

CONSTRAINING A GENERAL-RELATIVISTIC FRAME-DRAGGING MODEL FOR PULSED RADIATION FROM A POPULATION OF MILLISECOND PULSARS IN 47 TUCANAE USING *GLAST* LAT

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ABSTRACT

Although only 22 millisecond pulsars (MSPs) are currently known to exist in the globular cluster (GC) 47 Tucanae, this cluster may harbor 30–60 MSPs, or even up to ~ 200 . In this Letter, we model the pulsed curvature radiation (CR) gamma-ray flux expected from a population of MSPs in 47 Tucanae. These MSPs produce gamma rays in their magnetospheres via accelerated electron primaries which are moving along curved magnetic field lines. A GC such as 47 Tucanae containing a large number of MSPs provides the opportunity to study a randomized set of pulsar geometries. Geometry-averaged spectra make the testing of the underlying pulsar model more reliable, since in this case the relative flux uncertainty is reduced by 1 order of magnitude relative to the variation expected for individual pulsars (if the number of visible pulsars $N = 100$). Our predicted spectra violate the EGRET upper limit at 1 GeV, constraining the product of the number of visible pulsars N and the average integral flux above 1 GeV per pulsar. *GLAST* LAT should place even more stringent constraints on this product, and may also limit the maximum average accelerating potential by probing the CR spectral tail. For $N = 22$ – 200 , a *GLAST* LAT nondetection will lead to the constraints that the average integral flux per pulsar should be lower by factors of 0.03–0.003 than current model predictions.

Subject headings: gamma rays: theory — globular clusters: individual (47 Tucanae) — pulsars: general — radiation mechanisms: nonthermal

1. INTRODUCTION

Millisecond pulsars (MSPs) have been detected in several globular clusters (GCs), including the nearby 47 Tucanae (Camilo et al. 2000; Freire et al. 2003). Although only 22 MSPs are currently known to exist in 47 Tucanae (for a recent review, see Lorimer et al. 2003), it is expected that this cluster may harbor 30–60 MSPs (Camilo & Rasio 2005; Heinke et al. 2005), or even up to ~ 200 (Ivanova et al. 2005).

In this Letter, we model the pulsed curvature radiation (CR) gamma-ray flux expected from a population of MSPs in 47 Tucanae (§ 2), and compare it with the *GLAST* LAT sensitivity and EGRET upper limits. Our approach differs from similar recent studies (Bednarek & Sitarek 2007; Harding et al. 2005 [hereafter HUM05]; Cheng & Taam 2003) in a number of respects. First, we use a fully 3D general-relativistic (GR) polar cap (PC) pulsar model (see, e.g., Venter & de Jager 2005), sampling nonthermal gamma-ray radiation from a range of magnetic field lines above the PC. Next, our study involves a population of MSPs, and we sample randomly from this ensemble. We are therefore able to calculate absolute cumulative fluxes expected from the MSPs in 47 Tucanae, without needing to use approximate spectra or scale up single MSP predictions or single particle spectra. We are also specifically interested in the range of pulsed spectra expected due to the uncertain geometries and spread in values of the period P and its derivative \dot{P} of these pulsars.

X-rays from the MSPs in 47 Tucanae are believed to be of thermal origin, and are likely produced by a return current heating the pulsars' PCs (Bogdanov et al. 2006; Harding & Muslimov 2002; Cheng & Taam 2003). These X-rays may be

Compton upscattered to high-energy gamma rays. However, this inverse Compton scattering (ICS) component is believed to be dominated by the CR component (see, e.g., Fig. 3 of Bulik et al. 2000), and we therefore omitted it from our calculations below.

We lastly make some conclusions regarding the possibility of constraining a model of gamma-ray radiation from GCs (§ 4), since an ensemble of MSPs in a GC cluster provides a unique opportunity for specifically constraining the GR frame-dragging pulsar model (e.g., Harding & Muslimov 1998) in a geometry-independent way (§ 3).

2. PULSED GAMMA-RAY FLUX

The measured values of \dot{P} for GC pulsars are affected by their acceleration in the gravitational potential of the GC, and therefore need to be corrected to obtain intrinsic period derivatives \dot{P}_{int} . This was done for 47 Tucanae by Bogdanov et al. (2006) who derived spin-down luminosities \dot{E}_{rot} assuming a King model. From their Table 4, we selected 13 MSPs and calculated their corresponding \dot{P}_{int} using their periods P as given by Freire et al. (2003) and the expression $\dot{E}_{\text{rot}} = 4\pi^2 I_{\text{NS}} \dot{P}_{\text{int}} / P^3$, with I_{NS} the moment of inertia.

