DISCRETE DYNAMICS OF BANK CREDIT AND CAPITAL AND THEIR CYCLICALITY

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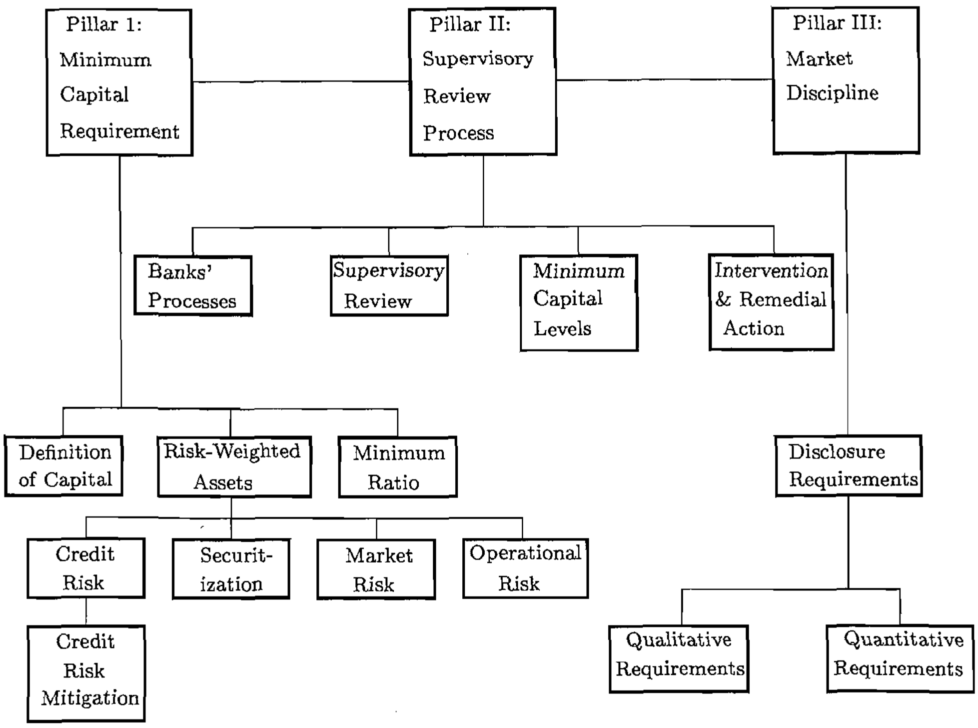

Figure 1: Overview of the Basel II Capital Accord

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Executive Summary

With the unraveling of the global subprime mortgage crisis (SMC) in 2007 and 2008 and the worldwide implementation of new banking regulation in the form of the Basel II Capital Accord, issues related to bank valuation and profitability have become even more topical. It is widely recognized that the new capital (the Basel II) accord will improve the prevention of individual bank failures, however concern has been expressed about some unintended consequences. These include issues such as procyclicality, changes in bank lending behavior and the ability of supervisors to regulate banks that use more sophisticated risk management techniques, among others. A motivation for studying profitability is the fact that it is a major indicator of financial crises for households, companies and financial institutions. An example of this from the SMC is the U.S. bank, Wachovia Corp., who reported a big loss as from the first quarter of 2007 and eventually was bought by the world's largest bank, Citigroup, on Monday, 29 September 2008.

In this dissertation, a first specific aim is to determine the value of a bank subject to Basel I and II capital requirements based on premiums for market, credit and operational risk. In order to accomplish this, we investigate the discrete-time dynamics of bank items such as assets, capital and bank profit when loan losses and macroeconomic conditions are explicitly considered. These models enable us to formulate an optimal bank valuation problem subject to cash flow, loan demand, financing and balance sheet constraints. However in the Basel I, we assume that risk-weights are constant. We also discuss the response of bank loans in the influence of the changes in the business cycle. Here operational risk is not considered as in the Basel II. Furthermore, we obtain results for the situation where loan losses and loan risk-weights are a function of the phases of the business cycle (i.e., risk-weights vary within the business cycle).

In this regard, the main achievement of this dissertation is the discrete-time maximization of bank value via optimal choices of loan rate and supply, This process leads to the establishment of corresponding optimal deposits, provisions for deposit withdrawals and bank profitability, as in [51]. The choice of the aforementioned loan rate provides a connection with the SMC and subsequent credit crunch. Some of our findings are corroborated by considering empirical data sourced from the South African Reserve Bank (SARB). Here, we consider the output gap as a proxy of the changes in the business cycle and the cyclicality of bank credit and capital. We present the output gap vs the business cycle in order to demonstrate that the output gap is a real proxy of the changes in the business cycle.

Opsomming

Met die ontrafeling van die Amerikaanse subprimaleningskrisis (SLK) in 2007 en 2008 en die wêreldwye implementering van nuwe bankregulasie in die vorm van die Basel II Kapitaalakkoord, het situasies wat betrekking het op bankwaardasie en -profyt meer aktueel geword. Ondanks die feit dat dit alombekend is dat die nuwe akkoord die bankrotskap van individuale banke sal verhoed, is daar kommer oor sekere onvoorsiene gevolge uitgespreek. Dit sluit in prosiklikaliteit, veranderinge in leningsgedrag en die vermoë van reguleerders om banke te reguleer wat meer ingewikkelde risikobestuur tegnieke gebruik. 'n Motivering om profyt te bestudeer is dat dit vir huishoudings, maatskappye en finansiële instansies 'n noemenswaardige indikator van finasiële krisisse is. 'n Voorbeeld hiervan is die Amerikaanse bank, Wachovia Corp., wat 'n groot verlies in die eerste kwartaal van 2007 gelei het en uiteindelik op Maandag, 28 September 2008 oorgekoop was deur die wêreld se grootste bank, Citigroup.

'n Eerste doelwit is om die waarde van 'n bank wat onderworpe is aan Basel I en II kapitaalvereistes op premiums vir mark-, krediet- en operasionele-risiko te bepaal. Om dit te behaal, ondersoek ons die diskrete-tyd dinamika van bankitems soos bates, kapitaal en profyt wanneer leningsverliese en makroekonomiese omstandighede eksplisiet in oorweging gebring word. Hierdie modelle stel ons in staat om 'n optimale bankwaarderingsprobleem onderworpe aan kasvloei, leningsaanvraag, finasiering en balansstaat beperkings, te formuleer. In die geval van Basel I neem ons aan dat die risiko-gewigte konstant is. Ons bespreek ook die uitwerking op die uitreiking van banklenings van veranderinge in die besigheidssiklusse. Hier word operasionele-risiko nie in ag geneem nie. Boonop, verkry ons resultate vir die situasie waar leningsverliese en leningsrisiko-gewigte 'n funksie van die fases van die besigheidsiklus is. (dws., risiko-gewigte verander tydens die besigheidssiklus).

In hierdie verband, is die hoofprestasie van die verhandeling die diskrete-tyd maksimalisering van bankwaarde deur middel van optimale keuses van leningskoerse en -verskaffing. Hiedie proses lei tot die daarstelling van ooreenstemmende optimale depositos, voorsiening vir deposito ontrekkings en bankprofyt soos in [51]. Die keuse van die bogenoemde leningskoers bied 'n aanknopingspunt met die SLK en kredietknoop. Van ons bevindinge word gestaaf deur empiriese data wat verkry was van die Suid-Afrikaanse Reserwebank (SARB). Hier, beskou ons die uitsetgaping as 'n volmag vir die verandering van die besigheidssiklus en die siklikaliteit van bankkrediet en -kapitaal. Ons stel die uitsetgaping teenoor die besigheidssiklus voor sodat ons kan aantoon dat die uitsetgaping 'n ware volmag van die verandering in die besigheidssiklus is.

Preface

One of the contributions made by the NWU-PC to the activities of the stochastic analysis community has been the establishment of an active research group that has an interest in institutional finance. In particular, this group has made contributions about modeling, optimization, regulation and risk management in insurance and banking. Students who have participated in projects in this programme under Prof. Petersen's supervision are listed below.

Level	Student	Graduation	Title
MSc	T Bosch	May 2003	Controllability of HJMM Interest Rate Models
MSc	CH Fouche	May 2006	Continuous-Time Stochastic Modelling of Capital Adequacy Ratios for Banks
MSc	MP Mulaudzi	May 2008	A Decision Making Problem in the Banking Industry
PhD	CH Fouche	May 2008	Dynamic Modeling of Banking Activities
PhD	F Gideon	Sept. 2008	Optimal Provisioning for Loan Losses and Deposit Withdrawals in the Banking Industry
PhD	T Bosch	May 2009	Optimal Auditing in the Banking Industry
MSc	MC Senosi	May 2009	Discrete Dynamics of Bank Credit and Capital and their Cyclicality
PhD	BA Tau	May 2009	Bank Loan Pricing and Profitability and Their Connections with Basel II and the Subprime Mortgage Crisis
PhD	MP Mulaudzi	Current	Mortgage Loan Securitization and the Subprime Mortgage Crisis
PhD	MC Senosi	Current	Discrete-Time Modeling of the Subprime Mortgage Crisis
PhD	S Thomas	Current	The Subprime Mortgage Crisis: Bank Profitability
Postdoc	J Mukuddem-Petersen	2006-8	Health Economics

Declaration

I declare that, apart from the assistance acknowledged, the work contained in this dissertation is my own. It is being submitted in partial fulfilment of the requirements for the degree of Master of Science in Applied Mathematics at the NWU-PC. It has not been submitted before for any degree or examination to any other university.

Nobody, including Prof. MA. Petersen (Supervisor), but myself is responsible for the final version of this dissertation.

Signature.................................

Date.......................................

Acknowledgements

Firstly, I would like to thank the Almighty for His grace in enabling me to complete this dissertation.

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I am indebted to my supervisor Prof. Mark A. Petersen of the School of Computer, Mathematical and Statistical Sciences at the NWU-PC, for the guidance provided during the completion of this dissertation. Also, I would like to thank the remaining members of staff in the Mathematics and Applied Mathematics Department at the NWU-PC (especially Dr. Ilse Schoeman) for the encouragement and support provided during my studies.

I am grateful to the National Research Foundation (NRF) for providing me with a Scarce Scholarship for Masters Studies as well as the Cannon and Collins Trust for financial support provided during the duration of my studies. Lastly, I would like to thank the Mathematics and Applied Mathematics Department at NWU-PC for additional financial support received.

List of Figures

List of Tables

Index of Abbreviations and Symbols

OECD - Organization for Economic Corporation and Development

IMA - Internal Models Approach

AMA - Advanced Measurement Approach

LLP - Loan Loss Provision

GDP - Gross Domestic Product

PD - Probability Default

LGD - Loss Given Default

TA - Total Assets

VaR - Value-at-Risk

TFP - Total Factor Productivity

HP - Hodrick-Prescott filter

JSE - Johannesburg Securities Exchange

CAR - Capital Adequacy Ratio

IRB - Internal Ratings Based

M - Remaining Maturity

SARB - South African Reserve Bank

Λ - Loans

$\mathtt{T}$ - Treasuries

R - Reserves

$\mathtt{K}$ - Total Bank Capital

D - Deposits

A - Assets

Γ - Total Liabilities

O - Subordinated Debt

E - Bank Equity

R^l - Loan Loss Reserve

L - Value of Loan Losses

r^d - Default Rate

γ - Reserve-Deposit Ratio

W - Sum of Treasuries and Reserves

Π - Profit

r^Λ - Loan Rate

$r^\mathtt{T}$ - Treasuries Rate

r^I - Rate of Return on Intangible Assets

$E(d)$ - Expected Loan Losses

k - Risk Premium

I_t - Intangible Assets

ω^I - Risk-Weights on Intangible Assets

ω^Λ - Risk-Weights on Loans

ω^T - Risk-Weights on Treasuries

ω^R - Risk-Weights on Reserves

$\mathtt{O}$ - Operational Risk

r^D - Deposit Rate

c^D - Cost of Deposit

$c^w(W_t)$ - Cost of Deposit Withdrawals

P - Total Provision

M_t - Level of Macroeconomic Activity

r^P - Penalty Rate on Deposit Withdrawals

ρ^τ - Relative Risk Ratio

q - One-Year Solvency Probability

Contents

Chapter 1

INTRODUCTION

In this dissertation, we mainly consider bank valuation[1] and profitability when loan losses and macroeconomic conditions are explicitly considered. We note that in the acquisition of bank equity, a valuation gives the stock analyst (possibly acting on behalf of a potential shareholder) an independent estimate of a fair price of the bank's shares. As far as profitability is concerned, we are motivated by the fact that it is a major indicator of financial crises for households, companies and financial institutions. An example of the latter from the subprime mortgage crisis (SMC) that became more apparent in 2007 and 2008, is that both the failure of the Lehmann Brothers investment bank and the acquisition in September 2008 of Merrill Lynch and Bear Stearns by Bank of America and JP Morgan Chase, respectively, were preceded by a decrease in profitability and an increase in the price of loans and loan losses (see Section 6.1 for a diagrammatic overview of the SMC). In this dissertation, we discuss the relationship between our banking models and the SMC as well as the subsequent credit crunch that has had a profound impact on the global banking industry from 2007 onwards. These connections are forged via the bank's risk premium, sensitivity of changes in capital to loan extension, Central Bank base rate, own loan rate, loan demand, loan losses and default rate, loan loss provisions, choice between raising deposits and interbank borrowing, liquidity, profit as well as bank valuation. In addition, we establish connections between our models and the Basel II capital accord. These associations are mainly determined via total bank capital, the bank capital constraint and the procyclicality of approaches to Basel II credit risk.

Loan pricing models usually have components related to the financial funding cost, a risk premium to compensate for the risk of default by the borrower, a premium reflecting market power exercised by the bank and the sensitivity of the cost of capital raised to changes in loans extended. On the other hand, loan losses can be associated with an offsetting expense called the *loan loss provision* (LLP) which is charged against nett profit. This offset will reduce reported income but has no impact on taxes, although when the assets are finally written off, a tax-deductible expense is created. An important factor influencing loan loss provisioning is regulation and supervision. Measures of capital adequacy are generally calculated using the book values of assets and equity. The provisioning of loans and their associated write offs will cause a decline in these capital adequacy measures, and may precipitate increased regulation by bank authorities. Greater levels of regulation generally entail additional costs for the bank. Currently, this regulation mainly takes the form of the Basel II Capital Accord (see Subsubsection 1.2.2.1 for a diagrammatic overview of Basel II; also [11] and [14]) that has been implemented on a worldwide basis since 2008.

[1]Bank value is commonly defined in terms of the market value of the investors equity (stock market capitalization if a company is quoted) plus the market value of the nett financial debt.

The impact of a risk-sensitive framework such as Basel II on macroeconomic stability of banks is an important issue. In this regard, we note that the 1996 Amendment's Internal Models Approach (IMA) determines the capital requirements on the basis of the institutions' internal risk measurement systems. The minimum capital requirement is then the sum of a premium to cover *credit risk*, general *market risk* and *operational risk*. The credit risk premium is made up of risk-weighted loans and the market risk premium is equal to a multiple of the average reported two-week VaRs in the preceding 60 trading days. Banks are required to report daily their value-at-risk (VaR) at a 99% confidence level over both a one day and two weeks (10 trading days) horizon. In order for a bank to determine their minimum capital requirements they will first decide on a planning horizon. This planning horizon is then partitioned into non-overlapping backtesting-periods, which is in turn divided into non-overlapping reporting periods. At the start of each reporting period the bank has to report its VaR for the current period and the actual loss from the previous period. The market risk premium for the current reporting period is then equal to the multiple m of the reported VaR. At the end of each backtesting period, the number of reporting periods in which actual loss exceeded VaR is counted and this determines the multiple m for the next backtesting period according to a given increasing scale. Usually the premium to cover operational risk equals the sum of the premiums for each of eight business lines. The operational risk premium is discussed further in Subsection 1.2.10.

A popular approach to the study of banking valuation and profitability involves a financial system that is assumed to be imperfectly competitive. As a consequence, profits (see, for instance, [90] and [91]) are ensured by virtue of the fact that the nett loan interest margin is greater than the marginal resource cost of deposits and loans. Besides competition policy, the decisions related to capital structure play a significant role in bank behavior. Here, the relationship between bank capital, credit and macroeconomic activity is of crucial importance. In this regard, it is a widely accepted fact that certain financial variables such as capital, credit, asset prices, profitability and provisioning (also bond spreads, ratings from credit rating agencies, leverage and risk-weighted capital adequacy ratios, other ratios such as write-off/loan ratios and perceived risk) exhibit cyclical tendencies. The cyclicality of a financial variable is related to its relationship with the business cycle or a proxy of the business cycle such as the output gap. Here the *output gap* is defined as the amount by which a country's output, or GDP, falls short of what it could be given its available resources. In particular, "procyclicality" has become a buzzword in discussions about banking regulation. In essence, the movement in a financial variable is said to be "procyclical" if it tends to amplify business cycle fluctuations. As such, procyclicality is an inherent property of any financial system. A consequence of procyclicality is that banks tend to restrict their lending activity during economic downturns because of their concern about loan quality and the probability of loan defaults. This exacerbates the recession since credit constrained businesses and individuals cut back on their investment activity. On the other hand, banks expand their lending activity during boom periods, thereby contributing to a possible overextension of the economy that may transform an economic expansion into an inflationary spiral. Our contribution emphasizes the cyclicality of bank profitability and provisioning for loan losses.

By way of addressing the issues raised above, we present a two-period discrete-time banking model involving on-balance sheet variables such as assets (cash, bonds, shares, loans, Treasuries and reserves), liabilities (deposits and interbank borrowing), bank capital (shareholder equity, subordinate debt and loan loss reserves) and off-balance sheet items such as intangible assets (see, for instance, [63] and [116]). In turn, the aforementioned models enable us to formulate an optimization problem that seeks to establish a maximal value of the bank by a stock analyst by choosing an appropriate loan rate and supply. Under cash flow, loan demand, financing and balance sheet constraints, the solution to this problem also yields a procedure for profit maximization in terms of the loan rate and deposits. Here profits are not only expressed as a function of loan losses but also depend heavily on provisions for loan losses.

1.1 RELATION TO PREVIOUS LITERATURE

In this section, we consider the association between our contribution and previous literature. The issues that we highlight include the role of bank capital, output gap, credit models for monetary policy, macroeconomic activity, cyclicality concerns and stochastic modeling and optimization.

1.1.1 Brief Literature Review of Output Gap

In our computation of the South African output gap, we make use of the actual GDP and potential GDP for the period 1990 to 2006. Although the output gap provided in our study is achieved by smoothing actual GDP via the Hodrick-Prescott filter method (see [66]), we also verified our findings by using the more complicated potential output method using a production function approach (see, for instance, [43]). Further discussions of output gap in a South African context is provided by [5], [6], [7] and [67]. The output gap for several OECD countries is investigated in [24], [25], [96] and [97].

As far as the calculation of output gap is concerned, [53] (see, also, [52] and [55]) reviews the methods used to estimate potential output and the resulting output gaps for the calculation of structural budget balances. The split time trend method for estimating trend output is compared with two alternative methods, viz., the Hodrick-Prescott filter and potential output methods. The conclusion is that the latter method is best for estimating output gaps and for calculating structural budget balances, with the results obtained by smoothing GDP providing a means of verification. The paper [5] (see, also, [68]) examines growth performance in post-apartheid South Africa within a growth accounting framework and assesses future growth prospects. Near-term prospects can be captured by potential output growth and the output gap. Furthermore, longer-term growth prospects can be determined on the basis of the full utilization of factors of production and the output gains that arise as these factors are more effectively utilized, for example, through structural reforms that enhance efficiency. The paper [6] provides estimates of potential output growth in South Africa since 1994 using both the time trend techniques and a production function approach. This paper claims that the output gap provides statistically significant information for forecasting inflation and could thus be used to formulate macroeconomic policy. Growth accounting and regression analysis suggest that an increase in trend GDP growth after the end of apartheid in 1994 is attributable to higher total factor productivity (TFP) growth driven by trade liberalization and greater private sector participation. The article [7] provides estimates of potential real GDP growth in South Africa based on alternative methodologies including a production function approach that is standard in the literature. The estimates suggest that during 1994-2001, potential output growth has been around 2.5% to 2.75% annually, and that in 2001 the output gap was around zero. The estimates of the output gap are reasonably correlated over time with inflation and capacity utilization, which are other indicators of the intensity of resource utilization. In order to elucidate South Africa's longer-term growth prospects, [7] analyzes the sources of real GDP growth in the country based on previous work in [67] (see, also, [96] and [97]). A striking fact is that the average annual growth rate of real GDP has increased significantly since 1994, rising from 1% in 1980-1993 to 2.7% in 1994-2001. The increase can mainly be attributed to TFP growth (improvements in efficiency and technology) rather than to increases in the factors of production (like employment). Also, this improvement can be associated with a number of factors including the structural change of the economy after the 1994 elections, the opening up of trade, tax relief and the gradual decrease in interest rates during that period, among other things. In particular, if the TFP growth rates experienced since 1994 are sustained and labor market rigidities are eased sufficiently so that employment rises in step with future increases in the labor force and increases in fixed capital formation, then the economy could achieve growth rates of around 5% over the longer term (see [7]). [81] predicts that the average projected growth in 2007-2012 would be slightly lower than the potential growth as the positive output gap closes over that period. This may be due to capacity constraints due to, for instance, the emerging scarcity of energy, cement and steel. These sentiments are also endorsed by the SA National Treasury (see [81] for more details).

1.1.2 Brief Literature Review of Bank Credit

[81] recognizes the developmental benefits of rapid credit expansion in South Africa. They, however, also caution against the risks that emanate from this expansion (see, for instance, [72]). Driven by buoyant credit and interest rate hikes in 2006 and 2007, the debt service-to-income ratio for South African households rose although it remained below historical highs (see [81]). While the majority of credit growth in South Africa takes the form of secured lending, the authorities have expressed concerns about the vulnerability of first-time and new borrowers and the moderate deterioration in asset quality (see, for instance, [65]). [72] identify that periods preceding financial instability share a number of common features. The first observation was that the lead up to each of the credit peaks was typified by an extended period of rapid economic growth. As a consequence, strong growth reinforced confidence and exacerbated booms.

1.1.3 Brief Literature Review of Bank Capital

The most important role of capital is to mitigate the moral hazard problem that results from asymmetric information between banks, depositors and borrowers. The Modigliani-Miller theorem forms the basis for modern thinking on capital structure (see [83]). In an efficient market, their basic result states that, in the absence of taxes, insolvency costs and asymmetric information, the bank value is unaffected by how it is financed. In this framework, it does not matter if bank capital is raised by issuing equity or selling debt or what the dividend policy is. By contrast, in our contribution, in the presence of loan market frictions, the bank value is dependent on its financial structure (see, for instance, [16], [40], [80] and [106] for banking). In this case, it is well-known that the bank's decisions about lending and other issues may be driven by the capital adequacy ratio (CAR) (see, for instance, [34], [35], [84], [105] and [107]). Further evidence of the impact of capital requirements on bank lending activities are provided by [61] and [111]. A new line of research into credit models for monetary policy has considered the association between bank capital and loan demand and supply (see, for instance, [2], [21], [28], [31], [110], [114] and [115]). This credit channel is commonly known as the *bank capital channel* and propagates that a change in interest rates can affect lending via bank capital. We also discuss the effect of macroeconomic activity on a bank's capital structure and lending activities (see, for instance, [60]). With regard to the latter, for instance, there is considerable evidence to suggest that the phases of the business cycle impact the probability of default and loss given default on loans (see, for instance, [3], [60] and [76]).

1.1.4 Brief Literature Review of Bank Profit

Profitability is a key source of information to understand the business. According to [102] cross-subsidization hypothesis, higher expected profits from relationship based lending creates an incentive to subsidize greater loan availability for new customers. As for the determinants of profitability, [118] argues that there has been relatively a limited overlap between the set of factors informed by the domestic bank profits literature and those based on the multinational bank profits literature to explain foreign bank profits. Several discussions related to bank profit are discussed in the papers [19], [21], [23], [28], [30] and [31]. On the other hand, [18] provide some evidence on the costs and profitability of relationship lending by commercial banks.

1.1.5 Brief Literature Review of Cyclicality of Credit and Capital

It is a widely accepted fact that certain financial variables (for instance, credit prices, bond spreads, ratings from credit rating agencies, provisioning, profitability, capital, leverage and risk weighted capital adequacy ratios, other ratios such as write-off/loan ratios and perceived risk) exhibit cyclical tendencies. In particular, "procyclicality" has become a buzzword in discussions around the new regulatory framework offered by the Basel II. In the sequel, the movement in a financial indicator is said to be "procyclical" if it tends to amplify business cycle fluctuations. A consequence of procyclicality is that banks tend to restrict their lending activity during economic downturns because of their concern about loan quality and the probability of loan defaults. This exacerbates the recession since credit constrained businesses and individuals cut back on their investment activity. On the other hand, banks expand their lending activity during boom periods, thereby contributing to a possible overextension of the economy that may transform an economic expansion into an inflationary spiral. The fact that provisioning (profitability) behaves procyclically by falling (rising) during economic booms and rising (falling) during recessions (see, for instance,

[19], [21], [23], [28], [30] and [31]) is incorporated in our models.

1.1.6 Brief Literature Review of Discrete-Time Optimization

Several discussions related to discrete-time optimization problems for banks have recently surfaced in the literature (see, for instance, [60], [80], [85] and [105]). Also, some recent papers using dynamic optimization methods in analyzing bank regulatory capital policies include [100] for the Basel II and [9], [32] and [79] for the Basel market risk capital requirements. In [105], a discrete-time dynamic banking model of imperfect competition is presented. For both these options, a maximization problem that involves the bank value for shareholders is formulated. On the other hand, [85] examines a problem related to the optimal risk management of banks in a continuous-time stochastic dynamic setting. In particular, the authors minimize market and capital adequacy risk that involves the safety of the assets held and the stability of sources of capital, respectively (see, also, [86]).

1.1.7 Brief Literature Review of Procyclicality

Existing studies of procyclicality in terms of the Basel II have focused on Pillar 1 of the Internal Rating Based (IRB) treatment of wholesale commercial loans. Regulatory capital charges are set at the individual loan level and are given by a formula with five inputs: the borrower's one-year probability of default (PD), the instrument's expected loss given default (ELGD) and remaining maturity (M), the asset-value correlation, ρ, which parameterizes dependence across borrowers, and a target one-year solvency probability (q) for the bank. Under the Basel II, q is fixed to 99.9% and (ρ) is specified as a decreasing function of PD (for more details, see [12]). The IRB capital formula is derived from the large-portfolio asymptotic dynamics of a Merton model with a single common risk-factor. As shown by [56], an instrument's marginal contribution to portfolio VaR converges in the asymptotic limit to its expected loss conditional on the common factor suffering a qth percentile stress event. In the IRB implementation, this conditional expected loss is approximated as a separable equation of three terms: ELGD; the one-year actuarial conditional expected loss; and a maturity adjustment that approximates the ratio of mark-to-market capital charges to actuarial-loss capital charges. Since the release of the Second Consultative Paper [11], many studies have assessed empirically the magnitude of procyclicality in the IRB capital formula. Both within and between studies, one sees a wide range of estimated response to a cyclical downturn. Required capital can double in some simulations, and in others can actually decline. The papet [71] points to differences across studies in sample and methodology that account for some of the differences in results. These include the average credit quality of the sample portfolio, and the country, time period and rating system from which historical experience was drawn. Some differences across studies may be attributed to successive revisions to the Basel II. [71] also identify methodological issues in survivorship bias and treatment of missing observations that appear to have not been handled properly in some earlier studies. Perhaps more important are two issues that can be cast as methodological, but have strong substantive implications. First, should measurement of changes over time in required capital be inclusive or exclusive of charge-offs due to defaulted loans? Second, should the simulated portfolio be passively or actively managed? A passively managed portfolio is fixed at the beginning of the simulation, and so shrinks as defaults accumulate. In an actively managed portfolio, defaulting or maturing loans are replaced with new loans. We can assume a simple time-invariant rule for new lending, or we can attempt to emulate the behavior of bank managers in loosening or tightening lending standards over the credit cycle.

As [71] observe, if we retain defaulted loans in the sample, then we measure the cumulative demands on bank capital over the simulated period. The accumulated charge-offs would have been incurred under the rules of the Basel II as well. From a policy perspective, we are interested only in the additional procyclicality associated with a change in capital regime. Furthermore, in the view of [71], it is somewhat misleading to include accumulated charge-offs without also imputing accumulated interest income net of dividend payments (see [71]). Dividends can be raised and lowered with the anticipated capital needs of the bank, so would need to be modeled endogenously. Therefore, we concur with [71] in preferring to measure capital changes exclusive of defaulted obligors. In most of their simulations, [71] find that roughly 40% to 60% of total change in capital (inclusive of defaulted loans) is attributable to accumulated charge-offs. When [29] similarly decompose total changes in required capital, they in some cases find that capital on the non-defaulting portfolio can decrease in a credit cycle downturn. By disregarding the riskiest borrowers, a stress event has the potential to improve the average credit quality in a portfolio. On the second issue, [71] favor a passive simulated portfolio because active management muddles together the direct effect of a tightened capital constraint with the bank's endogenous response. For example, suppose we look at the evolution of a bank's actively managed portfolio during a recession and find that average credit quality is roughly unchanged. Should we conclude from this that there is no cyclicality problem deserving of policymaker attention? Probably not-it may just be that the bank has reacted to a tightening capital constraint by cutting off credit to its riskier borrowers, which is precisely the policy problem that concerns us ([71]). [57] disagree with this view. They claim that if the goal is to estimate the additional procyclicality associated with the change in capital regime, then one needs to simulate active portfolio management as it occurs under the current regulatory regime. In short, [57] agree with [71] that portfolio management should not be made endogenous to the regulatory rule, but disagree on the appropriate alternative benchmark. Since the mid 1990s, large banks have been more or less unconstrained by regulatory capital requirements, so changes in lending standards over recent years ought to have been driven mainly by economic capital considerations. Such considerations are independent of regulatory regime, and thus would persist (though not necessarily as the binding constraint) into the Basel II. Empirical evidence is limited on this point, but suggests that banks currently do tighten lending standards during a recession and loosen lending standards in a boom. [15] show that the average quality of new loans decreased at the start of the recent recession, while the Federal Reserve Board's Senior Loan Officers Survey showed a commensurate tightening of lending standards. These results are not necessarily contradictory. Many or most nominally new loans are actually drawdowns under existing commitments. Tighter lending standards presumably increase the quality of newly established lending facilities. Thus, the change in new loan quality takes effect with a lag. Taking a more behavioral view of lending practices, [17] find evidence for a institutional memory hypothesis under which standards soften as time passes since a bank's most recent period of large credit losses. The ability to differentiate accurately between high risk and low risk borrowers deteriorates over time as loan officers forget the lessons of the last credit cycle. When large losses are again experienced, standards are tightened drastically, and the cycle begins again (see, for instance, [72]). Results in [70]) are consistent with this story. In addition to studying procyclicality on their simulated passive portfolios, [70] calculate the time-series of IRB required capital for Deutsche Bank's actual German commercial loan portfolio. The capital increase (excluding charge-offs) in this actively managed portfolio was only about one fourth of the increase in required capital for the passively managed, simulated KMV Germany portfolio. As the Deutsche Bank rating system is designed to be point-in-time like KMV, it is reasonable to attribute the difference to active management of lending standards. Thus, even under the risk-insensitive Basel I, internal management processes cause banks to respond to a downturn in a manner that would also serve to dampen the cyclicality of IRB capital requirements.

1.1.8 Brief Literature Review of the Subprime Mortgage Crisis

The working paper [36] explains the fundamentals of the SMC in some detail. A model that has become important during this crisis is the Diamond-Dybvig model (see, for instance, [37] and [38]). Despite the fact that these contributions consider a simpler model than ours, they are able to explain important features of bank liquidity that reflect reality. The quarterly reports [44] and [45] of the Federal Deposit Insurance Corporation (FDIC) intimate that profits decreased from \$ 35.6 billion to \$ 19.3 billion during the first quarter of 2008 versus the previous year, a decline of 46 %.

1.2 PRELIMINARIES

In this section, we provide some preliminaries on operational risk, the balance sheet, profit, macroeconomics, bank loans and bank capital, provisioning for deposit withdrawals and borrowing from other banks.

1.2.1 Preliminaries about Output Gap

In order to study the cyclicality of output, bank credit and capital, we acquired data from the SARB regarding financial variables such as household debt, mortgage loans, CARs and share capital. Information about the output gap was not provided by the SARB and was achieved via our own calculations. The measures of potential output are typically based on historical rates of factor utilization and total factor productivity, rather than on literal full employment, so that in the long run the average output gap is approximately zero. The implied output gap provides statistically significant information for predicting inflation and could thus provide valuable input for formulating macroeconomic policy. The gap can also be used for gauging the stance of fiscal policy in a cyclical context through its role in calculating structural measures of the fiscal balance. The policy outlook for a country depends importantly on both near- and long-term prospects for real output growth. Near-term prospects can be measured by potential output growth and the output gap. Longer-term growth prospects are based on the full utilization of factors of production and the output gains that arise as these factors are more efficiently utilized, for example, through structural reforms.

The output gap is measured as the percentage difference between actual GDP and estimated potential GDP. Symbolically this means that

$$\text{Output Gap} = \frac{\text{Actual Output - Potential Output}}{\text{Potential Output}} \times 100.$$

In other words, the output gap involves measuring the position of output in relation to potential. Output gaps are difficult to estimate and subject to margins of substantial error (see, for instance, [43] for more details on the South African situation).

1.2.2 Preliminaries about the Structure of the Basel II Capital Accord

This subsection is based on the discussion paper [64]. The most significant innovation of Basel II is the departure from a sole reliance on capital adequacy ratios. Basel II consists of three mutually reinforcing pillars (see the diagrammatic overview of Basel II in Figure 2 below), which together should contribute to safety and soundness in the financial system. To ensure that risks within an entire banking group are considered, Basel II is extended on a consolidated basis to holding companies of banking groups.

1.2.2.1 Diagrammatic Overview of the Basel II Capital Accord

Basel II provides a range of options for determining not only the capital requirements for credit and market risk but also operational risk. This allows banks and supervisors to select approaches that are most appropriate for their operations and their financial market infrastructure. Banks have to adopt an approach commensurate with their size and sophistication, and they will not be allowed to revert to a simpler approach once they have been approved for a more advanced approach without supervisory approval. Basel II allows, to a limited degree, for national discretion in the way in which each of the approaches is applied.

1.2.2.2 Pillar 1 - Minimum Capital Requirements

Pillar 1 sets out the minimum capital requirements. Basel I is based on a capital ratio where the numerator represents the amount of capital a bank has available and the denominator is a measure of the risks a bank faces and is referred to as risk-weighted assets.

Under Basel II, the regulations that define the numerator of the capital ratio essentially remain unchanged. Furthermore, the ratio for the minimum capital required has not changed. The modifications, therefore, lie in the definition of risk-weighted

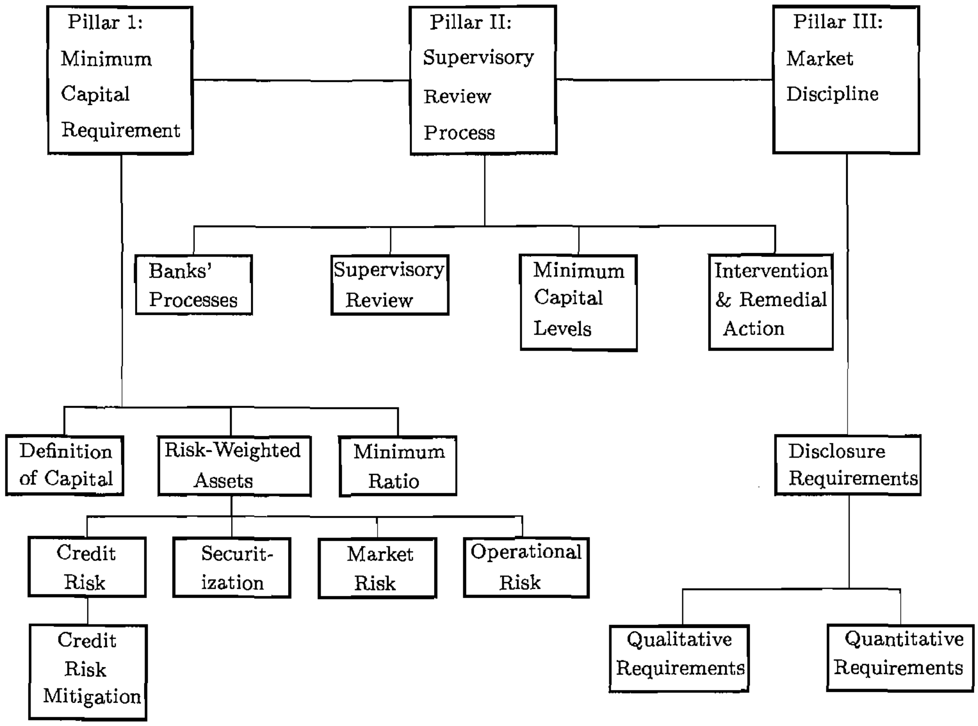

Figure 1.1: Diagrammatic Overview of the Basel II Capital Accord

assets. Whereas Basel I (as amended by the 1996 market risk amendment) explicitly covered only two types of risk in the definition of risk-weighted assets, namely credit and market risk, Basel II includes an explicit treatment of operational risk. Basel II introduced three distinct options (of increasing risk sensitivity) for the calculation of credit risk and three others for operational risk.

1.2.2.3 Credit Risk

In this subsection, we consider Basel II credit risk from the point of view of the standardized and internal-ratings based approaches. Firstly, we provide a diagrammatic overview of credit risk.

Standardized approach

The Standardized approach is similar to Basel I in that banks are required to slot their credit exposures into risk categories based on observable characteristics of the exposures. This approach establishes fixed risk weights corresponding to each risk category and makes use of external credit assessments to enhance risk sensitivity. The bank allocates a risk weight to each of its assets and off-balance sheet positions and produces a sum of risk-weighted asset values. A risk weight of 100 per cent

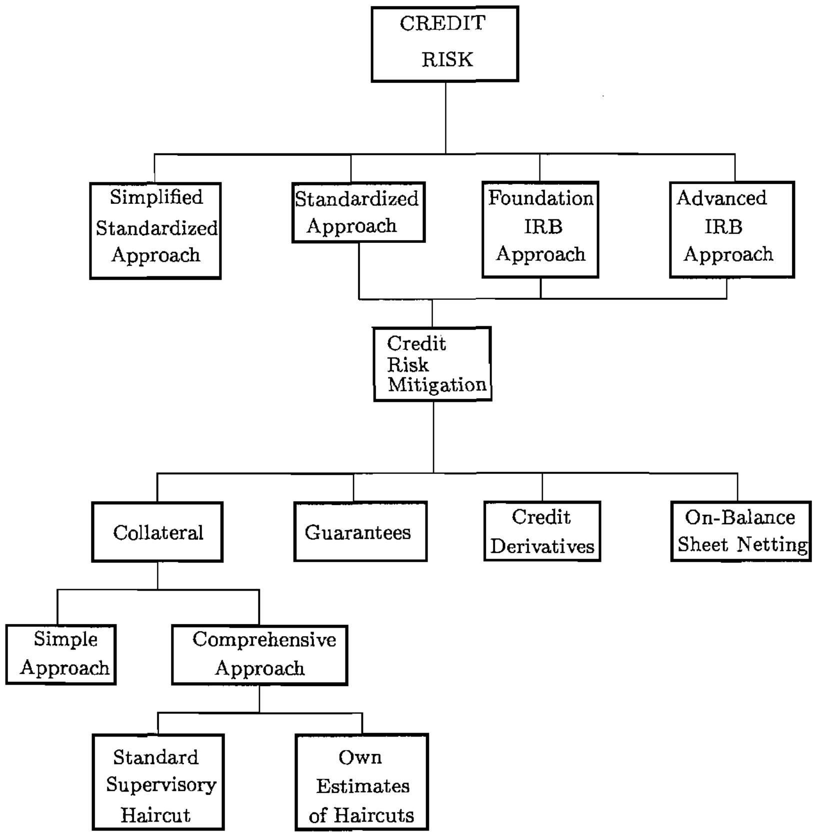

Figure 1.2: Diagrammatic Overview of Credit Risk

means that an exposure is included in the risk-weighted assets at its full value, which translates into a capital charge equal to 8 per cent of that value. The risk weights are refined by reference to a rating provided by an external credit assessment institution (ECAI)[2].

Internal ratings-based approach

[2] An external credit assessment institution (ECAI) has to comply with strict criteria to be recognised. National supervisors are responsible for determining whether an ECAI meets the criteria. Also, supervisors will be responsible for assigning eligible ECAI's assessments to the risk weights under the Standardized risk weighting framework. The mapping process should be objective. It also allows, to a limited degree, for national discretion in the way in which each of these options may be applied. Banks must have a robust system in place to validate the accuracy and consistency of rating systems, processes, and the estimation of all relevant risk components. A bank must demonstrate to its supervisor that the internal validation process enables it to consistently and meaningfully assess the performance of internal rating and risk estimation systems.

One of the most innovative (and problematic) aspects of Basel II is the IRB approach to credit risk, which includes two variants, namely a foundation (FIRB) and an advanced (AIRB) approach. The IRB approach differs significantly from the Standardised approach in that banks' internal assessments of key risk drivers serve as primary inputs to the capital calculation in the former approach (see, for instance, [108] for more information). As the approach is based on banks' own internal assessments, the potential for more risk-sensitive capital requirements is substantial. The IRB approach does not, however, allow banks themselves to determine all the elements needed to calculate their own capital requirements. Instead, the risk weights and thus capital charges are determined through the combination of quantitative inputs provided by banks and formulas specified by the Basel Committee. The formulas or risk weight functions translate a bank's inputs into a specific capital requirement and are based on modern risk management techniques that involve a statistical and thus quantitative assessment of risk.

1.2.2.4 Pillar 2 - Supervisory Review Process

The supervisory review process requires supervisors to ensure that each bank has sound internal processes in place to assess the adequacy of its capital based on a thorough evaluation of its risks. Basel II stresses the importance of bank management developing an internal capital assessment process and setting targets for capital that are commensurate with the bank's particular risk profile and control environment. Supervisors would be responsible for evaluating the extent to which banks are assessing their capital adequacy needs relative to their risks, and to intervene where necessary.

1.2.2.5 Pillar 3 - Market Discipline

The purpose of Pillar 3 is to complement the minimum capital requirements (Pillar 1) and the supervisory review process (Pillar 2) and aims to bolster market discipline (see, for instance, [93] and [99]) through enhanced disclosure by banks. Effective disclosure is essential to ensure that market participants can better understand banks' risk profiles and the adequacy of their capital positions.

1.2.3 Preliminaries about the Bank's Balance Sheet

Throughout, we suppose that $(\Omega, \mathbf{F}, (\mathcal{F}_t)_{t\geq 0}, \mathbf{P})$ is a filtered probability space. As is well-known, the bank balance sheet consists of assets (uses of funds) and liabilities (sources of funds) that are balanced by bank capital (see, for instance [40]) according to the well-known relation

$$\text{Total Assets } (A) = \text{Total Liabilities } (\Gamma) + \text{Total Bank Capital } (K). \qquad (1.1)$$

In period t, the main on-balance sheet items in (1.1) can specifically be identified as

$$A_t = \Lambda_t^m + W_t + C_t + S_t + B_t, \; W_t = \mathrm{T}_t + R_t; \; \Gamma_t = D_t; \; K_t = n_t E_{t-1} + O_t + R_t^l,$$

where Λ^m, C, S, B, T, R, D, n, E, O and R^l are the market value of loans, cash, shares, bonds, Treasuries, reserves, outstanding debt, number of shares, market price of the bank's common equity, subordinate debt and loan loss reserves, respectively.

1.2.4 Preliminaries about the Cyclicality of Credit and Capital

Next, we introduce the generic variable, M_t, that represents the level of macroeconomic activity in the bank's loan market. We suppose that $M = \{M_t\}_{t\geq 0}$ follows the first-order autoregressive stochastic process

$$M_{t+1} = \mu^M M_t + \sigma_{t+1}^M,$$

where σ_{t+1}^M denotes zero-mean stochastic shocks to macroeconomic activity.

The bank takes deposits, D_t, at a constant *marginal cost*, c^D, that may be associated with cheque clearing and bookkeeping. It is assumed that deposit taking is not interrupted even in times when the *interest rate on deposits* or *deposit rate*, r_t^D, is less than the *interest rate on Treasuries* or *bond rate*, r_t^{T}, We suppose that the dynamics of the deposit rate process, $r^D = \{r_t^D\}_{t\geq 0}$, is determined by the first-order autoregressive stochastic process

$$r_{t+1}^D = \mu^{r^D} r_t^D + \sigma_{t+1}^{r^D},$$

where $\sigma_{t+1}^{r^D}$ is zero-mean stochastic shocks to the deposit rate.

Remark 1.2.1 (Deposit Rate and Monetary Policy): *In some quarters, the deposit rate, r^D, is considered to be a strong approximation of bank monetary policy. Since such policy is usually affected by macroeconomic activity, M, we expect the aforementioned items to share an intimate connection. However, in our analysis, we assume that the shocks σ^D_{t+1} and σ^M_{t+1} to r^D and $\mathtt{M}$, respectively, are uncorrelated. Essentially, this means that a precise monetary policy is lacking in our bank model. This interesting relationship is the subject of further investigation.*

1.2.5 Preliminaries about Bank Credit

An observation is that loan losses are also dependent on macroeconomic activity. As a consequence, for the *value of loan losses*, L, and the *default rate*, r^d, we set

$$L(M_t) = r^d(M_t)\Lambda_t, \tag{1.2}$$

where $r^d \in [0, 1]$ increases when the phases of the business cycle deteriorate according to

$$0 \leq r^d(M_t) \leq 1, \quad \frac{\partial r^d(M_t)}{\partial M_t} < 0.$$

We note that the above description of the loan loss rate is consistent with empirical evidence that suggests that bank losses on loan portfolios are correlated with the business cycle under any capital adequacy regime (see, for instance, [19], [23], [30] and [77]).

Since it is uncommon for depositors to withdraw all of their funds simultaneously, only a portion of total deposits may be needed as reserves. As a result of this description, we may introduce a *reserve-deposit ratio*, γ, for which

$$R_t = \gamma D_t. \tag{1.3}$$

We suppose that, after providing liquidity, the bank lends in the form of t-th period loans, Λ_t, at the *bank's own loan rate*, r^Λ_t. This loan rate, for profit maximizing banks, is determined by the risk premium (or yield differential), given by

$$\varrho_t = r_t^{\Lambda} - r_t, \tag{1.4}$$

the industry's market power as determined by its concentration, N, the market elasticity of demand for loans, η, base rate, r_t, the marginal cost of raising funds in the secondary market, c^{rw}, and the product of the cost of elasticity (equity) raised, c^E, and the sensitivity of the required capital to changes in the amount of loans extended,

$$\frac{\partial K}{\partial \Lambda}. \tag{1.5}$$

In this situation, we may express the bank's own loan rate, r^{Λ}, as

$$r_t^{\Lambda} = \varrho + (1 + r_t)\frac{N}{\eta} + c^{rw} + c^E \frac{\partial K}{\partial \Lambda} + \mathbf{E}[l], \tag{1.6}$$

where

$$N = \sum_{i=1}^{n} S_i^2$$

is the Herfindahl-Hirschman index of the concentration in the loan market,

$$S_i = \frac{\Lambda_i}{\Lambda}$$

is the market share of bank i in the loan market, but in our contribution we only use one bank, therefore $N = 1$ and

$$\eta = -\frac{\partial \Lambda}{\partial r_t^{\Lambda}} \frac{r_t^{\Lambda}}{\Lambda}$$

is the elasticity of demand for loans. Also, in our model, besides the risk premium, we include $\mathbf{E}[l]$ which constitutes the amount of provisioning that is needed to match the average expected losses faced by the loans.

