

MODELLING THE CLIMATE IN A TRANSPORT CONTAINER

Corné Botha

North West University

Nov 2004

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Acknowledgements

This report presents the results of a project performed at Mathematics for Industry. The project was executed by Corné Botha with the help of Andriy Rychahivskyy. The problem is related to the climate control inside a transport container during transportation of fruit.

I would like to express my sincere gratitude to the problem owners of Agrotechnology and Food Innovations (A&FI), Dr. R.G.M. van der Sman and Ir. M. Vollebregt, for their suggestions and constant support through the project.

Also, I wish to thank Dr. Ir. A.A.F. van de Ven, a supervisor, and Dr. Ir. S.J.L. van Eijndhoven of the Technical University of Eindhoven (TU/e) for their guidance throughout the project, and their helpful comments.

Eindhoven
Nov 5, 2004

Corné Botha

Abstract

The transportation of agro-products is essential for all of us. The quality of food after transportation is very important - none of us will buy poor quality food. In September 2002, A&FI started the three year research project QUEST(Quality and Energy efficiency in the Storage and Transportation of Agro-materials), with the focus on the use of containers to transport products overseas. The aim of QUEST is to find ways to reduce energy consumption for climate conditioning during transportation of perishable goods, and to monitor the product quality in order to minimize product losses. To achieve these goals, predictive models that describe the climate in one box, in a layer consisting out of nine boxes, and in a stack (few layers on top of each other) should be developed. The transport containers used, have a cooling unit in the front and a sensor system measuring the temperature and humidity inside the container. The goods are stored in cardboard boxes that are stacked in piles on pallets. The climate inside the container can be controlled by the circulation of cooled air. The main goal of this project is to develop a two-dimensional model predicting the climate in one layer of boxes. The company A&FI already has a model for the whole container, and wants to plug my resulting model, for one layer of boxes, into their model.

First I model a box and a slit (space between the boxes) separately. Subsequently I derive a network model for one layer consisting of nine boxes. The idea of the network is to replace the temperature distribution by the averaged temperature related to each box and slit. Such a method is strongly based on the description of heat transfer using the analogy with electrical circuits.

My model makes it possible to determine the average temperatures at any moment of time, *i.e.*, predict the climate within a layer inside a transport container. My model brings A&FI another step closer in the process to have a global model for the whole container. I recommend further extension of our model to a stack of boxes (to three dimensions).

Nomenclature

Notation	Parameter	Value	Units
ε	porosity	4E-1	–
d	product diameter	6.5E-2	m
d_w	package thickness	3E-3	m
c_p^a	air specific heat capacity	1.005E3	J/kg K
L	characteristic length	4E-1	m
b	half-width of a box	1E-1	m
h	width of a slit	6E-3	m
M_w	molecular weight of the moisture vapour	1.8E-2	kg/mol
R	universal gas constant	8.31	J/mol K
T_h	initial layer temperature	290	K
T_c	temperature of cold air	280	K
κ^*	permeability	4.17E-6	m ²
λ_a	air thermal conductivity	2.5E-2	W/m K
ρ_a	air density	1.23	kg/m ³
μ	air dynamic viscosity	1.75E-5	Pa s
λ_w	thermal conductivity of the box wall	3E-2	W/m K
λ_{eff}	effective thermal conductivity	6.04E-2	W/m K
C_{eff}	effective heat capacity	1.05E6	J/kg K
r	latent heat of water	2.3E6	J/kg
β_m	moisture transfer coefficient	1E-5	m/s
O	vent hole ratio	5E-2	–
Q	heat source	1E1	W/m ³
c_0	initial moisture concentration	8.45E-3	kg/m ³
c_1	final moisture concentration	8.02E-3	kg/m ³
$c_{sat}(T_h)$	initial saturated moisture concentration	1.43E-2	kg/m ³
t_0	characteristic time	6E5	s
p_0	characteristic pressure difference through a box	3.33E-1	Pa
u_0	characteristic velocity	1.788E-1	m/s
$\bar{u}$	average dimensional velocity in a box	9.9286E-3	m/s
Fo_h	heat Fourier number	2.175E-1	–
Pe_h	heat Peclet number	1.464E3	–
Sh_h	heat Sherwood number	2.18E-1	–
Po	Pomerantsev number	2.649	–

Chapter 1

Management Introduction

1.1 Problem Background

Agriculture is one of the most important driving forces for the Dutch economy. The Netherlands are known worldwide as producer and exporter of tulip bulbs, potatoes, tomatoes, bell peppers, cheese, and many other agricultured products. To maintain the reputation of top-ranked exporter of agro-materials, the Netherlands must take care of the quality of the products they export. These products have to satisfy consumer requirements; they have to be fresh, tasty and good-looking. Therefore, the problem of maintaining quality of perishable goods during their transportation, is a major item.

Agrotechnology and Food Innovation (A&FI) is a company actively engaged in the problem of quality control of horticultural products. A&FI forms, together with the university department Agrotechnology and Food Sciences, the Agrotechnology & Food Sciences Group of the Wageningen University and Research Center. It performs research for agriculture and food industry. From September 2002, A&FI started the research project QUEST, which is an abbreviation for QUality and Energy efficiency in Storage and Transport of agromaterials. This research has been carried out within the department of Quality in Chains, consisting of the groups Production & Control Systems; Packaging, Transport & Logistics and Post harvest Quality Fresh Products. The following companies are partners in QUEST, besides A&FI: P&O Nedlloyd B.V. (container owner and producer), Carrier Transicold (refrigeration machine producer), the Greenery (marketing and sales organization for fresh produce representing Dutch growers), Frugi Venta (Dutch trade organization for fruit and vegetables), Haluco (marketing and distribution organization for fruit and vegetables), R&R Mechatronics (manufacturer of instruments for diagnostics and laboratory automation).

Within QUEST the transport overseas by containers is the main focus. The company deals with the question of regulating the climate inside the cool-storage containers with perishable goods during their transportation overseas. These containers have a cooling unit in the front (supplying air to the load), and a sensor system measuring the temperature and

humidity level in the container from the returned air. During transportation, the goods are stored in cardboard boxes, which are stacked in piles on the pallets. The climate in the container can be controlled. The method to influence the climate conditions (temperature and moisture concentration) in a container is by circulation of cooled air; as shown in Figure 1.1. The air is blown into the container through a T-bar floor and going up between the piles of packages and through the piles of packages. The main issue addressed is to find a relation between climate inside the boxes and the controlled air-inlet.

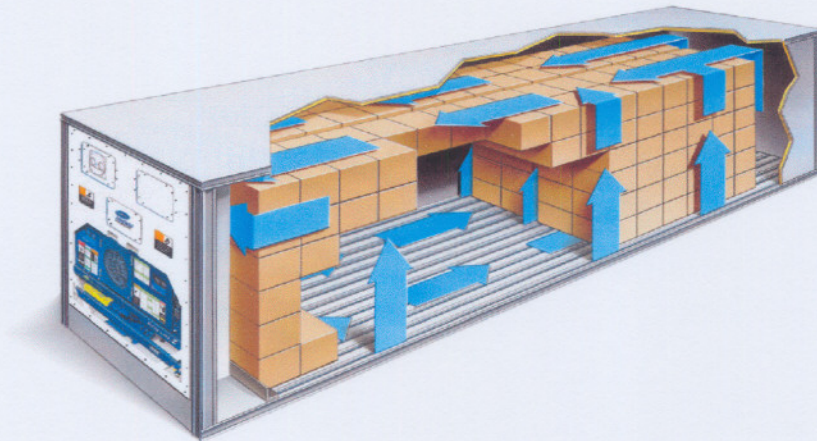


Figure 1.1: Circulation of the cooled air in a transport container

The overall aims of QUEST are:

- reduction of energy consumption for climate conditioning in transport of perishable goods
- monitoring the product quality to reduce product losses.

Predictive models for product behavior have already been developed. These models support the calculation of the appropriate climate settings. An interactive control system that fixes the optimal climate conditions based on the product states, as indicated by the product quality monitoring system and transport information, was developed and thereafter linked with a product quality monitoring system. However, the models for climate description on the scale of a layer (consisting of 9 boxes) and a stack (consisting of a few layers) should still be elaborated.

1.2 Problem Description and Specifications

Many physical processes play a role in maintaining quality of packed products during storage and transport in the container. The quality of the product is diminished due to drying and rotting. Rotting of products occurs due to high temperature, and on the other hand, low humidity and high temperature cause large evaporation rates, and thus drying of the products.

The primary physical processes of drying and rotting are influenced by the secondary processes of transfer of heat and moisture, evaporation and condensation. These secondary processes can be controlled. An appropriate controlled climate (temperature and moisture concentration) should be maintained in the container during transport of the perishable goods to avoid losses of the products, i.e to minimize quality decay.

Products in the container are stored in boxes. During storage and transportation, it is necessary to remove heat of respiration to avoid temperature rising in the box. Controlling the heat transfer in the box is very important in order to maintain the quality of the products. Therefore, a sufficient level of ventilation must be provided. To provide air circulation, packages (boxes) are made with vent holes, which allow air to flow in and out of the package. The airflow through the cardboard can be neglected. However, heat and moisture exchange is possible both through the package material and through the holes in the boxes. Increasing the number of vent holes, on one hand, stimulates the airflow, but on the other hand can cause an increase of moisture loss.

As already mentioned, the main technique to influence the climate in the container, is by circulation of air. The inner airflow I consider is a forced airflow. The air is blown into the container through a T-bar floor and then circulates between the boxes. In addition, the air enters and leaves each box through the circular shaped, oppositely placed vent holes in the middle of the smallest side walls of the box; see Figure (1.2). This air flow – through the vent holes in the longitudinal direction (direction of a vent hole) – is caused by the pressure difference at the opposite sides of the box induced by the air cross-flow along the holes.

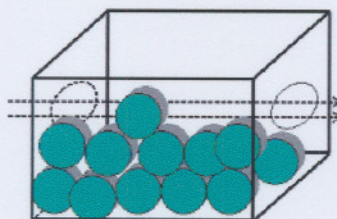


Figure 1.2: Flow of air through a vent-holed box

Various models were developed describing the airflow, heat and moisture transfer inside and outside the boxes. To control the climate, the controller requires a fast model to predict climate conditions inside the container based on a limited number of data from the cargo hold. Because of the limited calculation capacity of the controller, it is impossible

to perform detailed calculations. Therefore, the control model will be of the scale similar to or even bigger than the network-model developed by A&FI for the climate prediction.

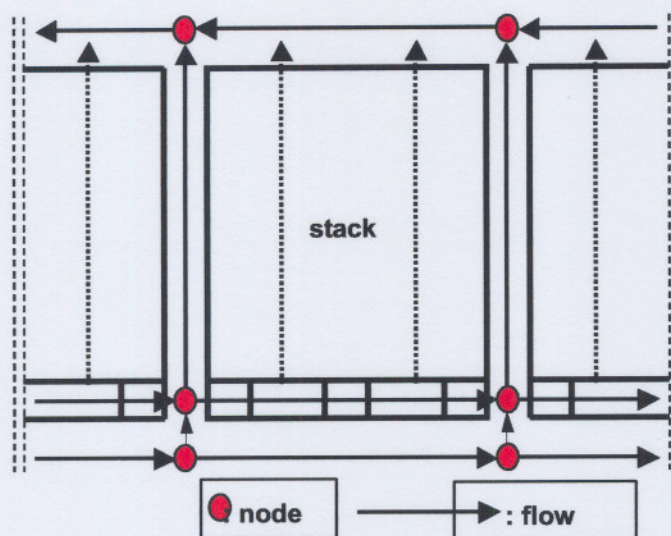


Figure 1.3: Scheme of network model (developed by Jasper Kelder (personal communication))

The network-model, partially sketched in Figure 1.3, describes the climate conditions at the representative points inside the container, making use of the product and package properties at stack scale, temperature at set points, and air inflow amount. The model predicts air velocities, air and product temperatures, and moisture amounts at these points. Each stack of boxes placed on a pallet is modeled as one node. From practice it is known that there can be significant differences in temperature in a stack of boxes, leading to quality variation.

Therefore, it is desired that the network-model can incorporate these differences by taking into account the effects of product and package properties. To extend this model, it is necessary to understand how the climate inside the box (local climate) is influenced by the climate outside the box (global climate).

To come from a local model describing the climate inside a box to a global and local climate, given product and package characteristics and the configuration of the network-model, the intermediate level from a box to one layer of boxes is the main goal of this project, which was carried out as a project for the post-graduate program Mathematics for Industry. Extending the global model with a better stack description will result in a more realistic model. Therefore, it will lead to the overall goal *i.e.*, controlling the climate conditions in transport of perishable goods to reduce product losses.

A box, filled with products, was modelled as a porous medium (Vollebregt, H.M., 2001), which is in local thermal equilibrium. Products were considered as a rigid solid, and the air was supposed to be a flowing fluid. The local thermal equilibrium model assumed the

solid-phase temperature to be equal to the fluid temperature, *i.e.*, local thermal equilibrium between the fluid and the solid-phase in any location of the porous medium.

The processes inside the box, *i.e.* convection of heat and moisture caused by the airflow, accompanied by respiration, evaporation, condensation, and diffusion at the surfaces of the products (assumed to have a spherical shape) and the box walls, have already been considered. (Nishchenko, 2004). The climate conditions – air pressure, temperature and moisture content – are prescribed at outside corner points of the stack. Each box in a layer interacts with the neighboring (adjacent) boxes through the vertical slits between the boxes.

The specifications of the problem are the following:

- The length×width dimensions of a rectangular box are 40×20 cm; the height of each box is 20 cm. The stack contains 80-100 boxes. I consider a 3x3 layer configuration with corresponding vent holes; see Figure 1.4.

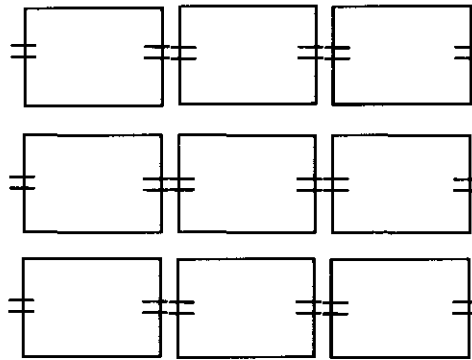


Figure 1.4: A 3x3 layer configuration

- The width of a slit can vary. I take its maximum value in one layer to be 0.6 cm.
- I make use of the parameters and coefficients collected in the section on Nomenclature (page iv).

My task is to predict the climate within the scope of one layer of boxes (within the narrow slits and the boxes) based on the volume averaging method (see chapter 4). The idea is to start by placing some representative points (I will call them nodes) both in the middle of each box and in the slits between the boxes over the layer. The so called network model sketched in Figure 1.5 must give the relation between the averaged slit and box quantities at the nodes.

