

# Monte Carlo Simulations of Compton Polarization in Astrophysical Sources

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# Abstract

The description of many high-energy astrophysical sources relies on the production of *relativistic jets* that are accompanied by the acceleration of particles up to very high energies, and the production of non-thermal radiation (e.g. active galactic nuclei (AGNs),  $\gamma$ -ray bursts (GRBs), *X*-ray binaries (XRBs), and  $\gamma$ -ray binaries). The radiation from these *jet-like* sources is characterized by their spectral energy distributions (SEDs), which can be modelled in many different ways, all of which are consistent with the spectral shape of the SED. Discriminating between different models is one of the main objectives in the field of high-energy astrophysics.

Compared with the orientation of the relativistic jet, the polarization from the high-energy radiation in astrophysical sources adds crucial knowledge of the jet-physics and jet-formation models. Even though high-energy polarization has remained largely unexplored, the future prospects of detecting polarization in *X*-rays/soft  $\gamma$ -rays from many astrophysical sources have renewed interest in model predictions of polarization in the high-energy regime. Linear polarization arises from synchrotron radiation of relativistic charged particles in ordered magnetic fields, while Compton scattering off relativistic electrons will reduce the degree of polarization to about half of the target photon polarization.

In a model where a thermal and a non-thermal particle distribution scatters an external radiation field, hard *X*-ray/ $\gamma$ -ray radiation results from relativistic electrons, and the radiation is predicted to be unpolarized. Contrarily, Ultraviolet (UV)/*X*-ray radiation, resulting from scattering by thermal electrons, is predicted to be polarized. This dissertation describes the development of a Monte Carlo code to study the degree and orientation of Compton polarization in the high-energy regime of jet-like astrophysical sources.

Keywords: Compton polarization, Monte Carlo, blazars, active galactic nuclei, gamma-ray bursts, *X*-ray binaries.



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# Nomenclature

## Superscripts

sc                Scattered quantities.

## Subscripts

e                Quantities in the electron rest frame

em              Quantities in the emission frame

i                Label of the current individual photon.

obs             Quantities in the observer frame

## Acronyms / Abbreviations

AGN            Active Galactic Nuclei

BH-XRBs      Black Hole *X*-ray Binaries

CDF            Cumulative Distribution Function

CV             Cataclysmic Variable

EM             Electromagnetic

FSRQ          Flat-spectrum Radio Quasars

GRB            Gamma Ray Burst

HBL            High-frequency-peaked BL Lac Object

HID            Hardness-Intensity-Diagram

HS             High/Soft State

|         |                                     |
|---------|-------------------------------------|
| HSP     | High-synchrotron-peaked             |
| IBL     | Intermediate BL Lac Object          |
| IR      | infrared                            |
| IS      | Intermediate State                  |
| LBL     | Low-frequency-peaked BL Lac Object  |
| LOS     | line-of-sight                       |
| LS      | Low/Hard State                      |
| LSP     | Low-synchrotron-peaked              |
| MDP     | Minimum Detectable Polarization     |
| NS-XRBs | Neutron Star <i>X</i> -ray Binaries |
| PA      | Polarization Angle                  |
| PDF     | Probability Density Function        |
| PD      | Polarization Degree                 |
| PRNG    | Pseudo Random Number Generator      |
| ISP     | Intermediate-synchrotron-peaked     |
| SED     | Spectral Energy Distribution        |
| SMBH    | Supermassive Black Hole             |
| UV      | Ultraviolet                         |
| XRB     | <i>X</i> -Ray Binary                |

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# Chapter 1

## Introduction

High energy astrophysical sources are astronomical objects with physical properties that result in the emission of  $X$ -rays and  $\gamma$ -rays. The description of many sources relies on the production of a relativistic outflow in a central compact object (i.e., *relativistic jets*), and dissipation of the outflow at large radii (e.g. active galactic nuclei (AGNs),  $\gamma$ -ray bursts (GRBs),  $X$ -ray binaries (XRBs), and  $\gamma$ -ray binaries). Relativistic jets are accompanied by the acceleration of particles up to very high energies, as well as the production of secondary non-thermal radiation. They are most likely powered by the rotational energy of the central object, or by the associated accretion disks. Understanding the particle acceleration, radiation mechanisms, and the magnetic field configuration of the jet structures are one of the main targets in the field of high energy astrophysics. Polarization carries important information about the astrophysical environment in terms of how the magnetic field configuration links into the dynamics and acceleration of the energetic particles (see [Trippe \(2014\)](#) for a review). High energy polarimetry measurements may provide unambiguous constraints on the geometry and structure of the astrophysical source by constraining the orientations of accretion disks with respect to our line-of-sight (LOS). Compared with the orientation of the relativistic jet, polarization adds crucial knowledge of jet-physics and jet-formation models (see [Rani et al. \(2019\)](#) for a review). This chapter discusses the main objective of this dissertation by supplying a short overview of how high energy polarization may be exploited in different jet-like astrophysical sources and concludes with an outline of the chapters that follow.

## 1.1 Exploiting High Energy Polarization in Astrophysical Sources

The radiation from astrophysical sources is characterized by its spectral energy distribution (SED), which is used to build physical models of the sources. The SEDs and variability of the sources can be modelled in many different ways, all of which are consistent with the spectral shape of the SEDs (Böttcher et al., 2012; Walcher et al., 2011). Additional constraints are therefore required in order to make a distinction between the existing models. The emission of  $X$ -rays and  $\gamma$ -rays from many astrophysical sources likely originates from relativistic jets, generated by a compact object which is located in the central engine of the source. The radiation mechanisms, particle acceleration, and the magnetic field configuration of relativistic jets provides additional information on jet-physics.

Spectral fitting and multi-wavelength light curves have been used to study the physics of relativistic jets, without regard to the magnetic field strength and morphology. Radio and optical polarization measurements have been a standard way to examine the jet magnetic field, since polarization measurements combined with the spectra and variability of the emission reveal significant insights about the magnetic field structure in the emission region. However, radio and optical polarization signatures often come from regions that do not emit strong, high energy radiation, whereas  $X$ -ray and  $\gamma$ -ray polarimetry can probe the most active jet regions with powerful particle acceleration (Böttcher, 2019; Zhang, 2017). Polarization in the high energy regime of astrophysical sources, which has been relatively unexplored, would add two essential parameters (the *polarization degree* (PD) and the *polarization angle* (PA)) to those already derived from spectra and variability. This will provide new and unique information about the fundamental physics and geometry of sources in most classes of interest. In the following section, a concise description of how high energy polarization can address open questions in jet-physics of different astrophysical sources will be discussed.

### 1.1.1 Active Galactic Nuclei

Most galaxies harbor supermassive black holes (SMBHs) at their central regions, many of which are very active in the accretion processes and, therefore, release tremendous amounts of energy. These galaxies are known as AGNs, some of the most luminous objects in the universe, often observed to host relativistic jets where the bulk energy is converted into kinetic

energy of electrons, multi-wavelength radiation, and possibly particle emission from ions and neutrinos. These particles and the radiation across the Electromagnetic (EM)) spectrum are the *messengers* of the mysterious conditions in the core of active galaxies and their jets.

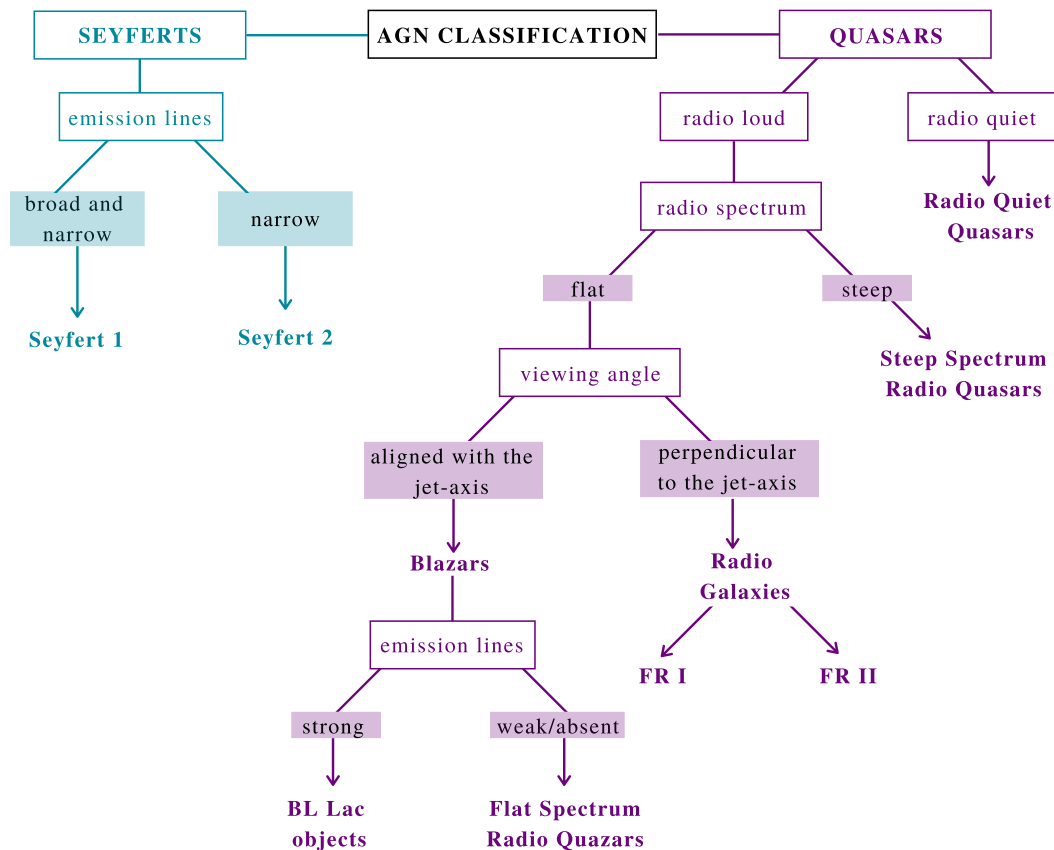


Fig. 1.1 The classification of AGNs: AGNs are classified into two main groups called Seyfert galaxies and quasars. Seyfert galaxies are further divided into two groups depending on their emission lines, while quasars are further classified depending on their radio emission.

AGNs are distinguished from other galaxies by their luminosity (sometimes over  $10^4$  times brighter than normal galaxies) as well as other features, which include their broad continuous spectra, strong (broad and/or narrow) emission lines, and polarized emission. Fig. 1.1 illustrates the AGN classification, and is sometimes referred to as the *AGN zoo*. They are classified by their observational properties, depending on how they were detected: In general, AGNs can be divided into two main groups, namely quasars and Seyfert galaxies. Seyfert galaxies are usually closer, less luminous, and account for 10% of all galaxies. They are further divided into Seyfert 1 and Seyfert 2 galaxies, in which the latter lacks broad emission lines. Quasars are usually further away than Seyfert galaxies, but they are considerably more powerful. Quasars are further

classified into subgroups depending on their radio emission (radio loud and radio quiet). The innermost regions of radio-quiet AGNs can be seen as scaled up versions of black hole systems with the hard Comptonization component produced from the thermal ultraviolet (UV) / soft X-ray disk component. In addition to the accretion disk, other reflecting regions are present, such as the dusty torus. A general consensus is that the differences in the phenomenology of AGNs is due to different viewing angles. A unification model (Urry and Padovani, 1995), as depicted in Fig. 1.2, suggests that the AGN harbors a SMBH with its accretion disk at its central engine, surrounded by the dusty torus. Jets are moving outward on both sides, while the broad line and narrow line region envelop the inner and outer part of the jet, respectively.

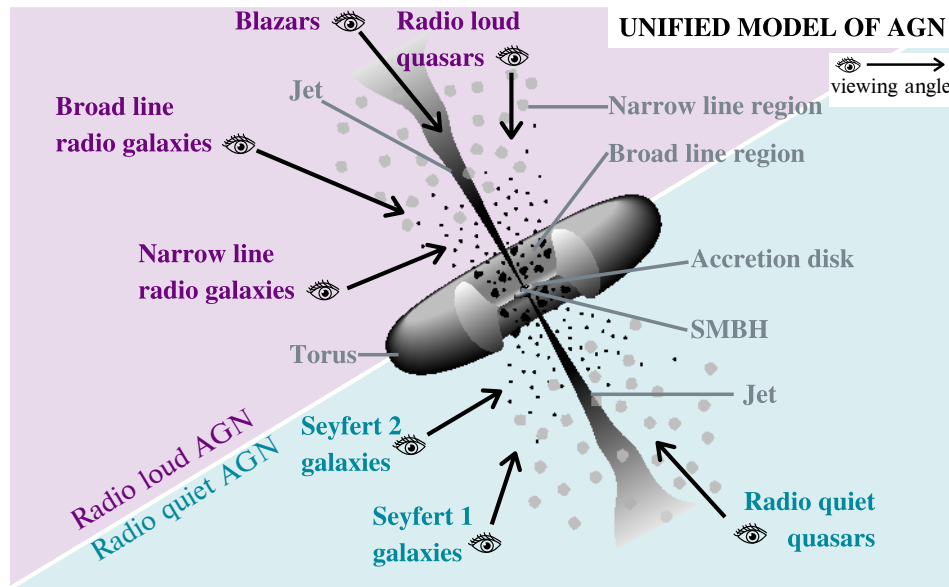


Fig. 1.2 An illustration of the unified model of AGNs: An AGN harbors a SMBH with its accretion disk at its central engine, surrounded by a dusty torus. Jets are moving outward on both sides, while the broad line region and narrow line region envelop the inner part and outer part of the jet, respectively. The different classes of AGNs are given with the corresponding viewing angles shown with black arrows. Adapted from Urry and Padovani (1995).

## 1. Blazars

Blazars are some of the most extreme classes of AGNs where, according to the unification model, the observer's LOS is closely aligned with the jet's axis. They are known to emit non-thermal dominated radiation throughout the entire EM spectrum, variable at all timescales, and are characterized by high and variable polarization, as well as superluminal motion (Böttcher et al., 2012). Such phenomena are likely to originate from the relativistic jet directed close to

our LOS. An illustration of the physical structure and emission regions of a blazar is shown in Fig. 1.3. The jet can be launched by the accretion disk as hydromagnetic flows, or electromagnetically when a rotating black hole is threaded by magnetic field lines (Blandford and Payne, 1982; Blandford and Znajek, 1977). The plasma itself is likely dominated by electrons, positrons, and possibly protons (Wardle et al., 1998). To understand blazar jet physics, several key processes need to be studied, in particular, the magnetic field evolution, the particle acceleration, and the radiation mechanisms. The physics of the jets and their relationship to the accretion disc comprise a key topic for polarimetry when studying Galactic sources like micro-quasars.

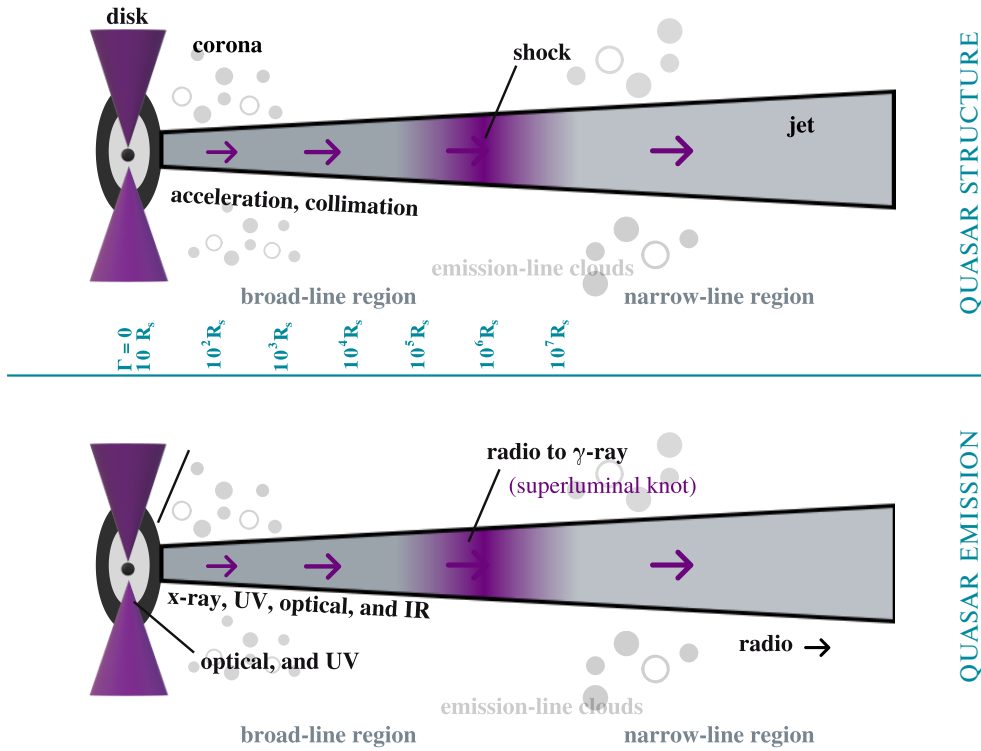


Fig. 1.3 An illustration of the structure and the emission regions of a quasar, not including the extended radio lobe at the end of the jet. Adapted from Marscher (2005).

Various properties of the radiation from blazars have been studied with multi-wavelength observations and spectral fitting, apart from the magnetic field evolution. Measurements of synchrotron polarization in the radio and optical emission from relativistic jet sources have been a standard way of assessing the degree of order and structure of the magnetic field. Observations of  $\gamma$ -ray flares with optical PA swings, and substantial variations of the PD, indicate that the

magnetic field plays an active role during flares (Abdo et al., 2010; Blinov et al., 2015). Several models have been put forward to explain these phenomena (Larionov et al., 2013; Marscher, 2013). Optical polarization, however, can often come from regions that do not emit high energy emission and, therefore, provides no clear view of the polarization signatures in the most active acceleration regions. High energy polarization can thus be used to probe the jet physics in the most active regions with the most active particles (Böttcher, 2019; Rani et al., 2019; Zhang, 2017).

#### (a) *Radiation mechanisms*

The SEDs of blazars are dominated by non-thermal emission across the entire EM spectrum, with two (low energy and high energy) peaks. The low energy peak is attributed to synchrotron emission from leptons, which provides a proper explanation for the spectrum in terms of both the spectral shape and the polarization signatures. Depending on the peak frequency of the low-frequency component, blazars are subdivided into low-synchrotron-peaked (LSP) blazars (consisting of flat-spectrum radio quasars (FSRQs) and low-frequency-peaked BL Lac objects (LBLs)), intermediate-synchrotron-peaked (ISP) blazars (generally intermediate BL Lac objects (IBLs)), and high-synchrotron-peaked (HSP) blazars (exclusively high-frequency-peaked BL Lac objects (HBLs)). Observations of FSRQs and LBLs at low energies (radio to optical) do confirm that the low energy peak is polarized as expected for synchrotron radiation (Zhang et al., 2014). HSP blazars have their first peak in the X-ray regime, and X-ray polarimetry can therefore be used to confirm the synchrotron nature of this peak (Zhang and Böttcher, 2013).

The origin of the high energy peak in the SEDs of blazars is less clear with two fundamentally different models that are both consistent with the spectral shape of the SEDs. The first *leptonic model* suggests that the radiative output throughout the EM spectrum is dominated by leptons (electrons and possibly positrons), while any protons (that are likely present in the outflow) are not accelerated to high enough energies to contribute significantly to the radiative output. The high energy component is then from [inverse] Compton scattering of synchrotron emission and/or external photons. The second *hadronic model* argues that both the primary electrons and protons are accelerated to ultra-relativistic energies, with the low energy component still dominated by synchrotron emission from the primary electrons. The high energy component is then dominated by synchrotron emission of ultra-relativistic protons and cascading secondary particles due to photo-hadronic processes (Böttcher et al., 2013). For a general review on the

features of these two models, see [Böttcher \(2010\)](#).

[Zhang and Böttcher \(2013\)](#) evaluated the expected high energy polarization signatures in leptonic and hadronic models, presenting the maximum achievable high energy PD in both models. Their results are shown in Fig. 1.4, with a representative blazar in each class; (a) 3C279 representing FSRQs, (b) OJ 287 representing LBLs, (c) 3C66A representing IBLs, and (d) RX J0648.7+1516 representing HBLs. In each case, the SED fits for hadronic (shown in purple) and leptonic models (shown in blue) are shown in the lower panels. The upper panels show the corresponding maximal PDs predicted for each model. For hadronic models, the PD is predicted to remain constant throughout the high energy regime for ISPs and HSPs, while continually increasing with photon energy in LSPs. Depending on the contribution of synchrotron emission to the X-ray emission in blazars (therefore, on the type of blazar being considered), leptonic models predict moderate X-ray polarization, and vanishing  $\gamma$ -ray polarization in LSPs, high soft X-ray polarization (rapidly decreasing with photon energies) for ISPs, and high X-ray polarization for HSPs. It is generally found that leptonic models predict X-ray and  $\gamma$ -ray  $PD \leq 40\%$ , while hadronic models predict very high degrees of maximum polarization ( $PD \geq 75\%$ ) for all classes of blazars. This is expected, since in the case of hadronic models the entire SED is dominated by synchrotron processes. These calculations assumed a perfectly ordered magnetic field, hence the representation of only upper limits to the PD actually expected. The observed PD in spectral regions that can confidently be described by synchrotron emission can thus be used to quantify the order of the magnetic field in the high-energy regime. The transition from low energy to high energy polarization may, therefore, be able to distinguish leptonic from hadronic high energy emission from blazars. Furthermore, even when accounting for the expected deviation from a perfectly ordered field, the predicted PD may be within reach of current and upcoming high energy polarimeters, which will be discussed in Chapter 2.

The production of high-energy neutrinos provides evidence for hadronic interactions. If blazars accelerate enough high-energy protons, the protons may interact with the local blazar radiation field and produce charged pions, which decay and emit neutrinos. The recent IceCube-170922A neutrino event, which was reported to coincide with the blazar TXS 0506+056 in flaring state ([Collaboration et al., 2018](#)), indicates that hadronic processes may operate in a blazar jet. Many models have been put forward to explain the corresponding SED of TXS 0506+056 during the neutrino alert (e.g. [Reimer et al. \(2019\)](#)), which can be categorized into two main groups: The first is a leptonic setup where inverse Compton dominates the high energy emission, and a

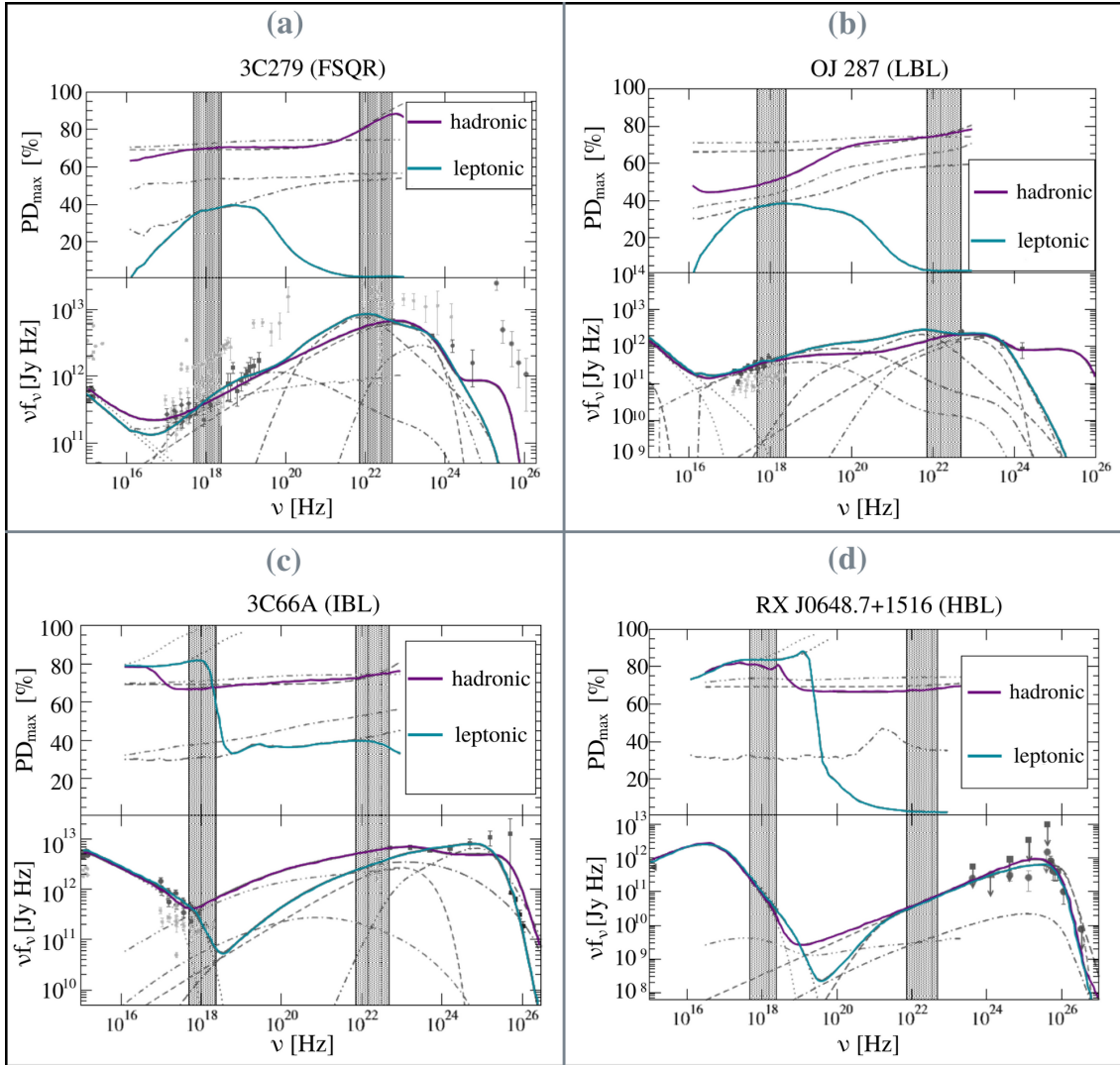


Fig. 1.4 The SED fits (lower panels) and maximal PDs (upper panels) of leptonic (blue) and hadronic models (purple) for different blazars. From [Zhang and Böttcher \(2013\)](#).

subdominant hadronic component produces the neutrinos and a considerable amount of X-rays through synchrotron emission from hadronically induced cascades. The second is a hadronic setup where the X-ray emission consists of both proton synchrotron and cascading synchrotron, and the  $\gamma$ -ray emission is dominated by proton synchrotron. Zhang et al. (2019) predicted the X-ray and  $\gamma$ -ray PDs based on TXS 0506+056 model parameters, shown in Fig. 1.5. The pure leptonic (i.e. inverse Compton, denoted *IC* in Fig.1.5) predicts PD  $\sim 5\%$  in the X-ray and [MeV]  $\gamma$ -ray bands because the inverse Compton processes generally reduces the PD to about half of the synchrotron PD. The proton synchrotron (denoted *PS* in Fig.1.5) and cascading synchrotron scenario predicts a higher PD  $\gtrsim 10\%$  in both the X-ray and [MeV]  $\gamma$ -ray bands, because of the synchrotron emission by primary protons or secondary cascading pairs. In the case of the inverse Compton scenario with a sub-dominant hadronic contribution, the X-ray band presents PD  $\gtrsim 10\%$ , but the [MeV]  $\gamma$ -ray bands show PD  $\sim 5\%$ . Therefore, while the X-ray PD can probe the secondary pair synchrotron contribution complementary to the neutrino detection,  $\gamma$ -ray polarization can be used to unambiguously distinguish between the inverse Compton and proton synchrotron scenario (Rani et al., 2019; Zhang et al., 2019).

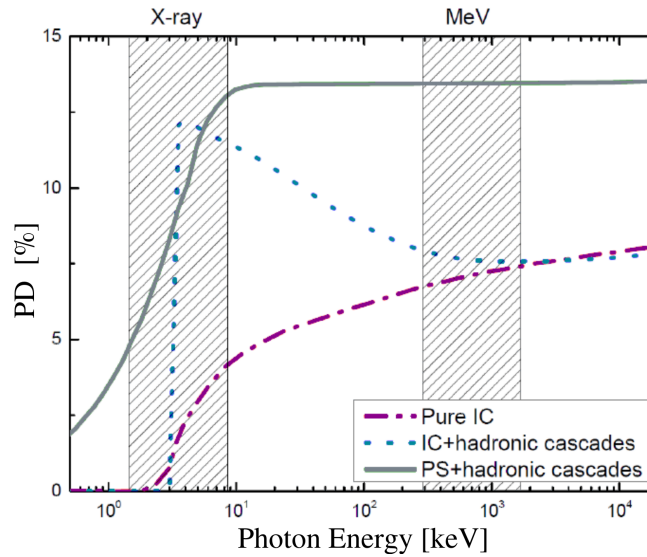


Fig. 1.5 The results from Zhang et al. (2019) of the high energy polarization degree (PD) of TXS 0506+056. The results here are based on three different radiation mechanisms: The inverse Compton scenario (shown in purple), the proton synchrotron and cascading synchrotron scenario (shown in grey), and the inverse Compton scenario with a sub-dominant hadronic contribution (shown in blue). From Rani et al. (2019).

(b) *Particle acceleration*

Blazars show multi-wavelength variations which indicate strong particle acceleration. The high energy emission in blazars likely originates in the most active acceleration regions (Böttcher et al., 2012), with  $\gamma$ -ray energies exhibiting the strongest flares. *Shocks* and *magnetic reconnection* have been suggested as candidates for the particle acceleration mechanisms, both able to produce acceptable fits to the SEDs. Shock models generally assume that the emission is consistent with low magnetization, and a significant amount of kinetic energy which can be converted into non-thermal particle energy through shock acceleration. A low magnetization is required for shocks being the dominant acceleration sites in blazars, since the presence of a dominant magnetic field suppresses efficient particle acceleration during strong shocks (Asano and Hayashida, 2018; Lemoine and Pelletier, 2011). Magnetic reconnection models accelerate the protons and electrons by converting the dominant magnetic field energy into particle kinetic energy in order to produce the non-thermal particle spectra inferred.

Shock and magnetic reconnection scenarios have, nonetheless, different magnetic field configurations. The demonstration that the magnetic field is actively evolving along the particle acceleration - through the observations of simultaneous PA swings and multi-wavelength flares (Abdo et al., 2010; Blinov et al., 2015) - is expected from both shock and magnetic reconnection scenarios. However, polarization properties of blazars indicate that the configuration of the magnetic field is not the same in the newly injected material for different epochs. Thus, the modeling of polarization needs to account for variability in order to distinguish between the two scenarios.

Zhang et al. (2016b) derived the time-dependent flux and polarization signatures of a lepto-hadronic blazar emission model (Diltz et al., 2015), where the magnetized compact region inside the relativistic jet carries a population of relativistic electrons and protons. Neutrinos are generated as part of the pion-decay chain in proton-photon interactions, while synchrotron-pair cascades of secondary particles and/or proton-synchrotron radiation are responsible for the high energy component of the SED. The time-dependent polarization signatures of the shock and magnetic reconnection scenarios are shown on the left-hand side of Fig. 1.6. In shocks, the magnetic field is compressed, thereby increasing the magnetic field while simultaneously producing a flare that may be accompanied by major polarization variations. In contrast to shocks, magnetic reconnection dissipates the magnetic energy through the topological rearrangements of the magnetic field. The topological arrangement of the magnetic field is, however, much faster than the global magnetic field diffusion time. Consequently, strong

changes in the polarization signatures are not expected during flares (Sironi et al., 2015; Zweibel and Yamada, 2016). The magnetic field evolution also depends on how strongly the jet is magnetized, with current jet models suggesting that the jet is highly magnetized close to the central engine. It is, therefore, important to determine whether the jet is dominated by kinetic or magnetic energy, and where most of the energy dissipates.

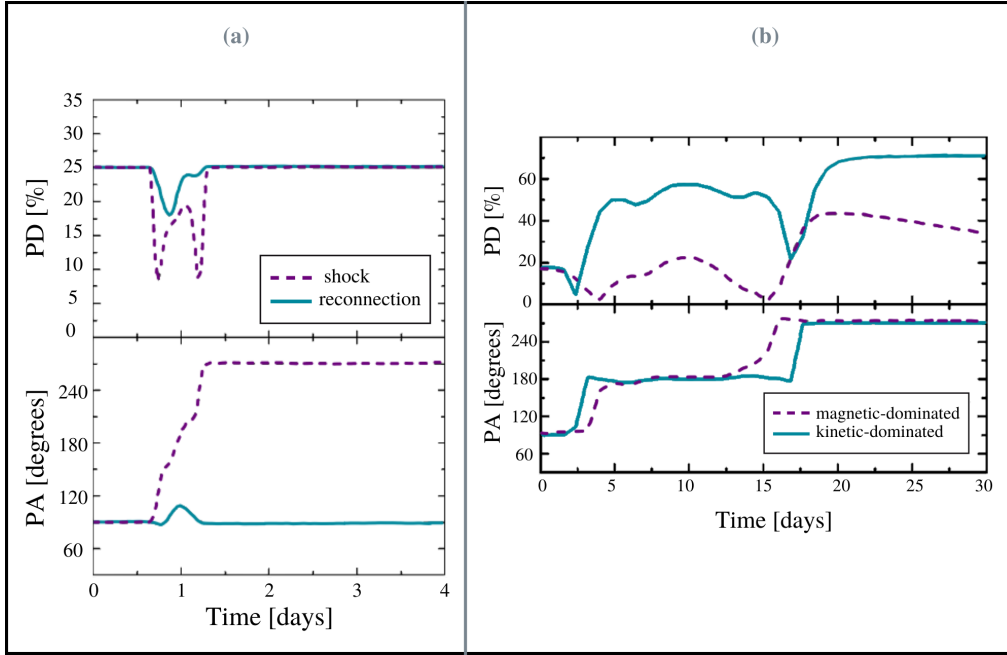


Fig. 1.6 (a) Time-dependent polarization signatures of shock (dashed purple) and magnetic reconnection (blue) scenarios. (b) Time-dependent polarization signatures of kinetic-dominated (blue) and magnetic-dominated (dashed purple) jet emission in blazars. From Zhang (2017).

Zhang et al. (2016a) presented polarization-dependent radiation modeling in the blazar emission region, based on relativistic magneto-hydrodynamic simulations of shocks in helical magnetic fields. The time-dependent polarization signatures of *kinetic-dominated* and *magnetic-dominated* jet emission are shown on the right-hand side of Fig. 1.6. In a kinetic-dominated shock, the shock compression at the shock front can be very strong compared to the magnetic pressure. This causes the magnetic field to become highly ordered, along with strong changes in the polarization signatures, as well as high PDs during flares. The magnetic field, strongly perturbed by the shock, will change considerably in the emission region and will take a long time to recover to its initial, partially ordered, state. As a result, the PD will also remain very high after the shock perturbation. Meanwhile, in a magnetic-dominated shock, the shock is relatively weak compared to the magnetic force, and the magnetic field is not expected to

change significantly during the shock perturbation. Consequently, the polarization signatures will stay approximately constant during flares with a clear restoration phase of the magnetic field and polarization signatures at the end of the flare (Zhang, 2017).

Optical polarization signatures favor a magnetized jet with  $PD \sim 10\%$  and variable polarization signatures restoring to the quiescent state relatively quickly. However, optical polarization signatures are inconclusive in regions with the most energetic particles. High energy polarization will, therefore, be useful to constrain the jet magnetization and where most of the energy in the blazar jet dissipates.

