
**MATHEMATICAL KNOWLEDGE
AND
SKILLS NEEDED
IN
PHYSICS EDUCATION
FOR
GRADES 11 AND 12**

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SUMMARY

The performance of mathematics and physical science students are very low in South Africa. These students lack algebraic knowledge and skills in physics education. They tend to treat mathematics and science as separate entities; to them the two subjects are not related. Even the teachers seem not to realise the interrelationship of the two subjects, because according to the research, they perpetuate this attitude. A possible reason could be that they are unfamiliar with common objectives and applications.

Knowledge of science is enhanced by the application of mathematics, but the role of mathematical knowledge and skills in the understanding of physical science is uncertain. Even in the new National Curriculum Statement (NCS) of South Africa the relationship between mathematics and physical science is not clearly indicated. Algebraic language is a main tool used in physics, but students still display a lack of understanding of mathematical concepts and problem solving skills.

The study was aimed at identifying the mathematical knowledge and skills that would enable students to solve physics problems in grades 11 and 12. The aim was also to identify the specific problems experienced by students in applying these skills and knowledge in physics at grades 11 and 12 level. The empirical study was conducted amongst a group of 120 students in four schools in the Rustenburg Region, North-West Province, South Africa and 28 teachers of which 10 were from these schools and 18 were teachers participating in the Sediba project of the North-West University. The investigation was done by means of a self constructed test and questionnaires. The results indicate that the biggest problem lies with a lack of conceptual knowledge, especially with a basic understanding of proportional reasoning. Other problems were identified and possible remedies proposed.

Key concepts for indexing: integration, interrelation, incorporating mathematics and science, teaching and learning, co-operative learning, skills and knowledge, conceptual and procedural knowledge, interconnection, curriculum, collaboration

OPSOMMING

Die wiskunde- en natuurwetenskap-prestasie van studente in Suid-Afrika is laag. Dit ontbreek by hierdie studente aan wiskunde-kennis en vaardighede in fisika-opleiding. Hulle neig om wiskunde en natuurwetenskap as aparte entiteite te behandel; vir hulle is die twee onderwerpe nie verwant nie. Selfs die onderwysers besef blykbaar nie die verwantskap tussen die twee vakke nie, omdat hulle volgens die navorsing hierdie houding versterk. 'n Moontlike rede kan wees dat hulle onbekend is met algemene doelwitte en toepassings.

Kennis van wetenskap word bevorder deur die toepassing van wiskunde, maar die rol van wiskundige kennis en vaardighede in die begrip van natuurwetenskap is onseker. Selfs in die nuwe Nasionale Kurrikulumverklaring van Suid-Afrika is die verwantskap tussen wiskunde en natuurwetenskap nie duidelik aangedui nie. Hoewel algebraïese taal een van die hoofwerktuie is wat in fisika gebruik word, toon studente steeds 'n gebrekkige begrip van wiskundige konsepte en probleemoplossings-vaardighede.

Hierdie studie het ten doel gehad om die wiskundige kennis en vaardighede te identifiseer wat studente in staat sal stel om fisika-probleme in grade 11 en 12 op te los. Die doel was ook om die bepaalde probleme te identifiseer wat studente ervaar in die toepassing van hierdie vaardighede en kennis in fisika in grade 11 en 12. Die empiriese studie is gedoen onder 'n groep van 120 studente in vier skole in die Rustenburg-streek, Noordwes Provinsie, Suid-Afrika, asook 28 wetenskap- en wiskunde-onderwysers waarvan 10 verbonde is aan die vier skole en 18 deel was van die Sediba-projek van die Noordwes-Universiteit. Die ondersoek is deur middel van 'n self-opgestelde toets en vraelyste gedoen. Die gevolgtrekkings toon dat die grootste probleem lê by 'n gebrek aan begripkennis, veral 'n basiese begrip van proporsionele redenering. Ander probleme is geïdentifiseer en moontlike oplossings is aan die hand gedoen.

Trefwoorde vir indeksering: integrasie, onderlinge verband, vereninging van wiskunde en wetenskap, onderrig en leer, samewerkende leer, vaardighede en kennis, konseptuele en prosedurekennis, onderlinge verbintenis, kurrikulum, samewerking

NOTES ON TERMINOLOGY

The researcher has chosen to use the terms “students” instead of “learners” and “teachers” instead of “educators”.

References to national curriculum statements, grades 11 and 12 and other school related statements refer to the South African situation, except where otherwise indicated.

The term “science” refers to physical science which includes physics and chemistry – one subject in grades 10 to 12 in South African schools.

ABBREVIATIONS

The following abbreviations are used in the text:

AAAS: American Association for the Advancement of Science

MSEB: Mathematics sciences education board

NCS: National Curriculum Statements

OBE: Outcomes-Based Education

PSSM: Principles and Standards for School Mathematics (USA)

NCTM: National Council of Teachers of Mathematics (USA)

SA: South Africa

DoE: Department of Education

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CHAPTER 1

PROBLEM ANALYSIS AND RESEARCH DESIGN

1.1 ORIENTIVE INTRODUCTION

Mathematics and science have many features in common. Both are trying to discover general patterns and relationships, and in this sense, they are part of the same endeavour (Pang & Good, 2000:73–82). Science provides mathematics with interesting problems to investigate, while mathematics provides science with powerful tools to analyse data. Skills such as problem solving, communication, reasoning, connections, estimations, measurements, patterns and relations are areas that are equally important in science and mathematics (Kaye & Mollie, 2000:149–167). However, in our present school situation students are still led to believe that science and mathematics are unrelated entities. Teachers often perpetuate this attitude, perhaps because they are unfamiliar with possible common objectives and applications.

Knowledge of physics is enhanced by the application of mathematics, but the contribution of mathematical knowledge plays an uncertain role in the developing of knowledge of physics (Noss, 1999:373). Physics teachers complain that students do not apply what they have learnt in their mathematics classes to their physics classes (Basson, 2002:679). An illustration of this problem of integration of mathematics and physics is the difficulties that students have in applying to physics what they have learnt in their study of mathematics, e.g. plotting of graphs and calculations (Breitenberger, 1990:318).

Another aspect is the level of mathematical skills development of students. Algebraic language is one of the main tools used in physics (Rebmann & Viennont, 1993:723), but students display a lack of understanding of modern mathematical concepts and a

lack of problem-solving skills (Breitenberger, 1990:318). They regard mathematics as a mechanical method, not as a constructive thinking process. Possible causes might be that students are able to manipulate formulas, but without real conceptual understanding.

According to Van de Walle (2004:36–40), what students learn is almost entirely dependent on the experiences that teachers provide every day in the classroom. Students learn when teachers' actions encourage them to think, question, solve problems and discuss their ideas, strategies and solutions. Teachers should have a deep understanding of the mathematics they are teaching. They must understand how students learn mathematics and should have a keen awareness of the individual mathematical development of their own students so that they can select instructional tasks and strategies that would enhance learning.

1.2 PROBLEM STATEMENT

From this background it can be expected that grades 11 and 12 students experience mathematical problems that prevent effective learning and teaching of physics. These problems are twofold:

- A lack of mathematical skills and knowledge in the manipulation of algebraic procedures as it occurs in physics; and
- problems with integration of mathematics into physics, that is, application of mathematical skills and knowledge in a physics situation.

1.3 AIMS OF THE STUDY

Based on the problem statement in paragraph 1.2, the focus of the study is to identify the mathematical problems that science students experience that restrict effective teaching and learning of physics at grades 11 and 12 level. In particular the research aims to:

- Identify the mathematical knowledge and skills that will enable students to solve physics problems in grades 11 and 12; and
- identify the specific problems experienced by students in applying these skills and knowledge in physics at grade 11 and 12 level.

1.4 RESEARCH DESIGN

1.4.1 LITERATURE STUDY

The research commenced with a literature review to gain an in-depth understanding of the mathematical problems encountered in solving physics problems. Study material was obtained in the library by a search on the EBSCOhost and RSAT databases of recent publications in scientific and educational journals. The following key words were used: integration, interrelation, incorporating mathematics and science, teaching and learning, co-operative learning, skills and knowledge.

1.4.2 EMPIRICAL RESEARCH

A qualitative research method in the form of a phenomenological study was used in the study (Leedy & Ormrod, 2001:153,154). The research instruments included a self-constructed test and questionnaires, which were followed up by interviews with selected students and teachers.

1.4.3 RESEARCH PROCEDURES

The following research procedures were used:

- An analysis was done of the National Curriculum Statement, textbooks and question papers of physics for grades 11 and 12 to identify the mathematical knowledge and skills involved in the solving of physics problems.

- A test was constructed to investigate the problems experienced by students in applying algebraic skills and knowledge to solve physics problems. The questions in the test were based on the analysis of the curriculum.
- A questionnaire was constructed in which teachers were asked to rate the performance of their students.
- Another questionnaire was constructed to determine to what extent the mathematics teacher co-operates (communicates) with the science teacher.
- Interviews were conducted with selected students and teachers to follow up on the responses to the questionnaire.

1.4.4 POPULATION

The study was conducted in four different secondary schools in the Rustenburg District in the North-West Province, South Africa, using 120 Grade 12 students who take mathematics and science as subjects. Only Grade 12 students were used because by the time the research was done, the Grade 11 students had not yet completed the relevant topics. So, by using Grade 12 students, the researcher ensured that they had completed the topics covered in the questionnaire.

1.5 CHAPTER OUTLINE

CHAPTER 1: PROBLEM ANALYSIS AND RESEARCH DESIGN

In this chapter a brief overview is given of the problems encountered that gave rise to the research questions of this study. The reader gets an idea of what to expect in the study by means of a brief literature study and an outline of the research design and procedure.

CHAPTER 2: TEACHING AND LEARNING IN MATHEMATICS AND SCIENCE

This chapter gives an overview of how to develop an understanding of mathematics and science. The researcher discusses the interrelation between mathematics and science and how both have moved from a traditional-formalistic approach to a constructivist-realistic approach.

CHAPTER 3: INTEGRATION OF MATHEMATICS AND SCIENCE

This chapter shows the importance of why the mathematics teacher must be aware of what the science teacher is doing. It encourages team teaching and opportunities for interrelations between mathematics and science. It gives an overall picture of the integration of mathematics and science, e.g. the meaning of integration, reasons for integration and different types of integration that we may engage in.

CHAPTER 4: KNOWLEDGE AND SKILLS NEEDED IN PHYSICAL SCIENCE FOR SOUTH AFRICA

In this chapter the algebraic knowledge and skills involved in science for grades 11 and 12 are summarised. This summary was done after an analysis of the different types of mathematical knowledge had been completed.

CHAPTER 5: EMPIRICAL STUDY AND DISCUSSION OF RESULTS

This chapter gives an overview of the empirical study and the discussion of the results. It deals with the institutions at which the research instruments were administered, the population, the development of the test and questionnaires, the results of the test, the discussion of the results and general problems identified.

CHAPTER 6: CONCLUSIONS AND RECOMMENDATIONS

1.6 THE VALUE OF THE STUDY

There is a serious mathematics related problem in physics at secondary school level. In South Africa there is generally a high rate of failure in mathematics and science. Students cannot apply what they have learnt in the mathematics class in a physical science class. This prevents the effectiveness of teaching and learning of physics. Research in this study will try to identify the mathematical problems that science students experience that hamper the learning of physics in grades 11 and 12.

CHAPTER 2

TEACHING AND LEARNING IN MATHEMATICS AND SCIENCE

2.1 INTRODUCTION

Physical science “focuses on investigating physical and chemical phenomena through scientific inquiry” (NCS, 2003:9). It explains and predicts events in our physical environment by applying scientific models, theories and laws. It deals with society’s desire to understand how the physical environment works and how to benefit from it and care for it responsibly (NCS, 2003:9). Mathematics is an exploratory science that seeks to understand every kind of pattern that occurs in nature, patterns invented by the human mind and even patterns created by other patterns.

Although both physical science and mathematics are trying to discover general patterns and relationships, the nature of science, fundamentally grounded in our physical world, is different from the nature of mathematics. Science seeks consistency with the natural/external world through empirical evidence; while mathematics seeks consistency within its internal system through logical deduction (Lederman & Niess, 1998:281–284). As a science of abstract objects, mathematics relies on logic rather than on observation as its standard of truth, while science employs observation, simulation and experimentation as a means of discovering truth (MSEB, 1989:31).

It is to be expected that the teaching and learning of mathematics and physical science will show many similarities. Both have moved from a traditional-formalistic approach (as featured in the previous content-based syllabus) to a constructivist, realistic approach as featured in the new OBE curriculum. This shift will be discussed in the following paragraphs.

2.2 TEACHING AND LEARNING APPROACHES IN MATHEMATICS

The relationship between teachers' knowledge and beliefs and their teaching practices has been one of the main areas of exploration in the mathematics and science education literature (Pang & Good, 2000:76). Ernest (1989:13–33) proposed the existence of three views of mathematics, namely the traditional-formalistic view, the relativist-dynamic view and the instrumentalist view. He links teachers' views of mathematics with their models of teaching and learning. He maintains that teachers' conception of the nature of mathematics form their philosophy of teaching and learning of mathematics, despite the fact that they may be unable to articulate their beliefs fully, as they are often implicitly held. In his research, he admits that there is a discrepancy and discontinuity between teachers' espoused beliefs and enacted practices, and suggests that the cause might be attributed to the negative effects of some social and educational contexts of learning.

2.2.1 TRADITIONAL-FORMALISTIC VIEW

In the traditional-formalistic approach (Ernest, 1989:13–33), mathematics is viewed as a “fixed” and static body of knowledge consisting of a “logical and meaningful network of interrelated truths (facts, rules, and algorithms)” (Nieuwoudt, 1998:69-76). Consequentially, the mathematics teacher is able to transfer these chunks of knowledge to the learner. These ideas have led to the popular social view of mathematics as a discipline dominated by computation, rules, method and reasons (Van de Walle, 2004:12). Many students therefore view mathematics as a “series of arbitrary rules handed down by the teacher” (Van de Walle, 2004:12-13). Dossey (as quoted in Nieuwoudt, 1998: 69-76), shows that this (rigid) view stems from Plato's learning, according to which the origin of mathematics is outside the individual in the “external world” of ideas. As such, it must be discovered by man, rather than produced (or “made”).

Traditionally, teachers represent the source of all that is to be known in mathematics. Instruction consists of showing the students how they are to conduct the exercises. The students' attention is on the teacher's directions, not on mathematical ideas. Answers are important, not mathematical ideas, concepts and knowledge. The traditional system rewards the learning of rules, but offers little opportunity to do mathematics (Van de Walle, 2004:12-13, 37).

2.2.2 THE RELATIVISTIC-DYNAMIC OR CONSTRUCTIVIST APPROACH

In contrast to the formalistic view, mathematics can also be viewed from a relativistic-dynamic perspective (Ernest, 1989:13–33). According to this view, mathematics is not viewed as a “finished product with its origin outside the individual, but remains ‘in the making’ in the individual’s mind” (Nieuwoudt, 1998:69-76). This is a problem-based approach, where mathematics is viewed from a “change and grow” perspective. Mathematics is viewed as a continually changing field of human activity, creativity and discovery, aimed at generating patterns through problem solving, which is then processed into mathematical knowledge (Van de Walle, 2004:13). Consequently, this view of mathematics bears a strong resemblance to Aristotle's experimentalist ideas about mathematics (Nieuwoudt, 1998:69-76).