The details of the implementation of our isolated MSP model, using the GR-framework of Harding, Muslimov, and Tsygan (e.g., Harding & Muslimov 1998), may be found in Venter & de Jager (2005), where we discuss the essential link between pulsar visibility and the assumed geometry, i.e., magnetic inclination angle χ and observer angle ζ (both measured with respect to the spin axis Ω).

HUM05 found that most MSPs are inefficiently screened by CR and ICS pairs. Screening caused by electron-positron pairs will lower the accelerating potential, leading to a lower pulsed CR spectral cutoff. Higher numbers of energetic electrons may however boost the unpulsed gamma-ray spectrum. The vast majority of MSPs in our population lie below the critical spin-

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down luminosity above which screening is expected to occur (Harding et al. 2002),

$$\dot{E}_{\text{rot,break}} \approx 1.4 \times 10^{34} \left(\frac{P}{B_{12}^2} \right)^{1/7} \text{ ergs s}^{-1}. \quad (1)$$

Here, $B_{12} = B/10^{12}$ G and P is the pulsar period in seconds. (Note the positive sign of the power of P .) This condition may be rewritten so that screening occurs when

$$\log_{10} \dot{P} > 2.625 \log_{10} P - 12.934, \quad (2)$$

assuming $I_{\text{NS}} = 0.4MR^2$, stellar radius $R = 10^6$ cm, and pulsar mass $M = 1.4 M_{\odot}$. We used an unscreened electrical potential for the bulk of the population, and used the approximation of a screened electric field given by Dyks & Rudak (2000) for MSPs with spin-down powers greater than this critical spin-down power (which was only the case for 47 Tuc U).

For each pulsar i in our population, we calculated phase-averaged CR spectra via (see also eq. [12] of Venter & de Jager 2005),

$$\frac{dN_{\gamma}^i}{dE}(E_{\gamma}, \chi, \zeta) = \frac{1}{2\pi d^2 \sin \zeta d\zeta dE_{\gamma}} \times \int \int_{\zeta}^{\zeta+d\zeta} \int_0^{2\pi} \frac{I(E_{\gamma}, E_{\gamma} + dE_{\gamma})}{E_{\gamma}^c} \left[\frac{dL_{\gamma}(\phi_L, \chi, \zeta, E_{\gamma})}{d\phi_L d\zeta dE_{\gamma}} \right] d\phi_L d\zeta dE_{\gamma}, \quad (3)$$

where ϕ_L is the phase angle and $I(E_{\gamma}, E_{\gamma} + dE_{\gamma})$ picks out the gamma-ray energies in the range $(E_{\gamma}, E_{\gamma} + dE_{\gamma})$. Primary electrons leave a stellar surface patch dS relativistically ($\beta_0 = v_0/c \sim 1$) at a rate of $d\dot{N}_e = \rho_e dS \beta_0 c/e$, where ρ_e is the charge density. These primaries radiate CR at a position (r, θ, ϕ) above the PC with a characteristic energy of $E_{\gamma}^c = 1.5\lambda_c \gamma^3 m_e c^2 \rho_c^{-1}$, where $\lambda_c = \hbar/(m_e c)$ is the Compton wavelength and $\rho_c(r, \theta, \phi)$ is the curvature radius of the GR-corrected dipolar magnetic field. The CR is associated with an incremental gamma-ray luminosity of $dL_{\gamma} = d\dot{N}_e \dot{E}_{\text{CR}} dt$, which is radiated in a time dt . The CR loss rate is given by $\dot{E}_{\text{CR}} \approx 2e^2 c \gamma^4 / (3\rho_c^2)$, where $\gamma(r, \theta, \phi)$ is the Lorentz factor of the electron primaries. We calculated CR photon spectra $dN_{\gamma}^i/dE(E_{\gamma}, \chi, \zeta)$ for $\chi = \zeta = 10^\circ, 20^\circ, \dots, 80^\circ$ and $i = 1, 2, \dots, 13$, assuming $R = 10^6$ cm, $M = 1.4 M_{\odot}$, and $I_{\text{NS}} = 0.4MR^2$. All spectra ($13 \times 8 \times 8 = 832$) were scaled to a distance of $d = 5$ kpc (Gratton et al. 2003).

