



The masses of Milky Way globular clusters using GAIA data

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Abstract

Globular clusters are gravitationally bound groups of stars that formed from a common molecular cloud. Globular clusters are, therefore, densely packed and have similar chemical composition, age and kinematics. They have been observed in different galaxy types, thus making them good natural laboratories for studying star formation. Stellar properties within globular clusters provide a reliable means of accurately determining various fundamental parameters associated with these clusters, including distances, ages, metallicities, sizes, and masses.

The Leonard-Merritt method is used in this study to measure the masses of the ten closest globular clusters. The mass distribution is modelled by fitting the photometric characteristics of each globular cluster to the King model. The gravitational potential is modelled using the proper motion data from the *Gaia* space telescope. The precise astrometric measurements from the *Gaia* mission allow for the most accurate proper motion measurement throughout the galaxy and enable measuring the masses of a vast number of globular clusters using a single data set and one mass measurement technique, providing a perfect point of reference for the Leonard-Merritt technique against the literature masses of this study's chosen globular clusters.

Keywords: *Gaia*; Globular clusters; King model; Leonard-Merritt; Mass.

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Table of abbreviations

A table containing a list of abbreviations used throughout the text.

AAO	Anglo-Australian Observatory
AC	Across-Scan
AGB	Asymptotic Giant Branch
AL	Along-Acan
AM-1	Arp-Madore 1
BBH	Binary Black Hole
BS	Blue Stragglers
CCD	Charge Coupled Device
CMD	Colour-Magnitude Diagram
COBE FIRAS	Cosmic Background Explorer Far-Infrared Absolute Spectrophotometer
DEC	Declination
DR1	Data Release 1
DR2	Data Release 2
DR3	Data Release 3
EDR3	Early Data Release 3
ESA	European Space Agency
ESO	European Southern Observatory
ETG	Early-Type Galaxies
Gaia	Global Astrometric Interferometer for Astrophysics
GMC	Giant Molecular Clouds
HB	Horizontal Branch
He	Helium
HR	Hertzsprung-Russell
HST	Hubble Space Telescope
IMF	Initial Mass Function
JWST	James Webb Space Telescope

MS	Main Sequence
MUSE	Multi-Unit Spectroscopic Explorer
MW	Milky Way
RA	Right Ascension
RGB	Red Giant Branch
RDP	Radial Density Profile
SB	Surface Brightness
SNR	Signal-to-Noise
VISTA	Visible and Infrared Survey Telescope for Astronomy
VLT	Very Large Telescope
VVV	VISTA Variable in the Vía Láctea

Chapter 1

Introduction

Globular clusters are known to be the building blocks of our Galaxy ([Meylan and Heggie, 1997](#)). The Milky Way (MW) harbours different types and populations of stars formed through the collapse of giant molecular clouds (GMC). Globular clusters differ based on age, signifying their formation history. The age discrepancy is reflected in their spatial distribution, which indicates their birthplace and subsequent motion within the galaxy. However, stars form within a cluster environment, where gravitational forces and interactions with neighboring stars play significant role in shaping their evolution ([Lada and Lada, 2003](#)). Star clusters, for example, have the potential to remain gravitationally bound for extended periods, from millions to billions of years ([Bruzual, 2009](#); [Li et al., 2017](#); [Krumholz et al., 2019](#)).

Globular clusters have been studied extensively in the MW as well as in nearby galaxies since observations made by Harlow Shapley (1885 – 1972), who used the Cepheids and W Virginis variable stars to accurately compute the distances to ninety-three globular clusters ([Shapley, 1918b](#); [Shapley and Curtis, 1921](#); [Mowla et al., 2022](#)). Initially, these clusters were perceived as relatively static stellar systems ([Forbes et al., 2018](#)), but they are far from stagnant ([Heggie, 1989](#); [D’Ercole et al., 2008](#)). Globular clusters are some of the oldest objects in the Universe, believed to have originated within one to two billion years after the Big bang, with a redshift estimated at $z \sim 5 - 10$ ([Kravtsov and Gnedin, 2005](#)). Although they may appear small because of their distance from the Earth, they are relatively large with a range of approximately 1 – 30 parsecs ([Carroll and Ostlie, 2017](#)). Some of the brightest globular clusters, such as ω Centauri, have

been subjected to observations (Poveda and Allen, 1975; Meylan, 1987; Carroll and Ostlie, 2017) for an extended period preceding the current state of astrophysics.

Globular clusters are significant because of how they are associated with galaxy formation, potentially contributing to the reionisation processes (Gratton et al., 2019) and the exotic stellar populations generated within them (Cassisi et al., 2008). According to Kravtsov and Gnedin (2005), there is compelling evidence to suggest that globular clusters formed before most stars in galaxies. As a result, the globular clusters have unique characteristics, such as uniform distribution of large stars, comparable ages, and consistent chemical compositions, especially regarding heavy elements (Forbes et al., 2018), make them invaluable for studying both star formation processes and the evolution of galaxies. The clusters provide a means to accurately estimate varying fundamental parameters, including distances, ages, metallicities, sizes, and masses (Krauss, 2000; Forbes et al., 2018). Furthermore, globular clusters are recognised as a prominent environment for exotic systems resulting from dynamical interactions such as pulsars, X-ray binaries, fast radio bursts, and merging binary black hole (BBH) systems (Fishbach and Fragione, 2023) that merge due to their close orbits, emitting gravitational waves within the timescale of Hubble expansion (Banerjee et al., 2010).

Astronomy has been revolutionised since the development of space telescopes, resulting in new and unexpected discoveries (Board et al., 2011). The enhanced instrumental capabilities provided by the Hubble and Chandra space observatories and numerous telescopes of the 10-metre class have provided a fresh perspective on the study of globular cluster dynamics (Haggard et al., 2009; Watkins et al., 2015b). The European Space Agency’s (ESA) Global Astrometric Interferometer for Astrophysics (*Gaia*) mission is widely regarded a very successful project (Brown, 2021). The spacecraft conducts measurements of stellar positions with unprecedented precision since *Gaia* provides astrometric solutions for sources brighter than ~ 20 mag (Gaia Collaboration et al., 2016; Lindegren et al., 2021), which provides an extreme advantage for analysing both internal and general proper motions for galactic globular clusters and distance estimations (Vasiliev and Baumgardt, 2021). With the large amount of accessible *Gaia* data, Brown (2021) states that *Gaia* has “*quickly become an indispensable part of the astronomical ecosystem*”, due to the amount of work done by scientists.

Extensive research on the spatial distribution of the clusters within the Local Group

led to the identification of multiple scaling relationships (Harris, 1996). Research also incorporates the analysis of globular cluster density distribution carried out within the framework of the King models (King, 1966). Furthermore, McLaughlin and van der Marel (2005) and Kuzma et al. (2018) demonstrated that these clusters extend more in the outer regions than the King models predict. Consequently, the power-law distribution model offers a more accurate representation of the outer regions (Carballo-Bello et al., 2012) since they demonstrate a more gradual decrease in density (Meylan and Heggie, 1997). Determining the properties of globular clusters, such as radial density distribution and the total number of stars, nevertheless, remains challenging due to their high central densities and the gradual transition of their outskirts merging with the background of the MW (Pieńkos and Hałas, 2015; Cheng and Jiang, 2023). Understanding the previously mentioned properties is, thus, crucial to accurately assess the physical conditions within the centre regions of the globular clusters. Because the systems are old, they can reach dynamic equilibrium and a stable symmetric radial distribution only if they have not collided with other globular clusters or experienced a significant disturbance when passing through the Galactic disk (Pieńkos and Hałas, 2015). The radial distribution of stars is crucial in analysing the dynamical properties of clusters (Sollima et al., 2015).

Objectives:

The main objective of this dissertation is to accurately determine the masses of ten globular clusters: BH 140, NGC 104, NGC 3201, NGC 6121, NGC 366, NGC 6367, NGC 6544, NGC 6656, NGC 6752, and NGC 6838 in the MW and using valuable dataset provided by the *Gaia* telescope, given its unparalleled accuracy and depth, allows for a comprehensive study of these stellar systems.

To achieve this objective, good qualitative data from *Gaia* is used, with a focus on parallax and proper motion measurements. Parallax data is used to estimate the distances to the globular clusters, forming the foundation for their spatial distribution analysis. Proper motion data is employed to work out the average velocity of the stars within each cluster and, through velocity dispersion, calculate the radial distribution of the stellar mass of the globular cluster using the Leonard-Merritt mass estimator.

The advantage of employing proper motion data lies in its capacity to provide velocity information for a multitude of stars, each with varying masses, positioned at various distances from the cluster centre. The data make it possible to study the dynamics and internal motions of globular clusters by deriving intrinsic dispersion in the proper motion components of member stars. Furthermore, the radial star density profiles are presented for each globular cluster, derived from resolved star counts. The profiles contribute to the determination of structural parameters, improving the understanding of the spatial distribution and characteristics of the studied clusters.

Chapter 2

Theoretical Background

This chapter provides a comprehensive review of the various types of galaxies, focussing on discussing what type the galaxy in the MW and the different types of star clusters are. The primary emphasis is on globular clusters, underlining their significance in astrophysics, and, in addition, an overview of crucial topics and open questions regarding globular clusters is presented. A more detailed background of each of the topics introduced is provided in the following chapter.

2.1 Galaxy Classifications

Galaxies are the fundamental building blocks of the Universe ([Mo et al., 2010](#)), encompassing a diverse array of stars, gas, and dark matter, possessing numerous observable characteristics that can be measured using a range of techniques, from gamma-ray to radio frequencies ([Conselice, 2006](#)). Interestingly, every galaxy in the local universe with stellar masses of $\sim 10^8 M_{\odot}$ is known to contain globular clusters ([De Lucia et al., 2023](#)). Before going into details about globular clusters, it is vital to understand what kind of galaxy the MW is.

Edwin Hubble introduced a classification of galaxies based on their morphology in 1926, and he depicted a figure known as the Hubble Tuning Fork because it resembled a tuning fork ([Hubble, 1926](#)). In his scheme, ellipticals and spirals were discovered to be the two primary classifications. Subsequently, two main subclasses emerged within the

spirals: Normal and Barred spirals (Hubble, 1979). Hubble also observed a small number of irregular galaxies that did not fit into a classification system. With more data becoming available and better telescopes developed, de Vaucouleurs (1959) expanded the classification system and integrated features such as rings, bars, or the structure of the spiral arms for consideration. The Hubble sequence, as illustrated in Figure 2.1, divides regular galaxies into three broad classes: elliptical; lenticular and spiral. Galaxies (elliptical, spiral, or lenticular), based on different parameters, depend on the extensive data sets obtained from photometric and spectroscopic observations (Cappellari et al., 2011) or multi-wavelength data (Buta et al., 2015). A background description of the individual galaxy is presented below (Royal Astronomical Society Article 93).

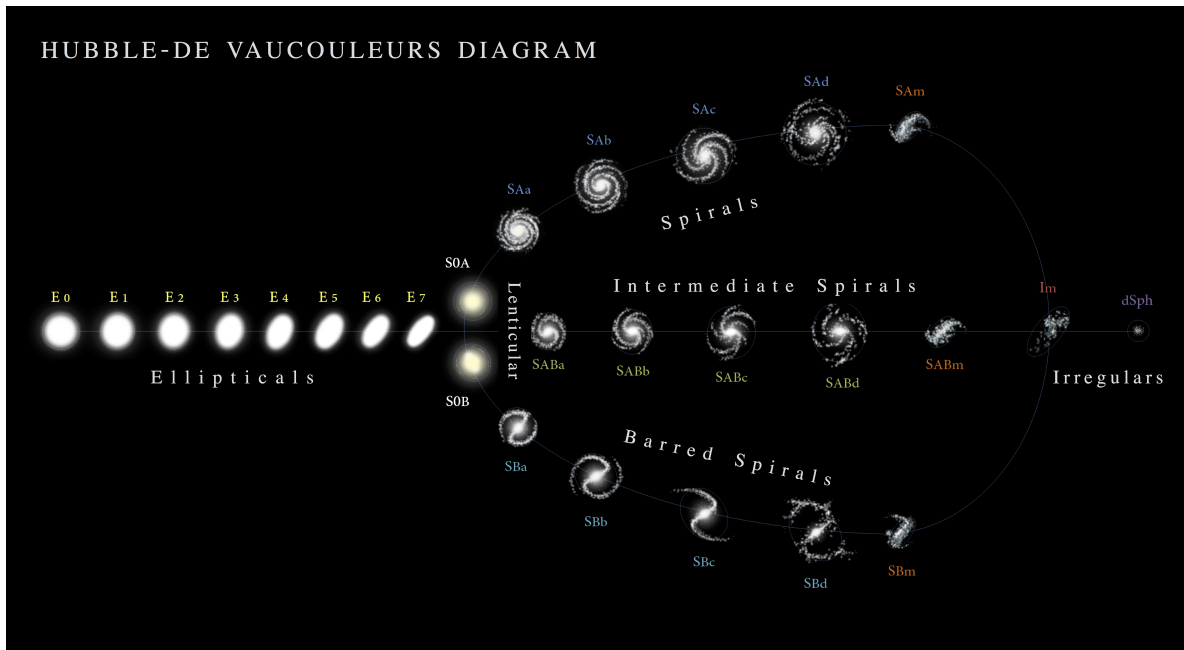


Figure 2.1: The Hubble-de Vaucouleurs Galaxy classification diagram with the addition of irregular and lenticular galaxies. Image Credit: Royal Astronomical Society Article 93: Astronomy - Redshift & the Non-expanding Universe - Cosmic Core Cosmic Times (https://www.cosmic-core.org/wp-content/uploads/2018/11/Hubble_-_de_Vaucouleurs_Galaxy_Morphology_Diagram.png).

Elliptical (E) galaxies

Elliptical galaxies lack spiral arms or galactic discs; aside from their dense central nucleus, they frequently show little to no internal structure. Comprising early-type galaxies (ETG) alongside lenticular galaxies, they contain old stellar populations believed to have originated in the early universe ([Kuntschner and Davies, 1998](#)). In addition, the majority of elliptical galaxies have little to no cool gas and dust and have a spheroidal distribution, which can be explained by a major merger with other galaxies ([Barnes and Hernquist, 1992](#)), or the collapse of non-rotating gas ([van Albada, 1982](#); [Burkert, 1994](#); [Ferreras, 2019](#)). The presence of a central black hole in an elliptical galaxy could explain why young stars do not form as frequently in these galaxies ([Ferreras, 2019](#)). The black hole is the result of the insufficient supply of dust and gas for new star formation since the black hole at the centre is believed to absorb the necessary raw materials. The classification is based on how they appear in the sky, with E0 denoting the most circular systems, E1 for flattened systems, and E7 for the most elongated ellipticals. Elliptical galaxies constitute approximately 12% of the total population of galaxies within the known universe ([Wilman and Erwin, 2012](#)).

Lenticular (S0) galaxies

Lenticular galaxies, regarded by Hubble as a class between spirals and ellipticals, possess characteristics of both spirals and ellipticals ([Seigar, 2017](#)). These disc-like galaxies lack prominent spiral arms in addition to the typical attributes of star formation, dust, or gas associated with spirals and other star-forming galaxies ([Conselice, 2006](#)). Lenticular galaxies contain a significant amount of dust ([Ferrari et al., 1999](#)), and approximately 20% of the galaxies in the known universe consist of lenticular galaxies ([Wilman and Erwin, 2012](#)).

Spiral (S) galaxies

The spiral galaxy, called a disc or late-type galaxy, is described as a flattened galactic disc with the presence of spiral arms, a central galactic bulge with a dense nucleus, and a large halo of faint old stars. Additionally, the spiral galaxy displays blue-white discs of stars, gas, and dust with yellowish bulges in their centres ([Seigar, 2017](#); [Carroll and](#)

[Ostlie, 2017](#)). A structure like this can form when the gas collapses under gravitational force. Compared to the elliptical galaxy, the disc galaxy has a more complex distribution of stellar populations, including ongoing star formation and multitudinous old stars. These galaxies are characterised as either typical or barred spiral galaxies ([Ferreras, 2019](#)). In barred spirals, the stellar bar traverses the centre bulge, and the spiral arms typically originate from the end of the bar rather than the bulge. Spiral galaxies, which actively produce stars, constitute a substantial portion, making up 60% of all galaxies in the nearby universes ([Loveday, 1996](#)).

Irregular (Irr) galaxies

Irregular galaxies lack symmetry, are neither disc-like nor elliptical because of the absent features such as spiral arms and central bulges ([Carroll and Ostlie, 2017](#)), and have relatively little dust. Star formation dominates the appearance and evolution of these galaxies because of the high amount of gas they contain. Irregular galaxies frequently have an ongoing merger or tidal interaction with a neighbouring galaxy, are generally abundant in interstellar matter, and are populated by bright, young blue stars ([Huchra, 1977](#)). Typically, irregular galaxies are smaller than standard galaxies with masses between $10^8 - 10^{10} M_{\odot}$.

Classification of the MW

The MW is a barred spiral galaxy, SBbc type ([197, 1970; Lin et al., 2008; Efremov, 2011](#)) and is estimated to contain approximately 10^{11} stars with a stellar mass of $\sim 5 \times 10^{10} M_{\odot}$. The MW galaxy spans several tens of kiloparsec (kpc) in diameter, and the Sun is far from its centre ([Carroll and Ostlie, 2017](#)). The galaxy comprises three elementary components: stellar discs, gas, and a central spherical bulge, all surrounded by a spherical halo of old stars, globular clusters, and a few satellite galaxies. Remarkably, the MW is estimated to have approximately 157 globular clusters ([Harris, 1996; Buser, 2000; Beasley et al., 2002; Portegies Zwart et al., 2010; Vasiliev, 2019; Ferreras, 2019](#)). Globular clusters are present in the thick disc, with many hidden behind the bulge and halo but non-existent in the thin disc.

2.2 Star Clusters

Over the past years, there have been varying definitions of what defines a star cluster. From the definition given by [Portegies Zwart et al. \(2010\)](#), a cluster is a collection of stars mutually attracted to one another through gravitational force. The definition provided by [Lada and Lada \(2003\)](#) characterises a star cluster as a collection of stars with a sufficiently high mass density ($\gtrsim 1 \text{ M}_\odot \text{ pc}^{-3}$) to withstand tidal disruption under Solar neighbourhood conditions, and comprise a significant number of stars to evade N -body evaporation for a minimum duration of 10 Myr. However, star clusters can be gravitationally bound or unbound, and young clusters can be a mixture.

Star clusters form from the same interstellar gas cloud under uniform environmental conditions, which collapse and form stars, and they are practically found in all star-forming regions ([Krause et al., 2020](#)). All cluster members are, therefore, comparable regarding age and chemical composition but different in mass ([Kroupa, 2005](#)). Star clusters provide a valuable testing ground for the stellar evolution theory through isochrone fitting and galactic structure ([Kalirai and Richer, 2010](#)). Using the definition of ‘bound’, these clusters are generally distinguished as open, globular, and young massive star clusters, as shown in Figure 2.2. Table 2.1 provides a summary of the key parameters of the three different populations of star clusters: globular, open and young massive clusters. These clusters exhibit variations in central density, mass, age, and metallicity, reflecting their diverse nature and evolutionary stages within the galactic environment.

Parameter	Globular clusters	Open clusters	Young massive clusters
Central density (ρ_0)	$8 \times 10^6 \text{ M}_\odot \text{ pc}^{-3}$	$10 \text{ M}_\odot \text{ pc}^{-3}$	$\gtrsim 10^3 \text{ M}_\odot \text{ pc}^{-3}$
Mass	$\gtrsim 10^4 \text{ M}_\odot$	$500 \text{ M}_\odot$	$10^4 - 10^7 \text{ M}_\odot$
Age	$\sim 10 - 13 \text{ Gyr}$	$\sim 300 \text{ Myr}$	$\gtrsim 100 \text{ Myr}$
Metallicity ($[\text{Fe}/\text{H}]$)	$-2.5 - 0.5$	$-0.7 - 0.2$	$\gtrsim 1$
Population	II (metal-poor)	I (metal-rich)	

Table 2.1: A comparison of various parameters of the galactic globular, open and young massive star cluster. The table was compiled using statistics obtained from different literature ([Harris \(1996\)](#); [Janes \(2001\)](#); [Portegies Zwart et al. \(2010\)](#); [Binney & Tremaine \(2011\)](#)).

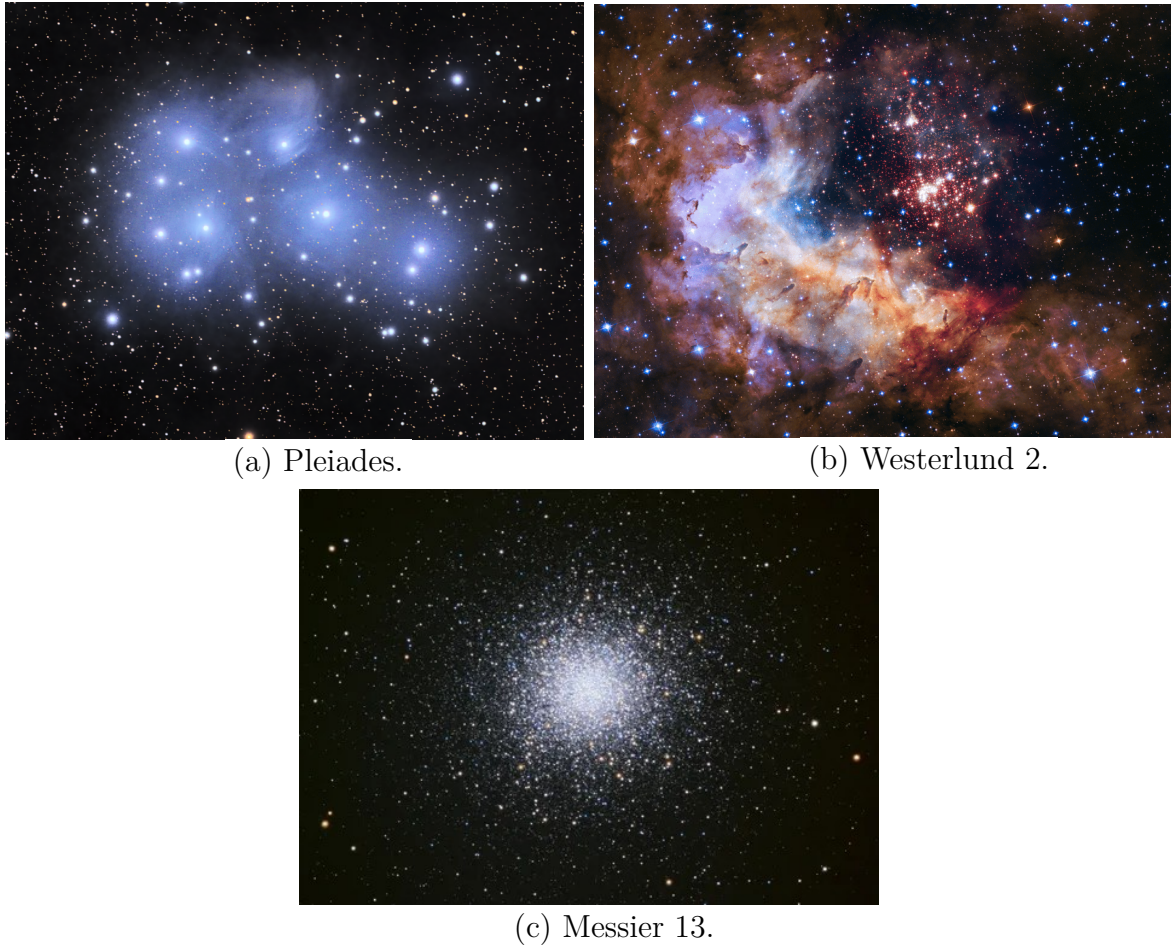


Figure 2.2: This figure shows the three different star clusters. (a) The top left image is an open cluster, the Pleiades cluster (M45), popularly known as the *Seven Sisters* and located 444 light-years from the Sun. Image credit: David Hoskin (<https://earthsky.org/astronomy-essentials/definition-examples-what-are-open-star-clusters/>). (b) The top right image shows a young massive cluster, Westerlund 2, located in the centre of the HII region RCW 49 observed by the Hubble Space Telescope (HST) in the optical and near-infrared. Image credit: NASA, ESA, Hubble Heritage Team (STScI/AURA), A. Nota (ESA/STScI) and the Westerlund 2 Science Team (https://www.researchgate.net/figure/The-Galactic-young-massive-star-cluster-Westerlund-2-in-the-center-of-the-HII-region_fig4_342021974). (c) The bottom image is the Messier 13 (NGC 6205) globular cluster, located at a distance 22.2 kly from the Sun. Image Credit: Sid Leach, Adam Block, and Mount Lemmon SkyCenter (<http://www.sidleach.com/m13.htm>).

Open clusters consist of stars from population I (metal-rich) with metallicity $[\text{Fe}/\text{H}] = -0.7 - 0.2$ (Janes, 2000), and it is possible to distinguish individual stars at the centre. They are relatively young objects with a mean age of ~ 300 Gyr and cluster masses of $\sim 500 M_{\odot}$ (Portegies Zwart et al., 2010). The clusters are small, ranging from $1 - 10$ pc and are only bound by gravity, eventually dispersing due to gravitational perturbations with other objects in the galaxy. Open clusters are located closer or in the galactic disc, thus making identification difficult.

Young massive clusters are dense aggregates of young stars of approximately $10^3 - 10^6$ and are characteristically portrayed by their high cluster mass of about $10^4 M_{\odot}$ (Portegies Zwart et al., 2010), but can be within the range of $10^4 - 10^8 M_{\odot}$ (Murray, 2009; Randriamanakoto et al., 2019). Young massive clusters have an age of $\gtrsim 100$ Myr, with total cluster mass and metallicity $[\text{Fe}/\text{H}] \gtrsim 1$ (Portegies Zwart et al., 2010). The age of young massive clusters is comparable to open clusters in the MW, but their central densities ($\gtrsim 10^3 M_{\odot} \text{pc}^{-3}$) and masses are several orders of magnitude higher.

