

**COLLABORATIVE TEACHING AND THE LEARNING
OF MATHEMATICS AT MATRIC LEVEL**

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ABSTRACT

Worldwide the teaching and learning of mathematics pose a great challenge to mathematics teachers as learners' performance in the subject leaves much to be desired. This is particularly the case in South Africa where there was a great disparity in the development of teachers in the past. Extensive research has shown that many teachers in South Africa are under-qualified, especially in the teaching of mathematics at secondary schools.

Those who are regarded as well qualified for teaching mathematics at secondary schools still experience problems in teaching certain sections of the syllabus, for example geometry, which is not offered at tertiary institutions. It is for this reason that the researcher, together with colleagues at an experimental school, joined forces to share the teaching of mathematics in what they referred to as "collaborative teaching". This work therefore involves a case study, which resulted after three teachers successfully achieved good matric results on employing this approach between 1993 and 1996.

The study is based on an experimental design where both quantitative and qualitative methods were used. The aim of the study was to measure the extent to which collaboration between teachers affects the learning of mathematics in Grades 12. Two schools, the experimental school and a control school were involved. Learners from the experimental school were taught according to a collaborative approach whereas learners at the control school were taught conventionally (one teacher teaching all sections alone). This happened over a period of six months in 2001. Learners who were taught collaboratively outperformed those who were taught conventionally especially in the most problematic areas of the syllabus, namely geometry and trigonometry.

The teachers who were involved in this approach, that is, collaborators, loved it to the extent that one of them applied it in another school where it improved their Grade 12 results tremendously. Learners who were taught according to this approach greatly appreciated it and wished they had been taught the same way in other subjects.

This approach did not, however, significantly influence learners in their problem solving and information processing skills. In addition, one of the most serious limitations of this approach is to find a substitute for a teacher who leaves the team.

Keywords for indexing:

Collaborative teaching; mathematics learning; mathematics teaching; teaching approach; learning task; learning theories; Grades 12; secondary school.

OPSOMMING

Samewerkende onderrig en die leer van wiskunde op matriekvlak

Die onderrig en leer van wiskunde daag onderwysers wêreldwyd uit om die swak prestasie van leerders in dié vak die hoof te bied. Hierdie situasie geld in die besonder in Suid-Afrika waar daar in die verlede groot ongelykhede ten opsigte van die ontwikkeling van onderwysers bestaan het. Uitvoerige navorsing toon dat baie Suid-Afrikaanse onderwysers ondergekwalfiseer is, veral in die geval van wiskunde op sekondêre skole.

Ook die wiskundeonderwysers, wat goedgekwalfiseer deurgaans, ondervind probleme met die onderrig van sekere gedeeltes van die kurrikulum. Meetkunde, wat oor die algemeen nie in onderwysers se naskoolse opleiding verdere aandag kry nie, is 'n belangrike voorbeeld van sodanige gedeelte. Die navorsers en kollegas by die eksperimentele skool in die ondersoek, het gevolglik saamgespan om die onderrigtaak in wiskunde onderling te deel in 'n benadering wat as "samewerkende onderrig" getipeer is. Hierdie studie sluit ondermeer 'n gevalle studie in van die implementering van die benadering deur drie onderwysers in die tydperk 1993 en 1996, wat goeie matriekuitslae gelever het.

Die studie is verder gegrond op 'n eksperimentele ontwerp wat kwantitatiewe sowel as kwalitatiewe metodes insluit. Die doel is om die mate waartoe samewerking tussen die onderwysers die leer van wiskunde in graad 12 beïnvloed, te bepaal. Twee skole, die eksperimentele skool en 'n kontrole skool, het aan die studie deelgeneem. In die eksperimentele skool is leerders volgens die samewerkende onderrigbenadering onderrig, terwyl die konvensionele onderrigbenadering waar een onderwyser alle afdelings onderrig, in die kontrole skool toegepas is. Die eksperiment is in 2001 oor 'n tydperk van ses maande uitgevoer. Leerders in die eksperimentele skool het beter as dié in die kontrole skool in die meeste afdelings van die kurrikulum, in die besonder die problematiese afdelings trigonometrie en meetkunde, presteer.

Die onderwysers wat aan die samewerkende benadering deelgeneem het, het tot sodanige mate van die benadering gehou dat een van hulle dit in 'n volgende skool toegepas het wat tot verbeterde uitslae aldaar gelei het. Leerders wat op hierdie wyse onderrig is, het hulle waardering daarteenoor uitgespreek en aangedui dat hulle ook in hulle ander vakke op hierdie wyse onderrig sou wou word.

Die benadering het egter nie leerders se probleemoplossing en inligting-verwerking betekenisvol beïnvloed nie. Daarby is bevind dat die probleem om 'n onderwyser wat die span verlaat te vervang, 'n ernstige beperking op die samewerkende benadering is.

Trefwoorde vir indeksering:

Samewerkende onderrig; wiskundeleer; wiskundeonderrig; onderrig-benadering; leertaak; leerteorie; graad 12; sekondêre skool.

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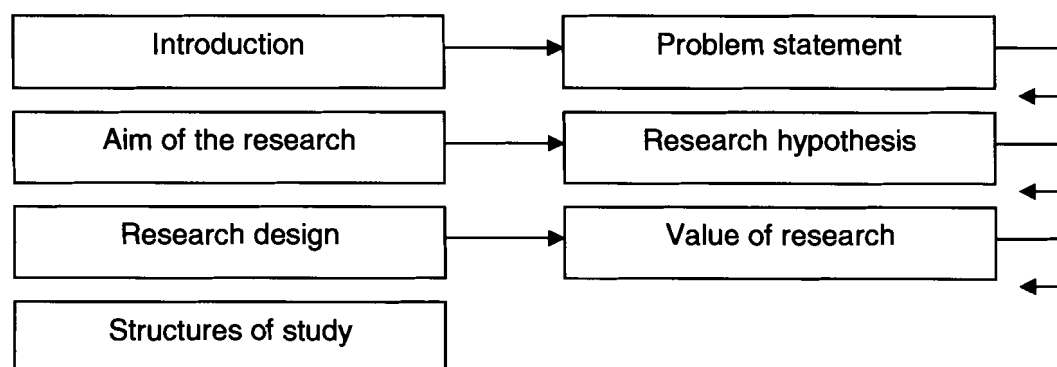
CHAPTER 1

STATEMENT OF THE PROBLEM AND PROGRAMME OF STUDY

1.1. INTRODUCTION

The teaching and learning of mathematics pose a great challenge to mathematics teachers, parents, learners, industry and governments worldwide. Extensive research is continuously being undertaken by scholars in the field of mathematics education to improve the teaching of the subject and the performance of learners in it. The purpose of this study is to join other researchers in trying to meet the challenges posed by mathematics at school level (in this instance at Grade 12). Section 1.7 gives an overview of the study.

The contents of this first chapter will be broken up into the following sections:



1.2. PROBLEM STATEMENT

Mathematics forms the heart of the science, technological and commercial faculties at tertiary institutions as well as the private sector. Unfortunately, fewer learners enrol for the subject at secondary schools than for other subjects such as biology, history and geography. Enrolment figures for the period 1982 – 1991 of the then Department of Education and Training already show that the enrolment for learners in mathematics ranged between 31.6 and 42.2 percent compared to 76.8

and 85.9 percent for biology and 42.2 and 67.4 percent for history (DoE, 1999). Recently only about 300 000 out of 700 000 (i.e. 42%) learners in South Africa took some form of mathematics at some level, including the functional and lower grades, at matric level (Laridon, 2000). Arguably, this trend can be linked to the comparative lack of success in school mathematics depicted for the years 1998 and 1999 in Table 1 below.

Table 1.1: National matric pass rate for selected subjects for 1998 and 1999 (DoE, 1999)

	Accounting	Biology	Business Economics	Geography	Mathematics	Physical Science
1998	65.9%	54.5%	65.3%	62.9%	41.9%	65.1%
1999	64.4%	52.2%	65.1%	60.0%	43.4%	63.9%

The trend is further illustrated in a report released by the Department of Education (DoE, 1999) which indicates that in 1998, 60 347 learners wrote higher grade (HG) mathematics and 26 640 passed, that is, 47.5 percent. In 1999, 27 187 out of 50 105 learners who wrote higher grade mathematics passed, that is, 54.3 percent (see Table 1.2).

Table 1.2: National matric pass rate and failure for mathematics higher grade for 1998 and 1999 (DoE, 1999)

	Failed	Passed (HG)	Passed (SG)	Total passed
1998	52.5	33.4	14.1	47.5
1999	45.7	39.6	14.6	54.3

The situation was even worse for learners taking mathematics on the standard grade in 1998: of the 219 390 learners that wrote standard grade mathematics, 88 572 passed (40%); in 1999, 231 199 learners wrote the corresponding examination of whom 95 038 passed (41.1%).

There are several factors that contribute to this state of affairs:

- Teacher conceptual development in mathematics (cf. Taylor & Vinjevold, 1999).
- Inconsistency in mathematics syllabi and lack of geometry training at tertiary level or for prospective mathematics teachers (cf. Laridon, 2000; Nieuwoudt, 1998: 256-7).
- Negative attitudes of teachers towards certain sections of mathematics, such as geometry, and socio-economic factors (cf. Laridon, 2000; Van der Walt, 2000).

In an interview, Laridon (2000), the first recipient of honorary life-long membership of the Association for Mathematics Education in South Africa (AMESA, 2000:5) for his contribution to mathematics education and research in South Africa, indicated that many mathematics teachers at matric level prefer to teach algebra instead of Euclidean geometry, calculus and linear programming (in the HG). Consequently, this leads to learners performing poorly in those sections of mathematics in the final examinations.

In the North West Province, Mulder (2000) and Van der Walt (2000), both examiners for Mathematics (HG) Paper 2, confirm that learners perform very poorly in mathematics, particularly geometry. The difference in performance, according to them, could be in the order of 10 to 20 percent. Their sentiments are echoed by Van Wyk (2000), who for some time has been the national moderator for Mathematics Paper 2 (HG) in South Africa. According to his experience over the years, there is a marked difference in performance between the two papers. Arguably, lack of effective teaching and learning of geometry, which forms about 20 percent of Paper 2 for both standard and higher grades, is impacting negatively on the final matric results in mathematics.

This situation is not peculiar to South Africa. According to Clements and Battista (1992:421), in the United States “elementary and middle school students are failing to learn basic geometric concepts and geometric

problem solving". They further indicate that teachers in the United States do not teach "even [an] improvised geometry curriculum that is available to them". As a result, only about a half of US high school learners take a geometry course. Unlike in South Africa, geometry is taught and learned independently from algebra in US schools and learners have the option to enrol for it or not. Accordingly, there are teachers specialising in geometry only, as well as in algebra only. Against this background, the teaching of mathematics, especially geometry, in South Africa is quite different from that in the US. However, there are some relevant lessons to be learnt from the US situation. Teachers in the US are engaged in programmes aimed at their professional development and competence, building on the principles and benefits of collaboration, partnerships and co-teaching (Duchardt, Marlow, Christensen & Reeves, 1999; Hobbs, Bullough, Kauchak, Crow & Stokes, 1998; Rottier, 2000; Sprague & Pennell, 2000).

One possible approach that may help to address and consequently minimise the problems of mathematics education in South Africa is collaborative teaching. Wiedemeyer and Lehman (1991, as quoted by Hewit & Whittler, 1997:155) describe this as "a co-operative and interactive process between teachers that allows them to develop creative solutions to mutual problems". Welch and Sheridan (1995:11) define collaboration as "a dynamic framework for efforts which endorses interdependence and parity during interactive exchange of resources between at least two partners who work together in a decision making process that is influenced by cultural and systematic factors to achieve common goals". Duchardt et al. (1999:186), at the end of a project by North Western State University in Louisiana, noticed nine positive outcomes of collaboration, and thus concluded: "all teachers in higher education, public schools and private schools can learn to develop a collaborative teaching environment that will benefit themselves and their students."

Collaborative teaching of matric mathematics was employed at Kwenya-Madiba High School (E) between 1993 and 1998, where three educators, including the researcher, shared the matric mathematics syllabus in the following manner: Educator X taught algebra and calculus for Paper 1; Educator Y taught Euclidean and analytical geometry and Educator Z (the researcher) taught trigonometry. The matric mathematics pass rates at E for 1993 and 1994 are given in Table 3; for comparison purposes the corresponding results of two similar neighbouring schools (C_1 , C_2), where conventional teaching took place, are indicated as well:

Table 1.3: Comparative matric pass rate for mathematics for 1993 and 1994 at schools E, C_1 and C_2 (data obtained directly from the three respective schools)

	Passed (HG)			Passed (SG)		
	E	C_1	C_2	E	C_1	C_2
1993	91.7%	73.2%	48.9%	78.6%	66.1%	67.1%
1994	79.2%	44.3%	39.5%	65.7%	51.8%	53.4%

A comparison of the results suggests that collaborative teaching, as a way of enhancing matric mathematics results, deserves investigation. Hence, this project/research/study hypothesises that collaborative teaching of mathematics at matric level can enhance both teacher and learner success in those areas where it is lacking.

This study seeks answers to the following research questions:

- What is collaborative teaching of matric mathematics, and how does it proceed?
- What are learners' perceptions of collaboration teaching and how does it impact on their affective, cognitive and contextual factors and performance in mathematics at grade 12 level?

- To what extent does the collaborative teaching approach address the problem of poor mathematics learning and, consequently, poor matric mathematics results?

1.3. AIM OF THE RESEARCH

The research aim is to analyse and describe the impact of collaborative teaching on poor mathematics teaching and learning in matric (Grade 12), that is, to answer the three research questions stated above.

1.4. RESEARCH HYPOTHESIS

There is a relationship between collaborative teaching and the learner's performance in mathematics at matric level.

1.5. RESEARCH DESIGN

1.5.1. Literature study

A DIALOG-search was done using the following keywords: cooperation; collaboration; learning; teaching; partnerships; team teaching; secondary school; mathematics; Grade 12.

Ample reading material was available in the Ferdinand Postma library and other libraries.

1.5.2. Experimental design

A pre-test/post-test experimental design, as indicated in the following diagram, was employed:

	Numbers	Pre-test (January 2001)	Intervention	Post-test (June 2001)
Experimental school (E)	Educators: 3 Learners: 75	SOM ^(a)	Collaborative teaching	SOM
Control school (C)	Educators: 1 Learners: 73	Math Test 1 ^(b)	Conventional teaching	Math Test 2 ^(b)

Notes:

(a) SOM: the Study Orientation in Mathematics Questionnaire, developed by the Human Sciences Research Council in South Africa (HSRC, 1997).

(b) Math Test 1 compiled by the researcher, based on concepts taught in Grade 11 (in both schools) and Math Test 2 on concepts taught during the first two quarters of Grade 12 (in both schools). The mathematics tests were extracted from the previous Grade 12 final examination question papers, with marks unchanged. These were moderated by experienced examiners (see Appendix A).

In addition, interviews were conducted with the mathematics educators involved at the two schools (see above diagram for numbers).

1.5.3. Population and sample

All 2001 Grade 12 mathematics learners and the mathematics educators involved at the two schools (see above diagram for tasks and approximate numbers).

1.5.4. Instruments

A Study Orientation in Mathematics (SOM) questionnaire, which is standardised for South African school mathematics learners (Grade 7 – 12), and self-constructed tests, interviews and observation schedules, as well as mathematics tasks report-style utilised as instruments.

1.5.5. Variables

Independent variable: Teaching approach, that is, collaborative or conventional. *Dependent variables:* Mathematics results obtained in examinations during 2001, as well as related cognitive, affective and contextual factors measured by the SOM (e.g. problem solving, math anxiety, study milieu).

1.6. VALUE OF THE RESEARCH

The current matric results, together with the performance of South African learners in the recent TIMSS (Howie, 1997) and UNESCO-sponsored (Strauss, 1999) surveys are just a few indicators of the alarming state of school mathematics education in South Africa. Research indicates that, for a variety of reasons, mathematics teachers often neglect some parts of the mathematics curriculum, for example geometry, leading to learners' poor understanding and performance in those parts of mathematics.

In focusing on a possible solution to this problem, this study can contribute towards proposing a practical teaching approach to enhance mathematics learners' understanding and performance in all aspects of the learning area curriculum. Internationally, the results may be useful for researchers, teachers and learners who have to deal with similar conditions and circumstances as in South Africa.

1.7. OVERVIEW OF STRUCTURE OF DISSERTATION

Chapter 1:

In this chapter the problem (which is poor mathematics teaching and learning) is stated and the way in which it is going to be researched in terms of aim, hypothesis, research design and structure, as well as the value of the research.

Chapter 2:

This chapter investigates the way in which people view mathematics according to certain belief systems as well as their understanding of what mathematics is. Learning of mathematics is analysed from different theoretical backgrounds; in particular, behavioural, cognitivistic and constructivistic. Learning tasks are analysed from both traditional and reformative viewpoints.

Chapter 3:

Although there may be several factors that affect the teaching and learning of mathematics, only the following factors were considered: Psychological, socio-cultural, the teacher's knowledge and teaching approaches.

Chapter 4:

This chapter outlines what a collaborative teaching approach is in the context of this study, and how it was employed. Team teaching is defined according to historical and current definitions. A distinction is made between a collaborative teaching approach and team teaching. The views of former learners taught by means of the collaborative method are recorded.

Chapter 5:

Results of both the experimental and control groups on mathematics tests (pre and post) are analysed and interpreted. A comparison is also made between experimental and control groups in terms of study orientation in the field of mathematics.

A collaborative teaching approach is thus regarded as having a positive impact on the teaching and learning of mathematics in Grade 12.

Chapter 6:

Chapter 6 gives a summary of the study. It draws critical deductions and makes recommendations on the basis of the findings of the study.

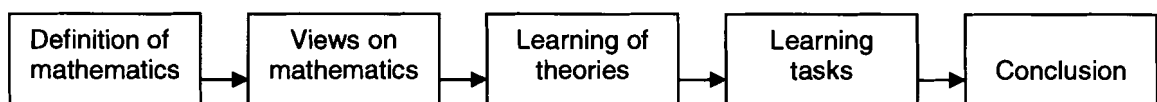
CHAPTER 2

THEORETICAL PERSPECTIVES ON MATHEMATICS LEARNING AND LEARNING TASKS

2.1 INTRODUCTION

Poor mathematic results at matric level can be associated with poor learning of the subject. Seeing that the aim of this study is to describe the impact of collaborative teaching on low mathematics achievement at matric level, this cannot be possible without first analysing how mathematics is learned. Consequently, some learning tasks and theories will be discussed. In addition, the nature and views of mathematics will be discussed because, as Nickson (1994:5) puts it, “the differing views held by teachers and pupils in relation to the nature of mathematical knowledge are important components in the culture of mathematics classroom since they are linked with the way mathematics is taught and received.”

The flow chart for describing the structure of the chapter will therefore be as follows:



These elements have been selected because they strongly influence the learning of mathematics.

2.2 THE NATURE OF MATHEMATICS

According to Baron (1976:24), mathematics seems to have grown as a human activity largely as a result of social needs, commerce, science and technology, as well as the intellectual need to connect together existing mathematics into a single logical framework or proof structure.

Mathematics, accordingly, can be used in two distinct and different senses viz. the methods used to discover certain truths and the usage of those truths that are discovered (Baron, 1976:23).

The Mathematical Sciences Education Board (1989:31) defines mathematics as “a science of pattern and order”. Its domain is not molecules or cells, but numbers, chance, form, algorithms and change. As a science of abstract objects, mathematics relies on logic rather than on observation as its standard of truth, and even experimentation as a means of discovering truth.

Van De Walle (2004:13) qualifies the above definition by stating that mathematics is a science of things that have a pattern of regularity and logical order. The pattern, he says, is not just in equations, but also in everything around us; in nature, art, buildings, music, commerce, science, medicine, manufacturing and sociology. Mathematics discovers this order, makes sense of it and uses it to improve human life and to expand knowledge.

Steen and Christie's definition becomes consolidated in Van de Walle's (2004:10) definition, which views mathematics “not as language, nor is it an object. It is a practice; the unseen work done by individuals and groups making sense of their lives, their territories, their histories, and economics through particular discourses which involve naming, ordering, recursion and valuing”.

Social and geographical factors and lifestyle will to a great extent influence people's understanding and views of mathematics. Baron (1976:24) indicates that Egyptian mathematics was largely practical in nature resulting in empirical formulae for mensuration. Greeks on the other hand were mainly concerned with the development of a unified proof structure and logical framework in terms of which all mathematical theorems could be expressed (Baron, 1976:24).

This leads us to examine how mathematics is viewed traditionally and currently.

2.3 VIEWS ON MATHEMATICS

The cultural beliefs that people hold about mathematics influence the way they view it and consequently the way mathematics will be taught and learned. Hanson (2004:161) refers to beliefs as myths and holds the following three as most common in the United Kingdom:

- Mathematics is difficult
- Mathematics is only for clever people
- Mathematics is a male domain

These beliefs cut across all cultures and, as a result, extensive research studies have been undertaken on them (Romberg, 1992; Fennema, Carpenter & Peterson, 1989). Chapter 3 of this study, which is about factors affecting the teaching and learning of mathematics, will deal with these in a fair amount of detail.

According to Dossey (1992:39) there are two contradicting schools of thought about mathematics: viz. that mathematics is a static discipline with a known set of concepts, principles and skills, and secondly, that mathematics is a growing, dynamic field of study. The former view is “traditional” and Romberg and Carpenter (1986) regard it as divorcing mathematics from reality. Accordingly, mathematics has been fragmented into concepts, facts, skills and procedures, which are then arranged into courses, topics and lessons. Knowledge of facts, concepts and procedures is thus regarded as key to knowledge of mathematics (Phye, 1997:346).

