviewed in Chapter 2 (empirical literature) and partly in Chapter 3 (theoretical literature). Chapter 3 presented the research methodology the research followed in responding to the research objectives as stipulated in Chapter 1. Data analysis and interpretation of results were presented in Chapter 4 of the study. Chapter 5 presented the summary of the findings, drew conclusions in line with the objectives of the research and suggested recommendations as well as areas for future study.

REFERENCES

ABDALLA, S. Z. S. (2012). Modeling exchange rate volatility using GARCH models: Empirical evidence from Arab countries. *International Journal of Economics and Finance*, 4, 216.

ADENOMON, M., OJEHOMON, V. T. & OYEJOLA, B. (2013). Modeling the dynamic relationship between rainfall and temperature time series data in Niger State, Nigeria.

AHMED, A. E. M. & SULIMAN, S. Z. (2011). Modeling stock market volatility using GARCH models evidence from Sudan. *International Journal of Business and Social Science*, 2.

AHMAD, M. H. & PING, P. Y. (2014). Modeling Malaysian gold using symmetric and asymmetric GARCH Models. *Applied Mathematical Sciences,* 8, 817-822.

AIT-SAHALIA, Y & KIMMEL, R. (2006). Maximum likelihood estimation of stochastic volatility models. *Journal of Financial Economics*, 83 (2007) 413–452

ALLEN, D., MCALEER, M., POWELL, R. J. & SINGH, A. K. (2015). A volatility impulse response analysis applying Multivariate GARCH models and news events around the GFC.

ASEMOTA, O. J. & EKEJIUBA, U. C. (2017). An Application of Asymmetric GARCH Models on volatility of Banks Equity in Nigeria's Stock Market.

ATHANASOPOULOS, G., DE CARVALHO GUILLÉN, O. T., ISSLER, J. V. & VAHID, F. (2011). Model selection, estimation and forecasting in VAR models with short-run and long-run restrictions. *Journal of Econometrics*, 164, 116-129.

ATOI, N. V. (2014). Testing volatility in Nigeria stock market using GARCH models *CBN Journal of Applied Statistics*, 5.

BABA, Y., ENGLE, R. F., KRAFT, D. & KRONER, K. (1990). Multivariate simultaneous generalized ARCH, unpublished manuscript, University of California, San Diego

BABU, C. N. & REDDY, B. E. (2015). Prediction of selected Indian stock using a partitioning–interpolation based ARIMA–GARCH model. *Applied Computing and Informatics*, 11, 130-143.

BACKUS, D. (1986). The Canadian-US exchange rate: Evidence from a vector autoregression. *The Review of Economics and Statistics*, 628-637.

BACHELIER, L. (1900). Théorie de la speculation. Annales Scientifiques de L'École Normale Supérieure, 17, pp.21–86. (English translation by A. J. Boness in Cootner, P.H. (Editor). 1964. The random character of stock market prices, Cambridge, MA: MIT Press).

BACKUS, D. K. & GREGORY, A. W. (1993). Theoretical relations between risk premiums and conditional variances. *Journal of Business and Economic Statistics*, 11(2), 177-185.

BACKUS, D. K., GREGORY, A. W. & ZIN, S. E. (1989). Risk premiums in the term structure: Evidence from artificial economies. J. Monetary Econ. 24, 371-99.

BAE, K.H., KAROLYI, G.A. & STULZ, R.M. (2003). A new approach to measuring financial contagion. *The Review of Financial Studies*, 16, 717-763.

BAGLIANO, F. C. & FAVERO, C. A. (1998). Measuring monetary policy with VAR models: An evaluation. *European Economic Review*, 42, 1069-1112.

BALA, D.A. & TAKIMOTO, T. (2017). Stock markets volatility spill-overs during financial crises: A DCC-Multivariate GARCH with skewed-t density approach. *Borsa Istanbul Review*, *17*(1), 25-48.

BALA, D. A. & ASEMOTA, J. O. (2013). Exchange-rates volatility in Nigeria: Application of GARCH models with exogenous break. *CBN Journal of Applied Statistics*, 4, 89-116

BASCI, E. S. & KARACA, S. S. (2013). The determinants of stock market index: VAR approach to Turkish stock market. *International Journal of Economics and Financial Issues*, 3(1), 163-171.

BAUWENS, L. & GIOT, P. (2003). Asymmetric ACD models: Introducing price information in ACD models with a two state transition model. *Empirical Economics*, 28(4).

BAYBOGAN, B. (2013). Empirical investigation of multivariate GARCH Models. *Journal of Statistical and Econometric Methods*, 2, 75-93.

BEHERA, H. K. (2011). Onshore and offshore market for Indian rupee: Recent evidence on volatility and shock spill-over. *Macroeconomics and Finance in Emerging Market Economies*, 4, 43-55.

BESSLER, D. A. (1984). Relative prices and money: A vector autoregression on Brazilian data. *American Journal of Agricultural Economics*, 66, 25-30.

BLACK, F. & SCHOLES, M.S. (1973). The pricing of options and corporate liabilities. *Journal of Political Economy*, 81, 637-659.

BOLLERSLEV, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics*, 31, 307-327.

BOLLERSLEV, T. (1990). Modeling the coherence in short-run nominal exchange rates: A Multivariate generalized ARCH model. *The review of Economics and Statistics*, 498-505.

BONGA-BONGA, L. & NLEYA, L. (2016). Assessing portfolio market risk in the BRICS economies: use of Multivariate GARCH models.

BREUSCH T. S. (1978). Testing for autocorrelation in dynamic linear models. *Australian Economic Papers*, 17, 334-355.

BUNNAG, T. (2016). Volatility transmission in crude oil, gold, S&P 500 and US Dollar Index futures using VAR-Multivariate GARCH model. *International Journal of Energy Economics and Policy*, 6.

CANDILA, V. (2013). A comparison of the forecasting performances of Multivariate volatility models. DiSES working Papers. Università degli Studi di Salerno. Via Ponte Don Melillo – 84084; Fisciano (SA) – Italy.

CAPORALE, G. M., SPAGNOLO, F. & SPAGNOLO, N. (2017). Macro news and exchange rates in the BRICS. Finance Research Letters.

CAPPIELLO, L., MCALEER, M. AND TANSUCHAT, R. (2003). Asymmetric dynamics in the correlations of global equity and bond returns. European Central Bank working paper No. 204.

CHAMALWA, H. & BAKARI, H. (2016). A vector autoregressive (VAR) cointegration and vector error correction model (VECM) approach for financial deepening indicators and economic growth in Nigeria. *American Journal of Mathematical Analysis*, 4, 1-6.

CHANG, C.L., GONZÁLEZ SERRANO, L. & JIMÉNEZ-MARTÍN, J.Á. (2012). Currency hedging strategies using dynamic Multivariate GARCH.

CHANG, C. C., HU, T. C., KAO, C. F. & CHANG, Y.C. (2015). Early warning signals using AVaRs of infinitely divisible GARCH models: Evidence from stock index markets. *Applied Economics*, 47, 4630-4652.

CHANG, C. L., MCALEER, M. & TANSUCHAT, R. (2011). Crude oil hedging strategies using dynamic Multivariate GARCH. *Energy Economics*, 912–923.

CHEN, J. (2015). Bayesian estimation of multivariate conditional correlation GARCH models and their application.

CHEN, R. & ZAPATA, H. O. (2015). Dynamics of price volatility in the China-US Hog industries. Annual Meeting, January 31-February 3, 2015, Atlanta, Georgia, 2015. Southern Agricultural Economics Association.

CHEVALLIER, J. (2012). Time-varying correlations in oil, gas and CO2 prices: an application using BEKK, CCC and DCC-Multivariate GARCH models. *Applied Economics*, *44*(32), 4257-4274.

CHOU, R. (1988). Volatility persistence and stock valuations: Some empirical evidence using GARCH. *Journal of Applied Econometrics*, 3, 279-294.

COLLIS, J. & HUSSEY, R. (2009). Business research: A practical guide for undergraduate and postgraduate students, 3rd edition, New York: *Palgrave Macmillan.*

CRESWELL, J. W. (2014). Research design: Qualitative, quantitative and mixed methods approaches. 4th edition. *Sage publication. California*, 169-177.

DANIEL, P. S. & SAM, A. G. (2011). Research methodology, Delhi: Kalpaz Publication, 91.

DHAMO, E., KOÇI, E., XHAJA, B. & ASIMI, A. (2012). Defects of fixed-line network modeling and prediction using ARIMA, GARCH models.

DICKEY, D. A. & FULLER, W. A. (1979). Distributions of the estimators for autoregressive time series with a unit root, *Journal of American Statistical Association*, 74, 427-481.

DICKEY, D. A. & FULLER, W. A. (1981). Likelihood ratio statistics for autoregressive time series with a unit root. *Econometrica*, 49, 1057-1072.

DICKEY, D. A. & PANTULA, S. G. (1987). Determining the order of differencing in autoregressive processes, *Journal of Business and Statistics*, 5, 455-61.

