

Chapter 1

Introduction

It is generally believed that massive stars ($M \gtrsim 8M_{\odot}$) will end their lives in a catastrophic supernova explosion (see, e.g., *Woosley et al.*, 2002; *Woosley and Janka*, 2005). The stellar material ejected during the explosion expands supersonically through the interstellar medium, leading to the formation of a shock wave that sweeps up and accelerates ambient matter. The accelerated particles radiate photons through various mechanisms, such as synchrotron radiation and inverse Compton scattering (see, e.g., *Reynolds*, 2008), and as the emission is most prominent at the shock, these supernova remnants are typically observed as shell-like structures. Notable examples of these remnants include Cassiopeia A and SN 1006.

Although often referred to as supernova remnants, pulsar wind nebulae are distinctly different from shell-type supernova remnants. As the name suggests, these nebulae are powered by a relativistic, magnetised particle outflow from a central pulsar. As a result of the continual injection of energy into the nebula by the pulsar, these systems have a filled morphology (see, e.g., *Weiler and Panagia*, 1980), in contrast to the shell-like structure of, e.g., SN 1006. Pulsar wind nebulae are often very luminous sources, radiating synchrotron emission that extends from radio to X-ray wavelengths, as well as radiating inverse Compton emission as high-energy and very-high-energy gamma-rays. The most notable example of a pulsar wind nebula is undoubtedly the well-studied Crab Nebula.

As the supernova explosion that leads to the formation of a shell-type remnant can also lead to the creation of a pulsar, it is only natural to expect that a number of systems should exist where a pulsar wind nebula is located within a shell-type remnant. These systems constitute the so-called composite remnants, with notable examples including the Vela remnant (see, e.g., *De Jager and Djannati-Ataï*, 2009) and G21.5-0.9 (see, e.g., *Slane et al.*, 2000).

Pulsar wind nebulae provide a rich environment for studying a large number of astrophysical problems, ranging from relativistic flows (see, e.g., *Coroniti*, 1990), to the possible origin of cosmic rays (see, e.g., *Bednarek and Bartosik*, 2003). These objects are therefore actively studied, both theoretically and observationally. One particular field of research focuses on the effect that the presence of a shell remnant has on the evolution of the pulsar wind nebula. For this specific purpose, time-dependent hydrodynamic models have been extensively employed, with some

examples including the spherically-symmetric, one-dimensional model of *Van der Swaluw et al.* (2001b), and the two-dimensional, two-fluid, axisymmetric model of *Blondin et al.* (2001). A sub-class of these models are the two-dimensional, axisymmetric simulations of *Bucciantini* (2002) and *Van der Swaluw et al.* (2003a, 2004) that focus on the evolution of a pulsar wind nebula containing a fast-moving pulsar.

In order to predict the synchrotron luminosity from pulsar wind nebulae, it is necessary to include a magnetic field in the evolutionary models. The aforementioned hydrodynamic simulations have therefore been extended to magnetohydrodynamic models. Examples include the two-dimensional, axisymmetric model of *Van der Swaluw* (2003b), the spherically-symmetric, relativistic model of *Bucciantini et al.* (2003, 2004b), as well as the two-dimensional, axisymmetric, relativistic models of *Del Zanna et al.* (2004), *Bogovalov et al.* (2005), and *Volpi et al.* (2008). Additionally, a number of two-dimensional, axisymmetric relativistic models that focus on simulating the observed jet and torus in the vicinity of the pulsar (see, e.g., *Weisskopf et al.*, 2000; *Helfand et al.*, 2001) have also been presented. These include the simulations of *Komissarov and Lyubarsky* (2003) and *Del Zanna et al.* (2006).

It may be argued that the (relativistic) magnetohydrodynamic models are an improvement over ordinary hydrodynamic models, but large computational resource requirements have limited the magnetohydrodynamic simulations to evolution time scales of 1000-1500 years, while pulsar wind nebulae are expected to reach ages of ~ 0.1 Myr (see, e.g., *De Jager*, 2008). This makes it difficult to perform parameter studies that focus on the effect that various parameters, such as the density of the interstellar medium and the energy loss rate of the pulsar, have on the long-term evolution of a composite remnant. For these studies, hydrodynamic models still represent the best option. However, to date no extensive study has been presented that investigates the role of the various parameters within this context.

The first aim of this study is therefore to investigate how a number of key parameters influence the evolution of a spherically-symmetric, composite remnant evolving into a homogeneous interstellar medium. For this purpose, an extended hydrodynamic model is used wherein the magnetic field is included in a kinematic fashion, as described by *Ferreira and de Jager* (2008). In this approach it is assumed that the ratio of magnetic to particle energy in the nebula is small, leading to a strictly one-sided interaction between the fluid and magnetic field, i.e. the magnetic field has no influence on the evolution of the fluid. Although this may seem a limiting assumption, it is expected to hold for a number of pulsar wind nebulae, and the present hydrodynamic model can thus also be used to calculate the evolution of the magnetic field.