This view emanates from the Platonist view, which sees mathematics as something fixed with axioms that have no connection whatsoever with the real world. Learners, according to this view, should see mathematics

as a mental game to play logically in arriving at mathematical results. The emphasis on mathematics learning is “agreed-on-axioms”.

Dossey’s (1992:42) second school of thought, which could still be regarded as current, sees mathematics as a dynamic human activity not governed by one school of thought. Mathematics here is seen as dealing with ideas and it is maintained that mathematical objects are invented or created by humans from already existing mathematical objects. Furthermore, mathematics is viewed as being guided by intuition, and the exploration of concepts and their interactions. Current views of mathematics, as stated in Principles and Standards (National Council of Teachers of Mathematics 2000), indicate that mathematics should be learned through exploration, conjecture, construction, investigation and verification so that learners are able to discover and formulate their own understanding in addition to what is known. In this way, as Van De Walle (2004:13) puts it, it will virtually be impossible for them to be passive observers.

Another view that has influenced the teaching and learning of mathematics, especially at secondary school level, is the formalist view, which is supported by a monological theory of mathematics learning. This view suggests that gaining knowledge in mathematics is essentially an individual activity involving one’s senses and that interaction with others is not a necessary feature of learning. Thus, mathematics becomes the memorisation of rules and procedures and knowing when to apply them (Phye, 1997:347). Earlier studies of this view by Nickson (1992:103) show that the foundations of mathematical knowledge were not seen to be social in origin, but that they were beyond human action in what was called “formalist heaven”. Mathematics was considered to consist of immutable truths and unquestionable certainty.

A “growth and change” view of mathematics is regarded as the foundation of a new direction for mathematics in the classroom. This is based on the idea of objective knowledge where knowledge is seen as

resulting from theories that are proposed, made public, and tested against other theories and held to be true until proven otherwise by other theories (Nickson, 1992:103). Following this view, mathematics is a static and bounded discipline that must be mastered and also that it consists of immutable truths. The fallibilist view sees mathematics as a social product and maintains that mathematical truth, like scientific truth, is subject to revision. According to this view, mathematical knowledge is arrived at in practice; conjecture, discussion, justification; refutation and modification all taking place in a social arena (Phye, 1997:347).

According to Phye (1997:347), individual mathematics does not create mathematics, rather, networking and communication of mathematics in arenas of conflict and cooperation, domination and subordination, create mathematics. Goodman (in Phye, 1997:348) contends that a mathematical theory, like any other theory, is a social product created and developed by the dialectical interplay of many minds, not just one mind. He further says that in each generation, mathematics rethinks the mathematics of the previous generation. This is in line with Freudenthal's (1987) realistic view that mathematics structures arise from reality, which is not fixed datum, but expands continuously in a person's individual and collective learning process (Phye, 1997:348).

Romberg (1992:226) argues that views similar to Platonist or formalist fail to see that mathematics is being discovered continuously and the types and variety of problems to which mathematics is being applied have grown at an unprecedented rate. Technology has generated an enormous wealth of ideas in several branches of mathematics. Romberg stresses that teachers and learners need to change the belief that mathematics is a set of rules and formalisms invented by experts and that everyone else should memorise them to obtain unique and correct answers. According to Lampert (Koehler & Grouws, 1992: 121), new mathematics is brought about through a process of conscious guessing about relationships among qualities and shapes, with proof following a

zigzag path starting from conjectures and moving to the examination of premises through the use of counter examples or refutations.

2.4 LEARNING THEORIES

2.4.1. Orientation

Goldin (2003:199) talks about mathematics education theories, which he refers to as traditional views and reform views. According to him, the traditional views advocate curriculum standards that stress specific, clearly identified mathematical skills at each grade level. Standards should be measurable and attention should be given to the correctness of the learners' responses and the mathematical validity of their methods. Furthermore, learners are seen as differing greatly in mathematical ability such that class groupings should be homogeneous by ability. Reform views, on the other hand, advocate curriculum standards in which high-level mathematical reasoning processes are central and universally expected. Proponents of these views lay emphasis on learners finding patterns, making connections, communicating mathematically and engaging in real-life conceptualised, and open-ended problem solving from the earliest grades. Children should be grouped heterogeneously to allow interaction among those with different learning styles (Goldin 2003:200).

In the following sections the learning theories of behaviourism, cognitivism (information processing) and constructivism will be discussed against the stated background. The reasons for choosing these theories are that they are prominent in mathematics education, and that each seems to be supported on the basis of the shortcomings of the others.

2.4.2. Behaviourism

Behaviourism was one of the dominant areas of research into learning throughout the twentieth century (Underwood, 2003). It is primarily associated with Pavlov (classical conditioning) and with Thorndike, Watson and Skinner (operant conditioning). Much behaviourist experimentation is undertaken with animals and then generalised. The basic mechanism of behaviourist learning is stimulus–response–reinforcement.

In educational settings, behaviourism implies the dominance of the teacher, as in behaviour modification programmes (Atherton, 2003:1). This dominance of the teacher in mathematics education is often seen in countries such as Japan where mathematics learning is dependent on the teacher (Woodward & Ono, 2004: 78). According to Tirosh (2003:231), learning environments designed according to behaviourist principles are organised with the goal of teachers, the source of knowledge, transmitting facts and procedural knowledge efficiently to learners.

Behaviourist psychologists reject on principle any incorporation of internal mental states, mental representations or cognitive models, thoughts, understanding or information gained through introspection into theory (Goldin, 2003:203). Behaviourists thus claim observed behaviour as the only means to study learning and view knowledge as an organised skills component (Tirosh, 2003: 231).

According to Underwood (2003:1), behaviourists stand firm in the tradition of “associationism” and they believe that there are three qualities from which this association arises: resemblance, contiguity in time or place, and cause and effect. Anderson and Bower (in Underwood: 2003:1) suggest the following four features of associationism:

- the notion that mental elements become associated through experience
- that complex ideas are reduced to a set of simple ideas
- that the simple ideas are sensations
- that simple additive rules are sufficient to predict properties of complex ideas from simple ideas

There is a maxim in the teaching of mathematics in South Africa that one should teach from the “simple to the complex”. To a great extent this maxim fits into the “associationism” principle. The teaching of fractions in elementary classes, as indicated by Orton (1990:70), assumes this principle when an apple is cut into two equal parts, each of which represents half, that is,

$$\begin{array}{ccccc}
 \text{Diagram of a semi-circle} & + & \text{Diagram of a semi-circle} & = & \text{Diagram of a circle} \\
 \frac{1}{2} & + & \frac{1}{2} & = & 1
 \end{array}$$

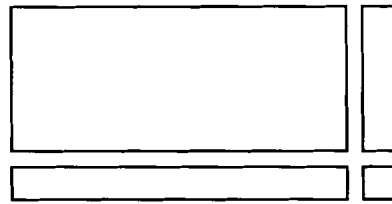
Understanding of this will later help the learner at junior secondary school to understand:

$$\frac{x}{2} + \frac{x}{2} = x,$$

Furthermore, “associationism” could be compared with Dienes theory of mathematics learning, which consists of

- the Multi-base Arithmetic Block (MAB) – for early learning
- the Algebraic Experience Material (AEM)
- the Equaliser (Dienes balanced)
- the logical blocks

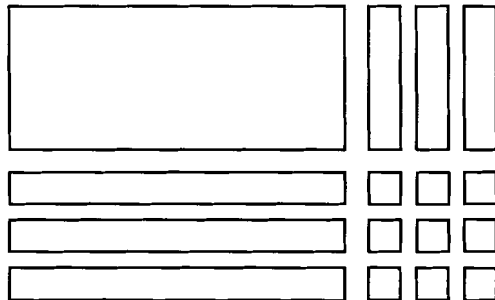
Consider for instance the application of Dienes AEM to promote understanding of $(x + 1)^2 = x^2 + 2x + 1$ and $(x + 3)^2 = x^2 + 6x + 9$



$$\text{as } (x+1)^2 = x^2 + 2x + 1,$$

which reads “one x square plus two x strips plus one unit square”.

Similarly,



$$\text{as } (x+3)^2 = x^2 + 6x + 9 \text{ (Orton, 1992: 70).}$$

2.4.2.1. Discussion

- In a behaviourist view, classrooms are viewed as a collection of individual learners who do not collaborate with each other.
- Instruction is often programmed and computer-based drill and practice programmes are designed.
- It describes in detail the succession of materials teachers present to their classes in order to provoke learning.
- Behaviourism cannot account for what is going on inside a learner's head (Tirosh, 2003; Goldin, 2003; Darby, 2003).

However, Darby (2003:16) has this positive comment to make about behaviourism: “It allows a natural progression of materials from concrete to abstract; it provides a contextual way into mathematics for those whose confidence and or ability is lower; it allows various areas of mathematics so that each does not exist in isolation; when up against inevitable time constraints, it provides a method for teaching to examination.”

2.4.2.2. Contextualisation to the South African situation

The teaching of mathematics in South Africa is generally behaviouristic. Studies conducted by Ensor (2000) indicate that although pre-service teachers knew reform views and recent theories in mathematics teaching and learning, they applied the traditional teaching styles dominated by the teacher. This was also observed by Taylor and Vinjevold (1999) in the President's Educational Initiative (report).

The educators and principals of science schools (refer to Chapter 1) were promised fringe benefits upon better results in mathematics and science by the Bophuthatswana government. The present government also wants to move in the same direction (*Sunday Times*, 2004). This is an indirect or direct application of Skinner's ideas of reinforcement. Moreover, learners who get better symbols in mathematics

- get easy access to tertiary institutions (see calendars of different institutions)
- are easily employed by the private sector on completion of Grade 12
- are considered intelligent by society (see 3.2.1)
- obtain bursaries to further their studies (see calendars of different institutions)
- enter careers considered prestigious, for example engineering, commerce and medicine (see calendars of different institutions)

2.4.3. Cognitivism

According to Underwood (2004:2), behaviourism was contested in the 1930s by a growing school of thought called Gestaltism, which believed that the object of instruction in mathematics should be helped by rich mental structures. This resulted in the conception of cognitive psychology, however, it was not until the latter part of the 20th century

that cognitive theory was born (Underwood, 2004:3). This theory refers to the brain as a mind, an information-processing machine. Selden and Selden (1997:2) talks about a “cognitive revolution” in which the mind was often regarded as a computer and consequently as an information processor. Mousley et al. (1992:112) talk about two types of cognitive theory viz. cognitive conflict and socio-cognitive conflict.

2.4.3.1. *Cognitive conflict theory*

This theory asserts that all humans learn by the twin processes of assimilation and accommodation (Piaget 1954; Sinclair 1990) with the world being assimilated in the mind, while the mind accommodates to the world. The cognitive conflict theory was propounded by Piaget who said that the teacher as an organiser is indispensable for creating the situations and constructing the initial devices, which present useful problems to the child.

The cognitive conflict theory is difficult to apply in the sense that the teacher is expected to make sure that when learners make systematic errors they are given tasks that are likely to make them aware that incorrect answers are inconsistent with other concepts and principles that they know and already understand (Mousley et al., 1992:112).

2.4.3.2. *Socio-cognitive conflict theory*

Social factors have an important influence on how children learn mathematics (Bell & Bassford, 1989; Doise, 1985; Light & Glychan, 1985). This will be discussed in more detail in the next chapter of this study. According to this theory of learning, children involved in problem-solving tasks in which they interact with peers are confronted with alternative and conflicting strategies. This is caused by the social context in which the interactions occur for they are of such a nature that most children feel inwardly compelled to take account of different solution strategies put forward by others (Mousley et al., 1992:113).

However, research conducted in the mid 1970s by Doise, Mugny and Clermond on these studies showed that under certain circumstances learners who participated in interaction sessions showed significantly more progress than those who did not have the opportunity to interact with peers. Furthermore, research done into the effects of cooperative mathematics learning by the Purdue Problem Centred Mathematics Project based at Purdue University strongly supports the potential of cooperative learning for school mathematics (Mousley et al., 1992:114). Also, cooperative learning is one of the recent learning strategies encouraged in mathematics learning by Curriculum 2005 in South Africa.

According to Selden and Selden (1997:4), there are two different perspectives found in education research that tend to reject the information-processing view of the mind. These are situated cognition and constructivism perspectives. Adherents of situated cognition focus on how individuals learn to participate within communities of practice and how their development is shaped by the perspective. They do not regard knowledge as being entirely in one's head, but suggest that attention should be paid to the way in which individuals interact with or function in various situations, often social situations. Taking interaction as a principle unit of analysis, this perspective believes that it is not particularly enlightening to look at what is in an individual's mind separate from the situation (Selden & Selden, 1997:5).

Indeed, in any teaching and learning situation contextual factors need to be taken into consideration. The next chapter deals with this in detail, while constructivism will shed some light on the learner in relation to the community.

2.4.3.3. Discussion

Information processing theory is credited with respecting the learner's capacity to apply the mind. However, it is not always made clear as to how learners should apply their minds in the classroom situation. Ohlsson (1990) feels that while information processing is good at encouraging skills acquisition, its view of the mind is limited. Furthermore, there are smaller theories within one big theory (information processing), viz. cognitive conflict, socio-cognitive conflict and situated cognitive theory, that seem to be opposing other theories, for example, socio-cognitive theory is in conflict with cooperative learning theory which is being encouraged worldwide. On the other hand, as Mousley et al. (1992) indicate, cognitive conflict theory is difficult to apply.

Of all smaller theories of cognitivism, situated cognition appears to be the most progressive in that it advocates the consideration of contextual factors in any teaching-learning situation.

2.4.3.4. Contextualisation to the South African situation

Historically, information processing (cognitivism) is relatively new (20th century) in education research, more so in the South African situation, which is undergoing transformation, educationally and politically. For this reason it has not been applied at the experimental school involved with collaborative teaching. If understood and well applied, the possibility of it impacting positively on mathematics learning in Grade 12 is great.

2.4.4. Constructivism

Constructivism was developed by Von Glasersfeld and it incorporates both Piaget's notions of assimilation and accommodation (Cobb & Yackel, 1998:159). According to Tirosh (2003:231), constructivism focuses on characterising the cognitive growth of children in conceptual

understanding. A basic assumption is that knowledge is not communicated, but constructed and reconstructed by unique individuals. This theory characterises learning as a process of self-organisation in which the individual reorganises his or her activity to eliminate perturbations. This occurs as the individual interacts with other members of a community. In this process of mutual adaptation, individuals negotiate meaning by continually modifying their interpretations. According to Von Glasersfeld, this is possible through communication (in Cobb & Yackel, 1998:160).

In constructivism therefore, negotiation becomes central in the teaching-learning situation. Newman, Griffin and Cole (1981) define negotiation, using socio-historical metaphor, as a process of mutual appropriation in which the teacher and learner continually use each other's contributions. In contrast, Bauersfeld (in Cobb & Yackel, 1998:161) uses an interactional metaphor whereby he characterises negotiation as a process of mutual adaptation in the cause of participants' interaction. In the former metaphor, this means that the teacher is said to appropriate learners' actions into the wider system of mathematical practices that he or she understands. In the latter metaphor, however, the local classroom microculture rather than mathematical practices is the primary point of departure.

According to Yackel, Cobb and Wood (1992:64) there are two basic principles of constructivism, viz. that knowledge is actively built up by the cognising subject and that the function of cognition is adaptive. These principles do not dictate specific teaching methods, thus they are, as a result, viewed as rubrics. Following the first principle, Von Glasersfeld (in Yackel et al., 1992:64) asserts that mathematical knowledge cannot be given ready-made to learners, instead, problem solving should be conceived as a crucial aspect of acquiring mathematical knowledge.

Children should be afforded learning opportunities to listen and to try and make sense of the solution methods of others; to give explanations and

question the explanations of others; to attempt to resolve conflicting points of view and to seek to develop a basis for collaborative activity. In such a classroom situation Cobb (1990) sees learning as an interactive activity (Yackel et al., 1992:65). According to Tirosh (2003:232), learning environments should be designed to provide learners with opportunities to construct conceptual understanding and to foster problem solving and reasoning abilities.

Although Von Glasersfeld (in Cobb & Yackel, 1998:160) defines learning as self-organisation, he acknowledges that constructive activity occurs as the cognising individual interacts with other members of a community. Bauersfeld (1980) complements Von Glasersfeld's cognitive focus by viewing communication as a process of mutual adaptation wherein individuals negotiate meanings by continually modifying their interpretations.

This aspect of communication leads to what Ernest (1991:42) refers to as social constructivism, which views mathematics as a social construction. It draws on conventionalism in accepting that human language, rules and agreement play a key role in establishing and justifying trust in mathematics. Social constructivism subscribes to the view that mathematical knowledge grows through conjectures and refutations utilising the logic of mathematical discovery.

The following are grounds for describing mathematical knowledge as a social construction:

- The basis of mathematical knowledge is linguistic knowledge, conversations and rules, and language is a social construction.
- Interpersonal social processes are required to turn an individual's subjective mathematical knowledge into accepted objective mathematical knowledge.
- Objectivity itself will be understood to be social (Ernest, 1991:42).

Mousley et al. (1991:109) present constructivism such that contextual factors are considered when they say: "It is impossible for mathematical concepts, or indeed any concepts, to be transmitted from one person to another by means of words alone. Inevitably both teachers and their students must become 'experiencing subjects' when they are involved with a mathematical task."

2.4.4.1. Discussion

The following points are noted in constructivism:

- The individual is unique and he or she has the potential to construct and reconstruct knowledge.
- The individual needs to communicate, interact, adapt and negotiate meaning with others.
- Problem solving as an instructional method becomes conspicuous.
- Cooperative learning is imperative.

Cobb (1990) suggests that a mathematics classroom environment should incorporate the following qualities:

- Learning as an interactive as well as a constructive activity.
- Presentation and discussion of conflicting points of view are encouraged.
- Reconstruction and verbalisation of mathematical ideas and solutions are commonplace.
- Learners and teachers learn to distance themselves from ongoing activities in order to understand alternative interpretations or solutions.
- The need to work towards consensus in which various mathematical ideas are coordinated is recognised.

2.4.4.2. *Contextualisation to the South African situation*

Thus far, the aspects of interaction, negotiation, cooperation, collaboration and communication are seen as key to constructivism. Language therefore becomes central. This was indicated by other researchers (Raghavan, 1994; Rakgokong, 1994) as posing a problem to the application of constructivism in a multilingual country such as South Africa.

The historical background of teacher development in South Africa reveals that few teachers, if any, have any idea about constructivism. Informing in-service teachers in schools about theories such as constructivism and its application in mathematical learning is of paramount importance.

2.4.5. General discussion

Every classroom situation consists of unique learners with different abilities in mathematics. Information processing requires the mathematics teacher to focus on each learner's mind. However, the minds of learners are different and this poses the teacher with the problem of how to teach them. On the other hand, constructivism encourages interaction, negotiation, adaptation and communication in an environment in which the individual constructs and reconstructs knowledge. However, in a real classroom situation there are learners who are introverts, learners who come from poor socio-economic backgrounds and some who are emotionally and spiritually abused. Under such conditions, it becomes difficult for the teacher to apply constructivism. Behaviourism is generally discredited and regarded as outdated. However, given the limitations that information processing and constructivism sometimes have, behaviourism remains an option.

No learning theory therefore will ever be superior to all the others. The approach, as suggested by Davis and Simmt (2003), of complexity

science appears to be the solution in the sense that it covers the three theories mentioned. This approach has an element of collaboration (internal diversity) in the sense that it accommodates all learners with different backgrounds.

The collaborating teachers (in this study) at the experimental school taught mathematics according to a behaviouristic approach. This was as a result of the way they were developed as teachers of mathematics. As a result of this their teaching was product based, that is, good results in Grade 12 in particular were the ultimate goal.

2.5. LEARNING TASKS

According to Doyle (1983:161), tasks form the basic treatment unit in classrooms. The accomplishment of academic tasks has two consequences, viz. a person will acquire the skills needed to accomplish the task and that a person will practise operations used to obtain or produce the information demanded by the task. "Task", accordingly, focuses attention on these three aspects of learner's work:

- the product learners are to formulate
- the operations that are to be used to generate the product
- the resources available to generate the product

According to Hudson et al (1995:3), a good learning task

- engages all the senses
- allows learners to construct and explore ideas
- has multiple paths to a valid outcome
- is not "over engineered"
- contains sound and significant mathematics or science

From the above it is imperative that, before a teacher can assign learners a task, there should be a well-defined goal (product) and that

learners should be provided not only with guidelines, but with resources as well. Furthermore, the task must consider the mental development of learners so that it is not above their thinking capacity and so that learners are active participants in the task.

Flewelling and Higgison (2002:130) talk about a “sense-making game” and “Rich Learning Tasks”. According to them, successful teachers provide learning tasks that will give their learners the opportunity to play the sense-making game over time so as to learn how to play this game at a progressively higher level. According to them, the sense-making game is about using knowledge and experience in integrated, creative, authentic and purposeful ways to solve problems, conduct inquiries, carry out investigations and perform experiments. Sense making in the mathematics classroom is about making sense when doing mathematics, and making sense of people’s actions and ideas. Learners have to play this game if they are to address the challenges and opportunities of life both in and outside school successfully (Flewelling & Higgison, 2002:131).

