

A multi-wavelength study of a sample of galaxy clusters

S.Wilson

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NORTH-WEST UNIVERSITY

Supervisor: Dr N. Oozeer

Co-supervisor: Dr S.I. Loubser

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Abstract

In this dissertation we aim to perform a multi-wavelength analysis of galaxy clusters. We discuss various methods for clustering in order to determine physical parameters of galaxy clusters required for this type of study. A selection of galaxy clusters was chosen from 4 papers, (Popesso et al. 2007b, Yoon et al. 2008, Loubser et al. 2008, Brownstein & Moffat 2006) and restricted by redshift and galactic latitude to reveal a sample of 40 galaxy clusters with $0.0 < z < 0.15$. Data mining using Virtual Observatory (VO) and a literature survey provided some background information about each of the galaxy clusters in our sample with respect to optical, radio and X-ray data. Using the Kayes Mixture Model (KMM) and the Gaussian Mixing Model (GMM), we determine the most likely cluster member candidates for each source in our sample. We compare the results obtained to SIMBADs method of hierarchy. We show that the GMM provides a very robust method to determine member candidates but in order to ensure that the right candidates are chosen we apply a select choice of outlier tests to our sources. We determine a method based on a combination of GMM, the QQ Plot and the Rosner test that provides a robust and consistent method for determining galaxy cluster members. Comparison between calculated physical parameters; velocity dispersion, radius, mass and temperature, and values obtained from literature show that for the majority of our galaxy clusters agree within 3σ range. Inconsistencies are thought to be due to dynamically active clusters that have substructure or are undergoing mergers, making galaxy member identification difficult. Six correlations between different physical parameters in the optical and X-ray wavelength were consistent with published results. Comparing the velocity dispersion with the X-ray temperature, we found a relation of $\sigma \sim T^{0.43}$ as compared to $\sigma \sim T^{0.5}$ obtained from Bird et al. (1995). X-ray luminosity temperature and X-ray luminosity velocity dispersion relations gave the results $L_X \sim T^{2.44}$ and $L_X \sim \sigma^{2.40}$ which lie within the uncertainty of results given by Rozgacheva & Kuvshinova (2010). These results all suggest that our method for determining galaxy cluster members is efficient and application to higher redshift sources can be considered. Further studies on galaxy clusters with substructure must be performed in order to improve this method. In future work, the physical parameters obtained here will be further compared to X-ray and radio properties in order to determine a link between bent radio sources and the galaxy cluster environment.

Keywords: Galaxy kinematics and dynamics, Galaxy Clusters, statistical analysis, clustering algorithms, Abell clusters, mass determination, multi-wavelength view, Kayes Mixing Model, Gaussian Mixture Model, multi-modality, radio galaxies, data mining, velocity dispersion, Kernel density estimation and outlier detection techniques.

Opsomming

In hierdie verhandeling bespreek ons verskeie metodes vir opeenhoping om die fisiese parameters van galaksieswerms te bepaal ten einde 'n multi-golflengte studie te verrig. Galaksieswerms is vanuit vier bronne verkies (Popesso et al. 2007b, Yoon et al. 2008, Loubser et al. 2008, Brownstein & Moffat 2006) en is beprek deur rooiverskuiwing en galaktiese breedtegraad om 'n steekproef van 40 galaksieswerms te verkry met $0.0 < z < 0.15$. Virtuele data ontginning en 'n literatuurstudie het agtergrond-inligting oor elke galaksieswerm in die optiese, radio en X-straal golflengte gebied verskaf. Ons bepaal die mees waarskynlikste galaksieswerm-lid kandidate vir elke swerm in ons steekproef deur van die “Kays Mixture Model” (KMM) en die “Gaussian Mixing Model” (GMM) gebruik te maak. Hierdie resultate word dan met die SIMBAD hierargie metode vergelyk. Ons bewys dat die GMM metode 'n baie standvastige metode is om swerm kandidate te kies, maar om te verseker dat die regte kandidate verkies word verrig ons ook 'n keuse van uitskieter toetse op ons bronne. Ons resultate bewys dat die “QQ Plot” en “Rosner” toets die mees effektiewe resultate vir ons doeleindes lewer. Ons bepaal 'n metode gebaseer op die kombinasie van die GMM, QQ Plot en Rosner toetse wat 'n konsistente metode lewer om galaksieswerm-lede vas te stel. 'n Vergelyking van fisiese parameters, snelheid dispersie, radius, massa en temperatuur, met waardes uit die literatuur wys dat die resultate binne die 3σ -vlak ooreenkom. Afwykings hiervan word moontlik toegeskryf aan dinamies aktiewe swerms wat sub-struktuur het of wat botsings met ander swerms ondergaan, wat swerm-lid identifikasie van vermoeilik. Ses verbande tussen verskillende fisiese parameters in die optiese en X-straal golflengte gebied stem ooreen met reeds gepubliseerde resultate. 'n Belangrike resultaat was die vergelyking van die snelheid dispersie met X-straal temperatuur, waar ons 'n verband $\sigma \sim T^{0.43}$ verkry het in vergelyking met $\sigma \sim T^{0.5}$ deur Bird et al. (1995). X-straal liggewendheid temperatuur en snelheid dispersie verbande lewer die resultate $L_X \sim T^{2.44}$ en $L_X \sim \sigma^{2.40}$ wat, foutgrense inagenome, ooreenstem met Rozgacheva & Kuvshinova (2010). Hierdie resultate suggereer dat ons metode om kandidate uit te kies effektief is en dat die toepassing daarvan by hoer rooiverskuiwings oorweeg kan word. 'n Verdere studie op galaksie-swerms met sub-struktuur sal onderneem moet word om hierdie metode te verfyn. In toekomstige werk sal die fisiese parameters wat verkry is met die X-straal en radio eienskappe vergelyk word met die hoop om 'n verband te vind tussen gebuigde radio bronne en hul galaksieswerm omgewing.

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“Most obstacles melt away when we make up our minds to boldly walk through them” - Orison Swett Marden

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Plagiarism Declaration

I, Susan Wilson, know the meaning of plagiarism and declare that all of the work in the document, save for that which is properly acknowledged, is my own.

Contents

1	Introduction	1
1.1	Galaxy Cluster Formation	1
1.2	Galaxy Clusters: A multi-wavelength overview	2
1.2.1	Optical	2
1.2.2	Radio	3
1.2.3	X-ray	4
1.3	Clustering Algorithms	5
1.3.1	Hierarchy	6
1.3.2	Partitioning	6
1.4	Statistical tools used for the detection of outliers	7
1.4.1	Distance Separation	7
1.4.2	Histogram	8
1.4.3	Kernel Density Estimate (KDE)	8
1.4.4	Mean	9
1.4.5	Quantile-Quantile Plot	10
1.4.6	Walsh Test	10
1.4.7	Rosner Test	10
1.5	Physical Parameters	11
1.5.1	Velocity Dispersion (σ)	11
1.5.2	Radius (R_{200})	11
1.5.3	Mass (M_{200})	12
1.5.4	Radio Properties	13
1.5.5	Temperature (T and T_{200})	13
1.6	Aims and Objectives	14
1.7	Report Layout	15
2	Cluster selection and Observations	17
2.1	Sample Selection	17
2.2	Multi-wavelength view of each cluster	21
2.2.1	Group 1 ($0.00 < z < 0.05$)	22
2.2.2	Group 2 ($0.05 < z < 0.10$)	27
2.2.3	Group 3 ($0.10 < z < 0.15$)	30
2.3	Conclusion	32

3	Methods and Analysis	37
3.1	Application of Clustering Algorithms	37
3.1.1	SIMBAD	37
3.1.2	Mixture model methods	38
3.1.3	Results	41
3.2	Application of outlier detection techniques	44
3.2.1	Distance Separation	44
3.2.2	Histogram	44
3.2.3	KDE	45
3.2.4	Mean	45
3.2.5	Quantile-Quantile Plot (QQ Plot)	46
3.2.6	Walsh Test	47
3.2.7	Rosner Test	47
3.2.8	Results	47
3.3	Final Method	50
3.4	Results	54
3.4.1	Velocity Dispersion (σ)	54
3.4.2	Radius (R_{200})	54
3.4.3	Mass (M_{200})	56
3.4.4	Temperature	57
3.5	Conclusion	58
4	Correlations of various physical parameters from multi-wavelength data	59
4.1	Velocity Dispersion (σ) vs Redshift (z)	59
4.2	Mass (M_{200}) vs Redshift (z)	60
4.3	X-ray Luminosity (L_X) vs Redshift (z)	61
4.4	X-ray Temperature (T_X) versus Velocity Dispersion (σ)	61
4.5	X-ray Luminosity (L_X) versus Velocity Dispersion (σ)	64
4.6	X-ray Luminosity (L_X) versus Temperature (T)	64
4.7	Conclusion	66
5	Conclusion and future work	67
5.1	Conclusions	67
5.2	Future Work	71
A	Clustering Algorithm Results	73
B	Outlier Techniques	79

List of Figures (Abbreviated title)

1.1	Synchrotron radiation	5
1.2	Histogram as used to detect outliers:	9
2.1	RA and DEC distribution of our sample	21
2.2	Redshift distribution of all the sources in our sample	21
2.3	Radio overlays - Group 1 ($0.00 < z < 0.05$)	22
2.4	Radio overlays - Group 1 continued	23
2.5	Radio overlays - Group 2 ($0.05 < z < 0.10$)	27
2.6	Radio overlays - Group 2	28
2.7	Radio overlays - Group 3 ($0.10 < z < 0.15$)	31
3.1	KDE analysis for A3581 data obtained from SIMBAD with outliers	37
3.2	KDE analysis for A3581 data obtained from SIMBAD without outliers	38
3.3	Flow diagram of Mixture Modeling Method	39
3.4	Redshift distribution of the dataset after the 3 Mpc search	39
3.5	Dendrogram of sample G0	40
3.6	Flow chart of KMM method	41
3.7	Box plots for the different options for determining possible cluster members.	42
3.8	Clustering algorithm results	43
3.9	Histograms obtained using different methods for calculating bin width.	44
3.10	KDE analysis as a tool for outlier detection.	45
3.11	Mean as a tool for outlier detection	46
3.12	QQ Plot as a tool for outlier detection	46
3.13	Outlier technique results	49
3.14	Our method for determining cluster members	50
3.15	Velocity dispersion comparison	54
3.16	Radius comparison	55
3.17	Mass comparison	56
3.18	Temperature comparison	57
4.1	σ vs z	59
4.2	M_{200} vs z	60
4.3	X-ray Luminosity vs z from Sommer et al. (2011)	61
4.4	L_X vs z	61
4.5	σ vs T_{lit}	62
4.6	σ vs T_{calc}	63
4.7	σ vs L_X	65

4.8	L_X vs T_X	66
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List of Tables (Abbreviated title)

1.1	Comparison of selection criteria for Abell and Zwicky catalogs	3
1.2	Classifications of galaxy clusters	4
2.1	Our Sample	19
2.2	References for Literature values	32
2.3	Galaxy cluster literature values	33
2.4	Galaxy cluster literature values continued	35
3.1	Physical parameters derived using our final method	52
4.1	Comparison of selection criteria for Abell and Zwicky catalogs obtained from Sarazin & Boller (1989)	65
A.1	SIMBAD Results	74
A.2	KMM Results	75
A.3	GMM Results	77
B.1	Outlier Tests – QQ Plot Results	80
B.2	Outlier Tests – Walsh Results	81
B.3	Outlier Tests – Rosner Results	82

Chapter 1

Introduction

The study of galaxy clusters has gained impetus with a huge amount of multi-wavelength data becoming available on-line. They are important as they provide a way to study galaxy formation and evolution, as well as large scale structure in the Universe. Galaxy clusters can be characterised via a set of parameters and knowing these parameters enable us to gain a better understanding of the cluster environment and the various processes within it. The cluster richness is one of the parameters most often used to characterise galaxy clusters. Determining the richness also enables us to obtain other properties such as the cluster size, mass and velocity dispersion. Clustering algorithms on the other hand, provide us with a statistical tool to group members sharing common characteristics. In this dissertation we use clustering algorithms to group together galaxies which eventually form a galaxy cluster. Once the galaxies are grouped together we can gather information that allows us to characterise the cluster.

In this chapter we will look at how galaxy clusters formed and why the study of them became important and what information we can gather about them from optical, radio and X-ray observations. We will introduce the clustering algorithms that will be tested and the physical parameters of interest that we will use to gain information about galaxy clusters.

1.1 Galaxy Cluster Formation

The formation of galaxy clusters is a complicated, non-linear process that is accompanied by a wide variety of physical phenomena on different scales. Therefore the exact method of the formation of galaxy clusters is unknown, but they are thought to form via a hierarchial sequence of mergers and accretion which is driven by gravity and dark matter (Kravtsov & Borgani 2012). One scenario involved the collapse of the largest gravitationally bound overdensities in the initial density field. Many theoretical models exist to try and explain different aspects of this formation. A very remarkable model is that suggested by Kaiser (1986) - the simple self-similar model of clusters. They model the baryonic processes that explain the observational properties, such as the galaxy cluster temperature. They make 3 main assumptions for this model (Kravtsov & Borgani 2012):

- i Clusters form via gravitational collapse of peaks in the initial density field in an Einstein - de - Sitter Universe, $\Omega_m = 1$
- ii Amplitude of the density fluctuations is a power law function of the size

iii The physical processes do not come from new scales in the problem

A review of different galaxy cluster formation models and an extension of the simple self-similar model is provided by Kravtsov & Borgani (2012).

1.2 Galaxy Clusters: A multi-wavelength overview

The study of galaxy clusters began in the 18th Century when Charles Messier and F. Wilhelm Herschel each produced their own catalogue of nebulae (Biviano 2000; Herschel 1864; Messier & Niles 1981) and noticed how they seem to concentrate. This sparked the interest of other astronomers who wanted to determine whether these concentrations of nebulae belonged to our Milky Way. In 1913 V.M. Slipher (Smith 1979; Slipher 1915) measured the radial velocity of the Andromeda nebulae for the first time using the spectral line shift. Slipher obtained a value of 300 km s^{-1} which was an order of magnitude higher than the measured radial velocities of the stars. Following this result, many investigations into the formation and evolution of galaxy clusters began, introducing the study of aspects such as the properties and distribution of galaxy clusters and their dynamical status. In the sections below we discuss how the study of galaxy clusters began in the optical, radio and X-ray wavelengths and what was initially discovered.

1.2.1 Optical

The night sky was originally studied in the optical waveband. In the study of galaxy clusters the two biggest and most-used catalogs were compiled by Abell (1958) and Zwicky et al. (1968). Abell's catalogue contained 2712 galaxy clusters and of those he analyzed the distribution of 1682. Zwicky's catalogue was more extensive and included information on the sizes of the largest galaxy clusters and the area of sky they covered (Abell 1975). At this time there was no standard definition of a cluster and therefore they each had their own set of criteria for what could be included in their catalog. Their criteria were based on magnitude, redshift and area within which these galaxies fell (Table 1.1).

From these catalogs astronomers tried to find a way to classify galaxy clusters in order to better understand them. Most of these classifications were based on the content of the galaxy clusters and their richness. The richness is a statistical measure of the number of galaxies within a cluster (Sarazin & Boller 1989). When looking at the content they studied the different types of galaxies in the cluster and which of these dominated. Different examples of classifications are summarised in Table 1.2.

However, due to a correlation between all of these classifications we can combine them and simply divide galaxy clusters into two main groups: regular and irregular. Regular clusters are defined as those which have a population greater than 10^3 in the interval of the 6 brightest magnitudes. These galaxy clusters have a central concentration of galaxies and show spherical symmetry. They are mostly constituted of E and S0 galaxies. On the other hand, irregular galaxy clusters vary from poor to relatively rich with little or no symmetry and no marked concentrations. They contain all types of galaxies, but spirals are the most common among the brighter galaxies.

The regular galaxy clusters are often dominated by a luminous cD galaxy or a pair of bright galaxies. cD galaxies are defined by Sarazin & Boller (1989) as “*galaxies with a nucleus of a very luminous elliptical galaxy embedded in an extended amorphous halo of surface brightness*”. After nuclear sources these are the most luminous with only a small dispersion in their magnitude. The main difference between cD and elliptical galaxies is that the core region of a cD galaxy is larger and it is embedded in a low surface brightness halo. It is found at rest in a gravitational potential well at the centre of compact galaxy clusters.

Table 1.1: Comparison of selection criteria for Abell and Zwicky catalogs (Sarazin & Boller 1989). In this table m_i refers to the magnitude of the i th brightest galaxy in the cluster and z is the redshift.

Abell	Zwicky
At least 50 galaxies with magnitude in the range m_1 to m_3	At least 50 galaxies in the range m_1 to m_1+3 must fall in the boundary
The galaxies must fall in a circle of radius $R_A = \frac{1.7}{z}$ arcminutes	The boundary of the cluster is the isopleth where the galaxy surface is twice the local background density
$0.02 \leq z \leq 0.20$	No distance criteria

1.2.2 Radio

In the beginning of the 1960’s an association between radio sources and galaxy clusters was found by Mills (1960) and van den Bergh (1961). Mills’ studies showed that the radio emission was found to be linked to individual galaxies or a pair of galaxies in the cluster. This was confirmed by van den Bergh a year later when he studied radio galaxies from the “3C” catalog (Edge et al. 1959, Bennett 1962) and compared them to the positions of rich galaxy clusters. The emission from these galaxies is known to be due to synchrotron radiation (Sarazin & Boller 1989).

Synchrotron radiation is caused by the acceleration of relativistic particles by a magnetic field. For non-relativistic particles this process is known as cyclotron radiation and the frequency at which the radiation is emitted is known as the frequency of gyration. For relativistic particles the frequency spectrum is more complex and the frequency is many times greater than the gyration frequency. The particle will undergo a combination of circular and uniform motion along the magnetic field which results in a helical motion as shown in Figure 1.1.

This emission is studied in many different frequencies with the most common being 1.4 GHz. Over the years the luminosities measured at these different frequencies have been used to try and successfully calculate the radio luminosity function. The approaches vary from the percentage luminosity function used by Owen (1975) to the volume averaged radio luminosity function used by Sommer et al. (2011).

Two main types of radio sources exist in galaxy clusters: single radio galaxies and cluster sized-radio halos. Isolated radio sources that are not in a cluster have a generally symmetric and simple structure. It is either a compact radio source associated with the nucleus of the galaxy or

Table 1.2: Classifications of galaxy clusters by different astronomers from Sarazin & Boller (1989) and Abell (1975).

Author	Classification	Explanation
Zwicky et al. (1968)	Compact	One pronounced concentration and >10 galaxies in contact
	Medium Compact	One pronounced concentration and >10 galaxies separated by several diameters or several pronounced concentrations
	Open	No pronounced peak of population
Bautz & Morgan (1970)	Type I	Cluster dominated by a cD galaxy
	Type II	Brightest galaxies are between a cD and giant ellipticals
	Type III	No dominating galaxies
Rood & Sastry (1971)	cD	Dominated by a central cD Galaxy
	B	Binary – dominated by a pair of luminous galaxies
	L	Line – at least 3 of brightest galaxies appear to be in a straight line
	C	Core – at least 4 of the 10 brightest galaxies form a cluster core
	F	Flat – brightest galaxies form a flattened distribution across the sky
	I	Irregular – brightest galaxies have an irregular distribution across the sky
Morgan (1961)	Type I	Contains many spirals
	Type II	Contains few spirals
Oemler (1974)	Spiral-Rich	Spirals most common
	Spiral-Poor	S0s most common
	cD	Dominated by a cD galaxy and most galaxies are S0 or elliptical

an extended source with double or single radio lobes. These lobes are in a straight line through the nucleus of the galaxy.

In galaxy clusters the radio sources have a more complex structure that lacks symmetry. The two main types are Wide Angle Tails (WAT) and Narrow Angle Tails (NAT) or Head-Tail (HT) galaxies. WATs are double lobed but the lobes are not aligned with the nucleus. They are generally associated with the optically dominant galaxies and are more luminous than the HT galaxies. For an HT galaxy all the emission lies in a tail on one side and the galaxy forms the head. They are often not associated with cD galaxies.

1.2.3 X-ray

Studies by the Uhuru satellite led to the discovery of five important properties of galaxy clusters (Sarazin & Boller 1989):

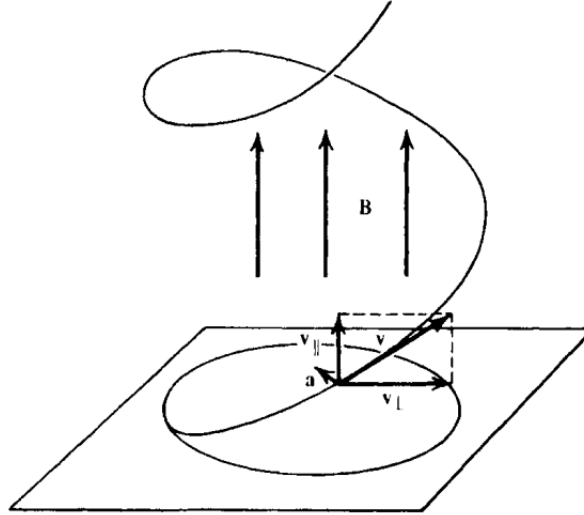


Figure 1.1: Synchrotron radiation showing the helical motion of a particle in a uniform magnetic field (Rybicki & Lightman 1986).

- i The most common bright extragalactic X-ray sources are due to galaxy clusters.
- ii Galaxy clusters are very luminous in the X-ray and are found over a wide range of luminosities.
- iii X-ray sources associated with galaxy clusters are extended (Kellogg et al. 1972).
- iv Observations of the X-ray spectra show no strong evidence for low photo-absorption.
- v X-ray emission is not time variable.

The last three of these suggest that the X-ray emission is diffuse. Improved telescopes such as ROSAT, Chandra and XMM have made many important discoveries such as the fact that X-ray emission is mostly due to thermal emission from hot intracluster gas. This is known as Bremsstrahlung radiation and provides the continuum component. Bremsstrahlung radiation is due to the acceleration of a particle by the field of another more massive particle. It is responsible for the X-ray emission from galaxy clusters. The non-thermal components of the X-ray emission are due to inverse Compton radiation and thought to be linked to the radio sources. Compton radiation is emitted when a collision occurs between a photon and an electron. The electron has more kinetic energy than the photon and therefore energy is transferred from the electron to the photon. X-ray properties are very helpful in the study of cosmology as they can be linked to the mass of the cluster via the virial theorem (Hill & Rines 2007).

1.3 Clustering Algorithms

In order to make use of all the multi-wavelength data obtained for galaxy clusters we need to determine robust criteria for the inclusion of a galaxy into a galaxy cluster. The physical parameters can be heavily influenced by the inclusion of galaxies which do not belong and this may lead to false conclusions. In the work we present here we look at clustering algorithms in order to determine a robust method for grouping galaxies together into clusters. Clustering algorithms can be divided into two main groups – hierarchy methods and partitioning methods.

1.3.1 Hierarchy

A hierarchical method is a statistical method used to build a cluster by arranging elements at various levels. This method can be either agglomerative or divisive. The agglomerative method is a bottom-up clustering method which starts with each object belonging to its own group and then merges them until one main group is formed. The divisive method is top-down and works in the opposite manner (Murtagh & Contreras 2011). We considered two hierarchical methods - the Dendrogram and SIMBAD - for further study.

Dendrogram

A dendrogram is a tree representation that splits the dataset into smaller and smaller groups until each group contains only one object (Sander et al. 1998). Each level will then represent a possible cluster. The height of the dendrogram shows the level of similarity that any two clusters are joined. The closer to the bottom they are the more similar the clusters are. Finding of groups from a dendrogram is not simple and is very often subjective. We choose a set level of similarity of about 50% of the height and then all lines which cross this level indicate a cluster. This method is combined into the partitioning methods to get starting points for the mixture modeling algorithms.

SIMBAD

The SIMBAD* astronomical database has a hierarchy method which uses information gained from bibliographic references and catalogues. This method has been updated on a regular basis since January 2008, however the system is not yet complete.

1.3.2 Partitioning

Partitioning algorithms make an initial division of the database and then use an iterative strategy to further divide it into sections (Sander et al. 1998).

Kaye's Mixture Model

Kaye's Mixture Model (KMM) algorithm (Ashman et al. 1994) is a mixture modeling code used to determine the likelihood of the underlying distribution being a single Gaussian versus a double Gaussian. This standard method assumes that the input data is a double Gaussian and then calculates the probability of each data point belonging to either of the modes. It calculates the Likelihood Ratio Test (LRT), using an approximation derived by Wolfe (1971) for the homoscedastic case, which determines if the data are best described by a double or single Gaussian. The ratio of maximum likelihoods is defined as $\lambda = \frac{L_{1,max}}{L_{2,max}}$ [†] and the statistic $-2\ln\lambda$ obeys the χ^2 distribution (Wolfe 1971; Muratov & Gnedin 2010). However, this is not a very reliable test as it depends on the modes having the same variance which is not often true of real data (Muratov & Gnedin 2010).

*<http://simbad.u-strasbg.fr/simbad/>

[†]Where $L_{1,max}$ and $L_{2,max}$ are the maximum likelihoods of group 1 and 2 respectively

Gaussian Mixture Model

Gaussian Mixture Model (GMM) is a general class of algorithms that KMM belongs to. The algorithm uses the expectation-maximization (EM) algorithm to maximise the likelihood of the data belonging to a specific group given all the parameters. For this method it is assumed that each mode is described by a Gaussian (Muratov & Gnedin 2010). The GMM algorithm is different from the KMM algorithm in that it uses three statistics of interest in order to compare the probability of bi-modality versus uni-modality: the LRT, the separation, and kurtosis (Muratov & Gnedin 2010). Therefore unlike KMM, the output result of GMM is independent of the variance of the groups. Also, when running GMM the homoscedastic and heteroscedastic cases are tested at the same time. However, the main issues at the moment are that the maximum number of groups is three and GMM will force the data into the number of groups specified. The code was obtained from Oleg Gnedin and is freely available on his webpage[†].

1.4 Statistical tools used for the detection of outliers

Apart from clustering potential galaxies to form a galaxy cluster, one should also ensure that the right candidates are chosen. Outlier detection is a full topic on its own. Various methods exist to search for outliers among a sample. We have used a selection of some of these which are discussed in this section.

1.4.1 Distance Separation

The average size of an Abell cluster is $1.5 h^{-1}$ Mpc (Dalton et al. 1997) which when using our chosen cosmology, $h=0.73$, gives a value of 2 Mpc. Assuming that the cluster is spherical with the BCG at the centre, we can calculate the distance between any galaxy and the BCG. If this value is greater than 2 Mpc then the galaxy is considered not to belong to the cluster. Since the right ascension (RA) and declination (DEC) gives us the location on the RA-DEC plane, the redshift provides us with depth information. We can calculate the 2D and 3D separation using:

$$Sep_{2D} = R_{tg} \times r_g \times \psi \quad (1.1)$$

where the scale factor is given by $R_{tg} = \frac{1}{1+z_g}$; z_g is the redshift of the galaxy, r_g is the co-moving distance of the galaxy and ψ is the angular distance between the galaxy and the BCG. Using spherical trigonometry ψ is given by

$$\cos\psi = \cos\theta_g \times \cos\theta_{BCG} + \sin\theta_g \times \sin\theta_{BCG} \times \cos(\phi_g - \phi_{BCG}). \quad (1.2)$$

The 3D Separation is calculated using

$$Sep_{3D} = \sqrt{(x_g - x_{BCG})^2 + (y_g - y_{BCG})^2 + (z_g - z_{BCG})^2} \quad (1.3)$$

where

$$x = R_{to} \times r \times \sin\theta \times \cos\phi, \quad (1.4)$$

[†]<http://dept.astro.lsa.umich.edu/~ognedin/>

$$y = R_{to} \times r \times \sin\theta \times \sin\phi, \quad (1.5)$$

$$z = R_{to} \times r \times \cos\theta. \quad (1.6)$$

R_{to} is the scale factor at the current epoch and is equal to 1. r is the co-moving distance, $\theta = (\frac{\pi}{2} - DEC)$ and $\phi = RA$ where RA and DEC are in radians.

1.4.2 Histogram

Histograms are non-parametric density estimators which are widely used to analyse data. However, we have to make a choice for two parameters: the origin and the bin width. The origin and bin width affect the structure and smoothness of the density estimate. When choosing a bin width we want to be able to see the major features while ignoring random fluctuations (Knuth 2006). If too few bins are chosen then we will have a large variance but if too many bins are chosen it will be too smooth and it will cause a large bias. As a rough estimate of the number of bins it is advised by a few authors (Scott (1979) and references therein) to use:

$$bins = 1 + 2 \times 2 \text{Log}_{10}(n) \quad (1.7)$$

where n is the number of sources in the sample. Scott (1979) suggests calculating the bin width, h , using

$$h = 3.49 \times s \times n^{-\frac{1}{3}} \quad (1.8)$$

where s is an estimate of the standard deviation. However, this works best for Gaussian data and gives overly large bin widths for non-Gaussian data (Izenman 1991). Freedman & Diaconis (1981) suggest a simple but robust method that calculates the bin width using

$$h = 2(IQR) n^{-\frac{1}{3}} \quad (1.9)$$

where IQR is the interquartile range. By plotting a histogram of the sample, it will show us how it is grouped. If any members do not belong to the main group these are considered outliers as shown in Figure 1.2.

1.4.3 Kernel Density Estimate (KDE)

A binned kernel density estimate (KDE) estimates the probability density function using

$$f(x) = \frac{1}{nh} \sum_{i=1}^n K \frac{x - X_i}{h} \quad (1.10)$$

where h is the bandwidth and K is the kernel function which satisfies

$$\int_{-\infty}^{\infty} K(x) dx = 1. \quad (1.11)$$

The KDE is closely related to the histogram but has properties such as smoothness or conti-

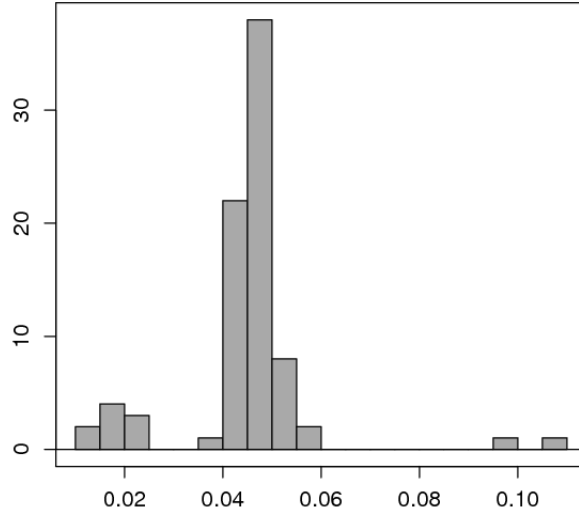


Figure 1.2: Histogram as used to detect outliers: We used the Freedman & Diaconis (1981) method to determine the bin width for the histogram of the redshift distribution for A1644. We can see that the main group lies at a redshift of 0.05 with possible outliers at $z \approx 0.02$ and $z \approx 0.1$.

nulty. For a histogram the horizontal axis is divided into bins which cover the range of the data, as explained in Section 1.4.2. The data points are then put into the relevant bins. The kernel estimator places a “bump” at each observation (Silverman 1986) and then they are summed to make the kernel density (Scott 1979). The shape of the “bumps” is determined by the kernel function and the width is determined by the bandwidth. The choice of bandwidth is the equivalent of choosing a bin width for a histogram.

