

Chapter 4

Distributed thermal modelling

This chapter starts off by discussing some of the background used to derive the distributed analytical thermal model. The derivation of a 2-D analytical distributed model for the PM is presented and verified using FEM. A 1-D analytical distributed model for the stator winding area is also derived and discussed.

4.1 Background

This section introduces some of the concepts needed to derive the 2-D analytic distributed thermal model for the PM found in the TWINS rotor.

4.1.1 Bessel functions

The origin of Bessel functions can be traced back to three independent problems. The first, investigated by Daniel Bernoulli in 1732, is the movement of a heavy chain hanging vertically. With the upper end fixed, the movement due to a disturbance at the lower end is explored. The second occurrence of the Bessel function is found in the heat flow in a solid cylinder, done by Joseph Fourier in 1822. This is also the application of Bessel functions in this thesis. The final problem is elliptical motion found in astronomy, solved by Friedrich Bessel in 1824 [128].

The second order differential equation, (4.1), is called the Bessel differential equation of order n and it is commonly found in cylindrical problems.

$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + (x^2 - n^2)y = 0 \quad (4.1)$$

The general solution of (4.1) is the sum of two linearly independent Bessel functions, each multiplied with an integration constant, or:

$$y(x) = C_1 J_n(x) + C_2 Y_n(x) \quad (4.2)$$

where J_n is the Bessel function of the first kind, Y_n is the Bessel function of the second kind and C_1, C_2 are constants. The modified Bessel function has a sign difference:

$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - (x^2 + n^2)y = 0. \quad (4.3)$$

The general solution of (4.3) is

$$y(x) = C_1 I_n(x) + C_2 K_n(x) \quad (4.4)$$

where I_n and K_n are the modified Bessel functions of the first and second kind, respectively. Figure 4.1 shows a plot of the Bessel functions. Note J and Y has multiple roots; K and I nears infinity at $x \rightarrow 0$ and $x \rightarrow \infty$, respectively. In developing the solution of the diffusion equation in the rz - plane, the Bessel equation will result from applying the separation of variables method, which is discussed next.

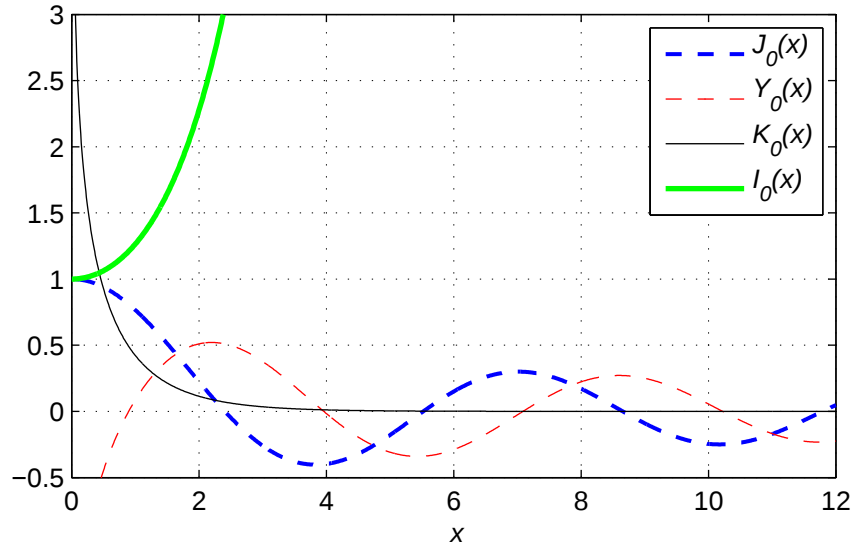


Figure 4.1: Bessel and modified Bessel functions of the first and second kind

4.1.2 Separation of variables

As stated previously, the diffusion equation describes the temperature in a solid. If it is assumed that steady state has been reached and the material is isotropic with no internal heat generation, the diffusion equation for the rz -plane reduces to

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = 0. \quad (4.5)$$

Separation of variables (SOV) is a technique that can be used to rewrite a partial differential equation (PDE) as multiple ordinary differential equations (ODEs) [129]. It is assumed in SOV that the temperature T can be written as the product of functions with each function only depending on one variable. Thus for (4.5):

$$T(r, z) = R(r)Z(z). \quad (4.6)$$

The diffusion equation then becomes (where $R = R(r)$ and $Z = Z(z)$):

$$\begin{aligned}\frac{\partial^2 R}{\partial r^2} Z + \frac{1}{r} \frac{\partial R}{\partial r} Z + \frac{\partial^2 Z}{\partial z^2} R &= 0 \\ \frac{\partial^2 R}{\partial r^2} Z + \frac{1}{r} \frac{\partial R}{\partial r} Z &= -\frac{\partial^2 Z}{\partial z^2} R \\ \frac{\partial^2 R}{\partial r^2} \frac{1}{R} + \frac{1}{r} \frac{\partial R}{\partial r} \frac{1}{R} &= -\frac{\partial^2 Z}{\partial z^2} \frac{1}{Z}.\end{aligned}\tag{4.7}$$