According to Collins, Brown and Newman (1989) in *The Math Forum* (1994–2003:2), in cognitive domains, drawing learners into a culture of experts involves teaching them to think like experts. Schoenfeld (1987) in *The Math Forum* (1994–2003:2), also emphasises the importance of creating a “microcosm of mathematical culture” in order to help learners think like expert mathematicians. Schoenfeld demonstrated this by solving mathematics problems alongside his learners. “Mathematics”, according to Sheffield (1989:213), “was the medium of exchange. We talked about mathematics, explained it to each other, share the mathematical people. By virtue of this cultural immersion, the learners experienced mathematics in a way that made sense.”

According to Flewelling and Higgison (2002:134), sense making, should be like a story with learners and teachers acting as authors and readers of and characters in the story. In contrast to a traditional mathematics

classroom culture, Flewelling and Higginson (2001:60) make the following comparison:

Sense-making (mathematics classroom) culture	Traditional (mathematics classroom) culture
1. Convincing	1. Unconvincing
2. The discipline as a way of thinking	2. The discipline as a collection of procedures
3. Working with things that make sense	3. Working with the inexplicable
4. Master	4. Slave
5. Address students need	5. Ignores needs of student
6. Known to be true	6. Accepted as true
7. Student active	7. Student passive
8. Validity by student	8. Validated by teacher
9. Student-owned	9. Teacher-owned
10. Student as rule maker	10. Student as rule taker
11. Described/explained in student language	11. Described/explained in teacher language
12. Teacher as educator	12. Teacher as inculcator
13. Independence/interdependence	13. Dependence
14. Develop by end of lesson	14. Presented at beginning of lesson
15. Connected	15. Isolated
16. Develop procedures	16. Follow procedures
17. A partnership	17. Master-slave relationship

As a solution, Flewelling and Higginson (2001: 60) suggest that to change a traditional mathematics classroom culture to a sense-making culture, both learners and teachers should work together and focus on, engage in and experience rich learning tasks. The two authors define a learning task as “rich” if the task gives the learner the opportunity to

- use (and learn to use) their knowledge in an integrated, creative and purposeful fashion
- acquire knowledge with understanding, and in the process
- develop the attitude and habits of life-long sense-makers

The following list compares rich learning tasks with traditional tasks:

Rich tasks	Traditional tasks
1. Prepare for success outside school	1. Prepare for success in school
2. Address relatively many learning outcomes	2. Address relatively few learning outcomes
3. Address discipline and cross-curricular learning outcomes	3. Address primarily learning outcomes of the discipline
4. Provide an opportunity to use broad range of skills in an integrated, often creative fashion, for a purpose	4. Isolate on the use of relatively few skills
5. Are authentic	5. Are more artificial
6. Are in context	6. Are usually an unbalanced use of actions
7. Encourage a balanced use of actions	7. Encourage an unbalanced use of actions
8. Are more like writing a story	8. Are more like writing a sentence
9. Emphasise problems solving	9. Emphasise procedures
10. Encourage more thinking, reflecting, and use of imagination	10. Encourage more recollection and practice

11.	Allow for demonstration of an a wide range of performance	11.	Allow for demonstration of narrow range of performance
12.	Need performance assessment strategies	12.	Need traditional assessment strategies
13.	Provide enrichment within the task	13.	Usually require enrichment to be added after the task
14.	Encourage the use of wide variety of teaching and learning strategies	14.	Permit the use of fewer teaching and learning strategies
15.	Encourage greater engagement of students and teachers in task	15.	Keep students and teachers distanced from the task
16.	Not a new/untried idea	16.	A much-applied idea (Fewelling & Higginson, 2001:60)

2.6 CONCLUSION

Analysis of mathematics learning and learning tasks was based on behaviourism, cognitivism and constructivism. Socio-cognitive conflict theory showed the importance of contextual social factors in mathematics learning. The TIMMS reports (1995; 1999) also consider these factors in their analysis of results. The next chapter deals with some of the factors affecting mathematics teaching and learning.

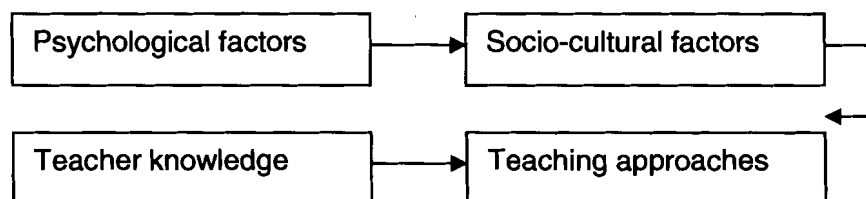
CHAPTER 3

FACTORS AFFECTING THE TEACHING AND LEARNING OF SENIOR SECONDARY MATHEMATICS IN SOUTH AFRICA

3.1. INTRODUCTION

The teaching and learning of mathematics (and of any subject) need to be viewed from the context in which they take place.

Surveys such as, TIMMS (1998); and research such as Tate (1997) and Stanic and Reyes (1998), have shown the extent to which contextual factors such as home, resources etc affect performance in mathematics and science. For this reason, this chapter seeks to analyse how some of these factors impact on mathematics learning and performance. Focus will be put on the following factors:



3.2. PSYCHOLOGICAL FACTORS

3.2.1. Mathematics beliefs

3.2.1.1. *Definition*

Schoenfeld (1992:359) defines beliefs as an individual's understanding and feelings that shape the way in which that individual conceptualises and engages in mathematical behaviour. Raymond (1997:552) defines mathematics beliefs as "personal judgements about mathematics formulated from experience in mathematics including beliefs about the nature of mathematics, learning mathematics and teaching mathematics".

Schoenfeld (1992:359) indicates that beliefs are influenced by factors such as the school and the society in which the individual grows up. The role that society plays in the teaching and learning of mathematics manifests itself socio-culturally, politically and psychologically. McLeod (1992:579) also contends that, in mathematics, most researchers agree that the development of beliefs about mathematics is heavily influenced by the cultural setting of the classroom.

3.2.1.2. *Teachers' beliefs*

Researchers, says Thompson (1992:135), have reported varying degrees of consistency between teachers' beliefs and teachers' instructional practice. Earlier studies on the relationship between mathematics beliefs and practice reported mathematics practices having a strong influence over mathematics beliefs (Thompson, 1992:135). However, studies conducted by Raymond (1995:571) show that mathematics beliefs have a strong influence on mathematics teaching practices. According to him (Raymond, 1997:571), traditionally teachers believed that mathematics was taught as an unrelated collection of facts, rules and skills, which are fixed, absolute and surprising. They (teachers) believed that their role was to dispense mathematical knowledge to learners who were supposed to receive it passively. Learners were further expected to learn as individuals and to memorise rules and algorithms.

The teaching of primary mathematics in the past in South Africa included what was called "mental arithmetic" where learners were required to recite tables. These tables were about multiples of different digits and only multiplication was recited. Learners were not told about the relationships between multiplication and division and between addition and subtraction. This type of approach has an impact on the learning of a section, for example "changing the subject of the formula" which is included in the Grade 10 to 12 syllabus.

Experience at the experimental school shows that learners do not understand terms such as “multiplicative inverse” and “additive inverse”. For example, “ $1/a$ ” (or “ a^{-1} ”) being the multiplicative inverse of “ a ” and “ $-a$ ” being the additive inverse of “ a ”.

Raymond (1997:556) suggests that mathematics should be regarded as dynamic, problem driven and continually expanding. According to him, the learner’s role should be that of autonomous explorer who learns cooperatively through problem-solving activities. The teacher’s role should be that of a facilitator and guide. Curriculum 2005 supports the role of teacher as mediator.

3.2.1.3. *Learners’ beliefs*

According to Garofalo (1989:50), traditionally learners believe that almost all mathematical problems can be solved by a direct application of the rules, facts and formulae, procedures and methods shown in the textbook and that only those sections meant for tests and examinations are important. There seems to be concurrence between Garofalo (1989) and Schoenfeld (1992: 358), who indicate that learners believe there is only one correct way to solve any mathematical problems – usually the rule the teacher has demonstrated. This results in learners believing, as Raymond (1997:571) puts it, that memorisation and mastery of algorithm signify learning.

Learners inherit their beliefs about mathematics mainly from their parents, and from their teachers, who teach for the purpose of completing the syllabus. Furthermore, teachers believe that they are the sole dispensers of information (Raymond, 1997:50) Consequently, learners believe in their teachers and the sources they use (textbooks usually) in the learning of mathematics. The teaching of mathematics in Grade 12 in South Africa is examination oriented. The teachers focus on the completion of the syllabus as a result, and less time is spent on the relevance of mathematics learning to everyday life. Various teaching and

learning strategies such as cooperative learning and problem solving need to be applied to bring about positive and productive beliefs on the nature of mathematics.

One of the characteristics of problem solving, as will be later noted in this chapter (see 3.5.4), is that it cultivates an inquiring mind in the learner (Bungane, 2002:33). According to Schoenfeld (1992:338), one of the goals of problem-solving strategies is to provide a new approach to remedial mathematics to induce critical thinking or analytical reasoning skills. The beliefs held by learners that mathematics is created by very prodigious and creative people (Garofalo, 1989:50) and that only geniuses can be creative in mathematics (McLeod, 1992:579) should be demystified.

To establish positive beliefs in the teaching and learning of mathematics in Grade 12 in South Africa, subscription to Kay's (in Schoenfeld, 1992:360) mathematics beliefs for both teachers and learners would be recommended. Kay believes that mathematics is a subject of ideas and mental processes that can best be understood by rediscovering its ideas. He recommends that the teacher should create and maintain an open and informal classroom atmosphere to insure the learners' freedom to ask questions and explore ideas. Moreover, the teacher should appeal to learners' intuition and experience when presenting the material in order to make it meaningful. Teachers' mathematics beliefs are very important for they influence the learner enormously. They might even affect other psychological factors that influence the learners' success in mathematics such as attitudes, interest, anxiety and motivation.

3.2.2. Attitudes

3.2.2.1. Definition

It is difficult to distinguish between attitudes and beliefs. Kretch et al. (1987) briefly define an attitude as "a like or dislike" that has behavioural

consequences". An attitude can be thought of as an emotion and an action tendency. From this definition, one can see a clear link between an attitude and a belief. McLeod (1992:581) defines attitudes as affective responses that involve positive feelings of moderate intensity and reasonable stability. McLeod (1992:581) notes that attitudes towards mathematics are not one-dimensional factors – there are many different kinds of mathematics as well as a variety of feelings about each type of mathematics. Most teachers and learners in South Africa have negative attitudes towards geometry (Laridon, 2000) and as a result Grade 12 mathematics learners perform poorly in Mathematics Paper 2 (Mulder, 2000; Van der Walt, 2000).

3.2.2.2. *Attitudes and mathematics achievement*

Research conducted by Ma and Kishor (1997:39) on the relationship between attitudes towards mathematics (ATM) and achievement in mathematics (AIM) reveal "ATM (cause) – AIM (effect)". This indicates that the effect of ATM on AIM is not strong and has no meaningful practical implication.

Ma and Kishor (1997:41) further note that the ATM–AIM relationship may not be strong at elementary school level but may be strong enough for practical consideration at the secondary level. They further suggest that the attitude measures need to be more age specific. This is due to the fact that as children grow older, they are better able to express their attitudes towards mathematics. Achievement in mathematics (AIM) should be area specific, that is, learners' performance should be measured in algebra or geometry or trigonometry. Furthermore, the inclusion of mathematics ability as a key variable between ATM and AIM is suggested. This was earlier also suggested by Schiefele (1995:163) who indicated that achievement in mathematics is strongly related to mathematics ability and that interest could account for a significant portion of achievement.

3.2.3. Interest

3.2.3.1. Definition

The concept "interest" has been viewed from various perspectives. It has been associated with psychological traits such as attitudes, motivation, feelings and attention. It is also seen to be as a result of environmental influences (Budhal, 1993:9). Budhal (1993:12) defines interest as the feeling of liking or disliking which a person experiences and the object or activity that a person likes or dislikes, while Alberts and Horn (in Budhal 1993:12) define interest as a relatively constant positive or negative attitude towards a particular familiar activity.

3.2.3.2. Interest and mathematics achievement

There are conflicting findings about the relationship between interest and mathematics achievement. Studies conducted by Vrey (1984) and Swanepoel (1982), Kidd et al. (1970), Husen (1967) and Moodley (1981) revealed that there is a positive correlation between interest in mathematics and achievement in mathematics. However, studies conducted by Köller and Baument (2001:468) and Budhal (1993:98) show that there is no significant positive correlation between interest in mathematics and achievement in mathematics.

Regarding the effect of prior achievement on interest, Köller and Baumert (2001:467) found inconsistent results. While achievement feedback enhanced interest during lower secondary school years (Grades 7 to 10), it did not have such an effect in upper secondary schools (Grades 10 to 12). The study therefore concludes that while there is no significant correlation between interest in mathematics and achievement in mathematics, interest in mathematics clearly decreases from Grade 7 to Grade 12 (Köller & Baument, 2001:468). They believe this trend is caused by a mismatch between the curriculum and the general interest of adolescents; the experience of learners and the field of experience outside the school,

which broadens considerably during adolescence, provide competing opportunities for interest development in mathematics.

The results of this investigation on interest and maths achievement present a strong challenge to mathematics teachers all over the world, and particularly to Grade 12 mathematics teachers in South Africa.

3.2.4. Motivation

3.2.4.1. Definition

Budhal (1993:48) defines motivation as a general process by which behaviour is initiated and directed towards a goal. According to him, when direction comes from the individual, it is called intrinsic motivation and this stems from interest and curiosity. On the other hand, when an individual does something in order to earn a reward, this kind of action is influenced by extrinsic motivation. In mathematics teaching, teachers can create intrinsic motivation by stimulating curiosity with their styles of teaching and can arouse extrinsic motivation by providing rewards and setting realistic goals for learners (Budhal, 1993:49).

3.2.4.2. Motivation and mathematics learning

Middleton and Spanias (1999:78) found that some children might begin to feel a lack of efficiency in mathematics as early as the third grade. They found that learners who like mathematics report that they developed the love for the subject from seventh grade. Equally, those who dislike mathematics from elementary to middle school where the level of mathematics and instruction patterns are higher seem to cause like or dislike for mathematics.

Research indicates that a person will pursue a task if he/she expects to be successful in that particular task, that is. extrinsic motivation results. Lack of success in mathematics is associated with lack of ability and this affects

the learner's self-image (Middleton & Spanias, 1999:79). Motivation as regards mathematics is developed early and is greatly influenced by teacher actions. It should be noted that, for learners to be motivated to learn mathematics, they depend, to a great extent, on the mathematics teacher. This state of affairs was also noted in mathematics beliefs. The present system of outcomes-based education requires the teacher to give projects, tests, tutorials and assignments. Learners have to work cooperatively to earn continuous assessment assignments (CASS) marks. This seems to have a positive effect on their learning and consequently motivates them. If well-structured, real-life problem situations are used in mathematics teaching, then learners uncover important and interesting knowledge that promotes understanding (Middleton & Spanias, 1999:79).

3.2.5. Mathematics anxiety

3.2.5.1. Definition

The term "mathematics anxiety" has been used to describe the panic, helplessness, paralysis and mental disorganisation that arise among some people when they are required to solve a mathematics problem (Dossel, 1993:4). Fiore (19903) describes mathematics anxiety as "an illness that is an emotional and a cognitive dread of mathematics".

Hawkey (1995:33) defines mathematics anxiety as "anxiety in the presence of mathematics". According to Richard and Sunn (in Hawkey, 1995:33), "mathematics anxiety involves feelings of tension and anxiety that interfere with the manipulations of numbers and the solving of mathematical problems in a wide of ordinary life and academic situation".

Mathematics anxiety is a real psychological response set off by the thought of doing mathematics. Individuals who suffer such anxiety become extremely nervous and go to great lengths to get away from the sources of their fear. These people learn about mathematics only under duress, which further increases fear and anxiety (Geldenhuys, 2000:24).

3.2.5.2. Mathematics anxiety and mathematics achievement

Research concerning mathematics anxiety and mathematics learning seems to indicate that high levels of anxiety hinder academic progress. Negative correlations have been found between mathematics anxiety ratings and performance on mathematics components of the Different Aptitude Test (Hawkey, 1995:37). Hembree (1990:42) reviewed 13 previous studies of the relationship between mathematics anxiety and performance. He reports that in all cases the low-anxious learners always performed better than those with high-anxiety levels. Furthermore, he discovered that reduction in mathematics anxiety was accompanied by a significant increase in mathematics tests scores.

On the question of mathematics anxiety depressing performance, he (Hembree) concludes:

- Reduction in mathematics anxiety results consistently in higher achievement.
- Treatment can restore the performance of formerly high-anxious learners to performance level associated with low mathematics anxiety.
- There is no compelling evidence that poor performance causes mathematics anxiety.

Ma (1997:535) argues that mathematics anxiety tends to rise in learners whose mathematics performance is improving. Her argument is that these learners (gifted or highly ambitious) are often unable to control their anxiety and channel it into the task because of their strong self-esteem and high levels of task-related confidence. This finding may be an eye-opener to the fact that mathematics anxiety can be useful in promoting mathematics achievement.

The conflicting research studies presented thus far illustrate the dynamic nature of the relationship between mathematics anxiety and mathematics achievement. Mathematics anxiety, therefore, may facilitate mathematics

performance, may debilitate mathematics performance or may be unassociated with mathematics performance (Ma, 1999:536).

In concluding remarks, Ma (1999:536) indicates that the relationships between mathematics anxiety and mathematics achievement can be understood as a psychological function of emotions, belief and attitude, which are major elements of the affective domain in the learning of mathematics.

3.3. SOCIO-CULTURAL FACTORS

3.3.1. Language

There is no doubt that language, as a means of communication, plays a vital role in teaching and learning in general. Luthuli (1992:27) says: "Spoken or written language is the main vehicle for communication of human thought. In a teaching-learning situation, pupils should be able to learn what the teacher intends them to learn without language itself getting in the way."

In mathematics, the focus should be on the role that the mother-tongue as a medium of instruction plays in determining the influence of language on mathematics learning. Cocking and Mestre (1988:28) ask these questions: "What is known about the dependence of mathematics upon language skills?" and "What is known about the effects of mathematics achievement?" Their studies reveal that algebraic word problems are solved with greater success when instruction focuses on meanings of the words and their translation into corresponding mathematics symbols. Results of these studies point to the strong effects of language on mathematics achievement.

In the USA, Bradby (1997:671) examined the demographic and language characteristics of Asians and Hispanics and related that information to their mathematics test scores. He further categorised

these language minority learners into the following four mathematics performance levels: basic, intermediate, advanced and below basic. He reports that minority learners who self-reported as Mexican, Cuban, Puerto Rican and Hispanic, and who were unable to attain the basic mathematics proficiency level, were 35 percent, 34 percent, 42 percent and 37 percent respectively. Accordingly, as English proficiency increased, the percentage of those below the basic mathematics level decreased. Specifically, 58 percent of the low-English proficient Hispanic learners failed to achieve the basic level of mathematics performance compared with 37 percent of the moderately proficient and 35 percent of the highly proficient (Tate, 1997:672). Bradby's (1997) conclusion thus is that the level of English proficiency for the Hispanic learners in particular, was positively related to mathematics achievement. Learners with a moderate level of English proficiency were better able to achieve a basic level of mathematics than learners classified as low-English proficient.

The National Council of Teachers of Mathematics (NCTM) suggests that increased opportunities for communication and mathematics learning must be given to learners so that they can

- model situations using oral, written, concrete, graphical and algebraic methods
- develop common understanding of mathematical ideas including the role of definitions
- reflect on and clarify their own thinking about mathematical ideas and situations
- use the skills of reading, listening and viewing to interpret and evaluate mathematical ideas
- discuss the mathematical ideas and make conjectures and convincing arguments
- appreciate the value of mathematical notation and its role in the development of mathematical ideas (Hayden & Cuevas, 1990:41)

Learning mathematics in a second language therefore affects achievement. Studies conducted by Secada (1992) on bilingual education and mathematics learning point to findings of a significant relationship between the development of language and achievement in mathematics (Setati, 2002:12). Mathematics in Grade 12 in South African schools is offered in either English or Afrikaans. No doubt this puts learners of white, coloured and Indian population groups, more so than African, at an advantage, consequently, whenever a learner of these racial groups does not understand a problem of mathematics in English, the Afrikaans version becomes an option and vice versa. For the African learner, both English and Afrikaans are second languages (Setati, 2002:7).

Consider the following Standard Grade Mathematics problem (Grade 12, November 2000): “Your school is raising funds to buy a school bus which costs R143 080. During the first month the school raised R280, during the second month it raised R560, and in the third month it raised R1120. That is, each month the school raised double the amount raised in the previous month. How long will it take to raise the full amount to buy the bus?” The key word in this passage is “raise” which is an equivalent of “increase” or “more”. This word appears seven times in the passage. The Afrikaans version (of the paper) has “insamel” as this key word. The same cannot be said about the Indian and particularly African learners. Collaboration between teachers of different racial groups especially on word problems, which are problematic in maths teaching, would minimise language problems especially for African learners.

3.3.2. Socio-economic factors

The problem of lack of resources in schools is very prevalent in communities where socio-economic status is low. In South Africa, this is a general outcry problem especially from schools that were disadvantaged politically and economically in the past. Says Thompson (NECC Mathematics Commission, 1993) “If one accepts the notion of

real world impacting on human beings, who in turn react thereto and that this reaction/action changes that real world, one could proceed to conclude that the development of mathematics is a socially motivated process.”