Globular clusters (discussed in the next section) typically appear to contain a sprinkling of bright, resolved stars from population II, superposed on a continuous background of unresolved fainter stars (Janes, 2000). Globular clusters are intermediate between open clusters and massive objects, such as young massive clusters or blue compact galaxies (Gratton et al., 2019). They have masses of $\gtrsim 10^4 M_{\odot}$ and ages around $10 - 13$ Gyr (Kharchenko et al., 2013).

2.3 Globular Clusters

Globular clusters are a massive stellar population (Paust et al., 2010), as seen in Figure 2.3 for the NGC 6441 globular cluster, with $10^4 - 10^6$ stars distributed spherically around the galactic centre (Lightman and Shapiro, 1978), and have excessively high inner densities of approximately 10^6 stars (Renaud et al., 2017; Vitral et al., 2022).

The historical narrative of globular clusters is connected with understanding the MW and other galaxies. William Herschel devised the term “globular cluster” in his catalogue of nebulae and clusters of stars (Herschel, 1786). Initially, globular clusters were assumed to be bright nebulae that did not contain stars. As observations advanced,

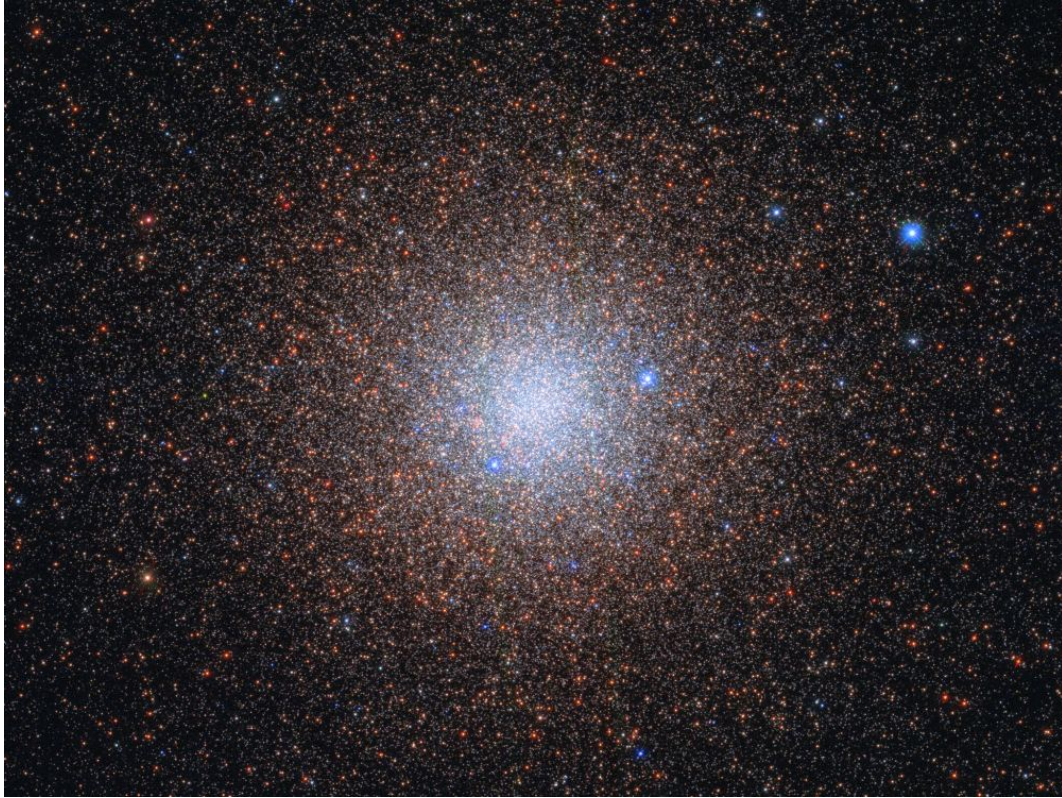


Figure 2.3: An image of the NGC 6441 globular cluster, recognised as one of the MW’s most luminous and massive globular clusters. Image credit: ESA/ Hubble/ NASA and G. Piotto (<https://www.sci.news/astrophysics/hubble-image-globular-cluster-ngc-6441-08484.html>).

however, they were recognised as massive spherical collections of stars (Vital, 2022), and most of the MW populations were identified by early 1900. Shapley (1918b) used globular clusters as indicators to map out the shape and size of the MW, paving the way for understanding galactic structures. Their studies in other galaxies progressed especially in the Andromeda Galaxy (M31), because of its size and scale compared to those of the MW (Barmby, 2001). Furthermore, globular clusters stand out as observable indicators of galaxy formation and evolution because of their spatial distributions, ages, metallicities, and kinematics (Meylan and Heggie, 1997).

2.3.1 Galactic Globular Clusters

Galactic globular clusters are observed in various regions of the MW, including the bulge, the thick disc, and the halo. Within the MW, the distribution of these clusters is

not uniform; the oldest globular clusters are spherically distributed around the galactic centre (Janes, 2000), as seen in Figure 2.4. In the figure, these clusters are represented as black circles overlaid on the COBE FIRAS (Cosmic Background Explorer Far-Infrared Absolute Spectrophotometer) micrometre map, capturing the far-infrared emission from various sources within the MW. Most of these clusters are found within a distance of 20 kpc from the galactic centre; only a few exist in more distant regions. The most remote cluster, Arp-Madore 1 (AM-1), is ~ 120 kpc away from the galactic centre (Aaronson et al., 1984).

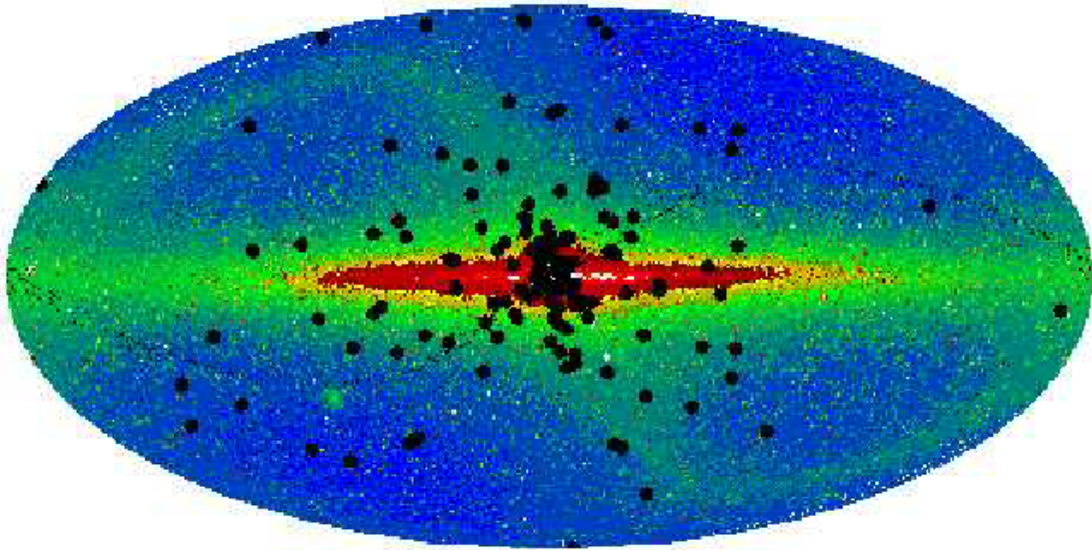


Figure 2.4: Illustration of the distribution of globular clusters in the MW, overlaid on the COBE FIRAS map serving as a backdrop for the globular clusters represented as black circles, which allows for a spatial correlation between the observed globular clusters and the broader far-infrared structure of the galaxy; the positions of the globular clusters are from the Harris (1996) catalogue. Image credit: https://www.researchgate.net/figure/Globular-cluster-distribution-about-the-galaxy-Positions-are-from-Harris-95-and-are_fig5_313506249.

Spatial distribution

The study of the spatial distribution of globular clusters began in neighbouring galaxies and giant elliptical galaxies from rich globular systems (Puzia et al., 2002; Puzia, 2003;

Arakelyan et al., 2018). Arakelyan et al. (2018) show that their spatial distribution within the MW is anisotropic for the distance range of $2 < R < 10$ kpc, indicating that they are unevenly distributed throughout the galaxy. The clusters disperse in a spherically symmetric manner about the galactic centre; their motion in space is not uniform in all directions but exhibits an ellipsoidal pattern, possibly influenced by the complex gravitational interactions and dynamics within the MW (Shapley, 1918b). Several factors are known to affect the spatial distribution of these systems. One factor is the initial conditions during the formation process, such as density, gas distribution, and dust (Keller et al., 2012; Sánchez-Salcedo and Lora, 2022). Another factor is the existence of dark matter haloes within dwarf spheroidal galaxies, which can cause globular clusters to migrate towards the galactic centre over time (Arakelyan, 2019). Additionally, the galactic environment contributes to understanding the distribution of globular clusters because it is closely related to the structure and evolution of the MW. The distributions can, therefore, offer insight into the history of accretion and the merger events of the MW (Hammer et al., 2023, 2024).

Metallicity

The study of these systems shows evidence of the two subpopulations, the metal-rich bulge and the metal-poor halo subpopulations. Initially, metal-rich globular clusters were assumed to be related to the disc (Zinn, 1985), particularly within the bulge region and exhibited similar chemical and kinematic properties (Minniti, 1995). On the other hand, metal-poor globular clusters are abundant in the thick disc and also in the halo and display very pronounced kinematic properties similar to those of the stellar halo population (Puzia, 2003), such as a hot system with some rotational or tangentially biased orbits (Mackey and Gilmore, 2004), where the orbits have non-zero eccentricities and their periapsis is not aligned with the apoapsis.

A study conducted on the halo of the giant elliptical galaxy NGC 5128 found that kinematics, such as rotation amplitude, rotation axis, and velocity dispersion of the metal-rich and metal-poor subpopulations of globular clusters are relatively similar (Woodley et al., 2007). The kinematic signatures, therefore, suggest that the subpopulation of globular clusters in the MW shares the kinematic properties of the significant Galactic subcomponents (Puzia, 2003).

Substantial evidence, however, supports that globular clusters consist of two subpopulations, a rapidly rotating disc and a slowly rotating halo subpopulations (Meylan and Heggie, 1997). Understanding these subpopulations in detail is vital because some internal properties, specifically the concentration parameter, exhibit an extensive correlation with global variables, such as the galactocentric distance (Meylan and Heggie, 1997; Turaev et al., 2023). The correlation implies that the halo subpopulation, found at greater galactocentric distances, exhibits a broader spatial distribution and lower concentration toward the galactic centre. In contrast, the disc subpopulation is characterised by higher concentration levels and a more centralised location within the galaxy. The observation implies that the internal dynamical evolution in the clusters are significantly influenced by certain external factors (Djorgovski, 1998).

Globular clusters have metallicities ranging from $[\text{Fe}/\text{H}] \sim -2.4$ to -0.5 dex (Paust et al., 2010). The metal-poor subpopulation, often referred to as the population II, has metallicities with the distribution peaks at $[\text{Fe}/\text{H}] \approx -1.5$ in the MW (Renaud et al., 2017). Metal-rich clusters in the galactic centre have metallicities around $[\text{Fe}/\text{H}] = -0.5$ and rotate with the galaxy (Renaud et al., 2017). The metallicity of the globular cluster presents a positive correlation with the total mass of a galaxy; the more massive the galaxy, the higher the metallicity of the average globular cluster. History has a share in shaping the globular cluster system since the galaxy's formation (Puzia, 2003).

2.3.2 Formation of Globular Clusters

There have been theories around the origin and formation of globular clusters that require a physical explanation within any cosmological framework. For decades, the formation of globular clusters has raised open questions about the formation of stars, the formation of galaxies, and their evolutionary mechanisms (Miocchi et al., 2013; Kruijssen, 2014).

Peebles and Dicke (1968) suggested that stellar systems may have originated as gravitationally bound gas clouds before galaxies formed, with their masses determined by the Jeans mass. Fall and Rees (1985) assumed globular clusters formed during the collapse of protogalaxies due to thermal instabilities in the hot gaseous haloes. Later, Larson (1996) believed that globular clusters formed within a massive star-forming sys-

tem, which underwent merging processes to form the present galaxies. However, none of these scenarios holds as the definitive explanation of the origin of most globular clusters.

Globular clusters are known to contain exotic stars such as pulsars, low-mass X-ray binaries, and blue stragglers that emerge due to the high density found in the central regions of the cluster and high rates of stellar interactions (Baumgardt, 2017). Globular clusters' positions offer valuable insight into both mass segregation and formation, such as the presence of pulsars that can serve as a means to investigate the cluster's gravitational potential (Phinney, 1992).

2.3.3 Evolution of Globular Clusters

Globular clusters are constantly disrupted by galactic tides caused by passages through both the bulge and disc, regardless of their formidable internal gravitational bounds (Dehnen et al., 2004; Piatti and Carballo-Bello, 2020). The clusters experience several evolutionary processes (tidal shocks, stellar evolution and evaporation, and dynamical friction), which can often be intricately interconnected. For clusters closer to the galactic centre, the influence of tidal shocks accelerates their internal evolution, potentially pushing them towards core collapse or even tidal disruption, resulting in a reduction in population size (Meylan and Heggie, 1997).

From a dynamic standpoint, the most significant effect of early evolution is the loss of mass, which can eventually lead to the disintegration of the cluster. Investigations carried out by Angeletti and Giannone (1977) and Applegate (1986) show the effects of mass loss towards the end of the main sequence that can result in a delay in the initial relaxation process or the cluster becomes exposed to disruption due to tidal shock. There is a belief that the stellar halo of the MW is primarily composed of remnants from disrupted star clusters and several dwarf satellite galaxies.

2.3.4 Cluster Masses

A typical galactic globular cluster has mass estimates of up to $10^6 M_{\odot}$ (Meylan and Heggie, 1997). However, not every globular cluster has a mass of about $10^6 M_{\odot}$, as some

range from $10^3 - 10^6 M_\odot$ (Carney and Harris, 2006; Gratton et al., 2019), considering that globular clusters within the MW differ regarding absolute magnitude, age, and total mass. The definition of a globular cluster, thus, cannot be confined to one parameter in any galaxy.

Globular clusters lose mass over time because of dynamical evolution. Therefore, the initial mass of the clusters is an important parameter that determines the chemical evolution within a cluster. In the case of typical halo globular clusters, characterised by a total integrated magnitude of $M_V(\text{tot}) = -12.9$ and a mass-to-light ratio of approximately $\Upsilon \approx 1.45 M_\odot L_\odot^{-1}$ (McLaughlin, 2000), the corresponding total mass is around $1.2 \times 10^7 M_\odot$ (Forbes and Bridges, 2010). According to estimates from Fall and Zhang (2001) and Bland-Hawthorn and Gerhard (2016), the total mass of the stellar halo within a radial range of $1 < R < 40 \text{ kpc}$ is approximately $4 - 8 \times 10^8 M_\odot$, and globular clusters constitute approximately 2 – 3% of the overall stellar mass (Forbes et al., 2018). The initial masses of the globular clusters published so far do not account for many processes, such as remnant retention and gas expulsion (Giersz et al., 2019), that occur during the early stages of evolution, which could contribute to the amount of mass loss. Consequently, the current mass may vary due to dynamical evolution, making it a potential indicator of age (van den Bergh and Lafontaine, 1984).

The initial mass function (IMF) of the globular clusters is defined by $\Phi(M) = dN/dM \propto M^{-2} \exp(-M/M_c)$ (Jordán et al., 2007). However, this does not agree with the mass function of old globular clusters

$$\frac{dN}{dM} \propto (M + \Delta M)^{-2} \exp\left(-\frac{M + \Delta M}{M_c}\right), \quad (2.1)$$

where ΔM is the average total mass lost in a cluster, and M_c is the high-mass cut-off (Jordán et al., 2007; Forbes et al., 2018). The integration of Equation 2.1 overall masses, with ΔM set to 0 for the initial mass, allows for determining the total current mass (M_{cur}) and the initial mass (M_{init}) of a globular cluster system.

2.3.5 Age and Distance of Globular Clusters

Age

Globular clusters are part of the oldest structures to have formed in the universe, and their age places constraints on the universe's age (Chaboyer et al., 1996; Chaboyer, 1996). However, their absolute age is still a cosmological problem, and Walker (1992) and Chaboyer et al. (1996) conclude that the oldest globular clusters have an age range of $11 - 12$ Gyrs. Old stellar populations are generally found in these clusters with an age $\tau \gtrsim 10$ Gyr (Kruijssen, 2014).

Although age is one parameter that defines a globular cluster, this no longer holds since rich stellar systems of $\tau < 10$ Gyr have been observed. For example, star clusters with ages $10^6 \lesssim \tau \lesssim 10^9$ have been observed in the Magellanic Clouds (Meylan and Heggie, 1997). Various methods have been employed to estimate globular cluster ages, including the analysis of the main-sequence turnoff luminosity (see Section 2.3.6), white dwarf cooling curves, and radioactive age dating of stars (Watkins et al., 2015b).

Distance

Globular clusters are observable at large distances from the galactic centre because of their high surface brightness. The nearest known globular cluster, NGC 6121, is located at a distance of 2 kpc from the Sun (Heggie and Giersz, 2008), and only a few are located within this proximity, such as NGC 6397 (Anthony-Twarog et al., 1992). The distance to the globular clusters plays a significant role for the clusters and the short end of the distance ladders (Shapley, 1918a; Watkins et al., 2015b; Baumgardt and Vasiliev, 2021). The distance serves as an additional dimension in the positional phase space of the globular clusters (Watkins et al., 2015b).

Distances are used to determine whether there is an association between clusters and substructures in the Galactic halo since they coincide (Mackey and Gilmore, 2004), and because tidal tails in some clusters can help constrain the cluster orbits (Küpper et al., 2015) and the shape of the inner Galactic halo (Watkins et al., 2015b). The globular cluster's distances were used by Shapley (1918b) to estimate the distance between the galactic centre and the Sun (~ 15 kpc); Harris (1976) also estimated the Sun's distance

to be approximately $R_0 = 8.5 \pm 1$ kpc.

Krauss and Chaboyer (2003) reviewed several methods to estimate globular cluster distances using kinematic and photometric data. Photometric distances are approximated from colour-magnitude diagrams by isochrone fitting (Baumgardt and Vasiliev, 2021), eclipsing binaries, and the distance modulus (Pogson law) using objects classified as “standard candles” such as RR Lyraes and Cepheids (Catelan et al., 2004). Kinematic distances are predicted by comparing line-of-sight velocity dispersions with velocity dispersions derived from proper motion measurements. It was challenging to assess the agreement between photometric and kinematic data previously because only a few clusters had measured internal proper motions for kinematical distances. Despite the different data sets and methods employed in kinematic and photometric distance determination, the estimates should ideally be similar for any cluster.

2.3.6 Colour-Magnitude Diagrams

Colour-magnitude diagrams (CMD) of globular clusters have been studied several times and represent luminous objects in the universe and compare them with one another. For example, the James Webb Space Telescope (JWST) was used to construct the first CMD for M92 and identified white dwarf candidates (Gratton et al., 2019; Nardiello et al., 2022). CMD were used to study stellar populations and star formation (JANG and SONG, 2015), providing information on various parameters such as age, metallicity, and distance modulus (Preet Kaur and Joshi, 2022). Additionally, CMD are used to determine the membership of stars in a cluster by comparing their positions in the observed CMD with the theoretical isochrones represent the expected positions of stars with different ages and metallicities (Gao, 2014; Sarro et al., 2014). Figure 2.5 shows the CMD of the globular cluster NGC 6809 (M55), with the components of stellar populations labelled throughout. The labels include the Main Sequence, the Horizontal Branch, the Red Giant Branch, the Asymptotic Giant Branch, and the Blue Stragglers. Globular clusters have a well-defined Main Sequence (MS) wherein most stars fall; it extends from the turn-off to fainter magnitudes and redder colours. MS contain only low-mass stars, with hydrogen burning in its core, resulting in low temperatures proportional to other stars. All high-mass stars leave the MS once they have spent their core hydrogen and evolved into red giants in the Red Giant Branch (RGB). The RGB

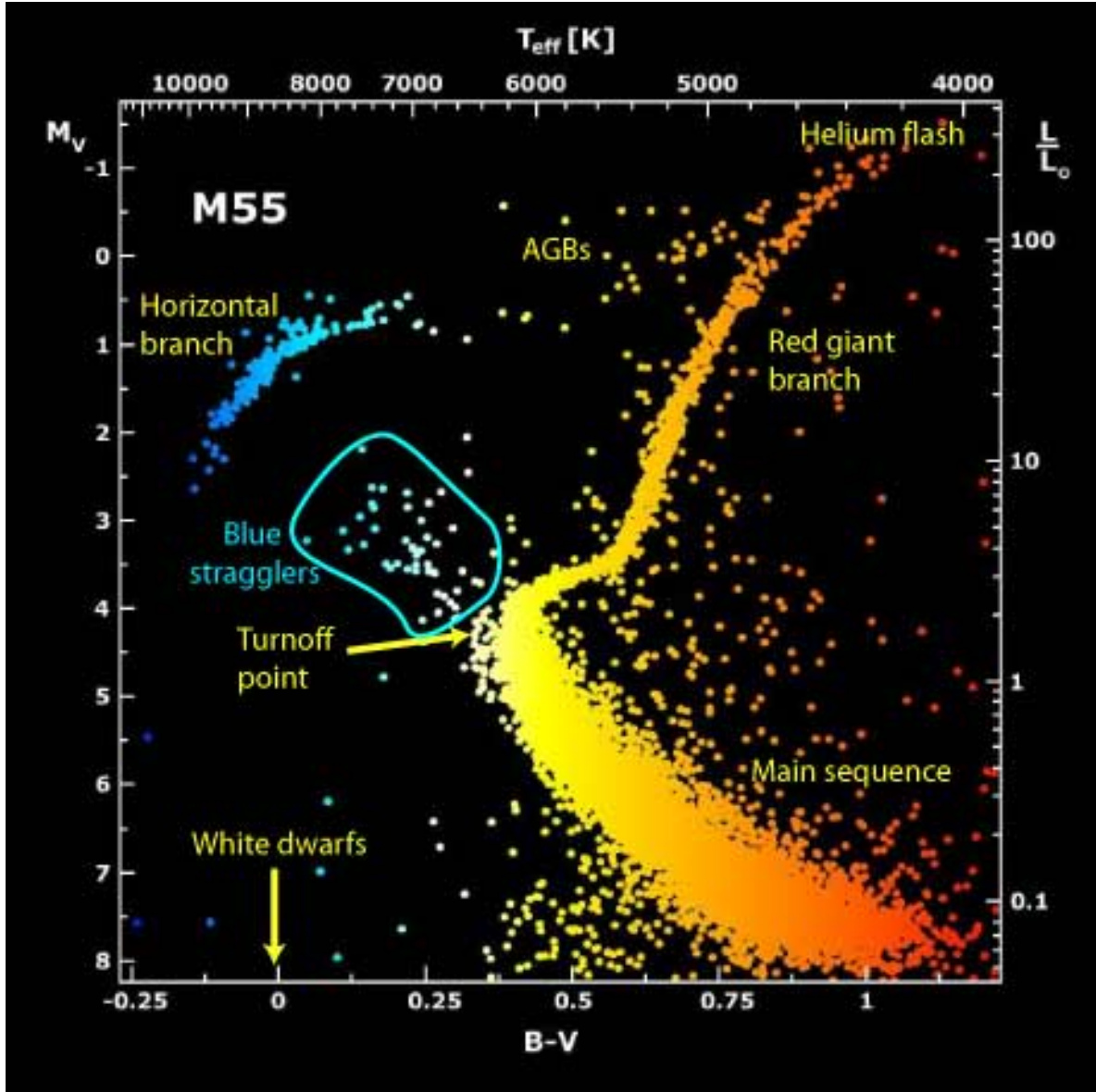


Figure 2.5: Illustration of colour-magnitude diagram of M55 globular cluster, indicating the position of the Main Sequence, the Blue Stragglers, the Red Giant Branch, the Horizontal Branch and the Asymptotic Giant Branch. Image Credit: <https://www.mso.anu.edu.au/~jerjen/researchprojects/bluestraggler/bluestraggler.html>.

is influenced by the extinction and age-metallicity of stars from the bulge (Ng, 1996). The precise positioning of the MS turn-off point is contingent on the metallicity and age of the cluster. The stars, located at this point, are now depleting the hydrogen in the core, therefore, their MS lifetimes equal the age of the cluster.

The Horizontal Branch (HB) exhibits additional indications of stellar interactions that can impact the evolutionary trajectory of a star. The HB stems from stars that have

undergone recent helium (He) flash and now exhibit core helium and shell hydrogen burning, transiting towards the red-giant region (Gratton et al., 2019). The HB also play a crucial role in studying RR-Lyrae variable stars, which can serve as a probe for distance measurements and globular cluster formation history. The colour of HB depends on the metallicity other than hydrogen or helium (Djorgovski, 1998). Metal-rich clusters have red stars, whereas metal-poor clusters contain blue stars. Stars on the HB and RGB are used to identify the cluster members photometrically.

After He has depleted the core, He begins to burn in the outer shells, increasing the luminosity and opacity of the star, and results in the star moving towards the giant region, asymptotically to the left side of the RGB. The newly formed branch is known as the Asymptotic Giant Branch (AGB) because the stars in this location experience substantial mass loss (Cohen, 1976).