Assuming a separation variable λ^2 exists, then the resulting two ODEs are:

$$\begin{aligned}\frac{\partial^2 R}{\partial r^2} \frac{1}{R} + \frac{1}{r} \frac{\partial R}{\partial r} \frac{1}{R} &= -\lambda^2 \\ \frac{d^2 R}{dr^2} + \frac{1}{r} \frac{dR}{dr} + R\lambda^2 &= 0\end{aligned}\tag{4.8}$$

and

$$\begin{aligned}-\frac{\partial^2 Z}{\partial z^2} \frac{1}{Z} &= -\lambda^2 \\ \frac{d^2 Z}{dz^2} - Z\lambda^2 &= 0.\end{aligned}\tag{4.9}$$

It is known that (4.8) is the Bessel differential equation of order zero, which has the solution:

$$R(r) = C_1 J_0(\lambda r) + C_2 Y_0(\lambda r)\tag{4.10}$$

where J is the Bessel function of the first kind and Y is the Bessel function of the second kind. It is also known that (4.9) has the solution:

$$Z(z) = C_3 e^{\lambda z} + C_4 e^{-\lambda z}.\tag{4.11}$$

The general solution of (4.5) is given in (4.12). The constants C_{1-4} and λ must be determined from the boundary conditions.

$$T(r, z) = [C_1 J_0(\lambda r) + C_2 Y_0(\lambda r)] [C_3 e^{\lambda z} + C_4 e^{-\lambda z}]\tag{4.12}$$

This is only one of the possible solutions of (4.5). Another is given in (4.13), where I is the modified Bessel function of the first kind and K is the modified Bessel function of the second kind. Carslaw and Jaeger states that (4.12) is used when the temperature is prescribed on a plane boundary (e.g. $z = 0$) and (4.13) when the temperature is prescribed on a circular boundary (e.g. $r = 5$) [5]. The different solutions are obtained by choosing a different sign for the separation variable.

$$T(r, z) = [C_1 I_0(\lambda r) + C_2 K_0(\lambda r)] [C_3 \cos \lambda z + C_4 \sin \lambda z]\tag{4.13}$$

4.1.3 Series expansions for thermal models

Series expansions is an important part of heat modelling. The widely used Fourier series were developed by Joseph Fourier to decompose a periodic boundary condition into a sum of sine and cosine functions. According to Fourier, any function $f(x)$ can be written in the form:

$$f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx) \quad (4.14)$$

where the Fourier coefficients (a_0 , a_n and b_n) can be calculated using:

$$\begin{aligned} a_0 &= \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) dx \\ a_n &= \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \\ b_n &= \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \end{aligned} \quad (4.15)$$

with $n = 1, 2, 3, \dots$. The periodic nature of the Bessel function of the first kind results in Fourier-Bessel series, where a function $f(x)$ can be written as a sum of Bessel functions:

$$\begin{aligned} f(x) &= \sum_{m=1}^{\infty} a_m J_n(k_{mn}x) \\ &= a_1 J_n(k_{1n}x) + a_2 J_n(k_{2n}x) + a_3 J_n(k_{3n}x) + \dots \end{aligned} \quad (4.16)$$

where n is the order of the Bessel function of the first kind and k_{mn} are constants. If the integral is taken over $0 \leq x \leq R$, the coefficients a_m can be calculated using:

$$a_m = \frac{2}{R^2 J_{n+1}^2(k_{mn}R)} \int_0^R x f(x) J_n(k_{mn}x) dx \quad (4.17)$$

where $m = 1, 2, 3, \dots$, due to the orthogonality of Bessel functions[130]. Another series that will be needed in this thesis when convection heat transfer occurs at a boundary, is:

$$f(x) = \sum_{n=1}^{\infty} A_n \left(\cos k_n x + \frac{h}{k_n} \sin k_n x \right) \quad (4.18)$$

where h is the convection constant and k_n are the positive roots [48]. According to Carslaw *et al.* the constants A_n are given by [5]:

$$A_n = \frac{2k_n^2}{(k_n^2 + h^2)l + 2h} \int_0^l f(x) \left[\cos k_n x + \frac{h}{k_n} \sin k_n x \right] dx \quad (4.19)$$

These series are needed when solving the constants of (4.12) and (4.13) when complex boundary conditions due to convection arise. Boundary conditions are discussed next.

4.1.4 Boundary conditions

Boundary conditions give a mathematical description of the temperature or heat flow on a boundary. The most simple thermal boundary condition is when a constant temperature (T_s) is prescribed, as shown in (4.20). This boundary is the least likely to occur in practical scenarios but is the most simple to use from a mathematical point of view.

$$T(r, z)|_{r=a} = T_s \quad (4.20)$$

The second boundary condition is when the heat flux (q'') is prescribed on the boundary as shown in (4.21), where k is the thermal conductivity of the material. When $q''_s = 0$, the boundary condition models an ideal thermal insulation or a symmetry plane.

$$-k \frac{\partial T}{\partial z}|_{z=l} = q''_s \quad (4.21)$$

The convection boundary condition occurs most frequently and is the most difficult to solve using exact analytical methods. The heat flux is dependent on the temperature difference between the surface ($T(b, z)$) and that of the surrounding fluid (T_∞):

$$k \frac{\partial T}{\partial r}|_{r=b} = h [T(b, z) - T_\infty] \quad (4.22)$$

where h is the convection coefficient. The boundary conditions are used to determine the constants in (4.12) or (4.13). This concludes the introduction of the methods that will be used to derive a 2-D analytical distributed model for the PM.

