

Chapter 7

A New Wavy Current Sheet Drift Model

7.1 Introduction

A potential drawback of the SDE approach is the inherent difficulty of the numerical scheme in handling discontinuous structures e.g. the HCS. Although some SDE models have included the wavy HCS structure [Miyake and Yanagita, 2005; Alanko-Huotari *et al.*, 2007], most models have focused on implementing the easier-to-handle flat HCS scenario with $\alpha = 0$ [Zhang, 1999; Florinski and Pogorelov, 2009; Pei *et al.*, 2010; Strauss *et al.*, 2011a], as has been done in the previous chapters. In this chapter, the 3D SDE model discussed in Chapter 4 is extended to include a 3D wavy HCS valid for $\alpha \in [0, \pi/2]$. Characteristics of the model are shown, as well as selected results, which include pseudo-particle traces illustrating drift along the HCS. The model is applied to the recent PAMELA proton and anti-proton observations as an illustration of charge-sign dependent modulation.

Results from this chapter are presented by Strauss *et al.* [2012a, b].

7.2 Drift Along a Wavy Heliospheric Current Sheet

The HCS drift velocity is directed parallel to the HCS and perpendicular to the HMF [e.g. Burger and Potgieter, 1989; Burger *et al.*, 1985]. In order to construct an expression for the HCS drift velocity $\vec{v}_{ns}$, a vector directed parallel to the HCS is defined as

$$\vec{p}(\epsilon) = \cos(\pm\epsilon) \mathbf{e}_r + \sin(\pm\epsilon) \mathbf{e}_\theta + \cos(\pm\epsilon) \mathbf{e}_\phi, \quad (7.1)$$

given in terms of an angle $\epsilon(r, \theta, \phi, \alpha) \in (-\pi/2, \pi/2)$. The geometry of this situation is shown in Fig. 7.1. A projection (at a constant value of ϕ) of the HCS is shown as the solid wavy line. Assume now a CR located at (r, θ, ϕ) . This CR will *sense* the effect of the HCS at a point (r', θ', ϕ') located on the HCS, with (r', θ', ϕ') being the closest point to (r, θ, ϕ) on the HCS. For the $A > 0$ cycle, this positively charged CR will drift *upwards* (i.e. towards the northern solar

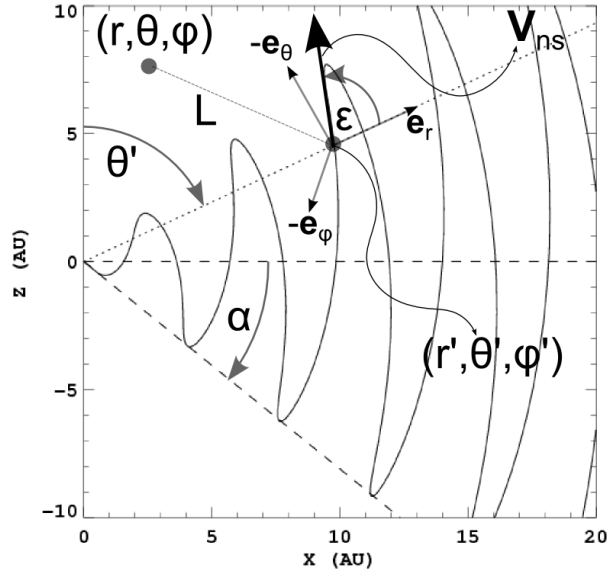


Figure 7.1: The geometry of the HCS. All labels refer to the notation used in this chapter.

pole) with a velocity $\vec{v}_{ns}$ as indicated on the figure. The angle ϵ is defined as the angle between $\vec{p}$ (which will be directed parallel to $\vec{v}_{ns}$) and the radial direction. This angle is calculated as

$$\tan^2 \epsilon = \left[-r \frac{\partial \theta'}{\partial r} \right]^2 + \left[\frac{1}{\sin \theta} \frac{\partial \theta'}{\partial \phi} \right]^2, \quad (7.2)$$

which reduces, for a Parker HMF geometry and assuming a constant solar wind speed, to

$$\tan(\pm \epsilon) = \pm \frac{\Omega r}{V_{sw}} \frac{1}{\sin \Psi} \frac{\sqrt{\sin^2 \alpha - \cos^2 \theta'}}{\sin \theta'}. \quad (7.3)$$

Similar expressions are given by *Caballero-Lopez and Moraal* [2003] and *Burger* [2012], but derived by using different expressions for the HCS's angular extent θ' . All of these approaches become identical when α becomes small, so that $\tan \alpha \approx \sin \alpha \approx \alpha$.

For a flat HCS, $\alpha = 0 \Rightarrow \epsilon = 0$, resulting in $\sin \epsilon = 0$ and $\cos \epsilon = 1$; the meaning of which will become clear in Eq. 7.6. The correct sign of ϵ also needs to be evaluated. This can be determined by examining the change of θ' with respect to the spatial coordinate r ,

$$\frac{\partial \theta'}{\partial r} \propto \cos \left(\phi + \phi_0 + \frac{\Omega r}{V_{sw}} \right). \quad (7.4)$$

When θ' increases with r , $\vec{p}$ will be directed upwards, leading to the relationships

$$\epsilon \begin{cases} < 0 : \frac{\partial \theta'}{\partial r} < 0 \\ > 0 : \frac{\partial \theta'}{\partial r} > 0. \end{cases} \quad (7.5)$$

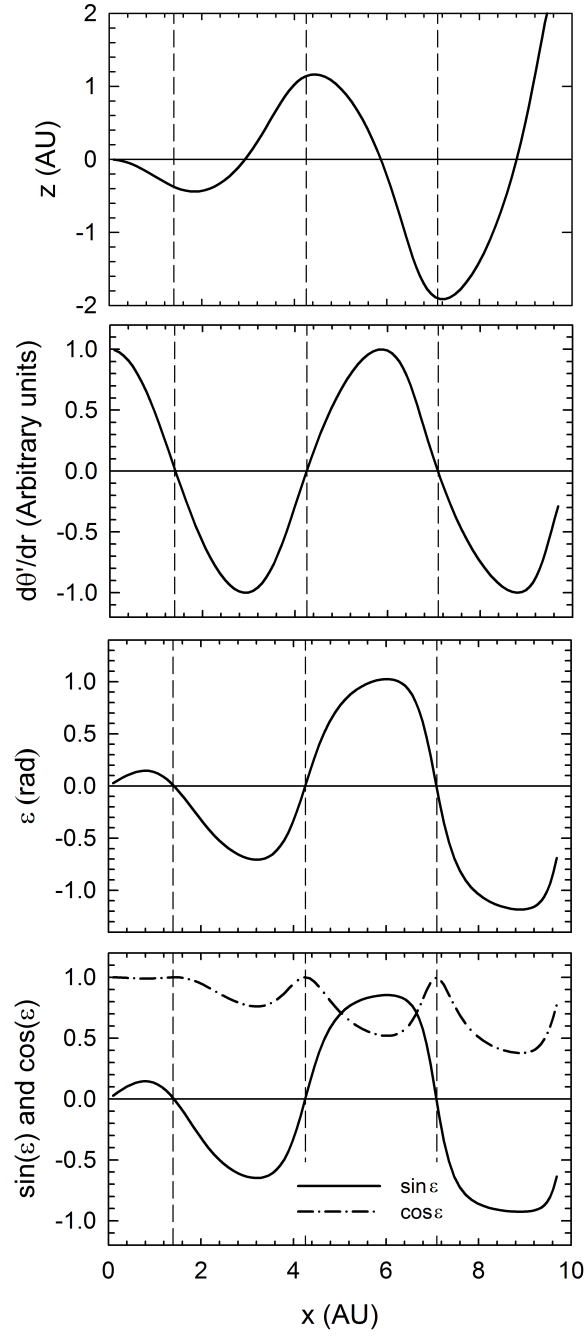


Figure 7.2: The top panel shows a projection of the HCS onto the meridional plane. The lower panels show the calculated values of $d\theta'/dr$, ϵ , $\sin \epsilon$ and $\cos \epsilon$, for this scenario, as used to determine the direction of $\vec{v}_{ns}$.

The calculation of ϵ is illustrated in Fig. 7.2, where the HCS, $\partial\theta'/\partial r$, $\cos \epsilon$ and $\sin \epsilon$ are shown. Note that $\cos \epsilon$ is always positive, while $\sin \epsilon$ changes sign with the *waviness* of the HCS.