There are many choices for the kernel function but for this dissertation we will use the Gaussian kernel function. The reason for this choice is because we expect that the distribution of the redshift of the galaxies in the cluster will follow a uni-modal Gaussian distribution. If this is not the case, it may suggest substructure in the cluster as discussed by Einasto et al. (2012b) and needs to be studied in more detail. Once the density has been calculated it can be used to plot contours. The contour levels used for this dissertation are 25, 50 and 75%. All galaxies which fall outside the contours are suspected outliers.

1.4.4 Mean

The mean is a measure of spread and is very sensitive to values that lie on the tails of the distribution and we can therefore use it to determine outliers. To do this, we change the significance of values on the tails and see if it greatly effects the mean. If it does not then the galaxies corresponding to these values are likely to belong to the galaxy cluster. One method for doing this is to apply a weighting so that values in the centre become more important than those at the tail by winsorizing the distribution. A percentage of the values which we believe are negatively influencing the mean are chosen and then the highest and lowest percentage of the scores are changed to the next smallest and biggest score respectively. Another method used is called trimming and for this method, the highest and lowest selected percentage are removed from the sample.

1.4.5 Quantile-Quantile Plot

A Quantile-Quantile (QQ) plot compares the variable values from our sample with the quantiles from a distribution selected by us. The points will form a straight line if they match the set distribution. It can be used to determine outliers via two methods. The first method calculates a value above which less than a specified number of points p are expected to occur, given the total size of the sample. The second method uses the fit residuals. A confidence level α is chosen and any values which have residuals above or below α are considered to be outliers. For this dissertation, we will use the second method and choose $\alpha = 0.1$.

1.4.6 Walsh Test

This test was developed by J.E. Walsh (Walsh 1950) to detect multiple outliers. It is a non-parametric test for large samples where $n > 60$. This test does not require an underlying normal distribution, however it declines or accepts the group of outliers as opposed to individual members. To perform the Walsh test the following calculations must be done (Messier & Niles 1981):

- Order the sample $X_1, X_2, \dots, X_n$ from smallest to largest,
- Identify the number of outliers, r , where $r \geq 1$,
- Calculate $c = \text{ceil}(\sqrt{2n})$, $k=r+c$, $b^2 = 1/\alpha$, $a = \frac{1+b\sqrt{\frac{c-b^2}{c-1}}}{c-b^2-1}$,

where α is the chosen level of significance. For $60 < n < 220$ we use $\alpha = 0.1$ and for $n > 220$ we use $\alpha = 0.05$. Then

- The r smallest points are outliers if $X_r - (1+a)X_{r+1} + aX_k < 0$
- The r largest points are outliers if $X_{n+1-r} - (1+a)X_{n-1} + aX_{n+1-k} < 0$

1.4.7 Rosner Test

The Rosner Test is a statistical method used to detect up to 10 outliers for a sample size of greater than 25. It can only be used for samples with an underlying normal distribution after the removal of the suspected outliers. To check for this we use the Shapiro-Francia test (Shapiro & Francia 1972) for normality. The test statistic W' is defined as $W' = \frac{(\sum_{i=1}^n b_i y_i)^2}{\sum_{i=1}^n (y_i - \bar{y})^3}$. y_i is a normal sample and is defined as $y_i = \mu + \sigma x_i^\dagger$ and $b' = (b_1, b_2, \dots, b_n) = \frac{m'}{\sqrt{m'm}} \ddagger$ (Shapiro & Francia 1972). This test was introduced as a modification of the well known Shapiro-Wilk test (Shapiro & Wilk 1965) with test statistic W defined as $W = \frac{(\sum_{i=1}^n a_i y_i)^2}{\sum_{i=1}^n (y_i - \bar{y})^2}$. Where $a' = (a_1, a_2, \dots, a_n) = \frac{m'V^{-1}}{\sqrt{m'V^{-1}V^{-1}m}}^\ddagger$. The main differences are that the Shapiro-Francia test statistic is easier to compute and it works for a larger sample. If the calculated p value is less than the chosen level of significance ($\alpha = 0.1$) then we reject the null hypothesis that the sample has a normal distribution. The test also calculates the W' statistic which has a value of approximately 1 for normal distributions. Unlike the Walsh test, this method rejects or accepts outliers individually. (Shapiro & Francia 1972)

[†]Where x_i is an ordered random sample of size n

^{††}Where m' is the vector of expected values of standard normal order statistics

[‡]Where V is the covariance matrix associated with m'

1.5 Physical Parameters

Physical parameters allow us to characterise and classify galaxy clusters. This section describes the parameters of interest which will be calculated in this dissertation.

1.5.1 Velocity Dispersion (σ)

The velocity dispersion is a crucial physical parameter for characterising galaxy clusters. It is closely related to the density and density profile of galaxy clusters as well as being an indicator for the dynamical state as explained in Section 4.4. To calculate it we use the following equation:

$$\sigma = \sqrt{\frac{\sum_{i=1}^N v_i^2}{N-1}} \text{ km s}^{-1} \quad (1.12)$$

where v_i is the peculiar velocity of each galaxy in the cluster and is given by:

$$v_i = c * \frac{z_i - z_{mean}}{1 + z_{mean}} \text{ km s}^{-1} \quad (1.13)$$

The error on the velocity dispersion can be calculated using error analysis and if one does not have the errors in the measured values (as in our case), the bootstrapping method can be used. Bootstrapping is a non-parametric method for calculating estimated standard errors (Andrae 2010). It follows three basic steps:

- i Re-sample a given data set a specific number of times.
- ii Calculate a specific statistic from each sample.
- iii Find the standard deviation from that statistic.

For example, we used the redshifts of each cluster member as our initial sample and calculated the velocity dispersion and its standard deviation using over a 1000 resamplings.

1.5.2 Radius (R_{200})

To obtain an approximate radius for our cluster we calculate the radius of the sphere centered on the cluster center containing a mean over-density of 200, R_{200} . Hoyle (2009) defines R_{200} as:

$$R_{200} = \frac{\sqrt{3}\sigma}{10H(z)} h^{-1} \text{ Mpc} \quad (1.14)$$

where $H(z) = H_0 \sqrt{\Omega_m(1+z)^3 + \Omega_R(1+z)^4 + \Omega_{vac}(1+z)^{3(1+w_0)}}$. w_0 is the equation of state of dark energy and is set to -1. Ω_R is the energy component of the radiation and reduces quickly as the Universe expands. The calculations for this thesis have been carried out using a cosmological model with $H_0 = 73 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $\Omega_M = 0.3$ and $\Omega_{vac} = 0.7$ unless otherwise specified. The error on the radius is calculated using error analysis and is given by:

$$R_{200}^{err} = \frac{\sqrt{3}\sigma_{err}}{10H(z)}h^{-1} \text{ Mpc} \quad (1.15)$$

where σ_{err} is the error on the velocity dispersion obtained from bootstrapping.

1.5.3 Mass (M_{200})

The mass of a cluster is quite difficult to calculate but it has been done in many ways. For this dissertation we calculate the mass by applying the virial theorem and making the following assumptions (Richmond 2012):

- i The system must be in equilibrium so that the relationship between the kinetic and gravitational energy holds,
- ii All galaxies have the same mass, and
- iii The system is not rotating.

The virial theorem is given by:

$$KE_{avg} = -\frac{1}{2}GPE_{avg} \quad (1.16)$$

The average kinetic energy is given by

$$KE_{avg} = \frac{1}{2}mv^2 \quad (1.17)$$

where $v^2 = 3\sigma_r^2$ if the motions of the galaxies are isotropic in the cluster and m is the mass of the individual galaxy. The average potential energy is then given by:

$$GPE_{avg} = -\frac{GMm}{r} \approx -\frac{GMm}{\frac{1}{2}R} \quad (1.18)$$

where M is the mass of the cluster, R is the radius of the cluster and r is the average distance between the centre of the cluster and a galaxy i.e. $\frac{R}{2}$. Combining Equations 1.17 and 1.18 we get

$$\frac{3}{2}m\sigma_r^2 \approx -\frac{GMm}{R}. \quad (1.19)$$

Rearranging and using the radius as calculated in Equation 1.14 we get an approximate formula for the mass in the sphere having an over-density of 200:

$$M_{200} \approx \frac{3\sigma^2 R_{200}}{2G} \times 10^{14} M_{\odot}. \quad (1.20)$$

Error analysis gives the error on the mass to be

$$M_{200}^{err} = \sqrt{2\left(\frac{\sigma_{err}}{\sigma}\right)^2 + \left(\frac{R_{200}^{err}}{R}\right)^2} M_{200} \times 10^{14} M_{\odot} \quad (1.21)$$

1.5.4 Radio Properties

In order to perform multi-wavelength analysis, we define here some of the physical parameters used in the radio regime.

Radio Luminosity ($L_{1.4GHz}$)

The luminosity can be observed in optical, radio and X-ray. The X-ray luminosity is determined from the amount of gas in the cluster and is measured in energy bands. The bolometric X-ray luminosity is the luminosity over all these bands. The radio luminosity is measured at different frequencies – the most common of these being at a frequency of 1.4 GHz. The luminosity at this frequency can be calculated using (Bornancini et al. 2010):

$$L_{1.4GHz} = 4\pi D_L^2 S_{1.4} (1+z)^{-(1+\alpha)} \text{ ergs s}^{-1} \quad (1.22)$$

where D_L is the luminosity distance in meters (m) and is dependent on the redshift, z , of the cluster. $S_{1.4}$ is the radio flux density at 1.4 GHz in $\text{ergs s}^{-1} \text{ m}^{-2} \text{ Hz}^{-1}$ and $(1+z)^{(1+\alpha)}$ is the standard k correction term. α is the spectral index and is a measure of how much the radiative flux density depends on the frequency, $S_\nu \propto \nu^\alpha$. If the radio luminosity is greater than $10^{23} \text{ W Hz}^{-1}$ then a source is considered to be radio loud else it is classified as radio quiet.

Radio Power ($P_{1.4GHz}$)

The radio power at 1.4 GHz can be calculated using

$$P_{1.4GHz} = 4\pi D_L^2 S_{1.4} (1+z) \text{ W Hz}^{-1} \quad (1.23)$$

where D_L , $S_{1.4}$ and z are defined as above. The radio power can be used to divide the radio galaxies into their respective Fanaroff-Riley (FR) (Fanaroff & Riley 1974) classes as discussed by Saripalli (2012). If the radio power at 1.4 GHz is less than $10^{25} \text{ W Hz}^{-1}$ then it is classified as an FRI galaxy. These galaxies have narrow emission lines, no polarization and weak variability. FRII galaxies are those with a power greater than $10^{25} \text{ W Hz}^{-1}$ with narrow and wide emission lines, no polarization and variability. These are the most powerful radio sources. FRI galaxies are more likely to be found in dense environments such as galaxy clusters than FRIIs.

1.5.5 Temperature (T and T_{200})

The temperature is an important parameter and for most galaxy clusters it is calculated in the X-ray since most of the radiation in X-rays comes from thermal radiation of the hot gas. Helsdon & Ponman (2003) found a relation between the X-ray temperature and the radius of a galaxy cluster using modeling

$$R_{200} = 1.14 \left[\frac{T}{\text{keV}} \right]^{\frac{1}{2}} h_{50}^{-1} E(z)^{-1} \text{ Mpc} \quad (1.24)$$

where $E(z) = \frac{H(z)}{H_0}$ where $H(z)$ and H_0 are defined in Section 1.5.2. By re-arranging this formula we can get

$$\frac{T}{\text{keV}} = \left[\frac{R_{200} E(z) h_{50}}{1.14} \right]^2 \text{ keV} \quad (1.25)$$

to calculate the temperature of the cluster. The error is given by

$$\frac{T_{err}}{keV} = \left[\frac{R_{200}^{err} E(z) h_{50}}{1.14} \right]^2 \text{ keV} \quad (1.26)$$

Navarro et al. (1995) gives

$$T_{200} = \frac{\mu m_p G M_{200}}{2k R_{200}} \text{ K} \quad (1.27)$$

where $\mu = 0.59$ is the mean molecular weight, m_p is the mass of a proton and k is the Boltzmann constant. M_{200} and R_{200} are used in kilograms (kg) and meters (m) respectively. This equation is not dependent on the chosen cosmology like Equation 1.26 but it is restricted to the temperature within R_{200} . The result is given in Kelvin and to convert to keV we divide by 1.17×10^7 . The error is given by

$$T_{200}^{err} = \sqrt{\left(\frac{M_{200}^{err}}{M_{200}} \right)^2 + \left(\frac{R_{200}^{err}}{R} \right)^2} \times T_{200} \text{ K} \quad (1.28)$$

1.6 Aims and Objectives

Almost four decades after the first cluster catalogue was published, galaxy clusters are becoming increasingly important, especially with the availability of huge amounts of multi-wavelength data. Multi-wavelength studies have been crucial in understanding the nature of galaxy clusters. Combining information from radio, X-ray and optical studies is necessary to get an overall picture of the galaxy clusters but can be time consuming. However, analysing the brightest cluster galaxies (BCGs) has been shown to be a very good starting point to trace the galaxy clusters and the cluster environment (Loubser et al. 2008).

The most massive of early-type galaxies (BCGs) are very unique, with extremely high luminosities, diffuse and extended structures, and dominant locations in galaxy clusters. Because of this special location, they are believed to be sites of very interesting evolutionary phenomena (for example dynamical friction, galactic cannibalism, cooling flows, etc.). This special class of objects may follow a separate evolutionary path from other massive early-type galaxies, one that is more influenced by their special location in the cluster. Studying BCGs gives us information about the formation of the galaxy clusters themselves (Loubser & Sánchez-Blázquez 2011).

The general process of galaxy formation via hierarchical merger is well accepted. However, details such as the impact of feedback sources on the cluster environment and radiative cooling in the cluster are not. Furthermore, the presence of radio loud galaxies and bent radio sources such as WAT and NAT radio galaxies are still not well understood. This project aims at understanding the nature and nurture of the BCGs and the galaxy cluster environment. To do so, it is important to develop ways of identifying and characterising a huge sample of galaxy clusters to complement the techniques that are already in use.

The availability of multi-wavelength data has rekindled an interest in the study of the structure and evolution of galaxy clusters. This project consists of building up a scientific tool and procedure to analyse multi-wavelength galaxy cluster data. To do this we need to

- understand galaxy clusters,
- characterise the physical properties of various types of galaxy clusters, and
- understand and address if there is any relationship between the presence of bent radio sources and the physical properties of the cluster environment.

When trying to understand the physical properties of galaxy clusters we discovered it was a far more complex task than originally thought. Determining the mass of a galaxy cluster is a tricky topic that has been approached in many ways, but we found that for many galaxy clusters it is still unknown. By using clustering algorithms we are able to specify which galaxies belong to a specific galaxy cluster, and from that calculate its velocity dispersion, radius and, the ultimate goal, mass. For this dissertation, we focus mainly on this and look at how effective a method it is for mass determination. We also touch on correlations between radio, optical and X-ray properties in the search for relationships between radio sources and physical properties of the cluster.

1.7 Report Layout

This thesis comprises of five chapters. The layout of each is summarised below:

- Chapter 2: In this chapter we look at the importance of sample selection and present our sample. We provide a brief overview of what is known about each cluster. We also carry out radio overlays to detect presence of radio emission from the BCG or in its surroundings.
- Chapter 3: This chapter discusses the methods we tested (mentioned in the introduction) with their results. Further detailed analysis used is also described and the results are summarised.
- Chapter 4: Here we present and discuss the results of the correlations of different physical properties.
- Chapter 5: This chapter includes a conclusion to this dissertation and possible future work.

Chapter 2

Cluster selection and Observations

An overview of the galaxy cluster sample will be given as well as a discussion of already known features in the selected clusters as obtained from a literature survey and Virtual Observatory (VO) data mining.

2.1 Sample Selection

In order to obtain meaningful statistical properties of astronomical groups we need to ensure that we have a complete sample. This means that we use certain criteria to select sources in a given area of the sky up to a depth limit. Our selection criteria will determine which class of object we wish to probe. Different selection criteria include physical parameters such as physical size, magnitude, redshift etc. However, the selection criteria can introduce bias. A common type of bias which arises from sample selection is known as Malmquist bias (Gould 1993). It refers to the bias that is introduced from magnitude and distance limited samples (Teerikorpi 1984).

Popesso et al. (2007b) studied 137 spectroscopically confirmed Abell clusters from the Sloan Digital Sky Survey (SDSS), with X-ray counterpart obtained from the ROSAT All Sky Survey (RASS) to try determine the reason for under-luminous X-ray clusters. They found that 40% of their sample was undetected or only marginally detected in the X-ray and that they did not follow the scaling relation between virial mass and X-ray luminosity as determined by the other clusters. The X-ray sample was obtained from the RASS-SDSS galaxy cluster catalog. The sample ranges from low-mass and X-ray/optically faint clusters to high mass and X-ray/optically bright clusters with a mean redshift of 0.1. The optical sample was selected from the Abell catalog and comprised of all the Abell clusters observed by SDSS DR3. This sample also covers the entire mass and luminosity range.

Yoon et al. (2008) tried to develop a density-measuring technique for measuring the 3D distribution of galaxies that will not be hampered by the incompleteness problem. From this they were able to determine densities both spectroscopically and from photometric data. They found 924 galaxy clusters from the SDSS DR5 database, 212 of which are new. They also use their data to calculate physical parameters such as R_{200} and velocity dispersion.

Loubser et al. (2008) compiled a series of papers where they looked at properties of a sample of BCGs and how they relate to the properties of the clusters. They concentrated mainly on

BCGs which had been classified as cD galaxies or those containing an optical halo. The sample was chosen using three methods. The first method used the catalogs of Hoessel et al. (1980) and Struble & Rood (1987). From these they identified the clusters known to obtain a cD galaxy. The second method used an all sky search in the HyperLEDA* database to obtain galaxies with a T-type of -3.7 and -4.3, an apparent B-magnitude brighter than 16, closer than 340 Mpc, and further than 15° from the galactic plane. The third method chose from a series of papers galaxies that had met two specific criteria – i) it had a a brightness profile of a cD galaxy and ii) at large radii it broke the de Vaucouleurs $r^{\frac{1}{4}}$ law. From these 3 methods they obtained a total of 63 galaxies.

Brownstein & Moffat (2006) attempted to explain the problem of X-ray galaxy clusters without exotic dark matter. Their sample was obtained by combining those used by Reiprich (2001) and Reiprich & Böhringer (2002). By using a gravity theory based on a metric-skew tensor, which led to a modified acceleration law, they were able to fit galaxy rotation curves and X-ray galaxy cluster mass profiles without introducing non-baryonic dark matter.

Our sample of galaxy clusters was compiled from these 4 papers. They are known to contain a wide range of sources in terms of luminosity and mass, however it does not constitute a complete sample. Therefore our results may be biased and this will be taken into consideration when doing our analysis. Further selection criteria is imposed in terms of redshift and galactic latitude. Due to the fact that nearby clusters are easier to observe, and therefore more is known about them, for comparison of our results we decided to exclude all the clusters with a redshift of greater than 0.15. The galactic plane can contaminate results. In order to avoid this we restrict the sample to all sources with a galactic latitude greater than 10° . This left a final sample of 40 clusters which was divided into 3 redshift groups, $z=0.0-0.05$, $z=0.05-0.10$ and $z=0.10-0.15$. The first, second and third group contain 18, 15 and 7 clusters respectively. The global properties of our sample are:

- Mass range: $10^{12} - 10^{15} M_\odot$
- Velocity Dispersion range: $200 - 1200 \text{ km s}^{-1}$
- Temperature range: $0.1 - 8 \text{ keV}$
- X-ray Luminosity: $10^{35} - 10^{38} \text{ W}$

Our final sample with position and redshift is summarised in Table 2.1. The RA, DEC and redshift distribution of our sample is shown in Figure 2.1 and Figure 2.2. Tables 2.3 and 2.4 show the literature values obtained for each cluster that we will use for comparisons. The references for each value are shown as subscripts and correlate to those found in Table 2.2. The values shown without subscripts are those that were given in the individual paragraphs on each cluster. The values in bold were calculated from known literature values using the equations given in Section 1.5.

*HyperLEDA website: <http://leda.univ-lyon1.fr/>

Table 2.1: Our Sample – Initial sample chosen from Popesso et al. (2007b), Yoon et al. (2008), Loubser et al. (2008), and Brownstein & Moffat (2006) after redshift and galactic latitude restrictions. The data are taken from NED. Columns (1) and (2) give the name of the cluster and the BCG. Columns (3) gives the Right Ascension, (4) gives the declinations and (5) gives the redshift of the cluster. Columns (6) - (9) give the position (J2000), redshift and magnitude of the BCG. N/A refers to data that are not available.

Cluster	BCG	RA _c [hh mm ss.s]	DEC _c [dd mm ss]	z _c	RA _{BCG} [hh mm ss.s]	DEC _{BCG} [dd mm ss]	z _{BCG}	M _{BCG}
A0595	MCG +09-13-062	07 48 50.5	+52 04 29	0.0670	07 49 27.2	+52 02 33	0.0680	16.1
A0610	B2 0756+27	07 59 15.6	+27 06 48	0.0950	07 59 16.0	+27 08 58	0.0980	18.7
A0628	2MASX J08100854+3516315	08 10 07.8	+35 13 07	0.0830	08 10 08.5	+35 16 31	0.0840	16.1
A0646	2MASX J08220955+4705529	08 22 09.6	+47 05 52	0.1290	08 22 09.6	+47 05 53	0.1270	17.1
A0690	B2 0836+29	08 39 14.3	+28 50 24	0.0790	08 39 15.8	+28 50 39	0.0790	15.3
A0779	NGC2832	09 19 50.8	+33 46 17	0.0220	09 19 46.9	+33 44 59	0.0230	12.9
A0858	2MASX J09431952+0553438	09 43 25.7	+05 53 14	0.0860	09 43 19.5	+05 53 44	0.0920	16.6
A1060	NGC3311	10 36 51.3	− 27 31 35	0.0130	10 36 42.8	− 27 31 42	0.0120	12.6
A1066	2MASX J10390665+0512353	10 39 23.9	+05 10 21	0.0700	10 39 06.6	+05 12 35	0.0680	15.9
A1080	2MASX J10435204+0103475	10 43 58.2	+01 05 14	0.1180	10 43 52.0	+01 03 42	0.1160	17.3
A1187	2MASX J11120447+3939433	11 11 39.7	+39 34 41	0.0750	11 12 04.5	+39 39 43	0.0680	16.7
A1346	4C 06.42	11 41 10.4	+05 41 22	0.0980	11 41 11.8	+05 44 05	0.0970	16.2
A1366	2MASX J11443683+6724211	11 44 57.3	+67 25 21	0.1170	11 44 36.8	+67 24 21	0.1160	16.4
A1367	NGC3842	11 44 29.5	+19 50 21	0.0220	11 44 02.2	+19 56 59	0.0210	12.7
A1516	2MASX J12185235+0514443	12 18 57.3	+05 14 21	0.0770	12 18 52.4	+05 14 44	0.0750	17.1
A1544	CGPG 1225.6+6338	12 27 42.2	+63 25 25	0.1460	12 27 54.2	+63 21 25	0.1450	N/A
A1559	VII Zw 470	12 33 05.6	+67 06 28	0.1070	12 33 14.1	+67 07 44	0.1050	17.7
A1644	PGC 044257	12 57 14.8	− 17 21 13	0.0470	12 57 11.6	− 17 24 34	0.0470	13.1
A1650	2MASX J12584149-0145410	12 58 46.2	− 01 45 11	0.0840	12 58 41.5	− 01 45 41	0.0850	16.0
A1656	NGC4889	12 59 48.7	+27 58 50	0.0230	13 00 08.1	+27 58 37	0.0220	12.5
A1809	2MASX J13530637+0508586	13 53 06.4	+05 08 59	0.0790	13 53 06.4	+05 08 59	0.0790	15.2
A1882	2MASX J14142405-0022395	14 14 39.9	− 00 19 57	0.1370	14 14 24.1	− 00 22 40	0.1550	17.1
A1890	NGC5539	14 17 34.3	+08 11 10	0.0570	14 17 37.8	+08 10 47	0.0580	14.2
A2029	IC 1101	15 10 56.0	+05 44 41	0.0770	15 10 56.1	+05 44 41	0.0780	14.7
A2052	UGC09799	15 16 45.5	+07 00 01	0.0350	15 16 44.5	+07 01 18	0.0340	13.9
A2092	2MASX J15331536+3108430	15 33 19.4	+31 08 58	0.0670	15 33 15.2	+31 08 41	0.0670	N/A
A2107	UGC09958	15 39 39.0	+21 47 00	0.0410	15 39 39.0	+21 46 58	0.0420	14.3
A2142	2MASX J15582002+2714000	15 58 16.1	+27 13 29	0.0910	15 58 20.0	+27 14 00	0.0900	16.2
A2147	UGC 10144	16 02 17.2	+15 53 43	0.0350	16 02 19.9	+16 20 46	0.0380	14.2
A2151	NGC6034	16 05 15.0	+17 44 55	0.0370	16 03 32.1	+17 11 55	0.0340	14.5
A2197	NGC6173	16 28 10.5	+40 54 26	0.0310	16 29 44.9	+40 48 42	0.0290	13.1

Continued on Next Page...

Table 2.1 – Continued

Cluster	BCG	RA _c [hh mm ss.s]	DEC _c [dd mm ss]	z _c	RA _{BCG} [hh mm ss.s]	DEC _{BCG} [dd mm ss]	z _{BCG}	M _{BCG}
A2199	NGC6166	16 28 38.5	+39 33 06	0.0300	16 28 38.5	+39 33 06	0.0300	12.7
A2255	ZwCl 1710.4+6401 A	17 12 31.0	+64 05 33	0.0810	17 12 28.8	+64 03 39	0.0740	15.5
A3526	NGC4696	12 48 51.8	− 41 18 21	0.0110	12 48 49.2	− 41 18 39	0.0100	11.3
A3571	PGC 048896	13 47 28.9	− 32 51 57	0.0390	13 47 28.4	− 32 51 54	0.0390	13.0
A3581	IC 4374	14 07 27.5	− 27 01 15	0.0230	14 07 29.8	− 27 01 04	0.0220	13.5
AWM5	NGC6269	16 58 02.4	+27 51 42	0.0340	16 57 58.1	+27 51 16	0.0350	13.3
MKW 4	NGC4073	12 03 57.7	+01 53 18	0.0200	12 04 27.1	+01 53 45	0.0200	12.9
MKW3	NGC5920	15 21 51.9	+07 42 31	0.0450	15 21 51.9	+07 42 32	0.0450	14.6
MKW8	NGC5718	14 40 43.1	+03 27 11	0.0270	14 40 42.8	+03 27 56	0.0270	13.8

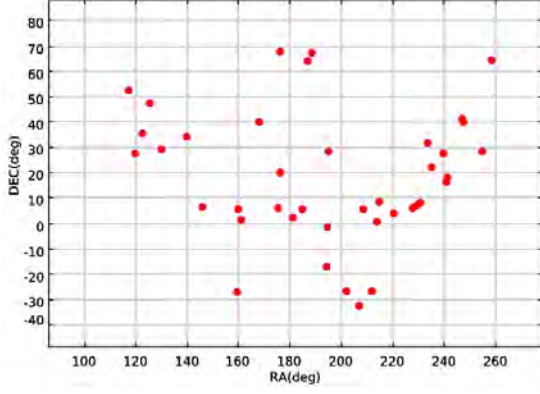


Figure 2.1: RA and DEC distribution of our sample.

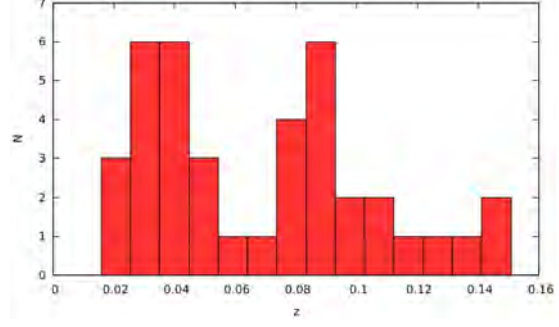


Figure 2.2: Redshift distribution of all the sources in our sample: z varies from 0.011 to 0.146 with a mean value of 0.063875. We divide our sample into three redshift groups – $0.00 < z < 0.05$, $0.05 < z < 0.10$ and $0.10 < z < 0.15$.

2.2 Multi-wavelength view of each cluster

In this section, we divide up the galaxy clusters into their redshift groups as explained in Section 2.1 and then give a brief review about the known optical, radio and X-ray properties of each redshift subsample. We can use this information to do comparisons with the our calculated values to check that our methods are accurate and robust. It also allows us to explain deviations from expected results for the correlations. The NVSS covers the all the sky with a declination greater than -40° at 1.4 GHz. The sources obtained from this survey numbers close to 2×10^6 with a flux greater than $S \approx 2.5$ mJy. The largest position uncertainties for the VLA survey are $\leq 7''$ which is sufficient for comparing with observations in other wavebands.

We have provided overlays of the optical and radio plate for each of our clusters in order to confirm the information obtained from papers with regards to the radio sources. The optical images are from the SDSS* (Richmond 1996, Abazajian et al. 2009) and the Digital Sky Survey (DSS) (The Catalogs and Surveys Group (CASG) 2012) and radio images from the NRAO VLA Sky Survey (NVSS)[†] (Condon et al. 1998). The NVSS with a moderately low resolution convolving beam of $45'' \times 45''$ allows low surface brightness features to be more discernible. The contour levels used for the radio images were at $(-1, 1, 2, 3, 4, 6, 8, \dots) \times 1 \text{ mJy beam}^{-1}$.