The history of education in South Africa has shown that schools from different economic backgrounds perform differently, especially in terms of Grade 12 examinations. Results from 1997 (Department of Education 1999 examination report) show that an economically viable province such as the Western Cape consistently attains an average percentage pass of 76 percent as compared to economically impoverished provinces such as the Eastern Cape and Northern Province, which attain average percentage passes of 41.3 and 30.8 respectively.

Thus, says Donovan (1990:17) “as teachers, we need to understand better the people and social groups who influence what happens in our classrooms, including the established structures and constraints that help determine access to and succession schooling”. Tate (1997:674) states that some educators and politicians have associated poor performance by learners of colour and low socio-economic status learners with a cultural deprivation argument.

A 1990 national assessment of an eighth grade mathematics programme in the USA found a strong relationship between residents’ economic status and the level of resources provided for their classroom experience. Eighty percent of teachers in schools with middle to upper socio-economic status learners received all materials and resources they needed for instructional purposes. Only 41 percent of teachers in schools with the largest concentration of low socio-economic status received all materials needed for instructional purposes. As a result, learners whose teachers reported limited materials or resources had lower mathematics achievement than those whose teachers indicated that their materials or resources were sufficient (Tate, 1997:674).

Rayes and Stannic (1998) give evidence of the positive relationship between socio-economic status (SES) and mathematics performance as found by Welch, Anderson and Harris (1982) who analysed data from National Assessment of Education (NAEP) and second mathematics assessment and achievement of 17-year-olds. Accordingly, the NAEP analysis found that high SES learners scored higher than low SES learners on number recognition tasks.

Anick et al. (1992:632) compared mathematics performance between a high metro and a low metro community. A high metro is a community whose learners' parents are employed in professional or managerial positions while a low metro is a community whose learners' parents are unemployed, on welfare or employed in factory or farm positions. They found that the average performance of the learners attending low metro schools was 9–13 percentage points below national average, whereas learners from high metro schools scored 8–10 percentage points above the national average.

Dossey et al. (1992:632) conducted studies on mathematics achievement as a function of four levels of parent education, viz. less than high school education; graduation from high school; some education beyond high school and graduation from college. It was found that for learners of all ages, the differences among the first three levels of parental education ranged from 7 to 19 points along a 500-point scale, the median difference was about 10 points. This implies that children whose parents have less education do not perform as well in mathematics than children whose parents have had a good education.

The Third International Mathematics and Science Study (TIMSS) (1996:38) reveals that South Africa is ranked 350 on 800-point scale in mathematics free response items. However, in addition, the following emerged from learners' questionnaires:

- The number of persons living in learners' home (commonly five or more) may not be conducive to study.

- There is a low level of achievement of the learning environment that the learners live in in terms of the limited number of books in the learners' homes. In South Africa about 60 percent of the learner population reports that they have less than 26 books in their homes. Books in the home serve as indicators of a home environment that values literacy and the acquisition of knowledge.
- There is a relatively high proportion of homes without electricity and other services.
- Most learners in South Africa reported that the highest level of education attained by either parents was to complete primary school. Studies show that parental education is often associated with learners' achievement.

The impact that SES has on mathematics performance can best be understood in a racially diversified society. As Reyes and Stanic (1988) point out: "We believe that differential performance in mathematics cannot be fully understood if factors on race, and socio-economic status are studied in isolation from each other. The two researchers made a comparative study of mathematics performance of Asian Americans and Hispanics who are low SES and white Americans who are high in SES. According to the 1973, 1978, 1982 and 1986 National Assessment of Education Progress (NAEP) tests, whites in America performed much better in mathematics than Hispanics, who in turn achieve slightly better than African Americans.

The South African situation is not much different from that of America. Historically (even now), white learners perform better in (Grade 12) examinations than learners of other racial groups, that is, blacks, coloureds and Indians. This is also the case in mathematics, as Hartshorne (1993:83) puts it: "In the 1989 Standard 10 classes, one third of the pupils took mathematics (at all levels), just over one fifth, physical science. In the final examination, less than one out of every twenty successful [Black] matriculants passed both mathematics and physical

science in higher grade. This should be contrasted with nearly 25 percent of White candidates who passed these two subjects with A, B and C symbols.”

The following table (e.g.) gives the overall results for all SA Grade 12 candidates for 1989:

Table 3.1 1989 Grade 12 mathematics results in South Africa

	Entries	HG passes	SG passes	Total passes	Failures
Black	209319	21357 10.2%	66153 31.6%	87510 41.8%	121809 58.2%
Coloured	22666	4044 17.8%	12431 54.8%	16475 62.6%	6191 37.4%
Indian	14191	5889 41.9%	7393 52.1%	13282 93.6%	909 6.4%
White	70666	29933 42.4%	37892 53.6%	67825 96.0%	2841 4.0%

As can be seen from Table 3.1, white and Indian candidates, who generally are of higher socio-economic status than coloured and black candidates have performed better (96% and 93.6%) compared to 62.6 percent and 41.8 percent for coloured and black candidates respectively. Fifteen years later, in the new dispensation, the same discrepancy is still haunting school mathematics education:

The 2003 and 2004 matric results for mathematics in the North West Province (North West Department of Education, 2005) show that the average for the standard grade was 28.07% and 28.18% respectively, whereas the average for the higher grade was 51.34% and 49.5% respectively. The vast majority of standard grade learners were black learners learning in English (as second language), while the vast

majority of the higher grade learners are white or Indian learners learning in their first language (Afrikaans or English).

3.4. TEACHER KNOWLEDGE

The history of teaching dates back to medieval times when universities regarded teaching as the most prestigious of the professions. The highest degrees awarded during this era were those of “master” and “doctor” which were traditionally interchangeable. Both words carried, by definition, the same meaning, viz. teacher. Teaching, during this era, was open to men and boys only. (Shulman, 1986:7). Content and pedagogy in teacher knowledge were part of one indistinguishable body of understanding (Shulman, 1986:6).

In the United States teacher examinations show that 90–95 percent of the tests were on content, the subject matter to be taught or on the knowledge base assumed to be needed by teachers. According to Shulman (1986:5), most states in the US around the eighties emphasised the capacity to teach as central in teacher knowledge. Later research classified “teaching effectiveness, process-product studies” or “teacher behaviour” as patterns of teacher knowledge that accounted for improved learner academic performance. Shulman (1986:6) criticises these research findings for leaving out subject matter. He refers to this as the “missing paradigm” problem.

Shulman (1986; 1987), who for now could be regarded as a pioneer of teacher knowledge research (his work is quoted by Marks, 1990; McEwan & Bull, 1991; Sherin & Madnes, 2000 and Kinach: 2002), suggests the following as categories of the knowledge base: content knowledge; curricular knowledge, knowledge of educational context and pedagogic content knowledge. A brief description of these types of knowledge follows:

3.4.1. Content knowledge

This refers to the amount and organisation of knowledge per se in the mind of the teacher. Teachers should not only be capable of defining for learners the accepted truth in a domain, they should also be able to explain why a particular proposition is deemed warranted and why it is worth knowing and how it relates to other propositions. In Shulman's words (1986:19), "we expect that the subject matter content understand that something is so: the teacher must further understand why it is so, on what grounds its warrant can be asserted").

Reality dictates that, in the South African context, this is not the case. This is due to teacher development happening at different types of institution where different examinations are being conducted. Collaboration, in-service workshops and networking are for now solutions, especially in mathematics teaching.

3.4.2. Curriculum knowledge

This is knowledge that is associated with a particular grasp of the materials and programmes that serve as "tools of the trade" for teachers (Shulman, 1987:8). According to Shulman (1986:10), the mature teacher is expected to have an understanding about the curricular alternatives available for instruction. He (Shulman) talks about lateral curriculum knowledge whereby the teacher is expected to be familiar with the curriculum materials of other subjects.

In mathematics teaching in Grade 11, the teaching of the cosine and sine graphs is closely linked with the teaching of waves in Grade 11 Physics. The terminology used, for example period, trough, crest and amplitude, is similar for both subjects. Knowledge of physics thus helps the teacher to enhance the teaching of mathematics.

On the vertical equivalent of the curriculum knowledge, Shulman (1986:10) talks about familiarity with the topics and issues that have been and will be taught in the same subject area during the preceding and later years in school. Mathematics teaching and learning entail continuity from the lower grades to the upper grades. If teachers in the lower grades do not teach effectively, this becomes a problem for teachers in the upper grades. In both the experimental and control groups, this is a serious problem because mathematics teachers in the middle schools, that is, Grades 5–9, either do not possess curriculum knowledge of the subject or do not teach effectively. Consequently, this impacts on learners in Grade 10. In most cases, teachers of mathematics in Grade 10 have to teach Grade 9 learning areas first before embarking on Grade 10 work.

3.4.3. Knowledge of educational context

According to P.L. Grossman (1990:9), knowledge of context includes knowledge of the district in which teachers work; knowledge of the school setting, including the school “culture”; departmental guidelines and other contextual factors at the school level; and knowledge of specific learners’ communities and learners’ backgrounds. Studies on socio-cognitive theory show the importance of contextual factors in mathematics learning.

Furthermore, a teacher has to belong to a teacher organisation or union. Knowledge of different teacher organisations is thus imperative.

3.4.4. Pedagogic content knowledge

This, according to Shulman (1986:9), goes beyond knowledge of the subject matter per se to the dimensions of subject matter knowledge for teaching. It is content knowledge that embodies the aspect of content most germane to its teachability. Shulman (1987:8) describes it as that special amalgam of content and pedagogy that is uniquely the province

of teachers, their special form of professional understanding. Within the category of pedagogical content knowledge, the following are included: the ways of presenting and formulating the subject that make it comprehensible to others; an understanding of what makes the learning of specific topics easy or difficult; the conceptions and preconceptions that learners of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons.

In this case, teaching and learning coincide. The teacher is therefore expected to have the didactics (in the South African context this is about method of teaching), the psychology of education to understand human development and learning, and the sociology of education to understand learners and their social backgrounds (refer to 3.2 and 3.3 of this chapter). Moreover, it is imperative for the teacher to have knowledge of views of mathematics, theories and approaches to mathematics learning and learning strategies. These were discussed in Chapter 2 of this study.

3.4.5. Application of pedagogic content knowledge (PCK) and its relation to other types of teacher knowledge

Shulman's study on pedagogic content knowledge (PCK) was taken further by Kinach (2002) in the US on mathematics pre-service teachers. According to Kinach (2002:51), it is documented that the procedural understanding of mathematics that pre-service teachers exhibit in university mathematics courses, method courses and other teacher education coursework, is not adequate to teach the reform mathematics curricular designed to implement the National Council of Teachers of Mathematics (NCTM) Principles and Standards of School Mathematics.

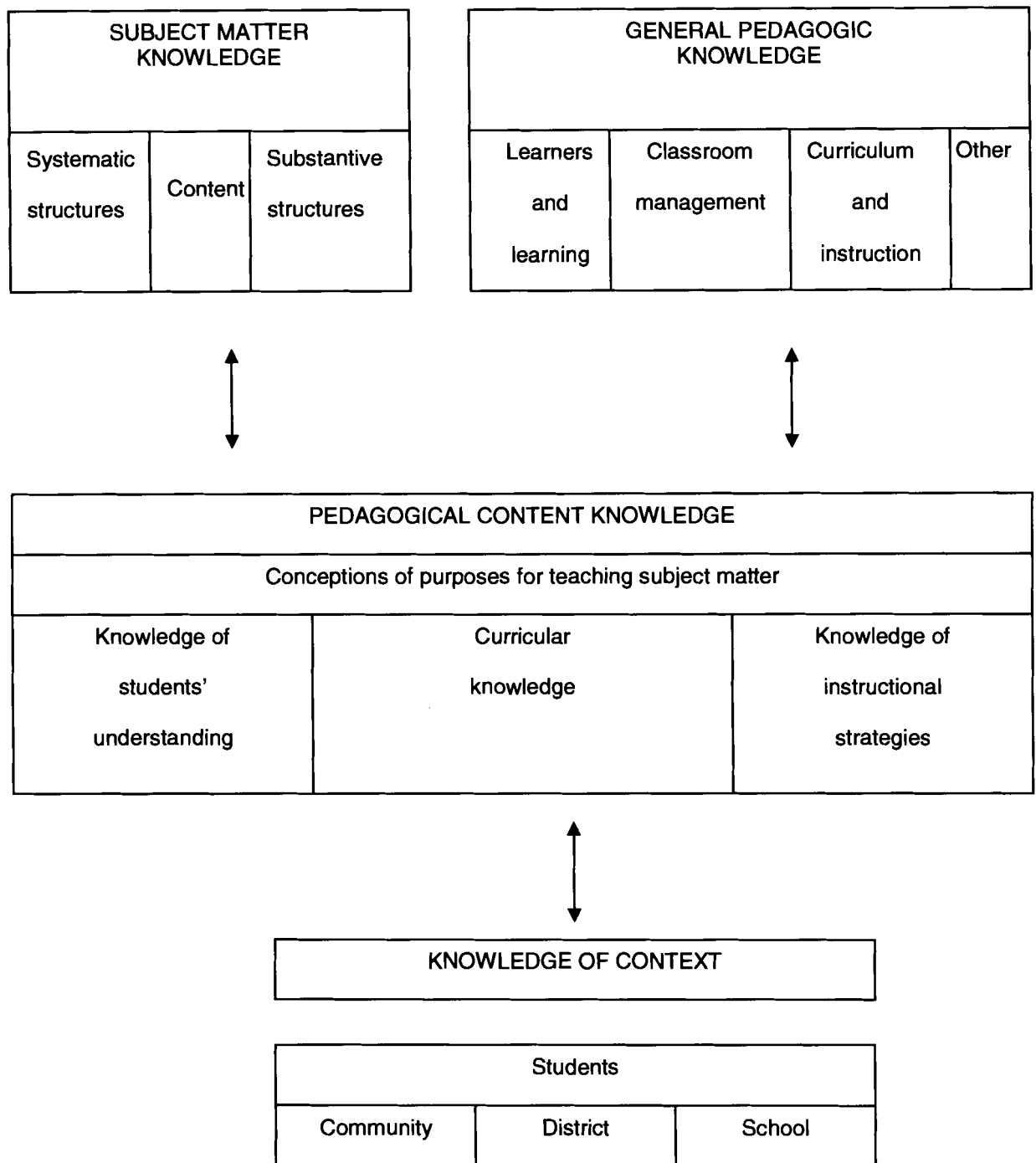
Kinach (2002:52) regards Shulman's (1986, 1987) pedagogic content knowledge as the cornerstone of his (Shulman) Knowledge Growth in Teaching Project. She regards the interface between subject-matter-knowledge (SMK) and pedagogic content knowledge (PCK) as central in transforming teacher knowledge, beliefs and thinking into a form

appropriate for teaching. According to Kinach (2002:54), there are two competing views of mathematics understanding viz. instrumental understanding which refers to the what and how of mathematics or rules without reasons, and relational understanding which includes insight into the why of mathematics or the reasons for the what and how. By comparison, she indicates, the relational approach broadens learner achievement more than instrumental understanding. Relational understanding further includes problem posing, critical contextual thinking and the ability to justify and represent one's mathematical thinking.

Kinach's studies found that prospective teachers' pedagogic content knowledge was instrumental by nature. Prospective teachers believe that knowing mathematics entails getting the answer; that teaching mathematics is a show-and-tell process of demonstrating for learners how to do something, and learning mathematics amounts to listening, copying and ultimately memorising the procedure received from the teacher (Kinach, 2002:64). This was as a result of the state of mathematics in the US at that time.

Grossman (1990:5) presents the following model on the relationships between different types of teacher knowledge:

Figure 3.1 Model of Teacher Knowledge (Grossman, 1990:5)



3.5. TEACHING APPROACHES

3.5.1. Orientation

In discussing the learning of mathematics in Chapter 2, mention was made of traditional views and reform views and, subsequently, theories

of learning in behaviourism, information processing and constructivism were analysed in relation to these views. For example, behaviourism is more traditional while information processing and constructivism are more reformative.

In a similar manner, the teaching of mathematics is divided into what is called product-directed and process-directed approaches. Based on these, mathematics teaching is divided further into the following four views:

- Content-centred mathematics teaching with the emphasis on achievement
- Classroom-centred mathematics teaching based on the effectiveness of the structure of the classroom
- Content-centred mathematics teaching with the emphasis on conceptual understanding
- Learner-centred mathematics teaching (Nieuwoudt, 1998:13)

Analysis of these views shows that they are based on behaviourism, cognitivism and constructivism.

In the section that follows, active mathematics teaching, realistic mathematics and cognitively guided instruction (CGI) will be discussed. These instructional approaches to the teaching of mathematics were preferred because they dovetail with the theories of teaching and learning mentioned above.

3.5.2. Cognitively guided instruction (CGI)

CGI was developed by Thomas Carpenter, Elizabeth Fennema and colleagues. It has the following instructional components:

- Instructional decisions are based on what is known about each child's cognition and knowledge.

- Instruction is organised to enable each child to construct and understand knowledge.
- Instruction stresses the relationships between concepts, problem solving and skills.
- Instruction is organised so that children are mentally involved and gain understanding so that teachers can assess children's cognition and knowledge.
- Instruction encourages children's monitoring of their own thinking and accepting responsibility for their own learning (Fennema, Carpenter & Peterson, 1989:182).

CGI follows the principles of cognitivism and constructivism where the child's mental and constructive capacities are considered. It focuses on facilitating the child's thinking and learning in mathematics by building instruction on what the child already knows. It should develop children's attitudes of self-reliance, confidence in learning mathematics and a belief that they are responsible for their own learning (Fennema et al. 1989:181).

Secada et al. (1989:2) who did studies on limited English proficient (LEP) learners indicate the following about CGI:

- Learners receive basic skills instruction in problem-solving context that is meaningful and fosters higher order thinking skills.
- Learners develop confidence and become proficient at problem solving.
- Problem solving motivates learners to stay on task since it is cognitively challenging.

Hiebert and Weame (1992) on cognition and task orientation instruction indicate that conceptually based instruction which emphasises understanding of underlying concepts in mathematics seems to be most appropriate for fostering tasks, deep-processing strategies and a task orientation. Similarly, Papert (1993) asserts that in learning learners

must be involved completely in the process of acquiring knowledge instead of being taught facts and formulas. Accordingly, this type of schooling could only occur if learners employ a task orientation and deep-processing strategies (Math Forum, 2004:2).

Secada et al. (1989:1) regard the following teacher competencies as basis for CGI:

- Teachers should know how specific mathematical content is organised in children's minds.
- Teachers should be able to make solving mathematical problems the content focus.
- Teachers should be able to assess in what way their learners are thinking about the content of mathematics.
- Teachers should be able to make instructional decisions based on their own knowledge of their learners' thinking.

CGI aims at facilitating the learners' thinking and learning in mathematics by building instruction on what the learner already knows. Teachers using CGI should try to understand how the learners think about and do mathematics. The emphasis should be on what the learner is thinking and doing, not what the teacher is to do (Nieuwoudt, 1998:39).

As it was demonstrated in Chapter 2 on cognitivism, CGI seems to be lacking on application in South African schools as researchers Vinjevold and Ensor (2000) have indicated. Should CGI be employed in a collaborative teaching approach, the researcher believes, this could even improve performance in mathematics in Grade 12 nationally and internationally.

3.5.3. Realistic mathematics education (RME)

This is a model that was developed by the Freudenthal Institute since 1971 in the Netherlands (Talati 2004; Wilson 2004; Zulkardi, 2004). It is an approach to teaching and learning in mathematics that purports that mathematics must be connected to reality and be seen as a human activity (Talati 2004:2). The word “realistic” refers not just to connection with the real world, but also to problem situations, which are real in the learners’ minds (Zulkardi 2004:3). This model is based on the principle that learners see meaning in schoolwork when they connect information with their own experience (Wilson 2004:2).

Eade (2004) outlines the key aspects of RME as follows:

- It is realistic
RME presents knowledge within a concrete context allowing learners to develop informal strategies and progress to formal, abstract and standard strategies.
- It involves “mathematisation”
This can be split into horizontal and vertical to characterise the learners’ discovery of mathematical tools that can help to organise and solve problems located in real-life situation. The latter refers to “building up” to reach more challenging mathematics and to a greater use of abstract strategies. Freudenthal (1991) states that horizontal mathematisation involves going from the world of life into the world of symbols, while vertical mathematisation means moving within the world of symbols.
- It stresses procedural understanding
RME stresses understanding processes rather than learning algorithms. Learners “discover” the mathematics for themselves, and so multiple solutions are encouraged and valued.
- It encourages “guided reinvention”

Because learners direct the course of lessons, RME requires a highly constructivist approach to teaching in which children are no longer seen as receivers of knowledge but the makers of it and the role of the teacher is that of a facilitator (Wilson 2004:3).

According to Zulkardi (2004:5), the following are the basic characteristics of RME:

- phenomenological explanation or use of contexts
- the use of models or bridging by vertical instruments
- the use of learners' own productions and constructions or learner contribution
- the interactive character of the teaching process or interactivity
- the intertwining of various learning standards

Zulkardi (2004:8) further states that RME is closely related to socio-constructivist theory in that: they are both developed independently of constructivism. In both approaches, learners are offered opportunities to share their experience with others and that both struggle with the idea that mathematics is a creative human activity.

It is apparent thus far that cooperative learning will be the order of the day where learners interact in the form of pairs and hold discussions, with some limited explanations from the teacher who should conduct the lessons by means of questions and task assignments. The teacher serves as a coordinator. According to Talati (2004:6), the method most used by learners for problem solving under RME is that of trial and error.