In the relativistic-dynamic approach, students take on very different roles as they strive to achieve complex learning outcomes. Students are challenged to reason mathematically, to explain and justify their mathematical reasoning, and to construct their mathematical knowledge through exploration and problem solving (Van de Walle, 2004:12–13). In this approach new goals have been set forth that include an emphasis on conceptual understanding, communicating, and learning through problem solving and inquiry (Pape & Smith, 2002:93-101).

In the relativistic-dynamic approach the teachers' role is to create this spirit of inquiry, trust and expectation. Within this environment, students are invited to do

mathematics. Problems are posed and students are actively figuring out, testing ideas and making conjectures, developing reasons and offering explanations. They work in groups, in pairs or individually; they are always sharing and discussing. Reasoning is celebrated as students defend their methods and justify their solutions (Van de Walle 2004:14).

Many school mathematics teachers still hold on to the traditional-formalistic view of mathematics and mathematics teaching (Van de Walle, 2004:37; Taylor & Vinjevold, 1999:142–143). Only a few are in favour of a dynamic alternative view of mathematics and its teaching. Research confirms that teachers' beliefs of mathematics teaching are deeply rooted and are not easy to change (Nieuwoudt, 1998: 69-76).

2.2.3 THE INSTRUMENTALIST VIEW

In the instrumentalist view of mathematics the utility value of mathematics is over-emphasised, while mathematics as a phenomenon of reality is reduced and narrowed down to a mere 'tool piece' (Nieuwoudt, 1998:69-76). The curriculum is pragmatically reduced to the useful elements of mathematics, which have to be drilled in through repeated practice and application. This is typically the view held from an engineer's perspective. This view is still prevalent in many schools where formulas are used without understanding (Breitenberger, 1990:318).

2.3 TEACHING AND LEARNING APPROACHES IN SCIENCE

In the same way as mathematics teaching and learning, science teaching and learning have moved from a traditional approach to a constructivist approach.

2.3.1 THE TRANSMISSION (TRADITIONAL) MODEL OF TEACHING

Initially, science was taught by means of lecture demonstrations, with the instructor performing experimental demonstrations and supplementing them with information from the textbooks. This method is known as the transmission (traditional) model of teaching (Wesi, 1997:56).

The transmission process is where knowledge is transferred from the teacher to the student; that is the student is the recipient of information from the active teacher (participant). The teacher and the textbook are the only sources of information. The students absorb the information by incorporating it in the same order and sequence as presented and again the teacher is responsible for the learning of the child. The consequence is that performance and motivation are influenced by the teacher's personality (Jacob, 1982:262).

In the transmission model, a well-prepared lesson is presented in a formal setting in a logical and clear manner. The teacher standing in front of a quietly seated and attentive class carries out an experimental demonstration. Rote learning and memorisation are all that is needed for the student to succeed in this approach to teaching. There is no internalisation of the content and understanding is not emphasised. This approach is aimed at enabling the student to remember information in order to pass the examination. There is no emphasis on the acquisition of problem-solving skills and logical reasoning.

This method yielded students that lack first-hand familiarity with science concepts and procedures, because students were not allowed to perform experiments on their own. Physics teaching needs to move beyond handing out notes, solving numerical exercises, and doing demonstrations and experiments. Learning improves when students become more aware of how they learn (Jacob, 1982:262). This involves greater knowledge of learning, increased awareness of the nature of learning tasks that will lead to greater control by students over their own learning.

2.3.2 THE HEURISTIC APPROACH

The ineffectiveness and the failures of the transmission model of teaching in sciences have led to the proposal of the heuristic approach to teaching in school sciences by Armstrong (in Wellington, 1989:36). The heuristic approach implies that students must be placed in the position of the original discoverer so that they discover information and knowledge for themselves. Armstrong's argument is that science teaching is about teaching scientific methods (processes), rather than merely teaching information and knowledge (content), as in the case of the transmission model.

The heuristic approach, which over-emphasised the scientific method over the learning of content, did not win much support in scientific and education circles, as some educationists still favoured the transmission approach as the best method of teaching science. The Piagetian developmental theory and theories on approaches to learning developed in the early sixties influenced science teaching towards observation, exploration and discovery approaches (Wellington, 1989:36). The method of learning by discovery became more favoured in the 1960s. In 1968, Ausubel proposed a learning theory called the constructivist learning theory (in Berlin, 1989:73-80), which followed on the heuristic approach.

2.3.3 THE CONSTRUCTIVIST APPROACH (THEORY)

Constructivism is the latest learning model (theory) that is based on several assumptions (Novodvorsky, 1997:242). The first assumption is that knowledge is constructed in the mind of the student. The second assumption is that students bring their prior knowledge about science into the classroom. The third assumption is that learning is a lifelong process. It is not confined to a specific period in the individuals' life, but is a continuous process and does not take place in stages. This implies that the learning process takes into consideration the characteristics of the student, his or her abilities, attitudes and perceptions of the world.

The individual's perception of the world is constructed as a result of observations made from the surroundings and personal experience with "the stuff of science" (Novodvorsky, 1997:242). In every individual's mind, there exists a contract of how the world operates and this influences the way incoming knowledge is interpreted and understood. It is possible that different people could interpret the same set of information differently.

This has implications for the teaching and learning of science and the role of the teacher in this process. Learning does not take place through transmission of knowledge from the teacher (the source) to the student (the receiver) (Wesi, 1997:56). Learning is not intended to drill information into the students' minds. In constructivism, the teacher is not the source of information and knowledge, but a facilitator or agent that guides students through the learning process. The teacher is responsible for supporting, nurturing and assessing students to help them improve (Novodvorsky, 1997:242).

Constructivism recognises that it is not practical to expect students to achieve success at the same rate (Jacob, 1982:268; Novodvorsky, 1997:242). The theory recognises that the individual's attitudes towards certain topics influence learning and that attitudes are guided by beliefs, value systems and the prior knowledge possessed by individuals. If students feel good about the learning task, they will have positive attitudes towards it.

Students bring their prior knowledge about science to the science classroom, which is referred to as preconception. This prior knowledge is seated within the mental structure of the student. The constructivist theory asserts that prior knowledge of the learner has a direct impact on his/her learning and should not be ignored (Novodvorsky, 1997:242). According to this theory, learning is not purely a receptive process. It is an active process where students construct their own knowledge. Students always create meaning for information presented to them. This meaning is always compared with already existing knowledge structures. The new knowledge

must be assimilated into the mental structures and prior knowledge that is still retained (Jacob, 1982:268).

Learning takes place if a student modifies his mental structures so as to match new knowledge with existing knowledge. The restructuring of existing structures is crucial to the learning process. Restructuring of the concepts is not an easy exercise. It is the most difficult aspect but yet the most essential aspect of learning (Jacob, 1982:242). If there is a match between the incoming knowledge and the existing knowledge, the new knowledge is incorporated in the mental structures and is understood. The knowledge is internalised and forms part of the individual's mental structures.

Understanding occurs when a fact, idea or procedure is part of a network of interconnected facts, ideas and procedures, and this network is connected to other networks in a meaningful way (Hiebert & Carpenter, 1992:65–70). Understanding is generative in that new connections are constructed. It promotes remembering as connections are formed between new and existing knowledge and transfer is enhanced as similarities and differences are noted in the connections. What is important is the links between the two, as both are essential.

2.4 THE ROLE OF PROBLEM-SOLVING SKILLS IN THE LEARNING OF SCIENCE AND MATHEMATICS

Problem solving is central to the teaching and learning of physical science and mathematics (NCS, 2003:13, Bybee *et al.*, 1997:328). By learning problem solving in science and mathematics, students acquire ways of thinking, habits of persistence and curiosity, and confidence in unfamiliar situations that will serve them well outside the classroom. Higher-order thinking and problem-solving skills are required to meet the demands of the labour market and for active citizenship within communities with increasingly complex technological, environmental and societal problems.

Problem solving involves identification and analysis of the problem you are dealing with and the design of procedures to reach solutions (NCS, 2003:13). Problem solving defines ways of thinking and knowing and encourages a certain stance toward learning (NCTM, 2000:52). Students should have frequent opportunities to solve complex problems that require a significant amount of effort. They should be encouraged to reflect on their thinking, enabling students to apply and adapt a variety of appropriate strategies to solve problems and to monitor and reflect on the process of mathematical thinking (NCTM, 2000:52).

Successful problem solvers are strategic in developing an understanding of a problem and forming a concrete or mental problem representation. In addition, problem representation depends on the co-ordination of reading comprehension strategies (Pape & Smith, 2002:93-101). How students go about this process impacts on their success. In the classroom, there are often two types of problem-solving behaviour. Some students use a direct translation approach, while others use a meaningful approach (Hegarty *et al.*, 1995:18–32). For example, they select numbers and relational terms from the text and translate it into arithmetic operations without constructing a mental model. In contrast, other students actively transform the information into an object-based representation or mental model of the problem situation. Students who use a meaningful approach experience more success. One of the goals of mathematics might be to help students learn when, what and how to monitor their progress in the domain of mathematics (Lester, 1994:660–675).

Researchers (Selden, quoted by McKittrick *et al.*, 1997) believe that current classroom instruction tends to create a culture that fosters algorithmic proficiency and a “machine-like” approach to the learning of mathematics and problem solving. They argue that mathematicians need to be aware of the distinction between knowing if a proof is true and explaining why it is true. It is then that the students will begin to acquire the mathematical knowledge to become better problem solvers. In mathematics, know-how is much more important than mere possession of information. This is the ability to solve problems, not merely routine problems, but problems requiring some degree of independence, judgement, originality and

creativity. Expert problem solvers are in possession of a better memory for important problem details. They are able to classify problems according to their underlying mathematical structure and not surface details. They can use forward chaining and working backwards in solving problems.

Expert problem solvers working on ill-structured problems possess a wide range of attributes, including domain-specific knowledge, problem-solving skills, a certain set of mathematical beliefs, meta-cognitive skills and aesthetic sensitivity (DeFranco, 1999:79–84). They tend to look for 'special features' of a problem and do not rely on algorithms to solve problems, i.e. to recognise that a problem belongs to a certain class of problems and then use a method of solution, which serves for any problem in that class.

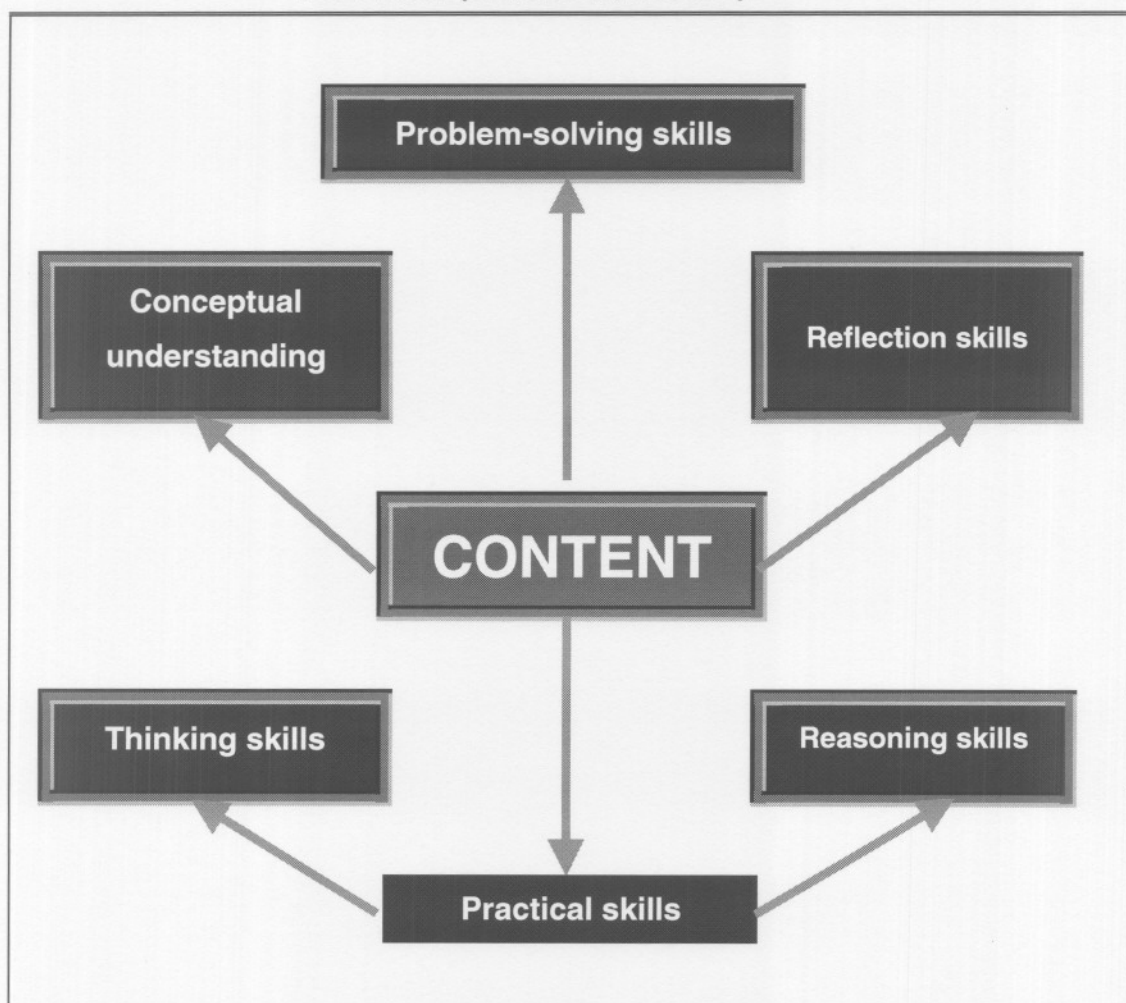
Van de Walle (2004:54) considers the following three areas as important for developing problem-solving skills: problem-solving strategies and processes, meta-cognitive habits of mentoring and regulating problem-solving activities and a positive disposition toward problem solving.

- **Strategies and processes** in problem solving refer to identifiable methods of approaching a task that are independent of the specific subject matter (Van de Walle, 2004:54–55). Strategies can be related to different phases of problem solving, namely understanding the problem, solving the problem and reflecting on the answer and solution.
- **Meta-cognition** refers to conscious monitoring and regulation of your own thought process. People that have the ability to solve problems (problem solvers) monitor their thinking regularly and automatically, hence they can recognise when they are stuck or do not fully understand. They are able to switch strategies when necessary, think about the problem again and search for related content knowledge that may help or they may simply start afresh (Van de Walle, 2004:54–55).

- **Positive deposition** refers to a student's positive attitudes and beliefs about mathematics and science. Students' beliefs in their ability to do mathematics and science have a significant effect on how they approach problems and ultimately on how well they succeed. Students that enjoy solving problems and feel satisfied at conquering a perplexing problem are likely to persevere, make second and third attempts and search out new related problems (Van de Walle, 2004:55).