### 1.1.2 Gamma-Ray Bursts

GRBs are the strongest explosions in the universe, revealing themselves as short-lived bursts of  $\gamma$ -rays. They are randomly distributed in the sky (Fishman and Meegan, 1995), with gradually decaying afterglows in X-rays, optical, and radio; some even extended (in time) emission in the high energy ( $E > 100\text{MeV}$ ) and very high energy ( $E > 100\text{GeV}$ )  $\gamma$ -rays (e.g. de Naurois (2019); Hurley et al. (1994)). GRBs and their afterglows are thought to be related to the formation of an ultra-relativistic jet. The GRB phenomenology may be separated into two phases. The initial burst of  $\gamma$ -rays (i.e., the *prompt emission*) lasts from a fraction of a second up to few hundred seconds, believed to originate in the early jet phase. The longer lasting (from days to weeks) *afterglow emission* is believed to originate in the later propagation phase. One of the most important open questions is the outflow composition. The energy may be carried out from the central source either as kinetic energy or in EM form.

There are at least two classes of GRBs depending on their duration  $t_{90}$  (i.e., the time between 5% and 95% of the  $\gamma$ -ray fluence received). Short GRBs have  $t_{90} \lesssim 2\text{s}$ , while long GRBs have  $t_{90} \gtrsim 2\text{s}$ , which last up to several minutes. The two classes also differ in the spectral hardness of their sub-MeV emission, where short GRBs typically have harder spectra (e.g. Dezalay et al. (1996)). Based on their host environments, the two classes are generally believed to have different progenitors. Long GRBs are often found in spiral arms (star forming regions) of very faint host galaxies while short GRBs are generally found far from star-forming regions. The leading model for long GRBs involves an explosion of a very massive ( $> 35M_{\odot}$ ) star where the core collapse leads to the direct formation of a black hole (the *supernova scenario*). The rest of the star is accreted onto the newly-formed black hole, resulting in the formation of an accretion disk which powers the ultra-relativistic jet. Short GRBs are likely caused by the merging of

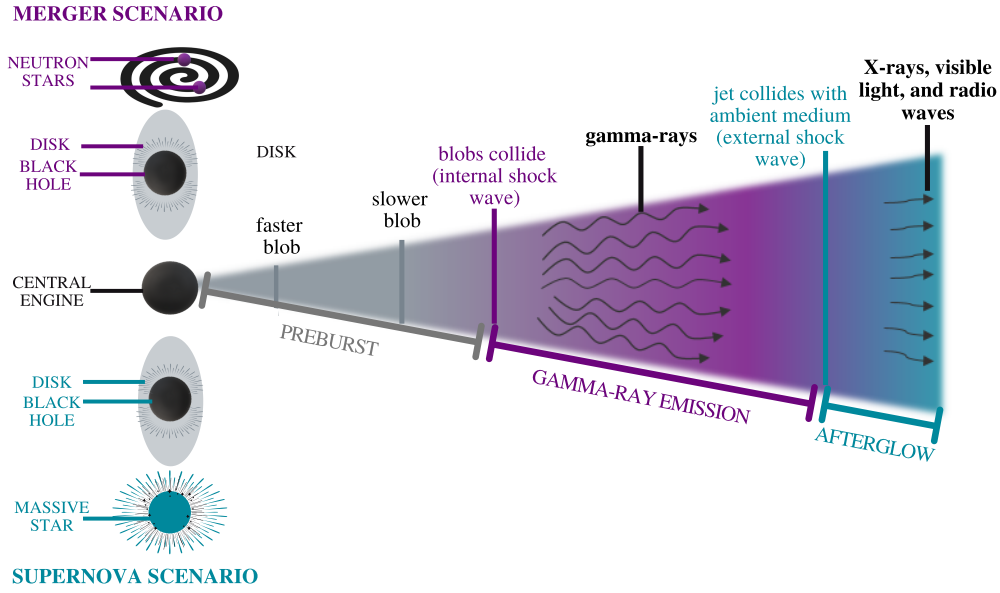


Fig. 1.7 The formation of a GRB could begin with a merger of two neutron stars (the *merger scenario*) or the collapse of a massive star (the *supernova scenario*). Both scenarios create a black hole with a disk of material around it, which ejects a jet that moves close to the speed of light. Shock waves within the jet (external/internal) give off radiation.

two compact objects (the *merging scenario*). The compact objects will be destroyed by tidal disruption onto the (potentially newly formed) black hole. This will lead to the formation of an accretion disk which powers the ultra-relativistic jets (as in the case of the supernova scenario). A summary of the two scenarios, along with the radiation emission of the jet, is shown in Fig. 1.7.

The outflow of the relativistic jet is composed by matter, magnetic fields, and photons: Photons decouple from the ejecta when it becomes transparent while the matter and magnetic flux are carried by the jet. More photons are generated in regions where kinetic or magnetic energy dissipates (in either shocks or magnetic reconnection regions) and escape without further coupling. Denoting the distribution of energy between the magnetic field and matter by the *magnetization factor*  $\sigma = B^2 / 4\pi\Gamma^2\rho^2$  (with  $B$  the magnetic field and  $\rho$  the matter density), two types of models currently exist based upon the composition of the outflow: the traditional *fireball model* (e.g. Piran (1999)) which is matter dominated (i.e.,  $\sigma \ll 1$ ) and *EM models* with strongly magnetized plasma ( $\sigma > 1$ ). The fireball shock model, also shown in Fig. 1.7, suggests that a relativistic jet is launched from the center of the explosion. The internal dissipation of the fireball leads to the high energy emission in the observed prompt emission. In EM models, the ejecta carries a globally ordered magnetic field, which is kinematically important. There are many scenarios explaining the prompt emission in EM models, one of which assumes that

the energy that powers the GRB comes from rotational kinetic energy of the central source extracted through the magnetic field. An important difference between EM models and the fireball model is how the energy is dissipated: While the fireball model argues that the energy is dissipated through shocks, EM models argue for the dissipation directly into the emitting particles through current-driven instabilities (Gomboc, 2012; McConnell and Ryan, 2004).

While the emission from GRBs has been well studied across the entire EM spectrum - thus providing a better understanding of the late stages of the jet evolution - there is only a limited understanding of the inner jet physics. The prompt emission spectrum may be empirically fitted with the Band function (Band et al., 1993), consisting of a broken power-law with a smooth break at a characteristic energy  $E_{peak}$ , that corresponds to the peak of the spectrum when plotted in terms of the energy output per decade of energy. The energy  $E_{peak}$  ranges from 10 keV up to at least 1 MeV with a broad peak near 200 keV. The precise nature of the emission is not well understood yet. Although synchrotron emission is believed to play a significant role (e.g. Rees and Meszaros (1994)), many aspects of the emission may also be explained by inverse Compton and thermal black body emission (e.g. Shaviv and Dar (1995)). The radiation process producing the prompt emission, the energetics of the explosions, and the role of the magnetic fields, therefore, remain largely unknown. In the last several decades, the spectra of the prompt  $\gamma$ -ray emission of GRBs has been extensively studied (e.g. Ryde and Pe  r (2009) and Zhang et al. (2007)). The total observable flux may be indistinguishable in both cases, but the polarization properties are expected to differ remarkably. The prompt emission and afterglow polarization properties are also powerful diagnostics of the jet geometry with distinct polarization predictions. The precise measurements of high energy polarization should provide crucial information about the inner structure of the jet, including the geometry and physical processes near the central engine (Toma et al., 2009).

## 1. High-energy polarization in GRBs

The expected level of polarization of the prompt  $\gamma$ -ray emission in GRBs has been estimated, assuming that in most cases the observed  $\gamma$ -ray emission is due to synchrotron radiation from relativistic electrons. To have a high radiative efficiency and to allow for the short time scale variability in the GRB light curves, these electrons have to be in a fast cooling regime. Their time-averaged distribution is most likely a broken power-law above the minimum Lorentz factor of the injected distribution of electrons. The maximum intrinsic polarization level of synchrotron radiation (Rybicki and Lightman, 2008) is in the order of  $PD \sim 75\%$  above, and

PD  $\sim 70\%$  below the synchrotron peak of electron spectrum. However, if inverse Compton scattering is the dominant radiative process, high PDs can also be reached (Eichler and Levinson, 2003).

Theoretical models have been developed in order to explain the already available polarization measurements (see Covino and Gotz (2016)). Most of the theoretical efforts have been applied to the standard fireball model (Meszaros and Rees, 1993; Piran, 1999), which offers the best interpretative scenario for polarimetric observations. The theoretical models either invoke a globally ordered magnetic field (Granot, 2003; Granot and Königl, 2003; Nakar et al., 2003), or a random magnetic field in the emission region. In the case of ordered magnetic fields, the electron synchrotron emission yields a net linear polarization, where the polarization properties are derived from the intrinsic characteristics, such as the the magnetic field geometry of the jet. These models apply to most observer's viewing angles, with PDs ranging from 20% up to  $\sim 60\%$ , and are characterized by a highly magnetized jet composition. The dissipation mechanism can either be magnetic reconnection or shocks, with the most probable emission mechanism being synchrotron radiation. When invoking a random magnetic field in the emission region (Lazzati et al., 2004), an optimal viewing direction is required in order to observe high degrees of polarization. No net polarization is detected along the jet, regardless of the radiation mechanism. If the viewing angle is near the edge of the jet – in particular about  $1/\Gamma$  outside the jet cone (where  $\Gamma$  is the bulk Lorentz factor of the jet) – a high PD results due to the loss of emission symmetry (Shaviv and Dar, 1995). These models are characterized by matter dominated outflow and shocks as the most likely dissipation mechanism (Lazzati and Begelman, 2009), with both synchrotron and inverse Compton being the possible radiation mechanisms. For most viewing angles,  $PD < 20\%$ , however, synchrotron emission may produce high PD as mentioned above, and Compton models can achieve  $PD \sim 100\%$  under favorable geometries.

A statistical study of GRB polarization properties could differentiate between models that invoke tangled or globally ordered magnetic fields. This provides a direct diagnostic of the magnetic field structure, the radiation mechanisms, and the geometric configuration of GRB jets. Assuming random viewing angles, Toma et al. (2009) simulated 10000 jets in order to study the distribution of GRB polarization for three generalized emission models: (1) synchrotron emission with ordered magnetic fields (SO model), (2) synchrotron emission in random magnetic fields (SR model), and (3) a Compton model with random magnetic fields (IC model). Each model predicts a different value for the maximum possible polarization that places constraints on the existing models. For example, the fraction of GRBs that have high PD is significantly

higher for models with ordered magnetic fields (SO model) than models invoking random magnetic fields in the emission region. A more powerful diagnostic can be seen in the distribution of PD for each simulated GRB event as a function of  $E_{peak}$ , shown in Fig. 1.8. Some models show very distinct structures in this parameter space, such as the correlation between  $E_{peak}$  and PD in the SR model shown in grey closed circles. The nature of GRB radiation mechanisms can also be derived from the energy-dependence of the polarization. The relative importance of synchrotron emission and inverse Compton emission may be distinguishable with energy dependent polarization measurements, since the various components have distinct polarization signatures.

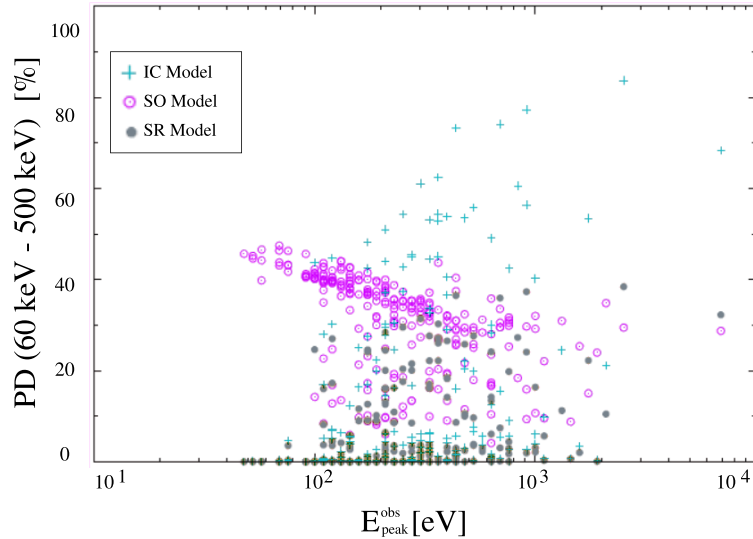


Fig. 1.8 The distribution of PD for 10000 simulated GRB events as a function of  $E_{peak}$  for three generalized models: an intrinsic model for synchrotron emission with ordered magnetic fields (SO model) shown in purple open circles, a geometric model for synchrotron emission in random magnetic fields (SR model) shown in grey closed circles, and a geometric Compton model (IC Model) shown in blue plus signs. From Toma et al. (2009).

### 1.1.3 X-ray and Gamma-Ray Binaries

XRBS are binary systems that emit X-rays. However, XRBS also emit energy in radio, infrared (IR), optical, UV, and sometimes,  $\gamma$ -ray emission as well. Essentially, XRBS consists of a compact star which produces a huge release in energy, with the material supplied from a companion star in a binary system. Matter from the companion star accretes onto the compact object which can be a neutron star (NS-XRBS), or a black hole (BH-XRBS). The classes of

XRBs are vast and varied: The first criteria, dividing XRBs into *high mass X-ray binaries* (HXRBS) and *low mass X-ray binaries* (LXRBS), is based on the mass of the companion stars: LXRBS have companion stars with  $M \ll 1M_{\odot}$ , while HXRBS are identifiable by a massive companion star with  $M \gg 1M_{\odot}$ . Fig. 1.9 illustrates the mass transfer mechanisms in HXRBS (left) and LXRBS (right): If the companion star fills its *Roche lobe* (in LMXRBs; see right-hand side of 1.9), the matter flows into the inner Lagrangian point and, approaching the compact object, forms an accretion disc around it due to angular momentum. If the companion star is contained within its Roche lobe (in HMXRBs; see left-hand side of 1.9), the matter accretes onto the compact object via a stellar wind (Moret et al., 2003; Savonije, 1978).

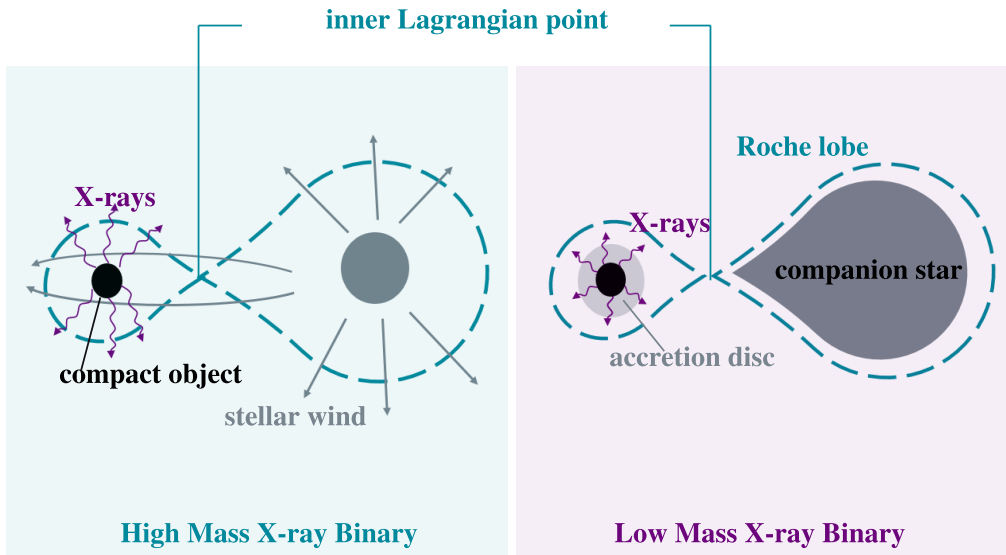


Fig. 1.9 Illustration of the mass transfer in HXRBS (left) and LXRBS (right). If the companion star fills its Roche lobe, the matter flows into the inner Lagrangian point and, approaching the compact object, forms an accretion disc around it due to angular momentum. If the companion star is contained within its Roche lobe, the matter accretes onto the compact object via a stellar wind. Adapted from Moret et al. (2003).

HXRBS can be further divided into *Hard X-ray Transient Sources*, with optical counterparts usually being main-sequence stars (with eccentric orbits), and *Permanent X-ray Sources* with optical counterparts usually being OB super-giant stars (with almost circular orbits). The optical companion in LXRBS is a late-type star which accretes onto the compact object by the Roche lobe overflow. They are often found in globular clusters and populate the Galactic Buldge, while HXRBS are more concentrated towards the Galactic Plane, which shows a clear signature of spiral structure in their spatial distribution (Grimm et al., 2002). Other sub-classes of XRBs

include the *Cataclysmic Variables* (CVs) in which the companion star is a low-mass-late-type star and the compact object is a white dwarf.

## 1. Spectral states and radiation mechanisms of XRB jets

The SEDs of some XRBs can be described by thermal emission from the companion star and the accretion disc, synchrotron emission from a jet, and Comptonization of the disk radiation in a hot corona. The companion star dominates the optical/near-IR emission, while the accretion disc dominates the emission at UV to X-ray wavelengths. The current observational picture of XRBs includes at least three *spectral states*: The *high/soft state* (HS), the *intermediate state* (IS), and the *low/hard state* (LS) (see Fender and Gallo (2014) for a review). The radio to IR spectrum of an XRB in the LS – which exists typically below a few percent of the Eddington luminosity  $L_{Edd}$  (e.g. Maccarone (2003); McClintock and Remillard (2006)) – is due to an almost flat, self absorbed, optically thick synchrotron spectrum, with a spectral index of  $\alpha \sim 0.0$ , where  $F_\nu \propto \nu^{-\alpha}$ . In BH-XRBs, synchrotron-emitting compact jets are formed during the spectral LS (Fender and Gallo, 2014). The spectrum of a BH-XRB jet, in the LS, is illustrated in Fig. 1.10. The higher-energy synchrotron emission arises from a small region of the jet near the compact object. Since the power-law index  $\alpha \gtrsim 0$ , most of the radiative power of the jet resides in the higher-energy synchrotron emission, with a *jet-break* in the IR regime. The flat spectral component breaks into an optically thin spectrum, where the jet becomes transparent, with a cut-off in the jet spectrum in the X-ray regime (Maitra et al., 2009). During the steady HS, the radio emission (and probably therefore the jet production) is strongly suppressed (Corbel et al., 2001; Fender et al., 1999; Gallo et al., 2003; Tananbaum et al., 1972).

Fender et al. (2004) presented a unified, semi-quantitative model for disc-jet coupling in BH-XRBs, as shown in Fig.1.11. An X-ray hardness-intensity diagram (HID) is shown above a schematic diagram of the bulk Lorentz factor  $\Gamma$  of the jet and inner accretion disc radius as a function of the X-ray hardness. The path of a typical X-ray transient is indicated by the solid arrows, and four sketches around the outside of the diagrams indicate the relative contributions of the jet (blue), corona (yellow) and accretion disc (red) at different phases (stage *i* - *iv*). At stage *i*, the sources are in the LS, which produces a steady jet, and probably extends down to very low luminosities. At stage *ii*, the motion of a XRB in the HID (for a typical outburst) is nearly vertical. There is a peak in the LS, after which the motion of the XRB in the HID becomes more horizontal (to the left) and the source moves into the IS. Despite this softening of the X-ray spectrum, the steady jet persists with a quantitatively similar *disc-jet*

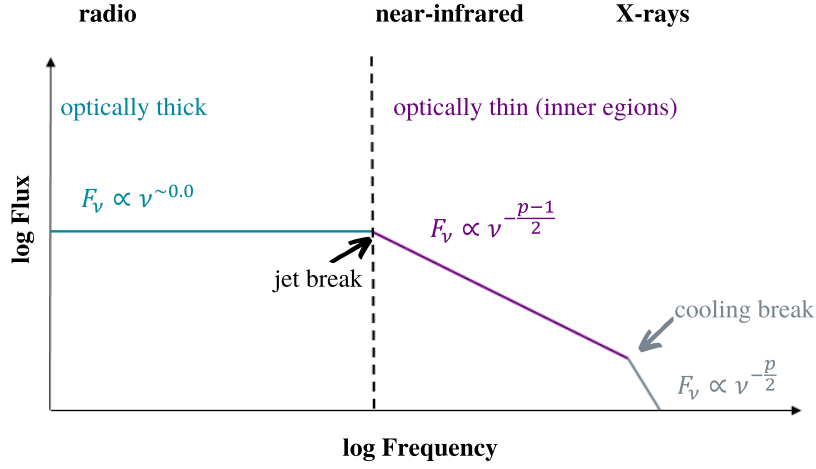


Fig. 1.10 A sketch of the spectrum of a steady jet in the LS of XRBs. The radio to near-IR is due to an almost flat, self-absorbed, optically thick synchrotron spectrum. One of the key spectral features is the break from optically thick to optically thin synchrotron emission (i.e., the *jet break*), between the mid-IR and near-IR wavelengths.

*coupling* to that of the LS. The source approaches the *jet-line* at stage iii in the HID between the jet-producing and the jet-free states. As this boundary is approached, the jet properties change, most noticeably the velocity of the jet changes. The final jet has the highest bulk Lorentz factor  $\Gamma$ , causing the propagation of the internal shock through the slower-moving outflow in front of it. At stage iv, the source is in the soft IS or the HS and no jet is produced, with fading optically thin emission observed from the optically thin shock.

It is, therefore, argued that during the rising phase of a black hole transient outburst, the steady jet (associated with the HS) persists, while the X-ray spectrum initially softens. Subsequently, the jet becomes unstable and an optically thin radio outburst is associated with a soft X-ray peak (which corresponds to a steep power-law state) at the end of this phase of softening. Furthermore, [Fender et al. \(2004\)](#) quantitatively demonstrates that the transient jets, which are associated with these optically thin events, are considerably more relativistic than those in the LS. This implies that  $\Gamma$  rapidly increases and results in an internal shock in the outflow, which is the cause of the optically thin radio emission.

In the HS, the emission is dominated by soft X-ray emission from the accretion disc, while the emission in the LS is possibly dominated by a non-thermal hard X-ray power-law. The corona is generally believed to produce this power-law due to Compton up-scattering of soft and hot electrons ([Fender and Gallo, 2014](#)). The origin of the X-ray emission in XRBs is still under

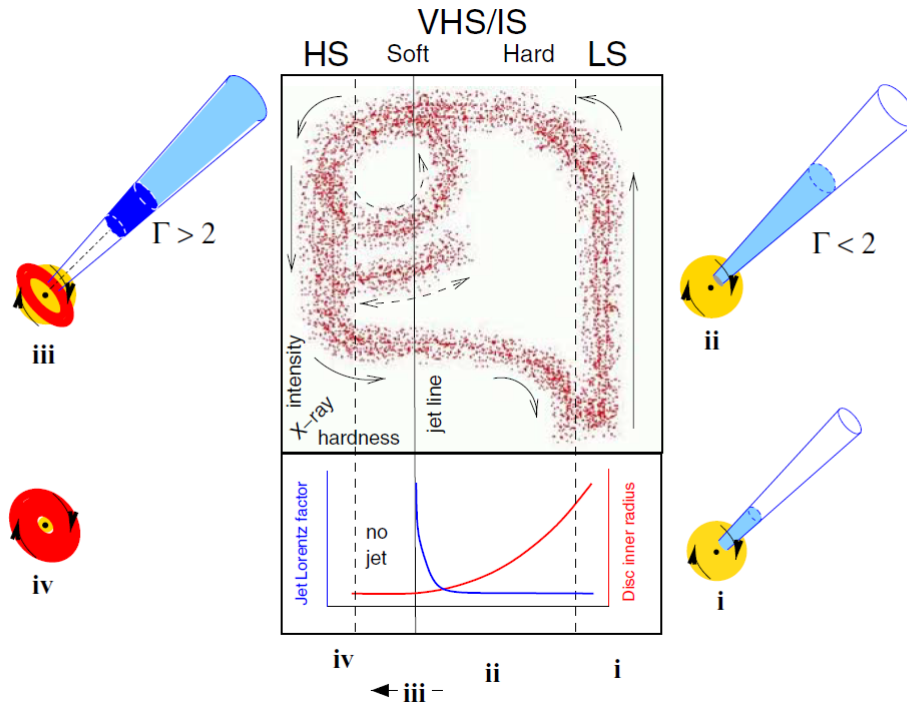


Fig. 1.11 A schematic for a simplified model for jet-disc coupling in black hole binaries. The upper central box panel represents an X-ray hardness intensity diagram (HID): The X-ray hardness increases to the right and intensity upwards. "HS" indicates the high/soft state, "VHS/IS" the high/intermediate state, and "LS" the low/hard state. The lower panel indicates the variation of the bulk Lorentz factor  $\Gamma$  of the outflow with hardness - in the LS and hard VHS/IS the jet is steady with an almost constant  $\Gamma < 2$ , progressing from state *i* to state *ii* as the luminosity increases. At some point (usually corresponding to the peak of the VHS/IS)  $\Gamma$  increases rapidly, producing an internal shock in the outflow (state *iii*), followed by the cessation of the jet production in the disc-dominated HS (state *iv*). At this stage, fading optically thin radio emission is only associated with a jet/shock physically decoupled from the central engine. The solid arrows indicate the track of a simple X-ray transient outburst with a single optically thin jet production episode. The dashed arrows indicate the paths that some transients take in repeatedly hardening, crossing the zone *iii* (the *jet-line*) from left to right, producing further optically thin radio bursts. The sketches around the outside illustrate a concept of the relative contributions of the jet (blue), corona (yellow) and accretion disc (red) at these different stages. From [Fender et al. \(2004\)](#).

investigation. [Markoff et al. \(2001\)](#) first proposed that the optically thin emission from the jet dominates the X-ray flux in BH-XRBs, based on radio to X-ray spectral modelling. The hard power-law at X-ray energies could then be explained by the optically thin jet emission extending from the optical regime. Similar models were developed to explain the SEDs of BH-XRBs in the LS, showing that the synchrotron component probably produces a significant fraction of the X-ray flux in the LS, but may not dominate (e.g. [Markoff et al. \(2003\)](#) and [Maitra et al. \(2009\)](#)). In particular, [Maitra et al. \(2009\)](#) gave empirical evidence for the jet producing the hard X-ray power-law in BH-XRBs, where the jet emission at IR frequencies was isolated from the accretion disc emission and the companion star. The spectral index of the jet component was found to be consistent with the optically thin emission during the fading LS of the outburst. The near IR emission from the jet was also found to be linearly proportional to the X-ray flux. Moreover, the spectral index between the X-ray and IR frequencies, the jet component in the IR regime, as well as the X-ray spectral index itself were all consistent with the same value. This implies that the broadband spectrum, from IR to X-ray energies, is consistent with the same power-law fading by one order of magnitude.

[Russell et al. \(2011\)](#) obtained similar results to those of [Maitra et al. \(2009\)](#), from multi-wavelength monitoring, where the optical jet emission was found to rise and fade during the LS. In some cases, a clear X-ray flare coexisted with the jet and had the same morphology in the light curve implying a common emission mechanism. The IR-X-ray spectral index was consistent with optically thin synchrotron emission, but the spectral properties before and during the flare were the same (within errors). This implies that either the corona and the jet have similar emitting properties, or while the X-ray emission is probably correlates to the jet, it may not be dominated by the jet. Evidence for a change in the X-ray radiation mechanism in the LS of some XRBs were also shown (e.g. [Rodriguez et al. \(2008\)](#); [Sobolewska et al. \(2011\)](#)), with two spectral components producing the X-ray power-law. A change in the X-ray emission mechanism was also implied by the emission becoming radiatively efficient above a critical X-ray luminosity ([Coriat et al., 2011](#)). All the evidence mentioned above still suggests that at some stages of a BH-XRB outburst, the majority of the X-ray flux originates in the optically thin synchrotron emission from compact jets in the system ([Russell, 2012](#)).

## 2. Polarization signatures in XRB jets

Polarimetry provides key physical information on the properties of interacting binary systems, which are sometimes difficult to obtain through any other types of observations. Radiation

processes, such as the scattering of free electrons in the hot plasma above the accretion discs, cyclotron emission by mildly-relativistic electrons in the accretion shocks on the surface of the highly magnetic white dwarfs, and the optically thin synchrotron emission from jets may be observed. The polarimetry of binary systems (in particular, CVs and BH-XRBs) has been well studied in the optical/near-IR regime, which allows estimations of the magnetic field strength in magnetic CVs, and determining the nature of the XRB jets. In optically thick jets the polarization vector is perpendicular to the projection of the magnetic field onto the sky, whereas, in the case of optically thin jets, the polarization vector will be parallel to the projection of the magnetic field. Optically thin synchrotron emission is intrinsically polarized, and a net polarization will be observed if the local magnetic field is ordered. If the magnetic field is tangled, different angles of the polarized light will decrease the average observed polarization.

BH-XRBs are particularly interesting candidates for polarization studies, due to their inherently high flux, which allows for the detection of clear signals. In the optical regime, polarization due to the scattering of intrinsically unpolarized thermal emission can be modulated on the orbital period, which can be used to constrain the physical and geometrical properties of the system (Dolan and Tapia, 1989; Gliozzi et al., 1998). At radio, and in some cases at IR frequencies, variable polarization has been detected. This may be due to the synchrotron emission from jets launched via the process of the accretion onto the black hole (e.g. Russell and Fender (2008)). When the radio emission is consistent with optically thin synchrotron emission (commonly occurring during the transition from the LS to the HS), a PD  $\lesssim 10\%$  is usually expected. Continuous compact jets produce optically thick and self-absorbed synchrotron jets at radio frequencies, while the optically thin emission can only be observed after the jet-break (see Fig. 1.10). As mentioned above, the synchrotron radiation from the jets in BH-XRBs can sometimes produce the X-ray power-law. Therefore, the same distribution of electrons possibly produce the IR to X-ray power-law, implying that the polarized emission is expected to be variable.

Near IR linear polarization was detected from some BH-XRBs (e.g. Russell and Fender (2008)), which was not consistent with an interstellar origin or due to scattering within the XRBs, since the PD did not increase with frequency. In some cases, the PA was perpendicular to the known jet axis, which implies that the magnetic field is parallel to the jet axis. Low PDs of  $\sim 1 - 7\%$  were measured, which implies a tangled magnetic field with rapid changes near the base of the jet. Measurements of polarization at X-ray energies for some BH-XRBs have been made. For instance, the PD of  $\sim 2.5\%$  for a BH-XRB (Cyg X-1) has been measured at  $2.5 - 5.3$  keV (Long et al., 1980). Using a simple phenomenological model, Russell and Shahbaz (2014)

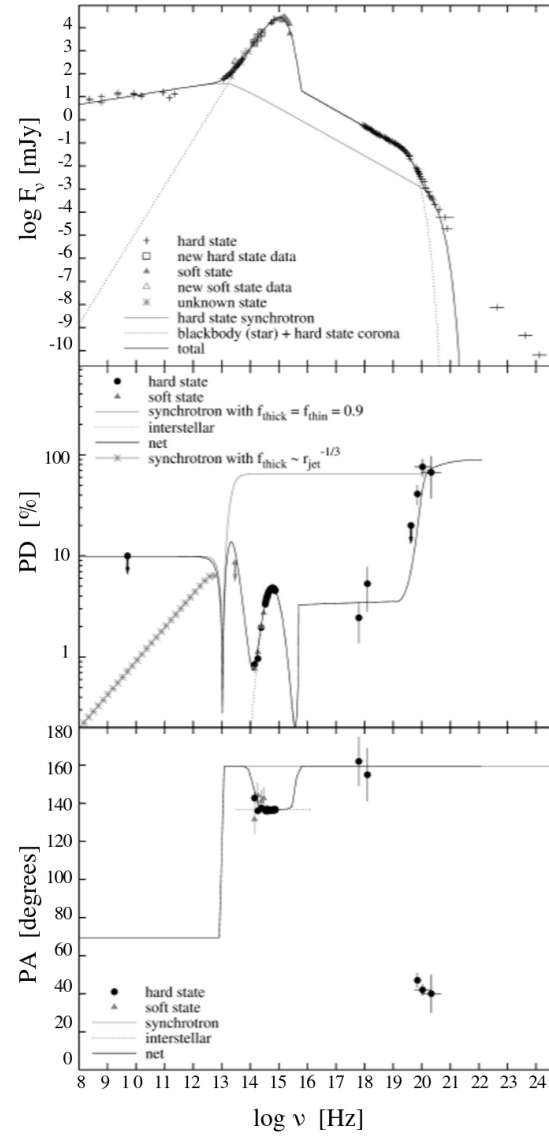


Fig. 1.12 The flux and polarization spectrum (radio to  $\gamma$ -rays) of Cyg X-1. The top panel shows the radio to  $\gamma$ -ray flux density, while the middle and bottom panel show the PD and PA as a function of the frequency, respectively. From [Russell and Shahbaz \(2014\)](#).

modelled the radio to  $\gamma$ -ray flux and polarization spectrum of Cyg X-1 in the LS. Their model consists of a strongly synchrotron polarized jet, an unpolarized Comptonized corona, and a moderately polarized interstellar component. The results, shown in Fig. 1.12, suggests that the origin of the  $\gamma$ -rays, X-rays, as well as some of the IR polarization is due to the optically thin synchrotron power-law from the inner regions of the jet. [Laurent et al. \(2011\)](#) claimed a PD of  $67 \pm 30\%$  in Cyg X-1 averaged over several years, which is consistent with an ordered and stable magnetic field, that misaligns with the jet axis. Details on the detection of X-ray and  $\gamma$ -ray polarization of various sources will be discussed in Chapter 2.

## 1.2 Dissertation Structure

This dissertation describes the development of a Monte Carlo code to study the degree and orientation of Compton polarization in the high-energy regime of jet-like astrophysical sources. A general discussion of astrophysical polarization will follow in Chapter 2, which includes the formalism for calculating the polarization in astrophysical sources, an introduction to synchrotron and Compton polarization, and the future prospects of detecting polarization in the high-energy regime of astrophysical sources. Chapter 3 expounds on how the Monte Carlo approach is used to simulate Comptonization, and concludes with a detailed description of the Monte Carlo code. The results is given in Chapter 4, followed by the conclusion and future outlook in Chapter 5.

# Chapter 2

## Astrophysical Polarization

Electromagnetic (EM) radiation refers to transverse waves and are made up of electric and magnetic fields perpendicular to the direction of propagation (see Fig. 2.1). Polarization is a typical property of EM radiation, defined by the direction of the EM wave's electric field, which is described by the electric field vector  $\vec{E} = (E_x, E_y, E_z)$ . The polarization of a single EM wave is illustrated in Fig. 2.2: EM radiation is *unpolarized* when the corresponding electric field vector oscillates in different directions with respect to the observer's line-of-sight (LOS). Contrarily, when the electric field vector oscillates in a single direction with respect to the LOS, the radiation is said to be *polarized*. This chapter expounds on the general formalism for calculating the polarization in astrophysical sources, including both the analytic and Stokes representation of the polarization signatures. Furthermore, this chapter also provides a brief introduction to the non-thermal polarization mechanisms.