4.2 Permanent magnet model

An accurate model of the PM is one of the main objectives of this thesis and the two dimensional (2-D) model will be discussed in this section. This 2-D model will give a more detailed thermal profile than the LP model which models only radial heat flow in the PM. The assumptions used during the derivation are discussed before the detail derivation is presented.

The shielding cylinder's purpose is to provide a preferred path for the eddy currents induced on the rotor. These eddy currents are due to the switching of the machine's drive and is at frequencies higher than the fundamental frequency. The penetration depth of the eddy current causing magnetic fields is thus very small, resulting in most of the losses being close to the surface of the rotor. It is assumed that the eddy current loss on the rotor is contained in the shielding cylinder and can thus be modelled as a surface heat source for the PM.

Section 3.1.1 discussed how the LP and distributed model will be combined. Figure 4.2 shows the boundary conditions of the PM section. The inner and outer radii of the cylinder are a and b , respectively. The cylinder has a length of l .

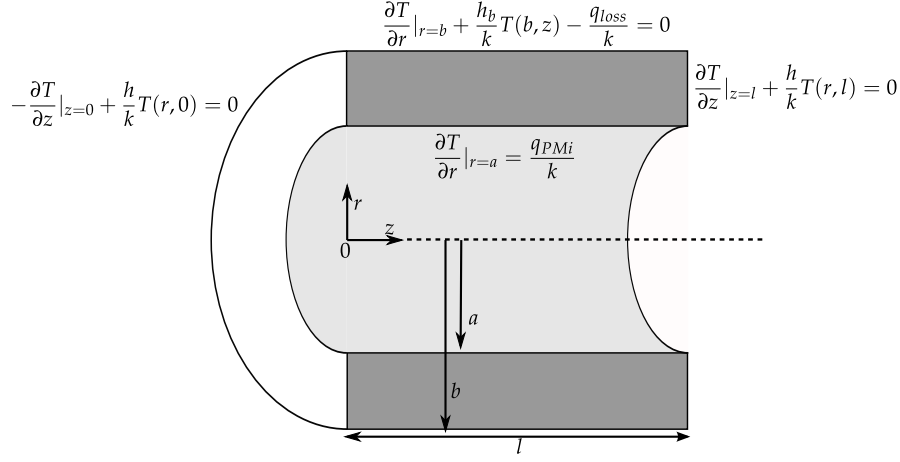


Figure 4.2: Boundary conditions of the PM

4.2.1 Boundary conditions of the PM

The boundary conditions is used to determine the constants of (4.13), which is the general solution for this senario. In order to simplify the boundary conditions, a linear transformation is used. If $t_{pm}(r, z)$ is the temperature distribution in the PM and the surrounding air temperature is T_∞ . Applying the linear transformation $T(r, z) = t_{pm}(r, z) - T_\infty$ the boundary where $z = 0$ becomes:

$$\begin{aligned} -k \frac{\partial t}{\partial z} \Big|_{z=0} + h [t(r, 0) - T_\infty] &= 0 \\ -\frac{\partial (T + T_\infty)}{\partial z} \Big|_{z=0} + \frac{h}{k} [T(r, 0) + T_\infty - T_\infty] &= 0 \\ -\frac{\partial T}{\partial z} \Big|_{z=0} + \frac{h}{k} T(r, 0) &= 0 \end{aligned} \quad (4.23)$$

where h is the convection constant at the side of the PM. The linear transformation is also applied to the other three boundary conditions as shown in (4.24) - (4.26).

$$\frac{\partial T}{\partial z} \Big|_{z=l} + \frac{h}{k} T(r, l) = 0 \quad (4.24)$$

$$\frac{\partial T}{\partial r} \Big|_{r=b} + \frac{h_b}{k} T(b, z) - \frac{q_{loss}}{k} = 0 \quad (4.25)$$

$$\frac{\partial T}{\partial r} \Big|_{r=a} = \frac{q_{PMi}}{k} \quad (4.26)$$

The convection constant on the outer part of the cylinder is h_b and the eddy current loss in the shielding cylinder is q_{loss} . The general solution of this senario is (4.27). The constants C_{1-4} are solved in the next section using the boundary conditions described in this section. Since k is a material dependent constant, it is assumed that $k = 1$ when determining the constants C_{1-4} . The actual thermal conductivity can be implemented in the final solution by dividing h, h_b, q_{loss} and q_{PMi} with k .

$$T(r, z) = [C_1 I_0(\lambda r) + C_2 K_0(\lambda r)] [C_3 \cos \lambda z + C_4 \sin \lambda z] \quad (4.27)$$