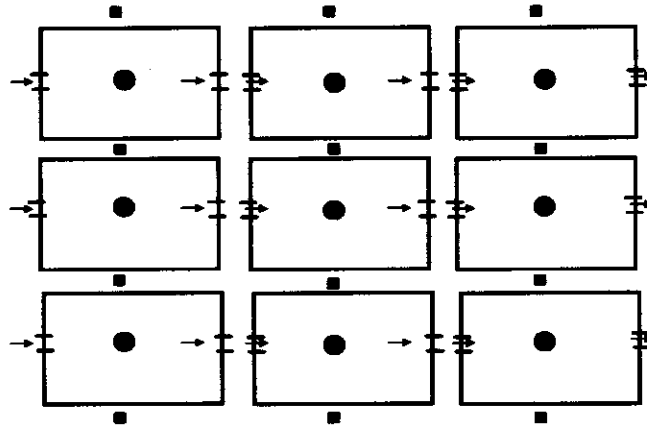


Figure 1.5: Top view of a layer: position of nodes

To get an idea of a network model, I refer the reader to electrical AC circuits: each node is modeled by a single heat capacitance connected by a heat resistance to other nodes. So, each box and slit is considered a resistance to the heat flow between two neighboring nodes. In general, my network model should result in a coupled set of first order differential equations enabling prediction of temperature and moisture contents at the nodes.

The relations for the 2D velocity field have already been derived. My goal is to model the relations between the temperature and moisture at the nodes for this two-dimensional case.

After that, I can extend the 2D-network model to the model of one layer of boxes. Reduction of the number of nodes, at which the network model is described is also an objective.

1.3 Assumptions

I assume that:

- An ideal situation exists inside the container - spaces between all the boxes are equal. The situation inside the container is symmetric (and remains like that throughout transportation).
- All boxes are filled with spherically shaped fruit.

To simplify the mathematical description of the air, heat, and moisture transfer, I assume:

- The air is an ideal gas with constant density.
- The air flow is stationary and incompressible having a Poiseuille profile.
- A quasi-static approximation of the convection process can be applied.

- The walls of each box are impervious to air flow, but allow heat conduction and moisture (mass) transfer by diffusion.
- There is a prescribed pressure drop in the horizontal-direction of a layer; see Figure 1.5. The air pressure in the vertical-direction is constant, *i.e.*, there is no air flow in this direction in the case of one layer of boxes.

1.4 Results

I have modeled the process of heat and moisture transfer through a layer consisting of boxes filled with spherical products and slits between them. By applying the method of volume averaging (also see Chapter 4) to each separate box and slit, I have reduced the set of the coupled boundary value problems to a set of Cauchy problems for the average temperatures. Assigning one node to each box and to each longitudinal slit within the layer and attaching the average temperatures of box and slit, respectively, to the nodes, I can predict the temperatures within a layer. My model is a network model in the sense that a matrix for the governing set of equations is of the form analogous to the matrices obtained when applying the Kirchhoff rules to electrical networks.

The main results are the following:

1. A simple configuration reveals that it takes approximately 3 seconds for the slits to cool down, and about 8 hours for the first column of boxes to cool down.
2. The model has been implemented in MATLAB®.
3. A less complicated model convenient for connection with the network-model using the method of volume averaging was constructed. The macroscopic ordinary differential equations for heat and moisture transfer were obtained by taking the average of the microscopic equations over the average volume of the box.

1.5 Conclusions & Recommendations

From my results I conclude that

1. It can be assumed that the temperature in the slits are equal to its final constant value T_c .
2. The predicted average temperature distribution is plausible since it is in accordance with the previous modelling results for a box. The model gives similar results in comparison with the results of FEM.

Recommendations to A&FI:

1. Reduce the number of nodes by using the first result and conclusion given above.
2. Extend this model to the case of a 3D layer of boxes.
3. Apply the volume averaging technique to different types of boxes to check its application range.

Chapter 2

Modelling a box with fruit

This chapter presents a mathematical model of the airflow, heat and mass transfer in a box filled with fruit that was developed. The total medium in the box, *i.e.* product and air, is treated as a porous medium, consisting of two components: a rigid solid matrix consisting of the spherical products, and an ideal gas, the air, flowing around the spheres. The air contains moisture. The product is considered rigid, but it can act as a source for heat or moisture, *e.g.*, due to evaporation.

2.1 Formulation of the problem

In this section specifically one box out of a stack of boxes in a container is considered. The box is filled with fruit, *e.g.* apples, and contains two circular vent holes. These vent holes are placed central in the left and right hand sides of the box; the diameter of the vent hole is d_h . The aim of the vent holes is to influence the climate inside the box by allowing air flow through the boxes; see also Figure 2.1. The dimensions of the box are: length L , width b , height h . Assigned to the box is a Cartesian coordinate system, with origin O in the center of the vent hole of the box, and the x -, y - and, z -axes in the length, width, and height directions of the box, respectively; see Figure 2.1.

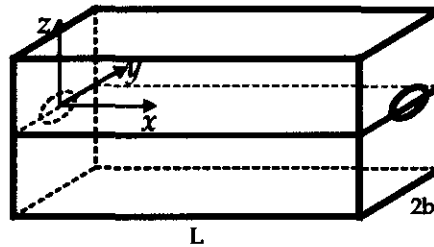


Figure 2.1: Geometry of the box

The vent holes are in the planes $x = 0$ and $x = L$. Consequently, the main flow through

the box is in the x -direction (for boxes of large height and high temperature differences also natural convection in the z -direction occurs). The inflow is through the vent hole at $x = 0$, and the air flows out of the box through the vent hole at $x = L$ (forced convection flow). The box is filled with fruit, say apples. There are relatively many apples (around 20-40) in the box, so the volume of one apple is small compared to the volume of the box. This makes it possible to define a characteristic volume element that is large with respect to the volume of the apple but small with respect to the volume of the box. Then, one may consider the medium inside the box as a continuum: a mixture of two components, apples and air, and we can model this mixture as a porous medium. (Dufr  che et al, 2003:623-639) Also see and Hsu, 2001. The mixture or porous medium consists of a rigid solid component, the apples, and a “fluid” component, the air, considered as an ideal gas. The air is humid due to moisture release of the apples. The humidity is characterized by the moisture content $c = c(\mathbf{x}; t)$, defined as the mass of the moisture contained in a unit of volume of air. Due to heat of evaporation or respiration, the apples also influence the temperature of the air in the boxes, and thus, the local temperature of the porous medium, $T = T(\mathbf{x}; t)$. The forced air flows through the boxes and causes convective flow of heat and moisture. These processes are, in general, non-stationary, however, over a long time, quasi-static approximations may become appropriate (this can be shown by scaling of the equations, and will be done further on in this chapter). The fundamental field variables that play a role in the behavior of the porous medium, described above, are successively: the density $\rho = \rho(\mathbf{x}; t)$, the velocity $\mathbf{u} = \mathbf{u}(\mathbf{x}; t)$, and the moisture content $c = c(\mathbf{x}; t)$ of the air, the pressure $p = p(\mathbf{x}; t)$ and temperature $T = T(\mathbf{x}; t)$ of the porous medium as a whole. These variables are described by well known local conservation equations together with constitutive laws, such as Fick’s law and Darcy’s law (these equations will be presented in the next section), and in Appendix C for the case when temperature of the solid differs from the temperature of the air. Moreover, we need explicit expressions for the source terms due to condensation, evaporation and respiration. A porous medium is characterized by material parameters such as the porosity ϵ and permeability κ . The porosity is a measure for the open space in the porous medium; it is defined as the ratio of the volume of air to the total volume of the medium. The permeability indicates the capability to flow through the pores of the porous medium; it is related to Darcy’s law, and a precise definition will be given when this law is introduced in the next section. Other relevant material coefficients are thermal (c_p, λ) and viscous (μ) coefficients and coefficients related to the sources (*e.g.*, β^*, β_m). Here, we consider the porous medium as a homogeneous medium, implying that the material coefficients cannot explicitly depend on position $\mathbf{x}$, nor on time t . However, as most of these coefficients depend on temperature and/or moisture content, both being functions of $\mathbf{x}$ and t , these coefficients can depend implicitly on $\mathbf{x}$ and t , and, therefore, they are not constant. For the practical situations we consider, the variations in both T and c are rather small, so that we can usually take the material coefficients as constant. The material coefficients for the individual components, apples and air are well-known, but for the porous medium as a whole, we need the so-called effective parameters. How these effective parameters can be derived from the individual ones will be explained in Section 2.4. The equations mentioned above as they will be presented in Section 2.2, constitute

a consistent system for the unknown variables $\rho, \mathbf{u}, c, p$ and T (a system of equations that can be used in case when temperature of the solid differ from the temperature of the air are presented in Appendix C). Besides these equations, we also need initial and boundary conditions. At the initial time $t = 0$, we assume that the state in the box is a quiescent one, and that all variables have prescribed initial values. However, if we only consider (quasi-) static states, the initial conditions become irrelevant. Boundary conditions must be described at the walls of the box, and, especially, at the vent holes. The walls are assumed impermeable for air, but permeable for heat and moisture. This means that at the (inner) walls of the box, apart from the vent holes, a no-slip conditions for the velocity holds. At the vent holes, the in- or out flow of air is prescribed (as related to the pressure drop over the length of the box). Moreover, perfect contact between air inside and outside the box at the vent holes is assumed, implying that c and T must be continuous over the vent holes. Finally, at the walls diffusive, Robin-type boundary condition for c and T apply (relating the c or T flux across the wall to their difference in- and outside the wall). Consequently, we can say that the boundary conditions at the vent holes are (mainly) of convective nature, whereas those for the walls of the box are of diffusive nature. In Section 2.5, the system of equations obtained thus far will be made dimensionless. Numerical values for the dimensionless coefficients are listed in the Nomenclature. From these values, certain effects can be shown to be irrelevant and, therefore, can be neglected.

2.2 System of governing equations

Transport phenomena in continuous media are governed by conservation principles (*e.g.*, for mass, momentum, and energy). The general local form of such a conservation law for an arbitrary (scalar or vector) field $\phi = \phi(\mathbf{x}; t)$ (*e.g.*, mass, velocity, or temperature) can be written as a local balance law of the following differential form

$$\rho \frac{d\phi}{dt} \equiv \frac{\partial \phi}{\partial t} + \nabla \cdot (\rho \phi \mathbf{u}) = \nabla \cdot (\rho D (\nabla \phi)^T) + Qv. \quad (2.1)$$

In (2.1), the second term on the left-hand side is the convective term (ρ is the density and $\mathbf{u}$ is the velocity), the first term on the right-hand side is the diffusive term, in which Fick's law is used already (D is the diffusion coefficient), and Qv is a source term.

From the general conservation equation (2.1), by specifying ϕ , successively the equation of continuity ($\phi = 1$), the energy balance or temperature equation ($\phi = E(T)$, E the internal energy density), the partial mass balance for the moisture ($\phi = c$) and the equation of motion ($\phi = \mathbf{u}$) can be derived.

• Equation of continuity

The equation of continuity for the gas can be derived from (2.1) by taking ϕ equal to 1, or $\rho\phi = \rho$, and $D = 0$, yielding the equation of continuity:

$$\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \mathbf{u}) = 0. \quad (2.2)$$

Here, ρ is the density of air and $\mathbf{u}$ is its velocity. In the sequel, we will see that the changes in ρ are extremely small, and therefore, we assume that ρ is constant, $\rho(\mathbf{x}; t) = \rho = \rho_0$, (the density in the quiescent state). As a consequence (2.2) reduces to the incompressibility condition

$$\operatorname{div} \mathbf{u} = 0. \quad (2.3)$$

For the numerical simulations, we will take this condition to hold (thus air is taken to be an incompressible fluid; an alternative could be to use a kind of Boussinesq equation for the pressure, but this is not done here).

• Energy balance

The balance equation for energy for a homogeneous porous medium follows from (2.1), with $\rho = E(T)$, by assuming that $E(T)$ is such that

$$\rho \frac{dE}{dt} = \frac{dE}{dT} \frac{dT}{dt} = \rho c_p \frac{dT}{dt}, \quad (2.4)$$

with $c_p(T) = dE/dT$.

In splitting this term for a porous medium into an nonstationary and a convective term, we must realize that the convection is only due to the flow of the gas. This leads us to the description

$$\rho c_p \frac{dT}{dt} \rightarrow C_{\text{eff}} \frac{\partial T}{\partial t} + \rho c_p^a \mathbf{u} \cdot \nabla T, \quad (2.5)$$

where C_{eff} is the effective heat capacity (per unit of volume, *i.e.* in J/Km^3) for the porous medium as a whole (see section 2.4), where as c_p^a is the heat capacity at constant pressure for the air (per unit of mass). In section 2.4, it will be derived that

$$C_{\text{eff}} = \rho_s c_p^s (1 - \epsilon) + \rho c_p^a \epsilon, \quad (2.6)$$

where ϵ is the porosity and c_p^s the heat capacity of the product (solid). Since $c_p^s \gg c_p^a$, C_{eff} is in good approximation equal to

$$C_{\text{eff}} = \rho_s c_p^s (1 - \epsilon). \quad (2.7)$$

Moreover, in (2.1) we replace D by the effective thermal conductivity λ_{eff} of the medium as a whole ($\rho D \rightarrow \lambda_{\text{eff}}$), and split Qv into $Q - rS$, where Q is the heat source term and r is the latent heat of water.

Because of all this, (2.1) can be formulated as a convection-diffusion equation for the temperature $T = T(\mathbf{x}, t)$ of the medium (i.e an averaged temperature for the product and air as a whole):

$$C_{\text{eff}} \frac{\partial T}{\partial t} + \rho c_p^a \mathbf{u} \cdot \nabla T = \lambda_{\text{eff}} \Delta T - rS + Q. \quad (2.8)$$

Here

$$S = \beta_m A_{\text{sp}} (c_{\text{sat}}(T) - c(T)), \quad (2.9)$$

is the evaporation and condensation source term, with β_m the mass transfer coefficient of water vapor from the product to the air, A_{sp} the specific surface area (see 2.11), and c_{sat} the temperature-dependent saturated moisture concentration. The latter can be expressed by the Tetens formula (Nishchenko, 2004) in the form

$$c_{\text{sat}}(T) = \frac{PM_w}{RT_h} \exp \left[17.27 \frac{T - 273.15}{T - 35.86} \right], \quad (2.10)$$

where $P = 611\text{Pa}$ is a constant, M_w is the molecular weight of moisture vapour, R is the universal gas constant, and T_h indicates the initial temperature. See figure 2.2.

The specific surface area A_{sp} for a spherical particle can be calculated as

$$A_{\text{sp}} = \frac{6(1 - \epsilon)}{d}, \quad (2.11)$$

where ϵ is the porosity and d is the diameter of the particle. Finally, $c(T)$ is the moisture content in the air, seen as a function of T . The volume source term Q , the heat generation due to respiration, is left unspecified for the time being. The first term on the right-hand side of (2.8) represents heat flow due to conduction (or diffusion), whereas the second term on the left-hand side accounts for convective heat transport by air flow (note $\mathbf{u}$ is the velocity of the air).