### 2.1 Polarization Theory

Polarization refers to the direction in which the electric field of an EM wave oscillates with respect to an observer's LOS (see Fig. 2.2). The polarization signatures observed, therefore, naturally carry important information about the astrophysical environment in terms of the magnetic field structure, and how the magnetic field configuration links into the dynamics and acceleration of the energetic particles. The polarization signature of the radiation contains two parts, the polarization degree (PD) and the polarization angle (PA). The former describes how much the radiation is polarized, commonly given in percentages;  $PD = 100\%$  means that the radiation is fully polarized, while  $PD = 0\%$  means that the radiation is unpolarized. The latter determines the direction of the polarization, and is, therefore, undefined for unpolarized radiation. The PA is determined by the position angle of the electric field vector and has a  $180^\circ$

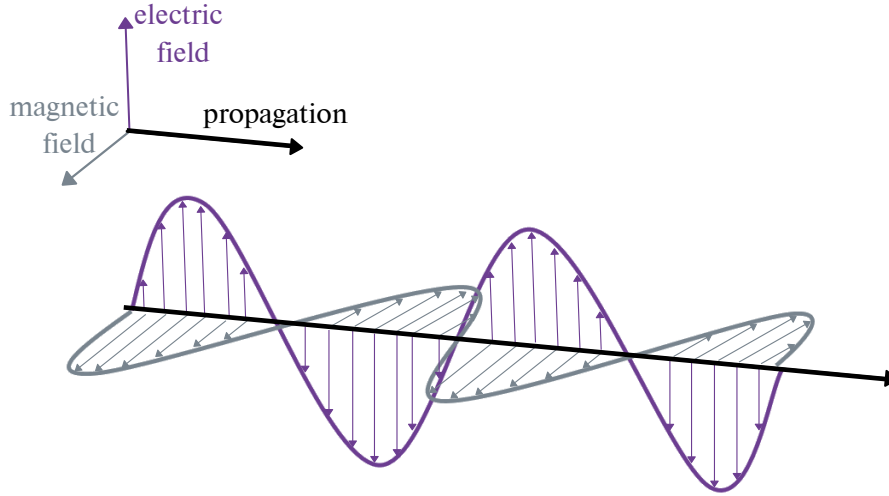


Fig. 2.1 An illustration of an EM wave, which consists of an electric field (purple) and a magnetic field (grey), perpendicular to the direction of propagation (black).

ambiguity, since electric fields that oscillate upwards and downwards are equivalent. The PD is generally given by

$$PD = \frac{|P_{\parallel} - P_{\perp}|}{P_{\parallel} + P_{\perp}} \quad (2.1)$$

where  $P_{\parallel}$  and  $P_{\perp}$  are the power components of the electric field vector, which are perpendicular and parallel to the projection of the magnetic field onto the plane of the sky, respectively (Rybicki and Lightman, 2008).

The formalism for calculating the polarization signatures, as seen above, may be used to fully describe the polarization in astrophysical sources. However, the so called *Stokes* formalism provides an easier representation of the polarization signatures in terms of four Stokes parameters.

### 2.1.1 Stokes Formalism

The Stokes parameters are a set of values that describe the polarization state of EM radiation (i.e., elliptical-, circular-, and linear polarization, shown in Fig. 2.3). They were originally formulated by George Gabriel Stokes (Stokes, 1851) and were intended for a more convenient mathematical alternative to the description of partially polarized light in terms of the total

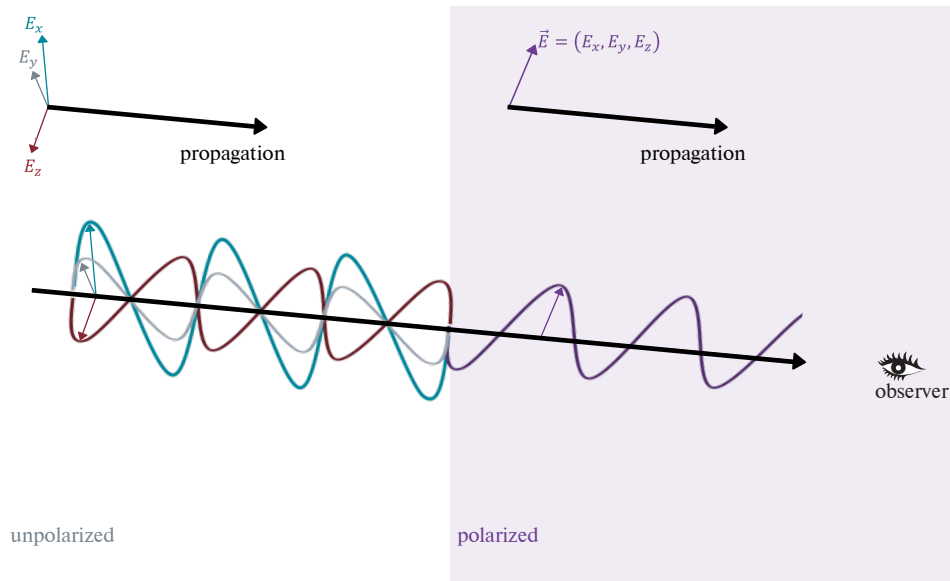


Fig. 2.2 An illustration of the polarization of a single EM wave. When the electric field of the EM wave  $\vec{E} = (E_x, E_y, E_z)$  oscillates in a single direction with respect to the observer's LOS (right side), the radiation is said to be polarized. Contrarily, when the electric field oscillates in different directions at the same time (left side), the radiation is unpolarized.

intensity ( $I$ ), the PD, and the shape parameters of the polarization ellipse, illustrated in Fig. 2.5.

Polarization states

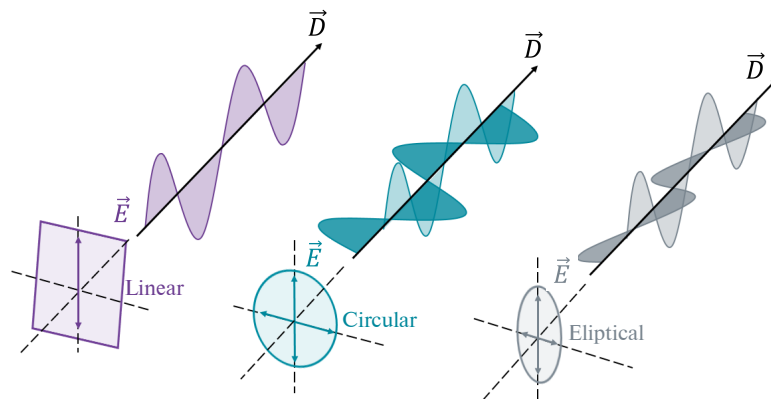


Fig. 2.3 Three different polarization states are given, with the direction of propagation  $\vec{D}$  to the LOS given in black. From the left, linear polarization is illustrated in purple, circular polarization in blue, and elliptical polarization is in grey.

An EM wave is specified by its propagation vector  $\vec{D}$  and electric field vector  $\vec{E}$ , with the polarization defined by the direction of  $\vec{E}$ , as illustrated in Fig. 2.1 and Fig. 2.2. Alternatively, the polarization can be described by a graphical tool in real three-dimensional space, known as the *Poincaré sphere* (shown in Fig. 2.4). The polarization state  $\psi$  of an EM wave is the curve traced out by its electric field as a function of time in a fixed plane. Three different polarization states are shown in Fig. 2.3, with the most common polarization states (linear and circular polarization) shown in purple and blue, respectively. The linear and circular polarization states are the degenerate cases of the most general, elliptical polarization state. The different polarization states can be uniquely represented by different points on the Poincaré sphere, shown in Fig. 2.4: The linear polarization states are located on the equator, while circular polarization states are located on the poles. The right-hand and left-hand elliptical polarization states are positioned on the northern and southern hemisphere, respectively. The intermediate elliptical polarization states are continuously distributed between the equator and poles. The coordinates of a specified point on the surface of the sphere are then defined by three, normalized Stokes parameters ( $Q$ ,  $U$ , and  $V$ ) which describe the state of the polarization.

The four Stokes parameters - denoted by  $I$ ,  $Q$ ,  $U$ , and  $V$  - may be defined in terms of the parameters of the polarization ellipse (i.e., the orientation  $\psi$  and the ellipticity  $\chi$ ), shown in Fig. 2.5, with

$$\begin{aligned}
 I &= X^2 + Y^2 \\
 Q &= (X^2 - Y^2) \cos(2\psi) \\
 U &= (X^2 - Y^2) \sin(2\psi) \\
 V &= 2XYH
 \end{aligned} \tag{2.2}$$

where  $X$  is the semi-major axes and  $Y$  the semi-minor axes of the polarization ellipse, and  $H = \text{sgn}(V)$ .

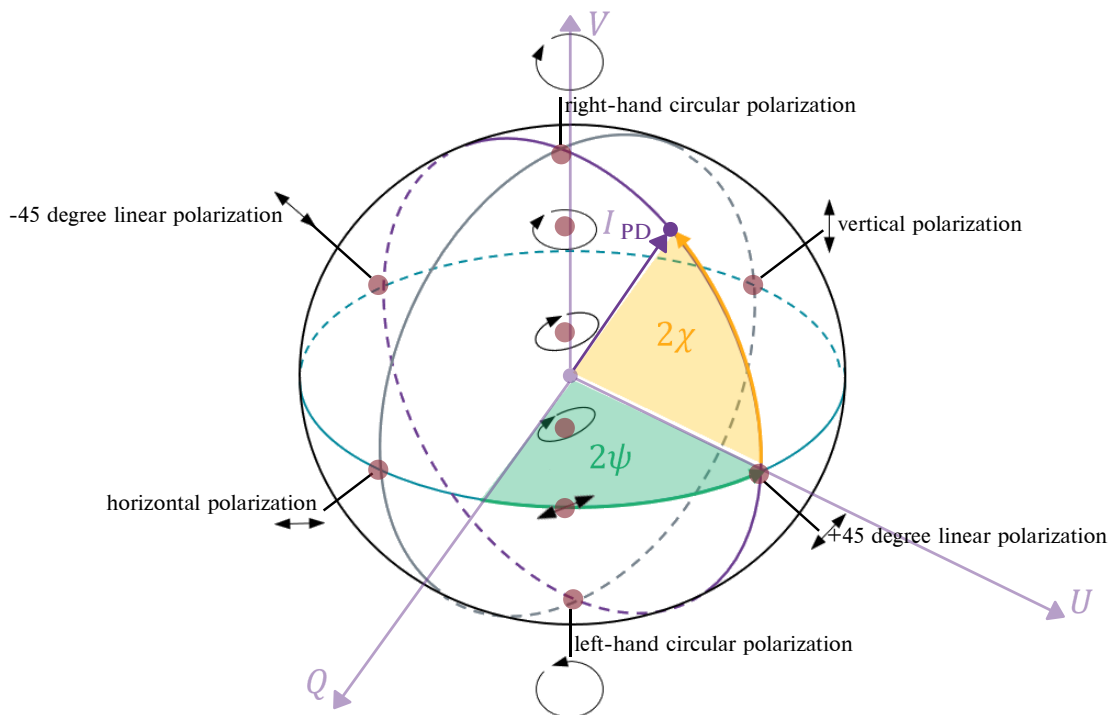


Fig. 2.4 The points on the surface of the Poincaré sphere represent the polarization of an EM wave. The linear polarization states are located on the equator, while circular polarization states are located on the poles. The intermediate, elliptical polarization states are continuously distributed between the equator and poles. The orientation angle  $\psi$  and ellipticity  $\chi$  are also shown with respect to a specified point on the sphere. The coordinates of this point is defined by the three normalized Stokes parameters namely  $Q$ ,  $U$ , and  $V$ .

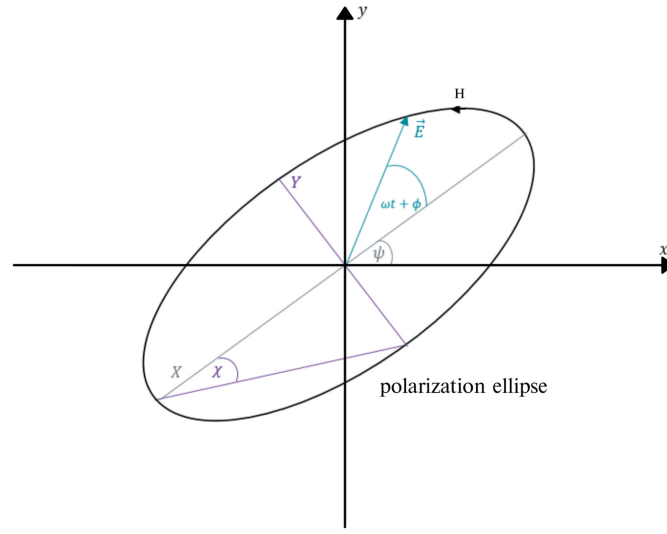


Fig. 2.5 The relationship of the polarization ellipse to the orientation angle  $\psi$  and the ellipticity  $\chi$  (the Poincaré sphere parameters), which are functions of the semi-major axes  $X$  and the semi-minor axes  $Y$ .

The PD, the orientation angle  $\psi$  (which defines the PA), and the ellipticity  $\chi$  of the polarization can thus be calculated with the Stokes parameters as follows:

$$\begin{aligned}
 \text{PD} &= \frac{\sqrt{Q^2 + U^2 + V^2}}{I} \\
 \psi &= \text{PA} = \frac{1}{2} \arctan \frac{U}{Q} \\
 \chi &= \frac{1}{2} \arctan \frac{V}{\sqrt{Q^2 + U^2}}.
 \end{aligned} \tag{2.3}$$

In this way the Stokes parameters provide an alternative description of a polarization state, where each parameter corresponds to the sum of (or difference between) measurable quantities. The Stokes formalism is therefore experimentally very convenient.

The signs of the Stokes parameters are defined by the helicity and orientation of the semi-major axis of the polarization, as shown in Fig. 2.6. Defining the standard Cartesian bases of the electric field vector as  $(\hat{x}', \hat{y}')$ , two additional bases with respect to  $(\hat{x}', \hat{y}')$  may be defined as follows: a Cartesian basis rotated by  $45^\circ$  as  $(\hat{x}'', \hat{y}'')$  and a circular basis as  $(\hat{l}, \hat{r})$ , so that

$\hat{l} = \frac{(\hat{x}' + i\hat{y}')}{\sqrt{2}}$  and  $\hat{r} = \frac{(\hat{x}' - i\hat{y}')}{\sqrt{2}}$ . The Stokes parameters are then given by

$$\begin{aligned} Q &= \langle E_{x'}^2 \rangle - \langle E_{y'}^2 \rangle \\ U &= \langle E_{x''}^2 \rangle - \langle E_{y''}^2 \rangle \\ V &= \langle E_l^2 \rangle - \langle E_r^2 \rangle \end{aligned} \quad (2.4)$$

with the total intensity

$$I = \langle E_{x'}^2 \rangle + \langle E_{y'}^2 \rangle = \langle E_{x''}^2 \rangle + \langle E_{y''}^2 \rangle = \langle E_l^2 \rangle + \langle E_r^2 \rangle. \quad (2.5)$$

Unpolarized light will have an intensity  $I > 0$ , while  $Q = U = V = 0$ , which indicates that no polarization type is dominant. The opposite would be true for 100% polarized light, where - in the case of linear polarization - the electric field vector oscillates in a single direction with respect to the LOS.

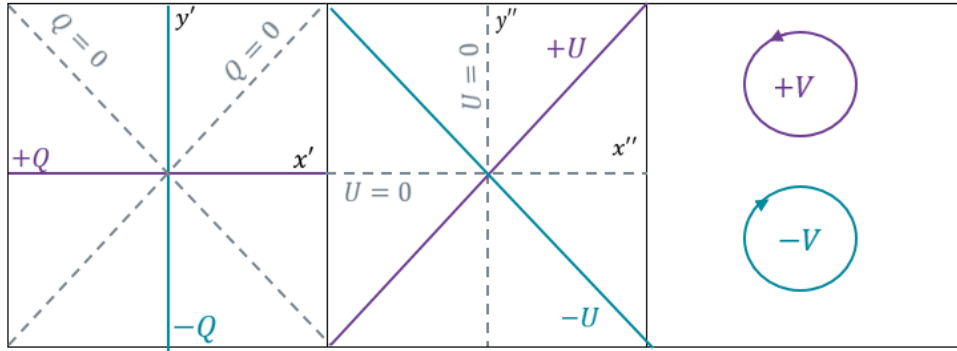


Fig. 2.6 An illustration of the Stokes parameters: The  $x'$  and  $y'$  axes are the standard Cartesian basis of the electric field vector, while  $x''$  and  $y''$  are the Cartesian basis rotated clockwise by  $\pi/4$  rad angle. The solid lines indicate where the Stokes parameters are at their degenerative states, with the signs determined by the helicity and orientation of the semi-major axis of the polarization ellipse.

When applying Stokes parameters, it is convenient to write them in a form of a four-vector

$$\begin{bmatrix} I \\ \vec{P} \end{bmatrix} = \begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix} \quad (2.6)$$

where  $\vec{P}$  refers to the Stokes vector that describes the polarization state. For unpolarized radiation

$$\vec{P}_U = \begin{bmatrix} Q \\ U \\ V \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}. \quad (2.7)$$

Linear polarization rotated by an angle  $\pm\pi/4$  rad (see Fig. 2.6) is given by

$$\vec{P}_{P,z} = \begin{bmatrix} Q \\ U \\ V \end{bmatrix} = \begin{bmatrix} \pm 1 \\ 0 \\ 0 \end{bmatrix} \quad \text{and} \quad \vec{P}_{P,x} = \begin{bmatrix} Q \\ U \\ V \end{bmatrix} = \begin{bmatrix} 0 \\ \pm 1 \\ 0 \end{bmatrix}. \quad (2.8)$$

Since the Stokes parameters are dependent on the choice of axes, a rotation matrix  $M$  relates the Stokes parameters in one coordinate system to another: The definition of the Stokes parameters given in Fig. 2.6 defines the standard Cartesian basis of the electric field vector given by the  $x'$  and  $y'$  axes. The  $x''$  and  $y''$  axes define a second coordinate system rotated about the direction of propagation at an angle  $\theta$  clockwise with respect to the standard coordinate system. If the Stokes parameters  $(I', Q', U', V')$  correspond to a photon in the standard coordinate system  $(x', y')$ , then the same photon in the  $(x'', y'')$  coordinate system may be described by the Stokes parameters

$$\begin{bmatrix} I'' \\ Q'' \\ U'' \\ V'' \end{bmatrix} = M \begin{bmatrix} I' \\ Q' \\ U' \\ V' \end{bmatrix} \quad (2.9)$$

where

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\theta & \sin 2\theta & 0 \\ 0 & -\sin 2\theta & \cos 2\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2.10)$$

for rotations about the axis represented by  $V$ . Thus, a photon with Stokes parameters of  $(I', Q', U', V')$ , may be represented in a new system of coordinates with Stokes parameters

$$\begin{bmatrix} I'' \\ Q'' \\ U'' \\ V'' \end{bmatrix} = \begin{bmatrix} I' \\ Q' \cos 2\theta + U' \sin 2\theta \\ -Q' \sin 2\theta + U' \cos 2\theta \\ V' \end{bmatrix} \quad (2.11)$$

where a rotation in the opposite direction will change the sign of the sine term. When photons undergo an interaction that is sensitive to polarization, the Stokes parameters of the initial beam are generally transformed into a new set of parameters. The relation between the two sets of Stokes parameters is given by a transformation matrix  $T$  characteristic of the interaction, with

$$\begin{bmatrix} I \\ \vec{P} \end{bmatrix} = T \begin{bmatrix} I_0 \\ \vec{P}_0 \end{bmatrix} \quad (2.12)$$

where  $\vec{P}_0$  and  $\vec{P}$  are the Stokes vectors that describe the polarization state before and after the interaction, respectively.

## 2.2 Polarization Mechanisms

A variety of physical phenomena may alter the polarization state of EM radiation, which includes the influence of the magnetic fields, general relativity effects, and the emission mechanisms. There are two types of EM emissions: *thermal* emission and *non-thermal* emission, depending on the particle population. The following section discusses the polarization-dependent formalism for both synchrotron emission and Compton scattering.

### 2.2.1 Synchrotron Polarization

When a relativistic particle moves in a magnetized region with a magnetic field  $B$ , the particle will follow a spiral trajectory. The radially accelerated motion causes a broad spectrum of *synchrotron radiation*. This section provides the calculations for the PD of synchrotron radiation in astrophysical applications based on the results from classical electrodynamics.

Consider a relativistic particle with a mass  $m$ , electric charge  $q$ , and a Lorentz factor  $\gamma$ . When the particle moves with a speed  $\beta c$  at a pitch angle  $\alpha$  with respect to an ordered magnetic field, the particle will lose energy (with the total energy  $E = \gamma mc^2$ ), due to its gyrational motion in

the magnetic field. The energy-loss rate may be written in terms of a change in the Lorentz factor as

$$\left(\frac{d\gamma}{dt}\right)_{sy} = -\frac{4}{3}c\sigma_T \frac{u_B}{m_e c^2} Z^4 \left(\frac{m_e}{m}\right)^3 \beta^2 \gamma^2 \quad (2.13)$$

where  $Z$  is the particle's charge in units of the elementary charge  $e$ ,

$$u_B = \frac{B^2}{8\pi} \quad (2.14)$$

is the energy density in the magnetic field and

$$\sigma_T = \frac{8\pi}{3} \left(\frac{e^2}{m_e c^2}\right)^2 \quad (2.15)$$

is the *Thomson* cross-section. Equation 2.13 reveals that the energy-loss rate of relativistic particles scales with mass as  $\frac{d\gamma}{dt} \propto -m^{-3}$ , which implies that electrons are the most efficient radiators (Böttcher et al., 2012).

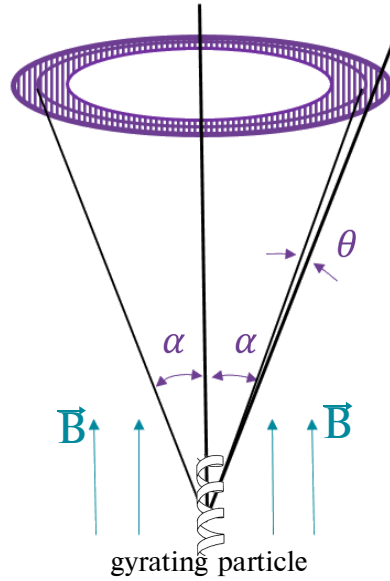


Fig. 2.7 Synchrotron emission from a particle with pitch angle  $\alpha$ , and an opening angle  $\theta = \frac{1}{\gamma}$ . The radiation is confined to the shaded solid angle around the magnetic field lines. Adapted from Rybicki and Lightman (2008).

Synchrotron radiation is expected to be strongly polarized, with the first optical polarization observed in the Crab Nebula by Vashakidze (1954) and Dombrovsky (1954). Synchrotron

emission, due to highly relativistic electrons directs emission patterns to highly collimated beams, is illustrated in Fig. 2.7. The radiation from a single electron is generally elliptically polarized with the sense of the polarization (right or left handed) determined by the observer's LOS. Therefore, when a distribution of electrons that varies smoothly with pitch angle  $\alpha$  is considered, the emission cones will contribute equally from both sides of the LOS, which results in linearly polarized radiation. Synchrotron radiation is thus said to be partially polarized, fully described by the power components

$$\begin{aligned} P_{\parallel} &= \frac{\sqrt{3}q^3 B \sin \alpha}{4\pi m_e c^2} [F(x) - G(x)] \\ P_{\perp} &= \frac{\sqrt{3}q^3 B \sin \alpha}{4\pi m_e c^2} [F(x) + G(x)] \end{aligned} \quad (2.16)$$

for a homogeneous, ordered magnetic field, where

$$F(x) = x \int_x^{\infty} K_{\frac{5}{3}}(\xi) d\xi \quad \text{and} \quad G(x) = x K_{\frac{2}{3}}(x). \quad (2.17)$$

Here,  $K_{\frac{5}{3}}$  and  $K_{\frac{2}{3}}$  are the modified Bessel functions of second kind of order 5/3 and 2/3 respectively; the angular frequency of the radiation is indicated with  $\omega$ , and

$$\omega_c = \frac{3qB}{2\pi mc} \gamma^2 \sin \alpha \quad (2.18)$$

is the critical frequency, with  $x = \frac{\omega}{\omega_c}$  and  $\xi = \omega_c t$  (Rybicki and Lightman, 2008).

The spectral power for a single particle  $F(x)$  is shown in Fig. 2.8, with asymptotes

$$F(x) \propto \begin{cases} x^{1/3} & \text{for } x \ll 1 \\ x^{1/2} & \text{for } x \gg 1. \end{cases} \quad (2.19)$$

The angle-averaged radiative synchrotron output can be evaluated - for a collection of particles with isotropic pitch angle distributions - by integrating

$$P_v^{sy}(\gamma) = \frac{1}{4\pi} \int_{4\pi} P_v^{sy}(\gamma) d\Omega \quad (2.20)$$

which yields

$$P_v^{sy}(\gamma) = \frac{\sqrt{3}q^3 B}{2m_e c^2} x C S(x) \quad (2.21)$$

with the function  $CS(x)$  given in terms of *Whittaker functions* (Crusius and Schlickeiser, 1986; Whittaker, 1903):

$$CS(x) = W_{0, \frac{3}{4}}(x)W_{0, \frac{1}{3}}(x) - W_{\frac{1}{2}, \frac{5}{6}}(x)W_{-\frac{1}{2}, \frac{5}{6}}(x). \quad (2.22)$$

The synchrotron PD, for linear polarization, is given by

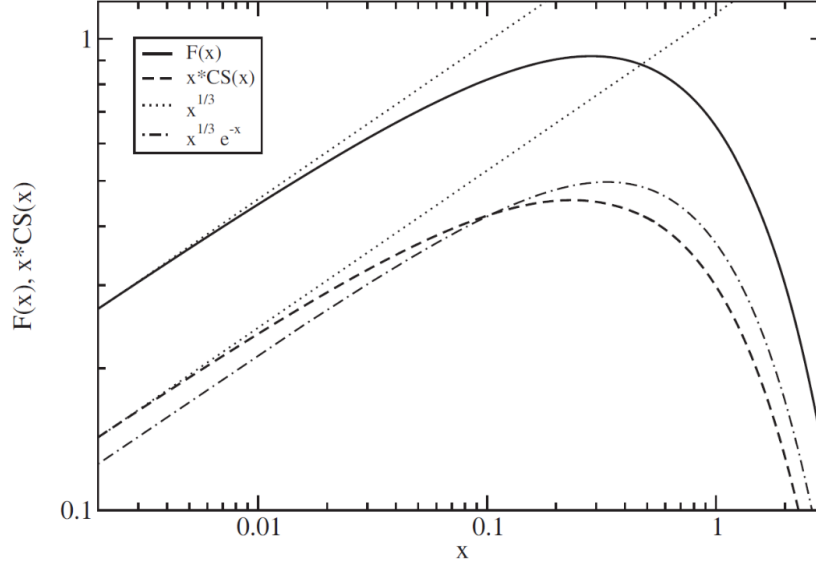


Fig. 2.8 The synchrotron power  $F(x)$  for a single particle, as described by  $F(x)$  and  $xCS(x)$  as a function of  $x = \frac{\omega}{\omega_c}$ , with  $\omega$  the angular frequency of the radiation. The low frequency asymptote is described by  $x^{1/3}$  with a  $x^{1/2}e^{-x}$  approximation for higher frequencies. From Böttcher et al. (2012).

$$PD^{sy} = \frac{P_{\perp} - P_{\parallel}}{P_{\perp} + P_{\parallel}} = F_B \cdot \frac{G(x)}{F(x)} \quad (2.23)$$

where  $F_B$  is a dimensionless quantity that indicates the order of the magnetic field. The maximum PD for a power-law distribution of electrons is given by

$$\begin{aligned} PD_{max, PL}^{sy} &= \frac{p+1}{p+\frac{7}{3}} \\ &= \frac{\alpha_{sy}+1}{\alpha_{sy}+\frac{5}{3}} \end{aligned} \quad (2.24)$$

with  $p$  the power-law index of the electron distribution, and  $\alpha_{sy} = \frac{p-1}{2}$  is the power-law index of the photon spectrum for  $F_{\nu} \propto \nu^{-\alpha_{sy}}$  (Rybicki and Lightman, 2008). The linear PD of synchrotron radiation will be a fraction of the total emission contributed by all the components

of the EM radiation observed. The total PD,

$$\text{PD}_{tot}^{sy} = \frac{\text{PD}^{sy} \cdot F_v^{sy}}{F_{total}} \quad (2.25)$$

will thus be a fraction of the synchrotron polarization, where  $F_v^{sy}$  is the synchrotron flux of the source. While the PD for synchrotron radiation is independent of the photon energy (for a straight power-law), the PD in the case of [inverse] Compton scattering is energy dependent. A discussion on the Compton polarization in the high-energy regime of astrophysical sources follows.

### 2.2.2 Compton Polarization

The *Compton effect* is an inelastic scattering between a photon and an electron (or any other charged particle) and was discovered by [Compton \(1923\)](#), earning him a Nobel prize in 1927. The Compton effect demonstrates that light cannot be treated as a wave when the target photon-energy in the electron-rest frame  $\epsilon_e = h\nu_e$  is comparable to the rest-energy of the electron  $m_e c^2$  (i.e., in the high-energy regime). Due to conservation of mass-energy and momentum, part of the target photon's energy is transferred to the electron, which recoils, and causes the scattered photon to be emitted in a different direction with a different frequency. In this section, the polarization signatures due to Compton scattering in the high-energy regime of astrophysical sources are discussed. Since the radiation is treated as a collection of particles, rather than an EM wave, only linear polarization will be considered. Furthermore, the emission regions for the jet-like sources considered are optically thin throughout most of the EM spectrum where the Compton scattering optical depth is typically very small (i.e.,  $\tau_C = nR\sigma_T \lesssim 10^{-5}$ ). Therefore, multiple scatterings of any individual photon are very unlikely ([Böttcher et al., 2012](#)), and the radiative diffusion aspects in relation to multiple Compton scatterings will not be considered.

The geometry of the Compton effect in the electron-rest frame is illustrated in [Fig.2.9](#). The Compton scattering between an electron and a photon is determined by the total Compton cross-section, which is sometimes referred to as the *Klein-Nishina* cross-section. The most convenient frame in which to evaluate the Klein-Nishina cross-section is the rest frame of the electron before the scattering event. The quantities in the electron rest frame are denoted by a subscript  $e$  and quantities after scattering are denoted by a superscript  $sc$ . The polarization

averaged differential Klein-Nishina cross-section is given by

$$\frac{d\sigma_{KN}}{d\Omega_e^{sc} dx_e^{sc}} = \frac{r_e^2}{2} \left( \frac{x_e^{sc}}{x_e} \right)^2 \left( \frac{x_e^{sc}}{x_e} + \frac{x_e}{x_e^{sc}} - \sin^2 \Theta_{sc} \right) \delta \left( x_e^{sc} - \frac{x_e}{1 + x_e[1 - \cos \Theta_{sc}]} \right) \quad (2.26)$$

where the  $\delta$  function results from the energy and momentum conservation. In Equation 2.26,  $x_e = \frac{\epsilon_e}{m_e c^2}$  is the dimensionless target photon energy,  $r_0$  is the classical electron radius,  $\Theta_e^{sc}$  is the angle between the target and the scattered photon (i.e., the polar scattering angle); and  $d\Omega_e^{sc} = d \cos \Theta_e^{sc} d\Phi_e^{sc}$  is a solid angle element of the scattered photon's direction of motion. The total scattering cross-section is obtained by integrating Equation 2.26 over all scattered photon energies  $x_e^{sc}$ , and the directions which correspond to the solid angles  $\Omega_e^{sc}$ ,

$$\sigma_{KN} = \sigma_T \frac{3}{4} \left\{ \frac{1 + x_e}{x_e^3} \left[ \frac{2x_e(1 + x_e)}{1 + 2x_e} - \ln(1 + 2x_e) \right] + \frac{1}{2x_e} \ln(1 + 2x_e) - \frac{1 + 3x_e}{(1 + 2x_e)^2} \right\} \quad (2.27)$$

where  $\sigma_T$  is the Thomson cross-section, defined in Equation 2.15.

Fig. 2.10 illustrates the dependence of the scattering cross-section on the dimensionless target photon-energy  $x_e$  in the electron-rest frame. In the Thomson regime ( $x_e \ll 1$ ), the cross-section assumes a constant value at the Thomson cross-section  $\sigma_T$ , and the recoil effect of the electron can be assumed to be negligible. The energy-exchange in this regime is therefore assumed to be elastic with  $x_e^{sc} \sim x_e$ . However, when  $x_e \ll 1$  (i.e., the Klein-Nishina regime), the energy exchange from the photon to the electron,

$$\epsilon_e^{sc} = \frac{\epsilon_e}{1 + x_e(1 - \cos \Theta_e^{sc})} \quad (2.28)$$

becomes substantial, reducing the scattering cross-section (Böttcher et al., 2012).

The scattering cross-section is generally dependent on the polarization through the angular distribution of the photons. The most familiar form of the polarization dependent cross-section is given by

$$\frac{d\sigma_{KN}}{d\Omega_e^{sc}} = \frac{1}{4} r_0^2 \left( \frac{x_e}{x_e^{sc}} \right)^2 \left[ \frac{x_e}{x_e^{sc}} + \frac{x_e^{sc}}{x_e} - 2 + 4 \cos^2 \theta_{pol} \right] \quad (2.29)$$

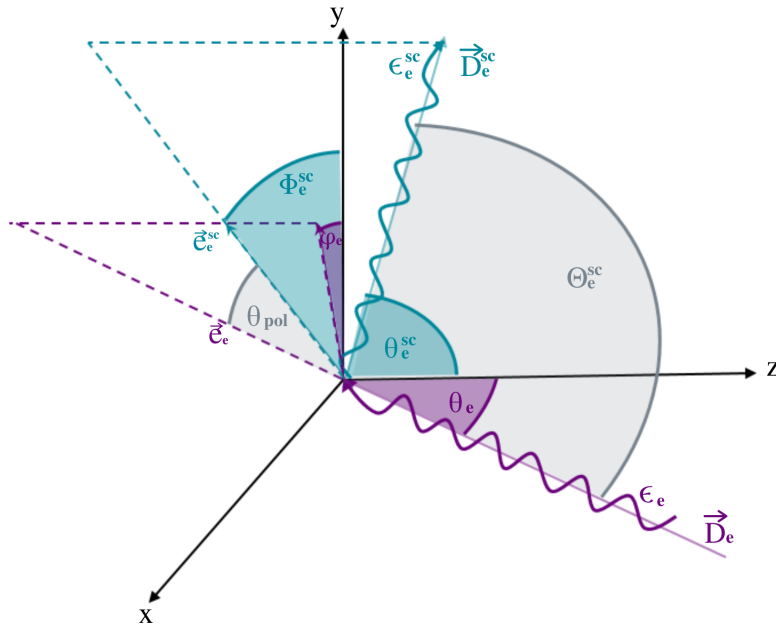


Fig. 2.9 The geometry of the Compton effect of a single photon in the electron-rest frame. The target photon is shown in purple with a direction  $\vec{D}_e$  and a polarization vector  $\vec{e}_e$ . The scattered photon is shown in blue with a direction  $\vec{D}_e^{sc}$  and a polarization vector  $\vec{e}_e^{sc}$ . The angle between the two polarization vectors  $\vec{e}_e$  and  $\vec{e}_e^{sc}$  is  $\theta_{pol}$ . The polar scattering angle  $\Theta_e^{sc}$  is the angle between the target and scattered photon, and the azimuthal scattering angle  $\Phi_e^{sc}$  is the angle between the polarization vector of the target photon and the plane of scattering.

