OPTIMISING UNDERGROUND MINE TRAIN SCHEDULING: A MIXED-INTEGER LINEAR PROGRAMMING APPROACH

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ABSTRACT

Underground mines utilise a single-lane, bidirectional transportation network during ore extraction. Once the underground train has reached the elevator shaft, the ore is dumped into a skip and lifted to ground level for further handling. Congestion on the single-lane network is a significant problem for underground mines. Several studies have addressed the congestion problem on a double-lane network, but the proposed models do not apply to the single-lane, bidirectional network. First, a brief background is provided regarding the related studies. Second, a mathematical programming model is proposed to schedule trains on a single-lane, bidirectional transportation network. The model aims to improve the network's transportation throughput. The formulation of the mathematical programming model is verified and validated using historical data. The main contribution of this study is the formulation of a mathematical programming model for scheduling underground mine trains on a single-lane, bidirectional network.

Keywords: Flow conservation, Optimisation, Single-lane bidirectional network

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1 INTRODUCTION

Operations research is essential for planning and scheduling in mines' transportation and production departments [1]. A detailed schedule should consider short-term and long-term requirements and daily operations. Block models commonly represent orebodies and are utilised in mathematical modelling to maximise the net present value of mine operations [1].

Underground mines extract ore from mineral deposits through drilling, blasting and crushing to transport the ore on a network of tunnels and shafts. The transportation network of the ore consists of various transportation methods, namely trackless vehicles, conveyor belt systems and rail bounded vehicles. The networks have very primitive communication and vehicle tracking systems leading to poor scheduling, causing inefficient and low transportation of ore across the network. The model proposed in this study aims to improve the transportation throughput to increase the mine's output capacity. The physical limitations and resource availability are crucial for understanding the transportation network and during the development of the mathematical model.

This study focuses on scheduling rail-bound vehicles on a single-lane, bidirectional transportation network to maximise the transportation throughput in tonnes. To simplify the mathematical model, only one level of an underground mine network is examined and scheduled. The personnel on the network is not part of the scheduling. The assumption is that the personnel's working hours are within the prescribed hours according to the labour and health and safety acts. The mine operates in shifts, and the change-overs between shifts are made seamlessly so that no disruption in the schedules occurs.

The rail-bound vehicles travel at 10km/h when loaded and have a top speed of 20km/h when empty. The locomotives have a load capacity of 35 tonnes for transporting ore. The transportation network's topology is reminiscent of an acyclic graph with a central line and branches connecting the extraction sites to the central line. The network is extended as the exploration expands and additional extraction sites are constructed. The mine supplies a list of coordinates regarding the nodes and edges of their network. Figure 1 visualises the network for a better understanding of the layout.

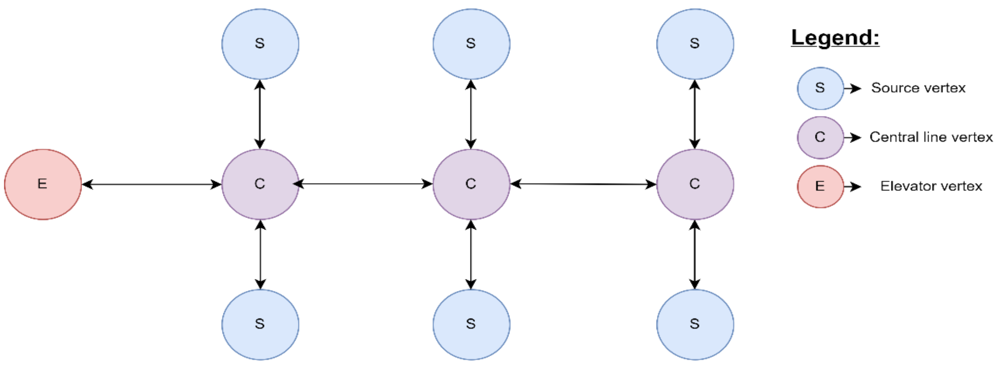

Figure 1: Visualisation of an underground mine network

The transportation network is constructed in table format from the coordinates supplied by the mine. The distance calculations of each arc are included within the input data, and the model calculates the travel durations per arc for loaded and empty trains.

Shift hours differ between mines. Generally, workers are present in a mine for the whole 24 hours. Mines operate approximately between 341 and 351 production days annually. Blasting and drilling operations are included in the shift operations, with two planned blasts per day, as in some mines [2]. The case mine utilised for the mathematical model's verification and validation had 263 production days and 102 days with production downtime. The daily ore transportation throughput is averaged at 66.12015 tonnes for the first 22 days, after which a 21-day downtime started. This study focuses solely on the transportation network of the underground mine. The supply capacity of the extraction site is assumed to be unlimited, ensuring that the transportation throughput is the limiting factor on the network. The profit or net present value (NPV) is not considered during the model formulation and calculations.

To address this problem experienced in the mining industry, this study proposes a mixed integer linear programming (MILP) model to determine the optimal train schedule on the single-lane, bidirectional network. The proposed model aims to maximise the transportation throughput of the network while considering the prevention of collisions on the network.

Scheduling the flow of trains in a single direction is computationally easier as combinations of trains are selected. For scheduling the overlapping flows of empty and loaded trains, the permutations of the different trains at different sections of the network are considered a feasible solution.

Following the introduction, the article briefly discusses related studies to the identified problem in Section 2. The mixed integer linear programming model is formulated in Section 3. Section 4 is the verification and validation of the model. In Section 5, the scalability of the model is examined, and the results are listed. The conclusion is given in Section 6, followed by a recommendation for future work in Section 7.

