Exact approaches towards solving generator maintenance scheduling problems

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Abstract

The holistic management of a national power system is faced with multiple long and short-term scheduling challenges. One of the key long-term scheduling problems is referred to as the generator maintenance scheduling problem (GMS). The GMS problem is a complex combinatorial scheduling problem with the main focus of finding an optimal schedule for execution of planned maintenance while satisfying operating, maintenance, financial and national grid demand constraints.

Three different deterministic mathematical modelling formulations were applied to try and solve realistic industry-sized GMS scenarios, within an acceptable time frame, while maximising net present value (NPV) over the selected planning horizon. The first being the well-known time index formulation which is frequently applied in the literature to solve GMS problems. The main drawback of employing mathematical deterministic approaches is increased computational complexity with enlarged solution search space. For this reason, network flow and graph theory formulations were considered as possible alternatives to the general time index formulation. Graph theory and network flow formulations are collectively known as resource flow formulations. A general resource flow formulation was employed which showed promising results for improved computational efficiency compared to the time index formulations, however, it was not capable of accounting for variability of resources, demand *etc.* over the planning horizon. To counteract this drawback a novel resource flow formulation was developed.

A comparative study was done where all three deterministic formulations were applied to solve multiple GMS case studies of varying size. Both the resource flow formulations were proven to be computationally superior in most cases. There were some instances where the time index formulations were computationally faster than the resource flow formulations, but ultimately the resource flow formulations were preferred when solving realistic industry-sized GMS scenarios.

Keywords
Optimisation, Scheduling, Generator, Maintenance, GMS

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List of Acronyms

ACO: Ant Colony Optimisation
DP: Dynamic Programming
ED: Economic Dispatch
GA: Genetic Algorithms
GAMS: Genetic Algebraic Modeling System
GMS: Generator Maintenance Scheduling
GO: General Overhaul
IEEE: Institute of Electrical and Electronic Engineers
IN: Interim Inspection
IP: Integer programming
LIFO: Last In First Out
LP: Linear Programming
MCF: Multi Commodity Flow
MIP: Mixed Integer Programming
MILP: Mixed Integer Linear Programming
MO: Multi Objective
NPV: Net Present Value
OEM: Original Equipment Manufacturer
PSO: Particle Swarm Optimisation
RAM: Random Access Memory
SA: South-Africa
TMS: Transmission Maintenance Scheduling
TS: Tabu Search
UC: Unit Commitment
VBA: Visual Basic for Application

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Chapter 1

Introduction

1.1 Background and rationale

Power generation forms the backbone of all world economies. Without electricity, economic development and growth will not be possible or may be hampered. With South-Africa experiencing a credit downgrade to junk status by two of the three major international credit rating agencies the growth and stability of the SA economy are clearly unstable. The South African economy decreased by 2.2 % for the first quarter of 2018 quarter-on-quarter while the electricity industry experienced negative growth of 0.5 % (Anon, 2018). These statistics paint a clear picture that the South African economy is on the edge of a financial and economic downfall and therefore all measures possible should be taken to try and avert the realization of economic collapse. Availability, sustainability and reliability of power supply are of key importance when considering improving the growth of the South African economy.

Power generation is the conversion of chemical and mechanical energy into electric energy. Electricity can be generated by utilising specialized process plants to convert the energy stored in coal, oil, gasoline, gas, uranium, wood, water, solar and wind into electricity. Coal fired power stations contribute to more than 40 % of global electricity generated (Linder, 2016). Although there is a big drive for renewable energy production, third world or developing countries like South-Africa still mainly rely on fossil fuel for power generation. Coal is a cheap fuel source and South-Africa use to have abundant amounts of coal reserves which prompted the construction of coal fired power stations to act as base load generating units. The chemical energy in coal is converted by means of combustion into heat energy which heats up steam to superheated conditions. The steam then flows through a turbine train where the thermal energy is converted into motion or mechanical energy in the form of rotational movement of the turbine blades. The turbine train drives a generator rotor within stator windings producing electricity. The electricity generated is then supplied via transmission lines and transformers to satisfy the electricity demand of the country. This conversion process from raw material to electricity is made possible by large fossil-fueled power plants. To ensure stable future electricity, countries like South-Africa should plan ahead to build and commission new power stations while executing effective planned maintenance strategies to ensure older stations can supply electricity as per design. South-Africa cannot only rely on new built power stations while neglecting maintenance on existing generating units to provide relief to the already constrained national grid.

From early 2008 to late 2015 the South-African power system was characterized by load shedding due to increased rates of asset failures and insufficient capacity planning (Anon, 2017). In some part policy and regulatory uncertainty also played a role in the realization of the energy crisis during 2008 (Joffe, 2012). Since then the South African power utility started construction on two new fossil fueled power stations, one pumped storage station, one wind, and one solar facility to restore the stability of the national power grid. However, some of these new stations under construction are behind schedule and significantly over budget (Goldberg, 2015). This creates a scenario where the existing generating capacity should be available and reliable at all times to prevent the risk of load shedding.

Figure 1.1 gives a representation of the current energy mix within South-Africa. Eskom currently supplies approximately 95% of the electricity demand in South-Africa and 45% of the electricity demand of the African continent. It is important to note that fossil fuel power generation units make up most of the generation mix and act as base load generating units. South-African coal fired power stations are responsible for 93% of electricity generated within the country (Linder, 2017). It is therefore critical that the coal fired base load units are available and reliable when required. If all generating units were in a working condition a total of 55 000MW could be available for production per day, however currently Eskom is struggling to reach the 30 000MW mark due to unforeseen breakdowns, process inefficiency and maintenance related load losses *etc.* (Fakir, 2018).

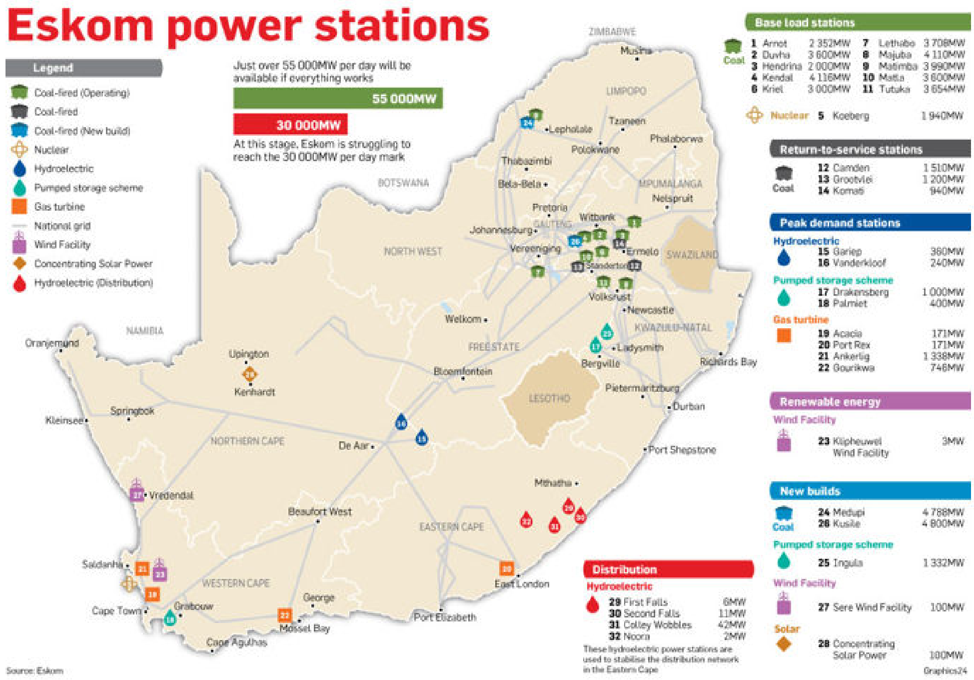

Figure 1.1: South African Eskom Power Mix (Fakir, 2018)

From Figure 1.2, which was presented in the medium-term system adequacy outlook for 2017 to 2021 by the SA power utility Eskom, it is clear that the demand has remained fairly constant from 2008. This can mainly be attributed to low economic growth, load shedding, system constraints, higher electricity prices, and energy efficiency measures implemented by industry. Even though the consumption of electricity has remained fairly constant from 2008 (Linder, 2017); a decline in reserve margin over the past few years from 15% to less than 8% was observed due to an increased rate of plant failures (Kolb, 2009). South-African power generation units are old which means increased levels of maintenance are required to ensure efficient operation. Reduced reserve margins and financial constraints result in planned maintenance being deferred which increase the possibility of unplanned failures causing grid planning instability and great financial loss to the power utility.

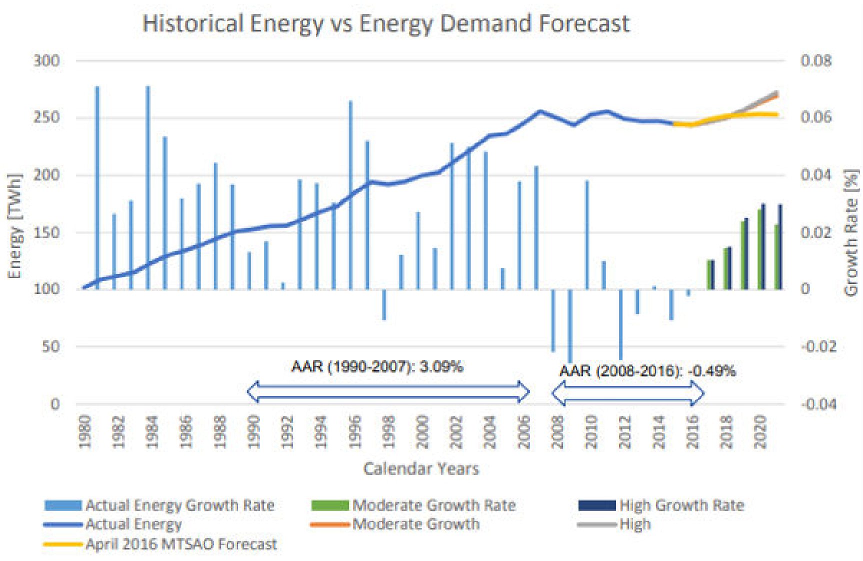

Figure 1.2: Historical energy demand vs future energy demand forecast for SA (Medium-term system adequacy outlook 2017 to 2021)

Grid planning instability in some instances force power generation units to operate at maximum load factors (percentage of maximum capable generation capacity) and when base load generating units cannot meet the demand more expensive peak load units *e.g.* gas turbines are committed to fill the gap (Linder *et al.*,2015). This puts a significant financial strain on the power utility and the South-African economy as a whole. If the SA power utility is not able to meet the national demand this may once again lead to shedding of load which can negatively impact industries such as mining, manufacturing, construction, steel making, food, financial sector *etc.* (Goldberg, 2015). The above-mentioned constraints further stress the importance of optimizing the generation fleet's planned maintenance schedules so as to ensure that the national demand is met while still having sufficient excess reserve available to execute planned maintenance (Linder *et al*, 2015; Goldberg, 2015).

Power plant manufacturers and original equipment manufacturers recommend preventative maintenance actions, inspections and maintenance intervals to prevent unforeseen failures of equipment that could result in downtime of generation units (Sergaki & Kalaitzakis, 2002). The major maintenance intervals are determined by the equivalent operating hours based on operating time, load cycles, start-up frequency, condition based and time-based activities (Sergaki & Kalaitzakis, 2002). Each power station develops its own maintenance philosophy to try and meet the energy availability factors, unplanned capability loss factors and planned capability loss factors targets set by generation. The planned maintenance opportunities also consider statutory inspections required and technical projects to improve the reliability and availability of plant assets. Planned maintenance is expensive as it requires skilled labour, shop facilities, spare parts, quality control and record keeping (Fetanat & Shafipour, 2011). The function of planned maintenance is to prevent avoidable breakdowns that may amount to significant financial expenditure compared to the actual cost of routine repairs. Planned maintenance focuses on ensuring availability and reliability of a plant while maximising production time (Fetanat & Shafipour, 2011; Yare & Venayagamoorthy, 2008).

Ensuring the smooth operation of a power utility is an intricate task that requires continuous development and innovation. Finding an optimal generator maintenance schedule for short and long-term maintenance planning of South-African generation units should be of utmost importance as it influences fuel scheduling, transmission line maintenance, unit commitment and power flow problems (Mollahassanipour *et al.*, 2013). A GMS is a timetable showing the allocated maintenance opportunities of each generating unit over a specific planning horizon (Firuzabad *et al.*, 2014). In general, a typical GMS schedule provides a one-year to five-year maintenance plan. These schedules may be amended based on available reserve, plant conditions, finances, maintenance crew, maintenance contracts and statutory requirements while aiming to maximise financial gain and improve availability and reliability. Improving planned maintenance schedules can be a great source of financial saving to the SA power utility Eskom and can allow proper planning to be done prior to maintenance execution. Moreover, an improved maintenance schedule can increase generator life which could in turn allow capital to be spent on building new power stations or development of renewable energy technology in order to meet the future national electricity demand requirements.

1.2 Research problem, research purpose and objectives

As motivated in the previous section, optimizing generator maintenance scheduling within the South-African context is paramount to ensuring sustainable growth and development of the SA economy. Due to the sheer size and complexity of the generator maintenance optimisation scheduling problem, innovation and advances in the field are continuously required. Innovation may refer to reducing solution times of algorithms used, increasing problem solution size and excluding assumptions to account for reality. This dissertation focuses on finding an optimal maintenance schedule for the well-known generator maintenance scheduling problem by utilising the time index and resource flow formulations while maximising net present value as the objective function. It is important to note that only coal fired power stations will be considered for the purpose of this dissertation as coal fired power stations make-up 93% of the generation capacity within South-Africa. Future attempts can be made to extend the model formulations to include the remaining 8% worth of generating capacity.

The following six research objectives are pursued in this dissertation:

1. To conduct a thorough literature survey related to the following:
    - Generator maintenance scheduling (GMS) overview.
    - General GMS model considerations which include model decision variables, parameters, constraints and objective functions
    - GMS problem solution methodologies.
2. To provide an overview of optimisation theory used to solve generator maintenance scheduling problems with focus on:
    - Linear programming.
    - Integer programming.
    - Graph theory and network flow formulations.
3. To formulate a general GMS optimisation model that accounts for most of the important objectives and constraints as identified in objective 1.
    - The proposed GMS optimisation model focuses on fossil fuel fired power stations within the South-African generation fleet.
    - The model should be capable of solving realistic industry-sized scheduling problems within an acceptable time frame.

4. To reduce the computational time required to finding an optimal generator maintenance schedule by utilising graph theory and network flow mathematical programming formulations.
5. To verify and validate all mathematical model formulations developed to satisfy objective 4.
6. To recommend further research endeavours and improvements based on the work presented in this dissertation.

The scope of this dissertation will be restricted to the generator maintenance scheduling problem, however, the unit commitment problem intrinsically forms part of the development process. The following exclusions apply:

- Transmission line maintenance and scheduling problem.
- Economic dispatch problem.
- Peak load power generation plants are not considered within the maintenance schedules.
- Nuclear power generation plants are not considered within the maintenance schedules.
- Renewable generation plants are not considered within the maintenance schedules.

1.3 Research methods

Multiple solution techniques have been utilised to try and solve the GMS problem. Heuristics techniques are generally used due to their simplicity and flexibility, however without an adequate first approximation to start the solution algorithm; these techniques can fall into a local minimum resulting in a sub-optimal solution. Heuristic algorithms use a trial and error method with significant operator input required which in some cases fail to produce feasible solutions (Dahal, & McDonald, 1998). Mathematical modeling approaches in literature include linear programming, integer programming, mix-integer programming, dynamic programming and the branch-and-bound algorithm. These mathematical modeling techniques provide exact optimal solutions when solved to optimality and can adequately be applied to describe small to medium sized problems (Khalid & Ioannis, 2012). An increase in problem size and constraints' complexity directly influence the computational time required to solve a GMS problem to optimality which in some cases can limit the application of mathematical algorithms. Metaheuristic approaches such as simulated annealing, artificial intelligence and genetic algorithms have been employed to try and overcome the time constraints experienced when solving exact mathematical formulations. From literature, genetic algorithms (GA) as well as GA hybrid techniques have been used with promising results (Khalid & Ioannis, 2012). For this dissertation mathematical formulation techniques will be considered with specific focus on developing a GMS optimisation model which can be applied to optimally solve realistic industry-sized problems within acceptable computing times.

The fossil fuel fired power generation units maintenance scheduling problem is a complex combinatorial optimisation problem with multiple conflicting objectives of which some include minimising maintenance and operating cost while ensuring reliability of power supply to the national grid (Goldberg, 2015). These types of objectives are typically accompanied by constraints which include maintenance window constraints which defines the earliest and latest scheduling times along with the maintenance duration, sequence or precedence constraints, non-simultaneous maintenance constrains, crew and resources distribution and national electricity demand constraints (Dahal, & McDonald, 1998). Multiple supplementary constraints can be found in literature depending on the model formulation and the type of generating units considered.

The methodology proposed to solve the generator maintenance scheduling problem, with reference to this dissertation, will include use of the well-known time index scheduling formulation as well as attempting to apply graph theory and network flow formulations. From the literature, the maintenance scheduling constraints naturally lend itself to the time index formulation while graph theory and network flow formulations have not yet been attempted, according to the knowledge of the author, to solve the GMS problem. A novel resource flow model is proposed to try and reduce the solution time required to solve realistic industry sized GMS problems using the time index formulation, while also possibly opening a new frontier for research to be conducted on applying resource flow formulations towards solving GMS problems. The mathematical models are developed and validated by means of CPLEX while using Excel as supplementary software.

1.4 Chapter Division

The introductory chapter is followed by six chapters and a bibliography. In Chapter 2 the generator maintenance scheduling problem is formally introduced by presenting a general literature overview followed by the latest mathematical advances in the generator maintenance scheduling field. The most common mathematical problem parameters, constraints and objective functions generally considered to develop a GMS optimisation model, as found in the literature, is reviewed after which the solution methodologies utilised are discussed. Chapter 2 ultimately provides insight into and clearly defines the scope of the generator maintenance scheduling problem while also identifying key gaps within the literature that will be addressed within the context of this dissertation.

Chapter 3 focuses on presenting a first principle overview of mathematical optimisation theory which includes linear, integer and mixed integer programming. Consideration will also be given to the fundamentals of graph theory and network flow concepts. The optimisation theory presented provides first principle insight into the solution algorithms applied within this dissertation to solve the GMS problem for a national power utility.

Chapter 4 presents the mathematical models developed to solve a realistic GMS problem while meeting the objectives as discussed in Section 1.3. Two different modeling approaches are considered which include the time index and resource flow formulations. Both models are formulated to find an optimal generator maintenance schedule while maximising net present value over the selected planning horizon. The models are systematically developed while introducing the constraints, variables and input parameters. The primary contribution of this chapter is the mathematical constraints introduced to the general resource flow project scheduling formulation to account for time dependency.

Verification of the time index and resource flow model formulations, as presented in Chapter 4, are firstly attempted in Chapter 5 by applying it to a simplistic 5 unit generator maintenance scheduling problem; after which a comparative study is embarked on in Chapter 6 to validate the models and proof as to how the objectives set out in Chapter 1 is satisfied. Chapter 6 also provides an in-depth discussion on the results obtained from the 5, 10, 20, 40 and 92 unit GMS scenarios considered.

Closing remarks along with an appraisal of the key research contributions are presented in Chapter 7. The chapter is concluded by proposing possible future research initiatives within the field of generator maintenance scheduling and possible areas for improvement based on the work presented in this dissertation.

Chapter 2

Literature review

This chapter presents a literature study applicable to the generator maintenance scheduling problem. First the different types of maintenance philosophies will be presented after which the generator maintenance scheduling problem along with relevant interconnected sub-problems will be defined. The GMS problem is complex in nature and forms part of a greater problem when considering optimizing the holistic management of a national power utility (Dahal *et al.*, 2000).

General GMS model formulations will be discussed which includes a review on the constraints, objective functions, decision variables and modeling parameters. The chapter closes with a review on the different modeling techniques and solution approaches applied within the literature to solve the GMS problem.

2.1 Power Plant Maintenance

Implementation and effective execution of maintenance strategies are the driving force for improved plant reliability, availability and significant cost saving (Ollila & Malmipuro, 1999; Ahuja & Khamba, 2008). There are various categories of maintenance philosophies within literature of which unplanned, planned and predictive maintenance are the main three (Linder, 2017). Planned, unplanned and predictive maintenance can further be subdivided into time based, run to failure or reactive and condition-based maintenance respectively (Linder, 2017). Another area of maintenance to consider is referred to as design out maintenance. Design out maintenance refers to capital improvement projects with the focus of increasing plant life, availability and reliability while addressing a plant specific problem.

The development of the various maintenance philosophies came about due to a change in the basic definition of maintenance and the improvement of technology. The initial perception was that maintenance only comprised of fixing broken plant, equipment or components (Ahuja & Khamba, 2008). Over the last few decades the definition of maintenance has changed to include all activities or tasks utilised to restore an item or plant to such a state in which it can perform its original function (Ahmad & Kamaruddin, 2012). The change in definition increased the scope of maintenance to include activities such as periodic inspections, preventative replacements and condition-based monitoring (Ahuja & Khamba, 2008). The different branches of maintenance are summarised in Figure 2.1.

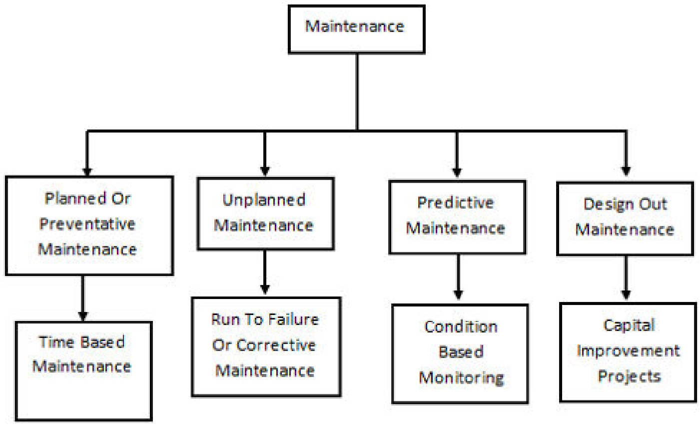

Figure 2.1: Maintenance Philosophies (Linder, 2017)

A basic overview of each of the different maintenance strategies will be considered below. The first of which is run to failure maintenance. This is the earliest type of maintenance considered in literature (Linder, 2017). Run to failure maintenance is also known as reactive maintenance, breakdown maintenance, corrective maintenance or unplanned maintenance (Froger *et al.*, 2015; Linder, 2017). This strategy focuses on restoring equipment to its original function after it has failed (Ahmad & Kamaruddin, 2012). This means that no preventative or planned maintenance is conducted to try and detect or avoid the onset of equipment failure (Al-Najjar & Alsyouf, 2003). Only a few scenarios exist in which it is cost effective to apply the run to failure philosophy (Al-Najjar & Alsyouf, 2003). A run to failure maintenance philosophy is usually applied to low cost low risk components where downtime is allowed, or no significant financial loss will be incurred due to equipment failure (Ahuja & Khamba, 2008; Dahal & Chakpitak, 2006). In some instances temporary repairs are made so that the facility can return to operation (Linder, 2017). This can lead to further damage to the plant if not correctly mitigated, and may result in increased financial expenditure or extended down time.

Contrary to the run to failure maintenance philosophy; planned maintenance includes all activities aimed at reducing the probability of unexpected plant failures (Or, 1993; Linder, 2017). Planned maintenance is usually based on a fixed schedule determined by means of considering plant running hours, plant risks, financial budgets, maintenance time durations, original equipment manufacturer (OEM) guidelines, national demand *etc.* (Ollila & Malmipuro, 1999). It is important to note that planned maintenance is usually conducted based on time requirements rather than based on the physical condition of the plant (Linder, 2017). Planned maintenance can also be referred to as preventative maintenance (Froger *et al.*, 2015). Activities that form the basis of preventative maintenance include lubrication of equipment, cleaning, replacement of worn components, periodic visual inspection for signs of deterioration, tightening, routine refurbishment and adjusting equipment clearances (Ahuja & Khamba, 2008). Planned maintenance, if executed properly, along with a well-defined scope of work can lead to significant savings as a result of reduced downtime, reduced component damage, increased plant efficiency, increased plant reliability and availability.

Predictive maintenance is a complimentary maintenance philosophy to planned or preventative maintenance. Predictive maintenance also known as condition based maintenance is used to predict component failure before it occurs (Ollila & Malmipuro, 1999; Al-Najjar & Alsyouf, 2003). Up to 99% of all mechanical failures are preceded by observable indicators such as temperature, vibrations, cracks, wall loss and noise depending upon the type of equipment and damage mechanisms present (Or, 1993; Linder, 2017; McKee *et al.*, 2014). Instruments and techniques used for detection of the preceding indicators include thermometric testing, thermography, non-contact proximity displacement probes to monitor shaft alignment, visual inspection, ultrasonic methods to locate cracks or degradation on piping, valves or pump casings, tribology testing, motor current signature analysis, wireless vibration sensors and microphones to detect cavitation or hydraulic disturbances (McKee et al., 2014). The data obtained from the above mentioned techniques can be used to predict component failures based on degradation rates, analysing indicator trends and assessing equipment condition. Predictive maintenance approaches allow for a sufficient planning buffer to source spare parts and allocate resources to execute the repair scope of work. This provides means by which components can be maintained only as and when required. If properly implemented this maintenance philosophy can prevent unnecessary expenditure while improving the reliability and availability of plant equipment.

In some cases, general planned, corrective or predictive maintenance is not sufficient to address equipment defects which are the result of inferior component design. Design out maintenance, also known as capital improvement projects, entail re-evaluation of a component's design along with the process conditions to establish the root cause of the continuous equipment failure. Based on the evaluation, the re-engineering process can be initiated to either replace a piece of equipment with an equivalent model or to evaluate different concepts to find a suitable solution.

The focus of this dissertation is to optimise the GMS problem for a national power utility. After considering the various maintenance definitions, the GMS problem considered in this dissertation can be classified as a planned preventative maintenance optimisation problem. Considerable expenditure is associated with planned preventative maintenance of generating units as it includes sourcing of spare parts and special tools, documenting findings and keeping records, stock taking, upkeep of maintenance facilities and employment of skilled labour, supervisors and contractors (Ahuja & Khamba, 2008). The cost associated with planned maintenance can easily be justified as it aims to improve efficiency, reliability, safety and availability of the plant while attempting to reduce unexpected downtime. Expenditure due to plant failures can amount to ten times the cost of component repairs (Linder, 2017).

2.2 Power System Optimisation Problem

A power system is a complex electricity network which is managed over various time-scales, with multiple uncertainties and dimensionality difficulties. The best approach to managing such a national power system is to divide it into sub-scheduling problems (Linder, 2017). The sub-scheduling problems that define the boundaries of a national power system include the generator maintenance scheduling problem (GMS), unit commitment problem (UC), economic generation dispatch problem (ED), generation expansion planning and transmission maintenance scheduling problem (TMS). Figure 2.2 is a graphical representation of all relevant input and output parameters associated with the power system sub-scheduling problems.

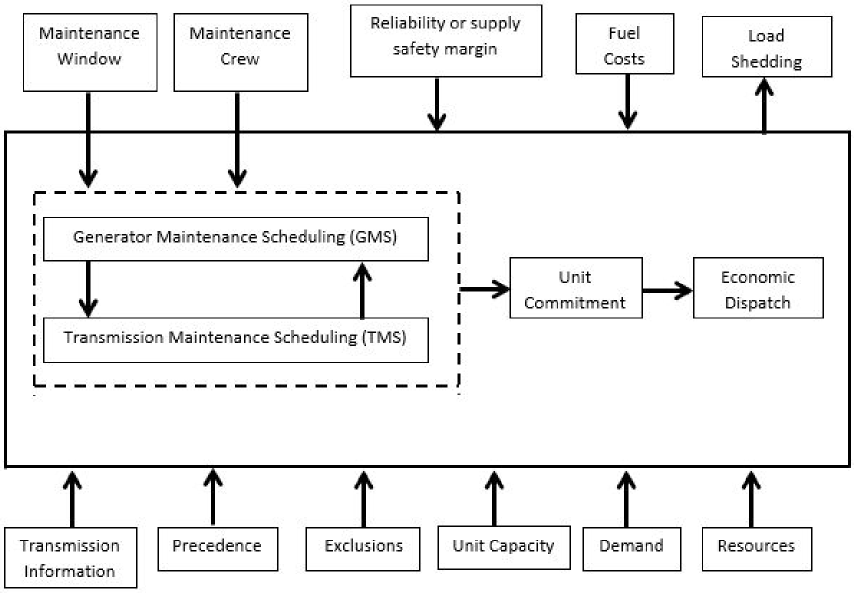

Figure 2.2: Power System Scheduling Problem (Schlunz, 2011)

From literature the GMS problem lies higher in the temporal operations scheduling hierarchy than the other sub-scheduling problems (Schlunz, 2011). The national generator maintenance schedule feeds into the other power system sub-scheduling problems and will therefore influence the holistic management of a national power system (Dahal *et al.*, 2000; Schlunz, 2011). A discussion on the various optimisation problems associated with a national power system will follow, clearly defining the boundaries, assumptions and scope associated to the GMS problem.

2.2.1 Generator maintenance scheduling problem

The GMS optimisation problem is a complex combinatorial problem that aims to find an optimal long-term preventative maintenance schedule for all generating units while reducing operational costs or ensuring improved reliability of supply to meet expected demand (Burke & Smith, 2000). Preventative maintenance is a prerequisite of efficient and cost-effective operation of a national power utility. The GMS problem usually considers a long term planning horizon ranging from 8 weeks up to 5 years (Fetanat & Shafipour, 2011; Samuel & Rajan, 2013; Linder, 2017; Conejo *et al.*, 2005). If preventative maintenance is neglected it could result in national power blackouts, unsafe operation of generating units, increased operating and maintenance expenditure due to reduced efficiency and increased downtime.

From the literature, generator maintenance schedules are subjected to constraints such as maintenance crew and resource allocation constraints, reserve margin constraints, precedence constraints, exclusion constraints, maintenance window and generation output constraints (Burke & Smith, 2000; Schlunz & Van Vuuren, 2013; Koay & Srinivasan, 2003; Samuel & Rajan, 2013; Conejo *et al*, 2005; Fetanat & Shafipour, 2011). An optimal generator maintenance schedule allows for proper long term planning to be done and could in turn reduce the excessive financial expenditure associated to planned maintenance. From the literature, various solution techniques have been applied to solve the GMS problem of which some include Linear programming (LP), Integer programming (IP) and Mixed integer programming (MIP), dynamic programming, simulated annealing, genetic algorithms, heuristics and multi stage approaches (Khalid & Ioannis, 2012).

2.2.2 Transmission Maintenance Scheduling Problem

Similar to the GMS problem, the transmission maintenance scheduling problem (TMS) involves the scheduling of transmission lines and substations for planned maintenance (Schlunz, 2011). Both generating units and transmission lines should be maintained regularly to ensure reliability and efficient operation (Marwali & Shahidehpour, 1998). The TMS problem is also a long-term scheduling problem with the planning horizon coinciding with that of the GMS problem. Transmission lines are used to distribute electricity from physical generating units over a vast spread national power grid.

The UC and ED problems are directly influenced by the TMS problem as seen from Figure 2.2. Transmission lines are associated with line losses, therefore when solving the ED and UC sub-problems the distribution distance and loss in power due to the supply network should be known to ensure the national demand can be satisfied at all times (Muckstadt & Koenig, 1977). The total energy produced, by the committed units, at any given instance should be equal to the actual national demand less the process and transmission losses (Muckstadt & Koenig, 1977). In addition to transmission line losses, the thermal capacity of transmission lines constrain the distribution of power. If the TMS problem is excluded from generator maintenance planning, it could lead to an optimistic maintenance schedule. Great care should therefore be given when selecting transmission line and generator pairs for planned maintenance to prevent line overloading problems (El-Sharkh & El-Keib, 2003).

The TMS problem should ideally be solved in parallel with the GMS problem to allow for effective maintenance planning (Linder, 2017). The long-term transmission and generator maintenance schedules then feed into the UC and ED problems as availability constraints.

2.2.3 Unit Commitment problem

South-Africa has a national electricity grid or power system supplying electricity to the entire country. Generating units are only committed to the national grid as and when required as per the national demand requirements. It is therefore a delicate balancing act to satisfy the national demand in a cost effective manner. From a planning perspective it would be preferable to know beforehand what the national demand will be a week ahead of time. This would allow accurate planning of unit commitment (UC) and economic dispatch (ED) which would in turn increase profit and reduced expenditure due to supply uncertainty. Load demand forecasting will not form part of the scope of this dissertation, but the forecasted demand is a key input into the unit commitment sub-scheduling problem (Schlunz, 2011).

The unit commitment problem (UC) seeks to determine which available generating units should be committed to the national grid so as to contribute to power generation (Carrion & Arroyo, 2006). The unit commitment problem is a short term scheduling problem with the planning horizon varying from a day to anything up to a week (Kazarlis *et al.*, 1996). Although the UC and GMS problems are quite similar with regards to the objective functions and constraints considered, the GMS problem usually spans over a larger planning horizon of up to five years (Linder, 2017). The shorter planning horizon UC problem dictates greater data accuracy with regards to load demand forecasting so as to improve planning and scheduling capabilities.

The main UC problem objective functions, as recorded in the literature, include minimising emissions, minimising operating cost (sum of fuel, start-up and shut-down costs, production cost and maintenance cost) or maximising power system reliability while satisfying generation constraints such as production limits, system reserve requirements, ramp rates, minimum up and down time, unit availability or status restrictions (scheduled for outage, available, fixed MW), Unit start-up and shut-down ramp rates, plant crew constraints and unit or plant fuel availability (Linder, 2017; Kazarlis *et al.*, 1996; Carrion & Arroyo, 2006; Schlunz, 2011). It is important to note that production cost, although used in the UC formulation, is most accurately determined by solving the economic dispatch (ED) problem which is a sub-problem of the UC problem (Linder, 2017).