As was mentioned before, the contribution [19] (see, also, [30] and [77]) highlights the fact that normally provisions for expected loan losses, $(\alpha\varrho+\mathbf{E}[l])\Lambda_t$, where $0 \leq \alpha \leq 1$ and ϱ is the risk premium from (1.4), and loan loss reserves, R^l, act as buffers against expected and unexpected loan losses, respectively. Firstly, we have to distinguish between total provisioning for loan losses, P, and loan loss reserves, R^l. Provisioning is a decision made by bank management about the size of the buffer that must be set aside in a particular time period in order to cover loan losses, L. However, not all of P may be used in a time period with the amount left over constituting *loan loss reserves*, R^l, so that for period t we have

$$R_t^l = P(M_t) - L(M_t), \ P > L.$$

Our model for provisioning in period $t+1$ can be taken to be

$$P(M_{t+1}) = \begin{cases} (\alpha\varrho + \mathbf{E}[l])\Lambda_t, & \text{for } P > L \ \text{ Expected Losses} \\ (\alpha\varrho + \mathbf{E}[l])\Lambda_t + R_{t+1}^l, & \text{for } P \leq L \ \text{ Expected Losses + Unexpected Losses,} \end{cases} \tag{1.7}$$

We note that our model determines the provisions for period $t+1$ in the t-th period which is a reasonable assumption. Our suspicion is that provisioning, P, is a decreasing function of current macroeconomic conditions, M, so that

$$\frac{\partial P(M_t)}{\partial M_t} < 0. \tag{1.8}$$

This claim has resonance with the idea of procyclicality where we expect the provisioning to decrease during booms, when macroeconomic activity increases. By contrast, provisioning may increase during recessions because of an elevated probability of default and/or loss given default on loans.

1.2.6 Preliminaries about Bank Capital

For the purposes of our study, we define *bank capital*, K, as the difference between the accounting value of the assets and liabilities. In the sequel, we take the bank capital, K, in period t to be

$$K_t = n_t E_{t-1} + O_t + R_t^l, \tag{1.9}$$

where $n_t E_{t-1}$ is the market value of the bank equity and constitutes Tier 1 capital in period $t-1$, O_t is the subordinate debt and is a constituent of Tier 2 capital as is the loan loss reserves, R_t^l. For K_t given by (1.9), we obtain the *balance sheet constraint*

$$W_t = D_t - \Lambda_t - C_t - B_t - S_t + K_t, \tag{1.10}$$

The *bank capital constraint* is defined by the inequality

$$K_t \geq \rho\Big(a_t + 12.5(mVaR + \mathtt{O})\Big), \tag{1.11}$$

where

$$a_t = \omega^I I_t + \omega^\Lambda \Lambda_t + \omega^{\mathrm{T}} \mathrm{T}_t + \omega^R R_t,$$

and ω^I, ω^Λ, ω^{T} and ω^R are the risk-weights for intangible assets, loans, Treasuries and reserves, respectively, $\rho = 0,08$, $mVaR$ is as described in Chapter 1 and $\mathtt{O}$ is the operational risk. If we assume that the risk-weights associated with intangible assets, Treasuries, reserves and loans may be taken to be $\omega^I \neq 0$, $\omega^{\mathrm{T}} = \omega^R = 0$ and $\omega^\Lambda = \omega(M_t)$, respectively, then equation (1.11) becomes

$$K_t \geq \rho\Big[\omega(M_t)\Lambda_t + \omega^I I_t + 12.5(mVaR + \mathtt{O})\Big]. \tag{1.12}$$

1.2.7 Preliminaries about Bank Profit

As far as profit, Π, is concerned, we closely follow the report [30] and use the basic fact that profits can be characterized as the difference between income and expenses that are reported in the bank's income statement. In our contribution, *income* is solely constituted by the return on intangible assets, $r_t^I I_t$, the return on loans, $r_t^\Lambda \Lambda_t$, and the return on Treasuries, $r_t^{\mathrm{T}} \mathrm{T}_t$. The rates of return on intangible assets, loans and Treasuries are denoted by r^I, r^Λ and r^{T} respectively. Furthermore, we assume that the level of macroeconomic activity is denoted by M_t. In our case we consider the cost of monitoring and screening of loans and capital, $c^\Lambda \Lambda_t$, interest paid to depositors, $r_t^D D_t$, the cost of taking deposits, $c^D D_t$, the cost of deposit withdrawals, $c^w(W_t)$, the value of loan losses, $L(M_t)$, and total loan loss provisions, $P(M_t)$ as *expenses*, in period t. Here r^D and c^D are the deposit rate and cost of deposits, respectively. Summing all the costs mentioned to operating costs and supposing that (1.2) holds and that $W_t = \mathrm{T}_t + \gamma D_t$, then the bank's profits are given by the expression

$$\begin{aligned} \Pi_t &= \left(r_t^{\Lambda} - c^{\Lambda} - r^d(M_t)\right)\Lambda_t + r_t^{\mathrm{T}} W_t + r_t^I I_t + r_t^C C_t + r_t^B B_t + r_t^S S_t \qquad (1.13) \\ &\quad -\left(r_t^D + c^D\right) D_t - c^w(W_t) - P(M_t) - r_t^{\mathrm{T}} \gamma D_t, \end{aligned}$$

where r^C, r^B and r^S are the rates of return of the cash, bonds and shares, respectively.

1.2.8 Preliminaries about Provisioning for Deposit Withdrawals

We have to consider the possibility that unanticipated deposit withdrawals will occur. By way of making provision for these withdrawals, the bank is inclined to hold Treasuries and reserves that are both very liquid. In our contribution, we assume that the *unanticipated deposit withdrawals*, x, originates from the *probability density function*, $f(x)$, that is independent of time. For sake of argument, we suppose that the unanticipated deposit withdrawals have a uniform distribution with support $[0, \overline{D}]$ so that the *cost of liquidation*, c^l, or additional external funding is a quadratic function of the sum of Treasuries and reserves, W. In addition, for any t, if we have that

$$x > W_t,$$

where $W_t = \mathrm{T}_t + R_t$, then bank assets are liquidated at some *penalty rate*, r_t^p. In this case, the *cost of deposit withdrawals* is

$$c^w(W_t) = r_t^p \int_{W_t}^{\infty} [x - W_t] f(x) dx = \frac{r_t^p}{2\overline{D}} [\overline{D} - W_t]^2.$$

Remark 1.2.2 (Deposit Withdrawals and Bank Liquidity): *A vital component of the process of deposit withdrawal is liquidity. The level of liquidity in the banking sector affects the ability of banks to meet commitments as they become due (such as deposit withdrawals) without incurring substantial losses from liquidating less liquid assets. Liquidity, therefore, provides the defensive cash or near-cash resources to cover banks' liabilities.*

1.2.9 Borrowing from Other Banks

Interbank borrowing including borrowing from the Central Bank provides a further source of funds. In the sequel, the amount borrowed from other banks is denoted by B, while the interbank borrowing rate (for instance, known as the Libor rate in the United Kingdom) and marginal borrowing costs are denoted by r^{B} and c^{B}, respectively. Of course, when our bank borrows from the Central Bank, we have $r^{\mathrm{B}} = r$, where r is the base rate appearing in (1.4). Another important issue here is the comparison between the cost of raising and holding deposits, $(r^D+c^D)D$, and the cost of interbank borrowing, $(r^{\mathrm{B}} + c^{\mathrm{B}})\mathrm{B}$. In this regard, a bank in need of capital would have to choose between raising deposits and borrowing from other banks on the basis of overall cost. In other words, the expression

$$\min\{(r^D + c^D)D,\ (r^{\mathrm{B}} + c^{\mathrm{B}})\mathrm{B}\} \tag{1.14}$$

is of some consequence. Under normal economic conditions, for $B \approx D$, we may have that

$$(r^D + c^D)D = \min\{(r^D + c^D)D,\ (r^{\mathrm{B}} + c^{\mathrm{B}})\mathrm{B}\}.$$

However, during the SMC, it is likely that $(r^{\mathrm{B}} + c^{\mathrm{B}})\mathrm{B} = \min\{(r^D + c^D)D,\ (r^{\mathrm{B}} + c^{\mathrm{B}})\mathrm{B}\}$.

1.2.10 Preliminaries about Operational Risk

In the Basel II framework, risk management is divided into credit, market as well as operational risk management. The premium to cover operational risk equals the sum of the premiums for each of eight business lines (corporate finance, trading and sales, retail banking, commercial banking, payment and settlement, agency services, asset management and retail brokerage). The Basel II framework outlines three quantitative approaches for determining an operational risk capital premium: the basic indicator approach, the standardized approach, and the advanced measurement approach. The basic indicator and the standardized approaches are simple and generate results on the basis of predetermined multipliers. More specifically, the capital premium for operational risk, under the Standardized Approach outlined in the Basel II, may be expressed as

$$\mathsf{O} = \max\left[\sum_{k=1}^{8} \beta_k g_k,\ 0\right],$$

where

g_{1-8} : Three-Year Average of Gross Income for Each of Eight Business Lines;

β_{1-8} : Fixed Percentage Relating Level of Required Capital to Level of Gross Income for Each of Eight Business Lines.

The β-values for operational risk are provided in the document [10]. The advanced measurement approach (AMA) differs from the basic indicator and the standardized approaches in that it explicitly attempts to estimate a bank's operational risk exposure (aggregate operational losses faced over a one-year period) at a soundness level consistent with a 99.9 % confidence level. Banks adopting an AMA will effectively calculate operational risk capital using a value at risk (VaR) approach common in both market risk and credit risk management. Each institution employing an AMA must use the following four elements in arriving at its operational risk capital estimate: internal data, external data, scenario analysis, and business environment and internal control factors. The operational risk capital estimate can be expressed as protection against expected and unexpected future losses at a selected confidence level, with some provisions for offsetting portions of this exposure through reserves or other permitted mitigation techniques.

1.3 OUTLINE OF THE DISSERTATION

The rest of dissertation is structured as follows.

1.3.1 Outline of Chapter 2

In Chapter 2, we discuss the discrete-time dynamics of bank items, represented the same as in [49]. These items include retained earnings (see Subsection 2.1.1), as well as loan supply and demand (see, for instance, Subsection 2.1.2), capital and bank profit under the influence of business cycle vagaries. These items enable us to formulate and solve an optimization bank valuation problem subject to capital, cash flow, loan demand, financing and balance sheet constraints (see Problem 2.2.1 and 2.3.1). The solution of the optimal bank valuation given in Theorem 2.3.1 is for the case when capital constraint is binding. For non-binding constraint, the solution is given in Corollary 2.3.2.

1.3.2 Outline of Chapter 3

Chapter 3 contains a discussion of the discrete-time dynamic model of bank items where, as in the Basel I, we assume that risk-weights are constant (see Section 3.1). In addition, borrowers are allowed to default on their principal repayments. Furthermore, we present a result about the effects of changes in the phases of the business cycle on bank capital and credit extension in the form of the quantity and price of loans (see Subsection 3.1.1). Moreover, we establish the impact of the changes in the business cycle on loans and loan rates under a non-binding as well as a binding capital constraint (see, for instance, Subsections 3.1.2 and 3.1.3, respectively, also compare with equation (1.11)). In Section 3.2, results analogous to those obtained in Section 3.1 are given for the situation where loan losses and loan risk-weights are a function of the phases of the business cycle (i.e., risk-weights vary with business cycle factors). This situation mimics the scenario covered by the Basel II. Furthermore, our analysis also involves a non-binding and binding capital constraint (see Subsections 3.2.2 and 3.2.3, respectively). In Subsection 3.2.4, we present cyclicality of loans and loan rates under the Basel II in future periods.

1.3.3 Outline of Chapter 4

Chapter 4 contains a discussion of the connection between procyclicality and financial stability in South Africa. We provide graphical representations of the interactions between the output gap as a proxy for the business cycle (see Section 4.1) and the cyclicality of bank credit and capital (see, for instance, Sections 4.2 and 4.3, respectively). The dynamics of these variables help to confirm the cyclicality of credit and capital in the South African context.

1.3.4 Outline of Chapter 5

An analysis of the cyclicality of bank credit and capital is given in Chapter 5. In Section 5.1, we discuss some of the issues related to the optimal bank valuation problem presented in Chapter 2 while in Section 5.2, we analyze the cyclicality of bank credit and capital under the Basel I (constant risk-weights) and the Basel II (varying risk-weights) presented in Chapter 3. In Section 5.3, we provide some comments about the numerical examples supporting our examples in Chapter 4.

1.3.5 Outline of Chapter 6

Aspects of the relationships between bank valuation and the SMC as well as Basel II are analyzed in Chapter 6. In Sections 6.1 and 6.2, we consider connections between the discrete-time stochastic models derived in the preceding discussions and the Subprime Mortgage Crisis (SMC) as well as the Basel II Capital Accord, respectively.

1.3.6 Outline of Chapter 7

Chapter 7 contains the conclusions that we can draw from the study. We also identify some of the research problems that may be addressed in future.

1.3.7 Outline of Chapter 8

Chapter 8 contains two tables outlining the main differences between the Basel I and the Basel II, summary of the main results of the cyclicality of bank credit and capital presented in Chapter 3 and the statements of the implicit function theorem and the Euler conditions, respectively.

1.3.8 Outline of Chapter 9

The bibliography in Chapter 9 lists the literature referred to throughout the dissertation.

Chapter 2

BANK VALUATION AND ITS OPTIMIZATION

Before examining the impact of the phases of the business cycle under the Basel I and the Basel II, we firstly introduce a few key concepts. Then we state and solve an optimal bank valuation problem. Throughout this section, we present most of the work already done in [49]. In this chapter, we assume that $r_t = r_t^{\mathrm{T}}$ and that

$$r_t^C C_t + r_t^B B_t + r_t^S S_t = 0$$

throughout (compare formula (1.13) for profit).

Problem 2.0.1 (Optimal Bank Valuation Problem): *Which decisions about loan rates, deposits and Treasuries must be made in order to attain an optimal bank value for a stockholder (possibly acting in the interests of a potential shareholder)?* (Theorem 2.3.1 in Chapter 2).

2.1 KEY BANKING CONCEPTS

In this section, we discuss banking concepts such as retained earnings and bank loan supply and demand.

2.1.1 Retained Earnings

We know that *bank profits*, Π_t, are used to meet the bank's commitments that include *dividend payments on equity*, $n_t d_t$, *interest and principal payments on subordinate debt*, $(1+r_t^O)O_t$, and *changes in loan-loss reserves*, $(1+r_t^{R^l})R_t^l$. The *retained earnings*, E_t^r, subsequent to these payments may be computed by using

$$\Pi_t = E_t^r + n_t d_t + (1 + r_t^O)O_t + (1 + r_t^{R^l})R_t^l. \tag{2.1}$$

In standard usage, retained earnings refer to earnings that are not paid out in dividends, interest or taxes. They represent wealth accumulating in the bank and should be capitalized in the value of the bank's equity. Retained earnings also are defined to include bank charter value income. Normally, *charter value* refers to the present value of anticipated profits from future lending. In each period, banks invest in fixed assets (including buildings and equipment) which we denote by F_t. The bank is assumed to maintain these assets throughout its existence so that the bank must only cover the costs related to the *depreciation of fixed assets*, ΔF_t. These activities are financed through retaining earnings and the eliciting of additional debt and equity, so that

$$\Delta F_t = E_t^r + (n_{t+1} - n_t)E_t + O_{t+1} + R_{t+1}^l. \tag{2.2}$$

We can use (2.1) and (2.2) to obtain an expression for bank capital of the form

$$K_{t+1} = n_t(d_t + E_t) + (1 + r_t^O)O_t + (1 + r_t^{R^l})R_t^l - \Pi_t + \Delta F_t, \tag{2.3}$$

where K_t is defined by (1.9).
If the expression for retained earnings given by (2.1) is substituted into (2.2), the *net cash flow generated by the bank* is given by

$$N_t = \Pi_t - \Delta F_t = n_t d_t + (1 + r_t^O)O_t + (1 + r_t^{R^l})R_t^l - K_{t+1} + n_t E_t, \tag{2.4}$$

2.1.2 Loan Supply and Demand

Taking our lead from the equilibrium arguments in [110], we denote loan demand and supply processes by $\Lambda = \{\Lambda_t\}_{t\geq 0}$. In this case, the bank faces a *Hicksian demand for loans* given by

$$\Lambda_t = l_0 - l_1 r_t^\Lambda + l_2 M_t + \sigma_t^\Lambda. \tag{2.5}$$

We note that the loan demand in (2.5) is an increasing function of M_t and a decreasing function of r_t^Λ. Further, we suppose that σ_t^Λ is the *random shock to the loan demand* with support $[\underline{\Lambda}, \overline{\Lambda}]$ that is independent of an exogenous stochastic variable, x_t. Also, we assume that the *loan supply process*, Λ, follows the first-order autoregressive stochastic process

$$\Lambda_{t+1} = \mu_t^\Lambda \Lambda_t + \sigma_{t+1}^\Lambda, \tag{2.6}$$

where $\mu_t^\Lambda = r_t^\Lambda - c^\Lambda - r^d(M_t)$ and σ_{t+1}^Λ denotes zero-mean stochastic shocks to loan supply.

2.2 STATEMENT OF OPTIMAL BANK VALUATION PROBLEM

Suppose that the bank's performance criterion, J, at t is given by

$$J_t = \Pi_t + l_t\Big[K_t - \rho\Big(\omega(M_t)\Lambda_t + \omega^I I_t + 12.5(mVaR + 0)\Big)\Big] - c_t^{dw}\Big[K_{t+1}\Big] + \mathbf{E}_t\Big[\delta_{t,1}V\Big(K_{t+1},\ x_{t+1}\Big)\Big], \tag{2.7}$$

where l_t is the Lagrangian multiplier for the total capital constraint, K_t is defined by (1.9), $\mathbf{E}_t\Big[\cdot\Big]$ is the expectation conditional on the bank's information at time t and x_t is the deposit withdrawals in period t with probability distribution $f(x_t)$. Also, c_t^{dw} is the deadweight cost of total capital consisting of equity, subordinate debt and loan loss reserves. The optimal bank valuation problem is to maximize the *bank value* given by

$$V_t = N_t + \mathbf{E}_t\Big[\sum_{j=1}^{\infty}\delta_{t,j}N_{t+j}\Big]. \tag{2.8}$$

We can now state the optimal valuation problem as follows.

Problem 2.2.1 (Statement of the Optimal Bank Valuation Problem): *Suppose that the total capital constraint and the performance criterion, J, are given by* (1.12) *and* (2.7), *respectively. The optimal bank valuation problem is to maximize the value of the bank given by* (2.8) *by choosing the loan rate, deposits and regulatory capital for*

$$V(K_t,\ x_t) = \max_{r_t^{\Lambda},\ D_t,\ K_t} J_t, \tag{2.9}$$

subject to the cash flow, balance sheet, financing constraint and loan demand given by (1.13), (1.10), (2.3) *and* (2.5) *respectively.*

2.3 SOLUTION TO THE OPTIMAL BANK VALUATION PROBLEM

In this section, we find a solution to Problem 2.2.1 when the capital constraint is binding as well as non-binding. In this regard, the main result can be stated and proved as follows.

Theorem 2.3.1 (Solution to the Optimal Bank Valuation Problem (Binding)): *Suppose that J and V are given by (2.7) and (2.9), respectively and $P(M_t) = \mathbf{E}(d)\Lambda_{t-1}$. When the capital constraint given by (1.12) is binding (i.e., $l_t > 0$), a solution to the optimal bank valuation problem yields an optimal bank loan supply and loan rate of the form*

$$\Lambda_t^* = \frac{K_t}{\rho\omega(M_t)} - \left(\frac{\omega^I I_t + 12.5(mVaR + 0)}{\omega(M_t)}\right) \tag{2.10}$$

and

$$r_t^{\Lambda *} = \frac{1}{l_1}\left(l_0 + l_2 M_t + \sigma_t^{\Lambda} - \left[\frac{K_t}{\rho\omega(M_t)} - \left(\frac{\omega^I I_t + 12.5(mVaR + 0)}{\omega(M_t)}\right)\right]\right), \tag{2.11}$$

respectively. In this case, the corresponding optimal deposits, provisions for deposit withdrawals and profits are given by

$$D_t^* = \overline{D} + \frac{\overline{D}(1-\gamma)}{r_t^p}\left[r_t^{\mathrm{T}} - \frac{(r_t^D + c^D)}{1-\gamma}\right] + \frac{K_t}{\rho\omega(M_t)} - \left(\frac{\omega^I I_t + 12.5(mVaR + 0)}{\omega(M_t)}\right) - K_t,$$

$$W_t^* = \overline{D} + \frac{\overline{D}(1-\gamma)}{r_t^p}\left[r_t^{\mathrm{T}} - \frac{(r_t^D + c^D)}{1-\gamma}\right]$$

and

$$\begin{aligned}\Pi_t^* &= \left[\frac{K_t}{\rho\omega(M_t)} - \left(\frac{\omega^I I_t + 12.5(mVaR+0)}{\omega(M_t)}\right)\right]\left\{\frac{1}{l_1}\left(l_0 - \frac{K_t}{\rho\omega(M_t)} + \left(\frac{\omega^I I_t + 12.5(mVaR+0)}{\omega(M_t)}\right)\right.\right.\\ &\left.\left. + l_2 M_t + \sigma_t^\Lambda\right) - \left(c^\Lambda + (r_t^D + c^D + r_t^T\gamma) + r^d(M_t)\right)\right\} + (r_t^D + c^D + r_t^T\gamma)K_t\\ &+ \overline{D}\left(r_t^T - (r_t^D + c^D + r_t^T\gamma)\right) + \left(r_t^T - \frac{(r_t^D + c^D)}{1-\gamma}\right)\left[\frac{\overline{D}(1-\gamma)(r_t^T - (r_t^D + c^D + r_t^T\gamma))}{r_t^P}\right]\\ &- c^w(W_t) - P(M_t) + r_t^I I_t,\end{aligned}$$

respectively.

Proof. An immediate consequence of the prerequisite that the total capital constraint from (1.12) is binding, is that loan supply is closely related to the capital adequacy constraint and is given by (2.10). Also, the dependence of changes in the loan rate on macroeconomic activity may be fixed as

$$\frac{\partial r_t^{\Lambda^*}}{\partial M_t} = \frac{l_2}{l_1}.$$

In (2.11) we substituted the optimal value for Λ_t into control law (2.5) to get the optimal default rate. We obtain the optimal W_t by using the following steps. Firstly, we rewrite (1.10) to make deposits the dependent variable so that

$$D_t = W_t + \Lambda_t - K_t.$$

Next, we note that the first order conditions are given by

$$\frac{\partial \Pi_t}{\partial r_t^\Lambda}\left[1 + c_t^{dw} - E_t\left\{\int_{\underline{\Lambda}}^{\overline{\Lambda}} \delta_{t,1}\frac{\partial V}{\partial K_{t+1}} dF(\sigma_{t+1}^\Lambda)\right\}\right] + l_1\rho^r\omega(M_t)l_t = 0; \tag{2.12}$$

$$\frac{\partial \Pi_t}{\partial D_t}\left[1 + c_t^{dw} - E_t\left\{\int_{\underline{\Lambda}}^{\overline{\Lambda}} \delta_{t,1}\frac{\partial V}{\partial K_{t+1}} dF(\sigma_{t+1}^\Lambda)\right\}\right] = 0; \tag{2.13}$$

$$\rho^r(12.5(mVaR+0) + \omega(M_t)\Lambda_t + \omega^I I_t) \leq K_t; \tag{2.14}$$

$$-c_t^{dw} + E_t\left\{\int_{\underline{\Lambda}}^{\overline{\Lambda}} \delta_{t,1}\frac{\partial V}{\partial K_{t+1}} dF(\sigma_{t+1}^\Lambda)\right\} = 0. \tag{2.15}$$

Here $F(\cdot)$ is the cumulative distribution of the shock to the loans. Using (2.15) we can see that (2.13) becomes

$$\frac{\partial \Pi_t}{\partial D_t} = 0.$$

Looking at the form of Π_t given in (1.13) and the equation

$$c^w(W_t) = \frac{r_t^p}{2\overline{D}}[\overline{D} - W_t]^2 \tag{2.16}$$

it follows that

$$\Pi_t = \left(r_t^{\Lambda} - c^{\Lambda} - r^d(M_t)\right)\Lambda_t + r_t^{\mathrm{T}} W_t + r_t^I I_t - \left(r_t^D + c^D\right) D_t \tag{2.17}$$
$$-\frac{r_t^p}{2\overline{D}}\left[\overline{D} - W_t\right]^2 - P(M_t) - r_t^{\mathrm{T}} \gamma D_t.$$

Therefore

$$\frac{\partial \Pi_t}{\partial D_t} = r_t^{\mathrm{T}} - (r_t^D + c^D) + \frac{r_t^p}{\overline{D}}\left(\overline{D} - W_t\right) - r_t^{\mathrm{T}}\gamma = 0. \tag{2.18}$$

This would then give us the optimal value for D_t. Using (1.10) and all the optimal values calculated to date, we can find optimal deposits, and the same goes for optimal profits. □

Corollary 2.3.2 (Solution to the Optimal Bank Valuation Problem (Non-Binding)): *Suppose that J and V are given by* (2.7) *and* (2.9), *respectively and $P(M_t) = \mathbf{E}(d)\Lambda_{t-1}$. When the capital constraint given by* (1.12) *is not binding (i.e., $l_t = 0$), a solution to the optimal bank valuation problem stated in Problem* 2.2.1 *yields the optimal bank loan supply and loan rate*

$$\Lambda_t^{*n} = \frac{1}{2}\left(l_0 + l_2 M_t + \sigma_t^{\Lambda}\right) - \frac{l_1}{2}\left(c^{\Lambda} + r^d(M_t) + (r_t^D + c^D) + r_t^{\mathrm{T}}\gamma\right) \tag{2.19}$$

and

$$r_t^{\Lambda*n} = \frac{1}{2l_1}\left(l_0 + l_2M_t + \sigma_t^{\Lambda}\right) + \frac{1}{2}\left(c^{\Lambda} + r^d(M_t) + (r_t^D + c^D) + r_t^{\mathrm{T}}\gamma\right), \tag{2.20}$$

respectively. In this case, the corresponding W_t, deposits and profits are given by

$$W_t^{*n} = \overline{D} + \frac{\overline{D}(1-\gamma)}{r_t^p}\left(r_t^{\mathrm{T}} - \frac{r_t^D + c^D}{1-\gamma}\right),$$

$$D_t^{*n} = \overline{D} + \frac{\overline{D}(1-\gamma)}{r_t^p}\left(r_t^{\mathrm{T}} - \frac{(r_t^D + c^D)}{1-\gamma}\right) + \Lambda_t^{*n} - K_t$$

and

$$\begin{aligned}\Pi_t^{*n} = &\frac{1}{2}\left(l_0 + l_2M_t + \sigma_t^{\Lambda}\right) - \frac{l_1}{2}\left(c^{\Lambda} + (r_t^D + c^D) + r^d(M_t) + r_t^{\mathrm{T}}(\gamma)\right)\Big\{\frac{1}{2l_1}\left(l_0 + l_2M_t + \sigma_t^{\Lambda}\right) \\ &-\frac{1}{2}\left(c^{\Lambda} + (r_t^D + c^D) + r^d(M_t) + r_t^{\mathrm{T}}\gamma\right)\Big\} + (r_t^D + c^D + r_t^{\mathrm{T}}\gamma)K_t + \overline{D}\Big(r_t^{\mathrm{T}} \\ &-(r_t^D + c^D + r_t^{\mathrm{T}}\gamma)\Big) + \left(r_t^{\mathrm{T}} - \frac{(r_t^D + c^D)}{1-\gamma}\right)\left[\frac{\overline{D}(1-\gamma)(r_t^{\mathrm{T}} - (r_t^D + c^D + r_t^{\mathrm{T}}\gamma))}{r_t^p}\right] - c^w(W_t) - P(M_t) + r^I I_t,\end{aligned}$$

respectively.

Proof. For the situation where capital constraint does not bind (i.e., $l_t = 0$), using equation (2.15) and the fact that $l_t = 0$, we can see that (2.12) becomes

$$\frac{\partial \Pi_t}{\partial r_t^{\Lambda}} = 0.$$

As in the proof of Theorem 2.3.1, looking at the form of Π_t given in (1.13) and (2.16), we have equation (2.17). Therefore

$$\frac{\partial \Pi_t}{\partial r_t^{\Lambda}} = \Lambda_t - l_1(r_t^{\Lambda} - c^{\Lambda} - r^d(M_t)) + l_1 r_t^{\mathrm{T}} + \frac{r_t^p}{\overline{D}}(\overline{D} - W_t)l_1 = 0. \tag{2.21}$$

By substituting (2.18) into (2.21) and using (2.5) would give us optimal loans and loan rate given by (2.19) and (2.20) respectively. Furthermore we can find the optimal deposit, deposit withdrawals and profits. □

Chapter 3

CYCLICALITY OF BANK CREDIT AND CAPITAL UNDER BASEL I AND II

In this chapter, we discuss the discrete-time dynamic model of bank items where, as in the Basel I, we assume that risk-weights are constant. Furthermore, (we) give the results for the situation where loan losses and loan risk-weights are a function of the phases of the business cycle (i.e., risk-weights vary with business cycle factors).

Problem 3.0.3 (Cyclicality of Bank Credit and Capital Under Basel I): *What is the effect of changes in the phases of the business cycle on bank credit and capital when asset risk-weights are constant (under Basel I) ? (see Section 3.1).*

Problem 3.0.4 (Cyclicality of Bank Credit and Capital Under Basel II): *What is the effect of changes in the phases of the business cycle on bank credit and capital when asset risk-weights vary with macroeconomic performance (under Basel II) ? (see Section 3.2).*

Assumption 3.0.5 *: It is assumed that banks make losses on loans but risk-weights are constant, as under the Basel I.*

Assumption 3.0.6 *: Asset risk-weights are assumed to vary over the business cycle, as under the Basel II.*

3.1 CYCLICALITY OF BANK CREDIT AND CAPITAL UNDER BASEL I (CONSTANT RISK-WEIGHTS)

In this section, we discuss the result of the model discussed in Chapter 2 under the assumption that bank loans may not be repaid and the response of bank loans in the influence of the changes in the business cycle. The risk-weight on loans is constant. In this case, the capital constraint that a bank faces when maximizing its profits is given by the following

$$K_t \geq \rho\left[\omega^\Lambda \Lambda_t + \omega^I I_t + 12.5mVaR\right]. \tag{3.1}$$

Here operational risk is not considered as in Basel II.

3.1.1 Cyclicality of the Quantity and Price of Loans and Bank Capital Under Basel I

In this subsection we examine how bank capital and loans are affected by changes in the phases of the business cycle.

Theorem 3.1.1 (Cyclicality of Bank Capital under Basel I): *Suppose that* $L(M_t) > 0$ *and* $\omega(M_t) = 1$ *(i.e risk-weights are constant). It follows that*

1. *if* $\frac{\partial \sigma_{t+1}^{\Lambda *}}{\partial M_t} < 0$ *then* $\frac{\partial K_{t+1}}{\partial M_t} > 0$.

2. *if* $\frac{\partial \sigma_{t+1}^{\Lambda *}}{\partial M_t} > 0$ *then* $\frac{\partial K_{t+1}}{\partial M_t} < 0$.

Proof. In order to prove Theorem 3.1.1, we need to use the implicit function theorem. Before we can use this theorem, we solve the critical shock to the loan demand, σ_t^{Λ}, such that the total capital constraint will just bind, i.e $\Lambda_t^{*n} = \Lambda_t^{*}$. By equating the optimal loans from the two problems (with $l_t = 0$ and $l_t > 0$), we obtain

$$\frac{1}{2}\left(l_0 + l_2 M_t + \sigma_t^{\Lambda}\right) - \frac{l_1}{2}\left(c^{\Lambda} + r^d(M_t) + (r_t^D + c^D) + r_t^{\mathrm{T}}\gamma\right) = \frac{K_t}{\rho} - \omega^I I_t - 12.5mVaR$$

Solving for σ_t^{Λ}, we get

$$\frac{1}{2}\sigma_t^{\Lambda} = -\frac{1}{2}\left(l_0 + l_2 M_t\right) + \frac{l_1}{2}\left(c^{\Lambda} + r^d(M_t) + (r_t^D + c^D) + r_t^{\mathrm{T}}\gamma\right) + \frac{K_t}{\rho} - \omega^I I_t - 12.5mVaR$$

therefore

$$\sigma_t^{\Lambda *} = 2\left(\frac{K_t}{\rho} - \omega^I I_t - 12.5mVaR\right) - \left(l_0 + l_2 M_t\right) + l_1\left(c^{\Lambda} + r^d(M_t) + (r_t^D + c^D) + r_t^{\mathrm{T}}\gamma\right).$$

Using the following equation

$$\Lambda_t - l_1(r_t^{\Lambda} - c^{\Lambda} - r^d(M_t)) + l_1 r_t^{\mathrm{T}} + l_1 \frac{r_t^p}{\overline{D}}(\overline{D} - W_t) + l_1 \rho l_t = 0, \tag{3.2}$$

and substituting provisions for deposit withdrawals we get

$$
\begin{aligned}
l_1 \rho l_t &= l_1(r_t^\Lambda - c^\Lambda - r^d(M_t)) - l_1 r_t^\mathrm{T} - l_1 \frac{r_t^p}{\overline{\overline{D}}}\left[\overline{D} - \left(\overline{D} + \frac{\overline{D}(1-\gamma)}{r_t^p}\left(r_t^\mathrm{T} - \frac{(r_t^D + c^D)}{1-\gamma}\right)\right)\right] - \Lambda_t \\
&= l_1(r_t^\Lambda - c^\Lambda - r^d(M_t)) - l_1 r_t^\mathrm{T} + l_1(1-\gamma)\left[r_t^\mathrm{T} - \frac{(r_t^D + c^D)}{1-\gamma}\right] - \Lambda_t \\
&= l_1(r_t^\Lambda - c^\Lambda - r^d(M_t) - (r_t^D + c^D + \gamma r_t^\mathrm{T})) - \Lambda_t.
\end{aligned}
$$

Substituting $r_t^{\Lambda^*}$ and Λ_t^* into the expression above, we obtain

$$
\begin{aligned}
l_1 \rho l_t = l_1 &\left[\frac{1}{l_1}\left((l_0 + l_2 M_t + \sigma_t^\Lambda) - \left[\frac{K_t}{\rho} - \omega^I I_t - 12.5 mVaR\right]\right) - c^\Lambda - r^d(M_t) - (r_t^D + c^D + \gamma r_t^\mathrm{T})\right] \\
&- \left(\frac{K_t}{\rho} - \omega^I I_t - 12.5 mVaR\right) \\
= (l_0 + l_2 M_t + \sigma_t^\Lambda) &- \left(\frac{K_t}{\rho} - \omega^I I_t - 12.5 mVaR\right) - l_1(c^\Lambda + r^d(M_t) + (r_t^D + c^D + \gamma r_t^\mathrm{T})) \\
&- \left(\frac{K_t}{\rho} - \omega^I I_t - 12.5 mVaR\right) \\
= \sigma_t^\Lambda - 2&\left(\frac{K_t}{\rho} - \omega^I I_t - 12.5 mVaR\right) + (l_0 + l_2 M_t) - l_1(c^\Lambda + r^d(M_t) + (r_t^D + c^D + \gamma r_t^\mathrm{T})) \\
= \sigma_t^\Lambda - &\left[2\left(\frac{K_t}{\rho} - \omega^I I_t - 12.5 mVaR\right) - (l_0 + l_2 M_t) + l_1(c^\Lambda + r^d(M_t) + (r_t^D + c^D + \gamma r_t^\mathrm{T}))\right] \\
&= \sigma_t^\Lambda - \sigma_t^{\Lambda *}.
\end{aligned}
$$

It follows that

$$
l_t^* = \frac{\sigma_t^\Lambda - \sigma_t^{\Lambda *}}{\rho l_1}, \qquad \sigma_t^{\Lambda *} \le \sigma_t^\Lambda \le \overline{\Lambda}.
$$

Using equation (2.9) to find the partial derivative of the value function with respect to bank capital we get

$$
\begin{aligned}
\frac{\partial V}{\partial K_t} &= l_t + (r_t^D + c^D + \gamma r_t^\mathrm{T}) \\
&= r_t^D + c^D + \gamma r_t^\mathrm{T}), \text{ for } \ \underline{\Lambda} \le \sigma_t^\Lambda \le \sigma_t^{\Lambda *} \\
&= (r_t^D + c^D + \gamma r_t^\mathrm{T}) + \frac{\sigma_t^\Lambda - \sigma_t^{\Lambda *}}{\rho l_1}, \text{ for } \ \sigma_t^{\Lambda *} \le \sigma_t^\Lambda \le \overline{\Lambda}.
\end{aligned}
$$

By substituting the above expression into the optimal condition for total capital (2.15), we get

$$c_t^{dw} - E_t\Big[\delta_{t,1}(r_t^D + c^D + \gamma r_t^{\mathrm{T}})\Big] - \frac{1}{\rho l_1} E_t\Big[\int_{\sigma_{t+1}^{\Lambda *}}^{\overline{\Lambda}} \delta_{t,1}(\sigma_{t+1}^{\Lambda} - \sigma_{t+1}^{\Lambda *}) dF(\sigma_{t+1}^{\Lambda})\Big] = 0.$$

We denote the left-hand side of the above expression by H, i.e.,

$$H = \frac{1}{\rho l_1} E_t\Big[\int_{\sigma_{t+1}^{\Lambda *}}^{\overline{\Lambda}} \delta_{t,1}(\sigma_{t+1}^{\Lambda} - \sigma_{t+1}^{\Lambda *}) dF(\sigma_{t+1}^{\Lambda})\Big] = 0. \tag{3.3}$$

By the implicit function theorem

$$\frac{\partial K_{t+1}}{\partial M_t} = -\frac{\partial H}{\partial M_t} \Big/ \frac{\partial H}{\partial K_{t+1}}.$$

To calculate for $\frac{\partial H}{\partial M_t}$, we use (3.3) and get

$$\frac{\partial H}{\partial M_t} = \frac{1}{\rho l_1}\Big\{\frac{\partial \sigma_{t+1}^{\Lambda *}}{\partial M_t} E_t\Big[\int_{\sigma_{t+1}^{\Lambda *}}^{\Lambda} \delta_{t,1} dF(\sigma_{t+1}^{\Lambda})\Big]\Big\},$$

where

$$\frac{\partial \sigma_{t+1}^{\Lambda *}}{\partial M_t} = -l_2 \mu^M + l_1 \mu^M \frac{\partial r^d}{\partial M_{t+1}}.$$

Therefore

$$\begin{aligned}\frac{\partial H}{\partial M_t} &= \frac{1}{\rho l_1}\Big\{\Big(-l_2\mu^M + l_1\mu^M \frac{\partial r^d}{\partial M_{t+1}}\Big) E_t\Big[\int_{\sigma_{t+1}^{\Lambda *}}^{\Lambda} \delta_{t,1} dF(\sigma_{t+1}^{\Lambda})\Big]\Big\} \\ &= \frac{\mu^M}{\rho}\Big(-\frac{l_2}{l_1} + \frac{\partial r^d}{\partial M_{t+1}}\Big) E_t\Big[\int_{\sigma_{t+1}^{\Lambda *}}^{\Lambda} \delta_{t,1} dF(\sigma_{t+1}^{\Lambda})\Big]\end{aligned}$$

and

$$\frac{\partial H}{\partial K_{t+1}} = \frac{2}{\rho}\frac{1}{\rho l_1}E_t\Big[\int_{\sigma^{\Lambda*}_{t+1}}^{\Lambda} \delta_{t,1} dF(\sigma^{\Lambda}_{t+1})\Big]$$
$$= \frac{2}{l_1\rho^2}E_t\Big[\int_{\sigma^{\Lambda*}_{t+1}}^{\Lambda} \delta_{t,1} dF(\sigma^{\Lambda}_{t+1})\Big].$$

Therefore

$$\frac{\partial K_{t+1}}{\partial M_t} = -\Big[\frac{\mu^M}{\rho}\Big(-\frac{l_2}{l_1}+\frac{\partial r^d}{\partial M_{t+1}}\Big)E_t\Big(\int_{\sigma^{\Lambda*}_{t+1}}^{\Lambda} \delta_{t,1} dF(\sigma^{\Lambda}_{t+1})\Big)\Big]\Big/\Big[\frac{2}{l_1\rho^2}E_t\Big(\int_{\sigma^{\Lambda*}_{t+1}}^{\Lambda} \delta_{t,1} dF(\sigma^{\Lambda}_{t+1})\Big)\Big]$$
$$= -\frac{\mu^M}{\rho}\Big(-\frac{l_2}{l_1}+\frac{\partial r^d}{\partial M_{t+1}}\Big)\Big/-(\frac{2}{l_1\rho^2}\Big)$$
$$= \frac{1}{2}l_1\rho\mu^M\Big(-\frac{l_2}{l_1}+\frac{\partial r^d}{\partial M_{t+1}}\Big)$$

$\frac{\partial K_{t+1}}{\partial M_t}$ is positive since $\frac{\partial \sigma^{\Lambda*}_{t+1}}{\partial M_t} < 0$. This concludes the proof of Theorem 3.1.1. □

Remark 3.1.2 (Cyclicality of Bank Capital under Basel I): *Under the assumption of positive loan losses and constant asset risk-weights, banks raise (decrease) their capital holdings in response to a positive (negative) shock in the current level of economic activity.*

3.1.2 Cyclicality of Loans and Loan Rates Under Basel I (Non-Binding Constraint)

In this subsection, we establish the impact of changes in M_t on loans and loan rates. We first present the results under a non-binding capital constraint.

Proposition 3.1.3 (Cyclicality of Loans and Loan Rates under Basel I (Non-Binding)): *If $l_t = 0$, then*

$$\frac{\partial \Lambda^{*n}{}_{t+j}}{\partial M_t} = \frac{1}{2}\mu^M_j\Big(l_2 - l_1\frac{\partial r^d(M_{t+j})}{\partial M_{t+j}}\Big)$$

and

$$\frac{\partial r^{\Lambda *n}_{t+j}}{\partial M_t} = \frac{1}{2}\mu^M_j \left(\frac{l_2}{l_1} + \frac{\partial r^d(M_{t+j})}{\partial M_{t+j}} \right).$$

Proof. In order to prove Proposition 3.1.3, we find the partial derivatives of the optimal loan supply and loan rate with respect to M_t of equations (2.19) and (2.20) respectively, from Chapter 2 under a non-binding constraint. But here we do not include operational risk. We also use the condition $\dfrac{\partial r^d(M_{t+j})}{\partial M_{t+j}} < 0$. Now we can calculate

$$\frac{\partial \Lambda^{*n}{}_{t+j}}{\partial M_t} \left(\frac{1}{2}(l_0 + l_2 M_t + \sigma^{\Lambda}_t) - \frac{l_1}{2}(c^{\Lambda} + r^d(M_t) + (r^D + c^D) + r^{\mathrm{T}}_t \gamma) \right)$$
$$= \frac{1}{2}\mu^M_j \left(l_2 - l_1 \frac{\partial r^d(M_{t+j})}{\partial M_{t+j}} \right)$$

and

$$\frac{\partial r^{\Lambda *n}_{t+j}}{\partial M_t} \left(\frac{1}{2l_1}(l_0 + l_2 M_t + \sigma^{\Lambda}_t) + \frac{1}{2}\left(c^{\Lambda} + r^d(M_t) + (r^D + c^D) + r^{\mathrm{T}}_t \gamma \right) \right)$$
$$= \frac{1}{2}\mu^M_j \left(\frac{l_2}{l_1} + \frac{\partial r^d(M_{t+j})}{\partial M_{t+j}} \right)$$

as required. □

Remark 3.1.4 (Cyclicality of Loans and Loan Rates under Basel I (Non-Binding)): *When the capital constraint (3.1) is non-binding, bank loans increase as a result of an improvement in macroeconomic activity, while the loan rate can either increase or decrease depending on the parameters characterizing the loan demand function and the loan default rate.*

3.1.3 Cyclicality of Loans and Loan Rates Under Basel I (Binding Constraint)

In this subsection, we consider the case in which the capital constraint binds.

Proposition 3.1.5 (Cyclicality of Loans and Loan Rates under Basel I (Binding)): *When $l_t > 0$ the loan supply is determined by the total capital constraint and is given by*

$$\Lambda_t^* = \frac{K_t}{\rho} - \omega^I I_t - 12.5mVaR$$

and

$$\frac{\partial \Lambda_{t+j}^*}{\partial M_t} = 0$$

while the loan rate response to changes in macroeconomic activity is

$$\frac{l_2}{l_1} = \frac{\partial r_{t+j}^{\Lambda^*}}{\partial M_t}.$$

Proof. In order to prove Proposition 3.1.5, we find the partial derivatives of the optimal loan supply and loan rate with respect to M_t of equations (2.10) and (2.11) to find $\frac{\partial \Lambda^*{}_{t+j}}{\partial M_t}$ and $\frac{\partial r_{t+j}^{\Lambda^*}}{\partial M_t}$ respectively. But here we do not include operational risk. Now we can calculate

$$\frac{\partial \Lambda_{t+j}^*}{\partial M_t}\left(\frac{K_t}{\rho\omega(M_t)} - \left[\frac{\omega^I I_t + 12.5mVaR}{\omega(M_t)}\right]\right) = 0$$

and

$$\frac{l_2}{l_1} = \frac{\partial r^{\Lambda_{t+j}^*}}{\partial M_t}\left(\frac{1}{l_1}\left(l_0 + l_2 M_t + \sigma_t^{\Lambda} - \frac{K_t}{\rho\omega(M_t)} + \left[\frac{\omega^I I_t + 12.5mVaR}{\omega(M_t)}\right]\right)\right).$$

□

Remark 3.1.6 (Cyclicality of Loans and Loan Rates under Basel I (Binding)): *When the capital constraint (3.1) is binding, current bank lending does not change in response to an improvement in the business cycle activity. The loan rate however increases as a consequence of a higher loan demand.*

3.2 CYCLICALITY OF BANK CREDIT AND CAPITAL UNDER BASEL II (VARYING RISK-WEIGHTS)

In this section, similar theorem and propositions to the ones in the previous section will be derived for a model where both loan losses and loan risk-weights are a function of the phases of the business cycle, M_t. The capital constraint is now described by the expression in (1.12), where the risk-weights on intangible assets $(\omega^I) \neq 0$ and the one on loans is a decreasing function of current phases of the business cycle, i.e, $\frac{\partial \omega(M_t)}{\partial M_t} < 0$.

3.2.1 Cyclicality of the Quantity and Price of Bank Loans and Capital Under Basel II

In this subsection, we examine how bank capital and loans are affected by changes in the phases of the business cycle when asset risk-weights are not constant.

Theorem 3.2.1 (Cyclicality of Bank Loans and Capital under Basel II): *Suppose that $L(M_t) > 0$ and risk-weights, $\omega(M_t)$ are not constant. In this case, we have that*

1. *if* $\frac{\partial \sigma_{t+1}^{\Lambda *}}{\partial M_t} < 0$ *then* $\frac{\partial K_{t+1}}{\partial M_t} > 0$;
2. *if* $\frac{\partial \sigma_{t+1}^{\Lambda *}}{\partial M_t} > 0$ *then* $\frac{\partial K_{t+1}}{\partial M_t} < 0$.

