COMPARING RISK AND REWARDS FOR PORTFOLIO SELECTION STRATEGIES

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The longest race we ever run -
The highest hill we climb -
The farthest journey that we make-
Comes one step at a time.

For I can do everything with the help of Christ who gives me the strength I need.
~ Philippians 4:13

OPSOMMING

Die doel van hierdie studie was om 'n doeltreffende oplossing vir die portefeulje seleksie probleem te vind. Verskillende risiko maatstawwe word gebruik. Daar is 'n "trade-off" tussen risiko en opbrengs. Ons will besluit hoe om 'n bedrag geld in 'n aantal fondse te belê, in terme van spesifieke risiko-opbrengs doelwitte. Dus, ons maksimeeer opbrengs vir 'n sekere risiko waarde of ons minimeer risiko vir 'n sekere opbrengs waarde.

Die vermoë om risiko met relatiewe akkuraatheid te meet is 'n belangrike komponent van doeltreffende risikobestuur. 'n Algemene manier om beleggings-risiko te beskryf, is om dit in verband te bring met onsekerheid of onbestendigheid in beleggings opbrengste, oor tyd. Byvoorbeeld, as 'n belegging se opbrengste tussen 0 en 2 persent wissel, is die belegging nie so onstabiel soos 'n belegging wat opbrengste tussen –10 en 12 persent toon nie (Clark, 1994:45). Risiko is dus belangrik, daarom gee die literatuur studie 'n oorsig van risiko en risikobestuur. Dit sluit die volgende in: bronne van beleggings risiko, beleggings risiko klassifikasies, verskillende risiko maatstawwe, die proses van risikobestuur en strategieë vir risikobestuur.

Verskillende beleggings strategieë word oor verskillende tyd periodes vergelyk, om die optimale strategie te bepaal ('n aantal risiko maatstawwe word gebruik om die beste strategie te bepaal). Twee benaderings word gevolg in hierdie studie, naamlik: onsekerheids modelle vir besluitneming en risiko modelle vir besluitneming. Die empiriese resultate van albei die benaderings word getoon in dié studie.

Die optimale strategie was die half jaarlikse pessimistiese Hurwicz kriterium strategie en vir die individuele fondse, S_3. Die belegger hoef egter nie altyd die optimale strategie te kies nie, maar kan ook 'n goeie model kies. Dit wil sê, 'n strategie wat 'n effens laer opbrengs lewer, maar wat ook laer risiko toon.

ABSTRACT

The purpose of this study is to obtain an effective solution to the portfolio selection problem, for different risk measures. There is a trade-off between risk and return. We want to determine how an investor should invest an amount of money, in a number of funds, in terms of specific risk-reward objectives. That is, to maximise return for a given risk level or to minimise risk for a given expected return level.

An important element of effectively managing risk is the ability to measure it with relative precision. One of the most common ways to describe investment risk is to relate it to the uncertainty or volatility of returns from an investment over time. For example, an investment whose returns range between 0 and 2 percent is less volatile than an investment whose returns range between –10 and 12 percent (Clark, 1994:45). Because risk is therefore important, the literature review presents an overview of risk and risk management. This includes the following: sources of investment risk, investment risk classifications, different risk measures, the risk management process and strategies for risk management.

In order to find the optimal or best strategy, different investment strategies are compared over different time horizons (comparison is done by using some risk measures, which are minimised, as criteria for selecting the best strategy. The investment strategies that are used include: models for decision making under uncertainty and decision making under risk. The empirical results of both these approaches are presented in this study.

The optimal strategy was the half yearly pessimistic Hurwicz criterion strategy and for the individual funds, S_3. The investor doesn't always have to select the optimal strategy, but he can also select a good model. Thus, a strategy that has a slightly lower return, but it also shows lower risk.

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CHAPTER 5

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CHAPTER 1
INTRODUCTION

1.1 Problem statement

An analytical study is done in search for an effective solution to the portfolio selection problem, for different measures of risk. This involves the following:

There is a trade-off between risk and return, thus, how should one invest an x amount of money in a market or fund to minimize the risk, subject to requiring the expected return to be at least as great as a prescribed return tolerance, over a single period of time? Additional constraints may be added to further restrict the investment choices.

Portfolio theory assumes that for a given level of risk, investors prefer higher returns to lower returns. Similarly, for a given level of expected return, investors prefer less risk to more risk. Therefore, we can assume that investors would like to invest in an efficient portfolio, that is, one in which there is no other portfolio that offers a greater return with the same or less risk, or less risk with the same or greater expected return (Coleman, 2001; Czyzyk *et al.*)

1.2 Research aim and objectives

The purpose of the study is to obtain an optimised portfolio. This will be done with regard to the following:

- Specific risk/reward objectives (i.e. to maximise return for a given risk level, or risk is minimised for a given expected return level).
- Reduction of overall portfolio risk, using different risk measures.

Thus, the aim is to use quantitative measures of risk and return to obtain a balance between these two factors that suits the individual investor.

1.3 Basic hypothesis

We will assume that an optimal investment strategy exists. In order to find the 'optimal' or best strategy, we will compare different investment strategies over different time horizons, which include:

- Models for decision-making under uncertainty,
- Monte Carlo simulation models and
- Optimisation models.

Transaction costs will be considered explicitly. The transaction costs used are as follows:

- Fund-manager fee (for making a switch) $\Rightarrow$ R100.
- To switch from one fund to another $\Rightarrow$ no charge.
- To stay in money market from month to month $\Rightarrow$ no charge.
- To switch from MM (money market) to a fund $\Rightarrow$ 0.25% of the investment + fund manager's fee.
- To switch from a fund to MM $\Rightarrow$ 0.40% of the investment + fund manager's fee.

1.4 Methodology

Two routes are followed in addressing the research problem. An analysis of literature and sources will be used as well as practical modelling. The method of investigation are structured as follows:

- Fund selection under uncertainty is done through statistical modelling.
- Linear and non-linear programming formulation and prediction are done through statistical modelling.
- Risk measurement and management are portrayed through the literature review.

For all the investment strategies shown in this study, investment will commence on Aug 86, as some of the strategies use the previous year's data to forecast the allocation of funds for the next year. Furthermore, the same initial investment amount of R50 000 will be used for all

strategies to allow switching between the MM and the other funds, and to effectively compare the results of the different strategies.

1.5 Source of data

Selected financial and stock market data were obtained from ABSA's actuarial division. In this analysis I use monthly percentage returns of eight funds: seven stock market funds and a MM fund are observed.

- Stock market funds – These dividend funds are funds of funds, which consist of investments in equity, bonds, cash, property, international equity, etc. The minimum amount that can be invested in a fund is R 50 000. In this study, the first fund will be referred to as S_1; the second fund will be S_2, etc.
- MM fund - The ABSA Money Market Fund is similar to ordinary unit trusts in that contributions from all clients are pooled, but these are now invested in highly specialised money market securities. These securities include bank deposits, wholesale call deposits, bridging bills, parastatal bills (such as Eskom), treasury bills and commercial papers issued by governments, banks and large corporations who need to borrow money on a short-term basis. The minimum amount that can be invested in this fund is R 20 000 (ABSA, 2005).

The data are from July 1985 to December 2003. Table 1.1 displays some statistics for all the monthly return data:

Stock/Fund	Mean (%)	Variance (%)	Min	Max	Skewness	Kurtosis
S_1	1.38%	3.51%	-18.87%	9.73%	-1.10	4.91
S_2	1.42%	4.01%	-22.63%	12.34%	-1.19	5.78
S_3	1.85%	3.63%	-12.87%	12.76%	-0.28	2.05
S_4	1.42%	3.52%	-18.87%	9.74%	-1.15	4.98
S_5	1.41%	3.61%	-20.58%	10.61%	-1.27	6.60
S_6	1.45%	4.32%	-28.06%	12.86%	-1.63	9.58
S_7	1.36%	3.88%	-23.61%	12.87%	-1.24	7.30
MM	1.14%	0.27%	0.71%	1.74%	0.28	-1.02

Table 1.1: Statistics of return data

1.6 Outline for rest of the study

Chapter 2

Chapter 2 present a literature review on different risk measures, types of risk and risk management.

Chapter 3

Chapter 3 represents the results of perfect information, thus it answers the question: 'What could have been the results, if we had perfect information about the future?' This chapter also presents the results of investment strategies not using risk measures, i.e. models for decision making under uncertainty.

Chapter 4

Chapter 4 focuses on investment strategies that use risk measures. This chapter includes the results of Monte Carlo simulation models and optimisation models with different risk measures.

Chapter 5

Chapter 5 provides a comparison of the strategies with and without risk measures that was presented in chapters 3 and 4.

Chapter 6

This chapter offers the summary and recommendations and suggestions for further work.

CHAPTER 2
LITERATURE REVIEW

2.1 Introduction

To most people "risk" evokes negative images, e.g. driving too fast or travelling alone to unknown places. If you mention risk in terms of investing, people might think about losing their life's savings or nest egg. Risk can be defined as the possibility that an investment may fluctuate in value, the possibility of loss or the uncertainty that the expected return will not be achieved (ABSA, 2005; Mayo, 2001: 181). In risk measurement (there is distinction between the identification and measurement of *ex ante* risk (involves the evaluation of risk before that risk is actually encountered) and *ex post* risk (i.e. analysis of risk after it has been taken, with the purpose of evaluating historical performance on a risk-adjusted basis).

All investments involve some element of risk, since financial decisions are made in the present but the results occur in the future. Risk is therefore important in financial decision-making, as the future is not certain; it is only expected. However, possible sources of risk can be identified and, to some extend, risk can be managed (Mayo, 2001:5). Thus, the following topics are addressed in this literature review:

- Risk (investment risk classifications, sources of investment risk and measures of risk).
- Risk management (process of risk management and strategies for risk management).
- A case study on asset allocation with linear programming.

2.2 Investment risk classifications

Investment risk comes in many forms and it can be broken down into specific classifications, which consist of the following:

2.2.1 Market risk

Market risk is also known as systematic risk and is the likelihood that the value of a security will move in tandem with the overall market. Hence, it is the risk that a security's value declines as a result of changes in some risk factor that affects all asset prices in some way (Christopher & Mensink, 1999:98).

2.2.2 Interest-rate risk

This type of risk is most often associated with fixed-income investments. AXA Advisors (2005) define interest-rate risk as the risk that the price of a bond or the price of a bond fund will fall with rising interest rates.

2.2.3 Inflation-rate risk

It is the chance that the purchasing power of an investment will be harmed by inflation. Stated differently, while an investment may show growth over time, it may actually be losing value if it does not keep tread with the rate of inflation (AXA Advisors, 2005). Inflation risk needs to be considered when evaluating conservative investments, for instance money market funds or bonds as long-term investments. However, even a highly diversified stock is expected to lose money relative to inflation about one-fifth of the time over a five year investment period (The Motley Fool, 2005).

2.2.4 Credit risk

Credit risk applies to bonds and bond funds. It refers to a bond issuer's ability to repay its debt as promised when the bond matures. Bonds and bond funds are given credit ratings: the higher the rating, the lower the credit risk (AXA Advisors, 2005).

2.3 Measures of risk

The following sections give an overview of the commonly used measures of risk.

2.3.1 Standard deviation and variance

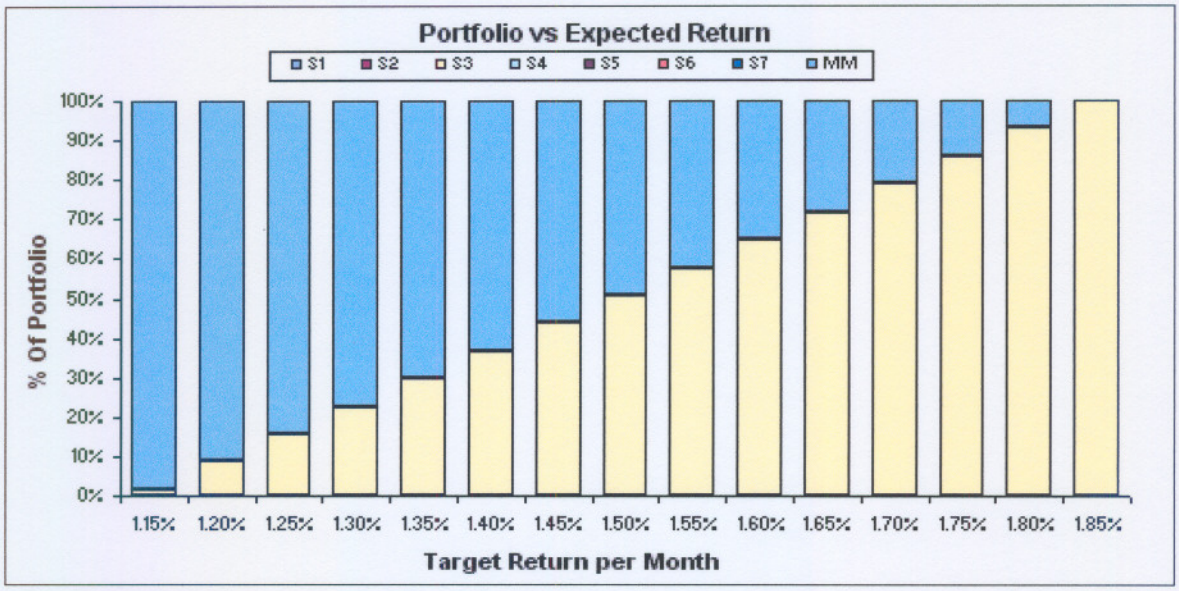
The variance or standard deviation represents the variability of the returns around the mean. The higher the level of standard deviation, the more variability there is in the probability distribution, i.e. fluctuations below and above the average return. Standard deviation as a measure of risk tends to work better when the probability distribution of returns are symmetric. When a distribution is skewed to one side or the other while another is symmetric around the mean, both might have the same standard deviation, but be perceived as having somewhat different levels of risk. E.g. a distribution skewed to the left is considered to have less likely but larger losses and more likely but smaller gains (with the opposite being true, if a distribution is skewed to the right). A risk-averse investor will thus prefer a right- skewed distribution to a left-skewed distribution (Clark, 1994:49). This is related to utility theory, i.e. the theory of economic behaviour that states that economic units will act to maximize their economic satisfaction or utility (Nawrocki, 2002:79). Clark (1994:51) states that standard deviation or variance can, lead to misleading results under the following circumstances:

- When the probability distribution is not symmetric- this can be induced by non-linear trading rules in the portfolio or it can be inherent in the asset itself.
- When a significant portion of the distribution lies in a range yielding negative returns.

2.3.1.1 Mean-variance Optimisation

An investor might be interested in the trade-off between the risk (variance) of the portfolio and its required expected return. Given a potential set of asset classes, the efficient frontier (a set of efficient portfolios, thus portfolios for which no greater average return can be found without a corresponding increase in risk, or instead no greater certainty of returns can be achieved without a decrease in expected return), can be found. The most common type of efficient frontier is the one popularised by Markowitz (which treats portfolio variance as a sufficient measure of risk). Portfolios on the mean-variance frontier are identified by portfolio optimisation, i.e. searching for the portfolio with the least variance given some minimum

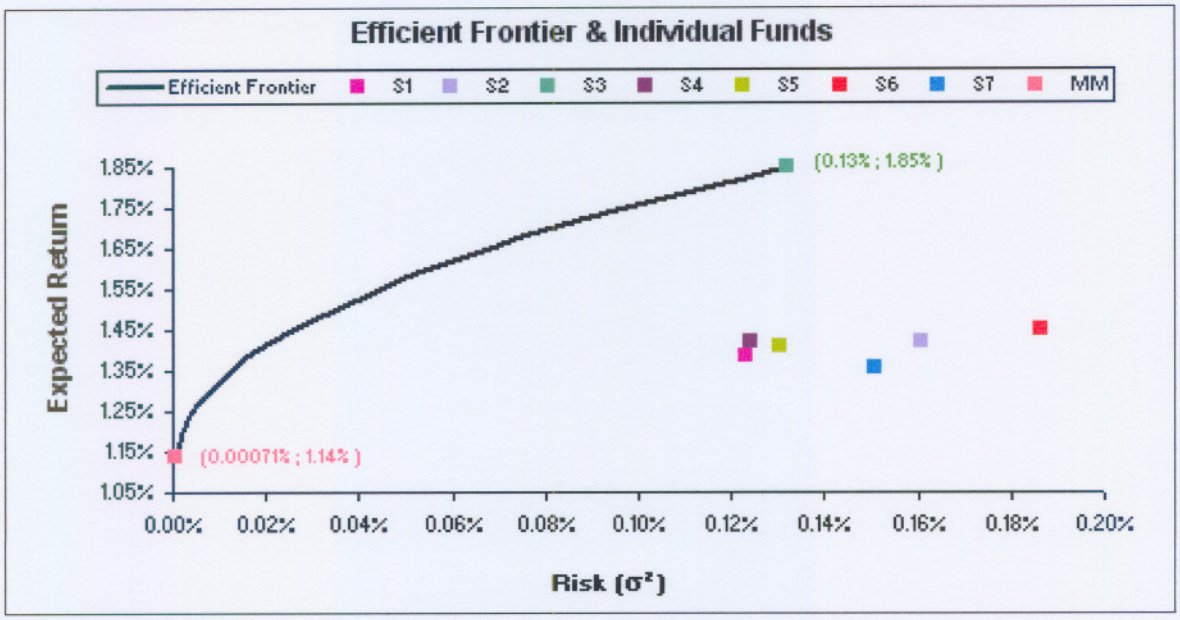
return. Repetition of this procedure for many return levels generates the efficient frontier (Christopher & Mensink, 1999:108). For example, if all the obtained historical data from July 1985 to December 2003 (with the MM included) are considered the fund allocation will be as follows:



Graph 2.1: Fund allocation for different return targets

15 different return targets are used, that range from 1.15% to 1.85%. S_3 and MM make up the portfolios. The percentage of the portfolio allocated to S_3 increases as the target return increases. All the funds are allocated to S_3 for the 1.85% target return.

If we solve the Markowitz portfolio for 15 different return targets (see graph 2.1 above) the efficient frontier, shown in graph 2.2, will be obtained. The concave line illustrated on the graph, is the efficient frontier. As the expected return increases, the risk (as measured by the variance) also increases. Again, as we saw in graph 2.1, the portfolios will consist of allocations made to S_3 and the MM fund. Points inside the frontier are inefficient because less return for the risk (high risk) is achieved. Points outside the frontier are more desirable, but are not obtainable (i.e. it is infeasible, as it is high returns at low risk). Thus, an increase in expected return goes hand in hand with an increase in risk (Christopher & Mensink, 1999:108).



Graph 2.2: Efficient frontier

2.3.1.2 Risk/reward analysis

- **The return to risk ratio**

The return to risk ratio is the ratio of average historical returns to the standard deviation of the manager's returns, where the sample average of the return on portfolio j is divided by the standard deviation of portfolio j's returns over the chosen period (Christopher & Mensink, 1999:110):

$$\frac{\overline{R}_j}{\sigma_j}$$

For example, the following risk/return analysis appeared in Australia Unity's product report as at 31 March 2004 (Australian Unity, 2004):

Measure	1 year	3 years
Total return (%)	9.75	5.82
Standard deviation (%pa)	1.68	2.09
Risk/Reward ratio	5.82	2.79

Table 2.1: Return/risk ratio calculations

Thus, the risk/return ratio is a measure of the return earned per unit of risk taken.

- **The coefficient of variation**

If two investments have the same expected returns but different standard deviations, most investors would choose the investment with the lower standard deviation (which implies lower risk). Likewise, if the two investments had the same standard deviations but different expected returns, investors would choose the investment with the higher return. However, a problem arises when investors have to choose between the following two alternatives: one investment has the higher expected return and the other investment has the lower standard deviation. For example, consider the return distributions that are illustrated in Figure 2.1.

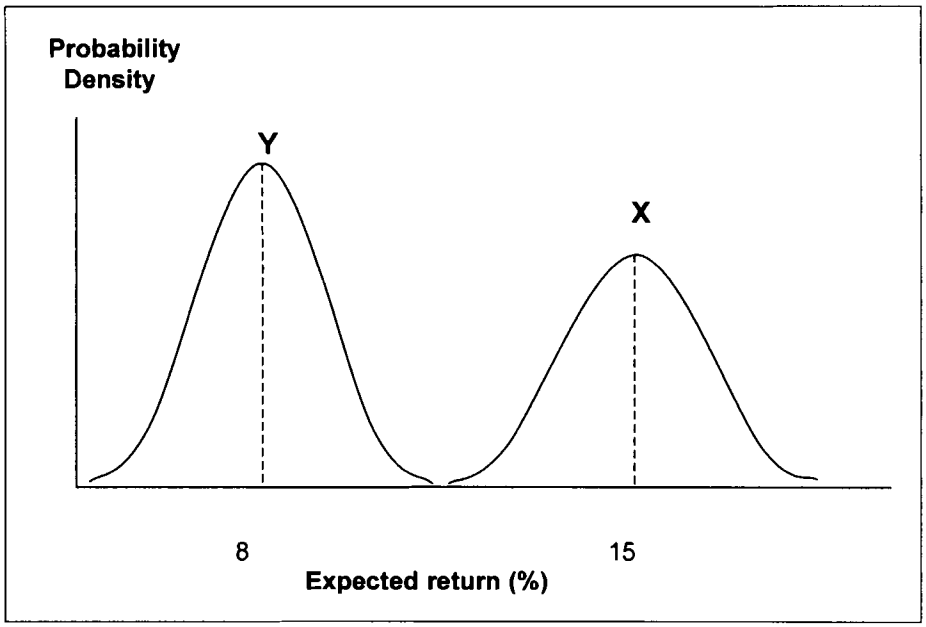


Figure 2.1: Distribution of returns

To choose between the two investment options, some basis of comparison is needed. The coefficient of variation (CV) can be used. Correia *et al.* (2003: 3-14) describes that the coefficient of variation relates the units of return to the units of risk. The coefficient of variation provides a meaningful basis of comparison when the expected returns of two (or more) alternatives are different. The units of risk per 1% of return are expressed as follows:

$$CV = \frac{\sigma_j}{\overline{R}_j}$$

Table 2.2 shows the CV calculations for the two investments from the example above (Brigham & Ehrhardt, 2002:208):

Measure	X	Y
Expected return (%)	15	8
Standard deviation (%)	3.75	3
CV	0.25	0.375

Table 2.2: Coefficient of variation calculations

Investment Y will be riskier than investment X (1.5 times more risky). Thus, even though investment Y has the lower standard deviation, it is riskier than investment X, according to the coefficient of variation.

2.3.2 β as a measure of risk

Beta (β) measures the volatility of a stock/fund relative to the overall market (measured by some index, e.g. ALSI). β is calculated as follows (Christopher & Mensink, 1999:102):

$$\beta = \frac{Cov\,(R_j, R_m)}{\sigma^2_m}$$

R_j is the expected return on stock *j*, R_m is the expected return on the market and σ^2_m is the variance of the market return. If $\beta < 1$ it implies that the stock is less volatile than the overall market; if $\beta = 1$ then the stock is as volatile as the overall market and if $\beta > 1$ then the stock is more volatile than the overall market.

2.3.3 Downside risk

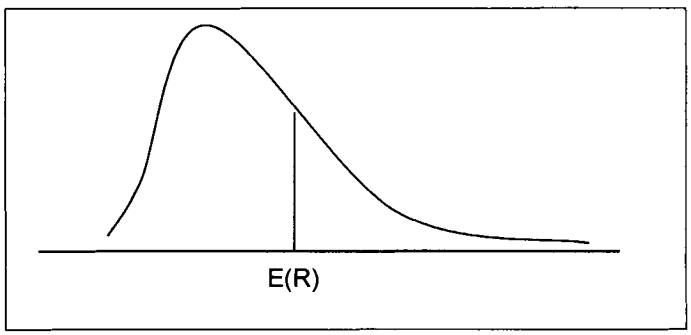
This is a popular measure of risk when portfolio returns are asymmetric. This summary of the total risk of a portfolio, is basically the risk that portfolio returns will fall below some specific target return level (Christopher & Mensink, 1999:102):

$$\int_{-\infty}^{T} (T - R)^z \, f_{j,t}\,(R)\, dR$$

T is the prespecified target return and z is a parameter for the particular risk statistic chosen.

Figures 2.2 and 2.3 display skewed probability distributions, which are mirror images of each other. They have equal means and variances but different downside risk. Thus, although the distributions have the same standard deviation, they have different levels of risk. A risk-averse investor would prefer the right skewed distribution (Clark, 1994: 50).

- **Probability distribution skewed to the right:**

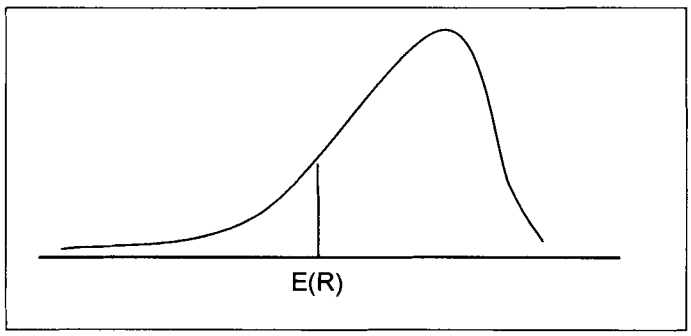


Characteristics:

- Less likely, but larger gains.
- More likely, but smaller losses.
- *Bad surprises* – more likely, but potentially smaller in magnitude.

Figure 2.2: Right skewed distribution

- **Probability distribution skewed to the left:**



Characteristics:

- More likely, but smaller gains.
- Less likely, but larger losses.
- *Bad surprises* – less likely, but potentially larger in magnitude.

Figure 2.3: Left skewed distribution

2.3.3.1 Below-target probability

When z = 0, the downside risk measure is called the below-target probability (BTP), which simply measures the probability of a shortfall in returns below some target T. This measure is criticized because it shows how likely returns are to fall below a chosen target, but it doesn't show the extent to which return shortfalls have occurred. The formula is given by (Christopher & Mensink, 1999:102):

$$BTP = \int_{-\infty}^{T} f_{j,t}(R)\, dR$$

2.3.3.2 Below-target variance

The above-mentioned problem is overcome by focusing on downside risk when z = 2. This new measure, is a measure of total risk called the below-target variance (BTV) or downside semivariance, which is represented by the following:

$$BTV = \int_{-\infty}^{T} (T - R)^2 f_{j,t}(R)\, dR$$

It measures the risk of a return shortfall below the target and gives more weight in the calculation of larger shortfalls. BTV's standard deviation is a measure called below-target risk (BTR). If the return distribution is symmetric however, the BTV is related to traditional portfolio variance. The sample statistic for BTV, for a time series with N observations is (Christopher & Mensink, 1999:103):

$$BTV = \left(\frac{1}{N+1}\right) \sum_{t=1}^{N} [\text{Max}\,(T - R_t, 0)]^2$$

2.3.3.3 Lower partial moment

The lower partial moment (LPM) measures the downside risk below some target return. The term *partial* indicates that the measure relates to only one side of the return distribution, relative to a target level. *Lower* implies that the side of interest is the downside, where most investors are the most sensitive to volatility. The LPM, for stock/fund *i* and for degree *n* is (Nawrocki, 2003:93):

$$LPM_{in} = \left(\frac{1}{m}\right) \sum_{t=1}^{m} [\text{Max}\ (0, T - R_t)]^n$$

Where

LPM_{in} = the lower partial moment for fund *i* and for degree *n*

t = 1, 2, ..., m observations

T = pre-specified target return

Max = maximisation function that selects the larger of the two values

R_t = return on the fund during time period t.

- **Range**

$LPM_{in} = (T - R_t)^n \Rightarrow$ over the range where $R_t < T$

$= 0 \Rightarrow$ over the range where $R_t \geq T$

- **Degree of n**

- If $n < 1 \Rightarrow$ the investor is a risk seeker.
- If $n = 1 \Rightarrow$ the investor is risk neutral. This is also called the expected shortfall.
- If $n > 1 \Rightarrow$ the investor is averse to risk. When n = 2, the measure is called semivariance.

2.3.4 Value at Risk

Embrechts (2000) describes Value at Risk (VaR) as a statistical estimation of a portfolio loss with the property that, with a given (small) probability, we can suffer that loss or more over a given holding period (which is typically short).

E.g. A 10-day, 99% VaR of R 1000 000

If we keep our portfolio position fixed for 10 days, there is a 1% probability that we experience a R1000000 or more loss, by the end of the 10 days. Also, there is a 99% chance that our loss will be bounded by R1000000.

2.4 Sources of investment risk

Clark (1994: 45) indicates that the most common sources of investment risk are: financial exposure to changes in interest rates, equity markets, inflation, foreign exchange rates, credit quality and commodity prices.

2.5 Risk management

Chong (2004: 10) defines risk management as the study and practice that offers some answers for choices in the financial markets.

2.5.1 The risk management process

Effective risk management (i.e. the formal measurement and control of risk) involves the following Clark (1994: 45):

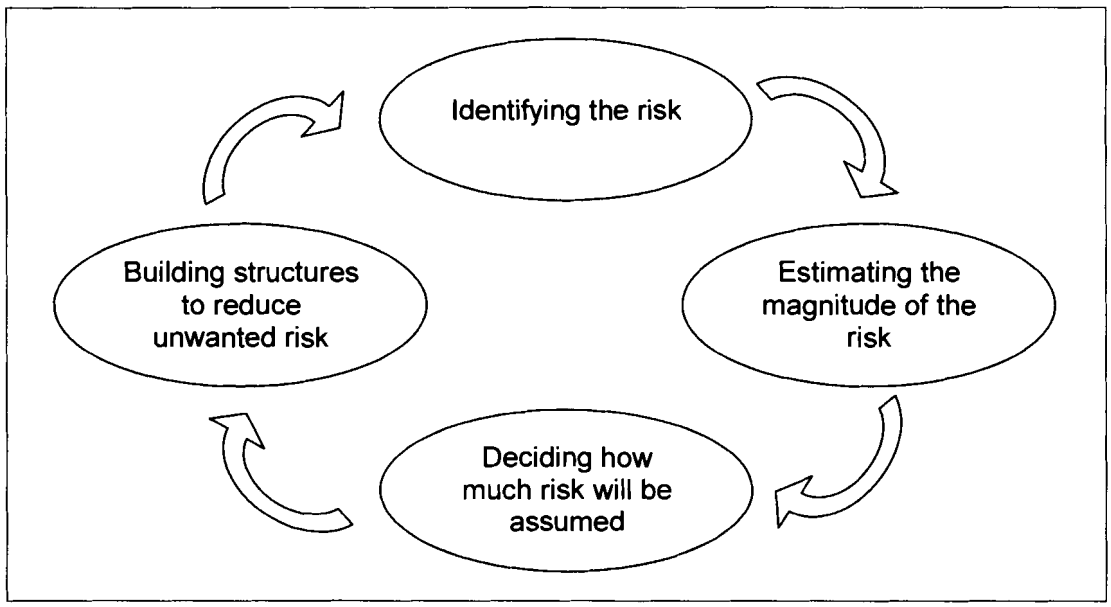


Figure 2.4: Risk management process

According to Christopher & Mensink (1999:126), any measure of risk that is based on historical returns on an actively managed portfolio is appropriate for performance evaluation, but not risk measurement for risk control. Historical returns on an actual actively managed portfolio are *ex post* measures of risk and may have no bearing on the risk of a position held from today through tomorrow. Risk measures used for risk management and control purposes may also be based on historical data, which lies at the core of any statistical inference problem, but that data, unlike data for performance evaluation, is tactics neutral. It should not and cannot depend on tactical management decisions, but should rather be confined to neutral market data about the risk inherent in instruments themselves. *Ex post*

measured risk helps determine how effective the other types of risk measures were, based on actual data.

Perhaps the best way to distinguish between measures of risk used for performance evaluation and those used for risk-control decisions, therefore, is to follow this rule: When evaluating performance, measure the risk of the manager based on his or her actual past performance; when evaluation risk going forward, measure the risk of the instruments currently held based on their actual past performance, regardless of who was trading them.

Risk measurement for asset allocation and risk management differ with regard to the following (Christopher & Mensink 1999:125):

- Asset allocation focuses on risk measurement exclusively at the level of asset classes, whereas most applications of VaR or BTR involve portfolio-specific or security-specific information. Returns on asset classes are usually sufficient to solve the asset allocation problem, but that is not always sufficient to engage in the monitoring of risk for risk-control purposes.
- The time horizon for the monitoring of trading activities is often much shorter, whereas asset allocation and rebalancing is costly and time consuming, e.g. large pension plans often have annual asset allocation, with quarterly or monthly rebalancing horizons.
- In most asset allocation problems, risk is solely defined as variance, because of the dominance of the Markowitz mean-variance portfolio optimisation paradigm. VaR and BTR can be calculated without optimisation for risk control and monitoring purposes, but for use in the efficient frontier, it is more resource intensive and unmanageable. As these measures are not necessarily variance based, it offers additional insights about downside risks that may not be sufficiently captured in a variance statistic.

2.5.2 Strategies for risk management

The key to dealing with investment risk is learning how to manage it. The following section presents a number of methods that investors can apply to manage risk:

2.5.2.1 Diversification

Investing in only one stock is usually more risky than investing in several stocks. By investing in different stocks in several industries, investors limit their exposure to a significant loss due to a price decline in only one stock (AXA Advisors, 2005).

2.5.2.2 Appropriate asset allocation

This refers to how an investor spread his portfolio among different types of investments, for instance stocks, bonds and money market funds. According to Howell (2002: 207) diversification across different asset classes is the accepted means of risk reduction.

2.5.2.3 Weathering market fluctuations

AXA Advisors (2005) states that staying invested through periods of market instability can also help manage risk or loss, as variability of returns tends to decrease over time.

2.5.2.4 Hedging

Webfinance (2005) describes hedging as an investment made in order to reduce the risk of adverse price movements in a security, by taking an offsetting position in a related security, such as an option or a short sale.

- **Option**

This is the right, but not the obligation, to buy or sell a specific amount of a given stock, commodity, currency or index at a specified price (the strike price) during a specified period of time. For stock options, the amount is usually 100 shares. Each option has a buyer, called the holder, and a seller, known as the writer. If the option contract is exercised, the writer is responsible for fulfilling the terms of the contract by delivering the shares to the appropriate party. For the holder, the potential loss is limited to the price paid to acquire the option. When an option is not exercised, it expires. No shares change hands and the money spent to purchase the option is lost. For the buyer, the upside is unlimited. For the writer, the potential loss is unlimited unless the contract is covered, meaning that the writer already

owns the security underlying the option. Options are most frequently as either leverage or protection. As leverage, options allow the holder to control equity in a limited capacity for a fraction of what the shares would cost. The difference can be invested elsewhere until the option is exercised (Investor words, 2005).

- **Short sale**

This refers to borrowing a security from a broker and selling it, with the understanding that it must later be bought back (hopefully at a lower price) and returned to the broker. Short selling is a technique used by investors who try to profit from the falling price of a stock. For example, consider an investor who wants to sell short 100 shares of a company, believing it is overpriced and will fall. The investor's broker will borrow the shares from someone who owns them with the promise that the investor will return them later. The investor immediately sells the borrowed shares at the current market price. If the price of the shares drops, he/she "covers the short position" by buying back the shares, and his/her broker returns them to the lender. The profit is the difference between the price at which the stock was sold and the cost to buy it back, minus commissions and expenses for borrowing the stock. But if the price of the shares increases, the potential losses are unlimited. The company's shares may go up and up, but at some point the investor has to replace the 100 shares he/she sold. In that case, the losses can mount without limit until the short position is covered. For this reason, short selling is a very risky technique (Investor words, 2005).

2.6 Linear programming in portfolio selection: case study

The following is a case study of asset allocation done in linear programming. This example appeared (Render *et al.*, 2003:301) as an example of linear programming as a financial application and is used as an illustration of the implementation of portfolio selection in linear programming (it was executed with QM for Windows).

Specific investments are selected from a large variety of alternatives. The objective is to maximize the expected return on the investment, given a set of risk constraints to which it is subjected.

For example a company invests in short-term trade credits, corporate bonds, gold stocks and construction loans. The board of directors has placed limits on the amount that can be committed to any one type of investment in order to encourage a diversified portfolio. R5000000 is available for investment and the company wants to achieve the following:

- Maximize the interest earned on the investments over the next six months, and
- Satisfy the diversification requirements that were set by the board of directors.

The specifics of the investment possibilities are as follow:

INVESTMENT	INTEREST EARNED (%)	MAX INVESTMENT (R millions)
Trade credit	7	1.0
Corporate bonds	11	2.5
Gold stocks	19	1.5
Construction loans	15	1.8

Table 2.3: Data for linear programming example

In addition, at least 55% of the funds invested must be in gold stocks and construction loans, and no less than 15% should be invested in trade credit.

The problem is formulated as follows:

X_1 is the amount invested in trade credit
X_2 is the amount invested in corporate bonds
X_3 is the amount invested in gold stocks
X_4 is the amount invested in construction loans

Objective:

Maximize interest earned = $0.07X_1 + 0.11X_2 + 0.19X_3 + 0.15X_4$

Subject to:

$X_1 \leq 1000000$

$X_2 \leq 2500000$

$X_3 \leq 1500000$

$X_4 \leq 1800000$

$X_3 + X_4 \geq 0.55(X_1 + X_2 + X_3 + X_4)$

$X_1 \geq 0.15(X_1 + X_2 + X_3 + X_4)$

$X_1 + X_2 + X_3 + X_4 \leq 5000000$

	X1	X2	X3	X4		RHS
Maximize	0.07	0.11	0.19	0.15		
Maximum investment in trade credit	1	0	0	0	<=	1000000
Maximum investment in corporate bonds	0	1	0	0	<=	2500000
Maximum investment in gold stocks	0	0	1	0	<=	1500000
Maximum investment in construction loans	0	0	0	1	<=	1800000
Min 55% invested in gold stocks & construction loans	-0.55	-0.55	0.45	0.45	>=	0
Min 15% invested in trade credit	0.85	-0.15	-0.15	-0.15	>=	0
Maximum amount to invest	1	1	1	1	<=	5000000
Solution->	**750000.1**	**950000**	**1500000**	**1800000**		**R712000**

Table 2.4: Output for linear programming example

The output in table 2.4 shows that the company should make the following investment to maximize its interest earned: X_1 = R750000, X_2 = R950000, X_3 = R1500000 and X_4 = R1800000, and the total interest earned will be R712000.

2.7 Conclusion

This chapter examined some measures of risk, sources of risk and classification of risk. The process of risk management was portrayed as well some methods for risk management. The investment management process can be viewed as a cycle in which asset allocation leads to performance evaluation, performance evaluation leads to risk management, and risk management leads to asset allocation.

CHAPTER 3
RESULTS OF STRATEGIES NOT USING RISK MEASURES IN THE DECISION MAKING

3.1 Introduction

In decision making under uncertainty, there are a number of possible outcomes for each alternative, and the decision maker does not know the probabilities of the various outcomes (Render *et al.*, 2003: 81). Risk measures are therefore not used in these investment strategies.

All the models without the MM fund included (except for perfect information) performed better; even before transaction costs were considered. Hence, only models without the MM are demonstrated in this chapter. This chapter presents the investment growth and spread of the obtained returns for the different decision making under uncertainty strategies.

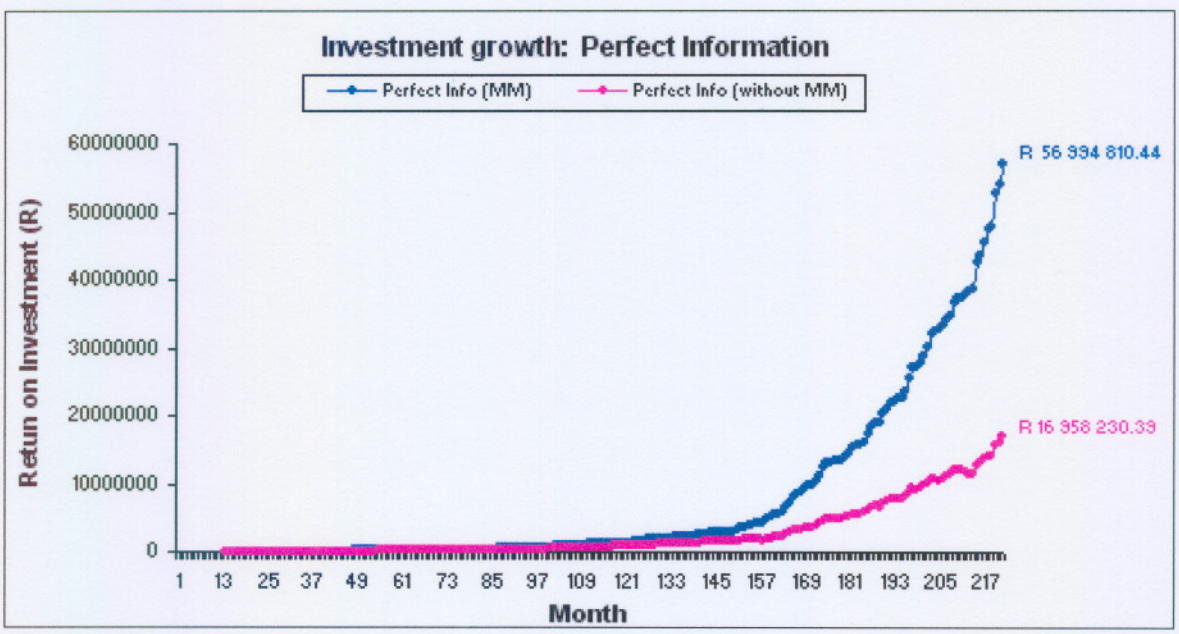
3.2 Perfect information

Perfect information implies that we know in advance what event will occur. Although we never have perfect information in practice, it is worth knowing how much we could improve the value of our decision if we had such information (Camm & Evans, 2000:265). Two perfect information strategies are applied:

- *Perfect information with MM* - Perfect information that includes the MM fund is obtained by selecting the fund that maximises the outcome for the current month. Thus, transaction costs are taken into consideration, to see whether it is worthwhile to make the switch from MM to a dividend fund or vice versa.
- *Perfect information without MM* - Perfect information that excludes the MM fund is obtained by selecting the fund with the maximum return for the present month.

3.2.1 Investment growth

The following graph shows a comparison between the two perfect information strategies. The two strategies grow concurrently (until around the 28th month). Then the perfect information strategy (with MM) begins to move above the perfect information strategy (without MM), i.e. grow at a faster rate. The perfect information (with MM) strategy's total growth is more than double the total growth of the perfect information (without MM) strategy.



Graph 3.1: Comparison between perfect information strategies

The perfect information (with MM) strategy shows an effective monthly interest rate of 3.43% and the perfect information (without MM) strategy shows an effective interest rate of 2.83% per month. The following formula is used to calculate the effective monthly interest rate (r):

$$FV = PV(1+r)^n$$

$$\therefore r = \left(\frac{FV}{PV}\right)^{\frac{1}{n}} - 1$$

Where

PV = the initial amount that is invested (present value)

FV = the final amount (final value)

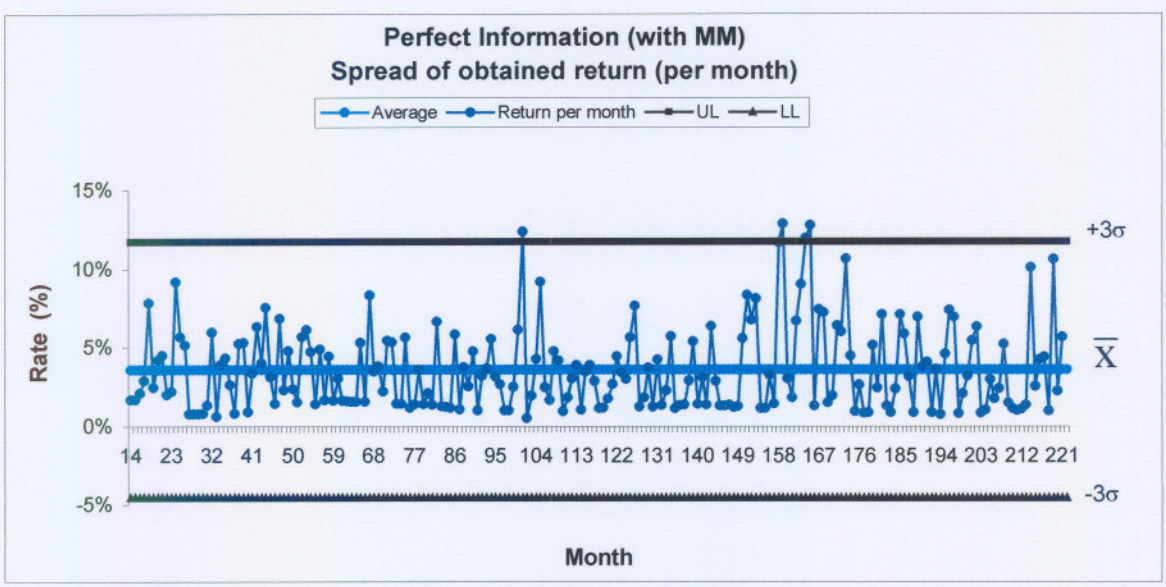
n = nr of months the money is invested

3.2.2 3-Sigma rule

The 3-sigma rule will be used to identify outliers (i.e. return values that is more than three standard deviations either below or above the mean).

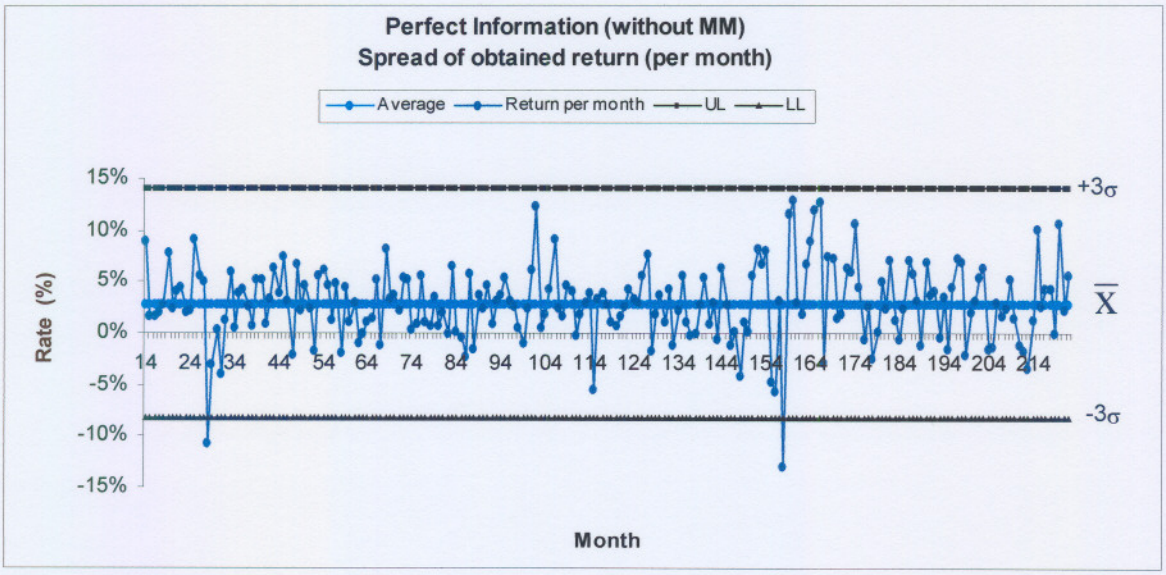
- **Perfect information with MM**

Graph 3.2 illustrates that none of the return values are below the lower limit (-4.54%) but 1.91% of the return values are above the upper limit (11.70%). 32.54% of the return values obtained with this strategy were below the target return of 1.60%. The average return that is achieved is 3.58% per month and the strategy's return/risk ratio is 1.3235. The return/risk ratio (return to risk ratio) is a measure of the return earned per unit of risk taken. The higher the value, the better. For instance, if an investor is willing to take some risk, a higher return for the amount of risk taken will be a more attractive (will make the risk taken meaningful). The standard deviation is 2.71%.



Graph 3.2: 3-Sigma rule for perfect information with MM

- **Perfect information without MM**



Graph 3.3: 3-Sigma rule for perfect information without MM

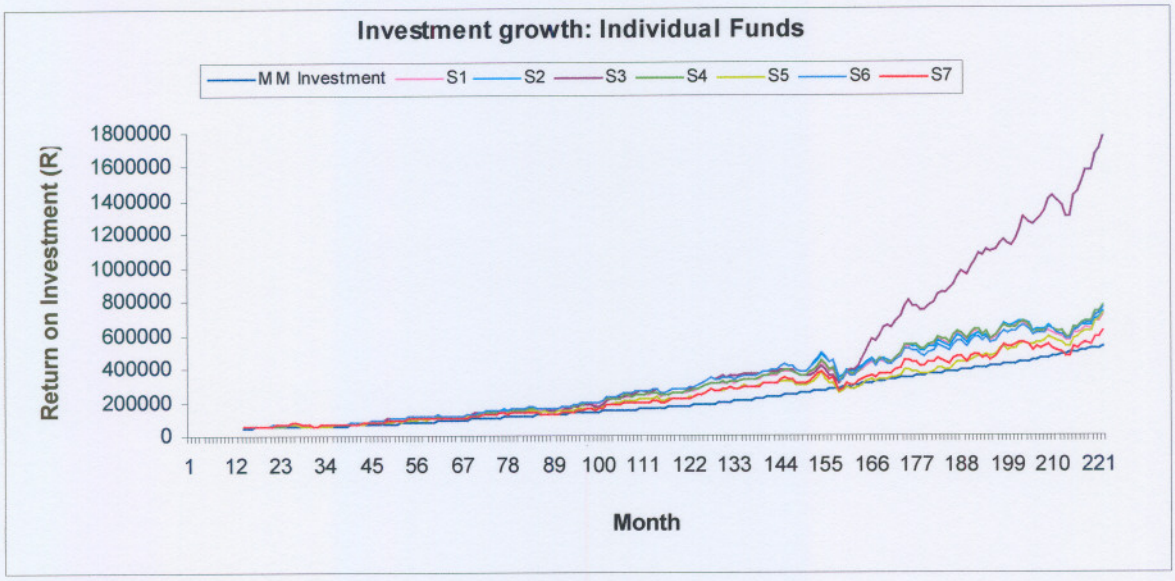
According to graph 3.3, 0.96% of the return values are below the lower limit (-8.22%) but none of the return values are above the upper limit (14.01%). 33.97% of the return values obtained with this strategy were below the target return of 1.60%. The average return is 2.89% per month and the return/risk ratio for this strategy is 0.7809. The standard deviation is 3.71%.

3.3 Investment in the individual funds

The following sections provide an overview of what the individual funds achieved. Thus, what is the outcome, if an investor invested in one fund over the entire investment period?

3.3.1 Investment growth

Graph 3.4 shows a comparison of the investment growths for the different funds. All the individual funds (from S_1 to S_7) performed better than the MM fund. From around the 165th month the S_3 fund investment grows at a much faster rate than the other funds. S_3 has the maximum investment growth and MM the minimum (see table 3.1 for specifics).



Graph 3.4: Comparison between individual funds

Table 3.1 displays the investment growth at the end of the investment period and the effective interest rate, for all eight the individual funds. The effective interest rate that is achieved per month, is determined by the following formula:

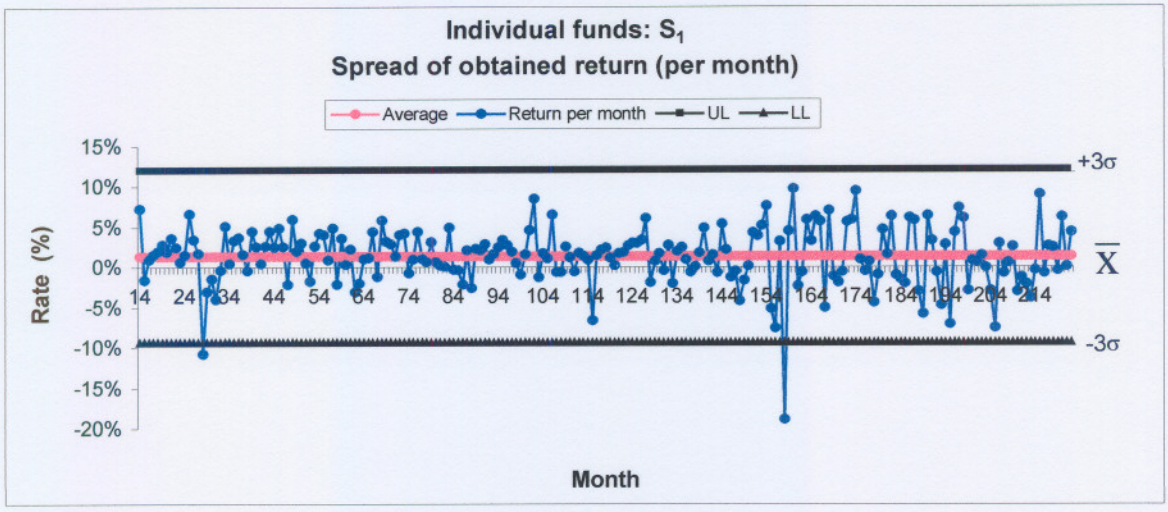
$$r = \left(\frac{FV}{PV}\right)^{\frac{1}{n}} - 1$$

Fund	Investment growth (R)	Effective interest rate (% per month)
S_1	R 705 741.01	1.27%
S_2	R 738 895.92	1.30%
S_3	R 1 765 255.32	1.72%
S_4	R 765 692.54	1.31%
S_5	R 722 140.53	1.29%
S_6	R 761 558.57	1.31%
S_7	R 622 990.30	1.21%
MM	R 524 554.48	1.13%

Table 3.1: Investment growth for individual funds

3.3.2 3-Sigma rule

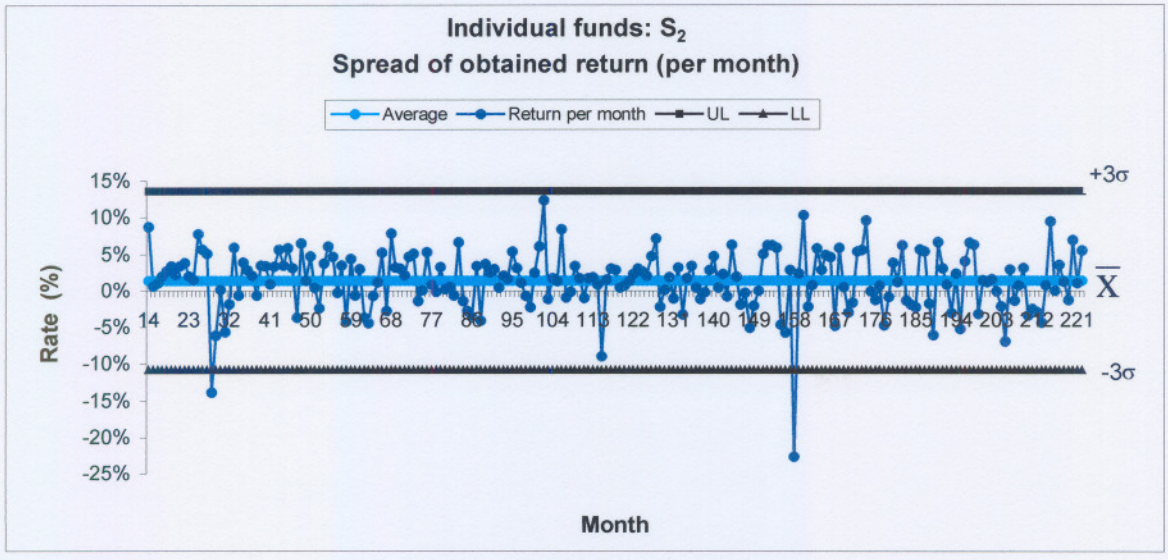
- **Fund S_1**



Graph 3.5: 3-Sigma rule for S_1

According to graph 3.5, 0.96% of the return values are below the lower limit (-9.36%) but none of the return values are above the upper limit (12.04%). 51.20% of the return values are below the target return of 1.60%. The average return is 1.34% per month and the return/risk ratio for this fund is 0.3754. The standard deviation is 3.57%.

- **Fund S_2**

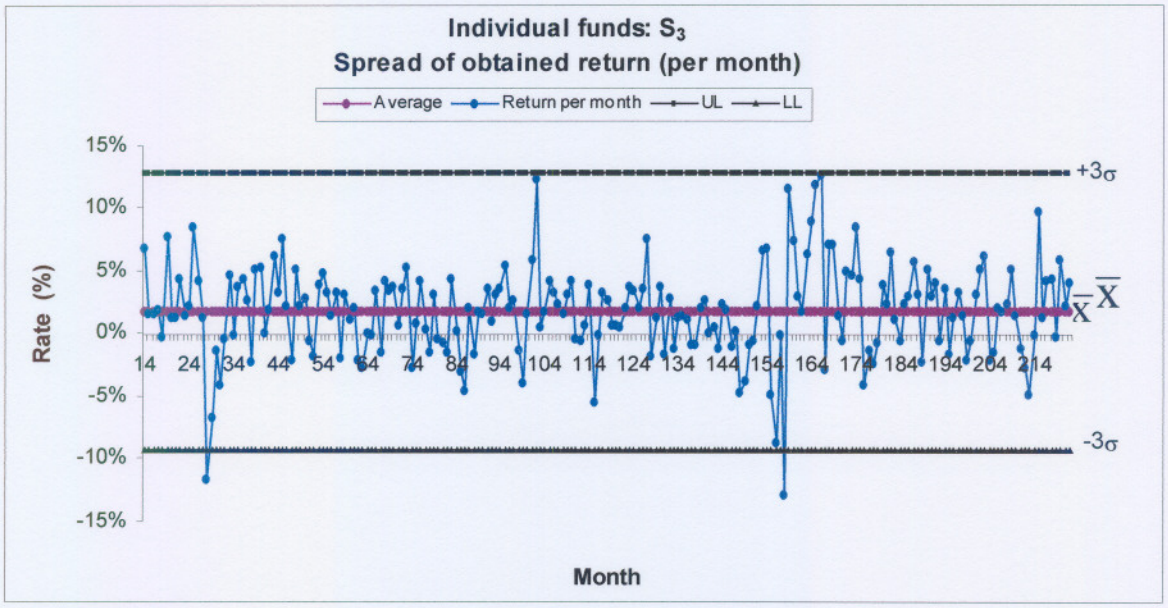


Graph 3.6: 3-Sigma rule for S_2

Only two return values (0.96%) are below the lower limit (-10.86%) and no return values are above the upper limit (13.62%). Less than half (49.28%) of the return values were below the target return of 1.60%. The average return is 1.38% per month and the return/risk ratio for this fund is 0.3387. The standard deviation for this strategy is 4.08%.

- **Fund S_3**

Graph 3.7 illustrates that 0.96% of the return values are below the lower limit (-9.24%) and no return values are above the upper limit (12.81%). S_3 has the lowest percentage of return values below the 1.60% target return, i.e. 45.93%. S_3 also attains the highest average return of 1.79% per month and the return/risk ratio for this fund is 0.4861. The standard deviation for this strategy is 3.68%.

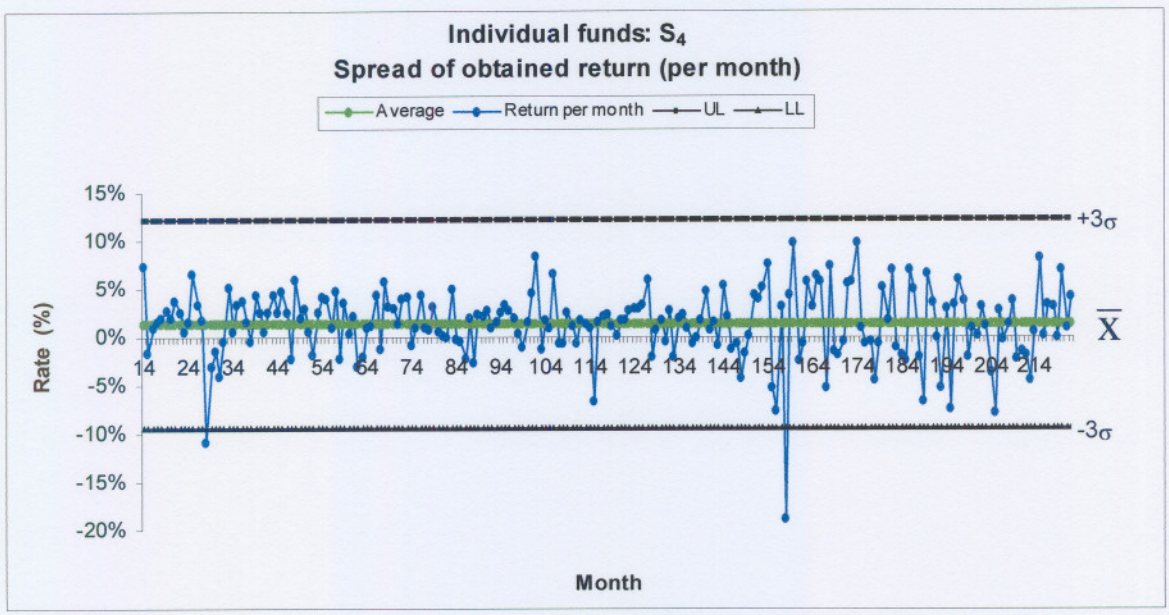


Graph 3.7: 3-Sigma rule for S_3

- **Fund S_4**

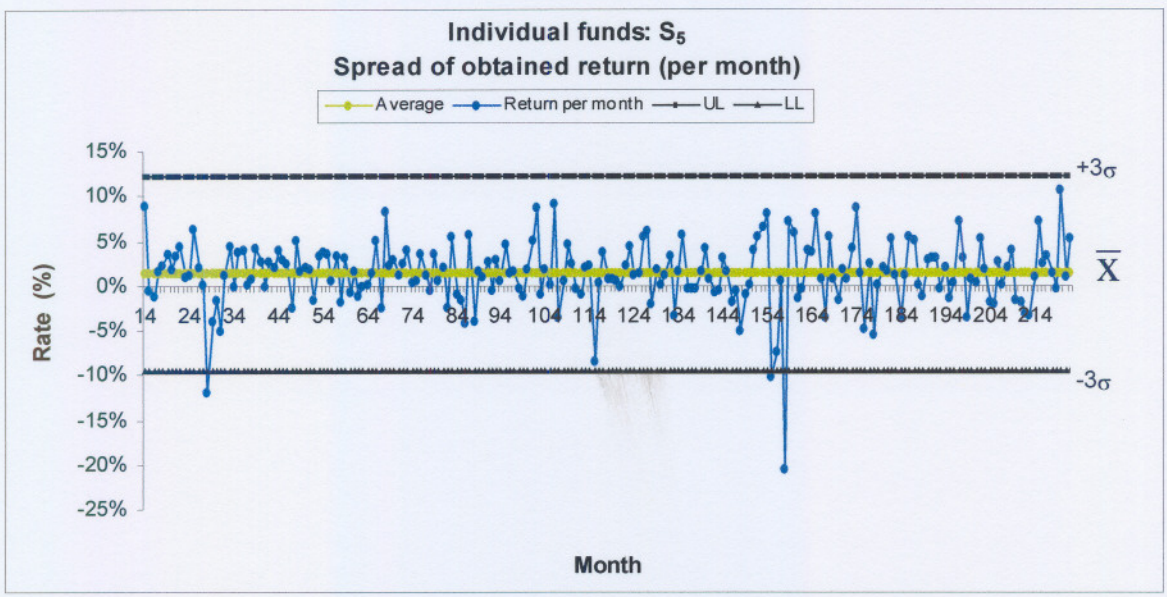
According to graph 3.8, 0.96% of the return values are below the lower limit (-9.35%) and none of the return values are above the upper limit (12.11%). More than half of the return values are below the 1.60% target return, namely 50.24% of the values. S_4 obtains an

average return of 1.38% per month and the return/risk ratio for this fund is 0.3854. The standard deviation for this strategy is 3.58%.



Graph 3.8: 3-Sigma rule for S_4

- **Fund S_5**

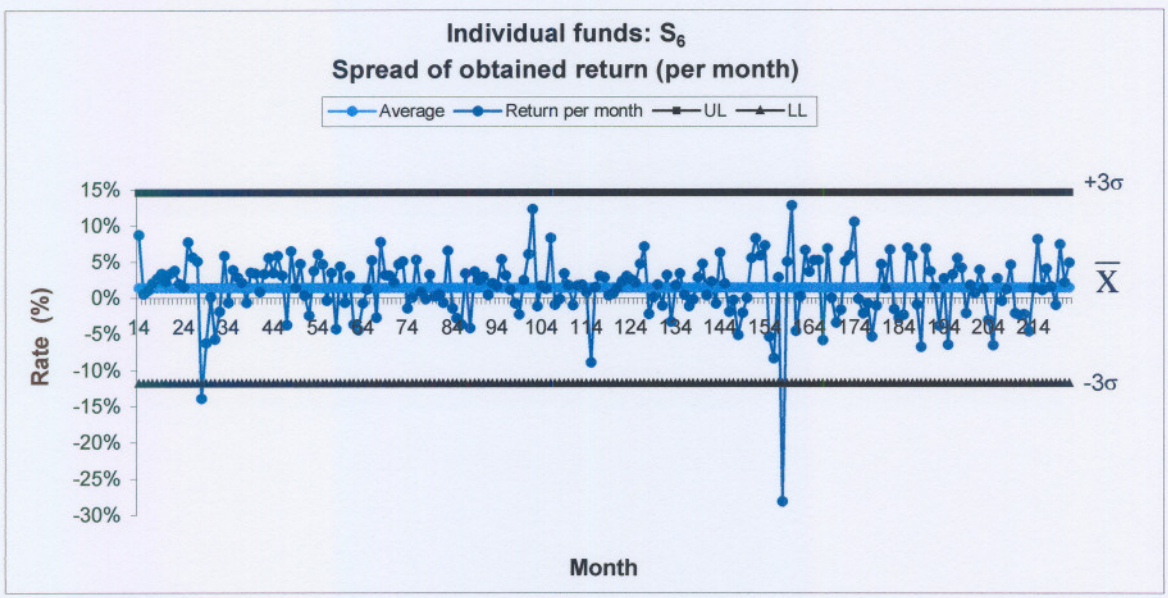


Graph 3.9: 3-Sigma rule for S_5

Graph 3.9, depicts that 1.44% of the return values are below the lower limit (-9.60%) and none of the return values are above the upper limit (12.31%). 51.20% of the return values are below the target return of 1.60%. The average return is 1.35% per month and the return/risk ratio for this fund is 0.3707. The standard deviation for this strategy is 3.65%.

- **Fund S_6**

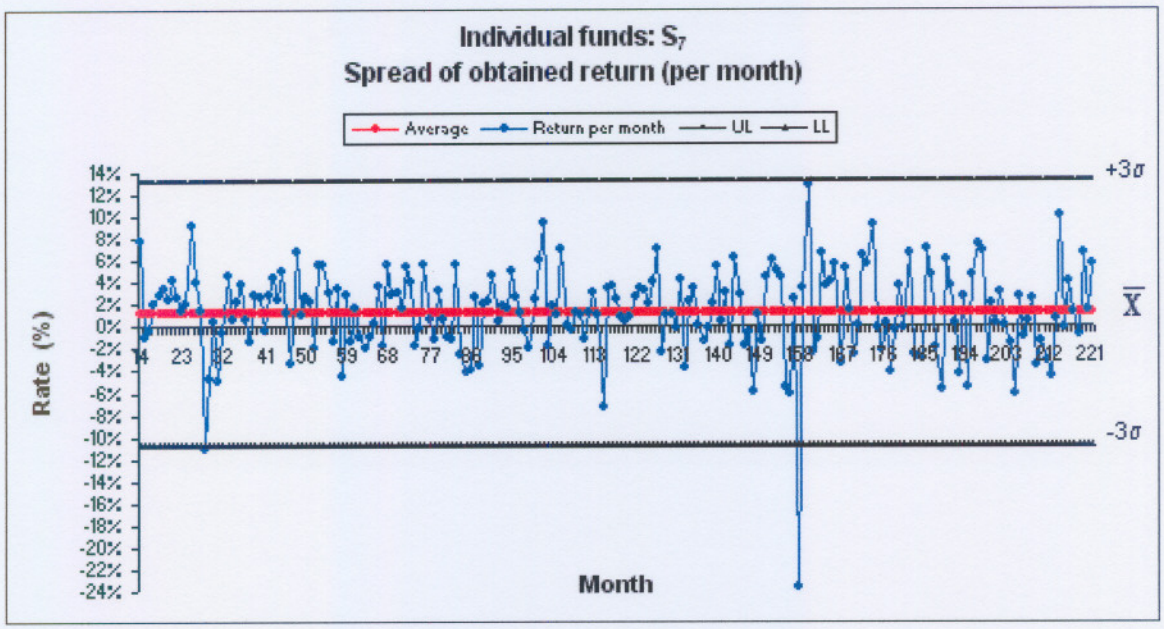
Graph 3.10 illustrates that 0.96% of the return values are below the lower limit (-11.79%) and no return values are above the upper limit (14.62%). 48.33% of the return values are below the 1.60% return target. S_4 attains the second highest average return of 1.41% per month and the return/risk ratio for this fund is 0.3209. The standard deviation is 4.40%.



Graph 3.10: 3-Sigma rule for S_6

- **Fund S_7**

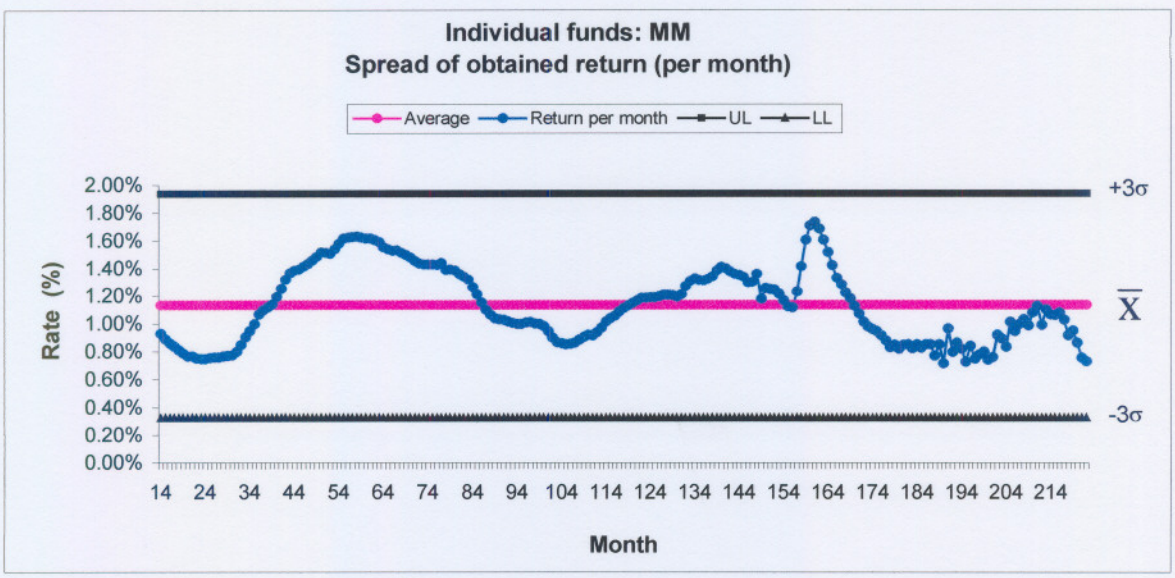
According to graph 3.11, 0.96% of the return values are below the lower limit (-10.58%) and 0% of the return values are above the upper limit (13.17%). More than half of the return values are below the 1.60% target return, namely 51.20% of the values. The average return for S_7 is of 1.29% per month and the return/risk ratio for this fund is 0.3270. The standard deviation for this strategy is 3.96%.



Graph 3.11: 3-Sigma rule for S_7

- **MM fund**

Graph 3.12, depicts that none of the return values are below the lower limit (0.33%) and none of the return values are above the upper limit (1.94%). 93.78% of the return values are below the target return of 1.60%. The MM fund has the lowest average return of 1.13% per month and the return/risk ratio for this fund is 4.2131. The standard deviation is 0.27%.



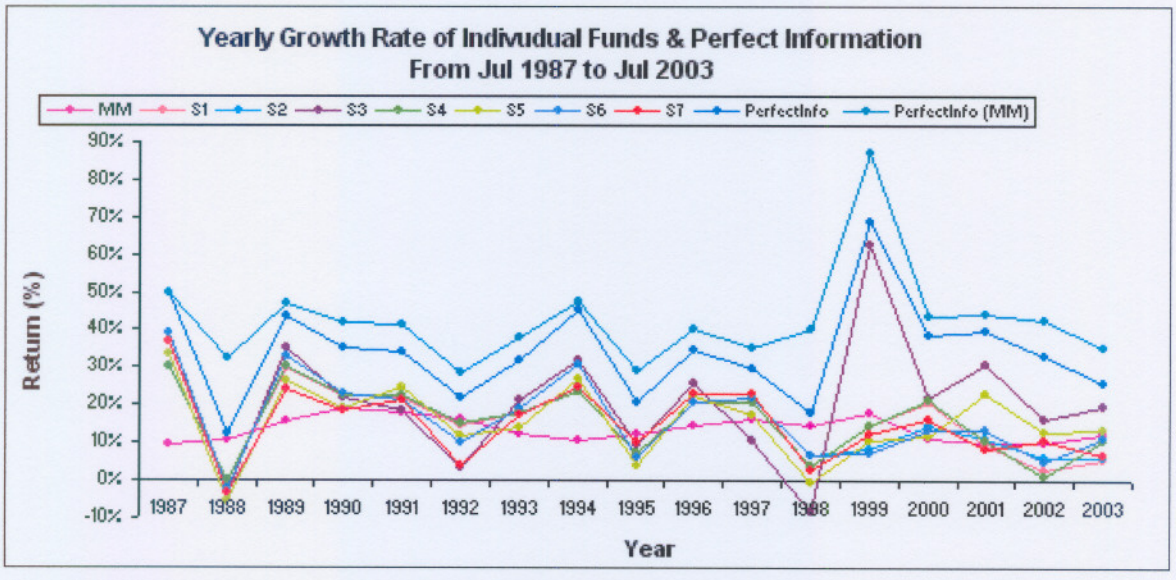
Graph 3.12: 3-Sigma rule for MM

The following table provides a summary of the information obtained from the 3-sigma rule graphs for perfect information and the individual funds ($\overline{X}\Rightarrow$ average return per month, $X \Rightarrow$ returns, UL $\Rightarrow$ upper limit and LL $\Rightarrow$ lower limit):

Fund	$\overline{X}$	X < 1.60% (%)	Return/Risk ratio	X > UL (%)	X < LL (%)
S_1	1.34%	51.20%	0.3754	0.00%	0.96%
S_2	1.38%	49.28%	0.3387	0.00%	0.96%
S_3	1.79%	45.93%	0.4861	0.00%	0.96%
S_4	1.38%	50.24%	0.3854	0.00%	0.96%
S_5	1.35%	51.20%	0.3707	0.00%	1.44%
S_6	1.41%	48.33%	0.3209	0.00%	0.96%
S_7	1.29%	51.20%	0.3270	0.00%	0.96%
MM	1.13%	93.78%	4.2131	0.00%	0.00%
Perfect information (without MM)	3.58%	32.54%	1.3235	1.91%	0.00%
Perfect information (with MM)	2.89%	33.97%	0.7809	0.00%	0.96%

Table 3.2: Summary of perfect information and funds information

3.4 Yearly growth



Graph 3.13: Yearly growth for individual funds and perfect information

Graph 3.13 presents the yearly growth rates for the individual funds and the perfect information strategies. Perfect information shows the highest growth rate, it is obtained in 1999 at 87.06%. S_3 also shows the highest yearly growth rate for the individual funds in 1999, namely 62.63%. The lowest yearly growth rate is obtained by S_3 in 1998, i.e. -8.36%. The following table presents the average yearly growth rates for the perfect information strategies and all the other individual funds.

Fund or strategy	Average yearly growth rate (%)
S_1	15.72%
S_2	16.20%
S_3	20.99%
S_4	16.07%
S_5	15.58%
S_6	16.47%
S_7	15.07%
MM	13.66%
Perfect information (without MM)	34.24%
Perfect information (with MM)	42.63%

Table 3.3: Average yearly growth rates for perfect information and funds

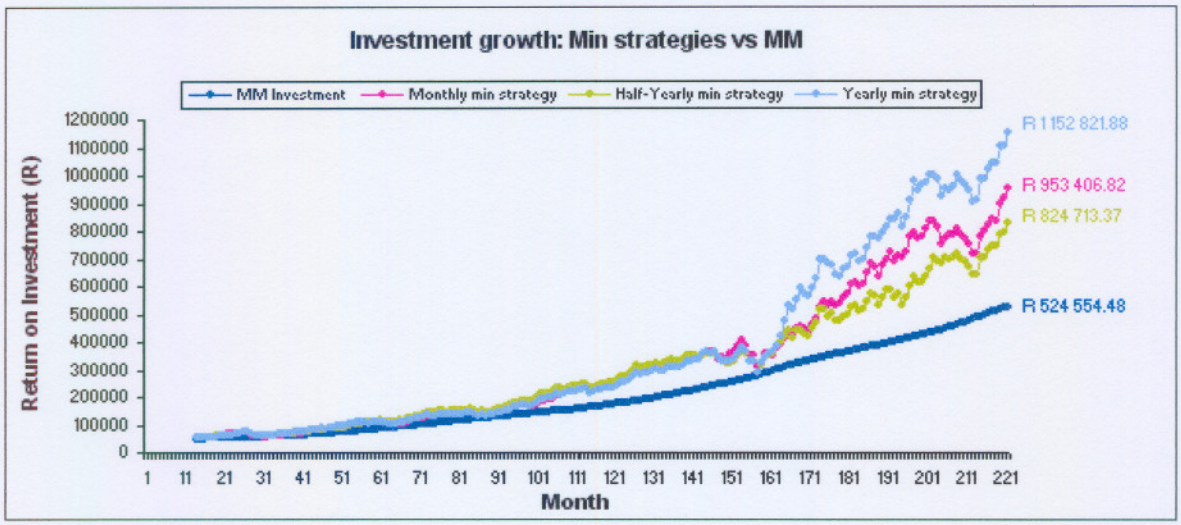
3.5 Min strategy

The min strategy is applied over three different time horizons, which include:

- *Monthly min strategy* - Invest in the fund that obtained the smallest return in the previous month.
- *Half yearly min strategy* - Invest in the fund that obtained the smallest average return over the past six months, for the next six months.
- *Yearly min strategy* - Invest in the fund that obtained the smallest average return over the past year, for the next year.

3.5.1 Investment growth

The following graph shows a comparison between the three min strategies and the MM:

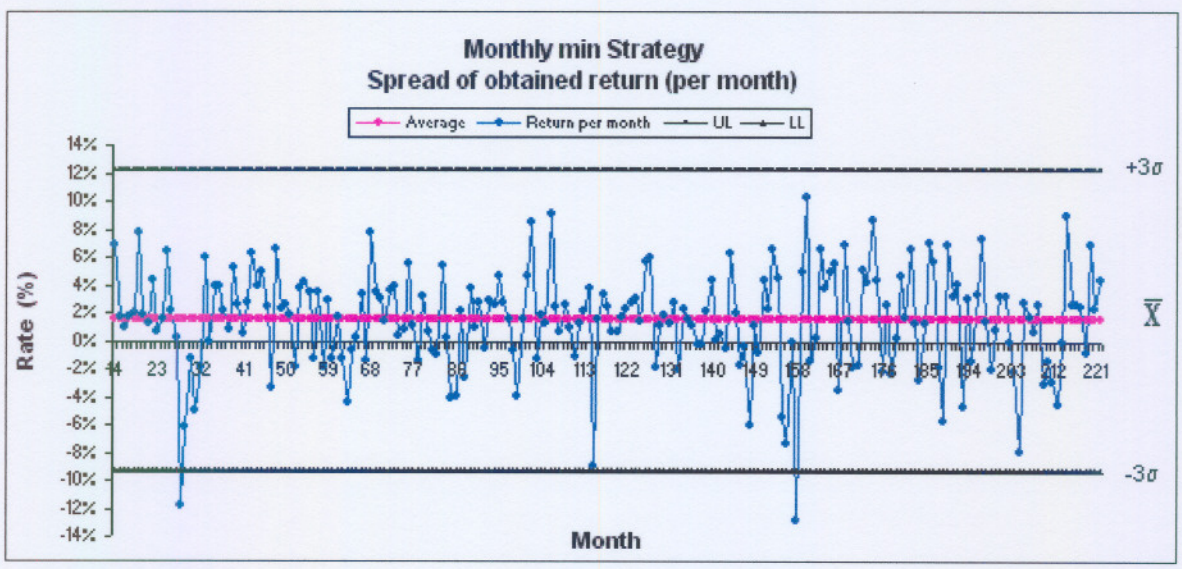


Graph 3.14: Comparison between min strategies

All three the min strategies performed better than the MM fund. The yearly min strategy performed the best, with an effective interest rate of 1.51% per month. The effective monthly interest rate for the monthly min strategy is 1.42% and for the half yearly min strategy it is 1.35%.

3.5.2 3-Sigma rule

- **Monthly min strategy**

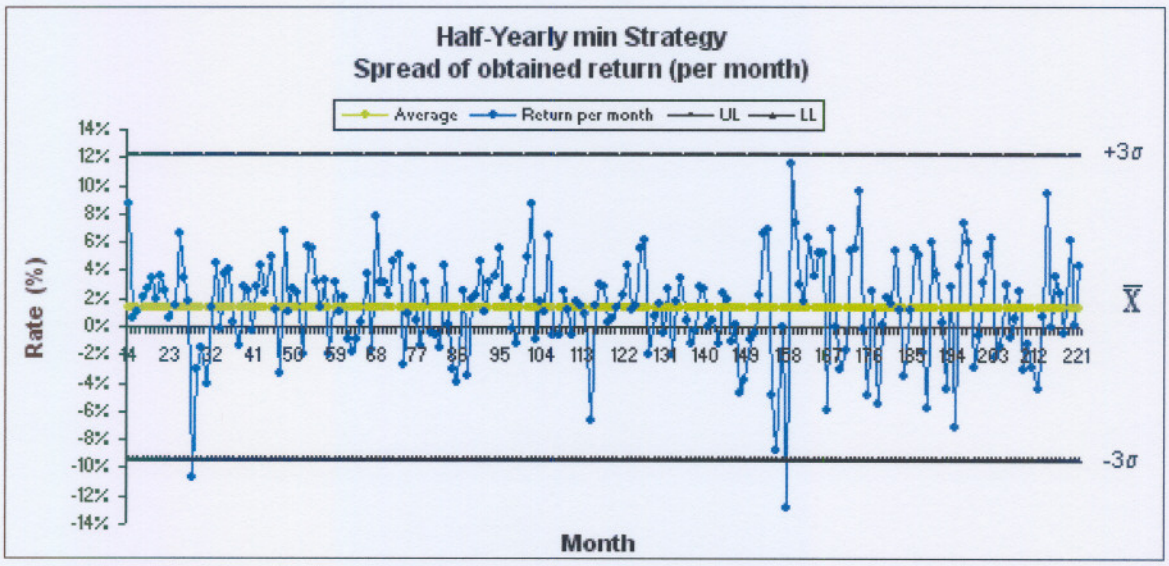


Graph 3.15: 3-Sigma rule for monthly min strategy

Graph 3.15 illustrates that 0.96% of the return values are below the lower limit (-9.32%). No return values are above the upper limit (12.29%). 47.37% of the return values obtained with this strategy were below the target return of 1.60%. The average return that is achieved per month is 1.49%, the strategy's return/risk ratio is 0.4123 and the standard deviation is 3.60%.

- **Half yearly min strategy**

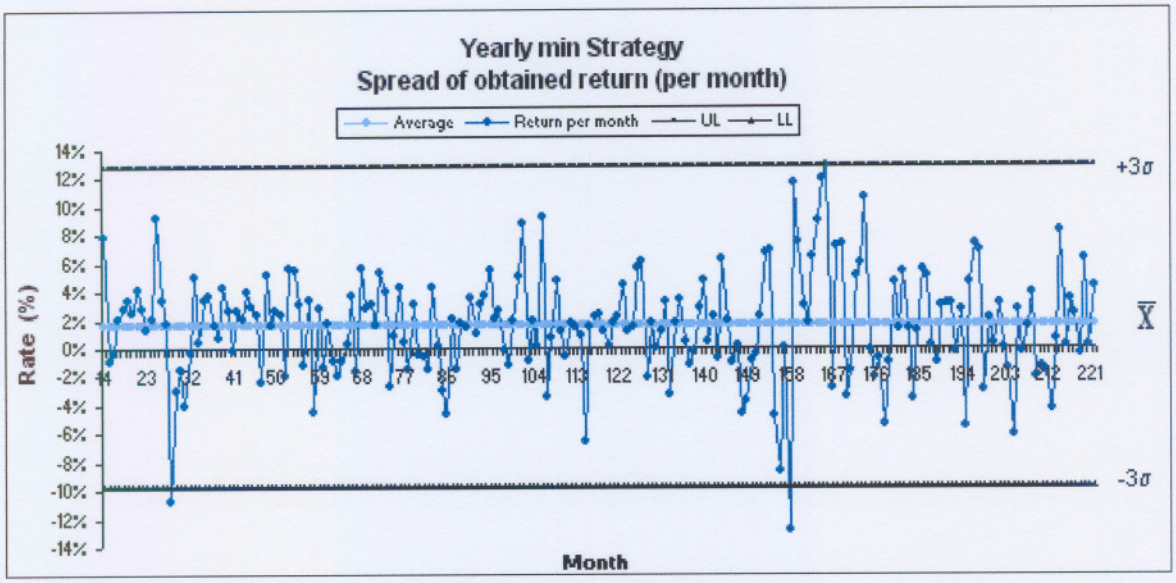
The following graph indicates that 0.96% of the return values are below the lower limit (-9.36%) but none of the return values are above the upper limit (12.19%). 49.76% of the return values obtained with this strategy are below the target return of 1.60%. The average return is 1.41% per month and the return/risk ratio for this strategy is 0.3939. The standard deviation for this strategy is 3.59%.