Most teachers agree that the skills surrounding the mathematics and science content are at least as important as the specific content. These skills are summarised in the table below (Grayson, 1995:54).

TABLE 2.1: THE MANY FACETS OF WHAT A STUDENT IS EXPECTED TO LEARN (GRAYSON, 1995:54)



2.5 SUMMARY OF CHAPTER

Mathematics and science show many similarities, as both try to discover general patterns and relationships. Teaching and learning in mathematics also show similarities as they followed the same trends of moving from a formalistic to a constructivist approach.

In the traditional-formalistic approach, ideas, concepts and knowledge are not important, as the emphasis is on the correct answer. This leads to students giving answers without real understanding. Learning is based on the memorisation and application of rules and formulas. Students are required to remember information in order to pass the examination. The acquisition of problem-solving skills and logical reasoning is not emphasised.

In a constructive approach, an individual's surroundings and personal experiences are highly regarded. Students are not seen as empty vessels that need to be filled by the teachers. They have an existing knowledge according to their world view. Knowledge is constructed and not transmitted by teachers drilling information. New knowledge must be incorporated into the existing mental structures and there must be an interconnection of facts, ideas and procedures in a meaningful way.

Procedures and concepts in mathematics and science can be taught through problem solving. In problem solving, students have the opportunity to exercise their own stance towards learning, their potentials and abilities. They get the opportunity to think and put all their efforts into solving complex problems, which encourages them to think critically and logically. Students exercise their independence, judgement, originality and creativity.

It seems that mathematics and science should be more closely linked in the curriculum and in teaching practice. Science can supply contexts for the learning of mathematics so that stronger connections can be made. Problem solving in

mathematics will help students to develop higher-order thinking skills that would serve them in solving science problems.

The question remains how mathematics and science can be integrated and what is meant by the integration of the two learning areas. This question will be addressed in the next chapter.

CHAPTER 3 **INTEGRATION OF LEARNING OF MATHEMATICS AND PHYSICAL SCIENCE AT SCHOOL LEVEL**

3.1 INTRODUCTION

From the previous chapter it can be concluded that the disciplines of mathematics and science are intertwined and that the pedagogy of the two disciplines is very similar. These similarities can be summarised as follows:

- They make similar attempts to discover patterns and relationships (AAAS quoted by Pang & Good., 2000:76).
- They are based on interdependent ways of knowing (Berlin & White, 1995:22–23; Pang & Good, 2000:76).
- They followed the same trends of moving from a formalistic to a constructivist approach (cf. Nieuwoudt, 1998: 69-76; Wesi,1997:56–62 and Pang & Good, 2000:75).
- They share similar scientific processes, such as inquiry and problem solving (Bybee *et al.*, 1997:328).
- They fundamentally require qualitative reasoning (Isaacs *et al.*, 1997:179–206).

It can be expected that the learning and teaching of mathematics and science should be integrated in teaching practice, but according to Jarvis (1987:2), there is a lack of liaison between mathematics and science departments in secondary schools in South Africa.

Another pervasive problem is that integration means different things to different teachers (Banks, 1993:22–28). According to Banks integration deals with the extent to which teachers use examples, data and information from a variety of disciplines and cultures to illustrate the key concepts, principles, generalisations and theories in their subject area or discipline. Other researchers (Miller *et al.*, 1993:327) emphasise the importance of an integrated curriculum for science and mathematics. In the following two paragraphs the needs for integration and the problems experienced with integration of learning of science and maths are discussed. Thereafter the question of an integrated curriculum is discussed in more detail and ideas for improvement of the situation are suggested.

3.2 THE NEED FOR INTEGRATED LEARNING OF PHYSICAL SCIENCE AND MATHEMATICS

The integration of mathematics and science offers a great opportunity to motivate students and create positive attitudes toward mathematics and science (Beane, 1995:616–662). When students see a relationship between what they are learning and their personal lives, their interest in learning increases. The key thought in this process is to develop relevancy and applicability of the discipline to existing student experiences. Students must see mathematics and science as relevant components of their world. Mathematics should no longer be seen as a discipline studied and applied for mathematics' sake, but rather to make sense out of some part of our world. It should be connected to real-life situations so that students learn and appreciate how different subjects are used together to solve an authentic problem. Mathematics, when integrated with science, provides opportunity for students to apply the discipline to real situations that are relevant to the students' world and are presented from the students' own perspective (Beane, 1995:616–662).

Integration also increases learner achievement in both disciplines. Mathematics can enable students to achieve a deeper understanding of science concepts by providing ways to quantify and explain science relationships (Bybee *et al.*, 1997:328). Science

activities illustrating mathematics concepts can provide relevancy and motivation for learning, for example, scientific concepts such as pressure, volume and temperature change and the use of the general gas laws provide opportunities for students to apply these mathematics operations. Manipulations of equations and formulae also require knowledge of mathematical procedures. This shows that mathematics has much to offer in science subjects. Science activities illustrating mathematics concepts can provide relevancy and motivation for the learning of mathematics (Bybee *et al.*, 1997:328).

Students of mathematics should have opportunities to apply their mathematical knowledge and skills to the solution of scientific problems. The use of inquiry methods in science teaching provides many opportunities for incorporating mathematics (Singh, 2000: 579–599). Mathematics is seen as a tool of science for quantifying and testing hypothesis. Students practise the skills learnt in their mathematics classes on a realistic and meaningful level. The difficulty of incorporating mathematics into science classes is compounded by extreme variations in mathematical ability among students. It is the duty of science teachers to convey an understanding of the role of mathematics in science.

3.3 PROBLEMS EXPERIENCED WITH INTEGRATED LEARNING OF PHYSICAL SCIENCE AND MATHEMATICS

Jacob (in Pang & Good, 2000:75) argued that any integrated approach should be based on an understanding of the nature of the disciplines involved in mathematics and science. However, Berlin (in Pang & Good, 2000:75) 'found that mathematics concepts were regarded as of primary importance in previous approaches to integration. Science instructional activities with ancillary related mathematics concepts were the dominant approach'. Many topics in mathematics and science are taught at surface level and few topics are covered or developed in much depth. The problem is that content coverage, not the provision of contextual understanding, has

been the valued mode in mathematics and science teaching. It is therefore difficult to integrate science and mathematics to make disciplines relevant and meaningful to students.

The mathematics teacher must recognise that mathematics in the school is not only a subject in its own right but that it links with other subjects, particularly physical science, and that these links must be explored. Usually, the mathematics teacher teaches for understanding, whereas the science teacher is more interested if the student 'can do it', i.e. if the students are able to apply their mathematics. Integration is based on science as inquiry and mathematics as problem solving (Bybee *et al.*, 1997:328).

Embedded in knowledge construction is the importance of authentic learning contexts and learning in social contexts. Situated knowledge is the result of such activities. It is believed that authentic relevant problem situations will influence knowledge construction and its transferability (Vail, 2002:68-83).

According to Huntley (1999:57–67), middle-school teachers who teach mathematics and science take a directive or modelling approach by providing students with explicit directions for tasks and by maintaining intellectual authority. In such a directive mode of instruction, the students do not get the opportunity to reason about and explore mathematical and scientific ideas, only to acquire procedural knowledge. Real integration requires full understanding of integration ideas. The argument of integration includes the effects of integration on students' conceptual development and the integrated approach at classroom level.

Students are unable to apply their mathematical knowledge and skills in the science classroom; they are even reluctant and unwilling to do so. "... Pupils, who appear to be perfectly familiar in a mathematical context, draw blank looks when they are asked to perform it in scientific calculations" (Jarvis, 1987:2). Teaching methods are vitally important for transfer between subjects, but it is the responsibility of the teacher to teach in such a way that a student's knowledge will be functional in new

situations. This is not possible if science and mathematics teachers work in isolation, each within their own framework, using different terminology, methods and approaches.

Many science teachers do not appreciate the difficulty that students have in learning mathematics. The assumption that mathematics is a tool composed of comparatively easily mastered skills leads to a gross mismatch between the demands of the mathematics and science lessons. In the mathematics lesson the student knows the skills, principles or strategies that must be applied to the given mathematics or word problem. What is assessed is students' ability to apply a skill, principle or strategy efficiently and accurately. In a science lesson, the student must first recognise what skill, principle or strategy to apply, which is frequently the most difficult part of solving a problem. There could then still be a possibility of the student not being able to apply the skill, principle or strategy correctly, especially if it has not been revised in the mathematics lesson recently. The question is: are science teachers aware that students experience difficulties in learning and understanding mathematics and how does the science teacher cope with the range of mathematical abilities that may be present in his/her science group?

3.4 FORMAL AND INFORMAL WAYS OF IMPLEMENTING INTEGRATED CURRICULA OF SCIENCE AND MATHEMATICS

According to Westbrook (in Pang & Good, 2000:75–76), who examined the effects of an integrated curriculum (USA) on students' conceptual organisations, students in an integrated algebra and physical science class used more conceptual linkages in constructing concept maps than did the students in a discipline-specific class. However, in South Africa we still lack formal integration of mathematics and science. One would expect more integration at school level, but even the NCS 2003 does not stipulate the integration of these two subjects clearly. The mathematics and science curricula are developed in isolation. In the content part of the maths curriculum, the

terminology and style used are often unfamiliar to the science teacher (Jarvis, 1987:2–4).

Researchers do not agree about the way that the maths and science curricula should be integrated. Isaacs *et al.* (1997:179–2006) suggest that mathematics should form the basis for an integrated mathematics and science curriculum, because of its inherently logical structure. In contrast to this, Lederman and Niess (1998:281–284) claim that science and mathematics have uniquely different perspectives and attempting to blur the disciplinary boundaries is not desirable. Furthermore, the development of an integrated curriculum is a complex process: “Exploring mathematics (or science) concepts that can be effectively learnt with the support of science (or mathematics) concepts or activities is relatively easy in comparison to constructing a conceptual interdisciplinary framework that includes mathematics and science” (Lederman & Niess, 1998:281–284).

Implicit understanding of the nature of mathematics and science need to be critically examined. The successful implementation of integrated curricula ultimately depends on whether teachers develop a solid understanding of subject matter and computational connection among the subjects (Underhill, 1994:1–2). Wicklein & Schell (1995:1-9) indicate that some elements influence the effectiveness of integrated curricula, which is administrator commitment, collaborative relationships among teachers who share similar integration ideas and significant curriculum changes or innovation.

Previously in the USA, science was taught in connection with an abbreviated mathematics, and the entire curriculum was taught sequentially with one topic preceding the other. (Miller *et al.*, 1993:3–7). Recently, national organisations have started to recognise the importance of the integration of mathematics and science teaching and learning: “students should have many opportunities to observe interaction of mathematics with other subjects and with everyday society” (NCTM, 1989:84). Throughout the NCTM standards (1989:548) the philosophy is that instruction needs “to emphasize exploring, investigating, reasoning and

communicating on the part of all students” and the integration of mathematics and science classes is one step toward such a goal.”

Miller *et al.* (1993:3–7) suggest the following ways in which an integrated curriculum can be implemented:

- ***Discipline-specific integration*** involves an activity that includes two or more different branches of mathematics and science. This type of integration requires a problem where students reach an informed decision based upon data analysis from all the disciplines and the use of critical thinking and problem-solving skills. Students learn that branches of mathematics and science are interrelated. The connections are prominent. However, there are times when mathematics and science must be taught separately for students to know basic concepts, procedures and skills. The standards of the NCTM state that students should be able to “apply mathematical thinking and modelling to solve problems that arise in other disciplines” (NCTM, 1989:84).
- ***Content-specific integration*** involves choosing an existing curriculum objective from mathematics and one from science. It conforms to the previously developed curriculum, infusing the objectives from each discipline. In the integration, the challenge to teachers is to weave together the existing programmes in science and mathematics with objectives from two separate and distinct curricula. The students explore the connection between mathematics in the reality of science, but not all mathematics and scientific concepts can be integrated. Basic mathematical and scientific concepts and processes may need to be taught first and sometimes separately. Specific integration activities may involve only surface-level organisation and development.
- ***Process integration*** involves the use of real-life activities in the classroom. By continuing experiments, collecting data, analysing the data and reporting the results, students experience the processes of science and perform the needed mathematics. Mathematics operations are performed for a purpose: to answer

questions that are of concern to the students about the problem under investigation and generally about real life.

- **Methodological integration** implies that “good” science methodology is integrated in “good” mathematical teaching. Integration of science and mathematics is accomplished by using science methods as the medium of integration, e.g. mathematics developed under the constructivist theory can use science discovery and inquiry teaching techniques. On the other hand, the learning cycle as a method of teaching science can be directly infused with the developments of teaching and learning models in mathematics.
- **Thematic integration** begins with a theme, which then becomes the medium, with all the disciplines interacting (Miller *et al.*, 1993:3–7). This integration was attempted in the initial outcome-based curriculum (Curriculum 2005), where the themes were introduced with the help of phase organisers.

3.5 PROBLEMS EXPERIENCED WITH THE IMPLEMENTATION OF INTEGRATED CURRICULA

While outcome-based education in South Africa is trying to integrate the learning fields in the system, there is still a lack of liaison between mathematics and science departments (Jarvis, 1987:2). No real co-operation exists between the teachers of the respective departments; each is still ignorant of the other’s work, needs and problems. According to Watanabe & Huntley (1998:19), classroom instruction that emphasises mathematics-science connections remains an exception rather than a norm. This might be the result of barriers in developing and implementing an integrating curriculum, such as the lack of high-quality materials and detailed guidance for actualisation in the professional literature.

At school level teachers of science and mathematics do not, as a rule, co-ordinate their syllabi, nor do they use each other’s disciplines in the planning of their schemes of work (Jarvis, 1987:2). Therefore, to make mathematics relevant to the science

being studied, increased co-operation and planning are needed between mathematics and science teachers to bridge the gap that now exists.

Mathematical skills, whether computational or conceptual, are used by a science syllabus without any reference to any scheme of work associated with the use of science-based activities to explore mathematical ideas. Mathematics learnt by science students seems to have little relevance to the mathematics used in their science courses. Despite all the rationales, the desire for integration remains unfulfilled; usually mathematics and science are taught in an unconnected way in most schools. Lonning and Defranco (1997:215) claim: "Integration of mathematics and science can be justified only when students' understanding of the mathematics and science concepts is enhanced."

According to Adams (1998:35–48), there is a lack of subject matter knowledge of elementary school mathematics teachers. This is a concern of how teachers can promote students' conceptual learning in integrated instruction. Many teachers have not studied each subject sufficiently to develop a sound conceptual foundation. Successful implementation of integrated curricula depends solemnly on whether teachers develop a solid understanding of subject matter and conceptualised interrelations among subjects (Underhill, 1994:1–2). There are already some educational programmes that have been designed to foster integration at elementary and secondary school levels in the United States of America (Lonning & Defranco, 1994:18–25). They address the subject matter content to be taught, ways of assessment, and students' attitudes towards integration.