2 RELATED STUDIES

Numerous problems exist in transportation networks, and congestion and energy consumption are some of the most popular issues addressed in various research papers [3]. Congestion is a complex problem with numerous aspects influencing schedules. Varying demand is one of the leading causes of congestion in train stations; Blanco et al. [4], Ying et al. [5], and Shahabi et al. [6] proposed three different solutions to improve the effectiveness of the model's solution against the variation in the demand.

A mixed integer linear programming (MILP) model was implemented with a constraint programming model to address periodic and non-periodic problems on a network [7]. A MILP model was developed and implemented with a stochastic programming model to prevent the Coronavirus disease at train stations [8].

Amaya and Uribe [9] proposed two models for improving train crew scheduling. The first model determines the train routes and schedules according to a network provided and loaded into the model. The second model focuses on scheduling the crew members according to the given law requirements, balancing the weekly load between drivers, and minimising the salary differences between drivers.

Marli'ere et al. [10] focused on a railway network's real-time traffic management problem. They proposed a constraint-based scheduling approach and compared its performance to the Recherche sur la Capacité d'Infrastructures Ferroviaires mixed integer linear programming algorithm. The study aimed to improve schedule changes caused by disruption to reduce passengers' dwell time. Giorgio et al. [10] proposed a mixed integer linear programming model with a dynamic graph for addressing the problem of period and non-periodic disturbances on the network.

Authors of [11], [12] and [13] attempted to reduce the disruptions in schedules caused by maintenance on the network. Zang et al. [11] combined a binary integer model and a dynamic programming model to schedule trains and planned maintenance on the network. Rokhforoz and Fink [12] focused on scheduling preventative maintenance on a network with multiple routes between stations, and a mixed integer linear programming model was proposed. Buriuly et al. [13] focused on dynamic programming and scheduling the downtime in the network.

3 MODEL FORMULATION

First, the model notation is given, and all the relevant sets, variables, and constants are defined. The model formulation follows in Section 3.2, where the objective function and all the required model constraints are given.

3.1 Model notation

The sets utilised in the mathematical model to formulate the scheduling problem are listed in Table 1.

Table 1: Sets implemented in the mathematical model

Symbol	Description
A	the set for all the arcs in the network
A_l	A subset of A, defining all the arcs; $A_l \subset A$
A_e	A subset of A, defining all the arcs; $A_e \subset A$
V	the set for all the vertices in the network
T	the set for all the trains in the network
E	Elevator vertices in the set of all vertices; $E \subset V$
C	Central line vertices in the set of all vertices; $C \subset V$
S	Supply vertices in the set of all vertices; $S \subset V$
B	Non-supply vertices in the set of all vertices; $B \subset V$

Set A is a superset of sets A_e and A_l, Which is utilised in the model for two graphs seen in Figures 2 and 3, respectively. The bidirectional graph is divided into two single-directional graphs representing the forward and backward flow across the network.

The graphs visualised in Figures 2 and 3 are relevant to the set functions in Table 2. The transportation network is a multi-layer graph consisting of G_o and G_1. Graph G_oConsist of all the arcs required for the forward flow. Graph G_1consist of all the arcs required for the backward flow.

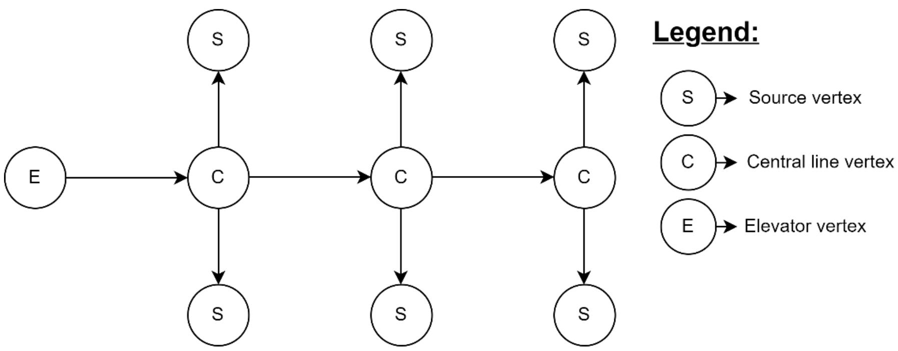

Figure 2: Visualisation of graph G_o with the forward flow

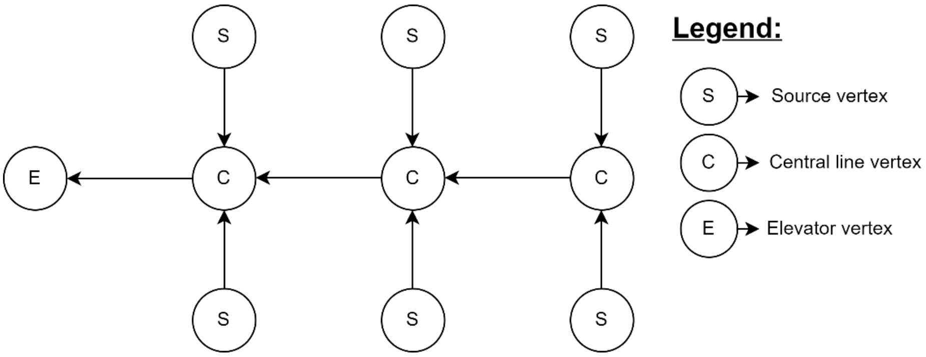

Figure 3: Visualisation of graph G_1 with the backward flow

Set functions are utilised in the model for specific constraints requiring a subset of particular vertex and arc sets. Table 2 lists all the set functions used within the model.