• Moisture balance

The mass balance for transport through a homogeneous porous medium follows from (2.1) by substituting c for ϕ , D_{eff} for D , and ρS for Qv , and taking ρ constant (so that we can divide by ρ) as

$$\frac{\partial c}{\partial t} + \mathbf{u} \cdot \nabla c = D_{\text{eff}} \Delta c + S. \quad (2.12)$$

Here, $c = c(\mathbf{x}; t)$ is the moisture concentration in the air (*i.e.* the mass of the moisture within a unit of volume of air), D_{eff} is the effective diffusion coefficient for moisture flow through the porous medium (see Section 4), and S is the same source term as in (2.8). Similar to the energy balance equation, the second term on the left-hand side accounts for convection by air flow. The first term on the right-hand side is the diffusion term.

• Equation of motion

The equation of motion in a porous medium follows in principle from (2.1) by substituting $\mathbf{u}$ for ϕ . However, we are dealing with a porous medium, in which only the air is flowing. The interaction with (or resistance of) the rigid solid pores is incorporated by a generalization of Darcy's law. The result is the Darcy-Forchheimer-Brinkmann (DFB) equation (Liu, 1999:229-252)

$$\frac{\rho}{\epsilon} \frac{\partial \mathbf{u}}{\partial t} + \frac{\rho}{\epsilon^2} \mathbf{u} \cdot \mathbf{u} = -\nabla p - \frac{\mu}{\kappa} \mathbf{u} - \rho \beta^* |\mathbf{u}| \mathbf{u} + \mu \beta^* \frac{\mathbf{u} \cdot \nabla \mathbf{u}}{|\mathbf{u}|} + \frac{\mu}{\epsilon} \Delta \mathbf{u}. \quad (2.13)$$

Here, $p = p(\mathbf{x}; t)$ is the pressure in the air, μ is the viscosity of the air, κ is the permeability of the porous medium, and β^* is the Forchheimer coefficient. The latter two coefficients account for the constraints on the air flow through the pores between the rigid solid particles (Darcy's law). For products of spherical shape, the permeability and the Forchheimer coefficient can be related to the geometric properties of the product via the so-called Ergun relations (Van der Sman, 2003b:49-57):

$$\frac{1}{\kappa} = \frac{180(1 - \epsilon^2)}{d^2 \epsilon^3}, \beta^* = \frac{1.75(1 - \epsilon)}{d \epsilon^3}. \quad (2.14)$$

The second term on the right-hand side of (2.13) is associated with the Stokes drag force; the third term, the Forchheimer term, accounts for the high-flow-rate inertial pressure losses; the last term, the viscous or Brinkmann term, is responsible for the appearance of a viscous boundary layer at the solid interface in the porous medium. (Beukema, 1980).

2.3 Boundary and initial conditions

In this section, we shall formulate the initial conditions at $t = 0$ and the boundary conditions at the walls Γ of the box. We split the wall in a fixed (closed) part Γ_f , where a no-slip condition holds for the flow, and the vent holes Γ_{vi}, Γ_{vo} (see figure below) in the boxes, which are in open contact with the environment, and where the in- (or out-) flow is prescribed.

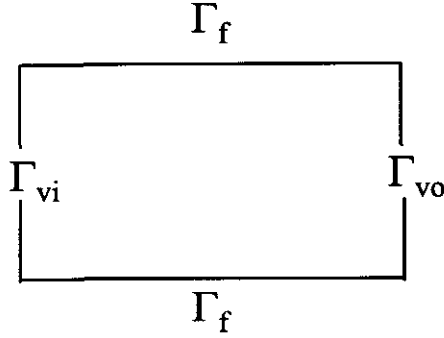


Figure 2.2: Geometry of the box

At the initial state $t = 0$, the air in the boxes is in a quiescent state, and the density, temperature, and moisture concentration in the gas equal their environmental values. This means that the initial conditions are (for $\mathbf{x} \in G$, the configuration of the box)

$$\begin{aligned}\rho(\mathbf{x}, 0) &= \rho_0, \\ T(\mathbf{x}, 0) &= T_0, \\ c(\mathbf{x}, 0) &= c_0,\end{aligned}\tag{2.15}$$

where ρ_0 , T_0 , and c_0 are the uniform initial density, temperature, and moisture concentration in the box, respectively.

We need boundary conditions for T , c , p and $\mathbf{u}$. I assume Γ_f semi-permeable for T and c , but not for the air. This leads to the following boundary conditions for $\mathbf{x} \in \Gamma_f$ and $t > 0$:

$$\begin{aligned}\mathbf{u}(\mathbf{x}, t) &= 0, \\ -\lambda_{\text{eff}} \frac{\partial T}{\partial n}(\mathbf{x}, t) &= \frac{\lambda_w}{d_w} (T(\mathbf{x}, t) - T_1), \\ -D_{\text{eff}} \frac{\partial c}{\partial n}(\mathbf{x}, t) &= \frac{D_w}{d_w} (c(\mathbf{x}, t) - c_1).\end{aligned}\tag{2.16}$$

Here, T_1 and c_1 are the uniform temperature and moisture concentrations in the environment; λ_w , D_w , and d_w are the thermal conductivity, diffusion coefficient for moisture and the thickness, respectively, of the wall of the box.

At the vent holes, $\mathbf{x} \in \Gamma_v$, the in- or out-flow of gas is prescribed, *i.e.*

$$\begin{aligned}p(\mathbf{x}, t) &= p_1, \mathbf{x} \in \Gamma_{vi}, \\ p(\mathbf{x}, t) &= p_2, \mathbf{x} \in \Gamma_{vo},\end{aligned}\tag{2.17}$$

where p_1 and p_2 are constant. The temperature and the moisture concentration at the vent holes are equal to their environmental values at the inlet and constant at the outlet, *i.e.*

$$\begin{aligned} T(\mathbf{x}, t) &= T_1, c(\mathbf{x}, t) = c_1, \mathbf{x} \in \Gamma_{vi}, \\ \frac{\partial T}{\partial n}(\mathbf{x}, t) &= 0, \frac{\partial c}{\partial n}(\mathbf{x}, t) = 0, \mathbf{x} \in \Gamma_{vo}. \end{aligned} \quad (2.18)$$

2.4 Effective parameters

The effective volumetric heat capacity c_{eff} can be determined from the sum of the heat capacities of the individual constituents according to,

$$C_{\text{eff}} = (\rho c_p)_s(1 - \epsilon) + (\rho c_p)_a \epsilon, \quad (2.19)$$

where the indices s and a indicate solid and air, respectively (Nejad).

The effective diffusivity is related to the fluid diffusivity by the following relation, (Nejad),

$$D_{\text{eff}} = \epsilon D_a. \quad (2.20)$$

A first order estimate of the effective thermal conductivity of a fluid-filled porous medium can be made by simply accounting for the volume fraction of each substance, giving the resulting relation based on the porosity and the thermal conductivity of each substance as

$$\lambda_{\text{eff}} = (1 - \epsilon)\lambda_s + \epsilon\lambda_a. \quad (2.21)$$

This equation, however, does not account for natural convection. The effective thermal conductivity can be calculated by using Zehner and Schlunder's equation (Pu, 1999:517-521)

$$\lambda_{\text{eff}} = \lambda_a \left(1 - \sqrt{1 - \epsilon} + 2 \frac{\sqrt{(1 - \epsilon)}}{\alpha B} \left(\frac{(1 - \alpha)B}{(1 - \alpha B)^2} \ln \frac{1}{\alpha B} - \frac{B + 1}{2} - \frac{B - 1}{1 - \alpha B} \right) \right), \quad (2.22)$$

with $\alpha = \lambda_a/\lambda_s$. The shape factor B for a packed bed consisting of uniform spheres is given by

$$B = 1.25 \left(\frac{1 - \epsilon}{\epsilon} \right)^{\frac{10}{9}}. \quad (2.23)$$

For pure conduction, estimates of the effective thermal conductivity can be obtained from Maxwell's model, which represents a heterogeneous system as a dispersion of spherical particles in a continuous phase (Quintard, 1997:77-94 and Beukema, 1980):

$$\lambda_{\text{eff}} = \lambda_a \left(\frac{3 - 2\epsilon(1 - \alpha)}{3\alpha + 2\epsilon(1 - \alpha)} \right). \quad (2.24)$$

This model was also used in as the most adequate model of effective thermal conductivity. (Cogne, 2003:331-341). Henceforth, I will also use formula (2.24) for λ_{eff} .

2.5 Dimensional analysis

To make the problem more accessible for a mathematical solution, we introduce dimensionless variables by

$$\hat{T} = \frac{T - T_1}{T_0 - T_1}, \hat{c} = \frac{c - c_0}{c_{\text{sat}}(T_0)}, \hat{c} = \frac{c_{\text{sat}} - c_0}{c_{\text{sat}}(T_0)}, \quad (2.25)$$

$$\hat{t} = \frac{t}{t_0}, \hat{x} = \frac{x}{L}, \hat{p} = \frac{p}{\Delta p}, \quad (2.26)$$

where T_0 is the initial temperature in the box, T_1 is the temperature of cold air, and also the uniform and constant temperature outside the box, c_0 is the uniform and constant moisture concentration outside the box, L is the length of the box, t_0 and u_0 are the characteristic time and characteristic velocity, respectively, and Δp is the pressure difference between the left and right sides of the box. For the specific time t_0 , I take a characteristic time for the transport duration, which I estimate at one week = 604800 $\approx 6.10^5$ sec. For the characteristic pressure difference Δp , I use a result found for the global network model. From this network model it is known that the pressure difference over a set of three boxes is characteristically 1Pa. This leads to an estimate for p_0 of 0.3Pa over one box. A specific value for u_0 will be determined later on. The density ρ in (2.2) is taken to be constant. Use of the scaling (2.26) in (2.8), (2.12), (2.13), omitting the hats, leads us to a dimensionless system of equations.

• Energy balance

Use of the scaling (2.26) in (2.8), with S eliminated from (2.9), leads to

$$C_{\text{eff}} \frac{T_0 - T_1}{t_0} \frac{\partial T}{\partial t} + \rho c_p^a u_0 \frac{T_0 - T_1}{L} \mathbf{u} \cdot \nabla T = \lambda_{\text{eff}} \frac{T_0 - T_1}{L^2} \Delta T - r \beta_m A_{\text{sp}} (c_{\text{sat}}(T) - c(T)) + Q. \quad (2.27)$$

Rearranging coefficients and using for A_{sp} the formula for spherical particles (2.11), give

$$\begin{aligned} & \frac{C_{eff}L^2}{\lambda_{eff}t_0} \frac{\partial T}{\partial t} + \dots \frac{\rho c_p^a u_0 L}{\lambda_{eff}} \mathbf{u} \cdot \nabla T = \\ \Delta T - 6 \frac{r\beta_m(1-\epsilon)L^2}{d\lambda_{eff}(T_0 - T_1)} c_{sat}(T_0)(c_{sat}(T) - c(T)) + \dots \frac{L^2}{\lambda_{eff}(T_0 - T_1)} Q. \end{aligned} \quad (2.28)$$

Introducing dimensionless numbers as defined in the Nomenclature, I can rewrite the energy balance in the form

$$\frac{1}{Fo_h} \frac{\partial T}{\partial t} + Pe_h \mathbf{u} \cdot \nabla T = \Delta T - \frac{6L}{d} Sh_h (c_{sat}(T) - c(T)) + Po. \quad (2.29)$$

• Moisture balance

Using the scaling parameters introduced in (2.26), I find from (2.12), with S eliminated from (2.9),

$$\begin{aligned} & \frac{c_{sat}(T_0)}{t_0} \frac{\partial c}{\partial t} + u_0 \frac{c_{sat}(T_0)}{L} \mathbf{u} \cdot \nabla c \\ & = D_{eff} \frac{c_{sat}(T_0)}{L^2} \Delta c + \beta_m A_{sp} (c_{sat}(T) - c(T)). \end{aligned} \quad (2.30)$$

Rearranging the coefficients and introducing dimensionless numbers as defined in the Nomenclature, I arrive at

$$\frac{1}{Fo_s} \frac{\partial c}{\partial t} + Pe_s \mathbf{u} \cdot \nabla c = \Delta c + \frac{6L}{d} (1-\epsilon) Sh_s (c_{sat}(T) - c(T)). \quad (2.31)$$

• Equation of motion

Using the scaling parameters introduced in (2.26), from (2.13), and rearranging the coefficients lead to

$$\frac{\kappa}{\epsilon v t_0} \frac{\partial \mathbf{u}}{\partial t} + \frac{u_0 \kappa}{\epsilon^2 v L} \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{\Delta p \kappa}{L \mu u_0} \nabla p - \mathbf{u} - \frac{\beta^* \kappa u_0}{v} \mathbf{u} \mathbf{u} + \frac{\beta^* \kappa}{L} \frac{\mathbf{u} \cdot \nabla \mathbf{u}}{\mathbf{u}} + \frac{\kappa}{\epsilon^2} \Delta \mathbf{u}. \quad (2.32)$$

where $v = \mu/\rho$, the kinematic viscosity of the air.

I also scale the initial and boundary conditions with the use of (2.26) and dimensionless numbers. This results, omitting the hats, for the

• initial conditions in (2.16)

$$\begin{aligned} T(\mathbf{x}, 0) &= 1, \\ c(\mathbf{x}, 0) &= 0; \end{aligned} \tag{2.33}$$

• boundary conditions for Γ_f in (2.17)

$$\begin{aligned} \mathbf{u}(\mathbf{x}, t) &= 0, \\ \frac{\partial T}{\partial n}(\mathbf{x}, t) &= -Bi_h T(\mathbf{x}, t), \\ \frac{\partial c}{\partial n}(\mathbf{x}, t) &= -Bi_s c(\mathbf{x}, t); \end{aligned} \tag{2.34}$$

• boundary conditions for Γ_{vi} and Γ_{vo} in (2.18)

$$\begin{aligned} T(\mathbf{x}, t) &= 0, c(\mathbf{x}, t) = 0, \\ p(0, t) &= \frac{p_1}{p_1 - p_2}, x \in \Gamma_{vi}; \end{aligned} \tag{2.35}$$

$$\begin{aligned} \frac{\partial T}{\partial n}(\mathbf{x}, t) &= 0, \frac{\partial c}{\partial n}(\mathbf{x}, t) = 0, \\ p(1, t) &= \frac{p_2}{p_1 - p_2}, x \in \Gamma_{vo}. \end{aligned} \tag{2.36}$$

Chapter 3

Mathematical Model

In this chapter, I construct a mathematical model for heat transfer within a horizontal (2-dimensional) layer consisting of nine boxes. First, I model this process for a box and a slit separately (Sections 3.2, 3.3), and then combine the models (Section 3.4).