*Information on the the data releases can be found on their website: <http://www.sdss.org/>

[†]Images taken from Montage (<http://hachi.ipac.caltech.edu:8080/montage/index.html>) and NED(<http://ned.ipac.caltech.edu/>)

2.2.1 Group 1 ($0.00 < z < 0.05$)

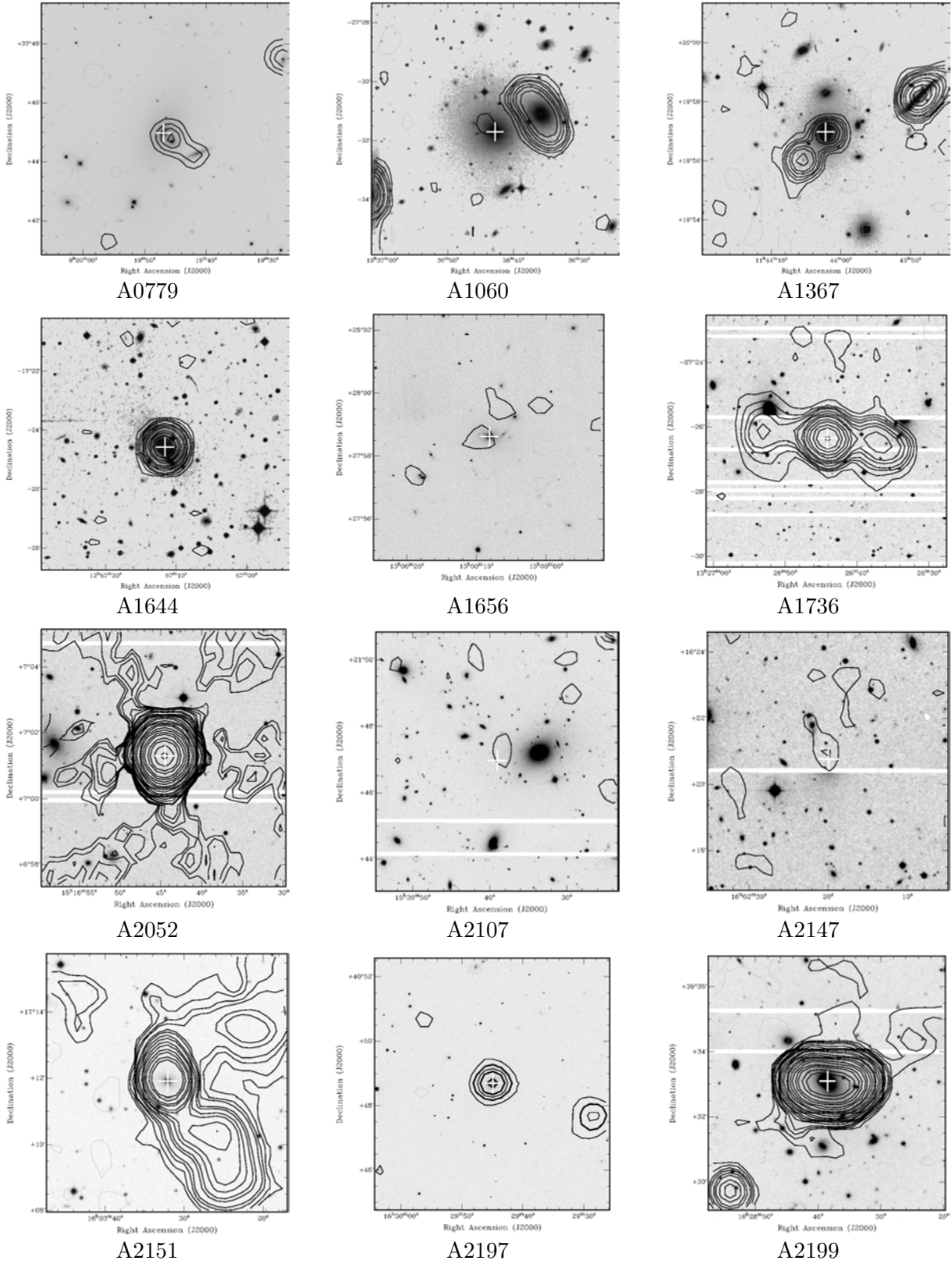


Figure 2.3: Radio overlays - Group 1: Each image shows the overlay of the NVSS radio contours onto the optical image obtained from DSS2 or SDSS for clusters with a redshift of less than 0.05.

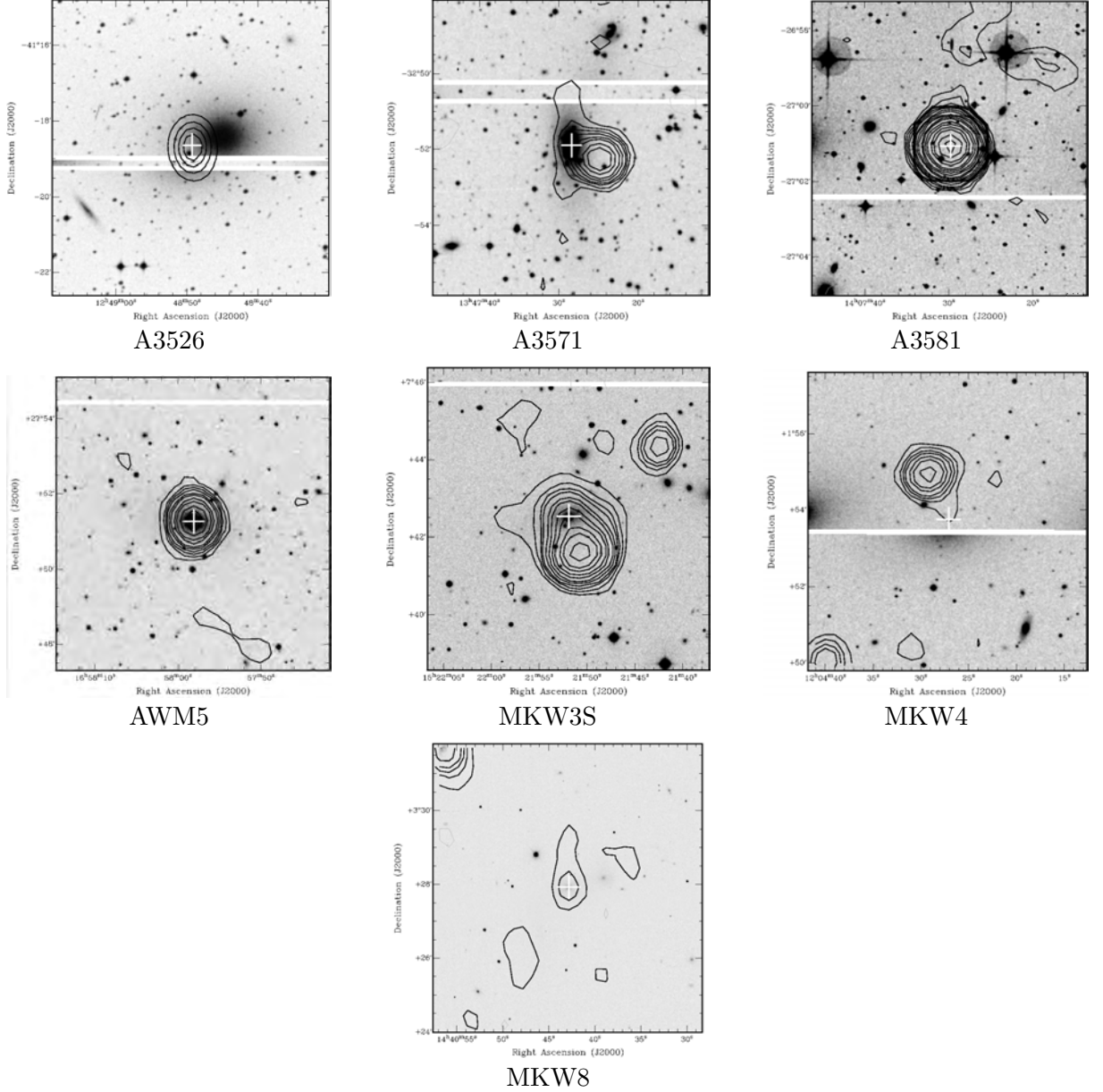


Figure 2.4: Radio overlays - Group 1 continued: The images in this figure show the overlay of the radio contours obtained from NVSS onto the optical image obtained from DSS2 or SDSS for clusters with a redshift of less than 0.05.

Abell 0779: This poor, faint cluster at a redshift of 0.023 has been studied at various wavelengths. Due to its small angular radius (only $75'$) it resembles a group rather than a cluster (Sreedhar et al. 2012) with more late-type galaxies than early-type. The BCG of this cluster is the cD galaxy NGC2832 which is at rest in the cluster potential (Hwang & Lee 2008). White et al. (1997) found this cluster to have a low luminosity in the X-ray with a gas temperature of only 1.5 keV. In the radio, Wilson & Vallee (1982) find a slightly extended source that lies in the NW-SE direction. The galaxies NGC2832 and NGC 2831 form a double system in a halo.

Abell 1060: Also known as Hydra I this cluster is located in the Southern Hemisphere at a distance of 50 Mpc and a redshift of 0.0144. It is a medium, compact cluster with a central cD galaxy NGC3311 (Arnaboldi et al. 2012). A giant elliptical NGC3309 lies only $1.7'$ away. This cluster is X-ray bright and is thought to be a relaxed system (Yamasaki et al. 2002) with

a smooth ICM distribution (Hayakawa et al. 2006). Tamura et al. (1996) found this cluster to have a constant temperature of 3.1 keV and a luminosity of 2×10^{43} ergs s⁻¹. The detected X-ray emission from NGC3311 and NGC3309 is made up of two components: the interstellar medium and the X-ray binaries (Yamasaki et al. 2002). From A1060 Figure 2.3 with a white cross centered on the BCG on the optical image we can see that NGC3311 has a lower radio flux than NGC3309, which has a flux density of 62.8 mJy (Condon et al. 1998).

Abell 1367: With a redshift of 0.022, this cluster lies at approximately the same distance as the well-known Coma cluster but has only half the Coma intracluster medium (ICM) (Scott et al. 2012). This spiral-rich cluster contains two subclusters of approximately the same mass which are in the process of forming from other smaller groups. These two subclusters appear to be in the beginning stages of a merger with A1367 forming at the intersection (Cortese et al. 2004). Forman et al. (2003) show, using XMM-Newton observations, that there is cool gas flowing into the centre from both of these subclusters. The idea that these clusters are merging is also suggested by the radio and optical data shown in A1367 in Figure 2.3 and discussed by Gavazzi et al. (1995). All this suggests that A1367 is a dynamically young cluster. The BCG for this cluster is NGC3842.

Abell 1644: This rich cluster, which is thought to be a double cluster at a redshift of 0.047, is found approximately 3 Mpc from the Shapley supercluster and most likely lies along one of its filaments (Johnson et al. 2010). Earlier Tustin et al. (2001) found this cluster to contain 141 galaxies with no significant evidence to suggest a double cluster. However, XMM-Newton observations by Reiprich et al. (2004) show a clear bimodal distribution. They suggested that the subcluster passed by the main cluster and some of its gas was stripped forming a warm ICM connecting the two. A1644 in Figure 2.3 shows the radio counterpart of the cD galaxy PGC04257.

Abell 1656: Also known as the Coma cluster this is one of the richest nearby clusters. It lies at a redshift of 0.023 and is ~ 100 Mpc away. It has a virial radius of 2.3 Mpc with an approximate mass of $10^{15} M_{\odot}$ and temperature of 8 keV. It has been studied in the optical, X-ray and radio (Keshet et al. 2012). Willson (1970) found that the cluster contained a giant radio halo and Giovannini et al. (1985) studied the extended radio source 1253+275 and suggested that it may be a radio relic. Both of these are studied in great detail by Brown & Rudnick (2011). It is known to contain subclusters and contains the 2 giant ellipticals NGC 4839 and NGC 4889 at its centre (West et al. 1995). X-ray studies by ROSAT and XMM-Newton show turbulence, in-falling galaxies and dynamical activity. A shock front from the giant radio halo is also visible in the X-ray (Planck Collaboration et al. 2012).

Abell 1736: This non-relaxed cluster is at a redshift of 0.045 and has $r_{500} = 916.9$ Kpc (Lau et al. 2012). Valentinuzzi et al. (2011) study this cluster with data obtained from the WIdE-field Nearby Galaxy-cluster Survey (WINGS) (Fasano et al. 2006) and find that it has a velocity dispersion of 853 km s⁻¹ and $\log(L_x) = 44.37 L_{\odot}$. Using Sunyaev–Zel’dovich (SZ) scaling relations and data obtained from Chandra telescope, Comis et al. (2011) find that this cluster has an angular diameter distance of 139 Mpc and a total mass of $0.137 \times 10^{14} M_{\odot}$. As shown in Figure 2.3, A1736 has a radio galaxy associated with the BCG, PGC047071.

Abell 2052: This moderately rich cluster sits at a redshift of 0.03549 has been observed in X-ray by Einstein, ASCA, Chandra, Suzaku and XMM-Newton and in radio by VLA. It has a central cD galaxy UGC09799 which hosts the radio galaxy 3C 317. It is a bright cool-core cluster that has shown AGN activity and H α regions (de Plaa et al. 2010). A spiral feature was seen with deep Chandra observation which is explained by a previous merger. Spectroscopy shows evidence of dynamical activity with the peculiar velocity of UGC09799 found to fairly large at $290 \pm 90 \text{ km s}^{-1}$ (Blanton et al. 2011).

Abell 2107: This nearby, isolated cluster with a cluster radius of $45'$ (Kalinkov et al. 2005). Girardi et al. (1997) found this regular cluster to host the central cD galaxy UGC09958. The X-ray images show an elongation in a East-West direction with the cD at the centre. Fujita et al. (2006) show using hardness ratio maps that the presumably coolest gas lies elongated in a North-South direction suggesting that the centre of the cluster is not in pressure equilibrium. A2107 in Figure 2.3 shows that UGC09958 has a low radio flux just above the 3σ detection from the NVSS.

Abell 2147: This cluster is part of the supercluster Hercules and is X-ray luminous, composed mostly of elliptical galaxies and dynamically evolved (Dickey 1997). It has a dense, hot ICM which is centered on UGC10143 which is one of the BCGs of this cluster. However, the main BCG is UGC10144 with a magnitude of 14.2 in the g band as compared to 14.4 for UGC10143. The optical and radio counterpart for UGC10144 is shown in A2147 (Figure 2.3).

Abell 2151: This young cluster is at a redshift of 0.037 and is part of the Hercules supercluster along with A2147 and A2152 (Cedr s et al. 2009). Dickey (1997) found an inhomogeneous galaxy distribution around A2151 and no trace of ICM suggesting that this cluster is not dynamically relaxed. The X-ray emission is found to be centered on NGC 6045 rather than the BCG NGC 6034 as one would expect to account for the morphology of the radio galaxy shown in A2151 in Figure 2.3. However, it may be due to the constant merging of galaxies occurring (Dickey 1997). S nchez-Janssen et al. (2005) show that A2151 has at least three subclusters.

Abell 2197: This cluster is slightly X-ray luminous and has an irregular structure (Muriel et al. 1996). It has two main concentrations suggesting substructure. The optical and X-ray data concentrate around the BCG NGC 6173 which has an elongation in the south-east direction. There is also a radio contribution as shown in A2197 in Figure 2.3.

Abell 2199: This cluster has a regular morphology and is a bright X-ray source at a redshift of 0.03 (Hwang et al. 2012). It forms part of a supercluster with the two concentrations in Abell 2197. The X-ray properties calculated by Muriel et al. (1996) suggest that the cluster is virialized. The massive cD galaxy NGC 6166 dominates the central region of the cluster (Kelson et al. 2002) and it is associated to the radio source 3C 338 (Figure 2.3). The X-ray emission is peaked on the cD galaxy which results in cooling flows.

Abell 3526: This cluster is also known as Centaurus and is at a redshift of 0.01. The BCG of this cluster is NGC4696 which is a giant elliptical galaxy (Farage et al. 2010). Mitchell et al.

(1975) found that the X-ray emission of this cool-core cluster was greater than calculated using the Uhuru isothermal gas sphere model and concluded that either the X-ray source was less elongated than previously thought or that NGC4696 is a compact source. There were no NVSS images available for this source as it has a declination of -41.3° which is too far south for VLA. Therefore for the overlay we used an image from the Sydney University Molonglo Sky Survey (SUMSS) which is a wide-field radio imaging survey of the Southern Sky at 843 MHz (Murphy et al. 2007) and this is shown in A3526 (Figure 2.4). SUMSS has detected a radio source associated with the BCG.

Abell 3571: This cluster is part of a complex of three clusters: A3571, A3572 and A3575 which is dominated by A3571 and forms part of the Shapley supercluster (Venturi et al. 2002). This cluster has a dominant cD galaxy at its centre, MGC-05-33-002 which is also known as PGC048896 or ESO 383-G 076. In the X-ray this cluster is bright and hot and a weak cooling flow is present (Nevalainen et al. 2000). It has a temperature of ≈ 8 keV and is well relaxed.

Abell 3581: The BCG for this cluster is IC4374, at $z=0.0218$, Smith et al. (2000). A3581 belongs to the supercluster Hydra-Centaurus which is about $40 \text{ h}^{-1} \text{ Mpc}$ away* and is situated $100 \text{ h}^{-1} \text{ Mpc}$ in front of the Shapley supercluster (Proust et al. 2006).

AWM 5 This is a poor, X-ray luminous cluster at a redshift of 0.0348. The cD galaxy NGC6269 lies at the centre of the cluster and at the peak of the X-ray emission (Baldi et al. 2009). NGC6269 is also associated with a low flux radio source as seen in AWM 5 (Figure 2.4).

MKW 3S: This cluster is found at a redshift of 0.0443 and has a dominant central galaxy, NGC5920 which has similar properties to a cD galaxy but is smaller (Krempec-Krygier & Krygier 1999). It is a poor cluster with a temperature of 3 keV (David et al. 1993).

MKW 4: This is a poor cluster of approximately 50 galaxies at a redshift of 0.02 (O’Sullivan et al. 2003). It is dominated by the cD galaxy NGC4073 and the cluster members are unambiguous suggesting no substructure is present (Koranyi & Geller 2002). MKW 4 in Figure 2.4 shows a faint radio source near NGC4073.

MKW 8: This poor cluster is at a redshift of 0.027 and shows little substructure in the X-ray. There are 2 bright galaxies in the centre, with the brightest one NGC5718 corresponding to the X-ray peak. The elongation in the X-ray extends towards the other galaxy in the East with a possible radio relic as seen in MKW 8 in Figure 2.4 (Hudson et al. 2010). It has a temperature of 3 keV and a virial radius of 1.17 Mpc (Raichoor & Andreon 2012).

*The Hubble constant is parameterized as $h=H_0/100$ by (Proust et al. 2006)

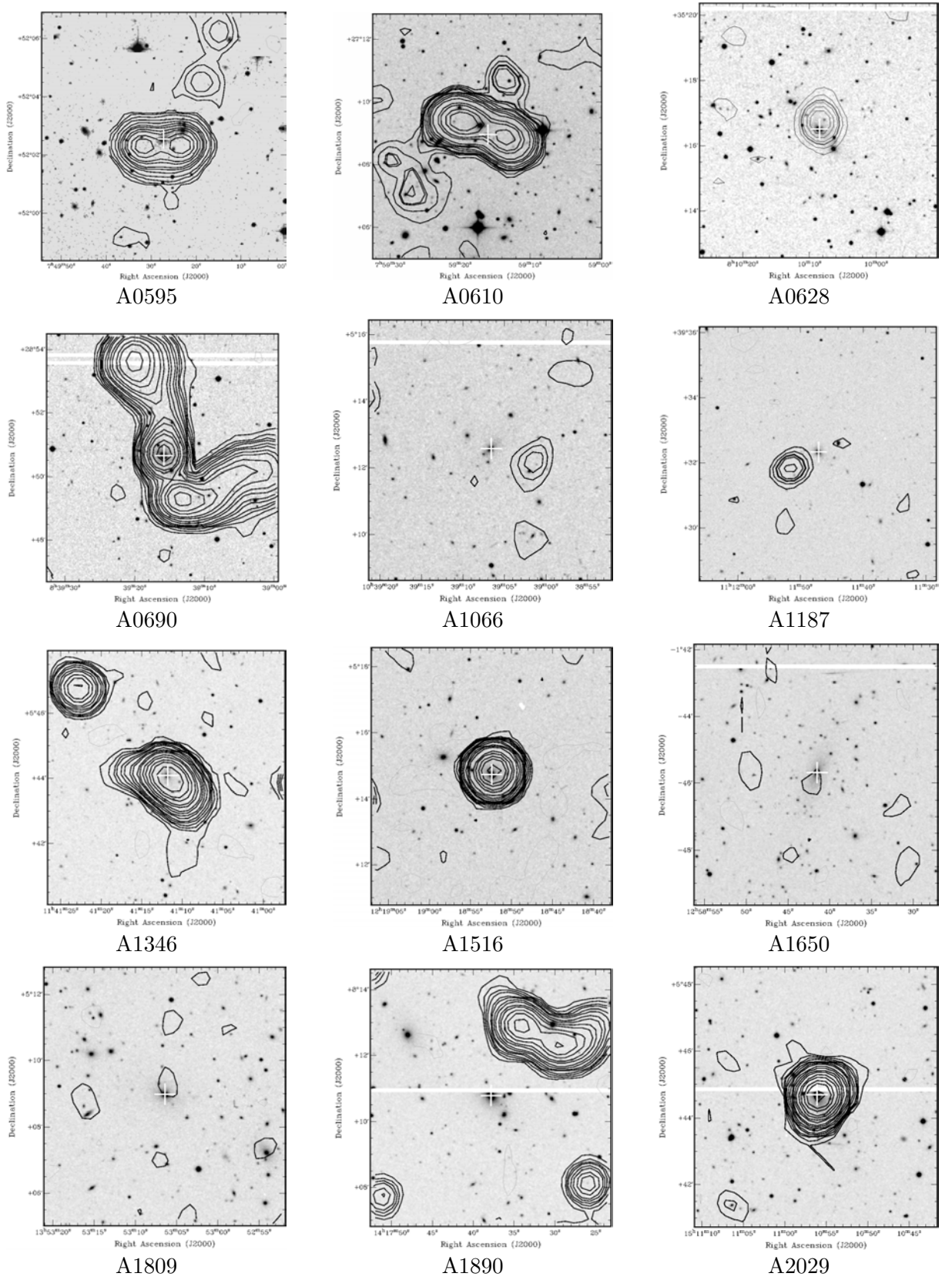
2.2.2 Group 2 ($0.05 < z < 0.10$)

Figure 2.5: Radio overlays - Group 2: These images show the overlay of the NVSS radio contours onto the optical image obtained from DSS2 or SDSS for clusters with a redshift between 0.05 and 0.10.

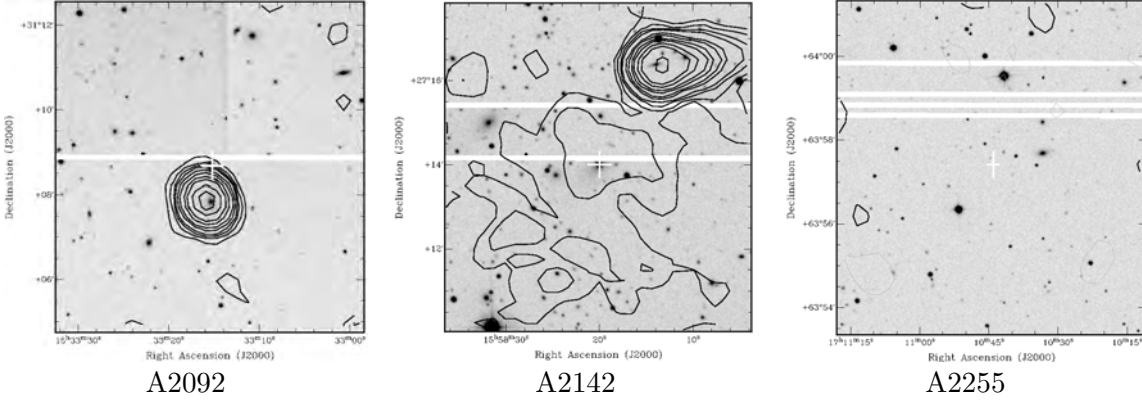


Figure 2.6: Radio overlays - Group 2 continued: The images in this figure show the overlay of the radio contours obtained from NVSS onto the optical image obtained from DSS2 or SDSS for clusters with a redshift between 0.05 and 0.10.

Abell 0595: This cluster at a redshift of 0.068 has a powerful radio source, 0745+521 (Jetha et al. 2006), associated with its BCG, MCG +09-13-062 as seen in A0595 (Figure 2.5). It has an observed flux of 520 mJy at a frequency of 1.4 GHz. The mass of the cluster is found to be $1.34 \times 10^{13} M_{\odot}$ (Deng et al. 2007).

Abell 0610: This poor cluster at a redshift of 0.0991 is situated behind the rich cluster ZwCl 0752.9+2833 which is at a redshift of 0.15. It has a WAT radio galaxy, B2 0756+27 situated with its centre (Valentijn 1979). Boschijn et al. (2008) calculate this cluster to have a velocity dispersion of 496 km s^{-1} and a virial mass of $2.3 \times 10^{14} M_{\odot}$.

Abell 0628: At a redshift of 0.0838, this cluster has between 61 and 65 members. It has a velocity dispersion of 555 km s^{-1} and a virial radius of $0.73 h^{-1} \text{ Mpc}$ (Einasto et al. 2012a). Popesso et al. (2007b) find the cluster to have a mass of $5.98 \times 10^{14} M_{\odot}$ and an optical luminosity of $2.81 \times 10^{12} L_{\odot}$. It has a compact radio source associated with the BCG (Figure 2.5).

Abell 0690: It is a rich cluster of galaxies at a redshift of 0.0788. The optical counterpart is dominated by a multiple-nuclei cD galaxy with the radio counterpart being the extended wide-angle tail galaxy B20836+29 as seen in A0690 Figure 2.5. It has a total radio power of $1.2 \times 10^{25} \text{ W Hz}$ and has two lobes. In the X-ray it is visible as extended emission from the intracluster gas it is buried in (Venturi et al. 1995).

Abell 1066: This rich cluster at a redshift of 0.07 is located in the Leo Sextans supercluster (Planck Collaboration et al. 2011). It has a complex distribution of its galaxy members in 3D. They form an hour-class shape (Einasto et al. 2010). Einasto et al. (2012b) find this to be an unimodal cluster at a distance of $207 h^{-1} \text{ Mpc}$ with a velocity dispersion of 748 km s^{-1} . A1066 in Figure 2.5 shows that the BCG does not have a radio galaxy associated with it.

Abell 1187: This cluster at a redshift of 0.0791 is classified by Rhee et al. (1991) as a RS type I and a BM class III which are explained in Table 1.2. It has a velocity dispersion of 1049 km s^{-1} and a X-ray luminosity of $0.093 \times 10^{44} \text{ ergs s}^{-1}$ (Plionis et al. 2009). As seen in A1187 (Figure 2.5) NVSS does not detect a radio galaxy associated with its BCG.

Abell 1346: This cluster at a redshift of 0.984 is a BM type II-III (Slingsend et al. 1998). Crawford et al. (2009) find this cluster to have a velocity dispersion of 732 km s^{-1} and a spectral index of -0.72. The BCG is associated with the possible WAT galaxy 1138+060 (Pinkney et al. 2000) as seen in A1346 in Figure 2.5.

Abell 1516: This cluster is located in the high-density core of supercluster SCl 111. The gradients of the peculiar velocities of the galaxies in this cluster suggest that it may be rotating. There is also a large fraction of red galaxies in this cluster. It is situated at a distance of $230.3 \text{ h}^{-1} \text{ Mpc}$ and has a velocity dispersion of 834 km s^{-1} (Einasto et al. 2010).

Abell 1650: This cluster at a redshift of 0.0845 is a BM I-II (Takahashi & Yamashita 2003). X-ray observations have been performed using ROSAT, ASCA and Einstein. White (2000) found that this cluster has a temperature of 6 keV and a metallicity of 0.3. The cD galaxy is 2MASX J12584149-0145410 and has a low radio flux as seen in A1650 (Figure 2.5).

Abell 1809: This is a rich cluster of redshift 0.0791 which is on the outskirts of supercluster SCl 126 and is at a distance of $236.5 \text{ h}^{-1} \text{ Mpc}$ (Einasto et al. 2010). This cluster has two components: the main central component and a poor component composed mostly of a few loose spirals. The main component is composed of most of the galaxies in the centre and it has a bimodal distribution suggesting that this cluster was formed by the merging of two groups with a third, the poor component, still in-falling. It has a high fraction of red galaxies and the multiple components suggest that this system is not virialized. Einasto et al. (2012a) find that this cluster has a virial radius of $0.44 \text{ h}^{-1} \text{ Mpc}$ and a velocity dispersion of 651 km s^{-1} . It is a Bautz-Morgan class II and does not have a cool core (Wojtak & Łokas 2010). From A1809 in Figure 2.5 we can see that the radio galaxy associated with the BCG is radio quiet.

Abell 1890: At a redshift of 0.058 this poor cluster hosts two giant ellipticals, NGC5539 and NGC 5532. NGC 5532 hosts the radio galaxy 3C269 and is at a redshift of 0.02357 (Hardcastle et al. 1997). It is a nearby FR I galaxy and has straight jets. NGC 5539 is a cD galaxy (Coziol et al. 2009) and dominates our cluster but as seen in A1890 in Figure 2.5 it has a low radio flux. The X-ray emission from this cluster is clumpy (Sakelliou & Merrifield 2000). It has a velocity dispersion of 508 km s^{-1} and a temperature of 2.9 keV (White et al. 1997).

Abell 2029: This is a relaxed cluster at a redshift of 0.0767 that has been widely studied in the X-ray (Walker et al. 2012). Uson et al. (1991) found this cluster to be possibly one of the most luminous and largest in the Universe. It has a large cD galaxy IC 1101 which is radio loud as shown in A2029 in Figure 2.5.

Abell 2092: This cluster is at a redshift of 0.0669 (Wen et al. 2010) with a cool temperature of 2-3 keV. It has a faint X-ray luminosity of only $1.9 \times 10^{43} \text{ ergs s}^{-1}$ (Burenin et al. 2007). It has a mass of $8.95 \times 10^{13} \text{ M}_{\odot}$ and a low radio luminosity of less than 22 W Hz^{-1} (Sun et al. 2009). The compact radio source is not associated with the BCG as seen in A2092 Figure 2.6.

Abell 2142: This X-ray luminous cluster at a redshift of 0.0909 has a high ICM temperature of 9 keV and was the first cluster in which cold fronts were detected (Akamatsu et al. 2011). This suggests that a merger is taking place in A2142 and evidence of a subcluster infall is present. Two large elliptical galaxies are found in the centre and are aligned with the X-ray brightness. A peak is visible in the X-ray image suggesting cooling flows (Markevitch et al. 2000). From A2142 in Figure 2.6 we can see that it appears to lie in the radio lobe of another radio source.

Abell 2255: This rich cluster at a redshift of 0.0806 has been studied in many wavelengths. X-ray observations by Sakelliou & Ponman (2006) show that is a non-relaxed cluster with a cool cluster core that is undergoing a merger. The bimodality and high dispersion in the velocity distribution found in optical observations by Yuan et al. (2003) confirm that the cluster is dynamically disturbed and suggests the possibility of substructure. Radio observations detect a diffuse radio halo, a relic source and seven extended head-tail radio galaxies (Pizzo et al. 2011). Harris et al. (1980) named these radio galaxies Goldfish, Double, Original TRG, Sidekick, Bean, Beaver and Embryo. The BCG of this cluster is ZwCl 1710.4+6401 A and is shown in A2255 in Figure 2.6.

2.2.3 Group 3 ($0.10 < z < 0.15$)

Abell 0646: This cluster at a redshift of 0.1303 has a luminosity of 44.54 W Hz^{-1} in the 0.1-2.4 keV band of the X-ray spectrum (Rudnick & Lemmerman 2009). It has the FR II radio galaxy, 3C197.1 at its centre (Saripalli & Subrahmanyam 2009) associated with the BCG as seen in A0646 in Figure 2.7. The radio power at 1.4 GHz is 25.79 W Hz^{-1} . The infra red luminosity is $1.49 \times 10^{44} \text{ ergs s}^{-1}$ (Quillen et al. 2009). The radius at an over-density of 200 is 2.034 Mpc (Barkhouse et al. 2007).