Finally, combining all of the results above, the HCS drift velocity is given by

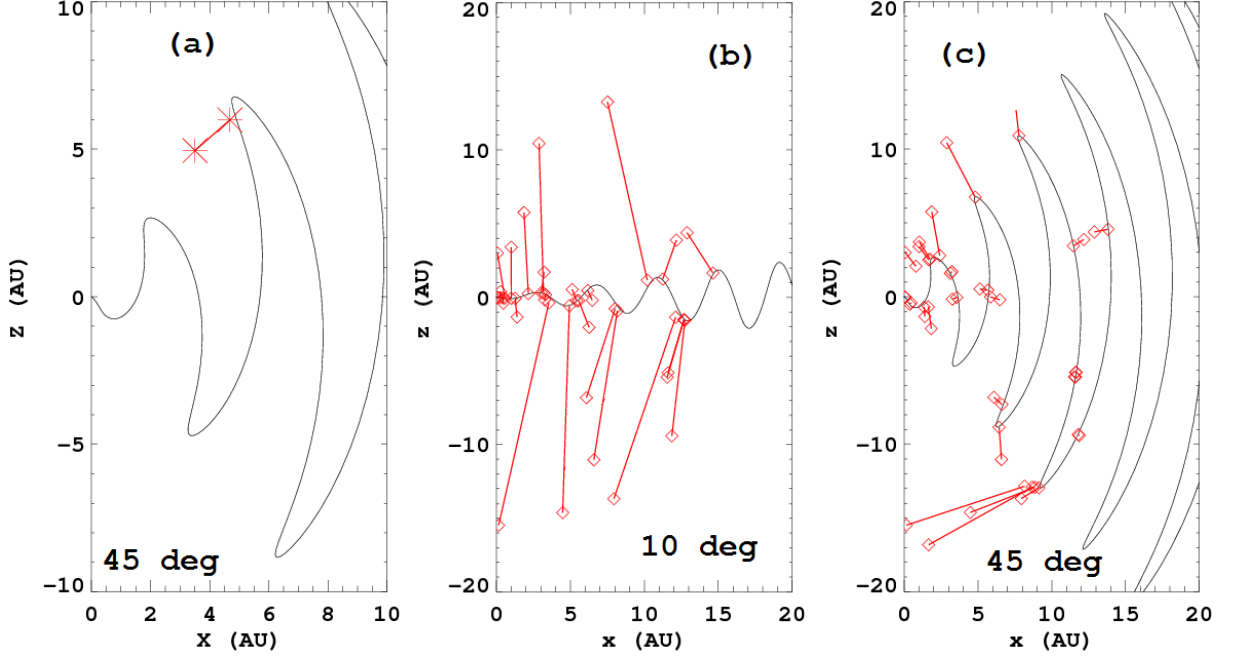


Figure 7.3: Panel (a) illustrates the optimization problem of incorporating a wavy HCS into the model. The left point is the position of a pseudo-particle (r, θ, ϕ) , while the right point is the corresponding closest point to the HCS (r', θ', ϕ') . Panels (b) and (c) show the same scenario, but for 20 randomly selected points with $\alpha = 10^\circ$ and $\alpha = 45^\circ$ respectively.

$$\vec{v}_{ns} = v_{ns} \{ \cos(\pm\epsilon) \sin \Psi \mathbf{e}_r + \sin(\pm\epsilon) \mathbf{e}_\theta + \cos(\pm\epsilon) \cos \Psi \mathbf{e}_\phi \} \cdot qA, \quad (7.6)$$

with v_{ns} the magnitude thereof, $q = \pm 1$ the charge of the CR population under consideration, and $A \pm 1$ indicating the HMF polarity. For v_{ns} , the approximation of *Burger et al.* [1985] is again used

$$v_{ns} = \left\{ 0.457 - 0.412 \frac{|L|}{r_L} + 0.0915 \frac{|L|^2}{r_L^2} \right\} v \quad (7.7)$$

where L is the smallest distance from the CR's current position to the HCS (as in Fig. 7.1). This equation is however only applied when

$$|L| \leq 2r_L. \quad (7.8)$$

The HCS is thus not added as a true discontinuity, but has an effective *thickness* of $4r_L$. It can also be proven that the condition $\nabla \cdot \vec{v}_d = 0$ is satisfied. Writing the divergence as

$$\nabla \cdot \vec{v}_d = \nabla \cdot \vec{v}_{gc} + \nabla \cdot \vec{v}_{ns}, \quad (7.9)$$

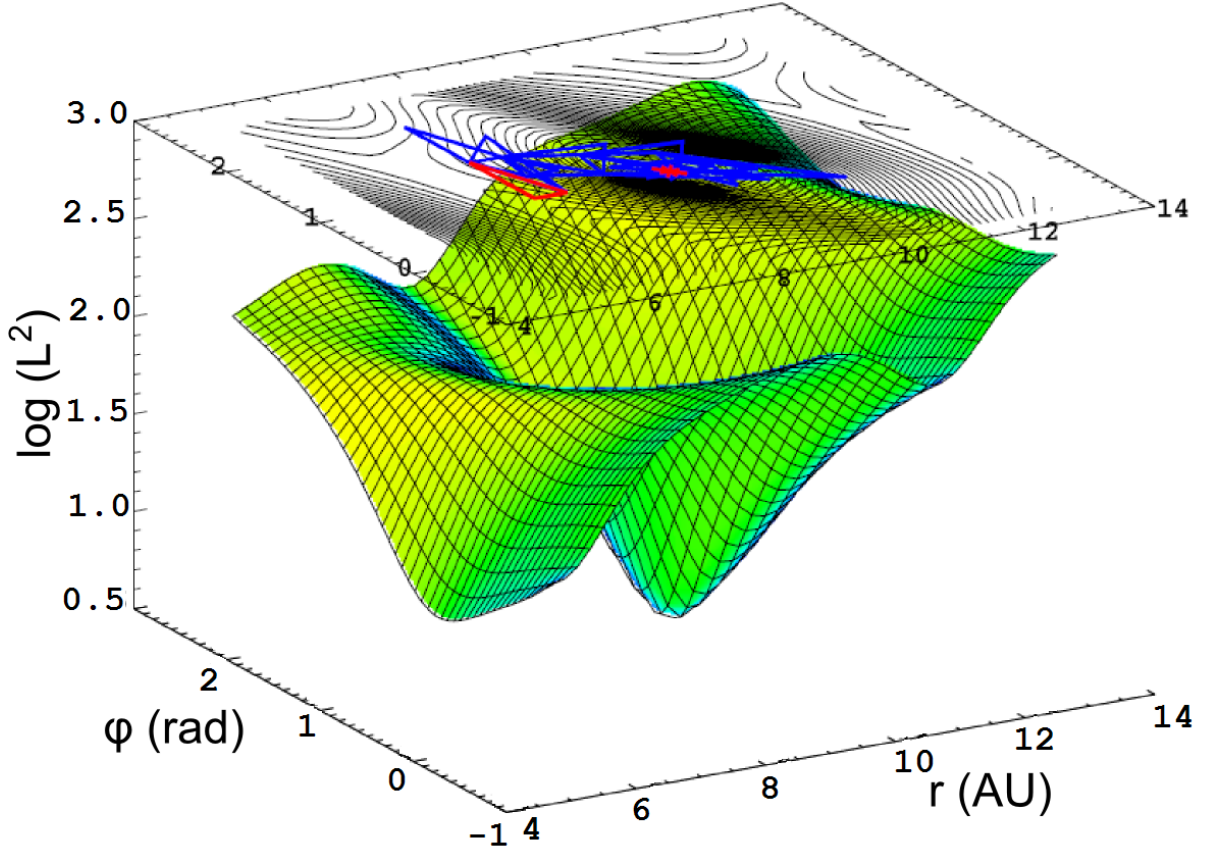


Figure 7.4: Illustrating the minimization of L as implemented in this chapter. The surface and contour graphs show $L(r', \phi')$ for the point (r, θ, ϕ) in panel (a) of Fig. 7.3. The series of triangles overlaid on the contour graph shows the Nelder-Mead optimization scheme at work in calculating the minima of L , indicated by the red asterisk.

the first term is zero due to the condition $\nabla \cdot \vec{B} = 0$, while the second term is zero, as the magnitude of $\vec{v}_{ns}$ only changes perpendicular to the HCS [Burger *et al.*, 1985; Burger and Potgieter, 1989].

It must be noted that Eq. 7.7 is derived for a locally flat HCS, meaning that CRs are assumed to interact with only a single fold of the HCS at a given time. The validity of this assumption has recently been challenged by the simulations of Florinski [2011]. This previously unexplored process (that can be called *drift short circuiting*) is however not yet understood, and as such, is not included in the present model.

A difficulty lies in calculating L , i.e. the minimum distance between (r, θ, ϕ) and (r', θ', ϕ') . The distance between these two points,

$$|L(r, \theta, \phi, r', \phi')|^2 \approx (\Delta l)^2 = (\Delta r)^2 + r^2 (\Delta \theta)^2 + r^2 \sin^2 \theta (\Delta \phi)^2, \quad (7.10)$$

with

$$\begin{aligned}
\Delta r(r, r') &= r - r' \\
\Delta \theta(\theta, r', \phi') &= \theta - \theta'(r', \phi') \\
\Delta \phi(\phi, \phi') &= \phi - \phi'
\end{aligned} \tag{7.11}$$

must thus be minimized, i.e. (r', θ', ϕ') must be the closest point to (r, θ, ϕ) on the HCS. The properties of the HCS at (r', θ', ϕ') will determine the direction of $\vec{v}_{ns}$, while L will be used to calculate its magnitude. Because θ' is a function of r' and ϕ' , $L = L(r', \phi')$ (being unique for each point (r, θ, ϕ)) and the minimization problem above reduces to a spatially 2D problem. Analytical solutions of Eq. 7.10 are impossible, and therefore a numerical minimization (optimization) scheme is adopted. Panel (a) of Fig. 7.3 illustrates the optimization problem of finding the minimum value of L : The left asterisk is an arbitrarily chosen point (r, θ, ϕ) , while the right asterisk is the corresponding closest point to the HCS, i.e. (r', θ', ϕ') . For this specific (left) point, $L(r', \phi')$ is shown in Fig. 7.4 as the surface and contour graphs. It is clear that a local minimum of $L(r', \phi')$ exists.