3.1 Problem Statement

The modelling of climate within even one layer of boxes requires understanding of heat and mass transfer processes within the fruit, between them and the surrounding medium (air), as well as between packaging material and both the fruits and the outside air. The company A&FI, which posed the problem, prefer a compromise between simplicity and accuracy, *i.e.*, some model incorporating the most important effects only.

Van der Sman developed (2002:49-57, 2003a:383-390) a simple model based on the porous medium approach. I also use this method to construct my network scheme.

The specifications of the problem are the following:

- The length×width dimensions of a rectangular box are 40×20 cm; the height of each box is 20 cm. The stack contains 80-100 boxes. I consider a 3x3 layer configuration with corresponding vent holes; see Figure 1.4.
- The width of a slit can vary. I take its maximum value in one layer to be 0.6 cm.
- I make use of the parameters and coefficients collected in Section Nomenclature (page iv).

As a basis, first I model the processes of heat and moisture transfer in each box and slit separately. In order to do that, I use the following governing equations and conditions:

– *Differential Equations*

- The balance equations for the temperature and moisture (energy and mass conservation).

– *Boundary Conditions*

1. *At the walls of each box apart from the vent holes:*

- Diffusion type boundary conditions for temperature and moisture (the flux is proportional to the difference of inside and outside boundary values of the temperature).

2. *At the vent holes of each box:*

- Continuity conditions for temperature and moisture (perfect contact of the influx).

3. *At the contact surfaces of the slit:*

- Diffusion type boundary conditions for temperature and moisture.

4. *At the beginning of the slit:*

- Temperature is prescribed.

– *Initial Conditions.*

- Temperature is prescribed at the initial time $t = 0$.

3.2 Model for a Box

My model for each separate box out of a layer is based on the assumption that packed fruits can be described as a single phase homogeneous porous medium (Vollebregt, 2001), in which heat transfer is by conduction through the solid phase (the product) and by convective flow in the fluid phase, i.e. air.

I consider the geometry of the box sketched in Figure 3.1. I assign a Cartesian coordinate system $Oxyz$ to the box, with the origin O placed at the center of the left vent hole. The air flow is directed along the x -axis.

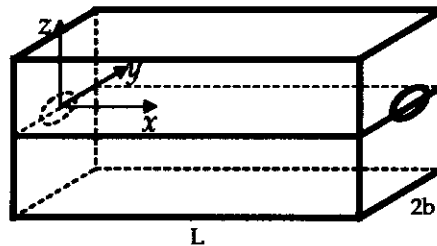


Figure 3.1: Geometry of the box

From now on, I restrict myself to considering a 2D climate problem in a horizontal cross-section of the box. Further on, if I refer to a box, it means the cross-section $\{x \in [0, L], y \in [-b, b], z = 0\}$. Such a problem has been considered by N. Nishchenko (2004) for a box filled with fruit of circular shape.

My modelling starts with the balance equation for the energy of a homogeneous porous medium contained in a box, as determined in the previous chapter:

$$C_{\text{eff}} \frac{\partial T}{\partial t} + \rho_a c_p^a \vec{u} \cdot \nabla T = \lambda_{\text{eff}} \Delta T - rS + Q, \quad \nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y} \right), \quad \Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}, \quad (3.1)$$

where C_{eff} is the effective (for the porous medium as a whole) heat capacity per unit of volume, λ_{eff} is the effective thermal conductivity, ρ_a is the air density, Q is a constant volume heat source term, c_p^a is the heat capacity at constant pressure per unit of mass, $T(x, y, t)$ refers to box temperature (t is the time), $\vec{u}(y)$ is the velocity of air flow through the box, r is the latent heat of water. S is given by,

$$S = \beta_m \frac{6(1 - \epsilon)}{d} [c_{\text{sat}}(T) - c(x, y, t)] \quad (3.2)$$

is the mass transfer source term, with $c(x, y, t)$ the moisture concentration in the air, β_m the mass transfer coefficient of the water vapour from the product to the air, ϵ is the porosity, d is the diameter of the product. $c_{\text{sat}}(T)$ is the temperature-dependent saturated moisture concentration, which can be expressed in the form of a Tetens function (Nishenko, 2004), as shown in the figure below:

$$c_{\text{sat}}(T) = \frac{PM_w}{RT_h} \exp \left[17.27 \frac{T - 273.15}{T - 35.86} \right]. \quad (3.3)$$

Here $P = 611\text{Pa}$ is a constant, M_w is the molecular weight of moisture vapor, R is the universal gas constant, and T_h indicates the initial temperature.

Note that equation (3.1) is non-linear due to the term in the right-hand side containing the saturated moisture concentration. I approximate the latter by a linear function:

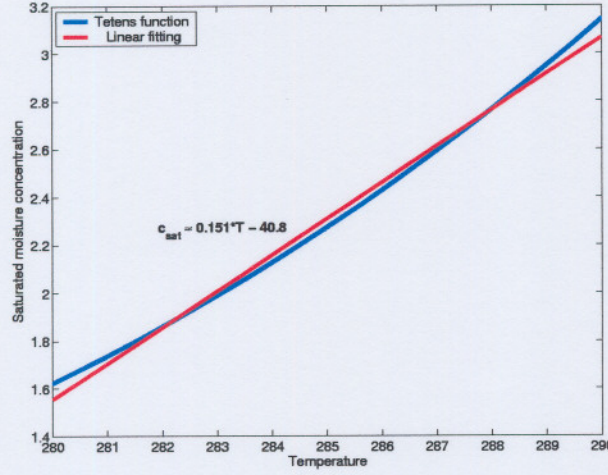


Figure 3.2: Linear approximation of the saturated moisture concentration

$$c_{\text{sat}}(T) = \frac{PM_w}{RT_h}(k'_1 T + k'_2). \quad (3.4)$$

I assume that initially the box is in a uniform quiescent state with

$$c(x, y, 0) \approx c_0; \quad T(x, y, 0) = T_h. \quad (3.5)$$

The results reported by N. Nishchenko (2003) in Figure 4.2 clearly show that the moisture contents reach its uniform and constant final state much faster than the temperature does (about 10^4 times faster). This means that on the time scale the temperature changes, but the moisture contents may be taken constant, *i.e.* equal to c_1 .

From now on, I will only consider the temperature problem. The walls of the box are assumed to be impervious to air flow and permeable to heat. This imposes the following boundary conditions on the box temperature:

$$\frac{\partial T(x, \pm b, t)}{\partial y} = \mp \frac{\lambda_w}{d_w \lambda_{\text{eff}}}(T^\pm - T_s^\mp); \quad (3.6)$$

$$T(0, y, t) = T_{\text{in}}, \quad \frac{\partial T(L, y, t)}{\partial x} = 0, \quad (3.7)$$

for $t > 0$, where λ_w represents the thermal conductivity of the box wall, d_w is the thickness of the wall, $T_s^\mp$ denote the temperatures of the adjacent imaginary (at the moment) upper and lower slits, respectively, $T^\pm \equiv T(x, \pm b, t)$, and T_{in} is the inlet temperature.

The choice of the second boundary condition of (3.7) is based on the following argument: since the convective flow at the outlet vent hole dominates the diffusion there, I assume that I may neglect the latter, which leads us to this boundary condition.

The volume averaging method, (see chapter 4), enables us to obtain the averaged temperature. This is done by first averaging over the y -direction and then over the x -

direction. Now I introduce the average temperature over the width of the box by

$$\tilde{T}(x, t) = \frac{1}{2b} \int_{-b}^b T(x, y, t) dy. \quad (3.8)$$

In order to express all the boundary temperatures in terms of the averaged values, I assume the temperature in the box to be a quadratic polynomial with respect to the width:

$$T(x, y, t) = T_0(x, t) + T_1(x, t) \frac{y}{b} + T_2(x, t) \left(\frac{y}{b} \right)^2, \quad (3.9)$$

where $T_i(x, t)$ ($i = 0, 1, 2$) are the coefficients expressed, by using formula (3.8), in terms of the boundary ($T^\pm$) and average temperature ($\tilde{T}$) values. Using the boundary conditions (3.6) and formulas (3.8), (3.9), I can express the boundary temperature values through the average values only; see Appendix A for more details.

By integrating equation (3.1) with respect to y , using the approximation (3.4), and taking the boundary conditions (3.6) into account, one arrives at the governing equation for $\tilde{T}(x, t)$ in the form

$$C_{\text{eff}} \frac{\partial \tilde{T}}{\partial t} + \rho_a c_p^a \bar{u} \frac{\partial \tilde{T}}{\partial x} - \lambda_{\text{eff}} \frac{\partial^2 \tilde{T}}{\partial x^2} - C_{\text{eff}} \sum_j B_{\bullet j} \tilde{T}_j + \tilde{Q} = 0, \quad (3.10)$$

where

$$\tilde{Q} = \frac{1}{2b} \int_{-b}^b (rS - Q) dy, \quad (3.11)$$

and summation over j refers to all the boxes and slits from the same column, $(B_{\bullet j})$ is a matrix containing all the coefficients, " $\bullet$ " refers to the specific box under consideration, *i.e.* to the corresponding row of the **B**-matrix; see Section 2.4 and Appendix B for more details.

The average velocity $\bar{u}$ of the air flow through the vent holes in the box is approximated by a generalized Darcy-Forchheimer equation (Vollbrecht, 2001):

$$\bar{u} = \frac{\kappa^* O}{\mu} \frac{p_0}{L}, \quad (3.12)$$

where κ^* is the permeability of the porous medium, O is the vent hole ratio (*i.e.* the ratio between the area of the vent hole and the area of the box side wall), μ is the dynamic viscosity of air, and p_0 is the characteristic pressure difference through a box.

Integrating equation (3.10) over the length of the box, and taking boundary conditions (2.7) into account, I arrive at the first-order ordinary differential equation for the average

temperature in the box $\bar{T}(t) = 1/L \int_0^L \tilde{T}(x, t) dx$:

$$C_{\text{eff}} \frac{d\bar{T}}{dt} + \frac{\rho_a c_p^a}{L} \bar{u} \left(\tilde{T}(L, t) - \tilde{T}(0, t) \right) - \frac{\lambda_{\text{eff}}}{L} \left(\frac{\partial \tilde{T}(L, t)}{\partial x} - \frac{\partial \tilde{T}(0, t)}{\partial x} \right) - C_{\text{eff}} \sum_j B_{\bullet j} \bar{T}_j + Q_0 = 0, \quad (3.13)$$

where

$$Q_0 = \frac{1}{C_{\text{eff}} L} \int_0^L \tilde{Q} dx. \quad (3.14)$$

At both the inlet, $x = 0$, and the outlet, $x = L$, the convection strongly dominates the diffusion, which implies that I may neglect the third term on the left-hand side of (3.13) with respect to the second one. Since $\tilde{T}(0, t) = T_{in}$ by (3.7)₁, we are left with only one unknown value, namely the box outlet temperature $\tilde{T}(L, t)$. For obtaining $\tilde{T}(L, t)$, I need a closure relation. For that I model the average temperature $\tilde{T}(x, t)$ by

$$\tilde{T}(x, t) = \begin{cases} T_c, & 0 \leq x \leq u_{\text{eff}} t - 2\sqrt{\kappa t}, \\ T_m, & u_{\text{eff}} t - 2\sqrt{\kappa t} < x \leq u_{\text{eff}} t + 2\sqrt{\kappa t}, \\ T_h, & u_{\text{eff}} t + 2\sqrt{\kappa t} < x \leq L, \end{cases} \quad (3.15)$$

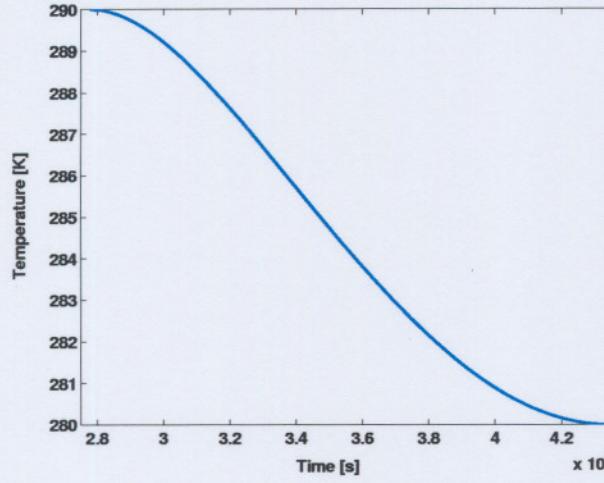
where

$$T_m(x, t) = T_c + (T_h - T_c) \left[\frac{1}{2} + \frac{3}{4} \frac{x - u_{\text{eff}} t}{2\sqrt{\kappa t}} \left(1 - \frac{1}{3} \left(\frac{x - u_{\text{eff}} t}{2\sqrt{\kappa t}} \right)^2 \right) \right] \quad (3.16)$$

and

$$\kappa = \frac{\lambda_{\text{eff}}}{C_{\text{eff}}}, \quad u_{\text{eff}} = \frac{\rho_a c_p^a}{C_{\text{eff}}} \bar{u}.$$

In (3.15), T_c represents the cold temperature in the front part of the box (near the inlet), $T_m(x, t)$ stands for the thermal front between the hot and the cold part, which is somewhat flattened due to the diffusion (increasing with time), and T_h represents the hot temperature in the back part of the box, near the outlet. For my purposes, because in this section I only consider the first column (see Figure 1.4) of three boxes in a layer consisting of nine boxes and because the sources are not too strong, I take $T_{in} = T_c$ and T_h equal to the initial temperature in the box. The distribution (3.15) holds over the whole box as long as t is less than t_1 , with t_1 such that $u_{\text{eff}} t_1 + 2\sqrt{\kappa t_1} = L$. When $t > t_1$, only part of the distribution (3.15) is inside the box. I emphasize here that (3.15) is NOT a proposal for the solution of (3.10), but only a model to derive an educated guess for $\tilde{T}(L, t)$.

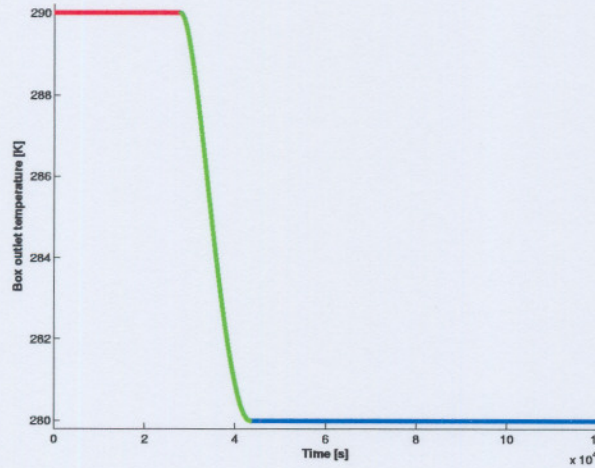
Figure 3.3: The thermal front T_m

The function (3.16) is continuously differentiable at $x = u_{\text{eff}}t \pm \xi(t)$ and satisfies equation (3.10) up to first order in $(x - u_{\text{eff}}t)/(2\sqrt{\kappa t})$.