4.2.2 Solving the constants C_{1-4}

Applying the first boundary condition (4.23) to the general solution (4.27) gives:

$$\begin{aligned}
 0 &= -\frac{\partial T}{\partial z}\bigg|_{z=0} + hT(r, 0) \\
 &= -\frac{\partial}{\partial z} ([C_1 I_0(\lambda r) + C_2 K_0(\lambda r)] [C_3 \cos \lambda z + C_4 \sin \lambda z]) \bigg|_{z=0} + \dots \\
 &\quad \dots + h [C_1 I_0(\lambda r) + C_2 K_0(\lambda r)] [C_3 \cos 0 + C_4 \sin 0] \\
 &= [C_3 \lambda \sin 0 - C_4 \lambda \cos 0] + hC_3 \\
 &= -C_4 \lambda + hC_3 \\
 C_4 &= \frac{hC_3}{\lambda}
 \end{aligned} \tag{4.28}$$

since $[C_1 I_0(\lambda r) + C_2 K_0(\lambda r)] = 0$ would result in $T(r, z) = 0$. The general solution then becomes:

$$T(r, z) = [C_1 I_0(\lambda r) + C_2 K_0(\lambda r)] \left[\cos \lambda z + \frac{h}{\lambda} \sin \lambda z \right] \tag{4.29}$$

Applying the second boundary condition (4.24) gives:

$$\begin{aligned}
 0 &= d\frac{\partial T}{\partial z}\bigg|_{z=l} + hT(r, l) \\
 &= \frac{\partial}{\partial z} \left([C_1 I_0(\lambda r) + C_2 K_0(\lambda r)] \left[\cos \lambda z + \frac{h}{\lambda} \sin \lambda z \right] \right) \bigg|_{z=l} + \dots \\
 &\quad \dots + h [C_1 I_0(\lambda r) + C_2 K_0(\lambda r)] \left[\cos \lambda z + \frac{h}{\lambda} \sin \lambda z \right] \\
 &= \frac{\partial}{\partial z} \left(\cos \lambda z + \frac{h}{\lambda} \sin \lambda z \right) \bigg|_{z=l} + h \left[\cos \lambda l + \frac{h}{\lambda} \sin \lambda l \right] \\
 &= -\lambda \sin \lambda l + h \cos \lambda l + h \cos \lambda l + \frac{h^2}{\lambda} \sin \lambda l \\
 &= -\lambda^2 \sin \lambda l + \lambda h \cos \lambda l + \lambda h \cos \lambda l + h^2 \sin \lambda l \\
 &= (-\lambda^2 + h^2) \sin \lambda l + 2\lambda h \cos \lambda l \\
 &= (-\lambda^2 + h^2) 2 \sin \frac{\lambda l}{2} \cos \frac{\lambda l}{2} + 2\lambda h \left(\cos^2 \frac{\lambda l}{2} - \sin^2 \frac{\lambda l}{2} \right) \\
 &= \lambda h \cos^2 \frac{\lambda l}{2} - (\lambda^2 - h^2) \sin \frac{\lambda l}{2} \cos \frac{\lambda l}{2} - \lambda h \sin^2 \frac{\lambda l}{2} \\
 &= \left(h \sin \frac{\lambda l}{2} + \lambda \cos \frac{\lambda l}{2} \right) \left(h \cos \frac{\lambda l}{2} - \lambda \sin \frac{\lambda l}{2} \right)
 \end{aligned} \tag{4.30}$$

The constant λ is the roots of (4.30) and can be written as λ_n with $n = 1, 2, 3, \dots$. This equation can also be rewritten:

$$\begin{aligned} 0 &= (-\lambda^2 + h^2) \sin \lambda l + 2\lambda h \cos \lambda l \\ \frac{\sin \lambda l}{\cos \lambda l} &= \frac{2\lambda h}{(\lambda^2 - h^2)} \\ \tan \lambda_n l &= \frac{2\lambda_n h}{(\lambda_n^2 - h^2)} \end{aligned} \quad (4.31)$$

where l only influences the left hand side and h only the right hand side. It is clear from (4.31) that l determines the period of λ_n . The effect of h can be seen in Figure 4.3. It is a hyperbola where the asymptotes are dependent on h . The general solution then becomes:

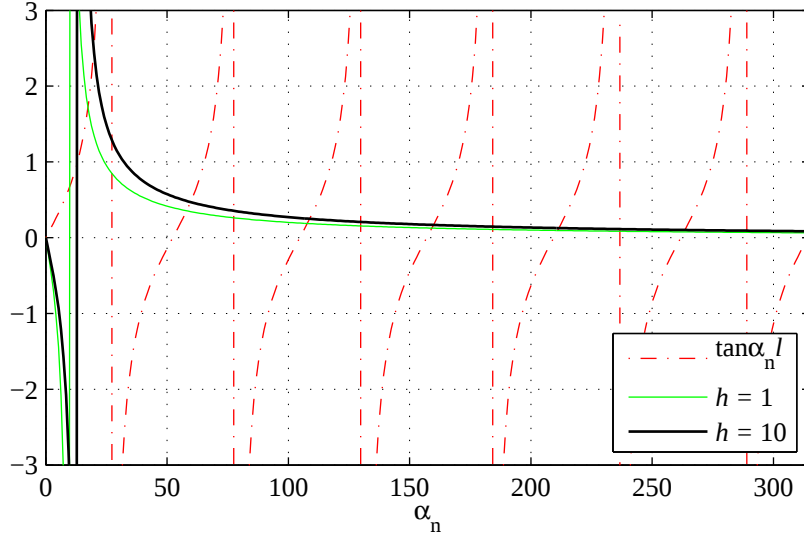
$$T(r, z) = \sum_{n=1}^{\infty} [C_1 I_0(\lambda_n r) + C_2 K_0(\lambda_n r)] \left[\cos \lambda_n z + \frac{h}{\lambda_n} \sin \lambda_n z \right] \quad (4.32)$$