Graph 3.16: 3-Sigma rule for half yearly min strategy

- **Yearly min strategy**

Graph 3.17 shows that 0.96% of the return values are below the lower limit (-9.62%). No return values are above the upper limit (12.78%). 47.85% of the return values obtained with this strategy are below the 1.60% target return. An average return of 1.58% is achieved per month. This strategy's return/risk ratio is 0.4236, which is the maximum for the min strategies. The standard deviation for this strategy is 3.73%.



Graph 3.17: 3-Sigma rule for yearly min strategy

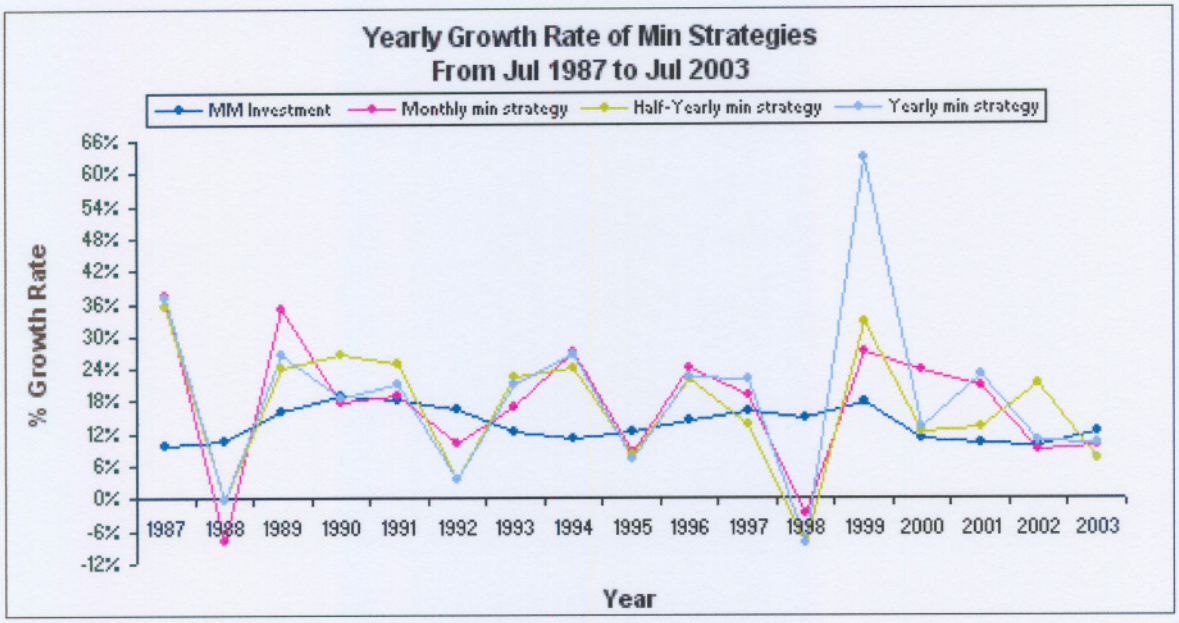
The following table provides a summary of the information obtained from the 3-sigma rule graphs for the min strategies ($\overline{X} \Rightarrow$ average return per month, $X \Rightarrow$ returns, UL $\Rightarrow$ upper limit and LL $\Rightarrow$ lower limit):

Strategy	$\overline{X}$	X < 1.60% (%)	Return/Risk ratio	X > UL (%)	X < LL (%)
Monthly min	1.49%	47.37%	0.4123	0.00%	0.96%
Half yearly min	1.41%	49.76%	0.3939	0.00%	0.96%
Yearly min	1.58%	47.85%	0.4236	0.00%	0.96%

Table 3.4: Summary of min strategies information

3.5.3 Yearly growth

Graph 3.18 portrays the yearly growth rates for the min strategies. The yearly min strategy shows the highest average yearly growth rate of 18.70%. The averages for the monthly and half yearly strategies are 17.37% and 16.64% correspondingly. The yearly min strategy obtained both the minimum (-8.36% in 1998) and the maximum (62.63% in 1999) yearly growth rates.



Graph 3.18: Yearly growth rates for min strategies

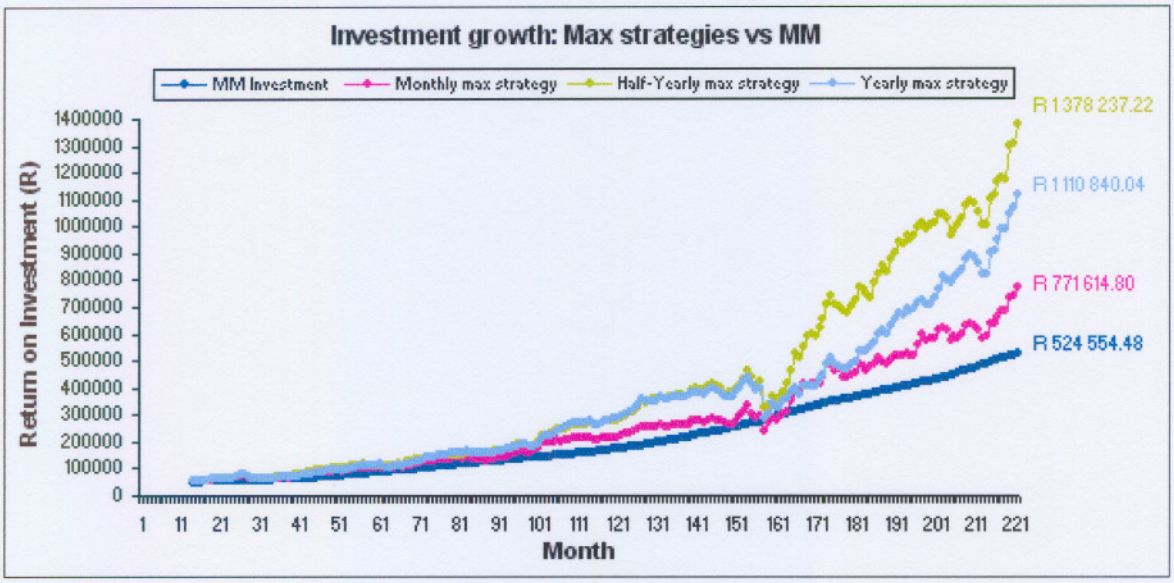
3.6 Max strategy

The max strategy is implemented for three different time horizons, which include:

- *Monthly max strategy* - Invest in the fund that obtained the largest return in the previous month.
- *Half yearly max strategy* - Invest in the fund that obtained the largest average return over the past six months, for the next six months.
- *Yearly max strategy* - Invest in the fund that obtained the largest average return over the past year, for the next year.

3.6.1 Investment growth

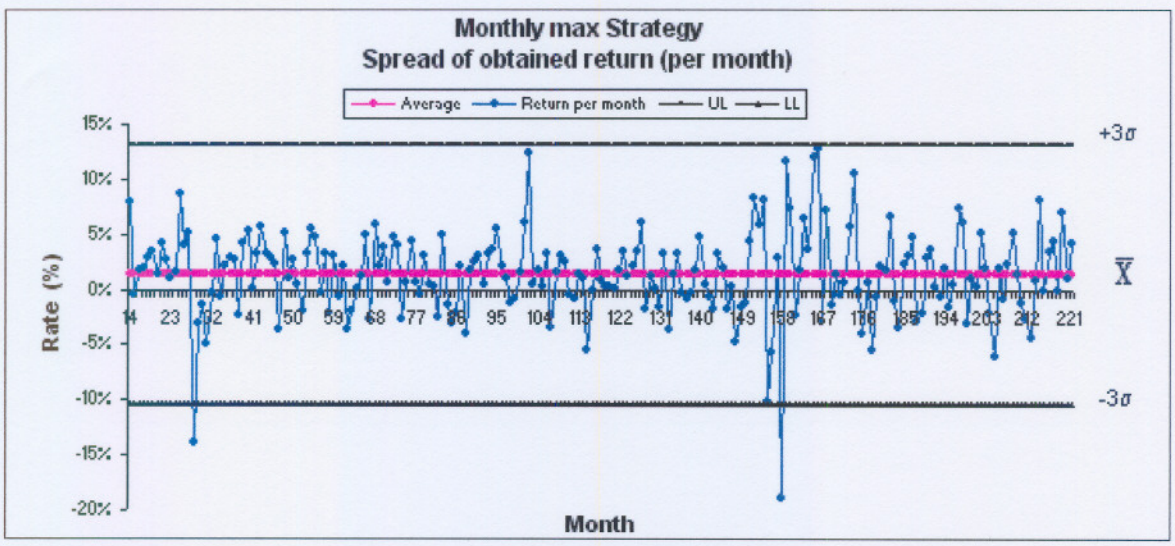
Graph 3.19 shows a comparison between the three max strategies and the MM. All three the max strategies performed better than the MM fund. The half yearly max strategy performed the best, with an effective interest rate of 1.60% per month. The effective monthly interest rate for the monthly max strategy is 1.32% and for the yearly max strategy it is 1.49%.



Graph 3.19: Comparison between max strategies

3.6.2 3-Sigma rule

- **Monthly max strategy**

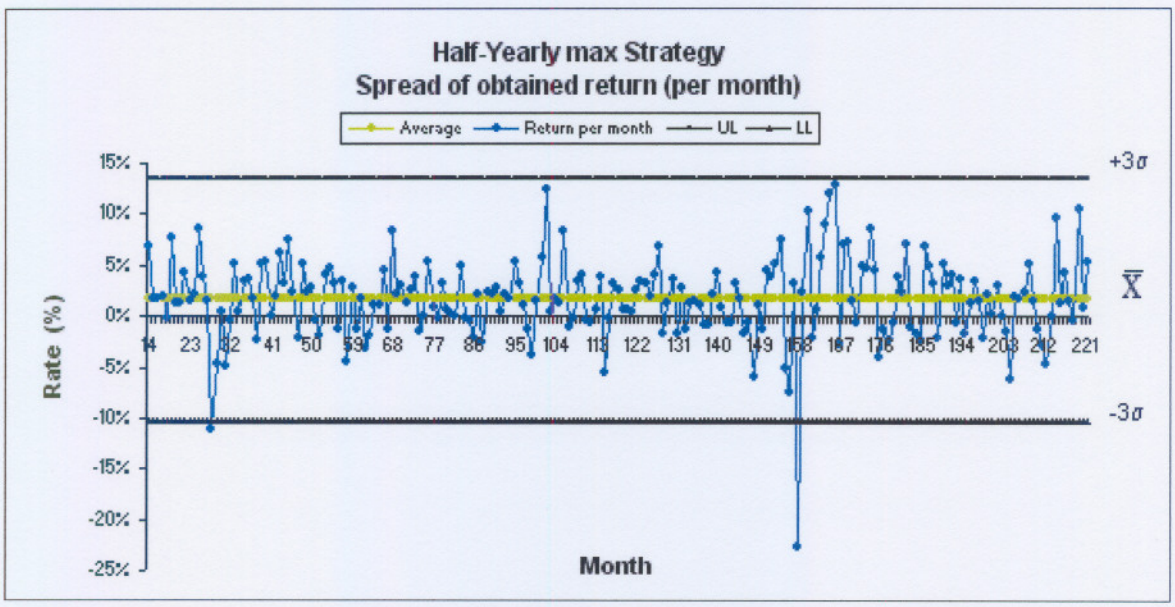


Graph 3.20: 3-Sigma rule for monthly max strategy

0.96% of the return values are below the lower limit (-10.41%), but no return values are above the upper limit (13.20%). More than half (51.67%) of the return values obtained with this

strategy are below the target return of 1.60%. The average return that is achieved per month is 1.40% and the strategy's return/risk ratio is 0.3547. The standard deviation is 3.93%.

- **Half yearly max strategy**



Graph 3.21: 3-Sigma rule for half yearly max strategy

According to graph 3.21, 0.96% of the return values are below the lower limit (-10.20%) but none of the return values are above the upper limit (13.56%). 49.28% of the return values obtained with this strategy are below the target return of 1.60%. The average return is 1.68% per month and the return/risk ratio for this strategy is 0.4238; both these values are the maximum for the max strategies. The standard deviation for this strategy is 3.96%.

- **Yearly max strategy**

Graph 3.22 shows that 0.96% of the return values are below the lower limit of –10.38%. None of the return values are above the upper limit of 13.54%. 44.98% of the return values obtained with this strategy are below the 1.60% target return. An average return of 1.58% is achieved per month. This strategy's return/risk ratio is 0.3959. The standard deviation for this strategy is 3.99%.



Graph 3.22: 3-Sigma rule for yearly max strategy

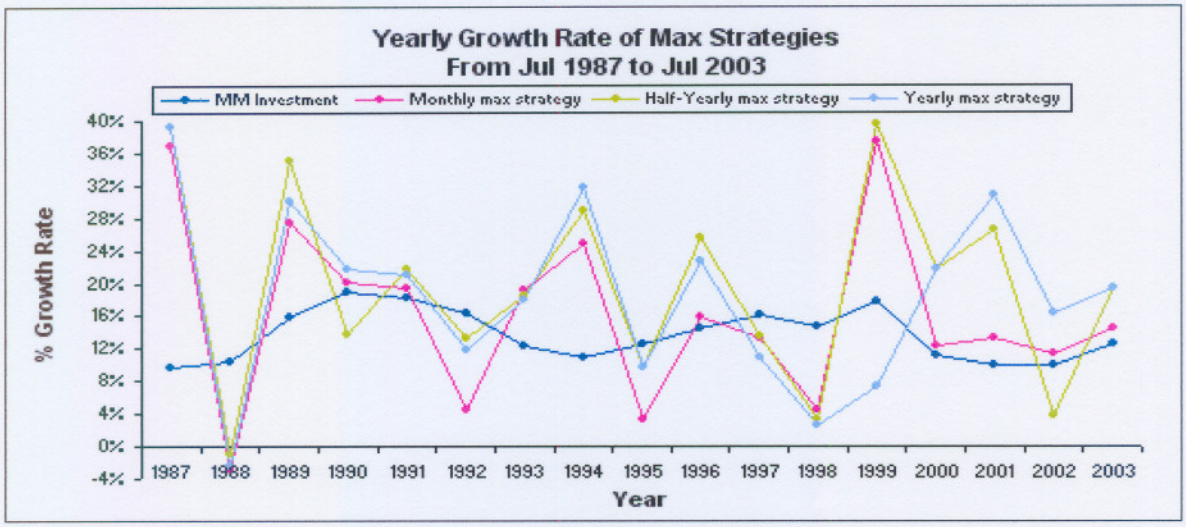
The following table provides a summary of the information obtained from the 3-sigma rule graphs for the max strategies ($\overline{X}$ ⇒ average return per month, X ⇒ returns, UL ⇒ upper limit and LL ⇒ lower limit):

Strategy	$\overline{X}$	X < 1.60% (%)	Return/Risk ratio	X > UL (%)	X < LL (%)
Monthly max	1.40%	51.67%	0.3547	0.00%	0.96%
Half yearly max	1.68%	49.28%	0.4238	0.00%	0.96%
Yearly max	1.58%	44.98%	0.3959	0.00%	0.96%

Table 3.5: Summary of max strategies information

3.6.3 Yearly growth

The following graph represents the yearly growth rates for the max strategies. The half yearly max strategy shows the highest average yearly growth rate of 19.57%. The averages for the monthly and yearly strategies are 16.20% and 18.43% respectively. The monthly max strategy obtained the lowest yearly growth rate of –2.86% in 1988 and highest yearly growth rate for this strategy is 39.18%, obtained in 1987%.

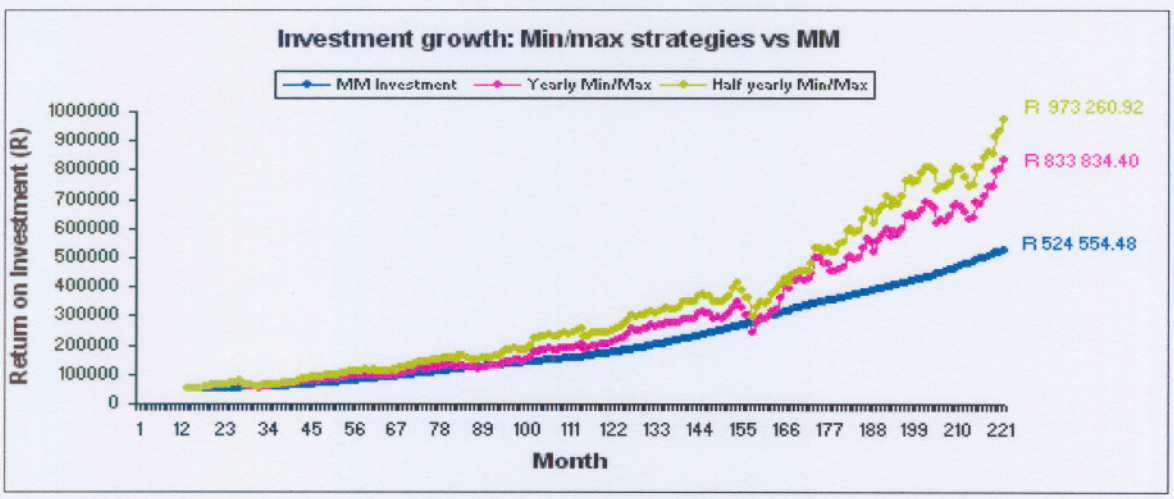


Graph 3.23: Yearly growth rates for max strategies

3.7 Min/Max strategy

The investor considers the average returns on the min strategy and the max strategy respectively, for the previous year. The investor then invests in the strategy that performed the best in the previous year, for the following year. This strategy is also performed half yearly.

3.7.1 Investment growth

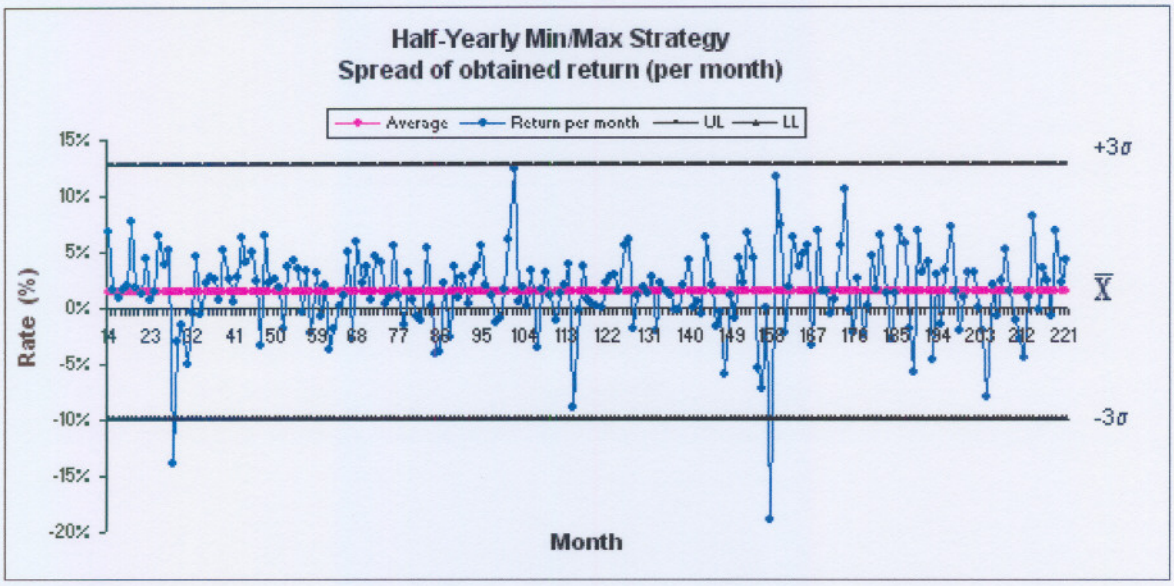


Graph 3.24: Comparison of min/max strategies

Graph 3.24 illustrates the comparison of the min/max strategies and the MM. Both the min/max strategies performed better than the MM fund. The half yearly min/max strategy shows the best effective interest rate, 1.43% per month. The effective monthly interest rate for the yearly min/max strategy is 1.36%.

3.7.2 3-Sigma rule

- **Half yearly min/max strategy**



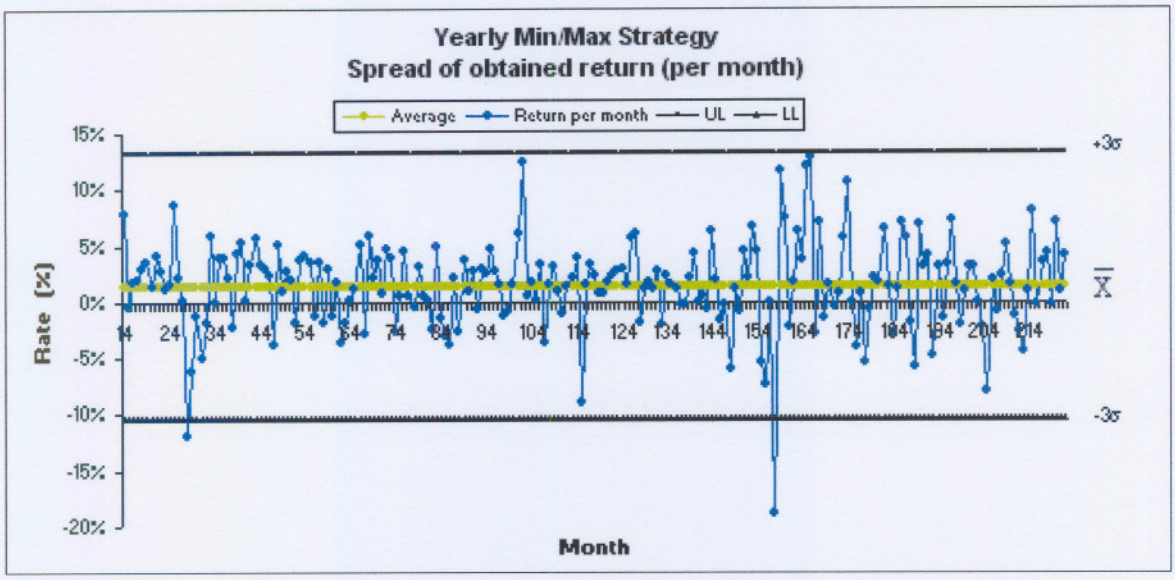
Graph 3.25: 3-Sigma rule for half yearly min/max strategy

As displayed by graph 3.25, 0.96% of the return values are below the lower limit of –9.86% but none of the return values are above the upper limit of 12.87%. 49.76% of the return values obtained with this strategy are below the target return of 1.60%. The average return per month is 1.50% and the return/risk ratio for this strategy is 0.3968. Both these values are higher than those obtained for the yearly min/max strategy. The standard deviation for this strategy is 3.79%.

- **Yearly min/max strategy**

Graph 3.26 shows that 0.96% of the return values are below the lower limit of –10.38%. None of the return values are above the upper limit of 13.24%. More than half, 50.24%, of

the return values obtained with this strategy are below the 1.60% target return. An average return of 1.43% is achieved per month. This strategy's return/risk ratio is 0.3641. The standard deviation for this strategy is 3.94%.



Graph 3.26: 3-Sigma rule for yearly min/max strategy

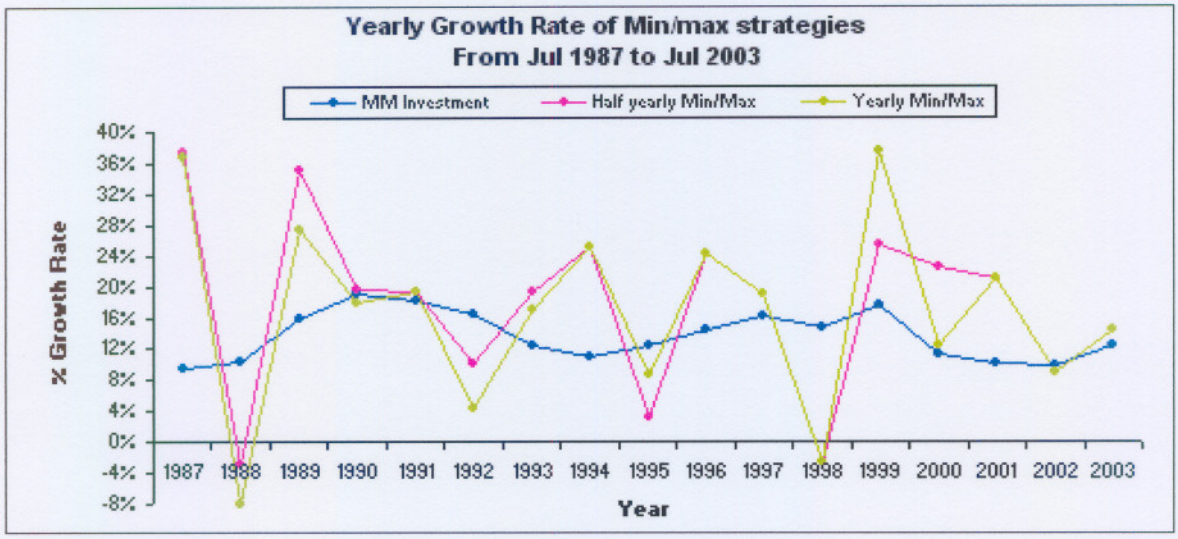
The following table provides a summary of the information obtained from the 3-sigma rule graphs for the min/max strategies:

Strategy	$\overline{X}$	X < 1.60% (%)	Return/Risk ratio	X > UL (%)	X < LL (%)
Half yearly min/max	1.50%	49.76%	0.3968	0.00%	0.96%
Yearly min/max	1.43%	50.24%	0.3641	0.00%	0.96%

Table 3.6: Summary of min/max strategies information

3.7.3 Yearly growth

The following graph indicates the yearly growth rates for the min/max strategies. The half yearly min/max strategy shows the highest average yearly growth rate of 17.59%, the average for the yearly min/max strategy is slightly lower, 16.66%. The half yearly min/max strategy obtained the highest yearly growth rate of 37.41% in 1987 and lowest yearly growth rate is obtained by the yearly min/max strategy (-7.90% in 1988).

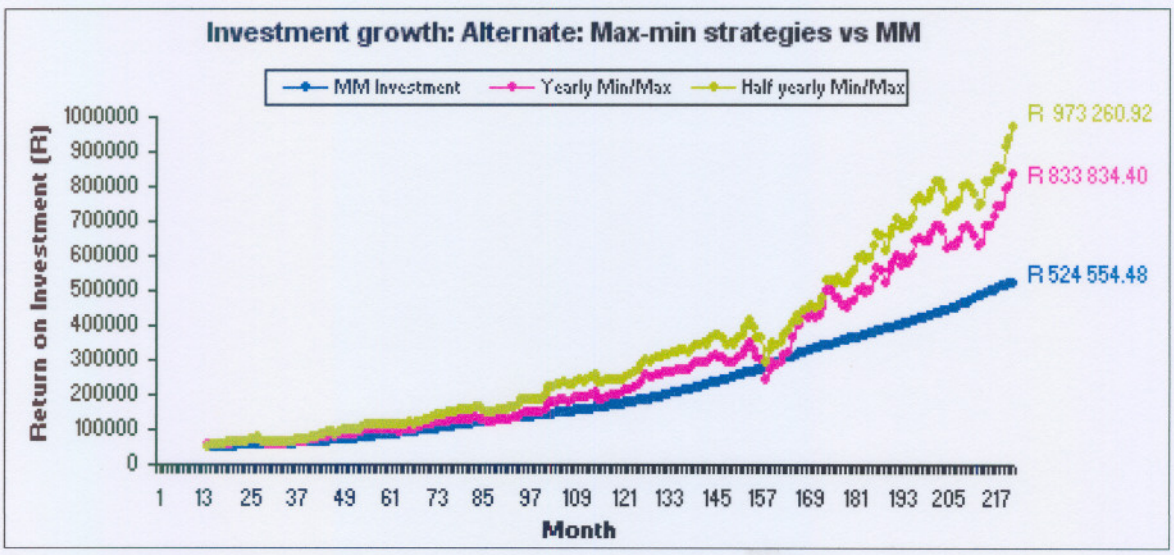


Graph 3.27: Yearly growth rates for min/max strategies

3.8 Alternate: Max/Min strategy

Invest in the max strategy and the min strategy in turn. This strategy is done yearly and half yearly.

3.8.1 Investment growth

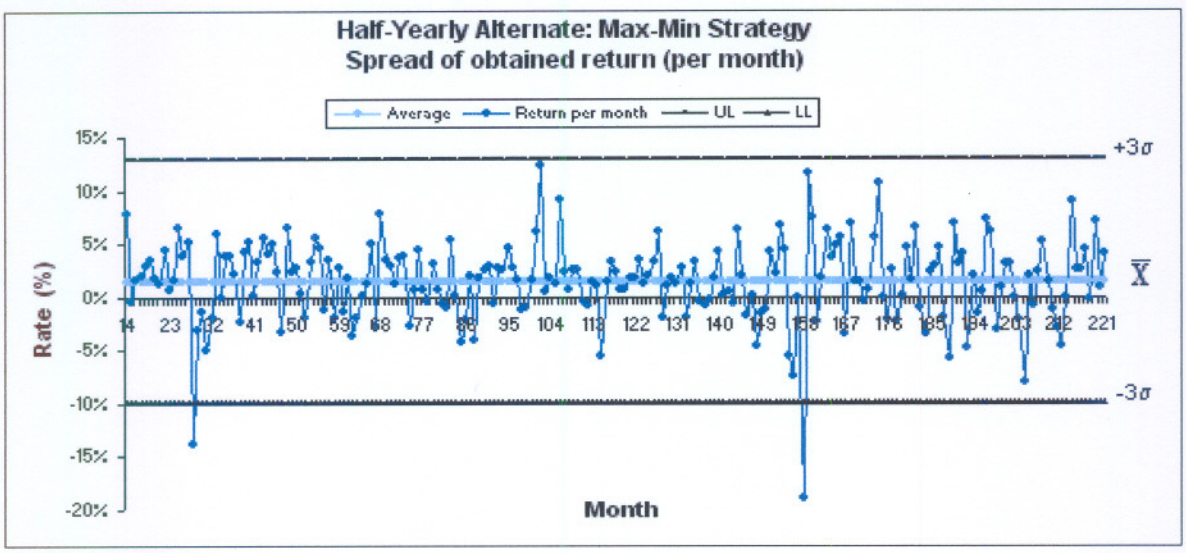


Graph 3.28: Comparison of Alternate: max/min strategies

Graph 3.28 illustrates the comparison of the alternate max/min strategies and the MM. Both the alternate max/min strategies achieved a higher investment growth than the MM fund. The half yearly alternate max/min strategy shows the best effective interest rate, 1.42% per month. The effective monthly interest rate for the yearly alternate max/min strategy is 1.37%.

3.8.2 3-Sigma rule

- **Half yearly alternate max/min strategy**



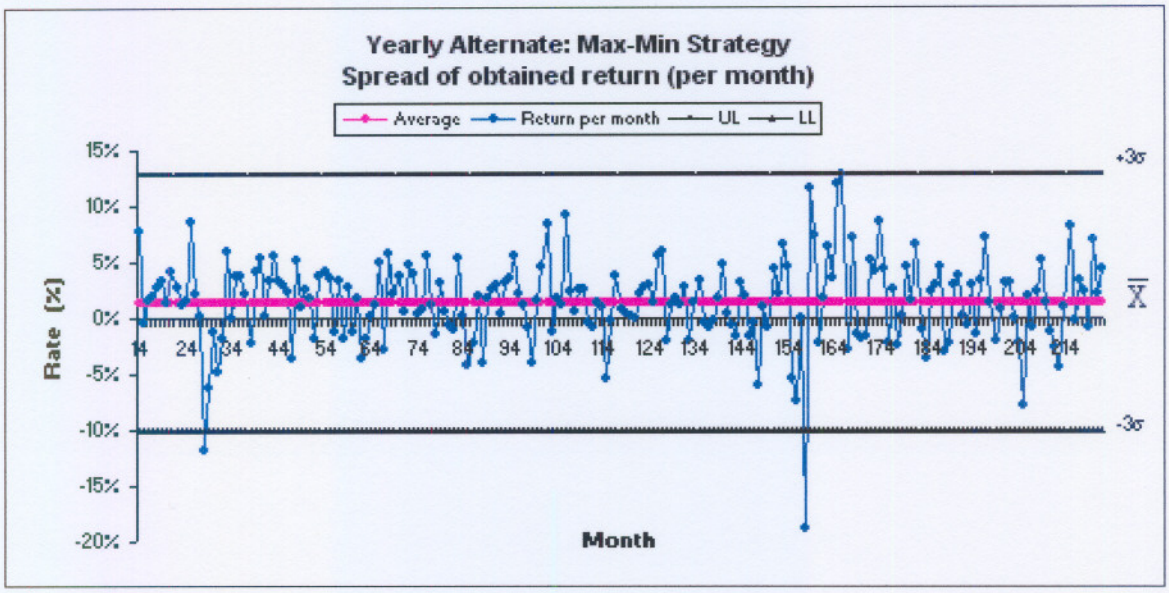
Graph 3.29: 3-Sigma rule for half yearly Alternate: max/min strategy

As displayed by graph 3.29 above, 0.96% of the return values are below the lower limit of –9.91% but none of the return values are above the upper limit of 12.97%. 48.33% of the return values obtained with this strategy are below the 1.60% target return. The average return per month is 1.49% and the return/risk ratio for this strategy is 0.3925. Both these values are higher than those obtained for the yearly alternate max/min strategy (another indication that this strategy is better than the yearly alternate max/min strategy). The standard deviation for this strategy is 3.80%.

- **Yearly alternate max/min strategy**

The following graph demonstrates that 0.96% of the return values are below the lower limit of –9.91%. None of the return values are above the upper limit of 12.97%. Nearly half, 49.28%,

of the return values obtained with this strategy are below the 1.60% target return. An average return of 1.44% is achieved per month. The return/risk ratio for this strategy is 0.3803. The standard deviation for this strategy is 3.78%.



Graph 3.30: 3-Sigma rule for yearly Alternate: max/min strategy

The following table provides a summary of the information obtained from the 3-sigma rule graphs for the alternate max/min strategies:

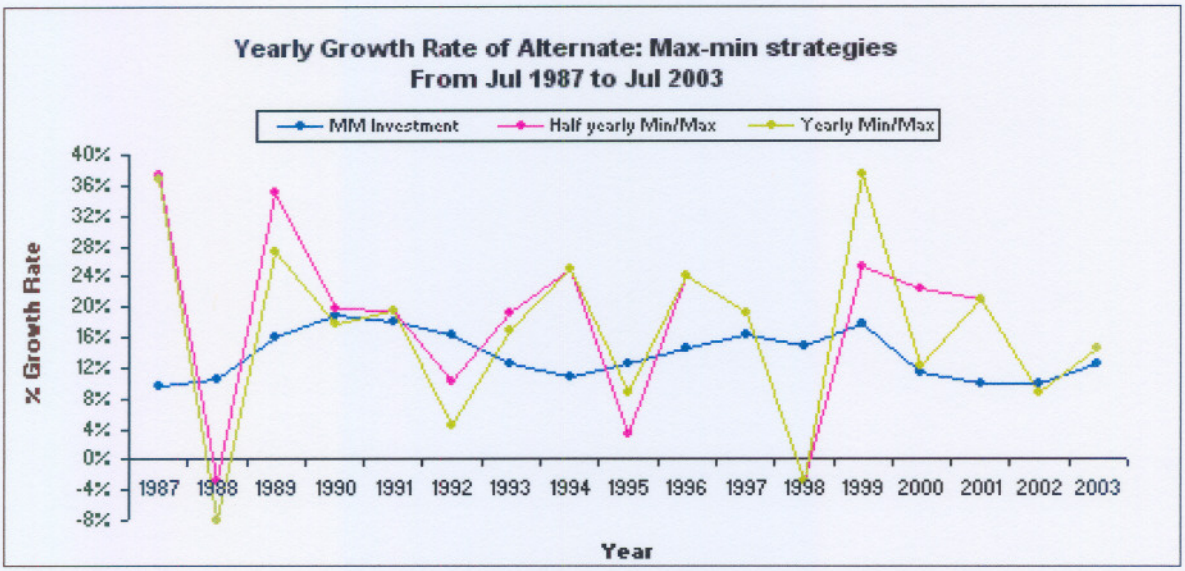
Strategy	$\overline{X}$	X < 1.60% (%)	Return/Risk ratio	X > UL (%)	X < LL (%)
Half yearly alternate max/min	1.49%	48.33%	0.3925	0.00%	0.96%
Yearly alternate max/min	1.44%	49.28%	0.3803	0.00%	0.96%

Table 3.7: Summary of alternate max/min strategies information

3.8.3 Yearly growth

The following graph denotes the yearly growth rates for the alternate max/min strategies. The half yearly alternate max/min strategy shows the highest average yearly growth rate of 17.59%, the average for the yearly alternate max/min strategy is slightly lower, 16.66%. The half yearly min/max strategy obtained the highest yearly growth rate of 37.41% in 1987 and

lowest yearly growth rate is obtained by the yearly alternate max/min strategy (-7.90% in 1988).



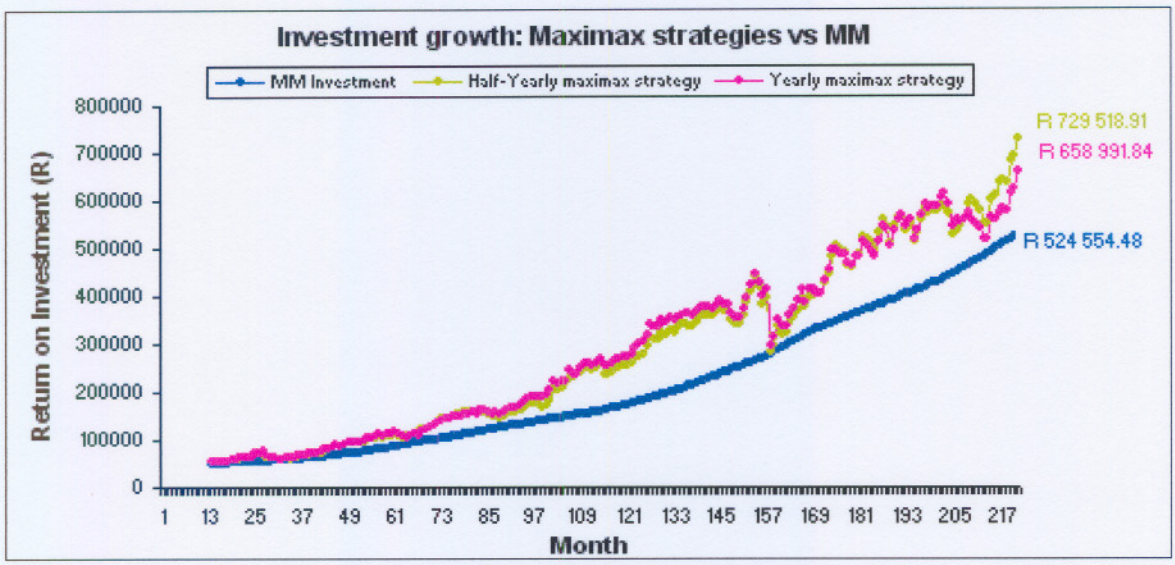
Graph 3.31: Yearly growth rates for Alternate: max/min strategies

3.9 Maximax strategy

This is an optimistic decision criterion that finds the alternative (fund) that maximizes the maximum payoff for each alternative (Render *et al.*, 2003:86). Thus, this decision-making strategy finds the alternative with the highest possible gain, in the previous year, to invest in that alternative for the following year. The strategy is also applied half yearly.

3.9.1 Investment growth

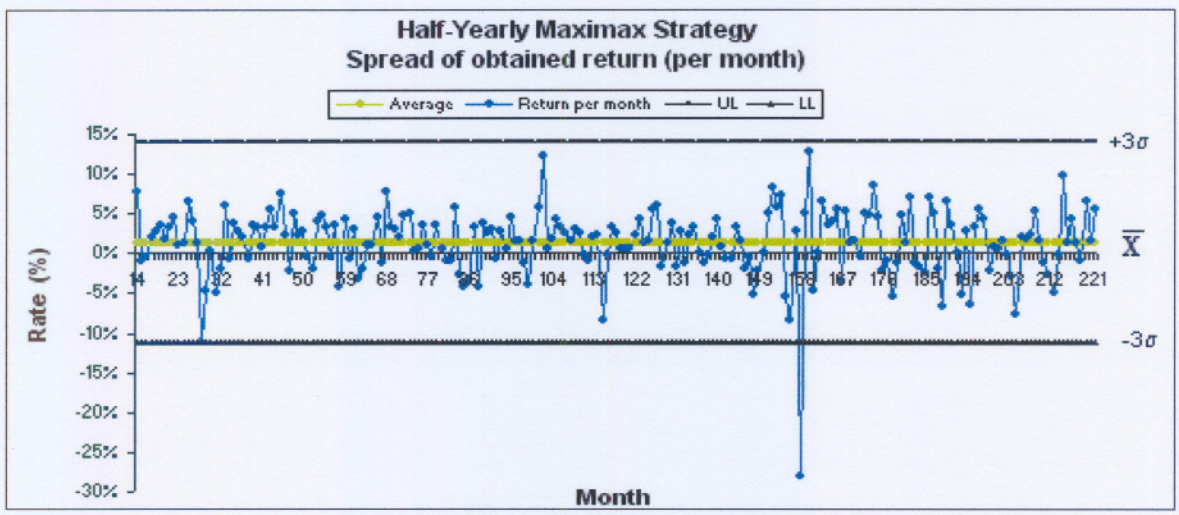
Graph 3.32 shows a comparison between the maximax strategies and the MM. Both the maximax strategies exceed the investment growth total of the MM fund. The half yearly maximax strategy shows the best effective interest rate, i.e. 1.29% per month and the effective monthly interest rate for the yearly maximax strategy is 1.24%.



Graph 3.32: Comparison between maximax strategies

3.9.2 3-Sigma rule

- **Half yearly maximax strategy**

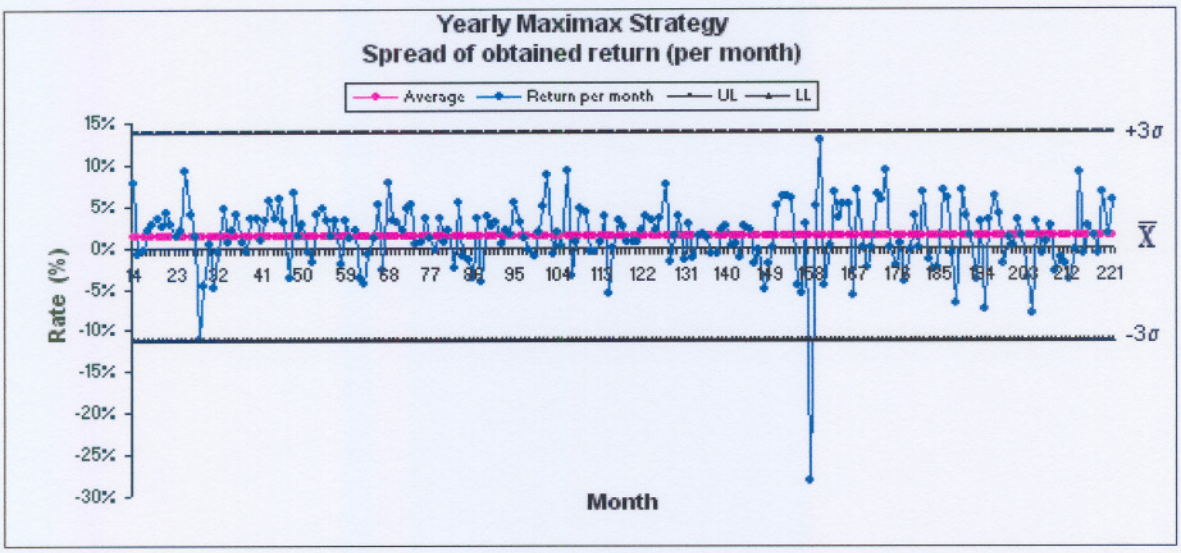


Graph 3.33: 3-Sigma rule for half yearly maximax strategy

No return values are above the upper limit of 13.89%, but 0.96% of the return values are below the lower limit of –11.13%. 49.48% of the return values are below the target return of 1.60%. The average return that is achieved per month is 1.38% and this strategy's return/risk ratio is 0.3314. The standard deviation for this strategy is 4.17%.

- **Yearly maximax strategy**

According to graph 3.34, 0.96% of the return values are below the lower limit (-11.06%) and none of the return values are above the upper limit (13.72%). More than half of the return values obtained with this strategy are below the target return of 1.60%, i.e. 51.67% of the return values. The average return is 1.33% per month and the return/risk ratio for this strategy is 0.3221. The standard deviation for this strategy is 4.13%.



Graph 3.34: 3-Sigma rule for yearly maximax strategy

The following table provides a summary of the information obtained from the 3-sigma rule graphs for the maximax strategies:

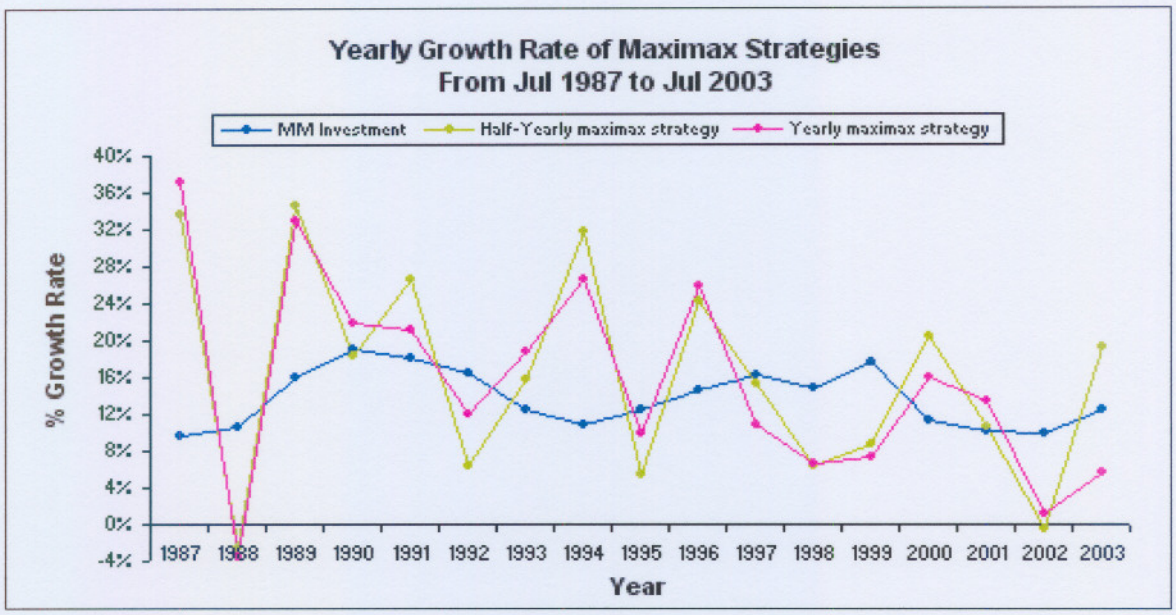
Strategy	$\overline{X}$	X < 1.60% (%)	Return/Risk ratio	X > UL (%)	X < LL (%)
Half yearly maximax	1.38%	49.48%	0.3314	0.00%	0.96%
Yearly maximax	1.33%	51.67%	0.3221	0.00%	0.96%

Table 3.8: Summary of maximax strategies information

3.9.3 Yearly growth

The following graph shows the yearly growth rates for the maximax strategies. The half yearly maximax strategy shows the highest average yearly growth rate of 16.14% and the

average for the yearly maximax strategy is 15.52%. The yearly maximax strategy obtained the lowest yearly growth rate of –3.50% in 1988 and highest yearly growth rate of 37.17% in 1998.



Graph 3.35: Yearly growth rates for maximax strategies

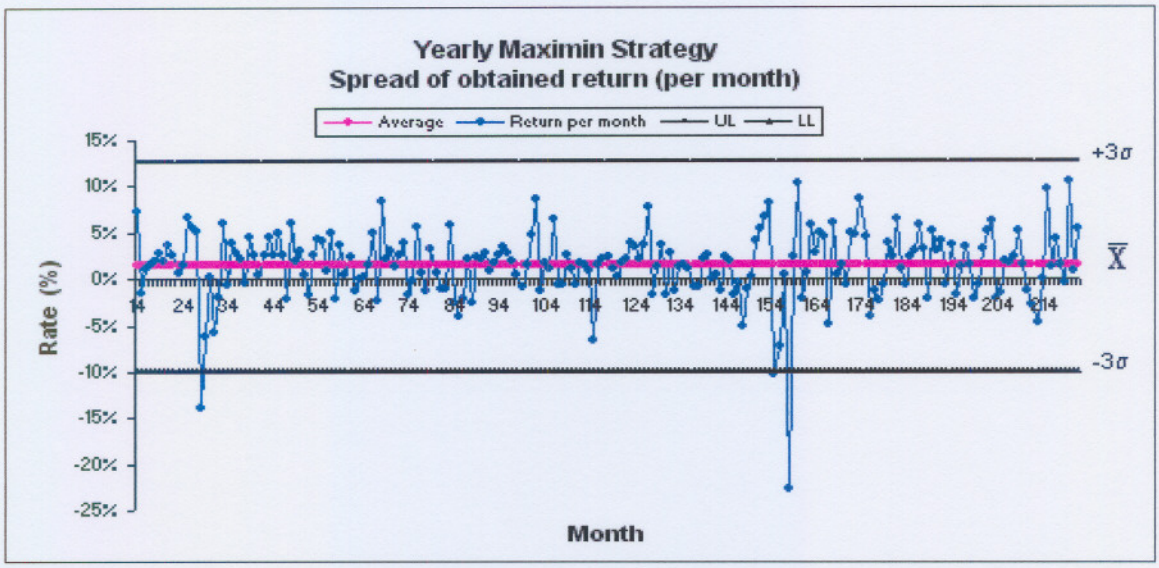
3.10 Maximin strategy

This is a pessimistic decision criterion that attains the alternative that maximizes the minimum payoff for each alternative (Render *et al.*, 2003:87). Thus, this decision-making strategy finds the alternative with the least possible loss, in the previous year, to invest in that alternative for the following year. The strategy is also applied half yearly.

3.10.1 Investment growth

The graph below illustrates the comparison between the maximin strategies and the MM. Both the maximin strategies surpass the investment growth total of the MM fund. The half yearly maximin strategy shows the best effective interest rate, i.e. 1.61% per month and the effective monthly interest rate for the yearly maximin strategy is 1.41%.

- **Yearly maximin strategy**



Graph 3.38: 3-Sigma rule for yearly maximin strategy

According to graph 3.38, 1.44% of the return values are below the lower limit (-9.77%) and none of the return values are above the upper limit (12.73%). 49.28% of the return values obtained with this strategy are below the target return of 1.60%. The average return is 1.48% per month and the return/risk ratio for this strategy is 0.3946. The standard deviation for this strategy is 3.75%.

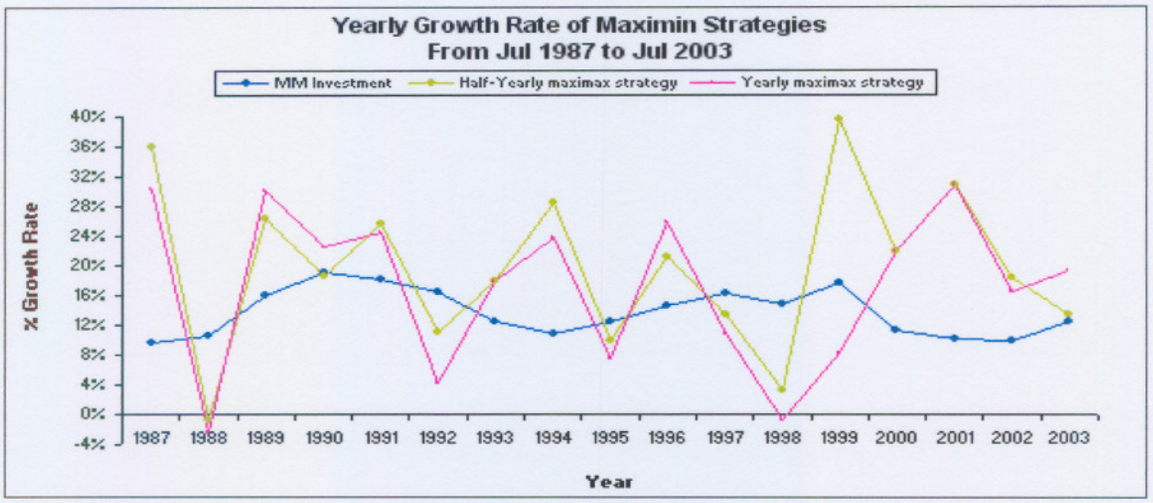
The following table provides a summary of the information obtained from the 3-sigma rule graphs for the maximin strategies:

Strategy	$\overline{X}$	X < 1.60% (%)	Return/Risk ratio	X > UL (%)	X < LL (%)
Half yearly maximin	1.69%	45.93%	0.4265	0.00%	0.96%
Yearly maximin	1.48%	49.28%	0.3946	0.00%	0.96%

Table 3.9: Summary of maximin strategies information

3.10.3 Yearly growth

Graph 3.39 shows the yearly growth rates for the maximin strategies. The half yearly maximin strategy shows the maximum average yearly growth rate of 19.70% and the average for the yearly maximax strategy is 17.12%. The yearly maximin strategy obtained the lowest yearly growth rate of –2.45% in 1988 and half yearly maximin strategy obtained the highest yearly growth rate of 39.56% in 1999.



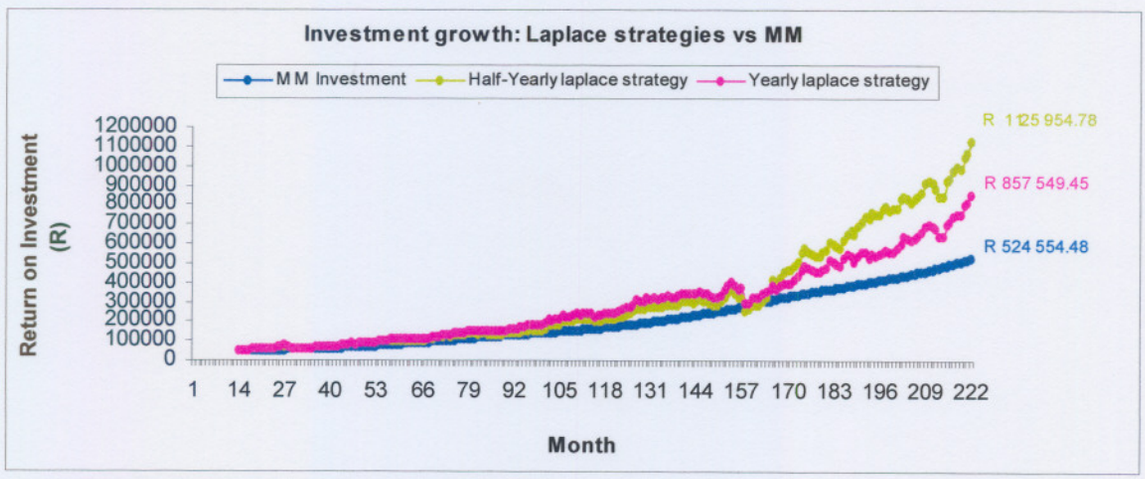
Graph 3.39: Yearly growth for maximin strategies

3.11 Laplace strategy

This is a decision approach that finds the alternative with the highest average payoff. This approach assumes that all probabilities of occurrence for the different outcomes are equal; therefore it is also called the equally likely strategy (Render *et al.*, 2003:88). This strategy is applied half yearly and yearly.

3.11.1 Investment growth

The graph below illustrates the comparison of the laplace strategies and the MM. Both the laplace strategies performed better than the MM fund. The half yearly laplace strategy shows the best effective interest rate, i.e. 1.50% per month and the effective monthly interest rate for the yearly laplace strategy is 1.37%.

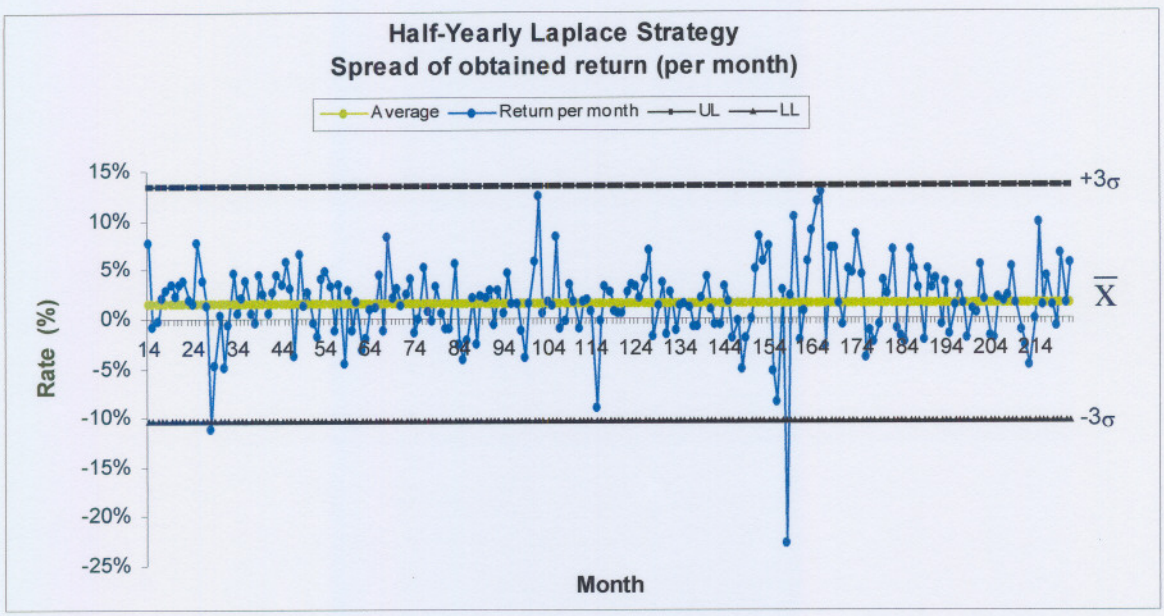


Graph 3.40: Comparison of laplace strategies

3.11.2 3-Sigma rule

- **Half yearly laplace strategy**

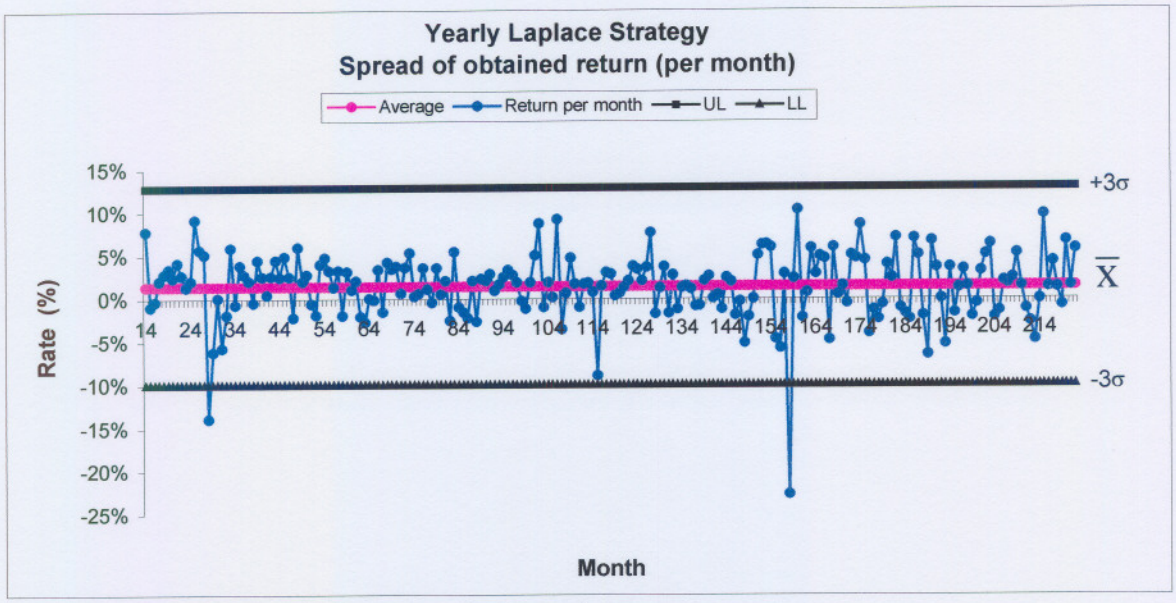
As displayed by graph 3.41, 0.96% of the return values are below the lower limit of –10.33% no return values are above the upper limit of 13.49%. 49.76% of the return values obtained with this strategy are below the target return of 1.60%. The average return per month is 1.58% and the return/risk ratio for this strategy is 0.3983. Both these values are higher than those obtained for the yearly laplace strategy. The standard deviation for this strategy is 3.97%.



Graph 3.41: 3-Sigma rule for half yearly laplace strategy

- **Yearly laplace strategy**

The following graph shows that 0.96% of the return values are below the lower limit of –9.94%. None of the return values are above the upper limit of 12.82%. 47.85%, of the return values obtained with this strategy are below the 1.60% target return. An average return of 1.44% is achieved per month and this strategy's return/risk ratio is 0.3804. The standard deviation for this strategy is 3.79%.



Graph 3.42: 3-Sigma rule for yearly laplace strategy

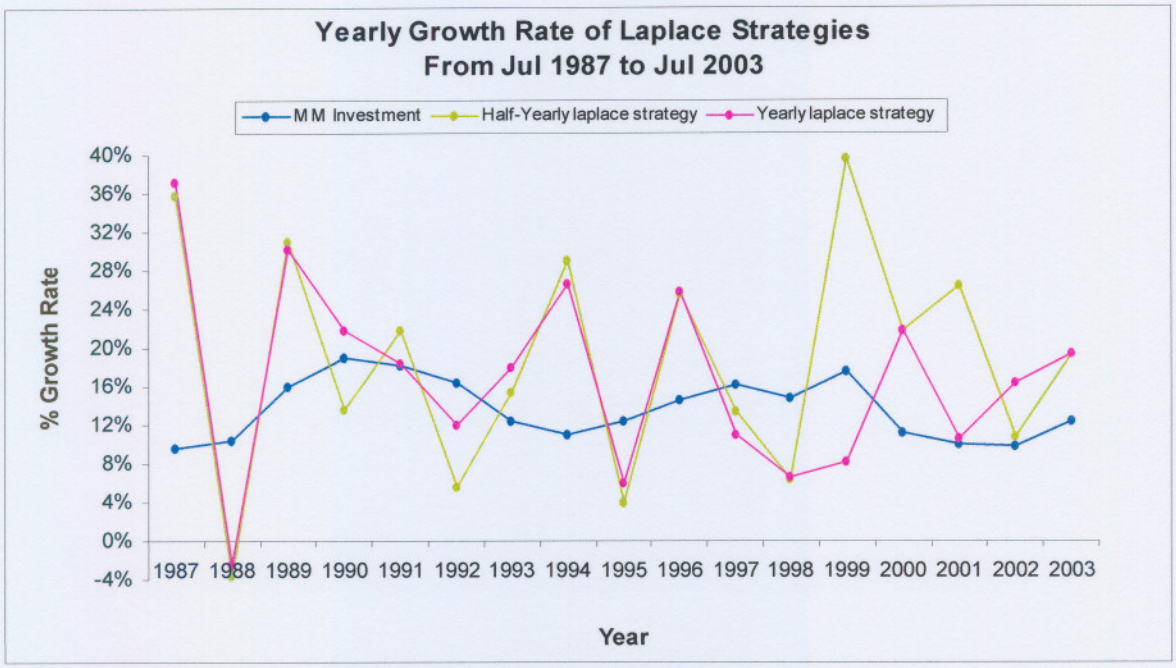
The following table provides a summary of the information obtained from the 3-sigma rule graphs for the laplace strategies:

Strategy	$\overline{X}$	X < 1.60% (%)	Return/Risk ratio	X > UL (%)	X < LL (%)
Half yearly maximin	1.69%	45.93%	0.4265	0.00%	0.96%
Yearly maximin	1.48%	49.28%	0.3946	0.00%	0.96%

Table 3.10: Summary of laplace strategies information

3.11.3 Yearly growth

The following graph portrays the yearly growth rates for the laplace strategies. The half yearly laplace strategy shows the highest average yearly growth rate of 18.60%, the average for the yearly laplace strategy is slightly lower at 16.90%. The half yearly laplace strategy obtained the highest yearly growth rate of 39.56% in 1999 and the lowest yearly growth rate of –3.50% in 1988.



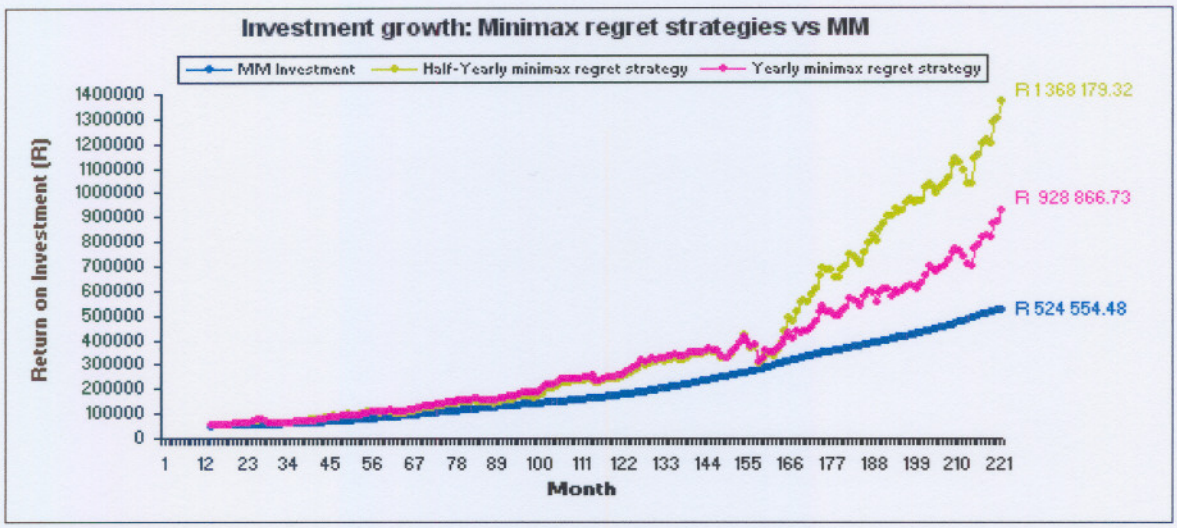
Graph 3.43: Yearly growth rates for laplace strategies

3.12 Minimax regret strategy

This decision criterion finds the alternative that minimizes the maximum opportunity loss within each alternative. Render *et al.* (2003: 88) states that the opportunity loss that is actually realized will be no more than the minimax value, as the investor finds the maximum opportunity loss within each alternative and then selects the alternative with the smallest value. This is performed yearly and half yearly.

3.12.1 Investment growth

The following graph illustrates the comparison between the minimax regret strategies and the MM. Both the minimax regret strategies surpass the investment growth total of the MM fund. The half yearly minimax regret strategy shows the best effective interest rate of 1.60% per month and the effective interest rate for the yearly minimax regret strategy is 1.41% per month.

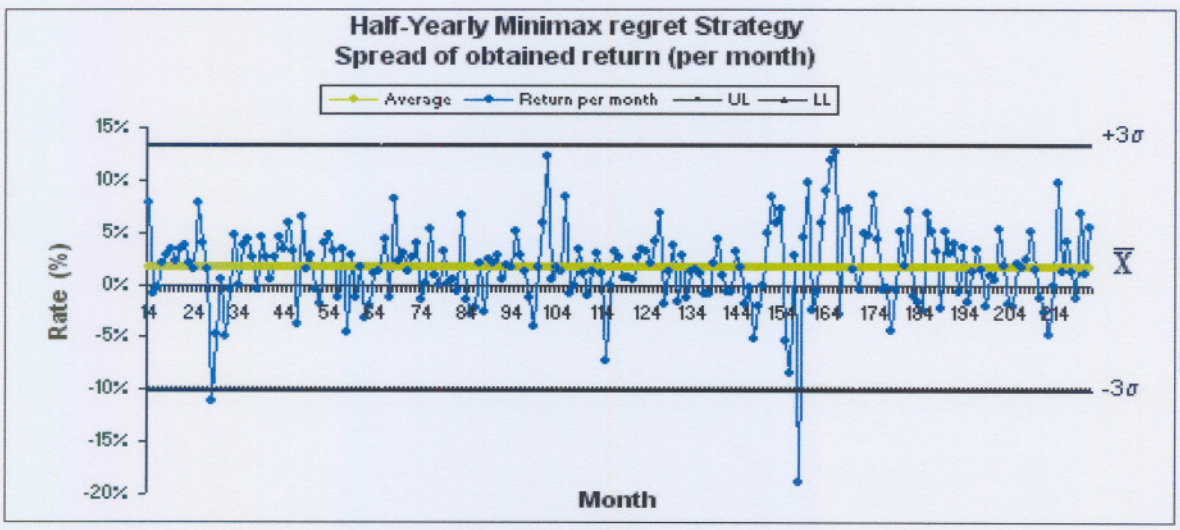


Graph 3.44: Comparison between minimax regret strategies

3.12.2 3-Sigma rule

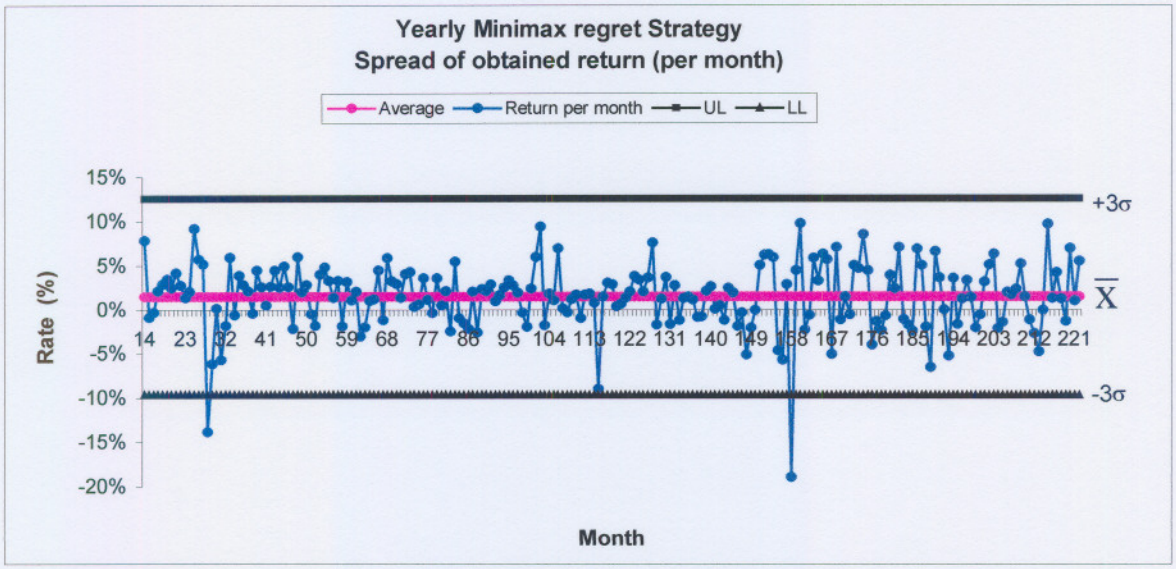
- **Half yearly minimax regret strategy**

According to graph 3.45, no return values are above the upper limit of 13.19%, but 0.96% of the return values are below the lower limit of –9.88%. The graph also shows that for this strategy 49.76% of the return values are below the target return of 1.60%. The average return is 1.67% per month the return/risk ratio is 0.4347. The standard deviation for this strategy is 3.84%.



Graph 3.45: 3-Sigma rule for half yearly minimax regret strategy

- **Yearly minimax regret strategy**



Graph 3.46: 3-Sigma rule for yearly minimax regret strategy

According to graph 3.46, 0.96% of the return values are below the lower limit (-9.62%) and none of the return values are above the upper limit (12.58%). 48.33% of the return values obtained with this strategy are below the target return of 1.60%. The average return is 1.48% per month and the return/risk ratio for this strategy is 0.3992. The standard deviation for this strategy is 3.70%.

The following table provides a summary of the information obtained from the 3-sigma rule graphs for the minimax regret strategies:

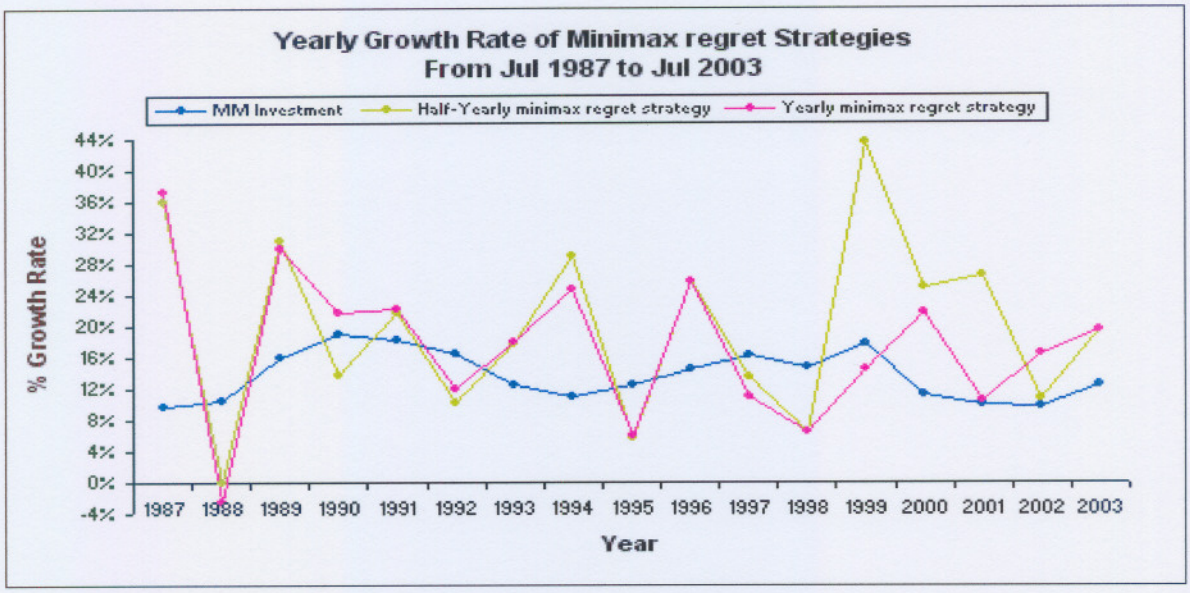
Strategy	$\overline{X}$	X < 1.60% (%)	Return/Risk ratio	X > UL (%)	X < LL (%)
Half yearly minimax regret	1.67%	49.76%	0.4347	0.00%	0.96%
Yearly minimax regret	1.48%	48.33%	0.3992	0.00%	0.96%

Table 3.11: Summary of minimax regret strategies information

3.12.3 Yearly growth

The following graph shows the yearly growth rates for the minimax regret strategies. The half yearly minimax regret strategy shows the maximum average yearly growth rate of 19.74% and the average for the yearly minimax regret strategy is 17.37%. The yearly minimax regret

strategy obtained the lowest yearly growth rate of –2.45% in 1988 and half yearly minimax regret strategy obtained the highest yearly growth rate of 43.53% in 1999.



Graph 3.47: Yearly growth rates for minimax regret strategies

3.13 Hurwicz criterion strategy

This is a weighted average approach that selects the alternative with the highest weighted average. A coefficient of realism (α) is selected; the coefficient is between 0 and 1. When α is close to 0, the decision maker is pessimistic about the future and when α is close to 1, the decision maker is optimistic about the future (Render *et al.*, 2003:87). This strategy is applied yearly and half yearly with both optimistic and pessimistic values for the criterion of realism.

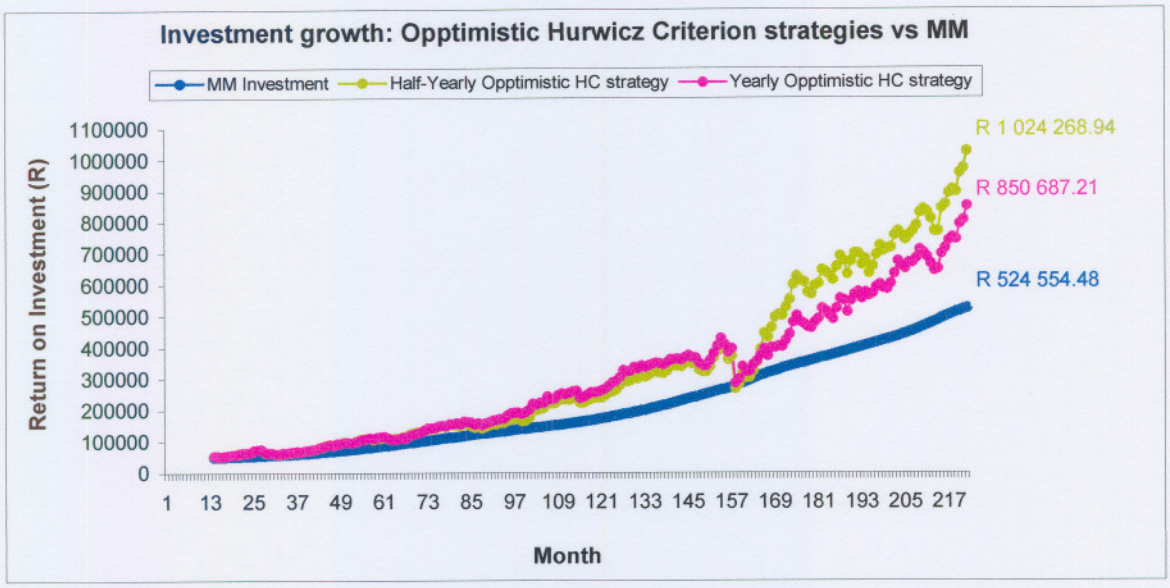
3.13.1 Optimistic Hurwicz criterion strategy

Set coefficient of realism, $\alpha = 0.8$.

3.13.1.1 Investment growth

The graph below illustrates the comparison of the optimistic Hurwicz criterion strategies and the MM. Both the optimistic Hurwicz criterion strategies performed better than the MM fund.

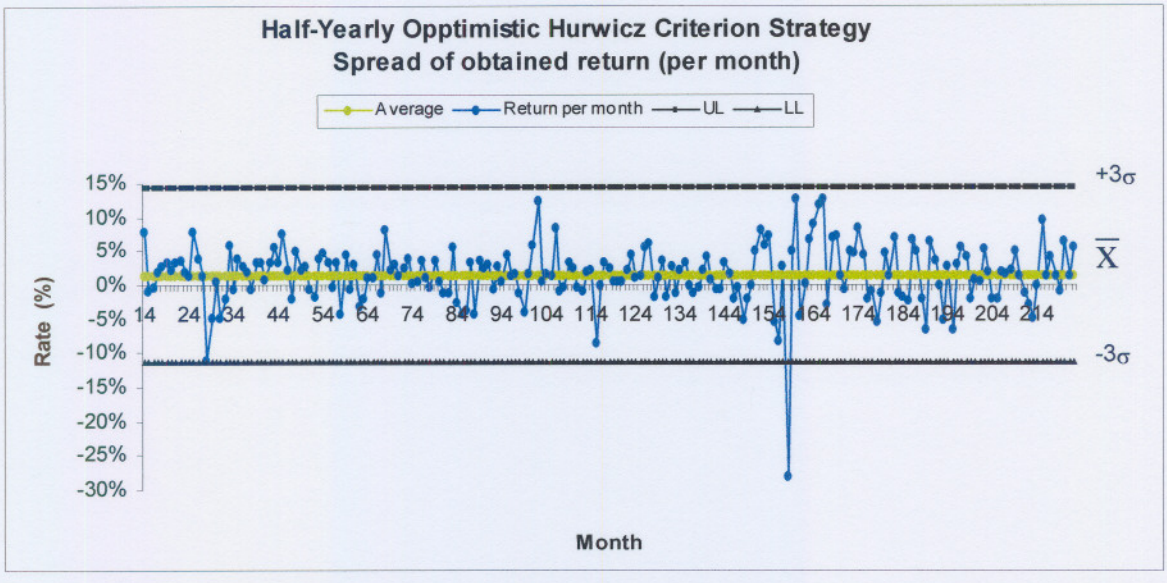
The half yearly optimistic Hurwicz criterion strategy shows the best effective interest rate of 1.46% per month and the effective interest rate for the yearly optimistic Hurwicz criterion strategy is 1.37% per month.



Graph 3.48: Comparison between optimistic Hurwicz criterion strategies

3.13.1.2 3-Sigma rule

- **Half yearly optimistic Hurwicz criterion strategy**

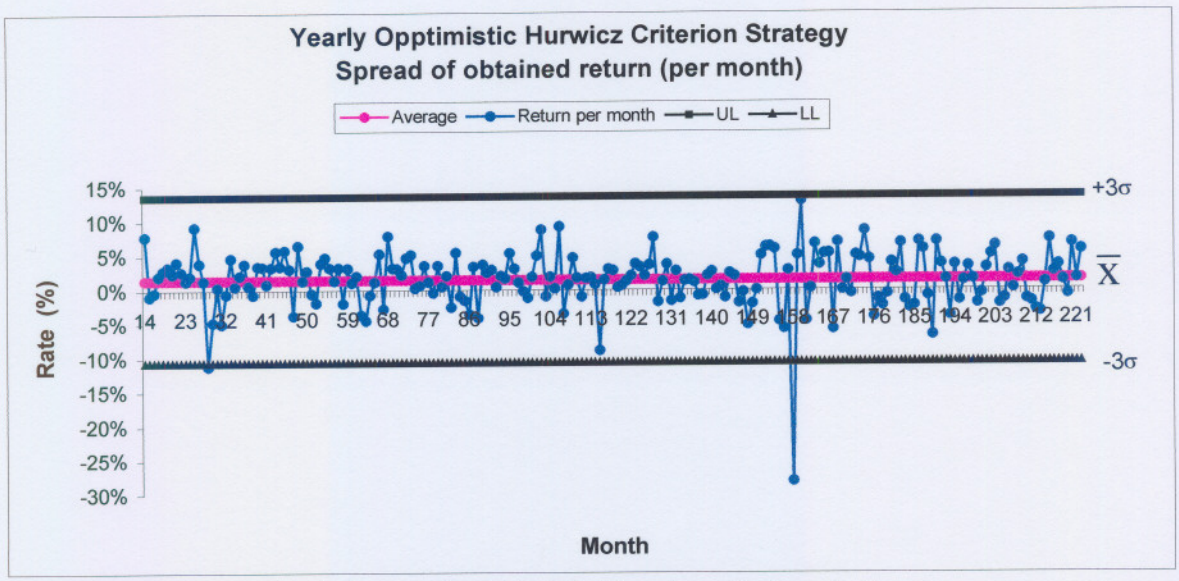


Graph 3.49: 3-Sigma rule for half yearly optimistic Hurwicz criterion strategy

As displayed by graph 3.49, 0.48% of the return values are below the lower limit of –11.45% no return values are above the upper limit of 14.56%. 49.28% of the return values obtained with this strategy are below the target return of 1.60%. The average return per month is 1.55% and the return/risk ratio for this strategy is 0.3582. The standard deviation is 4.33%.

- **Yearly optimistic Hurwicz criterion strategy**

The following graph shows that 0.96% of the return values are below the lower limit of –10.66%. None of the return values are above the upper limit of 13.56%. 49.28%, of the return values obtained with this strategy are below the 1.60% target return. An average return of 1.45% is achieved per month and this strategy's return/risk ratio is 0.3594. The standard deviation for this strategy is 4.04 %.