Blue Stragglers (BS) are stars above a certain mass and luminosity that have evolved off the MS. Yet, BS that fall above the cut-off remain in the MS. BS are frequently observed in most globular clusters and appears concentrated towards the central region of the clusters compared to the other visible stars. The observation implies that the BS masses are higher than the MS or red-giant stars, because more massive stars tend to gravitate towards the cluster centre (Djorgovski, 1998). Some BS are in nearby, eclipse-producing binaries. They are expected to dissipate enough of their orbital energy to merge.

2.3.7 Proper Motions and Radial Velocities

Proper motions are measured by their apparent motion in the sky. The measurements offer more information about the dynamics and orbits of globular clusters than radial velocities because of their two-dimensional nature, as opposed to the one-dimensional nature of the radial velocities (Binney and Mamon, 1982; Bellini et al., 2014). Radial velocities offer information about the speed of a star along the line of sight from Earth. Proper motions and positions on the sky provide a more comprehensive understanding of the cluster's motion in space. Proper motions, in particular, offer insight into the transverse motion of stars within the cluster, allowing the cluster's orbit to be reconstructed and understand its evolution within the gravitational potential of the galaxy

(Bellini et al., 2014). Moreover, combining proper motions with radial velocities enables the determination of full three-dimensional velocities, enhances the knowledge of the complex dynamics within globular clusters (Bianchini et al., 2018).

Previously, the internal dynamics of globular clusters were not extensively studied using their proper motions because it is difficult to accurately measure proper motion with required precision using ground-based telescopes (Bellini et al., 2014), attributed to the small size of space motion of the stars (Meylan, 1997; Bellini et al., 2009), particularly for faint stars in crowded fields.

With the development of telescopes with an accuracy of $1 - 2$ milliarcseconds/year (mas/yr) to 0.4 mas/yr, it is possible to use proper motion to study the dynamics and orbits of clusters within galactic potential models (Chemel et al., 2018). According to Zinn (1985), the metal-poor globular clusters do not show rotation, whereas the metal-rich globular clusters were supported by the rotation of the stellar disc.

Radial velocities, on the other hand, are vital in the analysis of properties of globular clusters. These velocities provide information about the motion of objects along the line of sight, indicating whether they are moving toward or away from the observer. Approximately 50% of the globular clusters lack spectroscopic measures of their metallicity, which is an essential parameter for understanding their composition and evolutionary history.

Vásquez et al. (2018) provided new radial velocities for $\sim 69\%$ MW globular clusters corresponding to those in the Harris catalogue, the data of which was collected using spectra obtained at the European Southern Observatory's Very Large Telescope (ESO's VLT). Many galactic globular clusters exhibit relatively high velocities. Their large radial velocities of $\sim 200 - 400$ km/s suggest that most of these clusters are engaged in highly eccentric elliptical orbits around the galactic centre, regardless of the inclination concerning the rotation of the galactic disc, compared to the typical velocities of stars with the galactic disc, which generally range from $\sim 20 - 250$ km/s (Gilmore et al., 2022). For instance, an examination of ω Centauri is based on individual star measurements of radial velocity, a decrease in velocity dispersion with radius, and rotation (Reijns et al., 2006).

2.3.8 Dynamical Models

Globular clusters become interesting from a dynamic view because of the large number of stars they contain. Over time, there have been developments in dynamical models of globular clusters to construct a range of simple mathematical representations of the cluster and to compare to observational data. Typically, these dynamical models can be divided into two categories, the static and evolutionary models (Heggie and Giersz, 2008), with the earliest successful dynamical models dating to the 1960s developed by Ivan King, Richard Michie, and Michel Henon.

Static Models

Much focus has been on static (equilibrium) models derived from the lowered Maxwellians. These dynamical models are elementary and cover the two-body relaxation, dynamical equilibrium, and tidal truncation that govern the structure of the globular cluster (Meylan and Heggie, 1997; Heggie and Giersz, 2008), such as the King model and its variants (discussed in detail in Section 3.3).

Static models are used to determine cluster properties such as mass segregation, mass-to-light ratio, and combined velocity dispersions from radial velocities and proper motion. The observational data is used to derive surface brightness (SB) profiles by employing density profiles from star counts and SB measurements, as well as velocity dispersion profiles. Various studies apply these methods, from Grillmair et al. (1995) studying the outer structures of 12 globular clusters through star count analysis to Illingworth (1976) determining masses of the clusters using velocity dispersions and mass-to-light ratios.

Despite the significance of accurately and precisely determining the properties of galactic globular clusters, the majority of their structural parameters are still obtained from SB profiles by using charge coupled device (CCD) images. Only a few of these parameters are derived by analysing stars on photographic plates (Trager et al., 1995; Miocchi et al., 2013). The presence of so many bright stars leads to SB profiles being biased. Instead, each star in the cluster has the same mass when constructing the number density profile; this eliminates any potential bias arising from sparsely distributed bright stars. However, despite the advantage of the star count profile, there is no known catalogue

sampling of the entire radial extension because it is not easy to thoroughly sample stars in the densely populated inner regions and the outermost regions of these clusters.

Evolutionary Models

Several evolutionary models were applied to a small number of globular clusters. These include steady-state models, the Monte Carlo model, the Fokker-Planck model, and the N-body numerical simulation model. Evolutionary models are crucial in the search for massive black holes within globular clusters ([Heggie and Giersz, 2008](#); [Baumgardt, 2017](#)). Numerous clusters have been subjected to N-body numerical simulations. For example, [Terlevich \(1987\)](#) indicated that the galactic gravitational field is responsible for the flattening of the globular clusters, and this is expected to be more significant in low-mass stars that are widely distributed, as opposed to high-mass stars that are more concentrated.

Chapter 3

Methods

In this chapter, the methods used to determine the masses are reviewed, the model is fit to the density profile, and the structural parameters of the globular clusters are estimated.

The mass of a star cluster is one of the crucial parameters describing such objects, and being able to estimate the masses of self-gravitating systems has been a significant topic in astronomy, considering its complexity from the observational and theoretical points of view, respectively. The mass of the cluster can be estimated using various techniques, but each has advantages and disadvantages compared to the other. In most cases, estimates of this nature rely on the spatial distribution and kinematics of a specific tracer population, such as stars, along with their corresponding radial velocities.

Using the mass estimator, namely the Leonard-Merritt method, allows for inference of the mass of a cluster based on the estimation of the stellar velocity dispersion and the average distances. Velocity dispersions derived from proper motions are advantageous because the centre of the MW has previously shown that proper motions are not subject to interference from unobserved binary systems ([Leonard and Merritt, 1989](#)) and provide a much more constrained mass estimate. In 1970, [Jones \(1970\)](#) incorporated proper motion data into the analysis of cluster dynamics; at the time, however, the analysis was limited by the data's quality. [Leonard and Merritt \(1989\)](#) showed that the proper motion data were sufficient to predict the dynamic mass of the cluster ([McNamara and Sekiguchi, 1986](#); [D'Souza and Rix, 2013](#)). The description of the Leonard-Merritt mass estimator is provided in the section below.

3.1 Leonard-Merritt Method

[Leonard and Merritt \(1989\)](#) makes use of components of proper motion and the angular (apparent) position of a star to determine the mass of a spherical system. The two-component velocity of proper motion derived in [section 3.2](#) makes it possible to directly determine the shape and amplitude of the distribution of velocity of stars at all radii, taking into account any anisotropy in the system. Understanding how to effectively use adequate kinematical data to constrain the gravitational potential $\Phi(r)$ also makes it possible to constrain the mass. This approach avoids the strict ad hoc assumptions regarding the shape of the mass distribution or the kinematics.

The Leonard-Merritt mass estimator directly exploits observable, projected properties of a self-gravitating system. It is both efficient and unbiased in acquiring mass estimates with finite variance. In practical terms, the estimators are derived by capturing moments of the Jeans equation for a spherical system ([D’Souza and Rix, 2013](#)). For a system with a constant mass-to-light ratio, the Leonard-Merritt estimator becomes the following:

$$M_T = \frac{32}{3\pi G} \langle R (2v_R^2 + v_T^2) \rangle, \quad (3.1)$$

where M_T is the total mass of the cluster, R is the projected distance and v_T and v_R are the tangential and radial proper motion velocities ([Leonard and Merritt, 1989](#)). Equation 3.1, like any other moment estimator, only provides limited information regarding the mass distribution.

The methodology described by [Jones \(1970\)](#), using proper motion data, is employed to determine the velocity dispersion; this is discussed in the subsequent section [3.2](#).

3.2 Velocity Dispersion

[McNamara and Sekiguchi \(1986\)](#) argue that the components of proper motion and their error estimates can be used to evaluate intrinsic dispersion because the observed dispersion in proper motion components is influenced by both intrinsic dispersion and dispersion resulting from errors such as instrumental limitations and uncertainties in the calibration of the observational data. The method described by [Jones \(1970\)](#) is used to estimate the intrinsic dispersion in the proper motion components. Assuming a

homogeneous set of motions, the observed proper motion dispersion in one coordinate is:

$$\sigma_0^2 = \frac{1}{(n-1)} \sum_{i=1}^n \mu_i^2, \quad (3.2)$$

where σ_0 is the observed dispersion, n is the sample size, and μ_i is the proper motion of star i in relation to the average motion of the cluster. For this analysis, the mean proper motion of the cluster, $\bar{\mu} = 0$. The errors must be corrected to obtain the intrinsic/true dispersion, as both the true dispersion and measurement errors contribute to σ_0 . Given the assumption that proper motion and error distributions follow a Gaussian distribution, the true dispersion σ_l is expressed as

$$\sigma_l^2 = \sigma_0^2 - \frac{1}{n} \sum_{i=1}^n \xi_i^2. \quad (3.3)$$

ξ_i presents the average error of the proper motion of the i th star. The total error in σ_l is:

$$\varepsilon(\sigma_l) = \left\{ \frac{1}{4\sigma_l^2} [\varepsilon^2(\sigma_0^2) + \varepsilon^2(\sigma_m^2)] \right\}, \quad (3.4)$$

with

$$\varepsilon^2(\sigma_0^2) = \sigma_0^2 \left(\frac{2}{n} \right)^{1/2}, \quad (3.5)$$

and

$$\varepsilon^2(\sigma_m^2) = \frac{\sqrt{2}}{n} \left(\sum_{i=1}^n \xi_i^4 / n_i \right)^{1/2}, \quad (3.6)$$

where n_i is the number of photographic plates wherein data related to star i have been recorded. The summation in the equation incorporates all stars from $i = 1$ to n , and the contribution of each star to the overall equation, takes both ξ_i and n_i , on which the star's data are present, into account.

The two components of velocity dispersion make it possible to derive a more precise mass estimator and only provide limited information about the form of the mass distribution, which is, furthermore, also only beneficial if the kinematical data is derived from a stellar sample that extends almost to the tidal radius of the cluster ([Leonard and Merritt, 1989](#)).

3.3 King Model

[King \(1962\)](#) investigated stellar clusters and dwarf elliptical galaxies, effecting the development of an empirical density law that successfully represents these systems. The law comprises a minimum number of parameters ([King, 1966](#)), resulting in similar systems within a scaling factor, suggesting that the systems underwent a relaxation process throughout their formation. Two mechanisms have the potential to explain the relaxation process. First, all stars are considered to have an equal mass, producing a uniform mass distribution, and the second generates a difference in distribution between the low- and high-mass stars. Globular clusters experience mass segregation, but the effect is noticeable in clusters with a high concentration level. Clusters with low concentrations have a nearly indistinguishable distribution of bright and faint stars.

Most observable stars in a globular cluster have a wide range of magnitudes and have undergone an evolutionary process in the MS around the turnoff, resulting in stars having the same mass. King's models assume that all stars have the same mass and represent stellar systems in quasi-equilibrium, a state in which the cluster's dynamical processes evolve towards a stable equilibrium configuration, even though the system has not yet fully reached that equilibrium, and are generally accepted as zero-order dynamical models. The King model and its variants are used in dynamical studies to identify physical mechanisms and give initial conditions in numerical simulations for the dynamical evolution of a cluster.

This study considers [King \(1962\)](#) empirical density law representing the stellar distribution in globular clusters of all degrees of central concentration. The previously mentioned means that the empirical density law applies to globular clusters regardless of how tightly or loosely packed their stars are towards the centre, describing the distribution of stars in globular clusters with varying levels of central concentration. The model, however, does not provide a practical solution to the collisionless Boltzmann equation because it does not consider the dynamic evolution and interactions between individual stars. The King model effectively portrays the density profile in a cluster ([Binney and Tremaine, 2011](#)).

The model mimics the observed surface density, $\Sigma(R)$, distributions of globular clusters in the MW (King, 1962; Pinfield et al., 1998), which accounts for both the inner (Equation 3.7a) and outer (Equation 3.7b) densities of the cluster and is defined by the two equations as:

$$\Sigma(R)_{\text{inner}} = \frac{\Sigma_0}{\left(1 + \left(\frac{r}{r_c}\right)^2\right)} \quad (3.7a)$$

$$\Sigma(R)_{\text{outer}} = \Sigma_1 \left(\frac{1}{r} - \frac{1}{r_t}\right)^2. \quad (3.7b)$$

The King empirical density distribution embodies the characteristics of both equations (3.7a and 3.7b) as

$$\Sigma(R) = k \left[\frac{1}{\sqrt{\left(1 + \left(\frac{r}{r_c}\right)^2\right)}} - \frac{1}{\sqrt{\left(1 + \left(\frac{r_t}{r_c}\right)^2\right)}} \right]^2, \quad (3.8)$$

where the core radius (r_c) is the distance from the centre where the density decreases to half its central value (if the tidal radius (r_t) is large), and serves as a measure of the extent of the region with a nearly constant density near the centre of the cluster (Peterson and King, 1975). The tidal radius (limiting radius) r_t is the distance at which the galaxy's gravitational potential tidally disperses the cluster, causing the density to drop to zero. The cluster cannot retain stars against the tidal forces exerted by the MW beyond this distance.

The parameter k , which determines the shape of the profile, is the normalisation constant that serves as a dimensionless depth of the gravitational potential and is related to the concentration of stars within the cluster and the central density (Pinfield et al., 1998; Piskunov et al., 2007).

The parameters, including k , are independent and fully describe the distribution of the projected density in a cluster. If $r_t \gg r_c$, Equation 3.8 falls into Equation 3.7a in the centre of the cluster, and if $r \rightarrow r_t$, Equation 3.8 converges into Equation 3.7b as $\sqrt{1 + \left(\frac{r}{r_c}\right)^2} \rightarrow \frac{r}{r_c}$ and $\sqrt{1 + \left(\frac{r_t}{r_c}\right)^2} \rightarrow \frac{r_t}{r_c}$, respectively. This behaviour reflects the adaptability of the King profile in capturing the density distribution in both the central and the outer regions of a cluster.

In a star count profile, the total number of stars in projection within a distance r from the centre is found by integrating $\Sigma(R)$ with respect to $2\pi dr$ between $r = 0$ and $r = r_t$, resulting in the following equation:

$$n(r) = \pi r_c^2 k \left[\ln(1 + (r/r_c)^2) - \frac{(3\sqrt{1 + (r/r_c)^2} - 1)(\sqrt{1 + (r/r_c)^2} - 1)}{1 + (r/r_c)^2} \right]. \quad (3.9)$$

Equation 3.8 represents a single-mass system. The determination of r_t depends on the cluster's mass, the mass of the MW within the orbit traced by the cluster around the galactic centre, and the radius of the orbit.

$$\frac{GM_g}{(R_g - r_t)^2} - \frac{GM_g}{R_g^2} = \frac{GM_C}{r_t^2}, \quad (3.10)$$

where G is the gravitational constant, M_c represents the mass of the cluster, and M_g signifies the mass of the galaxy within the cluster's orbital radius. When considering a scenario where $R_g \gg r_t$, the equation simplifies as described by [Von Hoerner \(1957\)](#). In this case, r_t is expressed by the following equation:

$$r_t = R_g \left(\frac{M_c}{2M_g} \right)^{1/3}. \quad (3.11)$$

The derivation of this formula involves substituting variables in terms of Oort's constants A and B. The constants are determined using proper motion and radial velocity data obtained from circular motion observation around the galactic centre. Specifically, for the angular velocity, $A - B = 26.4 \pm 1.9 \text{ km/s} \cdot \text{kpc}$ or $8.56 \times 10^{-16} / \text{s}$, r_t simplifies to

$$r_t = \left(\frac{GM_c}{2(A - B)^2} \right)^{1/3} = 1.46 M_c^{1/3}. \quad (3.12)$$

The profile of an individual group is characterised by a concentration $c \equiv \log(r_t/r_c)$. King model, with c between 0.75 and 1.75 fits globular clusters well ([Padmanabhan, 2001](#)). The half-light radii, r_h ([Šabalevičiūtė et al., 2007](#)) from the King profile parameters are computed as

$$r_h = 0.547 \cdot r_c \left(\frac{r_t}{r_c} \right)^{0.486}. \quad (3.13)$$

Chapter 4

Observatory, Data and Sample Selection.

In this chapter, the observatory from which the data was obtained, along with what the data constitutes and how sample selection for the globular cluster was accomplished.

4.1 *Gaia* Telescope

The *Gaia* Mission, launched by ESA in 2013 ([Gaia Collaboration et al., 2016](#)), is a space observatory designed to provide answers to a large number of open questions about the MW. It aims to shed light on the origin and evolution of the MW by analysing the kinematics and distribution of the luminous and dark mass.

The central goal of the *Gaia* mission is to produce an extensive and highly accurate three-dimensional map of the MW by performing a 1% survey of the galaxy's immense population of approximately 100 billion stars ([Gaia Collaboration et al., 2016](#)). *Gaia* is positioned in orbit around the Sun at a distance of around 1.5×10^6 km from Earth, specifically at the second Lagrange point L2, seen in Figure 4.1. This location offers a significant advantage because the L2 orbit provides continuous observations without interruptions caused by eclipses. Other missions, such as the Herschel and Planck space observatories, have also been deployed from the L2 point, and there are plans to position the Euclid observatory and the JWST there as well.

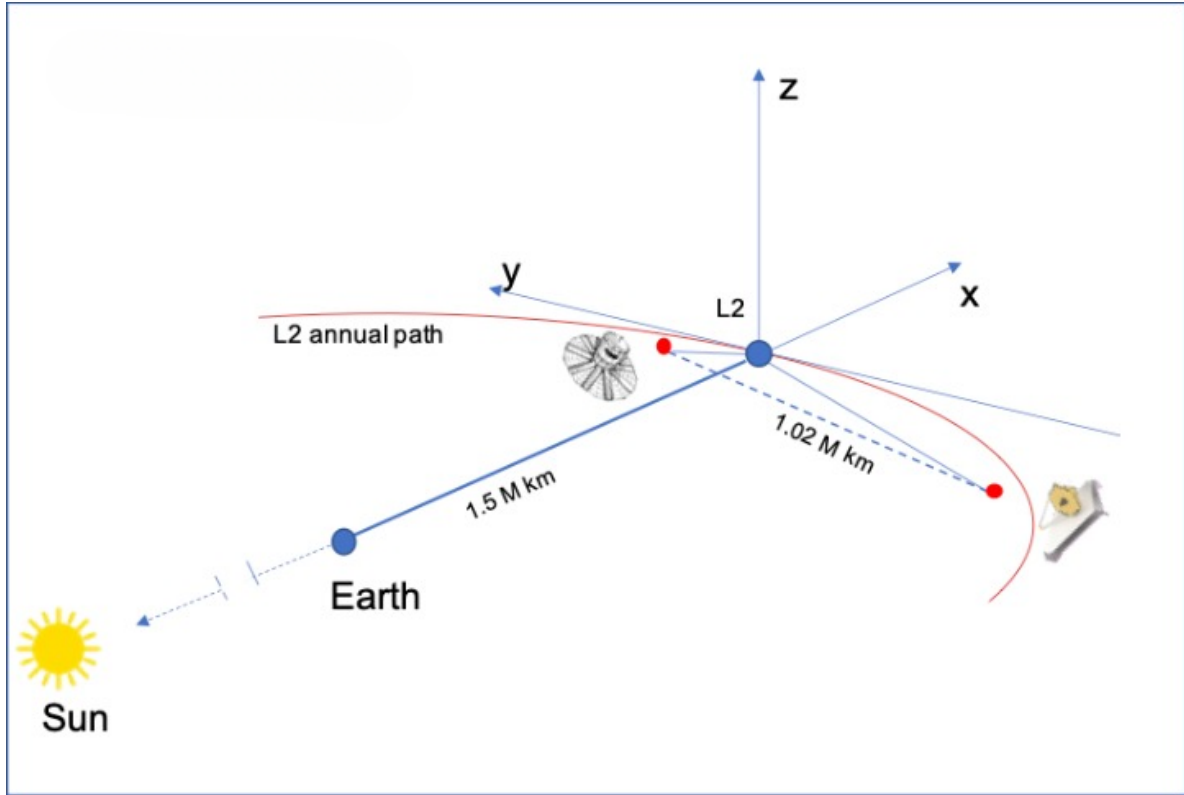


Figure 4.1: The *Gaia* astrometric satellite spacecraft and the L2 Lagrangian point. Image Credits: ESA (https://www.cosmos.esa.int/web/gaia/iow_20220316) .

Gaia contains two optical telescopes with a field of view separated by 106.5° that scan the entire sky simultaneously in an unbiased and accurate manner (Gaia Collaboration et al., 2018a). Figure 4.2 illustrates how astrometric observations are performed with *Gaia*. *Gaia* spins around the axis perpendicular to the spacecraft's line of sight every six hours. The projection in the sky of the focal plane through both fields of view is indicated with P and F for the preceding and following, respectively, separated by angle Γ in alignment with the spacecraft's spin direction (Brown, 2021). The system directs the light channelled from two apertures to a camera with 106 CCDs with approximately 1 billion individual pixels (Gaia Collaboration et al., 2016). The stars drift in a perpendicular direction known as across-scan (AC) and left to right across the field of view in an along-scan (AL) direction due to the spinning of the spacecraft. The array of CCDs is divided into separate regions, each of which independently measures its specific astrometric variables. On average, the astrometric data is gathered from two million stars per hour.

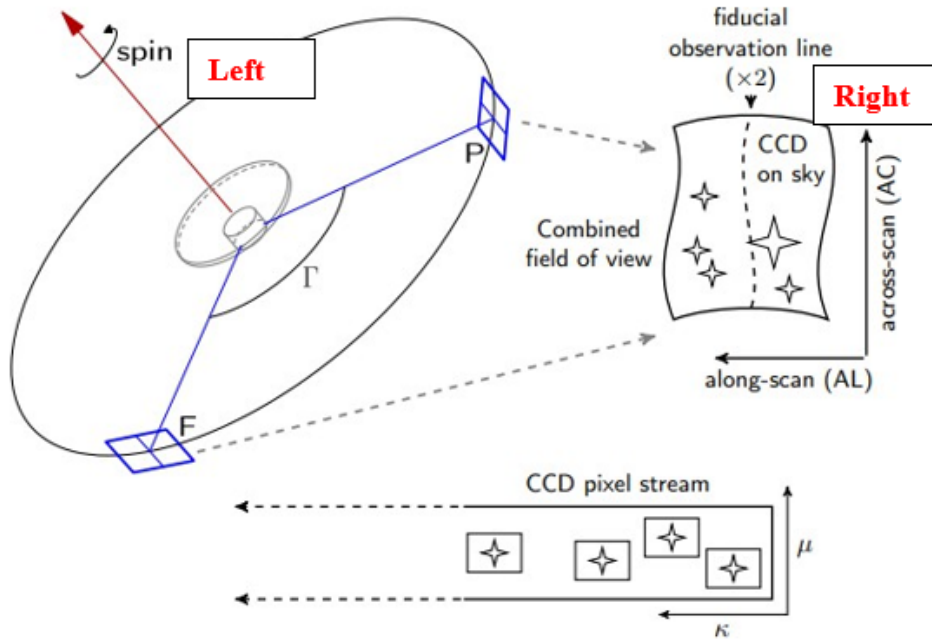


Figure 4.2: The *Gaia* telescopes, on the left, with schematically represented astrometric observations on the right. The spacecraft spin axis points away from the sun shield, with the line-of-sight for the two telescopes indicated. Image credit: [Brown \(2021\)](#).

The *Gaia* mission's data has been released in multiple stages, with data release 1 (DR1) in September 2016 ([Gaia Collaboration et al., 2016](#)), which contained information from two million stars with proper motion and parallax data from the Tycho-2 dataset, and elementary location and G-band magnitude information for almost a billion sources. The second data release (DR2) followed two years later, offering a remarkable level of precision in its astrometry that is superior, with higher-quality photometry ($G \sim 21$ mag) for 1.7 billion sources ([Gaia Collaboration et al., 2018a](#)). The releases facilitated the study of the MW dynamics, structure, and formation history and provided readily accessible data ([Gaia Collaboration et al., 2018b](#); [Vasiliev, 2019](#); [Helmi, 2020](#)). The third data release was halved and released in two stages: Early Data Release 3 (EDR3) and Full Data Release 3 (DR3) which was scheduled for 2022. *Gaia* EDR3 represents an improvement over the previous DR2 for the source list, astrometry ($\sim 80\%$ of sources), and broad-band photometry in the G, G_{BP} and G_{RP} bands. The subscripts BP and RP refer to different passbands within the photometric system employed by *Gaia*. The improvement encompasses a larger dataset for 1.8×10^9 sources without introducing

new data products (Vasiliev and Baumgardt, 2021). The statistical uncertainties are approximately two times better, providing improved precision in parallax and proper motion measurements.