In this chapter, $L(r', \theta')$ is optimized numerically by using the Nelder-Mead [Nelder and Mead, 1965] search algorithm (also referred to as the amoeba or downhill simplex method). In this approach, three initial points are chosen, forming the initial simplex (triangle). Through a process of elongation, reflection and contraction, the initial simplex is updated with a new simplex, but with the vertices thereof closer to the minima of the function being minimized. The process is then iterated until the final simplex is sufficiently close to the minimum of e.g. L . It is found that 10 iterations of the algorithm is sufficient to minimize L for a variety of heliospheric test cases. A visualization of the Nelder-Mead scheme is projected onto Fig. 7.4: The initial simplex is shown in red, all subsequent triangles by blue and the calculated minimum by the red asterisk. This numerically calculated minimum corresponds to the right asterisk shown in panel (a) of Fig. 7.3. Note that the HCS projections shown in Fig. 7.3 are done at an azimuthal angle of ϕ and do not precisely coincide with the actual position of ϕ' which would vary for each test point (r, θ, ϕ) . In panels (b) and (c) of this figure, L is minimized for 20 randomly selected points, with both (r, θ, ϕ) and (r', θ', ϕ') shown for each pair of points. Panel (b) is for $\alpha = 10^\circ$, while panel (c) is for $\alpha = 45^\circ$. From inspection, it is clear that the minimization scheme can in fact minimize L under a variety of conditions. The accuracy of this algorithm is however dependent on the choice of the initial simplex, and it is chosen to be close to (r, θ, ϕ) . This makes the convergence of the algorithm faster, but gives less accurate solutions if L gets very large. However, CRs do not *feel* the effect of the HCS for this scenario (see also Eq. 7.8).

The minimization scheme discussed here is valid for $\alpha \in (0, \pi/2]$. The case when $\alpha = 0$ (a flat, azimuthally independent HCS) is excluded, as L does not have a local minimum in terms of ϕ' . The scheme can, however, still be applied to this case, but with the restriction that

$$\phi' \equiv \phi : \alpha = 0, \tag{7.12}$$

reducing the Nelder-Mead search algorithm to a 1D line search in terms of r and r' .

7.3 Characteristics of the Model Solutions

Benchmarks of the new wavy HCS drift model with results from two other spatially 3D models, namely the models discussed by *Burger* [2012] and *Pei et al.* [2012], are presented in Fig. 7.5. What is also of interest, is that these models use different numerical schemes to solve the CR transport equation; *Pei et al.* [2012] used SDEs (similar to this model), while *Burger* [2012] uses a more traditional finite difference numerical method. Moreover, the drift field is also handled differently in these models, with the approach used here being close to the one adopted by *Pei et al.* [2012].

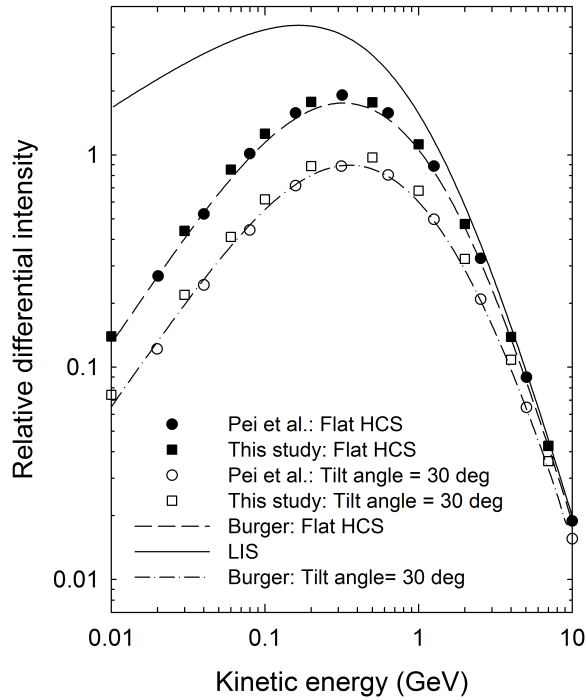


Figure 7.5: A comparison between modulated proton energy spectra as computed by various spatially 3D drift models, namely: This study (squares), *Pei et al.* [2012] (circles) and *Burger* [2012] (broken lines). Two modelled scenarios are shown; the case of a flat HCS ($\alpha = 0^\circ$) and an azimuthally varying HCS of $\alpha = 30^\circ$.

Fig. 7.5 shows a comparison between the results of these three models with respect to an assumed LIS. All simulations are for galactic proton energy spectra at Earth in the $A < 0$ drift cycle, using modulation parameters identical to those used by *Kóta and Jokipii* [1983]. Two model scenarios are shown, namely the case of a flat HCS ($\alpha = 0^\circ$) and when $\alpha = 30^\circ$. It is clear that the results from all three models agree well, vindicating the modelling approach (both the numerical scheme and the implementation of the drift field) as implemented in this chapter. There is, however, a small discrepancy between the results for the $\alpha = 30^\circ$ scenario. This is

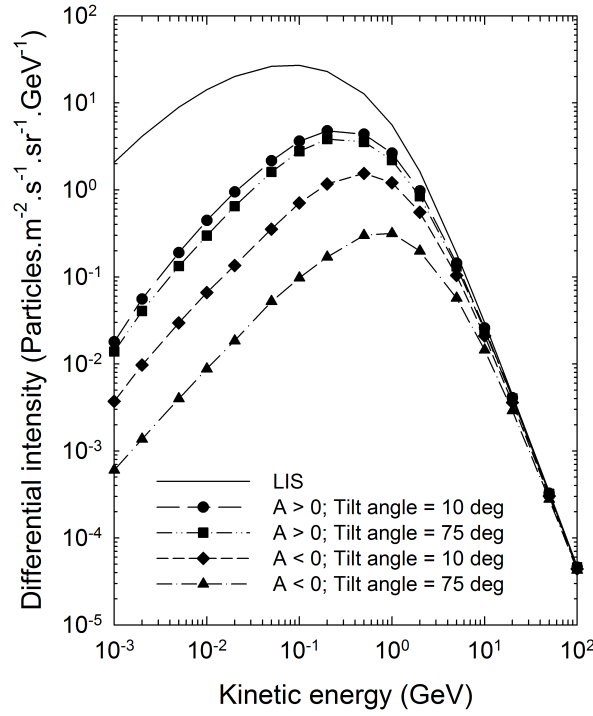


Figure 7.6: Computed energy spectra at Earth for both HMF cycles, for the cases of $\alpha = 10^\circ$ and $\alpha = 75^\circ$.

due to different expressions for θ' being used in the different models, with the discrepancy disappearing as $\alpha \rightarrow 0^\circ$.

The modelling results shown in this section assume a spherical heliosphere with the HP located at $r_{HP} = 100$ AU. Furthermore, $\vec{V}_{sw} = 400 \text{ km.s}^{-1} \mathbf{e}_r$ is used (constant over all latitudes) with a proton parallel mean free path of

$$\lambda_{||} = \lambda_0 \left(\frac{B}{B_0} \right) \left(\frac{P}{P_0} \right) \quad (7.13)$$

with $\lambda_0 = 0.05$ AU, $P_0 = 1$ GV and B_0 the HMF magnitude at Earth. Also, isotropic perpendicular diffusion with $\lambda_{\perp, \theta} = 0.02 \lambda_{||}$ is adopted [e.g. Giacalone and Jokipii, 1999]. These values are chosen for illustrative purposes, but also lead to proton intensities at Earth which are comparable to observed levels.