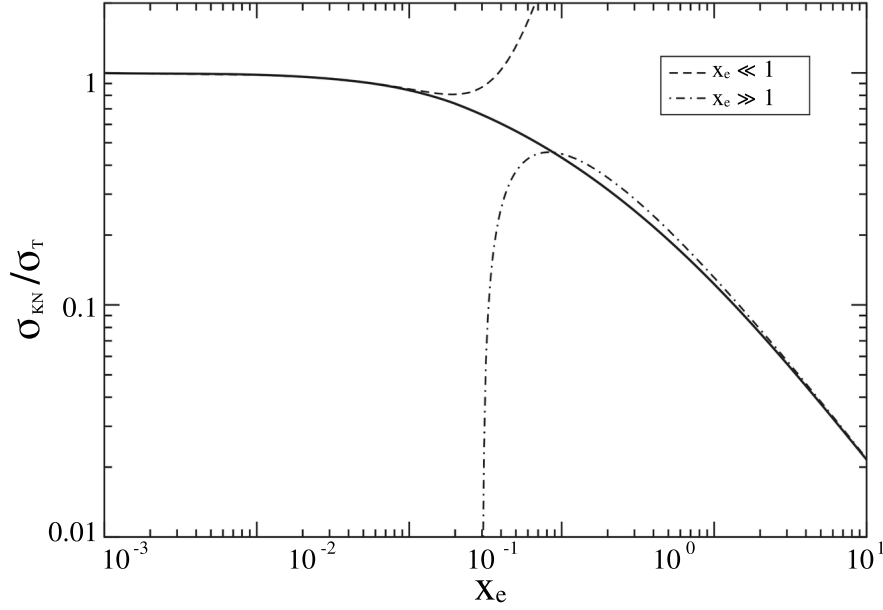


Fig. 2.10 The Compton cross-section as a function of the dimensionless target photon-energy  $x_e$  in the electron-rest frame. Also shown are the asymptotes for  $x_e \ll 1$  (dashed), and  $x_e \gg 1$  (dot-dashed). From [Böttcher et al. \(2012\)](#).

where  $\theta_{pol}$  is the angle between the polarization unit vector of the target photon  $\vec{e}_e$  and the scattered photon  $\vec{e}_e^{sc}$  (see Fig. 2.9). From Equation 2.29,

$$\sigma_{KN} \propto \frac{x_e}{x_e^{sc}} + \frac{x_e^{sc}}{x_e} - 2 + 4(\vec{e}_e \cdot \vec{e}_e^{sc}). \quad (2.30)$$

Therefore, polarization is expected to occur when photons are scattered in the Thomson regime (when  $x_e \ll 1$ ) with  $x_e \sim x_e^{sc}$  in the electron rest frame, since the polarization term in Equation 2.30 dominates so that  $\sigma_{KN} \propto 4(\vec{e}_e \cdot \vec{e}_e^{sc})$ . Contrarily, when  $x_e \gg 1$  in the Klein-Nishina regime, polarization is not expected to occur since the scattered photon energy  $x_e^{sc} \sim 1$ , and  $\sigma_{KN} \propto \frac{x_e}{1} + \frac{1}{x_e} - 2 + 4(\vec{e}_e \cdot \vec{e}_e^{sc})$ . Polarization can thus be induced when non-relativistic electrons scatter off an anisotropic radiation field, even if the target photons are unpolarized ([Bonometto and Saggion, 1973](#); [Celotti and Matt, 1994](#)). Compton scattering should then, in principle, include the polarization dependence of the angular distribution of the photons. If the target photons are unpolarized, the scattering process may produce linearly polarized photons, which produces an anisotropic azimuthal distribution of the scattered photons ([Matt et al., 1996](#)).

When photons undergo Compton scattering, the Stokes parameters of the target photon beam are transformed into a new set of parameters (see Equation 2.12). The relation between the two

Stokes vectors is given by a transformation matrix  $T$

$$T = \frac{1}{2}r_0^2 \left( \frac{x_e}{x_e^{sc}} \right)^2 \begin{bmatrix} T_{11} & T_{12} & 0 & T_{14} \\ T_{21} & T_{22} & 0 & T_{24} \\ 0 & 0 & T_{33} & T_{34} \\ T_{41} & T_{42} & T_{43} & T_{44} \end{bmatrix} \quad (2.31)$$

that characterize the Compton scattering event, where

$$\begin{aligned} T_{11} &= 1 + \cos^2 \Theta_e^{sc} + (x_e - x_e^{sc})(1 - \cos \Theta_e^{sc}) \\ T_{12} &= \sin^2 \Theta_e^{sc} \\ T_{14} &= -(1 - \cos \Theta_e^{sc}) \left( x_e \cos \Theta_e^{sc} + \vec{D}_e^{sc} \right) \cdot \vec{S} \\ T_{21} &= \sin^2 \Theta_e^{sc} \\ T_{22} &= 1 + \cos^2 \Theta_e^{sc} \\ T_{24} &= (1 - \cos \Theta_e^{sc}) (\vec{n}_e^{sc} \times \vec{n}_e) \cdot (\vec{D}_e \times \vec{S}) \\ T_{33} &= 2 \cos \Theta_e^{sc} \\ T_{34} &= (1 - \cos \Theta_e^{sc}) (\vec{D}_e \times \vec{n}_e^{sc}) \cdot \vec{S} \\ T_{41} &= -(1 - \cos \Theta_e^{sc}) (\vec{D}_e^{sc} \cos \Theta_e^{sc} + \vec{D}_e) \cdot \vec{S} \\ T_{42} &= (1 - \cos \Theta_e^{sc}) (\vec{n}_e \times \vec{n}_e^{sc}) \cdot (\vec{D}_e^{sc} \times \vec{S}) \\ T_{43} &= (1 - \cos \Theta_e^{sc}) (\vec{D}_e^{sc} \times \vec{n}_e^{sc}) \cdot \vec{S} \\ T_{44} &= 2 \cos \Theta_e^{sc} + (x_e - x_e^{sc})(1 - \cos \Theta_e^{sc}) \cos \Theta_e^{sc}. \end{aligned} \quad (2.32)$$

Here,  $\vec{n}_e$  and  $\vec{n}_e^{sc}$  are the unit vectors in the direction of the target photons  $\vec{D}_e$  and scattered photons  $\vec{D}_e^{sc}$ . Additionally,  $\vec{S}$  is the spin of the electron which occurs only in the fourth row and column; this means that the interaction between photons and electron spin occurs only with circularly polarized components of the target photon beam (McMaster, 1961). Since we are only considering linear polarization, we use the upper left  $3 \times 3$  submatrix of Equation 2.31. The transformation used for linear polarization due to Compton scattering is therefore

$$T_3 = \frac{1}{2}r_0^2 \left( \frac{x_e}{x_e^{sc}} \right)^2 \begin{bmatrix} \left( \frac{x_e}{x_e^{sc}} \right) + \left( \frac{x_e^{sc}}{x_e} \right) - \sin^2 \Theta_e^{sc} & \sin^2 \Theta_e^{sc} & 0 \\ \sin^2 \Theta_e^{sc} & 2 - \sin^2 \Theta_e^{sc} & 0 \\ 0 & 0 & 2 \cos \Theta_e^{sc} \end{bmatrix} \quad (2.33)$$

with

$$x_e - x_e^{sc} = \frac{(x_e)^2(1 - \cos \Theta_e^{sc})}{1 + x_e(1 - \cos \Theta_e^{sc})} \quad (2.34)$$

and

$$\frac{x_e}{x_e^{sc}} = (1 + x_e(1 - \cos \Theta_e^{sc}))^2 \quad (2.35)$$

from the equations governing the Compton effect. The target photon beam may be unpolarized, 100% polarized, or partially polarized. A partially polarized beam may, however, be represented by the linear superposition of an unpolarized and a 100% polarized beam, and it is sufficient to consider these two extreme cases (Matt et al., 1996).

### 1. Compton scattering of unpolarized target photons

The characterization of an unpolarized beam by the Stokes parameters is

$$\begin{bmatrix} I \\ Q \\ U \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad (2.36)$$

and undergoes a transformation after Compton scattering into

$$\begin{bmatrix} I^{sc} \\ Q^{sc} \\ U^{sc} \end{bmatrix} = T_3 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \sim \begin{bmatrix} \left(\frac{x_e}{x_e^{sc}}\right) + \left(\frac{x_e^{sc}}{x_e}\right) - \sin^2 \Theta_e^{sc} \\ \sin^2 \Theta_e^{sc} \\ 0 \end{bmatrix}. \quad (2.37)$$

The degree of polarization of an unpolarized beam is thus

$$\begin{aligned} \text{PD}_U^{sc} &= \frac{\sqrt{(Q^{sc})^2 + (U^{sc})^2}}{I^{sc}} \\ &= \frac{\sin^2 \Theta_e^{sc}}{\left(\frac{x_e}{x_e^{sc}}\right) + \left(\frac{x_e^{sc}}{x_e}\right) - \sin^2 \Theta_e^{sc}}. \end{aligned} \quad (2.38)$$

The ratio of intensities with opposite polarizations in the scattered beam is defined as

$$p_U^{sc} = \frac{d\sigma_{\perp}/d\Omega_e^{sc}}{d\sigma_{\parallel}/d\Omega_e^{sc}} \quad (2.39)$$

with

$$\frac{d\sigma_{\perp}}{d\Omega_e^{sc}} = \frac{1}{2} \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} T_3 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \frac{1}{4} r_0^2 \left( \frac{x_e}{x_e^{sc}} \right)^2 \left[ \frac{x_e}{x_e^{sc}} + \frac{x_e^{sc}}{x_e} \right] \quad (2.40)$$

and

$$\frac{d\sigma_{\parallel}}{d\Omega_e^{sc}} = \frac{1}{2} \begin{bmatrix} 1 & -1 & 0 \end{bmatrix} T_3 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \frac{1}{4} r_0^2 \left( \frac{x_e}{x_e^{sc}} \right)^2 \left[ \frac{x_e}{x_e^{sc}} + \frac{x_e^{sc}}{x_e} - 2 \sin^2 \Theta_e^{sc} \right] \quad (2.41)$$

where  $\perp$  and  $\parallel$  refer to plane polarization perpendicular and parallel to the plane of scattering, respectively (see Fig. 2.9). Therefore,

$$p_U^{sc} = \frac{[(x_e - x_e^{sc})(1 - \cos \Theta_e^{sc}) + 2]}{[(x_e - x_e^{sc})(1 - \cos \Theta_e^{sc}) + 2 \cos^2 \Theta_e^{sc}]}. \quad (2.42)$$

When  $p_u^{sc} = 1$  (i.e., when  $\Theta_e^{sc} = 0$  or  $\Theta_e^{sc} = \pi$ ), then the scattered beam is unpolarized. The scattering cross-section, in the case of unpolarized target photons, is given by the sum of  $\frac{d\sigma_{\perp}}{d\Omega_e^{sc}}$  and  $\frac{d\sigma_{\parallel}}{d\Omega_e^{sc}}$ , or

$$\frac{d\sigma_{KN,U}}{d\Omega_e^{sc}} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} T \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \frac{1}{2} r_0^2 \left( \frac{x_e}{x_e^{sc}} \right)^2 \left[ \frac{x_e}{x_e^{sc}} + \frac{x_e^{sc}}{x_e} - \sin^2 \Theta_e^{sc} \right]. \quad (2.43)$$

## 2. Compton scattering of polarized target photons

Positive values of the Stokes parameter  $Q$  refer to linear polarization perpendicular to the plane of scattering (along  $\vec{e}_{\perp}$ ), and the Stokes representation of a linearly polarized target photon is given by  $(I, Q, U) = (1, 1, 0)$  in a coordinate system that is rotated through  $\Phi_e^{sc}$  to the right of  $\vec{e}_{\perp}$ . Therefore, using the transformation matrix for Compton scattering given by Equation 2.12, the coordinate system needs to be rotated by  $\Phi_e^{sc}$  to the left using the matrix  $M$  (Equation 2.10) with the appropriate change of sign with the plane of scattering as a reference plane,

$$\begin{bmatrix} I' \\ Q' \\ U' \end{bmatrix} = M \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ \cos 2\Phi_e^{sc} \\ -\sin 2\Phi_e^{sc} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 - 2 \sin^2 \Phi_e^{sc} \\ -2 \sin \Phi_e^{sc} \cos \Phi_e^{sc} \end{bmatrix} \quad (2.44)$$

undergoing the transformation

$$\begin{bmatrix} I^{sc} \\ Q^{sc} \\ U^{sc} \end{bmatrix} = T_3 \begin{bmatrix} I' \\ Q' \\ U' \end{bmatrix} = \begin{bmatrix} \frac{x_e}{x_e^{sc}} + \frac{x_e^{sc}}{x_e} - 2 \sin^2 \Theta_e^{sc} \sin^2 \Phi_e^{sc} \\ 2 - 2 \sin^2 \Theta_e^{sc} \sin^2 \Phi_e^{sc} \\ -4 \cos \Theta_e^{sc} \sin \Phi_e^{sc} \cos \Phi_e^{sc} \end{bmatrix} \quad (2.45)$$

after the scattering event. The differential cross-section of scattering for a completely polarized target photon field is thus

$$\frac{d\sigma_{KN,P}}{d\Omega_e^{sc}} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} T_3 \begin{bmatrix} I' \\ Q' \\ U' \end{bmatrix} = \frac{1}{2} r_0^2 \left( \frac{x_e}{x_e^{sc}} \right)^2 \left[ \frac{x_e}{x_e^{sc}} + \frac{x_e^{sc}}{x_e} - \sin^2 \Theta_e^{sc} \sin^2 \Phi_e^{sc} \right]. \quad (2.46)$$

The probability for polarization orthogonal to the plane of scattering is

$$\frac{d\sigma_{\perp}}{d\Omega_e^{sc}} = \frac{1}{2} \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} T_3 \begin{bmatrix} I' \\ Q' \\ U' \end{bmatrix} = \frac{1}{4} r_0^2 \left( \frac{x_e}{x_e^{sc}} \right)^2 \left[ \frac{x_e}{x_e^{sc}} + \frac{x_e^{sc}}{x_e} + 2 + 4 \sin^2 \Phi_e^{sc} \right] \quad (2.47)$$

while the probability for polarization parallel to the plane of scattering is

$$\frac{d\sigma_{\parallel}}{d\Omega_e^{sc}} = \frac{1}{2} \begin{bmatrix} 1 & -1 & 0 \end{bmatrix} T_3 \begin{bmatrix} I' \\ Q' \\ U' \end{bmatrix} = \frac{1}{4} r_0^2 \left( \frac{x_e}{x_e^{sc}} \right)^2 \left[ \frac{x_e}{x_e^{sc}} + \frac{x_e^{sc}}{x_e} - 2 + 4 \cos^2 \Theta_e^{sc} \sin^2 \Phi_e^{sc} \right]. \quad (2.48)$$

The polarized target photons are therefore, from Equation 2.46, preferentially scattered in a plane perpendicular to the electric field vector, while the PD due to scattering is

$$PD_P^{sc} = \frac{\sqrt{(Q^{sc})^2 + (U^{sc})^2}}{I^{sc}} = 2 \frac{1 - \sin^2 \Theta_e^{sc} \cos^2 \Phi_e^{sc}}{\frac{\epsilon_e^{sc}}{\epsilon_e} + \frac{\epsilon_e}{\epsilon_e^{sc}} - 2 \sin^2 \Theta_e^{sc} \cos^2 \Phi_e^{sc}} \quad (2.49)$$

with  $PD_P^{sc} \sim 1$  in the Thomson regime ( $x_e \ll 1$ ). Therefore, when the target photons are completely polarized, the scattered photons will also be completely polarized. The  $PD_P^{sc}$  fraction of the scattered photons are thus polarized, with a polarization vector

$$\vec{P}_e^{sc} = \frac{1}{|\vec{P}_e^{sc}|} \left( \vec{P}_e \times \vec{D}_e^{sc} \right) \times \vec{D}_e^{sc} \quad (2.50)$$

where  $\vec{D}_e$  and  $\vec{D}_e^{sc}$  are the directions of the target and the scattered photons, respectively. The remaining  $1 - \text{PD}_p^{sc}$  photons are unpolarized with  $\vec{P}_e^{sc}$  randomly distributed in the plane normal to  $\vec{D}_e^{sc}$  (Matt et al., 1996).

In this dissertation, Monte Carlo methods are used to simulate anisotropic Compton scattering off thermal electrons, and a power-law tail of non-thermal electrons, in order to study the polarization signatures due to scattering. In Chapter 3, the polarized-dependent treatment of Compton scattering in Monte Carlo codes will be discussed.

## 2.3 Detection of Polarization in the High Energy Regime

Polarimetry is an observatory tool that is less advanced in terms of X-rays and  $\gamma$ -rays, compared to radio to optical emission; partly due to the difficulty in the detection of the polarization in the high energy regime (McConnell and Ryan, 2004). As discussed in Chapter 1, the measurement of linear polarization from high-energy radiation in astrophysical sources holds the promise to provide critical information about the innermost regions of the most powerful objects in the universe, and recent developments of new technologies will allow for the detection of high-energy polarization from many astrophysical sources. Several X-ray and soft  $\gamma$ -ray polarimeters are currently at various stages of planning, design, and operation - justifying the new interest in model predictions reported in the previous chapter. In this section, a basic overview of polarimetry will be given, in particular, the techniques used in the high-energy regime and the treatment of the relation between the typical high-energy formalism and Stokes parameters.

### 2.3.1 Polarimetry Basics and Techniques

The current technology available for high-energy polarimetry only considers linear polarization. A polarimeter is a detector that analyses different angular directions of radiation, and detects photons with respect to these directions. Fig. 2.11 illustrates the response of a polarimeter: If the radiation is not polarized, every angular direction  $\Phi$  has the same probability and the same number of photons is detected as a function of  $\Phi$ . The polarimeter's response is, therefore, flat (left-hand side of Fig. 2.11) for unpolarized radiation. If the radiation is polarized, a *modulation* arises (right-hand side of Fig. 2.11), and all values of  $\Phi$  will not have the same probability. The value of the modulation depends on the dipole interaction of the photon and the interacting electron of an atom in the sensitive volume of the detector. The *modulation factor* can be

defined as

$$N(\Phi) = A_p + B_p \cos^2(\Phi - \Phi_0) \quad (2.51)$$

with  $A$  the *flat* term, and  $B$  the *modulated* term (Fabiani, 2018). The *modulation fraction* is the amplitude of the azimuthal modulation measured for 100% polarized emission,

$$\mu = \frac{N_{100\%}^{max} - N_{100\%}^{min}}{N_{100\%}^{max} + N_{100\%}^{min}} = \frac{B_{100\%}}{2A_{100\%} + B_{100\%}} \quad (2.52)$$

and depends on the physics of the interaction and the properties of the polarimeter (Fabiani, 2018; Kislak et al., 2015). In equation 2.52,  $N_{100\%}^{max}$  and  $N_{100\%}^{min}$  is the maximum and minimum number of photons detected in the angular bins of the modulation histogram (for fully polarized radiation). The modulation term needed in order to derive a PD for radiation with an unknown polarization can be found from Equation 2.51, with

$$PD = \frac{1}{\mu} \frac{B_p}{2A_p + B_p} \quad (2.53)$$

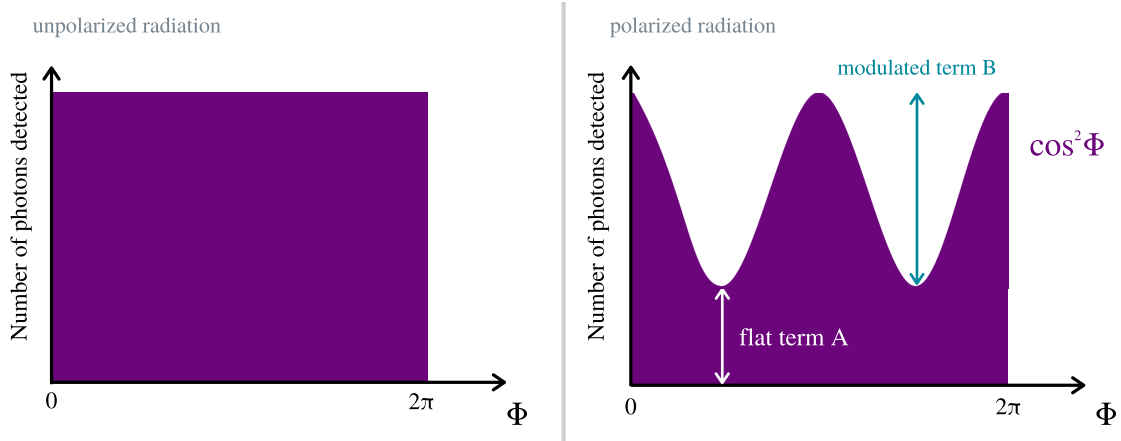


Fig. 2.11 An illustration of the general response of a polarimeter: For unpolarized radiation, every photon's angular direction  $\Phi$  has the same probability. The same number of photons as a function of  $\Phi$  is detected, and the detector response is flat (left-hand side). If the radiation is polarized, a modulation of  $\cos^2 \Phi$  arises (right-hand side). Adapted from Fabiani (2018).

The performance for different polarimeters is commonly characterized by the *Minimum Detectable Polarization* (MDP), defined as the minimum polarization that can be detected at a

confidence level of 99%,

$$\text{MDP}(99\%) = \frac{4.29}{\mu r} \sqrt{\frac{r+b}{t}} \quad (2.54)$$

where  $r$  is the photon count,  $b$  the background, and  $t$  is the observation time. When a signal's polarization is lower than the MDP, the signal is compatible with a statistical fluctuation and no positive detection of polarization can be claimed. The quality factor  $Q = \mu\sqrt{\epsilon}$  is a useful parameter to compare the sensitivity of different polarimeters, which can be derived from the MDP with

$$\text{MDP}(99\%) \propto \frac{1}{\mu\sqrt{\epsilon}\sqrt{F}} = \frac{1}{Q\sqrt{F}} \quad (2.55)$$

where  $\epsilon$  is the detector quantum efficiency and  $F$  is the source flux.

### 1. Polarimetry techniques and experiments

The physical processes present within the high-energy regime of EM radiation can be exploited to obtain polarization sensitivity. Polarimeters based on Bragg diffraction (i.e. diffraction on crystals, and more recently, the diffraction on multi-layer mirrors) are used below 1 keV. The photoelectric effect and Thomson scattering can be used at energies up to 100 keV, and Compton scattering in the 100 keV range.

#### (a) Bragg diffraction polarimeters

In a crystal with a lattice spacing  $d$ , diffraction may occur for an energy of  $E$  at an angle  $\theta$ , if the *Bragg formula*  $E = nhc/2d \sin \theta$  (where  $n$  is the diffraction order,  $h$  Planck's constant, and  $c$  the speed of light) is satisfied. Since the formula is only valid for a very narrow energy band, mosaic or bent crystals are commonly constructed to increase the average range of accepted angles. The incoming radiation consists of two components which are parallel and perpendicular to the incidence focal plane. If the diffraction angle is  $\pi/4$  rad, then only the component perpendicular to the incident focal plane survives and the transmitted beam is 100% polarized. By rotating the crystal around the beam axis, out-coming flux is modulated, since the modulation period is twice the rotation period. The technique is inefficient to measure the polarization of a continuous spectrum, since  $d$  and  $\theta$  has to be chosen to be as close to  $\pi/4$  rad as possible in order for the Bragg formula to be valid. Nevertheless, Bragg diffraction allows for the analysis of photons at energies  $\lesssim 1$  keV, and a large technological effort is focused on

developing multi-layer mirrors to exploit the Bragg diffraction (e.g. [Panini et al. \(2017\)](#)). The relevant future prospects for Bragg polarimeters are stipulated in Tab. 2.1, and include the Lightweight Asymmetry and Magnetism Probe (LAMP) ([She et al., 2015](#)), and the Rocket Experiment Demonstration of a Soft X-Ray Polarimeter (REDSOX) ([Marshall et al., 2017](#)).

#### (b) *Photoelectric effect polarimeters*

When a photon is absorbed via the photoelectric effect by an atom, a photo-electron is emitted. Photoelectric effect polarimeters track the direction of the photo-electrons which are preferentially emitted parallel to the electric field of the incoming photons. A gas detector is suitable to perform polarimetry by exploiting the photoelectric effect, since the photo-electron track-length is in the millimeter range in gas for absorbed photons with energies of 1 keV up to some tens of keV. Planned future photoelectric X-ray polarimetric missions (Tab. 2.1) include the Enhanced X-ray Timing and Polarimetry Mission (eXTP) and the Imaging X-ray Polarimetry Explorer (IXPE) ([Weisskopf, 2018](#)).

#### (c) *Scattering polarimeters*

The scattering of a photon by a free electron is described in section 2.2.2, and is determined by the Klein-Nishina cross-section given in Equation 2.26. Scattering polarimeters measure the direction into which the photons scatter, using the fact that the photons scatter preferentially perpendicular to the electric field direction of the incoming beam ([Kaaret, 2014](#); [Kislat et al., 2015](#)). According to Equation 2.26, a perfect polarization analyzer will be described for scattering at an angle  $\Theta_e^{sc} = \pi/2$  rad, with  $\epsilon_e \sim \epsilon_e^{sc}$ , where  $\epsilon_e$  is the target photon energy and  $\epsilon_e^{sc}$  the scattered photon energy (the Thomson limit; see figure 2.10). There is a wide range of polarimeters based on Compton and Thomson scattering, and a few examples are listed in Tab. 2.1: X-ray Polarimeter Experiment (POLIX) ([Paul et al., 2016](#)), POLAR ([Produit et al., 2018](#)), Polarised Gamma-Ray Observer (PoGOLite, and PoGO+) ([Chauvin et al., 2016](#); [Friis et al., 2018](#)), X-Calibur ([Guo et al., 2013](#)), AstroSat CZTI ([Vadawale et al., 2015](#)), Gamma-Ray Burst Polarimeter (GAP) ([Yonetoku et al., 2011](#)), Gamma Ray Polarimeter Experiment (GRAPE) ([McConnell et al., 2013](#)), Polarimetry for High ENergy X-ray (PHENEX) ([Kishimoto et al., 2008](#)), PolariS ([Hayashida et al., 2016](#)), and the Segmented Polarimeter for High eNergy X-rays (SPHiNX) ([Xie and Pearce, 2018](#)).

Table 2.1 Examples of polarimetry experiments and missions.

| Polarimetry technique | Examples of current and future missions                                                                                | Energy range (keV)   | Science objectives and source examples                   |
|-----------------------|------------------------------------------------------------------------------------------------------------------------|----------------------|----------------------------------------------------------|
| Bragg diffraction     | Lightweight Asymmetry and Magnetism Probe (LAMP) <a href="#">[assessment]</a>                                          | 0.25                 | Pulsars (thermal emission), and jet blazars              |
|                       | Rocket Experiment Demonstration of a Soft X-Ray Polarimeter (REDSOX) <a href="#">[development]</a>                     | 0.2 - 0.8            | AGNs and binaries (jets and discs), isolated pulsars     |
| Photoelectric effect  | Imaging X-ray Photoelectric Polarimeter (IXPE) <a href="#">[launch 2021]</a>                                           | 2 - 10               | AGNs , black holes, isolated and accreting neutron stars |
|                       | Enhanced X-ray Timing and Polarimetry Mission (eXTP) <a href="#">[launch before 2025]</a>                              | 2 - 10               | AGNs , black holes, isolated and accreting neutron stars |
| Thomson scattering    | The X-ray Polarimeter (POLIX) <a href="#">[launch with the X-ray Polarimetry Satellite mission (XPoSat) in 2021]</a>   | 5 - 30               | Accreting powered pulsars, black holes                   |
| Compton scattering    | POLAR <a href="#">[launched 2016]</a>                                                                                  | 50 - 500             | GRBs                                                     |
|                       | <i>Polarised Gamma-Ray Observer</i> (PoGOLite) <a href="#">[launched 2013]</a> (PoGO+) <a href="#">[launched 2013]</a> | 20 - 240<br>20 - 150 | Crab emission<br>Crab pulsar and nebula                  |
|                       | X-Calibur <a href="#">[launched 2014 and 2016]</a>                                                                     | 20 - 60              | Black-holes, AGN jets                                    |
|                       | AstroSat CZTI imager <a href="#">[launched 2015]</a>                                                                   | 100 - 380            | Crab Nebula                                              |
|                       | Gamma-Ray Burst Polarimeter (GAP) <a href="#">[launched 2010]</a>                                                      | 50- 300              | Crab pulsar and nebula, GRBs                             |
|                       | Gamma Ray Polarimeter Experiment (GRAPE) <a href="#">[launched 2014 and 2016]</a>                                      | 50 - 500             | Transient sources                                        |
|                       | Polarimetry for High ENERgy X-ray (PHENEX) <a href="#">[launched 2006]</a>                                             | 40 - 200             | Crab Nebula                                              |
|                       | Polarimetry Satellite (PolariS) <a href="#">[assessment]</a>                                                           | 10 - 80              | Black holes, GRBs, pulsars                               |
|                       | Segmented Polarimeter for High eNergy X-rays (SPHiNX) <a href="#">[assessment]</a>                                     | 50 - 500             | GRBs                                                     |

### 2.3.2 Analyzing Polarimetry Data with Stokes Parameters

The formalism described in section 2.3.1 is compatible with the Stokes formalism described in section 2.1.1. The Stokes formalism is typically used for analyzing the data in energy regimes for which the intensity of the radiation is studied instead of single photon counting. However, the data from photoelectric effect and scattering polarimeters consists of a distribution of emission angles of photo-electrons, and scattering angles of photons, respectively. These angles can be supplied as a list of angles  $\{\psi_k\}$  ( $k = 1, 2, \dots, K$ ), for each event (denoted with a subscript  $k$ ), related to the most likely azimuthal angle  $\Phi_k$  of the electric field vector. The Stokes parameters can be derived from the angle distributions and may consequently be used to analyze the data from both types of polarimeters (Kislat et al., 2015; Strohmayer and Kallman, 2013).

In the case of photo-electric effect polarimeters,  $\Phi_k = \psi_k$ , while  $\Phi_k = \psi_k - \frac{\pi}{2}$  rad for scattering polarimeters. The azimuthal angles  $\Phi_k$  exhibit a sinusoidal modulation

$$f(\Phi) = \frac{1}{2\pi} [1 - PD_0 \mu \cos(2(\Phi - \Phi_0))] \quad (2.56)$$

with a period of  $\pi$  rad (Kislat et al., 2015). In Equation 2.56,  $PD_0$  is the true polarization fraction,  $\Phi_0$  is the expected direction of the  $\Phi$ -distribution peaks, and  $\mu$  is the modulation factor given in Equation 2.52. The set of Stokes parameters can be defined, for each event, with

$$\begin{aligned} I_k &= 1 \\ Q_k &= \cos 2\Phi_k \\ U_k &= \sin 2\Phi_k \end{aligned} \quad (2.57)$$

where  $V_{j,k}$  is undefined since the polarimeter only constrains linear polarization. The Stokes parameters are, for the whole data set of  $K$  events detected,

$$\begin{aligned} I &= \sum_{k=1}^K I_k = K \\ Q &= \sum_{k=1}^K Q_k \\ U &= \sum_{k=1}^K U_k \end{aligned} \quad (2.58)$$

due to the additive quality of the Stokes parameters. Using the normalized Stokes parameters,  $\mathcal{Q} = Q/I$  and  $\mathcal{U} = U/I$ , the expected values for  $Q$  and  $U$  are

$$\begin{aligned}\langle \mathcal{Q} \rangle &= \int_0^{2\pi} \cos 2\Phi f(\Phi) d\Phi = \frac{1}{2} \text{PD}_0 \mu \cos 2\Phi_0 \\ \langle \mathcal{U} \rangle &= \int_0^{2\pi} \sin 2\Phi f(\Phi) d\Phi = \frac{1}{2} \text{PD}_0 \mu \sin 2\Phi_0\end{aligned}\quad (2.59)$$

so that

$$\begin{aligned}\langle \mathcal{Q} \rangle^2 + \langle \mathcal{U} \rangle^2 &= \frac{\text{PD}_0 \mu^2}{4} \\ \frac{\langle \mathcal{U} \rangle}{\langle \mathcal{Q} \rangle} &= \tan 2\Phi_0.\end{aligned}\quad (2.60)$$

It follows from Equations 2.59 and 2.60 that the reconstructed polarization fraction  $\text{PD}_r$  and position angle  $\Phi_r$  (i.e., the nominal values of  $\text{PD}_0$  and  $\Phi_0$ ) can be obtained from the Stokes parameters with

$$\begin{aligned}\text{PD}_r &= \frac{1}{2\mu} \sqrt{\mathcal{Q}^2 + \mathcal{U}^2} \\ \Phi_r &= \frac{1}{2} \arctan \frac{\mathcal{U}}{\mathcal{Q}}\end{aligned}\quad (2.61)$$

where the  $\frac{1}{2\mu}$  in the first equation comes from the fact that the Stokes parameters are derived from the distribution of angles measured, and are, for this reason, influenced by  $\mu$  of the polarimeter: Since the measured angles follow the sinusoidal distribution with a  $\pi$  rad period (given in Equation 2.52), the Stokes parameters derived from the angles are reduced by a factor of  $\frac{1}{2}$  compared to the true Stokes parameters of the incoming photons. The Stokes parameters are further reduced by a factor  $\mu$  due to the modulation of the instrument.

#### (a) Significance of a non-zero polarization detection

The values of  $\mathcal{Q}$  and  $\mathcal{U}$  are uncorrelated for the null hypothesis of no polarization (i.e.,  $\text{PD}_0 = 0$ ). According to the central limit theorem,  $\mathcal{Q}$  and  $\mathcal{U}$  are thus normally distributed for  $K \gg 1$  with a mean value of 0 and Gaussian width of  $\sigma_{\mathcal{Q}} = \sigma_{\mathcal{U}} = \frac{1}{2K}$ . The probability density function for  $\mathcal{Q}$  and  $\mathcal{U}$  is thus

$$f(\mathcal{Q}, \mathcal{U}) d\mathcal{Q} d\mathcal{U} = \frac{K}{\pi} \exp\{-K(\mathcal{Q}^2 + \mathcal{U}^2)\} d\mathcal{Q} d\mathcal{U} \quad (2.62)$$

under the null hypothesis. Therefore, if the detection of  $K$  events results in a  $\text{PD}_r$  (Equation 2.61), the probability to find a larger apparent polarization is

$$P = e^{-\frac{N}{4}\mu^2\text{PD}_r^2} \quad (2.63)$$

obtained by integrating  $f(\mathcal{Q}, \mathcal{U})$  over all values of  $\mathcal{Q}^2 + \mathcal{U}^2$  larger than the ones observed.

The MDP given in Equation 2.54 can be obtained as a function of  $K$  and  $\mu$  by solving Equation 2.63 for  $\text{PD}_r$ :

$$\text{MDP} \sim \frac{4.29}{\mu\sqrt{K}} \quad (2.64)$$

requiring that  $P = 1\%$ . The  $\text{MDP} \propto \mu^{-1}$ , which emphasises the importance of optimizing the modulation factor in polarimeters (Kislat et al., 2015). Since the Stokes parameters are reduced by a factor of  $1/\mu$  (see Equation 2.61), then reconstructed Stokes parameters

$$\mathcal{Q}_r = \frac{2}{\mu}\mathcal{Q} \quad (2.65)$$

$$\mathcal{U}_r = \frac{2}{\mu}\mathcal{U} \quad (2.66)$$

should be used in order to compare the results from different experiments and theoretical predictions. Hence,  $\text{PD}_r = \sqrt{\mathcal{Q}_r^2 + \mathcal{U}_r^2}$ , and the standard deviations of the Stokes parameters are given by

$$\begin{aligned} \sigma(\mathcal{Q}_r) &= \frac{2}{\mu} \sqrt{\frac{1}{K-1} \left( \frac{1}{2} - \mathcal{Q}_r^2 \right)} \\ \sigma(\mathcal{U}_r) &= \frac{2}{\mu} \sqrt{\frac{1}{K-1} \left( \frac{1}{2} - \mathcal{U}_r^2 \right)} \end{aligned} \quad (2.67)$$

since the post-measurement probability distributions of both  $\mathcal{Q}_r$  and  $\mathcal{U}_r$  are Gaussian distributions (Kislat et al., 2015; Strohmayer and Kallman, 2013). Therefore, thanks to their additivity and well-behaved probability distributions, Stokes parameters are useful tools for the analysis of data from high-energy polarimeters. Furthermore, Stokes parameters can be used to describe the background events (if present), and derive the uncertainty on the PD and PA detected. The results can then be used to fit model predictions to the experimental data. A full analysis on using the Stokes formalism for the statistical analysis of high-energy polarimetry is provided by Strohmayer and Kallman (2013) and Kislat et al. (2015).