The UC problem is a combinatorial problem. Some of the solution techniques attempted to solve this problem include dynamic programming, heuristics, simulated annealing, evolution-inspired approaches, branch-and-bound method, Lagrangian relaxation and artificial neural networks (Carrion & Arroyo, 2006; Linder, 2017).

Ideally the UC and GMS problems should be solved in parallel as they are intricately connected, however, this would escalate problem dimensionality which would increase problem complexity and computational time requirements (Schlunz, 2011). To reduce complexity, the UC and GMS problems are solved as independent scheduling problems. The GMS problem's solution can then be used as availability constraints for the UC problem (Linder, 2017). Ultimately, the results generated by solving the long term GMS problem cascades down to the short term UC planning problem as depicted in Figure 2.2.

2.2.4 Economic dispatch problem

The ED problem is a sub-problem of the well-known UC problem. The maintenance schedule produced by solving the GMS problem feeds into the UC problem and will ultimately determine which units are available (not scheduled for maintenance) to commit to the national power system. The ED problem then filters from the available units, to find the optimal units, based on the objectives and constraints, which will be committed to satisfy the national demand. The ED problem therefore determines what each unit's power output should be to minimise production cost (Al Farsi *et al.*, 2015). Some GMS problem formulations from literature intrinsically account for the ED problem without considering the short term planning detail. The ED problem, however, is a short term scheduling problem and in most cases the GMS and ED problems are segregated and approached as two separate scheduling problems (Schlunz, 2011).

Each generating unit has a specific heat rate [Btu/kWh] which is a measure of the thermal energy required as input into a generation system to produce 1 kWh of electrical energy. A smaller heat rate is an indication of greater unit efficiency. This means that for a smaller heat rate a larger portion of the thermal energy introduced into the generating system is converted into electrical energy. The relation between input fuel cost and power generated (MW) is known as the unit cost function (Abbas *et al.*, 2017; Walters & Sheble, 1993). Each generating unit has its own unit cost function based on the condition of the plant, unit design, process losses, process control *etc.* The unit cost function is a primary input into the ED optimisation problem. The optimised ED problem schedules the most cost effective and efficient generating units to satisfy the power system demand in order to minimise production cost or fuel cost (Abbas *et al.*, 2017; Bakirtzis *et al.*, 1994; Al Farsi *et al.*, 2015). The ED objective function can be formulated as a nonlinear, linear or a piecewise linear function based on the cost function and constraints considered (Abbas *et al.*, 2017). The nonlinear formulations of the ED problem can be solved utilising the lambda iteration method, Kuhn-Tucker method or dynamic programming, while the linear or piecewise linear formulations can be solved using standard LP, IP or MIP techniques (Linder, 2017).

2.3 GMS model considerations

There are various different GMS constraints considered in the literature. The type of constraints applied depends upon the scope of the scheduling problem under consideration. The primary constraints considered in the literature with reference to the GMS problem is presented below, but first some general notation used within this section will be defined. Let $\mathcal{N} = \{1, 2, ..., |\mathcal{N}|\}$

denote the index set of all generating units. Each power station $p \in \mathcal{P}$ has a specific number of power generating units. The subset $\mathcal{N}(p) \subseteq \mathcal{N}$ denotes the set of generating units belonging to power station $p \in \mathcal{P}$. Each generating unit $i \in \mathcal{N}$ is allowed only one maintenance opportunity throughout the planning horizon $\mathcal{T}$ where $\mathcal{T} = \{1, 2, ..., |\mathcal{T}|\}$. The binary decision variable x_{it} will take on the value of 1 when a unit $i \in \mathcal{N}$ is scheduled for planned maintenance during time period $t \in \mathcal{T}$ and 0 otherwise. Each unit i's maintenance opportunity lasts for a predetermined duration of d_i time periods. Continuity of maintenance is enforced by means of the continuity binary decision variable y_{it} which will take on the value of 1 when unit i is scheduled for panned maintenance during time period $t \in \mathcal{T}$. The binary decision variable y_{it} should be 1 for each time period $t \in \mathcal{T}$ for the total maintenance duration d_i of each unit $i \in \mathcal{N}$. The decision variables x_{it} and y_{it} may either be integer or binary decision variables, depending on the model formulation considered. When presenting the GMS constraints, as presented in the literate, appropriate distinction will be made as to which decision variable formulation is employed.

During maintenance execution each unit $i \in \mathcal{N}$ requires certain resources. The index set of all resources is denoted by $\mathcal{R} = \{1, 2, ..., |\mathcal{R}|\}$. Some GMS model formulations allocate resource requirements $r \in \mathcal{R}$ based on the various types of equipment to be considered during generator maintenance. The index set of all equipment types is defined as $\mathcal{E} = \{1, 2, ..., |\mathcal{E}|\}$. The type of resources $r \in \mathcal{R}$ and the quantities thereof may vary for each plant area or type of equipment e being maintained *e.g.* boiler, turbine, water plant, ash plant, draught groups, coal plant, sulphur plant *etc.*

Most of the general GMS modeling notation have been discussed, however there are still some constraint specific variables, notation and indices that will formally be defined in the remainder of this section.

One time maintenance and maintenance window constraints

All generating units within a power utility follow a specific maintenance philosophy (Mollahassanipour *et al.*, 2013). The purpose of the maintenance philosophy is to ensure the generating units are able to operate as per design while minimising downtime and wasteful expenditure due to reduced maintenance intervals. Most generator maintenance philosophies only allow units to be scheduled for planned maintenance once within the planning horizon (Mollahassanipour *et al.*, 2013). This allows planned maintenance of each unit to be spaced appropriately to maximise generator availability (Perez-Canto, 2014; Dahal, K.P. Chakpitak, N. 2006).

The one time maintenance constraint (2.1) is formulated to allow each unit $i \in \mathcal{N}$ to be scheduled for maintenanc only once within the planning horizon $\mathcal{T}$ (Mollahassanipour *et al.*, 2013). It should be noted that most GMS model formulations incorporate maintenance window constraints which pre-specify the earliest and latest possible maintenance starting times represented by e_i and l_i respectively (Linder, 2017). The maintenance window constraint is used to account for each unit's maintenance philosophy within the planning horizon $\mathcal{T}$ (Linder, 2017; Yare & Venayagamoorthy, 2008). When considering, for instance a 2 year planning horizon it would be wasteful to allow a unit to be maintained twice within a 6 month period. The maintenance window constraint therefore allows for adequate separation between successive maintenance opportunities:

$$\sum_{t \in \mathcal{T}_i} x_{it} = 1 \quad \forall i \in \mathcal{N} \tag{2.1}$$

where $\mathcal{T}_i = \{t \in \mathcal{T} | e_i \leq t \leq l_i\}$

Constraint (2.1) is applied to GMS formulations where binary (0/1) scheduling decision variables are used to indicate whether a unit is scheduled for maintenance or available to be committed to the national grid (Linder, 2017). When integer decision variables are used instead of binary decision variables, the maintenance window constraint can be formulated as

$$e_i \leq x_i \leq l_i \quad \forall i \in \mathcal{N} \tag{2.2}$$

where x_i is an integer maintenance scheduling decision variable. This means that x_{it} can take on any value of $t \in \mathcal{T}$ within the specified upper l_i and lower e_i time boundaries.

Maintenance duration and continues maintenance constraints

The maintenance duration of each unit $i \in \mathcal{N}$ varies based on the maintenance philosophy adopted (Perez-Canto, 2014). Some units might be due for an inspection outage (IN) and some for a general overhaul (GO). The maintenance durations d_i may vary between 14 to 120 days. The most frequent maintenance duration and consecutive maintenance constraints reported in literature (Mollahassanipour *et al.*, 2013; Linder, 2017) are formulated as

$$\sum_{t \in \mathcal{T}} y_{it} = d_i \quad \forall i \in \mathcal{N} \tag{2.3}$$

Supplementary to (2.3), constraints (2.4) and (2.5) are employed to ensure continuity of maintenance within the planning horizon $t \in \mathcal{T}$ (Mollahassanipour *et al.*, 2013; Linder 2017; Schlunz, 2011). Constraints (2.4) and (2.5) ensure activation of y_{it} for each time period t for the total duration of each unit's maintenance opportunity.

$$y_{it} - y_{i(t-1)} \leq x_{it} \quad \forall i \in \mathcal{N}, \forall t \in \mathcal{T} \tag{2.4}$$

$$y_{i1} \leq x_{i1} \quad \forall i \in \mathcal{N} \tag{2.5}$$

Constraint (2.6) is a variation on constraints (2.4) and (2.5) (Conejo *et al.*, 2005). It also aims to ensure continuity of maintenance while applying the same maintenance duration constraint as presented by (2.3).

$$y_{it} - y_{i(t-1)} \leq y_{i(t+d_i-1)} \quad \forall i \in \mathcal{N}, \forall t \in \mathcal{T} \tag{2.6}$$

In Fattahi *et al.* (2014), the binary maintenance execution decision variable x_{it} is added to the equality constraint presented in (2.3) and the constraint is formulated as an inequality so as to ensure that

$$\sum_{k=t}^{k=t+d_i-1} y_{ik} \geq d_i x_{it} \quad \forall i \in \mathcal{N}, t \in \mathcal{T} \tag{2.7}$$

When unit i is scheduled to start maintenance in time period t the binary decision variable x_{it} will take on the value 1 and therefore constraint (2.7) is simplified to

$$\sum_{k=t}^{k=t+d_i-1} y_{ik} = d_i \quad \forall i \in \mathcal{N}, t \in \mathcal{T} \tag{2.8}$$

which will ensure continuity of maintenance when initiated. In case integer decision variables are used the formulation can be modified to

$$\sum_{t=x_i}^{x_i+d_i-1} y_{it} = d_i \quad \forall i \in \mathcal{N} \tag{2.9}$$

where x_i is an integer decision variable representing the maintenance starting time of unit $i \in \mathcal{N}$ (Linder, 2017).

Resource allocation constraints

The purpose of the resource allocation constraints are to ensure that the total amount of resources required for execution of planned maintenance do not result in exceedance of the available resources. Let v_i be the quantity of resources being consumed during the maintenance of unit $i \in \mathcal{N}$. The amount of resources available is defined as U. Multiple resources such as spare parts, special tools, personnel, service budget *etc.* are required during the execution phase of planned maintenance (Linder, 2017; Lindner & Van Vuuren, 2014). There are two main resource allocation constraint formulations presented in literature. The first being

$$\sum_{i \in \mathcal{N}} v_i y_{it} \leq U \quad {\scriptstyle \forall t \in \mathcal{T}} \tag{2.10}$$

where (2.10) assumes that the resources v_i required by unit i remains constant throughout the maintenance planning horizon $t \in \mathcal{T}$. This formulation, however, over simplifies the resource requirements of each unit i as it is not an accurate representation of resource allocation for planned maintenance within the power generation environment. In reality, multiple different resources and resource quantities are required during maintenance execution as an array of equipment is simultaneously maintained during a planned maintenance opportunity. The formulation presented in (2.11) is an elegant practical approach which accounts for multiple resources $r \in \mathcal{R}$ as required during execution of maintenance on various types of equipment $e \in \mathcal{E}$ within a power station $p \in \mathcal{P}$ during time period $t \in \mathcal{T}$ (Linder, 2017). Let v_{iert} be the quantity of resources $r \in \mathcal{R}$ being consumed by unit $i \in \mathcal{N}(p)$ of power station $p \in \mathcal{P}$, to repair/replace equipment of type $e \in \mathcal{E}$ during time period $t \in \mathcal{T}$. The corresponding resource capacity is given by U_{ert}. The binary decision variable x_{iet} will take the value of 1 when equipment of type $e \in \mathcal{E}$ in generating unit $i \in \mathcal{N}(p)$ of power station $p \in \mathcal{P}$ during time period $t \in \mathcal{T}$ is not scheduled for maintenance and 0 otherwise (Alardhi & Labib, 2008).

$$\sum_{i \in \mathcal{N}(p)} v_{iert}(1 - x_{iet}) \leq U_{ert} \quad {\scriptstyle \forall p \in \mathcal{P}, \forall e \in \mathcal{E}, \forall t \in \mathcal{T}, \forall r \in \mathcal{R}} \tag{2.11}$$

A similar approach was employed by Yamayee *et al.* (1983) where the resource allocation constraint was also formulated to account for various different types of resources. The only difference being, the formulation was set up to accommodate unitized resource requirements without considering the different types of equipment located within each generating unit.

Precedence constraints

Precedence constraints are used to enforce maintenance priority between units. In some cases such as unit condition, safety risks, priority level *etc.* maintenance of unit i_1 should take priority to maintenance of unit i_2 (Linder,2017; Bisanovic *et al.*, 2011; Perez-Canto, 2014; Conejo *et al.*, 2005). This is achieved by means of enforcing (2.12) and (2.13).

$$\sum_{\tau=1}^{t} x_{i_1(\tau-1)} - x_{i_2 t} \geq 0 \quad {\scriptstyle \forall i \in \mathcal{N}, \forall t \in \mathcal{T}} \tag{2.12}$$

$$x_{i_1 t} + x_{i_2 t} \leq 1 \quad {\scriptstyle \forall i \in \mathcal{N}, \forall t \in \mathcal{T}} \tag{2.13}$$

When the GMS formulation utilises integer decision variables instead of binary decision variables, the formulation presented in (2.14) can be considered

$$x_{i_1} + d_{i_1} \leq x_{i_2} \tag{2.14}$$

where d_{i_1} represents the maintenance duration of unit i_1. Constraints (2.12) through (2.14) ensure that maintenance of unit i_2 will commence after the maintenance of unit i_1 has been completed. The precedence constraints can also be utilised to enforce power station specific or geographical scheduling requirements while providing means by which the operator of the models can manipulate the schedules when required to *e.g.* account for unplanned forced outages.

Maintenance crew constraints

One of the key constraints to consider within the GMS scheduling environment is the availability of the maintenance crew required to execute the planned maintenance scope according to the predetermined schedule. From literature, constraint (2.15) is the formulation generally applied to model the maintenance crew requirements m_i of each unit $i \in \mathcal{N}$. The formulation is based on the assumption that the maintenance crew m_i required for execution to maintain each unit $i \in \mathcal{N}$ remains constant throughout the planning horizon $\mathcal{T}$ where M_t denotes the total available maintenance crew for each time period t (Linder, 2017).

$$\sum_{i \in \mathcal{N}} m_i y_{it} \leq M_t \quad \forall t \in \mathcal{T} \tag{2.15}$$

In Yare & Venayagamoorthy (2008) a variation to the formulation presented in (2.15) is adopted while the maintenance crew requirement m_{it} is still assumed constant throughout the maintenance duration of each unit $i \in \mathcal{N}$ where $S(i,t) = \{k \in \mathcal{T}_i : t - d_i + 1 \leq k \leq t\}$ represents the set of starting time periods k in which a unit i can be scheduled for maintenance within the planning horizon $\mathcal{T}$ and $N(t) = \{i : t \in \mathcal{T}_i\}$ the set of units that are allowed to be scheduled for maintenance in time period t. The maintenance crew constraint proposed in Yare & Venayagamoorthy (2008) can be formulated as

$$\sum_{i \in \mathcal{N}(t)} \sum_{k \in \mathcal{S}(i,t)} x_{ik} m_{ik} \leq M_t \quad \forall t \in \mathcal{T} \tag{2.16}$$

In Perez-Canto (2014) the maintenance crew requirements are formulated as man hour constraints. This formulation aims to account for the availability of limited resources, materials and maintenance crew limitations as all these factors influence the total amount of man hours required h_i to execute the planned maintenance activities. H_t represents the total amount of working hours available for time period t (Perez-Canto, 2014).

$$\sum_{i \in \mathcal{N}} h_i y_{it} \leq H_t \quad \forall t \in \mathcal{T} \tag{2.17}$$

The assumption that the maintenance crew remains constant throughout the maintenance duration d_i of each unit i is, however, a simplification which does not represent the reality of the power industry. It is well known that the crew requirements for execution of planned maintenance differ throughout the maintenance duration d_i of each unit i. In general, when maintenance is initiated a larger maintenance crew is required than closer to the end of the maintenance duration. The approach presented in Schlunz & Van Vuuren (2013) is an elegant way of accounting for a variable maintenance crew m'_{cit} throughout a unit's maintenance duration.

$$\sum_{i \in \mathcal{N}} \sum_{c=1}^{t} m'_{cit} x_{ic} \leq M_t \quad \forall t \in \mathcal{T} \tag{2.18}$$

where $m'_{cit} = m_i^{(t-c+1)}$ if $t - c < d_i$ and 0 otherwise. m_i^u denotes the required maintenance crew of unit $i \in \mathcal{N}$ in its u^{th} period of maintenance.

Exclusion constraints

The exclusion constraint presented in Conejo *et al.* (2005) and Bisanovic *et al.* (2011) limits the number of units $i \in \mathcal{N}$ within some set $q \in \mathcal{Q}$ (power stations, geographical region *etc.*) to be simultaneously scheduled for planned maintenance within the planning horizon $t \in \mathcal{T}$. The purpose of exclusion constraints are mainly to avoid large transmission losses due to unbalanced power distribution, prevent reduced reserve margins within geographical regions while also ensuring that the minimum allowable number of available units within a power station is maintained. Constraint (2.19) is a general formulation of the GMS exclusion constraint, where G_{qt} represents the simultaneous scheduling limit within each subset $q \in \mathcal{Q}$ for time period t, and can be formulated as

$$\sum_{i \in \mathcal{N}} y_{iqt} \leq G_{qt} \quad {\scriptstyle \forall q \in \mathcal{Q}, \forall t \in \mathcal{T}} \tag{2.19}$$

The exclusion constraint can be simplified as presented in Perez-Canto (2014) to

$$y_{it} + y_{jt} \leq 1 \quad {\scriptstyle \forall t \in \mathcal{T}} \tag{2.20}$$

which prevents generating sets (i, j) from being simultaneously scheduled for maintenance during time period t. This formulation can be applied when (0/1) binary decision variables are used (Perez-Canto, 2014).

Reliability and load constraints

Reliability and load constraints are paramount during the development of a realistic generator maintenance schedule as the forecasted load demand L_t should be satisfied even though multiple units are scheduled for planned maintenance. This implies that only a finite amount of generating units can simultaneously be scheduled for preventative maintenance within the planning horizon $t \in \mathcal{T}$.

The binary decision variable o_{it}, as stated in (2.21), is an operations variable which will take the value of 1 when a unit is committed to supply the power grid and 0 otherwise. Each committed unit i will be loaded between its minimum and maximum allowable generation output limits $P_i^{(l)}$ and $P_i^{(d)}$, respectively, where each unit i's committed load is accounted for by the integer decision variable p_{it} for each time period t. The maximum load limit $P_i^{(d)}$ is usually associated to the unit's designed output capacity, however, this is subject to load restrictions due to high emissions, high condenser back pressure, draught group limitations, process inefficiencies *etc.* It is noteworthy that all national demand forecasts L_t have an error tolerance associated to it. For this reason a safety factor known as reserve margin R_t is added to the formulation to allow for a safety buffer with the main purpose of preventing load-shedding due to inadequate availability of generating capacity (Mollahassanipour *et al.*, 2013; Perez-Canto, 2014; Linder, 2017) . keeping this in mind the reliability and generator allowable output limit constraints can be formulated as

$$\sum_{i \in \mathcal{N}} o_{it} P_i^{(d)} \geq L_t + R_t \quad {\scriptstyle \forall t \in \mathcal{T}} \tag{2.21}$$

$$P_i^{(l)}(1 - y_{it}) \leq p_{it} \leq P_i^{(d)}(1 - y_{it}) \quad {\scriptstyle \forall i \in \mathcal{N}, \forall t \in \mathcal{T}} \tag{2.22}$$

$$\sum_{i \in \mathcal{N}} p_{it} = L_t \quad {\scriptstyle \forall t \in \mathcal{T}} \tag{2.23}$$

where o_{it} can also be substituted with $(1 - y_{it})$.

Another formulation of the reliability constraint is to subtract the summation of the generating capacity of the units scheduled for planned maintenance, according to the binary decision variable x_{it}, from the summation of the total generating capacity of all units within the national power utility (Yare & Venayagamoorthy, 2008). The inequalities

$$\sum_{i \in \mathcal{N}} P_i^{(d)} - \sum_{i \in \mathcal{N}(t)} \sum_{k \in \mathcal{S}(i,t)} x_{ik} P_i^{(d)} \geq L_t \quad \forall t \in \mathcal{T} \tag{2.24}$$

$$\sum_{i \in \mathcal{N}} P_i^{(d)} - \sum_{i \in \mathcal{N}(t)} \sum_{k \in \mathcal{S}(i,t)} x_{ik} P_i^{(d)} \geq L_t + R_t \quad \forall t \in \mathcal{T} \tag{2.25}$$

dictate that the difference in generating capacities should be larger than or equal to either the forcasted national demand L_t or the summation of the forecasted national demand L_t and the selected safety factor R_t (Dahal & Chakpitak, 2006). Where $S(i,t) = \{k \in \mathcal{T}_i : t - d_i + 1 \leq k \leq t\}$ is the set of starting time periods k during which unit i can be scheduled for planned maintenance.

Transmission or network availability constraints

Transmission line or power flow constraints are generally included in the GMS problem model formulation when the economic dispatch and transmission line scheduling problems are solved as sub-problems. The purpose of the transmission line constraints are to ensure the flow of power PL_{bt} through transmission line b during time period t is not exceeded. It also prevents power distribution through lines which have been scheduled for planned maintenance. The GMS and TMS problems should be solved in parallel with the ED problem being an important sub-scheduling problem to consider. By solving the ED problem the unit output loads can be determined with relative accuracy. If all the committed unit output loads p_{it} are known the load flow problem can be solved where PL_b is the maximum capacity of transmission line b and r_{bt} being a dummy unit vector to account for unsupplied energy during time period t (Kulkarni, R. 2017). Variable f denotes an active power flow vector, e the allowable level of unsupplied energy, s being the node-branch indice matrix, g the generated power vector and finally d the demand vector. The general transmission network constraints can then be formulated as

$$sf + g + r = d \quad \forall t \in \mathcal{T} \tag{2.26}$$

$$-PL_b \leq PL_{bt} \leq PL_b \quad \forall t \in \mathcal{T}, \forall b \in \mathcal{B} \tag{2.27}$$

$$\sum_{b \in \mathcal{B}_b} r_{bt} \leq e \quad \forall t \in \mathcal{T} \tag{2.28}$$

where (2.26) is known as the nodal balance or system constraint and considers the reliability of transmission systems (Kulkarni, 2017). It ensures a balance between demand and power generation at each node.

Maintenance and online status constraints

Solving the GMS problem is focused on finding an optimal planned maintenance schedule, however, in some instance where the ED and GMS problems are solved in conjunction with one another, a maintenance and online status constraint is required. This ensures that a unit i can either be scheduled for planned maintenance or committed to supply power to the national grid during time period t. A unit i is therefore not allowed to be scheduled for maintenance and supply power to the national grid at the same instance (Conejo *et al.*, 2005; Bisanovic *et al.*, 2011).

$$x_{it} + o_{it} \leq 1 \quad \forall i \in \mathcal{N}, \forall t \in \mathcal{T} \tag{2.29}$$

Constraint 2.29 is applied when binary decision variables are used where x_{it} is the binary maintenance scheduling variable and o_{it} the binary production scheduling variable (Conejo *et al.*, 2005; Bisanovic *et al.*, 2011).

2.3.1 GMS problem objective functions

Formulating suitable objective functions are crucial when attempting to optimise an industrial GMS problem. A variety of GMS problem objective functions exist within the literature. There are mainly three different categories of objective functions considered, namely, economic (Kumhar & Kumar, 2016; Mollahassanipour *et al.*, 2013; Perez Canto, 2014; Samuel & Rajan, 2013; Schlunz, 2011), reliability and suitability (Conejo *et al.*, 2005; Dahal & Chapitak, 2006; Anandhakumar *et al.*, 2011; Schlunz, 2011; Ekpenyong *et al.*, 2012; Foong *et al.*, 2008; Schunz & van Vuuren, 2013; Baskar *et al.*, 2003; Schunz & van Vuuren, 2012; Wang & Handschin, 2000; Linder, 2017; Perez Canto, 2014; Fattahi *et al.*, 2014) or convenience objectives (Kralj & Petrovic, 1995; Leou, 2006; Zurn & Quintana, 1977). Most model formulations only employ a single objective with appropriate accompanying constraints to model the GMS problem; however, formulations with multiple objective functions are also recorded in the literature (Linder, 2017). A discussion on the different objective function categories along with how they are implemented will follow.

Economic objective functions

Economic objective functions are one of the most frequently used objectives within the GMS environment. These types of objectives are usually partitioned into production and maintenance costs (Linder, 2017; Kumhar & Kumar, 2016; Mollahassanipour *et al.*, 2013; Perez Canto, 2014; Samuel & Rajan, 2013; Schlunz, 2011). Production cost can be subdivided into fuel cost, salaries and wages of production personnel, generator start-up and shut down cost, water cost, fuel oil cost, chemical costs *etc.* (Linder, 2017). Some formulations divide maintenance costs into either dependent or fixed maintenance costs (Linder, 2017), while others assume maintenance costs to remain constant for each unit over the planning horizon (Perez Canto, 2014). The type of formulation used is in some instances based on the industrial data available and the modeling approach considered. Unit operating data is usually easily obtainable in comparison to unit maintenance data. The model formulation employed is therefore dependent upon multiple factors.

Let $c_{it}^{(p)}$ and $c_{it}^{(m)}$ denote the production cost and maintenance cost of each unit i during time period t, while the binary decision variable y_{it} is the maintenance scheduling variable indicating which unit i is scheduled for maintenance during time period t. The variable p_{it} denotes the committed generator load of each unit i at time period t. The economic objective function, in most GMS formulations, aim to minimise the production and maintenance cost (collectively known as the total operating cost) of a national power utility over the planning horizon $\mathcal{T}$, *i.e.* to

$$\min \sum_{i \in \mathcal{N}} \sum_{t \in \mathcal{T}} (c_{it}^{(p)} p_{it} + c_{it}^{(m)} y_{it}). \tag{2.30}$$

According to (Linder, 2017; Perez Canto, 2014; Schlunz, 2011), other production costs are negligible compared to fuel cost and therefore, it is used in most economic objective function formulations as the main contributory cost. Based on the above, the production cost $c_{it}^{(p)}$, which is a function of unit load p_{it}, can be substituted with $c_{it}^{(f)}$ which denotes the fuel cost associated to each unit i during time period t. Fuel costs typically utilised in scheduling problems can be formulated as quadratic equations as shown in (2.31) (Mollahassanipour *et al.*, 2013).

$$c_{it}^{(f)}(p_{it}) = c_i + b_i p_{it} + a_i p_{it}^2 \tag{2.31}$$

The committed unit load p_{it} of each unit i can only be obtained by solving the ED problem in conjunction with the GMS problem. The formulation presented by constraint (2.30) and (2.31) is therefore only possible if the committed unit loads are known. Coefficients a_i, b_i and c_i are the quadratic cost coefficients of each generating unit i. From the literature (Linder, 2017; Mollahassanipour *et al.*, 2013) the quadratic fuel cost is generally approximated by piecewise linear segments as depicted in Figure 2.3. The error associated to the estimation method depends upon the number of linear segments considered. The larger the number of segments the closer the approximation to the quadratic function and the greater the accuracy.

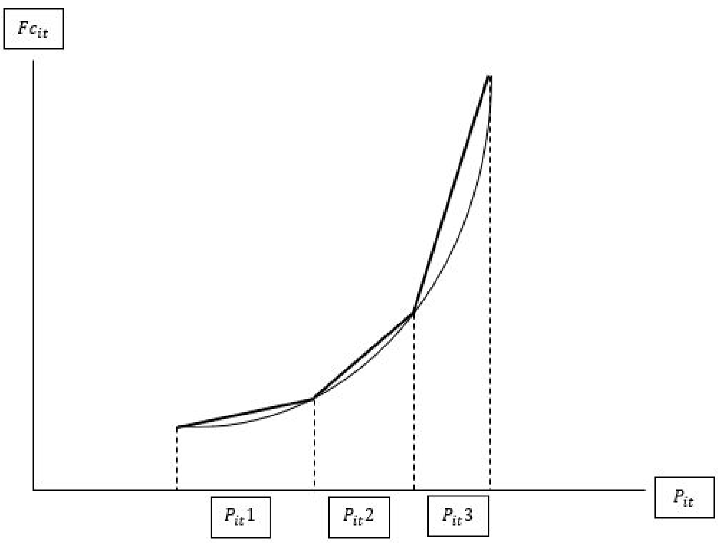

Figure 2.3: Unit fuel cost quadratic graph (Linder, 2017)

The formulation, presented in Saraiva *et al.* (2011) takes a more simplified approach to model the production cost. Saraiva et al. (2011) used a merit order dispatch approach to minimise the variable generation cost (production cost) of a power system while finding an optimal generator maintenance schedule. Merit order is a principle used to rank generating units in ascending order based on each unit's marginal costs of production. The merit order formulation, contrary to (2.30) and (2.31), assume that each station's operating costs are strictly proportional to its power output (Staffell, & Green, 2016). It is equivalent to a linear program. This implies that stations with lower variable costs will always be dispatched to satisfy the national grid before stations with a larger variable cost is committed. This merit order priority list can be determined by means of the commitment utilisation factor, the classic average full load cost method or a combination of the two (Sen & Kothari, 1998). The average full load cost method was initially used to approximate power station merit order cost. The short coming of the full load cost method was, however, that it only accounts for unit full load operation. This lead to the development of the commitment utilisation factor which accounts for intermittent unit operation (Voorspools & D D'haeseleer, 2003). The merit order unit commitment approach is the simplest form used in the literature to solve the UC problem (Sen & Kothari, 1998). Its simplicity and ease of application reduces the computational complexity of the model compared to using quadratic fuel cost functions or linearized approximations.

It is important to note that the merit order approach is based on multiple assumptions which could lead to sub-optimal solutions; however, in many situations these solutions are adequate for the model accuracy required (Sen & Kothari, 1998). The GMS problem is mainly focused on finding an optimal maintenance schedule while ensuring the national demand is satisfied. The GMS problem does not consider unit load scheduling of importance except if the ED problem is solved in conjunction with the GMS problem. Even though the main disadvantage of the merit order formulation is reduced operating cost accuracy, it can still be utilised during formulation of a combined GMS/UC problem to identify the available units that can be committed for power generation. The difference between UC and ED is that the UC problem focuses on identifying which units should be available for power supply to the national grid, while ED fits a given set of power plants into a certain power demand (Voorspools & D D'haeseleer, 2003).

The merit order economic objective function presented in Saraiva et al. (2011) aims to

$$\min \sum_{i\in\mathcal{N}}\sum_{t\in\mathcal{T}}\sum_{k\in\mathcal{K}} C_{ij}p_{ijt}\Delta T_j \tag{2.32}$$

Where C_{ij} is the merit order or variable generation cost for unit i in step j, in (€ /MWh) and p_{ijt} the power output (MW) of unit i in step j, in week t. This merit order formulation can be applied when solving both the GMS and UC problems simultaneously.

Supplementary to the production cost and maintenance costs identified in objective function (2.30); Perez Canto (2014) stated that there are three additional types of cost categories that can be considered when formulating an economic objective function. These three categories include fixed costs, start-up and shut-down costs. According to Perez Canto (2014) and Schlunz (2011), in comparison to start-up cost and production costs the fixed costs, shut-down costs and maintenance costs are considered to be insignificant and can be excluded from the economic objective function formulations (Perez Canto, 2014). This gives rise to an objective function which only accounts for start-up and operating cost where f_i is the start-up cost of each unit i and $c^{(s)}_{i,s,k,n}$ the start-up binary decision variable for power plant i, in sub-period n, period k and scenario s (Perez Canto, 2014). Variable C_i is the generation cost of electricity produced by power plant i in (€/MWh), p_{iskn} the generated power output of plant i, sub-period n, in period k, scenario s and τ_n the time duration of sub-period n in hours. The probability of the specific demand scenario s is represented by q_s (Perez Canto, 2014).

$$\sum_{i\in\mathcal{I}}\sum_{s\in\mathcal{S}}\sum_{k\in\mathcal{K}}\sum_{n\in\mathcal{N}} q_s(f_i c^{(s)}_{iskn} + C_i p_{iskn}\tau_n) \tag{2.33}$$

Objective function (2.33) excludes maintenance cost completely as it is considered insignificant in comparison to production and start-up costs; however, some formulations from literature, only consider maintenance costs. This is evident from the formulation presented in Schlunz (2011) and Linder (2017) where the objective aims to

$$\min \sum_{i\in\mathcal{N}}\sum_{t\in\mathcal{T}} c^{(m)}_{it} P^{(d)}_i y_{it}. \tag{2.34}$$

Objective function (2.34) is similar to that of the second term in (2.30), however, an extra variable $P^{(d)}_i$ is added to the formulation where $P^{(d)}_i$ is the design power output capacity of each unit i. The addition of the design power output variable serves as a penalty factor (Schlunz, 2011). Each unit i has a preferred maintenance slot where the maintenance cost of the unit will be minimised. From Linder (2017) and Schlunz (2011) this maintenance cost increase linearly with time and therefore, the objective function aims to give preference to the larger unit's preferred planned maintenance slots so as to minimise the objective function and optimise the planned generator maintenance schedule.

Reliability objective

The reliability of a power system is a measure of its capability to meet the national demand for each time period t. This is of great importance when solving the GMS problem, as it would be counter intuitive to have a cost effective schedule without being capable of satisfying the national demand. From literature, reliability of power supply is ensured by either utilising constraints or objective functions. The various types of reliability constraint formulations were presented in Section 2.3.1 while this section aims to provide an overview of the vast literature on reliability objective functions.