DO, H. Q., BHATTI, M. I. & KONYA, L. (2016). On Asean capital market and industry integration: A review. *Corporate Ownership and Control*, 2, 8-23.

DOLADO, J.J., JENKINSON, T. & SOSVILLA-RIVERO, S. (1990). Cointegration and unit roots. *Journal of Economic Surveys*, 4(3), 249-273.

E., M. I. R. (2012). Estimating portfolio value at risk with GARCH and Multivariate GARCH models. *Perfil de Coyuntura Económica*, 77-92.

EDGERTON, D. & SHUKUR, G. (1999). Testing autocorrelation in a system perspective. *Econometric Reviews*, 18(4), 343-86.

EFIMOVA, O. & SERLETIS, A. (2014). Energy markets volatility modeling using GARCH. *Energy Economics*, 43, 264-273.

EKLUND, B. (2007). Forecasting the Icelandic business cycle using vector autoregressive models. Central bank of Iceland working papers No. 36.

ELDER, J. & SERLETIS, A. (2010). Oil price uncertainty. *Journal of Money, Credit and Banking*, 42(6), 1137-1159.

ELLIOTT, G., ROTHENBERG, T.J & STOCK, J.H. (1996). Efficient tests for an autoregressive unit root, *Econometrica*, 64, 813-36.

ENDERS, W. & SANDLER, T. (1993). The effectiveness of antiterrorism policies: A vector autoregression intervention analysis. *American Political Science Review*, 87, 829-844.

ENGLE, R. F. (1982). Autoregressive conditional heteroskedasticity with estimates of the variance of United Kingdom inflation. *Econometrica*: *Journal of the Econometric Society*, 987-1007.

ENGLE, R. (2002). Dynamic conditional correlation: A simple class of Multivariate generalized autoregressive conditional heteroskedasticity models. *Journal of Business & Economic Statistics*, *20*(3), 339-350.

ENGLE, R. F. & KRONER, K. F. (1995). Multivariate simultaneous generalized ARCH. *Econometric theory*, 11(01), 122-150.

ENGLE, R., LILIEN, D. & ROBINS, R. (1987). Estimating time varying risk premia in the term structure: the ARCH–M model. *Econometrica*, 55, 387–401.

ENISAN, A. A. & OLUSAYO, A. O. (2009). Stock market development and economic growth: Evidence from seven sub-Sahara African countries. *Journal of Economics and Business*, 61(2), 162-171.

ERYıLMAZ, F. (2015). Modeling stock market volatility: The case of BIST-100. *Annals of the Constanfin Brâncuşi University of Târgu Jiu, Economy Series***,** 37-47.

ESTENSON, P. S. (1992). The Keynesian theory of the price level: an econometric evaluation using a vector autoregression model. *Journal of Post Keynesian Economics*, 14, 547-560.

FOUNTIS, N. G. & DICKEY, D. A. (1989). Testing for a unit root nonstationarity in multivariate autoregressive time series, *Annals of Statistics*, 419-28.

FREEMAN, J. R., WILLIAMS, J. T. & LIN, T.M. (1989). Vector autoregression and the study of politics. *American Journal of Political Science*, 842-877.

FRENCH, K. R., SCHWERT, G. & STAMBAUGH, R. F. (1987). Expected stock returns and volatility. *Journal of Financial Economics*, 19 (1), 3-29.

GABRIEL, A. S. (2012). Evaluating the forecasting performance of GARCH Models. Evidence from Romania. *Procedia-Social and Behavioral Sciences*, 62**,** 1006-1010.

GARDEBROEK, C., HERNANDEZ, M. A. & ROBLES, M. (2013). Market interdependence and volatility transmission among major crops.

GAU, Y. F. (2001). Time-varying conditional correlations and volatilities of stock index futures returns. Department of International Business Studies, National Chi Nan University.

GOODWIN, D. (2012). Modeling and forecasting volatility in copper price returns with GARCH Models. Unpublished thesis. Lund University.

GLOSTEN, L.R., JAGANNATHAN, R. & RUNKLE, D. (1993). On the relations between the expected value and the volatility of the normal excess return on stocks. *Journal of finance*, 48, 1779 – 1801.

GOYAL, A. (2000). Predictability of stock return volatility from GARCH models. Anderson Graduate School of Management, UCLA, Los Angeles, CA, May.

GREK, Å. & MANTALOS, P. (2014). Forecasting accuracy for ARCH models and GARCH (1,1) family – Which model does best capture the volatility of the Swedish stock market?

GUO, B. (2003). Currency risk hedging with time-varying correlations. UC Santa Cruz Economics working Paper No. 539.

GUO, H. (2012). Estimating volatilities by the GARCH and the EWMA model of PetroChina and TCL in the stock exchange market of China. 6^{th} International Scientific Conference Managing and Modeling of Financial Risks. VŠB-TU Ostrava, Faculty of Economics, Finance Department, 191-202.

HALL, A. (1989). Testing for a unit root in the presence of moving average errors. *Biometrika*, 76, 49-56.

HARRATHI, N., ALOUI, C., HOUFI, M. A. & MAJDOUB, J. (2016). Emerging equity markets connectedness, portfolio hedging strategies and effectiveness. *International Journal of Financial Research*, 7, 189.

HARTMAN, J. & SEDLAK, J. (2013). Forecasting conditional correlation for exchange rates using Multivariate GARCH models with historical value-at-risk application.

HENNING, E., VAN RENSBURG, W. & SMIT, B. (2004). Finding your way in qualitative research. Pretoria, South Africa: Van Schaik.

HERACLEOUS, M. S. (2003). Volatility modeling using the student's t distribution. Virginia Tech. Unpublished thesis. Faculty of the Virginia Polytechnic Institute and State University.

HONG, P. Y. (1991). The autocorrelation structure for the GARCH-M process. *Economics Letters*, 37, 129-132.

HOU, A. & SUARDI, S. (2012). A nonparametric GARCH model of crude oil price return volatility. *Energy Economics*, 618–626.

HSU KU, Y. H. (2008). Student-t distribution based VAR-Multivariate GARCH: An application of the DCC model on international portfolio risk management. *Applied Economics*, 40, 1685-1697.

HWANG, S. & SATCHELL, S. E. (1998). Implied volatility forecasting: A comparison of different procedures including fractionally integrated models with applications to UK equity options. In forecasting volatility in the financial markets. Edited by J. Knight and S. E. Satchell. Butterworth-Heinemann.

IJUMBA, C. (2013). Multivariate analysis of the BRICS financial markets. 1st edition. Pietermaritzburg: School of Mathematics, Statistics and Computer Science. Print.

ILTUZER, Z. & TAS, O. (2012). The analysis of bidirectional causality between stock market volatility and macroeconomic volatility. *International Journal of Business and Social Science*, 3.

JARQUE, C. & BERA, A. (1980). Efficient tests for normality homoscedasticity and serial independence of regression residuals. *Econometric Letters*, 6, 255–259.

KHALID, A. M. & RAJAGURU, G. (2006). Financial market contagion or spill-overs evidence from Asian crisis using Multivariate GARCH approach. Bond University Australia Seminar Paper.

KNIGHT, J. & SATCHELL, S. E. (1998). Forecasting volatility in the financial markets. Butterworth-Heinemann.

KOUKI, I., HARRATHI, N. & HAQUE, M. (2011). A volatility spill-over among sector index of international stock markets. *Journal of Money, Investment and Banking*, 22 (2011), 32-44.

KU, Y. H. H. (2008). Student-t distribution based VAR-Multivariate GARCH: an application of the DCC model on international portfolio risk management. *Applied Economics*, 40, 1685-1697.

KU, Y. H. H., & WANG, J. J. (2008). Estimating portfolio value-at-risk via dynamic conditional correlation Multivariate GARCH model: An empirical study on foreign exchange rates. *Applied Economics Letters*, *15*(7), 533-538.

KVASNAKOVA, K. (2009). Modeling dependence structure of the stock and bond market. Comenius University, Bratislava. Google Scholar.

LAMA, A., GIRISH K. JHA, G. K., GURUNG, B., PAUL, R. K., & KANCHAN SINHA, K. (2015). VAR-Multivariate GARCH models for volatility modeling pulses prices: An Application. *Journal of the Indian Society of Agricultural Statistics*, 70(2), 145–151.

LEAN, H.H., & TENG, K.T. (2013). Integration of world leaders and emerging powers into the Malaysian stock market: A DCC-Multivariate GARCH approach. *Economic Modeling*, 32, 333–342.

LEE, Y. H., FANG, H. & SU, W. F. (2014). Effectiveness of portfolio diversification and the dynamic relationship between stock and currency markets in the emerging Eastern European and Russian markets. *Czech Journal of Economics and Finance,* 296-311.

LEEVES, G. D. (2010). Declining US output volatility and its effect on labour flow volatility: A Multivariate GARCH analysis. *Applied Economics*, 42(20), 2553-2561.

