

Modelling the evolution of pulsar wind nebulae

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... *Man disobeying,*
Disloyal breaks his fealtie, and sinns
Against the high Supremacie of Heav'n,
Affecting God-head, and so loosing all,
To expiate his Treason hath naught left,
But to destruction sacred and devote,
He with his whole posteritie must dye,
Dye hee or Justice must; unless for him
Som other able, and as willing, pay
The rigid satisfaction, death for death.
Say Heav'nly Powers, where shall we find such love,
Which of ye will be mortal to redeem
Mans mortal crime, and just th' unjust to save,
Dwels in all Heaven charitie so deare?

He ask'd, but all the Heav'nly Quire stood mute,
And silence was in Heav'n: on mans behalf
Patron or Intercessor none appeerd,
Much less that durst upon his own head draw
The deadly forfeiture, and ransom set.
And now without redemption all mankind
Must have bin lost, adjudg'd to Death and Hell
By doom severe, had not the Son of God,
In whom the fulness dwells of love divine,
His dearest mediation thus renewd.

...

I offer, on mee let thine anger fall;
Account mee man; I for his sake will leave
Thy bosom, and this glorie next to thee
Freely put off, and for him lastly dye
Well pleas'd, on me let Death wreck all his rage;

Book III, Paradise Lost

- Milton

Abstract

This study focusses on modelling important aspects of the evolution of pulsar wind nebulae using two different approaches. The first uses a hydrodynamic model to simulate the morphological evolution of a spherically-symmetric composite supernova remnant that is expanding into a homogeneous interstellar medium. In order to extend this model, a magnetic field is included in a kinematic fashion, implying that the reaction of the fluid on the magnetic field is taken into account, while neglecting any counter-reaction of the field on the fluid. This approach is valid provided that the ratio of electromagnetic to particle energy in the nebula is small, or equivalently, for a large plasma β environment. This model therefore allows one to not only calculate the evolution of the convection velocity but also, for example, the evolution of the average magnetic field.

The second part of this study focusses on calculating the evolution of the energy spectra of the particles in the nebula using a number of particle evolution models. The first of these is a spatially independent temporal evolution model, similar to the models that can be found in the literature. While spatially independent models are useful, a large part of this study is devoted to developing spatially dependent models based on the Fokker-Planck transport equation. Two such models are developed, the first being a spherically-symmetric model that includes the processes of convection, diffusion, adiabatic losses, as well as the non-thermal energy loss processes of synchrotron radiation and inverse Compton scattering. As the magnetic field geometry can lead to the additional transport process of drift, the previous model is extended to an axisymmetric geometry, thereby allowing one to also include this process.

Keywords: Composite supernova remnants, morphological evolution, hydrodynamic, kinematic magnetic field, pulsar wind nebulae, particle evolution models, Fokker-Planck transport equation, spherically-symmetric, axisymmetric, diffusion, drift, energy losses

Opsomming

In dié studie word belangrike aspekte van die evolusie van pulsarwindnewels gemodelleer deur van twee verskillende benaderings gebruik te maak. Die eerste benadering gebruik 'n hidrodinamiese model om die morfologiese evolusie van 'n sferiese-simmetriese saamgestelde supernova-oorblyfsel wat in 'n homogene interstellêre medium uitsit te simuleer. Ten einde hierdie model uit te brei, word 'n magneetveld op 'n kinematiese wyse tot die model bygevoeg. Dit impliseer dat die effek van die fluïde op die magneetveld bereken word, maar dat die teen-reaksie van die magneetveld op die fluïde verwaarloos word. Hierdie benadering is voldoende wanneer die verhouding van die elektromagnetiese- en deeltjie-energie in die newel klein is, m.a.w. vir 'n hoë plasma β omgewing. Die model laat mens dus toe om nie net die evolusie van die fluïdegroothede te bereken nie, maar ook, bv., die evolusie van die gemiddelde magneetveld.

Die tweede deel van dié studie handel oor die evolusie van die deeltjies se energiespektrum in die newel. Dit word gedoen deur van 'n aantal deeltjie-evolusiemodelle gebruik te maak. Die eerste van hierdie modelle is 'n ruimtelike onafhanklike model, soortgelyk aan die modelle wat in die literatuur gevind kan word. Alhoewel 'n ruimtelike onafhanklike model onder sekere omstandighede nuttig is, word die grootste gedeelte van hierdie studie daaraan gewy om 'n ruimtelike model, wat op die Fokker-Planck transportvergelyking gebaseer is, te ontwikkel. In hierdie kategorie word twee modelle ontwikkel, waarvan die eerste 'n sferiese-simmetriese model is wat konveksie, diffusie, adiabatiese verliese, sowel as die nie-termiese energie verliesprosesses van sinkrotronstraling en omgekeerde Compton-verstrooiing insluit. Omdat die geometrie van die magneetveld aanleiding tot die addisionele transport proses van dryf kan gee, word die vorige model tot 'n aksiaal-simmetriese geometrie uitgebrei, wat voldoende is om die hoofaspekte van hierdie proses te ondersoek.

Keywords: Saamgestelde supernova-oorblyfsels, morfologiese evolusie, hidrodinamiese, kinematiese magneetveld pulsarwindnewels, deeltjie evolusiemodelle, Fokker-Planck transport vergelyking, sferiese-simmetries, aksiaal-simmetries, diffusie, dryf, energie verliese

Abbreviations

Listed below are the abbreviations used in the text. For the purpose of clarity, any such usage are written out in full when they first appear.

ADI	:	Alternating Direction Implicit
CMBR	:	cosmic microwave background radiation
IC	:	inverse Compton
ISM	:	interstellar medium
HD	:	hydrodynamic
K-N	:	Klein-Nishina
MHD	:	magnetohydrodynamic
PWN	:	pulsar wind nebula
SNR	:	supernova remnant
VHE	:	very high energy

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