Fig. 7.6 shows energy spectra at Earth as calculated by the modulation model for galactic protons. As input to the modulation model, the LIS of Moskalenko *et al.* [2002] is assumed. Solutions are shown for the $A > 0$ (squares and filled circles) and $A < 0$ (triangles and diamonds) HMF polarity cycles. Two pairs of solutions are shown, namely for minimum solar activity conditions (triangles and squares) assuming $\alpha = 10^\circ$, and for the case when $\alpha = 75^\circ$ (diamonds and filled circles). Note that only α and the HMF polarity are changed in the modulation model, neglecting the temporal behaviour of e.g. B , so that the $\alpha = 75^\circ$ model solution represents

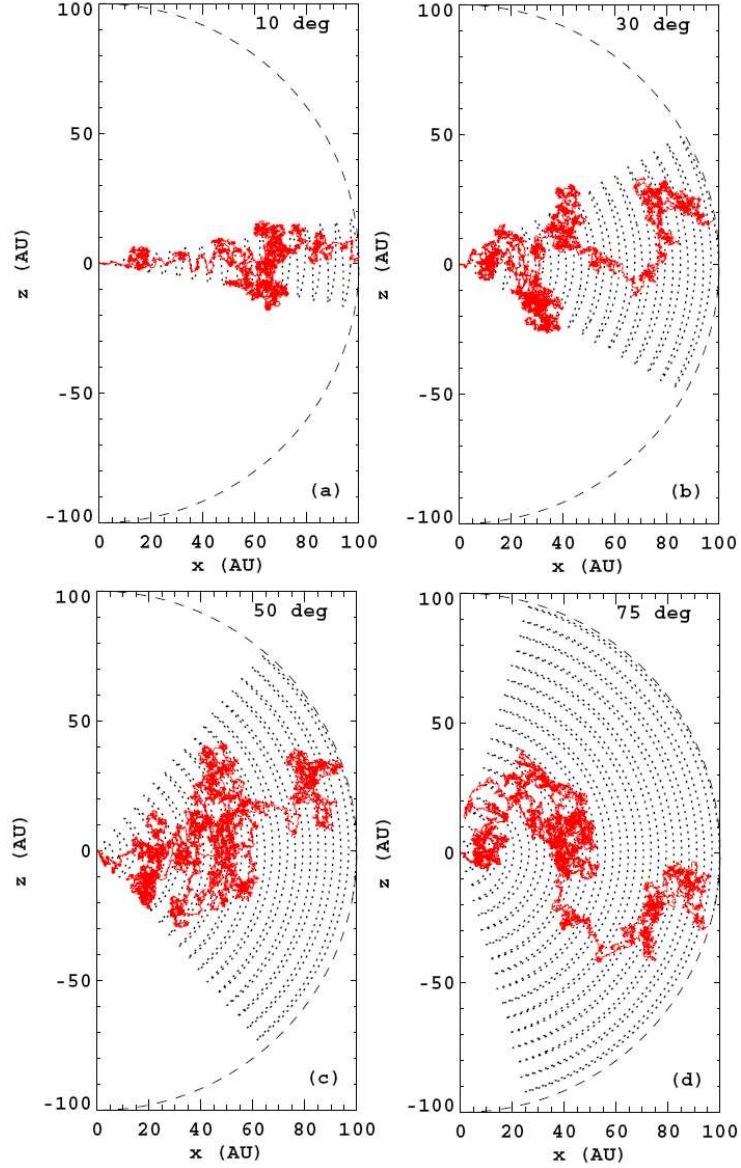


Figure 7.7: Pseudo-particle traces (trajectories) for galactic protons in the $A < 0$ HMF cycle projected onto the meridional plane for various values of α , shown as the dark lines. The HP position is indicated by the dashed line, while the dotted line shows a projection of the HCS onto the same plane. The simulations are done for $E = 100$ MeV.

increased solar activity and not actual solar maximum modulation conditions. Generally, the modelled $A > 0$ intensities are higher than those of the $A < 0$ cycle, except at the highest energies where the spectra cross [see also *Reinecke and Potgieter, 1994*]. This is in accordance with CR drifts: In the $A > 0$ cycle, protons reaching Earth drift primarily from the polar regions, while they drift inwards along the HCS in the $A < 0$ cycle, taking in general longer to do so, and thus also undergoing more modulation. The effect of increasing α is thus also much more pronounced in the $A < 0$ cycle, while the intensities of the $A > 0$ cycle are only marginally affected.

The SDE modelling approach allows for the visualization of the different drift patterns of the

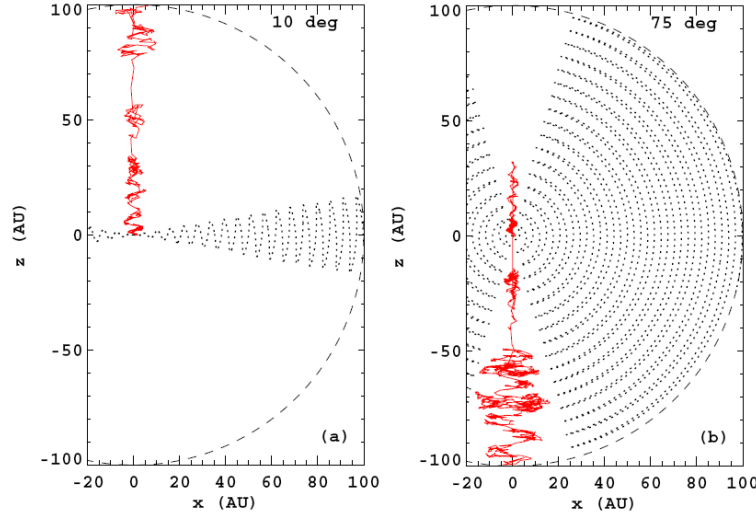


Figure 7.8: Similar to Fig. 7.7, but now for the $A > 0$ cycle, illustrating how effectively CRs drift into the inner heliosphere under these assumptions.

CRs in different drift cycles, as is done in Figs. 7.7 and 7.8. Fig. 7.7 shows pseudo-particle traces for protons in the $A < 0$ cycle and varying values of α , projected onto the meridional plane, as the dark solid lines. The dashed line shows the position of the HP, while the dotted line shows a projection of the HCS onto the same plane. What is clear is that these particles propagate mainly in latitudes covered by the HCS, and that their transport is greatly affected by different values of α . For small values of α (panels (a) and (b)) the CRs do, to a certain extent, follow the wavy HCS, although diffusion disrupts these *perfect* drift patterns. For larger values of α , diffusion almost wipes out the drift pattern (panel (d)). This topic will be discussed again later in Section 7.4. Fig. 7.8, on the other hand, is for the $A > 0$ cycle. Here, the protons drift from the polar regions to reach Earth and do not sample the HCS, except in the extreme case where α reaches its maximal value. This behaviour is characteristic of CR drifts in a purely Parkerian HMF and represents ideal drift conditions.

Fig. 7.9 shows the position (in terms of their polar and azimuthal angles) from which protons that reach Earth, enter at the HP. This is shown for the $A < 0$ (red circles) and $A > 0$ (blue squares) HMF cycles and varying values of α . The equatorial plane is located at $\theta = \pi/2$ (indicated by the dashed line), while the maximal latitudinal extent of the HCS ($\theta = \pi/2 \pm \alpha$) is indicated by the solid lines. As expected, the protons originate near the polar regions in the $A > 0$ cycle, while originating in the HCS regions in the $A < 0$ cycle. Note that for $\alpha > 45^\circ$, the CRs do not originate uniformly from the HCS regions, but tend to be clustered more or less near the equatorial regions; an effect investigated later in this chapter.

Fig. 7.10 shows the proton intensity response at Earth at $E = 1$ GeV to changing α for the different HMF cycles. For the $A > 0$ cycle, the intensities are affected very little by changing α , with the intensities slightly decreasing with increasing values of α . For the $A < 0$ cycle, however, the effect is much more pronounced. The CR intensities decrease sharply with in-