Abell 0858: This cluster is not very luminous in the X-ray with an X-ray luminosity of $0.539 \times 10^{44} \text{ ergs s}^{-1}$ (Plionis et al. 2009). It has a velocity dispersion of 727 km s^{-1} .

Abell 1080: This is a clumpy cluster at a redshift of 0.116 (Bahcall et al. 2003). A1080 in Figure 2.7 shows that the radio source associated with the BCG is small and quiet.

Abell 1366: This cluster at a redshift of 0.1164 has a velocity dispersion of 788 km s^{-1} (Miller et al. 2005). It is moderately bright in the optical and X-ray with luminosities of $2.61 \times 10^{12} L_{\odot}$ and $1.55 \times 10^{44} \text{ ergs s}^{-1}$ respectively (Popesso et al. 2007b). The mass of the cluster within the radius r_{200} is $7.72 \times 10^{14} M_{\odot}$. It has a radio loud galaxy associated with the BCG at the centre.

Abell 1544: This poor cluster is at a redshift of 0.1459 (Sakelliou & Merrifield 2000) and has a velocity dispersion of 556 km s^{-1} (Miller et al. 2005). It does not have a radio source associated directly with the BCG but there is one slightly to the North as shown in A1544 in Figure 2.7.

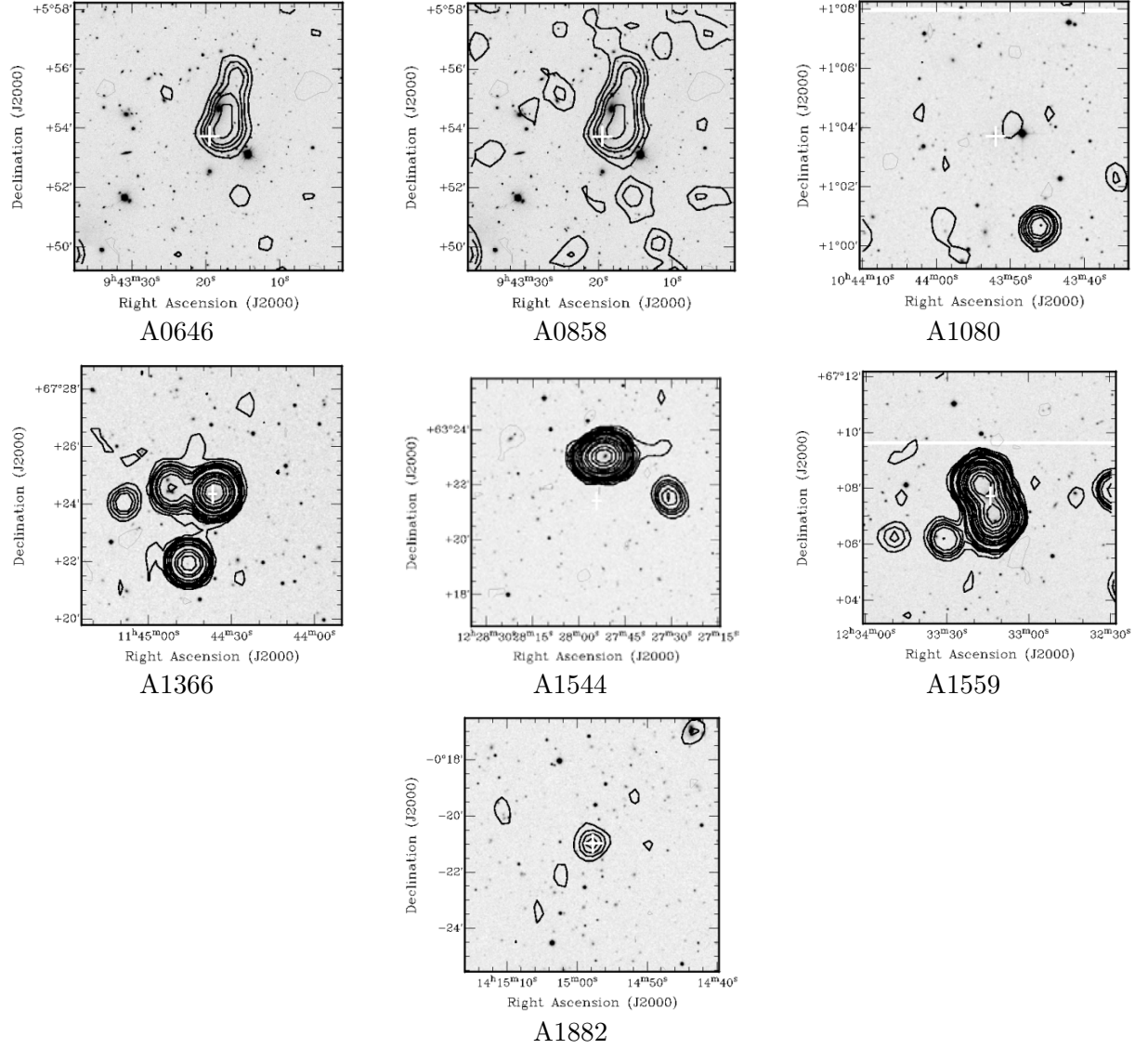


Figure 2.7: Radio overlays - Group 3: The images in this figure show the overlay of the radio contours obtained from NVSS onto the optical image obtained from DSS2 or SDSS for clusters with a redshift between 0.10 and 0.15.

Table 2.2: References for the literature values in Tables 2.3 and 2.4

1. Yoon et al. (2008)	2. Lopes et al. (2009)	3. Hernandez-Fernandez et al. (2012)
4. Rines & Diaferio (2006)	5. Popesso et al. (2007a)	6. Deng et al. (2007)
7. Coenda & Muriel (2009)	8. Hudson et al. (2001)	9. Fukazawa et al. (2004)
10. Prugniel & Simien (1996)	11. Szabo et al. (2011)	12. Wen et al. (2012)
13. Flesch (2010)	14. Sun (2009)	15. Ebeling et al. (1998)
16. Owen et al. (1982)	17. White & Becker (1992)	18. Condon & Kaplan (1998)
19. Vollmer (2009)	20. Healey et al. (2007)	21. Laine et al. (2003)
22. Lawrence et al. (1983)	23. Ledlow et al. (2003)	24. Becker et al. (2012)
25. Shen et al. (2008)	26. Popesso et al. (2004)	27. Brownstein & Moffat (2006)
28. Aguerri et al. (2007)	29. White et al. (1997)	30. David et al. (1993)

Abell 1559: This cluster sits at a redshift of 0.1058 and has the strong radio source, 1231+674 at its center associated with the BCG (Jetha et al. 2006). Hardcastle & Sakelliou (2004) suggest that it is a NAT that is falling into the potential well of the galaxy. The optical counterpart is a dumbbell galaxy orientated in a E-W direction. At 1.4 GHz it has a radio flux of 900 mJy. It has a velocity dispersion of 863 km s^{-1} (Popesso et al. 2007b) and a mass of $14.06 \times 10^{14} M_{\odot}$ with a radius of 1.27 Mpc (Poggianti et al. 2006).

Abell 1882: This cluster at a redshift of 0.1396 has three extended sources visible in both optical and X-ray (Dietrich et al. 2007). In a R_{200} radius of 1.9 Mpc it has a mass of $7.44 \times 10^{14} M_{\odot}$ and a velocity dispersion of 733 km s^{-1} . It is very bright in the optical with a luminosity of $13.29 \times 10^{12} L_{\odot}$ (Popesso et al. 2007b). A1882 in Figure 2.7 shows that the radio source associated with the BCG is compact.

2.3 Conclusion

In this chapter we looked at the selection criteria we used to obtain our sample and gave its global properties. Data mining using VO and a literature survey provided some background information about each of the galaxy clusters in our sample with respect to optical, radio and X-ray data. We present overlays of the radio contours on the optical image to compare with what is known about the radio sources in our galaxy clusters. Finally we summarised the known results in Tables 2.3 and 2.4.

Table 2.3: Galaxy cluster literature values: Column 1 and 2 gives the cluster and BCG name respectively with the redshift given in column 3. The physical parameters velocity dispersion, radius, mass and temperature are given in columns 3 - 6. Column 7 and 8 are the calculated temperatures using the equations given in Section 1.5.5. The subscript gives the reference as shown in Table 2.2 and values in **bold** have been calculated given the equations in Section 1.5.

Cluster Name	BCG Name	z	σ (km s ⁻¹)	Radius (Mpc)	Mass (10 ¹⁴ M _o)	T (keV)	T ₁ (keV)	T ₂ (keV)
A0595	SDSS J074927.24+520232.5	0.0698	349 ¹	0.789 ¹	0.130	-	0.251	0.218
A0610	B2 0756+27	0.0991	496	0.932 ¹	2.300	-	0.360	3.270
A0628	2MASX J08100854+3516315	0.0838	555	0.850 ¹	5.980	-	0.295	9.322
A0646	2MASX J08220955+4705529	0.1303	927 ⁹	1.669 ⁷	2.034	6.0 ⁹	1.190	1.615
A0690	B2 0836+29	0.0788	380 ¹	0.542 ¹	0.273	-	0.119	0.667
A0779	NGC2832	0.0255 ¹²	503 ⁹	0.960 ¹²	2.150 ²	1.5	0.449	2.968
A0858	2MASXJ09431952+0553438	0.0855 ¹	727	0.664 ¹	1.233	-	0.180	2.441
A1060	NGC3311	0.0144	630 ⁹	2.032	2.810	3.1	1.577	1.832
A1066	2MASX J10390665+0512353	0.0700	748	1.194 ¹	2.328	-	0.574	2.584
A1080	2MASX J10435204+0103475	0.1160	420 ⁷	1.038 ⁷	1.400 ⁷	-	0.454	1.787
A1187	2MASXJ11114710+3932202	0.0791	1049	0.685 ¹	2.626	-	0.191	5.080
A1346	4C06.42	0.0982	732	0.969 ¹	1.809	-	0.388	2.474
A1366	2MASX J11443683+6724211	0.1164	788	1.122 ⁷	7.720	5.6 ¹⁵	0.530	9.117
A1367	NGC3842	0.0220	500 ⁸	1.607	1.400	3.4 ²⁷	0.993	1.154
A1516	2MASXJ12185235+0514443	0.0769 ¹	834	0.749 ¹	8.300 ⁵	-	0.227	14.684
A1544	CGPG 1225.6+6338	0.1459	556	1.370 ¹²	1.476	-	0.815	1.428
A1559	VII Zw 470	0.1058	863	1.270	14.060	-	0.672	14.670
A1644	PGC044257	0.0470	933 ⁹	2.964	8.990	4.7 ⁹	3.459	4.019
A1650	2MASXJ12584149-0145410	0.0845	740 ⁷	1.833 ⁷	7.690 ⁷	6.0	1.372	5.559
A1656	NGC4889	0.0230	1010 ⁹	2.300	10.000	8.0	2.037	5.761
A1736	PGC047071	0.0450	853	2.712	0.137	4.6 ¹⁵	2.891	0.067
A1809	2MASX J13530637+0508586	0.0791	651	1.869 ⁷	5.790 ⁷	2.9 ²	1.418	4.105
A1882	2MASXJ14145769-0020589	0.1396	733 ⁵	1.900	7.440	-	1.557	5.189
A1890	NGC5539	0.0500	661	0.871 ¹	1.326	2.9	0.300	2.017
A2029	IC 1101	0.0767	786 ⁹	1.580 ¹²	3.401	7.8 ⁹	1.011	2.852
A2052	UGC09799	0.0355	750 ⁹	2.395	4.694	3.1 ⁹	2.234	2.597
A2092	2MASX J15331536+3108430	0.0669	458 ²⁸	0.679 ¹	0.496	2.5	0.185	0.968
A2107	UGC09958	0.0501 ¹²	577 ⁹	1.640 ¹²	1.902	4.2 ⁹	1.062	1.537
A2142	2MASX J15582002+2714000	0.0909	1295 ⁹	2.918 ¹	16.930 ²	9.0	3.498	7.709
A2147	UGC10144	0.0353 ⁹	1148 ⁹	3.667	16.838	4.3 ⁹	5.237	6.085
A2151	NGC6034	0.0370	827 ⁹	2.640	6.291	2.1 ⁹	2.719	3.158

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Table 2.3 – Continued								
Cluster Name	BCG Name	z	σ (km s $^{-1}$)	Radius (Mpc)	Mass (10^{14} M $_{\odot}$)	T (keV)	T $_1$ (keV)	T $_2$ (keV)
A2197	NGC6173	0.0308	564 ⁹	1.883	2.087	1.6 ⁹	1.375	1.469
A2199	NGC6166	0.0300	756 ³	1.830 ³	9.810 ²	4.5 ⁹	1.298	7.103
A2255	ZwCl 1710.4+6401	0.0806	883 ²	2.785 ¹	13.000 ²	5.4 ²	3.154	6.185
A3571	ESO383-G076	0.039 ⁹	1150 ⁹	3.667	16.897	8.0	5.255	6.106
A3581	IC4374	0.0218	654 ⁹	2.102	3.132	1.7 ⁹	1.699	1.974
AWM5	NGC6269	0.0348	218 ¹⁰	0.697	0.115	-	0.189	0.219
MKW3S	NGC5920	0.0443	678 ⁹	1.380 ¹²	2.210	3.0	0.748	2.122
MKW4	NGC4073	0.0200	392 ²	1.261	2.040 ²	1.5 ²	0.611	2.144
MKW8	NGC5718	0.0270	420 ²	1.170	3.040 ²	3.0	0.529	3.443

Table 2.4: Galaxy cluster literature values continued: Column 1 gives the cluster name. The physical parameters radio flux density at frequency 1.4 GHz, spectral index and X-ray luminosity are given in columns 2, 3 and 7 respectively. These were calculated using different methods explained in their respective literature papers. The calculated radio luminosity and radio power at frequency 1.4 GHz using equations as given in Section 1.5.4 are given in column 4 and 5. Column 6 gives the Fanaroff-Riley classification of the radio galaxy based on its radio power. The subscript gives the reference as shown in Table 2.2 and values in bold have been calculated given the equations in Section 1.5.

Cluster Name	S1400 (mJy)	α	$L_{1.4}$ (10^{24} ergs s^{-1})	$P_{1.4}$ (10^{25} W)	FR Type	$L_x(10^{37}$ W)
A0595	520 ¹⁶	-0.67 ¹⁹	5.02	0.5370	I	-
A0610	490 ¹⁶	-0.84 ¹⁹	9.16	1.0100	II	-
A0628	16 ¹⁸	-	-	0.0237	I	0.52 ²⁵
A0646	820 ¹⁶	-0.45 ²⁰	25.40	2.8700	II	4.54 ¹⁵
A0690	877 ¹⁷	-0.60 ¹⁷	10.70	1.1500	II	0.04 ²³
A0779	5 ¹⁸	-0.59 ²⁰	0.51	0.0633	I	0.20 ²³
A0858	112 ¹⁸	-	-	0.1720	I	0.54
A1060	63 ¹⁸	-0.82 ²⁰	0.03	0.0029	I	0.30 ²³
A1066	5 ¹⁸	-0.31 ²¹	0.05	0.0052	I	0.17 ²³
A1080	-	-	-	-	-	-
A1187	7 ¹⁷	-	-	0.0093	I	0.09
A1346	690 ¹⁶	-0.72	12.70	1.3900	II	0.39 ²⁴
A1366	380 ¹⁶	-0.90 ¹⁹	9.57	1.0700	II	1.55
A1367	-	-	-	-	-	0.23 ²³
A1516	579 ¹⁷	-0.65 ¹⁹	6.72	0.7240	I	0.36 ²⁴
A1544	233 ¹⁸	-0.89 ¹⁹	8.86	1.0200	II	2.15 ²⁶
A1559	101 ¹⁷	-1.05 ¹⁹	2.13	0.2360	I	-
A1644	-	-	-	-	-	5.16 ²⁹
A1650	1 ²⁹	-	-	0.0015	I	-
A1656	-	-	-	-	-	-
A1736	181.1 ¹⁸	-	-	0.0787	I	2.37 ¹⁵
A1809	-	-	-	-	-	0.30 ²³
A1882	11 ¹⁸	-	-	0.0440	I	-
A1890	1 ²⁹	-	-	0.0005	I	1.07 ²⁹
A2029	590 ¹⁶	-1.39 ¹⁹	6.82	0.7340	I	2.60 ²³
A2052	5499.3 ¹⁸	-	-	1.4900	II	1.39 ³⁰
A2092	47 ¹⁸	-	-	0.0447	I	0.19
A2107	-	-	-	-	-	2.00 ⁹
A2142	-	-	-	-	-	58.59 ²⁹
A2147	7 ¹⁸	-	-	0.0019	I	4.23 ²⁹

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Table 2.4 – Continued

Cluster Name	S1400 (mJy)	α	$L_{1.4}$ (10^{24} ergs s $^{-1}$)	$P_{1.4}$ (10^{25} W Hz $^{-1}$)	FR Type	L_x (10^{37} W)
A2151	1450 ¹⁶	-0.64 ¹⁹	4.12	0.4280	I	1.46 ²⁹
A2197	-	-	-	-	-	-
A2199	3480 ¹⁶	-1.45 ¹⁹	6.58	0.6770	I	2.90 ³⁰
A2255	1 ¹⁶ -	-	0.0014	I	4.87 ⁵	
A3571	30 ¹⁸	-1.10 ²⁰	0.09	0.0098	I	7.36 ¹⁵
A3581	646 ¹⁸	-0.53 ¹⁹	0.65	0.0667	I	0.63 ⁹
AWM5	-	-	-	-	-	-
MKW3S	126 ¹⁷	-1.38 ²²	0.51	0.0531	I	2.68 ¹⁵
MKW4	-	-	-	-	-	-
MKW8	3 ¹⁸	-	-	0.0005	I	0.32 ²⁵

Chapter 3

Methods and Analysis

Data mining is an important tool for analysing large amounts of astronomical data (Zhang & Zhao 2004). With the improvement of technology over the years, more sensitive telescopes are coming on-line and all these data are becoming available to the public. We wish to come up with an efficient method to use and obtain the information, such as the physical parameters described in Section 1.5. We test each of our methods on data obtained for the cluster Abell 3581 as a test case. Once the procedures have been tested, they are then applied to all the sources in our sample. The code used for these methods used a combination of previously written codes and subroutines as well as my own work using the R package and Python.

3.1 Application of Clustering Algorithms

In this section, the three methods of clustering are discussed.

3.1.1 SIMBAD

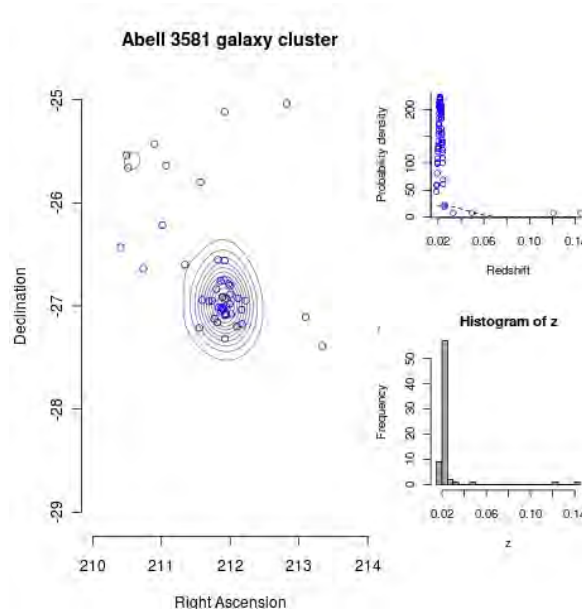


Figure 3.1: KDE analysis for A3581 data obtained from SIMBAD. Clockwise from left, the RA-DEC scatter plot, KDE, and histogram of the full SIMBAD/NED search results. The contours are 25, 50 and 75%.

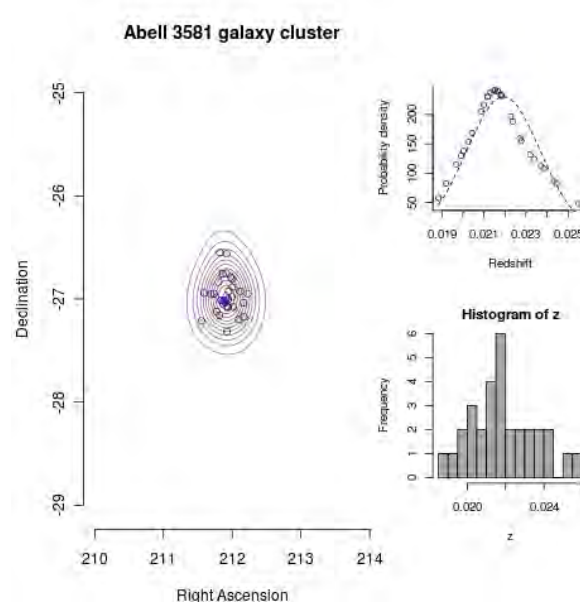


Figure 3.2: KDE analysis for A3581 data obtained from SIMBAD. The same as in Figure 3.1 but with the sources lying outside the contours removed. The distributions are now unimodal indicating that all sources considered here are part of the same cluster.

Searching the SIMBAD database for the cluster of galaxies A3581 yielded 107 possible galaxy members and their positions. The NASA/IPAC Extragalactic Database (NED) was used to search for redshifts for each galaxy. A binned kernel density estimate (KDE) (Section 3.2.3) is then applied to this cluster in order to estimate the probability density function and is shown in Figure 3.1. Many of the sources lie outside the contours and are thus less likely to belong to the cluster and are removed (Figure 3.2). This method was applied to all the sources from our sample in order to obtain galaxy members and their redshifts. These were then used to calculate the physical parameters discussed in Section 1.5 (Table A.1).

3.1.2 Mixture model methods

In Section 1.3.2 we discuss the GMM and KMM algorithms and how they work. A flow diagram showing the procedure we followed to determine cluster members using these algorithms is given in Figure 3.3. A 3 Mpc search around the BCG of the cluster of interest was found from two catalogues: Tago et al. (2010) and Jones et al. (2009). For A3581 the histogram of the redshift revealed basic possible groups within the 3 Mpc sample (Figure 3.4). For this cluster, the BCG has a redshift of $z=0.0218$ and we therefore select the group in this area (Sample G0). A dendrogram of this region of interest shows the possibility of 2 groups as seen in Figure 3.5. The algorithm was then applied to sample G0 and the group with a redshift similar to the BCG was chosen. The redshift of this group was then used to refine the search to 2 Mpc (Sample G1). Application of the algorithm to sample G1 gives our final sample.

KMM

When the KMM algorithm was applied to sample G0 for A3581 it showed that this sample contained three populations. Out of these three groups the one of interest had a 98.6% probability of containing 57 sources with a mean redshift of 0.0217. Using this to refine the search to 2 Mpc

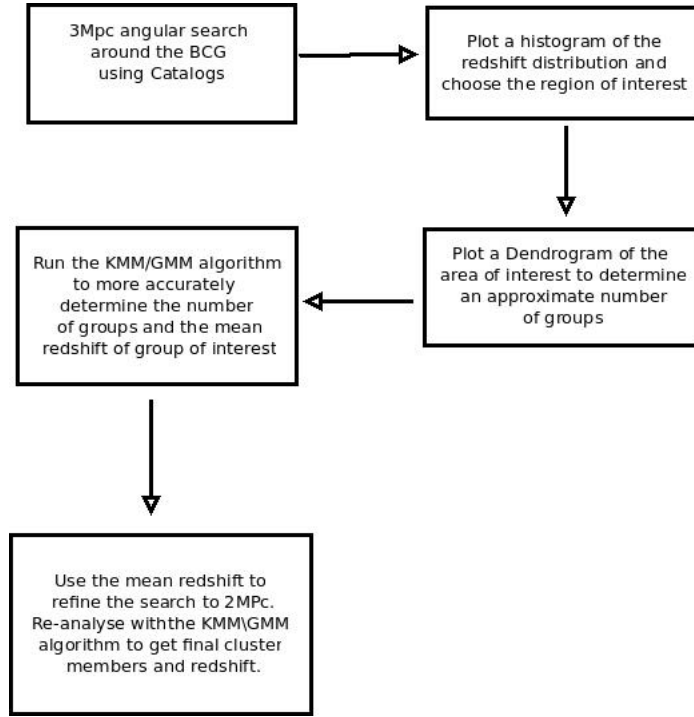


Figure 3.3: Flow diagram of Mixture Modeling Method showing the basic method followed for applying GMM and KMM in order to determine the number of galaxy cluster members.

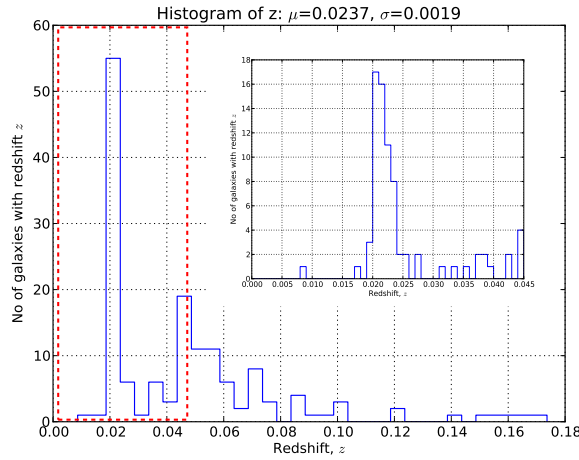


Figure 3.4: Redshift distribution of the dataset after the 3 Mpc search: This histogram shows the possibility of groups within this sample. A zoom in on the region (Sample G0) around the BCG is shown in the inset and suggests the possibility of more than one group.

and applying KMM revealed three populations with our group of interest having a 94.1% probability of containing 57 sources with a mean redshift of 0.0259. However, this redshift differed from that of the BCG by 16% and the velocity dispersion for this sample was calculated to be $2369.7 \pm 265.4 \text{ km s}^{-1}$. This high velocity dispersion raised concerns as to the correctness of this approach.

After performing the 2 Mpc search and running the KMM algorithm a modified Levene test was performed on the sample. The Brown-Forsyth Levene test method was used with a confidence level of 95%. The test gave $p < 0.05$, implying that each group had a different

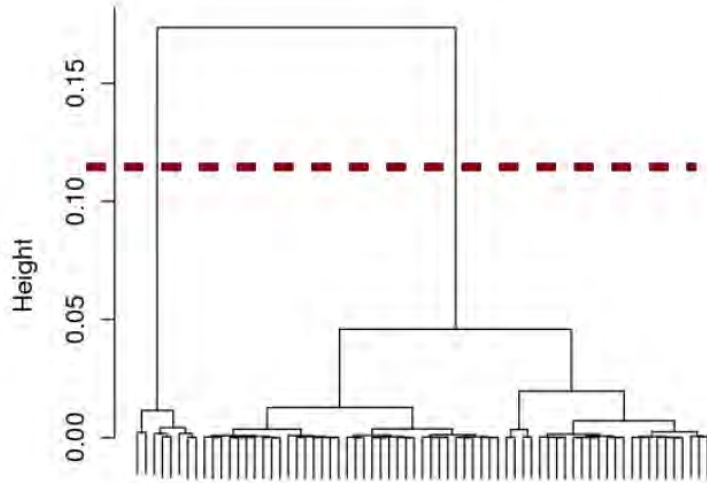


Figure 3.5: Dendrogram of sample G0: This shows that the sample has two populations at a 50% level of similarity as shown by the red dotted line.

variance. As mentioned earlier, the KMM algorithm assumes homoscedastic samples and errors due to differences in variances of the groups may cause some discrepancies. The variance of each group was determined and then KMM was re-run using these variances as a starting point. This now revealed our group to have 45 members with a mean redshift $z=0.0216$. One problem found with the KMM algorithm is that if the variances are selected to be different then it will not determine groups with the same variance. Therefore, after applying the KMM algorithm with different variances it must be re-applied to the group of interest, specifying homogeneity to obtain the final sample. For A3581 the final sample contained 28 members with a redshift of 0.021 and a velocity dispersion of $190.5 \pm 17.3 \text{ km s}^{-1}$. This modified method is shown in a flow diagram in Figure 3.6 and the results are given in Table A.2.

GMM

When applying GMM to sample G0 it revealed 3 populations with our population of interest containing 54 sources with a mean redshift of 0.022. This was used to refine the search radius to 2 Mpc. GMM was then run again to reveal 3 groups with different variances. Our group of interest contained 54 sources with a mean redshift of 0.022. One of the issues with GMM is that unlike KMM, GMM does not specify which group each source belongs to; it merely gives a probability of a source to belong to each group. We used the following criteria for separating the potential candidates of each group:

1. Highest probability to belong to a particular group
2. 95% confidence level only
3. Redshift restriction only using $\pm 1\sigma$ around the mean
4. 95% confidence level and redshift restriction.

Box plots (Figure 3.7) revealed that option 1 can be excluded as it contained outliers. Option 2 and 4 gave the same result of 50 sources and option 3 gave 52 sources. Option 3 was thought to give our best results as it included two extra sources which fell within an Abell cluster size.

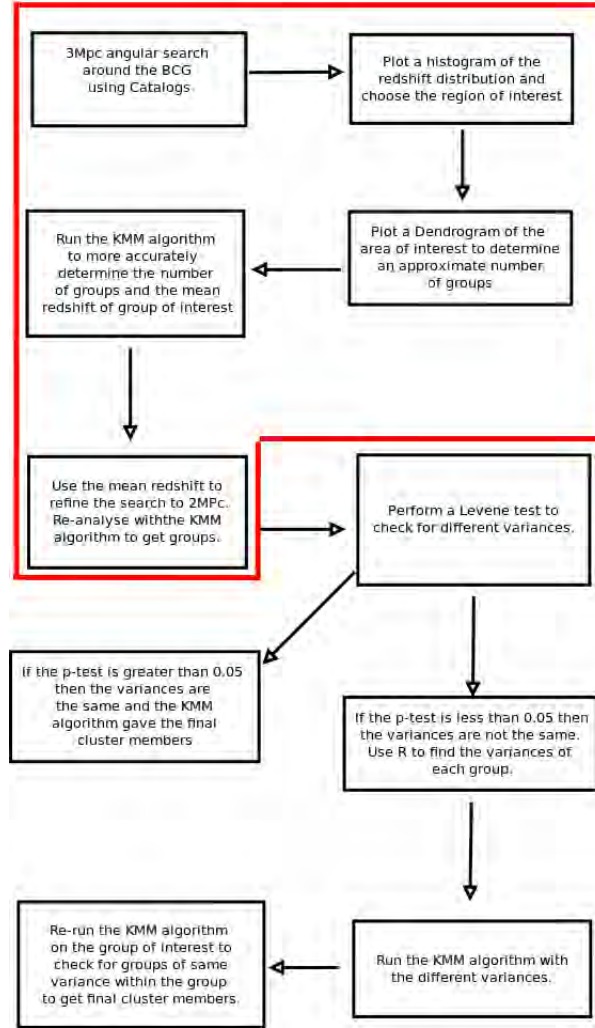


Figure 3.6: Flow chart of KMM method: This shows the method used to determine cluster members using KMM. The red block shows the original method as given in Figure 3.3 which was then extended to allow for heteroscedastic samples.