The third boundary condition (4.25) is applied next.

$$\begin{aligned} 0 &= \frac{\partial T}{\partial r} \Big|_{r=b} + h_b T(b, z) - q_{loss} \\ q_{loss} &= \frac{\partial}{\partial r} \left\{ [C_1 I_0(\lambda_n r) + C_2 K_0(\lambda_n r)] \left[\cos \lambda_n z + \frac{h}{\lambda_n} \sin \lambda_n z \right] \right\} \Big|_{r=b} + \dots \\ &\quad \dots + h_b \left\{ [C_1 I_0(\lambda_n b) + C_2 K_0(\lambda_n b)] \left[\cos \lambda_n z + \frac{h}{\lambda_n} \sin \lambda_n z \right] \right\} \\ q_{loss} &= \left[\frac{\partial}{\partial r} ([C_1 I_0(\lambda_n r) + C_2 K_0(\lambda_n r)]) \Big|_{r=b} + h_b ([C_1 I_0(\lambda_n b) + C_2 K_0(\lambda_n b)]) \right] \times \dots \quad (4.33) \\ &\quad \dots \times \left[\cos \lambda_n z + \frac{h}{\lambda_n} \sin \lambda_n z \right] \\ &= [\lambda_n C_1 I_1(\lambda_n b) - \lambda_n C_2 K_1(\lambda_n b) + h_b C_1 I_0(\lambda_n b) + h_b C_2 K_0(\lambda_n b)] \times \dots \\ &\quad \dots \times \left[\cos \lambda_n z + \frac{h}{\lambda_n} \sin \lambda_n z \right] \end{aligned}$$

This can be rewritten in the form of (4.18):

$$\begin{aligned} f(r) &= \sum_{n=1}^{\infty} A_n \left[\cos \lambda_n z + \frac{h}{\lambda_n} \sin \lambda_n z \right] \\ q_{loss} &= \sum_{n=1}^{\infty} [\lambda_n C_1 I_1(\lambda_n b) - \lambda_n C_2 K_1(\lambda_n b) + h_b C_1 I_0(\lambda_n b) + h_b C_2 K_0(\lambda_n b)] \times \dots \quad (4.34) \\ &\quad \dots \times \left[\cos \lambda_n z + \frac{h}{\lambda_n} \sin \lambda_n z \right] \end{aligned}$$

Figure 4.3: Values of λ_n ; $l = 0.06$

Using (4.19) then gives:

$$\begin{aligned}
 A_n &= \frac{2\lambda_n^2}{(\lambda_n^2 + h^2)l + 2h} \int_0^l f(x) X_n dx \\
 \lambda_n C_1 I_1(\lambda_n b) - \lambda_n C_2 K_1(\lambda_n b) + \dots \\
 \dots + h_b C_1 I_0(\lambda_n b) + h_b C_2 K_0(\lambda_n b) &= \frac{2\lambda_n^2}{(\lambda_n^2 + h^2)l + 2h} \int_0^l q_{loss} \left[\cos \lambda_n z + \frac{h}{\lambda_n} \sin \lambda_n z \right] dz \\
 C_1 [\lambda_n I_1(\lambda_n b) + h_b I_0(\lambda_n b)] + \dots \\
 \dots + C_2 [-\lambda_n K_1(\lambda_n b) + h_b K_0(\lambda_n b)] &= \frac{2q_{loss}\lambda_n^2}{(\lambda_n^2 + h^2)l + 2h} \left[\frac{1}{\lambda_n} \sin \lambda_n z - \frac{h}{\lambda_n^2} \cos \lambda_n z \right]_0^l \\
 &= \frac{2q_{loss}\lambda_n^2}{(\lambda_n^2 + h^2)l + 2h} \left[\frac{1}{\lambda_n} \sin \lambda_n l - \frac{h}{\lambda_n^2} \cos \lambda_n l + \frac{h}{\lambda_n^2} \right] \\
 &= \frac{2q_{loss}}{(\lambda_n^2 + h^2)l + 2h} [\lambda_n \sin \lambda_n l - h \cos \lambda_n l + h].
 \end{aligned} \tag{4.35}$$