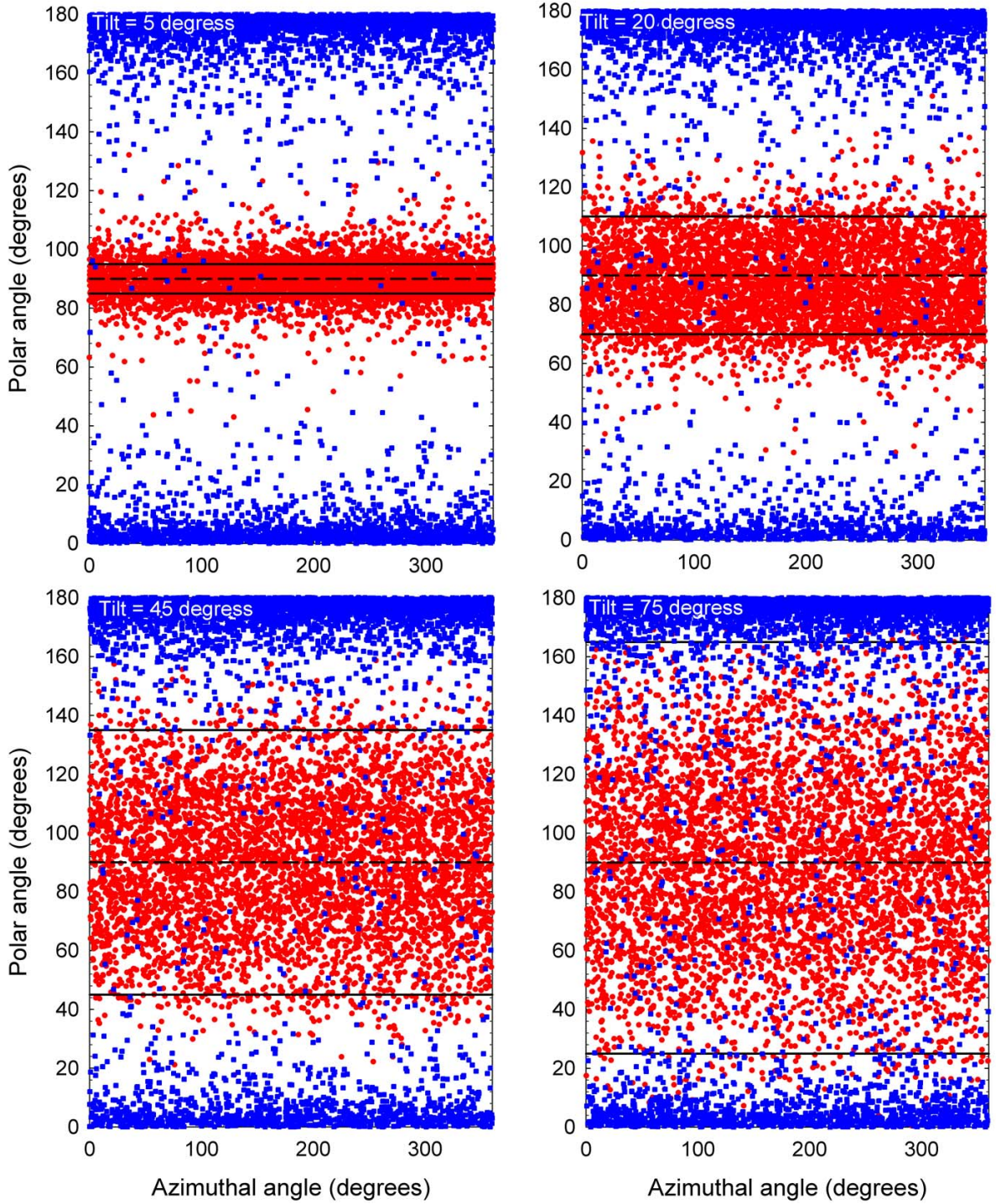


Figure 7.9: The position (in terms of the polar and azimuthal angles) at which protons, that reach the Earth, enter the heliosphere. These are shown for the $A < 0$ (red circles) and $A > 0$ (blue squares) HMF cycles and for four values of α : 5° , 20° , 45° and 75° . The dashed line indicates the equatorial plane, while the solid lines show the maximum latitudinal extent of the HCS.

creasing α up to $\alpha \sim 45^\circ$. For $\alpha > 45^\circ$, the intensities maintain relatively constant levels. The reason behind this behaviour is discussed in the next section. Note that when $\alpha = 0$, the pro-

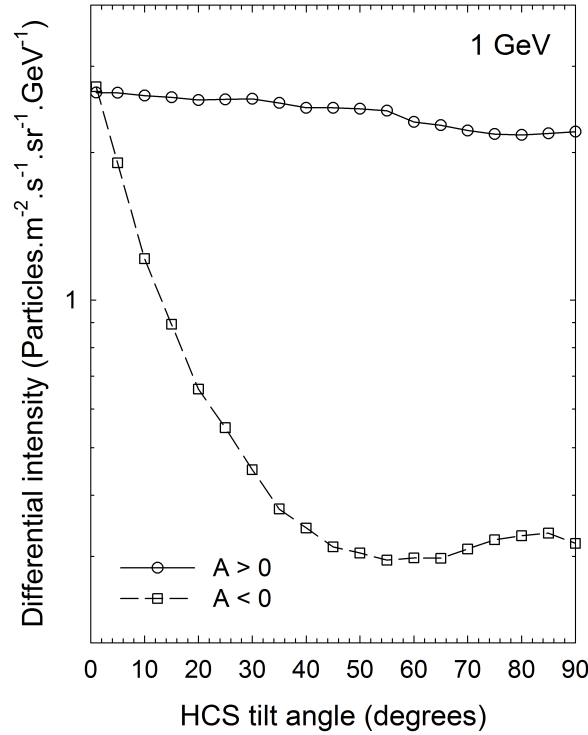


Figure 7.10: The response of the 1 GeV proton differential intensity at Earth to changing values of α for both HMF cycles.

ton intensities are highest in the $A < 0$ cycle; characteristic of CR drift models [see e.g. *Reinecke and Potgieter, 1994*]. Qualitatively, the modelling results shown here are consistent with results from previous drift dominated finite differences modulation models [e.g. *Jokipii et al., 1977; Potgieter and Moraal, 1985; Webber et al., 1990; Burger, 1999*], but with the additional insights obtained from Figs. 7.7 - 7.9.

A generic solar cycle is modelled in Fig. 7.11. The value of α is changed from $\alpha = 75^\circ$ to $\alpha = 10^\circ$ and back to $\alpha = 75^\circ$, as shown in the bottom panel of the figure. This is, however, not done time dependently (where the changing value of α is propagated with the solar wind speed from the Sun to the HP), but as a series of steady-state solutions where α is changed instantaneously throughout the heliosphere (see *le Roux and Potgieter [1990]* for a discussion of both of these approaches and how the resulting intensities differ). By changing α , the resulting intensities at Earth at $E = 100$ MeV are shown in the top and middle panels of Fig. 7.11. As already discussed, the $A > 0$ intensities are relatively insensitive to changing values of α , while the $A < 0$ intensities are sensitive. The resulting temporal profile for the $A < 0$ cycle has a peaked form, while it is relatively flat for the $A > 0$ cycle. This is in accordance with NM and space based observations of the ~ 22 year CR cycle [e.g. *Strauss et al., 2012c*].

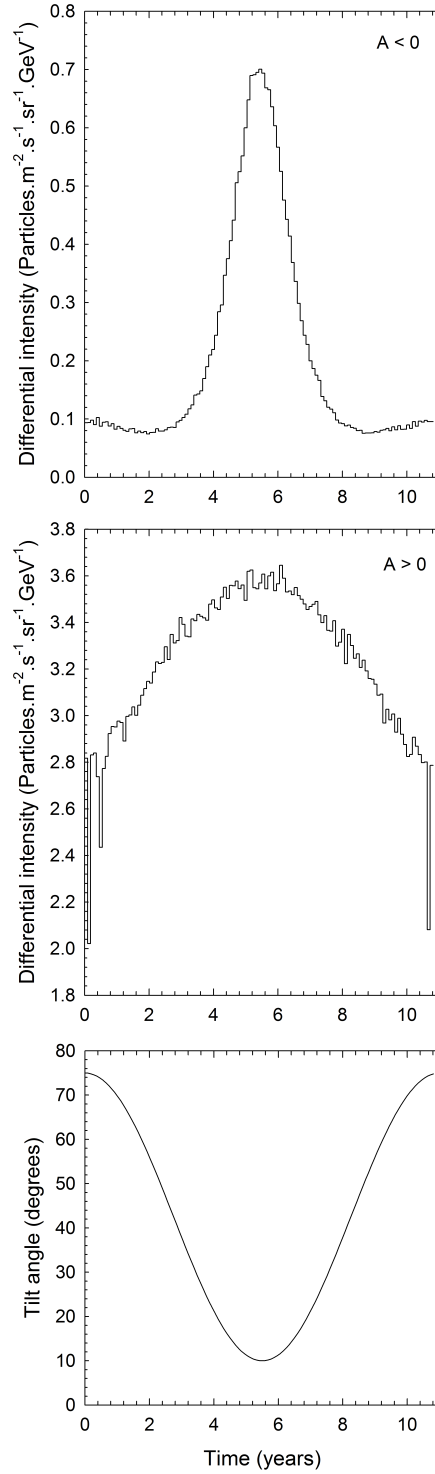


Figure 7.11: A modelled generic solar cycle. The HCS tilt angle is changed as shown in the bottom panel, while the modelled intensities are shown for the $A < 0$ (top panel) and $A > 0$ (middle panel) HMF cycles. The proton intensities are shown at Earth with $E = 100$ MeV.