Graph 3.50: 3-Sigma rule for yearly optimistic Hurwicz criterion strategy

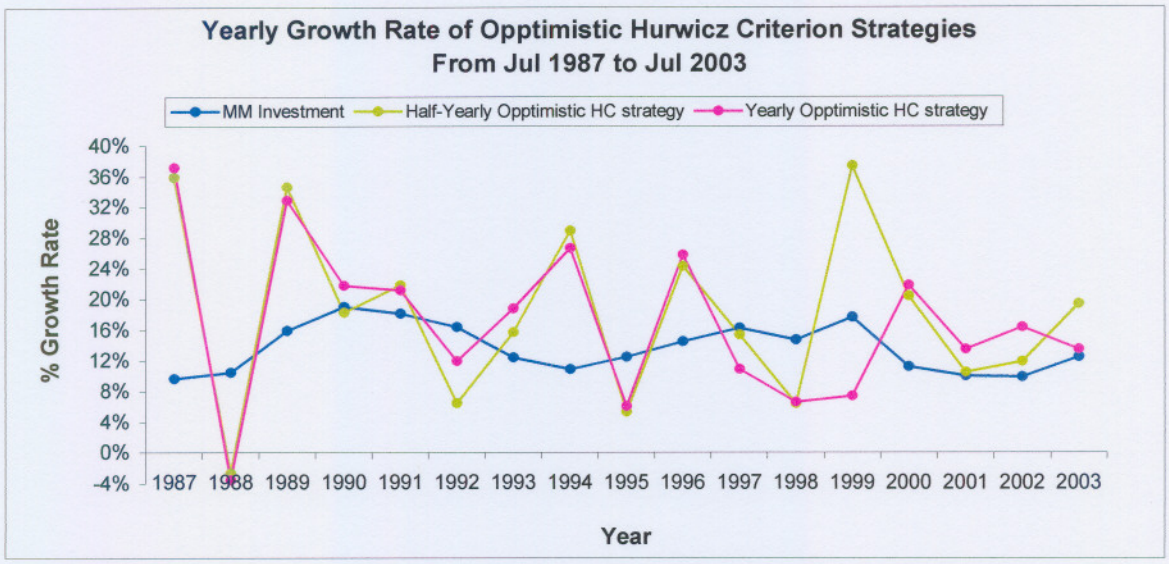
The following table provides a summary of the information obtained from the 3-sigma rule graphs for the optimistic Hurwicz criterion strategies:

Strategy	$\overline{X}$	X < 1.60% (%)	Return/Risk ratio	X > UL (%)	X < LL (%)
Half yearly optimistic Hurwicz criterion	1.55%	49.28%	0.3582	0.00%	0.48%
Yearly optimistic Hurwicz criterion	1.45%	49.28%	0.3594	0.00%	0.96%

Table 3.12: Summary of optimistic Hurwicz criterion strategies information

3.13.1.3 Yearly growth

The following graph portrays the yearly growth rates for the optimistic Hurwicz criterion strategies:



Graph 3.51: Yearly growth rates for optimistic Hurwicz criterion strategies

The half yearly optimistic Hurwicz criterion strategy shows the highest average yearly growth rate of 18.24%, the average for the yearly optimistic Hurwicz criterion strategy is 16.99%. The half yearly optimistic Hurwicz criterion strategy obtained the highest yearly growth rate of 37.38% in 1999 and the yearly optimistic Hurwicz criterion strategy obtained the lowest yearly growth rate of –3.50% in 1988.

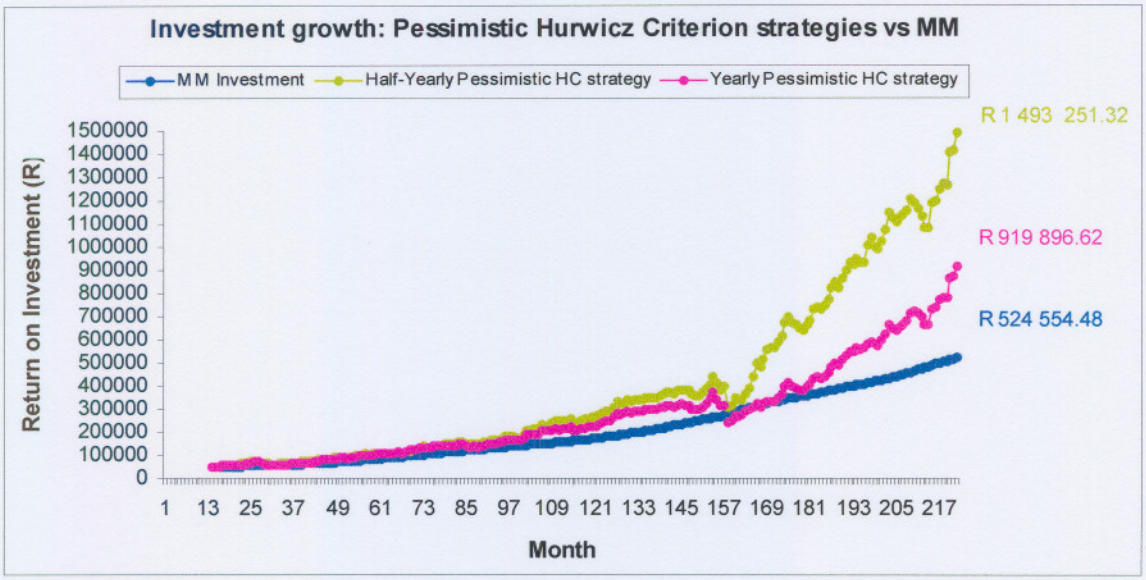
3.13.2 Pessimistic Hurwicz criterion strategy

Set coefficient of realism, $\alpha = 0.1$.

3.13.2.1 Investment growth

Graph 3.52 illustrates the comparison of the pessimistic Hurwicz criterion strategies and the MM. Both the pessimistic Hurwicz criterion strategies performed better than the MM fund. The half yearly pessimistic Hurwicz criterion strategy shows the best effective interest rate of

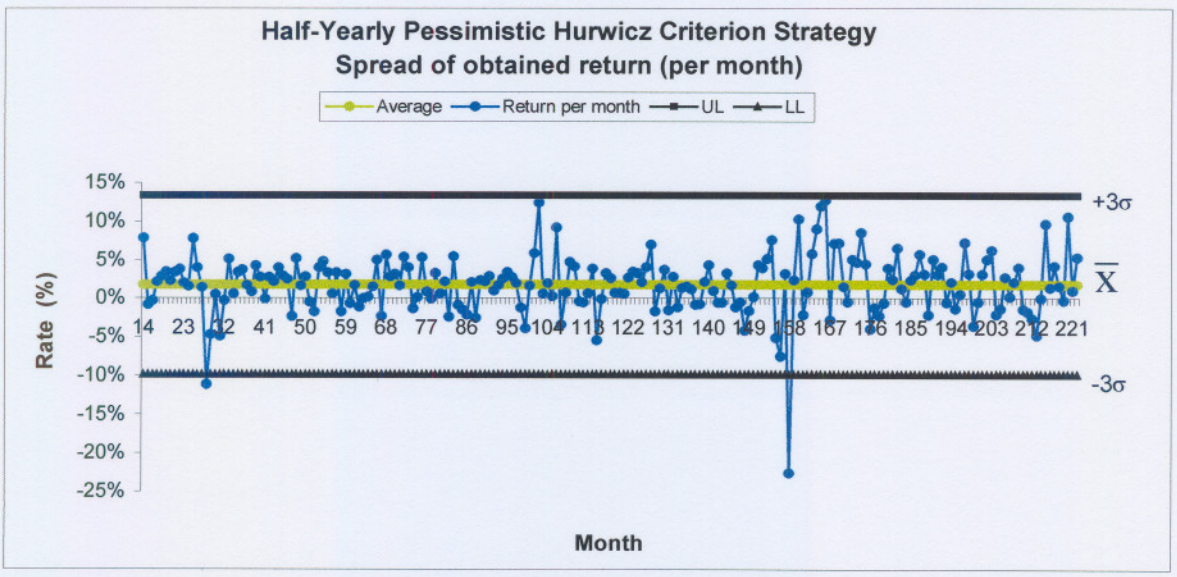
1.64% per month and the effective interest rate for the yearly pessimistic Hurwicz criterion strategy is 1.40% per month.



Graph 3.52: Comparison between pessimistic Hurwicz criterion strategies

3.13.2.2 3-Sigma rule

- **Half yearly pessimistic Hurwicz criterion strategy**

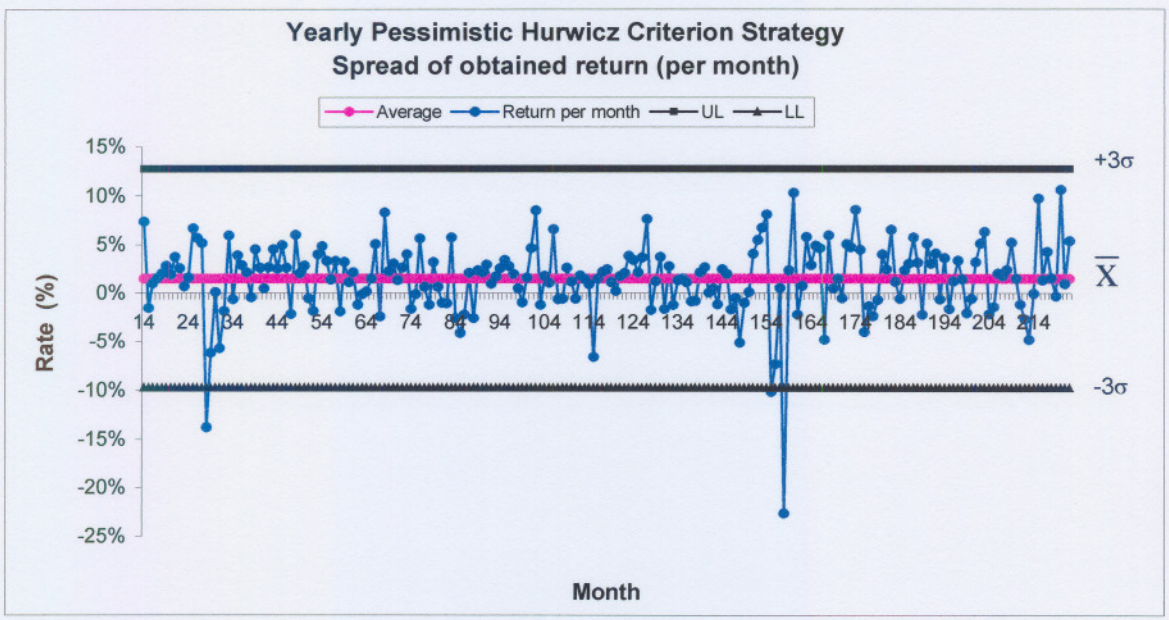


Graph 3.53: 3-Sigma rule for half yearly pessimistic Hurwicz criterion strategy

As displayed in graph 3.53, 0.96% of the return values are below the lower limit (–9.88%) and no return values are above the upper limit (13.31%). 46.41% of the return values obtained with this strategy are below the target return of 1.60%. The average return per month is 1.71% and the return/risk ratio for this strategy is 0.4435. Both these values are higher for the half yearly strategy than for the yearly strategy. The standard deviation for this strategy is 3.86 %.

- **Yearly pessimistic Hurwicz criterion strategy**

Graph 3.54 shows that 1.44% of the return values are below the lower limit of –9.76%. None of the return values are above the upper limit of 12.71%. 49.28%, of the return values obtained with the pessimistic Hurwicz criterion strategy are below the 1.60% target return. An average return of 1.48% is achieved per month and this strategy's return/risk ratio is 0.3939. The standard deviation for this strategy is 3.75 %.



Graph 3.54: 3-Sigma rule for yearly pessimistic Hurwicz criterion strategy

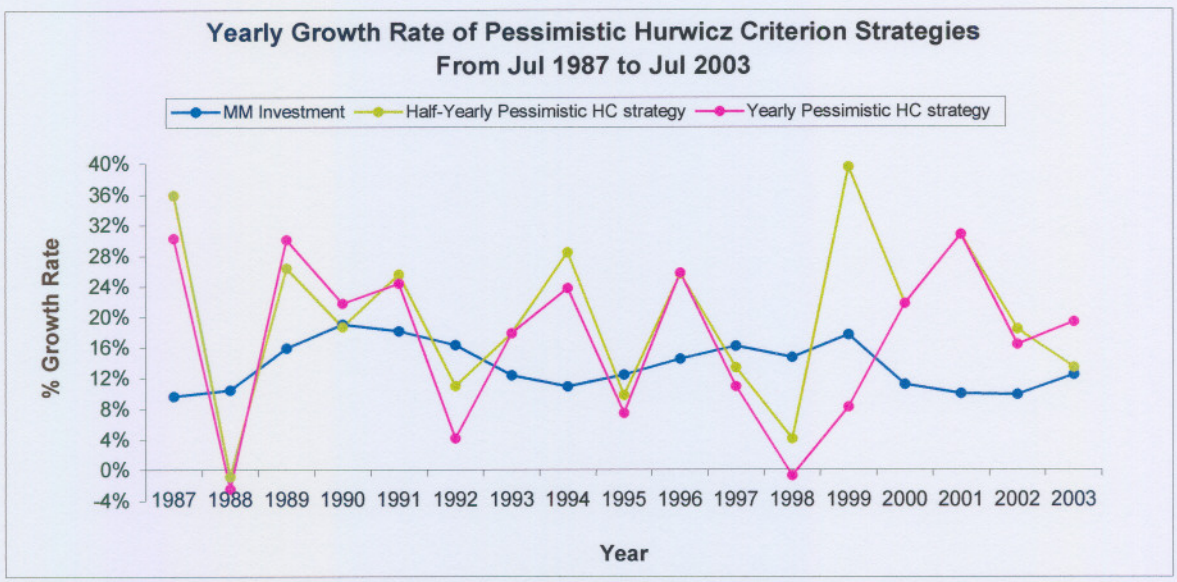
The following table provides a summary of the information obtained from the 3-sigma rule graphs for the pessimistic Hurwicz criterion strategies:

Strategy	$\overline{X}$	X < 1.60% (%)	Return/Risk ratio	X > UL (%)	X < LL (%)
Half yearly pessimistic Hurwicz criterion	1.71%	46.41%	0.4435	0.00%	0.96%
Yearly pessimistic Hurwicz criterion	1.48%	49.28%	0.3594	0.00%	1.44%

Table 3.13: Summary of pessimistic Hurwicz criterion strategies information

3.13.2.3 Yearly growth

Graph 3.55 portrays the yearly growth rates for the pessimistic Hurwicz criterion strategies. The half yearly pessimistic Hurwicz criterion strategy shows the highest average yearly growth rate of 20.10%, the average for the yearly pessimistic Hurwicz criterion strategy is 17.07%. The half yearly pessimistic Hurwicz criterion strategy obtained the highest yearly growth rate of 39.56% in 1999 and the yearly pessimistic Hurwicz criterion strategy obtained the lowest yearly growth rate of –2.45% in 1988.



Graph 3.55: Yearly growth rates for pessimistic Hurwicz criterion strategies

3.14 Conclusion

This chapter investigated some decision-making strategies that do not use risk measures in the decision making process, i.e. decision making under uncertainty. The half yearly pessimistic Hurwicz criterion strategy performed the best, in terms of total investment growth.

S_3 achieved the best result for investment in the individual funds, thus if an investor decided to stay in one fund over the entire investment period. The results for the perfect information models are also represented.

CHAPTER 4
RESULTS OF STRATEGIES USING RISK MEASURES IN THE DECISION MAKING

4.1 Introduction

Render *et al.* (2003:81) indicates that in decision-making under risk, there are a number of possible outcomes for each alternative, and the decision maker knows the probabilities of the various outcomes.

This chapter presents the investment growth and spread of the obtained returns for the different decision making under risk strategies, which include:

- Monte Carlo simulation models.
- Non-linear optimisation models (Markowitz portfolio model).
- Linear programming models, with:
  - β as a risk measure and
  - Downside risk as a risk measure.

Almost all the models without the MM fund performed better - even before transaction costs were considered. Therefore, only models without the MM are demonstrated in this chapter, except where the models with the MM performed better.

4.2 Monte Carlo simulation models

Understanding risk and expressing it in meaningful terms is important in making good decisions. Therefore, according to Camm and Evans (2000:286) simulation is a powerful technique for analysing models involving probabilistic assumptions. The basis of Monte Carlo simulation is experimentation on the chance (or probabilistic) elements through random sampling. The five steps of Monte Carlo simulation is as follows (Render *et al.*, 2003: 604):

1. Set up a probability distribution for the important variables.
2. Construct a cumulative probability distribution for each of these variables.
3. Determine an interval of random numbers for each variable.
4. Generate random numbers.
5. Simulate a series of trials.

4.2.1 Historical probability distribution strategy

We acquire the probability distribution by observing how many times a fund was the best fund the previous year. The probability distribution is updated each year. Hence, all the historical data (up to that point) are used each time, when setting up the probability distribution for the next year. The historical probability distribution strategy is implemented for three different time horizons, which include:

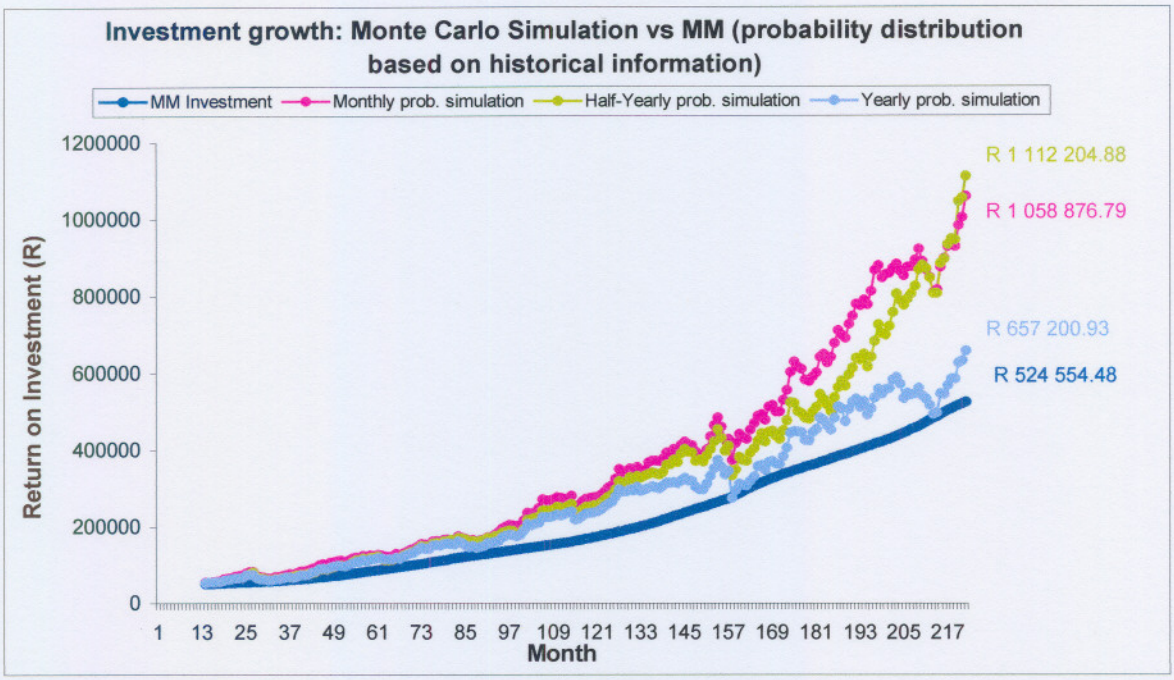
- *Monthly historical probability distribution strategy* – A random number is generated each month, to choose the fund to invest in for that month (according to the previous year's historical probability distribution).
- *Half yearly historical probability distribution strategy* - A random number is generated every six months, to choose the fund to invest in for the next six months (according to the previous year's historical probability distribution).
- *Yearly historical probability distribution strategy* - A random number is generated once a year, to choose the fund to invest in for the next year (according to the previous year's historical probability distribution).

4.2.1.1 Investment growth

Graph 4.1 shows a comparison between the three historical probability distribution strategies and the MM. All three the strategies performed better than the MM fund. The half yearly historical probability distribution strategy performed the best; with an effective interest rate of 1.50% per month and the effective monthly interest rates for the monthly and yearly historical probability distribution strategies are 1.47% and 1.24% per month, respectively.

The effective monthly interest rate is determined with the following formula:

$$r = \left(\frac{FV}{PV}\right)^{\frac{1}{n}} - 1$$

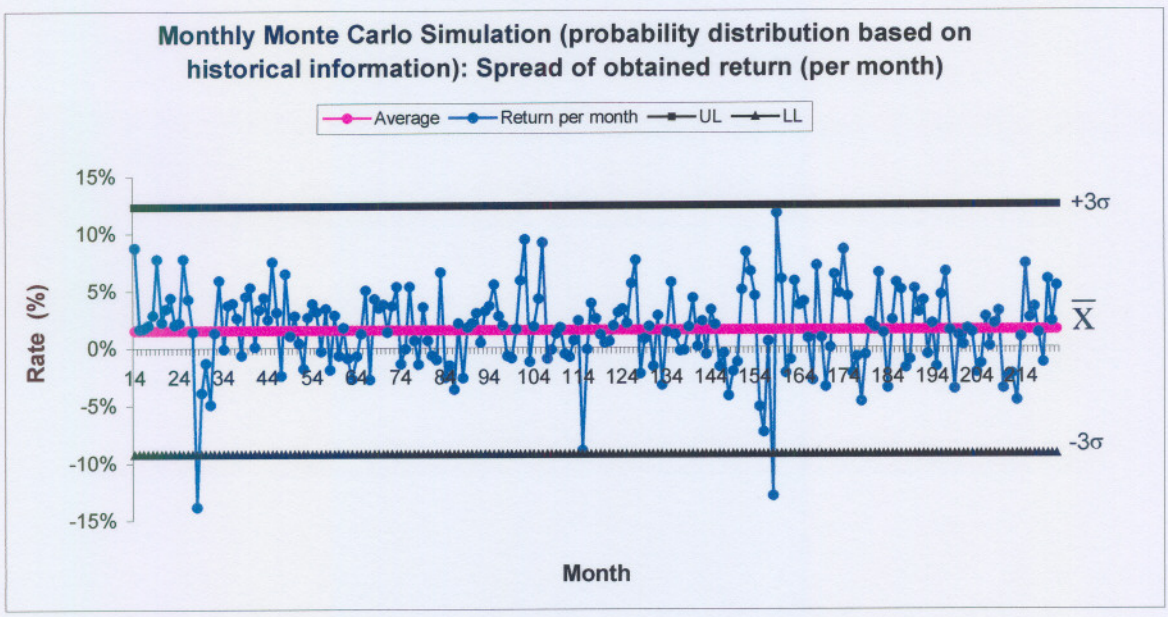


Graph 4.1: Comparison between historical probability distribution strategies

4.2.1.2 3-Sigma rule

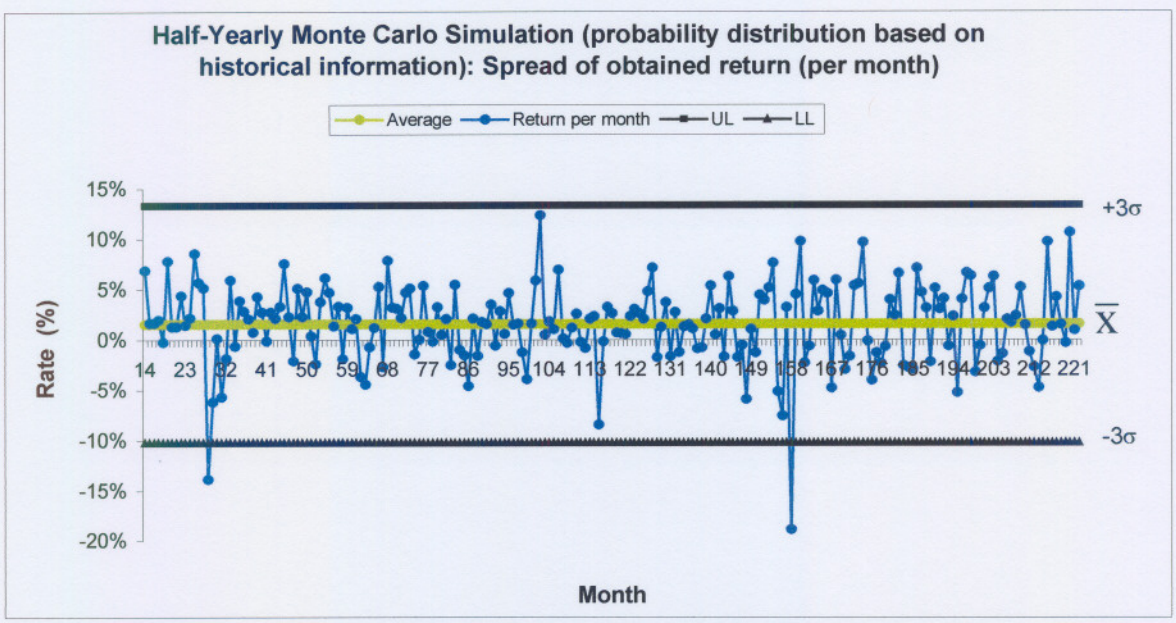
- **Monthly historical probability distribution strategy**

The following graph illustrates that 0.96% of the return values are below the lower limit of -9.27%. No return values are above the upper limit of 12.34%. 47.37% of the return values obtained with this strategy were below the target return of 1.60%. The average return that is achieved per month is 1.54% and the strategy's return/risk ratio is 0.4265. The return/risk ratio (return to risk ratio) is a measure of the return earned per unit of risk. The standard deviation for this strategy is 3.60%.



Graph 4.2: 3-Sigma rule for monthly historical probability distribution strategy

- **Half yearly historical probability distribution strategy**



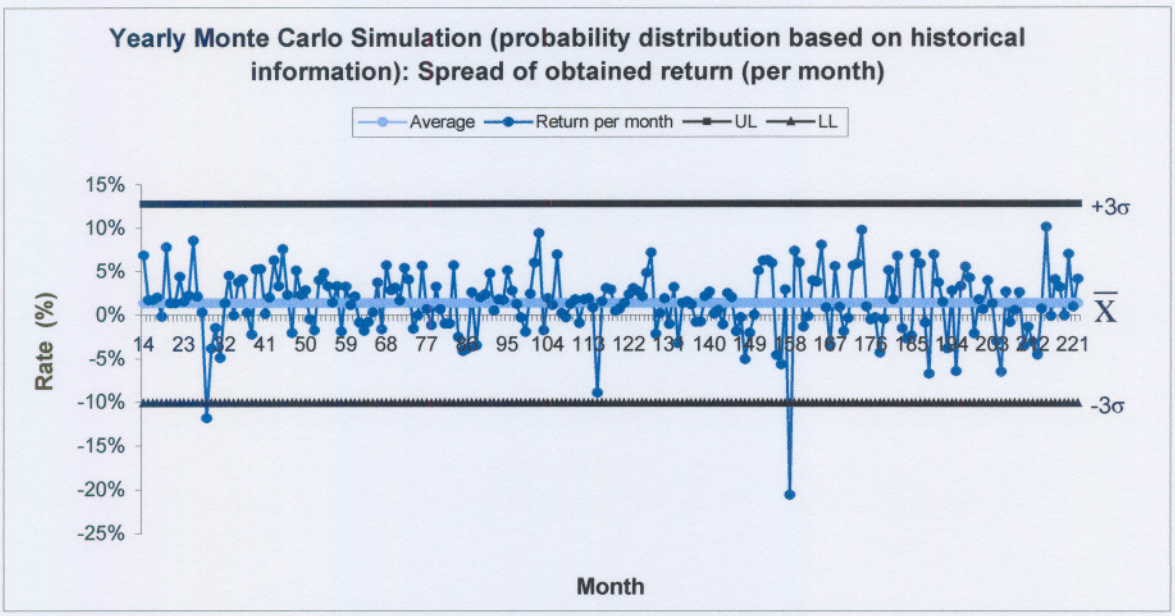
Graph 4.3: 3-Sigma rule for half yearly historical probability distribution strategy

Graph 4.3 displays that 0.96% of the return values are below the lower limit of –10.18% but none of the return values are above the upper limit of 13.33%. 47.85% of the return values obtained with this strategy are below the target return of 1.60%. The average return is 1.57%

per month and the return/risk ratio for this strategy is 0.4014. The standard deviation for this strategy is 3.60%. The standard deviation for this strategy is 3.60%.

- **Yearly historical probability distribution strategy**

The following graph indicates that 0.96% of the return values are below the lower limit of -10.11%. No return values are above the upper limit of 12.74%. The graph also shows that more than half (51.20%) of the return values are below the 1.60% target return. An average return of 1.31% is achieved per month. This strategy's return/risk ratio is 0.3450. The standard deviation for this strategy is 3.81%.



Graph 4.4: 3-Sigma rule for yearly historical probability distribution strategy

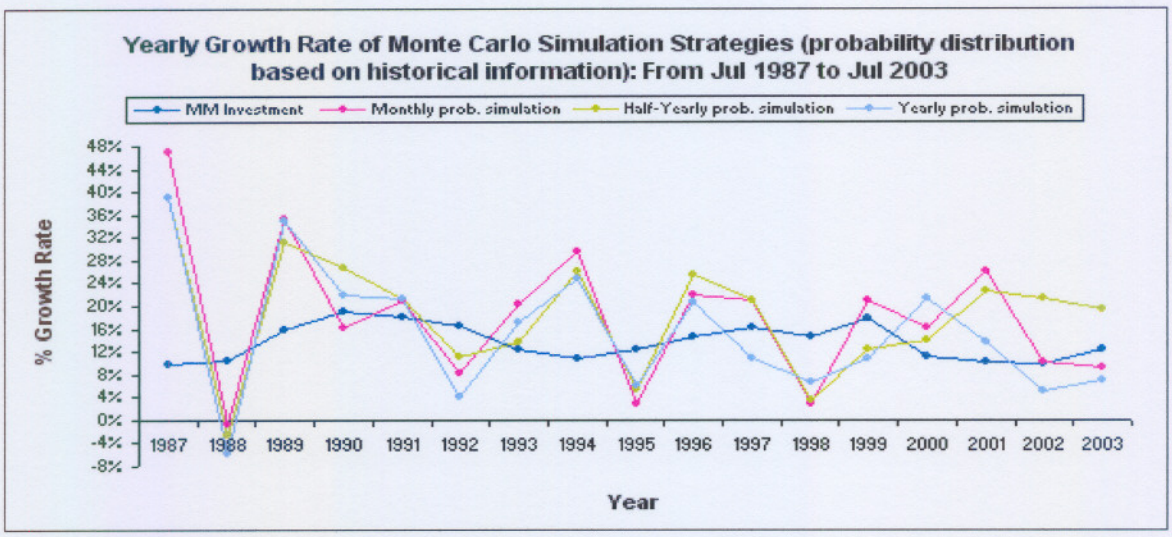
The following table provides a summary of the information obtained from the 3-sigma rule graphs for the historical probability distribution strategies ($\overline{X}$ ⇒ average return per month, X ⇒ returns, UL ⇒ upper limit and LL ⇒ lower limit):

Strategy	$\overline{X}$	X < 1.60% (%)	Return/Risk ratio	X > UL (%)	X < LL (%)
Monthly historical probability	1.54%	47.37%	0.4265	0.00%	0.96%
Half yearly historical probability	1.57%	47.85%	0.4014	0.00%	0.96%
Yearly historical probability	1.31%	51.20%	0.3450	0.00%	0.96%

Table 4.1: Summary of historical probability distribution strategies information

4.2.1.3 Yearly growth

Graph 4.5 portrays the yearly growth rates for the historical probability distribution strategies. The half yearly historical probability distribution strategy shows the highest average yearly growth rate of 18.27%. The averages for the monthly and the yearly strategies are 18.09% and 15.26% respectively. The yearly historical probability distribution strategy obtained the minimum yearly growth rate of –5.79% in 1998 and the monthly historical probability distribution strategy obtained the maximum yearly growth rate of 46.91% in 1987.



Graph 4.5: Yearly growth rates for historical probability distribution strategies

4.2.2 Even or equally likely probability strategy

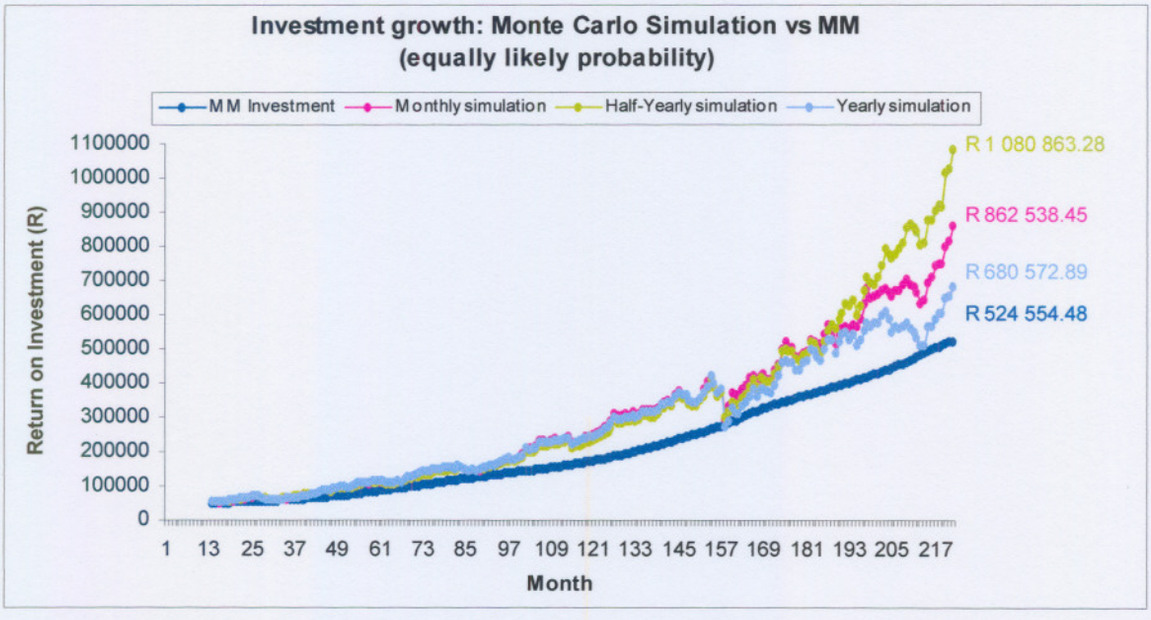
The probability for each event occurring, i.e. the chance of a fund being chosen, is 1/7 (= 0.1429 or 14.29%), since each event is equally likely to occur (Giordano *et al.*, 20003:187). The even probability strategy is implemented for three different time horizons, which include:

- *Monthly even probability strategy* – A random number is generated each month, to choose the fund to invest in for that month according to the probability distribution.
- *Half yearly even probability strategy* - A random number is generated every six months, to choose the fund to invest in for the next six months according to the probability distribution).

- *Yearly even probability strategy* - A random number is generated once a year, to choose the fund to invest in for the next year according to the probability distribution.

4.2.2.1 Investment growth

Graph 4.6 illustrates a comparison between the three even probability strategies and the MM. All three the strategies performed better than the MM fund. The half yearly even probability strategy performed the best, with a monthly effective interest rate of 1.48%. The effective interest rates for the monthly and yearly even probability strategies are 1.37% and 1.26% correspondingly.



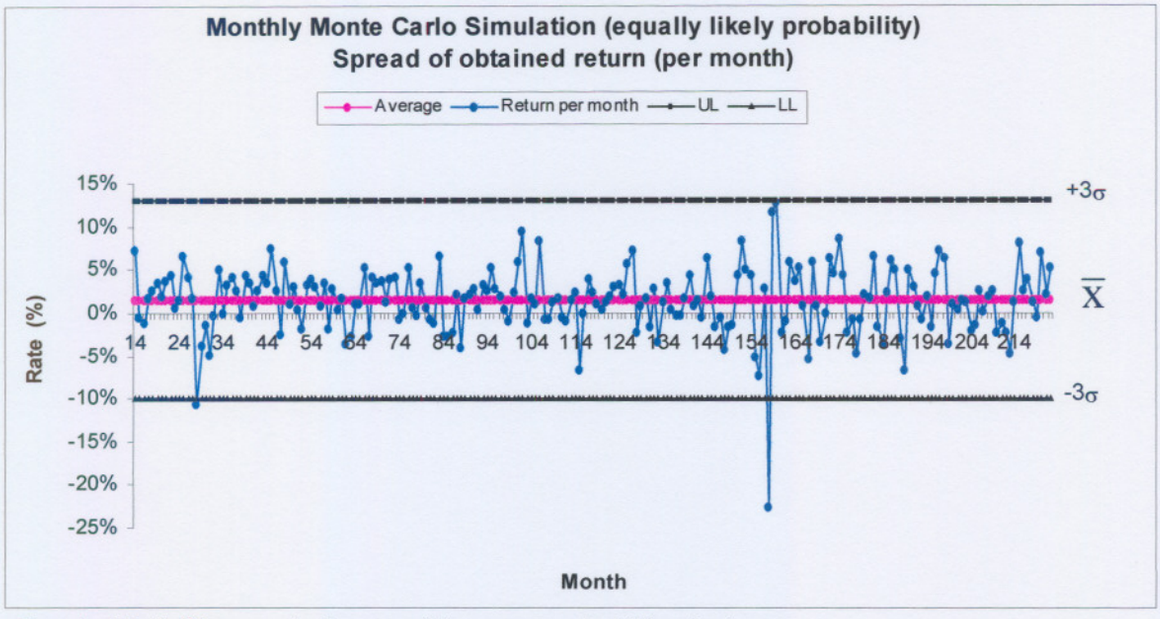
Graph 4.6: Comparison between even probability strategies

4.2.2.2 3-Sigma rule

- **Monthly even probability strategy**

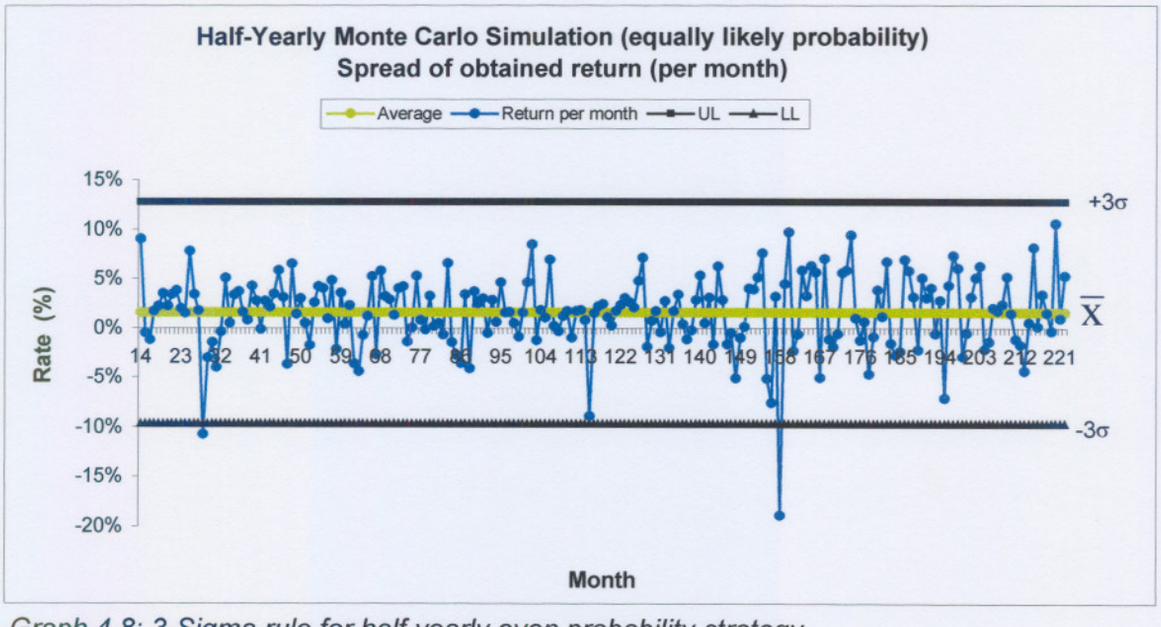
The following graph illustrates that 0.96% of the return values are below the lower limit (-10.07%) and none of the return values are above the upper limit (12.96%). The graph shows that 49.76% of the return values were below the target return of 1.60%. The average

return that is achieved per month is 1.45% and the strategy's return/risk ratio is 0.3769. The standard deviation is 3.84%.



Graph 4.7: 3-Sigma rule for monthly even probability strategy

- **Half yearly even probability strategy**

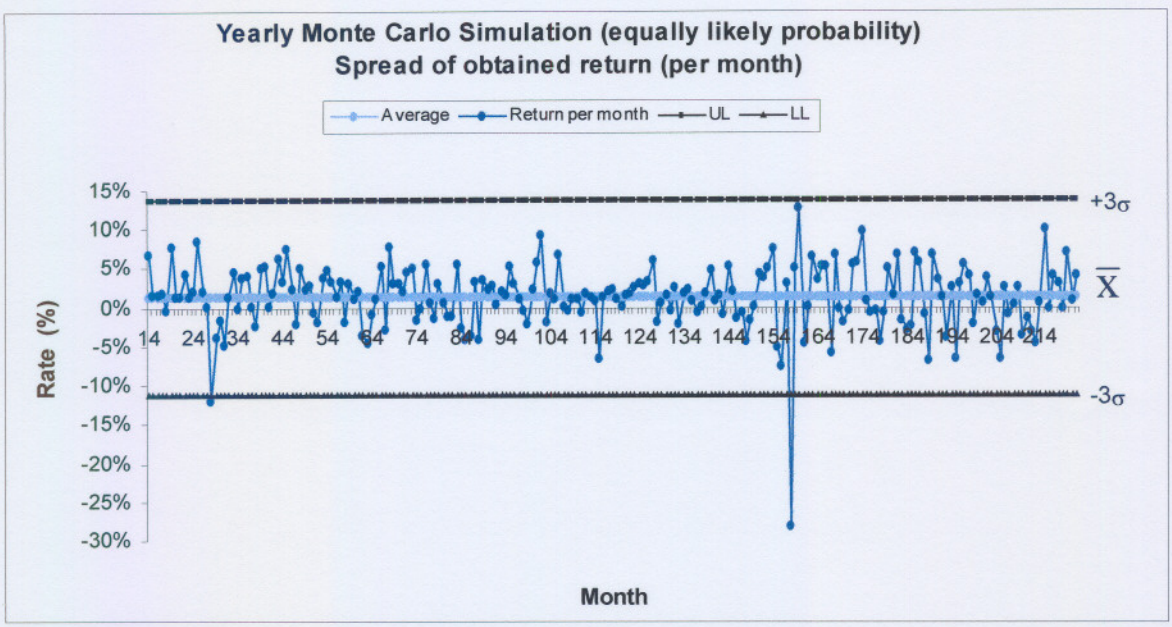


Graph 4.8: 3-Sigma rule for half yearly even probability strategy

Graph 4.8 presents that 0.96% of the return values are below the lower limit of –9.63% and none of the return values are above the upper limit of 12.73%. 47.85% of the return values are below the target return of 1.60%. The average return is 1.55% per month and the return/risk ratio for this strategy is 0.4162. The standard deviation for this strategy is 3.73%.

- **Yearly even probability strategy**

Graph 4.9 indicates that 0.96% of the return values are below the lower limit (-11.13%). No return values are above the upper limit (13.83%). The graph shows that more than half (50.24%) of the return values are below the 1.60% target return. An average return of 1.35% is achieved per month and the strategy's return/risk ratio is 0.3239. The standard deviation for this strategy is 4.16%.



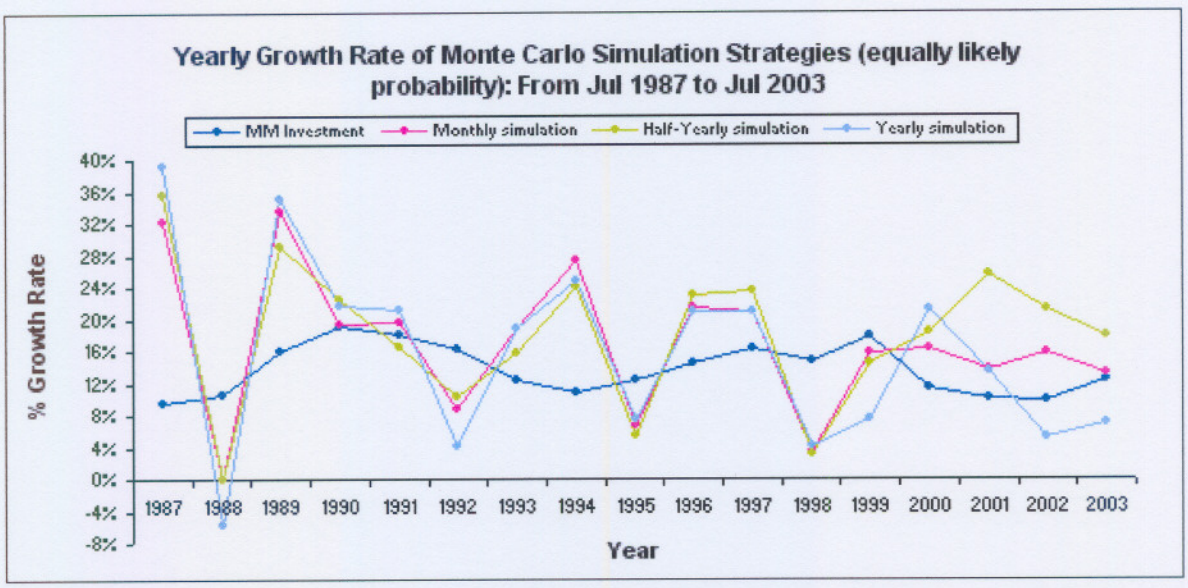
Graph 4.9: 3-Sigma rule for yearly even probability strategy

Table 4.2 presents a summary of the information obtained from the 3-sigma rule graphs for the even probability strategies ($\overline{X}$ ⇒ average return per month, X ⇒ returns, UL ⇒ upper limit and LL ⇒ lower limit):

Strategy	$\overline{X}$	X < 1.60% (%)	Return/Risk ratio	X > UL (%)	X < LL (%)
Monthly even probability	1.45%	49.76%	0.3769	0.00%	0.96%
Half yearly even probability	1.55%	47.85%	0.4162	0.00%	0.96%
Yearly even probability	1.35%	50.24%	0.3239	0.00%	0.96%

Table 4.2: Summary of even probability strategies information

4.2.2.3 Yearly growth



Graph 4.10: Yearly growth rates for even probability strategies

The graph above displays the yearly growth rates for the even probability strategies. The half yearly even probability strategy shows the highest average yearly growth rate of 18.01%. The averages for the monthly and the yearly strategies are 16.89% and 15.68% respectively. The yearly even probability strategy obtained the minimum and the maximum yearly growth rates of –5.79% in 1998 and 39.18% in 1987, correspondingly.

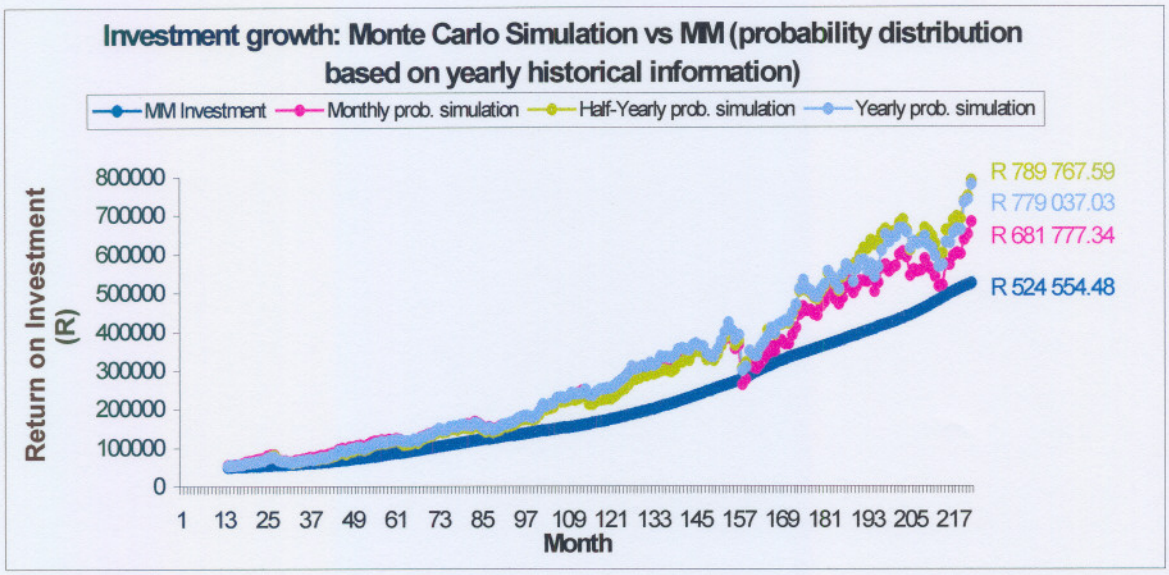
4.2.3 Previous year probability distribution strategy

We acquire the probability distribution by observing how many times a fund was the best fund the previous year. Unlike the historical probability distribution strategy, the probability distribution is not updated each year. Hence, only the previous year's data are used each

time, when setting up the probability distribution for the next year. The previous year probability distribution strategy is applied for three different time horizons, which include:

- *Monthly previous year probability distribution strategy* – A random number is generated each month, to choose the fund to invest in for that month according to the previous year's probability distribution.
- *Half yearly previous year probability distribution strategy* - A random number is generated every six months, to choose the fund to invest in for the next six months according to the previous year's probability distribution).
- *Yearly previous year probability distribution strategy* - A random number is generated once a year, to choose the fund to invest in for the next year according to the previous year's probability distribution.

4.2.3.1 Investment growth



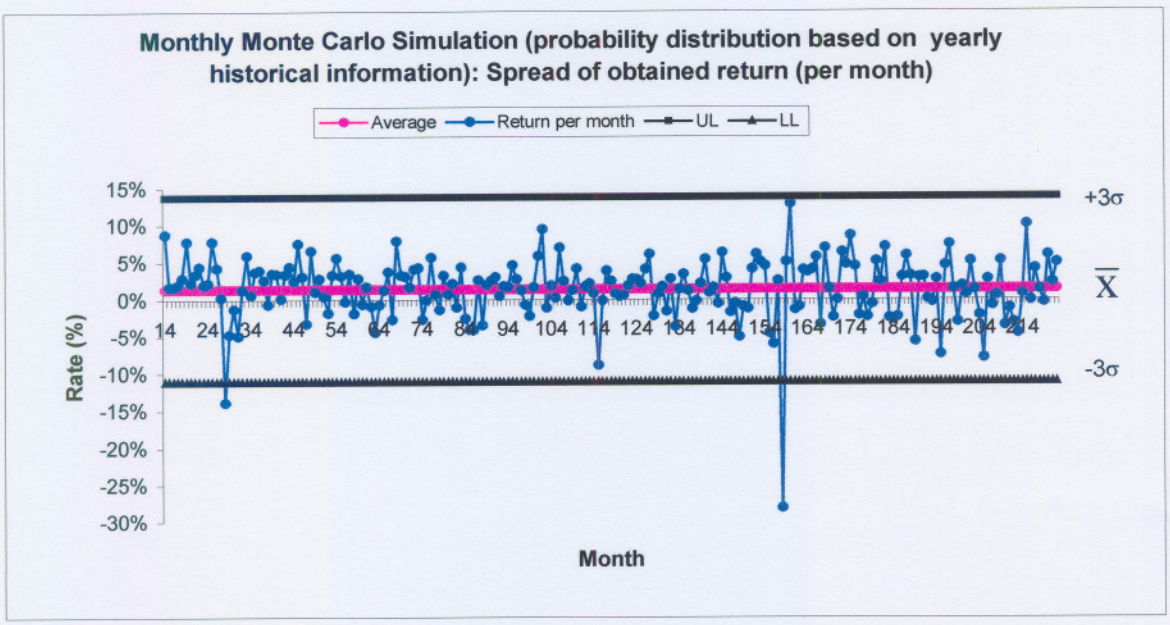
Graph 4.11: Comparison between previous year probability distribution strategies

Graph 4.11 shows a comparison between the three previous year probability distribution strategies and the MM. All three the strategies performed better than the MM fund. The half yearly previous year probability distribution strategy performed the best, with an effective

interest rate of 1.33% per month. The monthly effective interest rates for the monthly and yearly previous year probability distribution strategies are 1.26% and 1.32% correspondingly.

4.2.3.2 3-Sigma rule

- **Monthly previous year probability distribution strategy**

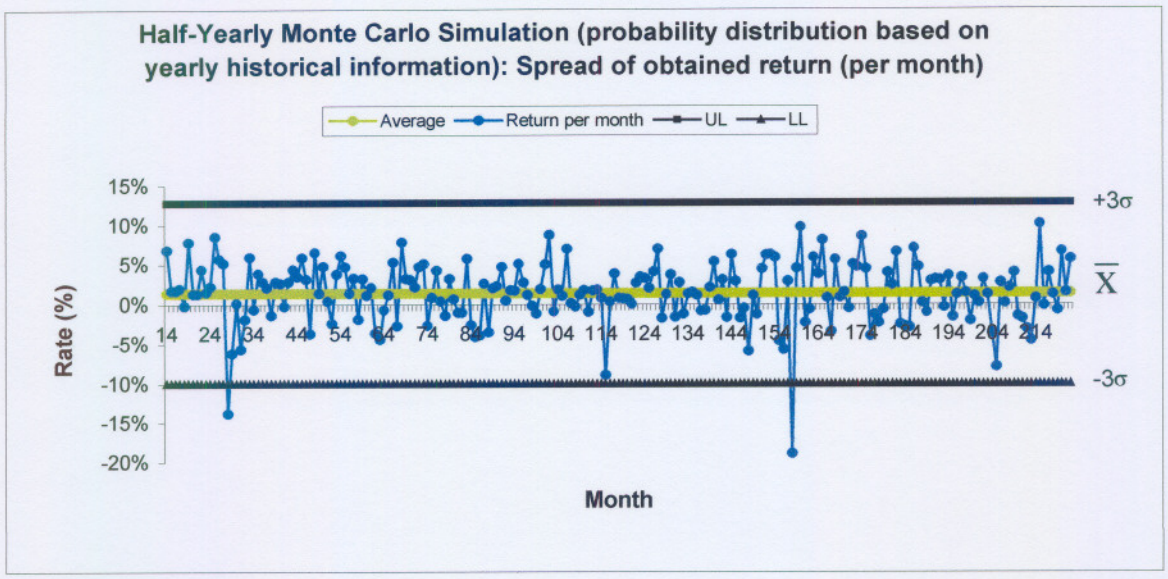


Graph 4.12: 3-Sigma rule for monthly previous year probability distribution strategy

Graph 4.12 shows that 0.96% of the return values are below the lower limit (-11.08%) and none of the return values are above the upper limit (13.78%). 46.89% of the return values are below the target return of 1.60%. The average return is 1.35% per month and the strategy's return/risk ratio is 0.3253. The standard deviation for this strategy is 4.14%.

- **Half yearly previous year probability distribution strategy**

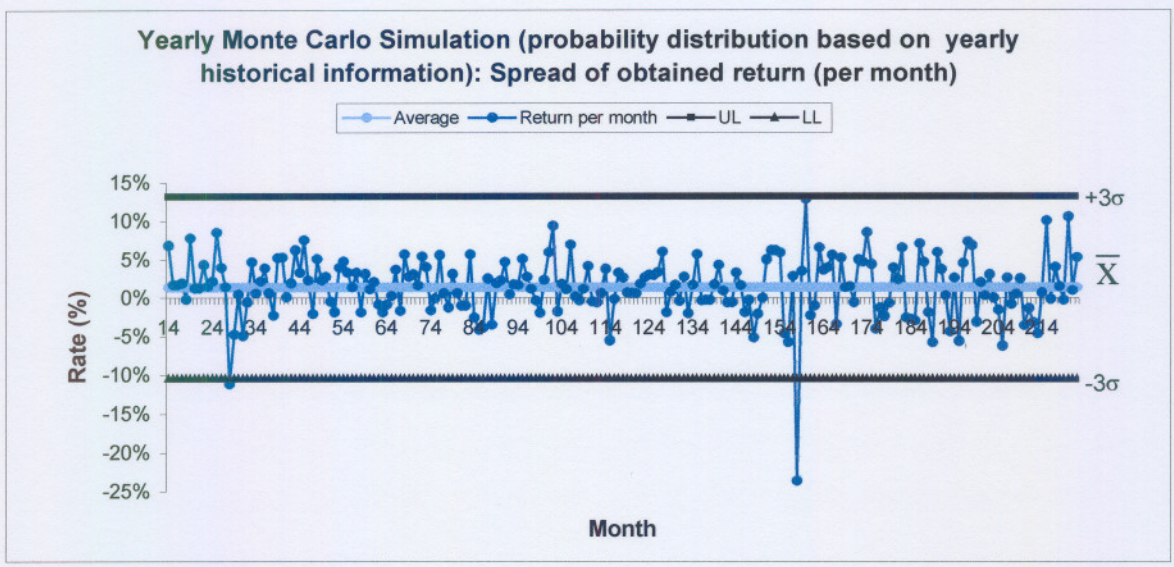
Graph 4.13 portrays that 0.96% of the return values are below the lower limit of –10.01% and none of the return values are above the upper limit of 12.81%. More than half of the return values, i.e. 51.20%, are below the target return of 1.60%. The average return is 1.40% per month and the return/risk ratio for this strategy is 0.3687. The standard deviation for this strategy is 3.80%.



Graph 4.13: 3-Sigma rule for half yearly previous year probability distribution strategy

- **Yearly previous year probability distribution strategy**

The following graph illustrates that 0.96% of the return values are below the lower limit of -10.40% and none of the return values are above the upper limit of 13.21%. More than half (50.24%) of the return values are below the 1.60% target return. An average return of 1.40% is achieved per month. The return/risk ratio for the strategy is 0.3562. The standard deviation for this strategy is 3.93%.



Graph 4.14: 3-Sigma rule for yearly previous year probability distribution strategy

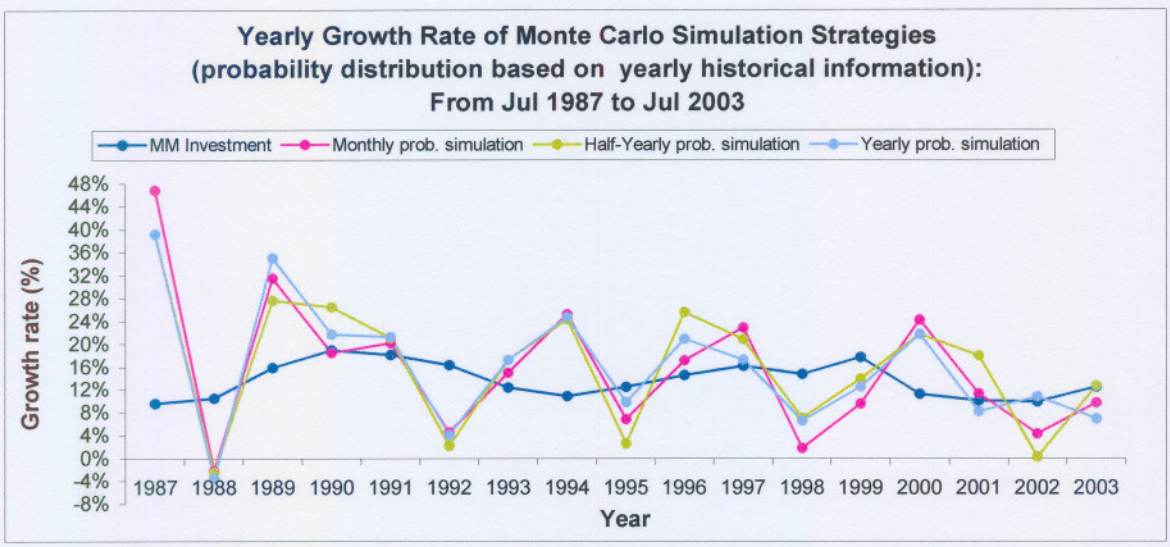
The following table provides a summary of the information obtained from the 3-sigma rule graphs for the previous year probability distribution strategies:

Strategy	$\overline{X}$	X < 1.60% (%)	Return/Risk ratio	X > UL (%)	X < LL (%)
Monthly previous year probability	1.35%	46.89%	0.3253	0.00%	0.96%
Half yearly previous year probability	1.40%	51.20%	0.3487	0.00%	0.96%
Yearly previous year probability	1.40%	50.24%	0.3562	0.00%	0.96%

Table 4.3: Summary of previous year probability distribution strategies information

4.2.3.3 Yearly growth

The following graph displays the yearly growth rates for the previous year probability strategies:



Graph 4.15: Yearly growth rates for previous year probability distribution strategies

The half yearly previous year probability distribution strategy shows the highest average yearly growth rate of 16.40%. The averages for the monthly and yearly strategies are 15.75% and 16.16% respectively. The monthly previous year probability distribution strategy obtained the maximum yearly growth rate of 46.91% in 1987 and the yearly previous year probability distribution strategy realized the minimum yearly growth rate of –3.50% in 1988.

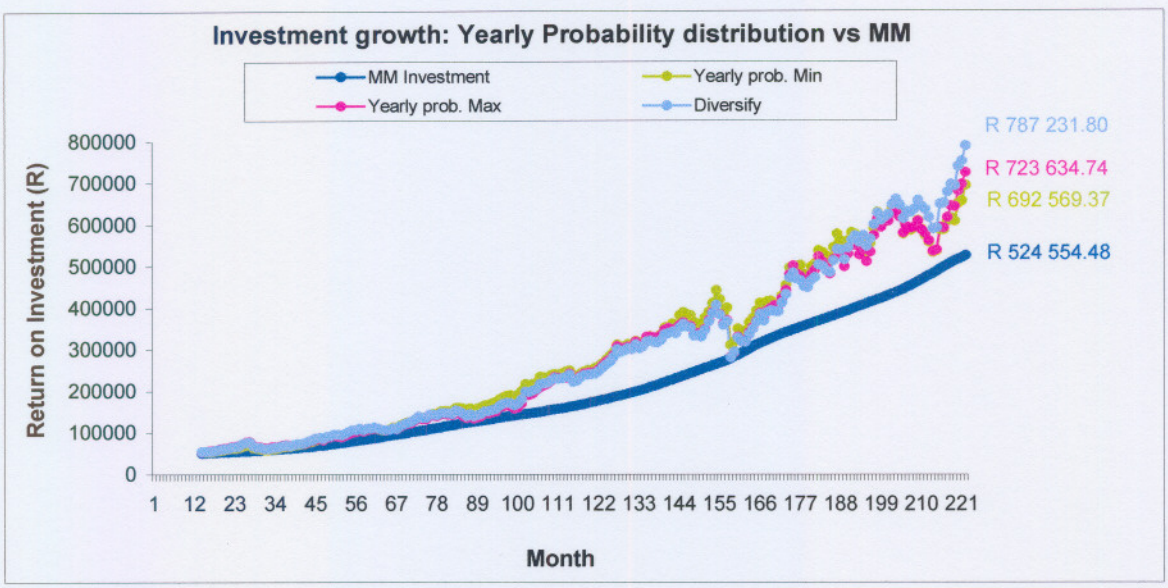
4.3 Strategies that use the probability distribution to make decisions

The probability distribution is obtained by observing how many times a fund was the best fund the previous year. The following three strategies are applied on a yearly basis. The strategies use the previous year's probability distribution, to select the funds to invest in.

- *Diversify strategy* - Invest in the funds according to their probability distribution of the previous year, for the next year.
- *Max probability strategy* - Invest in the fund (for the next year) that had the maximum probability, according to the probability distribution of the previous year.
- *Min probability strategy* - Invest in the fund (for the next year) that had the minimum probability, according to the probability distribution of the past year.

4.3.1 Investment growth

The graph below shows a comparison between the three probability distribution strategies and the MM:



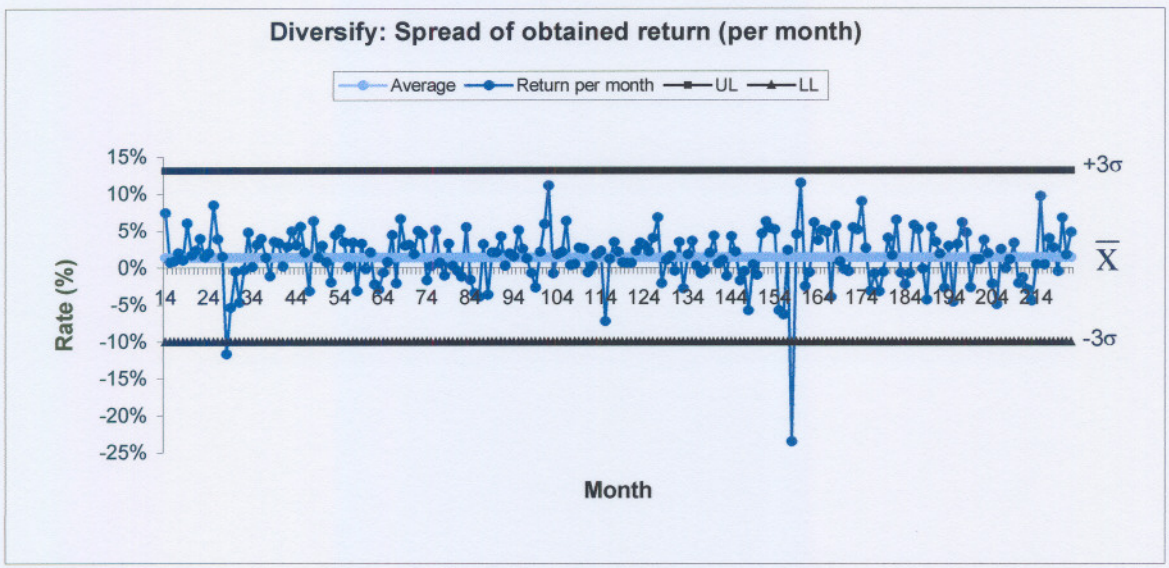
Graph 4.16: Comparison between probability distribution strategies

All three the probability distribution strategies performed better than the MM fund. The diversify strategy achieve the best result, with a monthly effective interest rate of 1.33%. The max probability strategy and the min probability strategy show effective interest rates of 1.29% and 1.27% per month, respectively.

4.3.2 3-Sigma rule

- **Diversify strategy**

Graph 4.17 illustrates that 0.96% of the return values are below the lower limit (-9.98%). No return values are above the upper limit (12.78%). 48.33% of the return values are below the target return of 1.60%. The average return that is achieved per month is 1.40% and the strategy's return/risk ratio is 0.3695. The standard deviation for this strategy is 3.79%.

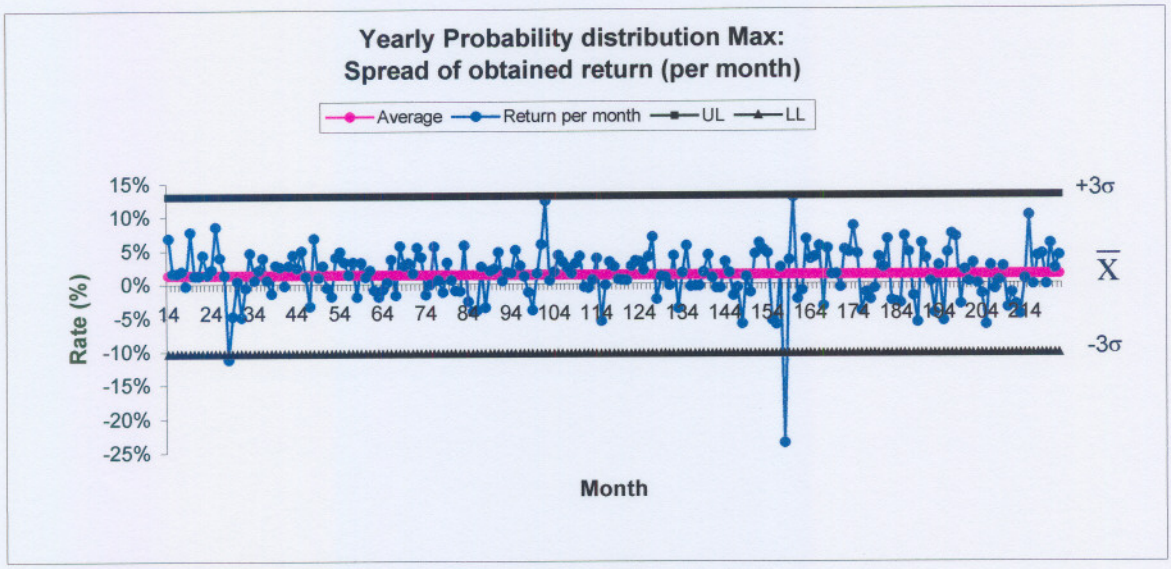


Graph 4.17: 3-Sigma rule for diversify strategy

- **Max probability strategy**

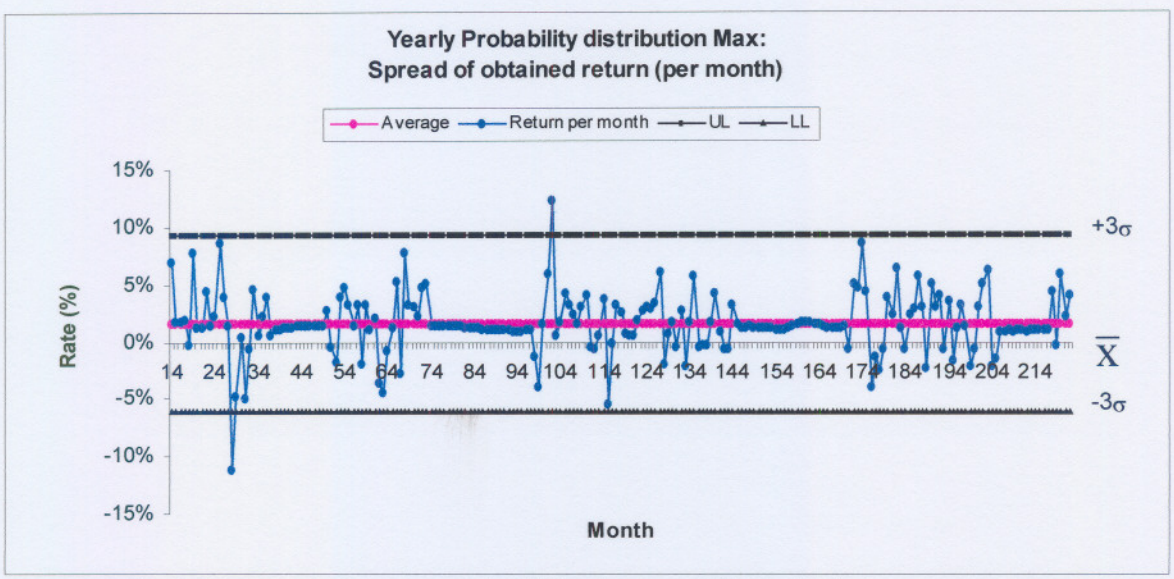
According to the following graph, 0.96% of the return values are below the lower limit of –10.37% but none of the return values are above the upper limit of 13.10%. 48.80% of the return values that are obtained with this strategy are below the target return of 1.60%. The

average return is 1.37% per month and the return/risk ratio for this strategy is 0.3488. The standard deviation for this strategy is 3.91%.



Graph 4.18: 3-Sigma rule for max probability strategy

The max probability strategy shows a higher return when the MM is included, even when transaction costs are included. This strategy has a monthly effective interest rate of 1.56%.

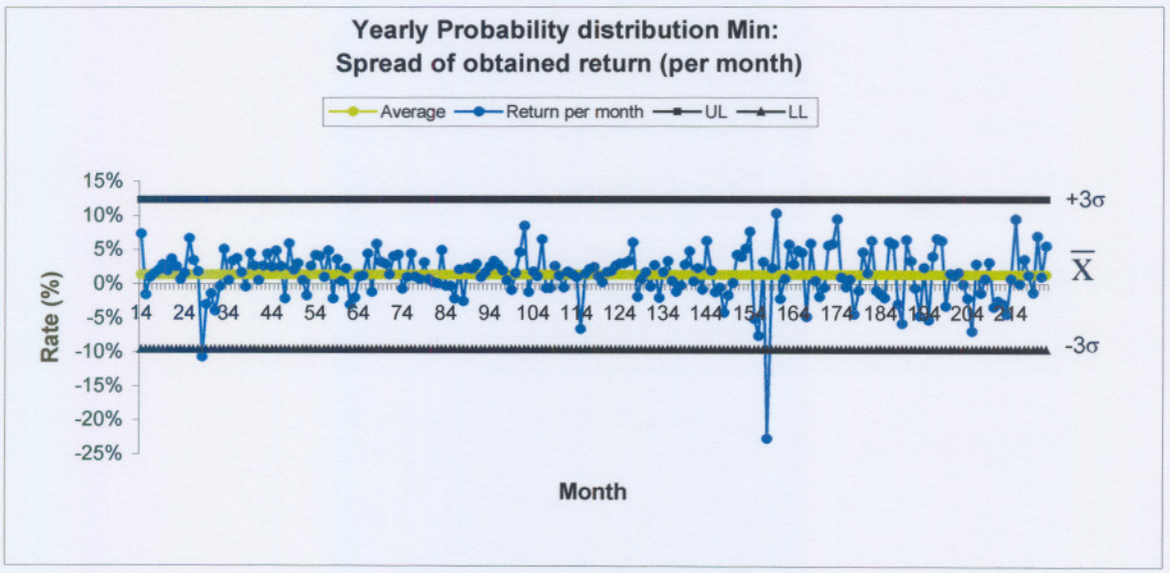


Graph 4.19: 3-Sigma rule for max probability strategy (MM included)

Graph 4.19 illustrates that one of the return values (0.48%) are below the lower limit of –6.14% and one of the return values (0.48%) are above the upper limit of 9.33%. More than half (61.24%) of the return values that are obtained with this strategy are below the target return of 1.60%. The average return is 1.60% per month and the return/risk ratio for this strategy is 0.6192. The standard deviation for this strategy is 2.58%.

- **Min probability strategy**

The following graph illustrates that 0.96% of the return values are below the lower limit of -9.64% and no return values are above the upper limit of 12.31%. More than half (50.72%) of the return values obtained with this strategy are below the 1.60% target return. An average return of 1.33% is attained per month. This strategy's return/risk ratio is 0.3647, which is the lowest for these probability distribution strategies. The standard deviation for this strategy is 3.66%.



Graph 4.20: 3-Sigma rule for min probability strategy

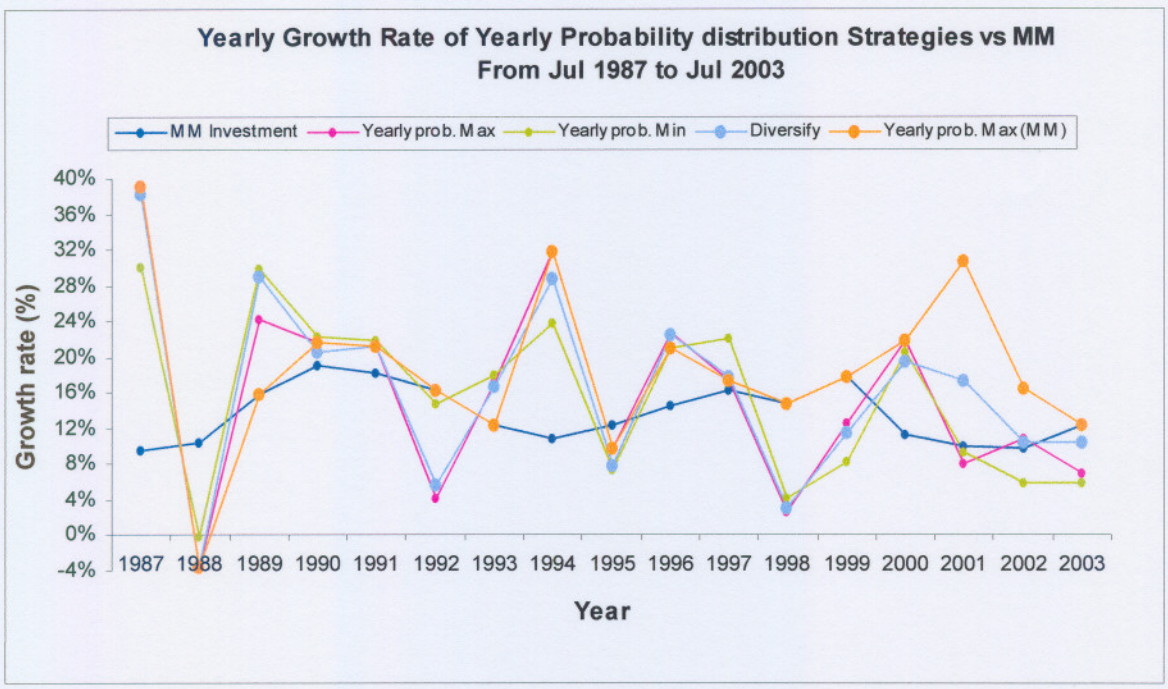
The following table provides a summary of the information obtained from the 3-sigma rule graphs for the strategies that use the probability distribution to make decisions:

Strategy	$\overline{X}$	X < 1.60% (%)	Return/Risk ratio	X > UL (%)	X < LL (%)
Diversify	1.40%	48.33%	0.3695	0.00%	0.96%
Max probability	1.37%	48.80%	0.3488	0.00%	0.96%
Max probability (with MM)	1.60%	61.24%	0.6192	0.48%	0.48%
Min probability	1.33%	50.72%	0.3647	0.00%	0.96%

Table 4.4: Summary of probability distribution strategies information

4.3.3 Yearly growth

Graph 4.21 portrays the yearly growth rates for the probability distribution strategies. The max probability strategy (with MM) shows the highest average yearly growth rate of 18.65%. The averages for the diversify, max probability distribution, and min probability distribution strategies are 16.32%, 15.81% and 15.61% respectively. The diversify strategy obtained lowest yearly growth rate of –3.68% in 1998 and the probability max (and probability max with MM) strategy obtained the highest yearly growth rate 39.18% in 1987.



Graph 4.21: Yearly growth rates for probability distribution strategies

4.4 Markowitz portfolio model

This non-linear optimisation model searches for a portfolio of funds that minimises risk, as measured by its variance. The investor has a fixed amount to invest and would like to know what percentage of the investment budget to allocate to each fund (Camm & Evans, 2000:221). The results of a second model that restricts the allocation of funds to 50% (i.e. no more than 50% of the portfolio may be invested in any one fund) are also presented. The models are implemented for two different time horizons, which include:

- *Yearly Markowitz strategy* – A Markowitz portfolio model is executed once a year to determine the portfolio of funds to invest in for the next year (based on the expected return, variance and covariance data of the previous year).
- *Half yearly Markowitz strategy* - A Markowitz portfolio model is executed every six months to determine the portfolio of funds to invest in for the next six months (based on the expected return, variance and covariance data of the past six months).

The Markowitz portfolio model has the following structure (Camm & Evans, 2000:222):

Objective:

Minimise Portfolio variance = $\sum_{i=1}^{k} s_i^2 x_i^2 + \sum_{i=1}^{k} \sum_{j>i} 2 s_{ij} x_i x_j$

Where:

x_i = fraction of the portfolio to invest in fund *i*

s_i^2 = the sample variance in the return of fund *i*

s_{ij} = the sample covariance between funds *i* and *j*

Constraints:

$\sum_{i=1}^{k} x_i = 1$ (Invest 100% of the investment budget)

$\sum_{i=1}^{k} r_i x_i \geq T$ (Expected return of the portfolio must meet or exceed some target return)

$x_1, x_2, \ldots x_k \geq 0$ (Non-negativity constraint, cannot invest a negative amount)

Where:

r_i = the expected return on fund *i*

T = the target return (for all the models the target return is set at 1.60% per month)

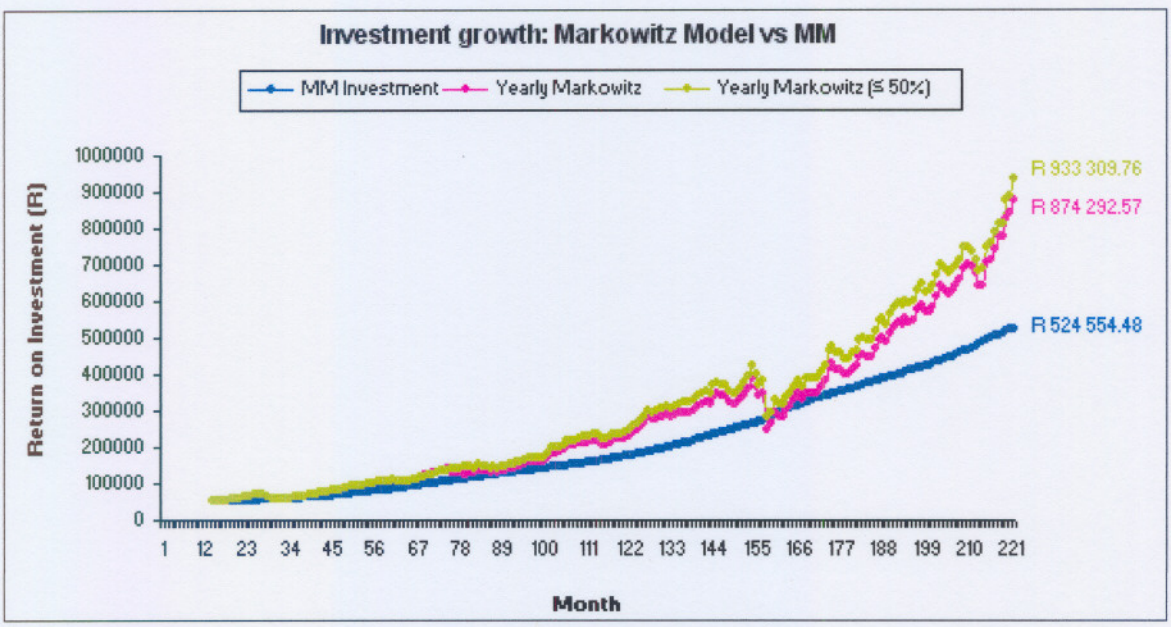
The second model attempts to diversify the portfolio more, by limiting the allocation to 50%, for any one fund. The following constraint is added for the second model:

$x_1, x_2, \ldots x_k \leq 0.5$

4.4.1 Yearly Markowitz strategy

4.4.1.1 Investment growth

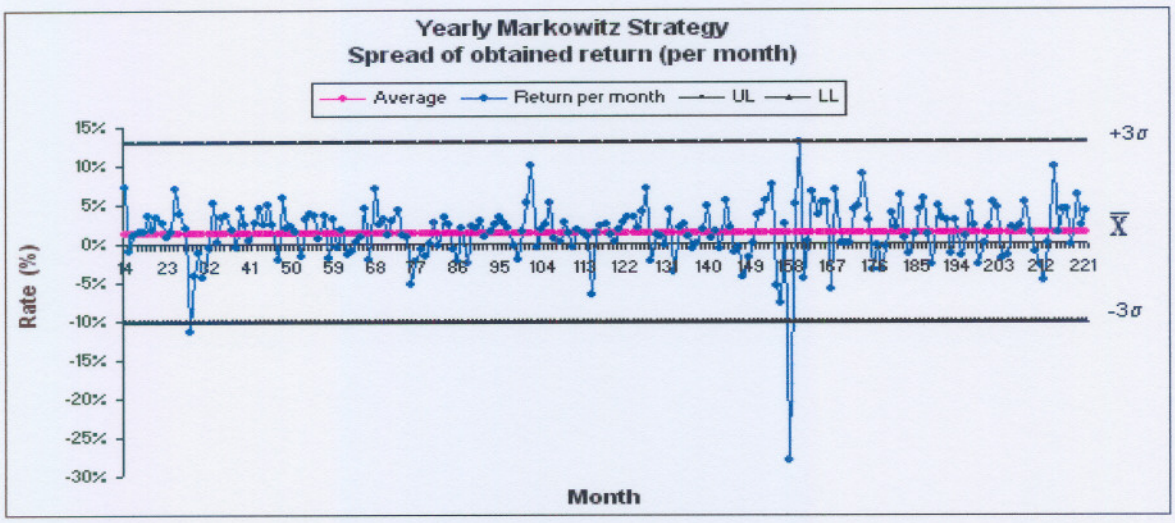
Graph 4.22 illustrates a comparison between the two yearly Markowitz strategies and the MM. Both the strategies performed better than the MM fund. The second Markowitz strategy performed the best, with an effective interest rate of 1.41% per month. The monthly effective interest rate for the first strategy is 1.38%.