# Chapter 3

## Monte Carlo Simulations

Monte Carlo simulations are used to model the probability of different outcomes of a phenomenon, by means of computational algorithms (i.e., the Monte Carlo *methods*) that rely on the randomness and repetitive nature of a process to produce numerical results. Monte Carlo methods were conceptualized by [Metropolis and Ulam \(1949\)](#). Ulam later collaborated with John von Neumann to run the Monte Carlo simulations required for the Manhattan project (a research and development project during World War II that produced the first nuclear weapons). This chapter supplies a brief overview of the Monte Carlo methods that are essential to this dissertation, followed by a discussion of how the Monte Carlo approach can be used to simulate Comptonization in astrophysics. The chapter is concluded by a formal description of the Monte Carlo code developed in this dissertation.

### 3.1 Monte Carlo Methods

There is a broad range of Monte Carlo methods, and they all share one trait: They mainly rely on the random number generation to solve deterministic problems. The general problem in Monte Carlo codes is to obtain the relation between random numbers and the quantities to be determined. In probability theory, the *Probability Density Function* (PDF) is used to describe the relative likelihood for a random variable to take on a specified value. The *Cumulative Distribution Function* (CDF) describes the probability that a random variable  $x$ , with a given probability distribution, will be found at a value less than or equal to another variable  $a$ ,

$$P(x \leq a) \equiv F(a) = \int_{-\infty}^a f(r) dr \quad (3.1)$$

where  $f(r)$  is the PDF of the physical quantity  $r$ . The probability of extracting the variable  $r$  within an interval  $r \in [r_{min}, r_{max}]$  is thus given by the CDF

$$P(r \leq r_0) \equiv F(r_0) = \frac{\int_{r_{min}}^{r_0} f(r') dr'}{\int_{r_{min}}^{r_{max}} f(r') dr'} \quad (3.2)$$

with the denominator being the normalization factor assuring that the total cumulative probability is equal to one.

Fig. 3.1 illustrates how a value  $r$ , that is distributed according to the PDF  $f(r)$ , may be extracted by means of the *inverse transform method* and *interpolation*. Suppose a uniformly distributed random variable  $\xi \in [0, 1]$  is generated. A unique value of  $r$  can be chosen from the PDF  $f(r)$  for a given value of  $\xi$  with the inversion transform method (illustrated in the top panel of Fig. 3.1) by setting  $\xi = F(r)$  (provided that the CDF is *invertible*), so that  $r = F^{-1}(\xi)$ . In the case where the CDF  $F(r)$  is a discrete distribution,  $F(r)$  will have a discontinuous step size  $f(r_k)$  at each allowed value of  $r_k$  ( $k = 1, 2, \dots$ ). The value  $r$  can then be chosen by interpolation (illustrated in the bottom panel of Fig. 3.1) where grid of values  $f(r_k)$  is built. A uniformly distributed random variable  $\xi \in [0, 1]$  is associated with the closest value of  $f(r_k)$  for which  $F(r_{k-1}) \leq \xi \leq F(r_k)$ , so that  $r = r_k$ .

When the PDF  $f(r)$  is not integrable, the so called *rejection technique* is often used in order to find  $r \in [r_{min}, r_{max}]$ , illustrated in Fig. 3.2: An integrable PDF is chosen within the limits  $r_{min}$  and  $r_{max}$  with a peak  $f_{max}$  that corresponds to the maximum value of  $f(r)$ . A random variable  $\xi$  is extracted between  $r_{min}$  and  $r_{max}$ , so that  $\xi = r_{min} + \xi_1(r_{max} - r_{min})$ , with the PDF  $f(\xi)$ , where  $\xi_1 \in [0, 1]$  is a uniformly distributed. Another random number  $\xi_2 \in [0, 1]$  is drawn, and a random probability  $f_\xi = \xi_2 f_{max}$  is calculated. If  $f_\xi < f(\xi)$ , then the random variable  $\xi$  is accepted so that  $\xi = r$ . Otherwise,  $\xi$  is drawn again and the procedure is repeated. There are many more algorithms and numerical procedures that fall into the class of Monte Carlo methods (see e.g. Sobol (1994)). The methods described in this section will be used in the code developed in this project in order to simulate Comptonization, and study the Compton polarization in the high-energy regime of jet-like astrophysical sources. The application of the Monte Carlo approach in astrophysics will be discussed in the next section.

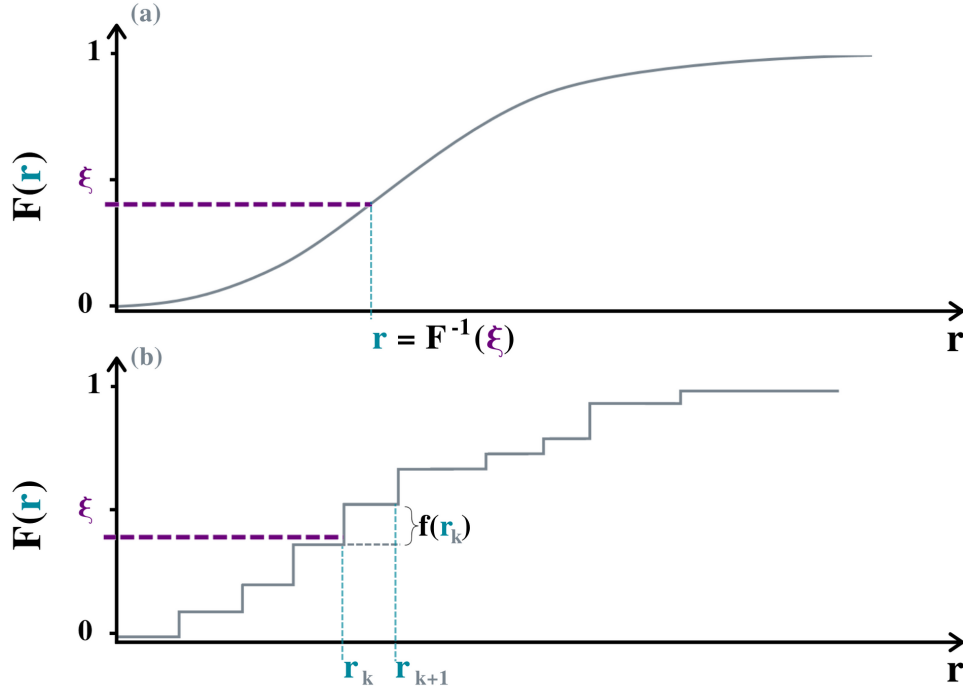


Fig. 3.1 An illustration of choosing a value  $r$  which is distributed according to the PDF  $f(r)$  by means of (a) the inverse transform method and (b) interpolation.

## 3.2 The Monte Carlo Approach in Astrophysics

The Comptonization problem in astrophysics has been addressed in different ways. One method is to solve the nonlinear radiative transfer equations under certain approximations and assumptions, while using a semi-analytical approach for consecutive scattering orders. The methods used in the semi-analytical approach have the advantage of being able to build a grid of solutions for a large range of free parameters in order to fit the data. Another approach is using Monte Carlo methods in order to simulate the Comptonization process, which consists of tracking the photons in a geometrical setup, while recording the photons' history as they are scattered. The advantage of Monte Carlo methods lies in its generality; complex problems can be solved without any specific limitations in relation to the characteristics of the Comptonizing medium, and particle distributions (see [Andreio \(1991\)](#) for a review).

Monte Carlo simulations are highly compatible with the semi-analytic approach (see e.g. [Stern et al. \(1995\)](#)), yet more rigorous, with a significant amount of computational time needed in order to achieve precision. However, the stochastic approach of the Monte Carlo methods

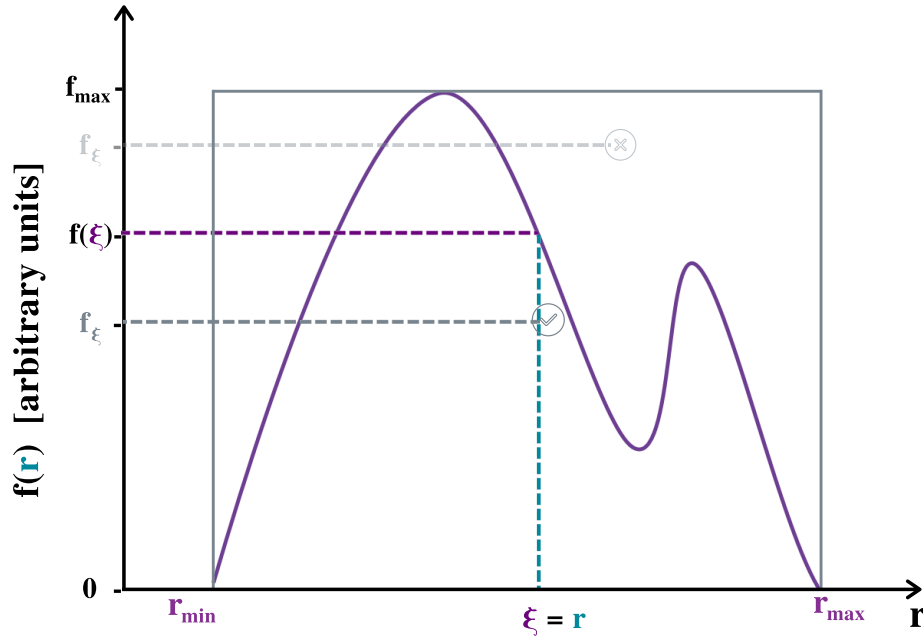


Fig. 3.2 Illustration of the rejection technique used to draw a random variable  $r$  with the PDF  $f(r)$ .

allows for a geometrical description of the radiative process, free from the restrictions of solving the radiative transfer equations. This means that the semi-analytical approach is less flexible, compared to the Monte Carlo approach, for specific combinations of free parameters, while only consisting of a mono-dimensional description of the radiative process (Poutanen and Svensson, 1996; Poutanen and Vilhu, 1993). Notable work on the Monte Carlo approach was done by Pozdnyakov et al. (1977), Sunyaev and Titarchuk (1984), and an extensive review is provided by Pozdnyakov et al. (1983). Polarization of the radiation is, however, generally not included in these works, with the exception of Sunyaev and Titarchuk (1984), who provided calculations of the polarization degree (PD) in the Thomson regime (i.e., elastic scattering). More recent work on implementing the Monte Carlo approach in Comptonization problems does include the polarization signatures. This is due to recent developments of new technology and could make high-energy polarization testable in the future.

There are different ways to implement Monte Carlo methods in the simulations, two of which are the *photon packet* and the *single photon* approach. A flow diagram of the procedure is provided in Fig. 3.3, for the most general case of simulating Compton scattering, using the photon packet approach (left-hand side) and the single photon approach (right-hand side). In the case of the photon packet approach, an ensemble of the total number of photons is followed, and

a *statistical weight* is assigned to every packet, after which a distribution of electrons is drawn. In the single photon approach, every photon is followed individually and a random electron from a distribution of electrons is assigned to a scattering event. In both cases, the probability of scattering is calculated (obtained from the Compton cross-section) in order to determine whether the interaction will take place. However, the photon packet approach requires the evaluation of the area under the probability distribution (i.e. the CDF), where the single photon approach only requires one effective extraction of a random number in order to use the Monte Carlo methods described in Section 3.1. When multiple scattering is included, the probability of scattering is evaluated multiple times in the photon packet approach, until the weight of the packet is below a cut-off weight defined. In the single photon approach, another electron is drawn for every photon multiple times before the probability of scattering can be calculated. Another subtle difference between the single photon approach and the photon packet approach is the treatment of polarized photons in the simulation. The polarization of the photons can be obtained, in both cases, using the Stokes formalism given in section 2.1.1. However, in the single photon approach, every individual photon must be treated as being 100% polarized, even if the radiation is unpolarized.

The photon packet approach is more commonly used in astrophysics in order to save computational time. For instance, [Schnittman and Krolik \(2010\)](#) adopted the photon packet approach to simulate Comptonization, which can mostly be used in the Thomson regime, with the energy exchange only taken into account by transforming the photons between different frames of references. [Beheshtipour et al. \(2017\)](#) expanded on the work of [Krawczynski \(2012\)](#), and managed to include the Klein-Nishina treatment for Comptonization, as well as the contribution of non-thermal electrons in the emission region, also following the photon packet approach as in [Schnittman and Krolik \(2010\)](#). However, the single photon approach enables the exploration of the whole space of free parameters that characterize the Comptonizing medium without particular limitations, while also being able to include the Klein-Nishina treatment in order to study the polarization of high-energy radiation. [Tamborra et al. \(2018\)](#) developed a Monte Carlo code for Comptonization of accretion in astrophysics (MoCA) while following the single photon approach. The MoCA code is devoted to the spectrum and polarization signals in the X-ray band for accretion systems, in order to study the geometry of the corona, and better understand its origins. Following the treatment of Compton scattering of linearly polarized photons in Monte Carlo codes by [Matt et al. \(1996\)](#), the contribution to the Stokes parameters is registered for every photon. In this way, the polarization of the radiation is obtained by the

summation of the contributions of all the photons in a specified direction.

Hence, the Monte Carlo approach is very flexible in the description of the geometry of the Comptonizing medium, as well as the distribution of the particles involved. The single photon approach also allows more adaptability in terms of the free parameters used in the simulations. Monte Carlo methods are, therefore, particularly favorable when considering anisotropic scattering off thermal electrons with different energy distributions, in order to study the polarization of the high-energy radiation. Furthermore, the approach can easily be adapted to different energy regimes in order to study the transition from high-energy to low-energy polarization in various astrophysical sources.

The Monte Carlo simulations used to develop the code in this dissertation consist of tracking the photons in a geometrical setup, while following the single photon approach. The polarization of the photon is calculated before and after the scattering event using the Stokes formalism discussed in Section 2.1.1. A detailed discussion of the code given in the following section.

### 3.3 The Monte Carlo Code

A flow diagram of the code developed in this project is given in Fig. 3.4: The code is divided into two programs, the main program written in the C++ (object orientated) programming language, and a supplementary *py*-program, also written in C++, with an embedded *python* code in order to visualize the results.

The main program consists of two source files; the first includes the main function, as well as the two *void* functions which perform the Monte Carlo simulation, depending on the electron distribution considered (either a purely thermal (Maxwell) distribution, or a hybrid (Maxwell/power-law) distribution). The latter contains an *[ICpol]* class which defines every photon's properties as they are being transported through the computational domain. The free parameters are specified in the main function, after which an *instance* (a user-defined object) of the ICpol class is created that represents an individual photon. The Monte Carlo simulation is then performed, while following a single photon approach in order to delineate an emission region without specific limitations. The photons are, therefore, tracked individually over a *for-loop* that repeats  $N_{phot}$  times. After evaluating the scattering cross-section between the photon and electron considered, the code will continue with only the photons that undergo Compton scattering. The properties of the target photon, electron, and the scattered photon are registered

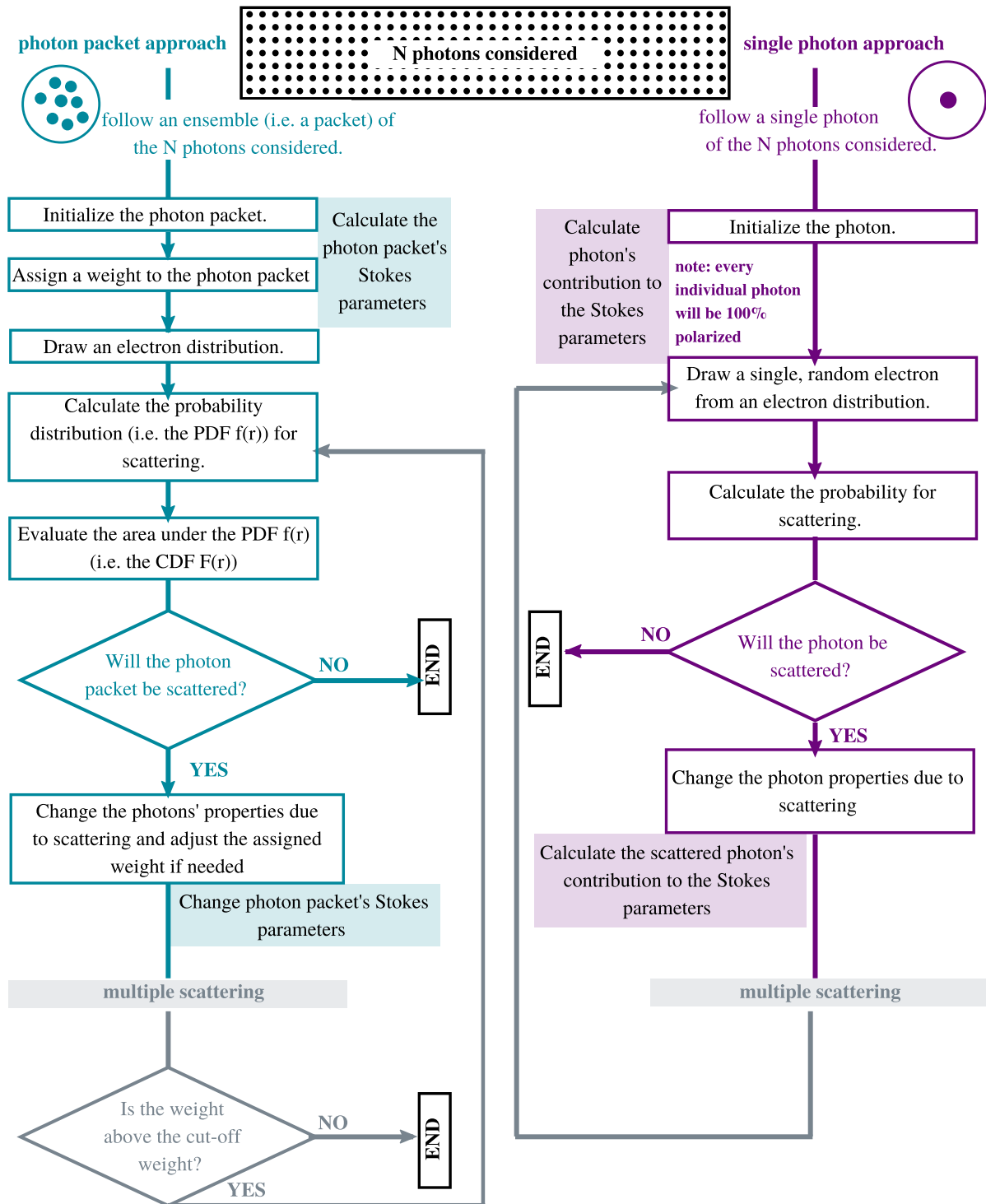


Fig. 3.3 A flow diagram of using the the Monte Carlo approach to simulate Comptonization. The photon packet approach is shown on the left-hand side, where an ensemble of the total number of photons is followed. The single photon approach is shown on the right-hand side where every photon is followed individually.

and saved into two files (*SeedData.txt* for the properties before scattering and *ICData.txt* for the properties after scattering). The data files may then be imported into the second *py*-program which bins the properties of the single photons in order to visualize the results. This section presents a detailed description of the code and the astrophysical environment that the code is applied to, starting with the model setup and free parameters.

### 3.3.1 Model Setup and Free Parameters

The code is applied to astrophysical sources that are associated with relativistic jets, where the emission region moves along the jet with a Lorentz factor  $\Gamma$ . The emission region is assumed to be well localized and optically thin throughout the electromagnetic (EM) spectrum. The electrons are assumed to be isotropically distributed in the emission-rest frame, with energies drawn from either a purely thermal (Maxwell) distribution, or a hybrid (Maxwell/power-law) distribution. The target photons are assumed to be unpolarized and isotropically distributed in the frame of the observer with a black body spectrum. Since the Compton scattering optical depth is typically very small in these environments, multiple scatterings of any individual photon are unlikely and are not considered in the simulations. Following the treatment of Compton scattering of linearly polarized photons in Monte Carlo methods by [Matt et al. \(1996\)](#), the contribution of every photon to the Stokes parameters is registered individually before and after the scattering event. The polarization signatures of the radiation are determined after the Monte Carlo simulation is complete, by summing the contributions of the Stokes parameters over all the photons in the specified direction.

The free parameters determine the characteristics of the emission-region, the target photon-energy distribution, and the electron-energy distributions. The values of the free parameters are given in Tab. 3.1: The emission region is assumed to move along the jet with a Lorentz factor  $\Gamma$  of order 10. A total of  $N_{phot} = 10^8$  target photons are considered, in three different photon-energy regimes with temperatures  $kT_{rad}$  that correspond to *X*-rays ( $kT_{rad} = 0.5$  keV), hard *X*-rays ( $kT_{rad} = 50$  keV), and  $\gamma$ -rays ( $kT_{rad} = 500$  keV). The target photons undergo anisotropic scattering off electrons with thermal temperatures  $kT_e$ ; non-relativistic ( $kT_e = 50$  keV), mildly-relativistic ( $kT_e = 500$  keV), and relativistic electrons ( $kT_e = 5000$  keV) are considered.

Two electron-energy distributions are considered, namely a Maxwellian distribution and a hybrid distribution. In the latter case, a fraction  $f_{nth} = 0.02$  of the electrons is assumed to be

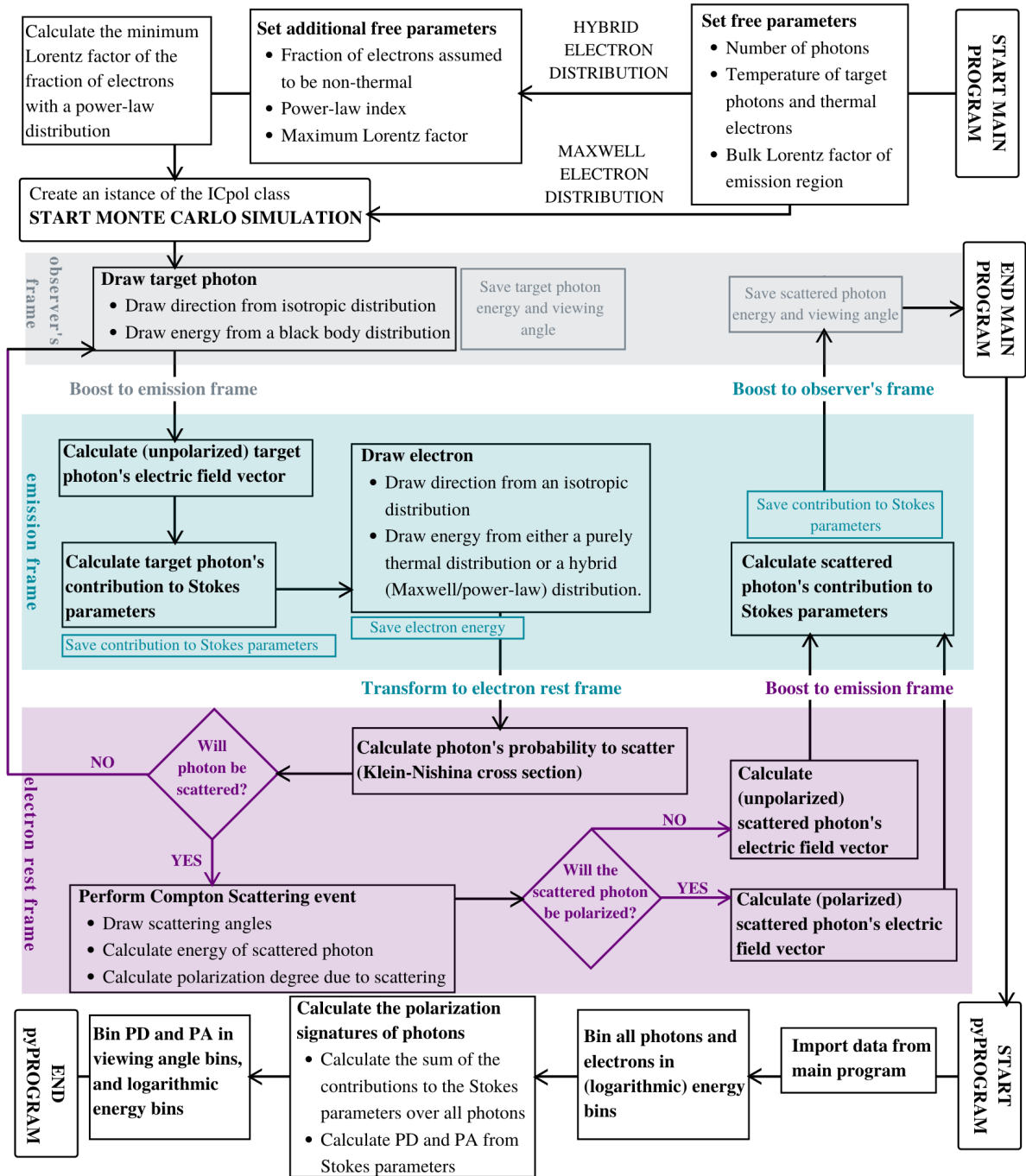


Fig. 3.4 A flow diagram of the Monte Carlo code. The free parameters are specified in the main program, after which the Monte Carlo simulation is performed: A single photon approach is followed where every photon is tracked individually. The photon is transformed between the observer's frame (grey shaded area), emission frame (blue shaded area), and the electron rest frame (purple shaded area) during the simulation. The properties of the electrons, target photons, and scattered photons are registered throughout, and saved into two data files: *SeedData.txt* and *ICData.txt*. After the Monte Carlo simulation, the *py*-program may be used to calculate the polarization signatures of the photons, bin the data, and visualize the results.

Table 3.1 The values and description of the free parameters considered in the code.

| Description                                                                                          | Symbol            | Value                        |
|------------------------------------------------------------------------------------------------------|-------------------|------------------------------|
| The Lorentz factor of the emission region                                                            | $\Gamma$          | 10                           |
| <b>Target photons</b>                                                                                |                   |                              |
| The number of the target photons that are considered                                                 | $N_{\text{phot}}$ | $10^8$                       |
| The target photon energies considered<br>soft X-rays<br>hard X-rays<br>$\gamma$ -rays                | $kT_{\text{rad}}$ | 0.5 keV<br>50 keV<br>500 keV |
| <b>Electrons</b>                                                                                     |                   |                              |
| The thermal temperatures of the electrons<br>non-relativistic<br>mildly-relativistic<br>relativistic | $kT_e$            | 50 keV<br>500 keV<br>500 keV |
| hybrid distribution of electrons                                                                     |                   |                              |
| Fraction of the electrons that is assumed to be non-thermal                                          | $f_{\text{nth}}$  | 0.02                         |
| Power-law index                                                                                      | $p$               | 2.5                          |
| Lorentz factor that corresponds to the cut-off energy of the power-law distribution of the electrons | $\gamma_2$        | $10^4$                       |

non-thermal, drawn from a power-law distribution  $n_{pl} \propto \gamma^{-p}$  where  $p = 2.5$  is the power-law index. The power-law tail has a cut-off energy, with a Lorentz factor of  $\gamma_2$  assumed to be  $10^4$ . The Lorentz factor  $\gamma_1$ , that corresponds to where the power-law tail begins, is determined by iteration until the fraction of electrons that is assumed to be non-thermal,

$$f_{nth} = \frac{n_{pl}}{n_{th} + n_{pl}} \quad (3.3)$$

is equal to 0.02 as given in Tab. 3.1. In equation 3.3,

$$n_{pl} = N_{pl} \int_{\gamma_1}^{\gamma_2} \tilde{\gamma}^{-p} d\tilde{\gamma} \quad (3.4)$$

is the power-law tail with  $p = 2.5$  the power-law index and  $N_{pl}$  the normalization constant. The thermal distribution is given by

$$n_{th} = N_{th} \int_1^{\gamma_1} \tilde{\gamma}^2 \tilde{\beta} e^{-\frac{\tilde{\gamma}}{\Theta}} d\tilde{\gamma} \quad (3.5)$$

where  $\tilde{\beta} = \sqrt{1 - \tilde{\gamma}^{-2}}$ ,  $\Theta = \frac{KT_e}{m_e c^2}$ , and  $N_{th} = 1$  the [arbitrary] normalization constant. At the point of intersection (where  $n_{th} = n_{pl}$ ), we have that

$$N_{pl} \gamma_1^{-p} = N_{th} \gamma_1^2 \beta_1 e^{-\frac{\gamma_1}{\Theta}}. \quad (3.6)$$

The fraction  $f_{nth}$  is thus calculated from Equation 3.3, using Equations 3.5 and 3.4, for different values of  $\gamma_1$ , until  $f_{nth} = 0.02$ , when the corresponding value of  $\gamma_1$  is accepted as an input parameter. All the parameters are then given as input arguments for the Monte Carlo simulation that follows.

### 3.3.2 The Monte Carlo Simulation

The Monte Carlo simulation is performed by calling one of two functions defined in the main program. The functions correspond to simulating anisotropic Compton scattering off electrons drawn from a purely thermal (Maxwellian) distribution and a hybrid (Maxwell/power-law) distribution, respectively. Both functions follow exactly the same procedure with the electron distribution being the only difference between the two. The free parameters (and the input parameter  $\gamma_1$ , in the case of electrons with a hybrid distribution) are given as input arguments for the functions, and are used to define a unique object that corresponds to an individual photon. This object is defined by creating an instance of the ICpol class which contains all the

properties of the photon as it is transported through the computational domain.

### 1. The ICpol class

In the C++ programming language, a *class* is a user-defined data-type which holds its own data members (member variables) and member functions, all of which may be accessed by the user by creating an instance of the class in the main program. The ICpol class defines an object in the simulation that corresponds to one individual photon. The member variables corresponds to every individual photon's properties, which makes every photon unique and accessible to the main program. The member functions of the class can either be public (accessible to the main program) or private (only used within the class).

The private member functions include a function that draws a random number between two specified limits. The random numbers are drawn with the *Mersenne Twister* pseudo random number generator (PRNG), developed by [Matsumoto and Nishimura \(1998\)](#). The public member functions include a constructor (called by default when an instance of the class is created) which initializes all the member variables to zero, and a de-structor that deletes the member variables from memory once the simulation is complete. After an instance of the class is created, the main *for-loop* begins in which the member variables are changed according to the input arguments given. The remaining member functions perform certain tasks in order to manipulate and change the photon's properties in different frames of reference. In what follows, the standard definition of spherical coordinates is used, quantities in the observer frame are denoted with a subscript *obs*, while quantities in the emission frame and the electron rest frame are denoted with the subscripts *em* and *e*, respectively. While implementing the single photon approach, an additional subscript *i* is used to label the current individual photon that is being followed. The scattered quantities are denoted with a superscript *sc* and all random numbers are denoted by  $\xi$ , drawn between zero and one, unless specified otherwise.

#### (a) *The target photons*

The target photons are drawn in the observer frame and isotropically distributed, with a black body spectrum that corresponds to temperatures  $kT_{\text{rad}}$ . Following a single photon, the polar angle  $\Theta_{i,obs}$  and azimuth angle  $\Phi_{i,obs}$  of the target photon are drawn from an isotropic

distribution in the observer frame with respect to the jet-axis (see Fig. 3.6), where

$$\begin{aligned}\Theta_{i,obs} &= \arccos(2\xi_1 - 1) \\ \Phi_{i,obs} &= 2\pi\xi_2.\end{aligned}\tag{3.7}$$

The energy of the photon is drawn from a black body distribution,

$$B_\nu(T_{rad}) = \frac{2h\nu^3}{c^2} \frac{1}{e^{\frac{h\nu}{kT_{rad}}} - 1}\tag{3.8}$$

corresponding to the photon temperature  $T_{rad}$  where  $\nu$  is the frequency of the photons, and  $h$  the Planck constant. This is done by following an algorithm given by [Pozdnyakov et al. \(1983\)](#): The number density of the photons having the energy  $\epsilon_{obs} = h\nu$  is expressed by

$$p(\epsilon_{obs}) = \frac{1}{2\zeta(3)} \left( \frac{1}{kT_{rad}} \right)^3 \epsilon^2 \left( e^{\frac{\epsilon}{kT_{rad}}} - 1 \right)^{-1}\tag{3.9}$$

where  $\zeta(3) = \sum_{m=1}^{\infty} m^{-3} = 1.202$  is the Riemann zeta function. The function  $\left( e^{\frac{\epsilon}{kT_{rad}}} - 1 \right)^{-1}$  can easily be expanded in powers of  $e^{-\frac{m\epsilon}{kT_{rad}}}$ , written as

$$p(\epsilon_{obs}) = (1.202)^{-1} \sum_{m=1}^{\infty} m^{-3} p_m(\epsilon_{obs})\tag{3.10}$$

with  $p_m(\epsilon_{obs}) = \frac{1}{2} \left( \frac{m}{kT_{rad}} \right)^3 \epsilon^2 e^{-\frac{m\epsilon}{kT_{rad}}}$  the normalized densities, which represents the gamma-distribution. Therefore,  $p(\epsilon_{obs})$  may be modelled with the inversion method. In order to draw a value for the target photon energy  $\epsilon_{i,obs} = h\nu_i$ , four random numbers ( $\xi_1, \xi_2, \xi_3, \xi_4$ ) are first drawn. From  $\xi_1$ , an auxiliary random number  $\alpha$  is defined with

$$\alpha = \begin{cases} 1 & \text{if } 1,202\xi_1 < 1 \\ m & \text{if } \sum_{j=1}^{m-1} j^{-3} \leq 1,202\xi_1 \leq \sum_{j=1}^m j^{-3} \end{cases}$$

where  $m = (2, 3, \dots)$ . The energy is then set to  $\epsilon_{i,obs} = -\left( \frac{kT_{rad}}{\alpha} \right) \ln(\xi_2 \xi_3 \xi_4)$ . After the simulation is complete, the supplementary *py*-program may be used to bin the photon count into logarithmic energy bins in order to visualize the energy distribution of the photons. The energy distribution of the  $N_{phot}$  target photons is shown in Fig. 3.5, where  $kT_{rad} = 0.5$  keV is shown in purple,  $kT_{rad} = 50$  keV is shown in blue, and  $kT_{rad} = 500$  keV is shown in grey.

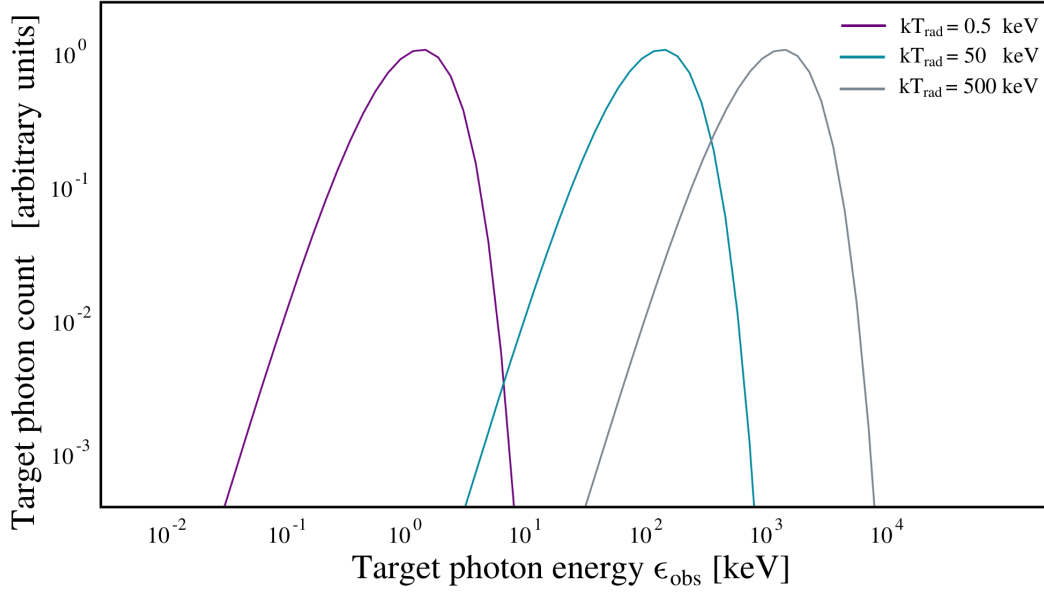


Fig. 3.5 The photon count as a function of the target photon energy. Three different photon-temperatures are shown which correspond to soft X-rays ( $kT_{\text{rad}} = 0.5$  keV) shown in purple, hard X-rays ( $kT_{\text{rad}} = 50$  keV) shown in blue, and  $\gamma$ -rays ( $kT_{\text{rad}} = 500$  keV) shown in grey.