Lehman (1994) shows that pre-service teachers tend to express positive attitudes toward integration, 'whereas in-service teachers display reluctance, partly because of their subject-oriented preparations. It seems that the teacher who does not have the underlying foundational knowledge of other disciplines can at most facilitate superficial connections among disciplines' (Pang & Good, 2000:77). Many of their studies show a lack of subject matter knowledge within mathematics and science.

Sufficient co-ordination between the mathematics and physical science syllabi has not yet been established. OBE is trying to liaise the core syllabi of mathematics and science, but they are still largely developed in isolation and are subject-based (Jarvis, 1987:2–4). It is not yet clear that they have integrated them. For instance, the requirements of the physical science courses are seldom considered in the design of the mathematics syllabi or vice versa. The NCS (2003) is trying to co-ordinate the syllabi, but the mathematical skills, whether computational or conceptual, are still used by science syllabi without any references to any scheme of work associated with the students' experiments in mathematics. And mathematics topics are still denying the use of science-based activities to explore mathematical ideas.

The language of mathematics and the approaches and methods used in the teaching of the subject, are undergoing a change in the NCS (2003) However, many science teachers are unaware of the changing nature of mathematics, the content of the mathematics syllabi, the terminology used and the style in which they are delivered (Jarvis, 1987:2–4).

3.6 REMEDIES FOR THE PROBLEMS

Although the transfer of knowledge is not easy, teaching must be aimed at transfer. It does not take place automatically; it is the responsibility of the teachers to teach in such a way that students' knowledge will be functional in new situations. However, this would not be possible if the mathematics and science teacher still work in isolation, each within their own framework, using different terminology, methods and approaches (Jarvis, 1987:2–4). The question is: how can we facilitate and ensure more effective transfer of mathematical knowledge, skills and strategies from the mathematics classroom to the science classroom?

The solution lies in closer co-operation between teachers of mathematics and physical science. But how close the co-operation and working relationships is established and the best way to promote better understanding and effective planning

is still a problem. "Science is one of the 'users' of mathematics and it is essential that the physical science teacher does not work in isolation and keeps up to date with developments in mathematics teaching" (Jarvis, 1987:2). Many teachers in smaller schools are responsible for teaching both mathematics and science classes, which helps the teacher to be aware of all the disciplines involved.

Mathematics can also gain greatly from science subjects if the teachers of science and mathematics make an effort to plan and standardise the notation system they use. If, for example, mathematics teachers in their applications use science examples and science teachers apply the mathematics at every opportunity in working science problems, students would understand that mathematics and science are inseparable entities and highly important to scientific endeavour (Jarvis, 1987:2–4).

Integration could serve as a foundation to overcome the difficulties (Berlin quoted by Pang & Good, 2000:93–82) dealing with the nature of mathematics and science and their comparisons. Possible solutions are the development of curricula materials and instructional models for integration, connections between teacher education programmes for integration and teachers' subsequent classroom teaching practices, changing perceptions of integration on the part of teaching as well as teachers and the effect of technology-based curriculum progress on students' understanding of mathematics and science.

The following are ways of enhancing co-operation between mathematics and science as recommended by Jarvis (1987:2–4) and Pang & Good (2000:93–82):

- ***Representatives of syllabi committees:*** There should be a representative from the physical science syllabus committee on the mathematical syllabus committee and vice versa. These representatives could provide the necessary link between the two disciplines at syllabus committee meetings. They could note developments, syllabus changes and trends in each other's work and

inform each other of the developments in the respective subjects and their needs and requirements.

- **Co-ordination of syllabi:** The problem identified as far as the lack of co-ordination of the mathematics and physical science syllabi are concerned, is that the area of overlap has not been defined and the respective syllabi have not been sequenced to fit each other's work. The area of overlap needs to be defined, especially when the same topic is taught in the mathematics and science departments (Jarvis, 1987:2–4).
- **Appointment of a mathematics co-ordinator:** Schools have heads of departments that have not necessarily majored with mathematics and science on tertiary level. For instance, a head of department in Natural Sciences covers Mathematics, Science, Agricultural Sciences and Biology, in most cases he or she majored with two subjects among the four; hence it is important to appoint a co-ordinator in the secondary school. This co-ordinator should be vested with the responsibility and authority to co-ordinate and oversee the work in mathematics throughout the school.
- **Dialogue between departments:** Formal inter-departmental meetings and discussions could promote better understanding of each other's problems and demands and bring more effective planning when dialogue occurs between mathematics and science teachers. They would learn some of the language and structures of each other's syllabi. The apparatus and methods used in the teaching of the two subjects and mutual problems may be noted. The science teacher could demonstrate how laboratory equipment are used, discuss the results expected to be arrived at and the mathematical skills and knowledge required to be able to handle the results (Jarvis, 1987:2–4).
- **Integrated work:** If possible, students who take physical science as subject should be grouped in the same mathematics class. Flexible time-tabling allowing for separate mathematics and science lessons and periods for integrated work would be ideal.

- **Exploring teacher education programmes for integration:** The relationships between teachers' knowledge and beliefs and their actual teaching practices have been one of the main areas of exploration in the mathematics and science education literature. Kennedy (as quoted by Pang & Good, 2000:77) suggests that clear evidence is needed of the ways that the characteristics of knowledge and beliefs on the part of teachers actually contribute to their teaching practices. Considering that the influence of teachers on classroom practices is mediated by complex contextual variables, it seems clear that mathematics and science teachers should extend their concerns beyond linear relationships between teachers' variables and their teaching practices. Some studies explore teacher education programmes designed to foster integrated teaching at secondary school levels. These studies address the subject matter content to be taught, ways of assessment, and students' attitudes toward integration (Lonning & Defranco, 1994:18–25). This requires further consideration of developing integrated mathematics and science (methods) sources in teacher education programmes.
- **Using technology for integration:** Technology has been incorporated in an attempt to integrate instruction between mathematics and science (Pang & Good, 2000:77–78) and to facilitate collaboration among users. Integration can be achieved by the use of current technology and curriculum resources (Roth, 1992:293–318). According to Berlin (1990:254–257), recent technological advances provide students with a means for direct involvement with science and mathematical ideas in a computer-based laboratory environment. The integrated methods class will enable students to see the strengths and importance of each discipline and expose them to the appropriate use of the technology and curriculum resources that already exist.

3.7 CONCLUSION

There is a need for the integration of the mathematics and physical science curriculum. Linkages are needed between the disciplines in terms of content,

processes involved, methodology and themes. Mathematics and science must be relevant to the students' world; something that can be applied in real-life situations. If mathematics and science were integrated, students would have the opportunity to apply what they have learnt in the mathematics class in the science class.

A few researchers have discussed substantive psychological reasons why integrating mathematics and science education would improve students' understanding. It follows that the interconnectedness of teaching and learning of mathematics and science should benefit students. Therefore, teachers must be aware of the uses of mathematics in the science class at a particular grade in order to point out the possible application to students.

A reason why mathematics and science have not yet been integrated, is that both are aimed at content coverage, but not at contextual understanding. The interconnection solely depends on whether teachers develop a solid understanding of the subject matter and the connection among the subjects.

The use of different technologies is perceived as powerful for effective integration. The potential value of technology can be realised by analysing how the characteristics of the curriculum it delivers promote the students' understanding of science and mathematics.

CHAPTER 4 **ANALYSIS OF MATHEMATICAL KNOWLEDGE AND SKILLS NEEDED IN PHYSICS FOR GRADES 11 AND 12**

4.1 INTRODUCTION

In the previous chapter the need for integration of science and mathematics was emphasised. Underhill *et al.* (1994:1–2) suggest that the successful implementation of integrated curricula ultimately depends on whether teachers have a solid understanding of the subject matter and have conceptualised the connections among subjects. In this chapter the types of mathematical knowledge and skills needed for physics in grades 11 and 12 are analysed in terms of research on what constitutes mathematical knowledge.

Bell (1991:108–110) divides mathematical knowledge into four different mathematical objects that can be linked to the different levels of Bloom’s taxonomy of educational objectives. Hiebert and Carpenter (1992:65–97) divide mathematical knowledge into procedural and conceptual knowledge. The division into mathematical objects (Bell, 1991: 108–110) can be connected to the more formalistic view of mathematics, while the division into procedural and conceptual knowledge (Van de Walle, 2004:27–28) is more in line with constructivist views of mathematics. Both these divisions are discussed in more detail in the next paragraphs. Thereafter the knowledge and skills needed in physics for grades 11 and 12 are categorised in terms of conceptual and procedural knowledge.

4.2 THE OBJECTS OF MATHEMATICS LEARNING

From a formalistic point of view, mathematics can be viewed as a fixed body of knowledge that can be divided into mathematical objects. The objects of mathematics learning consists of direct and indirect objects that teachers want students to learn in mathematics (Bell, 1991: 108–110). Direct mathematical learning is facts, skills, concepts and principles. Indirect mathematical objects are transfer of learning, inquiry ability, problem solving, self-discipline and appreciation for the structure of mathematics.

4.2.1 DIRECT OBJECTS OF MATHEMATICS LEARNING

The direct objects of mathematical learning can be summarised as follows:

- **Mathematical facts** are those arbitrary conventions in mathematics such as symbols of mathematics (Bell, 1991: 108–110). Facts are learnt by memorisation, drilling, practice, timed tests, games and contests, i.e. rote learning. Students have learnt a fact if they can state it and make appropriate use of it in different situations. Van de Walle (2004:156) defines a fact in a more constructivist manner as something a student has to master without resorting to non-efficient methods, e.g. finding the product of 7×8 . Bell (1991: 108–110) refers to this procedure as a skill.
- **Mathematical skills** are operations and procedures that students are expected to carry out with speed and accuracy (Bell, 1991: 108–110). Many skills are specified by sets of rules and instructions; ordered sequences of specific procedures (algorithms). Students are expected to master mathematical skills such as long division, addition of fractions, multiplication of decimal fractions, constructing right angles, bisecting angles, and finding intersections of sets of objects. Skills are learnt through demonstrations and various types of drilling and practice (e.g. worksheets, chalkboard work, group activities and games).

Students are considered to have mastered a mathematical skill if they can correctly demonstrate the skill by solving different problems requiring the skill or by applying the skill to different situations.

- **Mathematical concepts** are abstract ideas (Bell, 1991: 108–110). Sets, subsets, equality, inequality, triangle, cube, radius and exponent are all examples of concepts. Concepts can be defined through definitions. It can be learnt by hearing, seeing, handling, discussing, or thinking about a variety of examples and non-examples of the concept and by constructing the examples and non-examples. A student has learnt a concept if he or she is able to separate examples of the concept from non-examples.

Cangelosi (2003:173–179) defines conceptual knowledge in terms of specifics. Real life is composed of 'specifics' that students detect with their empirical senses. There are numerous specifics to think about as a unique entity, hence it is categorised and sub-categorised according to certain commonalities or attributes (Cangelosi, 2003: 173–179). The process that a student uses to group specifics to construct a mental category is called conceptualisation. Constructing a concept is an abstraction that enables students to extend what they understand beyond the specific situations they have experienced in the past. According to the constructivist view, concepts are the building blocks of mathematical knowledge.

- **Mathematical principles** are the most complex mathematical objects (Bell, 1991: 108–110). They are sequences of concepts together with relationships among these concepts. Principles involve several concepts and relations among these concepts. A concept is a single mediator that represents a class of stimuli or objects, whereas a principle is a sequence of mediators (Bell, 1991: 108–110). Cangelosi (2003: 173–179) prefers to use the term relationships instead of principles. Students discover relationships through inductive reasoning. That is, students form hypotheses or formulate a proposition from their experiments with specifics (Cangelosi, 2003: 173–179).

The objects of mathematics (facts, skills, concepts and principles) are taught at all grade levels (Bell, 1991: 108–110). Facts and skills are emphasised in lower grades, concepts in intermediate grade levels and principles in higher grades. Grades 6-9 students tend to be concrete operational thinkers, whereas grades 10-12 students tend to be formal operational thinkers. Most secondary school students learn better if the new concepts and principles are illustrated by concrete representations. Facts and skills are taught by using expository methods, demonstrations and individualised models. Concepts and principles are approached through discovery, inquiry and laboratory models.

4.2.2 EDUCATIONAL OBJECTIVES

Bloom and his associates published their taxonomy of educational objectives in 1956 (see Bell, 1991:167–182). There are three types of educational objectives, i.e. cognitive, affective and manipulative objectives. *Cognitive objectives* specify behaviour that indicates the functioning of and changes in various mental processes. *Affective objectives* specify behaviours that indicate changes in attitudes, whereas *manipulative objectives* specify behaviour which shows that students have learnt certain physical manipulative skills.

The purpose of this hierarchical classification system is to categorise the cognitive changes produced in students as a result of the goals and methods of our formal educational system; changes that can be inferred from problem- solving, testing and observations (Bell, 1991: 167–182). Teachers can use this taxonomy as an aid in formulating instructional objectives, selecting teaching methods and designing tests and activities to determine students' learning. The hierarchy of Bloom's taxonomy according to the complexity of observable behaviour is knowledge, comprehension, application, analysis, synthesis and evaluation. This is based on objectives and application from secondary school mathematics.

-
- **Simple knowledge** objectives emphasise the mental processes of remembering information in the same way that it was presented. Recalling knowledge requires little more than bringing to mind relevant material. The knowledge category does not include any degree of understanding mathematics (Bell, 1991: 167–182). According to Cangelosi (2003:167), students achieve an objective at the simple-knowledge learning level by remembering a specific response to a specific stimulus.

In the school mathematics course, we want students to remember mathematical symbols, facts, skills and principles (Bell, 1991: 167–182). They must remember symbols for addition, subtraction, multiplication and division. They must be able to define natural, rational and real numbers; to recall facts for one-digit numbers; to remember the procedures for carrying out long divisions and extracting square roots; and to state principles such as the Pythagorean Theorem, the factor theorem, and the distributive law for multiplication over division.

- **Comprehension:** Students comprehend a mathematical idea if they can use it without relating it to other ideas or understanding all its implications (Bell, 1978:170). Cangelosi (2003:159–162) defines comprehension as extracting and interpreting meaning from an expression, using the language of mathematics and communicating with and about mathematics. One of the aspects of comprehension (Bell, 1978:170) is the ability to *translate* verbal statements or problems into mathematical symbolism or vice versa. Another aspect is *interpretation*, which is the ability to formulate new viewpoints of material. Many of the behaviours expected of students in graphing functions involve interpreting data in different ways or forms (Bell, 1978:170). “Activities such as sketching graphs, understanding graphs and charts and interpreting lists of data have the objective of comprehending activities of translation, interpreting and extrapolating” (Bell, 1991:170).