LI, S. (2012). Risk neutral measures and GARCH model calibration. Unpublished thesis. University of Calgary

LI, Y. & BEGUM, D. M. (2013). GARCH models for forecasting volatilities of three major stock indexes: using both frequentist and Bayesian approach.

LIM, C. M. & SEK, S. K. (2013). Comparing the performance of GARCH-type models in capturing the stock market volatility in Malaysia. *International Conference on Applied Economics: Procedia and Finance*, 5, 478 – 487.

LIN, B. & LI, J. (2015). The spill-over effects across natural gas and oil markets: Based on the VEC-Multivariate GARCH framework. *Applied Energy*, 155, 229–241.

LIU, C. (2011). The dynamic relationship of China's Stock Markets: A VAR-Multivariate GARCH model. Business Computing and Global Informatization (BCGIN), International Conference on, 2011. IEEE, 166-169.

LU, M. (2001). Vector autoregression (VAR): An approach to dynamic analysis of geographic processes. Geografiska Annaler: Series B, *Human Geography*, 83, 67-78.

LUCCHETTI, R. (2002). Analytical Score for Multivariate GARCH models. *Computational Economics*, 133–143.

LUTKEPOHL, H. (1991). Introduction to multiple time series analysis. Springer-Verlag, Berlin.

LÜTKEPOHL, H. (2007). New introduction to multiple time series analysis. 1st edition. Berlin: Springer, 576.

MAD'AR, M.M. (2014). Multivariate GARCH**.** Unpublished thesis. Univerzita Karlova v Praze

MALO, P. & KANTO, A. (2006). Evaluating Multivariate GARCH models in the Nordic electricity markets. *Communications in Statistics: Simulation and Computational Economics*, 35, 117-148.

MATHOERA, M. (2016). Does any model beat the GARCH (1.1)? A forecast comparison of volatility models through option prices.

MIKKONEN, T. (2017). Time-varying conditional correlation: Effect on international portfolio diversification in Southeast Asia. Unpublished thesis. University of Jyvaskyla.

MIRON, D. & TUDOR, C. (2010). Asymmetric conditional volatility models: Empirical estimation and comparison of forecasting accuracy. *Romanian Journal of Economic Forecasting*, 13**,** 74-92.

MCLEOD, A. I. & LI, W. K. (1983). Diagnostic checking ARIMA time series models using squared residual autocorrelations. *J. Time Series Anal*, 4, 269-273.

MCMILLIN, W. D. (1991). The velocity of M1 in the 1980s: Evidence from a Multivariate time series model. *Southern Economic Journal*, 634-648.

MINOVIĆ, J. (2017). Application and diagnostic checking of univariate and Multivariate GARCH Models in Serbian financial market. *Economic analysis*, 41, 73-87.

MINOVIĆ, J. & SIMEUNOVIĆ, I. (2009). Applying Multivariate GARCH models in finance. 633-641.

MOHANASUNDARAM, T. & KARTHIKEYAN, P. (2015). Cointegration and stock market interdependence: Evidence from South Africa, India and the USA. *South African Journal of Economic and Management Sciences*, 18, 475-485.

MOHD, M. A., NAWAWI, A. H. M., HUSSIN, S. A. S. & RAMDZAN, S. N. A. (2016). Currency hedging strategies using Multivariate GARCH models. *Journal of Technology*.

MOKOMA, T. J. & MOROKE, N. D. (2014). Exchange rate volatility in South Africa: A comparative analysis of the ARCH and GARCH models.

MOULE, P. & GOODMAN, M. (2009). Nursing research: An introduction. Sage, Los Angeles.

MUKHERJEE, P. (2011). An exploration on volatility across India and some developed and emerging equity markets. Asia-Pacific Development, 11.

MUSTAFA, A, T. (2008). A survey of multivariate GARCH models, a dissertation submitted to the Bilkent University, Ankara.

MWITA, P. N. & NASSIUMA, D. K. (2015). Volatility estimation of stock prices using GARCH method. *Kabarak Journal of Research & Innovation,* 3, 48-53.

NELSON, D. B. (1991). Conditional heteroskedasticity in asset returns: A new approach, *Econometrica*, 59, 347-370.

NELSON, D. B. & CAO, C. Q. (1992). Inequality constraints in the univariate GARCH model. *Journal of Business & Economic Statistics,* 10, 229-235.

NELSON. C. R. & PLOSSER, C. L. (1982). Trends and random walks in macroeconomic time series: Some evidence and implications, *Journal of Monetary Economics*, 10, 139-162.

NEUMAN, W. L. (2011). Social Research Methods: Quantitative and qualitative approaches. 7th edition. Boston: Allyn and Bacon.

NEWEY, W. K. & WEST, K. D. (1987). A simple, positive semidefinite, heteroskedasticity and autocorrelation consistent covariance matrix. *Econometric*, 55, 703-708.

NORTEY, E. N., NGOH, D. D., DOKU-AMPONSAH, K. & OFORI-BOATENG, K. (2015). Modeling inflation rates and exchange rates in Ghana: Application of Multivariate GARCH models.

OBENG, P. (2012). Testing the predictive power of various exchange rate models in forecasting the volatility of exchange rate. University of Lethbridge. Unpublished thesis. Faculty of Arts and Science.

ONO, S. (2011). Oil price shocks and stock markets in BRICS. *The European Journal of Comparative Economics*, 8(1), 29- 45.

ORSKAUG, E. (2009). Multivariate DCC-GARCH Model: With various error distributions. Unpublished thesis, Institute for Mathematics.

PAGAN, A. & SCHWERT, G. W. (1990). Alternative models for conditional stock volatility. *Journal of Econometrics*, 45, 267-290.

PALM, F.C. (1996). 7 GARCH models of volatility, Handbook of statistics, 14, 209,240

PARADZA, A. & ERICSCHALING, P. (2015). The efficient market hypothesis in developing economies: an investigation of the monday effect and january effect on the zimbabwe stock exchange post the multi-currency system (2009-2013). A GARCH approach analysis.

PHILLIPS, P.C.B., & PERRON, P. (1988). Testing for a unit root in time series regression. *Biometrika*, 75, 335-346.

POLASEK, W. & REN, L. (1999). A Multivariate GARCH-M model for exchange rates in the US, Germany and Japan. 1-11.

PREDESCU, O. M. & STANCU, S. (2011). Portfolio risk analysis using ARCH and GARCH models in the context of the global financial crisis. *Theoretical & Applied Economics*, 18.

SANTOS, A. A., NOGALES, F. J. & RUIZ, E. (2012). Comparing univariate and Multivariate models to forecast portfolio value-at-risk. *Journal of financial econometrics*, 11, 400-441.

SAUNDERS, M. N. K. & BEZZINA, F. (2015). Reflections on conceptions of research methodology among management academics. *European Management Journal*, 33(5), 297–304.

SAUNDERS, M., LEWIS, P. & THORNHILL, A. (2009) Research methods for business students, 5th edition. Harlow, Pearson Education.

SAUNDERS, M., LEWIS, P. AND THORNHILL, A. (2012). Research methods for business students, 6th edition. New York: *Pearson Education*, Inc. 19.

SAVVA, C. S., OSBORN, D. R. & GILL, L. (2005). Volatility, spill-over effects and correlations in U.S. and major European markets. Working Paper, University of Manchester.

SCHWERT, G. W. (1989). Stock market volatility, *Financial Analysts Journal*, 23

SCHWERT, G. W. (1990). Stock volatility and the crash of 87. *The review of Financial Studies*, 3(1), 77-102.

SELMI, N. & HACHICHA, N. (2014). Were oil price markets the source of credit crisis in European Countries? Evidence using a VAR-Multivariate GARCH-DCC model. *International Journal of Energy Economics and Policy*, 4, 169.

SERRANO, J. E. B. (2009). Hedging interest rate risk and foreign exchange risk for a bank in the British market with a simultaneous hedging strategy. Dissertation submitted to Erasmus University Rotterdam.

SHEPARD, N. (1996). Statistical aspects of ARCH and stochastic volatility. In D.V. Lindley D. R. Cox and O. E. Barndorff-Nielse, editors, time series models in econometrics, finance, and other fields. Chapman-Hall, London.

SHERAFATMAND, H., YAZDANI, S. & MOGHADDASI, R. (2014). Futures markets development as a price risk strategy in Iran's dates. *European Journal of Experimental Biology*, 4, 327-333.

SHEU, H.J. & CHENG, C.L. (2011). A study of U.S. and China's volatility spill-over effects on Hong Kong and Taiwan. *African Journal of Business Management,* 5, 5232-5240.

SIAW, R. O. (2014). Investment portfolio optimization with GARCH models. Unpublished thesis. University of Ghana.

SIMS, C. A. (1980). Macroeconomics and reality. *Econometrica: Journal of the Econometric Society*, 1-48.

SJÖHOLM, S. (2015). Heteroskedasticity models and their forecasting performance. Unpublished thesis. Uppsala University.