The definition of the transformations between the observer frame and emission frame, along with an illustration of the light emitted in each frame, are shown in Fig. 3.6. The photon is thus boosted to the emission frame with bulk boost equations (Böttcher et al., 2012), where the energy and the polar distribution of the photon is given in the emission frame by

$$\epsilon_{i,em} = \Gamma \epsilon_{i,obs} (1 - \beta_{\Gamma} \cos \Theta_{i,obs}) \quad (3.11)$$

$$\cos \Theta_{i,em} = \frac{\cos \Theta_{i,obs} - \beta_{\Gamma}}{1 - \beta_{\Gamma} \cos \Theta_{i,obs}} \quad (3.12)$$

where  $\beta_{\Gamma} = \sqrt{1 - \Gamma^{-2}}$  is the normalized velocity of the emission region, assumed to move along the jet with a Lorentz factor of  $\Gamma$ . The azimuthal angle remains unchanged,  $\Phi_{i,obs} = \Phi_{i,em}$  as shown in Fig. 3.6 (Böttcher et al., 2012).

Once in the emission frame, the direction of the photon  $\vec{D}_{i,em}$  is calculated, using spherical coordinates, with the components given by

$$\begin{aligned} D_{i,em,x} &= \sin \Theta_{i,em} \cos \Phi_{i,em} \\ D_{i,em,y} &= \sin \Theta_{i,em} \sin \Phi_{i,em} \\ D_{i,em,z} &= \cos \Theta_{i,em}. \end{aligned} \quad (3.13)$$

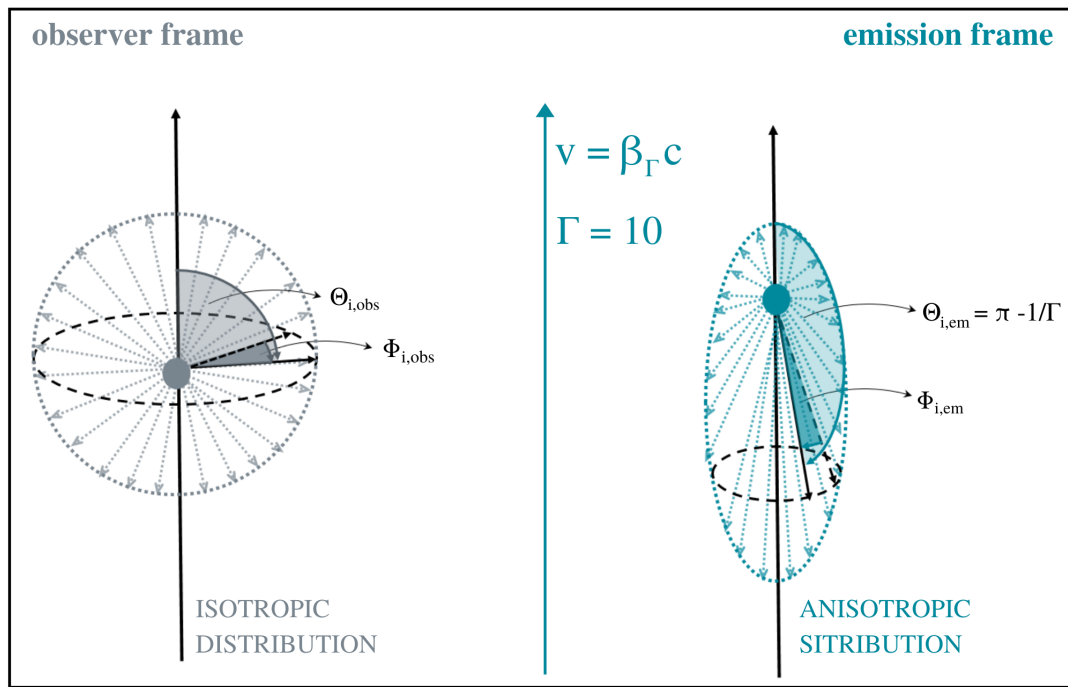


Fig. 3.6 The definition or the transformation between the observer frame (on the left panel), and the emission frame (on the right panel). The emission region moves with a velocity  $v = \beta_{\Gamma} c$ , along the jet with a Lorentz factor  $\Gamma$  of order 10. The length of grey and blue arrows in the respective frames of reference represent the intensity of the radiation in the specified direction. The light moving from a relativistic source in the observer's frame is beamed into a narrow cone of an opening angle of  $\Theta_{em} = \frac{1}{\Gamma}$ . Adapted from [Böttcher et al. \(2012\)](#).

The target photons are assumed to be unpolarized, with the polarization signatures obtained by using the Stokes formalism, discussed in Section 2.1.1. Since the target photon is assumed to be unpolarized, the polarization vector  $\vec{P}_{i,em}$  – which represents in what direction the electric field of the electromagnetic wave is oscillating – is randomly distributed in the plane normal to  $\vec{D}_{i,em}$ . This may be calculated using auxiliary vectors (Matt et al., 1996)

$$\begin{aligned}\vec{Q}_{i,+} &= \frac{1}{\sqrt{1-D_{i,em,z}^2}}(-D_{i,em,y}, D_{i,em,x}, 0) \\ \vec{Q}_{i,-} &= \frac{1}{\sqrt{1-D_{i,em,z}^2}}(-D_{i,em,x}D_{i,em,z}, -D_{i,em,y}D_{i,em,z}, 1-D_{i,em,z})\end{aligned}\quad (3.14)$$

that define the  $x'$  and  $y'$  axes of a reference frame in the plane normal to the photon direction  $\vec{D}_{i,em}$  (see Fig. 3.7). The  $y'$  axis is defined by  $\vec{Q}_{i,-}$ , which is the projection of the  $z$ -axis of the photon direction. The  $x'$ -axis defined by  $\vec{Q}_{i,+}$ , which is the intersection of the  $x-y$  plane of the photon direction, obtained by a  $\frac{\pi}{2}$  rad anti-clockwise rotation of  $\vec{Q}_{i,-}$ . The polarization vector of the unpolarized photon may then be obtained by a random linear combination of  $\vec{Q}_{i,+}$  and  $\vec{Q}_{i,-}$

$$\vec{P}_{i,em} = \sin \alpha_r \vec{Q}_{i,-} + \cos \alpha_r \vec{Q}_{i,+} \quad (3.15)$$

with  $\alpha_r$  a random angle between 0 and  $2\pi$  (Tamborra, 2013).

Once the polarization vector is known, it is possible to obtain the contribution of the current photon to the Stokes parameters (discussed in Section 2.1.1). Another set of auxiliary vectors (additionally to the auxiliary vectors given in Equation 3.14),

$$\begin{aligned}\vec{U}_{i,+} &= \frac{1}{N_{U\pm}}(-D_{i,em,y} - D_{i,em,x}D_{i,em,z}, D_{i,em,x} - D_{i,em,y}D_{i,em,z}, 1-D_{i,em,z}^2) \\ \vec{U}_{i,-} &= \frac{1}{N_{U\pm}}(D_{i,em,y} - D_{i,em,x}D_{i,em,z}, -D_{i,em,x} - D_{i,em,y}D_{i,em,z}, 1-D_{i,em,z}^2)\end{aligned}\quad (3.16)$$

is calculated, where  $N_{U\pm} = \sqrt{2(1-D_{i,em,z}^2)}$ . These vectors define another reference frame normal to the photon direction, obtained by a  $\frac{\pi}{4}$  rad rotation of the of  $\vec{Q}_{i,+}$  and  $\vec{Q}_{i,-}$  (see Fig. 3.7). From Equation 2.4, the contributions of the  $i^{th}$  photon to the second ( $Q$ ) and third ( $U$ )

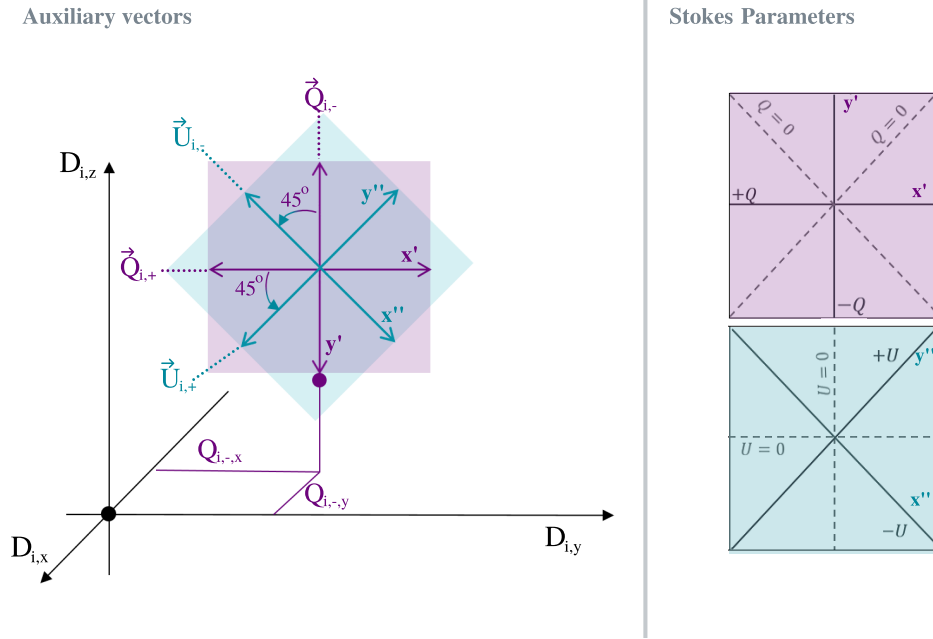


Fig. 3.7 On the left hand-side, an illustration of the auxiliary vectors  $\vec{Q}_{i,+}$ ,  $\vec{Q}_{i,-}$ ,  $\vec{U}_{i,+}$  and  $\vec{U}_{i,-}$  is given. These vectors define two reference frames normal to the photon direction. The  $x'$ -axis of the first reference frame (shown in purple) is defined by  $\vec{Q}_{i,+}$  which is the intersection of the  $x - y$  plane, obtained by a  $\frac{\pi}{2}$  rad anti-clockwise rotation of  $y'$  axis (the projection to the  $z$ -axis) denoted by  $\vec{Q}_{i,-}$ . The  $x''$  and  $y''$  axis of the second reference frame (shown in blue) is a  $\frac{\pi}{4}$  rad rotation of  $\vec{Q}_{i,+}$  and  $\vec{Q}_{i,-}$ . A representation of the Stokes parameters is given in the right-hand side for reference.

Stokes parameters are given by

$$\begin{aligned} Q_i &= \left( \vec{Q}_{i,+} \cdot \vec{P}_{i,em} \right)^2 - \left( \vec{Q}_{i,-} \cdot \vec{P}_{i,em} \right)^2 \\ U_i &= \left( \vec{U}_{i,+} \cdot \vec{P}_{i,em} \right)^2 - \left( \vec{U}_{i,-} \cdot \vec{P}_{i,em} \right)^2. \end{aligned} \quad (3.17)$$

The contribution of the current photon to the Stokes parameters  $Q_i$  and  $U_i$  is registered individually for every photon (Matt et al., 1996; Tamborra, 2013). After the simulation,  $Q_i$  and  $U_i$  are summed over all the  $N_{phot}$  photons considered to obtain the Stokes parameters  $Q$  and  $U$  (this is done with the *py*-program as shown in Fig. 3.4). From Equation 2.3, PA and PD is obtained, shown as a function of the polar angle  $\Theta_{obs}$  in Fig. 3.8, and energy  $\epsilon_{obs}$  in Fig. 3.9. Since the target photons are unpolarized, the PD is zero for all angles  $\Theta_{obs} \in [0, \pi]$  rad, and energy regimes. The PA randomly assumes all possible values  $PA \in [0, \pi]$  rad in both cases, and is therefore undefined, as expected for unpolarized radiation.

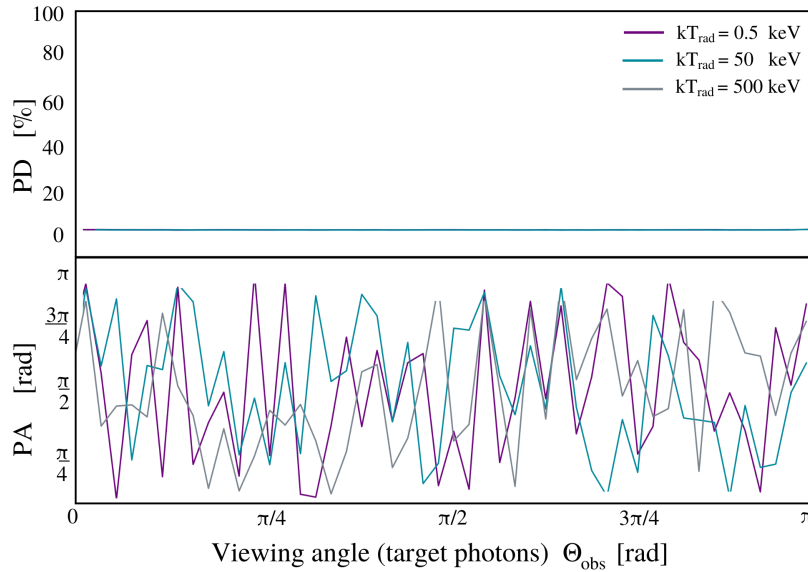


Fig. 3.8 The PD (upper panel) and PA (lower panel) of the target photons – for  $kT_{rad} = 0.5$  keV (shown in purple),  $kT_{rad} = 50$  keV (shown in blue), and  $kT_{rad} = 500$  keV (shown in grey) – binned in polar angle bins. The PD is zero for all values of polar angles  $\Theta_{obs} \in [0, \pi]$  rad. The PA randomly assumes all possible values  $\in [0, \pi]$  and is therefore undefined for the unpolarized target photons.

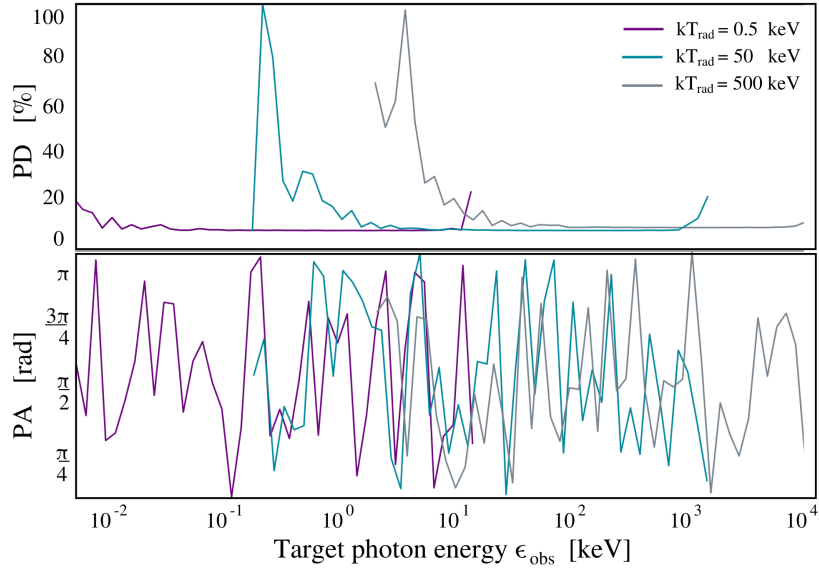


Fig. 3.9 The PD (upper panel) and PA (lower panel) of the target photons – for  $kT_{\text{rad}} = 0.5$  keV (shown in purple),  $kT_{\text{rad}} = 50$  keV (shown in blue), and  $kT_{\text{rad}} = 500$  keV (shown in grey) – binned in logarithmic energy bins. The PD is zero for all energies; because of low statistics (Monte Carlo noise) is why the spikes are in some of the lower energy bins. The PA randomly assumes all possible values  $\in [0, \pi]$  and is therefore undefined for the unpolarized target photons.

(b) *The electrons*

The emission region is assumed to be filled with electrons with thermal temperatures of  $kT_e$ , assumed to be isotropically distributed in the emission-region rest frame. The electron energy can either be drawn from a purely thermal (Maxwell) distribution, or from a hybrid (Maxwell/power-law) distribution. Since every photon is followed individually, a Lorentz factor  $\gamma_i$  that corresponds to a single electron from the electron distribution considered, is randomly drawn and assigned to the current photon considered. Considering the thermal distribution of electrons (see Equation 3.5), the momentum of an electron is drawn by following algorithms by Pozdnyakov et al. (1983) and Sobol (1973): The number of the Maxwellian electrons having the momentum  $\vec{p}$  is given by

$$N(\vec{p})d^3\vec{p} = e \frac{-(p^2c^2 + m_e^2c^4)^{1/2}}{kT_e} d^3\vec{p} \quad (3.18)$$

where  $\vec{p}$  is the momentum of the electron,  $m_e$  is the mass of the electron, and  $c$  the speed of light. If all the directions of  $\vec{p}$  are equally probable, the density of the dimensionless momentum

$\eta = \frac{p}{m_e c}$  is given by

$$p(\eta) = \frac{N_{th}}{K_2\left(\frac{1}{\Theta}\right)} \eta^2 e^{-\frac{1}{\Theta} \sqrt{1+\eta^2}} \quad (3.19)$$

where  $\Theta = \frac{kT_e}{m_e c^2}$ ,  $K_2\left(\frac{1}{\Theta}\right)$  is the modified Bessel function of second kind, and  $N_{th}$  the normalization constant. A rejection technique for Equation 3.19 is constructed where four random numbers  $\xi_1, \xi_2, \xi_3$  and  $\xi_4$  are drawn; and two quantities  $\eta' = -\Theta \ln(\xi_1 \xi_2 \xi_3)$  and  $\eta'' = -\Theta \ln(\xi_1 \xi_2 \xi_3 \xi_4)$  are calculated. If  $(\eta'')^2 - (\eta')^2 < 1$ , the dimensionless momentum is set as  $\eta = \eta'$ .

The Lorentz factor of a single electron  $\gamma_i$ , drawn from a thermal distribution, is then obtained from  $\eta$  where

$$\gamma_i = \gamma_{i,th} = \sqrt{\eta^2 + 1}. \quad (3.20)$$

If a hybrid distribution of electrons are considered,  $\gamma_i$  is drawn from either a thermal distribution described above, or from a non-thermal distribution with a power-law tail (see Equation 3.4). In the latter case,

$$\gamma_i = \gamma_{i,pl} = \left[ \xi \left( \gamma_2^{1-p} - \gamma_1^{1-p} \right) + \gamma_1^{1-p} \right]^{\frac{1}{1-p}} \quad (3.21)$$

where  $p = 2.5$  is the power-law index, and  $\xi \in [0, 1]$  is a random number. To determine from which distribution the electron – that is assigned to the current photon – is drawn, another random number  $\xi_f \in [0, 1]$  is drawn, and compared to the fraction of electrons that is assumed to be non-thermal  $f_{nth}$ ,

$$\gamma_i = \begin{cases} \gamma_{i,th} & \text{if } \xi_f > f_{nth} \\ \gamma_{i,pl} & \text{if } \xi_f < f_{nth}. \end{cases} \quad (3.22)$$

Like in the case of the target photons, the electrons are binned in logarithmic energy bins after the simulation is complete. The energy distributions of the electrons are shown in Fig. 3.10, with thermal temperatures decreasing with the shade of purple;  $KT_e = 50$  keV (non-relativistic),  $KT_e = 500$  keV (mildly-relativistic),  $KT_e = 5000$  keV (relativistic). The solid lines represent the purely thermal electrons, while the electrons drawn from the hybrid distribution are shown in dashed lines. The normalized velocity of the electron  $\beta_i = \sqrt{1 - \gamma_i^{-2}}$  is derived from  $\gamma_i$ , along with a random direction  $\vec{\beta}_i = (\beta_{i,x}, \beta_{i,y}, \beta_{i,z})$ , assuming the electrons do not have

a preferential motion with respect to the photons in the emission-region rest frame. Each electron is assigned to the current photon, where the possible interaction is determined by the Klein-Nishina scattering cross-section  $\sigma_{KN}$ .

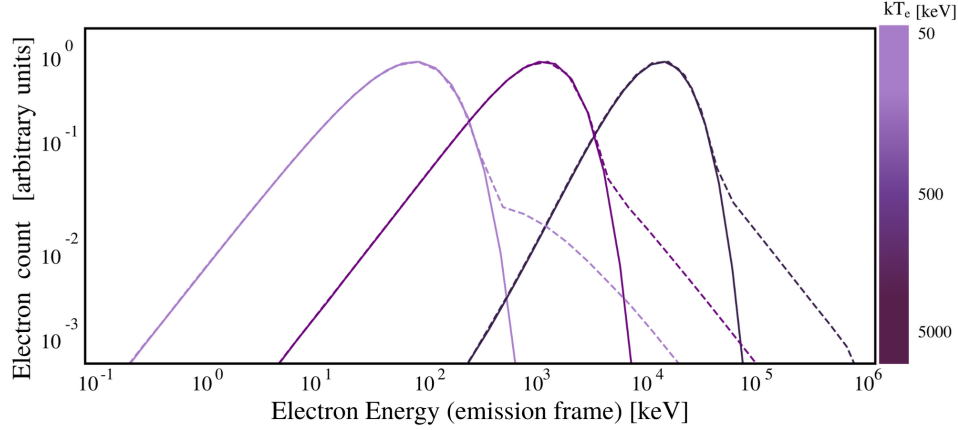


Fig. 3.10 The electron energy drawn from either a purely thermal (Maxwell) distribution (solid lines) and a hybrid (Maxwell/power-law) distribution (dashed lines). In the case of the hybrid distribution, a fraction  $f_{nth} = 0.02$  of the electrons is assumed to be non-thermal with a power-law  $n_{pl} \propto \gamma^{-p}$  tail, where  $p = 2.5$  is the power-law index. Three thermal temperatures are shown ( $KT_e = 50$  keV,  $KT_e = 500$  keV, and  $KT_e = 5000$  keV) which increases with the shade of purple.

(c) *The Compton scattering event*

The probability that the target photons will undergo Compton scattering is given by the Compton cross-section, most conveniently evaluated in the electron rest frame. The target photons are transformed into the electron rest frame, by using the Lorentz matrix,

$$\Lambda = \begin{pmatrix} \gamma_i & -\beta_{i,x}\gamma_i & -\beta_{i,y}\gamma_i & -\beta_{i,z}\gamma_i \\ -\beta_{i,x}\gamma_i & 1 + (\gamma_i - 1)\frac{\beta_{i,x}^2}{\beta_i^2} & (1 - \gamma_i)\frac{\beta_{i,x}\beta_{i,y}}{\beta_i^2} & (1 - \gamma_i)\frac{\beta_{i,x}\beta_{i,z}}{\beta_i^2} \\ -\beta_{i,y}\gamma_i & (1 - \gamma_i)\frac{\beta_{i,x}\beta_{i,y}}{\beta_i^2} & 1 + (\gamma_i - 1)\frac{\beta_{i,y}^2}{\beta_i^2} & (1 - \gamma_i)\frac{\beta_{i,y}\beta_{i,z}}{\beta_i^2} \\ -\beta_{i,z}\gamma_i & (1 - \gamma_i)\frac{\beta_{i,x}\beta_{i,z}}{\beta_i^2} & (1 - \gamma_i)\frac{\beta_{i,y}\beta_{i,z}}{\beta_i^2} & 1 + (\gamma_i - 1)\frac{\beta_{i,z}^2}{\beta_i^2} \end{pmatrix} \quad (3.23)$$

which requires the wave four-vector

$$\mathbf{K}_{i,em} = \frac{\epsilon_{i,em}}{2\pi\hbar c} (1, \vec{D}_{i,em}) \quad (3.24)$$

and the polarization four-vector  $\mathbf{P}_{i,em}$  of the current photon. The polarization four-vector is obtained by setting the temporal component to zero, so that  $\mathbf{P}_{i,em} = (0, \vec{P}_{i,em})$ .

The wave four-vector of the photon, in the electron rest frame,  $\mathbf{K}_{i,e}$  is obtained by a simple Lorentz transformation of Equation 3.24

$$\mathbf{K}_{i,e} = \Lambda \mathbf{K}_{i,em} \quad (3.25)$$

with the energy of the target photon  $\epsilon_{i,e}$  contained within the temporal component of  $\mathbf{K}_{i,e}$ , which is statistically increased by a factor of  $\gamma_i$ .

In order to transform the polarization four-vector  $\mathbf{P}_{i,em}$  to the electron rest frame, a temporary polarization four-vector  $\mathbf{P}_{i,temp}$  is first set as the Lorentz transformed polarization four-vector ( $\mathbf{P}_{i,temp} = \Lambda \mathbf{P}_{i,em}$ ). The polarization four-vector in the electron rest frame  $\mathbf{P}_{i,e}$  is then obtained with

$$\mathbf{P}_{i,e} = \left( 0, \mathbf{P}_{i,temp,j} - \frac{P_{i,temp,t} K_{e,j}}{K_{e,t}} \right), \quad j = x, y, z \quad (3.26)$$

where  $K_{e,t}$  and  $P_{i,temp,t}$  are the temporal components of  $\mathbf{K}_{i,e}$  and  $\mathbf{P}_{i,temp}$  (Tamborra, 2013).

Once in the electron rest frame, the full Klein-Nishina cross-section given in Equation 2.27, is calculated for the current photon,

$$\sigma_{i,KN} = \sigma_T \frac{3}{4} \left\{ \frac{1+x_{i,e}}{x_{i,e}^3} \left[ \frac{2x_{i,e}(1+x_{i,e})}{1+2x_{i,e}} - \ln(1+2x_{i,e}) \right] + \frac{1}{2x_{i,e}} \ln(1+2x_{i,e}) - \frac{1+3x_e}{(1+2x_{i,e})^2} \right\} \quad (3.27)$$

where is  $x_{i,e} = \frac{\epsilon_{i,e}}{m_e c^2}$  the ratio between target photon energy  $\epsilon_{i,e}$  and the electron rest mass energy. The probability of scattering  $P_{i,scatt}$  is obtained accordingly by the ratio between  $\sigma_{i,KN}$  and the Thomson cross-section  $\sigma_T = 6.65 \times 10^{-25}$ ,

$$P_{i,scatt} = \frac{\sigma_{i,KN}}{\sigma_T}. \quad (3.28)$$

The dependence of the cross-section to the dimensionless photon energy (in the electron rest frame)  $x_e$  is shown in Fig. 3.11, where the  $\sigma_{KN}/\sigma_T$  is plotted as a function of the target photon energy  $x_e$  for the first  $10^3$  photons in the simulation. As discussed in section 2.2.2, the cross-section assumes a constant value in the Thomson regime ( $x_e \ll 1$ ), and decreases as

the target photon energy approaches the Klein-Nishina limit ( $x_e \gg 1$ ). The target photons are expected to undergo Compton scattering in the Thomson regime with the scattered energies  $x_e^{sc}$  comparable to the target photon energies  $x_e$  (i.e.,  $x_{i,e}^{sc} \sim x_{i,e}$ ) in the electron rest frame. If the photon does undergo Compton scattering in the Klein-Nishina regime, the energy exchange between the electron and target photon will become substantial, with scattered photon energies  $x_{i,e}^{sc} \sim 1$ .

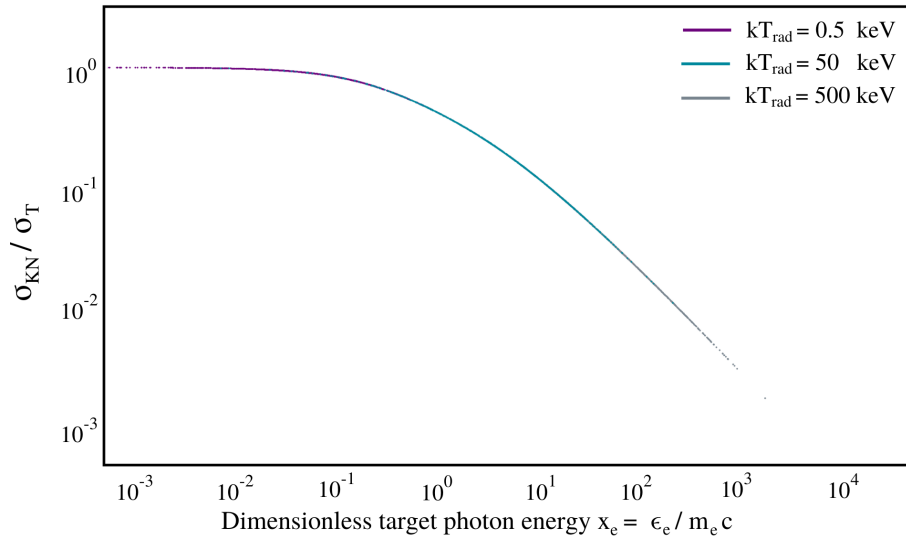


Fig. 3.11 The probability of scattering (Compton cross-section) as a function of the dimensionless target photon energy for  $kT_{\text{rad}} = 0.5$  keV (shown in purple),  $kT_{\text{rad}} = 50$  keV (shown in blue), and  $kT_{\text{rad}} = 500$  keV (shown in grey). In the Thomson regime (target photon-energies shown in purple and blue where  $x_e < 10^{-1}$ ) the cross-section assumes a constant value. The cross-section decreases as the target photon energies increase into the Klein-Nishina regime (target photon-energies shown in grey where  $x_e > 10^{-1}$ ).

To determine whether the photon will undergo Compton scattering, a random number  $\xi \in [0, 1]$  is drawn and compared to  $P_{i,\text{scatt}}$ . If  $\xi$  is larger than  $P_{i,\text{scatt}}$  the photon will continue in the same direction without scattering. If  $\xi$  is smaller than  $P_{i,\text{scatt}}$  the current photon and the electron (that is assigned to the current photon) will interact and the simulation will continue with the scattered photons. The energy of the scattered photons is obtained according to Equation 2.28, which is a function of the polar scattering angle  $\Theta_e^{sc}$  (the angle between the target and scattered photons). Furthermore, in order to calculate the PD due to scattering from Equation 2.38, the azimuth scattering angle  $\Phi_e^{sc}$  (the angle between the plane of scattering and the polarization vector of the target photons) has to be known. The complete independence of  $\Theta_e^{sc}$  and  $\Phi_e^{sc}$

allows separate calculations of  $\Theta_{i,e}^{sc}$  and  $\Phi_{i,e}^{sc}$  for the current photon considered.

The formula for obtaining the polar scattering angle  $\Theta_e^{sc}$  (see Fig. 2.9) can be calculated from the differential scattering cross-section, given in Equation 2.26 using interpolation. The probability of the photon to have a scattering angle,  $\Theta_{i,e}^{sc}$ , is given by

$$P(\xi < \Theta_{i,e}^{sc}) = \frac{1}{\frac{2}{3} + 2x_{i,e} + \frac{1}{x_{i,e}} \ln(1 + 2x_{i,e})} \left[ x_{i,e} \left( \frac{3}{2} + \cos \Theta_{i,e}^{sc} - \frac{1}{2} \cos^2 \Theta_{i,e}^{sc} \right) + \frac{1}{3} (1 + \cos^3 \Theta_{i,e}^{sc}) - \frac{1}{x_{i,e}} \{ \ln [1 + x_{i,e} (1 - \cos \Theta_{i,e}^{sc})] - \ln (1 + 2x_{i,e}) \} \right]. \quad (3.29)$$

Since this Equation is not analytically invertable, a numerical approach is needed in order to obtain  $\Theta_{i,e}^{sc}$  for every photon: A grid of values for  $P(\Theta_{i,e}^{sc})$  is built, by letting  $\Theta_{i,e}^{sc} \in [0, \pi]$  rad range from 0 to  $\pi$  in  $10^3$  steps. A random number  $\xi \in [0, 1]$  is then associated to the closest value of  $P(\xi < \Theta_{i,e}^{sc})$ , and the corresponding  $\Theta_{i,e}^{sc}$  is taken as the polar scattering angle of the current photon (Tamborra, 2013).

As mentioned in section 2.2.2, the azimuthal distribution of the photons is dependent of the polarization status of the target photons (Matt et al., 1996) (see Fig. 2.9). When the target photons are unpolarized, the azimuth scattering angle  $\Phi_e^{sc}$  can be assumed to be isotropically distributed in the electron rest frame. Although, it is important to note that even though the target photons are assumed to be unpolarized, the current photon needs to be treated as being 100% polarized, since the photons are transported individually through the computational domain (and a single photon is 100% polarized). Thus, in a single photon approach, all the individual photons are fully polarized and the unpolarized radiation refers to the net polarization of the target photons which is zero (Tamborra et al., 2018). Consequently, the azimuth scattering angle cannot be drawn isotropically as in the case of unpolarized radiation. The probability of the individual photon to have an angle  $\Phi_{i,e}^{sc}$  is obtained, from Equation 2.46, as

$$P(\xi < \Phi_{i,e}^{sc}) = \frac{\Phi_{i,e}^{sc}}{2\pi} - \frac{\sin^2 \Theta_{i,e}^{sc} \sin \Phi_{i,e}^{sc} \cos \Phi_{i,e}^{sc}}{2\pi \left( \frac{\epsilon'}{\epsilon} + \frac{\epsilon}{\epsilon'} - \sin^2 \Theta_{sc} \right)}. \quad (3.30)$$

The same numerical approach followed for obtaining  $\Theta_{i,e}^{sc}$  is then used in order to obtain  $\Phi_{i,e}^{sc} \in [0, 2\pi]$  rad. In Fig. 3.12, the normalized photon count is shown as a function of  $\Theta_e^{sc}$  (top panel) and  $\Phi_e^{sc}$  (bottom panel), for the case of Compton scattering off mildly-relativistic

( $kT_e = 500$  keV) electrons. The modulation of the  $\Phi_e^{sc}$  is depended on the target photon energy, as well as the polar scattering angle  $\Theta_e^{sc}$ : When  $\Theta_e^{sc}$  is zero, the modulation of  $\Phi_e^{sc}$  decreases, and (if viewed together with the probability of scattering shown in Fig. 3.11) the effect is less for photons that have a larger probability of being scattered forward (i.e., target photons with  $kT_{rad} = 500$  keV shown in grey).