Gaia conducts astrometry, photometry, and spectroscopy, offering positional, kinematic, and stellar properties (Brown, 2021). *Gaia* measures positions precisely, allowing the study of shapes and possibly atmospheres of the Kuiper Belt objects through stellar occultations (Brown, 2021). Moreover, *Gaia* provides parallaxes with unmatched accuracy. The improvements of parallaxes by *Gaia* enable the application of more observational constraints to various stellar models during varying stages of their evolutionary processes. The extensive parallax measurements for large numbers of stars reveal subtle yet unexplained features on the observational Hertzsprung-Russell (HR) diagram. Additionally, they allow for a profound exploration of the interiors of white dwarfs, offering an insight into the stellar remnants (Brown, 2021).

Photometry is available for all sources that observe astrometrically. The radial velocity catalogue is estimated to have a substantial number of about 150 million celestial objects regarding spectroscopy; this observing approach facilitates the exploration of numerous additional scientific domains beyond the MW, making it possible to study the structure, dynamics, and star formation history of the MW (Helmi, 2020).

The *Gaia* mission has extended its impact beyond its initial goals. Its high spatial resolution has opened up opportunities for studying multiple stellar systems and binaries, allowing larger sample sizes and enhanced precision in determining orbital parameters. *Gaia* was primarily designed to identify point sources, mainly stars, but it can also detect other point-like sources such as solar system bodies, and quasars due to its detection algorithm (Prusti, 2012).

Like any telescope, however, *Gaia* has certain limitations and is currently limited to measuring the instantaneous velocities of stars without the capability to determine the accelerations resulting from the galactic gravitational potential. These accelerations are far too small, meaning the gravitational field of the galaxy can only be deduced, not measured (McMillan, 2017).

4.2 Astrometric and Photometric Data

The dataset from EDR3 comprises five astrometric solutions, each depicted by a set of key parameters. These parameters consist of stellar positions in terms of Right Ascension (RA) and Declination (DEC) (α, δ), proper motions ($\mu_\alpha \cos \delta, \mu_\delta$), parallax angles (ϖ) and their error measurements. The dataset has a $G = 21$ mag limit (Lindgren et al., 2018; Gaia Collaboration et al., 2021).

For sources falling within the magnitude range of $G \leq 15$ mag and $G \sim 17$ mag, the errors in the parallax are up to $0.02 - 0.03$ milliarcseconds (mas) and 0.07 mas, respectively. Additionally, the uncertainties associated with the proper motion components are within the range of $0.01 - 0.02$ mas/yr for objects with a magnitude of $G \leq 15$ and roughly 0.05 mas/yr for objects with a magnitude of $G \sim 20$ mag.

4.3 Sample Selection

The sample was obtained by searching for a catalogue of globular clusters within the MW with good-quality data, precise positions, sufficient proper motion, and reliable parallax measurements from *Gaia* EDR3. The catalogue found is by Vasiliev and Baumgardt (2021), hereafter VB21, which comprised 120 potential globular cluster candidates obtained using the *Gaia* archive. VB21 went a step further and determined the memberships of stars within each clusters, using parallax and proper motion data. This use of proper motion data enables the determination of spatial motions, which, in turn, facilitates the identification of probable members within a cluster (VB21). A multi-Gaussian mixture modelling approach was employed to categorise stars into cluster members or background stars. The mean cluster parameters and their associated uncertainties were established through multiple iterations using statistically generated member catalogues.

The sample chosen is based on

- The globular cluster distance is $\ll 5$ kpc because sources beyond these distances have higher uncertainty compared to the local velocity dispersion, which could potentially lead to underestimation of errors (Vitrail, 2022).

- Each cluster has membership probabilities of stars $\geq 99\%$ (indicating high-confidence cluster members).
- The signal-to-noise (SNR) of parallax measurements ≥ 10 for error measurements.

4.3.1 Target Globular Clusters

From the sample, only ten globular clusters were considered based on the sample selection criteria. Figure 4.3 shows the ten globular clusters sourced from the globular cluster data. A detailed description of each cluster, in terms of distance, constellation, metallicity, age, radius, apparent magnitude, and their discovery is thus provided.

BH 140

BH 140 is a cluster located in the Musca constellation at a distance of 4180 ± 250 pc from the sun. The cluster is associated with the Galactic disk and likely formed in situ (Ishchenko et al., 2023) and has an estimated apparent magnitude in the V-band of 9.14 ± 0.12 mag. This globular cluster has limited information on its properties, which could suggest that it may be one of the newly discovered clusters.

47 Tuc. NGC 104

NGC 104 (47 Tuc) is one of the most massive globular clusters surrounding the MW. It contains at least 500,000 stars and is situated in the southern constellation of Tucana. Its estimated distance is about $D = 4.5 \pm 0.01$ kpc (15,000 light-years) (VB21), and a radius of 18.40 pc. Forbes and Bridges (2010) report an age and metallicity [Fe/H] of 13.06 Gyrs and -0.78 dex, respectively. As the second brightest globular cluster in the Southern Hemisphere, it can be observed with the naked eye having an apparent magnitude of 4.1 mag. Nicolas-Louis de Lacaille, conducting observations from South Africa, discovered this cluster in the 1750s (Seligman, 2015). It has an extremely dense core believed to house an immense black hole.



Figure 4.3: Illustrates all ten target globular clusters for this study in the MW taken from the globular cluster database. Image Credits: ESO Online Digitized Sky Survey, Bernhard Hubl, ESO, ESO Imaging Survey, PanSTARRS DR1, PS1 Science Consortium (<https://people.smp.uq.edu.au/HolgerBaumgardt/globular/>).

NGC 3201

NGC 3201, also known as Caldwell 79, is located in the southern constellation of Vela. With a radius of 12.26 pc (Seligman, 2015), this cluster is positioned at relatively low galactic latitudes. Hosting approximately 13,000 stars, NGC 3201 is situated at a distance of $D = 5.0 \pm 0.01$ kpc (16,300 light-years) (Paust et al., 2010). Its estimated age is around 12.2 ± 0.5 Gyr (Monty et al., 2018). This cluster was discovered by James Dunlop and listed in his 1827 catalogue (Seligman, 2015). The core concentration of stars is meagre (Webb et al., 2006) and the metallicity is about ~ -1.24 dex (Forbes and Bridges, 2010) and an apparent magnitude of 6.77 ± 0.04 mag in the V-band.

Messier 4. NGC 6121

NGC 6121 (M4) is a globular cluster in the Scorpius constellation. Philippe Loys de Chéseaux first identified it in 1745, and Charles Messier officially catalogued it in 1764 (Figuera Jaimes et al., 2016). It is estimated to host 30,000 stars. Located at a distance of approximately $D = 1.999$ kpc (Gaia Collaboration et al., 2018b)(VB21) (7,000 light-years), NGC 6121 has a radius of 10.731 pc, making it the closest globular cluster to the Earth. With an estimated age of 12.2 ± 0.2 Gyr (Caputo et al., 1985), a metallicity of $[\text{Fe}/\text{H}] = -1.07 \pm 0.01$ (Marino et al., 2008) or -1.5 dex reported by (Forbes and Bridges, 2010) and an apparent magnitude $m_V = 5.6$ mag, NGC 6121 allows a higher fraction of stars to be accurately measured in terms of their proper motions, with measurement errors lower than the dispersion of the globular cluster velocity by a factor of 10.

NGC 6366

The NGC 6366 globular cluster is faintly scattered, reddened by galactic dust, and found in the constellation Ophiuchus. Friedrich August Theodor Winnecke first discovered NGC 6366 in 1860 (Seligman, 2015). It is located at a distance $D = 3.6$ kpc $\approx 11,400$ light-years (Harris, 1996), with a radius of 0.72 pc (Seligman, 2015). Being the fifth closest to the Sun, NGC 6366 is a metal-rich halo globular cluster with mean metallicity $[\text{Fe}/\text{H}] = -0.60 \pm 0.03$ dex (Puls et al., 2018). It has an apparent magnitude of 9.5 mag in the V-band (Seligman, 2015) and a similar composition to other globular

clusters, such as NGC 6342.

NGC 6397

NGC 6397, located in the constellation Ara, is believed to have approximately 400,000 stars. It is situated at a distance of 2.5 kpc (VB21) ($\sim 7,800$ light-years), with a radius of 10.42 pc. NGC 6397 is one of the two closest globular clusters to Earth, along with NGC 6121. As part of the 20 globular clusters of the MW that have undergone a core collapse, NGC 6397 has a dense stellar agglomeration at its core. Its estimated age is 13.4 ± 0.8 Gyr, a magnitude of 6.68 mag in the V-band and metallicity $[\text{Fe}/\text{H}] = -1.76$ (Forbes and Bridges, 2010). The cluster was discovered by James Dunlop in 1826 and forms part of the metal-rich thick disk of globular clusters (Zinn, 1985).

NGC 6544

NGC 6544, a globular cluster in the Sagittarius constellation with tightly bound stars, was first discovered by William Herschel in 1784 using an 18.4-inch reflector (Seligman, 2015). Its distance is estimated at $D = 2.6$ kpc (9,458 light-years) (VB21) from the Sun and a radius of 3.2 pc, making it also one of the smaller globular clusters known. Densely populated with approximately 10,000 stars, it has an apparent magnitude in the V-band, age, and metallicity of 9 mag, 11–13 Gyr and -1.5 dex, respectively (Contreras Ramos et al., 2017). Its metallicity classifies it as a halo globular cluster. Whether it formed with the Milky Way or originated from elsewhere is unclear. However, the discovery of the VVV-CL160 globular cluster, believed to be associated with NGC 6544 due to its similar location, metallicity, and trajectory, was reported by Minniti et al. (2021).

Messier 22. NGC 6656

NGC 6656 is an elliptical globular cluster composed of a tightly packed group of stars ($\sim 70,000$ stars) in the Sagittarius constellation, close to the galactic bulge. The globular cluster NGC 6656 was discovered by Abraham Ihle and was among the earliest identified and subsequently listed in Charles Messier’s catalogue of comet-like objects

in 1764 (Monaco et al., 2004). This cluster is considered one of the brightest clusters visible in the sky and has a dense core (Shapley, 1930). Positioned at a relatively close distance of $D = 3.0 \pm 0.03$ kpc (10,600 light-years) (Monaco et al., 2004), with a radius of 15.33 ± 1.5 pc, it has an estimated age of 12 Gyr (Gaudi, 2002), a metallicity of $[\text{Fe}/\text{H}] = -1.49$ dex (Forbes and Bridges, 2010), and an apparent magnitude of $m_V = 5.1$ mag (Castellani, 2009).

NGC 6752

The NGC 6752 globular cluster is located in the Pavo constellation (Chadwick and Cooper, 2012) at a distance $D = 4.0$ kpc (13,000 light-years) (Boyles et al., 2011). It is considered the fourth brightest globular cluster, behind ω Centauri, 47 Tucanae, and M22, respectively (Privett and Parsons, 2001). It was discovered by James Dunlop in 1826 (Seligman, 2015). It exhibits a densely concentrated core. Its age is estimated to be 11.78 Gyr, metallicity $[\text{Fe}/\text{H}] = -1.24$ dex (Forbes and Bridges, 2010) and apparent magnitude $m_V = 5.4$ mag (Seligman, 2015).

Messier 71. NGC 6838

NGC 6838 is a loose globular cluster in the small constellation Sagitta. The NGC 6838 discovery dates back to 1745 by Philippe Loys de Chéseaux and earned a place in Charles Messier’s catalogue of celestial objects unrelated to comets in 1780 (Seligman, 2015). Positioned at a distance of $D = 4.0$ kpc (13,000 light-years) (Hessels et al., 2007) from the Earth, the cluster has a radius of approximately 3.99 pc. It has an estimated age and metallicity, and apparent magnitude in the V-band are 9 – 10 Gyr, -0.78 dex (Boyles et al., 2011), and 8.2 mag, respectively.

4.4 Methodology

4.4.1 Distance Determination

The *Gaia* telescope provides precise astrometric measurement and does not directly measure individual stellar distances. The parallax is measured accurately because of precise astrometric measurements. However, it is plausible to approximate that all stars in a cluster are equally spaced because of the much smaller distances between stars within a cluster compared to the distance from the cluster to the Sun (Weintraub, 2010); this is an advantage because the distance to globular clusters can be determined directly and without requiring an intermediate distance method. Additionally, the reddening of the clusters does not affect the parallaxes. Two key points must be considered when measuring the distance to the clusters: ensure that the distance is measured accurately and the different distance measurements must agree so that the methods are not biased.

To determine the distances of the clusters, Gaussian distribution functions were fitted to the distributions of member stars. The fitting enabled the construction of a trigonometric parallax histogram that determined the mean parallax value. χ^2 minimisation was performed out to optimise the Gaussian fit. The mean trigonometric parallax was converted to the trigonometric parallax distance (D_{ϖ}) using the linear equation $D(\text{pc}) = 1/\varpi$ (in mas). Baumgardt and Vasiliev (2021) found parallax correction that overcorrects the *Gaia* EDR3 data by $\Delta\varpi \sim 0.007$ mas following the Lindegren et al. (2021) method. Additionally, a systematic uncertainty of $\epsilon_{\varpi} \sim 0.01$ mas was assumed to account for the parallax bias in determining the distances.

4.4.2 Star Count Profiles

Cluster centres and cluster size

To effectively identify and describe a stellar cluster, it is crucial to have, at minimum, its centre position and angular size. Accurate estimation of the central coordinates of a cluster is essential for establishing the radius, radial density profile, and the cluster's mass distribution. While literature offers only a limited number of methods for centre determination, Picard and Johnston (1994) investigated the relative accuracy of differ-

ent methods of central position determination in globular clusters through a testbed of artificial clusters. The artificial clusters are simulated clusters that mimic the characteristics of real globular clusters and systematically allow the comparison of various methods for determining the central position in a controlled and reproducible manner.

The method applied in this work for determining the central coordinates is similar to that of [Maia et al. \(2014\)](#), using initial estimates of the cluster centre and radius and averaging the positions of the stars weighted by the stellar densities in their proximity. The cluster centre (C_{centre}) is assumed to be the location of the maximum stellar density in the cluster area. C_{centre} is computed from star counts to avoid any potential bias that may arise from the presence of several bright stars. Gaussian distribution functions are fitted to the star frequency distributions of RA and DEC to determine the mean positions of RA and DEC within a circle of maximum radius (r_{max}).

The value of r_{max} is an essential parameter, often referred to as the limiting radius, and it is calculated based on the farthest members from the centre. According to [Harris \(1996\)](#), the value of r_{max} consistently exceeds the core radius, indicating its sensitivity to the region of the profile where the slope changes and the density becomes less homogeneous.

χ^2 minimisation is employed to enhance the accuracy of the Gaussian fit and quantify the distance between the observed data. This minimisation process is expressed as:

$$\chi^2 = \sum_i \frac{(y_i - f(x_i))^2}{\sigma_i^2}, \quad (4.1)$$

where $f(x)$ is the model (the fitted Gaussian function), χ^2 is the independent variable, and y_i and σ_i are the measurements and their uncertainties, respectively. This methodology ensures the precise determination of the central coordinates and, consequently, the accurate characterisation of stellar clusters.

Radial density profile

Radial density profiles (RDP) are constructed around the estimated central coordinate of globular clusters to understand their structural characteristics and spatial extent. The underlying assumption is that stars with measured kinematics provide an accept-

able representation of the entire population and contribute to the density profile denoted as $\Sigma(R)$. The density profile is computed by dividing the complete data set into N concentric annuli around the cluster's centre. Each globular cluster's radius is first divided into annuli with a fixed radial-bin width in the units kpc, and as a result, the number of radial bins may differ from one cluster to another. The star counts are divided by the respective areas of each annular ring and are represented as $\Sigma_i = N_i / \pi(R_{i+1}^2 - R_i^2)$ for each i^{th} ring. In the previous equation, N_i corresponds to the number of falling stars within a specific ring. R_i and R_{i+1} represent the inner and outer radii of the ring, respectively. The uncertainties (assumed to be $1/\sqrt{N}$ in origin) associated with the profile measurements are estimated by dividing each annulus into several regions and then calculating the formal errors concerning the whole annulus results (Chernoff and Djorgovski, 1989). Since star count profiles are susceptible to crowding in the central regions, crowding correction is considered. The corrections should be viewed as identifying locations where the counts become unreliable rather than as adjustments that yield a meaningful star density value at those locations.

The observed stellar density profiles in logarithmic scale are plotted against the radial distance for the cluster and fitted with the King (1962) model. The fitting is done over the range $[r_{\min}, r_{\max}]$ using a least-square technique, weighting the points by the observational determined uncertainties. Since the projected King model has three parameters, the fit is carried out by choosing k, r_c and r_t and then adjusting the $\sigma_{\text{King}}(0)$ to minimize χ^2 , where

$$\chi^2 = \sum_i \left[\frac{\sigma(r_i) - \sigma_{\text{King}}(r_i)}{\delta\sigma(r_i)} \right]^2. \quad (4.2)$$

In the sum, i runs over the data point with $r_{\min} < r_i < r_{\max}$, where $\sigma(r_i)$ is the measured value, $\delta\sigma(r_i)$ is the measuring uncertainty, and σ_{King} is the King model. The King (1962) model (Equation 3.3) in the isotropic, spherical, and single-mass approximation is accounted for to determine the structural parameters.

Chapter 5

Result: Distances, Mass and Structural Parameters of Globular Clusters.

The *Gaia* space mission measures the astrometry of stars in the MW, thus allowing for an accurate measurement of the proper motions of stars ([Gaia Collaboration et al., 2016](#)) and is used to measure the mass of globular clusters via the Leonard-Merritt method ([Leonard and Merritt, 1989](#)). Previously, the mass of these systems was determined by the Virial theorem, based on radial velocity measurements ([Michie, 1961](#)). Radial velocity measurements, however, provide limited information on spatial distribution and true motion ([Meylan and Heggie, 1997](#); [Rosenthal et al., 2019](#)); it is, therefore, vital to use proper motion or photometric data to estimate the mass of globular clusters.

Additionally, the star's radial velocity is measured by spectroscopy, a more involved process that consumes significant telescope time ([Lindegren and Dravins, 2003](#); [Bellini et al., 2014](#)). Therefore, it is crucial to use independent techniques when determining the distances of globular clusters, especially those not affected by reddening. This method is the first step before applying the Leonard-Merritt method to calculate the mass of these stellar systems.

5.1 Cluster's Distances

The results for the mean parallax (ϖ) and distances (D_ϖ) for each globular cluster calculated in this dissertation are represented in Table 5.1. For BH 140 (one of the poorly studied globular clusters), the derived distance is $D_\varpi = 4.541 \pm 0.612$ kpc for $\varpi = 0.227 \pm 0.005$ mas. NGC 104 remains one of the most extensively studied clusters within the MW. As a result, there are many distance estimates in the literature. The best-fitting distance for NGC 104 is $D_\varpi = 4.425 \pm 0.461$ kpc, which corresponds to $\varpi = 0.233 \pm 0.020$ mas. The value is consistent with other distance determination methods, including the value $D_\varpi = 4.42 \pm 0.20$ kpc, determined from *Gaia* EDR3 (Maíz Apellániz et al., 2021). The value is also consistent with the distance of $D = 4.50 \pm 0.03$ kpc, derived from eclipsing binaries reported by Thompson et al. (2020). The previously mentioned is in agreement, as the value of Watkins et al. (2015a) falls within 1σ of the value in this work, where $D = 4.15 \pm 0.08$ kpc. The slight discrepancy may arise because the HST proper motion dispersion profile only accounts for velocities within the central $100''$, and stars located further away from the central region are not considered.

This work		
Cluster	ϖ (mas)	D_ϖ (kpc)
BH 140	0.227 ± 0.005	4.541 ± 0.612
NGC 104 (47 Tuc)	0.233 ± 0.020	4.425 ± 0.461
NGC 3201	0.224 ± 0.023	4.611 ± 0.551
NGC 6121 (M4)	0.573 ± 0.046	1.768 ± 0.149
NGC 6366	0.297 ± 0.024	3.447 ± 0.320
NGC 6397	0.426 ± 0.035	2.388 ± 0.210
NGC 6544	0.407 ± 0.033	2.501 ± 0.222
NGC 6656 (M22)	0.325 ± 0.032	3.148 ± 0.340
NGC 6752	0.262 ± 0.024	3.929 ± 0.421
NGC 6838 (M71)	0.251 ± 0.023	4.107 ± 0.440

Table 5.1: The clusters in this work are in column 1, the parallax (ϖ) calculated is in column 2, and the distance derived from parallaxes (D_ϖ) is in column 3.

For parallax $\varpi = 0.224 \pm 0.023$ mas, the estimated distance for NGC 3201 is $D = 4.611 \pm 0.551$ kpc, in agreement with the value $D = 4.7 \pm 0.2$ kpc reported by [Layden and Sarajedini \(2003\)](#) using σ Scutti and RR Lyrae stars. For NGC 6121, the parallax-based distance estimation of $D_{\varpi} = 1.768 \pm 0.149$ kpc corresponds to a parallax of $\varpi = 0.573 \pm 0.046$ mas. This distance is slightly lower than the distance of $D = 1.80 \pm 0.05$ kpc ([Hendricks et al., 2012](#)) using the luminosity of HB. NGC 6366 is well-known to be the fifth closest globular cluster to the Sun ([Harris, 1996](#)), with $D = 3.5$ kpc. For $\varpi = 0.297 \pm 0.024$ mas, this is in agreement with this study's best-fitting D_{ϖ} value of 3.45 ± 0.32 kpc. NGC 6366, which is recognised as the fifth closest globular cluster to the Sun, according to [Harris \(1996\)](#), is positioned at a distance of $D = 3.5$ kpc. The analysis, considering a parallax of $\varpi = 0.297 \pm 0.024$ mas, remarkably aligns with this value, resulting in a best-fitting D_{ϖ} measurement of 3.45 ± 0.32 kpc.

The globular clusters NGC 6656 and NGC 6397 are the closest to the Sun and are extensively studied due to their proximity and minimal reddening, resulting in more observational data. For NGC 6397, the analysis produces a derived distance of $D_{\varpi} = 2.388 \pm 0.210$ kpc, with $\varpi = 0.573 \pm 0.046$ mas. The measurement makes NGC 6397 the second-closest cluster to the Sun. The result is consistent with the value $D = 2.39^{+0.13}_{-0.11}$ kpc, obtained by [Watkins et al. \(2015a\)](#) using proper motions and line-of-sight velocities. [Gratton et al. \(2003\)](#) also derive a distance value $D = 2.54^{+0.10}_{-0.09}$ kpc, based on MS fitting. The best-fitting distance for the cluster NGC 6656 is $D_{\varpi} = 3.148 \pm 0.340$ kpc for $\varpi = 0.325 \pm 0.032$ mas. For this cluster, [Cudworth \(1986\)](#) and [Watkins et al. \(2015a\)](#) estimate a distance of 3.2 ± 0.03 kpc from the HB fitting and 2.84 ± 0.16 kpc by combining proper motion and line-of-sight velocity dispersion, respectively.

In the NGC 6752 cluster, a parallax value of $\varpi = 0.262 \pm 0.024$ mas leads to a distance of $D_{\varpi} = 3.929 \pm 0.421$ kpc. This distance derived is closely aligned with the reported value of $D_{\varpi} = 3.92 \pm 0.16$ kpc, documented in [Maíz Apellániz et al. \(2021\)](#). NGC 6544 is relatively less studied among globular clusters, despite its proximity in the direction of the galactic bulge. The estimated distance for this cluster is $D_{\varpi} = 2.501 \pm 0.222$ kpc, with a corresponding parallax value of $\varpi = 0.407 \pm 0.033$ mas. [Cohen et al. \(2014\)](#) derived a distance of $D = 2.46 \pm 0.05$ kpc by combining optical and near-infrared photometry using HST and the Visible and Infrared Survey Telescope for Astronomy (VISTA) variables in the Vía Láctea (VVV) survey. For NGC 6838, this study's best-fit distance estimation is $D_{\varpi} = 4.107 \pm 0.440$ kpc, with $\varpi = 0.251 \pm 0.023$ mas. This dis-

tance is slightly larger than other measurements, typically indicating a cluster distance of around $D = 4.0$ kpc, as reported by [Hessels et al. \(2007\)](#).

5.2 Cluster’s Centre, Spatial Distributions, Masses and Structural Parameters

It is easier to implement the mass estimator, as described in Equation 3.1, to determine the mass and, in addition, to derive structural parameters for each cluster and fit in a King model to the density profile once the distance measurements have been acquired. However, before proceeding, it is essential to have an accurately determined centre of each cluster, as this step is critical in deriving the structural parameters and characterising the mass distribution within these globular clusters.

5.2.1 Centre Determination

A well-defined estimate of the central position of a cluster is equally crucial when accurately determining the density profile because using an incorrect centre can result in a shallower inner profile. Table 5.2 lists the centre coordinates for the ten target globular clusters from the Gaussian fitting and the χ^2 minimisation. The fitted Gaussian distribution function provides an estimate of the clusters’ centres based on the peak of the density distribution. Using the χ^2 minimisation to optimise the Gaussian fit, the minimised Gaussian fit accurately represents the observed data distribution, emphasising the true centre of the globular cluster. The minimised parameters yield more reliable information about the centre of the cluster compared to the initial Gaussian fit. The results reveal that these clusters exhibit a dense central concentration of stars. A comparison of the coordinates obtained through these methods showed that the differences there are an association in uncertainties in both RA and DEC. Therefore, “Gaussian fitting” and using “ χ^2 minimisation” to optimise the results are both reliable techniques for centre determination in globular clusters. Furthermore, comparing the results with the literature values of [VB21](#) is in good agreement, confirming the precision and reliability of the measurements.