One of the most commonly used reliability objective functions is to minimise the sum of square reserve r_t over the planning horizon $t \in \mathcal{T}$ where the reserve is calculated by subtracting the required power demand L_t from the available generating capacity (installed capacity $P_i^{(d)}$ less the capacity lost due to planned maintenance or process inefficiencies) (Schlunz, 2011). The reserve capacity acts as a safety margin to ensure the national demand can be met for each time period t. Various safety margins are recorded in literature where Schlunz & van Vuuren (2013) utilised a 15% reserve margin, Perez Canto (2014) a reserve margin of 10% above the load demand, Fettahi *et al.* (2014) a reserve of 5% above the load demand and Linder (2017) enforced a constant 250MW reserve level for each week (time period).

Equations (2.35) and (2.36) aims to level the reserve margin over the planning horizon $\mathcal{T}$ which means it aims to eliminate the deviation between the available generation capacity and the expected demand L_t in time period t. The sum of square reserve r_t^2, as presented in Equation (2.36), is specifically employed as it severely penalises outliers in comparison to the general reserve formulation r_t (Schlunz, 2011; Linder, 2017).

$$r_t = \sum_{i \in \mathcal{N}} P_i^{(d)}(1 - y_{it}) - L_t \quad \forall t \in \mathcal{T} \tag{2.35}$$

$$\min \sum_{t \in \mathcal{T}} r_t^2 \tag{2.36}$$

A modified version of Equation (2.36) is presented by Linder (2017) and Schlunz (2011) where r_{min} represents a constant minimum reserve requirement that should be met for each time period t. The objective function aims to direct the system reserve r_t over the planning horizon $\mathcal{T}$ as close as possible to the set minimum reserve requirement. This formulation is similar to selecting a fixed reserve safety margin S above the required load demand L_t as employed by (Schlunz & van Vuuren, 2013).

$$\min \sum_{t \in \mathcal{T}} (r_t - r_{min})^2 \tag{2.37}$$

Another approach is to minimise the reserve according to Equation (2.39) where $\bar{r}$ represents the average reserve margin over the planning horizon $\mathcal{T}$ and r_t is determined by Equation (2.38) for each time period t (Schlunz, 2011; Dahal & Chakpitak, 2006).

$$r_t = \sum_{i \in \mathcal{N}} P_i^{(d)} - \sum_{i \in \mathcal{N}} P_{it} y_{it} - L_t \tag{2.38}$$

$$\min \sum_{t \in \mathcal{T}} (\bar{r} - r_t)^2 \tag{2.39}$$

The average reserve is determined by means of (2.40).

$$\bar{r} = \frac{1}{m} \sum_{t \in \mathcal{T}} r_t \tag{2.40}$$

Yet another formulation presented in Schlunz (2011) and Linder (2017) follows a different approach where the objective function is used to maximise the minimum reserve margin during time period t. This formulation allows a power utility to maintain the larges possible reserve margin for each time period t.

$$\max \min_{\forall t \in \mathcal{T}} (\sum_{i \in \mathcal{N}} P_i^{(d)} - \sum_{i \in \mathcal{N}} P_i^{(d)} y_{it} - L_t) \tag{2.41}$$

A reliability index approach is employed by Conejo *et al.* (2005) to evaluate the power network's supply security. The reliability index is defined as the ratio of the net reserve divided by the gross reserve. The gross reserve is the total power system generating capacity less the power demand while the net reserve is the gross reserve less the generating capacity scheduled for planned maintenance (Conejo *et al.*, 2005). The variable L_{tb} and P_{ij} denotes the power demand for time period t and subperiod b and the output capacity of unit i of producer j.

$$r_{tb} = \frac{\sum_{i=1}^{N} \sum_{j \in \mathcal{G}_i} P_{ij}(1 - x_{ijt}) - L_{tb}}{\sum_{i=1}^{N} \sum_{j \in \mathcal{G}_i} P_{ij} - L_{tb}} \tag{2.42}$$

Conejo et al. (2005) employs the reliability index formulation (2.42) in conjunction with (2.43), to maximise the reliability of the power system over the planning horizon $\mathcal{T}$. The objective function (2.43) is the summation of each reliability index r_{tb} over each time period t and sub-time period b. Where $\mathcal{B} = \{1, 2, ..., |\mathcal{B}|\}$ denotes the number of sub-time periods and $\mathcal{T} = \{1, 2, ..., |\mathcal{T}|\}$ the number of time periods. The gross reserve remains constant over the planning horizon $\mathcal{T}$ while the objective function (2.43) adjusts the net reserve to find the optimal maintenance schedule with the greatest reserve margin.

$$max \frac{1}{T \times B} \sum_{t=1}^{T} \sum_{b=1}^{B} r_{tb} \tag{2.43}$$

The type of reliability objective function used is dependent upon the modeling approach considered and the industrial data available. Even though there are multiple different reliability objective functions recorded in literature, their purpose remain the same throughout. It is used to ensure security of power supply while attempting to optimise the planned GMS problem.

Suitability or convenience objective

Convenience or suitability objectives are another class of objectives used within the multi-objective (MO) GMS modeling environment with the aim being to assess the suitability of maintenance schedules by attempting to minimise the degree of constraint violations or minimise any deviation from an ideal maintenance schedule (Linder, 2017; Kralj & Petrovic, 1995). Generally GMS models utilise one dominant criterion as the optimisation criteria and all other remaining requirements are formulated as constraints (Kralj & Petrovic, 1995). When multiple conflicting objectives (MO) are employed it could lead to difficulty in solving the GMS model within an acceptable computational time while satisfying all GMS constraints (Linder, 2017; Leou, 2006). For this reason the suitability objectives are used to allow a certain violation tolerance (Leou, 2006).

In Kralj & Petrovic (1993) a three vector objective function is proposed where the objective function includes economic considerations (system fuel cost), reliability considerations (expected unserved energy) and a convenience or penalty function expressing constraint violations. Similarly Leou (2006) implemented a MO function considering reliability and cost indices, where a reliability index is supplementary terminology used to describe suitability or convenience objectives. In Leou (2006), all supplementary GMS constraints including spinning reserve, maintenance crew, line flow limitations and maintenance duration were allowed a certain violation tolerance d_i according to the reliability index. Although some form of violation is allowed, the convenience objective function aims to minimise these violations while ensuring a feasible optimal solution is achieved. Similar to what was proposed in Leou (2006), Zurn & Quintana (1975) also discussed constraint violations using a penalty criterion as a means to relax some GMS constraints.

According to Zurn & Quintana (1975), four main objective function properties are encompassed by the maintenance scheduling deviation criterion or suitability objective. The first two objectives being maintenance urgency and ideal maintenance frequency which aims to minimise any deviations from acceptable standards or recommended preventative maintenance guidelines. The last two aiming to minimise deviation from ideal maintenance sequences of multiple units as well as any changes to previously established maintenance schedules as a result of *e.g.* forced outages or unforeseen restrictions.

It is not known that convenience or suitability objectives have been implemented as a single objective within the GMS modelling environment (Linder, 2017). These types of objectives are usually supplementary to the economic or reliability objectives as recorded in the previous two sections (Linder, 2017; Kralj & Petrovic, 1993; Leou, 2006).

Net present value objective

Although net present value also known as discounted cash flow is not generally utilised within the GMS literature as an objective function, it is a good measure of a project's overall profitability over a long term planning horizon. In essence, planned generator maintenance is a long term project which influences a power utility's return on investment. Each power station can be subdivided into power generating units. Each generating unit produces a certain design capacity of power. Profit gained from each unit is dependent upon the cost of fuel oil, coal, water, plant efficiency, electricity selling price, economic stability *etc.* The main focus of solving the GMS problem is to find a maintenance schedule that will not only ensure maximum return on investment in the short term, but also improve future returns. Successful management of a national power utility requires a long term financial planning approach.

The value of money decrease with each time period t (Yang et al., 1993; Myers, 1984). If we consider an investment scenario over a time horizon $t \in \mathcal{T}$ where $\mathcal{T} = \{t_0, t_1, ..., t_{n-1}, t_n\}$,the value of for *e.g.* one rand in time period t_0 is much more worth than one rand in time period t_n due to the discount rate or rate of inflation $\mathcal{I}$ (Myers, 1984). If a capital investment was made in time period t it could have grown with interest or the capital could have been invested in another venture. This is, however, not possible with capital only obtained in time period t_n. It is therefore more profitable to increase the return on investment of a project closer to time period t_0. Equation (2.44) represents the net present value formulation where PV is the present market value of the investment and C_t the projected capital gain or capital return during time period t.

$$PV = \sum_{t=1}^{\mathcal{T}} \frac{C_t}{(1+\mathcal{I})^t} \tag{2.44}$$

Equation (2.44) is a summation of the total individual discounted present value capital flows at each time period t (Rahimi & Goodarzi, 2015). The NPV formulation can be used to determine the net gain or intrinsic investment value of a project or investment. It is important to note that the NPV formulation generally follows a nonlinear trend over the time horizon $\mathcal{T}$. With reference to the GMS scheduling problem, different maintenance schedules can deliver different NPV projections. The NPV formulation along with appropriate constraints can therefore adequately be implemented to determine which project or maintenance schedule will deliver the greatest intrinsic investment value over a time horizon $\mathcal{T}$. This can be achieved by maximising NPV over the time horizon $\mathcal{T}$.

2.4 GMS solution approaches

A review of the solution methodologies, applied to solve the GMS problem, are provided next, with the main focus being to identify suitable gaps in literature that can be addressed within the context of this dissertation.

2.4.1 Mathematical programming

Linear, integer and mixed integer programming

Mathematical optimisation techniques such as linear, integer and mixed integer programming are deterministic in nature and have successfully been applied within the GMS modeling environment (Al-Arfaj *et al.*, 2015; Kumhas & Kumar, 2016; Bisanovic *et al.*, 2011; Mollahassanipour *et al.*, 2013; Perez Canto, 2014; Garver, 1962; Chattopadhyay, 2001; Ahamad & Kothari, 2000; Barot & Bhattacharya, 2008; Froger *et al.*, 2015). Mathematical programming approaches, when solved utilising an appropriate solver algorithm, provide an exact optimal solution for small problems; however as the problem size increase the solution space expands which leads to computational restrictions and increased model complexity (Al-Arfaj *et al.*, 2015).

In general linear programming (LP) problems are solved using the simplex algorithm while integer programming (IP) and mixed integer programming (MIP) problems are solved by means of applying either the branch-and-bound, variations on the branch-and-bound or cutting plane algorithms (Linder, 2017). Binary 0/1 integer programming is also widely applied to model the GMS problem where binary IPs are usually solved utilising implicit enumeration techniques (Linder, 2017). The below text aims to only provide an overview to the reader on the vast array of mathematical formulations and its application.

An integer programming approach was proposed by Garver (1962) to model the GMS problem. The dual method of linear programming modified to produce integer values also known as the dual Euclidean method was recommended to solve the IP problem formulation.

The simplex method was applied by Ahamad & Kothari (2000) to find the continuous optimum solution of an Institute of Electrical and Electronics Engineers (IEEE) 14 bus mixed integer linear programming problem (four thermal and two hydro units) after which the branch-and-bound procedure was implemented to find the desired integer solution. The MILP formulation proposed by Ahamad & Kothari (2000) included the added complexity of transmission constraints to try and model a realistic power system. The formulation was also applied to an Indian power system containing 63 major buses and 104 power generating units (70 thermal and 34 hydro units) of which an execution time of 667 seconds was recorded to find an optimal solution with a monthly planning calendar considered.

In Bisanovic *et al.* (2011), a 0/1 MILP approach was utilised to model the GMS problem of 22 thermal generating units over a 52 week planning horizon within competitive markets. The model proposed in Bisanovic *et al.* (2011) was solved utilising the homogeneous and self-dual interior point method for linear programming with a branch-and-bound optimiser for the binary sections. The model was successfully applied to solve the industry specific case study with great accuracy and computational efficiency.

Similarly, Mollahassanipour *et al.* (2013) proposed a new linearized cost based preventative maintenance scheduling formulation which was developed as a MILP problem. The new MILP formulation was applied to the IEEE reliability test system which is composed of 26 units with the generating capacities ranging between 12 and 400 MW. Several case studies were solved where it was found that suitable reserve allotment could lead to a decline in total system cost while also reducing the total reserve margin for each time period. This preventative maintenance scheduling considering reserve assessment scheduling approach however ultimately increased overall operating cost. The method proposed to solve the MILP was to relax some coupling constraints and divide it into multiple sub-problems after which it was solved utilising GAMS and CPLEX.

The 0/1 mixed integer linear programming model proposed by Perez Canto (2014) is applied to a power system of 90 generating units including 45 thermal, 8 nuclear, 20 hydro and 17 wind stations over 13 periods and 6 sub-periods (52 weeks) planning horizon. A model solution time of 117.58 seconds was recorded by employing a general algebraic modeling system (GAMS). It is important to note that the reduced computational time can mainly be attributed to the periodic planning horizon approach utilised. A 52-week planning horizon was divided into 13 periods with each period representing 4 weeks. The 13 periods were selected based on the assumption that an average maintenance duration lasts 4 weeks. Although the model formulation clearly reduced the computational requirements; some planning detail is lost compared to a daily planning calendar.

It is clear that there is a vast amount of literature available on the formulation and application of mathematical programming methodologies to describe the GMS problem. The main factor that makes these formulations so attractive is their exact deterministic nature. The drawback of increased computational time with increased solution space is, however, still a hindrance that should be addressed (Khalid & Ioannis, 2012). Some mathematical formulations, as presented above, have been applied to describe realistic GMS instances with great success, but in general assumptions are made, detail omitted, planning calendars reduced or constraints relaxed to ensure a suitable schedule can be obtained within a realistic computational time. It is clear from literature that there is still room for innovation and improvement and, therefore, research focus should be shifted to reduce modeling complexity and to allow development of new modeling formulations which will permit the application of mathematical models to realistic industry-sized problems while ensuring improved computational efficiency.

Dynamic programming

Dynamic programming (DP) is classified as a mathematical programming approach generally applied up to the 1990s to solve the GMS problem (Froger *et al.*, 2015). DP is a complete enumeration scheme applied to solve combinatorial optimisation problems (Al-Farsi *et al.*, 2015). The DP algorithms follow a divide and conquer approach to reduce the amount of computations required while solving a series of sub problems until the solution of the original problem is obtained. According to Yamayee *et al.* (1983) dynamic programming is ideal for the GMS environment due to the sequential nature of the problem.

Dynamic programming with successive approximations has been employed by Yamayee *et al.* (1983) to solve a 21 unit power system GMS problem with a one year planning horizon. Two case studies were utilised of which the first considered a production cost objective function and the second a LOLP (reliability) objective function. Similarly Zurn & Quintana (1977) also applied dynamic programming with successive approximations to solve the GMS problem of 20 fossil fuel units over a 12 month planning horizon. utilising successive approximations address some aspects of the dimensionality computational restrictions caused by industry sized realistic GMS problems (Yamayee *et al.*, 1983).

2.4.2 Heuristics

Heuristics is defined as any practical approach to problem solving that can be used to achieve an immediate goal. From Khalid & Ioannis (2012), heuristics may not lead to the global optimal when applied to a complex problem and usually falls into a local minimum when a sub-optimal starting point is selected. Heuristic methods are trial and error approaches which may include rule of thumb and technical guesses (Linder, 2017; Dahal, 2004). These methods are usually applied in centralized power systems due to their simplicity and flexibility (Khalid & Ioannis, 2012). Applying heuristics within a centralized power system require significant input by the power system operator which means it is highly dependent upon technical expertise (Dahal, 2004). This may result in heuristics not being able to reach the global minimum of a GMS problem and therefore other optimisation techniques could provide more suitable optimal solutions.

In Chattopadhyay (2001) it was proposed to utilise a combination of LP with a heuristic to develop an operations planning model for the Northern regional electricity board of India. The formulation consists of mainly three computational steps which include solving the relaxed LP, ordering binary maintenance variable by means of the heuristic and lastly solving the continuous LP after fixing the maintenance plan. Random outages of generators are evaluated by applying the Monte Carlo algorithm. A combined GAMS language and spreadsheet interface was used to solve the proposed model. Although there are some instances where heuristics are applied to solve the GMS problem; in general mathematical programming or metaheuristic algorithms are preferred.

2.4.3 Meta-heuristics

Metaheuristics are search and optimisation algorithms or procedures used to solve NP-hard GMS problems where the problem size precludes traditional approaches (Khalid & Ioannis, 2012; Linder, 2017). All metaheuristics have three main common features of which include diversification, intensification and memory (Petrowski *et al.*, 2006). Diversification forces these types of algorithms to explore regions of the solution search space rarely assessed by other methods while intensification ensures that a thorough search is conducted when a promising solution space is identified (Petrowski *et al.*, 2006). Metaheuristics usually use some memory features to archive the best solution found during the searching process and is updated as and when a more optimal solution is discovered (Petrowski *et al.*, 2006). The biggest drawback of these types of algorithms is the sensitivity of its tuning parameters and the fact that it is not guaranteed to find an optimal solution. The most common metaheuristic algorithms employed in literature are simulated annealing, Tabu search, ant colony, particle swarm, evolutionary and genetic algorithms (Foong *et al.*, 2008; Anandhakumar *et al.*, 2011; Wang & Handschin, 2000; Volkanovski *et al.*, 2008; Dahal, & McDonald, 1997; Samuel & Rajan, 2013; Saraiva *et al.*, 2011; Satoh & Nara, 1991; El-Amin *et al.*, 2000). A brief overview of the implementation of each of these solution approaches will be provided as recorded in literature along with their advantages and shortcomings.

Simulated annealing

Simulated annealing (SA) is one of the preferred metaheuristics implemented within the GMS environment due to the algorithm's ability to incorporate discrete variables and parameters, the ease of implementation and the good results recorded in literature (Saraiva *et al.*, 2011). SA follows a systematic approach by starting with evaluation of an initial maintenance schedule. Evaluation of a schedule is done by firstly calculating the generation cost of the schedule along the time horizon $\mathcal{T}$ after which all GMS constraints are checked for any violations. Violation of constraints results in penalties being added to the evaluation function. A new solution is then identified in the vicinity of the current one and the evaluation process re-executed to evaluate the new solution. Upon completion of the evaluation a decision can be made whether or not to accept the new solution. Application of the SA algorithm is discussed in detail by (Saraiva et al., 2011). The SA metaheuristic proposed by Saraiva et al. (2011) was applied to a mixed integer problem case study of 29 units (8 coal fired, 8 CCGT, 11 fuel oil and 2 diesel generating units). Two different simulations were conducted on the selected case study with the total amount of iterations for the first simulation being 14.130 and for the second 23.997. The solution time was recorded as approximately 15 minutes. From the results presented by Saraiva *et al.* (2011) the proposed SA model produced an adequate maintenance schedule with reduced computational time requirements.

Similarly, Satoh & Nara (1991) employed SA to solve three different cases of the GMS problem which was formulated as a mixed integer programming problem. The three GMS scenarios were categorized into small, medium and large sized scheduling problems. The small GMS problem scenario considered 15 units, the medium 30 units and the large scenario 60 units over a planning horizon of 25, 40 and 52 weeks, respectively. A comparison was done between two solution techniques where the problem was firstly formulated as a 0-1 integer programming problem and solved by the implicit enumeration method and secondly as a mixed integer programming problem solved utilising the simulated annealing algorithm. From the results obtained it was clear that the SA approach was far superior with reference to the computational time required to find an optimal solution. The large GMS scenario of 60 units was solved within 1295.9 minutes using the SA approach while a solution was not possible with the general integer mathematical programming method employed. Satoh & Nara (1991) proved that although mathematical programming methods ensure optimality there are some computational restrictions.

Tabu Search

The Tabu search (TS) method is an iterative search algorithm which is inspired by mechanisms of human memory (Linder, 2017; Petrowski *et al.*, 2006). Tabu search mainly differ from SA on the basis that TS records recent solution space search history in a short term memory known as a tabu list (El-Amin *et al.*, 2000). The TS method, similar to SA generates a neighbourhood of solutions from the current solution (Linder, 2017; El-Amin *et al.*, 2000). The best neighbouring solution replaces the current solution even if the current solution is superior. This search approach will prevent the algorithm from getting stuck in a local optimum (Linder, 2017; El-Amin *et al.*, 2000). TS has not been extensively applied to solve the GMS problem, however, El-Amin *et al.*. (2000) utilised a Tabu search algorithm to solve a 4 unit and 22 unit GMS scenario after which the results were validated by the implicit enumeration algorithm. It was noted that the TS algorithm offers a promising viable approach to solving the GMS problem (El-Amin *et al.*, 2000).

Genetic and swarm intelligent algorithms

Genetic algorithms (GA) were inspired by the biological process of evolution and inheritance phenomena of living things in nature (Wang & Handschin, 2000; Linder, 2017). GA's can be classified as a multi starting point and multi searching route stochastic heuristic optimisation algorithm (Wang & Handschin, 2000). It is well known from literature that GA's are simple and robust, but has a short coming in the sense that the algorithm proceeds very slowly (Wang & Handschin, 2000). This is mainly due to the complicated decoding computation and heavy fitness evaluation computation required (Wang & Handschin, 2000). GA's are widely implemented to solve the GMS problem *e.g.* Wang & Handschin (2000) employed binary and a new integral GA approach to solve a 30 unit GMS instance over a five year planning horizon. Similarly Volkanovski *et al.* (2008) also developed a genetic optimisation algorithm which was successfully applied to a Macedonian power system containing 29 units over a one year planning horizon with the main focus being to minimise the annual loss of load expectation.

In Dahal, & McDonald (1997), three different types of GA's were employed of which include binary GA's, binary for integer GA's and integer GA's. A 21 unit power system over a 52 week planning horizon was considered with the solution time for each GA algorithm being 45.01s, 21.07s and 16.81s, respectively. It is a significant achievement to find a suitable solution to a complex industrial sized GMS problem; however, Dahal, & McDonald (1997) also made it clear that it cannot guarantee that a global optimum will be reached when applying these type of solution methods.

Another form of metaheuristics that have been employed to solve the GMS problem is ant colony and particle swarm optimisation also known as population based metaheuristics (Foong *et al.*, 2008; Samuel & Rajan, 2013). PSO and ACO metaheuristics exhibit many similarities to genetic algorithms (Linder, 2017). These algorithms are collectively known as swarm intelligence algorithms and are both inspired by the behaviour of biological species such as ants, bees, fish, wasps and birds (Linder, 2017).

A hybrid particle swarm optimisation model was employed by Samuel & Rajan (2013) to solve an IEEE 24 bus system with 32 generating units over a planning horizon of 52 weeks. This proposed method was tested against Lagrangian relaxation, dynamic programming, genetic algorithms, particle swarm optimisation and evolutionary programming after which it was concluded that the proposed hybrid PSO model provided the best results with the fastest computation time. PSO and ACO methods have only been applied in a few instances to solve the GMS problem (Linder, 2017).

It is clear that metaheuristics is seen, by many researchers, as the future for optimisation of the GMS problem as it addresses some critical computational limitations experienced by mathematical programming techniques. It is, however, important to be reminded that metaheuristics also have some shortcomings in the sense that these methods cannot assure global optimality. Multiple advances have been made in the field of maintenance scheduling since the development of these methods, but innovation and progress is a continuous process and it is expected that these methods along with their hybrids will be used in future research endeavours to improve GMS.

2.5 Summary

Chapter 2 provided the scope of the generator maintenance scheduling problem along with the general modeling constraints and objective functions employed in the literature to simulate realistic industry sized scheduling problems. An overview of solution techniques recorded in the literature was presented with the main focus being the identification of gaps that can be addressed within the context of this dissertation. From the literature presented it can be concluded that mathematical models are preferred for their deterministic exact nature, however, the computational restrictions imposed by an increased solution space limits its application. One of the key objectives of this dissertation, as outlined in Section 1.2, is to propose mathematical modeling approaches which can be applied to solve realistic industry sized GMS problems within reasonable computing times.

Chapter 3

Overview of optimisation theory

Operations research originated during the Second World War when British military forces sought to optimise deployment strategies, management of convoys, troops, bombing and allocation of resources (Winston & Goldberg, 2004). After the war these methods attracted the attention of industrial engineers and researchers who focused on optimizing industrial and business processes. Operations research has been called many different names since its development of which decision science, quantitative management and management science, are a few examples. The main focus of optimisation theory has, however, remained the same. The function of operations research was to aid and optimise decision making processes by maximising or minimising certain objectives *e.g.* finances, durations, reliability indices *etc.* Some problems are to complex too simply analyze data in order to determine the optimal decision to be made. The approach used to optimise decision making was to utilise mathematical models to try and describe realistic real life problems or scenarios. Mathematical models along with solver algorithms, used to solve the models, ensure that optimal decisions can be made while satisfying all relevant problem constraints.

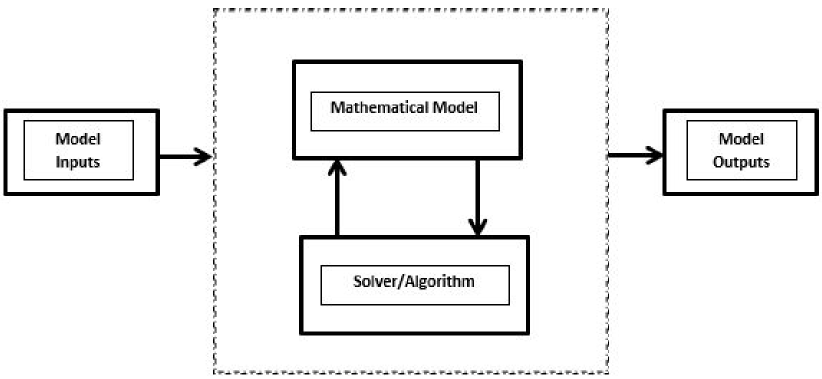

Figure 3.1: Optimisation process diagram

The optimisation process is best explained with reference to Figure 3.1. An optimisation problem mainly consists out of 4 main components:

- Input data such as process data, financial data, information data *etc.*
- A realistic mathematical model to abstract a problem or a specific scenario.
- A solver algorithm to adequately solve the mathematical model in order to produce a solution.

- Output that either assists in a decision making process, provides answers to a question or provides an optimised schedule.

The focus of this dissertation is to develop mathematical models to find an optimal generator maintenance schedule for a national power utility. It is however good practice to have some fundamental knowledge with regards to which modeling and solution techniques are suitable for the problem under consideration before a mathematical model can be developed.

Section 2.4 only provided a broad overview of GMS modeling and solution techniques applied to solve the GMS problem and identified key gaps in literature that should be addressed. As a deterministic mathematical modeling approach is adopted within this dissertation it would be preferable to have a deeper fundamental understanding of some of the relevant modeling and solution techniques presented in Section 2.4 before the GMS models can be formulated. The mathematical formulation structures most frequently used in literature to describe the GMS problem, with applicability to this dissertation, include linear programming (LP), integer programming (IP) and mixed integer programming (MIP). The accompanying solution techniques for each of the above mentioned mathematical formulation structures, as mentioned in Section 2.4, are the primal simplex and the branch-and-bound algorithms. Another general modeling technique found in the literature, although not yet applied within the GMS modeling environment, is graph theory and network flow formulations.

Chapter 3 will be dedicated to expanding the readers knowledge on the above mentioned subjects to ensure a firm understanding of mathematical GMS optimisation theory is gained before the proposed deterministic GMS mathematical models will be developed in Chapter 4.

3.1 Linear programming

Linear programming (LP) is a tool used to describe real life scenarios as optimisation problems. This method of solving optimisation problems were developed in 1947 by an American mathematical scientist George Dantzig (Winston & Goldberg, 2004; Bixby, 2012). LP is widely utilised throughout all industries including banking, petroleum, supply chain, forestry, mining *etc.* as it is one of the simplest ways to perform optimisation. The text to follow will focus on the standard form for LP problems while also presenting a small practical example to explain the fundamental principles behind linear programming.

3.1.1 LP standard form formulation

Firstly, we will look at two definitions that define the fundamental concepts behind linear programming problems (Winston & Goldberg, 2004). The first definition being that:

Definition 3.1.1. A function $f(x_1, x_2, ..., x_n)$ of x_1, x_2,...,x_n is a linear function if and only if for a set of constants c_1,c_2,...,c_n, $f(x_1, x_2, ..., x_n) = c_1x_1 + c_2x_2 + ... + c_nx_n$. This means that the constants along with the variables form a linear relationship to one another. For example, the function $f(x) = 2x_1 + 4x_2$ is a linear function while $f(x) = 2x_1^2 + 4x_2$ is a non-linear function.

Definition 3.1.2. The second definition is defined as for any linear function $f(x_1, x_2, ..., x_n)$ and any number b, the inequalities $f(x_1, x_2, ..., x_n) \leq b$ and $f(x_1, x_2, ..., x_n) \geq$ b are linear inequalities. This means that an inequality such as $2x_1 + 4x_2 \leq 4$ or $2x_1 + 4x_2 \geq 4$ is linear however $2x_1^2 + 4x_2 \leq 4$ is a non-linear inequality.

utilising the above definitions, we can now formally define an LP problem as an optimisation problem where the focus is to either maximise or minimise a linear objective function of decision variables (Winston & Goldberg, 2004). The decision variables must satisfy a set of constraints where these constraints should be linear such as the inequalities defined above. It is also important to note that each variable used to formulate an LP problem is sign restricted. The variables can either be non-negative or unrestricted, depending on the problem formulation. To allow clear understanding of the LP standard form formulation, a practical example will be considered. Firstly, the problem statement will be presented after which the LP mathematical model will be developed.

Consider a valve manufacturer with two different types of valves A and B (Winston & Goldberg, 2004). Valve A sells for R10 000.00 and uses R7000.00 worth of raw material to construct. A total of 3 hours is required for manufacturing of valve A. Valve B sells for R15 000.00 and uses R6000.00 worth of raw material to construct. A total of 5 hours is required for manufacturing of valve B. Within one week a maximum of 35 labour hours are available for manufacturing. The demand for valve A is unlimited, but at most, based on historic data, only 4 of valve B is bought per week. The valve manufacturer wants to optimise weekly profit (revenues - costs).

The first step in formulating the LP problem is to define the relevant decision variables and formulate the objective function. The function of the decision variables is to completely describe the choices to be made while maximising weekly profit. The main choice to be made is to determine the number of each valve to be manufactured and sold per week to maximise profit.

From the problem statement the objective function can be defined as (Weekly revenue) - (raw material purchased).

Where Weekly profit = (Price for valve A)*(number of valve A sold per week) + (Price for valve B)*(Number of valve B sold per week) - (Cost of construction for valve A)*(number of valve A sold per week) - (Cost of construction for valve B)*(number of valve B sold per week)

Mathematically the above can be formulated as $10000x_1 + 15000x_2 - 7000x_1 - 6000x_2 =$ Weekly revenue, where weekly revenue should be maximised. It is also important to note from the problem statement that there are mainly two constraints that should be considered. The first being a limited amount of available manufacturing hours within one week and secondly a limitation on the number of valves to be sold with reference to valve B. A constraint should be formulated for each limitation to ensure that the total amount of manhours and valve sales are not exceeded. Mathematically the construction hours constraint can be defined as

$$3x_1 + 5x_2 \leq 40 \tag{3.1}$$

While the valve sales constraint can be formulated as:

$$x_2 \leq 5 \tag{3.2}$$

The last step in formulating the LP problem is to introduce sign restrictions. The sign restrictions ensure logic answers as an output. These answers can then be used to guide the valve manufacturer as to how many valves of each type A or B should be manufactured to maximise profit. The sign restrictions can be formulated as

$$x_1 \geq 0 \tag{3.3}$$

$$x_2 \geq 0 \tag{3.4}$$

The inequalities and equalities presented above clearly describe the problem statement as a mathematical model and can thus be used to optimise the problem. It is noteworthy that all the inequalities and equalities are linear as per the definitions provided earlier in this chapter. An assumption of divisibility is also made which assumes for a specific linear problem the decision variables x_1 and x_2 can take a fractional value although it is known that *e.g.* one cannot make 1.5 valves. A LP problem for which the variables are restricted to be non-negative integers is known as integer or mixed integer programming. This will be further discussed in Section 3.3 of this chapter (Winston & Goldberg, 2004).

Small LP problems comprising only two decision variables can be solved using a graphical solution method, however, larger problems that have multiple variables and inequalities require appropriate numerical procedures. The most common methods used to solve LP problems are the Simplex algorithm and interior point methods. For the purpose of this dissertation the simplex algorithm will be considered. The simplex algorithm can only be applied to solve the LP problem if the LP mathematical model is in its standard form. The LP standard form ensures that all inequality constraints are changed to equality constraints and all variables are non-negative. To change inequalities to equalities, a slack variable s_i is added to each inequality and the s_i variables are constrained to be greater than zero. When considering the mathematical model presented above, the LP standard form can be formulated as follow:

maximise the objective function

$$w_r = 10000x_1 + 15000x_2 - 7000x_1 - 6000x_2 \tag{3.5}$$

Subjected to

$$3x_1 + 5x_2 + s_1 = 40 \tag{3.6}$$

$$x_2 + s_2 = 5 \tag{3.7}$$

$$x_1, x_2, s_1, s_2 \leq 0 \tag{3.8}$$

The simplex algorithm can now be used to find the optimal LP problem solution.

3.1.2 Primal simplex algorithm

From section 3.1 it is clear that a mathematical model along with a solver algorithm is required to solve an optimisation problem. The first application of the simplex algorithm, developed by Dantzig, was to solve a 21 constraint, 77 variable instance for the classic Stigler diet problem (Bixby, 2012). It was reported that it took up to 120 man days to solve the problem (Bixby, 2012). The simplex algorithm has been extensively studied and improved since then and is still used to this day as a powerful mathematical tool to solve linear optimisation problems.