Proof. As in the proof of Theorem 3.1.1, we equate the optimal loans from the two problems (with $l_t = 0$ and $l_t > 0$) and obtain

$$\frac{1}{2}\left(l_0 + l_2 M_t + \sigma_t^{\Lambda}\right) - \frac{l_1}{2}\left(c^{\Lambda} + r^d(M_t) + (r_t^D + c^D) + r_t^{T}\gamma\right) = \frac{K_t}{\rho\omega(M_t)} - \left[\frac{\omega^I I_t + 12.5(mVaR + 0)}{\omega(M_t)}\right].$$

Solving for σ_t^{Λ}, we get

$$\frac{1}{2}\sigma_t^{\Lambda} = -\frac{1}{2}\left(l_0 + l_2 M_t\right) + \frac{l_1}{2}\left(c^{\Lambda} + r^d(M_t) + (r_t^D + c^D) + r_t^{T}\gamma\right) + \frac{K_t}{\rho\omega(M_t)} - \left[\frac{\omega^I I_t + 12.5(mVaR + 0)}{\omega(M_t)}\right]$$

therefore

$$\sigma_t^{\Lambda*} = 2\left(\frac{K_t}{\rho\omega(M_t)} - \left[\frac{\omega^I I_t + 12.5(mVaR + 0)}{\omega(M_t)}\right]\right) - \left(l_0 + l_2 M_t\right) + l_1\left(c^\Lambda + r^d(M_t) + (r_t^D + c^D) + r_t^{\mathrm{T}}\gamma\right).$$

Using equation (3.2) and substituting provisions for the deposit withdrawals we get

$$\begin{aligned}
l_1\rho\omega(M_t)l_t &= l_1(r_t^\Lambda - c^\Lambda - r^d(M_t)) - l_1 r_t^{\mathrm{T}} - l_1\frac{r_t^p}{\overline{D}}\left[\overline{D} - \left(\overline{D} + \frac{\overline{D}(1-\gamma)}{r_t^p}\left(r_t^{\mathrm{T}} - \frac{(r_t^D + c^D)}{1-\gamma}\right)\right)\right] - \Lambda_t \\
&= l_1(r_t^\Lambda - c^\Lambda - r^d(M_t)) - l_1 r_t^{\mathrm{T}} + l_1(1-\gamma)\left[r_t^{\mathrm{T}} - \frac{(r_t^D + c^D)}{1-\gamma}\right] - \Lambda_t \\
&= l_1(r_t^\Lambda - c^\Lambda - r^d(M_t) - (r_t^D + c^D + \gamma r_t^{\mathrm{T}})) - \Lambda_t.
\end{aligned}$$

Substitute $r_t^{\Lambda^*}$ and Λ_t^* into the expression above to obtain

$$\begin{aligned}
l_1\rho\omega(M_t)l_t &= l_1\left[\frac{1}{l_1}\left((l_0 + l_2 M_t + \sigma_t^\Lambda) - \frac{K_t}{\rho\omega(M_t)} + \left[\frac{\omega^I I_t + 12.5(mVaR + 0)}{\omega(M_t)}\right]\right) - c^\Lambda - r^d(M_t) - (r_t^D + c^D + \gamma r_t^{\mathrm{T}})\right] \\
&\qquad - \frac{K_t}{\rho\omega(M_t)} + \left[\frac{\omega^I I_t + 12.5(mVaR + 0)}{\omega(M_t)}\right] \\
&= (l_0 + l_2 M_t + \sigma_t^\Lambda) - \frac{K_t}{\rho\omega(M_t)} + \left[\frac{\omega^I I_t + 12.5(mVaR + 0)}{\omega(M_t)}\right] - l_1(c^\Lambda + r^d(M_t) + (r_t^D + c^D + \gamma r_t^{\mathrm{T}})) \\
&\qquad - \frac{K_t}{\rho\omega(M_t)} + \left[\frac{\omega^I I_t + 12.5(mVaR + 0)}{\omega(M_t)}\right] \\
&= \sigma_t^\Lambda - 2\left(\frac{K_t}{\rho\omega(M_t)} - \left[\frac{\omega^I I_t + 12.5(mVaR + 0)}{\omega(M_t)}\right]\right) + \left(l_0 + l_2 M_t\right) - l_1\left(c^\Lambda + r^d(M_t) + (r_t^D + c^D + \gamma r_t^{\mathrm{T}})\right) \\
&= \sigma_t^\Lambda - \left[2\left(\frac{K_t}{\rho\omega(M_t)} - \left[\frac{\omega^I I_t + 12.5(mVaR + 0)}{\omega(M_t)}\right]\right) - \left(l_0 + l_2 M_t\right) + l_1\left(c^\Lambda + r^d(M_t) + (r_t^D + c^D + \gamma r_t^{\mathrm{T}})\right)\right] \\
&= \sigma_t^\Lambda - \sigma_t^{\Lambda*}
\end{aligned}$$

therefore

$$l_t^* = \frac{\sigma_t^\Lambda - \sigma_t^{\Lambda*}}{\omega(M_t)\rho l_1}.$$

Using equation (2.9) to find the partial derivative of the value function with respect to bank capital we get

$$\begin{aligned}\frac{\partial V}{\partial K_t} &= l_t + (r_t^D + c^D + \gamma r_t^{\mathrm{T}}) \\ &= (r_t^D + c^D + \gamma r_t^{\mathrm{T}}), \quad \text{for} \quad \underline{\Lambda} \leq \sigma_t^{\Lambda} \leq \sigma_t^{\Lambda*} \\ &= (r_t^D + c^D + \gamma r_t^{\mathrm{T}}) + \frac{\sigma_t^{\Lambda} - \sigma_t^{\Lambda*}}{\omega(M_t)\rho l_1}, \quad \text{for} \quad \sigma_t^{\Lambda*} \leq \sigma_t^{\Lambda} \leq \overline{\Lambda}.\end{aligned}$$

By substituting the above expression into the optimal condition for total capital (2.15), we get

$$c_t^{dw} - E_t\left[\delta_{t,1}(r_t^D + c^D + \gamma r_t^{\mathrm{T}})\right] - \frac{1}{\omega(M_{t+1})\rho l_1} E_t\left\{ \int_{\sigma_{t+1}^{\Lambda*}}^{\overline{\Lambda}} \delta_{t,1}(\sigma_{t+1}^{\Lambda} - \sigma_{t+1}^{\Lambda*}) dF(\sigma_{t+1}^{\Lambda}) \right\} = 0.$$

We denote the left-hand side of the above expression by H, i.e,

$$H = \frac{1}{\omega(M_{t+1})\rho l_1} E_t\left\{ \int_{\sigma_{t+1}^{\Lambda*}}^{\overline{\Lambda}} \delta_{t,1}(\sigma_{t+1}^{\Lambda} - \sigma_{t+1}^{\Lambda*}) dF(\sigma_{t+1}^{\Lambda}) \right\}. \tag{3.4}$$

By the implicit function theorem

$$\frac{\partial K_{t+1}}{\partial M_t} = -\frac{\partial H}{\partial M_t} \Big/ \frac{\partial H}{\partial K_{t+1}}$$

To calculate for $\dfrac{\partial H}{\partial M_t}$, we use equation (3.4) and get

$$\begin{aligned}\frac{\partial H}{\partial M_t} = &-\frac{1}{\rho l_1} \frac{-\mu^M \frac{\partial \omega}{\partial M_{t+1}}}{[\omega(M_{t+1})]^2} E_t\left\{ \int_{\sigma_{t+1}^{\Lambda*}}^{\Lambda} \delta_{t,1}(\sigma_{t+1}^{\Lambda} - \sigma_{t+1}^{\Lambda*}) dF(\sigma_{t+1}^{\Lambda}) \right\} \\ &- \frac{1}{\rho l_1 \omega(M_{t+1})} \frac{\partial \sigma_{t+1}^{\Lambda*}}{\partial M_t} E_t\left\{ \int_{\sigma_{t+1}^{\Lambda*}}^{\Lambda} \delta_{t,1} dF(\sigma_{t+1}^{\Lambda}) \right\}\end{aligned}$$

where

$$\frac{\partial \sigma_{t+1}^{\Lambda*}}{\partial M_t} = -\frac{2}{\rho}\left(\frac{K_t - \rho(12.5(mVaR + 0) + \omega^I I_t)}{[\omega(M_{t+1})]^2}\right)\mu^M \frac{\partial \omega}{\partial M_{t+1}} - l_2\mu^M + l_1\mu^M \frac{\partial r^d}{\partial M_{t+1}}.$$

Therefore, we have that

$$\frac{\partial H}{\partial M_t} = -\frac{1}{\rho l_1}\frac{-\mu^M \frac{\partial \omega}{\partial M_{t+1}}}{[\omega(M_{t+1})]^2}E_t\Big\{\int_{\sigma^{\Lambda*}_{t+1}}^{\Lambda} \delta_{t,1}(\sigma^{\Lambda}_{t+1}-\sigma^{\Lambda*}_{t+1})dF(\sigma^{\Lambda}_{t+1})\Big\}$$
$$+\frac{1}{\omega(M_{t+1})\rho l_1}\left(\frac{2}{\rho}\right)\left(\frac{K_t-\rho(12.5(mVaR+0)+\omega^I I_t)}{[\omega(M_{t+1})]^2}\right)\mu^M\frac{\partial \omega}{\partial M_{t+1}}E_t\Big\{\int_{\sigma^{\Lambda*}_{t+1}}^{\Lambda} \delta_{t,1}dF(\sigma^{\Lambda}_{t+1})\Big\}$$
$$-\frac{1}{\omega(M_{t+1})\rho l_1}\left(-l_2\mu^M + l_1\mu^M\frac{\partial r^d}{\partial M_{t+1}}\right)E_t\Big\{\int_{\sigma^{\Lambda*}_{t+1}}^{\Lambda} \delta_{t,1}dF(\sigma^{\Lambda}_{t+1})\Big\}$$

and

$$\frac{\partial H}{\partial K_{t+1}} = \frac{2}{\omega(M_{t+1})\rho}\frac{1}{\omega(M_{t+1})\rho l_1}E_t\Big\{\int_{\sigma^{\Lambda*}_{t+1}}^{\Lambda} \delta_{t,1}dF(\sigma^{\Lambda}_{t+1})\Big\}$$
$$= \frac{2}{l_1[\omega(M_{t+1})\rho]^2}E_t\Big\{\int_{\sigma^{\Lambda*}_{t+1}}^{\Lambda} \delta_{t,1}dF(\sigma^{\Lambda}_{t+1})\Big\}.$$

Therefore, we have that

$$\frac{\partial K_{t+1}}{\partial M_t} = -\frac{\partial H}{\partial M_t}\Big/\frac{\partial H}{\partial K_{t+1}}$$
$$= \frac{-\frac{1}{\rho l_1}\Big\{\frac{-\mu^M\frac{\partial\omega}{\partial M_{t+1}}}{[\omega(M_{t+1})]^2}E_t\Big[\int_{\sigma^{\Lambda*}_{t+1}}^{\Lambda}\delta_{t,1}(\sigma^{\Lambda}_{t+1}-\sigma^{\Lambda*}_{t+1})dF(\sigma^{\Lambda}_{t+1})\Big]-\frac{1}{\omega(M_{t+1})}\frac{\partial\sigma^{\Lambda*}_{t+1}}{\partial M_t}E_t\Big[\int_{\sigma^{\Lambda*}_{t+1}}^{\Lambda}\delta_{t,1}dF(\sigma^{\Lambda}_{t+1})\Big]\Big\}}{\frac{2}{l_1[\omega(M_{t+1})\rho]^2}E_t\Big[\int_{\sigma^{\Lambda*}_{t+1}}^{\Lambda}\delta_{t,1}dF(\sigma^{\Lambda}_{t+1})\Big]}$$

is positive only if $\frac{\partial\sigma^{\Lambda*}_{t+1}}{\partial M_t} < 0$. □

Remark 3.2.2 (Cyclicality of Bank Loans and Capital under Basel II) *Under the assumptions of positive loan losses and a risk-sensitive capital constraint, banks can either raise or lower their capital holdings in response to a positive shock in the current level of economic activity. Their choice depends on the effect that the changes in M_t has on the likelihood of the capital constraint binding next period.*

3.2.2 Cyclicality of Loans and Loan Rates Under Basel II (Non-Binding Constraint)

We now turn to the effect of a shock to M_t on bank loans and the loan rate and analyze the case, when the capital constraint is not binding.

Proposition 3.2.3 (Cyclicality of Bank Loans under Basel II (Non-Binding)): *Under the same assumptions of Theorem 3.2.1, and when* $l_t = 0$

$$\frac{\partial \Lambda^{*n}{}_{t+j}}{\partial M_t} = \frac{1}{2}\mu_j^M \left(l_2 - l_1 \frac{\partial r^d(M_{t+j})}{\partial M_{t+j}} \right)$$

and

$$\frac{\partial r_{t+j}^{\Lambda *n}}{\partial M_t} = \frac{1}{2}\mu_j^M \left(\frac{l_2}{l_1} + \frac{\partial r^d(M_{t+j})}{\partial M_{t+j}} \right).$$

Proof. In order to prove Proposition 3.2.3, we find the partial derivatives of the optimal loan supply and loan rate with respect to M_t. We use equations (2.19), (2.20) and the condition $\frac{\partial r^d(M_{t+j})}{\partial M_{t+j}} < 0$. Now we can calculate

$$\begin{aligned} \frac{\partial \Lambda^{*n}{}_{t+j}}{\partial M_t} &\left(\frac{1}{2}(l_0 + l_2 M_t + \sigma_t^\Lambda) - \frac{l_1}{2}(c^\Lambda + r^d(M_t) + (r_t^D + c^D) + r_t^{\mathrm{T}}\gamma) \right) \\ &= \frac{1}{2}\mu_j^M \left(l_2 - l_1 \frac{\partial r^d(M_{t+j})}{\partial M_{t+j}} \right) \end{aligned}$$

and

$$\frac{\partial r_{t+j}^{\Lambda *n}}{\partial M_t} \left(\frac{1}{2l_1}(l_0 + l_2 M_t + \sigma_t^\Lambda) + \frac{1}{2}\left(c^\Lambda + r^d(M_t) + (r_t^D + c^D) + r_t^{\mathrm{T}}\gamma \right) \right)$$

$$= \frac{1}{2}\mu_j^M \left(\frac{l_2}{l_1} + \frac{\partial r^d(M_{t+j})}{\partial M_{t+j}} \right)$$

as required.

□

It can be seen that when the capital constraint is non-binding, the models with constant and non-constant risk-weights yield the same results.

Remark 3.2.4 (Cyclicality of Bank Loans under Basel II (Non-Binding)): *Under the same assumptions of Theorem 3.2.1, and when the capital constraint (1.12) is non-binding, bank loans increase as a result of an improvement in the business cycle* *activity. The loan rate can either increase or decrease depending on the parameters characterizing the loan default rate and the loan demand function.*

3.2.3 Cyclicality of Loans and Loan Rates under Basel II (Binding Constraint)

We now present the result of the effect of the changes in the business cycle on bank loans when the capital constraint is binding.

Proposition 3.2.5 (Cyclicality of Bank Loans under Basel II (Binding))*: Under the same assumptions of Theorem 3.2.1, and if $l_t > 0$, then by taking the first derivatives of the following equation with respect to M_t,*

$$\Lambda_t^* = \frac{K_t}{\omega(M_t)\rho} - \left(\frac{\omega^I I_t + 12.5(mVaR + 0)}{\omega(M_t)}\right)$$

we get

$$\frac{\partial \Lambda_t^*}{\partial(M_t)} = -\frac{K_t - \rho(12.5(mVaR + 0) + \omega^I I_t)}{[\omega(M_t)]^2 \rho} \frac{\partial \omega(M_t)}{\partial(M_t)}.$$

while the loan rate response to the changes in the business cycle is

$$\frac{\partial r_{t+j}^{\Lambda^*}}{\partial M_t} = \frac{l_2}{l_1} + \frac{K_t - \rho(12.5(mVaR + 0) + \omega^I I_t)}{[\omega(M_t)]^2 \rho l_1} \frac{\partial \omega(M_t)}{\partial(M_t)}.$$

Proof. In order to prove Proposition 3.2.5, we find the partial derivatives of the optimal loan supply and loan rate with respect to M_t of equations (2.10) and (2.11) and the condition $\frac{\partial \omega(M_{t+j})}{\partial M_{t+j}} < 0$ to find $\frac{\partial \Lambda^*{}_{t+j}}{\partial M_t}$ and $\frac{\partial r_{t+j}^{\Lambda^*}}{\partial M_t}$ respectively. Now we can calculate

$$\frac{\partial \Lambda^*{}_{t+j}}{\partial M_t}\left(\frac{K_t}{\rho\omega(M_t)} - \left[\frac{\omega^I I_t + 12.5(mVaR + 0)}{\omega(M_t)}\right]\right)$$
$$= -\frac{K_t - \rho(12.5(mVaR + 0) + \omega^I I_t)}{[\omega(M_t)]^2 \rho} \frac{\partial \omega(M_t)}{\partial(M_t)}$$

and

$$\frac{\partial r_{t+j}^{\Lambda*}}{\partial M_t}\left(\frac{1}{l_1}\left(l_0 + l_2 M_t + \sigma_t^{\Lambda} - \frac{K_t}{\rho\omega(M_t)} + \frac{\omega^I I_t + 12.5(mVaR + 0)}{\omega(M_t)}\right)\right)$$
$$= \frac{l_2}{l_1} + \frac{K_t - \rho(12.5(mVaR + 0) + \omega^I I_t)}{[\omega(M_t)]^2 \rho l_1} \frac{\partial \omega(M_t)}{\partial (M_t)}$$

as required. □

Remark 3.2.6 (Cyclicality of Bank Loans under Basel II (Binding)): *Under the same assumptions of Theorem 3.2.1, and when the capital constraint (1.12) is binding, current bank lending increases in response to an improvement in business cycle activity. The loan rate can either increase or decrease depending on the parameters characterizing the loan demand and the risk-weights.*

3.2.4 Cyclicality of Loans and Loan Rates Under Basel II (Future Periods)

In what follows we examine the effect of a current shock in future periods to the phases of the business cycle on bank loans and loan rates. Even here we look at two scenarios where the capital constraint binds or not.

3.2.4.1 Non-Binding Constraint

If the capital constraint does not bind, the response of loans and loan rates in period $j \geq 1$ to a current business cycle fluctuations is described by Theorem 3.2.1. However as time goes on, the impact of the shock dies out since $\mu_j^M < 1$.

3.2.4.2 Binding Constraint

In the future, if the capital constraint binds, then response of loans and loan rates to a change in M_t is described by

$$\frac{\partial \Lambda^*_{t+j}}{\partial (M_t)} = \frac{\mu^M_{j-1}}{\omega(M_{t+j})\rho}\left[\frac{\partial\Big(K_{t+j} - \rho(12.5(mVaR + 0) + \omega^I I_{t+j})\Big)}{\partial(M_{t-1+j})}\right] \tag{3.5}$$
$$-\frac{\mu^M_{j-1}}{\omega(M_{t+j})\rho}\left[\frac{\mu^M}{\omega(M_{t+j})}\Big(K_{t+j} - \rho(12.5(mVaR + 0) + \omega^I I_{t+j})\Big)\frac{\partial\omega(M_{t+j})}{\partial(M_{t+j})}\right]$$

and

$$\frac{\partial r^{\Lambda^*}_{t+j}}{\partial M_t} = \frac{l_2}{l_1}\mu^M_j - \frac{\mu^M_{j-1}}{\omega(M_{t+j})\rho l_1}\frac{\partial\Big(K_{t+j} - \rho(12.5(mVaR + 0) + \omega^I I_{t+j})\Big)}{\partial(M_{t-1+j})} \tag{3.6}$$
$$+\frac{\mu^M_j}{[\omega(M_{t+j})]^2\rho}\Big(K_{t+j} - \rho(12.5(mVaR + 0) + \omega^I I_{t+j})\Big)\frac{\partial\omega(M_{t+j})}{\partial(M_{t+j})}.$$

From the equation 3.5, it can be seen that future loans can either increase or decrease in response booms depending on the relative magnitudes of the terms in that equation. If capital decreases due to boom phases, loans can decrease provided that the effect of the shock on capital dominates the effect of the shock on loan risk-weights.

Chapter 4

NUMERICAL EXAMPLES

In this chapter, we consider the output gap as a proxy of the changes in the business cycle and the cyclicality of bank credit and capital. We present the output gap vs the business cycle in order to demonstrate that the output gap is a real proxy of the changes in the business cyclce. In addition, we present household debt, total private credit-to-GDP ratio, mortgage loans and interest (repo) rate vs output gap in order to demonstrate that bank credit is procyclical. Furthermore, we present issued primary share capital and capital adequacy ratios vs output gap in order to demonstrate that bank capital is acyclical. The data used in this chapter is sourced from the South African Reserve Bank (SARB).

The main problems that we address in this chapter are related to the cyclical effect of the dynamics of financial variables such as output gap, bank credit and capital.

Problem 4.0.7 (Relationship between Output Gap, Business Cycle and Bank Credit and Capital): *Can we establish a relationship between the South African output gap and composite business cycles ? (see 4.1).*

Problem 4.0.8 (Output Gap as a True Measure of Cyclicality): *Can we conjecture that output gap is a true measure of cyclicality by confirming that the business cycle and output gap in South Africa is strongly positively correlated ? (see 4.1).*

Problem 4.0.9 (Cyclicality of Bank Credit and Capital): *Can we draw conslusions about the cyclicality of bank credit and capital ? (see 4.2 and 4.3).*

4.1 OUTPUT GAP AND THE BUSINESS CYCLE

In Figure 4.1, we present the output gap vs the business cycle in order to demonstrate that the output gap is a real proxy of the changes in the business cycle.

For the period under investigation, viz., 1990 to 2006, South Africa has experienced three business cycles (see Table 1 and [113]). These may be identified as one major cycle for August 1992-November 1998 (peak in January 1995) and two minor ones for December 1998-August 2001 (turning point reached in February 2000) and September 2001-May 2003 (peak in April 2002). The same periods of cyclicality are discernible for the output gap. These trends are reflected in the graph below, where the output gap and business cycle are compared with each other. Figure 4.1 clearly demonstrates that a stronger positive correlation exists between the South African business cycles and output gap.

As a result of the above discussion, output gap may be considered to be a proxy for the business cycle. In replacing the business cycle with the output gap, we are able to better compare our study with that of others. Also, business cycles can only be determined subsequent to its realization while the output gap can be computed in real time and has a predictive quality.

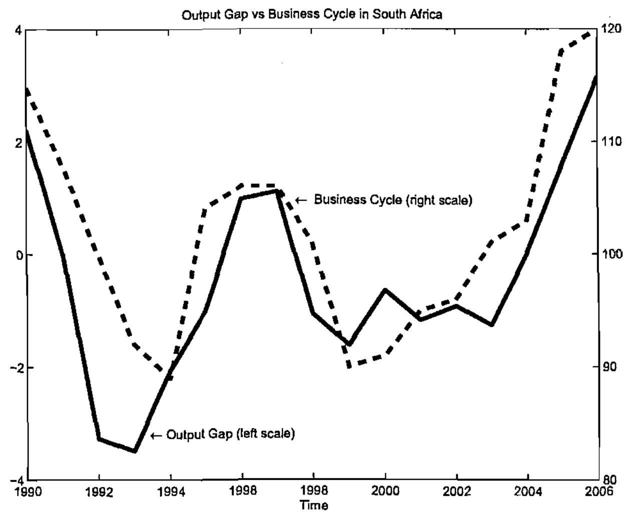

Figure 4.1: Output Gap vs Business Cycle

4.2 CYCLICALITY OF BANK CREDIT

In Figure 4.2, we present household debt vs output gap in order to demonstrate that bank credit is procyclical.

In Figure 4.3, we present total private credit-to-GDP ratio vs output gap in order to demonstrate that bank credit is procyclical.

From Figures 4.2 and 4.3, it is clear that household debt and total private sector credit-to-GDP ratio follows the output gap rather closely and as a result are both strongly procyclical. We note that since 1999, there has been a dramatic increase in the aforementioned ratio in South Africa.

In Figure 4.4, we present mortgage loans vs output gap in order to demonstrate that bank credit is procyclical.

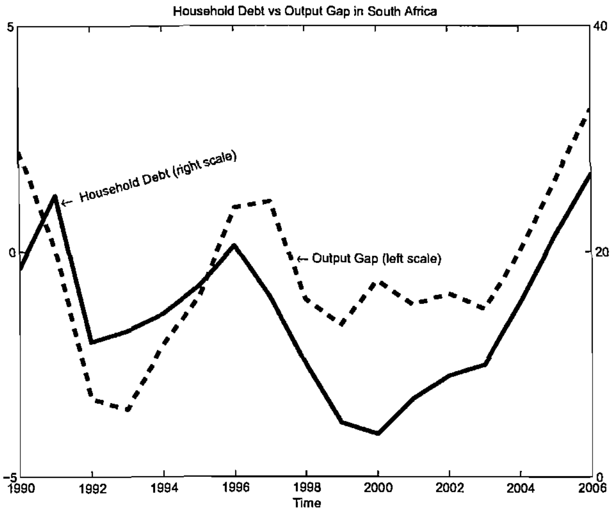

Figure 4.2: Household Debt vs Output Gap

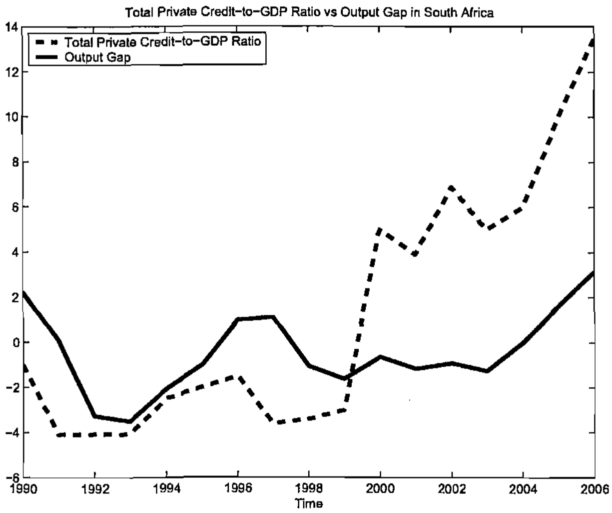

Figure 4.3: Total Private Credit-to-GDP Ratio vs Output Gap

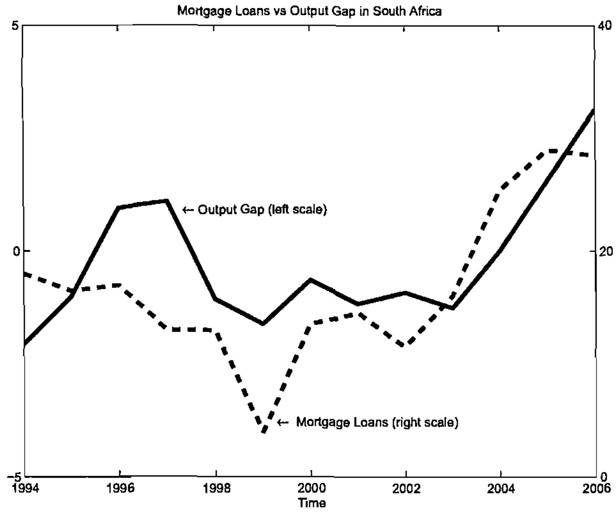

Figure 4.4: Mortgage Loans vs Output Gap

Figure 4.4 intimates that credit extension by financial institutions is strongly procyclical. In particular, annual growth in mortgage loans is procyclical. Mortgage loans are related to the level of the loan rate.

In Figure 4.5, we present interest (repo) rate vs output gap in order to demonstrate that bank credit is procyclical.

4.3 CYCLICALITY OF BANK CAPITAL

In Figure 4.6, we present issued primary share capital vs output gap in order to demonstrate that bank capital is acyclical.

In Figure 4.7, we present capital adequacy ratios vs output gap in order to demonstrate that bank capital is acyclical.

Figure 4.7 provides clear evidence that the Capital-to-Total Assets Ratio (CTAR) is acyclical for all three business cycles and beyond. On the other hand, the Capital-to-Risk Weighted Assets Ratio (CRWAR) appears to be weakly countercyclical for the August 1992-November 1998 business cycle and acyclical for the December 1998-August 2001 and September 2001-May 2003 cycles and beyond. Any discussion of the movements of South African CARs must take the increase from 8% to 10% in the minimum capital requirement from October 2001 onwards into account.

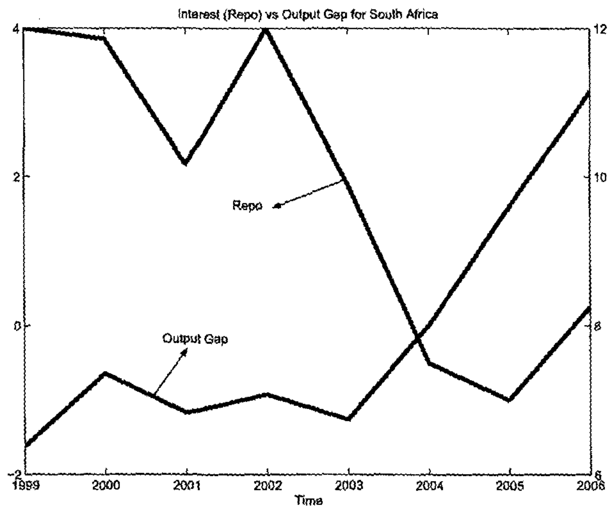

Figure 4.5: Interest (Repo) Rate vs Output Gap

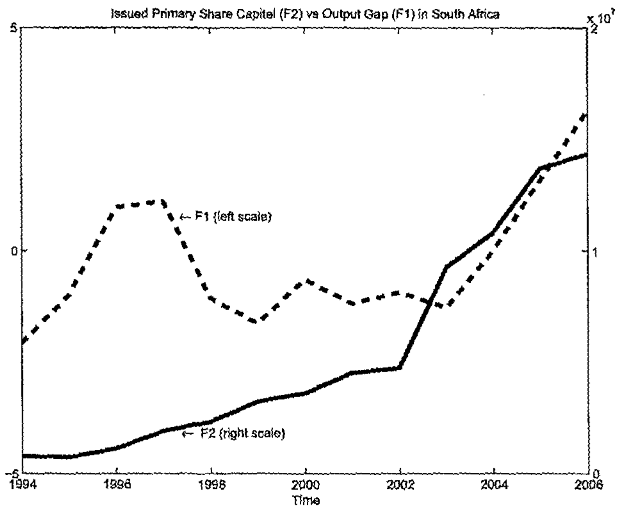

Figure 4.6: Issued Primary Share Capital vs Output Gap

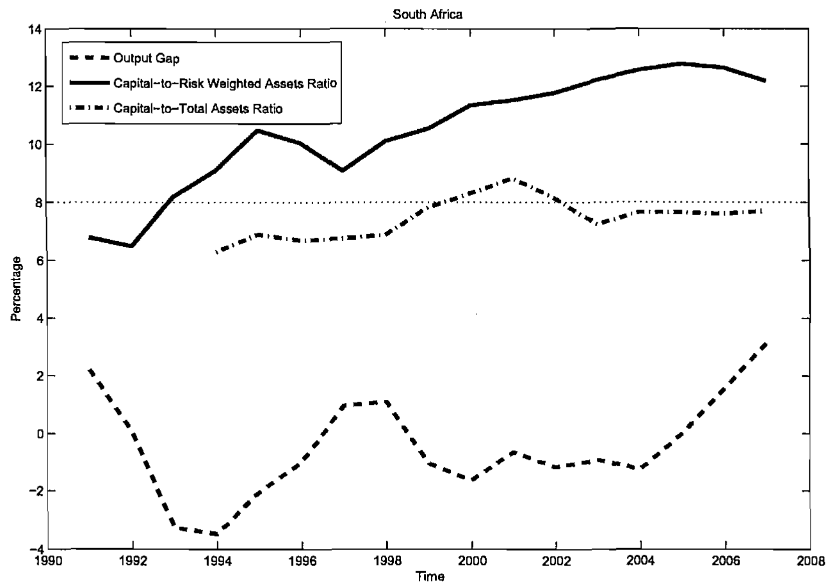

Figure 4.7: CARs vs Output Gap

Chapter 5

ANALYSIS OF CYCLICALITY OF BANK CREDIT AND CAPITAL

There are two versions of the model represented throughout our dissertation, the Basel I and Basel II. Both versions consist of two cases, binding constraint as well as non-binding constraint. In this regard, we analyze some of the main issues raised in Chapter 3. But first we have to discuss an optimal bank valuation problem presented in Chapter 2. Furthermore, we would like to extend the discussion on the connection between the cyclicality of output gap, bank credit and bank capital in South Africa represented in Chapter 4.

The issue of procyclical effects of the Basel II capital requirements is recently the subject of an extreme discussion in the financial and supervisory community. This dissertation presents important contributions to the discussion. We analyzed the effect of macroeconomic shocks on models for bank's lending under the two capital adequacy regimes (the Basel I and Basel II) in discrete-time. One of the result of the dissertation is that the response of banks to shocks that affect loan demand differs when the minimum capital requirements are calculated with risk-weights that are sensitive to the phases of the business cycle. In particular, bank capital is less volatile than under capital requirements with constant risk-weights. In this dissertation, we discussed procyclicality in the South African financial system. In particular, we have analyzed the effect that cyclicality in bank credit and bank capital has on financial stability. Our evidence is consistent with the view that the procyclicality of bank credit and bank capital might be a cause of financial distress in the South African system.

5.1 BANK VALUATION AND ITS OPTIMIZATION

In this section, we discuss some of the issues related to the optimal bank valuation problem presented in Chapter 2 when capital constraint is binding and non-binding.

5.1.1 Key Banking Concepts

In this subsection, we discuss all the banking concepts discussed in Chapter 2.

5.1.1.1 Retained Earnings

As far as the bank valuation is concerned, an interesting scenario from Subsection 2.1.1 to consider, is when $\Delta F_t = 0$ in (2.4). This provides another expression for profit of the form

$$N_t = \Pi_t = E_t^r + n_t d_t + (1 + r_t^O)O_t + (1 + r_t^{R^l})R_t^l.$$

If, in addition, $(1 + r_t^O)O_t = O_{t+1}$ and $(1 + r_t^{R^l})R_t^l = R_{t+1}^l$ then we may conclude that

$$\Pi_t = E_t^r + n_t d_t + O_{t+1} + R_{t+1}^l.$$

In turn, this results in the inequalities

$$\Pi_t > n_t d_t \Rightarrow n_{t+1} E_t < n_t E_t \text{ and } \Pi_t < n_t d_t \Rightarrow n_{t+1} E_t > n_t E_t.$$

Essentially, under the assumption that $\Delta F_t = 0$, the first statement implies that if the profit exceeds the dividends in period t then there may be a decline in the period $t + 1$ shareholder equity when compared with period t equity. The opposite is true for the second statement.

5.1.1.2 Loan Supply and Demand

Subsection 2.1.2 of Section 2.1 provides us with a description of the main components of a bank's lending activities. Banks respond differently to shocks that affect loan demand, Λ, when the minimum capital requirements are calculated by using risk-weighted assets. In the Hicksian case, these responses are usually sensitive to the phases of the business cycle that are related to the term $l_2 M_t$ in (2.5). Loan defaults are independent of the capital adequacy paradigm that is chosen. In this regard, empirical evidence supports the opinion that better phases of the business cycle reduce the loan default rate and thus the loan marginal cost.

5.1.2 Statement of the Optimal Bank Valuation Problem

Problem 2.2.1 in Chapter 2 addresses a very important issue in bank operations that is related to the optimal implementation of financial economic principles under regulatory constraint. In this regard, our investigation is largely motivated by the need to maximize profits. As far as the optimal loan rate and general interest rate decisions are concerned, market, credit (see, for instance, [69] and [85]) and interest rate risk are the main risks to be taken care of.

5.1.3 Solution to the Optimal Bank Valuation Problem

If we substitute the optimal bank dividends given by (2.15) in Section 2.3 into the optimal decisions for the loan rate and deposits represented by (2.12) and (2.13), respectively, we can obtain a time-independent solution for the optimal bank valuation problem. This leads to a significant reduction in the technical difficulty of the procedure.

5.2 CYCLICALITY OF BANK CREDIT AND CAPITAL UNDER BASEL I AND BASEL II

In this section, we discuss cyclicality of bank credit and capital under the Basel I and the Basel II presented in Chapter 3.

5.2.1 Cyclicality of Bank Credit and Capital Under Basel I (Constant Risk-Weights)

In this subsection, we analyze bank loan responses to the changes in the business cycle under the Basel I (constant risk-weights) presented in Section 3.1 when capital constraint is binding and non-binding.

When the capital constraint is not binding, bank loans and loan rate increase as a result of boom phases. However if the capital constraint binds, an increase in the loan demand will result in an increase in the loan rate, which will leave the loan supply unchanged. While booms increases the chances of bank capital binding in the future, banks increase the amount of capital they hold.

5.2.2 Cyclicality of Bank Credit and Capital Under Basel II (Varying Risk-Weights)

In this subsection, we analyze bank loan responses to the changes in the business cycle under the Basel II (risk-weights varying) presented in Section 3.2 when capital constraint is binding and non-binding.

In the Basel II model, a shock to current phases of the business cycle does not only affect the loan demand but also the risk-weights in bank's capital to asset ratios. If the capital constraint is not binding, bank loans increase as in the Basel I model, whereas if the capital constraint binds, banks can still expand their credit supply, but by a smaller amount compared to when the capital constraint is non-binding. They are able to do so because boom phases causes lower risk-weights and for that reason a slackening of the capital constraint. The lower rate can either increase or decrease, depending on the relative size of the change in loan demand and capital ratio. Similarly, recessionary phases results in a possibly greater reduction of credit than in the Basel I model because of both a decrease on loan demand as well as a tightening of the capital constraint. Furthermore, under the Basel II, the sign of the change in bank's capital holding is undetermined because boom phases has two counteracting effects on the equilibrium values of bank capital. On the one hand, booms has a persistent positive effect on loan demand and so raises the probability of the capital constraint binding in the future. At the same time, though, the capital ratio increases so that there are chances that the capital constraint might become lower.

5.3 NUMERICAL EXAMPLES

In this section, we provide some comments about the numerical examples supporting our examples in Chapter 4.

5.3.1 Discussion of Output Gap and Business Cycle

By considering Section 4.1, we have that the correlation between the composite business cycles and output gap may be summarized as follows.

FIGURE	CORRELATION	Full and Partial Business Cycles			
		08/92-11/98	12/98-08/01	09/01-05/03	06/03-12/06
Figure 4.1: Business Cycles vs Output Gap	Business Cycles and Output Gap	Strongly Positive	Strongly Positive	Strongly Positive	Strongly Positive

Table 5.1: Summary of Relationships between the Business Cycle and Output Gap in South Africa

The importance of Figure 4.1 is that it shows that the South African output gap is a good proxy for the composite business cycles determined by [113]. This has many advantages such as making country-to-country comparisons possible since smoothing actual GDP via the Hodrick-Prescott filter is an easy procedure to perform under almost any national economic dispensation.

5.3.2 Discussion of Bank Credit

In our opinion, the risk perceptions, the financial accelerator, excessive optimism and pessimism, individual economic agents and new market and credit risk all have a role to play in explaining the procyclicality of credit in South Africa. In this regard, by considering Section 4.2, we have that the cyclicality of credit and its correlation with the output gap may be summarized as in Table 5.2.

FIGURE	**VARIABLE**	**Full and Partial Business Cycles**			
		08/92-11/98	12/98-08/01	09/01-05/03	06/03-12/06
Figure 4.2: Annual Percentage Change in Household Debt vs Output Gap	Private Credit	Strongly Procyclical Strong Positive Correlation	Strongly Procyclical Strong Positive Correlation	Strongly Procyclical Strong Positive Correlation	Strongly Procyclical Strong Positive Correlation
Figure 4.3: Total Private Credit-to-GDP Ratio vs Output Gap	Private Credit	Strongly Procyclical Strong Positive Correlation	Strongly Procyclical Strong Positive Correlation	Strongly Procyclical Strong Positive Correlation	Strongly Procyclical Strong Positive Correlation
Figure 4.4: Annual Percentage Change in Mortgage Loans vs Output Gap	Credit Extension by Financial Institutions	Strongly Procyclical Strong Positive Correlation	Strongly Procyclical Strong Positive Correlation	Strongly Procyclical Strong Positive Correlation	Strongly Procyclical Strong Positive Correlation

Table 5.2: Summary of the Cyclicality of Bank Credit in South Africa

It appears that measures of credit risk behave as if such risk declined during booms and rose only close to the peak as the recession set in. The relaxation of credit limitations can have both a direct or indirect effect on valuations. Indirectly, expenditures on goods and services tend to generate an upswing in economic activity.

5.3.3 Discussion of Bank Capital

From Section 4.3, we have that the cyclicality of capital requirements and its correlation with the output gap may be summarized as in Table 5.3.

FIGURE	VARIABLE	Full and Partial Business Cycles			
		08/92-11/98	12/98-08/01	09/01-05/03	06/03-12/06
Figure 4.6: Issued Primary Share Capital vs Output Gap	Bank Capital	Acyclical No Correlation	Acyclical No Correlation	Acyclical No Correlation	Pro-cyclical Positive Correlation
Figure 4.7: Capital-to-Total Assets Ratio and Capital-to-Risk Weighted Assets Ratio vs Output Gap	CTARs and CRWARs	CTAR: Acyclical No Correlation CRWAR: Weakly Counter cyclical Weak Positive Correlation	CTAR: Acyclical No Correlation CRWAR: Acyclical No Correlation	CTAR: Acyclical No Correlation CRWAR: Acyclical No Correlation	CTAR: Acyclical No Correlation CRWAR: Acyclical No Correlation

Table 5.3: Summary of the Cyclicality of Bank Capital in South Africa

The features of Figure 4.7 of Section 4.3 as summarized in Table 5.3, suggest that an uncertain relationship exists between bank capital and the output gap. We also note that banks have to comply to minimum regulatory capital requirements irrespective of the phase of the business cycle. In particular, we note that the minimum requirement increased from 8% to 10% from October 2001 onwards. Figure 4.7 suggests that, in South Africa, CRWARs were weakly countercyclical for the August 1992-November

1998 business cycle and acyclical thereafter. This may, also, be due to the fact that banks have to satisfy minimum requirements irrespective of the phase of the business cycle. Also, for the boom period August 2002-January 1995, banks improved their capital position as measured by the CRWARs with the overall objective of repairing their balance sheets. This behavior was partly fueled by pressure from rating agencies and the financial markets. In addition, CTARs appear to be acyclical for more or less the same reasons.

However, in principle, the Basel II dictates that a macroeconomic shock should influence the loan risk-weights in the CRWAR with the bank capital being affected in the following ways (see [23]). Generally, a negative (positive) shock results in the tightening (loosening) of the capital constraints. As a consequence, in terms of a possible binding capital constraint, banks are at liberty to increase (decrease) the loan supply when the phases of the business cycle improve (deteriorate). On the other hand, if the risk-weights are constant, a shock does not affect the loan supply but rather results in a change in the loan rate when the capital constraint binds. It is not always true that the Basel II risk-sensitive weights lead to an increase (decrease) in bank capital when macroeconomic activity in the loan market increases (decreases). A simple explanation for this is that the phases of the business cycle do not necessarily only affect loan demand but also influences the total capital constraint. Furthermore, banks do not necessarily need to raise new capital to expand their loan supply, since boom phases result in a decrease in the RWAs with a corresponding increase in CRWARs. Similarly, banks are not compelled to decrease their capital when the loan demand decreases since the capital constraint usually tightens in response to a recessionary phases. A further complication is that an improvement in economic conditions may result in an increase in the loan demand and, as a consequence, an increase in the probability that the capital constraint will be binding. Banks may react to this situation by increasing capital to maximize profits (compare the return on equity (ROE)).

Historical data confirms that, in general, there has been a sharp decline in CARs from 1990 to 1992 before a steady increase thereafter. However, the task of detecting cyclicality is made difficult by the introduction of the Basel I in 1988, which may have caused a structural change in CARs in some countries (for a survey of the impact of the Basel I see, for instance, [64]. Nevertheless, global historical data do not suggest a strong business cycle effect in CARs (see [23]). Furthermore, in South Africa, the cyclicality of the CRWAR has been much more pronounced than the cyclicality of the CTARs. This reflects the fact that subsequent to the banking crises in the late 1990's, RWAs fell more strongly as banks shifted their portfolios away from relatively high risk-weighted commercial lending towards residential mortgages and public sector securities (both of which have relatively low risk-weights).

Chapter 6

BANK VALUATION AND ITS CONNECTIONS WITH THE SMC AND BASEL II

In this chapter, we consider connections between the discrete-time stochastic models derived in the preceding discussions and the Subprime Mortgage Crisis (SMC) as well as the Basel II Capital Accord.

6.1 BANK VALUATION AND THE SUBPRIME MORTGAGE CRISIS

The SMC is an ongoing crisis characterized by shrinking liquidity in global credit markets and banking systems. A downturn in the U.S. housing market, risky practices by lenders and borrowers and excessive individual and corporate debt levels have affected the world economy adversely on a number of levels. The SMC has exposed pervasive weaknesses in the global financial system and regulatory framework. The connections between this crisis and our banking models are mainly forged via the bank's

- risk premium, ϱ, from (1.4) in Subsection 1.2.5 (see HM1 and HM2 in Figure 6.1 below);
- required capital sensitivity to changes in the amount of loans extended as given by (1.5) in Subsection 1.2.5 (see FM2 in Figure 6.1 below);
- base rate, r, from (1.6) in Subsection 1.2.5 decided upon by Central Banks as well as interbank loan rates (see HM1, HM3, FM4 and GIR1 in Figure 6.1 below);
- own loan rate, r^{Λ}, from (1.6) and (2.11) and (2.20) in Subsection 1.2.5 and Section 2.3, respectively (see HM1, HM3, FM4 and GIR1 in Figure 6.1 below);
- loan demand represented by (2.5) in Subsection 2.1.2 (see HM3, FM4, GIR1 and GIR5 in Figure 6.1 below);
- loan losses and default rate given by (1.2) in Subsection 1.2.5 (see HM4, FM1, FM3, GIR3 and GIR5 in Figure 6.1 below);
- loan loss provisions from (1.7) and (1.8) in Subsection 1.2.5 (see HM4, FM1, FM3, GIR3 and GIR5 in Figure 6.1 below);
- choice between raising deposits and interbank borrowing (including borrowing from the Central Bank) as reflected by (1.14) in Subsection 1.2.9 (see GIR1 in Figure 6.1 below);

- liquidity as described by Remark 1.2.2 (see FM4 in Figure 6.1 below);
- profit, Π, given by (1.13) in Subsection 1.2.7 (see HM2, HM4, FM1, FM3, GIR1 and GIR5 in Figure 6.1 below);
- bank valuation performance criterion, J, given by (2.7) in Section 2.2 (see HM4, HM6, FM1, FM2 and FM3 in Figure 6.1 below).

6.1.1 Diagrammatic Overview of the Subprime Mortgage Crisis

A diagrammatic overview of the SMC (see, for instance, [117]) may be represented as follows.