Graph 4.22: Comparison between yearly Markowitz strategies

4.4.1.2 3-Sigma rule

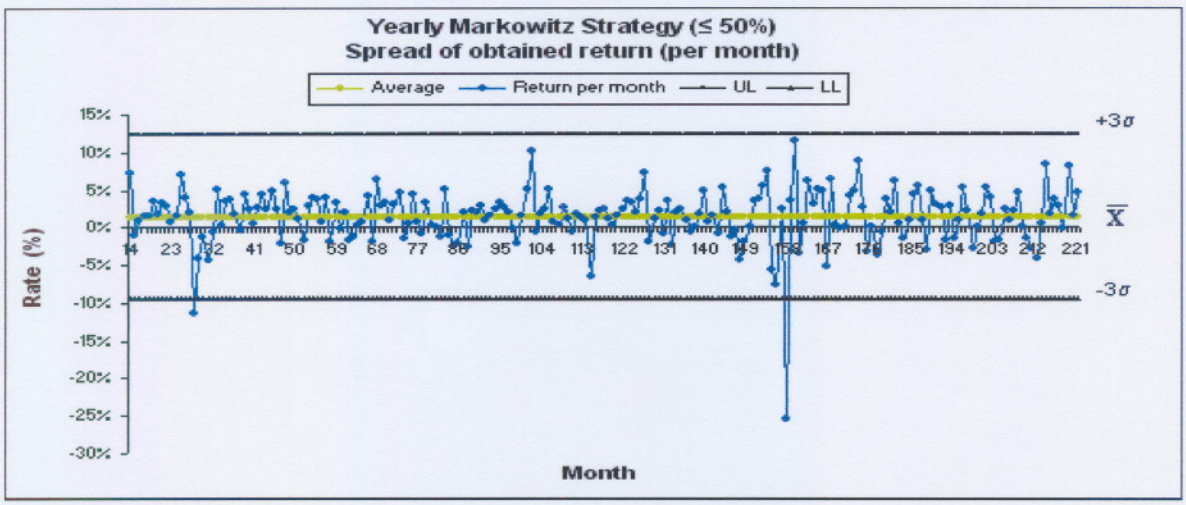
- **Yearly Markowitz strategy 1**



Graph 4.23: 3-Sigma rule for yearly Markowitz strategy 1

Graph 4.23 shows that 0.96% of the return values are below the lower limit (-9.94%) and one of the return values are above the upper limit (12.85%). 45.93% of the return values are below the target return of 1.60%. The average return that is achieved per month is 1.45% and the strategy's return/risk ratio is 0.3830. The standard deviation for this strategy is 3.80%.

- **Yearly Markowitz strategy 2**



Graph 4.24: 3-Sigma rule for yearly Markowitz strategy 2

Graph 4.24 displays that 0.96% of the return values are below the lower limit of –9.43% and no return values are above the upper limit of 12.39%. 45.93% of the return values are below the target return of 1.60%. The average return is 1.48% per month and the return/risk ratio for this strategy is 0.4066. The standard deviation for this strategy is 3.64%.

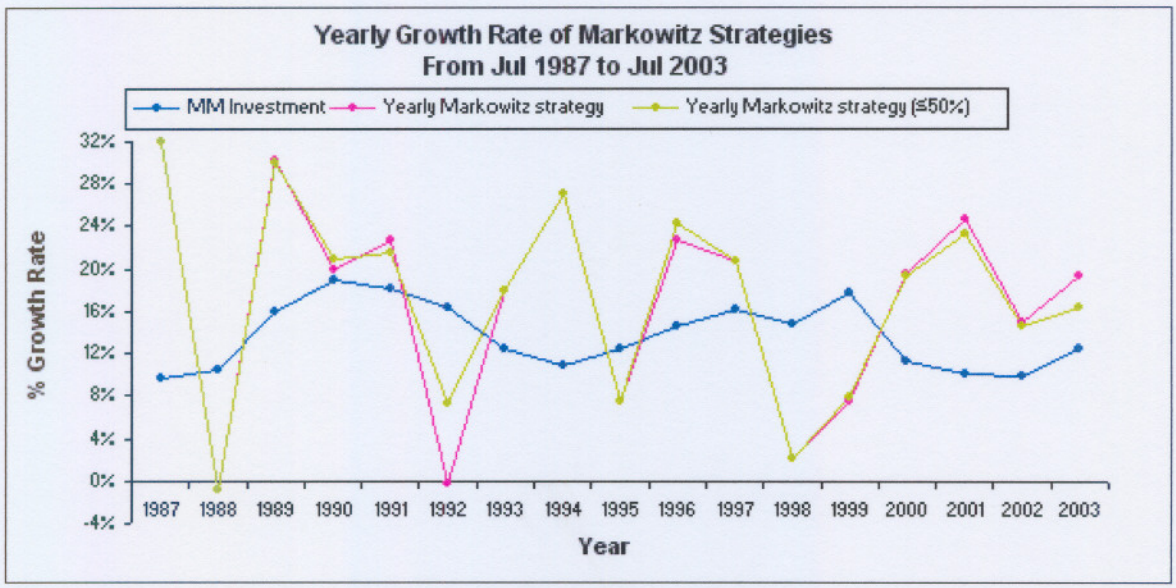
The following table provides a summary of the information obtained from the 3-sigma rule graphs for the yearly Markowitz strategies:

Strategy	$\overline{X}$	X < 1.60% (%)	Return/Risk ratio	X > UL (%)	X < LL (%)
Yearly Markowitz 1	1.45%	45.93%	0.3830	0.00%	0.96%
Yearly Markowitz 2	1.48%	45.93%	0.4066	0.00%	0.96%

Table 4.5: Summary of yearly Markowitz strategies information

4.4.1.3 Yearly growth

The following graph displays the yearly growth rates for the two yearly Markowitz strategies. Yearly Markowitz strategy 2 shows the highest average yearly growth rate of 17.17%, the yearly growth rate for the first strategy is 16.91%. The second strategy shows the maximum yearly growth rate of 31.92% in 1987 and the minimum yearly growth rate of –0.93% in 1988.

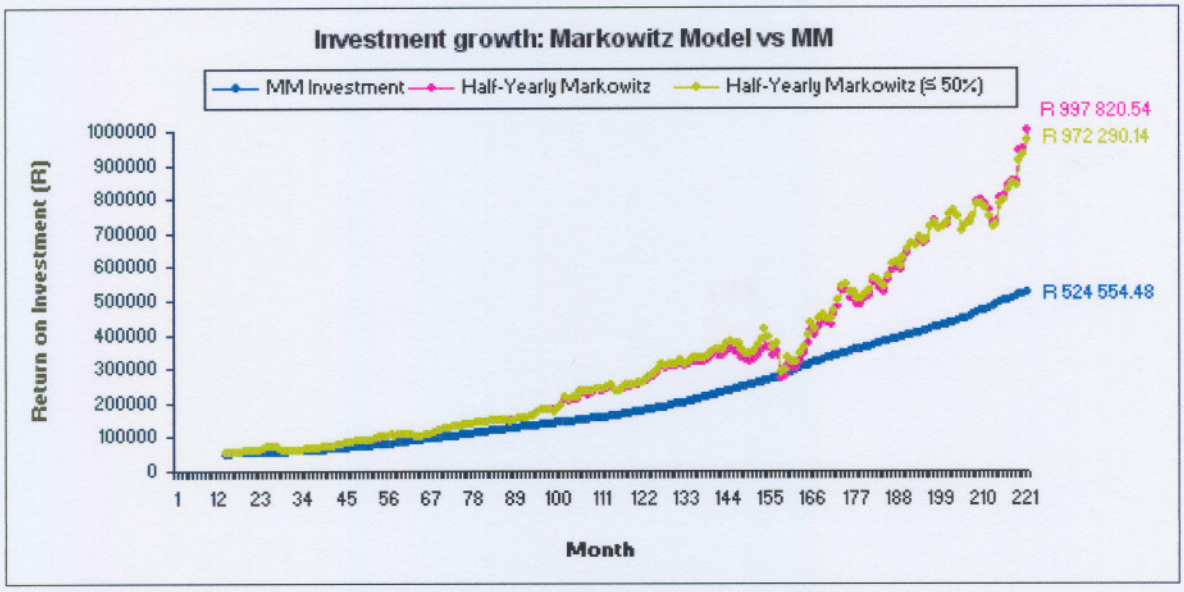


Graph 4.25: Yearly growth rates for yearly Markowitz strategies

4.4.2 Half yearly Markowitz strategy

4.4.2.1 Investment growth

The following graph illustrates the comparison between the two half yearly Markowitz strategies and the MM. Both the strategies performed better than the MM fund. Half yearly Markowitz strategy 1 performed the best, with an effective interest rate of 1.44% per month. The monthly effective interest rate for the half yearly Markowitz strategy 2 is 1.43%.

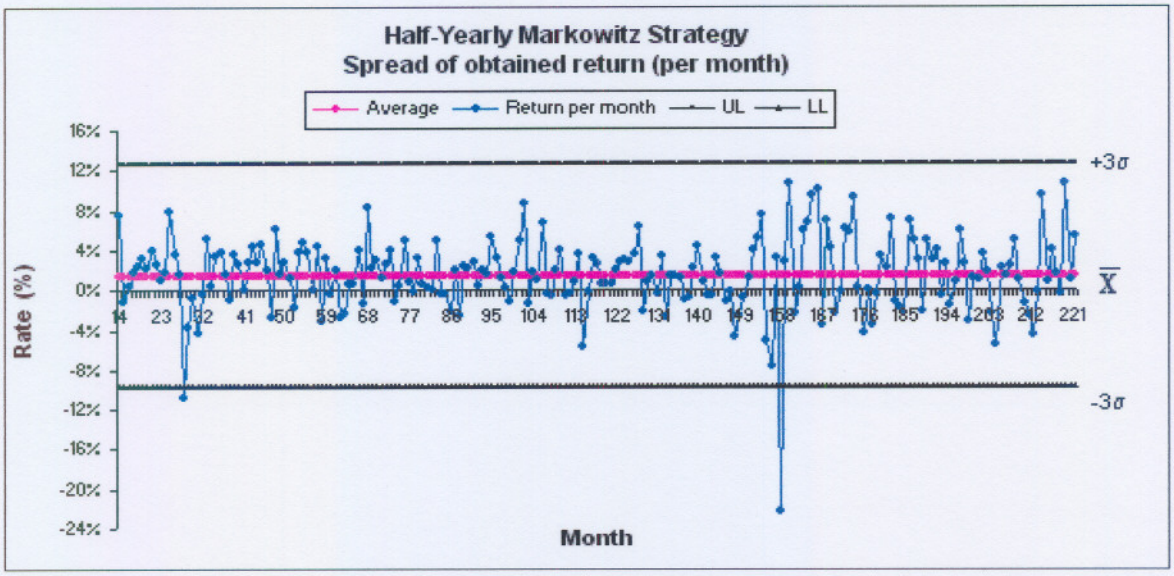


Graph 4.26: Comparison between half yearly Markowitz strategies

4.4.2.2 3-Sigma rule

- **Half yearly Markowitz strategy 1**

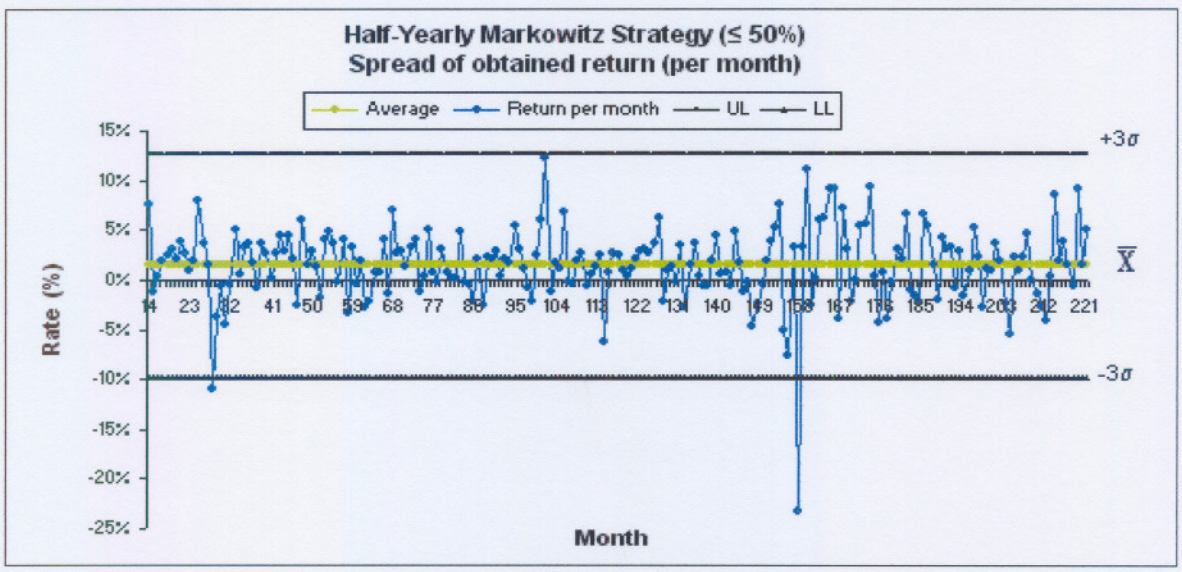
Graph 4.27 shows that 0.96% of the return values for the half yearly Markowitz strategy 1 are below the lower limit (-9.62%) and none of the return values are above the upper limit (12.65%). More than half of the return values are below the target return of 1.60%, i.e. 50.24% of the return values. The average return that is achieved per month is 1.51%. The return/risk ratio for this strategy is 0.4075. The standard deviation for this strategy is 3.71%.



Graph 4.27: 3-Sigma rule for half yearly Markowitz strategy 1

- **Half yearly Markowitz strategy 2**

The following graph shows that 0.96% of the return values are below the lower limit of –9.72%, but no return values are above the upper limit of 12.72%. 49.76% of the return values are below the target return of 1.60%. The average return is 1.50% per month and the return/risk ratio for this strategy is 0.4014. The standard deviation for this strategy is 3.74%.



Graph 4.28: 3-Sigma rule for half yearly Markowitz strategy 2

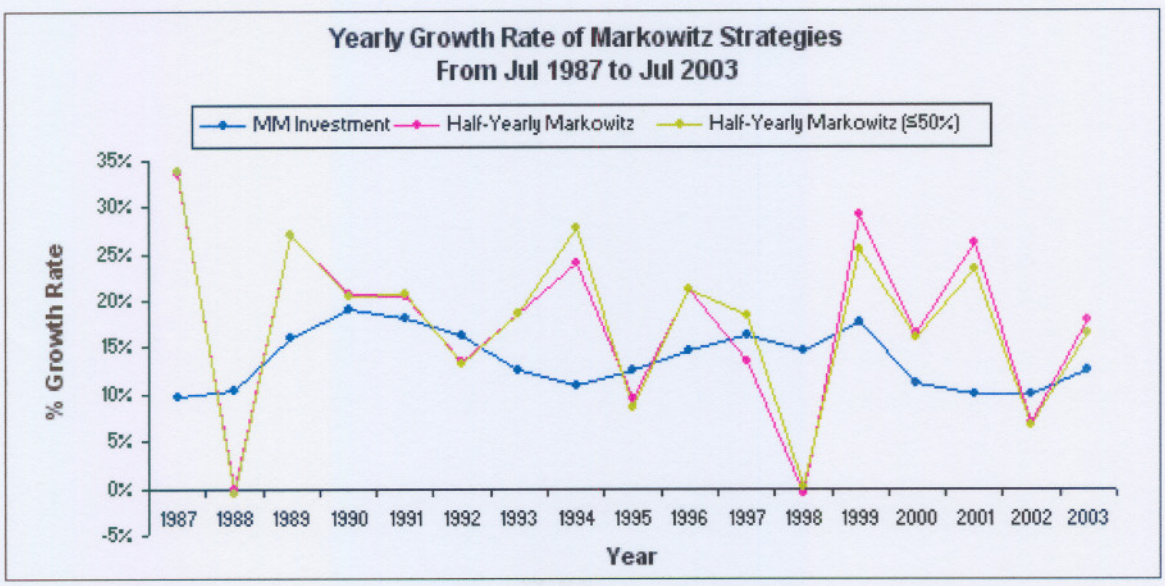
Table 4.6 portrays a summary of the information obtained from the 3-sigma rule graphs for the half yearly Markowitz strategies:

Strategy	$\overline{X}$	X < 1.60% (%)	Return/Risk ratio	X > UL (%)	X < LL (%)
Half yearly Markowitz 1	1.51%	50.24%	0.4075	0.00%	0.96%
Half yearly Markowitz 2	1.50%	49.76%	0.4014	0.00%	0.96%

Table 4.6: Summary of half yearly Markowitz strategies information

4.4.2.3 Yearly growth

The following graph displays the yearly growth rates for the two half yearly Markowitz strategies. Half yearly Markowitz strategy 1 has the highest average yearly growth rate of 17.53%, the half yearly growth rate for the second strategy is 17.48%. The second strategy shows the highest yearly growth rate (33.75%) in 1987 and the first strategy shows the lowest yearly growth rate (-0.64%) in 1998.



Graph 4.29: Yearly growth rates for half yearly Markowitz strategies

4.5 Linear programming models: β as a risk measure

This strategy involves risk-reward trade-off. This implies that although the expected return is important in the investments, the risk as measured by the β of the funds, is also important. A

high value of β indicates that the fund has a relatively high risk (Render *et al.*, 2003:321). Three models are presented. The linear programming (LP) model searches for a portfolio of funds that minimises the portfolio risk and meet or exceed a certain return target. Similar to the Markowitz model, this model also determines the percentage of the investment budget to allocate to each fund. The second model restricts the allocation of funds to 50%. The third model maximises return, while the portfolio risk should not exceed some specified target (two targets are set). The models are implemented for two different time horizons, which include:

- *Yearly LP (β) strategy* – The LP model is executed once a year to determine the portfolio of funds to invest in for the next year (based on the expected return and risk data of the previous year).
- *Half yearly LP (β) strategy* - The LP model is executed every six months to determine the portfolio of funds to invest in for the next six months (based on the expected return and risk data of the past six months).

The LP (β) model has the following structure (Render *et al.*, 2003:321):

Objective:

Minimise Portfolio $\beta = \sum_{i=1}^{k} \beta_i x_i$

Where:

β_i = the β-value of fund *i*

x_i = fraction of the portfolio to invest in fund *i*

Constraints:

$\sum_{i=1}^{k} x_i = 1$ (Invest 100% of the investment budget)

$\sum_{i=1}^{k} r_i x_i \geq T$ (Expected return of the portfolio must meet or exceed a target return)

$x_1, x_2, \ldots x_k \geq 0$ (Non-negativity constraint)

Where:

r_i = the expected return on fund *i*

T = the target return (for all the models the target return is set at 1.60% per month)

The second model attempts to diversify the portfolio more, by limiting the allocation to 50%, for any one fund. The following constraint is added for the second model: $x_1, x_2, \ldots x_k \leq 0.5$

The third LP (β) model has the following structure (the variables are the same as for the model above):

Objective:

Maximise Expected return of the portfolio = $\sum_{i=1}^{k} r_i x_i$

Constraints:

$\sum_{i=1}^{k} x_i = 1$ (Invest 100% of the investment budget)

$\sum_{i=1}^{k} \beta_i x_i \leq \text{Target } \beta$ (Portfolio β should be no more than the target β)

$x_1, x_2, \ldots x_k \geq 0$ (Non-negativity constraint)

Where:

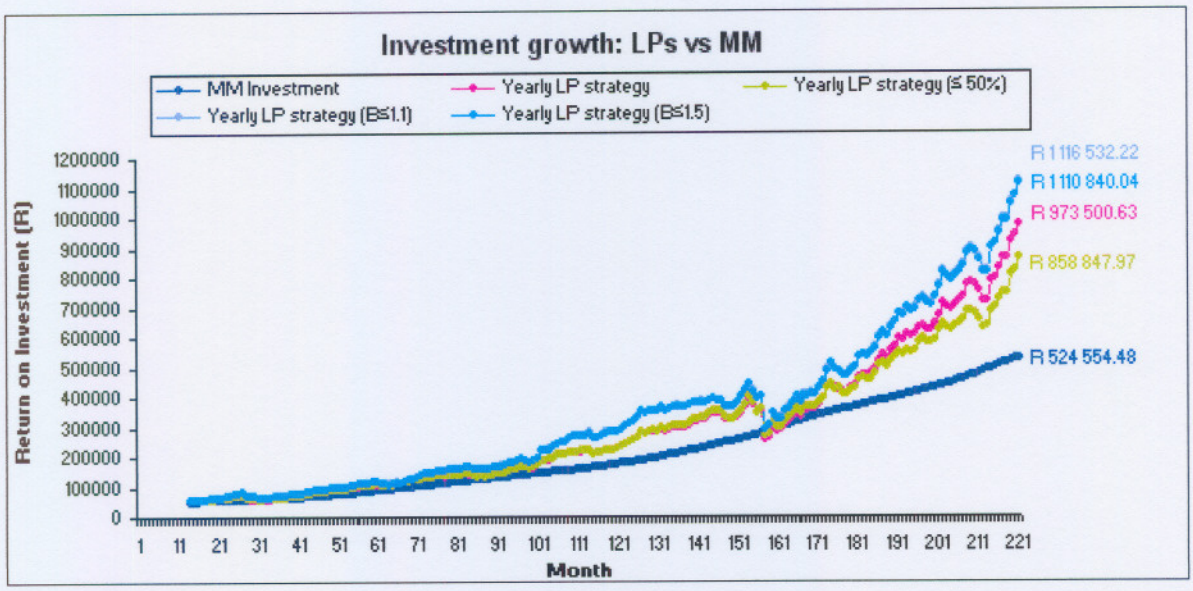
Target β = the target set for the β–value of the portfolio

The results of two models will be shown (two target β–values): $\beta \leq 1.1$ and $\beta \leq 1.5$.

4.5.1 Yearly LP (β) strategy

4.5.1.1 Investment growth

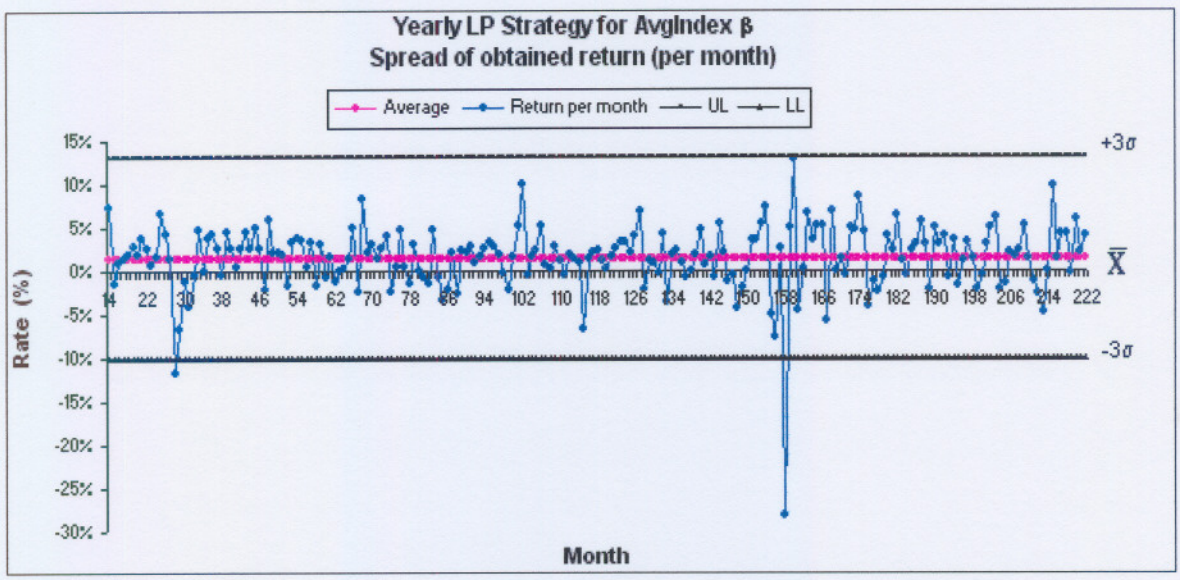
Graph 4.30 illustrates a comparison between the four yearly LP (β) strategies and the MM. All the strategies performed better than the MM fund. The yearly ($\beta \leq 1.1$) strategy performed the best, with an effective interest rate of 1.50% per month. The monthly effective interest rate for the other strategies are 1.43%, 1.37% and 1.49% correspondingly.



Graph 4.30: Comparison between yearly LP (β) strategies

4.5.1.2 3-Sigma rule

- **Yearly LP (β) strategy 1**



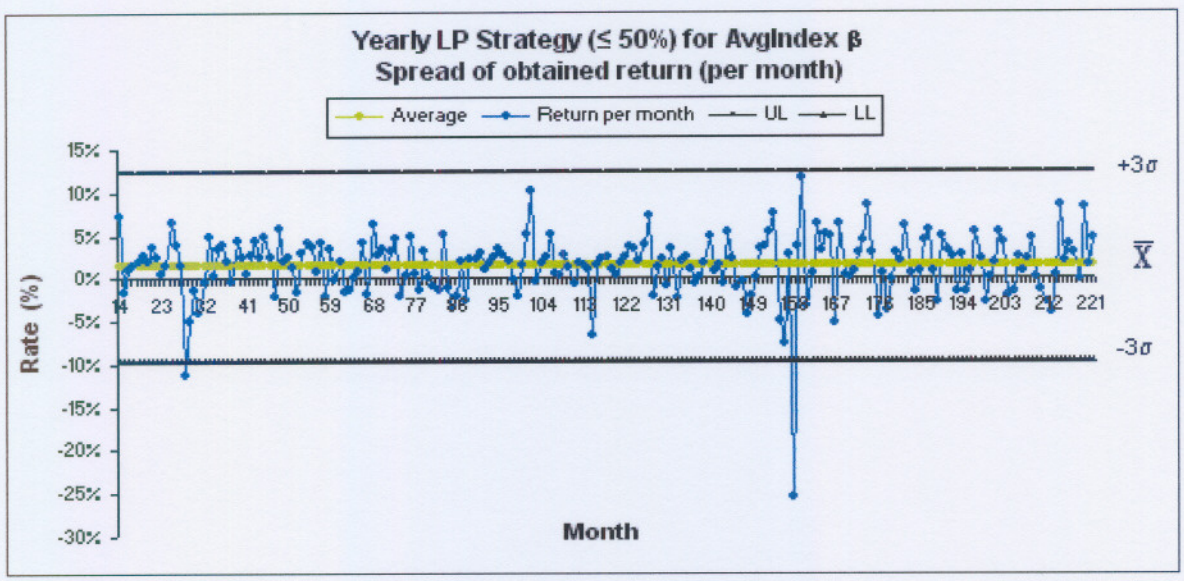
Graph 4.31: 3-Sigma rule for yearly LP (β) strategy 1

Graph 4.31 shows that 0.96% of the return values are below the lower limit (-9.99%) and none of the return values are above the upper limit (13.01%). 46.89% of the return values

are below the target return of 1.60%. The average return that is achieved per month is 1.51% and the strategy's return/risk ratio is 0.3934. The standard deviation for this strategy is 3.83%.

- **Yearly LP (β) strategy 2**

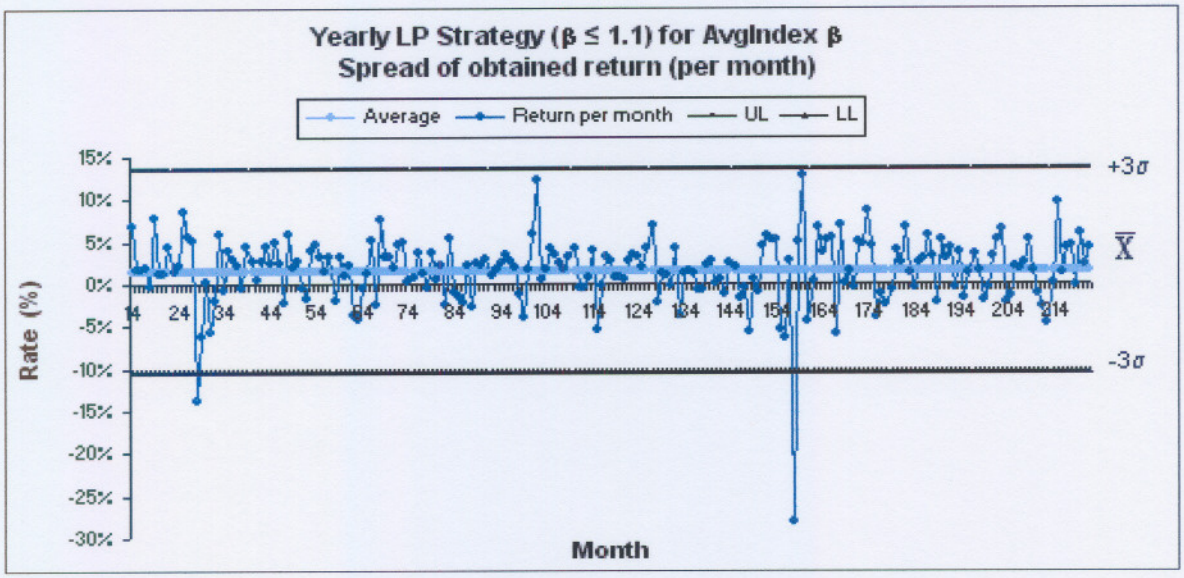
Graph 4.32 displays that 0.96% of the return values are below the lower limit of –9.49% and no return values are above the upper limit of 12.37%. 47.39% of the return values are below the target return of 1.60%. The average return is 1.44% per month and the return/risk ratio for this strategy is 0.3949. The standard deviation for this strategy is 3.64%.



Graph 4.32: 3-Sigma rule for yearly LP (β) strategy 2

- **Yearly LP (β) strategy 3**

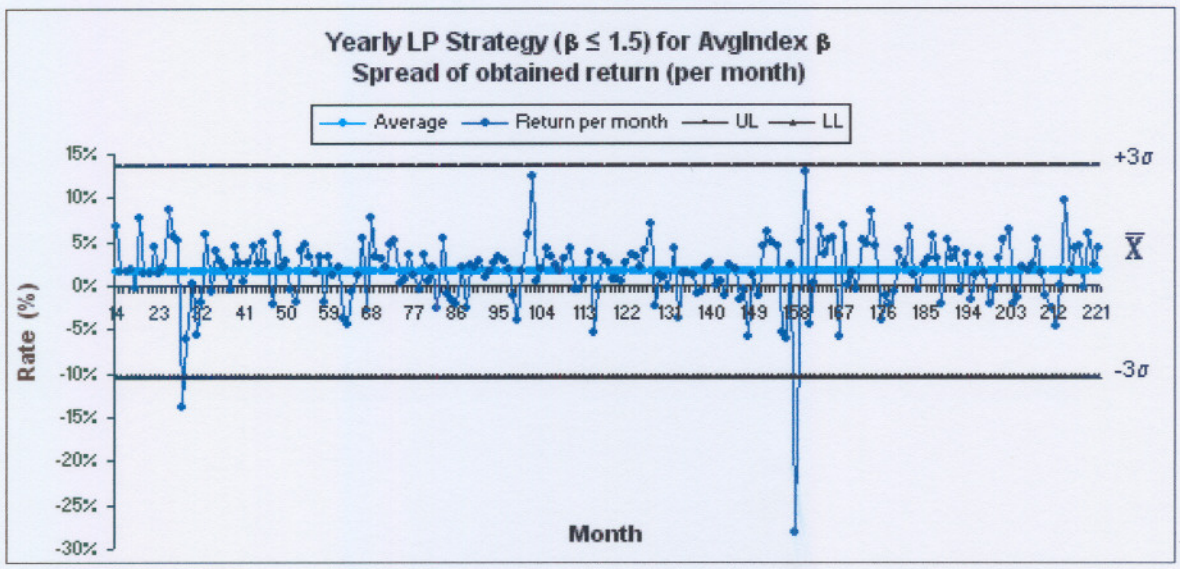
The following graph shows that 0.96% of the return values are below the lower limit (-10.35%) and no return values are above the upper limit (13.51%). 44.98% of the return values are below the target return of 1.60%. The average return that is achieved per month is 1.58% and the strategy's return/risk ratio is 0.3975. The standard deviation for this strategy is 3.98%.



Graph 4.33: 3-Sigma rule for yearly LP (β) strategy 3

- **Yearly LP (β) strategy 4**

The following graph displays that 0.96% of the return values are below the lower limit of –10.38% and no return values are above the upper limit of 13.54%. 44.98% of the return values are below the target return of 1.60%. The average return is 1.58% per month and the return/risk ratio for this strategy is 0.3959. The standard deviation for this strategy is 3.99%.



Graph 4.34: 3-Sigma rule for yearly LP (β) strategy 4

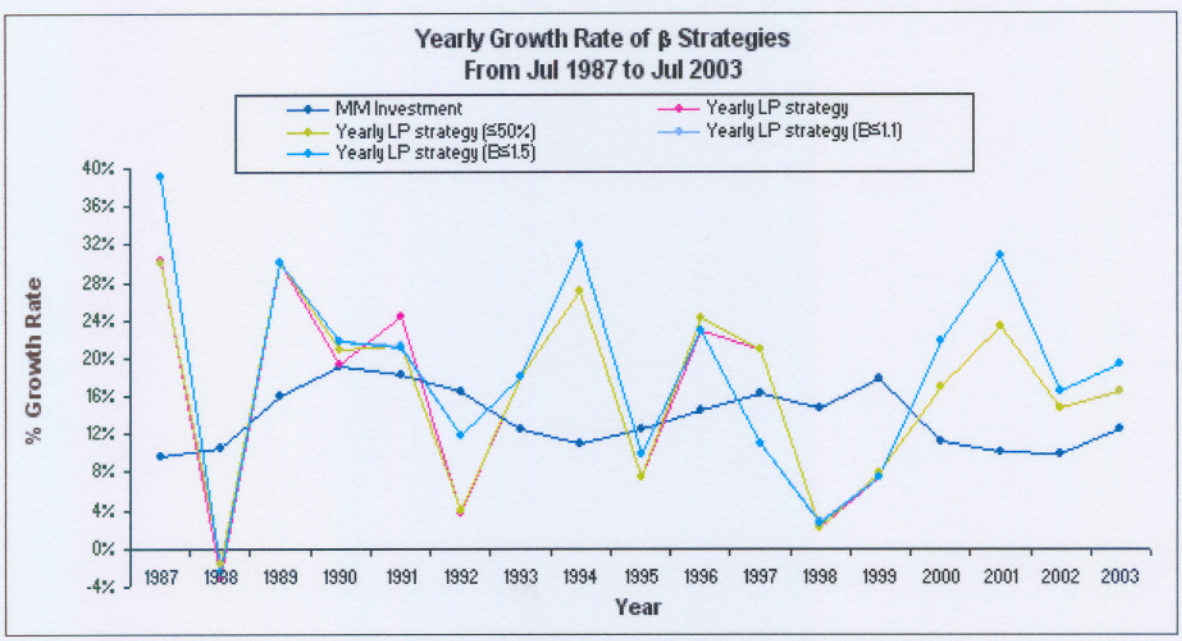
Table 4.7 portrays a summary of the information obtained from the 3-sigma rule graphs for the yearly LP (β) strategies:

Strategy	$\overline{X}$	X < 1.60% (%)	Return/Risk ratio	X > UL (%)	X < LL (%)
Yearly LP (β) 1	1.51%	46.89%	0.3934	0.00%	0.96%
Yearly LP (β) 2	1.44%	47.37%	0.3949	0.00%	0.96%
Yearly LP (β) 3	1.58%	44.98%	0.3975	0.00%	0.96%
Yearly LP (β) 4	1.58%	44.98%	0.3959	0.00%	0.96%

Table 4.7: Summary of yearly LP (β) strategies information

4.5.1.3 Yearly growth

Graph 4.35 displays the yearly growth rates for the four yearly LP (β) strategies. The yearly ($\beta \leq 1.1$) strategy shows the highest average yearly growth rate of 18.46%, the yearly growth rates for the other strategies are 17.57%, 16.67% and 18.43% correspondingly. The first yearly LP (β) strategy shows the lowest yearly growth rate of –3.14% in 1988 and the yearly ($\beta \leq 1.5$) strategy reached the highest yearly growth rate of 39.18% is in 1987.

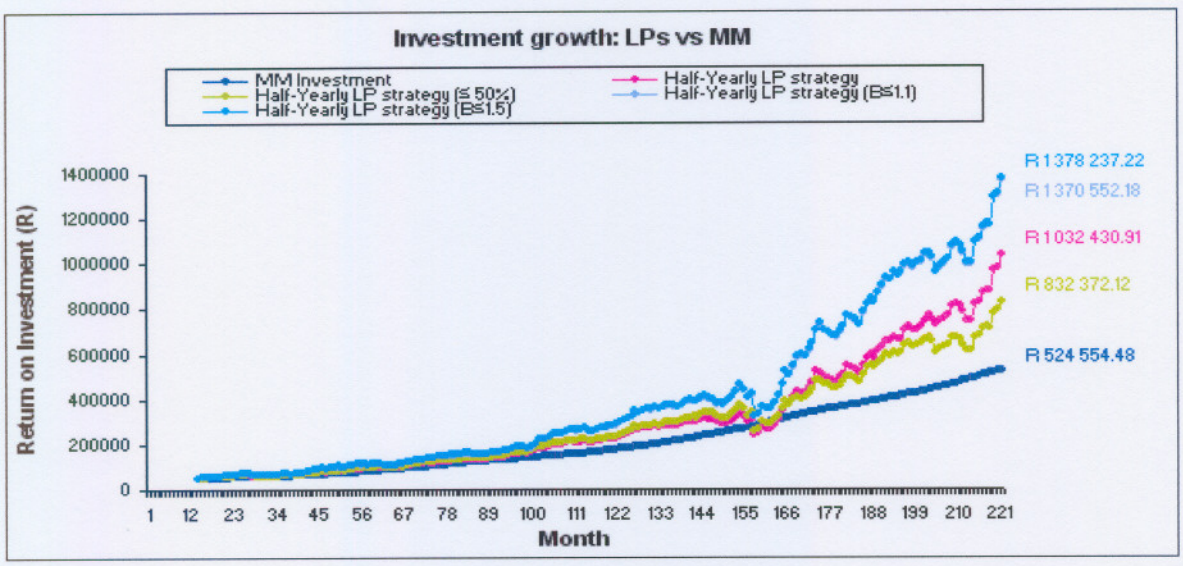


Graph 4.35: Yearly growth rates for yearly LP (β) strategies

4.5.2 Half yearly LP (β) strategy

4.5.2.1 Investment growth

The following graph illustrates a comparison between the four half yearly LP (β) strategies and the MM. All the strategies performed better than the MM fund. The half yearly ($\beta \leq 1.1$) strategy performed the best (the total investment growth is slightly higher than that of the third strategy). The half yearly ($\beta \leq 1.1$) has an effective interest rate of 1.60% per month. The monthly effective interest rate for the other strategies are 1.46%, 1.35% and 1.60% correspondingly.

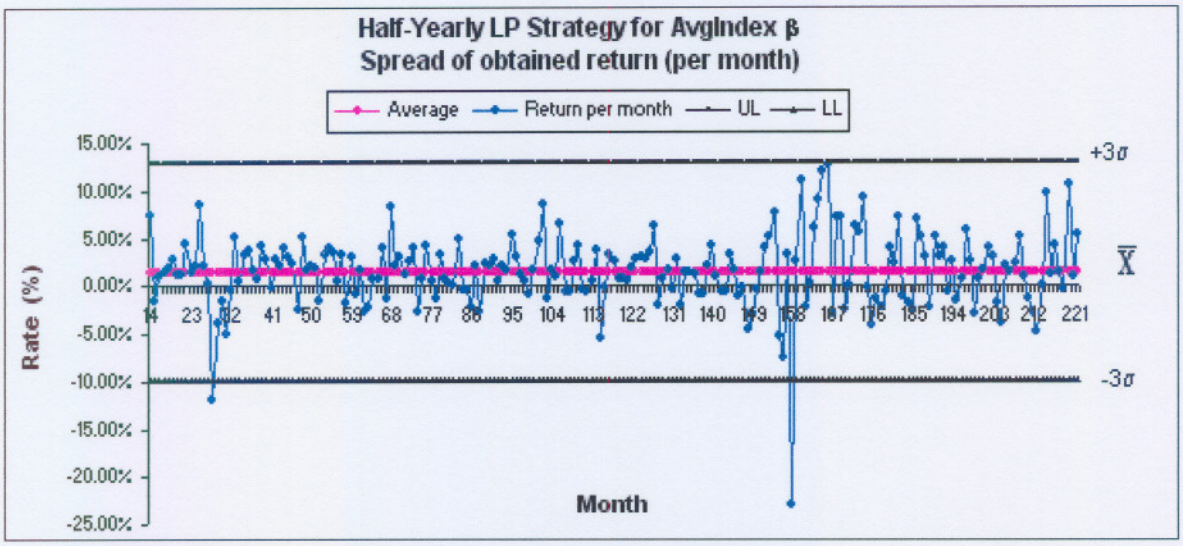


Graph 4.36: Comparison between half yearly LP (β) strategies

4.5.2.2 3-Sigma rule

- **Half yearly LP (β) strategy 1**

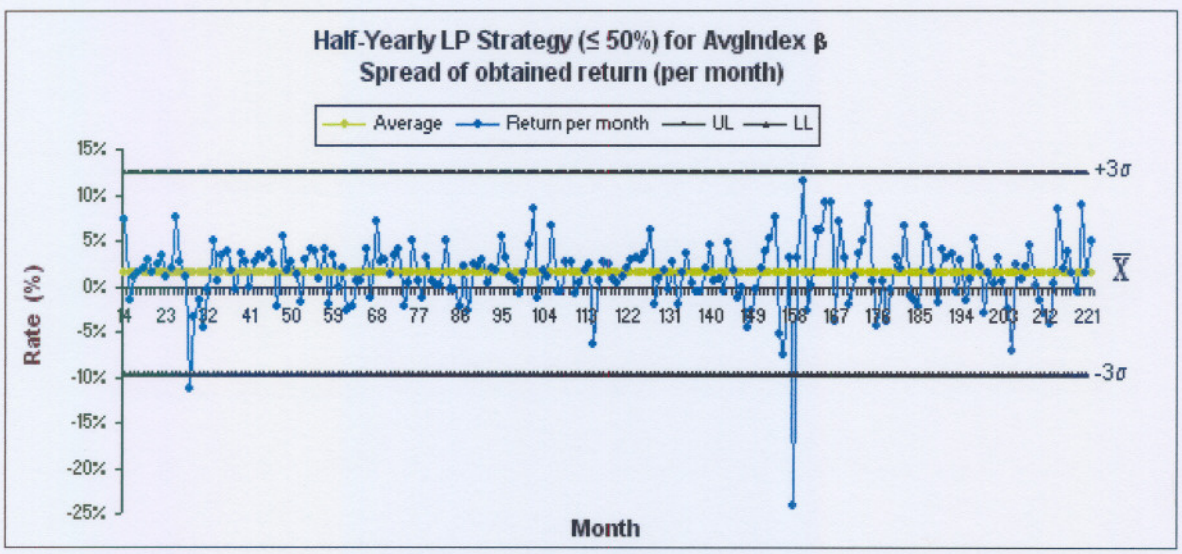
The following graph above shows that 0.96% of the return values are below the lower limit (-9.92%) and none of the return values are above the upper limit (12.98%). More than half (50.72%) of the return values are below the target return of 1.60%. The average return that is achieved per month is 1.53% and the strategy's return/risk ratio is 0.4016. The standard deviation for this strategy is 3.82%.



Graph 4.37: 3-Sigma rule for half yearly LP (β) strategy 1

- **Half yearly LP (β) strategy 2**

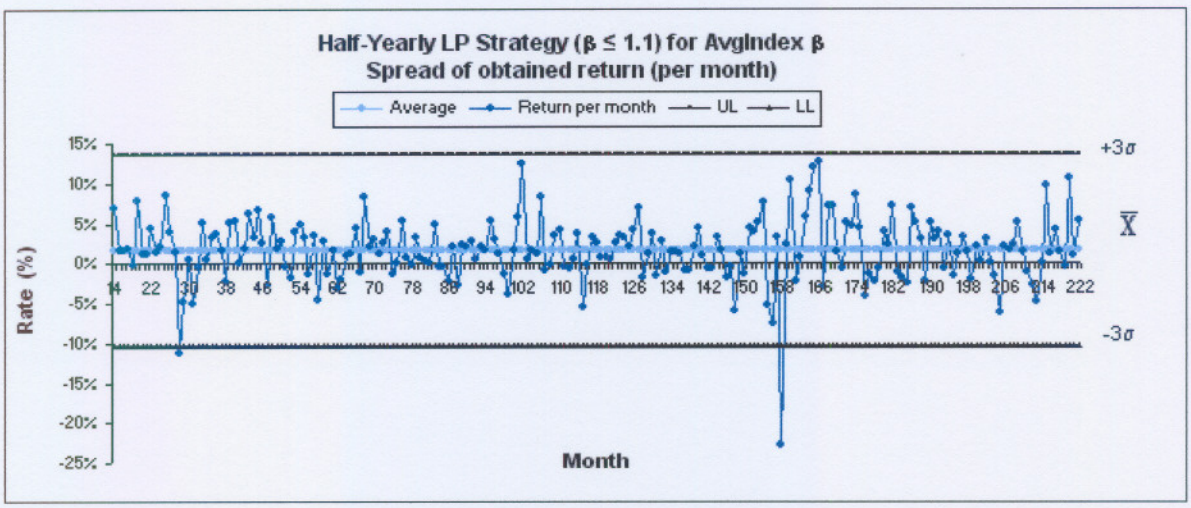
Graph 4.38 displays that 0.96% of the return values are below the lower limit of –9.63% and no return values are above the upper limit of 12.48%. 49.28% of the return values are below the target return of 1.60%. The average return is 1.42% per month and the return/risk ratio for this strategy is 0.3867. The standard deviation for this strategy is 3.68%.



Graph 4.38: 3-Sigma rule for half yearly LP (β) strategy 2

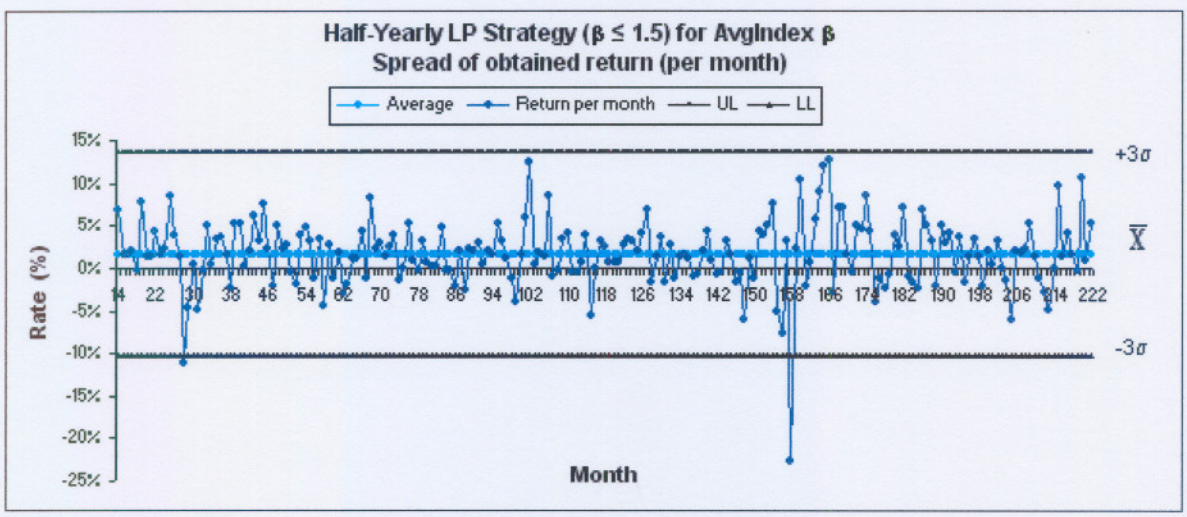
- **Half yearly LP (β) strategy 3**

The following graph shows that 0.96% of the return values are below the lower limit (-10.21%) and no return values are above the upper limit (13.56%). 49.28% of the return values are below the target return of 1.60%. The average return that is achieved per month is 1.68% and the strategy's return/risk ratio is 0.4230. The standard deviation for this strategy is 3.96%.



Graph 4.39: 3-Sigma rule for half yearly LP (β) strategy 3

- **Half yearly LP (β) strategy 4**



Graph 4.40: 3-Sigma rule for half yearly LP (β) strategy 4

Graph 4.40 indicates that 0.96% of the return values are below the lower limit of –10.20%, but none of the return values are above the upper limit of 13.56%. 49.28% of the return values are below the target return of 1.60%. The average return for this strategy is 1.68% per month. The return/risk ratio for this strategy is 0.4238. The standard deviation is 3.96%.

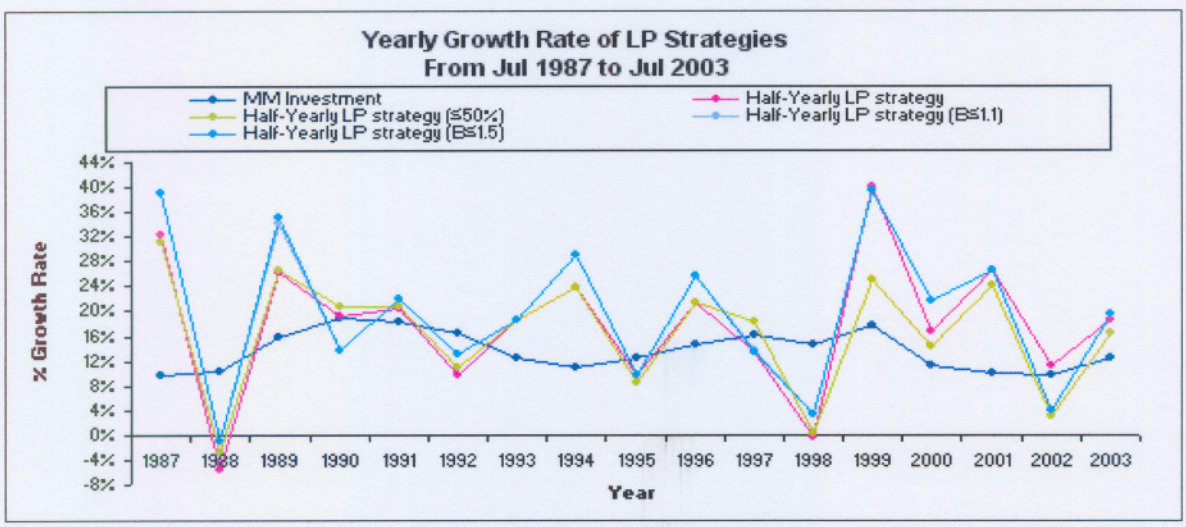
The following table provides a summary of the information obtained from the 3-sigma rule graphs for the half yearly LP (β) strategies:

Strategy	$\overline{X}$	X < 1.60% (%)	Return/Risk ratio	X > UL (%)	X < LL (%)
Half yearly LP (β) 1	1.53%	50.72%	0.4016	0.00%	0.96%
Half yearly LP (β) 2	1.42%	49.28%	0.3867	0.00%	0.96%
Half yearly LP (β) 3	1.68%	49.28%	0.4230	0.00%	0.96%
Half yearly LP (β) 4	1.68%	49.28%	0.4238	0.00%	0.96%

Table 4.8: Summary of half yearly LP (β) strategies information

4.5.2.3 Yearly growth

The following graph displays the yearly growth rates for the four half yearly LP (β) strategies. The half yearly ($\beta \leq 1.5$) strategy shows the highest average yearly growth rate of 19.57%, the yearly growth rates for the other strategies are 17.78%, 16.53% and 19.54% correspondingly. The first yearly LP (β) strategy realized both the lowest yearly growth rate of –5.58% in 1988 and the highest yearly growth rate of 40.07% is in 1999.



Graph 4.41: Yearly growth rates for half yearly LP (β) strategies

4.6 Linear programming models: LPM as a risk measure

The goal is to find a portfolio of funds that minimises the portfolio risk (downside risk, i.e. lower partial moment is used as the risk measure), while a specified return target is met or exceeded. The second model restricts the allocation of funds to 50%.

The models are implemented for three different degrees of the LPM (thus, for three different values of n), which include:

- $n = 0.5 \Rightarrow$ Risk seeking investor.
- $n = 1 \Rightarrow$ Risk neutral investor.
- $n = 2 \Rightarrow$ Risk averse investor.

The models are executed for two different time horizons, which include:

- *Yearly LP (LPM) strategy* – The LP model is executed once a year to determine the portfolio of funds to invest in for the next year, using the expected return and risk data of the previous year.
- *Half yearly LP (LPM) strategy* - The LP model is executed every six months to determine the portfolio of funds to invest in for the next six months, using the expected return and risk data of the past six months.

The LP (LPM) model has the same structure as the LP (β) model, but the portfolio risk is different:

Objective:

Minimise Portfolio risk = $\sum_{i=1}^{k} LPM_i x_i$

Where:

LPM_i = the LPM-value of fund *i*

x_i = fraction of the portfolio to invest in fund *i*

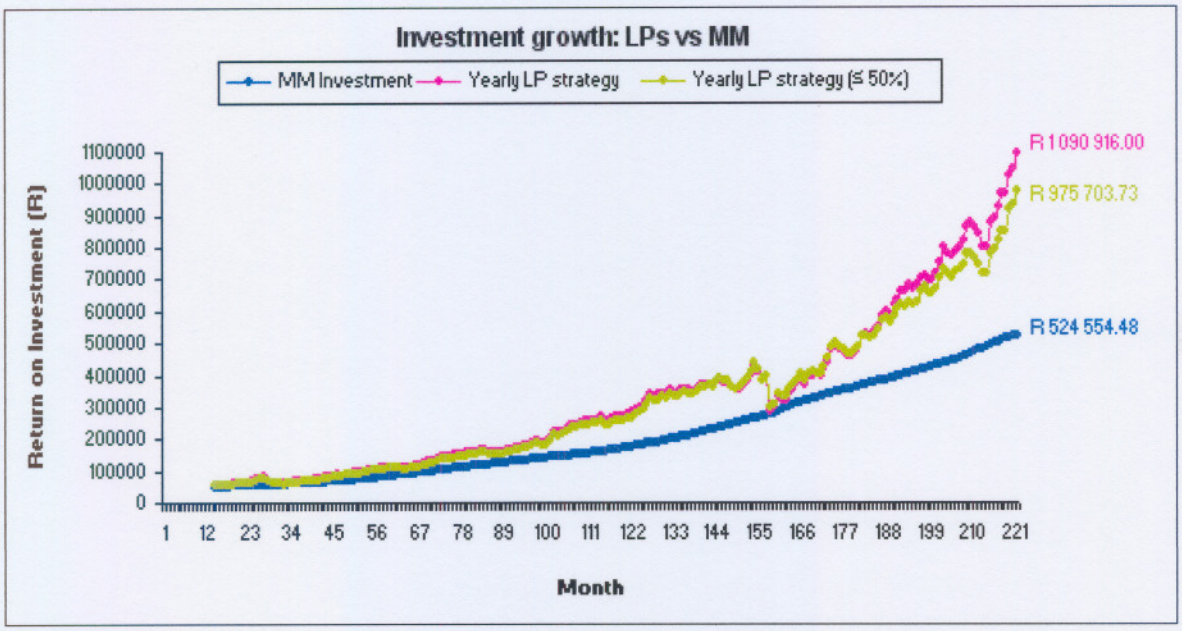
Constraints: Equivalent to LP (β) model constraints, for strategy 1 and strategy 2.

4.6.1 Yearly LP (LPM) strategy

4.6.1.1 Investment growth

- **Yearly LP (LPM) strategy (n = 0.5)**

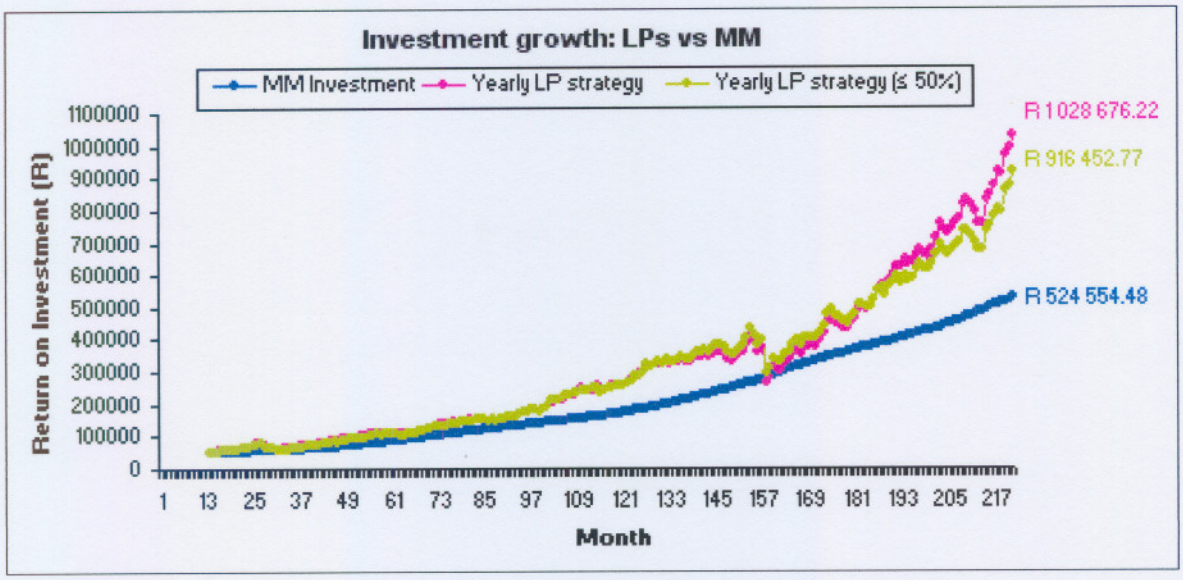
The following graph illustrates a comparison between the two yearly LP (LPM) strategies (for n = 0.5) and the MM. All the strategies surpass the MM fund. The yearly (LPM) strategy 1 performed the best, with an effective interest rate of 1.49% per month. The monthly effective interest rate for the other strategy is 1.43%.



Graph 4.42: Comparison between yearly LP (LPM) strategies (n = 0.5)

- **Yearly LP (LPM) strategy (n = 1)**

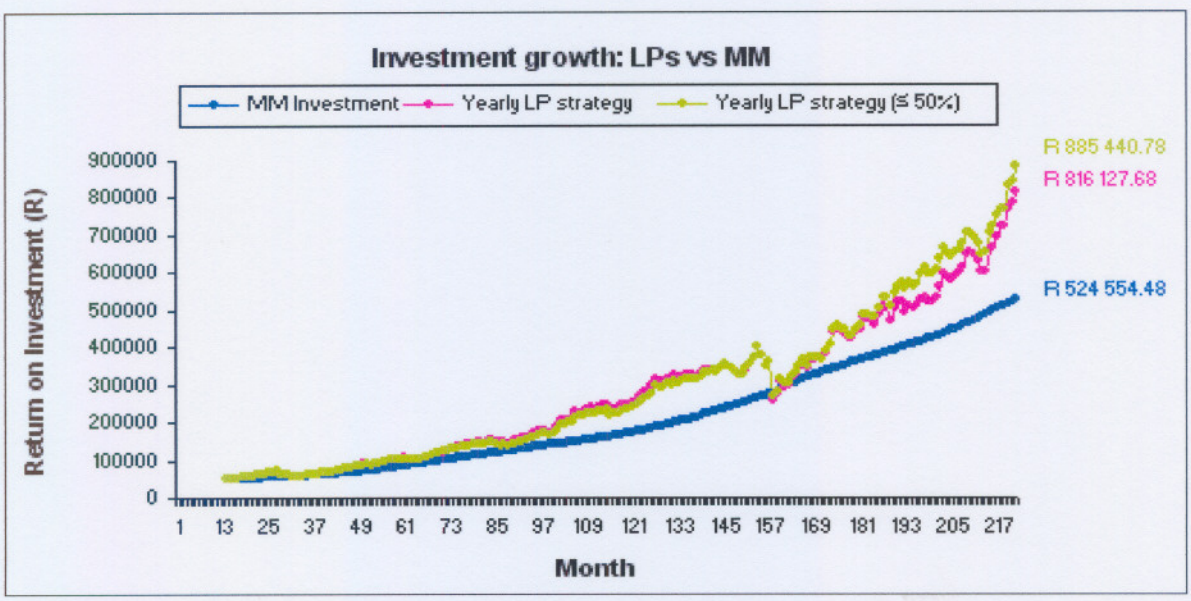
Graph 4.43 indicates a comparison between the two yearly LP (LPM) strategies (for n = 1) and the MM. All the strategies exceed the MM fund's investment growth. The yearly (LPM) strategy 1 performed the best, with a monthly effective interest rate of 1.46%. The monthly effective interest rate for yearly (LPM) strategy 2 is 1.40%.



Graph 4.43: Comparison between yearly LP (LPM) strategies (n = 1)

- **Yearly LP (LPM) strategy (n = 2)**

The following graph shows a comparison between the two yearly LP (LPM) strategies (for n = 2) and the MM. All the strategies surpass the MM fund's investment growth. The yearly (LPM) strategy 2 performed the best, with an effective interest rate of 1.38%. The monthly effective interest rate for the other strategy is 1.35%.

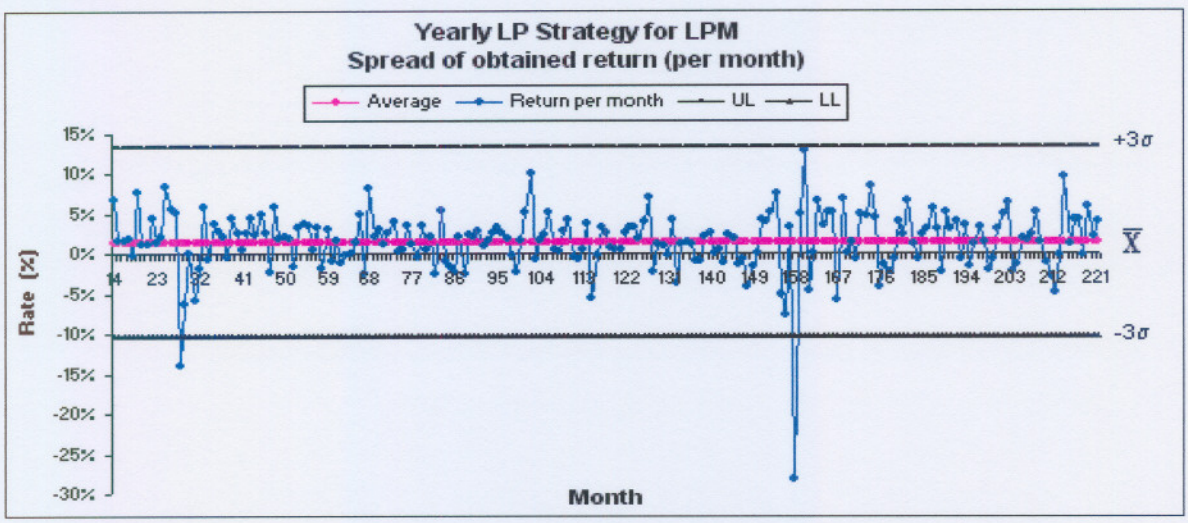


Graph 4.44: Comparison between yearly LP (LPM) strategies (n = 2)

4.6.1.2 3-Sigma rule

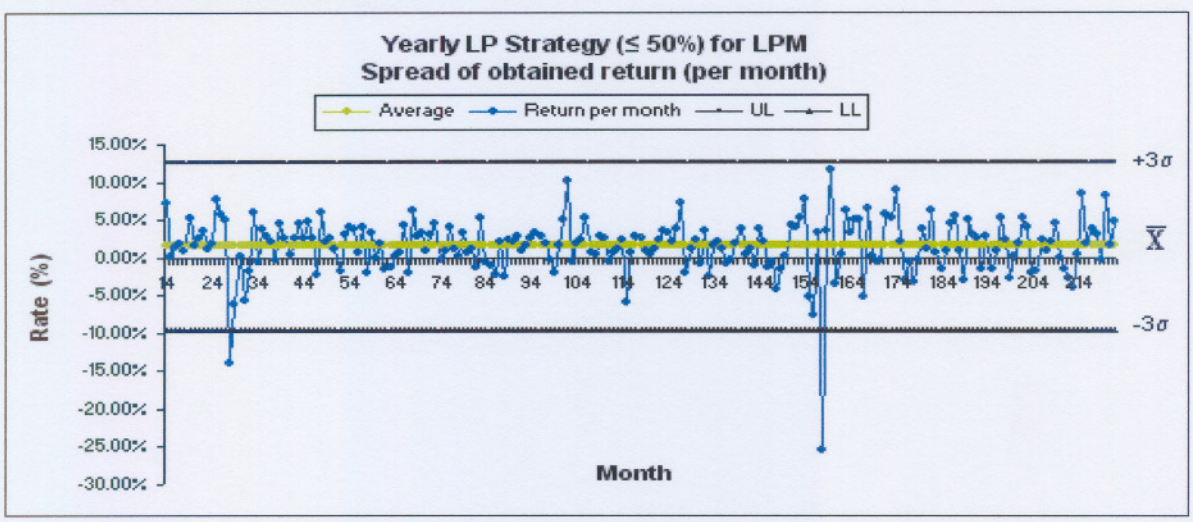
- **Yearly LP (LPM) strategy 1 (n = 0.5)**

The following graph shows that 0.96% of the return values are below the lower limit (-10.15%) and none of the return values are above the upper limit (13.28%). 46.41% of the return values are below the target return of 1.60%. The average return that is achieved per month is 1.57% and the strategy's return/risk ratio is 0.4010. The standard deviation for this strategy is 3.91%.



Graph 4.45: 3-Sigma rule for yearly LP (LPM) strategy 1 (n = 0.5)

- **Yearly LP (LPM) strategy 2 (n = 0.5)**

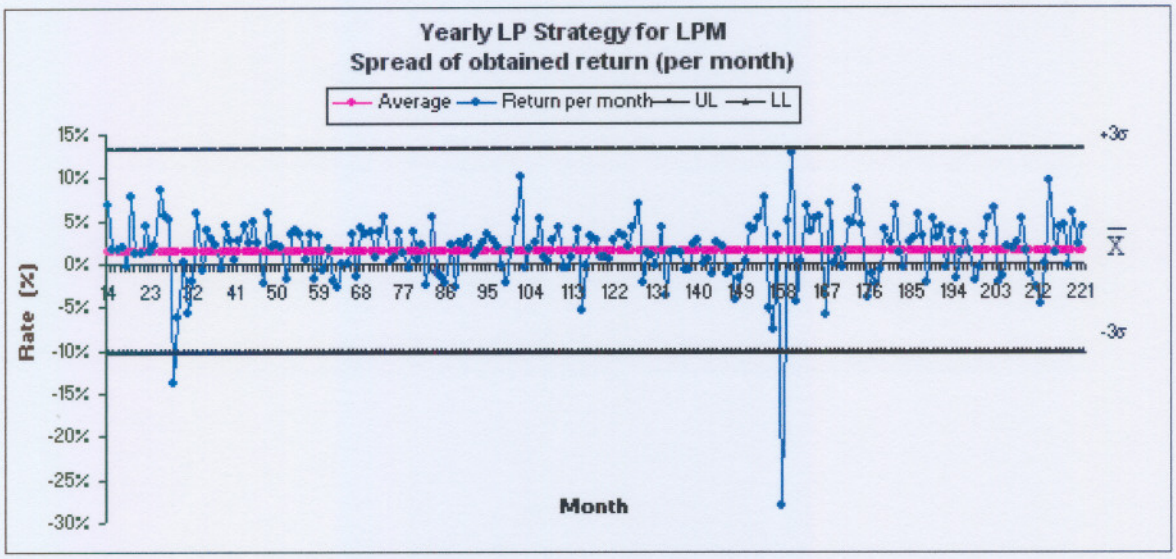


Graph 4.46: 3-Sigma rule for yearly LP (LPM) strategy 2 (n = 0.5)

Graph 4.46 displays that 0.96% of the return values are below the lower limit of –9.64% and no return values are above the upper limit of 12.64%. 46.89% of the return values are below the target return of 1.60%. The average return is 1.50% per month and the return/risk ratio for this strategy is 0.4050. The standard deviation for this strategy is 3.71%.

- **Yearly LP (LPM) strategy 1 (n = 1)**

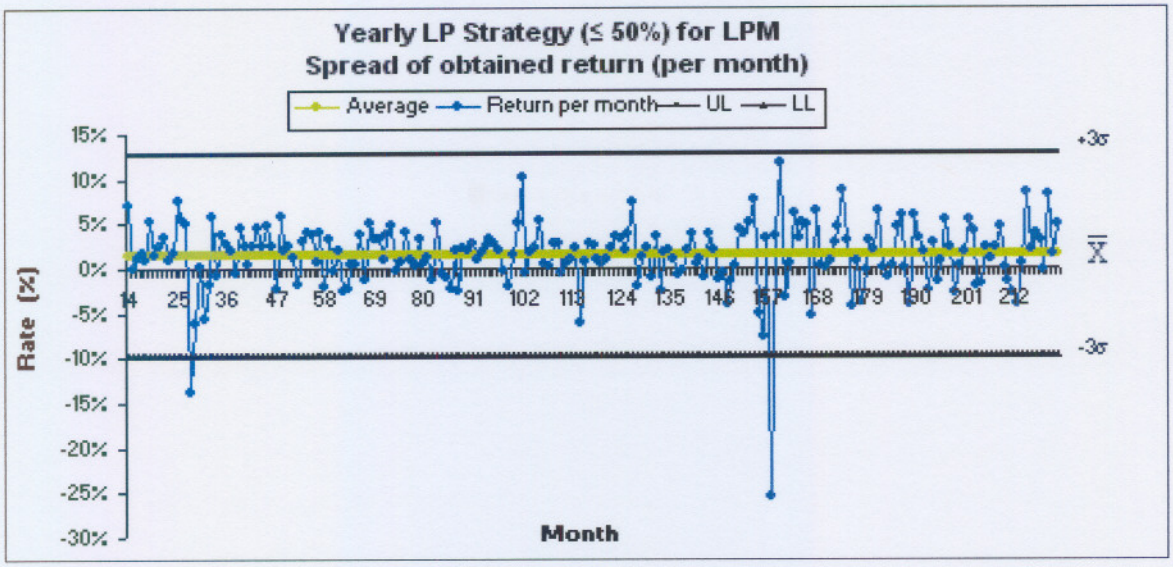
The following graph shows that 0.96% of the return values are below the lower limit (-10.16%) and none of the return values are above the upper limit (13.23%). 46.41% of the return values are below the target return of 1.60%. The average return that is attained per month is 1.54% and the strategy's return/risk ratio is 0.3944. The standard deviation is 3.90%.



Graph 4.47: 3-Sigma rule for yearly LP (LPM) strategy 1 (n = 1)

- **Yearly LP (LPM) strategy 2 (n = 1)**

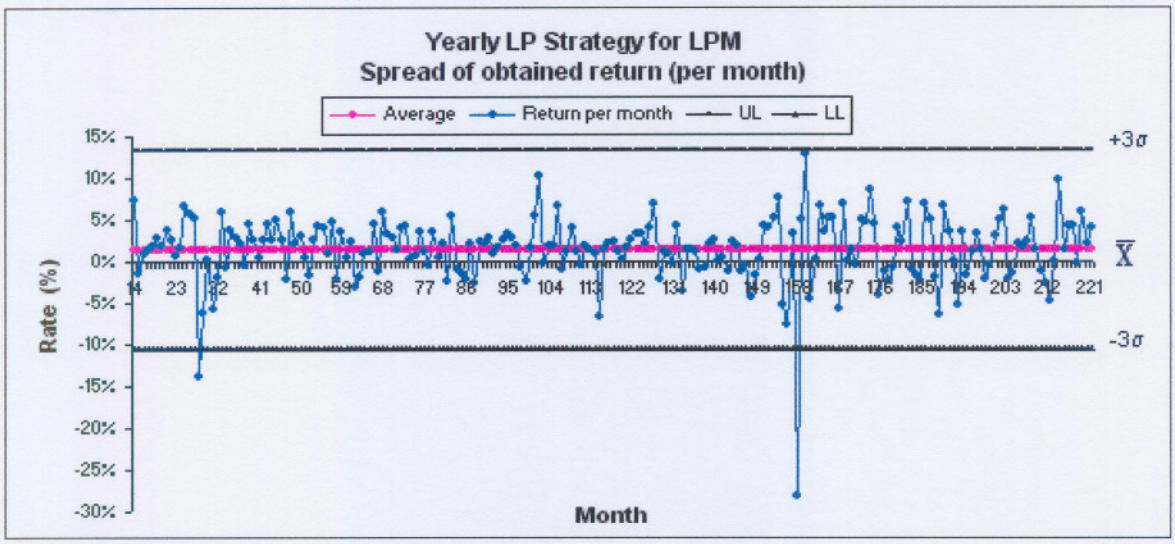
Graph 4.48 illustrates that 0.96% of the return values are below the lower limit of –9.70%. None of the return values are above the upper limit of 12.65%. 46.41% of the return values are below the target return of 1.60%. The strategy obtained an average return of 1.47% per month. The return/risk ratio for this strategy is 0.3956. The standard deviation for this strategy is 3.72%.



Graph 4.48: 3-Sigma rule for yearly LP (LPM) strategy 2 (n = 1)

- **Yearly LP (LPM) strategy 1 (n = 2)**

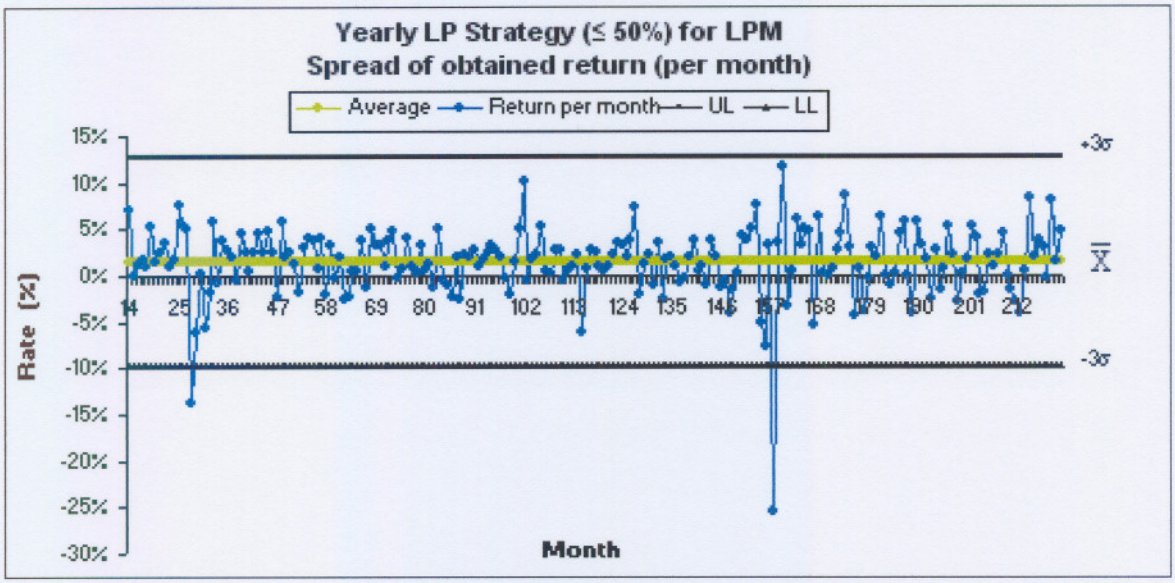
The following graph shows that 0.96% of the return values are below the lower limit (-10.55%) and none of the return values are above the upper limit (13.41%). 48.33% of the return values are below the target return of 1.60%. The average return that is obtained per month is 1.43% and the strategy's return/risk ratio is 0.3579. The standard deviation is 3.99%.



Graph 4.49: 3-Sigma rule for yearly LP (LPM) strategy 1 (n = 2)

- **Yearly LP (LPM) strategy 2 (n = 2)**

The following graph illustrates that 0.96% of the return values are below the lower limit of –9.82%. No return values are above the upper limit of 12.74%. 46.89% of the return values are below the target return of 1.60%. The average return is 1.46% per month and the return/risk ratio for this strategy is 0.3879. The standard deviation for this strategy is 3.76%.



Graph 4.50: 3-Sigma rule for yearly LP (LPM) strategy 2 (n = 2)

The following table provides a summary of the information obtained from the 3-sigma rule graphs for the yearly LP (LPM) strategies:

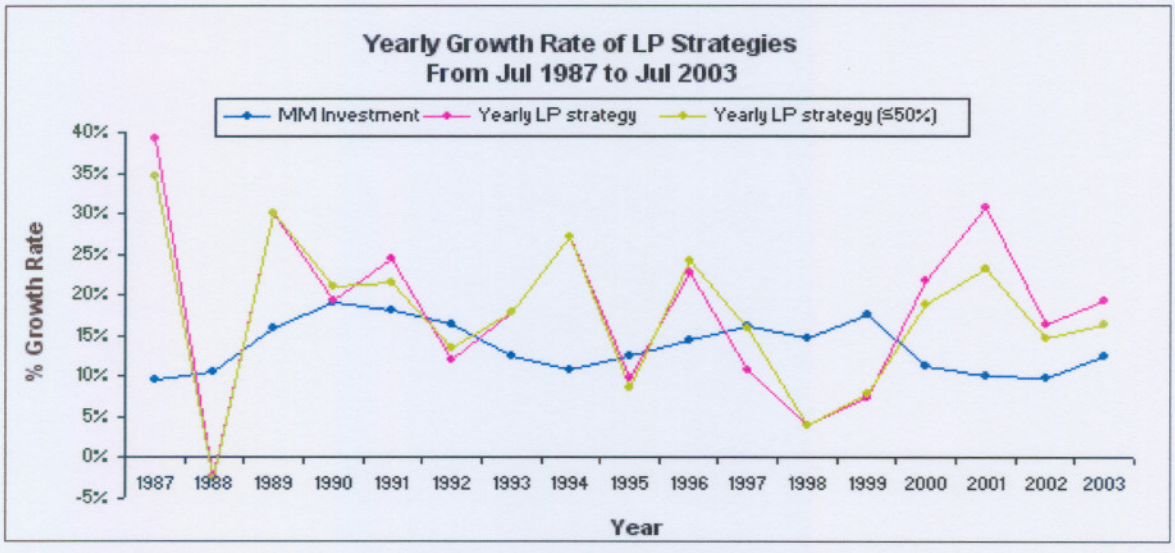
Strategy	$\overline{X}$	X < 1.60% (%)	Return/Risk ratio	X > UL (%)	X < LL (%)
Yearly LP (LPM, n= 0.5) 1	1.57%	46.41%	0.4010	0.00%	0.96%
Yearly LP (LPM, n= 0.5) 2	1.50%	46.89%	0.4050	0.00%	0.96%
Yearly LP (LPM, n= 1) 1	1.54%	46.41%	0.3944	0.00%	0.96%
Yearly LP (LPM, n= 1) 2	1.47%	46.41%	0.3956	0.00%	0.96%
Yearly LP (LPM, n= 2) 1	1.43%	48.33%	0.3579	0.00%	0.96%
Yearly LP (LPM, n= 2) 2	1.46%	46.89%	0.3879	0.00%	0.96%

Table 4.9: Summary of yearly LP (LPM) strategies information

4.6.1.3 Yearly growth

- **Yearly LP (LPM) strategy (n = 0.5)**

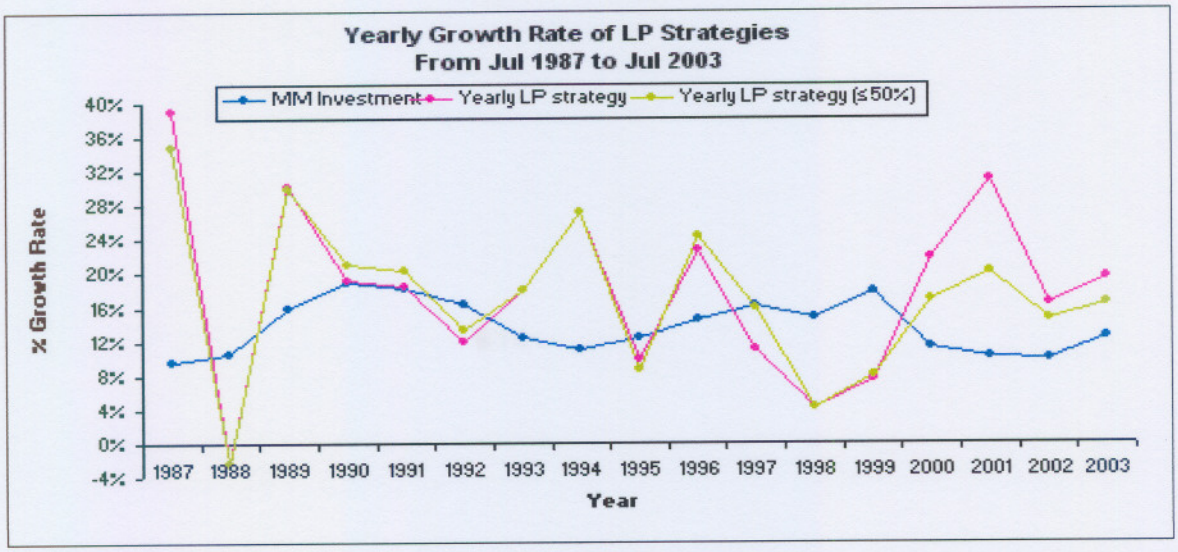
Graph 4.51 displays the yearly growth rates for the two yearly LP (LPM) strategies, where n = 0.5. Yearly (LPM) strategy 1 shows the highest average yearly growth rate of 18.29%, the average yearly growth rate for yearly (LPM) strategy 2 is 17.47%. The yearly (LPM) strategy 1 shows the maximum yearly growth rate of 39.18% in 1987 and both the strategies obtained the minimum yearly growth rate of –2.45% is in 1988.



Graph 4.51: Yearly growth rates for yearly LP (LPM) strategies (n = 0.5)

- **Yearly LP (LPM) strategy (n = 1)**

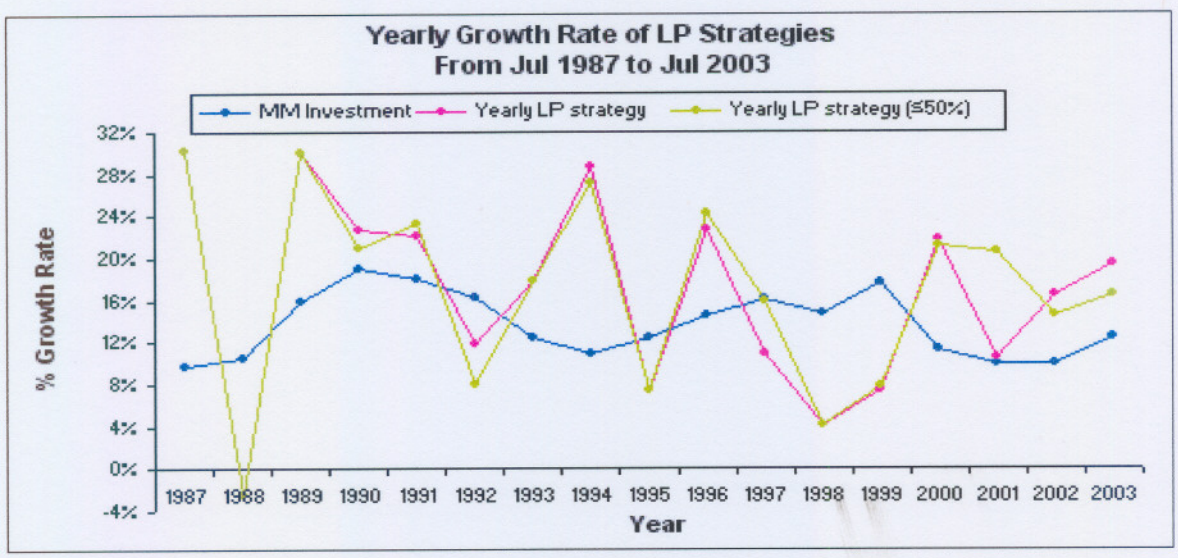
Graph 4.52 displays the yearly growth rates for the two yearly LP (LPM) strategies, where n = 1. Yearly (LPM) strategy 1 shows the highest average yearly growth rate of 17.93%, the average yearly growth rate for yearly (LPM) strategy 2 is 17.10%. The yearly (LPM) strategy 1 shows the highest yearly growth rate of 39.18% in 1987 and both the strategies obtained the lowest yearly growth rate of –2.45% is in 1988.



Graph 4.52: Yearly growth rates for yearly LP (LPM) strategies (n = 1)

- **Yearly LP (LPM) strategy (n = 2)**

The following graph illustrates the yearly growth rates for the two yearly LP (LPM) strategies, where n = 2. Yearly (LPM) strategy 2 shows the highest average yearly growth rate of 16.91%, the average yearly growth rate for yearly (LPM) strategy 1 is 16.60%. The yearly (LPM) strategy 2 shows the maximum yearly growth rate of 30.20% in 1987 and both the strategies obtained the minimum yearly growth rate of –2.45% is in 1988.