Later in this chapter, I will explain the above modelling of inlet- and outlet box temperatures in more detail. Here, I only use formulas (3.16) and (3.17) to obtain

$$\tilde{T}(0, t) = T_c, \tilde{T}(L, t) = \begin{cases} T_h, & 0 \leq t \leq t_1^{(1)}, \\ T_m(L, t), & t_1^{(1)} < t \leq t_2^{(1)}, \\ T_c, & t > t_2^{(1)}, \end{cases}$$

where $t_1^{(1)}$ and $t_2^{(1)}$ are solutions of the equation $L - u_{\text{eff}}t = \pm 2\sqrt{\kappa t}$.

Figure 3.4: Model of the outlet box temperature $\tilde{T}(L, t)$

Then, (3.13) yields

$$\frac{d\bar{T}}{dt} = \sum_j B_{\bullet j} \bar{T}_j - Q_0 - \frac{\rho_a c_p^a \bar{u}}{C_{\text{eff}} L} [\tilde{T}(L, t) - T_c]. \quad (3.19)$$

The initial condition is given, according to (3.5)₂, by

$$\bar{T}(0) = T_h. \quad (3.20)$$

To complete this section, I perform the dimensional analysis of the governing equation (3.1), which can be easily written down in a dimensionless form as

$$\frac{1}{\text{Fo}_h} \frac{\partial \hat{T}}{\partial \hat{t}} + \text{Pe}_h \bar{u} \frac{\partial \hat{T}}{\partial \hat{x}} = \Delta \hat{T} - \frac{6L}{d} (1 - \varepsilon) \text{Sh}_h [\hat{c}_{\text{sat}} - \hat{c}] + \text{Po}, \quad (3.21)$$

if I introduce the dimensionless variables

$$\begin{aligned} \hat{T} &= \frac{T - T_c}{T_c - T_h}, \quad \hat{c} = \frac{c - c_0}{c_{\text{sat}}(T_h)}, \quad \hat{c}_{\text{sat}} = \frac{c_{\text{sat}} - c_0}{c_{\text{sat}}(T_h)}, \\ \hat{t} &= \frac{t}{t_0}, \quad \hat{x} = \frac{x}{L}, \quad \hat{y} = \frac{y}{b}, \quad \hat{u} = \frac{u}{u_0}, \end{aligned}$$

where t_0 is a characteristic time, which is here related to the total transport time.

With the help of the values from Nomenclature, I determine the order of magnitude of the coefficients of (3.21), yielding

$$\frac{1}{\text{Fo}_h} = \mathcal{O}(1), \quad \text{Pe}_h \bar{u} = \mathcal{O}(10^1), \quad \frac{6L}{d} (1 - \varepsilon) \text{Sh}_h = \mathcal{O}(1), \quad \text{Po} = \mathcal{O}(1).$$

The conclusion is that neither the convective process nor the diffusive one in the heat exchange can be neglected.

3.3 Model for a Slit

Let us consider a horizontal slit (the slit in the x -direction) between two boxes in the first column of one layer. I model the slit by a narrow strip $\{[0, L] \times [0, h]\}$ ($h/L \ll 1$) in the xy -plane; see Figure 3.5.

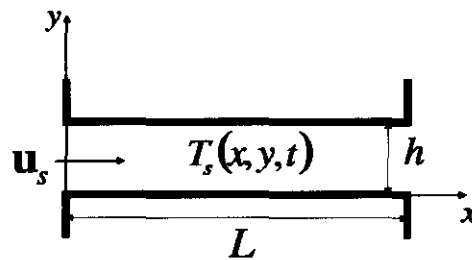


Figure 3.5: Geometry of the slit

The equation governing the heat transfer in a slit due to the air flow in the x -direction and the diffusion through the walls of the boxes has the form (Chadwick, 1976)

$$\frac{\partial T_s}{\partial t} + u_s(y) \frac{\partial T_s}{\partial x} = \tilde{D} \frac{\partial^2 T_s}{\partial y^2}, \quad (3.21)$$

where $T_s = T_s(x, y, t)$ denotes the temperature in a slit, $\tilde{D} = \lambda_a / (\rho_a c_p^a)$ is the diffusion coefficient, $u_s(y) = p_0(hy - y^2)/(2\mu)$ is the longitudinal velocity in the slit (the air flow has a Poiseuille profile). Note that the heat transfer equation (3.21) does not contain the second x -derivative of the temperature, *i.e.* the diffusion in the longitudinal direction is neglected, because the slit is very narrow ($h \ll L$). The consequence of such an assumption is that only one boundary condition can be prescribed at either the inlet or outlet of the slit. Since the flow enters the slit at $x = 0$, I choose

$$T_s(0, y, t) = T_s^{in}, \quad t > 0. \quad (3.22)$$

The boundary conditions at the walls of the slit are given by

$$\lambda_a \frac{\partial T_s(x, 0, t)}{\partial y} = \frac{\lambda_w}{d_w} (T_s - T^-), \quad \lambda_a \frac{\partial T_s(x, h, t)}{\partial y} = -\frac{\lambda_w}{d_w} (T_s - T^+), \quad (3.23)$$

where λ_a is the air thermal conductivity and $T^\pm$ corresponds to the temperatures at the walls of the upper and lower boxes, respectively.

To find a relation for $T^\pm$, I approximate the temperature to be a quadratic function in y . Using a similar approach as for the box, I obtain

$$T_s(x, y, t) = T_s^- + 2 \left(3\tilde{T}_s - T_s^+ - 2T_s^- \right) \frac{y}{h} - 3 \left(2\tilde{T}_s - T_s^+ - T_s^- \right) \left(\frac{y}{h} \right)^2.$$

For expressing the $T_s^\pm$ -values in terms of the averaged values, I use the boundary conditions (3.23); see Appendix A for more details.

By averaging equation (3.21) over the width of the slit, I arrive at the equation

$$\frac{\partial \tilde{T}_s}{\partial t} + \frac{1}{h} \int_0^h u_s(y) \frac{\partial T_s}{\partial x} dy = \frac{\tilde{D}}{h} \int_0^h \frac{\partial^2 T_s}{\partial y^2} dy, \quad (3.25)$$

where $\tilde{T}_s(x, t) = 1/h \int_0^h T_s(x, y, t) dy$. Then averaging (3.25) with respect to x , I obtain the ordinary differential equation for the volume averaged temperature $\bar{T}_s(t) = 1/L \int_0^L \tilde{T}_s(x, t) dx$

$$\frac{d\bar{T}_s}{dt} - \sum_j B_{\bullet,j} \bar{T}_j = -\frac{U}{L} \left[\tilde{T}_s(L, t) - \tilde{T}_s(0, t) \right], \quad (3.26)$$

where

$$U = \frac{1}{h} \int_0^h u_s(y) dy \equiv \frac{p_0 h^2}{12L\mu},$$

and "•" refers to the specific slit under consideration.

Equation (3.27) is complemented by the initial condition

$$\bar{T}_s(0) = T_h, \quad (3.29)$$

following from (3.20).

Here also I need a closure relation, since I do not know the temperature at slit's edge $\tilde{T}_s(L, t)$. The cooling process of the slits are much faster than for the boxes. In fact it takes approximately 2.8 seconds for the slits to cool down. For simplicity I take the temperature in the slits to be T_h (hot air) for the first 2.8 seconds, and after that it immediately 'jumps' to T_c (cooled air). Therefore I model it by a step function; see Figure 3.6.

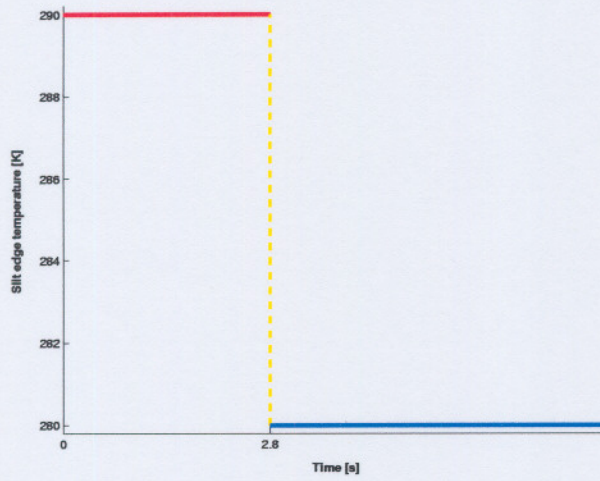


Figure 3.6: The averaged temperature at slit's edge

To scale equation (3.22), I introduce the dimensionless variables

$$\hat{T}_s = \frac{T_s - T_c}{T_c - T_h}, \quad \hat{t} = \frac{t}{t_0}, \quad \hat{x} = \frac{x}{L}, \quad \hat{y} = \frac{y}{h}, \quad \hat{u}_s = \frac{u_s}{u_s^0}, \quad (3.30)$$

where u_s^0 is a characteristic velocity in the slit. Then equation (3.22) takes the form (the hats are omitted)

$$\frac{h^2}{\tilde{D}t_0} \frac{\partial T_s}{\partial t} + \frac{u_s^0 h^2}{\tilde{D}L} u_s \frac{\partial T_s}{\partial x} = \frac{\partial^2 T_s}{\partial y^2}. \quad (3.31)$$

By selecting $u_s^0 = p_0 h^2 / (L\mu)$ and substituting this into (3.31), I get

$$\frac{h^2}{\tilde{D}t_0} \frac{\partial T_s}{\partial t} + \frac{p_0 h^4}{\mu \tilde{D}L^2} u_s \frac{\partial T_s}{\partial x} = \frac{\partial^2 T_s}{\partial y^2}.$$

Dimensional analysis shows that

$$\frac{h^2}{\tilde{D}t_0} = \mathcal{O}(10^{-6}), \quad \frac{p_0 h^4}{\mu \tilde{D} L^2} u_s = \mathcal{O}(1).$$

The conclusion is that I can neglect the first term in (3.31) (however, I did not neglect the non-stationary term). If you were to neglect the non-stationary term, you would get

$$\frac{p_0 h^4}{\mu \tilde{D} L^2} u_s \frac{\partial T_s}{\partial x} = \frac{\partial^2 T_s}{\partial y^2}. \quad (3.34)$$

3.4 2D Layer

In this section, I combine the models obtained in the previous section for a separate box and a separate slit to assemble the network scheme describing the heat exchange in one two-dimensional layer of boxes. As a result of the constructed network model, the average temperature can be predicted within the network provided the initial temperature values are prescribed. In fact, by doing that, I replace the temperature distribution through the whole layer by the average temperatures, which are time dependent.

I model each box and longitudinal slit in a layer by one node; see Figure 3.7.

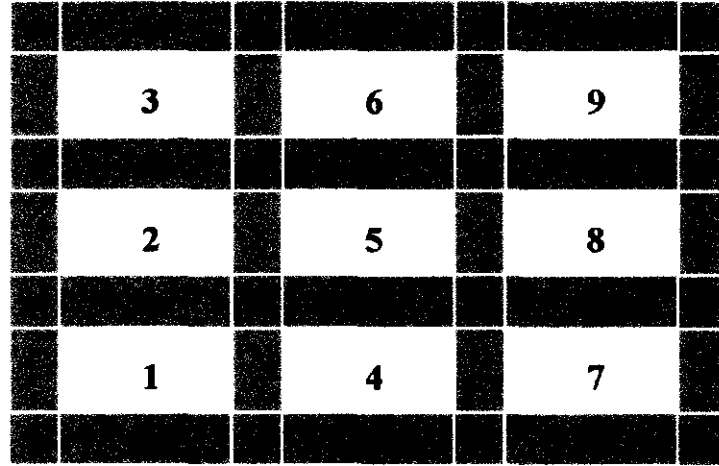


Figure 3.7: Numbering of nodes for a layer of nine boxes

Using equations (3.19) and (3.27) for every box node and for every slit h -node, respectively, I arrive at the coupled set of 21 network equations for 21 volume averaged temperatures.

I number the volume averaged temperatures with i , $i = \overline{1, 21}$, where $i = \overline{1, 9}$ stands for the boxes, and $i = \overline{10, 21} \equiv \overline{h_1, h_{12}}$, stands for the longitudinal slits. The nodes $\overline{a_1, a_{16}}$ are

just auxiliary nodes, and will not enter my final equations explicitly.
So, for each of the boxes, I get the equation below for $i = \overline{1, 9}$ (also see (3.18)).

$$\frac{d\bar{T}_i}{dt} = \sum_{j=1}^{21} B_{ij} \bar{T}_j - \frac{q_0}{C_{\text{eff}}} - \frac{\rho_a c_p^a \bar{u}}{C_{\text{eff}} L} [\tilde{T}_i(L, t) - \tilde{T}_i(0, t)]. \quad (3.34)$$

In the first column of boxes with $i = \overline{1, 3}$, I have (here $T_m(t) = T_m(L, t)$, according to (3.16))

$$\tilde{T}_i(L, t) - \tilde{T}_i(0, t) = \begin{cases} T_h - T_c, & t \leq t_1^{(1)}, \\ T_m(L, t) - T_c, & t_1^{(1)} < t \leq t_2^{(1)}, \\ 0, & t \geq t_2^{(1)}. \end{cases} \quad (3.35)$$

In the second column with $i = \overline{4, 6}$, I have

$$\tilde{T}_i(L, t) - \tilde{T}_i(0, t) = \begin{cases} 0, & t \leq t_1^{(1)}, \\ T_h - T_m(t), & t_1^{(1)} < t \leq t_2^{(1)}, \\ T_h - T_c, & t_2^{(1)} < t \leq t_1^{(2)}, \\ T_m(2L, t) - T_c, & t_1^{(2)} < t \leq t_2^{(2)}, \\ 0, & t > t_2^{(2)}, \end{cases} \quad (3.36)$$

and in the third column of boxes with $i = \overline{7, 9}$,

$$\tilde{T}_i(L, t) - \tilde{T}_i(0, t) = \begin{cases} 0, & t \leq t_1^{(2)}, \\ T_h - T_m(2L, t), & t_1^{(2)} < t \leq t_2^{(2)}, \\ T_h - T_c, & t_2^{(2)} < t \leq t_1^{(3)}, \\ T_m(t - 2t_1) - T_c, & t_1^{(3)} < t \leq t_2^{(3)}, \\ 0, & t > t_2^{(3)}. \end{cases} \quad (3.37)$$

where $t_i^{(1)}$ and $t_i^{(2)}$ are solutions of the equation $iL - u_{\text{eff}}t = \pm 2\sqrt{\kappa t}$, $i = 2, 3$.

Note that in my modelling I assume that the cooled air out of each box enters directly the vent hole of the adjacent box from the next column.