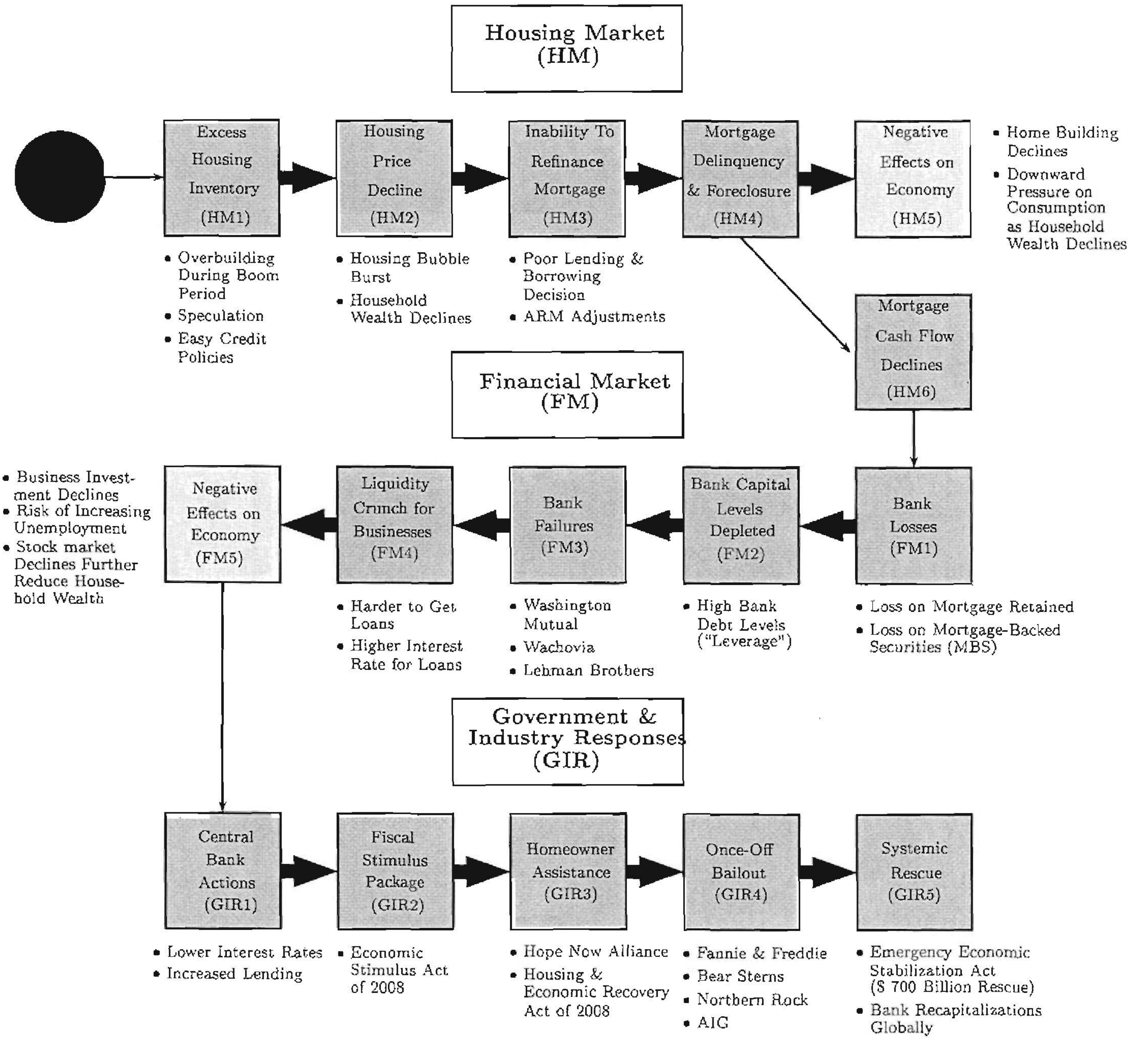

Figure 6.1: Diagrammatic Overview of the Subprime Mortgage Crisis (compare [117])

6.1.2 Background to the Subprime Mortgage Crisis

Most of the information contained in this subsection was sourced from [117]. The SMC was initiated by the deflation of the United States housing bubble (see, for instance, [20] and [78]) and high default rates on subprime and adjustable rate mortgages (ARM). Loan incentives, such as easy initial terms and low loan rates, in combination with escalating housing prices encouraged borrowers to assume difficult mortgages on the belief they would be able to quickly refinance at more favorable terms (see HM1 in Figure 6.1). Great concern was also expressed about the rapid growth in business loans at commercial banks with excessively easy credit standards. Some analysts claim that competition for lenders had greatly increased, causing banks to reduce loan rates and ease credit standards in order to issue new credit. Others are of the opinion that as the economic expansion continued and past loan losses were forgotten, banks exhibited a greater propensity for risk. However, once U.S. housing prices started to fall moderately in 2006-2007, refinancing became more difficult (see HM2 and HM3 in Figure 6.1). Defaults and foreclosure activity increased dramatically, as easy initial terms expired, home prices failed to go up as anticipated, and ARM interest rates reset higher. Foreclosures accelerated in the U.S. in late 2006 and triggered a global financial crisis through 2007 and 2008. During 2007, nearly 1.3 million U.S. housing properties were subjected to foreclosure activity; up 79 % from 2006 (see [104] for more details; also HM4 in Figure 6.1). The mortgage lenders that retained credit risk were the first to be affected, as borrowers became unable or unwilling to make payments (see HM5 and HM6 in Figure 6.1).

Major banks and other financial institutions globally had reported losses of approximately \$ 435 billion from SMC-related activities by Thursday, 17 July 2008 (see [47] and [94]; also FM1 in Figure 6.1). By using securitization strategies, many mortgage lenders passed the rights to the mortgage payments and related credit risk to third-party investors via mortgage-backed securities (MBS) and collateralized debt obligations (CDO). Corporate, individual and institutional investors holding MBS or CDO suffered significant losses, as the underlying mortgage asset value decreased. Stock markets in many countries declined significantly (see FM2 in Figure 6.1). The broader international financial sector first began to experience the fallout from the SMC in February 2007 with the \$ 10.5 billion writedown of HSBC, which was the first major CDO or Mortgage Bankers Association (MBA) related loss to be reported. During 2007, at least 100 mortgage companies had either failed, suspended operations or been sold. Top management did not escape without blemish, as the CEOs of Merrill Lynch and Citigroup were forced to resign within a week of each other. Subsequently, merger deals were struck by many institutions. In addition, Northern Rock and Bear

Stearns required emergency assistance from central banks. IndyMac was shut down by the FDIC on Sunday, 11 July 2008. Moreover, on Sunday, 14 September 2008, after performing banking duties for more than 150 years, Lehman Brothers filed for bankruptcy as a consequence of losses stemming from the SMC (see FM3 in Figure 6.1). Subsequent to this many U.S. and other banks throughout the world also failed. The widespread dispersion of credit risk and the unclear effect on financial institutions caused reduced lending activity and increased spreads on higher interest rates. Similarly, the ability of corporations to obtain funds through the issuance of commercial paper was affected. This aspect of the crisis is consistent with a credit crunch. There are a number of reasons why banks may suddenly make obtaining a loan more difficult or increase the costs of obtaining a loan. This may be due to an anticipated decline in the value of the collateral used by the banks when issuing loans; an increased perception of risk regarding the solvency of other banks within the banking system; a change in monetary conditions (for example, where the central bank suddenly and unexpectedly raises interest rates or capital requirements); the central government imposing direct credit controls and instructing the banks not to engage in further lending activity. The subprime crisis has adversely affected several inputs in the economy, resulting in downward pressure on economic growth. Fewer and more expensive loans tend to result in decreased business investment and consumer spending (see FM4 and FM5 in Figure 6.1).

Liquidity concerns drove central banks around the world to take action to provide funds to member banks to encourage lending to worthy borrowers and to restore faith in the commercial paper markets (see GIR1 in Figure 6.1). With interest rates on a large number of subprime and other ARM due to adjust upward during the 2008 period, U.S. legislators, the U.S. Treasury Department, and financial institutions took action. A systematic program to limit or defer interest rate adjustments was implemented to reduce the effect. In addition, lenders and borrowers facing defaults have been encouraged to cooperate to enable borrowers to stay in their homes. Banks have sought and received over $ 250 billion in additional funds from investors to offset losses (see [82] for more information). The risks to the broader economy created by the financial market crisis and housing market downturn were primary factors in several decisions by the U.S. Federal Reserve to cut interest rates and the Economic Stimulus Package (ESP) passed by Congress and signed by President Bush on Wednesday, 13 February 2008 (see, for instance, [8], [46] and [112]; also GIR2 in Figure 6.1). Bush also announced a plan voluntarily and temporarily to freeze the mortgages of a limited number of mortgage debtors holding ARMs. A refinancing facility called FHA-Secure was also created. This action is part of an ongoing collaborative effort between the U.S. government and private industry to help some subprime borrowers called the

Hope Now Alliance (see GIR3 in Figure 6.1). The U.S. government also bailed-out key financial institutions, assuming significant additional financial commitments (see GIR4 in Figure 6.1). Following a series of ad-hoc market interventions to bailout particular firms, a $ 700 billion systemic rescue plan was accepted by the U.S. House of Representatives on Friday, 3 October 2008. These actions are designed to stimulate economic growth and inspire confidence in the financial markets (see GIR5 in Figure 6.1). By November 2008, banks in Europe, Asia, Australia and South America had followed the example of the U.S. government by putting rescue plans in place.

6.1.3 Connections Between Our Models and the Subprime Mortgage Crisis

The connections between our banking models and the SMC are complicated. However, a first step towards understanding this relationship entails identifying the problematic loan subportfolios. In this regard, it is possible to decompose the (total) loans, Λ, discussed above as

$$\Lambda = \sum_{i=1}^{m} \Lambda_i,$$

where m is the *number of loan subportfolios* with i being the *index of each loan subportfolio* so that $i = 1, 2, \ldots, m$. We note that the Basel II IRB approach to credit risk (see [14]) dictates that $m \leq 15$. In particular, this approach identifies 15 credit risk exposure types (loan subportfolios) that may be listed as

$i = 1:$ Project Finance (PF);
$i = 2:$ Object Finance (OF);
$i = 3:$ Commodities Finance (CF);
$i = 4:$ Income Producing Real Estate (IPRE);
$i = 5:$ Specialized Lending High Volatility Commercial Real Estate (SLHVCRE)
$i = 6:$ Specialized Lending Not Including High Volatility Commercial Real Estate (SLNIHVCRE)
$i = 7:$ Bank Exposure (BE);
$i = 8:$ Sovereign Exposure (SE);
$i = 9:$ Retail Residential Mortgage (RRM);
$i = 10:$ Home Equity Line of Credit (HELOC);
$i = 11:$ Other Retail Exposure (ORE);
$i = 12:$ Qualifying Revolving Retail Exposure (QRRE);
$i = 13:$ Small to Medium Size Enterprises with Corporate Treatment (SMECT)
$i = 14:$ *Small to Medium Size Enterprises with Retail Treatment* (SMERT)
$i = 15:$ Equity Exposure Not Held in the Trading Book (EENHTB)

with i = 1-6 and i = 9-12 constituting corporate and retail exposures, respectively. As a result of the SMC, market trade in almost all of these loan subportfolios became sluggish. In particular, a problematic loan subportfolio corresponds to $i = 4$ which encompasses income producing real estate. Furthermore, in commercial real estate (i.e., $i = 5$ and $i = 6$) there is a slowing due to the tightening credit and slowing growth, the former a direct result of the SMC. Although capital remains available for residential loans, the credit crunch is pronounced in commercial lending. Moreover, $i = 7$ which corresponds to interbank lending had been identified as a problematic subportfolio. This is because banks were not lending to each other during the early stages of the crisis (see FM4 in Figure 6.1).

The risk premium, ϱ, from (1.4) has had a part to play in the SMC (see HM1 in Figure 6.1). This premium is part of the expression for the bank's own loan rate, r^{Λ}, and its size is an indication of perceived credit risk. A study by the U.S. Federal Reserve indicated that the average difference in mortgage interest rates between subprime ("subprime markup") and prime mortgages declined from $\varrho = 2.8$ percentage points (280 basis points) in 2001, to $\varrho = 1.3$ percentage points in 2007. In other words, the risk premium required by lenders to offer a subprime loan declined dramatically during the aforementioned period. This occurred even though subprime borrower and loan quality declined overall during the 2001-2006 period. In fact, this state of affairs should have had the opposite effect. However, it is clear that such conditions are consistent with classic boom and recession credit cycles (see [36]).

The sensitivity of the required capital to changes in the amount of loans extended from (1.5) is an important issue in the SMC. Bank capital levels were depleted with banks experiencing high debt levels (see FM2 in Figure 6.1). As a consequence, for many banks (1.5) begun to take on negative values during the SMC.

The Central Bank base rate, r, from (1.6) in Subsection 1.2 has also had a role to play in trying to alleviate global economic pressures during the SMC. For instance, on Wednesday, 8 October 2008, central banks in the U.S. (U.S. Federal Reserve), England, China, Canada, Sweden, Switzerland and the European Central Bank cut rates in a coordinated effort to assist the world economy. This rate will be lowered when it is necessary to stimulate growth and provide liquidity.

Despite the discussion above, the most significant relationships between our models and the SMC are established via the bank's own loan rate, r^{Λ}, given by (1.6) (see also (2.11) and (2.20) as well as HM1, HM3, FM4 and GIR1 in Figure 6.1). As we have noted in (1.6), this loan rate may be expressed as

$$r_t^{\Lambda} = \varrho + (1 + r_t)\frac{N}{\eta} + c^{rw} + c^E \frac{\partial K}{\partial \Lambda} + \mathbf{E}[l].$$

In general terms, r^{Λ} can be written as a function of credit risk, market structure (power), marginal cost of debt, marginal cost of equity and the sensitivity of capital to loans extended. The representation of the bank's interest setting intimates that banks will experience positive returns in good times when the *actual rate of default*, r^d, is lower than the provisioning for expected losses, $\alpha\varrho + \mathbf{E}[l]$, and may not be able to cover their expected losses when $r^d > \alpha\varrho + \mathbf{E}[l]$ (see equation (1.2) for loan losses and default rate). In the latter case, bank capital may be needed to cover these excess (and unexpected) losses. If this capital is not enough then the bank will face insolvency. During the latter half of 2008, we have seen a decline in such capital.

A combination of factors resulting from the SMC have led to problems in the commercial real estate market as regards loan demand (see (2.5); also HM3, FM4, GIR1 and GIR5 in Figure 6.1). According to the National Association of Realtors (NAR) there is a slowing in commercial real estate due to the tightening credit and slowing growth, the former a direct result of the SMC. Although capital remains available for residential loans, the credit crunch is pronounced in commercial lending. Patricia Nooney, chairperson of the Realtors Commercial Alliance Committee (RCAC), portrayed the decline as unusual since

> "transactions are being curtailed not for lack of demand, but for serious challenges in obtaining financing." (see [75] for more information).

Next, we discuss loan losses and default rate given by (1.2) in Subsection 1.2.5 (see HM4, FM1, FM3, GIR3 and GIR5 in Figure 6.1). An acceleration in loan growth, as was experienced prior to the SMC, eventually leads to a surge in loan losses (see, for instance, (1.2) for loan losses and default rate) resulting in reduced bank profits (see equation (1.13) for bank profit) and precipitating a new round of bank failures. As experience during the mortgage crisis has shown, such a slump in banking could not only threaten the deposit insurance fund but also slow the economy by entrenching credit crunches. The view that faster loan growth leads to higher loan losses should not be taken lightly; nor should it be accepted without question. If loan growth increases because banks become more willing to lend, credit standards should fall and loan losses should eventually rise. For many distressed banks suffering because of the SMC, increased loan loss provisions from (1.7) and (1.8) translate to a decrease in earnings.

Usually, when capital is needed, banks have to choose between raising and holding deposits and borrowing from the Central Bank as reflected by (1.14) in Subsection 1.2.9.

Contracted liquidity in the global credit markets and banking system (as described by Remark 1.2.2; see also FM4 in Figure 6.1) is an ongoing economic problem. These liquidity concerns drove central banks around the world to take action to provide funds to member banks to encourage lending to worthy borrowers and to restore faith in the commercial paper markets (see GIR1 in Figure 6.1).

We recall that the bank's profits are given by (1.13) in the form

$$\begin{aligned}\Pi_t &= \left(r_t^{\Lambda} - c_t^{\Lambda} - r^d(\mathtt{M}_t)\right)\Lambda_t + r_t^{\mathtt{T}}W_t + r_t^I I_t + r_t^C C_t + r_t^B B_t + r_t^S S_t \\ &\quad -\left(r_t^{\mathtt{B}} + c_t^{\mathtt{B}}\right)\mathtt{B}_t - \left(r_t^D + c_t^D\right)D_t - c^w(W_t) - P(\mathtt{M}_t) - r_t^{\mathtt{T}}\gamma D_t,\end{aligned}$$

It is important to note that this expression represents the bank's profits from unsecuritized loans. However, in the SMC, the attention has been on borrowing under securitization strategies (see Figure 8.2 in Subsection 8.5.1). In principle, we can provide an analogue of (1.13) in a securitization context by introducing additional variables. In this regard, suppose that r^r is the average residual rate in securitized transactions, p^{rc} the average expected fraction of loan losses on securitized debt, c^{li} the marginal cost of mortgage loan insurance under securitization, c^{mw} the marginal cost to monitor a loan that was securitized, c^{tw} the marginal transaction costs to securitize a loan amortized over a loan term, p^w the fraction of the loan amount securitized and $\widetilde{p}^w$ the fraction of the mortgage loan principal, Λ_0, realized as a new loan in securitization. Then the bank's profits under securitization, Π_t^w, may be presented in the form

$$\begin{aligned}\Pi_t^w &= (r_t^r - p_t^{rc} - c_t^{li} - c_t^{mw} - c_t^{tw})p_t^w\widetilde{p}_t^w\Lambda_t + (r_t^\Lambda - c_t^{lc} - c_t^{tw})p_t^w(1-\widetilde{p}_t^w)\Lambda_t \\ &+(1-p_t^w)\Bigg[\Big(r_t^\Lambda - c_t^\Lambda - r^d(\mathtt{M}_t)\Big)\Lambda_t + r_t^{\mathrm{T}}W_t + r_t^I I_t + r_t^C C_t + r_t^B B_t + r_t^S S_t \quad (6.1)\\ &-\Big(r_t^{\mathrm{B}} + c_t^{\mathrm{B}}\Big)\mathrm{B}_t - \Big(r_t^D + c_t^D\Big)D_t - c^w(W_t) - P(\mathtt{M}_t) - r_t^{\mathrm{T}}\gamma D_t\Bigg] - V_t^a - \mathtt{E}_t - \mathtt{F}_t.\end{aligned}$$

Here we note that, in terms of the organization of the special purposes vehicle (SPV), there are different states that can be associated with the different possible corporate forms of the SPV. The SPV can be an LLC ($\mathtt{E}^1$), LLP ($\mathtt{E}^2$), C-corporation ($\mathtt{E}^3$) or a trust ($\mathtt{E}^4$). Each of these SPVs has its own unique tax benefits and problems as well as degree of asset protection and limited liability under legal prescripts. Furthermore, we denote the optimal state during the lifetime of the SPV by $\mathtt{E}^*$, with deviations from $\mathtt{E}^*$ being given by $|\mathtt{E}^* - \mathtt{E}_t^i|$, where $i = 1,\ 2,\ 3, \ldots,\ 4$. These deviations measure the loss and opportunity costs arising from the use of suboptimal corporate structures as SPVs. Usually, such loss manifests as elevated legal fees, increased losses due to low limited-liability and/or the value of additional time spent in dispute resolution. In this case, the formula for $\mathtt{E}$ in (6.1) may be given by

$$\mathtt{E}_t = \max\{|\mathtt{E}^* - \mathtt{E}_t^1|,\ |\mathtt{E}^* - \mathtt{E}_t^2|,\ |\mathtt{E}^* - \mathtt{E}_t^3|,\ |\mathtt{E}^* - \mathtt{E}_t^4|,\ 0\}.$$

Furthermore, in the securitization context of (6.1), we assume that financing can take the form of $\mathtt{F}^f$, where the transaction is an assignment and $\mathtt{F}^s$, where the transaction is a true sale. In the former case, the SPV is not subsidized by the sponsor. Alternatively, the impaired collateral is only substituted by the sponsor. In this situation, where $\mathtt{F}^f$ is the optimal state but $\mathtt{F}^s$ is being used instead, the transaction incurs losses and vice versa. On the other hand, where the correct transaction format is used, there is neither a gain nor a loss. Hence the value of the transaction format choice (compare with (6.1) where F is used) is

$$\mathtt{F}_t = \max\{|\mathtt{F}_t^f - \mathtt{F}_t^s|,\ 0\}.$$

Furthermore, we denote the value of the adverse selection problem by V^a. In this case, the transaction is an assignment and the sponsor is the servicer. Here, the sponsor will typically substitute impaired collateral, and hence is incentivized not to prefer the best collateral as a replacement. In this regard, we denote impaired collateral, value of replacement collateral and the value of average-quality collateral available for replacement by C^0, $C^i \dots C^n$ and C^a, respectively. We assume that collateral consists of units of the same or similar sizes where $\mathbf{P}^i$ is the probability of there being impaired collateral and $\mathbf{P}^a$ the probability that the servicer/sponsor will offer medium- or low-quality collateral as replacement for the impaired collateral. In this situation, it follows that V^a in (6.1) may be represented by

$$V^a = \int_0^t \left\{ \sum C^a * \mathbf{P}^i(1 - \mathbf{P}^a) \right\}.$$

Another remark about profitability is that during the SMC, profits at the 8 533 U.S. banks insured by the FDIC fell from \$ 35.2 billion to \$ 646 million (effectively by 89 %) during Quarter 4 of 2007 when compared with the previous year. This was largely due to escalating loan losses and provisions for loan losses. This decline in profits contributed to the worst bank and thrift quarterly performance since 1990. In 2007, these banks earned approximately \$ 100 billion, which represented a decline of 31 % from the record profit of \$ 145 billion in 2006. Profits decreased from \$ 35.6 billion to \$ 19.3 billion during the first quarter of 2008 versus the previous year, a decline of 46 % (see [44] and [45] for more detail). The contribution [19] considers the following strategy to be optimal for banks to shield their profits from loan losses. The loan loss reserves, R^l, is built up in every period that $P > L$. On the other hand, when $P \leq L$ the bank is allowed to draw on R^l from the current period. When R^l or other capital becomes inadequate, at some point, the bank will face insolvency.

Bank valuations as realized by (2.7) in Section 2.2 have decline dramatically during the SMC as a direct result of sharp decreases in profitability and bank capital.

6.2 BANK VALUATION AND BASEL II

In this section, we analyze the connections between our banking model and the Basel II capital accord (see Chapter 1 for a diagrammatic overview of Basel II; also [11] and [14]). These relationships are forged via the

- description of total bank capital given by (1.9) in Subsection 1.2.6;
- description of the bank capital constraint given by (1.11) and (1.12) in Subsection 1.2.6.

6.2.1 Bank Regulatory Capital

Bank capital is notoriously difficult to define, monitor and measure. For instance, the measurement of equity depends on how all of a bank's financial instruments and other assets are valued. The description of the shareholder equity component of bank capital, E, given by (1.9), is largely motivated by the following two observations. Firstly, it is meant to reflect the nature of the book value of equity. Our intention is also to recognize that the book and market value of equity is highly correlated.

Under Basel II, bank capital requirements have replaced reserve requirements as the main constraint on the behavior of banks (see, for instance, [22]). A first motivation for this is that bank capital has a major role to play in overcoming the moral hazard problem arising from asymmetric information between banks, creditors and debtors. Also, bank regulators require capital to be held to protect themselves against the costs of financial distress, agency problems and the reduction of market discipline caused by the safety net. Subsection 1.2.6 suggests that a close relationship exists between bank capital holding and macroeconomic activity in the loan market (see equation (1.12) describing the bank capital constraint).

6.2.2 Procyclicality of Basel II Regulation

The Standardized approach to credit risk differentiates assets not only according to the borrower but according to riskiness proxied by credit rating agencies' assessment of the borrower. This approach represents an improvement in the sense that corporations are rated. However, it is deficient with respect to the viewpoint that ratings properly reflect risk and the inducing of procyclical capital charges which will lead to overlending during booms and underlending during recessions. Also, the capital formula for the IRB approach to credit risk is problematic. In short, the IRB capital formula is deduced by considering the large-portfolio asymptotic dynamics of a Merton model with a single common risk-factor. Many empirical studies have confirmed that this formula gives rise to procyclicality.

In addition, Basel II dictates that a macroeconomic shock will affect the loan risk-weights in the CAR. In general, a negative (positive) shock results in the tightening (loosening) of the capital constraint (see equation (1.12) describing the bank capital constraint). As a consequence, in terms of a possible binding capital constraint, banks are free to increase (decrease) the loan supply when macroeconomic conditions M_t improve (deteriorate). On the other hand, if the risk-weights are constant, a shock does not affect the loan supply but rather results in a change in the loan rate when the capital constraint binds. It is not always true that Basel II risk-sensitive weights lead to an increase (decrease) in bank capital when macroeconomic activity in the loan market increases (decreases). A simple explanation for this is that macroeconomic conditions do not necessarily only affect loan demand but also influences the total capital constraint (see equation (1.12) describing the bank capital constraint). Furthermore, banks do not necessarily need to raise new capital to expand their loan supply, since a positive macroeconomic shock may result in a decrease in the RWAs with a commensurate increase in CARs (compare the minimum capital constraint as expressed in (1.11) and (1.12)). Similarly, banks are not compelled to decrease their capital when the loan demand decreases since the capital constraint usually tightens in response to a negative macroeconomic shock. A further complication is that an improvement in the latter conditions may result in an increase in the loan demand and, as a consequence, an increase in the probability that the capital constraint may be binding (see equations (2.5) and (1.12) for the loan demand and bank capital constraint, respectively). Banks may react to this situation by increasing capital to maximize profits (compare the definition of the return on equity (ROE) measure of profitability). Our main conclusion is that bank capital is procyclical because it is dependent on fluctuations in loan demand which, in turn, is reliant on macroeconomic activity (see equation (1.12) describing the bank capital constraint).

Chapter 7

CONCLUDING REMARKS AND FUTURE DIRECTIONS

In this section, we make a few concluding remarks and comment about possible future research projects.

7.1 CONCLUDING REMARKS

The issue of procyclical effects of the Basel II capital requirements is currently the subject of an extreme discussion in the financial and supervisory community. This dissertation presents important contributions to the discussion. We analyzed the effect of macroeconomic shocks on models for bank's lending under the two capital adequacy regimes (the Basel I and Basel II) in discrete-time. One of the results of the dissertation is that the response of banks to shocks that affect loan demand differs when the minimum capital requirements are calculated with risk-weights that are sensitive to the phases of the business cycle. In particular, bank capital is less volatile than under capital requirements with constant risk-weights. In this dissertation, we discussed procyclicality in the South African financial system. In particular, we have analyzed the effect that cyclicality in bank credit and bank capital has on financial stability. Our evidence is consistent with the view that the procyclicality of bank credit and bank capital might be a cause of financial distress in the South African system.

In this paper, we have forged connections between discrete-time stochastic banking models and macroeconomic activity as well as the SMC and Basel II Capital Accord. Furthermore, we solved an optimal bank valuation problem that, amongst other things, maximized profit under several realistic banking constraints. A comprehensive illustration of some of the concepts discussed in the main body of the paper was provided. The discussion of procyclicality confirms that loan loss provisions increase as world economies move further into recession. Also, we demonstrate that banks increase loan loss provisions as loan problems become more apparent. In short, this contribution represents a starting point for studying modeling issues related to the SMC and, to a lesser extent, Basel II. We envisage that our study on the modeling of the SMC will be extended in several directions in the near future.

7.2 FUTURE DIRECTIONS

Future modeling research should consider the bank's lending responses to business cycles as it pertains to the nexus between the proximity to its capital constraints and its forthrightness in recognizing loan losses. This relationship may not be constant over time. Further research should also establish a solid basis for comparison of extant valuation practice and demonstrate the superiority of our model to another model. In particular, we need to learn more about the inadequacies of current practice as a basis for substantiating the need for our modeling paradigm. Moreover, future reserach should provide empirical support to demonstrate the superiority of our model over other models or the accuracy of its performance relative to any market benchmark. Hence, even if one could utilize the model in some valuation project to estimate the value of the bank's common shareholder's equity, we have no idea of how close we might be to the truth with respect to market values. Since fair market values are of great interest to valuators, a test of reasonable congruence with such values would seem to be prerequisite to the use of the valuation model for purposes of business valuation. Although large sample empirical data would be preferred, smaller sample analysis using recent actual acquisitions might also be helpful. All these facts have to be incorporated in future modeling programmes.

Another research topic will involve complex models of bank items driven by Lévy processes (see, for instance, Protter in [103, Chapter I, Section 4]). Such processes have an advantage over the more traditional modelling tools such as Brownian motion because the behavior of bank loans, wealth, capital and CARs are characterized by jumps. As a result of this, recent research has strived to replace the existing Brownian motion-based bank models (see, for instance, [35], [50], [80], [90] and [107]) by systems driven by more general processes. Also, a study of the optimal capital structure should ideally involve the consideration of taxes and costs of financial distress, transformation costs, asymmetric bank information and the regulatory safety net. Another research area that is of ongoing interest is the (credit, market, operational, liquidity, securitization) risk minimization of bank operations within a regulatory framework (see, for instance, [85] and [89]). Another risk that becomes important is *interest rate risk* at the point of loan issuing. For instance, an alternative optimization problem would be to maximize the risk-free rate of interest in order to provide a shareholder with an incentive to invest money. An example of a over-simplified model that has become more important during the current SMC and global credit crunch is the Diamond-Dybvig model (see, for instance, [37] and [38]). Their contributions proposes a much simpler model than ours but yet explains key aspects of what bank liquidity means in reality *ceteris parabis*. In future, it will be worthwhile to explore connections with the work of Diamond and Dybvig on liquidity.

Chapter 8

APPENDIX

This appendix contains tables outlining the main differences between the Basel I and Basel II and statement of the implicit function theorem. We also discuss the formal derivation of the first order conditions (2.12) to (2.15). Furthermore, we provide diagrammatic overviews of the subprime mortgage crisis and Basel II credit risk (see Figure 8.1 below) as well as borrowing under securitization strategies (see Figure 8.2 below) and the financial leverage profit engine (see Figure 8.3 below).

8.1 THE MAIN DIFFERENCES BETWEEN BASEL I AND BASEL II

Focus	Basel I	Basel II
Risk Measure	Single Risk Measure	Counterparty & Transaction Specific Risk Measures
Risk Sensitivity	Broad Brush Approach	Granularity and Risk Sensitivity
Credit Risk Mitigation	Limited Recognition	Comprehensive Recognition
Operational Risk	Excluded	Included
Flexibility	One Size Fits All	Menu of Approaches
Supervisory Review	Implicit	Explicit
Market Discipline	Not Addressed	Supervisory Role Conferred on Market
Incentives	Not Addressed	Explicit and Well Defined
Economic Capital	Divergence	Convergence

Table 8.1: Main Differences between Basel I and Basel II

Basel I	Basel II
Not Risk Sensitive; No Reliance on Risk Ratings or Maturity.	Risk Sensitive
Based on Simplistic or Crude Categories of Obligors and Assets with Fixed Risk-Weighted Asset Percentages; No Minimal Recognition of Risk Mitigation	Based on Risk Ratings, Maturity Substantial of Recognition Risk Mitigation
Calculated in Aggregate	Calculated at Facility/Transaction Level
No Acknowledgment of Operational Risk as a Separate Risk Discipline/Category	Operational Risk as a Separate Discipline/Category
No Relationship with Risk Capital	Alignment with Risk Capital, Advanced Risk Management Concepts
Largely Finance-Driven Off of Finance Systems	Operationally Intense - Places Major Emphasis on Rigorous Risk Management Processes, Models Systems/Technology and Integration of Risk/Finance Systems and Processes

Table 8.2: Main Differences between Basel I and Basel II (continued)

8.2 STATEMENT OF THE IMPLICIT FUNCTION THEOREM

Suppose that (a, b) is a point on the curve $H(M_t, K_{t+1}) = 0$. Suppose that we can solve the equation for K_{t+1} as a function of M_t for all (M_t, K_{t+1}) sufficiently near (a, b). This part of the curve is the graph of a function $K_{t+1} = \eta(M_t)$ on some interval $|M_t - a| < h$ with $\eta(a) = b$. If $\eta'(M_t)$ exists, we can compute it by differentiating both sides of the equation $H(M_t, \eta(M_t)) = 0$ with respect to M_t to get

$$\frac{\partial H}{\partial M_t}(M_t, \eta(M_t)) + \frac{\partial H}{\partial K_{t+1}}(M_t, \eta(M_t))\eta'(M_t) = 0.$$

providing that the partial derivatives exist. If $\frac{\partial H}{\partial K_{t+1}}(M_t, \eta(M_t)) \neq 0$, we can solve for $\eta'(M_t)$ and obtain the well known formula

$$\eta'(M_t) = -\frac{\frac{\partial H}{\partial M_t}(M_t, \eta(M_t))}{\frac{\partial H}{\partial K_{t+1}}(M_t, \eta(M_t))},$$

or, more precisely,

$$\frac{\partial K_{t+1}}{\partial M_t} = -\frac{\frac{\partial H}{\partial M_t}}{\frac{\partial H}{\partial K_{t+1}}}.$$

8.3 DERIVATION OF FIRST ORDER CONDITIONS (2.12) to (2.15)

To derive equations (2.12) to (2.15), we rewrite equation (2.9) to become

$$\begin{aligned} V(K_t, x_t) &= \max_{r_t^\Lambda,\ D_t,\ \Pi_t} \Big\{ \Pi_t + l_t \Big[K_t - \rho \Big(\omega(M_t)\Lambda_t + \omega^I I_t + 12.5(mVaR + 0) \Big) \Big] \\ &\quad - c_t^{dw} \Big[K_{t+1} \Big] + \mathbf{E}_t \Big[\delta_{t,1} V \Big(K_{t+1},\ x_{t+1} \Big) \Big] \Big\}, \end{aligned} \tag{8.1}$$

but

$$\Pi_t = n_t(d_t + E_t) + (1 + r_t^O)O_t - K_{t+1} + \Delta F_t. \tag{8.2}$$

By substituting equations (2.5) and (8.2), equation (8.1) becomes

$$\begin{aligned} V(K_t, x_t) &= \max_{r_t^\Lambda,\ D_t,\ \Pi_t} \Big\{ \Big(n_t(d_t + E_t) + (1 + r_t^O)O_t - K_{t+1} + \Delta F_t \Big) \\ &\quad + l_t \Big[K_t - \rho \Big[\omega(M_t) \Big(l_o - l_1 r_t^\Lambda + l_2 M_t + \sigma_t^\Lambda \Big) + \omega^I I_t + 12.5(mVaR + 0) \Big] \Big] \\ &\quad - c_t^{dw} \Big[K_{t+1} \Big] + \mathbf{E}_t \Big[\delta_{t,1} V \Big(K_{t+1},\ x_{t+1} \Big) \Big] \Big\}. \end{aligned} \tag{8.3}$$

Finding the partial derivative of the bank value in (8.3), with respect to the capital constraint, K_{t+1}, we have

$$\frac{\partial V}{\partial K_{t+1}} = -1 - c_t^{dw} + E_t\left[\int_{\underline{\Lambda}}^{\overline{\Lambda}} \delta_{t,1}\frac{\partial V}{\partial K_{t+1}}dF\left(\sigma_{t+1}^{\Lambda}\right)\right]. \tag{8.4}$$

8.3.1 First Order Condition (2.12)

Choosing the loan rate, r_t^{Λ}, from equation (8.3) and using equation (8.4) above, the first order condition (2.12) for Problem 2.2.1 is

$$\frac{\partial \Pi_t}{\partial r_t^{\Lambda}}\left[1 + c_t^{dw} - \mathbf{E}_t\left\{\int_{\underline{\Lambda}}^{\overline{\Lambda}} \delta_{t,1}\frac{\partial V}{\partial K_{t+1}}dF(\sigma_{t+1}^{\Lambda})\right\}\right] + l_t\rho l_1\omega(M_t) = 0$$

8.3.2 First Order Condition (2.13)

Choosing the deposits, D_t, from equation (8.3) and using equation (8.4) above, the first order condition (2.13) for Problem 2.2.1 is

$$\frac{\partial \Pi_t}{\partial D_t}\left[1 + c_t^{dw} - \mathbf{E}_t\left\{\int_{\underline{\Lambda}}^{\overline{\Lambda}} \delta_{t,1}\frac{\partial V}{\partial K_{t+1}}dF(\sigma_{t+1}^{\Lambda})\right\}\right] = 0$$

8.3.3 First Order Condition (2.14)

We now find the partial derivative of the bank value in (8.3) with respect to the Lagrangian multiplier, l_t.

$$\frac{\partial V}{\partial l_t} = K_t - \rho\left[\omega(M_t)\Lambda_t + \omega^I I_t + 12.5(mVaR + 0)\right].$$

Therefore the first order condition (2.14) for Problem 2.2.1 is

$$\rho\left[\omega(M_t)\Lambda_t + \omega^I I_t + 12.5(mVaR + 0)\right] \leq K_t.$$

8.3.4 First Order Condition (2.15)

Choosing the regulatory capital, Π_t, from equation (8.3) and using equation (8.4) above, the first order condition (2.15) for Problem 2.2.1 is

$$-1 - c_t^{dw} + \mathbf{E}_t\left\{ \int_{\underline{\Lambda}}^{\overline{\Lambda}} \delta_{t,1} \frac{\partial V}{\partial K_{t+1}} dF(\sigma_{t+1}^{\Lambda}) \right\} + 1 = 0,$$

which is the same as

$$-c_t^{dw} + \mathbf{E}_t\left\{ \int_{\underline{\Lambda}}^{\overline{\Lambda}} \delta_{t,1} \frac{\partial V}{\partial K_{t+1}} dF(\sigma_{t+1}^{\Lambda}) \right\} = 0,$$

8.4 DIAGRAMMATIC OVERVIEW OF BASEL II CREDIT RISK

In this appendix, we provide a diagrammatic overview of Basel II credit risk.

8.5 SECURITIZATION

In this section, we provide more information about securitization with regard to borrowing under securitization strategies (see Figure 8.2 below) and the financial leverage profit engine (see Figure 8.3 below) as presented on the website [117].

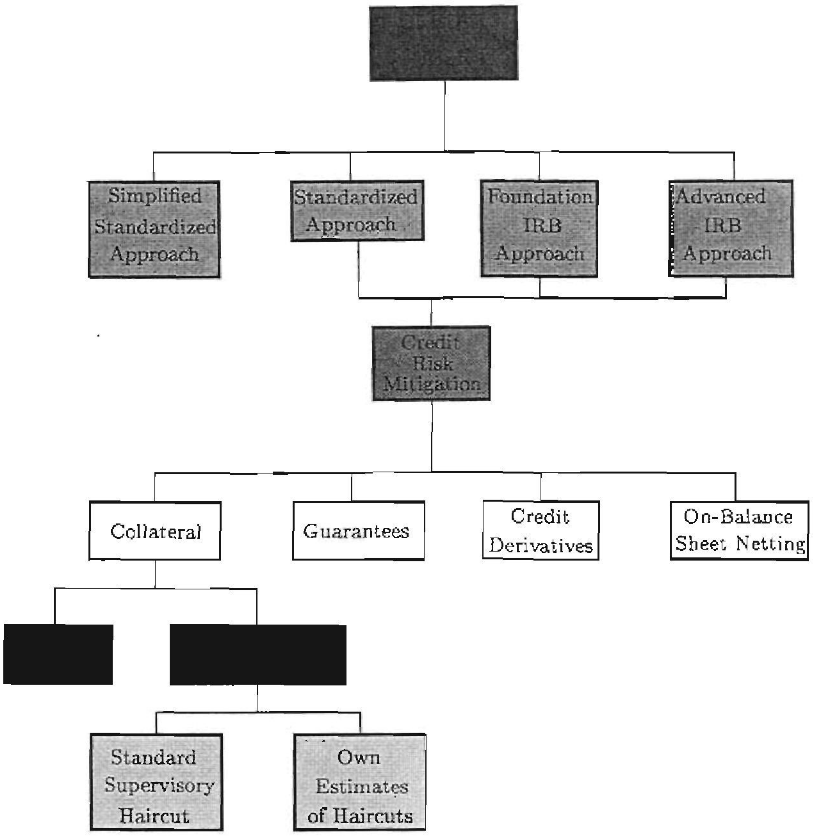

Figure 8.1: Diagrammatic Overview of Basel II Credit Risk

8.5.1 Borrowing Under Securitization Strategies

A diagrammatic overview of borrowing under securitization strategies may be represented as follows.

A simpler procedure for borrowing under an MBS strategy may be summarized as follows. The bank lends money to the borrower (homeowner). The bank creates an asset pool from a group of equivalent assets and sells CMO certificates to investors. The mortgage holders pay periodic payments to the bank. The bank collects mortgage payments, extracts servicing fees, pays guarantee fees to the trustees and passes the rest of the payment on to the investors. In the case where the mortgage holder

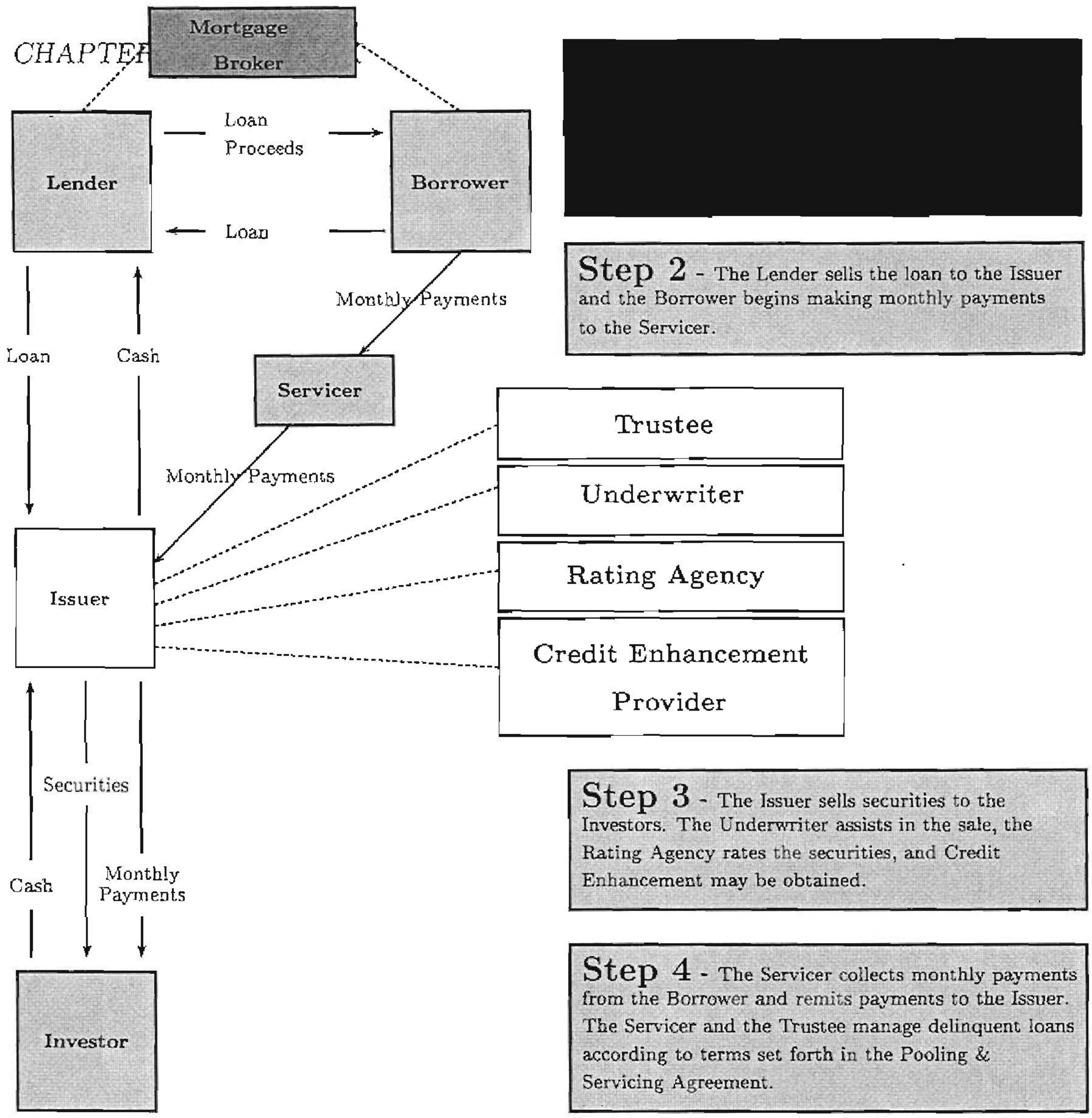

Figure 8.2: Diagrammatic Overview of Borrowing Under Securitization Strategies (compare [117])

defaults the trustee pays the remaining mortgage balance to the investors. Next, a diagrammatic overview of the financial leverage profit engine that is closely related to borrowing under securitization strategies will be given.

8.5.2 The Financial Leverage Profit Engine

Further insight may be gained about securitization by considering the following illustration (compare with Figure 8.3). Let us assume that an investment bank borrows money from an investor or money market fund and agrees to pay, for instance, 4 %

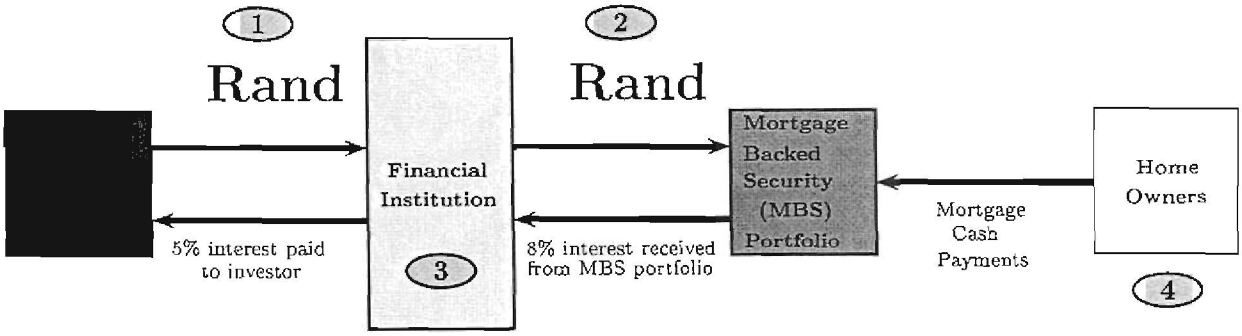

Figure 8.3: Diagrammatic Overview of the Financial Leverage Profit Engine (compare [117])

interest rate. The MBS portfolio is collateral, which the investors can seize in the event of a default on interest payments. The investment bank uses the funds to expand its MBS portfolio, which is paying, for instance, 7 % interest rate. The 3 % rate difference between the amounts is called the spread. In our case, for every R 100.00 invested in this manner, the investment bank makes R 3.00 profit margin. This provides an incentive to borrow and invest as much as possible, known as leveraging. This was considered safe during the early 2007 housing boom, as MBS portfolios typically received high credit ratings and defaults were minimal. Since investment banks do not have the same capital reserve requirements as depository banks, many borrowed and lent amounts exceeding 30 times their net worth. By contrast, depository banks rarely lend more than 15 times their net worth. Freddie Mac was leveraged nearly 70 times its net worth. With increasing delinquencies and foreclosures during 2007-2008, the value of the MBS portfolios declined. Investors became concerned and in some cases demanded their money back, resulting in margin calls (need to sell/liquidate the MBS portfolios) to pay them. At such a high leverage amount, many investment banks and mortgage companies suffered huge losses, bankruptcy, or merged with other institutions. Because MBS securities became "toxic" due to uncertainty in the housing market, they became illiquid and their values dropped. The market value is penalized by the inability to sell the MBS; it may be less than the value the actual cash inflow would merit. The ability of financial institutions to obtain funds in this manner via MBS has been dramatically curtailed. Spreads have narrowed, as investors are demanding higher returns to lend money to highly leveraged institutions.