Although (magneto)hydrodynamic models have played a considerable role in the development of the theory on pulsar wind nebulae, it is difficult to describe the evolution of the particle energy spectrum within the framework of these simulations. This is a relevant calculation as energy losses and diffusion can lead to a modification of this spectrum, which in turn determines the spectrum of the observed non-thermal emission. For this purpose, a number of approaches have been used. Examples include the leaky-box model of *Zhang et al.* (2008), or

the model of *Tanaka and Takahara* (2010, 2011) in which the authors solve a transport equation in time and energy. Another popular approach is particle-counting models, used by, e.g., *Gelfand et al.* (2009). If it is known how many particles are injected into the nebula in a given time and energy interval, and the energy loss rate of the particles can be calculated, then it is possible to construct the evolution of the particle spectrum.

The common factor shared by the above-mentioned particle evolution models is that these simulations are all spatially independent. These models are undoubtedly useful in their own right, but are limited in scope as a spatially independent approach implicitly assumes that the particle source is uniformly distributed throughout the nebula. This is contrary to the nature of pulsar wind nebulae, as a defining feature of these systems is the continual injection of particles into the system by the central pulsar. A number of spatially dependent models have therefore been presented, including the well-known model of *Kennel and Coroniti* (1984a,b), and the particle-counting model of *Schöck et al.* (2010). Again these particle models are limited in scope as they do not include any time-dependence. Only a small number of models have been presented that calculate the evolution of the particle spectrum in both space and time, including the particle-counting model of *Van Etten and Romani* (2011). However, this model focuses on reproducing the non-thermal emission of one specific nebula, HESS J1825–137, without investigating the evolution of the particle spectrum.

The second aim of this study is therefore to develop a model that can be used to calculate the evolution of the particle spectrum in the nebula as a function of both position and time. In order to achieve this, a Fokker-Planck type transport equation is used, similar to the transport equation used to describe the propagation of cosmic rays in the Galaxy (*Ginzburg and Syrovatskii*, 1964), and to the Parker transport equation, used to describe the modulation of cosmic rays in the heliosphere (*Parker*, 1965). This work is an extension of the results presented by *Lerche and Schlickeiser* (1981), in which the authors investigated the effect of convection, diffusion, and energy changes on the evolution of a spectrum originating from point and diffuse sources. These solutions were, however, obtained for an axisymmetric, cylindrical coordinate system, with the limitation that the convection velocity must either be constant, or increase linearly from the source. This limitation was primarily dictated by the analytical nature of the solution scheme. Although useful as a starting point, these analytical solutions require an unphysical behaviour for pulsar wind nebulae as the convection velocity decreases as a function of distance. In the present study, the work of *Lerche and Schlickeiser* (1981) is extended by solving the transport equation in a spherical system. As a numerical solution technique is used, the present model has the advantage in that it does not place any limitation on the convection velocity. In addition, the effect of the magnetic field structure on the evolution of the particle spectrum is also investigated. This requires that the transport equation be solved in at least two spatial dimensions.

Although the focus is on constructing a particle model for pulsar wind nebulae, the approach is equally valid for any similar central-source system, such as globular star clusters. It is be-

lieved that non-thermal particles are injected into the cluster by millisecond pulsars located at its centre. The particles propagate away from the centre, producing synchrotron and inverse Compton emission. The large size of the cluster compared to the compact core of pulsars makes it possible to approximate the central pulsars as a single source (see, e.g., *Venter et al.*, 2009). A further possible application of the particle model is to the bubbles observed below and above the Galactic plane by the *Wilkinson Microwave Anisotropy Probe* (*Dobler and Finkbeiner*, 2008), *Fermi* (*Su et al.*, 2010), and more recently by *Planck* (*Planck Collaboration, P.A.R. Ade et al.*, 2013). Various explanations for the observed emission have been proposed, ranging from dark matter annihilation to emission from cosmic-ray electrons. If the latter are the cause of the emission, then the present model can be used to explain some of the observed characteristics of the bubbles.

The advantage of the present study is that it represents a self-consistent approach to the modelling of pulsar wind nebulae. The kinematic hydrodynamic simulations can be used to calculate the velocity and magnetic field in the nebula. In turn, these results can be used in the particle transport code as the transport and energy loss processes depend on these fluid quantities.

The outline of the study is as follows: Chapter 2 gives a succinct overview of shell-remnants, pulsars, and pulsar wind nebulae, with emphasis on the latter. Chapter 3 briefly discusses the hydrodynamic model, with the largest part of the chapter focusing on the results obtained. As an introduction to the spatially dependent transport models developed in this study, Chapter 4 first presents a spatially independent particle evolution model that is representative of current pulsar wind nebula models. The strong points of this type of model are highlighted, as well as its inherent limitations. The focus of the study subsequently shifts to the solutions of spatially dependent particle models based on a Fokker-Planck transport equation. In Chapter 5 the solutions obtained from a spherically-symmetric model are presented and discussed, while Chapter 6 deals with the solutions of the transport equation in a steady-state, axisymmetric system. The last chapter summarises the salient results obtained from the present study, as well as discussing possible future avenues of research.

Large parts of the research presented in this study have been published in *Vorster et al.* (2013a), *Vorster et al.* (2013b), and *Vorster and Moraal* (2013).