The simplex algorithm can be divided into the following basic steps (Winston Goldberg, 2004):

1. Convert the LP problem into its standard form.
2. Choose an initial basis.
3. Find a basic feasible solution from the LP standard form.
4. Determine whether the basic feasible solution is optimal or not based on stopping criteria.
5. If the basic feasible solution is not optimal, determine the entering variable (from the non-basis variables) to enter the basis and the basis variable to leave the basis.
6. Execute row operation based on the variable changes identified.
7. Update the basis and repeat from step 3

Each step of the algorithm will be explained by utilising first principle theory while also partially solving the simple LP optimisation problem presented in Section 3.1.1.

Consider a generalized form of a LP problem:

maximise the object function

$$cx \tag{3.9}$$

Subjected to

$$Ax \leq b \tag{3.10}$$

$$x \geq 0 \tag{3.11}$$

Where row vector $c \in \mathcal{R}^n$, matrix $A \in \mathcal{R}^{(m \times n)}$ and column vectors $x \in \mathcal{R}^n$ and $b \in \mathcal{R}^m$. The general formulation of these vectors and matrices can be represented as follow:

$$\text{A} = \begin{bmatrix} a_{11} & a_{12} & ... & a_{1n} \\ a_{21} & a_{22} & ... & a_{2n} \\ \\ a_{m1} & a_{m2} & ... & a_{mn} \end{bmatrix}, b = \begin{bmatrix} b_1 \\ b_2 \\ \\ b_m \end{bmatrix}, \ c = \begin{bmatrix} c_1 & c_2 & ... & c_n \end{bmatrix}$$

$$\text{x} = \begin{bmatrix} x_1 \\ x_2 \\ \\ x_m \end{bmatrix}, 0 = \begin{bmatrix} 0 \\ 0 \\ \\ 0 \end{bmatrix}$$

The first step of the simplex algorithm was completed in Section 3.1.1 where the optimisation inequalities and constraints were converted to the LP standard form. The augmented form of these constraints with the added slack variables can be written as:

$$\text{x}_s = \begin{bmatrix} x_{n+1} \\ x_{n+2} \\ \\ x_{n+m} \end{bmatrix}$$

The next step is to select an initial basis. The number or basis variables are determined by $(n - m)$. In this case there will be 2 basis and 2 non-basis variables. The slack variables s_1 and s_2 are initially selected as the base variables with x_1 and x_2 the non-basis variables. The basis variables are collectively referred to as x_B and the non-basis variables as x_N.

The third step of the algorithm is to find an initial feasible solution. This can be done by using the following equation:

$$x_B = B^{-1}b - B^{-1}Nx_N \tag{3.12}$$

In order to find a feasible solution the non-basis variables are set to zero which simplifies equation (3.12) to

$$x_B = B^{-1}b \tag{3.13}$$

The values of the initial basis variables can, therefore, be determined by equation (3.13). For the problem statement presented in section 3.1.1 the initial feasible solution is found to be:

$$x_B = \begin{bmatrix} 40 \\ 5 \end{bmatrix} \tag{3.14}$$

where $x_1 = x_2 = 0$

The stopping criteria for the simplex algorithm, applied to a maximisation problem, are met when the following inequality is satisfied:

$$(c_N - c_B B^{-1} N) < 0 \tag{3.15}$$

The same applies to a minimisation problem, however, the inequality should be larger than 0. By applying inequality (3.15) to the optimisation problem the following is found:

$$(c_N - c_B B^{-1} N) = (\begin{bmatrix} 3 & 5 \end{bmatrix} - \begin{bmatrix} 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 5 \\ 0 & 1 \end{bmatrix}) \tag{3.16}$$

$$(c_N - c_B B^{-1} N) = (\begin{bmatrix} 3 & 5 \end{bmatrix}) \tag{3.17}$$

When evaluating the stopping criteria it is clear that the problem under consideration is not at its optimal solution and therefore step 5 of the algorithm should be executed. Step 5 involves selecting one exiting basis variable to leave the basis and one entering non-basis variable to enter the basis. From equation (3.17), the largest non-basis variable will enter into the basis. From the non-basis variables x_1 and x_2, x_2 is the largest and will become the entering variable. The entering variable should replace an existing basis variable. In order to determine which of the basis variables should leave, the following ratio test is done:

$$\frac{x_b}{B^{-1} A_i} \tag{3.18}$$

Where A_i is the column vector from A that corresponds to the entering variable which is x_2 in this case. The variable that will leave the basis is determined as follow:

$$\frac{\begin{bmatrix} S_1 \\ s_2 \end{bmatrix}}{\begin{bmatrix} 5 \\ 1 \end{bmatrix}} = \frac{\begin{bmatrix} 40 \\ 5 \end{bmatrix}}{\begin{bmatrix} 5 \\ 1 \end{bmatrix}} = \frac{8}{5} \tag{3.19}$$

With the result that s_2 will leave the basis. As it is now known which variable will enter and exit the basis, the basis can be updated accordingly. The last step of the algorithm is to execute row operations based on the new entering variable selected. The entering variable should become a basic variable in a row which wins the ratio test. For the optimisation problem under consideration, the entering variable will be made a basic variable in row 2. This means that elementary row operations will be done on row 2 to make x_2 a coefficient of 1 in row 2 and a coefficient of 0 in all the other rows.

After executing the simplex algorithm once, a better feasible solution is found, however, the fact that the stopping criteria is not satisfied after the first iteration indicates that the optimal solution is yet to be discovered. In order to find the optimal solution an iterative process is employed until the stopping criteria is met. The completed iterative solution process will not be presented as the purpose of the example is just to give a clear understanding on how the simplex algorithm is utilised to solve LP problems. This was achieved by analyzing a single iteration of the algorithm.

For the purpose of completeness the definition of feasible region and optimal solution for a LP problem is presented below as found in (Winston & Goldberg, 2004) to ensure a clear understating on the terminology used within the optimisation framework:

- A feasible region for a LP problem is defined as the set of all points that satisfy the LP problem constraints and sign restrictions.
- The optimal solution to a maximization or minimization LP problem is the point, within the feasible solution region, which gives the larges or smallest objective function value respectively.

3.2 Integer programming

After development of the iconic simplex algorithm, used to solve LP problems, it became evident that adding integrality constraints to such linear formulations could greatly increase the application of such optimisation methods within industry. There was, however, no algorithm available to solve these types of problems (Junger *et al.*, 2010). It was not until 1958 when Ralph Gomory published a small paper on the modifications required to Dantzig's simplex algorithm in order to provide a finite algorithm for finding an optimal solution to a LP problem. Ralpoh Gomory made some of the first founding contributions to the field of integer programming which sparked interest, innovation and development in the field. IP in conjunction with development of computers at the end of the decade of the 50's gave rise to endless possibilities with reference to optimizing business processes and implementation of IP technology.

IP is used to solve LP optimisation problems with the added complexity where some or all variables are constrained to be integers. There are mainly three major types of integer programming formulations used as presented in the literature which include pure integer programming (IP), MIP and $(0-1)$ integer programming $(0-1IP)$ (Winston & Goldberg, 2004). Pure IP is a problem where all decision variables are constrained to be pure integers. Contrary to IP problems, MIP problems only require some variables to be integers and finally $(0-1IP)$ problems use integer variables where the variables are constrained to either 0 or 1. IP is used in multiple industries including production planning, scheduling, telecommunications networks, supply chain *etc.* The main function of IP techniques is to solve discrete problems and allow for proper output information which can guide decision making processes.

3.2.1 Logical modeling with binary variables

Integer or also known as binary variables can be used to include logical conditions or constraints into a mathematical problem formulation. IP where decision variables are constrained between 0 and 1 form the basis to formulating logical constraints/ conditions (Fourer *et al.*, 2003). LP on its own is not capable of simulating logical decision making processes of most industries and therefore binary decision variables are included into constraints and object functions. Logical modeling can best be explained by means of a simple example.

If we consider the following investment problem where the aim is to invest R15 000.00 in order to maximise annual interest gained. Investment 1 requires R 10 000.00 input capital with an 8% interest rate, investment 2 requires input capital of R8 000.00 with a capital gain of 7.4% and investment 3 requires a R4 000.00 input cost with interest rate of 9%. In most cases the optimal solution would be to invest the money in the investment option with the greatest capital growth potential. For the purpose of this example it is assumed that the money can only be invested by splitting it between investments. No fractional investment is allowed (Sherali & Driscoll, 2000).

With most problems there are logical conditions to consider such as the following:

- Only 2 investments can be made in total.
- If investment 1 is made, then investment 3 should not be considered.

IP allows means by which such a decision making problem can adequately be described. The IP constraints and objective function for the above described problem is formulated below.

Maximise the objective function

$$8x_1 + 7.4x_2 + 9x_3 \tag{3.20}$$

Subject to

$$10000x_1 + 8000x_2 + 4000x_3 \leq 15000 \tag{3.21}$$

$$x_j = 0 \ or \ 1 \tag{3.22}$$

Where the logical constraints can now be modelled as:

$$x_1 + x_2 + x_3 \leq 2 \tag{3.23}$$

$$x_1 + x_3 \leq 1 \tag{3.24}$$

The decision variables for pure IP, with reference to the above problem, may either be 0 or 1. Where 0 indicates that the investment will not be considered and 1 that the investment will be made. Logical constraints allow structure to be added to an optimisation problem that otherwise would not have been possible. Constraints (3.23) and (3.24) are used to include logical structure to the problem where constraint (3.23) prevents more than two investments from being made while constraint (3.24) ensures that if investment 1 is considered, investment 3 will be excluded as an option. Logical modeling with binary variables create an opportunity to describe large decision-making processes as mathematical models. These models can then in turn be used to determine which decision is optimal based on the results of the objective function.

3.2.2 IP and MIP standard form formulation

Similar to LP problems, IP problems also have a standard form to which it should adhere before it can be solved using a mathematical solver. The standard IP form is the following:

Minimise the object function

$$c^T x \tag{3.25}$$

Subject to

$$Ax + s = b \tag{3.26}$$

$$x \geq 0 \quad {\scriptstyle \forall x \in \mathcal{Z}^n} \tag{3.27}$$

$$s \geq 0 \tag{3.28}$$

Where b and c are vectors, A is a matrix, x represents the decision variable and where all entries are integers with linear constraints and an objective function. From the above mathematical definition, the IP problem is converted to its standard form by adding sign restricted slack variables to eliminate all inequalities. The standard form for the IP and MIP is similar to that of LP problems. The only difference between the MIP and IP formulations is the fact that not all variables are required to be integers (Cornuejols & Tutuncu, 2006).

Another important concept to understand with reference to solving IP and MIP problems are the relaxation form of such problems. Relaxation of an IP or MIP problem can be defined as omitting all integers or 0-1 constraints imposed on the IP problem variables (dropping the integrality restrictions). An IP or MIP problem can, therefore, be viewed as an LP relaxation problem with the addition of constraints to define which variables should be integers.

When referring to the LP relaxation form of an IP or MIP problem the following is true:

- The optimal objective function value for a relaxed LP problem is less or equal to the optimal for the IP/MIP problem.
- If the relaxed LP problem is infeasible then the IP/MIP problem will also be infeasible.
- If the optimal solution of the relaxed LP problem satisfies the constraints to which the decision variables are subjected (pure integer or mixed integer constraints) then the optimal solution of the relaxed LP solution is also the optimal solution of the IP/MIP problem.
- The relaxed LP formulation provides information on the bound of the optimal value. By rounding the optimal of the relaxation will in general not give the optimal IP/MIP solution.

Standard IP/MIP problems can be solved using an algorithm such as the branch-and-bound method. The fundamental principles of the branch-and-bound algorithm will be presented in Section 3.2.3. It is important to note that the branch-and-bound method for IP and MIP problems differ, therefore, the method to solve IP problems will be presented first along with a practical example after which the slight modifications required to solve MIP problems will be stated.

3.2.3 The branch-and-bound method for pure and mixed integer problems

The branch-and-bound method is focused on finding an optimal solution by dividing the total feasible set of solutions into partitions after which a systematic approach is followed to evaluate each subset until the optimal solution is found (Taylor, 2006). IP and MIP problems are mainly solved using the branch-and-bound method (Winston & Goldberg, 2004). The branch-and-bound method is, however, not restricted to only integer programming problems and can be applied to solve a wide range of optimisation problems (Taylor, 2006).

The method was first presented in 1960 in a paper written by Ailsa Land and Alison Doig titled an automatic method of solving discrete programming problems. Since then the method has been extensively used within operations research to solve discrete programming problems (Land & Doig, 1960).

The branch-and-bound method for pure integer programming can be divided into the following steps (with reference to a maximization problem with smaller than or equal to constraints) (Taylor, 2006):

1. The solution method is initiated by solving the LP relaxation of the IP problem. Meaning relaxing the integer restrictions.
2. If the solution of the LP relaxation is all pure integers then the optimal solution of the LP problem is also the optimal solution of the IP problem. If the solution of the LP relaxation problem is non-integers, the objective function value of the LP solution forms the upper bound for the optimisation problem while the rounded down solution forms the lower bound.
3. Create two solution subsets from the initial relaxed solution. This is done by evaluating the relaxed LP solution. The variable with the greatest fraction portion will branch on.

4. Two constraint sets are developed to define the new search area of the branched variable. Each constraint set represents a new node.
5. The solutions obtained from each solved relaxed LP node/subset will have an upper and a lower bound. The relaxed solution forms the upper bound while the existing maximum integer solution forms the lower bound.
6. If the solutions obtained are not pure integers the process of branching should once again continue. The node/subset with the highest upper bound (with reference to the object function) will be selected for branching.
7. This process will continue until a feasible integer solution is found. A feasible integer solution is reached when an integer solution is generated at a node and the upper bound of the specific node is greater or equal to any node at the end of a branch.
8. The only difference between maximisation and minimisation pure integer problems is the fact that for a minimisation problem the relaxed solutions are rounded up and the upper and lower bounds are reversed.

Consider the following practical example on how the branch-and-bound method is applied to solve an IP problem. Woody Corporation is a manufacturer that designs and constructs bed frames and antique furniture sets. A bed frame requires 1 hour of labour and 9 crates of raw material while an antique furniture set requires 1 hour of labour and 5 crates of raw material. A profit of R8.00 is made for a single bed frame and R5.00 for an antique furniture set. This problem is clearly an IP problem as one cannot manufacture and sell only half of a bed frame, even though the optimal solution could be fractional values. The branch-and-bound method should, therefore, be used to find the optimal IP solution to the problem presented. The problem is a variation on an IP problem found in (Winston & Goldberg, 2004).

The first step is to formulate the problem statement as a mathematical model after which the LP relaxation of the IP problem should be solved.

Let x_1 denote the number of bed frames and x_2 the number of antique furniture sets.

The problem statement gives rise to the following mathematical model:

maximise the object function

$$y = 8x_1 + 5x_2 \tag{3.29}$$

Subjected to

$$x_1 + x_2 \leq 6 \tag{3.30}$$

$$9x_1 + 5x_2 \leq 25 \tag{3.31}$$

$$x_1, x_2 \geq 0 \tag{3.32}$$

Where x_1, x_2 are integers.

The primal simplex algorithm as stated in section 3.1.2 can be used to solve the LP relaxation of the IP. The results for the LP relaxation are y = 41.25, $x_1 = 3.75$ and $x_2 = 2.25$. It can be concluded from the results that the optimal solution to the LP relaxation will not be the optimal solution to the IP problem; it is, however, the upper bound for the objective function. The optimal y value for the IP will be smaller or equal to the optimal y value for the LP relaxation. As the optimal IP solution has not been identified after execution of the first two steps of the branch-and-bound algorithm the third step can be initiated. Two new solution subsets will be created from the initial relaxation solution. The variable with the greatest fraction portion will be selected to be branched on. As x_1 has the largest fraction portion of 0.75 two new solution subsets will be formed by branching on x_1. The two new branched sub-problems can be defined as:

- Sub set problem 2 = sub set problem 1 + constraint $x_1 \geq 4$
- Sub set problem 3 = sub set problem 1 + constraint $x_1 \leq 3$

The branch-and-bound method completely excludes the possibility that the previous LP relaxation optimal solution can be found as the next optimal solution for one of the branched sub set problems. The method effectively searches the feasible solution area to find the most optimal integer solution. By arbitrarily selecting sub set problem 2 and applying the simplex algorithm to solve the LP relaxation of the sub set a solution is obtained where y = 41, x_1 = 4 and x_2 = 1.8. The group of sub sets created is called a tree, while each sub problem is referred to as a node and the connecting lines between the nodes are known as arcs. The constraints of each node will include the constraints of the previous node connected by the arc line along with the constraints for the LP relaxation. The chronological solving pattern is identified by the label T as can be seen in Figure 3.2. The solution tree development can be summarized as presented in Figure 3.2.

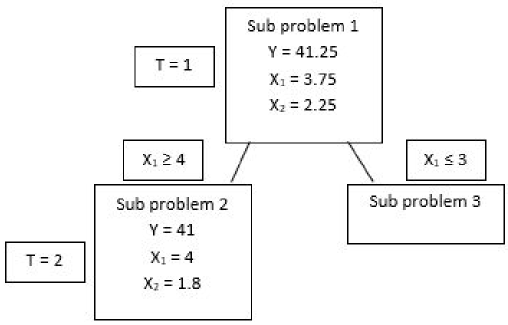

Figure 3.2: Branch-and-bound sub problem tree - first iteration (Winston & Goldberg, 2004)

As no optimal integer solution was found by solving sub problem 2 another branch should be added to the solution tree according to step 6 of the branch-and-bound algorithm. A rule exist which is known as the last in first out rule or LIFO. This implies that the newest branches should be solved first before returning to the other side of the solution tree. From the solution of sub problem 2; only variable x_2 has a fractional value. The next branch is therefore branched from sub problem 2 to create two new sub-problems. The two preceding sub problems are defined as follow:

- Sub set problem 4 = sub set problem 2 + constraint $x_2 \geq 2$
- Sub set problem 5 = sub set problem 2 + constraint $x_2 \leq 1$

By applying the LIFO rule it is clear that either sub set problem 4 or 5 should be solved next. Sub set problem 4 is selected arbitrarily as the node to be solved next. After careful consideration it is found that sub set problem 4 is infeasible and will therefore not yield any useful information. There is no need to branch further from sub set problem 4 as the sub set is fathomed. According to the LIFO rule sub set problem 5 should be considered next. After applying the primal simplex method to the LP relaxation of sub set problem 5 a solution of $y = 40.55$, $x_1 = 4.44$ and $x_2 = 1$ is obtained. Although the solution of sub set problem 5 did not produce the optimal integer solution its solution is still feasible and the process of branching can be once again executed by branching from sub set 5 to form sub set 6 and 7. The solution process will continue based on the branch-and-bound algorithm until the optimal IP solution as presented in Figure 3.3 is attained.

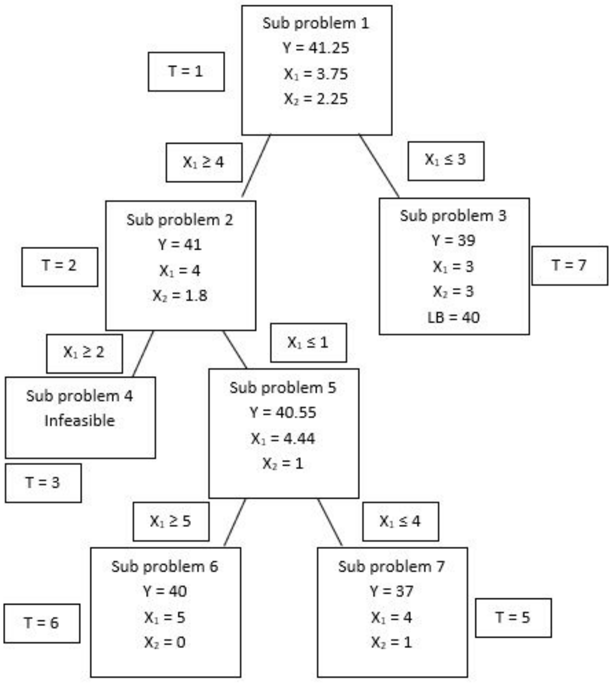

Figure 3.3: Branch-and-bound sub problem tree - final iteration (Winston & Goldberg, 2004)

For the purpose of explaining the branch-and-bound method; the iterations discussed is sufficient to gain insight into how the method is applied in order to find an optimal solution for an IP problem. The branch-and-bound method is an efficient solution algorithm that allows large optimisation problems to be solved. This method of solving IP problems has made it possible to formulate logical industry specific models that can be used to optimise business and decision processes.

The above mentioned procedure is, however, only applicable to pure IP problems where it is required that all variables should be integers. When considering MIP problems the branch-and-bound method can still be used as a solution method, but with a modification to the solution process. The only modification between the IP and MIP branch-and-bound methods being that with the MIP method branching only occurs on variables that are required to be integers. The rest of the algorithm remains the same.

3.3 Graph theory and network flows

The basic concepts of graph theory found its origin in 1735 when Euler studied the notable mathematical problem of Koinsberg Bridge, after which he created the Eulerian graph concept. Multiple researchers after Euler such as Gustav kirchoff (1845), Thomas Gutherie (1852), Thomas P Kirkman and William R. Hamilton (1856), H. Dudency (1913), G.J. Minty (1960

1966), L.R. Ford and D.R. Fulkerson (1962) and C. Berg (1962) have all contributed, following the development of the Eulerian graph concept, to the development and improvement of the different branches within the field of graph theory and network flow to what it is today (Shirinivas *et al.*, 2010; Iri, 1996).

Graph theory is applied in multiple fields such as chemistry, sociology, transport, biology, telecommunications, logistics, engineering, manufacturing science and operations research (Babonneau, 2006; Iri, 1996). Graph theory is applied in operations research to solve problems such as the traveling salesman, to model transport networks and used to solve combinatorial problems (Shirinivas et al., 2010). It is also a powerful tool used to optimise project planning and scheduling problems.

The concept of graph theory will be explained with reference to Figure 3.4. A graph can be defined as a pair of sets $\mathcal{G} = (\mathcal{V}, \mathcal{A})$ where $\mathcal{V}$ represents vertices or nodes $\mathcal{V} = \{1, 2, ..., n\}$ connected by means of arcs or edges defined as $\mathcal{A} = \{(1, 2), (1, 3), ...\}$.

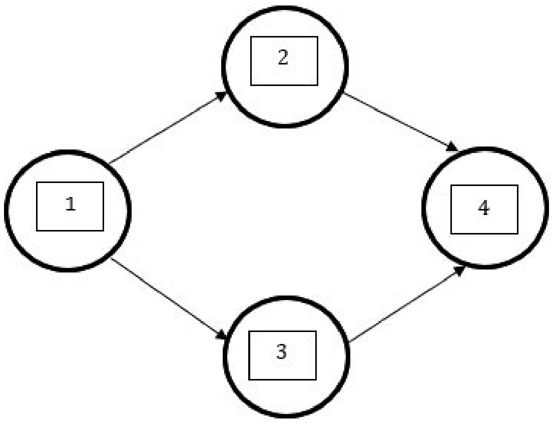

Figure 3.4: Graph theory and network flow optimisation

The basic definitions of graph theory can be summarized as follow (Winston & Goldberg, 2004):

Definition 3.3.1. Arcs or edges are ordered pairs of vertices and can indicate possible direction of motion between nodes.

Definition 3.3.2. A collection of arcs where the preceding arc has only one node in common with the previous arc is known as a chain.

Definition 3.3.3. In graph theory a path is a chain of arcs where the terminal node of each arc is identical to the initial node of the preceding arc.

There are different types of graphs within graph theory. By only slightly changing the definition of a graph the following variations are obtained (Harris *et al.*, 2008):

Definition 3.3.4. - Undirected and directed graphs: In mathematics the set of arcs or edges are unordered meaning that $(1, 2)$ is the same as $(2, 1)$. Unordered sets, therefore, do not have direction. Graphs with ordered sets of edges are referred to as directed graphs.

Definition 3.3.5. - Multigraph: A multigraph is a graph with repeated elements in its set of edges, however, in a multigraph no loops are allowed.

Definition 3.3.6. - Pseudo-graph: A pseudo-graph refers to multiple edges that connect nodes or vertices to itself by allowing the edges to loop between nodes.

Definition 3.3.7. - Hypergraph:

When edges are arbitrary subsets of vertices it is referred to as hypergraphs.

Definition 3.3.8. - Infinite graphs:

An infinite graph is a graph which has V or E infinite sets.

There are multiple types of network flow/graph theory problems from literature of which some include:

- The shortest path problem.
- Maximum flow problems.
- Project and activity scheduling problems.
- Minimum cost network flow/tension problems.
- Minimum spanning tree problems.

The study and application of network flow techniques and graph theory have a long history and is still a field of continuous development (Iri, 1996). It is one of the widest fields of study within the optimisation environment. It is, therefore, noteworthy to provide some background on graph theory and network flow first principles while also presenting the basics on how it can be applied to solve *e.g.* the single and multi-commodity network flow problems as similar concepts will be utilised during development of the GMS resource flow optimisation models presented in Chapter 4.

3.3.1 The single commodity network flow problem

The single commodity network flow problem is a problem which simulates flow of a single commodity from a single source to a single destination through a network of nodes (Du & Chandrasekaran, 2006). A simple example can be used to describe a single commodity flow problem. A company wants to transport the maximum number of a single product between a factory s (source) and a warehouse t (destination) within a certain time frame. The maximum quantity of the product, which can be transported within the given time frame, is governed by the transportation network capacity (Fletcher & Xu, 2009). There may be various transportation routes from the factory to the warehouse. The purpose of the single commodity flow problem, with reference to the above example, is to find the optimal route while adhering to the arc capacities and either minimising or maximising the objective function.

Mathematically the single commodity max flow problem can be defined as a directed flow network with graph $\mathcal{G} = (\mathcal{V}, \mathcal{A})$ where $i \in \mathcal{V} = \{0, 1, ..., N\}$ is the vertices or nodes and $\mathcal{A}$ the arcs $(i, j) \in \mathcal{A}$ dictating the possible flow directions between the vertices. The index 0 denotes the source node and N the sink node. The notation $(i, j) \in \mathcal{A}(i)$ represents a set of arcs where node i denotes the source and $(i, j) \in \mathcal{A}(j)$ represents a set of arcs where node j denotes the target. Each arc has a capacity t_{ij} dictating the maximum allowed flow x_{ij} via each arc (i, j). The flow between nodes is represented by x_{ij} and the cost of flow between arcs is denoted by the unit cost variable r_{ij}. The single commodity maximum flow problem aims to maximise flow or minimise cost (3.43):

$$Max \;\; f_x = \sum_{(s,j)\in\mathcal{A}} x_{sj} \qquad or \quad Min \;\; z = \sum_{(i,j)\in\mathcal{A}} r_{ij}x_{ij}, \tag{3.33}$$

Subjected to

$$\sum_{(i,j)\in\mathcal{A}(i)} x_{ij} - \sum_{(i,j)\in\mathcal{A}(j)} x_{ij} = 0, \quad {\scriptstyle \forall i\in\mathcal{N}}, i \neq 0, i \neq N, \tag{3.34}$$

$$l_{ij} \leq x_{ij} \leq t_{ij}, \quad {\scriptstyle \forall (i,j)\in\mathcal{A}}. \tag{3.35}$$

Where constraint (3.34) is a single commodity conservation constraint that should be satisfied for all nodes except the source and sink nodes where n_i is the general notation for all nodes within the flow network and n_s and n_t the source and sink nodes respectively. The flow x_{ij} between arcs (i, j) are bounded by constraint (3.35) to ensure that the allowable flow capacity between arcs (i, j) is not exceeded where l_{ij} is the lower bound and t_{ij} the upper bound.

The single commodity network flow problem will now be expanded in Section 3.3.3 to include multiple commodities.

3.3.2 The multi commodity network flow problem

The multi commodity network flow problem is a problem which simulates flow of multiple commodities between various nodes within a network. A simple example to describe this flow of commodities can be defined as there are multiple factories which produce various products. The products must be transported to multiple warehouses, each with a different demand. The factories and warehouses represent the nodes within the network and the paths of transport from the factories to the warehouses the arcs between the nodes (dictating the possible capacity for each arc) (Fletcher & Xu, 2009).

The simplest form of the multi commodity flow problem is referred to as the maximum multi commodity flow problem (MCF). The objective of this type of MCF problem is focused on maximising the flows between the nodes (Madry, 2010). Another variation on the maximum flow MCF problem is the maximum weighted MCF problem. Weights are introduced into the problem formulation while the objective is only slightly changed to maximise the weighted sum of the flows (Madry, 2010). In some instances such as the maximum concurrent flow problem the flow of commodities are in one direction (between source and sink nodes) with the focus being to find a multi commodity flow that is feasible while maximising flow between the source and sink nodes (Madry, 2010). A variation on the concurrent flow problem is known as the minimum cost concurrent flow problem. A cost function associated to the arcs are introduced where the objective is to find a maximum concurrent flow between all nodes while ensuring that the sum of the costs incurred due to the flow on each arc meets a specific target or is minimised. For the purpose of this dissertation, a general multi commodity flow problem will be presented in the text to follow. This is to provide insight into the basic formulation and purpose of utilising multi commodity flow problems.

When considering a basic directed multi commodity flow network $\mathcal{G}$ $(\mathcal{V}, \mathcal{A})$ with $i \in \mathcal{V} = \{0, 1, ..., N\}$ the vertices and $\mathcal{A}$ the edges or arcs $(i, j) \in \mathcal{A}$ each arc (i, j) has a maximum capacity t_{ij} dictating the total flow $\sum_{k\in\mathcal{K}} x^k_{ij}$ allowed via each arc. The variable t^k_{ij} denotes the upper bound on the flow x^k_{ij} for each commodity $k \in \mathcal{K}$ along each arc (i, j). The definition of the capacity variable t^k_{ij} depends upon the type of network problem under consideration *e.g.* for a shortest path problem each arc or edge represents a distance, for the maximum flow problem each arc has a maximum capacity *etc.* The flow variable x_{ij}^k simulates the consumption or utilisation of the total capacity t_{ij} of each arc by each commodity k. In an MCF problem commodities compete for mutual arc capacities (Babonneau, 2006). The formulation of the flow variable, similar to the capacity variable, also depends upon the type of network problem considered. A multi commodity flow problem can be defined for k commodities where $\mathcal{K} = \{1, 2, ..., |\mathcal{K}|\}$ and r_{ij}^k denotes the cost of flow of the k^{th} commodity over each arc.

The function of the multi commodity flow problem is to find a flow pattern which satisfies all the below constraints while minimising or maximising an objective function. The formulation presented below is a linear multi commodity flow problem and it is focused on minimising cost or maximising flow (3.36). It is however important to note that the formulations may vary based on the type of multi commodity network flow problem considered.

$$Max \;\; f_x = \sum_{k \in \mathcal{K}} \sum_{(s,j) \in \mathcal{A}} x_{sj}^k \qquad or \quad Min \;\; z = \sum_{k \in \mathcal{K}} \sum_{(i,j) \in \mathcal{A}} r_{ij}^k x_{ij}^k, \tag{3.36}$$

Subjected to

$$\sum_{(i,j) \in \mathcal{A}(i)} x_{ij}^k - \sum_{(i,j) \in \mathcal{A}(j)} x_{ij}^k = 0, \quad \forall k \in \mathcal{K}, \forall i \in \mathcal{N}, i \neq 0, i \neq N, \tag{3.37}$$

$$l_{ij}^k \leq x_{ij}^k \leq t_{ij}^k, \quad \forall (i,j) \in \mathcal{A}, \forall k \in \mathcal{K}, \tag{3.38}$$

$$\sum_{k \in \mathcal{K}} x_{ij}^k \leq t_{ij}, \quad \forall (i,j) \in \mathcal{A}. \tag{3.39}$$

Constraint (3.37) is referred to as the conservation constraint. The conservation constraint ensures conservation of flow between all nodes excluding the source and sink nodes (Karsten *et al.*, 2015; Hall *et al.*, 2007). Constraint (3.38) constrains the flows of each individual commodity to a lower and an upper bound while Constraint (3.39) has the function of placing a common flow upper bound for the sum of the multiple commodities on each arc (i, j) (Karsten *et al.*, 2015; Hall *et al.*, 2007). It is important to note that a multi commodity flow problem with $\mathcal{K}$ commodities is about the size of an $\mathcal{K}$ single commodity flow problem. Constraint (3.39) therefore connects all the $\mathcal{K}$ single commodity problems in a single network to form a multi commodity flow problem.

3.3.3 Summary

This chapter was focused on presenting fundamental optimisation theory with reference to linear and integer programming as well as graph theory and network flow optimisation concepts. Linear and integer programming are key to development of the generator maintenance scheduling models with specific reference to the time index formulations that will be presented in Chapter 4.

Graph theory and network flow optimisation concepts form the foundation for the resource flow model formulations used to optimise schedules and improve project planning. Although the application of graph theory, to solve the GMS problem, has not yet been attempted in literature; these concepts will be used as the basis for formulating the GMS resource flow and novel resource flow models as presented in Chapter 4.

Chapter 4

Mathematical Models

The GMS optimisation problem entails finding an optimal maintenance schedule while both minimising maintenance and operating cost, maximising reliability of electricity supply to satisfy the national demand or any combination of the above-mentioned. The most common mathematical modelling approach used to solve the GMS problem is the time index formulation. Contrary to the literature, the time index formulation presented in this chapter maximises NPV while satisfying all operating, maintenance and financial constraints. It should be noted that the time index formulation naturally lends itself to utilising NPV as an objective function. The aim of this dissertation is to improve the computational efficiency while also reducing the modelling complexity of the model formulations used to solve the GMS problem. This will in turn allow the mathematical models presented in this Chapter to be applied to solve realistic industrial sized GMS problems.