SOHN, B. (2010). Stock market volatility and trading strategy based factors. Unpublished paper, Georgetown University.

TASHAKKORI, A. & CRESWELL, J.W. (2007). Exploring the nature of research questions in mixed methods research. *Journal of Mixed Methods Research*, 1(3), 207–211.

TAS, M. A. (2008). A survey of Multivariate GARCH models. Unpublished thesis. Bilkent University

TASTAN, H. (2006). Estimating time-varying conditional correlations between stock and foreign exchange markets. *Physica A: Statistical Mechanics and its Applications*, 360, 445-458.

TERÄSVIRTA, T. (2009). An introduction to univariate GARCH models. Handbook of financial time series. Springer.

TSAY, R. S. (2005). Analysis of financial time series, volume 543. Wiley.com, 2nd edition.

TSE, Y.K. & A.K.C. TSUI. (2002). A Multivariate generalized autoregressive conditional heteroskedasticity model with time-varying correlations, *Journal of Business and Economic Statistics 20,* 351-362.

TÜRKYILMAZ, S. & BALIBEY, M. (2014). The relationships among interest rate, exchange rate and stock price: A BEKK-Multivariate GARCH approach. *International Journal of Economics, Finance and Management Sciences*, 1, 166.

WANG, J. & ZIVOT, E. (2006). Modeling financial time series with S-PLUS. Springer

WAHAB, M. (2012). Asymmetric effects of US stock returns on European equities. *International Review of Economics & Finance*, 21, 156-172.

WANG, Y. & WU, C. (2012). Forecasting energy market volatility using GARCH models: Can Multivariate models beat univariate models? *Energy Economics*, 2167–2181.

WEGNER, T. (2016). Applied business statistics: Methods and excel based applications. 4th edition. Cape Town: Juta.

WEI, C. C. (2016). Modeling and analysing the mean and volatility relationship between electricity price returns and fuel market returns. *International Journal of Economics and Finance*, 8, 55-70.

WEI, Y., WANG, Y. & HUANG, D. (2010). Forecasting crude oil market volatility: Further evidence using GARCH-class models. *Energy Economics*, 1477–1484.

WENNSTRÖM, A. (2014). Volatility forecasting performance: Evaluation of GARCH type volatility models on Nordic equity indices.

YI, Z., HENG, C. & WONG, W.K. (2009). China’s stock market integration with a leading power and a close neighbor. *Journal of Risk and Financial Management*, 2, 38-74.

ZAKOIAN, J. M. (1994). Threshold heteroskedastic models. *Journal of Economic Dynamics and Control*, 18(5), 931-955.

ZEITLBERGER, A. C. & BRAUNEIS, A. (2016). Modeling carbon spot and futures price returns with GARCH and Markov switching GARCH models. *Central European Journal of Operations Research*, 24, 149-176.

ZHANG, T., ZHOU, F., ZHANG, X & LI, X. (2016). Multivariate time series analysis on the dynamic relationship between Class B notifiable diseases and gross domestic product (GDP) in China. Scientific Reports, 6 (29). DOI:10.1038/s41598-016-0020-5

ZHAO, H. (2010). Dynamic relationship between exchange rate and stock price: Evidence from China. Research. *International Business and Finance*, 103–112.

ZHOU, B. & WU, C. (2014).The dynamic relationships between stock index futures and stock index markets: Evidence from China. Management Science & Engineering (ICMSE), International Conference on, 2014. IEEE, 1442-1450.

ZHOU, B. & WU, C. (2016). Intraday dynamic relationships between CSI 300 index futures and spot markets: A high-frequency analysis. *Neural Computing and Applications*, 27(4), 1007-1017.

APPENDICES

Appendix A: VAR model

```
$`selection`
AIC(n)  HQ(n)  SC(n) FPE(n) 
     2      2      1      2 

$criteria
                   1             2             3             4             5
6
AIC(n) -3.865276e+01 -3.908252e+01 -3.892197e+01 -3.874913e+01 -3.875103e+01
-3.882765e+01
HQ(n)  -3.836211e+01 -3.854967e+01 -3.814691e+01 -3.773186e+01 -3.749156e+01
-3.732596e+01
SC(n)  -3.793669e+01 -3.776973e+01 -3.701245e+01 -3.624289e+01 -3.564807e+01
-3.512796e+01
FPE(n)  1.635033e-17  1.066474e-17  1.259932e-17  1.515174e-17  1.541353e-17
1.469017e-17

VAR Estimation Results:
========================= 
Endogenous variables: Brazil, China, India, Russia, SouthAfrica 
Deterministic variables: const 
Sample size: 120 
Log Likelihood: 1491.225 
Roots of the characteristic polynomial:
0.9812 0.9812 0.9136 0.9136 0.7821
Call:
VAR(y = data11, p = 1)


Estimation results for equation Brazil: 
======================================= 
Brazil = Brazil.l1 + China.l1 + India.l1 + Russia.l1 + SouthAfrica.l1 + const 

               Estimate Std. Error t value Pr(>|t|)    
Brazil.l1       0.90557    0.07536  12.017  < 2e-16 ***
China.l1       -0.37489    0.11886  -3.154  0.00206 ** 
India.l1       -0.15395    0.08220  -1.873  0.06364 .  
Russia.l1       0.02977    0.03276   0.909  0.36534    
SouthAfrica.l1  0.13685    0.07284   1.879  0.06284 .  
const           0.98192    0.44703   2.197  0.03008 *  
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1


Residual standard error: 0.03732 on 114 degrees of freedom
Multiple R-Squared: 0.9827,	Adjusted R-squared: 0.9819 
F-statistic:  1295 on 5 and 114 DF,  p-value: < 2.2e-16 
```

```
Estimation results for equation China:
======================================
China = Brazil.l1 + China.l1 + India.l1 + Russia.l1 + SouthAfrica.l1 + const

                Estimate Std. Error t value Pr(>|t|)
Brazil.l1       0.009313   0.012581   0.740    0.461
China.l1        0.967983   0.019843  48.782   <2e-16 ***
India.l1       -0.004202   0.013722  -0.306    0.760
Russia.l1       0.001163   0.005469   0.213    0.832
SouthAfrica.l1 -0.002287   0.012161  -0.188    0.851
const           0.069153   0.074631   0.927    0.356
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1


Residual standard error: 0.006231 on 114 degrees of freedom
Multiple R-Squared: 0.9812,   Adjusted R-squared: 0.9804
F-statistic:  1193 on 5 and 114 DF,  p-value: < 2.2e-16


Estimation results for equation India:
======================================
India = Brazil.l1 + China.l1 + India.l1 + Russia.l1 + SouthAfrica.l1 + const

                Estimate Std. Error t value Pr(>|t|)
Brazil.l1      -0.020307   0.038748  -0.524 0.601237
China.l1       -0.214121   0.061116  -3.504 0.000657 ***
India.l1        0.842828   0.042264  19.942  < 2e-16 ***
Russia.l1      -0.001928   0.016844  -0.114 0.909068
SouthAfrica.l1  0.098048   0.037455   2.618 0.010052 *
const           0.834889   0.229859   3.632 0.000423 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.01919 on 114 degrees of freedom
Multiple R-Squared: 0.9861,   Adjusted R-squared: 0.9855
F-statistic:  1620 on 5 and 114 DF,  p-value: < 2.2e-16


Estimation results for equation Russia:
=======================================
Russia = Brazil.l1 + China.l1 + India.l1 + Russia.l1 + SouthAfrica.l1 + const

               Estimate Std. Error t value Pr(>|t|)
Brazil.l1       0.05187    0.09180   0.565   0.5732
China.l1       -0.36439    0.14479  -2.517   0.0132 *
India.l1       -0.16225    0.10013  -1.620   0.1079
Russia.l1       0.92272    0.03991  23.123   <2e-16 ***
SouthAfrica.l1  0.12656    0.08874   1.426   0.1565
const           1.28946    0.54458   2.368   0.0196 *
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1


Residual standard error: 0.04547 on 114 degrees of freedom
Multiple R-Squared: 0.9833,   Adjusted R-squared: 0.9826
F-statistic:  1341 on 5 and 114 DF,  p-value: < 2.2e-16
```

```
Estimation results for equation SouthAfrica:
============================================
SouthAfrica = Brazil.l1 + China.l1 + India.l1 + Russia.l1 + SouthAfrica.l1 +
const

               Estimate Std. Error t value Pr(>|t|)    
Brazil.l1       0.14635    0.07637   1.916  0.05782 .  
China.l1       -0.29745    0.12045  -2.469  0.01501 *  
India.l1       -0.11666    0.08330  -1.400  0.16409    
Russia.l1      -0.03474    0.03320  -1.047  0.29750    
SouthAfrica.l1  0.93042    0.07382  12.604  < 2e-16 ***
const           1.19329    0.45303   2.634  0.00961 ** 
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1


Residual standard error: 0.03782 on 114 degrees of freedom
Multiple R-Squared: 0.9789,   Adjusted R-squared: 0.978 
F-statistic:  1059 on 5 and 114 DF,  p-value: < 2.2e-16 



Covariance matrix of residuals:
               Brazil     China     India    Russia SouthAfrica
Brazil      1.393e-03 5.877e-05 0.0004356 6.990e-04   9.318e-04
China       5.877e-05 3.882e-05 0.0000138 7.618e-05   4.497e-05
India       4.356e-04 1.380e-05 0.0003683 2.545e-04   4.135e-04
Russia      6.990e-04 7.618e-05 0.0002545 2.067e-03   5.456e-04
SouthAfrica 9.318e-04 4.497e-05 0.0004135 5.456e-04   1.431e-03

Correlation matrix of residuals:
            Brazil  China  India Russia SouthAfrica
Brazil      1.0000 0.2527 0.6081 0.4119      0.6601
China       0.2527 1.0000 0.1154 0.2689      0.1908
India       0.6081 0.1154 1.0000 0.2917      0.5697
Russia      0.4119 0.2689 0.2917 1.0000      0.3173
SouthAfrica 0.6601 0.1908 0.5697 0.3173      1.0000
```