Cluster	Gaussian Fitting		χ^2	
	RA (α)	DEC (δ)	RA (α)	DEC (δ)
	(deg)	(deg)	(deg)	(deg)
BH 140	193.426 $\pm$ 0.191	-67.192 $\pm$ 0.040	193.402 $\pm$ 0.205	-67.184 $\pm$ 0.130
NGC 104 (47 Tuc)	6.014 $\pm$ 0.332	-72.084 $\pm$ 0.099	6.020 $\pm$ 0.428	-72.082 $\pm$ 0.129
NGC 3201	154.402 $\pm$ 0.077	-46.408 $\pm$ 0.049	154.395 $\pm$ 0.150	-46.403 $\pm$ 0.098
NGC 6121 (M4)	245.901 $\pm$ 0.073	-26.519 $\pm$ 0.057	245.907 $\pm$ 0.106	-26.515 $\pm$ 0.092
NGC 6366	261.939 $\pm$ 0.460	-5.077 $\pm$ 0.042	261.942 $\pm$ 0.134	-5.075 $\pm$ 0.051
NGC 6397	265.179 $\pm$ 0.109	-53.674 $\pm$ 0.064	265.184 $\pm$ 0.163	-53.673 $\pm$ 0.093
NGC 6544	271.843 $\pm$ 0.044	-25.001 $\pm$ 0.044	271.849 $\pm$ 0.053	-25.004 $\pm$ 0.053
NGC 6656 (M22)	279.100 $\pm$ 0.069	-23.911 $\pm$ 0.063	279.097 $\pm$ 0.096	-23.912 $\pm$ 0.084
NGC 6752	287.700 $\pm$ 0.129	-59.987 $\pm$ 0.058	287.700 $\pm$ 0.184	-59.990 $\pm$ 0.090
NGC 6838 (M71)	298.442 $\pm$ 0.032	18.778 $\pm$ 0.031	298.447 $\pm$ 0.127	18.780 $\pm$ 0.037

Table 5.2: Comparison cluster centre coordinates determining the fitting of Gaussian functions and χ^2 minimization.

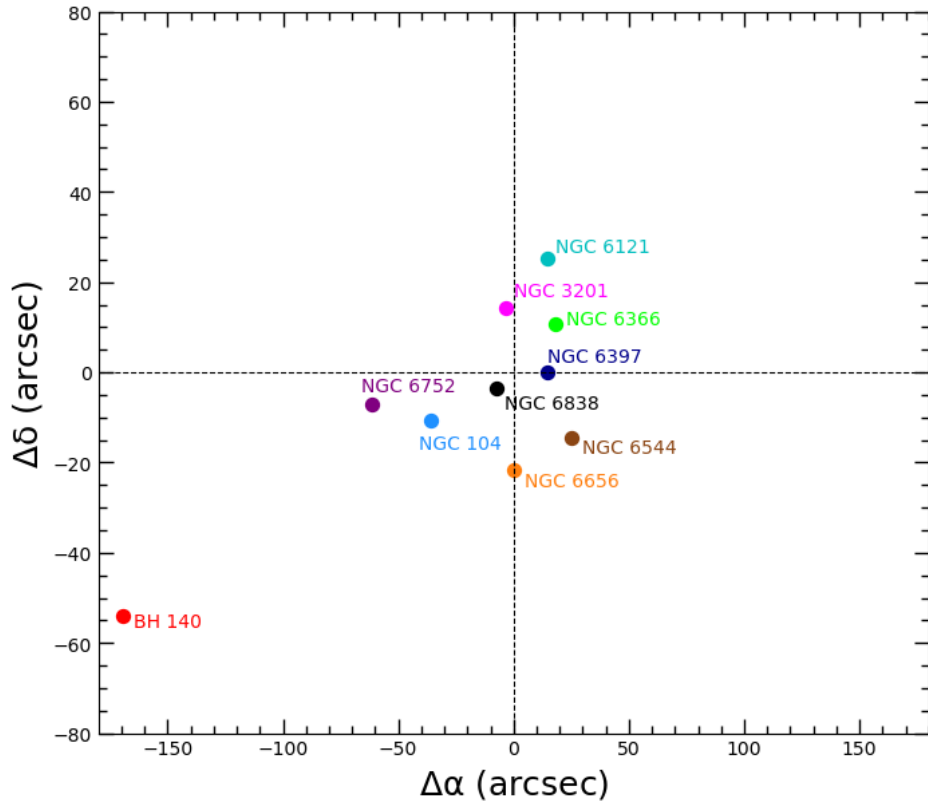


Figure 5.1: Comparison of the differences between this study's determination of the centre coordinates in Table 5.2 and those quoted by VB21. $\Delta\alpha$ and $\Delta\delta$ represent the change in RA and DEC (in arcseconds).

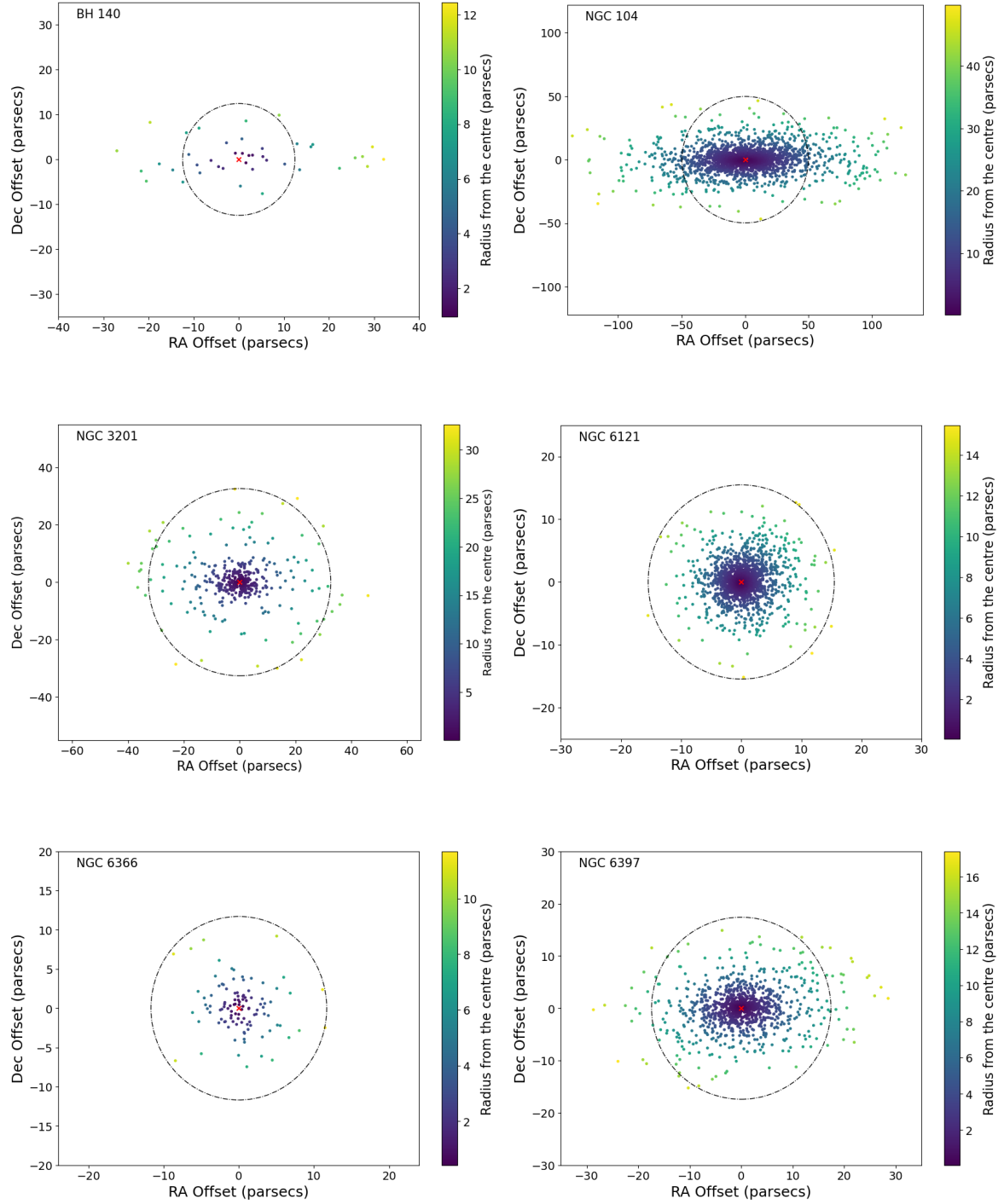
Figure 5.1 compares the differences between the centre coordinates presented in Table 5.2 and those cited in VB21. Most clusters exhibit differences in both RA and DEC that are consistently smaller than $\simeq 60''$, reflecting the overall accuracy and precision of the measurements. The BH 140 globular cluster exhibits significant deviations compared to the other clusters. BH 140 has relatively few stars observed, suggesting a low stellar density in the core region of the cluster. It is, therefore, difficult to accurately determine the central coordinates of the cluster, as established by the large uncertainty of $\sigma_{\alpha,\delta} \geq 125.4''$. This case is a valuable reminder of the importance of robust data acquisition and analysis, particularly for clusters with sparse observational data.

5.2.2 Spatial Distribution of Stars

With the highly-precise centre coordinates obtained in the previous section, constructing the spatial distribution of stars within the globular clusters became more straightforward. The spatial distribution is illustrated in Figure 5.2. The analysis of the stellar distribution, as depicted in the plots, offers valuable insight into the structure and stellar arrangements within the clusters. In these plots, the centre coordinates are identified by an ‘x’ mark and $r_{\max}$ by the circle based on the outermost member.

The stellar distribution within these clusters shows a characteristic core-halo structure, where stars are densely concentrated in the centre, and their density decreases as one moves toward the outer regions. This stellar distribution is because of dynamical evolution, where older stars are situated in the core region and younger stars in the outer region because of gravitational interaction and evaporation over time. However, for globular cluster BH 140, only a few stars are observed and are sparsely distributed. NGC 3201, NGC 6366, NGC 6544 and NGC 6838 clusters have a milder concentration of stars at the centre, while NGC 104, NGC 6121, NGC 6397, and NGC 6656 are more densely concentrated. The NGC 104 cluster stands out as the densest and most massive. From the plots, most stars in globular clusters are found within $r_{\max}$, except for NGC 104, because the cluster is massive and elongated, suggesting that the stars are not uniformly distributed in a spherically symmetric manner, but instead exhibit a certain degree of anisotropy in their spatial distribution. Such an elongation can arise due to a combination of internal dynamics, such as mass distribution and stellar interactions, and external influences, including interactions with the surrounding environment.

Internally, the mass distribution and gravitational interactions within the cluster can lead to deviations from spherical symmetry, resulting in elongation or asymmetry in its shape. Additionally, external factors such as tidal interactions with the host galaxy or encounters with neighboring objects can further exacerbate these deviations, shaping the cluster's morphology over time.



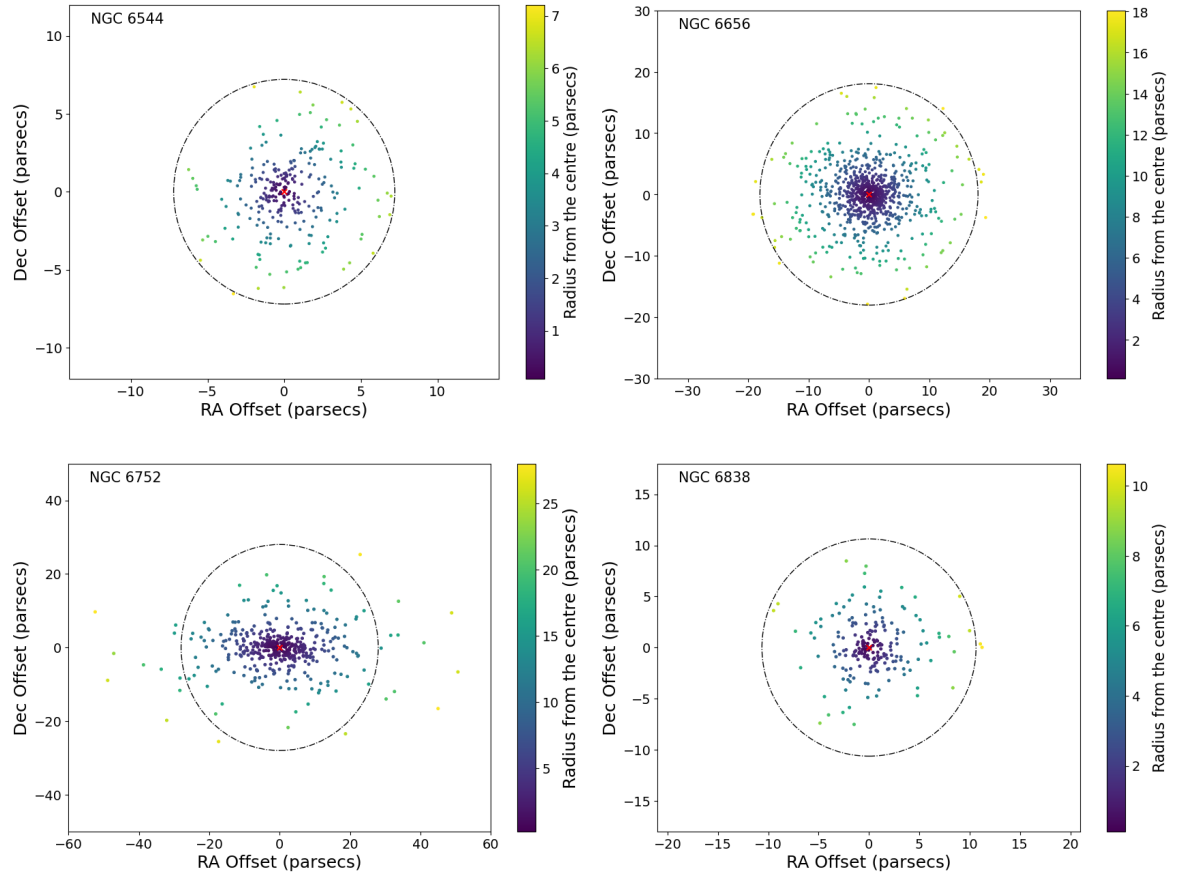


Figure 5.2: Illustrates the stellar distribution of stars within the globular clusters. The red ‘x’ mark represents the central coordinates of the clusters, and the dotted line extends to showcase the maximum radius r_{max} from the centre.

5.2.3 Mass and Structural Parameters

The observed star count density profiles for each globular cluster are presented below, calculated considering uncertainties in each ring for the globular cluster, accompanied by the best-fit King profile depicted as a solid line. In this analysis, the [King \(1962\)](#) model is applied while considering isotropy, sphericity, and a single-mass distribution as the underlying assumptions. The results of this analysis, including the mass and structural parameters, are detailed in Table 5.3. The parameters: the number of stars (N_{mem}), core radius (r_c), tidal radius (r_t), half-light radius (r_h), concentration (c), derived distance (D) and mass are presented for each cluster.

This study							
Cluster	N_{mem}	r_c (pc)	r_t (pc)	r_h (pc)	c	D (kpc)	Mass ($M_\odot$)
BH 140	41	5.71 ± 0.500	54.96 ± 0.0016	9.39 ± 0.82	0.98 ± 0.09	4.541 ± 0.612	$5.34 \pm 4.60 \cdot 10^4$
NGC 104 (47 Tuc)	1937	0.47 ± 0.006	130.60 ± 0.0005	3.94 ± 0.05	2.45 ± 0.01	4.425 ± 0.461	$7.16 \pm 8.39 \cdot 10^5$
NGC 3201	368	2.33 ± 0.127	79.12 ± 0.0013	7.07 ± 1.51	1.53 ± 0.21	4.611 ± 0.551	$1.59 \pm 7.73 \cdot 10^5$
NGC 6121 (M 4)	1897	1.22 ± 0.049	54.53 ± 0.0003	4.23 ± 0.170	1.65 ± 0.040	1.768 ± 0.149	$5.21 \pm 0.97 \cdot 10^4$
NGC 6366	104	2.09 ± 0.234	55.57 ± 0.0007	5.63 ± 0.64	1.42 ± 0.19	3.447 ± 0.320	$5.51 \pm 1.99 \cdot 10^4$
NGC 6397	1097	0.30 ± 0.001	61.38 ± 0.0004	2.17 ± 0.01	2.23 ± 0.003	2.388 ± 0.210	$7.43 \pm 1.47 \cdot 10^4$
NGC 6544	265	0.33 ± 0.004	81.27 ± 0.0007	2.64 ± 0.03	2.39 ± 0.02	2.501 ± 0.222	$1.72 \pm 4.73 \cdot 10^5$
NGC 6656 (M 22)	824	2.08 ± 0.074	101.63 ± 0.003	7.54 ± 0.02	1.69 ± 0.04	3.148 ± 0.340	$3.37 \pm 2.69 \cdot 10^5$
NGC 6752	410	0.36 ± 0.002	81.51 ± 0.0009	2.76 ± 0.01	2.35 ± 0.006	3.929 ± 0.421	$1.74 \pm 5.82 \cdot 10^5$
NGC 6838 (M 71)	190	2.04 ± 0.196	61.18 ± 0.0004	5.83 ± 0.56	1.48 ± 0.10	4.107 ± 0.440	$7.36 \pm 1.60 \cdot 10^4$

Table 5.3: The names of the clusters are in column 1 and the number of stars in each cluster in column 2. The derived parameters are: core radius (r_c) in column 3, tidal radius (r_t) in column 4, half-light radius (r_h) in column 5, concentration (c) in column 6, and distance and mass of the clusters in columns 7 and 8 for each globular cluster.

BH 140

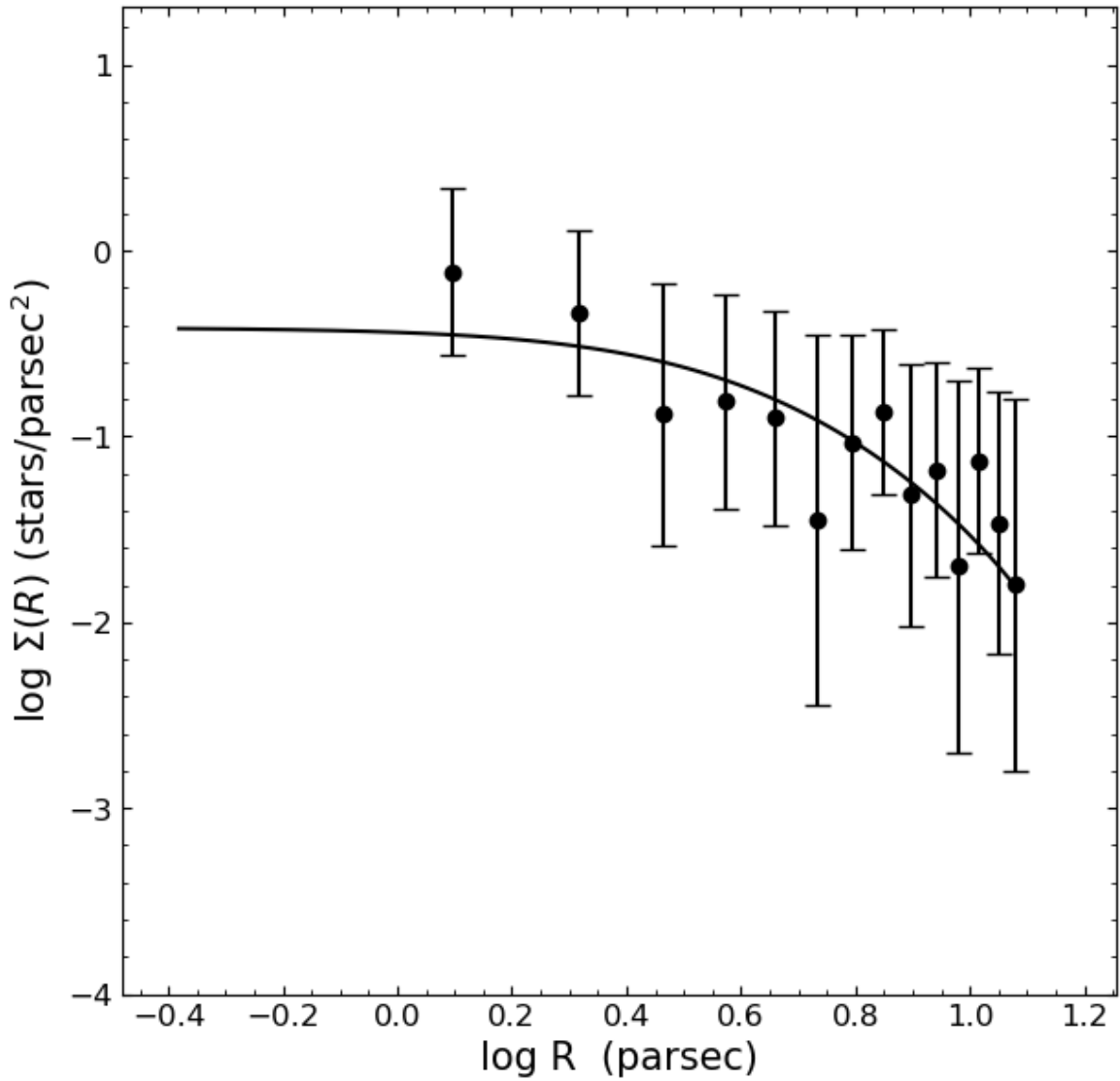


Figure 5.3: The density distribution in radial terms of the cluster member characterised using the King model fitting for BH 140.

The estimated mass for BH 140 is $5.31 \pm 4.60 \times 10^4 M_{\odot}$, revealing a slightly smaller value compared to the mass of $6.1 \times 10^4 M_{\odot}$ reported by [Hilker et al. \(2020\)](#), hereafter [BHS23](#) catalogue, the latest version ¹. Delving into the structural parameters of the BH 140 globular cluster, this study's analysis yields values of $r_c = 5.71 \pm 0.500$ pc, $r_t = 54.96 \pm 0.0016$ pc, $r_h = 9.39 \pm 0.82$ pc, and $c = 0.98 \pm 0.09$. The [BHS23](#) catalogue provides the parameters for r_c , r_t , and r_h as 4.81 pc, 51.77 pc, and 6.22 pc, respectively, which are very close to the values derived in this work. In [Figure 5.3](#), the King model offers a compelling match with the observed stellar density up to 12 pc. Notably, a flat

isothermal core extends up to the core radius, followed by a decline in density in the outer region, reaching a density of 0.0159 stars/pc².

NGC 104 (47 Tuc)

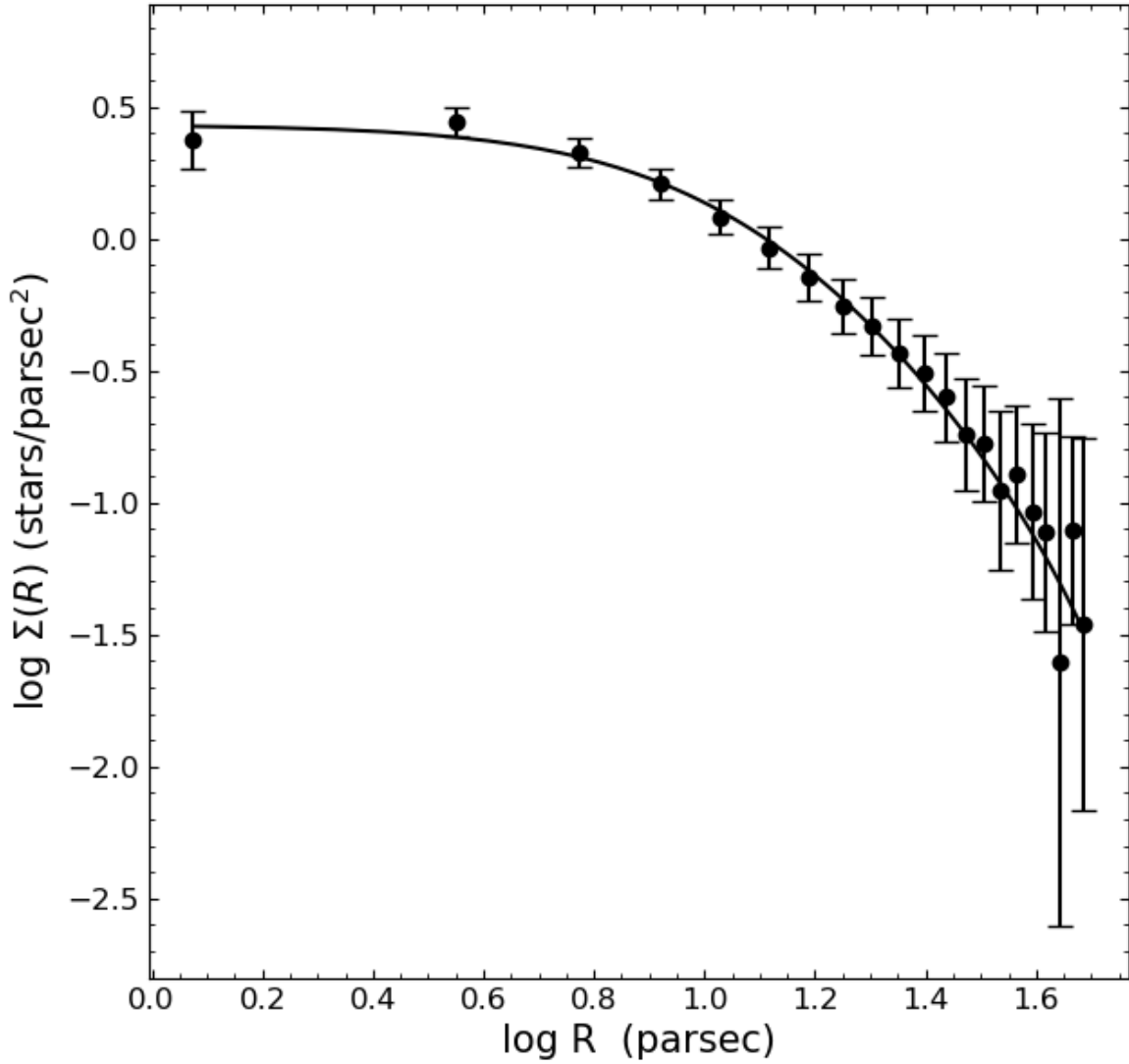


Figure 5.4: The density distribution in radial terms of the cluster member characterised using the King model fitting for NGC 104.