From Chapter 3 it is clear that graph theory or network flow formulations can also be used to solve scheduling problems. These formulations are collectively known as resource flow formulations. The resource flow formulations presented in this chapter also consider NPV as its objective function. The resource flow framework does, however, not naturally accommodate NPV due to its non-linear nature and, therefore, a piecewise linearisation method is employed to approximate the NPV objective function. Two variations of the resource flow formulation are presented in this chapter. The first being a general resource flow formulation as usually applied in literature to optimise scheduling problems. Due to limitations identified with regards to the time dependency of the general resource flow formulation, a novel approach is proposed which allows the resource flow formulation to account for time dependent constraints. It is believed that the novel approach can be used to solve realistic industry sized GMS problems with improved computational efficiency compared to the time index modelling framework.

4.1 Model 1: Time Index GMS Model

The time index GMS model can best be understood by referring to the visual maintenance scheduling scenarios as depicted in Figure 4.1. The time index model, presented in this section, is formulated in such a manner to account for each scheduling scenario. By considering each scheduling scenario within the mathematical formulation of the time index model realistic industry relevant GMS problems can be solved. The GMS time index problem formulation along with all relevant constraints considered are discussed below.

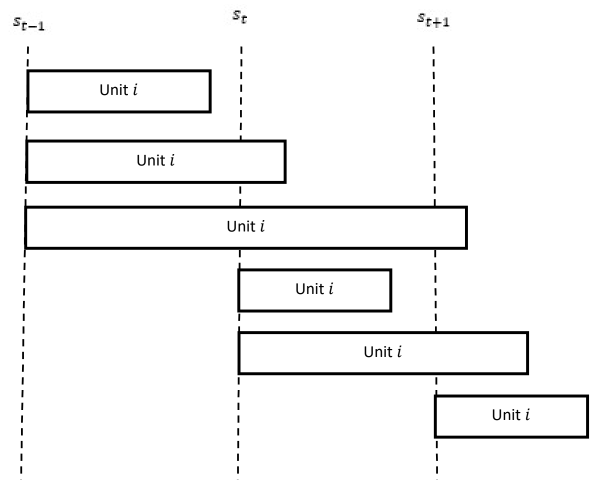

Figure 4.1: Possible time index scheduling scenarios (Terblanche, 2017)

Let $\mathcal{T}$ = {1, 2, ...,|T|} denote the time period indices for the time index formulation. When considering the GMS problem the planning horizon can be divided into a daily, weekly or monthly planning calendar. A 365 day planning horizon with a daily scheduling calendar is considered.

In general the greatest cost associated to power generation is unit operating cost, which simplistically refers to each unit's electricity generation cost or fuel cost. By maximising NPV the most expensive units will be scheduled for maintenance first before the cheaper units are considered. The variable $\mathcal{C}_i^o$ represents the operating cost of each unit i where $i \in \mathcal{N} = \{1, 2..., |\mathcal{N}|\}$. NPV is the difference between the present value of capital inflow and the present value of capital outflow over a period of time t. To determine the value of capital a period of time into the future, inflation $\mathcal{I}$ should also be considered.

The binary decision variable $x_{it} \in \{0, 1\}$ also known as the scheduling variable takes the value of 1 if a unit i is scheduled for maintenance and 0 otherwise. It is important to note that each unit is subjected to a fixed maintenance philosophy based on turbine running hours. The earliest and latest starting time variables T_i^b and T_i^e, therefore, dictate the specific time period in which each unit $i \in \mathcal{N}$ can be considered for maintenance. This will ensure adherence to each unit's maintenance philosophy while preventing premature maintenance scheduling within the planning horizon $\mathcal{T}$.

The binary decision variable $y_{it} \in \{0, 1\}$ is used to enforce a continues maintenance duration d_i for each unit i over the planning horizon $\mathcal{T}$. The maintenance duration d_i is dependent on the type of scheduled maintenance opportunity under consideration *e.g.* IN, IR or GO. The binary variable y_{it} takes the value of 1 if unit i is scheduled for maintenance and 0 otherwise. Activation of decision variable y_{it} will follow the activation of decision variable x_{it}. Maintenance of a power generating unit i, when initiated by decision variable x_{it}, should be completed before it can be committed to return to service and, therefore, the variable y_{it} prevents discontinuity within the scheduling formulation.

For maintenance to occur, multiple resources can be required *e.g.* maintenance crew, spares, special tools *etc.* Within the GMS environment these types of resources are common among most generation units. Units can therefore only be scheduled for maintenance if the required resources are available for the execution phase. v_i represents the number of resources required for maintaining each unit i over the planning horizon $\mathcal{T}$. Resource v_i is only consumed when unit i is scheduled for maintenance as per activation of the binary decision variable y_{it}. This means that for each time index t the sum of the resources v_i required by each unit i should be equal to or less that the available resources U_r. This will prevent resources from being over committed for each time index t.

A GMS model should intrinsically take into consideration the UC problem. The UC problem involves scheduling units for operation to meet the national electricity demand over each time index t. Generally, units that are not scheduled for maintenance, according to the binary decision variable x_{it}, can be committed for power generation to meet the national demand; however, combining both the UC and GMS problems within the GMS modelling framework increases model complexity and computational time requirements. The time index formulation presented in this section differs from what is commonly found in literature. The proposed model formulation rather utilises the available excess reserve margin as key scheduling criterion rather than directly considering the national demand. Excess reserve is calculated as the total generation capacity of the fleet less the national demand and a selected safety factor. Each unit i has a maximum design electricity generation output L_i^d. When a unit is scheduled for maintenance, according to decision variable y_{it}, it cannot contribute to ensuring the national demand is satisfied. The time index formulation presented in this section ensures that the generation capacity L_i^d, removed from the national grid, for all units i that are scheduled for maintenance during time index t may not exceed the available excess reserve margin L_t^e. In this way the national demand requirement is intrinsically accounted for while reducing the complexity of the scheduling formulation.

It is important to note that not all units can simultaneously be scheduled for maintenance within the same power station or geological region as *e.g.* auxiliary steam is used for start-up of units within a power station and there are geological transmission constraints that should not be exceeded. The set of arcs $\mathcal{Z} = \{(1,2),(1,3),...\}$ is employed to account for all precedence or geological transmission requirements.

The objective of the time index GMS formulation when maximising NPV is to

$$maximise \sum_{i\in\mathcal{N}}\sum_{t\in\mathcal{T}}\frac{(C_i^o)}{(1\ +\ \mathcal{I})^t}x_{it} \tag{4.1}$$

subjected to

$$\sum_{\substack{t\in\mathcal{T}\\ t\geq T_i^b\\ t\leq T_i^e}} x_{it} = 1, \quad \forall i\in\mathcal{N}, \tag{4.2}$$

$$\sum_{t\in\mathcal{T}} y_{it} = d_i, \quad \forall i\in\mathcal{N}, \tag{4.3}$$

$$\sum_{\substack{k\in\mathcal{T}\\ k\geq t\\ k\leq(t+di-1)}} y_{ik} \geq d_i x_{it} \quad \forall i\in\mathcal{N},\forall t\in\mathcal{T}, \tag{4.4}$$

$$\sum_{i\in\mathcal{N}} v_i y_{it} \leq U_r, \quad \forall t\in\mathcal{T}, \tag{4.5}$$

$$\sum_{i\in\mathcal{N}} L_i^d y_{it} \leq\ L_t^e, \quad \forall t\in\mathcal{T} \tag{4.6}$$

$$\sum_{t\in\mathcal{T}} t x_{ot} \geq \sum_{t\in\mathcal{T}}(t x_{it} + (d_i - 1)x_{it}), \quad \forall (i,o)\in\mathcal{Z} \tag{4.7}$$

The objective function (4.1) maximises NPV while intuitively committing the cheapest units to satisfy the national electricity demand. Constraint (4.2) ensures that each unit i is only allowed to be maintained once within the planning horizon $\mathcal{T}$. Constraint (4.3) ensures that the maintenance duration is met while constraint (4.4) prevents discontinuity within the maintenance schedule. Constraint (4.5) is utilised to efficiently distribute resources between generating units while preventing resources from being over-committed for each time index t. Constraint (4.6) ensures that units can only be scheduled for maintenance based on the excess reserve L_t^e available for each time index t. Constraint (4.7) also known as the precedence constraint can be utilised to either force units on unplanned maintenance or ensure a specific unit's maintenance takes precedence over others. Constraint (4.7) can also be used to manipulate the maintenance schedule to ensure power station specific or geological region requirements are satisfied.

4.2 Model 2: General resource flow GMS model

Contrary to the time index formulation, the resource flow formulation utilises a graph $\mathcal{G}(\mathcal{N}, \mathcal{A})$ with the set of arcs $\mathcal{A}$ representing the flow of resources $r \in \mathcal{R} = \{1, 2, ..., |\mathcal{R}|\}$ among the nodes $\mathcal{N}$. For the purpose of the resource flow formulations each node $i \in \mathcal{N}$ represents a power generation unit within the generation fleet where $\mathcal{N} = \{0, 1, ..., N\}$.The index 0 is used to define the source unit and the index N the sink unit. The subset $\mathcal{N}'$ is defined as $\mathcal{N}' \subseteq \mathcal{N} \backslash \{0, N\}$. The notation $(i, j) \in \mathcal{A}(i)$ represents a set of arcs where node i denotes the source and $(i, j) \in \mathcal{A}(j)$ represents a set of arcs where node j denotes the target. Each unit i has a specific resource requirement v_{ir} for each resource r within the planning horizon $\mathcal{T} = \{1, 2, ..., |\mathcal{T}|\}$. The resource requirement v_{ir} should be met before a unit can be scheduled for maintenance where $s_i \in \{0, 365\}$ represents each unit's maintenance execution start date. The balance of resource flows dictate that the source and sink node resource requirements v_{0r} and v_{Nr}, respectively, should be set equal to the availability of the resources U_r for all $r \in \mathcal{R}$. Units can only be scheduled for maintenance based on the distribution of the available resources U_r among the unit's requirements v_{ir}. The decision variable f_{ijr} represent the flow of resources $r \in \mathcal{R}$ between each arc (i, j). Allocation of resources v_{ir} among units are dictated by the binary scheduling decision variable z_{ij}. When flow of resources from node i to j is permitted, z_{ij} will take the value of 1 and 0 otherwise.

Figure 4.2 is a basic graphical representation of a typical resource flow problem formulation with a source and sink node n_0 and n_n respectively. The intermittent nodes n_1 through $n_3$represent 3 generating units. The resource flow formulation is set up in such a manner so as to ensure each unit's resource requirements v_{ir} are satisfied within the planning horizon $\mathcal{T}$ while not exceeding the availability of resources U_r. The arrows in Figure 4.2 depict the possible resource flow directions $(i, j) \in \mathcal{A}$ that can be considered as per the predefined set of arcs $\mathcal{A}$. The solid arrows represent the flow of resource 1 while the dashed arrows represent the flow of resource 2. Multiple resources can be required for maintenance execution. Although various flow directions are possible as defined by the set of arcs $\mathcal{A}$, the final resource flow directions $(i, j) \in \mathcal{A}$ will ultimately be dependent upon the availability of resources U_r, additional problem constraints used to define the GMS problem and the optimality of the scheduling solution proposed.

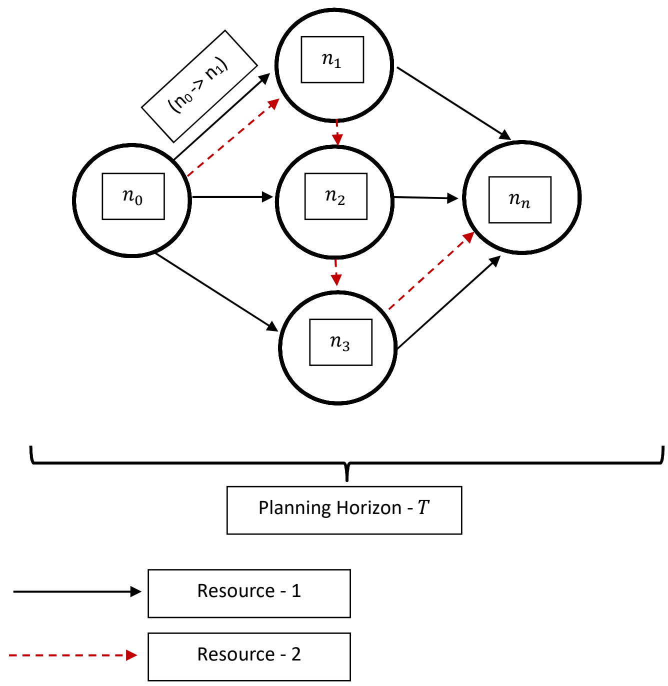

Figure 4.2: General resource flow formulation

To allow comparison between the time index and resource flow models, NPV is considered as the objective function. It is, however, evident that the time index model as presented in section 4.1 naturally lends itself to a linear formulation of the NPV objective function. This is, however, not the case with the resource flow formulation since the scheduling decision variable s_i used in the resource flow formulation of the NPV objective function introduces non-linearity. When considering the resource flow model the approach presented in (Terblanche, 2017) is used to approximate the non-linear NPV objective function as a piece-wise linear function. The non-linear NPV objective function can be written as follow:

$$\max \sum_{i \in \mathcal{N}} f_i(s_i) \tag{4.8}$$

where

$$f_i(s_i) = e^{-\mathcal{I} \times s_i} c_i. \tag{4.9}$$

The piecewise approximation method is explained by the graphical representation presented in Figure 4.3. The data points (k_{iv}, h_{iv}) are the vertices of the piecewise linear approximation of the non-linear NPV objective function (4.9) where $v \in \mathcal{V} = \{0, 1, ..., V-1\}$ represents the piecewise intervals in which the non-linear NPV function is divided. The accuracy of the piece-wise approximation can be manipulated by changing the number of linear segments $v \in \mathcal{V}$. As model accuracy and computational solution time are influenced by the number of linear segments a balance between the two should be established.

Two auxiliary decision variables $\lambda_{iv} \geq 0$, $v \in \mathcal{V}$ and $l_{iv} \in \{0, 1\}$, $v \in \mathcal{V} \backslash \{0\}$ are introduced into the resource flow formulation. The former allows the decision variable s_i to be defined as the combination of the data points k_{iv} and h_{iv}, respectively, while the latter is a decision variable which indicates the appropriate line segment selection with reference to the objective function (4.9). The decision variable n_i represents the NPV for each unit $i \in \mathcal{N}$ as determined by the piecewise linear approximation method. The decision variable l_{iv} takes the value of 1 when a line segment has been selected and 0 otherwise.

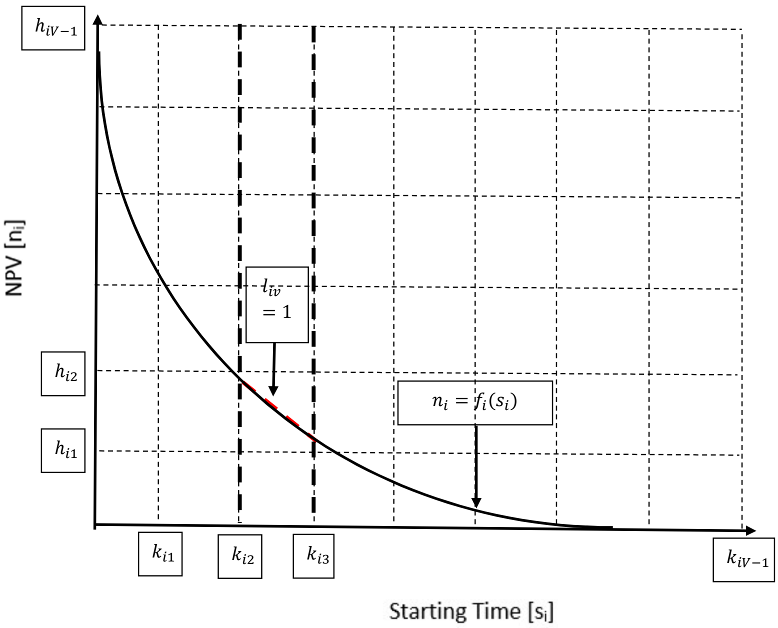

Figure 4.3: Linear approximation of NPV vs. maintenance start time

Similar to the time index formulation presented in section 4.1; the resource flow formulation should also account for unplanned unit maintenance or allow specific units to take precedence over others. The precedence arcs are represented by (i,o) $\in \mathcal{Z}$ where the maintenance start time s_i of unit i should precede that of unit o. Not only should precedence be accounted for but also the unit maintenance philosophies and previous unit maintenance end dates. This is achieved by allowing each unit i's maintenance starting time s_i to be constrained between an earliest and latest starting time T_i^b and T_i^e, respectively.

The objective of the general resource flow GMS formulation when maximising NPV is to

$$maximise \sum_{i \in \mathcal{N}'} n_i, \tag{4.10}$$

Subjected to

$$s_j - s_i - (d_i + M)z_{(i,j)} \geq -M, \quad \forall (i,j) \in \mathcal{A}, \tag{4.11}$$

$$\sum_{(i,j) \in \mathcal{A}(i)} f_{ijr} = v_{ir}, \quad \forall i \in \mathcal{N}, \forall r \in \mathcal{R}, \tag{4.12}$$

$$\sum_{(i,j) \in \mathcal{A}(j)} f_{ijr} = v_{jr}, \quad \forall j \in \mathcal{N}, \forall r \in \mathcal{R}, \tag{4.13}$$

$$f_{ijr} - \min\{v_{ir}, v_{jr}\} z_{(i,j)} \leq 0, \quad \forall (i,j) \in \mathcal{A}, \forall r \in \mathcal{R}, \tag{4.14}$$

$$s_i = \sum_{v \in \mathcal{V}} k_{iv} \lambda_{iv}, \quad \forall i \in \mathcal{N}', \tag{4.15}$$

$$n_i = \sum_{v \in \mathcal{V}} h_{iv} \lambda_{iv}, \quad \forall i \in \mathcal{N}', \tag{4.16}$$

$$\sum_{v \in \mathcal{V}} \lambda_{iv}, = 1, \quad \forall i \in \mathcal{N}', \tag{4.17}$$

$$\lambda_{i0} - l_{i1} \leq 0, \quad \forall i \in \mathcal{N}', \tag{4.18}$$

$$\lambda_{iv} - l_{iv} + l_{i(v+1)} \leq 0, \quad \forall i \in \mathcal{N}', \forall v \in \mathcal{V} \setminus \{0, V-1\}, \tag{4.19}$$

$$\lambda_{i(V-1)} - l_{i(V-1)} \leq 0, \quad \forall i \in \mathcal{N}', \tag{4.20}$$

$$s_o \geq (s_i + d_i - 1), \quad \forall (i,o) \in \mathcal{Z}, \tag{4.21}$$

$$s_i \geq T_i^b, \quad \forall i \in \mathcal{N}', \tag{4.22}$$

$$s_i \leq T_i^e, \quad \forall i \in \mathcal{N}'. \tag{4.23}$$

The objective function (4.10) allows NPV to be maximised while constraint (4.11) along with the linear ordering variable z_{ij} ensures that resources can only be allocated to maintain unit i after the preceding unit j's maintenance has been completed according to $s_i + d_i$. The variable M in (4.10) can be selected as the latest possible finishing time of the scheduling calendar. Constraint (4.11) dictates the possible flow distribution patterns of resources r among all units. Constraint (4.12) represent the flow of resources v_{ir} from the source nodes i while constraint (4.13) represent the flow of resources v_{ir} to the target nodes j. Constraint sets (4.12) through (4.14) therefore allow balance of resources between units while preventing resources from being over commitment. Constraint sets (4.15) and (4.16) express s_i and n_i as a convex combinations of the linear piecewise segments of $f_i(s_i)$ for all $i \in \mathcal{N}$. Constraint (4.17) enforces convexity while constraints (4.18) through (4.20) allow activation of decision variable λ_{iv} to take appropriate values based on the linear line segment selection as dictated by the binary decision variable l_{iv}. Constraint (4.21) is used to enforce precedence between units while constraint (4.22) and (4.23) are used to account for each unit's maintenance philosophy by enforcing an earliest and latest possible starting time T_i^b and T_i^e, respectively.

The resource flow formulation presented in this section has a significant limitation in that the available resources U_r that can be distributed between each unit i, with reference to Figure 4.2, are fixed over the planning horizon $\mathcal{T}$. When considering the excess demand available as discussed in section 4.1 it is clear that for the time index formulation, the excess demand can be varied for each time index t over the planning horizon $\mathcal{T}$ which is a more realistic approach than selecting a fixed amount of resources available U_r over the planning horizon $\mathcal{T}$ as is currently the case in the above formulation. This limitation is addressed by the proposed novel resource flow formulation presented in section 4.3.

4.3 Model 3: Novel Resource flow GMS model

In the section to follow only the variations between the general and the novel resource flow formulations will be discussed. The same piecewise approximation method as discussed in section 4.2 will be used to account for non-linearity of the NPV objective function formulation. In order to address the limitations of the resource flow model, as presented in section 4.2, the following changes and additions were made to the resource flow formulation.

A binary decision variable x_{ig} is introduced into the model formulation. The function of the binary decision variable x_{ig} can best be explained with reference to Figure 4.4. From Figure 4.4 it is clear that the proposed novel resource flow model allows the planning horizon $\mathcal{T}$ to be divided into sub-periods $g \in \mathcal{G} = \{1, 2, ..., |G|\}$. Resource availability for a resource $r \in \mathcal{R}$ in time period $g \in \mathcal{G}$, is defined as U_{rg}.

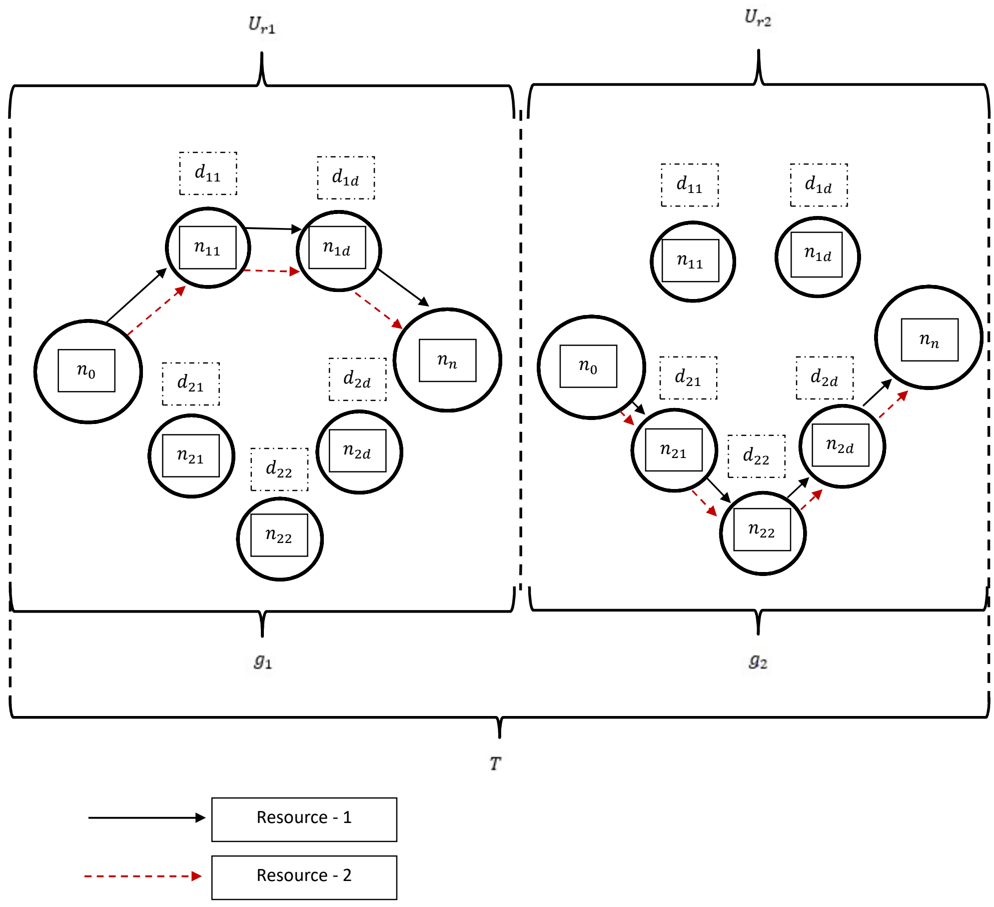

Figure 4.4: Novel resource flow formulation

Contrary to the formulation discussed in section 4.2, the maintenance duration of each unit is divided into sub nodes as presented in Figure 4.4. The reason for sub-dividing the maintenance duration of each unit i is to accommodate the multiple resource availability U_{rg}, for each time period $g \in \mathcal{G}$. More specifically, if $x_{ig} = 1$ for a given $g \in \mathcal{G}$, then the corresponding resource availability U_{rg} applies. According to Figure 4.4, unit 1's maintenance schedule fits into one sub-period. Where the resource availability U_{r1} applies in the first sub-period, and U_{r2} applies in the second sub-period. There may, however, exist scenarios as presented in Figure 4.5 where the maintenance duration of unit i can continue into a subsequent time interval. Figure 4.5 represents the theoretical outage schedule for two units, unit 1 and 2 with the planning horizon

$\mathcal{T}$ divided into two sub time periods g_1 and g_2, respectively. It is important to note that the number of sub-periods $g \in \mathcal{G}$, into which the planning horizon $\mathcal{T}$ can be divided, is not fixed and can be varied to find an optimal balance between model accuracy and solution time. When considering the scheduling scenario depicted by Figure 4.5, it is clear that unit 1's planned maintenance is scheduled to start on day s_{11} and will last a duration of d_{11} where d_i refers to the maintenance duration for each sub-divided node as presented in Figure 4.4. Unit 1's maintenance starts in time interval g_1, but due to the maintenance duration d_{11}, will continue into time interval g_2. The formulation presented in section 4.2 only accounts for the starting times s_i and for this reason although the maintenance of unit 1 extends to time interval g_2, the binary scheduling variable x_{ig} will only take the value of 1 for time interval g_1 and 0 for time interval g_2 as it is linked to the starting maintenance times s_i. This creates the possibility to over-commit the available resources U_{rg} for a specific time interval $g \in \mathcal{G}$. As the new resource flow formulation should be capable of simulating different time periods; dividing the unit maintenance durations into small sub-sections and limiting the flow of resources, between nodes, based on the principle presented in Table 4.1 ensures that even if the duration extends to the next sub time period, interval g_2, the total amount of available resources U_{rg} will not be over-committed.

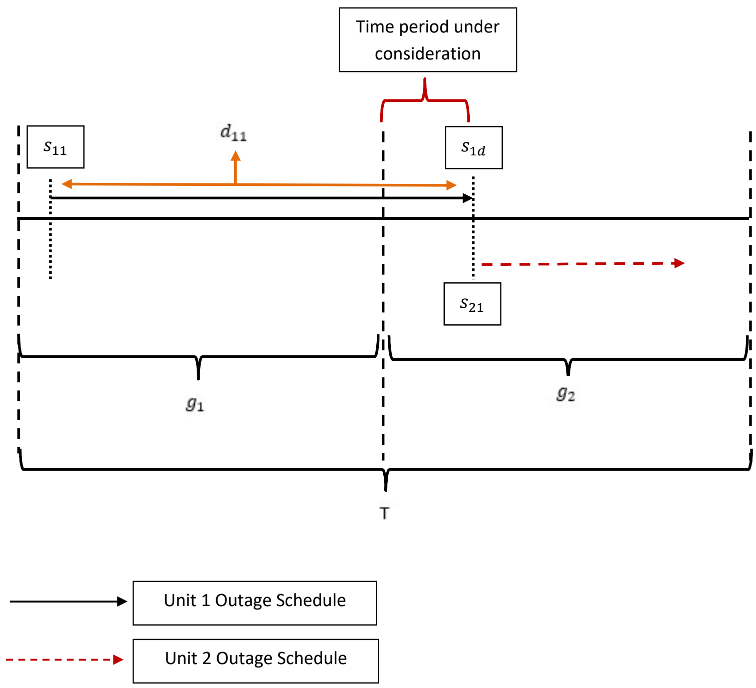

Figure 4.5: Sub time interval resource flow scheduling scenario

From Table 4.1 each unit has a dummy node n_{id}. The function of the dummy node is to represent the end dates of each unit's maintenance opportunity and therefore, no actual maintenance will be conducted based on the activation of the dummy node's scheduling variable x_{ig}. This allows means by which the consumption of resources can be tracked over each simulated time interval g. The maintenance duration d_i for the dummy node n_{id} is set to 0.

By sub-dividing the maintenance durations of each unit i into multiple sub-nodes, introducing dummy nodes and the binary scheduling decision variable x_{ig} into the general resource flow formulation the time dependency limitations identified in Section 4.2 is adequately addressed. Variability of the available resources U_{rg}, for each time interval g, lends the resource flow formulation to the capability of solving realistic industry sized generator maintenance scheduling problems with greater accuracy.

Problem Description		Allowed Flow of Resources Between Nodes $z_{(i,j)}$	
Nodes	Units	Source Nodes	Target Nodes
n_0	Source Node	n_0	n_{11}
n_{11}	Unit 1	n_0	n_{1d}
n_{1d}		n_0	n_n
n_{21}	Unit 2	n_0	n_{21}
n_{22}		n_0	n_{22}
n_{2d}		n_0	n_{2d}
n_n	Sink Node	n_{11}	n_{1d}
		n_{11}	n_n
		n_{1d}	n_n
		n_{1d}	n_{21}
		n_{21}	n_{22}
		n_{21}	n_n
		n_{22}	n_{2d}
		n_{22}	n_n
		n_{2d}	n_n
		n_{2d}	n_{11}
Problem Description		Connecting unit specific nodes to simulate a single unit $w_{(i,j)}$	
Nodes	Units	n_{11}	n_{1d}
n_0	Source Node	n_{21}	n_{22}
n_{11}	Unit 1	n_{22}	n_{2d}
n_{1d}			
n_{21}	Unit 2		
n_{22}			
n_{2d}			
n_n	Sink Node		

Table 4.1: Flow of resources for a two-unit scheduling example

Dividing the resource flow model formulation into time intervals dictates the use of the binary decision variable $o_{ig} \in \{0,1\}$ which is used to link the consumption of resources within a specific time intervals g of each unit i with the maintenance scheduling variable x_{ig}. For each resource $r \in \mathcal{R}$ added to the resource flow formulation a resource consumption linkage decision variable, a flow distribution decision variable and resource requirement variable should be added to the formulation. For the purpose of this section let f^1_{ijg} denote the added flow distribution variable and v^1_{ig} the added resource requirement variable. If unit i is scheduled to start its maintenance duration in time interval g_1, all of its resource requirements should be satisfied in the same time interval *e.g.* distribution of excess reserve, maintenance crew *etc.* These decision variables prevent *e.g.* unit i's excess reserve requirement to be satisfied in time period g_1 while it's maintenance crew requirement is satisfied in time interval g_2 according to Figure 4.4. A start and end time T^b_g and T^e_g, respectively, are allocated to each time interval g according to Figure 4.7. This means that *e.g.* time period g_1 represents a time interval which spans from day 0 to day 28.08 while chronologically time period g_2 will start at day 28.08 and end on day 56.15. Each unit i's maintenance starting time s_i should correspond to a specific time interval g within the planning horizon $\mathcal{T}$. This formulation of the resource flow model can best be explained with reference to Figure 4.7 where unit 1's starting time s_1 falls within the start and end times T^b_1 and T^e_1, respectively, for time interval g_1. The binary maintenance scheduling decision variable $x_{(11)(g_1)}$, therefore, takes the value of 1 for time interval g_1. Unit 1 is sub-divided into two nodes as presented in Figure 4.4 which includes a dummy node n_{1d}. This means that the end date for maintenance completion is accounted for by the dummy node. The end date for the maintenance opportunity on Unit 1, according to Figure 4.6, falls within time interval g_2 and therefore, the binary decision variable $x_{(1d)(g_2)}$ will also take the value of 1 for time interval g_2. This unique formulation of the resource flow model solves the time dependent resource limitations that were identified in Section 4.2.