This equation contains two unknowns C_1 and C_2 , thus another equation is needed to solve the constants. Applying the final boundary condition gives:

$$\begin{aligned}
 \frac{\partial T}{\partial r} \bigg|_{r=a} &= q_{PMi} \\
 \frac{\partial}{\partial r} \left\{ \sum_{n=1}^{\infty} [C_1 I_0(\lambda_n r) + C_2 K_0(\lambda_n r)] \left[\cos \lambda_n z + \frac{h}{\lambda_n} \sin \lambda_n z \right] \right\} \bigg|_{r=a} &= q_{PMi} \\
 \sum_{n=1}^{\infty} \left[\cos \lambda_n z + \frac{h}{\lambda_n} \sin \lambda_n z \right] [\lambda_n C_1 I_1(\lambda_n a) - \lambda_n C_2 K_1(\lambda_n a)] &= q_{PMi}
 \end{aligned} \tag{4.36}$$

This can also be rewritten using (4.19):

$$\begin{aligned}
A_n &= \frac{2\lambda_n^2}{(\lambda_n^2 + h^2)l + 2h} \int_0^l f(x) X_n dx \\
\lambda_n C_1 I_1(\lambda_n a) - \lambda_n C_2 K_1(\lambda_n a) &= \frac{2\lambda_n^2}{(\lambda_n^2 + h^2)l + 2h} \int_0^l (q_{PMi}) \left[\cos \lambda_n z + \frac{h}{\lambda_n} \sin \lambda_n z \right] dz \\
&= \frac{2\lambda_n^2 (q_{PMi})}{(\lambda_n^2 + h^2)l + 2h} \int_0^l \left[\cos \lambda_n z + \frac{h}{\lambda_n} \sin \lambda_n z \right] dz \\
&= \frac{2 (q_{PMi})}{(\lambda_n^2 + h^2)l + 2h} [\lambda_n \sin \lambda_n l - h \cos \lambda_n l + h]
\end{aligned} \tag{4.37}$$

The constants C_1 and C_2 can be calculated using (4.36) and (4.37). When implementing the solution, the summation cannot be done to infinity, thus m is introduced as shown in (4.38).

$$T(r, z) = \sum_{n=1}^m [C_1 I_0(\alpha_n r) + C_2 K_0(\alpha_n r)] \left[\cos \alpha_n z + \frac{h}{\alpha_n} \sin \alpha_n z \right] \tag{4.38}$$

4.2.3 Permanent magnet model verification using FEM

Before using a derived model, its validity should be investigated. The analytical model of the PM will form part of the LP model, but its accuracy can be verified separately. The analytical model of the PM is verified with COMSOL Multiphysics 4[®] which uses the FEM for numerical solution of the diffusion equation. Assuming the FEM model is correctly implemented, the following errors can be identified when comparing the results from the two methods:

Error in the derivation Discrepancies in the results could point to an error in the derivation of the analytical model.

Implementation Even if the model derivation is correct, errors can occur during the implementation of the model in a computer solution product. Using different software products can point out these implementation errors.

Some errors in the derived model cannot be identified when using one software technique to verify another. These include assumptions of the material properties, thermal constants and geometrical irregularities. If there is an error in the approximation of the reality, the results of both techniques will correlate, but still not give an accurate model of reality.

FEM and the analytical model are used to solve the boundary value problem shown in Figure 4.2. The following constants are used: $a = 30$ mm, $b = 80$ mm, an axial length of 60 mm, $q_{loss} = 10$ W loss on the outside surface and $q_{PMi} = 2.5$ W flowing through the inside surface. Convection heat flux, with a convection coefficient of $100 \text{ W}/(\text{m}^2 \cdot \text{K})$, is assumed at three of the boundaries. Implementation of the distributed model using MATLAB[®] gives a surface plot as shown in Figure 4.4.

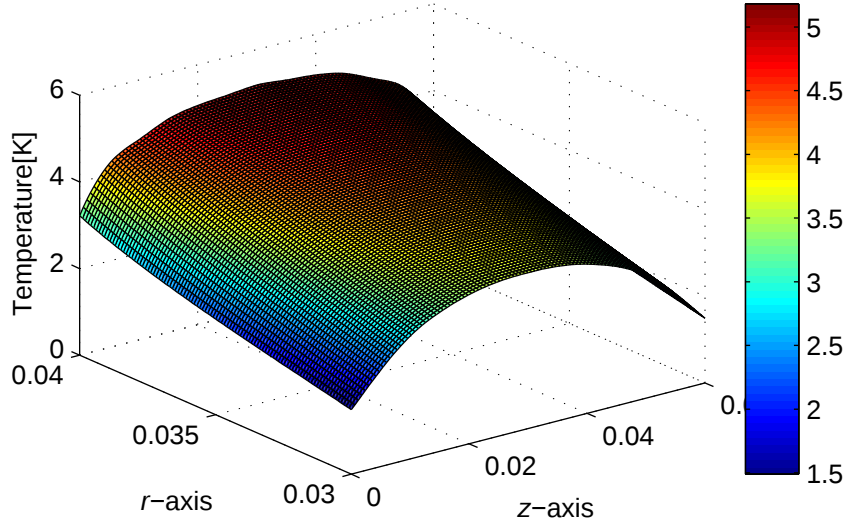


Figure 4.4: Implementation of PM 2-D analytical model in MATLAB®

The percentage difference between the results obtained with FEM and the analytical method is shown in Figure 4.5 if m in (4.38) equals 50. The approximation of (4.19) can clearly be seen. Figure 4.6 shows the percentage error for increasing m . The maximum error settles at 1.5 % and only a small decrease in error is evident when m is larger than 10. The solution time is directly proportional to m . Thus increasing m from 10 to 50, will result in the solution time increasing five times, but the error decreases less than 1 %. It can be concluded that the 2-D analytical model of the PM temperature derived in this section will accurately predict the temperature and heat flow.