The total cluster mass estimated for NGC 104 is $7.16 \pm 8.39 \times 10^5 M_{\odot}$, aligning closely with the findings in Baumgardt (2017), reporting $7.00 \pm 0.06 \times 10^5 M_{\odot}$ and $6.5^{+1.1}_{-0.9} \times$

¹<https://people.smp.uq.edu.au/HolgerBaumgardt/globular/parameter.html>, link to the BHS23 catalogue. (©Baumgardt H, Sollima A, Hilker M, Bellini A, Vasiliev E, Henault-Brunet V and Dickson N. March 2023).

$10^5 M_{\odot}$, from [Kimmig et al. \(2015\)](#). However, it is smaller than the mass value of $1.3 \times 10^6 M_{\odot}$ determined by [Meylan and Mayor \(1986\)](#). The derived structural parameters for NGC 104 include $r_t = 130.60 \pm 0.0005$ pc, $r_c = 0.47 \pm 0.006$ pc, $r_h = 3.94 \pm 0.05$ pc, and $c = 2.45 \pm 0.01$. Interestingly, [Webbink \(1985\)](#) reports values of 60 pc, 0.52 pc, and 2.08 for r_t , r_c and c , and r_t , respectively, with the r_t derived in this work significantly greater. The smaller value of r_c indicates that the cluster has a more compact core and captures the observed central density peak, while the estimated c is also greater than the values obtained by [McLaughlin and van der Marel \(2005\)](#); [Kaluzny et al. \(2013\)](#) (1.76 ± 0.02 and 2.01 ± 0.03).

In Figure 5.4, the King model fitting demonstrates agreement with the observed density profile, evident in the flattening observed in the central region and the steep envelope in the outer region. The surface density decreased from 2.368 stars/pc² to 0.034 stars/pc². The figure also indicates that the cluster is more self-contained within its tidal radius; thus, the King model cannot reproduce the density beyond the tidal radius due to the elongation of the cluster, where there are some limitations.

NGC 3201

The mass of the cluster is estimated to be $1.59 \pm 7.73 \times 10^5 M_{\odot}$, demonstrating good agreement with the value $1.58 \pm 0.11 \times 10^5 M_{\odot}$, reported in [Baumgardt \(2017\)](#) and $1.57 \times 10^5 M_{\odot}$ by [Vital et al. \(2022\)](#). In Figure 5.5, the density profile for NGC 3201 reveals structural parameters of $r_t = 79.12 \pm 0.0013$ pc, $r_c = 2.33 \pm 0.127$ pc, $r_h = 7.07 \pm 1.51$ pc and $c = 1.53 \pm 0.21$. While the tidal radius aligns with the values of $r_t = 82.34$ pc, the half-light radius is bulkier than $r_h = 5.17$ pc reported in the [BHS23](#) catalogue. Additionally, [Baumgardt and Hilker \(2018\)](#) report a core radius value of 1.91 pc, slightly lower than the core radius derived in this work.

The larger tidal radius allows the King model to extend its influence over a greater radial distance, influencing the decline in density beyond the core. In Figure 5.5, the fitted King's model exhibits a slight flattening in the outer region, with a gradual decline in density as expected from the centre of the cluster. The larger r_h also indicates a more extended distribution of stars in the cluster. [Harris \(1996\)](#) and [Webbink \(1985\)](#) consistently present concentration values of 1.29 and 1.55, respectively, which align with this study's estimated concentration value.

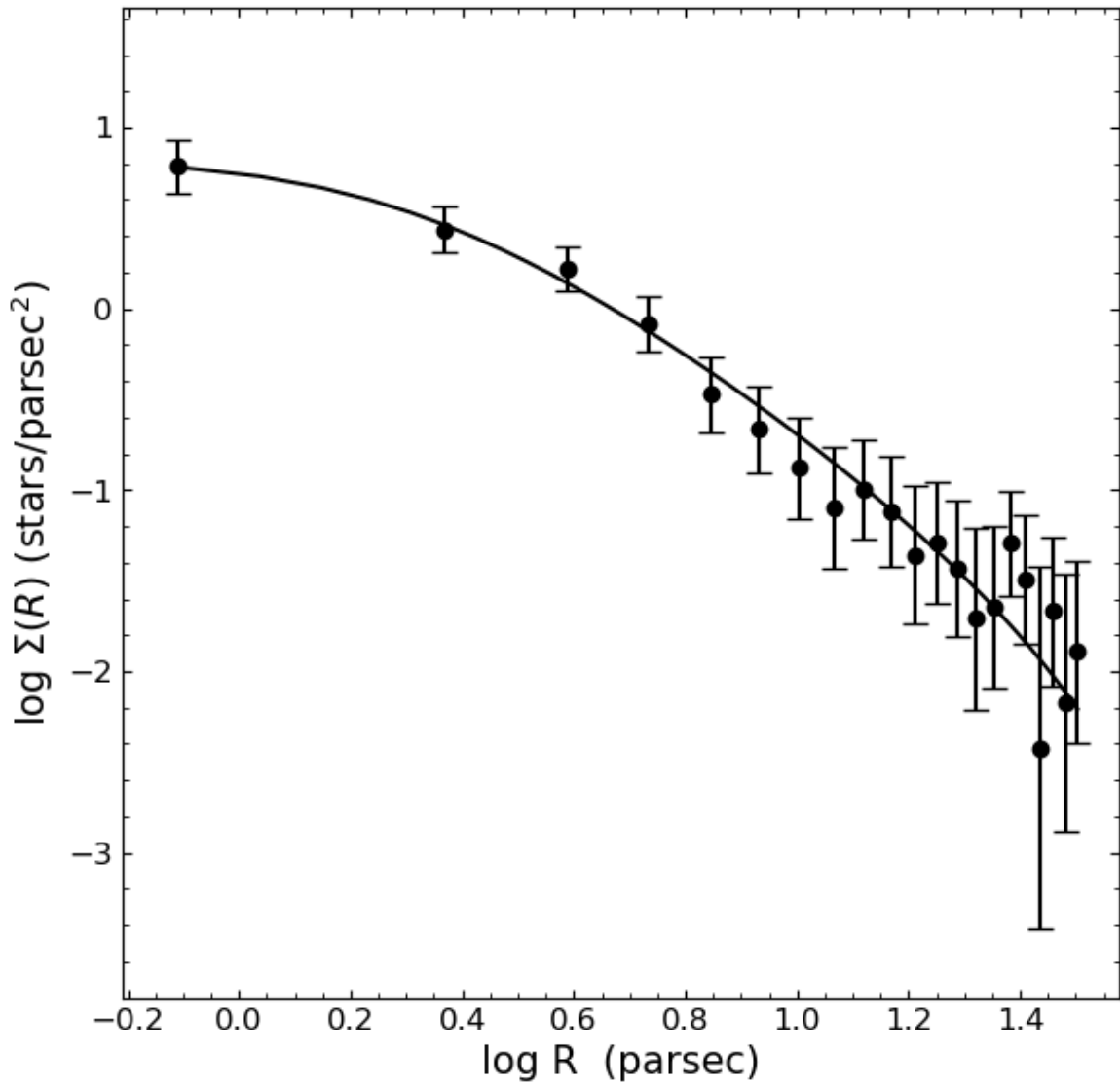


Figure 5.5: The density distribution in radial terms of the cluster member characterised using the King model fitting for NGC 3201.

NGC 6121 (M4)

The estimated mass of NGC 6121 is $5.21 \pm 0.97 \times 10^4 M_{\odot}$. The value is smaller than the mass estimate of $9.04 \pm 0.11 \times 10^4 M_{\odot}$ from the [BHS23](#) catalogue and $8.04 \times 10^4 M_{\odot}$ from [Vital et al. \(2023\)](#). In the analysis of the NGC 6121 profile, presented in Figure 5.6, it is evident that the King model fitting exhibits agreement with the observed density profile, and the model accurately captures the peak density of 134.75 stars/pc² near the core. A gradual decline in density is observed in the outer regions, reaching 0.078 stars/pc².

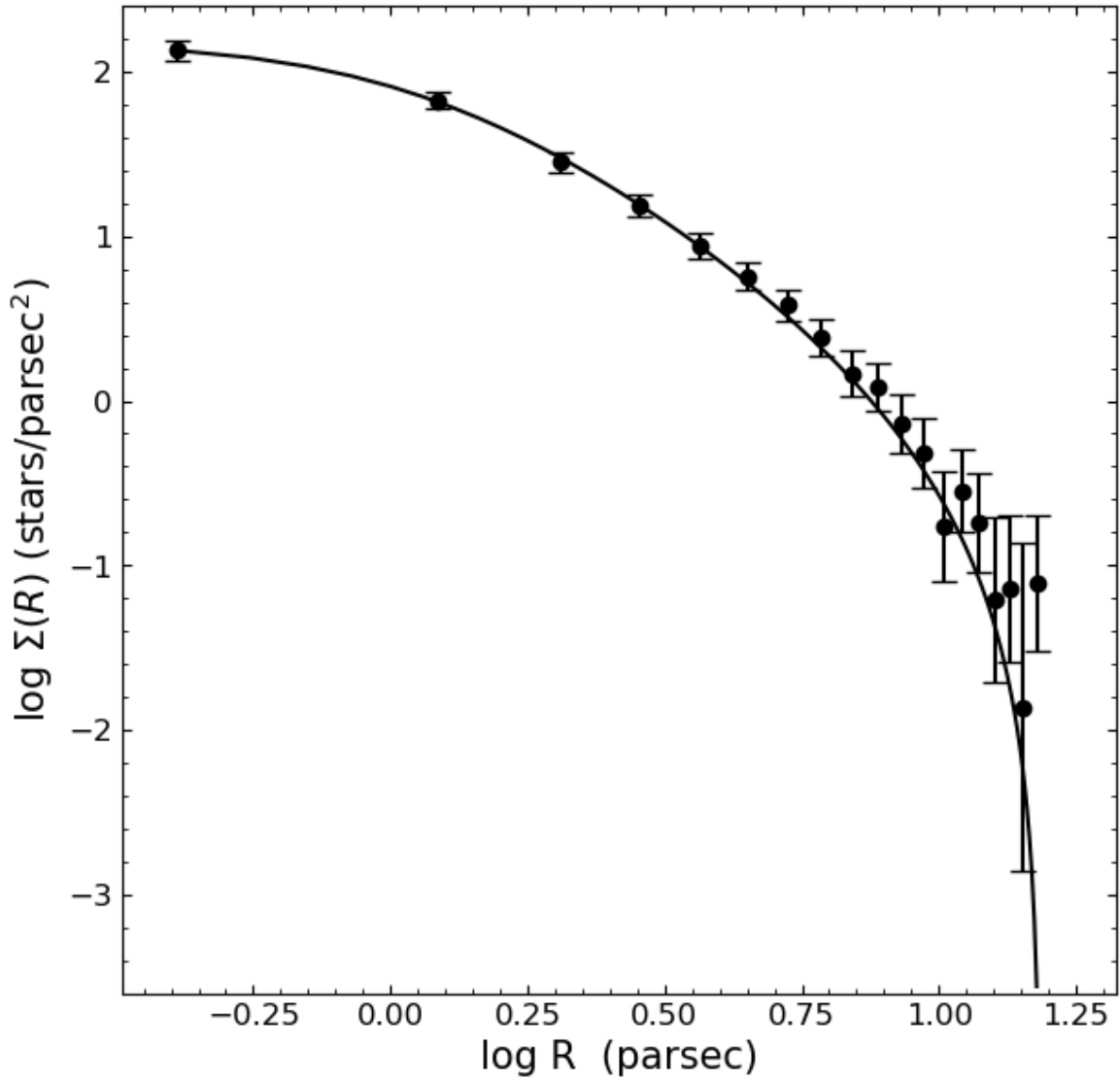


Figure 5.6: The density distribution in radial terms of the cluster member characterised using the King model fitting for NGC 6121.

The estimated parameters are $r_t = 54.53 \pm 0.0003$ pc and $r_c = 1.22 \pm 0.049$ pc, $r_h = 4.23 \pm 0.170$ pc and $c = 1.65 \pm 0.040$. While the values of r_c and r_t differ considerably from those reported in [Webbink \(1985\)](#) (0.75 pc, 26.5 pc, respectively), the value of c is consistent with the reported value $c = 1.55$ with only 0.1 pc difference. The larger values of r_t and c suggest more of a diffuse central region.

The King model fitting provides a good fit for the observed density profile. However, it extends beyond the observed density profile, especially in the outer regions. This suggests that the model extends its influence beyond the last observed point in the profile and predicts the density distribution even in a region where observational data

may not be available, indicating a potential impact of external tidal forces on the cluster's structure beyond the observed range.

NGC 6366

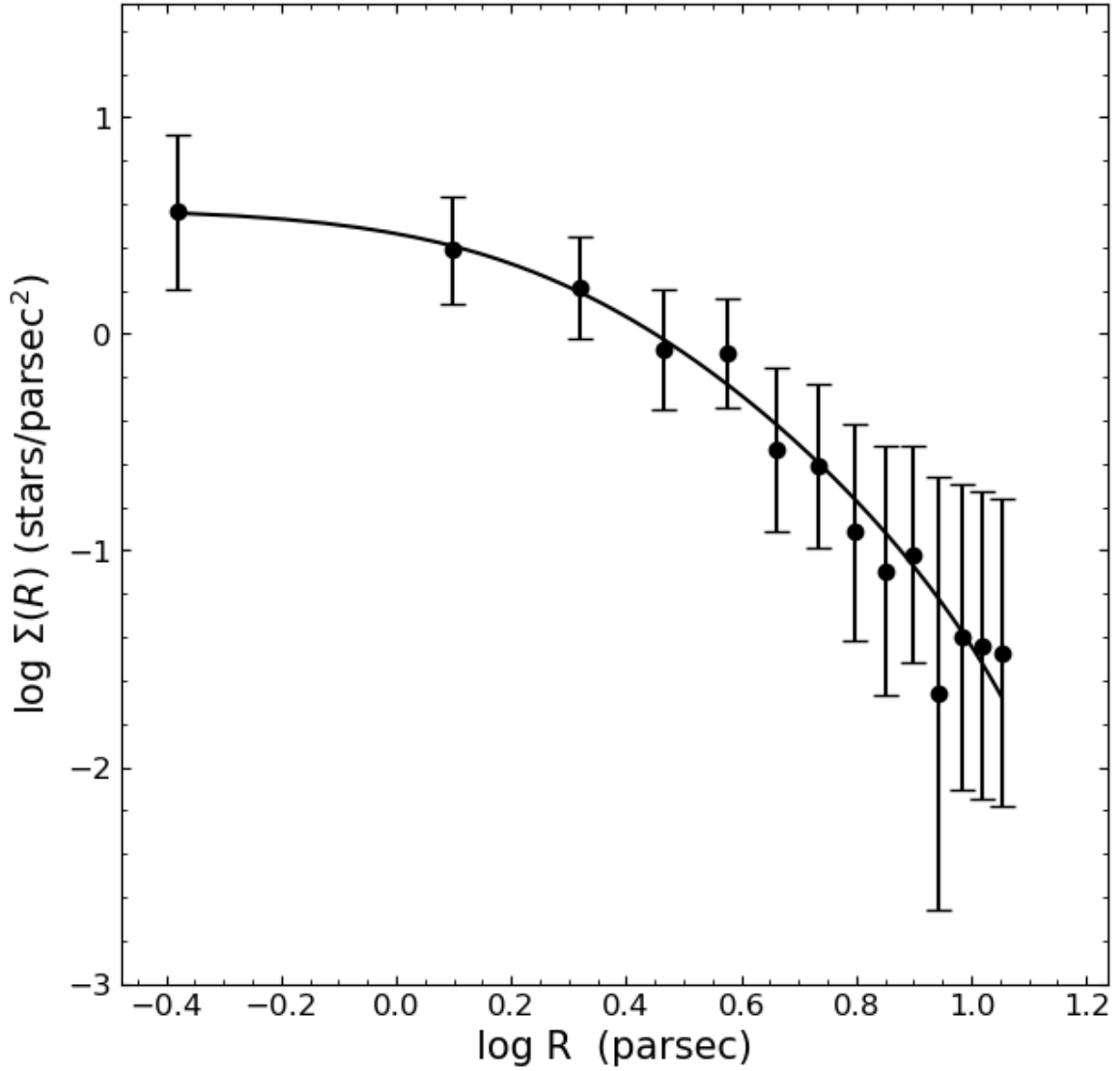


Figure 5.7: The density distribution in radial terms of the cluster member characterised using the King model fitting for NGC 6366.

The calculated total mass for NGC 6366 is $5.51 \pm 1.99 \times 10^4 M_{\odot}$. The value aligns with the reported mass of $4.73 \pm 0.56 \times 10^4 M_{\odot}$ by [Baumgardt and Hilker \(2018\)](#). Regarding the NGC 6366 cluster, the surface density profile shown in Figure 5.7 displays uniformity before and after the core radius. The King model effectively fits the observed density

as the count of star members decreases significantly and declines in density in the surroundings at approximately 0.033 stars/pc².

The King model, however, faces challenges in accurately reproducing the density beyond the tidal radius $r_t = 55.57 \pm 0.0007$ pc. The obtained r_t parameter differs considerably by twice the values from those reported in [de Boer et al. \(2019\)](#) (21.00 ± 0.51 pc) and [BHS23](#) catalogue (37.85 pc) by approximately 34.57 pc and 17.72 pc, respectively. Additionally, [Baumgardt and Hilker \(2018\)](#) report values for r_c and r_h as 0.27 pc and 2.55 pc, respectively, which are larger than the derived parameters $r_h = 5.63 \pm 0.64$ pc and $r_c = 2.09 \pm 0.234$ pc. The computed $c = 1.42 \pm 0.190$ in this work exhibits a slight discrepancy from the values $c = 0.74$ reported by [Harris \(1996\)](#) and 0.94 by [Webbink \(1985\)](#). Considering that NGC 6366 is assumed to be heavily stripped ([Paust et al., 2009](#)), it is plausible that the stripping process contributes to an increase in the core radius and the expansion of the cluster's tidal radius, as the cluster's outer parts decimate.

NGC 6397

The mass of the cluster of NGC 6397 is estimated at $7.7 \pm 1.47 \times 10^4 M_\odot$. The value is in agreement with the mass of $1.14 \times 10^5 M_\odot$ reported by [Vital et al. \(2022\)](#) but is smaller than the value of $9.40 \pm 0.32 \times 10^4 M_\odot$ from [Baumgardt \(2017\)](#). The density profile of NGC 6397, illustrated in Figure 5.8, shows a steep inner profile and flattening of the King model at the outer region of the cluster. The density gradually decreases from 8.932 stars/pc² to 0.140 stars/pc².

The density profile of the NGC 6397 has a truncated radius $r_t = 61.38 \pm 0.0004$ pc, a core radius of $r_c = 0.30 \pm 0.001$ pc, half-light radius $r_h = 2.17 \pm 0.01$ pc, and a concentration of $c = 2.23 \pm 0.003$. The r_t value exceeds the r_t values reported by [de Boer et al. \(2019\)](#) (43.00 ± 3.08 pc and 32.16 ± 0.45). The r_c and r_h are also different from the reported value of 0.08 pc by [Baumgardt and Hilker \(2018\)](#), although the half-light radius aligns closely with the value of 2.19 pc. The smaller values of r_c and r_h emphasise the compact nature of the cluster and are further supported by the estimated concentration.

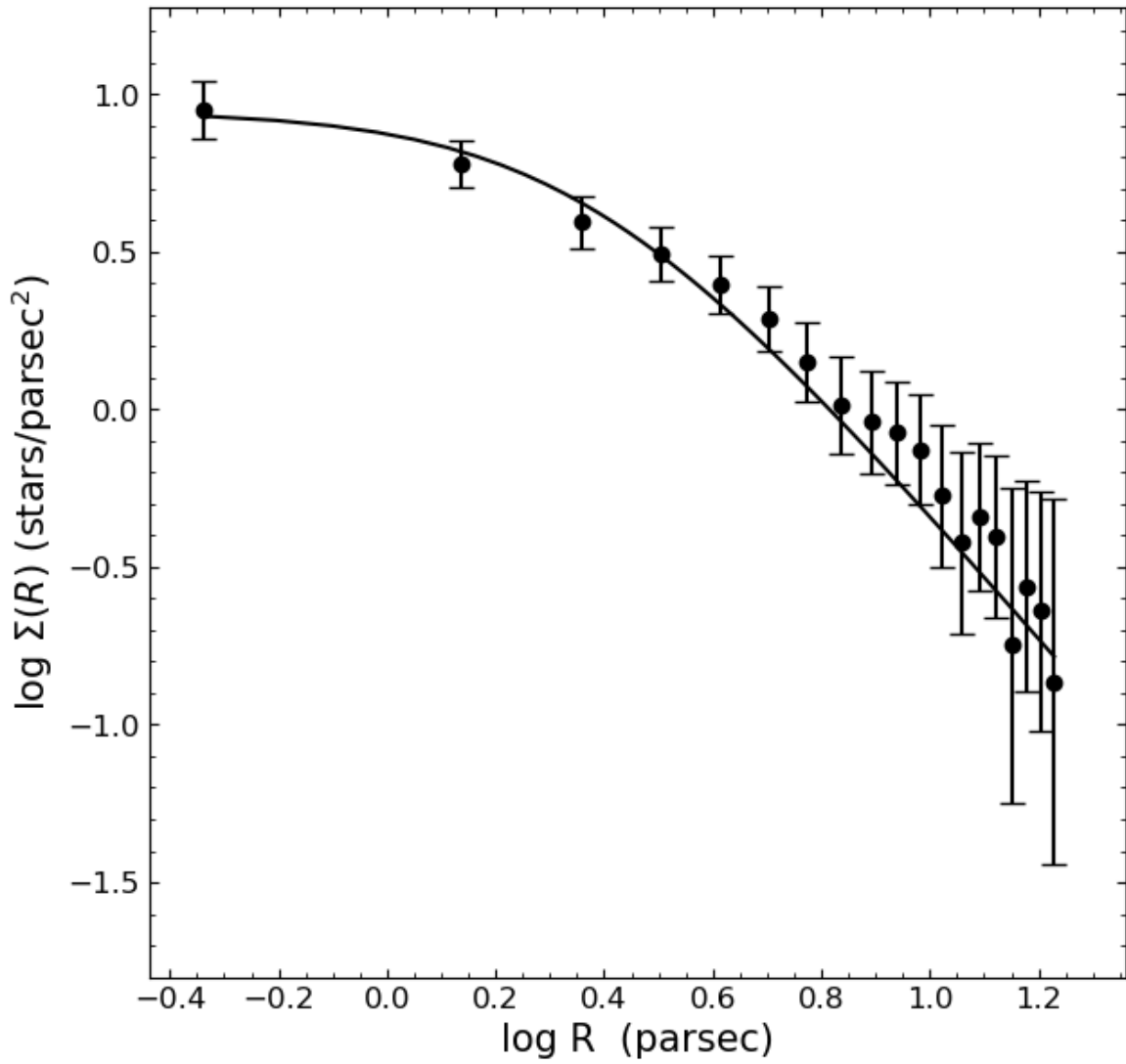


Figure 5.8: The density distribution in radial terms of the cluster member characterised using the King model fitting for NGC 6397.

NGC 6544

The total cluster mass for NGC 6544 is estimated at $1.72 \pm 4.73 \times 10^5 M_{\odot}$. The value surpasses the mass values of $6.34 \pm 3.12 \times 10^4 M_{\odot}$ reported by [Baumgardt and Hilker \(2018\)](#). The derived parameters values, for $r_h = 2.64 \pm 0.03$ pc and $r_c = 0.33 \pm 0.004$, are larger than the values 1.37 pc and 0.08 pc, respectively, as reported by [Baumgardt and Hilker \(2018\)](#).