Appendix B1: GARCH model for Brazil

```
*---------------------------------*
*    GARCH Model Fit: Brazil      *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : sGARCH(1,1)
Mean Model      : ARFIMA(0,0,0)
Distribution    : std

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu     -0.001070    0.003324 -0.32177 0.747624
omega   0.000827    0.000233  3.54979 0.000386
alpha1  0.454875    0.225914  2.01349 0.044063
beta1   0.013612    0.102551  0.13273 0.894405
shape   7.817511    5.269664  1.48349 0.137943

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu     -0.001070    0.004182 -0.25578 0.798120
omega   0.000827    0.000206  4.00280 0.000063
alpha1  0.454875    0.189182  2.40443 0.016198
beta1   0.013612    0.050227  0.27101 0.786385
shape   7.817511    4.815920  1.62326 0.104533

LogLikelihood : 232.166

Information Criteria
------------------------------------

Akaike       -3.7861
Bayes        -3.6700
Shibata      -3.7894
Hannan-Quinn -3.7389

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic   p-value
Lag[1]                      17.09 3.567e-05
Lag[2*(p+q)+(p+q)-1][2]     17.16 2.476e-05
Lag[4*(p+q)+(p+q)-1][5]     17.47 9.574e-05
d.o.f=0
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                      2.377  0.1232
Lag[2*(p+q)+(p+q)-1][5]     3.513  0.3213
Lag[4*(p+q)+(p+q)-1][9]     4.031  0.5836
d.o.f=2
```

```
Weighted ARCH LM Tests
------------------------------------
             Statistic Shape Scale P-Value
ARCH Lag[3]     0.3834 0.500 2.000  0.5358
ARCH Lag[5]     0.4387 1.440 1.667  0.9016
ARCH Lag[7]     0.7229 2.315 1.543  0.9540

Nyblom stability test
------------------------------------
Joint Statistic:  1.5596
Individual Statistics:
mu     0.61877
omega  0.07220
alpha1 0.05813
beta1  0.11461
shape  0.23307

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:        1.28 1.47 1.88
Individual Statistic:   0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value   prob sig
Sign Bias           1.2507 0.2136
Negative Sign Bias  0.2564 0.7981
Positive Sign Bias  0.2952 0.7683
Joint Effect        4.3939 0.2220


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     15.67       0.6794
2    30     37.00       0.1462
3    40     40.00       0.4256
4    50     60.00       0.1349
```

Appendix B2: GARCH model for China

```
*---------------------------------*
*     GARCH Model Fit: China      *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : sGARCH(1,1)
Mean Model      : ARFIMA(0,0,0)
Distribution    : std

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu     -0.000521    0.000254 -2.05328 0.040046
omega   0.000001    0.000003  0.30423 0.760955
alpha1  0.408172    0.134150  3.04265 0.002345
```

```
beta1   0.590828    0.094868  6.22788 0.000000
shape   3.911562    0.802769  4.87259 0.000001

Robust Standard Errors:
        Estimate  Std. Error   t value Pr(>|t|)
mu     -0.000521    0.000856 -0.608085 0.543131
omega   0.000001    0.000028  0.032771 0.973857
alpha1  0.408172    0.111089  3.674274 0.000239
beta1   0.590828    0.352552  1.675860 0.093766
shape   3.911562    1.707327  2.291045 0.021961

LogLikelihood : 481.8181

Information Criteria
------------------------------------

Akaike       -7.9470
Bayes        -7.8308
Shibata      -7.9503
Hannan-Quinn -7.8998

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                      3.365 0.06659
Lag[2*(p+q)+(p+q)-1][2]     3.664 0.09287
Lag[4*(p+q)+(p+q)-1][5]     5.322 0.12928
d.o.f=0
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                    0.02973  0.8631
Lag[2*(p+q)+(p+q)-1][5]   0.07502  0.9989
Lag[4*(p+q)+(p+q)-1][9]   0.10906  1.0000
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]   0.02676 0.500 2.000  0.8701
ARCH Lag[5]   0.02916 1.440 1.667  0.9977
ARCH Lag[7]   0.05508 2.315 1.543  0.9998

Nyblom stability test
------------------------------------
Joint Statistic:  4.5629
Individual Statistics:
mu     0.19504
omega  1.87963
alpha1 0.14339
beta1  0.50500
shape  0.07488

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:        1.28 1.47 1.88
```

```
Individual Statistic:   0.35 0.47 0.75

Sign Bias Test
------------------------------------
                    t-value   prob sig
Sign Bias          1.006816 0.3161
Negative Sign Bias 0.005298 0.9958
Positive Sign Bias 0.810827 0.4191
Joint Effect       1.607956 0.6576


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     19.33      0.43565
2    30     30.50      0.38937
3    40     46.00      0.20494
4    50     68.33      0.03529
```

Appendix B3: GARCH model for India

```
*---------------------------------*
*     GARCH Model Fit: India      *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : sGARCH(1,1)
Mean Model      : ARFIMA(0,0,0)
Distribution    : std

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001299    0.001458  0.89043 0.373235
omega   0.000026    0.000031  0.83643 0.402913
alpha1  0.200918    0.133420  1.50590 0.132093
beta1   0.753180    0.142642  5.28020 0.000000
shape   5.268026    2.600110  2.02608 0.042757

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001299    0.001513  0.85825 0.390752
omega   0.000026    0.000029  0.88187 0.377850
alpha1  0.200918    0.119309  1.68402 0.092179
beta1   0.753180    0.136201  5.52992 0.000000
shape   5.268026    2.123987  2.48025 0.013129

LogLikelihood : 307.0985

Information Criteria
------------------------------------

Akaike       -5.0350
Bayes        -4.9188
Shibata      -5.0383
```

```
Hannan-Quinn -4.9878

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic  p-value
Lag[1]                      7.111 0.007661
Lag[2*(p+q)+(p+q)-1][2]     7.113 0.011122
Lag[4*(p+q)+(p+q)-1][5]     7.288 0.044129
d.o.f=0
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                     0.1424  0.7060
Lag[2*(p+q)+(p+q)-1][5]    1.6949  0.6920
Lag[4*(p+q)+(p+q)-1][9]    2.6286  0.8183
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]     1.884 0.500 2.000  0.1699
ARCH Lag[5]     2.688 1.440 1.667  0.3383
ARCH Lag[7]     2.943 2.315 1.543  0.5243

Nyblom stability test
------------------------------------
Joint Statistic:  0.8095
Individual Statistics:
mu     0.18568
omega  0.38806
alpha1 0.20132
beta1  0.33370
shape  0.08995

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:        1.28 1.47 1.88
Individual Statistic:   0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value   prob sig
Sign Bias           0.9987 0.3200
Negative Sign Bias  0.4139 0.6797
Positive Sign Bias  0.1544 0.8776
Joint Effect        1.2547 0.7399


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     8.667       0.9786
2    30    12.500       0.9967
3    40    20.000       0.9950
4    50    25.000       0.9983
```