Our final sample contained 52 sources with a mean redshift of 0.022. The results for all clusters are shown in Table A.3.

3.1.3 Results

We wish to compare our results that we obtained for each of these three clustering algorithms. Since all properties can be calculated from the velocity dispersion based on the equations given in Section 1.5, we will compare the results for this parameter. A graphical representation is given in Figure 3.8. The first obvious conclusion that we can make is that the results obtained using SIMBAD are in general between 10^3 to 3×10^4 times greater than the values obtained by KMM and GMM and the literature values (Figure 3.8(a)). This is because the redshift range of the galaxies which SIMBAD labels as members is very large. This results in large errors when calculating the velocity dispersion. Another readily apparent problem is that, for many of the clusters, results were not obtained from SIMBAD. We therefore conclude that this method is not an effective nor robust way to determine cluster members.

Looking at Figure 3.8(a) it appears as if the results obtained for KMM and GMM are very

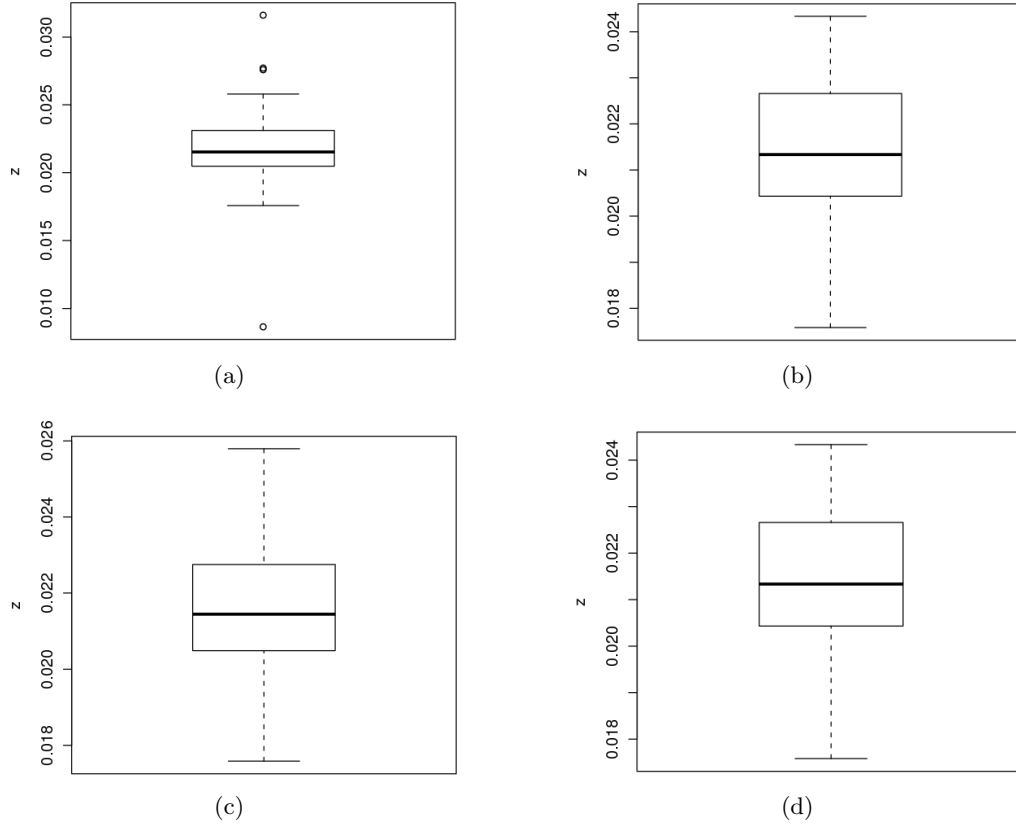
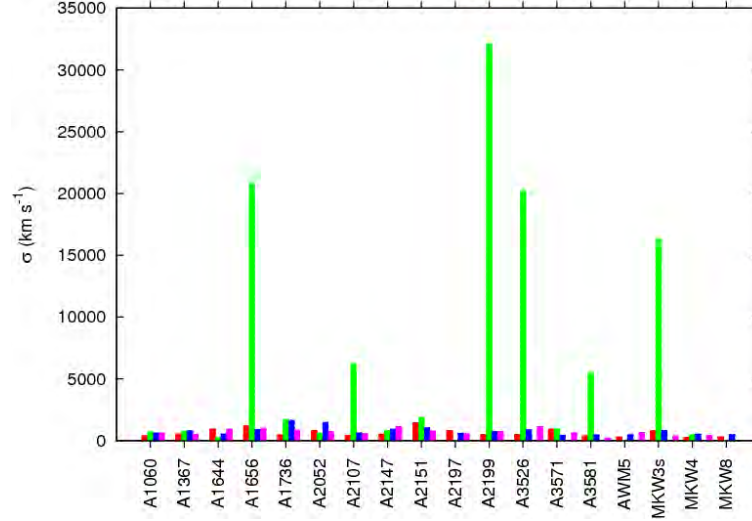


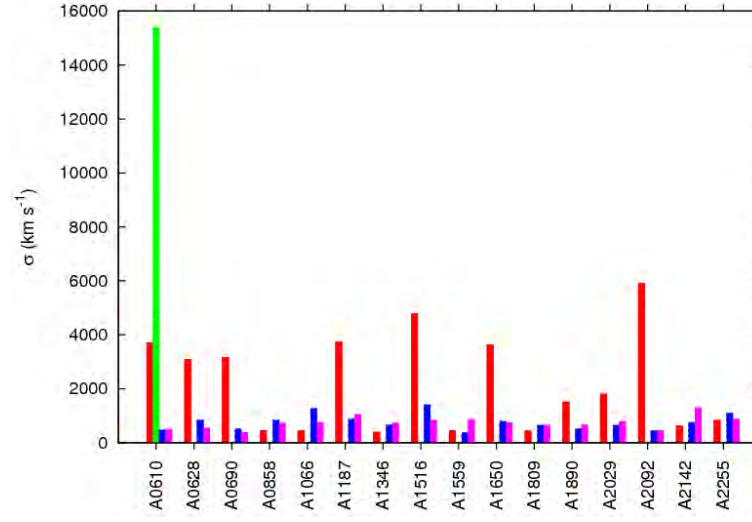
Figure 3.7: Box plots showing the results from each set of criteria for determining possible cluster members. The criteria used are (a) the highest probability, (b) a 95% confidence level only, (c) a redshift restriction of within 1σ of the mean, and (d) both a 95% confidence level and a 1σ redshift restriction.

similar but this is mostly due to the large scale caused by the SIMBAD results. Looking at Figures 3.8(b) and 3.8(c) where there are few or no SIMBAD results, we see that in general the results obtained using the KMM algorithm are much larger than GMM and the literature value. For example, for cluster A2092 using KMM we find a velocity dispersion of 5909 km s^{-1} as opposed to the literature value of 458 km s^{-1} . This is most likely due to the fact that KMM has included galaxies that do not belong to this cluster. A1890 is a poor cluster with a velocity dispersion of only 508 km s^{-1} but KMM returns a result of 1506 km s^{-1} . Due to the fact that it is a poor cluster, KMM may not be able to distinguish it from the surrounding galaxies resulting in a larger velocity dispersion. Although in general this method is better than SIMBAD, it still returns some abnormalities such as mentioned above and has the problem of requiring a homoscedastic sample in order to obtain the most accurate results.

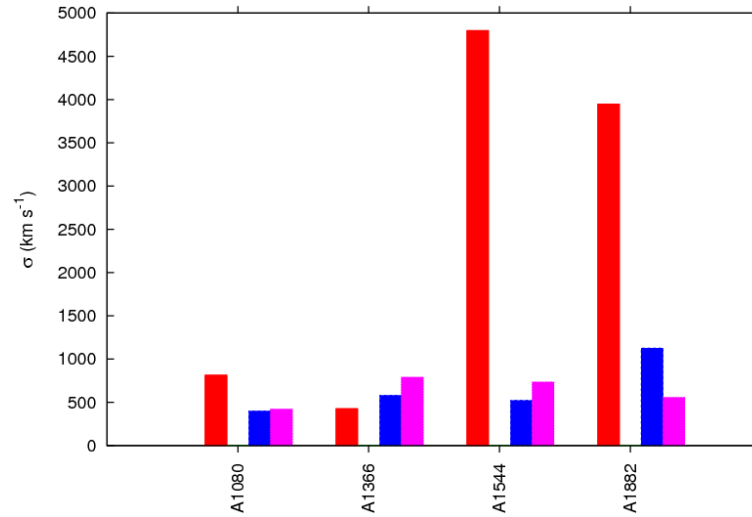
From all the plots in Figure 3.8 it can be seen that GMM gives the results closest to the literature value. Although there are still some clusters with a much larger than normal velocity dispersion, they are in the minority. Therefore, out of the three clustering algorithms tested we identify GMM as producing the best results. Focusing on this method, we now look at various methods to detect outliers in order to remove the inconsistencies.



(a) Group 1



(b) Group 2



(c) Group 3

Figure 3.8: Clustering algorithm results: These plots show the results obtained for the velocity dispersion using different clustering algorithms. To make it easier to see they have been plotted in their redshift groups. The red bar shows the results obtained from KMM, the green is using SIMBAD and the blue is from GMM. The magenta shows the expected results obtained from the literature (Table 2.3).

3.2 Application of outlier detection techniques

Here we investigate and compare the seven different methods for identifying outliers, as introduced in Section 3.2, after application of the GMM algorithm.

3.2.1 Distance Separation

The separation of the possible galaxy members for A3581 from the BCG, IC4374 with a redshift of $z=0.022$ was calculated as a test case. The 2D separation resulted in only one galaxy at a distance of further than 2 Mpc. The 3D separation yielded on average a 10% larger separation. The error on the 3D separation is larger than that on the 2D one as it includes the error in redshift as discussed by Harris (2012). Therefore to reduce error we use the 2D separation to remove the sources further than an Abell radius from the BCG.

3.2.2 Histogram

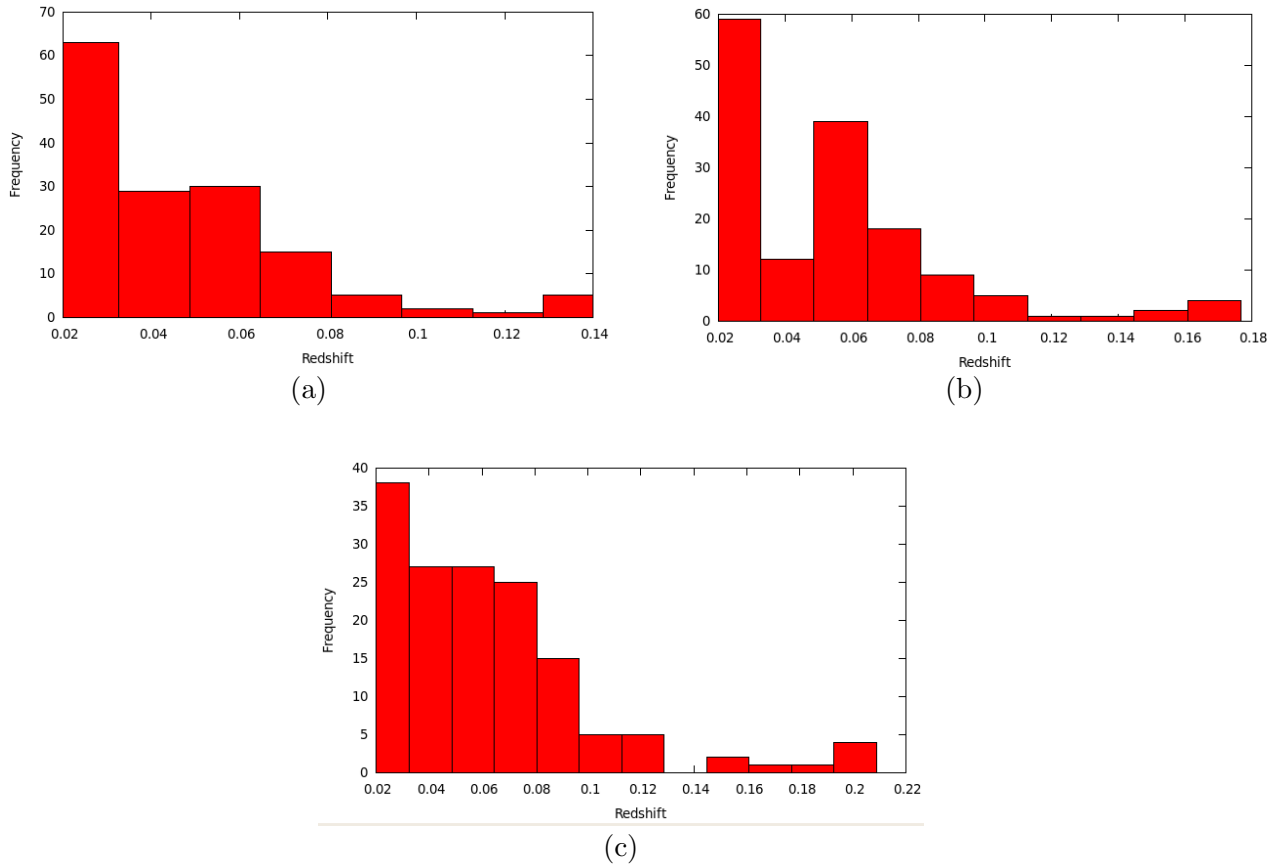


Figure 3.9: Histograms showing the change in redshift distribution depending on the method used to determine the number of bins. The bin widths were calculated using (a) the optimum number of bins prescribed by Scott (1979), (b) the optimum bin width suggested by Scott (1979), and (c) the optimum bin width as suggested by Freedman & Diaconis (1981). The formula for methods (a)-(c) are given in Equations 1.7 to 1.9 and for our A3581 sample resulted in 10, 8 and 12 bins respectively. For discussion of the results see the text in Section 3.2.2.

In Section 1.4.2 we discussed the various methods for determining the bin size used for plotting a histogram and the effect of this choice. In order to determine which method is the best we apply them to the sample obtained after the 3 Mpc search around the BCG of A3581. The

reason for this is that we know how many groups are in this sample from application of GMM. The histograms are shown in Figure 3.9.

This sample is known to contain 2 groups which is clearly shown in Figure 3.9(c). (a) is the worst method out of these three as it has two few bins resulting in a large variance and the loss of major features. (b) shows a dip in probability at $z \approx 0.14$ which gives the indication that a feature exists there. Therefore Equation 1.9 gives the best choice for the bin size. The choice of origin effects the structure and therefore we chose it to be the lowest redshift in the dataset. Histograms are inefficient for methods such as cluster analysis (Silverman 1986) and outlier identification and many other options are available and therefore this will not be used.

3.2.3 KDE

The KDE method as described in Section 1.4.3 was applied to A3581 as seen in Figure 3.10(a). For this example it is easy to identify and remove the outlier and from that this was initially considered to be a good method for determining outliers. However when applying this method to other clusters in our sample it becomes more difficult. The reason for this is because some of the galaxies are very close to each other and lie close to or on the outermost contour. It is hard to determine the exact RA and DEC of the sources from the plot and this requires one to make a subjective choice as to which source we wish to delete as seen in Figure 3.10(b). Also, when sources are removed the density changes, therefore the contours change, making it hard to determine which sources should be removed. Thus in order to avoid bias we looked at other methods to determine outliers.

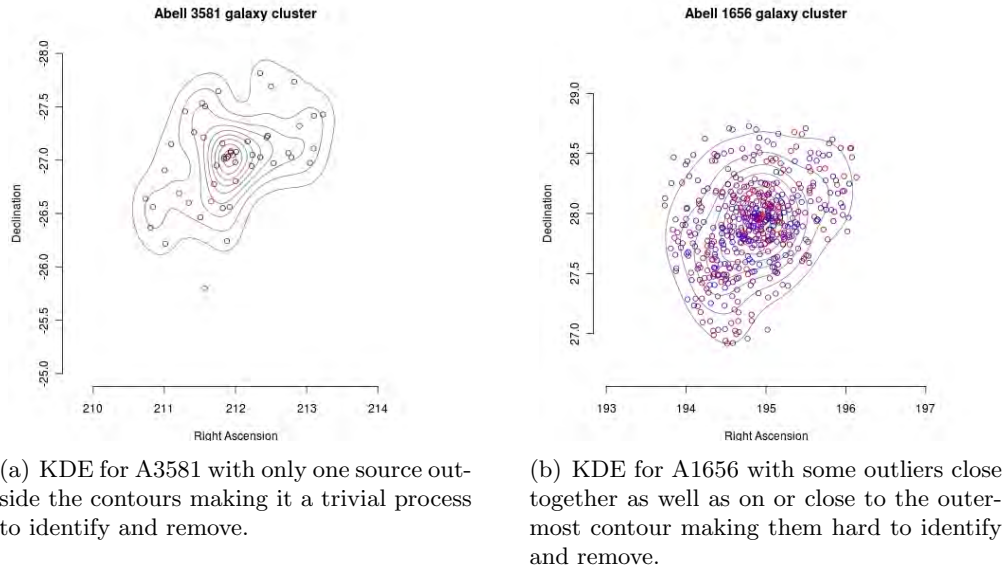


Figure 3.10: KDE analysis as a tool for outlier detection.

3.2.4 Mean

We applied the idea of using means to determine outliers to A3581. The normal mean of the sample obtained after application of GMM was found to be 0.02160. The original sample contains 52 sources and therefore to calculate the winsorized mean the 5 highest and 5 lowest sources were changed to the 6th highest and 6th lowest value respectively and a value of 0.02157

was obtained. Removal of the 5 highest and 5 lowest sources gave us the trimmed mean with a value of 0.02155. Figure 3.11 shows the results and we can conclude that these tail galaxies do not greatly effect the mean value and are therefore not likely to be outliers. However, due to the high sensitivity of the mean to possible outliers this method was found not to be very reliable and therefore is not used in our source selection.

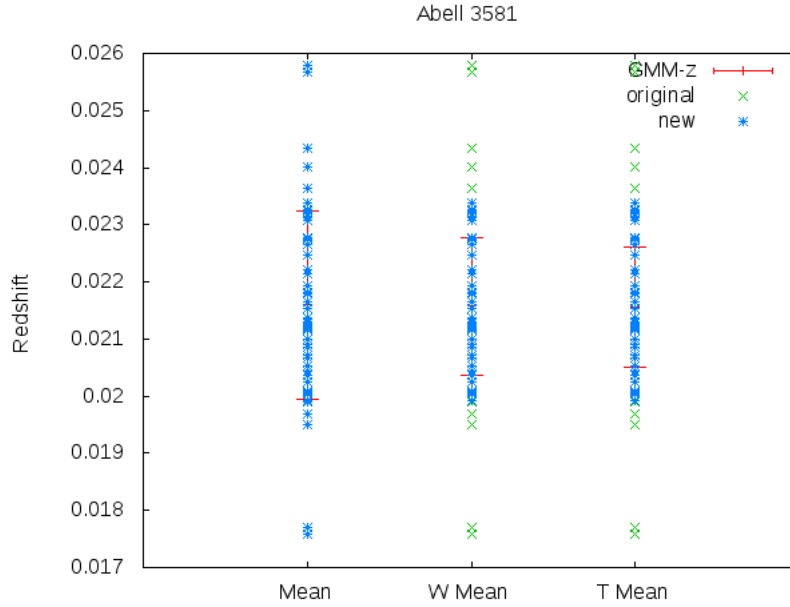


Figure 3.11: Mean as a tool for outlier detection: This plot shows the arithmetic, winsorized (W) and trimmed (T) mean for Abell 3581 showing the original points in the sample and those which were changed or removed for the winsorized and trimmed mean respectively.

3.2.5 Quantile-Quantile Plot (QQ Plot)

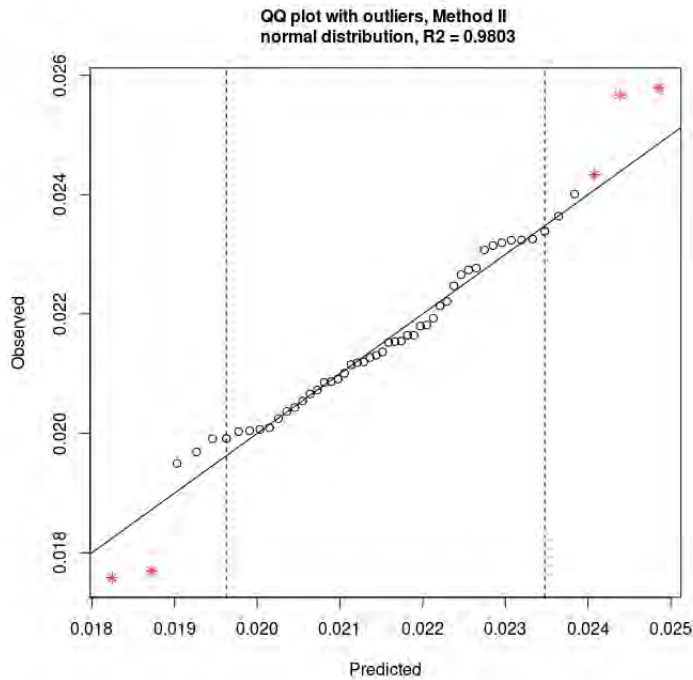


Figure 3.12: QQ Plot as a tool for outlier detection: This QQ Plot for the redshift of A3581 shows 5 possible outliers with residuals above or below the confidence level.

A QQ Plot for the redshift of the sample obtained for A3581 after the application of GMM using a confidence level of 0.1 was plotted and is shown in Figure 3.12. This revealed 5 outliers, 2 galaxies with residuals below 0.1 and three with residuals above this limit. When applying this to the rest of our clusters we found that some did not have any outliers. For those which did, we removed the outliers and recalculated the physical parameters and these are shown in Table B.1. In general this method improved our results, however, for some clusters such as A3581 it appears that this method removes too many outliers, making the velocity dispersion lower than expected. In order to avoid error due to this we use these outliers as starting points in two statistical tests: the Walsh test and the Rosner test.

3.2.6 Walsh Test

This test could not be applied to the cluster A3581 as it did not meet the requirement of having more than 60 sources, however it was applied to 11 of our clusters which did. The results are shown in Table B.2. The largest error resulted for cluster A1066 as the Walsh test indicated that the 8 outliers detected by the QQ Plot were not in fact outliers. This problem is due to the fact that the Walsh test declines or accepts outliers as a group rather than individually.

3.2.7 Rosner Test

In order to apply the Rosner test, the three conditions described in Section 1.4.7 must be met. Firstly, it can only be applied to samples than have greater than 25 members, which excludes A1080 and A1544. Secondly, this test only works to test for less than 10 outliers, which excludes A0779, A2151, MKW4 and MKW8. The final condition is that the sample must follow a normal distribution and we therefore use the Shapiro-Francia test, after removing the suspected outliers determined by QQ Plot. This process excluded clusters A2197, A2199 and A2255 as they were not normally distributed after the removal of the outliers. We expect a normal distribution if the galaxies all belong to one cluster. A2199, as stated in Section 2.2, forms part of a supercluster with A2197. When performing the GMM test it may be including galaxies that belong to the other cluster resulting in it not following a normal distribution. A2197 also has two concentrations and A2255 shows evidence of substructure which would account for their non-normal distributions. This leaves us with a sample of 11 clusters on which we can apply the Rosner test, the results of which are shown in Table B.3.

3.2.8 Results

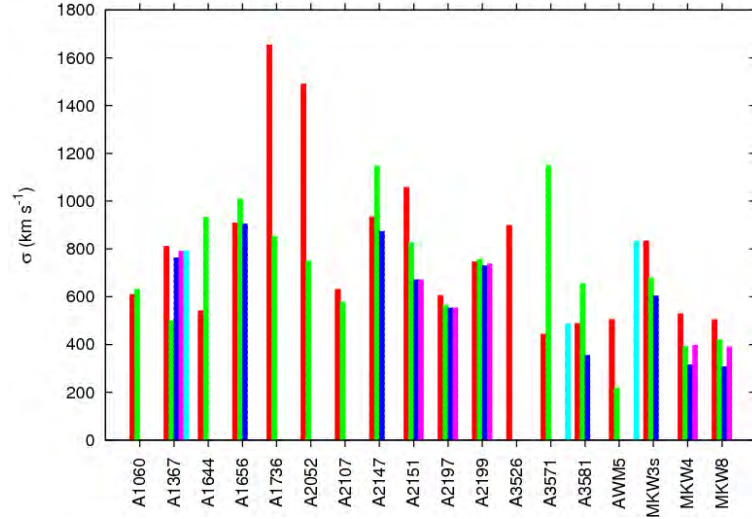
From the discussions above we can effectively rule out histograms, the mean and the KDE as effective outlier detection techniques. The 2D separation was calculated for all galaxies in each galaxy cluster and all those with a separation of greater than 2 Mpc were removed. The 3 main techniques of interest are the QQ Plot, the Walsh test and the Rosner test. For these we show a plot of the velocity dispersion and compare it to the results we got from GMM and the literature result (Figure 3.13).

The QQ Plot is used to provide an estimate of the number of outliers which can be used by the more robust statistical outlier tests, Rosner and Walsh. For those which these tests can't

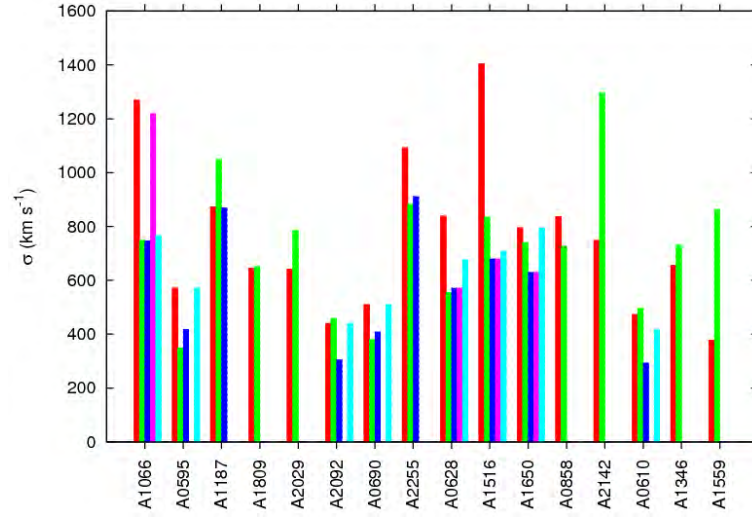
be applied, the QQ Plot gives a good enough result that it can be used alone.

It can be seen that in general the application of the outlier tests improves the result obtained from GMM. For the few clusters than the Walsh test can be applied to it gives the worst results out of three tests. This is because it accepts or rejects the outliers as a group rather than individually. Therefore some true outliers may not be rejected, or non-outliers are rejected.

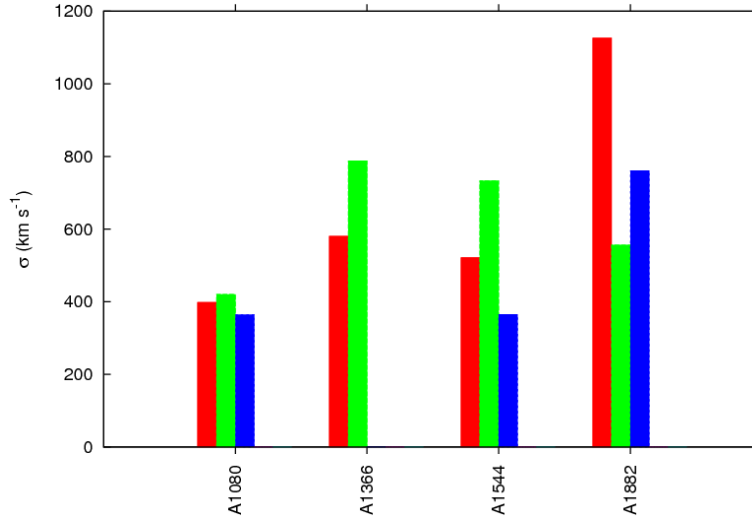
Our investigations show that the Rosner test is a robust and simple method giving accurate results. The largest error occurs for A0595 where the calculated velocity dispersion is 571 km s^{-1} as opposed to 349 km s^{-1} . The reason for this error may not lie in the Rosner test however, as the literature value quoted from Tago et al. (2010) is the velocity dispersion of galaxies within R_{200} . Our result does not have this restriction and we therefore obtain a higher value. Although it only applies to a few of our clusters, the Rosner test provided the most accurate results and will be used in the final method.



(a) Group 1



(b) Group 2



(c) Group 3

Figure 3.13: Outlier technique results: These plots show the results obtained for the velocity dispersion using different outlier techniques. They have been plotted in their redshift groups. The red bar shows the results obtained from GMM and the green shows the expected results obtained from literature (Table 2.3). The blue, magenta and cyan correspond to the QQ Plot, Walsh test and Rosner test.

3.3 Final Method

After studying each of the above methods for clustering and outlier determination, we determined a method which we believe to be robust and that can accurately calculate galaxy members and certain physical parameters. This method is shown as a flow chart in Figure 3.14.

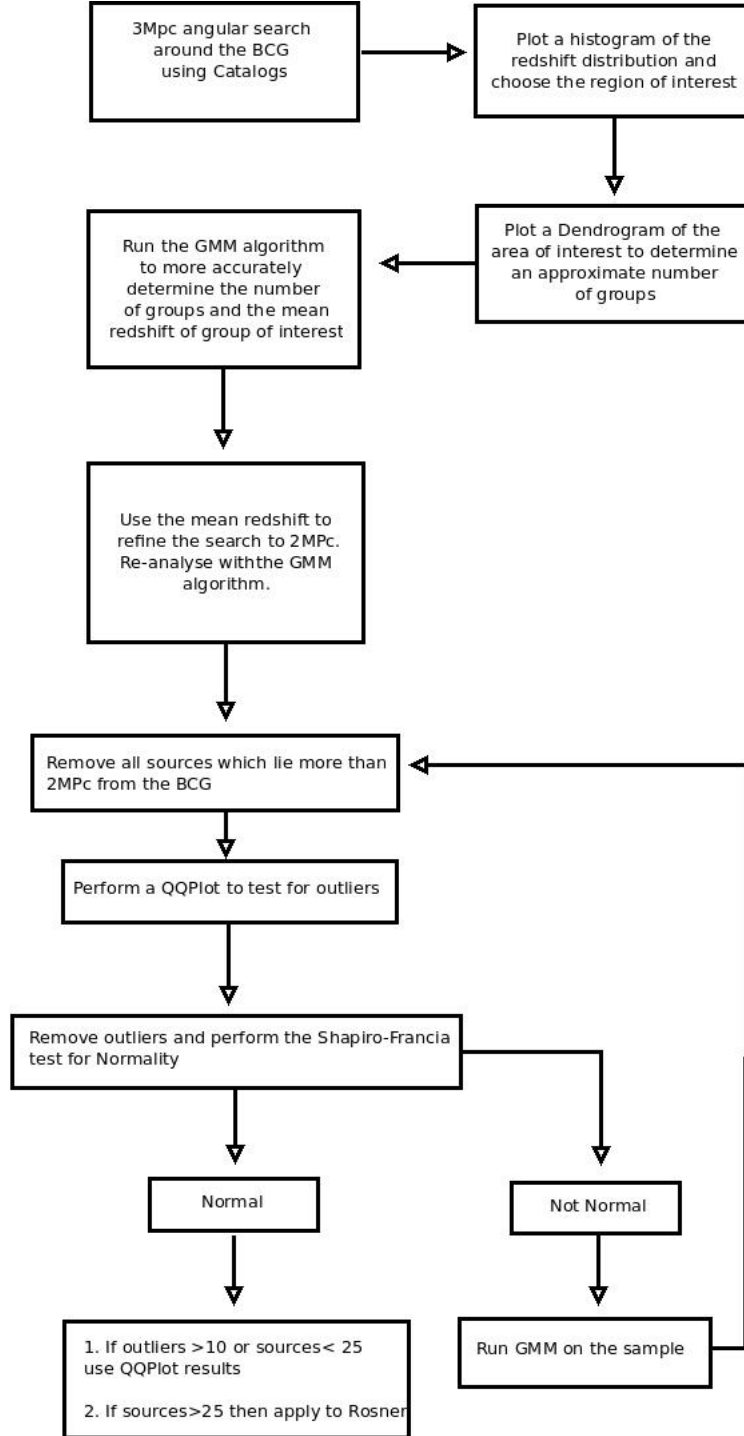


Figure 3.14: Our method for determining cluster members: This flow diagram shows the final method chosen to determine cluster members including outlier detection.