The equations for the average temperatures in the slits have the form ($i = \overline{10, 21} \equiv \overline{h_1, h_{12}}$)

$$\frac{d\bar{T}_i}{dt} = \sum_{j=1}^{21} B_{ij} \bar{T}_j - \frac{U}{L} [\tilde{T}_i(L, t) - \tilde{T}_i(0, t)]. \quad (3.38)$$

According to Figure 2.6, the term in the brackets: $\tilde{T}_i(L, t) - \tilde{T}_i(0, t)$ is given by

$$\tilde{T}_i(L, t) - \tilde{T}_i(0, t) = \begin{cases} T_h - T_c, & t \leq \frac{L}{U} \\ 0, & t \geq \frac{L}{U} \end{cases}, \quad i = \overline{10, 13}; \quad (3.39)$$

$$\tilde{T}_i(L, t) - \tilde{T}_i(0, t) = \begin{cases} 0, & t \leq \frac{L}{U} \\ T_h - T_c, & \frac{L}{U} < t \leq \frac{2L}{U} \\ 0, & t > \frac{2L}{U} \end{cases}, \quad i = \overline{14, 17}; \quad (3.40)$$

$$\tilde{T}_i(L, t) - \tilde{T}_i(0, t) = \begin{cases} 0, & t \leq \frac{2L}{U} \\ T_h - T_c, & \frac{2L}{U} < t \leq \frac{3L}{U} \\ 0, & t > \frac{3L}{U} \end{cases}, \quad i = \overline{17, 21}. \quad (3.41)$$

My system (2.27), (2.31) can be presented in the symbolic form

$$\dot{\mathbf{T}} = \mathbf{B}\mathbf{T} + \mathbf{c}, \quad (3.42)$$

where $\mathbf{T} = (T_1 \dots T_9 \ T_{h_1} \dots T_{h_{12}})^T$ is a vector consisting of 21 temperatures, $\mathbf{B} = (B_{ij})_{i,j=\overline{1,21}}$ is a matrix consisting of 21 rows and 21 columns (see Appendix B for the explicit expressions for the coefficients): $\mathbf{c} = (c_i)_{i=\overline{1,21}}$. By taking (3.35)–(3.37) and (3.39)–(3.41) into account, I arrive at the following expressions for the coefficients c_i ,

$$c_i = \begin{cases} -\frac{q_0}{C_{\text{eff}}} - \frac{\rho_a c_p^a}{C_{\text{eff}} L} \bar{u} [\tilde{T}_i(L, t) - \tilde{T}_i(0, t)], & i = \overline{1, 9} \\ -\frac{U}{L} [\tilde{T}_i(L, t) - \tilde{T}_i(0, t)], & i = \overline{10, 21}. \end{cases} \quad (3.43)$$

According to formulas (3.19) and (3.28), the initial conditions are given by

$$\bar{T}_i(0) = T_h, \quad i = \overline{1, 21}. \quad (3.44)$$

In the next chapter, I solve the system (3.42)–(3.44), and present the results, specifically for the volume averaged temperatures in the boxes.

Chapter 4

Volume Averaging

My ultimate aim is the modelling of the climate in a stack of boxes. Each stack consists of 80-100 single boxes. For such a big and complex system it is not really practical to create a very detailed model of the climate in every single box, since it takes a lot of computer memory and it is computationally difficult to implement in practice. Even for one layer (9 boxes) it will be a cumbersome job. It seems to be more realistic to model every box in the layer as one node, having as average characteristics: velocity, temperature and moisture concentration. In order to do so the method of volume averaging is used.

4.1 Volume averages of heat and moisture content

The volume averaging technique I will use here was developed by S. Whitaker (1999). This method considers a representative elementary volume in the domain under study, and the local conservation equations are integrated over this volume providing averaged macroscopic transport equations valid in the whole domain. The macroscopic ordinary differential equations are obtained by averaging of the microscopic equations over the average volume, (volume of the box), and by using some closing assumptions (to be introduced further on). For my purposes, this average volume will be taken equal to one box. Microscopic equations are the equations of heat (2.8) and moisture content (2.12). For an arbitrary variable F the average quantity is defined by

$$\bar{F} = \frac{1}{V} \int_V F dV, \quad (4.1)$$

where V is the volume of the box. In the derivation below, the following conditions will be used:

$$\begin{aligned}
\operatorname{div} \mathbf{u} &= 0, \\
\mathbf{u} \cdot \nabla f &= \operatorname{div}(\mathbf{u}f) - f, \\
\Delta f &= \operatorname{div}(\nabla f).
\end{aligned} \tag{4.2}$$

With the use of the properties (4.2), the energy balance equation (2.8) can be written in the form

$$C_{\text{eff}} \frac{\partial T}{\partial t} = \operatorname{div}(-\rho c_p^a \mathbf{u}T + \lambda_{\text{eff}} \nabla T) - rS + Q. \tag{4.3}$$

By applying to (4.3) the volume averaging (4.1) and using the divergence theorem

$$\int_V (\operatorname{div} \mathbf{q}) dV = \oint_{\Gamma} (\mathbf{q} \cdot \mathbf{n}) d\sigma, \tag{4.4}$$

where Γ is the boundary surface of V and $\mathbf{n}$ is the unit vector normal to the surface Γ , taken to face out of V everywhere on its boundary Γ , I find

$$C_{\text{eff}} \frac{d\bar{T}}{dt} = \frac{1}{V} \int_{\Gamma} [(-\rho c_p^a \mathbf{u}T + \lambda_{\text{eff}} \nabla T) \cdot \mathbf{n}] d\sigma + \bar{Q} = \bar{J} + \bar{Q}, \tag{4.5}$$

where $\bar{J}$ is the average heat flux over Γ , and

$$\bar{Q} = \frac{1}{V} \int_V (-rS + Q) dV \tag{4.6}$$

is the average total source term.

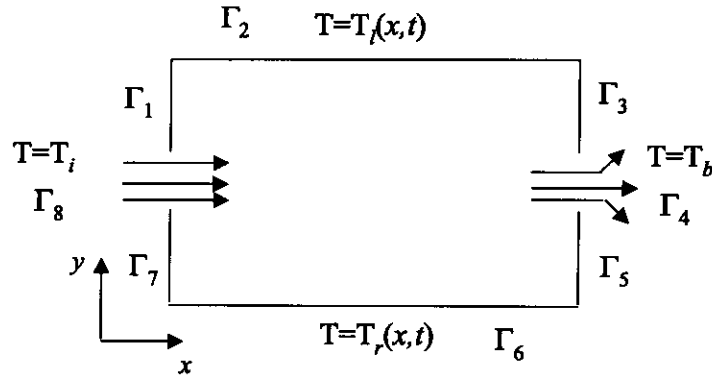


Figure 4.1: Boundaries $\Gamma_1 - \Gamma_8$ of the vent-holed box

In order to calculate the average heat flux $\bar{J}$, I split the boundary of the box Γ into eight parts: $\Gamma = \Gamma_1 \cup \Gamma_2 \cup \dots \cup \Gamma_7 \cup \Gamma_8$, as depicted in the figure above. Note that on the vent holes Γ_4 and Γ_8 the convective flux is dominant over the diffusive flux, whereas on the walls of the box, $(\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_5, \Gamma_6, \Gamma_7)$, the diffusive flux is dominant (in fact, there is no convective flux at all there, since on the walls $\mathbf{u} \cdot \mathbf{n} = 0$). Therefore, the average heat flux $\bar{J}$ can be represented in the form

$$\bar{J} = \frac{1}{V} \left[- \int_{\Gamma_4} \rho c_p^a T (\mathbf{u} \cdot \mathbf{n}) d\sigma - \int_{\Gamma_8} \rho c_p^a T (\mathbf{u} \cdot \mathbf{n}) d\sigma + \sum_{i \neq 4,8} \int_{\Gamma_i} \lambda_{\text{eff}} (\nabla T \cdot \mathbf{n}) d\sigma \right]. \quad (4.7)$$

Here, the heat fluxes through the holes can be evaluated by

$$\bar{J}^{(4)} = - \frac{1}{V} \int_{\Gamma_4} \rho c_p^a T (\mathbf{u} \cdot \mathbf{n}) d\sigma = - \frac{d_h}{LV} \rho c_p^a T_b \bar{u}, \quad (4.8)$$

$$\bar{J}^{(8)} = - \frac{1}{V} \int_{\Gamma_8} \rho c_p^a T (\mathbf{u} \cdot \mathbf{n}) d\sigma = \frac{d_h}{LV} \rho c_p^a T_i \bar{u}, \quad (4.9)$$

where T_i is the inlet temperature and T_b the outlet temperature. Note that the outlet temperature T_b is still unknown. Any finite set of transport equations is insufficient to provide a closed set of equations, and therefore it is necessary to use a closure relation, namely, to introduce an approximation scheme to eliminate some of the variables or to express some of the variables in terms of the others. In my case, the unknowns are outlet temperature and temperature gradient at the wall.

Therefore, I shall in a first attempt assume that in a first order approximation $T_b = \bar{T}$; Then, equation (4.8) can be rewritten as

$$\bar{J}^{(4)} = - \frac{d_h}{LV} \rho c_p^a \bar{T} \bar{u}. \quad (4.10)$$

The average velocity of the air flow through the vent holes into the box can be approximated by the generalized Darcy-Forchheimer equation, as

$$\bar{u} = \frac{\kappa O}{\mu} \frac{\Delta p}{L}, \quad (4.11)$$

where O is the vent hole ratio (ratio between area of the vent hole and area of the box side wall), κ is permeability, μ is viscosity, $\Delta p/L$ is the pressure gradient over the box. The heat flux through the walls of the box is ($i \neq 4,8$)

$$\bar{J}^{(i)} = \frac{1}{V} \int_{\Gamma_i} \lambda_{\text{eff}} (\nabla T \cdot \mathbf{n}) d\sigma = \frac{\lambda_{\text{eff}}}{V} \int_{\Gamma_i} \frac{\partial T}{\partial n} d\sigma. \quad (4.12)$$

As the next step, the following approximation for the temperature gradient at the wall is made:

$$\frac{\partial T}{\partial n} = \frac{T_w - \bar{T}}{d_c}. \quad (4.13)$$

Here, T_w is the temperature at the wall inside the box and d_c is a parameter, called the effective (diffusion) distance. The unknown temperature T_w can be found as a solution of (4.13) and boundary conditions at the wall, stating that (see 2.16)

$$\lambda_{\text{eff}} \frac{\partial T}{\partial n} = \frac{\lambda_w}{d_w} (T_s - T_w), \quad (4.14)$$

where T_s is the temperature at the wall outside the box (temperature in the slit). This solution reads

$$T_w = \frac{1}{1+B} (\bar{T} + B T_s), \quad (4.15)$$

where $B = \lambda_w d_c / \lambda_{\text{eff}} d_w$ is a constant. With the results derived above, (A1) can be rewritten as

$$C_{\text{eff}} \frac{d\bar{T}}{dt} = \frac{d_h}{LV} \rho c_p^a \bar{u} (T_i - \bar{T}) + \sum_{i \neq 4,8} \frac{L \lambda_w}{d_w V (1 + B^{(i)})} (T_s^i - \bar{T}) + \bar{Q}. \quad (4.16)$$

using short-hand notations for the coefficients, I can write (4.16) in a more concise form

$$\frac{d\bar{T}}{dt} + \Lambda_1 \bar{T} = \Lambda_2, \quad (4.17)$$

where

$$\begin{aligned} \Lambda_1 &= \frac{1}{C_{\text{eff}}} \left(\frac{d_h \rho c_p^a}{LV} \bar{u} + \sum_{i \neq 4,8} \frac{L \lambda_w}{d_w V (1 + B^{(i)})} \right), \\ \Lambda_2 &= \frac{1}{C_{\text{eff}}} \left(\frac{d_h \rho c_p^a}{LV} \bar{u} T_i + \sum_{i \neq 4,8} \frac{L \lambda_w T_s^i}{d_w V (1 + B^{(i)})} + Q \right). \end{aligned} \quad (4.18)$$

The coefficient Λ_1 is clearly a constant, but Λ_2 is not, because the temperature in the slit will in general be a function of time t , and possibly the source term $\bar{Q}$ will be so. Hence, $\Lambda_2 = \Lambda_2(t)$. The ordinary differential equation (4.17) can be solved analytically. With $\bar{T}(0) = T_0$ (the initial temperature inside the box), its solution is given by

$$\bar{T} = T_0 \exp^{-\Lambda_1 t} + \int_0^t \Lambda_2(\tau) \exp -\Lambda_1(t - \tau) d\tau. \quad (4.19)$$

The same procedure can be followed to find an equation for the volume averaged moisture content $\bar{c}(t)$ in the box. The result reads

$$\frac{d\bar{c}}{dt} = \frac{d_h}{LV} \bar{u}(c_i - \bar{c}) + \sum_{i \neq 4,8} \frac{LD_w}{d_w V(1 + F^{(i)})} (c_s^i - \bar{c}) + \bar{S}. \quad (4.20)$$

Here, $F = D_w f_c / d_w D_{\text{eff}}, d_m$ is a parameter, comparable with d_c .

Chapter 5

Results

By applying the Runge-Kutta method, the system of equations (3.43), (3.45) is solved, and thus the average temperature distribution in time is calculated at nodes. The plots represented in Fig.5.1 and 5.2 illustrate the results.

I start by investigating the average temperature behavior in the boxes due to: the flow of cooled air through the boxes, the internal source ($Q_0 = 0$), and the interaction with the adjacent slits. The latter means that I have to take the matrix $\mathbf{B}$ identically equal to zero (because in this model $\lambda_w = 0$). This yields, for $i = \overline{1, 9}$,

$$\frac{d\bar{T}}{dt} = -\frac{\rho_a c_p^a \bar{u}}{C_{\text{eff}} L} [\tilde{T}_i(L, t) - \tilde{T}_i(0, t)], \quad (5.1)$$

with

$$\bar{T}_i(0) = T_h. \quad (5.2)$$

Figure 5.1 depicts the decreasing behavior of the average box temperatures according to my layout (the top view of one layer of boxes) - in three columns of boxes - the case of neglecting all the source terms and interaction with slits.