- **Application:** Knowledge is useless unless it is applied to problem solving. It should involve the use of acquired knowledge in different situations (areas), applying the acquired theories and practices to solve problems, and bringing general principles to bear on new questions (Van den Aardweg & Van den Aardweg, 1988:32–33). To demonstrate the ability to apply a mathematical abstraction, students must select it and use it correctly. The ability to select the appropriate mathematical techniques, postulates and theorems to prove a new theorem is an example of an application of mathematics. Students can usually comprehend mathematical ideas well, but apply them badly (Bell, 1991:170).
- **Analysis** involves the breaking down of the whole to clarify relationships and constituent parts, finding relationships among the parts, and observing the organisation of the parts. It involves the ability to differentiate between hypotheses and facts, finding themes and patterns, identifying hidden meanings and understanding the system of organisation (Van den Aardweg & Van den Aardweg, 1988:32–33). In general, analysing mathematical ideas or structures requires a higher order of comprehension than just applying that structure. In solving problems in arithmetic, algebra, trigonometry, and calculus, students must analyse the relationships among the unknowns and the given information (Bell, 1991:170).
- **Synthesis** is the process of creating or recombining elements to form a new whole; rearranging or reclassifying to make a new structure. It involves organising a set of ideas to form a new structure, developing plans to test a new hypothesis, creating a new form of classifying data, discovering new alternatives and changing them, and improving alternatives (Van den Aardweg & Van den Aardweg, 1988:32–33).
- **Evaluation** is making judgements about the value of ideas, methods and creations. This is the highest type of educational objective because it involves the use of knowledge, comprehension, new applications, and unique methods of analysis and synthesis (Bell, 1991:170). This is a process of appraisal,

assessing or criticising on the basis of specific of logical accuracy and consistency, verifying the value of the evidence, evaluating according to specific criteria, comparing and discriminating between the theories and generalisations, and assessing work. (Van den Aardweg & Van den Aardweg, 1988:32–33).

Concept learning includes knowledge, comprehension and application, while analysis objectives are the appropriate objectives for higher-level concept formation. Cognitive objectives for principles are analysis and synthesis. Evaluation is an appropriate objective when sets of principles are to be compared or structured into mathematical systems (Bell, 1991: 170–171).

Some teachers think that there is an over-emphasis on knowledge and comprehension in mathematics textbooks, teaching methods, homework assignments and tests (Bell, 1991: 170–171). A large proportion of text of textbook problems and test questions require only the lower-level cognitive activities of knowledge and comprehension. This indicates that high school mathematics students are infrequently required to practise synthesis and evaluation. Many tasks require application, while analysis is contrived and lacking in relevance.

This problem was probably one of the motivations for the introduction of the constructivist approach to mathematics learning. According to this approach, mathematical knowledge is divided into conceptual and procedural knowledge.

4.3 CONCEPTUAL AND PROCEDURAL KNOWLEDGE IN MATHEMATICS

The division of mathematics into conceptual knowledge and procedural knowledge is based on the assumption that all knowledge consists of internal or mental representations of ideas that the mind has constructed (Hiebert & Lindquist, 1990:17–36). Mathematics with its tightly structured and clearly defined content has

provided an arena for much discussion of conceptual knowledge and procedural knowledge.

The tools of mathematics are abstraction, symbolic representation and symbolic manipulation. However, being trained in the use of these tools does not mean that one thinks mathematically, just as knowing how to use shop tools does not make a craftsman (Schoenfeld, 1994:60). Learning to think mathematically means the development of a mathematical point of view and valuing the processes of mathematisation and abstraction. It also means having the predilection to apply them and developing competence with the tools of the trade. It entails using the tools in the service of the goal, namely understanding structure and mathematical sense-making (Schoenfeld, 1994:60).

4.3.1 CONCEPTUAL KNOWLEDGE

Conceptual knowledge can be roughly defined as knowledge that is understood (Hiebert & Carpenter, 1992:65–97). Conceptual knowledge is characterised as knowledge that is rich in relationships, i.e. it is a connected web of knowledge, a network of linking relationships between the discrete pieces of information (Hiebert & Lefevre, 1986:3). Its development is achieved by constructing relationships between the existing knowledge stored in memory and the one that is newly learnt. Insights are the basis of discovery (Bruner, 1961:21–32) and they lead to an increase in conceptual knowledge. Relationships can tie together small pieces of information or larger pieces that are themselves a network of some sorts. Understanding is the term used most often to describe the station of knowledge when new mathematical information is connected appropriately to existing knowledge. According to Piaget (Hiebert & Carpenter, 1992:65–97), the heart of the process involves assimilating the new information into appropriate knowledge networks or structures. The results are that the new knowledge must become part of an existing network.

4.3.2 PROCEDURAL KNOWLEDGE

Procedural knowledge consists of two distinct parts, i.e. formal language (symbol representation system of mathematics) and algorithms, or rules for completing mathematics. Formal language includes a familiarity with the symbols used to represent mathematical ideas and an awareness of the syntactic rules for writing symbols in an acceptable form (Hiebert & Lefevre, 1986:7). For example, $3, 5 \times \square = 2, 71$ are syntactically acceptable, but $6+ = 2 \square$ is not acceptable.

Rules, algorithms or procedures are used to solve mathematics tasks. They are step-by-step instructions that prescribe how to complete tasks. They are executed in a predetermined linear sequence. That is, for the completion of a task, the initial procedure operates on the input and produces an outcome that is recognised by the next procedure in the sequence.

Procedural knowledge is important in both learning and in doing mathematics. Algorithmic procedures help to do routine task easily and free the mind to concentrate on more important tasks (Van de Walle, 2004:28). Symbolism is a powerful mechanism for conveying mathematical ideas and for playing around with an idea. It is important to note that procedural rules should never be learnt in the absence of a concept (Van de Walle, 2004:28).

At school, mathematics students are presented with problems in the form of symbol expressions, such as adding whole numbers, translating from common to decimal fraction notation, or solving algebraic equations. Some procedures manipulate written mathematics symbols, whereas others operate on concrete objects, visual diagrams, or other entities (Hiebert & Lefevre, 1986:3). The important feature of the procedural system is that it is structured. Procedural knowledge also includes strategies for solving problems that do not operate directly on symbols.

4.3.3 MEANINGFUL LEARNING

The biggest difference between procedural knowledge and conceptual knowledge is that the primary relationship in procedural knowledge is 'after' which is used to sequence different procedures linearly. In contrast conceptual knowledge is saturated with relationships of many kinds (Hiebert & Lefevre, 1986:7). Meaning is generated as relationships between units of knowledge that are recognised or created. Conceptual knowledge must be learnt meaningfully. Procedures can be learnt without meaning, however without meaning transfer would be difficult to achieve. Procedures that are learnt meaningfully are those that are linked to conceptual knowledge.

4.3.4 ROTE LEARNING

Rote learning produces learning that is absent in relationships and is tied closely to the context in which it is learnt. The knowledge that is learnt from rote learning is not linked with other knowledge and therefore cannot be generalised to other situations (Hiebert & Lefevre, 1986:2–9). It can be accessed and applied only in those contexts that look very much like the original. Conceptual learning cannot be generated directly by rote learning. Facts and propositions learnt by rote are stored in the memory as isolated bits of information and are not linked to any conceptual network. It might happen that a student later recognises or constructs relationships between isolated pieces of information. Conceptual knowledge is then created from information that was initially learnt by rote.

Procedures can be learnt by rote. They can be acquired and executed even if they are linked tightly to surface characteristics of the original context. Procedures, especially those that operate on symbol patterns, are triggered by surface features similar to those of the original context. Procedures are often learnt by rote, since they seem susceptible to this form of instruction. It is possible to consider procedures without concepts, but as mentioned before, the procedure will be tied to the context

in which it has been learnt and cannot easily be transferred to other situations (Hiebert & Lefevre, 1986:9).

4.3.5 RELATIONSHIPS BETWEEN CONCEPTUAL AND PROCEDURAL LEARNING

Mathematical knowledge includes fundamental relationships between conceptual knowledge and procedural knowledge. Students are not fully competent in mathematics if either kind of knowledge is deficient or if both kinds have been acquired but remains as separate entities (Hiebert & Lefevre, 1986:9). When concepts and procedures are not connected, students may have a good intuitive feeling for mathematics, but may not be able to solve problems. Building relationships between conceptual knowledge and the formal system of mathematics is the process of giving meaning to the symbols. Building relationships between conceptual knowledge and the procedures of mathematics contributes to memory of procedures of mathematics and to their effective use (Hiebert & Lefevre, 1986:9).

Procedures translate conceptual knowledge into something observable. Conceptual knowledge linked to a procedure can monitor the selection and use of procedural outcomes. It can also evaluate the reasonable procedural outcome (Hiebert & Lefevre, 1986:9). Conceptual knowledge informs us that a procedure is inappropriate. This happens when the procedure is violating a conceptual principle. It also acts as a screening agent to reject inappropriate procedures and warns students if their answer is inappropriate. Conceptual knowledge thus plays a strong role in strategic decision making.

Conceptual knowledge releases the procedure from the surface context in which it was learnt. It encourages its use in other structurally similar problems. From this, one can conclude that without conceptual knowledge, a student will not be able to solve problems in a real-life context (Hiebert & Lefevre, 1986:9).

4.4 KNOWLEDGE AND SKILLS NEEDED FOR PHYSICAL SCIENCE IN GRADES 11 AND 12

In this section the mathematical skills and knowledge needed for physical science are analysed against the background of the learning outcomes for physics in grades 11 and 12 (NCS, 2003). Firstly, the outcomes are discussed and then the mathematical skills and knowledge needed are summarised in terms of procedural and conceptual knowledge.

4.4.1 OUTCOMES FOR PHYSICAL SCIENCE IN GRADES 11 AND 12

Physical science outcomes are aligned to three focus areas, i.e. they aim to develop the abilities of *doing* (skills), *applying* (knowledge) and lastly *being and becoming* (values and attitudes) (NCS, 2003:6). The following are four knowledge areas in physics (grades 11 and 12), which needs mathematical skills and knowledge:

- Mechanics
- Waves, sound, and light
- Electricity and magnetism
- Matter and material (integrated with chemistry)

Students demonstrate achievement of the learning outcomes if they are able to conduct an investigation, interpret data and draw conclusions, solve problems, communicate and present information and scientific arguments (NCS, 2003:6). They also demonstrate the achievement of outcomes if they are able to recall and state specified concepts, and are able to indicate and explain relationships. They should also be able to apply the scientific knowledge gained.

Students display their level of achievement if they can evaluate knowledge claims and science's inability to stand in isolation from other fields. They should also be able to evaluate the impact of science on human development. They display it if they can evaluate science's impact on the environment and sustainable development (NCS, 2003:6).

If we take equations of motion as an example, students need to consider the 'physics' of the problem rather than blindly plugging numbers into a formula. Students often struggle to 'unpack' a question. They have difficulty in extracting the information that has been given to them and what they have been asked to calculate. In some cases, the problem lies with the wording of a question. Take the example of a body accelerating from rest (cf. Basson, 2002, NCS, 2003:108). To distinguish between initial and final velocity remains a serious obstacle in problem solving. Since acceleration in most cases is positive, students tend to generalise that the final velocity is the one with greater magnitude. Students have to be clear about the fact that the initial velocity is the velocity at the beginning of the motion under consideration, while the final velocity is the velocity at the end.

4.4.2 SUMMARY OF MATHEMATICAL KNOWLEDGE AND SKILLS NEEDED IN GRADES 11 AND 12 PHYSICAL SCIENCE

After analysis of the National Curriculum Statement, textbooks and question papers of physics for grades 11 and 12, the researcher identified the mathematical knowledge and skills involved in the solving of physics problems. This knowledge and skills were categorised by using the four headings of mathematical objects according to Bell (1991:108), namely facts, skills, concepts and principles (see paragraph 4.2.1). To make the classification easier to work with and more in line with the constructivist approach, the researcher has chosen to include alternative headings, namely simple knowledge, procedures, understanding and relations. The heading *simple knowledge* and *procedures* can also be seen as procedural knowledge, while *understanding* and *relations* can be seen as referring to conceptual

knowledge. The meaning of the terms conceptual knowledge and procedural knowledge has been discussed in paragraphs 4.3.1 and 4.3.2. The classification is summarised in table 4.1.

TABLE 4.1: MATHEMATICAL KNOWLEDGE AND SKILLS NEEDED IN PHYSICS EDUCATION FOR GRADES 11 AND 12

Facts/Simple knowledge	Skills/Procedures	Concepts/ Understanding	Principles/ Relationships
CONVERSION OF UNITS			
Knowing conversion factors of units e.g. 100 cm = 1 m	Doing simple conversions quickly e.g. 2000 g = 2 kg	Direct and indirect proportions e.g. 10 g = 0,01 kg	Double conversions e.g. 10 m/s = 36 km/h
SCIENTIFIC NOTATION			
Knowing conversion factors for powers: e.g. 1000 = 10 ³	Doing simple conversions to powers quickly: e.g. 2000 = 2 x 10 ³	Understanding the conversion of fraction to power: e.g. $\frac{1}{10} = 10^{-1}$ because $\frac{1}{10} = 0,1$	Using laws of exponents to simplify in scientific notation: e.g. (2 x 10 ³)(3 x 10 ⁴) = 6 x 10 ⁷
CHANGING THE SUBJECT OF THE FORMULA			
Subtraction is inverse of addition Division is inverse of multiplication	Changing simple equations e.g. If $F = ma$ then $m = \frac{F}{a}$	Understanding proportions e.g. if $F = \frac{Gm_1m_2}{r^2}$, then if distance doubles the force is reduced by factor 4	Changing more complex equations e.g. If $v = u + at$ then $a = \frac{v-u}{t}$

Facts/Simple knowledge	Skills/Procedures	Concepts/ Understanding	Principles/ Relationships
USING FORMULAE TO CALCULATE A VALUE			
Substitution i.e. replacing symbols by numbers i.e. F by 10 N	Using calculator to find value	Understanding symbols	Changing the subject of formula to get correct physical quantity
DRAWING AND INTERPRETATION OF GRAPHS			
(x, y) represent a point on graph	Draw points (x, y)	gradient = $\frac{\Delta y}{\Delta x}$	Connecting slope to rate of change e.g. velocity or acceleration
SOLVING REAL-LIFE PROBLEMS			
Knowing formulae involved	Making a schematic diagram Substituting values into formulae Calculations	Understand relevant formula i.e. choose the correct formula	Interpretation of data Unpacking the given data Connecting given data to the formula
USING TRIGONOMETRIC RATIOS AND LAWS			
Knowing trigonometric ratios and law of sine and cosine	Doing calculations with trigonometric ratios and law of sine and cosine	Understanding trigonometric ratios and law of sine and cosine	Using trigonometric ratios and law of sine and cosine in solving real life problems

MEASUREMENT			
Knowing units of measurement e.g. length in cm	Draw lines and angles	Understand concept of an angle	Make scale drawings

4.5 SUMMARY

One of the aims was to identify the algebraic knowledge and skills in physics education, this was done in terms of procedural and conceptual knowledge. In mathematics, linking conceptual knowledge and procedural knowledge has many advantages, especially that of procedural knowledge, because it is informed by conceptual knowledge. Conceptual knowledge results in symbols that have meaning and procedures that can be remembered better and used in different situations. Procedural knowledge provides students with a formal language and sequences that raise the level and applicability of conceptual knowledge.

Relationship between conceptual knowledge and procedural knowledge holds the key to many learning processes and problems. What is important is to understand the acquisition of this knowledge. This knowledge will help students to unlock doors that have hidden significant learning problems.

If we combine the teaching of knowledge with the teaching of skills needed, we will help to develop students that have knowledge, skills and understanding. Students will be in a strong position to be fully participating, contributing members of a democratic society. They will be able to cope with the changing nature of employment in the future.