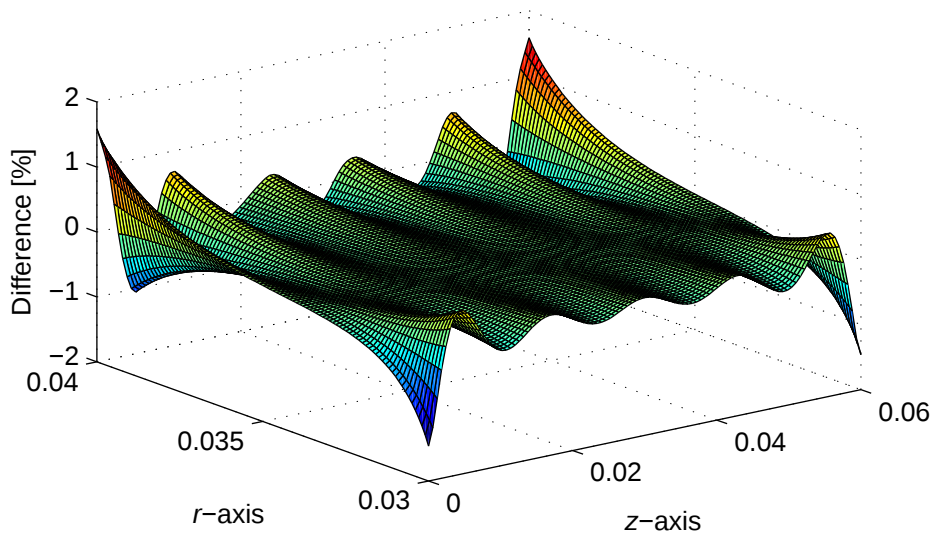
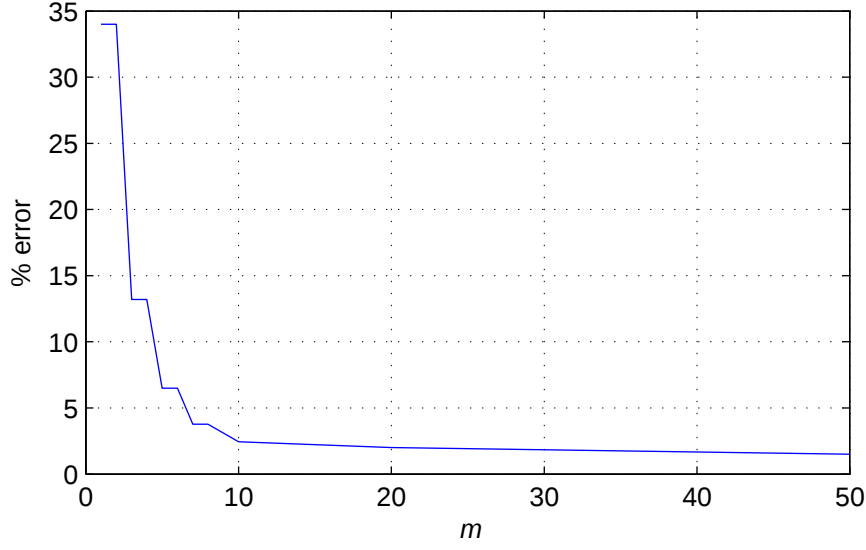


Figure 4.5: Percentage difference, FEM and analytical method

Figure 4.6: Percentage difference for m upper range

4.3 Stator winding model

In all electric machines, stator winding overheating will cause insulation failure, destroying the machine. The thermal modelling of the stator winding is thus very important. The steady state diffusion equation for the stator winding in the rz - plane is:

$$\frac{1}{r} \frac{\partial}{\partial r} \left(k_r r \frac{\partial t}{\partial r} \right) + \frac{\partial}{\partial z} \left(k_z \frac{\partial t}{\partial z} \right) + \dot{q} = 0 \quad (4.39)$$

where the axial and radial conduction coefficients are k_z and k_r , respectively. The anisotropic nature of the winding necessitates this, since the winding consists of conductors and insulating material. The heat generated inside the winding (due to I^2R loss) is $\dot{q}$. This is a Poisson equation which cannot be solved using SOV directly since it is inhomogeneous. The boundary conditions of the winding and end winding areas differ since forced convection cooling is applied to the outside of the latter in the TWINS.

Modelling the stator winding of the TWINS in 1-D instead of 2-D can be motivated by:

Anisotropic conductivity The copper conductors point in an axial direction inside the winding. The good conductivity of the copper will dominate the heat flow in the axial direction, resulting in a small change in temperature in the axial direction. In the radial direction, the insulation will dominate the heat flow, thus resulting in large temperature differences in this direction.