8.6 KEY DEFINITIONS

In this section, we present the definitions of a few key concepts as provided by, for instance, [117].

Credit crunch is a term used to describe a sudden reduction in the general availability of loans (or credit) or sudden increase in the cost of obtaining loans from banks (by raising interest rates).

Special Purpose Vehicle (SPV): A SPV is a body corporate (usually a limited company of some type or, sometimes, a limited partnership) created to fulfill narrow, specific or temporary objectives, primarily to isolate financial risk, usually bankruptcy but sometimes a specific taxation or regulatory risk. A SPV may be owned by one or more other entities and certain jurisdictions may require ownership by certain parties in specific percentages. Often it is important that the SPV not be owned by the entity on whose behalf the SPV is being set up by the sponsor. For example, in the context of a loan securitization, if the SPV securitization vehicle were owned or controlled by the bank whose loans were to be secured, the SPV would be consolidated with the rest of the bank's group for regulatory, accounting, and bankruptcy purposes, which would defeat the point of the securitization. Therefore many SPVs are set up as "orphan" companies with their shares settled on charitable trust and with professional directors provided by an administration company to ensure there is no connection with the sponsor.

Mortgage-Backed Security: A mortgage-backed security (MBS) is an asset-backed security whose cash flows are backed by the principal and interest payments of a set of mortgage loans. Payments are typically made monthly over the lifetime of the underlying loans. Residential mortgages in the United States have the option to pay more than the required monthly payment (curtailment) or to pay off the loan in its entirety (prepayment). Because curtailment and prepayment affect the remaining loan principal, the monthly cash flow of an MBS is not known in advance, and therefore presents an additional risk to MBS investors. Commercial mortgage-backed securities (CMBS) are secured by commercial and multifamily properties (such as apartment buildings, retail or office properties, hotels, schools, industrial properties and other commercial sites). The properties of these loans vary, with longer-term loans (5 years or longer) often being at fixed interest rates and having restrictions on prepayment, while shorter-term loans (1-3 years) are usually at variable rates and freely prepayable.

Subprime lending is the practice of making loans to borrowers who do not qualify for market interest rates owing to various risk factors, such as income level, size of the down payment made, credit history and employment status.

Project finance (PF) is a method of funding in which the lender looks primarily to the revenues generated by a single project, both as the source of repayment and as security for the exposure.

Object Finance (OF) refers to a method of funding the acquisition of physical assets (e.g. ships, aircraft, satellites, railcars, and fleets) where the repayment of the exposure is dependent on the cash flow generated by the specific assets that have been financed and pledged or assigned to a lender. A primary source of these cash flows might be rental or lease contracts with one or several parties.

Commodities finance (CF) refers to structured short-term lending to finance reserves, inventories, or receivable of exchange-traded commodities (e.g. crude oil, metals, or crops), where the exposure will be repaid from the proceeds of the sale of the commodity and the borrower has no independent capacity to repay the exposure. This is the case when the borrower has no other activities and no other material assets on its balance sheet.

Income producing real estate (IPRE) refers to a method of providing funding to real estate (such as, office buildings to let, retail space, multifamily residential buildings, industrial or warehouse space and hotels) where the prospects for repayment and recovery on the exposure depend primarily on the cash flow generated by the asset. The primary source of these cash flows would generally be lease or rental payments or the sale of the asset.

High volatility commercial real estate (HVCRE) lending is the financing of commercial real estate that exhibits higher loss rate volatility (e.g., higher asset correlation).

Specialized lending high volatility commercial real estate (SLHVCRE) includes commercial real estate exposures secured by properties of types that are categorized by the national supervisor as sharing higher volatilities in portfolio defaults rates; loans financing any of the land acquisition, development and construction (ADC) phases for properties of those types in such jurisdiction; loans financing ADC of any other properties where the source of repayment at origination of the exposure is either the future uncertain sale of the property or cash flows whose source of repayment is substantially uncertain (e.g the property has not yet been leased to the occupancy rate prevailing in that geographic markets for that type of commercial real estate), unless the borrower has substantial equity at risk.

The asset class, *bank exposure* (BE), covers exposures to banks and securities firms that are subject to supervisory and regulatory arrangements comparable to those under Basel II. Bank exposure also includes claims on domestic public sector entities

(PSEs) that are treated like claims on banks under the Standardized approach, and multilateral development banks (MDBs) that do not meet the criteria for 0 % risk weight under the Standardized approach.

Sovereign exposure (SE), covers all exposures to counterparties treated as sovereign under the Standardized approach. This includes sovereigns (and their central banks), certain PSEs identified as sovereigns in the Standardized approach, MDBs that meet the criteria for 0 % risk weight under the Standardized approach.

Retail residential mortgage (RRM) loans (including first and subsequent liens[1], term loans and revolving (HELOC) home equity lines of credit) are eligible for retail treatment regardless of exposure size so long as the credit is extended to an individual that is an owner-occupier of the property.

[1]In law, a lien is a form of security interest granted over an item of property to secure the payment of a debt or performance of some other obligation. The owner of the property, who grants the lien, is referred to as the lienor and the person who has the benefit of the lien is referred to as the lienee.

Chapter 9

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Chapter 1

INTRODUCTION

In this dissertation, we mainly consider bank valuation[1] and profitability when loan losses and macroeconomic conditions are explicitly considered. We note that in the acquisition of bank equity, a valuation gives the stock analyst (possibly acting on behalf of a potential shareholder) an independent estimate of a fair price of the bank's shares. As far as profitability is concerned, we are motivated by the fact that it is a major indicator of financial crises for households, companies and financial institutions. An example of the latter from the subprime mortgage crisis (SMC) that became more apparent in 2007 and 2008, is that both the failure of the Lehmann Brothers investment bank and the acquisition in September 2008 of Merrill Lynch and Bear Stearns by Bank of America and JP Morgan Chase, respectively, were preceded by a decrease in profitability and an increase in the price of loans and loan losses (see Section 6.1 for a diagrammatic overview of the SMC). In this dissertation, we discuss the relationship between our banking models and the SMC as well as the subsequent credit crunch that has had a profound impact on the global banking industry from 2007 onwards. These connections are forged via the bank's risk premium, sensitivity of changes in capital to loan extension, Central Bank base rate, own loan rate, loan demand, loan losses and default rate, loan loss provisions, choice between raising deposits and interbank borrowing, liquidity, profit as well as bank valuation. In addition, we establish connections between our models and the Basel II capital accord. These associations are mainly determined via total bank capital, the bank capital constraint and the procyclicality of approaches to Basel II credit risk.

Loan pricing models usually have components related to the financial funding cost, a risk premium to compensate for the risk of default by the borrower, a premium reflecting market power exercised by the bank and the sensitivity of the cost of capital raised to changes in loans extended. On the other hand, loan losses can be associated with an offsetting expense called the *loan loss provision* (LLP) which is charged against nett profit. This offset will reduce reported income but has no impact on taxes, although when the assets are finally written off, a tax-deductible expense is created. An important factor influencing loan loss provisioning is regulation and supervision. Measures of capital adequacy are generally calculated using the book values of assets and equity. The provisioning of loans and their associated write offs will cause a decline in these capital adequacy measures, and may precipitate increased regulation by bank authorities. Greater levels of regulation generally entail additional costs for the bank. Currently, this regulation mainly takes the form of the Basel II Capital Accord (see Subsubsection 1.2.2.1 for a diagrammatic overview of Basel II; also [11] and [14]) that has been implemented on a worldwide basis since 2008.

[1]Bank value is commonly defined in terms of the market value of the investors equity (stock market capitalization if a company is quoted) plus the market value of the nett financial debt.

The impact of a risk-sensitive framework such as Basel II on macroeconomic stability of banks is an important issue. In this regard, we note that the 1996 Amendment's Internal Models Approach (IMA) determines the capital requirements on the basis of the institutions' internal risk measurement systems. The minimum capital requirement is then the sum of a premium to cover *credit risk*, general *market risk* and *operational risk*. The credit risk premium is made up of risk-weighted loans and the market risk premium is equal to a multiple of the average reported two-week VaRs in the preceding 60 trading days. Banks are required to report daily their value-at-risk (VaR) at a 99% confidence level over both a one day and two weeks (10 trading days) horizon. In order for a bank to determine their minimum capital requirements they will first decide on a planning horizon. This planning horizon is then partitioned into non-overlapping backtesting-periods, which is in turn divided into non-overlapping reporting periods. At the start of each reporting period the bank has to report its VaR for the current period and the actual loss from the previous period. The market risk premium for the current reporting period is then equal to the multiple m of the reported VaR. At the end of each backtesting period, the number of reporting periods in which actual loss exceeded VaR is counted and this determines the multiple m for the next backtesting period according to a given increasing scale. Usually the premium to cover operational risk equals the sum of the premiums for each of eight business lines. The operational risk premium is discussed further in Subsection 1.2.10.

A popular approach to the study of banking valuation and profitability involves a financial system that is assumed to be imperfectly competitive. As a consequence, profits (see, for instance, [90] and [91]) are ensured by virtue of the fact that the nett loan interest margin is greater than the marginal resource cost of deposits and loans. Besides competition policy, the decisions related to capital structure play a significant role in bank behavior. Here, the relationship between bank capital, credit and macroeconomic activity is of crucial importance. In this regard, it is a widely accepted fact that certain financial variables such as capital, credit, asset prices, profitability and provisioning (also bond spreads, ratings from credit rating agencies, leverage and risk-weighted capital adequacy ratios, other ratios such as write-off/loan ratios and perceived risk) exhibit cyclical tendencies. The cyclicality of a financial variable is related to its relationship with the business cycle or a proxy of the business cycle such as the output gap. Here the *output gap* is defined as the amount by which a country's output, or GDP, falls short of what it could be given its available resources. In particular, "procyclicality" has become a buzzword in discussions about banking regulation. In essence, the movement in a financial variable is said to be "procyclical" if it tends to amplify business cycle fluctuations. As such, procyclicality is an inherent property of any financial system. A consequence of procyclicality is that banks tend to restrict their lending activity during economic downturns because of their concern about loan quality and the probability of loan defaults. This exacerbates the recession since credit constrained businesses and individuals cut back on their investment activity. On the other hand, banks expand their lending activity during boom periods, thereby contributing to a possible overextension of the economy that may transform an economic expansion into an inflationary spiral. Our contribution emphasizes the cyclicality of bank profitability and provisioning for loan losses.

By way of addressing the issues raised above, we present a two-period discrete-time banking model involving on-balance sheet variables such as assets (cash, bonds, shares, loans, Treasuries and reserves), liabilities (deposits and interbank borrowing), bank capital (shareholder equity, subordinate debt and loan loss reserves) and off-balance sheet items such as intangible assets (see, for instance, [63] and [116]). In turn, the aforementioned models enable us to formulate an optimization problem that seeks to establish a maximal value of the bank by a stock analyst by choosing an appropriate loan rate and supply. Under cash flow, loan demand, financing and balance sheet constraints, the solution to this problem also yields a procedure for profit maximization in terms of the loan rate and deposits. Here profits are not only expressed as a function of loan losses but also depend heavily on provisions for loan losses.

1.1 RELATION TO PREVIOUS LITERATURE

In this section, we consider the association between our contribution and previous literature. The issues that we highlight include the role of bank capital, output gap, credit models for monetary policy, macroeconomic activity, cyclicality concerns and stochastic modeling and optimization.

1.1.1 Brief Literature Review of Output Gap

In our computation of the South African output gap, we make use of the actual GDP and potential GDP for the period 1990 to 2006. Although the output gap provided in our study is achieved by smoothing actual GDP via the Hodrick-Prescott filter method (see [66]), we also verified our findings by using the more complicated potential output method using a production function approach (see, for instance, [43]). Further discussions of output gap in a South African context is provided by [5], [6], [7] and [67]. The output gap for several OECD countries is investigated in [24], [25], [96] and [97].

As far as the calculation of output gap is concerned, [53] (see, also, [52] and [55]) reviews the methods used to estimate potential output and the resulting output gaps for the calculation of structural budget balances. The split time trend method for estimating trend output is compared with two alternative methods, viz., the Hodrick-Prescott filter and potential output methods. The conclusion is that the latter method is best for estimating output gaps and for calculating structural budget balances, with the results obtained by smoothing GDP providing a means of verification. The paper [5] (see, also, [68]) examines growth performance in post-apartheid South Africa within a growth accounting framework and assesses future growth prospects. Near-term prospects can be captured by potential output growth and the output gap. Furthermore, longer-term growth prospects can be determined on the basis of the full utilization of factors of production and the output gains that arise as these factors are more effectively utilized, for example, through structural reforms that enhance efficiency. The paper [6] provides estimates of potential output growth in South Africa since 1994 using both the time trend techniques and a production function approach. This paper claims that the output gap provides statistically significant information for forecasting inflation and could thus be used to formulate macroeconomic policy. Growth accounting and regression analysis suggest that an increase in trend GDP growth after the end of apartheid in 1994 is attributable to higher total factor productivity (TFP) growth driven by trade liberalization and greater private sector participation. The article [7] provides estimates of potential real GDP growth in South Africa based on alternative methodologies including a production function approach that is standard in the literature. The estimates suggest that during 1994-2001, potential output growth has been around 2.5% to 2.75% annually, and that in 2001 the output gap was around zero. The estimates of the output gap are reasonably correlated over time with inflation and capacity utilization, which are other indicators of the intensity of resource utilization. In order to elucidate South Africa's longer-term growth prospects, [7] analyzes the sources of real GDP growth in the country based on previous work in [67] (see, also, [96] and [97]). A striking fact is that the average annual growth rate of real GDP has increased significantly since 1994, rising from 1% in 1980-1993 to 2.7% in 1994-2001. The increase can mainly be attributed to TFP growth (improvements in efficiency and technology) rather than to increases in the factors of production (like employment). Also, this improvement can be associated with a number of factors including the structural change of the economy after the 1994 elections, the opening up of trade, tax relief and the gradual decrease in interest rates during that period, among other things. In particular, if the TFP growth rates experienced since 1994 are sustained and labor market rigidities are eased sufficiently so that employment rises in step with future increases in the labor force and increases in fixed capital formation, then the economy could achieve growth rates of around 5% over the longer term (see [7]). [81] predicts that the average projected growth in 2007-2012 would be slightly lower than the potential growth as the positive output gap closes over that period. This may be due to capacity constraints due to, for instance, the emerging scarcity of energy, cement and steel. These sentiments are also endorsed by the SA National Treasury (see [81] for more details).

1.1.2 Brief Literature Review of Bank Credit

[81] recognizes the developmental benefits of rapid credit expansion in South Africa. They, however, also caution against the risks that emanate from this expansion (see, for instance, [72]). Driven by buoyant credit and interest rate hikes in 2006 and 2007, the debt service-to-income ratio for South African households rose although it remained below historical highs (see [81]). While the majority of credit growth in South Africa takes the form of secured lending, the authorities have expressed concerns about the vulnerability of first-time and new borrowers and the moderate deterioration in asset quality (see, for instance, [65]). [72] identify that periods preceding financial instability share a number of common features. The first observation was that the lead up to each of the credit peaks was typified by an extended period of rapid economic growth. As a consequence, strong growth reinforced confidence and exacerbated booms.

1.1.3 Brief Literature Review of Bank Capital

The most important role of capital is to mitigate the moral hazard problem that results from asymmetric information between banks, depositors and borrowers. The Modigliani-Miller theorem forms the basis for modern thinking on capital structure (see [83]). In an efficient market, their basic result states that, in the absence of taxes, insolvency costs and asymmetric information, the bank value is unaffected by how it is financed. In this framework, it does not matter if bank capital is raised by issuing equity or selling debt or what the dividend policy is. By contrast, in our contribution, in the presence of loan market frictions, the bank value is dependent on its financial structure (see, for instance, [16], [40], [80] and [106] for banking). In this case, it is well-known that the bank's decisions about lending and other issues may be driven by the capital adequacy ratio (CAR) (see, for instance, [34], [35], [84], [105] and [107]). Further evidence of the impact of capital requirements on bank lending activities are provided by [61] and [111]. A new line of research into credit models for monetary policy has considered the association between bank capital and loan demand and supply (see, for instance, [2], [21], [28], [31], [110], [114] and [115]). This credit channel is commonly known as the *bank capital channel* and propagates that a change in interest rates can affect lending via bank capital. We also discuss the effect of macroeconomic activity on a bank's capital structure and lending activities (see, for instance, [60]). With regard to the latter, for instance, there is considerable evidence to suggest that the phases of the business cycle impact the probability of default and loss given default on loans (see, for instance, [3], [60] and [76]).

1.1.4 Brief Literature Review of Bank Profit

Profitability is a key source of information to understand the business. According to [102] cross-subsidization hypothesis, higher expected profits from relationship based lending creates an incentive to subsidize greater loan availability for new customers. As for the determinants of profitability, [118] argues that there has been relatively a limited overlap between the set of factors informed by the domestic bank profits literature and those based on the multinational bank profits literature to explain foreign bank profits. Several discussions related to bank profit are discussed in the papers [19], [21], [23], [28], [30] and [31]. On the other hand, [18] provide some evidence on the costs and profitability of relationship lending by commercial banks.

1.1.5 Brief Literature Review of Cyclicality of Credit and Capital

It is a widely accepted fact that certain financial variables (for instance, credit prices, bond spreads, ratings from credit rating agencies, provisioning, profitability, capital, leverage and risk weighted capital adequacy ratios, other ratios such as write-off/loan ratios and perceived risk) exhibit cyclical tendencies. In particular, "procyclicality" has become a buzzword in discussions around the new regulatory framework offered by the Basel II. In the sequel, the movement in a financial indicator is said to be "procyclical" if it tends to amplify business cycle fluctuations. A consequence of procyclicality is that banks tend to restrict their lending activity during economic downturns because of their concern about loan quality and the probability of loan defaults. This exacerbates the recession since credit constrained businesses and individuals cut back on their investment activity. On the other hand, banks expand their lending activity during boom periods, thereby contributing to a possible overextension of the economy that may transform an economic expansion into an inflationary spiral. The fact that provisioning (profitability) behaves procyclically by falling (rising) during economic booms and rising (falling) during recessions (see, for instance,

[19], [21], [23], [28], [30] and [31]) is incorporated in our models.

1.1.6 Brief Literature Review of Discrete-Time Optimization

Several discussions related to discrete-time optimization problems for banks have recently surfaced in the literature (see, for instance, [60], [80], [85] and [105]). Also, some recent papers using dynamic optimization methods in analyzing bank regulatory capital policies include [100] for the Basel II and [9], [32] and [79] for the Basel market risk capital requirements. In [105], a discrete-time dynamic banking model of imperfect competition is presented. For both these options, a maximization problem that involves the bank value for shareholders is formulated. On the other hand, [85] examines a problem related to the optimal risk management of banks in a continuous-time stochastic dynamic setting. In particular, the authors minimize market and capital adequacy risk that involves the safety of the assets held and the stability of sources of capital, respectively (see, also, [86]).

1.1.7 Brief Literature Review of Procyclicality

Existing studies of procyclicality in terms of the Basel II have focused on Pillar 1 of the Internal Rating Based (IRB) treatment of wholesale commercial loans. Regulatory capital charges are set at the individual loan level and are given by a formula with five inputs: the borrower's one-year probability of default (PD), the instrument's expected loss given default (ELGD) and remaining maturity (M), the asset-value correlation, ρ, which parameterizes dependence across borrowers, and a target one-year solvency probability (q) for the bank. Under the Basel II, q is fixed to 99.9% and (ρ) is specified as a decreasing function of PD (for more details, see [12]). The IRB capital formula is derived from the large-portfolio asymptotic dynamics of a Merton model with a single common risk-factor. As shown by [56], an instrument's marginal contribution to portfolio VaR converges in the asymptotic limit to its expected loss conditional on the common factor suffering a qth percentile stress event. In the IRB implementation, this conditional expected loss is approximated as a separable equation of three terms: ELGD; the one-year actuarial conditional expected loss; and a maturity adjustment that approximates the ratio of mark-to-market capital charges to actuarial-loss capital charges. Since the release of the Second Consultative Paper [11], many studies have assessed empirically the magnitude of procyclicality in the IRB capital formula. Both within and between studies, one sees a wide range of estimated response to a cyclical downturn. Required capital can double in some simulations, and in others can actually decline. The papet [71] points to differences across studies in sample and methodology that account for some of the differences in results. These include the average credit quality of the sample portfolio, and the country, time period and rating system from which historical experience was drawn. Some differences across studies may be attributed to successive revisions to the Basel II. [71] also identify methodological issues in survivorship bias and treatment of missing observations that appear to have not been handled properly in some earlier studies. Perhaps more important are two issues that can be cast as methodological, but have strong substantive implications. First, should measurement of changes over time in required capital be inclusive or exclusive of charge-offs due to defaulted loans? Second, should the simulated portfolio be passively or actively managed? A passively managed portfolio is fixed at the beginning of the simulation, and so shrinks as defaults accumulate. In an actively managed portfolio, defaulting or maturing loans are replaced with new loans. We can assume a simple time-invariant rule for new lending, or we can attempt to emulate the behavior of bank managers in loosening or tightening lending standards over the credit cycle.

As [71] observe, if we retain defaulted loans in the sample, then we measure the cumulative demands on bank capital over the simulated period. The accumulated charge-offs would have been incurred under the rules of the Basel II as well. From a policy perspective, we are interested only in the additional procyclicality associated with a change in capital regime. Furthermore, in the view of [71], it is somewhat misleading to include accumulated charge-offs without also imputing accumulated interest income net of dividend payments (see [71]). Dividends can be raised and lowered with the anticipated capital needs of the bank, so would need to be modeled endogenously. Therefore, we concur with [71] in preferring to measure capital changes exclusive of defaulted obligors. In most of their simulations, [71] find that roughly 40% to 60% of total change in capital (inclusive of defaulted loans) is attributable to accumulated charge-offs. When [29] similarly decompose total changes in required capital, they in some cases find that capital on the non-defaulting portfolio can decrease in a credit cycle downturn. By disregarding the riskiest borrowers, a stress event has the potential to improve the average credit quality in a portfolio. On the second issue, [71] favor a passive simulated portfolio because active management muddles together the direct effect of a tightened capital constraint with the bank's endogenous response. For example, suppose we look at the evolution of a bank's actively managed portfolio during a recession and find that average credit quality is roughly unchanged. Should we conclude from this that there is no cyclicality problem deserving of policymaker attention? Probably not-it may just be that the bank has reacted to a tightening capital constraint by cutting off credit to its riskier borrowers, which is precisely the policy problem that concerns us ([71]). [57] disagree with this view. They claim that if the goal is to estimate the additional procyclicality associated with the change in capital regime, then one needs to simulate active portfolio management as it occurs under the current regulatory regime. In short, [57] agree with [71] that portfolio management should not be made endogenous to the regulatory rule, but disagree on the appropriate alternative benchmark. Since the mid 1990s, large banks have been more or less unconstrained by regulatory capital requirements, so changes in lending standards over recent years ought to have been driven mainly by economic capital considerations. Such considerations are independent of regulatory regime, and thus would persist (though not necessarily as the binding constraint) into the Basel II. Empirical evidence is limited on this point, but suggests that banks currently do tighten lending standards during a recession and loosen lending standards in a boom. [15] show that the average quality of new loans decreased at the start of the recent recession, while the Federal Reserve Board's Senior Loan Officers Survey showed a commensurate tightening of lending standards. These results are not necessarily contradictory. Many or most nominally new loans are actually drawdowns under existing commitments. Tighter lending standards presumably increase the quality of newly established lending facilities. Thus, the change in new loan quality takes effect with a lag. Taking a more behavioral view of lending practices, [17] find evidence for a institutional memory hypothesis under which standards soften as time passes since a bank's most recent period of large credit losses. The ability to differentiate accurately between high risk and low risk borrowers deteriorates over time as loan officers forget the lessons of the last credit cycle. When large losses are again experienced, standards are tightened drastically, and the cycle begins again (see, for instance, [72]). Results in [70]) are consistent with this story. In addition to studying procyclicality on their simulated passive portfolios, [70] calculate the time-series of IRB required capital for Deutsche Bank's actual German commercial loan portfolio. The capital increase (excluding charge-offs) in this actively managed portfolio was only about one fourth of the increase in required capital for the passively managed, simulated KMV Germany portfolio. As the Deutsche Bank rating system is designed to be point-in-time like KMV, it is reasonable to attribute the difference to active management of lending standards. Thus, even under the risk-insensitive Basel I, internal management processes cause banks to respond to a downturn in a manner that would also serve to dampen the cyclicality of IRB capital requirements.

1.1.8 Brief Literature Review of the Subprime Mortgage Crisis

The working paper [36] explains the fundamentals of the SMC in some detail. A model that has become important during this crisis is the Diamond-Dybvig model (see, for instance, [37] and [38]). Despite the fact that these contributions consider a simpler model than ours, they are able to explain important features of bank liquidity that reflect reality. The quarterly reports [44] and [45] of the Federal Deposit Insurance Corporation (FDIC) intimate that profits decreased from \$ 35.6 billion to \$ 19.3 billion during the first quarter of 2008 versus the previous year, a decline of 46 %.

1.2 PRELIMINARIES

In this section, we provide some preliminaries on operational risk, the balance sheet, profit, macroeconomics, bank loans and bank capital, provisioning for deposit withdrawals and borrowing from other banks.

1.2.1 Preliminaries about Output Gap

In order to study the cyclicality of output, bank credit and capital, we acquired data from the SARB regarding financial variables such as household debt, mortgage loans, CARs and share capital. Information about the output gap was not provided by the SARB and was achieved via our own calculations. The measures of potential output are typically based on historical rates of factor utilization and total factor productivity, rather than on literal full employment, so that in the long run the average output gap is approximately zero. The implied output gap provides statistically significant information for predicting inflation and could thus provide valuable input for formulating macroeconomic policy. The gap can also be used for gauging the stance of fiscal policy in a cyclical context through its role in calculating structural measures of the fiscal balance. The policy outlook for a country depends importantly on both near- and long-term prospects for real output growth. Near-term prospects can be measured by potential output growth and the output gap. Longer-term growth prospects are based on the full utilization of factors of production and the output gains that arise as these factors are more efficiently utilized, for example, through structural reforms.

The output gap is measured as the percentage difference between actual GDP and estimated potential GDP. Symbolically this means that

$$\text{Output Gap} = \frac{\text{Actual Output - Potential Output}}{\text{Potential Output}} \times 100.$$

In other words, the output gap involves measuring the position of output in relation to potential. Output gaps are difficult to estimate and subject to margins of substantial error (see, for instance, [43] for more details on the South African situation).

1.2.2 Preliminaries about the Structure of the Basel II Capital Accord

This subsection is based on the discussion paper [64]. The most significant innovation of Basel II is the departure from a sole reliance on capital adequacy ratios. Basel II consists of three mutually reinforcing pillars (see the diagrammatic overview of Basel II in Figure 2 below), which together should contribute to safety and soundness in the financial system. To ensure that risks within an entire banking group are considered, Basel II is extended on a consolidated basis to holding companies of banking groups.

1.2.2.1 Diagrammatic Overview of the Basel II Capital Accord

Basel II provides a range of options for determining not only the capital requirements for credit and market risk but also operational risk. This allows banks and supervisors to select approaches that are most appropriate for their operations and their financial market infrastructure. Banks have to adopt an approach commensurate with their size and sophistication, and they will not be allowed to revert to a simpler approach once they have been approved for a more advanced approach without supervisory approval. Basel II allows, to a limited degree, for national discretion in the way in which each of the approaches is applied.

1.2.2.2 Pillar 1 - Minimum Capital Requirements

Pillar 1 sets out the minimum capital requirements. Basel I is based on a capital ratio where the numerator represents the amount of capital a bank has available and the denominator is a measure of the risks a bank faces and is referred to as risk-weighted assets.

Under Basel II, the regulations that define the numerator of the capital ratio essentially remain unchanged. Furthermore, the ratio for the minimum capital required has not changed. The modifications, therefore, lie in the definition of risk-weighted

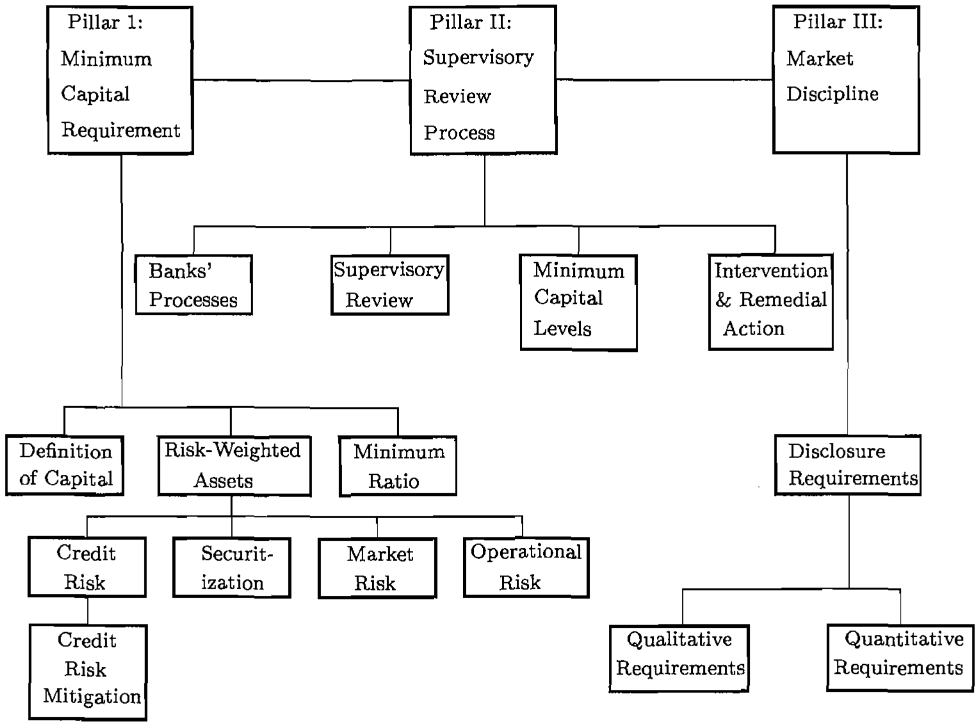

Figure 1.1: Diagrammatic Overview of the Basel II Capital Accord

assets. Whereas Basel I (as amended by the 1996 market risk amendment) explicitly covered only two types of risk in the definition of risk-weighted assets, namely credit and market risk, Basel II includes an explicit treatment of operational risk. Basel II introduced three distinct options (of increasing risk sensitivity) for the calculation of credit risk and three others for operational risk.

1.2.2.3 Credit Risk

In this subsection, we consider Basel II credit risk from the point of view of the standardized and internal-ratings based approaches. Firstly, we provide a diagrammatic overview of credit risk.

Standardized approach

The Standardized approach is similar to Basel I in that banks are required to slot their credit exposures into risk categories based on observable characteristics of the exposures. This approach establishes fixed risk weights corresponding to each risk category and makes use of external credit assessments to enhance risk sensitivity. The bank allocates a risk weight to each of its assets and off-balance sheet positions and produces a sum of risk-weighted asset values. A risk weight of 100 per cent

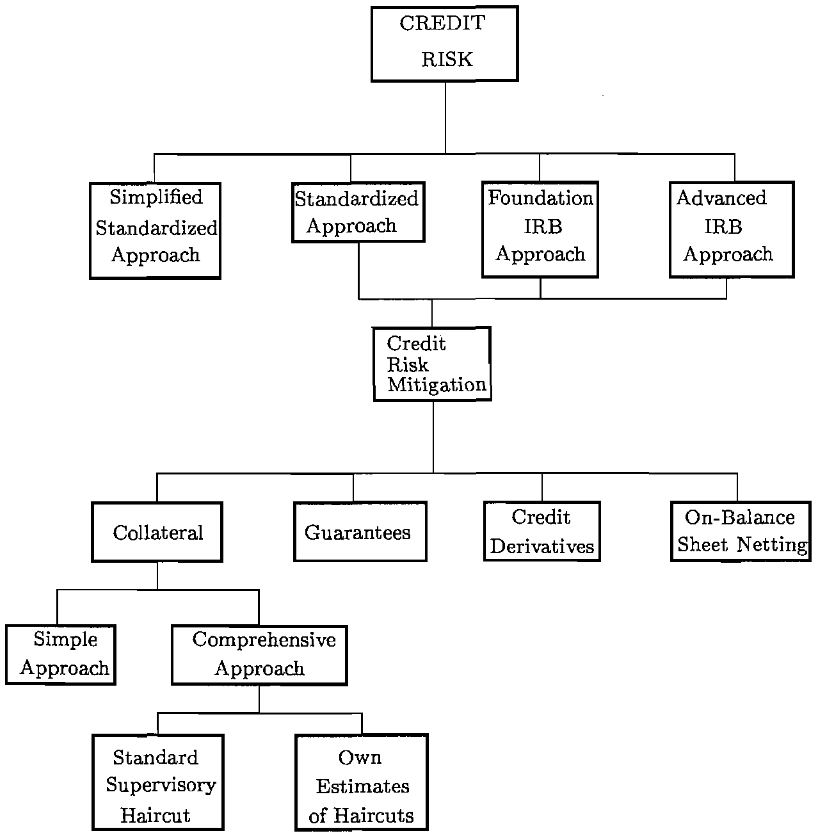

Figure 1.2: Diagrammatic Overview of Credit Risk

means that an exposure is included in the risk-weighted assets at its full value, which translates into a capital charge equal to 8 per cent of that value. The risk weights are refined by reference to a rating provided by an external credit assessment institution (ECAI)[2].

Internal ratings-based approach

[2]An external credit assessment institution (ECAI) has to comply with strict criteria to be recognised. National supervisors are responsible for determining whether an ECAI meets the criteria. Also, supervisors will be responsible for assigning eligible ECAI's assessments to the risk weights under the Standardized risk weighting framework. The mapping process should be objective. It also allows, to a limited degree, for national discretion in the way in which each of these options may be applied. Banks must have a robust system in place to validate the accuracy and consistency of rating systems, processes, and the estimation of all relevant risk components. A bank must demonstrate to its supervisor that the internal validation process enables it to consistently and meaningfully assess the performance of internal rating and risk estimation systems.

One of the most innovative (and problematic) aspects of Basel II is the IRB approach to credit risk, which includes two variants, namely a foundation (FIRB) and an advanced (AIRB) approach. The IRB approach differs significantly from the Standardised approach in that banks' internal assessments of key risk drivers serve as primary inputs to the capital calculation in the former approach (see, for instance, [108] for more information). As the approach is based on banks' own internal assessments, the potential for more risk-sensitive capital requirements is substantial. The IRB approach does not, however, allow banks themselves to determine all the elements needed to calculate their own capital requirements. Instead, the risk weights and thus capital charges are determined through the combination of quantitative inputs provided by banks and formulas specified by the Basel Committee. The formulas or risk weight functions translate a bank's inputs into a specific capital requirement and are based on modern risk management techniques that involve a statistical and thus quantitative assessment of risk.

1.2.2.4 Pillar 2 - Supervisory Review Process

The supervisory review process requires supervisors to ensure that each bank has sound internal processes in place to assess the adequacy of its capital based on a thorough evaluation of its risks. Basel II stresses the importance of bank management developing an internal capital assessment process and setting targets for capital that are commensurate with the bank's particular risk profile and control environment. Supervisors would be responsible for evaluating the extent to which banks are assessing their capital adequacy needs relative to their risks, and to intervene where necessary.

1.2.2.5 Pillar 3 - Market Discipline

The purpose of Pillar 3 is to complement the minimum capital requirements (Pillar 1) and the supervisory review process (Pillar 2) and aims to bolster market discipline (see, for instance, [93] and [99]) through enhanced disclosure by banks. Effective disclosure is essential to ensure that market participants can better understand banks' risk profiles and the adequacy of their capital positions.

1.2.3 Preliminaries about the Bank's Balance Sheet

Throughout, we suppose that $(\Omega, \mathbf{F}, (\mathcal{F}_t)_{t\geq 0}, \mathbf{P})$ is a filtered probability space. As is well-known, the bank balance sheet consists of assets (uses of funds) and liabilities (sources of funds) that are balanced by bank capital (see, for instance [40]) according to the well-known relation

$$\text{Total Assets } (A) = \text{Total Liabilities } (\Gamma) + \text{Total Bank Capital } (K). \qquad (1.1)$$

In period t, the main on-balance sheet items in (1.1) can specifically be identified as

$$A_t = \Lambda_t^m + W_t + C_t + S_t + B_t,\ W_t = \mathrm{T}_t + R_t;\ \Gamma_t = D_t;\ K_t = n_t E_{t-1} + O_t + R_t^l,$$

where Λ^m, C, S, B, T, R, D, n, E, O and R^l are the market value of loans, cash, shares, bonds, Treasuries, reserves, outstanding debt, number of shares, market price of the bank's common equity, subordinate debt and loan loss reserves, respectively.

1.2.4 Preliminaries about the Cyclicality of Credit and Capital

Next, we introduce the generic variable, M_t, that represents the level of macroeconomic activity in the bank's loan market. We suppose that $M = \{M_t\}_{t\geq 0}$ follows the first-order autoregressive stochastic process

$$M_{t+1} = \mu^M M_t + \sigma_{t+1}^M,$$

where σ_{t+1}^M denotes zero-mean stochastic shocks to macroeconomic activity.

The bank takes deposits, D_t, at a constant *marginal cost*, c^D, that may be associated with cheque clearing and bookkeeping. It is assumed that deposit taking is not interrupted even in times when the *interest rate on deposits* or *deposit rate*, r_t^D, is less than the *interest rate on Treasuries* or *bond rate*, r_t^{T}, We suppose that the dynamics of the deposit rate process, $r^D = \{r_t^D\}_{t\geq 0}$, is determined by the first-order autoregressive stochastic process

$$r_{t+1}^D = \mu^{r^D} r_t^D + \sigma_{t+1}^{r^D},$$

where $\sigma_{t+1}^{r^D}$ is zero-mean stochastic shocks to the deposit rate.

Remark 1.2.1 (Deposit Rate and Monetary Policy): *In some quarters, the deposit rate, r^D, is considered to be a strong approximation of bank monetary policy. Since such policy is usually affected by macroeconomic activity, M, we expect the aforementioned items to share an intimate connection. However, in our analysis, we assume that the shocks σ^D_{t+1} and σ^M_{t+1} to r^D and* $\mathtt{M}$*, respectively, are uncorrelated. Essentially, this means that a precise monetary policy is lacking in our bank model. This interesting relationship is the subject of further investigation.*

1.2.5 Preliminaries about Bank Credit

An observation is that loan losses are also dependent on macroeconomic activity. As a consequence, for the *value of loan losses*, L, and the *default rate*, r^d, we set

$$L(M_t) = r^d(M_t)\Lambda_t, \tag{1.2}$$

where $r^d \in [0, 1]$ increases when the phases of the business cycle deteriorate according to

$$0 \leq r^d(M_t) \leq 1, \quad \frac{\partial r^d(M_t)}{\partial M_t} < 0.$$

We note that the above description of the loan loss rate is consistent with empirical evidence that suggests that bank losses on loan portfolios are correlated with the business cycle under any capital adequacy regime (see, for instance, [19], [23], [30] and [77]).

Since it is uncommon for depositors to withdraw all of their funds simultaneously, only a portion of total deposits may be needed as reserves. As a result of this description, we may introduce a *reserve-deposit ratio*, γ, for which

$$R_t = \gamma D_t. \tag{1.3}$$

We suppose that, after providing liquidity, the bank lends in the form of t-th period loans, Λ_t, at the *bank's own loan rate*, r^Λ_t. This loan rate, for profit maximizing banks, is determined by the risk premium (or yield differential), given by

$$\varrho_t = r_t^{\Lambda} - r_t, \tag{1.4}$$

the industry's market power as determined by its concentration, N, the market elasticity of demand for loans, η, base rate, r_t, the marginal cost of raising funds in the secondary market, c^{rw}, and the product of the cost of elasticity (equity) raised, c^E, and the sensitivity of the required capital to changes in the amount of loans extended,

$$\frac{\partial K}{\partial \Lambda}. \tag{1.5}$$

In this situation, we may express the bank's own loan rate, r^{Λ}, as

$$r_t^{\Lambda} = \varrho + (1 + r_t)\frac{N}{\eta} + c^{rw} + c^E \frac{\partial K}{\partial \Lambda} + \mathbf{E}[l], \tag{1.6}$$

where

$$N = \sum_{i=1}^{n} S_i^2$$

is the Herfindahl-Hirschman index of the concentration in the loan market,

$$S_i = \frac{\Lambda_i}{\Lambda}$$

is the market share of bank i in the loan market, but in our contribution we only use one bank, therefore $N = 1$ and

$$\eta = -\frac{\partial \Lambda}{\partial r_t^{\Lambda}} \frac{r_t^{\Lambda}}{\Lambda}$$

is the elasticity of demand for loans. Also, in our model, besides the risk premium, we include $\mathbf{E}[l]$ which constitutes the amount of provisioning that is needed to match the average expected losses faced by the loans.

As was mentioned before, the contribution [19] (see, also, [30] and [77]) highlights the fact that normally provisions for expected loan losses, $(\alpha\varrho+\mathbf{E}[l])\Lambda_t$, where $0 \leq \alpha \leq 1$ and ϱ is the risk premium from (1.4), and loan loss reserves, R^l, act as buffers against expected and unexpected loan losses, respectively. Firstly, we have to distinguish between total provisioning for loan losses, P, and loan loss reserves, R^l. Provisioning is a decision made by bank management about the size of the buffer that must be set aside in a particular time period in order to cover loan losses, L. However, not all of P may be used in a time period with the amount left over constituting *loan loss reserves*, R^l, so that for period t we have

$$R^l_t = P(M_t) - L(M_t), \; P > L.$$

Our model for provisioning in period $t+1$ can be taken to be

$$P(M_{t+1}) = \begin{cases} (\alpha\varrho + \mathbf{E}[l])\Lambda_t, & \text{for } P > L \quad \text{Expected Losses} \\ (\alpha\varrho + \mathbf{E}[l])\Lambda_t + R^l_{t+1}, & \text{for } P \leq L \quad \text{Expected Losses + Unexpected Losses,} \end{cases} \tag{1.7}$$

We note that our model determines the provisions for period $t+1$ in the t-th period which is a reasonable assumption. Our suspicion is that provisioning, P, is a decreasing function of current macroeconomic conditions, M, so that

$$\frac{\partial P(M_t)}{\partial M_t} < 0. \tag{1.8}$$

This claim has resonance with the idea of procyclicality where we expect the provisioning to decrease during booms, when macroeconomic activity increases. By contrast, provisioning may increase during recessions because of an elevated probability of default and/or loss given default on loans.

1.2.6 Preliminaries about Bank Capital

For the purposes of our study, we define *bank capital*, K, as the difference between the accounting value of the assets and liabilities. In the sequel, we take the bank capital, K, in period t to be

$$K_t = n_t E_{t-1} + O_t + R^l_t, \tag{1.9}$$

where $n_t E_{t-1}$ is the market value of the bank equity and constitutes Tier 1 capital in period $t-1$, O_t is the subordinate debt and is a constituent of Tier 2 capital as is the loan loss reserves, R_t^l. For K_t given by (1.9), we obtain the *balance sheet constraint*

$$W_t = D_t - \Lambda_t - C_t - B_t - S_t + K_t, \tag{1.10}$$

The *bank capital constraint* is defined by the inequality

$$K_t \geq \rho\left(a_t + 12.5(mVaR + \mathtt{O})\right), \tag{1.11}$$

where

$$a_t = \omega^I I_t + \omega^\Lambda \Lambda_t + \omega^{\mathrm{T}} \mathrm{T}_t + \omega^R R_t,$$

and ω^I, ω^Λ, ω^{T} and ω^R are the risk-weights for intangible assets, loans, Treasuries and reserves, respectively, $\rho = 0,08$, $mVaR$ is as described in Chapter 1 and $\mathtt{O}$ is the operational risk. If we assume that the risk-weights associated with intangible assets, Treasuries, reserves and loans may be taken to be $\omega^I \neq 0$, $\omega^{\mathrm{T}} = \omega^R = 0$ and $\omega^\Lambda = \omega(M_t)$, respectively, then equation (1.11) becomes

$$K_t \geq \rho\left[\omega(M_t)\Lambda_t + \omega^I I_t + 12.5(mVaR + \mathtt{O})\right]. \tag{1.12}$$

1.2.7 Preliminaries about Bank Profit

As far as profit, Π, is concerned, we closely follow the report [30] and use the basic fact that profits can be characterized as the difference between income and expenses that are reported in the bank's income statement. In our contribution, *income* is solely constituted by the return on intangible assets, $r_t^I I_t$, the return on loans, $r_t^\Lambda \Lambda_t$, and the return on Treasuries, $r_t^{\mathrm{T}} \mathrm{T}_t$. The rates of return on intangible assets, loans and Treasuries are denoted by r^I, r^Λ and r^{T} respectively. Furthermore, we assume that the level of macroeconomic activity is denoted by M_t. In our case we consider the cost of monitoring and screening of loans and capital, $c^\Lambda \Lambda_t$, interest paid to depositors, $r_t^D D_t$, the cost of taking deposits, $c^D D_t$, the cost of deposit withdrawals, $c^w(W_t)$, the value of loan losses, $L(M_t)$, and total loan loss provisions, $P(M_t)$ as *expenses*, in period t. Here r^D and c^D are the deposit rate and cost of deposits, respectively. Summing all the costs mentioned to operating costs and supposing that (1.2) holds and that $W_t = \mathrm{T}_t + \gamma D_t$, then the bank's profits are given by the expression

$$\begin{aligned} \Pi_t &= \left(r_t^{\Lambda} - c^{\Lambda} - r^d(M_t)\right)\Lambda_t + r_t^{\mathrm{T}} W_t + r_t^I I_t + r_t^C C_t + r_t^B B_t + r_t^S S_t \qquad (1.13)\\ &\quad -\left(r_t^D + c^D\right)D_t - c^w(W_t) - P(M_t) - r_t^{\mathrm{T}}\gamma D_t, \end{aligned}$$

where r^C, r^B and r^S are the rates of return of the cash, bonds and shares, respectively.