The table 4.1 (pp. 47 - 48) outlines the relationship between conceptual and procedural knowledge needed in the teaching and learning of physics in grades 11 and 12..

CHAPTER 5 **EMPIRICAL STUDY AND DISCUSSION OF THE RESULTS**

5.1 INTRODUCTION

The aim of this study was to identify the mathematical knowledge and procedures used in physics at school level (grades 11 and 12) and the extent to which students understand and master these procedures. In Chapter 4 the mathematical knowledge and skills needed for physics in grades 11 and 12 were determined and classified in terms of conceptual and procedural knowledge (see Table 4.1).

The aim of the empirical study was to identify the mathematical problems of science students that restrict effective teaching and learning of physics at grade 11 and 12 levels. This was done by means of a test to determine the main trends of students' common practices.

5.2 DESIGN OF QUALITATIVE STUDY

A qualitative research method in the form of a phenomenological study was used in the research (Leedy & Ormrod, 2001:153,154). Qualitative research involves an interpretative, naturalistic approach to its subject. It is an inquiry process of understanding based on methodological traditions of inquiry that explore a social or human problem (Creswell, 1997:402). It attempts to understand students' perception, perspectives and understanding of a particular situation. The researcher then makes generalisations of what something is like from an insider's perspective (Leedy & Ormrod, 2001:153,154). As a phenomenological researcher, the researcher depended on the test given to the students on mathematical skills and knowledge in physics and the 'interview' questionnaires of teachers about their students'

performance to determine the students' ability to use mathematics in solving physics problems.

5.3 PROCEDURES USED IN EMPIRICAL STUDY

The following steps were followed in the empirical study:

- Construction of the test for students from information summarised in Chapter 4.
- Piloting the test on Grade 11 students
- Administering of test to students in Grade 12
- Processing of test
- Constructing two questionnaires for teachers
- Processing of teachers' questionnaires
- Comparison of teachers' questionnaire results with students' test results

5.4 MEASURING INSTRUMENTS

The empirical investigation was done by means of a test for students and two questionnaires for teachers.

Test for students:

The information summarised in Table 4.1 in Chapter 4 was used to construct a test to investigate the students' ability to use mathematics in solving physics problems (appendix A). It was decided to restrict the test to algebraic skills and knowledge. The Grade 11 class that did not form part of the target population was used to pilot the questionnaire. This ensured that the items used in the questionnaire were at the level of Grade 12 students' comprehension.

Questionnaires for teachers:

A questionnaire was constructed asking teachers to rate the algebraic performance of students in their classes in various topics in physics (appendix B). These topics corresponded with the topics in the test for the students. Another questionnaire was compiled to retrieve information about collaboration between science and maths teachers (appendix C).

5.5 POPULATION

The students involved in the survey were all in Grade 12 and enrolled for both physical science and mathematics. In total 120 students from four different high schools in the Rustenburg Region in the North-West Province, South Africa, were involved in the survey. Two of the high schools are in the township and the other two are in rural areas. Approximately 75% of the students were from rural areas, while the majority of the students travelled from their homes to the townships for 'better teaching and learning'.

Twenty-eight (28) teachers were involved in the teachers' questionnaire. Eighteen (18) of these teachers were enrolled by the project called SEDIBA (*an upgrading programme for science and mathematics teachers*) at the Potchefstroom Campus of the North-West University. The project is aimed at improving the standard of effective teaching and learning in the North-West Province. The other 10 teachers were selected from the four high schools in Rustenburg mentioned in the paragraph above, as well as some neighbouring schools.

5.6 TEST FOR STUDENTS

5.6.1 CONSTRUCTION OF TEST FOR STUDENTS

The test (see Appendix A) consisted of 17 items. These items focussed on the following topics: decimal fractions, scientific notation, proportionality, changing the subject of the formula, equations of motion and interpretation of graphs as samples of physics topics from mechanics that need algebraic skills and knowledge. The items were selected from previous Grade 12 examination papers and study guides (appendix D), therefore there was no need to seek moderations for standardisation.

5.6.2 PILOTING THE TEST FOR STUDENTS

A selection of questions applicable to Grade 11 students was used to pilot the test on a Grade 11 class taking both mathematics and physical science. This was done in July 2005, when most of the Rustenburg Region high schools (North-West Province, South Africa) were expected to have finished their physics part of the curriculum, as they were writing a common paper for the region. The reason for using Grade 11 students was to avoid contamination of the test for Grade 12 students.

5.6.3 ADMINISTERING THE TEST FOR STUDENTS

All students that were present on the day that the questionnaire was administered completed it. Students completed this questionnaire under normal test procedures (strict supervision). The test was to contribute to their continuous assessment mark. This was done in order to get a true reflection of their level of understanding and existing knowledge base. They were not warned about the tests beforehand, since the aim was to test their existing knowledge and skills. They were given ample time to write until they had completed the test.

5.6.4 PROCESSING OF THE TEST FOR STUDENTS

Analysis of the students' responses to the test was done according to a procedure described by Gilbert (1991:73–79). According to this procedure, the responses were first read through to get an overview of the ideas and opinions held by the students and to identify the specific problems that the students experienced when trying to solve the problems. The second reading involved the actual marking of the test and the collection of statistics.

In the collection of the statistics the students' performances were analysed in three ways: firstly according to each question set, then according to the different topics and lastly according to the different mathematical objects. The aim of this analysis was to get more insight into the origin of the problem. Through this analysis it will be possible to see if the problem is topic-related or to see if it is due to a lack of skills or a lack of conceptual knowledge.

5.6.5 RESULTS AND DISCUSSION OF THE TEST FOR STUDENTS

The results are summarised in tables 5.1, 5.2 and 5.3. Table 5.1 provides statistics per question, Table 5.2 gives the statistics per sub-topic and Table 5.3 gives an overview of the performance for different mathematical objects.

5.6.5.1 Discussion of results per question

TABLE 5.1: PERFORMANCE OF STUDENTS PER QUESTION

NUMBER OF QUESTION	STUDENTS PASSED	%	STUDENTS FAILED	%	STUDENTS THAT MADE NO ATTEMPT	%
DECIMAL CONVERSIONS						
1	24	20,0	12	10,0	84	70,0
2	54	45,0	66	55,0	0	0
3	42	35,0	78	65,0	0	0
4	32	26,7	88	73,3	0	0
SCIENTIFIC NOTATION						
5a	83	69,2	37	30,8	0	0
5b	68	56,7	52	43,3	0	0
6a	47	39,2	72	60,0	1	0,8
6b	48	40,0	72	60,0	0	0
7a	48	40,0	72	60,0	0	0
7b	64	53,4	55	45,8	1	0,8
8	27	22,5	93	77,5	0	0

NUMBER OF QUESTION	STUDENTS PASSED	%	STUDENTS FAILED	%	STUDENTS THAT MADE NO ATTEMPT	%
PROPORTIONALITY						
9a	22	18,3	98	81,7	0	0
9b	21	17,5	99	82,5	0	0
9c	18	15,0	102	85,0	0	0
9d	2	1,7	118	98,3	0	0
9e	2	1,7	118	98,3	0	0
9f	2	1,7	118	98,3	0	0
9g	5	4,2	115	95,8	0	0
CHANGE THE SUBJECT OF THE FORMULA						
10	86	71,6	32	26,7	2	1,7
11	63	52,5	54	45,0	3	2,5
12a	30	25,0	88	73,3	2	1,7
12b	48	40,0	70	58,3	2	1,7
EQUATIONS OF MOTION						
13(v)	81	67,5	39	32,5	0	0
13(t)	69	57,5	51	42,5	0	0
13(a)	60	50,0	60	50,0	0	0
13(s)	32	26,7	88	73,3	0	0
13(a2)	28	23,4	91	75,8	1	0,8
13(t2)	33	27,5	82	68,3	5	4,2

NUMBER OF QUESTION	STUDENTS PASSED	%	STUDENTS FAILED	%	STUDENTS THAT MADE NO ATTEMPT	%
INTERPRETATION AND APPLICATION OF GRAPHS						
14ai	87	72,5	31	25,8	2	1,7
14aii	1	0,8	117	97,5	2	1,7
14aiii	76	63,3	42	35,0	2	1,7
14aiv	3	2,5	114	95,0	3	2,5
14b	26	21,7	93	77,5	1	0,8
14c	23	19,2	96	80,0	1	0,8
14d	39	32,5	81	76,5	0	0
14e	20	16,7	100	83,3	0	0
14f(AB)	3	2,5	117	97,5	0	0
14f(BC)	3	2,5	117	97,5	0	0
14f(graph)	0	0	120	100	0	0

ITEM 1: DECIMAL CONVERSION

1. How many centimetres are there in 50 km?

The majority of students did not make an attempt to answer this question. Apparently the double conversion factor was a problem. They could not convert centimetres to kilometres. The majority of those who attempted it failed because they used 1000 instead of 100 000 (double conversion). A few did not do any calculation but just gave an answer, which was incorrect.

2. How many kilograms are there in 50 grams?

All students attempted this question. Some 54 of the students managed to get the correct solution, 66 got it wrong, while 33 of the students divided by 100 instead of 1000. This indicates a lack of simple knowledge. Twenty (20) students did not know whether to multiply or to divide to convert from kilograms to grams, which indicates a lack of conceptual knowledge about indirect proportions. Thirteen (13) students just wrote down incorrect solutions, which show a lack application of an algebraic procedure.

3. Convert 3 minutes to hours

All students realised that to change from minutes to hours or vice versa the number 60 was involved. The problem was when to divide or multiply. They lacked conceptual knowledge of proportional reasoning to solve the problem. Some 42 of them got it right, while 78 answered incorrectly.

4. Convert 60 km/h to m/s

Most students realised that they had to multiply and divide and that 60 and 1000 were the values to be used, but how to use it was the problem. Eighty-eight (88) of the students failed and 32 passed. Fifty-two (52) of the 88 students knew that they had to use 60 and 1000, but lacked the proportional reasoning skills to apply these numbers correctly. Thirty-two (32) students managed to multiply and divide correctly.

ITEM 2: SCIENTIFIC NOTATION

5. Write each number as a power of 10:

a. $100 = \dots\dots\dots$ and b. $\frac{1}{10} = \dots\dots\dots$

All of them attempted this question: 83 found the answer correctly, 37 were wrong. This result was not too bad. Those who got it wrong lacked the simple knowledge that $100 = 10^2$ and $\frac{1}{10} = 10^{-1}$.

6. Write each constant in scientific notation:

speed of light in vacuum: 299792458 m/s =

wave length of a red light ray: 0, 00000075m =

All except one student attempted this question, 47 got it right and 72 got it wrong. The students seemed to have a problem working with very large and very small numbers. They did not know where to put a comma and did not count the digits correctly. It seems that they find conversion to decimal fractions difficult.

7. Use your calculator to simplify the following and write the answer in scientific notation:

a. $0,000024 \times 6000$

b. $60\ 000\ 000 - 0,000012$

All of them answered the question. Only 48 managed to pass and 72 failed. They could use their calculator correctly, but the scientific notation part seemed to be the problem. The skill of using a calculator was not a problem, but they could not put their answer into scientific notation.

8. Simplify without using a calculator: $\frac{(4 \times 10^3) \times (2 \times 10^9) \times (3 \times 10^{-6})}{(3 \times 10^2) \times (6 \times 10^{-4}) \times (3 \times 10^4)}$

No less than 93 students failed because they did not adhere to the instruction of not using a calculator. They lacked a relational understanding as they did not understand

the connection between the example and the rules of exponents, i.e. they could not do division and multiplication with exponents. Only 27 answered correctly.

ITEM 3 PROPORTIONALITY

9. Given: $F = \frac{Gm_1m_2}{r^2}$, with F the gravitational force between two masses m_1 and m_2 separated by a distance of r . In each of the following, give only the answer (no calculations are required). How does F change if:
- a. Both masses are doubled
 - b. Both masses are halved
 - c. One of the masses is halved while the other is doubled
 - d. The distance between the masses is halved
 - e. The distance between the masses is doubled
 - f. Both masses are doubled and the distance between them is halved.
 - g. Both masses as well as the distance are doubled.

This question was based on pure proportional reasoning without the help of numbers. It included 'relationships' between quantities, i.e. how radius and the two masses affect the force if they change. The ideas of directly proportional and inversely proportional were important here. On average 95% of them did not know what to do. The answers they gave indicate that they lack the concept of proportional reasoning, because they used the trial and error method to answer this question.

ITEM 4 CHANGING THE SUBJECT OF THE FORMULA

10. Given: $P_1V_1 = P_2V_2$

Use the following values to find P_2 :

$$P_1 = 3 \times 10^{-4}, \quad V_1 = 6 \times 10^{-8}, \quad V_2 = 9 \times 10^2$$

This question was well-attempted, and 86 answered correctly. The majority of the students who got it wrong still managed to make the correct substitutions. The procedural knowledge involved to make P_2 the subject of the formula was the problem. Some 'plugged' in the values correctly without changing the original equation, but could not find the correct solution.

11. Given: $v = u + at$

Make a the subject of the formula

Just more than half, namely 63 out of 120 students, got it right. Some 34 of the 63 students did not do any calculations but just wrote down the correct formula. A possible explanation is prior recognition, as science teachers frequently use $a = \frac{v-u}{t}$ in class calculations. This shows rote learning or that they have encountered the problem somewhere before, because the question did not require them to show their calculation steps. Fifty-four (54) students struggled to get the correct solution. They understood the question; but lacked the procedural knowledge to find a formula for a . Three (3) students did not write down anything.

12. Given: $F = \frac{Gm_1m_2}{r^2}$,

a. Make r the subject of the formula

Students struggled to apply their conceptual understanding of division and multiplication to make r the subject of the formula. Only 30 students managed to get it right, while 88 got it wrong. Many of these just wrote down $r = \dots$. Others made more of an effort, e.g. $r^2 = \frac{Gm_1m_2}{F}$, $r = \frac{Gm_1m_2}{r^2}$ and $r = \frac{Gm_1m_2}{F}$.

b. Calculate r if the gravitational force between two large ships with masses of 6×10^4 kg and 2×10^5 kg respectively is $1,64 \times 10^{-2}$ N

Some 48 students passed and 70 failed. Two did not make any attempt at all. The 30 students who could arrive at the correct formula for r in 12a managed to plug in the given values and find the correct answer. Eighteen (18) out of 48 used the original formula $F = \frac{Gm_1m_2}{r^2}$, substituted their values correctly and made the correct calculation. Their problem might have been a lack of understanding of deductive or abstract reasoning.