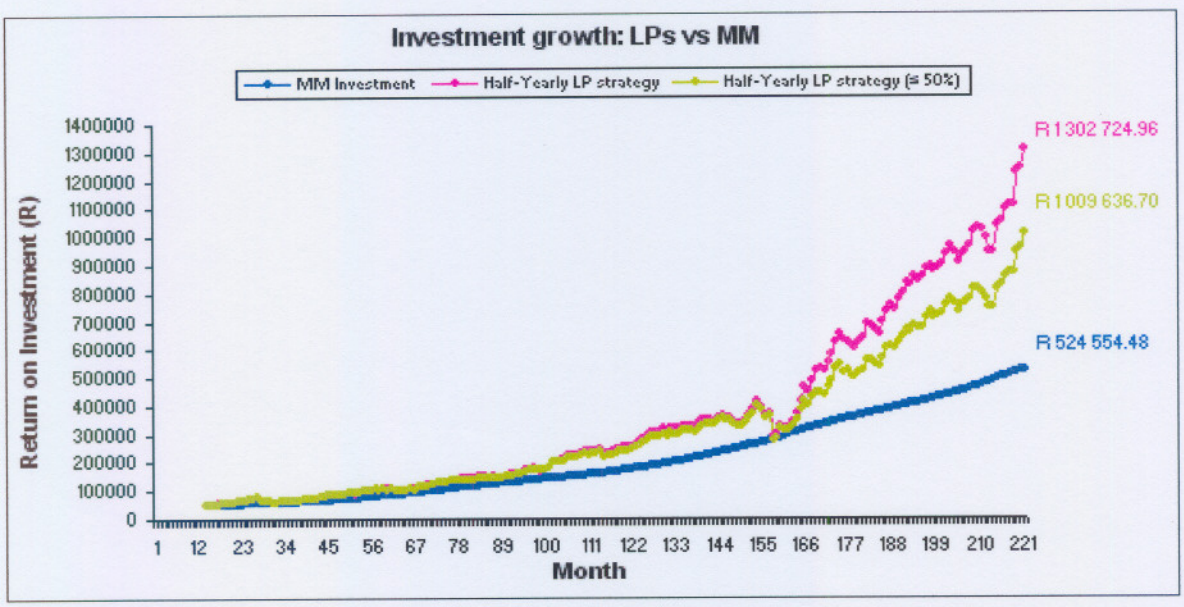
Graph 4.53: Yearly growth rates for yearly LP (LPM) strategies (n = 2)

4.6.2 Half yearly LP (LPM) strategy

4.6.2.1 Investment growth

- **Half yearly LP (LPM) strategy (n = 0.5)**

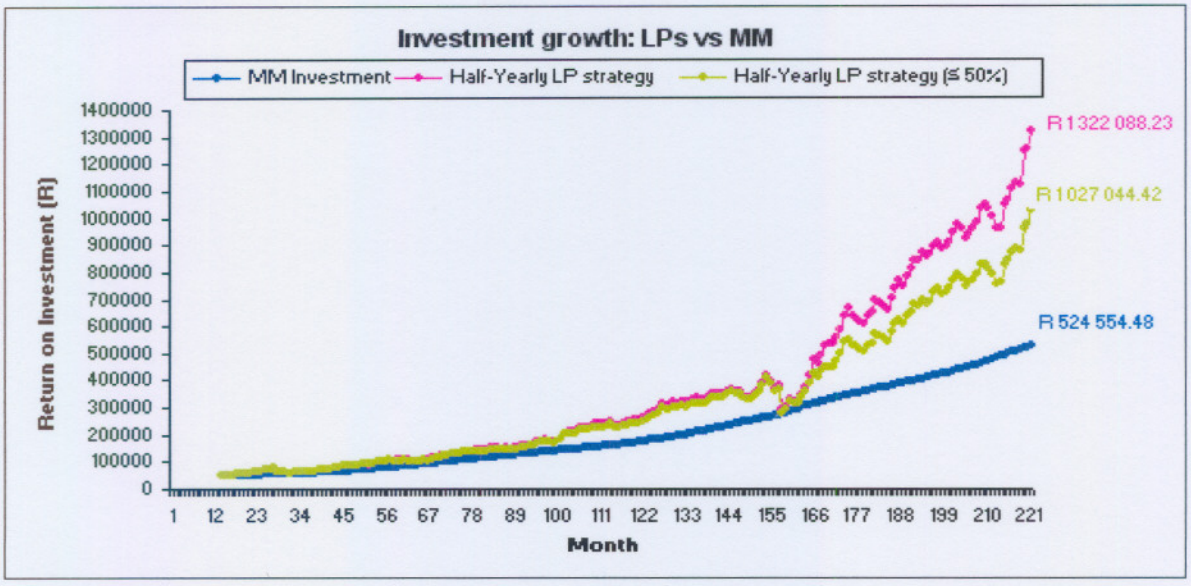
Graph 4.54 illustrates a comparison between the two half yearly LP (LPM) strategies (for n = 0.5) and the MM. All the strategies surpass the MM fund. The half yearly (LPM) strategy 1 performed the best, with an effective monthly interest rate of 1.57%. The effective monthly interest rate for half yearly (LPM) strategy 2 is 1.45%.



Graph 4.54: Comparison between half yearly LP (LPM) strategies (n = 0.5)

- **Half yearly LP (LPM) strategy (n = 1)**

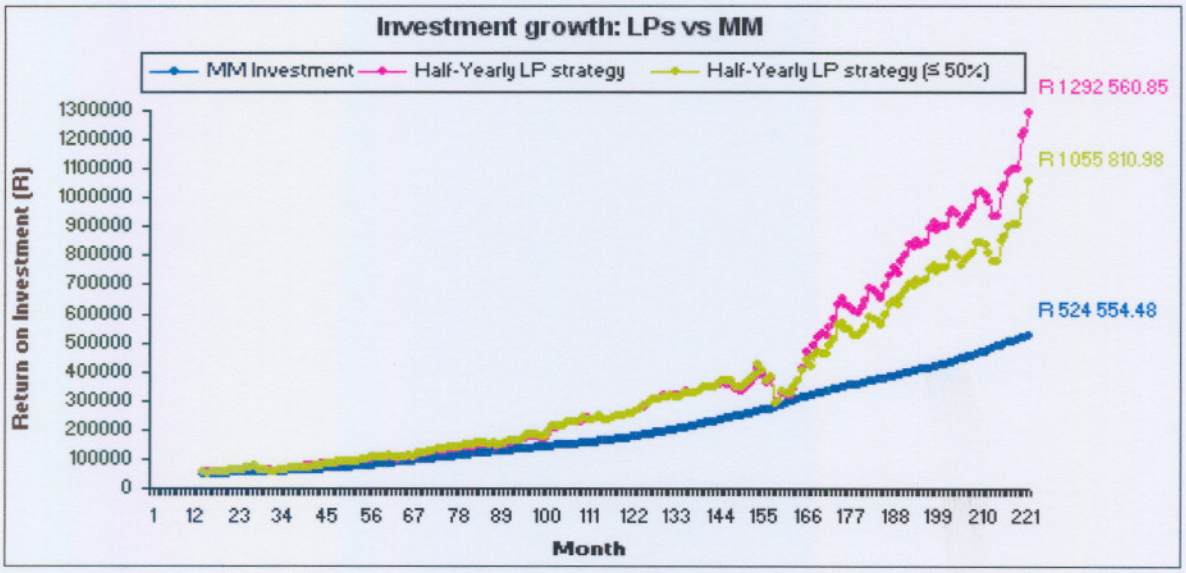
Graph 4.55 indicates a comparison between the two half yearly LP (LPM) strategies (for n = 1) and the MM. All the strategies exceed the MM fund's investment growth. The half yearly (LPM) strategy 1 performed the best, with an effective monthly interest rate of 1.58%. The effective monthly interest rate for half yearly (LPM) strategy 2 is 1.46%.



Graph 4.55: Comparison between half yearly LP (LPM) strategies (n = 1)

- **Half yearly LP (LPM) strategy (n = 2)**

The following graph shows a comparison between the two half yearly LP (LPM) strategies (for n = 2) and the MM. All the strategies surpass the MM fund's investment growth. The half yearly (LPM) strategy 1 performed the best, with an effective monthly interest rate of 1.57%. The effective monthly interest rate for half yearly (LPM) strategy 2 is 1.47%.

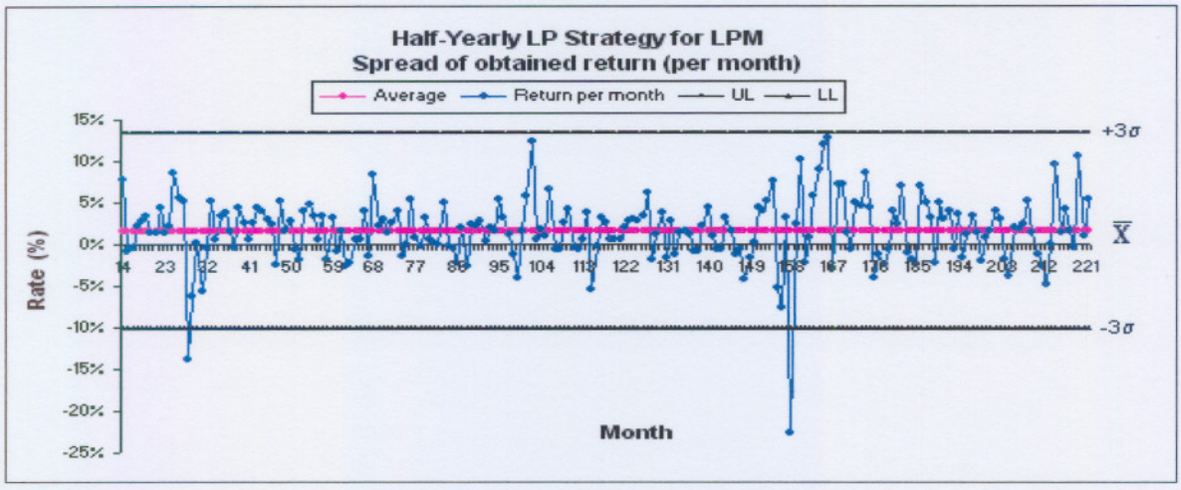


Graph 4.56: Comparison between half yearly LP (LPM) strategies (n = 2)

4.6.2.2 3-Sigma rule

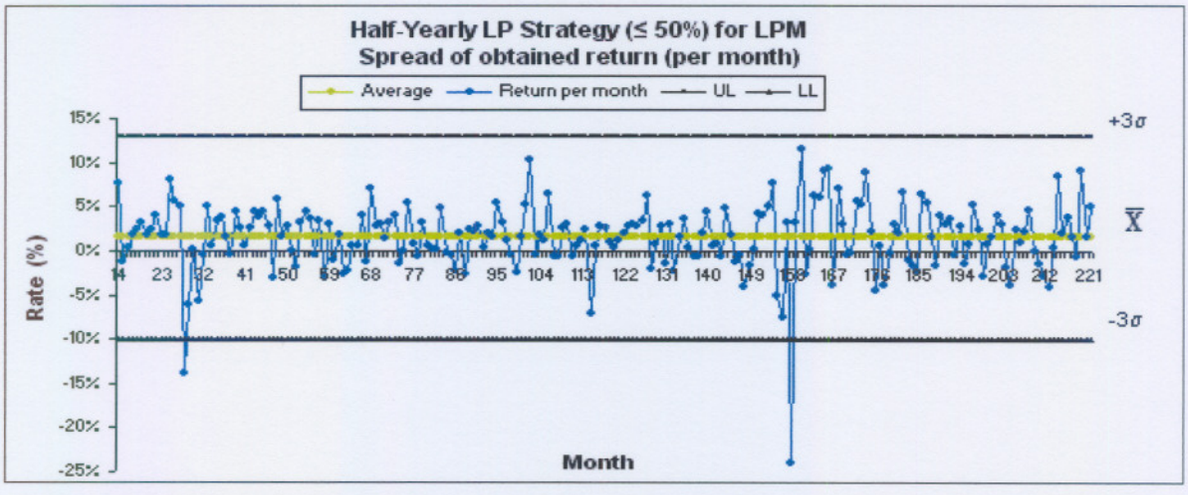
- **Half yearly LP (LPM) strategy 1 (n = 0.5)**

Graph 4.57 shows that 0.96% of the return values are below the lower limit (-10.09%) and none of the return values are above the upper limit (13.39%). 48.80% of the return values are below the target return of 1.60%. The average return that is achieved per month is 1.65% and the strategy's return/risk ratio is 0.4215. The standard deviation for this strategy is 3.91%.



Graph 4.57: 3-Sigma rule for half yearly LP (LPM) strategy 1 (n = 0.5)

- **Half yearly LP (LPM) strategy 2 (n = 0.5)**

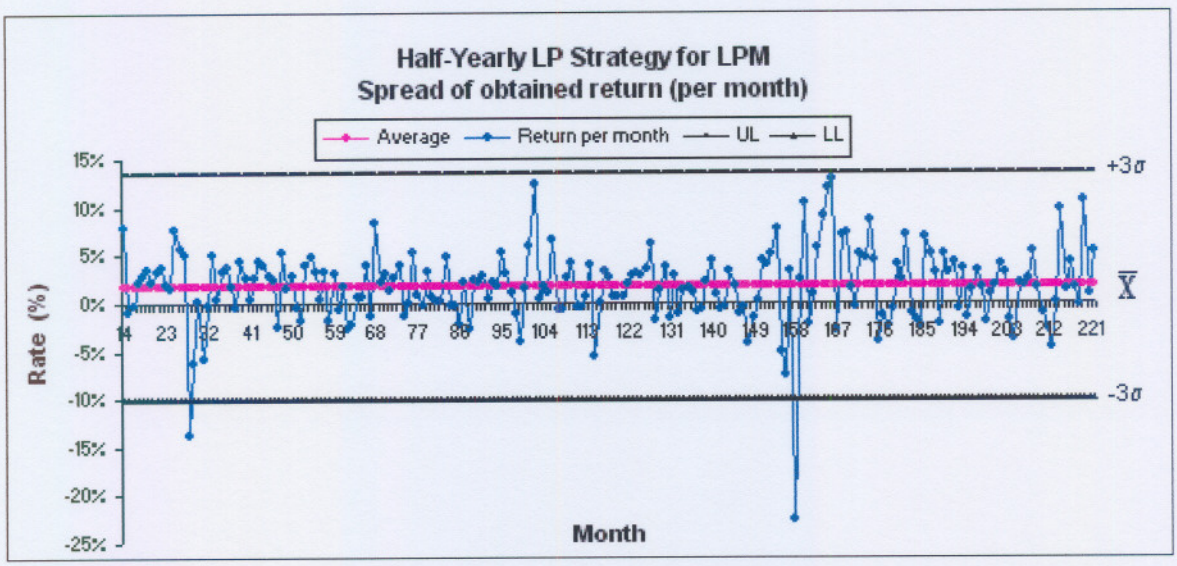


Graph 4.58: 3-Sigma rule for half yearly LP (LPM) strategy 2 (n = 0.5)

Graph 4.58 displays that 0.96% of the return values are below the lower limit of –9.96% and no return values are above the upper limit of 13.00%. 46.89% of the return values are below the target return of 1.60%. The average return is 1.52% per month and the return/risk ratio for this strategy is 0.3982. The standard deviation for this strategy is 3.83%.

- **Half yearly LP (LPM) strategy 1 (n = 1)**

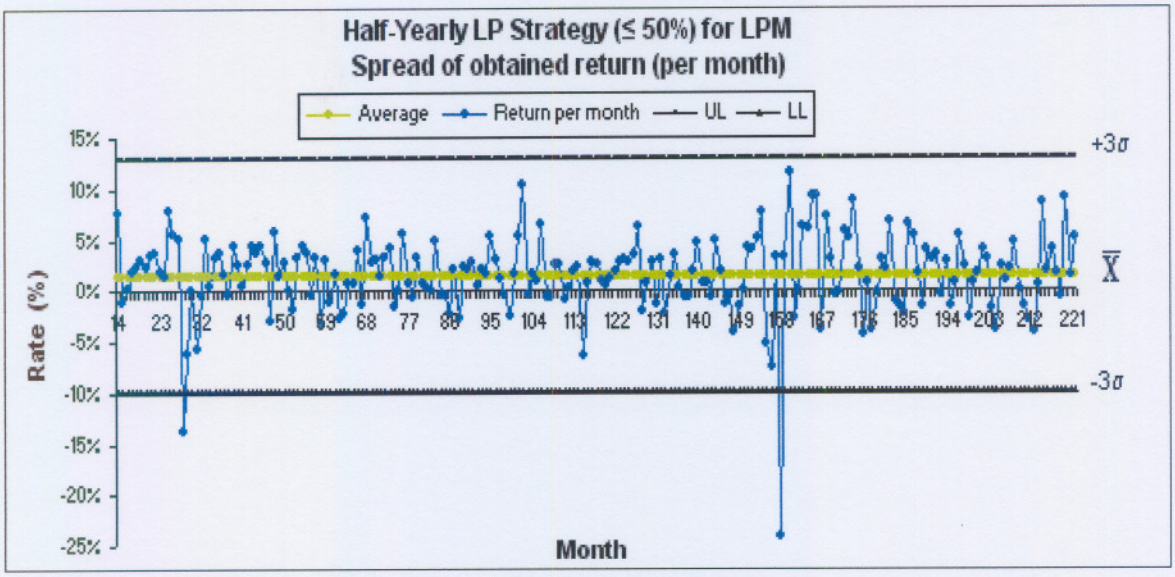
The following graph shows that 0.96% of the return values are below the lower limit (-10.07%) and none of the return values are above the upper limit (13.38%). 47.85% of the return values are below the target return of 1.60%. The average return that is attained per month is 1.66% and the strategy's return/risk ratio is 0.4240. The standard deviation for this strategy is 3.91%.



Graph 4.59: 3-Sigma rule for half yearly LP (LPM) strategy 1 (n = 1)

- **Half yearly LP (LPM) strategy 2 (n = 1)**

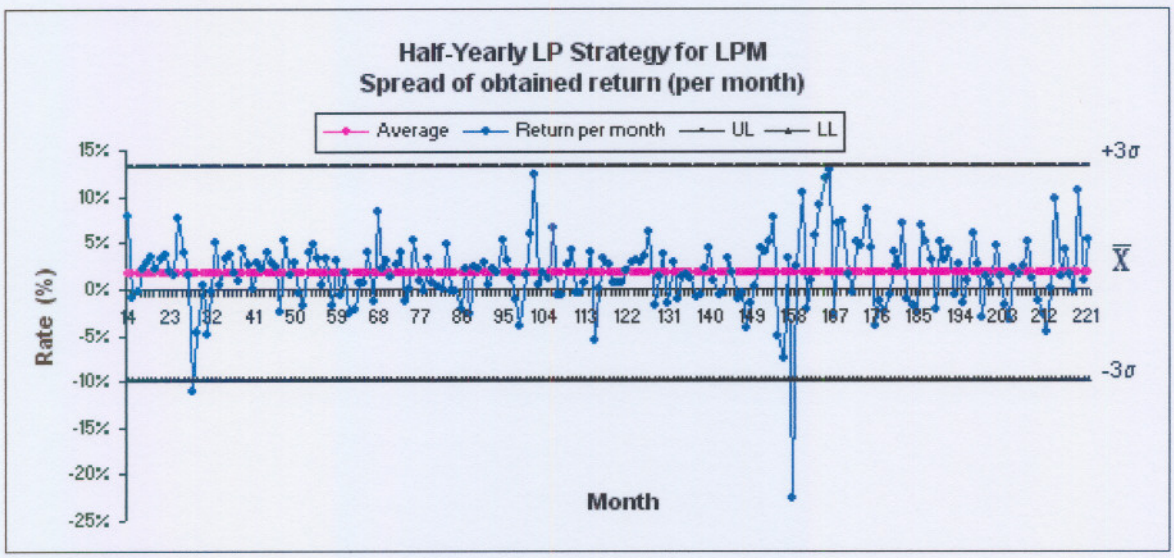
Graph 4.60 illustrates that 0.96% of the return values are below the lower limit of –9.92%. No return values are above the upper limit of 12.98%. 46.89% of the return values are below the target return of 1.60%. The strategy obtained an average return of 1.53% per month. The return/risk ratio for this strategy is 0.4014. The standard deviation for this strategy is 3.82%.



Graph 4.60: 3-Sigma rule for half yearly LP (LPM) strategy 2 (n = 1)

- **Half yearly LP (LPM) strategy 1 (n = 2)**

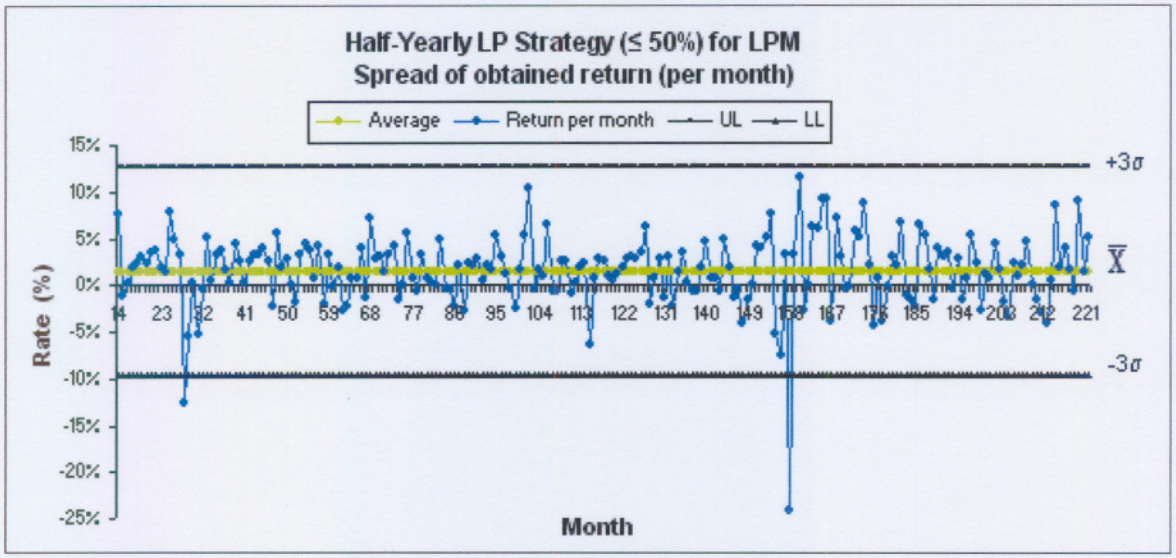
Graph 4.61 below, shows that 0.96% of the return values are below the lower limit (-9.85%) and none of the return values are above the upper limit (13.14%). 49.28% of the return values are below the target return of 1.60%. The average return that is obtained per month is 1.64% and the strategy's return/risk ratio is 0.4288. The standard deviation is 3.83%.



Graph 4.61: 3-Sigma rule for half yearly LP (LPM) strategy 1 (n = 2)

- **Half yearly LP (LPM) strategy 2 (n = 2)**

The following graph illustrates that 0.96% of the return values are below the lower limit of –9.70%. None of the return values are above the upper limit of 12.78%. 46.89% of the return values are below the target return of 1.60%. The average return is 1.54% per month and the return/risk ratio for this strategy is 0.4166. The standard deviation for this strategy is 3.75%.



Graph 4.62: 3-Sigma rule for half yearly LP (LPM) strategy 2 (n = 2)

Table 4.10 below provides a summary of the information obtained from the 3-sigma rule graphs for the half yearly LP (LPM) strategies:

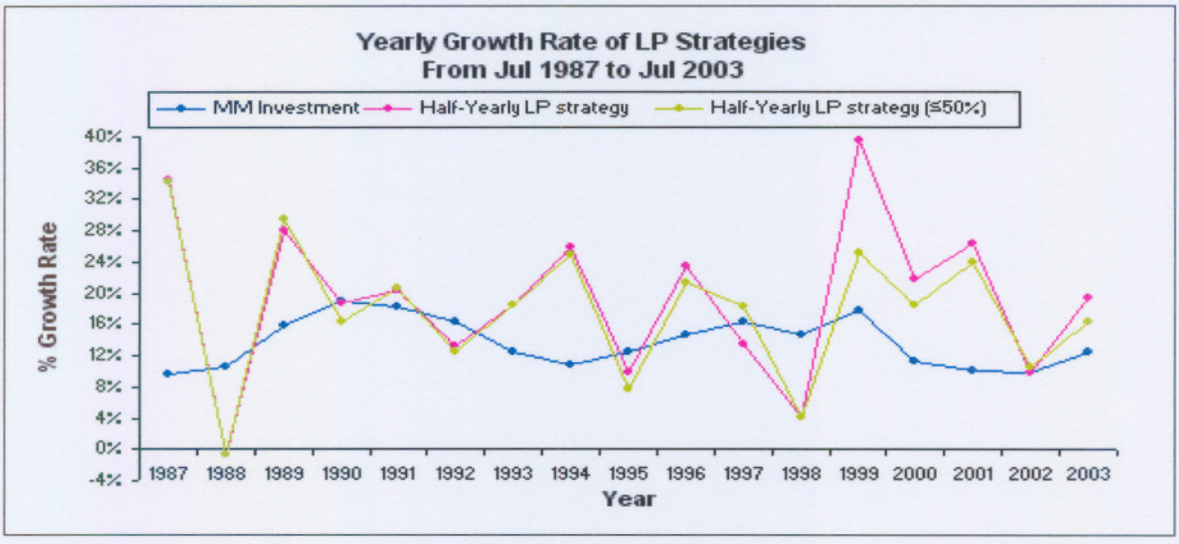
Strategy	$\overline{X}$	X < 1.60% (%)	Return/Risk ratio	X > UL (%)	X < LL (%)
Half yearly LP (LPM, n= 0.5) 1	1.65%	48.80%	0.4215	0.00%	0.96%
Half yearly LP (LPM, n= 0.5) 2	1.52%	46.89%	0.3982	0.00%	0.96%
Half yearly LP (LPM, n= 1) 1	1.66%	47.85%	0.4240	0.00%	0.96%
Half yearly LP (LPM, n= 1) 2	1.53%	46.89%	0.4014	0.00%	0.96%
Half yearly LP (LPM, n= 2) 1	1.64%	49.28%	0.4288	0.00%	0.96%
Half yearly LP (LPM, n= 2) 2	1.54%	46.89%	0.4166	0.00%	0.96%

Table 4.10: Summary of half yearly LP (LPM) strategies information

4.6.2.3 Yearly growth

- **Half yearly LP (LPM) strategy (n = 0.5)**

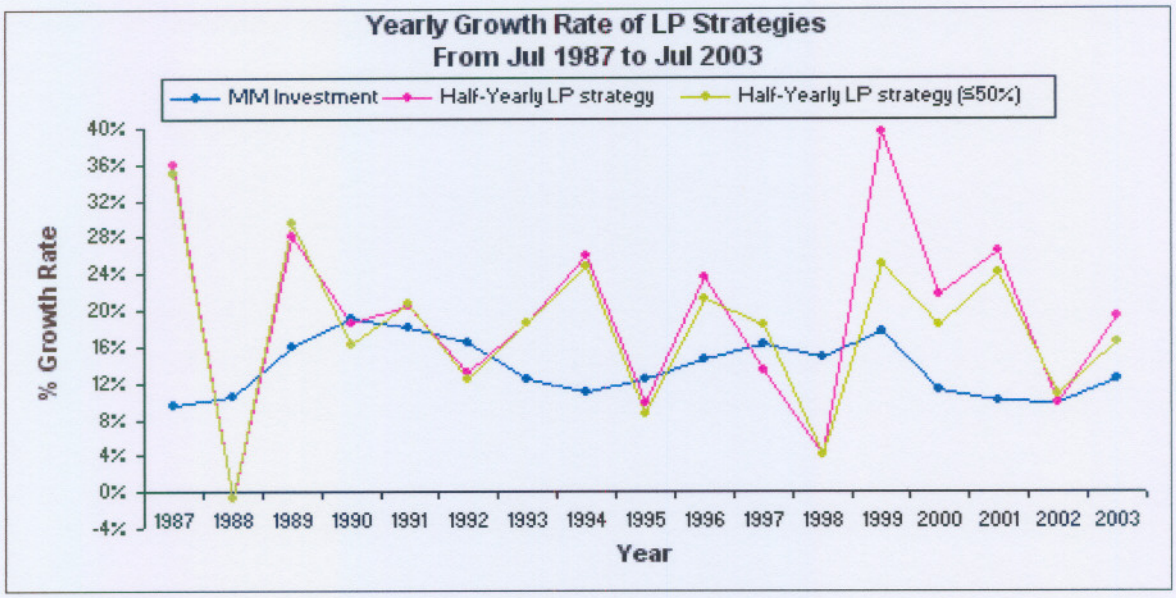
Graph 4.63 displays the yearly growth rates for the two half yearly LP (LPM) strategies, where n = 0.5. Half Yearly (LPM) strategy 1 shows the highest average yearly growth rate of 19.22%, the average yearly growth rate for half yearly (LPM) strategy 2 is 17.75%. The half yearly (LPM) strategy 1 shows the maximum yearly growth rate of 39.56% in 1999 and half yearly (LPM) strategy 2 obtained the minimum yearly growth rate of –0.70% is in 1988.



Graph 4.63: Yearly growth rates for half yearly LP (LPM) strategies (n = 0.5)

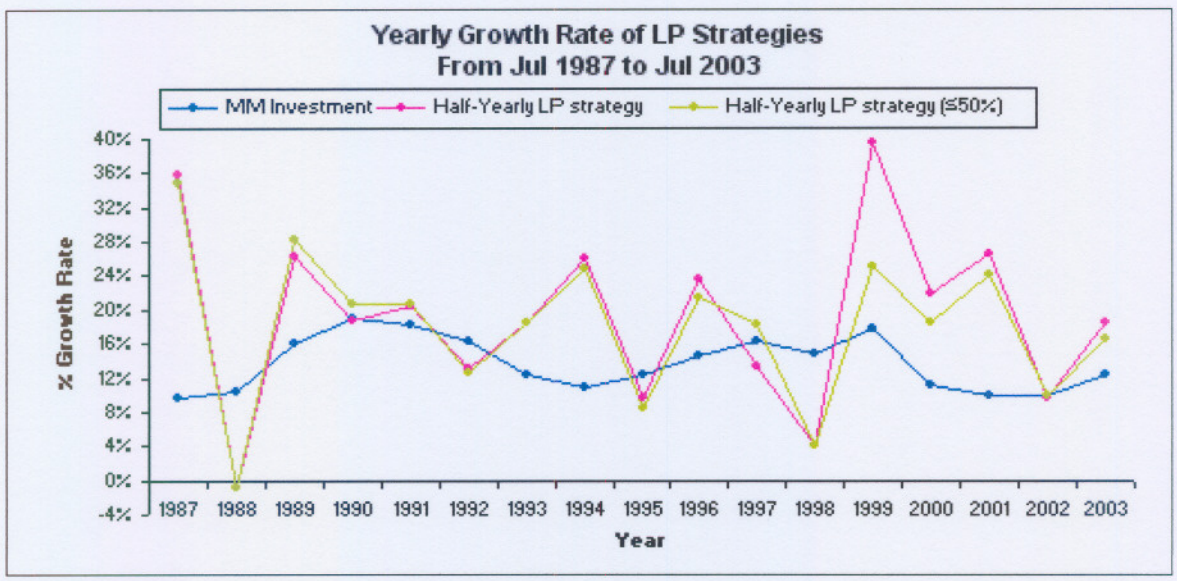
- **Half yearly LP (LPM) strategy (n = 1)**

Graph 4.64 displays the yearly growth rates for the two half yearly LP (LPM) strategies, where n = 1. Half yearly (LPM) strategy 1 shows the highest average yearly growth rate of 19.30%, the average yearly growth rate for half yearly (LPM) strategy 2 is 17.85%. The half yearly (LPM) strategy 1 shows the highest yearly growth rate of 39.56% in 1999 and half yearly (LPM) strategy 2 obtained the lowest yearly growth rate of –0.70% is in 1988.



Graph 4.64: Yearly growth rates for half yearly LP (LPM) strategies (n = 1)

- **Half yearly LP (LPM) strategy (n = 2)**



Graph 4.65: Yearly growth rates for half yearly LP (LPM) strategies (n = 2)

Graph 4.65 illustrates the yearly growth rates for the two half yearly LP (LPM) strategies, where n = 2. Half yearly (LPM) strategy 1 shows the highest average yearly growth rate of 19.13%, the average yearly growth rate for half yearly (LPM) strategy 2 is 17.98%. The half

yearly (LPM) strategy 1 shows both the maximum yearly growth rate of 39.56% in 1999, and the minimum yearly growth rate of –0.85% is in 1988.

4.7 Conclusion

This chapter examined some decision-making strategies that use risk measures in the decision making process, i.e. decision making under risk. Monte Carlo simulation models and optimisation models (with different measures of risk) are represented. The non-linear optimisation model (Markowitz portfolio model) uses variance as a risk measure. The linear programming models are performed with two risk measures, namely downside risk and β. The half yearly LP model that maximised the return and set $\beta \leq 1.5$, performed the best, in terms of total investment growth.

CHAPTER 5
COMPARISON OF STRATEGIES WITH MONEY MARKET

5.1 Introduction

This chapter presents a comparison between the strategies for decision-making under uncertainty and the strategies for decision-making under risk, in terms of risk. The chapter portrays the return of the strategies (per month) vs. the return of the MM (per month). The following risk measures are used as criteria for selecting an optimal strategy in terms of risk (minimise risk).

- The variance of a strategy's return from the Money Market's return (yearly averages are used) as given by: $\frac{\sum (X - MM)^2}{n}$, where X is the actual returns obtained for a strategy and MM is the returns for the Money Market.
- The number of times that the strategy performed worst that the MM, i.e. the estimated probability that the strategy showed a lower return than the MM: P (X<MM), where X is the actual returns obtained for a strategy and MM is the returns for the Money Market.
- The standard deviation (s) for the strategies (shown in chapters 3 and 4).
- The expected shortfall (yearly averages are used):

$$LPM_{in} = \left(\frac{1}{m}\right) \sum_{t=1}^{m} [Max\ (0, X - MM)]^n$$

Where

LPM_{in} = the lower partial moment for fund *i* and for degree *n* (for expected shortfall n = 1)

t = 1, 2, ..., m observations

MM = return on the Money Market

Max = maximisation function that selects the larger of the two values

X = return on the strategy during time period t.

The range are as follows (Clark, 1994:54):

$LPM_{in} = (X - MM)^n \Rightarrow$ over the range where $X < MM$

$= 0 \Rightarrow$ over the range where $X \geq MM$

The following reward measures are used as criteria for selecting an optimal strategy in terms of return (maximise return).

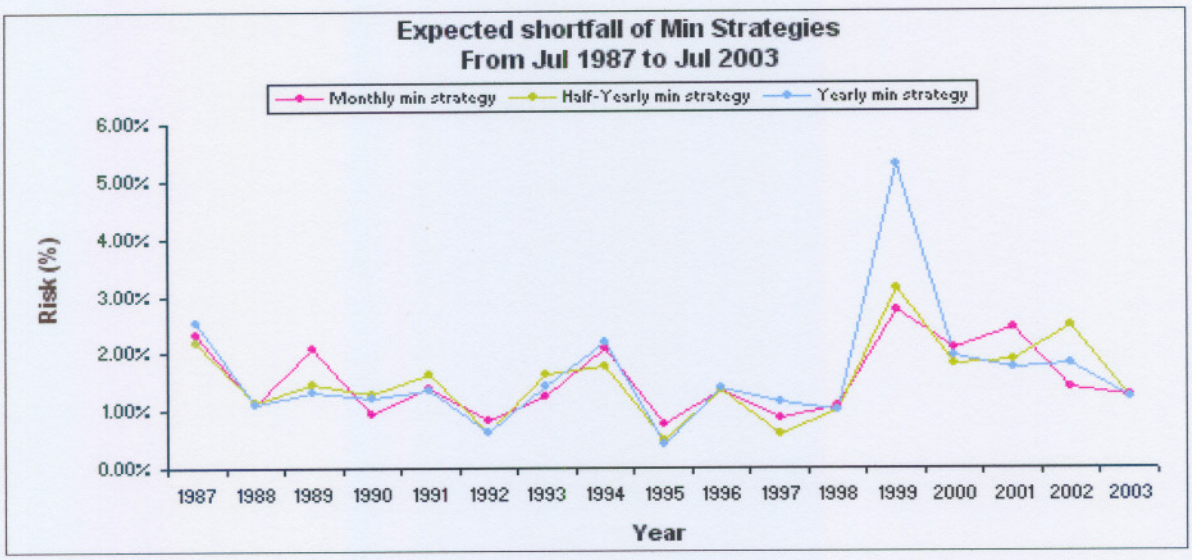
- The average return $\overline{X}$ for the strategies.
- The average deviation, measured as the difference between the actual returns obtained for a strategy and the Money Market's return (yearly averages are used), given by: $\overline{X} - \overline{MM}$, where $\overline{X}$ is the average return on a strategy and $\overline{MM}$ is the average return on the Money Market (this is the relative return).

5.2 Decision making under uncertainty

5.2.1 Min strategies

- **Expected shortfall**

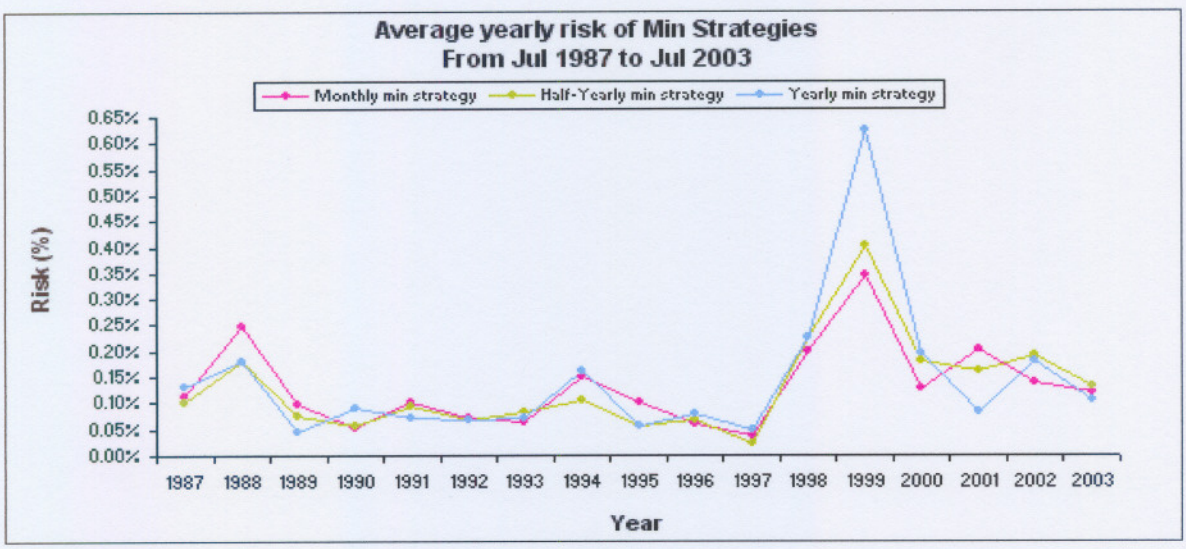
The following graph shows the comparison between the expected shortfall risk measures for the different min strategies. The yearly min strategy obtained the maximum risk value (5.28% in 1999) and the minimum risk value (0.42% in 1995). The half yearly min strategy attained the lowest yearly risk average of 0.1.5064%. The values for the monthly and yearly min strategies are 1.5268% and 1.6320% respectively.



Graph 5.1: Expected shortfall for min strategies

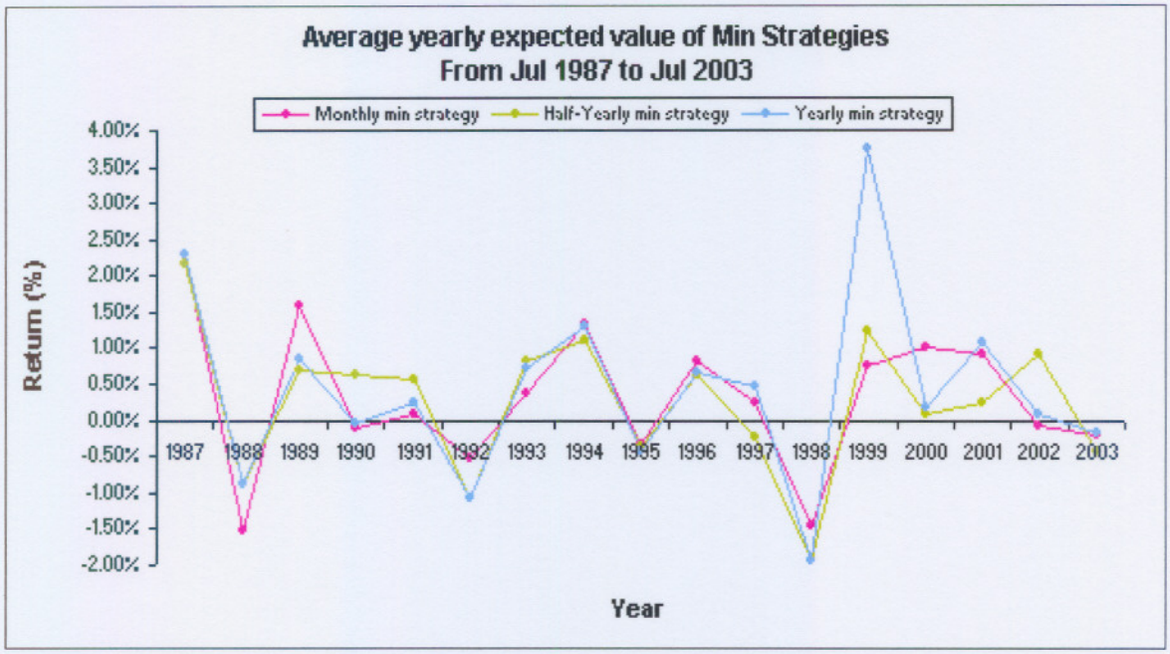
- **Variance**

Graph 5.2 shows the comparison between the variance risk measures for the different min strategies. The yearly min strategy obtained the maximum risk value (0.62% in 1999) and the half yearly min strategy obtained the minimum risk value (0.02% in 1997). The half yearly min strategy attained the lowest yearly risk average of 0.1296%. The values for the monthly and yearly min strategies are 0.1312% and 0.1416% respectively.



Graph 5.2: Variance comparison between min strategies

- **Average deviation**



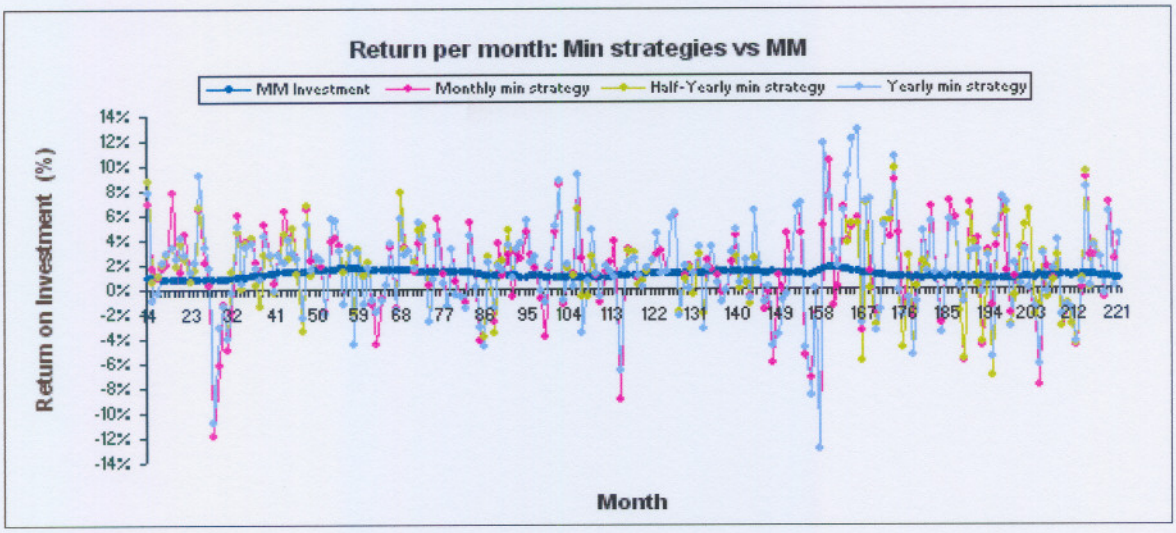
Graph 5.3: Expected value comparison between min strategies

The graph above shows the comparison between the average deviation return measures for the different min strategies. The yearly min strategy obtained the maximum risk value of 3.74% in 1999. Both the half yearly and yearly min strategy obtained the minimum risk value of -1.92% in 1998. The half yearly min strategy attained the lowest yearly average of 0.2485%. The values for the monthly and yearly min strategies are 0.3090% and 0.4198% respectively.

- **Return per month**

Graph 5.4 shows the comparison between the obtained returns of the min strategies and the MM (per month). The returns for the min strategies fluctuate around the MM. P (X<MM) for the min strategies is as follows:

- Monthly min strategy → 40.19%.
- Half yearly min strategy → 44.02%.
- Yearly min strategy → 41.63%.

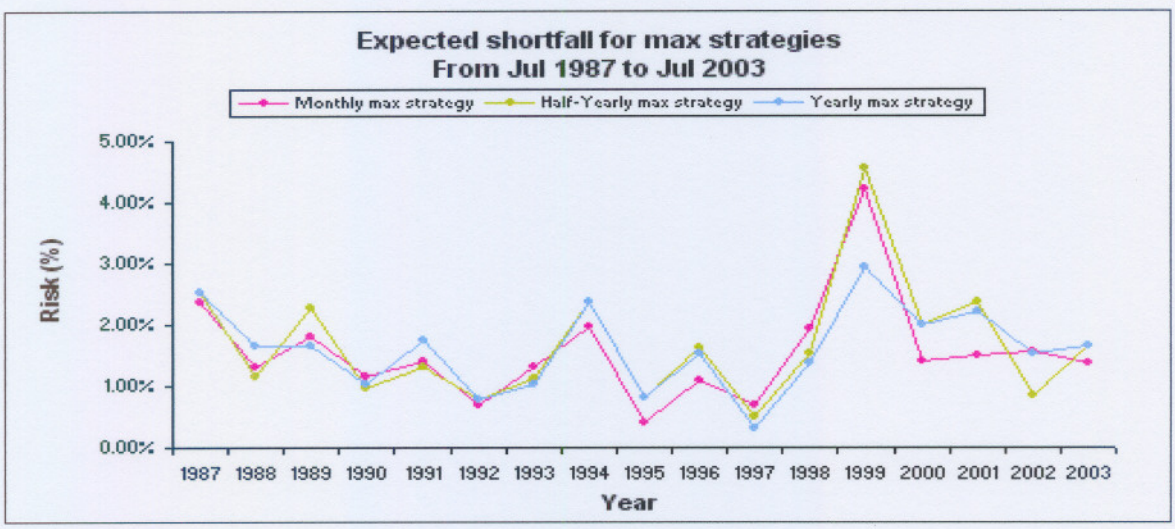


Graph 5.4: Return comparison between min strategies and MM

5.2.2 Max strategies

- **Expected shortfall**

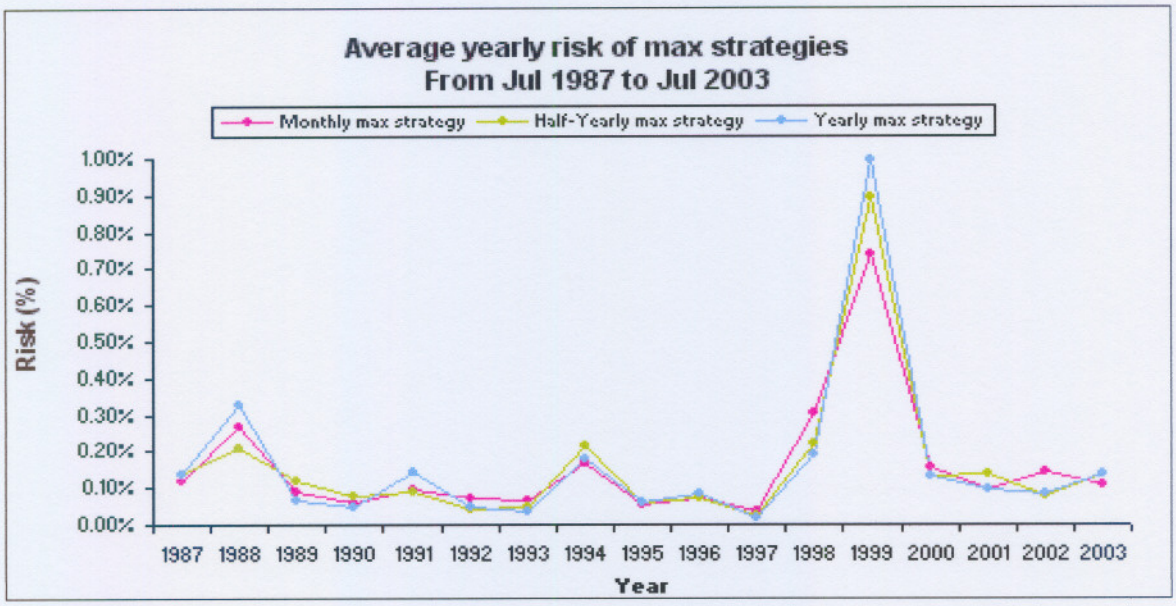
The following graph shows the comparison between the expected shortfall risk measures for the different min strategies. The half yearly max strategy obtained the maximum risk value (4.58% in 1999) and the yearly max strategy obtained the minimum risk value (0.33% in 1997). The monthly max strategy attained the lowest yearly risk average of 0.15435%. The values for the half yearly and yearly max strategies are 1.6725% and 1.6019% respectively.



Graph 5.5: Expected shortfall for max strategies

- **Variance**

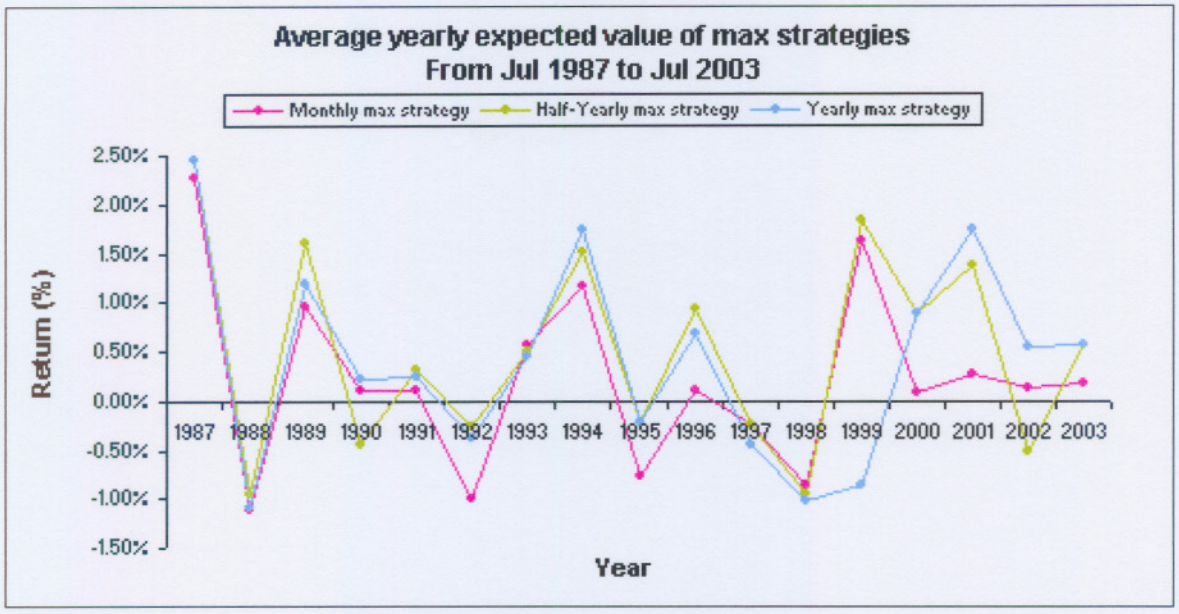
The following graph indicates the comparison between the variance risk measures for the different max strategies. The yearly max strategy obtained both the highest risk value of 0.99% in 1999 and the lowest risk value 0.02% in 1997. The monthly max strategy attained the lowest yearly risk average of 0.1553%. The values for the half yearly and yearly max strategies are 0.1580% and 0.1631% correspondingly.



Graph 5.6: Variance comparison between max strategies

- **Average deviation**

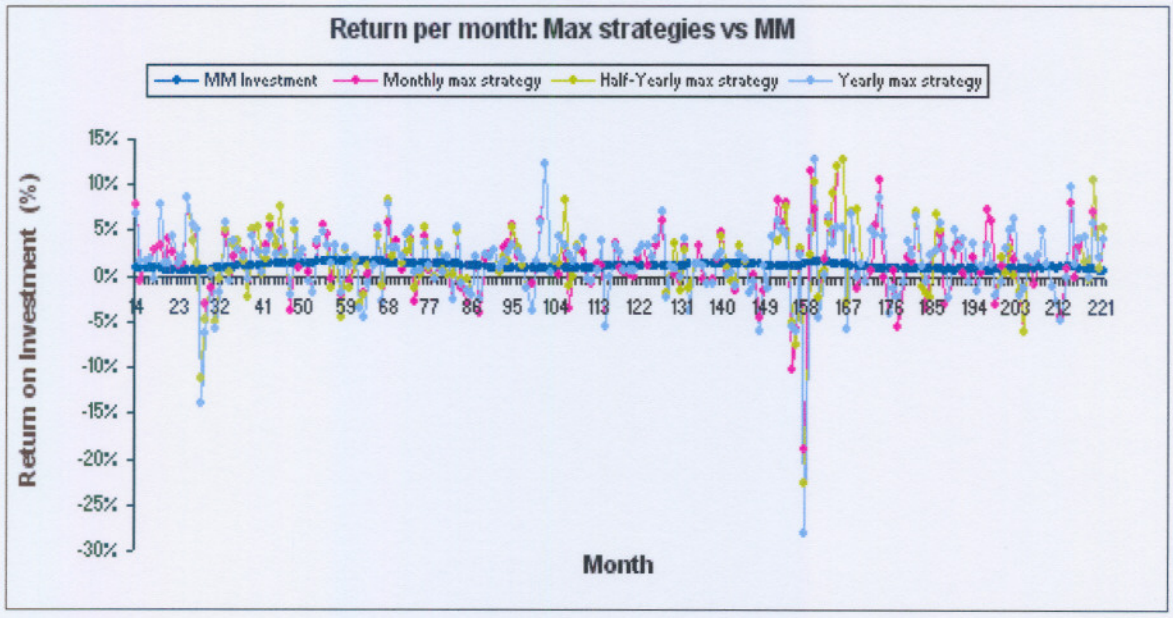
Graph 5.7 shows a comparison between the average deviation return measures for the different max strategies. The yearly max and the half yearly max strategies obtained the maximum risk value of 2.46% in 1987. The monthly max strategy obtained the minimum value of -1.11% in 1988. The monthly max strategy attained the lowest yearly risk average of 0.2113%. The values for the half yearly and yearly max strategies are 0.4926% and 0.3975% correspondingly.



Graph 5.7: Expected value comparison between max strategies

- **Return per month**

The following graph shows the comparison between the obtained returns of the max strategies and the MM (per month). The returns for the max strategies vary around the MM.



Graph 5.8: Return comparison between max strategies and MM

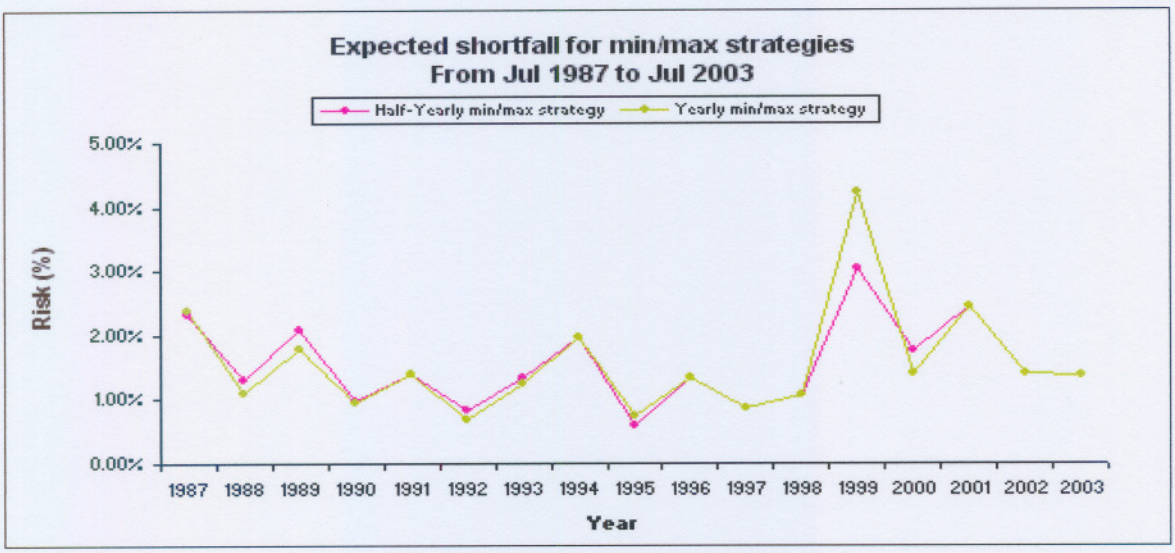
P (X<MM) for the max strategies is as follows:

- Monthly max strategy → 44.98%.
- Half yearly max strategy → 41.63%.
- Yearly max strategy → 39.71%.

5.2.3 Min/max strategies

- **Expected shortfall**

The following graph shows the comparison between the expected shortfall risk measures for the different min strategies. The yearly min/max strategy obtained the maximum risk value of 4.22% in 1999 and half yearly min/max strategy attained the minimum risk value of 0.59% in 1995. The half yearly min/max strategy attained the lowest yearly risk average of 0.15392%. The value for the yearly min/max strategy is 1.5537%.

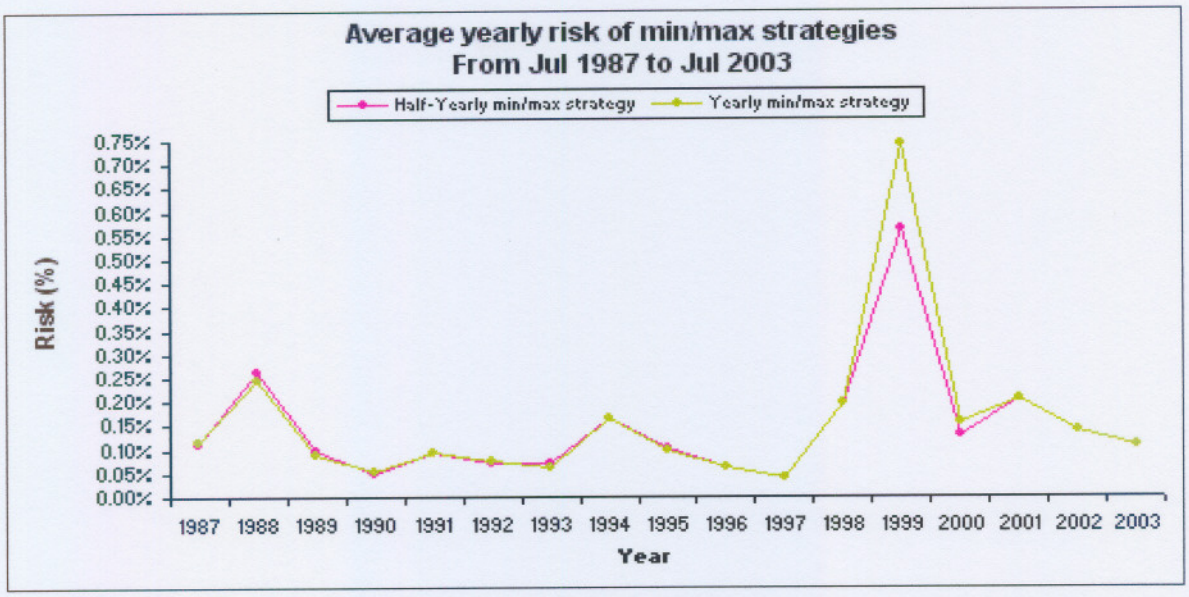


Graph 5.9: Expected shortfall for min/max strategies

- **Variance**

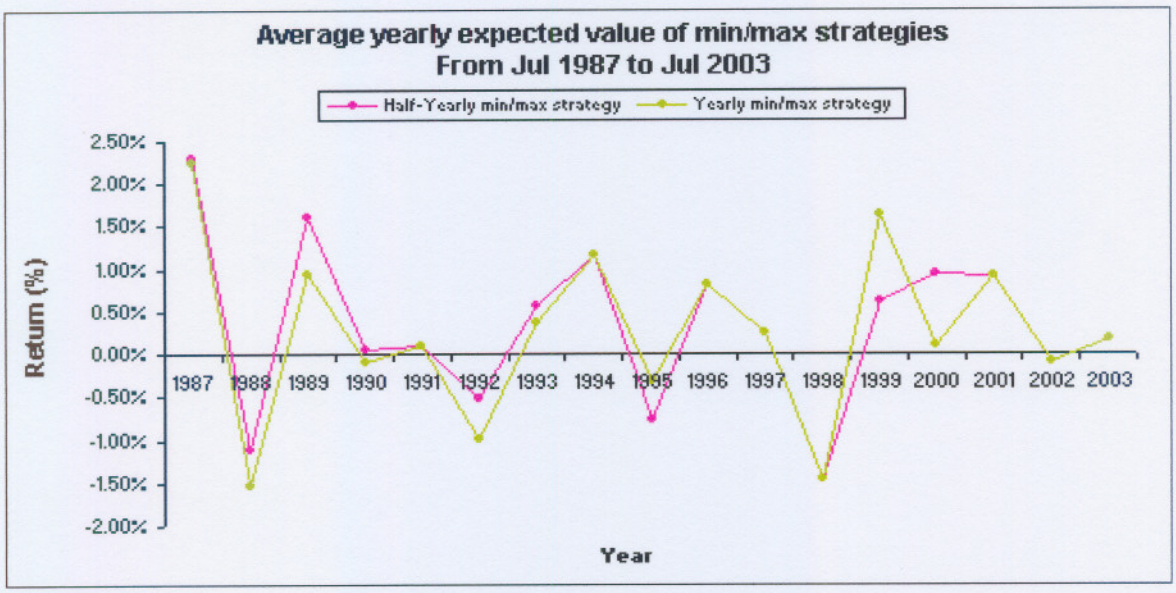
Graph 5.10 illustrates the comparison between the variance risk measures for the different min/max strategies. The half yearly and yearly min/max strategies obtained the lowest risk

value of 0.04% in 1997 and the yearly min/max strategy attained the highest risk value of 0.74% in 1999. The yearly risk averages for the half yearly and yearly min/max strategies are 0.1453% and 0.1558%, respectively



Graph 5.10: Variance comparison between min/max strategies

- **Average deviation**



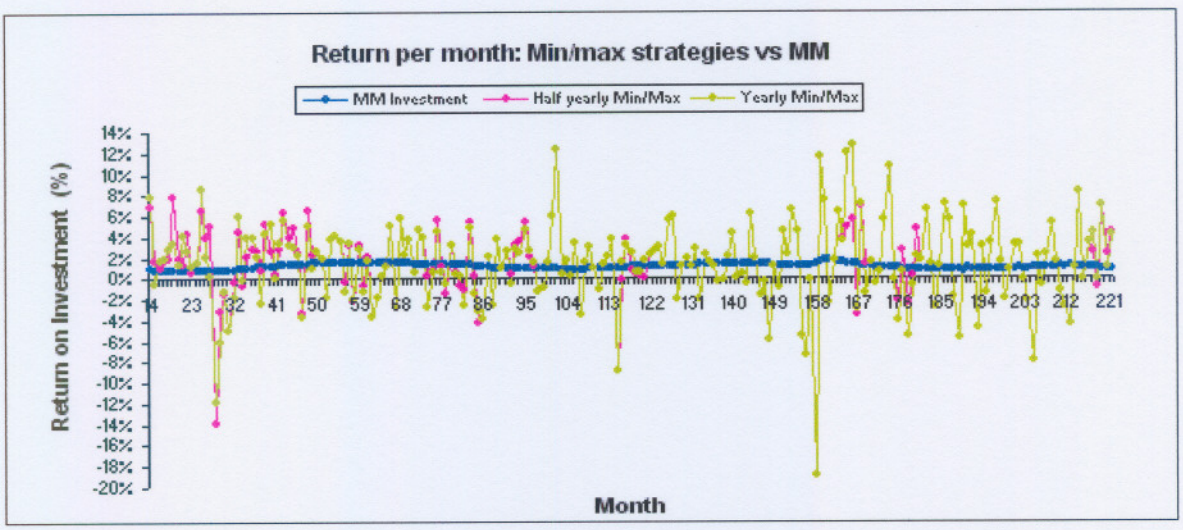
Graph 5.11: Expected value comparison between min/max strategies

Graph 5.11 shows a comparison between the average deviation return measures for the different min/max strategies. The half yearly min/max strategy obtained the highest risk value of 2.31% in 1987. The yearly min/max strategy obtained the lowest risk value of –1.53% in 1988. The yearly risk average for the half yearly min/max strategy is 0.3271% and for the yearly min/max strategy it is 0.2499%

- **Return per month**

The following graph shows the comparison between the acquired returns of the min/max strategies and the MM. The returns for the max strategies vary around the MM. P (X<MM) for the min/max strategies are as follows:

- Half yearly min/max strategy → 41.63%.
- Yearly min/max strategy → 42.11%.



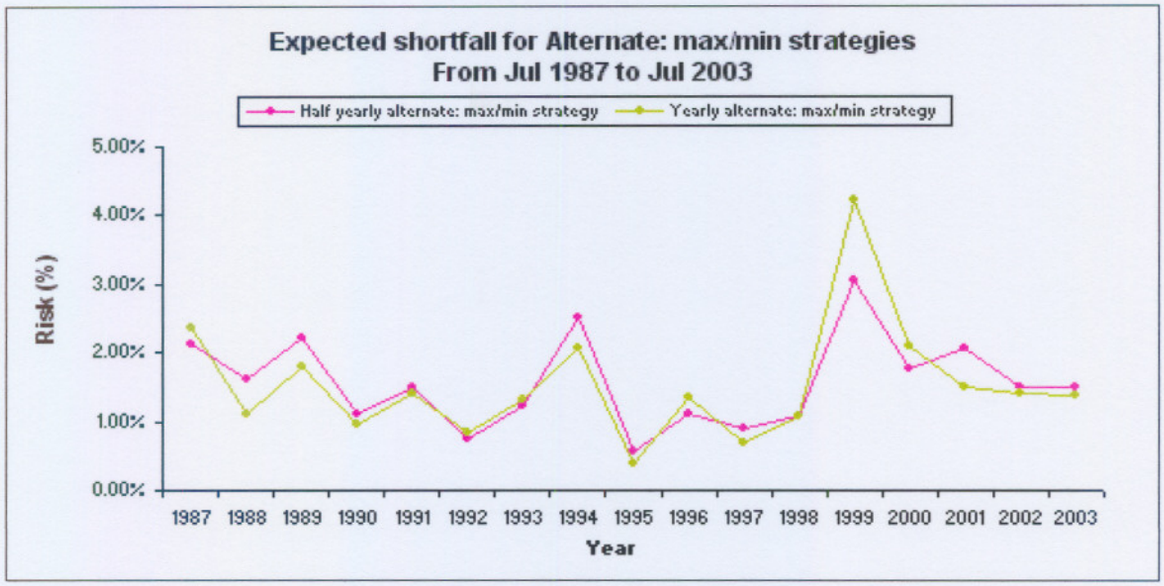
Graph 5.12: Return comparison between min/max strategies and MM

5.2.4 Alternate max/min strategies

- **Expected shortfall**

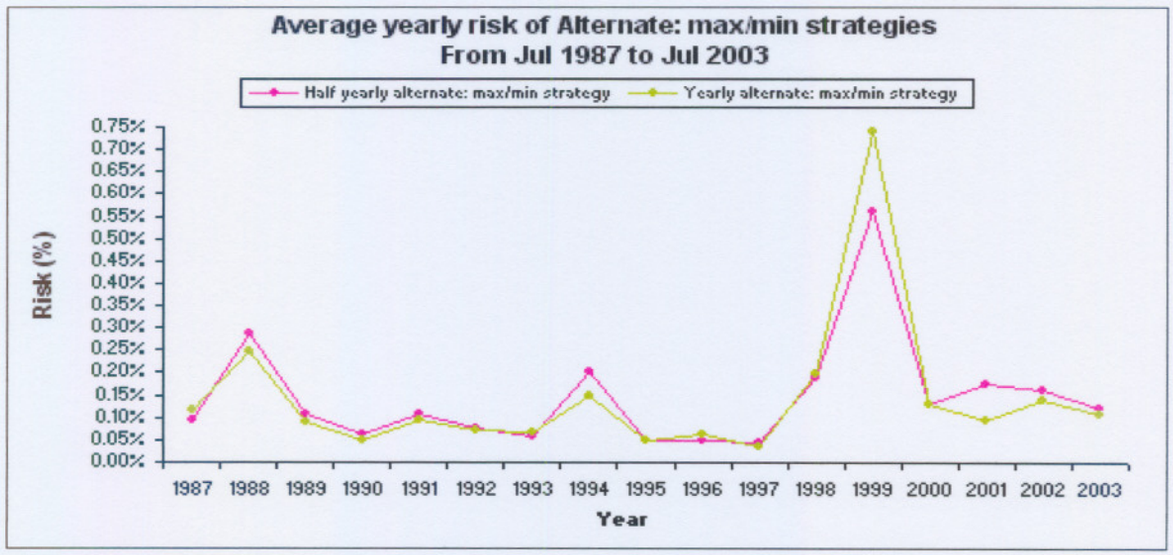
The following graph shows the comparison between the expected shortfall risk measures for the different alternate min/max strategies. The yearly alternate min/max strategy obtained

the maximum risk value (4.22% in 1999) and the minimum risk value (0.40% in 1995). The yearly alternate min/max strategy attained the lowest yearly risk average of 1.5290%. The yearly risk average for the half yearly alternate min/max strategy is 1.5633%.



Graph 5.13: Expected shortfall for alternate max/min strategies

- **Variance**

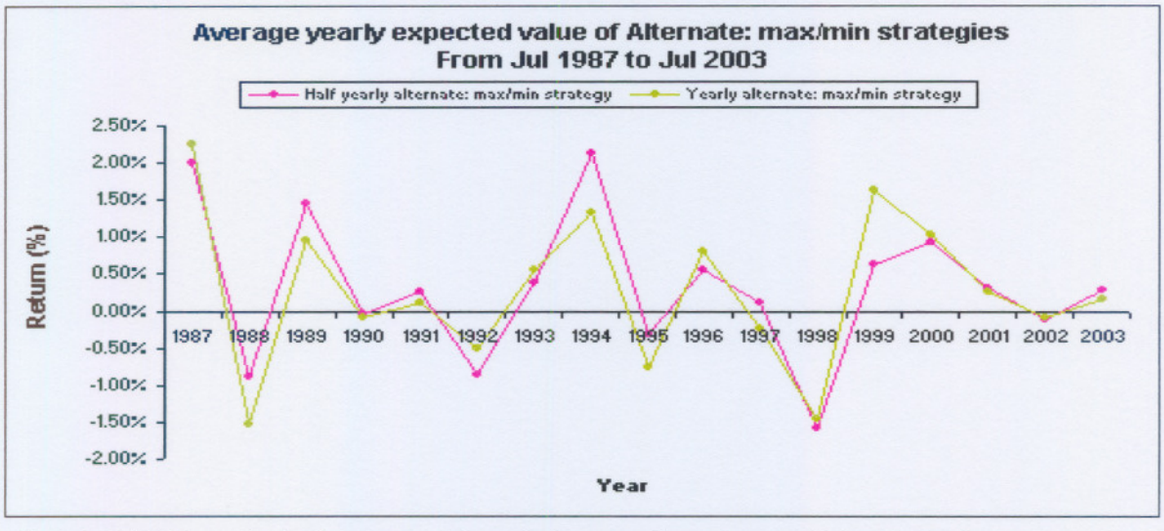


Graph 5.14: Variance comparison between alternate: max/min strategies

Graph 5.14 presents the comparison between the variance risk measures for the different alternate max/min strategies. The half yearly and yearly alternate max/min strategies obtained the lowest risk value of 0.04% in 1997 and the yearly alternate max/min strategy attained the highest risk value of 0.74% in 1999. The yearly risk averages for the half yearly and yearly alternate max/min strategies are 0.1461% and 0.1441%, respectively.

- **Average deviation**

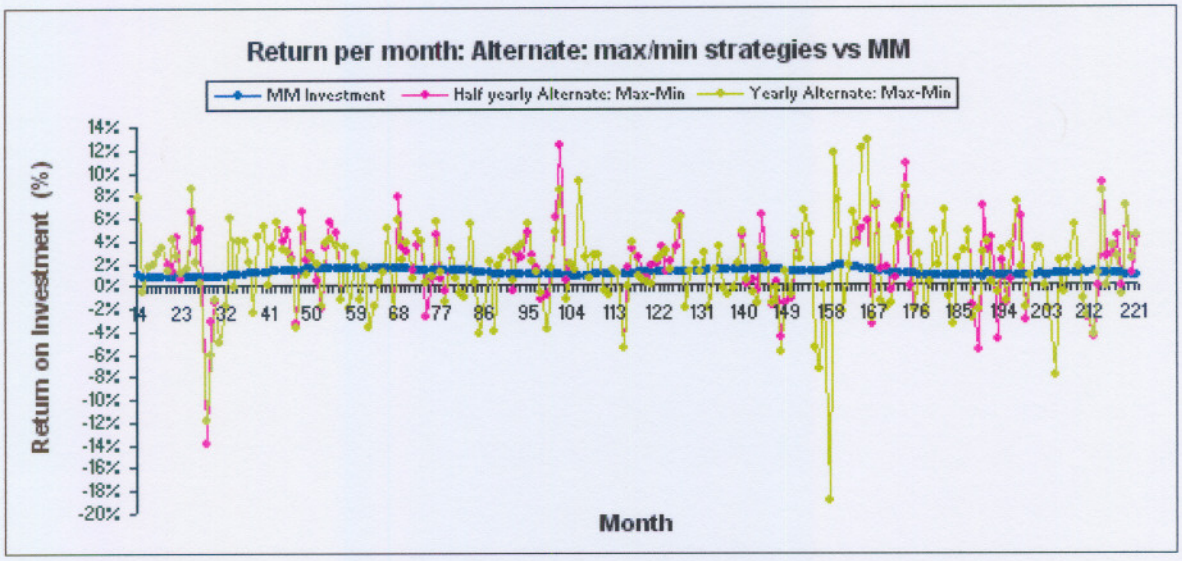
The following graph shows a comparison between the average deviation return measures for the different alternate: max/min strategies. The yearly alternate: max/min strategy obtained the highest risk value of 2.26% in 1987. The half yearly alternate: max/min strategy obtained the lowest risk value of –1.58% in 1998. The yearly average for the half yearly alternate: max/min strategy is 0.3098% and for the yearly alternate: max/min strategy it is 0.2611%.



Graph 5.15: Expected value comparison between alternate: max/min strategies

- **Return per month**

The following graph shows the comparison between the acquired returns of the alternate: max/min strategies and the MM. The returns for the alternate: max/min strategies fluctuate around the MM.



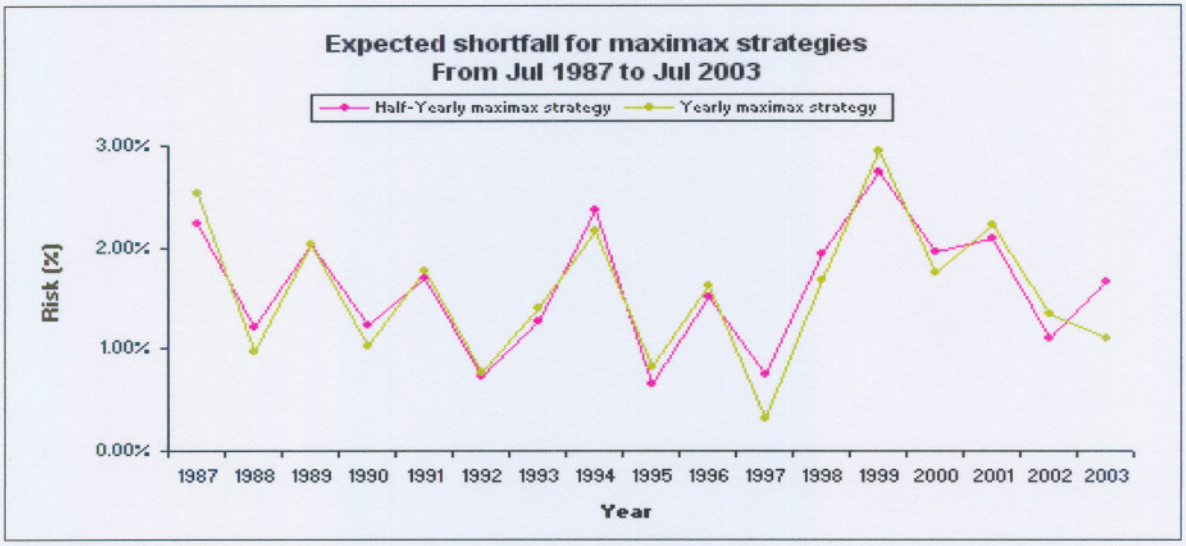
Graph 5.16: Return comparison between alternate: max/min strategies and MM

P (X<MM) for the alternate: max/min strategies are as follows:

- Half yearly alternate: max/min strategy → 41.15%.
- Yearly alternate: max/min strategy → 43.06%.

5.2.5 Maximax strategies

- **Expected shortfall**

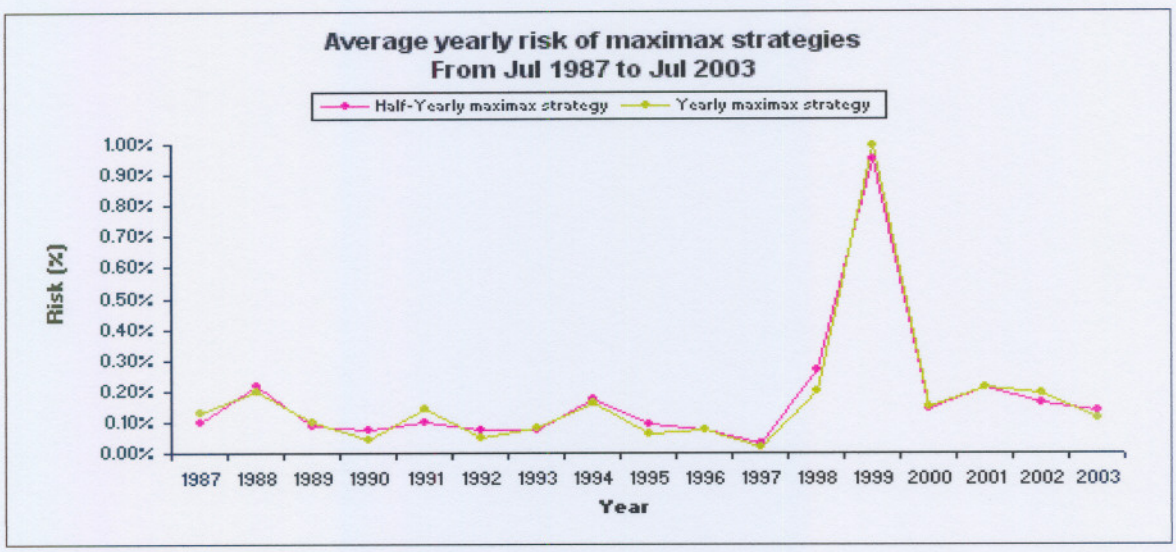


Graph 5.17: Expected shortfall for maximax strategies

Graph 5.17 shows the comparison between the expected shortfall risk measures for the different maximax strategies. The yearly maximax strategy obtained both the highest risk value of 2.94% in 1999 and the lowest risk value of 0.33% in 1997. The yearly maximax strategy attained the lowest yearly risk average of 1.5571%. The yearly risk average for the half yearly maximax strategy is 1.5970%.

- **Variance**

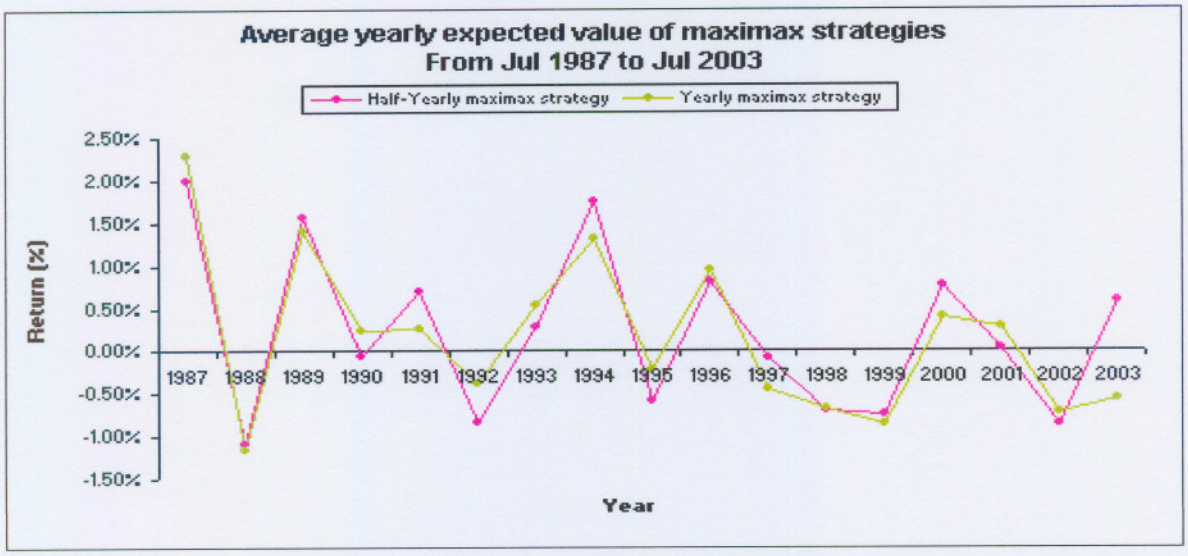
The following graph indicates the comparison between the variance risk measures for the different maximax strategies. The yearly maximax strategy obtained both the highest risk value of 0.99% in 1999 and the lowest risk value 0.02% in 1997. The yearly maximax strategy attained the lowest yearly risk average of 0.1715%. The half yearly risk average is 0.1748%.



Graph 5.18: Variance comparison between maximax strategies

- **Average deviation**

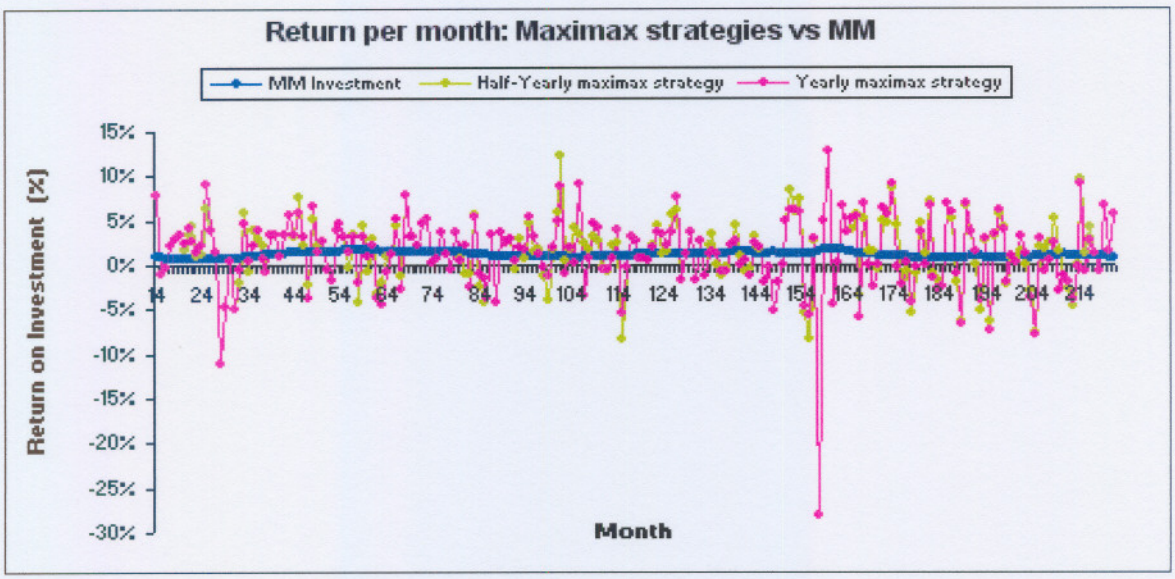
The following graph shows a comparison between the average deviation return measures for the different maximax strategies. The yearly maximax strategy obtained the maximum risk value of 2.29% in 1987. The yearly maximax strategy obtained the minimum value of -1.17% in 1988. The yearly average for the half yearly maximax strategy is 0.2067% and for the yearly maximax strategy it is 0.1545%.



Graph 5.19: Expected value comparison between maximax strategies

- **Return per month**

The following graph shows the comparison between the obtained returns of the maximax strategies and the MM (per month). The returns for the maximax strategies vary around the MM.