1.2.8 Preliminaries about Provisioning for Deposit Withdrawals

We have to consider the possibility that unanticipated deposit withdrawals will occur. By way of making provision for these withdrawals, the bank is inclined to hold Treasuries and reserves that are both very liquid. In our contribution, we assume that the *unanticipated deposit withdrawals*, x, originates from the *probability density function*, $f(x)$, that is independent of time. For sake of argument, we suppose that the unanticipated deposit withdrawals have a uniform distribution with support $[0, \overline{D}]$ so that the *cost of liquidation*, c^l, or additional external funding is a quadratic function of the sum of Treasuries and reserves, W. In addition, for any t, if we have that

$$x > W_t,$$

where $W_t = \mathrm{T}_t + R_t$, then bank assets are liquidated at some *penalty rate*, r_t^p. In this case, the *cost of deposit withdrawals* is

$$c^w(W_t) = r_t^p \int_{W_t}^{\infty} [x - W_t] f(x)dx = \frac{r_t^p}{2\overline{D}}[\overline{D} - W_t]^2.$$

Remark 1.2.2 (Deposit Withdrawals and Bank Liquidity): *A vital component of the process of deposit withdrawal is liquidity. The level of liquidity in the banking sector affects the ability of banks to meet commitments as they become due (such as deposit withdrawals) without incurring substantial losses from liquidating less liquid assets. Liquidity, therefore, provides the defensive cash or near-cash resources to cover banks' liabilities.*

1.2.9 Borrowing from Other Banks

Interbank borrowing including borrowing from the Central Bank provides a further source of funds. In the sequel, the amount borrowed from other banks is denoted by B, while the interbank borrowing rate (for instance, known as the Libor rate in the United Kingdom) and marginal borrowing costs are denoted by r^{B} and c^{B}, respectively. Of course, when our bank borrows from the Central Bank, we have $r^{\mathrm{B}} = r$, where r is the base rate appearing in (1.4). Another important issue here is the comparison between the cost of raising and holding deposits, $(r^D + c^D)D$, and the cost of interbank borrowing, $(r^{\mathrm{B}} + c^{\mathrm{B}})\mathrm{B}$. In this regard, a bank in need of capital would have to choose between raising deposits and borrowing from other banks on the basis of overall cost. In other words, the expression

$$\min\{(r^D + c^D)D,\ (r^{\mathrm{B}} + c^{\mathrm{B}})\mathrm{B}\} \tag{1.14}$$

is of some consequence. Under normal economic conditions, for $B \approx D$, we may have that

$$(r^D + c^D)D = \min\{(r^D + c^D)D,\ (r^{\mathrm{B}} + c^{\mathrm{B}})\mathrm{B}\}.$$

However, during the SMC, it is likely that $(r^{\mathrm{B}} + c^{\mathrm{B}})\mathrm{B} = \min\{(r^D + c^D)D,\ (r^{\mathrm{B}} + c^{\mathrm{B}})\mathrm{B}\}$.

1.2.10 Preliminaries about Operational Risk

In the Basel II framework, risk management is divided into credit, market as well as operational risk management. The premium to cover operational risk equals the sum of the premiums for each of eight business lines (corporate finance, trading and sales, retail banking, commercial banking, payment and settlement, agency services, asset management and retail brokerage). The Basel II framework outlines three quantitative approaches for determining an operational risk capital premium: the basic indicator approach, the standardized approach, and the advanced measurement approach. The basic indicator and the standardized approaches are simple and generate results on the basis of predetermined multipliers. More specifically, the capital premium for operational risk, under the Standardized Approach outlined in the Basel II, may be expressed as

$$\mathbb{O} = \max\left[\sum_{k=1}^{8} \beta_k g_k,\ 0\right],$$

where

g_{1-8} : Three-Year Average of Gross Income for Each of Eight Business Lines;

β_{1-8} : Fixed Percentage Relating Level of Required Capital to Level of Gross Income for Each of Eight Business Lines.

The β-values for operational risk are provided in the document [10]. The advanced measurement approach (AMA) differs from the basic indicator and the standardized approaches in that it explicitly attempts to estimate a bank's operational risk exposure (aggregate operational losses faced over a one-year period) at a soundness level consistent with a 99.9 % confidence level. Banks adopting an AMA will effectively calculate operational risk capital using a value at risk (VaR) approach common in both market risk and credit risk management. Each institution employing an AMA must use the following four elements in arriving at its operational risk capital estimate: internal data, external data, scenario analysis, and business environment and internal control factors. The operational risk capital estimate can be expressed as protection against expected and unexpected future losses at a selected confidence level, with some provisions for offsetting portions of this exposure through reserves or other permitted mitigation techniques.

1.3 OUTLINE OF THE DISSERTATION

The rest of dissertation is structured as follows.

1.3.1 Outline of Chapter 2

In Chapter 2, we discuss the discrete-time dynamics of bank items, represented the same as in [49]. These items include retained earnings (see Subsection 2.1.1), as well as loan supply and demand (see, for instance, Subsection 2.1.2), capital and bank profit under the influence of business cycle vagaries. These items enable us to formulate and solve an optimization bank valuation problem subject to capital, cash flow, loan demand, financing and balance sheet constraints (see Problem 2.2.1 and 2.3.1). The solution of the optimal bank valuation given in Theorem 2.3.1 is for the case when capital constraint is binding. For non-binding constraint, the solution is given in Corollary 2.3.2.

1.3.2 Outline of Chapter 3

Chapter 3 contains a discussion of the discrete-time dynamic model of bank items where, as in the Basel I, we assume that risk-weights are constant (see Section 3.1). In addition, borrowers are allowed to default on their principal repayments. Furthermore, we present a result about the effects of changes in the phases of the business cycle on bank capital and credit extension in the form of the quantity and price of loans (see Subsection 3.1.1). Moreover, we establish the impact of the changes in the business cycle on loans and loan rates under a non-binding as well as a binding capital constraint (see, for instance, Subsections 3.1.2 and 3.1.3, respectively, also compare with equation (1.11)). In Section 3.2, results analogous to those obtained in Section 3.1 are given for the situation where loan losses and loan risk-weights are a function of the phases of the business cycle (i.e., risk-weights vary with business cycle factors). This situation mimics the scenario covered by the Basel II. Furthermore, our analysis also involves a non-binding and binding capital constraint (see Subsections 3.2.2 and 3.2.3, respectively). In Subsection 3.2.4, we present cyclicality of loans and loan rates under the Basel II in future periods.

1.3.3 Outline of Chapter 4

Chapter 4 contains a discussion of the connection between procyclicality and financial stability in South Africa. We provide graphical representations of the interactions between the output gap as a proxy for the business cycle (see Section 4.1) and the cyclicality of bank credit and capital (see, for instance, Sections 4.2 and 4.3, respectively). The dynamics of these variables help to confirm the cyclicality of credit and capital in the South African context.

1.3.4 Outline of Chapter 5

An analysis of the cyclicality of bank credit and capital is given in Chapter 5. In Section 5.1, we discuss some of the issues related to the optimal bank valuation problem presented in Chapter 2 while in Section 5.2, we analyze the cyclicality of bank credit and capital under the Basel I (constant risk-weights) and the Basel II (varying risk-weights) presented in Chapter 3. In Section 5.3, we provide some comments about the numerical examples supporting our examples in Chapter 4.

1.3.5 Outline of Chapter 6

Aspects of the relationships between bank valuation and the SMC as well as Basel II are analyzed in Chapter 6. In Sections 6.1 and 6.2, we consider connections between the discrete-time stochastic models derived in the preceding discussions and the Subprime Mortgage Crisis (SMC) as well as the Basel II Capital Accord, respectively.

1.3.6 Outline of Chapter 7

Chapter 7 contains the conclusions that we can draw from the study. We also identify some of the research problems that may be addressed in future.

1.3.7 Outline of Chapter 8

Chapter 8 contains two tables outlining the main differences between the Basel I and the Basel II, summary of the main results of the cyclicality of bank credit and capital presented in Chapter 3 and the statements of the implicit function theorem and the Euler conditions, respectively.

1.3.8 Outline of Chapter 9

The bibliography in Chapter 9 lists the literature referred to throughout the dissertation.

Chapter 2

BANK VALUATION AND ITS OPTIMIZATION

Before examining the impact of the phases of the business cycle under the Basel I and the Basel II, we firstly introduce a few key concepts. Then we state and solve an optimal bank valuation problem. Throughout this section, we present most of the work already done in [49]. In this chapter, we assume that $r_t = r_t^{\mathrm{T}}$ and that

$$r_t^C C_t + r_t^B B_t + r_t^S S_t = 0$$

throughout (compare formula (1.13) for profit).

Problem 2.0.1 (Optimal Bank Valuation Problem): *Which decisions about loan rates, deposits and Treasuries must be made in order to attain an optimal bank value for a stockholder (possibly acting in the interests of a potential shareholder)?* (Theorem 2.3.1 in Chapter 2).

2.1 KEY BANKING CONCEPTS

In this section, we discuss banking concepts such as retained earnings and bank loan supply and demand.

2.1.1 Retained Earnings

We know that *bank profits*, Π_t, are used to meet the bank's commitments that include *dividend payments on equity*, $n_t d_t$, *interest and principal payments on subordinate debt*, $(1+r_t^O)O_t$, and *changes in loan-loss reserves*, $(1+r_t^{R^l})R_t^l$. The *retained earnings*, E_t^r, subsequent to these payments may be computed by using

$$\Pi_t = E_t^r + n_t d_t + (1 + r_t^O)O_t + (1 + r_t^{R^l})R_t^l. \tag{2.1}$$

In standard usage, retained earnings refer to earnings that are not paid out in dividends, interest or taxes. They represent wealth accumulating in the bank and should be capitalized in the value of the bank's equity. Retained earnings also are defined to include bank charter value income. Normally, *charter value* refers to the present value of anticipated profits from future lending. In each period, banks invest in fixed assets (including buildings and equipment) which we denote by F_t. The bank is assumed to maintain these assets throughout its existence so that the bank must only cover the costs related to the *depreciation of fixed assets*, ΔF_t. These activities are financed through retaining earnings and the eliciting of additional debt and equity, so that

$$\Delta F_t = E_t^r + (n_{t+1} - n_t)E_t + O_{t+1} + R_{t+1}^l. \tag{2.2}$$

We can use (2.1) and (2.2) to obtain an expression for bank capital of the form

$$K_{t+1} = n_t(d_t + E_t) + (1 + r_t^O)O_t + (1 + r_t^{R^l})R_t^l - \Pi_t + \Delta F_t, \tag{2.3}$$

where K_t is defined by (1.9).
If the expression for retained earnings given by (2.1) is substituted into (2.2), the *net cash flow generated by the bank* is given by

$$N_t = \Pi_t - \Delta F_t = n_t d_t + (1 + r_t^O)O_t + (1 + r_t^{R^l})R_t^l - K_{t+1} + n_t E_t, \tag{2.4}$$

2.1.2 Loan Supply and Demand

Taking our lead from the equilibrium arguments in [110], we denote loan demand and supply processes by $\Lambda = \{\Lambda_t\}_{t\geq 0}$. In this case, the bank faces a *Hicksian demand for loans* given by

$$\Lambda_t = l_0 - l_1 r_t^\Lambda + l_2 M_t + \sigma_t^\Lambda. \tag{2.5}$$

We note that the loan demand in (2.5) is an increasing function of M_t and a decreasing function of r_t^Λ. Further, we suppose that σ_t^Λ is the *random shock to the loan demand* with support $[\underline{\Lambda}, \overline{\Lambda}]$ that is independent of an exogenous stochastic variable, x_t. Also, we assume that the *loan supply process*, Λ, follows the first-order autoregressive stochastic process

$$\Lambda_{t+1} = \mu_t^\Lambda \Lambda_t + \sigma_{t+1}^\Lambda, \tag{2.6}$$

where $\mu_t^\Lambda = r_t^\Lambda - c^\Lambda - r^d(M_t)$ and σ_{t+1}^Λ denotes zero-mean stochastic shocks to loan supply.

2.2 STATEMENT OF OPTIMAL BANK VALUATION PROBLEM

Suppose that the bank's performance criterion, J, at t is given by

$$J_t = \Pi_t + l_t\Big[K_t - \rho\Big(\omega(M_t)\Lambda_t + \omega^I I_t + 12.5(mVaR + 0)\Big)\Big] - c_t^{dw}\Big[K_{t+1}\Big] \\ +\mathbf{E}_t\Big[\delta_{t,1}V\Big(K_{t+1},\ x_{t+1}\Big)\Big], \tag{2.7}$$

where l_t is the Lagrangian multiplier for the total capital constraint, K_t is defined by (1.9), $\mathbf{E}_t\Big[\cdot\Big]$ is the expectation conditional on the bank's information at time t and x_t is the deposit withdrawals in period t with probability distribution $f(x_t)$. Also, c_t^{dw} is the deadweight cost of total capital consisting of equity, subordinate debt and loan loss reserves. The optimal bank valuation problem is to maximize the *bank value* given by

$$V_t = N_t + \mathbf{E}_t\Big[\sum_{j=1}^{\infty}\delta_{t,j}N_{t+j}\Big]. \tag{2.8}$$

We can now state the optimal valuation problem as follows.

Problem 2.2.1 (Statement of the Optimal Bank Valuation Problem): *Suppose that the total capital constraint and the performance criterion, J, are given by* (1.12) *and* (2.7)*, respectively. The optimal bank valuation problem is to maximize the value of the bank given by* (2.8) *by choosing the loan rate, deposits and regulatory capital for*

$$V(K_t,\ x_t) \quad = \quad \max_{r_t^{\Lambda},\ D_t,\ K_t} J_t, \tag{2.9}$$

subject to the cash flow, balance sheet, financing constraint and loan demand given by (1.13), (1.10), (2.3) *and* (2.5) *respectively.*

2.3 SOLUTION TO THE OPTIMAL BANK VALUATION PROBLEM

In this section, we find a solution to Problem 2.2.1 when the capital constraint is binding as well as non-binding. In this regard, the main result can be stated and proved as follows.

Theorem 2.3.1 (Solution to the Optimal Bank Valuation Problem (Binding)): *Suppose that J and V are given by (2.7) and (2.9), respectively and $P(M_t) = \mathbf{E}(d)\Lambda_{t-1}$. When the capital constraint given by (1.12) is binding (i.e., $l_t > 0$), a solution to the optimal bank valuation problem yields an optimal bank loan supply and loan rate of the form*

$$\Lambda_t^* = \frac{K_t}{\rho\omega(M_t)} - \left(\frac{\omega^I I_t + 12.5(mVaR + 0)}{\omega(M_t)}\right) \tag{2.10}$$

and

$$r_t^{\Lambda *} = \frac{1}{l_1}\left(l_0 + l_2 M_t + \sigma_t^{\Lambda} - \left[\frac{K_t}{\rho\omega(M_t)} - \left(\frac{\omega^I I_t + 12.5(mVaR + 0)}{\omega(M_t)}\right)\right]\right), \tag{2.11}$$

respectively. In this case, the corresponding optimal deposits, provisions for deposit withdrawals and profits are given by

$$D_t^* = \overline{D} + \frac{\overline{D}(1-\gamma)}{r_t^p}\left[r_t^{\mathrm{T}} - \frac{(r_t^D + c^D)}{1-\gamma}\right] + \frac{K_t}{\rho\omega(M_t)} - \left(\frac{\omega^I I_t + 12.5(mVaR + 0)}{\omega(M_t)}\right) - K_t,$$

$$W_t^* = \overline{D} + \frac{\overline{D}(1-\gamma)}{r_t^p}\left[r_t^{\mathrm{T}} - \frac{(r_t^D + c^D)}{1-\gamma}\right]$$

and

$$\begin{aligned}\Pi_t^* &= \left[\frac{K_t}{\rho\omega(M_t)} - \left(\frac{\omega^I I_t + 12.5(mVaR+0)}{\omega(M_t)}\right)\right]\left\{\frac{1}{l_1}\left(l_0 - \frac{K_t}{\rho\omega(M_t)} + \left(\frac{\omega^I I_t + 12.5(mVaR+0)}{\omega(M_t)}\right)\right.\right. \\ &\left.\left. + l_2 M_t + \sigma_t^\Lambda\right) - \left(c^\Lambda + (r_t^D + c^D + r_t^\mathrm{T}\gamma) + r^d(M_t)\right)\right\} + (r_t^D + c^D + r_t^\mathrm{T}\gamma)K_t \\ &+ \overline{D}\left(r_t^\mathrm{T} - (r_t^D + c^D + r_t^\mathrm{T}\gamma)\right) + \left(r_t^\mathrm{T} - \frac{(r_t^D + c^D)}{1-\gamma}\right)\left[\frac{\overline{D}(1-\gamma)(r_t^\mathrm{T} - (r_t^D + c^D + r_t^\mathrm{T}\gamma))}{r_t^P}\right] \\ &- c^w(W_t) - P(M_t) + r_t^I I_t,\end{aligned}$$

respectively.

Proof. An immediate consequence of the prerequisite that the total capital constraint from (1.12) is binding, is that loan supply is closely related to the capital adequacy constraint and is given by (2.10). Also, the dependence of changes in the loan rate on macroeconomic activity may be fixed as

$$\frac{\partial r_t^{\Lambda^*}}{\partial M_t} = \frac{l_2}{l_1}.$$

In (2.11) we substituted the optimal value for Λ_t into control law (2.5) to get the optimal default rate. We obtain the optimal W_t by using the following steps. Firstly, we rewrite (1.10) to make deposits the dependent variable so that

$$D_t = W_t + \Lambda_t - K_t.$$

Next, we note that the first order conditions are given by

$$\frac{\partial \Pi_t}{\partial r_t^\Lambda}\left[1 + c_t^{dw} - E_t\left\{\int_{\underline{\Lambda}}^{\overline{\Lambda}} \delta_{t,1}\frac{\partial V}{\partial K_{t+1}} dF(\sigma_{t+1}^\Lambda)\right\}\right] + l_1\rho^r\omega(M_t)l_t = 0; \quad (2.12)$$

$$\frac{\partial \Pi_t}{\partial D_t}\left[1 + c_t^{dw} - E_t\left\{\int_{\underline{\Lambda}}^{\overline{\Lambda}} \delta_{t,1}\frac{\partial V}{\partial K_{t+1}} dF(\sigma_{t+1}^\Lambda)\right\}\right] = 0; \quad (2.13)$$

$$\rho^r(12.5(mVaR+0) + \omega(M_t)\Lambda_t + \omega^I I_t) \leq K_t; \quad (2.14)$$

$$-c_t^{dw} + E_t\left\{\int_{\underline{\Lambda}}^{\overline{\Lambda}} \delta_{t,1}\frac{\partial V}{\partial K_{t+1}} dF(\sigma_{t+1}^\Lambda)\right\} = 0. \quad (2.15)$$

Here $F(\cdot)$ is the cumulative distribution of the shock to the loans. Using (2.15) we can see that (2.13) becomes

$$\frac{\partial \Pi_t}{\partial D_t} = 0.$$

Looking at the form of Π_t given in (1.13) and the equation

$$c^w(W_t) = \frac{r_t^p}{2\overline{D}}[\overline{D} - W_t]^2 \tag{2.16}$$

it follows that

$$\Pi_t = \left(r_t^\Lambda - c^\Lambda - r^d(M_t)\right)\Lambda_t + r_t^{\mathrm{T}} W_t + r_t^I I_t - \left(r_t^D + c^D\right) D_t \tag{2.17}$$
$$-\frac{r_t^p}{2\overline{D}}\left[\overline{D} - W_t\right]^2 - P(M_t) - r_t^{\mathrm{T}}\gamma D_t.$$

Therefore

$$\frac{\partial \Pi_t}{\partial D_t} = r_t^{\mathrm{T}} - (r_t^D + c^D) + \frac{r_t^p}{\overline{D}}\left(\overline{D} - W_t\right) - r_t^{\mathrm{T}}\gamma = 0. \tag{2.18}$$

This would then give us the optimal value for D_t. Using (1.10) and all the optimal values calculated to date, we can find optimal deposits, and the same goes for optimal profits. □

Corollary 2.3.2 (Solution to the Optimal Bank Valuation Problem (Non-Binding)): *Suppose that J and V are given by* (2.7) *and* (2.9), *respectively and $P(M_t) = \mathbf{E}(d)\Lambda_{t-1}$. When the capital constraint given by* (1.12) *is not binding (i.e., $l_t = 0$), a solution to the optimal bank valuation problem stated in Problem* 2.2.1 *yields the optimal bank loan supply and loan rate*

$$\Lambda_t^{*n} = \frac{1}{2}\left(l_0 + l_2 M_t + \sigma_t^\Lambda\right) - \frac{l_1}{2}\left(c^\Lambda + r^d(M_t) + (r_t^D + c^D) + r_t^{\mathrm{T}}\gamma\right) \tag{2.19}$$

and

$$r_t^{\Lambda*n} = \frac{1}{2l_1}\Big(l_0 + l_2M_t + \sigma_t^{\Lambda}\Big) + \frac{1}{2}\Big(c^{\Lambda} + r^d(M_t) + (r_t^D + c^D) + r_t^{\mathrm{T}}\gamma\Big), \qquad (2.20)$$

respectively. In this case, the corresponding W_t, deposits and profits are given by

$$W_t^{*n} = \overline{D} + \frac{\overline{D}(1-\gamma)}{r_t^p}\left(r_t^{\mathrm{T}} - \frac{r_t^D + c^D}{1-\gamma}\right),$$

$$D_t^{*n} = \overline{D} + \frac{\overline{D}(1-\gamma)}{r_t^p}\left(r_t^{\mathrm{T}} - \frac{(r_t^D + c^D)}{1-\gamma}\right) + \Lambda_t^{*n} - K_t$$

and

$$\begin{aligned}\Pi_t^{*n} = {} & \frac{1}{2}\Big(l_0 + l_2M_t + \sigma_t^{\Lambda}\Big) - \frac{l_1}{2}\Big(c^{\Lambda} + (r_t^D + c^D) + r^d(M_t) + r_t^{\mathrm{T}}(\gamma)\Big)\Big\{\frac{1}{2l_1}\Big(l_0 + l_2M_t + \sigma_t^{\Lambda}\Big) \\ & -\frac{1}{2}\Big(c^{\Lambda} + (r_t^D + c^D) + r^d(M_t) + r_t^{\mathrm{T}}\gamma\Big)\Big\} + (r_t^D + c^D + r_t^{\mathrm{T}}\gamma)K_t + \overline{D}\Big(r_t^{\mathrm{T}} \\ & -(r_t^D + c^D + r_t^{\mathrm{T}}\gamma)\Big) + \left(r_t^{\mathrm{T}} - \frac{(r_t^D + c^D)}{1-\gamma}\right)\left[\frac{\overline{D}(1-\gamma)(r_t^{\mathrm{T}} - (r_t^D + c^D + r_t^{\mathrm{T}}\gamma))}{r_t^p}\right] - c^w(W_t) - P(M_t) + r^I I_t,\end{aligned}$$

respectively.

Proof. For the situation where capital constraint does not bind (i.e., $l_t = 0$), using equation (2.15) and the fact that $l_t = 0$, we can see that (2.12) becomes

$$\frac{\partial \Pi_t}{\partial r_t^{\Lambda}} = 0.$$

As in the proof of Theorem 2.3.1, looking at the form of Π_t given in (1.13) and (2.16), we have equation (2.17). Therefore

$$\frac{\partial \Pi_t}{\partial r_t^{\Lambda}} = \Lambda_t - l_1(r_t^{\Lambda} - c^{\Lambda} - r^d(M_t)) + l_1 r_t^{\mathrm{T}} + \frac{r_t^p}{\overline{D}}(\overline{D} - W_t)l_1 = 0. \qquad (2.21)$$

By substituting (2.18) into (2.21) and using (2.5) would give us optimal loans and loan rate given by (2.19) and (2.20) respectively. Furthermore we can find the optimal deposit, deposit withdrawals and profits. □

Chapter 3

CYCLICALITY OF BANK CREDIT AND CAPITAL UNDER BASEL I AND II

In this chapter, we discuss the discrete-time dynamic model of bank items where, as in the Basel I, we assume that risk-weights are constant. Furthermore, (we) give the results for the situation where loan losses and loan risk-weights are a function of the phases of the business cycle (i.e., risk-weights vary with business cycle factors).

Problem 3.0.3 (Cyclicality of Bank Credit and Capital Under Basel I): *What is the effect of changes in the phases of the business cycle on bank credit and capital when asset risk-weights are constant (under Basel I) ? (see Section 3.1).*

Problem 3.0.4 (Cyclicality of Bank Credit and Capital Under Basel II): *What is the effect of changes in the phases of the business cycle on bank credit and capital when asset risk-weights vary with macroeconomic performance (under Basel II) ? (see Section 3.2).*

Assumption 3.0.5 *: It is assumed that banks make losses on loans but risk-weights are constant, as under the Basel I.*

Assumption 3.0.6 *: Asset risk-weights are assumed to vary over the business cycle, as under the Basel II.*

3.1 CYCLICALITY OF BANK CREDIT AND CAPITAL UNDER BASEL I (CONSTANT RISK-WEIGHTS)

In this section, we discuss the result of the model discussed in Chapter 2 under the assumption that bank loans may not be repaid and the response of bank loans in the influence of the changes in the business cycle. The risk-weight on loans is constant. In this case, the capital constraint that a bank faces when maximizing its profits is given by the following

$$K_t \geq \rho\left[\omega^{\Lambda}\Lambda_t + \omega^{I}I_t + 12.5mVaR\right]. \tag{3.1}$$

Here operational risk is not considered as in Basel II.

3.1.1 Cyclicality of the Quantity and Price of Loans and Bank Capital Under Basel I

In this subsection we examine how bank capital and loans are affected by changes in the phases of the business cycle.

Theorem 3.1.1 (Cyclicality of Bank Capital under Basel I): *Suppose that $L(M_t) > 0$ and $\omega(M_t) = 1$ (i.e risk-weights are constant). It follows that*

1. *if* $\frac{\partial \sigma_{t+1}^{\Lambda *}}{\partial M_t} < 0$ *then* $\frac{\partial K_{t+1}}{\partial M_t} > 0$.
2. *if* $\frac{\partial \sigma_{t+1}^{\Lambda *}}{\partial M_t} > 0$ *then* $\frac{\partial K_{t+1}}{\partial M_t} < 0$.

Proof. In order to prove Theorem 3.1.1, we need to use the implicit function theorem. Before we can use this theorem, we solve the critical shock to the loan demand, σ_t^{Λ}, such that the total capital constraint will just bind, i.e $\Lambda_t^{*n} = \Lambda_t^*$. By equating the optimal loans from the two problems (with $l_t = 0$ and $l_t > 0$), we obtain

$$\frac{1}{2}\left(l_0 + l_2 M_t + \sigma_t^{\Lambda}\right) - \frac{l_1}{2}\left(c^{\Lambda} + r^d(M_t) + (r_t^D + c^D) + r_t^{\mathrm{T}}\gamma\right) = \frac{K_t}{\rho} - \omega^I I_t - 12.5mVaR$$

Solving for σ_t^{Λ}, we get

$$\frac{1}{2}\sigma_t^{\Lambda} = -\frac{1}{2}\left(l_0 + l_2 M_t\right) + \frac{l_1}{2}\left(c^{\Lambda} + r^d(M_t) + (r_t^D + c^D) + r_t^{\mathrm{T}}\gamma\right) + \frac{K_t}{\rho} - \omega^I I_t - 12.5mVaR$$

therefore

$$\sigma_t^{\Lambda *} = 2\left(\frac{K_t}{\rho} - \omega^I I_t - 12.5mVaR\right) - \left(l_0 + l_2 M_t\right) + l_1\left(c^{\Lambda} + r^d(M_t) + (r_t^D + c^D) + r_t^{\mathrm{T}}\gamma\right).$$

Using the following equation

$$\Lambda_t - l_1(r_t^{\Lambda} - c^{\Lambda} - r^d(M_t)) + l_1 r_t^{\mathrm{T}} + l_1 \frac{r_t^p}{\overline{D}}(\overline{D} - W_t) + l_1 \rho l_t = 0, \tag{3.2}$$

and substituting provisions for deposit withdrawals we get

$$
\begin{aligned}
l_1\rho l_t &= l_1(r_t^\Lambda - c^\Lambda - r^d(M_t)) - l_1 r_t^\mathrm{T} - l_1\frac{r_t^p}{\overline{D}}\left[\overline{D} - \left(\overline{D} + \frac{\overline{D}(1-\gamma)}{r_t^p}\left(r_t^\mathrm{T} - \frac{(r_t^D + c^D)}{1-\gamma}\right)\right)\right] - \Lambda_t \\
&= l_1(r_t^\Lambda - c^\Lambda - r^d(M_t)) - l_1 r_t^\mathrm{T} + l_1(1-\gamma)\left[r_t^\mathrm{T} - \frac{(r_t^D + c^D)}{1-\gamma}\right] - \Lambda_t \\
&= l_1(r_t^\Lambda - c^\Lambda - r^d(M_t) - (r_t^D + c^D + \gamma r_t^\mathrm{T})) - \Lambda_t.
\end{aligned}
$$

Substituting $r_t^{\Lambda*}$ and Λ_t^* into the expression above, we obtain

$$
\begin{aligned}
l_1\rho l_t &= l_1\left[\frac{1}{l_1}\left((l_0 + l_2M_t + \sigma_t^\Lambda) - \left[\frac{K_t}{\rho} - \omega^I I_t - 12.5mVaR\right]\right) - c^\Lambda - r^d(M_t) - (r_t^D + c^D + \gamma r_t^\mathrm{T})\right] \\
&\qquad - \left(\frac{K_t}{\rho} - \omega^I I_t - 12.5mVaR\right) \\
&= (l_0 + l_2M_t + \sigma_t^\Lambda) - \left(\frac{K_t}{\rho} - \omega^I I_t - 12.5mVaR\right) - l_1(c^\Lambda + r^d(M_t) + (r_t^D + c^D + \gamma r_t^\mathrm{T})) \\
&\qquad - \left(\frac{K_t}{\rho} - \omega^I I_t - 12.5mVaR\right) \\
&= \sigma_t^\Lambda - 2\left(\frac{K_t}{\rho} - \omega^I I_t - 12.5mVaR\right) + (l_0 + l_2M_t) - l_1(c^\Lambda + r^d(M_t) + (r_t^D + c^D + \gamma r_t^\mathrm{T})) \\
&= \sigma_t^\Lambda - \left[2\left(\frac{K_t}{\rho} - \omega^I I_t - 12.5mVaR\right) - (l_0 + l_2M_t) + l_1(c^\Lambda + r^d(M_t) + (r_t^D + c^D + \gamma r_t^\mathrm{T}))\right] \\
&= \sigma_t^\Lambda - \sigma_t^{\Lambda*}.
\end{aligned}
$$

It follows that

$$
l_t^* = \frac{\sigma_t^\Lambda - \sigma_t^{\Lambda*}}{\rho l_1}, \qquad \sigma_t^{\Lambda*} \le \sigma_t^\Lambda \le \overline{\Lambda}.
$$

Using equation (2.9) to find the partial derivative of the value function with respect to bank capital we get

$$
\begin{aligned}
\frac{\partial V}{\partial K_t} &= l_t + (r_t^D + c^D + \gamma r_t^\mathrm{T}) \\
&= r_t^D + c^D + \gamma r_t^\mathrm{T}), \text{ for } \underline{\Lambda} \le \sigma_t^\Lambda \le \sigma_t^{\Lambda*} \\
&= (r_t^D + c^D + \gamma r_t^\mathrm{T}) + \frac{\sigma_t^\Lambda - \sigma_t^{\Lambda*}}{\rho l_1}, \text{ for } \sigma_t^{\Lambda*} \le \sigma_t^\Lambda \le \overline{\Lambda}.
\end{aligned}
$$

By substituting the above expression into the optimal condition for total capital (2.15), we get

$$c_t^{dw} - E_t\left[\delta_{t,1}(r_t^D + c^D + \gamma r_t^{\mathrm{T}})\right] - \frac{1}{\rho l_1}E_t\left[\int_{\sigma_{t+1}^{\Lambda*}}^{\overline{\Lambda}} \delta_{t,1}(\sigma_{t+1}^{\Lambda} - \sigma_{t+1}^{\Lambda*})dF(\sigma_{t+1}^{\Lambda})\right] = 0.$$

We denote the left-hand side of the above expression by H, i.e.,

$$H = \frac{1}{\rho l_1}E_t\left[\int_{\sigma_{t+1}^{\Lambda*}}^{\overline{\Lambda}} \delta_{t,1}(\sigma_{t+1}^{\Lambda} - \sigma_{t+1}^{\Lambda*})dF(\sigma_{t+1}^{\Lambda})\right] = 0. \tag{3.3}$$

By the implicit function theorem

$$\frac{\partial K_{t+1}}{\partial M_t} = -\frac{\partial H}{\partial M_t}\Big/\frac{\partial H}{\partial K_{t+1}}.$$

To calculate for $\frac{\partial H}{\partial M_t}$, we use (3.3) and get

$$\frac{\partial H}{\partial M_t} = \frac{1}{\rho l_1}\left\{\frac{\partial \sigma_{t+1}^{\Lambda*}}{\partial M_t}E_t\left[\int_{\sigma_{t+1}^{\Lambda*}}^{\Lambda} \delta_{t,1}dF(\sigma_{t+1}^{\Lambda})\right]\right\},$$

where

$$\frac{\partial \sigma_{t+1}^{\Lambda*}}{\partial M_t} = -l_2\mu^M + l_1\mu^M\frac{\partial r^d}{\partial M_{t+1}}.$$

Therefore

$$\begin{aligned}\frac{\partial H}{\partial M_t} &= \frac{1}{\rho l_1}\left\{\left(-l_2\mu^M + l_1\mu^M\frac{\partial r^d}{\partial M_{t+1}}\right)E_t\left[\int_{\sigma_{t+1}^{\Lambda*}}^{\Lambda} \delta_{t,1}dF(\sigma_{t+1}^{\Lambda})\right]\right\}\\ &= \frac{\mu^M}{\rho}\left(-\frac{l_2}{l_1} + \frac{\partial r^d}{\partial M_{t+1}}\right)E_t\left[\int_{\sigma_{t+1}^{\Lambda*}}^{\Lambda} \delta_{t,1}dF(\sigma_{t+1}^{\Lambda})\right]\end{aligned}$$

and

$$\begin{aligned}\frac{\partial H}{\partial K_{t+1}} &= \frac{2}{\rho}\frac{1}{\rho l_1}E_t\Big[\int_{\sigma^{\Lambda*}_{t+1}}^{\Lambda} \delta_{t,1}dF(\sigma^{\Lambda}_{t+1})\Big] \\ &= \frac{2}{l_1\rho^2}E_t\Big[\int_{\sigma^{\Lambda*}_{t+1}}^{\Lambda} \delta_{t,1}dF(\sigma^{\Lambda}_{t+1})\Big].\end{aligned}$$

Therefore

$$\begin{aligned}\frac{\partial K_{t+1}}{\partial M_t} &= -\Big[\frac{\mu^M}{\rho}\Big(-\frac{l_2}{l_1}+\frac{\partial r^d}{\partial M_{t+1}}\Big)E_t\Big(\int_{\sigma^{\Lambda*}_{t+1}}^{\Lambda} \delta_{t,1}dF(\sigma^{\Lambda}_{t+1})\Big)\Big]\Big/\Big[\frac{2}{l_1\rho^2}E_t\Big(\int_{\sigma^{\Lambda*}_{t+1}}^{\Lambda} \delta_{t,1}dF(\sigma^{\Lambda}_{t+1})\Big)\Big] \\ &= -\frac{\mu^M}{\rho}\Big(-\frac{l_2}{l_1}+\frac{\partial r^d}{\partial M_{t+1}}\Big)\Big/-\big(\frac{2}{l_1\rho^2}\Big) \\ &= \frac{1}{2}l_1\rho\mu^M\Big(-\frac{l_2}{l_1}+\frac{\partial r^d}{\partial M_{t+1}}\Big)\end{aligned}$$

$\frac{\partial K_{t+1}}{\partial M_t}$ is positive since $\frac{\partial \sigma^{\Lambda*}_{t+1}}{\partial M_t} < 0$. This concludes the proof of Theorem 3.1.1. □

Remark 3.1.2 (Cyclicality of Bank Capital under Basel I): *Under the assumption of positive loan losses and constant asset risk-weights, banks raise (decrease) their capital holdings in response to a positive (negative) shock in the current level of economic activity.*

3.1.2 Cyclicality of Loans and Loan Rates Under Basel I (Non-Binding Constraint)

In this subsection, we establish the impact of changes in M_t on loans and loan rates. We first present the results under a non-binding capital constraint.

Proposition 3.1.3 (Cyclicality of Loans and Loan Rates under Basel I (Non-Binding)): *If* $l_t = 0$, *then*

$$\frac{\partial {\Lambda^{*n}}_{t+j}}{\partial M_t} = \frac{1}{2}\mu^M_j\Big(l_2 - l_1\frac{\partial r^d(M_{t+j})}{\partial M_{t+j}}\Big)$$

and

$$\frac{\partial r_{t+j}^{\Lambda *n}}{\partial M_t} = \frac{1}{2}\mu_j^M \left(\frac{l_2}{l_1} + \frac{\partial r^d(M_{t+j})}{\partial M_{t+j}} \right).$$

Proof. In order to prove Proposition 3.1.3, we find the partial derivatives of the optimal loan supply and loan rate with respect to M_t of equations (2.19) and (2.20) respectively, from Chapter 2 under a non-binding constraint. But here we do not include operational risk. We also use the condition $\dfrac{\partial r^d(M_{t+j})}{\partial M_{t+j}} < 0$. Now we can calculate

$$\frac{\partial \Lambda^{*n}{}_{t+j}}{\partial M_t} \left(\frac{1}{2}(l_0 + l_2 M_t + \sigma_t^{\Lambda}) - \frac{l_1}{2}(c^{\Lambda} + r^d(M_t) + (r^D + c^D) + r_t^{\mathrm{T}}\gamma) \right)$$
$$= \frac{1}{2}\mu_j^M \left(l_2 - l_1 \frac{\partial r^d(M_{t+j})}{\partial M_{t+j}} \right)$$

and

$$\frac{\partial r_{t+j}^{\Lambda *n}}{\partial M_t} \left(\frac{1}{2l_1}(l_0 + l_2 M_t + \sigma_t^{\Lambda}) + \frac{1}{2}\left(c^{\Lambda} + r^d(M_t) + (r^D + c^D) + r_t^{\mathrm{T}}\gamma \right) \right)$$
$$= \frac{1}{2}\mu_j^M \left(\frac{l_2}{l_1} + \frac{\partial r^d(M_{t+j})}{\partial M_{t+j}} \right)$$

as required. □

Remark 3.1.4 (Cyclicality of Loans and Loan Rates under Basel I (Non-Binding)): *When the capital constraint (3.1) is non-binding, bank loans increase as a result of an improvement in macroeconomic activity, while the loan rate can either increase or decrease depending on the parameters characterizing the loan demand function and the loan default rate.*

3.1.3 Cyclicality of Loans and Loan Rates Under Basel I (Binding Constraint)

In this subsection, we consider the case in which the capital constraint binds.

Proposition 3.1.5 (Cyclicality of Loans and Loan Rates under Basel I (Binding)): *When $l_t > 0$ the loan supply is determined by the total capital constraint and is given by*

$$\Lambda_t^* = \frac{K_t}{\rho} - \omega^I I_t - 12.5mVaR$$

and

$$\frac{\partial \Lambda_{t+j}^*}{\partial M_t} = 0$$

while the loan rate response to changes in macroeconomic activity is

$$\frac{l_2}{l_1} = \frac{\partial r_{t+j}^{\Lambda^*}}{\partial M_t}.$$

Proof. In order to prove Proposition 3.1.5, we find the partial derivatives of the optimal loan supply and loan rate with respect to M_t of equations (2.10) and (2.11) to find $\frac{\partial \Lambda^*{}_{t+j}}{\partial M_t}$ and $\frac{\partial r_{t+j}^{\Lambda^*}}{\partial M_t}$ respectively. But here we do not include operational risk. Now we can calculate

$$\frac{\partial \Lambda_{t+j}^*}{\partial M_t}\left(\frac{K_t}{\rho\omega(M_t)} - \left[\frac{\omega^I I_t + 12.5mVaR}{\omega(M_t)}\right]\right) = 0$$

and

$$\frac{l_2}{l_1} = \frac{\partial r^{\Lambda_{t+j}^*}}{\partial M_t}\left(\frac{1}{l_1}\left(l_0 + l_2 M_t + \sigma_t^{\Lambda} - \frac{K_t}{\rho\omega(M_t)} + \left[\frac{\omega^I I_t + 12.5mVaR}{\omega(M_t)}\right]\right)\right).$$

□

Remark 3.1.6 (Cyclicality of Loans and Loan Rates under Basel I (Binding)): *When the capital constraint (3.1) is binding, current bank lending does not change in response to an improvement in the business cycle activity. The loan rate however increases as a consequence of a higher loan demand.*

3.2 CYCLICALITY OF BANK CREDIT AND CAPITAL UNDER BASEL II (VARYING RISK-WEIGHTS)

In this section, similar theorem and propositions to the ones in the previous section will be derived for a model where both loan losses and loan risk-weights are a function of the phases of the business cycle, M_t. The capital constraint is now described by the expression in (1.12), where the risk-weights on intangible assets $(\omega^I) \neq 0$ and the one on loans is a decreasing function of current phases of the business cycle, i.e, $\frac{\partial \omega(M_t)}{\partial M_t} < 0$.

3.2.1 Cyclicality of the Quantity and Price of Bank Loans and Capital Under Basel II

In this subsection, we examine how bank capital and loans are affected by changes in the phases of the business cycle when asset risk-weights are not constant.

Theorem 3.2.1 (Cyclicality of Bank Loans and Capital under Basel II): *Suppose that $L(M_t) > 0$ and risk-weights, $\omega(M_t)$ are not constant. In this case, we have that*

1. *if* $\frac{\partial \sigma_{t+1}^{\Lambda *}}{\partial M_t} < 0$ *then* $\frac{\partial K_{t+1}}{\partial M_t} > 0$;
2. *if* $\frac{\partial \sigma_{t+1}^{\Lambda *}}{\partial M_t} > 0$ *then* $\frac{\partial K_{t+1}}{\partial M_t} < 0$.