Thus Watson's and Thorndike's behavioural theory of trial and error becomes applicable. RME caters for team teaching or collaboration where two or more teachers can present lessons to two or more classes in a rational procedure (as happened at the experimental school of this study) (Wilson 2004:4).

However, Wilson (2004:5) observes that good discussions require skilful maintenance by the teacher. Furthermore, the following hinder the application of RME:

- It demands a lot from the teacher.
- The variety of learner response may be difficult to understand.
- There is resistance to formal method.
- Differentiation of learners is difficult.
- Progression is very slow.
- Lack of ability to articulate is an obstacle to learners.

Although South Africa has recently become one of the countries that practise RME (Zulkardi 2004:3), its application will not be smooth in all schools, especially those in cosmopolitan areas (such as squatter camps) where different languages are being spoken. RME centres around communication, and the teaching of mathematics in a second language even now is a problem for both teachers and learners.

3.5.4. Problem solving

Problem solving is defined differently by different scholars. Kontowski (1977) defines it as “a situation for which the individual confronting it has no readily accessible algorithm that will guarantee a solution”. Trisman (1988) defines it as “what you do when you don’t know what to do”, while Lesh (1981) says, “problem solving is primarily a way of thinking, of analysing a situation, of using reasoning skills not learned through memorisation of specific facts, but by immersing oneself in the problem solving process and applying both past experience and knowledge to the experience and knowledge to the problem at hand”. According to NCTM (1989), “problem solving is the process by which students experience the power and usefulness of mathematics in the world around them”.

Schoenfeld (1992) regards problem solving as the heart of mathematics if not mathematics itself. He further indicates that he believes that

problems are the heart of mathematics and that teachers, in the classrooms, in seminars, and in books and articles, will emphasise that learners should be taught to become better problem posers and problem solvers.

According to Bungane (2002:33), the following are characteristics of problem solving:

- It cultivates an enquiring mind in the learner.
- It enables learners to generate their own meaning around the context they are exposed to do.
- It encourages learners not to take any one particular thing for granted.
- It is process oriented rather than product oriented.

Schoenfeld (1992:338) does not regard problem solving as a goal in itself, but as facilitator in the achievement of other goals. He suggests the following goals for problem solving:

- to train learners to think creatively and/or develop their problem solving ability
- to prepare learners for problem competitions such as national or Mathematics Olympics
- to provide potential teachers with instruction in a narrow band of heuristic strategies
- to provide a new approach for remedial mathematics or to induce critical thinking or analytical resourcing skills

Problem solving, in the South African context, is a critical outcome (CO) for both the General Education and Training Certificate (GETC) and the Further Education and Training Certificate (FETC).

A comprehensive problem-solving approach is given by Jones (2000:2), who uses cognitive theory that embraces the works of Piaget, Bruner and Ausubel. Piaget believes that acquisition of knowledge is a

continuous self-construction process. This means, as Driscoll (1994) puts it, “knowledge is invented and reinvented as the child develops and interacts with the world surrounding her”. The information processing theory is concerned with how a learner takes in knowledge, stores it, and retrieves it from memory. Bruner (1964) presents the concept of discovering learning, which refers to the acquisition of knowledge for purposes of individual achievement.

According to Jones (2000:3), a good problem-solving environment needs to be created. He suggests the following three roles for the teacher:

- modelling aspects of problem solving
- teaching some aspects of problem solving directly
- facilitating some aspects of problem solving

He regards the first role as the most important in the sense that learners need to be shown the thinking and the struggle through which the teacher goes in solving a problem. In solving a problem, Polya (1962), the father of the problem solving approach, expected learners to engage in thinking about the various tactics, techniques and strategies available to them. Dewey (1933) emphasised self-reflection in the solution of problems. This, according to Polya (1973), includes examination of the solution by such activities as checking the results, checking the argument, deriving the result differently, using the result for some other problems, interpreting the result, extending the process or extending the solution (Jones, 2000:3).

Jones (2000) further suggests the use of cognitivism together with technology in teaching problem solving. The cognitive approach and the NCTM standards both support the integration of technology into the mathematics classroom in order to improve problem solving. An example is the GPTutor computer programme which was created to provide learners with computer-based tutoring in geometry while introducing proofs. This programme is said to allow the teacher freedom to circulate

among the terminals and provide assistance to those in need. Further technological tools suggested in this regard are the Problem Solving Assistance (PSA) and Tutoring in Problem Solving (TIPS).

It is suggested that a cooperative learning approach be used through three boards: (1) a Planning Board, where important information and ideas about the problem are recorded, (2) a Representation Board, where diagrams illustrating the problems are drawn, and (3) a Doing Board, where appropriate equations are developed and the problem is solved.

According to North West Province (DoE) FET Transition Guidelines for Mathematics (2004:10), the following are implications for teaching problem solving:

- Learners are exposed to non-routine problems, for example word problems, open-ended problems and geometry riders.
- Development of logical reasoning skills and problem solving strategies, for example, entails four steps:
 - reading with understanding
 - planning a strategy
 - implementing the plan
 - checking/evaluating the solution

3.5.5. Discussion

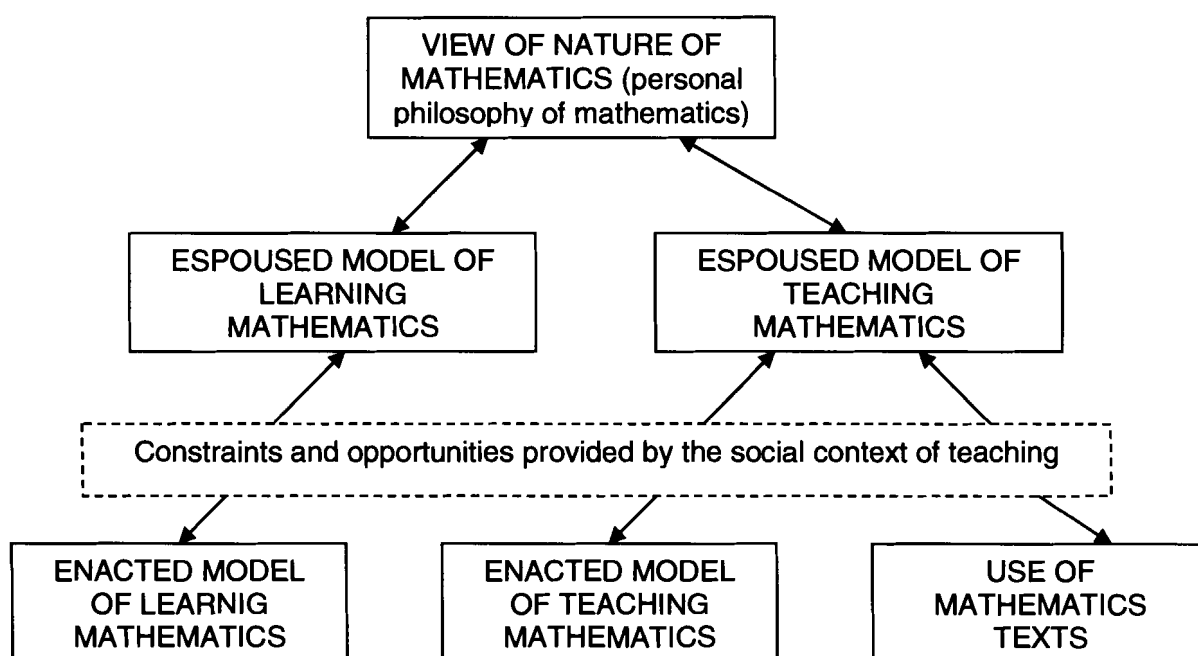
There is a close relationship between problem solving, CGI and a realistic mathematics education approach. All of them encourage learner participation in the form of cooperative learning. In all of them, the teacher serves as the facilitator.

However, of enormous importance to teaching approaches and learning theories that are available is the teacher's belief system in mathematics.

According to Ernest (1994:1), the belief components of the mathematics teacher are

- the view or conception of the nature of mathematics
- model or view of the nature of mathematics teaching
- model or view of the process of learning mathematics

Figure 3.2 The relationship between beliefs and their impact on mathematics teaching and learning practice (after Ernest, 1994)



The model above shows that mathematics teaching and learning are intertwined. Teachers of mathematics in Grade 12 need to change their beliefs about mathematics (often traditional) and adapt to new theories and approaches.

3.5.6. Implications for S.A teacher education

For decades, teacher education in South Africa took place at colleges of education, technikons and universities. Teachers therefore wrote different examinations with different standards and content. This has

caused huge disparities in terms of pedagogic content knowledge among teachers, especially in mathematics teaching. A council of mathematics teachers, just like the National Council of Teachers in the United States, is needed to address this problem.

3.6. Conclusion

Factors discussed in this chapter are just a few of those that affect mathematics teaching and learning. More research studies need to be conducted, especially on psychological factors and mathematics achievement. There are some inclusive findings, as was indicated, on factors such as attitudes, interest and motivation on mathematics achievement. From a layman's point of view, if a person is less interested, is demotivated or has a negative attitude to something, performance is bound to be affected. This was not necessarily the case in this study. However, it has been shown that factors such as language and socio-economic status affect mathematics achievement. In the case of this research study, both the experimental and the control groups have the same socio-economic status and they both learn mathematics in a second language.

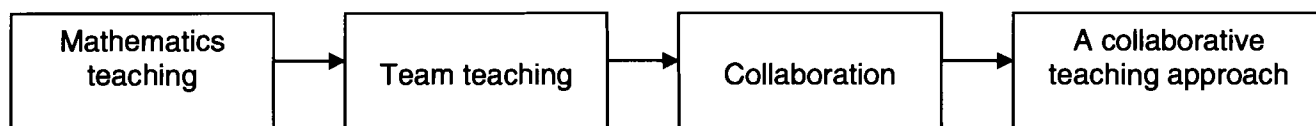
Teacher knowledge as was discussed in this chapter is broad and diverse. It is imperative that teacher development in South Africa be centralised so that it is uniform. As was discussed in Chapter 2, mathematics learning will only be effective if teacher education is of a good quality and a high standard.

CHAPTER 4

A COLLABORATIVE TEACHING APPROACH

4.1. INTRODUCTION

The purpose of this chapter is to define and present a collaborative teaching approach as a case study at the experimental school. However, the chapter will begin with a brief outline of a “good” mathematics teacher as an extension of teacher knowledge discussed in the previous chapter. The flowchart of the chapter will thus be:



4.2. MATHEMATICS TEACHING

The previous chapter contained a discussion on teachers' knowledge, and a great deal of literature was referred to in this regard. In this section, the researcher looks into the general trend of mathematics teaching in the world (although attention is paid particularly to a few of the countries discussed in the 1995 TIMMS report). Furthermore, from the limited literature available, the researcher will consider some characteristics of a “good” mathematics teacher.

The 1995 TIMMS report looked at mathematics teachers' instruction in terms of how much time they spent on introducing new content, practising new content or reviewing earlier content. American teachers spent more time reviewing old material than anyone else except the Czechs. None of the countries spent a great deal of time practising new material except Japanese teachers, who spent 60 percent of their time on introducing new content (Bracey, 2003:253).

In studying the 1995 TIMMS report, the researchers made comparisons for mathematics teaching by different countries based on stating concepts; using procedures or making concepts between mathematics and facts. It was found that American teachers spent a large amount of time (60%) focusing on procedures. However, they were exceeded in this activity by Hong Kong (84%) and the Czech Republic (77%). Japanese teachers spent the least amount of time focusing on procedures (41%), while they spent the most time (54%) making connections between mathematical facts (Bracey, 2003:254).

The 1999 TIMMS Video Study finds that teachers dominate classroom discussions worldwide. American teachers spoke eight words for every word the learners spoke while in Hong Kong the ratio is 16:1 and in Japan 13:1. Japan and Hong Kong had the highest percentages, with teachers speaking 25 or more words with no interruption from learners (Bracey, 2003: 253)

Sanders (2002: 181) asks “What do schools think makes a good mathematics teacher?” Research studies conducted in the UK regard the following as attributes that make a good mathematics teacher: qualifications (44%); teaching skills (41%); professional qualities (32%); personal qualities (94%) and “value-added-ness” (81%) (Sanders, 2002: 182).

Tables 4.1 and 4.2 unpack personal qualities and value-added-ness, which are the most highly regarded qualities of a good mathematics teacher (Sanders, 2002: 185).

Table 4.1: Preferred personal qualities of a mathematics teacher

Personal quality	% of schools mentioning
Ability to work in a team	54
Enthusiasm	48
Good communication	28
Good interpersonal skills	19
Energetic	16
Hardworking	14
Positive attitude	13
Good sense of humour	13

According to Sanders (2002: 185), the ability to work in a team is mentioned in the threshold standards (Department for Education and Standards, 2001).

Table 4.2: Preferred value-added-ness of a mathematics teacher

Skills being sought	% of schools mentioning
- able to contribute to the wider life of the school	
through extracurricular activities	30
- willing to take on pastoral role	24
- knowledge of issues to do with equality and	
multicultural nature of UK society	21
- supportive of the ethos of the school	16

One notices that some (if not most) of the qualities expected from a mathematics teacher have nothing to do with mathematics as a subject. This also pertains to the Department of Education in South Africa, which requires of every teacher, irrespective of the subject they teach, to play the following roles: learning mediator; interpreter and designer of learning programmes and materials; leader; administrator and manager; community, citizenship and pastoral role; scholar, researcher and

lifelong learner; assessor; learning area phase specialist (NEPA, 1996: A-47).

A brief analysis of some of the roles is as follows.

Learning mediators

An educator is expected to use his/her practical competences and

- use the language of instruction to describe and discuss key concepts in conversational styles.
- prepare thoroughly and thoughtfully for teaching.
- use key teaching strategies such as higher level questioning, problem-based tasks and projects and appropriate use of group work, whole class teaching and individual self-study.
- adjust teaching strategies to cater for different learning styles and match the developmental stages of learners.
- create a learning environment in which learners develop strong internal discipline and are encouraged to be critical and creative thinkers (NEPA, 1996:A-48)

Community, citizenship and pastoral role

An educator is expected to use his/her practical competences to

- develop life-skills, work skills, a critical, ethical and a healthy lifestyle in learners
- provide guidance to learners about work and study possibilities
- show an appreciation and respect for people of different values, beliefs, practices and cultures
- counsel and or tutor learners with social or learning problems
- demonstrate caring, ethical professional behaviour, protection of children and the development of the whole person (NEPA, 1996: A-50)

Interpreter and designer of learning programmes

An educator is expected to use his/her foundational competences and

- understand the principles of curriculum: how decisions are made; who makes decisions, on what basis and in whose interests they are made
- understand various approaches to curriculum and programme design
- understand the learning area to be taught, including appropriate content knowledge, pedagogic content knowledge with other subjects (NEPA, 1996: A-49)

Scholar, researcher and lifelong learner

An educator is expected to use his/her foundational competences to

- understand current thinking about technological, numerical and media literacies with particular reference to educators in a diverse and developing country like South Africa
- Understand the reasons and uses for and various approaches to educational research (NEPA, 1996: A-51)

Qualities of the professional mathematics teacher

The following is a list of some of the qualities of the professional mathematics teacher:

- Zweng (1983) defines a good mathematics teacher as “one who uses his knowledge and respect for his students to lead students to enjoy the studies of mathematics”. The 1981 Mathematics Association of South Africa found professional teachers of mathematics should possess at least the following four qualities:
 - a broad mathematical background, encouraging the range of contents and activities in present and future curricula

- a thorough knowledge and understanding of teaching strategies and techniques
- proficiencies in the design and production of suitable curriculum material, the interpretation of given syllabus and identification and remediation of problems which may be experienced by learners with respect to mathematics (MASA, 1981)

Following Leinhard (1986: 30) and Hendricks, (1990: 20), South Africa needs teachers with a deeper knowledge of mathematics; who are aware of the role of mathematics in society; who understand and communicate with their learners; who are able to teach capacity; who lead their learners to goals of independence and self-esteem, and who will provide moral and intellectual role models for their learners.

According to Botha (2000: 139) the “good” mathematics teacher is one who

- is successful in improving learners’ scholastic achievements
- commands a range of effective teaching skills and teaching methodologies such as discussion and problem solving, as well as explanatory teaching and reinforcement skills

It is very clear from the large number of attributes of a good mathematics teacher cited so far, that no single person could possess all of them. For this reason, the researcher believes “the ability to work in a team” as quoted by schools in the UK, to be the single most important attribute which a good mathematics teacher should possess.

4.3. TEAM TEACHING

According to Shaplin and Olds (1964: 4), team teaching began in America in 1954 as an extremely controversial subject with ardent supporters and equally determined detractors. No definite definition of

team teaching was given during this period, however, Shaplin and Olds (1964: 10) describe team teaching as an approach whereby two or more teachers work together loosely as associates, meeting occasionally and dividing up the responsibility for instruction. They further indicate that team teaching projects started as an “experiment” and their objectives were labelled “hypotheses” (Shaplin & Olds, 1964:7)

According to Freeman (1969:24), English, Social Studies and Physical Education were the subjects found more frequently in team-teaching programmes than Science and Mathematics. The following were “claimed” advantages of team teaching:

- Fewer teachers are required per number of learners.
- More economic use is possible of time and money.
- Small groups permit much greater intensity of work and enable remedial work to be effective.
- Teachers in teams learn more from each other.
- Working with and in front of colleagues stimulate teachers to prepare better (Freeman, 1969:37).

The following are objections to team teaching:

- Team teaching demands more staff.
- Very large groups are difficult to control and pose problems of method.
- Teachers are trained to handle single classes, not large classes by a group of teachers as team teaching would require.
- Not all teacher can fit into a team (Freeman, 1969:38)

Team teaching still exists and there is recent literature on this subject. However, most literature modifies the old type of team teaching to a collaborative approach (Goetz, 2000).

Shafer (2001:1) has the following three suggestions about the presentation of team teaching:

- Two or more instructors from closely allied disciplines or from different fields get involved to teach a group of learners or students.
- All instructors or teachers are jointly responsible for course content presentation and grading, interacting in front of a class or taking turns to present lessons.
- One coordinator alone is responsible for course content and grading but regularly inviting guest lecturers and panels.

The first two suggestions were applied in a case study involving eight teachers who taught one theme at Cactus High School (in the US). The title of the theme “The American Identity” consisted of US history, American literature, and American art history. The eight teachers had a combination of following subjects: English and Art, World History and World Literature. The teachers had experience in teaching ranging from two to 20 years. They taught the theme in pairs and each pair took its turn on a different day (Murata, 2002:69). There are benefits reported from this case study, one of which is that there was a development of rapport among learners and teachers that was fostered by curriculum and pedagogy.

A broader research study on team teaching was undertaken by Goetz (2000). He divided team teaching into two broad categories viz. Category A: where two or more instructors taught the same students or learners at the same time within the same classroom, and Category B: where instructors or teachers worked together but did not necessarily teach the same group of students or learners nor necessarily teach at the same time.

One notices a common line between Shafer’s (2000) suggestions (first two) and Goetz categories, that is, learners can be taught as one unit at the same time or can be taught by a team of teachers at different periods of time.

Goetz (2000) furthermore subdivided their categories as follows:

CATEGORY A

- **Traditional team teaching**

In this case, the teachers actively share instruction of content and skills with all students.

- **Collaborative teaching**

This describes a traditional form of team teaching in which the team works together in designing the course and teaches the material by exchanging and discussing ideas and theories in front of the learners. Cooperative learning strategies are being used in this regard.

- **Complimentary/supportive team teaching**

This occurs when one teacher is responsible for teaching the content while the other teacher takes charge of providing follow-up activities related to the lesson.

- **Parallel instruction**

In this setting, the class is divided into groups and each teacher is responsible for teaching the same material to his or her smaller group

- **Differentiated split class**

This involves dividing the class into smaller groups according to learning needs and each teacher provides the respective group with instruction to meet their learning needs.

- **Monitoring teacher**

This occurs when one teacher teaches the entire class while the other teacher monitors learners' understanding and behaviour.

CATEGORY B

- Team members meet to share ideas and resources but function independently

This cooperative teaching occurs weekly when teachers meet once to discuss the concepts to be taught and to share ideas.

- Teams of teachers sharing a common resource centre.

In this form, teachers teach classes independently, but share resource materials such as lesson plans, supplementary textbooks and exercise problems.

- Teaching different sub-groups within the whole group

Team members share a common group of students or learners and common planning for instruction.

- One individual plans the instructional activities for the entire team

This model is used particularly if there are financial constraints

- The team members share planning but each member teaches his or her own specialised skills area to the whole group of students or learners.

An example is given here of seven teachers of mathematics teaching seven different topics to seven different classes and rotating throughout the duration of the course (Goetz, 2000:2)

This last model is similar to the one applied at the experimental school in this research study (referred to as a collaborative teaching approach)

where three teachers teach two grades, viz. Grade 12, Algebra, Geometry and Trigonometry in rotation (more on this later). There are two outstanding features in this model, which are absent in other models, viz.

- That teachers teach their specialised skill area
- That they rotate between separate classes or grades

Unlike other forms of team teaching this model

- is easily applied and saves time
- requires the teachers to teach the areas they specialise in
- does not require the teacher to teach in front of other colleagues thereby having to compete with them indirectly
- fits easily in the school timetable, as it will be demonstrated in the next section of this study

4.4. COLLABORATION

4.4.1. Orientation

In the previous section, the terms “team” and “collaboration” were often used interchangeably. However, it must be noted that the scope of “team”, as in team teaching, is narrower, whereas that of “collaboration” is broader. Whereas team teaching was briefly outlined in the previous section, it is found, in addition, that it is being practised mainly in the middle and junior secondary schools (especially in the US) (Kruise, 1997; Murata, 2002). Team teaching is also concerned with matters pertaining to teaching and learning within particular schools. Collaboration on the other hand takes place: within a school between teachers of the same or different subject(s); between two institutions (school and school, school and university and academic school and vocational) (Inger, 1993; Magolda, 2001) and in a classroom between a teacher (stage director) and learners (actors) (Sawyer, 2004:18).