We next randomly chose $N = 100$ visible MSPs (with random χ and ζ), and summed their pulsed spectra to obtain a million cumulative spectra from the Monte Carlo process:

$$\left(\frac{dN_{\gamma}}{dE} \right)_{\text{cum}}^j = \sum_{k=1}^{N=100} \frac{dN_{\gamma}^k}{dE}(E_{\gamma}, \chi, \zeta), \quad (4)$$

where for each index $j = 1, 2, \dots, N_i = 10^6$, a total of $N = 100$ spectra (randomly sampled from the above-mentioned 832 spectra) were summed. Therefore, j corresponds to a specific choice of χ, ζ , and k -values when oversampling from 13 to 100 pulsar spectra. This procedure was repeated for 1 MSP instead of 100, i.e., setting $N = 1$. Upon comparison of these results, one would expect that the mean values of the single MSP differential spectra would scale with N , and the standard deviations with \sqrt{N} . In Figure 1a the mean values of

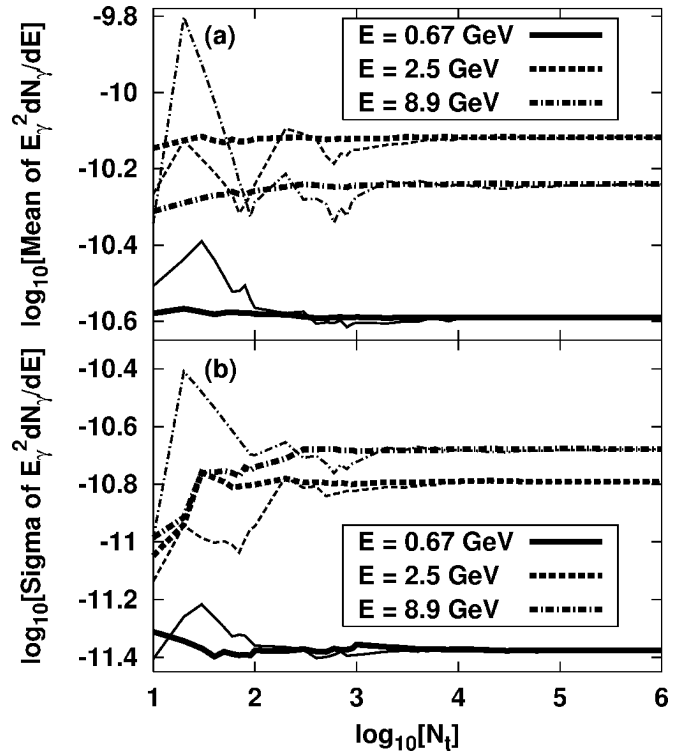


FIG. 1.—(a) Mean and (b) standard deviation σ of $E_{\gamma}^2 dN_{\gamma}/dE$ (in units of $\text{ergs s}^{-1} \text{cm}^{-2}$) vs. N_i , with N_i the number of values of $E_{\gamma}^2 dN_{\gamma}/dE$ used to calculate the mean and σ -values. Line types correspond to different energies as shown in the legend. Thick lines are for values of $E_{\gamma}^2 dN_{\gamma}/dE$ summed over $N = 100$ randomly chosen visible pulsars, while thin lines are for values of $E_{\gamma}^2 dN_{\gamma}/dE$ for single ($N = 1$) randomly chosen pulsars. The latter values were scaled by a factor 100 in (a), and a factor $\sqrt{100} = 10$ in (b). Results converge beyond $N_i \sim 10^4$.

$E_{\gamma}^2 dN_{\gamma}/dE$ at three different photon energies E_{γ} are shown as a function of the number N_i of values used to calculate the mean (with a maximum of $N_i = 10^6$). The thick lines indicate results when $N = 100$, while the thin lines indicate results for $N = 1$, scaled by a factor 100. Similarly, Figure 1b indicates the standard deviation of $E_{\gamma}^2 dN_{\gamma}/dE$ at three different photon energies versus N_i . The thick lines again indicate results when $N = 100$, while the thin lines indicate results for $N = 1$ scaled by a factor 10. It is clear that the mean and σ -values converge beyond $N_i \sim 10^4$, for both $N = 1$ and $N = 100$. The $N = 1$ results are more erratic at small N_i values. The relative error on the mean values are therefore a factor 10 lower when using a population of 100 MSPs, as opposed to the much larger error obtained for single pulsars. Figure 2 depicts average cumulative pulsed differential spectra $\langle E_{\gamma}^2 dN_{\gamma}/dE \rangle_{\text{cum}}$ for $N = 100$ and $N_i = 10^6$, as well as 2σ bands. Also shown are the *GLAST* LAT sensitivity curve (HUM05) and EGRET upper limits (Fierro et al. 1995), as well as other predictions of pulsed gamma-ray radiation from 47 Tucanae (HUM05).