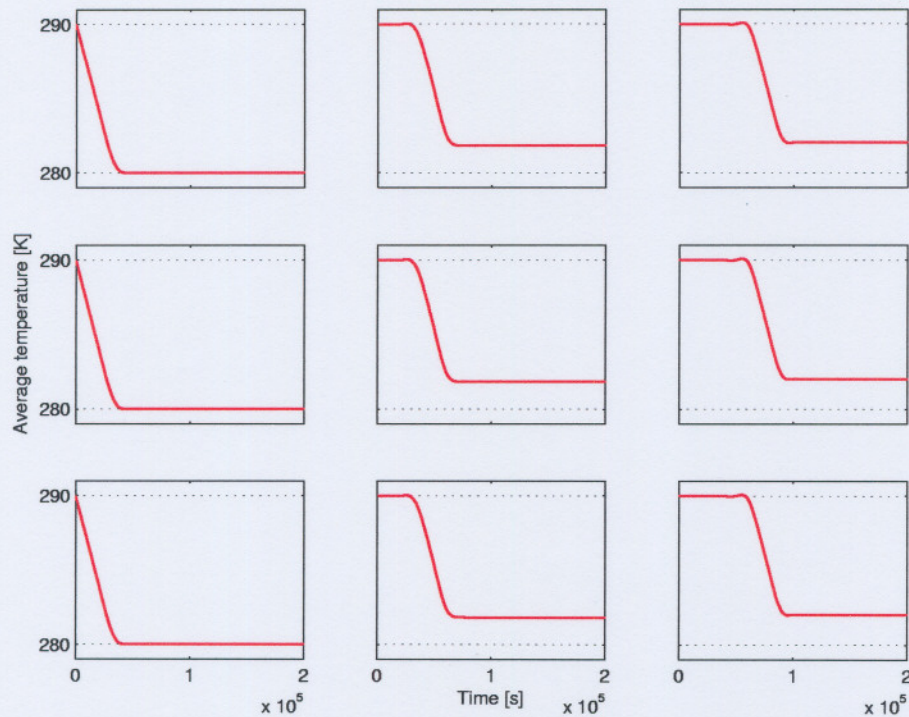


Figure 5.1: Results for the case of no sources

Within the first-column boxes, the average temperature starts its decrease immediately. Clear delay phases can be observed for the second and third columns. By this I mean that it takes longer for the cooled air to reach and cool down the second and third columns of boxes, than it takes to cool down the first column. To be more specific, it takes approximately half a day (twelve hours) till the porous medium within the first-column boxes cools down completely, to the temperature of T_c . Within first eight hours the average temperature in the second column remains constant. The boxes in the second column are cooled down after twenty hours. The delay phase for the third-column boxes is approximately sixteen hours, therefore it takes twenty-eight hours to cool down.

My results for the first-column boxes agree with the numerical solution of the climate problem in a separate box computed by the method of finite elements using the FEMLAB[®] software package; see (Nishenko, 2004) for the details.

Figure 5.2 shows that for the case of constant slit temperature, starting from the moment of time $t = L/U$, I have the following results:

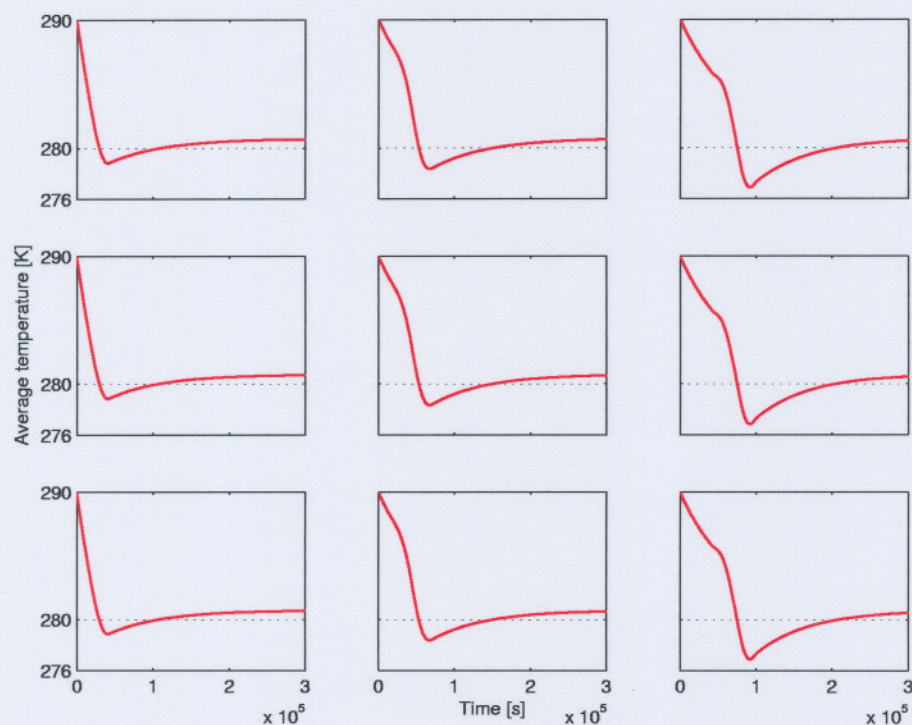


Figure 5.2: Results for the case of constant slit temperature

The plots demonstrate the behavior of the averaged temperature field similar to the case of neglecting all the sources. The main distinction from the previous case is that for all three columns of boxes, the average temperature immediately starts decreasing. However, the decrease in the second and third columns is less sharp (much slower) than in the first column. This can be explained by the interaction of boxes with the adjacent slits, because the slits take off some heat from the boxes due to diffusion through the walls, so the slits are the negative sources.

Chapter 6

Conclusions and Recommendations

This project has investigated the processes of heat and mass transfer within a layer consisting of nine boxes separated by slits.

The main results are the following:

1. The process of mass transfer through the boxes is much more rapid than the process of heat transfer.
2. The convective flux along the longitudinal slits is much faster than the convective and diffusive fluxes through the boxes. The model predicts that it takes approximately 3 seconds for the slits to cool down (reach T_c), and about 8 hours for the first column of boxes to reach T_c .
3. I found that if I look at the time scales for moisture and temperature, the time scale for moisture is much smaller than the time scale for temperature.
4. It is clear from my numerical results (see matrix B in Appendix B), that the coefficients for the adjacent slits and boxes are of order one ($\mathcal{O}(1)$), while the other interactions are of order $\mathcal{O}(10^{-6})$.
5. A simple 2D network which can predict the average temperatures within a 3x3 layer of boxes has been constructed. The model is a realization of a 2D model which allows easy extension to 3D.
6. The model has been implemented in MATLAB[®].
7. The results, *i.e.* the predicted average temperature values, appear to be plausible in comparison with previous results.

From my results I conclude the following:

1. In considering the processes of heat and mass transfer, we can neglect the mass transfer and restrict ourselves to modelling heat transfer only.

2. In determining the temperature distribution within the boxes, we can assume the temperature within the slits to be equal to its final constant value T_c .
3. From the third main result I conclude that one can take the moisture c as constant.
4. In the network model I only consider the relevant interaction between adjacent slits and boxes.
5. From results 3 and 4 I can draw a very important conclusion: we can take the rows to be equal which makes averaging over the columns a lot easier. However, for the columns the situation is different, because of a time delay due to
 - convection, and
 - diffusion in the x -direction

Recommendations to A&FI:

1. The reduction of nodes is possible. If we model each row as one node. The assumptions that the temperature in the slits is equal to T_c , and that the air that leaves the box enters the next box directly, can be used.
2. The above model for the case of a 3D layer of boxes, can be extended by taking the natural convection flow in boxes in vertical direction into account.
3. In this given model I assumed a perfectly symmetric situation inside the container, but in reality it is not true. Therefore I recommend to allow for randomly varying width of the slits to obtain more accurate results.

Appendix A

In this appendix, I consider a simple model of one box between two isolated outer slits; see Figure 1. I will provide a detailed explanation of how to express the box and slit boundary temperature values in terms of the appropriate averaged values.

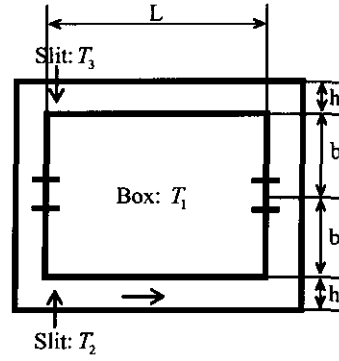


Figure 1: One box between two slits

I first consider one of the two slits along the longitudinal sides of the boxes. In these slits, $x \in [0, L]$, $y \in [0, h]$; the common sides with the boxes exchange heat with the boxes, the opposites are isolated. The slits in front and on the back of the box have uniform temperature; in these slits, no convection takes place and diffusion is neglected.

The equation governing the heat transfer in each slit due to the air flow in the horizontal direction and the diffusion through the walls of the boxes has the form

$$\frac{\partial T_i}{\partial t} + u_s(y) \frac{\partial T_i}{\partial x} = \tilde{D} \frac{\partial^2 T_i}{\partial y^2}, \quad i = 2, 3. \quad (1)$$

At the common sides, I apply the same boundary conditions as in Section 2.3, whereas the outer boundaries are thermally insulated; this yields

$$\begin{aligned} \lambda_a \frac{\partial T_2(x, h, t)}{\partial y} &= -\frac{\lambda_w}{d_w} [T_2(x, h, t) - T_1(x, -b, t)] = -\frac{\lambda_w}{d_w} (T_2^+ - T_1^-), \\ \lambda_a \frac{\partial T_3(x, 0, t)}{\partial y} &= \frac{\lambda_w}{d_w} [T_3(x, 0, t) - T_1(x, b, t)] = \frac{\lambda_w}{d_w} (T_3^- - T_1^+); \end{aligned} \quad (2)$$

$$\frac{\partial T_2(x, 0, t)}{\partial y} = \frac{\partial T_3(x, b, t)}{\partial y} = 0. \quad (3)$$

I will derive equations for the volume averaged temperatures in the box and two side slits in two steps: first, I average over the thickness direction, and after that, over the length direction.

This is explained for slit 2: Defining the width averaged temperature $\tilde{T}_2(x, t)$ by

$$\tilde{T}_2(x, t) = \frac{1}{h} \int_0^h T_2(x, y, t) dy, \quad (4)$$

and applying this averaging to (1) for $i = 2$, we obtain

$$\frac{\partial \tilde{T}_2}{\partial t} + \frac{\partial}{\partial x} \left[\frac{1}{h} \int_0^h u(y) T_2(x, y, t) dy \right] = \tilde{D} \frac{\partial T_2}{\partial y} \Big|_{y=0}^h = \tilde{D} \frac{\partial T_2(x, h, t)}{\partial y} = -\frac{\tilde{D} \lambda_w}{d_w \lambda_a} (T_2^+ - T_1^-), \quad (5)$$

where in the latter two steps I have used (3).

This relation still contains the unknown temperature T_2^+ , which we, at this point, want to express in terms of $\tilde{T}_2$ and T_1^- ; further on, we shall express T^- in terms of $\tilde{T}_i$, $i = 1, 2, 3$ also.

For this, I approximate the temperature in the slit to be a quadratic polynomial in y , say

$$T_2(x, y, t) = T_{2,0}(x, t) + T_{2,1}(x, t) \frac{y}{h} + T_{2,2}(x, t) \left(\frac{y}{h} \right)^2 = \frac{1}{2} (3\tilde{T}_2 - T_2^+) - \frac{3}{2} (\tilde{T}_2 - T_2^+) \left(\frac{y}{h} \right)^2, \quad (6)$$

where I have used the boundary conditions (3) and applied an averaging over the width. From this expression there follows

$$\frac{\partial T(x, h, t)}{\partial y} = -\frac{3}{h} (\tilde{T}_2 - T_2^+), \quad (7)$$

and from this by use of the first boundary condition of (3), we have

$$T_2^+ = \frac{1}{1 + B_s} \tilde{T}_2 + \frac{B_s}{1 + B_s} T_1^-, \quad (8)$$

where $B_s = \lambda_w h / (3\lambda_a d_w)$.

Analogously I derive for slit 3

$$T_3^- = \frac{1}{1 + B_s} \tilde{T}_3 + \frac{B_s}{1 + B_s} T_1^+. \quad (9)$$

Also for the box I proceed in a similar way. Approximating T_1 by the quadratic polynomial

$$T_1(x, y, t) = \left[\frac{3}{2} \tilde{T}_1 - \frac{1}{4} (T_1^+ + T_1^-) \right] + \frac{1}{2} (T_1^+ - T_1^-) \frac{y}{b} + \left[-\frac{3}{2} \tilde{T}_1 + \frac{3}{4} (T_1^+ + T_1^-) \right] \left(\frac{y}{b} \right)^2, \quad (10)$$

inserting this expression into the boundary conditions for the side walls of the box (2), and using (8) and (9) for T_2^+ and T_3^- , respectively, there follows

$$T_1^+ = \frac{1}{\hat{B} + 3} \left[3\tilde{T}_1 - \frac{\hat{B}}{\hat{B} + 1} \tilde{T}_2 + \frac{\hat{B}(\hat{B} + 2)}{\hat{B} + 1} \tilde{T}_3 \right], T_1^- = \frac{1}{\hat{B} + 3} \left[3\tilde{T}_1 + \frac{\hat{B}(\hat{B} + 2)}{\hat{B} + 1} \tilde{T}_2 - \frac{\hat{B}}{\hat{B} + 1} \tilde{T}_3 \right], \quad (11)$$

where $\hat{B} = \lambda_w b / (\lambda_{\text{eff}} d_w (1 + B_s))$.

Using this result in (8) and (9), we can also express T_2^+ and T_3^- in terms of $\tilde{T}_i$, $i = 1, 2, 3$. The total result can be schematically written as

$$\begin{bmatrix} T_1^+ \\ T_1^- \\ T_2^+ \\ T_3^- \end{bmatrix} = \tilde{\mathbf{B}} \begin{bmatrix} \tilde{T}_1 \\ \tilde{T}_2 \\ \tilde{T}_3 \end{bmatrix}, \quad (12)$$

where $\tilde{\mathbf{B}}$ is a 4×3 matrix, the entries B_{kl} , $k = 1, \dots, 4$, $l = 1, 2, 3$ of which follow from the results (8)–(11).

I will now show how these results can be used in, first, the y -averaged equations (for $\tilde{T}_i$) and, finally, in the volume averaged ones.

Start with the box, by averaging (3.1) over the width of the box, leading to

$$C_{\text{eff}} \frac{\partial \tilde{T}_1}{\partial t} + \rho c_p \bar{u} \frac{\partial \tilde{T}_1}{\partial x} - \lambda_{\text{eff}} \frac{\partial^2 \tilde{T}_1}{\partial x^2} - \lambda_{\text{eff}} \left[\frac{\partial \tilde{T}_1}{\partial y}(x, b, t) - \frac{\partial \tilde{T}_1}{\partial y}(x, -b, t) \right] + \tilde{Q} = 0, \quad (13)$$

with ($\tilde{T}_1$ according to (3.8), and $\bar{u}$ to (3.12)) and where

$$\tilde{Q} = \tilde{Q}(x, t) = \frac{1}{2b} \int_{-b}^b (rS - Q)(x, y, t) dy. \quad (14)$$

Using the boundary conditions at the side walls of the box; see (3.6), the fourth term in (13) becomes

$$\begin{aligned} -\lambda_{\text{eff}} \left[\frac{\partial \tilde{T}_1}{\partial y}(x, b, t) - \frac{\partial \tilde{T}_1}{\partial y}(x, -b, t) \right] &= \\ &= \frac{\lambda_w}{d_w} \left[T_1^+ + T_1^- - T_2^+ - T_3^- \right] = \\ &= \frac{\lambda_w}{d_w} \sum_{i=1}^3 \left(\tilde{B}_{1i} + \tilde{B}_{2i} - \tilde{B}_{3i} - \tilde{B}_{4i} \right) \tilde{T}_i = C_{\text{eff}} \sum_{i=1}^3 B_{1i} \tilde{T}_i, \end{aligned} \quad (15)$$

where, in the third step, I have used (13).