Appendix B4: GARCH model for Russia

```
*---------------------------------*
*     GARCH Model Fit: Russia     *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : sGARCH(1,1)
Mean Model      : ARFIMA(0,0,0)
Distribution    : std

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu     -0.001759    0.002757 -0.63819 0.523350
omega   0.000328    0.000128  2.55313 0.010676
alpha1  0.712692    0.224933  3.16846 0.001532
beta1   0.240389    0.125103  1.92152 0.054666
shape  12.959598   14.357271  0.90265 0.366711

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu     -0.001759    0.003052  -0.5765 0.564275
omega   0.000328    0.000096   3.4139 0.000640
alpha1  0.712692    0.178036   4.0031 0.000063
beta1   0.240389    0.113567   2.1167 0.034284
shape  12.959598   12.700417   1.0204 0.307535

LogLikelihood : 225.9144

Information Criteria
------------------------------------

Akaike       -3.6819
Bayes        -3.5658
Shibata      -3.6852
Hannan-Quinn -3.6347

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic   p-value
Lag[1]                      11.26 0.0007913
Lag[2*(p+q)+(p+q)-1][2]     11.34 0.0008470
Lag[4*(p+q)+(p+q)-1][5]     11.84 0.0030654
d.o.f=0
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                    0.04377  0.8343
Lag[2*(p+q)+(p+q)-1][5]   1.18716  0.8161
Lag[4*(p+q)+(p+q)-1][9]   2.34077  0.8607
d.o.f=2
```

```
Weighted ARCH LM Tests
------------------------------------
             Statistic Shape Scale P-Value
ARCH Lag[3]     0.9278 0.500 2.000  0.3354
ARCH Lag[5]     0.9785 1.440 1.667  0.7393
ARCH Lag[7]     1.5768 2.315 1.543  0.8058

Nyblom stability test
------------------------------------
Joint Statistic:  0.8696
Individual Statistics:
mu     0.15988
omega  0.03245
alpha1 0.07448
beta1  0.09165
shape  0.24428

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:        1.28 1.47 1.88
Individual Statistic:   0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value   prob sig
Sign Bias           0.7754 0.4397
Negative Sign Bias  0.1700 0.8653
Positive Sign Bias  0.5105 0.6107
Joint Effect        1.0313 0.7937


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     20.67      0.35551
2    30     40.00      0.08394
3    40     30.67      0.82732
4    50     54.17      0.28384
```

Appendix B5: GARCH model for South Africa

```
*---------------------------------*
* GARCH Model Fit: South Africa   *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : sGARCH(1,1)
Mean Model      : ARFIMA(0,0,0)
Distribution    : std

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.004834    0.003124  1.54731 0.121788
omega   0.000266    0.000373  0.71428 0.475053
alpha1  0.149614    0.177695  0.84197 0.399804
```

```
beta1   0.652905    0.381659  1.71070 0.087136
shape   6.003666    2.636806  2.27687 0.022794

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.004834    0.004090  1.18191 0.237241
omega   0.000266    0.000638  0.41726 0.676487
alpha1  0.149614    0.246221  0.60764 0.543425
beta1   0.652905    0.662405  0.98566 0.324300
shape   6.003666    3.063207  1.95993 0.050004

LogLikelihood : 231.5065

Information Criteria
------------------------------------

Akaike       -3.7751
Bayes        -3.6590
Shibata      -3.7784
Hannan-Quinn -3.7279

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                      5.650 0.01746
Lag[2*(p+q)+(p+q)-1][2]     5.894 0.02348
Lag[4*(p+q)+(p+q)-1][5]     6.312 0.07589
d.o.f=0
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                     0.1077  0.7428
Lag[2*(p+q)+(p+q)-1][5]    0.9248  0.8765
Lag[4*(p+q)+(p+q)-1][9]    7.8862  0.1356
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]    0.1504 0.500 2.000 0.69819
ARCH Lag[5]    0.4768 1.440 1.667 0.89046
ARCH Lag[7]    8.4168 2.315 1.543 0.04235

Nyblom stability test
------------------------------------
Joint Statistic:  1.9205
Individual Statistics:
mu     0.2197
omega  0.1444
alpha1 0.1435
beta1  0.1038
shape  0.4087

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:        1.28 1.47 1.88
```

```
Individual Statistic:   0.35 0.47 0.75
Sign Bias Test
------------------------------------
                   t-value   prob sig
Sign Bias           0.8116 0.4187    
Negative Sign Bias  0.3588 0.7204    
Positive Sign Bias  0.8991 0.3705    
Joint Effect        3.6979 0.2960    


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     23.67       0.2093
2    30     34.00       0.2393
3    40     40.00       0.4256
4    50     47.50       0.5341
```

Appendix C1: TGARCH model for Brazil

```
*---------------------------------*
*     GARCH Model Fit: Brazil     *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : gjrGARCH(1,1)
Mean Model      : ARFIMA(0,0,0)
Distribution    : std

Optimal Parameters
------------------------------------
        Estimate  Std. Error    t value Pr(>|t|)
mu      0.001322    0.000045    29.1573  0.00000
omega   0.000103    0.000008    12.3827  0.00000
alpha1  0.077878    0.014669     5.3090  0.00000
beta1   0.934771    0.000067 13928.3379  0.00000
gamma1 -0.297479    0.025660   -11.5932  0.00000
shape   6.872372    3.318569     2.0709  0.03837

Robust Standard Errors:
        Estimate  Std. Error    t value Pr(>|t|)
mu      0.001322    0.000068    19.4643 0.000000
omega   0.000103    0.000007    15.5486 0.000000
alpha1  0.077878    0.008436     9.2311 0.000000
beta1   0.934771    0.000072 12956.5741 0.000000
gamma1 -0.297479    0.020682   -14.3834 0.000000
shape   6.872372    3.532613     1.9454 0.051726

LogLikelihood : 233.111

Information Criteria
------------------------------------
Akaike       -3.7852
Bayes        -3.6458
Shibata      -3.7899
```

```
Hannan-Quinn -3.7286

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic   p-value
Lag[1]                      18.69 1.538e-05
Lag[2*(p+q)+(p+q)-1][2]     18.74 9.512e-06
Lag[4*(p+q)+(p+q)-1][5]     18.96 3.751e-05
d.o.f=0
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic  p-value
Lag[1]                      8.068 0.004506
Lag[2*(p+q)+(p+q)-1][5]     9.694 0.011025
Lag[4*(p+q)+(p+q)-1][9]    10.347 0.042250
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]    0.7918 0.500 2.000  0.3735
ARCH Lag[5]    1.3948 1.440 1.667  0.6204
ARCH Lag[7]    1.6157 2.315 1.543  0.7979

Nyblom stability test
------------------------------------
Joint Statistic:  1.1226
Individual Statistics:
mu     0.21102
omega  0.09120
alpha1 0.06168
beta1  0.07575
gamma1 0.06585
shape  0.25745

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:        1.49 1.68 2.12
Individual Statistic:   0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value    prob sig
Sign Bias           0.5546 0.58023
Negative Sign Bias  0.2101 0.83395
Positive Sign Bias  1.8418 0.06808   *
Joint Effect        7.3562 0.06137   *


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     23.67       0.2093
2    30     37.50       0.1339
3    40     45.33       0.2248
4    50     45.00       0.6360
```

Appendix C2: TGARCH model for China

```
*---------------------------------*
*      GARCH Model Fit: China     *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : gjrGARCH(1,1)
Mean Model      : ARFIMA(0,0,0)
Distribution    : std

Optimal Parameters
------------------------------------
        Estimate  Std. Error    t value Pr(>|t|)
mu     -0.001393    0.000332   -4.19475 0.000027
omega   0.000000    0.000001    0.11436 0.908951
alpha1  0.060262    0.005030   11.98152 0.000000
beta1   0.997415    0.000204 4886.00515 0.000000
gamma1 -0.123810    0.005928  -20.88465 0.000000
shape   4.286401    1.129936    3.79349 0.000149

Robust Standard Errors:
        Estimate  Std. Error   t value Pr(>|t|)
mu     -0.001393    0.010680 -0.130464  0.89620
omega   0.000000    0.000049  0.003332  0.99734
alpha1  0.060262    0.083745  0.719590  0.47178
beta1   0.997415    0.011797 84.550391  0.00000
gamma1 -0.123810    0.274340 -0.451302  0.65177
shape   4.286401    7.274776  0.589214  0.55572

LogLikelihood : 478.3175

Information Criteria
------------------------------------

Akaike       -7.8720
Bayes        -7.7326
Shibata      -7.8766
Hannan-Quinn -7.8154

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic   p-value
Lag[1]                      15.69 7.470e-05
Lag[2*(p+q)+(p+q)-1][2]     17.00 2.727e-05
Lag[4*(p+q)+(p+q)-1][5]     23.20 2.533e-06
d.o.f=0
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                   0.003951  0.9499
Lag[2*(p+q)+(p+q)-1][5]  0.196460  0.9928
Lag[4*(p+q)+(p+q)-1][9]  0.424827  0.9991
d.o.f=2
```

```
Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]   0.06435 0.500 2.000  0.7997
ARCH Lag[5]   0.24829 1.440 1.667  0.9537
ARCH Lag[7]   0.35740 2.315 1.543  0.9894

Nyblom stability test
------------------------------------
Joint Statistic:  15.3329
Individual Statistics:
mu     0.11001
omega  3.83469
alpha1 0.04141
beta1  0.04478
gamma1 0.04344
shape  0.12058

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:        1.49 1.68 2.12
Individual Statistic:   0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value   prob sig
Sign Bias           0.6397 0.5236
Negative Sign Bias  0.3265 0.7446
Positive Sign Bias  0.2750 0.7838
Joint Effect        0.4135 0.9374


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     34.00      0.01838
2    30     40.50      0.07609
3    40     52.00      0.07957
4    50     71.67      0.01903
```