Proof. As in the proof of Theorem 3.1.1, we equate the optimal loans from the two problems (with $l_t = 0$ and $l_t > 0$) and obtain

$$\frac{1}{2}\left(l_0 + l_2 M_t + \sigma_t^{\Lambda}\right) - \frac{l_1}{2}\left(c^{\Lambda} + r^d(M_t) + (r_t^D + c^D) + r_t^{\mathrm{T}}\gamma\right) = \frac{K_t}{\rho\omega(M_t)} - \left[\frac{\omega^I I_t + 12.5(mVaR + 0)}{\omega(M_t)}\right].$$

Solving for σ_t^{Λ}, we get

$$\frac{1}{2}\sigma_t^{\Lambda} = -\frac{1}{2}\left(l_0 + l_2 M_t\right) + \frac{l_1}{2}\left(c^{\Lambda} + r^d(M_t) + (r_t^D + c^D) + r_t^{\mathrm{T}}\gamma\right) + \frac{K_t}{\rho\omega(M_t)} - \left[\frac{\omega^I I_t + 12.5(mVaR + 0)}{\omega(M_t)}\right]$$

therefore

$$\sigma_t^{\Lambda *} = 2\Big(\frac{K_t}{\rho\omega(M_t)} - \Big[\frac{\omega^I I_t + 12.5(mVaR + 0)}{\omega(M_t)}\Big]\Big) - \Big(l_0 + l_2 M_t\Big) + l_1\Big(c^\Lambda + r^d(M_t) + (r_t^D + c^D) + r_t^{\mathrm{T}}\gamma\Big).$$

Using equation (3.2) and substituting provisions for the deposit withdrawals we get

$$\begin{aligned} l_1\rho\omega(M_t)l_t &= l_1(r_t^\Lambda - c^\Lambda - r^d(M_t)) - l_1 r_t^{\mathrm{T}} - l_1\frac{r_t^p}{\overline{D}}\Big[\overline{D} - \Big(\overline{D} + \frac{\overline{D}(1-\gamma)}{r_t^p}\Big(r_t^{\mathrm{T}} - \frac{(r_t^D + c^D)}{1-\gamma}\Big)\Big)\Big] - \Lambda_t \\ &= l_1(r_t^\Lambda - c^\Lambda - r^d(M_t)) - l_1 r_t^{\mathrm{T}} + l_1(1-\gamma)\Big[r_t^{\mathrm{T}} - \frac{(r_t^D + c^D)}{1-\gamma}\Big] - \Lambda_t \\ &= l_1(r_t^\Lambda - c^\Lambda - r^d(M_t) - (r_t^D + c^D + \gamma r_t^{\mathrm{T}})) - \Lambda_t. \end{aligned}$$

Substitute $r_t^{\Lambda^*}$ and Λ_t^* into the expression above to obtain

$$\begin{aligned} l_1\rho\omega(M_t)l_t &= l_1\Big[\frac{1}{l_1}\Big((l_0 + l_2 M_t + \sigma_t^\Lambda) - \frac{K_t}{\rho\omega(M_t)} + \Big[\frac{\omega^I I_t + 12.5(mVaR + 0)}{\omega(M_t)}\Big]\Big) - c^\Lambda - r^d(M_t) - (r_t^D + c^D + \gamma r_t^{\mathrm{T}})\Big] \\ &\qquad - \frac{K_t}{\rho\omega(M_t)} + \Big[\frac{\omega^I I_t + 12.5(mVaR + 0)}{\omega(M_t)}\Big] \\ &= (l_0 + l_2 M_t + \sigma_t^\Lambda) - \frac{K_t}{\rho\omega(M_t)} + \Big[\frac{\omega^I I_t + 12.5(mVaR + 0)}{\omega(M_t)}\Big] - l_1(c^\Lambda + r^d(M_t) + (r_t^D + c^D + \gamma r_t^{\mathrm{T}})) \\ &\qquad - \frac{K_t}{\rho\omega(M_t)} + \Big[\frac{\omega^I I_t + 12.5(mVaR + 0)}{\omega(M_t)}\Big] \\ &= \sigma_t^\Lambda - 2\Big(\frac{K_t}{\rho\omega(M_t)} - \Big[\frac{\omega^I I_t + 12.5(mVaR + 0)}{\omega(M_t)}\Big]\Big) + \Big(l_0 + l_2 M_t\Big) - l_1\Big(c^\Lambda + r^d(M_t) + (r_t^D + c^D + \gamma r_t^{\mathrm{T}})\Big) \\ &= \sigma_t^\Lambda - \Big[2\Big(\frac{K_t}{\rho\omega(M_t)} - \Big[\frac{\omega^I I_t + 12.5(mVaR + 0)}{\omega(M_t)}\Big]\Big) - \Big(l_0 + l_2 M_t\Big) + l_1\Big(c^\Lambda + r^d(M_t) + (r_t^D + c^D + \gamma r_t^{\mathrm{T}})\Big)\Big] \\ &= \sigma_t^\Lambda - \sigma_t^{\Lambda *} \end{aligned}$$

therefore

$$l_t^* = \frac{\sigma_t^\Lambda - \sigma_t^{\Lambda *}}{\omega(M_t)\rho l_1}.$$

Using equation (2.9) to find the partial derivative of the value function with respect to bank capital we get

$$\begin{aligned}\frac{\partial V}{\partial K_t} &= l_t + (r_t^D + c^D + \gamma r_t^{\text{T}}) \\ &= (r_t^D + c^D + \gamma r_t^{\text{T}}), \quad \text{for} \quad \underline{\Lambda} \le \sigma_t^{\Lambda} \le \sigma_t^{\Lambda *} \\ &= (r_t^D + c^D + \gamma r_t^{\text{T}}) + \frac{\sigma_t^{\Lambda} - \sigma_t^{\Lambda *}}{\omega(M_t)\rho l_1}, \quad \text{for} \quad \sigma_t^{\Lambda *} \le \sigma_t^{\Lambda} \le \overline{\Lambda}.\end{aligned}$$

By substituting the above expression into the optimal condition for total capital (2.15), we get

$$c_t^{dw} - E_t\Big[\delta_{t,1}(r_t^D + c^D + \gamma r_t^{\text{T}})\Big] - \frac{1}{\omega(M_{t+1})\rho l_1}E_t\Big\{\int_{\sigma_{t+1}^{\Lambda *}}^{\overline{\Lambda}} \delta_{t,1}(\sigma_{t+1}^{\Lambda} - \sigma_{t+1}^{\Lambda *})dF(\sigma_{t+1}^{\Lambda})\Big\} = 0.$$

We denote the left-hand side of the above expression by H, i.e,

$$H = \frac{1}{\omega(M_{t+1})\rho l_1}E_t\Big\{\int_{\sigma_{t+1}^{\Lambda *}}^{\overline{\Lambda}} \delta_{t,1}(\sigma_{t+1}^{\Lambda} - \sigma_{t+1}^{\Lambda *})dF(\sigma_{t+1}^{\Lambda})\Big\}. \tag{3.4}$$

By the implicit function theorem

$$\frac{\partial K_{t+1}}{\partial M_t} = -\frac{\partial H}{\partial M_t}\Big/\frac{\partial H}{\partial K_{t+1}}$$

To calculate for $\frac{\partial H}{\partial M_t}$, we use equation (3.4) and get

$$\begin{aligned}\frac{\partial H}{\partial M_t} = &-\frac{1}{\rho l_1}\frac{-\mu^M \frac{\partial \omega}{\partial M_{t+1}}}{[\omega(M_{t+1})]^2}E_t\Big\{\int_{\sigma_{t+1}^{\Lambda *}}^{\Lambda} \delta_{t,1}(\sigma_{t+1}^{\Lambda} - \sigma_{t+1}^{\Lambda *})dF(\sigma_{t+1}^{\Lambda})\Big\} \\ &-\frac{1}{\rho l_1 \omega(M_{t+1})}\frac{\partial \sigma_{t+1}^{\Lambda *}}{\partial M_t}E_t\Big\{\int_{\sigma_{t+1}^{\Lambda *}}^{\Lambda} \delta_{t,1}dF(\sigma_{t+1}^{\Lambda})\Big\}\end{aligned}$$

where

$$\frac{\partial \sigma_{t+1}^{\Lambda *}}{\partial M_t} = -\frac{2}{\rho}\left(\frac{K_t - \rho(12.5(mVaR + 0) + \omega^I I_t)}{[\omega(M_{t+1})]^2}\right)\mu^M\frac{\partial \omega}{\partial M_{t+1}} - l_2\mu^M + l_1\mu^M\frac{\partial r^d}{\partial M_{t+1}}.$$

Therefore, we have that

$$\frac{\partial H}{\partial M_t} = -\frac{1}{\rho l_1}\frac{-\mu^M \frac{\partial \omega}{\partial M_{t+1}}}{[\omega(M_{t+1})]^2} E_t\Big\{\int_{\sigma^{\Lambda*}_{t+1}}^{\Lambda} \delta_{t,1}(\sigma^{\Lambda}_{t+1} - \sigma^{\Lambda*}_{t+1}) dF(\sigma^{\Lambda}_{t+1})\Big\}$$
$$+\frac{1}{\omega(M_{t+1})\rho l_1}\Big(\frac{2}{\rho}\Big)\Big(\frac{K_t - \rho(12.5(mVaR+0) + \omega^I I_t)}{[\omega(M_{t+1})]^2}\Big)\mu^M \frac{\partial \omega}{\partial M_{t+1}} E_t\Big\{\int_{\sigma^{\Lambda*}_{t+1}}^{\Lambda} \delta_{t,1} dF(\sigma^{\Lambda}_{t+1})\Big\}$$
$$-\frac{1}{\omega(M_{t+1})\rho l_1}\Big(-l_2\mu^M + l_1\mu^M \frac{\partial r^d}{\partial M_{t+1}}\Big) E_t\Big\{\int_{\sigma^{\Lambda*}_{t+1}}^{\Lambda} \delta_{t,1} dF(\sigma^{\Lambda}_{t+1})\Big\}$$

and

$$\frac{\partial H}{\partial K_{t+1}} = \frac{2}{\omega(M_{t+1})\rho}\frac{1}{\omega(M_{t+1})\rho l_1} E_t\Big\{\int_{\sigma^{\Lambda*}_{t+1}}^{\Lambda} \delta_{t,1} dF(\sigma^{\Lambda}_{t+1})\Big\}$$
$$= \frac{2}{l_1[\omega(M_{t+1})\rho]^2} E_t\Big\{\int_{\sigma^{\Lambda*}_{t+1}}^{\Lambda} \delta_{t,1} dF(\sigma^{\Lambda}_{t+1})\Big\}.$$

Therefore, we have that

$$\frac{\partial K_{t+1}}{\partial M_t} = -\frac{\partial H}{\partial M_t}\Big/\frac{\partial H}{\partial K_{t+1}}$$
$$= \frac{-\frac{1}{\rho l_1}\Big\{\frac{-\mu^M \frac{\partial \omega}{\partial M_{t+1}}}{[\omega(M_{t+1})]^2} E_t\Big[\int_{\sigma^{\Lambda*}_{t+1}}^{\Lambda} \delta_{t,1}(\sigma^{\Lambda}_{t+1} - \sigma^{\Lambda*}_{t+1}) dF(\sigma^{\Lambda}_{t+1})\Big] - \frac{1}{\omega(M_{t+1})}\frac{\partial \sigma^{\Lambda*}_{t+1}}{\partial M_t} E_t\Big[\int_{\sigma^{\Lambda*}_{t+1}}^{\Lambda} \delta_{t,1} dF(\sigma^{\Lambda}_{t+1})\Big]\Big\}}{\frac{2}{l_1[\omega(M_{t+1})\rho]^2} E_t\Big[\int_{\sigma^{\Lambda*}_{t+1}}^{\Lambda} \delta_{t,1} dF(\sigma^{\Lambda}_{t+1})\Big]}$$

is positive only if $\frac{\partial \sigma^{\Lambda*}_{t+1}}{\partial M_t} < 0$. □

Remark 3.2.2 (Cyclicality of Bank Loans and Capital under Basel II) *Under the assumptions of positive loan losses and a risk-sensitive capital constraint, banks can either raise or lower their capital holdings in response to a positive shock in the current level of economic activity. Their choice depends on the effect that the changes in M_t has on the likelihood of the capital constraint binding next period.*

3.2.2 Cyclicality of Loans and Loan Rates Under Basel II (Non-Binding Constraint)

We now turn to the effect of a shock to M_t on bank loans and the loan rate and analyze the case, when the capital constraint is not binding.

Proposition 3.2.3 (Cyclicality of Bank Loans under Basel II (Non-Binding)): *Under the same assumptions of Theorem 3.2.1, and when* $l_t = 0$

$$\frac{\partial \Lambda^{*n}{}_{t+j}}{\partial M_t} = \frac{1}{2}\mu_j^M \left(l_2 - l_1 \frac{\partial r^d(M_{t+j})}{\partial M_{t+j}} \right)$$

and

$$\frac{\partial r_{t+j}^{\Lambda*n}}{\partial M_t} = \frac{1}{2}\mu_j^M \left(\frac{l_2}{l_1} + \frac{\partial r^d(M_{t+j})}{\partial M_{t+j}} \right).$$

Proof. In order to prove Proposition 3.2.3, we find the partial derivatives of the optimal loan supply and loan rate with respect to M_t. We use equations (2.19), (2.20) and the condition $\frac{\partial r^d(M_{t+j})}{\partial M_{t+j}} < 0$. Now we can calculate

$$\begin{aligned} \frac{\partial \Lambda^{*n}{}_{t+j}}{\partial M_t} \left(\frac{1}{2}(l_0 + l_2 M_t + \sigma_t^\Lambda) - \frac{l_1}{2}(c^\Lambda + r^d(M_t) + (r_t^D + c^D) + r_t^{\mathrm{T}}\gamma) \right) \\ = \frac{1}{2}\mu_j^M \left(l_2 - l_1 \frac{\partial r^d(M_{t+j})}{\partial M_{t+j}} \right) \end{aligned}$$

and

$$\frac{\partial r_{t+j}^{\Lambda*n}}{\partial M_t} \left(\frac{1}{2l_1}(l_0 + l_2 M_t + \sigma_t^\Lambda) + \frac{1}{2}\left(c^\Lambda + r^d(M_t) + (r_t^D + c^D) + r_t^{\mathrm{T}}\gamma \right) \right)$$

$$= \frac{1}{2}\mu_j^M \left(\frac{l_2}{l_1} + \frac{\partial r^d(M_{t+j})}{\partial M_{t+j}} \right)$$

as required.

□

It can be seen that when the capital constraint is non-binding, the models with constant and non-constant risk-weights yield the same results.

Remark 3.2.4 (Cyclicality of Bank Loans under Basel II (Non-Binding)): *Under the same assumptions of Theorem 3.2.1, and when the capital constraint (1.12) is non-binding, bank loans increase as a result of an improvement in the business cycle* *activity. The loan rate can either increase or decrease depending on the parameters characterizing the loan default rate and the loan demand function.*

3.2.3 Cyclicality of Loans and Loan Rates under Basel II (Binding Constraint)

We now present the result of the effect of the changes in the business cycle on bank loans when the capital constraint is binding.

Proposition 3.2.5 (Cyclicality of Bank Loans under Basel II (Binding)): *Under the same assumptions of Theorem 3.2.1, and if $l_t > 0$, then by taking the first derivatives of the following equation with respect to M_t,*

$$\Lambda_t^* = \frac{K_t}{\omega(M_t)\rho} - \left(\frac{\omega^I I_t + 12.5(mVaR + 0)}{\omega(M_t)}\right)$$

we get

$$\frac{\partial \Lambda_t^*}{\partial (M_t)} = -\frac{K_t - \rho(12.5(mVaR + 0) + \omega^I I_t)}{[\omega(M_t)]^2 \rho} \frac{\partial \omega(M_t)}{\partial (M_t)}.$$

while the loan rate response to the changes in the business cycle is

$$\frac{\partial r_{t+j}^{\Lambda^*}}{\partial M_t} = \frac{l_2}{l_1} + \frac{K_t - \rho(12.5(mVaR + 0) + \omega^I I_t)}{[\omega(M_t)]^2 \rho l_1} \frac{\partial \omega(M_t)}{\partial (M_t)}.$$

Proof. In order to prove Proposition 3.2.5, we find the partial derivatives of the optimal loan supply and loan rate with respect to M_t of equations (2.10) and (2.11) and the condition $\frac{\partial \omega(M_{t+j})}{\partial M_{t+j}} < 0$ to find $\frac{\partial \Lambda^*{}_{t+j}}{\partial M_t}$ and $\frac{\partial r_{t+j}^{\Lambda^*}}{\partial M_t}$ respectively. Now we can calculate

$$\frac{\partial \Lambda^*{}_{t+j}}{\partial M_t}\left(\frac{K_t}{\rho\omega(M_t)} - \left[\frac{\omega^I I_t + 12.5(mVaR + 0)}{\omega(M_t)}\right]\right)$$
$$= -\frac{K_t - \rho(12.5(mVaR + 0) + \omega^I I_t)}{[\omega(M_t)]^2 \rho} \frac{\partial \omega(M_t)}{\partial (M_t)}$$

and

$$\frac{\partial r^{\Lambda *}_{t+j}}{\partial M_t}\left(\frac{1}{l_1}\left(l_0 + l_2 M_t + \sigma^{\Lambda}_t - \frac{K_t}{\rho\omega(M_t)} + \frac{\omega^I I_t + 12.5(mVaR + 0)}{\omega(M_t)}\right)\right)$$
$$= \frac{l_2}{l_1} + \frac{K_t - \rho(12.5(mVaR + 0) + \omega^I I_t)}{[\omega(M_t)]^2 \rho l_1}\frac{\partial\omega(M_t)}{\partial(M_t)}$$

as required. □

Remark 3.2.6 (Cyclicality of Bank Loans under Basel II (Binding)): *Under the same assumptions of Theorem 3.2.1, and when the capital constraint (1.12) is binding, current bank lending increases in response to an improvement in business cycle activity. The loan rate can either increase or decrease depending on the parameters characterizing the loan demand and the risk-weights.*

3.2.4 Cyclicality of Loans and Loan Rates Under Basel II (Future Periods)

In what follows we examine the effect of a current shock in future periods to the phases of the business cycle on bank loans and loan rates. Even here we look at two scenarios where the capital constraint binds or not.

3.2.4.1 Non-Binding Constraint

If the capital constraint does not bind, the response of loans and loan rates in period $j \geq 1$ to a current business cycle fluctuations is described by Theorem 3.2.1. However as time goes on, the impact of the shock dies out since $\mu^M_j < 1$.

3.2.4.2 Binding Constraint

In the future, if the capital constraint binds, then response of loans and loan rates to a change in M_t is described by

$$\frac{\partial\Lambda^*_{t+j}}{\partial(M_t)} = \frac{\mu^M_{j-1}}{\omega(M_{t+j})\rho}\left[\frac{\partial\Big(K_{t+j} - \rho(12.5(mVaR + 0) + \omega^I I_{t+j})\Big)}{\partial(M_{t-1+j})}\right] \tag{3.5}$$
$$-\frac{\mu^M_{j-1}}{\omega(M_{t+j})\rho}\left[\frac{\mu^M}{\omega(M_{t+j})}\Big(K_{t+j} - \rho(12.5(mVaR + 0) + \omega^I I_{t+j})\Big)\frac{\partial\omega(M_{t+j})}{\partial(M_{t+j})}\right]$$

and

$$\frac{\partial r^{\Lambda^*}_{t+j}}{\partial M_t} = \frac{l_2}{l_1}\mu^M_j - \frac{\mu^M_{j-1}}{\omega(M_{t+j})\rho l_1}\frac{\partial\Big(K_{t+j} - \rho(12.5(mVaR + 0) + \omega^I I_{t+j})\Big)}{\partial(M_{t-1+j})} \tag{3.6}$$
$$+\frac{\mu^M_j}{[\omega(M_{t+j})]^2\rho}\Big(K_{t+j} - \rho(12.5(mVaR + 0) + \omega^I I_{t+j})\Big)\frac{\partial\omega(M_{t+j})}{\partial(M_{t+j})}.$$

From the equation 3.5, it can be seen that future loans can either increase or decrease in response booms depending on the relative magnitudes of the terms in that equation. If capital decreases due to boom phases, loans can decrease provided that the effect of the shock on capital dominates the effect of the shock on loan risk-weights.

Chapter 4

NUMERICAL EXAMPLES

4.1 OUTPUT GAP AND THE BUSINESS CYCLE

4.2 CYCLICALITY OF BANK CREDIT

4.3 CYCLICALITY OF BANK CAPITAL

In this chapter, we consider the output gap as a proxy of the changes in the business cycle and the cyclicality of bank credit and capital. We present the output gap vs the business cycle in order to demonstrate that the output gap is a real proxy of the changes in the business cyclce. In addition, we present household debt, total private credit-to-GDP ratio, mortgage loans and interest (repo) rate vs output gap in order to demonstrate that bank credit is procyclical. Furthermore, we present issued primary share capital and capital adequacy ratios vs output gap in order to demonstrate that bank capital is acyclical. The data used in this chapter is sourced from the South African Reserve Bank (SARB).

The main problems that we address in this chapter are related to the cyclical effect of the dynamics of financial variables such as output gap, bank credit and capital.

Problem 4.0.7 (Relationship between Output Gap, Business Cycle and Bank Credit and Capital): *Can we establish a relationship between the South African output gap and composite business cycles ? (see 4.1).*

Problem 4.0.8 (Output Gap as a True Measure of Cyclicality): *Can we conjecture that output gap is a true measure of cyclicality by confirming that the business cycle and output gap in South Africa is strongly positively correlated ? (see 4.1).*

Problem 4.0.9 (Cyclicality of Bank Credit and Capital): *Can we draw conslusions about the cyclicality of bank credit and capital ? (see 4.2 and 4.3).*

4.1 OUTPUT GAP AND THE BUSINESS CYCLE

In Figure 4.1, we present the output gap vs the business cycle in order to demonstrate that the output gap is a real proxy of the changes in the business cycle.

For the period under investigation, viz., 1990 to 2006, South Africa has experienced three business cycles (see Table 1 and [113]). These may be identified as one major cycle for August 1992-November 1998 (peak in January 1995) and two minor ones for December 1998-August 2001 (turning point reached in February 2000) and September 2001-May 2003 (peak in April 2002). The same periods of cyclicality are discernible for the output gap. These trends are reflected in the graph below, where the output gap and business cycle are compared with each other. Figure 4.1 clearly demonstrates that a stronger positive correlation exists between the South African business cycles and output gap.

As a result of the above discussion, output gap may be considered to be a proxy for the business cycle. In replacing the business cycle with the output gap, we are able to better compare our study with that of others. Also, business cycles can only be determined subsequent to its realization while the output gap can be computed in real time and has a predictive quality.

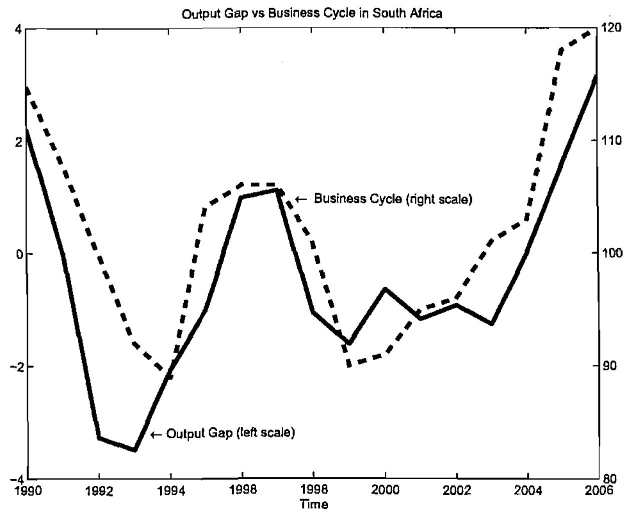

Figure 4.1: Output Gap vs Business Cycle

4.2 CYCLICALITY OF BANK CREDIT

In Figure 4.2, we present household debt vs output gap in order to demonstrate that bank credit is procyclical.

In Figure 4.3, we present total private credit-to-GDP ratio vs output gap in order to demonstrate that bank credit is procyclical.

From Figures 4.2 and 4.3, it is clear that household debt and total private sector credit-to-GDP ratio follows the output gap rather closely and as a result are both strongly procyclical. We note that since 1999, there has been a dramatic increase in the aforementioned ratio in South Africa.

In Figure 4.4, we present mortgage loans vs output gap in order to demonstrate that bank credit is procyclical.

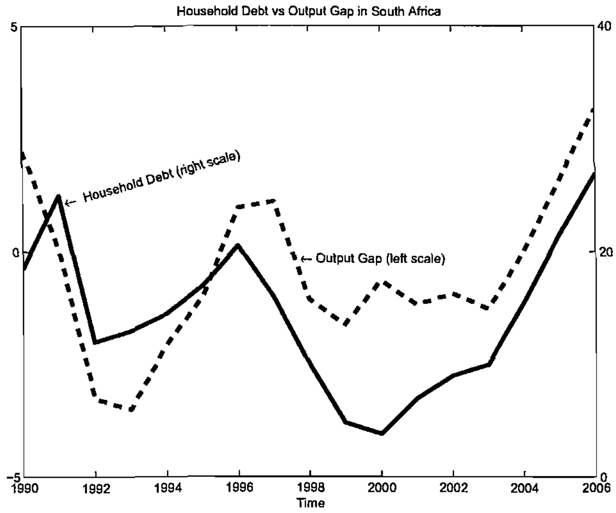

Figure 4.2: Household Debt vs Output Gap

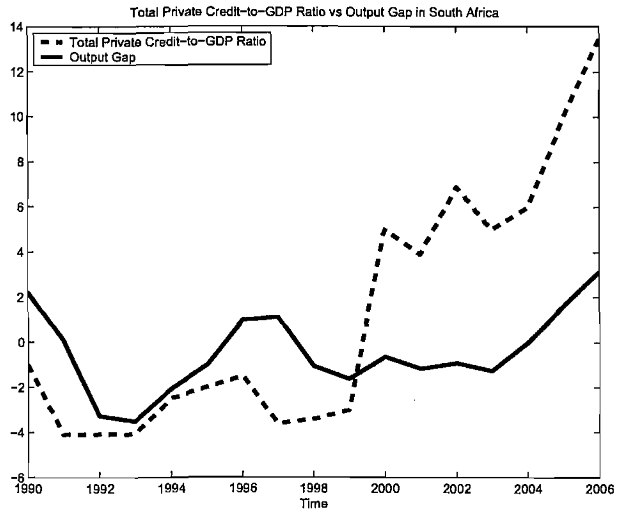

Figure 4.3: Total Private Credit-to-GDP Ratio vs Output Gap

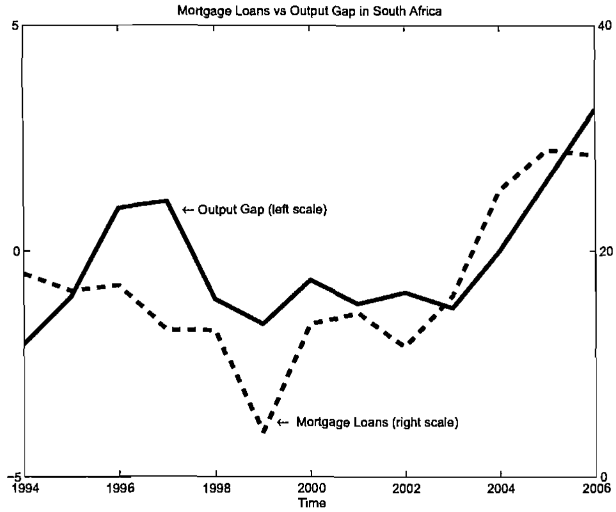

Figure 4.4: Mortgage Loans vs Output Gap

Figure 4.4 intimates that credit extension by financial institutions is strongly procyclical. In particular, annual growth in mortgage loans is procyclical. Mortgage loans are related to the level of the loan rate.

In Figure 4.5, we present interest (repo) rate vs output gap in order to demonstrate that bank credit is procyclical.

4.3 CYCLICALITY OF BANK CAPITAL

In Figure 4.6, we present issued primary share capital vs output gap in order to demonstrate that bank capital is acyclical.

In Figure 4.7, we present capital adequacy ratios vs output gap in order to demonstrate that bank capital is acyclical.

Figure 4.7 provides clear evidence that the Capital-to-Total Assets Ratio (CTAR) is acyclical for all three business cycles and beyond. On the other hand, the Capital-to-Risk Weighted Assets Ratio (CRWAR) appears to be weakly countercyclical for the August 1992-November 1998 business cycle and acyclical for the December 1998-August 2001 and September 2001-May 2003 cycles and beyond. Any discussion of the movements of South African CARs must take the increase from 8% to 10% in the minimum capital requirement from October 2001 onwards into account.

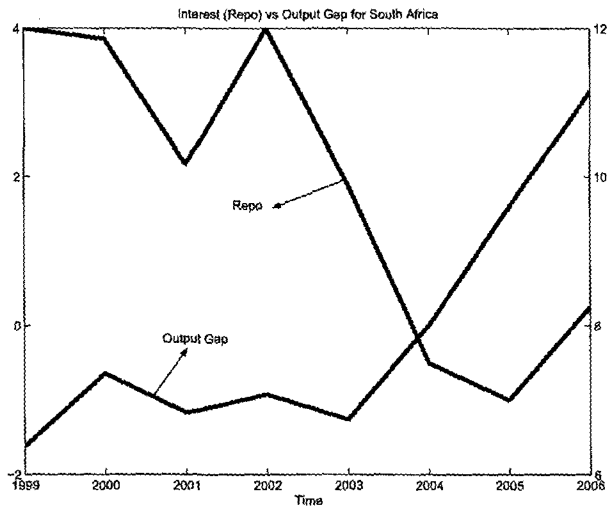

Figure 4.5: Interest (Repo) Rate vs Output Gap

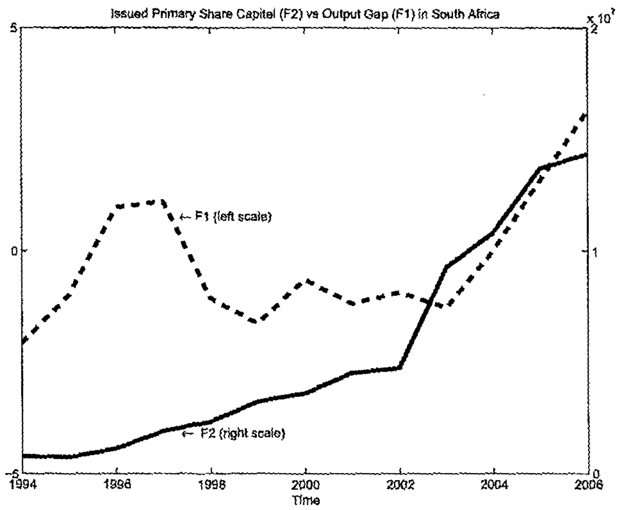

Figure 4.6: Issued Primary Share Capital vs Output Gap

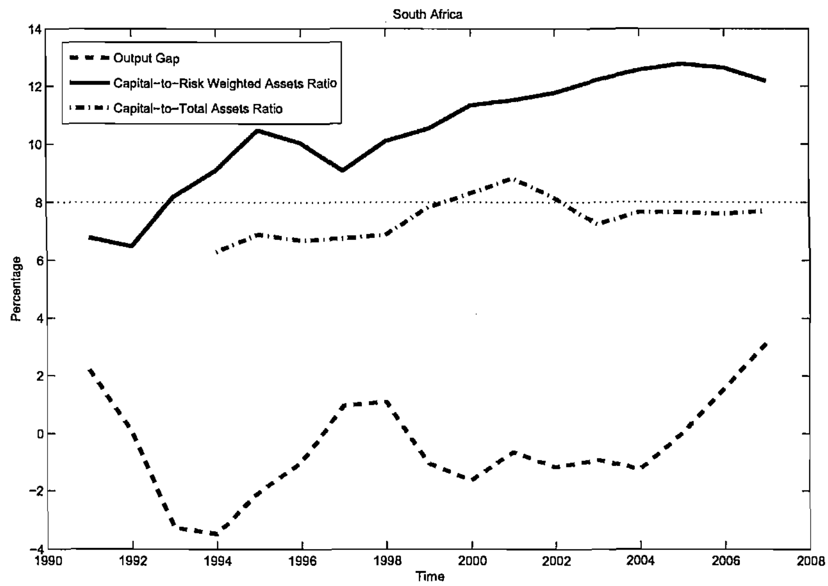

Figure 4.7: CARs vs Output Gap

Chapter 5

ANALYSIS OF CYCLICALITY OF BANK CREDIT AND CAPITAL

There are two versions of the model represented throughout our dissertation, the Basel I and Basel II. Both versions consist of two cases, binding constraint as well as non-binding constraint. In this regard, we analyze some of the main issues raised in Chapter 3. But first we have to discuss an optimal bank valuation problem presented in Chapter 2. Furthermore, we would like to extend the discussion on the connection between the cyclicality of output gap, bank credit and bank capital in South Africa represented in Chapter 4.

The issue of procyclical effects of the Basel II capital requirements is recently the subject of an extreme discussion in the financial and supervisory community. This dissertation presents important contributions to the discussion. We analyzed the effect of macroeconomic shocks on models for bank's lending under the two capital adequacy regimes (the Basel I and Basel II) in discrete-time. One of the result of the dissertation is that the response of banks to shocks that affect loan demand differs when the minimum capital requirements are calculated with risk-weights that are sensitive to the phases of the business cycle. In particular, bank capital is less volatile than under capital requirements with constant risk-weights. In this dissertation, we discussed procyclicality in the South African financial system. In particular, we have analyzed the effect that cyclicality in bank credit and bank capital has on financial stability. Our evidence is consistent with the view that the procyclicality of bank credit and bank capital might be a cause of financial distress in the South African system.

5.1 BANK VALUATION AND ITS OPTIMIZATION

In this section, we discuss some of the issues related to the optimal bank valuation problem presented in Chapter 2 when capital constraint is binding and non-binding.

5.1.1 Key Banking Concepts

In this subsection, we discuss all the banking concepts discussed in Chapter 2.

5.1.1.1 Retained Earnings

As far as the bank valuation is concerned, an interesting scenario from Subsection 2.1.1 to consider, is when $\Delta F_t = 0$ in (2.4). This provides another expression for profit of the form

$$N_t = \Pi_t = E_t^r + n_t d_t + (1 + r_t^O)O_t + (1 + r_t^{R^l})R_t^l.$$

If, in addition, $(1+r_t^O)O_t = O_{t+1}$ and $(1+r_t^{R^l})R_t^l = R_{t+1}^l$ then we may conclude that

$$\Pi_t = E_t^r + n_t d_t + O_{t+1} + R_{t+1}^l.$$

In turn, this results in the inequalities

$$\Pi_t > n_t d_t \Rightarrow n_{t+1} E_t < n_t E_t \text{ and } \Pi_t < n_t d_t \Rightarrow n_{t+1} E_t > n_t E_t.$$

Essentially, under the assumption that $\Delta F_t = 0$, the first statement implies that if the profit exceeds the dividends in period t then there may be a decline in the period $t+1$ shareholder equity when compared with period t equity. The opposite is true for the second statement.

5.1.1.2 Loan Supply and Demand

Subsection 2.1.2 of Section 2.1 provides us with a description of the main components of a bank's lending activities. Banks respond differently to shocks that affect loan demand, Λ, when the minimum capital requirements are calculated by using risk-weighted assets. In the Hicksian case, these responses are usually sensitive to the phases of the business cycle that are related to the term $l_2 M_t$ in (2.5). Loan defaults are independent of the capital adequacy paradigm that is chosen. In this regard, empirical evidence supports the opinion that better phases of the business cycle reduce the loan default rate and thus the loan marginal cost.

5.1.2 Statement of the Optimal Bank Valuation Problem

Problem 2.2.1 in Chapter 2 addresses a very important issue in bank operations that is related to the optimal implementation of financial economic principles under regulatory constraint. In this regard, our investigation is largely motivated by the need to maximize profits. As far as the optimal loan rate and general interest rate decisions are concerned, market, credit (see, for instance, [69] and [85]) and interest rate risk are the main risks to be taken care of.

5.1.3 Solution to the Optimal Bank Valuation Problem

If we substitute the optimal bank dividends given by (2.15) in Section 2.3 into the optimal decisions for the loan rate and deposits represented by (2.12) and (2.13), respectively, we can obtain a time-independent solution for the optimal bank valuation problem. This leads to a significant reduction in the technical difficulty of the procedure.

5.2 CYCLICALITY OF BANK CREDIT AND CAPITAL UNDER BASEL I AND BASEL II

In this section, we discuss cyclicality of bank credit and capital under the Basel I and the Basel II presented in Chapter 3.

5.2.1 Cyclicality of Bank Credit and Capital Under Basel I (Constant Risk-Weights)

In this subsection, we analyze bank loan responses to the changes in the business cycle under the Basel I (constant risk-weights) presented in Section 3.1 when capital constraint is binding and non-binding.

When the capital constraint is not binding, bank loans and loan rate increase as a result of boom phases. However if the capital constraint binds, an increase in the loan demand will result in an increase in the loan rate, which will leave the loan supply unchanged. While booms increases the chances of bank capital binding in the future, banks increase the amount of capital they hold.

5.2.2 Cyclicality of Bank Credit and Capital Under Basel II (Varying Risk-Weights)

In this subsection, we analyze bank loan responses to the changes in the business cycle under the Basel II (risk-weights varying) presented in Section 3.2 when capital constraint is binding and non-binding.

In the Basel II model, a shock to current phases of the business cycle does not only affect the loan demand but also the risk-weights in bank's capital to asset ratios. If the capital constraint is not binding, bank loans increase as in the Basel I model, whereas if the capital constraint binds, banks can still expand their credit supply, but by a smaller amount compared to when the capital constraint is non-binding. They are able to do so because boom phases causes lower risk-weights and for that reason a slackening of the capital constraint. The lower rate can either increase or decrease, depending on the relative size of the change in loan demand and capital ratio. Similarly, recessionary phases results in a possibly greater reduction of credit than in the Basel I model because of both a decrease on loan demand as well as a tightening of the capital constraint. Furthermore, under the Basel II, the sign of the change in bank's capital holding is undetermined because boom phases has two counteracting effects on the equilibrium values of bank capital. On the one hand, booms has a persistent positive effect on loan demand and so raises the probability of the capital constraint binding in the future. At the same time, though, the capital ratio increases so that there are chances that the capital constraint might become lower.

5.3 NUMERICAL EXAMPLES

In this section, we provide some comments about the numerical examples supporting our examples in Chapter 4.

5.3.1 Discussion of Output Gap and Business Cycle

By considering Section 4.1, we have that the correlation between the composite business cycles and output gap may be summarized as follows.

FIGURE	CORRELATION	Full and Partial Business Cycles			
		08/92-11/98	12/98-08/01	09/01-05/03	06/03-12/06
Figure 4.1: Business Cycles vs Output Gap	Business Cycles and Output Gap	Strongly Positive	Strongly Positive	Strongly Positive	Strongly Positive

Table 5.1: Summary of Relationships between the Business Cycle and Output Gap in South Africa

The importance of Figure 4.1 is that it shows that the South African output gap is a good proxy for the composite business cycles determined by [113]. This has many advantages such as making country-to-country comparisons possible since smoothing actual GDP via the Hodrick-Prescott filter is an easy procedure to perform under almost any national economic dispensation.

5.3.2 Discussion of Bank Credit

In our opinion, the risk perceptions, the financial accelerator, excessive optimism and pessimism, individual economic agents and new market and credit risk all have a role to play in explaining the procyclicality of credit in South Africa. In this regard, by considering Section 4.2, we have that the cyclicality of credit and its correlation with the output gap may be summarized as in Table 5.2.

FIGURE	VARIABLE	Full and Partial Business Cycles			
		08/92-11/98	12/98-08/01	09/01-05/03	06/03-12/06
Figure 4.2: Annual Percentage Change in Household Debt vs Output Gap	Private Credit	Strongly Procyclical Strong Positive Correlation	Strongly Procyclical Strong Positive Correlation	Strongly Procyclical Strong Positive Correlation	Strongly Procyclical Strong Positive Correlation
Figure 4.3: Total Private Credit-to-GDP Ratio vs Output Gap	Private Credit	Strongly Procyclical Strong Positive Correlation	Strongly Procyclical Strong Positive Correlation	Strongly Procyclical Strong Positive Correlation	Strongly Procyclical Strong Positive Correlation
Figure 4.4: Annual Percentage Change in Mortgage Loans vs Output Gap	Credit Extension by Financial Institutions	Strongly Procyclical Strong Positive Correlation	Strongly Procyclical Strong Positive Correlation	Strongly Procyclical Strong Positive Correlation	Strongly Procyclical Strong Positive Correlation

Table 5.2: Summary of the Cyclicality of Bank Credit in South Africa

It appears that measures of credit risk behave as if such risk declined during booms and rose only close to the peak as the recession set in. The relaxation of credit limitations can have both a direct or indirect effect on valuations. Indirectly, expenditures on goods and services tend to generate an upswing in economic activity.

5.3.3 Discussion of Bank Capital

From Section 4.3, we have that the cyclicality of capital requirements and its correlation with the output gap may be summarized as in Table 5.3.

FIGURE	VARIABLE	Full and Partial Business Cycles			
		08/92-11/98	12/98-08/01	09/01-05/03	06/03-12/06
Figure 4.6: Issued Primary Share Capital vs Output Gap	Bank Capital	Acyclical No Correlation	Acyclical No Correlation	Acyclical No Correlation	Pro-cyclical Positive Correlation
Figure 4.7: Capital-to-Total Assets Ratio and Capital-to-Risk Weighted Assets Ratio vs Output Gap	CTARs and CRWARs	CTAR: Acyclical No Correlation CRWAR: Weakly Counter cyclical Weak Positive Correlation	CTAR: Acyclical No Correlation CRWAR: Acyclical No Correlation	CTAR: Acyclical No Correlation CRWAR: Acyclical No Correlation	CTAR: Acyclical No Correlation CRWAR: Acyclical No Correlation

Table 5.3: Summary of the Cyclicality of Bank Capital in South Africa

The features of Figure 4.7 of Section 4.3 as summarized in Table 5.3, suggest that an uncertain relationship exists between bank capital and the output gap. We also note that banks have to comply to minimum regulatory capital requirements irrespective of the phase of the business cycle. In particular, we note that the minimum requirement increased from 8% to 10% from October 2001 onwards. Figure 4.7 suggests that, in South Africa, CRWARs were weakly countercyclical for the August 1992-November

1998 business cycle and acyclical thereafter. This may, also, be due to the fact that banks have to satisfy minimum requirements irrespective of the phase of the business cycle. Also, for the boom period August 2002-January 1995, banks improved their capital position as measured by the CRWARs with the overall objective of repairing their balance sheets. This behavior was partly fueled by pressure from rating agencies and the financial markets. In addition, CTARs appear to be acyclical for more or less the same reasons.

However, in principle, the Basel II dictates that a macroeconomic shock should influence the loan risk-weights in the CRWAR with the bank capital being affected in the following ways (see [23]). Generally, a negative (positive) shock results in the tightening (loosening) of the capital constraints. As a consequence, in terms of a possible binding capital constraint, banks are at liberty to increase (decrease) the loan supply when the phases of the business cycle improve (deteriorate). On the other hand, if the risk-weights are constant, a shock does not affect the loan supply but rather results in a change in the loan rate when the capital constraint binds. It is not always true that the Basel II risk-sensitive weights lead to an increase (decrease) in bank capital when macroeconomic activity in the loan market increases (decreases). A simple explanation for this is that the phases of the business cycle do not necessarily only affect loan demand but also influences the total capital constraint. Furthermore, banks do not necessarily need to raise new capital to expand their loan supply, since boom phases result in a decrease in the RWAs with a corresponding increase in CRWARs. Similarly, banks are not compelled to decrease their capital when the loan demand decreases since the capital constraint usually tightens in response to a recessionary phases. A further complication is that an improvement in economic conditions may result in an increase in the loan demand and, as a consequence, an increase in the probability that the capital constraint will be binding. Banks may react to this situation by increasing capital to maximize profits (compare the return on equity (ROE)).

Historical data confirms that, in general, there has been a sharp decline in CARs from 1990 to 1992 before a steady increase thereafter. However, the task of detecting cyclicality is made difficult by the introduction of the Basel I in 1988, which may have caused a structural change in CARs in some countries (for a survey of the impact of the Basel I see, for instance, [64]. Nevertheless, global historical data do not suggest a strong business cycle effect in CARs (see [23]). Furthermore, in South Africa, the cyclicality of the CRWAR has been much more pronounced than the cyclicality of the CTARs. This reflects the fact that subsequent to the banking crises in the late 1990's, RWAs fell more strongly as banks shifted their portfolios away from relatively high risk-weighted commercial lending towards residential mortgages and public sector securities (both of which have relatively low risk-weights).

Chapter 6

BANK VALUATION AND ITS CONNECTIONS WITH THE SMC AND BASEL II

6.1 BANK VALUATION AND THE SUBPRIME MORTGAGE CRISIS

6.1.1 Diagrammatic Overview of the Subprime Mortgage Crisis

6.1.2 Background to the Subprime Mortgage Crisis

6.1.3 Connections Between our Models and the Subprime Mortgage Crisis

6.2 BANK VALUATION AND BASEL II

6.2.1 Bank Regulatory Capital

6.2.2 Procyclicality of Basel II Regulation

In this chapter, we consider connections between the discrete-time stochastic models derived in the preceding discussions and the Subprime Mortgage Crisis (SMC) as well as the Basel II Capital Accord.

6.1 BANK VALUATION AND THE SUBPRIME MORTGAGE CRISIS

The SMC is an ongoing crisis characterized by *shrinking liquidity in global credit* markets and banking systems. A downturn in the U.S. housing market, risky practices by lenders and borrowers and excessive individual and corporate debt levels have affected the world economy adversely on a number of levels. The SMC has exposed pervasive weaknesses in the global financial system and regulatory framework. The connections between this crisis and our banking models are mainly forged via the bank's

- risk premium, ϱ, from (1.4) in Subsection 1.2.5 (see HM1 and HM2 in Figure 6.1 below);
- required capital sensitivity to changes in the amount of loans extended as given by (1.5) in Subsection 1.2.5 (see FM2 in Figure 6.1 below);
- base rate, r, from (1.6) in Subsection 1.2.5 decided upon by Central Banks as well as interbank loan rates (see HM1, HM3, FM4 and GIR1 in Figure 6.1 below);
- own loan rate, r^{Λ}, from (1.6) and (2.11) and (2.20) in Subsection 1.2.5 and Section 2.3, respectively (see HM1, HM3, FM4 and GIR1 in Figure 6.1 below);
- loan demand represented by (2.5) in Subsection 2.1.2 (see HM3, FM4, GIR1 and GIR5 in Figure 6.1 below);
- loan losses and default rate given by (1.2) in Subsection 1.2.5 (see HM4, FM1, FM3, GIR3 and GIR5 in Figure 6.1 below);
- loan loss provisions from (1.7) and (1.8) in Subsection 1.2.5 (see HM4, FM1, FM3, GIR3 and GIR5 in Figure 6.1 below);
- choice between raising deposits and interbank borrowing (including borrowing from the Central Bank) as reflected by (1.14) in Subsection 1.2.9 (see GIR1 in Figure 6.1 below);

- liquidity as described by Remark 1.2.2 (see FM4 in Figure 6.1 below);
- profit, Π, given by (1.13) in Subsection 1.2.7 (see HM2, HM4, FM1, FM3, GIR1 and GIR5 in Figure 6.1 below);
- bank valuation performance criterion, J, given by (2.7) in Section 2.2 (see HM4, HM6, FM1, FM2 and FM3 in Figure 6.1 below).

6.1.1 Diagrammatic Overview of the Subprime Mortgage Crisis

A diagrammatic overview of the SMC (see, for instance, [117]) may be represented as follows.

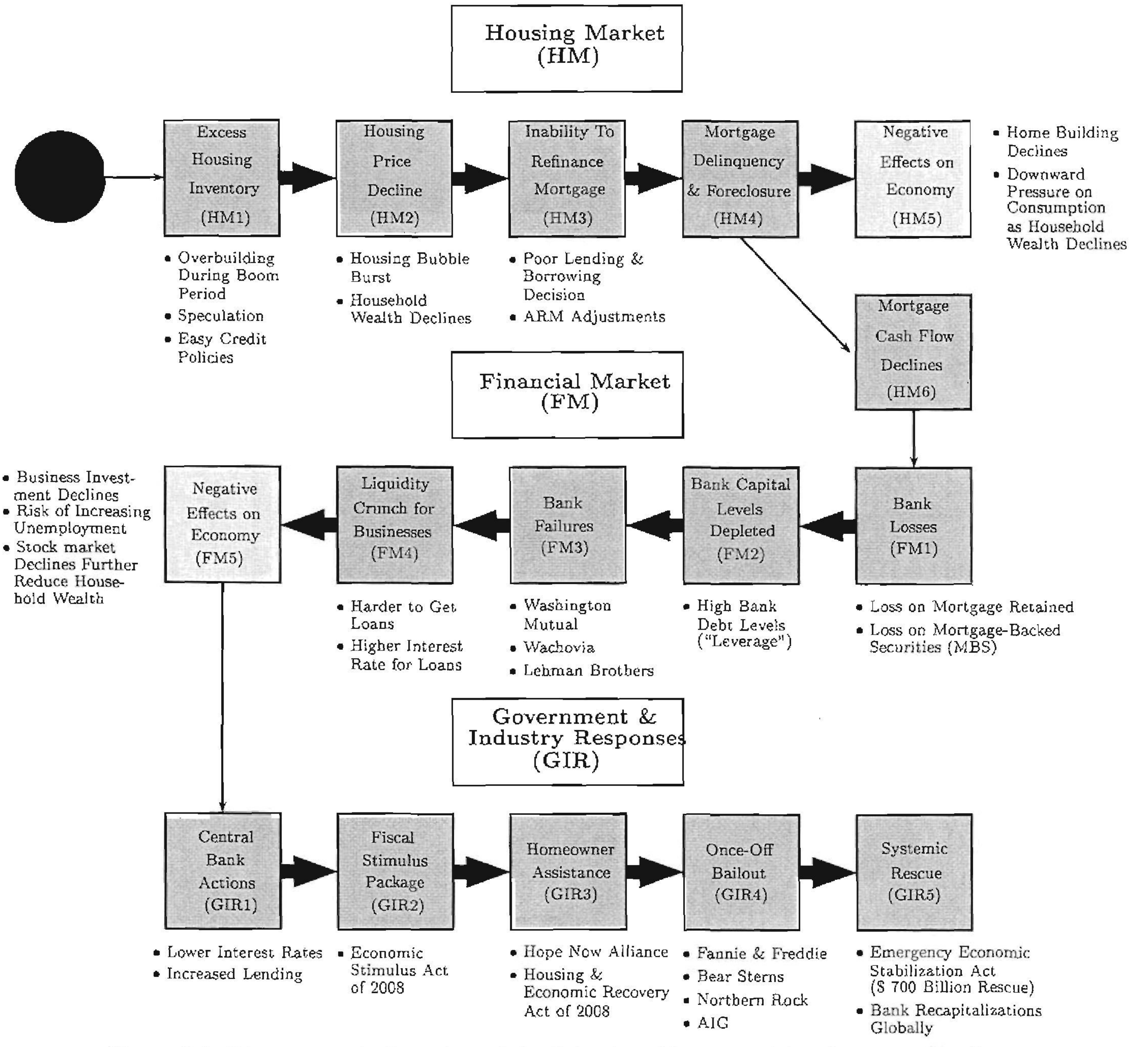

Figure 6.1: Diagrammatic Overview of the Subprime Mortgage Crisis (compare [117])

6.1.2 Background to the Subprime Mortgage Crisis

Most of the information contained in this subsection was sourced from [117]. The SMC was initiated by the deflation of the United States housing bubble (see, for instance, [20] and [78]) and high default rates on subprime and adjustable rate mortgages (ARM). Loan incentives, such as easy initial terms and low loan rates, in combination with escalating housing prices encouraged borrowers to assume difficult mortgages on the belief they would be able to quickly refinance at more favorable terms (see HM1 in Figure 6.1). Great concern was also expressed about the rapid growth in business loans at commercial banks with excessively easy credit standards. Some analysts claim that competition for lenders had greatly increased, causing banks to reduce loan rates and ease credit standards in order to issue new credit. Others are of the opinion that as the economic expansion continued and past loan losses were forgotten, banks exhibited a greater propensity for risk. However, once U.S. housing prices started to fall moderately in 2006-2007, refinancing became more difficult (see HM2 and HM3 in Figure 6.1). Defaults and foreclosure activity increased dramatically, as easy initial terms expired, home prices failed to go up as anticipated, and ARM interest rates reset higher. Foreclosures accelerated in the U.S. in late 2006 and triggered a global financial crisis through 2007 and 2008. During 2007, nearly 1.3 million U.S. housing properties were subjected to foreclosure activity; up 79 % from 2006 (see [104] for more details; also HM4 in Figure 6.1). The mortgage lenders that retained credit risk were the first to be affected, as borrowers became unable or unwilling to make payments (see HM5 and HM6 in Figure 6.1).