In an analogous way, I obtain for slit 2 from (5), (here, I have replaced $u(y)$ by the mean velocity U through the slit)

$$\frac{\partial \tilde{T}_2}{\partial t} + U \frac{\partial \tilde{T}_2}{\partial x} = \sum_{i=1}^3 B_{2i} \tilde{T}_i = 0, \quad (16)$$

where

$$B_{2i} = \frac{\tilde{D} \lambda_w}{d_w \lambda_a} \sum_{i=1}^3 \left(\tilde{B}_{3i} + \tilde{B}_{2i} \right). \quad (17)$$

An analogous formula for $\tilde{T}_3$ can be found, with

$$B_{3i} = \frac{\tilde{D} \lambda_w}{d_w \lambda_a} \sum_{i=1}^3 \left(\tilde{B}_{4i} + \tilde{B}_{1i} \right). \quad (18)$$

We can now derive the volume averaged equations:

Starting with (13), with (15) substituted into it, and averaging this relation in the x -direction, we obtain

$$\frac{d\tilde{T}_1}{dt} + \frac{\rho c_p^a \bar{u}}{C_{\text{eff}} L} \left[\tilde{T}_1(L, t) - \tilde{T}_1(0, t) \right] - \frac{\lambda_{\text{eff}}}{C_{\text{eff}}} \left[\frac{\partial \tilde{T}}{\partial x}(L, t) - \frac{\partial \tilde{T}}{\partial x}(0, t) \right] + \sum_{i=1}^3 B_{1i} \tilde{T}_i + Q_0 = 0, \quad (19)$$

where

$$Q_0 = \frac{1}{C_{\text{eff}}} \int_0^L \tilde{Q}(x, t) dt. \quad (20)$$

As already done in Section 2.2 (see (3.13) and further), (19) can be further evaluated (compare with (3.18)) to yield

$$\frac{d\tilde{T}_1}{dt} + \sum_{i=1}^3 B_{1i} \tilde{T}_i = -\frac{\rho c_p^a \bar{u}}{C_{\text{eff}} L} \left[\tilde{T}_1(L, t) - T_c - Q_0, \right. \quad (21)$$

with

$$\tilde{T}_1(L, t) = \begin{cases} T_h, & 0 \leq t \leq t_1^{(1)}, \\ T_m(L, t), & t_1^{(1)} < t \leq t_2^{(1)}, \\ T_c, & t > t_2^{(1)}, \end{cases} \quad (22)$$

For slit 2 or 3, we obtain in analogy with (3.26)

$$\frac{d\bar{T}_i}{dt} + \sum_{j=1}^3 B_{ij} \tilde{T}_j = -\frac{U}{L} \left[\tilde{T}_i(L, t) - T_c \right], \quad (23)$$

for $i = 2, 3$, and $\tilde{T}_i(L, t)$ according to Figure 3.6, as

$$\tilde{T}_i(L, t) = \begin{cases} T_h, & 0 \leq t \leq \frac{U}{L}, \\ T_c, & t > \frac{U}{L}. \end{cases} \quad (24)$$

The overall initial condition is $T_i(0) = T_h$ for $i = \overline{1, 3}$.

Appendix B

-1.4E-05	-3.6E-07	-8.9E-09	0	0	0	0	0	0	6.51E-06	8.77E-08	1.11E-07	1.78E-09	0	0	0	0	0	0	0	0
-3.6E-07	-1.4E-05	-3.6E-07	0	0	0	0	0	0	1.87E-07	8.77E-08	8.77E-08	1.87E-07	0	0	0	0	0	0	0	0
-8.9E-09	-3.6E-07	-1.4E-05	0	0	0	0	0	0	1.78E-09	1.11E-07	8.77E-08	8.51E-08	0	0	0	0	0	0	0	0
0	0	0	-1.4E-05	-3.6E-07	-8.9E-09	0	0	0	0	0	0	0	6.51E-06	8.77E-08	1.11E-07	1.78E-09	0	0	0	0
0	0	0	-3.6E-07	-1.4E-05	-3.6E-07	0	0	0	0	0	0	0	1.87E-07	8.77E-08	8.77E-08	1.87E-07	0	0	0	0
0	0	0	-8.9E-09	-3.6E-07	-1.4E-05	0	0	0	0	0	0	0	1.78E-09	1.11E-07	8.77E-08	8.51E-08	0	0	0	0
0	0	0	0	0	0	-1.4E-05	-3.6E-07	-8.9E-09	0	0	0	0	0	0	0	0	6.51E-06	8.77E-08	1.11E-07	1.78E-09
0	0	0	0	0	0	-3.6E-07	-1.4E-05	-3.6E-07	0	0	0	0	0	0	0	0	1.87E-07	8.77E-08	8.77E-08	1.87E-07
0	0	0	0	0	0	-8.9E-09	-3.6E-07	-1.4E-05	0	0	0	0	0	0	0	0	1.78E-09	1.11E-07	8.77E-08	8.51E-08
0.184337	0.003033	4.89E-05	0	0	0	0	0	0	-0.12867	-0.06759	-0.00086	-1.5E-05	0	0	0	0	0	0	0	0
0.191545	0.191683	0.003132	0	0	0	0	0	0	-0.06759	-0.06759	-0.00086	-0.00086	0	0	0	0	0	0	0	0
0.003132	0.191683	0.191545	0	0	0	0	0	0	-0.00086	-0.00086	-0.06759	-0.06759	0	0	0	0	0	0	0	0
4.89E-05	0.003033	0.184337	0	0	0	0	0	0	-1.5E-05	-0.00086	-0.06759	-0.12867	0	0	0	0	0	0	0	0
0	0	0	0.184337	0.003033	4.89E-05	0	0	0	0	0	0	0	-0.12867	-0.06759	-0.00086	-1.5E-05	0	0	0	0
0	0	0	0.191545	0.191683	0.003132	0	0	0	0	0	0	0	-0.06759	-0.06759	-0.00086	-0.00086	0	0	0	0
0	0	0	0.003132	0.191683	0.191545	0	0	0	0	0	0	0	-0.00086	-0.00086	-0.06759	-0.06759	0	0	0	0
0	0	0	4.89E-05	0.003033	0.184337	0	0	0	0	0	0	0	-1.5E-05	-0.00086	-0.06759	-0.12867	0	0	0	0
0	0	0	0	0	0	0.184337	0.003033	4.89E-05	0	0	0	0	0	0	0	0	-0.12867	-0.06759	-0.00086	-1.5E-05
0	0	0	0	0	0	0.191545	0.191683	0.003132	0	0	0	0	0	0	0	0	-0.06759	-0.06759	-0.00086	-0.00086
0	0	0	0	0	0	0.003132	0.191683	0.191545	0	0	0	0	0	0	0	0	-0.00086	-0.00086	-0.06759	-0.06759
0	0	0	0	0	0	4.89E-05	0.003033	0.184337	0	0	0	0	0	0	0	0	-1.5E-05	-0.00086	-0.06759	-0.12867

Figure 2: B-matrix

Appendix C

Double-Phase Approximation

When air temperature and temperature of products differ, a two-phase approach is needed to account for this temperature difference. In this case we assumed that the spherical product is homogeneously dispersed as a solid phase in the air phase. Several assumptions for the two-phase model are made:

1. temperature gradients in products are negligible;
2. heat conduction between products is negligible;
3. heat transfer is described with the energy balance equation for air phase and product phase.

The quantity per unit mass of fluid ϕ in the general conservation equation (2.1) now becomes $\epsilon\phi$ for the air phase and $(1 - \epsilon)\phi$ for the solid phase. The governing equations for the dry air are:

$$\epsilon\rho_a c_p^a \frac{\partial T_p}{\partial t} + \epsilon\rho_a c_p^a \mathbf{u} \nabla T_a = \dots \epsilon\lambda_a \Delta T_a + h_T A_{sp}(T_p - T_a), \quad (25)$$

$$\epsilon \frac{\partial c}{\partial t} + \epsilon \nabla c = \epsilon D_a \Delta c + S. \quad (26)$$

For the product phase we have

$$(1 - \epsilon)\rho_p c_p^p \frac{\partial T_p}{\partial t} = (1 - \epsilon)\lambda_p \Delta T_p - \dots h_T A_{sp}(T_p - T_a) - rS + (1 - \epsilon)Q. \quad (27)$$

In equations (25), (26) and (27) h_T is the surface convective heat transfer coefficient. The second term on the right hand side in (25) and (27) is the heat, which removed from the product phase and added to the air phase. To make the problem non-dimensional let us introduce scaling parameters for the temperature of the air and the product

$$\hat{T}_a = \frac{T_a - T_0}{T_0}, \hat{T}_p = \frac{T_p - T_0}{T_0}. \quad (28)$$

For the rest of scaling parameters we will use parameters introduced in (2.25). Rearranging of the coefficients and omitting hats gives us

- dry air phase

$$\frac{1}{Fo_{\text{ha}}} \frac{\partial T_a}{\partial t} + \dots Pe_{\text{ha}} \mathbf{u} \cdot \nabla T_a = \Delta T_a + \dots 6h_T \frac{1-\epsilon}{\epsilon} \frac{L^2}{d\lambda_a} (T_p - T_a); \quad (29)$$

$$\frac{1}{Fo_{\text{sa}}} \frac{\partial c}{\partial t} + \dots Pe_{\text{sa}} \mathbf{u} \cdot \nabla c = \Delta c + \dots \frac{6L}{d} \frac{1-\epsilon}{\epsilon} Sh_{\text{sa}} (c_{\text{sat}}(T) - c(T)). \quad (30)$$

- product phase

$$\frac{1}{Fo_p} \frac{\partial T_p}{\partial t} = \Delta T_p - \dots 6h_T \frac{L^2}{d\lambda_p} (T_p - T_a) - \dots 6Sh_p \frac{L}{d} (c_{\text{sat}}(T) - c(T)) + Po_p. \quad (31)$$

Here,

$$\begin{aligned} Fo_{\text{ha}} &= \frac{\lambda_a t_0}{\rho_a c_p^a L^2}, Pe_{\text{ha}} = \dots \rho_a c_p^a \frac{u_0 L}{\lambda_a}; \\ Fo_{\text{sa}} &= \frac{D_a t_0}{L^2}, Pe_{\text{sa}} = \frac{u_0 L}{D_a}, Sh_{\text{sa}} = \dots \frac{\beta_m L}{D_a}; \\ Fo_p &= \frac{\lambda_p t_0}{\rho_p c_p^p L^2}, Sh_p = \dots \frac{r L \beta_m c_{\text{sat}}(T_0)}{\lambda_p} T_p, \end{aligned} \quad (32)$$

are the heat Fourier and Peclet numbers for air, the species Fourier and Sherwood numbers for the products, respectively.

Bibliography

- [1] ALTEVOGT, A.S., ROLSTON, D.E., AND WHITAKER, S. 2003. *New equation for binary gas transport in porous media. Part 1: equation development*, Advances in Water Resources, Vol.26, 695–715.
- [2] BEUKEMA, K.J. 1980. *Heat and Mass Transfer during Cooling and Storage of Agricultural Products as Influenced by Natural Convection*, PhD Thesis Wageningen.
- [3] BOUSQUET-MELOU, P., GOYEAU, B., QUINTARD, M., FICHOT, F. AND GOBIN, D. 2002. *Average Momentum Equation for Interdendritic Flow in a Solidifying Columnar Mushy Zone*, International Journal of Heat and Mass Transfer, Vol.45, 3651-3665.
- [4] CHADWICK, P. 1976. *Continuum Mechanics: Concise Theory and Problems*, Allen and Unwin, London.
- [5] COGNE, C., ANDRIEU, J., LAURENT, P., BESSON, A. AND NOCQUET, J. 2003. *Experimental data and modelling of thermal conductivities of ice creams*, Journal of Food Engineering, Vol.58, 331-341.
- [6] CORREA, E.J. AND COTTA, R.M. 1998. *Enhanced lumped-differential formulations of diffusion problems*, Applied Mathematical Modelling, Vol.22, 137–152.
- [7] DUFRÊCHE, J., PRAT, M. AND SCHMITZ, P. 2003. *A Two-Scale Domain Decomposition Method for Computing the Flow Through a Porous Layer Limited by a Perforated Plate*, International Journal for Numerical Methods in Fluids, Vol.42, 623-633.
- [8] HSU, C.T. AND FU, H. 2001. *Oscillating Flows and Enhanced Heat Transfer in Porous Media*, The Hong Kong Uni. of Science and Technology.
- [9] LIU, S. AND MASLIYAH, J.H. 1999. *Non-linear Flows in Porous Media*, Journal of Non-Newtonian Fluid Mechanics, Vol.86, 229-252.
- [10] NEJAD, M., SAGHIR, M.Z., VAN VAERENBERGH, S. AND LEGROS, J.C., *Numerical Study of Convection and Diffusion in Porous Media for Binary System*.
- [11] NISHCHENKO, N. 2004. *Climate Modelling in a Transport Container*, Final report of “Mathematics for Industry”, Stan Ackermans Institute, Eindhoven.
- [12] PU, W.L., CHENG, P. AND ZHAO, T.S. 1999. *An Experimental Study of Mixed Convection Heat Transfer in Vertical Packed Channel*, Journal of Thermophysics and Heat Transfer, Vol.13, 517-521.

- [13] QUINTARD, M., KAVIANY, M. AND WHITAKER, S. 1997. *Two-medium Treatment of Heat Transfer in Porous Media: Numerical Results for Effective Properties*, Advances in Water Resources, Vol.20, 77-94.
- [14] QUINTARD, M., KAVIANY, M. AND WHITAKER, S. 1997. *Two-medium treatment of heat transfer in porous media: numerical results for effective properties*, Advances in Water Resources, Vol.20, 77-94.
- [15] VAN DER SMAN, R.G.M. 2002. *Prediction of airflow through a vented box by the Darcy-Forchheimer equation*, Journal of Food Engineering, Vol.55, 49-57.
- [16] VAN DER SMAN, R.G.M. 2003a. *Simple model for estimating heat and mass transfer in regular-shaped foods*, Journal of Food Engineering, Vol.60, 383-390.
- [17] VAN DER SMAN, R.G.M. 2003b. *Simple Model for Estimating Heat and Mass Transfer in Regular-Shaped Foods*, Journal of Food Engineering, in press, 49-57.
- [18] VOLLEBREGT, H.M. 2001. *Modelling the Effect of Cross-Flow on the Climate in Transport Container for Perishable Goods*, Final report of "Mathematics for Industry", Stan Ackermans Institute, Eindhoven.
- [19] WHITAKER, S. 1999. *The Method of Volume Averaging*, Kluwer Academic Publishers.
- [20] YOO, JOO-SIK. 2003. *Thermal convection in a vertical porous slot with spatially periodic boundary temperatures: low Ra flow*, International Journal of Heat and Mass Transfer, Vol.46, 381-384.