After obtaining the results from GMM using the method described in Section 3.1.2 we calculate the 2D distance separation of each galaxy from the BCG. All galaxies that fall more than an

Abell radius from the BCG are removed. A QQ Plot reveals any possible outliers in each cluster of galaxies and after removing these a Shapiro-Francia test for normality is applied. If the QQ Plot revealed no outliers, this is where the process stops for these clusters of galaxies. If our sample contains only one cluster of galaxies then it will follow a normal uni-modal distribution.

If it does not, it suggests the possibility of substructure or contamination by a nearby cluster and we then re-apply the GMM algorithm to try and remove the contamination. A2197, A2255, A1736, A2052, A3526, A1656, MKW8 and A2142 were still not normally distributed after applying GMM for the second time. The reasons for this have already been given for A2197 and A2255 in Section 3.2.7. As for the rest of these clusters, A1736 is known to be a non-relaxed cluster that is still undergoing formation and evolution and A2052 shows signs of a merger and dynamical activity. A3526 and A2142 show evidence of a cool-core and cold fronts which suggest mergers. This may cause these clusters not to follow a normal distribution as these processes cause substructure, as is seen in A1656. MKW 8 is a poor cluster which makes selection of members a difficult task and contamination from surrounding galaxies may be causing this cluster to have a non-normal distribution.

For galaxy clusters with a normal distribution and less than 25 sources or more than 10 outliers the QQ Plot was taken as the final test and the physical parameters were calculated. Otherwise the Rosner test was applied before calculating the desired parameters.

Table 3.1: This table shows the physical parameters calculated after performing the method shown in Figure 3.14. Column 1 gives the galaxy cluster name, column 2 is the redshift and n is the number of galaxies in the galaxy cluster. Columns 4 - 8 give the physical parameters we calculated. Column 7 and 8 are the temperatures calculated using the equations given by Helsdon & Ponman (2003) and Navarro et al. (1995) respectively.

Cluster Name	z	n	σ (km s $^{-1}$)	Radius (Mpc)	Mass ($10^{14} M_{\odot}$)	T ₁ (KeV)	T ₂ (KeV)
A0595	0.069	51	571 $\pm$ 60	1.794 $\pm$ 0.190	2.040 $\pm$ 0.372	1.407 $\pm$ 0.298	1.507 $\pm$ 0.318
A0610	0.097	37	417 $\pm$ 52	1.293 $\pm$ 0.161	0.784 $\pm$ 0.169	0.751 $\pm$ 0.188	0.803 $\pm$ 0.200
A0628	0.084	60	676 $\pm$ 68	2.110 $\pm$ 0.211	3.362 $\pm$ 0.585	1.976 $\pm$ 0.395	2.111 $\pm$ 0.424
A0646	0.126	25	772 $\pm$ 85	2.361 $\pm$ 0.260	4.907 $\pm$ 0.936	2.578 $\pm$ 0.568	2.754 $\pm$ 0.607
A0690	0.080	55	510 $\pm$ 46	1.596 $\pm$ 0.143	1.448 $\pm$ 0.226	1.125 $\pm$ 0.202	1.202 $\pm$ 0.216
A0779	0.023	110	308 $\pm$ 15	0.989 $\pm$ 0.049	0.327 $\pm$ 0.028	0.410 $\pm$ 0.041	0.438 $\pm$ 0.043
A0858	0.088	34	837 $\pm$ 67	2.606 $\pm$ 0.209	6.366 $\pm$ 0.883	3.024 $\pm$ 0.485	3.237 $\pm$ 0.519
A1060	0.012	172	609 $\pm$ 30	1.968 $\pm$ 0.096	2.545 $\pm$ 0.216	1.605 $\pm$ 0.157	1.714 $\pm$ 0.168
A1066	0.069	97	766 $\pm$ 47	2.408 $\pm$ 0.147	4.927 $\pm$ 0.523	2.536 $\pm$ 0.310	2.711 $\pm$ 0.332
A1080	0.117	21	364 $\pm$ 47	1.117 $\pm$ 0.143	0.516 $\pm$ 0.115	0.572 $\pm$ 0.147	0.612 $\pm$ 0.157
A1187	0.075	29	226 $\pm$ 23	0.708 $\pm$ 0.072	0.126 $\pm$ 0.022	0.220 $\pm$ 0.045	0.236 $\pm$ 0.048
A1346	0.098	64	656 $\pm$ 47	2.034 $\pm$ 0.146	3.052 $\pm$ 0.379	1.861 $\pm$ 0.267	1.988 $\pm$ 0.285
A1366	0.116	36	580 $\pm$ 57	1.781 $\pm$ 0.174	2.089 $\pm$ 0.355	1.453 $\pm$ 0.284	1.554 $\pm$ 0.305
A1367	0.022	156	791 $\pm$ 42	2.543 $\pm$ 0.136	5.548 $\pm$ 0.511	2.706 $\pm$ 0.289	2.891 $\pm$ 0.308
A1516	0.076	68	708 $\pm$ 57	2.217 $\pm$ 0.178	3.875 $\pm$ 0.540	2.164 $\pm$ 0.347	2.316 $\pm$ 0.372
A1544	0.145	14	365 $\pm$ 63	1.106 $\pm$ 0.191	0.514 $\pm$ 0.154	0.577 $\pm$ 0.199	0.616 $\pm$ 0.213
A1559	0.105	15	139 $\pm$ 17	0.430 $\pm$ 0.053	0.029 $\pm$ 0.006	0.084 $\pm$ 0.021	0.089 $\pm$ 0.022
A1644	0.048	23	220 $\pm$ 21	0.699 $\pm$ 0.066	0.118 $\pm$ 0.019	0.209 $\pm$ 0.040	0.224 $\pm$ 0.042
A1650	0.084	67	794 $\pm$ 74	2.479 $\pm$ 0.232	5.450 $\pm$ 0.881	2.726 $\pm$ 0.510	2.913 $\pm$ 0.544
A1809	0.080	88	645 $\pm$ 40	2.016 $\pm$ 0.124	2.925 $\pm$ 0.313	1.797 $\pm$ 0.220	1.923 $\pm$ 0.237
A1882	0.139	35	368 $\pm$ 41	1.117 $\pm$ 0.125	0.527 $\pm$ 0.102	0.585 $\pm$ 0.131	0.625 $\pm$ 0.140
A1890	0.058	78	509 $\pm$ 35	1.608 $\pm$ 0.110	1.453 $\pm$ 0.173	1.119 $\pm$ 0.153	1.197 $\pm$ 0.164
A2029	0.078	41	642 $\pm$ 53	2.009 $\pm$ 0.166	2.887 $\pm$ 0.413	1.780 $\pm$ 0.294	1.904 $\pm$ 0.315
A2092	0.067	56	440 $\pm$ 41	1.384 $\pm$ 0.128	0.934 $\pm$ 0.150	0.836 $\pm$ 0.155	0.894 $\pm$ 0.166
A2107	0.041	73	630 $\pm$ 47	2.007 $\pm$ 0.148	2.778 $\pm$ 0.358	1.716 $\pm$ 0.254	1.834 $\pm$ 0.272
A2147	0.037	337	934 $\pm$ 32	2.980 $\pm$ 0.101	9.065 $\pm$ 0.536	3.768 $\pm$ 0.255	4.031 $\pm$ 0.275
A2151	0.035	222	671 $\pm$ 30	2.143 $\pm$ 0.095	3.365 $\pm$ 0.260	1.944 $\pm$ 0.172	2.081 $\pm$ 0.185
A2199	0.031	282	746 $\pm$ 30	2.388 $\pm$ 0.095	4.634 $\pm$ 0.322	2.406 $\pm$ 0.192	2.571 $\pm$ 0.206
A3571	0.039	54	443 $\pm$ 34	1.411 $\pm$ 0.108	0.966 $\pm$ 0.128	0.847 $\pm$ 0.129	0.907 $\pm$ 0.139
A3581	0.022	52	487 $\pm$ 53	1.564 $\pm$ 0.169	1.293 $\pm$ 0.243	1.023 $\pm$ 0.222	1.095 $\pm$ 0.237
AWM5	0.035	40	504 $\pm$ 37	1.611 $\pm$ 0.119	1.427 $\pm$ 0.182	1.099 $\pm$ 0.162	1.174 $\pm$ 0.173

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Table 3.1 – Continued

Cluster Name	z	n	σ (km s $^{-1}$)	Radius (Mpc)	Mass ($10^{14} M_{\odot}$)	T ₁ (KeV)	T ₂ (KeV)
MKW3S	0.044	59	833 ± 90	2.650 ± 0.286	6.412 ± 1.199	2.999 ± 0.647	3.206 ± 0.692
MKW4	0.020	85	315 ± 21	1.013 ± 0.069	0.351 ± 0.041	0.429 ± 0.058	0.459 ± 0.062

3.4 Results

The final results from our clustering and outlier techniques are shown in Table 3.1. Figures 3.15 to 3.18 are graphical representations of these results and we discuss them in more detail here.

3.4.1 Velocity Dispersion (σ)

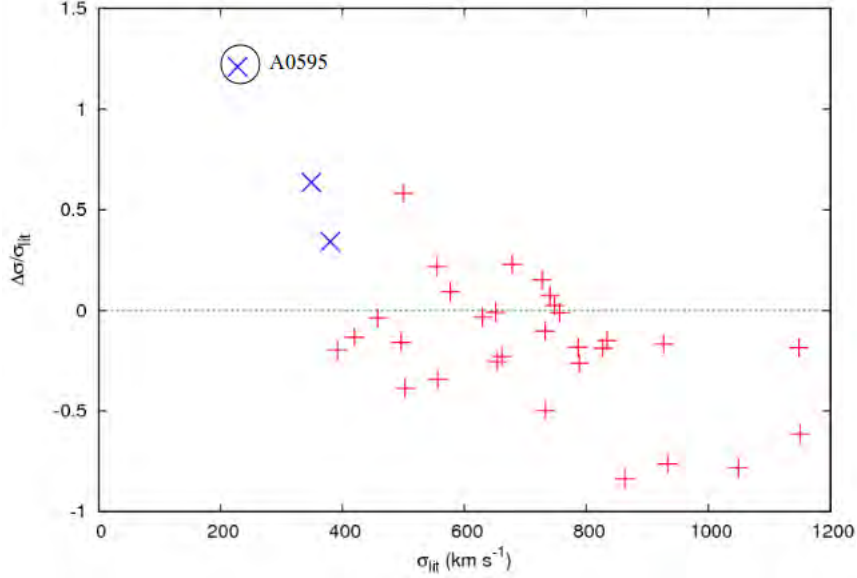


Figure 3.15: Velocity dispersion comparison. This graph shows the variation of our calculated values from those obtained from literature. The points shown in blue are those for which the literature quoted a central velocity dispersion or in the case A0595 the velocity dispersion within R_{200} .

The first physical parameter of interest that we calculated was the velocity dispersion and is shown in Figure 3.15. For this graph we plotted the ratio of the difference in velocity dispersion versus the literature values. The green line shows where we expect the points to fall if our measured velocity dispersions are the same as the literature values. All the points above the green line show the galaxy clusters for which we obtained a higher velocity dispersion than the literature values. For the points in blue, the central velocity dispersions, this is expected as we calculate the velocity dispersion for the entire cluster and not just the central region, therefore we include more galaxies which results in a higher velocity dispersion. From the graph it can be seen that we underestimate the velocity dispersion for most of the clusters, however this is a result we expect as we are constraining the system better, assuming an Abell radius of 2 Mpc. Due to the fact that more redshift values have become available we are able to include more galaxies in our studies of the cluster making the results statistically better.

3.4.2 Radius (R_{200})

The second parameter we looked at is the radius within an over-density of 200 as given by Equation 1.14. It is dependent on the velocity dispersion and the redshift of the cluster and the results are shown in Figure 3.16. The calculated value also depends on the cosmology chosen as it is dependent on $H(z)$ and for most of our literature values they use the cosmology $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $\Omega_M = 0.3$ and $\Omega_{vac} = 0.7$ as opposed to our choice of $H_0 = 73 \text{ km s}^{-1} \text{ Mpc}^{-1}$. All of our galaxy clusters, except A1644 and A3571, fall within $\pm 3\sigma$ of the mean value

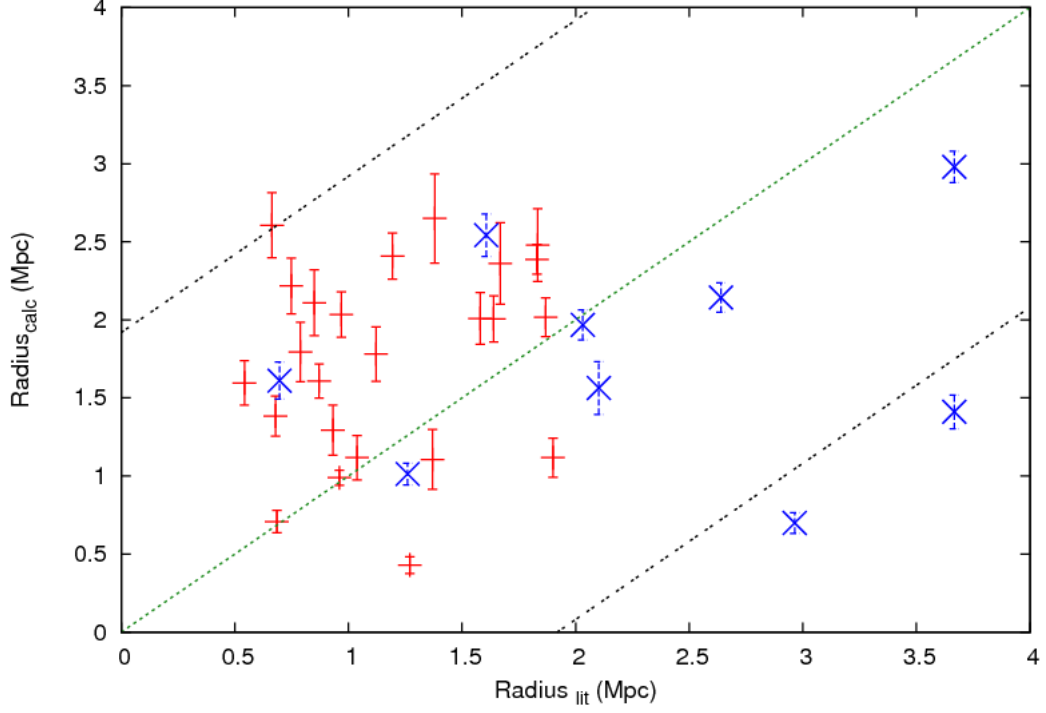


Figure 3.16: Radius comparison: The calculated radius is plotted against the values obtained from literature to show variation. The blue points are those for which literature values were not found and therefore they were calculated using the velocity dispersion from the literature. The green line shows where we expect the points to fall and the black lines represent $\pm 3\sigma$.

for our calculated values, showing that our results are in good agreement with the literature. The scatter can be attributed to differences in cosmology and velocity dispersion. A1644 and A3571 have calculated radii that are much less than the “literature values” which we calculated for them using the given velocity dispersion. Therefore, when looking for a reason for the unexpected low values we compare the velocity dispersions we calculated versus the literature values. For A1644 we calculated a velocity dispersion of 220 km s^{-1} which was much lower than the literature value of 933 km s^{-1} . The result obtained for A3571 of 443 km s^{-1} is also much lower than the expected 1150 km s^{-1} .

From Section 2.2 we know that A1644 is a double cluster with a main and subcluster and it belongs to the Shapely supercluster. After the first application of the GMM algorithm it was found that the velocity dispersion was lower than expected with a value of only 533 km s^{-1} as opposed to 933 km s^{-1} . When applying the Shapiro-Francia test for normality we found that this cluster did not follow a normal distribution, but this was corrected after a second application of the GMM algorithm. One of the reasons for the lower than expected velocity dispersion is because the GMM algorithm removed the subcluster part of this double cluster in an attempt to get a uni-modal distribution, thereby reducing the galaxy members.

A3571 is also part of the Shapely Supercluster and is connected to two other clusters, A3572 and A3575. The GMM algorithm was only applied to this cluster once and the QQ Plot revealed no outliers. The reason for the lower than expected velocity distribution could be due to GMM’s inability to successfully separate nearby clusters that are all connected. The velocity dispersion quoted from Kuvshinova & Rozgacheva (2010) could also be for the entire complex of three

clusters as opposed to just A3571 which we have calculated.

3.4.3 Mass (M_{200})

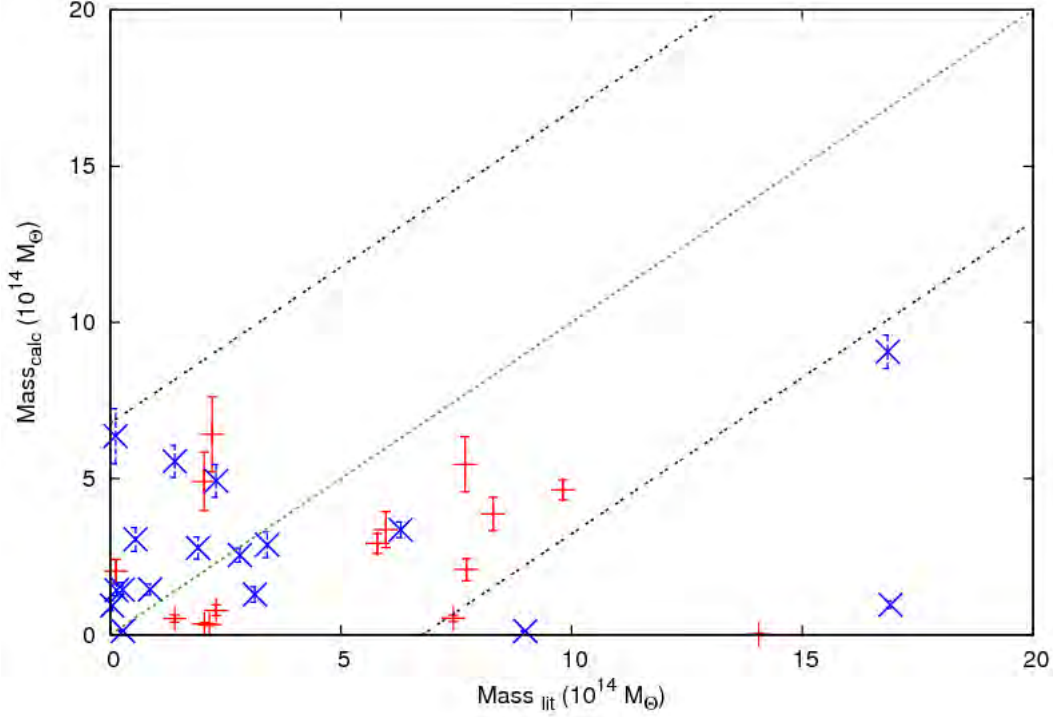


Figure 3.17: Mass comparison: The calculated mass is plotted against the values obtained from literature to show variation. The blue points are those for which literature values were not found and therefore they were calculated using the velocity dispersion and radius from the literature. The green line shows where we expect the points to fall and the black lines represent $\pm 3 \sigma$.

The mass within R_{200} is shown in Figure 3.17. The blue points are the calculated values for the literature values using Equation 1.20, and we know that they are dependent on the velocity dispersion and radius of the cluster and therefore we expect a larger error on the mass. The majority of the literature values obtained for the galaxy clusters show good agreement with the calculated values obtained using our final method and fall within 3σ ($6.761 \times 10^{14} M_{\odot}$). Only 4 clusters, (A2147, A3571, A1644 and A1559) fall outside this region.

As discussed above A3571 and A1644 have a lower than expected radius and velocity dispersion and therefore we expect that since the mass is dependent on these values the mass would be lower than the literature value. For A2147 although it is not an extreme outlier in the velocity dispersion or the radius, differing by only 214 km s^{-1} and 0.7 Mpc respectively when this is combined in the equation for the mass, it results in a much bigger difference putting it just outside the 3σ region. A1559 is a cluster in our third redshift group, $z = 0.1058$, and therefore had fewer initial sources than clusters in the first 2 groups. This cluster also showed a non-normal distribution and GMM was applied for a second time. This left us with a sample size of only 15 as compared to Popesso et al. (2007) who used a sample size of 45 which explains the lower than expected velocity dispersion. The calculated radius was 0.43 Mpc as compared with the literature value of 1.27 Mpc . Combining these gives as expected a lower mass than predicted by literature.

By statistically confirming the cluster members of a cluster, we derived the velocity dispersion and from this we can calculate the virial radius as well as the mass of the cluster which is basically proportional to the square of the velocity dispersion, as shown in Section 1.5.3. We assumed here the cluster is relaxed and is in virialised state i.e. in dynamic equilibrium.

3.4.4 Temperature

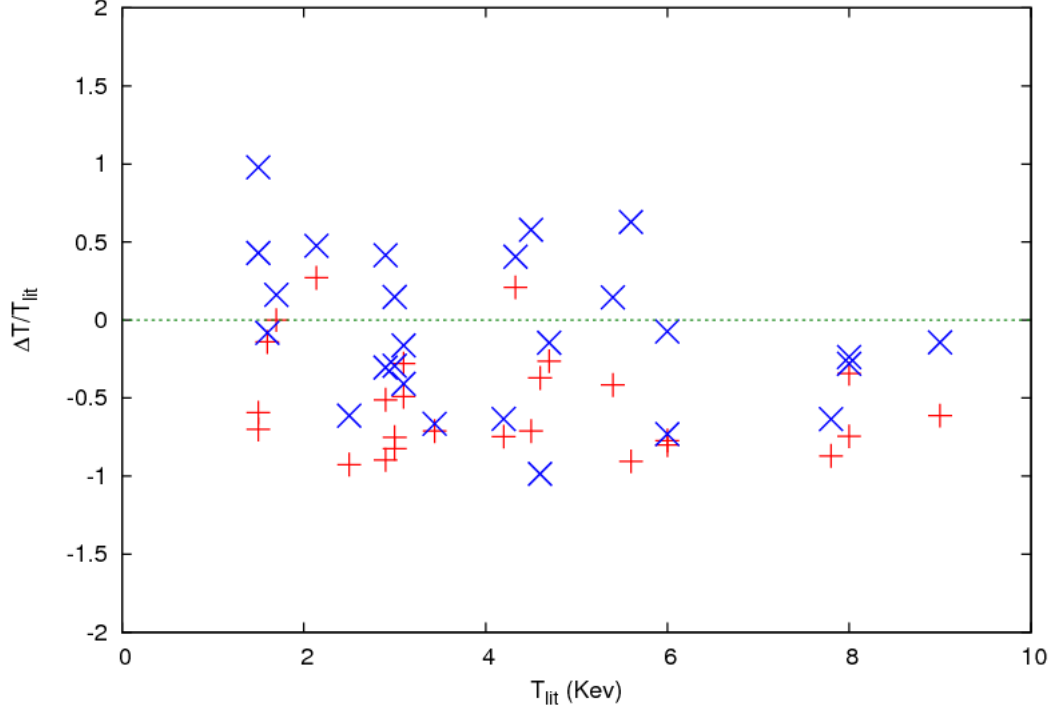


Figure 3.18: Temperature comparison: This plot shows the ratio of the difference in temperature to the literature value versus the literature value for our two methods as discussed in Section 1.5.5. The points in red are those calculated using the Helsdon & Ponman (2003) formula obtained from fitting and the blue points are from the formula given by Navarro et al. (1995).

The two methods for calculating the temperature as discussed in Section 1.5.5 are compared both with each other as well as the literature values in Figure 3.18. For the formula given by Helsdon & Ponman (2003) the temperature is dependent on the radius, redshift and chosen cosmology. When calculating the temperatures, we used $h=0.73$ as opposed to $h=0.5$ as used by Helsdon & Ponman (2003) since we wished to be consistent with our calculations. The formula given by Navarro et al. (1995) does not directly depend on the cosmology but rather on the mass and radius. We wished to determine which of these two methods is more reliable and therefore for Figure 3.18 we calculated the temperatures for each method from the literature values to avoid bias from our calculations of the mass and radius. Note that the temperatures we are calculating are for the kinetic and gravitational energy from the cluster and the literature values are the X-ray temperatures. We are thus not looking at whether our results are similar to the literature values, but rather which method give the most consistent results. From Figure 3.18, it can be seen that the results using Helsdon & Ponman (2003) are generally an underestimation of the temperature, whereas those from Navarro et al. (1995) are scattered equally above and below the expected result. Due to the fact that a change of cosmology may affect the former,

as well as to avoid the error that may result from this we choose the method given by Navarro et al. (1995) for further comparisons.

3.5 Conclusion

In this chapter we discussed three clustering algorithm methods; SIMBAD, KMM and GMM, that could be used to determine which galaxies belong to our galaxy clusters. Out of these three methods, GMM provided the most consistent and accurate results when we compared the physical parameters to those obtained from literature. In order to ensure that the right candidates were chosen we also look at a few outlier tests and determined that the QQ Plot and the Rosner test were the most effective for our sample. A combination of GMM, QQ Plot and the Rosner test were used to determine our final results. We made comparisons between our calculated parameters and those obtained from literature and found that within 3σ the results were mostly consistent. A few clusters did not give the desired results but the reasons for this were explained.

Chapter 4

Correlations of various physical parameters from multi-wavelength data

By studying the physical parameters of galaxy clusters and looking for correlations we can determine information about cluster evolution and formation as well as the dynamical state of the cluster. Many astronomers have studied this in detail and below we compare our results with published results.

4.1 Velocity Dispersion (σ) vs Redshift (z)

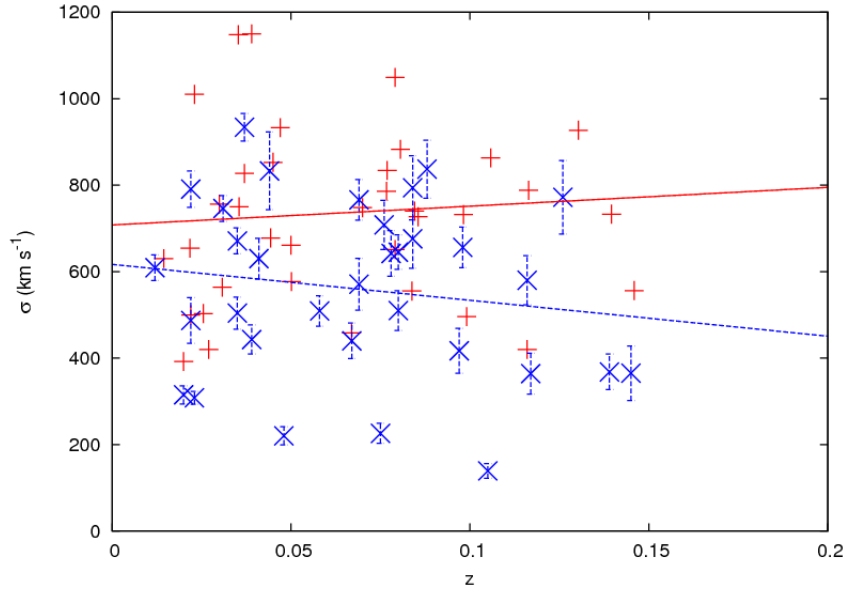


Figure 4.1: σ vs z : This plot shows the relation between the velocity dispersion of the cluster and the redshift. The points in red shown the values obtained from literature (Table 2.3) and those in blue are the values that we have calculated in Section 3.3. The red and blue lines show the line of best fit for the literature and calculated values respectively.

In this section we look at the relation between a galaxy cluster's velocity dispersion and its redshift. We would assume that the velocity dispersion would increase with redshift as at an earlier time the universe was less stable and galaxies were moving at faster speeds. By finding the best fit line for the literature values, $\sigma = (708 \pm 74.5) + (434 \pm 996)z$, it appears as if

it follows this trend. Due to the large scatter in the data the best fit line has extremely large errors and the $\chi^2=48150$. Using our calculated data the chi-squared value is slightly reduced, $\chi^2=41096$, but nowhere near one. Therefore, although it appears that our literature and calculated values are giving contradictory results, due to the large errors we cannot accurately detect a trend of either an increase or decrease in the velocity dispersion with redshift.

Girardi & Mezzetti (2001) studied the evolution of the velocity dispersion function with redshift and found that at high redshift the velocity dispersion was similar to clusters at low redshift. In the central region of the cluster the velocity anisotropies play a more active role and cause large changes to the velocity dispersion profile but as one moves out towards the outer regions of the cluster they flatten out. Therefore if we were to measure the central velocity dispersion of our clusters we may find that there is a relation to the redshift. It appears that our results are consistent with the findings of Girardi & Mezzetti (2001) as our clusters don't follow a specific trend of either increasing or decreasing with redshift.