Graph 5.20: Return comparison between maximax strategies and MM

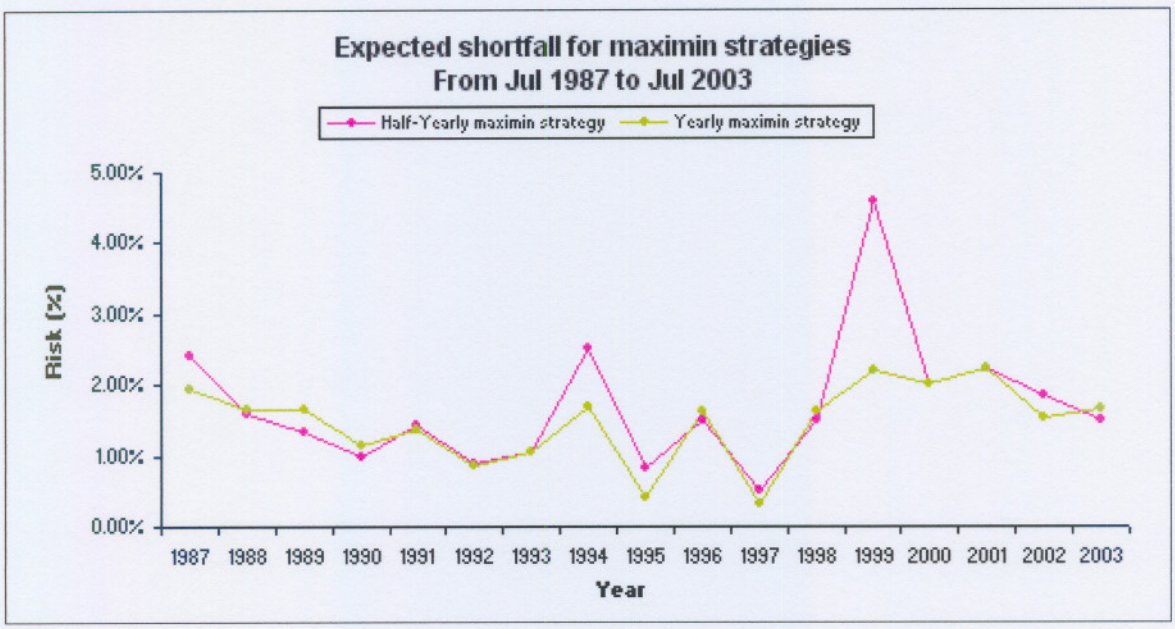
P (X<MM) for the maximax strategies is as follows:

- Half yearly maximax strategy → 41.63%.
- Yearly maximax strategy → 46.41%.

5.2.6 Maximin strategies

- **Expected shortfall**

Graph 5.21 portrays the comparison between the expected shortfall risk measures for the different maximin strategies. The half yearly maximin strategy obtained the maximum risk value (4.58% in 1999) and the yearly maximin strategy obtained minimum risk value (0.33% in 1997). The yearly maximin strategy attained the lowest yearly risk average of 1.4681%. The value for half yearly maximin strategy is 1.6873%.

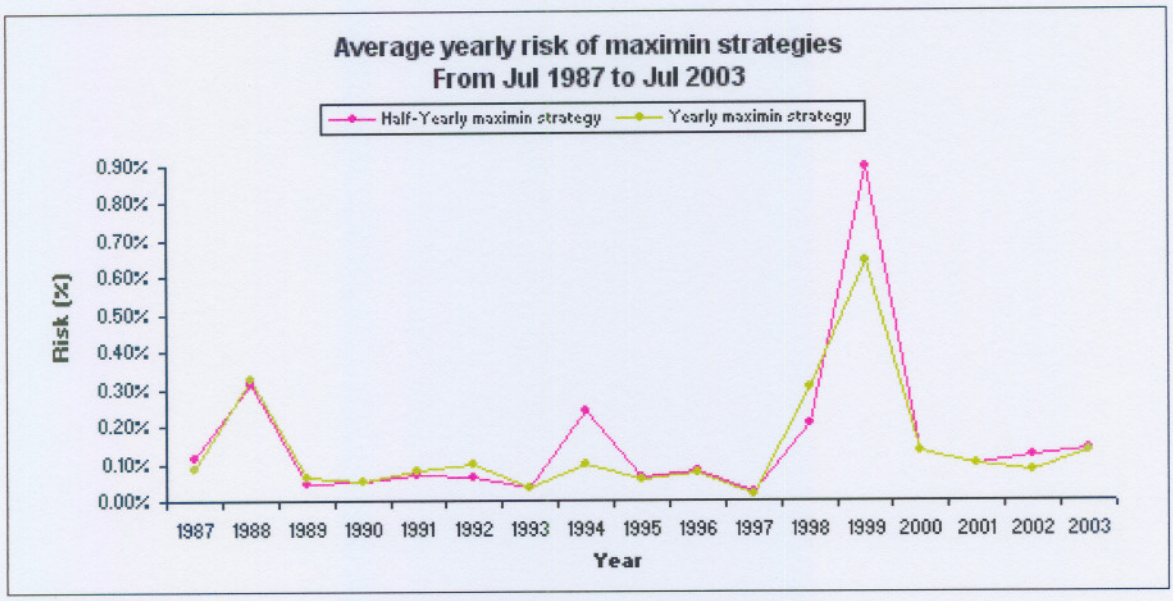


Graph 5.21: Expected shortfall for maximin strategies

- **Variance**

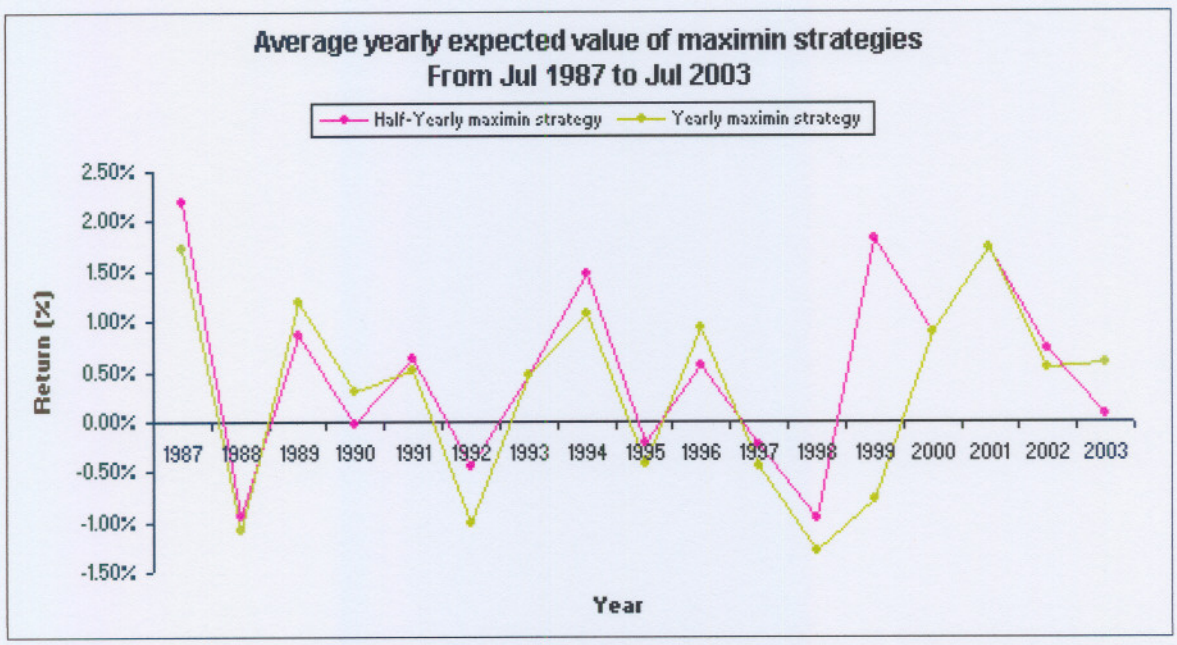
The following graph indicates the comparison between the variance risk measures for the different maximin strategies. The half yearly maximin strategy obtained the highest risk value of 0.89% in 1999 and both the strategies attained the lowest risk value of 0.02% in 1997. The

yearly maximin strategy attained the lowest yearly risk average of 0.1397%. The half yearly strategy's risk average is 0.1583%.



Graph 5.22: Variance comparison between maximin strategies

- **Average deviation**



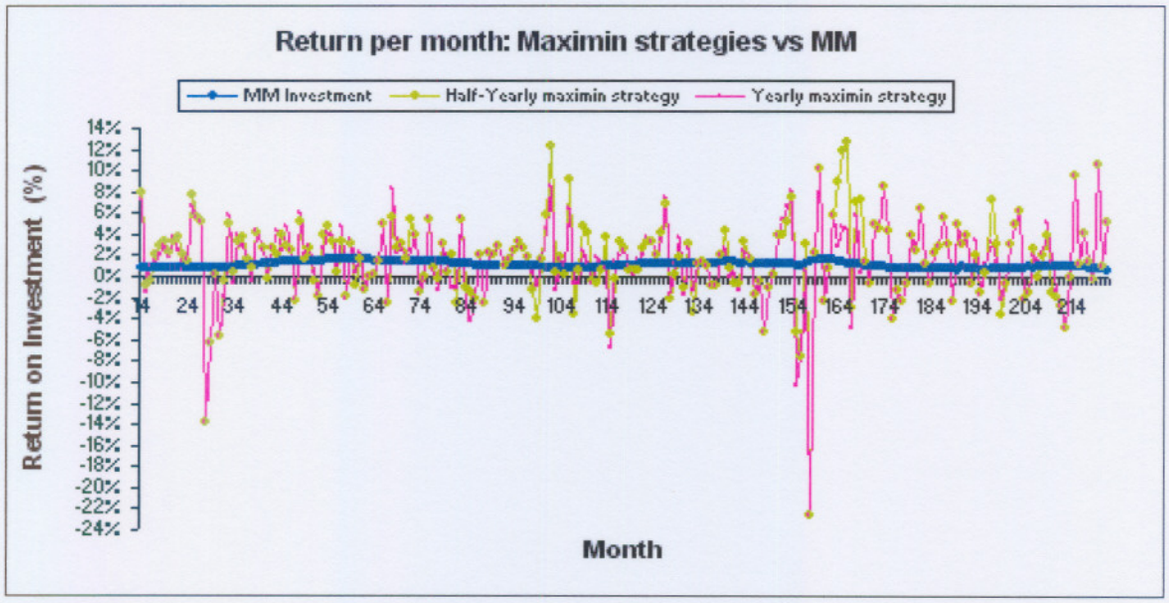
Graph 5.23: Expected value comparison between maximin strategies

Graph 5.23 shows a comparison between the average deviation return measures for the different maximin strategies. The half yearly maximin strategy obtained the maximum risk value of 2.19% in 1987. The yearly maximin strategy obtained the minimum value of -1.28% in 1998. The yearly averages for the half yearly and yearly maximin strategies are 0.5031% and 0.2885%, respectively.

- **Return per month**

Graph 5.24 displays the comparison between the obtained returns of the maximin strategies and the MM (per month). The returns for the maximin strategies fluctuate around the MM. P (X<MM) for the maximin strategies is as follows:

- Half yearly maximin strategy → 42.11%.
- Yearly maximin strategy → 39.71%.



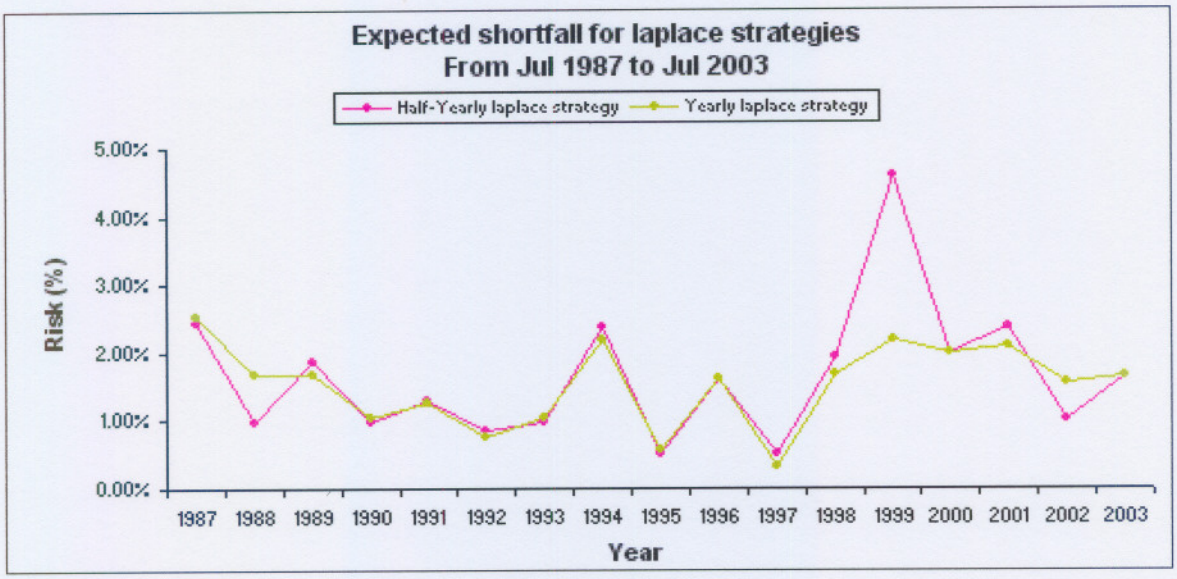
Graph 5.24: Return comparison between maximin strategies and MM

5.2.7 Laplace strategies

- **Expected shortfall**

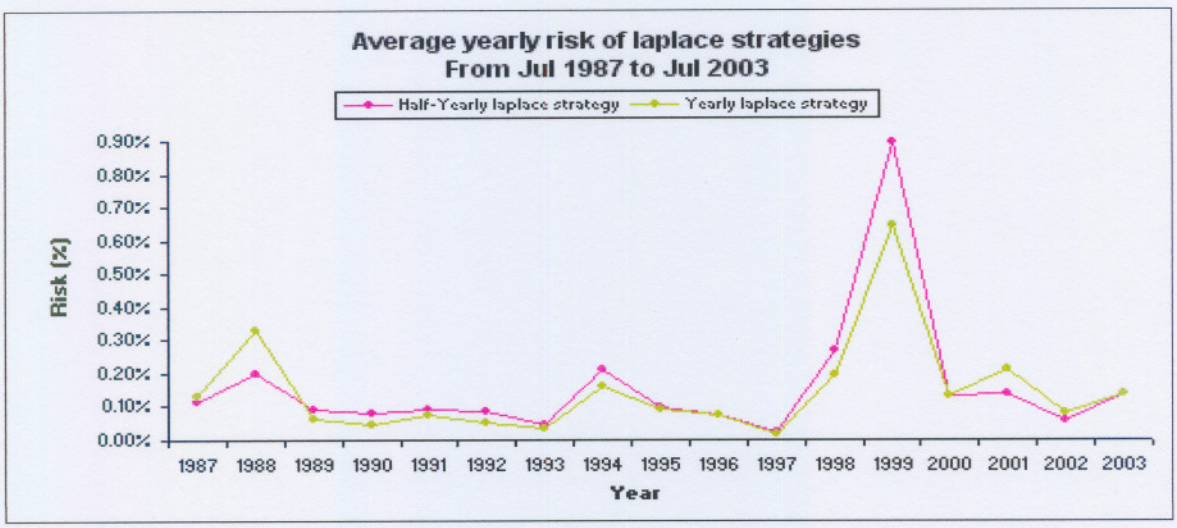
The following graph illustrates a comparison between the expected shortfall risk measures for the different laplace strategies. The half yearly laplace strategy obtained the maximum risk

value of 4.58% in 1999. The yearly laplace strategy obtained the minimum risk value of 0.33% in 1997. The yearly risk averages for the half yearly and yearly laplace strategies are 1.6455% and 1.5181%, respectively.



Graph 5.25: Expected shortfall for laplace strategies

- **Variance**



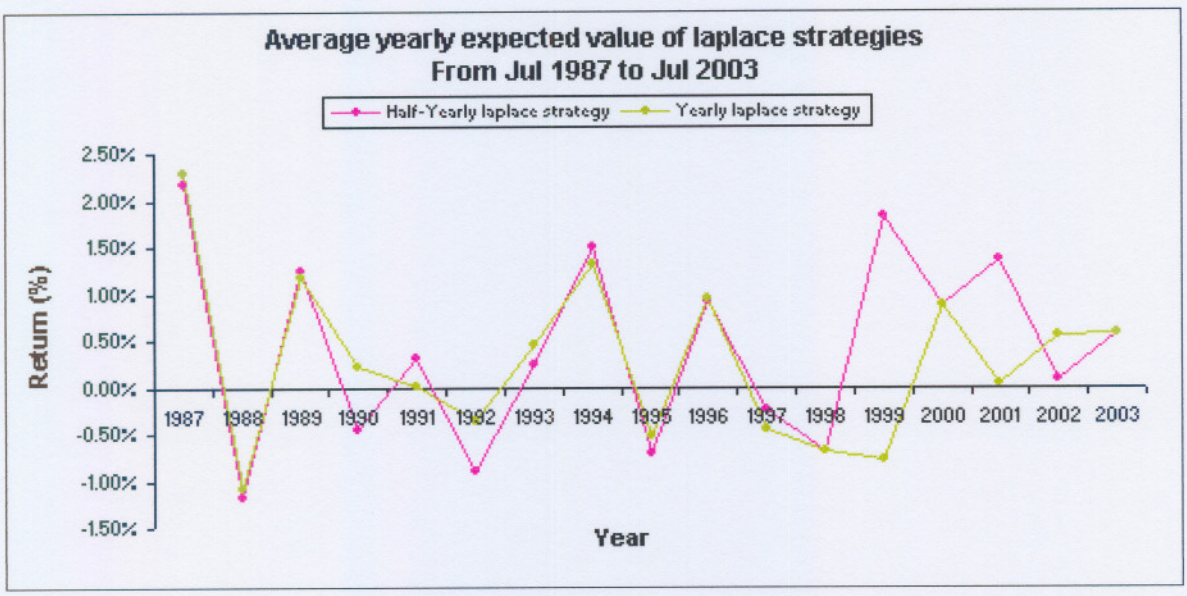
Graph 5.26: Variance comparison between laplace strategies

Graph 5.26 indicates the comparison between the variance risk measures for the different laplace strategies. The half yearly laplace strategy realized the maximum risk value of 0.89%

in 1999 and both the strategies obtained the minimum risk value of 0.02% in 1997. The yearly laplace strategy attained the lowest yearly risk average of 0.1454% and the half yearly strategy's risk average is 0.1604%.

- **Average deviation**

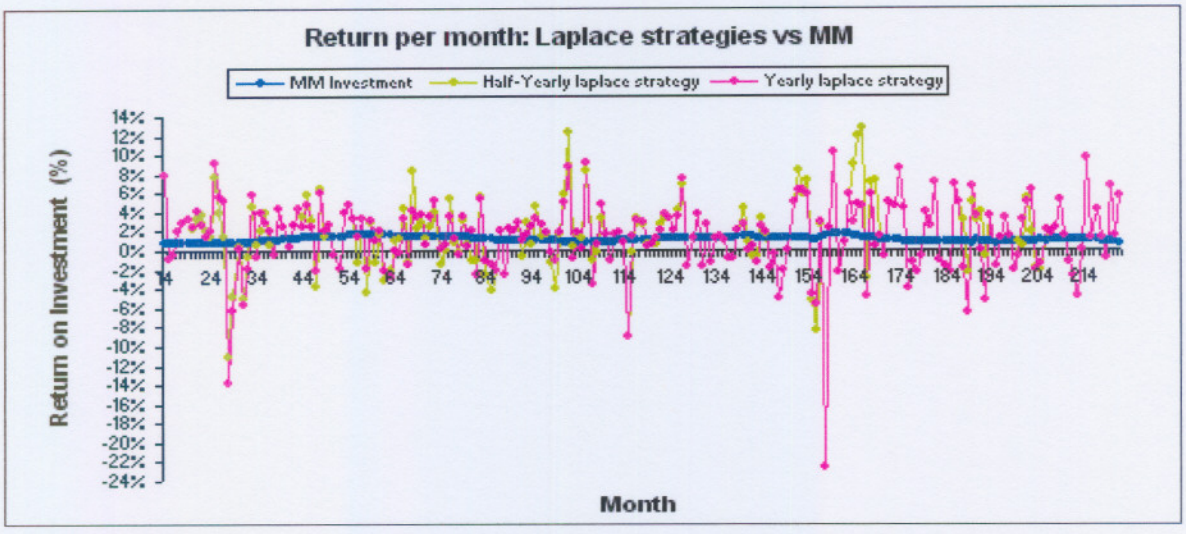
The following graph shows a comparison between the average deviation return measures for the different laplace strategies. The half yearly laplace strategy obtained the minimum risk value of –1.17% in 1988. The yearly laplace strategy obtained the maximum value of 2.29% in 1987. The yearly averages for the half yearly and yearly laplace strategies are 0.4112% and 0.2697%, correspondingly.



Graph 5.27: Expected value comparison between laplace strategies

- **Return per month**

The following graph portrays the comparison between the monthly returns of the laplace strategies and the MM. The returns for the laplace strategies fluctuate around the MM.



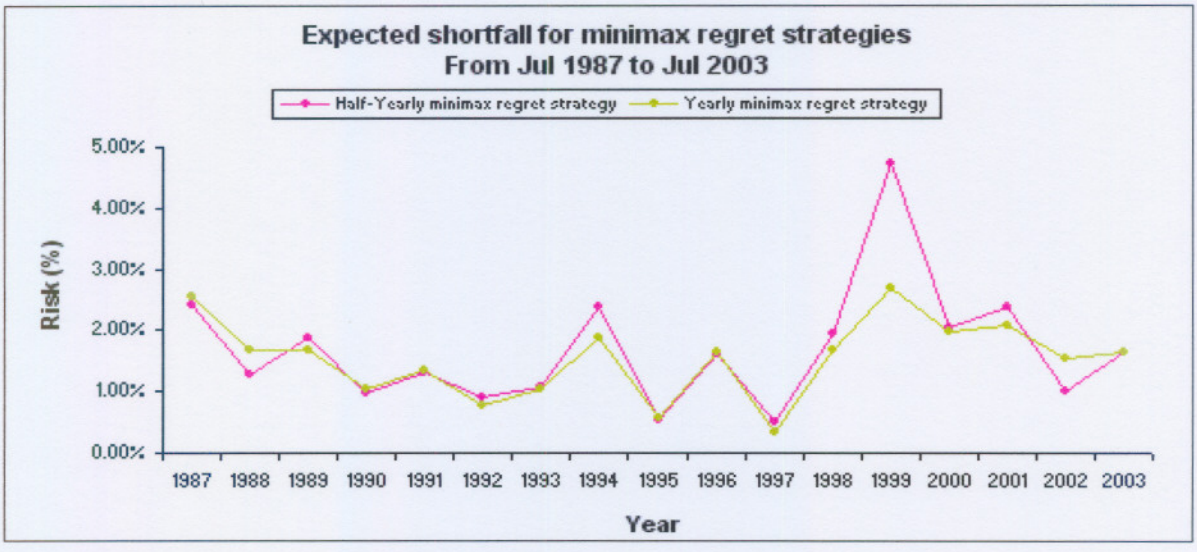
Graph 5.28: Return comparison between laplace strategies and MM

P (X<MM) for the laplace strategies is as follows:

- Half yearly laplace strategy → 42.58%.
- Yearly laplace strategy → 41.63%.

5.2.8 Minimax regret strategies

- **Expected shortfall**

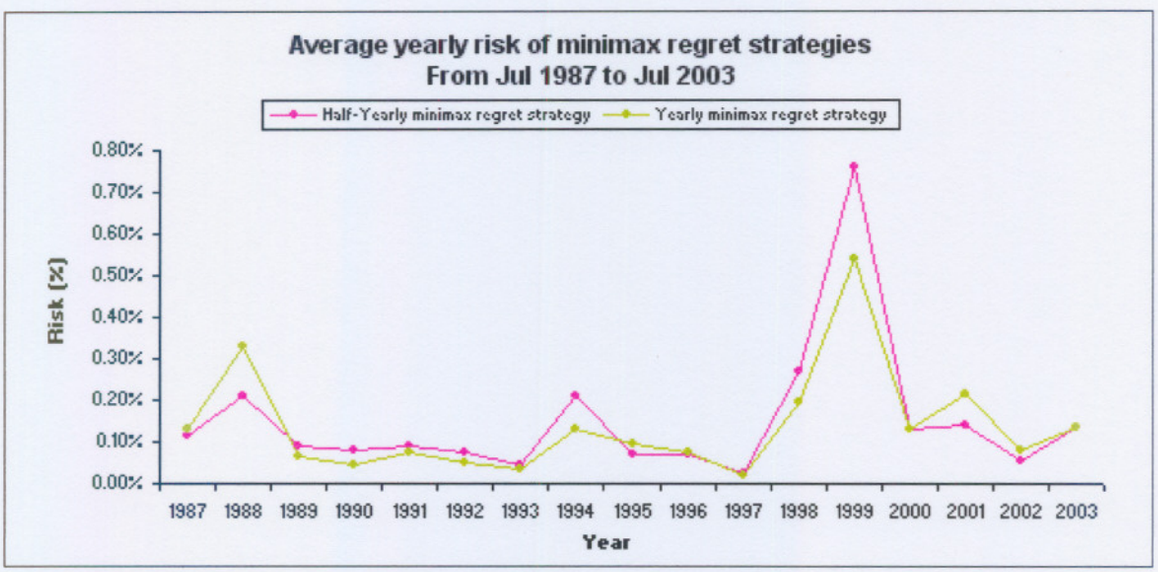


Graph 5.29: Expected shortfall for minimax regret strategies

Graph 5.29 illustrates a comparison between the expected shortfall risk measures for the different minimax regret strategies. The half yearly minimax regret strategy obtained the maximum risk value of 4.72% in 1999. The yearly minimax regret strategy obtained the minimum risk value of 0.33% in 1997. The yearly risk averages for the half yearly and yearly minimax regret strategies are 1.6823% and 1.5338%, respectively.

- **Variance**

The following graph shows the comparison between the variance risk measures for the different minimax regret strategies. The half yearly minimax regret strategy obtained the highest risk value of 0.76% in 1999 and both the strategies obtained the lowest risk value of 0.02% in 1997. The yearly minimax regret strategy attained the lowest yearly risk average of 0.1380%. The yearly risk average for the half yearly strategy is 0.1507%.

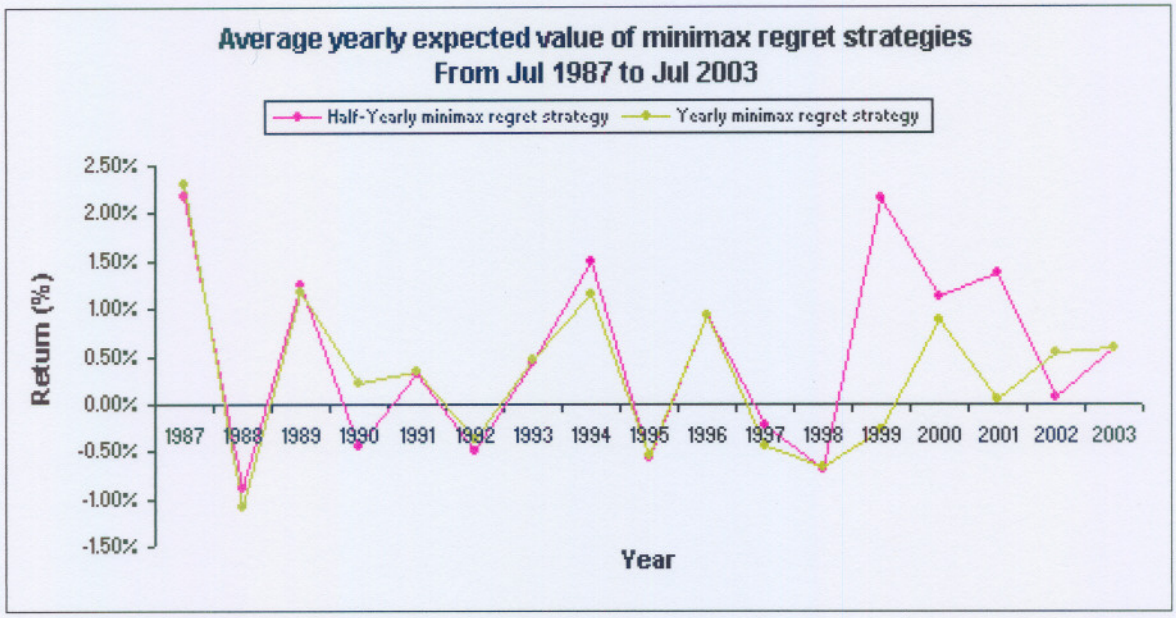


Graph 5.30: Variance comparison between minimax regret strategies

- **Average deviation**

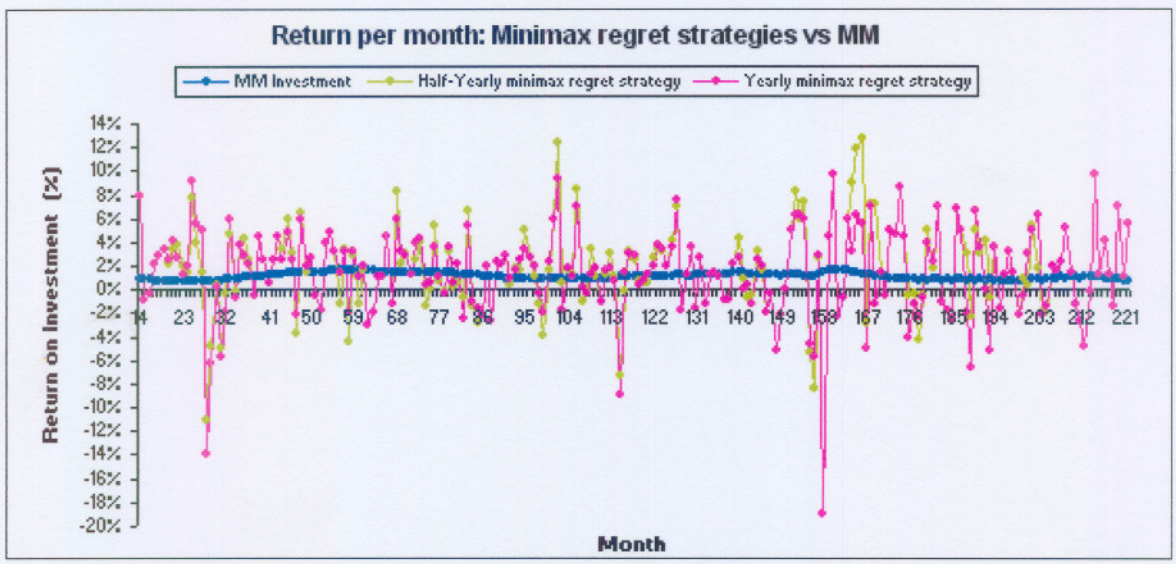
Graph 5.31 shows a comparison between the average deviation return measures for the different minimax regret strategies. The yearly minimax regret strategy obtained the minimum risk value of –1.08% in 1988 and the maximum value of 2.29% in 1987. The yearly

averages for the half yearly and yearly minimax regret strategies are 0.5062% and 0.3090%, correspondingly.



Graph 5.31: Expected value comparison between minimax regret strategies

- **Return per month**



Graph 5.32: Return comparison between minimax regret strategies and MM

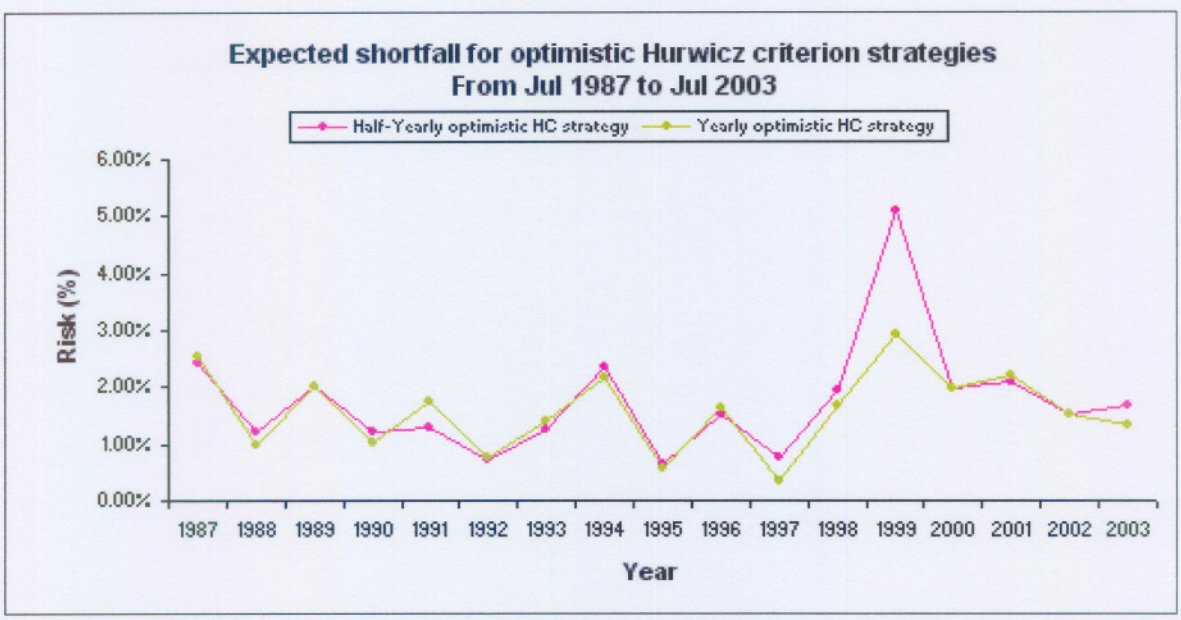
Graph 5.32 shows the comparison between the monthly returns of the minimax regret strategies and the MM. The returns for the minimax regret strategies vary around the MM. P (X<MM) for the minimax regret strategies is as follows:

- Half yearly minimax regret strategy → 41.15%.
- Yearly minimax regret strategy → 41.15%.

5.2.9 Optimistic Hurwicz criterion strategies

- **Expected shortfall**

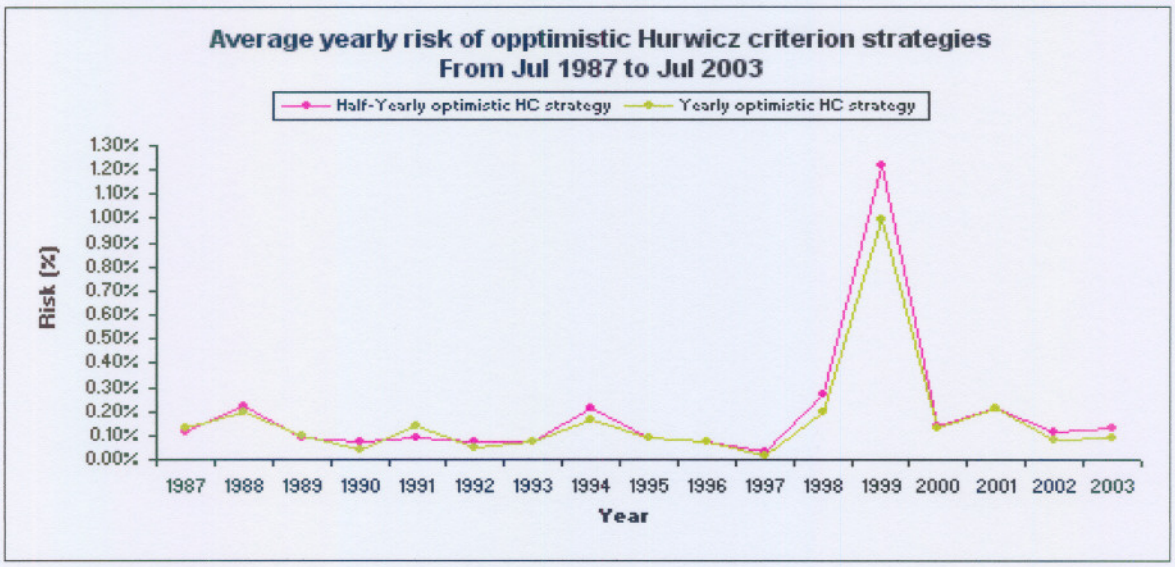
Graph 5.33 illustrates a comparison between the expected shortfall risk measures for the different optimistic Hurwicz criterion strategies. The half yearly optimistic Hurwicz criterion strategy obtained the maximum risk value of 5.09% in 1999. The yearly optimistic Hurwicz criterion strategy obtained the minimum risk value of 0.33% in 1997. The yearly risk averages for the half yearly and yearly optimistic Hurwicz criterion strategies are 1.7480% and 1.5815%, respectively.



Graph 5.33: Expected shortfall for optimistic Hurwicz criterion strategies

- **Variance**

The following graph shows the comparison between the variance risk measures for the different optimistic Hurwicz criterion strategies. The half yearly optimistic Hurwicz criterion strategy obtained the highest risk value of 1.21% in 1999 and yearly optimistic Hurwicz criterion strategy obtained the lowest risk value of 0.02% in 1997. The yearly optimistic Hurwicz criterion strategy attained the lowest yearly risk average of 0.1647%. The yearly risk average for the half yearly strategy is 0.1901%.

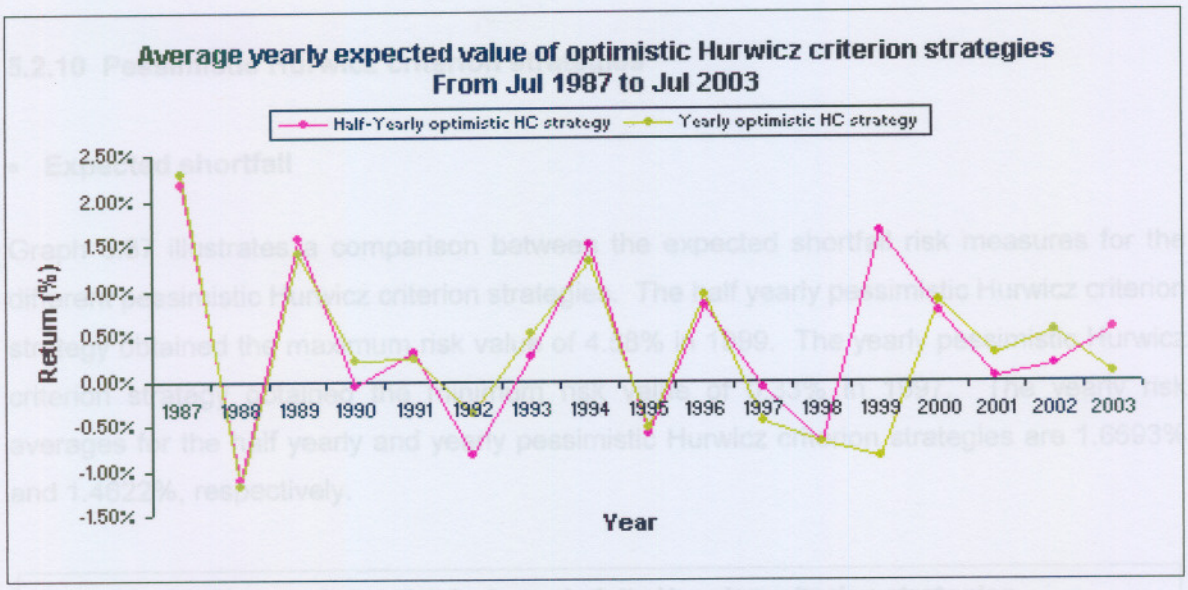


Graph 5.34: Variance comparison between optimistic Hurwicz criterion strategies

- **Average deviation**

Graph 5.35 shows a comparison between the average deviation return measures for the different optimistic Hurwicz criterion strategies. The yearly optimistic Hurwicz criterion strategy obtained both the minimum value (–1.17% in 1988) and the maximum value (2.29% in 1987). The yearly averages for the half yearly and yearly optimistic Hurwicz criterion strategies are 0.3819% and 0.2774%, respectively. P (X<MM) for the optimistic Hurwicz criterion strategies is as follows:

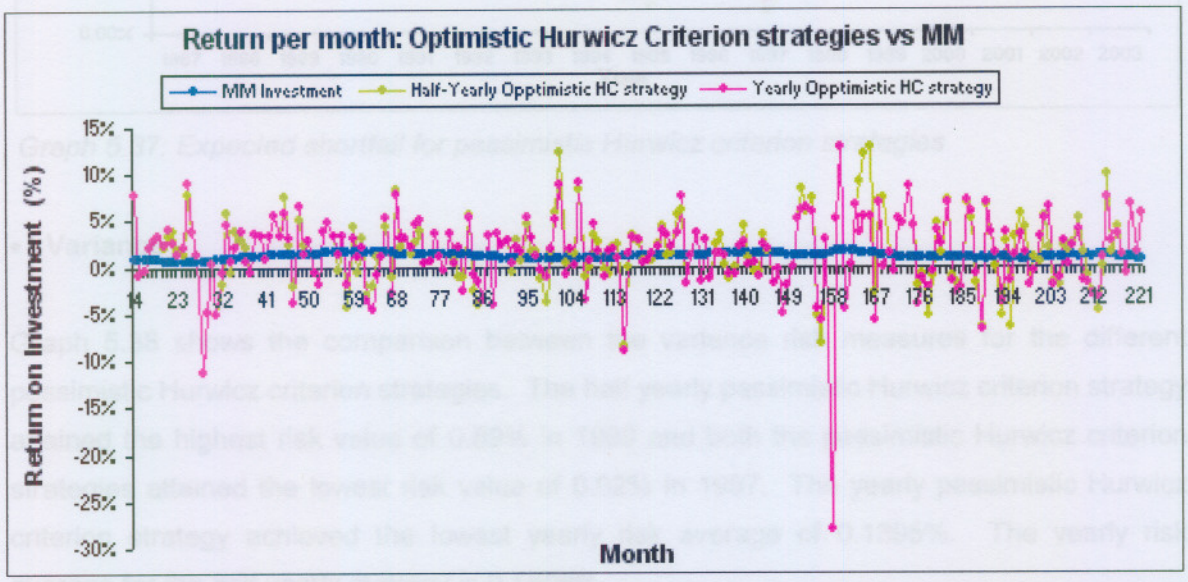
- Half yearly optimistic Hurwicz criterion strategy → 42.58%.
- Yearly optimistic Hurwicz criterion strategy → 42.58%.



Graph 5.35: Expected value comparison of optimistic Hurwicz criterion strategies

- **Return per month**

Graph 5.36 illustrates the comparison between the monthly returns of the optimistic Hurwicz criterion strategies and the MM. The returns for the optimistic Hurwicz criterion strategies fluctuate around the MM.

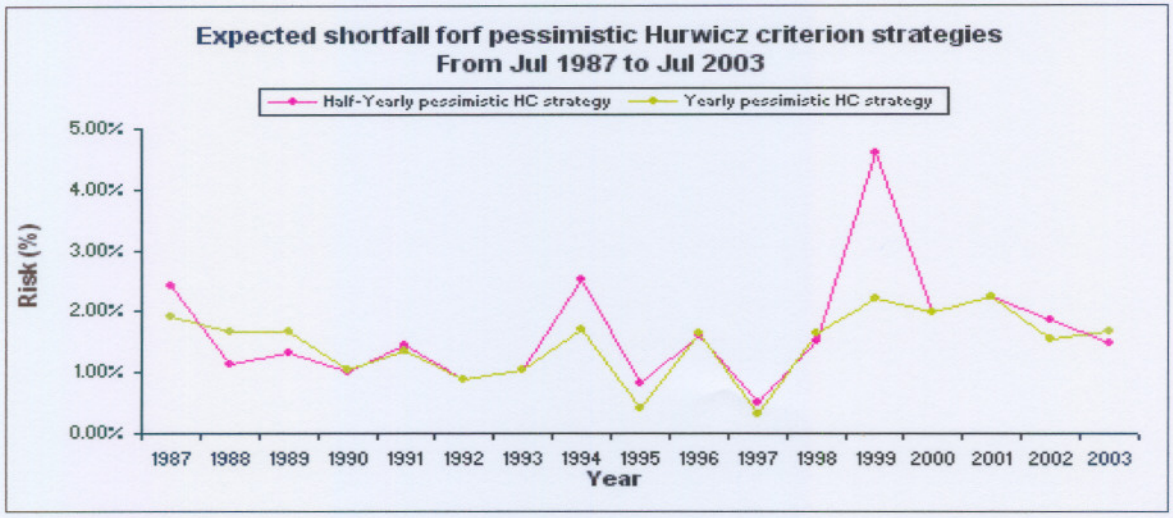


Graph 5.36: Return comparison between optimistic Hurwicz criterion strategies and MM

5.2.10 Pessimistic Hurwicz criterion strategies

- **Expected shortfall**

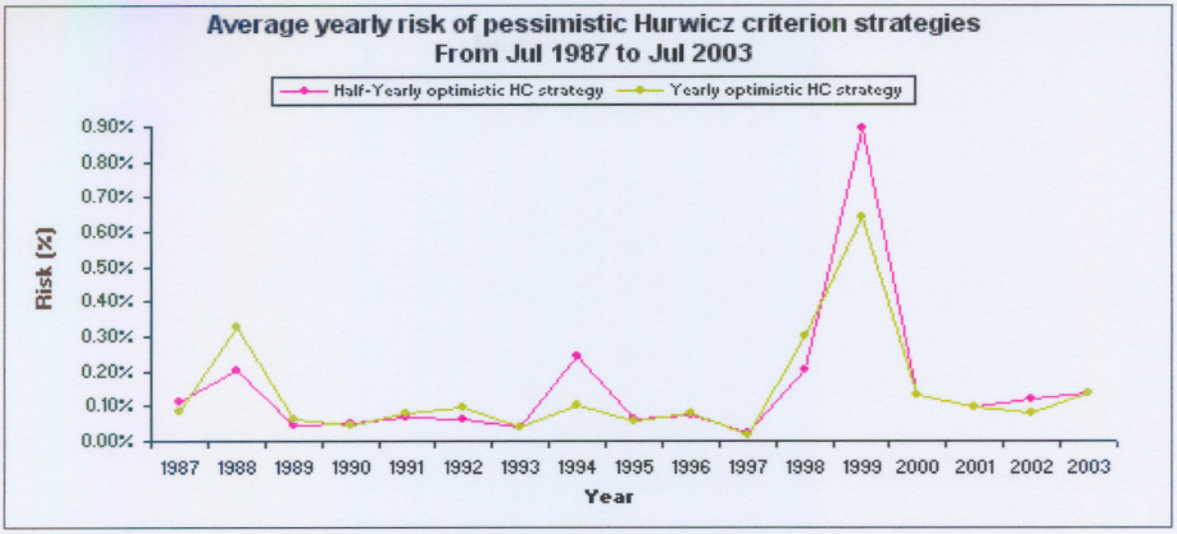
Graph 5.37 illustrates a comparison between the expected shortfall risk measures for the different pessimistic Hurwicz criterion strategies. The half yearly pessimistic Hurwicz criterion strategy obtained the maximum risk value of 4.58% in 1999. The yearly pessimistic Hurwicz criterion strategy obtained the minimum risk value of 0.33% in 1997. The yearly risk averages for the half yearly and yearly pessimistic Hurwicz criterion strategies are 1.6693% and 1.4622%, respectively.



Graph 5.37: Expected shortfall for pessimistic Hurwicz criterion strategies

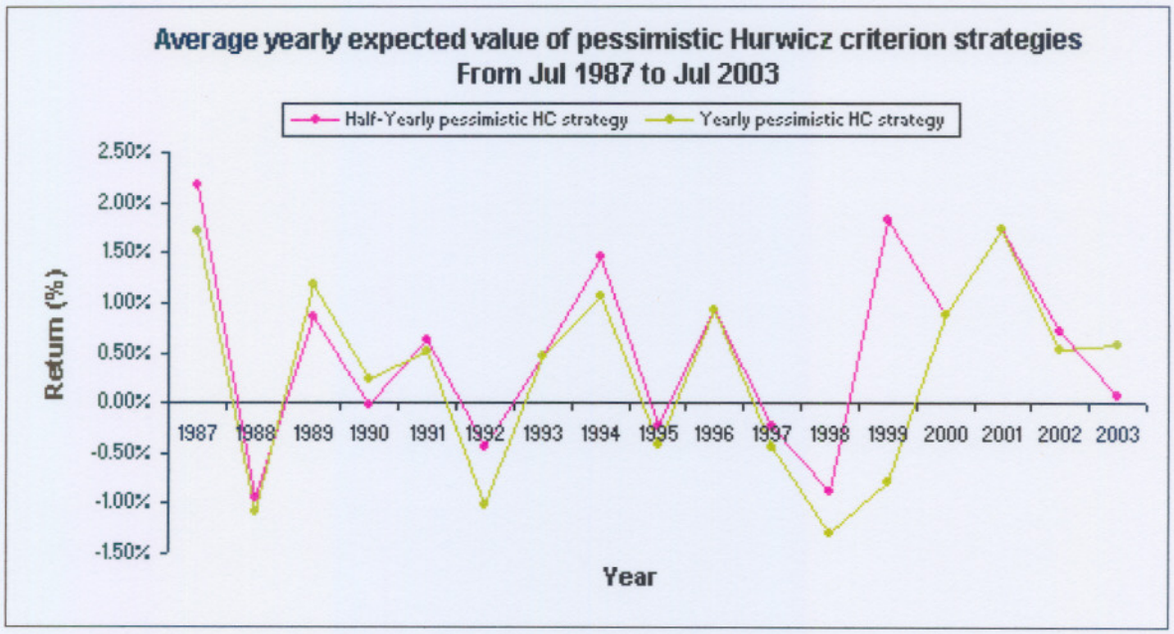
- **Variance**

Graph 5.38 shows the comparison between the variance risk measures for the different pessimistic Hurwicz criterion strategies. The half yearly pessimistic Hurwicz criterion strategy attained the highest risk value of 0.89% in 1999 and both the pessimistic Hurwicz criterion strategies attained the lowest risk value of 0.02% in 1997. The yearly pessimistic Hurwicz criterion strategy achieved the lowest yearly risk average of 0.1395%. The yearly risk average for the half yearly strategy is 0.1508%.



Graph 5.38: Variance comparison between pessimistic Hurwicz criterion strategies

- **Average deviation**



Graph 5.39: Expected value comparison of pessimistic Hurwicz criterion strategies

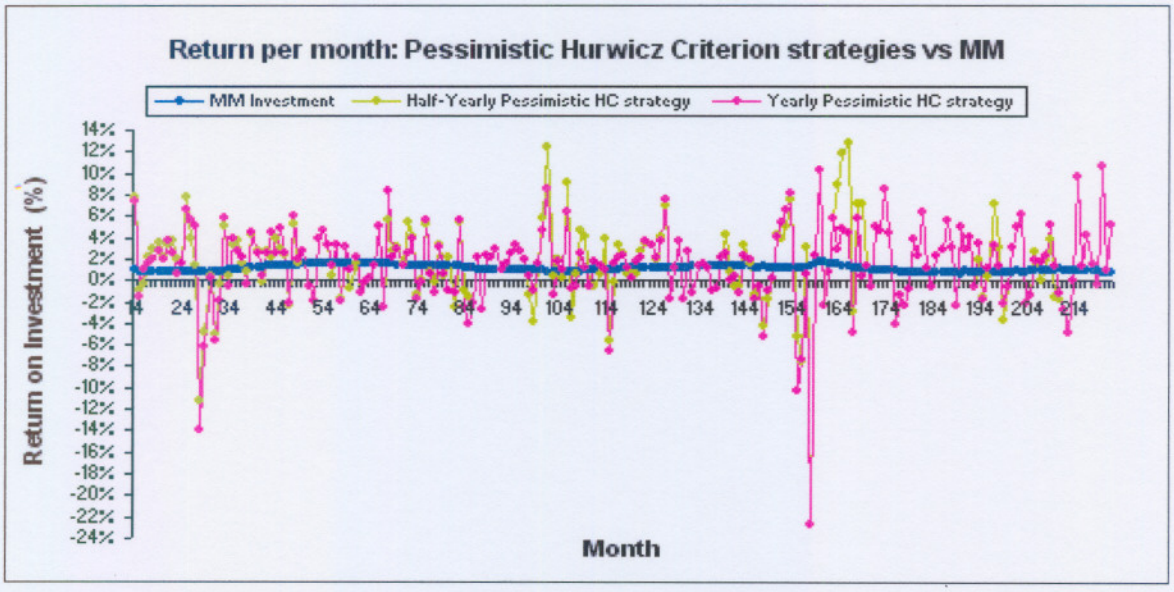
Graph 5.39 shows a comparison between the average deviation return measures for the different pessimistic Hurwicz criterion strategies. The yearly pessimistic Hurwicz criterion strategy obtained the minimum value (–1.28% in 1998) and the half yearly pessimistic Hurwicz criterion strategy obtained the maximum value (2.19% in 1987). The yearly

averages for the half yearly and yearly pessimistic Hurwicz criterion strategies are 0.5287% and 0.2842%, respectively.

- **Return per month**

Graph 5.40 illustrates the comparison between the monthly returns of the pessimistic Hurwicz criterion strategies and the MM. The returns for the pessimistic Hurwicz criterion strategies fluctuate around the MM. P (X<MM) for the pessimistic Hurwicz criterion strategies is as follows:

- Half yearly pessimistic Hurwicz criterion strategy → 41.63%.
- Yearly pessimistic Hurwicz criterion strategy → 39.71%.



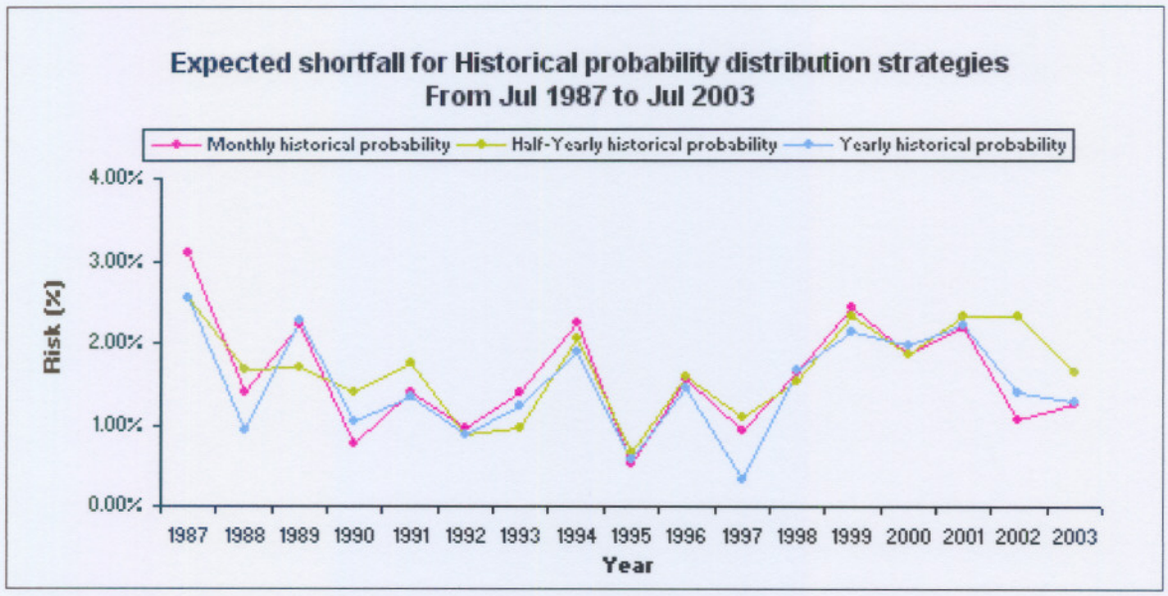
Graph 5.40: Return comparison between pessimistic Hurwicz criterion strategies and MM

5.3 Decision making under risk

5.3.1 Historical probability distribution strategies

- **Expected shortfall**

The following graph illustrates a comparison between the expected shortfall risk measures for the different historical probability distribution strategies. The monthly historical probability distribution strategy obtained the maximum risk value of 3.10% in 1987. The yearly historical probability distribution strategy obtained the minimum risk value of 0.33% in 1987. The yearly risk averages for the monthly, half yearly and yearly historical probability distribution strategies are 1.5861% and 1.6662% and 1.4799% respectively.

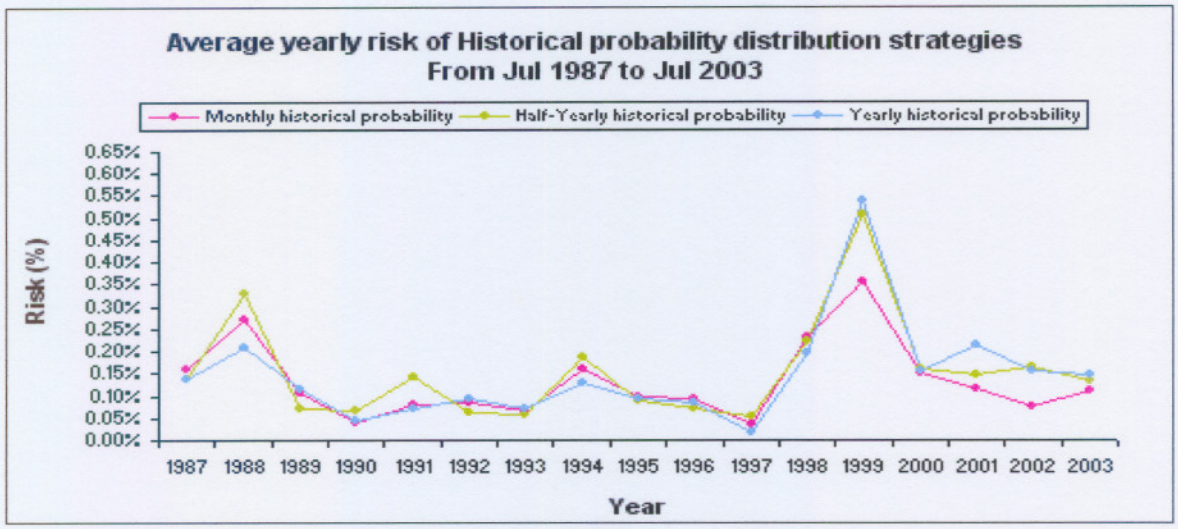


Graph 5.41: Expected shortfall for historical probability distribution strategies

- **Variance**

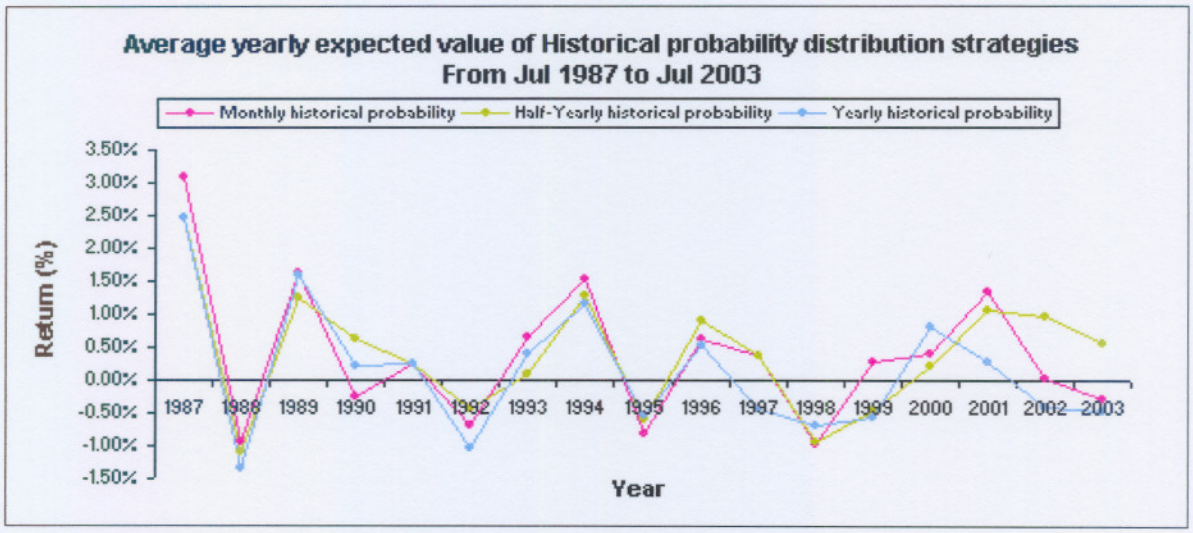
The following graph shows the comparison between the variance risk measures for the different historical probability distribution strategies. The historical probability distribution strategy attained the highest risk value of 0.54% in 1999 and the lowest risk value of 0.02% in 1997. The monthly historical probability distribution strategy achieved the lowest yearly risk

average of 0.1322%. The yearly risk averages for the half yearly and yearly strategies are 0.1535% and 0.1458%, respectively.



Graph 5.42: Variance comparison between historical probability distribution strategies

- **Average deviation**

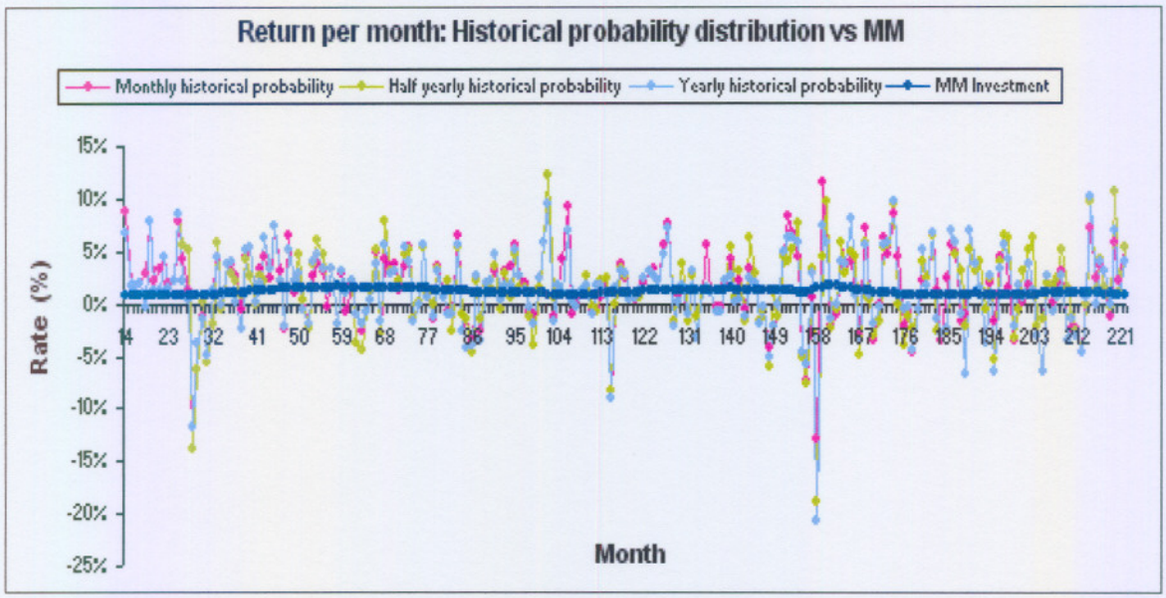


Graph 5.43: Expected value comparison of historical probability distribution strategies

Graph 5.43 shows a comparison between the average deviation return measures for the different historical probability distribution strategies. The yearly historical probability distribution strategy obtained the minimum value (–1.36% in 1988) and the monthly historical

probability distribution strategy obtained the maximum value (3.10% in 1987). The yearly averages for the monthly, half yearly and yearly historical probability distribution strategies are 0.3694%, 0.3839% and 0.1335%, respectively.

- **Return per month**



Graph 5.44: Return comparison between historical probability distribution strategies and MM

Graph 5.44 illustrates the comparison between the monthly returns of the historical probability distribution strategies and the MM. The returns for the historical probability distribution strategies fluctuate around the MM. P (X<MM) for the historical probability distribution strategies is as follows:

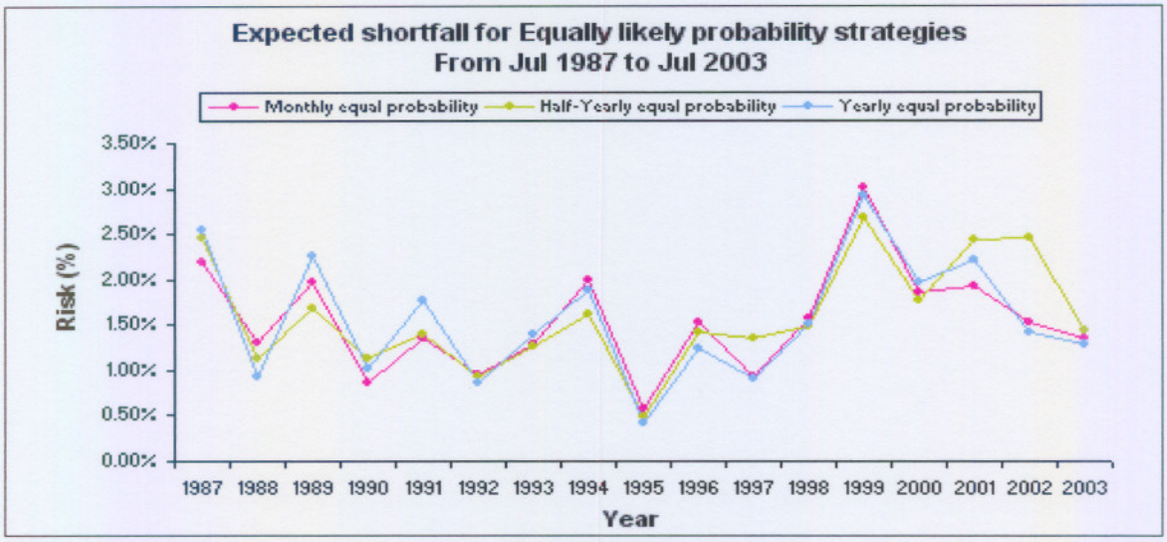
- Monthly historical probability distribution strategy → 42.58%.
- Half yearly historical probability distribution strategy → 41.15%.
- Yearly historical probability distribution strategy → 44.02%.

5.3.2 Even probability strategies

- **Expected shortfall**

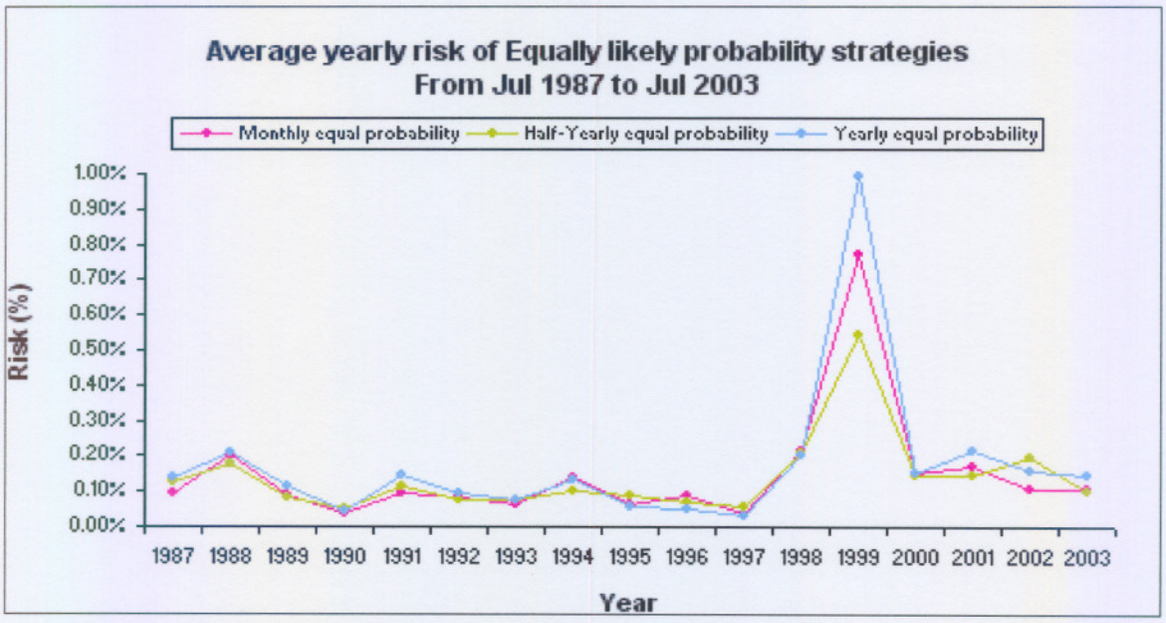
The following graph illustrates a comparison between the expected shortfall risk measures for the different even probability strategies. The monthly even probability strategy obtained the

highest risk value of 3.01% in 1999. The yearly even probability strategy obtained the lowest risk value of 0.42% in 1995. The yearly risk averages for the monthly, half yearly and yearly even probability strategies are 1.5426%, 1.5930%,and 1.5633% correspondingly.



Graph 5.45: Expected shortfall for even probability strategies

- **Variance**

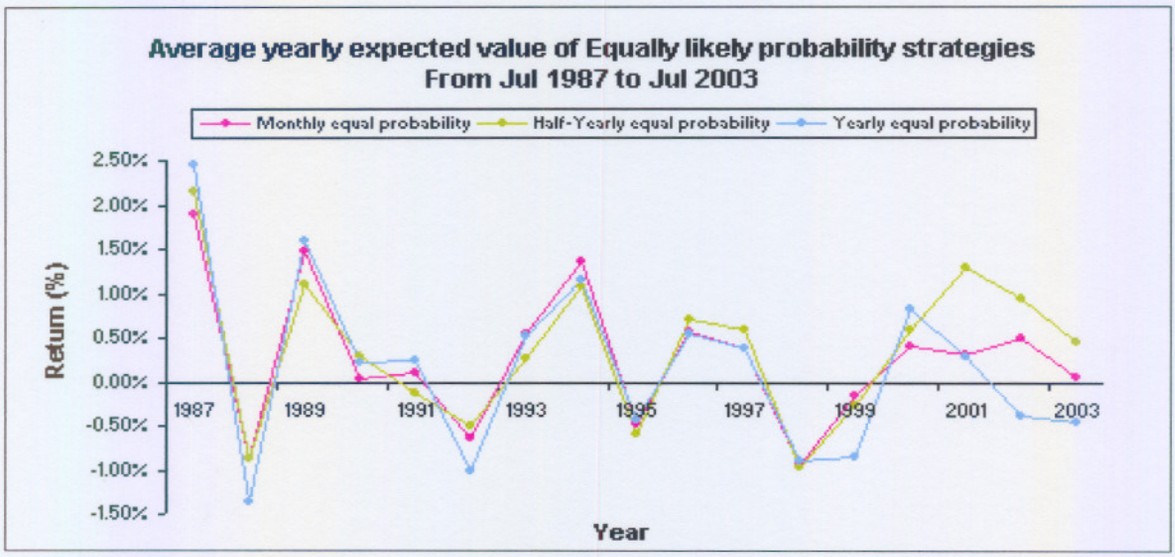


Graph 5.46: Variance comparison between even probability strategies

Graph 4.45 shows the comparison between the variance risk measures for the different even probability strategies. The yearly even probability strategy attained the highest risk value of 0.99% in 1999 and lowest risk value of 0.03% in 1997. The half yearly even probability strategy achieved the lowest yearly risk average of 0.1386%. The yearly risk averages for the monthly and yearly strategies are 0.1483% and 0.1744%, correspondingly.

- **Average deviation**

Graph 5.46, below shows a comparison between the average deviation return measures for the different even probability strategies. The yearly even probability strategy obtained the minimum risk value of –1.36% in 1988 and the maximum value of 2.46% in 1987. The yearly even probability strategy achieved the lowest yearly average of 0.1680%. The yearly averages for the monthly and half yearly strategies are 0.2687% and 0.3622%, correspondingly.



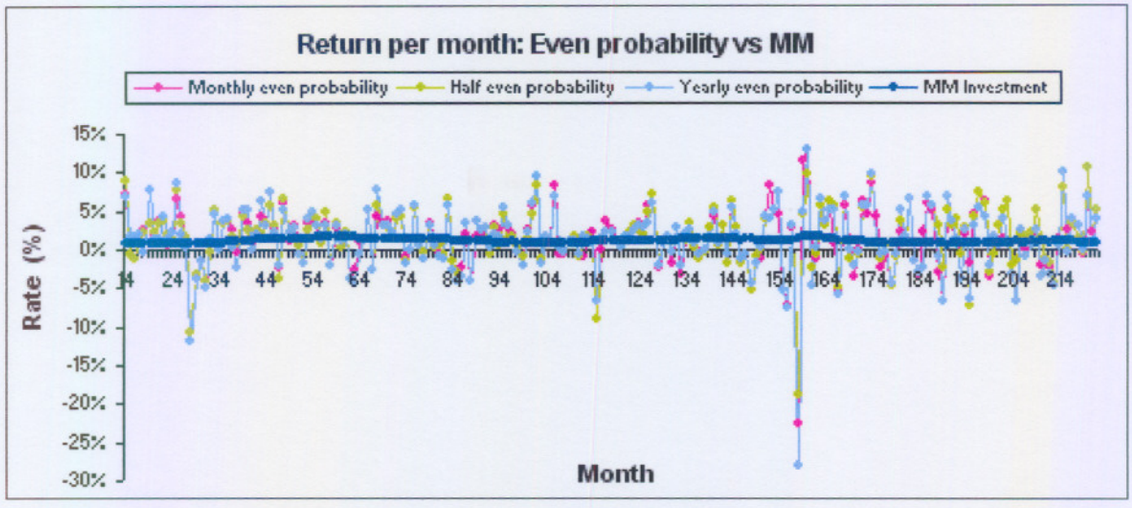
Graph 5.47: Expected value comparison of even probability strategies

- **Return per month**

Graph 5.47 illustrates the comparison between the monthly returns of the even probability strategies and the MM. The returns for the even probability strategies fluctuate around the MM. P (X<MM) for the even probability strategies is as follows:

➢ Monthly even probability strategy → 43.06%.

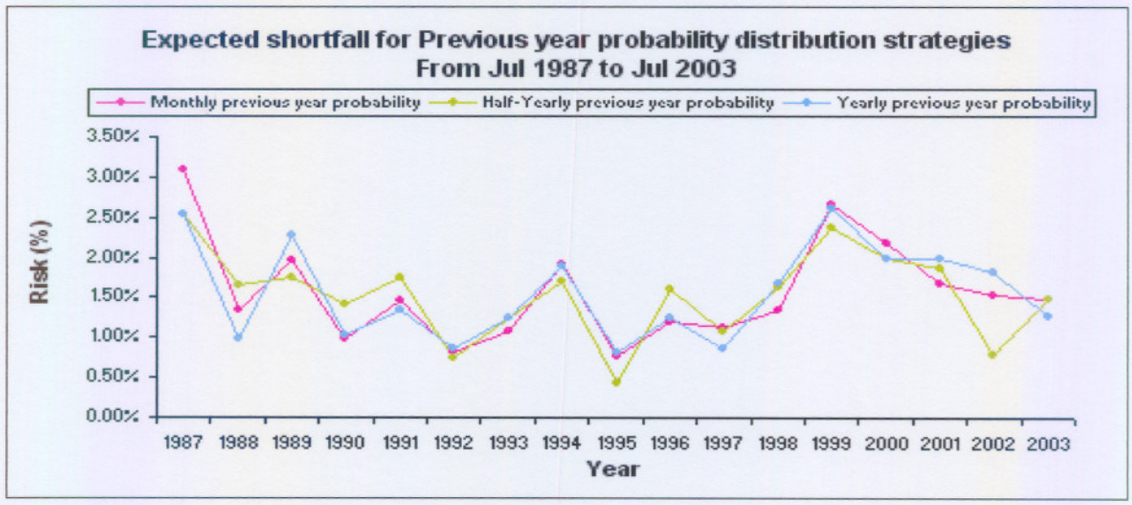
- Half yearly even probability strategy → 42.11%.
- Yearly even probability strategy → 42.58%.



Graph 5.48: Return comparison between even probability strategies and MM

5.3.3 Previous year probability distribution strategies

- **Expected shortfall**



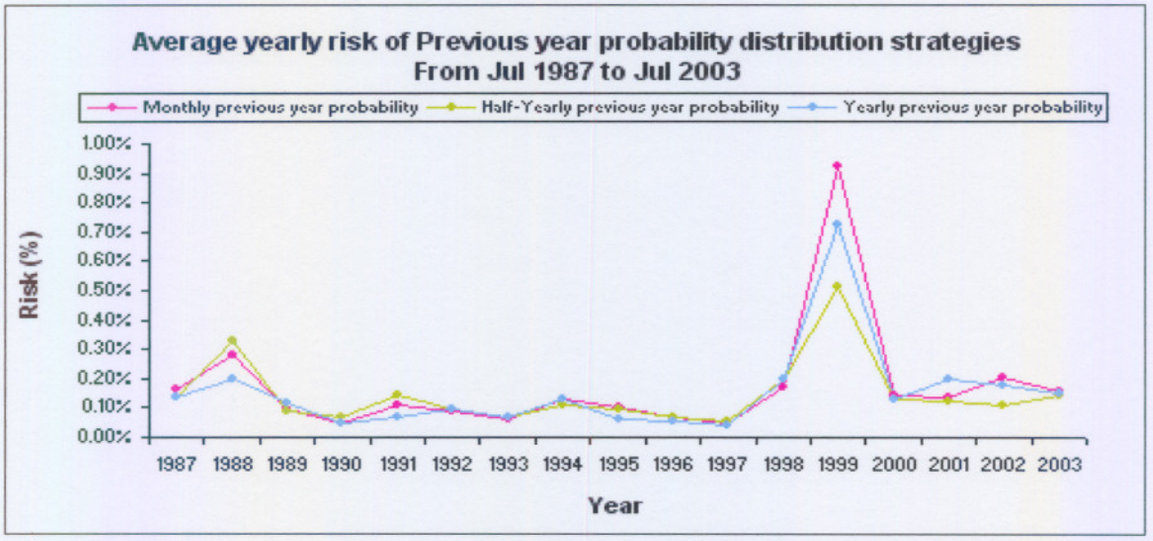
Graph 5.49: Expected shortfall for previous year probability distribution strategies

Graph 5.49 illustrates a comparison between the expected shortfall risk measures for the different previous year probability distribution strategies. The monthly previous year

probability distribution strategy obtained the maximum risk value of 3.10% in 1987. The half yearly previous year probability distribution strategy obtained the minimum risk value of 0.44% in 1995. The half yearly previous year probability distribution obtained the minimum yearly risk average for the previous year probability distribution strategies (1.5360%). The yearly risk average for the monthly and yearly previous year probability distribution strategies are 0.1.56661% and 0.1.5543%, correspondingly.

- **Variance**

Graph 5.50 illustrates the comparison between the variance risk measures for the different previous year probability distribution strategies. The monthly previous year probability distribution strategy attained the highest risk value of 0.93% in 1999 and the yearly, previous year probability distribution strategy attained the lowest risk value of 0.04% in 1997. The half yearly previous year probability distribution strategy achieved the lowest yearly risk average of 0.1453%. The yearly risk average for the monthly strategy is 0.1740% and the yearly risk average for the yearly strategy is 0.1531%.

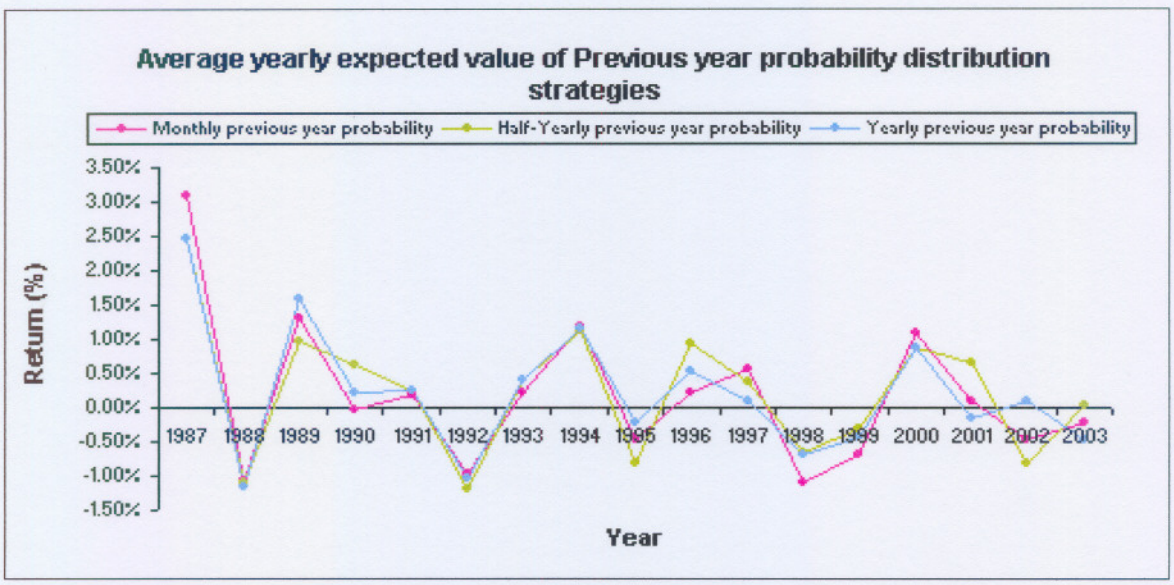


Graph 5.50: Variance comparison of previous year probability distribution strategies

- **Average deviation**

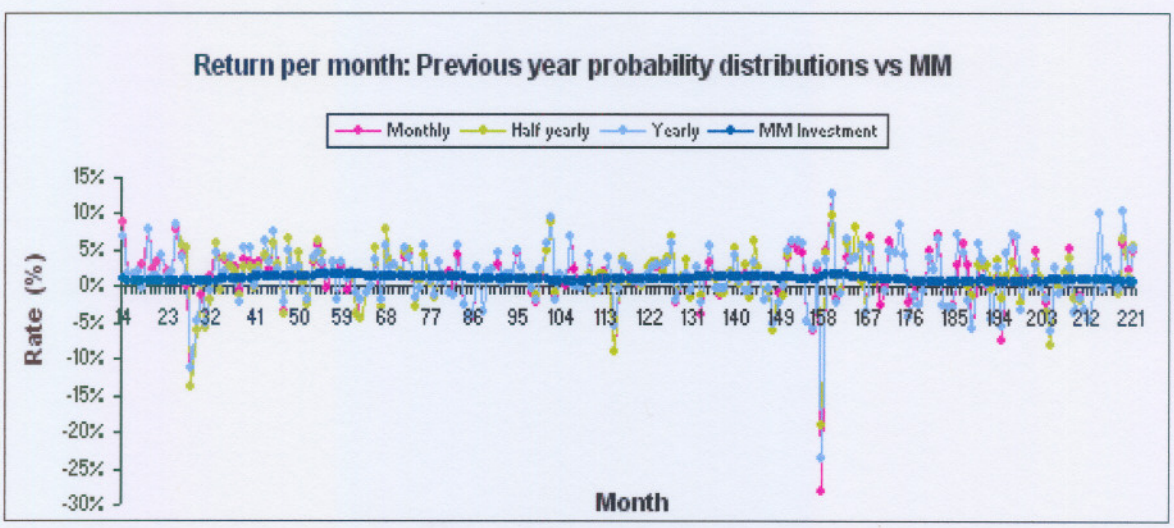
Graph 5.51 shows a comparison between the average deviation return measures for the different previous year probability distribution strategies. The half yearly previous year

probability distribution strategy obtained the minimum value (–1.18% in 1992) and the monthly previous year probability distribution strategy obtained the maximum risk value (3.10% in 1987). The yearly averages for the monthly, half yearly and yearly previous year probability distribution strategies are 0.1744%, 0.2282%, and 0.2085% respectively. Thus, the monthly previous year probability distribution strategy obtained the lowest yearly risk average.