ITEM 5: EQUATIONS OF MOTION (SUBSTITUTION)

13. Complete the table (do the necessary calculations at the bottom of the page)

Use the following equations:

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

$$s = \frac{1}{2}(v + u)t$$

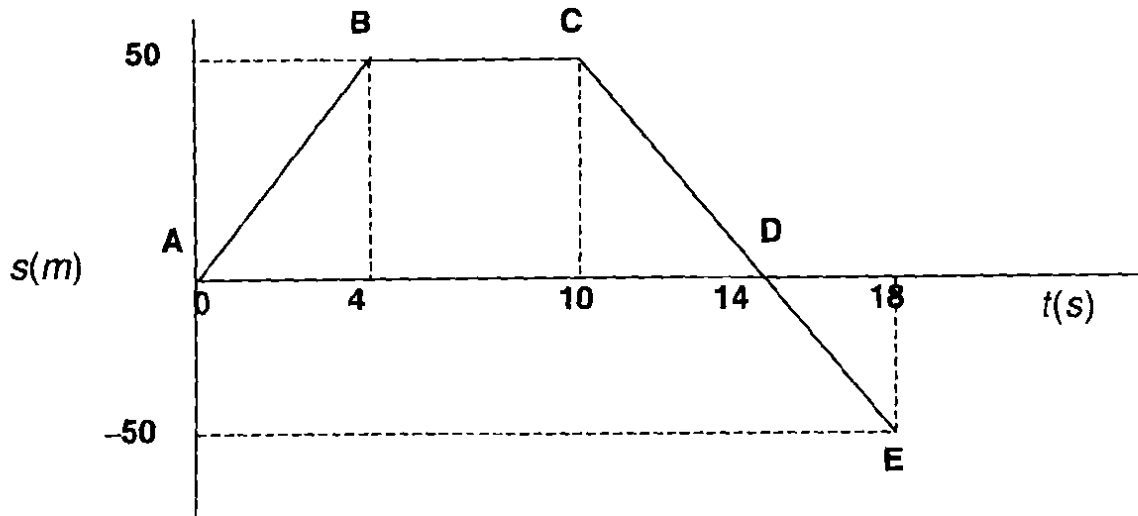
u	v	a	s	t
0 m.s^{-1}		2 m.s^{-2}	36 m	
4 m.s^{-1}	64 m.s^{-1}			25 s
15 m.s^{-1}	7 m.s^{-1}		90 m	

All the students attempted this question. They did not show their calculations as the question instructed them to complete the table. Generally, they understood how to apply the equations of motion and they were able to do substitution if numbers and formulae were supplied.

The calculation of v (in the first row) involved a direct calculation from the equation $v^2 = u^2 + 2as$ and 67,5% of students achieved this. The calculation of t (in the first row) involved changing the subject of the formula. As seen in Question 11, they were able to do this and 57,5% passed this question. The calculation of a in row 2 involved the same equation, but in this case only 50% succeeded. This is difficult to explain. The calculation of s in rows 2 and 3 required the use of the equations $s = \frac{1}{2}(v + u)t$ and $v^2 = u^2 + 2as$ respectively. The achievements were 26,7% and 23,3% respectively, indicating unfamiliarity with the use of these equations. The same applied to the calculation of t in the third row, where 68,3% failed to find the correct answer from the equation $s = \frac{1}{2}(v + u)t$.

ITEM 6: INTERPRETATION AND APPLICATION OF GRAPHS

14. Examine the following displacement-time graph for a car that moves on a straight road in an *easterly direction*



- 14a Which parts of the graph (e.g. CD or DE) represent the following?

- i. Forward motion

14a needed an interpretation of the graph. Some 87 students got it right and 31 got it wrong. Only 2 students did not attempt it. The students managed to spot the area where the car was moving in a forward motion.

- ii. Backward motion

Only one answered this question correctly, 2 students did not attempt it at all and 117 answered incorrectly. A possible explanation is that they interpreted the graph as a velocity versus time graph; hence they chose DE as the answer. This could be due to prior recognition, as science teachers frequently use the v versus t graph in class

and in examinations. Even textbooks mostly use the velocity versus time graph in order to illustrate the concept of acceleration as the slope of this graph.

iii. State of rest

Some 76 students out of 120 got it right, 42 students failed and 2 did not answer it at all. Again some students interpreted the graph as a velocity versus time graph and chose D as the answer. Another explanation might be a problem with interpretation of the formulation of the question. According to them, a moving car cannot experience a state of rest.

iv. Negative slope

All of them attempted this question: 3 students got it right and 114 answered it incorrectly. It seems that only one of the students that answered correctly made the connection between a negative slope and backward motion (see 14a.ii). A possible explanation could be that students chose DE as a negative slope, this means they just took the negative part of the graph and did not interpret the slope correctly.

14b What is the total distance that the car covered?

Only 26 students answered this question correctly and 93 got it wrong, and one did not attempt it. The majority of those who got it wrong interpreted the graph as a velocity versus time graph and attempted to use the area under the curve or the formula $\Delta s = v\Delta t$ to calculate the distance. Nobody confused distance with displacement in this question, as none of them chose 50 m as the total distance.

14c What is the car's total displacement?

All except one student attempted to answer it: 23 students got it right and 96 wrong. Some 42 students did not show an understanding of the difference between distance and displacement and chose 150 m again. Fifty-four (54) students interpreted the graph as a velocity versus time graph and calculated s by using $\Delta s = v\Delta t$.

14d Determine the car's velocity in reverse gear.

All of them answered this question, but only 39 got it right. More students could find the velocity in reverse gear than those that could identify the part of the graph that represented backward motion. This can be explained by the fact that the slope of DE (wrong choice for backward motion) and CE (correct answer for backward motion) is the same. In some cases, language could be a problem if the students did not understand what a reverse gear was.

14e What is the car's velocity at D?

All students attempted this question. Only 20 got it right and 100 wrong. The velocity at D was a big problem for them. They interpreted it as being zero, as they thought it was a velocity versus time graph. Some used $v = s/t$, the formula that are mostly used in calculations of a ticker timer, with $s = 0$, which gave an answer of zero. The problem seemed to be the interpretation of the graph.

14 f Determine the car's velocity for sections AB and BC and sketch a velocity-time graph for the car's motion.

Three students managed to calculate the velocity correctly, but they could not draw a graph using these points. Some 117 students could not calculate the velocity. It seems as if 111 of the students interpreted the graph as a velocity-time graph and just repeated the given graph. Less than 97 students did not draw anything at all. Some only drew a few points, using their calculations

The latter part of the question requested the students to draw the velocity-time graph. No less than 120 got it wrong. Thus 100% of the students did not know how to sketch the graph, because they did not have the values to use. Even those who had them did not know the correct value to draw the graph. Some tried to re-calculate velocity at different levels, but did not plot them correctly.

5.6.5.2 Discussion of results per sub-topic

The percentages in the table were calculated using the actual marks that the students got for each question in the test.

TABLE 5.2: PERFORMANCE OF STUDENTS PER SUB-TOPIC

TOPICS	PERFORMANCE OF THE STUDENTS
	(%)
Conversions	25,2
Scientific notation	41,9
Proportionality	8,6
Changing the subject of the formula	37,8
Equations of the motion	42,1
Interpretation and application of graphs	7,4

ITEM 1: DECIMAL CONVERSIONS

This was supposed to be the easiest part of the test, but the average performance on this sub-item was only 25,2%. The problem seemed to be specifically with conversions requiring two or more steps, e.g. the conversions in questions 1 and 4. In Question 1 they first had to convert from centimetres to metres and then from metres to kilometres. In Question 4 they had to convert from km/h to m/s. This meant that they had to change kilometres to metres and at the same time change hours to seconds. This indicates unfamiliarity with proportional reasoning.

ITEM 2: SCIENTIFIC NOTATION

This item, together with item 5, were answered best of all items. Still, only 41,9% managed to get it right. Question 8 was a problem. Some students did not follow the

instruction, as they were instructed not to use a calculator, while others did not realise they had to apply exponential laws. This indicates that their knowledge was not connected or integrated.

ITEM 3: PROPORTIONALITY

The average mark for this sub-section was 8,6%, indicating a complete lack of ability to reason proportionally. Generally, students did not show that they understood the relationships at all. They experienced problems with direct and especially with inverse proportionality, which should be a serious concern for the teachers.

ITEM 4: CHANGING THE SUBJECT OF THE FORMULA

The average mark for this sub-section was 37,8%. It seems as if they could not use the basics of mathematics, i.e. addition, subtraction, multiplication and division correctly. A more detailed analysis shows that when numbers are involved (e.g. Question 10 in table 5.1) 71,6% of the students were able to change the subject of the formula. This also explains the improvement from 25% to 40% in questions 12a to 12b (see table 5.1). However, when they had to work with variables, they struggled to reason proportionally. The better performance in Question 11 can be ascribed to prior knowledge, as they knew the formula for acceleration.

ITEM 5: EQUATION OF MOTIONS

This item, together with item 2, were answered best of all items. The average performance for this sub-section was 42,1%. It involved the application of procedural knowledge (refer to paragraph 4.3.2). They had to select the correct formula from the given list, make the substitution and do the calculation.

ITEM 6: INTERPRETATION AND APPLICATION OF GRAPHS

This section was a disaster, with an average performance of 7,4%. It seems as if students only looked at the shape of the graph and interpreted it as the velocity-time

graph normally used in class and in the textbooks instead of a displacement-time graph.

5.6.5.3 Discussion of results for different mathematical objects

TABLE 5.3: PERFORMANCE LEVEL FOR DIFFERENT MATHEMATICAL OBJECTS

Mathematical objects	% of performance
Skills	23,9
Facts	44,2
Concepts	36,4
Principles	7,2

As expected, the performance rate on mathematical facts or simple knowledge was the highest (44,2%). What were not expected were the skills (23,9%) to be lower than the concepts (36,4%). However, both yielded low performance rates. The extremely low percentage for principles (7,2%) indicates that students' conceptual knowledge was poor in relation to procedural knowledge. The implication is that students will have difficulty solving real-life problems (refer to paragraph 4.3).

5.7 QUESTIONNAIRES FOR TEACHERS

5.7.1 QUESTIONNAIRE FOR TEACHERS TO RATE STUDENTS' PERFORMANCE

A questionnaire (Appendix B) was drafted asking teachers to rate their students' performance on topics in mathematics that correspond with the sub-topics of the students' test (see tables 5.1 and 5.2). The 10 teachers were selected from the four high schools used in the research and 18 teachers from the SEDIBA project (Potchefstroom Campus) were also used. The results are summarised as percentages in Table 5.4.

TABLE 5.4: RESULTS OF THE TEACHERS' QUESTIONNAIRE IN PERCENTAGES

	Very poor	Poor	Good	Very good
	(%)	(%)	(%)	(%)
a. Decimal conversion	6,7	43,4	49,9	0
b. Scientific notation	9,7	55,5	34,8	0
c. Changing subject of formula	7,5	29,7	52,7	11,1
d. Equations of motions	4,75	34,7	40,6	19,9
e. Proportionality	23,1	38,5	37,7	2,5
f. Interpretation and application of graphs	18,8	36,3	30,6	15,3

A percentage of 50,1% say their students performed poorly in decimal conversions, while 49,9% say students performed well. According to the students' questionnaire, they did not manage this item well. Some 63,8% of the teachers thought that students did well in changing the subject of the formula and 60,5% in equations of the motion. The other topics scored less than 50%. This collaborates with the results for students (Table 5.2.), where the best results were obtained for changing the subject of the formula and the equations of the motion. Students did better in scientific notation than the teachers' expected, but far worse in the interpretation of graphs.

When comparing the performance statistics of the students and the expectation of the teachers, there is a significant difference. This could be an indication that in their classes they performed in accordance with the statistics of their teachers, indicating rote learning without understanding, with no meaningful learning that could be applied in real-life situations. These statistics indicate that there is no effective teaching and learning in physical science and mathematics classes.

5.7.2 QUESTIONNAIRE FOR TEACHERS ON CO-OPERATION AMONG SCIENCE AND MATHEMATICS TEACHERS

A questionnaire was drawn up to elicit teachers' views on co-operation among science and maths teachers. The ten (10) teachers from the chosen schools to follow up on their responses to the questions. A summary of the responses the teachers gave is given in Table 5.5 below. These responses are about the co-operation among teachers of science and mathematics, their concern about the level of integration of mathematics and science curricula and the level of assistance they receive from the Department of Education or NGOs (Non Government Organisations) to improve the level of understanding of the subject matter. This was investigated in the questionnaire for teachers reported in the next paragraph.

TABLE 5.5: SUMMARY OF TEACHERS' QUESTIONNAIRE

ITEMS	NO. OF TEACHERS	AGREE	DISAGREE
1. CO-OPERATION	10	2	8
2. INTEGRATION: CURRICULUM	10	9	1
3. SUBJECT MATTER	10	8	2
4. WORKSHOPS & IN-SERVICE TRAINING	10	5	5
5. CAN INTEGRATION PLAY A ROLE	10	8	2

The responses of the teachers are discussed in more detail below:

ITEM 1: Is there any co-operation between mathematics and physical science teachers (subject preparation/-consultation)? Elaborate:

Eight (8) of the teachers said no and 2 said yes. Mathematics and science teachers are working independently of each other. Usually, preparations are done separately

with minimum consultation. Every teacher concentrates on the subject that he or she teaches. Even though these teachers are teaching both mathematics and science, they do not use examples of science in mathematics classes or vice versa. There is virtually no interrelation of the subjects.

ITEM 2: Do you think there is any integration of the mathematics and science curricula? Your comments:

Nine (9) teachers believe there is an integration of mathematics and science curriculum in the sense that the two subjects have an interrelationship. The problem is that it is not clearly stated in their separate subject curricula. One science teacher said that in his presentation of physical science examples he tried to show the interrelationship between mathematics and science, but even if this were the case, he did not use it frequently.

ITEM 3: Is the level of understanding of the subject matter not a contributory factor? Your comments:

Eight (8) of the teachers said the level of understanding of the subject matter was still a contributory factor. They believed that teachers were not necessarily experts in the field of mathematics and science and that the majority of teachers did not have an in-depth knowledge of these subjects. Two said that it was not a problem.

ITEM 4: Do the workshops and the in-service training in place for mathematics and physical science do enough to bridge the gap? Elaborate:

Five (5) of the teachers agreed that the workshops and in-service training in place tried to bridge the gap, even though it was not enough. Five (5) said they did not receive relevant workshops and in-service training, hence the problem of science and mathematics recurred. They believed that Government was not doing enough

research on the problem of poor performance in mathematics and science so as to give teachers proper/relevant assistance.

ITEM 5: Your general comments about the integration of mathematics and science curricula. How can integration play a role in improving the level of performance in our country?

Eight (8) of the teachers believed that integration of mathematics and science could increase the level of understanding in their schools and could increase the performance as expected in the country. The prerequisite was that teachers of mathematics should be reasonably educated in the science field and vice versa so that there was reinforcement between the two subjects. This could provide students with the opportunity to develop a range of skills that they could use and apply throughout their lives.

ITEM 6: What can be done to improve the poor performance in our country?

Teachers' comments on the poor performance of mathematics and science subject were that teachers had to improve their qualifications in science and mathematics, which would automatically improve the level of understanding. If there were an improvement in the methods used for teaching and learning from grades 1 to 12 and if the overlap of subject matter could be displayed between the grades, there would be an improvement in performance. Sufficient teaching and learning materials had to support this. There is also no co-operation of mathematics teachers and the science teachers when planning learning experiences. Even worse, it seems as if a science teacher cannot use examples from mathematics as part of the learning experience.