4.2 Mass (M_{200}) vs Redshift (z)

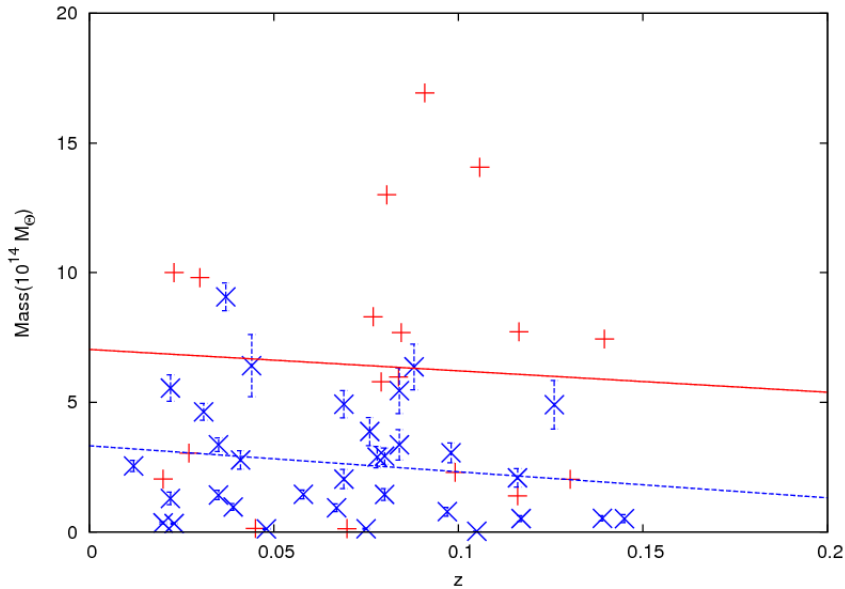


Figure 4.2: M_{200} vs z : We compare the relation between the literature mass of the cluster versus redshift to that obtained for our calculated values. The points in red are the literature values given in Section 2.2 and those in blue are the values calculated using the equations in Section 1.5. The lines of best fit for each are shown in red and blue with a $\chi^2 = 25$ and $\chi^2 = 5$ respectively.

The most accepted model for our universe, Λ CDM, adopts a hierarchical method for the formation of our Universe. This means that matter started off small and over time they became grouped together forming larger and larger objects. Therefore, we would expect that the higher redshift we go to, the smaller the galaxy clusters will become and the less massive we expect them to be. However, recently, very massive galaxy clusters have been detected at high redshift ($z > 1$) using the South Pole Telescope (Brodwin et al. 2010).

When looking at the relation of the galaxy mass versus redshift for the literature and calculated values, we observe that the mass seems to decrease with the redshift. The gradient

obtained for our calculated values is slightly steeper, $m = -10$, than the $m = -8$ obtained for the literature values. This can be accounted for by the fact that our calculated values are more constrained and therefore have less scatter than the literature values.

4.3 X-ray Luminosity (L_X) vs Redshift (z)

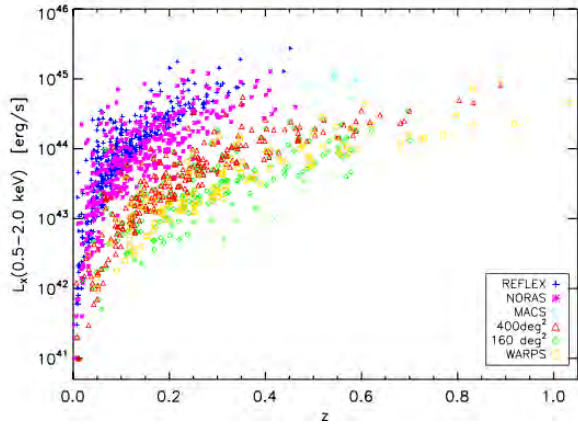


Figure 4.3: X-ray Luminosity vs z from Sommer et al. (2011): This shows the distribution of X-ray clusters in the L_X - z plane.

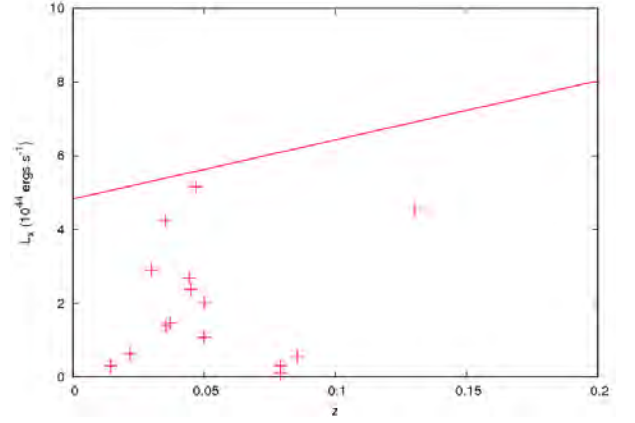


Figure 4.4: L_X vs z : This plot shows the relation between the X-ray luminosity in ergs s^{-1} and the redshift. Due to the fact that X-ray luminosity is dependent on the cosmology chosen and we wanted to avoid bias due to this we included only those with $H_0 = 50 \text{ km s}^{-1} \text{ Mpc}^{-1}$ which greatly reduced our sample. Due to this the range of luminosities was quite large and to more accurately depict the relation we zoomed in on the region from 10^{37} to 10^{38} W .

Since the X-ray luminosity is dependent on cosmology, we extract a subsample from our original sample that contains the most galaxy clusters with the same cosmology, $H_0 = 50 \text{ km s}^{-1} \text{ Mpc}^{-1}$. This left us with only 17 galaxy clusters and therefore we can not accurately estimate the relation between these two properties. Results obtained by Sommer et al. (2011) (Figure 4.3) show an increase in X-ray luminosity with redshift and in future the use of a bigger sample will hopefully show agreement with this.

4.4 X-ray Temperature (T_X) versus Velocity Dispersion (σ)

This correlation is a robust way to test for dynamical properties of the cluster (Wu et al. 1998). Bird et al. (1995) explains that by making the following assumptions:

- i Galaxy orbits are isotropic;
- ii Gas and galaxies occupy the same potential well;
- iii Gravity is the only source of energy for either the gas or the galaxies.

and using the virial theorem the correlation between the X-ray temperature and the velocity dispersion of the cluster is given by $\sigma \sim T^{0.5}$. If the data deviates from this result then it

suggests that the sample is not in hydrostatic equilibrium. However, calculations by Wu et al. (1998), Bird et al. (1995) and Girardi et al. (1996) found a value closer to $\sigma \sim T^{0.6}$. Deviation mechanisms that could account for this are:

- Anisotropy of galaxy velocity dispersions
- Protogalactic winds which heat the intercluster medium
- Velocity bias between galaxies and dark matter particles
- Asymmetric mass distributions
- Effect of cooling flows

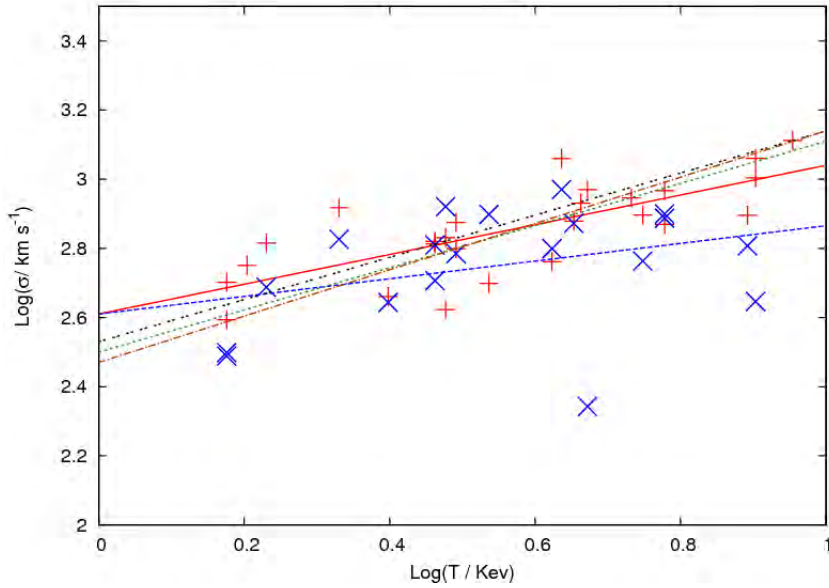


Figure 4.5: σ vs T_{lit} : This plot shows the relation between the velocity dispersion and the temperature obtained from literature. The points and line shown in red are for the velocity dispersion values obtained from literature and the best fit line of the data. The points and line shown in blue are for the velocity dispersion values that we calculated and the best fit line of the data. The green, black and orange lines show the fits from Bird et al. (1995), Girardi et al. (1996) and Wu et al. (1998) respectively.

In Figure 4.5 we compare the fit of our data to those found by Wu et al. (1998), Bird et al. (1995) and Girardi et al. (1996). Using the velocity dispersion from literature we obtain a relation of $\sigma \sim T^{0.43}$ and for our calculated values we get $\sigma \sim T^{0.26}$. In both cases we obtain a shallower result than that found in literature.

Another way to characterize the dynamical properties is to calculate the ratio of the specific kinetic energy in the galaxies to that in the gas. This is known as the β parameter and is calculated by

$$\beta = \frac{\sigma^2}{kT/\mu m_p} \quad (4.1)$$

where $\mu=0.59$ is the mean molecular weight and m_p is the mass of a proton. If the cluster is in hydrostatic equilibrium, i.e. there is a perfect energy equipartition between the galaxies and

the gas in the clusters, then $\beta=1$. If a result close to unity is obtained then deviation in the correlation between T_X and σ may be explained by the reasons above. For the velocity dispersion obtained from literature we calculate an average value of $\beta = 0.928 \pm 0.403$ but when using our calculated values we get $\beta = 0.679 \pm 0.383$. This tells us that the clusters in our sample are in hydrostatic equilibrium but our process of choosing cluster members is resulting in us rejecting or accepting galaxies which change this equilibrium. This is particularly apparent in clusters A0779, A1366 and A3571 who all have a β of less than 0.4. On the other extreme MKW3S has a $\beta = 1.425$.

For galaxy cluster A0779, the QQ Plot detected more than 10 outliers. Due to the fact that the Rosner test can not be used in this case we accepted the result that we obtained from QQ Plot, therefore some of these galaxies may in fact have belonged to the cluster and by removing them, the hydrostatic equilibrium was destroyed.

It is already known that A3571 has a lower than expected velocity dispersion (Section 3.3) and therefore a low β is expected. When using the literature value of 788 km s^{-1} and a temperature of 5.6 KeV we obtain $\beta = 0.683$. Using our calculated value of 580 km s^{-1} this value drops to $\beta = 0.370$. This drop is expected however as the calculation of β is dependent on σ^2 and therefore, although the velocity dispersion falls within 3σ of the literature value, the value for β drops by 0.313. In this case, our choice of method is not at fault. A similar inconsistency has occurred for the cluster MKW3S but in this case we have overestimated the velocity dispersion.

Therefore, we can conclude that our sample of galaxy clusters are in hydrostatic equilibrium and the deviation from the results obtained by Bird et al. (1995), Girardi et al. (1996) and Wu et al. (1998) are for the reasons given above.

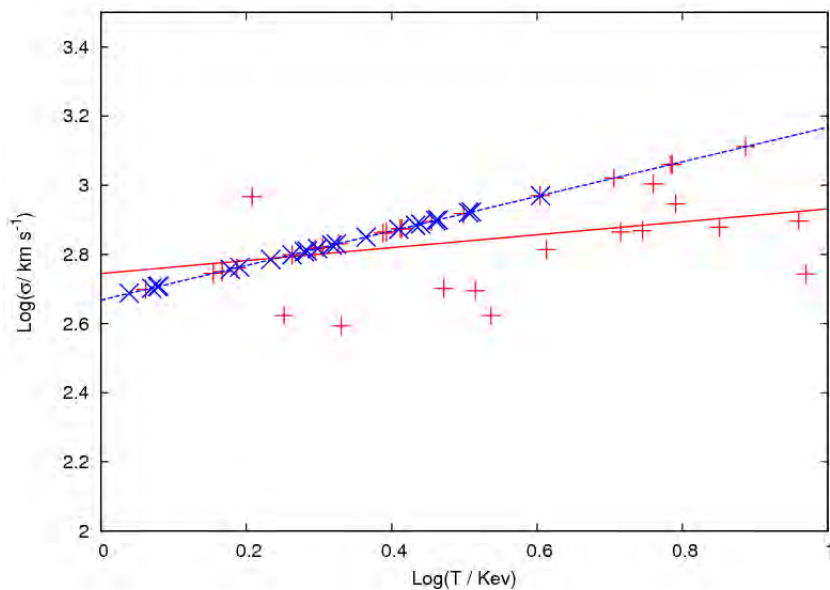


Figure 4.6: σ vs T_{calc} : In this plot we look for a relation between the velocity dispersion and T_{200} calculated using the formula given by Navarro et al. (1995). The points in blue are those for which the calculated velocity dispersion are used. The points in red show those for which the literature values were used. The blue and red line show the best fit for the two datasets.

We compared the velocity dispersion against our calculated temperature, T_{200} . Since the calculation of T_{200} depends on M_{200} and R_{200} we would expect a strong correlation as shown by the blue points and line in Figure 4.6. When using the literature velocity dispersions, we get a relation of $\sigma \sim T^{0.19}$. In this case, we do not expect to obtain a result similar to the literature as the temperature we are calculating is not the thermal X-ray temperature but rather the summation of the kinetic and gravitational energy with R_{200} .

4.5 X-ray Luminosity (L_X) versus Velocity Dispersion (σ)

Quintana & Melnick (1982) found that the X-ray luminosity and velocity dispersion were related by $L_x \sim \sigma^4$. Dell’Antonio et al. (1994) found a similar result for rich clusters but found that for clusters with $\sigma < 300 \text{ km s}^{-1}$ the slope flattened. They suggested that the reason for this was the extended emission associated with individual galaxies in the cluster which adds to the total X-ray luminosity. They found that for these clusters the relation became $L_x \sim \sigma^{2.7}$. Mahdavi et al. (1997) gets results similar to this. Diaferio et al. (1995) used simulations for clusters with more than 4 members and $\sigma < 300 \text{ km s}^{-1}$ and found that for compact groups $L_x \sim \sigma^{0.48}$. They suggested that the reasons for the flattened slope are:

- Groups of galaxies are rarely virialised;
- The velocity dispersion of a group is similar to the internal velocity dispersion of the galaxies in it;
- Galaxy-galaxy interaction leads to a merging instability and departure from equilibrium.

Our result is compared to that by Rozgacheva & Kuvshinova (2010) who use the same cosmology but a larger sample, 156 sources, and get a relation of $L_x \sim \sigma^{2.93}$ (Figure 4.7). Using the literature values we get $L_x \sim \sigma^{3.81}$ and using the calculated values we get $L_x \sim \sigma^{2.40}$. When determining the relation for the calculated values, we omitted A3571 and A1644 because, as explained in Section 3.3, the velocity dispersion for these clusters is much lower than expected and they lay far from the other points when plotting the relation. By doing this, we can see that our results are consistent with those obtained in the literature.

4.6 X-ray Luminosity (L_X) versus Temperature (T)

The L_X refers to the X-ray bolometric luminosity which is the luminosity in all bands of the X-ray and it is determined by the mass of gas in the intracluster medium (ICM). The temperature is determined by the total, gravitating mass of the cluster. Therefore the L_X vs T relation is an important way to probe variations in the gas fraction i.e.. $\frac{ICM_{gas}mass}{TotalMass}$ (Arnaud & Evrard 1999).

David et al. (1993) found the relation to have the form $L_X \sim T^3$ but Navarro et al. (1995) find $L_X \sim T^2$ using simulations. This difference may be due to the fact that Navarro et al. (1995) use models with no segregation of gaseous and dark matter which suggests that they are distributed and evolve differently. Arnaud & Evrard (1999) find agreement with Navarro et al. (1995) when they make the following assumptions during their derivation:

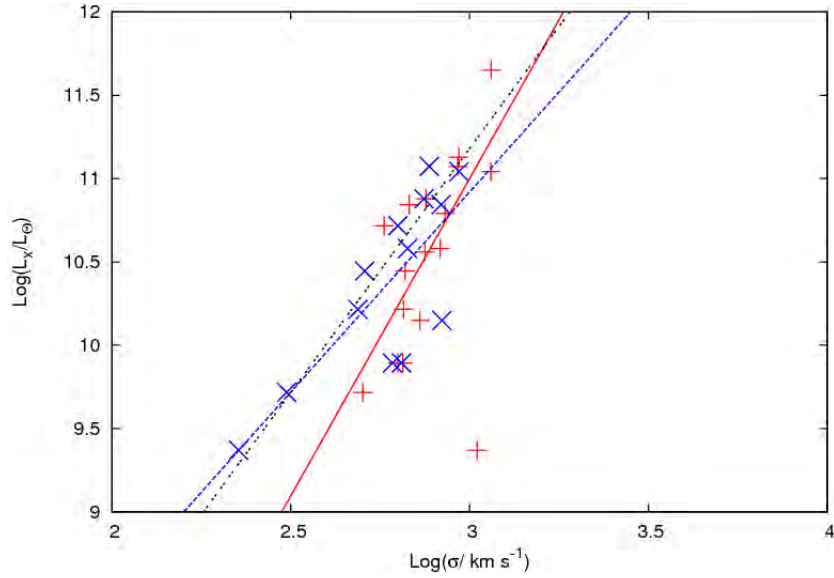


Figure 4.7: σ vs L_X : This plot shows the relation between the X-ray luminosity and the velocity dispersion. The red points and line show that obtained for the literature values. The blue points and line are for the calculated velocity dispersion and the black dashed line shows the fit obtained by Rozgacheva & Kuvshinova (2010).

- Cluster density profiles and gas fractions are temperature dependent;
- Clusters are internally isothermal;
- The emission is purely Bremsstrahlung;
- The cluster is in virial equilibrium;
- The clusters are structurally identical;
- The gas fraction is constant.

Fabian et al. (1994) found that clusters with a cooling flow core will have a different relation to those without or with a weak cooling flow. Table 4.1 shows the slope of the L_X vs T relation for clusters without cooling flows. We compare our results against Rozgacheva & Kuvshinova (2010) as they use the same cosmology of $H_0 = 50 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (Figure 4.8). Even using our reduced sample we obtain a fit of $\text{Log} \left(\frac{L_X}{L_\odot} \right) = (9.309 \pm 0.227) + (2.444 \pm 0.362) \text{Log} \left(\frac{T}{\text{KeV}} \right)$ with $\chi^2 = 0.15$, which is in good agreement with the published results.

Table 4.1: Comparison of selection criteria for Abell and Zwicky catalogs obtained from Sarazin & Boller (1989)

Author	Result
Arnaud & Evrard (1999)	2.88 ± 0.15
Markevitch (1998)	2.64 ± 0.27
Allen & Fabian (1998)	2.90 ± 0.30
Rozgacheva & Kuvshinova (2010)	2.57 ± 0.10

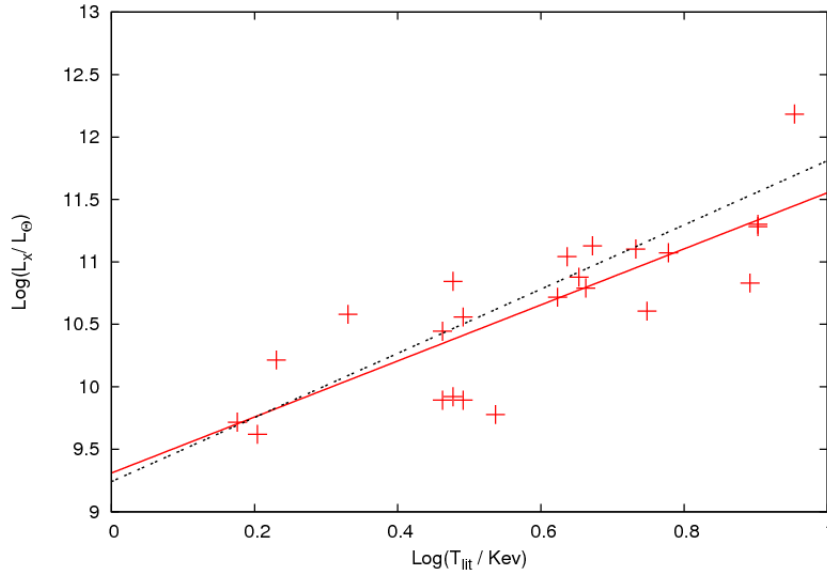


Figure 4.8: L_X vs T_X : In this plot we show the relation between the X-ray luminosity and the X-ray temperature obtained from literature (Table 2.3). The points in red show the data and its best fit line. The black line shows the relation obtained by Rozgacheva & Kuvshinova (2010).

4.7 Conclusion

In this chapter we looked at six correlations between different physical parameters and found in general a good agreement with published results. We looked for correlations of various physical parameters, σ , M_{200} and L_X , with redshift (z). We were unable to come up with a concrete correlation between σ and z due to the large scatter but Girardi & Mezzetti (2001) find that the velocity dispersion at high redshift is similar to that at low redshift. It was found that the mass of the galaxy clusters decreases with redshift as is expected by the Λ CDM model of the universe. Due to our small sample size of galaxy clusters with a X-ray luminosity, using the same cosmology, we could not accurately obtain a relation, but it did follow a similar trend of increasing with redshift as predicted.

We also compared the correlation between the X-ray temperature and the velocity dispersion with literature and found a good match showing that the galaxy clusters are in hydrostatic equilibrium. Since the velocity dispersion is used to calculate the temperature through the radius and mass, we cannot accurately conclude if there is a relation between the σ and T_{200} . A comparison of the X-ray luminosity and the velocity dispersion was in agreement with literature and has a relation close to the order of $L_X \propto \sigma^3$. The X-ray luminosity-temperature relation gave results very similar to that found in literature and provides proof that our method for determining galaxy cluster members is accurate.

Chapter 5

Conclusion and future work

In this dissertation we aimed to gain an understanding of galaxy clusters by doing a multi-wavelength study. In this chapter we give the main conclusions we can draw from our study and then present the scope for future work.

5.1 Conclusions

We provided a multi-wavelength overview of the study of galaxy clusters. From the optical data, astronomers were able to catalogue galaxy clusters, the most notable of these being the Abell and Zwicky catalogues. These were then used to classify the clusters into groups dependent on types of galaxies, magnitude and concentrations of galaxies. The broad classification that was determined from this was the division between regular and irregular galaxy clusters. Studies in the radio wavelength revealed a link between radio galaxies and galaxy clusters for the first time in the 1960's with the emission from these galaxies being due to synchrotron radiation. Further studies revealed that two main types of radio sources exist in galaxy clusters, single radio galaxies and giant radio haloes. The single radio galaxies can be further divided into WATs and NATs. The Uhuru satellite studied galaxy clusters in the X-ray wavelength and found that it was diffuse and further studies discovered that it was due to Bremsstrahlung radiation. Since the X-ray properties can be linked to the mass of the cluster via the virial theorem, they are very important in the study of cosmology.

Our interests then lay in determining the physical properties of galaxy clusters, which led to building a sample. Ideally, we want to use a sample that includes a wide range of luminosities and masses in an attempt to remove most of the bias. We combined samples from 4 papers (Popesso et al. 2007b, Yoon et al. 2008, Loubser et al. 2008, Brownstein & Moffat 2006) and removed all galaxy clusters with a galactic latitude of less than 10° to avoid contamination from the galactic plane. A redshift restriction of $z < 0.15$ was also placed on the sample as we wished to study nearby galaxy clusters as these have the most information available. This left us with a sample of 40 galaxy clusters. We did a literature review on each of the galaxy clusters in our sample and provided a brief overview of the important properties of each. An overlay of the radio contours from NVSS on the optical image from SDSS or DSS allowed us to compare the literature information with an image and to look for interesting formations of the radio sources.

When searching for the physical properties of galaxy clusters to be used for the correlations,

it was found that for many of the galaxy clusters in our sample we could not find the mass or the temperature. It also became apparent that the methods for finding the physical properties such as the velocity dispersion and radius were different for all the astronomers who published the results. Therefore we went in search of a method to determine physical parameters for all our galaxy clusters in a consistent and robust way. We determined the following properties; velocity dispersion (σ), radius (R_{200}), mass (M_{200}) and temperature (T_{200}).

The radius, mass and temperature can all be calculated once the velocity dispersion of the cluster is known. In order to calculate the velocity dispersion, we need to know the peculiar velocity of all the galaxies in the galaxy cluster. In order to obtain accurate results for the radius, mass and temperature we need to constrain the error on the velocity dispersion. $R_{200} \propto \sigma$ and therefore proportional to the error on velocity dispersion. The mass is dependent on σ^2 and R_{200} so the error is effectively tripled. For the temperature, we obtained two formulas to test. Helsdon & Ponman (2003) found a fitting that depends only on R_{200}^2 but the formula given by Navarro et al. (1995) is also dependent on M_{200} . Therefore it is very important that we find a very effective method of determining which galaxies belong to our galaxy clusters in order to determine an accurate result for the velocity dispersion. This led to the study of clustering algorithms. We compared three well-known clustering algorithms, the SIMBAD hierarchical database, the Kaye's Mixture Model (KMM) and the Gaussian Mixture Model (GMM).

The SIMBAD astronomical database has a method of hierarchy which they use to group galaxies into their respective galaxy clusters based on references in catalogues and papers. The main problem with using this method to find galaxy cluster members is that it is not complete and therefore it did not provide information for more than 50% of our sample. The redshift range of the galaxies belonging to a specific cluster is large which results in large peculiar velocities and therefore large velocity dispersions. This method was ruled out as an effective way to determine galaxy cluster members.

The KMM algorithm uses mixture modeling code to determine the likelihood that a sample is a single Gaussian as opposed to a double. It assumes the sample is a double Gaussian and assigns each source in the sample the probability of belonging to each of the modes. The main problem with this method is that it works best on homoscedastic data which is not true for most real data. Therefore we modified this method and applied a Levene test to check for different variances and then re-applied the KMM algorithm. Even with this modified method we found that the results were too inconsistent and could not be used to effectively calculate the velocity dispersion.

The GMM algorithm uses the expectation-maximization algorithm to determine the maximum probability of a galaxy belonging to a group. It assumes that each mode is described by a Gaussian, like KMM, but it does not assume that the variance is the same for each mode. However, at present the GMM algorithm works only on a maximum of 3 groups and the data in the sample will be forced into the number of groups you specify. Also, unlike KMM, it does not specify which mode each source of the sample belongs to it just gives a percentage probability. Therefore, we included sources which fell within 1 standard deviation of the mean of the group

given by GMM. Out of the three methods discussed this gave the most consistent results and those most similar to literature. Therefore we used this method to determine the galaxies that belonged to each of our galaxy clusters. However, to ensure that we did not include any galaxies which did not belong we looked at a few outlier tests.

We considered using the histogram to determine outliers but due to its susceptibility to give different results dependent on the choice of bin size and origin we ruled it out as a consistent method. The mean and KDE were also disregarded as suitable options due to the difficulty in determining outliers from the results. This left us 4 main options, distance separation, QQ Plot, Walsh test and Rosner test.

The distance separation technique was used to remove all sources that fell greater than an Abell radius from the BCG. We assumed that the clusters had a spherical nature and that the BCG was at the centre and used a 2D separation. A Quantile-Quantile (QQ) plot compares the variable values from our sample with the quantiles from a distribution selected by us. We used fit residuals and a confidence level of $\alpha = 0.1$ to determine outliers. This method provides consistent results but for a few of our galaxy clusters it appeared that it may be removing too many outliers so we decided to use it as a starting point for the Walsh and Rosner test.

The Walsh test requires that the sample have more than 60 members and therefore this ruled out most of our galaxy clusters. It also excludes or removes outliers as a group and this appeared to cause inconsistencies when comparing our results to literature. Therefore this method was ruled as an effective method to detect outliers. The Rosner test requires more than 25 sources in a sample and an underlying normal distribution. We used the Francia-Wilk test to determine if our galaxy clusters had an underlying normal distribution and if it did we applied the Rosner test and the results obtained were very consistent with the literature values.

Therefore we came up with a simple and consistent method for determining which galaxies belong to each of our galaxy clusters that used a combination of the GMM algorithm, the 2D separation, the QQ Plot and the Rosner test. Eight of the galaxy clusters in our sample showed a non-normal distribution and upon further investigation these were found to be dynamically active with ongoing mergers and galaxy evolution which has resulted in substructure. Future work will involve studying clusters such as these in more depth and looking for a solution to determining galaxy cluster members. We compared each of the calculated physical parameters to those obtained from literature, and if literature values were not available we calculated them using the formulas given in Section 1.5.

When comparing our velocity dispersion with that obtained from literature we found that they were in good agreement and most of them fell within 3 standard deviations. Discrepancies arose for the literature values where central velocity dispersions were calculated as we obtained larger values but this is expected. It appeared that we underestimated the velocity dispersions of many of the clusters but this does not concern us as we known that due to us using a more galaxies in our determination of the velocity dispersion our results are statistically better.

The radius we calculated was found to be in good agreement with the literature values for all but two of our galaxy clusters, A1644 and A3571. These clusters were found to have much lower than expected velocity dispersions due to the fact that the GMM algorithm doesn't effectively calculate their members due to the fact that they belong to the Shapley supercluster and have substructure. When comparing our masses, as expected these two clusters fall outside 3 standard deviations due to the fact that the error gets propagated. Two other clusters, A2147 and A1559, are also effected by this propagation and although they fall within 3 standard deviations for the velocity dispersion and the radius they are outliers in the mass comparison.

When calculating the temperature we used two methods. The first was dependent on the cosmology and the second was dependent on the mass. The first method seemed to underestimate the temperature as compared to the second method which was scattered equally above and below the literature value. Therefore we chose the second method as the one to be used for all other comparisons. However, this comparison was not completely accurate as we were comparing the temperature within a R_{200} to an X-ray temperature and therefore no further comparison about similarity can be done.

In order to further test our results, we used some known correlations between physical properties of galaxy clusters and tested to see if our calculated values gave the same results. We tested a total of 6 correlations and found good agreement with published results for most of these. The following summarises our main results:

- i We would expect that the velocity dispersion would increase with velocity dispersion but due to the large scatter in our data we were not able to concretely determine a specific trend. Girardi & Mezzetti (2001) found by studying the velocity dispersion profile of galaxy clusters that there is no difference between those at low and high redshift and therefore maybe the reason we found no trend is due to the fact that one doesn't exist.
- ii Comparison of the mass versus redshift finds that it decreases with redshift as is expected by lambda CDM model of the Universe. However, high redshift massive galaxy clusters have been found suggesting that we may need to reconsider the model of the Universe.
- iii Due to the fact that X-ray luminosity is dependent on the cosmological model we had to extract the sources from our sample which had the same, most common cosmology, $H_0=50 \text{ km s}^{-1} \text{ Mpc}^{-1}$. This left us with a very small sample and therefore we were not able to accurately confirm a relation between the X-ray luminosity and the redshift. However, we did find that it seemed to follow the same trend of increasing with redshift as seen in published results.
- iv The correlation between the X-ray temperature and the velocity dispersion is very important for determining dynamical properties of a cluster such as hydrostatic equilibrium. We found a relation of $T_X \sim \sigma^{0.4}$ which is close to the accepted value of $T_X \sim \sigma^{0.5}$. We also found that our galaxy clusters are in hydrostatic equilibrium by using the β parameter. Comparison of T_{200} with the velocity dispersion revealed a tight correlation which we expect because T_{200} is calculated from the radius and mass which are dependent on the velocity dispersion.

v We found the relation between the X-ray luminosity and the velocity dispersion to be of the order $L_X \sim \sigma^{2.40}$ as compared to the literature value of $L_X \sim \sigma^{2.93}$. This discrepancy may be accounted for by our small sample size used for to find the correlation. Comparison of the X-ray luminosity to the X-ray temperature also provided an excellent comparison to published with results with us obtaining a relation of $L_X \sim T^{2.44}$ compared to the literature result of $L_X \sim T^{2.57}$.