Graph 5.51: Expected value comparison of previous year probability distribution strategies

- **Return per month**



Graph 5.52: Return comparison of previous year probability distribution strategies and MM

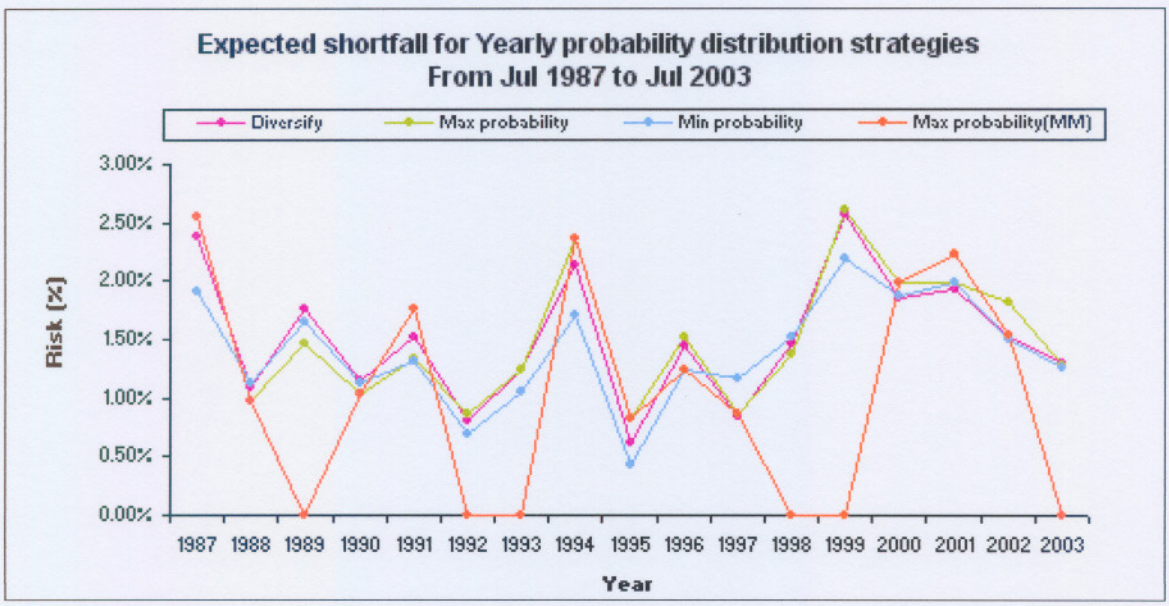
Graph 5.52 illustrates the comparison between the monthly returns of the previous year probability distribution strategies and the MM. The returns for the previous year probability distribution strategies fluctuate around the MM. P (X<MM) for the previous year probability distribution strategies is as follows:

- Monthly previous year probability distribution strategy → 42.58%.
- Half yearly previous year probability distribution strategy → 43.54%.
- Yearly previous year probability distribution strategy → 44.98%.

5.3.4 Probability distribution strategies

- **Expected shortfall**

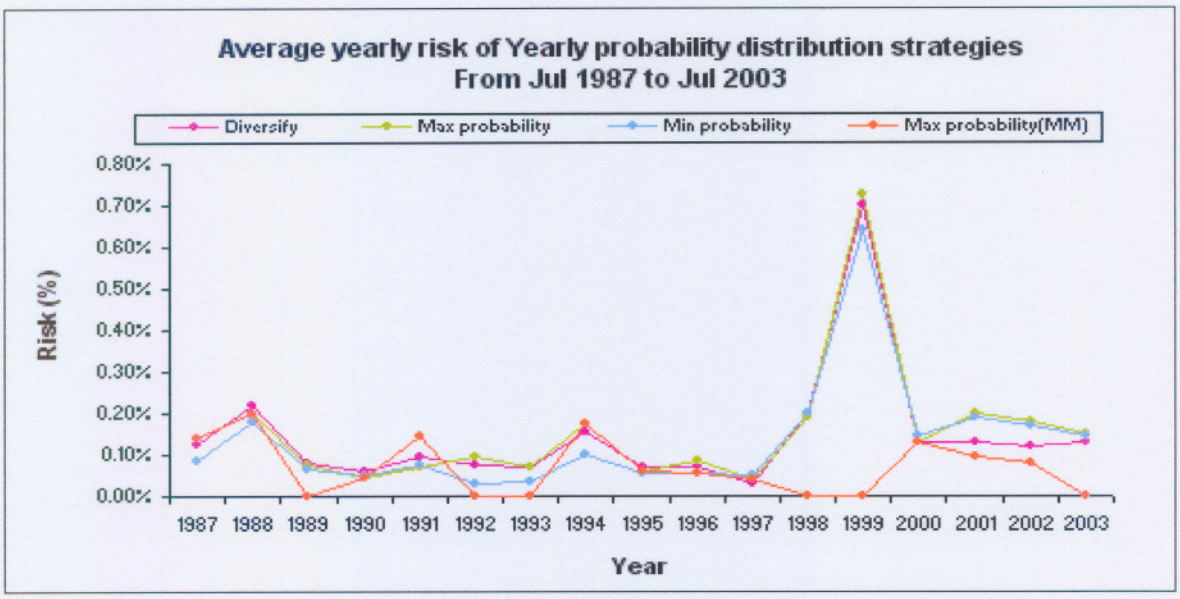
The following graph shows a comparison between the expected shortfall risk measures for the different probability distribution strategies. The max probability distribution strategy obtained the maximum risk value of 2.60% in 1999. The min probability distribution strategy obtained the minimum risk value of 0.42% in 1995. The yearly risk averages for the diversify, max probability, max probability (with MM) and min probability strategies are 1.5054%, 1.5340%, 1.0208% and 1.3942%, correspondingly.



Graph 5.53: Expected shortfall for probability distribution strategies

- **Variance**

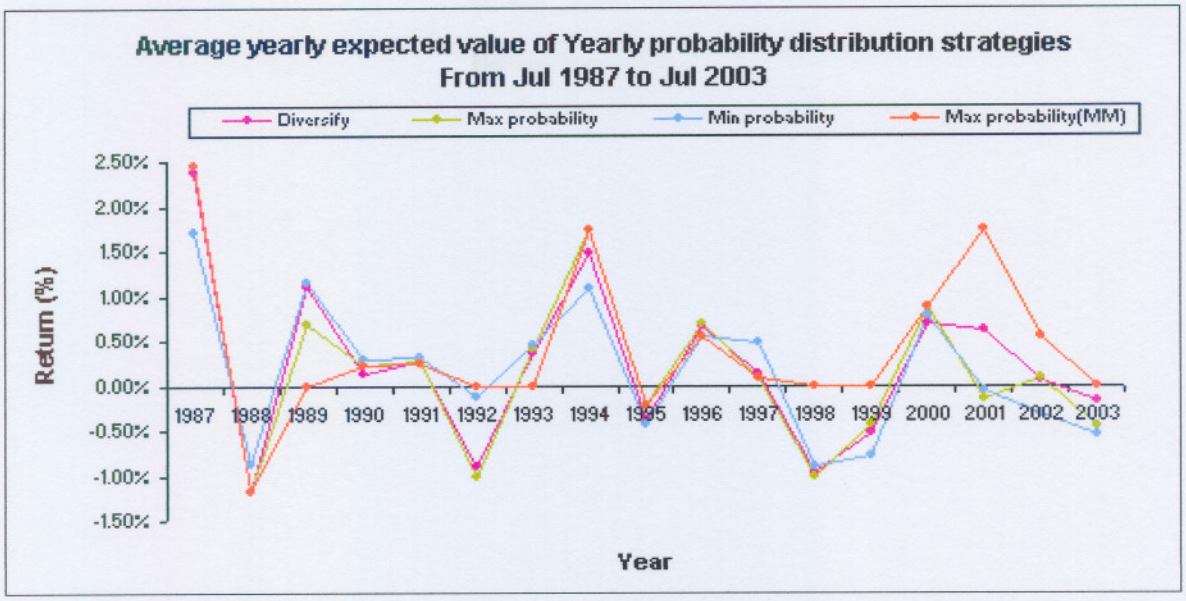
The following graph shows the comparison between the variance risk measures for the different probability distribution strategies. The max probability strategy attained the highest risk value of 0.72% in 1999 and max probability (with MM) strategy attained the lowest risk value of 0.0% (this occurred several times). The max probability (with MM) strategy achieved the lowest yearly risk average of 0.0687%. The yearly risk averages for the other strategies is: diversify (0.1453%), max probability (0.1548%) and the min probability (0.1335%).



Graph 5.54: Variance comparison between probability distribution strategies

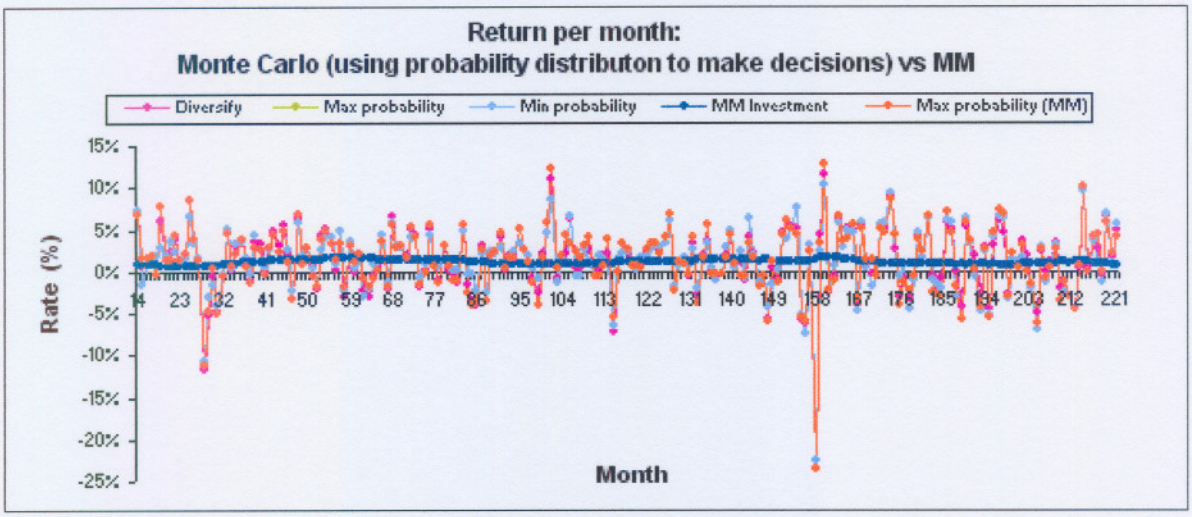
- **Average deviation**

Graph 5.55 shows a comparison between the average deviation return measures for the different probability distribution strategies. The diversify strategy obtained the minimum value of –1.18% in 1988 and the max probability strategy obtained the maximum value of 2.46% in 1987. The min probability strategy shows the lowest yearly average of 0.1625%. The yearly averages for the other strategies is: diversify (0.2215%), max probability (0.1792%) and the max probability with MM (0.4160%).



Graph 5.55 Expected value comparison of probability distribution strategies

- **Return per month**



Graph 5.56: Return comparison between probability distribution strategies and MM

Graph 5.56 illustrates the comparison between the monthly returns of the probability distribution strategies and the MM. The returns for the probability distribution strategies fluctuate around the MM. P (X<MM) for the probability distribution strategies is as follows:

➢ Diversify strategy → 43.06%.

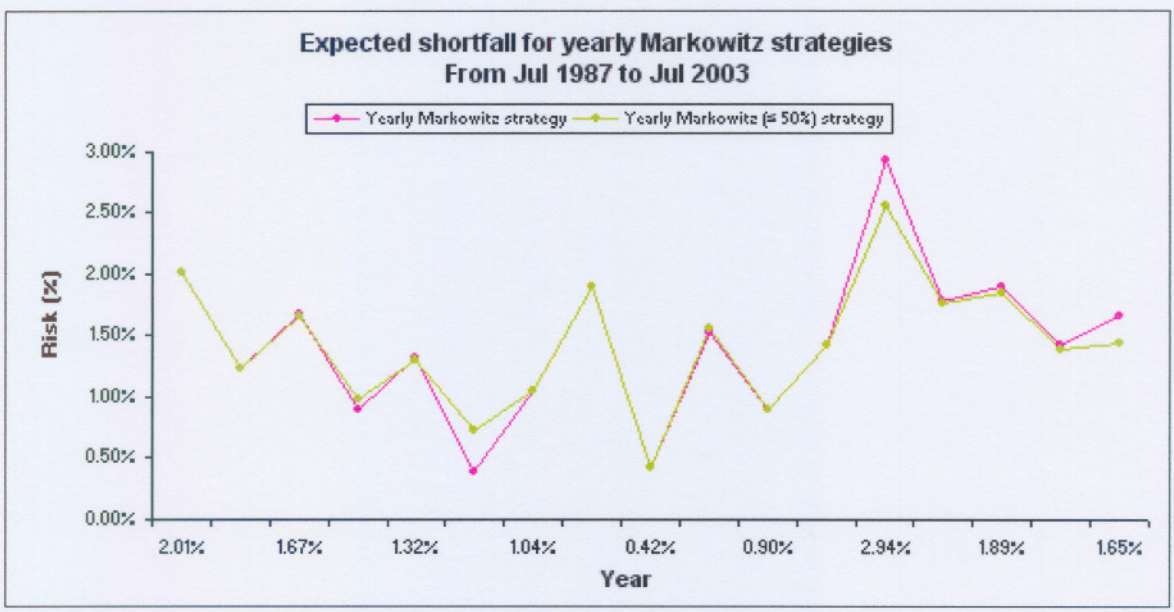
- Max probability strategy → 45.45%.
- Max probability strategy → 25.36%.
- Min probability strategy → 43.54%.

5.3.5 Yearly Markowitz strategies

- Markowitz strategy 1 ⇒ 100% investment in funds (can invest all in one fund), min variance.
- Markowitz strategy 2 ⇒ no more than 50% can be invested in a fund, minimise variance.

- **Expected shortfall**

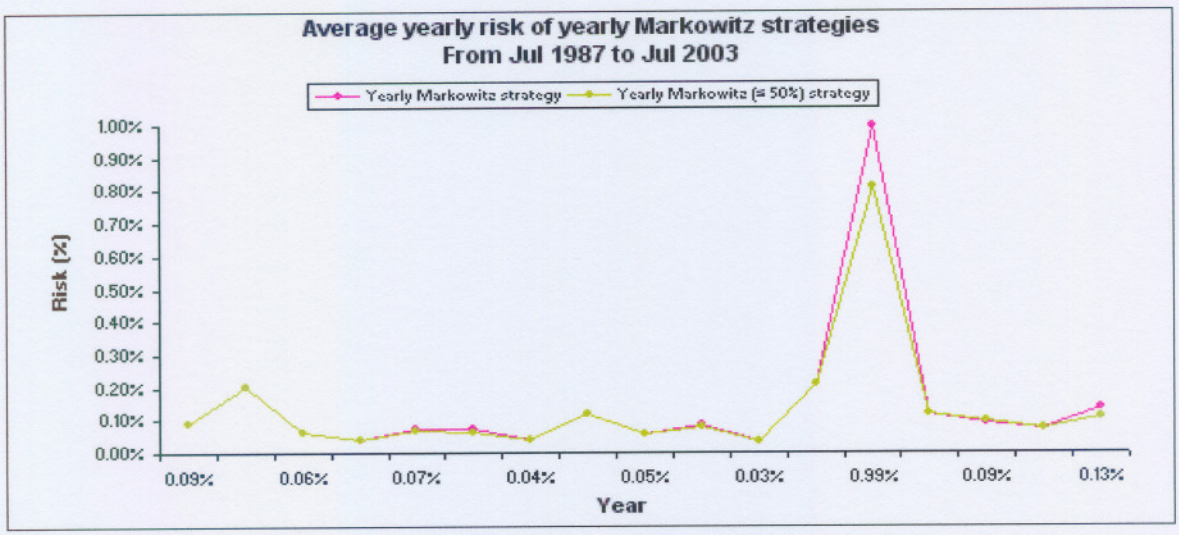
The following graph shows a comparison between the expected shortfall risk measures for the different yearly Markowitz strategies. The yearly Markowitz strategy 1 obtained the maximum risk value of 2.94% in 1999. The yearly yearly Markowitz strategy 1 obtained the minimum risk value of 0.39% in 1992. The yearly risk averages for yearly Markowitz strategy 1 and yearly yearly Markowitz strategy 1 are 1.4349% and 1.4213%, respectively.



Graph 5.57: Expected shortfall for yearly Markowitz strategies

- **Variance**

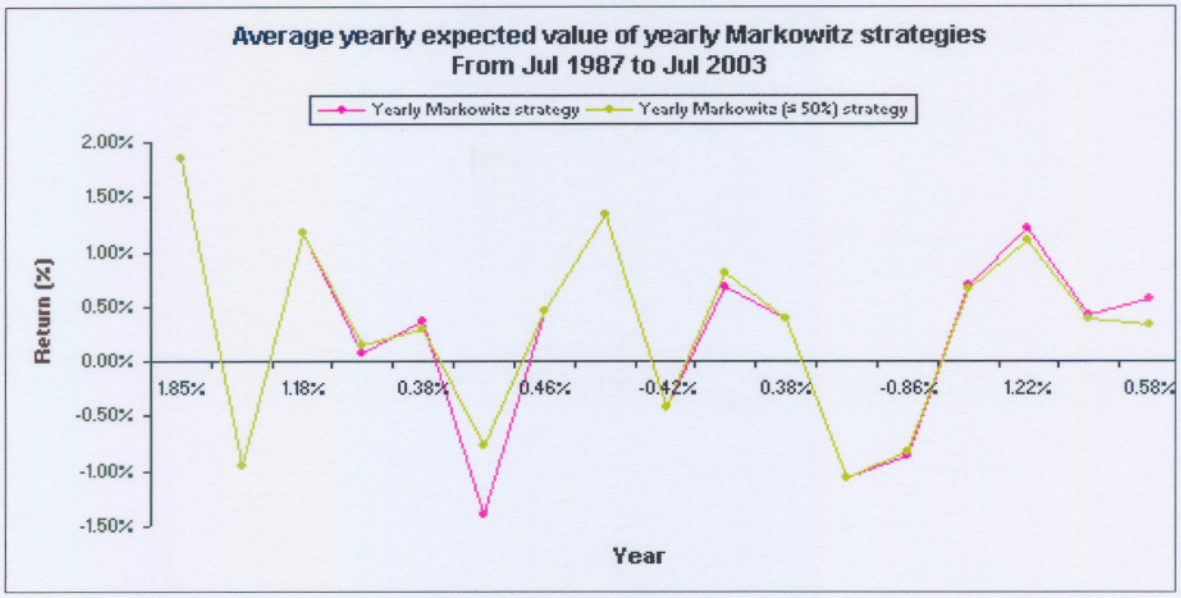
The following graph shows the comparison between the variance risk measures for the yearly Markowitz strategies. The first Markowitz strategy attained the highest risk value of 0.99% in 1999 and both the strategies attained the lowest risk value of 0.02% in 1997. The second yearly Markowitz strategy achieved the lowest yearly risk average of 0.1331%. The yearly risk average for the first strategy is 0.1466%.



Graph 5.58: Variance comparison between yearly Markowitz strategies

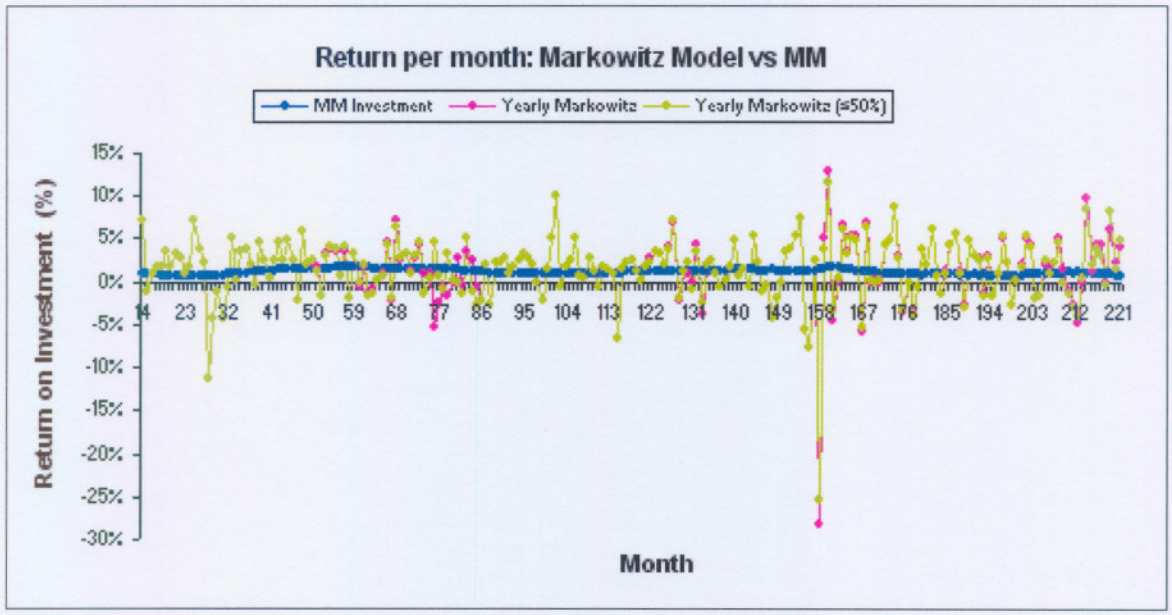
- **Average deviation**

Graph 5.59 shows a comparison between the average deviation return measures for the yearly Markowitz strategies. Both yearly Markowitz strategies obtained the minimum value of –1.08% in 1988 and the first Markowitz strategy obtained the maximum value of 1.72% in 1987. The yearly averages for the yearly Markowitz strategies are 0.2707% and 0.2920%, respectively. Thus, the first strategy (where there is no restriction on the percentage that you can invest in any one fund) shows the minimum yearly average.



Graph 5.59: Expected value comparison of yearly Markowitz strategies

- **Return per month**



Graph 5.60: Return comparison between yearly Markowitz strategies and MM

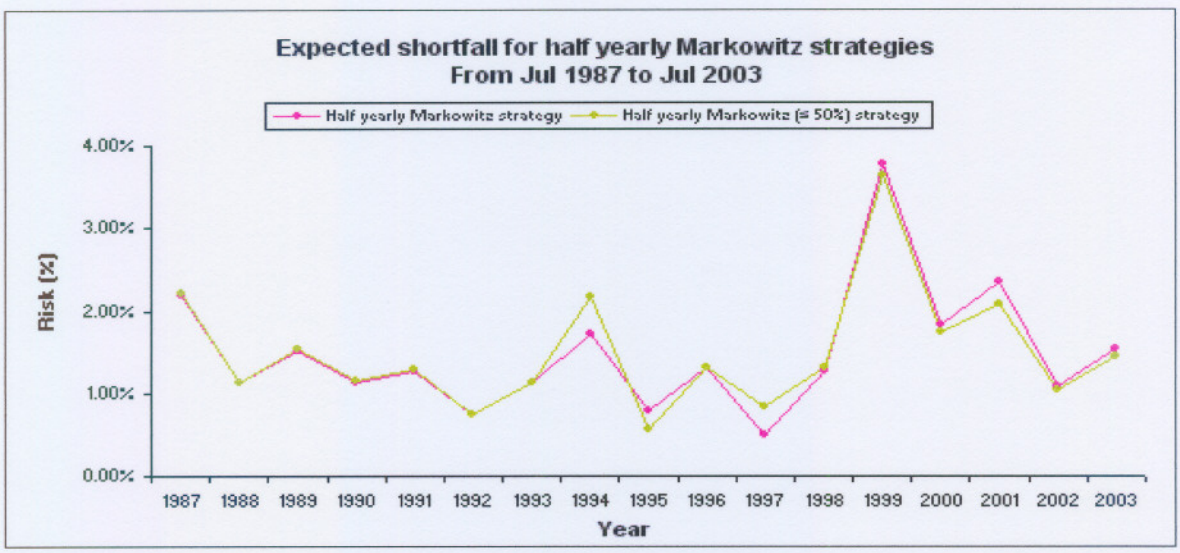
The graph above illustrates the comparison between the monthly returns of the yearly Markowitz strategies and the MM. The returns for the yearly Markowitz strategies vary around the MM. P (X<MM) for the yearly Markowitz strategies is as follows:

- Yearly Markowitz strategy 1 → 40.19%.
- Yearly Markowitz strategy 2 → 41.15%.

5.3.6 Half yearly Markowitz strategies

- **Expected shortfall**

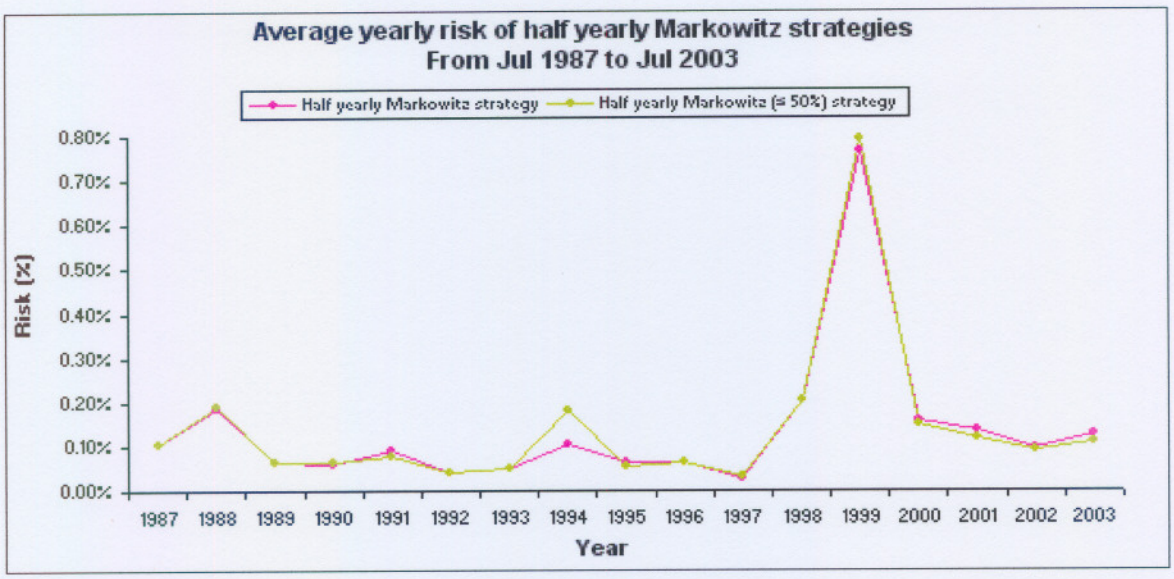
The following graph shows a comparison between the expected shortfall risk measures for the different half yearly Markowitz strategies. The half yearly Markowitz strategy 1 obtained the maximum risk value of 3.78% in 1999 and the minimum risk value of 0.51% in 1997. The yearly risk averages for half yearly Markowitz strategy 1 and half yearly Markowitz strategy 2 are 1.49090% and 1.4901%, respectively.



Graph 5.61: Expected shortfall for half yearly Markowitz strategies

- **Variance**

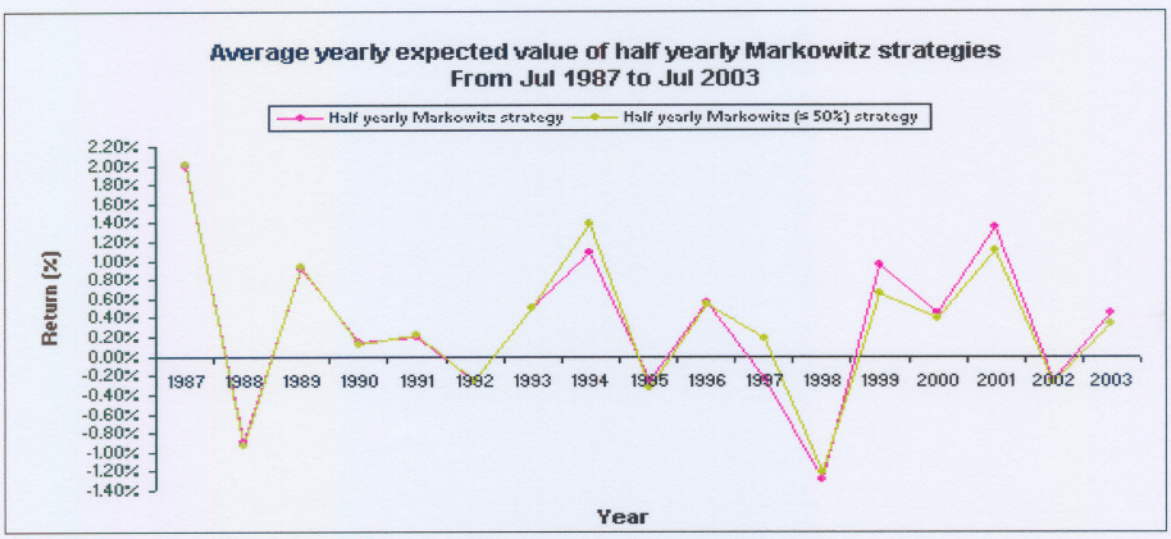
The following graph shows the comparison between the variance risk measures for the half yearly Markowitz strategies. The first Markowitz strategy attained the highest risk value (0.89% in 1999) and the lowest risk value (0.02% in 1997). The first half yearly Markowitz strategy achieved the minimum yearly risk average of 0.1367%. The yearly risk average for the second strategy is 0.1401%.



Graph 5.62: Variance comparison between half yearly Markowitz strategies

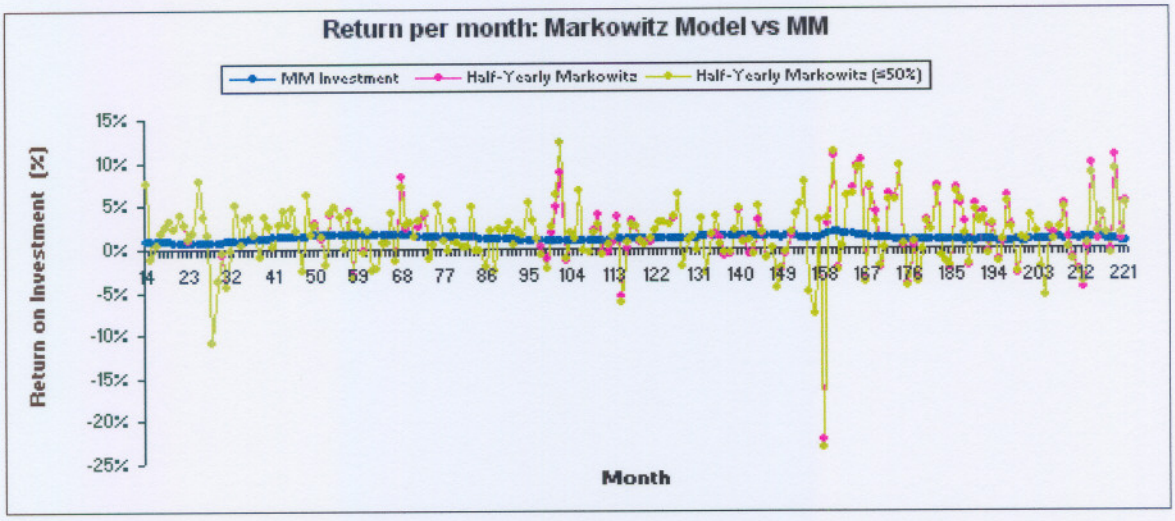
- **Average deviation**

Graph 5.63 shows a comparison between the average deviation return measures for the half yearly Markowitz strategies. The first Markowitz strategy attained the highest value (2.19% in 1987) and the lowest value (-0.95% in 1988). The second half yearly Markowitz strategy achieved the minimum yearly average of 0.3183%. The yearly average for the first strategy is 0.3224%.



Graph 5.63: Expected value comparison of half yearly Markowitz strategies

- **Return per month**



Graph 5.64: Return comparison between half yearly Markowitz strategies

Graph 5.64 illustrates the comparison between the monthly returns of the half yearly Markowitz strategies and the MM. The returns for the half yearly Markowitz strategies fluctuate around the MM.

P (X<MM) for the half yearly Markowitz strategies is as follows:

- Half yearly Markowitz strategy 1→ 44.02%.
- Half yearly Markowitz strategy 2 → 43.06%.

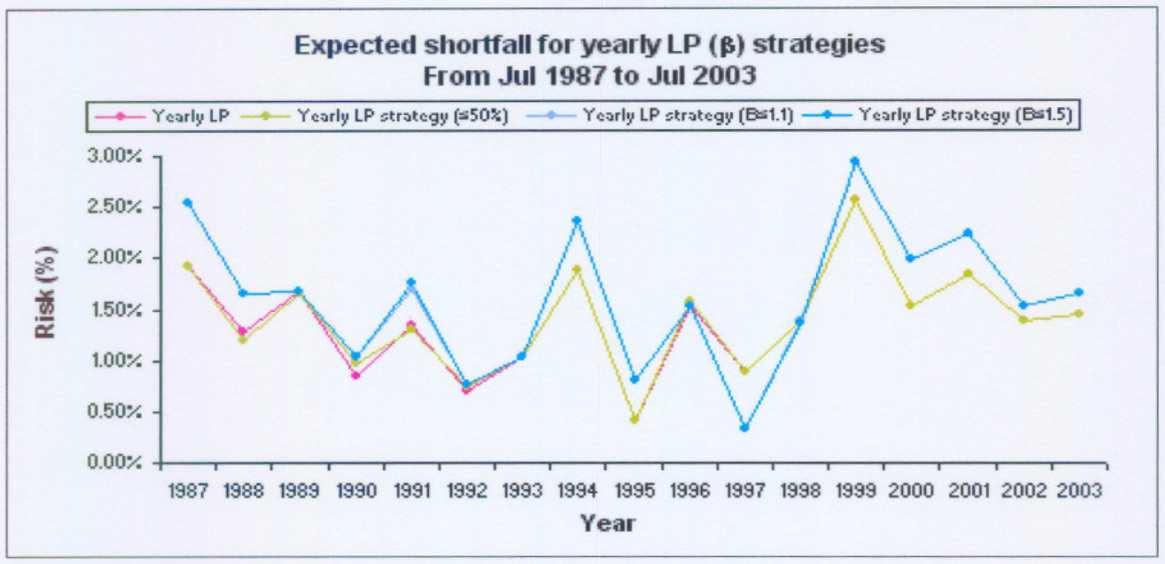
5.3.7 Yearly LP (β) strategies

- LP (β) strategy 1 ⇒ 100% investment in funds (can invest all in one fund), minimise risk.
- LP (β) strategy 2⇒ no more than 50% can be invested in a fund, minimise risk.
- LP (β) strategy 3 ⇒ $\beta \leq 1.1$, maximise return.
- LP (β) strategy 4 ⇒ $\beta \leq 1.5$, maximise return.

- **Expected shortfall**

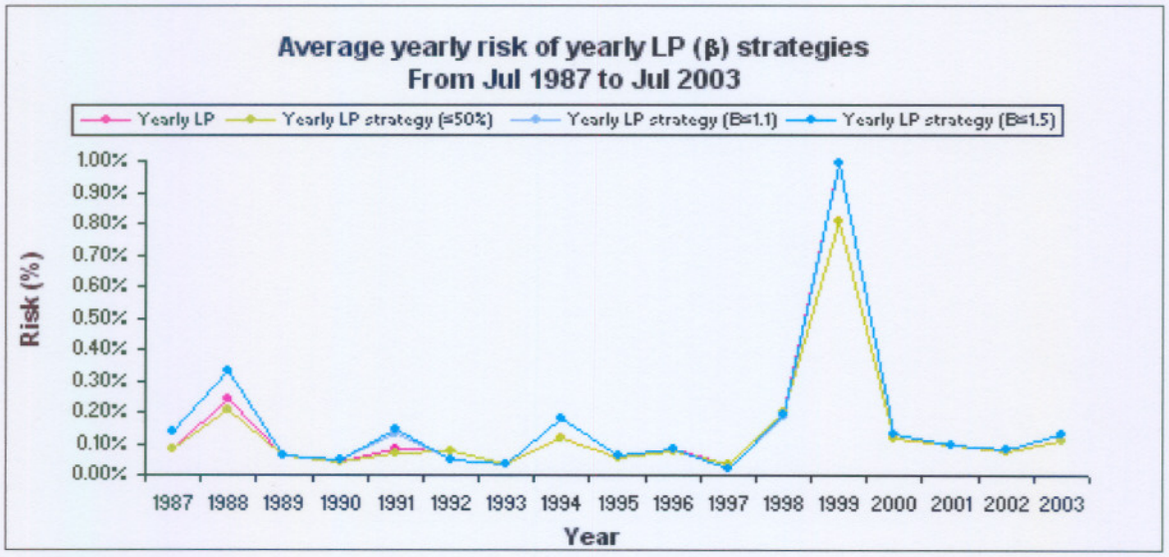
The following graph shows a comparison between the expected shortfall risk measures for the different yearly LP (β) strategies. The yearly LP (β) strategies 1, 3 and 4 obtained the maximum risk value of 2.94% in 1999 and the yearly LP (β) strategies 3 and 4 minimum risk

value of 0.331% in 1997. The yearly risk averages for yearly LP (β) strategies are 1.4893%, 1.4006%, 1.5996% and 1.6019%%, correspondingly.



Graph 5.65: Expected shortfall for yearly LP (β) strategies

- **Variance**

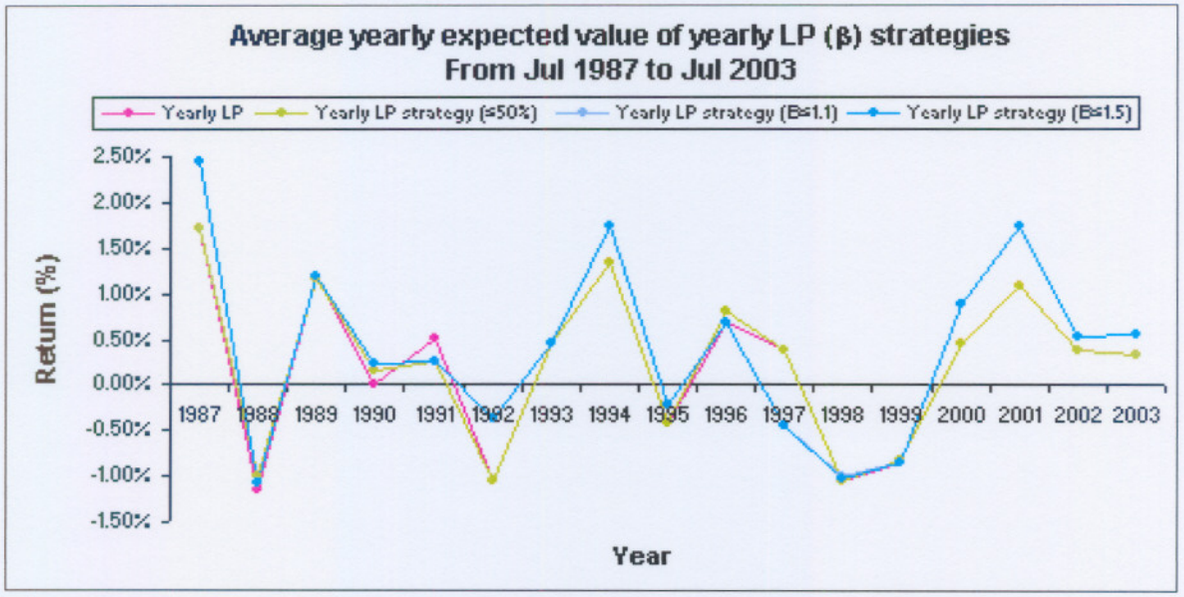


Graph 5.66: Variance comparison between yearly LP (β) strategies

Graph 5.66 shows the comparison between the variance risk measures for the yearly LP (β) strategies. The LP (β) strategy 4 attained the highest risk value (0.99% in 1999) and the LP (β) strategies 3 and 4 obtained the lowest risk value (0.02% in 1997). The second LP (β)

strategy achieved the minimum yearly risk average of 0.1332%. The yearly risk averages for the other LP (β) strategies are 0.1496%, 0.1622% and 0.1631% respectively.

- **Average deviation**



Graph 5.67: Expected value comparison of yearly LP (β) strategies

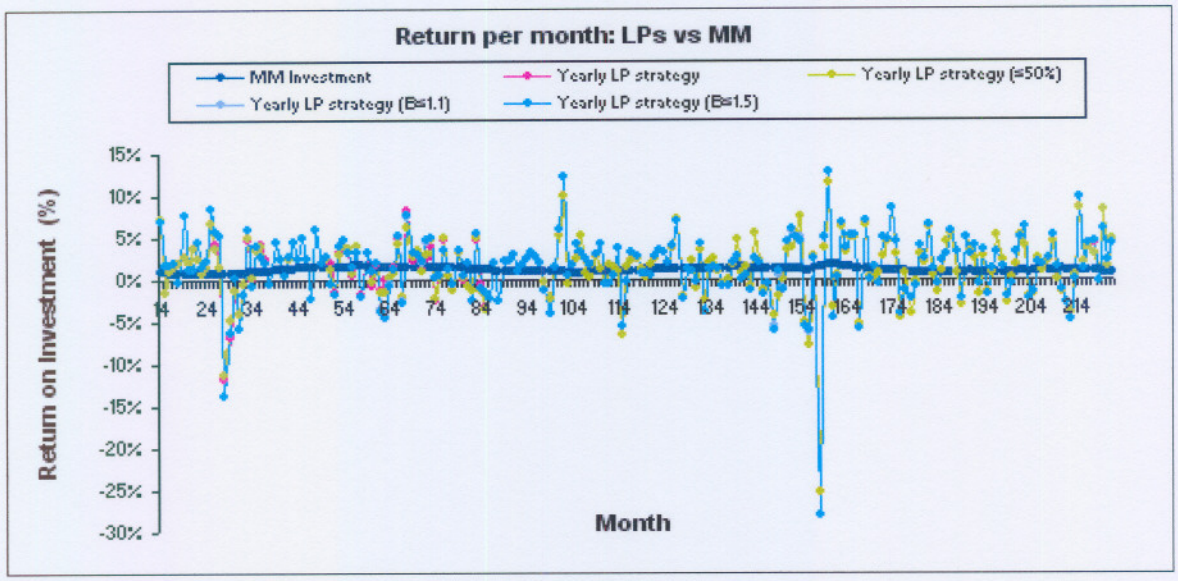
Graph 5.67 shows a comparison between the average deviation return measures for the different yearly LP (β) strategies. The yearly LP (β) strategies 3 and 4 attained the highest value of 2.46% in 1987 and the first yearly LP (β) strategy attained the lowest value of –1.14% in 1988. Yearly LP (β) strategy 2 achieved the lowest yearly average of 0.2509%. The yearly averages for the other yearly LP (β) strategies are: 0.3256%, 0.3997% and 0.3975%, respectively.

- **Return per month**

Graph 5.68 illustrates the comparison between the monthly returns of the yearly LP (β) strategies and the MM. The returns for the LP (β) strategies fluctuate around the MM. P (X<MM) for the LP (β) strategies is as follows:

➢ LP (β) strategy 1 ⇒ 39.23%.

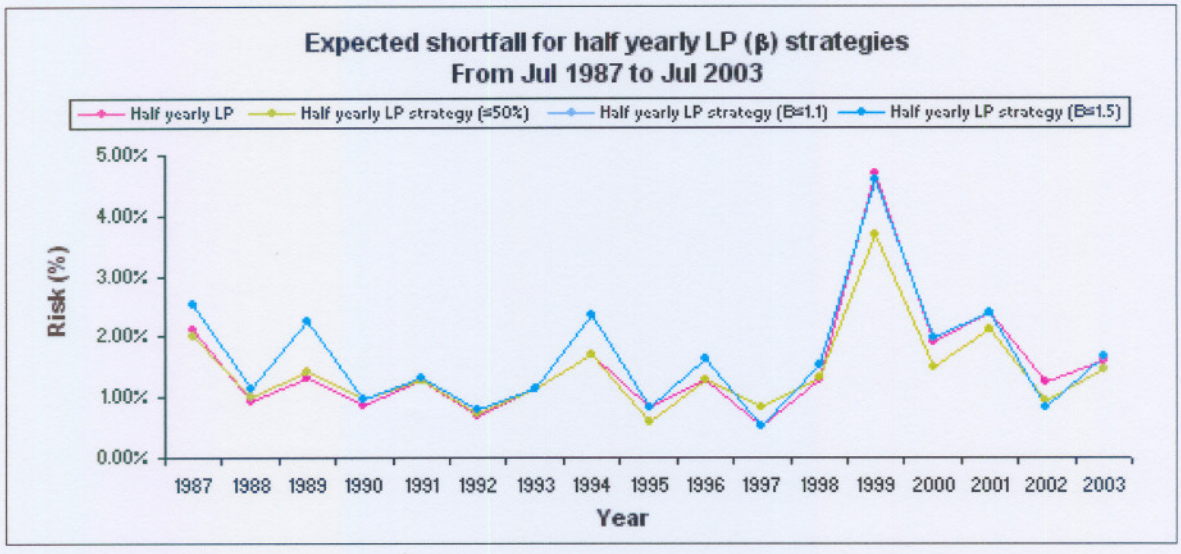
- LP (β) strategy 2⇒ 41.15%.
- LP (β) strategy 3 ⇒ 39.71%.
- LP (β) strategy 4 ⇒ 39.71%.



Graph 5.68: Return comparison between yearly LP (β) strategies and MM

5.3.8 Half yearly LP (β) strategies

- **Expected shortfall**

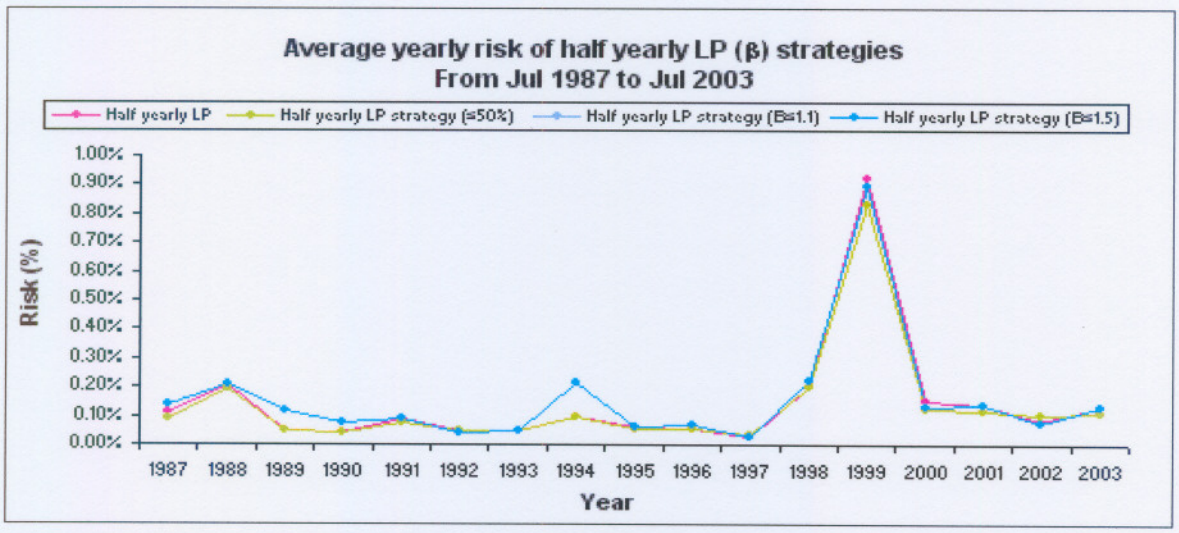


Graph 5.69: Expected shortfall for half yearly LP (β) strategies

Graph 5.69 shows a comparison between the expected shortfall risk measures for the different half yearly LP (β) strategies. The half yearly LP (β) strategy 1 obtained the maximum risk value of 4.69% in 1999. Both half yearly LP (β) strategies 1 and 4 obtained the minimum risk value of 0.51% in 1997. The yearly risk averages for the half yearly LP (β) strategies are 1.5124%, 1.4075%, 1.6728% and 1.6725%, respectively.

- **Variance**

Graph 5.70 shows the comparison between the variance risk measures for the different half yearly LP (β) strategies. The first half yearly LP (β) strategy attained the highest risk value of 0.92% in 1999 and both the first and fourth strategies obtained the lowest risk value of 0.02% in 1997. Half yearly LP (β) strategy 2 achieved the lowest yearly risk average of 0.1349%. The yearly risk averages for the other half yearly LP (β) strategies are: 0.1449%, 0.1581% and 0.1580%, correspondingly.

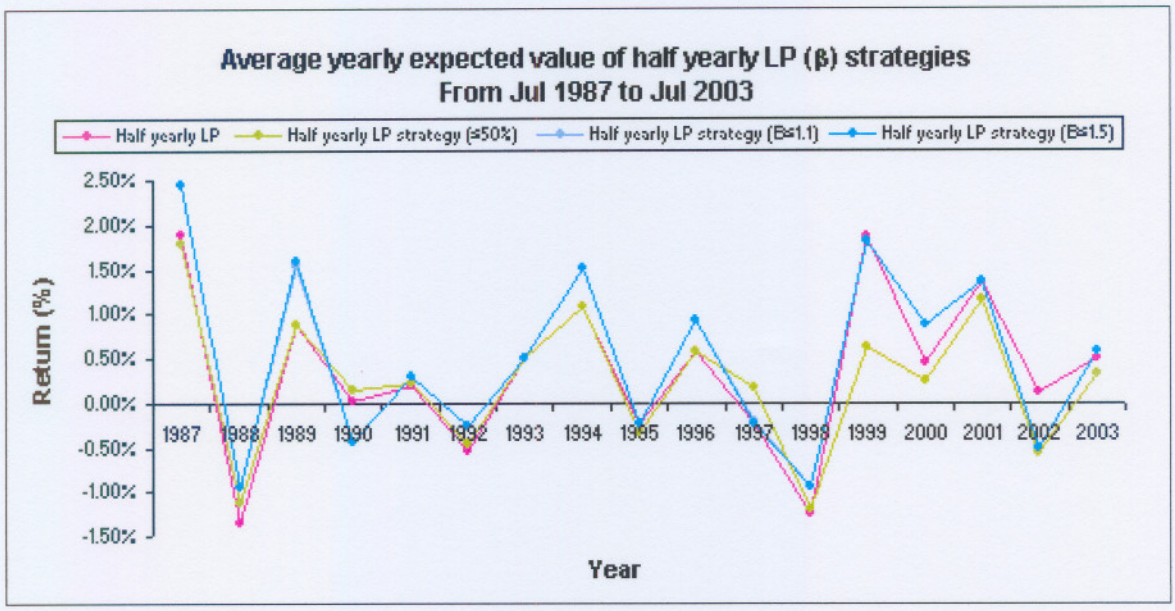


Graph 5.70: Variance comparison between half yearly LP (β) strategies

- **Average deviation**

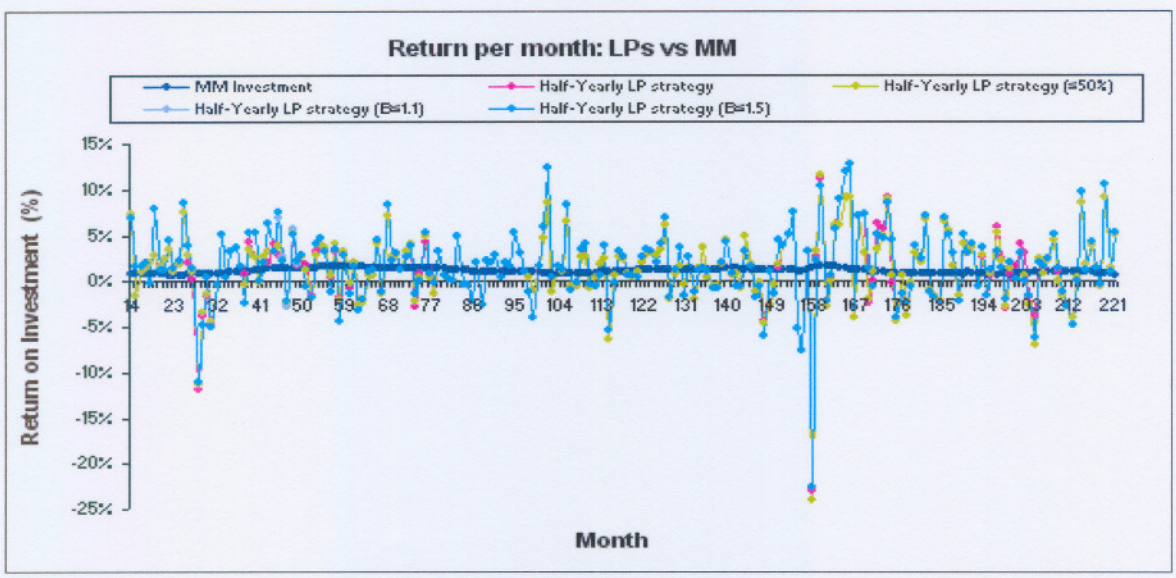
The following graph presents a comparison between the average deviation return measures for the different half yearly LP (β) strategies. Half yearly LP (β) strategy 3 obtained the highest value of 2.46% in 1987 and the half yearly LP (β) strategy 1 attained the lowest value of –1.34% in 1988. Half yearly LP (β) strategy 2 achieved the lowest yearly average of

0.2394%. The yearly averages for the other half yearly LP (β) strategies are: 0.3434%, 0.4899% and 0.4926%, correspondingly.



Graph 5.71: Expected value comparison of half yearly LP (β) strategies

- **Return per month**



Graph 5.72: Return comparison between half yearly LP (β) strategies and MM

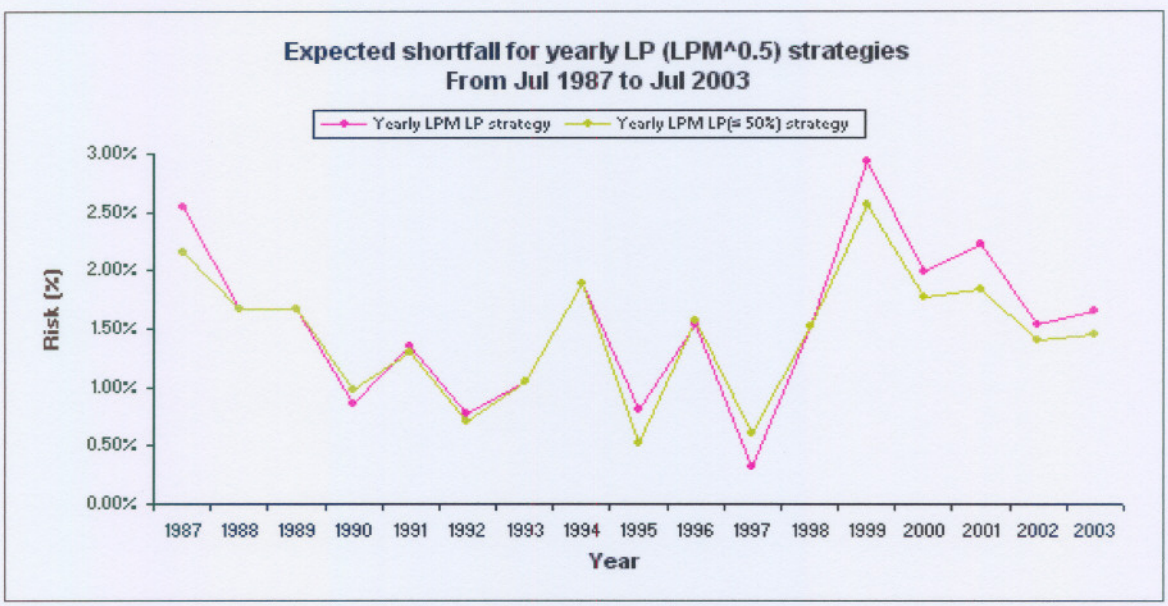
Graph 5.72 illustrates the comparison between the monthly returns of the half yearly LP (β) strategies and the MM. The returns for the half yearly LP (β) strategies fluctuate around the MM. P (X<MM) for the half yearly LP (β) strategies is as follows:

- LP (β) strategy 1 ⇒ 43.06%.
- LP (β) strategy 2⇒ 43.54%.
- LP (β) strategy 3 ⇒ 41.63%.
- LP (β) strategy 4 ⇒ 41.63%.

5.3.9 Yearly LP (LPM) strategies (n = 0.5)

- LP (LPM) strategy 1 ⇒ 100% investment in funds (can invest all in one fund), minimise risk.
- LP (LPM) strategy 2⇒ no more than 50% can be invested in a fund, minimise risk.

- **Expected shortfall**



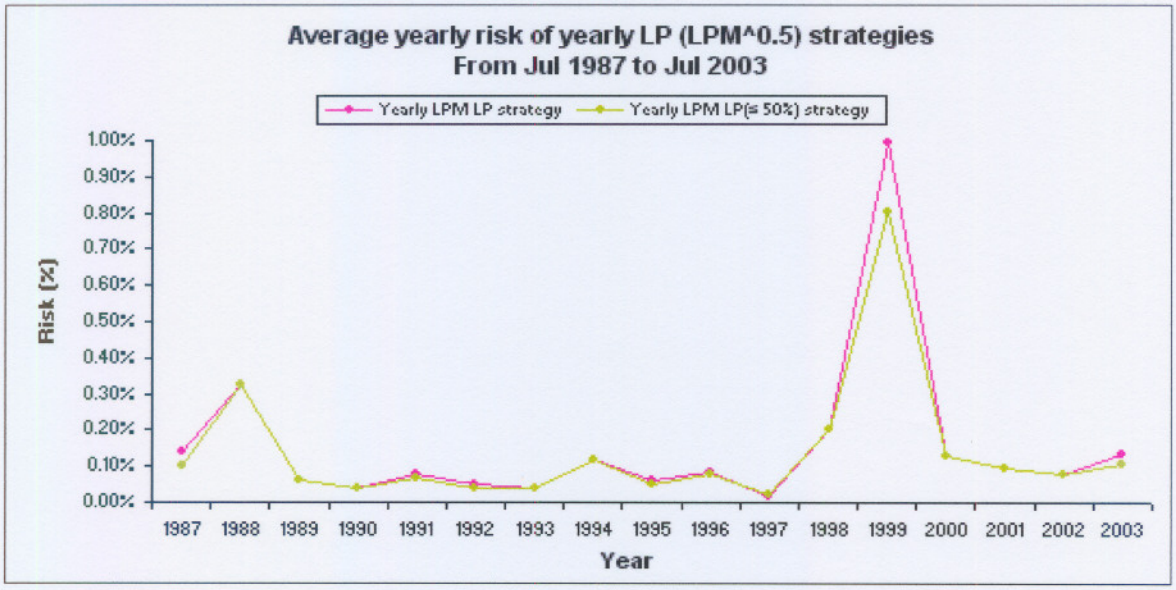
Graph 5.73: Expected shortfall for yearly LP (LPM, n = 0.5) strategies

Graph 5.73 shows a comparison between the expected shortfall risk measures for the different LP (LPM) strategies. Yearly LP (LPM) strategy 1 obtained both the maximum risk

value of 2.94% in 1999 and the minimum risk value of 0.33% in 1997. The yearly risk averages for the yearly LP (LPM) strategies are 1.5481% and 1. 4485%, respectively.

- **Variance**

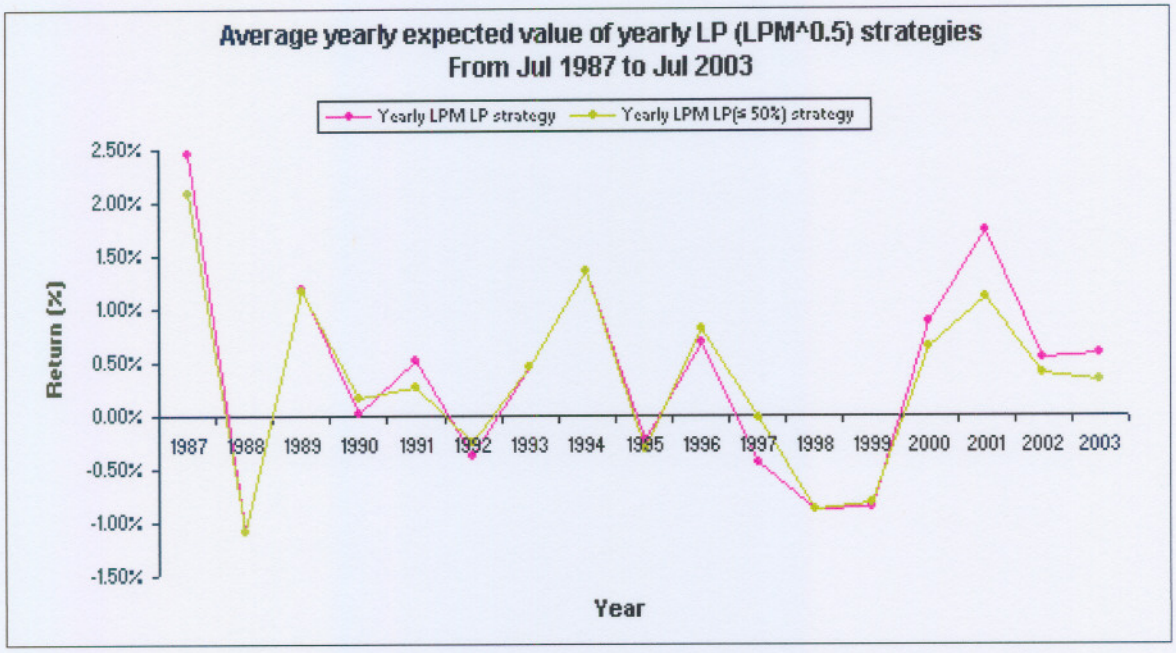
The following graph shows the comparison between the variance risk measures for the different yearly LP (LPM, n = 0.5) strategies. The yearly LP (LPM) strategy 1 attained the highest risk value of 0.99% in 1999 and both the strategies obtained the lowest risk value of 0.02% in 1997. Yearly LP (LPM) strategy 2 achieved the lowest yearly risk average of 0.1390%. The yearly risk averages for yearly LP (LPM) strategy 1 is 0.1562%.



Graph 5.74: Variance comparison between yearly LP (LPM, n = 0.5) strategies

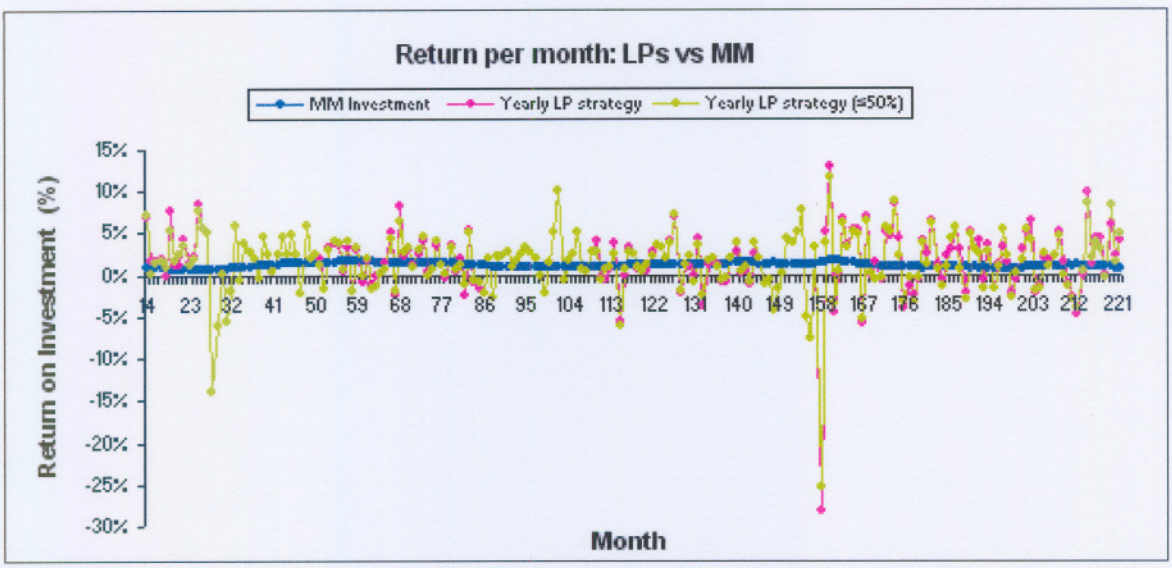
- **Average deviation**

The graph below illustrates a comparison between the average deviation return measures for the yearly LP (LPM) strategies. The yearly LP (LPM) strategy 1 attained the highest value of 2.46 % in 1987 and both the strategies obtained the lowest value of –1.08% in 1988. Yearly LP (LPM) strategy 2 achieved the lowest yearly average (0.3171%). The yearly average for yearly LP (LPM) strategy 1 is 0.3854%.



Graph 5.75: Expected value comparison of yearly LP (LPM, n = 0.5) strategies

- **Return per month**



Graph 5.76: Return comparison between yearly LP (LPM, n = 0.5) strategies and MM

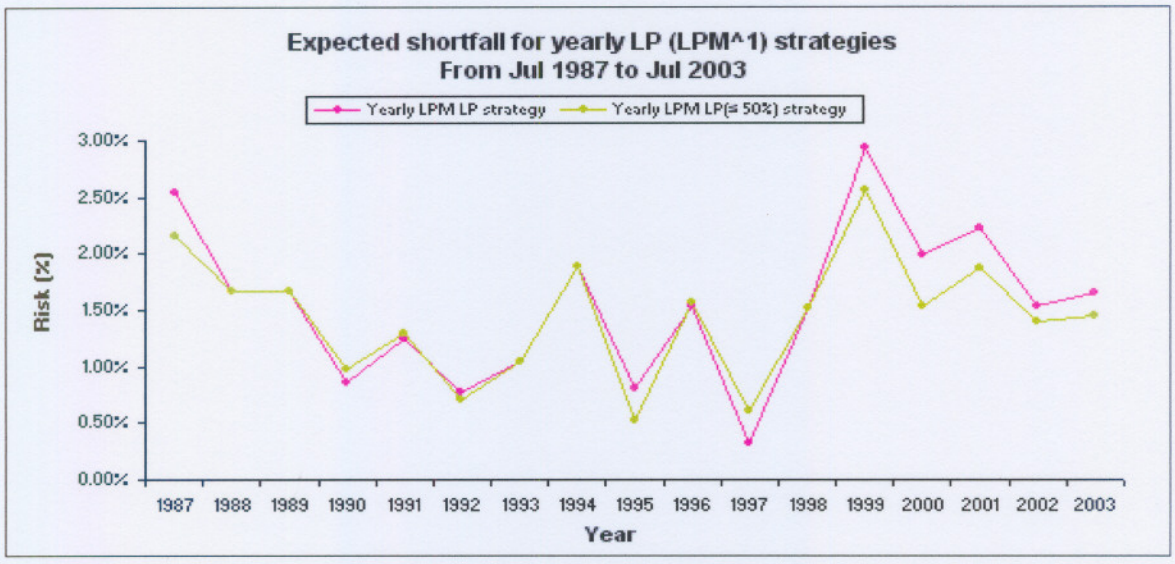
Graph 5.76 illustrates the comparison between the monthly returns of the yearly LP (LPM) strategies and the MM. The returns for the yearly LP (LPM) strategies fluctuate around the MM. P (X<MM) for the pessimistic yearly LP (LPM) strategies is as follows:

- Yearly LP (LPM) strategy 1 → 40.67%.
- Yearly LP (LPM) strategy 2 → 42.58%.

5.3.10 Yearly LP (LPM) strategies (n = 1)

- **Expected shortfall**

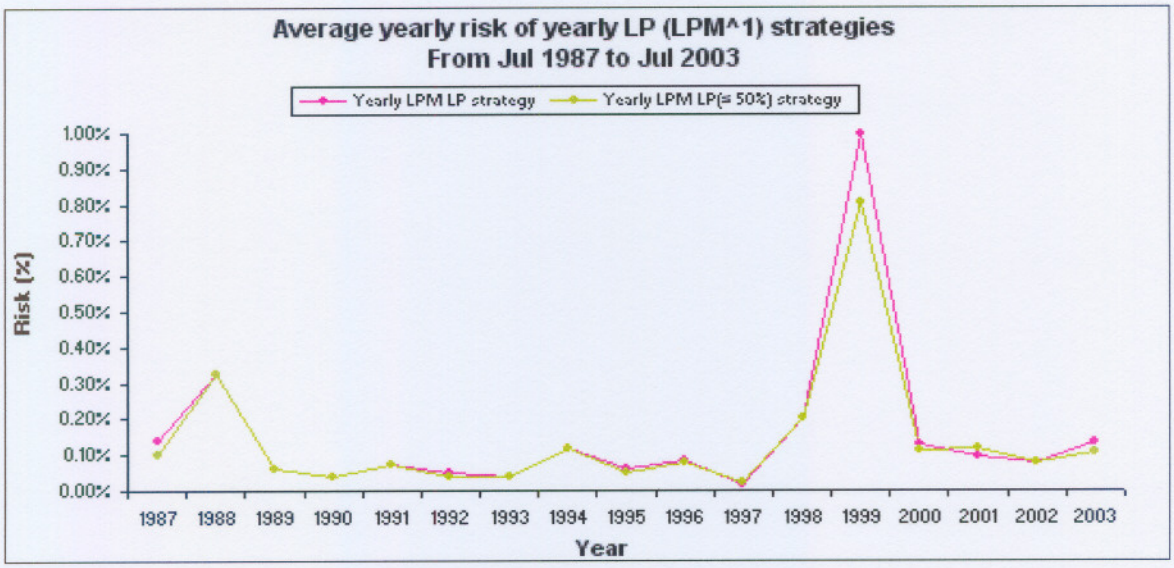
The following graph shows a comparison between the expected shortfall risk measures for the different yearly LP (LPM) strategies. The yearly LP (LPM) strategy 1 obtained the highest risk value of 2.94% in 1999. The yearly LP (LPM) strategy 1 also obtained the lowest risk value of 0.33% in 1997. The yearly risk averages for the yearly LP (LPM) strategies are 1.5421% and 1.4355%, respectively.



Graph 5.77: Expected shortfall for yearly LP (LPM, n = 1) strategies

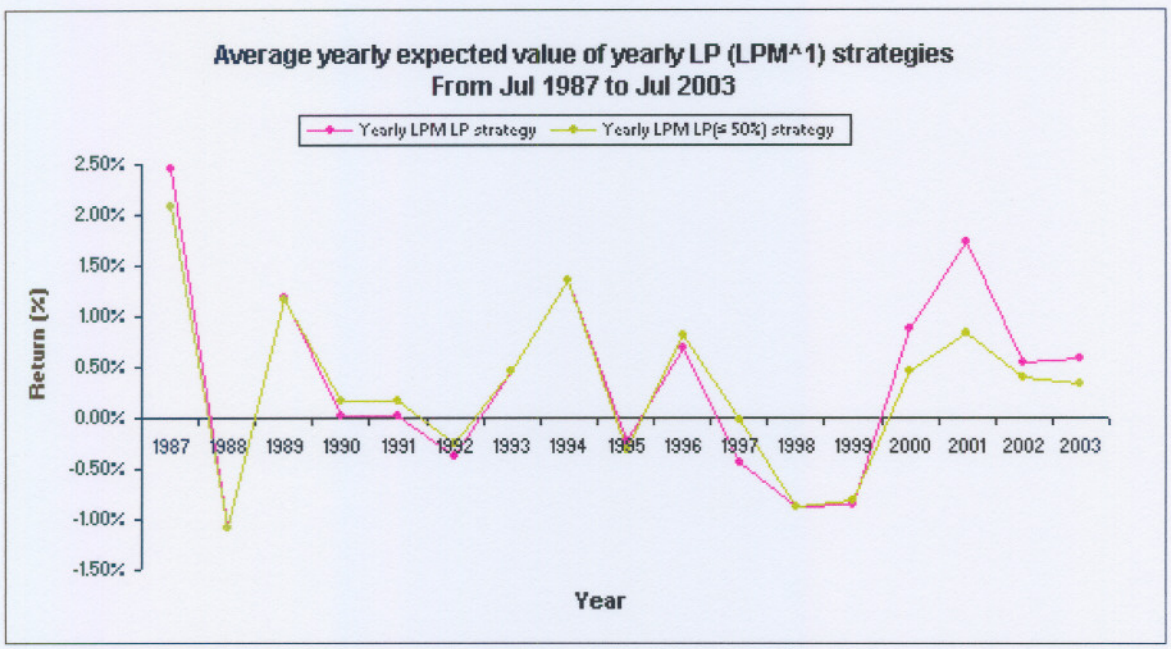
- **Variance**

The following graph shows the comparison between the variance risk measures for the yearly LP (LPM, n = 1) strategies. The yearly LP (LPM) strategy 1 attained the highest risk value of 0.99% in 1999 and both the strategies obtained the lowest risk value of 0.02% in 1997. Yearly LP (LPM) strategy 2 achieved the lowest yearly risk average of 0.1396%. The yearly risk average for yearly LP (LPM) strategy 1 is 0.1557%.



Graph 5.78: Variance comparison between yearly LP (LPM, n = 1) strategies

- **Average deviation**



Graph 5.79: Expected value comparison of yearly LP (LPM, n = 1) strategies

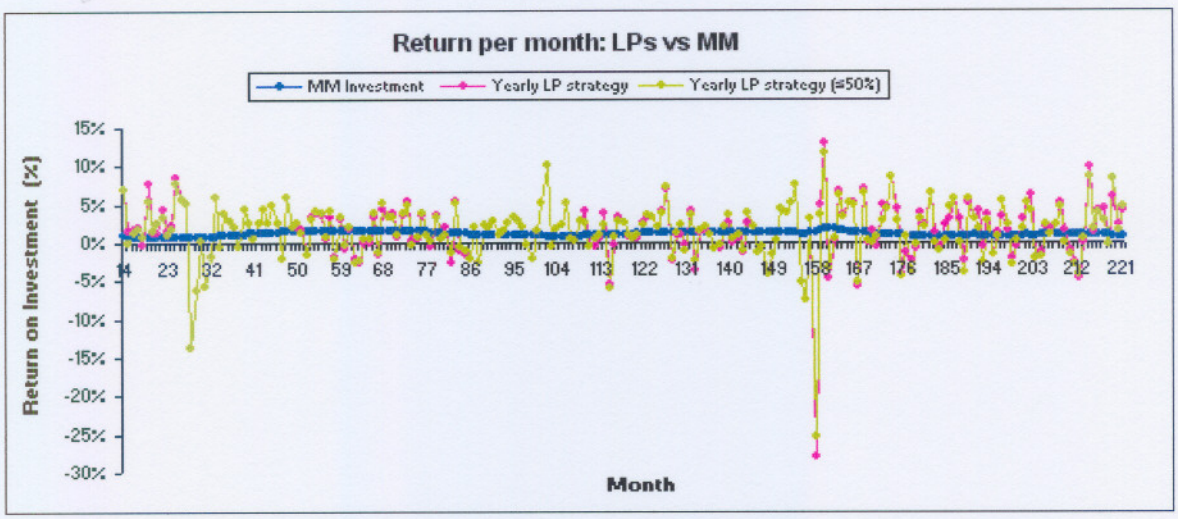
The graph above illustrates a comparison between the average deviation return measures for the yearly LP (LPM) strategies. The yearly LP (LPM) strategy 1 attained the highest risk value of 2.46 % in 1987 and both the strategies obtained the lowest risk value of –1.08% in

1988. Yearly LP (LPM) strategy 2 achieved the lowest yearly risk average (0.2865%). The yearly risk average for yearly LP (LPM) strategy 1 is 0.3559%.

- **Return per month**

Graph 5.80 illustrates the comparison between the monthly returns of the yearly LP (LPM) strategies and the MM. The returns for the yearly LP (LPM) strategies fluctuate around the MM. P (X<MM) for the yearly LP (LPM) strategies is as follows:

- Yearly LP (LPM) strategy 1 → 40.67%.
- Yearly LP (LPM) strategy 2 → 43.54%.

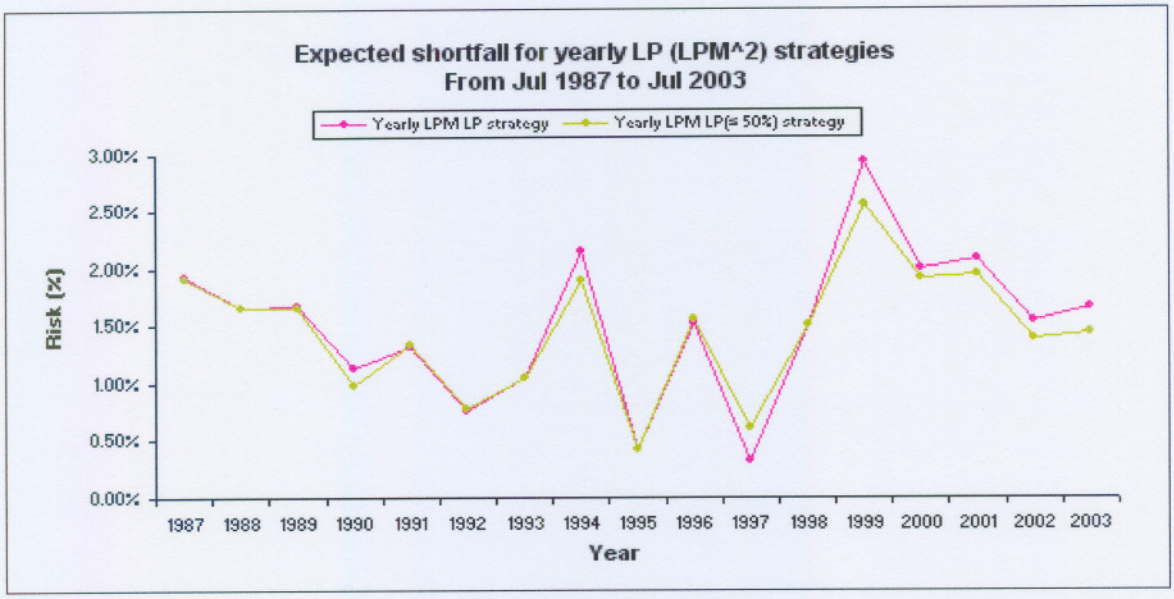


Graph 5.80: Return comparison between yearly LP (LPM, n = 1) strategies and MM

5.3.11 Yearly LP (LPM) strategies (n = 2)

- **Expected shortfall**

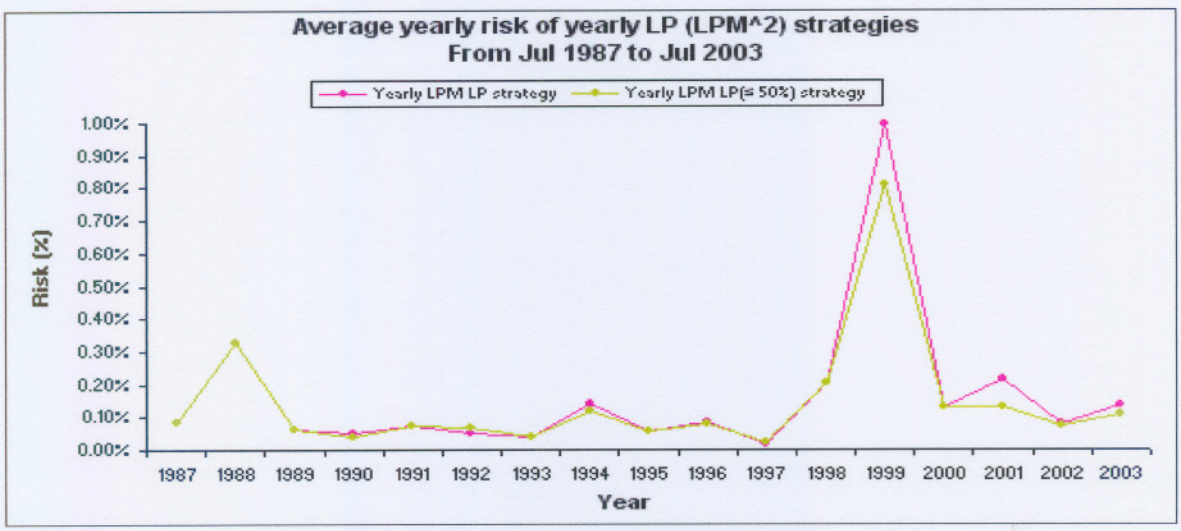
The following graph shows a comparison between the expected shortfall risk measures for the different yearly LP (LPM) strategies. The yearly LP (LPM) strategy 1 obtained the highest risk value of 2.94% in 1999. The yearly LP (LPM) strategy 1 also obtained the lowest risk value of 0.33% in 1997. The yearly risk averages for the yearly LP (LPM) strategies are 1.5108% and 1.4509%, respectively.



Graph 5.81: Expected shortfall for yearly LP (LPM, n = 2) strategies

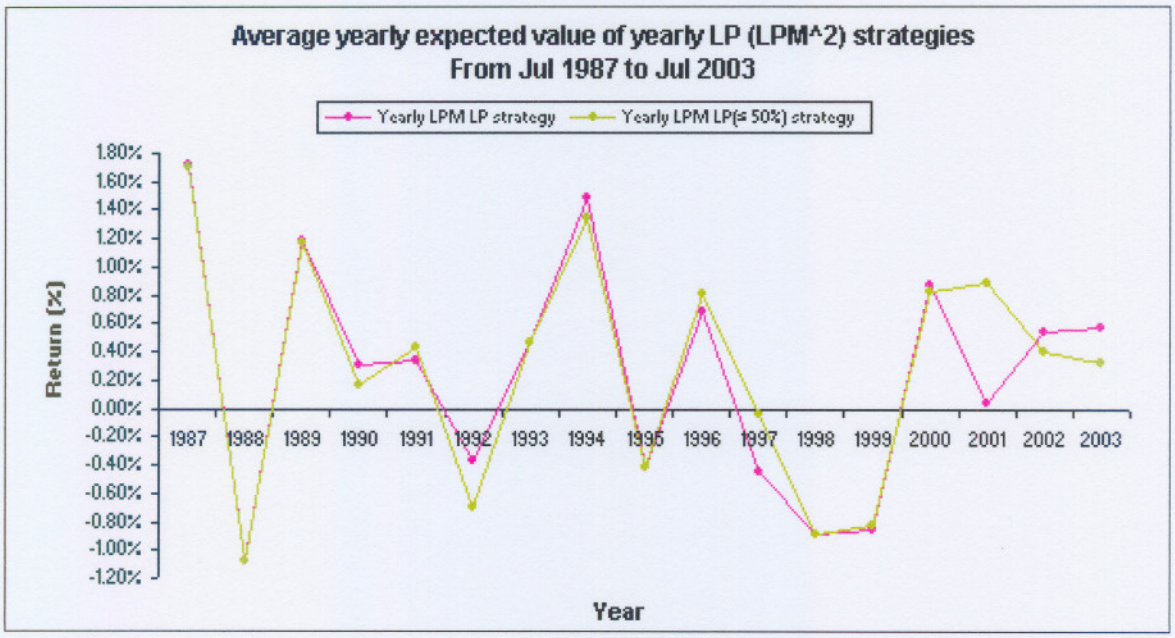
- **Variance**

The following graph shows the comparison between the variance risk measures for the yearly LP (LPM, n = 2) strategies. The yearly LP (LPM) strategy 1 attained the maximum risk value of 0.99% in 1999 and both the strategies obtained the minimum risk value of 0.02% in 1997. Yearly LP (LPM) strategy 2 achieved the lowest yearly risk average of 0.1421%. The yearly risk average for yearly LP (LPM) strategy 1 is 0.1612%.



Graph 5.82: Variance comparison between yearly LP (LPM, n = 2) strategies

- **Average deviation**



Graph 5.83: Expected value comparison of yearly LP (LPM, n = 2) strategies

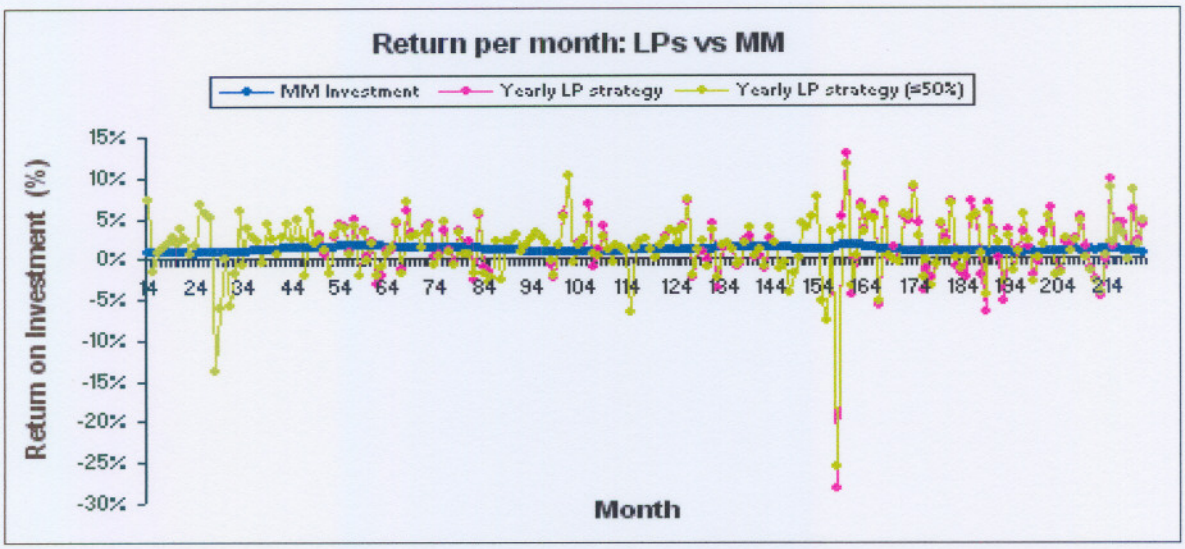
The graph above illustrates a comparison between the average deviation return measures for the yearly LP (LPM) strategies. The yearly LP (LPM) strategy 1 attained the highest value of 1.72 % in 1987 and both the strategies obtained the lowest value of –1.08% in 1988. Yearly LP (LPM) strategy 1 achieved the lowest yearly average (0.2447%). The yearly average for yearly LP (LPM) strategy 2 is 0.2707%.

- **Return per month**

The following graph indicates the comparison between the monthly returns of the yearly LP (LPM) strategies and the MM. The returns for the yearly LP (LPM) strategies vary around the MM.