Appendix C3: TGARCH model for India

```
*---------------------------------*
*      GARCH Model Fit: India     *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model    : gjrGARCH(1,1)
Mean Model     : ARFIMA(0,0,0)
Distribution   : std

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001760    0.001470   1.1966 0.231468
omega   0.000032    0.000029   1.1198 0.262807
```

```
alpha1  0.248497    0.148550   1.6728 0.094363
beta1   0.777343    0.116530   6.6707 0.000000
gamma1 -0.254775    0.161548  -1.5771 0.114775
shape   5.980018    3.306799   1.8084 0.070544

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001760    0.001528  1.15187 0.249372
omega   0.000032    0.000035  0.91565 0.359849
alpha1  0.248497    0.151695  1.63814 0.101393
beta1   0.777343    0.127563  6.09380 0.000000
gamma1 -0.254775    0.193141 -1.31911 0.187131
shape   5.980018    2.610441  2.29081 0.021975

LogLikelihood : 308.7052

Information Criteria
------------------------------------

Akaike       -5.0451
Bayes        -4.9057
Shibata      -5.0498
Hannan-Quinn -4.9885

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                      6.144 0.01319
Lag[2*(p+q)+(p+q)-1][2]     6.155 0.02000
Lag[4*(p+q)+(p+q)-1][5]     6.304 0.07623
d.o.f=0
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                     0.2175  0.6410
Lag[2*(p+q)+(p+q)-1][5]    2.2937  0.5508
Lag[4*(p+q)+(p+q)-1][9]    3.3752  0.6957
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]     2.607 0.500 2.000  0.1064
ARCH Lag[5]     3.410 1.440 1.667  0.2356
ARCH Lag[7]     3.561 2.315 1.543  0.4143

Nyblom stability test
------------------------------------
Joint Statistic:  1.2399
Individual Statistics:
mu     0.1408
omega  0.6434
alpha1 0.2966
beta1  0.4604
gamma1 0.2978
```

```
shape  0.1519

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:        1.49 1.68 2.12
Individual Statistic:   0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value   prob sig
Sign Bias           0.8578 0.3928
Negative Sign Bias  0.8668 0.3879
Positive Sign Bias  0.2312 0.8176
Joint Effect        0.9426 0.8151


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     13.00       0.8386
2    30     16.00       0.9755
3    40     29.33       0.8694
4    50     28.33       0.9921
```

Appendix C4: TGARCH model for Russia

```
*---------------------------------*
*      GARCH Model Fit: Russia    *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : gjrGARCH(1,1)
Mean Model      : ARFIMA(0,0,0)
Distribution    : std

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu     -0.000652    0.002939 -0.22172 0.824535
omega   0.000327    0.000125  2.61808 0.008843
alpha1  0.852982    0.311625  2.73720 0.006196
beta1   0.254378    0.126508  2.01077 0.044350
gamma1 -0.394124    0.370771 -1.06298 0.287789
shape  12.293000   12.140148  1.01259 0.311256

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu     -0.000652    0.003498 -0.18631 0.852203
omega   0.000327    0.000101  3.22424 0.001263
alpha1  0.852982    0.242924  3.51132 0.000446
beta1   0.254378    0.119740  2.12443 0.033634
gamma1 -0.394124    0.415136 -0.94938 0.342425
shape  12.293000   11.070887  1.11039 0.266831

LogLikelihood : 226.457
```

```
Information Criteria
------------------------------------

Akaike       -3.6743
Bayes        -3.5349
Shibata      -3.6790
Hannan-Quinn -3.6177

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic  p-value
Lag[1]                      9.356 0.002223
Lag[2*(p+q)+(p+q)-1][2]     9.428 0.002708
Lag[4*(p+q)+(p+q)-1][5]    10.042 0.008982
d.o.f=0
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                     0.2649  0.6068
Lag[2*(p+q)+(p+q)-1][5]    1.5910  0.7175
Lag[4*(p+q)+(p+q)-1][9]    2.7179  0.8045
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]    0.9126 0.500 2.000  0.3394
ARCH Lag[5]    1.0635 1.440 1.667  0.7142
ARCH Lag[7]    1.5679 2.315 1.543  0.8077

Nyblom stability test
------------------------------------
Joint Statistic:  0.8961
Individual Statistics:
mu     0.14370
omega  0.03215
alpha1 0.05656
beta1  0.09217
gamma1 0.14423
shape  0.24055

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:        1.49 1.68 2.12
Individual Statistic:   0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value   prob sig
Sign Bias           0.5520 0.5820    
Negative Sign Bias  0.2909 0.7717    
Positive Sign Bias  0.6313 0.5291    
Joint Effect        0.5008 0.9187    
```

```
Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     13.00       0.8386
2    30     22.00       0.8202
3    40     32.67       0.7528
4    50     52.50       0.3400
```

Appendix C5: TGARCH model for South Africa

```
*---------------------------------*
* GARCH Model Fit: South Africa   *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : gjrGARCH(1,1)
Mean Model      : ARFIMA(0,0,0)
Distribution    : std

Optimal Parameters
------------------------------------
        Estimate  Std. Error     t value Pr(>|t|)
mu      0.004497    0.002495     1.80213 0.071525
omega   0.000003    0.000007     0.46519 0.641793
alpha1  0.039216    0.003016    13.00265 0.000000
beta1   1.000000    0.000062 16044.98314 0.000000
gamma1 -0.102231    0.007482   -13.66366 0.000000
shape   8.221502    4.559447     1.80318 0.071360

Robust Standard Errors:
        Estimate  Std. Error     t value Pr(>|t|)
mu      0.004497    0.003148     1.42867 0.153099
omega   0.000003    0.000015     0.21948 0.826274
alpha1  0.039216    0.002029    19.32657 0.000000
beta1   1.000000    0.000034 29284.09181 0.000000
gamma1 -0.102231    0.011203    -9.12499 0.000000
shape   8.221502    4.815584     1.70727 0.087772

LogLikelihood : 233.2828

Information Criteria
------------------------------------

Akaike       -3.7880
Bayes        -3.6487
Shibata      -3.7927
Hannan-Quinn -3.7314

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic  p-value
Lag[1]                      9.105 0.002549
Lag[2*(p+q)+(p+q)-1][2]     9.276 0.002970
Lag[4*(p+q)+(p+q)-1][5]     9.591 0.011716
d.o.f=0
```

```
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                     0.9062  0.3411
Lag[2*(p+q)+(p+q)-1][5]    2.1332  0.5873
Lag[4*(p+q)+(p+q)-1][9]    5.2507  0.3935
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]    0.3894 0.500 2.000  0.5326
ARCH Lag[5]    0.8141 1.440 1.667  0.7888
ARCH Lag[7]    3.5885 2.315 1.543  0.4098

Nyblom stability test
------------------------------------
Joint Statistic:  16.3751
Individual Statistics:
mu     0.1924
omega  0.4328
alpha1 0.1228
beta1  0.1200
gamma1 0.1180
shape  0.1482

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:        1.49 1.68 2.12
Individual Statistic:   0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value    prob sig
Sign Bias            1.157 0.24987
Negative Sign Bias   1.700 0.09184   *
Positive Sign Bias   1.080 0.28234
Joint Effect         5.677 0.12844


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     13.67       0.8028
2    30     25.50       0.6521
3    40     32.00       0.7790
4    50     41.67       0.7621
```

Appendix D: BEKK-GARCH

Parameter estimation matrix

```
$`1`
         [,1]      [,2]       [,3]        [,4]       [,5]
[1,] 1.258629 0.6045632 0.15224174 -18.3413390 17.2490356
[2,] 0.000000 1.0693520 0.07560597 -12.5425507 11.8776165
[3,] 0.000000 0.0000000 1.06682811  -0.4994388  0.7053325
[4,] 0.000000 0.0000000 0.00000000   6.1418023 -5.5259687
[5,] 0.000000 0.0000000 0.00000000   0.0000000 71.6646071

$`2`
           [,1]       [,2]         [,3]     [,4]      [,5]
[1,] -0.2205181 -0.1168006  0.008345951 11.44731 -10.42351
[2,] -0.6084166 -0.3541207 -0.087007243 25.21703 -23.21204
[3,] -1.4167295 -0.8781611 -0.303736812 53.81105 -49.75205
[4,] -1.2870620 -0.8034353 -0.275918553 49.27553 -45.53842
[5,] -0.7682484 -0.4668279 -0.135356454 30.83388 -28.41677

$`3`
           [,1]       [,2]        [,3]     [,4]      [,5]
[1,] -0.5419929 -0.2363545  0.21022259 12.84784 -11.73842
[2,] -0.3523372 -0.1135014  0.10393443 12.84953 -11.74214
[3,] -0.1874210 -0.1082707  0.08778779 12.88525 -11.70807
[4,] -0.3204004 -0.2070425 -0.10134119 11.77130 -10.98825
[5,] -0.3208925 -0.2139901 -0.12093428 11.90720 -11.12645
```