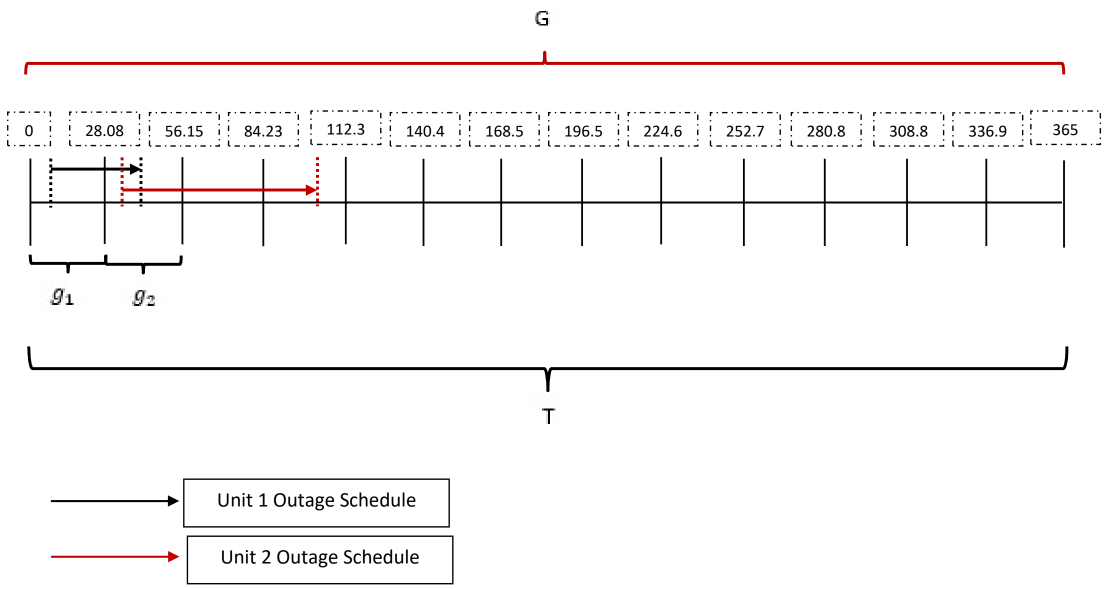

Figure 4.6: Simulating time dependency

The objective of the resource flow GMS formulation when time-dependant resource availability is considered, is to

$$Maximise \sum_{i \in \mathcal{N}'} n_i, \tag{4.24}$$

Subjected to the constraints

$$s_j - s_i - (d_i + M)z_{(i,j)} \geq -M, \quad \forall (i,j) \in \mathcal{A}, \tag{4.25}$$

$$\sum_{(i,j) \in \mathcal{A}(i)} f_{ijg} = v_{ig} x_{ig}, \quad \forall i \in \mathcal{N}, \forall g \in \mathcal{G}, \tag{4.26}$$

$$\sum_{(i,j) \in \mathcal{A}(j)} f_{ijg} = v_{jg} x_{ig}, \quad \forall j \in \mathcal{N}, \forall g \in \mathcal{G}, \tag{4.27}$$

$$f_{ijg} - \min\{v_{ig}, v_{jg}\} z_{(i,j)} \leq 0, \quad \forall (i,j) \in \mathcal{A}, \forall g \in \mathcal{G}, \tag{4.28}$$

$$x_{ig} = o_{ig}, \quad \forall i \in \mathcal{N}, \forall g \in \mathcal{G}, \tag{4.29}$$

$$\sum_{(i,j) \in \mathcal{A}(i)} f^1_{ijg} = v^1_{ig} o_{ig}, \quad \forall i \in \mathcal{N}, \forall g \in \mathcal{G}, \tag{4.30}$$

$$\sum_{(i,j)\in\mathcal{A}(j)} f^1_{ijg} = v^1_{jg} o_{ig}, \quad \forall j\in\mathcal{N}, \forall g\in\mathcal{G}, \tag{4.31}$$

$$f^1_{ijg} - \min\{v^1_{ig}, v^1_{jg}\} z_{(i,j)} \leq 0, \quad \forall (i,j)\in\mathcal{A}, \forall g\in\mathcal{G}, \tag{4.32}$$

$$s_q \geq (s_i + d_i - 1), \quad \forall (i,q)\in\mathcal{Z}, \tag{4.33}$$

$$s_q \geq (s_i + d_i), \quad \forall (i,q)\in\mathcal{W}, \tag{4.34}$$

$$s_i \leq (T^e_g + (1 + x_{ig}) \times 365), \quad \forall j\in\mathcal{N}', \forall g\in\mathcal{G}, \tag{4.35}$$

$$s_i \geq (T^b_g - (1 - x_{ig}) \times 365), \quad \forall j\in\mathcal{N}', \forall g\in\mathcal{G}, \tag{4.36}$$

$$\sum_{g\in\mathcal{G}} x_{ig} = 1, \quad \forall j\in\mathcal{N}', \tag{4.37}$$

$$s_i = \sum_{v\in\mathcal{V}} k_{iv}\lambda_{iv}, \quad \forall i\in\mathcal{N}', \tag{4.38}$$

$$n_i = \sum_{v\in\mathcal{V}} h_{iv}\lambda_{iv}, \quad \forall i\in\mathcal{N}', \tag{4.39}$$

$$\sum_{v\in\mathcal{V}} \lambda_{iv} = 1, \quad \forall i\in\mathcal{N}', \tag{4.40}$$

$$\lambda_{i0} - l_{i1} \leq 0, \quad \forall i\in\mathcal{N}', \tag{4.41}$$

$$\lambda_{iv} - l_{iv} + l_{i(v+1)} \leq 0, \quad \forall i\in\mathcal{N}', \forall v\in\mathcal{V}\backslash\{0,V-1\}, \tag{4.42}$$

$$\lambda_{i(V-1)} - l_{i(V-1)} \leq 0, \quad \forall i\in\mathcal{N}', \tag{4.43}$$

$$s_i \geq T^b_i, \quad \forall i\in\mathcal{N}', \tag{4.44}$$

$$s_i \leq T^e_i, \quad \forall i\in\mathcal{N}'. \tag{4.45}$$

The novel resource flow formulation maximises the summation of each unit's NPV (4.24) while satisfying constraints (4.25) to (4.45). Constraint sets (4.25) through (4.28) are similar to which was presented in section 4.2 with the addition of the binary maintenance scheduling variable x_{ig}. The function of the constraints being to balance flow of resources between units while also simulating time dependency within the resource flow formulation. The function of constraints (4.29) through (4.32) are to align the consumption of resources within each time interval g. A new resource r can be added to the novel resource flow model formulation by utilising a set of constraints similar to constraints (4.29) to (4.32). Constraints (4.29) to (4.32) are used to account for each unit's maintenance crew requirements. These types of balancing constraints along with the binary scheduling decision variable x_{ig} and binary resource consumption linking variable o_{ig} effectively simulate consumption of resources within the resource flow formulation for each time interval g. This also allows means by which the available resources U_{rg} for each simulated time period g can be varied. Constraint (4.33) is used to enforce precedence in order to account for unit maintenance philosophy, unplanned maintenance, geographical or power station specific scheduling requirements while constraint (4.34) is utilised to regulate flow of resources according to Figure 4.6 respectively. Constraint sets (4.35) and (4.36) along with the binary scheduling decision variable x_{ig} and the resource flow precedence arcs (i,j) $\in \mathcal{W}$, Figure 4.6, allow means by which time dependency can be introduced into the resource flow formulation. Constraints (4.37) ensures that each unit i is only allowed to be maintained once within the planning horizon $\mathcal{T}$. Similar to the general resource flow model as presented in section 4.2, constraints (4.38) through (4.43) is used to approximate the non-linear NPV objective function with a piecewise linear solution method. Constraint (4.44) and (4.45) are used to account for each unit's maintenance philosophy by enforcing an earliest and latest possible starting time T_i^b and T_i^e respectively.

4.4 Summary

The GMS problem entails finding the optimal maintenance schedule for a power generation fleet while taking into consideration operating, maintenance and financial constraints. In this chapter three mathematical models were proposed to solve the GMS problem. The first being a variation on the time index formulation as generally recorded in the literature. Most of the time index formulations from literature usually minimise either maintenance cost, operating cost or in some cases maximises power supply reliability. There are also instances where multiple objective functions are considered in a single scheduling model. Contrary to general GMS literature the time index model proposed maximises NPV. Intrinsically by maximising NPV the maintenance schedule minimises operating cost which is the greatest contributor to generation expenditure.

Two resource flow models are also proposed to solve the GMS problem. From the literature, no evidence is available that shows how the resource flow model formulation is applied to solve the GMS problem. The first model is a general formulation of the resource flow model as presented in (Terblanche, 2017). Both the resource flow models minimise NPV. The time index formulation naturally lends itself to a linear formulation of the NPV objective function, while the piecewise formulation method is implemented to account for the non-linearity introduced by both the resource flow formulations. The general resource flow formulation has a significant drawback in that the available resources U_r *e.g.* available excess reserve and maintenance crew cannot be varied over the planning horizon $\mathcal{T}$ as discussed in section 4.2. This limits the implementation of the model to solve realistic GMS problems. The limitation was resolved by the novel resource flow model approach proposed in Section 4.3.

Chapter 5

GMS Models Verification

This chapter will be dedicated to the verification of the models developed in Chapter 4. The purpose of model verification is to proof the functioning of the modeling constraints while also critically evaluating the proposed optimised maintenance schedules generated by each model formulation. The time index and resource flow model formulations will be verified by applying it to solve a simple 5 unit GMS scenario, but first the modeling assumptions, input data sets and the computational hardware specifications will be discussed to give a clear picture on the modeling background.

5.1 GMS Modeling Assumptions

The purpose of a mathematical model is to try and accurately describe a real life scenario or problem. A model can be used to predict, estimate, optimise or solve problems. In some instances modeling a realistic problem is a complex task which may also result in significant computational requirements. It may therefore, be necessary to make appropriate assumptions in order to simplify the models while still addressing the key aspects of the problem under consideration. The following exclusions and assumptions were made during development of the time index and resource flow models:

- The GMS models were set up for fossil fuel fired power stations. Hydro, pumped storage, wind, solar, nuclear, gas and diesel generating units and their associated constraints were not considered during formulation of the proposed GMS models.
- A 365 day's planning horizon was selected which was divided into a daily planning calendar.
- Five different scheduling scenarios have been developed. This includes GMS scenarios for 5, 10, 20, 40 and 92 generating units. The 5, 10, 20 and 40 unit scenarios are scaled based on the realistic industry sized 92 units scheduling scenario.
- An annual maintenance philosophy is assumed for each generating unit. This means that each generating unit should be maintained once within a 365 days planning calendar.
- The model formulations, contrary to the literature, utilise excess reserve margin instead of national demand as the main scheduling criterion. The model formulations' planning horizon is divided into time intervals as can be seen from Appendix A and B. Each time interval is made up of approximately 24 days. The reason for this is to be able to compare the results of the time index and resource flow formulations with one another. The time index formulation naturally accommodates a daily planning calendar with a variable national demand requirement for each day, while the resource flow formulations had to be specifically altered to apply it to the GMS problem. Scheduling is done based on the

worst case scenario minimum available excess reserve margin for each time period. This will in essence give a worst case scenario maintenance schedule which in some cases could be adjusted due to the availability of larger reserve margins.

- Predicted national demand data was used to populate the input excess reserve margins for each scheduling scenario.
- The model can accommodate long term and forced maintenance opportunities, but does not account for unplanned failures. Unplanned maintenance opportunities will manually have to be considered by means of manipulating the input data and correctly applying the precedence constraints.
- Variable maintenance crew requirements throughout each unit's outage duration have been excluded from the model formulations.
- The GMS problem is considered in isolation. From Chapter 2 it is clear that a Power system consists out of multiple sub-scheduling problems. The focus of this dissertation is, however, solely placed upon solving the GMS problem. The UC problem was considered with reference to setting up the constraints as ultimately the optimised generator maintenance schedules should feed into the various sub-problems as availability constraints.
- The merit order cost ranking was used to approximate each generating unit's operating costs. This reduced model complexity due to exclusion of non-linear heat rate cost functions from the model formulations. The NPV objective functions employed within each model formulation, based on the merit order cost ranking, is not a true reflection of actual financial expenditure for the 365 days planning calendar, but can effectively be used to prioritise maintenance scheduling.
- It is assumed that all outages will be executed within the allocated outage durations. The objective function is not penalised based on outage slips. If an outage slip occurs the model should be executed with the applicable changes made to the input data.
- A 6% annual inflation is assumed which is divided over the daily planning calendar.
- An 8% safety margin is assumed to ensure sufficient units will be available to meet the national demand for each time period.

5.2 GMS data sets and scheduling scenarios

Five equivalent GMS scheduling scenarios were developed based on historic and predicted data from the South African National power utility Eskom. It is important to note that the input data sets presented in Appendix A is only equivalent to what is observed in industry as this type of information is confidential to Eskom. The input data generated corresponds well with the outage planning tool Tetris used by the National Power utility to plan maintenance opportunities based on the grid constraints.

The national power utility's installed capacity, including all categories of generating units, approximates to 48 000 MW. The predicted maximum and minimum peak load demands, over a 3 year planning horizon, with a daily planning calendar is approximately 72.00% and 49.00% of the maximum total installed generation capacity. Five scheduling scenarios (5, 10, 20, 40 and 92 units) are developed with the above as a guideline. The 92 generating units scheduling scenario, as presented in Appendix A and B, is the basis from which all the other scheduling scenarios are scaled. An installed capacity of 42 088MW was selected for the 92 fossil fuel generating unit scheduling scenario, which is calculated as the summation of all the fossil fuel generating units' design output capacities. The upper and lower reserve margin boundaries were selected as 61.00% and 54% of the installed generating capacity, respectively. A slightly less constrained national grid, compared to the Tetris data, was selected to firstly test the basic operation of the model formulations after which a more constrained national grid was considered to test the robustness and performance of the proposed models under realistic conditions.

The excess reserve margin for each time period was calculated as the difference between the installed capacity less the national demand and the selected safety margin. The national demand was randomly populated using an Excel VBA function. The randomiser was applied within the above stated reserve margin boundaries.

Equivalent financial data to Eskom's merit order cost ranking was used as input data. The merit order ranking is in general used by the national power utility to try and optimise UC in a cost effective manner in order to meet the national electricity demand.

The remaining input data such as planned outage durations, earliest and latest outage philosophy starting times, precedence arcs and maintenance crew requirements were arbitrarily populated to generate random but industry equivalent GMS scenarios.

5.3 Computational hardware

The models were solved by means of CPLEX while using Excel as a supplementary data input tool. The models were executed on an Inspiron 15 5000 Series Dell laptop with the following specifications:

- Processor: Intel(R) Core(TM) i7-7500U CPU @ 2.750GHz 290GHz.
- Operating System: Windows 10, 64 bit.
- Installed memory (RAM): 6.00 GB.

The above information is provided to give a clear background on the processing unit employed to solve the models and to make it possible to replicate the results if required.

5.4 GMS Model Verification - 5 Unit Scheduling Scenario

Verification of the time index, general resource flow and novel resource flow formulations will be applied to solve a 5 generating unit maintenance scheduling scenario. The data sets used as input into the scheduling scenario is recorded in Appendix A Tables A.1 and A.2. The 5 unit scheduling scenario was selected based on its simplicity. Model simplicity allows means by which the influence of each constraint and objective function can critical be evaluated. The formulation and technical background of each constraint is clearly discussed in Chapters 2 and 4 and it will therefore not be duplicated in this chapter. This chapter will mainly focus on evaluating and discussing the efficiency and effectiveness of the different constraint formulations with reference to the results presented in Table 5.1.

Table 5.1: Verification - 5 Unit Scheduling Scenario

	Time index - Constant excess reserve	**Time index - Variable excess reserve**	**Resource flow - Constant excess reserve**	**Resource flow - Variable excess reserve**
NPV [R/MWh]	1164.40	1164.40	1164.50	1164.60
Computational Time [s]	02:62	02:59	01:10	01:65
Units	Maintenance Schedule - 365 Days Planning Horizon			
1	5 - 34	5 - 34	5 - 34	5 - 34
2	40 - 55	40 - 55	40 - 55	40 - 55
3	56 - 90	56 - 90	56 - 90	56 - 90
4	91 - 150	91 - 150	91 - 150	91 - 150
5	240 - 254	240 - 254	240 - 254	240 - 254

The computational time in Table 5.1 refers to the time that was required for the model formulation to find the optimal solution to the problem considered. The NPV is only a financial indicator used as objective function to schedule units for maintenance based on cost. In order to determine the total cost of the schedule, the ED and UC should be solved as sub-scheduling problems. The maintenance schedules proposed for each model formulation, as recorded in Table 5.1, is a representation of each unit's maintenance interval. The proposed schedule indicates each unit's starting and ending date.

Once off and continuous maintenance constraint

The once off and continuous maintenance constraints is enforced to ensure each generating unit is scheduled for maintenance only once within the 365 days planning horizon, while also ensuring maintenance continuity. Three different model formulations were proposed in Chapter 4. A once off and continuous maintenance constraint was developed for each model formulation. Constraint (4.2) was used within the time index environment to enforce the once off maintenance philosophy while constraints (4.3) and (4.4) ensure maintenance continuity. The general resource flow formulation intrinsically ensures once off maintenance while maintenance continuity is enforced by the linear ordering constraint (4.11). Contrary to the general resource flow formulation, constraint (4.37) was employed to satisfy the once off maintenance constraint within the novel resource flow formulation while similar to the general formulation the linear scheduling constraint (4.25) was utilised to ensure continuous maintenance.

It is important to note that, depending on the input data, the time index and general resource flow model formulations should give the same answers while the time index and novel resource flow model formulations should give the same answers. If the same input data is used and the same problem is solved, in general, only one optimal solution should exist.

From Table 5.1 none of the units were scheduled for maintenance more than once within the 365 days planning horizon. Also the continuity constraints clearly ensured that when a unit's maintenance opportunity is initiated it will continue for its duration. According to Table A.1 unit 1 and 3 have maintenance durations of 30 and 35 days, respectively. The optimal schedule proposed by the various models, as indicated by Table 5.1, shows that unit 1 should be scheduled to start its maintenance opportunity from day 5 for a 30 day period which would then stipulate a maintenance end date of day 34. Similarly, if unit 3's maintenance is initiated on day 56 it will continue for 35 days which would dictate an end date of day 90. Units 1 and 3 are only used to demonstrate the validity of the constraints. Upon investigation, the scheduling results for units 2, 4 and 5 also satisfy these two constraints.

Outage philosophy constraint

The outage philosophy constraint is used in combination with the once off maintenance constraints to prevent units from being maintained before the 365 days or one year outage philosophy has been exceeded. If this constraint was excluded from the formulations, units could be maintained more often than required which would result in unwanted financial expenditure. The user of the model will select the current financial year's allowable earliest and latest starting times based on the previous maintenance cycle's execution dates. The time index formulation makes use of constraint (4.2) to ensure the outage philosophy is enforced while the general resource flow formulation make use of constraints (4.22) and (4.24) and the novel resource flow model constraints (4.44) and (4.46).

From Table A.1 in Appendix A units 1, 2, 3, 4 and 5 are only allowed to be scheduled for maintenance after time periods 5, 40, 1, 1 and 240, respectively. The results recorded in Table 5.1 clearly substantiate the fact that the outage philosophy constraints adequately prevent premature outage scheduling. All units 1 through 5 are scheduled for maintenance either on the earliest allowable maintenance start date or sometime afterward. Units 1, 2 and 5 are scheduled to start their maintenance opportunities on the earliest allowable time period, while units 3 and 4 only start their maintenance a few time periods after the earliest allowable maintenance start dates. The scheduling scenario will determine the optimal maintenance schedule.

Precedence constraint

The precedence constraint formulations can be used to fulfill multiple functions of which some include forcing units on outage based on condition, preventing a number of units within the same power station or geological region to be schedule for maintenance during the same time period and finally to allow means by which the user can manipulate some aspects of the maintenance schedule if and when required. Constraint (4.7) is employed within the time index environment to enforce a predetermine precedence arc while constraints (4.21) and (4.33) are applied within the general and novel resource flow formulations, respectively.

From Appendix A, unit 1 should take precedence over unit 2 for the 5 generating unit scheduling scenario. The optimised maintenance schedule as recorded in Table 5.1 for all four GMS models indicate that unit 1 will start its maintenance on day 5 and will continue up to day 34 while unit 2 will only start its maintenance on day 40. Although the results presented in Table 5.1 satisfy the precedence requirements, the constraint can only properly be evaluated if for example unit 2 should take precedence over unit 1 as this will prevent the outage philosophy constraint from basically bypassing the precedence requirements. The optimised maintenance schedule as recorded in Table 5.2, where unit 2 should take precedence over unit 1 clearly proof the correctness of the precedence constraints.

Table 5.2: Verification - 5 Unit Scheduling Scenario to proof precedence constraints

	Time index - Constant excess reserve	**Time index - Variable excess reserve**	**Resource flow - Constant excess reserve**	**Resource flow - Variable excess reserve**
NPV [R/MWh]	1163.90	1163.90	1164.00	1164.20
Computational Time [s]	02:28	02:15	01:09	01:37
Units	Maintenance Schedule - 365 Days Planning Horizon			
1	56 - 85	56 - 85	56 - 85	56 - 85
2	40 - 55	40 - 55	40 - 55	40 - 55
3	1 - 35	1 - 35	1 - 35	1 - 35
4	86 - 145	86 - 145	86 - 145	86 - 145
5	240 - 254	240 - 254	240 - 254	240 - 254

From the adjusted optimised maintenance schedule, as per Table 5.2, unit 2's maintenance execution date precedes that of unit 1 even though the outage philosophy constraint allows unit 1 to be schedule for maintenance from time period 5. The precedence constraints provide modeling versatility to account for multiple scheduling criteria.

Excess reserve margin constraint

As discussed in Chapter 2, 4 and the start of this chapter, excess reserve is used as one of the main scheduling criteria when considering whether a unit can be scheduled for maintenance. A unit can only be scheduled for maintenance if the national demand can be satisfied for each time period while maintaining a predetermined safety margin. The time index model naturally lends itself to a simplistic reserve margin constraint formulation as shown by in constraint (4.6). The resource flow models, however, are more complicated as the constraints take a flow distribution approach. Constraints (4.13) through (4.15) are employed by the general resource flow formulation to govern the distribution of available excess reserve between all units throughout each time period. A similar approach is utilised by the novel resource flow formulation with the main difference being the addition of a binary scheduling variable x_{ig} which allows means by which the resource flow formulation can account for variability in time dependent constraints.

As discussed in Chapter 4, the novel approach can accommodate variability in excess reserve over multiple time periods while the general resource flow formulation can only accommodate a constant excess reserve through the planning horizon. The general formulation lacks the capability to account for time dependency.

From Table A.2 and the data presented in Appendix A it is clear that no two units can simultaneously be schedule for maintenance within the planning horizon due to available excess reserve. The design output capacities of units 1 and 2 are 686MW while units 3 to 5 are 618 MW. No matter how the units are paired it will always exceed the available reserve margin as recorded in Table A.2. This means that maintenance durations of any of the 5 generating units cannot overlap within the 365 days planning horizon. From Table 5.1, the maintenance slots for each unit never overlap which therefore substantiates the constraints validity and effectiveness.

Resource constraint

The resource allocation constraint follows an identical approach to the excess reserve margin constraint. In order to effectively evaluate the function of the resource allocation constraint, the excess reserve and precedence constraints were removed from the model formulations. Any influence on the maintenance schedule is therefore determined by the outage philosophy and resource allocation constraints.

Table 5.3: Verification - 5 Unit Scheduling Scenario - Resource constraints

	Time index - Constant excess reserve	**Time index - Variable excess reserve**	**Resource flow - Constant excess reserve**	**Resource flow - Variable excess reserve**
NPV [R/MWh]	1167.30	1167.30	1167.40	1167.50
Computational Time [s]	01:48	01:53	01:01	01:39
Units	Maintenance Schedule - 365 Days Planning Horizon			
1	5 - 34	5 - 34	5 - 34	5 - 34
2	40 - 55	40 - 55	40 - 55	40 - 55
3	1 - 35	1 - 35	1 - 35	1 - 35
4	56 - 115	56 - 115	56 - 115	56 - 115
5	240 - 254	240 - 254	240 - 254	240 - 254

From Appendix A, the 5 unit scheduling scenario has a total of 100 crew members that can be distributed between units 1 through 5 as and when maintenance is initiated. When considering the outage philosophy constraint it is clear that units 3 and 4 are both allowed to start maintenance from day 1 of the planning horizon. From Table 5.3 it is seen that unit 2's maintenance is, however, only initiated on day 56. This can be attributed to the influence of the resource allocation constraint. Unit 3 requires 29 crew members and unit 4 requires 85 to execute the maintenance scope. If both units' maintenance opportunities were to overlap the resource allocation constraint would be violated as a total of 114 crew members would be required. It can, therefore, be concluded that the resource allocation constraint correctly governs resource distribution throughout each time period.

Supplementary constraints

Some supplementary constraints were required to formulate the novel resource flow model as the general resource flow model has some critical limitations when accounting for variable time dependent constraints. Constraints (4.34), (4.35) and (3.36) were added to the novel resource flow model formulation to overcome the time dependent limitation. The function of the above constraints is to allow unit maintenance to stretch continuously over multiple time periods, without any discontinuity. Each time period was selected to represent approximately 24 days. As discussed in Chapter 4 each unit's maintenance duration can be sub-divided into multiple nodes. This means that different nodes can be activated for maintenance within different time intervals. From Table A.2 it is clear that the first two time intervals of the planning horizon span from 1-24 days and 25-48 days, respectively. Constraint (4.34) ensures that when a unit is scheduled for maintenance, unit specific nodes are activated according to the precedence flow arcs as discussed in Chapter 4. Constraints (4.35) and (4.36) allow the activation of each unit specific node to occur within one of the time intervals as stipulated in Table A.2. When considering the results of unit 1 in Table 5.1 its scheduled maintenance duration span from time period 5 to 34, which means it overlaps the two first time intervals. If Constraints (4.34) through (4.36) were not added to the novel resource flow formulation discontinuity would exist and the results as presented in Table 5.1 would not have been possible.

NPV objective function

NPV is employed as the objective function to optimise the 5 unit GMS scenario. As discussed in Chapter 4 the time index formulation naturally lends itself to a linear formulation of the NPV objective function as can be seen from constraint (4.1). The resource flow formulations, however, require a segmented linearized approach to approximate the non-linear NPV objective function. Objective function (4.10) along with constraints (4.16) through (4.21) are used to approximate the non-linear NPV objective function. Similarly, for the novel resource flow formulation objective function (4.24) along with constraints (4.38) through (4.43) are used to approximate the non-linear NPV function over the 365 days planning horizon.

The constraints were clearly defined in Chapter 4 and therefore, it will not be repeated in this chapter, however, it is noteworthy to mention that the function of the NPV objective function is to maximise NPV over the planning horizon. The merit order operating cost is used as the financial indicator for the maintenance schedules, which intuitively means, that the model effectively maximises the operating expenses. The reasoning behind this is to propose a schedule where the most expensive units are scheduled for maintenance first while the cheaper units are used to supply demand. This will increase the NPV of the schedule and will allow for greater capital gain earlier within the planning horizon. From Table A.1 it can be noted that units 1 and 2 have a merit order cost of 291 R/MWh while units 3, 4 and 5 have a cost of 199 R/MWh. As the objective functions are formulated to maximise NPV it would be expected that both units 1 and 2 would firstly be scheduled for maintenance followed by the cheaper units. This expectation is confirmed by the results recorded in Table 5.1. Even though units 3 and 4 are allowed to start their maintenance from day 1 of the planning horizon, based on the outage philosophy constraint, their maintenance opportunities are only initiated during time period 56 and 91, respectively, so as to maximise the NPV of the maintenance schedules.

It is also noteworthy from Table 5.1 that the NPV for the two time index formulations are slightly different from which was recorded for the resource flow formulations. This can be explained due to the segmented linearization method used to approximate the non-linear NPV objective function. The input data, time intervals *etc.* all influence the accuracy of the linearisation method. It can, however, be concluded, from the results recorded in Table 5.1, that the linearisation method can effectively be applied to approximate the non-linear NPV objective function while allowing the resource flow formulations to efficiently evaluate the solution search space in order to find the optimal maintenance schedule.

5.5 Summary

The focus of chapter 5 was on verification of the time index, general resource flow and novel resource flow models by means of solving a simplistic 5 unit GMS problem instance. The proposed optimised schedules were critically evaluated by means of comparing the results of all the models with the expected intent of the constraints presented in Chapter 4. It was concluded from the verification process that all constraints function as expected allowing the models to be further scrutinised by applying it to realistic problems of a larger size. Chapter 6 will focus on validating the three proposed GMS models by evaluating their performance against the objectives proposed in Chapter 1 of this dissertation. The models will be used to solve GMS instances for 5, 10, 20, 40 and 92 generating units while also evaluating each models capability to accommodate various reserve margins.

Chapter 6

GMS Model Validation

Chapter 6 will aim to validate whether objectives 3 and 4, as presented in Section 1.2 of Chapter 1, has been achieved by utilising the time index, general resource flow and novel resource flow formulations to solve instances for 5, 10, 20, 40 and 92 generating unit GMS scenarios. The results of the GMS scenarios are presented in Appendix B as no value would be added by repeating the same discussion for the various maintenance schedules. What would, however, add value is to critically evaluate the computational requirements of each proposed model formulation for each GMS scenario. Two different instances of the GMS problem will be considered with the first being GMS scenarios with sufficient excess reserve margins and, secondly, GMS scenarios with a constrained national grid. The sufficient excess reserve margin scenarios assume a upper and lower reserve margin of 61% and 54% of the total installed capacity, while the constrained national grid scenarios assume a 72% upper and 49% lower reserve margin, as recorded in Appendix A. The sufficient excess reserve instances will be presented first followed by the constrained GMS scenarios.

6.1 Models Validation

The unconstrained national grid GMS scheduling scenarios consider an upper and lower excess reserve margin of 61% and 54% of the total installed capacity. As discussed in Chapter 5, the 92 unit scheduling scenario presented in Appendix A forms the basis from which the 5, 10, 20 and 40 unit instances are scaled.

The two main objectives of this dissertation is to develop a mathematical formulation that can be used to solve realistic industry-sized GMS scenarios with reduced computational time requirements compared to general modeling approached used in the literature. Tables 6.1 and 6.2 provide a summary of the important indicators used to evaluate the efficiency and validity of each model formulation proposed in Chapter 4. The NPV recorded in Table 6.1 is a measure of the optimality of the generator maintenance schedules proposed by each model formulation. It is intuitive that as the size of the scheduling scenarios increase so will the order of magnitude of the NPV objective function values. This is substantiated by the results presented in Table 6.1. When evaluating the results presented in Tables 6.1 and 6.2, the results of the constant excess reserve models should be compared with one another while the variable excess reserve models should be compared. It is noteworthy that most of the NPV values, as per Table 6.1, have some marginal differences. These differences are however, negligible as it varies between 0.00% and 0.10%. It can therefore be justified, from a modeling point of view, that although there is marginal differences in the objective function results the resource flow model formulations compared to the time index formulations are able to provide equivalent optimised maintenance schedules.

The differences in NPV between the time index and resource flow formulations are as a result of the segmented linearisation method used to approximate the non-linear NPV objective function for the resource flow formulations which were extensively discussed in Chapter 4. The difference in NPV also increases with an increase in the number of units considered. This can be explained by the number of NPV graphs introduced into the problem formulation which was used for the purpose of linearisation. It is important to remember that although a discrete modeling approach is considered, CPLEX still applies some numerical methods to find solutions to the discrete models. An increase in the number of NPV graphs with an increase in problem size give room for numerical errors to occur and therefore, some differences in the results can be expected especially with the larger problem instances.

Differences in the NPV results infer that there will be some small differences in the proposed maintenance schedules. These differences are, however, acceptable and do not influence the validity of the models in any way.

Table 6.1: Unconstrained GMS Scenarios Financial Indicators

	Time index - Constant excess reserve [R/MWh]	**Time index - Variable excess reserve [R/MWh]**	**Resource flow - Constant excess reserve [R/MWh]**	**Resource flow - Variable excess reserve [R/MWh]**
Units	GMS Models Financial Indicators			
5 Units	1164.40	1164.40	1164.50	1164.60
10 Units	2810.80	2810.80	2811.20	2811.50
20 Units	6731.50	6731.40	6732.80	6732.70
40 Units	14 204.00	14 204.00	14 207.00	14 207.00
92 Units	30 229.00	30 229.00	30 235.00	30 234.00

Although solution optimality of the various model formulations are quite important, the computational time required to solve these models to optimality is of greater interest within the context of this dissertation. Table 6.2 along with Figures 6.1 and 6.2 provide a summary of the computational requirements of each model for the various scheduling scenarios.

Table 6.2: Unconstrained GMS Scenarios Computational Time

	Time index - Constant excess reserve [min]	**Time index - Variable excess reserve [min]**	**Resource flow - Constant excess reserve [min]**	**Resource flow - Variable excess reserve [min]**
Units	GMS Models Computational Time			
5 Units	00:02:62	00:02:59	00:01:10	00:01:65
10 Units	00:03:84	00:03:76	00:02:20	00:02:28
20 Units	00:07:29	00:07:07	00:03:92	00:09:19
40 Units	00:22:86	00:17:49	00:02:88	00:05:71
92 Units	00:44:50	00:54:17	00:04:37	00:26:29

Figure 6.1 is a comparison of the computational requirements of the time index and general resource flow formulations with a constant excess reserve margin as input. From Figure 6.1 it is clear that for all the scheduling scenarios from 5 up to 92 units; the general resource flow model formulation is significantly faster in finding the optimal generator maintenance schedule. On the contrary the computational time of the time index formulation is almost directly proportional to the amount of units considered. This is proof that for a fairly unconstrained national grid the

general resource flow formulation can effectively be used to solve realistic industry sized GMS scenarios with reduced computational requirements

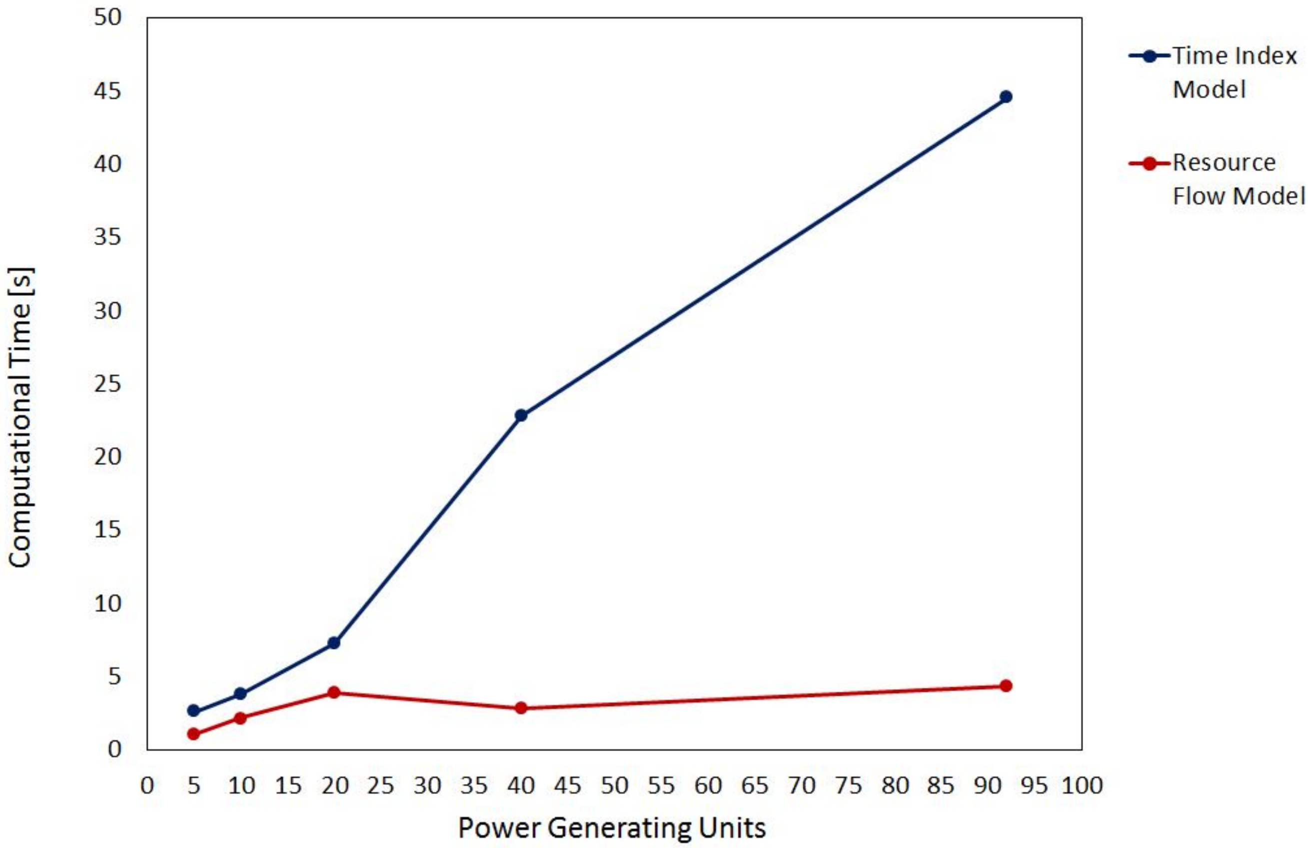

Figure 6.1: Time index vs general resource flow GMS models - Unconstrained reserve margin

The comparison of the time index and novel resource flow formulations with variable excess reserve margin as input, as presented in Figure 6.2 show a similar trend to what is depicted in Figure 6.1. In general the novel resource flow formulation is more efficient in finding the optimal maintenance schedules of the various GMS scenarios considered. It should, however, be noted that for the 20 units scheduling scenario the novel resource flow formulation is slightly slower computationally than the time index formulation. Although the computational time of the general resource flow formulation for the 20 unit scenario, presented in Figure 6.1, is faster than the time index formulation, a similar spike in the computational time is observed.