All of the conclusions show that the method we have used to determine our galaxy cluster members is robust and provides results that are in agreement with published values. However, improvements can always be made.

5.2 Future Work

For cluster surveys to reach their full potential, we must understand the relationships between their observable properties (e.g. optical richness, X-ray luminosity or temperature, SZ signal) and mass, as it is the latter which is needed to compare the observations with cosmological models. This requires an improved understanding of the physics of galaxy clusters (e.g. the impact of heating by active galactic nuclei (AGN) and supernovae within galaxies on the intracluster medium; the influence of dynamical disturbance through recent mergers), which can be obtained by measuring the evolution of the mass–observable relations with redshift and comparison with numerical simulations. In this dissertation, we gain an understanding of galaxy clusters by performing a multi-wavelength study and looking at scaling relations for nearby clusters. However, the evolution of the scaling relations to high redshift ($z > 0.5$) is largely unexplored. We may expect evolution to occur, because star formation and AGN activity increase by an order of magnitude by $z \sim 1$ (Silverman et al. 2005, Magnelli et al. 2009). In addition, the frequency of cluster mergers is expected to increase with redshift.

To address this situation, we will perform a detailed study of the gas physics and dynamical status of distant ($z > 0.5$) clusters, using a multi-wavelength approach, in order to measure the evolution of their mass-scaling relations with redshift. This work requires new, large samples of high redshift clusters, which are being obtained within the framework of two major international collaborations conducting cluster surveys: the Atacama Cosmology Telescope (ACT) project (Marriage et al. 2011), and the XMM Cluster Survey (Lloyd-Davies et al. 2011). ACT is one of the first searches for clusters using the SZ effect, and is revealing a population of very massive, high redshift, clusters for the first time. XCS is a serendipitous search for clusters using archival X-ray data from the XMM-Newton satellite. The XMM Cluster Survey (XCS) contains far more high redshift clusters than previous generation X-ray surveys, and the data are of sufficient quality to measure gas temperatures for many of the clusters. The first XCS cluster catalogue, released earlier this year*, is the largest sample of X-ray clusters with temperature measurements assembled to date (Mehrtens et al. 2012).

We will use in-hand, proprietary optical spectroscopic data obtained with the 8m Gemini telescopes (PI: M. Hilton; 180 hours over the last four semesters) for a sample of 15, $z > 0.5$,

*<http://www.xcs-home.org/datareleases>

X-ray clusters to measure their dynamical masses, and measure the evolution of the scaling relations between mass and X-ray luminosity and temperature. The optical spectroscopic data will also allow us to classify clusters as either dynamically disturbed or relaxed (by e.g. identifying evidence of substructure using the Dressler-Shectman method), allowing us to see if the disturbed clusters follow the same relations as the relaxed objects. We will interpret our results in the context of hydrodynamical simulations, applying the same techniques we use to analyse the data to the simulated samples (Hilton et al. 2012). This will be the first such study using a homogeneous cluster sample in this redshift range. Note that in addition to the results of this work being able to inform future X-ray surveys such as eROSITA, a number of objects in the cluster sample have been selected to lie in the Dark Energy Survey (DES) region, and so it will also be possible to use these objects to help calibrate a richness–mass relation for the DES optical cluster survey.

We will similarly investigate the scaling of dynamical mass with SZ-signal (Sifon et al. 2012), using a sample drawn from ACT. The ACT receiver is currently being upgraded and a new survey covering several thousand degrees to much better sensitivity than the current ACT maps will begin in late 2012 (Niemack et al. 2010). The spectroscopic data for the study of ACT/ACTPol clusters will be obtained using the Southern African Large Telescope (SALT); we currently have SALT data on one ACT cluster on which to develop the necessary data reduction and analysis techniques. The ACT sample is highly complementary to the XCS sample, as XCS typically probes lower mass clusters, while ACT detects only the most massive objects. There is also the potential to combine the two projects, by stacking on low mass XCS clusters in the ACTPol maps, which will enable a measurement of X-ray – SZ-signal scaling relations beyond current limits.

We will also perform radio observations of ACT and XCS clusters, which can provide complementary information on the dynamical state: radio relics and halos are often found in clusters which have undergone a recent merger (Ensslin & Biermann 1998). KAT-7 is the precursor to the Karoo Array Telescope (MeerKAT) and although it is not a cutting edge telescope, it provides us with very good sensitivity and short spacings to detect extended diffuse emission. A series of potential halos and relics have already been observed and others will continue to be observed as part of the KAT-7 science verification programme. These halos and relics will be further imaged using higher resolution instruments to understand how the dynamics of the observed clusters and the presence of halos and relics scales with redshift.

As mentioned in Section 1.5.4 radio galaxies can be broken up into two classes, FRI and FRII. Owen & White (1991) and Ledlow & Owen (1996) found that the break between these two classes depends on both the radio power and optical luminosity of the host galaxy. This shows that there is at least one correlation between different types of bent radio sources and their optical counterparts, however we wish to determine if any others exist. These types of sources can also be used to complement X-ray observations as a probe of the density of the intracluster medium, e.g. through measuring the curvature of the jets (Freeland et al. 2008).

Appendix A

Clustering Algorithm Results

In this chapter we provide tables of the results obtained using the 3 different clustering algorithms discussed in Section 3.1. In each table we give the number of potential members and the physical parameters calculated using the equations given in Section 1.5.

Table A.1: SIMBAD Results: This table shows the physical parameters calculated after using the SIMBAD database to obtain galaxy cluster members. Column 1 gives the galaxy cluster name, column 2 is the redshift and n is the number of galaxies in the galaxy cluster. Columns 4 - 8 give the physical parameters we calculated. Column 7 and 8 are the temperatures calculated using the equations given by Helsdon & Ponman (2003) and Navarro et al. (1995) respectively.

Cluster Name	z	n	σ (km s ⁻¹)	Radius (Mpc)	Mass ($10^{14} M_{\odot}$)	T ₁ (KeV)	T ₂ (KeV)
A0779	0.023	14	475 ± 88	1.526 ± 0.283	1.200 ± 0.385	0.955 ± 0.354	1.042 ± 0.386
A1060	0.011	260	709 ± 35	2.290 ± 0.113	4.011 ± 0.343	2.867 ± 0.283	2.321 ± 0.229
A1367	0.022	168	761 ± 48	2.446 ± 0.154	4.935 ± 0.539	2.732 ± 0.345	2.673 ± 0.337
A1644	0.047	305	250 ± 14	0.794 ± 0.044	0.173 ± 0.017	0.300 ± 0.034	0.289 ± 0.033
A1656	0.054	770	20724 ± 1263	65.617 ± 3.999	98189.573 ± 10364.719	1839.363 ± 224.196	1982.883 ± 241.690
A1736	0.042	108	1695 ± 64	5.397 ± 0.204	54.025 ± 3.534	456.021 ± 34.437	13.264 ± 1.002
A2052	0.035	58	605 ± 56	1.933 ± 0.179	2.465 ± 0.395	1.874 ± 0.347	1.690 ± 0.313
A2107	0.047	68	6226 ± 1371	19.778 ± 4.355	2671.176 ± 1018.788	191.130 ± 84.176	178.965 ± 78.816
A2147	0.037	87	832 ± 62	2.656 ± 0.198	6.406 ± 0.827	135.258 ± 20.159	3.196 ± 0.476
A2151	0.037	83	1859 ± 751	5.933 ± 2.397	71.439 ± 49.988	17.549 ± 14.179	15.955 ± 12.892
A2199	0.119	566	32080 ± 1017	98.392 ± 3.119	352801.433 ± 19371.677	49461.174 ± 3136.036	4751.368 ± 301.245
A3526	0.058	386	20234 ± 779	63.944 ± 2.462	91214.761 ± 6082.643	36329.066 ± 2797.306	1890.225 ± 145.551
A3571	0.039	98	994 ± 62	3.170 ± 0.198	10.913 ± 1.180	54.155 ± 6.756	4.562 ± 0.570
A3581	0.025	72	5474 ± 1985	17.569 ± 6.371	1834.250 ± 1152.064	153.841 ± 111.572	138.344 ± 100.334
A0610	0.091	153	15389 ± 2415	47.861 ± 7.511	39491.570 ± 10734.328	113507.745 ± 35625.603	1093.379 ± 343.172
MKW3s	0.078	93	16329 ± 1796	51.108 ± 5.621	47479.911 ± 9045.028	198836.202 ± 43739.337	1231.031 ± 270.791
MKW4	0.020	39	506 ± 65	1.628 ± 0.209	1.453 ± 0.323	95.382 ± 24.505	1.183 ± 0.304

Table A.2: KMM Results: This table shows the physical parameters calculated after using the method shown in Figure 3.6 to obtain galaxy cluster members. Column 1 gives the galaxy cluster name, column 2 is the redshift and n is the number of galaxies in the galaxy cluster. Columns 4 - 8 give the physical parameters we calculated. Column 7 and 8 are the temperatures calculated using the equations given by Helsdon & Ponman (2003) and Navarro et al. (1995) respectively.

Cluster Name	z	n	σ (km s ⁻¹)	Radius (Mpc)	Mass ($10^{14} M_{\odot}$)	T ₁ (KeV)	T ₂ (KeV)
A0595	0.069	34	224 ± 23	0.704 ± 0.072	0.123 ± 0.022	0.217 ± 0.045	0.232 ± 0.048
A0610	0.095	50	3702 ± 813	11.491 ± 2.524	548.697 ± 208.724	59.226 ± 26.013	63.274 ± 27.794
A0628	0.083	71	3095 ± 525	9.663 ± 1.639	322.504 ± 94.751	41.396 ± 14.044	44.225 ± 15.003
A0646	0.129	29	2746 ± 888	8.380 ± 2.710	220.164 ± 123.317	32.587 ± 21.076	34.814 ± 22.516
A0690	0.082	77	3161 ± 366	9.874 ± 1.143	343.751 ± 68.933	43.180 ± 9.999	46.132 ± 10.682
A0779	0.023	126	658 ± 79	2.114 ± 0.254	3.189 ± 0.663	1.871 ± 0.449	1.999 ± 0.480
A0858	0.091	20	452 ± 63	1.406 ± 0.196	1.001 ± 0.242	0.883 ± 0.246	0.943 ± 0.263
A1060	0.012	120	396 ± 38	1.279 ± 0.123	0.699 ± 0.116	0.678 ± 0.130	0.724 ± 0.139
A1066	0.066	34	437 ± 37	1.376 ± 0.116	0.916 ± 0.134	0.825 ± 0.140	0.882 ± 0.149
A1080	0.118	26	813 ± 172	2.495 ± 0.528	5.746 ± 2.106	2.856 ± 1.209	3.052 ± 1.292
A1187	0.077	84	3739 ± 492	11.708 ± 1.541	570.290 ± 129.988	60.415 ± 15.900	64.545 ± 16.989
A1346	0.097	43	387 ± 36	1.200 ± 0.112	0.626 ± 0.101	0.647 ± 0.120	0.691 ± 0.129
A1366	0.117	28	426 ± 58	1.308 ± 0.178	0.827 ± 0.195	0.784 ± 0.214	0.838 ± 0.228
A1367	0.021	109	532 ± 37	1.711 ± 0.119	1.687 ± 0.203	1.223 ± 0.170	1.307 ± 0.182
A1516	0.083	106	4787 ± 490	14.946 ± 1.530	1193.311 ± 211.572	99.029 ± 20.273	105.798 ± 21.660
A1644	0.047	63	938 ± 92	2.980 ± 0.292	9.135 ± 1.551	3.802 ± 0.746	4.062 ± 0.796
A1650	0.086	76	3626 ± 669	11.305 ± 2.086	517.879 ± 165.502	56.819 ± 20.966	60.702 ± 22.400
A1656	0.024	610	1213 ± 68	3.895 ± 0.218	19.968 ± 1.938	6.359 ± 0.713	6.793 ± 0.761
A1544	0.134	22	4799 ± 516	14.608 ± 1.571	1172.179 ± 218.314	99.526 ± 21.403	106.329 ± 22.868
A1559	0.105	34	455 ± 54	1.405 ± 0.167	1.013 ± 0.208	0.895 ± 0.212	0.955 ± 0.227
A1736	0.035	38	464 ± 63	1.482 ± 0.201	1.112 ± 0.261	0.930 ± 0.253	0.994 ± 0.270
A1809	0.077	48	433 ± 49	1.356 ± 0.153	0.886 ± 0.173	0.810 ± 0.183	0.866 ± 0.195
A1882	0.143	66	3950 ± 573	11.968 ± 1.736	650.605 ± 163.465	67.427 ± 19.562	72.035 ± 20.899
A1890	0.058	85	1506 ± 405	4.759 ± 1.280	37.607 ± 17.518	9.801 ± 5.272	10.471 ± 5.632
A2029	0.080	67	1800 ± 182	5.628 ± 0.569	63.533 ± 11.126	14.002 ± 2.831	14.959 ± 3.025
A2052	0.034	92	829 ± 149	2.650 ± 0.476	6.345 ± 1.975	2.970 ± 1.068	3.173 ± 1.140
A2092	0.081	109	5909 ± 271	18.468 ± 0.847	2246.724 ± 178.471	150.891 ± 13.840	161.205 ± 14.787
A2107	0.042	56	420 ± 33	1.337 ± 0.105	0.822 ± 0.112	0.762 ± 0.120	0.815 ± 0.128
A2142	0.093	84	619 ± 42	1.923 ± 0.130	2.567 ± 0.301	1.656 ± 0.225	1.769 ± 0.239
A2147	0.037	216	522 ± 19	1.666 ± 0.061	1.582 ± 0.100	1.178 ± 0.086	1.258 ± 0.092
A2151	0.037	313	1452 ± 89	4.634 ± 0.284	34.040 ± 3.614	9.111 ± 1.117	9.734 ± 1.193

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Table A.2 – Continued

Cluster Name	z	n	σ (km s $^{-1}$)	Radius (Mpc)	Mass ($10^{14} M_{\odot}$)	T_1 (KeV)	T_2 (KeV)
A2197	0.031	255	825 ± 97	2.641 ± 0.310	6.263 ± 1.275	2.941 ± 0.692	3.142 ± 0.738
A2199	0.032	170	498 ± 34	1.593 ± 0.109	1.377 ± 0.163	1.072 ± 0.146	1.145 ± 0.157
A2255	0.082	47	825 ± 113	2.577 ± 0.353	6.111 ± 1.450	2.941 ± 0.806	3.142 ± 0.861
A3526	0.009	47	491 ± 39	1.588 ± 0.126	1.334 ± 0.183	1.042 ± 0.166	1.113 ± 0.176
A3571	0.039	59	932 ± 89	2.972 ± 0.284	8.995 ± 1.488	3.754 ± 0.717	4.011 ± 0.766
A3581	0.022	33	384 ± 23	1.234 ± 0.074	0.634 ± 0.066	0.637 ± 0.076	0.681 ± 0.082
AWM5	0.034	20	295 ± 50	0.943 ± 0.160	0.286 ± 0.084	0.376 ± 0.127	0.402 ± 0.136
MKW3s	0.045	58	803 ± 99	2.553 ± 0.315	5.736 ± 1.225	2.787 ± 0.687	2.977 ± 0.734
MKW4	0.020	80	275 ± 17	0.885 ± 0.055	0.233 ± 0.025	0.327 ± 0.040	0.349 ± 0.043
MKW8	0.028	86	297 ± 31	0.952 ± 0.099	0.293 ± 0.053	0.381 ± 0.080	0.408 ± 0.085

Table A.3: GMM Results: This table shows the physical parameters calculated after using the GMM algorithm to obtain galaxy cluster members. Column 1 gives the galaxy cluster name, column 2 is the redshift and n is the number of galaxies in the galaxy cluster. Columns 4 - 8 give the physical parameters we calculated. Column 7 and 8 are the temperatures calculated using the equations given by Helsdon & Ponman (2003) and Navarro et al. (1995) respectively.

Cluster Name	z	n	σ (km s ⁻¹)	Radius (Mpc)	Mass ($10^{14} M_{\odot}$)	T ₁ (KeV)	T ₂ (KeV)
A0595	0.069	51	571 ± 61	1.795 ± 0.192	2.039 ± 0.377	1.409 ± 0.301	1.505 ± 0.322
A0610	0.097	38	473 ± 67	1.467 ± 0.208	1.144 ± 0.281	0.967 ± 0.274	1.033 ± 0.293
A0628	0.084	63	839 ± 93	2.618 ± 0.290	6.421 ± 1.232	3.042 ± 0.674	3.250 ± 0.720
A0646	0.126	25	772 ± 85	2.359 ± 0.260	4.899 ± 0.934	2.576 ± 0.567	2.752 ± 0.606
A0690	0.080	55	510 ± 46	1.595 ± 0.144	1.445 ± 0.226	1.124 ± 0.203	1.201 ± 0.217
A0779	0.023	129	651 ± 78	2.091 ± 0.251	3.088 ± 0.641	1.831 ± 0.439	1.957 ± 0.469
A0858	0.088	34	837 ± 67	2.607 ± 0.209	6.363 ± 0.883	3.028 ± 0.485	3.234 ± 0.518
A1060	0.012	172	609 ± 30	1.966 ± 0.097	2.541 ± 0.217	1.603 ± 0.158	1.712 ± 0.169
A1066	0.070	105	1269 ± 132	3.987 ± 0.415	22.370 ± 4.031	6.959 ± 1.448	7.435 ± 1.547
A1080	0.117	22	398 ± 53	1.222 ± 0.163	0.674 ± 0.156	0.685 ± 0.182	0.731 ± 0.195
A1187	0.075	65	872 ± 68	2.733 ± 0.213	7.241 ± 0.978	3.286 ± 0.512	3.511 ± 0.547
A1346	0.098	64	656 ± 47	2.033 ± 0.146	3.048 ± 0.379	1.860 ± 0.266	1.987 ± 0.285
A1366	0.116	36	580 ± 57	1.782 ± 0.175	2.089 ± 0.355	1.454 ± 0.286	1.553 ± 0.305
A1367	0.022	158	812 ± 44	2.610 ± 0.141	5.996 ± 0.562	2.849 ± 0.309	3.044 ± 0.329
A1516	0.078	74	1404 ± 209	4.394 ± 0.654	30.178 ± 7.781	8.519 ± 2.536	9.101 ± 2.709
A1544	0.145	16	521 ± 98	1.577 ± 0.297	1.491 ± 0.486	1.173 ± 0.441	1.253 ± 0.472
A1559	0.105	30	378 ± 43	1.167 ± 0.133	0.581 ± 0.115	0.617 ± 0.140	0.660 ± 0.150
A1644	0.047	48	541 ± 35	1.719 ± 0.111	1.753 ± 0.196	1.265 ± 0.164	1.351 ± 0.175
A1650	0.084	70	795 ± 70	2.481 ± 0.218	5.463 ± 0.833	2.731 ± 0.481	2.918 ± 0.513
A1656	0.023	556	909 ± 23	2.920 ± 0.074	8.406 ± 0.369	3.571 ± 0.181	3.815 ± 0.193
A1736	0.040	116	1654 ± 64	5.272 ± 0.204	50.251 ± 3.368	11.822 ± 0.915	12.631 ± 0.977
A1809	0.080	88	645 ± 40	2.017 ± 0.125	2.924 ± 0.314	1.798 ± 0.223	1.921 ± 0.238
A1882	0.140	47	1126 ± 180	3.417 ± 0.546	15.095 ± 4.179	5.479 ± 1.752	5.854 ± 1.871
A1890	0.058	78	509 ± 35	1.609 ± 0.111	1.452 ± 0.173	1.120 ± 0.154	1.196 ± 0.165
A2029	0.078	41	642 ± 53	2.009 ± 0.166	2.885 ± 0.413	1.781 ± 0.294	1.903 ± 0.314
A2052	0.038	127	1490 ± 65	4.753 ± 0.207	36.766 ± 2.776	9.594 ± 0.837	10.250 ± 0.894
A2092	0.067	56	440 ± 41	1.385 ± 0.129	0.934 ± 0.151	0.837 ± 0.156	0.894 ± 0.167
A2107	0.041	73	630 ± 47	2.007 ± 0.150	2.775 ± 0.359	1.715 ± 0.256	1.832 ± 0.274
A2142	0.091	141	748 ± 38	2.326 ± 0.118	4.534 ± 0.399	2.418 ± 0.246	2.583 ± 0.262
A2147	0.037	337	934 ± 32	2.981 ± 0.102	9.061 ± 0.537	3.770 ± 0.258	4.028 ± 0.276
A2151	0.036	266	1057 ± 43	3.375 ± 0.137	13.138 ± 0.925	4.828 ± 0.393	5.158 ± 0.419

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Table A.3 – Continued

Cluster Name	z	n	σ (km s ⁻¹)	Radius (Mpc)	Mass ($10^{14} M_{\odot}$)	T ₁ (KeV)	T ₂ (KeV)
A2197	0.030	252	605 ± 29	1.937 ± 0.093	2.470 ± 0.205	1.582 ± 0.152	1.690 ± 0.162
A2199	0.031	282	746 ± 30	2.388 ± 0.096	4.630 ± 0.322	2.405 ± 0.193	2.569 ± 0.207
A2255	0.081	57	1092 ± 103	3.413 ± 0.322	14.180 ± 2.317	5.153 ± 0.972	5.505 ± 1.039
A3526	0.012	178	898 ± 41	2.899 ± 0.132	8.145 ± 0.644	3.485 ± 0.318	3.723 ± 0.340
A3571	0.039	54	443 ± 33	1.413 ± 0.105	0.966 ± 0.125	0.848 ± 0.126	0.906 ± 0.135
A3581	0.022	52	487 ± 53	1.565 ± 0.170	1.293 ± 0.244	1.025 ± 0.223	1.095 ± 0.238
AWM5	0.035	40	504 ± 37	1.610 ± 0.118	1.425 ± 0.181	1.098 ± 0.161	1.173 ± 0.172
MKW3s	0.044	59	833 ± 90	2.650 ± 0.286	6.407 ± 1.198	2.999 ± 0.648	3.204 ± 0.692
MKW4	0.020	102	528 ± 49	1.699 ± 0.158	1.650 ± 0.265	1.205 ± 0.224	1.287 ± 0.239
MKW8	0.027	150	503 ± 38	1.613 ± 0.122	1.422 ± 0.186	1.093 ± 0.165	1.168 ± 0.177

Appendix B

Outlier Techniques

In this chapter we provide tables of the results obtained using the different outlier techniques discussed in Section 3.2. In each table we give the number of potential members and the physical parameters calculated using the equations given in Section 1.5.

Table B.1: Outlier Tests – QQ Plot Results: This table shows the physical parameters calculated after removing the outliers detected using the QQ Plot. Column 1 gives the galaxy cluster name, column 2 is the redshift and n is the number of galaxies in the galaxy cluster. Columns 4 - 8 give the physical parameters we calculated. Column 7 and 8 are the temperatures calculated using the equations given by Helsdon & Ponman (2003) and Navarro et al. (1995) respectively.

Cluster Name	z	n	σ (km s ⁻¹)	Radius (Mpc)	Mass ($10^{14} M_{\odot}$)	T ₁ (KeV)	T ₂ (KeV)
A0595	0.069	46	418 ± 41	1.314 ± 0.129	0.800 ± 0.136	0.755 ± 0.148	0.807 ± 0.158
A0610	0.097	32	293 ± 25	0.909 ± 0.078	0.272 ± 0.040	0.371 ± 0.063	0.396 ± 0.068
A0628	0.083	57	571 ± 43	1.783 ± 0.134	2.025 ± 0.264	1.409 ± 0.212	1.505 ± 0.226
A0690	0.080	50	408 ± 32	1.276 ± 0.100	0.740 ± 0.101	0.719 ± 0.113	0.769 ± 0.121
A0779	0.023	110	308 ± 15	0.989 ± 0.048	0.327 ± 0.028	0.410 ± 0.040	0.438 ± 0.043
A1066	0.069	96	747 ± 45	2.348 ± 0.141	4.565 ± 0.476	2.411 ± 0.291	2.576 ± 0.310
A1080	0.117	21	364 ± 47	1.118 ± 0.144	0.516 ± 0.115	0.573 ± 0.148	0.612 ± 0.158
A1187	0.074	70	869 ± 61	2.725 ± 0.191	7.170 ± 0.871	3.263 ± 0.458	3.486 ± 0.489
A1367	0.022	154	763 ± 39	2.452 ± 0.125	4.974 ± 0.440	2.516 ± 0.257	2.688 ± 0.274
A1516	0.076	67	679 ± 53	2.127 ± 0.166	3.417 ± 0.462	1.992 ± 0.311	2.129 ± 0.332
A1544	0.145	14	365 ± 63	1.105 ± 0.191	0.513 ± 0.153	0.576 ± 0.199	0.615 ± 0.212
A1650	0.084	61	629 ± 57	1.963 ± 0.178	2.706 ± 0.425	1.710 ± 0.310	1.827 ± 0.331
A1656	0.023	547	904 ± 24	2.904 ± 0.077	8.269 ± 0.380	3.532 ± 0.188	3.773 ± 0.200
A1882	0.140	50	761 ± 66	2.309 ± 0.200	4.659 ± 0.700	2.503 ± 0.434	2.674 ± 0.463
A2092	0.066	48	305 ± 24	0.960 ± 0.076	0.311 ± 0.042	0.402 ± 0.063	0.429 ± 0.068
A2147	0.037	323	873 ± 31	2.786 ± 0.099	7.398 ± 0.455	3.294 ± 0.234	3.519 ± 0.250
A2151	0.035	222	671 ± 30	2.144 ± 0.096	3.363 ± 0.261	1.946 ± 0.174	2.079 ± 0.186
A2197	0.030	246	553 ± 21	1.771 ± 0.067	1.887 ± 0.124	1.322 ± 0.100	1.412 ± 0.107
A2199	0.031	278	729 ± 29	2.333 ± 0.093	4.320 ± 0.298	2.297 ± 0.183	2.454 ± 0.195
A2255	0.081	53	911 ± 75	2.847 ± 0.234	8.232 ± 1.173	3.587 ± 0.591	3.832 ± 0.630
A3581	0.022	47	354 ± 26	1.138 ± 0.084	0.497 ± 0.063	0.542 ± 0.080	0.579 ± 0.085
MKW3S	0.044	54	604 ± 60	1.921 ± 0.191	2.442 ± 0.420	1.577 ± 0.313	1.684 ± 0.335
MKW4	0.020	85	315 ± 21	1.013 ± 0.068	0.350 ± 0.041	0.429 ± 0.057	0.458 ± 0.061
MKW8	0.027	123	307 ± 14	0.984 ± 0.045	0.323 ± 0.026	0.407 ± 0.037	0.435 ± 0.040

Table B.2: Outlier Tests – Walsh Results: This table shows the physical parameters calculated after removing the outliers detected using the Walsh test. Column 1 gives the galaxy cluster name, column 2 is the redshift and n is the number of galaxies in the galaxy cluster. Columns 4 - 8 give the physical parameters we calculated. Column 7 and 8 are the temperatures calculated using the equations given by Helsdon & Ponman (2003) and Navarro et al. (1995) respectively.

Cluster Name	z	n	σ (km s ⁻¹)	Radius (Mpc)	Mass ($10^{14}M_{\odot}$)	T ₁ (KeV)	T ₂ (KeV)
A0628	0.083	57	571 ± 43	1.783 ± 0.134	2.025 ± 0.264	1.409 ± 0.212	1.505 ± 0.226
A0779	0.023	117	415 ± 40	1.333 ± 0.129	0.800 ± 0.134	0.744 ± 0.143	0.795 ± 0.154
A1066	0.070	104	1219 ± 132	3.830 ± 0.415	19.829 ± 3.720	6.422 ± 1.391	6.861 ± 1.486
A1367	0.022	156	791 ± 42	2.542 ± 0.135	5.542 ± 0.510	2.704 ± 0.287	2.889 ± 0.307
A1516	0.076	67	679 ± 53	2.127 ± 0.166	3.417 ± 0.462	1.992 ± 0.311	2.129 ± 0.332
A1650	0.084	61	629 ± 57	1.963 ± 0.178	2.706 ± 0.425	1.710 ± 0.310	1.827 ± 0.331
A2151	0.035	222	671 ± 30	2.144 ± 0.096	3.363 ± 0.261	1.946 ± 0.174	2.079 ± 0.186
A2197	0.030	246	553 ± 21	1.771 ± 0.067	1.887 ± 0.124	1.322 ± 0.100	1.412 ± 0.107
A2199	0.031	279	738 ± 29	2.362 ± 0.093	4.482 ± 0.305	2.354 ± 0.185	2.515 ± 0.198
MKW4	0.020	91	397 ± 35	1.277 ± 0.113	0.701 ± 0.107	0.681 ± 0.120	0.728 ± 0.129
MKW8	0.027	137	390 ± 22	1.251 ± 0.071	0.663 ± 0.065	0.657 ± 0.074	0.702 ± 0.079

Table B.3: Outlier Tests – Rosner Results: This table shows the physical parameters calculated after removing the outliers detected using the Rosner test. Column 1 gives the galaxy cluster name, column 2 is the redshift and n is the number of galaxies in the galaxy cluster. Columns 4 - 8 give the physical parameters we calculated. Column 7 and 8 are the temperatures calculated using the equations given by Helsdon & Ponman (2003) and Navarro et al. (1995) respectively.

Cluster Name	z	n	σ (km s ⁻¹)	Radius (Mpc)	Mass ($10^{14} M_{\odot}$)	T ₁ (KeV)	T ₂ (KeV)
A0595	0.069	51	571 ± 60	1.795 ± 0.189	2.039 ± 0.371	1.409 ± 0.296	1.505 ± 0.316
A0610	0.097	37	417 ± 52	1.293 ± 0.161	0.783 ± 0.169	0.751 ± 0.187	0.802 ± 0.200
A0628	0.084	60	676 ± 68	2.110 ± 0.212	3.360 ± 0.585	1.975 ± 0.397	2.110 ± 0.424
A0690	0.080	55	510 ± 45	1.595 ± 0.141	1.445 ± 0.221	1.124 ± 0.198	1.200 ± 0.212
A1066	0.069	97	766 ± 47	2.408 ± 0.148	4.923 ± 0.523	2.536 ± 0.311	2.709 ± 0.332
A1367	0.022	156	791 ± 42	2.542 ± 0.135	5.542 ± 0.510	2.704 ± 0.287	2.889 ± 0.307
A1516	0.076	68	708 ± 57	2.218 ± 0.179	3.874 ± 0.541	2.166 ± 0.349	2.314 ± 0.373
A1650	0.084	67	794 ± 74	2.478 ± 0.231	5.443 ± 0.879	2.724 ± 0.508	2.911 ± 0.543
A3581	0.022	52	487 ± 53	1.565 ± 0.170	1.293 ± 0.244	1.025 ± 0.223	1.095 ± 0.238
A2092	0.067	56	440 ± 41	1.385 ± 0.129	0.934 ± 0.151	0.837 ± 0.156	0.894 ± 0.167
MKW3S	0.044	59	833 ± 90	2.650 ± 0.286	6.407 ± 1.198	2.999 ± 0.648	3.204 ± 0.692

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