4.4.2. Definition of collaboration

Christiansen et al. (1997:9) define collaboration as “the explicit agreement among two or more persons to meet and accomplish a particular goal or goals”. Stewart (1997:31) defines collaboration as “a relationship involving equal partners working on an ongoing basis to achieve mutually beneficial goals.” On the other hand, Dutchard et al. (1999:186) define collaboration in teaching as sharing expertise in delivering a lesson, solving a problem or working on a project.

From the three definitions outlined, one can conclude that in collaboration, partners agree to participate on an equal footing and to share their expertise. This is as far as teaching is concerned. The situation may be different in a collaboration that takes place between two or more institutions, which are not on the same level, for example a university and a school. In this sense, the university provides resources, facilities and expertise.

4.4.3. The improvisation of classroom collaboration

An effective classroom discussion emerges from classroom discourse and is not scripted by the lesson plan or by the teacher’s predetermined agenda. Sawyer (2003) refers to this as collaborative emergence where classroom discussion is emergent and is collaborative because no single participant can control what emerges and the outcome is collectively determined by all participants (Sawyer, 2004:13)

According to Sawyer (2004:13), the basic insight of constructivism is that learning is a creative improvisational process. Both neo-Piagetian social constructivist and Vygotskian inspired socio-culturalist focus on how knowledge is learned in and by groups (Palinscar, 1998; Rogoff, 1998; Verba, 1994). Furthermore, socio-cultural studies have demonstrated the importance of social interaction in groups and have shown that focus on improvised interactional process can reveal many insights into how

learning takes place. These studies hold that groups can be said to learn as collectives, and that knowledge can be a possession or property of a group, not only of the individual participants in the group (Sawyer, 2004:14)

Accordingly, in socio-cultural and social constructivist theory, effective teaching must be improvisational because if the classroom is scripted and directed by the teacher, the learners cannot construct their own knowledge. The socio-cultural perspective implies that the entire classroom is improvising together and that the most effective learning results when the classroom proceeds in an open, improvisational fashion as children are allowed to experiment, interact and participate in the collaboration construction of their own knowledge. In improvisational teaching therefore, learning is a shared social activity and is collectively managed by all participants not only by the teacher. Cobb (1995) argues that this interaction is multivocal, containing multiple perspectives rather than the single perspective of the teacher (Sawyer, 2004:14).

In conclusion, Sawyer (2004:16) indicates that collaborative learning can only work in a give and take interaction. Research has shown that this form of collaborative practice is uniquely beneficial to learning in a wide range of content areas. Moreover, research, according to Sawyer (2004:17) has shown that the most effective collaborating groups are those that are partially structured in careful ways by the teacher.

4.5. A COLLABORATIVE TEACHING APPROACH

In 1991 the then Bophuthatswana Government established what was called science schools. It was realised then that schools offering general streams encouraged less learners to pursue science and mathematics. The experimental school in this study was one of the science schools. Incentives, in terms of equipping such schools with science kits and offering hard-working teachers and principals scholarships, were promised. Principals and teachers at such schools had to be innovative

in their instructional methods. Consequently, three teachers at an experimental school met to discuss the means by which they could improve mathematics results especially in Grade 12. The collaborative teaching approach, which was initially called team teaching, was born with teacher X offering algebra and calculus; teacher Y offering geometry and the researcher offering trigonometry.

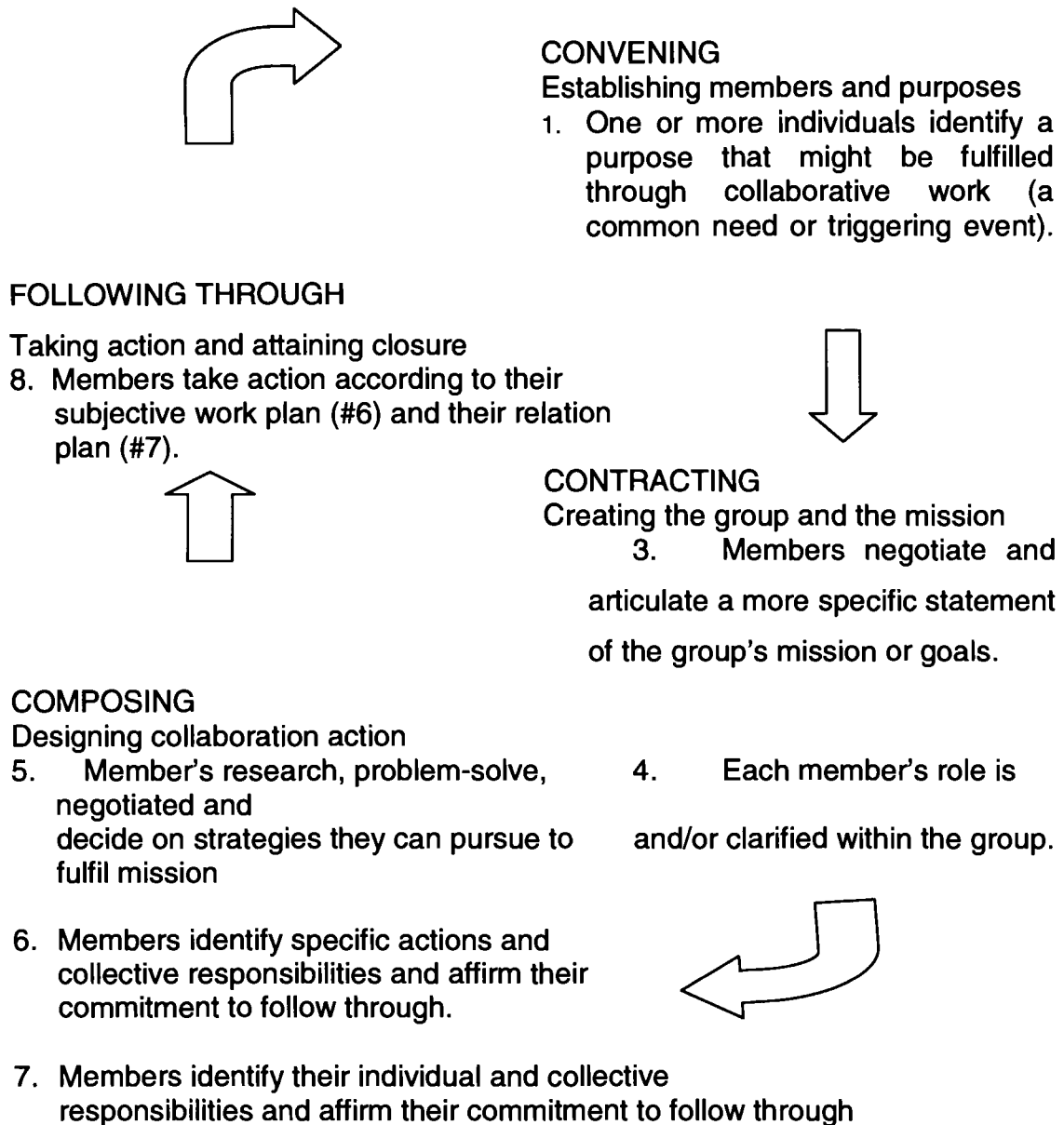
Table 4.3 Time-table for collaborating teachers at the experimental school (E)

FEBRUARY

WEEK	GRADE	TEACHER	SECTION
1	12	X	ALGEBRA
2	12		
1	11	Y	GEOMETRY
2	11	Z	TRIGONOMETRY
3	12	Y	GEOMETRY
3	11	X	ALGEBRA
4	12	Z	TRIGONOMETRY
4	11	X	ALGEBRA

This collaborative approach is similar to that suggested by Donaldson and Sanderson (1998:9), as depicted in Figure 4.1.

Figure 4.1: Framework for the process of working together



The three teachers offered the said sections in Grade 12 out of interest and ability. They believed one could effectively teach a section of mathematics well if one has interest and ability in it. To a greater extent this was true because their Grade 12 results for 1993 and 1994 in particular were as follows:

In 1996 teacher X got a promotion to teach in another school. She introduced a collaborative teaching approach there and says at the end of that year the school's Grade 12 mathematics improved from 37% to

76%. She attributes this great improvement to a collaborative teaching approach.

In the meantime, because teachers X and Y left the experimental school due to promotion, the team was dismantled and consequently the Grade 12 mathematics results at the experimental school have been dropping since 1996.

4.6. CONCLUSION

A collaborative teaching approach appears to be preferred by learners. Given their responses to questions on these methods and their comments about them, one can conclude that it may well be a solution to mathematics teaching and learning in South Africa. However, collaborating teachers need to apply learning theories such as Information Processing and constructivism to achieve maximum results. While collaborating, a complexity science (Davis & Simmit, 2003) learning style would enhance this approach.

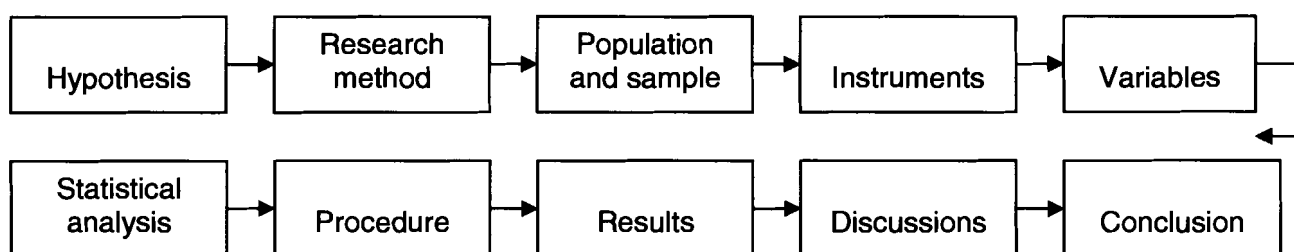
CHAPTER 5

THE EMPIRICAL STUDY

5.1. INTRODUCTION

The aim of the empirical research was to analyse and describe the impact of a collaborative teaching approach on the quality of mathematics teaching and learning in Grade 12.

The contents of this chapter are thus presented as follows:



5.2. RESEARCH HYPOTHESIS

There is a relationship between collaborative teaching and the learner's performance in mathematics in Grade 12.

5.3. RESEARCH METHOD

A quantitative research method was employed (see 1.5.2). In employing quantitative research, the standardised Study Orientation in Mathematics questionnaire (SOM) (Maree, 1997) pre- and post-tests were given to both the experimental and the control groups of the two high schools, in the same area (rural), offering the same curriculum (mathematics, science and commerce). The SOM test was complemented by mathematics content pre and post tests based on sections of the Grade 12 syllabus. The sections embraced algebra, trigonometry and geometry (see Appendices A and B for the tests).

5.4. STUDY POPULATION AND SAMPLE

All Grade 12 mathematics learners as well as mathematics educators involved at the two schools. Experimental school had three Grade 12 classes with a total of 75 learners. The control school had two Grade 12 classes with a total of 73 learners.

5.5. INSTRUMENTS

A standardised instrument the Study Orientation in Mathematics Questionnaire (SOM) (Maree, 1997) was used to measure cognitive and psychological learning factors (see 5.9 for more information about the SOM instrument). The SOM was chosen on the basis that it measures relevant learning factors (study attitude, mathematics anxiety, study habits, problem solving behaviour, study milieu, information processing), which contribute toward conceptualisation and achievement in school mathematics (Maree et al., 1997: 2), and proves to be a reliable measure for South African school learners in Grades 7 to 12, with reliability coefficients (r_{tt}) varying between 0.69 and 0.97 (Maree et al., 1997: 26). Mathematics tests set by the researcher were used to measure mathematics achievement (see Appendices A and B). The mathematics tests were moderated by two teachers at each of school E and C, and accordingly adjusted, to increase their face validity and reliability. In addition, a self-constructed questionnaire and interview schedule were used to gather information about the collaborative teaching approach from former learners who had been exposed to the approach in school E.

5.6. STUDY VARIABLES

The independent variable in the study was the collaborative teaching approach employed at school E, and compared to conventional teaching approach employed in school C.

The dependent variables in the study were mathematics achievement, as indicated by the mathematics test results, and the SOM's cognitive, affective and contextual learning factors (attitudes, anxiety, study milieu, problem solving, study habits and information processing).

5.7. STATISTICAL TECHNIQUES

The results of the SOM test (pre and post) were processed at the Statistical Consultation Services of the Potchefstroom Campus of the North West University, using the SAS Statistical package (SAS Institute, 1988).

Effect sizes, particularly Cohen's Criterion d , were used to determine whether the differences between the experimental and control groups were of practical significance (Steyn, 1999:3). The following formula was used to calculate d -values:

$$d = \left| \frac{\bar{x}_1 - \bar{x}_2}{\text{larger std dev.}} \right|$$

If $0.2 \leq d < 0.5$, the difference is of a small effect

If $0.5 \leq d < 0.8$, the difference is of a medium effect, which could possibly indicate a practically significant difference

If $d \geq 0.8$ the difference is of a large effect and practically significant.

The practical significance of the difference between the pre-post test of both the experimental and control groups was determined by using the above formula.

5.8. PROCEDURE

The researcher approached the Grade 12 mathematics teachers at the control school to enquire about the amount of work Grade 12 had

covered. This was towards the end of January 2001. The SOM tests were thus given to both the experimental and control groups at the beginning of February 2001 and a mathematics test based mainly on Grade 11 syllabus was given to the two groups (E and C). The educator at the control school was requested to go on teaching the way she had been teaching up to June 2001. In other words, she was requested not to add or apply any external factors that would impact on the performance of her learners.

At the experimental school meantime, the researcher and teacher X taught collaboratively, that is, they shared the work. The researcher taught the learners trigonometry and geometry while teacher X taught algebra only.

In June 2001, once more the SOM tests were given to the two groups, followed by the mathematics test (see Appendices A and B for the mathematics tests).

5.9. STUDY ORIENTATION IN MATHEMATICS

Du Toit (1970) defines study orientation in Mathematics (SOM) as follows: "Relatively protracted application to a topic or problem for the purpose of learning about the topic, solving the problem, or memorising part or all of the presented material."

This definition requires further explanation and an attempt to simplify the instrument would be, according to the designers of the SOM (Maree et al., 1997): "It is a South African invented instrument used to measure or determine to what extent social, affective and cognitive factors influence achievement in mathematics learning." Hence, the instrument closely relate to the learning factors identified in chapter 3.

SOM consists of the following fields (Maree et al., 1997: 6-9):

- study attitude (SA)

- mathematics anxiety (MA)
- study habits (SH)
- problem-solving behaviour (PSB)
- study milieu
- information processing

A SOM questionnaire consisting of 92 questions, which cover sub-questions from each field, is generally administered to learners (Maree, 1997). The results of such a questionnaire are then analysed by statisticians to obtain a learning profile of mathematics learners, which can be used to explain critical aspects of their achievement. In this study, such an analysis was done by the Statistical Advisory Services at the Potchefstroom Campus of the North-West University.

The preliminary questionnaire of SOM was administered between August 1994 and March 1995. The Human Sciences Research Council was consulted for an education database. The reliability co-efficiency for the different fields and the questionnaire as a whole were determined with the aid of Ferguson's adaptation of the Kuder-Richardson Formula 20.

According to the developers (Maree et al., 1997: 25-32) appropriate steps were taken to ensure content and construct validity, as well as reliability of the SOM questionnaire.

5.10 RESULTS

5.10.1. Experimental group

5.10.1.1. *SOM means of subscales - pre-test*

Cognitive factors have an impact on learning in general, and particularly in school mathematics (Maree et al., 1997). Pre-testing was necessary

for the experimental group to check, for instance, whether they have positive study attitudes, skills in problem solving and information processing, and to determine their level of mathematics anxiety. These will later be compared with those of the control group (see 5.10.3). For the sake of profiling, the means obtained are related to percentile ranks, with 70% a threshold for a positive profile (Maree et al., 1997: 15).

Table 5.1: Pre-test means and percentile ranks of SOM subscales at school E

Variables	No.	Mean	Std. Dev.	Percentile Rank	Minimum	Maximum
Study attitude	70	40.65	8.39	60	17.00	55.00
Mathematics anxiety	70	37.76	7.81	35	19.00	52.00
Study habits	70	38.11	10.33	35	4.67	54.13
Problem solving	70	32.54	9.37	40	7.47	56.00
Study milieu	70	38.90	7.84	45	16.15	52.00
Information processing	70	40.43	10.93	55	5.00	60.00

From the above table, it is clear that in all fields the participating learners demonstrated an inadequate study orientation profile at the pre-testing stage.

5.10.1.2. SOM means of subscales – post-test

After six months of collaborative teaching, there was no significant improvement on these learning factors. In fact, the study orientation profile seems to have dropped somewhat (albeit not significantly) during the experiment. In particular, the experimental group's study habits, problem solving and mathematics anxiety remained relatively constant. This lack of improvement may be contributed to the fact that the collaborating teachers still employed a traditional teaching approach in keeping learners passive while presenting mathematics lessons.

Contextual factors (e.g. socio-economic status) relating to the learners may have had some influence as well.

Table 5.2: Post-test means and percentile ranks of SOM subscales at school E

Variables	No.	Mean	Std. Dev.	Percentile Rank	Minimum	Maximum
Study attitude	66	34.62	9.26	35	11.00	48.00
Mathematics anxiety	66	37.10	8.47	30	7.00	51.00
Study habits	66	33.50	11.09	25	5.00	54.13
Problem solving	66	28.86	10.77	25	6.53	54.13
Study milieu	66	37.69	9.04	40	15.08	53.85
Information processing	66	37.81	9.86	50	12.00	58.00

Further discussion on these tables follows under 5.11.

5.10.2. Control group

5.10.2.1. SOM means of subscales – pre-test

it was necessary that pre-testing should be conducted in the six learning factors as it was the case with the experimental group. This was to check whether the two groups were on the same level of mathematics performance. The results show that to be the case, indeed.

Table 5.3: Pre-test means and percentile ranks of SOM subscales at school C

Variables	No.	Mean	Std. Dev.	Percentile Rank	Minimum	Maximum
Study attitude	64	41.53	6.65	65	25.00	56.00
Mathematics anxiety	64	36.68	8.04	30	14.00	53.00
Study habits	64	36.27	8.87	30	18.66	54.13
Problem solving	64	31.84	9.15	35	13.07	53.20
Study milieu	64	35.93	9.19	35	14.00	54.92
Information processing	64	37.33	11.32	45	10.00	57.00

Again, it is evident that the control learners were at the time of pre-testing at an inadequate level of study orientation in all fields.

5.10.2.2. SOM means of subscales – post-test

Table 5.4: Post-test means and percentile ranks of SOM subscales at school C

Variables	No.	Mean	Std. Dev.	Percentile Rank	Minimum	Maximum
Study attitude	69	37.65	7.53	50	21.53	53.00
Mathematics anxiety	69	37.74	6.60	35	18.00	52.00
Study habits	69	32.31	8.73	20	13.07	53.20
Problem solving	69	28.60	9.40	25	5.00	51.00
Study milieu	69	36.82	8.91	35	11.85	56.00
Information processing	69	36.36	10.70	40	7.00	60.00

The control group did not demonstrate any improvement on the six learning factors either. In fact, they too seem to have dropped somewhat in study orientation profile (albeit not significantly). Their problem solving

and information processing skills did not improve after the six months of being taught by the same teacher, using a traditional approach, neither did their study habits nor mathematics anxiety get any better.

Further discussion on these tables follows under 5.11.

5.10.3. SOM mean of difference by group between pre and post test

5.10.3.1. *Experimental group*

The following table reflects the results of the comparison of the pre- and post-test SOM results of the experimental group. Evidently, the experimental group's profile regarding study attitudes dropped almost practically significantly during the study, while the drop in their study habit profile could be significant. In all other instances no significant changes occurred.

Table 5.5: Means of difference for SOM subscales at school E

Variables	No.	Mean	Std. Dev.	Effect size (d)	Minimum	Maximum
Study attitude	63	-6.18	8.63	0.71	-33.00	23.00
Mathematics anxiety	63	-0.96	8.02	0.12	-24.00	15.00
Study habits	63	-4.97	9.89	0.50	-32.00	41.00
Problem solving	63	-3.70	10.23	0.36	-29.00	46.67
Study milieu	63	-1.54	6.95	0.22	-19.38	11.85
Information processing	63	-2.82	9.76	0.29	-23.00	18.00

5.10.3.2. Control group

Table 5.6: Means of difference for SOM subscales at school C

Variables	No.	Mean	Std. Dev.	Effect size (d)	Minimum	Maximum
Study attitude	58	-3.54	6.34	0.56	-18.00	8.00
Mathematics anxiety	58	1.22	7.67	0.16	-14.00	28.00
Study habits	58	-3.90	8.08	0.48	-26.20	9.33
Problem solving	58	-2.85	9.23	0.30	-33.27	14.00
Study milieu	58	1.00	8.85	0.11	-24.77	32.31
Information processing	58	-1.88	8.12	0.23	-18.00	19.00

Again, as with the experimental group, no significant changes occurred with regard to the control group's study orientation profile. However, in the case of their study attitude there may have been some significant drop, as with the experimental group.

Further discussion based on these tables follows under 5.11.