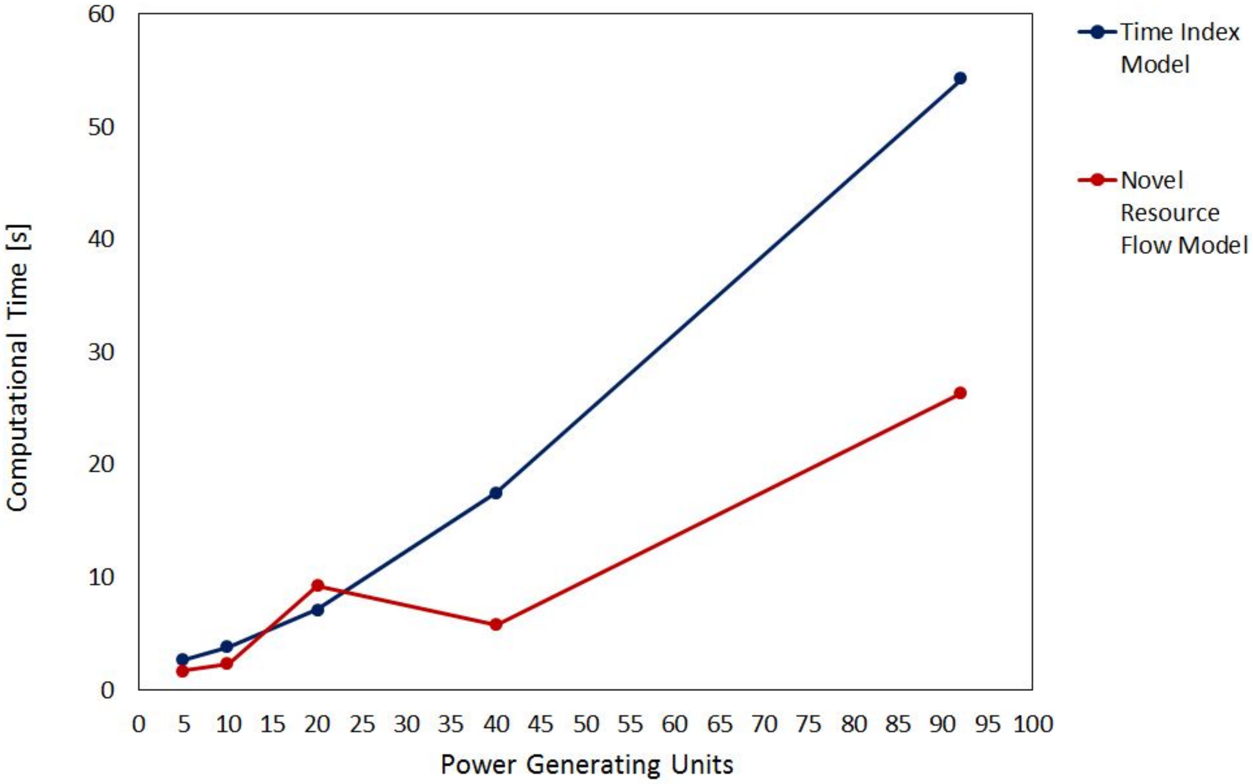

Figure 6.2: Time index vs novel resource flow GMS models - Unconstrained reserve margin

The spike in computational time can be explained with reference to Table 6.3. The results presented in Table 6.3 are for the exact same 20 unit scheduling scenario as presented in Appendix A. The only difference being the formulation of the novel resource flow model. The models used to generate the results presented in Figure 6.1 and 6.2 utilised a 365 days planning horizon divided into 15 time intervals while the results recorded in Table 6.3 was generated by a resource flow model utilising a 365 days planning horizon divided into quarters. The reason for reducing the time intervals is to prove the influence of the time intervals on the resource flow models' computational efficiency and model accuracy.

A larger number of time intervals dictate a greater solution search space that should be evaluated, however, this may increase the model accuracy. The computational time of the novel resource flow model can, therefore, to a certain extent be optimised by means of adjusting the number of time intervals considered. With reference to Figure 4.3 in Chapter 4, it is important to realise that a balance is required between the number of time intervals used versus the accuracy of the model; speed without accuracy is also pointless. A reduced number of time intervals will result in the piecewise approximation of the non-linear NPV objective function to be less accurate. From the results presented in Tables B.3 and 6.3 it is clear that the quarterly time interval structured novel resource flow model is more efficient in finding the optimal maintenance schedule, due to its reduced computational time, while the 15 time interval structured model provides a more accurate solution when taking the equivalent time index formulation as the basis. This statement is validated with reference to the maintenance schedule proposed for unit 2 in Tables 6.3 and B.3. The maintenance schedule recorded in Table B.3 for the 15 time interval structured resource flow model formulation, directly correspond with the schedule proposed by the time index model formulation with variable excess reserve. The only drawback of the 15 time interval structured resource flow formulation being the increased computational time. On the contrary the quarterly structured resource flow model proposes a slightly different maintenance schedule as reported in Table 6.3, but with reduced computational requirements. It can, therefore, be concluded that the number of time intervals employed influence computational efficiency and model accuracy. The maintenance schedule presented in Table B.3, as proposed by the novel resource flow formulation, shows a small difference in the schedule starting dates for units 2, 16, 10 and 17 compared to the schedule proposed in Table 6.3. All of these discrepancies can be attributed to the reduced accuracy of the quarterly structured resource flow model.

Table 6.3: GMS - 20 Unit Scheduling Scenario.

	Time index model constant excess reserve	**Time index Variable excess reserve**	**Resource flow constant excess reserve**	**Resource flow Variable excess reserve**
NPV [R/MWh]	6731.50	6731.40	6732.80	6732.70
Computational Time [s]	07:29	07:07	03:92	06:17
Units	Maintenance Schedule - 365 Days Planning Horizon			
1	5 - 34	5 - 34	5 - 34	5 - 34
2	55 - 74	56 - 75	55 - 74	55 - 77
3	1 - 35	1 - 35	1 - 35	1 - 35
4	67 - 126	67 - 126	67 - 126	67 - 126
5	240 - 254	240 - 254	240 - 254	240 - 254
6	90 - 124	90 - 124	90 - 124	90 - 124
7	42 - 55	42 - 55	42 - 55	42 - 55
8	1 - 36	1 - 36	1 - 36	1 - 36
9	25 - 96	25 - 96	25 - 96	25 - 96
10	75 - 99	76 - 100	75 - 99	75 - 99
11	35 - 66	35 - 66	35 - 66	35 - 66
12	36 - 54	36 - 54	36 - 54	36 - 54
13	10 - 88	10 - 88	10 - 88	10 - 88
14	1 - 20	1 - 20	1 - 20	1 - 20
15	145 - 176	145 - 176	145 - 176	145 - 176
16	56 - 117	55 - 116	56 - 117	56 - 117
17	100 - 173	100 - 173	100 - 173	101 - 174
18	20 - 49	20 - 49	20 - 49	20 - 49
19	190 - 219	190 - 219	190 - 219	190 - 219
20	40 - 100	40 - 100	40 - 100	40 - 100

From the above-mentioned it can be concluded that the resource flow formulations are computationally superior to the time index formulations with reference to its efficiency in finding an optimal maintenance schedule. The above statement is, however, only true with reference to an unconstrained national grid. The resource flow formulations should, therefore, also be tested with an input data set which is equivalent to a realistic industry sized problem with the available excess reserve margins being a true representation of the current national grid as recorded in the Eskom Tetris planning tool. It is important to once again note that the type of input data and model structure greatly influence the computational time and the proposed optimal maintenance schedules of these models. The performance of the resource flow models, for a realistic constrained scheduling scenarios, can be seen by means of evaluating the results for the 20, 40 and 92 units scheduling scenarios, with an upper and lower national demand of 72% and 49% of the installed design capacity respectively.

The constrained input excess reserve margin data is presented in Appendix A Tables A.12 through A.14.

Table 6.4: Constrained GMS scenarios Financial Indicators

	Time index - Constant excess reserve [R/MWh]	**Time index - Variable excess reserve [R/MWh]**	**Resource flow - Constant excess reserve [R/MWh]**	**Resource flow - Variable excess reserve [R/MWh]**
Units	GMS Models Financial Indicators			
20 Units	6726.50	6726.80	6727.90	6728.30
40 Units	14 200.00	14 200.00	14 202.00	14 203.00
92 Units	30 227.00	30 228.00	30 232.00	30 232.00

Table 6.5: Constrained GMS scenarios Computational Time

	Time index - Constant excess reserve [min]	**Time index - Variable excess reserve [min]**	**Resource flow - Constant excess reserve [min]**	**Resource flow - Variable excess reserve [min]**
Units	GMS Models Computational Time			
20 Units	01:03:31	00:59:94	05:26:42	03:43:28
40 Units	05:08:44	11:31:81	55:18:74	38:22:62
92 Units	00:49:37	00:42:35	00:29:53	00:28:82

By evaluating the results recorded in Table 6.5, Figures 6.3 and 6.4 an interesting phenomena is observed. For the constrained 20 and 40 units scheduling instances, the resource flow formulations are computationally slower than the time index formulations. A sharp increase in computational time is observed from 20 to 40 units. Contrary to the 20 and 40 unit instances, the resource flow formulations are once again computationally superior when considering the results of the constrained 92 unit scheduling scenario. The solution time for the 92 unit scheduling scenario (when considering the constant excess reserve instances) for the time index and resource flow model formulations are 0.49s and 0.29s, respectively. When considering the variable excess reserve instances, the time index and novel resource flow model formulations' solution times were recorded as 0.42s and 0.28s, respectively. It is clear that an equilibrium exist between the number of units, the size of the planning horizon considered and the availability of excess reserve where the time index models will outperform the resource flow models.

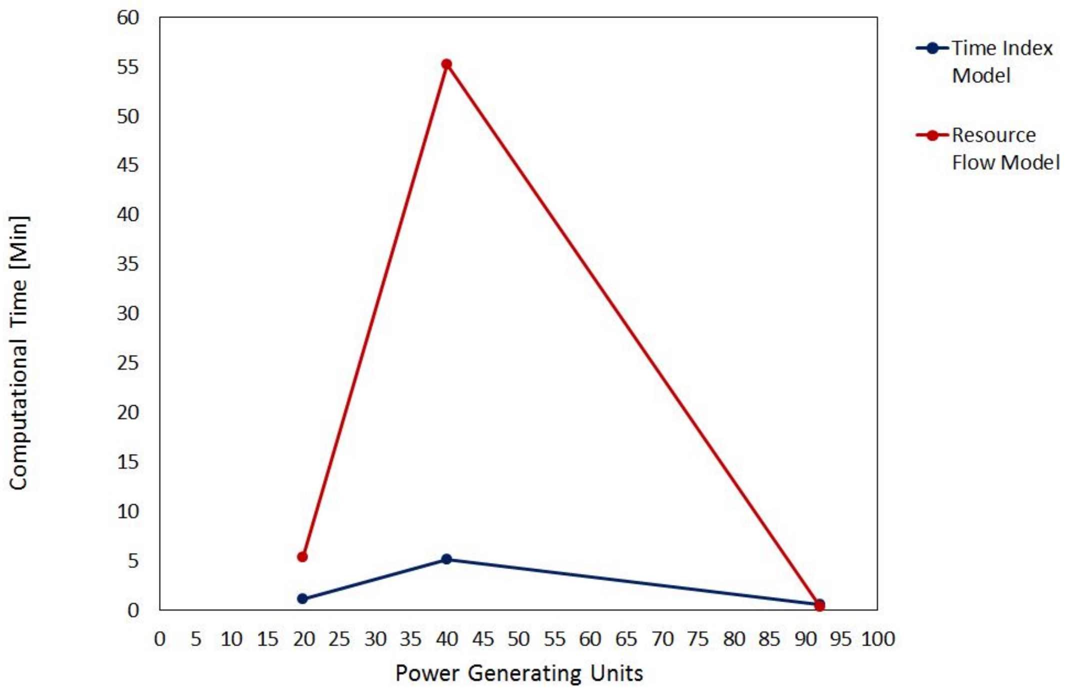

Figure 6.3: Time index vs general resource flow GMS models - Constrained reserve margin

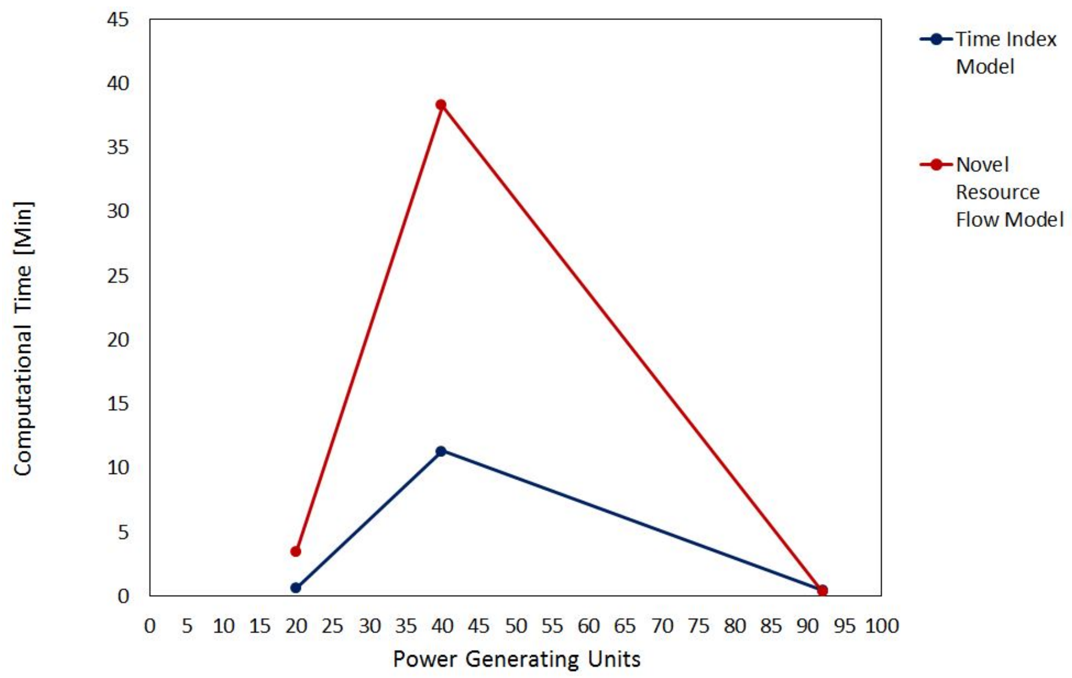

Figure 6.4: Time index vs novel resource flow GMS models - Constrained reserve margin

For certain scheduling scenarios with a constrained national grid; the time index formulation provides a faster solution to the GMS problem instances. Although the resource flow model's performance drastically deteriorates with small constrained problem instances, the pros of this type of formulation when considering larger scheduling scenarios outweigh the cons.

The results presented in this chapter clearly prompt further investigation into the resource flow formulations as means by which GMS problems can be solved. The development of the novel resource flow formulation opens a new world of possibilities within the field of generator maintenance scheduling.

6.2 Summary

This chapter was dedicated to validating the time index, general resource flow and novel resource flow formulations developed in Chapter 4 so as to determine if the objectives set out in Chapter 1 was achieved.

It is clear from the results presented in this Chapter that the proposed resource flow formulations can be applied to solve realistic industry-sized GMS instances with greater computational efficiency than the time index formulations. The objectives set out in Chapter 1 have, therefore, been satisfied by the results provided while also urging further investigation into applying the novel resource flow formulation within the GMS problem environment. The proposed novel resource flow model shows promising results which could lead to improving the way GMS is approached.

Chapter 7

Summary and conclusion

GMS is imperative to effective and efficient management of a national power utility. Sub-optimal generator maintenance schedules may result in unwanted financial expenditure due to the cascaded planning nature of a power system. The GMS problem is a complex combinatorial problem. Solving realistic industry sized GMS problems can be a daunting task due to computational power limitations, the vast nature of the problem constraints and the interconnecting structure of a national power system. It is, therefore, imperative that the field of GMS be approached with an innovative, creative and a questioning attitude so as to ensure continuous development and improvement.

In general, from a mathematical deterministic modelling point of view, the time index formulation has always been used as basis for solving most generator maintenance scheduling problems. The nature of the time index modelling formulation naturally lends itself to time dependant constraints. Most researchers attempt to use variations of the time index formulation and it is seldom that other GMS mathematical modelling approaches are observed in literature. However, one of the main objectives of this dissertation was to challenge this notion by attempting to use graph theory and network flow modelling structures to solve the GMS problem.

This conclusion chapter will focus on providing a summary of the content presented in this dissertation which will include the key research contributions made to the field of GMS after which the chapter will be concluded with a discussion on possible future research initiatives to consider as a result of the work done in this dissertation

7.1 Chapter summaries

The first chapter of this dissertation focused on providing context for the GMS problem while motivating the value and necessity of improving the models and algorithms used to solve these complex combinatorial scheduling problems. The primary objective of this dissertation was to apply graph theory and network flow concepts to develop a model that would outperform the general time index formulations without compromising on accuracy. Due to the sheer size and complexity of GMS problems, finding an optimised schedule, within an acceptable time frame, for a realistic industry sized GMS problem was identified as the secondary objective.

Chapter 2 was dedicated to providing a literature overview about maintenance within the power generation industry after which the scope of the GMS problem was clearly defined within the context of a power system. All relevant GMS constraints and objective functions as employed in literature were presented, after which the chapter was concluded with a discussion on the solution algorithms generally applied to solving the GMS problems, while noting some gaps that could be addressed by means of the research presented in this dissertation.

A theoretical overview on the mathematical algorithms used to generally solve GMS problems was presented in Chapter 3 with the objective to give a thorough background on optimisation theory applicable to GMS. The fundamental principles of LP, the simplex algorithm, IP, the branch-and-bound algorithm, graph theory, single and multi-commodity flow problems were discussed.

Chapter 4 presented the formulation and development of three different types of GMS models. The first being the general time index model, secondly a general resource flow model and finally the novel resource flow model. A key contribution of this chapter is the application of the resource flow modelling structure in order to solve the GMS problem. Another key contribution is the development of the novel resource flow model formulation which is capable of accounting for time dependant constraints. This allows application of resource flow models to industry size GMS problems. This is a key contribution as the resource flow formulation has not been applied previously to solve the GMS problem.

Verification of the proposed models presented in Chapter 5 was done by means of solving a 5 generating unit scheduling problem while critically evaluating each constraint to proof its correctness. A key contribution was made by demonstrating that the resource flow formulations as presented in Chapter 4 can be used to accurately solve GMS problems. The resource flow models were found to propose similar or equivalent optimised maintenance schedules compared to the time index formulations.

Chapter 6 was dedicated to presenting the results of the 5, 10, 20, 40 and 92 units scheduling scenarios so as to demonstrate that the resource flow formulations can be used to solve GMS problems with greater computational efficiency while still being able to provide accurate comparable results to that of the time index formulations. The key contribution of this chapter was the solution of a realist industry sized 92 units scheduling scenario while also proving that for all the above scheduling scenarios the resource flow formulations are computationally more efficient. The models were scrutinised to evaluate its robustness.

The heart of this dissertation is the novel resource flow formulation. It can be concluded from the results presented in Chapter 5 and 6 that the novel resource flow model formulation has met all the main objectives as set out in the introductory chapter of this dissertation and has opened new frontiers with regards to solving the GMS problem.

7.2 Future research initiatives

The development and implementation of the novel resource flow GMS model demonstrated that it is computationally more efficient in finding optimal maintenance schedules. This allows for a new horizon of possibilities that should be investigated. The following may be considered for furthering the work presented in this dissertation:

- **Reduction of modeling assumptions**: GMS relevant constraints, not considered within this dissertation, can be added to the model formulation in order to reduce the number of assumptions made while attempting to model reality.

- **Increasing the planning horizon**: Apply the novel resource flow model to larger planning horizons while utilising predicted national demand as input so as to optimise the maintenance life cycle management of a national power utility.

- **Expanding the novel resource flow model to include all generating categories**: The main focus of this dissertation was to utilise graph theory and network flow concepts to develop a model that can be used to solve the GMS problem with great computational efficiency. Within the scope of this dissertation only fossil fuel power stations were considered. It is, therefore, proposed to include renewable stations, peak load stations and nuclear stations into future resource flow model improvements.

- **Power system management**: Incorporate the novel resource flow model formulation into a power system scheduling framework to optimise the holistic management of a national power utility.

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Appendix A

GMS Model Data Sets

The input data for the mathematical models, as developed in Chapter 4, is presented below in tables A.1 through A.14. Unconstrained reserve margin GMS scheduling scenarios were populated and solved using the time index, resource flow and novel resource flow model formulations for 5, 10, 20, 40 and 92 generating units. The model formulations were also applied to solve constrained reserve margin scheduling scenarios for 20, 40 and 92 generating units. The constrained 92 unit scheduling scenario simulates a realistic industry sized scheduling problem.

The national demand and excess reserve data was generated using an excel VBA randomize function with the recorded Eskom Tetris scheduling tool data as a guideline. The input data, as required by the GMS model formulations developed in Chapter 4, include the design output capacity of each generating unit, the planned maintenance duration, the earliest and latest maintenance philosophy starting times, the power generation merit order cost and the maintenance crew requirements .

Firstly the unconstrained data sets will be presented followed by the constrained sets.

Unconstrained reserve margin GMS scenarios

Table A.1: GMS - 5 Unit Unconstrained Reserve Margin Scheduling Scenario

Generating Unit	**Design Output [MW]**	**Outage Duration [Days]**	**Earliest Execution Date**	**Latest Execution Date**	**Merit Order Cost [R/MWh]**	**Crew Requirements**
1	686	30	5	365	291	45
2	686	16	40	365	291	36
3	618	35	1	365	199	29
4	618	60	1	365	199	85
5	618	15	240	365	199	7

The data recorded in Table A.2 represents the available excess reserve for each time period within the planning horizon. Two of the model formulations (time index and general resource flow) were solved by allowing a constant excess reserve margin available over the planning horizon while two of the model formulations (time index and novel resource flow) were solved allowing variability in the excess reserve margin over each time period.

Table A.2: GMS - 5 Unit Unconstrained Reserve Margin Scheduling Scenario

Time Interval	**Variable Excess Reserve [MW]**	**Constant Excess Reserve [MW]**
1 - 24	1109.20	1100.56
25 - 48	1100.56	1100.56
49 - 72	1104.88	1100.56
73 - 96	1120.00	1100.56
97 - 120	1104.88	1100.56
121 - 144	1117.84	1100.56
145 - 168	1117.84	1100.56
169 - 192	1101.64	1100.56
193 - 216	1100.56	1100.56
217 - 240	1110.28	1100.56
241 - 264	1103.80	1100.56
265 - 288	1107.04	1100.56
289 - 312	1107.04	1100.56
313 - 336	1111.36	1100.56
337 - 365	1116.76	1100.56

All the basic input information required to solve the 5 unit GMS scenario has been captured in Tables A.1 and A.2. There are however some supplementary information required of which include:

- Available maintenance crew: 100
- Scheduling precedence considerations:
  - Unit 1 should take precedence over Unit 2.
- Total installed capacity for the 5 unit GMS scenario: 3226 MW
- Annual inflation: 6% distributed over a 365 days planning horizon.
- National demand upper boundary is calculated as a fraction of the total installed capacity: 61.00%.
- National demand lower boundary is calculated as a fraction of the total installed capacity: 54.00%.

Table A.3: GMS - 10 Unit Unconstrained Reserve Margin Scheduling Scenario

Generating Unit	**Design Output [MW]**	**Outage Duration [Days]**	**Earliest Execution Date**	**Latest Execution Date**	**Merit Order Cost [R/MWh]**	**Crew Requirements**
1	686	30	5	365	291	45
2	686	20	40	365	291	36
3	618	35	1	365	199	29
4	618	60	1	365	199	85
5	618	15	240	365	199	7
6	665	35	90	365	368	9
7	665	14	42	365	368	2
8	665	36	1	365	368	23
9	716	72	25	365	368	13
10	794	25	45	365	194	44

The data recorded in Table A.4 represents the available excess reserve for each time period within the planning horizon. Two of the model formulations (time index and general resource flow) were solved by allowing a constant excess reserve available over the planning horizon while two of the model formulations (time index and novel resource flow) were solved allowing variability in excess reserve over each time period.

Table A.4: GMS - 10 Unit Unconstrained Reserve Margin Scheduling Scenario

Time Interval	**Variable Excess Reserve [MW]**	**Constant Excess Reserve [MW]**
1 - 24	2304.08	2299.76
25 - 48	2300.84	2299.76
49 - 72	2306.24	2299.76
73 - 96	2299.76	2299.76
97 - 120	2321.36	2299.76
121 - 144	2304.08	2299.76
145 - 168	2304.08	2299.76
169 - 192	2313.80	2299.76
193 - 216	2319.2	2299.76
217 - 240	2321.36	2299.76
241 - 264	2309.48	2299.76
265 - 288	2306.24	2299.76
289 - 312	2314.88	2299.76
313 - 336	2310.56	2299.76
337 - 365	2315.96	2299.76

All the basic input information required to solve the 10 unit GMS scenario has been captured in Tables A.3 and A.4. There are however some supplementary information required of which include:

- Available maintenance crew: 200
- Scheduling precedence considerations:
  - Unit 1 should take precedence over Unit 2.
  - Unit 6 should take precedence over Unit 7.
  - Unit 6 should take precedence over Unit 7.
  - Unit 9 should take precedence over Unit 7.
  - Unit 9 should take precedence over Unit 8.
- Total installed capacity for the 5 unit GMS scenario: 6731 MW
- Annual inflation: 6% distributed over a 365 days planning horizon.
- National demand upper boundary is calculated as a fraction of the total installed capacity: 61.00%.
- National demand lower boundary is calculated as a fraction of the total installed capacity: 54.00%.

Table A.5: GMS - 20 Unit Unconstrained Reserve Margin Scheduling Scenario

Generating Unit	**Design Output [MW]**	**Outage Duration [Days]**	**Earliest Execution Date**	**Latest Execution Date**	**Merit Order Cost [R/MWh]**	**Crew Requirements**
1	686	30	5	365	291	45
2	686	20	40	365	300	36
3	618	35	1	365	199	29
4	618	60	1	365	199	85
5	618	15	240	365	199	7
6	665	35	90	365	368	9
7	665	14	42	365	368	2
8	665	36	1	365	368	40
9	716	72	25	365	368	13
10	794	25	45	365	194	44
11	609	32	30	365	405	20
12	609	19	36	365	405	16
13	500	79	10	365	329	51
14	400	20	1	365	329	14
15	500	32	145	365	329	20
16	500	62	53	365	351	21
17	100	74	60	365	449	60
18	100	30	1	365	449	54
19	200	30	190	365	449	22
20	100	61	40	365	449	45

The data recorded in Table A.6 represent the available excess reserve for each time period within the planning horizon. Two of the model formulations (time index and general resource flow) were solved by allowing a constant excess reserve available over the planning horizon while two of the model formulations (time index and novel resource flow) were solved allowing variability in excess reserve over each time period.

Table A.6: GMS - 20 Unit Unconstrained Reserve Margin Scheduling Scenario

Time Interval	**Variable Excess Reserve [MW]**	**Constant Excess Reserve [MW]**
1 - 24	3628.16	3532.04
25 - 48	3536.36	3532.04
49 - 72	3539.60	3532.04
73 - 96	3569.84	3532.04
97 - 120	3533.12	3532.04
121 - 144	3543.92	3532.04
145 - 168	3534.20	3532.04
169 - 192	3573.08	3532.04
193 - 216	3554.72	3532.04
217 - 240	3594.68	3532.04
241 - 264	3587.12	3532.04
265 - 288	3533.12	3532.04
289 - 312	3580.64	3532.04
313 - 336	3541.76	3532.04
337 - 365	3532.04	3532.04

All the basic input information required to solve the 20 unit GMS scenario has been captured in Tables A.5 and A.6. There are however some supplementary information required of which include:

- Available maintenance crew: 430
- Scheduling precedence considerations:
  - Unit 1 should take precedence over Unit 2.
  - Unit 20 should take precedence over Unit 17.
  - Unit 7 should take precedence over Unit 6.
  - Unit 14 should take precedence over Unit 18.
  - Unit 3 should take precedence over Unit 4.
- Total installed capacity for the 5 unit GMS scenario: 10 349 MW
- Annual inflation: 6% distributed over a 365 days planning horizon.
- National demand upper boundary is calculated as a fraction of the total installed capacity: 61.00%.
- National demand lower boundary is calculated as a fraction of the total installed capacity: 54.00%.

Table A.7: GMS - 40 Unit Unconstrained Reserve Margin Scheduling Scenario

Generating Unit	Design Output [MW]	Outage Duration [Days]	Earliest Execution Date	Latest Execution Date	Merit Order Cost [R/MWh]	Crew Requirements
1	370	29	1	365	348	12
2	390	40	20	365	348	40
3	396	35	100	365	348	28
4	396	66	160	365	348	60
5	400	14	200	365	348	15
6	400	10	15	365	348	4
7	200	6	40	365	392	10
8	200	26	60	365	392	13
9	200	47	300	365	392	36
10	196	120	1	365	392	90
11	195	18	40	365	392	14
12	195	12	27	365	392	10
13	190	8	1	365	329	6
14	185	96	54	365	329	49
15	600	35	5	365	188	32
16	600	100	200	365	188	84
17	600	10	1	365	188	3
18	600	21	29	365	188	19
19	600	120	60	365	188	105
20	200	28	3	365	444	23
21	200	62	280	365	444	58
22	200	45	40	365	444	46
23	200	24	23	365	444	12
24	200	28	30	365	444	30
25	200	20	1	365	444	6
26	200	34	70	365	408	27
27	200	14	31	365	408	9
28	200	70	1	365	408	54
29	200	12	84	365	408	11
30	200	4	60	365	408	4
31	200	28	79	365	408	9
32	200	28	160	365	408	9
33	200	35	36	365	408	18
34	200	7	1	365	408	3
35	200	14	250	365	408	7
36	686	120	5	365	291	110
37	686	52	40	365	291	36
38	686	18	21	365	291	4
39	686	15	50	365	291	19
40	686	21	1	365	291	33

The data recorded in Table A.8 represents the available excess reserve for each time period within the planning horizon. Two of the model formulations (time index and general resource flow) were solved by allowing a constant excess reserve available over the planning horizon while two of the model formulations (time index and novel resource flow) were solved allowing variability in excess reserve over each time period.

Table A.8: GMS - 40 Unit Unconstrained Reserve Margin Scheduling Scenario

Time Interval	**Variable Excess Reserve [MW]**	**Constant Excess Reserve [MW]**
1 - 24	4660.00	4621.12
25 - 48	4621.12	4621.12
49 - 72	4641.64	4621.12
73 - 96	4703.20	4621.12
97 - 120	4641.64	4621.12
121 - 144	4693.48	4621.12
145 - 168	4694.56	4621.12
169 - 192	4626.52	4621.12
193 - 216	4624.36	4621.12
217 - 240	4664.32	4621.12
241 - 264	4635.16	4621.12
265 - 288	4648.12	4621.12
289 - 312	4648.12	4621.12
313 - 336	4667.56	4621.12
337 - 365	4690.24	4621.12

All the basic input information required to solve the 40 unit GMS scenario has been captured in Tables A.7 and A.8. There are however some supplementary information required of which include:

- Available maintenance crew: 800
- Scheduling precedence considerations:
  - Unit 1 should take precedence over Unit 2.
  - Unit 3 should take precedence over Unit 4.
  - Unit 15 should take precedence over Unit 16.
  - Unit 22 should take precedence over Unit 21.
  - Unit 33 should take precedence over Unit 34.
- Total installed capacity for the 5 unit GMS scenario: 13 543 MW
- Annual inflation: 6% distributed over a 365 days planning horizon.
- National demand upper boundary is calculated as a fraction of the total installed capacity: 61.00%.
- National demand lower boundary is calculated as a fraction of the total installed capacity: 54.00%.