Table 2: Set functions utilised in the mathematical model

Symbol	Description
$IE(v)$	A set function that returns all incident arcs for vertex v in graph $G_o = (V, A_e)$
$OE(v)$	A set function that returns all emanating arcs for vertex v in graph $G_o = (V, A_e)$
$I(v)$	A set function that returns all incident arcs for vertex v in graph $G_1 = (V, A_l)$
$O(v)$	A set function that returns all emanating arcs for vertex v in graph $G_1 = (V, A_l)$
$\sigma(a)$	A set function returning source vertex of an arc a in graph $G_1 = (V, A_l)$
$\beta(a)$	A set function returning source vertex of an arc a in graph $G_o = (V, A_e)$

Table 3 lists the relevant variables utilised in the model, along with the domain, indices, and description. The first four variables are the time variables allocated in the model to capture the time of each train at each interval. The rest are decision variables utilised in the model.

Table 3: Notation of variables utilised in the mathematical model

Symbol	Domain	Index sets	Description
γ_{vt}	$\mathbb{R}_+$	$v \in V, t \in T$	Time variable for backward flow entering vertex
δ_{vt}	$\mathbb{R}_+$	$v \in V, t \in T$	Time variable for backward flow exiting vertex
ζ_{vt}	$\mathbb{R}_+$	$v \in V, t \in T$	Time variable for forward flow entering vertex
ϵ_{vt}	$\mathbb{R}_+$	$v \in V, t \in T$	Time variable for forward flow exiting vertex
x_{at}	$\{0,1\}$	$a \in A, t \in T$	Backward flow decision variable
y_{vtk}	$\{0,1\}$	$v \in V, t \in T, k \in T$	Sequence of the flow decision variable
w_{vtk}	$\{0,1\}$	$v \in V, t \in T, k \in T$	Second sequence of flow decision variable

z_{vt}	{0,1}	$v \in V, t \in T$	Decision variable of vertices used in the model
b_{at}	{0,1}	$a \in A, t \in T$	Forward flow decision variable
d_t	{0,1}	$t \in T$	Train usage decision variable

Table 4 lists all the relevant constants utilised in the mathematical model within the objective function and constraints.

Table 4: Notation of constant utilised in the mathematical model

Symbol	Index sets	Description
ρ_a	$a \in A$	Travel duration for loaded trains in minutes
κ_a	$a \in A$	Travel duration for empty trains in minutes
M	Scalar	Large value for Big-M constraints
μ	Scalar	Arc load capacity in tonnes
P	Scalar	Time duration of a train passing a vertex
ω	Scalar	Schedule interval duration

The travel duration per arc for the loaded and empty trains is represented by ρ_a and κ_a respectively. The value of the Big-M constant, M, was determined as 1440. The arc capacities, μ, is utilised in the objective function for the throughput calculations. The constant P is utilised in the model to implement a time duration between the start and end of a train passing a vertex. The time variables are constrained to an upper bound, ω, which is predetermined as the schedule duration.

3.2 The model is formulated as follows:

Maximise

$$\sum_{t \in T} \sum_{e \in E} \mu z_{et} \tag{3.1}$$

The objective function aims to maximise the product of the binary flow across the elevator vertex, z_{et}, and the train capacity, μ.

subject to

{Forward Flow Constraints}

$$\sum_{s \in S} \sum_{a \in O(s)} x_{at} = \sum_{e \in E} \sum_{k \in I(e)} x_{kt}, \quad \forall t \in T \tag{3.2}$$

$$\sum_{a \in O(c)} x_{at} = \sum_{k \in I(c)} x_{kt}, \quad \forall c \in C, t \in T \tag{3.3}$$

Constraint (3.2) ensures that the sum of the loaded trains exiting the elevator vertices equals the sum of the loaded trains entering the supply vertices. Constraint (3.3) implements the conservation of the flow of the loaded trains across the vertices in the central line of the network.

$$\sum_{a \in I(v)} x_{at} = z_{vt}, \quad \forall v \in V, t \in T \tag{3.4}$$

Constraint (3.4) links the decision variables of the flow of loaded trains to the decision variable of vertices utilised in the model. The z_{vt} decision variable is utilised in the objective function and other constraints.

{Backward Flow Constraints}

$$\sum_{e \in E} \sum_{a \in OE(e)} b_{at} = \sum_{s \in S} \sum_{k \in IE(s)} b_{kt}, \quad \forall t \in T \tag{3.5}$$

$$\sum_{a \in OE(c)} b_{at} = \sum_{k \in IE(c)} b_{kt}, \quad \forall c \in C, t \in T \tag{3.6}$$

Constraint (3.5) ensures that the sum of the empty trains exiting the elevator vertices equals the sum of the empty trains entering the supply vertices. Constraint (3.6) implements the conservation of the flow of the empty trains across the vertices in the central line of the network.

$$x_{at} = b_{at}, \quad \forall s \in S, a \in IE(s), t \in T \tag{3.7}$$

$$\sum_{e \in E} \sum_{a \in OE(e)} b_{at} = \sum_{e \in E} \sum_{k \in I(e)} x_{kt}, \quad \forall t \in T \tag{3.8}$$

Constraint (3.7) ensures that the flow of empty trains at the supply vertices has a corresponding flow of loaded trains at these vertices. Constraint (3.8) provides that the sum of the empty trains exiting the elevator vertices equals the sum of the loaded trains entering the elevator vertices.