5.8 GENERAL PROBLEMS IDENTIFIED THROUGH THE EMPIRICAL STUDY

The level of understanding of the students portrays what is happening in classes throughout South Africa (cf. Taylor & Vinjevold, 1999: 142–162). The country is performing below what is expected. The report of matric results of 2004 showed that the performance mathematics and physical science in this country was still a major problem. Students lack mathematical knowledge and skills, especially conceptual knowledge. They cannot apply relevant algebraic procedures gained in mathematics in physical science.

Integration of mathematics and science is still a problem. Science and mathematics teachers are still not collaborating sufficiently in the planning of learning experiences. The general principles of the National Curriculum Statements (NCS) advocate collaboration between mathematics and science, but it is not clearly spelt out in the learning outcomes and assessment standards how this should be done in practice.

Students can manipulate or use the equations of motion if numbers and formulae were given (refer question 13 in paragraph 5.6.5). That is, they work easily with numbers, but they experience some difficulties when variables are used. They lack conceptual and procedural knowledge. They cannot generalise, that is, they cannot work inductively. Abstract thinking is still a problem with students. Students display a lack of proportional reasoning skills. This is in line with the findings of Singh's study on proportional reasoning done on grade 9 students in Malaysia (Singh, 2000: 579–599).

CHAPTER 6 **CONCLUSIONS AND RECOMMENDATIONS**

6.1 INTRODUCTION

This chapter consist of conclusions and recommendations that are based on the results of the study. The research aimed to identify the algebraic knowledge and skills that would enable students to solve physics problems in grades 11 and 12 and to identify the specific problems experienced by students in applying these skills and knowledge in physics at grade 11 and 12 level.

6.2 RECOMMENDATIONS

The research indicates that students in grades 11 and 12 experience mathematical problems that prevent effective learning and teaching of physics. This confirms the hypotheses of the research that there is a lack of mathematical skills and knowledge in the manipulation of algebraic procedures in physics education for grades 11 and 12. There is also a lack of integration of mathematics into physics, that is, applying mathematical skills and knowledge in physics education.

It is recommended that different types of mathematical knowledge be observed, that is procedural knowledge and conceptual knowledge in teaching and learning physics, algebraic procedures and manipulation. Different mathematical skills as outlined by Grayson (1995:54–57) should be part of learning.

Objects of mathematics play a major role in teaching and learning. It needs to be considered, especially the direct objects (facts, skills, procedures and principles). Bloom's taxonomy is another aspect that plays a major role in learning, specifically cognitive objectives as important variables in teaching and learning.

Constructivism (discussed in chapter 2) is the common approach researched by both mathematics and science education researchers and is seen to be good thus far, where pre-knowledge is taken into account. It stipulates that the student must be actively involved in the learning process. This method is highly supported by the Outcomes-Based Education (OBE) system in South Africa. OBE emphasises hands-on activities, where the teacher acts as facilitator of learning and not as knowledge provider.

6.3 INTERVENTION STRATEGIES

Science is one of the main 'users' of mathematics and it is important that the physical science teacher does not work in isolation and keeps up to date with developments in teaching mathematics and vice versa. The question is, how could this be best achieved? It appears that students are not only unable to apply their mathematical knowledge and skills in the science class, but are also reluctant if not unwilling to do so.

Proper planning is needed for the integration of mathematics education and physical science education, since there is a lack of co-operation. The National Curriculum Statement (DoE) of South Africa needs to be clear in their documents of how to integrate the mathematics education with science education in teaching and learning. Teaching strategies and approaches (methods) need thorough adjustments to fit the system of the country (South Africa). Exploring teacher education programmes for integration and the use of technology shows a greater impact in effective teaching and learning.

6.4 RECOMMENDATION FOR FURTHER STUDY

Integration of mathematics and science education involves many factors. Further research is needed to investigate the factors that hamper the effectiveness of

integration of mathematics and science education. For example, problem solving contexts using vectors.

6.5 CONCLUSIONS

Mathematics learnt by science students seems to have little relevance to the mathematics used in their science courses. Therefore, to make the mathematics relevant to the science being studied, increased co-operation and planning is needed between mathematics and science teachers to bridge the gap that now exists. Concepts such as pressure, volume and temperature change and the use of the general gas laws provide opportunities for students to apply these mathematics operations.

Students cannot apply what they have learnt in the mathematics class in a physical science class. This prevents the effectiveness of teaching and learning of physics.

Team-teaching arrangements between science and mathematics teachers can afford many opportunities to interrelate the two disciplines. Many teachers are trained equally well in both areas and have teaching responsibilities in both areas, hence maximum effectiveness should be achieved.

The Department of Education in South Africa is trying to integrate the subjects, but there is still a lot to be done. For instance, there is a lack of liaison between mathematics and science departments. There is no co-ordination of syllabi. The terminology, methods and approaches used for science and mathematics in the NCS have not yet been fully aligned.

The recommendations mentioned in this study is not a permanent solution to effective teaching and learning in mathematics and science education, but are positive suggestions on how to improve our teaching and learning towards mathematical skills and knowledge needed in physics education. Efforts to integrate

the two subjects need to be developed and researched until there is a greater improvement in the performance of mathematics and science.

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NCS **see** SOUTH AFRICA Department of Education

NCTM **see** NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

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APPENDIX A

QUESTIONNAIRE

MATHEMATICAL SKILLS

NAME:

SCHOOL:

DATE:

DECIMAL CONVERSIONS

1. How many centimetres are there in 50 km?

2. How many kilograms are there in 50 grams?

3. Convert 3 minutes to hours.

4. Convert 60 km/h to m/s.

SCIENTIFIC NOTATION

5. Write each number as a power of 10:

a. $100 = \dots\dots\dots$

b. $\frac{1}{10} = \dots\dots\dots$

6. Write each constant in scientific notation:

speed of light in vacuum: $299792458 \text{ m/s} = \dots\dots\dots$

wave length of a red light ray: $0,00000075 \text{ m} = \dots\dots\dots$

7. Use your calculator to simplify the following and write the answer in scientific notation:

a. $0,000024 \times 6000$

b. $60\,000\,000 - 0,000012$

8. Simplify without using calculator: $\frac{(4 \times 10^3) \times (2 \times 10^9) \times (3 \times 10^{-6})}{(3 \times 10^2) \times (6 \times 10^{-4}) \times (3 \times 10^4)}$

PROPORTIONALITY

9. Given: $F = \frac{Gm_1m_2}{r^2}$, with F the magnitude of the gravitational force between two masses m_1 and m_2 separated by a distance of r .

In each of the following, give only the answer (no calculations are required).

How does F change if:

- a. Both masses are doubled

- b. Both masses are halved

- c. One of the masses is halved while the other is doubled

- d. The distance between the masses is halved

e. The distance between the masses is doubled

f. Both masses are doubled and the distance between them is halved.

g. Both masses as well as the distance are doubled.

CHANGING THE SUBJECT OF THE FORMULA

10. Given: $P_1V_1 = P_2V_2$

Use the following values to find P_2 :

$$P_1 = 3 \times 10^{-4} \text{ kPa}, V_1 = 6 \times 10^{-8} \text{ cm}^3 \quad V_2 = 9 \times 10^2 \text{ cm}^3$$

11. Given: $v = u + at$

Make a the subject of the formula.

12. Given: $F = \frac{Gm_1m_2}{r^2}$,

a. Make r the subject of the formula.

b. Calculate r if the gravitational force between two large ships with masses of 6×10^4 kg and 2×10^5 kg respectively is $1,64 \times 10^{-2}$ N and $G = 6,67 \times 10^{-11}$ N.m².kg⁻².

EQUATIONS OF MOTION (SUBSTITUTION)

13. Complete the table (do the necessary calculations at the bottom of page).

Use the following equations:

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

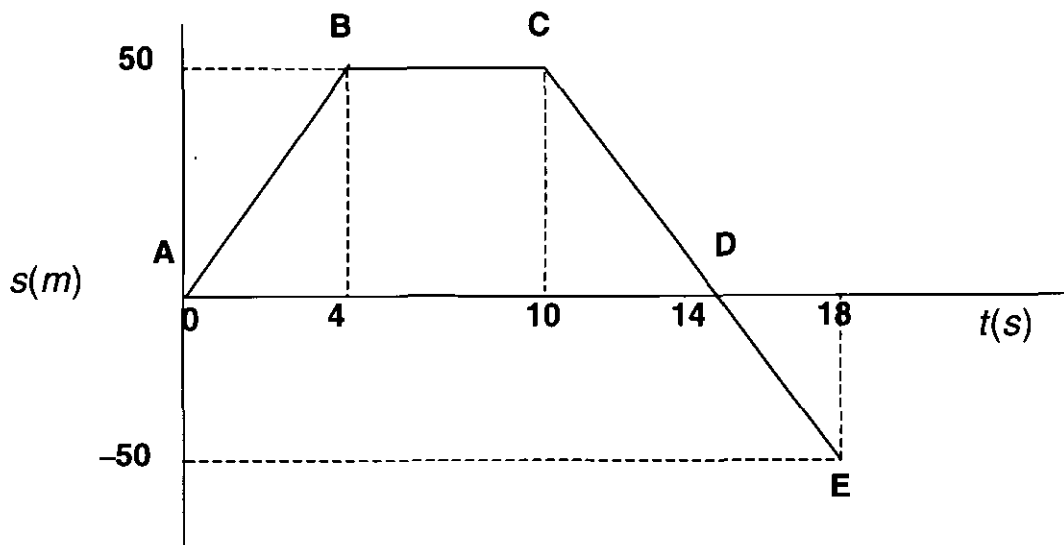
$$v^2 = u^2 + 2as$$

$$s = \frac{1}{2}(v + u)t$$

u	v	a	s	t
0 m.s^{-1}		2 m.s^{-2}	36 m	
4 m.s^{-1}	64 m.s^{-1}			25 s
15 m.s^{-1}	7 m.s^{-1}		90 m	

INTEPRETATION AND APPLICATIONS OF GRAPHS

14. Examine the following displacement-time graph for a car that moves on a straight road in an *easterly direction*.



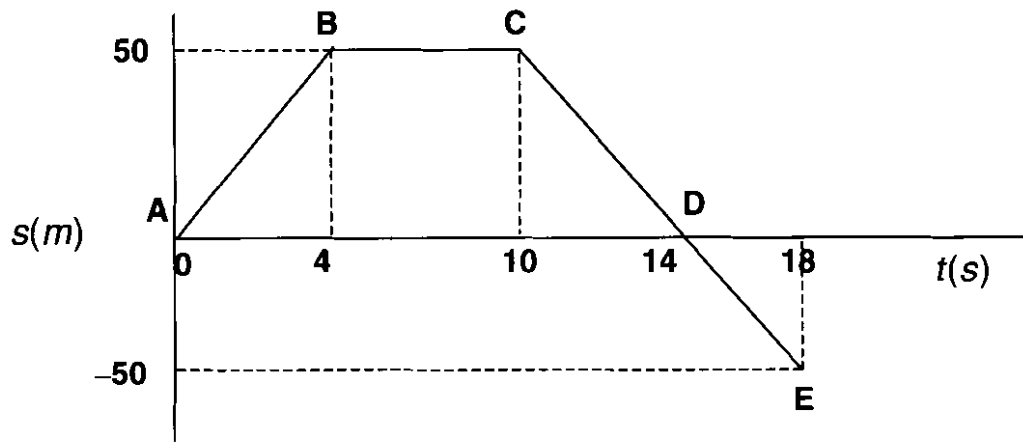
- a. Which parts on the graph (e.g. CD or DE) represent the following?

Forward motion _____

Backward motion _____

State of rest _____

Negative slope _____



b. What is the total distance the car covered?

c. What is the car's total displacement?

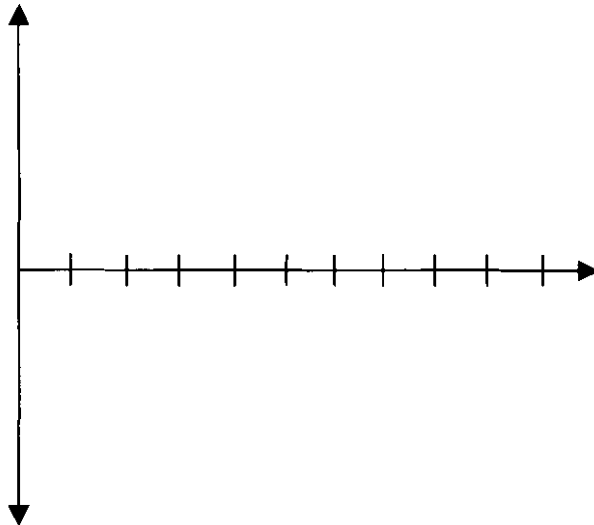
d. Determine the car's velocity in reverse gear.

e. What is the car's velocity at D?

- f. Determine the car's velocity for sections AB and BC and sketch a velocity-time graph for the car's motion.

AB: _____

BC: _____



APPENDIX B

MATHEMATICAL SKILLS AND KNOWLEDGE IN SCIENCE

Science teacher:

Maths teacher:

Grade 11

Grade 11

Grade 12

Grade 12

1. With reference to the mathematical skills and knowledge below, how do you rate your **science** students' performance? Use an X to mark your choice.

	Very poor	Poor	Good	Very good
1. decimal conversion				
2. scientific notation				
3. changing subject of formula				
4. equations of motions (substitution)				
5. proportionality				
6. application of trigonometry				
7. problem-solving skills				

APPENDIX C

THERE IS A POOR PERFORMANCE OF MATHEMATICS AND SCIENCE SUBJECTS IN OUR COUNTRY. STUDENTS DO NOT PERFORM ACCORDING TO THE EXPECTED STANDARD. THEY PERFORM BELOW AVERAGE. WHAT CAN BE THE RATIONALE BEHIND THIS?

- Is there any co-operation between mathematics and physical science teachers (subject preparation/consultation)? Elaborate:

- Do you think there is any integration of the mathematics and science curriculum? Your comments:

- Is the level of understanding of the subject matter not a contributory factor? Your comments:

-
-
- Do the workshops and the in-service training in place for mathematics and physical science do enough to bridge the gap? Elaborate:

- Your general comments about the integration of mathematics and science curriculum, how could integration play a role in improving the level of performance in our country?

- What could be done to improve the poor performance in our country?

APPENDIX D

LIST OF TEXTBOOKS AND STUDY GUIDES USED

- ✓ X-kit (The definitive exam preparation kit) HG and SG. Grade 11 and 12 . Longman: 2003.
- ✓ Study and Master. Physical science. Grade 11 and 12. Roedurico: 2004.
- ✓ Physical Science developed by the ESST for Amplats adopted schools.
- ✓ Physical Science: standard 9. Student workbook developed by Jonathan Clark. PROTEC.
- ✓ Physics grade 11 and 12 (Mechanics) compiled by A. Olivier. Rustenburg: Areivilo Publishers.

LIST OF QUESTION PAPERS

- ✓ Impala enrichment project (question paper grade 11 and 12), 2004.
- ✓ Rustenburg Regional paper: physical science : paper 1 , grade 11, 2004.