The calculated concentration, $c = 2.39 \pm 0.02$, is also bulkier than $c = 1.63$ provided by [Harris \(1996\)](#), and the tidal radius $r_t = 81.27 \pm 0.0007$ pc, as provided in the [BHS23](#)

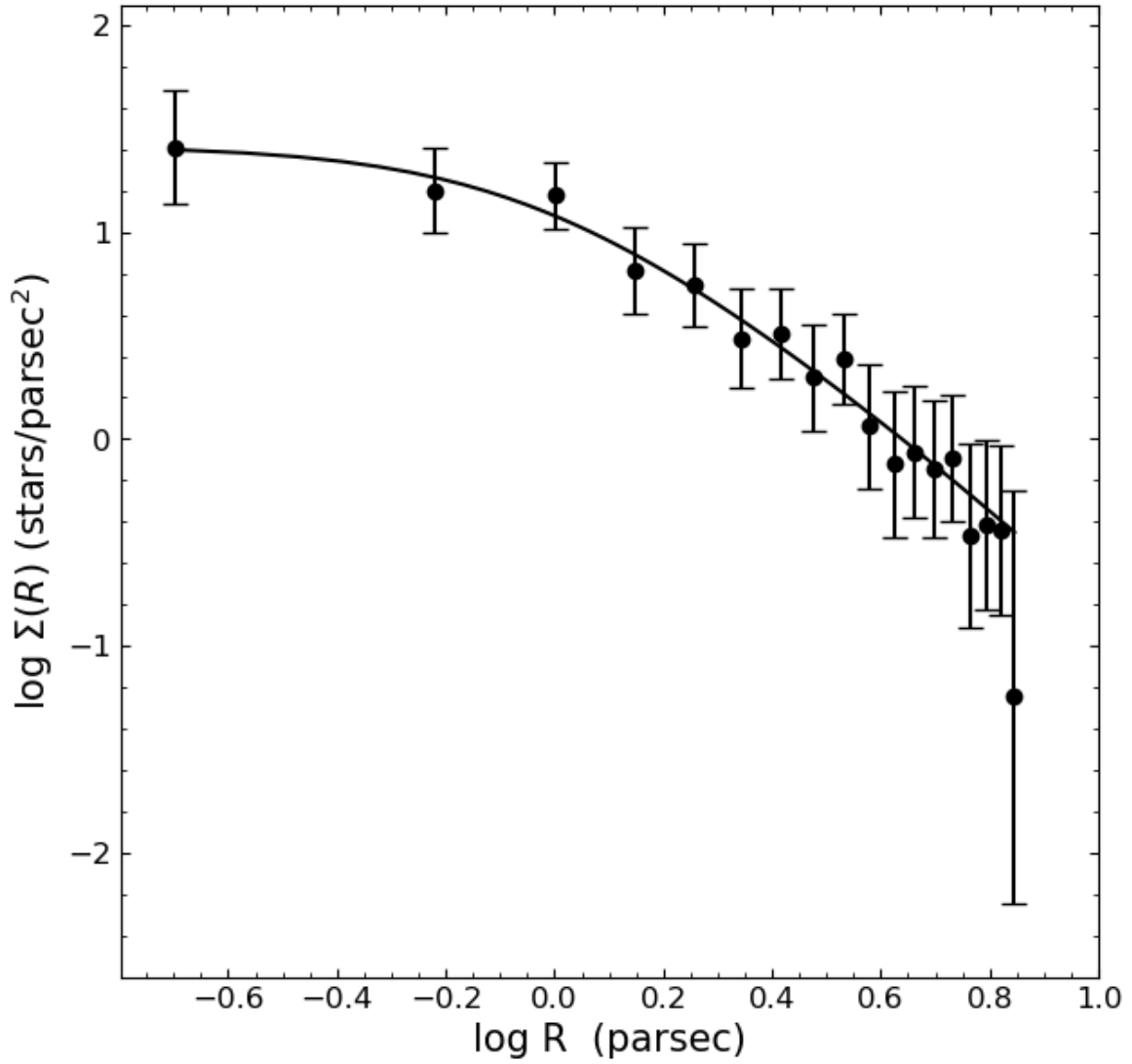


Figure 5.9: The density distribution in radial terms of the cluster member characterised using the King model fitting for NGC 6544.

catalogue. In the density profile of NGC 6544, as depicted in Figure 5.9, the observed density in the central region decreased from 6.396 stars/pc² to 0.194 stars/pc². There is also a presence of a flat core describing the model. NGC 6544 is also assumed to have undergone tidal stripping since it is located in the halo and is currently passing close to the galactic disc and projected towards the bulge (Bica et al., 2016). This could explain the flattening in the King model presented in the density profile resulting from decreased stars in the outer regions.

NGC 6656 (M22)

The estimated total mass of NGC 6656 is $3.37 \pm 2.69 \times 10^5 M_\odot$ and this falls within the range of previously reported masses, with [Marks and Kroupa \(2010\)](#) providing a dynamical mass of $2.9 \times 10^5 M_\odot$ within a tidal radius of 65 pc, and [Kimmig et al. \(2015\)](#) estimating a mass of approximately $3.9^{+1.38}_{-0.72} \times 10^5 M_\odot$. Notably, the mass estimate in this study aligns closely with the value reported by [Kimmig et al. \(2015\)](#).

Examining the density profile of NGC 6656, shown in Figure 5.10, the fitting reveals a core radius of $r_c = 2.08 \pm 0.074$ pc, the tidal radius $r_t = 101.63 \pm 0.003$ pc and half-light

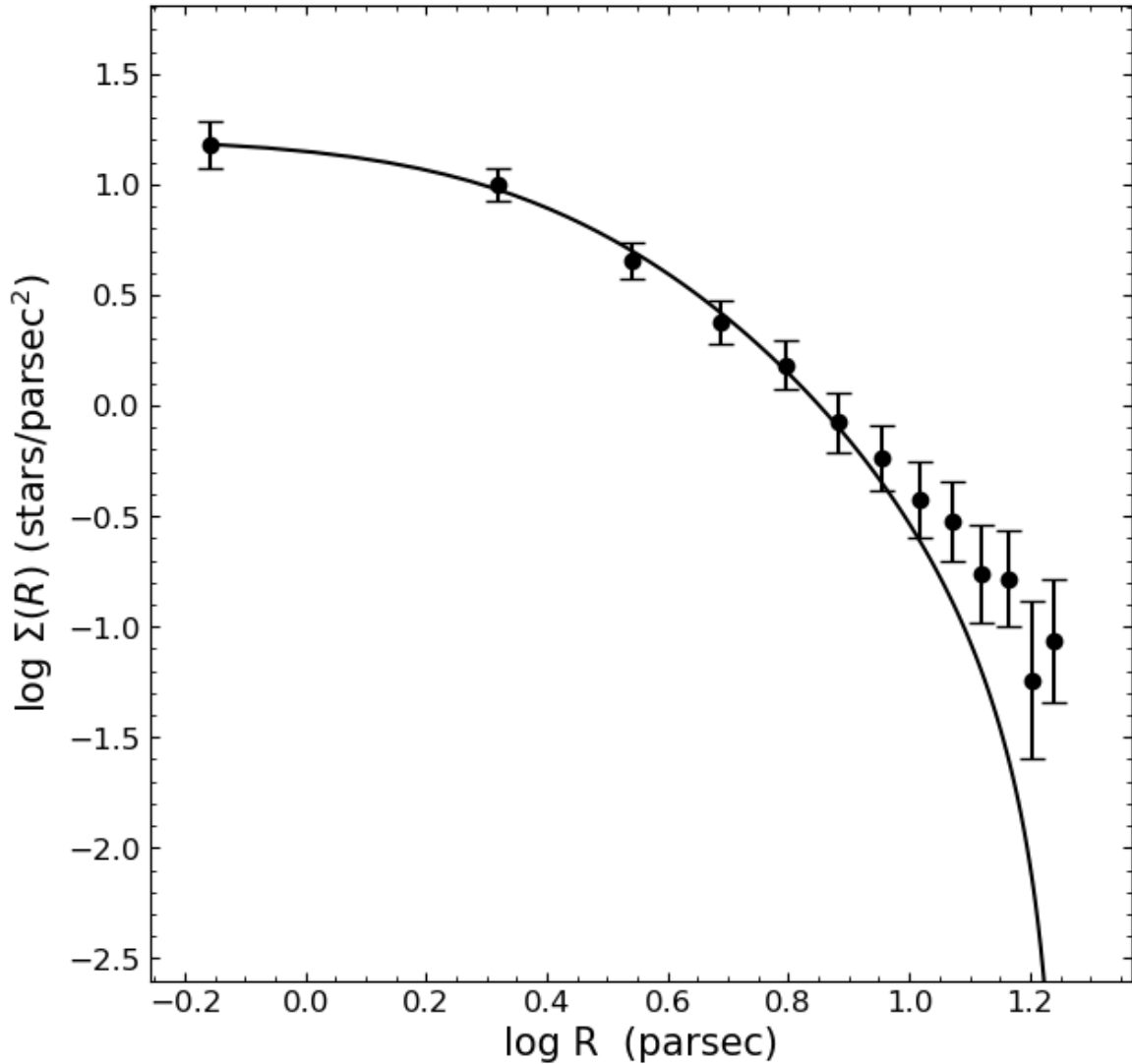


Figure 5.10: The density distribution in radial terms of the cluster member characterised using the King model fitting for NGC 6656.

radius $r_h = 7.54 \pm 0.02$ pc. A comparison with [Webbink \(1985\)](#) shows a reported r_c of 1.1 pc, smaller than the estimated value of the core radius in this work. Additionally, values from the [BHS23](#) catalogue for r_t and r_h are 76.82 pc and 3.46 pc, respectively, both lower than the estimates in this study. The concentration parameter is confirmed as 1.69 ± 0.04 , closely resembling the value $c = 1.38 \pm 0.04$ reported by [McLaughlin and van der Marel \(2005\)](#).

The King model fitting of the density profile shows a gradual decline, consistent with the expectations from the observed data in this study. Interestingly, the model extrapolates the density distribution into regions where observational data is unavailable. The extrapolation assumes that the King model remains valid in areas beyond the observed range, which could also be explained by the larger tidal radius.

NGC 6752

The total calculated mass for NGC 6752 is $1.74 \times 10^5 M_\odot$. Previous studies, such as [Marks and Kroupa \(2010\)](#), reported a mass of $1.4 \times 10^5 M_\odot$, showcasing an alignment between the estimates. The analysis of the density profile, illustrated in Figure 5.11, reveals a consistent decrease in observed density from a central peak of 5.00 stars/pc². The King model fitting displays an extrapolation beyond the observed density profile, likely influenced by the bulky tidal radius of $r_t = 81.51 \pm 0.00009$. The fitting effectively captures the overall profile of NGC 6752 and shows a shallow envelope.

[de Boer et al. \(2019\)](#)), on the contrary, conclude that NGC 6752 has a resolved tidal radius of $r_t = 32.19 \pm 0.31$ pc, which is inconsistent with the derived tidal radius. On the other hand, [Baumgardt and Hilker \(2018\)](#) report core and half-light radii values of 0.19 pc and 2.40 pc, respectively, which are slightly lower but still in good agreement with the values of $r_c = 0.36$ pc and $r_h = 2.76$ pc. Both values of r_c and r_h are lower, indicating a compact distribution of stars. The computed concentration value is approximately 2.35 ± 0.006 , slightly lower than $c = 2.5$ in [Harris \(1996\)](#). Despite [Djorgovski and King \(1986\)](#) suggesting a post-core collapse state for the cluster, the reasonable fit of the King model implies that NGC 6752 may not necessarily be in a post-collapsed state, offering a more conservative interpretation.

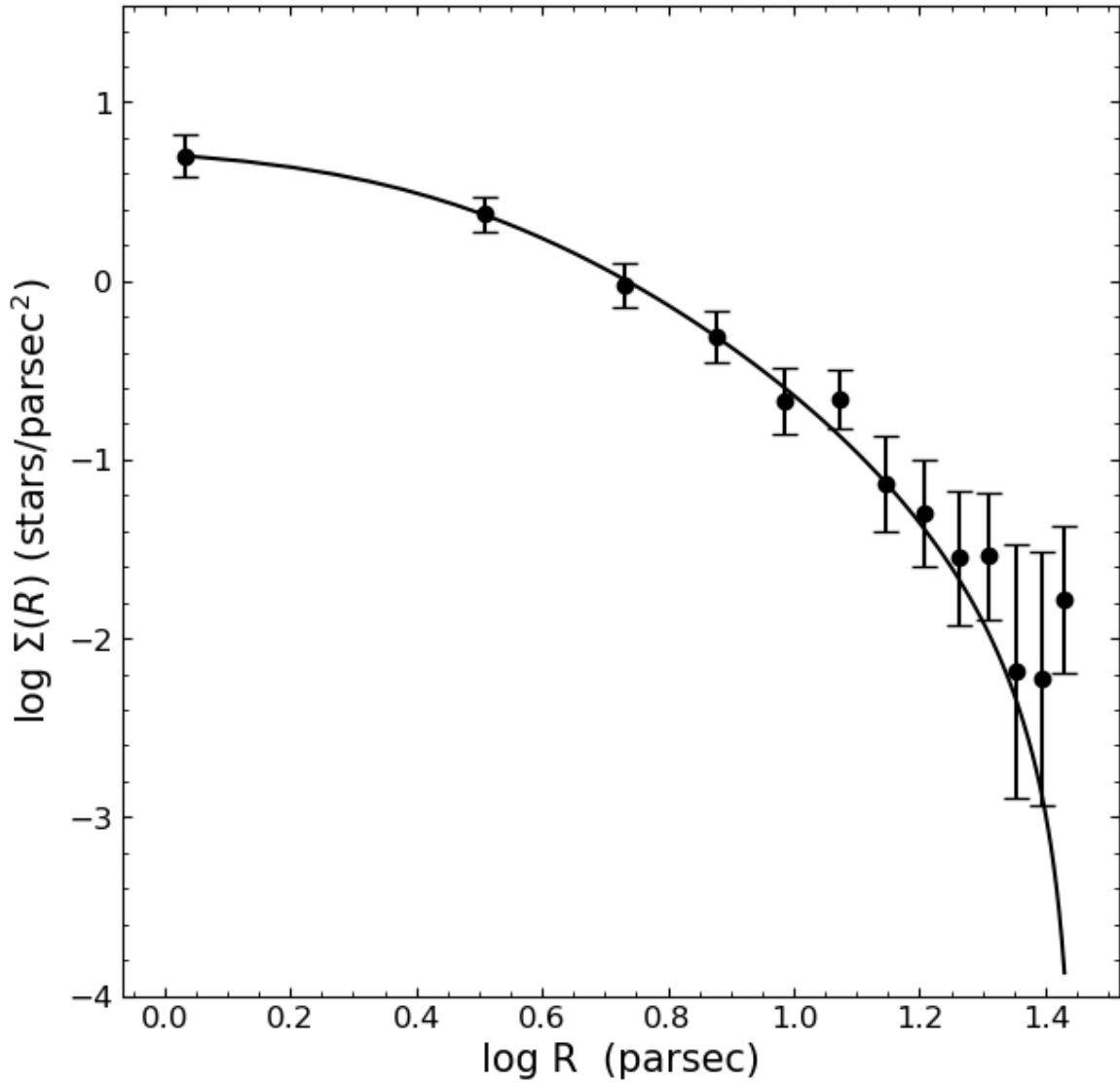


Figure 5.11: The density distribution in radial terms of the cluster member characterised using the King model fitting for NGC 6752.

NGC 6838 (M71)

The estimated mass for the NGC 6838 cluster is approximately $7.03 \pm 2.79 \times 10^4 M_{\odot}$. This value is higher than the mass $M = 2.0_{-0.9}^{+1.6} \times 10^4 M_{\odot}$ from [Kimmig et al. \(2015\)](#) and $1.7 \times 10^4 M_{\odot}$ from [Marks and Kroupa \(2010\)](#). [Cadelano et al. \(2017\)](#) suggest that NGC 6838 has lost a significant amount of its original mass due to environmental factors interacting with the inner galactic disc over the last 3 Gyr.

The NGC 6838 profiles in Figure 5.12 illustrate a gradual decline in the inner region up to $r_c = 2.04 \pm 0.196 \text{ pc}$, with surface density decreasing from 8.089 stars/pc^2 in

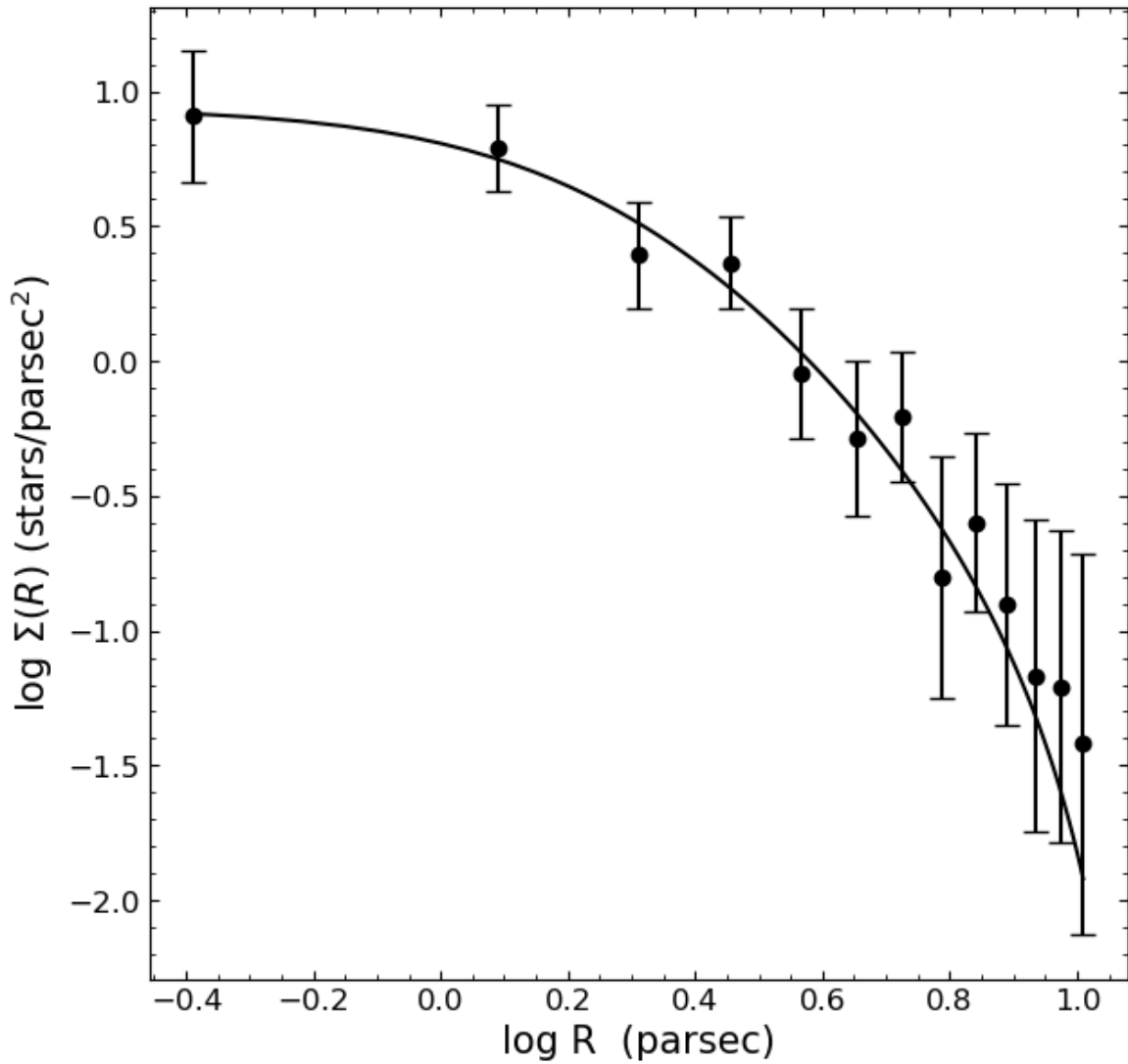


Figure 5.12: The density distribution in radial terms of the cluster member characterised using the King model fitting for NGC 6838.

the cluster centre to 0.038 stars/pc^2 . The spatial distribution shows that the King model fits the projected radial distribution of stars well because most of the stars in NGC 6838 are within the tidal radius. The estimated tidal radius, the half-light radius and concentration are $r_t = 61.18 \pm 0.0004 \text{ pc}$, $r_h = 5.83 \pm 0.560 \text{ pc}$, and 1.48 ± 0.096 , respectively. In comparison, [Cadelano et al. \(2017\)](#) report a core radius of $r_c = 56.2^{+4.5}_{-4.0} \text{ arcsec}$, a truncation radius of $r_t = 871.8^{+247}_{-164} \text{ arcsec}$ (equivalent to $r_t = 17.35 \text{ pc}$, and a core radius of $r_c = 1.1 \text{ pc}$), with a concentration parameter $c = 1.9$. While the tidal radius and core radius obtained in this work are bulkier than the reported values, there is a slight discrepancy in the computed concentration value.

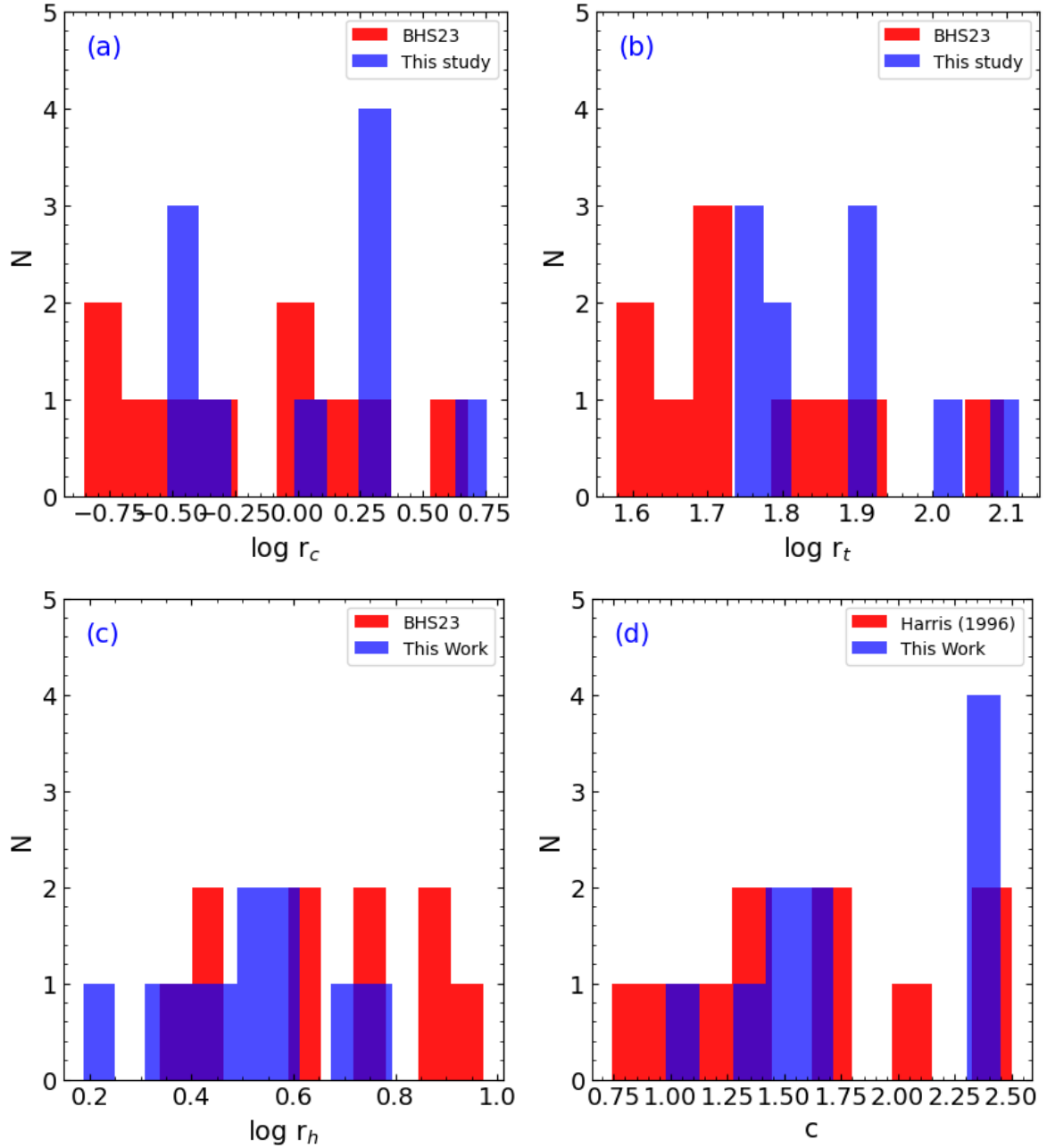


Figure 5.13: Comparison of the distribution of the King radii and concentration calculated in this work and literature values from the [BHS23](#) and [Harris \(1996\)](#) catalogues for the ten globular clusters. N is the number of globular clusters on the y-axis. The two top plots, (a) and (b), represent the core radius r_c and tidal radius r_t in parsecs. The bottom left plot, (c), is for the half-light radius r_h (in parsecs), and the concentration c presented at the bottom right (d).

Figure 5.13 shows the number of globular clusters, N , compared to the structural parameters of $\log r_c$, $\log r_t$, $\log r_h$, and the concentration parameter, c , derived in this dissertation, and the literature values. The values on the x-axis represent the range within which this study's structural parameters for the ten globular clusters fall. The concentration parameter is calculated from the $\log(r_t/r_c)$. In comparing the structural parameters from this study's sample in Table 5.3 and those presented in previous studies displayed in Figure 5.13, the estimated average accuracy of the c measurement is approximately 0.2 in comparison to the Harris (1996) catalogue; the measurements of $\log r_c$ approximately 0.56; and for $\log r_t$ and $\log r_h$ approximately, 0.21 and 0.34 from the BHS23 catalogue. The cluster BH 140 has the lowest concentration value because it is less concentrated, and the stars are distributed more uniformly across the clusters, with less pronounced differences in density between the central and the outer regions. Also, a smaller value of c means that the r_c is significantly smaller than the tidal radius r_t , as can be observed from this study's results. For clusters with smaller r_t , this shows a higher stellar density in the cluster's central region, a compact inner region, and enhanced stellar interactions.

Chapter 6

Discussion and Conclusion.

In this study, it is established that the Leonard-Merritt method ([Leonard and Merritt, 1989](#)) can be used to determine the masses of the globular clusters. The results are within 1σ of the masses determined by [Baumgardt and Hilker \(2018\)](#) using the N -body modelling and the [BHS23](#) catalogue obtained from various literature.

Consider, for instance, the number of stars and distances in this study's clusters tends to be lower than in literature, which, on the face of it, leads to limitations in statistical precision. According to the Leonard-Merritt equation, lower distances correlate with lower mass determination. Employing the literature distances for mass determination results in better agreement between determined masses and the literature masses, suggesting that the differences in the masses determined in this study can be partly explained by the differences in the distance determination.

The use of different centre determination techniques to determine the centres is another potential cause of the differences. The differences fall within 1σ and thus would not lead to a concern of the globular clusters not coinciding with the clusters in literature.