5.10.4. Comparison between experimental and control groups on post-test SOM variables

From the table below it is evident that no significant differences occurred between the two groups as a result of collaborative teaching. Again, the predominantly traditional teaching approach in both classes may be the reason for lack of any favourable changes with regards to study orientation being affected. Again, the equal contextual factors (e.g. socio-economic status) of the learners in the two groups may have had some influence in this regard as well.

Table 5.7: Comparison between school E and school C on post-test SOM subscales

Variable	Effect size (d)
Study attitude	0.31
Mathematics anxiety	0.03
Study habits	0.11
Problem solving behaviour	0.08
Study milieu	0.06
Information processing	0.10

5.10.5. Comparison between experimental and control groups on mathematics achievement test

5.10.5.1. Mean of test mark by group: pre-test

Experimental group

Table 5.8: Pre-test means of mathematics test for school E

Variables	No.	Mean	Std. Dev.	Minimum	Maximum
Algebra	75	25.20	13.46	2.00	72.00
Geometry	75	6.77	7.64	2.00	34.00

Control group

Table 5.9: Pre-test means of mathematics test for school C

Variables	No	Mean	Std Dev.	Minimum	Maximum
Algebra	73	26.27	15.98	2.00	72.00
Geometry	73	8.19	7.36	2.00	30.00

From the above two tables follow:

Effect size with regard to the algebra mark: $d_a = 0.07$

Effect size with regard to the geometry mark: $d_g = 0.19$

Hence, at pre-test level, both the E and C groups were equivalent on mathematics achievement for both algebra and geometry.

5.10.5.2. Mean of test mark by group: post-test

Experimental group

Table 5.10: Post-test means of mathematics test for school E

Variables	No.	Mean	Std. Dev.	Minimum	Maximum
Algebra	75	35.95	24.74	2.00	88.00
Geometry	75	23.28	20.53	2.00	90.00

Control group

Table 5.11: Post-test means of mathematics test for school C

Variables	No	Mean	Std Dev.	Minimum	Maximum
Algebra	73	30.99	23.25	4.00	86.00
Geometry	73	11.42	14.61	2.00	68.00

From the above two tables follow:

Effect size with regard to the algebra mark: $d_a = 0.20$

Effect size with regard to the geometry mark: $d_g = 0.58$

Hence, at post-test level, no significant differences occurred between the two groups with regard to the algebra mark. However, the experimental group could possibly have achieved practically significantly better than the control group in geometry.

5.10.5.3. Mean of difference by group

In this section a somewhat stronger statistical measure of difference, namely mean of difference, is applied to assess the effect of the collaborative approach on learners' mathematics achievement.

Experimental group

Table 5.12: Mean of difference between pre- and post-test marks at school E

Variables	No.	Mean	Std. Dev.	Minimum	Maximum
Algebra	75	10.75	19.50	-24.00	62.00
Geometry	75	16.51	16.40	-6.00	70.00

From the above table follows:

Effective size with regard to the algebra mark: $d_a = 0.55$

Effect size with regard to the geometry mark: $d_g = 1.01$

Hence, during the experiment the experimental group's achievement with regard to geometry increased with practical significance, while the increase in the algebra achievement could also be practically significant.

Control group

Table 5.13: Mean of difference between pre- and post-test marks at school C

Variables	No	Mean	Std Dev.	Minimum	Maximum
Algebra	73	4.71	16.59	-38.00	42.00
Geometry	73	3.23	10.19	-16.00	40.00

From the above table follows:

Effect size with regard to the algebra mark: $d_a = 0.28$

Effect size with regard to the geometry mark: $d_g = 0.32$

Hence, during the experiment the control group's achievement with regard to algebra as well as geometry did not show any significant difference.

Hence, the experimental group achieved significantly better in the post-test geometry, and possibly algebra too, than in the pre-test, whereas the control group did not show any significant change in achievement at all. Given the equivalence with regard to other measured factors between the two groups, it seems logical to conclude that the application of collaborative teaching may have affected better achievement in those learners' mathematics marks. Hence, the collaborative approach does seem to have some significant achievement benefit compared to traditional teaching.

When comparing the post-test marks of the experimental and the control group by mean of difference, the following results were obtained:

Table 5.14: Mean difference between marks at school E and school C

Variables	Mean of difference	Stand. Dev.	Effect size
Algebra	6.03	19.50	0.31
Geometry	13.28	16.40	0.81

Hence, there was no significant difference between the two groups with regard to their algebra achievement. However, the experimental group on a practically significant level outperformed the control group in geometry.

5.11. DISCUSSION

5.11.1. Discussion on study orientation factors (SOM)

5.11.1.1. Study attitude

A collaborative teaching approach does not have an impact on learners' study attitudes. Attitudes towards mathematics were discussed in Chapter 3 of this study. Although it is found that most teachers and learners in South Africa have a negative attitude towards geometry (Laridon, 2000; Mulder, 2000; Van der Walt, 2000) and consequently they perform poorly in Grade 12 mathematics. Research conducted by Ma and Kishor (1997:3) found that attitudes toward mathematics (ATM) have no influence on achievement in mathematics (AIM). In this study, teaching learners mathematics in a collaborative approach did not affect their attitudes towards the subject.

5.11.1.2. Mathematics anxiety

A collaborative teaching approach does not have an influence on mathematics anxiety of learners. Section 3.2.5.2 of this study reveals conflicting results on mathematics anxiety affecting mathematics achievement. Studies conducted by Hembree (1990) indicated that once there is reduction in mathematics anxiety there will be higher achievement. However, studies conducted by Resnick, Viehe and Segal (in Ma, 1999:535) indicate that a decrease in mathematics anxiety is not associated with improvement in mathematics performance.

5.11.1.3. Study habits

A collaborative teaching approach does not change the study habits of learners. Although teachers collaborated in teaching they did not influence learners to do likewise. As a result, the experimental group was no better than the control group. This shows that although teachers

(at the experimental school) shared their skills collaboratively, they still used the conventional method of teaching where learners are generally passive. This is in line with the 1999 TIMMS Video Study.

5.11.1.4. Problem solving

While problem solving is a very important skill to be taught to learners especially in mathematics teaching, this was not the case with the collaborators at the experimental school. As a result, this did not influence learners in their problem-solving behaviour, indicating that the ideas of Sawyer (2004), discussed under 4.4.3, did not apply here.

5.11.1.5. Study milieu

Learners of both the experimental and control groups resided in an equal socio-economic status environment. A collaborative teaching approach did not affect the study milieu of learners.

5.11.1.6. Information processing

The artificial intelligence of computer programs, which influenced the behaviourist to rely on performance behaviour, stimulated Schoenfeld (1985) to examine human problem solving. This examining of problem-solving process and strategies became known as Information Process (IP) (Leder & Forgasz, 1992: 11). Beilin (1985: 382) says, "in applying information processing models, emphasis is on how problems are best represented, the nature of the task environment and the optimal organisation of task instructions". Information processing theory recognises that learning is a constructive process, that is, new knowledge must be connected to establish knowledge structures in order to construct new relationships between them (Leder & Forgasz, 1992: 13).

The collaborative teaching approach did not specifically consider this (IP) theory because as such it did not impact on the experimental group's behaviour in their information process.

5.11.2. Discussion on mathematics test results

A comparison between the experimental and the control groups on Algebra and Geometry performance reveals that the effect size with regard to Algebra is 0.31 while that with regard to Geometry is 0.82. The latter indicates a practically significant improvement by the experimental group in Geometry.

Findings from Chapter 1 show that learners, especially in Grade 12 final examinations, perform poorly in Mathematics Paper 2 (Geometry, Trigonometry and Analytical Geometry). Results from this study show that when learners are taught collaboratively, the situation will change. A comparison of the 1993 and 1994 Grade 12 mathematics results of the experimental school with those of other schools (see Table 3 of Chapter 1) shows the experimental school (E) performing better than C. Evidently, this was as a result of a collaborative teaching approach.

5.11.3 Analysis of former learners' responses on the collaborative teaching approach

Sixty questionnaires were sent to former learners and 48 (80%) were returned. The questionnaire consists of 18 items as can be seen in Appendix C. From these items, only the following were analysed quantitatively: 6, 7, 8, 9, 10, 12 and 13. These are the items that best express the opinions of learners about the collaborative teaching approach (see Table 5.14).

Table 5.15: Responses of former learners on the collaborative teaching approach

Item	Frequency					Avg.
	SA	A	N	D	SD	
6. I attributed my performance in Grade 12 to the method that was employed.	28	16	4	0	0	1.5
7. One maths teacher masters all sections well.	0	0	16	12	20	4.1
8. There are certain sections of maths e.g. trigonometry, where one maths teacher would fail to teach as they should.	28	4	8	8	0	1.9
9. It is not good to be taught different sections of maths by different teachers in Std 9 and Std 10.	4	8	4	8	24	3.8
10. When we are taught different sections of maths by different teachers we understand all maths better.	16	8	8	8	8	2.6
12. We get confused when we are taught by different teachers.	0	8	8	8	24	4.0
13. I wish learners at Std 9 and Std 10 were taught the way we were taught.	20	8	12	4	4	2.3

Discussion of the responses to the individual items follows:

Item 6:

Former learners agree that a collaborative teaching approach has helped them pass Grade 12. Most learners indicated in item 4 of the questionnaire that their performance was either good or average, which suggests that they passed mathematics in Grade 12.

Item 7:

Former learners disagree that one mathematics teacher will master all the sections of mathematics equally well. One learner is of the opinion that “being taught by one teacher in all sections of mathematics is deadly.” The learners seem to have come to the conclusion that in the prevailing school conditions one teacher would not be able to teach all sections of mathematics as effectively as a team of teachers.

Item 8:

Former learners agree that there are certain sections of mathematics which one mathematics teacher would not teach well. This is based on the experience they have had at middle school and in Grade 10, where some of the main sections e.g. trigonometry were not taught by the same teacher of mathematics.

Item 9:

Former learners disagree that it is not good to be taught different sections of mathematics by different teachers. The question was asked in the negative form. The positive form would have been: "It is good to be taught different sections of mathematics by different teachers." The purpose of this question was to ascertain consistency with item question 8 above.

Item 10:

Former learners were neutral on this item. This could have been caused by the ambiguity of the question about "different" teachers. This question did not specify "different" teachers; as a result, former learners might have thought it referred to any mathematics teacher other than those involved in a collaborative teaching approach.

Item 12:

Former learners disagree that they get confused when different teachers teach them mathematics. It appears in this item that "different" teachers was understood in the context of collaborative approach. This is unlike earlier in item 10.

Item 13:

Former learners wish a collaborative teaching approach could be continued at the experimental school.

Next, individual former learners' comments about their assessment of a collaborative teaching approach are summarised below. Learners' ideas are expressed unedited in their own words (no inverted commas used).

Learner 1:

I was taught very well as I had different teachers in different sections, so I got all different methods because each one had his own method of teaching and explaining, so I got the chance to pick up where the other teacher left out.

Advantages:

- *You link easily to the sections.*
- *You get the chance to understand each section easily.*
- *You catch up a lot at a time.*

Disadvantages:

- *You have a lot of work at a time.*
- *Makes you practise everyday (like it or not).*
- *If you lose a lesson, you'll be lost unless you ask the teacher to go back.*

I recommend the method of different teachers per section. It is very good.

Learner 2:

It was excellent to have different teachers teaching different sections of the same subjects, helps learners to perform good on the subject in totality.

Different teachers apply different approaches, which might be good for learners who don't understand the other teacher. Therefore it balances the performance of that learner.

Recommendations:

- *Have two or more teachers on one subject e.g. maths and science*
- *Keep close monitoring and evaluation on the performance of teacher to check section of excellence.*

Learner 3:

Being confined to a single teacher in a subject as complex and essential as mathematics is deadly. Time has proved this method of teaching to be ineffective, when collaborative teaching proved effective in that different teachers of different attitudes expose you to different methods of teaching.

Having been a victim of circumstances of one teacher teaching a particular subject from beginning to end, I encourage that collaborative teaching be adopted as a standard practice not in mathematics only but all subjects that are as important physical science, geography and biology.

Learner 4:

It was superb until we lost our best maths teacher and after my classmates and I were excluded in the classroom. In other words, I could say I haven't had a science and maths teacher.

Advantage: *Best 'cause the teacher knows you better and what you master or not.*

Disadvantage: *The teacher does not have enough time (maths too demanding), he teaches what he/she masters.*

Recommendation: *Teachers must be upgraded (education wise)*

Learner 5:

It was quite interesting with different teachers bringing different ideas on how to solve maths problems especially geometry and trigonometry.

- Advantage:** *The more the teachers specialises on a subject, it is easier to deliver to the pupils*
- Disadvantage:** *The workload might cause frustration and can be difficult for pupils to understand.*
- Recommendations:** *If only the teachers are committed more enough to their job, it matters not the workload & they will deliver to the best of their ability.*

Learner 6:

It was the first time I encountered that method in Std 9 when I joined Kwenya-ya-Madiba High, and their teaching method was exceptional. I completely disagree with this one [referring to one teacher teaching all sections of mathematics] because, let's say for example, if you have a problem with the maths teacher, then your maths is doomed because you'll hate the teacher concerned and maths also. Another point to ponder: One teacher will not be able to complete the whole syllabus of algebra, geometry, calculus and trigonometry.

If a teacher understands one section of maths e.g. geometry, he/she might make it horrible for a learner to clearly understand the other sections of maths e.g. algebra.

I experienced a completely revamped type of learning, where I had three teachers who taught me maths and they continued with me to Standard 10 from Standard 9. Today, which is 10 years later, I still understand maths and I am happy to teach a class of pupils.

The most important factor that influenced my learning is the relationship I had with my teachers of different subjects. They have really motivated and encouraged me even in personal capacity. I am what I am today because of them.

I haven't seen a disadvantage of collaborative learning to date; hence I will briefly indicate some of the advantages that I discovered about myself.

- It encourages one to open up to others, by absorbing their methodologies of learning, and also influences team work, which a requirement in today's world of work.*
- It helps one to quickly identify their deficits and respond that one can benefit.*
- It also encourages caring about one another; you will know others difficulties and you will strive to help them so that they can also like you!*

I sincerely recommend that Mr. Ranamane's maths teaching method be introduced to other schools so that they can have immediate benefit and their results will surely improve immediately. These methods can also be introduced to physical science and chemistry and you will surely reap the benefits in short to long term. These methods will also create jobs for maths teaching in the entire country.

Thank you for making me a part of this discovery.

Learner 7:

Teaching was motivational and encouraged competition as learners were told of their quarterly performance.

Advantage: *Every teacher specialises in his/her section.*

Disadvantage: *Teachers might not be aware of the weakness or strength of learners in other sections of maths.*

Recommendations: *Teachers must not overload learners in order to complete syllabus, rather spend some extra time on a chapter and thus ensuring that it is clearly mastered by both fast learners and slow learners.*

Learner 8:

Most of our contents teachers didn't give themselves enough time to teach properly.

The disadvantages are the fact that one teacher will only concentrate on the section that he excel in and if there are more teachers, each will concentrate on the section that he excel in.

I recommended that collaboration teaching be practised in subject such as maths, science and biology.

Discussion of the responses

Former learners seem to have enjoyed being taught collaboratively, while feeling that they have benefited from the teaching approach too. Comments such as “it was excellent to have different teachers teaching different sections of the same subject” (Learner 2) support the finding, as well as to indicate that the benefit could go beyond mathematics. From Learner 3's and Learner 4's responses it seems as if the approach was both interesting and novel to the learners, and that they appreciate the value of the approach highly.

One particular former learner, who is an electronic engineer working for Telkom, indicates that “corporate business survive on collaboration” and that the experience at school thus helped him to adjust to being taught in one subject by different lecturers at university, as well as to work effectively in a corporate working environment.

Evidently, the former learners hold the collaborative teaching approach (under scrutiny in this study) in high esteem and recommend the continued and expanded use thereof in more schools and subjects. However, few of them have cited the following limitations, which need to be read with the few limitation listed in 4.6:

- Learners complain of being overloaded with work as each sectional teacher gives classwork, homework and tests separately from other collaborating teachers.
- Collaborating teachers fail to link the sections they teach with those they do not teach.

5.12. CONCLUSION

Overall collaborative teaching, compared to traditional teaching, seems to have had a positive effect on learning achievement in mathematics at Grade 12 level. In particular, it had a practically significant effect ($d=0.8$) on the learning of geometry and trigonometry. This approach appears to offer some solution to an old problem of poor performance by Grade 12's, particularly in Paper 2 (geometry and trigonometry). However, whilst collaborative teaching is being used, mathematics teachers need to apply other learning strategies such as problem solving, cooperative learning and information processing as well. Furthermore, educators need to understand new developments on theories of teaching and learning. All this put together could improve mathematics teaching and learning in South Africa.

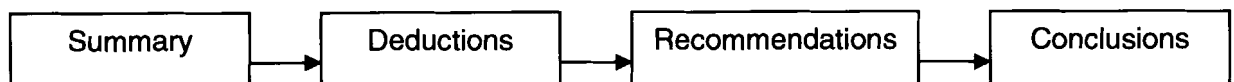
In reference to the stated hypothesis (see 5.2), the obtained results do not conclusively and unconditionally support the (statistical) acceptance of the hypothesis. However, there are indications that the approach may have some added value with regard to learning achievement when compared to the traditional approach, provided certain additional conditions of learning in mathematics classes are met too. The same might be true of learners' study orientation, but further investigation is needed to come to a conclusion in this respect.

CHAPTER 6

DEDUCTION, RECOMMENDATIONS AND CONCLUSION

6.1. INTRODUCTION

In this chapter, consolidation of the study will be undertaken to check whether the three questions posed in Chapter 1 (refer to section 1.2) have been answered. As a result, a summary of the chapters will be given and the deductions and recommendations will follow. This chapter will thus flow as follows:



6.2. SUMMARY

The research study involved three teachers who taught mathematics collaboratively in Grade 12. One teacher (x) taught algebra and calculus, another (y) taught geometry and the researcher taught trigonometry. Refer to page 80 where a schematic diagram appears about this teaching approach. This happened at a science school whose purpose was to encourage more learners to study mathematics and science. This is therefore a case-study that resulted from the success of this collaborative teaching approach which did produce good Grade 12 examination results in 1993 and 1994.

The study therefore aimed at answering the following questions:

- What is collaborative teaching and how does it proceed?
- How does it address the teaching and learning of mathematics in Grade 12?
- How does it affect learner performance in mathematics in Grade 12?

In attempting to answer these questions, Chapter 2 of the study analysed mathematics learning and learning tasks in Grade 12. The chapter looked into the nature of mathematics, views of mathematics and learning theories. It was found that the way people view mathematics influences the way they learn it and thus the way they will teach it. Three learning theories viz. behaviourism, cognitivism and constructivism were analysed and it is recommended that they be used concurrently where possible. This is because the researcher feels that none of them is superior to the others. Mathematics learning tasks were also analysed. Rich and traditional learning tasks were compared and it was found that rich learning tasks are oriented towards cognitivism and constructivism, while traditional learning tasks are more inclined towards behaviouristic thinking.

The collaborators in this study were behaviourists in their teaching styles. Collaboration in cognitivism and constructivistic styles needs further investigation.

In Chapter 3 the focus was on factors affecting the teaching and learning of mathematics in South Africa. Although South Africa was the focal point, it was not possible to isolate it from the rest of the world, that is, factors such as language, socio-economic status, teacher knowledge and teaching approaches are not peculiar to South Africa only, they affect mathematics teaching and learning internationally.

While cultural and socio-economic factors among others affect the teaching and learning of mathematics positively or negatively in terms of learner performance, there were (and still are) inconclusive findings on psychological factors such as interests, motivation, attitudes and anxiety. More research is needed, as there are contradictions in this regard. Teacher knowledge and teaching approaches impact on the teaching and learning of mathematics, for example knowledge of learning theories influences teaching.

Chapter 4 explains in detail how the collaborative teaching approach came about. The chapter begins with a definition and explanation of team teaching, which may be confused with collaborative teaching approach. This is followed by the definition and explanation of collaboration, which leads to the full explanation of a collaborative teaching approach. A questionnaire on this approach was sent to former learners to elicit their opinions about the approach, that is, collaborative teaching. It is generally found that former learners are happy with this teaching approach. However, a few of its limitations are cited.

Chapter 5 is an empirical study based on the performance of the experimental and control groups on mathematics pre- and post-tests. The experimental research method was used and the results were analysed by the Statistical Services of the Potchefstroom campus of the University of the North West. The results of this investigation therefore show that a collaborative teaching approach has a significant impact on the learning of mathematics, especially on geometry and trigonometry. This therefore answers questions raised in Chapter 1 about the effectiveness of a collaborative teaching approach on the learning of mathematics in Grade 12.

6.3. DEDUCTIONS

6.3.1. Mathematics learning and teaching

Mathematics is influenced by views on the nature of mathematics. Those who believe mathematics is fixed and governed by rules will learn and teach it according to the way they perceive it. Similarly, those who regard mathematics as dynamic, forming a pattern and needing to be discovered, will learn and teach it accordingly.

Both traditional and current approaches to mathematics teaching and learning need to be used interchangeably because of the dynamic nature of the classroom situation.

School mathematics education differs significantly worldwide. In Chapter 2, the study of Japanese school mathematics teaches us that parental involvement in schooling is essential. Furthermore, teacher development and the teaching profession need to be taken seriously by those involved, that is, the teachers themselves, policy makers and the community as a whole.