Major banks and other financial institutions globally had reported losses of approximately $ 435 billion from SMC-related activities by Thursday, 17 July 2008 (see [47] and [94]; also FM1 in Figure 6.1). By using securitization strategies, many mortgage lenders passed the rights to the mortgage payments and related credit risk to third-party investors via mortgage-backed securities (MBS) and collateralized debt obligations (CDO). Corporate, individual and institutional investors holding MBS or CDO suffered significant losses, as the underlying mortgage asset value decreased. Stock markets in many countries declined significantly (see FM2 in Figure 6.1). The broader international financial sector first began to experience the fallout from the SMC in February 2007 with the $ 10.5 billion writedown of HSBC, which was the first major CDO or Mortgage Bankers Association (MBA) related loss to be reported. During 2007, at least 100 mortgage companies had either failed, suspended operations or been sold. Top management did not escape without blemish, as the CEOs of Merrill Lynch and Citigroup were forced to resign within a week of each other. Subsequently, merger deals were struck by many institutions. In addition, Northern Rock and Bear

Stearns required emergency assistance from central banks. IndyMac was shut down by the FDIC on Sunday, 11 July 2008. Moreover, on Sunday, 14 September 2008, after performing banking duties for more than 150 years, Lehman Brothers filed for bankruptcy as a consequence of losses stemming from the SMC (see FM3 in Figure 6.1). Subsequent to this many U.S. and other banks throughout the world also failed. The widespread dispersion of credit risk and the unclear effect on financial institutions caused reduced lending activity and increased spreads on higher interest rates. Similarly, the ability of corporations to obtain funds through the issuance of commercial paper was affected. This aspect of the crisis is consistent with a credit crunch. There are a number of reasons why banks may suddenly make obtaining a loan more difficult or increase the costs of obtaining a loan. This may be due to an anticipated decline in the value of the collateral used by the banks when issuing loans; an increased perception of risk regarding the solvency of other banks within the banking system; a change in monetary conditions (for example, where the central bank suddenly and unexpectedly raises interest rates or capital requirements); the central government imposing direct credit controls and instructing the banks not to engage in further lending activity. The subprime crisis has adversely affected several inputs in the economy, resulting in downward pressure on economic growth. Fewer and more expensive loans tend to result in decreased business investment and consumer spending (see FM4 and FM5 in Figure 6.1).

Liquidity concerns drove central banks around the world to take action to provide funds to member banks to encourage lending to worthy borrowers and to restore faith in the commercial paper markets (see GIR1 in Figure 6.1). With interest rates on a large number of subprime and other ARM due to adjust upward during the 2008 period, U.S. legislators, the U.S. Treasury Department, and financial institutions took action. A systematic program to limit or defer interest rate adjustments was implemented to reduce the effect. In addition, lenders and borrowers facing defaults have been encouraged to cooperate to enable borrowers to stay in their homes. Banks have sought and received over $ 250 billion in additional funds from investors to offset losses (see [82] for more information). The risks to the broader economy created by the financial market crisis and housing market downturn were primary factors in several decisions by the U.S. Federal Reserve to cut interest rates and the Economic Stimulus Package (ESP) passed by Congress and signed by President Bush on Wednesday, 13 February 2008 (see, for instance, [8], [46] and [112]; also GIR2 in Figure 6.1). Bush also announced a plan voluntarily and temporarily to freeze the mortgages of a limited number of mortgage debtors holding ARMs. A refinancing facility called FHA-Secure was also created. This action is part of an ongoing collaborative effort between the U.S. government and private industry to help some subprime borrowers called the

Hope Now Alliance (see GIR3 in Figure 6.1). The U.S. government also bailed-out key financial institutions, assuming significant additional financial commitments (see GIR4 in Figure 6.1). Following a series of ad-hoc market interventions to bailout particular firms, a \$ 700 billion systemic rescue plan was accepted by the U.S. House of Representatives on Friday, 3 October 2008. These actions are designed to stimulate economic growth and inspire confidence in the financial markets (see GIR5 in Figure 6.1). By November 2008, banks in Europe, Asia, Australia and South America had followed the example of the U.S. government by putting rescue plans in place.

6.1.3 Connections Between Our Models and the Subprime Mortgage Crisis

The connections between our banking models and the SMC are complicated. However, a first step towards understanding this relationship entails identifying the problematic loan subportfolios. In this regard, it is possible to decompose the (total) loans, Λ, discussed above as

$$\Lambda = \sum_{i=1}^{m} \Lambda_i,$$

where m is the *number of loan subportfolios* with i being the *index of each loan subportfolio* so that $i = 1, 2, \ldots, m$. We note that the Basel II IRB approach to credit risk (see [14]) dictates that $m \leq 15$. In particular, this approach identifies 15 credit risk exposure types (loan subportfolios) that may be listed as

$i = 1:$ Project Finance (PF);
$i = 2:$ Object Finance (OF);
$i = 3:$ Commodities Finance (CF);
$i = 4:$ Income Producing Real Estate (IPRE);
$i = 5:$ Specialized Lending High Volatility Commercial Real Estate (SLHVCRE)
$i = 6:$ Specialized Lending Not Including High Volatility Commercial Real Estate (SLNIHVCRE)
$i = 7:$ Bank Exposure (BE);
$i = 8:$ Sovereign Exposure (SE);
$i = 9:$ Retail Residential Mortgage (RRM);
$i = 10:$ Home Equity Line of Credit (HELOC);
$i = 11:$ Other Retail Exposure (ORE);
$i = 12:$ Qualifying Revolving Retail Exposure (QRRE);
$i = 13:$ Small to Medium Size Enterprises with Corporate Treatment (SMECT)
$i = 14:$ *Small to Medium Size Enterprises with Retail Treatment* (SMERT)
$i = 15:$ Equity Exposure Not Held in the Trading Book (EENHTB)

with i = 1-6 and i = 9-12 constituting corporate and retail exposures, respectively. As a result of the SMC, market trade in almost all of these loan subportfolios became sluggish. In particular, a problematic loan subportfolio corresponds to $i = 4$ which encompasses income producing real estate. Furthermore, in commercial real estate (i.e., $i = 5$ and $i = 6$) there is a slowing due to the tightening credit and slowing growth, the former a direct result of the SMC. Although capital remains available for residential loans, the credit crunch is pronounced in commercial lending. Moreover, $i = 7$ which corresponds to interbank lending had been identified as a problematic subportfolio. This is because banks were not lending to each other during the early stages of the crisis (see FM4 in Figure 6.1).

The risk premium, ϱ, from (1.4) has had a part to play in the SMC (see HM1 in Figure 6.1). This premium is part of the expression for the bank's own loan rate, r^{Λ}, and its size is an indication of perceived credit risk. A study by the U.S. Federal Reserve indicated that the average difference in mortgage interest rates between subprime ("subprime markup") and prime mortgages declined from $\varrho = 2.8$ percentage points (280 basis points) in 2001, to $\varrho = 1.3$ percentage points in 2007. In other words, the risk premium required by lenders to offer a subprime loan declined dramatically during the aforementioned period. This occurred even though subprime borrower and loan quality declined overall during the 2001-2006 period. In fact, this state of affairs should have had the opposite effect. However, it is clear that such conditions are consistent with classic boom and recession credit cycles (see [36]).

The sensitivity of the required capital to changes in the amount of loans extended from (1.5) is an important issue in the SMC. Bank capital levels were depleted with banks experiencing high debt levels (see FM2 in Figure 6.1). As a consequence, for many banks (1.5) begun to take on negative values during the SMC.

The Central Bank base rate, r, from (1.6) in Subsection 1.2 has also had a role to play in trying to alleviate global economic pressures during the SMC. For instance, on Wednesday, 8 October 2008, central banks in the U.S. (U.S. Federal Reserve), England, China, Canada, Sweden, Switzerland and the European Central Bank cut rates in a coordinated effort to assist the world economy. This rate will be lowered when it is necessary to stimulate growth and provide liquidity.

Despite the discussion above, the most significant relationships between our models and the SMC are established via the bank's own loan rate, r^{Λ}, given by (1.6) (see also (2.11) and (2.20) as well as HM1, HM3, FM4 and GIR1 in Figure 6.1). As we have noted in (1.6), this loan rate may be expressed as

$$r_t^{\Lambda} = \varrho + (1 + r_t)\frac{N}{\eta} + c^{rw} + c^E \frac{\partial K}{\partial \Lambda} + \mathbf{E}[l].$$

In general terms, r^{Λ} can be written as a function of credit risk, market structure (power), marginal cost of debt, marginal cost of equity and the sensitivity of capital to loans extended. The representation of the bank's interest setting intimates that banks will experience positive returns in good times when the *actual rate of default*, r^d, is lower than the provisioning for expected losses, $\alpha\varrho + \mathbf{E}[l]$, and may not be able to cover their expected losses when $r^d > \alpha\varrho + \mathbf{E}[l]$ (see equation (1.2) for loan losses and default rate). In the latter case, bank capital may be needed to cover these excess (and unexpected) losses. If this capital is not enough then the bank will face insolvency. During the latter half of 2008, we have seen a decline in such capital.

A combination of factors resulting from the SMC have led to problems in the commercial real estate market as regards loan demand (see (2.5); also HM3, FM4, GIR1 and GIR5 in Figure 6.1). According to the National Association of Realtors (NAR) there is a slowing in commercial real estate due to the tightening credit and slowing growth, the former a direct result of the SMC. Although capital remains available for residential loans, the credit crunch is pronounced in commercial lending. Patricia Nooney, chairperson of the Realtors Commercial Alliance Committee (RCAC), portrayed the decline as unusual since

> "transactions are being curtailed not for lack of demand, but for serious challenges in obtaining financing." (see [75] for more information).

Next, we discuss loan losses and default rate given by (1.2) in Subsection 1.2.5 (see HM4, FM1, FM3, GIR3 and GIR5 in Figure 6.1). An acceleration in loan growth, as was experienced prior to the SMC, eventually leads to a surge in loan losses (see, for instance, (1.2) for loan losses and default rate) resulting in reduced bank profits (see equation (1.13) for bank profit) and precipitating a new round of bank failures. As experience during the mortgage crisis has shown, such a slump in banking could not only threaten the deposit insurance fund but also slow the economy by entrenching credit crunches. The view that faster loan growth leads to higher loan losses should not be taken lightly; nor should it be accepted without question. If loan growth increases because banks become more willing to lend, credit standards should fall and loan losses should eventually rise. For many distressed banks suffering because of the SMC, increased loan loss provisions from (1.7) and (1.8) translate to a decrease in earnings.

Usually, when capital is needed, banks have to choose between raising and holding deposits and borrowing from the Central Bank as reflected by (1.14) in Subsection 1.2.9.

Contracted liquidity in the global credit markets and banking system (as described by Remark 1.2.2; see also FM4 in Figure 6.1) is an ongoing economic problem. These liquidity concerns drove central banks around the world to take action to provide funds to member banks to encourage lending to worthy borrowers and to restore faith in the commercial paper markets (see GIR1 in Figure 6.1).

We recall that the bank's profits are given by (1.13) in the form

$$\begin{aligned}\Pi_t &= \left(r_t^{\Lambda} - c_t^{\Lambda} - r^d(\mathtt{M}_t)\right)\Lambda_t + r_t^{\mathtt{T}} W_t + r_t^I I_t + r_t^C C_t + r_t^B B_t + r_t^S S_t \\ &\quad - \left(r_t^{\mathtt{B}} + c_t^{\mathtt{B}}\right)\mathtt{B}_t - \left(r_t^D + c_t^D\right) D_t - c^w(W_t) - P(\mathtt{M}_t) - r_t^{\mathtt{T}}\gamma D_t,\end{aligned}$$

It is important to note that this expression represents the bank's profits from unsecuritized loans. However, in the SMC, the attention has been on borrowing under securitization strategies (see Figure 8.2 in Subsection 8.5.1). In principle, we can provide an analogue of (1.13) in a securitization context by introducing additional variables. In this regard, suppose that r^r is the average residual rate in securitized transactions, p^{rc} the average expected fraction of loan losses on securitized debt, c^{li} the marginal cost of mortgage loan insurance under securitization, c^{mw} the marginal cost to monitor a loan that was securitized, c^{tw} the marginal transaction costs to securitize a loan amortized over a loan term, p^w the fraction of the loan amount securitized and $\widetilde{p}^w$ the fraction of the mortgage loan principal, Λ_0, realized as a new loan in securitization. Then the bank's profits under securitization, Π^w_t, may be presented in the form

$$\begin{aligned}\Pi^w_t &= (r^r_t - p^{rc}_t - c^{li}_t - c^{mw}_t - c^{tw}_t)p^w_t\widetilde{p}^w_t\Lambda_t + (r^\Lambda_t - c^{lc}_t - c^{tw}_t)p^w_t(1-\widetilde{p}^w_t)\Lambda_t \\ &+(1-p^w_t)\Bigg[\left(r^\Lambda_t - c^\Lambda_t - r^d(\mathtt{M}_t)\right)\Lambda_t + r^{\mathtt{T}}_t W_t + r^I_t I_t + r^C_t C_t + r^B_t B_t + r^S_t S_t \quad (6.1)\\ &-\left(r^{\mathtt{B}}_t + c^{\mathtt{B}}_t\right)\mathtt{B}_t - \left(r^D_t + c^D_t\right)D_t - c^w(W_t) - P(\mathtt{M}_t) - r^{\mathtt{T}}_t\gamma D_t\Bigg] - V^a_t - \mathtt{E}_t - \mathtt{F}_t.\end{aligned}$$

Here we note that, in terms of the organization of the special purposes vehicle (SPV), there are different states that can be associated with the different possible corporate forms of the SPV. The SPV can be an LLC ($\mathtt{E}^1$), LLP ($\mathtt{E}^2$), C-corporation ($\mathtt{E}^3$) or a trust ($\mathtt{E}^4$). Each of these SPVs has its own unique tax benefits and problems as well as degree of asset protection and limited liability under legal prescripts. Furthermore, we denote the optimal state during the lifetime of the SPV by $\mathtt{E}^*$, with deviations from $\mathtt{E}^*$ being given by $|\mathtt{E}^* - \mathtt{E}^i_t|$, where $i = 1,\ 2,\ 3, \ldots,\ 4$. These deviations measure the loss and opportunity costs arising from the use of suboptimal corporate structures as SPVs. Usually, such loss manifests as elevated legal fees, increased losses due to low limited-liability and/or the value of additional time spent in dispute resolution. In this case, the formula for $\mathtt{E}$ in (6.1) may be given by

$$\mathtt{E}_t = \max\{|\mathtt{E}^* - \mathtt{E}^1_t|,\ |\mathtt{E}^* - \mathtt{E}^2_t|,\ |\mathtt{E}^* - \mathtt{E}^3_t|,\ |\mathtt{E}^* - \mathtt{E}^4_t|,\ 0\}.$$

Furthermore, in the securitization context of (6.1), we assume that financing can take the form of $\mathtt{F}^f$, where the transaction is an assignment and $\mathtt{F}^s$, where the transaction is a true sale. In the former case, the SPV is not subsidized by the sponsor. Alternatively, the impaired collateral is only substituted by the sponsor. In this situation, where $\mathtt{F}^f$ is the optimal state but $\mathtt{F}^s$ is being used instead, the transaction incurs losses and vice versa. On the other hand, where the correct transaction format is used, there is neither a gain nor a loss. Hence the value of the transaction format choice (compare with (6.1) where F is used) is

$$\mathtt{F}_t = \max\{|\mathtt{F}_t^f - \mathtt{F}_t^s|,\ 0\}.$$

Furthermore, we denote the value of the adverse selection problem by V^a. In this case, the transaction is an assignment and the sponsor is the servicer. Here, the sponsor will typically substitute impaired collateral, and hence is incentivized not to prefer the best collateral as a replacement. In this regard, we denote impaired collateral, value of replacement collateral and the value of average-quality collateral available for replacement by C^0, $C^i \dots C^n$ and C^a, respectively. We assume that collateral consists of units of the same or similar sizes where $\mathbf{P}^i$ is the probability of there being impaired collateral and $\mathbf{P}^a$ the probability that the servicer/sponsor will offer medium- or low-quality collateral as replacement for the impaired collateral. In this situation, it follows that V^a in (6.1) may be represented by

$$V^a = \int_0^t \left\{ \sum C^a * \mathbf{P}^i(1 - \mathbf{P}^a) \right\}.$$

Another remark about profitability is that during the SMC, profits at the 8 533 U.S. banks insured by the FDIC fell from \$ 35.2 billion to \$ 646 million (effectively by 89 %) during Quarter 4 of 2007 when compared with the previous year. This was largely due to escalating loan losses and provisions for loan losses. This decline in profits contributed to the worst bank and thrift quarterly performance since 1990. In 2007, these banks earned approximately \$ 100 billion, which represented a decline of 31 % from the record profit of \$ 145 billion in 2006. Profits decreased from \$ 35.6 billion to \$ 19.3 billion during the first quarter of 2008 versus the previous year, a decline of 46 % (see [44] and [45] for more detail). The contribution [19] considers the following strategy to be optimal for banks to shield their profits from loan losses. The loan loss reserves, R^l, is built up in every period that $P > L$. On the other hand, when $P \leq L$ the bank is allowed to draw on R^l from the current period. When R^l or other capital becomes inadequate, at some point, the bank will face insolvency.

Bank valuations as realized by (2.7) in Section 2.2 have decline dramatically during the SMC as a direct result of sharp decreases in profitability and bank capital.

6.2 BANK VALUATION AND BASEL II

In this section, we analyze the connections between our banking model and the Basel II capital accord (see Chapter 1 for a diagrammatic overview of Basel II; also [11] and [14]). These relationships are forged via the

- description of total bank capital given by (1.9) in Subsection 1.2.6;
- description of the bank capital constraint given by (1.11) and (1.12) in Subsection 1.2.6.

6.2.1 Bank Regulatory Capital

Bank capital is notoriously difficult to define, monitor and measure. For instance, the measurement of equity depends on how all of a bank's financial instruments and other assets are valued. The description of the shareholder equity component of bank capital, E, given by (1.9), is largely motivated by the following two observations. Firstly, it is meant to reflect the nature of the book value of equity. Our intention is also to recognize that the book and market value of equity is highly correlated.

Under Basel II, bank capital requirements have replaced reserve requirements as the main constraint on the behavior of banks (see, for instance, [22]). A first motivation for this is that bank capital has a major role to play in overcoming the moral hazard problem arising from asymmetric information between banks, creditors and debtors. Also, bank regulators require capital to be held to protect themselves against the costs of financial distress, agency problems and the reduction of market discipline caused by the safety net. Subsection 1.2.6 suggests that a close relationship exists between bank capital holding and macroeconomic activity in the loan market (see equation (1.12) describing the bank capital constraint).

6.2.2 Procyclicality of Basel II Regulation

The Standardized approach to credit risk differentiates assets not only according to the borrower but according to riskiness proxied by credit rating agencies' assessment of the borrower. This approach represents an improvement in the sense that corporations are rated. However, it is deficient with respect to the viewpoint that ratings properly reflect risk and the inducing of procyclical capital charges which will lead to overlending during booms and underlending during recessions. Also, the capital formula for the IRB approach to credit risk is problematic. In short, the IRB capital formula is deduced by considering the large-portfolio asymptotic dynamics of a Merton model with a single common risk-factor. Many empirical studies have confirmed that this formula gives rise to procyclicality.

In addition, Basel II dictates that a macroeconomic shock will affect the loan risk-weights in the CAR. In general, a negative (positive) shock results in the tightening (loosening) of the capital constraint (see equation (1.12) describing the bank capital constraint). As a consequence, in terms of a possible binding capital constraint, banks are free to increase (decrease) the loan supply when macroeconomic conditions M_t improve (deteriorate). On the other hand, if the risk-weights are constant, a shock does not affect the loan supply but rather results in a change in the loan rate when the capital constraint binds. It is not always true that Basel II risk-sensitive weights lead to an increase (decrease) in bank capital when macroeconomic activity in the loan market increases (decreases). A simple explanation for this is that macroeconomic conditions do not necessarily only affect loan demand but also influences the total capital constraint (see equation (1.12) describing the bank capital constraint). Furthermore, banks do not necessarily need to raise new capital to expand their loan supply, since a positive macroeconomic shock may result in a decrease in the RWAs with a commensurate increase in CARs (compare the minimum capital constraint as expressed in (1.11) and (1.12)). Similarly, banks are not compelled to decrease their capital when the loan demand decreases since the capital constraint usually tightens in response to a negative macroeconomic shock. A further complication is that an improvement in the latter conditions may result in an increase in the loan demand and, as a consequence, an increase in the probability that the capital constraint may be binding (see equations (2.5) and (1.12) for the loan demand and bank capital constraint, respectively). Banks may react to this situation by increasing capital to maximize profits (compare the definition of the return on equity (ROE) measure of profitability). Our main conclusion is that bank capital is procyclical because it is dependent on fluctuations in loan demand which, in turn, is reliant on macroeconomic activity (see equation (1.12) describing the bank capital constraint).

Chapter 7

CONCLUDING REMARKS AND FUTURE DIRECTIONS

In this section, we make a few concluding remarks and comment about possible future research projects.

7.1 CONCLUDING REMARKS

The issue of procyclical effects of the Basel II capital requirements is currently the subject of an extreme discussion in the financial and supervisory community. This dissertation presents important contributions to the discussion. We analyzed the effect of macroeconomic shocks on models for bank's lending under the two capital adequacy regimes (the Basel I and Basel II) in discrete-time. One of the results of the dissertation is that the response of banks to shocks that affect loan demand differs when the minimum capital requirements are calculated with risk-weights that are sensitive to the phases of the business cycle. In particular, bank capital is less volatile than under capital requirements with constant risk-weights. In this dissertation, we discussed procyclicality in the South African financial system. In particular, we have analyzed the effect that cyclicality in bank credit and bank capital has on financial stability. Our evidence is consistent with the view that the procyclicality of bank credit and bank capital might be a cause of financial distress in the South African system.

In this paper, we have forged connections between discrete-time stochastic banking models and macroeconomic activity as well as the SMC and Basel II Capital Accord. Furthermore, we solved an optimal bank valuation problem that, amongst other things, maximized profit under several realistic banking constraints. A comprehensive illustration of some of the concepts discussed in the main body of the paper was provided. The discussion of procyclicality confirms that loan loss provisions increase as world economies move further into recession. Also, we demonstrate that banks increase loan loss provisions as loan problems become more apparent. In short, this contribution represents a starting point for studying modeling issues related to the SMC and, to a lesser extent, Basel II. We envisage that our study on the modeling of the SMC will be extended in several directions in the near future.

7.2 FUTURE DIRECTIONS

Future modeling research should consider the bank's lending responses to business cycles as it pertains to the nexus between the proximity to its capital constraints and its forthrightness in recognizing loan losses. This relationship may not be constant over time. Further research should also establish a solid basis for comparison of extant valuation practice and demonstrate the superiority of our model to another model. In particular, we need to learn more about the inadequacies of current practice as a basis for substantiating the need for our modeling paradigm. Moreover, future reserach should provide empirical support to demonstrate the superiority of our model over other models or the accuracy of its performance relative to any market benchmark. Hence, even if one could utilize the model in some valuation project to estimate the value of the bank's common shareholder's equity, we have no idea of how close we might be to the truth with respect to market values. Since fair market values are of great interest to valuators, a test of reasonable congruence with such values would seem to be prerequisite to the use of the valuation model for purposes of business valuation. Although large sample empirical data would be preferred, smaller sample analysis using recent actual acquisitions might also be helpful. All these facts have to be incorporated in future modeling programmes.

Another research topic will involve complex models of bank items driven by Lévy processes (see, for instance, Protter in [103, Chapter I, Section 4]). Such processes have an advantage over the more traditional modelling tools such as Brownian motion because the behavior of bank loans, wealth, capital and CARs are characterized by jumps. As a result of this, recent research has strived to replace the existing Brownian motion-based bank models (see, for instance, [35], [50], [80], [90] and [107]) by systems driven by more general processes. Also, a study of the optimal capital structure should ideally involve the consideration of taxes and costs of financial distress, transformation costs, asymmetric bank information and the regulatory safety net. Another research area that is of ongoing interest is the (credit, market, operational, liquidity, securitization) risk minimization of bank operations within a regulatory framework (see, for instance, [85] and [89]). Another risk that becomes important is *interest rate risk* at the point of loan issuing. For instance, an alternative optimization problem would be to maximize the risk-free rate of interest in order to provide a shareholder with an incentive to invest money. An example of a over-simplified model that has become more important during the current SMC and global credit crunch is the Diamond-Dybvig model (see, for instance, [37] and [38]). Their contributions proposes a much simpler model than ours but yet explains key aspects of what bank liquidity means in reality *ceteris parabis.* In future, it will be worthwhile to explore connections with the work of Diamond and Dybvig on liquidity.

Chapter 8

APPENDIX

This appendix contains tables outlining the main differences between the Basel I and Basel II and statement of the implicit function theorem. We also discuss the formal derivation of the first order conditions (2.12) to (2.15). Furthermore, we provide diagrammatic overviews of the subprime mortgage crisis and Basel II credit risk (see Figure 8.1 below) as well as borrowing under securitization strategies (see Figure 8.2 below) and the financial leverage profit engine (see Figure 8.3 below).

8.1 THE MAIN DIFFERENCES BETWEEN BASEL I AND BASEL II

Focus	**Basel I**	**Basel II**
Risk Measure	Single Risk Measure	Counterparty & Transaction Specific Risk Measures
Risk Sensitivity	Broad Brush Approach	Granularity and Risk Sensitivity
Credit Risk Mitigation	Limited Recognition	Comprehensive Recognition
Operational Risk	Excluded	Included
Flexibility	One Size Fits All	Menu of Approaches
Supervisory Review	Implicit	Explicit
Market Discipline	Not Addressed	Supervisory Role Conferred on Market
Incentives	Not Addressed	Explicit and Well Defined
Economic Capital	Divergence	Convergence

Table 8.1: Main Differences between Basel I and Basel II

Basel I	Basel II
Not Risk Sensitive; No Reliance on Risk Ratings or Maturity.	Risk Sensitive
Based on Simplistic or Crude Categories of Obligors and Assets with Fixed Risk-Weighted Asset Percentages; No Minimal Recognition of Risk Mitigation	Based on Risk Ratings, Maturity Substantial of Recognition Risk Mitigation
Calculated in Aggregate	Calculated at Facility/Transaction Level
No Acknowledgment of Operational Risk as a Separate Risk Discipline/Category	Operational Risk as a Separate Discipline/Category
No Relationship with Risk Capital	Alignment with Risk Capital, Advanced Risk Management Concepts
Largely Finance-Driven Off of Finance Systems	Operationally Intense - Places Major Emphasis on Rigorous Risk Management Processes, Models Systems/Technology and Integration of Risk/Finance Systems and Processes

Table 8.2: Main Differences between Basel I and Basel II (continued)

8.2 STATEMENT OF THE IMPLICIT FUNCTION THEOREM

Suppose that (a, b) is a point on the curve $H(M_t, K_{t+1}) = 0$. Suppose that we can solve the equation for K_{t+1} as a function of M_t for all (M_t, K_{t+1}) sufficiently near (a, b). This part of the curve is the graph of a function $K_{t+1} = \eta(M_t)$ on some interval $|M_t - a| < h$ with $\eta(a) = b$. If $\eta'(M_t)$ exists, we can compute it by differentiating both sides of the equation $H(M_t, \eta(M_t)) = 0$ with respect to M_t to get

$$\frac{\partial H}{\partial M_t}(M_t, \eta(M_t)) + \frac{\partial H}{\partial K_{t+1}}(M_t, \eta(M_t))\eta'(M_t) = 0.$$

providing that the partial derivatives exist. If $\frac{\partial H}{\partial K_{t+1}}(M_t, \eta(M_t)) \neq 0$, we can solve for $\eta'(M_t)$ and obtain the well known formula

$$\eta'(M_t) = -\frac{\frac{\partial H}{\partial M_t}(M_t, \eta(M_t))}{\frac{\partial H}{\partial K_{t+1}}(M_t, \eta(M_t))},$$

or, more precisely,

$$\frac{\partial K_{t+1}}{\partial M_t} = -\frac{\frac{\partial H}{\partial M_t}}{\frac{\partial H}{\partial K_{t+1}}}.$$

8.3 DERIVATION OF FIRST ORDER CONDITIONS (2.12) to (2.15)

To derive equations (2.12) to (2.15), we rewrite equation (2.9) to become

$$\begin{aligned} V(K_t, x_t) &= \max_{r_t^\Lambda,\ D_t,\ \Pi_t} \Big\{ \Pi_t + l_t \Big[K_t - \rho \Big(\omega(M_t)\Lambda_t + \omega^I I_t + 12.5(mVaR + 0) \Big) \Big] \\ &\quad -c_t^{dw} \Big[K_{t+1} \Big] + \mathbf{E}_t \Big[\delta_{t,1} V \Big(K_{t+1},\ x_{t+1} \Big) \Big] \Big\}, \end{aligned} \tag{8.1}$$

but

$$\Pi_t = n_t(d_t + E_t) + (1 + r_t^O)O_t - K_{t+1} + \Delta F_t. \tag{8.2}$$

By substituting equations (2.5) and (8.2), equation (8.1) becomes

$$\begin{aligned} V(K_t, x_t) &= \max_{r_t^\Lambda,\ D_t,\ \Pi_t} \Big\{ \Big(n_t(d_t + E_t) + (1 + r_t^O)O_t - K_{t+1} + \Delta F_t \Big) \\ &\quad + l_t \Big[K_t - \rho \Big[\omega(M_t) \Big(l_o - l_1 r_t^\Lambda + l_2 M_t + \sigma_t^\Lambda \Big) + \omega^I I_t + 12.5(mVaR + 0) \Big] \Big] \\ &\quad -c_t^{dw} \Big[K_{t+1} \Big] + \mathbf{E}_t \Big[\delta_{t,1} V \Big(K_{t+1},\ x_{t+1} \Big) \Big] \Big\}. \end{aligned} \tag{8.3}$$

Finding the partial derivative of the bank value in (8.3), with respect to the capital constraint, K_{t+1}, we have

$$\frac{\partial V}{\partial K_{t+1}} = -1 - c_t^{dw} + E_t\left[\int_{\underline{\Lambda}}^{\overline{\Lambda}} \delta_{t,1} \frac{\partial V}{\partial K_{t+1}} dF\left(\sigma_{t+1}^{\Lambda}\right)\right]. \tag{8.4}$$

8.3.1 First Order Condition (2.12)

Choosing the loan rate, r_t^{Λ}, from equation (8.3) and using equation (8.4) above, the first order condition (2.12) for Problem 2.2.1 is

$$\frac{\partial \Pi_t}{\partial r_t^{\Lambda}}\left[1 + c_t^{dw} - \mathbf{E}_t\left\{\int_{\underline{\Lambda}}^{\overline{\Lambda}} \delta_{t,1} \frac{\partial V}{\partial K_{t+1}} dF(\sigma_{t+1}^{\Lambda})\right\}\right] + l_t \rho l_1 \omega(M_t) = 0$$

8.3.2 First Order Condition (2.13)

Choosing the deposits, D_t, from equation (8.3) and using equation (8.4) above, the first order condition (2.13) for Problem 2.2.1 is

$$\frac{\partial \Pi_t}{\partial D_t}\left[1 + c_t^{dw} - \mathbf{E}_t\left\{\int_{\underline{\Lambda}}^{\overline{\Lambda}} \delta_{t,1} \frac{\partial V}{\partial K_{t+1}} dF(\sigma_{t+1}^{\Lambda})\right\}\right] = 0$$

8.3.3 First Order Condition (2.14)

We now find the partial derivative of the bank value in (8.3) with respect to the Lagrangian multiplier, l_t.

$$\frac{\partial V}{\partial l_t} = K_t - \rho\left[\omega(M_t)\Lambda_t + \omega^I I_t + 12.5(mVaR + 0)\right].$$

Therefore the first order condition (2.14) for Problem 2.2.1 is

$$\rho\left[\omega(M_t)\Lambda_t + \omega^I I_t + 12.5(mVaR + 0)\right] \leq K_t.$$

8.3.4 First Order Condition (2.15)

Choosing the regulatory capital, Π_t, from equation (8.3) and using equation (8.4) above, the first order condition (2.15) for Problem 2.2.1 is

$$-1 - c_t^{dw} + \mathbf{E}_t\left\{ \int_{\underline{\Lambda}}^{\overline{\Lambda}} \delta_{t,1} \frac{\partial V}{\partial K_{t+1}} dF(\sigma_{t+1}^{\Lambda}) \right\} + 1 = 0,$$

which is the same as

$$-c_t^{dw} + \mathbf{E}_t\left\{ \int_{\underline{\Lambda}}^{\overline{\Lambda}} \delta_{t,1} \frac{\partial V}{\partial K_{t+1}} dF(\sigma_{t+1}^{\Lambda}) \right\} = 0,$$

8.4 DIAGRAMMATIC OVERVIEW OF BASEL II CREDIT RISK

In this appendix, we provide a diagrammatic overview of Basel II credit risk.

8.5 SECURITIZATION

In this section, we provide more information about securitization with regard to borrowing under securitization strategies (see Figure 8.2 below) and the financial leverage profit engine (see Figure 8.3 below) as presented on the website [117].

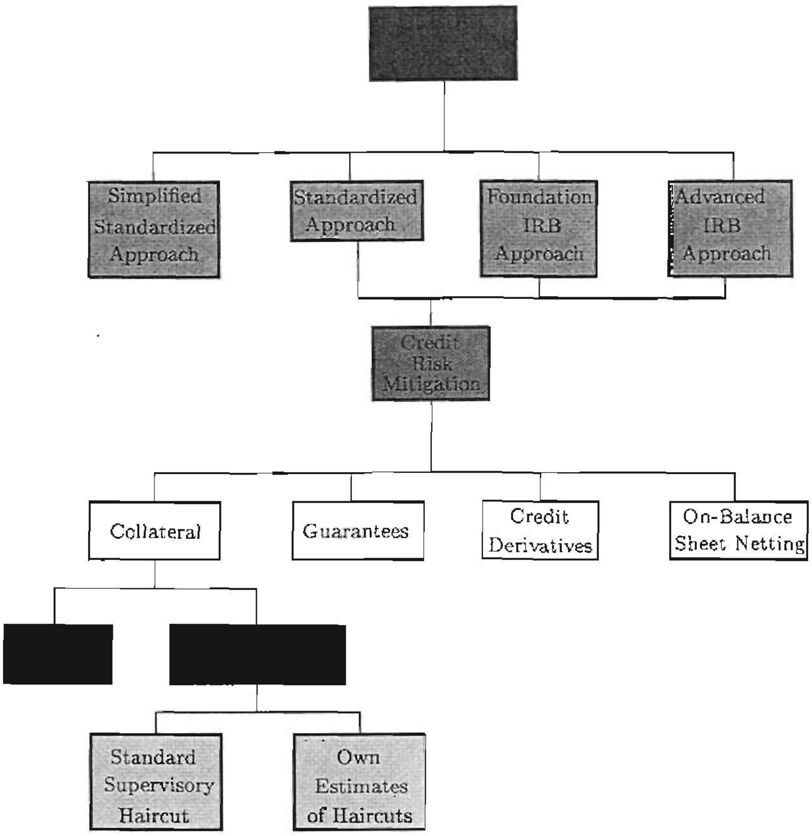

Figure 8.1: Diagrammatic Overview of Basel II Credit Risk

8.5.1 Borrowing Under Securitization Strategies

A diagrammatic overview of borrowing under securitization strategies may be represented as follows.

A simpler procedure for borrowing under an MBS strategy may be summarized as follows. The bank lends money to the borrower (homeowner). The bank creates an asset pool from a group of equivalent assets and sells CMO certificates to investors. The mortgage holders pay periodic payments to the bank. The bank collects mortgage payments, extracts servicing fees, pays guarantee fees to the trustees and passes the rest of the payment on to the investors. In the case where the mortgage holder

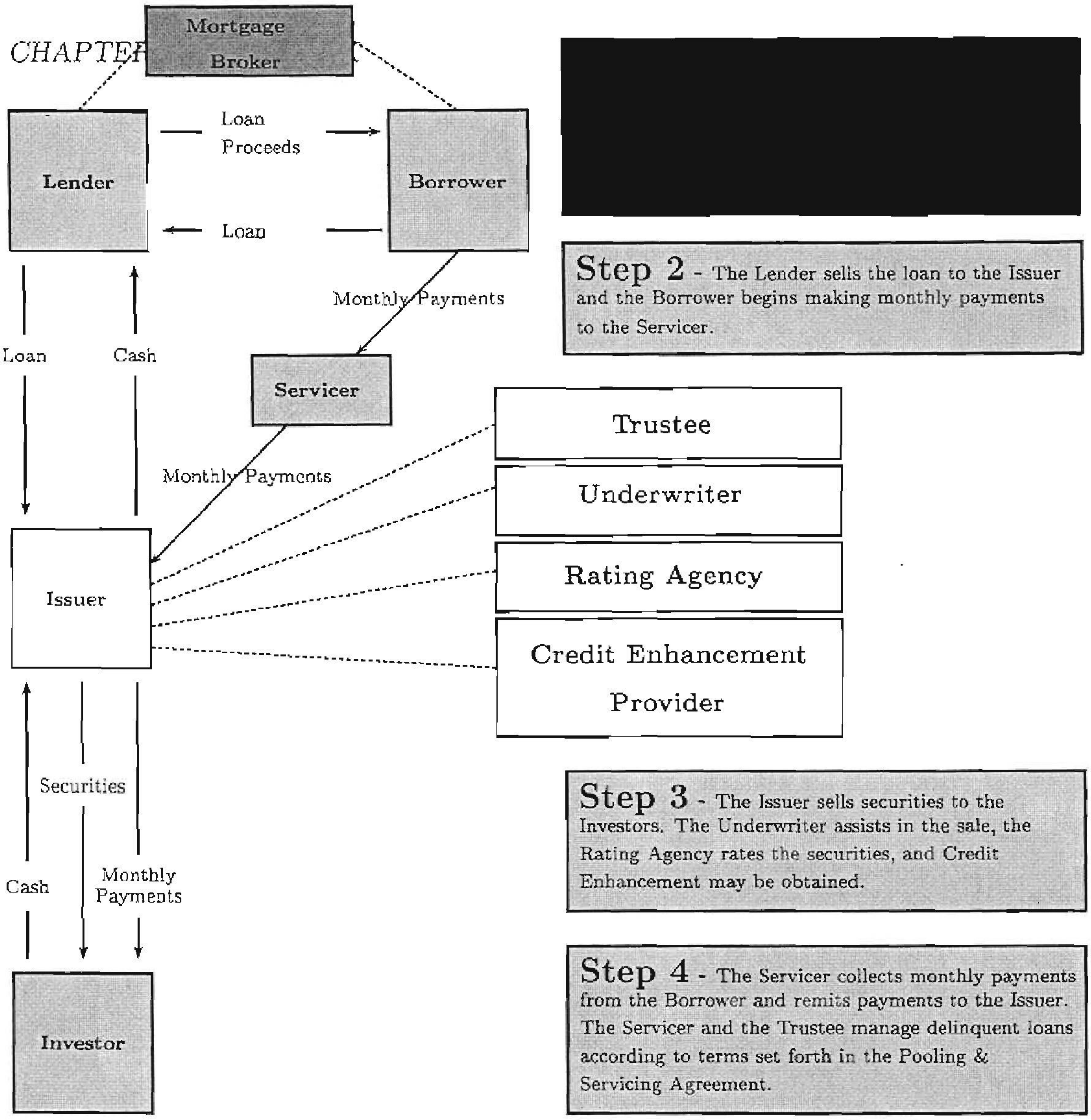

Figure 8.2: Diagrammatic Overview of Borrowing Under Securitization Strategies (compare [117])

defaults the trustee pays the remaining mortgage balance to the investors. Next, a diagrammatic overview of the financial leverage profit engine that is closely related to borrowing under securitization strategies will be given.

8.5.2 The Financial Leverage Profit Engine

Further insight may be gained about securitization by considering the following illustration (compare with Figure 8.3). Let us assume that an investment bank borrows money from an investor or money market fund and agrees to pay, for instance, 4 %

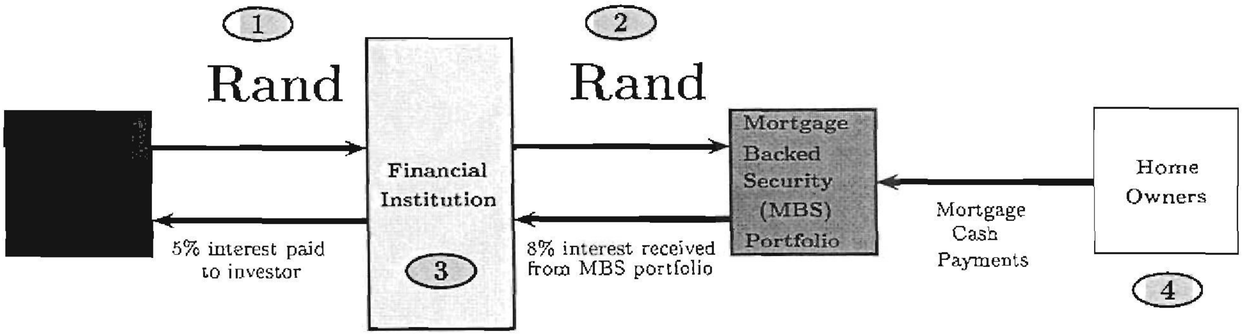

Figure 8.3: Diagrammatic Overview of the Financial Leverage Profit Engine (compare [117])

interest rate. The MBS portfolio is collateral, which the investors can seize in the event of a default on interest payments. The investment bank uses the funds to expand its MBS portfolio, which is paying, for instance, 7 % interest rate. The 3 % rate difference between the amounts is called the spread. In our case, for every R 100.00 invested in this manner, the investment bank makes R 3.00 profit margin. This provides an incentive to borrow and invest as much as possible, known as leveraging. This was considered safe during the early 2007 housing boom, as MBS portfolios typically received high credit ratings and defaults were minimal. Since investment banks do not have the same capital reserve requirements as depository banks, many borrowed and lent amounts exceeding 30 times their net worth. By contrast, depository banks rarely lend more than 15 times their net worth. Freddie Mac was leveraged nearly 70 times its net worth. With increasing delinquencies and foreclosures during 2007-2008, the value of the MBS portfolios declined. Investors became concerned and in some cases demanded their money back, resulting in margin calls (need to sell/liquidate the MBS portfolios) to pay them. At such a high leverage amount, many investment banks and mortgage companies suffered huge losses, bankruptcy, or merged with other institutions. Because MBS securities became "toxic" due to uncertainty in the housing market, they became illiquid and their values dropped. The market value is penalized by the inability to sell the MBS; it may be less than the value the actual cash inflow would merit. The ability of financial institutions to obtain funds in this manner via MBS has been dramatically curtailed. Spreads have narrowed, as investors are demanding higher returns to lend money to highly leveraged institutions.

8.6 KEY DEFINITIONS

In this section, we present the definitions of a few key concepts as provided by, for instance, [117].

Credit crunch is a term used to describe a sudden reduction in the general availability of loans (or credit) or sudden increase in the cost of obtaining loans from banks (by raising interest rates).

Special Purpose Vehicle (SPV): A SPV is a body corporate (usually a limited company of some type or, sometimes, a limited partnership) created to fulfill narrow, specific or temporary objectives, primarily to isolate financial risk, usually bankruptcy but sometimes a specific taxation or regulatory risk. A SPV may be owned by one or more other entities and certain jurisdictions may require ownership by certain parties in specific percentages. Often it is important that the SPV not be owned by the entity on whose behalf the SPV is being set up by the sponsor. For example, in the context of a loan securitization, if the SPV securitization vehicle were owned or controlled by the bank whose loans were to be secured, the SPV would be consolidated with the rest of the bank's group for regulatory, accounting, and bankruptcy purposes, which would defeat the point of the securitization. Therefore many SPVs are set up as "orphan" companies with their shares settled on charitable trust and with professional directors provided by an administration company to ensure there is no connection with the sponsor.

Mortgage-Backed Security: A mortgage-backed security (MBS) is an asset-backed security whose cash flows are backed by the principal and interest payments of a set of mortgage loans. Payments are typically made monthly over the lifetime of the underlying loans. Residential mortgages in the United States have the option to pay more than the required monthly payment (curtailment) or to pay off the loan in its entirety (prepayment). Because curtailment and prepayment affect the remaining loan principal, the monthly cash flow of an MBS is not known in advance, and therefore presents an additional risk to MBS investors. Commercial mortgage-backed securities (CMBS) are secured by commercial and multifamily properties (such as apartment buildings, retail or office properties, hotels, schools, industrial properties and other commercial sites). The properties of these loans vary, with longer-term loans (5 years or longer) often being at fixed interest rates and having restrictions on prepayment, while shorter-term loans (1-3 years) are usually at variable rates and freely prepayable.

Subprime lending is the practice of making loans to borrowers who do not qualify for market interest rates owing to various risk factors, such as income level, size of the down payment made, credit history and employment status.

Project finance (PF) is a method of funding in which the lender looks primarily to the revenues generated by a single project, both as the source of repayment and as security for the exposure.

Object Finance (OF) refers to a method of funding the acquisition of physical assets (e.g. ships, aircraft, satellites, railcars, and fleets) where the repayment of the exposure is dependent on the cash flow generated by the specific assets that have been financed and pledged or assigned to a lender. A primary source of these cash flows might be rental or lease contracts with one or several parties.

Commodities finance (CF) refers to structured short-term lending to finance reserves, inventories, or receivable of exchange-traded commodities (e.g. crude oil, metals, or crops), where the exposure will be repaid from the proceeds of the sale of the commodity and the borrower has no independent capacity to repay the exposure. This is the case when the borrower has no other activities and no other material assets on its balance sheet.

Income producing real estate (IPRE) refers to a method of providing funding to real estate (such as, office buildings to let, retail space, multifamily residential buildings, industrial or warehouse space and hotels) where the prospects for repayment and recovery on the exposure depend primarily on the cash flow generated by the asset. The primary source of these cash flows would generally be lease or rental payments or the sale of the asset.

High volatility commercial real estate (HVCRE) lending is the financing of commercial real estate that exhibits higher loss rate volatility (e.g., higher asset correlation).

Specialized lending high volatility commercial real estate (SLHVCRE) includes commercial real estate exposures secured by properties of types that are categorized by the national supervisor as sharing higher volatilities in portfolio defaults rates; loans financing any of the land acquisition, development and construction (ADC) phases for properties of those types in such jurisdiction; loans financing ADC of any other properties where the source of repayment at origination of the exposure is either the future uncertain sale of the property or cash flows whose source of repayment is substantially uncertain (e.g the property has not yet been leased to the occupancy rate prevailing in that geographic markets for that type of commercial real estate), unless the borrower has substantial equity at risk.

The asset class, *bank exposure* (BE), covers exposures to banks and securities firms that are subject to supervisory and regulatory arrangements comparable to those under Basel II. Bank exposure also includes claims on domestic public sector entities

(PSEs) that are treated like claims on banks under the Standardized approach, and multilateral development banks (MDBs) that do not meet the criteria for 0 % risk weight under the Standardized approach.

Sovereign exposure (SE), covers all exposures to counterparties treated as sovereign under the Standardized approach. This includes sovereigns (and their central banks), certain PSEs identified as sovereigns in the Standardized approach, MDBs that meet the criteria for 0 % risk weight under the Standardized approach.

Retail residential mortgage (RRM) loans (including first and subsequent liens[1], term loans and revolving (HELOC) home equity lines of credit) are eligible for retail treatment regardless of exposure size so long as the credit is extended to an individual that is an owner-occupier of the property.

[1] In law, a lien is a form of security interest granted over an item of property to secure the payment of a debt or performance of some other obligation. The owner of the property, who grants the lien, is referred to as the lienor and the person who has the benefit of the lien is referred to as the lienee.

Chapter 9

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