{*Time Constraints*}

$$\gamma_{vt} \le \omega, \quad \forall v \in V, t \in T \tag{3.9}$$

$$\delta_{vt} \le \omega, \quad \forall v \in V, t \in T \tag{3.10}$$

$$\zeta_{vt} \le \omega, \quad \forall v \in V, t \in T \tag{3.11}$$

$$\epsilon_{vt} \le \omega, \quad \forall v \in V, t \in T \tag{3.12}$$

Constraints (3.9-3.12) are included to set the upper bound of the time variables.

$$\gamma_{st} \ge \rho_a x_{at} + \gamma_{\sigma(a)t} + M[x_{at} - 1], \quad \forall s \in B, a \in I(s), t \in T \tag{3.13}$$

$$\gamma_{st} \le \rho_a x_{at} + \gamma_{\sigma(a)t} + M[1 - x_{at}], \quad \forall s \in B, a \in I(s), t \in T \tag{3.14}$$

Constraints (3.13) and (3.14) are conditional equalities; the condition is determined by the value of x_{at}. If x_{at} is equal to one the time variable, γ_{st}, The loaded train's time value at the preceding vertex should equal the sum of the travel duration between the two vertices.

$$\gamma_{vt} = \delta_{vt} - P, \quad \forall v \in V, t \in T \tag{3.15}$$

$$\gamma_{st} = \zeta_{st} + \lambda, \quad \forall s \in S, a \in IE(s), t \in T \tag{3.16}$$

Constraint (3.15) links each loaded train's entering and existing times. Constraint (3.16) links the time variable of the empty trains with the loaded trains at the supply vertices.

$$\zeta_{vt} \ge \kappa_a b_{at} + \zeta_{\beta(a)t} + M[b_{at} - 1], \quad \forall v \in V \setminus \{e\}, e \in E, a \in IE(v), t \in T \tag{3.17}$$

$$\zeta_{vt} \le \kappa_a b_{at} + \zeta_{\beta(a)t} + M[1 - b_{at}], \quad \forall v \in V \setminus \{e\}, e \in E, a \in IE(v), t \in T \tag{3.18}$$

Constraints (3.17) and (3.18) are conditional equalities, with the condition determined by the value of b_{at}. If b_{at} is equal to one the time variable, ζ_{vt}, should be equal to the sum of the time value of the empty train at the preceding vertex and the travel duration between the two said vertices.

$$\zeta_{vt} = \epsilon_{vt} - P, \quad \forall v \in V, t \in T \tag{3.19}$$

Constraint (3.19) links each loaded train's entering and existing times.

$$\zeta_{vt} \le \zeta_{vk} - \tau + M[1 - w_{vkt}], \quad \forall v \in V, t \in T, k \in T \setminus \{t\} \tag{3.20}$$

$$\zeta_{vt} \ge \zeta_{vk} + \tau + P - Mw_{vkt}, \quad \forall v \in V, t \in T, k \in T \setminus \{t\} \tag{3.21}$$

$$w_{vtk} + w_{vkt} = 1, \quad \forall v \in V, t \in T, k \in T \setminus \{t\} \tag{3.22}$$

Constraints (3.20-3.22) are conditional equalities; the conditions are dependent on the value of w_{vtk} determining the sequence of the empty trains passing each vertex in the network.

{*Constraint Propagation*}

The set T is defined as follows

$T = \{1, 2, \cdots, |T|\}$

$$\sum_{e \in E} \sum_{a \in I(e)} x_{at} \leq 1, \quad \forall t \in T \tag{3.23}$$

$$\sum_{e \in E} \sum_{a \in I(e)} x_{at} \leq \sum_{e \in E} \sum_{a \in I(e)} x_{at-1}, \quad \forall t \in T \setminus \{1\} \tag{3.24}$$

$$\sum_{e \in E} \sum_{a \in OE(e)} b_{at} \leq 1, \quad \forall t \in T \tag{3.25}$$

$$\sum_{e \in E} \sum_{a \in OE(e)} b_{at} \leq \sum_{e \in E} \sum_{a \in OE(e)} b_{at-1}, \quad \forall t \in T \setminus \{1\} \tag{3.26}$$

$$\sum_{s \in S} \sum_{a \in O(s)} x_{at} \leq 1, \quad \forall t \in T \tag{3.27}$$

$$\sum_{s \in S} \sum_{a \in O(s)} x_{at} \leq \sum_{s \in S} \sum_{a \in O(s)} x_{at-1}, \quad \forall t \in T \setminus \{1\} \tag{3.28}$$

$$\sum_{s \in S} \sum_{a \in IE(s)} b_{at} \leq 1, \quad \forall t \in T \tag{3.29}$$

$$\sum_{s \in S} \sum_{a \in IE(s)} b_{at} \leq \sum_{s \in S} \sum_{a \in IE(s)} b_{at-1}, \quad \forall t \in T \setminus \{1\} \tag{3.30}$$

Constraints (3.23) and (3.25) and constraints (3.27) and (3.29) ensure that a train can only enter one elevator and supply vertex for the loaded and empty trains, respectively. Constraints (3.24) and (3.28) and constraints (3.26) and (3.30) ensure the schedule utilises the trains in chronological order, therefore reducing the combinations of feasible solutions with different trains and with the same throughput.

$$\gamma_{st} + M[1 - x_{at}] \geq \gamma_{kt-1}, \quad \forall s \in S, a \in O(s), k \in S, c \in O(k), t \in T \setminus \{1\} \tag{3.31}$$

$$\zeta_{st} + M[1 - b_{at}] \geq \zeta_{kt-1}, \quad \forall s \in S, a \in IE(s), k \in S, c \in IE(k), t \in T \setminus \{1\} \tag{3.32}$$

$$\zeta_{et} + M[1 - b_{at}] \geq \zeta_{kt-1}, \quad \forall e \in E, a \in IE(e), k \in E, c \in IE(k), t \in T \setminus \{1\} \tag{3.33}$$

Constraint (3.31) ensures the loaded trains are scheduled chronologically at the supply vertices. The big M constraint ensures that loaded trains can flow from different supply vertices and are not constrained to one supply vertex. If trains flow to different supply vertices, the constraint is unbounded. Constraints (3.32) and (3.33) ensure that the empty trains are scheduled chronologically at the supply and elevator vertices, respectively. The big M constraints ensure that empty trains can flow to different supply or elevator vertices and are not constrained to one supply or elevator vertex.