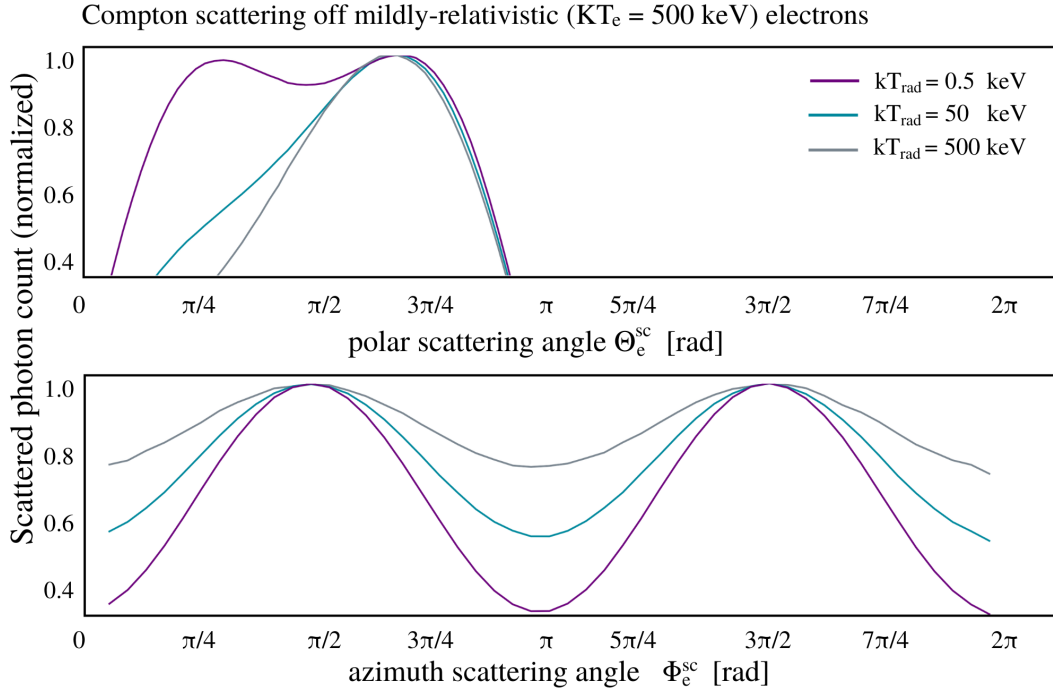


Fig. 3.12 The photon count (normalized) as a function of the polar scattering angle  $\Theta_e^{sc}$  (top panel) and azimuth scattering angle  $\Phi_e^{sc}$  (bottom panel) in the electron rest frame. The different target photon-energies are shown in the same color as before:  $kT_{rad} = 0.5$  keV (shown in purple),  $kT_{rad} = 50$  keV (shown in blue), and  $kT_{rad} = 500$  keV (shown in grey).

Having both the scattering angles  $\Theta_{i,e}^{sc}$  and  $\Phi_{i,e}^{sc}$ , the energy of the scattered photon

$$\epsilon_{i,e}^{sc} = \frac{\epsilon_{i,e}}{1 + x_{i,e} \left( 1 - \cos \Theta_{i,e}^{sc} \right)} \quad (3.31)$$

is calculated for every photon, and the direction of the scattered photon  $\vec{D}_{i,e}^{sc}$  is obtained with the components

$$\begin{aligned} D_{i,x,e}^{sc} &= D_{i,x,e} \cos \Theta_{i,e}^{sc} + P_{i,x,e} \sin \Theta_{i,e}^{sc} \cos \Phi_{i,e}^{sc} + \sin \Theta_{i,e}^{sc} \sin \Phi_{i,e}^{sc} (D_{i,y,e} P_{i,z,e} - D_{i,z,e} P_{i,y,e}) \\ D_{i,y,e}^{sc} &= D_{i,y,e} \cos \Theta_{i,e}^{sc} + P_{i,y,e} \sin \Theta_{i,e}^{sc} \cos \Phi_{i,e}^{sc} + \sin \Theta_{i,e}^{sc} \sin \Phi_{i,e}^{sc} (D_{i,z,e} P_{i,x,e} - D_{i,x,e} P_{i,z,e}) \\ D_{i,z,e}^{sc} &= D_{i,z,e} \cos \Theta_{i,e}^{sc} + P_{i,z,e} \sin \Theta_{i,e}^{sc} \cos \Phi_{i,e}^{sc} + \sin \Theta_{i,e}^{sc} \sin \Phi_{i,e}^{sc} (D_{i,x,e} P_{i,y,e} - D_{i,y,e} P_{i,x,e}) \end{aligned} \quad (3.32)$$

where  $\vec{P}_{i,e} = (P_{i,x,e}, P_{i,y,e}, P_{i,z,e})$  is the polarization vector of the target photon in the electron rest frame, contained in the spatial component of the polarization four-vector  $\mathbf{P}_{i,e}$  (Matt et al., 1996; Tamborra, 2013). Thereupon, the polarization vector of the photon is calculated depending on whether the photon will be polarized after the scattering event. The probability of whether the scattered photon will be polarized, is determined by the PD due to scattering  $\text{PD}^{sc}$ , given in Equation 2.38 and Equation 2.49 for unpolarized and polarized target photons, respectively. As motioned above, every individual photon needs to be treated as being 100% polarized. The PD due to scattering for the current photon is, therefore, calculated with

$$\text{PD}_{i,P}^{sc} = 2 \left[ \frac{1 - \sin^2 \Theta_{i,e}^{sc} \cos^2 \Phi_{i,e}^{sc}}{\frac{\epsilon_{i,e}^{sc}}{\epsilon_{i,e}} + \frac{\epsilon_{i,e}}{\epsilon_{i,e}^{sc}} - 2 \sin^2 \Theta_{i,e}^{sc} \cos^2 \Phi_{i,e}^{sc}} \right]. \quad (3.33)$$

The probability of the scattered photons to be polarized, given by Equation 3.33, decreases with the increase of the target photon-energies. When viewed together with the cross-section in Fig. 3.11, the photons that are scattered in the Thomson regime are expected to be polarized, while photons approaching the Klein-Nishina limit are not expected to be polarized along with the dilution of the scattering cross-section.

A random number  $\xi \in [0, 1]$  is drawn and compared to  $\text{PD}_{i,P}^{sc}$ ; if  $\xi$  is smaller than  $\text{PD}_{i,P}^{sc}$ , then the scattered photon is polarized, and the polarization vector,

$$\vec{P}_{i,e}^{sc} = \vec{D}_{i,e}^{sc} \sin \Theta_{i,e}^{sc} \cos \Phi_{i,e}^{sc} - \vec{P}_{i,e} \quad (3.34)$$

is determined from Equation 2.50. Otherwise, the scattered photon is unpolarized with the polarization vector randomly drawn in the plane normal to  $\vec{D}_{i,e}^{sc}$ , analogous to Equation 3.15, with

$$\vec{P}_{i,e}^{sc} = \sin \alpha_r \vec{Q}_{i,-}^{sc} + \cos \alpha_r \vec{Q}_{i,+}^{sc} \quad (3.35)$$

where  $\alpha_r \in [0, \pi]$  is a random angle, and

$$\begin{aligned}\vec{Q}_{i,+}^{sc} &= \frac{1}{\sqrt{1 - (D_{i,e,z}^{sc})^2}} (-D_{i,e,y}^{sc}, D_{i,e,x}^{sc}, 0) \\ \vec{Q}_{i,-}^{sc} &= \frac{1}{\sqrt{1 - (D_{i,e,z}^{sc})^2}} (-D_{i,e,x}^{sc} D_{i,e,z}^{sc}, -D_{i,e,y}^{sc} D_{i,e,z}^{sc}, 1 - D_{i,e,z}^{sc})\end{aligned}\quad (3.36)$$

which describes a reference frame normal to the scattered photon, obtained in the same way as  $\vec{Q}_{i,+}$  and  $\vec{Q}_{i,-}$  in Equation 3.14 (see Fig. 3.7).

Like in the case of the target photons, the polarization properties of the scattered photons are to be calculated in the moving frame of the emission region. The current scattered photon is therefore transformed back into the emission frame by using the same Lorentz matrix as given in Equation 3.23, while substituting the electron direction  $\vec{\beta}_i$  with its additive inverse  $-\vec{\beta}_i$ . Analogous to Equation 3.24, the wave-four-vector of the scattered photon  $\mathbf{K}_{i,e}^{sc}$  is built and transformed to the emission frame with  $\mathbf{K}_{i,em} = \Lambda \mathbf{K}_{i,e}$ . The polarization four-vector,  $\mathbf{P}_{i,e}^{sc} = (0, \vec{P}_{i,e}^{sc})$ , is transformed to the emission frame in the same way described in Equation 3.26: A temporary polarization four-vector  $\mathbf{P}_{i,temp}^{sc} = \Lambda \mathbf{P}_{i,e}^{sc}$  is set, and

$$\mathbf{P}_{i,em}^{sc} = \left( 0, \mathbf{P}_{i,temp,j}^{sc} - \frac{P_{i,temp,t}^{sc} \mathbf{K}_{e,j}^{sc}}{\mathbf{K}_{e,t}^{sc}} \right), \quad j = x, y, z \quad (3.37)$$

Once in the emission rest frame, the scattered photon's contribution to the Stokes parameters  $Q^{sc}$  and  $U^{sc}$  are calculated, by following the same procedure as in the case of the target photons, with

$$\begin{aligned}Q_i^{sc} &= \left( \vec{Q}_{i,+}^{sc} \cdot \vec{P}_{i,em}^{sc} \right)^2 - \left( \vec{Q}_{i,-}^{sc} \cdot \vec{P}_{i,em}^{sc} \right)^2 \\ U_i^{sc} &= \left( \vec{U}_{i,+}^{sc} \cdot \vec{P}_{i,em}^{sc} \right)^2 - \left( \vec{U}_{i,-}^{sc} \cdot \vec{P}_{i,em}^{sc} \right)^2\end{aligned}\quad (3.38)$$

where

$$\begin{aligned}\vec{U}_{i,+}^{sc} &= \frac{1}{N_{U_{\pm}^{sc}}} \left( -D_{i,em,y}^{sc} - D_{i,em,x}^{sc} D_{i,em,z}^{sc}, D_{i,em,x}^{sc} - D_{i,em,y}^{sc} D_{i,em,z}^{sc}, 1 - (D_{i,em,z}^{sc})^2 \right) \\ \vec{U}_{i,-}^{sc} &= \frac{1}{N_{U_{\pm}^{sc}}} \left( D_{i,em,y}^{sc} - D_{i,em,x}^{sc} D_{i,em,z}^{sc}, -D_{i,em,x}^{sc} - D_{i,em,y}^{sc} D_{i,em,z}^{sc}, 1 - (D_{i,em,z}^{sc})^2 \right)\end{aligned}\quad (3.39)$$

$$\text{and } N_{U_{\pm}^{sc}} = \sqrt{2 \left( 1 - \left( D_{i,em,z}^{sc} \right)^2 \right)}.$$

Finally, the energy and direction of the scattered photon is Doppler boosted back into the observer frame, according to the bulk boost equations (Böttcher et al., 2012)

$$\begin{aligned} \epsilon_{i,obs}^{sc} &= \epsilon_{i,em}^{sc} \Gamma (1 + \beta_{\Gamma} D_{i,em,z}^{sc}) \\ \cos \Theta_{i,obs}^{sc} &= \frac{D_{i,em,z}^{sc} + \beta_{\Gamma}}{1 + \beta_{\Gamma} D_{i,em,z}^{sc}} \end{aligned} \quad (3.40)$$

where  $D_{i,em,z}^{sc} = \cos \Theta_{i,em}^{sc}$ , in order to visualize the results with respect to the observer's line of sight. Throughout the Monte Carlo simulation, the properties of the target photons, electrons, and the scattered photons are saved into two data files – *SeedData.txt* for the properties before scattering and *ICData.txt* for the properties after scattering – and imported into the supplementary *py*-program in order to bin the data. The results will be discussed in the following chapter.

### 3.4 Comparison to Previous Results

The code developed by Krawczynski (2011) can be used to numerically compute the polarization due to Compton scattering in the Thomson and Klein-Nishina regimes. They compared the numerical results of the Compton polarization in the Thomson regime to analytical results on the basis of quantum mechanical scattering calculations from Bonometto et al. (1970), and used the numerical formulation to study the polarization of Compton radiation emitted in the Klein Nishina regime. In what follows, the code developed in this project is compared to the results of Krawczynski (2011) for polarization due to unpolarized target photons, scattered in the Thomson regime, and polarized target photons scattered in the Klein-Nishina regime.

#### 1. The polarization degree of unpolarized target photons in the Thomson regime.

The Monte Carlo simulations of Krawczynski (2011) is very consistent with the analytical calculations of Bonometto et al. (1970) in the Thomson regime. An important implication of the analytical calculations of Bonometto et al. (1970) is that the PD vanishes for unpolarized target photons scattered by electrons with Lorentz factors  $\gamma \gtrsim 10$ . Krawczynski (2011) tested this prediction by generating an isotropic distribution of unpolarized photons with a frequency of  $4.8 \times 10^{11} \text{ Hz}$  ( $\sim 2 \text{ keV}$ ). They found a net polarization of  $\text{PD} \sim 0.26\%$  due to scattering of

mono-energetic, isotropic electrons with Lorentz factors of  $\gamma = 10^3$ . McNamara et al. (2009), on the other hand, reported PDs of  $\sim 20\%$  of Compton scattering of unpolarized photons in AGN jets, inconsistent with results of both Bonometto et al. (1970) and Krawczynski (2011).

The code developed in this dissertation was used to simulate Comptonization with initial conditions similar to that of Krawczynski (2011). An isotropic distribution of  $10^8$  mono-energetic, unpolarized target photons is generated in the frame of the observer with  $kT_{\text{rad}} = 1.9$  keV. The emission region moves along the jet with a bulk Lorentz factor of  $\Gamma = 5$ , with mono-energetic ( $\gamma = 10^3$ ) electrons isotropically distributed in the emission frame. The simulated events were divided into 100 subsets for which the Q and U values exhibited mean values of 0 consistent with unpolarized scattered photons, with a total net PD  $\sim 0.9\%$ . The results are therefore consistent with those of Krawczynski (2011) which do not confirm PD  $\sim 20\%$  predicted by McNamara et al. (2009).

## 2. The polarization degree of polarized target photons in the Klein-Nishina regime.

The numerical results of Krawczynski (2011) for Compton emission in the Klein-Nishina regime are given in Fig. 3.13. They considered mono-energetic ( $kT_{\text{rad}} \sim 1.3$  keV), polarized target photons, with initial Stokes vectors  $(I, Q, U) = (1, 1, 0)$ . The target photons are considered to be uni-directional in the frame of the observer, scattering off mono-energetic electrons with Lorentz factors between 0 and 62500, isotropically distributed in the emission frame. The intensity and PD in the upper panel of Fig. 3.13 are shown as a function of the energy of the target photons  $\epsilon_{\text{obs}}^{\text{sc}}$  in units of the maximum energy allowed kinematically where

$$x_{\text{obs},\text{max}}^{\text{sc}} = \frac{4\gamma x_{\text{obs}}}{1 + 4\gamma x_{\text{obs}}} \quad (3.41)$$

with  $x_{\text{obs}} = \frac{\epsilon_{\text{obs}}}{m_e c^2}$  is the dimensionless target photon energy in the frame of the observer. The net polarization is given as a function of the Lorentz factor  $\gamma \in [10, 10^6]$  in the lower panel of Fig. 3.13. The red dashed line displays the function  $PD = \frac{0.5}{1+x_e}$  where  $x_e = \frac{\epsilon_e}{m_e c^2}$  is the dimensionless target photon energy in the electron rest frame.

A similar setup was used to test the code developed in this dissertation for Compton scattering in the Klein-Nishina regime. The jet is orientated at an angle of  $\Theta \sim 1.4$  rad towards the LOS with a bulk Lorentz factor of  $\Gamma = 5$ . The target photons are considered to be polarized (with the initial Stokes vector  $(I, Q, U) = (1, 1, 0)$ ), mono-energetic ( $kT_{\text{rad}} = 5.1$  keV), and

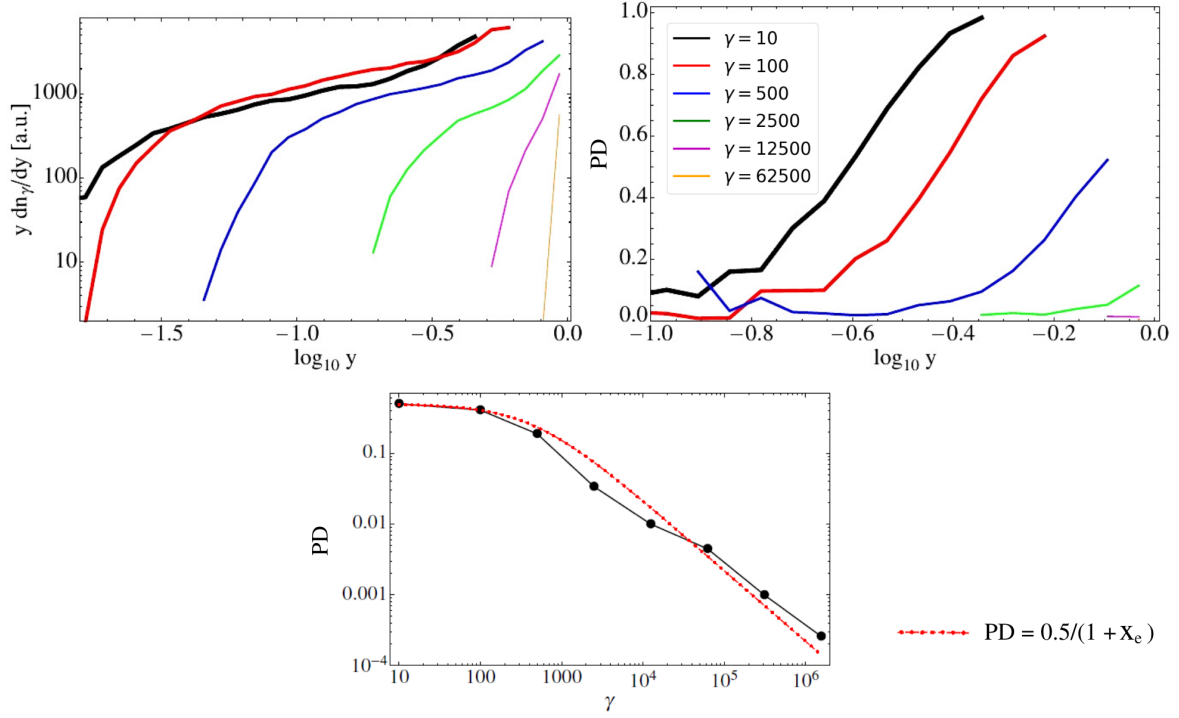


Fig. 3.13 Results from Krawczynski (2011) of the intensity and polarization from Compton emission in the Klein-Nishina regime. The target photons are assumed to be unidirectional in the observer frame with mono-energetic energies of  $x_{obs} = \epsilon_{obs}/m_e c^2 = 0.0025$ , scattered by isotropically distributed electrons with Lorentz factors of  $\gamma = 10, 100, 500, 2500, 12500$  and  $62500$ . The intensity (upper panel, left) and the PD (upper panel, right) is given as a function of the scattered photon energy in units of the maximum kinematically allowed energy  $y = x_{obs}^{sc}/x_{obs,max}$ . The net polarization of the Compton emission is given as a function of the Lorentz factor  $\gamma$  (additional simulations for Lorentz factors of  $3.125 \times 10^5$  and  $1.5652 \times 10^6$  was added in the results shown in the lower panel). The red dashed line shows the function  $PD = 0.5/(1 + x_e)$ . From Krawczynski (2011).

unidirectional in the direction of the jet with  $(\Theta_{obs}, \Phi_{obs}) = (1.4, 0)$  rad. The intensity and the PD target photons, due to scattering off isotropic, mono-energetic electrons with Lorentz factors  $\gamma = (10, 100, 500, 2500, 12500, 62500)$  is given on the upper panel of Fig. 3.14.

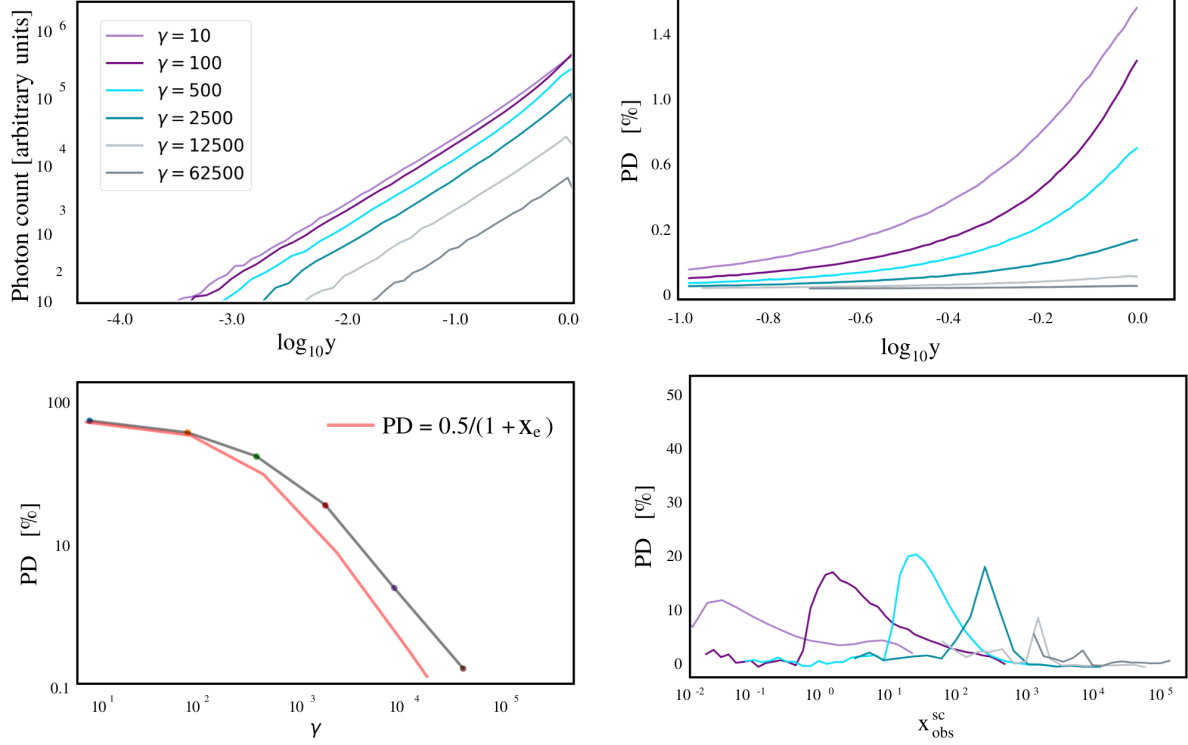


Fig. 3.14 Results from the code developed in this dissertation of the intensity and polarization from Compton emission in the Klein-Nishina regime. The target photons are assumed to be unidirectional in the observer frame and are mono-energetic with  $x_{obs} = \epsilon_{obs}/m_e c^2 = 0.0025$ , scattered by isotropically distributed electrons with Lorentz factors of  $\gamma = 10, 100, 500, 2500, 12500$  and  $62500$ . The intensity (upper panel, left) and the PD (upper panel, right) is given as a function of the scattered photon energy in units of the maximum kinematically allowed energy  $y = x_{obs}^{sc}/x_{obs,max}$  where  $x_{obs}^{sc} = \epsilon_{obs}^{sc}/m_e c^2$ . The PD is shown as a function of the  $x_{obs}^{sc}$  in the lower, right panel. The net polarization of the Compton emission is given as a function of the Lorentz factor  $\gamma$  in the lower, left panel where the red line shows the function  $PD = 0.5/(1 + x_e)$ .

The results given in Fig. 3.14 are overall consistent with those of Krawczynski (2011). In both cases, the intensity of the scattered photons shifts to higher energies, and peak towards  $x_{obs,max}$  deeper in the Klein-Nishina regime (i.e. for larger Lorentz factors), with the polarization strongly suppressed for  $\gamma \gg 10$ . The net polarization (given in the lower panel of Fig. 3.13 and the lower, left panel of Fig. 3.14) decreases approximately as the inverse of the target photon energy in the electron rest frame  $x_e$ . The analytical prediction of the function  $PD = 0.5/(1 + x_e)$  agrees well with the numerical results in the Thomson regime, but deviates in the Klein-Nishina

regime.

An important difference between the numerical calculations of [Krawczynski \(2011\)](#) and the code used in this dissertation is that [Krawczynski \(2011\)](#) only considered one scattering angle, while the results given in Fig. 3.14 is averaged over all scattering angles which results in lower values of the PD in each case.

# Chapter 4

## Results and Interpretation

The code discussed in the previous chapter uses Monte Carlo methods in order to simulate the anisotropic Compton scattering off thermal electrons and a non-thermal power-law tail of relativistic electrons. In a model where a thermal and a non-thermal particle distribution scatters an external radiation field, the hard  $X$ -ray/ $\gamma$ -ray radiation results from scattering by relativistic electrons and is, therefore, predicted to be unpolarized. Soft  $X$ -ray radiation, on the other hand, results from thermal electrons and is predicted to be polarized. In this chapter the effects of Compton scattering and the polarization due to the scattering event are discussed. As mentioned before, the Compton scattering optical depth of the emission regions in jet-like astrophysical sources is very small, and multiple scattering of any individual photon is thus very unlikely (Böttcher et al., 2012). Therefore, the Monte Carlo code simulates only one single scattering event, and the effects of multiple scattering are not included.

The results are shown for the combination of free parameters given in Tab. 3.1. In all the figures discussed, results for three different target photon temperatures are presented, with  $kT_{\text{rad}} = 0.5$  keV shown in purple,  $kT_{\text{rad}} = 50$  keV shown in blue, and  $kT_{\text{rad}} = 500$  keV shown in grey. The results for scattering off electrons drawn from a purely thermal (Maxwell) distribution are shown in solid lines, and those for electrons drawn from a hybrid (Maxwell/power-law) distribution are shown in dashed lines. The thermal temperatures of the electrons  $kT_e$  increase, in all the figures, with the shade of the color that corresponds to the photon energy considered.

### 4.1 Compton Scattering

In this section, the effects of Compton scattering is explored. The scattering event itself is evaluated in the electron rest frame, as discussed in section 2.2.2. Compton scattering in

the Thomson regime is almost elastic in the electron rest frame, while the energy exchange between the photon and electron becomes substantial in the Klein Nishina regime, along with a reduction of the scattering cross-section. While the electron rest frame is the most convenient frame to evaluate the Compton cross-section, the electrons and the target photons are both specified in a certain reference frame. The scattered photons are thus transformed into the emission frame and then Doppler boosted to the observer frame with Equation 3.40. After the simulation is complete, the scattered photons are binned into [logarithmic] energy bins.

The energy distributions of the scattered photons are shown in Fig. 4.1 with  $kT_{\text{rad}} = 0.5$  keV shown in purple (top panel),  $kT_{\text{rad}} = 50$  keV shown in blue (middle panel), and  $kT_{\text{rad}} = 500$  keV shown in grey (bottom panel). The results are shown for three electron temperatures which increase with the shade of the color that corresponds to the photon energy considered. Due to relativistic boosting, the photons that are scattered in the Thomson regime will have energies  $\epsilon_{\text{obs}}^{\text{sc}} \sim \gamma^2 \Gamma^2 \epsilon_{\text{obs}}$  (where  $\gamma$  is the Lorentz factor of the electrons, and  $\Gamma$  the Lorentz factor of the emission region) in the frame of the observer. Compton scattering off a power-law distribution of non-thermal electrons will result in a power-law distribution of scattered photons (indicated with dashed lines in all figures). The photons that are scattered in the Klein-Nishina regime have cut-off energies that correspond to the reduction of the cross-section.

For electrons with thermal temperatures of  $kT_e = 50$  keV, all the electrons are non-relativistic. Photons with  $kT_{\text{rad}} = 0.5$  keV and  $kT_{\text{rad}} = 50$  keV are scattered in the Thomson regime, with energies of order  $\sim \gamma^2 \Gamma^2$  higher than the target photon energies. For mildly-relativistic electrons with thermal temperatures of  $kT_e = 500$  keV (see Fig. 4.2), the peak of the electron distribution (top panel of Fig. 4.2) is around  $\gamma \sim 2$ . Photons with  $kT_{\text{rad}} = 50$  keV and  $kT_{\text{rad}} = 500$  keV are thus scattered in the Klein Nishina regime. The high-energy spectra for scattering of  $kT_{\text{rad}} = 50$  keV and  $kT_{\text{rad}} = 500$  keV are, therefore, very similar in the bottom panel of Fig. 4.2.

## 4.2 Compton Polarization

The scattering cross-section is generally dependent on polarization through the angular distribution of the photons as given by Equation 2.30. Polarization is expected to arise in the Thomson regime since the polarization term in Equation 2.30 dominates for  $x_e \ll 1$ . Polarization is not expected to be induced for photons scattered in the Klein Nishina regime since the polarization

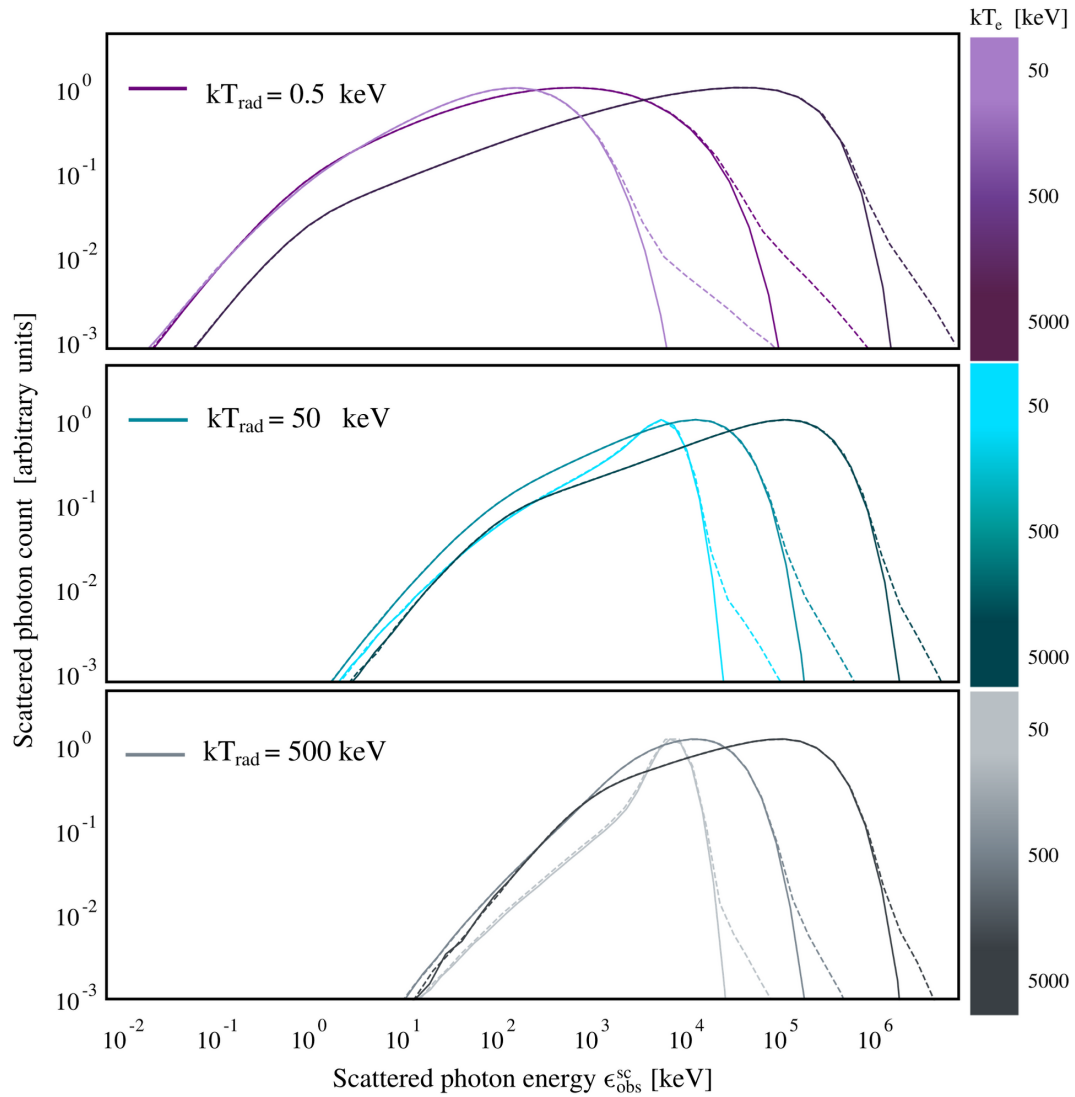


Fig. 4.1 The scattered photon energy distributions for Compton scattering between target photons with three different temperatures –  $kT_{\text{rad}} = 0.5$  keV shown in purple (top panel),  $kT_{\text{rad}} = 50$  keV shown in blue (middle panel), and  $kT_{\text{rad}} = 500$  keV shown in grey (bottom panel) – and electrons with three different thermal temperatures, which increase with the shade of purple (top panel), blue (middle panel), and grey (bottom panel). The target photons are scattered by electrons drawn from either a Maxwell distribution (solid lines) or a hybrid (Maxwell/power-law) distribution (dashed lines).

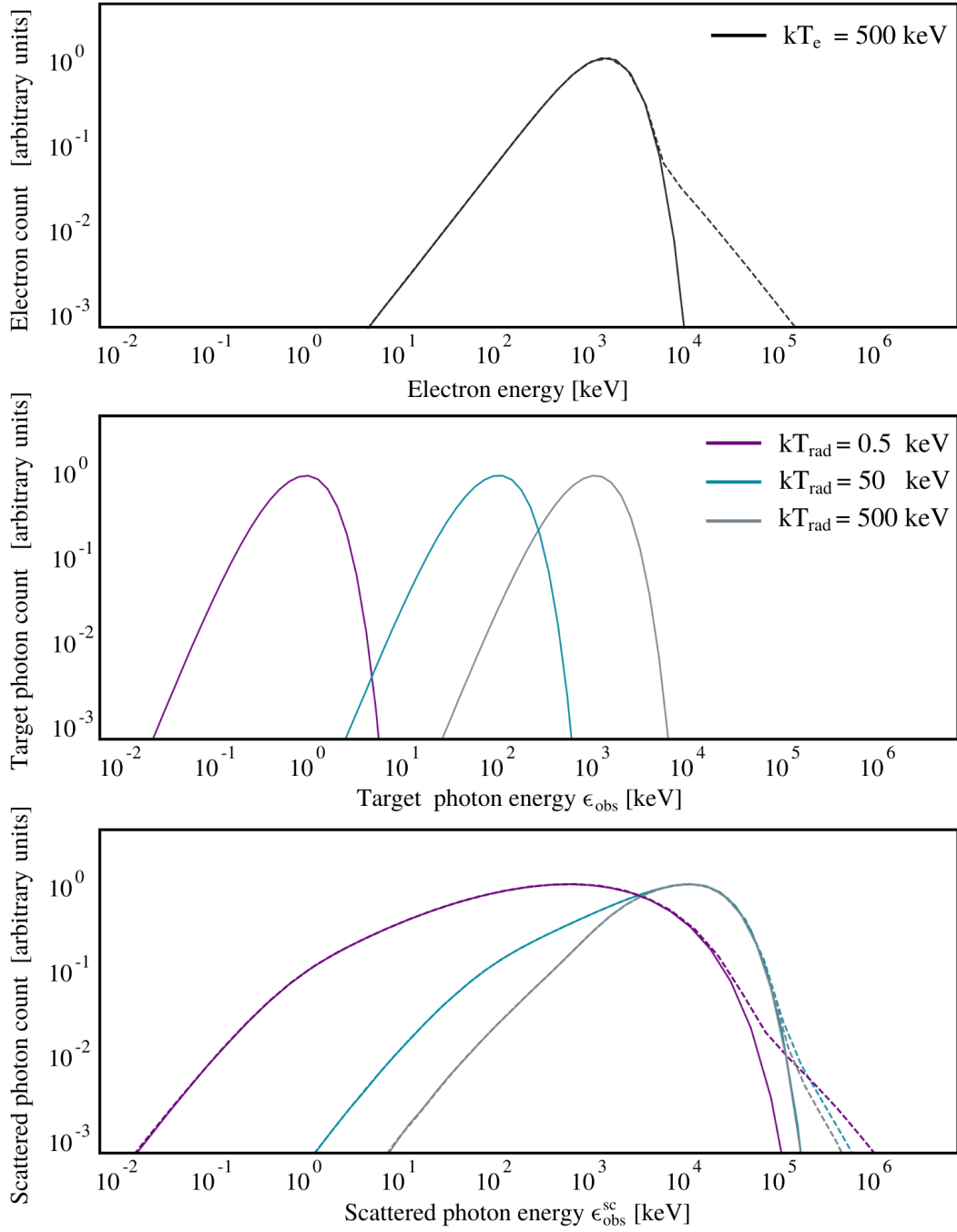


Fig. 4.2 The electron, target photon, and scattered photon energy distributions for Compton scattering between photons and mildly-relativistic ( $kT_e = 500$  keV) electrons. The electron spectrum is shown in the top panel, drawn from a purely thermal (Maxwell) distribution shown with a solid line, and a hybrid (Maxwell/power-law) distribution, shown with a dashed line. The target photon energy distributions for three different target photon temperatures ( $kT_{\text{rad}} = 0.5$  keV shown in purple,  $kT_{\text{rad}} = 50$  keV shown in blue, and  $kT_{\text{rad}} = 500$  keV shown in grey) are shown in the middle panel, with the corresponding scattered photon energies in the bottom panel.

term in Equation 2.30 becomes negligible for  $x_e \gg 1$ . The polarization signatures are calculated by summing every scattered photon's contribution to the second ( $Q_i$ ) and third ( $U_i$ ) Stokes parameters over all the photons in the specified direction. The polarization degree (PD) and polarization angle (PA) are obtained from Equation 2.3, and binned into 50 viewing angle  $\Theta_{obs}^{sc}$  bins, as well as 50 [logarithmic] energy  $\epsilon_{obs}^{sc}$  bins.