P (X<MM) for the yearly LP (LPM) strategies is as follows:

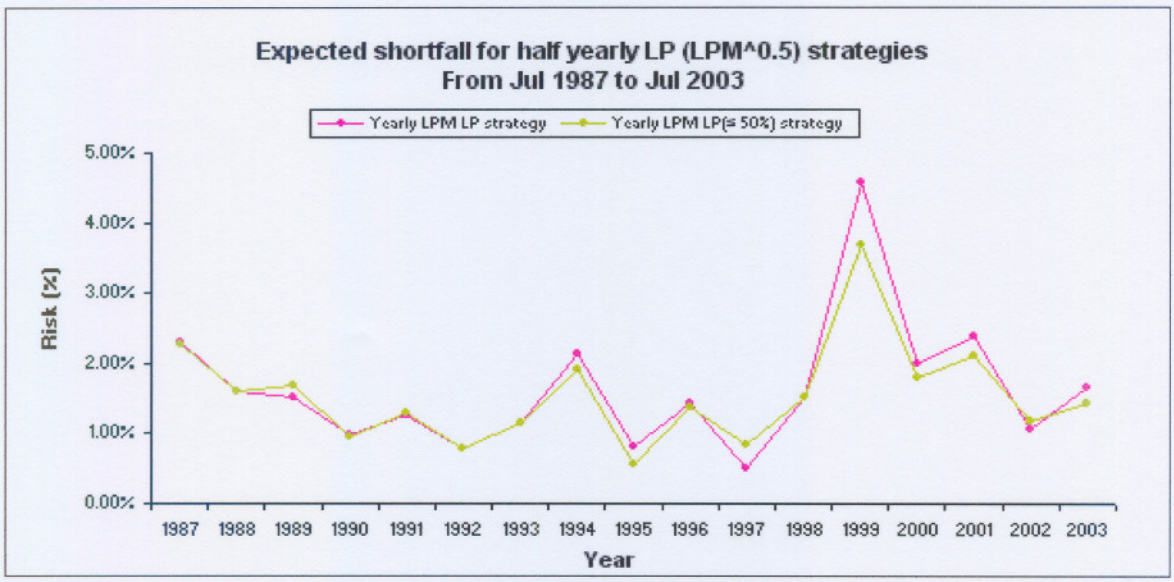
- Yearly LP (LPM) strategy 1 → 41.15%.
- Yearly LP (LPM) strategy 2 → 42.58%.



Graph 5.84: Return comparison between yearly LP (LPM, n = 2) strategies and MM

5.3.12 Half yearly LP (LPM) strategies (n = 0.5)

- **Expected shortfall**



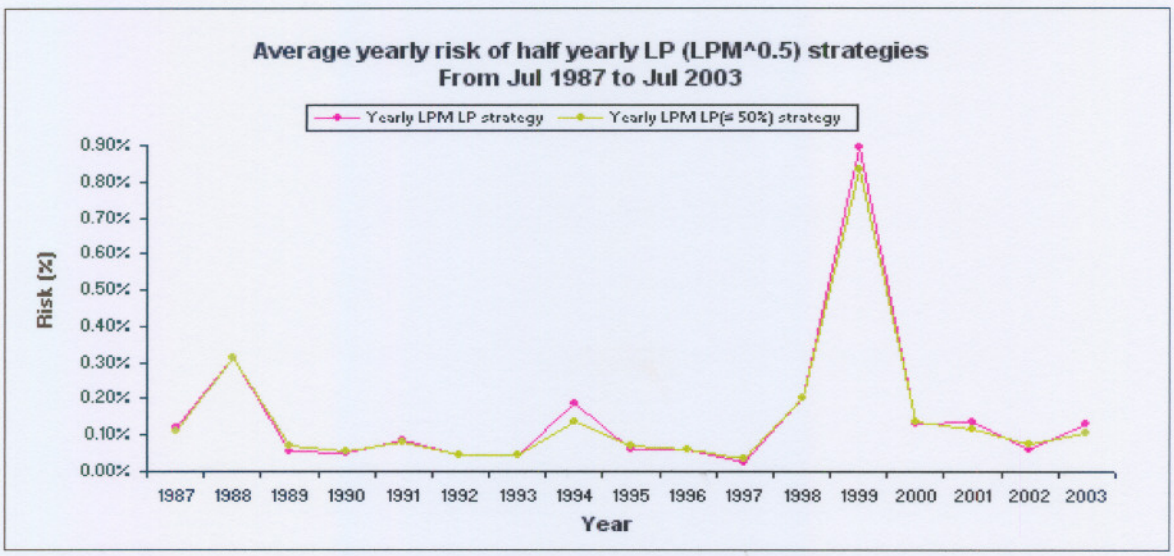
Graph 5.85: Expected shortfall for half yearly LP (LPM, n = 0.5) strategies

Graph 5.85 shows a comparison between the expected shortfall risk measures for the different half yearly LP (LPM) strategies. The half yearly LP (LPM) strategy 1 obtained the

highest risk value of 4.58% in 1999. The half yearly LP (LPM) strategy 1 also obtained the lowest risk value of 0.51% in 1997. The yearly risk averages for the half yearly LP (LPM) strategies are 1.6281% and 1.5357%, respectively.

- **Variance**

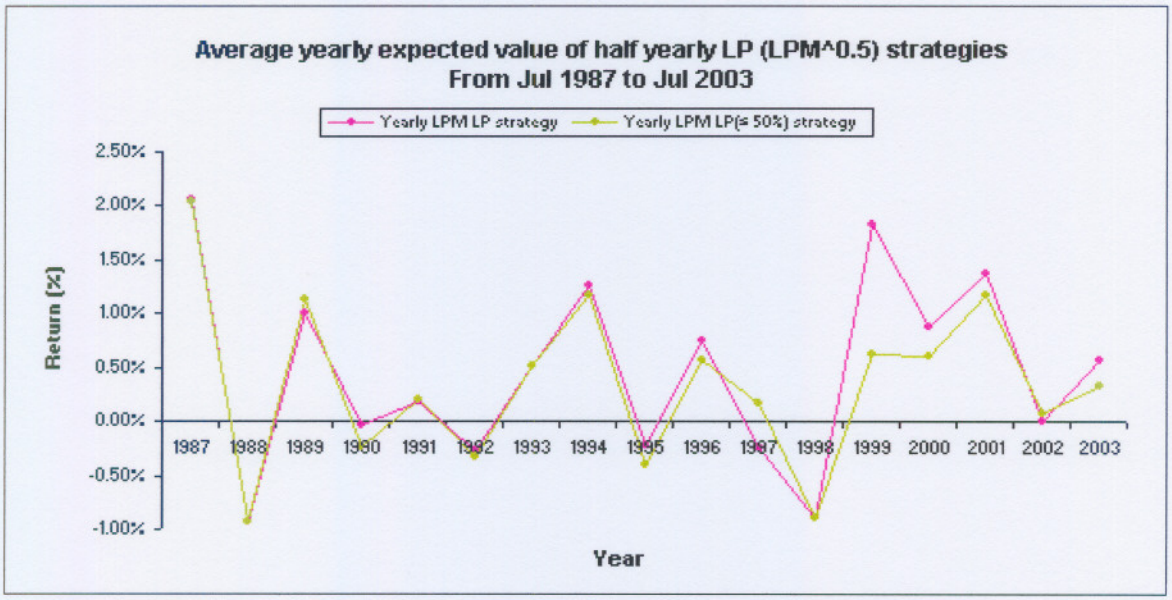
The following graph shows the comparison between the variance risk measures for the half yearly LP (LPM, n = 0.5) strategies. The half yearly LP (LPM) strategy 1 attained the maximum risk value of 0.89% in 1999 and the minimum risk value of 0.02% in 1997. Half yearly LP (LPM) strategy 2 achieved the lowest yearly risk average of 0.1471%. The yearly risk average for half yearly LP (LPM) strategy 1 is 0.1537%.



Graph 5.86: Variance comparison between half yearly LP (LPM, n = 0.5) strategies

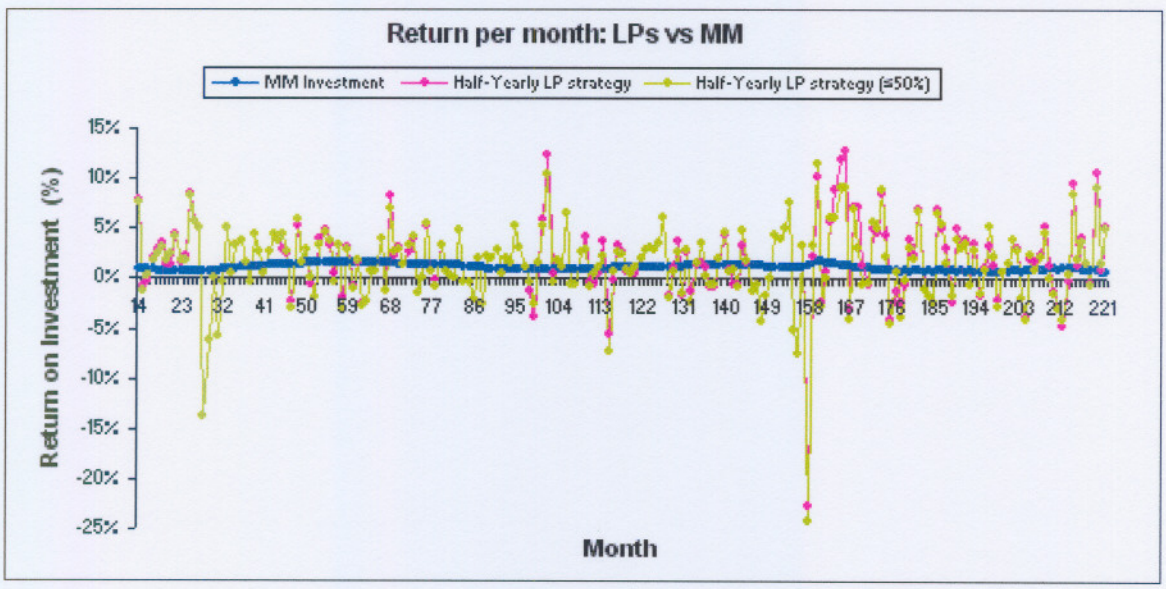
- **Average deviation**

The following graph illustrates a comparison between the average deviation return measures for the half yearly LP (LPM) strategies. The half yearly LP (LPM) strategy 1 attained the highest value of 2.06 % in 1987 and both the strategies obtained the lowest value of –0.93% in 1988. Half yearly LP (LPM) strategy 1 achieved the lowest yearly average (0.3410%). The yearly average for half yearly LP (LPM) strategy 2 is 0.4701%.



Graph 5.87: Expected value comparison of half yearly LP (LPM, n = 0.5) strategies

- **Return per month**



Graph 5.88: Return comparison between half yearly LP (LPM, n = 0.5) strategies and MM

Graph 5.88 indicates the comparison between the monthly returns of the half yearly LP (LPM) strategies and the MM. The returns for the half yearly LP (LPM) strategies vary around the MM.

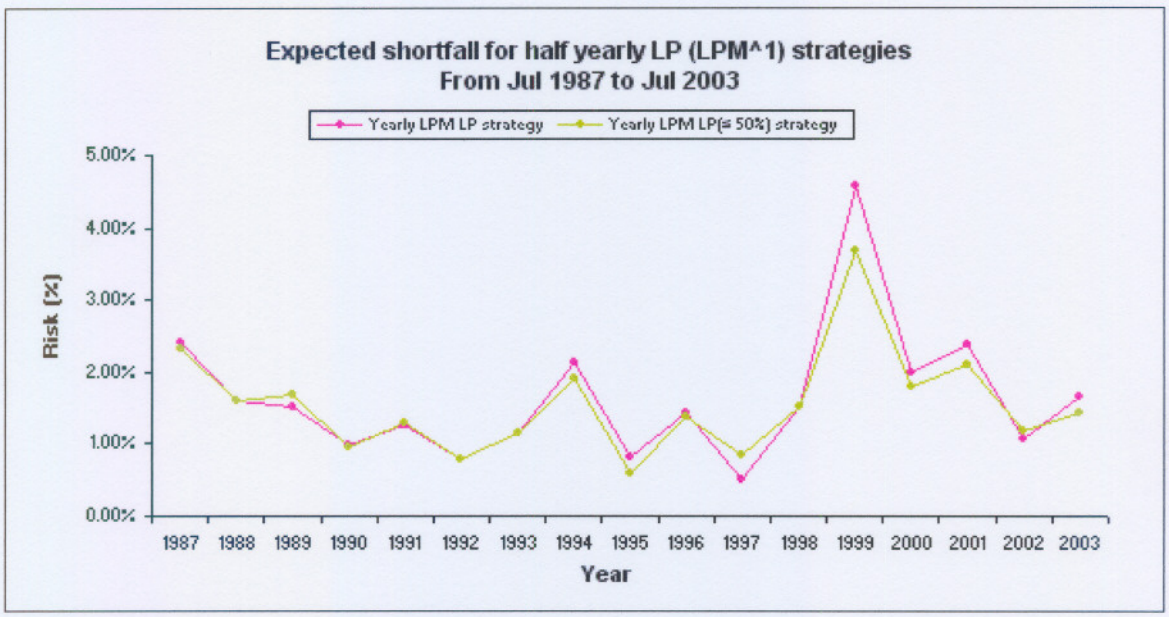
P (X<MM) for the yearly LP (LPM) strategies is as follows:

- Half yearly LP (LPM) strategy 1 → 41.15%.
- Half yearly LP (LPM) strategy 2 → 42.58%.

5.3.13 Half yearly LP (LPM) strategies (n = 1)

- **Expected shortfall**

The following graph displays a comparison between the expected shortfall risk measures for the different half yearly LP (LPM) strategies. The half yearly LP (LPM) strategy 1 obtained the highest risk value of 4.58% in 1999. The half yearly LP (LPM) strategy 1 also obtained the lowest risk value of 0.51% in 1997. The yearly risk averages for the half yearly LP (LPM) strategies are 1.6353% and 1.5410%, respectively.

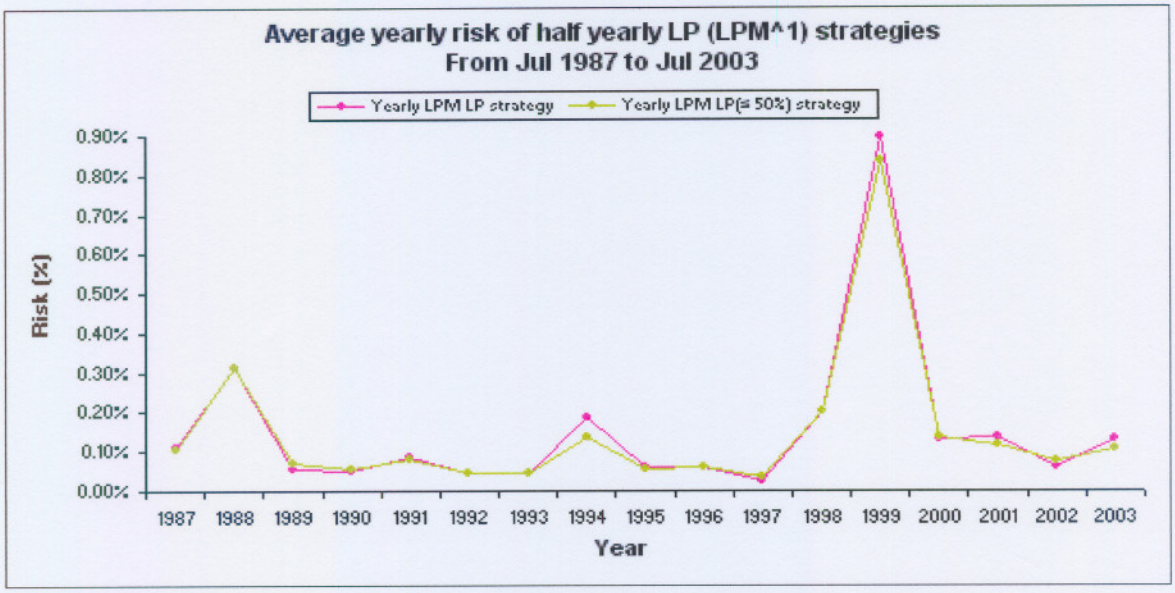


Graph 5.89: Expected shortfall for half yearly LP (LPM, n = 1) strategies

- **Variance**

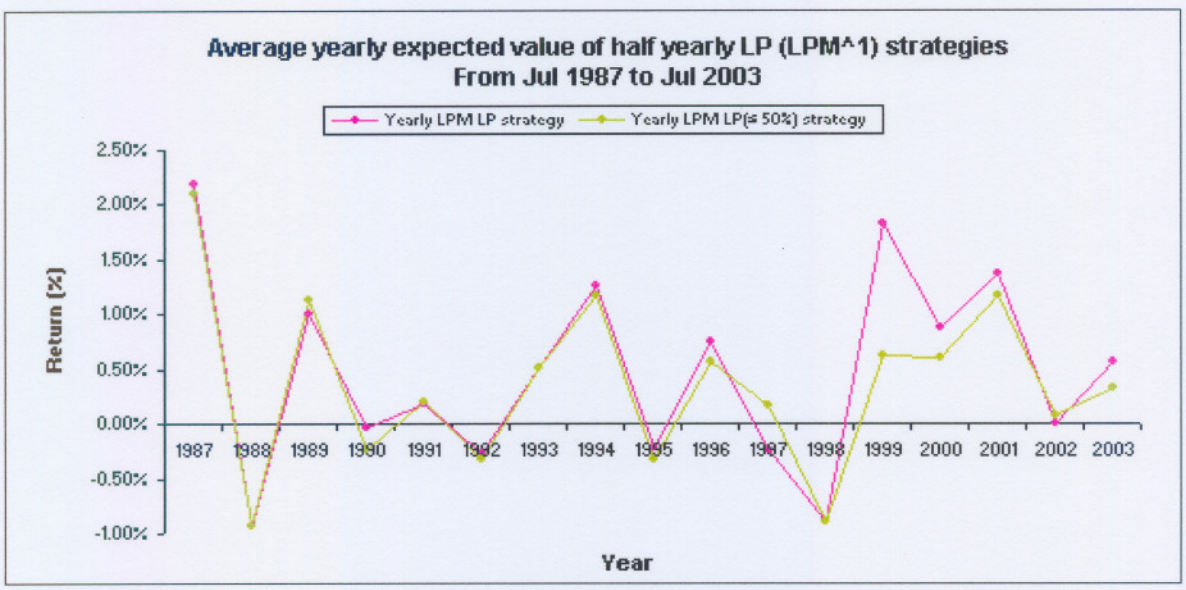
The following graph shows the comparison between the variance risk measures for the half yearly LP (LPM, n = 1) strategies. The half yearly LP (LPM) strategy 1 attained the maximum risk value of 0.89% in 1999 and the minimum risk value of 0.02% in 1997. Half yearly LP

(LPM) strategy 2 achieved the lowest yearly risk average of 0.1464%. The yearly risk average for half yearly LP (LPM) strategy 1 is 0.1534%.



Graph 5.90: Variance comparison between half yearly LP (LPM, n = 1) strategies

- **Average deviation**



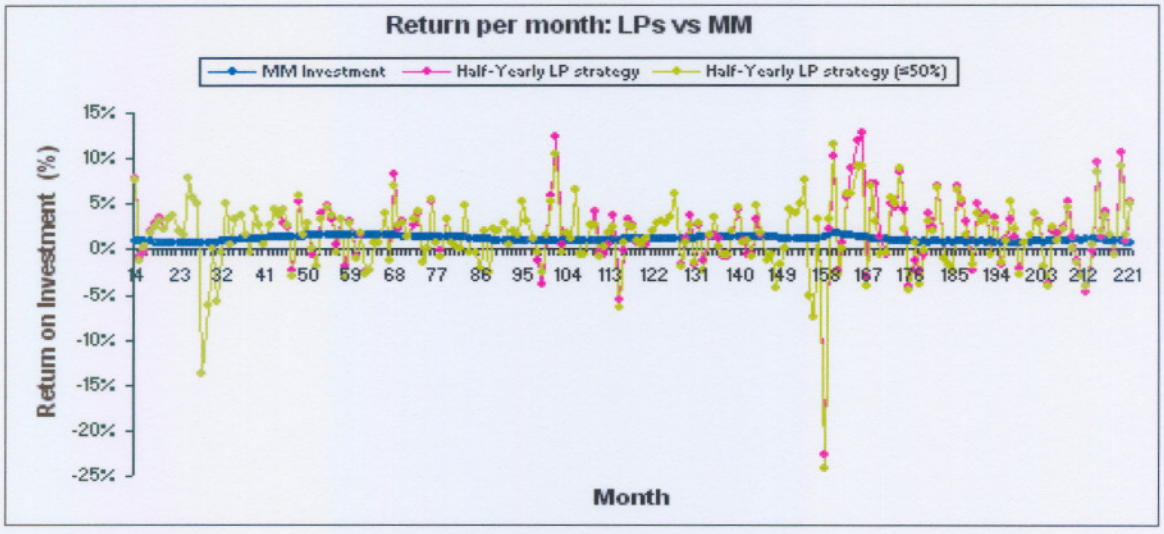
Graph 5.91: Expected value comparison of half yearly LP (LPM, n = 1) strategies

Graph 5.91 illustrates a comparison between the average deviation return measures for the half yearly LP (LPM) strategies. The half yearly LP (LPM) strategy 1 attained the highest value of 2.19 % in 1987 and both the strategies obtained the lowest value of –0.93% in 1988. Half yearly LP (LPM) strategy 2 achieved the lowest yearly average of 0.3491%. The yearly average for half yearly LP (LPM) strategy 1 is 0.4701%.

- **Return per month**

Graph 5.92 indicates the comparison between the monthly returns of the half yearly LP (LPM) strategies and the MM. The returns for the half yearly LP (LPM) strategies vary around the MM. P (X<MM) for the half yearly LP (LPM) strategies is as follows:

- Half yearly LP (LPM) strategy 1 → 41.15%.
- Half yearly LP (LPM) strategy 2 → 42.58%.



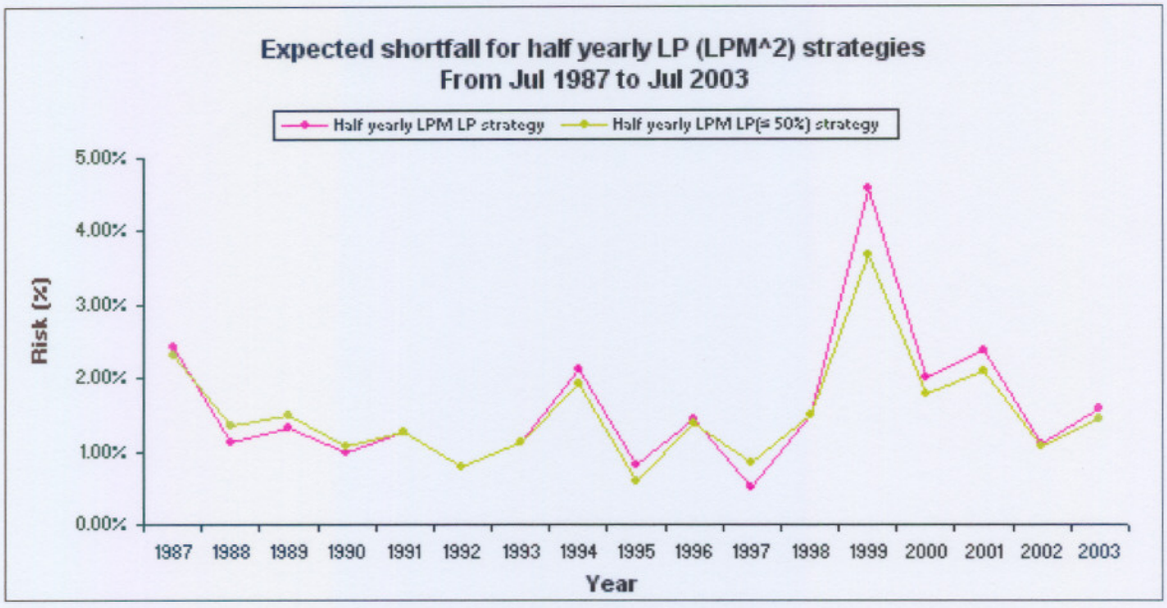
Graph 5.92: Return comparison between half yearly LP (LPM, n = 1) strategies and MM

5.3.14 Half yearly LP (LPM) strategies (n = 2)

- **Expected shortfall**

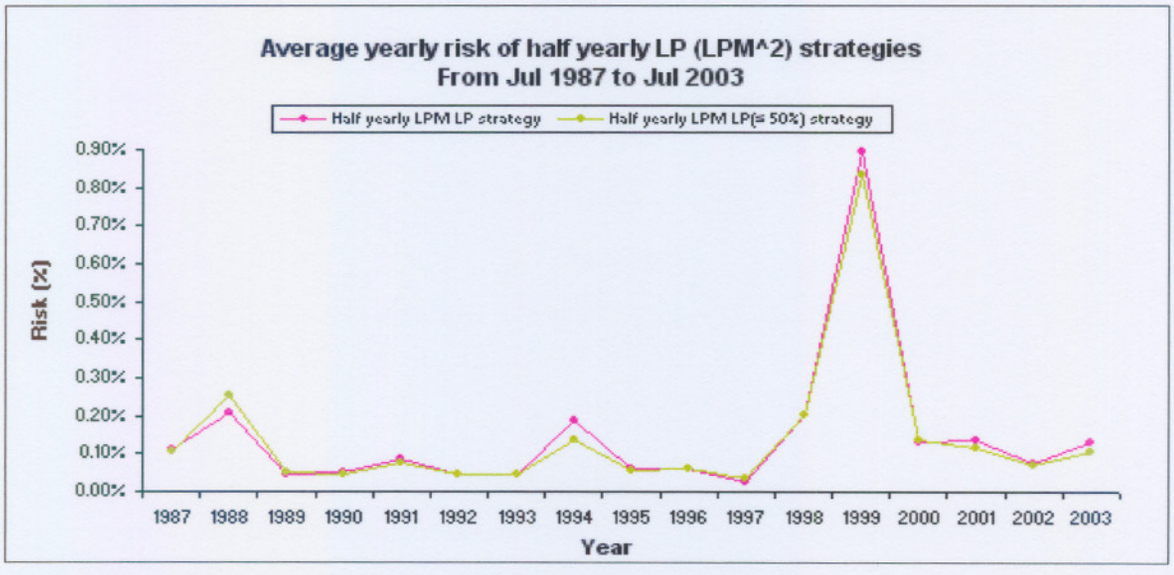
The following graph displays a comparison between the expected shortfall risk measures for the different half yearly LP (LPM) strategies. The half yearly LP (LPM) strategy 1 obtained

the highest risk value of 4.58% in 1999. The half yearly LP (LPM) strategy 1 also obtained the lowest risk value of 0.51% in 1997. The yearly risk averages for the half yearly LP (LPM) strategies are 1.5966% and 1.5165%, respectively.



Graph 5.93: Expected shortfall for half yearly LP (LPM, n = 2) strategies

- **Variance**

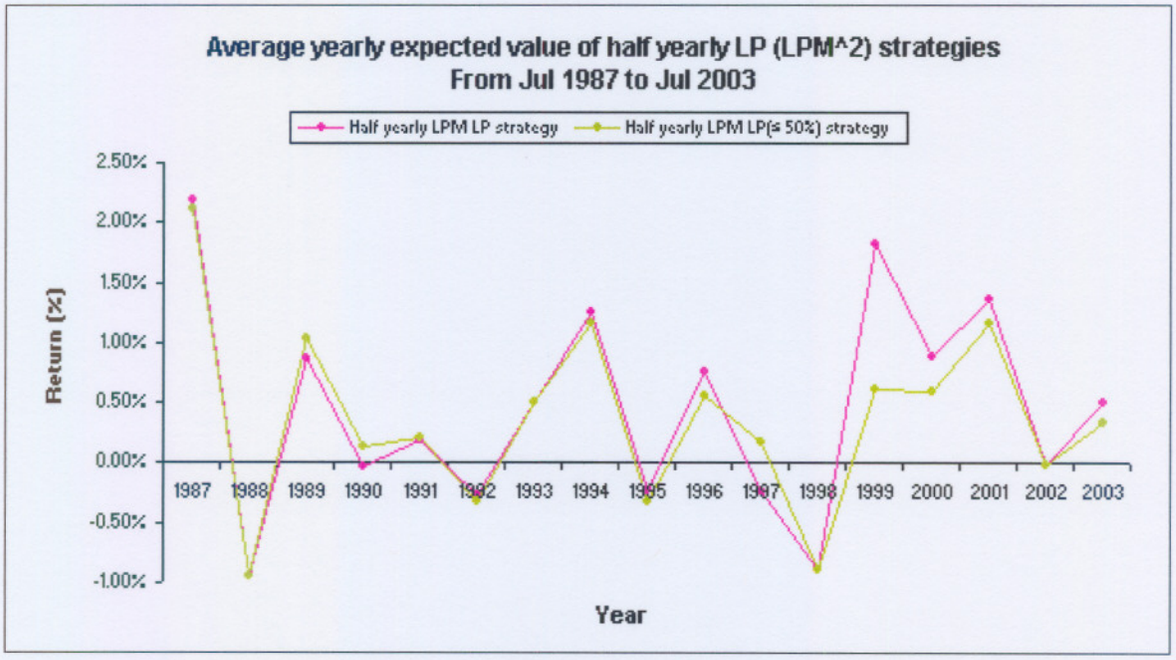


Graph 5.94: Variance comparison between half yearly LP (LPM, n = 2) strategies

Graph 5.94 shows the comparison between the variance risk measures for the half yearly LP (LPM, n = 2) strategies. The half yearly LP (LPM) strategy 1 attained the maximum risk value of 0.89% in 1999 and the minimum risk value of 0.02% in 1997. Half yearly LP (LPM) strategy 2 achieved the lowest yearly risk average of 0.1409%. The yearly risk average for half yearly LP (LPM) strategy 1 is 0.1472%.

- **Average deviation**

The following graph illustrates a comparison between the average deviation return measures for the half yearly LP (LPM) strategies. The half yearly LP (LPM) strategy 1 attained the highest value of 2.06 % in 1987 and both the strategies obtained the lowest value of –0.93% in 1988. Half yearly LP (LPM) strategy 2 achieved the lowest yearly average of 0.3600%. The yearly risk average for half yearly LP (LPM) strategy 1 is 0.4555%.

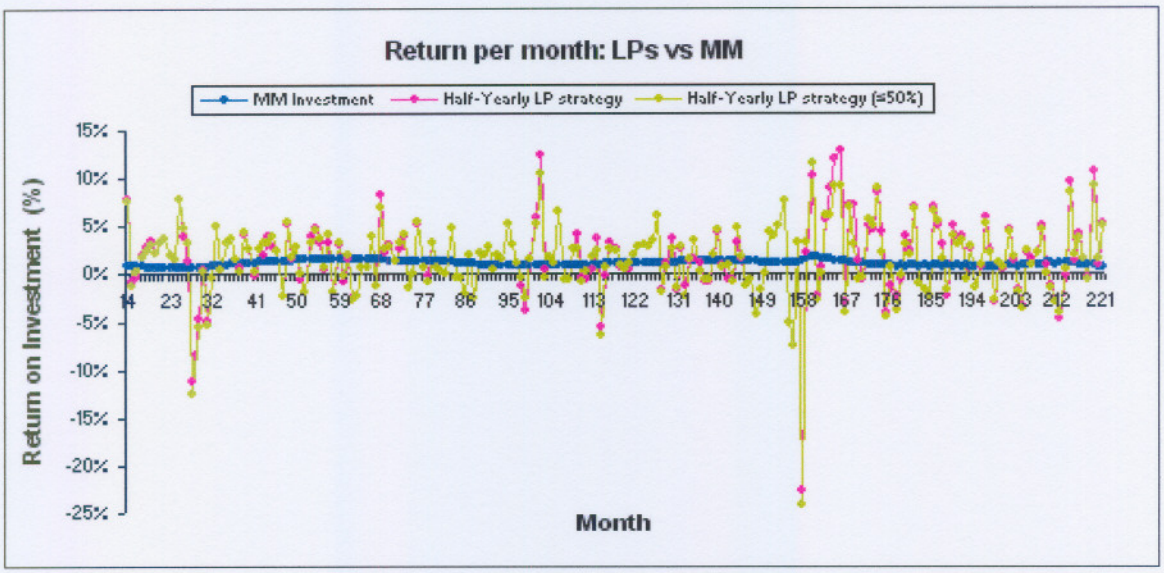


Graph 5.95: Expected value comparison of half yearly LP (LPM, n = 2) strategies

- **Return per month**

Graph 5.96 indicates the comparison between the monthly returns of the half yearly LP (LPM) strategies and the MM. The returns for the half yearly LP (LPM) strategies fluctuate around the MM. P (X<MM) for the half yearly LP (LPM) strategies is as follows:

- Half yearly LP (LPM) strategy 1 → 42.58%.
- Half yearly LP (LPM) strategy 2 → 43.06%.



Graph 5.96: Return comparison between half yearly LP (LPM, n = 2) strategies and MM

5.4 Summary: Decision making under uncertainty criteria

- **s and $\overline{X}$ as decision criteria**

Table 5.1 provides a summary for the monthly averages and the standard deviation for the decision making under uncertainty strategies.

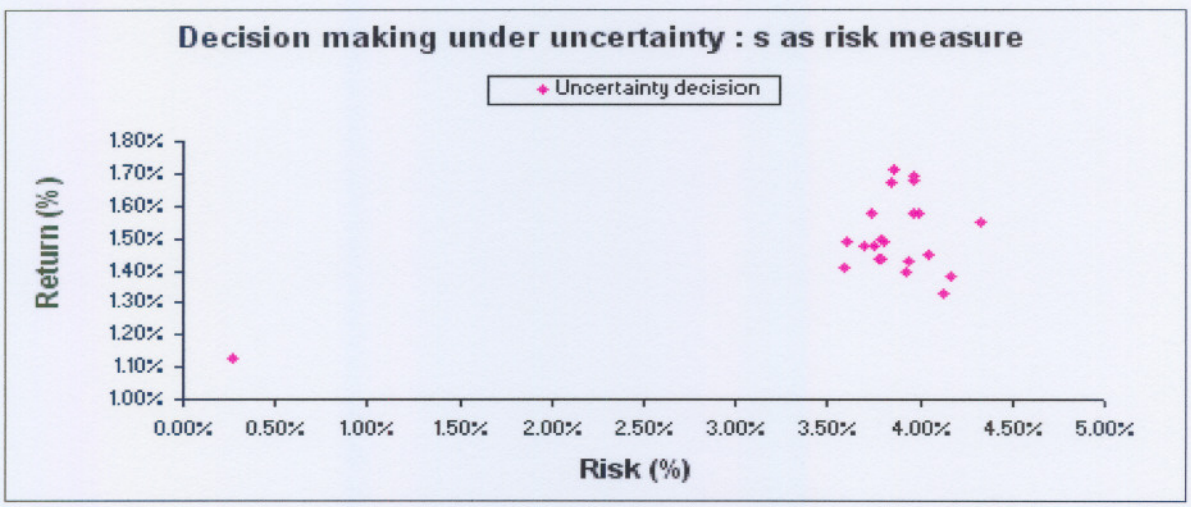
- If the average monthly return is used as criterion, then the objective will be to maximise the value, i.e. the strategy that has the highest value will be selected. The half yearly pessimistic Hurwicz criterion strategy (where α = 0.1) shows the highest value. Thus, this strategy will be selected as the optimal strategy for decision making under uncertainty, if this criterion was preferred.

- If the standard deviation (s in the table) is used as criterion, then the objective will be to minimise this risk value, i.e. the strategy that has the lowest value will be selected. The MM fund shows the lowest value. Thus, this strategy will be selected as the optimal strategy for decision making under uncertainty, if this criterion was chosen.

Strategy	$\overline{X}$	s
Money Market	1.13%	0.27%
Monthly min	1.49%	3.60%
Half yearly min	1.41%	3.59%
Yearly min	1.58%	3.73%
Monthly max	1.40%	3.93%
Half yearly max	1.68%	3.96%
Yearly max	1.58%	3.99%
Half yearly min/max	1.50%	3.79%
Yearly min/max	1.43%	3.94%
Half yearly Alternate: max/min	1.49%	3.80%
Yearly Alternate: max/min	1.44%	3.78%
Half yearly maximax	1.38%	4.17%
Yearly maximax	1.33%	4.13%
Half yearly maximin	1.69%	3.96%
Yearly maximin	1.48%	3.75%
Half yearly laplace	1.58%	3.97%
Yearly laplace	1.44%	3.79%
Half yearly minimax regret	1.67%	3.84%
Yearly minimax regret	1.48%	3.70%
Half yearly optimistic Hurwicz criterion	1.55%	4.33%
Yearly optimistic Hurwicz criterion	1.45%	4.04%
Half yearly pessimistic Hurwicz criterion	1.71%	3.86%
Yearly pessimistic Hurwicz criterion	1.48%	3.75%

Table 5.1: s *Decision criterion for decision making under uncertainty*

The following graph displays the risk (as measured by s) vs. the average monthly returns:



Graph 5.97: Risk (s risk measure) vs. return

The MM has the minimum risk, but it also has the lowest return value. All the other strategies have monthly averages of 1.33% and above, but these strategies have higher risk. From the graph we can see that if an investor e.g. specify that he require the risk to be 3.85% or less, the best strategy will be half yearly minimax regret strategy. The average is the highest for this option, and the risk restriction is satisfied. The half yearly pessimistic Hurwicz criterion won't be selected (although it has the highest average), since it does not satisfy the risk constraint, the risk for this option is 3.86%.

- **P(X<MM) as decision criterion**

Table 5.2 provides a summary for the monthly averages and the estimated probability that X will be less than MM for the decision making under uncertainty strategies.

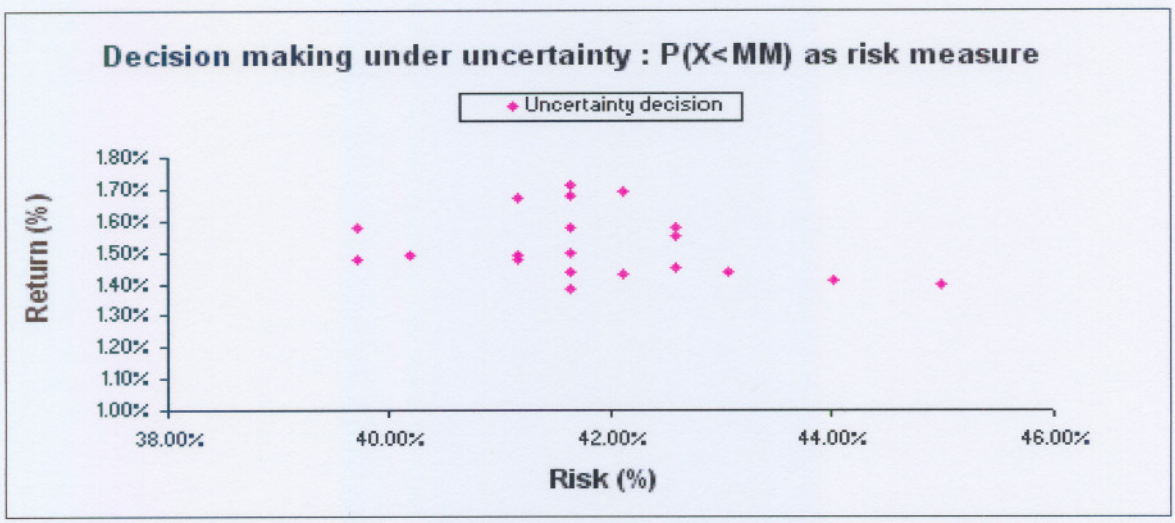
Strategy	$\overline{X}$	P(X<MM)
Money Market	1.13%	N/A
Monthly min	1.49%	40.19%
Half yearly min	1.41%	44.02%
Yearly min	1.58%	41.63%
Monthly max	1.40%	44.98%
Half yearly max	1.68%	41.63%
Yearly max	1.58%	39.71%
Half yearly min/max	1.50%	41.63%
Yearly min/max	1.43%	42.11%
Half yearly Alternate: max/min	1.49%	41.15%
Yearly Alternate: max/min	1.44%	43.06%
Half yearly maximax	1.38%	41.63%
Yearly maximax	1.33%	46.41%
Half yearly maximin	1.69%	42.11%
Yearly maximin	1.48%	39.71%
Half yearly laplace	1.58%	42.58%
Yearly laplace	1.44%	41.63%
Half yearly minimax regret	1.67%	41.15%
Yearly minimax regret	1.48%	41.15%
Half yearly optimistic Hurwicz criterion	1.55%	42.58%
Yearly optimistic Hurwicz criterion	1.45%	42.58%
Half yearly pessimistic Hurwicz criterion	1.71%	41.63%
Yearly pessimistic Hurwicz criterion	1.48%	39.71%

Table 5.2: P(X<MM) as decision criterion for decision making under uncertainty

If the probability that the returns is less than the MM (Estimated probability ⇒ P(X<MM)), is used as criterion, then the objective will be to minimise this risk value (the strategy with has the lowest value will be selected). Three strategies showed the same minimum value for this criterion:

- Yearly max strategy
- Yearly maximin strategy
- Yearly pessimistic Hurwicz criterion strategy

If the average highest average return is also taken into consideration, then the yearly max strategy will be selected as the optimal strategy for decision-making under uncertainty (if this criterion was chosen). This is illustrated in the following graph that displays the risk vs. the average monthly returns for all the strategies for this criterion.



*Graph 5.98: Risk (*P(X<MM) *risk measure) vs. return*

- **Expected shortfall as decision criterion**

Table 5.3 provides a summary for the monthly averages and the expected shortfall for the decision making under uncertainty strategies. Graph 5.99 displays the risk vs. the average monthly returns for all the strategies for the expected shortfall criterion.

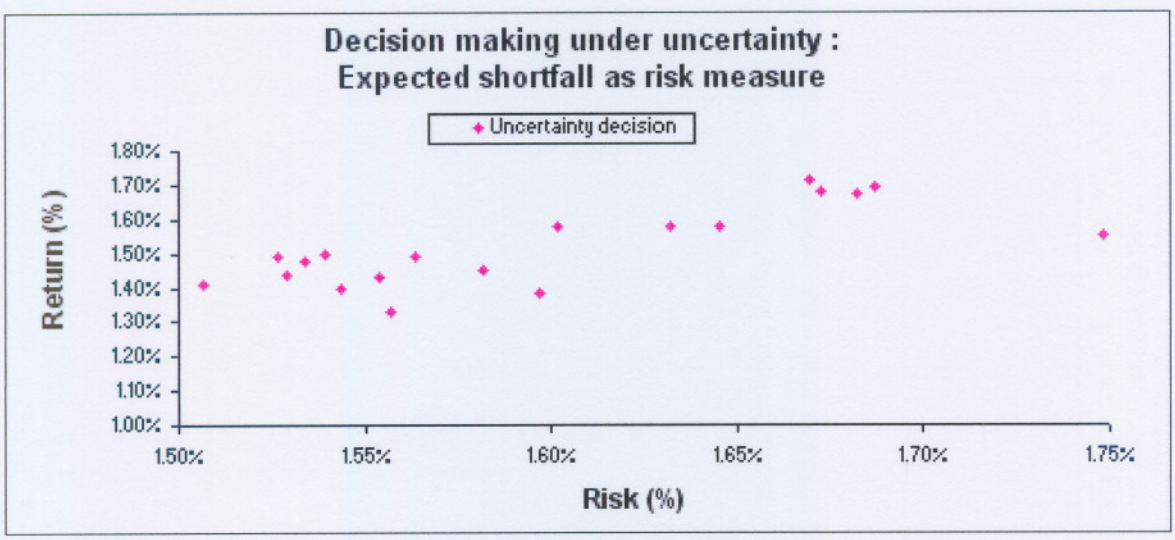
If the expected shortfall is used, as criterion, then the objective will be to minimise this risk value. The yearly laplace strategy shows the lowest value. Thus, this strategy will be

selected as the optimal strategy for decision making under uncertainty, if this criterion was chosen.

Strategy	$\overline{X}$	Expected shortfall
Money Market	1.13%	N/A
Monthly min	1.49%	1.5268%
Half yearly min	1.41%	1.5064%
Yearly min	1.58%	1.6320%
Monthly max	1.40%	1.5435%
Half yearly max	1.68%	1.6725%
Yearly max	1.58%	1.6019%
Half yearly min/max	1.50%	1.5392%
Yearly min/max	1.43%	1.5537%
Half yearly Alternate: max/min	1.49%	1.5633%
Yearly Alternate: max/min	1.44%	1.5290%
Half yearly maximax	1.38%	1.5970%
Yearly maximax	1.33%	1.5571%
Half yearly maximin	1.69%	1.6873%
Yearly maximin	1.48%	1.4681%
Half yearly laplace	1.58%	1.6455%
Yearly laplace	1.44%	1.1581%
Half yearly minimax regret	1.67%	1.6823%
Yearly minimax regret	1.48%	1.5338%
Half yearly optimistic Hurwicz criterion	1.55%	1.7480%
Yearly optimistic Hurwicz criterion	1.45%	1.5815%
Half yearly pessimistic Hurwicz criterion	1.71%	1.6693%
Yearly pessimistic Hurwicz criterion	1.48%	1.4622%

This strategy shows a very high risk, but the return are below the 1.60% target. The half yearly pessimistic Hurwicz criterion strategy, shows the highest return and the risk is high, but not as high as for some of the other strategies with lower risk.

Table 5.3: Expected shortfall as decision criterion for decision making under uncertainty



Graph 5.99: Risk (expected shortfall as risk measure) vs. return

- **Average deviation and variance as decision criteria**

Table 5.4 provides a summary for the average deviation and the variance values for the decision making under uncertainty strategies. Graph 5.100 displays the risk vs. the average monthly returns for all the strategies for these criteria.

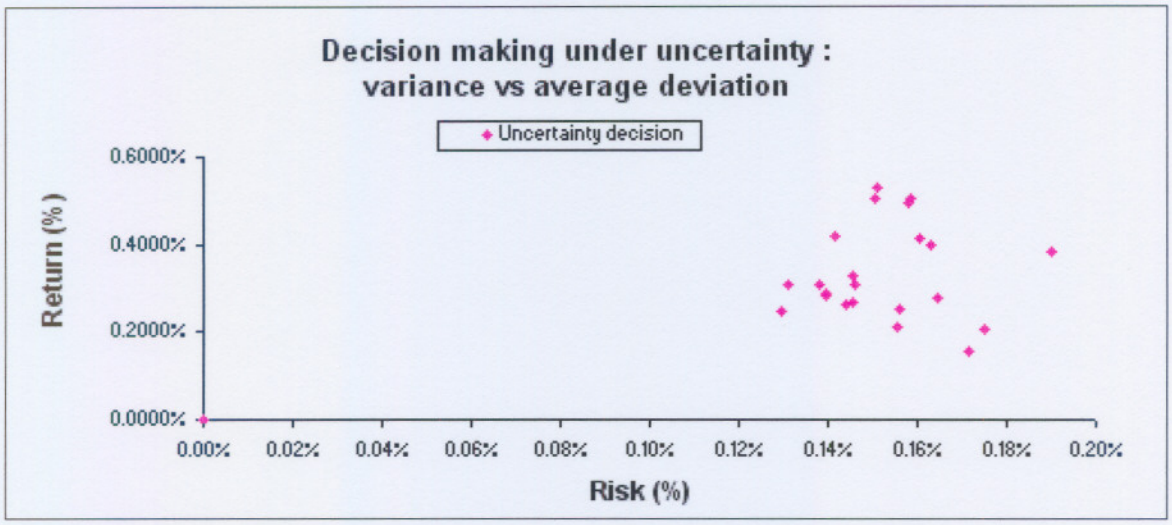
Strategy	Variance	Average deviation
Money Market	0.0000%	0.0000%
Monthly min	0.1312%	0.3090%
Half yearly min	0.1296%	0.2485%
Yearly min	0.1416%	0.4198%
Monthly max	0.1553%	0.2113%
Half yearly max	0.1580%	0.4926%
Yearly max	0.1631%	0.3975%
Half yearly min/max	0.1453%	0.3271%
Yearly min/max	0.1558%	0.2499%
Half yearly Alternate: max/min	0.1461%	0.3098%
Yearly Alternate: max/min	0.1441%	0.2611%
Half yearly maximax	0.1748%	0.2067%
Yearly maximax	0.1715%	0.1545%
Half yearly maximin	0.1583%	0.5031%
Yearly maximin	0.1397%	0.2885%
Half yearly laplace	0.1604%	0.4112%
Yearly laplace	0.1454%	0.2697%
Half yearly minimax regret	0.1507%	0.5062%
Yearly minimax regret	0.1380%	0.3090%
Half yearly optimistic Hurwicz criterion	0.1901%	0.3819%
Yearly optimistic Hurwicz criterion	0.1647%	0.2774%
Half yearly pessimistic Hurwicz criterion	0.1508%	0.5287%
Yearly pessimistic Hurwicz criterion	0.1395%	0.2842%

Table 5.4: Average deviation and variance as decision criteria for decision making under uncertainty

- If the variance: $\frac{\sum (X - MM)^2}{n}$ is used, as criterion, then the objective will be to minimise this risk value. The MM fund shows the lowest value. Thus, this strategy will be selected as the optimal strategy for decision making under uncertainty, if this criterion was chosen.

- If the average deviation: $\overline{X} - \overline{MM}$ is used, as criterion, then the objective will be to maximise this return measure. The half yearly pessimistic Hurwicz criterion strategy

shows the highest value. Thus, this strategy will be selected as the optimal strategy for decision making under uncertainty, if this criterion was preferred.



Graph 5.100: Risk (variance) vs. return (average deviation)

5.5 Summary: Decision making under risk criteria

- **s and $\overline{X}$ as decision criteria**

Table 5.5 provides a summary for the monthly averages and the standard deviation for the decision making under risk strategies.

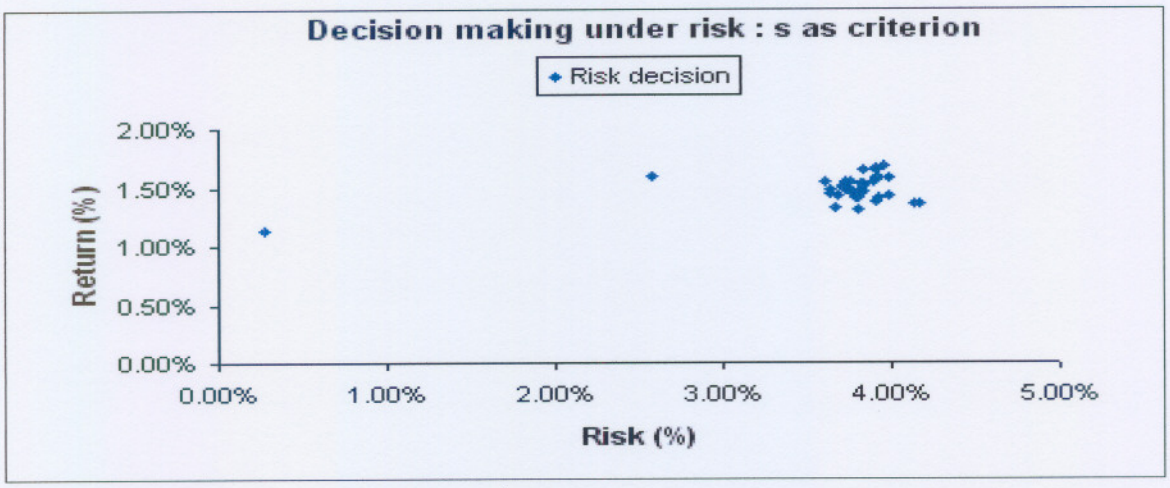
- If the average monthly return is used as criterion, then the objective will be to maximise the value, i.e. the strategy that has the highest value will be selected. Half yearly LP (β) strategies 3 and 4 show the highest values, i.e. 1.68%. These strategies will be selected as the optimal strategies for decision making under uncertainty, if this criterion was preferred.

- If the standard deviation (s in the table) is used as criterion, then the objective will be to minimise this risk value, i.e. the strategy that has the lowest value will be selected. The MM fund shows the lowest value. Thus, this strategy will be selected as the optimal strategy for decision making under uncertainty, if this criterion was chosen.

Strategy	$\overline{X}$	s
Money Market	1.13%	0.27%
Monthly historical probability distribution	1.54%	3.60%
Half yearly historical probability distribution	1.57%	3.92%
Yearly historical probability distribution	1.31%	3.81%
Monthly even probability	1.45%	3.84%
Half yearly even probability	1.55%	3.73%
Yearly even probability	1.35%	4.16%
Monthly previous year probability distribution	1.35%	4.14%
Half yearly previous year probability distribution	1.40%	3.80%
Yearly previous year probability distribution	1.40%	3.93%
Diversify	1.40%	3.79%
Max probability	1.37%	3.91%
Max probability (with MM)	1.60%	2.58%
Min probability	1.33%	3.66%
Yearly Markowitz 1	1.45%	3.80%
Yearly Markowitz 2	1.48%	3.64%
Half yearly Markowitz 1	1.51%	3.71%
Half yearly Markowitz 2	1.50%	3.74%
Yearly LP (β) 1	1.51%	3.83%
Yearly LP (β) 2	1.44%	3.64%
Yearly LP (β) 3	1.58%	3.98%
Yearly LP (β) 4	1.58%	3.99%
Half yearly LP (β) 1	1.53%	3.82%
Half yearly LP (β) 2	1.42%	3.68%
Half yearly LP (β) 3	1.68%	3.96%
Half yearly LP (β) 4	1.68%	3.96%
Yearly LP (LPM) 1, n=0.5	1.57%	3.91%
Yearly LP (LPM) 2, n=0.5	1.50%	3.71%
Yearly LP (LPM) 1, n=1	1.54%	3.90%
Yearly LP (LPM) 2, n=1	1.47%	3.72%
Yearly LP (LPM) 1, n=2	1.43%	3.99%
Yearly LP (LPM) 2, n=2	1.46%	3.76%
Half yearly LP (LPM) 1, n=0.5	1.65%	3.91%
Half yearly LP (LPM) 2, n=0.5	1.52%	3.83%
Half yearly LP (LPM) 1, n=1	1.66%	3.91%
Half yearly LP (LPM) 2, n=1	1.53%	3.82%
Half yearly LP (LPM) 1, n=2	1.64%	3.83%
Half yearly LP (LPM) 2, n=2	1.54%	3.75%

Table 5.5: s *Decision criterion for decision making under risk*

The following graph displays the risk (as measured by s) vs. the average monthly returns:



Graph 5.101: Risk (s risk measure) vs. return

The MM has the minimum risk, but it also has the lowest return value. All the other strategies have monthly averages of 1.31% and above, but these strategies have higher risk.

The max probability (with MM) is good strategy because it has a low risk value (2.58%) compared to the other strategies, and the target return of 1.60% per month is met.

- **P(X<MM) as decision criterion**

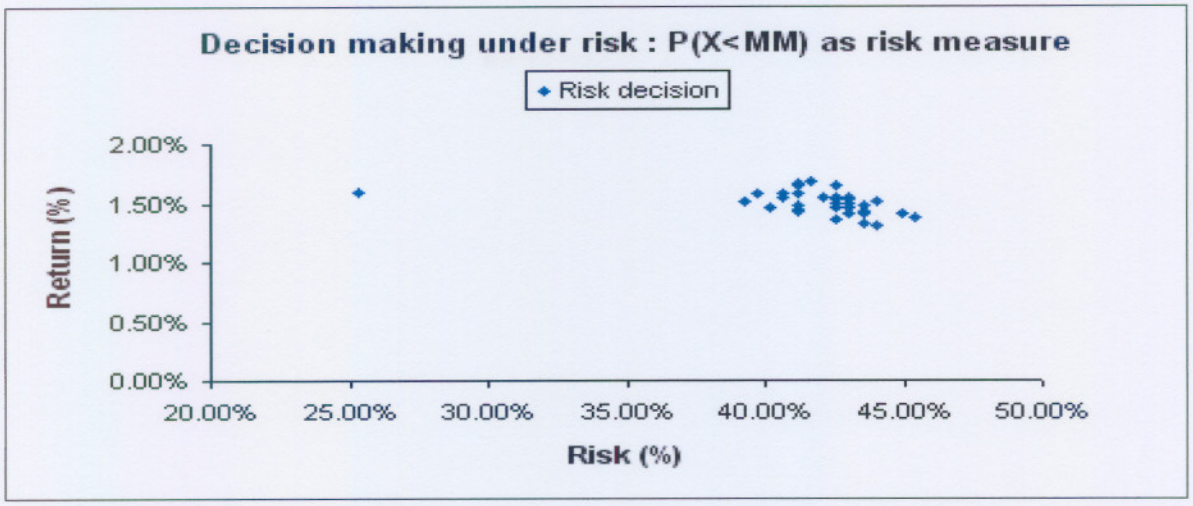
Table 5.6 provides a summary for the monthly averages and the estimated probability that X will be less than MM for the decision making under risk strategies.

Strategy	$\overline{X}$	P(X<MM)
Money Market	1.13%	N/A
Monthly historical probability distribution	1.54%	42.58%
Half yearly historical probability distribution	1.57%	41.15%
Yearly historical probability distribution	1.31%	44.02%
Monthly even probability	1.45%	43.06%
Half yearly even probability	1.55%	42.11%
Yearly even probability	1.35%	42.58%
Monthly previous year probability distribution	1.35%	42.58%
Half yearly previous year probability distribution	1.40%	43.54%
Yearly previous year probability distribution	1.40%	44.98%
Diversify	1.40%	43.06%
Max probability	1.37%	45.45%
Max probability (with MM)	1.60%	25.36%
Min probability	1.33%	43.54%

Yearly Markowitz 1	1.45%	40.19%
Yearly Markowitz 2	1.48%	41.15%
Half yearly Markowitz 1	1.51%	44.02%
Half yearly Markowitz 2	1.50%	43.06%
Yearly LP (β) 1	1.51%	39.23%
Yearly LP (β) 2	1.44%	41.15%
Yearly LP (β) 3	1.58%	39.71%
Yearly LP (β) 4	1.58%	39.71%
Half yearly LP (β) 1	1.53%	43.06%
Half yearly LP (β) 2	1.42%	43.54%
Half yearly LP (β) 3	1.68%	41.63%
Half yearly LP (β) 4	1.68%	41.63%
Yearly LP (LPM) 1, n=0.5	1.57%	40.67%
Yearly LP (LPM) 2, n=0.5	1.50%	42.58%
Yearly LP (LPM) 1, n=1	1.54%	40.67%
Yearly LP (LPM) 2, n=1	1.47%	43.54%
Yearly LP (LPM) 1, n=2	1.43%	41.15%
Yearly LP (LPM) 2, n=2	1.46%	42.58%
Half yearly LP (LPM) 1, n=0.5	1.65%	41.15%
Half yearly LP (LPM) 2, n=0.5	1.52%	42.58%
Half yearly LP (LPM) 1, n=1	1.66%	41.15%
Half yearly LP (LPM) 2, n=1	1.53%	42.58%
Half yearly LP (LPM) 1, n=2	1.64%	42.58%
Half yearly LP (LPM) 2, n=2	1.54%	43.06%

Table 5.6: P(X<MM) as decision criterion for decision making under risk

If the probability that the returns is less than the MM (Estimated probability $\Rightarrow$ P(X<MM)), is used as criterion, then the objective will be to minimise this risk value (the strategy with has the lowest value will be selected). The max probability (with MM) will be selected; this strategy shows a minimum value of 25.36%. This is very low, compared to the other strategies, the target return of 1.60% per month is also achieved. This is illustrated in the following graph that displays the risk vs. the average monthly returns for all the strategies for this criterion.



Graph 5.102: Risk (P(X<MM) risk measure) vs. return

- **Expected shortfall as decision criterion**

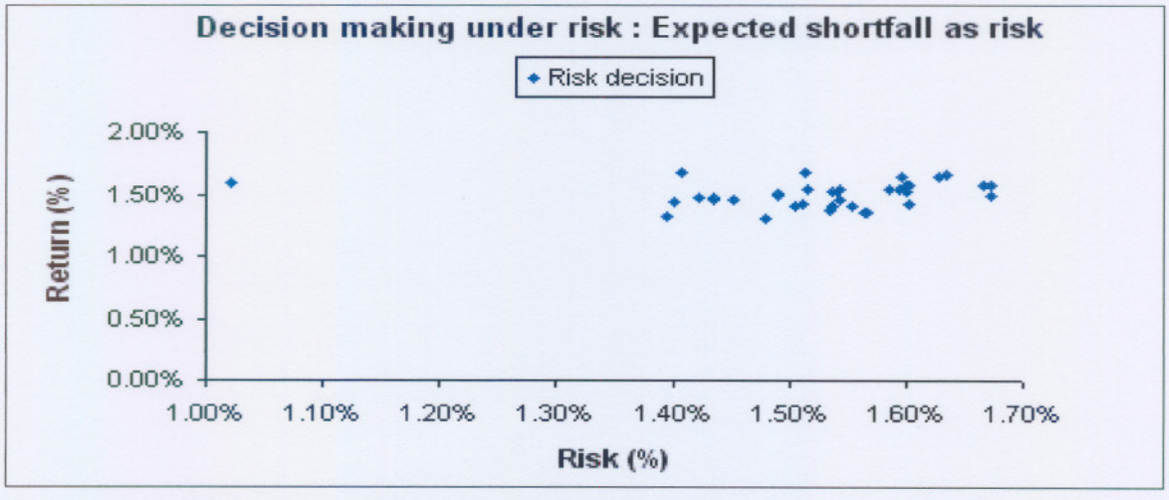
Table 5.7 provides a summary for the monthly averages and the expected shortfall for the decision making under risk strategies. Graph 5.103 displays the risk vs. the average monthly returns for all the strategies for the expected shortfall criterion.

If the expected shortfall is used, as criterion, then the objective will be to minimise this risk value. The probability (with MM) strategy shows the lowest value. Thus, this strategy will be selected as the optimal strategy for decision making under uncertainty, if this criterion was used.

Strategy	$\overline{X}$	Expected Shortfall
Money Market	1.13%	N/A
Monthly historical probability distribution	1.54%	1.5861%
Half yearly historical probability distribution	1.57%	1.6662%
Yearly historical probability distribution	1.31%	1.4799%
Monthly even probability	1.45%	1.5426%
Half yearly even probability	1.55%	1.5930%
Yearly even probability	1.35%	1.5633%
Monthly previous year probability distribution	1.35%	1.5661%
Half yearly previous year probability distribution	1.40%	1.5360%
Yearly previous year probability distribution	1.40%	1.5543%
Diversify	1.40%	1.5054%
Max probability	1.37%	1.5340%
Max probability (with MM)	1.60%	1.0208%

Min probability	1.33%	1.3943%
Yearly Markowitz 1	1.45%	1.4349%
Yearly Markowitz 2	1.48%	1.4213%
Half yearly Markowitz 1	1.51%	1.4909%
Half yearly Markowitz 2	1.50%	1.4901%
Yearly LP (β) 1	1.51%	1.4893%
Yearly LP (β) 2	1.44%	1.4006%
Yearly LP (β) 3	1.58%	1.5996%
Yearly LP (β) 4	1.58%	1.6019%
Half yearly LP (β) 1	1.53%	1.5996%
Half yearly LP (β) 2	1.42%	1.6019%
Half yearly LP (β) 3	1.68%	1.5124%
Half yearly LP (β) 4	1.68%	1.4075%
Yearly LP (LPM) 1, n=0.5	1.57%	1.6728%
Yearly LP (LPM) 2, n=0.5	1.50%	1.6725%
Yearly LP (LPM) 1, n=1	1.54%	1.5421%
Yearly LP (LPM) 2, n=1	1.47%	1.4355%
Yearly LP (LPM) 1, n=2	1.43%	1.5108%
Yearly LP (LPM) 2, n=2	1.46%	1.4509%
Half yearly LP (LPM) 1, n=0.5	1.65%	1.6281%
Half yearly LP (LPM) 2, n=0.5	1.52%	1.5357%
Half yearly LP (LPM) 1, n=1	1.66%	1.6353%
Half yearly LP (LPM) 2, n=1	1.53%	1.5410%
Half yearly LP (LPM) 1, n=2	1.64%	1.5966%
Half yearly LP (LPM) 2, n=2	1.54%	1.5165%

Table 5.7: Expected shortfall as decision criterion for decision making under risk



Graph 5.103: Risk (expected shortfall as risk measure) vs. return

- **Average deviation and variance as decision criteria**

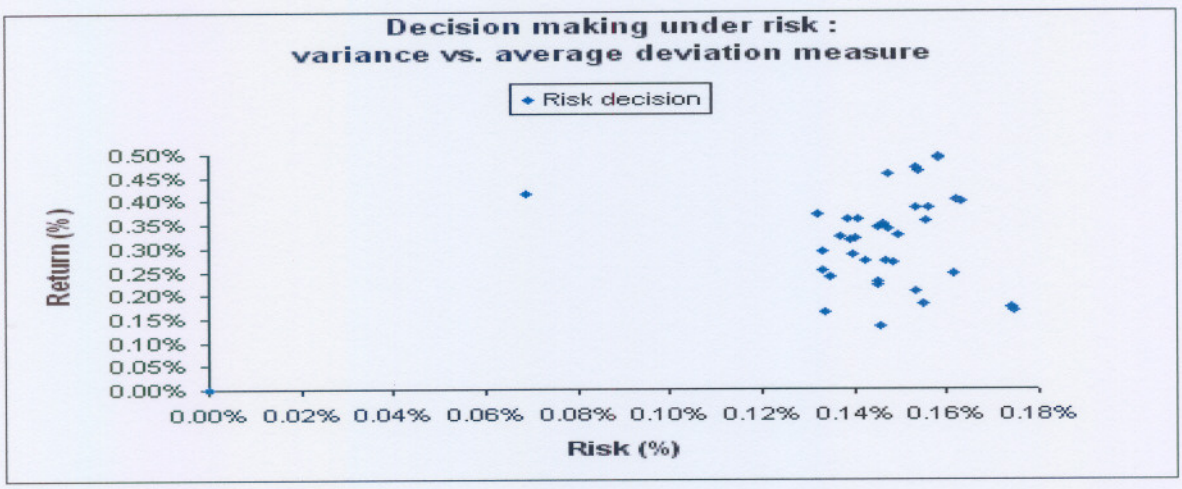
Table 5.8 provides a summary for the average deviation and the variance values for the decision making under risk strategies. Graph 5.104 displays the risk vs. the average monthly returns for all the strategies for these criteria.

Strategy	Variance	Average deviation
Money Market	0.0000%	0.0000%
Monthly historical probability distribution	0.1322%	0.3694%
Half yearly historical probability distribution	0.1535%	0.3839%
Yearly historical probability distribution	0.1458%	0.1335%
Monthly even probability	0.1483%	0.2687%
Half yearly even probability	0.1386%	0.3622%
Yearly even probability	0.1744%	0.1680%
Monthly previous year probability distribution	0.1740%	0.1744%
Half yearly previous year probability distribution	0.1453%	0.2282%
Yearly previous year probability distribution	0.1531%	0.2085%
Diversify	0.1453%	0.2215%
Max probability	0.1548%	0.1792%
Max probability (with MM)	0.0687%	0.4160%
Min probability	0.1335%	0.1625%
Yearly Markowitz 1	0.1466%	0.2707%
Yearly Markowitz 2	0.1331%	0.2920%
Half yearly Markowitz 1	0.1367%	0.3224%
Half yearly Markowitz 2	0.1401%	0.3183%
Yearly LP (β) 1	0.1496%	0.3256%
Yearly LP (β) 2	0.1332%	0.2509%
Yearly LP (β) 3	0.1622%	0.3997%
Yearly LP (β) 4	0.1631%	0.3975%
Half yearly LP (β) 1	0.1449%	0.3434%
Half yearly LP (β) 2	0.1349%	0.2394%
Half yearly LP (β) 3	0.1581%	0.4899%
Half yearly LP (β) 4	0.1580%	0.4926%
Yearly LP (LPM) 1, n=0.5	0.1562%	0.3854%
Yearly LP (LPM) 2, n=0.5	0.1390%	0.3171%
Yearly LP (LPM) 1, n=1	0.1557%	0.3559%
Yearly LP (LPM) 2, n=1	0.1396%	0.2865%
Yearly LP (LPM) 1, n=2	0.1612%	0.2447%
Yearly LP (LPM) 2, n=2	0.1421%	0.2707%
Half yearly LP (LPM) 1, n=0.5	0.1537%	0.4630%
Half yearly LP (LPM) 2, n=0.5	0.1471%	0.3410%
Half yearly LP (LPM) 1, n=1	0.1534%	0.4701%
Half yearly LP (LPM) 2, n=1	0.1464%	0.3491%
Half yearly LP (LPM) 1, n=2	0.1472%	0.4555%
Half yearly LP (LPM) 2, n=2	0.1409%	0.3600%

Table 5.8: Average deviation and variance as decision criteria for decision making under risk

- If the variance: $\frac{\sum (X - MM)^2}{n}$ is used, as criterion, then the objective will be to minimise this risk value. The MM fund shows the lowest value. Thus, this strategy will be selected as the optimal strategy for decision making under uncertainty, if this criterion was preferred.

- If the average deviation: $\overline{X} - \overline{MM}$ is used, as criterion, then the objective will be to maximise this return measure. The half yearly LP(β) strategy 4, where return is maximised and the β-value is set to be no more than 1.5, shows the highest value. Thus, this strategy will be selected as the optimal strategy for decision making under uncertainty, if this criterion was preferred.



Graph 5.104: Risk (variance) vs. return (average deviation)

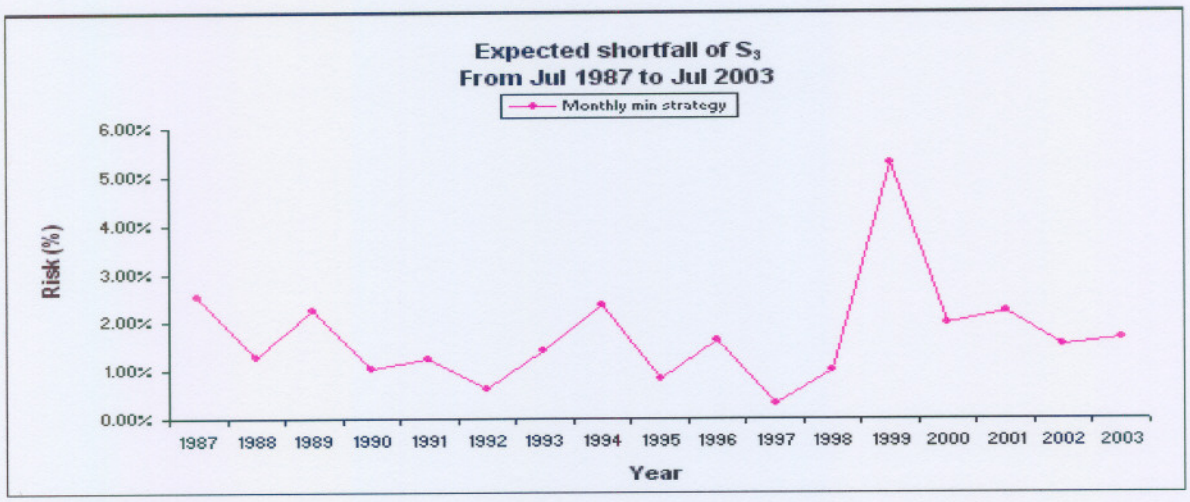
Again, the max probability (with MM) is a good strategy, it meets the 1.60% target and it has much lower variance than the other strategies.

5.6 Fund S_3

S_3 performed the best for the individual funds, thus the risk measures presented in this chapter is also applied to this fund.

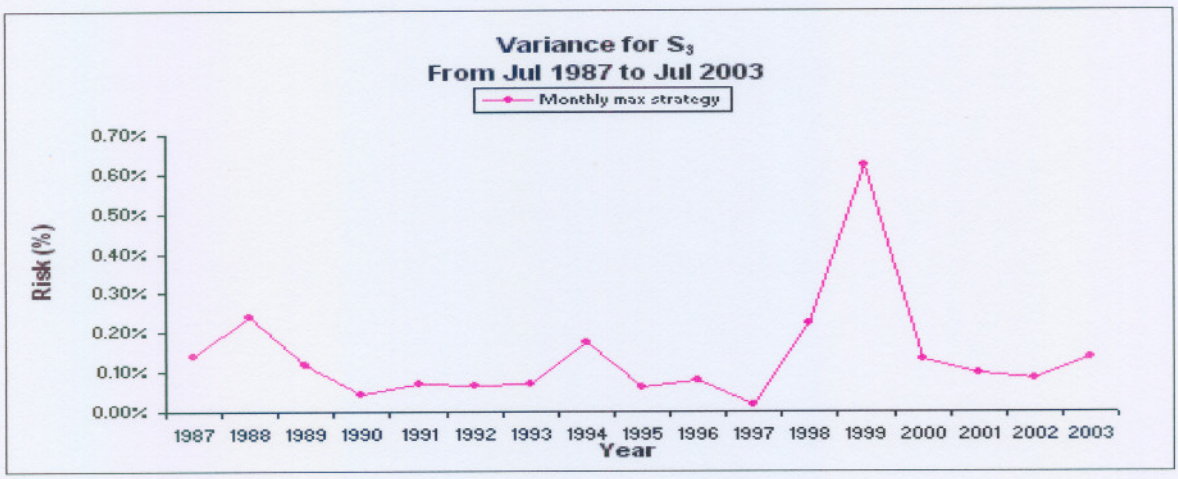
- **Expected shortfall**

The following graph displays the expected shortfall risk measure for the S_3 fund. The fund obtained the highest risk value of 5.28% in 1999. The lowest risk value of 0.33% was attained in 1997. The yearly risk average for the fund is 1.7220%.



Graph 5.105: Expected shortfall for fund S_3

- **Variance**

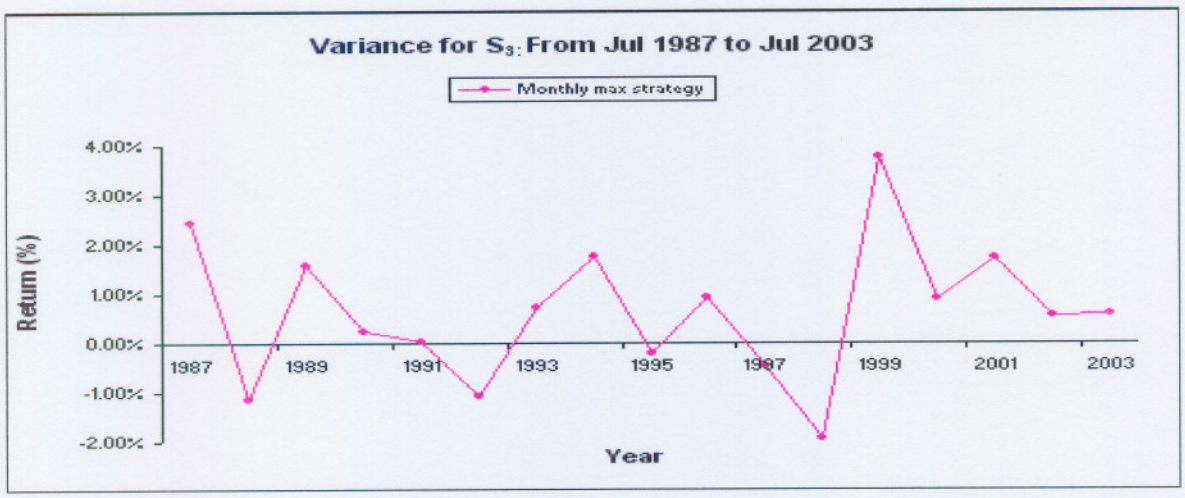


Graph 5.106: Variance for fund S_3

Graph 5.106 shows the variance risk measures for S_3. The fund obtained the maximum risk value of 0.62% in 1999 and the minimum risk value of 0.02% in 1997. The yearly risk average is 0.1397%.

- **Average deviation**

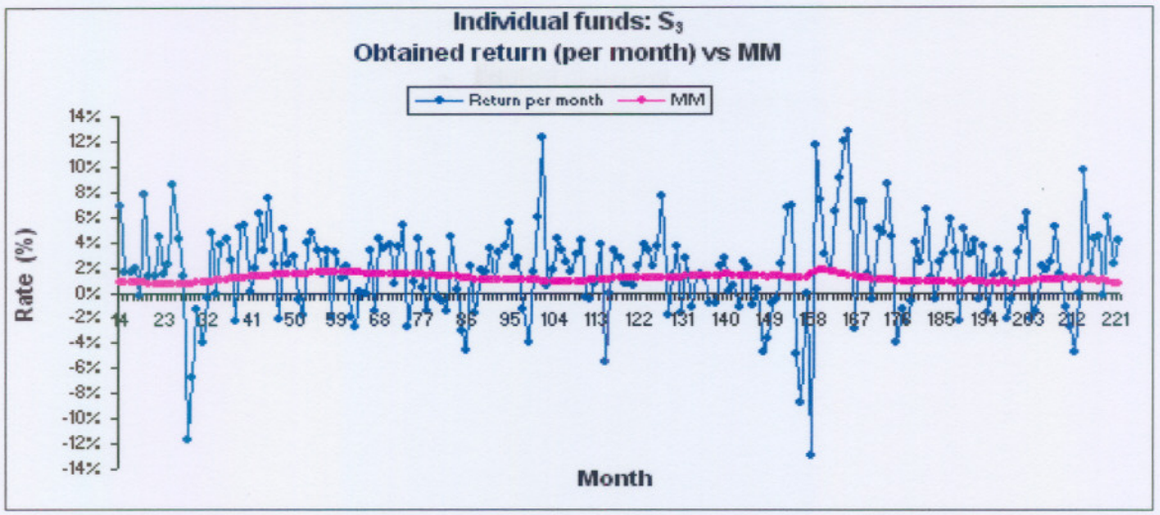
The following graph illustrates the average deviation return measure for S_3. The fund attained the highest value of 3.74 % in 1999 and the lowest value of —1.92% in 1988. The yearly return average is 0.6110%.



Graph 5.107: Expected value of fund S_3

- **Return per month**

Graph 5.108 indicates the comparison between the monthly returns of S_3 and the MM. The returns for S_3 fluctuate around the MM. P (X<MM) for S_3 is 39.23%. The standard deviation is 3.68%. The monthly average return is 1.79%, and a monthly effective interest rate of 1.72%.



Graph 5.108: Return comparison between S_3 and MM

5.7 Conclusion

This chapter examined some risk measures for the two decision-making approaches that were followed, namely decision making under risk and decision making under uncertainty.

The decision criteria depends on the individual investor, whether he wants to minimise risk or be more aggressive and choose to maximise return, subject to some risk constraint. The investment strategies that had a high monthly average showed the largest investment growth.

The strategy that performed the best overall was the half yearly pessimistic Hurwicz criterion, a decision-making strategy for decision making under uncertainty. This strategy had the highest average monthly return of 1.71%, and an effective interest rate of 1.64% per month. For investment in the individual funds, is S_3.

The max probability (with MM) strategy that chooses from MM or the dividend funds was a good strategy, with several of the criteria, because of its low risk. But this strategy also meets the 1.60% monthly target, on average.

MM has the lowest risk, but this option also has the lowest return from all the strategies.

CHAPTER 6
CONCLUSIONS AND RECOMMENDATIONS

6.1 Summary

A study was done in search for an effective solution to the portfolio selection problem, for different measures of risk. The overall objective of the study was to compare different investment strategies over different time horizons, in order to find the optimal strategy. A number of risk measures were used as criteria for selecting the optimal strategy (the risk were minimised). The maximum average return (i.e. maximise reward) was also employed as a criterion for selecting the best strategy.

Chapter two provided the background of this study. This chapter presented a literature review on risk. The following topics were discussed:

- Definition of risk.
- Risk classifications.
- Sources of risk.
- Risk measures.
- Risk management and techniques for risk management.

Chapter three looked at perfect information (what could the maximum outcome have been, if we knew the future) and investment in the individual funds. This chapter also presented the results of strategies for decision making under uncertainty. These models included:

- **Min strategy**

Monthly → invest in the fund that obtained the smallest return in the previous month.
Half yearly → invest in the fund that obtained the smallest average return over the past six months, for the next six months.

Yearly → invest in the fund that obtained the smallest average return over the past year, for the next year.

- **Max strategy**

Monthly → invest in the fund that obtained the largest return in the previous month.
Half yearly → invest in the fund that obtained the largest average return over the past six months, for the next six months.
Yearly → invest in the fund that obtained the largest average return over the past year, for the next year.

- **Min/Max strategy**

Consider the average returns on the min strategy and the max strategy respectively, for the previous year. Then, invests in the strategy that performed the best in the previous year, for the following year. This strategy was also executed half yearly.

- **Alternate Max/Min strategy**

Invest in the max strategy and the min strategy in turn. This was done yearly and half yearly.

- **Maximax strategy**

This is an optimistic decision criterion that finds the alternative (fund) that maximizes the maximum payoff for each alternative (Render *et al.*, 2003:86). This was carried out yearly and half yearly.

- **Maximin strategy**

This is a pessimistic decision criterion that attains the alternative that maximizes the minimum payoff for each alternative (Render *et al.*, 2003:87). This was carried out yearly and half yearly.

- **Equally likely (Laplace) strategy**

This is a decision approach that finds the alternative with the highest average payoff (Render *et al.*, 2003:88). This was performed yearly and half yearly.

- **Minimax regret strategy**

This decision criterion finds the alternative that minimizes the maximum opportunity loss within each alternative (Render *et al.*, 2003:88). This was performed yearly and half yearly.

- **Hurwicz criterion strategy**

This is a weighted average approach that selects the alternative with the highest weighted average (Render *et al.*, 2003:87). This was done yearly and half yearly with both optimistic and pessimistic values for the criterion of realism.

Chapter four presented the results of strategies for decision making under risk. The models that were used, include:

- **Monte Carlo simulation models**

These models selected funds according to some probability distribution. Four models were used:

- Historical probability distribution strategy (executed monthly, half yearly and yearly).
- Even probability strategy (performed monthly, half yearly and yearly).
- Previous year probability distribution strategy (executed monthly, half yearly and yearly).
- Strategies that use the previous year's probability distribution to make decisions (three models were done : diversify, max probability and min probability).

- **Markowitz portfolio model**

The goal is to find a portfolio of funds that minimises risk (measured by the variance of the portfolio return), subject to some linear constraints, among others; the expected return of the

portfolio must meet or exceed a certain target (Camm & Evans, 2000:77). A second model that restricts the allocation of funds to a certain percentage was also be considered. These models were executed yearly and half yearly.

- **LP models**

This strategy involves risk-reward trade-off, for different risk measures:

➢ *β as a measure of risk*

Three models was performed. The first model's goal is to find a portfolio of funds that minimises portfolio risk and meet or exceed a certain return target. The second model restricts the percentage of each fund to be invested, to a certain percentage. The third model looked at, maximises return, while the portfolio risk needs to be less than or equal to some specified target. Different portfolio risk targets were set. These models was performed yearly and half yearly.

➢ *Downside risk as a risk measure*

Two models was considered. The goal is to find a portfolio of funds that minimises portfolio risk, while a specified return target is met or exceeded. The second model restricts the allocation of funds to a certain percentage. These models were executed yearly and half yearly.

Chapter five presented a comparison between the strategies that use risk measures and those that do not, thus the strategies presented in chapters 3 and 4, respectively.

The criteria that were used to obtain the optimal strategy are:

- Average return (maximise this reward value).
- Average deviation (maximise this value).
- Expected shortfall (minimise this risk measure).
- Probability that the obtained return was below the MM (minimise this risk measure).
- Standard deviation (minimise this risk measure).

- Variance of a strategy's return from the Money Market's return (minimise this risk measure).

Therefore, it is concluded that this study indicated different criteria for selecting the optimal strategy that would be preferable to an investor. That is, should the return be maximised or should some risk measure be minimised. The strategy that performed the best (in terms of average return per month) was the half yearly pessimistic Hurwicz criterion. For investing in the individual funds S_3 performed the best. Overall the MM showed the lowest average return per month. The max probability strategy (with MM) showed the lowest risk with several criteria (apart from the MM) and the target return of 1.60% per month is met. Therefore, this is a *good* strategy. The investor doesn't always have to select the best strategy, but he can also select a good strategy. Thus, a model that has a slightly lower return, but it also shows lower risk. In conclusion, the selection of an investment strategy depends on an individual investor's risk/reward objectives. If an investor is willing to take more risk, one decision option might be more attractive than another.

6.2 Methodological reflection

Two routes were followed in addressing the research problem. An analysis of literature and sources were used as well as practical modelling. The method of investigation are structured as follows:

- Fund selection under uncertainty was done through statistical modelling.
- Linear and non-linear programming formulation and prediction and Monte Carlo simulation were done through statistical modelling.
- Risk measurement and management were portrayed through the literature review.

6.3 Recommendations for further research

The results of the models utilized in this study invite further research in the development of even better investment strategies, in terms of risk/reward trade-off. Different risk measures can also be considered.

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