3. DISCUSSION

The gamma-ray visibility of an isolated MSP depends extremely sensitively on the assumed pulsar geometry. Inclination and observer angles χ and ζ are usually estimated from radio polarization measurements (e.g., Manchester & Johnston 1995), involving the rotating vector model. In our MSP model, the electric potential boundary condition of $\Phi(\xi = 1) = 0$ is used (Muslimov & Harding 1997) as the last open magnetic field lines ($\xi = 1$) are treated as equipotentials. In addition, the

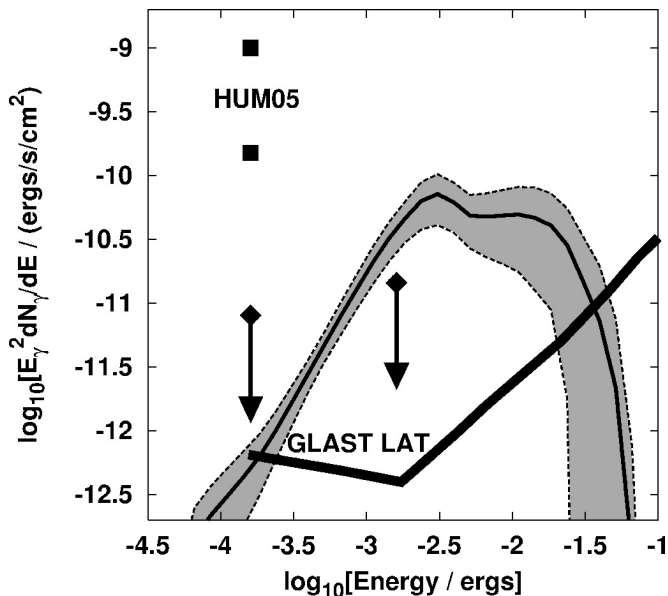


Fig. 2.—Average differential pulsed gamma-ray spectrum (*solid line*) with 2σ error bands (*shaded band between dashed lines*) for $N = 100$ and $N_i = 10^6$, with smoothed high-energy tails. Furthermore we indicate predictions of HUM05 (lower square for 15 MSPs, upper square scaled to 100 MSPs). The predictions from HUM05 involve estimates of single-particle spectra scaled using the Goldreich-Julian PC current (Goldreich & Julian 1969) and arbitrary beaming angles. Our spectra include variations across the PC (sampling different magnetic field lines) as well as variations due to various inclination angles and observer geometries (with beaming angles calculated implicitly). The *GLAST LAT* sensitivity (HUM05) as well as EGRET upper limits (*diamonds*; Fierro et al. 1995) are also shown (the EGRET upper limits and HUM05 results were converted to differential values assuming an E^{-2} spectrum).

space-charge-limited nature of this model requires that the accelerating electric field parallel to the magnetic field (E_{\parallel}) should be zero at the stellar surface ($r = R$). This means that on-beam radiation (when $\chi \sim \zeta$) will have a spectral cutoff which roughly scales with the maximum potential Φ_{\max} . Off-beam radiation will however rapidly decrease as the impact angle $\beta = \zeta - \chi$ increases, because this radiation is produced along field lines where the electric potential Φ , and hence E_{\parallel} , has dropped significantly.

An ensemble of MSPs provides the unique opportunity to sidestep the issue of pulsar geometry (which is crucial when modeling a single MSP) by averaging over numerous spectra and geometries, which results in a “geometry averaged” spectrum with relatively small uncertainty. We also exploit this scenario in the sense that we use results from the corotating frames of the MSPs, since we expect that the effect of Lorentz transformations to the observer frame will average out over the ensemble. *We find that the relative spread of ensemble values for the bolometric gamma-ray and electron luminosities as well as the spin-down power and average gamma-ray spectra are typically one order smaller than for the same quantities averaged as single MSP quantities (i.e., they scale with $N^{-1/2}$).* This illustrates the importance of testing pulsar models by observing GCs. (This conclusion follows when using a constant equation of state [EOS]. Variations in the EOS have a much smaller effect than that of differing pulsar geometries and are therefore not included in this discussion; see § 4.)

Due to the lower relative ensemble errors, one should therefore expect to be able to constrain average pulsar parameters using average spectra and their errors shown in Figure 2. This would not be the case if ensemble values did not converge

(Fig. 1) and had relatively small errors. Furthermore, the fact that the ratios of average cumulative luminosities and spin-down powers to their single MSP counterparts are very close to $N = 100$ gives us confidence in our sampling algorithm.