{*Collision Prevention Constraints*}

$$\sum_{a \in A} x_{at} \geq d_t, \quad \forall t \in T \tag{3.34}$$

$$x_{at} \leq d_t, \quad \forall a \in A, t \in T \tag{3.35}$$

$$y_{vtk} + y_{vkt} = 1, \quad \forall v \in V, t \in T, k \in T \setminus \{t\} \tag{3.36}$$

$$\gamma_{vt} \leq \zeta_{vk} - Pd_t - \rho_a x_{at} - \kappa_c b_{ck} - \tau + M[1 - y_{vkt}] + M[1 - b_{ck}], \quad \forall v \in C, t \in T, k \in T \setminus \{t\}, a \in IE(v), c \in I(v) \tag{3.37}$$

$$\gamma_{v,t} \geq \epsilon_{vk} + Pd_t + \rho_a x_{at} + \kappa_c b_{ck} + \tau - My_{vkt} + M[b_{ck} - 1], \quad \forall v \in C, t \in T, k \in T \setminus \{t\}, a \in IE(v), c \in I(v) \tag{3.38}$$

Constraints (3.34) and (3.35) ensure that the required decision variables are linked to implementation in constraints (3.37) and (3.38). In constraint (3.36), the decision variables are constrained to one, ensuring that a loaded train can pass a vertex before or after the empty trains, but not both conditions can be true. Collisions on the network are prevented by constraints (3.37) and (3.38), which determine the time of the loaded and empty trains as well the sequence of flow of the load and empty trains across the network. The constraints ensure the loaded and empty trains are not simultaneously located at the same vertex or arc. The big M constraints are useful for conditional equality, which is crucial to scheduling with the flow conservation approach. The big M constraints extend the model bounds and reduce the model tightness. This causes the model to take longer to solve for optimality than a tight model.

4 VERIFICATION AND VALIDATION

In this section, the formulation of the mathematical model is verified. The results obtained from the model are validated to ensure the effectiveness of the model's schedule.

4.1 Verification

A critical evaluation of the constraints (3.2) to (3.38) is conducted using the Elytica platform and HiGHS 1.6.0 solver to verify the model. During verification, each constraint's impact on the model's results is evaluated to determine if the model is correctly formulated and implemented. A small-scale data set is used in the verification phase. The trains are scheduled for a 24-hour duration.

4.1.1 Objective function

The model verification starts with the objective function without including any constraints. A baseline is created to assess the impact of each constraint on the model.

Table 5: Verification result of the objective function

Description	Value
Objective value	210
Binary decision variables: $x_{at}, \forall a \in A, t \in T$	Either 1 or 0, have no influence and is randomly allocated
Binary decision variables: $b_{at}, \forall a \in A, t \in T$	Either 1 or 0, have no influence and is randomly allocated
Binary decision variables: $d_t, \forall t \in T$	Either 1 or 0, have no influence and is randomly allocated

Table 5 provides the decision variables, and the model yields 210 tonnes. This is expected as the model is limited to 6 trains with a capacity of 35 tonnes each; therefore, the solution found is the maximum throughput of the model. The flow parameter $z_{et}, \forall e \in E, \forall t \in T$ is maximised in the objective function, which is defined between 0 and 1. With six available trains in the model, the maximum possible objective value is 210 tonnes.

4.1.2 Flow constraints

Constraints (3.2) to (3.8) are included in the model to verify its flow. The flow constraints do not limit the model's results, as flows are created without any time duration or constraining factor except for the train's load capacity.

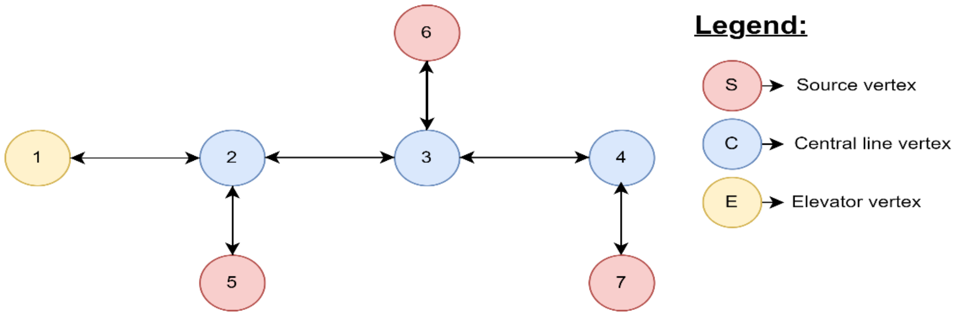

Figure 4: The small-scale dataset network topology

Figure 4 visualises the network utilised in the verification of the model.