7.4 Propagation Times and Energy Losses

In order to investigate the effectiveness of HCS drifts, the relationship between galactic proton propagation times in the $A < 0$ cycle and the effective length of the HCS is investigated in this

section. The length of the HCS at a constant azimuthal angle (i.e. $d\phi = 0$) is calculated as the line integral

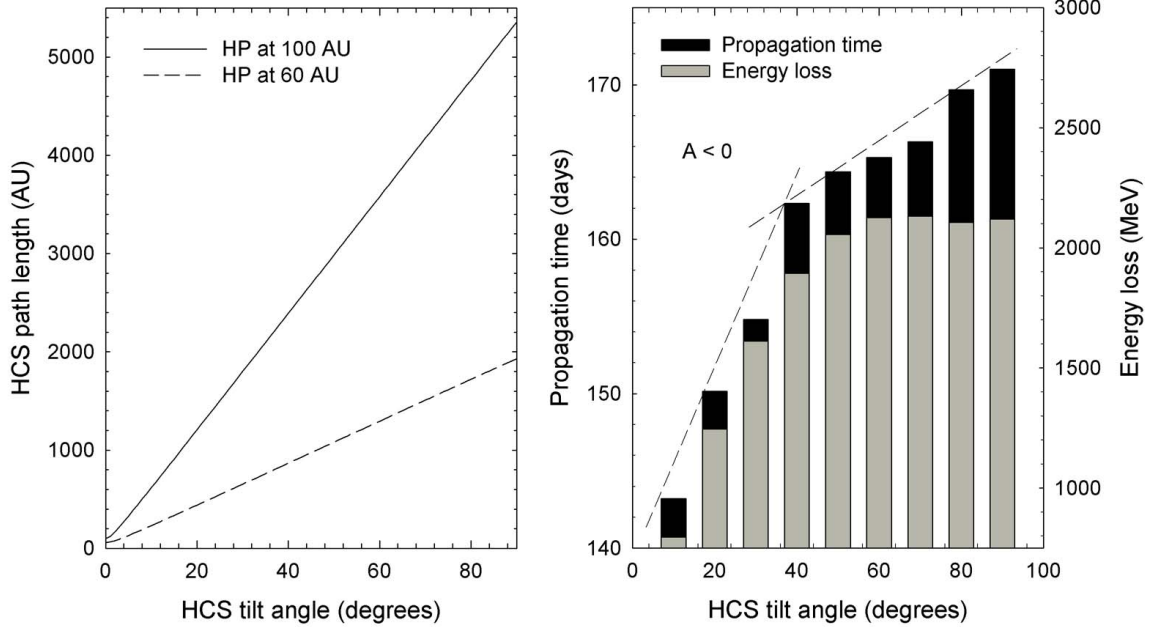


Figure 7.12: The left panel shows the HCS path length, calculated for $r_{HP} = 100$ AU (solid line) and $r_{HP} = 60$ AU (dashed line), as a function of α for illustrative purposes. The right panel shows the calculated $\langle\tau\rangle$ (black fill) and $\langle\Delta E\rangle$ (grey fill) for galactic protons of 100 MeV in the $A < 0$ HMF cycle.

$$l = \left[\left(\int_{r_a}^{r_b} dr \right)^2 + \left(\int_{r_a}^{r_b} r d\theta_{ns}(r, \alpha) \right)^2 \right]^{\frac{1}{2}}. \quad (7.14)$$

For a flat HCS, $\alpha = 0$ ($\theta_{ns} = \pi/2 \Rightarrow d\theta_{ns} = 0$), the HCS obtains its minimum length, $l_{min} = r_b - r_a$. For the present situation, this minimum length is $l_{min} \approx r_{HP}$ ($r_a = 1$ AU at Earth and $r_b = r_{HP}$). For $\alpha > 0$, Eq. 7.14 is integrated numerically, with the calculated HCS path length shown in the left panel of Fig. 7.12. Two illustrative scenarios are shown, namely $r_{HP} = 100$ AU (solid line; as used in this study) and $r_{HP} = 60$ AU (dashed line; shown for illustrative purposes only). For these calculations, where α is assumed to be independent of r , the HCS path length is proportional to α , i.e. $l \propto \alpha$. If galactic protons would drift very effectively along the HCS towards Earth in the $A < 0$ cycle, it is expected that their propagation time $\langle\tau\rangle$ should increase linearly with increasing l , or equivalent $\langle\tau\rangle \propto \alpha$. The relationship between CR intensities and l was investigated previously by *Williams and Potgieter* [1991]. Finite differences modulation models can, however, not calculate $\langle\tau\rangle$, whereas with the SDE approach, it is possible.

The right panel of Fig. 7.12 shows $\langle\tau\rangle$, calculated at $E = 100$ MeV, for the $A < 0$ HMF cycle as a function of α . The black fills show $\langle\tau\rangle$, while the grey fills show $\langle\Delta E\rangle$. As expected, $\langle\tau\rangle$ increases with increasing α . Below $\alpha = 40^\circ$, $\langle\tau\rangle$ increases linearly with α . This is in line with

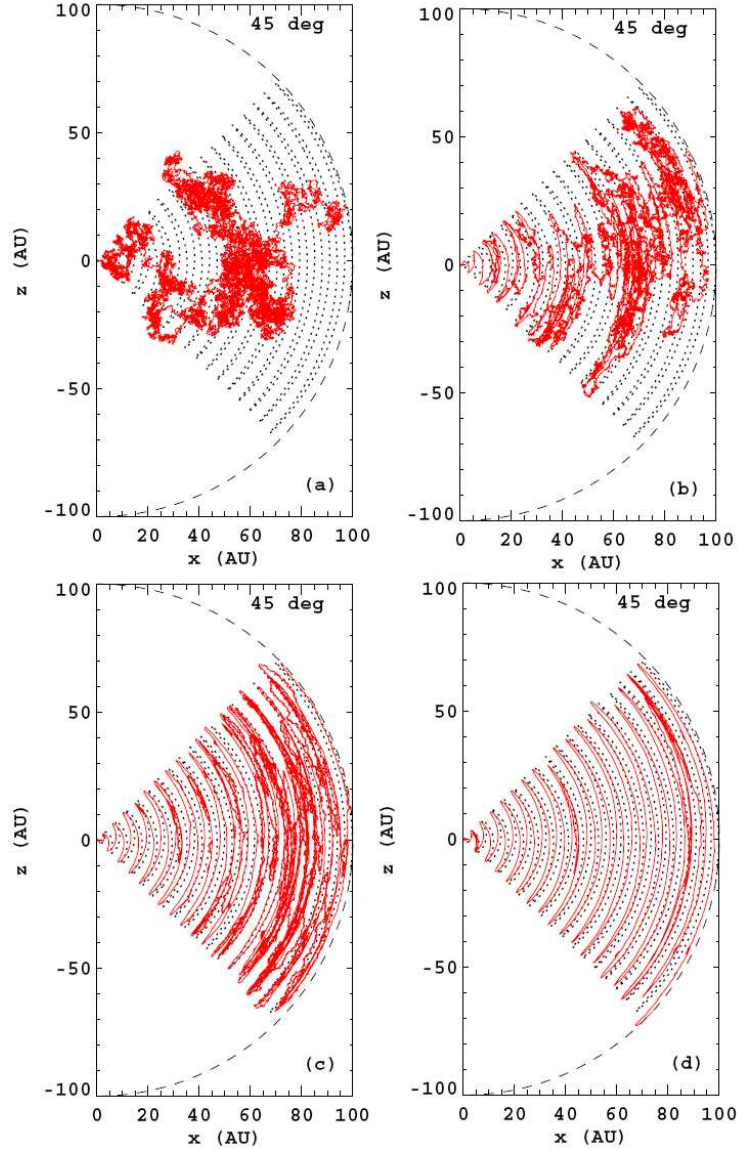


Figure 7.13: Pseudo-particle traces of 100 MeV galactic protons, similar to those shown in Fig. 7.7, but for a constant value of $\alpha = 45^\circ$ and different choices of the magnitude of $\mathbf{K}$. Panel (a) shows the resulting particle trajectory for $\mathbf{K}$ as used throughout this section, while consecutive panels show the results when $\mathbf{K}$ is divided by an additional factor of 10 each time.

the *drift dominated* transport picture discussed above, leading to the conclusion that the CRs drift quite effectively along the HCS. Above $\alpha = 40^\circ$, the relationship $\langle \tau \rangle \propto \alpha$ breaks down, forming a second linearity, but with a completely different slope. This is because the transport of the CRs becomes increasingly *diffusion dominated*, i.e. the characteristic effects of drifts on the modelled CR intensities diminish. When α is small, HCS drift is dominated by the $\hat{r}$ component of $\vec{v}_{ns}$, leading to effective transport of CRs towards the inner heliosphere (smaller values of r). However, if α becomes large, the $\hat{\theta}$ component of $\vec{v}_{ns}$ becomes increasingly dominant, and the CRs are transported very effectively towards the Earth via diffusion (mainly by means of cross field diffusion, $\kappa_{\perp r}$). The CRs in the latter scenario thus largely *ignore* the HCS's

structure. This effect is also visible in the pseudo-particle traces shown in Fig. 7.7: When α gets large, diffusion in essence disrupts the drift of CRs along the HCS. This can also explain the response of CR intensities with changing values of α shown in Fig. 7.10: Above $\alpha = 40^\circ$, the modelled intensities, for the $A < 0$ cycle, become relatively insensitive to changing α due to their transport becoming increasingly diffusion dominated. For both HMF cycles, $\langle \Delta E \rangle$ follows $\langle \tau \rangle$.

A question arises: Under which conditions do CRs not follow the HCS?

Fig. 7.13 shows pseudo-particle traces for galactic protons in the $A < 0$ cycle with $E = 100$ MeV and $\alpha = 45^\circ$. Panel (a) shows the resulting particle trajectory using Eq. 7.13 for $\mathbf{K}$ (where $\mathbf{K}$ represents the diffusion tensor), panel (b) for $\mathbf{K}/10$, panel (c) for $\mathbf{K}/100$ and panel (d) for $\mathbf{K}/1000$. With decreasing magnitude of $\mathbf{K}$, it appears that the CRs follow the HCS more effectively, i.e. the CRs *stick* better to the HCS and are less scattered away from it. In the extreme case of panel (d) an almost perfect drift pattern is obtained. This choice of $\mathbf{K}$ is, however, completely unrealistic. Perfect drift dominated CR transport (where drifts are the dominant transport process) thus does not exist, and it must be remembered that CR modulation is essentially a convection-diffusion process. Some studies completely neglect diffusion when modelling CR transport and focus only on drifts [e.g. *Robert, 2011*]; a scenario which can now be proved to be completely unjustified and unrealistic.