Distances

In Table 5.1, the photometric distances derived in this study are systematically low for most globular clusters compared to the distances from Baumgardt and Hilker (2018, hereafter BH18) and Baumgardt and Vasiliev (2021, hereafter BV21) provided in Table 6.1. The distances for the globular clusters from BH18 are the kinematic distances, determined using line-of-sight velocities derived from archival spectra from the ESO and Keck data archives, for the globular clusters NGC 104, NGC 6397, NGC 6656, and NGC 6752. The best-fitting distance is determined by fitting an N -body simulation grid to each cluster (BH18). The distance estimates for the other five clusters (NGC 3201, NGC 6121, NGC 6366, NGC 6544, and NGC 6838) are taken from the literature (BH18). The BH 140 globular cluster is not in BH18.

ESO/VLT and Keck Spectral				Literature	
Cluster	N_{RV}	D	Mass	D	Mass
	(kpc)	($M_{\odot}$)	(kpc)	($M_{\odot}$)	
BH 140	–	–	–	$4.808^{+0.258}_{-0.233}$	$6.12 \pm 0.51 \cdot 10^4$
NGC 104	2896	4.41	$7.79 \pm 0.05 \cdot 10^5$	4.521 ± 0.031	$8.53 \pm 0.05 \cdot 10^5$
NGC 3201	690	4.70	$1.49 \pm 0.09 \cdot 10^5$	$4.737^{+0.043}_{-0.042}$	$1.93 \pm 0.03 \cdot 10^5$
NGC 6121	2981	2.14	$9.69 \pm 0.26 \cdot 10^4$	$1.851^{+0.016}_{-0.015}$	$9.04 \pm 0.11 \cdot 10^4$
NGC 6366	220	3.30	$4.73 \pm 0.56 \cdot 10^4$	$3.444^{+0.051}_{-0.050}$	$4.66 \pm 0.19 \cdot 10^4$
NGC 6397	2945	2.48	$8.89 \pm 0.16 \cdot 10^4$	2.482 ± 0.019	$8.24 \pm 0.14 \cdot 10^4$
NGC 6544	10	2.46	$6.34 \pm 3.12 \cdot 10^4$	$2.582^{+0.059}_{-0.057}$	$8.15 \pm 0.56 \cdot 10^4$
NGC 6656	720	3.10	$4.16 \pm 0.05 \cdot 10^5$	3.303 ± 0.037	$4.70 \pm 0.06 \cdot 10^5$
NGC 6752	1095	4.30	$2.39 \pm 0.04 \cdot 10^5$	4.125 ± 0.041	$2.61 \pm 0.03 \cdot 10^5$
NGC 6838	237	3.86	$4.91 \pm 0.47 \cdot 10^4$	4.001 ± 0.050	$3.78 \pm 0.14 \cdot 10^4$

Table 6.1: Shows the cluster name in column 1, the number of stars with radial velocity in column 2 and parameters from BH18 displayed in column 3 for the distance and the mass of the cluster in column 4 derived parameters from ESO/VLT and Keck spectral archival using radial velocity. The distances in column 5, taken from BV21, are derived using Gaia archival data and the mass in column 6, taken from the BHS23 catalogue.

BV21 determines the mean distances to the globular clusters using five different distance estimates: cluster parallax distances, kinematic distances, subdwarf distances, star count distances, and literature distances. The mean parallax, derived from VB21, found by averaging the cluster parallax for individual member stars from the *Gaia* EDR3 catalogue, is considered for the parallax distances, taking the parallax overcorrection and systematic error ($\Delta\varpi = 0.007$ mas and $\epsilon_{\varpi} \sim 0.01$ mas) suggested by VB21 into account. The kinematic distances are derived from line-of-sight and proper motion velocity dispersions. Line-of-sight velocity dispersions are taken from BH18 and Baumgardt et al. (2019a,b), calculated using *Gaia* DR2 and the Anglo-Australian Observatory (AAO) data archive, and based on Multi-Unit Spectroscopic Explorer (MUSE) data from Kamann et al. (2018). Proper motion velocity dispersions are derived from data taken from the *Gaia* EDR3 and the *HST*. The subdwarf star distances are obtained using *Gaia* EDR3 parallaxes from nearby stars. The star count distances are derived by combining the photometric data from *HST* obtained through the MS fitting with the Advanced Camera for Surveys (*ACS*)/*WFC* in the F606W and F814W bands.

Error propagation is used to compute the ratio of those uncertainties, taking into account the distance errors derived in this work because BH18 provides values for distances but not their uncertainties. Figure 6.1 shows the average relationship between the distances obtained in this study and the distance estimates presented in BV21, denoted as $D_{\text{This study}}/D_{\text{BV21}}$, and in BH18, denoted as $D_{\text{This study}}/D_{\text{BH18}}$, as a function of the distance estimates. The average ratio of the distances in this study to those in the BV21 catalogue is $D_{\text{This study}}/D_{\text{BV21}} = 0.97 \pm 0.04$, while it is $D_{\text{This study}}/D_{\text{BH18}} = 0.98 \pm 0.05$ for the BH18 catalogue. Notably, the unweighted mean of the offsets is very close to unity, indicating a high degree of agreement between this study's distance estimates and those of BV21 and BH18. Generally, this study's estimates are approximately 2.85% and 2.28% smaller, respectively, in comparison to the other distances mentioned, and thus, the study's method does not exhibit significant biases.

While the results align well with the distance estimates from the literature, it is noteworthy that Bailer-Jones (2015) raised a critical point regarding the challenge of determining distances from parallax measurements, specifically when the fractional parallax error exceeds roughly 20%. A higher fractional parallax error introduces significant uncertainty in determining the true distance to the celestial object. For example, for a star whose measured parallax has a higher fractional error than 20%, the calculated

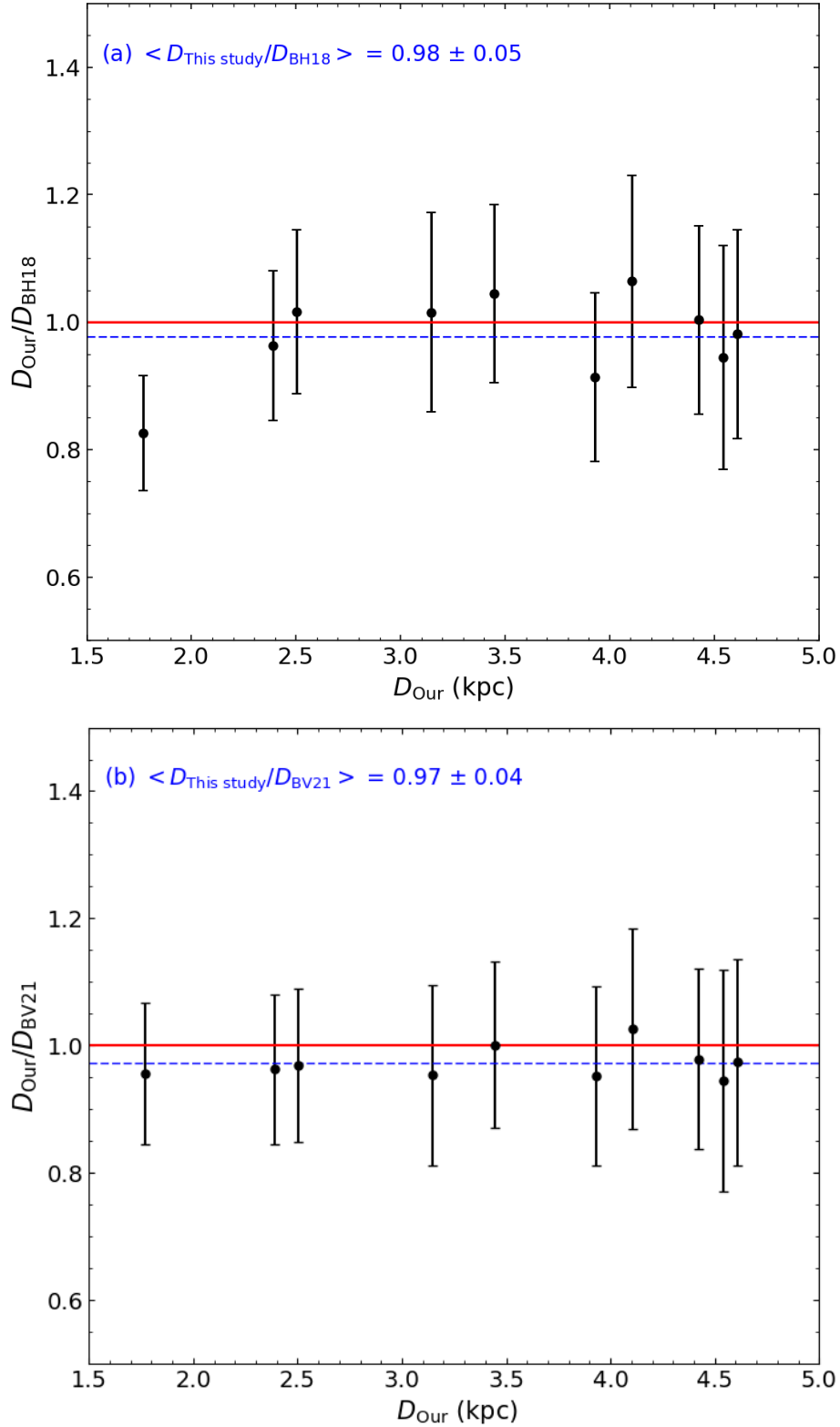


Figure 6.1: Comparison of the optimal cluster distance, obtained from trigonometric parallax distance (this work), with kinematic distances given by BH18 in panel (a) and photometric distances derived from BV21 in panel (b). The black points indicate the fractional difference in distance, relative to this study’s distance estimates, for each globular cluster. The dashed lines represent the mean distance offsets, and the solid line highlights one. Overall, the agreement between the distances is excellent.

distance to that star may result in a higher degree of uncertainty. This uncertainty, in turn, can affect subsequent analyses and interpretations that rely on accurate distance information, such as estimating the star’s luminosity, size, or other physical properties. This situation is particularly relevant, affecting roughly 80% of the stars in the *Gaia* catalogue. The issue arises because parallax measurements inherently carry an expected systematic uncertainty of 0.01 mas, as reported in [BV21](#). Although there is a straightforward relationship between parallax and distance, the process of inverting a parallax to obtain a distance is valid only in the absence of measurement errors. Determining the distance from a parallax becomes an inference problem, given that measurement errors are always present.

Centre determination

Comparison of this study’s centre coordinates with the [VB21](#) dataset provides confidence in the precision of this study’s centre determination methods for globular clusters. In particular, the centre coordinates in the study exhibit only a negligible overall percentage difference of 0.02% compared to those in [VB21](#), thus affirming the effectiveness of Gaussian fitting for centre determination in globular clusters. Although there is a high degree of consistency with literature values, the deviations observed in the case of BH 140 emphasise the need for meticulous data collection and cautious interpretation, especially for clusters with limited observational data.

The distance and centre determination results in this study align well with previous studies. Despite the differences in distances, the results are effective, as reported by [Watkins et al. \(2013\)](#) representing a case wherein the photometric and kinematic distances can differ. Notably, the distances derived in this study are consistently low.

Mass, density profiles and structural parameters

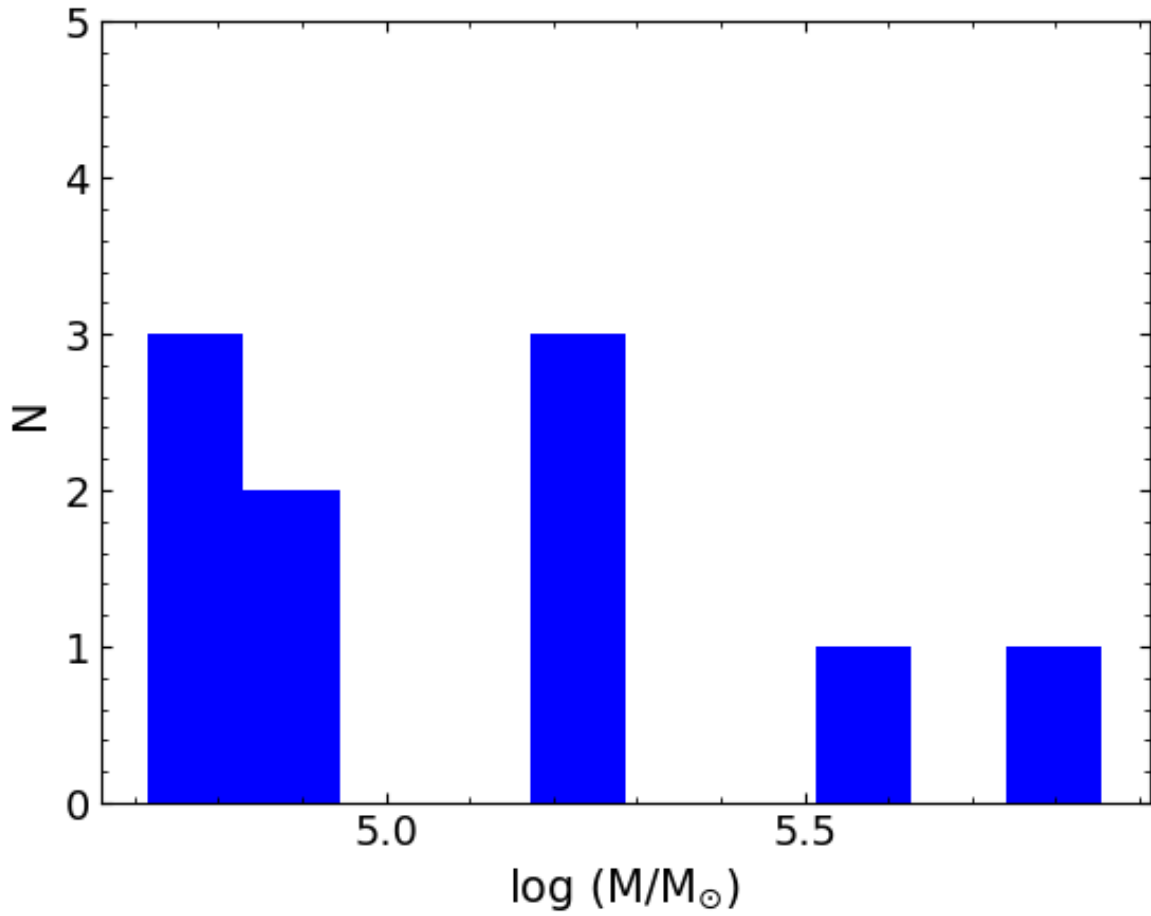


Figure 6.2: Illustrates the distribution of the globular cluster masses. N is the number of globular clusters in this study's sample compared to the logarithm of the mass, estimated in $M_\odot$.

The results in Table 5.3 reveal that the estimated masses of the globular clusters exhibit a notable diversity, ranging between $\log(M/M_\odot) \simeq 5 - 6$ in Figure 6.2. The NGC 104 cluster with a mass of $7.16 \times 10^5 M_\odot$ stands out as order of magnitude more massive than others. On the other hand, the clusters (BH 140, NGC 6121, NGC 6366 and NGC 6397) with masses $\sim 5 \times 10^4 M_\odot$, are among the least massive in the sample. The more massive clusters have stronger gravitational binding, which helps to hold stars within the cluster together without being stripped away and resisting disruptive forces, which is not the case in lower-mass clusters. From Table 5.3, it is also evident

that the relationship between the distance, representing the position of the globular cluster, and its mass is not straightforward. The mass of the clusters is not solely determined by its spatial location. Several factors, such as diverse stellar populations, dynamical and stellar evolution, and the initial mass (some clusters may have formed with smaller initial masses, while others had more massive progenitors ([von Hoerner, 1957](#); [Kruijssen, 2014](#); [Webb and Leigh, 2015](#); [Choksi and Kruijssen, 2021](#))) and can influence the observed mass of a cluster, and these factors may not necessarily correlate with the cluster's distance.

The Leonard-Merritt technique provides very good agreement with the literature values for the globular clusters in this study. By conducting a thorough comparison with the masses reported by [BH18](#), derived using N -body modelling by comparing the velocity dispersion profiles based on line-of-sight velocities with SB profiles and mass-to-light ratios, as well as masses from the [BHS23](#) catalogue, obtained from various literature given in Table 6.1, the estimates demonstrate an agreement of 75.64% and 72.50%, respectively. Specifically, the mass measurements for nine out of ten globular clusters studied fall within 1σ of the mass estimates from [BH18](#) and the [BHS23](#) catalogues. The percentages indicate a significant level of consistency in which this study's calculated masses align closely with those reported in the referenced literature for the ten globular clusters. The Leonard-Merritt method and the velocity dispersion from the line-of-sight velocities method both include the variation of velocity dispersion as a function of the radius from the cluster centre. The velocity dispersion is expected to be higher in the central region of a cluster, primarily due to the higher density and the presence of more massive stars. As one moves towards the outer region, the velocity dispersion decreases gradually ([BH18](#)). The Leonard-Merritt method, however, like any other modelling approach, does not capture all features of a globular cluster: (1) the distribution of stellar mass within the cluster and energy equipartition, (2) the rotation of the cluster, and (3) the degree of anisotropy in the velocity distribution ([Leonard and Merritt, 1989](#)).

Using the distance provided in Table 6.1 for [BH18](#) and [BV21](#) to estimate the masses, based on the Leonard-Merritt technique, reveals a notable correlation; as the distance from the cluster increases, the mass of the globular cluster also increases. The overall agreement percentage between the masses in this study and the masses derived using distances from [BH18](#) is approximately 78.52%, and for the mass estimates from the

BHS23 catalogue, it stands at 74.51%. Nine out of ten globular clusters exhibits the mass measurements falling within the 1σ range of the literature values from BH18 and the BHS23 catalogue. The percentages succinctly reflect the level of agreement between the masses, affirming the consistency of the results. The observation highlights the direct influence of distance on the derived mass of the cluster. The preference for the Leonard-Merritt mass model in this study is grounded in its superior accuracy in estimating mass (D’Souza and Rix, 2013).

Examination of the star count density profiles for all ten globular clusters reveals a drop in density in the outer envelope of the clusters, as expected, and that the King model consistently provides a good fit to the observed RDPs. Globular clusters were previously classified into two groups: (i) King model clusters with a flat isothermal core and steep envelope, and (ii) collapsed core clusters that follow a pure power law with an exponent of -1, with 20% of the galactic globular clusters belonging to the second division (Djorgovski and King, 1986; Chernoff and Djorgovski, 1989; Trager et al., 1995). Any globular cluster with a concentration of $c \gtrsim 2 - 2.5$ is considered collapsed, on the verge of collapsing, or just beyond collapse (Meylan and Heggie, 1997).

For 70% of this study’s sample, the King model provides a satisfactory fit, as demonstrated by BH 140 and NGC 104 in Figures 5.3 and 5.4, indicating a flat isothermal core and a steep envelope. Furthermore, NGC 6121, NGC 6656, and NGC 6752 exhibit an extrapolation of the King model, coupled with a uniform gradual decline or shallow envelope in the observed density profile for the NGC 6366 and NGC 6838 clusters. The remaining 20% of the sample (represented by NGC 6397 and NGC 6544) displays a slight flattening in the core, predominantly pronounced in the outer region, and suggests that the cluster’s density profile is influenced by complex interactions, which indicates that the King model successfully captures the structure of the cluster. In contrast, NGC 3201 shows an explicit deviation from a flat core, indicating a tendency to follow a power-law distribution similar to the SB profile of Terzan 2, as documented in Meylan and Heggie (1997). NGC 6397, NGC 6544, and NGC 6752 have $c \gtrsim 2$, signifying core-collapsed clusters, and are expected to follow a power law. In these cases, the King model may not fully represent the clusters. Harris (1996) also categorises these clusters as core collapsed, except that the concentration value of NGC 6752, approximately 1.6, diverges from this study’s derived c value. NGC 104, despite the concentration falling within the collapsed cluster range, is well defined by the King model, potentially due

to its short dynamical times leading to collapse within a fraction of the Hubble time (Meylan and Heggie, 1997).

For structural parameters, the discrepancies may be attributed to the different methodologies applied to determine the parameters and the number of stars. The structural parameters might be affected by various processes that influence their size, density distribution, and overall dynamics. These processes include tidal stripping, which can modify the tidal radius to be larger and affect the overall shape and extent of the cluster (Dehnen et al., 2004; Ramos et al., 2015); core collapse triggered by various dynamical interactions between stars within the cluster, like mass segregation, whereby, over time, more massive stars tend to sink towards the centre due to interactions with other cluster members and gravitational forces, and low-mass stars becoming distributed in the outer parts of the cluster (Madrid et al., 2012). Figure 6.3 illustrates the process of mass segregation for NGC 104 and NGC 6752 clusters. The first observation was made by Paresce et al. (1995) for NGC 104 and Shara et al. (1995) for NGC 6752. Mass segregation influences the parameters (central density, core radius, and tidal radius) by resulting in a higher central density and a smaller core radius due to the concentration of massive stars in the central region. Globular clusters such as NGC 104 and NGC 6121, with a more pronounced central concentration and a denser core, as indicated in their spatial distribution are often indicative of mass segregation, which is expected in star counts, since they measure different radial profiles because of this phenomenon.

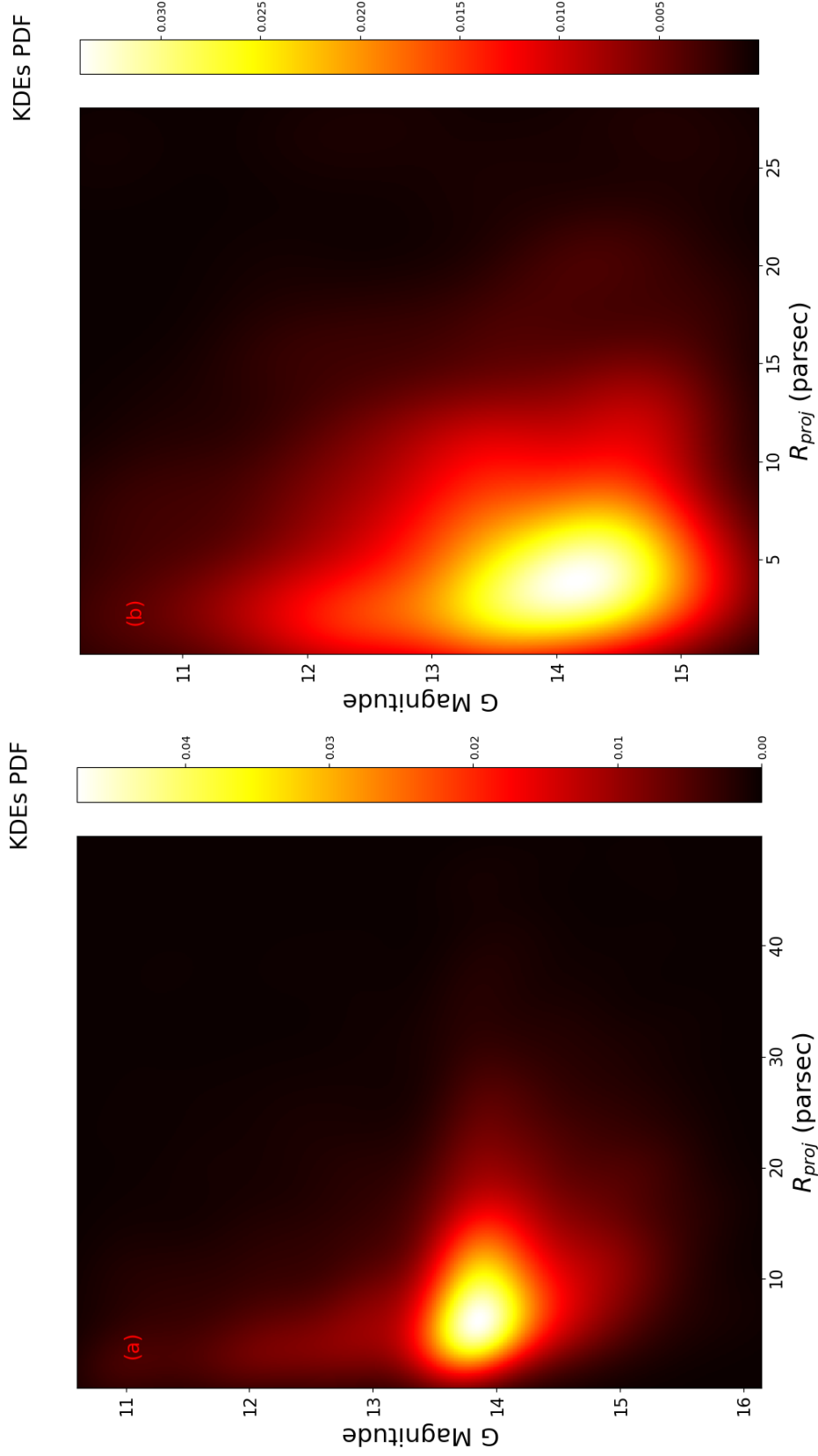


Figure 6.3: Illustration of G magnitude against projected distance in parsecs from the centre of the NGC 104, panel (a), and NGC 6752 in panel (b) globular clusters. The plot shows that more massive stars, indicated by smaller magnitudes, are concentrated in the inner regions of the clusters. The colour bar accompanying the plot represents the Probability Distribution Function (PDF) of the data computed through *Kernel Density Estimation* (KDE). Darker tones on the colour bar correspond to less dense regions, while brighter tones signify denser areas.

In conclusion, the globular cluster property analysis provides valuable insights into their diverse nature and evolutionary histories. Comparisons with previous studies show good agreement with these estimates, emphasising the accuracy of the Leonard-Merritt mass model. Analysing star count density profiles reveals that the King model generally provides an excellent fit for observed RDP. Most clusters exhibit characteristics of the King model clusters, with only a few deviating slightly, suggesting complex interactions. Structural parameter discrepancies may arise from processes such as tidal stripping and mass segregation that influence size, density distribution, and overall dynamics. The comprehensive analysis contributes to understanding globular cluster diversity, shedding light on the intricate interplay of mass, structure, and evolutionary processes.

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