Table 6: Verification results of the model with constraints (3.2 - 3.8)

Description	Value
Objective value	210
Binary decision variables: x_{1t} *and* $x_{6t}, \forall t \in T$	1
Binary decision variables: $x_{at}, \forall a \in A\backslash\{1,6\} t \in T$	0
Binary decision variables: b_{1t} *and* $b_{6t}, \forall t \in T$	1
Binary decision variables: $b_{at}, \forall a \in A\backslash\{1,6\} t \in T$	0
Binary decision variables: $d_t, \forall t \in T$	Either 1 or 0 has no influence and is randomly allocated

Table 6 provides the model results; all the trains are scheduled to and from mine site 1 at vertex 7. Results obtained from the model are as expected, with the maximum possible flow of 210 tonnes obtained. The distance and time duration of the trains do not influence the model results. Therefore, one of five possible routes, along with its relevant combination of decision variables.

Table 7: Combination of decision variables according to mine sites

Flow to the mine site	**Decision variables set to 1 $\forall t \in T$**
Mine site 1: Vertex 7	x_{1t} and x_{6t}
Mine site 2: Vertex 8	x_{1t} , x_{2t} and x_{7t}
Mine site 3: Vertex 9	x_{1t} , x_{2t} , x_{3t} and x_{8t}
Mine site 4: Vertex 10	x_{1t} , x_{2t} , x_{3t} , x_{4t} and x_{9t}
Mine site 5: Vertex 11	x_{1t} , x_{2t} , x_{3t} , x_{4t} , x_{5t} and x_{10t}

These combinations of the binary decision variable x_{at} will be selected randomly as the model determines a specific mine site to retrieve the ore. The decision variable b_{at} will be selected for the same arcs and trains as the model should send an empty train for a loaded train to be able to return.

4.1.3 Time constraints

The time constraints (3.9 - 3.22) are added to the model for the following stage of the verification phase. All the time constraints are included in the model as constraints interlink.

Table 8: Verification results of the model with constraints (3.2 - 3.22)

Description	Value
Objective value	210
Binary decision variables: x_{1t}, x_{2t} *and* x_{7t}, $\forall t \in T$	1
Binary decision variables: $x_{at}, \forall a \in A\backslash\{1,2,7\} t \in T$	0

Binary decision variables: b_{1t}, b_{2t} and b_{7t}, $\forall t \in T$	1
Binary decision variables: $b_{at}, \forall a \in A\backslash\{1,2,7\} t \in T$	0
Binary decision variables: $d_t, \forall t \in T$	Either 1 or 0 has no influence and is randomly allocated

The time constraints link the time variables to the flow variables, respectively. Intersecting flows of loaded and empty trains are not prevented by time constraints. Therefore, the results obtained in Table 8 are similar to what is expected. The decision variable d_t is selected randomly as the variable is still unbounded by the included constraints.

4.1.4 Constraint Propagation

Constraints (3.23 – 3.32) are included in the model to implement constraint propagation and improve its calculation time. The train passes are scheduled chronologically to reduce the combinations in the feasible solution region.

Table 9: Verification results of the model with constraints (3.2 - 3.32)

Description	**Value**
Objective value	210
Binary decision variables: x_{1t}, x_{2t} and x_{7t}, $\forall t \in T$	1
Binary decision variables: $x_{at}, \forall a \in A\backslash\{1,2,7\} t \in T$	0
Binary decision variables: b_{1t}, b_{2t} and b_{7t}, $\forall t \in T$	1
Binary decision variables: $b_{at}, \forall a \in A\backslash\{1,2,7\} t \in T$	0
Binary decision variables: $d_t, \forall t \in T$	Either 1 or 0 has no influence and is randomly allocated

The results obtained from the model in Table 9 correlate with the expected results. The maximum possible trains are scheduled chronologically, with empty and loaded trains still colliding.

4.1.5 Collision Prevention Constraints

The last seven constraints (3.32 – 3.38) are included in the model to ensure that collisions are prevented on the network. The binary decision variable d_t is linked with the flow of the loaded trains in constraints (3.34) and (3.35). The purpose of the binary decision variable d_t is to remove the influence of the passing duration constant, P, and relaxing the constraint. A test case was constructed to compare the model results. Table 10 below provides the times of each vertex visited per train.

Table 10: Results of calculations in Microsoft Excel

Description		Train 1	Train 2	Train 3	Train 4	Train 5	Train 6
Empty Train	Vertex 1	0	277	554	831	1108	1385
Empty Train	Vertex 2	5	282	559	836	1113	1390
Empty Train	Vertex 7	14	291	568	845	1122	1399

Loaded train	Vertex 7	214	491	768	1045	1322	1599
Loaded train	Vertex 2	226	503	780	1057	1334	1611
Loaded train	Vertex 1	276	553	830	1107	1384	1661

As seen in Table 10, only five trains can be scheduled to retrieve ore from a mine site and return to the elevator node within the 1440-minute time frame. These results will be compared with the model results, including all the constraints.

Table 11: Verification results of the model with constraints (3.32 - 3.38)

Description	Value
Objective value	175
Binary decision variables: x_{1t} and $x_{6\,t}$, $\forall t \in T/\{6\}$	1
Binary decision variables: $x_{at}, \forall a \in A\backslash\{1,6\} t \in T$	0
Binary decision variables: b_{1t} and b_{6t}, $\forall t \in T$	1
Binary decision variables: $b_{at}, \forall a \in A\backslash\{1,6\} t \in T$	0
Binary decision variables: $d_t, \forall t \in T/\{6\}$	1
Binary decision variables: d_6	0

Table 11 provides results from the model that are as expected, with constraints (3.2) to (3.38) included; the correctness of this model is verified in this section.