7.5 Charge-sign Dependent Modulation: Protons and Anti-protons

This section aims to illustrate the ability of the SDE model to reproduce and interpret charge-sign dependent modulation. As such, a more fundamental approach to the transport coefficients is taken and the results compared to recent proton and anti-proton measurements. For the parallel mean free path, the analytical form of *Engelbrecht [2008]* and *Burger et al. [2008]* is used, which is based on results from quasi-linear theory as well as a dynamical model for slab turbulence discussed by *Teufel and Schlickeiser [2003]*. For the mean free paths directed perpendicular to the HMF, the non-linear guiding centre results of *Shalchi et al. [2004]* are approximated as

$$\lambda_{\perp} = a\lambda_{\parallel} \cdot \left[\frac{P}{P_0} \right]^{-b}, \quad (7.15)$$

with $a = 0.01$ [e.g. *Giacalone and Jokipii, 1999*], $P_0 = 1$ GV and $b = 1/3$ to produce an almost energy dependent form for $\lambda_{\perp}$ below ~ 5 GV. Note that isotropic perpendicular diffusion is assumed with $\lambda_{\perp} = \lambda_{\perp r} = \lambda_{\perp \theta}$.

The drift scale, defined by

$$\lambda_d = \frac{(\omega\tau_d)^2}{1 + (\omega\tau_d)^2} r_L, \quad (7.16)$$

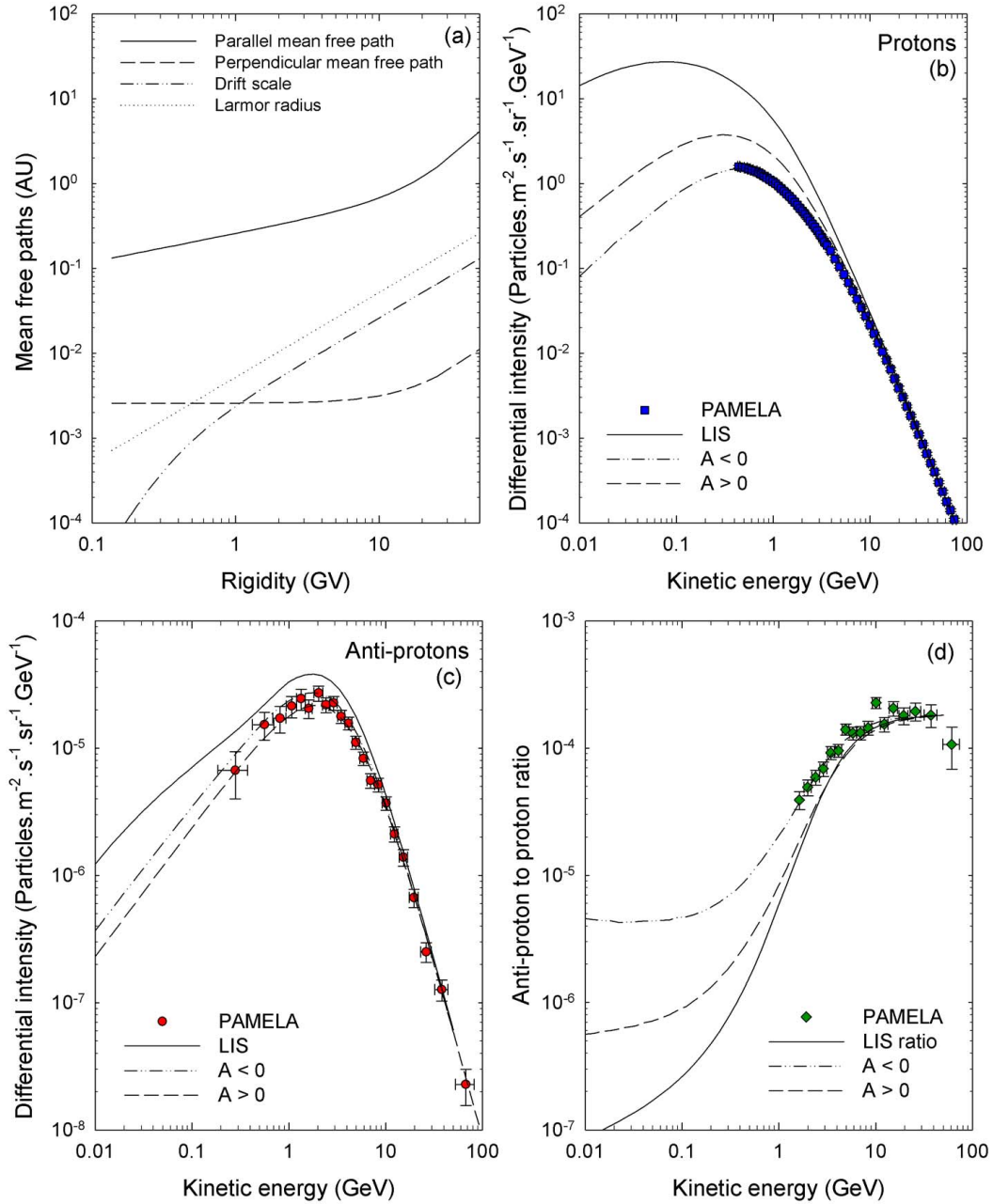


Figure 7.14: Panel (a) shows the mean free paths at Earth in the equatorial plane used in this section; $\lambda_{\parallel}$ as the solid line, $\lambda_{\perp\theta,r}$ as the dashed line and λ_d by the dashed-dotted line. Panel (b) shows calculated galactic proton energy spectra for the previous $A < 0$ solar minimum (dashed-dotted line), compared to the PAMELA proton spectrum of *Adriani et al.* [2011]. The predicted $A > 0$ spectrum (dashed line) for the next solar minimum in ~ 2021 should be considered as an upper limit. Panel (c) shows equivalent modelling results for galactic anti-protons, compared to the PAMELA anti-proton spectrum of *Adriani et al.* [2010]. Panel (d) shows modelled results for the anti-proton to proton ratio (dashed-dotted line for the past $A < 0$ cycle and the dashed line for the nearing $A > 0$ minimum). The solid line is the ratio of the two LIS, while the PAMELA observations are taken from *Adriani et al.* [2010].

with r_L the Larmor radius and $\omega\tau_d$ the so-called scattering parameter, is introduced to account for the suppression of drifts due to the presence of turbulent scattering [e.g. *Minnie et al.*, 2007].