Table A.9: GMS - 92 Unit Unconstrained Reserve Margin Scheduling Scenario - Units 1 - 40

Generating Unit	**Design Output [MW]**	**Outage Duration [Days]**	**Earliest Execution Date**	**Latest Execution Date**	**Merit Order Cost [R/MWh]**	**Crew Requirements**
1	370	29	1	365	348	12
2	390	40	20	365	348	40
3	396	35	100	365	348	28
4	396	66	160	365	348	60
5	400	14	200	365	348	15
6	400	15	15	365	348	6
7	200	6	40	365	392	10
8	200	26	60	365	392	13
9	200	47	300	365	392	36
10	196	120	1	365	392	90
11	195	18	40	365	392	14
12	195	12	27	365	392	10
13	190	20	1	365	329	8
14	185	96	54	365	329	49
15	600	35	5	365	188	32
16	600	100	200	365	188	84
17	600	17	1	365	188	4
18	600	21	29	365	188	19
19	600	120	60	365	188	105
20	200	28	3	365	444	23
21	200	62	280	365	444	58
22	200	45	40	365	444	46
23	200	24	23	365	444	12
24	200	28	30	365	444	30
25	200	5	1	365	444	2
26	200	34	70	365	408	27
27	200	14	31	365	408	9
28	200	70	1	365	408	54
29	200	12	84	365	408	11
30	200	4	60	365	408	4
31	200	28	79	365	408	9
32	200	28	160	365	408	9
33	200	35	36	365	408	18
34	200	7	1	365	408	8
35	200	14	250	365	408	7
36	686	120	5	365	291	110
37	686	52	40	365	291	36
38	686	6	21	365	291	3
39	686	15	50	365	291	19
40	686	21	1	365	291	33

Table A.10: GMS - 92 Unit Unconstrained Reserve Margin Scheduling Scenario - Units 41 - 92

Generating Unit	Design Output [MW]	Outage Duration [Days]	Earliest Execution Date	Latest Execution Date	Merit Order Cost [R/MWh]	Crew Requirements
41	686	41	76	365	291	45
42	100	74	60	365	449	60
43	100	7	1	365	449	2
44	100	30	190	365	449	22
45	100	61	40	365	449	45
46	100	50	34	365	449	41
47	125	46	16	365	349	38
48	125	26	91	365	449	16
49	125	17	200	365	449	12
50	125	36	29	365	449	27
51	500	79	10	365	329	51
52	500	14	1	365	329	4
53	500	32	145	365	329	20
54	500	62	53	365	351	21
55	500	37	24	365	351	33
56	500	60	30	365	351	41
57	618	10	1	365	199	7
58	618	10	1	365	199	5
59	618	14	240	365	125	6
60	618	120	60	365	199	99
61	618	4	1	365	199	15
62	618	60	40	365	199	46
63	665	17	90	365	368	9
64	665	3	42	365	368	2
65	665	30	1	365	368	16
66	716	21	25	365	368	13
67	716	35	39	365	368	14
68	716	47	6	365	368	23
69	665	28	111	365	195	10
70	665	19	290	365	195	11
71	665	42	130	365	195	29
72	665	7	1	365	195	8
73	665	36	180	365	195	21
74	665	12	1	365	195	7
75	600	32	132	365	283	17
76	600	45	29	365	283	24
77	600	91	210	365	283	40
78	600	18	170	365	283	9
79	600	17	1	365	283	9
80	609	85	123	365	283	55
81	609	16	240	365	405	5
82	609	43	70	365	405	60
83	609	16	1	365	405	3
84	609	32	30	365	405	20
85	609	19	36	365	405	16
86	609	20	1	365	405	24
87	794	8	180	365	194	3
88	794	42	130	365	194	23
89	794	28	210	365	194	20
90	794	31	45	365	194	44
91	794	23	20	365	194	16
92	794	34	1	365	194	22

The data recorded in Table A.11 represents the available excess reserve for each time period within the planning horizon. Two of the model formulations (time index and general resource flow) were solved by allowing a constant excess reserve available over the planning horizon while two of the model formulations (time index and novel resource flow) were solved allowing variability in excess reserve over each time period.

Table A.11: GMS - 92 Unit Unconstrained Reserve Margin Scheduling Scenario

Time Interval	Variable Excess Reserve [MW]	Constant Excess Reserve [MW]
1 - 24	14481.00	14361.00
25 - 48	14361.00	14361.00
49 - 72	14424.00	14361.00
73 - 96	14616.00	14361.00
97 - 120	14426.00	14361.00
121 - 144	14586.00	14361.00
145 - 168	14590.00	14361.00
169 - 192	14380.00	14361.00
193 - 216	14370.00	14361.00
217 - 240	14497.00	14361.00
241 - 264	14407.00	14361.00
265 - 288	14445.00	14361.00
289 - 312	14446.00	14361.00
313 - 336	14506.00	14361.00
337 - 365	14577.00	14361.00

All the basic input information required to solve the 92 unit GMS scenario has been captured in Tables A.9, A10 and A.11. There are however some supplementary information required of which include:

- Available maintenance crew: 1800
- Scheduling precedence considerations:
  - Unit 1 should take precedence over Unit 2.
  - Unit 8 should take precedence over Unit 7.
  - Unit 19 should take precedence over Unit 18.
  - Unit 24 should take precedence over Unit 23.
  - Unit 29 should take precedence over Unit 30.
  - Unit 26 should take precedence over Unit 27.
  - Unit 42 should take precedence over Unit 45.
  - Unit 48 should take precedence over Unit 50.
  - Unit 79 should take precedence over Unit 82.
  - Unit 91 should take precedence over Unit 90.
- Total installed capacity for the 5 unit GMS scenario: 42 088 MW
- Annual inflation: 6% distributed over a 365 days planning horizon.
- National demand upper boundary is calculated as a fraction of the total installed capacity: 61.00%.
- National demand lower boundary is calculated as a fraction of the total installed capacity: 54.00%.

Constrained reserve margin GMS scenarios

When considering the constrained data sets, all the input data remain the same as for the unconstrained sets, the only difference being the excess reserve margin considered. For this reason only the changes in excess reserve margin for the 20, 40 and 92 generating unit scheduling scenarios will be presented in the text to follow:

Table A.12: GMS - 20 Unit Constrained Reserve Margin Scheduling Scenario

Time Interval	Variable Excess Reserve [MW]	Constant Excess Reserve [MW]
1 - 24	2693.56	2601.00
25 - 48	2601.00	2601.00
49 - 72	2648.84	2601.00
73 - 96	2798.60	2601.00
97 - 120	2650.92	2601.00
121 - 144	2775.72	2601.00
145 - 168	2778.84	2601.00
169 - 192	2614.52	2601.00
193 - 216	2607.24	2601.00
217 - 240	2706.04	2601.00
241 - 264	2635.32	2601.00
265 - 288	2666.52	2601.00
289 - 312	2666.52	2601.00
313 - 336	2712.28	2601.00
337 - 365	2768.44	2601.00

Table A.13: GMS - 40 Unit Constrained Reserve Margin Scheduling Scenario

Time Interval	Variable Excess Reserve [MW]	Constant Excess Reserve [MW]
1 - 24	3524.68	3403.00
25 - 48	3403.00	3403.00
49 - 72	3466.44	3403.00
73 - 96	3663.00	3403.00
97 - 120	3469.56	3403.00
121 - 144	3631.80	3403.00
145 - 168	3635.96	3403.00
169 - 192	3421.72	3403.00
193 - 216	3412.36	3403.00
217 - 240	3541.32	3403.00
241 - 264	3448.76	3403.00
265 - 288	3489.32	3403.00
289 - 312	3490.36	3403.00
313 - 336	3549.64	3403.00
337 - 365	3623.48	3403.00

Table A.14: GMS - 92 Unit Constrained Reserve Margin Scheduling Scenario

Time Interval	**Variable Excess Reserve [MW]**	**Constant Excess Reserve [MW]**
1 - 24	11511.00	10574.00
25 - 48	10574.00	10574.00
49 - 72	11006.00	10574.00
73 - 96	10632.00	10574.00
97 - 120	10731.00	10574.00
121 - 144	11016.00	10574.00
145 - 168	10813.00	10574.00
169 - 192	11050.00	10574.00
193 - 216	10682.00	10574.00
217 - 240	11238.00	10574.00
241 - 264	10586.00	10574.00
265 - 288	10618.00	10574.00
289 - 312	10682.00	10574.00
313 - 336	10983.00	10574.00
337 - 365	11053.00	10574.00

Appendix B

GMS Model Results

Eight scheduling scenarios (five unconstrained reserve margin and three constrained reserve margin scheduling scenarios) as presented in Appendix A, were solved utilising the three different model formulations as presented in Chapter 4. The results presented in the tables below include the NPV for the proposed maintenance schedules, the computational time required for each model to reach optimality and finally the maintenance execution start and completion dates. The GMS scenarios were scaled starting from a 5 unit scheduling scenario up to a realistic industry sized GMS problem of 92 generating units. The various GMS scenarios were used to test the robustness of the developed models while also providing means by which the key thesis objective of improving computational time requirements could be assessed. The results for the unconstrained 5, 10, 20, 40 and 92 scheduling scenarios will be presented first followed by the constrained 20, 40 and 92 generating unit scheduling scenarios.

Results of unconstrained Reserve margin GMS scenarios

Table B.1: GMS - 5 Unit Unconstrained Excess reserve Scheduling Scenario

	Time index model constant excess reserve	**Time index Variable excess reserve**	**Resource flow constant excess reserve**	**Resource flow Variable excess reserve**
NPV [R/MWh]	1164.40	1164.40	1164.50	1164.60
Computational Time [s]	02:62	02:59	01:10	01:65
Units	Maintenance Schedule - 365 Days Planning Horizon			
1	5 - 34	5 - 34	5 - 34	5 - 34
2	40 - 55	40 - 55	40 - 55	40 - 55
3	56 - 90	56 - 90	56 - 90	56 - 90
4	91 - 150	91 - 150	91 - 150	91 - 150
5	240 - 254	240 - 254	240 - 254	240 - 254

Table B.2: GMS - 10 Unit Unconstrained Excess Reserve Scheduling Scenario

	Time index model constant excess reserve	**Time index Variable excess reserve**	**Resource flow constant excess reserve**	**Resource flow Variable excess reserve**
NPV [R/MWh]	2810.80	2810.80	2811.20	2811.50
Computational Time [s]	03:84	03:76	02:20	02:28
Units	Maintenance Schedule - 365 Days Planning Horizon			
1	5 - 34	5 - 34	5 - 34	5 - 34
2	40 - 59	40 - 59	40 - 59	40 - 59
3	1 - 35	1 - 35	1 - 35	1 - 35
4	1 - 60	1 - 60	1 - 60	1 - 60
5	240 - 254	240 - 254	240 - 254	240 - 254
6	90 - 124	90 - 124	90 - 124	90 - 124
7	124 - 137	124 - 137	124 - 137	124 - 137
8	124 - 159	124 - 159	124 - 159	124 - 159
9	35 - 106	35 - 106	35 - 106	35 - 106
10	60 - 84	60 - 84	60 - 84	60 - 84

Table B.3: GMS - 20 Unit Unconstrained Excess Reserve Scheduling Scenario

	Time index model constant excess reserve	**Time index Variable excess reserve**	**Resource flow constant excess reserve**	**Resource flow Variable excess reserve**
NPV [R/MWh]	6731.50	6731.40	6732.80	6732.70
Computational Time [s]	07:29	07:07	03:92	09:19
Units	Maintenance Schedule - 365 Days Planning Horizon			
1	5 - 34	5 - 34	5 - 34	5 - 34
2	55 - 74	56 - 75	55 - 74	56 - 75
3	1 - 35	1 - 35	1 - 35	1 - 35
4	67 - 126	67 - 126	67 - 126	67 - 126
5	240 - 254	240 - 254	240 - 254	240 - 254
6	90 - 124	90 - 124	90 - 124	90 - 124
7	42 - 55	42 - 55	42 - 55	42 - 55
8	1 - 36	1 - 36	1 - 36	1 - 36
9	25 - 96	25 - 96	25 - 96	25 - 96
10	75 - 99	76 - 100	75 - 99	76 - 100
11	35 - 66	35 - 66	35 - 66	35 - 66
12	36 - 54	36 - 54	36 - 54	36 - 54
13	10 - 88	10 - 88	10 - 88	10 - 88
14	1 - 20	1 - 20	1 - 20	1 - 20
15	145 - 176	145 - 176	145 - 176	145 - 176
16	56 - 117	55 - 116	56 - 117	55 - 116
17	100 - 173	100 - 173	100 - 173	100 - 173
18	20 - 49	20 - 49	20 - 49	20 - 49
19	190 - 219	190 - 219	190 - 219	190 - 219
20	40 - 100	40 - 100	40 - 100	40 - 100

Table B.4: GMS - 40 Unit Unconstrained Excess Reserve Scheduling Scenario

	Time index model constant excess reserve	**Time index Variable excess reserve**	**Resource flow constant excess reserve**	**Resource flow Variable excess reserve**
NPV [R/MWh]	14204.00	14204.00	14207.00	14207.00
Computational Time [s]	22:86	17:49	02:88	05:71
Units	Maintenance Schedule - 365 Days Planning Horizon			
1	1 - 29	1 - 29	1 - 29	1 - 29
2	29 - 68	29 - 68	29 - 68	29 - 68
3	100 - 134	100 - 134	100 - 134	100 - 134
4	160 - 225	160 - 225	160 - 225	160 - 225
5	200 - 213	200 - 213	200 - 213	200 - 213
6	15 - 24	15 - 24	15 - 24	15 - 24
7	40 - 45	40 - 45	40 - 45	40 - 45
8	60 - 85	60 - 85	60 - 85	60 - 85
9	300 - 346	300 - 346	300 - 346	300 - 346
10	1 - 120	1 - 120	1 - 120	1 - 120
11	40 - 57	40 - 57	40 - 57	40 - 57
12	27 - 38	27 - 38	27 - 38	27 - 38
13	1 - 8	1 - 8	1 - 8	1 - 8
14	54 - 149	54 - 149	54 - 149	54 - 149
15	5 - 39	5 - 39	5 - 39	5 - 39
16	200 - 299	200 - 299	200 - 299	200 - 299
17	1 - 10	1 - 10	1 - 10	1 - 10
18	29 - 49	29 - 49	29 - 49	29 - 49
19	60 - 179	60 - 179	60 - 179	60 - 179
20	3 - 30	3 - 30	3 - 30	3 - 30
21	280 - 341	280 - 341	280 - 341	280 - 341
22	40 - 84	40 - 84	40 - 84	40 - 84
23	23 - 46	23 - 46	23 - 46	23 - 46
24	30 - 57	30 - 57	30 - 57	30 - 57
25	1 - 20	1 - 20	1 - 20	1 - 20
26	70 - 103	70 - 103	70 - 103	70 - 103
27	31 - 44	31 - 44	31 - 44	31 - 44
28	1 - 70	1 - 70	1 - 70	1 - 70
29	84 - 95	84 - 95	84 - 95	84 - 95
30	60 - 63	60 - 63	60 - 63	60 - 63
31	79 - 106	79 - 106	79 - 106	79 - 106
32	160 - 187	160 - 187	160 - 187	160 - 187
33	36 - 70	36 - 70	36 - 70	36 - 70
34	70 - 76	70 - 76	70 - 76	70 - 76
35	250 - 263	250 - 263	250 - 263	250 - 263
36	5 - 124	5 - 124	5 - 124	5 - 124
37	40 - 91	40 - 91	40 - 91	40 - 91
38	21 - 38	21 - 38	21 - 38	21 - 38
39	50 - 64	50 - 64	50 - 64	50 - 64
40	1 - 21	1 - 21	1 - 21	1 - 21

Table B.5: GMS - 92 Unit Unconstrained Excess Reserve Scheduling Scenario - Units 1 - 40

	Time index model constant excess reserve	**Time index Variable excess reserve**	**Resource flow constant excess reserve**	**Resource flow Variable excess reserve**
NPV [R/MWh]	30229.00	30229.00	30235.00	30234.00
Computational Time [s]	44:50	54:17	04:37	26:29
Units	Maintenance Schedule - 365 Days Planning Horizon			
1	1 - 29	1 - 29	1 - 29	1 - 29
2	29 - 68	29 - 68	29 - 68	29 - 68
3	100 - 134	100 - 134	100 - 134	100 - 134
4	160 - 225	160 - 225	160 - 225	160 - 225
5	200 - 213	200 - 213	200 - 213	200 - 213
6	15 - 29	15 - 29	15 - 29	15 - 29
7	85 - 90	85 - 90	85 - 90	85 - 90
8	60 - 85	60 - 85	60 - 85	60 - 85
9	300 - 346	300 - 346	300 - 346	300 - 346
10	1- 120	1 - 120	1 - 120	1 - 120
11	40 - 57	40 - 57	40 - 57	40 - 57
12	27 - 38	27 - 38	27 - 38	27 - 38
13	1 - 20	1 - 20	1 - 20	1 - 20
14	54 - 149	54 - 149	54 - 149	54 - 149
15	5 - 39	5 - 39	5 - 39	5 - 39
16	200 - 299	200 - 299	200 - 299	200 - 299
17	1- 17	1 - 17	1 - 17	1 - 17
18	179 - 199	179 - 199	179 - 199	179 - 199
19	60 - 179	60 - 179	60 - 179	60 - 179
20	3 - 30	3 - 30	3 - 30	3 - 30
21	280 - 341	280 - 341	280 - 341	280 - 341
22	40 - 84	40 - 84	40 - 84	40 - 84
23	57 - 80	57 - 80	57 - 80	57 - 80
24	30 - 57	30 - 57	30 - 57	30 - 57
25	1 - 5	1 - 5	1 - 5	1 - 5
26	70 - 103	70 - 103	70 - 103	70 - 103
27	103 - 116	103 - 116	103 - 116	103 - 116
28	1 - 70	1 - 70	1 - 70	1 - 70
29	84 - 95	84 - 95	84 - 95	84 - 95
30	95 - 98	95 - 98	95 - 98	95 - 98
31	79 - 106	79 - 106	79 - 106	79 - 106
32	160 - 187	160 - 187	160 - 187	160 - 187
33	36 - 70	36 - 70	36 - 70	36 - 70
34	1 - 7	1 - 7	1 - 7	1 - 7
35	250 - 263	250 - 263	250 - 263	250 - 263
36	5 - 124	5 - 124	5 - 124	5 - 124
37	40 - 91	40 - 91	40 - 91	40 - 91
38	21 - 26	21 - 26	21 - 26	21 - 26
39	50 - 64	50 - 64	50 - 64	50 - 64
40	1 - 21	1 - 21	1 - 21	1 - 21

Table B.6: GMS - 92 Unit Unconstrained Excess Reserve Scheduling Scenario - Units 41 - 92

	Time index model constant excess reserve	**Time index Variable excess reserve**	**Resource flow constant excess reserve**	**Resource flow Variable excess reserve**
NPV [R/MWh]	30229.00	30229.00	30235.00	30234.00
Computational Time [s]	44:50	54:17	04:37	26:29
Units	Maintenance Schedule - 365 Days Planning Horizon			
41	76 - 116	76 - 116	76 - 116	76 - 116
42	60 - 133	60 - 133	60 - 133	60 - 133
43	1 - 7	1 - 7	1 - 7	1 - 7
44	190 - 219	190 - 219	190 - 219	190 - 219
45	133 - 193	133 - 193	133 - 193	133 - 193
46	34 - 83	34 - 83	34 - 83	34 - 83
47	16 - 61	16 - 61	16 - 61	16 - 61
48	91 - 116	91 - 116	91 - 116	91 - 116
49	200 - 216	200 - 216	200 - 216	200 - 216
50	116 - 151	116 - 151	116 - 151	116 - 151
51	10 - 88	10 - 88	10 - 88	15 - 93
52	1 - 14	1 - 14	1 - 14	1 - 14
53	145 - 176	145 - 176	145 - 176	145 - 176
54	53 - 114	53 - 114	53 - 114	53 - 114
55	24 - 60	24 - 60	24 - 60	24 - 60
56	30 - 89	30 - 89	30 - 89	30 - 89
57	1 - 10	1 - 10	1 - 10	1 - 10
58	1 - 10	1 - 10	1 - 10	13 - 22
59	240 - 253	240 - 253	240 - 253	240 - 253
60	60 - 179	60 - 179	60 - 179	60 - 179
61	1 - 4	1 - 4	1 - 4	1 - 4
62	40 - 99	40 - 99	40 - 99	40 - 99
63	90 - 106	90 - 106	90 - 106	90 - 106
64	42 - 44	42 - 44	42 - 44	42 - 44
65	1 - 30	1 - 30	1 - 30	1 - 30
66	25 - 45	25 - 45	25 - 45	27 - 47
67	39 - 73	39 - 73	39 - 73	39 - 73
68	6 - 52	6 - 52	6 - 52	6 - 52
69	111 - 138	111 - 138	111 - 138	111 - 138
70	290 - 308	290 - 308	290 - 308	290 - 308
71	130 - 171	130 - 171	130 - 171	130 - 171
72	1 - 7	1 - 7	1 - 7	1 - 7
73	180 - 215	180 - 215	180 - 215	180 - 215
74	1 - 12	1 - 12	1 - 12	1 - 12
75	132 - 163	132 - 163	132 - 163	132 - 163
76	29 - 73	29 - 73	29 - 73	29 - 73
77	210 - 300	210 - 300	210 - 300	210 - 300
78	170 - 187	170 - 187	170 - 187	170 - 187
79	1 - 17	1 - 17	1 - 17	1 - 17
80	123 - 207	123 - 207	123 - 207	123 - 207
81	240 - 255	240 - 255	240 - 255	240 - 255
82	70 - 112	70 - 112	70 - 112	70 - 112
83	1 - 16	1 - 16	1 - 16	5 - 20
84	30 - 61	30 - 61	30 - 61	30 - 61
85	36 - 54	36 - 54	36 - 54	36 - 54
86	1- 20	1- 20	1- 20	1- 20
87	180 - 187	180 - 187	180 - 187	180 - 187
88	130 - 171	130 - 171	130 - 171	130 - 171
89	210 - 237	210 - 237	210 - 237	210 - 237
90	45 - 75	45 - 75	45 - 75	45 - 75
91	20 - 42	20 - 42	20 - 42	20 - 42
92	1 - 34	1 - 34	1 - 34	1 - 34

Results of constrained reserve margin GMS scenarios

Table B.7: GMS - 20 Unit Constrained Excess Reserve Scheduling Scenario

	Time index model constant excess reserve	**Time index Variable excess reserve**	**Resource flow constant excess reserve**	**Resource flow Variable excess reserve**
NPV [R/MWh]	6726.50	6726.80	6727.90	6728.30
Computational Time [Min]	01:03:31	00:59:94	05:26:42	03:43:28
Units	Maintenance Schedule - 365 Days Planning Horizon			
1	5 - 34	5 - 34	5 - 34	5 - 34
2	67 - 86	55 - 74	67 - 86	55 - 74
3	1 - 35	1 - 35	1 - 35	1 - 35
4	101 - 160	101 - 160	101 - 160	101 - 160
5	240 - 254	240 - 254	240 - 254	240 - 254
6	90 - 124	92 - 126	90 - 124	92 - 126
7	42 - 55	42 - 55	42 - 55	42 - 55
8	1 - 36	1 - 36	1 - 36	1 - 36
9	81 - 152	75 - 146	81 - 152	75 - 146
10	56 - 80	67 - 91	56 - 80	67 - 91
11	35 - 66	35 - 66	35 - 66	35 - 66
12	36 - 54	36 - 54	36 - 54	36 - 54
13	21 - 99	21 - 99	21 - 99	21 - 99
14	1 - 20	1 - 20	1 - 20	1 - 20
15	145 - 176	145 - 176	145 - 176	145 - 176
16	55 - 116	57 - 118	55 - 116	56 - 117
17	100 - 173	100 - 173	100 - 173	100 - 173
18	20 - 49	21 - 50	20 - 49	20 - 49
19	190 - 219	190 - 219	190 - 219	190 - 219
20	40 - 100	40 - 100	40 - 100	40 - 100

Table B.8: GMS - 40 Unit Constrained Excess Reserve Scheduling Scenario

	Time index model constant excess reserve	**Time index Variable excess reserve**	**Resource flow constant excess reserve**	**Resource flow Variable excess reserve**
NPV [R/MWh]	14200.00	14200.00	14202.00	14203.00
Computational Time [Min]	05:08:44	11:31:81	55:18:74	38:22:62
Units	Maintenance Schedule - 365 Days Planning Horizon			
1	1 - 29	1 - 29	1 - 29	1 - 29
2	29 - 68	39 - 78	39 - 78	39 - 78
3	100 - 134	100 - 134	100 - 134	100 - 134
4	160 - 225	160 - 225	160 - 225	160 - 225
5	200 - 213	200 - 213	200 - 213	200 - 213
6	15 - 24	15 - 24	15 - 24	15 - 24
7	40 - 45	40 - 45	40 - 45	40 - 45
8	60 - 85	60 - 85	60 - 85	60 - 85
9	300 - 346	300 - 346	300 - 346	300 - 346
10	1 - 120	1 - 120	1 - 120	1 - 120
11	40 - 57	40 - 57	40 - 57	40 - 57
12	27 - 38	27 - 38	27 - 38	27 - 38
13	1 - 8	1 - 8	1 - 8	1 - 8
14	54 - 149	54 - 149	54 - 149	54 - 149
15	5 - 39	5 - 39	5 - 39	5 - 39
16	200 - 299	200 - 299	200 - 299	200 - 299
17	1 - 10	1 - 10	1 - 10	1 - 10
18	39 - 59	29 - 49	29 - 49	29 - 49
19	71 - 190	73 - 192	77 - 196	73 - 192
20	3 - 30	3 - 30	3 - 30	3 - 30
21	280 - 341	280 - 341	280 - 341	280 - 341
22	40 - 84	40 - 84	40 - 84	40 - 84
23	23 - 46	23 - 46	23 - 46	23 - 46
24	30 - 57	30 - 57	30 - 57	30 - 57
25	1 - 20	1 - 20	1 - 20	1 - 20
26	70 - 103	70 - 103	70 - 103	70 - 103
27	31 - 44	31 - 44	31 - 44	31 - 44
28	1 - 70	1 - 70	1 - 70	1 - 70
29	84 - 95	84 - 95	84 - 95	84 - 95
30	60 - 63	60 - 63	60 - 63	60 - 63
31	79 - 106	79 - 106	79 - 106	79 - 106
32	160 - 187	160 - 187	160 - 187	160 - 187
33	36 - 70	36 - 70	36 - 70	36 - 70
34	70 - 76	70 - 76	70 - 76	70 - 76
35	250 - 263	250 - 263	250 - 263	250 - 263
36	58 - 177	65 - 184	65 - 184	45 - 164
37	65 - 116	45 - 96	45 - 96	65 - 116
38	21 - 38	21 - 38	21 - 38	21 - 38
39	50 - 64	50 - 64	50 - 64	50 - 64
40	1 - 21	1 - 21	1 - 21	1 - 21

Table B.9: GMS - 92 Unit Constrained Excess Reserve Scheduling Scenario - Units 1 - 40

	Time index model constant excess reserve	**Time index Variable excess reserve**	**Resource flow constant excess reserve**	**Resource flow Variable excess reserve**
NPV [R/MWh]	30227.00	30228.00	30232.00	30232.00
Computational Time [Min]	00:49:37	00:42:35	00:29:53	00:28:82
Units	Maintenance Schedule - 365 Days Planning Horizon			
1	1 - 29	1 - 29	1 - 29	1 - 29
2	29 - 68	29 - 68	29 - 68	29 - 68
3	100 - 134	100 - 134	100 - 134	100 - 134
4	160 - 225	160 - 225	160 - 225	160 - 225
5	200 - 213	200 - 213	200 - 213	200 - 213
6	15 - 29	15 - 29	15 - 29	15 - 29
7	85 - 90	85 - 90	85 - 90	85 - 90
8	60 - 85	60 - 85	60 - 85	60 - 85
9	300 - 346	300 - 346	300 - 346	300 - 346
10	1- 120	1 - 120	1 - 120	1 - 120
11	40 - 57	40 - 57	62 - 79	40 - 57
12	27 - 38	27 - 38	27 - 38	27 - 38
13	1 - 20	1 - 20	1 - 20	1 - 20
14	54 - 149	54 - 149	62 - 157	54 - 149
15	5 - 39	5 - 39	5 - 39	8 - 42
16	200 - 299	200 - 299	200 - 299	200 - 299
17	1- 17	1 - 17	1 - 17	1 - 17
18	179 - 199	179 - 199	179 - 199	181 - 201
19	60 - 179	60 - 179	60 - 179	62 - 181
20	3 - 30	3 - 30	3 - 30	3 - 30
21	280 - 341	280 - 341	280 - 341	280 - 341
22	40 - 84	40 - 84	40 - 84	43 - 87
23	57 - 80	57 - 80	57 - 80	60 - 83
24	30 - 57	30 - 57	30 - 57	30 - 57
25	1 - 5	1 - 5	1 - 5	8 - 12
26	70 - 103	70 - 103	70 - 103	70 - 103
27	103 - 116	103 - 116	103 - 116	103 - 116
28	1 - 70	1 - 70	1 - 70	1 - 70
29	84 - 95	84 - 95	84 - 95	84 - 95
30	95 - 98	95 - 98	95 - 98	95 - 98
31	79 - 106	79 - 106	79 - 106	79 - 106
32	160 - 187	160 - 187	160 - 187	160 - 187
33	36 - 70	36 - 70	36 - 70	36 - 70
34	1 - 7	1 - 7	1 - 7	1 - 7
35	250 - 263	250 - 263	250 - 263	250 - 263
36	5 - 124	5 - 124	5 - 124	5 - 124
37	40 - 91	43 - 94	40 - 91	40 - 91
38	21 - 26	21 - 26	21 - 26	21 - 26
39	50 - 64	50 - 64	50 - 64	50 - 64
40	1 - 21	1 - 21	1 - 21	1 - 21

Table B.10: GMS - 92 Unit Constrained Excess Reserve Scheduling Scenario - Units 41 - 92

	Time index model constant excess reserve	**Time index Variable excess reserve**	**Resource flow constant excess reserve**	**Resource flow Variable excess reserve**
NPV [R/MWh]	30227.00	30228.00	30232.00	30232.00
Computational Time [Min]	00:49:37	00:42:35	00:29:53	00:28:82
Units	Maintenance Schedule - 365 Days Planning Horizon			
41	76 - 116	76 - 116	76 - 116	76 - 116
42	60 - 133	60 - 133	60 - 133	60 - 133
43	1 - 7	1 - 7	1 - 7	1 - 7
44	190 - 219	190 - 219	190 - 219	190 - 219
45	133 - 193	133 - 193	133 - 193	133 - 193
46	34 - 83	34 - 83	34 - 83	34 - 83
47	16 - 61	16 - 61	16 - 61	16 - 61
48	91 - 116	91 - 116	91 - 116	91 - 116
49	200 - 216	200 - 216	200 - 216	200 - 216
50	116 - 151	116 - 151	120 - 155	116 - 151
51	10 - 88	10 - 88	10 - 88	10 - 88
52	1 - 14	1 - 14	1 - 14	1 - 14
53	145 - 176	145 - 176	145 - 176	145 - 176
54	53 - 114	53 - 114	58 - 119	55 - 116
55	24 - 60	24 - 60	24 - 60	24 - 60
56	43 - 102	30 - 89	30 - 89	30 - 89
57	1 - 10	1 - 10	1 - 10	1 - 10
58	1 - 10	1 - 10	1 - 10	1 - 10
59	240 - 253	240 - 253	240 - 253	240 - 253
60	61 - 180	60 - 179	60 - 179	60 - 179
61	1 - 4	1 - 4	1 - 4	1 - 4
62	40 - 99	40 - 99	40 - 99	43 - 102
63	90 - 106	90 - 106	90 - 106	90 - 106
64	42 - 44	42 - 44	42 - 44	42 - 44
65	1 - 30	1 - 30	1 - 30	5 - 34
66	25 - 45	25 - 45	24 - 44	25 - 45
67	39 - 73	39 - 73	39 - 73	46 - 80
68	6 - 52	6 - 52	11 - 57	13 - 59
69	111 - 138	111 - 138	111 - 138	111 - 138
70	290 - 308	290 - 308	290 - 308	290 - 308
71	130 - 171	130 - 171	130 - 171	130 - 171
72	1 - 7	1 - 7	1 - 7	1 - 7
73	180 - 215	180 - 215	180 - 215	180 - 215
74	1 - 12	1 - 12	1 - 12	1 - 12
75	132 - 163	132 - 163	132 - 163	132 - 163
76	29 - 73	29 - 73	29 - 73	29 - 73
77	210 - 300	210 - 300	210 - 300	210 - 300
78	170 - 187	170 - 187	170 - 187	170 - 187
79	1 - 17	1 - 17	1 - 17	1 - 17
80	123 - 207	123 - 207	123 - 207	123 - 207
81	240 - 255	240 - 255	240 - 255	240 - 255
82	70 - 112	70 - 112	70 - 112	70 - 112
83	1 - 16	1 - 16	1 - 16	1 - 16
84	30 - 61	30 - 61	30 - 61	30 - 61
85	36 - 54	36 - 54	36 - 54	36 - 54
86	11- 30	1- 20	1- 20	1- 20
87	180- 187	180- 187	180- 187	180- 187
88	130- 171	130- 171	130- 171	130- 171
89	210 - 237	210 - 237	210 - 237	210 - 237
90	45 - 75	45 - 75	45 - 75	45 - 75
91	20 - 42	20 - 42	20 - 42	20 - 42
92	1 - 34	1 - 34	1 - 34	1 - 34