4.2 Validation

Section 4.1 verified a model for scheduling trains on a single-lane, bidirectional network with small-scale data; ensuring that the formulation of the model is correct. The validation of the model ensures that the result from the model is an improvement on the current state of the scenario. The model was run with a historical data set to compare the results with the historical data results.

The model schedules the trains within a 1440-minute, similar to the verification model. The historical data consists of averaged data used for comparison, which is the daily supply of the mine network. The data is averaged, meaning the daily supply was 66.12015 tonnes on operating days and zero tonnes produced on shutdown days. The model aims to schedule trains on the days the mine operates and have a supply greater than zero tonnes per day. The optimal objective value found by the mixed integer linear programming model was 175 tonnes per day. The model results are listed in Table 13.

Table 12: Model results with historical input data

Description	Value
Objective value	175
Binary decision variables: x_{6t} and x_{8t}, $\forall t \in T/\{7\}$	1
Binary decision variables: $x_{at}, \forall a \in A\backslash\{6,8\} t \in T$	0
Binary decision variables: b_{6t} and b_{8t}, $\forall t \in T$	1

Binary decision variables: $b_{at}, \forall a \in A\backslash\{6,8\} t \in T$	0
Binary decision variables: $d_t, \forall t \in T/\{7\}$	1
Binary decision variables: d_7	0

The results obtained for the mathematical model improve the daily supply by 66.120 tonnes. The model also supplies the schedule's time parameters listed below.

Table 13: Empty trains scheduled time [Minutes]

Vertex	Train 1	Train 2	Train 3	Train 4	Train 5
1	0	285.04	570.08	855.12	1140.16
2	1.51	286.55	571.59	856.63	1141.67
3	33.02	318.06	603.10	888.14	1173.18
4	34.53	319.57	604.61	889.65	1174.69
8	36.04	321.08	606.12	891.16	1176.20
5	36.17	321.21	606.25	891.29	1176.33
7	36.33	321.37	606.41	891.45	1176.49

The times for the loaded trains returning from the supply vertices.

Table 14: Loaded trains scheduled time [Minutes]

Vertex	Train 1	Train 2	Train 3	Train 4	Train 5
7	236.33	521.37	806.41	1091.45	1421.29
5	236.65	521.69	806.73	1091.77	1421.61
8	236.92	521.96	807.00	1092.04	1421.88
4	239.95	524.99	810.03	1095.07	1421.29
3	242.98	528.02	813.06	1098.10	1427.94
2	246.01	531.05	816.09	1101.13	1430.97
1	249.03	534.07	819.11	1104.15	1433.99

The model scheduled two trains to operate in a relay or handover manner. Here's an explanation of the order of operations for the trains: Train A will leave the elevator vertex as soon as the loaded train B enters it. Train B will start to unload, and as soon as train A enters the elevator vertex, train B will leave to retrieve the ore. In this manner, the trains can transport 175 tonnes of ore daily.

The model is an ideal solution, but it did not consider breakdowns or unscheduled maintenance on the network, varying supply capacities, or downtime at supply. These disruptions to the network will influence the schedule and will be deemed to happen sometime during the mine's operations. Therefore, the case study considered uses empirical data. To prove optimality and validate the model Figure 5 is included in the validation to visualise the bottleneck forming at Vertex 1, the elevator vertex.

If the trains can retrieve ore from different source vertices, no congestion will occur at the source vertices. However, the elevator vertex will experience congestion once more than one train is sent to retrieve ore from various source vertices. Two trains will enter the elevator vertex to unload, and additional trains will have to wait on the network until these trains finish unloading at elevator vertices and travel across the network to load ore at a source vertex. Thus, the same amount of ore will be transported across the network as the model schedules. The only difference between the schedules is that more trains will be utilised, and loaded trains will have to wait on the network.