6.3.2. Factors affecting the teaching and learning of mathematics

Psychological factors affect the teaching and learning of mathematics differently. Some enhance mathematics learning while others depress learning and consequently performance. Beliefs of teachers, learners and parents about the nature of mathematics have similar results to views on mathematics as cited in Chapter 2. Beliefs (according to researchers' deductions) influence other psychological factors such as mathematics anxiety, attitude, interest and motivation.

There are conflicting research studies on factors such as attitudes (refer to 3.2.2.2) and interest (3.2.3.2), as well as motivation (3.2.4) and mathematics anxiety (3.2.5.2) affecting mathematics achievement. Extensive research is needed on this so that people are aware of the role these factors play in mathematics performance.

Social factors, such as language and socio-economic status, affect mathematics learning in particular. The comparative studies in the TIMSS reports on mathematics teaching and learning, for example in Japan and the USA, indicate the extent to which these factors affect mathematics learning. The South African situation is also a clear example of how these factors influence mathematics learning. Research on this confirms the conclusion.

To a great extent one could conclude that teacher knowledge influences the teaching and learning of mathematics. More research is needed on this. However, lack of knowledge on teaching and learning theories, for example, will affect instruction and consequently learners' performance. Moreover, Japanese quality of teaching and teacher qualification can be linked to the high standard of Japanese learner performance in mathematics.

6.3.3. The collaborative teaching approach

Collaboration happens worldwide in different forms. Countries, all over the world, collaborate socially (e.g. sport) and economically. In industry, this phenomenon is more explicit. Different forms of collaboration in this chapter were cited with emphasis on mathematics teaching and learning as a case study. A collaborative teaching approach works and will continue being necessary especially in this era of information explosion we live in.

Today's methods of instruction and, consequently, theories of teaching and learning, will be outdated in the next ten years and yet those who begin teaching today, will still be in the system then. This will make it necessary for the old teachers to orientate the new ones while new teachers will also help old ones with new technological information especially.

The researcher (of this study) is excited because the mathematics teachers at the control school involved in this study have also decided to use this approach at their school. They are two female teachers sharing Grade 12 mathematics syllabuses in the same way as it was done at the experimental school. They believe this will improve their Grade 12 mathematics results, which are expected at the end of 2004. Note must be taken that already teacher X, who was involved in this approach at the experimental school, has applied it at another school where she was head of department. She claims the results improved from 23% to 72%.

6.3.4. Empirical study

Chapter 5 helped in linking cognitive factors (SOM) to mathematics learning. Although some psychological factors, for example mathematics anxiety and attitudes towards mathematics, were stated in Chapter 2 of this study, they were quantified in this chapter. A factor such as study milieu can be associated with socio-economic status (stated in 3.3.2.), which affects mathematics learning.

Through statistical analysis, this chapter proved that collaborative teaching is a possible approach for improving mathematics results, especially in geometry and trigonometry. This appears to be a breakthrough, not only in the South African situation, but also in the world as a whole as research shows learners performing poorly in geometry.

6.4. Recommendations

6.4.1. Recommendation 1

Results with regard to the study milieu of both the experimental and the control group did not show any stimulation from home. Hence, in conjunction with Hawkey's (1995) finding, it is suggested that parents and teachers need to impress upon learners that mathematics is exciting, not difficult, needs to be discovered and that anybody can do it irrespective of their gender, race and social status. A close link between the school and the home (as in Japan) is strongly recommended and a collaborative approach may afford a sound basis for such cooperation to take place. Hence, it is recommended that the current views and beliefs about school mathematics in South Africa need to be addressed to affect change that can support relevant and appropriate mathematics teaching and learning practices at all school levels, particularly at matric level.

6.4.2. Recommendation 2

In Chapter 3 of this study, conflicting findings on interest and anxiety affecting achievement in mathematics were reported, a situation that requires further investigation in close cooperation between psychologists and mathematics educators. Further investigation on psychological factors (interest, motivation and anxiety) influencing mathematics performance is therefore recommended, particularly in the kind of teaching-learning conditions that applied to the schools involved in this study.

6.4.3. Recommendation 3

Section 3.3.1 has shown the importance of language as factor in the learning and teaching of mathematics. For instance, the Grade 12 mathematics examination paper puts the first language speaking (Afrikaans and English) learners at an advantage because it caters for both English and Afrikaans. Better results therefore attained by white and Indian learners (historically) could also be attributed to the language factor, besides socio-economic-status. Therefore, learners should be taught mathematics in the language they understand best, keeping in mind that a collaborative may afford teachers and learners a valuable opportunity to utilize (e.g.) code-switching more effectively to improve the quality of mathematics learning.

6.4.4. Recommendation 4

The current status is that the teaching of languages (understandably, in view of 6.4.3 above) is given more time than the teaching of content subjects, that is, in nine periods of 35 minutes each a day, language is allocated 245 minutes a week whereas content subjects (such as mathematics) are allocated 210 minutes. In view of the nature and importance of mathematics in the school curriculum, the same guideline should be applied to mathematics as well. Once more, schools in Japan

serve as a good example where learners spend more time learning mathematics and thus carrying out instructions in mathematics (see Chapter 2). Therefore, mathematics teaching and learning should be allocated more time in school timetables, and a collaborative approach may afford the opportunity to utilize the allocated time more effectively.

6.4.5. Recommendation 5

It has been proved beyond reasonable doubt that, in general, when people collaborate, production will increase. This is despite setbacks which arise as people collaborate. Collaboration can be extended further to include two or more schools in the subject, particularly mathematics where two mathematics teachers of two schools could collaborate to teach two different sections of mathematics at their schools. This type of approach could also be applied to physical science as well between physics teachers and chemistry teachers. Therefore, collaborative teaching should be considered for introduction as one of the teaching approaches in mathematics, after appropriate development thereof in the prevailing school conditions in South Africa.

6.5 Limitations of the study

This research study has the following limitations:

- Teaching one section of mathematics over an extended period may cause one to rust in the other sections, for example teaching trigonometry only for five years for instance might affect one's knowledge and ability to teach algebra and geometry.
- In a case where one of the collaborators leaves for one reason or another it becomes difficult to get a replacement for the same section taught by the teacher who left.

- “Good” mathematics results attained in this instance are a group effort, no individual teacher can claim them as his or hers alone.
- When one of the collaborators is transferred or promoted to another school, the set up changes and it becomes difficult to adjust to the new set up.

6.6. FINAL CONCLUSION

The teaching and learning of mathematics pose a serious challenge to educators, parents, policy makers and researchers all over the world. This study has to some meaningful extent proved that collaborative teaching in mathematics can help in improving learning achievement, and with some further development, learning performance in the subject. The findings in this study justify further research, especially with regard to cognitive factors affecting the teaching and learning of mathematics.

Furthermore, mathematics teacher development needs serious attention from policy makers. The recommendations made in this study, if heeded, have the potential for improvements in the teaching and learning of mathematics in our schools. In this endeavour collaboration may prove to be instrumental to address critical issues relating to the effective teaching and learning of mathematics in South African schools, particularly in those historically disadvantaged regions that still suffer most from lack of relevant learning performance and achievement.

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APPENDIX A

Pre Mathematics Test

Mathematics Test

Examiner: Ranamane NS

Grade : 11

Date : 06 March 2001

Duration : 1 ½

SECTION A

Choose the correct answer

1. If $x + y = -7$ and $x - y = 5$, then
 $x^2 - y^2 =$
A. -35
B. -3
C. -143
D. 25

2. Quadratic equation with roots -3 and 4 , is =
A. $y^2 + y - 7 = 0$
B. $y^2 + y = 12$
C. $y^2 - y = 12$
D. $y^2 - 7y - 12 = 0$

3. The domain of $\frac{x-3}{\sqrt{x+2}}$ is ...
A. $x = -2$
B. $x < -2$
C. $x > -2$
D. $x = 3$

4. If $(x - 3)(y + 6) = 0$ and $x = 3$, the value of y will be equal to:

- A. 6
- B. -6
- C. 3
- D. 0

5. The sum of the roots of $2x^2 + 5x - 3 = 0$ is:

- A. $\frac{5}{5}$
- B. $\frac{5}{2}$
- C. -3
- D. $\frac{7}{2}$

SECTION B

1. Solve for x :

1.1. $5x^2 + 6x - 7 = 0$ (4)

1.2. $\frac{6}{x+1} + \frac{3x}{x-1} = \frac{2x}{x-1}$ (5)

2. Let $(2x + 1)(x - 2) = k$. Solve for x if

2.1.1. $K = x - 2$ (5)

2.1.2. $K = 1$ (approximate to two decimal digits) (5)

2.2. Calculate the value(s) of m for which the equation $x^2 - 3x + 5m = 0$ has

2.2.1. One root with the value of -2 (3)

2.2.2. Equal roots (4)

3.

3.1. Solve for x and y in the simultaneous equation:

$$x - 2y = -3$$

$$xy = 20$$
 (6)

3.2. If $f(x) = x^3 + kx^2 - kx - 4$ is divided by $(x - 5)$, the remainder is $9k$. Calculate k . (4)

3.3. Two factors of

$f(x) = x^3 + mx^3 - nx + 8$ are
 $(x - 1)$ and $(x + 4)$

3.3.1. By using the factor theorem, calculate the values of m and n (6)

3.3.2. Calculate the third factor (2)

4. The graphs of a parabola and a straight line are shown in the sketch.

Using the information given in the diagram, determine:

4.1. The Equation of G (4)

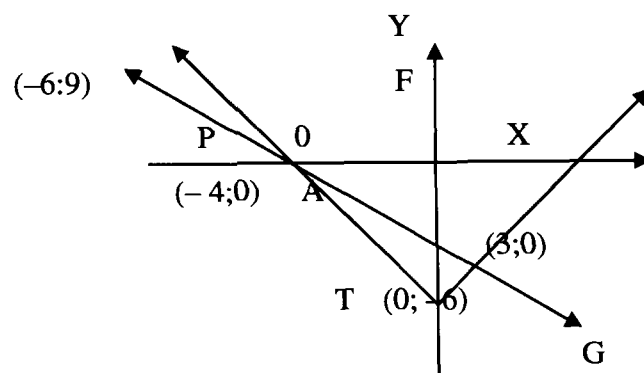
4.2. The Equation of F (4)

4.3. The Co-Ordinates of A , The X -Intercept of G (2)

4.4. The Co-Ordinates of T , The Turning Point of F (4)

4.5. The Range of F (2)

4.6. The Element of $F \cap G$ (2)



SECTION C

1. If $5 \sin x = 3$ and $x + y = 90^\circ$, calculate without using a calculator, the value of:

$$\frac{\tan x + \operatorname{cosec} y}{\tan y + \operatorname{cosec} x} \quad (7)$$

2. Simply to an expression containing one trigonometric ratio of ϕ only:

$$\frac{\tan \phi + \cos \sec \phi}{\cot \phi} \quad (7)$$

3.

If $x = 124,30^\circ$ and $y = 153,40^\circ$, calculate, rounded off to one decimal digit, the value of:

3.1. $\sin x + y$ (2)

3.2. $2\cos(y - x) \cot x$ (2)

4. Use trigonometric identities and prove that:

4.1. $\cos x + \tan x \cdot \sin x = \sec x$ (5)

4.2. $\sin^2 \phi - \cos^2 \phi = 2\sin^2 \phi - 1$ (5)

Total: [100]

APPENDIX B

Section A

Post Mathematics Test

Question 1

1.1. If $f(x) = 12x^3 - 25x^2 - 4x + 12$, show that $(x - 2)$ is a factor of $f(x)$ (3)

1.2. Hence solve for x if

$$12x^3 - 25x^2 - 4x + 12 \quad (4)$$

1.3. When $x^3 - 3x - 8$ is divided by $(x - 3)$ the remainder is 10, use the fact that $x^3 - 3x + k = (x^3 - 3x - 8) + 8 + k$ to find the value of k if $x^3 - 3x + k$ is divisible by $(x - 3)$ (2)

Question 2

2.1. Solve for x if $\frac{x - 5}{3x} = \frac{2}{3}$ (5)

2.2. Solve for y if $y + 1 - \frac{5}{3y + 3} = \frac{2}{3}$ (3)

2.3. Solve for a and b simultaneously:

$$a + b = 10$$

$$a^2 + ab - b^2 = 4 \quad (9)$$

Question 3

3.1. Simplify the following WITHOUT using a calculator:

3.1.1. $\frac{6^{a+3} \cdot 2^{a-1}}{12^{a+2}}$ (5)

3.1.2. $\frac{10g^2 - 10g}{10g^2 + 10g}$ (3)

3.2. Solve x

3.2.1. $\frac{(1)^{y-2}}{3} = 9$ (2)

3.2.2. $Y^3 = 2$ (2)

$$3.2.3. \quad 10g_3(x+1) = 10g_3 2 = (x > -1) \quad (4)$$

Question 4

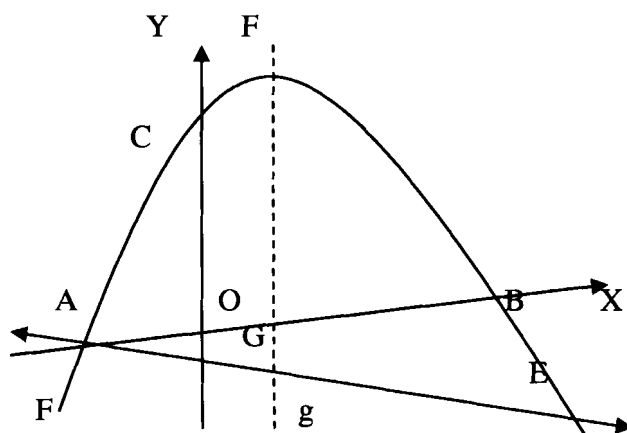
The diagram represents the graphs of

$$f(x) = -2x^2 + 8x + 10$$

and

$$g(x) = -2x - 2$$

FG is the axis of symmetry of the parabola and F is the turning point.



Determine:

$$4.1. \quad \text{The length of OA and OB} \quad (4)$$

$$4.2. \quad \text{The co-ordinates of F} \quad (4)$$

SECTION B

Question 1

Simplify the following to a single trigonometric ratio of ϕ :

$$1.1. \quad \frac{\cos(180^\circ + \phi)}{\sin(180^\circ - \phi)} \quad (3)$$

$$1.2. \quad 1 - \sin^2 \phi \quad (1)$$

Question 2

$$2.1. \quad \text{If given that } \cos x = \frac{\sqrt{3}}{2} \text{ and}$$

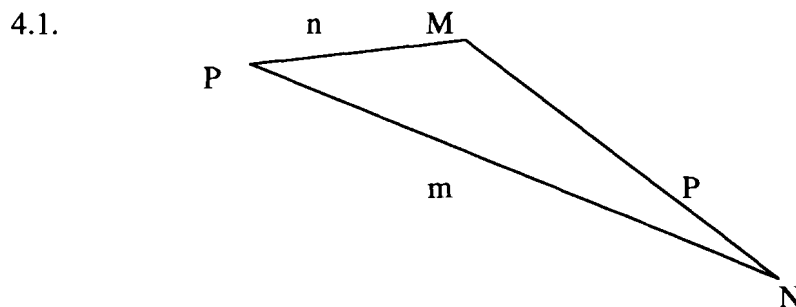
$$0^\circ < x < 180^\circ, \text{ determine } x \quad (2)$$

- 2.2. If given that law $B = -\sqrt{3}$ and $0^\circ < B < 180^\circ$, determine B (2)
- 2.3. Hence determine the value of $\sin(x + B)$
[Do not use a calculator]

Question 3

- 3.1. Solve for x WITHOUT using a calculator $\tan(x + 25^\circ) = \cot(x + 11^\circ)$; $x \in [0^\circ; 90^\circ]$ (3)
- 3.2. Given $f: x \rightarrow \sin 2x$
and $g: x \rightarrow 2\cos x$

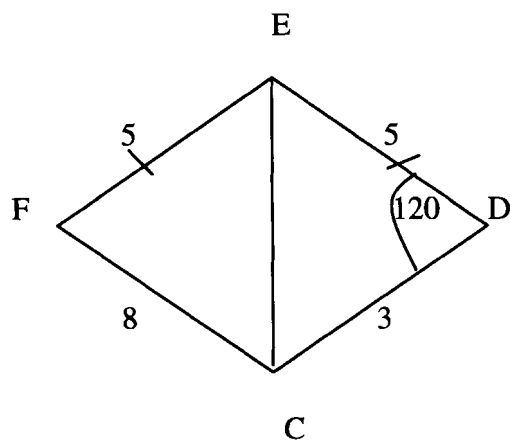
Draw on the same set of axis not sketches graphs of f and g for the interval $[0^\circ; 180^\circ]$. indicate clearly the turning points and intercepts with the axis (8)



In the figure, MNP is any triangle. Use the figure to show the following and $\sin$

- $p = \sin(m + n)$ (2)
- 4.4.2. $M = p \sin M$ (1)
 $\sin(m + N)$
- 4.4.3. Area of $\triangle MNP = \frac{1}{2} p^2 \sin M \sin N$ (2)
 $2 \sin(M + N)$
- 4.4.4. If $MN = 4,3$ units, $M = 65^\circ$ and $N = 40^\circ$, calculate the area of $\triangle MNP$ (2)

5. In the figure, CDEF is a quadrilateral with $CD = 3$ units, $CF = 8$ units, $DE = EF = 5$ units and $\angle CDE = 120^\circ$



- 5.1. Calculate the length of CE (3)
- 5.2. Calculate the size of F (3)
- 5.3. Hence state, giving reasons what type of quadrilateral CDEF is (2)

6. Complete without proof

- 6.1. The sum of the opposite angle of a cyclic quadrilateral is (1)
- 6.2. The size of an angle at the circumference of a circle which is subtended by the same arc as an angle at the centre of the circle is (1)
- 6.3. In the figure, ABCD is a cyclic quadrilateral of a circle with centre O $BC = CD$ and $\angle A = 80^\circ$

6.3.1 $\angle BCD$

6.3.2 $\angle CBD$

6.3.3 $\angle COD$

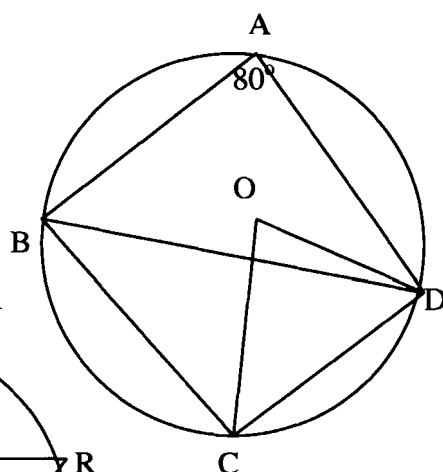


Figure 1

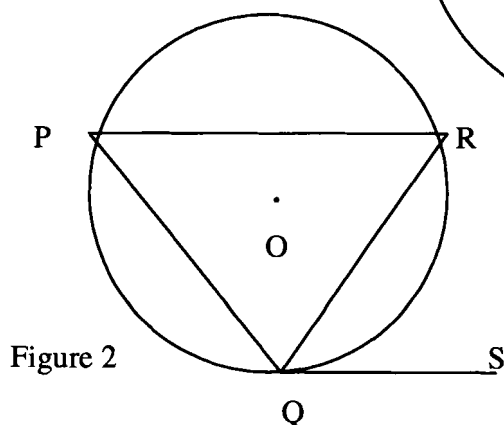


Figure 2

- 6.4. In the figure 2 QS is a tangent to circle with centre O. prove that the theorem which state that $\angle RQS = \angle QPR$ (5)

.....

APPENDIX C

The following questionnaire was sent to former learners:

PLEASE RESPOND TO THE ITEMS BELOW ACCORDING TO THE FOLLOWING KEY:

- 1: SA - I Strong Agree
2: A - I Agree
3: N - I am Neutral
4: D - I Disagree
5: SD - I Strongly Disagree

1. Algebra and calculus are easy to understand.

SA	A	N	D	SD
----	---	---	---	----

2. Geometry is easy to understand.

SA	A	N	D	SD
----	---	---	---	----

3. Trigonometry is easy to understand.

SA	A	N	D	SD
----	---	---	---	----

4. My performance in maths in Std 10 was Good: Average: Bad

SA	A	N	D	SD
----	---	---	---	----

5. In Std 10 (final exams), I believe I performed better in Paper 1 than in Paper 2.

SA	A	N	D	SD
----	---	---	---	----

6. I attribute my performance in Std 10 to the method that was employed.

SA	A	N	D	SD
----	---	---	---	----

7. One maths teacher masters all sections well.

SA	A	N	D	SD
----	---	---	---	----

8. There are certain sections of maths e.g. trigonometry, where one maths teacher would fail to teach as they should.

SA	A	N	D	SD
----	---	---	---	----

9. It is not good to be taught different sections of maths by different teachers in Std 9 and Std 10.

SA	A	N	D	SD
----	---	---	---	----

10. When we are taught different sections of maths by different teachers we understand all maths better.

SA	A	N	D	SD
----	---	---	---	----

11. It is boring to be taught by one maths teacher from Std 8 – 10.

SA	A	N	D	SD
----	---	---	---	----

12. We get confused when different teachers teach us.

SA	A	N	D	SD
----	---	---	---	----

13. I wish learners in Std 9 and Std 10 were taught the way we were taught.

SA	A	N	D	SD
----	---	---	---	----

13. The symbol I got in maths as Std 10 has helped be in the career I'm pursuing.

SA	A	N	D	SD
----	---	---	---	----

14. My career is

15. In your own words, elaborate on the way we taught you in Std 9 and Std 10.

.....
.....
.....

16. State advantages and disadvantages of collaborative teaching.

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.....
.....

17. Make some recommendations.

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.....
.....

Thank you