### 1. Polarization degree in the Thomson and Klein Nishina regimes

The PD is plotted as a function of  $\Theta_{obs}^{sc}$  in Fig. 4.3, and as a function of  $\epsilon_{obs}^{sc}$  in Fig. 4.4. The results are given for three target photon energies with  $kT_{rad} = 0.5$  keV shown in purple (top panel),  $kT_{rad} = 50$  keV shown in blue (middle panel), and  $kT_{rad} = 500$  keV shown in grey (bottom panel). The electron temperatures increase with the shade of the color that corresponds to the photon energy considered. The PDs decrease with increasing photon energies, and further decrease for higher electron temperatures. This indicates, once again, that polarization from Comptonized photons are only expected to arise in the Thomson regime, and no polarization is expected to be induced in the Klein Nishina regime. For photons that are scattered in the Thomson regime, the maximum PD occurs where thermal non-relativistic electrons scatter the target photons (i.e., to an energy of  $\Gamma^2 kT_{rad}$ ). For electron temperatures of  $kT_e = 5000$  keV, essentially all the electrons are highly relativistic with  $\gamma \gtrsim 10$ , and no Compton polarization is induced irrespective of whether the photons are scattered in the Thomson or Klein Nishina regime.

Since the orientation of the polarization does not change significantly for different electron distributions, the PD as a function of  $\Theta_{obs}^{sc}$  is similar for the case of purely thermal electrons (solid lines) and a power-law distribution of non-thermal electrons (dash lines). The PD as a function of  $\epsilon_{obs}^{sc}$  shows that there are more photons available in higher energies for a power-law distribution of non-thermal electrons (dash lines) than in the case of purely thermal electrons (solid lines), but these photons are unpolarized since the non-thermal electrons are relativistic.

### 2. Polarization signatures in the Thomson regime

The polarization signatures due to Compton scattering off non-relativistic and mildly-relativistic electrons are given as a function of  $\Theta_{obs}^{sc}$  in Fig. 4.5 and Fig. 4.6. The same polarization signatures is given as a function of  $\epsilon_{obs}^{sc}$  in Fig. 4.7 (for non-relativistic electrons) and Fig. 4.8 (for mildly-relativistic electrons). The results are shown for three different target photon energies

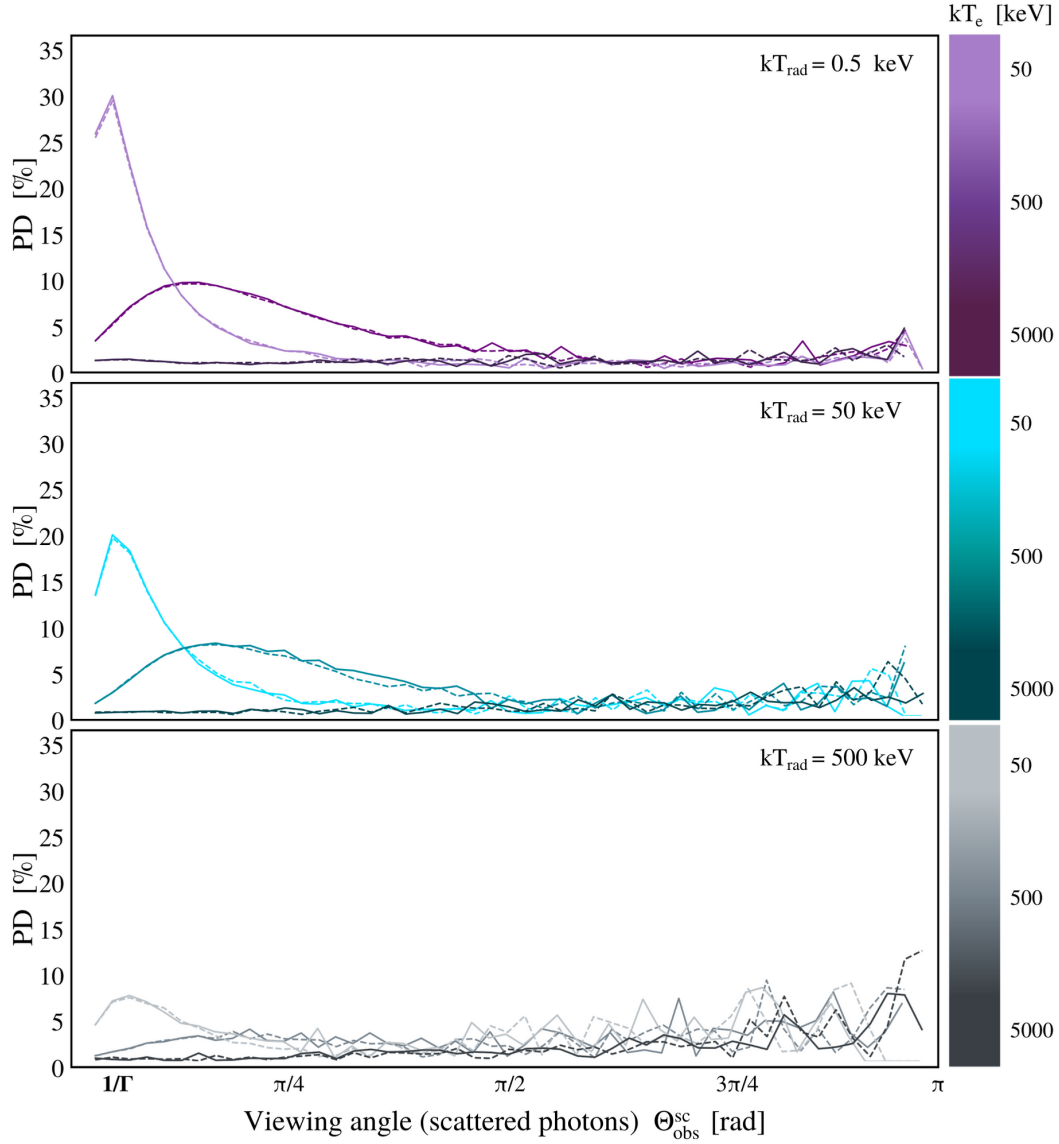


Fig. 4.3 The PD of the scattered photons as a function of the viewing angle. Three different target photon temperatures are given, with  $kT_{\text{rad}} = 0.5$  keV shown in purple (top panel),  $kT_{\text{rad}} = 50$  keV shown in blue (middle panel), and  $kT_{\text{rad}} = 500$  keV shown in grey (bottom panel). The results are shown for Compton scattering off non-relativistic ( $kT_e = 50$  keV), mildly-relativistic electrons ( $kT_e = 500$  keV), and relativistic electrons ( $kT_e = 5000$  keV), with the electron temperatures increasing with the shade of purple, blue, and grey in each case. The target photons are scattered by electrons drawn from either a Maxwell distribution (solid lines) or a hybrid (Maxwell/power-law) distribution (dashed lines).

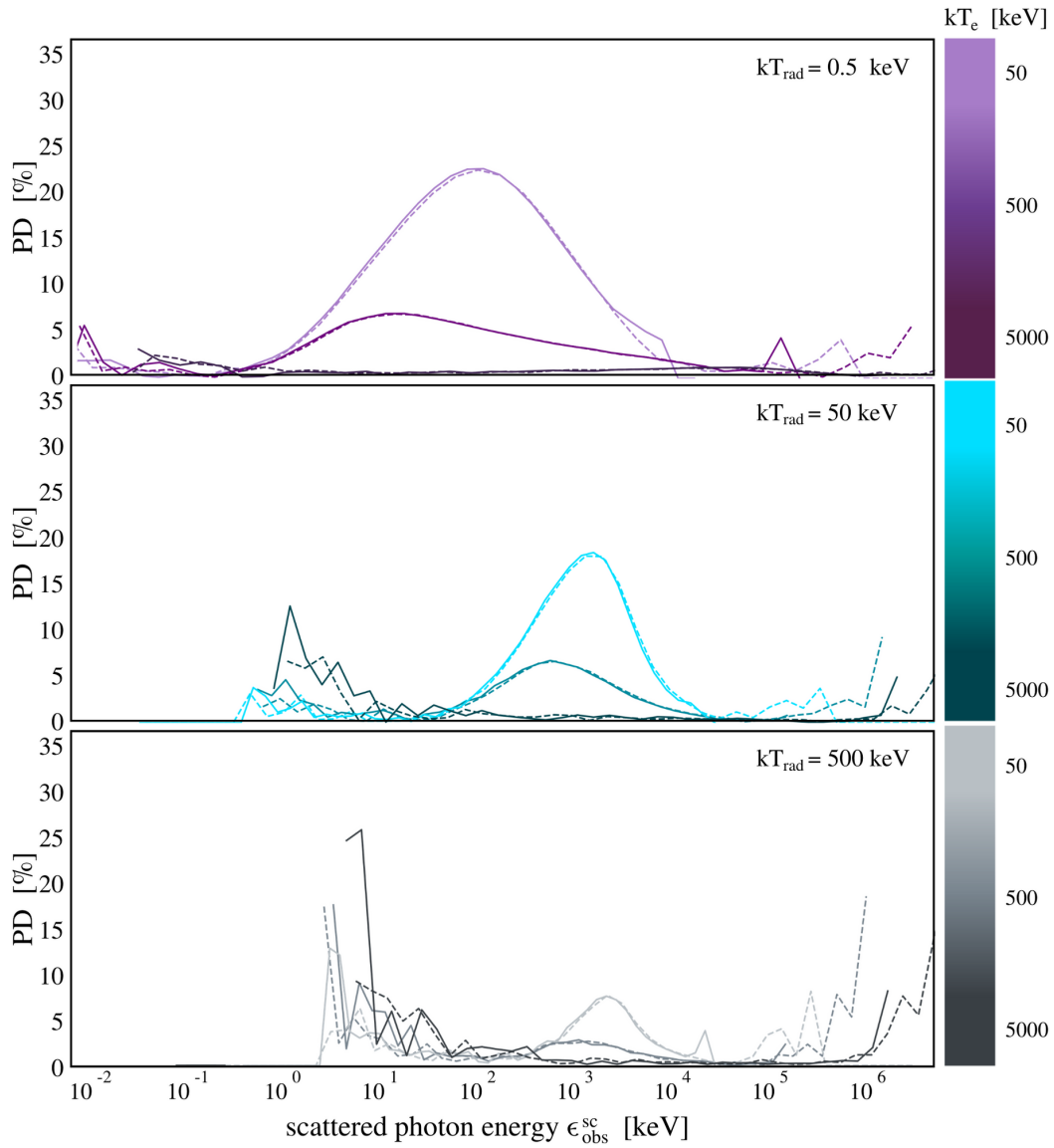


Fig. 4.4 The PD of the scattered photons as a function of the scattered photon energy. Three different target photon temperatures are given, with  $kT_{\text{rad}} = 0.5$  keV shown in purple (top panel),  $kT_{\text{rad}} = 50$  keV shown in blue (middle panel), and  $kT_{\text{rad}} = 500$  keV shown in grey (bottom panel). The results are shown for Compton scattering off non-relativistic ( $kT_e = 50$  keV), mildly-relativistic electrons ( $kT_e = 500$  keV), and relativistic electrons ( $kT_e = 5000$  keV), with the electron temperatures increasing with the shade of purple, blue, and grey in each case. The target photons are scattered by electrons drawn from either a Maxwell distribution (solid lines) or a hybrid (Maxwell/power-law) distribution (dashed lines).

with  $kT_{\text{rad}} = 0.5$  keV shown in purple,  $kT_{\text{rad}} = 50$  keV shown in blue, and  $kT_{\text{rad}} = 500$  keV shown in grey. In each case, the PD (top panels) and PA (bottom panels) are shown for all scattering directions.

The PD as a function of  $\Theta_{\text{obs}}^{\text{sc}}$  indicates at which angles the maximum polarization occurs. In the emission-region rest frame, almost all the photons move in the negative jet direction with  $\epsilon_e = \epsilon_{\text{em}}\gamma(1 - \beta \cos \theta)$  (where  $\theta$  is the angle between the electron and photon directions of motion). For the photons scattered in the Thomson regime, the maximum PDs occur at the right angles of  $\Theta_e^{\text{sc}} \sim \frac{\pi}{2}$  rad in the electron rest frame, which are essentially the same in the emission-rest frame for non-relativistic electrons. Boosting to the observer frame for  $\Gamma \gg 1$ , the photons at the right angles of  $\Theta_e^{\text{sc}} \sim \frac{\pi}{2}$  rad in the emission-rest frame, will be observed at angles of  $\Theta_{\text{obs}}^{\text{sc}} \sim \frac{1}{\Gamma}$  rad (see equation 3.40). The maximum PDs for non-relativistic electrons occur thus at angles of  $\Theta_{\text{obs}}^{\text{sc}} \sim \frac{1}{\Gamma}$  rad (indicated with a green line in Fig. 4.5), which correspond to the *edge-on* view with respect to the jet.

Considering Compton scattering off mildly-relativistic electrons with  $kT_e = 500$  keV, the peak of the electron distribution is around  $\gamma \sim 2$ . Assuming the emission region moves with a Lorentz factor of  $\Gamma = 10$ , the target photons with  $kT_{\text{rad}} = 50$  keV are boosted to  $kT_{\text{rad}} \sim 50\Gamma$  keV  $\sim 500$  keV in the emission frame. The peak of the black body spectrum thus appears at  $\Gamma \frac{2.8 kT_{\text{rad}}}{m_e c^2} \sim 2.7$ . For scattering to happen in the Thomson regime ( $\epsilon_e < 1$ ), the electrons have to move in the same direction as the target photons (backwards in the jet). The scattered photons move perpendicular to their incoming direction in the electron rest frame, which appear at an angle  $\sim \frac{1}{\gamma}$  rad with respect to the backward direction, so that the scattered photons have an angle of  $\Theta_{\text{em}}^{\text{sc}} = \left(\pi - \frac{1}{\gamma}\right)$  rad in the emission frame. Relativistic aberration into the observer's frame causes the maximum PD to occur in angles (indicated with green in Fig. 4.6) that are larger than those in the case of non-relativistic electrons ( $\Theta_{\text{obs}}^{\text{sc}} \sim \frac{1}{\Gamma}$  rad, indicated with green line in Fig. 4.5). For example, if an electron with  $\gamma = 3$  scatters a photon at a right angle with respect to its incoming direction (in the electron rest frame), then the scattered photon will move in the emission frame with an angle of  $\Theta_{\text{em}}^{\text{sc}} \sim 2.8$  rad. The photon is boosted to the observer frame with Equation 3.40 which results in an angle of  $\Theta_{\text{obs}}^{\text{sc}} \sim 0.6$  rad.

The PD as a function of  $\epsilon_{\text{obs}}^{\text{sc}}$  indicates in which energy regime the maximum PD occur. Considering the detection of polarization, this allows for the evaluation of which energy band provides the most promising prospects for the detection of high-energy polarization in astrophysical sources. Fig. 4.3 clearly shows that the energy regime of the maximum PD becomes smaller,

and shifts to higher energies, for higher photon energies. In each case, the energy regime also becomes smaller for higher electron temperatures, and almost diminishes for Compton scattering off relativistic electrons. For the photons scattered in the Thomson regime by non-relativistic electrons (Fig. 4.7), the maximum PD appears at  $\epsilon_{obs}^{sc} \sim 2.8kT_{rad}\Gamma^2$ . For mildly-relativistic electrons the energy of the maximum PD (Fig. 4.8) is significantly lower than that since the electrons move backwards in the jet. The energy of the target photons in the electron rest frame is  $\epsilon_e \sim 2.8kT_{rad}\frac{\Gamma}{\gamma}$  which is equal to  $\epsilon_e^{sc}$  in the Thomson regime. This means that the energy of the photons in the co-moving frame of the emission region is  $\sim 2.8kT_{rad}\Gamma$  but the boost to the frame of the observer is not just by a factor of  $\Gamma$  since the scattered photons are moving backwards in the emission region.

As mentioned in Section 3.3, the PA for unpolarized radiation is undefined, and randomly assumes all possible values for  $PA \in [0, \pi]$  rad. For the fraction of the scattered photons that is polarized, the PA assumes a constant value at  $PA = \frac{\pi}{2}$  rad (*vertical* polarization), for both Compton scattering off non-relativistic and mildly-relativistic electrons.

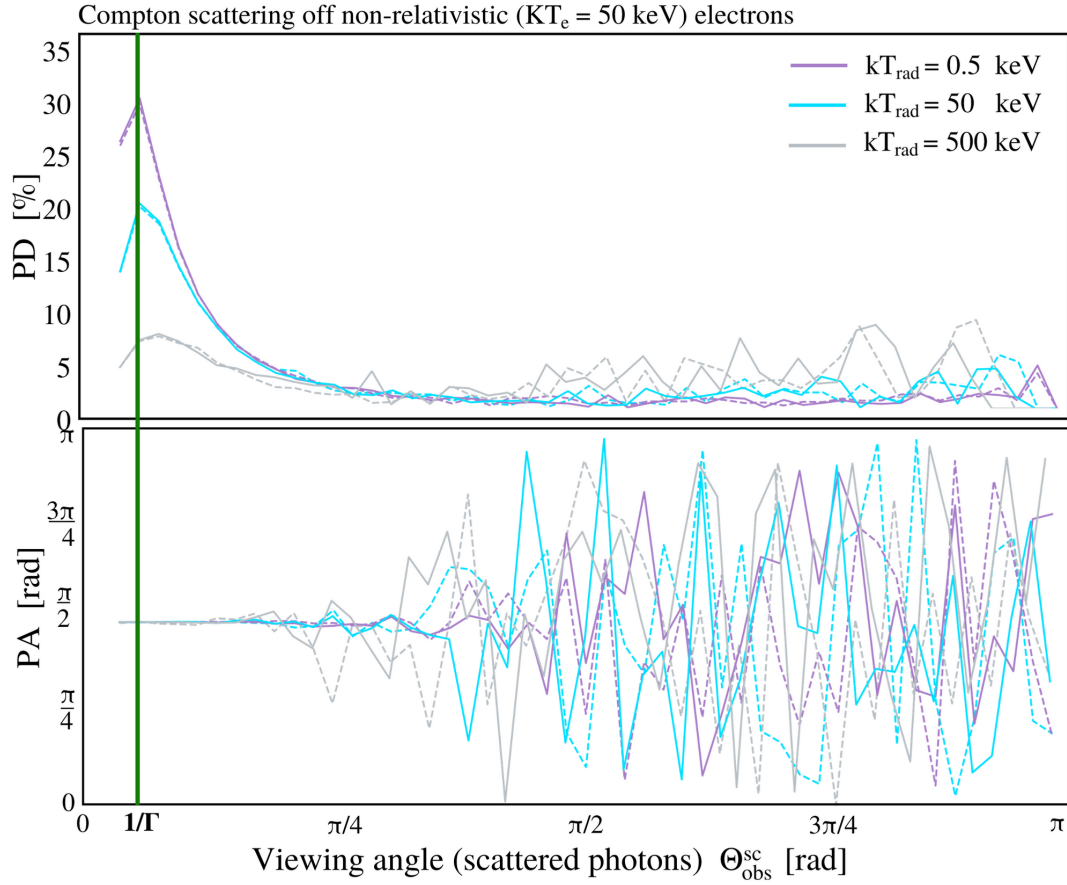


Fig. 4.5 The PD (top panel) and PA (bottom panel) of the scattered photons— due to Compton scattering off non-relativistic electrons ( $kT_e = 50$  keV) – as a function of the viewing angle. Three different target photon temperatures are given, with  $kT_{rad} = 0.5$  keV shown in purple,  $kT_{rad} = 50$  keV shown in blue, and  $kT_{rad} = 500$  keV shown in grey. The maximum PD occurs at angles of  $\Theta_{obs}^{sc} \sim \frac{1}{\Gamma}$  rad shown in a green line.

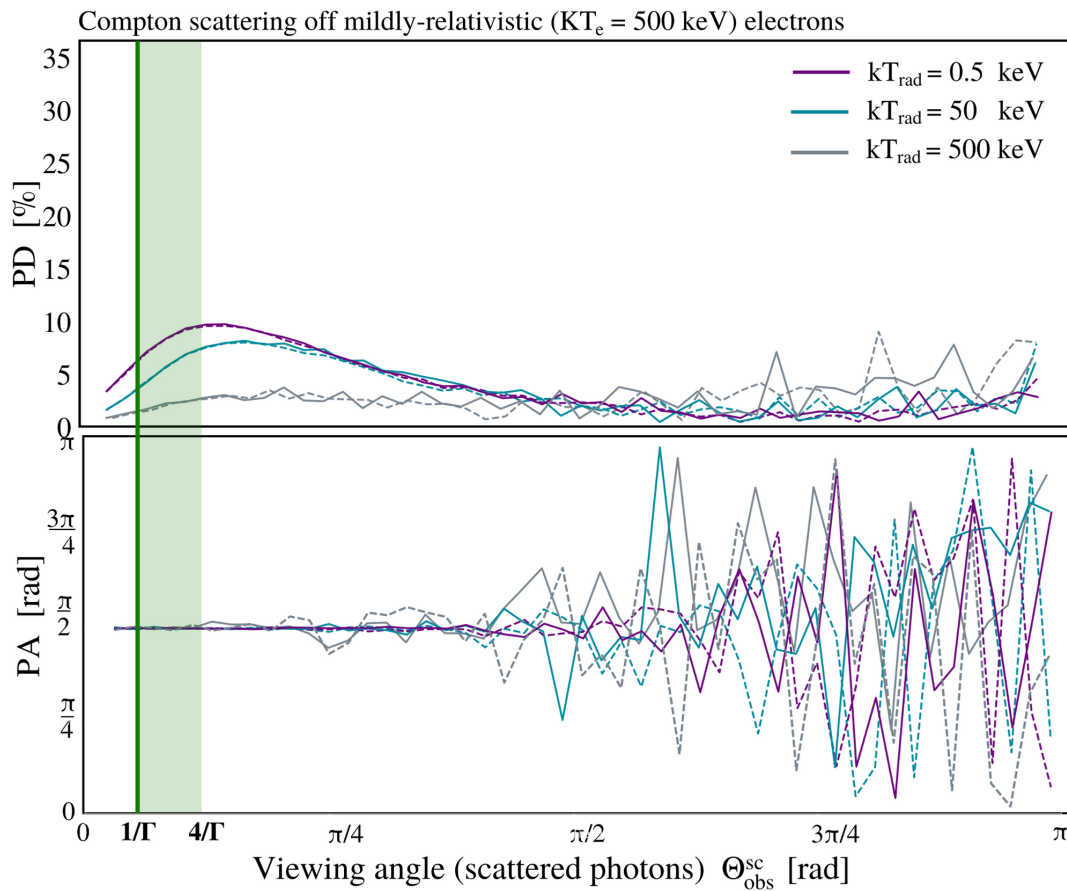


Fig. 4.6 The PD (top panel) and PA (bottom panel) of the scattered photons— due to Compton scattering off non-relativistic electrons ( $kT_e = 50$  keV) – as a function of the viewing angle. Three different target photon temperatures are given, with  $kT_{rad} = 0.5$  keV shown in purple,  $kT_{rad} = 50$  keV shown in blue, and  $kT_{rad} = 500$  keV shown in grey. The target photons are scattered by electrons drawn from either a Maxwell distribution (solid lines) or a hybrid (Maxwell/power-law) distribution (dashed lines). The maximum PD occurs at angles larger (indicated with the green shaded area) than those for non-relativistic electrons ( $\Theta_{obs}^{sc} \sim \frac{1}{\Gamma}$  rad, shown with a green line).

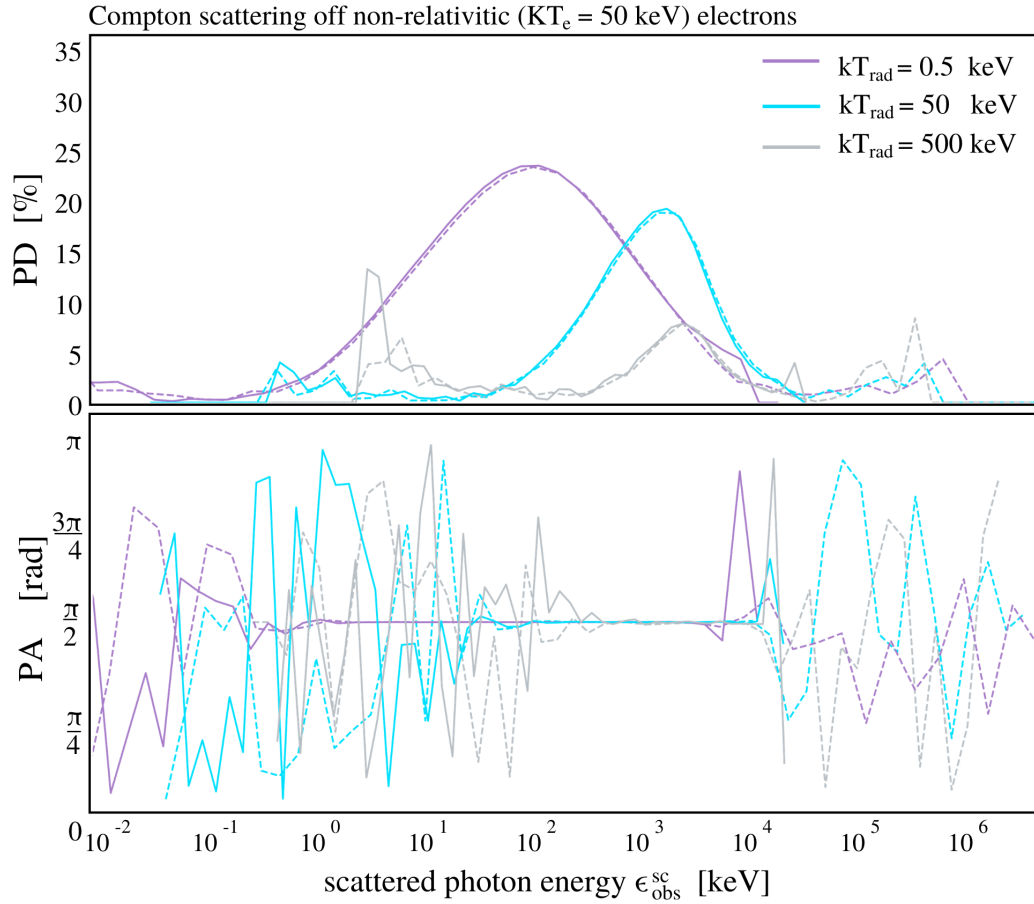


Fig. 4.7 The PD (top panel) and PA (bottom panel) of the scattered photons— due to Compton scattering off non-relativistic electrons ( $kT_e = 50$  keV) – as a function of the scattered photon energy. Three different target photon temperatures are given, with  $kT_{\text{rad}} = 0.5$  keV shown in purple,  $kT_{\text{rad}} = 50$  keV shown in blue, and  $kT_{\text{rad}} = 500$  keV shown in grey. The target photons are scattered by electrons drawn from either a Maxwell distribution (solid lines) or a hybrid (Maxwell/power-law) distribution (dashed lines).

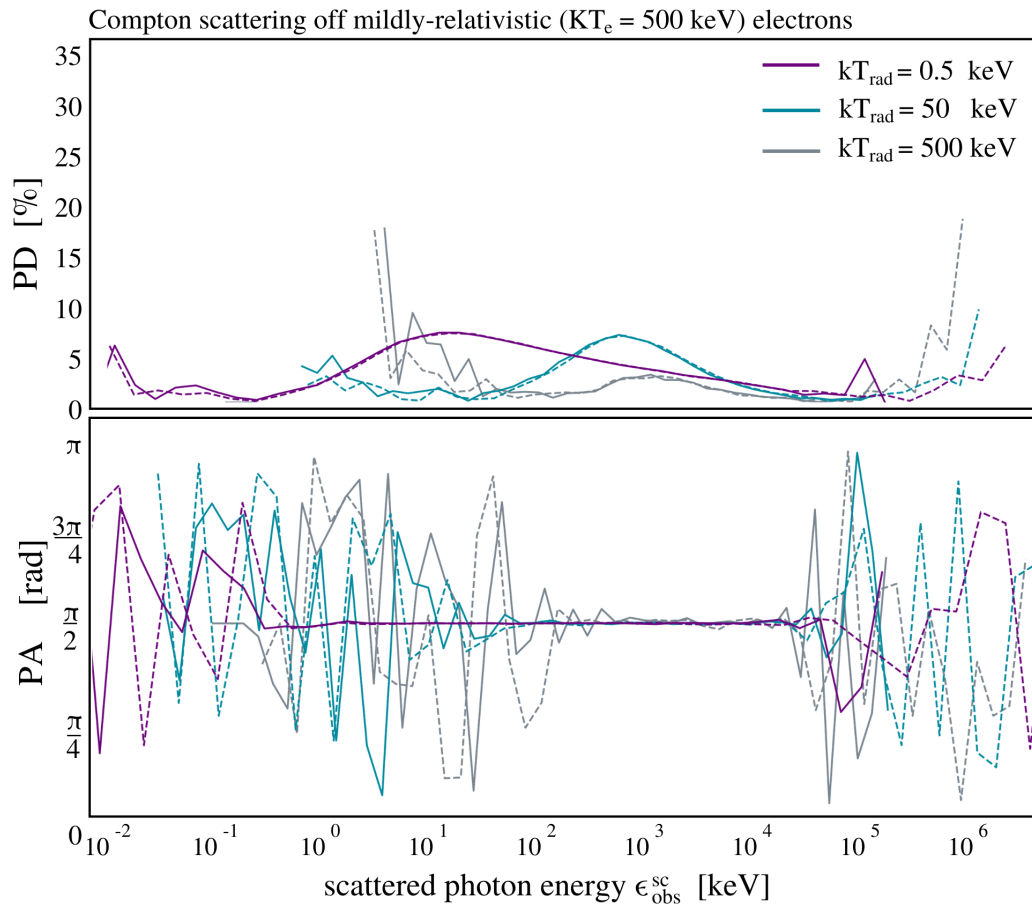


Fig. 4.8 The PD (top panel) and PA (bottom panel) of the scattered photons— due to Compton scattering off non-relativistic electrons ( $KT_e = 50$  keV) – as a function of the scattered photon energy. Three different target photon temperatures are given, with  $kT_{\text{rad}} = 0.5$  keV shown in purple,  $kT_{\text{rad}} = 50$  keV shown in blue, and  $kT_{\text{rad}} = 500$  keV shown in grey. The target photons are scattered by electrons drawn from either a Maxwell distribution (solid lines) or a hybrid (Maxwell/power-law) distribution (dashed lines).



# Chapter 5

## Conclusion and Future Outlook

The code described in section 3.3 is capable of predicting the Compton polarization in different jet-like astrophysical sources for different photon energies and electron temperatures. The effects of Compton scattering between photons and electrons depend on the target photon energy, the Lorentz factor of the electrons  $\gamma$  (thus the thermal temperatures and the energy distribution of the electrons), as well as the bulk Lorentz factor  $\Gamma$  of the emission region. In general, the target photons scatter to higher energies with a factor of  $\gamma^2\Gamma^2$  in the Thomson regime, and have cut-off energies that correspond to the reduction of the cross-section in the Klein-Nishina regime.

The polarization degree (PD) of the scattered photons depends on the effects of the Compton scattering due to the polarization dependence of the Klein-Nishina cross-section discussed in section 2.2.2 (see Equation 2.30). The polarization angle (PA) that corresponds to the maximum PD has a constant value of  $PA \sim \frac{\pi}{2}$  rad that corresponds to *vertical* polarization, regardless of the photon energy or electron temperatures.

The PD decreases with increasing photon energies, and decreases further with higher electron temperatures. The energy regimes with non-negligible PDs shift to higher energies for higher target photon energies. The energy regimes also become smaller for higher photon energies, further narrowing for higher electron temperatures. Thus, polarization is expected to arise in the Thomson regime, while no polarization is expected to be induced for photons scattered in the Klein-Nishina regime. The energy and viewing angle where the maximum PD occurs suggests that Compton polarization is sensitive to relativistic aberration for mildly-relativistic electrons with Lorentz factors of  $\gamma \gtrsim 2$ .

## 1. Future Outlook

The code developed in this project simulates Comptonization with unpolarized target photons and is restricted to study the high-energy polarization from different astrophysical sources. Compton scattering of an external field by thermal electrons produces a bulk Compton feature in the spectral energy distributions (SEDs) of some blazars. The first application of the code will be to simulate the polarization signatures from a model where the soft X-ray excess (Big Blue Bump) in blazar spectra arises from the bulk Compton process, as proposed in e.g., [Böttcher et al. \(2017\)](#) for the BL Lac object AO 0235+164.

Furthermore, by adding synchrotron polarized target photons, the code will be used to study the transition from low-energy to high-energy polarization in synchrotron self Compton (SSC) scenarios. This will reinforce future prospects of using measurements of polarization signatures to suggest observational strategies – combining optical, X-ray, and  $\gamma$ -ray polarimetry – to determine the degree of ordering of the magnetic field, and to use high-energy polarization as a diagnostic to distinguish leptonic from hadronic high-energy emission from blazars.

The Imaging X-ray Polarimetry Explorer (IXPE) ([Weisskopf et al., 2016](#)) is scheduled for launch in early 2021 (see Tab. 2.1). IXPE can deliver scientifically meaningful polarimetric images of the brightest X-ray sources, including Active Galactic Nuclei (AGN) jets. Applying the code developed in this project to different jet-like astrophysical sources can provide information about the expected polarization characteristics (e.g. how the polarization changes as a function of the viewing angle and which energy regime is the most convenient in terms of the polarization amplitude) in the sources of interest.

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