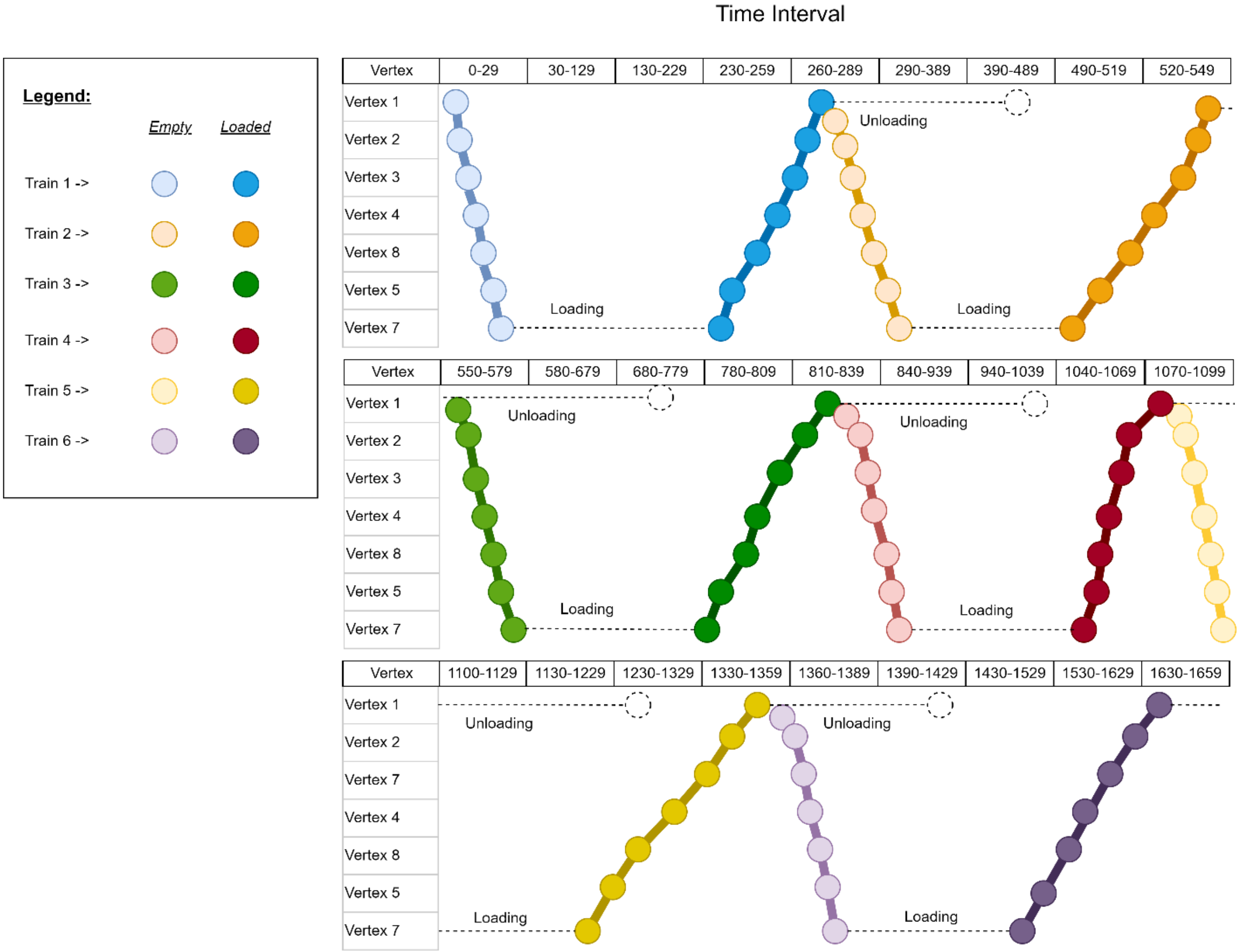

Figure 5: Visualisation of train position over time

5 RESULTS

The mathematical model was developed and implemented on the Elytica platform using a HiGHS solver algorithm, version 1.6.0. The model calculations were conducted on 60 vCPUs. The mathematical model is scoped to schedule trains on the transportation network for only 24 hours, reducing the computational power required and calculation time.

Table 16 lists the relevant information regarding the scalability testing of the model.

Table 15: Scalability results of the mathematical model

Iteration	Total number of arcs	Total number of nodes	Number of supply nodes	Number of available trains	Optimal solution found (in tonnes)	Average solving time (in seconds)	Standard deviation (in seconds)
1	7	8	1	7	175	0.390	0.066
2	9	10	2	7	175	5.130	0.905
3	11	12	3	7	175	60.619	19.531
4	13	14	4	7	175	582.889	169.380
5	15	16	5	7	175	3987.632	617.336

6	17	18	6	7	175	20221.405	3985.580
7	21	20	7	7	175	>43200	-

The HiGHS solver algorithm utilises random seeding of solutions within the feasible region as part of its exploration and solving process. The average and standard deviation of the solving time for ten iterations were calculated and included in Table 16.

As seen with the iterations of different network sizes and linear increase in supply nodes, the computational time of solving the model grew exponentially. The cutoff for the dataset size is a time limit of 43 200 seconds or 12 hours, as it is the general shift duration. The model should be flexible enough to adjust after the first shift and recalculate the schedule for the following day. The ideal is a shorter computational duration so that the model is more responsive towards disruptions on the network. The model reached the time limit with the 7th iteration; therefore no standard deviation is included for the 7th iteration.

The optimal solution obtained by the mathematical model was included in the results. The optimal solution was consistent between the iterations as the given case study used in the scaling has one path towards the single elevator on the network and the simultaneous movement of trains clash.

6 CONCLUSION

The demand for operations research is exceptionally high in business, as mathematical optimisation is highly diverse and valuable. The mining sector has various opportunities for optimization. This study found that scheduling trains more efficiently can significantly improve the transportation network.

The mine network consists of various transportation systems for different mine sections. This study focuses on the rail-bounded transportation section in the transportation network of underground mines. Trackless vehicles load the train wagons at the supply vertices from which the trains transport the ore to elevator shafts. The elevator raises the ore to the surface for further processing or stockpiling.

Given the constraints and requirements, this study proposes a mixed integer linear programming model that maximises the transportation throughput of the rail-bounded vehicle network. The model quantifies the time duration for traversing the network in a loaded and empty train and uses the values to schedule the trains. The mathematical formulation of the mixed integer linear programming model is verified to ensure that it is correctly formulated and coded in Elytica. Historical data is used to validate the results obtained from the model. The model was solved using an open-source solver, but commercial solvers will perform better.

7 FUTURE RECOMMENDATIONS

Future research should further investigate the model's response by comparing the simulated results to those obtained for the practical case study.

This study schedules trains without uncertainty in any parameter, making the schedule ideal but impractical. For future research, uncertainty within the schedule will be recommended to determine a more practical and applicable day-to-day schedule. The unscheduled maintenance and breakdowns will increase the complexity of the model; a stochastic model will incorporate the probability of unscheduled breakdowns on the network and downtime in the supply.

As seen in Table 16, the model’s scalability is poor and impractical for large data sets. An exact or heuristic model with greater bounds is recommended for solving in a shorter time, which will improve the model’s practicality for real-time solutions.

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