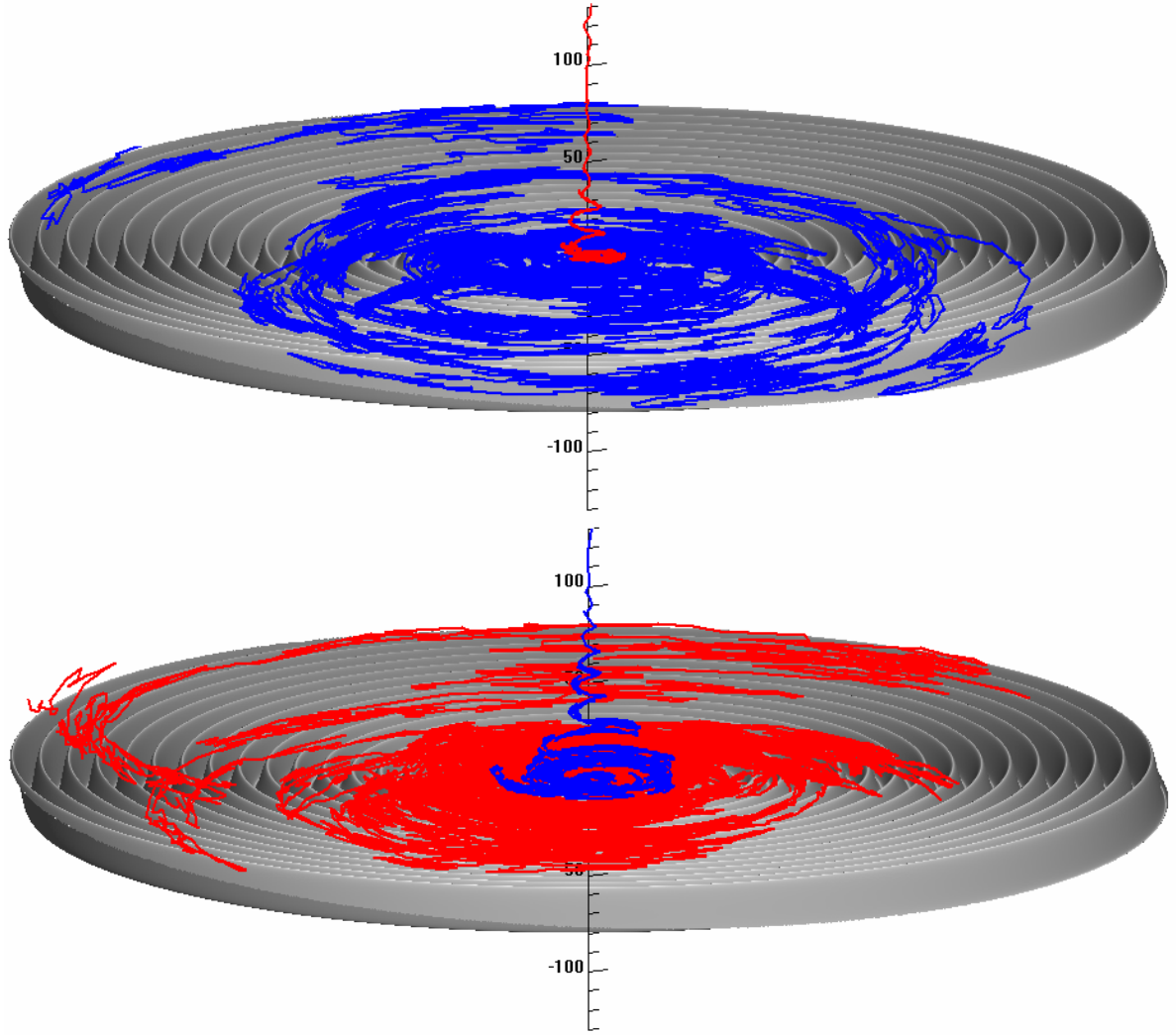


Figure 7.15: Pseudo-particle traces of protons (blue) and anti-protons (red) in the $A < 0$ (top panel) and the $A > 0$ (bottom panel) HMF polarity cycles. The grey surface shows the wavy HCS. The polar axis is also shown and labelled in AU.

Here, the approach of *Burger et al.* [2000] is followed, where $\omega\tau_d \sim P$ [see also *Langner, 2004*]. This drift suppression factor causes λ_d to be decreased at low energies. Note that, in the absence of scattering, $\lambda_d = r_L$. Both λ_d and r_L are shown in panel (a) of Fig. 7.14 as a function of rigidity together with $\lambda_{\parallel}$ and $\lambda_{\perp\theta/r}$. The solar wind speed is assumed to be latitude dependent in accordance with the Ulysses observations discussed in *Phillips et al.* [1995]. The HP is assumed to be located at $r_{HP} = 130$ AU. For protons and anti-protons, the LIS of *Moskalenko et al.* [2002] is used. For all simulations, a constant HCS tilt angle of $\alpha = 5^\circ$ is assumed, while an unmodified Parker HMF is implemented, normalized to a magnitude of 4 nT at Earth.

Modelled energy spectra for protons and anti-protons ($\bar{p}$) are shown in panels (b) and (c) of Fig. 7.14 respectively. For protons, the modelled $A < 0$ spectra are compared to the observed PAMELA spectrum of the past solar minimum. The proton $A > 0$ results were obtained by

using the same transport parameters, only switching the HMF polarity. As a Parker HMF is used, this spectrum should be interpreted as an upper limit for the proton flux in the next $A > 0$ solar minimum. The modelled $\bar{p}$ spectrum is also consistent with the PAMELA observations, using exactly the same drift and diffusion coefficients used for protons (except of course for the charge). Note that, for both protons and $\bar{p}$, the energy spectra at low energies exhibit the well known $j \propto E$ adiabatic range. For $\bar{p}$, the LIS is also quite close to this spectral shape, so that the $\bar{p}$ spectra exhibit the $j \propto E$ limit very quickly. This produces very little difference between the $\bar{p}$ intensities for the two drift cycles, as compared to that of protons.

The modelled anti-proton to proton ratio is shown in panel (d) of Fig. 7.14 for both HMF cycles at Earth. These results are also compared successfully to PAMELA observations. At very low energies the ratio becomes independent of energy due to the adiabatic effect discussed above. The ratio of anti-protons to protons differs by a factor of ~ 10 between drift cycles at 10 MeV, with the difference decreasing with increasing energy. Beyond ~ 30 GeV the ratio becomes independent of drifts (and also modulation) as a high energy *no modulation* regime is reached, where the mean free path becomes equivalent to the size of the heliosphere. Because the modelled $A > 0$ spectrum is an upper limit, the modelled ratio for the $A > 0$ cycle should be considered as a lower limit prediction for the next solar minimum.

Fig. 7.15 shows pseudo-particle traces for protons (blue) and anti-protons (red) in the $A < 0$ (top panel) and $A > 0$ (bottom panel) HMF cycles. The shaded surface shows the HCS. These trajectories show the well-known charge-sign dependent difference in drift directions, with protons and $\bar{p}$ sampling different regions of the heliosphere in the same drift cycle. In the $A > 0$ cycle, protons reach Earth by drifting from the polar regions, while they reach Earth from the equatorial (HCS) region in the $A < 0$ cycle. For $\bar{p}$ these drift directions reverse. For further discussions regarding charge-sign dependent modulation, see also the modelling done by Heber and Potgieter [2006], Potgieter and Moraal [1985], Langner and Potgieter [2004a], as well as the $\bar{p}$ and proton modelling of Webber and Potgieter [1989], Potgieter [1995], Langner and Potgieter [2004b].

The previous $A < 0$ solar minimum of 2009 is generally referred to as being *unusual*. Observations by NMs, as well as space borne instruments, were at the highest levels ever recorded [Heber et al., 2009; Mewaldt et al., 2010; McDonald et al., 2010; Moraal and Stoker, 2010; Vos, 2011], while the Sun was particularly quiet for a prolonged period of time [Kane, 2010]. It should be noted that if the same heliospheric conditions persist in the following $A > 0$ solar minimum of ~ 2021 , even higher fluxes (at least for positively charged CRs) will be recorded, due to drift effects, as illustrated in Fig. 7.14. The highest levels of CR flux might therefore be yet to come [see also Potgieter et al., 2013].

7.6 Summary and Conclusions

A new numerical approach to implementing drifts, especially focusing on current sheet drifts in a spatially 3D CR modulation model was discussed. This approach allows for the incorporation of a full 3D wavy HCS and is valid up to the theoretical maximum of $\alpha = 90^\circ$. Moreover, because the model uses SDEs to solve the relevant transport equation, the modulation model remains numerically stable, even for these extreme scenarios.

Benchmarks for the modulation model, and more importantly the drift field as used in the model, were presented and showed excellent agreement between the present model and results from two contemporary spatially 3D numerical models, vindicating the modelling approach used here.

Modelled CR intensities behave qualitatively similar to first generation modulation models for different drift cycles and different values of α . In accordance with general drift considerations, galactic proton intensities are found to be relatively insensitive to changing values of α in the $A > 0$ drift cycle. For the $A < 0$ HMF cycle however, the effect of changing α is very pronounced with proton intensities decreasing sharply with increasing α . Examining pseudo-particle trajectories of protons in the two drift cycles, it was clearly illustrated why this is the case: In the $A > 0$ cycle, protons drift from the polar regions to reach Earth and are less influenced by the HCS, as these CRs only rarely interact with it. This is not the case for the $A < 0$ cycle, where the protons drift along the HCS, and thus sample it extensively. Although these considerations are by no means new, the SDE approach allows for the visualization of this effect.

The SDE approach allows for the calculation of CR propagation times, something not possible to calculate analytically, or by means of finite different modulation models. The effectiveness of HCS drifts in the $A < 0$ HMF cycle is investigated by calculating τ for different values of α and comparing these values to the total path length of the HCS. For small values of α , CRs generally follow, or trace, the HCS quite effectively. It may be said that they *stick* to the HCS fairly well. However, when α gets large, diffusion disrupts these perfect drift patterns so that the CRs do not effectively follow the HCS. The result is that the CR intensities become independent of α , when $\alpha > 45^\circ$, for the $A < 0$ drift cycle.

As an illustration, the SDE model was applied to the modulation of galactic protons and anti-protons. Charge-sign dependent modulation, being very evident when studying the modulation of these CR species, was discussed and illustrated. The modelled energy spectra (for both protons and anti-protons) for the $A < 0$ HMF cycle were compared successfully to recent PAMELA observations, while upper limits for the fluxes in the next ($A > 0$) cycle were given. The same was done for the anti-proton to proton ratio. From this ratio, it is evident that drift effects are present up to ~ 10 GeV. Drift effects are, however, most prominent at low energies, at 10 MeV the anti-proton to proton ratio differs by a factor of ~ 10 between different drift cycles, while these effects diminish at the higher energies. With the upcoming solar minimum

proton and $\bar{p}$ results from the AMS-2 mission for the $A > 0$ cycle eminent, a value for the anti-proton to proton ratio is predicted that is lower than measured in the present $A < 0$ cycle. If the modulation conditions during the next solar minimum (in ~ 2021) would be the same as in the $A < 0$ minimum of 2009 (coined an unusual solar minimum with record breaking levels of CRs observed), galactic proton intensities will be even higher due to drift effects. The SDE model has therefore been successfully applied to charge-sign dependent modulation.