Shape The stator winding has a long and thin cylindrical shape. The ratio of length and width is 18.75 in the TWINS. The heat flow from the end winding will thus be mostly in the radial direction since this surface is much larger than the axial surface area.

4.3.1 Derivation of a 1-D solution for the TWINS stator winding

When Joulean heat generation occurs due to the current flow, the heat generation $\dot{q}$ can be written as $\dot{q} = I^2\rho$, where ρ is the electrical resistivity and I is electrical current. The resistivity is linearly dependent on the temperature, or: $\rho = \rho_0(1 + \alpha_0 t)$. The heat generated can thus be written as:

$$\dot{q} = I^2\rho(1 + \alpha_0 t) \quad (4.40)$$

where α_0 is the temperature coefficient [48]. Applying the linear transformation:

$$\begin{aligned} T &= I^2\rho(1 + \alpha_0 t) \\ &= a + bt \end{aligned} \quad (4.41)$$

the diffusion equation (4.39) reduces to a Bessel function of order zero.

$$r^2 \frac{\partial^2 T}{\partial r^2} + r \frac{\partial T}{\partial r} + br^2 T = 0 \quad (4.42)$$

This has the general solution:

$$T(r) = C_1 J_0(r\sqrt{b}) + C_2 Y_0(r\sqrt{b}) \quad (4.43)$$

where C_{1-2} are constants that can be determined from the boundary conditions. For the winding section, the inside and outside temperatures can be determined from the LP model, thus if the boundary conditions are: $t(r_1) = t_1$ and $t(r_2) = t_2$, the constants are:

$$C_1 = \frac{a + bt_1 - C_2 Y_0(r_1\sqrt{b})}{J_0(r_1\sqrt{b})} \quad (4.44)$$

$$C_2 = \frac{(a + bt_2) J_0(r_1\sqrt{b}) - (a + bt_1) J_0(r_2\sqrt{b})}{Y_0(r_2\sqrt{b}) J_0(r_1\sqrt{b}) - J_0(r_2\sqrt{b}) Y_0(r_1\sqrt{b})} \quad (4.45)$$

4.3.2 Verification of 1-D stator winding model

To verify the 1-D stator model COMSOL Multiphysics® is used. A cylinder with OD of 82 mm, ID of 66 mm, conduction coefficient of 1 W/(m.K) and internal heat generation of 10 W is used. The inside and outside surfaces are kept at 297.15 K and 298.15 K respectively. The left part of Figure 4.7 shows the temperature in the winding for the senario described above. There is a very small difference between the temperatures calculated using the two methods, as shown in Figure 4.7 on the right. This discrepancy can be attributed to the discretization error found in the FEM results.

4.4 Conclusion

This chapter described the derivation of a 2-D distributed thermal model for a cylinder with convection heat flow on three sides and a heat source on the outer surface. This surface is where

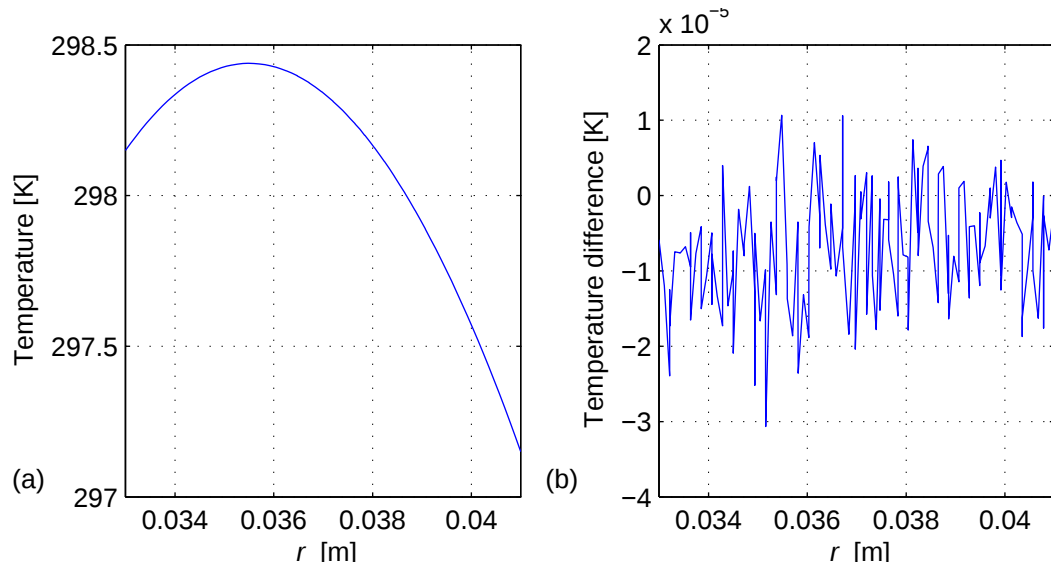


Figure 4.7: Winding temperature (a) temperature; and (b) difference between LP and distributed models

shielding cylinder eddy current loss occurs due to high frequency stator currents. The model was verified using FEM and a good correlation found. A 1-D model for the winding was also derived and verified with FEM.