In our model, the predicted average single pulsar efficiencies for electron and gamma-ray production are typically $\sim 2\%$ and $\sim 7\%$, depending on the population of MSPs used. For our 47 Tucanae MSP population (with $\langle \dot{E}_{\text{rot}} \rangle = 2.2 \times 10^{34}$ ergs s^{-1}), we found 0.74% and 6.1%, respectively. This should be compared to the estimate of Bednarek & Sitarek (2007) of 1% for the first value, and to that of Harding et al. (2002) of $\sim 10\%$ for the latter quantity. Venter & de Jager (2005) quoted values of 1%–2.5% and 2%–9% for these quantities for the case of the pulsar PSR J0437–4715.

In Figure 2 we converted integral EGRET upper limits (Fierro et al. 1995) and model predictions of HUM05 to differential values assuming a E^{-2} spectrum. Our predicted pulsed gamma-ray flux at 100 MeV is below the differential EGRET upper limit, much lower than that of HUM05. Their prediction of the integral flux above 100 MeV exceeds the EGRET upper limit by a factor $f \sim 19$ for 15 MSPs, and $f \sim 125$ for 100 MSPs. (This seemingly overprediction of integral flux was also found in the case of PSR J0437–4715 [Venter & de Jager 2005]. HUM05’s prediction is a factor $f \sim 35$ above the EGRET upper limit for this MSP.) However, our summed pulsed spectra violate the EGRET upper limit at 1 GeV (Fierro et al. 1995), providing constraints on the low-energy tail of the average pulsed spectrum. *GLAST LAT* observations will however constrain the predicted spectrum at all energies of relevance, which should provide a meaningful constraint on the product of visible pulsars N , and the average integral flux above 1 GeV per pulsar.

4. CONCLUSIONS

In this Letter, we modeled the pulsed gamma-ray flux expected from a population of MSPs in 47 Tucanae. We chose a number of $N = 100$ visible members, but the spectra can easily be (linearly) scaled to any other reasonable number. We were especially interested in obtaining errors on the pulsed gamma-ray spectrum which would reflect the uncertainty in the geometries, as well as the spread in P and \dot{P} , of the individual pulsars. In this way, we wanted to see whether it would be possible to constrain the underlying pulsar model independently of the assumed pulsar geometry. It remains to be shown whether the spread of the 13 selected pulsars represent the true inherent spread of MSPs in 47 Tucanae.

For the sake of completeness, we furthermore investigated the effect of varying the EOS (M , R , and I). By choosing $I_{\text{NS}} = 0.4MR^2$ and letting M and R vary between $M/M_{\odot} = 1.4, 1.5, 1.6$ and $R_6 = R/10^6 \text{ cm} = 1.0, 1.2, 1.4, 1.6$, we found that the maximum integral flux did not vary by more than $\sim 4\%$ (although it increases with both M and R ; the spectral cutoff energy furthermore did not vary by more than $\sim 13\%$). Since this variation will not significantly influence our constraints derived below, we used fixed values of $M = 1.4 M_{\odot}$ and $R_6 = 1$ throughout.

Our average pulsed spectrum, including errors, exceed the EGRET upper limit at 1 GeV (for the average integral flux spectrum, by a factor of ~ 3.9). Assuming that this model predicts the gamma-ray flux correctly, this EGRET upper limit constrains the number of GC pulsars to $100/3.9 \approx 25$. Gen-

erally, the constraints derived may be summarized as follows:

$$\frac{N\langle F_{\text{obs}} \rangle}{\langle F_{\text{model}} \rangle} \leq \frac{25}{q}, \quad (5)$$

with $\langle F_{\text{obs}} \rangle$ and $\langle F_{\text{model}} \rangle$ the average observed and predicted integral flux above 1 GeV per pulsar, and q a factor indicating telescope sensitivity. For EGRET, $q = 1$, while $q = 40$ for *GLAST* LAT. For example, setting $N = 22, 100$, and 200 , we find that $\langle F_{\text{obs}} \rangle / \langle F_{\text{model}} \rangle \leq 1.1, 0.25, 0.13$ for EGRET. This indicates that the EGRET limit is obeyed for $N = 22$, but that larger values of N imply a reduction in the model-predicted

integral flux (by, e.g., factors of 4 and 8 for $N = 100$ and 200). In the case of a *GLAST* LAT nondetection, we find $\langle F_{\text{obs}} \rangle / \langle F_{\text{model}} \rangle \leq 0.03, 0.006, 0.003$ for $N = 22, 100$, and 200 . *GLAST* LAT might lastly even constrain the maximum average electric potential per pulsar $\langle \Phi_{\text{max}} \rangle$ via the CR cutoff energy. It should also in principle be possible to look for the radio periods corresponding to the 22 known MSPs in 47 Tucanae in the gamma-ray data (or detect new gamma-ray periods if beaming properties are different), should *GLAST* LAT detect the cumulative pulsed CR spectrum.

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