Standard error matrix

```
[[1]]
          [,1]      [,2]       [,3]      [,4]      [,5]
[1,] 0.5058607 0.3840273 0.41130463 19.348612 20.125567
[2,] 0.0000000 0.2014449 0.33629328  3.515898  6.583834
[3,] 0.0000000 0.0000000 0.02290674  1.535342  5.041910
[4,] 0.0000000 0.0000000 0.00000000  3.139861  5.806061
[5,] 0.0000000 0.0000000 0.00000000  0.000000  5.786452

[[2]]
         [,1]     [,2]      [,3]      [,4]      [,5]
[1,] 2.884949 2.942644  9.405675 136.47726 169.35367
[2,] 2.861711 3.682686  6.896283  68.08442  91.52203
[3,] 3.845125 1.329433  5.334055 167.69530  66.33279
[4,] 3.146547 1.392712  4.491787 134.23042 136.80664
[5,] 3.705519 5.440044 10.354135  58.63277  72.30788


[[3]]
            [,1]        [,2]        [,3]         [,4]        [,5]
[1,] 1.756545840 1.039498858 0.301532693  71.47037677  67.6708881
[2,] 2.991304658 1.821690404 0.696606546 114.64187493 107.8838651
[3,] 0.724985245 0.614975758 0.715707476  21.56661785  19.9390360
[4,] 0.001529862 0.002131053 0.002617643   0.09737697   0.1233668
[5,] 0.002095179 0.002643720 0.007583297   0.22630395   0.2805168
```

Appendix E: DCC-GARCH

```
*---------------------------------*
*          DCC GARCH Fit          *
*---------------------------------*

Distribution         :  mvnorm
Model                :  DCC(1,1)
No. Parameters       :  32
[VAR GARCH DCC UncQ] : [0+20+2+10]
No. Series           :  5
No. Obs.             :  121
Log-Likelihood       :  940.5562
Av.Log-Likelihood    :  7.77

Optimal Parameters
-----------------------------------
                      Estimate  Std. Error    t value Pr(>|t|)
[Brazil].mu           0.708628    0.024745  28.636732 0.000000
[Brazil].omega        0.000629    0.000610   1.031464 0.302323
[Brazil].alpha1       0.999000    0.412899   2.419476 0.015543
[Brazil].beta1        0.000000    0.426870   0.000000 1.000000
[China].mu            1.921591    0.026027  73.830937 0.000000
[China].omega         0.000000    0.000067   0.005933 0.995266
[China].alpha1        0.851563    0.327067   2.603631 0.009224
[China].beta1         0.147437    1.752588   0.084125 0.932957
[India].mu            4.162412    0.022262 186.972624 0.000000
[India].omega         0.000040    0.000025   1.592291 0.111319
[India].alpha1        0.774195    0.088243   8.773453 0.000000
[India].beta1         0.207625    0.081166   2.558033 0.010527
[Russia].mu           3.431745    0.011814 290.481812 0.000000
[Russia].omega        0.000359    0.000169   2.123780 0.033689
[Russia].alpha1       0.819648    0.093982   8.721305 0.000000
[Russia].beta1        0.179352    0.108625   1.651102 0.098718
[SouthAfrica].mu      2.120388    0.014052 150.892236 0.000000
[SouthAfrica].omega   0.001375    0.002265   0.606965 0.543874
[SouthAfrica].alpha1  0.961544    0.159187   6.040330 0.000000
[SouthAfrica].beta1   0.037456    0.179114   0.209120 0.834355
[Joint]dcca1          0.356853    0.369526   0.965704 0.334192
[Joint]dccb1          0.606797    0.464972   1.305020 0.191886
```

Appendix F: VAR BEKK-GARCH

Parameter estimation matrix

```
$`1`
            [,1]          [,2]         [,3]         [,4]         [,5]
[1,] -0.01912828 -0.0001574078 -0.012536064 -0.020163956 -0.004028948
[2,]  0.00000000  0.0027885161  0.001115385  0.013421225 -0.005319645
[3,]  0.00000000  0.0000000000  0.008224805 -0.002537465  0.008187020
[4,]  0.00000000  0.0000000000  0.000000000 -0.019698943 -0.012522213
[5,]  0.00000000  0.0000000000  0.000000000  0.000000000 -0.007978029
```

```
$`2`
             [,1]         [,2]         [,3]        [,4]       [,5]
[1,] -0.004180791 -0.093049059  0.020026095 -0.50230222 -0.1124241
[2,]  1.384856340  1.100435965  0.470575843  0.15377948  1.5772917
[3,]  0.421081709  0.029197059  0.303303417 -0.02589204  0.1462563
[4,]  0.057668865 -0.004364877 -0.009542355  0.79817372  0.1290635
[5,]  0.059691906  0.064480506  0.012157876  0.18988256  0.3090632

$`3`
            [,1]         [,2]         [,3]         [,4]        [,5]
[1,]  0.73173271  0.011160846  0.346011215  0.065605133  0.93550670
[2,] -0.72017572 -0.135851952 -0.805727248 -0.223949134 -1.12369683
[3,] -1.62892692 -0.076855128 -0.248382660 -0.430978475 -1.24889934
[4,]  0.22515712 -0.012050375 -0.074585550  0.114356950 -0.08462755
[5,] -0.01555523 -0.008614049  0.004621272  0.003342927 -0.03332108
```

Standard Error matrix

```
[[1]]
            [,1]         [,2]        [,3]        [,4]        [,5]
[1,] 0.003317533 0.0004828107 0.002522816 0.006721071 0.005406749
[2,] 0.000000000 0.0008577034 0.005135022 0.005803735 0.012238797
[3,] 0.000000000 0.0000000000 0.002285613 0.007137678 0.005429777
[4,] 0.000000000 0.0000000000 0.000000000 0.006546077 0.005636804
[5,] 0.000000000 0.0000000000 0.000000000 0.000000000 0.008834079

[[2]]
          [,1]       [,2]       [,3]      [,4]      [,5]
[1,] 0.1967203 0.03742934 0.08044675 0.2329274 0.2136607
[2,] 1.5140355 0.19018854 0.55646211 1.5839952 1.2863301
[3,] 0.3482938 0.02676628 0.15310430 0.3362152 0.3186783
[4,] 0.1269729 0.01062231 0.05908677 0.2337250 0.1142913
[5,] 0.1570718 0.06478055 0.08048170 0.3955683 0.2639648

[[3]]
           [,1]       [,2]       [,3]       [,4]      [,5]
[1,] 0.09428781 0.03187365 0.14106687 0.17655704 0.4679342
[2,] 0.84049860 0.27249059 0.50750052 0.33773066 0.4453029
[3,] 0.26640510 0.10211973 0.20512127 0.34187952 0.3966827
[4,] 0.12566688 0.02493838 0.05985864 0.10132247 0.2536405
[5,] 0.16854562 0.01230791 0.19244213 0.04986071 0.4243480
```

Appendix G: VAR DCC-GARCH

```
*---------------------------------*
*          DCC GARCH Fit          *
*---------------------------------*

Distribution         :  mvnorm
Model                :  DCC(1,1)
No. Parameters       :  32
[VAR GARCH DCC UncQ] : [0+20+2+10]
No. Series           :  5
No. Obs.             :  120
Log-Likelihood       :  1407.269
```

```
Av.Log-Likelihood    :  11.73

Optimal Parameters
-----------------------------------
                      Estimate  Std. Error   t value Pr(>|t|)
[Brazil].mu           0.000000    0.007728  0.000000 1.000000
[Brazil].omega        0.000001    0.000002  0.543623 0.586701
[Brazil].alpha1       0.050673    0.045022  1.125501 0.260377
[Brazil].beta1        0.900209    0.095425  9.433675 0.000000
[China].mu            0.000000    0.003296  0.000000 1.000000
[China].omega         0.000000    0.000024  0.001566 0.998751
[China].alpha1        0.050006    0.074349  0.672583 0.501213
[China].beta1         0.899999    0.177062  5.082958 0.000000
[India].mu            0.000000    0.001940  0.000000 1.000000
[India].omega         0.000001    0.000014  0.050378 0.959821
[India].alpha1        0.122029    0.144458  0.844741 0.398255
[India].beta1         0.874822    0.128347  6.816057 0.000000
[Russia].mu           0.000000    0.010158  0.000000 1.000000
[Russia].omega        0.000002    0.000004  0.541638 0.588068
[Russia].alpha1       0.051407    0.036514  1.407847 0.159176
[Russia].beta1        0.899891    0.083270 10.806905 0.000000
[SouthAfrica].mu      0.000000    0.007017  0.000000 1.000000
[SouthAfrica].omega   0.000001    0.000007  0.199877 0.841577
[SouthAfrica].alpha1  0.051161    0.136437  0.374978 0.707677
[SouthAfrica].beta1   0.900424    0.290650  3.097968 0.001949
[Joint]dcca1          0.047638    0.016403  2.904229 0.003682
[Joint]dccb1          0.000000    0.475000  0.000000 1.000000
```